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NUMERO: **2531A**

ANNO: 2022

APPUNTI

STUDENTE: Sobrero Giovanni

MATERIA: Advanced topics of engineering thermodynamics -
Prof. Asinari, Prof. Chiavazzo

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ATTENZIONE: QUESTI APPUNTI SONO FATTI DA STUDENTI E NON SONO STATI VISIONATI DAL DOCENTE.
IL NOME DEL PROFESSORE, SERVE SOLO PER IDENTIFICARE IL CORSO.

Advanced Topics of Engineering Thermodynamics (AET) – Numerical Modelling (NM) (10 cfu)

Professors

AET: Prof. Pietro Asinari (2019) Prof. Eliodoro Chiavazzo (2020); Dr. M. Fasano & Dr. M. Morciano

NM: Prof. Claudio Canuto; MATLAB: Prof. Paolo Bardella & Dr. Lorenzo Columbo

(Ref.: Video-lectures ITA 2017 of prof. P. Asinari and C. Canuto)

Giovanni Soderro's Schemes

A.A. 2018 – 2019 / 2019 -2020 / 2020-2021/2021-2022 ←

Program

Advanced Topics of Engineering Thermodynamics (AET)

CLASSICAL MOLECULAR DYNAMICS and KINETIC THEORY

Introduction to classical molecular dynamics. Bond and non-bond interactions. Force fields. Elementary numerical schemes (Verlet integration). Elementary statistical ensembles: Thermostats and barostats. Practical examples. Large systems approaching the local equilibrium: Maxwellian distribution function. The distribution function dynamics. Linear relaxation towards the local equilibrium: Bhatnagar–Gross–Krook (BGK) model. [Modelling photos transport](#). Practical examples.

CONTINUUM THERMO-MECHANICS

Deduction of the equation of mass and momentum conservation by both kinetic local equilibrium and by elementary control volume. Deduction of the wave equation. Small deviations from the conditions of local equilibrium. Phenomenological relations in Navier-Stokes-Fourier equations: Stress tensor and thermal flux. Generalization of the results obtained by the ideal gas to other types of fluids. Dimensionless equations. Meaning of dimensionless numbers. Incompressible limit. Equation for kinetic energy and enthalpy. First principle of thermodynamics. Generalization of entropy for continuous body. Generalization of Gibbs's correlation. The second principle of thermodynamics for a continuous body. Work, heat and the thermodynamics of irreversible processes.

THERMAL DESIGN

Deduction of the integral equations for closed systems and open systems. Technical formulation of integral equations. Physical meaning of irreversibility. Correct calculation of irreversibility by practical formulas. Turbulence and turbulent flows. Characteristic scales of the phenomenon, deduction of the equations for the average quantities and the closure problem. Artificial viscosity induced by turbulence and modelling. Exergy balance in a reversible system. Exergy and internal exergy for an ideal gas. The theorem of Guy-Stodola. Physical meaning of exergy. Efficiency according to the second principle. Examples of exergy analysis. Exergy diagrams. Thermodynamic diagrams.

ACOUSTICS

Deduction of the wave equation. Introduction, elastic, plane, longitudinal and progressive waves. Propagation speed of elastic waves; sound speed of air. Mechanical power transported by sound wave, wave intensity, resistance and effective pressure. Acoustic intensity and acoustic feeling: Law of Weber-Fechner. Diagram of the normal acoustic response. Acoustic field, feeling and the intensity level, decibels. Iso-phon curves. Frequency bands, level of pressure, interpolating weight curve A. Interaction between elastic waves and materials, factors of reflection, transmission, absorption, apparent absorption. Effect of frequency. Apparent absorption factor of several walls. Acoustics in open environments. Open field. Sound tail. Acoustic energy balance and reverberation, reverberation time by conventional formula of Sabine. Sound insulation; sound proofing; plain wall and law of mass and frequency; case study for a pipe.

LIGHTING

Deduction of the radiative transfer equation (RTE) from kinetic theory. The light, electromagnetic radiation, main features, diffuse radiation. Visual perception and photometric system. Definition of physical units of measured quantities. Point source. Light intensity. Indicator of emission. Light flux emitted from a point source with a given indicator of emission. The first formula of Lambert. Linear source, linear luminance, and lighting calculations on surface. Surface source, luminance, and lighting calculation on a surface. The second law of Lambert. Lambert emitter. Efficiency of a light bulb.

Numerical Modelling (NM)

INTRODUCTORY PART

General concepts about partial differential equations; boundary and initial conditions; properties of solutions. Basic concepts of numerical methods.

STEADY-STATE PROBLEMS

Elliptic problems; the steady diffusion and the membrane equilibrium examples; discretization by centered finite differences; variational formulation; discretization by finite elements. Implementation of Dirichlet, Neumann and Robin boundary conditions. Reduction of the discrete problem to an algebraic problem; properties of the corresponding matrices; techniques for solving large systems of algebraic equations. Mathematical properties of consistency, stability and convergence of the numerical schemes. Modal analysis; the free vibration of a membrane; discretization of eigenvalue problems.

TIME-DEPENDENT PROBLEMS

Formulation and discretization of evolutionary problems; parabolic and hyperbolic equations; the heat equation, the wave equation; mass lumping; time advancing techniques; asymptotic stability and choice of the time step; rate of convergence in space and time. Convection-diffusion problems; mesh Peclet number; centered versus upwind discretizations. Conservation and balance laws; characteristics; integral formulation; discretization by finite volumes; cell averages and numerical fluxes; review of the main classical methods; relation with finite differences; Courant number and CFL condition; numerical diffusion and dispersion; stability and convergence.

Course Structure

In addition to lessons, the following activities are provided.

Concerning the first part of Applied Engineering Thermodynamics (AET), students are expected to develop a project. Students are divided into 5 teams, as many as the number of applications. For each theme, they must provide (a) calculation of an off-design condition, (b) exergetic analysis and (c) all the technical details related to the design performed. To develop the project, specific notes are made available on the "Portale della Didattica". In addition, some lectures are focused on the presentation of the guidelines for the project developments and practical examples.

Concerning the part on applied acoustics, a practical application in class is developed, aiming at the evaluation of acoustic behavior of the room. In particular, three different analyses are performed: evaluation of the acoustic field, measurement of the reverberation time and measurements of the acoustic pressure.

Concerning the part on Numerical Modelling (NM), the following exercises and laboratory activity is developed: Mesh generation; construction of mass and stiffness matrices in various situations; iterative solution of large algebraic systems with sparse matrices; computation of the equilibrium configuration of several physical problems; analysis of the behavior of the spatial discretization error. Implementation of eigenvalue problems and modal analysis. Implementation of time advancing techniques; investigation on the stability of the schemes and the behavior of the temporal error; computation of the evolution of the temperature of a conducting body, and of the propagation of waves in an elastic body. Implementation of numerical schemes for scalar conservation laws and experimental investigation on their behavior.

Reading Material

- P. Asinari, E. Chiavazzo, *An Introduction to Multiscale Modelling with Applications*, Società Editrice Esculapio, Bologna 2013.
- M. Calì, P. Gregorio, "Termodinamica" Esculapio, Bologna 1997.
- A. Bejan, "Advanced Engineering Thermodynamic" John Wiley & Sons 1997.
- G. Guglielmini, C. Pisoni, *Introduzione alla trasmissione del calore*, Casa Editrice Ambrosiana, 2002.
- G. Comini, G. Cortella, *Fondamenti di trasmissione del calore*, Servizi Grafici Editoriali, 2001.
- C. Canuto, "Metodi e Modelli Numerici ", note delle lezioni con esercizi, disponibile online sul Portale della Didattica.
- A. Quarteroni, "Numerical Models for Differential Problems", Springer 2014.

Assessment and Grading Criteria

Exam:

- Computer lab-based test;
- Written test;
- Compulsory oral exam;**
- Group project;**

The exam consists of both a written part, related to the module of Numerical Modelling, and an oral part, related to the module of Advanced Engineering Thermodynamics. The final evaluation of the exam consists of the arithmetic mean (rounded up) of the two partial scores obtained in the two modules.

With regards to the module on Numerical Modelling, the evaluation procedure is based on the following tests:

- a) Solving some exercises on the main topics covered in the module (available time: 60 minutes);
- b) Answering a series of multiple-choice questions with the help of MATLAB (available time: 45 minutes).

No educational material is allowed in these tests.

The mark of the written part will also take into account c) the optional preparation of a computational project during the semester, carried out by small groups of students on a numerical topic related to the group project developed in the advanced engineering thermodynamics module, and evaluated on the basis of the individual contribution. Tests a) and b) have a relative weight of 2/3 and 1/3, and overall they allow the student to obtain up to 28 points, whereas the computational project allows the student to obtain a higher mark than 28, including the laude. The mark of the written part will be communicated to students through the Portale della Didattica, together with the indication of when and where they can meet the teacher and check the results of their tests.

Consistently with the expected learning outcomes, the written part of the exam aims to ensure the achievement of the following objectives:

1. In-depth knowledge of the main methods to numerically discretise a mathematical model and to translate it into a system of algebraic equations. This is accomplished by test a).
2. Ability to implement in MATLAB(r) the numerical models presented in class. This is mainly established through test b), but also through test c).
3. Ability to apply the numerical tools to the simulation of the behaviour of physical problems of simple thermodynamic interest. This is established through test c).

As for the module on Advanced Engineering Thermodynamics, the exam is oral and is conducted as follows. Each student will have to answer a first question on a theoretical topic discussed during the semester. The answer to the first question is written and discussed immediately afterwards through a direct interaction with the examiner, so that it is possible to assess that the student has correctly learned the fundamentals of thermomechanical continuous media, thermodynamics, fluid dynamics as well as environmental acoustics and lighting. This part will last about forty five minutes with no educational material allowed. Subsequently, the student will have to demonstrate that he/she has actively contributed to the group project, answering a second oral question by the examiner. Here, by focussing on a realistic energy conversion system, it is possible to assess if the student correctly developed sufficient skills for thermal, energy, exergy analysis of energy conversion devices. Alternatively, at the choice of the examiner, this second question may possibly focus on the applied acoustics part. About thirty minutes will be given to reply to the second question. During this part, it is allowed the use of educational material.

A partial score for the **Advanced Engineering Thermodynamics** module only is established, which is given by the arithmetic mean of the marks assigned by the examiner to the first and second answers.

