1D Judge - GS1 US
Optical Design and User Methodology

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1) Lens Calculation (N lens assembly)

$$X_I(X_O, X_L, f) := \left| \begin{array}{l} \text{"Calculate Image X coordinate for a lens"} \\ \text{"X.O = object coordinate in front of the lens"} \\ \text{"X.L = lens coordinate"} \\ XI \leftarrow \dfrac{X_O \cdot X_L - X_L^{\,2} + X_O \cdot f}{X_O - X_L + f} \end{array} \right.$$

$$\frac{1}{X_I - X_L} + \frac{1}{X_L - X_O} = \frac{1}{f}$$

$$MAG(X_I, X_L, f) := \left| \begin{array}{l} \text{"Calculate Image Magnification for a lens"} \\ \text{"X.I = image coordinate past the lens"} \\ \text{"X.L = lens coordinate"} \\ MAG \leftarrow \dfrac{f - (X_I - X_L)}{f} \end{array} \right.$$

$$mag = \frac{f - (X_I - X_L)}{f}$$

$$XHIM(X_O, H_O, MLA) := \left| \begin{array}{l} \text{"X.O = object X coordinate in front of the lens"} \\ \text{"H.O = object height"} \\ \text{"MLA = Matrix Lens Assembly"} \\ \text{"Output: Subsequent Intermediary Images X, H coordinates "} \\ \text{"Last Image is the one on the CCD - if in focus"} \\ XL \leftarrow MLA^{\langle 0 \rangle} \\ F \leftarrow MLA^{\langle 1 \rangle} \\ XI_0 \leftarrow X_I(X_O, XL_0, F_0) \\ HI_0 \leftarrow H_O \cdot MAG(XI_0, XL_0, F_0) \\ \text{for } I \in 1..\text{rows}(MLA) - 1 \\ \quad \left| \begin{array}{l} XI_I \leftarrow X_I(XI_{I-1}, XL_I, F_I) \\ HI_I \leftarrow HI_{I-1} \cdot MAG(XI_I, XL_I, F_I) \end{array} \right. \\ XH \leftarrow \text{augment}(XI, HI) \end{array} \right.$$

$OPTICS(MLA) :=$

"Optical Characteristics"

$XL \leftarrow MLA^{\langle 0 \rangle}$

$F \leftarrow MLA^{\langle 1 \rangle}$

$N \leftarrow rows(MLA)$

"HP and X.O are arbirary values"

$$HP \leftarrow \frac{F_0}{16}$$

$$XO \leftarrow \frac{F_0}{2}$$

"Back Focal Point BFP from 0 to N-1"

$XBFP_0 \leftarrow XL_0 + F_0$

for $I \in 1 .. rows(MLA) - 1$

$\quad XBFP_I \leftarrow X_I\left(XBFP_{I-1}, XL_I, F_I\right)$

"Image of Lens position"

$XLI_0 \leftarrow XL_0$

for $I \in 1 .. rows(MLA) - 1$

$\quad XLI_I \leftarrow X_I\left(XLI_{I-1}, XL_I, F_I\right)$

"Parallel ray (from infinite) at the pupil at lens 0,1, etc..)"

$H_0 \leftarrow HP$

for $I \in 1 .. rows(MLA) - 1$

$$H_I \leftarrow H_{I-1} - \frac{H_{I-1} \cdot \left(XL_I - XL_{I-1}\right)}{XBFP_{I-1} - XL_{I-1}}$$

"Effective Focal Length EFL"

" XHIM Output: Subsequent Intermediary Images X, H coordinates "

"XL(N-1) and N(N-1) position of last lenth and height of the ray"

$$EFL \leftarrow H_0 \cdot \frac{\left[\left(XHIM\left(XO, H_0, MLA\right)^{\langle 0 \rangle}\right)_{N-1} - XL_{N-1}\right]}{\left(XHIM\left(XO, H_0, MLA\right)^{\langle 1 \rangle}\right)_{N-1} - H_{N-1}}$$

$augment(XBFP, H)$

$$\begin{pmatrix} EFL \\ XBFP \\ H \end{pmatrix}$$

2) Miscellaneous Graphing

$$
\mathrm{GRALENS}(MLA, pc, daa) := \left|
\begin{array}{l}
\text{"Graph Lens and Apertures position"} \\
\text{"Front limiting and lens clear"} \\
DiaLens \leftarrow 1.035\cdot in \\
\text{"daa distance lens apex - HA"} \\
\text{"HA limiting aperture"} \\
\text{"dfa = distance flange - lens apex - given by manufacturer"} \\
dfa \leftarrow 4.8\cdot mm \\
HA \leftarrow pc\cdot\dfrac{DiaLens}{2} \\
\text{"HP = lens clear aperture - given by manufacturer "} \\
HP \leftarrow 0.9\cdot\dfrac{DiaLens}{2} \\
XL \leftarrow MLA^{\langle 0\rangle} \\
out \leftarrow 2\cdot HP \\
PUL_0 \leftarrow \begin{pmatrix}
XL_0 - daa & -HA \\
XL_0 - daa & -out \\
XL_0 - dfa & -out \\
XL_0 - dfa & -HP \\
XL_0 - dfa & -out \\
XL_0 & -out \\
XL_0 & out \\
XL_0 - dfa & out \\
XL_0 - dfa & HP \\
XL_0 - dfa & out \\
XL_0 - daa & out \\
XL_0 - daa & HA \\
XL_0 - daa & out
\end{pmatrix} \\
\text{for } I \in 0 .. rows(MLA) - 2 \\
\quad \left|
\begin{array}{l}
LEX_{2\cdot I, 0} \leftarrow XL_{I+1} \\
LEX_{2\cdot I+1, 0} \leftarrow XL_{I+1} \\
LEX_{2\cdot I, 1} \leftarrow (-1)^{I}\cdot out \\
LEX_{2\cdot I+1, 1} \leftarrow (-1)^{I+1}\cdot out
\end{array}\right. \\
\mathrm{stack}(PUL_0, LEX)
\end{array}\right.
$$

