

# Using the Timoshenko Beam Bond Model: Example Problem

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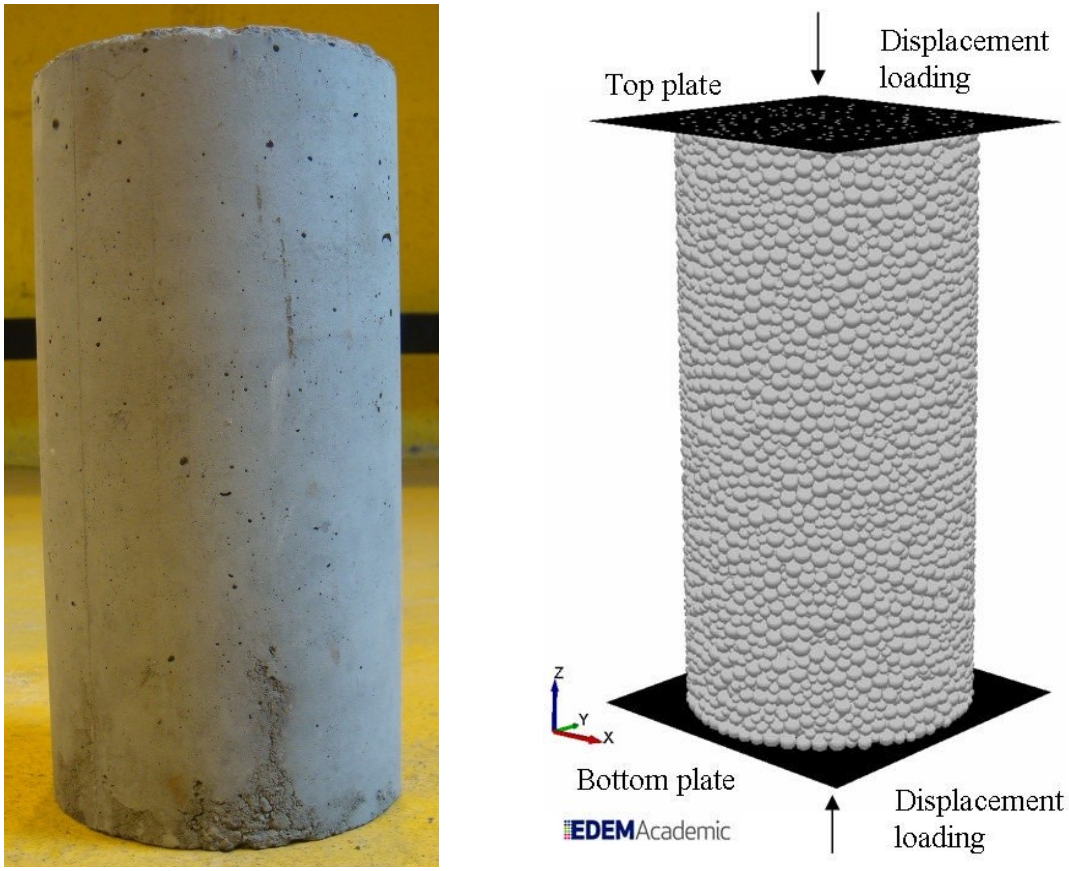
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# 1 UNI-AXIAL COMPRESSION OF CEMENTED CYLINDRICAL SPECIMEN

This guide provides a reference for how to successfully use the Timoshenko Beam Bond Model (TBBM) in EDEM® to model the three dimensional response of a cemented granular material under loading. A reference manual, which includes the theory of the TBBM and a description of the input parameters required to run the model, is available separately.

The numerical simulations used as the example problem in this guide mimic physical tests of concrete cylinders under uni-axial compression as shown in Figure 1.1.




**(a)**Physical cylindrical specimen **(b)**DEM specimen

**FIGURE 1.1:** Physical specimen compared to DEM representation

In this example problem a particle assembly is created and bonded together to form a specimen of "*concrete*". After displacement loading is applied, the important bulk (macroscopic) characteristics of the sample specimen are determined.

# 2 SPECIMEN GENERATION

The TBBM is only compatible with spherical particles which can be generated either using EDEM® or third party software. To obtain a stiffness and strength that is comparable to a real bonded cementitous material, a particle assembly with a high solid fraction is suggested. The properties of the cylindrical specimen used in this example are shown in Table 2.1.

In order for bonds to be formed between particles that are not in physical contact, EDEM® requires the contact radius of those particles to be greater than their physical radius. For the example problem particles of a uniform type (e.g. type "A") and poly-disperse size range are employed. The contact radius multiplier, $\eta$ was set as to be 1.1 times greater than the physical radius of each particle. This can be achieved in the particle creator section of EDEM® (See Reference Manual for details).

**TABLE 2.1:** Specimen properties for the example problem

| Parameter | Description | Value |
|---|---|---|
| $h_0$ | Initial specimen height (mm) | 200 |
| $A_{ratio}$ | Aspect ratio – height to diameter | 2:1 |
| $N_P$ | Total number of particles | 28,982 |
| $\eta$ | Contact radius multiplier | 1.1 |
| $r_{min}$ | Minimum particle radius (mm) | 1.15 |
| $r_{max}$ | Maximum particle radius (mm) | 2.71 |
| $n$ | Porosity | 0.37 |
| $\rho_p$ | Particle density (kg.m$^{-3}$) | 2700 |

The porosity shown in Table 2.1 relates to the tightness of the particle packing achieved by the particle generation technique. It does not directly relate to the porosity of the subject material, which for concrete would be expected to be much lower.