Consistently with the expected learning outcomes, the oral part of the exam aims to ensure the achievement of the following objectives:

1. In-depth knowledge of the theoretical notions on thermo-mechanics, continuum theory and thermodynamic. This is accomplished by the first theoretical question;
2. Ability to use the theoretical tools provided in the subject energy and exergetic design and analysis to study real/complex systems involving energy transformation processes. This is established both through the first theoretical question and through the implementation of the group project;
3. Ability to properly interpret the regulations and to perform estimates in the field of lighting and applied acoustic. This is mainly determined by the implementation of the group project and the report on applied acoustic.

O. TENSOR NOTATION

[REF. 2021-2022]

O.1 MEANINGFUL QUANTITIES

○ SCALAR QUANTITIES ($p, P, T, E_k, h, s, \text{EXERGY}$)

$a, b \in \mathbb{R}$ (NO NEED OF INDEXES)

○ VECTORIAL QUANTITIES (γ, M, \dots)

$\underline{a}, \vec{a} = \begin{Bmatrix} a_1 \\ \vdots \\ a_D \end{Bmatrix} = \{a_i\} \in \mathbb{R}^D \leftarrow \text{COMPACT NOTATION} \quad (\text{NOTE: } \{\}) \text{ CAN BE OMITTED}$
 $\underline{a} \leftarrow \text{CARTESIAN REPR.}$

○ TENSORIAL QUANTITIES

$$\underline{\underline{A}}, \underline{\underline{\underline{B}}} = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1D} \\ B_{21} & B_{22} & \dots & B_{2D} \\ \vdots & \vdots & \ddots & \vdots \\ B_{D1} & B_{D2} & \dots & B_{DD} \end{bmatrix} = \{B_{ij}\} \in \mathbb{R}^{D \times D}$$

O.2 PRODUCTS

○ INNER PRODUCT (ORDER REDUCTION $\rightarrow \Sigma$)

$$1) \underline{a}, \underline{b} \in \mathbb{R}^D \rightarrow c = \underline{a} \cdot \underline{b} = \sum_{i=1}^D a_i b_i = a_1 b_1 + a_2 b_2 + \dots + a_D b_D \in \mathbb{R}$$

$$2) \underline{\underline{A}} \in \mathbb{R}^{D \times D}, \underline{b} \in \mathbb{R}^D \rightarrow c = \underline{\underline{A}} : \underline{b} = \sum_{i=1}^D \underline{A}_{ii} b_i = \begin{bmatrix} A_{11} b_1 + A_{12} b_2 + \dots + A_{1D} b_D \end{bmatrix} \in \mathbb{R}^D$$

$$3) \underline{\underline{A}}, \underline{\underline{B}} \in \mathbb{R}^{D \times D} \rightarrow c = \underline{\underline{A}} : \underline{\underline{B}} = \sum_{i=1}^D \sum_{j=1}^D \underline{A}_{ij} \underline{B}_{jj} \in \mathbb{R} \quad A_{11} B_{11} + A_{12} B_{22} + \dots + A_{DD} B_{DD}$$

$$\begin{bmatrix} A_{11} & A_{12} & \dots & A_{1D} \\ \vdots & \vdots & \ddots & \vdots \\ A_{D1} & A_{D2} & \dots & A_{DD} \end{bmatrix} : \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1D} \\ \vdots & \vdots & \ddots & \vdots \\ B_{D1} & B_{D2} & \dots & B_{DD} \end{bmatrix} = \begin{bmatrix} A_{11} B_{11} + A_{12} B_{12} + \dots + A_{1D} B_{1D} \\ \vdots \\ A_{D1} B_{D1} + A_{D2} B_{D2} + \dots + A_{DD} B_{DD} \end{bmatrix} \in \mathbb{R}$$

NW: ":" GIVES A VECTOR;
":" GIVES A SCALAR.

○ OUTER PRODUCT (OR DYADIC) (ORDER INCREASING)

$$4) \underline{a}, \underline{b} \in \mathbb{R}^D \rightarrow \underline{\underline{c}} = \{c_{ij}\} = \{a_i b_j\} = \underline{a} \otimes \underline{b} = \begin{bmatrix} a_1 b_1 & a_1 b_2 & \dots & a_1 b_D \\ a_2 b_1 & a_2 b_2 & \dots & a_2 b_D \\ \vdots & \vdots & \ddots & \vdots \\ a_D b_1 & a_D b_2 & \dots & a_D b_D \end{bmatrix} \in \mathbb{R}^{D \times D}$$

O.3 OPERATORS (DIFFERENTIAL)

○ GRADIENT (∇) (ORDER INCREASING)

$$a \in \mathbb{R} \rightarrow \nabla a = \left\{ \frac{\partial a}{\partial x_i} \right\} = \left\{ \frac{\partial a}{\partial x_1}, \frac{\partial a}{\partial x_2}, \dots, \frac{\partial a}{\partial x_D} \right\} \in \mathbb{R}^D$$

$$a \in \mathbb{R}^D \rightarrow \nabla a = \left\{ \frac{\partial a_i}{\partial x_j} \right\} = \begin{bmatrix} \frac{\partial a_1}{\partial x_1} & \frac{\partial a_1}{\partial x_2} & \dots & \frac{\partial a_1}{\partial x_D} \\ \frac{\partial a_2}{\partial x_1} & \frac{\partial a_2}{\partial x_2} & \dots & \frac{\partial a_2}{\partial x_D} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial a_D}{\partial x_1} & \frac{\partial a_D}{\partial x_2} & \dots & \frac{\partial a_D}{\partial x_D} \end{bmatrix} \in \mathbb{R}^{D \times D}$$

○ DIVERGENCE ($\nabla \cdot \underline{a}$) (ORDER REDUCTION $\rightarrow \Sigma$)

$$a \in \mathbb{R}^D \rightarrow \nabla \cdot \underline{a} = \sum_{i=1}^D \frac{\partial a_i}{\partial x_i} = \frac{\partial a_1}{\partial x_1} + \frac{\partial a_2}{\partial x_2} + \dots + \frac{\partial a_D}{\partial x_D} \in \mathbb{R}$$

$$\underline{\underline{A}} \in \mathbb{R}^{D \times D} \rightarrow \nabla \cdot \underline{\underline{A}} = \left\{ \sum_{i=1}^D \frac{\partial A_{ij}}{\partial x_i} \right\} = \begin{bmatrix} \frac{\partial A_{11}}{\partial x_1} + \frac{\partial A_{12}}{\partial x_2} + \dots + \frac{\partial A_{1D}}{\partial x_D} \\ \frac{\partial A_{21}}{\partial x_1} + \frac{\partial A_{22}}{\partial x_2} + \dots + \frac{\partial A_{2D}}{\partial x_D} \\ \vdots \\ \frac{\partial A_{D1}}{\partial x_1} + \frac{\partial A_{D2}}{\partial x_2} + \dots + \frac{\partial A_{DD}}{\partial x_D} \end{bmatrix} \in \mathbb{R}^D$$

○ LAPLACIAN ($\nabla^2 = \nabla \cdot \nabla$) (DIVERGENCE OF GRADIENT) (NOT INCR. NOT RED.)

$$a \in \mathbb{R} \rightarrow \nabla^2 a = \sum_{i=1}^D \frac{\partial^2 a}{\partial x_i^2} = \sum_{i=1}^D \frac{\partial^2 a}{\partial x_1^2} = \frac{\partial^2 a}{\partial x_1^2} + \frac{\partial^2 a}{\partial x_2^2} + \dots + \frac{\partial^2 a}{\partial x_D^2} \in \mathbb{R}$$

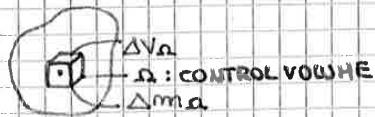
$$a \in \mathbb{R}^D \rightarrow \nabla^2 a = \left\{ \sum_{i=1}^D \frac{\partial^2 a_i}{\partial x_i^2} \right\} = \left\{ \sum_{i=1}^D \frac{\partial^2 a_i}{\partial x_1^2} \right\} = \begin{bmatrix} \frac{\partial^2 a_1}{\partial x_1^2} + \frac{\partial^2 a_2}{\partial x_2^2} + \dots + \frac{\partial^2 a_D}{\partial x_D^2} \\ \frac{\partial^2 a_2}{\partial x_1^2} + \frac{\partial^2 a_3}{\partial x_2^2} + \dots + \frac{\partial^2 a_D}{\partial x_D^2} \\ \vdots \\ \frac{\partial^2 a_D}{\partial x_1^2} + \frac{\partial^2 a_1}{\partial x_2^2} + \dots + \frac{\partial^2 a_D}{\partial x_D^2} \end{bmatrix} \in \mathbb{R}^D$$

①

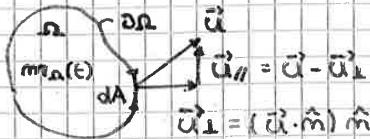
1. THERMO MECHANICS (HEURISTIC INTRODUCTION)

1.1 CONSERVED QUANTITIES (MASS, MOMENTUM, ENERGY)

I MASS CONSERVATION (1.1.1)



$$\rho(p) = \lim_{\Delta V_0 \rightarrow 0} \frac{\Delta m_0}{\Delta V_0} : \text{DEF OF CONTINUUM}$$



$$dV = (\vec{u} \cdot \hat{n}) dA$$

$d\dot{m} = \rho dV = \rho \vec{u} \cdot \hat{n} dA$: VARIATION OF MASS THROUGH dA (IN THE UNIT OF TIME)

$$\frac{dm_{\Omega}}{dt} = \oint_{\partial\Omega} \rho \vec{u} \cdot \hat{n} dA : \text{VARIATION OF TOTAL MASS}$$

CONSIDERING THAT: $m_{\Omega} = \int dm = \int \rho dV$

$$\frac{dm_{\Omega}}{dt} = \frac{1}{\Delta t} \int_{\Omega} \rho dV = \int_{\Omega} \frac{\partial \rho}{\partial t} dV = - \oint_{\partial\Omega} \rho \vec{u} \cdot \hat{n} dA, \quad \rho \vec{u} : \text{MASS FLUX}$$

$$\oint_{\partial\Omega} \frac{\partial \rho}{\partial t} dV + \oint_{\partial\Omega} \rho \vec{u} \cdot \hat{n} dA = 0 : \text{MASS CONSERVATION LAW}$$