$$
\mathrm{GRASUP}\left(X_O, pc, daa, MLA\right) :=
\left|
\begin{array}{l}
\text{""Graph WP Points and Pseudo Marginal Rays at focus"} \\
N \leftarrow \mathrm{rows}(MLA) \\
DiaLens \leftarrow 1.035 \cdot in \\
out \leftarrow DiaLens \\
HA \leftarrow pc \cdot \dfrac{DiaLens}{2} \\
HAp \leftarrow \dfrac{HA \cdot X_O}{X_O} \\
ZOB \leftarrow \begin{bmatrix}
X_O & 0 \cdot mm \\
X_O & out \\
X_O & -out \\
X_O & 0 \cdot mm \\
\left(MLA^{\langle 0 \rangle}\right)_0 & \dfrac{HAp \cdot X_O}{X_O + daa} \\
\left(MLA^{\langle 0 \rangle}\right)_{N-1} & \dfrac{HAp \cdot X_O}{X_O + daa} \\
\left(MLA^{\langle 0 \rangle}\right)_{N-1} - X_O & 0 \cdot mm \\
\left(MLA^{\langle 0 \rangle}\right)_{N-1} - X_O & out \\
\left(MLA^{\langle 0 \rangle}\right)_{N-1} - X_O & -out \\
\left(MLA^{\langle 0 \rangle}\right)_{N-1} - X_O & 0 \cdot mm \\
\left(MLA^{\langle 0 \rangle}\right)_{N-1} & -\dfrac{HAp \cdot X_O}{X_O + daa} \\
\left(MLA^{\langle 0 \rangle}\right)_0 & -\dfrac{HAp \cdot X_O}{X_O + daa} \\
X_O & 0 \cdot mm
\end{bmatrix} \\
\text{""} \\
LRAYS \leftarrow ZOB
\end{array}
\right.
$$

$$APIM(X, H_O) := \left| \begin{array}{l} \text{"Graph Back Aperture and Sensor"} \\ DiaLens \leftarrow 1.035 \cdot in \\ HP \leftarrow 0.9 \cdot \frac{DiaLens}{2} \\ out \leftarrow 2 \cdot HP \\ \begin{pmatrix} X & -H_O \\ X & -out \\ X + 8 \cdot mm & -out \\ X + 8 \cdot mm & out \\ X & out \\ X & H_O \end{pmatrix} \end{array} \right.$$

$$MOBIM(X_O, H_O) := \begin{pmatrix} X_O & -H_O \\ X_O & H_O \\ X_O - \frac{H_O}{16} & H_O \cdot \frac{3}{4} \\ X_O + \frac{H_O}{16} & H_O \cdot \frac{3}{4} \\ X_O & H_O \end{pmatrix}$$

3) Graphing Chief Rays

$$RAYS(X_O, H_O, \alpha, MLA) := \left| \begin{array}{l} \text{"Light Rays from the tip of the object passing through Lens 0 at height } \alpha \text{ H.O"} \\ \text{""} \\ XL \leftarrow MLA^{\langle 0 \rangle} \\ F \leftarrow MLA^{\langle 1 \rangle} \\ N \leftarrow rows(MLA) \\ XI \leftarrow XHIM(X_O, H_O, MLA)^{\langle 0 \rangle} \\ HI \leftarrow XHIM(X_O, H_O, MLA)^{\langle 1 \rangle} \\ \text{"======================================"} \\ HA_0 \leftarrow H_O \cdot \alpha \\ \text{for } I \in 1 .. rows(MLA) - 1 \\ \quad HA_I \leftarrow HA_{I-1} + \frac{HI_{I-1} - HA_{I-1}}{XI_{I-1} - XL_{I-1}} \cdot (XL_I - XL_{I-1}) \\ XLHA \leftarrow augment(XL, HA) \\ OBJECT \leftarrow (X_O \ \ H_O) \\ IMAGE \leftarrow (XI_{N-1} \ \ HI_{N-1}) \\ stack(OBJECT, XLHA, IMAGE) \end{array} \right.$$

$$\text{FUNRAYS}(X_O, H_O, pc, daa, MLA) := \left| \begin{array}{l} \text{""} \\ DiaLens \leftarrow 1.035 \cdot in \\ \text{"For CHIEF RAYS"} \\ \alpha_{A1} \leftarrow 1 + \left(\frac{pc}{2} \cdot \frac{DiaLens}{H_O} - 1\right) \cdot \frac{X_O}{X_O + daa} \\ \alpha_{A2} \leftarrow 1 - \left(\frac{pc}{2} \cdot \frac{DiaLens}{H_O} + 1\right) \cdot \frac{X_O}{X_O + daa} \\ ARAYS \leftarrow stack(RAYS(X_O, H_O, \alpha_{A1}, MLA), reverse(RAYS(X_O, -H_O, \alpha_{A1}, MLA))) \\ BRAYS \leftarrow stack(RAYS(X_O, H_O, \alpha_{A2}, MLA), reverse(RAYS(X_O, -H_O, \alpha_{A2}, MLA))) \\ \text{""} \\ \text{"For MARGINAL RAYS (Formula to be checked"} \\ \varepsilon \leftarrow 0.001 \\ \alpha_M \leftarrow \left(\frac{pc}{2} \cdot \frac{DiaLens}{H_O}\right) \cdot \frac{X_O}{X_O + daa} \cdot \frac{1}{\varepsilon} \\ MRAYS \leftarrow stack(RAYS(X_O, H_O \cdot \varepsilon, \alpha_M, MLA), reverse(RAYS(X_O, -H_O \cdot \varepsilon, \alpha_M, MLA))) \\ SUMRAYS \leftarrow stack(ARAYS, BRAYS, MRAYS) \end{array} \right.$$

