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# **A P P U N T I**

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# BASICS OF DESIGN

## 9/03/20 (Giordano) Lesson I

The first question we will answer is:

### ■ What is a bridge?

A bridge is a structure built to span a valley, road, river, body of water, or any other physical obstacle.

Designed to carry their own weight (dead load) and people/traffic loads (live loads) and to resist natural forces, such as wind and earthquakes.

You must consider effects of contraction or expansion due to temperature changes and, in case of concrete bridges, contraction due to shrinkage and creep.

Designs of bridges vary depending on the function of the bridge and the nature of the area where the bridge is to be constructed.

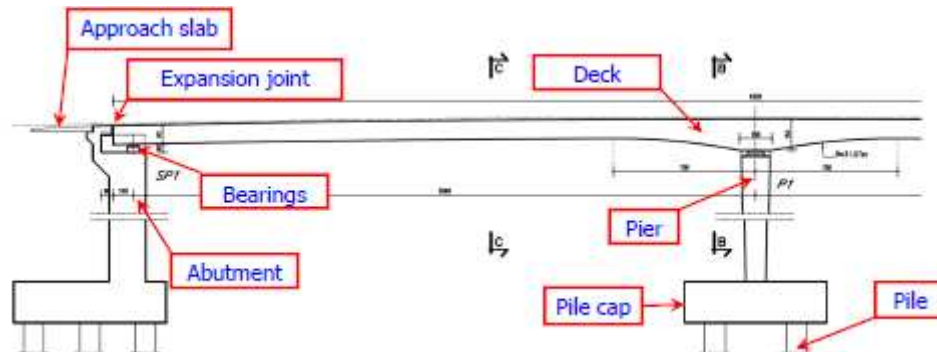
From this answer we first understand that bridges are subjected to different types of loads, and live loads can be very big loads above all if we talk about rail bridges, but also for road bridges.

In every case dead load is always a very important portion of total load. With increasing of the span dead load can easily become 70% or 80% of the total load. When someone say that a bridge collapse because of the increasing of traffic load it's not totally the true, because live loads are only a little part of loads: if you increase the live loads of 25% or 30% actually you increase the total load of about 5% and it is difficult to think that this increase can cause the collapse of the structure, so the reason is probably in another part.

The first concept to highlight is the concept of **expansion and contraction of the bridge due to temperature variation**: temperature variation, and of course shrinkage and creep because the contraction or the expansion of a bridge, is an issue that can be very important in bridge design. Every structure is subjected to temperature action and if the structure is made of concrete it will move due to shrinkage, and if the concrete is pre-stressed it will move due to creep; of course shrinkage is a decrease of volume of the concrete body in the time and it is mainly due to the evaporation of the water, if the body is mainly developed in the longitudinal direction, so the decrease of volume will be a shortening of the body. On the other hand creep is a phenomena where the body is subjected to a constant stress in time, then deformation is not constant in time but it

## TERMINOLOGY – LONGITUDINAL DIRECTION

This is a longitudinal section of a bridge:



The most important parts of a bridge are indicated:

The deck is the most important part of the bridge in absolute and it is supported by the piers and the abutments; between the supports and the deck we can see bearings; at the base of the piers there are the foundations, generally in bridge fields we have a very large forces, so we will generally use deep foundations like piles and between piles and piers we have a pile cap and at the ends of the deck we can see expansion joints and finally at the side of the bridge we could find an approach slab that sometimes is called transition slab: it is a structural element that permits the transition between the embankment and the deck. Generally these structural elements are divided between two types:

- **Superstructures** → all the elements that are above the bearings;
- **Substructures** → all the structural elements that we can find below the bearings;

The bearings are usually a part of the superstructure.

The **deck** is the roadway portion of a bridge that directly supports vehicular and pedestrian or railway traffic; we have to design the material to use, to construct, the shape and eventually the transversal section can change in the longitudinal direction as for example is shown in the picture where we can see that the depth of the section isn't constant in longitudinal direction but it increase in correspondence of the pier as we have a continuous static scheme and this section is the most solicited one.

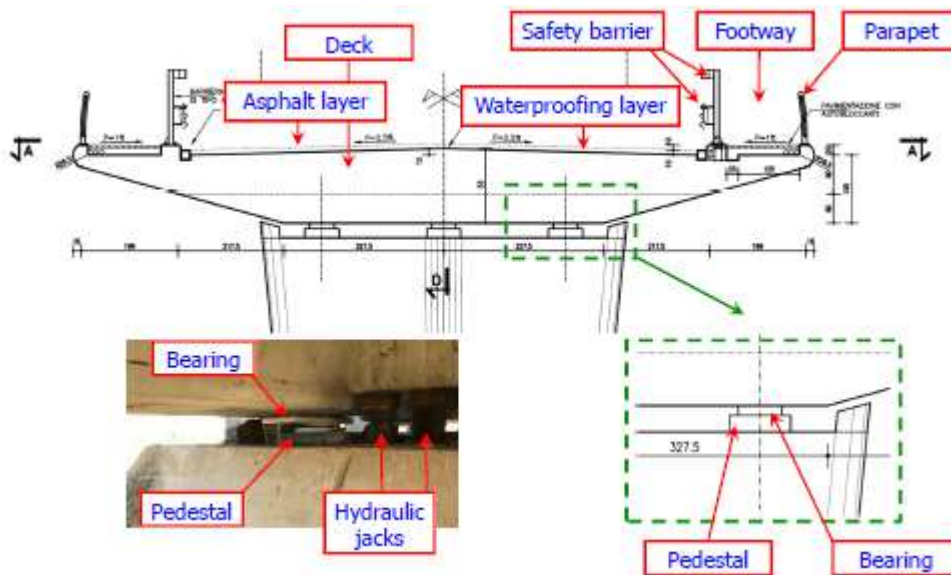
The **piers** are intermediate supports of bridge superstructure and they transfer the load (vertical and horizontal) to the foundations; in case of structures in a seismic area piers are the only resisting

**Bearings** are devices that bring the substructure and the superstructure to transfer the applied vertical and horizontal loads; in addition they allow relative movements and translational movements in longitudinal and/or transverse direction.

**Expansion joints** are devices used to allow for continuous traffic between structures while accommodating movement due to temperature variations, horizontal actions, shrinkage, creep etc.

The **approach slab** isn't really a structural element in fact it is a slab supported directly by the soil used to provide a smooth and structurally sound transition from the roadway pavement to the bridge (the area between the roadway embankment and the bridge frequently experiences differential settlement).

### TERMINOLOGY – TRANSVERSE DIRECTION



In this direction we have other typical elements in bridge field: at the top surface of the deck we have the **waterproofing layer** and the **asphalt layer** in fact the deck have to be protected from water; it is achieved by the prevision of adequate drainage and waterproofing the upper surface of the bridge waterproofing layer has to be rough enough to maintain adherence between the deck and the vehicle. At the side of the deck we have a **safety barrier** that is used to keep vehicles inside the roadway and eventually to protect pedestrians. During the crash the safety barrier transmits big point loads at the connection with the deck, so a larger local stress has to be considered during the design and of course this is not a problem of stability of the bridge but it is a local verification; generally the verification is made with a capacity design criteria that is the action used to evaluate the reinforcement or the internal action of the bridge evaluated using the bending resisting moment of the safety barrier: when a crash occurs we have damage of barrier but not the damage of the

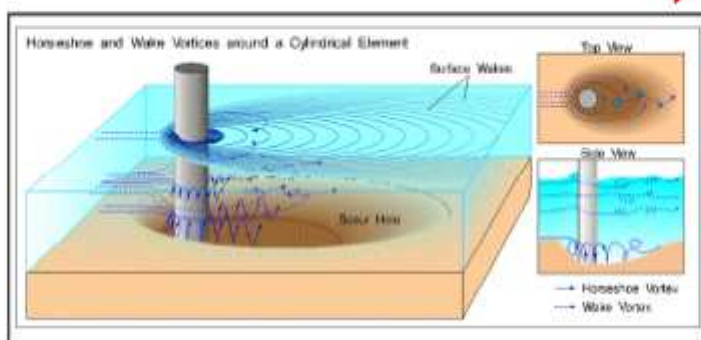
## INPUT DATA

Of course any design approach needs of input data that someone else have to give to the designer; in the case of the bridge design this input data are the road project and the boundary conditions in terms of geotechnical, hydraulic and geological data. In the next slide we can see the input of the road project:

⇒ Planimetric configuration	Layout (Radius of curvature, span...) Obstacles to be crossed (rivers, roads, railways, valleys...)
⇒ Altimetric configuration	Max and min depth, slope,... Curvature radius in the vertical plane
⇒ Transverse section geometry	Live load (train, truck...) Lane number and width Carriageway number (single, multiple) Sidewalk, services...

For the correct foundation design we need to report the following input data:

⇒ Foundation	Geological and geotechnical investigations Stratigraphies Landslides (old landslides, landslides in action...) Hydraulic studies (max flood level, scour)
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## PRE-STRESSING

Pre-stressing is a technique that introduces compressed stress in the fibre in which the designer is expected to have tensile stress in service conditions; so after the application of service live loads the structures should be anchored in addition in such way we can use reinforced concrete structures high strain steel; in fact without prestressing we couldn't use high strain steel as we would have too wide cracks in service conditions; in reinforced concrete structures the crack width depends mainly by the tensile stress in the reinforcement so also if we use a strain with a yield strength of 100 MPa for instance, in order to control the crack width, the service stress should be limited to 300 or 350 MPa so prestressing make impossible to use high strength steel.

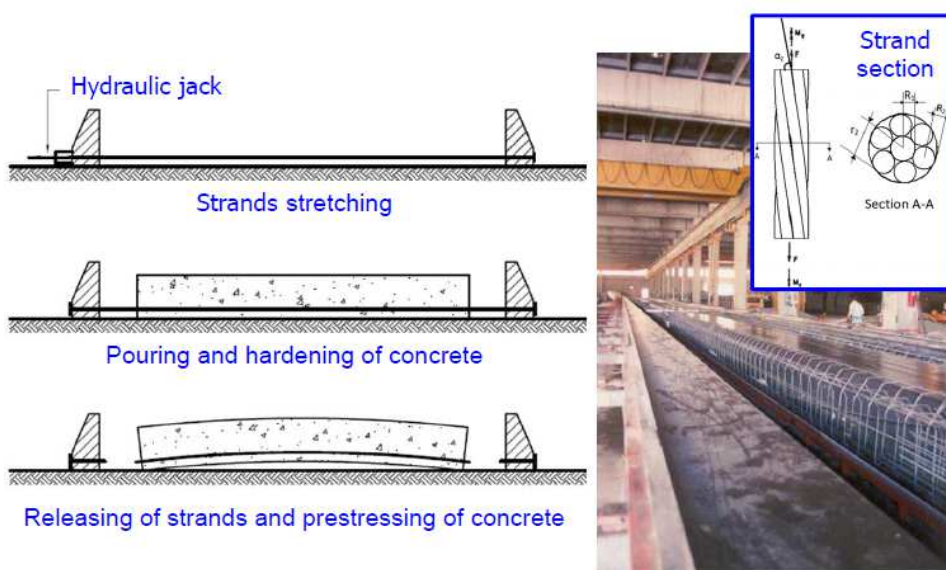
Prestressing can be introduced in a structure in three different ways:

### PRE-TENSIONING

Pre-tensioning is realized by stretching strands between two anchorages prior to placing concrete as shown in the next figure. The concrete is then placed and strands become bonded to concrete throughout their length.

After concrete has hardened, the strands are released by cutting them at the anchorages. The strands tend to regain their original length by shortening and in this process transfer through bond a compressive stress to the concrete.

The strands are usually stressed by the use of hydraulic jacks. The stress in strands is maintained during the placing and curing of concrete by anchoring the ends of the strands to abutments that may be as much as 200m apart. The abutments and other formwork used in this procedure are called prestressing bench or bed.





## UNBONDED (OR EXTERNAL) PRESTRESSING

As for post-tensioning but with tendons located outside the section of a structural member (or inside the hollow space of a box girder), only connected to the member through deviators and end-anchorages. Because of the absence of bond between the tendon and structure, external prestressing allows the removal and replacement of one or two tendons at a time so that the bridge could be retrofitted in the event of deterioration and their capacity could be also increased easily.



### Advantages:

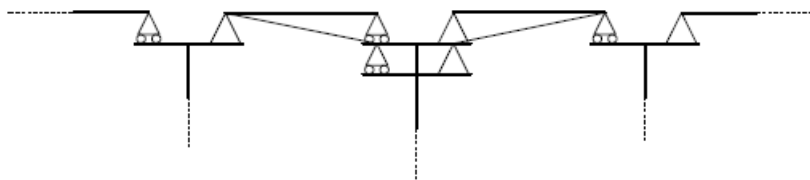
- ❖ Possibility of inspection, maintenance and possible replacement of some tendons, even in service conditions

### Disadvantages:

- ❖ Anchorage zones are critical areas. In the case of internal tendons with grouting, the long-term failure of anchor head has limited consequences because prestressing may be transferred to the structure by bond. In the case of external tendons, the behavior of anchor heads is much more critical because a anchor head failure makes the tendon completely ineffective
- ❖ Needs of more prestressing tendons due to the lack of bond with concrete
- ❖ At ultimate limit states, failure may happen with little warning due to insufficient ductility, above all in case of precast segments where there is no ordinary reinforcement crossing the joint

In the picture we can see other solutions to realize a simply supported girder, generally used when the pier cup has large size: with this solution we double the number of expansion joints as we have two expansion joints for each pier; in particular the picture on the left is used to make room at the end of the girder to easily performing the post-tension of the prestressing tendons; on the other hand the other solution was used with a three dimensional framed piers, also here we have two joints for each pier.

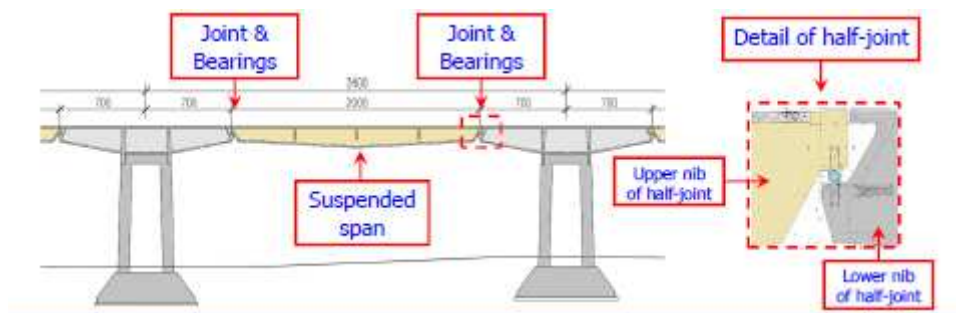
This solution was used a lot in the past because of its simplicity both for realization and for calculation, in fact the calculation is quite simple, actually is not so simple because its not a simply supported beam but a simply supported girder (a set of beams and cross beams that work together and transversely share loads to transfer it to the bearings and we will analyse that in two ways: Courbon solution and numerical calculation). With this solution and Courbon calculation was quite easy to perform an hand calculation so this structures were wide used in the past; the simplicity of construction of course plays an important role because if a structure is easy to calculate it is difficult to realize you'll never use it; this solution is well suited to prefabrication and has a good speed in construction: if we have to build hundred and hundred kilometres as it happened in 50's and 60's in the last century. Another advantage of this solution is the not sensitivity to differential settlements at support; that is if we have a vertical displacement at the support we have a rigid body motion and we don't have any internal action:



As we can see we have a rigid rotation of the span and then we don't have any internal action in the girders, so from this point of view is a very good solution; the same thing appends with temperature gradient, also in this case we don't have internal actions but we have only deformation of the structure.

But this solution has of course also a lot of disadvantages: first of all it is limited in length, even with pre-stressed concrete solutions it's difficult to have span length bigger than 40 or 45 m, also for the transportation problem of the precast elements. In addition the real limitation of this solution is a large use of components with short service life: in particular bearings have a service life of 30 years and joints of 10 years if they don't come in contact with ice and salt; for instance also the removal of the snow causes big damages to the joints so they become permeable and water can enter in them and go to the bearings. A big

This is a real case designed by Riccardo Morandi in 1945 where we can see that with the use of a simply supported beam of 20 m so called suspended span, we can cover a span length of 44m; in the detail on the right we can see how is realized the half joint: we have a lower bearing part as so called “lower nib of half joint” and “upper nib of half joint”, in the middle there is a bearing that in this case is realized with a steel cylinder that allows the longitudinal displacement; of course this is an old structure in which there isn't a pedestal and this means that bearing replacement is very difficult; of course the design of this zone is not simple because it is a discontinuity region and then the reinforcement arrangement is complex, the number of reinforcement bars is large; the strengthening of this zone is very difficult too. Therefore in the end we can say that this solution has shown in time a lot of weak point so has been faced out for many years; unfortunately there are many old bridges realized in this way and they have to be repaired and it's quite complex.



Advantages and disadvantages of half supported bridges are quite the same of simply supported bridges, the only difference is related to the use of the material that in this case is better comparing with the simply supported solution as we can take advantages from the negative bending resistance of a section, but the reality is that this solution give more durability problems than the previous one and this is the way because it isn't used nowadays.

Some typical effects of combination of broken expansion joint and the use of the icing salt: we have deep damage of the concrete cover in the joint areas and this damage is also for reinforcing bars.

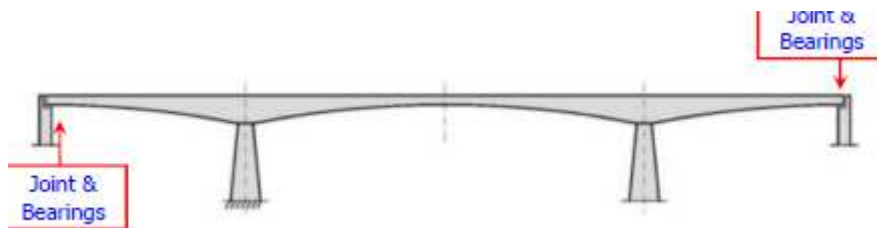
structural inertia because we increase the depth of the section and consequently we increase constraint stresses.

As we have said, with this solution we solved some durability problems, but some other durability problems will increase: precast elements have to be assembled together with prestressing, so we have to do a good grouting of tendons and this introduces other problems of durability and other calculations.

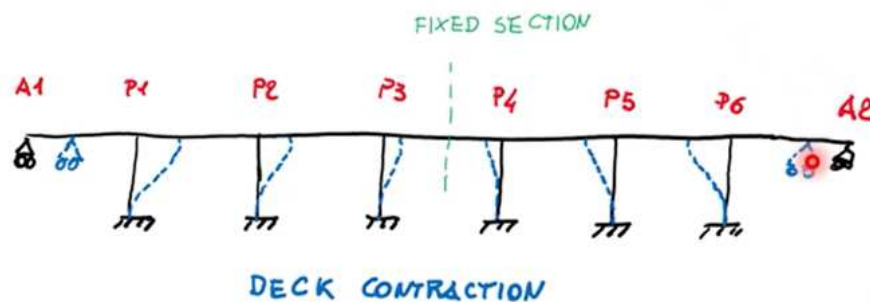
The connection between different elements is simple in steel construction, in fact connections can be realized through welding or bolting (using plate with a small thickness).

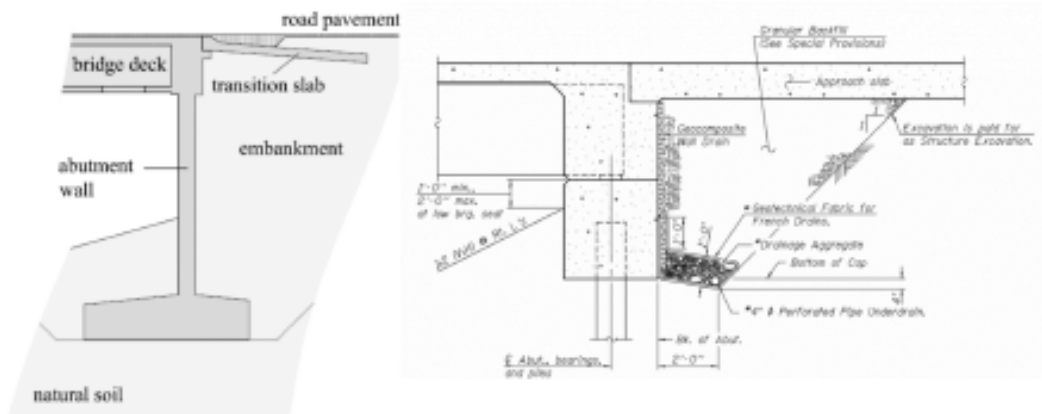
- **Framed girder bridges**

If we want to continue to reduce the number of components with a short design life we could use framed girder bridges; in the previous solution the connection between the pier and the deck wasn't able to transmit longitudinal bending moment. Vice versa if we realize a fixed connection between the pier and deck we can transfer this action removing in this way all the pier bearings; this solution is not so used because of the difficulty of the realization of the connection zone and in addition if the bridge is in seismic zone the connection zone can be easily damaged while with the previous solution the damaged zone expected is only the base of the pier.



From a calculation point of view the constant temperature variation, and in general the expansion and the contraction of the deck give rise to internal action. In this configuration the deck and the pier have the same displacements and this leads to a bending of the pier.





We have only spoken about girder bridges, that will be the object of this course but there are also other types of bridges that we can realize.

- **Trestle bridges** (*ponte a cavalletto*)

It is actually a generalization of framed bridges; in this following case the piers that sometimes are called legs, are vertically inclined in the longitudinal direction and generally inched at the bases.



The hinge connection can be done in two ways: it can be cylindrical that means that the rotation about the axis coming out from the plane it's free whereas the rotation about the longitudinal axis of the leg is fixed; or can be done with a spherical hinges and it means that both the rotations can be free.

The deck is supported by the arch and slender columns placed at shorter distance; greater is the distance of the columns and greater would be the depth of the deck.

The designer should change this parameters (distance between the columns) in order to have a good static appearance and a good structural behaviour.

Because of live loads can be everywhere in the deck, the arch won't be purely compressed but it will be also subjected to bending moment.

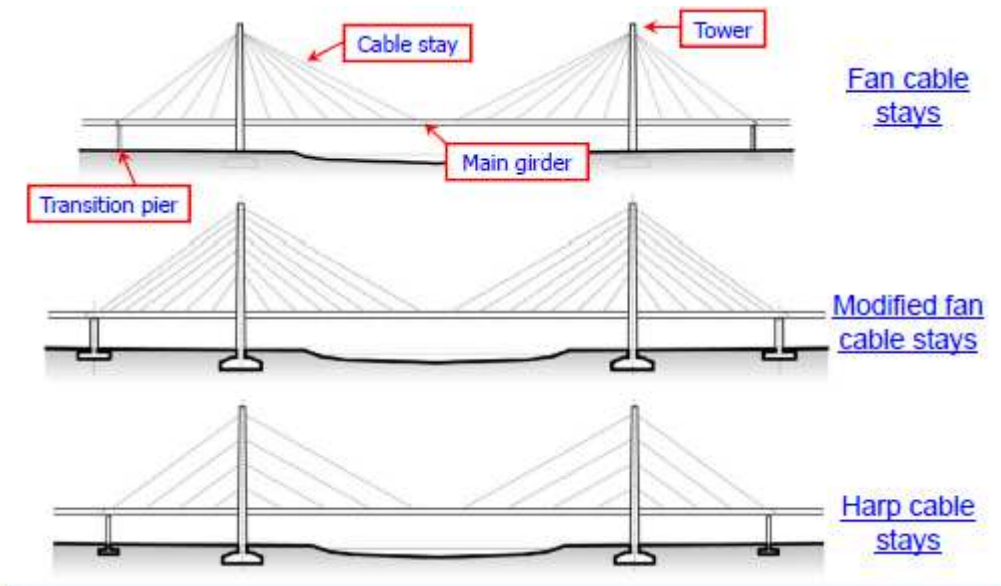
Particular attention should be played for arch buckling, especially for the out of plane direction.

2. The second type of arch bridge are **through arch bridges** (*arco a spinta eliminata*) in which the deck is below the arch; the Italian name explain well the static behaviour of this bridge, that can be translated in English as “arch with eliminated push”.

Practically the horizontal component of the arch compressive force at the connection with the deck is taken from the deck that act like a chain.

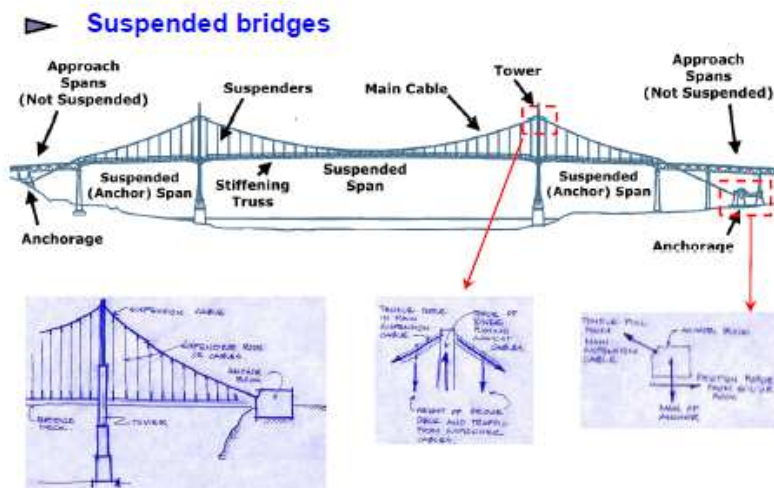


The deck is mainly subjected to torsion and it is suspended with vertical or sub-vertical steel elements called suspender (*pendini*); the distance between the suspenders is about 8-12 m and influences the bending moment in the deck because of course the deck is subjected also to bending moment due to live loads; in addition the distance between the suspenders is limited in order to make possible the substitution of a single supporter in every situation; from a maintenance point of view in fact there is the possibility to change one suspender per time and in this way the distance between the suspender should not be so high.

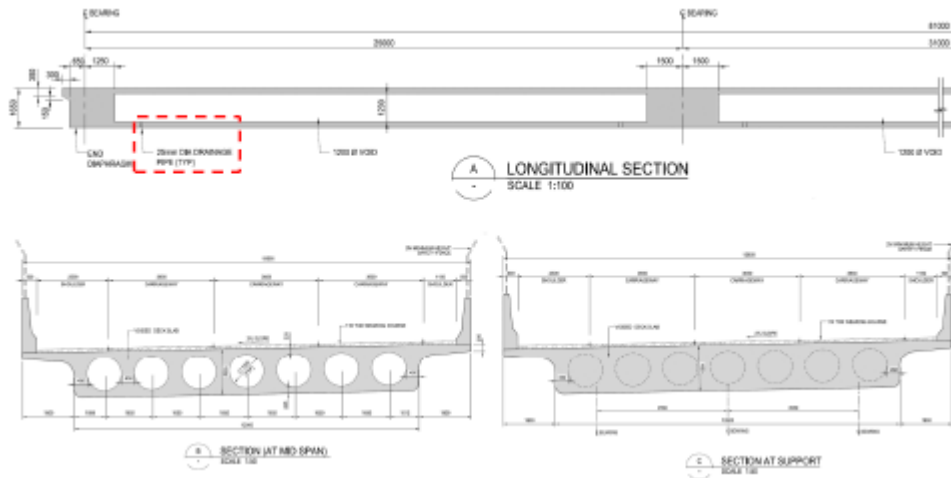


- **Suspended bridges**

The main structural component of this type of bridge is the stiffening truss which is this longitudinal element which acts like a chord and sure aerodynamyc stability of the bridge; then we have also some cables which support the stiffening truss like suspenders for arch bridges and the main cable transfers loads to the tower; this last tranfers bridge load to the soil.



- **Voided slab deck (rc/prc):  $h > \sim 100$  cm**



This other solution is adopted when the slab's thickness is more or equal to 100 cm, because the dead load in this case becomes more important and to light this we introduce voids that can be circular or rectangular and are putted near the neutral axis of the slab.

If the depth and the width of voids are less than 60% of the overall depth, their effect on stiffness is small and the deck behaviour is like a solid slab so like a plate.

Voided slab's depth are usually realized with cast in situ concrete with permanent voided forms; they can be also made with pre-cast allow-core beams but in this case the problem become the continuity of the transversal section as we can't precast an element that as the total width of the bridge so we have to connect the precast parts to each other and generally this is done with transversal elements.

If the voids are in excess of 60% the behaviour becomes cellular and it is different from the plate one. Of course incorporating voids in the deck increases the cost due to complexity in the reinforcement, designed to resist also to resist to transversal bending.

As water can enter in the voids, durability problems can born, so it is recommended to guarantee the water drainage in each void in order to eliminate the water as we can see in the picture in the ditched line; as we can see voids aren't present at the supports abutment and at the piers.

If we want to summarize the advantages of this solution we have that the formwork is simpler and less costly compared to other solutions in particular the cast in place girder with T-beams, the thickness of the deck can be kept quite small at least in prestressing solution, in this case the economic advantage is not in material cost so not due to less use of concrete but in the eight of the embankment and so in the cost of the approaching retaining walls. Also the reinforcement layout is



This types of deck have been used sometimes in the past but not so much nowadays. The connection between transversal elements can be realised in different ways, for instance with an on-site concrete casting or using shear keys o through reinforcement.

Shear keys are in general technical solutions used to provide connection between two different bodies.

- **cast in place girder  $L \leq \sim 40$  m (PRC solution)**

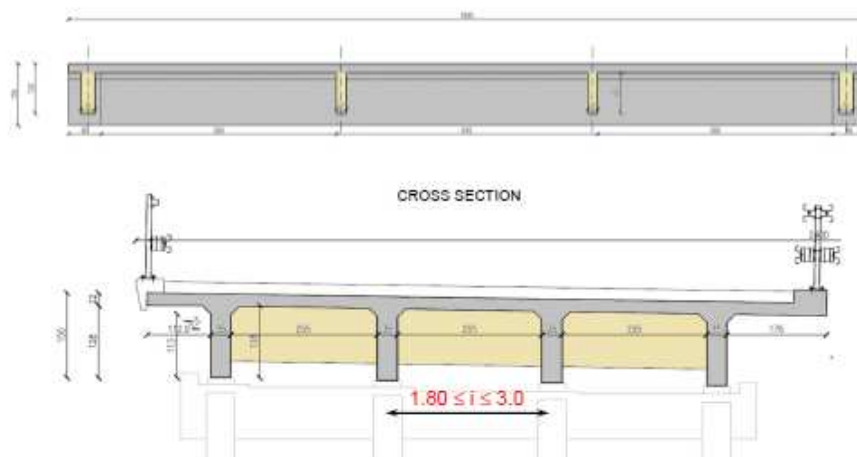
The structure can be realized using reinforced concrete or reinforced pre-stressed concrete, of course using post tensioning because this is a cast in place solution.

With reinforced concrete the maximum length that we can reach is about 20m, even if we can reach 30m both increasing the depth and the beam's number, but it isn't an economic solution.

With a pre-stressed solution we can get up to 40 m if we use a simply supported structural system or more than 40 m if we use a continuous structural system, but in this last case the issue becomes the construction system. In addition, using a continuous girder doesn't permit to give the correct bending resistance in the hogging zone (*zona soggetta a momento negativo*) as we have a small compressed area.

This solution requires more complicated formwork in particular for skew bridges, compared to slab bridges, but it permit to reach bigger distances. The beam's web thickness usually varies from 35 to 55 cm and it is controlled from the required horizontal spacing.

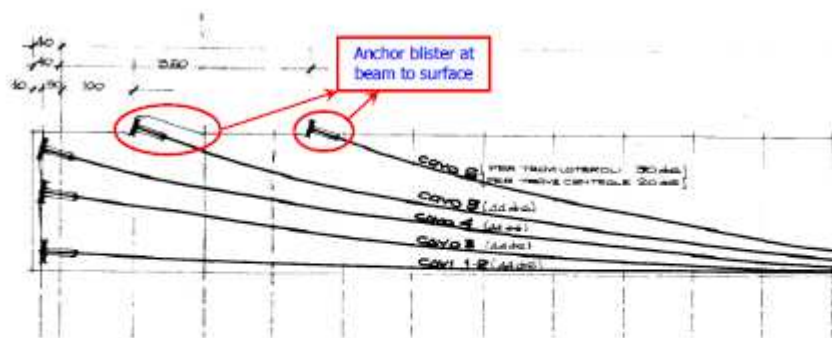
The web thickness should be changed both in longitudinal direction and in transverse direction to give more room to reinforcement, but doing this would complicate the formwork and this is the reason why generally cast in situ girder present a constant transverse section like this one:



They can be realized both in pre-tensioning and post-tensioning or with a mixed solution in which we can have a first prestressing with pre-tensioning and then complete the pre-stressing on site with post-tensioning.

Sometimes also the concreting of the slab can be realised in different phases in order to anchor additional tendons in blisters realised at beam to surface.

In this picture there is an old representation of those blisters (*lesene*) in which additional tendons have been putted:

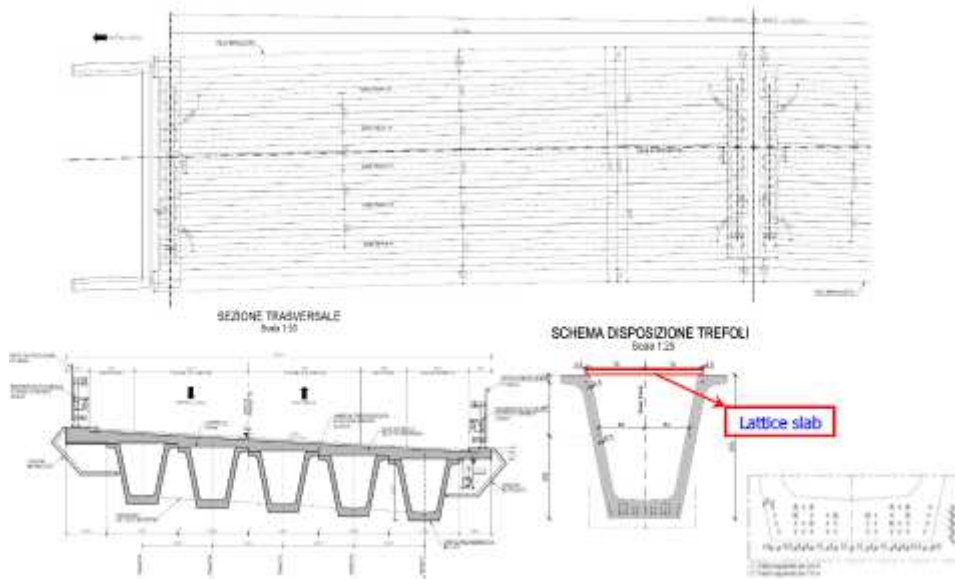


**Solution with durability problem!!!**

Tendons 5 and 6 are tensed in a second moment after the concreting; in any case this solution of tendons anchored in the top surface of the slab creates some durability problems because it is more exposed to icing salt that can corrode the steel.

Another issue of this solution is in the casting of cross beams: the major advantage in use of precast beams is the simplification of the formworks as we don't need it for the beams in place; in addition we don't need falsework as the slab's formwork is carried by the precast beams; but for cross beams unfortunately we need falsework. Also the connection between the slab and the cross beams is a problem because in this case we have to connect a precast element with a cast in place one. The better way to solve this is putting some reinforcing bars that come out from the beam for a length equal of the overlapping length. Another way to solve the problem is having holes in the web of the precast element and in the cross beam section and then putting there reinforcement or better pre-stressed reinforcement to connect them (this is the case that we can see in the picture:

placing them into a plastic shape; in this way the pre-stress comes into the beam more gaudery and we can control the stresses at the end in a better way.



With the upper strands we can reduce the global eccentricity of the pre-stressing and reduce the moment due to the pre-stress.

This type of deck is largely used in rail-bridges thanks to the torsional stiffness of his beams and of the reliability of the prestressing system which don't need on site grouting.

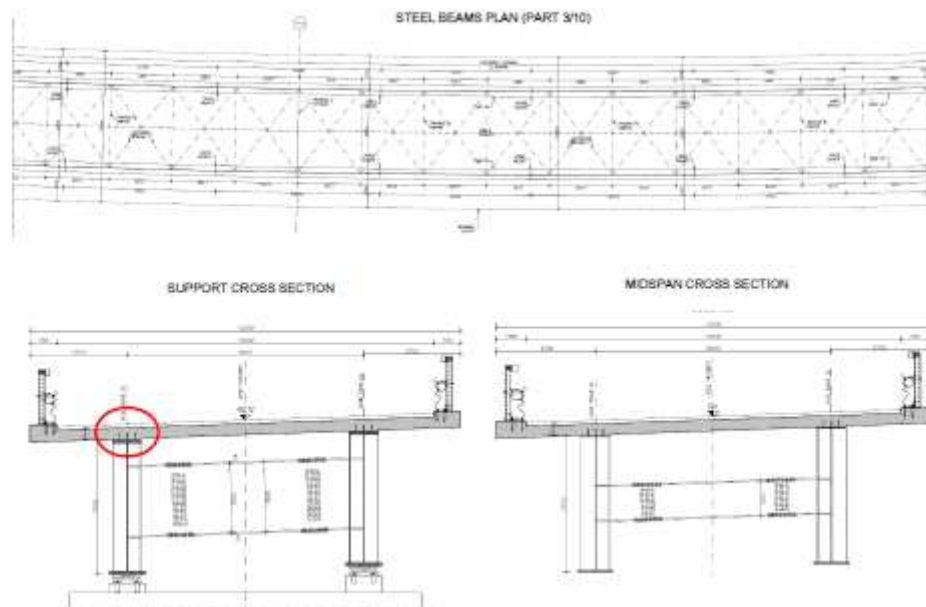
About the rail bridges we have to say that simply supported solution is the most used one in fact for this kind of bridges an important issue is the interaction between the longitudinal movement of the deck due to temperature, shrinkage, creep and braking force too and the track, as the track has an infinite length because there aren't expansion joints for many and many kilometres, the longitudinal movements of the deck lead to additional stresses in the track in particular at the bridge's expansion joints. In order to reduce this stress problem, the distance between the bridge's expansion joints has to be limited to decines of meters, better if 30 or 33 m maximum. In rail bridges iced salt are not used and so there are less durability problems than in road bridges.

slab will be in tension and the upper part of the I steel beams will be in compression; in fact these areas are the most stressed ones, and a lot of attention has to be put in service conditions, in fact slab cracks can affect the durability of the bridge, as are closed to the water on the slab's surface. In any case in a continuous beam the hogging zone is quite small (20% of the span length, 10% at each part of the support) and continuous structure systems are very used nowadays because with them we can control cracking in a more efficient way.