/ CONTINUITY EQ. IN GLOBAL/INTEGRAL FORM

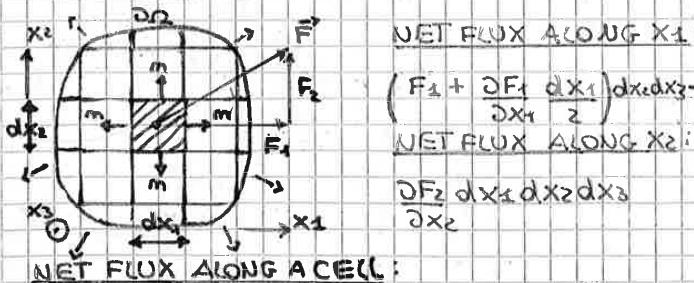
WE CAN TRANSFORM IT IN A LOCAL FORMULATION (THAT WE CAN USE IN CFD CODE)

THROUGH THE USE OF:

GAUSS/DIVERGENCE THEOREM (1.1.2)

$$\oint_{\partial\Omega} \vec{F} \cdot \hat{n} dA = \int_{\Omega} \nabla \cdot \vec{F} dV$$

TO UNDERSTAND ITS RATIONALE LET'S CONSIDER THE 3D DOMAIN:



$$\left(\frac{\partial F_1}{\partial x_1} + \frac{\partial F_2}{\partial x_2} + \frac{\partial F_3}{\partial x_3} \right) dx_1 dx_2 dx_3 = \nabla \cdot \vec{F} dV$$

$$\sum_{\text{CELLS}} (\nabla \cdot \vec{F}) dV = \oint_{\text{BORDO}} \vec{F} \cdot \hat{n} dA \leftrightarrow \int_{\Omega} \nabla \cdot \vec{F} dV = \oint_{\partial\Omega} \vec{F} \cdot \hat{n} dA$$

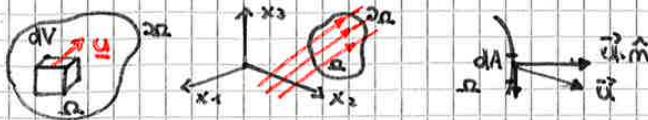
CONTINUITY EQUATION (1.1.3)

$$\int_{\Omega} \frac{\partial \rho}{\partial t} dV + \oint_{\partial\Omega} \rho \vec{u} \cdot \hat{n} dA = 0 \text{ BECOMES: } (\vec{u} \text{ OR } \underline{u})$$

$$\int_{\Omega} \frac{\partial \rho}{\partial t} dV + \int_{\Omega} \nabla \cdot (\rho \vec{u}) dV = 0 \leftrightarrow \int_{\Omega} [\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u})] dV = 0 \quad (\Omega \text{ DOESN'T CHANGE})$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 : \text{LOCAL FORMULATION OF CONTINUITY LAW}$$

II BALANCE EQUATION FOR MOMENTUM (1.1.6)



$$\text{MOMENTUM: } (\rho \mathbf{u}) \cdot \hat{\mathbf{u}}$$

$$\text{TOTAL MOMENTUM: } \int_{\Omega} \rho \mathbf{u}^2 dV$$

$$\frac{d(\text{MOMENTUM})}{dt} = \frac{d}{dt} \int_{\Omega} \rho \mathbf{u}^2 dV = \int_{\Omega} \frac{\partial \rho \mathbf{u}}{\partial t} \cdot \hat{\mathbf{u}} dV : \text{MOMENTUM RATE OF CHANGE}$$

$$\frac{d(i\text{-th COMP.})}{dt} = \int_{\Omega} \frac{\partial \rho u_i}{\partial t} dV : \text{MOMENTUM RATE OF CHANGE (DEF)}$$

$\rho u_i (\hat{\mathbf{u}} \cdot \hat{\mathbf{n}}) dA$: SMALL CONTR. TO VARIATION OF MOM. (ALONG i) (IN THE UNIT OF TIME)

$-\oint \rho u_i (\hat{\mathbf{u}} \cdot \hat{\mathbf{n}}) dA$: TOTAL ADVECTIVE CONTR. TO VARIATION OF MOM. (\leftarrow WE HAVE ONLY THIS FOR MASS M.)

$$\frac{d(i\text{-th COMP.})}{dt} = \int_{\Omega} \frac{\partial \rho u_i}{\partial t} dV = - \oint_{\partial\Omega} \rho u_i (\hat{\mathbf{u}} \cdot \hat{\mathbf{n}}) dA + \dots \text{ (OTHER CONTR.)}$$

$$\int_{\Omega} \frac{\partial \rho u_i}{\partial t} dV + \oint_{\partial\Omega} \rho u_i (\hat{\mathbf{u}} \cdot \hat{\mathbf{n}}) dA = \dots$$

USING THE EINSTEIN NOTATION:

$$\int_{\Omega} \frac{\partial \rho u_i}{\partial t} dV + \oint_{\partial\Omega} \underbrace{\rho u_i u_j m_j}_{\text{TENSOR}} dA = \dots$$

$$\int_{\Omega} \frac{\partial \rho \mathbf{u}}{\partial t} dV + \oint_{\partial\Omega} \underbrace{(\rho \mathbf{u} \otimes \mathbf{u}) \cdot \hat{\mathbf{n}}}_{\text{FLUX OF A TENSOR.}} dA = \dots$$

USING THE GAUSS THEOREM:

$$\int_{\Omega} \frac{\partial \rho \mathbf{u}}{\partial t} dV + \int_{\Omega} \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) dV = \dots$$

$$\int_{\Omega} \left[\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) \right] dV = \dots$$

$\underbrace{\quad}_{\text{EULERIAN DERIVATIVE}} \quad \underbrace{\quad}_{\text{ADVECTIVE FLUX}}$

WHAT OTHER CAUSES ARE RESPONSABLE OF CHANGE IN MOMENTUM IN Ω ?

- SURFACE FORCES

$$-\oint \underline{\underline{\sigma}} \cdot \hat{\mathbf{n}} dA ; \text{ BECAUSE } d\mathbf{F} = \underline{\underline{\sigma}} \cdot \hat{\mathbf{n}} \cdot dA \quad dA \rightarrow \hat{\mathbf{n}} \cdot dF, \underline{\underline{\sigma}} : \text{STRESS TENSOR}$$

- EXTERNAL VOLUME FORCES

$$+\int_{\Omega} \rho g dV, [g] = [\text{N/kg}]$$

THEREFORE:



$$\int_{\Omega} \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) dV = - \oint \underline{\underline{\sigma}} \cdot \hat{\mathbf{n}} dA + \int_{\Omega} \rho g dV$$

$$\int_{\Omega} \left[\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) \right] dV = - \int_{\Omega} \nabla \cdot \underline{\underline{\sigma}} dV + \int_{\Omega} \rho g dV$$

$$\int_{\Omega} \left[\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} + \underline{\underline{\sigma}}) \right] dV = \int_{\Omega} \rho g dV : \text{M. BALANCE EQ. - GLOBAL FORM}$$

V Ω

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} + \underline{\underline{\sigma}}) = \rho g : \text{STANDARD FORM}, \rho g : \text{SOURCE TERM}$$

IF NO EXT. V.F. $\rho g = 0$: BALANCE EQ. \rightarrow CONSERV. EQ. (5)

B) FLUID NOT AT REST

hp¹: $S_{ij} \propto \frac{\partial u_i}{\partial x_j}$ (THIS DOESN'T RECOVER THE HYDR. CASE)

hp²: $S_{ij} - p\delta_{ij} \propto \frac{\partial u_i}{\partial x_j}$

hp³: $S_{ij} - p\delta_{ij} \propto \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$

$$\text{Tr}(S_{ij} - p\delta_{ij}) = 3p - 3p = 0 \neq \text{Tr}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} = 2\nabla \cdot \bar{u} \quad \text{THEN:}$$

$$\text{Tr}\left(-\frac{2}{3}\nabla \cdot \bar{u} S_{ij}\right) = -2\nabla \cdot \bar{u}$$

$$hp^4: S_{ij} - p\delta_{ij} \propto \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\nabla \cdot \bar{u} \delta_{ij}$$

[IN ORDER TO TRANSFORM "OC" INTO "=" WE CONSIDER:

$$\left[\frac{N}{m^2}\right] = [X] \left[\frac{m}{s} \frac{1}{m}\right] \Rightarrow [X] = \left[\frac{kg \cdot m}{s^2 \cdot m^2}\right] \left[\frac{m}{m}\right] = \underbrace{\left[\frac{kg}{m^3}\right]}_P \underbrace{\left[\frac{m^2}{s}\right]}_V$$

THEN, WE CAN WRITE:

$$S_{ij} - p\delta_{ij} = \pm \rho v \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\nabla \cdot \bar{u} \delta_{ij} \right]$$

$$S_{ij} = p\delta_{ij} \pm \rho v \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\nabla \cdot \bar{u} \delta_{ij} \right]$$

COMPACT NOTATION:

$$\underline{S} = \underbrace{p \underline{\underline{I}}}_{\substack{\text{HYDROSTATIC} \\ \text{TERM}}} \pm \underbrace{\rho v \left[\nabla \bar{u} + \nabla \bar{u}^T - \frac{2}{3}\nabla \cdot \bar{u} \underline{\underline{I}} \right]}_{\substack{\text{VISCOUS} \\ \text{TERM}}} = S_H - S_V, \quad S_V = +\rho v \left[\nabla \bar{u} + \nabla \bar{u}^T - \frac{2}{3}\nabla \cdot \bar{u} \underline{\underline{I}} \right] \quad \text{OR } \underline{\underline{\Pi}} = p \underline{\underline{I}} - \Pi_V$$

THEREFORE:

$$\frac{\partial p}{\partial t} + \nabla \cdot (p\bar{u}) = 0$$

$$\frac{\partial p\bar{u}}{\partial t} + \nabla \cdot (p\bar{u} \otimes \bar{u}) = -\nabla \cdot \underline{S} + (p\bar{u})$$

$$\underline{S} = p \underline{\underline{I}} \pm \rho v \left[\nabla \bar{u} + \nabla \bar{u}^T - \frac{2}{3}\nabla \cdot \bar{u} \underline{\underline{I}} \right]$$