$$\text{CALE}(X_O, MLA) := \left| \begin{array}{l} \text{"Calculate Lens Characteristics:"} \\ \text{"Effective Focal Length"} \\ \text{"Back lens apex to CCD distance"} \\ \text{"Magnification"} \\ XL \leftarrow MLA^{\langle 0 \rangle} \\ F \leftarrow MLA^{\langle 1 \rangle} \\ N \leftarrow rows(MLA) \\ H_O \leftarrow 15 \cdot mm \\ X_I \leftarrow \left(XHIM(X_O, H_O, MLA)^{\langle 0 \rangle}\right)_{N-1} \\ H_I \leftarrow \left(XHIM(X_O, H_O, MLA)^{\langle 1 \rangle}\right)_{N-1} \\ F_L \leftarrow -OPTICS(MLA)_0 \\ \begin{bmatrix} \text{"Effective Focal Length "} & \left(\frac{F_L}{mm}\right) & \text{"mm"} \\ \text{"Back lens apex to Image"} & \left(\frac{X_I - XL_{N-1}}{mm}\right) & \text{"mm"} \\ \text{"Magnification "} & \frac{H_I}{H_O} & \text{""""} \end{bmatrix} \end{array} \right.$$

4) Managing Graphs

$$GOLIR(X_O, H_O, pc, daa, MLA) :=$$

""Graph Object, Lens, Image, and Light Rays"

$HP \leftarrow H_O$

$N \leftarrow rows(MLA)$

$XL \leftarrow MLA^{\langle 0\rangle}$

$F \leftarrow MLA^{\langle 1\rangle}$

$X_{OINF} \leftarrow -50 \cdot m$

"Normalize pupil size at H.O"

$$H \leftarrow OPTICS(MLA)_2 \cdot \left(\frac{16}{F_0} \cdot H_O\right)$$

$RAYP \leftarrow augment(XL, H)$

$XBFP \leftarrow XHIM(X_{OINF}, H_O, MLA)^{\langle 0\rangle}$

$XBIM \leftarrow XHIM(X_O, H_O, MLA)^{\langle 0\rangle}$

$HI \leftarrow XHIM(X_O, H_O, MLA)^{\langle 1\rangle}$

$POB \leftarrow \begin{pmatrix} X_O & H_O \end{pmatrix}$

$RAYP \leftarrow augment(XL, H)$

$$IMAGE \leftarrow \begin{pmatrix} XBIM_{N-1} & HI_{N-1} \\ XBIM_{N-1} & -HI_{N-1} \end{pmatrix}$$

$RAYN \leftarrow augment(reverse(XL), -reverse(H))$

$NOB \leftarrow \begin{pmatrix} X_O & -H_O \end{pmatrix}$

$$CENLIN \leftarrow \begin{pmatrix} X_O & 0 \\ XBIM_{N-1} & 0 \\ X_O & 0 \end{pmatrix}$$