**NOTE 1**: SIMULATION DECK

An EDEM® simulation deck which includes this particle assembly is freely available through the DEM Solutions website. As such users can progress to Section 3 to set up this simulation.

In this example problem, particles and bonds represent the structure of the subject material at the mesoscopic scale. Particles and bonds do not directly represent individual grains and cement interfaces, but rather, represent the constituent parts of the subject material and their interactive properties at the mesoscopic scale.

The numerical concrete specimen will be loaded through the displacement of platens. The dimensions of these plates are not critical but they should be greater than or equal in size to the cylinder diameter and placed to form the boundaries of the specimen (see Figure 1.1b).

# 3 SIMULATION PARAMETERS

The particle information for the example problem is introduced into EDEM® as a new deck with the simulation time set to zero. The TBBM should be loaded into EDEM® in three locations:

1. As a custom particle-particle model
2. As a custom particle-geometry model
3. As a custom body force model

The preference file titled "*BondedParameters.txt*" must be located in the same folder as the contact model library file. The input parameters for the example simulation can be broken into three categories and explained independently. The categories are: Global parameters, Bond parameter and Non-bond parameters. The parameters for each category are discussed in the following sections.

## 3.1 GLOBAL PARAMETERS

There are a number of simulation parameters that need to be considered when assessing the numerical stability of a simulation. For the example loading problem these are the time step $\Delta t$, the bond time $t_{bond}$, the global damping $\iota_d$ and the loading rate $L_r$. Unlike the other global parameters the time step needs to be calculated by the user rather than defined. This can only be done when bonds have formed, as the critical time step for bonded contacts as well as for non-bonded contacts (calculated in EDEM) needs to be considered, as shown in Equation (3.1).

$$\Delta t = \xi min\left(\Delta t_{b,crit}, \Delta t_{HM,crit}\right) \tag{3.1}$$

In Equation (3.1) $\xi$ is a factor that should be kept in the range of zero to one. This parameter is similar to the one set as a percentage of the Rayleigh time step, which is normally used in EDEM. Before running the main loading simulation with an estimated time step it is suggested that a dummy simulation is run. This dummy simulation can be used to determine the critical time step for bonded contacts, which is related to the stiffness of the bonds, as shown in Equation (3.2). The critical time step is therefore determined for each bonded contact using the smallest particle mass $m_{p\ min}$ and the largest bond stiffness component $K_{b\ max}$ for that contact.

$$\Delta t_{b,crit} = 2sqrt\frac{m_{p\ min}}{K_{b\ max}} \tag{3.2}$$

In the dummy simulation the time step is first set to safe percentage of the Rayleigh critical

time step. The simulation is started, but the velocities and rotations of all particles in the simulation are capped at zero, which essentially fixes the particles in place. The simulation is allowed to continue beyond the time at which bonds are formed (see Section 4.1) at which point the simulation is stopped. As none of the particles will have moved, the bonds will have been formed without any disturbance to the particle assembly. At this point the value for axial bond stiffness (which is the largest stiffness in the stiffness matrix, $K_{b\ max}$) and bond length for each bond is exported for inspection.

The unknown from Equation (3.2) for each bond is the mass of the smallest particle. Fortunately the unknown mass can be determined by exporting axial bond stiffness and bond length for each bond. Equation (3.2) can be rearranged as Equation (3.3), with the full rearrangement shown in Appendix A.

$$\Delta t_{b,crit} = 2\sqrt{\frac{\rho_p \frac{4}{3}\pi \left(\frac{L_b K_1}{E_b \lambda \pi}\right)}{K_1}} \tag{3.3}$$

The parameters particle density $\rho_p$, Young's modulus for the bond $E_b$ and bond radius multiplier $\lambda$ are all parameters that are specified by the user. The time step $\Delta t$, can then be calculated using Equation (3.1).

In this example problem $\xi$ was set at a conservative 0.05 (5% of the critical timestep), which gives a time-step for the main simulation of $1x10^{-7}$ seconds. The simulation time can now be reset to zero and the newly determined time step included.

The global damping parameter, $\iota_d$, is set by the user. As global damping is not included in EDEM® the value for global damping is imported from the bonded particle preference file, as shown highlighted in red in Figure 3.1, where it is the first item in the simulation properties section. In this example problem a value of 0.5 for global damping is used.

**NOTE 2**: GLOBAL DAMPING USAGE

Global damping is a non-viscous damping applied to particle accelerations and is best suited for static and quasi-static problems.

In dynamic problems, such as breakage in a crusher or impacts, where the initial acceleration of the particles is an important aspect to capture this should be set to zero. Very small amounts may be acceptable on a per-use basis and should be checked by the user.

The bond time is also set by the user in the preference file; it is the second item in the simulation properties section, as shown highlighted in green in Figure 3.1. This is the time

that the bond initialisation procedure is triggered, a description of which is included in Section 4.1.