CONSIDERING THAT:

$$\underline{S} = \underline{S}_H + \underline{S}_V \rightarrow \nabla \cdot \underline{S} = \nabla \cdot \underline{S}_H - \nabla \cdot \underline{S}_V = \nabla \cdot (p \underline{\underline{I}}) - \nabla \cdot \left\{ \rho v \left[\nabla \bar{u} + \nabla \bar{u}^T - \frac{2}{3}\nabla \cdot \bar{u} \underline{\underline{I}} \right] \right\}$$

$$\nabla \cdot (p \underline{\underline{I}}) = \nabla p$$

THEREFORE:

$$\frac{\partial p}{\partial t} + \nabla \cdot (p\bar{u}) = 0$$

$$\frac{\partial p\bar{u}}{\partial t} + \nabla \cdot (p\bar{u} \otimes \bar{u}) + \nabla p = +\nabla \cdot \underline{S}_V + (p\bar{u})$$

$$\underline{S} = \underline{S}_H \pm \underline{S}_V$$

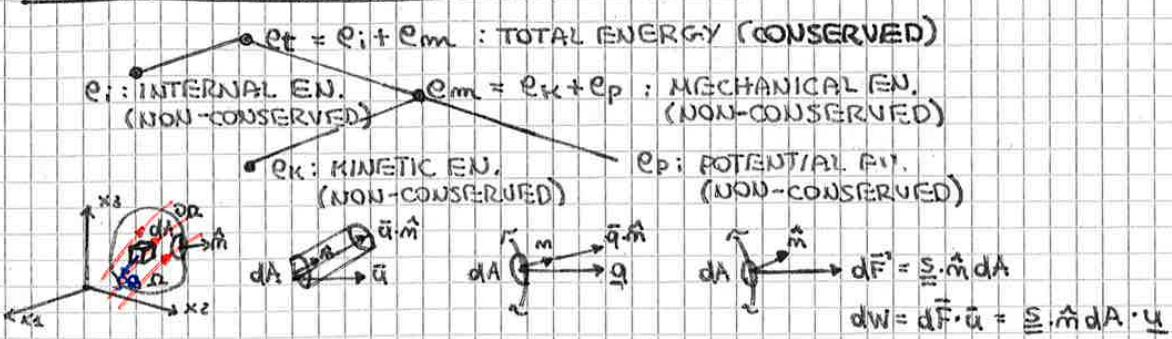
$$\frac{\partial p}{\partial t} = \frac{\partial p}{\partial t} + \bar{u} \cdot \nabla p = -\rho \nabla \cdot \bar{u}$$

$$\rho \frac{\partial \bar{u}}{\partial t} = \rho \left(\frac{\partial \bar{u}}{\partial t} + \bar{u} \cdot \nabla \bar{u} \right) = -\nabla \cdot \underline{S} + (p\bar{u})$$

$$\frac{\partial p}{\partial t} = \frac{\partial p}{\partial t} + \bar{u} \cdot \nabla p = -\rho \nabla \cdot \bar{u}$$

$$\rho \frac{\partial \bar{u}}{\partial t} = \rho \left(\frac{\partial \bar{u}}{\partial t} + \bar{u} \cdot \nabla \bar{u} \right) = -\nabla \cdot \underline{S} + (p\bar{u})$$

III CONSERVATION OF ENERGY (1.1.7)



$$E_t = \int_V \rho dV \cdot E_t$$

$$[\text{KJ}] \quad [\text{Kg}] \cdot [\text{KJ/Kg}] \quad ([\text{KJ}/\text{s}] = [\text{KW}])$$

$$\frac{\partial E_t}{\partial t} = \frac{\partial}{\partial t} \int_V \rho E_t dV = - \oint_{\partial V} \underline{\underline{S}} \cdot \hat{n} dA$$

GAUSS THEOREM:

$$\int_V \frac{\partial \rho E_t}{\partial t} dV + \int_V \nabla \cdot \underline{\underline{S}} dV = 0$$

 $\underline{\underline{S}}$: FLUX OF TOTAL ENERGY

$$\underline{\underline{S}}(\rho E_t) = \underline{\underline{S}_{ADV}} + \underline{\underline{S}_{TH}} + \underline{\underline{S}_{NECH}}$$

$$\cdot \underline{\underline{S}_{ADV}} = \rho E_t \underline{u} : \text{ADVECTIVE FLUX } [\text{W/m}^2]$$

$$\cdot \underline{\underline{S}_{TH}} = \underline{q} = -\lambda \nabla T : \text{HEAT FLUX}, \lambda: \text{HT. CONDUCTIVITY } [\text{W/mK}]$$

$$\left(\lambda = \rho \alpha C_p, \alpha = \frac{\lambda}{\rho C_p}, \text{HT. DIFFUSIVITY} ; \Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{\lambda}, \nu: \text{KINETIC VISCOSITY} \right)$$

$$\underline{\underline{S}_{NECH}} = \underline{\underline{S}} \cdot \underline{u} \quad \left[\frac{\text{W}}{\text{m}^2} \right] \quad \left[\frac{\text{J}}{\text{kg} \cdot \text{m}} \right]$$

THEREFORE: $\left[\frac{\text{W}}{\text{s}} \right]$

$$\frac{\partial E_t}{\partial t} = \int_V \frac{\partial \rho E_t}{\partial t} dV = - \oint_{\partial V} \rho E_t \underline{u} \cdot \hat{n} dA - \oint_{\partial V} q \cdot \hat{n} dA - \oint_{\partial V} \underline{\underline{S}} \cdot \hat{n} dA + \left(\int_V \rho \underline{a} \cdot \underline{u} dV \right)$$

USING GAUSS THEOREM:

(TO ADD IF \underline{a} IS NON-CONS.) EXT. V. FORCE FIELD CONTRIB. \rightarrow NON-CONS. EQ.
TOTAL ENERGY BALANCE: EQ.
INTEGRAL/GLOBAL FORM.

THEN THE LOCAL FORM RESULTS TO BE:

$$\frac{\partial \rho E_t}{\partial t} + \nabla \cdot (\rho E_t \underline{u} + \underline{q} + \underline{\underline{S}} \cdot \underline{u}) = 0$$

CONSIDERING THAT:

$$\underline{\underline{S}} = P \underline{\underline{I}} - S_v \underline{\underline{u}} \Rightarrow \nabla \cdot \underline{\underline{S}} \cdot \underline{u} = \nabla \cdot P \underline{u} - \nabla \cdot (S_v \underline{u})$$

WE GET:

$$\frac{\partial \rho E_t}{\partial t} + \nabla \cdot (\rho E_t \underline{u} + \underline{q} + P \underline{u}) = \nabla \cdot (S_v \underline{u})$$

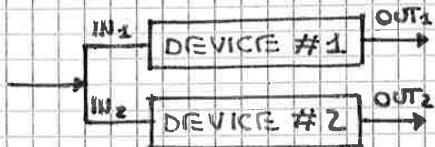
NOTES:

$$\nabla \cdot (P \underline{\underline{I}}) = \nabla P$$

$$\nabla \cdot (P \underline{\underline{I}}) \cdot \underline{u} = \nabla \cdot (P \underline{\underline{I}} \cdot \underline{u}) = \nabla \cdot P \underline{u}$$

$$\underline{\underline{I}} : \nabla \underline{u} = \underline{\underline{u}} \quad | \quad (\underline{\underline{I}} \cdot \underline{u} = \underline{u})$$

1.2 NON-CONSERVED QUANTITIES



WHICH IS THE MOST PERFORMING? ?

THE COMPARISON NEEDS TO BE EVALUATED IN TERMS OF NON-CONSERV. QTY'S
(IN TERMS OF \dot{Q} WE CANNOT DO THE COMPARISON)

CONSIDERING A GENERIC QUANTITY THAT IS CONSERVED (ϕ):

$$\frac{\partial \phi}{\partial t} + \nabla \cdot \vec{s}(\phi) = 0$$

$\vec{s} = 0$ (STEADY STATE C.)

$$\int_{\Omega} \nabla \cdot \vec{s}(\phi) dV = 0 \leftrightarrow \oint_{\partial\Omega} \vec{s}(\phi) \cdot \hat{n} dA = 0 \leftrightarrow \oint_{\partial\Omega} \vec{s}(\phi) \cdot \hat{n} dA + \oint_{\partial\Omega_{\text{OUT}}} \vec{s}(\phi) \cdot \hat{n} dA = 0$$

WE CONCLUDE:

$$\oint_{\partial\Omega_{\text{IN}}} \vec{s}(\phi) \cdot \hat{n} dA = - \oint_{\partial\Omega_{\text{OUT}}} \vec{s}(\phi) \cdot \hat{n} dA$$

THIS CONCLUSION DOES NOT DEPEND ON DEVICE #1, #2

WE CANNOT PERFORM THE COMPARISON!

CONSIDERING :

$$\frac{\partial \phi}{\partial t} + \nabla \cdot \vec{s}(\phi) = S(\phi), S(\phi): \text{SOURCE TERM, IT IS DEVICE DEPENDENT}$$

IF DEVICE #1 PRODUCES MORE ENTROPY ($S(\phi)$), #2 IS MORE PERFORMING.

INTERNAL ENERGY (e_i) (1.2.2)

$$e_i = e_t - e_m$$

$$\frac{\partial(\rho e_t)}{\partial t} + \nabla \cdot (\rho e_t \underline{u} + g + \underline{\underline{S}} \cdot \underline{u}) = 0 \quad ; \quad p \frac{\partial e_t}{\partial t} = \frac{\partial(\rho e_t)}{\partial t} + \nabla \cdot (\rho e_t \underline{u}) = -\nabla \cdot (g + \underline{\underline{S}} \cdot \underline{u})$$

$$\frac{\partial(\rho e_m)}{\partial t} + \nabla \cdot (\rho e_m \underline{u} + \underline{\underline{S}} \cdot \underline{u}) = \underline{\underline{S}} : \nabla \underline{u} \quad ; \quad p \frac{\partial e_m}{\partial t} = \frac{\partial(\rho e_m)}{\partial t} + \nabla \cdot (\rho e_m \underline{u}) = -(\nabla \cdot \underline{\underline{S}}) \cdot \underline{u}$$

THEREFORE: SINK TERM L $p \left(\frac{\partial e_m}{\partial t} + \underline{u} \cdot \nabla e_m \right)$

$$\frac{\partial(p e_i)}{\partial t} + \nabla \cdot (p e_i \underline{u} + g) = -\underline{\underline{S}} : \nabla \underline{u} \quad (\text{THIS DESTROYED HIGH. EN. BECOMES INT. EN.})$$

LAGRANGIAN FORM: SOURCE TERM

$$p \frac{\partial e_i}{\partial t} = p \left[\frac{\partial e_i}{\partial t} + \underline{u} \cdot \nabla e_i \right] = -\nabla \cdot g - \underline{\underline{S}} : \nabla \underline{u} = -\nabla \cdot g - p \nabla \cdot \underline{u} + \underline{\underline{S}} : \nabla \underline{u}$$

BEING:

$$\underline{\underline{S}} = p \underline{\underline{I}} - \underline{\underline{S}}_v : - p \underline{\underline{I}} : \nabla \underline{u} = - p \nabla \cdot \underline{u}$$