$LRAYS_1 \leftarrow stack(POB, RAYP, IMAGE, RAYN, NOB, CENLIN)$

$LRAYS_2 \leftarrow FUNRAYS(X_O, H_O, pc, daa, MLA)$

$MOB \leftarrow MOBIM(X_O, H_O)$

$MIM \leftarrow MOBIM(XBIM_{N-1}, HI_{N-1})$

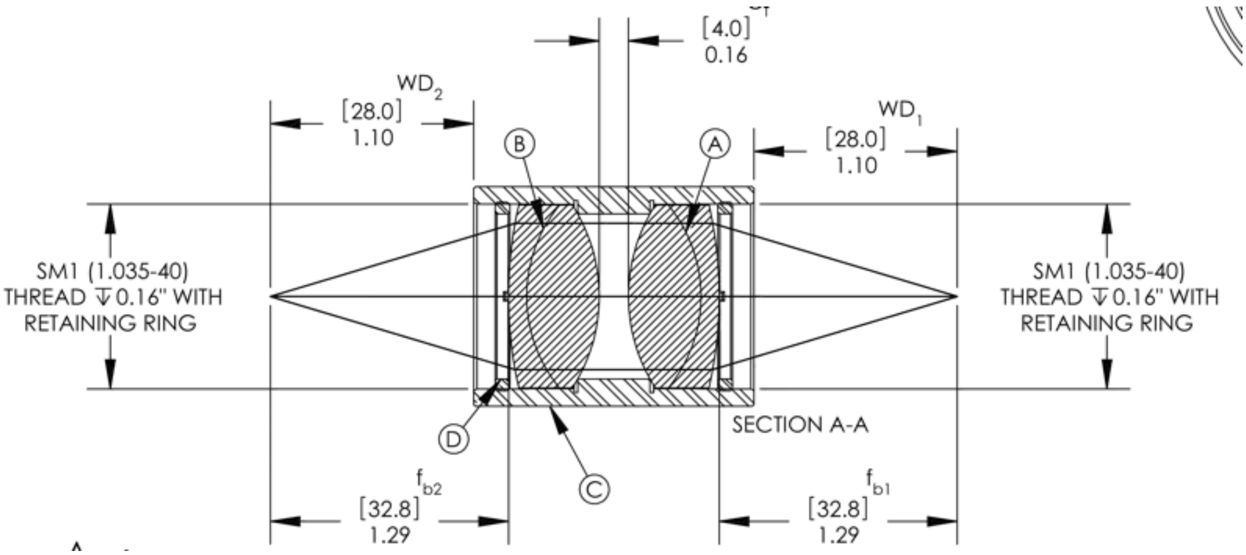
$LENS \leftarrow GRALENS(MLA, pc, daa)$

$X \leftarrow XBIM_{N-1}$

$$\begin{pmatrix} LENS \\ LRAYS_1 \\ MOB \\ MIM \\ LRAYS_2 \end{pmatrix}$$

$$\text{GOTALL}\left(f_b, x_{22}, H_O, H_{AP}, pc, daa, MLA_1, \Delta X_O, \Delta X_{AP}\right) := \left| \begin{array}{l} \text{"Graph ALL"} \\ X_O \leftarrow -f_b - \Delta X_O \\ \begin{pmatrix} ML \\ MR \\ MOB \\ MIM \\ MRA \end{pmatrix} \leftarrow \text{GOLIR}\left(X_O, H_O, pc, daa, MLA_1\right) \\ DFW \leftarrow \text{GRASUP}\left(-f_b, pc, daa, MLA_1\right) \\ AS \leftarrow \text{APIM}\left(f_b + x_{22} + \Delta X_{AP}, H_{AP}\right) \\ \begin{pmatrix} ML \\ MR \\ MOB \\ MIM \\ DFW \\ AS \\ MRA \end{pmatrix} \end{array} \right.$$

5) The THORSLABS INC. Paired Doublets



From the data sheet part number MAP104040-A

D = flange to flange distance = 38.6 mm (1.52 in)

WD = working distance = 28 mm **(That's the distance from back flange to image or front flange to object)**

ΔD = flange to apex = 4.8 mm

f1 = focal length of doublet #1 = 40 mm
f2 = focal length of doublet #2= 40 mm

Apex to apex = 29.0 mm

Lens thickness d.L= 12.5 mm
Lens separation d.s = 4 mm

The Thorlabs achromatic pair is made of 2 doublets (doublet 1 & 2), each having a focal length f = 40 mm and being separated by a distance $d._S$ ...

Each doublet is represented by 2 thin lenses separated by a distance $d._L$ that is the thickness of the doublet at the apex.

$$\frac{1}{f_1} = \frac{1}{f_{11}} + \frac{1}{f_{12}} - \frac{d_L}{f_{11} \cdot f_{12}}$$

$f := 40 \cdot mm$ $d_L := 12.5 \cdot mm$ $d_s := 4.mm$ $\beta \equiv \frac{1}{6}$ $f_b := 32.8 \cdot mm$

$$f_{11} := f \cdot \frac{\left[2 \cdot f + (\beta - 2) \cdot d_L\right]}{\left[f + (\beta - 1) \cdot d_L\right]}$$

$$f_{12} := 2 \cdot f + (\beta - 1) \cdot d_L$$

$$f_c(f_{11}, f_{12}, d_L) := \left(\frac{1}{f_{11}} + \frac{1}{f_{12}} - \frac{d_L}{f_{11} \cdot f_{12}}\right)^{-1}$$

$f_c(f_{11}, f_{12}, d_L) = 40 \cdot mm$ $DiaLens := 1.035 \cdot in$

Position of the 4 thin lenses: (lens # 0 at position 0 mm)

$x_{11} := 0 \cdot mm \quad x_{12} := d_L \quad x_{21} := d_L + d_s \quad x_{22} := d_L + d_s + d_L$

$$MLA_1 := \begin{pmatrix} x_{11} & f_{11} \\ x_{12} & f_{12} \\ x_{21} & f_{12} \\ x_{22} & f_{11} \end{pmatrix} \qquad MLA_1 = \begin{pmatrix} 0 & 77.1831 \\ 12.5 & 69.58333 \\ 16.5 & 69.58333 \\ 29 & 77.1831 \end{pmatrix} \cdot mm \qquad pc := \frac{2 \cdot f_b}{DiaLens} \cdot \tan\left(\frac{7.5}{180} \cdot \pi\right)$$