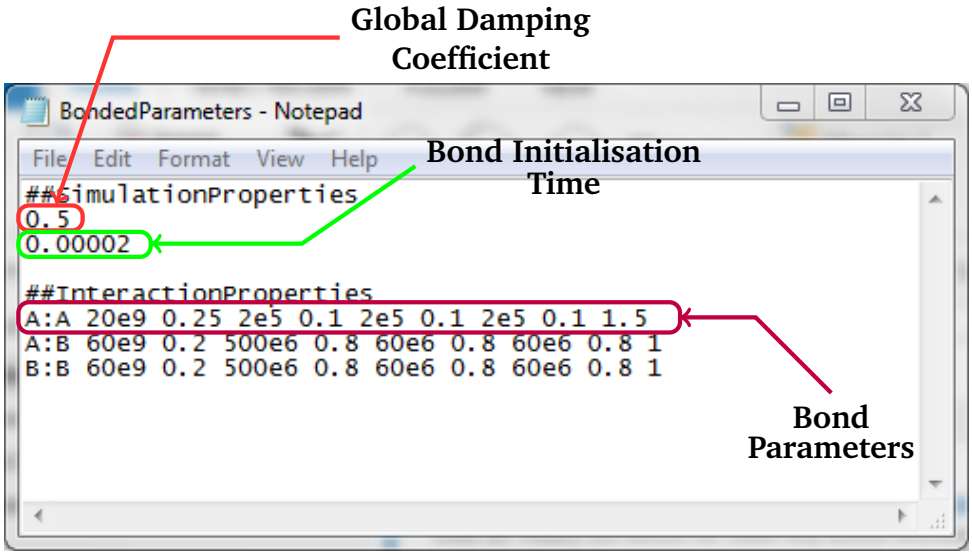



**FIGURE 3.1:** Global parameters in the preference file

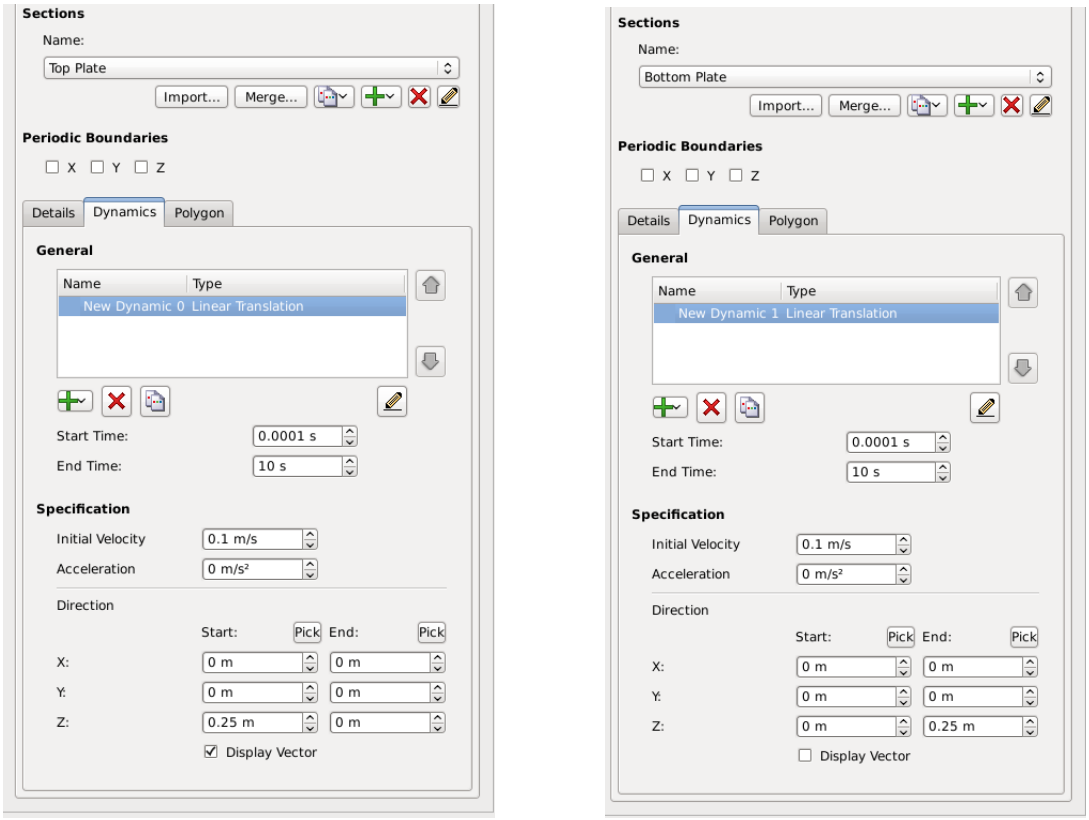
The loading rate is also set by the user; in the simulation this is achieved by applying motion to the geometries that were created as the top and bottom plates. Instructions on how to apply motion to geometries are well documented in the EDEM® user manual and not introduced in detail here. The loading rate used in the simulation needs to be sufficiently low to provide a static solution, whist not requiring an unreasonable computational time. The rate for a displacement controlled machine in physical tests is usually around 0.02 $mm.s^{-1}$. In the example problem a loading rate this low would be impractical as the computational time would be excessive. The loading rate used in the example problem will be 200 $mm.s^{-1}$ (each plate displacing at 100 $mm.s^{-1}$). The geometry dynamics setup is shown in Figure 3.2.

For a numerical simulation this loading rate is acceptable as a time step of $1x10^{-7}$s means that 10,000 calculation steps need to be computed for the specimen to displace 1 mm. The global parameters for the example problem are summarised in Table 3.1.