ENTROPY (1.2.G)

$$\delta q - \delta p = d\epsilon_i + d\epsilon_m : \text{1st LAW OF THERMODYNAMICS}$$

(δ INDICATES INEXACT DIFFERENTIALS; q, p ARE NOT STATE F. BUT DEP. ON TRANSF.)

$$-\delta p = -pdV + d\epsilon_m + \delta \epsilon_a : \text{BERNOULLI'S EQ.}$$

(WITH $\delta \epsilon_a$: FRICTION WORK.)

$$\delta q = TdS - \delta \epsilon_a : \text{2nd LAW OF THERMODYNAMICS FOR CLOSED SYSTEMS}$$

BY SUBSTITUTING THE LAST 2 Eqs IN THE FIRST :

$$TdS - \delta \epsilon_a - pdV + d\epsilon_m + \delta \epsilon_a = d\epsilon_i + d\epsilon_m$$

$$\left\{ \begin{array}{l} TdS - pdV = d\epsilon_i : \text{1st GIBB'S EQUATION} ; h = e_i + pv \Rightarrow e_i = h - pv \Rightarrow d\epsilon_i = dh - pdV - vdp \end{array} \right.$$

$$\left\{ \begin{array}{l} TdS + vdp = dh : \text{2nd GIBB'S EQUATION} \quad (\leftarrow \text{NOT SEEN DURING LESSONS}) \end{array} \right.$$

* CONSIDERING THE 1st GIBB'S R. FOR CLOSED SYSTEM (\leftarrow PROF. WAY)

$$TdS = pdV + d\epsilon_i$$

$$\frac{\partial S}{\partial E} = \frac{p \cdot \frac{\partial V}{\partial E} + \frac{\partial \epsilon_i}{\partial E}}{\frac{\partial E}{\partial T}} \quad (\text{WRITTEN FOR THE LAGRANGIAN OBSERVER; FOR HIM SYST. IS CLOSE})$$

$$\frac{\partial S}{\partial T} = -\frac{p}{T^2} \frac{\partial \frac{\partial V}{\partial E}}{\partial T} + \frac{\partial \epsilon_i}{\partial T}$$

BY MULTIPLYING PER P BOTH SIDES:

$$p \frac{\partial S}{\partial E} = -\frac{p}{T} \frac{\partial \frac{\partial V}{\partial E}}{\partial T} + \frac{p \partial \epsilon_i}{\partial T}$$

RECALCULATING:

$$\frac{p \partial \epsilon_i}{\partial T} = \frac{\partial(p \epsilon_i)}{\partial T} + \nabla \cdot (\frac{\partial(p \epsilon_i)}{\partial x}) = -\nabla \cdot \underline{q} - \underline{s} : \nabla \underline{u}$$

AND CONSIDERING:

$$-\underline{s} : \nabla \underline{u} = \frac{p}{T} \frac{\partial \frac{\partial V}{\partial E}}{\partial T} + \underline{\epsilon}_v \Rightarrow \frac{p \partial \epsilon_i}{\partial T} = -\nabla \cdot \underline{q} + \frac{p}{T} \frac{\partial \frac{\partial V}{\partial E}}{\partial T} + \underline{\epsilon}_v$$

BY SUBSTITUTING:

$$p \frac{\partial S}{\partial E} = -\nabla \cdot \underline{q} + \underline{\epsilon}_v$$

DIVIDING PER T:

$$\frac{p \frac{\partial S}{\partial E}}{T} = -\frac{1}{T} \nabla \cdot \underline{q} + \frac{\underline{\epsilon}_v}{T}$$

USING EINSTEIN NOTATION:

$$\nabla \cdot \left(\frac{q_i}{T} \right) = \frac{\partial}{\partial x_i} \frac{q_i}{T} = \frac{1}{T} \frac{\partial q_i}{\partial x_i} + q_i \frac{\partial(\frac{1}{T})}{\partial x_i} = \frac{1}{T} \frac{\partial q_i}{\partial x_i} - \frac{q_i}{T^2} \frac{\partial T}{\partial x_i} = \frac{1}{T} \nabla \cdot \underline{q} - \frac{1}{T^2} \underline{q} \cdot \nabla T =$$

$$= \frac{1}{T} \nabla \cdot \underline{q} - \frac{1}{T^2} (-2 \nabla T) \cdot \nabla T = \frac{1}{T} \nabla \cdot \underline{q} + \frac{1}{T} \underbrace{\left(\frac{2 \nabla T \cdot \nabla T}{T} \right)}_{\Sigma_{\alpha} > 0 \text{ (ALWAYS)}} = \frac{1}{T} \nabla \cdot \underline{q} + \frac{\Sigma_{\alpha}}{T}$$

$$\Rightarrow \frac{1}{T} \nabla \cdot \underline{q} = \nabla \cdot \left(\frac{\underline{q}}{T} \right) - \frac{\Sigma_{\alpha}}{T}$$

THEREFORE:

$$\frac{p \frac{\partial S}{\partial E}}{T} = -\nabla \cdot \left(\frac{\underline{q}}{T} \right) + \frac{\Sigma_{\alpha}}{T} + \frac{\underline{\epsilon}_v}{T}$$

RE-WRITTEN:

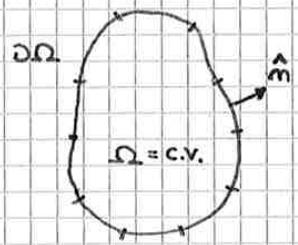
$$\frac{p \frac{\partial S}{\partial E}}{T} + \nabla \cdot \left(\frac{\underline{q}}{T} \right) = \frac{\Sigma_{\alpha}}{T} + \frac{\underline{\epsilon}_v}{T} \quad \text{TRANSPORT ENTROPY EQ.}$$

1.7 INTEGRAL EQUATIONS

THE BOUNDARY OF CAN BE ARBITRARILY DIVIDED IN :

M SECTIONS FOR HEAT FLUXES (Φ_i) : $\partial\Omega = \bigcup_{j=1}^M \partial\Omega_j$

N SECTIONS FOR MASS FLUXES (G_i): $\partial\Omega = \bigcup C_i$



INTEGRAL FORM OF THE 1ST LAW OF THERMODYNAMICS (1.7.1)

STARTING FROM THE EULERIAN FORMULATION OF THE TOTAL ENERGY Eq:

$$\underline{\underline{\sigma}}(\rho e \underline{v}) + \nabla \cdot (\rho e \underline{v} \underline{u} + q + \underline{\underline{S}} \cdot \underline{u}) = 0$$

$$\underline{S} = \rho \underline{\underline{I}} - \underline{\underline{S}_{vv}}; \nabla \cdot \underline{\underline{S}} \cdot \underline{u} = \nabla \cdot \rho \underline{\underline{I}} \cdot \underline{u} - \nabla \cdot \underline{\underline{S}_{vv}} \underline{u} = \nabla \cdot \rho \underline{u} - \nabla \cdot \underline{\underline{S}_{vv}} \underline{u} (\rho \cdot \underline{\underline{S}_{vv}})$$

$$\frac{\partial (\rho e)}{\partial t} + \nabla \cdot (\rho \mathbf{v} e + p \rho \mathbf{v} \cdot \mathbf{u}) = \nabla \cdot (-\mathbf{q} + S_{\mathbf{v}} \cdot \mathbf{u})$$

$$\frac{\partial \psi}{\partial t} + \nabla \cdot [\rho (e_m + e_i + p v) \psi] = \nabla \cdot (-\underline{q}) + \nabla \cdot (\underline{\underline{v}} \cdot \underline{\underline{\psi}})$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot [\rho(e_m + h) \mathbf{u}] = \nabla \cdot (-\mathbf{q}) + \nabla \cdot (\frac{S}{\rho} \mathbf{v} \cdot \mathbf{u})$$

LOCAL \rightarrow (S, dV) \rightarrow GLOBAL:

$$\int_{\Omega} \frac{\partial (CSE)}{\partial E} dV + \int_{\Omega} \nabla \cdot [j(Cem + h)u] dV = \int_{\Omega} \nabla \cdot (-q) dV + \int_{\Omega} \nabla \cdot (\underline{S}v \cdot u) dV$$

USING GAUSS THEOREM ($\int_S \nabla \cdot (\mathbf{A}) dV = \oint_{\partial S} (\mathbf{A} \cdot \hat{n}) dA$):

$$\frac{d}{dt} \int_{\Omega} \rho e dV + \int_{\partial\Omega} \rho (\text{emth}) u \cdot \hat{n} dA = f - q \cdot \hat{n} dA + \int_{\partial\Omega} S_v u \cdot \hat{n} dA$$

$$\frac{d}{dt} \int_{\Omega} p(e + e_m) dV + \sum_{i=1}^N \int_{\partial\Omega; i^*} p(e_m + h) u_i \hat{n}_i dA = \sum_{j=1}^N \int_{\partial\Omega; j^*} q_j \hat{n}_j dA + f \int_{\Omega} u \hat{m} dA,$$

NC: n. OF COND.
NP: n. OF PERM.
PARTS

CONSIDERING:

$$\int_{\partial P; F} \rho u \hat{m} dA = G_i : \text{MASS FLOW RATE} ; \int_{\partial P; \Phi} -q \cdot \hat{n} dA = \Phi_i : \text{HEAT FLUX}$$

(>> 0 IF EXITING AS W_i) (>> 0 IF ENTERING)

THEREFORE

$$\frac{d}{dt} \underbrace{\int_{\Omega} p(e_i + e_m) dV}_{E_i + E_M} + \sum_{i=1}^{N_C} G_i (e_m + h)_i = \sum_{j=1}^{N_P} \Phi_j + \underbrace{\oint_{\partial\Omega} \underline{S} \cdot \underline{v} \cdot \underline{u} \cdot \hat{n} dV}_{= -Wt}$$

THEN

$$\frac{d}{dt} \left(E_i + E_m \right) + \sum_{i=1}^{N_C} G_i (e_m + h)_i = \sum_{i=1}^{N_P} (\Phi_i - W_i)_i^*, \quad W_i = W_i^* - P_o \frac{dV}{dt}$$

RE-WRITTEN:

$$\sum_{i=1}^{N_c} Q_i - W_F = \frac{d}{dt} (E_i + E_m)_n + \sum_{i=1}^{N_c} G_i (h + e_m)_i$$