The real advantage in this solution is the easy way to connect several parts of the beam: in fact in pre-cast concrete elements the connection can be made only with post-tensioning moreover with a lot of difficult, instead in composite structure is quite easy and can be realized both with bolted solution and with welded connections. In any case the beams are divided in segments with a maximum length so that the transport is very easy.

So the construction method involves in assembling on site the steel beam segment, the launch of the steel beam (*varo della trave di acciaio*), the installation of the precast concrete slab and then the curing of concrete.

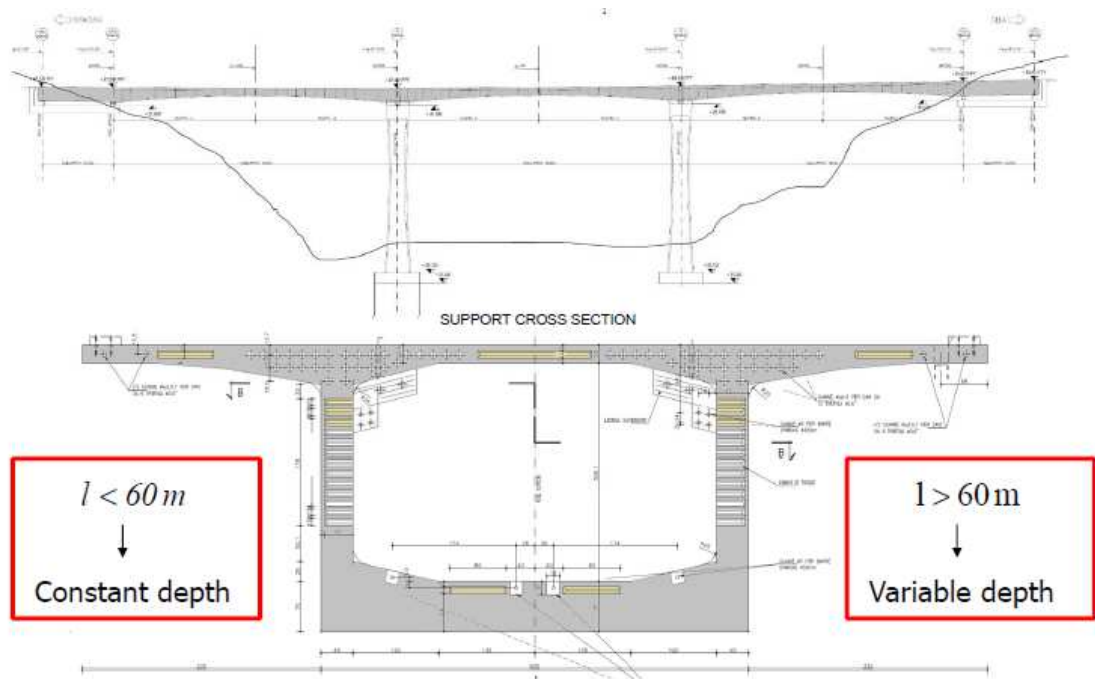
When the span length becomes greater than 40m it's better to use a solution with only two beams connected by a cross beam; this solution is used for span length of about 70m:



and the stiffeners s shown in the picture on the right low; increasing the number of the stiffeners of course we increase also the effective section but we also increase the section of welding and this can be a problem in fatigue verifications. The web longitudinal stiffener has the same function due to the buckling pretension for both normal and tangential stresses and reduces the shear lag effect; in the picture we can see a t shaped stiffener but other shapes are used and we're going to see other sections talking about orthotropic diaphragms .

- **intermediate diaphragm** is used to improve the behaviour of the box beam regarding torsion: in fact as the closed section has a great torsional inertia, the box beam will attract great torsional moment; the torsional behaviour of a thin walled cross section is quite complex: in fact if the cross section isn't circular torsion implies an out of plan deformation of the cross section; this behaviour happen for any thin walled section open or closed but actually in closed cross section it is negligible, but in close section there is also another issue related to the distortion of the cross section: an initial rectangular section becomes to be a parallelogram for example as we can see in the picture; the distortion leads to a further longitudinal stress and to a long transverse behaviour, so in order to prevent the cross section distorsion a number of intermediate diaphragms are place in the beam at a distance variable form one to 3 section's depth
- **support diaphragms** have to transform the support torsional reaction into 2 vertical forces in the bearings
- **vertical stiffeners** they avoid the local buckling of the greater reaction forces

in cross sections we have to make inspection possible, and this can be done maintain a main big hole in the bottom flange that make possible to get in the box and by means by diaphragms holes that allow to walk along the bridge inside the box section.

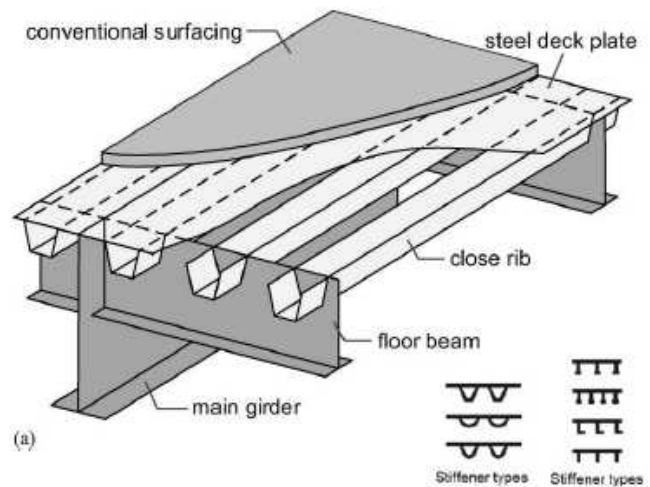


Generally the web's thicknesses are constant along the span, as well as the thickness of the top slab that is constant along the span but variable in the transverse section with a maximum in the web in order to accommodate the longitudinal tendons.

Vice versa the bottom slab thickness varies from the middle to a maximum at the support because in the hogging the bottom slab is compressed and so we need a lot of area. Generally the cantilever of the transversal section has a depth of less than 3 m.

The high torsional resistance of the box section makes it particularly suitable for curved alignments. But also this type of sections has the torsional problems we have discussed before regarding the steel closed sections but the situation is better thanks to the bending rigidity of the corner of the cross section in fact thanks to the thicknesses of the web and of the slab it is possible to transfer bending moment from one element to another one, in other words, the steel section because of small thickness of the web and flange can easily lose its section than a concrete box section. Therefore in the end in concrete box section we don't need intermediate diaphragms but we need support diaphragms that allow the transformation of torsional moment in vertical forces just like for steel box sections.

- **Orthotropic steel deck**



This is the only deck in which the traffic load is not applied to a concrete deck slab but is applied to a steel plate; to increase to plate's rigidity there are some stiffeners and longitudinal elements; in the picture we can see the two elements that are used to stiff the upper plate: ribs are longitudinal elements and transversal elements are floor beams. The longitudinal elements can have different shapes and so can be classified in two typologies: closed ribs and open ribs. Both steel plate and ribs have usually a small thickness of 10 or 12 mm; the rib's spacing in the transverse direction is usually 30 or 40 cm in case of open ribs, in case of closed ones it can be doubled.

They are also subjected to several states of stress: they transfer the point load from the point of application to the floor beams, so they work as local elements and they are subjected to local stresses, but in addition they work with the main girder to transfer the load to the support working in the main structural system, for instance like a continuous girder and so there is an additional stress due to the global behaviour. Actually the behaviour of these elements (ribs and steel plate) is very complex because they don't have a bending behaviour only but have also a plate behaviour, so there are also axial forces due to the second order effects. As a result in this kind of bridges we don't have problems of strength but problems of deformation and fatigue at webbing.

This solution becomes an economical alternative only when it is important to reduce the mass or to have a thinner section and a rapid bridge installation.

- **Column pier (hammerhead pier)**

This type of piers are often found in urban areas, where space limitation is a concern. They are used to support steel girders or precast prestressed concrete superstructures; they generally occupy less space while they provide more room for traffic under them; the pier cup works like a cantilever and generally have a variable shape as it is subjected to a negative bending moment and in some cases it may be prestressed with straight tendons in the upper part of the cup in order to work to absorb the negative bending moment.

The columns cross sections are usually circular or rectangular even if another section with a transversal resistance greater than the longitudinal one is the best solution as the column is very stressed in the transversal direction because of the eccentricity of live loads.



Now let's move on to column bent piers, they consist in an upper beam and a pier system forming a frame; they can be plane or 3D.

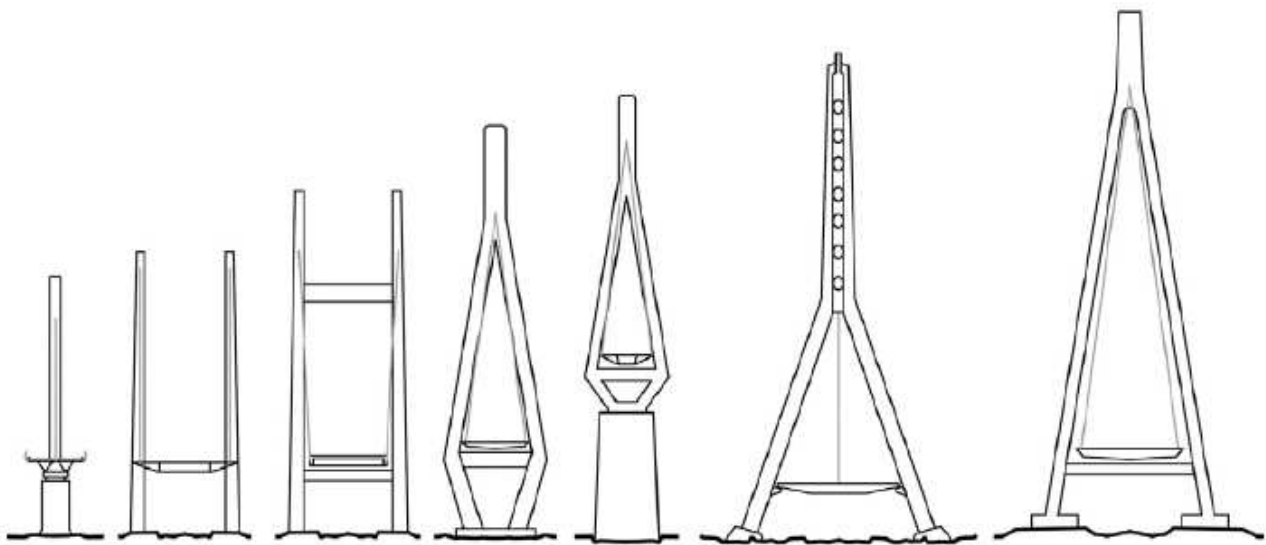


→ **special piers**

we talk about piers of stay-cable bridges or suspended bridges, they are usually called towers or pylon (generally pylon is used for stay-cable bridges and for suspended we use the term tower).

Tower is the most visible element of long span bridges, so their appearance must be important in the design of this kind of bridges. they can be defined as the vertical steel or concrete structures supporting cables and carrying the forces to which the bridge is subjected to the ground.

Towers of cable stayed bridges can have a wide choice of form and height as we can see below:



Types of towers for cable stayed bridges.

The height of the stay cable tower can be assumed to be about 30% of the main span length, to this value we have to add the structural depth of the girder and the clearance to the foundations in order to determine the total tower height.

The simplest tower form has a single vertical shaft and it is the first one on the left.

An interesting option from the static point of view is the I framed tower in which the two planes of the stay-cable are inclined outwards producing a compression component across the deck support system.

The form of towers of cables stay bridges below the road way is also important for both aesthetic and costs: if the shaft of the towers forms an inverted Y frame, the foundation may become very wide and costly.

So sometimes the lower shaft are inclined under the road way producing a modified diamond.

**Fixed type (F)**



**Guided sliding type (UL/UT)**



**Free sliding type (M)**



**Reinforced elastomeric bearings**



Pot bearings can be

→ **fixed type**

it constrain all horizontal movements through a contact between the upper plate and the pot

→ **guided sliding type**

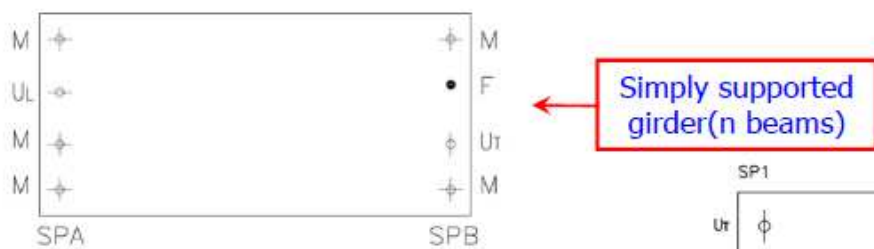
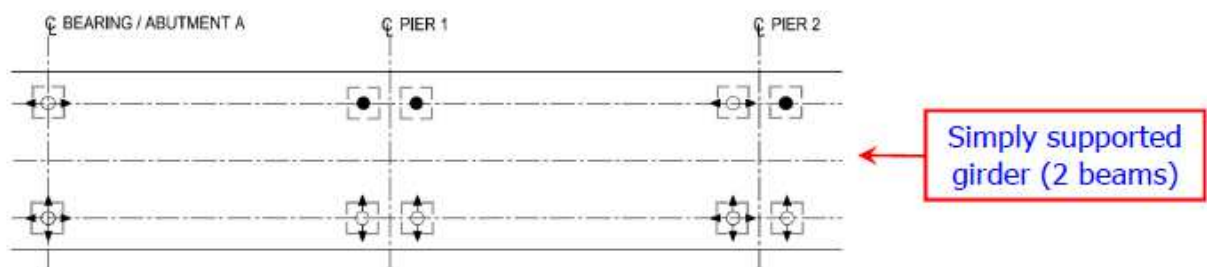
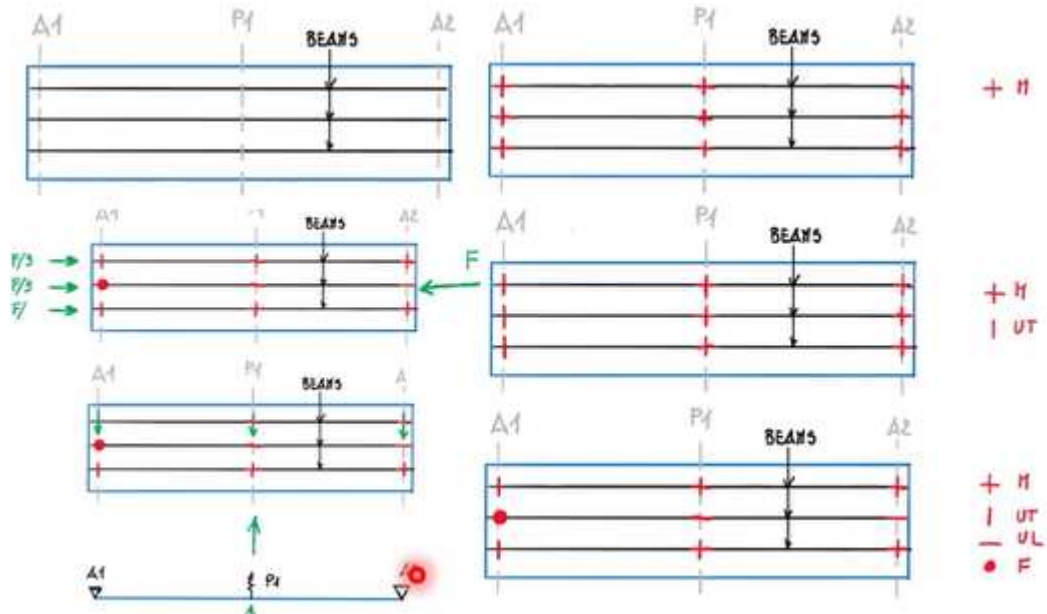
it constraints only one horizontal movement through contact between the upper plate and a contact with a guided placed in the pot; the upper plate is shaped to realize a male/female joint. Depending by the direction of the guide we can allow longitudinal movement or transversal one.

→ **free sliding type**

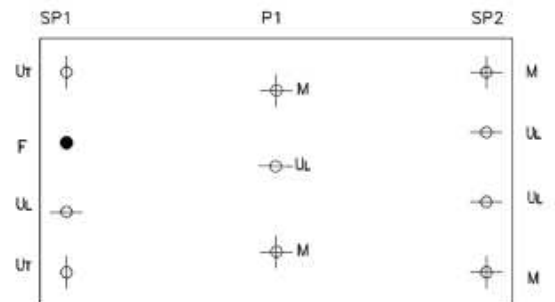
- **reinforced elastomeric bearings**

it is realized with an elastomeric core in which one or more thin steel plates are insert in the hot vulcanized rubber and that is that the steel plate in putted in the rubber by means of hit (???) steel plate give vertifacl stiffening.

With the chosen bearing articulation the longitudinal forces are divided in equal parts among the three bearings and the abutment one, whereas the transversal force is not divided in equal parts.



Continues slab



#### **4. Dead load**

In buildings typically span range from few meters to 10 to 15 meters and as the self-weight is proportional to the span length and so it is small, in bridges the span can vary from about 30 m to 100 or 150 m and this means having a big self-weight.

#### **5. Live load**

Live loads in buildings are small, of about 2 or 6  $\left[\frac{kN}{m^2}\right]$  whereas in bridges they reach a lot of tons.

All these reasons make the bridge building a very difficult exercise, overtime designers and construction companies have developed new construction methods in order to reduce construction cost and time.

Today very often modern construction system require equipment that can cost more than a single bridge and that therefore to be economically sustainable require construction of a large number of structures.

Finally the economy and the speed of construction are not always synonymous of durability: the construction companies tend always to look at the cost aspect but the bridge owner and designer must also consider the durability of the structure, because the total cost of the structure is given by the sum of the construction cost and maintenance cost.

Obviously therefore the designer must indicate in addition to the material to be used, to the dimension of the structural element, the construction details, also the construction system to be use and the necessary equipment to do this.

- **I beams (precast concrete or steel)**
  - **Erection from the ground with cranes**

This construction method is suitable for launching single concrete I beams or steel ones; steel beams are more sensitive to torsion and therefore they are coupled to prevent lateral torsional buckling; of course this is made possible by the relative lightness of the steel beams which permit to lift and launch two beams together. Concrete I beams have an inertia so that lateral buckling is not a problem and so it is possible to lift only one of it per time. this can be done placing one or two cranes on the ground, but no more than two

- **Launching girder**

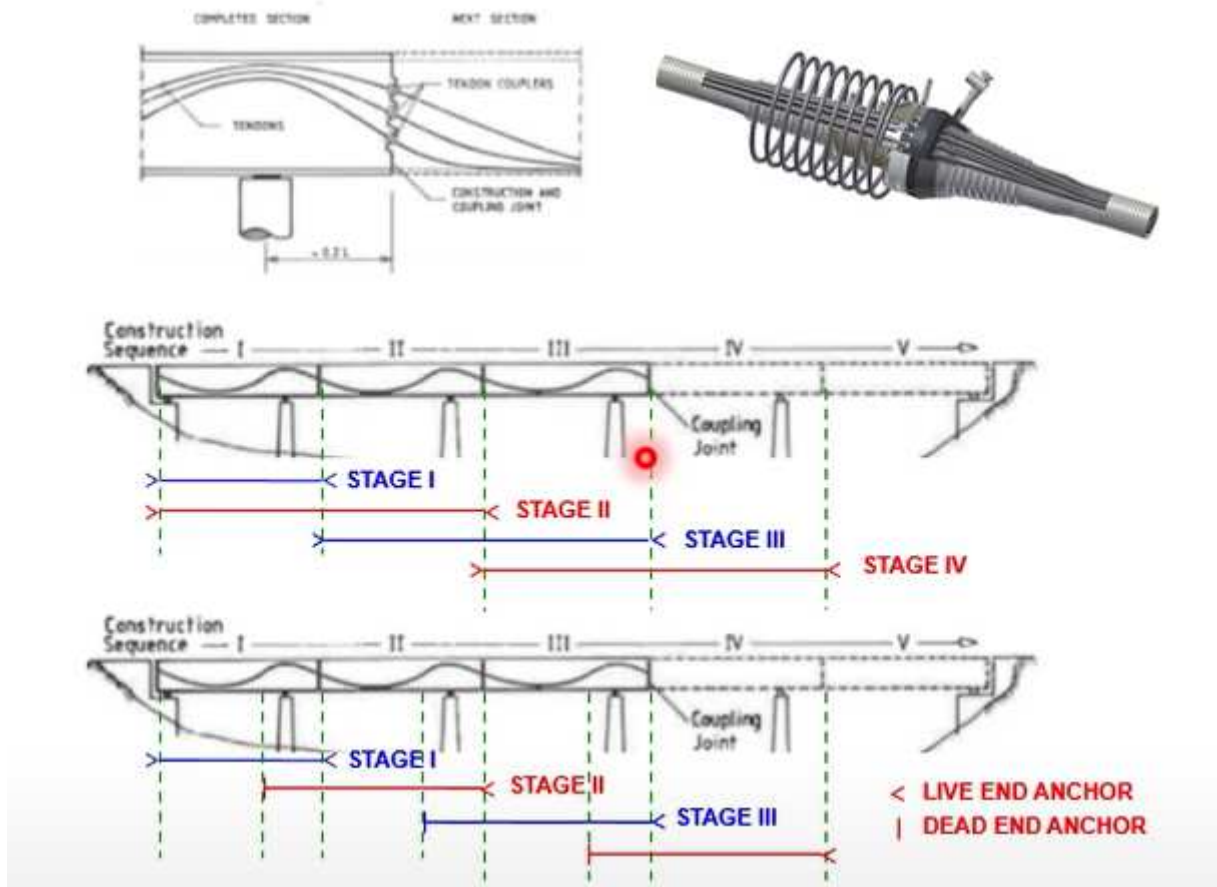
<https://www.youtube.com/watch?v=CCguT7FrsP4>

when the height of the piers becomes too large to be able to use cranes in an efficient way, we can use a launching girder;

in the first part of the video we can see the transport of single beams elements in a dimension to be easily transported on tracks of therefore about 12 m; then they are assembled on side by bolting in case of steel beams or with post-tensioning if they are realized in concrete. The first spans are low and therefore the individual beams can be launched by cranes as we have seen before; when the height of the piers doesn't allow the use of cranes we move on the launching girder system. First of all we can see the assembly of the launching girder that has a length of about two times the span length so that it can move cantilever up the next pier without overturning. Each single beam is carried by the launching girder and placed starting from one side of the deck and ending at the opposite site of the deck; the supply of the beams is made in part of the bridge that is already built. As usual, once beams are placed, the lattice slabs are placed on the girder and the slab is cast in place.

- **Moving scaffolding (cast-in-place of a complete span)**

This construction method can be used for construction of cast in place concrete slabs. With this system we build one span at a time using scaffolding that are placed at the spans under construction and then moved to the next span after the hardening of concrete. The formworks can be supported by fixed falsework or mobile falsework; the first one have to be assembled and disassembled every time whereas the mobile one slides from one span to the next one but in both cases we have to pay attention to the falsework safety as the falsework carry the weight of fresh concrete which can be very high. If we realize temporary foundation to carry the falsework we have to check not only the bearing capacity but also the displacement of the foundation because the formwork will have the same displacement of the ground and then the fresh concrete will have the same deformation of the falsework.



We have a bridge with five spans and it is realized with five castings and of course I have also a lot of stages of prestressing: the first stage I which I pre-stress the first span; in the second stage in order to avoid to have all tendons coupled in this section I use a set of pre-stressed tension that go from the start section of the bridge till the second joint construction, so in the first construction joint I have only the 50% of total tendons that are coupled.

Another option is to overlap the tendons placing that anchor in the portion already built and tensioning only to the other hand.

<https://www.youtube.com/watch?v=sA2vWMOc5Sw>

of course falsework can be supported in several ways as we can see in this video; in particular the horizontal structure that supports the formworks change depending by the distance between the vertical supports of the formworks. With the increasing of the distance we have also to increase the temporary foundation.



- **Free cantilevering of cast-in-place-segments**

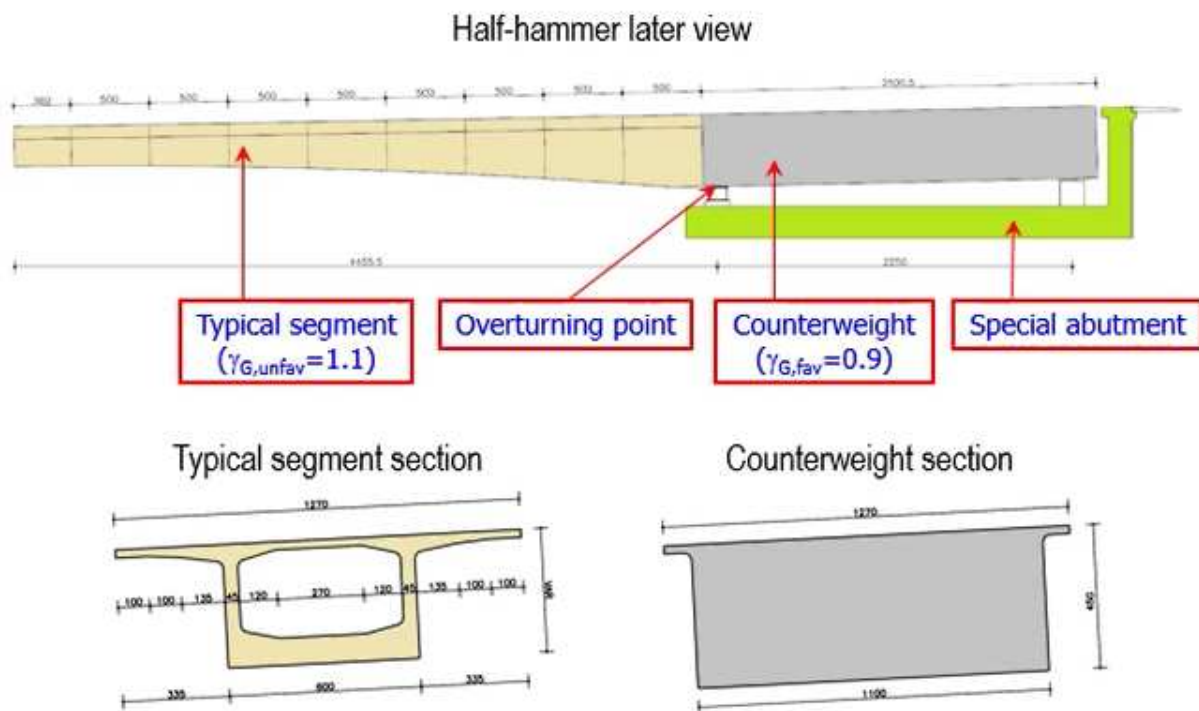
Even this system is used for cast in place structures; but this time the single span is not built with a single concrete but with a multiple concreting. Actually we don't cast a span an hammer consisting of two half spans starting from one pier. The two half spans are realized with segments and that is with slices of bridge with a length of about four or five meters. In this way the equipment built during construction don't have to carry the total weight of the span but only the weight of a single segment. This method is mainly used for bridges of spanning deep valleys.

If we have to build a lot of bridges in the same motorway for example it is better to use another type of building construction system because free cantilevering is a good method for only one bridge to build for speed reasons and also because this method requires expensive equipment that can be adsorbed in cost only if we build more bridges at the same time.

An optimal span suitable for free cantilevering is in the range from 60 m to 150 m even if bridges with greater length have been built in this way.

this because we always cast the left segment before the right one. The hammer overturning is avoided thanks to a temporary connection between the hammer and the pier realized with temporary tendons as we are going to see in the next picture.

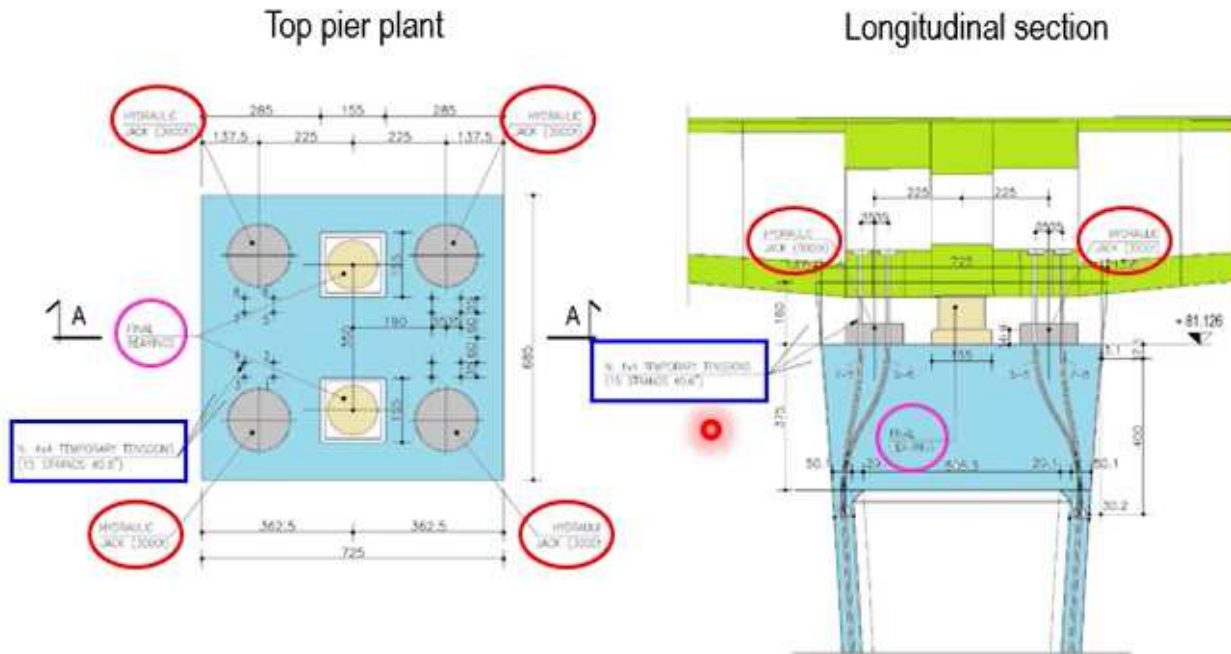
In the case of half hammer the construction is not self-balanced until we realize a counterweight and that is a large block of concrete which prevent overturning with its mass, so the counterweight is designed in order to prevent the overturning of the half hammer. The overturning point is the axis of the bearing on the left. The figure shown an half hammer and to avoid the overturning the box cross section is transformed in a full concrete cross section. this is a typical situation in which we have to perform an USL verification considering two different safety factor for the self-weight: for the counterweight we have to use the favourable coefficient and for the cantilever part we have to use th unfavourable one.



In any case, not every situation need of counterweight of the abutment; we need of counterweight if we want spans of equal length, but if the first or last span is equal to an half of the intermediate spans then we don't need of counterweight as we can see in the following picture:

From <http://en.vsl.cz/bridges/>



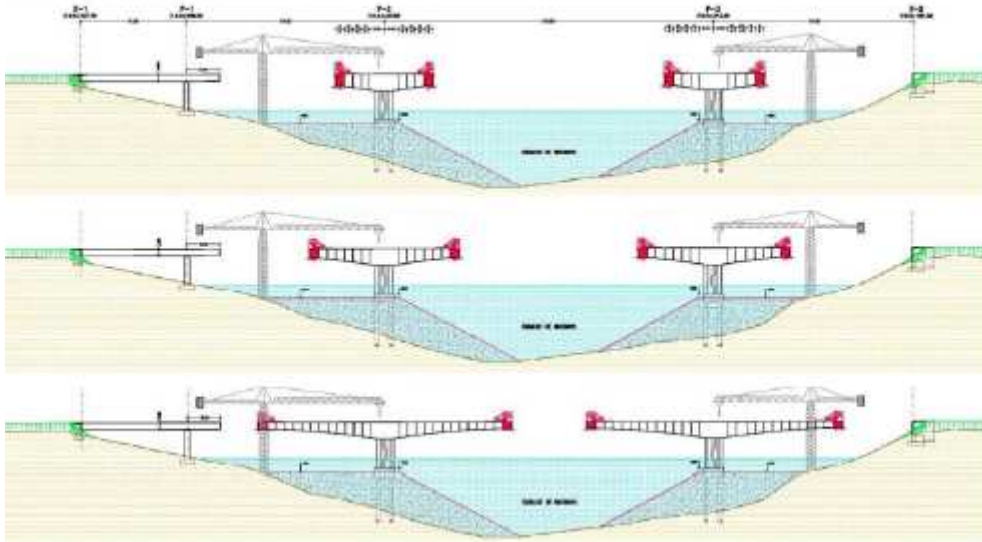


Finally we have prestressing tendons for each corner, totally we have sixteen tendons of fifteen strand each that press the deck and the pier in order to take the tensile force given by the overturning moment of the hammer; we can have this because for instance we can build a segment on the left immediately before to cast the other one on the right side, and so temporary we can have more weight on the left side and have an overturning, so we have to press the deck and the pier.

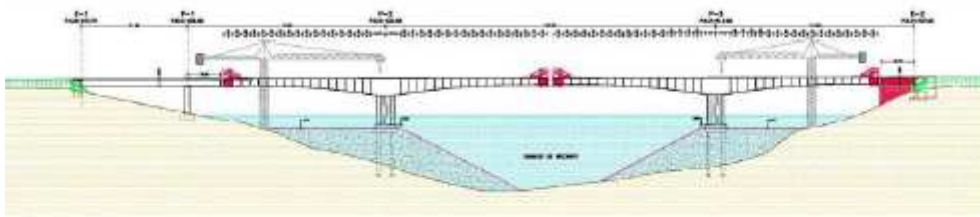
Now we will see the construction stages of a real bridge, and we want to point out in this case that the first span on the left and a portion of the second span has been realized by means of a scaffolding and so we start with the construction of the cantilever.

We don't have to think at this construction method in a rigid way but we have to look at it with a sense that we can mix it up in several ways in order to achieve the goal, that in this case is to build the bridge in the most simple and speed way.

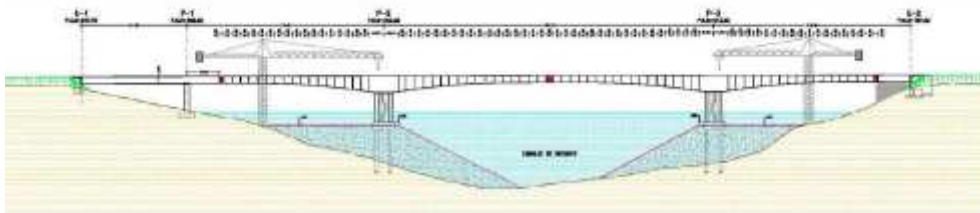
### 5) Concreting of following segments



### 6) Hammers completion



### 7) Concreting of key segments

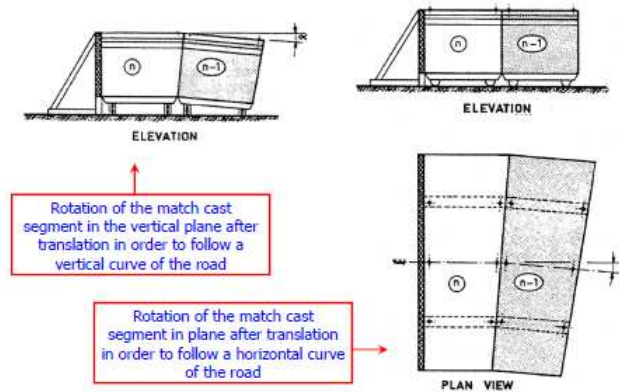


- Construction with precast segments

The most important disadvantages we find with the free cantilever cast in place method is the timing. In fact, after the casting of a segment we have to wait more or less at least three days to wait the hardening of it and connect it with the part of the bridge that we have already realized. Moreover when we apply loads to a young concrete the creep effect is greater and may give some problems in pre-stress losses.

To avoid these problems we can use construction with precast segments; the use of precast segments has the advantages that the superstructure can be erected in a faster

Usually a bridge is not straight because very often we have curved layout in the horizontal or in the vertical plane so when we build a segment we have to follow exactly the road layout, so after the casting of the previous segment we have to modify its inclination in the vertical or horizontal plane in order to follow the curve; so we have a rotation of the match cast segments.



The other construction method is the so called long-line method; in this method all segments of a span are manufactured on a fixed casting bed (or soffit) with the formwork moving along the bed for successive casting operation. Geometry control is established by adjusting the side forms and the soffit. Variable depth structures may be cast by varying the elevation of the soffit.

In the picture we can see the typical construction phases of a span: the span  $n$  is the span between the pier  $i$  and the pier  $i+1$ ; in our example the span is realized using 11 segments plus the pier segments that are not included in the example. The construction starts with casting of the segment  $s_1$  of the span  $n$  at day 1, at day 2 we cast the segment 2 but using segment 2 as match segment and at day 3 we cast the segment 3 using the segment 2 as casting segment in the same way we already said for the short-line method. Therefore we can move the segment 1 has have been three days since the casting and the concrete has more or less the 50% of his characteristic strength and can carry its self-weight; in the same day we cast the segment  $s_4$  as usual for the segment 3 as match cast segment. We can jump at day six when we move the segment three at the storage cast the segment six and in addition we start to cast the segment 1 of the span  $n+1$ ; in this way from now on we build two segments per day, so we build two span in 11 days.