**TABLE 3.1:** Global parameters for the example problem

| Parameter | Description | Value |
|---|---|---|
| $\Delta t$ | Time step (s) | $1x10^{-7}$ |
| $t_{bond}$ | Bond time (s) | 0.001 |
| $L_r$ | Loading rate ($mm.s^{-1}$) | 200 |
| $\iota_d$ | Global damping | 0.5 |

In addition to the global parameters described above the use of gravity in simulations is something that should be considered. The inclusion of gravity may be influenced by the particle generation technique used. In the example problem no gravity is considered.




**(a)**Top Plate Dynamic **(b)**Bottom Plate Dynamic

**FIGURE 3.2:** Geometry dynamics

## 3.2 BONDED PARAMETERS

The theory describing the behaviour of particles at bonded contacts is described elsewhere in the Reference Manual. The base properties for the bonds are contained in the interaction properties section of the TBBM preference file, as shown in Figure 3.1.

The bonded contact parameters are shown for the example problem in Table 3.2. Definitions for each bonded contact parameter are described in the accompanying reference manual.

**TABLE 3.2:** Bonded contact parameters for type A:A interactions

| Parameter | Description | Value |
|---|---|---|
| $E_b$ | Young's modulus (GPa) | 35 |
| $\upsilon_b$ | Poisson's ratio | 0.20 |
| $S_C$ | Mean compressive strength (MPa) | 500 |
| $\zeta_C$ | Coefficient of variation of compressive strength | 0.0 |
| $S_T$ | Mean tensile strength (MPa) | 60 |
| $\zeta_T$ | Coefficient of variation of tensile strength | 0.8 |
| $S_s$ | Mean shear strength (MPa) | 60 |
| $\zeta_S$ | Coefficient of variation of shear strength | 0.8 |
| $\lambda$ | Bond radius multiplier | 1 |

## 3.3 NON-BONDED PARAMETERS

The Hertz-Mindlin (no-slip) contact model is used to describe the behaviour at non bonded contacts. The non-bonded parameters for the example problem are shown in Table 3.3.

**TABLE 3.3:** Particle and boundary model parameters for the reference case

| Parameter | Description | Value |
|---|---|---|
| $E_p$ | Particle Young's modulus (GPa) | 70 |
| $\upsilon_{pp}$ | Particle Poisson's ratio | 0.25 |
| $e_{pp}$ | Particle-particle coefficient of restitution | 0.5 |
| $\mu_{sP}$ | Particle-particle coefficient of static friction | 0.5 |
| $\mu_{rP}$ | Particle-particle coefficient of rolling friction | 0.5 |
| $E_g$ | Plate Young's modulus (GPa) | 200 |
| $\upsilon_g$ | Plate Poisson's ratio | 0.30 |
| $e_{pg}$ | Platen-particle coefficient of restitution | 0.0001 |
| $\mu_{sg}$ | Platen-particle coefficient of static friction | 1 |
| $\mu_{rg}$ | Platen-particle coefficient of rolling friction | 0 |

# 4 STARTING SIMULATION

It is important that the specimen is in a state of equilibrium before bonding or loading. The TBBM will automatically create a static state during the bond initialisation procedure. Therefore, if a user is struggling to achieve equilibrium, for example due to changing the contact model between particles, they can simply cap the velocities and rotations of the particles to zero until after the bond initialisation time. If particles velocities are capped it is important to un-cap them before loading.

## 4.1 BOND INITIALISATION

The bond initialisation procedure is triggered when the simulation time, $t$, exceeds the bond time $t_{bond}$ for the first time. During the bond initialisation procedure, bonds will be inserted between particles if the two particles contact radii overlap and they are allowed to bond i.e. that there is a bond type for that interaction in the preference file. In the example problem the generated particles are all given the type "*A*" so that there are bond properties defined for them in the particle preference file.

At the end of the computational time step where bonds have been initialised a static assembly of bonded particles with zero overlap and no contact force at the start of the loading phase of the simulation is created. Following bonding all capped velocities and rotations should be removed.

**NOTE 3**: CAPPED LIMITS

If capped limits (velocity and/or rotation) are not removed, it is likely that no bonds in the assembly will fail during loading.

Please remember to remove all capped limits immediately after the bond initialisation time before loading begins.

## 4.2 LOADING

For the example problem the specimen is loaded by displacing the two pieces of geometry. In physical experiments usually one platen would move whilst the other is static. In the example problem both plates are moved. As described in Section 3.1 the loading rate for the specimen is 200 $mm.s^{-1}$. It should be remembered that loading should not be commenced before the bonds have been initialised. As the bond time has been set in the preference file as 0.001 seconds the loading time could be set at a time of 0.002 seconds.

# 5 EXAMPLE RESULTS AND ANALYSIS

Two of the most desirable properties for a sample of concrete are the ultimate compressive strength and the secant modulus of elasticity. These properties can be determined by plotting the stress strain curve of the cylinder under loading. This section will describe how to plot the stress strain curve, as shown in Figure 5.1, as well as show how to plot some additional properties of interest such as the progression of broken bonds.