1st LAW OF THERMODYNAMICS
FOR OPEN SYSTEMS

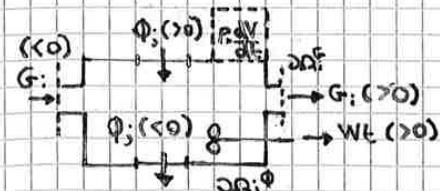
$$E_i - W_F = \frac{d}{dt} (E_i + E_m + P\Delta V) + \sum_i G_i (h + e_m)_i$$

$$\Delta Q - \Delta S_W = d\Delta U + \Delta e_m$$

$$\sum \Phi_i - W_E = \frac{d}{dt} (E_i + E_{m,i} + p_0 V) + \sum_{i=1}^n G_i (h + e_{m,i})$$

FOR OPEN SYSTEMS
[$\sum q_i - SW = dE_i + dE_{m,i}$]

(IN OUR COURSE WE DON'T DEAL WITH VARIATION OF V ($dV/dt = 0$))



NOTE: { **CLOSED SYST**: IF EXCH. ONLY Φ , WE
OPEN SYST: IF EXCH. ALSO $G(h+em)$

EULERIAN TOTAL ENERGY EQ. - HEAT DIFFUSION IN SOLIDS [T] ← [SEEN DURING LESSONS]

$$\frac{\partial \rho e_t}{\partial t} + \nabla \cdot (\rho e_t \mathbf{u} + \mathbf{q} + \frac{\rho}{2} \mathbf{u}^2) = 0$$

CONSIDERING A SOLID \Rightarrow WE HAVE NO MORE A SYSTEM CROSSED BY FLUID FLOW:

$$\mathbf{u} = 0$$

$$\frac{\partial \rho e_t}{\partial t} + \nabla \cdot \mathbf{q} = 0$$

CONSIDERING THAT:

$$\left\{ \begin{array}{l} e_t = e_i + e_m = e_i + e_k + e_p = (cT + e_{i0}) + \frac{u^2}{2} + q_T \\ q = -\lambda \nabla T \end{array} \right. \quad \text{NEGLIGENCE}$$

$$\left\{ \begin{array}{l} q = -\lambda \nabla T \\ H_P: \rho = \text{const}, \lambda = \text{const}, c = \text{const} = \text{THERMAL CAPACITY?} \end{array} \right.$$

WE CAN WRITE:

$$\rho c \frac{\partial T}{\partial t} + \nabla \cdot (-\lambda \nabla T) = 0$$

$$\rho c \frac{\partial T}{\partial t} - \lambda \nabla \cdot \nabla T = \rho c \frac{\partial T}{\partial t} - \lambda \nabla^2 T = \frac{\partial T}{\partial t} - \frac{\lambda}{\rho c} \nabla^2 T = \frac{\partial T}{\partial t} - \alpha \nabla^2 T = 0$$

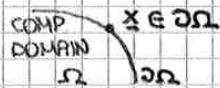
THEREFORE:

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T : \text{HEAT DIFFUSION EQ. (IN SOLIDS)} \quad (\text{HEAT CONDUCTION FOR DIFF.})$$

$$\text{WITH } \alpha = \frac{\lambda}{\rho c} \quad [\frac{m^2}{s}] : \text{THERMAL DIFFUSIVITY} ; \lambda \quad [\frac{W}{mK}] : \text{THERMAL CONDUCTIVITY}$$

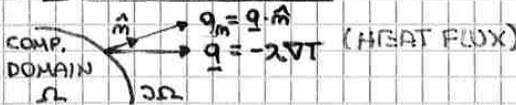
IN ORDER TO SOLVE IT WE'VE TO ADD THE BOUNDARY CONDITIONS (B.C.)

1) DIRICHLET B.C.



$$T(x, t) = \bar{T} \quad (\text{D.C.})$$

2) NEUMANN B.C.



$$-\lambda \nabla T \cdot \hat{n} = q_m(x, t) \quad (\text{N.C.}) \quad \text{ADIABATIC CONDITION:}$$

$$-\lambda \frac{\partial T}{\partial n} = q_m(x, t) \quad (\text{N.C.}) \quad q_m = 0 : \text{(HOMOGENEOUS N.C.)}$$

3) ROBIN B.C.

$$\left. \begin{array}{l} (T_{\infty}) \quad \phi \\ \text{FLUID} \quad h \\ \text{SOLID} \quad \frac{\partial T}{\partial n} \\ \left(\frac{\partial T}{\partial n} = \alpha \nabla T \right) \end{array} \right\} \begin{array}{l} \text{WE'VE TO KNOW THE HEAT TRANSF. (h), } T_{\infty} : \text{T OF THE FLUID (CONST)} \\ \left\{ \begin{array}{l} (-\lambda \nabla T \cdot \hat{n}) \cdot A : \text{FOURIER LAW} \quad (\rightarrow \text{LOOKS AT SOLID REGION}) \\ h(T - T_{\infty}) \cdot A : \text{NEWTON LAW} \quad (\rightarrow \text{LOOKS AT FLUID REGION}) \end{array} \right. \end{array}$$

FOR CONTINUITY:

$$-\lambda \nabla T \cdot \hat{n} \cdot A = h(T - T_{\infty}) \cdot A \quad (\text{R.C.})$$

$$-\lambda \frac{\partial T}{\partial n} = h(T - T_{\infty}) \quad \text{OR} \quad (\text{R.C.})$$

NOTE: $\alpha = h$

CONSIDERING:

$$c : \text{THERMAL CAPACITY} \quad [\frac{J}{kgK}]$$

$$\lambda : \text{THERMAL CONDUCTIVITY} \quad [\frac{W}{mK}]$$

$$h = \frac{\lambda}{c} : \text{THERMAL CONDUCTIVITY} \quad [\frac{W}{m^2K}]$$

$$\alpha = \frac{\lambda}{\rho c} : \text{THERMAL DIFFUSIVITY} \quad [\frac{m^2}{s}]$$

2. HEAT EXCHANGERS

2.1 INTRODUCTION

- DOUBLE PIPE H.E.
- SHELL AND TUBE H.E.

2.2 DOUBLE PIPE HEAT EXCHANGER

- IT'S SIMPLE AND CHEAP
- INNER PIPE = INTERFACE BETWEEN THE 2 FLUIDS
- PARALLEL FLOW CONFIGURATION (2.2.1)



- SAME DIRECTION, SAME VERSUS; h_p : ADIABATIC TOWARD THE ENV.
- EXCHANGE PROCESS BEGINS AT INLET AND ENDS AT OUTLET

CONSIDERING THE HOT FLUID (OPERATING IN STEADY-STATE C.):

$$\delta q - \delta e = d\epsilon_i + d\epsilon_m \xrightarrow{\text{NEGL.}} \text{1st LAW}, \epsilon_i = h - pV \quad \text{T OR } t : \text{TEMP.}$$

$$q = \Delta h = C_p(T_{in} - T_{out}) : \text{HEAT}, C_p \rightarrow \text{ISOBARIC PROCESS}$$

$$\begin{cases} \dot{Q} = -m\dot{q} = -m\Delta h = -mC_p(T_{in} - T_f) : \text{THERMAL POWER, EXCHANGED BY HOT FLUID} \\ \dot{Q} = m'C_p(T_{in} - T_{out}) : \text{THERMAL POWER, ABSORBED BY COLD FLUID} \end{cases} =$$

THE FOLLOWING EQ. HOLDS:

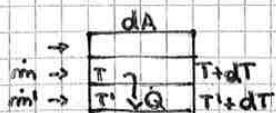
$$(PREVIEW) \quad Q = KA_t (T - T')_{pm} \quad dQ = k dA (T - T')$$

DESIGN TARGET $\rightarrow A_t = \frac{Q}{k(T - T')}$ (DESIRER \dot{Q}) K COMES FROM THE THERMAL EXCHANGE THEORY

TEST TARGET $\rightarrow \dot{Q} = \dot{s}(A_t)$

LIMINAL CONTR. 1
CONVECTIVE CONTR.
LIMINAL CONTR. 2

CONSIDERING AN INFINITESIMAL PORTION:



$$d\dot{Q} = -mC_p dT \Rightarrow dT = -d\dot{Q} \cdot \left(\frac{1}{mC_p} \right) : \text{FOR HOT FLUID}$$

$$d\dot{Q} = m'C_p dT' \Rightarrow dT' = d\dot{Q} \cdot \left(\frac{1}{m'C_p} \right) : \text{FOR COLD FLUID}$$

$$\Rightarrow d(T - T') = -d\dot{Q} \left(\frac{1}{mC_p} + \frac{1}{m'C_p} \right) = -K dA (T - T') \left(\frac{1}{mC_p} + \frac{1}{m'C_p} \right)$$

CONSIDERING: $\theta = T - T'$; $\theta_0 \approx (T - T')$ INLET:

$$\frac{d\theta}{\theta} = -K \left(\frac{1}{mC_p} + \frac{1}{m'C_p} \right) dA \rightarrow \int_{\theta_0}^{\theta} \frac{d\theta}{\theta} = - \int_A A' K \left(\frac{1}{mC_p} + \frac{1}{m'C_p} \right) dA$$

$$\theta = \int_{\theta_0}^{\theta} -K \left(\frac{1}{mC_p} + \frac{1}{m'C_p} \right) dA$$

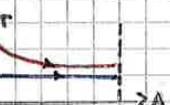
THEREFORE:

$$\theta = \theta_0 e^{-K \left(\frac{1}{mC_p} + \frac{1}{m'C_p} \right) A} = \theta_0 e^{-\alpha A} \quad \alpha = K \left(\frac{1}{mC_p} + \frac{1}{m'C_p} \right)$$

hp: STATE-CHANGING

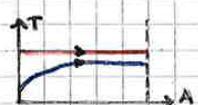
- IF COLD FLUID IS EVAPORATING:

$$m'C_p \rightarrow \infty \quad (\text{BECAUSE } C_p \rightarrow \infty)$$



- IF HOT FLUID IS CONDENSING:

$$dT \rightarrow 0$$



IF $T - T_0 > T_0' - T_0'$

$$\Rightarrow mC_p < m'C_p$$

2.3 GENERAL APPROACH

$$\Theta = \Theta_0 e^{-K \left(\frac{1}{m'c_p} + \frac{1}{mc_p} \right) A}$$

↓ TIN-TOUT

$$d\dot{Q} = K dA (T - T')$$
 (. $dQ = m' c_p dT$)

$$\dot{Q} = \int_{\Theta_0}^{\Theta_t} K \Theta d\Theta = \int_{\Theta_0}^{\Theta_t} K \Theta_0 e^{-K \left(\frac{1}{m'c_p} + \frac{1}{mc_p} \right) A} d\Theta = K \Theta_0 \left[e^{-K \left(\frac{1}{m'c_p} + \frac{1}{mc_p} \right) A} \right]_{\Theta_0}^{\Theta_t} = K A t \frac{\Theta_t - \Theta_0}{-K \left(\frac{1}{m'c_p} + \frac{1}{mc_p} \right)}$$

$$\dot{Q} = K A t \frac{\Theta_t - \Theta_0}{\ln \frac{\Theta_t}{\Theta_0}} = K A t (t - t')_{em}, (t - t')_{em} = \frac{\Theta_t - \Theta_0}{\ln \frac{\Theta_t}{\Theta_0}} : \begin{array}{l} \text{LOGARYTHMIC} \\ \text{MEAN T-DIFF.} \end{array}$$