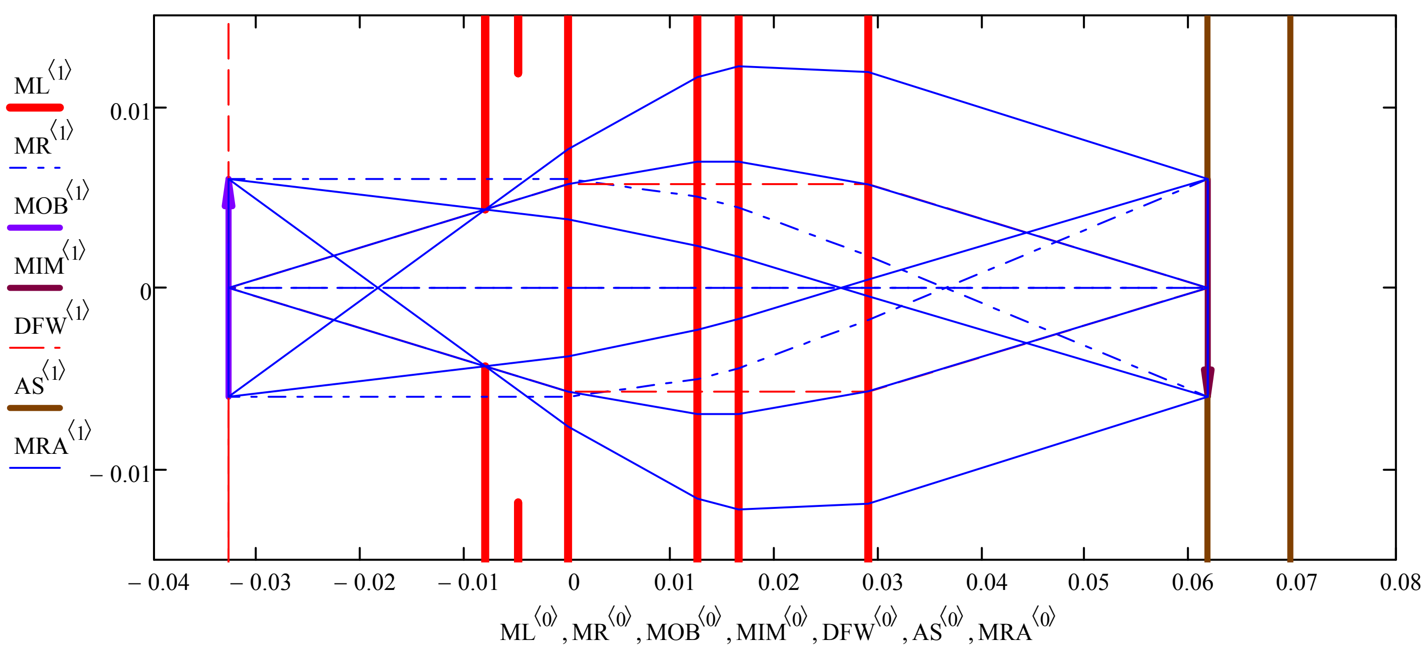
6) The Functioning System

$H_O := 6 \cdot mm \quad H_{AP} := 6 \cdot mm \qquad \Delta X_O := -0mm \qquad \Delta X_{AP} := -0 \cdot mm \qquad pc = 0.329 \qquad daa := 8 \cdot mm$

$X_O = -f_b - \Delta X_O$

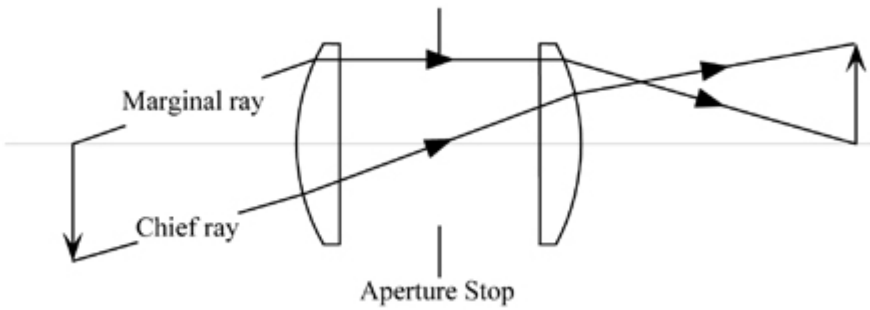
$$\begin{pmatrix} ML \\ MR \\ MOB \\ MIM \\ DFW \\ AS \\ MRA \end{pmatrix} := GOTALL\left(f_b, x_{22}, H_O, H_{AP}, pc, daa, MLA_1, \Delta X_O, \Delta X_{AP}\right)$$

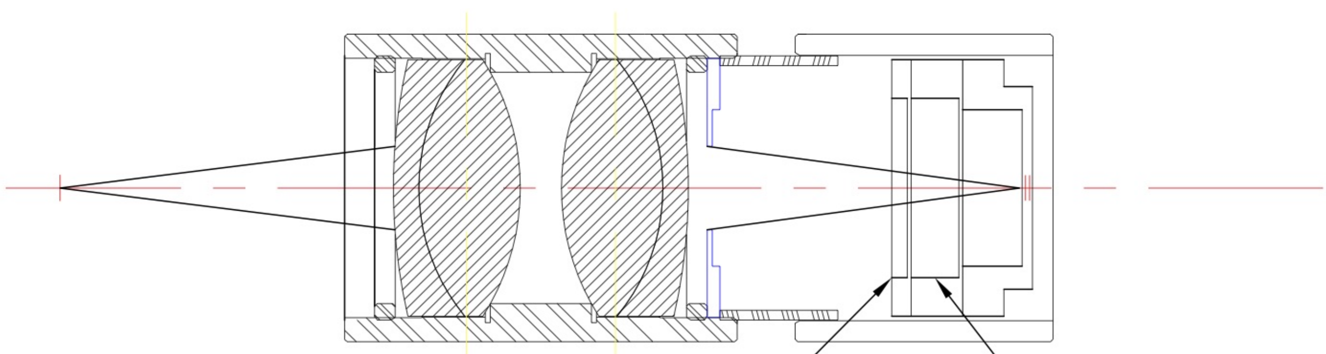
$$CALE\left(-f_b + \Delta X_O, MLA_1\right) = \begin{pmatrix} \text{"Effective Focal Length "} & 25.379 & \text{"mm"} \\ \text{"Back lens apex to Image"} & 32.829 & \text{"mm"} \\ \text{"Magnification "} & -1.001 & \text{""""} \end{pmatrix}$$