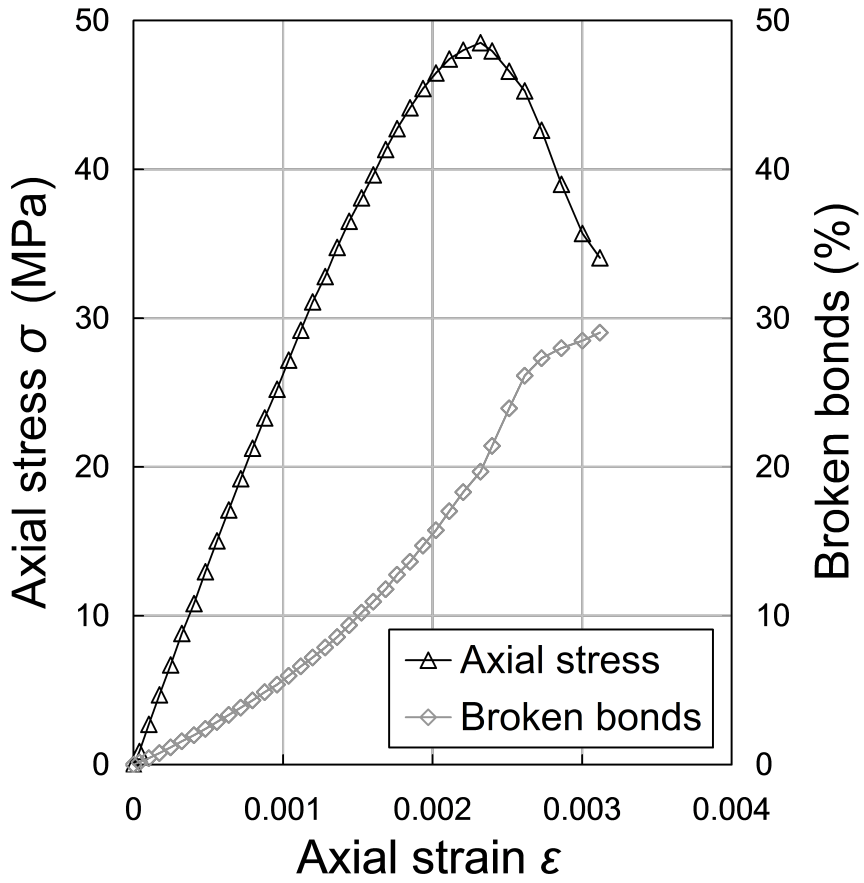



**FIGURE 5.1:** Axial stress and broken bonds against axial strain

## 5.1 PLOTTING STRESS AND STRAIN

The compressive stress $\sigma$ is calculated using the compressive forces acting on the top and bottom loading plates. In theory the forces acting on each plate should be equal, assuming that the area of the top plate in contact with specimen is the same as area of the specimen in contact with the bottom plate. However, as this cannot be guaranteed in the numerical simulation the average force should be used when plotting the stress, which can be calculated as:

$$\sigma = \frac{4(F_T + F_B)}{\pi d_g^2} \tag{5.1}$$

where $F_T$ and $F_B$ are the total compressive forces acting on the top and bottom loading plates respectively and $d_g$ is the diameter of either loading plate in contact with the specimen, which are assumed to be equal. The forces acting on the plates, $F_T$ and $F_B$, are determined by exporting the magnitude of the total forces acting on the two different geometries.

The axial strain $\varepsilon$ can be determined from the full height of the specimen, such that:

$$\varepsilon = \frac{(h_0 + h)}{h_0} \tag{5.2}$$

where $h$ is the current height of the specimen and $h_0$ is the initial height of the specimen.

**NOTE 4**: SIGN CONVENTION

Note that under compressive loading the specimen experiences axial contraction and is considered as a positive strain in this example.

The height of the specimen is defined as the maximum vertical distance between the top and bottom surfaces of the specimen, as shown in Figure 5.2. This definition differs from simply assessing the axial strain from the positions of the loading plates, as it takes into consideration the maximum overlap that develops between particles and the loading plates to remove the influence of the rough top surface of the specimen.

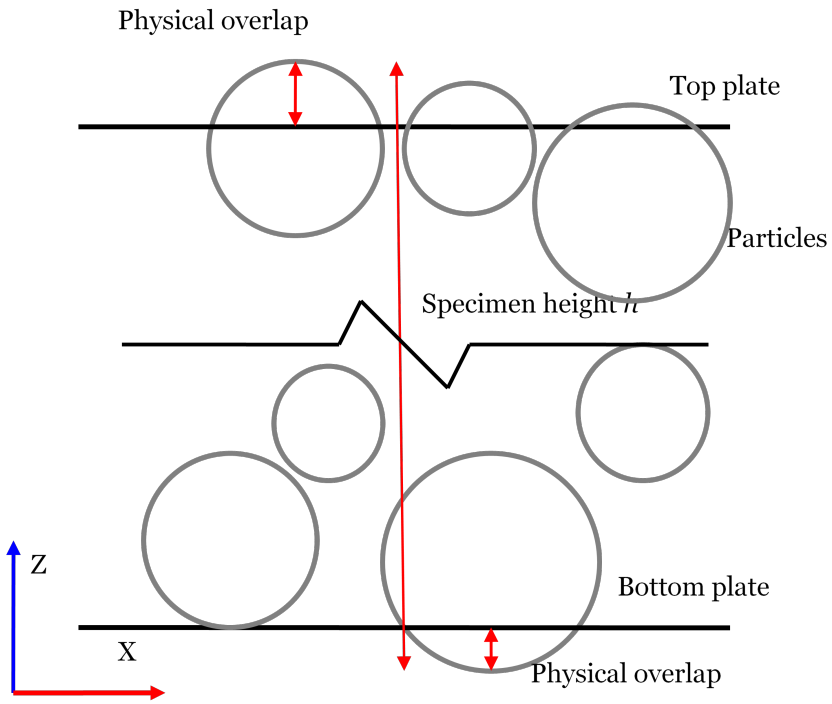



**FIGURE 5.2:** Calculation of the specimen height

This is particularly important in stiff, bonded assemblies where significant forces develop

over very small strains. Neglecting the maximum overlap, simulation results showed non-linear initial loading arising from the particles coming into contact with the loading plates. Although initial non-linear response is sometimes seen in physical tests, the degree shown in the numerical response was unrealistic.