THE NUMERICAL DESIGN APPROACHES (TO H.E.) ARE:

- 1) LOGARYTHMIC MEAN T-DIFF. METHOD
- 2) E-NTU EFFICIENCY METHOD

2.4 LOGARYTHMIC MEAN T-DIFF. METHOD

IT REQUIRES TO KNOW:

$$\left\{ \begin{array}{l} K, A t, (m, c_p, m', c'_p) \\ t_i, t'_i \end{array} \right.$$

$$\Theta_0 = t_i - t'_i, \Theta_t = \Theta_0 e^{-K \left(\frac{1}{m'c_p} + \frac{1}{mc_p} \right) A t} \Rightarrow \Theta_0, \Theta_t \text{ COMPUTED } \checkmark \Rightarrow \dot{Q}$$

$$\dot{Q} = K A t (t - t')_{em} = K A t \frac{\Theta_t - \Theta_0}{\ln \frac{\Theta_t}{\Theta_0}} \checkmark$$

IN CASE OF COUNTERFLOW CONFIGURATION:

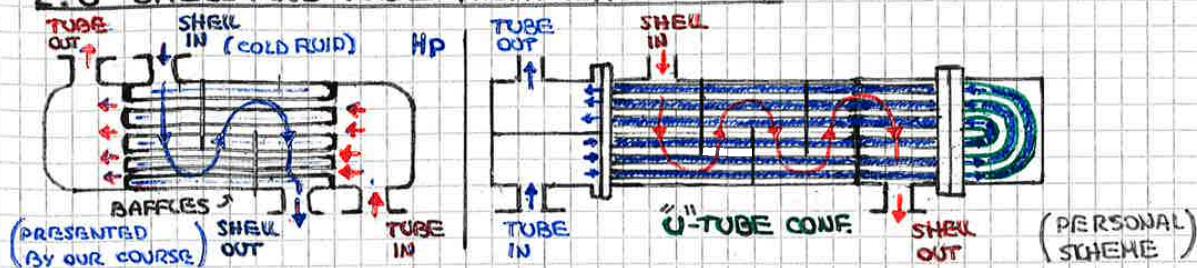
$\Theta_0 = t_i - t'_0$ BUT t'_0 IS NOT INITIALLY KNOWN \Rightarrow ITERATIVE PROCEDURE :

- WE ASSUME A VALUE OF t'_0

- WE DERIVE THE PARAMETERS OF H.E.

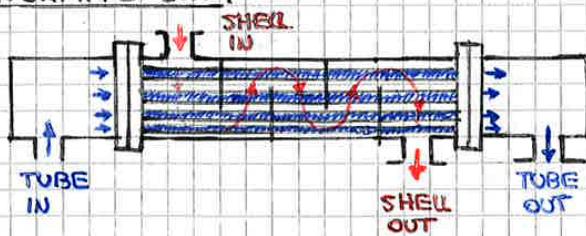
- IF THEY ARE CONSISTENT $\Rightarrow t'_0$ IS CORRECT (AND SO THE CORRESPONDING \dot{Q})

2.6 SHELL AND TUBE HEAT EXCHANGER



- THIS LAYOUT MAKES THE H.E. MORE COMPACT.
- IF IT IS PROVIDED OF SINGLE INLET AND SINGLE OUTLET PORTS, THEN THE ANALYTICAL APPROACH OF DOUBLE PIPE H.E. CAN BE APPLIED.
- BEST WORKING CONDITION: VARIOUS MASS FLOW RATES ARE EQUAL.
IN ORDER TO GET THIS CONDITION: EACH RESISTANCE TO THE FLOW OF TUBES MUST BE HIGHER THAN THE ONE OF THE MIXING CHAMBER => IN THIS WAY THE FLUID PARTICLES WILL NOT EXPERIENCE A PREFERRED TRAJECTORY, AND THEY WILL DISTRIBUTE EQUALLY AT INLET SECTION.
- AN IDEAL PARALLEL FLOW CONFIGURATION MUST BE CREATED.
TO THIS END, SPARERS ARE OFTEN INSERTED IN THE SHELL IN ORDER TO FORCE A "S-SHAPE" TRAJECTORY OF THE FLUID.
- "U-TUBE" CONFIGURATION: ONE OF THE MOST COMMON LAYOUT FOR ITS EASY ASSEMBLY AND ACCESS. IT IS CHARACTERIZED BY PROPER C, E-NTU DIAGRAM.

NORMAL CONF.:



3. EXERGY

- IT'S A UNIQUER COMBINATION OF 1st AND 2nd LAWS OF THERMODYNAMICS
- IT GIVES THE MAXIMUM WORK THAT WE CAN EXTRACT BY THE SYSTEM, LETTING THE SYSTEM RELAXING FROM INITIAL CONDITIONS TO A REFERENCE STATE (DEAD STATE; IT SHOULD BE SUBSTITUTED AS THE ENVIRONMENT STATE)

INTERNAL EXERGY: ENTHALPIC EXERGY

$$a = e_i - T_0 s$$

$$b = h - T_0 s \quad | \quad h = e_i + pV \Rightarrow e_i = h - pV \Rightarrow a = b - pV$$

TOTAL INTERNAL EX.: TOTAL ENTHALPIC EX. / TOTAL EX. / STAGNATION EX.:

$$a_T = e_i + e_m - T_0 s$$

$$b_T = h + e_m - T_0 s$$

3.1 EXERGY BALANCE EQUATION

1st LAW OF THERMODYNAMICS FOR OPEN SYSTEMS :

$$\sum_{j=1}^{NP} \Phi_j - W_t = \frac{d}{dt} (E_i + E_m + p_0 V) + \sum_{i=1}^{NC} G_i (h_i + e_m)_i$$

$$\Phi_j = \int \frac{\partial q}{\partial \theta_j} \hat{m} dA$$

$$W_t = W_t^* - p_0 \frac{dV}{dt} = - \int \underline{q} \cdot \underline{v} \cdot \hat{m} dV - p_0 \frac{dV}{dt}$$

$$E_i = \int \underline{p} \underline{e}_i dV$$

$$E_m = \int \underline{p} \underline{e}_m dV$$

$$G_i = \int \underline{p} \underline{u} \cdot \hat{m} dA ; (h_i + e_m)_i = \frac{1}{G_i} \int \underline{p} (h + e_m) \underline{u} \cdot \hat{m} dA$$

2nd LAW OF THERMODYNAMICS FOR OPEN SYSTEMS

$$\sum_{j=1}^{NP} \frac{\Phi_j}{T_j} + \Psi_{IRR} = \left(\frac{dS}{dt} \right)_D + \sum_{i=1}^{NC} G_i S_i$$

$$\int \Psi_{IRR} = \int \frac{\sum G_i + E_m}{T} dV \quad \text{or} \quad E_{IRR} = \int G_i + E_m dV$$

$$S = \int \underline{p} \underline{s} dV ; S_i = \frac{1}{G_i} \int \underline{p} \underline{s} \underline{u} \cdot \hat{m} dA$$

BY DOING (1st) - $T_0 \cdot$ (2nd):

$$\sum_{j=1}^{NP} \Phi_j \left(1 - \frac{T_0}{T_j} \right) - W_t - T_0 \Psi_{IRR} = \frac{d}{dt} (E_i + E_m + p_0 V - T_0 S) + \sum_{i=1}^{NC} G_i (h_i + e_m - T_0 s)_i$$

RE-WRITTEN:

$$\sum_{j=1}^{NP} \Phi_j \Theta_j - W_t = \frac{d}{dt} (A_t + p_0 V) + \sum_{i=1}^{NC} G_i (b_i + e_m)_i + T_0 \Psi_{IRR} : \text{EXERGY BALANCE EQ.}$$

$$\Theta_j = 1 - \frac{T_0}{T_j} : \text{CARNOT FACTOR} (\# C. EFF.) \quad \text{or} \quad \eta_j$$

T_0 : DEAD STATE.

$$T_0 \Psi_{IRR} = W_{loss} : \text{SINK T.} = \text{LOST WORK/DESTROYED EXERGY (GOULD-STODOLA)} \quad \text{or} \quad T_0 E_{loss}$$

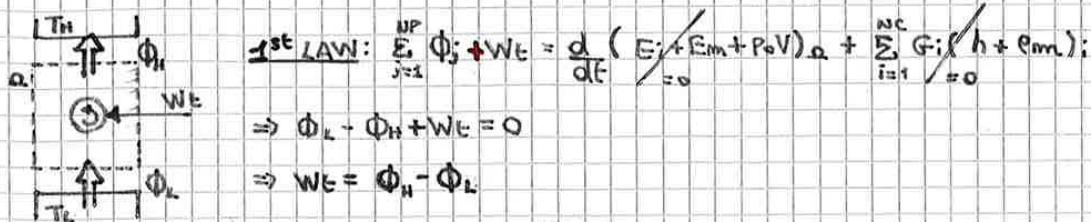
- MOTOR MACHINE (WE GET USEFUL WORK):

$$\sum_{j=1}^{NP} \Phi_j \Theta_j - W_t = \frac{d}{dt} (A_t + p_0 V) + \sum_{i=1}^{NC} G_i (b_i)_i + T_0 \Psi_{IRR} ; W_t = W_t^{\text{REV}} - T_0 \Psi_{IRR} ; W_t^{\text{REV}} = \sum_{j=1}^{NP} \Phi_j \Theta_j - \frac{d}{dt} (A_t + p_0 V)$$

- OPERATING MACHINE (WE PROVIDE WORK TO THE MACHINE):

$$\sum_{j=1}^{NP} \Phi_j \Theta_j + W_t = \frac{d}{dt} (A_t + p_0 V) + \sum_{i=1}^{NC} G_i (b_i)_i + T_0 \Psi_{IRR} ; W_t = W_t^{\text{REV}} + T_0 \Psi_{IRR} ; W_t^{\text{REV}} = \sum_{j=1}^{NP} \Phi_j \Theta_j + \frac{d}{dt} (A_t + p_0 V)$$