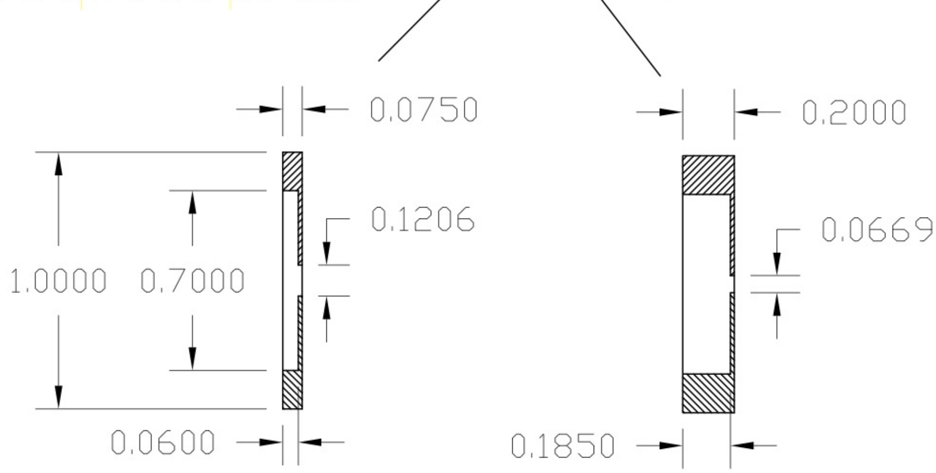
The red vertical lines are located at the apexes of the lens (front and back). I moved the blue object on the left and calculated the position of the brown image and the magnification. Placing the apex of the first lens at zero, I found that for the object position at -32.8 mm (as specify by the manufacturer) I have the sensor surface position at 32.829 and a magnification of -1.001.

That proves that the Working Distance is indeed the distance fb (from back lens apex to sensor surface and from front end apex to sample). Thorlabs gives 32.8 mm...





Aperture 0.325 “ in front and back close to the lenses..



7) Error Analysis

(misplacement of sample and master aperture: variation of the brightness of the spot)

$$ERANA\left(f_b, x_{22}, H_O, H_{AP}, pc, daa, MLA_1, \Delta X_O, \Delta X_{AP}\right) :=$$

"Use the outer Chief Ray passing through the aperture"

"Calculate the brightness of the spot"

"=="

$$X_O \leftarrow -f_b + \Delta X_O$$

$$XL \leftarrow MLA_1^{\langle 0 \rangle}$$

$$N \leftarrow rows\left(MLA_1\right)$$

$$XI \leftarrow XHIM\left(X_O, H_O, MLA_1\right)^{\langle 0 \rangle}$$

$$HI \leftarrow XHIM\left(X_O, H_O, MLA_1\right)^{\langle 1 \rangle}$$

"=="

$$\alpha_{A2} \leftarrow 1 - \left(\frac{pc}{2} \cdot \frac{DiaLens}{H_O} + 1\right) \cdot \frac{X_O}{X_O + daa}$$

$$HA_0 \leftarrow H_O \cdot \alpha_{A2}$$

$$\text{for } I \in 1 .. rows\left(MLA_1\right) - 1$$

$$HA_I \leftarrow HA_{I-1} + \frac{HI_{I-1} - HA_{I-1}}{XI_{I-1} - XL_{I-1}} \cdot \left(XL_I - XL_{I-1}\right)$$

$$XLHA \leftarrow augment(XL, HA)$$

"Outer Chief Ray crossing last lens)"

$$CRCL \leftarrow submatrix(XLHA, N-1, N-1, 0, 1)$$

$$APERT \leftarrow \left(x_{22} + f_b + \Delta X_{AP} \quad H_{AP}\right)$$

$$IMAGE \leftarrow \left(XI_{N-1} \quad HI_{N-1}\right)$$

$$SL \leftarrow \frac{IMAGE_{0,1} - CRCL_{0,1}}{IMAGE_{0,0} - CRCL_{0,0}}$$

$$HPR \leftarrow sign\left(CRCL_{0,1}\right) \cdot CRCL_{0,1} + SL \cdot \left(CRCL_{0,0} - APERT_{0,0}\right)$$

$$\left(\frac{HPR}{HI_{N-1}}\right)^2$$

$$\Delta X_O := -2 \cdot mm$$

$$ERANA\left(f_b, x_{22}, H_O, H_{AP}, pc, daa, MLA_1, \Delta X_O, \Delta X_{AP}\right) = 0.88608$$

$$\Delta X_O := 2 \cdot mm$$

$$ERANA\left(f_b, x_{22}, H_O, H_{AP}, pc, daa, MLA_1, \Delta X_O, \Delta X_{AP}\right) = 1.12551$$

There is NO maximum of brightness of the spot at focus...
A "working" system has to be built according to the specifications.

8) User Methodology (a CZC suggestion)

Bar code and blur with circular aperture simulation

$$
\begin{aligned}
\mathrm{BLUR}(IMA, ND) := \;& \text{"Blur IMA with disk dimeter ND"} \\
& MASK \leftarrow IMA\cdot 0 \\
& R \leftarrow \mathrm{rows}(IMA) - 1 \\
& C \leftarrow \mathrm{cols}(IMA) - 1 \\
& \text{for } I \in 0..ND-1 \\
& \quad \text{for } J \in 0..ND-1 \\
& \qquad MASK_{I,J} \leftarrow \Phi\left[\frac{ND}{2}\cdot\left(\frac{ND}{2}+1\right) - \left[\left(I-\frac{ND}{2}\right)^2 + \left(J-\frac{ND}{2}\right)^2\right]\right] \\
& FIMA \leftarrow \overrightarrow{(\mathrm{cfft}(IMA)\cdot\mathrm{cfft}(MASK))} \\
& AVMA \leftarrow \mathrm{icfft}(FIMA) \\
& AVIMA \leftarrow \frac{AVMA}{\mathrm{mean}(AVMA)}\cdot\mathrm{mean}(IMA) \\
& MAV \leftarrow \overrightarrow{(\mathrm{Re}(AVIMA)\cdot\Phi(\mathrm{Re}(AVIMA)))}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{BARSPA}(M, N, WL, BL) := \;& \text{"Simple Bar - Space Image"} \\
& BB_{M-1,N-1} \leftarrow 0 \\
& BB \leftarrow BB + BL \\
& WB_{M-1,N-1} \leftarrow 0 \\
& WB \leftarrow WB + WL \\
& IMA \leftarrow \mathrm{augment}(BB, WB, BB, WB, BB, WB, BB, WB, BB)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{NOISE}(IMA, NL) := \;& \text{for } i \in 0..\mathrm{rows}(IMA) - 1 \\
& \quad \text{for } j \in 0..\mathrm{cols}(IMA) - 1 \\
& \qquad NIMA_{i,j} \leftarrow \mathrm{rnd}(NL) - \frac{NL}{2} \\
& IMA + NIMA
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{MAGNIF}(MAT, K) := \;& M \leftarrow \mathrm{rows}(MAT) \\
& N \leftarrow \mathrm{cols}(MAT) \\
& \text{for } i \in 0..K\cdot M - 1 \\
& \quad \text{for } j \in 0..K\cdot N - 1 \\
& \qquad MAGMA_{i,j} \leftarrow MAT_{\mathrm{floor}\left(\frac{i}{K}\right), \mathrm{floor}\left(\frac{j}{K}\right)} \\
& MAGMA
\end{aligned}
$$