The specimen height $h$ is therefore calculated such that:

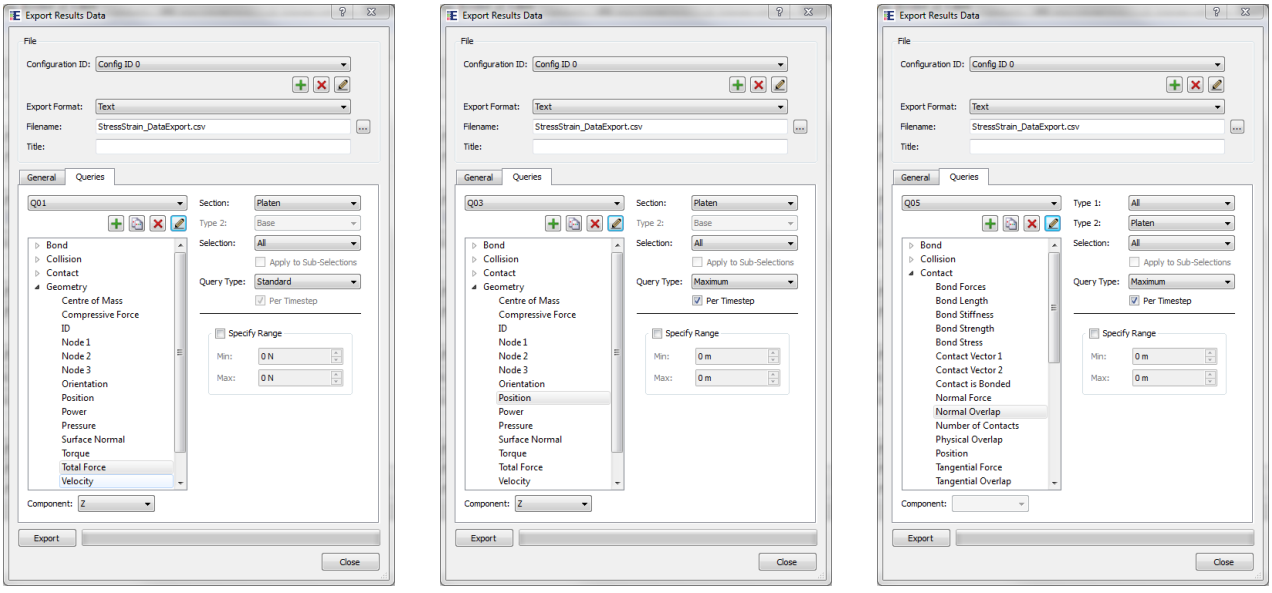
$$h = Z_t - Z_B + \delta_T + \delta_B \tag{5.3}$$

Numerical stability of this equation leads to a gradual reduction in $h$, as this is reliant on quasi-static conditions such that the maximum overlaps do not change significantly between the time steps of the DEM simulation.

In summary, to plot stress against strain the following six properties must be exported from EDEM®:

- $F_T$ = Total contact force, particles and top plate
- $F_B$ = Total contact force, particles and bottom plate
- $Z_T$ = Pos Z, top loading plate
- $Z_B$ = Pos Z, bottom loading plate
- $\delta_T$ = Max overlap, particles and top loading plate
- $\delta_B$ = Max overlap, particles and bottom loading plate

These required data queries are shown in Figure 5.3.




**(a)**Platen Total Force **(b)**Platen Position **(c)**Maximum Normal Overlap

**FIGURE 5.3: Exporting Stress-Strain Data** - EDEM data queries during export

The stress-strain curve shown in Figure 5.1 indicates a close to linear but gradually softening ascending branch with a loss of stiffness noted after approximately $\varepsilon = 0.0008$ (43% of the strength) when 5% of the bonds have failed. This is consistent with physical experiments on concrete where under loading up to approximately 30% to 40% of the ultimate compressive strength. The loss of stiffness increased with an increase in the number of broken bonds, until the peak stress was reached. This loss of stiffness was caused by the continual failure of bonds, resulting in a reduction in the overall stiffness of the bond network.

## 5.2 ANALYSING BOND BREAKAGE

To determine how damage propagates in the specimen the failure mode of bonds over time can be plotted. In Figure 5.4 the number of broken bonds as a percentage of total bonds in the specimen has been plotted against axial strain. Axial strain can be determined as shown in Section 5.1.