- ✓ We have low maintenance cost due to improved quality
- ✓ Speed of construction as in site we can assemble the bridge in a very speed way

But of course we have also disadvantages because

- ✗ High construction loading or high technology is uses
- ✗ Need high safety precautions during construction
- ✗ Extra costs (due to more prestressing required)

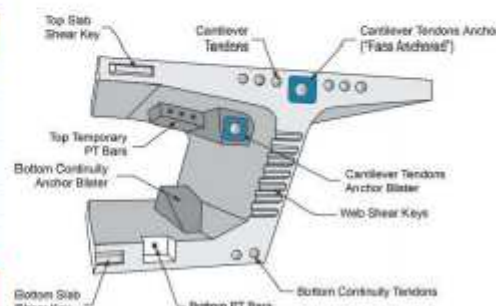
• **Balanced cantilever erection with cranes**

From a conceptual point of view this method looks like the free cantilever method with the difference that we use precast segments. Segments can be erected by cranes or by launching; the first one of course is used when piers aren't so tall and when segments can easily be erected in the vertical direction: <https://www.youtube.com/watch?v=If7MS7ygzcY>. The video starts with a track carrying the pier segment and the segment is erected by a crane, but as we can see the pier segment has a diaphragm due to the greater reaction forces during service and we can see some adjustment jacks to take the deck in the correct position and there are also temporary tendons to connect the deck and the pier. Every segment is temporary connected to the previous one by means of prestressing bars that in this video are called dywidag bars. From now the construction follows the phases of the free cantilever method and we can see that tendons are anchored in special pockets in the top slab that are grouted in place and this solution is dangerous as the anchorage is in the upper surface is near the water; there are other solutions that are more safety. The key element is the last element and it has to be casted in place. After the casting of the key tendons we can pre-stress the bottom tendons as now the structure will have also positive bending moment.



From <http://en.vsl.cz/bridges/>

Typical Erection Cycle Description	Duration in Shifts							
	1	2	3	4	5	6	7	8
Installation of Pier Segment Support Brackets								
Installation of Pier Segment								
Segment Erection - Pair 1-3								
Segment Erection - Pair 4-6								
Segment Erection - Pair 7-9								
Segment Erection - Pair 10-13								



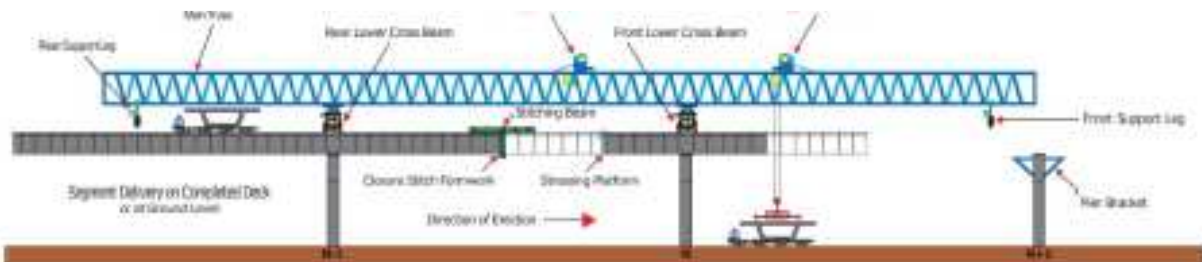
Then the segment is lifted up under the portion of the deck already built and moved in the longitudinal direction at the end of the hammer and then it is connected to the hummer.

With respect to the previous technique the construction time increases.

• **Balanced cantilever erection with launching gantry**

When the pier is quite tall or it is difficult to carry segments at the base of the pier, we use launching gantry.

In the first picture we can see two ways to use this way of construction:



From <http://en.vsl.cz/bridges/>



Typical Erection Cycle	Duration in Shifts					
	1	2	3	4	5	6
Span N-1						
Closing at Slab (Overnight)						
Continuity P.T.						
Span N						
Launch Gantry to Span N						
Segment Erection Span N						
Ditch N to N-1						
Span N+1						
Erect Pier Segment						
Align Pier Segment						
Place Reinforcement						
Place Formwork						
Cast In situ Diaphragm						
Gantry Piers/Culverts Jctd						

As we can see on the right the segment is uplift from the ground, but as we see on the left, the segment is taken from the part of the girder already built and this is the most common way to use this construction system.

A critical aspect of this method is the safety verification of the gantry from wind loads; in fact overturning of the gantry is a more common situation than you can think.

- **Full span precast method**

This is a fast construction method and it is very simple from the theoretical point of view as we can precast a whole span, but the problem becomes the launching of the precast beam and so how to transport the whole span and how to erect it.

Talking about timing we can erect a span per day and so this is the faster construction method until now.

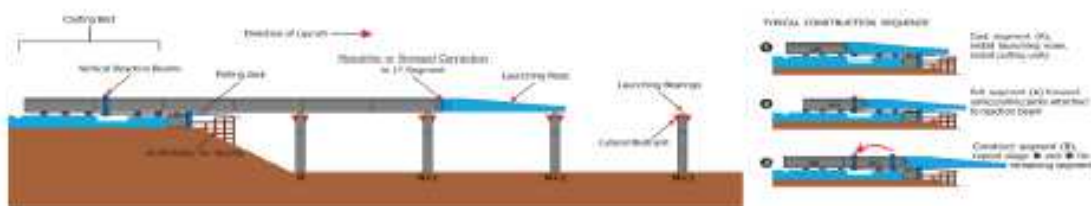
<https://www.youtube.com/watch?v=zvWAwizxU9o>

the girder must be able to carry itself, the erection machine weight and the weight of another span; in order to launch the new span, the machine has to reach the next span cantilevering and the overturning is avoided with the launched span.

Also in this method it is easy to realize simply supported girders whereas it is difficult to realize continuous girders.

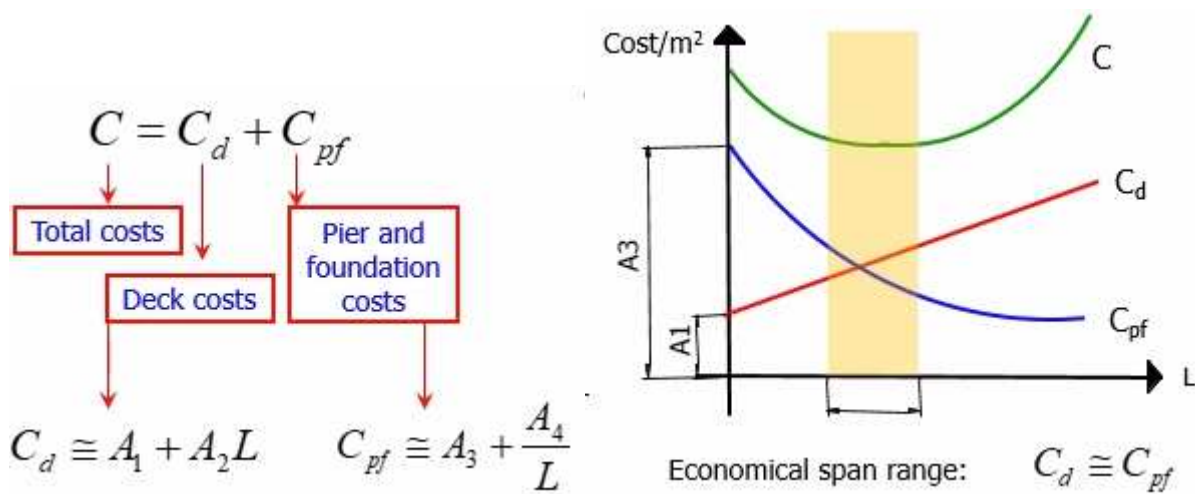
- **Incremental launching construction**

The principle of the incrementally launched bridge consists of building the superstructure segments in a casting yard located behind the bridge abutment. Each segment is matchcast against the previous one and prestressed to the section of superstructure already built. The entire superstructure is then jacked forward a distance equal to the length of this segment. This process is repeated until the bridge is in its final position. This form of construction can be used for bridges having constant cross sectional shape throughout their length. The bridge should be straight or have a constant horizontal and vertical curvature.



From <http://en.vsl.cz/bridges/>

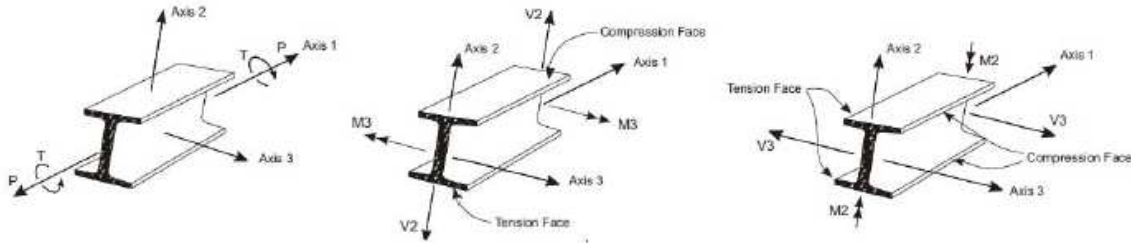
<https://www.youtube.com/watch?v=S3Kf9e6JgF4>



The total cost has a minimum at more or less the intersection of the red line and the blue curve, that is we have the minimum cost when superstructure's cost is equal to the substructure cost. So the designer should try to find the span following these parameters.

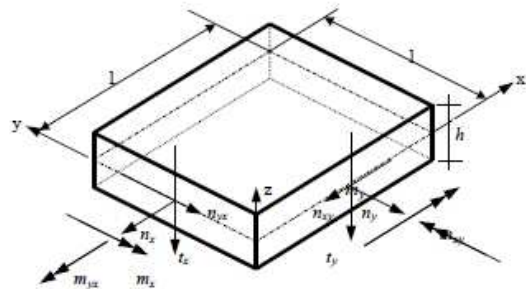
3. This is an aesthetic parameter because the span length should be greater than the height of the pier.

➔ **Beam analysis: 6 internal actions (axial force, 2 bending moments, 2 shear forces, torsional moment)**



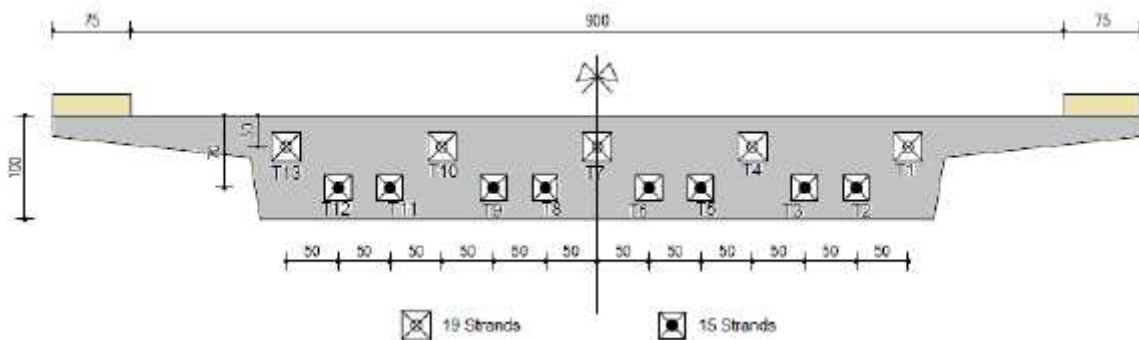
➔ **Slab analysis: 8 internal actions (3 membrane forces  $n_x - n_y - n_{xy}$ , 3 plate moments  $m_x - m_y - m_{xy}$ , 2 out of plane shear forces  $t_x - t_y$ )**

- ⊙ The internal forces and moments are per unit length
- ⊙ For regular slab shape and if  $L/B > 2$  then  $m_y - t_y - n_y - n_{xy}$  are little and beam analysis may be performed (assuming x as longitudinal direction). In other words, principal direction are more or less parallel to the edges
- ⊙ In addition, if the slab is not longitudinally prestressed, also  $n_x$  may be neglected



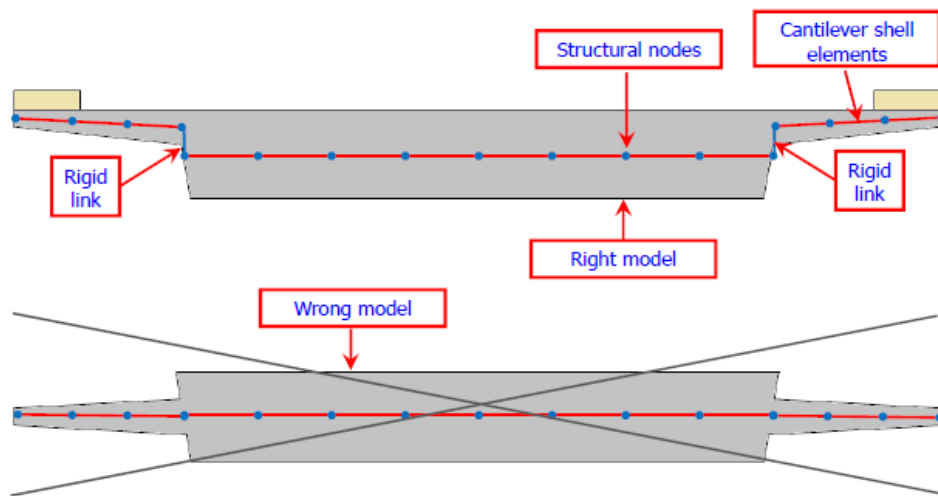
If the span length is less 15 m then we can use a reinforced concrete solution without prestressing while we should use small distance between reinforcement bars to control crack opening. Generally we're going to use bar diameters quite big ( from 20 to 26 mm) and in order to control crack opening we have to put bars at a small distance.

And if span length is bigger or equal to 15 m, we should use prestressing reinforced concrete solution. In this case is better to use small tendons at a small distance that helps to reduce the diffusion region. But for small widths the transversal prestressing is not necessary, but id the width becomes bigger than 12 m it is useful to use it in order to avoid the longitudinal cracks due to concentrated loads; but it is quite difficult from a construction point of view to insert transversal tendons in a slab bridge because we have to perform a lot of prestressing stages.





➔ In the finite element model with shell element, we have to consider the transversal cantilevers in their actual position



So if I want to use a finite element model with shell elements, in order to evaluate the internal actions in the structure, I have to consider the actual geometry of the deck: considering this deck shown, we have two transversal cantilevers in order to reduce the weight of the deck and I have to consider the shell element that models the cantilever in the middle of the cantilever and the shell element that model the central part of the deck in the middle of this part and I have to connect these two part of the deck with rigid links.

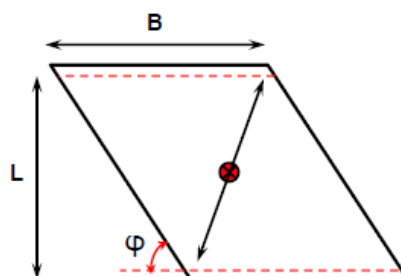
If we put all the element in the same plane, we model another structure that is not our structure.

Shear verification generally is not a problem because shear verification is usually satisfied without shear reinforcement as we have a big depth.

In any case we have to provide the minimum shear reinforcement especially at the supports where we have great forces that have to be carried.

### SKEW SLABS

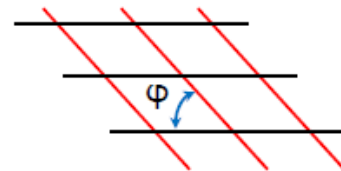
These slabs have the two edges not perpendicular each other and in particular we talk about slabs with a skewness angle  $\varphi$ . This angle represents the irregularity of our slab.



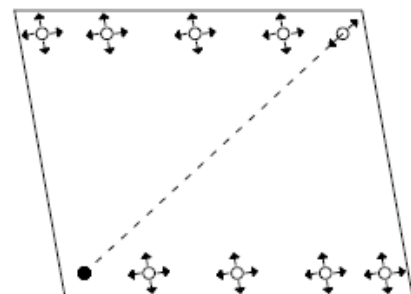
If we consider rigid bearings that have zero displacements, we can have the reactions that are written. The reaction on the main left is much bigger with respect to the one on the right and also reactions in the middle have strange values. If we want to solve this problem and reduce the big reaction on the left avoiding the uplift of the slab, we can reduce the number of bearings so that the distribution of reaction forces will be more standard, but if bearings are at a greater length we'll need more reinforcement; another way is consider elastic bearings like in the second analysis with high rigidity.

➔ With  $\varphi \geq 60^\circ$  longitudinal and transverse reinforcement generally parallel to edges and that is they are not perpendicular to each other. This may give calculation problems. Place minimum shear reinforcement.

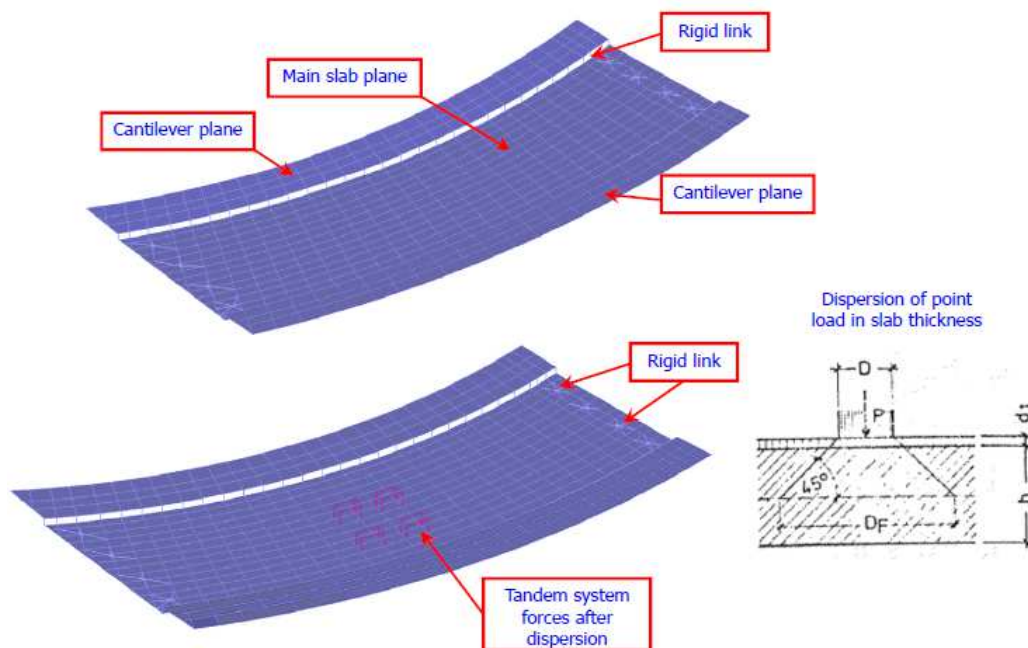
➔ With  $\varphi < 60^\circ$  the two reinforcement layers parallel to the edge would be too skew. Then we use a orthogonal reinforcement mesh in longitudinal direction and in the perpendicular one. Along the free edge provide additional reinforcement parallel to edges



➔ A possible bearing articulation is shown in the figure, with the fixed bearing at the obtuse angle, a unidirectional bearing at the other obtuse angle with guide place along the line between the two obtuse angles and mobile bearings for the other ones.



We can see the analytical model used to evaluate internal actions in which the cantilever plane is not coincident with the main slab plane as we discussed before and we can see rigid links used both to link the main deck and the cantilevers and also the other in the transverse direction that represents the bearing used to reduce a great stress concentration at the point where we insert the restraint; in the reality in fact we don't have a point load but a distribution of the reaction on an area and that is why we use these rigid links.



The transverse beams distribution depends on many factors and the main ones are:

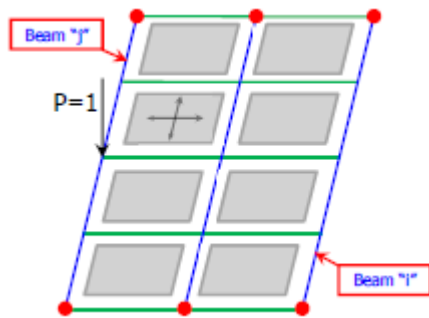
- beams torsional stiffness;
- cross beams bending stiffness;
- slab characteristic in transverse direction.

of course we have to generalize the problem and we have to consider the point load  $P$  applied in every zone of the slab and not only on one beam.

The main girders (beams and cross beams) may be realized both in concrete or steel.

**Concrete longitudinal girders** can be cast in place or precast, this can be done using or not prestress and using I or V cross section; whereas concrete transverse girders are usually cast in place and connected to the longitudinal beams by means of mild or prestressing steel, generally if the deck is completely realized with a cast in place solution beam and cross beams are connected by mild steel (that is non prestressed steel) whereas if the longitudinal beams are precast or prestressed the connection will be surely realized using prestressing. Cross beams can be connected to the slab or not, surely they will be connected to the longitudinal beams but the slab can not be connected to cross beams and this depends by many factors but generally speaking connecting the slab and the cross beams is more difficult when we use prestressed beams. In any case the connection between longitudinal and transversal girders is quite difficult and so the difficulty of construction and connection suggests using few cross beams: certainly at supports one or two along the span; the use of few cross beams implies that the slab is very involved in the transverse load distribution because as we said before, the transverse load distribution depend on the torsional stiffness of the beams, on the bending stiffness of the beams and on the slabs characteristics in the transverse direction, so if we use a few cross beams then the slab is very stressed in transverse direction.

Instead talking about **composite solutions**, steel longitudinal girders can be shaped with I or box cross sections. The different segments of longitudinal girders can be connected using bolted or welded splices. Transverse girders can be realized with I cross section members or with a truss system; once again the transversal girders can be connected with the slab or not and the simplicity of the construction and connection suggest to use many transverse girder this time and the slab is not so much stressed in the transverse direction. The top slab is always realized with cast in place solution on lattice precast slab but it can also be cast on classic formworks that we reuse after the removal of the formwork.



It should result:  

$$\sum_{i=1}^n \rho_{ij} = 1$$
 (vertical equilibrium condition)  
 For  $P \neq 1$  the principle of superposition may be used and then  

$$P_i = \rho_{ij} P$$

I want to know the coefficient  $\rho_{ij}$  and so the percentage of the force  $P$  that is carried by the beam “i” so another beam respect to the beam “j” on which the load is applied.

Of course if the applied load  $P$  is equal to 1, the sum of the transverse distribution coefficients will be 1 as the load for the vertical equilibrium condition to be respected. Instead for a general case in which the value of the force  $P$  is different form one, we may use the superposition principle and then we can say that the force  $P$  carried by the beam “i” will be equal to the transversal repartition coefficient multiplied by the value of the load  $P$ .

In other words, in order to determine the most unfavourable position of live loads for each structural element and for each internal action we have to know the influence line of an internal action, of a reaction, of a settlement for our structural element.

The transversal distribution coefficients depend on the beams torsional stiffness  $\Upsilon_p$  and the cross beam bending stiffness  $\rho_e$ .

Given that slab contribution is considered within the cross beam, we can analyse some liti situations:

- Cross beam without bending stiffness or connected by means of hinges to the beams**

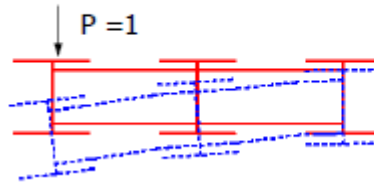
$\rho_{11} = 1$   
 $\rho_{12} = \rho_{13} = 0$

In this case if the cross beam doesn't have bending stiffness or is connected to the beams by means of hinges, if we apply a force  $P=1$  on the beam 1, this load is carried only by beam 1 and we don't have transversal distribution so the beam 2, 3 anf thrro others are generally unloaded; so in this case the transversal repartition coefficients

- Cross beam with finite bending stiffness and beams with finite torsional stiffness

$$\gamma_p \neq 0$$

$$\rho_E \neq 0$$



This is the actual situation for a normal bridge because it has always a finite stiffness both for bending and for torsion.

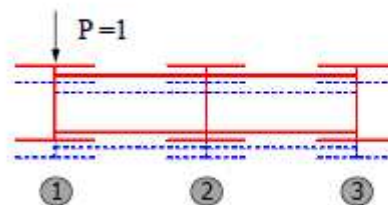
- Cross beam with infinite bending stiffness and beams with infinite torsional stiffness

$$\gamma_p = \infty$$

$$\rho_p = \infty$$

$$\rho_{1,1} = \rho_{1,2} = \rho_{1,3} = 1/3$$

(uniform distribution)



Here we also have a rigid body motion but without a rotation. In the starting hypothesis where we had cross beams without bending stiffness and the load that we applied on beam one was carried only by beam one, this one is the opposite situation in which a unit force applied on beam one is carried equally by the different longitudinal beams.

Actually the real solution is in between these two limit situations.

The main analysis procedure that we can think to use to find the internal reaction, displacement and so on of our structural members are three:

1. **Courbon method:** for its simplicity it was widely used in the past;
2. **Massonet method:** is a general method that transforms girder in an orthotropic slabs (a slab that has two behaviour in the two direction perpendicular to each other) and gives the solution in tabular form. It is quite used in the past.
3. **Finite element method**

We're going to see the Courbon method and the finite element method. In fact, the first one may be used for preliminary design, but also for detailed design (if the hypotheses are true the solution has acceptable approximation). The second one is a general method that can be used in all case.

Global displacement of transverse beam  $\Rightarrow$  Rotation around G( $\varphi$ ) + displacement ( $\delta$ )

$$\delta_i = \delta + \varphi \cdot d_i$$

Force in the general spring:  $P_i = K_i \delta_i = K_i (\delta + \varphi \cdot d_i)$

Equilibrium in vertical direction:  $\sum_{i=1}^n P_i = 1 \Rightarrow \sum_{i=1}^n (K_i \cdot \delta + K_i \cdot \varphi \cdot d_i) = \sum_{i=1}^n K_i \cdot \delta + \varphi \sum_{i=1}^n K_i \cdot d_i = 1$

$$\delta = \frac{1}{\sum_{i=1}^n K_i}$$

$= 0$  because G is the centroid of rigidities

Equilibrium to rotation:  $\sum_{i=1}^n P_i \cdot d_i = 1 \cdot e = \sum_{i=1}^n (K_i \cdot \delta + K_i \cdot \varphi \cdot d_i) d_i = \delta \sum_{i=1}^n K_i \cdot d_i + \varphi \sum_{i=1}^n K_i \cdot d_i^2$

$$\varphi = \frac{e}{\sum_{i=1}^n K_i \cdot d_i^2}$$

Finally  $P_i = \rho_{i,e} = \frac{K_i}{\sum_{i=1}^n K_i} + \frac{K_i e d_i}{\sum_{i=1}^n K_i d_i^2}$       " $e$ " and " $d_i$ " may be  $>0$  or  $<0$

Percentage of load  $P = 1$  with eccentricity " $e$ " acting on beam " $i$ "

+ for " $e$ " and " $d_i$ " with the same sign  
- for " $e$ " and " $d_i$ " with different sign

If the beams are identical and identically restrained, it results:

$$K_i = K = \text{cost} \quad \text{then} \quad \rho_{i,e} = \frac{K}{nK} + \frac{K e d_i}{K \sum_{i=1}^n d_i^2} = \frac{1}{n} + \frac{e d_i}{\sum_{i=1}^n d_i^2} \quad \Leftarrow \text{Courbon}$$

For a given beam ( $d_i = \text{cost}$ ) the influence line of  $\rho_{i,e}$  with varying " $e$ " can be drawn




Load carried by beam " $i$ " for a variable position of acting load  $P=1$

So this is the influence line of the load carried by beam 1 for any position of the unit force along the deck.

For the Betti Maxwell theorem we have that  $P_{1,2} = P_{2,1}$  so I can change the index and this means that this picture also say me that when I have a force on beam one, the 70% of it is carried by beam one, the 40% is carried by beam two, the 10% by beam three and -20% by beam four.

We can perform the same thing for beams two and three, always equally for the symmetry of the deck again.

Beams 2 e 3 
$$\rho_{2,e} = \frac{1}{4} + \frac{0.5 * e}{2(1.5^2 + 0.5^2)} = 0.25 + 0.1e$$

$$\left. \begin{aligned} \rho_{2,1} &= 0.25 + 0.1 * 1.5 = 0.40 \\ \rho_{2,2} &= 0.25 + 0.1 * 0.5 = 0.30 \\ \rho_{2,3} &= 0.25 - 0.1 * 0.5 = 0.20 \\ \rho_{2,4} &= 0.25 - 0.1 * 1.5 = 0.10 \end{aligned} \right\} \sum = 1 \xrightarrow[\text{Theorem}]{\text{Betti Maxwell}}$$


Within the Courbon formula it can be introduced  $e = e' b_0$  and  $d_i = d'_i b_0$  then:

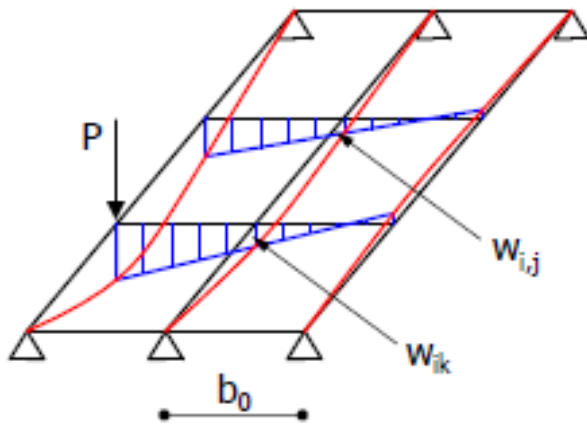
$$\rho_{i,e} = \frac{1}{n} + \frac{e' b_0 d'_i b_0}{\sum_{i=1}^n d_i'^2 b_0^2} = \frac{1}{n} + \frac{e' d'_i}{\underbrace{\sum_{i=1}^n d_i'^2}_{\text{independent by } b_0}}$$

Of course if I apply the forces in the middle of the deck, due to the infinite bending stiffness of the cross beams, all the longitudinal beams would have the same foces and that is the 25% of the applied forces.

I can also simplify the previous equation multiplying the distance between beams for another value  $e'$  and replacing in the first equation; doing the same for  $d_i$ , I will find an equation that will be independent by  $b_0$



➔ Full deck behavior: as the loaded transverse beam is linear, the unloaded one is linear too → the transverse beam "j" follows the deformation of the "k" one, remaining linear.



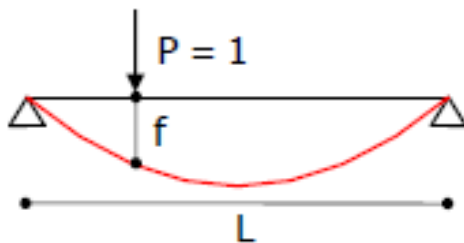
For a general beam "i":

$$\frac{W_{ij}}{W_{ik}} = \text{cost. } \forall i$$

That is if I apply a load on the second cross beam as in the picture upside, I have a certain transversal deformation of deck, and the same I have on the other cross beam.

➔ Applicability of the method

- The spring stiffness depends on the transverse beam position



$$f = c \frac{1 \cdot L^3}{EI_{fb}} \rightarrow K = \frac{EI_{fb}}{c \cdot L^3}$$

The spring stiffness that I have to consider in the evaluation of transverse distribution coefficients depends on transverse beam position; the two hinges that I can see in the picture upside are the longitudinal beams, so we are assuming that the bridge is a simply supported girder and that is the longitudinal beams are simply supported.

When I load the beam in a generic position with a unit load, I have a displacement  $f$  in the point of application of the force; the opposite of this equation is the stiffness  $K$ .

We call "a" the distance between the support and the point of application of the load.

If I consider the midspan, the value  $a$  is equal to half of  $L$  and this means that we will have the smallest stiffness because here we have the maximum displacement.

So we have to keep in mind that also if Z coefficient isn't an infinite value, but it is big enough, the Courbon solution can be considered as a good approximation for our bridge.

### EVALUATION OF INTERNAL ACTIONS

➔ Beams internal actions

⊙ A. Case of infinite number (or high number) of rigid transverse beam (Courbon / Albenga)

n° of beams	beam	Lc			
		1	2	3	4
2	1	1	0	0	0
3	1	0,833	0,333	-0,166	0
	2	0,333	0,333	0,333	0
4	1	0,7	0,4	0,1	-0,2
	2	0,4	0,3	0,2	0,1

A generic load applied on a beam is distributed with the same law on the other beams, proportionally to the transverse distribution coefficients.

(Rough hypothesis but good approximation in the result)

For the evaluation of internal actions in beams we have to consider two different situations: the first one is related to the case of infinite numbers of rigid transverse beams also called Courbon /Albenga Method.

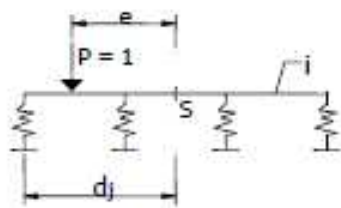
To show the method we are going to consider this girder bridge with three longitudinal beams and a lot of cross beams. We suppose that the beam one is loaded with a uniformly distributed load  $q$ . considering the table seen before, we are in the highlighted situation upside for the transverse repartition coefficient. so the 83% of the load is carried by the first beam, the 33% of the load is carried by the second and finally -16% of the load makes the third beam uplift. That is we can evaluate internal action considering that a generic load applied on a beam is distributed within the same law on the other beams, proportionally to the transverse distribution coefficients. So If I have the beam one alone, I have the bending moment in the midspan equal to  $\frac{ql^2}{8}$ ; but I have three beams in the deck and the bending moment on them in the midspan will be proportional to this value, according to the transverse repartition coefficient in the table.

- If the number of transverse beams is  $\geq 3$  the differences between the previous approaches are negligible. Any case the Engesser method is closer to the reality for the shear evaluation

➔ Transverse beams internal actions

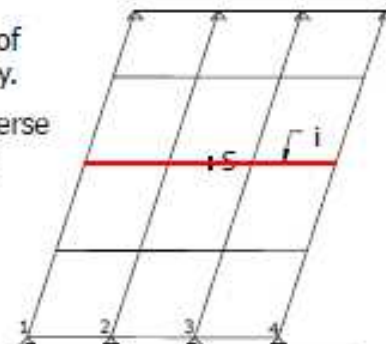
Once known the coefficients  $\rho_{ij}$ , the evaluation of internal actions in transverse beams is quite easy.

- The load is moved transversely along the transverse beam: the influence line of bending moment in section S is to be evaluated.



$$M_s = \sum_{j=1}^n \rho_{ij} d_j - 1 e$$

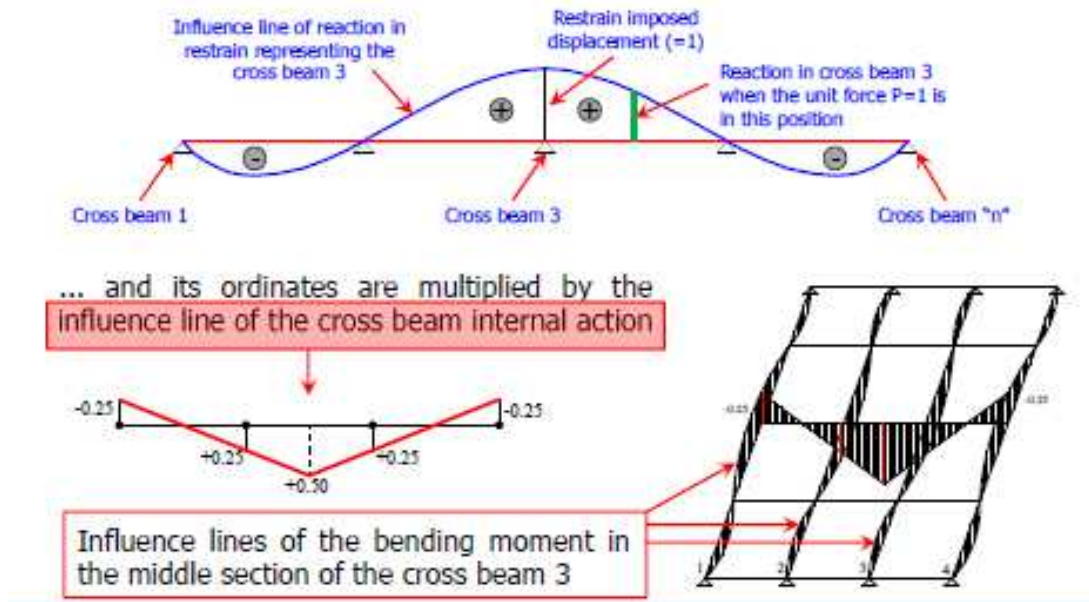
n° of beams	beam	1	2	3	4	Lc
2	1	1	0			
	1	0,833	0,333	-0,166		
	2	0,333	0,333	0,333		
3	1	0,7	0,4	0,1	-0,2	
	2	0,4	0,3	0,2	0,1	



So now I want to evaluate the internal action on the transverse beams; once known the coefficients  $\rho_{ij}$  the evaluation is very simple because first I consider the load moving transversely along the transverse beam, then I can evaluate in a direct way the influence line of bending moment in any section S; for instance we consider the central cross beam i as in the picture upside, and we want to **evaluate the bending moment in the central section S**; so for every position of this force  $P=1$  we want to know the moment in the section S; this is quite simple if we know the reaction in the springs because then the moment in S will be the sum of the reactions (represented by  $\rho_{ij}$ ) multiplied for the distance d from the section we want to analyse and then subtract the eccentricity of the load from the section multiplied for the value of the load itself that in this case is equal to one for sake of simplicity.

As we can see down in the next picture, if I have a force on the first longitudinal beam, we have the same coefficient we find in the Courbon example. Then applying the relation we have just said we can find the bending moment  $M_S^1$  that is equal to -0,25 and this means that here I have a negative bending moment; the same thing can be done for the load applied on the second longitudinal beam, considering that this time the  $\rho_{ij}$  coefficients will be different because of the change of the load position, so we calculate  $M_S^2$  that time will be positive and equal to 0,25. As the influence line is linear because the cross beam has an infinite bending stiffness, then two point are enough for the evaluation of the influence line. The meaning of this influence line is that when the unit load is for instance on longitudinal beam one, the influence line give the value of the moment in the section considered.

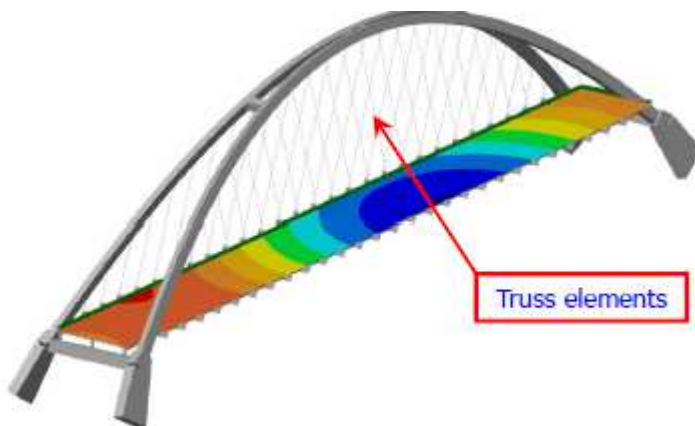
In practice it's evaluated the influence line of reactions in a continuous beam on fixed bearings...