$ZIMA := BARSPA(200, 80, 200, 50)$

ZIMA

$MAZIMA := MAGNIF(ZIMA, 5)$

$MABIM := BLUR(MAZIMA, 5 \cdot D_A)$

$BIM := MAGNIF\left(MABIM, \frac{1}{5}\right)$

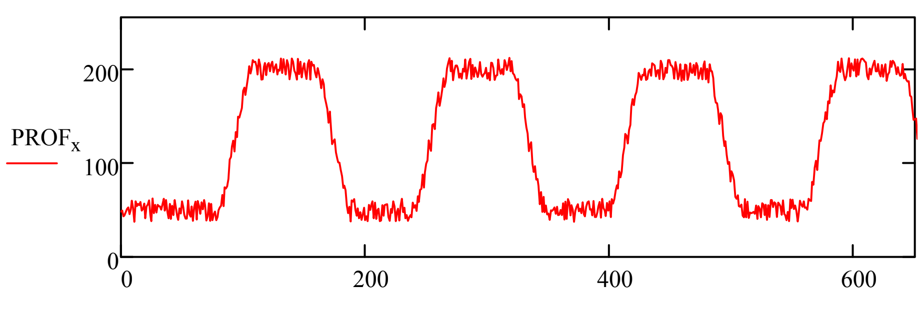
BIM

$x := 0 .. cols(BIM) - 1$

D = diameter of aperture $D_A \equiv 28$

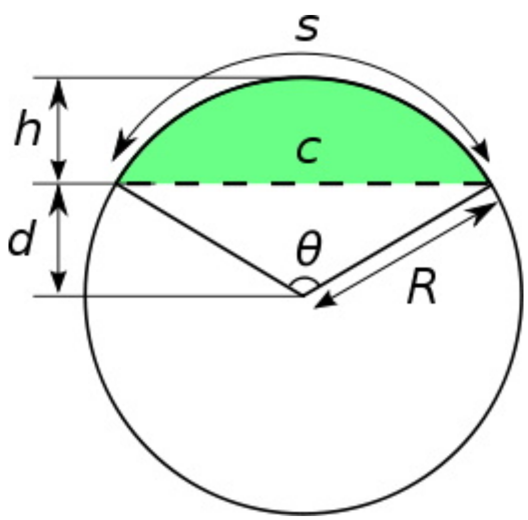
$NBIM := NOISE(BIM, 25)$

$PROF_x := NBIM_{100, x}$



Bar Code edge due to circular blur:

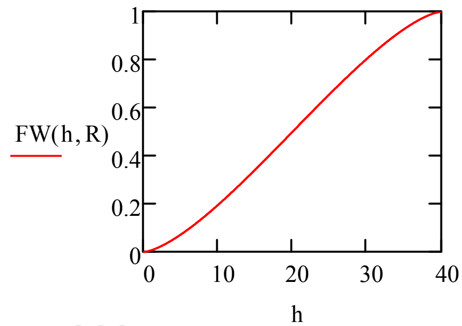
The area of the circular segment is equal to the area of the circular sector minus the area of the triangular portion. We are looking for a function of R and h



$$\theta(h,R) := 2\cdot\mathrm{acos}\left(1 - \frac{h}{R}\right)$$

$$FW(h,R) := \frac{1}{2\cdot\pi}\cdot(\theta(h,R) - \sin(\theta(h,R)))$$

$R := 20 \qquad h := 0, 0.001 .. 2\cdot R$



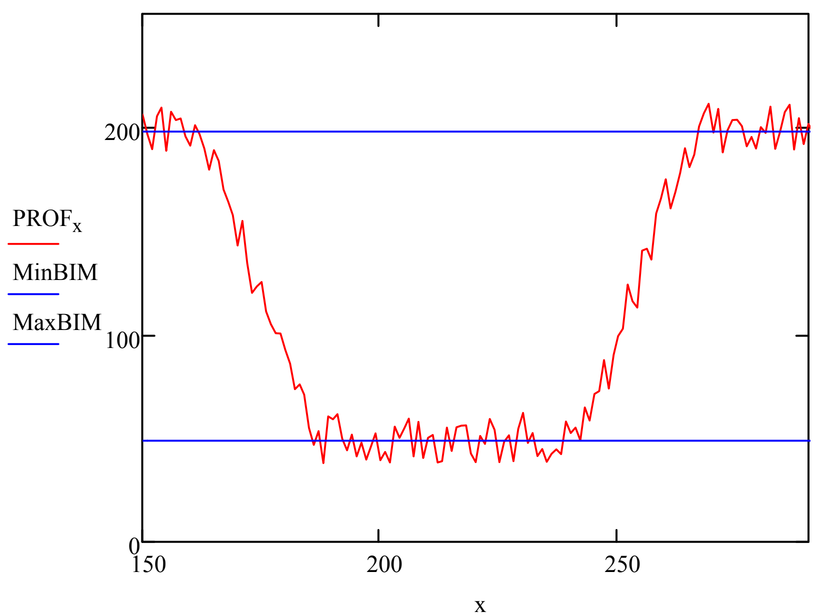
Calculation of the effective aperture and bar width