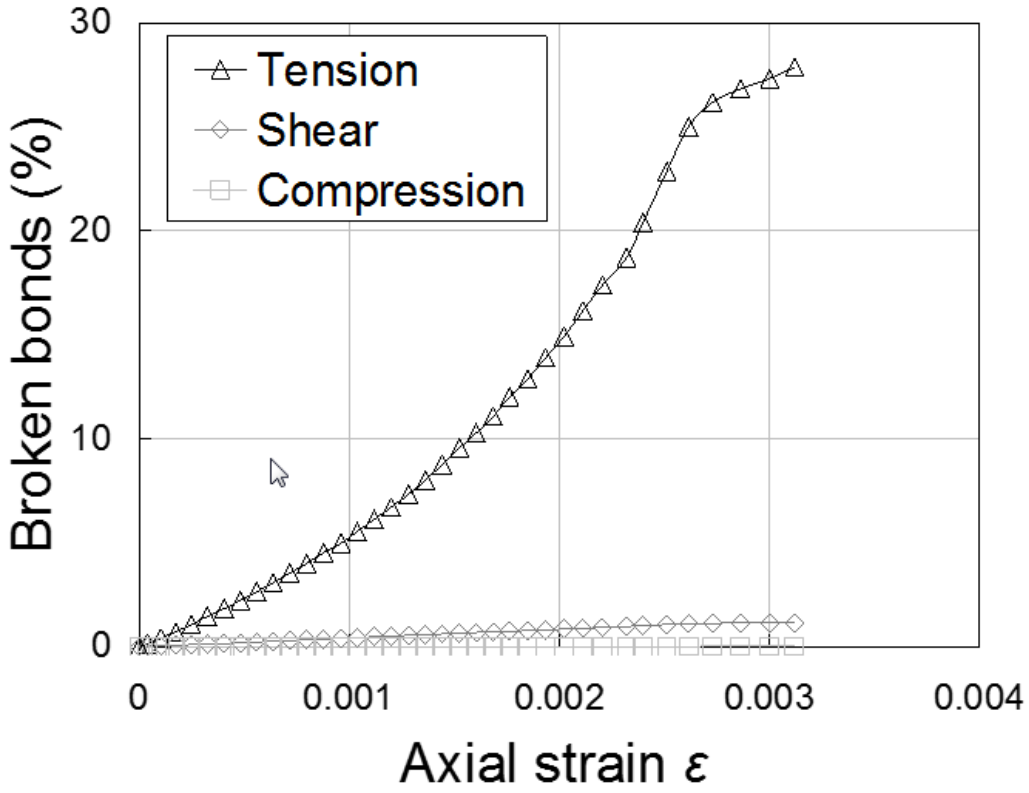



**FIGURE 5.4:** Type of broken bonds vs axial strain

In the TBBM, bonds fail because either their compressive strength, tensile strength or shear strength is exceeded by the respective stresses. To plot broken bonds the failure mode can be exported from EDEM® from each time step. The TBBM is implemented in EDEM® as a custom contact model. As such, when using the EDEM® Analyst the user should ensure to look at the properties of interest under the "contacts" category rather than the "*bond*" category.

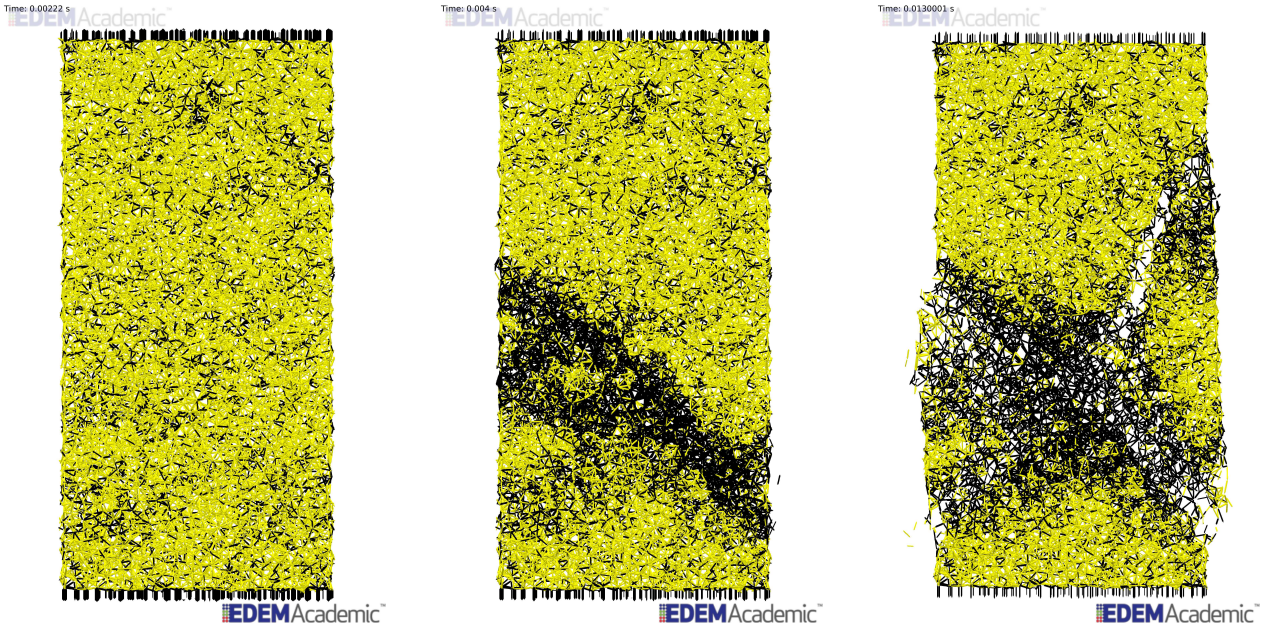
To determine the number of bonds that have broken through each failure mode export the property failure mode, where 0,1 and 2 refer to the failure mode in compression, tension and shear respectively. As can be seen in Figure 5.4 the majority of bonds in the specimen failed because their tensile strength was exceeded. The rate of bond failure increased as the loading continued towards the peak failure and after the peak stress, the rate began to decrease in the softening regime.