In the picture we can see a longitudinal beam of length  $L$  and it is restrained with a rigid bearing at the intersection between the beam and the cross beam. So I have 5 restraints because in the girder I have 5 cross beams; then I have to draw the influence line of the reaction in the restraint corresponding to the cross beam under consideration; that is if I am considering the cross beam three, I have to draw the influence line of the reaction in this support and in order to evaluate this reaction I have to impose in that point by imposing a vertical displacement equal to one; so I can draw the influence line in the restraint representing the cross beam three and its vertical displacement in a certain position represents the reaction in cross beam three when the load  $P$  is in that point.

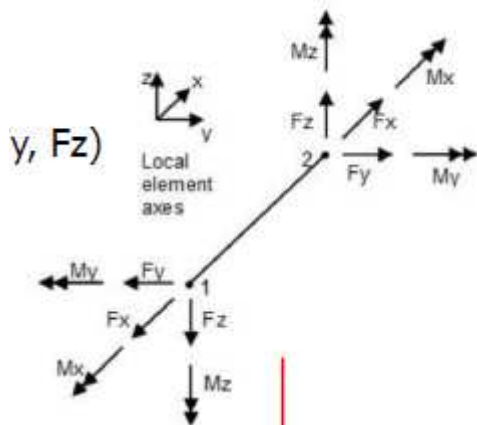
Knowing the influence line of reaction in a continuous beam on a fixed bearing, we have to multiply this value by the influence line of cross beam internal action.

element as the cross section area. We can think to use this element to model suspender in an arch bridge, bracing in a composite bridge and cable stayed.

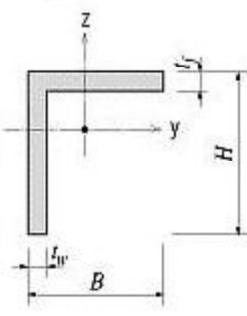
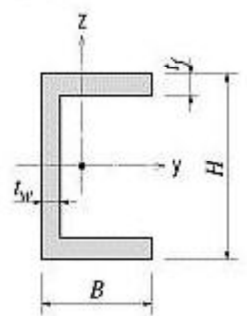
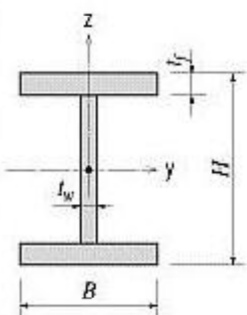
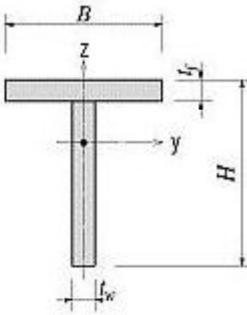
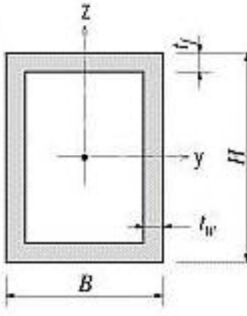
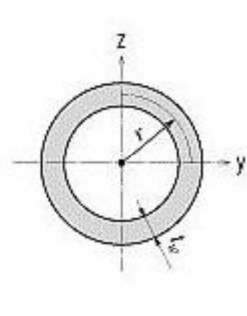
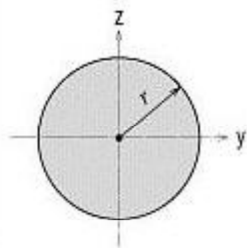
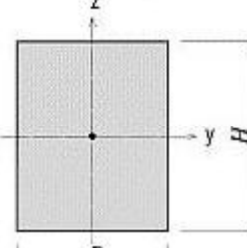


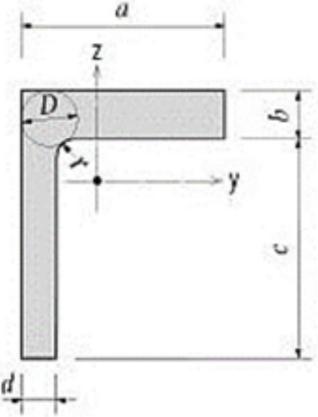
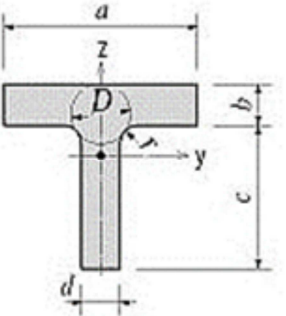
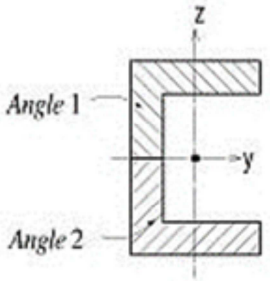
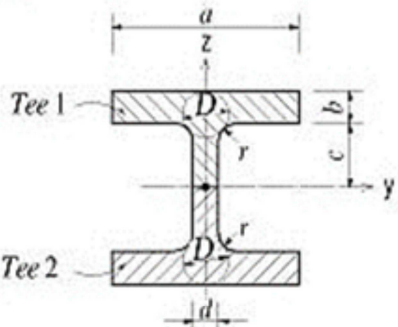
A bar element is shown represented by a single line. A node is located at each end of the line, one node labeled 1 and the other node labeled 2. Arrows representing axial force are shown at each node and are oriented along the axis of the bar element. The arrows point outwards away from the bar element and are labeled  $F_x$ . A local element axis is included in the figure, with the  $x$  axis in the same arbitrary direction as the bar element and the  $y$  and  $z$  axes oriented in the horizontal and vertical directions of the page, respectively.

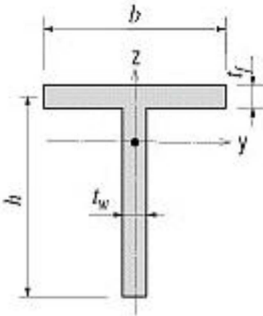
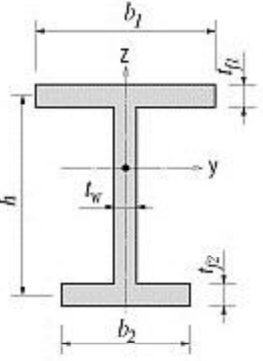
- **Beam (or Frame) elements**



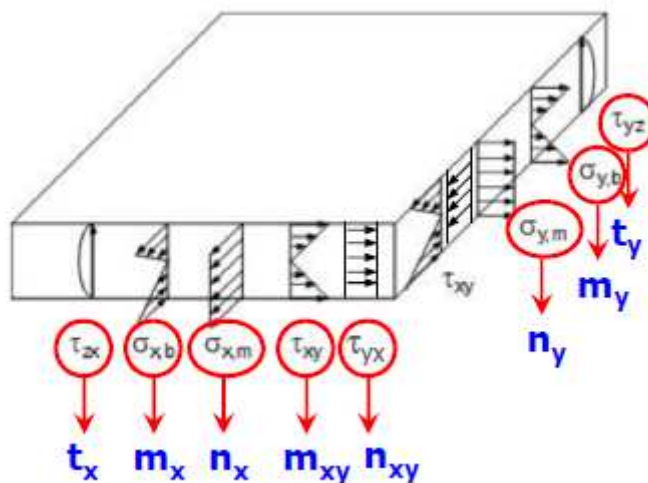
Also in this case we have a line element, but now we have six degrees of freedom for each node and particular  $u_x$ ,  $u_y$ ,  $u_z$ ,  $r_x$ ,  $r_y$  and  $r_z$ . The element output is made of six internal actions, in particular we have the axial force  $F_x$ , the shear forces in  $y$  and  $z$  directions  $F_y$  and  $F_z$ , a torsional moment  $M_x$  and finally the two bending moments  $M_y$  and  $M_z$ . The complete definition of the beam element need the definition of the cross section properties as the area  $A$  as for the truss element, but also the moments of inertia. In bridge field it is the most used element to model structural elements subjected to bending as beam elements.

Section Shape	Effective Shear Area	Section Shape	Effective Shear Area
<p>1. <i>Angle</i></p> 	$A_{sy} = \frac{5}{6} B \times t_f$ $A_{sz} = \frac{5}{6} H \times t_w$	<p>2. <i>Channel</i></p> 	$A_{sy} = \frac{5}{6} (2 \times B \times t_f)$ $A_{sz} = H \times t_w$
<p>3. <i>I - Section</i></p> 	$A_{sy} = \frac{5}{6} (2 \times B \times t_f)$ $A_{sz} = H \times t_w$	<p>4. <i>Tee</i></p> 	$A_{sy} = \frac{5}{6} B \times t_f$ $A_{sz} = H \times t_w$
<p>5. <i>Thin Walled Tube</i></p> 	$A_{sy} = 2 \times B \times t_f$ $A_{sz} = 2 \times H \times t_w$	<p>6. <i>Thin Walled Pipe</i></p> 	$A_{sy} = \pi \times r \times t_w$ $A_{sz} = \pi \times r \times t_w$
<p>7. <i>Solid Round Bar</i></p> 	$A_{sy} = 0.9\pi r^2$ $A_{sz} = 0.9\pi r^2$	<p>8. <i>Solid Rectangular Bar</i></p> 	$A_{sy} = \frac{5}{6} BH$ $A_{sz} = \frac{5}{6} BH$

Section Shape	Torsional Resistance
<p>1. Angle</p> 	$I_{xx} = I_1 + I_2 + \alpha D^4$ $I_1 = ab^3 \left[ \frac{1}{3} - 0.21 \frac{b}{a} \left( 1 - \frac{b^4}{12a^4} \right) \right]$ $I_2 = cd^3 \left[ \frac{1}{3} - 0.105 \frac{d}{c} \left( 1 - \frac{d^4}{192c^4} \right) \right]$ $\alpha = \frac{d}{b} \left( 0.07 + 0.076 \frac{r}{b} \right)$ $D = 2 \left[ d + b + 3r - \sqrt{2(2r + b)(2r + d)} \right]$ <p>(where, <math>b &lt; 2(d + r)</math>)</p>
<p>2. Tee</p>  <p>IF <math>b &lt; d</math> : <math>t = b, t_1 = d</math>  IF <math>b &gt; d</math> : <math>t = d, t_1 = b</math></p>	$I_{xx} = I_1 + I_2 + \alpha D^4$ $I_1 = ab^3 \left[ \frac{1}{3} - 0.21 \frac{b}{a} \left( 1 - \frac{b^4}{12a^4} \right) \right]$ $I_2 = cd^3 \left[ \frac{1}{3} - 0.105 \frac{d}{c} \left( 1 - \frac{d^4}{192c^4} \right) \right]$ $\alpha = \frac{t}{t_1} \left( 0.15 + 0.10 \frac{r}{b} \right)$ $D = \frac{(b + r)^2 + rd + \frac{d^2}{4}}{(2r + b)}$ <p>(where, <math>d &lt; 2(b + r)</math>)</p>
<p>3. Channel</p> 	<p>Sum of Torsional Stiffnesses of 2 angles</p>
<p>4. I-Section</p>  <p>IF <math>b &lt; d</math> : <math>t = b, t_1 = d</math>  IF <math>b &gt; d</math> : <math>t = d, t_1 = b</math></p>	$I_{xx} = 2I_1 + I_2 + 2\alpha D^4$ $I_1 = ab^3 \left[ \frac{1}{3} - 0.21 \frac{b}{a} \left( 1 - \frac{b^4}{12a^4} \right) \right]$ $I_2 = \frac{1}{3} cd^3$ $\alpha = \frac{t}{t_1} \left( 0.15 + 0.10 \frac{r}{b} \right)$ $D = \frac{(b + r)^2 + rd + \frac{d^2}{4}}{(2r + b)}$ <p>(where, <math>d &lt; 2(d + r)</math>)</p>

<p>4. Tee</p> 	$I_{xx} = \frac{1}{3} (h \times t_w^3 + b \times t_f^3)$
<p>5. I-Section</p> 	$I_{xx} = \frac{1}{3} (h \times t_w^3 + b_1 \times t_{f1}^3 + b_2 \times t_{f2}^3)$

- Shell elements

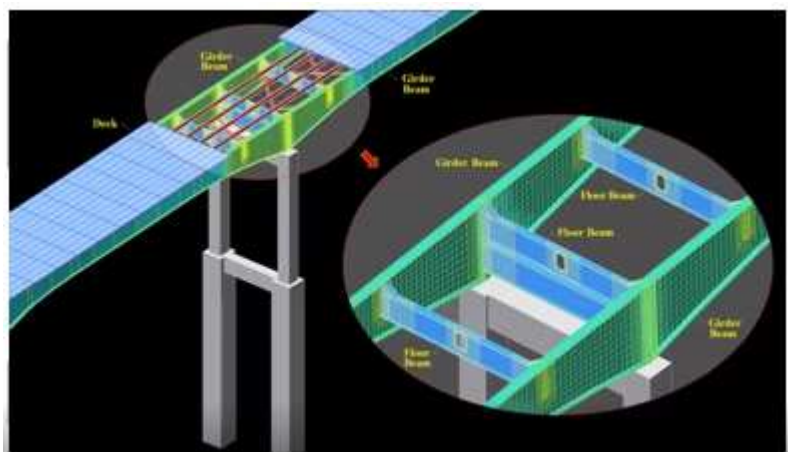


They are bi-dimensional elements and so we need at least three points to define them as we can have a triangular shell element or a quadratic shell element and in this case we need at least four points to define it. For triangular shell element I need a linear equation to find nodal point displacements where I need quadratic equations to find them for quadratic shell



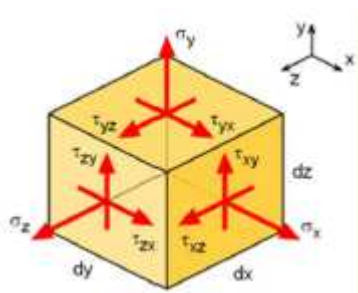
In the design of bridges we can use those elements in order to realize local models as when we want to evaluate the stress distribution at the intersection of two beams also considering the stiffeners, for example in a fatigue verification. But these are not useful for the design of the bridge where we should use mostly beam elements.

Another possible use of these elements is for validation of simple models generally in the design I try to use a simple model so in this way I can check the results and control the design; then I can create a more refined model using 2D elements, but I won't use it as first model in order to design, because the possibility to make mistake would be very high.



- **3D elements**

In this case the shape of the elements is something like this:



Each node has three degrees of freedom that are the three translation in the respective three directions. In this case I don't need any additional input because element describes the volume of the body. The output are the global stress tensor, and that is I have the three normal stresses and the three tangential ones. In bridge structures we can think to use these type of elements in massive elements like masonry arch bridges for example.

Which element type should be used?

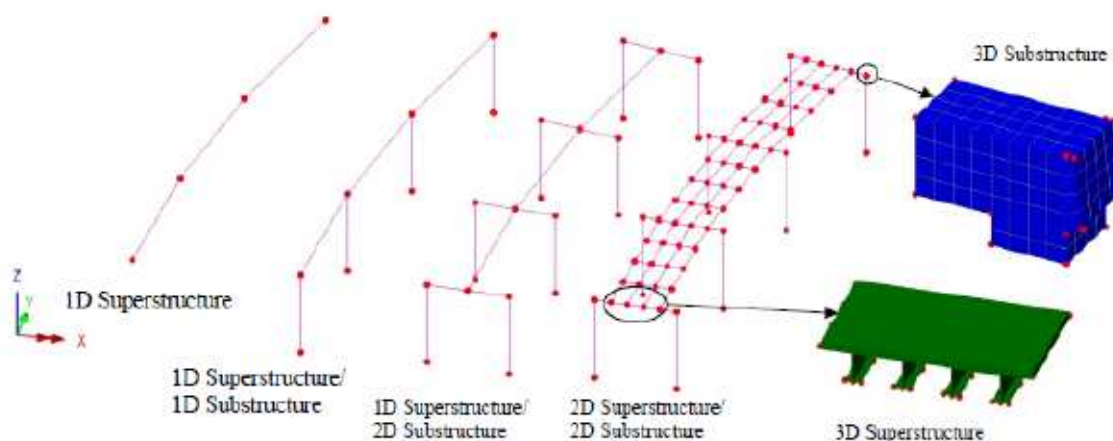
It depends on what you are looking for: FEA and FEM are tools to get the answers, but the answer depends on the question; if the question is wrong, the answer will be wrong too;

- If we want to design a structure (evaluate a reinforcement, a resistance, define a tendon layout, etc.) it's easy manage truss or beam elements, is more difficult manage 2D/shell elements, it's impossible manage 3D elements

- If we want to validate simpler model, we can use 2D/3D/shell elements as we don't use the result to design. Usually we consider few load conditions and compare the displacements, transverse deformation, eigenmodes with the results of the simpler model

- If we want to evaluate local stress, maybe even in non-linear field, we can use 2D/3D/shell elements considering sub-models loaded at the boundary with the internal action evaluated with a simpler model

- In general we should consider many models with increasing complexity, using the simpler one to understand the structural behaviour and the magnitude of the interesting output

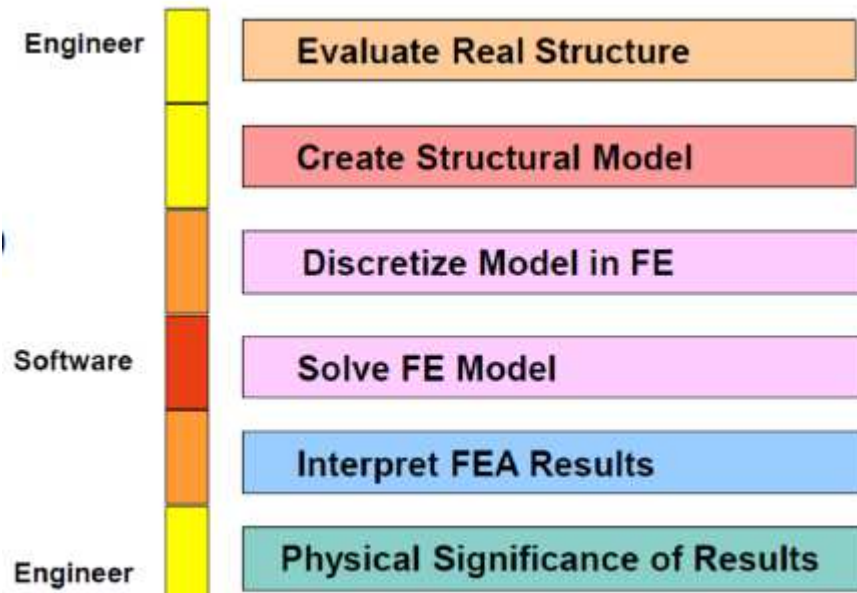


Here we can see a typical example of increasing complex of the model: first of all we can make a 1D model of the superstructure; this model should be able to capture the behaviour of the bridge for only vertical actions and not of course horizontal actions. Then we can go to a 1D superstructure and substructure model, so I will be able to capture the horizontal forces as wind, brake and seismic action.

But if my piers are framed piers of course in this case the substructure model will be a 2D one while the superstructure model will be a 1D model.

We can also think to refine the model of the superstructure too and so to be able to know the transverse behaviour of the bridge and not only the longitudinal one.

- After running the analysis you have to check if the outputs are only numbers or are also results (V.I. Carbone)



Here we can summarize the process that we have to follow to perform a finite element analysis:

1. We have to **prepare the finite elements model** discretizing (and so meshing) the structure; but we have also to prescribe loads and supports.
2. We have to choose the type of analysis that we want to perform and then we have to solve the problem and this means doing these steps that are made by the software:
  - Generate stiffness matrix ( $k$ ) for each element
  - Connect elements (assemble  $k$ )
  - Assemble loads (into a vector  $R$ )
  - Impose support conditions
  - Solve equations ( $KD = R$ ) for displacements
3. Post process the results

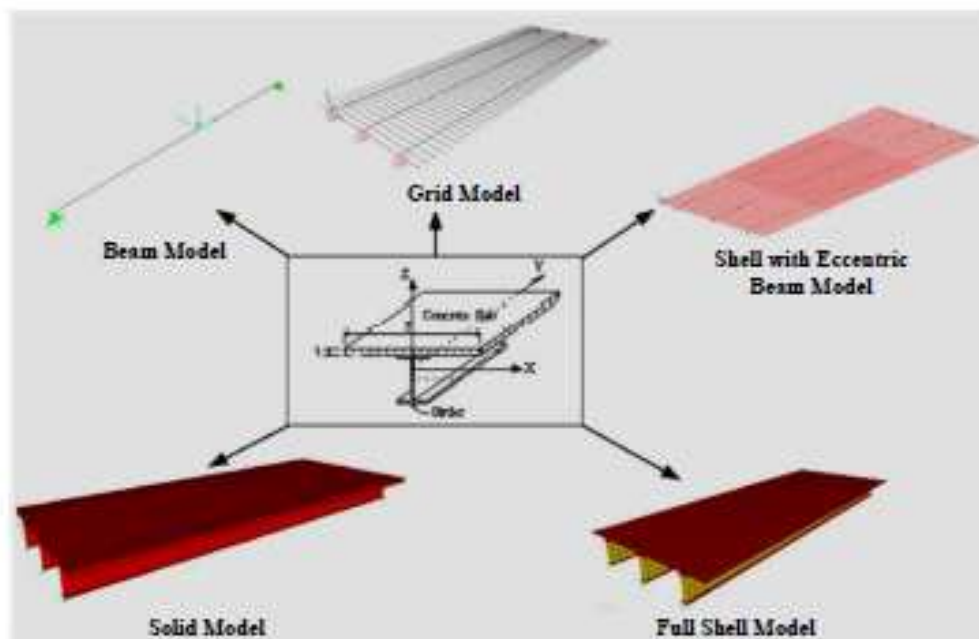
girders. Then the effect of these cracks is to lengthen the length of this portion of the slab between girders, but actually this increase of length is restrained by the longitudinal girders that don't want to lengthen each other and then in this part of the slab will arise a compressive force that increases the bending resistance of the slab and this is what we call compressive membrane action of the slab. It is difficult to evaluate this contribution to resistance but it exists.

- The orthotropic nature of the slab stiffness due to different reinforcement ratio in longitudinal and transverse direction because of course this slab will be reinforced in the two different directions and then this slab will have an orthotropic behaviour that is not easy to capture.
- The effect of slab cracking, the effect of the longitudinal compressive force in the slab on its flexural behaviour

Taking into account all these effects is a very difficult issue and in some cases we will disregard a lot of them. But if we want to perform a complete analysis we should take into account all of them.

## DECK MODELLING OPTIONS

When we want to model a bridge or a structure in general, we have a lot of options.



For instance we can do a very simple model using a beam element in the transverse direction, so the transverse behaviour of the deck is represented by only one beam element.

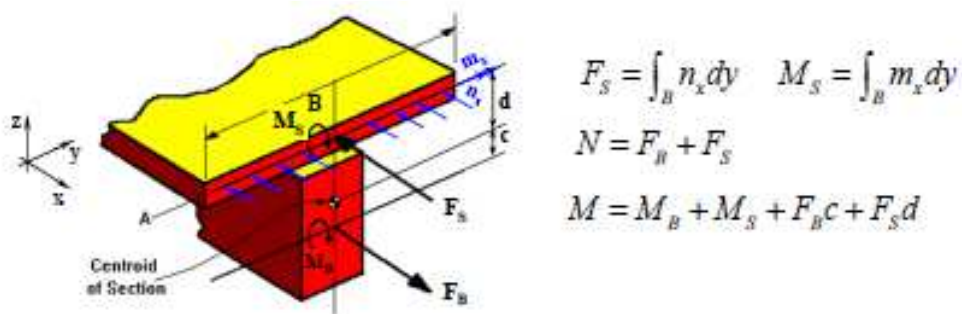
We can see in the picture upward, a typical concrete steel deck with steel girders in the longitudinal direction with I cross section; these longitudinal beams are connected with a concrete slab. I can represent this structure by using beam elements representing the longitudinal girders and a number of shell elements representing the concrete slab. In this case the beam has been considered in the centroid of the longitudinal girders. To these elements the cross section of the beam element has been assigned. the shell elements are placed in the middle of the beams and the thickness of this shell element is the thickness of the slab; the thickness variation is followed assigning different thicknesses to the different shell elements. The nodes of the beams and the nodes of the shell are connected with rigid links. Another rigid link is used to connect the centroid of the beam element with the actual position of the bearings.

With this kind of model we can separate material properties that can be assigned to the girders and to the slab, as they are made of different materials.

We can also vary the section properties in the girders with stepwise section properties in the model.

Bridges of arbitrary cross-section and geometry can be handled in this way but we can have difficulty in applying moving loads because applying them on shell elements is not easy.

But the most important problem for this kind of model is to act the analysis to design the structure, because the design force outputs have to be added and translated in the composite section centroid in order to evaluate the resistance of the girder + slab section in the correct way.



$$F_S = \int_B n_x dy \quad M_S = \int_B m_x dy$$

$$N = F_B + F_S$$

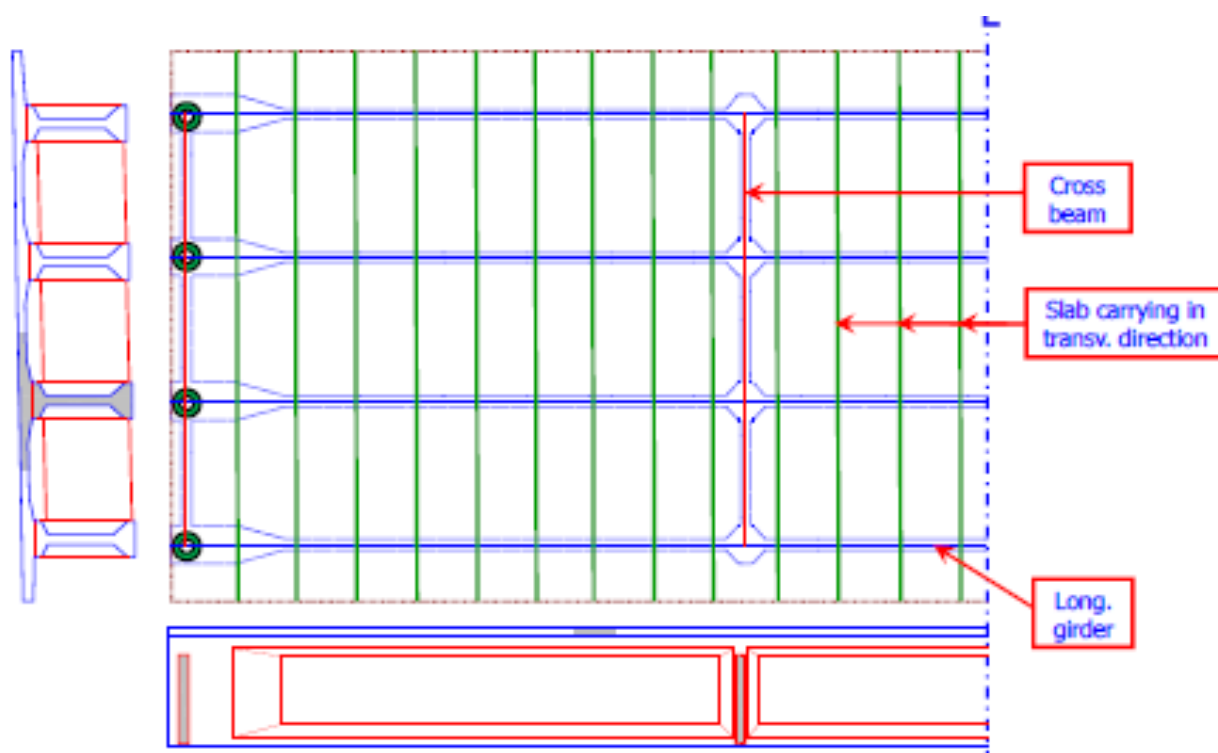
$$M = M_B + M_S + F_B c + F_S d$$

As the rule that I use to design the structure is not divided considering beam and slab, but I'm going to consider a section made by the beam and by the slab with a total centroid, I have to sum the internal actions evaluated in the shell and in the longitudinal girder; this means that I have to integrate all the tensions that are the output of the analysis in the shell elements.

## GRID MODEL (GRILLAGE)

This is the type of model we are going to use more often. In this type the longitudinal and transverse stiffness properties of the bridge structure are concentrated in the grid of beams living on the same plane; in this way we don't consider the distance between the distance between the centroid of the slab and the centroid of the girder but we compress all the elements in a single plane.

In this example we have a girder bridge realized with four precast concrete longitudinal beams connected by a cast in place slab and we have also four cross beams.



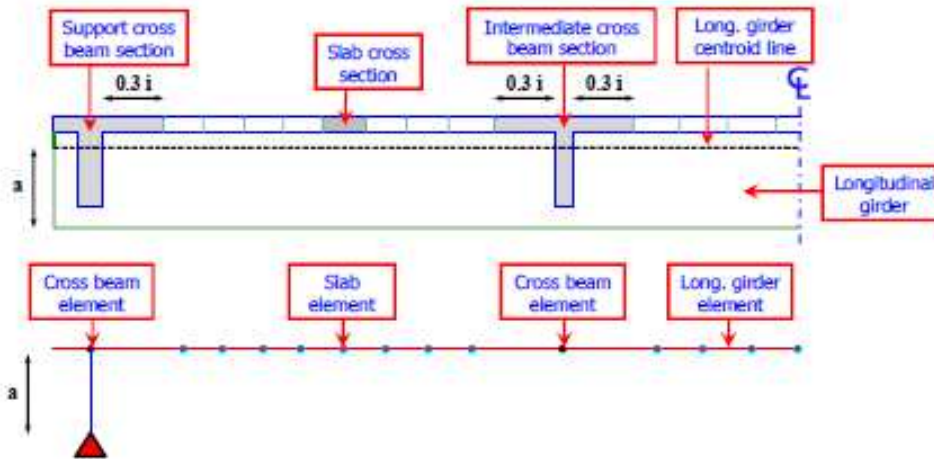
In order to analyse this bridge first of all we can consider the longitudinal beams and then we can consider a set of longitudinal beam elements representing the longitudinal girders plus the slab that is working with them and that is the slab with the effective width; so first of all I have to evaluate the effective width and then the properties of the total section.

At the main left we can see the bearing alignment; these bearings restraint of course the vertical displacements but also some of them should restraint also horizontal displacement.

Considering the transverse girder we have a set of cross beams, that in this case as we can see from the transversal section, are not connected to the slab, so their area won't be related to an effective width.

## Section properties: **transverse elements** and **slab**

- Section properties are calculated similarly to the longitudinal element properties
- If the cross beam is connected to the slab, consider an effective width equal to 0.3 of the distance between longitudinal members



In the previous example the cross beams were not connected to the slab, but in some cases it is connected. If they are connected we can use an effective width of 0,3 times the distance between longitudinal members.

**Loading** can be applied to the transverse elements (slab elements and cross beams elements) or to dummy longitudinal elements and that is should consider in addition to the elements I described, other longitudinal elements that are fictitious and used only to apply loads, but these elements will be without any stiffness in order not to modify the behaviour of the bridge.

NB. This kind of model is able to evaluate the internal action on longitudinal girders or transverse girder, but it is not able to evaluate internal actions on the slab, so I need another kind of model to evaluate the internal action on the slab. We're going to talk about this aspect talking about local effect.

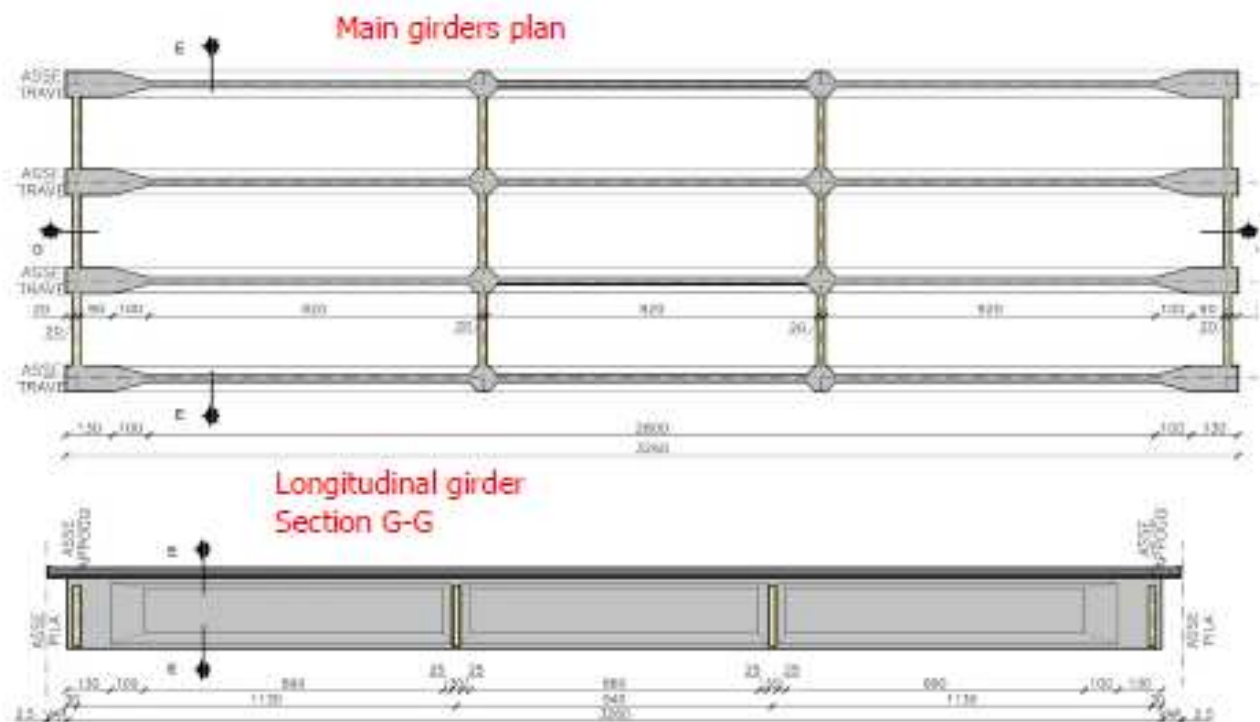
### **Skew deck**

In this kind of element, the skew deck can be analysed in different ways: first of all the orientation of transverse lines preferably should be normal to the longitudinal lines of the grid and that is the slab elements and the longitudinal beam elements should be perpendicular to each other; when the angle of skew is less than 20 degrees, transverse lines

## Response to modelling issues in bridge deck

- ⊗ The eccentricity between the deck slab and the longitudinal girder cross sectional centroid
- ⊗ The eccentricity of the girder cross sectional centroid from the bearing top level
- ⊗ The effective width of the slab carrying the wheel loads
- ⊗ The effects of deck skew, and non-rectangular slab
- ⊗ The effect of the diaphragm connection with longitudinal girder
- ⊗ The shear connection between the slab and longitudinal girder
- ⊗ The torsional stiffness of the slab
- ⊗ The compressive membrane action in the slab between the girders
- ⊗ The orthotropic nature of the slab stiffness due to different reinforcement ratio in longitudinal and transverse direction
- ⊗ The effect of slab cracking, the effect of the longitudinal compressive force in the slab on its flexural behavior

### ➔ Example 1: PRC girder bridge with I beams



This is a prestressed reinforced concrete girder bridge with I beams. We have 4 beams and 4 cross beams.

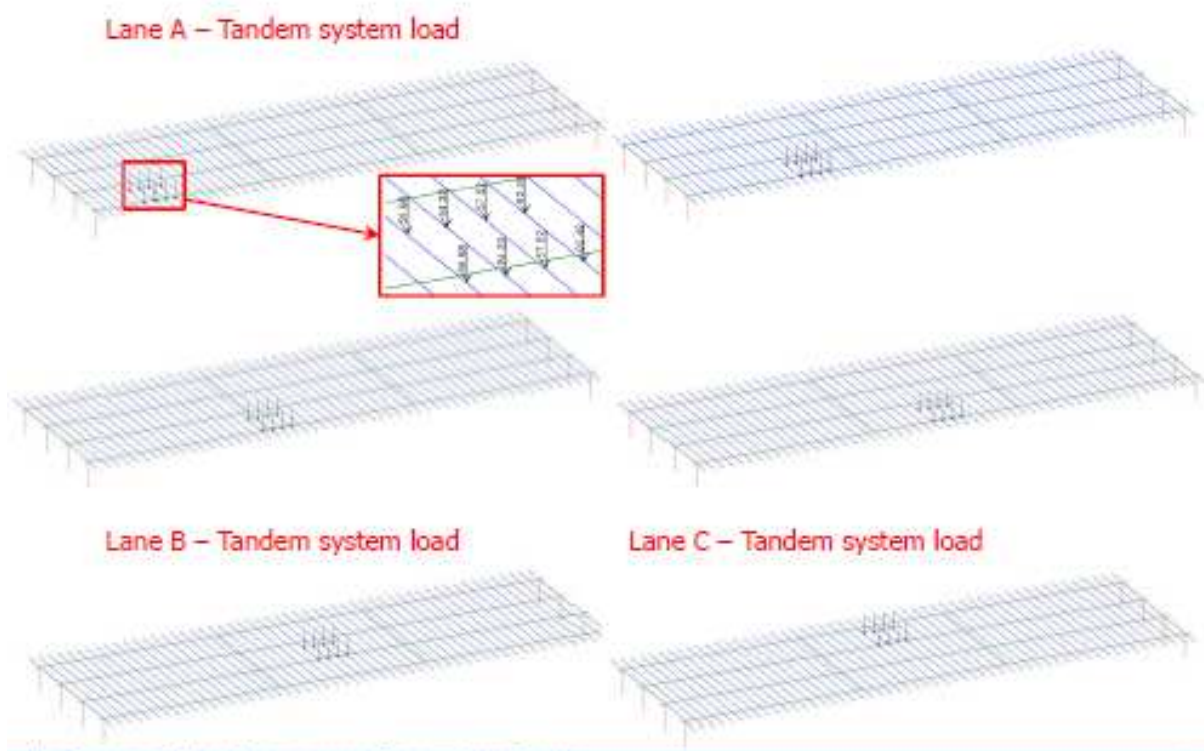


times, because if we consider it into longitudinal beams we don't have to consider it applying it to the transverse element representing the slab.