1) Averaging noise (on top and bottom):

$$MinBIM := \frac{1}{31}\cdot\sum_{y=200}^{230} PROF_y \qquad MaxBIM := \frac{1}{31}\cdot\sum_{y=275}^{305} PROF_y$$

(Has to be generalized and optimized for automation)

$y := 200 .. 300 \qquad y_0 := 220$



$MinBIM = 48.991$

$MaxBIM = 198.295$

2) Very simple piece wise function must be fitted:

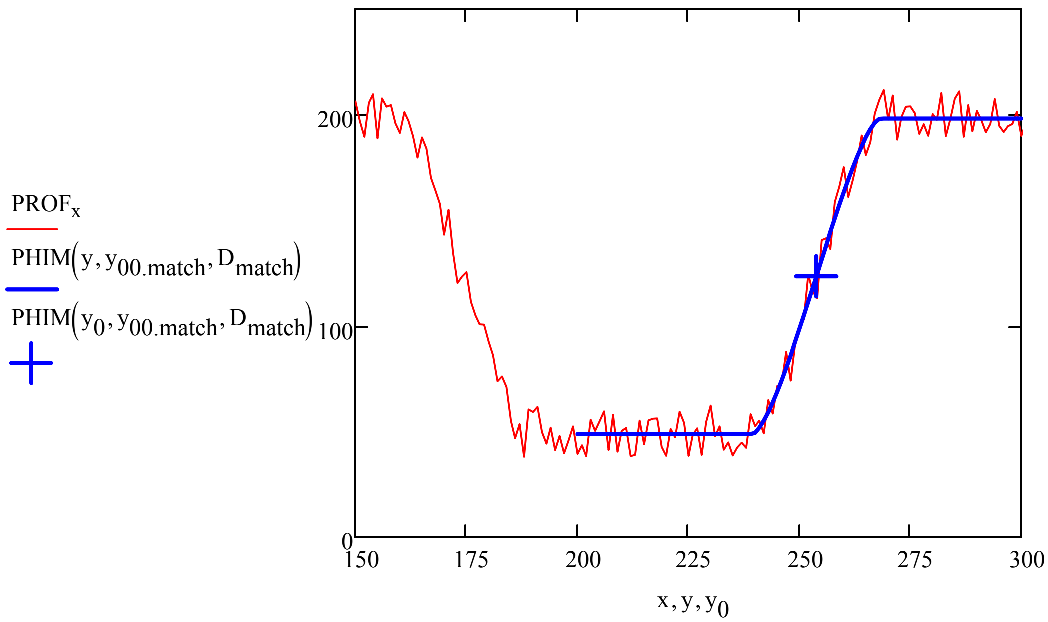
$$PHIM(y, y_0, D) := MinBIM + \left(\Phi(y - y_0)\cdot\Phi(y_0 + D - y)\cdot FW\left(y - y_0, \frac{D}{2}\right) + \Phi(y - y_0 - D)\right)\cdot(MaxBIM - MinBIM)$$

3) Very simple procedure:

$$
\mathrm{RESOLVE}(PROF, x_{min}, x_{max}, D_{min}, D_{max}) := \left|\begin{array}{l}
\text{""} \\
y_0 \leftarrow \dfrac{x_{min} + x_{max}}{2} \\
\text{for } i \in 0..500 \\
\quad \left|\begin{array}{l}
D_i \leftarrow D_{min} + \dfrac{(D_{max} - D_{min}) \cdot i}{500} \\
y00_i \leftarrow \mathrm{root}\left(\displaystyle\sum_{x = x_{min}}^{x_{max}} PROF_x - \sum_{x = x_{min}}^{x_{max}} \mathrm{PHIM}(x, y_0, D_i), y_0\right) \\
VAR_i \leftarrow \displaystyle\sum_{x = x_{min}}^{x_{max}} \left(PROF_x - \mathrm{PHIM}(x, y00_i, D_i)\right)^2
\end{array}\right. \\
n_{match} \leftarrow \mathrm{match}(\min(VAR), VAR)_0 \\
D_{n_{match}} \\
y_0 \leftarrow \mathrm{root}\left(\mathrm{PHIM}(y_0, y00_{n_{match}}, D_{n_{match}}) - \dfrac{MinBIM + MaxBIM}{2}, y_0\right) \\
\text{"y.0 is coordinate where to measure the bar width "} \\
\begin{pmatrix} D_{n_{match}} \\ y00_{n_{match}} \\ y_0 \end{pmatrix}
\end{array}\right.
$$

$$
\begin{pmatrix} D_{match} \\ y_{00.match} \\ y_0 \end{pmatrix} := \mathrm{RESOLVE}(PROF, 200, 300, 20, 40)
$$

$D_{match} = 28.56$ $D_A = 28$



That's just an example of fitting one side or a bar. Similar calculation can be done for the other side to measure the bar width. Note that the value of the measured effective aperture, as well as the measured width of the bar are very sensitive to the noise. Multiple profiles with enough data points on the side of the bar have to be used in conjunction with a statistical analysis.

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