EXAMPLE : REFRIGERATION MACHINE (O.M) (STADY STATE)



2nd LAW: $\sum_{j=1}^{NP} \frac{\dot{\Phi}_j}{T_j} + \dot{\Psi}_{IRR} = \left(\frac{d, S}{d, t} \right)_{\infty} + \sum_{i=1}^{NC} \dot{G}_i / s_i = 0$

$\Rightarrow \frac{\dot{\Phi}_L}{T_L} - \frac{\dot{\Phi}_H}{T_H} + \dot{\Psi}_{IRR} = 0$

$\Rightarrow \dot{\Psi}_{IRR} = \frac{\dot{\Phi}_H}{T_H} - \frac{\dot{\Phi}_L}{T_L} > 0 \quad T_0 = T_H$

O.M.: $\dot{W}_E = \dot{W}_E^{\text{REV}} + T_0 \dot{\Psi}_{IRR} \Rightarrow \dot{W}_E^{\text{REV}} = \dot{W}_E - T_0 \dot{\Psi}_{IRR} = \dot{\Phi}_H - \dot{\Phi}_L - \dot{\Phi}_H + \dot{\Phi}_L \frac{T_H}{T_L} = \dot{\Phi}_L \left(\frac{T_H}{T_L} - 1 \right) = \dot{\Phi}_L \frac{1}{\eta_{10}^{\text{II}}}$

EXERGY ANALYSIS: $\sum_{j=1}^{NP} \dot{\Phi}_j \theta_j + \dot{W}_E = \frac{d}{dt} (\dot{A}_I + \dot{P}_V V)_{\infty} + \sum_{i=1}^{NC} \dot{G}_i / (b_i)_{i} + T_0 \dot{\Psi}_{IRR}$

$\dot{\Phi}_L \theta_L - \dot{\Phi}_H \theta_H + \dot{W}_E = T_0 \dot{\Psi}_{IRR}$

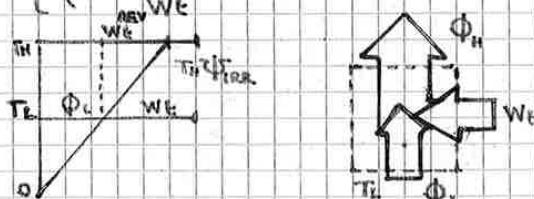
$\dot{W}_E = \dot{\Phi}_L \theta_H - \dot{\Phi}_L \theta_L + T_0 \dot{\Psi}_{IRR} = \dot{\Phi}_L \left(1 - \frac{1}{\eta_{10}^{\text{II}}} \right) - \dot{\Phi}_L \left(1 - \frac{T_0}{T_L} \right) + T_0 \dot{\Psi}_{IRR} = \dot{\Phi}_L \left(\frac{T_0}{T_L} - 1 \right) + T_0 \dot{\Psi}_{IRR}$

$\dot{W}_E^{\text{REV}} = \dot{\Phi}_L \left(\frac{T_0}{T_L} - 1 \right) = \dot{\Phi}_L \frac{1}{\eta_{10}^{\text{II}}}$

$\eta_{10}^{\text{I}} = \frac{\dot{\Phi}_L}{\dot{W}_E^{\text{REV}}} = 1 / \left(\frac{T_0}{T_L} - 1 \right)$

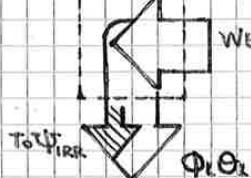
COP = $\frac{|\dot{\Phi}_H|}{|\dot{W}_E|} = \eta_{10}^{\text{I}} \cdot \eta_{10}^{\text{II}} = \frac{\dot{\Phi}_L}{\dot{W}_E^{\text{REV}}} \cdot \frac{\dot{W}_E^{\text{REV}}}{\dot{W}_E} : \text{COEFF. OF PERFORMANCE}$

$\eta_{10}^{\text{II}} = \frac{\dot{W}_E^{\text{REV}}}{\dot{W}_E} : \text{II LAW EFF.}$

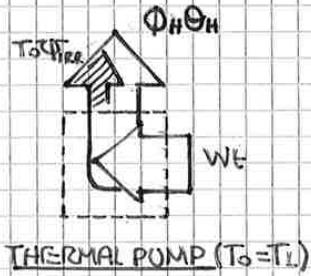


ENERGY FLOW
(FROM 1st LAW)
 $\dot{W}_E = \dot{\Phi}_H - \dot{\Phi}_L$

REFRIGER. M ($T_0 = T_H$)



EXERGY FLOW
(FROM EXERGY ANALYSIS)
 $\dot{W}_E = -\dot{\Phi}_L \theta_L + T_0 \dot{\Psi}_{IRR}$



THERMAL PUMP ($T_0 = T_L$)

IN CASE OF A THERMAL PUMP (STILL INVERSE CYCLE):

- IT TAKES THERMAL POWER FROM ENV. AND RELEASES IN A CONTROLLED ENV. AT SUPERIOR T.
- FROM ENERGETIC P.O.V. NOTHING CHANGES.
- FROM EXERGETIC P.O.V. : THE DESTINATION OF EXERGY CHANGES

$T_0 = T_L$ (TO CHECK; I THINK IT'S CORRECT)

$\dot{\Phi}_L \theta_L - \dot{\Phi}_H \theta_H + \dot{W}_E = T_0 \dot{\Psi}_{IRR} = \dot{\Phi}_H \left(1 - \frac{T_0}{T_H} \right) = \dot{\Phi}_H \left(1 - \frac{T_0}{T_H} \right) + T_0 \dot{\Psi}_{IRR}$

$\dot{W}_E = \dot{\Phi}_H \theta_H - \dot{\Phi}_L \theta_L + T_0 \dot{\Psi}_{IRR} = \dot{\Phi}_H \left(1 - \frac{T_L}{T_H} \right) - \dot{\Phi}_L \left(1 - \frac{T_L}{T_H} \right) + T_0 \dot{\Psi}_{IRR}$

$\dot{W}_E^{\text{REV}} = \dot{\Phi}_H \left(1 - \frac{T_L}{T_H} \right) = \dot{\Phi}_H \frac{1}{\eta_{10}^{\text{II}}}$

$\eta_{10}^{\text{I}} = \frac{|\dot{\Phi}_H|}{|\dot{W}_E^{\text{REV}}|} = \eta_{10}^{\text{I}} \cdot \eta_{10}^{\text{II}}$

$\eta_{10}^{\text{II}} = \frac{\dot{W}_E^{\text{REV}}}{\dot{W}_E}$

2) IDEAL GAS

$$TdS + VdP = dh : \text{2nd GIBB'S R.} ; dh = C_p dT$$

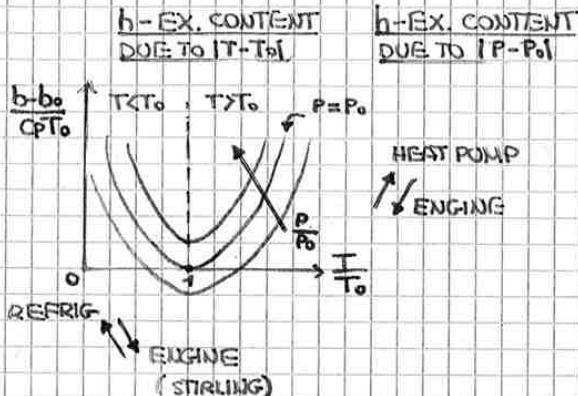
$$dS = \frac{dh}{T} - \frac{VdP}{T}, PV = RT \Rightarrow \frac{V}{T} = \frac{R}{P}$$

$$dS = \frac{dh}{T} - \frac{RdP}{P} \Rightarrow S - S_0 = \int dS = \int \frac{dh}{T} - \int \frac{RdP}{P} = \int \frac{C_p dT}{T} - R \int \frac{dP}{P} = C_p \ln \frac{T}{T_0} - R \ln \frac{P}{P_0}$$

$$b - b_0 = (h - h_0) - T_0(S - S_0) = C_p(T - T_0) - T_0 C_p \ln \frac{T}{T_0} + T_0 R \ln \frac{P}{P_0} = \\ = C_p T_0 \left(\frac{T}{T_0} - 1 - \ln \frac{T}{T_0} + \frac{R \ln \frac{P}{P_0}}{C_p} \right), \frac{R}{C_p} = \frac{k-1}{k}$$

MAXIMUM WORK
UNDER IDEAL CONDITIONS

$$\frac{b - b_0}{C_p T_0} = \underbrace{\frac{T}{T_0} - 1 - \ln \frac{T}{T_0}}_{h-\text{EX. CONTENT DUE TO } (T-T_0)} + \underbrace{\frac{k-1}{k} \ln \frac{P}{P_0}}_{h-\text{EX. CONTENT DUE TO } (P-P_0)} : \text{ENTHALMIC EXERGY CONTENT OF AN IDEAL GAS}$$



PERSONAL CONSIDERATION (TO CHECK):

$$a = e_i - T_0 s, h = e_i + PV \Rightarrow e_i = h - PV$$

$$b = h - T_0 s$$

$$a = (b - PV) - T_0 s = b - PV$$

$$\begin{cases} a = b - PV \\ a_0 = b_0 - P_0 V_0 \end{cases}$$

$$a - a_0 = b - b_0 - (PV - P_0 V_0)$$

THEREFORE:

$$\frac{a - a_0}{C_p T} = \frac{b - b_0}{C_p T} - \frac{(PV - P_0 V_0)}{C_p T} \quad (\text{I THINK IT'S CORRECT})$$

3.4 EXERGY ANALYSIS FOR SOLAR SYSTEM : SOLAR ENERGY → SOLAR EXERGY

$$\eta_p = \frac{\text{EXERGY}}{\text{ENERGY}} = \left(\frac{\text{EXERGY IN TIME}}{\text{POWER}} \right)$$

$$\Rightarrow \text{EXERGY} = \eta_p \cdot \text{ENERGY}$$

$$\left\{ \begin{array}{l} \eta_p = 1 - \frac{4}{3} \frac{T_0}{T_1} + \frac{1}{3} \left(\frac{T_0}{T_1} \right)^4 : \text{PETERLA FORMULA (1960)} \\ \eta_p = 1 - \frac{4}{3} \frac{T_0}{T_1} : \text{SPANNER FORMULA (1964)} \end{array} \right.$$

$$\left\{ \begin{array}{l} \eta_p = 1 - \frac{T_0}{T_1} : \text{JETTER FORMULA (1981)} \end{array} \right.$$

$T_1 = 5762 \text{ [K]}$: APPARENT SUN T