Vice versa for the cross beam elements, the weight is not included and I have to apply it to the model.

In these picture below we can see the tandem system loads: the first four picture are related to the tandem system applied on lane A and we can see that the tandem system is related to four forces of 150 kN in this case we can see that we have 8 forces, because the single force of 150 kN is in general not applied on the transverse element, but between two of them and so we divide the force.

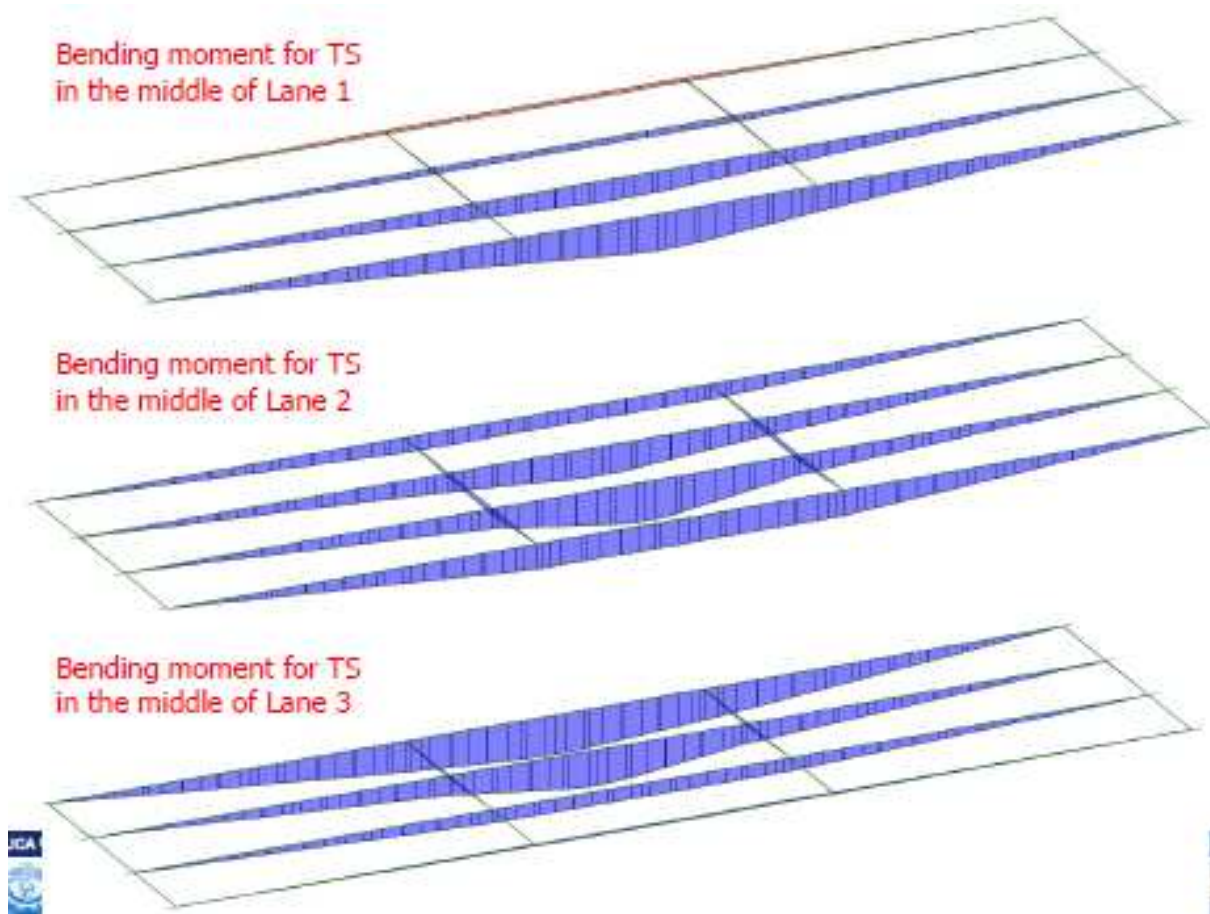
About the lane A we consider a lot of positions of these forces, for instance the followings:



Of course I can consider other positions but they are symmetric positions of these and so it is not useful. Then only one position of the tandem system for the other two lanes is represented.

Next we can see the loading related to the uniform distributed load applied on the three lanes. As we can see UDL and tandem systems are applied on transverse elements representing the slab, that carry this load to the beams and then the beams carry the load to the supports.

For the third case we can see it is correct using the Courbon hypothesis too.



Here we can see the bending moment distributions. The bending moment related to the tandem system in the middle of lane 1 shows that we have a triangular distribution of bending that is the same also on the other three beams, but in the last one we can see an uplift.

In the second picture we already said that Courbon approach should not be applied, in fact the distribution of bending moment is not triangular.

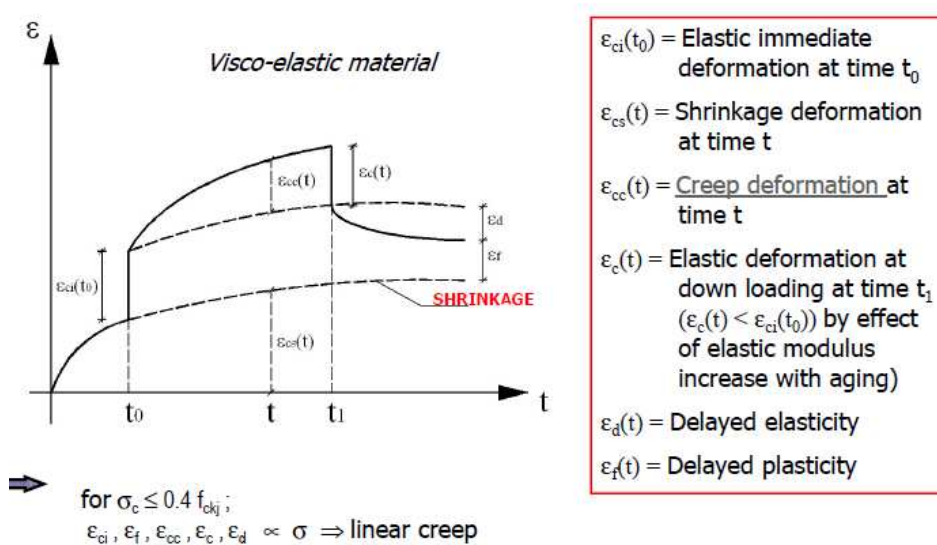
## CREEP EFFECTS

### Lesson XIV (Costanza Anerdi)

In current situation reinforced concrete structures are assumed to have a linear elastic behaviour while in the reality they provide inelastic responses. This hypothesis allows us the adoption of simple calculation models but leads to underestimate or ignore some phenomena like the increasing deformation or long term prestressing losses. The importance of this independent behaviour was highlighted with the more and more frequent use of prestressing and in case of bridges of cantilevered constructions which have staged construction.

Let's imagine of taking two specimen of concrete subjecting them to different treatment while maintaining the same environmental conditions like constant temperature and constant humidity. The specimens are exactly the same and they are made of the same material characteristics and same dimensions.

For the first specimen we are going to measure what is the variation in length of the bar exclusively due to the shrinkage act.



As we can see in the  $\epsilon$  diagram in function of time  $t$  we appreciate the increase of deformation which gradually tends to an asymptote; both profiles are subjected to the same initial treatment but for the second one something different will happen in  $t_0$  : while the first specimen continues to undergo the same conditions due to shrinkage, the second one will be loaded with an axial load (vertical line at  $t_0$ .) applied in a short time like some seconds and this can be considered an instantaneous application with respect to the scale time of several years. Due to the application of this axial load an elastic deformation occurs  $\epsilon_{ci}(t_0)$ . If there were no viscos deformations we should find in the second specimen a behaviour similar to the first unloaded specimen. We should basically find two parallel curves. But in the reality that's not happening and for the second specimen we measure some additional deformations. That means that we have a lot of deformation

To confirm that hypothesis we will talk about an important principle: the Mac-Henry Principle.

The importance of this principle comes from the fact that viscos deformation and the non-linear behaviour of concrete seems to be challenging in analysing the concrete structures because the basis of our elastic theory cannot be validate anymore and the existence of non-linearity implies the non-applicability of superimposition principle, thing that complicates a lot the structural analysis as in presence of more than one actions their effects cannot be summed anymore, but have to be taken into account separately.

The mac-henry principle overcomes this problem by a quite simple consideration that takes back the analysis of the structures in the field of applicability of the superimposition principle.

In order to have a better representation of the problem we decided to diagram only the viscos deformation, so we disregard the linear function.

Mac-Henry has taken a certain number of specimen and he analysed the first and the second one, which are identical.

At time  $t_0$  he loaded the first specimen with a constant load placed in a permanent way on the specimen. The elastic deformation is not reported but we can see the consequent viscos deformation that occurs overtime following that curve. We can see that the deformation rapidly increases at the beginning and tends to an asymptote overtime.

Arrived in  $t_1$  we have two things to do: the first is to download the first specimen and the first is apply the load to the second specimen and we can see that the first specimen stops to give back a part of deformation and continues overtime while the second specimen begins to accumulate creep deformation. But this deformation is smaller respect to the first one.

So we can firstly conclude that viscos deformation is function of the age of the specimen uploading

That consideration is true but it is not compatible with real life as the loading process starts as soon the resistance in concrete is developed and so a not real big time.

A second evaluation specifically related to Mac-Henry principle is that if I look at what happens at generic time  $t$  is that the quantity of deformation given back by the first specimen (segment b) measured by the point if the specimen was not unloaded at  $t_1$  is the same as the quantity a of deformation accumulated by the second specimen.

That consideration means that for certain stress variations applied at time  $t_1$  has the same effect whichever is the age at loading and the sign of the stress variation.

Practically I can analyse stress history and its effects by summing the effects of variation produced by every single interval in which the stress is constant. We have so came back to condition of applicability of superposition principle.

$$\varepsilon_{c\sigma}(t, t_0) = \sigma_c(t_0) \left[ \frac{1}{E_c(t_0)} + \frac{\phi(t, t_0)}{E_{ci}} \right] = \sigma_c(t_0) J(t, t_0)$$

**J = creep function** [F<sup>-1</sup>L<sup>2</sup>] → total deformation at time t by effect of a unit stress

The product should give a non-dimensional parameter.

Disregarding the effect of environmental conditions (T[°], RH[%]) and considering only the effect of stress history by applying the *superposition principle* and the *linearity hypothesis*, the evolution law of total deformation may be expressed as follows (sum of a first part due to stresses and possible imposed deformation  $\varepsilon_{cn}(t)$ ):

$$\varepsilon_c(t) = \varepsilon_{cn}(t) + \int_0^t J(t, \tau) \frac{\partial \sigma_c(\tau)}{\partial \tau} \partial \tau$$

$\tau$  = time to which the stress variation  $\partial\sigma/\partial\tau$  is applied.

This equation allows me to evaluate the total deformation of concrete summed to the imposed deformation for a variable stress application law.

Generally the application of loads cannot occur at time zero because concrete is not consistent, but we have to consider an interval of time to do this. So considering  $\tau$  corresponds to the time in which the structure is loaded we can assume that  $t = \tau$  so  $\sigma(t) = \sigma(\tau)$  and if we assume that  $\varepsilon_{cn}(t_0) = 0$  it will result:

$$\varepsilon_c(t, t_0) = \sigma_c(t_0) J(t, t_0) + \int_{t_0}^t J(t, \tau) \frac{\partial \sigma_c(\tau)}{\partial \tau} \partial \tau + \varepsilon_{cn}(t) \quad (1)$$

The first part of this sum describes what happens in  $t_0$  and the second part that describes what happens after  $t_0$ .

If the stress variation is applied by means of finite intervals, it results:

$$\varepsilon_c(t, t_0) = \sigma_c(t_0) J(t, t_0) + \sum_{i=1}^n J(t, t_i) \Delta\sigma(t_i) + \varepsilon_{cn}(t)$$

And so the integral will become a summatory.

Imagine now to invert the process. With the previous expression we have evaluated the effects in term of deformation for a given stress history.

Let's start now on stresses given by deformation history:

Volterra integral equation is a special type of convolution integral equation which is usually solved in a numerical way as it is very difficult to solve.

A different situation is the one in which we have the creep function and an assigned stress history because the deformation is outside the integral and so we can solve it by a simple calculation.

As we were saying J and R describe the same phenomena, in fact the rheological behaviour of concrete is characterized both by the availability of them.

The consequent relationship between the two functions can be obtained introducing in the integral law for creep a deformation history characterized by a single step:

$$\begin{aligned} \text{for } t < t_0 & \quad \varepsilon_c(t) - \varepsilon_{cn}(t) = 0 \\ \text{for } t \geq t_0 & \quad \varepsilon_c(t) - \varepsilon_{cn}(t) = 1 \end{aligned}$$

From (2) it results:  $\sigma(t, t_0) = R(t, t_0)$  and substituting in (1), considering that  $R(t, t_0) = E_c(t_0)$ , it can be obtained:

$$1 = J(t, t_0)E_c(t_0) + \int_{t_0}^t J(t, \tau) \frac{\partial R(\tau, t_0)}{\partial \tau} \partial \tau$$

Such equation has been solved numerically, once known the  $J(t, t_0)$  values, and the function  $R(t, t_0)$  has been tabled.

The equation contains both the functions J and R but why it is so important to have a relation that puts them together? The answer is in real life: for example we know that experimentally we are able to measure deformation coming from stress application, we are not able to measure the variation of stress coming from an imposed deformation so the relaxation function is more difficult to be measured than the creep function.

A simplified evaluation of relaxation function can be obtained by means of the semi-empirical expression (with an error smaller than 10%):

$$R(t, t_0) = \frac{1 - 0,008}{J(t, t_0)} - \frac{0,115}{J(t, t-1)} \left[ \frac{J(t-\Delta, t_0)}{J(t, t_0 + \Delta)} - 1 \right] \geq 0$$

with  $D = (t-t_0)/2$

For the practical applications, within the *linear creep* field, the structures can be divided into the following four groups:

- **homogeneous structures with rigid (elastic) restraints**
- **structures with constant restraint conditions**
- **heterogeneous structures with rigid (elastic) restraints**
- **structures subjected to variation of the static scheme**

The problems involving homogeneous structures may be easily solved by use of J and R functions.

The problems involving heterogeneous structures are controlled by one or more integral equations.

## **THEOREM OF ISOMORPHISM (SECOND PRINCIPLE OF LINEAR CREEP)**

A viscous-elastic body, homogeneous and with rigid restraints is subjected to imposed deformations irrespective of internal and external compatibility  $\bar{\epsilon}_A$ .

Total deformation:  $\bar{\epsilon}_A + \epsilon_A \rightarrow$  internally and externally compatible

$\epsilon_A$ : complementary elastic deformation producing one self-equilibrated stress system  $s_A$ .

Imagine now to have a cylinder of concrete and we want to apply an imposed deformation able to push down the upper face of about 1 mm, what happens? We are asking to the free surface to move and to compenetrare inside the rest of the material. This implies the arise of a deformation which is not compatible.

If the specimen doesn't break what happen is that all the elementary part of the cylinder will absorb a little amount of the deformation by shortening. To allow that process  $\epsilon_a$  must arise and accompany the imposed deformation, so the sum of the two will be compatible.

If elastic deformation arises, I will also have a system of  $\sigma_A$  but those are not given by external forces and have to be self-equilibrated.

Now we will add to the body another system of imposed deformation  $\bar{\epsilon}_B$  proportional to the pre-existing  $\epsilon_a$ .

What will happen now is that the new imposed deformation won't be compatible because it is proportional to  $\epsilon_a$  that is not compatible it-self.

That is the reason why we have to introduce another system of elastic deformation  $\epsilon_b$  to be put inside of  $\bar{\epsilon}_B$ . So now the compatibility is ensured.

The elastic deformations will give rise to a system of stresses  $\sigma_A + \sigma_B$ .

The total deformation can be expressed as the sum of al those deformations.

One add now a system of  $\epsilon_B$  proportional to  $\epsilon_A$  ( $\epsilon_B = k\epsilon_A$ ) than irrespective of internal and external compatibility; as a consequence a further system of complementary elastic deformation  $\epsilon_B$  arises, so that  $\bar{\epsilon}_B + \epsilon_B$  respect the internal and external compatibility.

$\epsilon_B$  involve the arising of self-equilibrated  $s_B$ .

⇒ Overall deformation (respective of internal and external compatibility)

$$\mathcal{E} = \bar{\epsilon}_A + \epsilon_A + \bar{\epsilon}_B + \epsilon_B$$

⇒ Total stress (self-equilibrated):

$$\sigma = \sigma_A + \sigma_B$$

From a quantitative point of view:

$U_i^{el}(t)$  = deformation state of an elastic, homogeneous, rigidly restrained structure deriving by imposed deformations  $\epsilon_{cn}(t)$ .

$$\bar{\epsilon}_B \equiv \text{linear creep}$$

Following the application of an *isomorph deformation* we get:

$$U_i(t) = U_i^{el}(t)$$

$$\sigma_c(t) = \int_0^t R(t, \tau) \frac{\partial(\epsilon_c(\tau) - \epsilon_{cn}(\tau))}{\partial \tau} \partial \tau = \frac{1}{E_{co}} \int_0^t R(t, \tau) \frac{\partial \sigma_c}{\partial \tau} \partial \tau$$

If, on the opposite,  $s_c^{el}(t)$  is the stress state deriving by an equilibrated system of forces, it results:

$$s_c(t) = s_c^{el}(t) \quad U_i(t) = \int_0^t J(t, \tau) \frac{\partial \sigma(\tau)}{\partial \tau} \partial \tau = E_{co} \int_0^t J(t, \tau) dU_i^{el}(\tau)$$

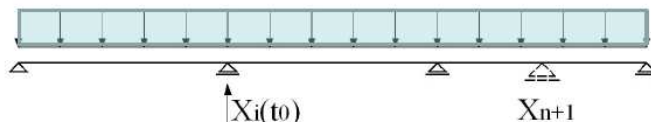
We have seen with the two linear creep principles the two phenomena that describe quite the opposite: we have an increasing of deformation and a non-variation of stress and in the other side a non-variation of stresses while a variation of deformation.

### Lesson XV (Costanza Anerdi)

## THIRD PRINCIPLE OF LINEAR CREEP

### PRINCIPLE OF REINTRODUCTION OF DELAYED RESTRAINTS

An elastic body, homogeneous and with  $n$  rigid restraints is subjected to  $X_i(t_0)$  reactions by effect of constant forces  $F$  applied at time  $t_0$ .



Immediately after the application of the load a further restraint is introduced, in which obviously at time  $t_0$  the reaction is  $X_{n+1}(t_0) = 0$ . Let's analyze the evolution of all the reactions by effect of creep. Suppose to introduce the  $n+1$  restraint before the application of the load; consequently a reaction will arise in it :  $X_{n+1}(t_0)$ , and all other reactions will have variations  $DX_i(t_0)$ .

This principle has been demonstrated by Levi and it is a fundamental principle every time we have a variation in our static scheme.

Let's imagine to have a structure which is represented in the figure with a continuous beam, an homogenous elastic structure rigidly restrained subjected to particular conditions: for example we have four bearings with reaction equal to  $X_i(t_0)$  and for the moment we will disregard the fifth one. At time  $t_0$  we will apply a uniform permanent load to the whole structure. We can consider for example the self-weight and the structure will deform; we can consider for example the deformed configuration that will imply the presence of deflection; while this elastic deformation occurs immediately after the application of the load another restraint is



The equations that represent the external reactions are

$$\begin{aligned} \text{in } n+1: \quad & \underbrace{X_{n+1}(t_0)}_A - \underbrace{X_{n+1}(t_0)}_B = 0 \\ \text{in } i, \text{ in } t_0: \quad & \underbrace{X_i(t_0)}_C + \underbrace{\Delta X_i(t_0)}_D - \underbrace{\Delta X_i(t_0)}_E = X_i(t_0) \end{aligned}$$

where:

- A : effect of forces, than constant with time
- B : effect of imposed deformation, then variable in time with relaxation law
- C : effect of forces
- D : effect of forces
- E : effect of imposed deformation

In n+1 when I apply the load the bearing already exist so the reaction A will arise, then when I apply the settlement and move the bearing, the reaction goes back to zero, so I have B= -A. so the total reaction in n+1 will be equal to zero and also in the other bearing the situation will be the same.

The A term born by the effect of forces is invariant over time by the application of the first principle of linear creep, so it means this term born in an hyperstatic structure for the effect of the creep.

The B term otherwise derives from the introduction of an imposed deformation and varies in time and as we will see it will reduce progressively.

Looking at the firs picture, in i at time t<sub>0</sub> when we apply the load a reaction C will arise, that is the one I would have had in four bearings configuration, but a variation D occurs that follows the introduction of the fifth bearing before the application of the load. E is the part that arises when I introduce the elastic settlement. In analogy with what was said before the first two terms derive from the application of load by effect of forces and they are constant in time while the third term is coming from the application of an imposed deformation and so it varies overtime.

So now we know what happen in t<sub>0</sub> but we have to evaluate what happen in t.

Considering the nature of different contributions, at time t, it will result:

$$\begin{aligned} X_{n+1}(t) &= X_{n+1}(t_0) - X_{n+1}(t_0) \frac{R(t,t_0)}{E_c} = X_{n+1}(t_0) \left[ 1 - \frac{R(t,t_0)}{E_c} \right] \\ X_i(t) &= X_i(t_0) + \Delta X_i(t_0) - \Delta X_i(t_0) \frac{R(t,t_0)}{E_c} = X_i(t_0) + \Delta X_i(t_0) \left[ 1 - \frac{R(t,t_0)}{E_c} \right] \end{aligned}$$

As for t<sub>0</sub> = 28 days it results R(t,t<sub>0</sub>)/E<sub>c</sub> = 0,15÷0,30, for t = ∞ it results:

$$X_{n+1}(t) = (0,70 \div 0,85) X_{n+1}(t_0)$$

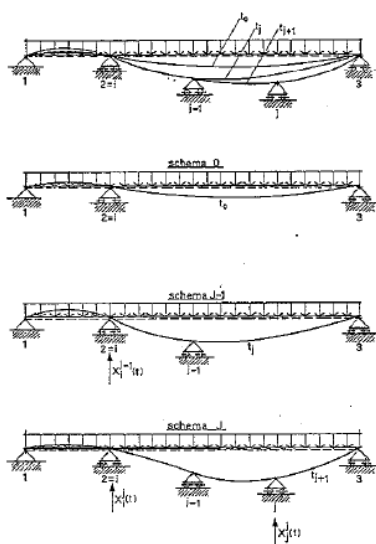
The final value of reaction in restraints n+1 is very close to the value that corresponds to the restraints introduced before the application of the load.

Several construction procedures imply variation of static scheme, but with time t<sub>1</sub> of introduction of new restraints sometimes very different from t<sub>0</sub>.

Then the previous principle should be generalized introducing the variable t<sub>1</sub> > t<sub>0</sub>

The solution is obtained by imposing the congruence starting from  $t_1$  then in the range  $t_0$   $t_1$  the reactions  $X_i(t)$  are the  $m$  pre-existing restraints of the scheme 1 due to the application of load at time  $t_0$  are constant, while the deformations increase proportionally to the creep function, so we have to solve reactions deriving by forces elastic by the effect of the first principle they are constant. This is the effect of scheme 1 and only the deformation increase. Then we introduce a new function  $\xi(t, t_1, t_0)$  that is equal to the integral from  $t_1$  to  $t$  of the relaxation function. This function measures the portion due to creep of difference between the reaction part corresponding to the application of load in scheme 2 at  $t_1$  for load applied in  $t_0$ .

### FIFTH PRINCIPLE OF LINEAR CREEP



The integral function  $\xi(t, t_1, t_0)$  may also be used for homogeneous structures subjected to several variations of static scheme.

The fifth principle is a further generalization and we will introduce several variation of static scheme at different times.

As we can see in the figure we'll have scheme 0 that will be just after the application of the load, then in the scheme  $j-1$  we will have deformation than the application of restraint at time  $t_1$  evaluated in  $t$  and we will have the application of another restraint at time  $t_j$  and so we will have a complex situation to evaluate, but in reality we are quite always in the fifth principle situation.

RH: ambient relative humidity (%)  $f_{cm}$ : compressive mean strength AT 28 days  
 RH<sub>0</sub>: 100%  $f_{cm0}$ : 10 MPa  
 h: hydraulic radius of the element  $t_0$ : age at loading (days)  
 $h = 2Ac/u$   $t_1$ : 1 day  
 with A: transverse section area  
 u: element perimeter in contact with the atmosphere  
 h<sub>0</sub>: 100 mm  
**b<sub>c</sub>(t-t<sub>0</sub>)**: evolution in time of creep

$$\beta_c(t-t_0) = \left[ \frac{(t-t_0)/t_1}{\beta_H + (t-t_0)/t_1} \right]^{0.3}$$

$$\beta_H = 150 \left[ 1 + \left( 1.2 \frac{RH}{RH_0} \right)^{18} \right] \frac{h}{h_0} + 250 \leq 1500$$

Some problems of definition can occur when for example we have a box section bridge: the external perimeter is in contact with atmosphere except for the upper surface which is protected with waterproofing. But should internal perimeter be considered in contact or not? For reason of inspection is possible to access to the box and the air is circulating inside the box but is not the same with external perimeter. As compromise we calculate half of the internal perimeter.

The influence of cement type corresponds to a correction of age at loading  $t_0$

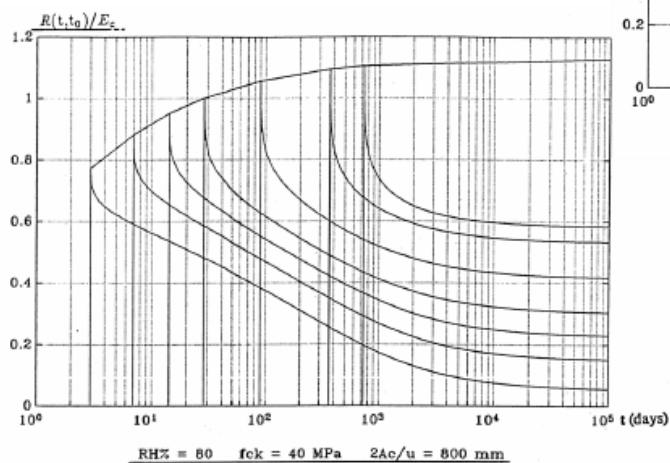
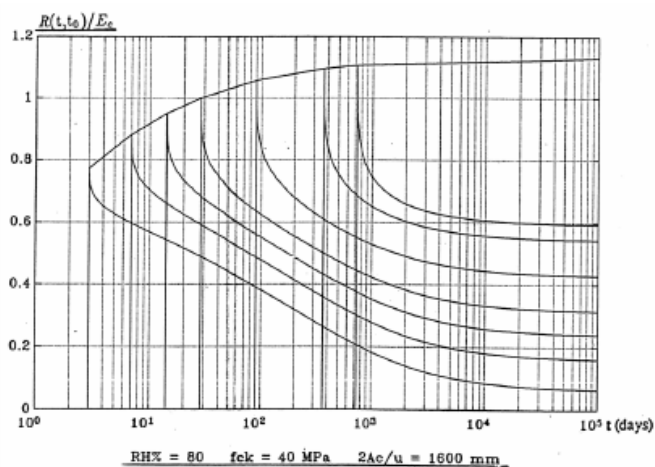
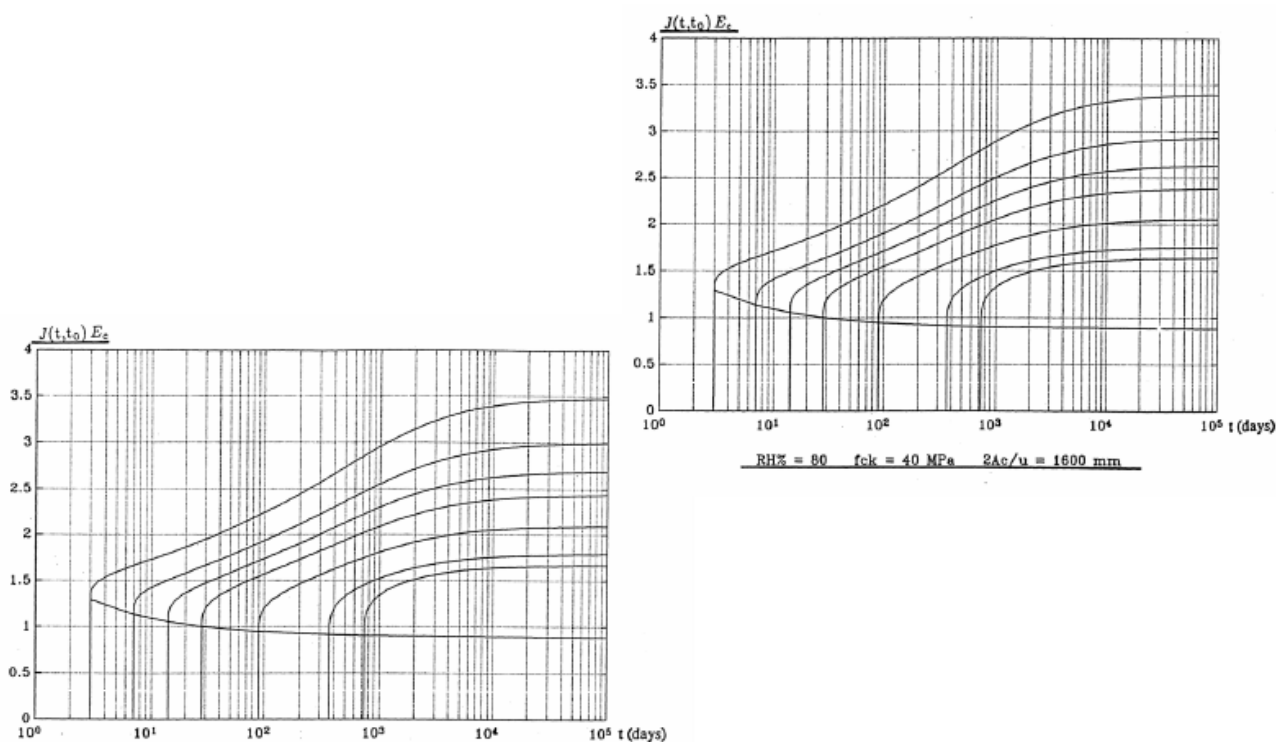
$$t_0 = t_{0,T} \left[ \frac{9}{2 + (t_{0,T}/t_{1,T})^{0.5}} + 1 \right]^\alpha \geq 0,5 \text{ days}$$

with

$$t_{0,T} = \sum_{i=1}^n \Delta t_i \exp \left[ 13,65 - \frac{4000}{273 + T(\Delta T_i)/T_0} \right]$$

Dt<sub>i</sub>: number of days in which T is prevailing  
 T(DT<sub>i</sub>): curing temperature  
 T<sub>0</sub>: 1 °C  
 t<sub>1t</sub>: 1 day  
 a: = -1 slow hardening (i.e. pozzolanic) cement  
 = 0 normal and rapid cement (325/425)  
 = 1 rapid and high strength cement (525)

In term of graphic representation we can see the parameters we have described:



In the first diagram we have the J creep function multiplied by the E elastic modulus in order to have a non-dimensional value as J is the inverse of E. this is a logarithmic scale to have a better representation. Each curve is a different age of loading. **The higher is age at loading, the smaller is the creep deformation** as

i: initial static scheme	M <sub>f</sub> : moment evaluated in the final static scheme
f: final static scheme	M <sub>i</sub> : moment evaluated in the initial static scheme
	ΔM <sub>i</sub> : M <sub>f</sub> - M <sub>i</sub>

For  $t_1 \equiv t_0$  → *3<sup>d</sup> principle of linear creep*

$$\begin{aligned}
 M_\infty &= M_i + \Delta M_i \left(1 - \frac{R(\infty, t_0)}{E_c}\right) = M_i + \Delta M_i - \Delta M_i \frac{R(\infty, t_0)}{E_c} = \\
 &= M_f - \Delta M_i \frac{R(\infty, t_0)}{E_c} = M_f - M_f \frac{R(\infty, t_0)}{E_c} - M_i \frac{R(\infty, t_0)}{E_c} = M_f \left(1 - \frac{R(\infty, t_0)}{E_c}\right) + M_i \frac{R(\infty, t_0)}{E_c}
 \end{aligned}$$

For  $t_1 \neq t_0$  → *4<sup>th</sup> principle of linear creep*

$$M_\infty = M_i + \xi(\infty, t_1, t_0) \Delta M_i$$

then 
$$M_\infty = M_f \xi(\infty, t_1, t_0) + M_i [1 - \xi(\infty, t_1, t_0)]$$

A significant migration of actions forces us to preview a higher resistance compared to the necessary one if the structure was built from the beginning in the final static scheme.

The third principle illustrates that if in a structure at time  $t_0$  just after the application of load and internal or external restraints are added, the previous state of stress changes.

The first extension of the third principle is the fourth principle of linear creep and this one provides a generalization considering the application of the restrains at time  $t_1$  following  $t_0$ . In this case it is necessary to introduce a redistribution function  $\xi$  which expresses the percentage of acquisition of the stresses on the modified static scheme. This function can be evaluated through the numerical solution of the Volterra integration. So the principle of partial acquisition of the modified static scheme is valid for structures considered homogeneous. As we said ordinary structures can be considered homogeneous neglecting the local heterogeneity due to limited variation in the characteristics material.

Then a third generalization takes us from the fourth principle to the fifth principle considering not only one modification of the static scheme but more changes make at different times. Generally we are in the field of fifth principle application.

We see for example for the moment that considering moment  $M_f$  (evaluated in the final static scheme) and  $M_i$  (evaluated on the initial static scheme) and the difference between them  $\Delta M_i$  we can apply the third principle to the evaluation of bending moment.

The same can be done with the fourth principle at time  $t_1$  different than time  $t_0$ .

An important consideration to notice is that according to the third principle and following the second principle of linear creep, the application of impressed deformations on the final static scheme, doesn't change the deformation, but only the stresses. So the deformation that we will have in the final scheme are the same one of the deformation of the first static scheme.

Using the computer analysis the structure may be completely described by the beginning, activating step by step the part of the structure effectively built, in the actual static scheme.

Small altimetric irregularities may be regularized before concreting the key segments introducing impressed deformations to the opposite parts, which static effect will be reduced during the time with relaxation law.

Using prefabricated segments the hammers are built on oleodynamic jacks, to allow the regulation of the deck (both planimetric and altimetric) before fixing it on the definitive bearings.

In case of realization of cast in situ segments, the geometry may be progressively corrected during construction.

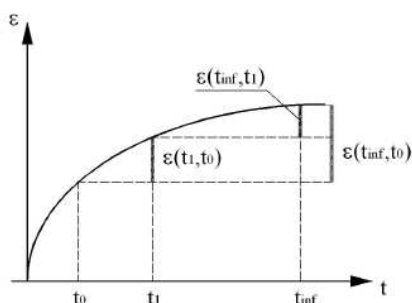
At the moment, the most used procedure is match cast segmental construction

## DEFORMATION AND CUMBERING

Permanents actions:

- Selfweight, top prestressing } initial static schemes (e.g. Hammer construction)
- Bottom prestressing } intermediate schemes (e.g. closing of midspan key segments)
- Pavement, kerbs, barriers etc. } final static scheme

Variable actions: final static scheme



$\delta'$ : deformations in the initial static scheme ( $t_0 < t < t_1$ )

$\delta''$ : deformations in the final static scheme ( $t_1 < t < \infty$ )

It can be written:

$$\delta(t) = \delta' \varphi(t_1, t_0) + \delta'' \varphi(t, t_1)$$

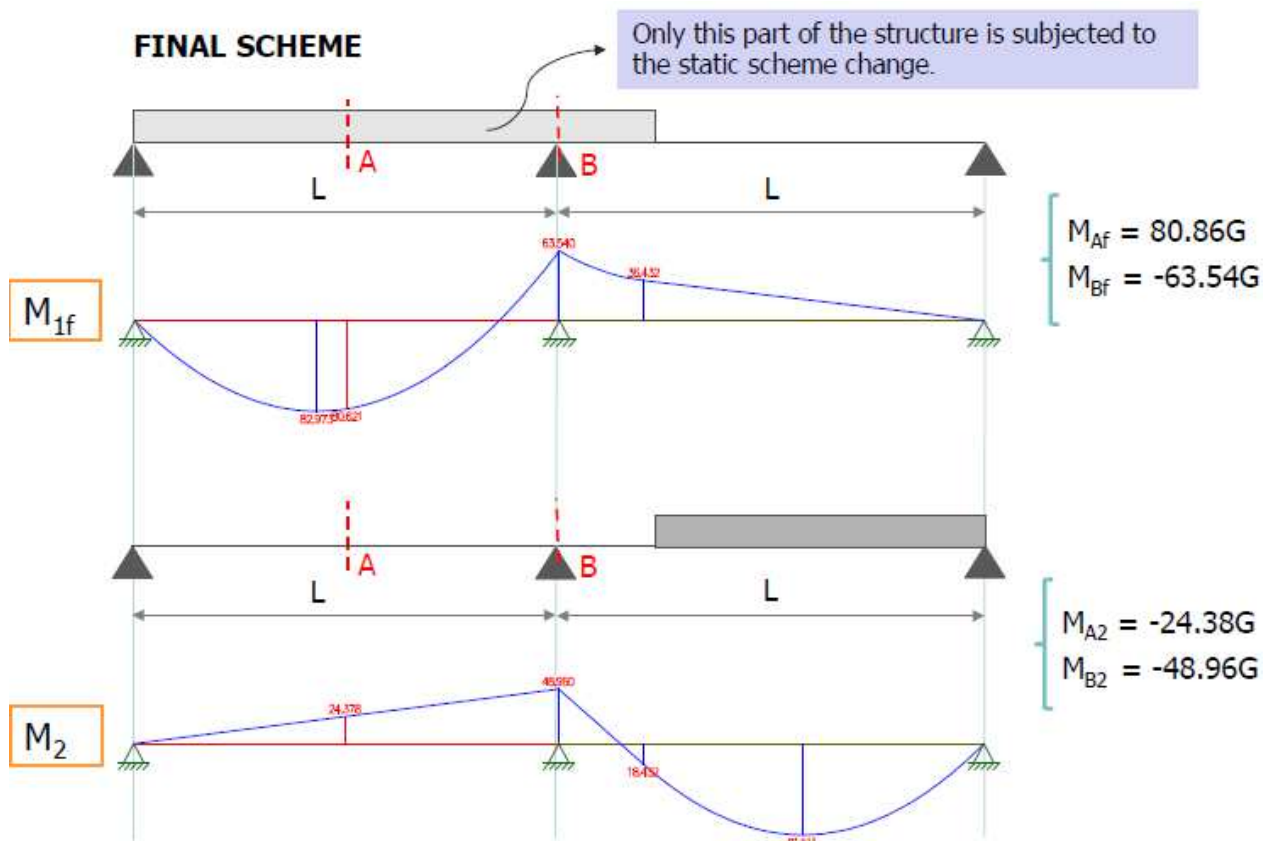
$$\delta_{\infty} = \delta' \varphi(t_1, t_0) + \delta'' \varphi(t_{\infty}, t_1)$$

We introduce the concept of the imposed deformation to the structure to prevent the effect of the viscoelasticity in time, called cumbering.