## 5.3 VISUALISING BREAKAGE

Sections 5.1 and 5.2 have shown how to extract prominent results such as peak failure strength or peak strain from the bonded assembly. However, the many custom contact properties are also available for visualisation within EDEM analyst.

Figures 5.5 and 5.6 use EDEM's capability to slice through the assembly to examine the internal structure of your assembly. Both figures plot results at three different stage of loading in Figure 5.1 - the peak strain, the post peak value at approximately $\varepsilon = 0.0035$, and at a much larger strain before the sample collapses, which is not included in that figure.

In Figure 5.5 the custom contact property of "*Bonded*" (See Reference Manual for details) is plotted to show the evolution of bond failure in the sample. Intact bonds are shown in yellow, while non-bonded contacts are shown as black. Non-bonded contacts can be either broken bonds where the particles are now in contact or contacts between particles and other elements that were never bonded.



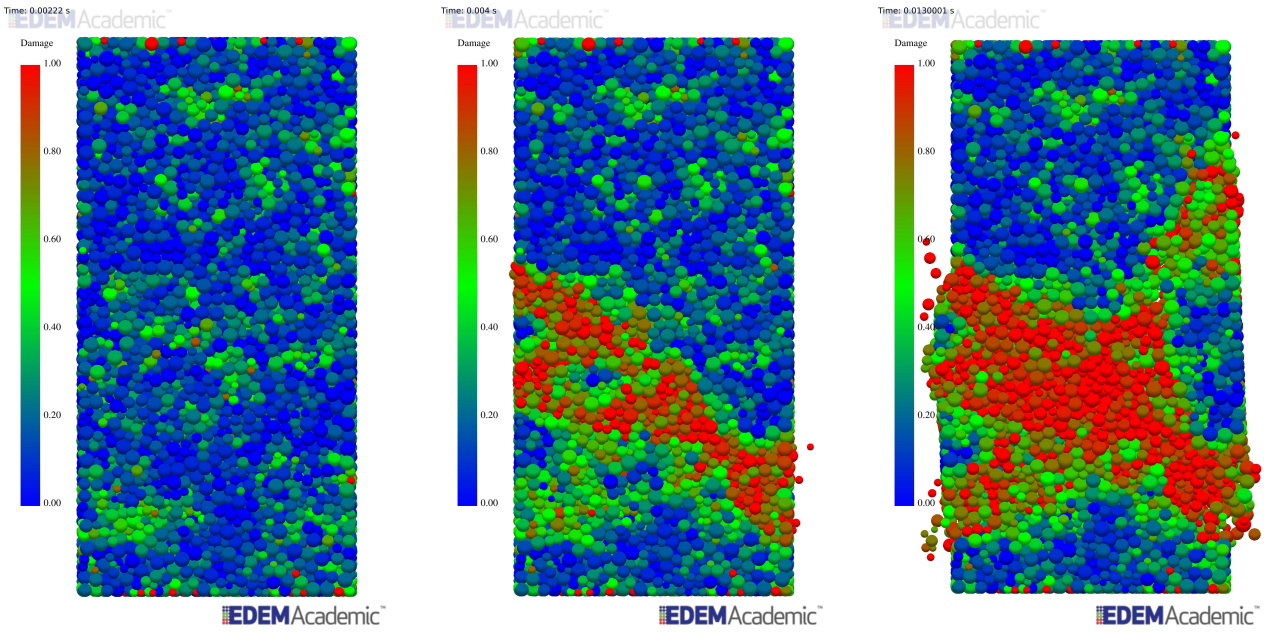

**(a)**At Peak **(b)**Post-Peak **(c)**Before collapse

**FIGURE 5.5: Visualising bond breakage** - Broken bonds at various stages of loading

At the peak strain no clear shear pattern is observed from the broken contacts, but as the

sample continues to be loaded, a clear shear band develops post peak as the number of broken bonds continues to increase. Finally, large cracking and separation is observed at large strains that are significantly past the peak.

In Figure 5.6 the custom particle property of "*Damage*" (See Reference Manual for details) is plotted for the same three time-steps. Damage is the ratio of broken bond to the initial number of bonds a particle had. A ratio of 1 means all of the original bonds that particle formed are now broken. A similar pattern is observed for the sliced assembly, with the advantage that the particles can still be visualised once the contact no longer exists following breakage.




**(a)**At Peak **(b)**Post-Peak **(c)**Before collapse

**FIGURE 5.6: Visualising damage** - Damage ratio at various stages of loading

# A TIMESTEP CALCULATIONS

Calculations to determine the mass of the smallest particle in a bonded contact:

$$K_1 = \frac{E_b A_b}{L_b} \tag{A.1}$$

$$A_b = \pi R_b^2 \lambda \tag{A.2}$$

Therefore, by rearranging Equation (A.2) and substituting for $A_b$ in Equation (A.1):

$$r_b = \sqrt{\frac{L_b K_1}{E_b \pi \lambda}} \tag{A.3}$$

The mass of a particle can be calculated as:

$$m_p = \rho_p \frac{4}{3} \pi r_b^3 \tag{A.4}$$

By substituting Equation (A.3) into Equation (A.4):

$$m_p = \rho_p \frac{4}{3} \pi \left( \frac{L_b K_1}{E_b \pi \lambda} \right)^2 \tag{A.5}$$