The permanent action acting on the structure are the self-weight, the top prestressing in the initial scheme, for example in the balanced cantilever technique, then the bottom prestressing to close the structure in the change of static scheme, and the kerbs, the pavement and the barriers are permanent action on the final static scheme, so they are not subjected to the delayed restrain effects.

We can see in the diagram that the sum of the deformations in  $t_1$  and  $t_0$  plus the deformation in  $t_{inf}$  and  $t_1$  is equal to the total deformation evaluated at time  $t_{inf}$  for a load applied at time  $t_0$ .

Following those considerations we can write that:

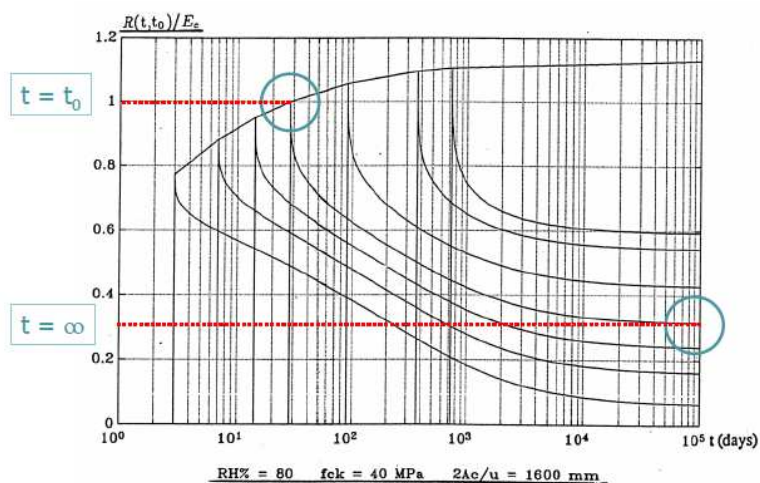


Then we consider the bending moment in the initial scheme but with the introduction of the third support.

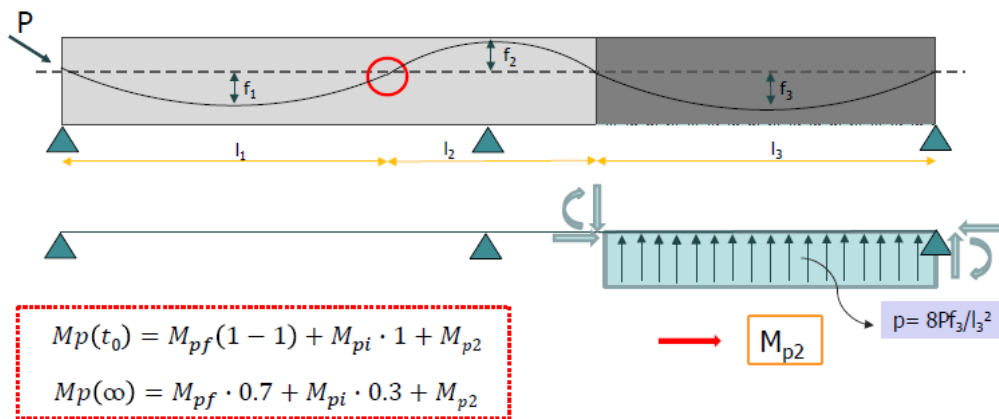
The moments  $M_{1f}$  that we obtain are related only to the part of the first static scheme because the second part of the structure will be born in the final static scheme so will not be subjected to this static scheme, and this is very important to understand.

Then we have to calculate the bending moment arising in the first span but by the effect of the application of the second load because all the effects have to be summed together.

## THE RELAXATION FUNCTION – MC90



Applying the **second phase prestressing**:

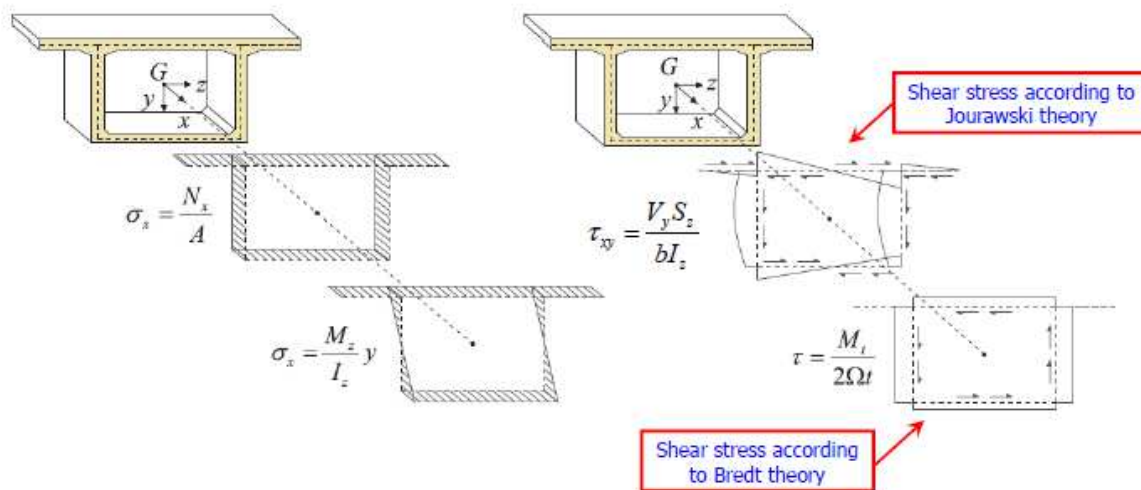


The second phase prestressing, as to the second phase self-weight, is not subjected to the variation of the static scheme.

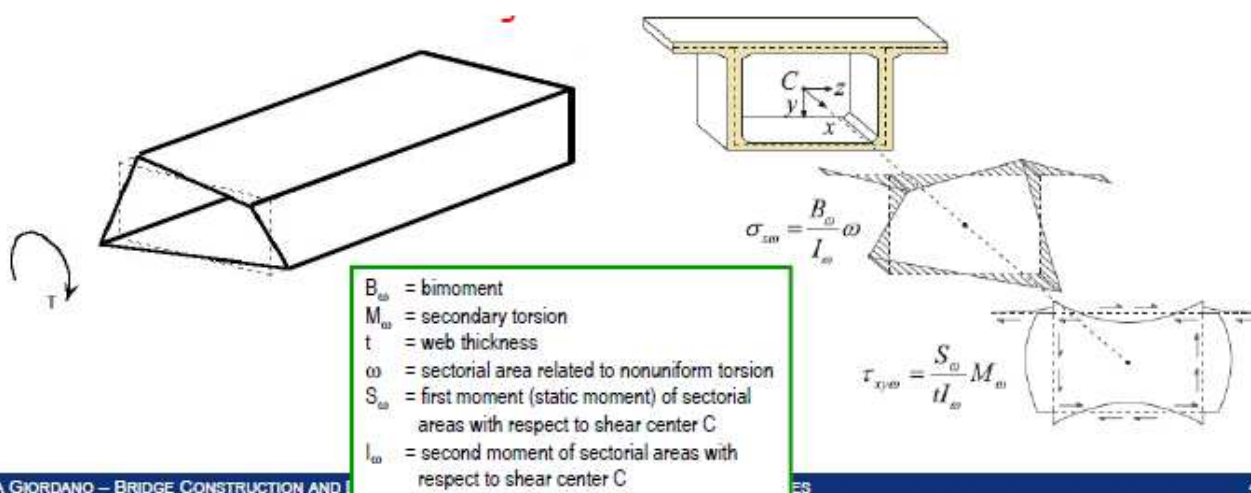
The application of prestressing automatically removes the formwork by prevailing against the self-weight.



On the right we have the distribution of the shear stresses and in particular we have the shear stress according to the Jourawsky theory and that is the shear force  $V_y$ ; the last part of the picture is related to the shear stresses given by the torque  $M_t$  for which the Bredt's theory can be used.



But in addition to the stress state just shown we have other states of stress related with eccentric loads. In particular the first additional state of stress is the **warping stresses** related to the longitudinal displacement that we have in a general section subjected to torque moment; in fact pure torsion of a thin walled section will produce a warping of the cross-section as is shown in the figure for a rectangular section that is free to warp at its ends. However, in practice box section is not subject to pure torsion; wherever there is a change of torque (at a point of application of load or at a torsional restraint) there is restraint to warping, because the 'free' warping displacements due to the different torques would be different. Such restraint gives rise to longitudinal warping stresses and associated shear stresses in each wall of the box



So warping in box section can be neglected in fact the box section has an high torsional rigidity, so warping results little compared with other stresses, for instance the tangential stress due to Bredt's behaviour.

The warping stress is relevant in open sections.

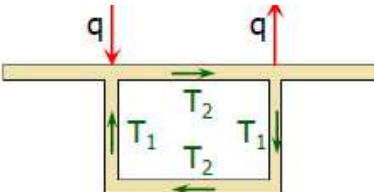
in order to simplify the problem I can think that this load condition is given by the sum of two different load conditions:

1. in the first load condition I apply a force  $q$  that is an half of the original one, one on the left web and one on the right web; so in this case my structure will be subjected only to bending and shear.
2. in the second load condition, in order to have the original load condition I have to add a downwards load  $q$  on the left and an upwards load  $q$  on the right web.

The sum of those two load conditions gives the original load condition but in this way I decomposed the initial internal action in two sets of internal actions.

The subject of this lesson is to study the torsion and the way in which the box section answer to the application of this torsion.

We can study the structure's response using the Bredt's formulation according to which we have a shear flow in the wall of the section and this shear flow can be called with  $t$  and is constant in several walls and is given by the torque divided by two times the area inside the mean line.

$$\begin{aligned}
 t &= \tau_i \cdot s_i = \frac{M_x}{2 \cdot b \cdot h} && \leftarrow \text{Shear flow} \\
 T_1 &= t \cdot h = M_x / 2b && \leftarrow \text{Wall shear forces} \\
 T_2 &= t \cdot b = M_x / 2h = T_1 b / h
 \end{aligned}$$


The shear flow is also equal to the tangential stress  $\tau_i$  where  $i$  is the number of the wall that I am considering multiplied by the thickness of the single wall.

If we know the shear flow we can evaluate the wall shear forces and we can find the forces  $T_1$  on the web and the forces  $T_2$  on the slabs. Of course in this case the two webs have the same length so the forces are equal and the same for the slabs.

Let's consider now an elementary segment of length  $dx$  and put in evidence the differential of the internal actions due to an infinitesimal variation of the torque. And that is on this element I apply a force  $qdx$  downwards on one side and upwards on the other side and so there is a variation of the torque along the length  $dx$ . This variation is  $dM_x$  we can also evaluate the differential between the internal forces  $dT_1$  and  $dT_2$ .

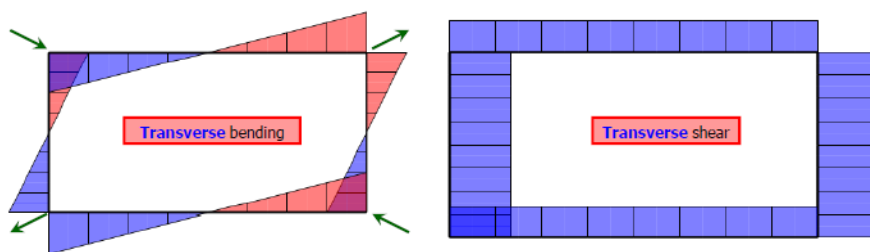
If I consider a single wall, for instance the left web, I can see that on this web I apply a downwards force  $qdx$  and an upwards force  $dT_1$ .

As we can see the system A corresponds to apply the torsion (Bredt solution) but with every plane element equilibrated and so we don't have distortion because the forces are applied exactly how they are with the Bredt solution.

The system B is instead related to a self-equilibrated system of forces, in fact its resultant is equal to zero both for vertical and horizontal direction, but these forces cause distortion.

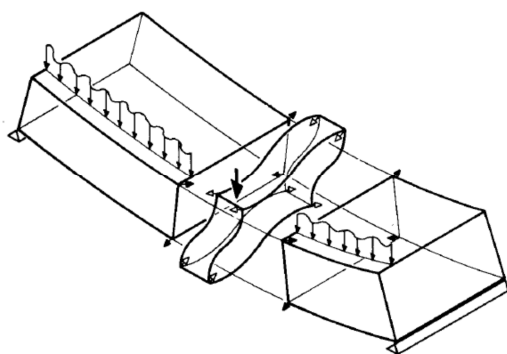
The resistant mechanism B is a combination of bearing capacity both in longitudinal and transverse direction, and that is, in addition to longitudinal stress  $\sigma_x$ , we have also transverse stress ( $\sigma_y$  in the web and  $\sigma_z$  in the slabs) due to transverse bending moment.

In fact, if we consider a bridge segment cut off from the remaining part of the bridge and loaded with the self-equilibrated diagonal forces, we get the following internal actions



As we can see we have a tensile stress at the bottom fibre of the bottom slab and compressive  $\sigma_z$  stresses at the top fibre of the bottom slab.

But actually the segment is not isolated from the remaining part of the bridge. If we have a concentrated wheel load, the unloaded part of the bridge wouldn't want to distort its self but it will because of displacement compatibility.



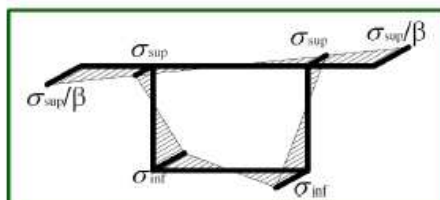
So if I think that this segment is isolated by the remaining part of the bridge, it will have this deformation, but actually the segment is continuous with the remaining part of the bridge that in part will constraint this deformation but in part this deformation happens. So a part of the distortional forces (the diagonal forces) will be beard in the transverse direction from the transverse frames shown above, and the remaining part will be beard in the longitudinal direction.

Then the transverse internal action will be less than the one evaluated before and that is considering only the transverse frame, but we also have longitudinal stresses and the evaluation of the part of the load that is carried in longitudinal direction and the part carried in the transverse direction is absolutely not easy.

- ➔ The equation of BEF can be solved finding the displacement  $y_A(x)$  and the bending moment  $M(x)$
- ➔ Normal stresses at the top and bottom of webs are

$$\sigma_{sup}(x) = -\frac{M(x)}{I_w} \frac{\beta^2(\alpha_s + 3)}{\alpha_s + \alpha_i\beta^2 + 6\beta^2} h$$

$$\sigma_{inf}(x) = \frac{M(x)}{I_w} \frac{(\alpha_s + 3\beta^2)}{\alpha_s + \alpha_i\beta^2 + 6\beta^2} h$$

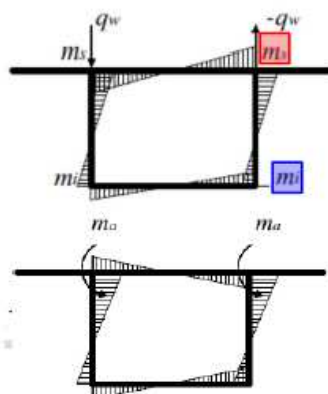


from which longitudinal normal stress diagram of box section is known

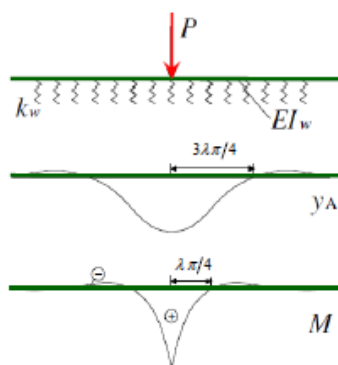
- ➔ Transverse bending moment are

$$m_s(x) = -h \rho \frac{3+r_s}{6+r_s+r_i} y_A(x)$$

$$m_i(x) = h \rho \frac{3+r_i}{6+r_s+r_i} y_A(x)$$



According to the solution of a BEF (see for example Barber J. "Beams on Elastic Foundations" in "Intermediate Mechanics of Materials"), the load effects are maximum at the point load and decrease very quickly. Consequently external restraints (and that is diaphragms) affect the solution only if their center to center distance is comparable with the characteristic length  $\lambda$ .



$$\lambda = \sqrt[4]{\frac{4EI_w}{k_w}}$$

Characteristic length [kN/m/m]

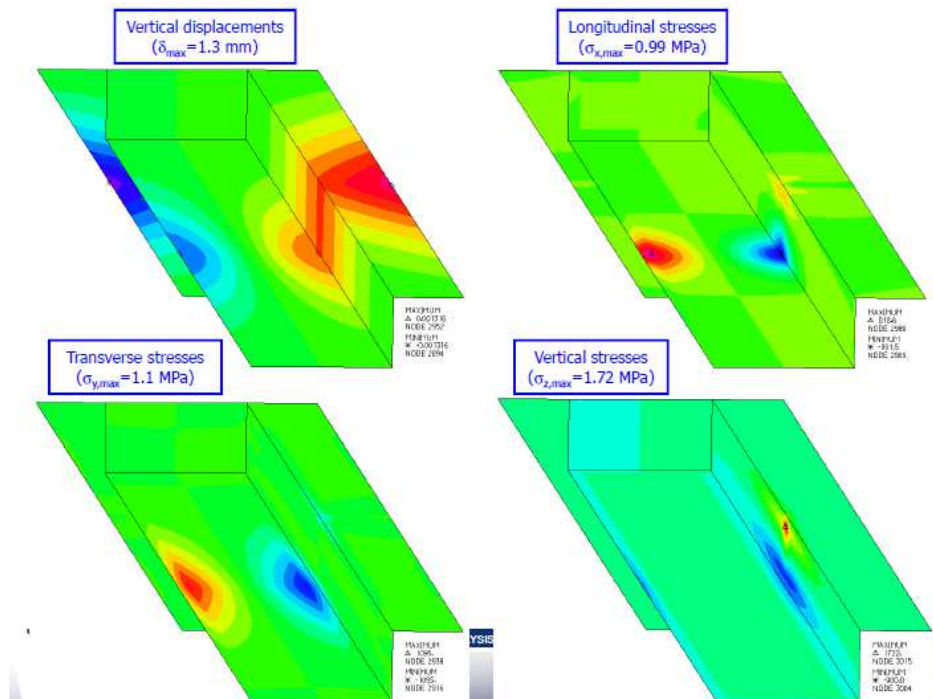
This approach can be used to evaluate the influence of many parameters. In the following we will refer to FEA of typical situation in order to highlight the behavior of the different box girder solutions

Form a quantitative point of view we have the displacements and the stresses given here:

The maximum vertical displacement is of course at the application point and will be negative on the right web and negative on the left web.

The longitudinal stresses of course see the maximum value at the application point. While transverse stresses are the horizontal stresses in the bottom slab and in the upper slab, and the maximum is at the bottom.

Finally the maximum vertical stress is on the web.



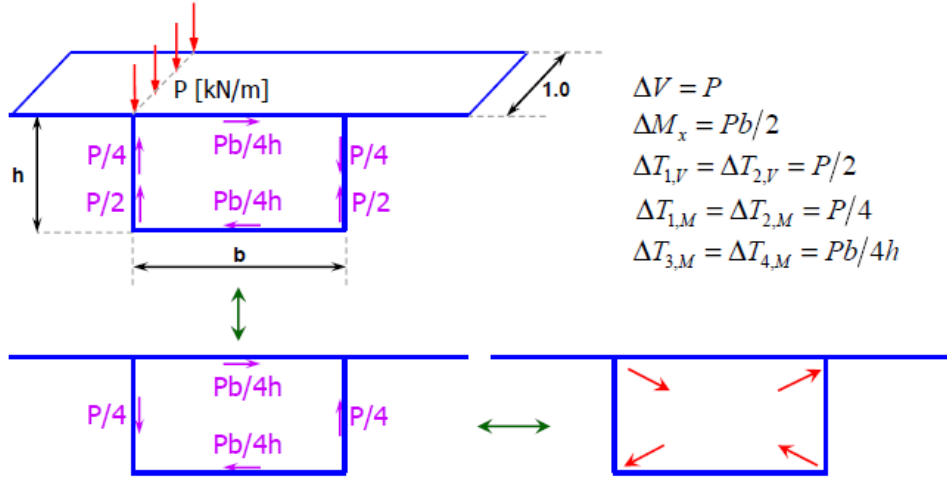
If we change the point of application of the force and for instance we apply the force at  $x = 0,25 L$  (15m from the support) we obtain similar results because the longitudinal stress will be this time equal to 1,04 Mpa that is more or less the same.

Of course in this case we see only the effect of the tandem system and we should add other loads, so we will appreciate that we will have more or less half than this stress.

So in order to simplify the design we can propose a procedure without using the beam on elastic foundation and without using a FEM approach with shell elements that is very complicate to manage in real life. From the numerical example we can say that longitudinal stresses  $\sigma_x$  can be neglected considering that maximum torque and maximum longitudinal bending moment are related to different combination of actions. And that is when I have the maximum bending moment (maximum longitudinal stress) I don't have a big torque because I have to consider the load on the complete width on the deck.

❖ Solve the model and evaluate the transverse reinforcements from the transverse internal actions

➔ Application to a simple load condition



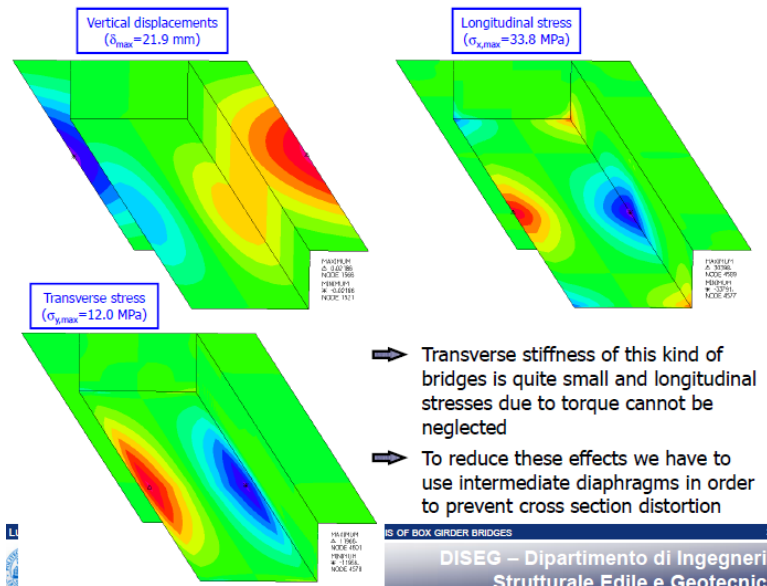
### OPEN STEEL BOX GIRDER WITH RC SLAB

We can consider the same structure of the previous chapter, but this time the webs and the bottom plates are realized with structural steel with a thickness of 20 mm. the tandem system is on the lane 1 with an eccentricity of 4 m and the notional lane with an eccentricity of 1 m, we have so a maximum applied torque of

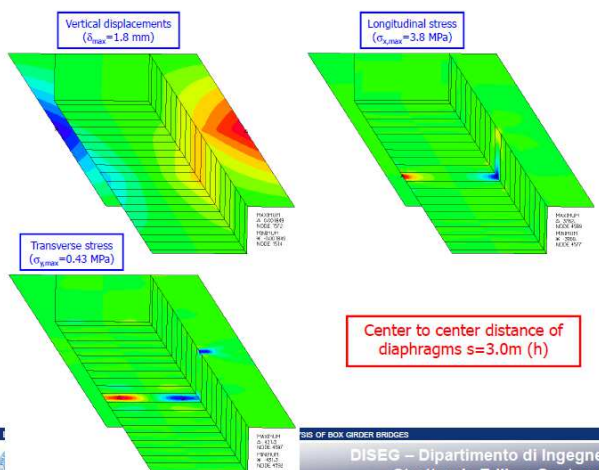
$$T = 600 \cdot 4 + 400 \cdot 1 = 2800 \text{ kNm}$$

To simulate this torque we consider a force qa of 310 kN/m on a length of 1,5 m applied at  $x=L/2$ .

We are going to see the differences of solution compared with the RC solution seen before.

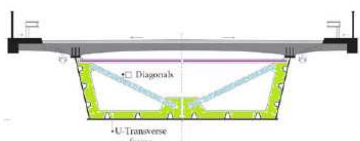


Now we reduce the centre to centre distance of the diaphragms:



The vertical displacement is more or less the same we had in the previous case, but the longitudinal stresses are different because now the maximum is 3,8 MPa.

Usually intermediate diaphragms are realized with **U-transverse frame** and **diagonals**. Furthermore, the **upper steel member** used to improve the torsional behavior before the concreting of the slab can be used to complete a steel transverse frame. In this way the transverse slab is not used to prevent the distortion but only to bear the local vertical loads



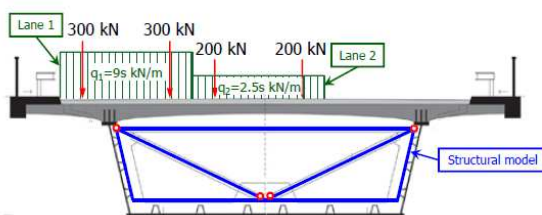
In conclusion the design procedure can be summarized in the following points:

- ❖ Using diaphragms with center to center distance  $s = h + 2h$ , longitudinal stresses due to distortion may be neglected
- ❖ RC slab can be considered simply supported by the main steel beam and transverse reinforcement can be evaluated as in girder bridges

❖ Diaphragms elements (U-frame, diagonals, etc.) can be calculated considering the variation of longitudinal shear ( $\Delta V$ ) and torque ( $\Delta M_x$ ) due to the loads applied on a segment with length  $s$  (center to center distance of diaphragms)

$$\Delta V = \sum P_i + w \cdot s \sum q_j + \Delta G \quad \Delta M_x = \sum P_i \cdot y_i + w \cdot s \sum q_j \cdot y_j$$

( $i=1$ +number of point forces,  $j=1$ +number of lanes,  $w$ =lane width,  $\Delta G$ =segment self-weight + permanent loads,  $y$ =load eccentricity)

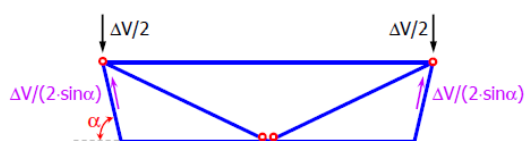


To highlight the difference with the rc solution, we have considered a segment with unit length because we considered the dispersion of the point force on the slab; here we don't consider the dispersion of it.

❖ To emphasize some particular behavior, let's consider the structural model with  $\Delta V$  and then with  $\Delta M_x$ .

Considering  $\Delta V$  it can be appreciated that, if the webs are not vertical, variation of web shear forces are greater than  $\Delta V/2$  and then we have to take this into account in the longitudinal shear verification (of course also the web length is greater than the cross section height).

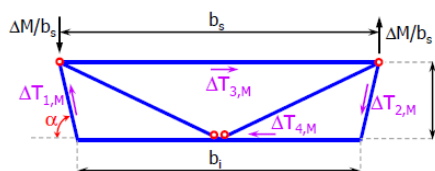
Furthermore, the top horizontal member is subjected to traction and the bottom one to compression



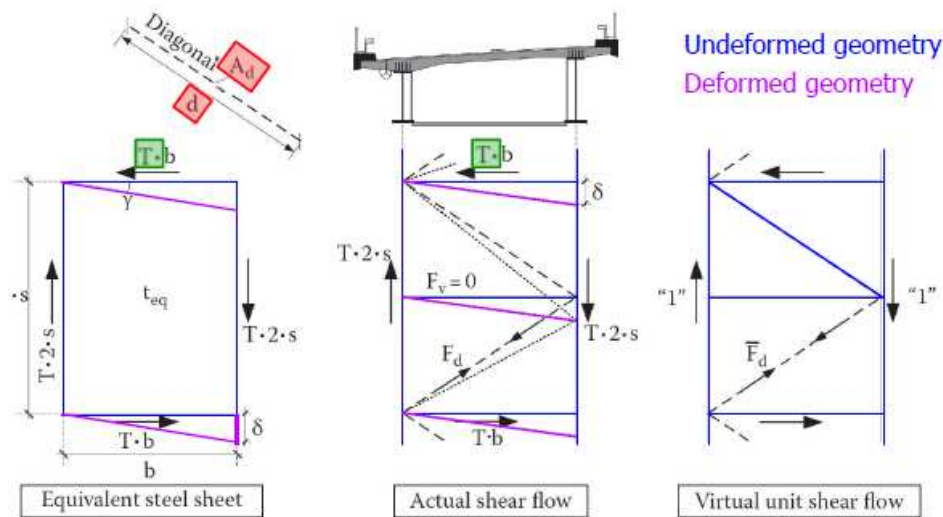
Considering  $\Delta M_x$  we can follow the procedure already used for concrete box cross section

$$b = (b_s + b_i)/2 \quad \Delta T_{3,M} = \Delta M_x \cdot b_s / (2bh)$$

$$\Delta T_{1,M} = \Delta T_{2,M} = \Delta M_x / (2b \sin \alpha) \quad \Delta T_{4,M} = \Delta M_x \cdot b_i / (2bh)$$

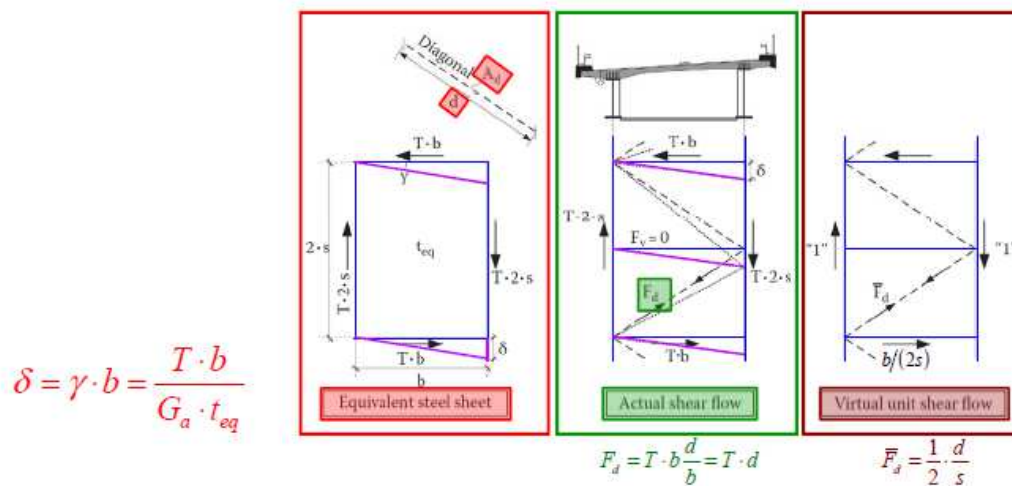


Solving the structural model, the internal actions in intermediate diaphragms members are evaluated



This real structure in the middle is transformed in an equivalent steel sheet on the left with a thickness  $t_{eq}$ . In this solution we have a shear flow  $T$  and then the shear forces in this plate are equivalent to  $T$  multiplied by the width of the element  $T \cdot b$  and  $T \cdot 2s$  where  $s$  is the distance between the diaphragms. This plate subjected to this shear flow has a certain angular deformation  $\gamma$  due to this shear flow. This shear deformation on the actual structure leads to a modification of the length of the diagonal, the upper one will be lengthen because it will be in tension in this case whereas the lower will be shorten because subjected to compression.

In order to evaluate these forces in the diagonals and the equivalent thickness in the steel sheet we use the virtual work principle and that is we use a virtual structure (the one on the right) subjected to a virtual shear flow: on the longitudinal element we use a virtual force equal to 1.



$$\delta = \gamma \cdot b = \frac{T \cdot b}{G_a \cdot t_{eq}}$$

$$1 \cdot \delta = \sum \bar{F}_d \cdot \delta_d = \sum \bar{F}_d \frac{F_d}{E_a \cdot A_d} \cdot d_i = \bar{F}_d \frac{F_d}{E_a \cdot A_d} \cdot 2d \rightarrow \delta = \frac{T \cdot d^3}{E_a \cdot A_d \cdot s}$$

$$\frac{T \cdot b}{G_a \cdot t_{eq}} = \frac{T \cdot d^3}{E_a \cdot A_d \cdot s} \rightarrow t_{eq} = \frac{E_a \cdot A_d \cdot s \cdot b}{G_a \cdot d^3}$$

Equation valid only when  $A_y \gg A_d$   
( $A_y$  = bottom flange area)



All the solutions that we can have with box girder have been analysed, as for orthotropic box girders we can apply the same solution that we found on the open steel box with RC slab, in fact also for orthotropic steel decks we need intermediate diaphragms at a centre to centre distance between  $h \div 2h$ .

### INTERACTION BETWEEN LONGITUDINAL SHEAR AND TRANSVERSE BENDING IN THE DESIGN OF THE WEBS

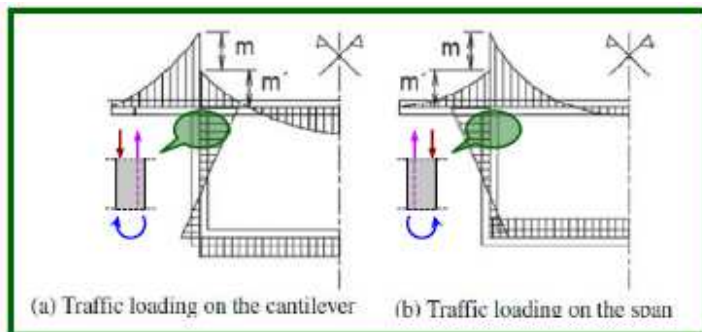
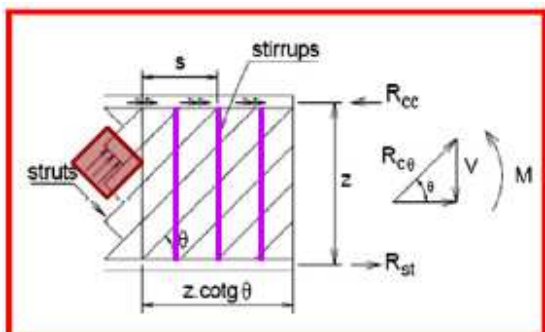
This is an interesting problem in concrete box girder, in particular this problem is related to the interaction between internal actions coming from different types of analysis. In fact, in order to simplify the design, box girders are analysed with two different models:

- the longitudinal model that allows us to evaluate axial forces, bending moments, longitudinal shear forces and torsion on the section;
- the transverse behaviour is analysed with a second model in order to find the transverse bending moment.

Both these effects lead to a vertical reinforcement and concrete stress. In fact due to the longitudinal shear we need of vertical reinforcements in the webs and that is we need stirrups, but this vertical reinforcement is used also to carry the transverse bending moment. As we can see in the two pictures on the right below, the distribution of transverse bending moment in a box section depends on the position of loads on the top slab, and so we can have tension part inside the box or at the external part; if the load is mostly on the cantilever the tension part is inside the box, if the load is mostly between the webs is the opposite. Then the stirrups are subjected also to traction due to the transverse bending moment. About concrete we have that due to the longitudinal shear we have an inclined stress in concrete of the angle  $\theta$  that can be evaluated with the resisting model that we used to carry the longitudinal shear. But also transverse bending gives compression in the web and that is we have two states of stress in the web:

- the first one related to a compressive stress inclined of  $\theta$  due to the longitudinal shear
- the second is related to a vertical stress due to transverse bending moment

of course these two states of stress work together and we have to consider the interaction between them.



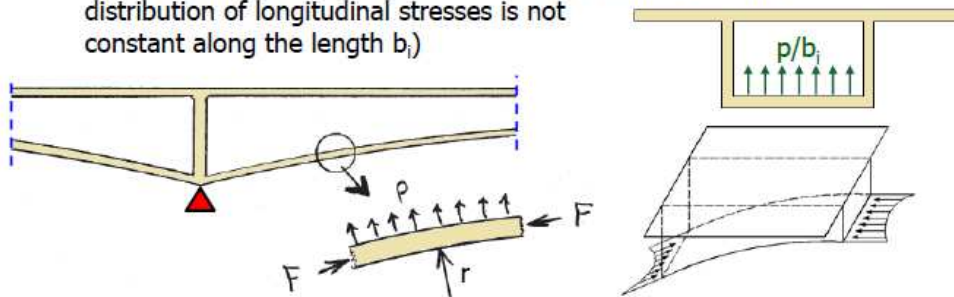
We can introduce in the web a non-symmetric reinforcement, but usually we use a symmetric one.

## TRANSVERSAL ACTIONS INDUCED ON THE BOX GIRDERS AT THE VARIABLE DEPTH

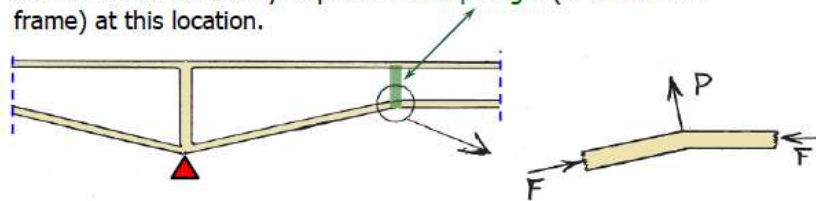
Another particular issue related to the transverse analysis of a concrete box girder is this one.

When a bridge deck has variable depth, the bottom slab will be subject to out-of-plane forces.

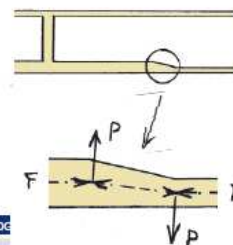
If the variation of depth is circular or parabolic, the compression in the bottom slab creates a distributed force normal to the slab  $p = F/r$  where  $F$  is the force in the slab due to all loads including prestress, and  $r$  is the local radius of curvature of the bottom slab. This force must be carried in transverse bending of the slab (actually, the distribution of longitudinal stresses is not constant along the length  $b_i$ )



If the bridge deck has a linearly haunched profile, the forces in the bottom slab either side of the angle change give rise to an out-of-plane resultant. It is quite likely that the bottom slab will not be strong enough to carry this force in transverse bending, in which case it will be necessary to provide a **diaphragm** (or a stiffened frame) at this location.



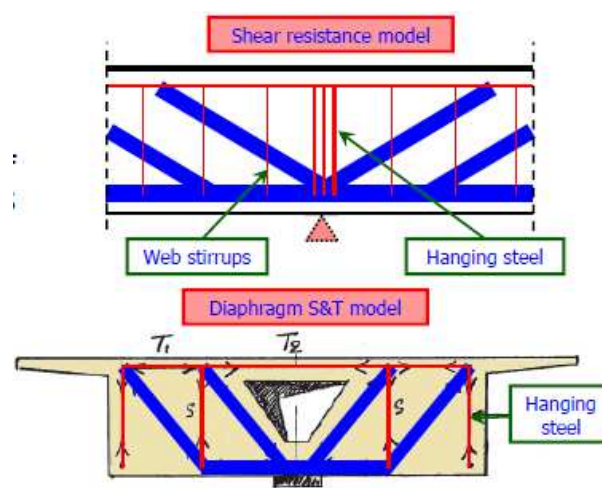
Finally, if the bridge soffit is flat but changes in thickness, out-of-plane local forces in the bottom slab are created at changes in its thickness



If we use the solution of the central column we don't have a torsional constraint in the section and so the rotation about the longitudinal axis of the section is free, whereas in the other solution or when we have two bearings inside the webs, the rotation about the longitudinal axis is not free and we have an external torsional constraint.

At the USL shear force is carried by webs according to the truss model. The shear force is carried by the webs and at the internal support we have our inclined compressive stress field that go to the support. If the support is not at the webs the diaphragm need to traslate the shear that are at the webs to the position of the web. To do this we need of a hanging steel and that is the shear force on the left and on the right of the bearing has to be hanged by means of these hanging steel ans do we need of freinforcement, because the area of the central reinforcement (the vertical one in the second picture) is given by the sum of the shear force on the left nd on the right divided by the yielding strenght of the reinforcement. .so we have a lot of steel area in a very limited portion of the structure, so we have a lot of reinforced bars. When this force is hanged by this hanging steel we have this force on the top of the support diaphragm and we can use the strut and tie model that we have in the picture. The strut brings the force to the central bearing but we need of ties in the upper part of the diaphragms. The tension forces  $t_1$  and  $t_2$  may be carried by the reinforcement or by prestressing if they are too big to be carried by ordinary reinforcement (remember that reinforcement has to be anchored behind the nodes of the truss).

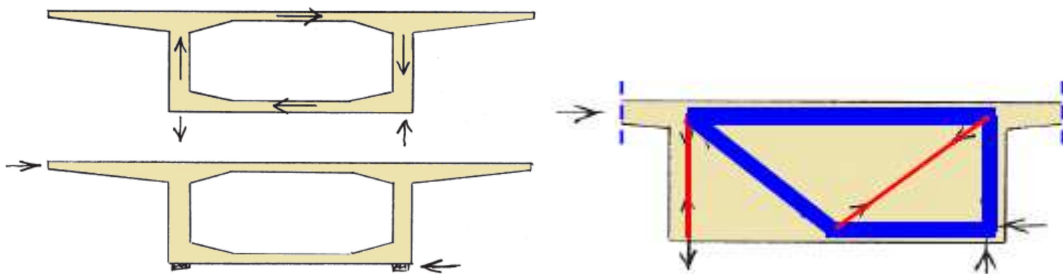
The tension force  $S$  represents the shear in the diaèhragm and requires reinforcement that is the shear stirrups.



### TRANSFER TORQUE AND HORIZONTAL FORCES TO THE BEARINGS

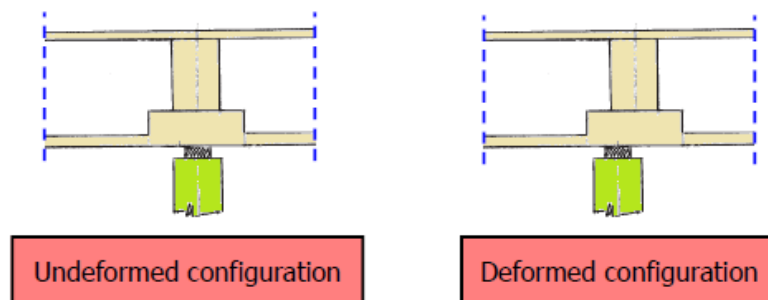
Finally the support diaphragms are used to transfer torque moments and horizontal forces to the bearings, the web shear forces due to torque can be added to the web shear forces due to the shear; then the design procedure is the same as the previous one.

The horizontal components in the slabs tend to cause the box to sway lateral forces such as wind and earthquake acting on the deck and on traffic, and traffic-induced loading such as centrifugal force also cause sway forces that may cumulate with the torque. In other words we need a strut and tie that brings the horizontal force to the bearing in the bottom part.



### LONGITUDINAL DISPLACEMENTS

The longitudinal displacements can affect the design in fact for long continuous bridge it is essential to take in consideration the movement of the bridge deck due to the length changes caused by temperature, creep and shrinkage. These displace the diaphragm with respect to the bearings, and require either that the diaphragm is thickened so that a sufficient width always remains engaged. And that is I have a undeformed configuration in this situation on the left that is the initial theory situation, then when I have the longitudinal displacement the bearing remains in the same position but the position of the diaphragm changes and the reaction in outside the diaphragm, so it should be thick enough that the reaction remains inside the support diaphragm.



⇒ With a back-analysis of realized bridges, we have an estimation of the material quantities:

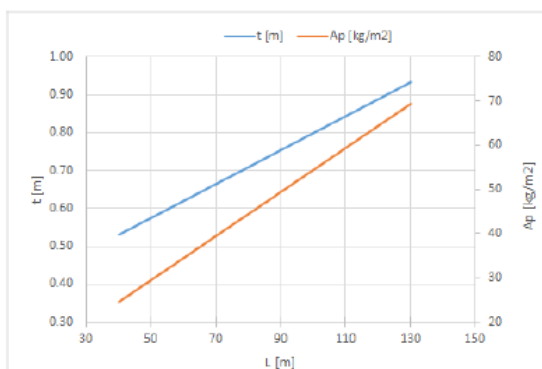
❖ Concrete (thickness of equivalent slab, L in m)

$$t = 0.35 + \frac{0.45L}{100} \quad [m^3/m^2]$$

❖ Prestressing steel (L in m)

$$A_p = 4.5 + 0.5L \quad [kg/m^2]$$

❖ Ordinary reinforcement:  
150 kg/m<sup>2</sup>



We can see from the upward picture that for a typical span length equal to 50m we have an equivalent thickness of 57cm and for a span length of 130m we have an equivalent thickness of more or less 95cm with a linear variation. Also the amount of prestressing steel presents a linear variation with the increasing length of the span.

The amount of the ordinary reinforcement is not a function of the span length because the most part of the ordinary reinforcement is placed in transverse direction.

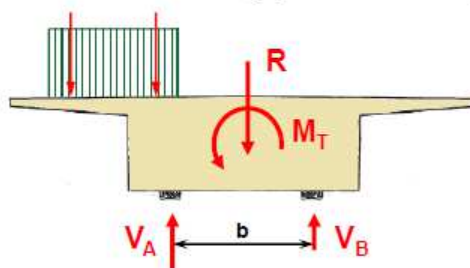
⇒ Definition of span length: let  $L_1$ =lateral span length,  $L_i$ =intermediate span length

❖ Optimum ratio between lateral and internal span:  $L_1/L_i = 0.75$   
(sagging moment in lateral and intermediate span roughly equal, as well as hogging moment at the first and intermediate piers)

❖ If  $L_1/L_i < 0.5$  check uplift of abutment bearings, also considering torque

$$V_A = R/2 + M_T/b$$

$$V_B = R/2 - M_T/b$$

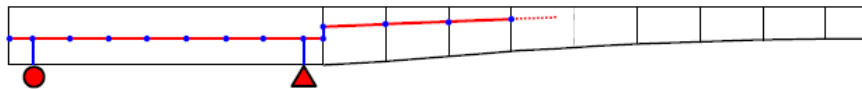


❖ If  $V_B < 0$  modify the piers distribution or consider a counterweight or increase b

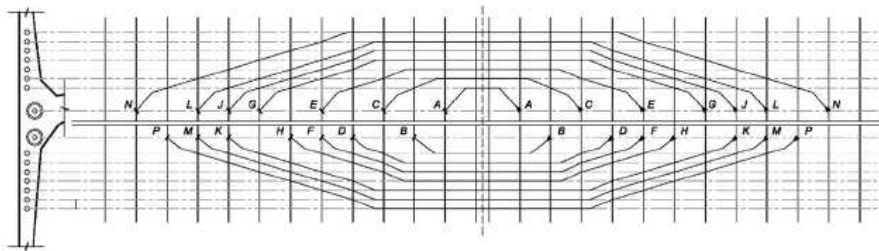
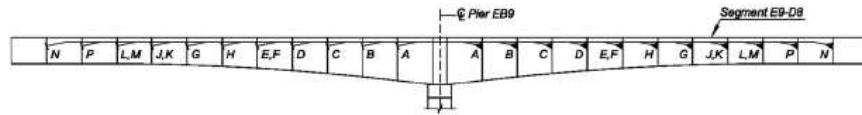
Particular attention should be paid when the ratio between  $L_1$  and  $L_i$  is lower than 0,5 because in this case an uplift of abutment bearings may occur; in fact at the abutment bearings we have a vertical reaction and a torque one, this vertical reaction is divided in two equal parts between the bearings if we are in a symmetric condition whereas the torque reaction is adsorbed with two vertical reactions of opposite directions, one upwards and the second downwards. In this way we have those two values for the bigger and the lower reactions. If  $M_T$  is big enough compared to R then  $V_B$  can be less then zero, but

## LONGITUDINAL DESIGN OF SINGLE-CELL BOX

- ➔ The FEA is performed with a single-line elements representing the whole cross section at the centroid line
- ➔ In segmental construction consider at least a finite element for each segment
- ➔ Deck with variable depth can be analyzed using the mean section properties of each element (but resistance have to be evaluated considering the actual depth of the section)
- ➔ Sudden variation of geometry should be taken into account with rigid link, as well as the actual bearings position



- ➔ Number and layout of tendons should be designed to meet the safety verification both during construction and service, considering ULS and SLS
- ➔ Pay attention to friction losses in precast segmental construction: tendons have little vertical deviations but large horizontal deviations that have to be added to the vertical one



For this kind of construction method tendons have little vertical deviation but have horizontal variation that has to be added to the vertical one.

In the picture we can see a typical layout of tendons in a cantilever construction. This is an hammer with their top tendons. As we can see the vertical deviation is quite little, but in addition we have a deviation in the horizontal plane (the plane of the slab). So in this kind of section the horizontal deviation is more important than the vertical one and both of them have to be considered.

Live loads should be applied considering their actual eccentricity in such a way that torque is properly evaluated, for instance if we have this box section and this finite element placed at the centroid of the cross section, then if I have this point load with an eccentricity  $i$ , I will apply on my model a vertical force  $P$  and a torque moment  $M_x$  equal to  $P$  multiplied by  $e$ , so in this way I can evaluate in a correct way the distribution of torque on the structure. Then I have to give a torsional constant to this finite element that is the one written.

## PRESTRESSING

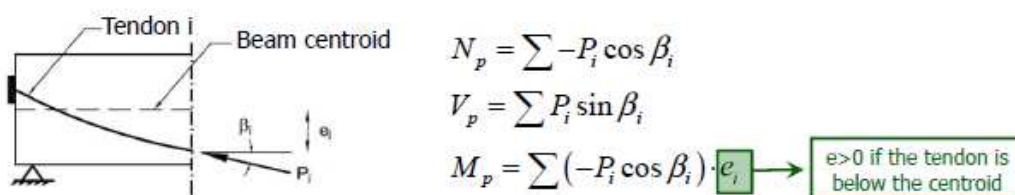
One of the most important issue in longitudinal analysis is the evaluation of total prestressing area and tendons layout. Prestressing must be design in order to satisfy the structural verification both during construction and in service.

Usually, prestress is evaluated in order to satisfy the design criteria at SLS (for instance fixing  $\sigma_{ct} \leq f_{ctm}$  at the frequent combination of action). Then, using the prestress evaluate at SLS, verification at ULS is performed, eventually adding ordinary reinforcement to meet the strength verification

This procedure cannot be applied in precast segmental construction, as there is not ordinary reinforcement at joints; generally, for this type of bridges, the governing design scenario is the ultimate one

In general, internal actions due to prestress consist of two components: primary component and secondary component.

Primary component can be evaluated knowing the force  $P$  and the position (and inclination) of the tendons (remember that  $P$  changes section by section and in time). So we have to perform this verification for each section and for initial and final time.



In determinate structure we have only primary effects and then internal actions due to prestress can be easily evaluated. Because knowing the position and the force in each tendon I can evaluate the primary effect of prestressing, that in determinate structure is the only one.

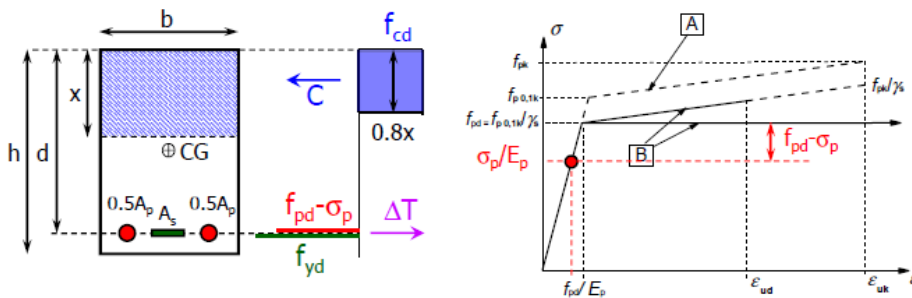
In indeterminate structures, prestress results in displacements that are not compatible with the restraint; then external reactions and a new distribution of bending moment take place. Although these additional moments are termed 'secondary', they cannot be ignored as they can often have a substantial effect on the distribution of stress in a continuous member.

➔ About the ultimate moment capacity, is more convenient consider the effects of prestress within the effects of loads ( $N_p=N_{p1}+N_{p2}$ ,  $M_p=M_{p1}+M_{p2}$ ):

$$N_{Ed} = \gamma_G N_G + \gamma_P N_P + \gamma_{Q,1} N_{Q,1} + \sum_{j>1} \gamma_{Q,j} \psi_{0,j} N_{Q,j}$$

$$M_{Ed} = \gamma_G M_G + \gamma_P M_P + \gamma_{Q,1} M_{Q,1} + \sum_{j>1} \gamma_{Q,j} \psi_{0,j} M_{Q,j}$$

and evaluate the moment capacity considering the further strength of the prestressing steel beyond the tension  $\sigma_p$  that give  $N_p$  and  $M_p$



Assuming that: the ULS deformation is at the concrete side ( $\epsilon_c = \epsilon_{cu2}$ ); the prestressing steel is yield ( $\epsilon_p = \sigma_p/E_p + \Delta\epsilon_p \geq f_{pd}/E_p$  where  $\Delta\epsilon_p = \epsilon_{cu2} \cdot (d-x)/x$ ); the ordinary steel is yield; and neglecting the stress increment due to strain hardening (curve B of the previous slide) we have:

$$C + \Delta T = \frac{N_{Ed}}{0.8 \cdot b \cdot x \cdot f_{cd}} \rightarrow x = \frac{\Delta T - N_{Ed}}{0.8 \cdot b \cdot f_{cd}}$$

$$\Delta T = A_p \cdot (f_{pd} - \sigma_p) + A_s \cdot f_{yd}$$

$N_{Ed} < 0$  for compression

$$M_{Rd} = C \left( \frac{h}{2} - 0.4x \right) + \Delta T \left( d - \frac{h}{2} \right)$$

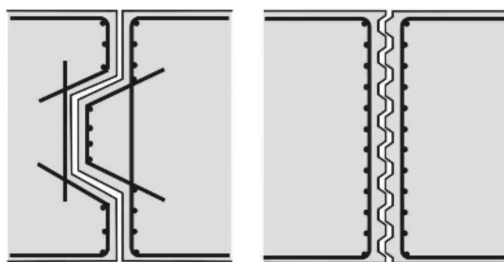
➔ Some further considerations should be done for unbonded tendons. In this case  $\Delta\epsilon_p \neq \epsilon_{cu2} \cdot (d-x)/x$  and is difficult to evaluate. EC2 suggests to use 100 MPa or to neglect the resistance contribution of unbonded tendons. In any case  $x$  will be less than for bonded tendons, as well as  $M_{Rd}$ . In conclusion, at the ULS, unbonded prestressing is less efficient than the bonded one (but allows inspection, maintenance and replacement of tendons)

## JOINT DESING

In precast segmental bridges, the connection interface between segments can be achieved using different types of joints. This element is of critical importance for the following reasons:

- geometric accuracy (the segments fit accurately with the alignment once erected)
- structural safety (load transfer between adjacent elements)
- durability (e.g. watertightness)



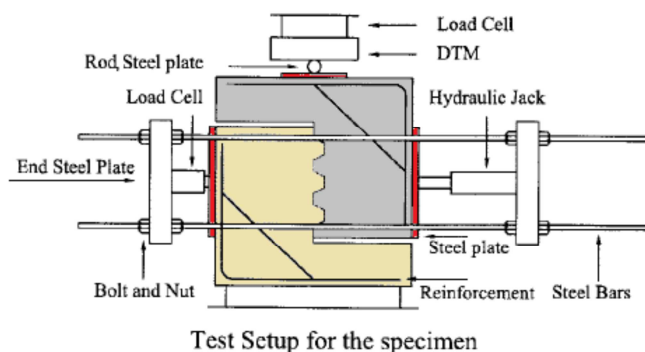


Early segmental bridges were built with a few single large reinforced keys in the web section. The main disadvantage of this joint type is the concentrated load transfer and the resulting high local stresses.

Current practice is to use of multiple shear keys distributed over the whole height of the webs and flanges. The fine indentation of the segment surface gives a smooth load transfer and avoids high concentrated force. A perfect fit of the end of adjacent segments is required.

Regarding joints shear capacity, experimental tests show that the shear capacity:

- Increased as confining pressure increased
- Epoxied joints have consistently higher shear strength than dry joints, but the failure is more brittle than dry joints
- The average shear strength for a key in multiple-keyed dry joints is less than those in single-keyed dry joints due to imperfections in fitting of keys and sequential failure
- The shear strength of keys in multiple-keyed epoxied joints is similar to those in single-keyed joints, indicating epoxy mitigated the fixing imperfections.



In the next graph, the normalized shear stress-relative displacements curves are represented for three epoxide joints; the first highlighted joint is relative to the confining stress and the second highlighted one is related to the epoxy thickness. As we can see the shear capacity of the specimen with 1mm of epoxy thickness is bigger than the one of a specimen with 2 or 3mm of epoxy thickness.

Then on the right we can see the crack formation sequence in the shear key.

But considerable errors that are not on the safe side are for multiple dry joints and it means we have on over- estimation of shear resistance. The same is for joints with 3mm of epoxy.

Then we have the second formulation which gives me the tangential stress and so in order to obtain the shear resistance we have to multiply it for the area of the joint.

⇒ MC2010 formulation (usually on the side of safety)

$$\tau_{Rd1} = c_a \cdot f_{ctd} + \mu \cdot \sigma_n \leq 0.5 \cdot v \cdot f_{ct} \quad (7.3-50)$$

where:

- $c_a$  is the coefficient for the adhesive bond;
- $\mu$  is the friction coefficient from Table 7.3-2;
- $\sigma_n$  is the (lowest expected) compressive stress resulting from an eventual normal force acting on the interface.

The adhesion factor  $c_a$  depends on the roughness of the interface (see Table 7.3-1;  $R_f$  is derived from the sand patch method).

**Table 7.3-1:** Coefficients for the adhesive bond resistance

Surface characteristics of interface	$c_a$
Very rough (including shear keys) $R_f \geq 3.0$ mm	0.5
Rough (strongly roughened surface) $R_f \geq 1.5$ mm	0.40
Smooth (concrete surface without treatment after vibration or slightly roughened when cast against formwork)	0.20
Very smooth (steel, plastic, timber formwork)	0.025

Under fatigue or dynamic loads the values for  $c_a$  as found in Table 7.3-1 have to be reduced to 50%.

**Table 7.3-2:** Coefficients for different surface roughness

Surface Roughness	$\mu$	
	$f_{ck} \geq 20$	$f_{ck} \geq 35$
Very rough* $R_f \geq 3.0$ mm	0.8	1.0
Rough $R_f \geq 1.5$ mm	0.7	
Smooth	0.6	
Very smooth	0.5	

\* valid also for shear keys

### INCREMENTAL LAUNCHING

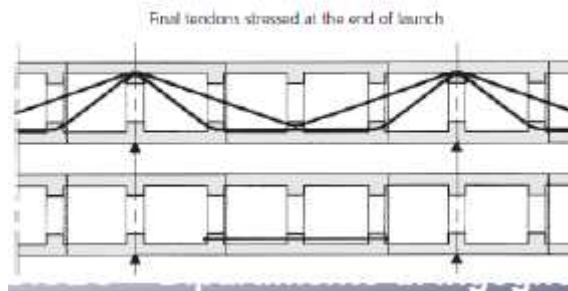
The principle of the incrementally launched bridge consists of building the superstructure segments in a casting yard located behind the bridge abutment

Each segment is matchcast against the previous one and prestressed to the section of superstructure already built. The entire superstructure is then jacked forward a distance equal to the length of this segment.

This process is repeated until the bridge is in its final position. The additional prestress is then installed and the temporary bearings are replaced by the permanent bearings.

This form of construction can be used for bridges having constant cross sectional shape throughout their length. The bridge should be straight or have a constant horizontal and vertical curvature.

The most characteristic aspect of the design of a launched bridge is the need to resist the stresses due to the transient support configurations assumed by the deck during launch. Every crosssection of the deck passes cyclically in midspan and above the piers, and is therefore subject to the maximum positive moment, the maximum negative moment, and the maximum shear. Each crosssection has to resist high self-weight



Here we can see an example of temporary tendons. In the first picture we have the final tendons stressed before the launch, so they are used to give the correct bending moment capacity to the structure in the final configuration. In the third picture we can see the final tendons stressed at the end of launch. Also they are used to give to the structure the correct ultimate capacity but we also need of temporary tendons that are stressed before the launch and that are removed after the launch as we can see in the second picture. So we have three different set of tendons, some are stressed before and some stressed after the launch,

About temporary prestressing, the evolution of the incremental launching construction method has seen several schemes:

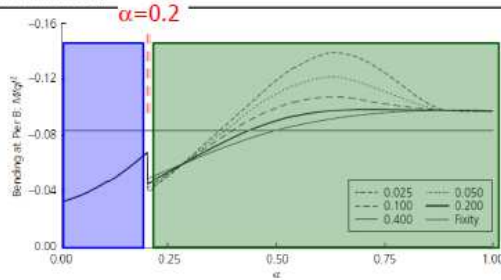
- permanent internal rectilinear tendons spliced by couplers at the construction joints between subsequent deck segments;
- temporary or permanent internal rectilinear tendons anchored in accessible blisters and spliced by overlapping (the temporary tendons are typically in the top slab in the midspan and in the bottom slab in the support regions);
- temporary or permanent external rectilinear tendons, either together with one of the previous types or alone;
- permanent draped tendons, either internal (parabolic) or external (polygonal), along with temporary antagonist tendons that make the resultant prestressing force rectilinear – in most cases the resultant force is at the middle of the depth of the cross-section, and additional temporary external rectilinear tendons are used to lift the resultant force to the deck gravity axis.

Modern launched bridges use external tendons for integrative end-of launch prestressing.

In general during the launch of a concrete box girder we should have only an axial force due to prestressing and we don't want to have bending moment due to prestressing as this bending moment is constant in time, but the external bending moment changes in time because follows the configuration of the bridge. So it's better to not have bending moment due to prestressing, but only an axial force.

We can see in this picture the variation of the moment at the pier divided by the factor  $q \cdot l^2$  in function of the value of  $\alpha = x/l$  and so  $\alpha$  changes from 0 to 1. This diagram is divided in two area by a value of  $\alpha$  that in this case is 0,2 but this diagram is related to a ration between the nose length and the span length equal to 0,8 and a ratio between the weight of the nose and the eight of the deck between 0,1 and we are going to plot the variation of bending moment related to different values of the ratio between the nose stiffness and the deck stiffness.

Figure 2.43 Progression of  $M_B$  for  $L_n/L = 0.8$  and  $q_n/q = 0.1$  in relation to  $E_n I_n/EI$ . (Reproduced with permission from ASCE)

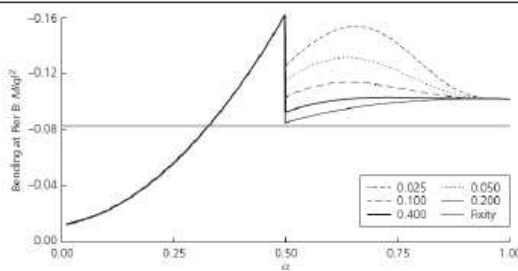


Prior to landing at pier A ( $\alpha < 0.2$ ),  $M_B$  grows with  $\alpha^2$  irrespective of  $E_n I_n/EI$ . Upon reaching pier A, the positive moment generated by deflection recovery reduces  $M_B$  and as launch continues,  $M_B$  tends to an end-of-launch value ( $\alpha = 1.0$ ). For  $\alpha > 0.9$ ,  $M_B$  does not depend on  $E_n I_n/EI$ .

Therefore, the flexural stiffness of the launch nose can be used only to prevent the peak negative bending after landing and until stabilisation of the end-of-launch value ( $0.2 < \alpha < 0.9$ ) from exceeding the greater of the moments at landing and at the end of launch. The optimum stiffness turns out to be around  $E_n I_n/EI = 0.2$ . Greater stiffness is pointless, as it cannot prevent reaching the end-of-launch moment. Lower stiffness increases the negative design moment in the front deck region. Finally,  $M_B$  at landing is lower than  $M_B$  EOL, landing is uselessly anticipated, and the launch nose may be shortened. So a nose length  $L_n = 0.8 \cdot L$  is too great

Then here we can see the sae diagrams as before, but with different ratio

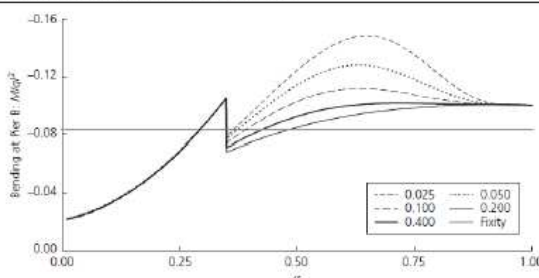
Figure 2.44 Progression of  $M_B$  for  $L_n/L = 0.5$  and  $q_n/q = 0.1$  in relation to  $E_n I_n/EI$ . (Reproduced with permission from ASCE)



Nose length  $L_n = 0.5 \cdot L$  is too short ( $M_B$  for  $\alpha = 0.5$  greater of the EOL value)

In this case the bending moment at pier B and for  $\alpha = 0.5$  is greater than the same moment at the end of launching and so this nose length is too short, because this moment governs the design.

Figure 2.45 With  $q_n/q = 0.1$ , negative bending at landing is equal to the end-of-launch moment for  $L_n/L = 0.65$ . (Reproduced with permission from ASCE)



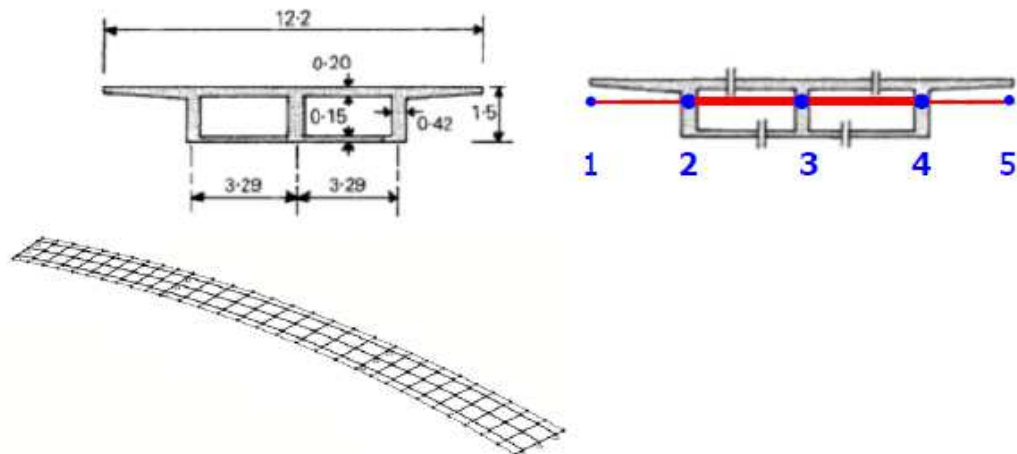
Optimum nose length:  $L_n = 0.65 \cdot L$  ( $M_B$  for  $\alpha = 0.35$  equal to the EOL value considering  $E_n I_n/EI = 0.2$ )

BRIDGE BRIDGES

1. the first deformation is related to a vertical displacement of the free webs, that in this case is equal for each webs and that is related to a longitudinal bending like if this transverse section has an infinite stiffness.
2. The second is the transverse bending deformation related to a curvature of the top slab and the bottom slab.
3. Another deformation is related to a torsional mode and in this case we have a rotation about the longitudinal axis of the bridge, but with a rigid rotation of the longitudinal axis about the bridge,
4. Finally we have a distortion of the section due to a load applied in a generic position.

The sum of these four contribution give the total deformation of the structure. Of course internal action in each portion of the bridge will be related to this displacement, and so I have to evaluate this displacement in a correct way in order to evaluate internal actions in each element, and that is in each web but also in the top and bottom slab.

- ➡ The four modes can be captured using a grid model (see Chapter 5 of E.C. Hambly's book "Bridge Deck Behavior")
- ➡ Grid method subdivide the box girder cross section into a series of longitudinal and transverse beam elements with characteristics such that, when analyzed together, replicate bridge deck behavior with appropriate accuracy

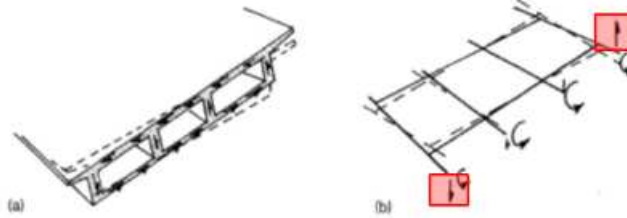


to model this two-cell box five longitudinal lines elements can be used and in particular we are going to consider the element one and the element five on the tip of the wing cantilever and the elements two, three and element four at the webs. Of course element one and five are only dummy element because I use them to apply the loads in a more convenient way, but the resisting elements are two, three and four.

- ❖ **Shear areas of transverse members on cantilever.** We consider a rectangular section with unit width

$$a_s = \frac{5}{6} d_1 \quad \rightarrow \quad a_s = \frac{5}{6} \times 0.20 = 0.167 \text{ m}^2 / \text{m}$$

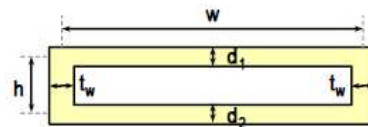
- ❖ **Torsional stiffeners of longitudinal members 2-3-4.** A part of the torque is carried by the longitudinal beams with **opposite shear forces**



Then the torsional constant take into account only the top and bottom slabs and can be evaluated as half the torsional constant of a box section with  $w \gg h$

$$J = \frac{4A_0^2}{\int \frac{ds}{t(s)}} = \frac{4w^2h^2}{\frac{w}{d_1} + \frac{w}{d_2} + 2\frac{h}{t_w}} \approx \frac{4w^2h^2}{\frac{w}{d_1} + \frac{w}{d_2}} = \frac{4wh^2d_1d_2}{d_1 + d_2}$$

$$j = \frac{1}{2} \frac{J}{w} = \frac{2h^2d_1d_2}{d_1 + d_2}$$



For our example is 
$$j = \frac{2 \times 1.325^2 \times 0.2 \times 0.15}{(0.2 + 0.15)} = 0.30 \text{ m}^4 / \text{m}$$

and considering the cell width ( $w_2 = w_4 = 3.29/2$ ,  $w_3 = 3.29$ ) we have

$$J_2 = J_4 = 0.49 \text{ m}^4 \quad J_3 = 0.99 \text{ m}^4$$

- ❖ **Torsional constant of longitudinal members 1-5.** Let's consider the section properties of half the cantilever

$$J_1 = J_5 = \frac{bd_1^3}{3} = \frac{2.81}{2} \times \frac{0.20^3}{6} = 0.0019 \text{ m}^4 / \text{m}$$

- ❖ **Torsional constant of transverse members between webs.** Let's consider the equation already used for longitudinal members 2-3-4

$$j = \frac{2h^2d_1d_2}{d_1 + d_2} \quad \rightarrow \quad j = 0.30 \text{ m}^4 / \text{m}$$

- ❖ **Torsional constant of transverse members on cantilever.** Let's consider a rectangular section with unit width

$$j = \frac{d_1^3}{6} \quad \rightarrow \quad j = \frac{0.20^3}{6} = 0.00134 \text{ m}^4 / \text{m}$$

The previous system can be rewritten as

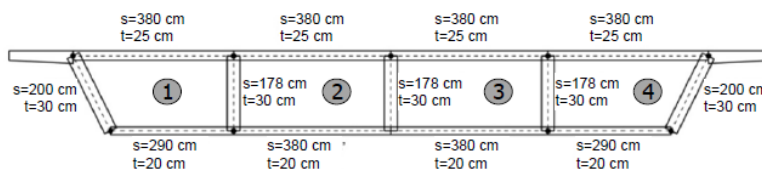
$$\begin{bmatrix} \delta_1 & \delta_{12} \\ \delta_{21} & \delta_2 \end{bmatrix} \cdot \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = 2G\theta \begin{Bmatrix} A_{0,1} \\ A_{0,2} \end{Bmatrix}$$

then the shear flows,  $q_1$  and  $q_2$ , can be solved in terms of the constant  $G\theta$ .

The constant  $G\theta$  may be evaluated remembering that  $M_T = GJ\theta$  where  $J$  is evaluated considering that

$$M_T = GJ\theta = 2 \sum_{i=1}^n A_{0,i} q_i$$

➔ Considering for example the following section



The areas  $A_{0,i}$  and the coefficients  $\delta_{ij}$  are

$A_{0,1}$	5.963	[m <sup>2</sup> ]
$A_{0,2}$	6.764	[m <sup>2</sup> ]
$A_{0,3}$	6.764	[m <sup>2</sup> ]
$A_{0,4}$	5.963	[m <sup>2</sup> ]

Cell 1-4		
s	t	s/t
2.00	0.30	6.667
2.90	0.20	14.500
1.78	0.30	5.933
3.80	0.25	15.200
$\delta_1 = \delta_4 =$		42.300

Cell 2-3		
s	t	s/t
1.78	0.30	5.933
3.80	0.25	15.200
1.78	0.30	5.933
3.80	0.25	15.200
$\delta_2 = \delta_3 =$		42.267

The coefficient  $\delta_{ij}$  is equal for each  $i$  and  $j$  as the common members are equal for each cell:  $\delta_{ij} = -5.933$

Then the matrix  $[\delta]$  and the shear flows are:

$$[\delta] = \begin{bmatrix} 42.300 & -5.933 & 0.000 & 0.000 \\ -5.933 & 42.267 & -5.933 & 0.000 \\ 0.000 & -5.933 & 42.267 & -5.933 \\ 0.000 & 0.000 & -5.933 & 42.300 \end{bmatrix} \quad \{q\} = \begin{bmatrix} 0.342 \\ 0.428 \\ 0.428 \\ 0.342 \end{bmatrix} \times G\theta$$

The torsional constant is  $J = 19.74 \text{ m}^4$  considering that

$$M_T = GJ\theta = 2 \sum_{i=1}^n A_{0,i} q_i \rightarrow GJ\theta = 2 \times (2 \times 5.963 \times 0.342 + 2 \times 6.764 \times 0.428) \times G\theta$$

So, for a given  $M_T$  we can evaluate  $\theta$  and consequently  $q_i$

You can see that, considering only one cell (and that is neglecting the 3 vertical central members), is  $J = 18.36 \text{ m}^4$  with difference of about 7%

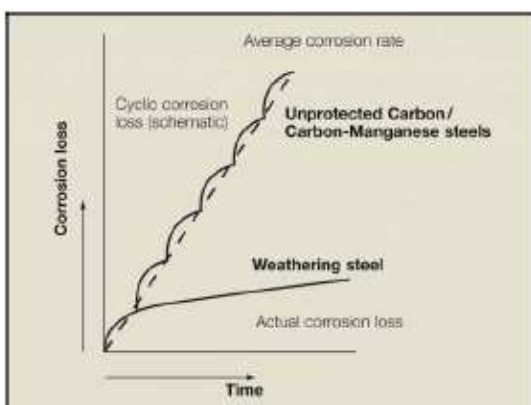
Another advantage is related to the assembly capability on site, and then this means transport elements with a lower weight and then a lower transport and lifting costs; in this way also a more flexible site planning can be prepared and in general we don't need propping (puntellamento) during construction because the steel girder gives the support for formwork for the casting of concrete slab and so we don't have any traffic interaction and a partial elimination of formworks.

From the composite concrete steel bridges we can have a maximum of prefabrication because the steel is realized in a carpentry and then we have high quality of the single part, fewer cast in place activities and of course from this we'll have high speed of construction and then also low labour cost. Of course this advantage compared to the concrete solution is true when we consider a cast in place concrete solution, but it is less important when we consider a complete precast segmental construction.

Of course we have also disadvantages in composite concrete steel bridges and the main one is that structural steel in bridge construction is susceptible to corrosion and then it needs painting and future repainting (every 5 or 10 years more or less) with protective coatings, or we need to plan the use of weathering steel, that is a particular steel that we can use in bridge construction using structural steel.

A further disadvantage is related to fire, in fact composite bridge deck is more sensitive to fire than a concrete deck because steel mechanical properties are more sensitive to fire than concrete mechanical property, of course related to time; the time of exposure to fire decreases also concrete properties. In general structural steel has a decrease of the strength with temperature greater than 400°C and at this temperature we have the start of degradation of the resistance, even if the high reduction is for temperatures greater than 550°C. But bridge degradation by fire has a low probability and temperatures reached by steel are dangerous only if the deck is few meters above the flame and this is not a usual condition. So actually we don't have a lot of problems with fire.

Weathering steel is often referred to COR-TEN steel (corrosion resistance and tensile strength) and this is a particular type of alloyed (*lega*) steel that forms adherent protective rust (*patina*) that inhibits further corrosion.

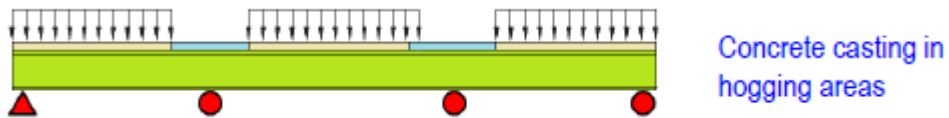


The rust layers formed on most ordinary structural steels are porous and detach from the metal surface after a certain time, and the corrosion cycle commences again. Hence, the rusting rate progresses as a series of incremental curves approximating to a straight line, the slope of which depends on the aggressiveness of the environment.

With weathering steel, the rusting process is initiated in the same way, but the specific alloying elements in the steel produce a stable rust layer that adheres to the base metal, and is much less porous. This rust 'patina' develops under conditions of alternate wetting and drying to produce a protective barrier that impedes further access of oxygen, moisture, and pollutants. The result is a much lower corrosion rate than would be found on ordinary structural steel.



We can think to precast the part of concrete subjected to positive bending moment and after the casting of this part of concrete we can think to apply a preloading on this part of the structure; after that we can cast the concrete in the hogging areas

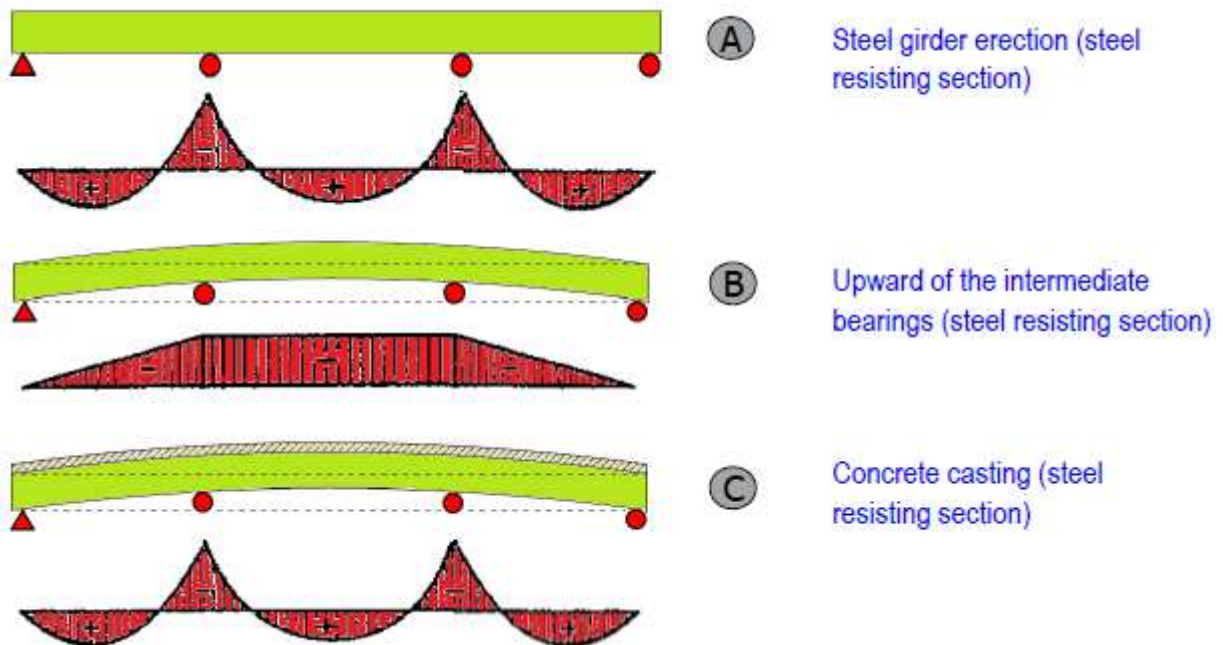


and after this we can remove preloading, this will induce a positive bending moment in the zone of the supports and then induce a sort of prestressing of the structure.



Actually is not easy to realize this type of structure as this load is obtained with ballast and we need a lot of it and it is not easy from the practical point of view.

- Another way to improve performance of the composite structure in hogging areas is by an application of imposed deformations at supports.



The structure is the same of the previous case then the first part is a steel girder erection and then we have a typical distribution of bending moment as the one we can see. After we apply an upward deformation at the intermediate bearing and in this way we impose on the structure the second distribution of bending moment which is negative in the whole section. Of course the resisting section is the steel one. The next phase is related to the concrete casting and so we start the pouring of concrete, the resisting section also in this case is the steel one because concrete has no strength yet

The most used solutions with concrete-steel bridges are related to the use of only two main girders. The two main girders can be used in two different ways:

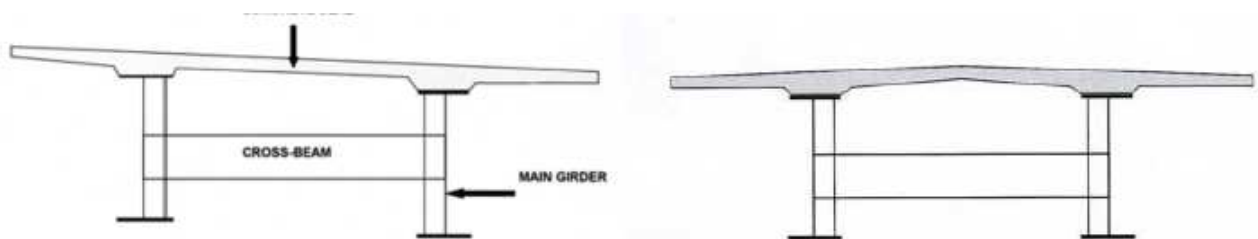
- I **Twin girder bridges:** in this solution the concrete slab is supported by the two main girders connected with the deck slab through connectors and the girders are connected by cross beams and they are not connected with the slab. The main advantage with twin girder is that there are fewere girders to erect and then we have a great speed of construction; the main disadvantage on the other side is the lack of robustness, in fact if a girder is damaged, internal forces cannot be redistributed to an adjacent beam and collapse is highly possible.

Main girders are fabricated by shop welding into I sections and that is are forms by free plates, the top flange, the bottom flange are welded together. In cases of curved bridges, main girders are realized connecting a series of straight sections, so I don't have a continuous curve for the main girders but main girders are realized with straight sections connected together generally welded together. For the flanges, plates wider than 600 mm (and up to 1200mm) and thicker than 60mm (and up to 120mm) are not rare (born in mind steel's strength reduction in cases of thicker plates).

Cross girders at spans are 500–700 mm high and are placed at a constant distance of no more than 8 m in order to give an adequate stability during erection.

Cross girders at piers are stiffer than those at spans with depth ranging from 600 to 1600 mm. In continuous bridges, high values of hogging moments at piers can result in lateral torsional buckling of bottom flanges, the compress flange in continuous beam in hogging areas. In such cases, the cross girders spacing may be reduced to 4 or 5 or 6 m .

- II **Ladder deck bridges:** in this type of bridges, cross girders are rigidly connected with the deck slab. Cross girders are normally spaced every 3.0–3.5 m so that a depth of 25 cm for the deck slab can be suitable.



The main girders and the cross girders effectively act as supports for the deck slab allowing an economical reinforcing against global and local bending because the slab is subjected to global internal actions due to deformation of the vertical displacement of the main girders that becomes also the vertical displacement of the slab and it also subjected to local bending and that is the bending due to the force directly applied on the slab, so I have to evaluate internal actions coming from global and local analysis. In this kind of solution I have to add these internal actions because the global analysis give internal actions in the slab in longitudinal direction only and the local analysis gives

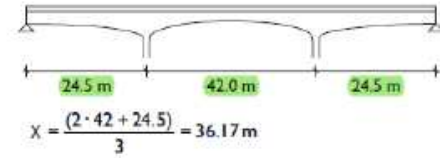
The preliminary design is based on two variables that are the width of the deck that is indicated as  $L_t$  and  $x$  that is the length of the span.

➔ Preliminary design for twin-girder bridges

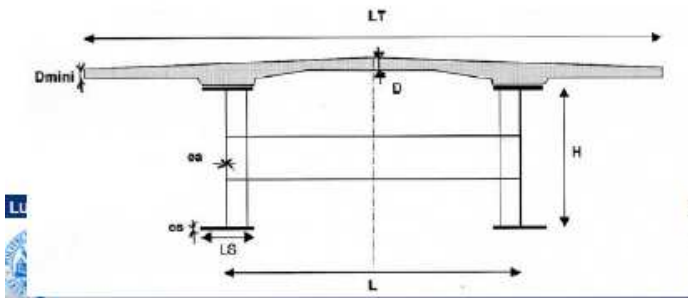
Main girder depth $H$	$\text{Max} \left( \frac{X}{28} \left( \frac{L_t}{12} \right)^{0.45}, 0.40 + \frac{X}{35} \right)$ for a constant depth deck. <small>X/24 at pier and X/36 at mid-span for a variable depth deck with more than 2 spans.</small>
Main girder c/c distance	$L = \text{approx. } 0.55 L_t$
Bottom flange width (Binfl)	$\left( 0.25 + \frac{L_t}{40} + \frac{X}{125} \right) \left( 0.92 + \frac{L_t}{150} \right)$
Top flange width (Bsup)	Binfl - 0.100 for a 2-lane deck Binfl - 0.200 for a 4-lane deck
Standard cross-beams	IPE500 to IPE700 standard section or equivalent
Steelwork tonnage	$63 + 0.9 X^{1.2} \left( 1.34 - \frac{L_t}{40} \right) + 0.26 X \text{ in kg/m}^2$
Slab thickness	$0.13 + \frac{(L_t - L)}{26}$ at main girders $0.12 + \frac{L}{50}$ at deck centre
Slab reinforcement ratio	Approx. 260 kg/m <sup>2</sup>

**Example**

A three-span bridge with a deck slab width:  $L_t = 12.3 \text{ m}$



- Girder spacing (L) 8.42 m
- Main girder depth (H) 1.51 m at pier  
1.51 m at midspan
- Slab thickness (D) 0.39 m at main girders  
0.29 m at deck center
- Bottom flange width (LS) 0.94 m
- Top flange width 0.84 m
- Steel consumption 135.9 kg/m<sup>2</sup>



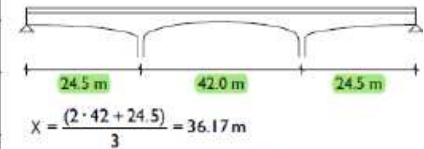
For isostatic bridge,  $X=1.4L$ . For continuous bridge,  $X$  is the standard span length or, for unequal spans, the weighted length of the two longest consecutive spans  $X=(2L_i+L_{i+1})/3$  with  $L_i > L_{i+1}$ ; end span lengths are multiplied by 1.25

➔ Preliminary design for ladder deck

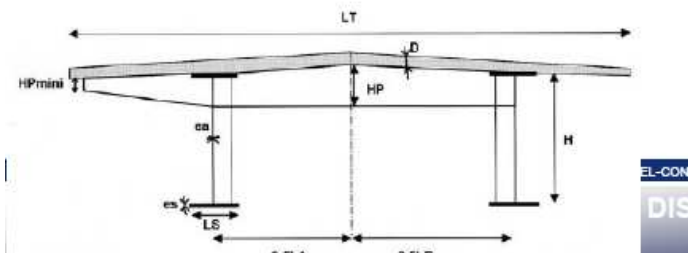
Main girder depth $H$	$\text{Max} \left( \frac{X}{28} \left( \frac{L_t}{12} \right)^{0.333}, 0.40 + \frac{X}{35} \right)$ for a constant depth deck. <small>X/24 at pier and X/36 at mid-span for a variable depth deck.</small>
Main girder c/c distance	$L_A = \text{approx. } 0.55 L_t$ ← with cantilever $L_B = L_t - 4 \text{ m}$ ← without cantilever
Bottom flange width (Binfl)	$0.25 + \frac{L_t}{40} + \frac{X}{125}$
Top flange width (Bsup)	Binfl - 0.100 for a 2-lane deck Binfl - 0.200 for a 4-lane deck
Directly supporting cross-beam depth	HP = approx. 1/11 <sup>th</sup> of $L_A$ or $L_B$ . HPmini = approx. 300 mm.
Steelwork tonnage	$65 + 0.9 X^{1.2} \left( 1.43 - \frac{L_t}{30} \right) + 2 L_t + 0.22 X \text{ in kg/m}^2$
Slab thickness	24 to 26 cm
Slab reinforcement ratio	Approx. 275 kg/m <sup>2</sup>

**Example**

A three-span bridge with a deck slab width:  $L_t = 12.3 \text{ m}$



- Girder spacing (L) 8.42 m  
8.3 m with cantilever
- Main girder depth (H) 1.51 m at pier  
1.51 m at midspan
- Cross girder (HP) 750 mm
- Slab thickness (D) 24-26 cm
- Bottom flange width (LS) 0.92 m
- Top flange width 0.82 m
- Steel consumption 164.9 kg/m<sup>2</sup>

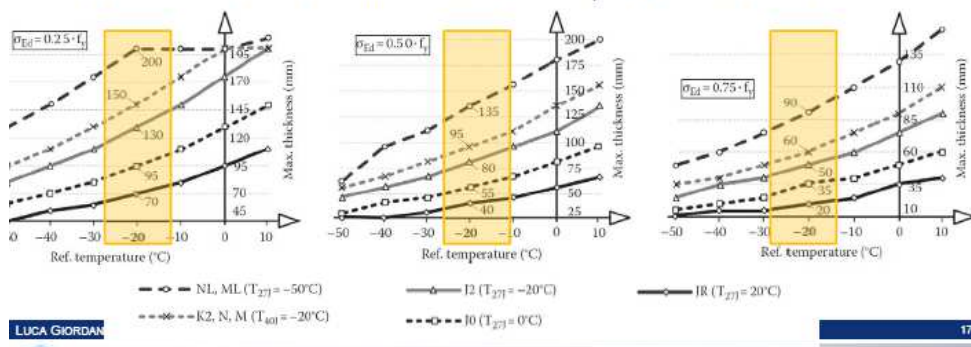


For isostatic bridge,  $X=1.4L$ . For continuous bridge,  $X$  is the standard span length or, for unequal spans, the weighted length of the two longest consecutive spans  $X=(2L_i+L_{i+1})/3$  with  $L_i > L_{i+1}$ ; end span lengths are multiplied by 1.25

In particular using V-notched Charpy test we can classify different types of material related to toughness; we can have the toughness plus jR that gives a temperature of 20°C and a Charpy impact energy of 27 J; if we want the same Charpy impact energy but a temperature of 0°C, then the steel is a type j0; a type j2 is a steel that gives Charpy impact energy of 27 J at a temperature of -20°C. Finally if we want the temperature -20°C with a Charpy impact energy then we have a K2 steel.

There are also other types of steel related to different toughness. In general the lower is the temperature and the more is difficult to have a correct impact energy and then a correct impact energy.

- ➔ Actually, material toughness depends on: the lowest service temperature, the element thickness, the type of loading (static or dynamic), the intensity of the applied stresses due to external loading, the intensity of the residual stresses due to restraint or fabrication and the construction detail in reference to stress concentrations and weld details
- ➔ Charts below show maximum plate thickness given by EC3-1-10 function of the minimum service temperature and the stress  $\sigma_{Ed}$  in the accidental design combination ( $G_k + \psi_1 \cdot Q_k$ ). Brittle fracture refers mainly to tension stresses. For plates in compression  $\sigma_{Ed} = 0.25 \cdot f_y$  may be considered



From a design point of view we should evaluate the material related to the following parameters: the lowest service temperature, the thickness of the plate and the stress in the plate in the accidental design situation.

In the accidental design combination the  $\psi_1$  parameter transform the characteristic value of the variable load into the frequent one.

So the first chart is related to a  $\sigma_{Ed} = 0,25 \cdot f_y$  (yield strength) , the second is related to an half of the yield strength and the third to 0,75 times the yield strength. In particular for the different types of materials that we have with the Charpy test, these charts give the maximum thickness for different temperatures. Generally the reference temperature is -20°C and the diagram says that if the stress is equal to the 25% of the yield strength, then the material Jr can be used until to a plate thickness of 70 mm, but if the stress in the accidental design combination is equal to an half of the yield strength, then the same material Jr can be used with a maximum thickness of 40mm; if our plate has a thickness bigger than this one and for instance 50 mm I can't use this type of material but I have to use for instance the j0 material. If the thickness is 90mm then I have to use K2 steel, because in this case the maximum thickness is 95 mm.

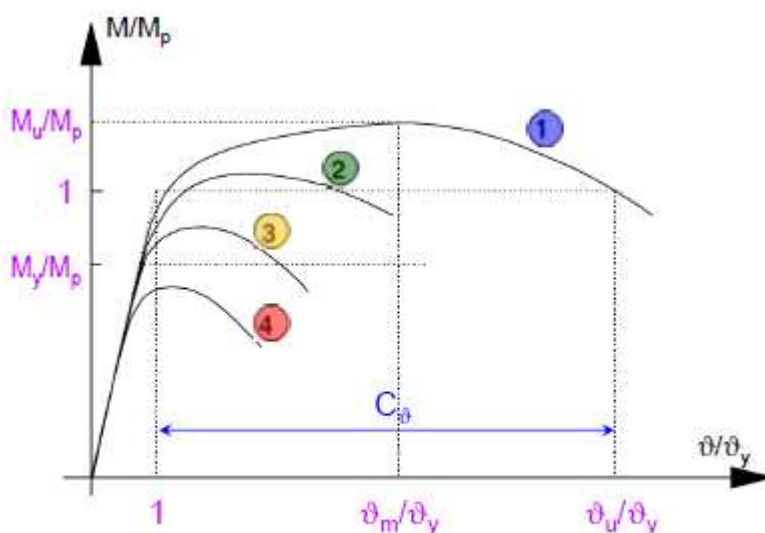
## CROSS SECTION CLASSIFICATION

Cross section classification deal with structural activity, in particular steel cross section can be classified according to their rotation capacity. So the cross section classification gives the rotation capacity of the steel cross section.

Steel is a very ductile material and the ultimate deformation of steel is greater than 10% and so we have a very high material ductility, but to ensure a sufficient rotation capacity the material ductility is not enough and we need other requirements to the cross section and in particular we need the extreme fibres should be able to sustain a very large strains without any drop-off in resistance. Usually in tension the usual steel grades have sufficient ductility to allow for the desired amount of tensile strains. But without compressive stresses it is not so much a question of material ductility as of ability to sustain these stresses without local buckling.

So we classify the steel cross section in four classes with a different rotation capacity. To explain this different behaviour of the steel cross section we can plot the bending moment  $M/M_p$  as a function of the section rotation  $\vartheta_p$  divided by the rotation of the section at the first yielding and that is when the extreme fibre reaches the yielding stress.

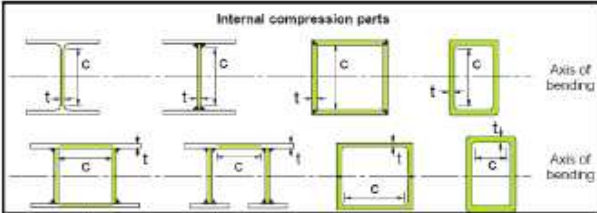
So in this diagram 1 is the yield rotation while 1 on the ordinates is the plastic bending moment. Other particular points are related to the moment  $M_u$  that is the maximum moment that the cross section can carry that is grater than the plastic moment because the class 1 section can use the hardening of the behaviour of steel, and that is it can work with a stress greater than the yielding one. The last point is related to the maximum rotation  $\vartheta_u$ ; the rotation related to the maximum moment is called  $\vartheta_m$



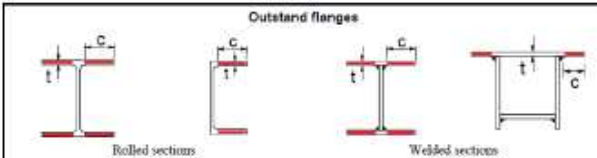
The Class 1 cross section is related to ductile or plastic cross sections; this cross section can develop a plastic hinge with sufficient rotation capacity to allow redistribution of bending moments in the structure ( $C_\vartheta \geq 3$ )

EC3 gives us some rules to evaluate the classification of the cross section, the first table is related to internal component whereas the second is related to outstand components.

■ **Internal components**



■ **Outstand components**



Class	Internal compression parts		Outstand flanges			
	Part subject to bending	Part subject to compression	Part subject to compression	Part subject to bending and compression		
1	$c/t \leq 7.2\epsilon$	$c/t \leq 3.3\epsilon$	$c/t \leq \frac{9\epsilon}{\alpha}$	$c/t \leq \frac{9\epsilon}{\alpha/\alpha}$		
2	$c/t \leq 8.3\epsilon$	$c/t \leq 3.8\epsilon$	$c/t \leq \frac{10\epsilon}{\alpha}$	$c/t \leq \frac{10\epsilon}{\alpha/\alpha}$		
3	$c/t \leq 12.4\epsilon$	$c/t \leq 4.2\epsilon$	$c/t \leq 14\epsilon$	$c/t \leq 21\epsilon \sqrt{k_1}$		
For $k_1$ , see EN 1993-1-5						
$\epsilon = \sqrt{235/f_y}$	$f_y$	235	275	355	420	460
	$\epsilon$	1.00	0.92	0.81	0.75	0.71

\*)  $\psi \leq -1$  applies where either the compression stress  $\sigma < -f_c$  or the tensile strain  $\epsilon_s > f_t/E$

1. Evaluate the position on plastic neutral axis
2. Check if class section is 1 or 2
3. If the c/t limit of class 2 is exceeded, evaluate the position of elastic neutral axis
4. If the c/t limit of Class 3 is not exceeded, section is in Class 3, otherwise is in Class 4

For each table we have the indication of the length of the component c and the thickness of component t. the tables give the limit value of the limit ratio between c and t in order to classify the section as class 1,2,3 or 4.

We can see that the limit value depends on the distribution of the stresses in the component, so we have the part related to bending moment, the one subjected to compression and the third column related to both compression and bending moment.

Consider that the positive tension in this picture are compressive tensions.

Pay attention that the one we see are not the stress distribution in the whole section but are stress distribution of the part that I am taking into consideration. For instance I have a section which is subjected to bending moment and I have two internal parts: the webs and the flanges; if I have a bending moment that acts on the square section, the webs will be subjected to the stress distribution of the first column and the upper flange, if the bending moment is positive, will be subjected to the second column type of stress distribution; so in order to classify this type of cross section I have to evaluate the classification for the web, for the compressive flange and the classification of the cross section will be equal to the worst classification of the single part.

The limit slenderness value depends on distribution of stresses but also on  $\epsilon$  parameter.

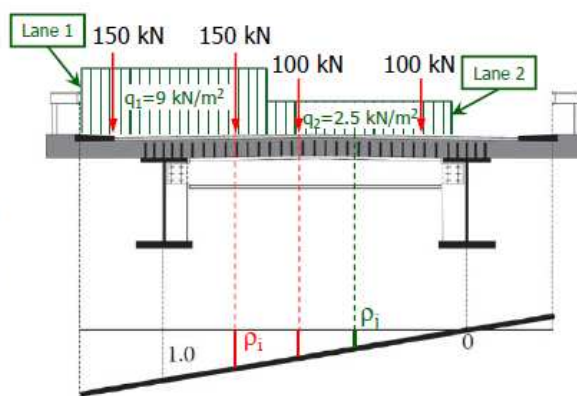
## GLOBAL ANALYSIS

- ➔ Global analysis may be performed using a grid model with some special attention as indicated in the following (slab effective width, concrete cracking and creep effects)
- ➔ In composite bridges, usually
  - ❖  $L/b_0 > 10$  ( $L$ =span length,  $b_0$ =main girder center to-center distance) so the system deck slab-cross girders can be assumed as infinite rigid
  - ❖ the torsional stiffness of main girder is small
 and then, if horizontal curvature and skew can be disregarded, a Courbon solution can be used, avoiding the modelling of the complete bridge

➔ For twin-girder we have:

$$F = \sum P_i \rho_i \quad q = \sum q_j w_j \rho_j$$

So, we can consider a single-line model representing a main girder loaded with forces  $F$  and UDL  $q$  (in the figure below the load condition that gives maximum bending moment in the first span)



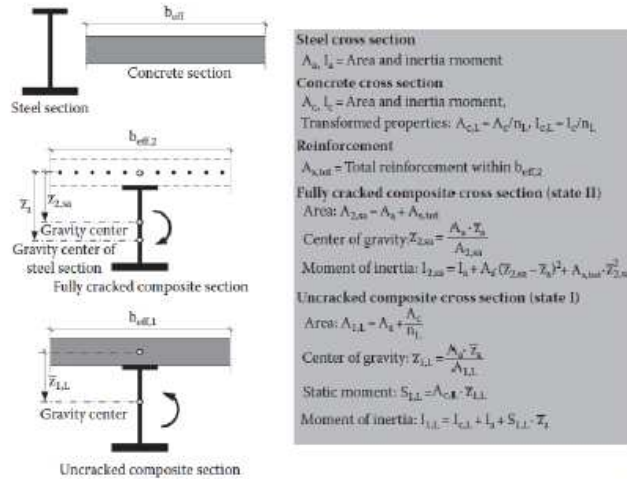
In this case the influence line of the transverse behaviour is very easy because I have 1 at bema 1 and 0 at beam 2 and that is when the load is applied on the beam 1 the load is completely carried by the beam 1 and the beam 2 is unloaded; when the load in the middle of the deck the load is carried at 50% by main girders and that is if I have this distribution of the notional lanes as four point forces of 150 kN and an uniform distributed load of 9 kN/m<sup>2</sup> and on the second lane I have four point forces of 100 kN and an udl of 2,5 kN/m<sup>2</sup> then the loads carried by the beam one will be equal to the equations that we can see here:

$$F = \sum P_i \rho_i \quad q = \sum q_j w_j \rho_j$$

So we can consider a single line model representing a main girder loaded with forces  $F$  and udl  $q$  as indicated in the picture where we can see the load condition that gives the maximum bending moment in the first span. In this way we can avoid the develop of the model for the whole bridge considering the two main girders, the cross beam, the slabs and so on and then the model is more simple and more easy to evaluate the internal actions and displacements using this configuration.

➔ EC4 suggests the following procedure:

1. Evaluate the equivalent cross section of steel both for uncracked composite section and fully cracked composite section. For uncracked sections reduce the concrete area with the modular ratio  $n_L = E_a / E_c^*$  where  $E_c^*$  is a concrete fictitious Young modulus that take into account creep effects as described in the next points



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2. Evaluate the envelope of the internal actions in the characteristic combinations (including long-term effects as indicated in the following slides) using the un-cracked cross sections properties on the whole structure (un-cracked analysis)
3. In regions where the extreme fibre tensile stress in the concrete due to the envelope of global effects exceeds twice the strength  $f_{ctm}$ , the fully cracked cross section properties should be used
4. This distribution of stiffness may be used for ultimate limit states and for serviceability limit states
5. A new distribution of internal actions is then determined by re-analysis (cracked analysis)

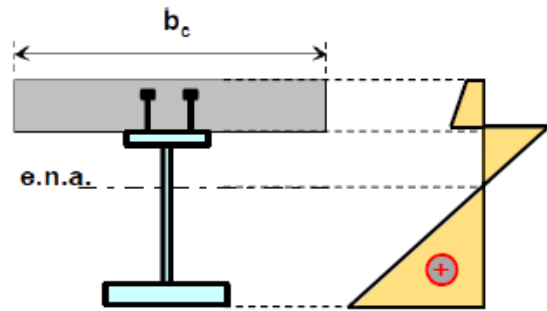
For continuous composite beams with the concrete slab above the steel section and not prestressed the following simplified method may be used. Where all the ratios of the length of adjacent continuous spans (shorter/longer) between supports are at least 0.6, the effect of cracking may be taken into account by using the cracked properties over 15% of the span on each side of each internal support, and the un-cracked values elsewhere



3. Add the contribution of step 1 and step 2

$$\sigma_c = E_c \epsilon_{sh} + \frac{1}{n_L} \left( -\frac{N_{sh}}{A_{1,L}} + \frac{N_{sh} \cdot e}{I_{1,L}} y \right)$$

$$\sigma_a = -\frac{N_{sh}}{A_{1,L}} + \frac{N_{sh} \cdot e}{I_{1,L}} y$$



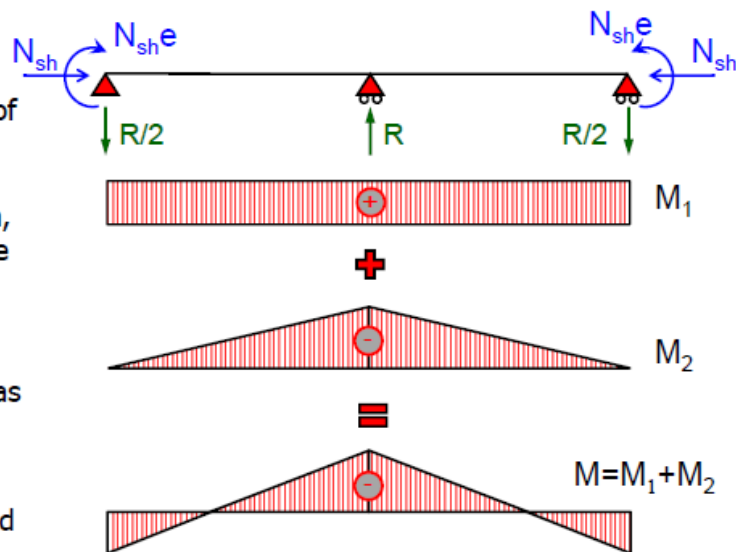
➔ Note that:

- ❖ eccentric force  $N_{sh}$  increased the deflection
- ❖ Tension stresses in concrete may results in cracking in the quasi-permanent situation (concrete slab is not stressed by self-weight), in particular close to the support
- ❖ Steel stresses are increased

**Shrinkage of concrete: statically indeterminate system**

➔ In statically indeterminate systems, the state of stress A can be evaluated as in determinate system, whereas the reverse of the force  $N_{sh}$  on the composite structures gives secondary effects (as for prestress)

➔ Actually, shrinkage should be considered only in un-cracked areas



So we have:

$$E_c^* = \frac{E_c}{1 + \psi_L \cdot \varphi(t, t_0)}$$

For variable actions  
is  $\psi_L = 0$

Type of action	Description	$\psi_L$
Permanent (P)	Permanent actions invariant in time (e.g., self weights)	1.10
Secondary effects (PT)	Secondary effects of creep and shrinkage	0.55
Imposed deformations (D)	Prestressing by imposed deformation (e.g., support settlement)	1.50

Source: EN 1994-2, Design of composite steel and concrete structures, Part 2: Rules for bridges, 2005.

In particular, if I consider a permanent action P then the coefficient  $\psi_L$  is equal to 1,1 and for instance the permanent actions of course a self-weight, but if I consider the shrinkage effect or the secondary effect of creep, the coefficient changes and becomes 0,55; finally for imposed deformations it becomes 1,5. Of course for variable action this value is 0 and for variable actions that are instantaneous actions and that is are not affected by creep effects, in this case the fictitious elastic modulus becomes equal to the real one of concrete.

The modular ratio  $n_L$  that we use to evaluate the geometric properties depends by the type of loads and it is equal to

$$n_L = n_0 \cdot [1 + \psi_L \cdot \varphi(t, t_0)]$$

Where  $n_0$  is the ratio between the elastic modulus of steel and the actual elastic modulus of concrete ( $n_0 = E_a/E_{cm}$ ). consequently we have:

$n_L = n_P$  when we evaluate permanent actions

$n_L = n_{PT}$  when we evaluate rheological effects

$n_L = n_D$  for imposed deformations

$n_L = n_0$  for variable actions

and that is the geometric properties of the cross section are different and depends on the type of action.

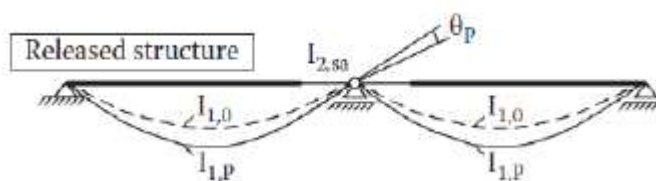
Effects of creep on composite girders at sagging moment areas for a given bending moment  $M_L$  are shown in the picture: due to creep, time-dependent cross-sectional forces are developed that redistribute stresses from concrete to steel; thus, concrete stresses become lower and steel stresses higher.

For instance we can see the stress distribution in concrete and in steel at initial time when we apply the moment  $M_L$  for instance due to permanent actions, we have the blue distribution that in time when creep effect is develop we have a variation of stress distribution in the section and stresses become the one in red



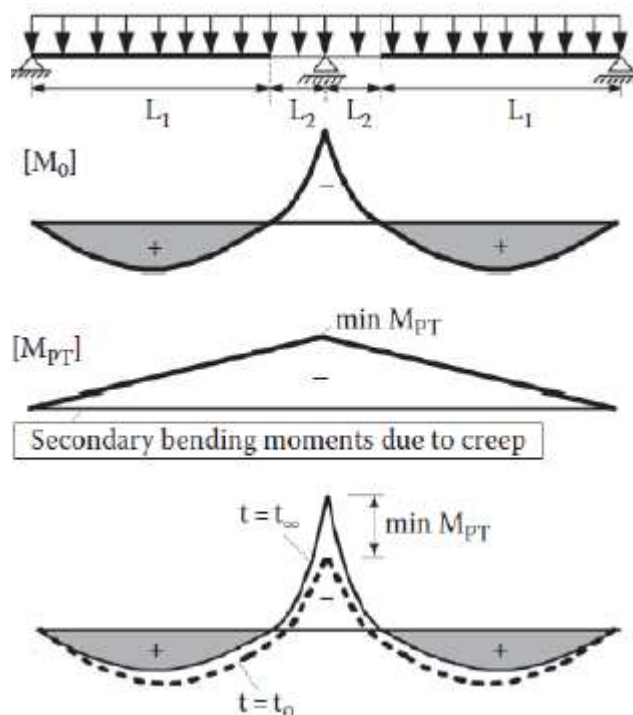
□  $M_0$  is the initial bending moment evaluated using the modular ratio  $n_0$  ( $\psi_L=0$ ) in un-cracked areas

But due to creep,  $I_{1,0}$  will decrease to  $I_{1,p}$  and that is the inertia evaluated considering  $\psi_p=1.1$ . If I consider the released structure and that is an equivalent determined structure getting introduced an internal hinge in the intermediate support, then this decrease of inertia in the uncracked regions results in an additional rotation  $\theta_p$  in the internal hinge, of course results in a further deformation of structure in time and this also means an increase of the rotation.



□ Due to creep,  $I_{1,0}$  will decrease to  $I_{1,p}$  ( $\psi_p=1.1$ ,  $n=n_p$ ). This results in an additional rotation  $\theta_p$  in the released structure

Of course The rotation  $\theta_p$  is inconsistent with the real structure (with the statically indeterminate structure) and then additional moments  $M_{PT}$  is developed and the additional moment due to creep we can see that is negative bending moment and then increases the initial bending moment in the hogging sections and decreases the initial bending moment in the sagging zones:



## USL VERIFICATION

### RESISTANCE TO VERTICAL SHEAR (EC3)

It can be performed in two different ways: using a plastic design or an elastic design, but in general we proceed by using a plastic design.



(1) The design value of the shear force  $V_{Ed}$  at each cross section shall satisfy:

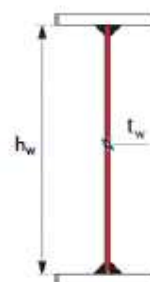
EC3-1-1

$$\frac{V_{Ed}}{V_{c,Rd}} \leq 1,0 \tag{6.17}$$

where  $V_{c,Rd}$  is the design shear resistance. For plastic design  $V_{c,Rd}$  is the design plastic shear resistance  $V_{pl,Rd}$  as given in (2). For elastic design  $V_{c,Rd}$  is the design elastic shear resistance calculated using (4) and (5).

Plastic design  $\rightarrow V_{pl,Rd} = \frac{\eta \sum (h_w t_w) A_v (f_y / \sqrt{3})}{\gamma_{M0}}$

$\eta = 1,20$  for steel grades up to and including S460  
 $\eta = 1,00$  for higher steel grades



Elastic design  $\rightarrow \frac{\tau_{Ed}}{f_y / (\sqrt{3} \gamma_{M0})} \leq 1,0$

$$\tau_{Ed} = \frac{V_{Ed} S}{I t}$$

$V_{Ed}$  is the design value of the shear force  
 $S$  is first moment of the area above the examined point  
 $I$  is second moment of area of the whole cross section  
 $t$  is the thickness at the examined point

Nb. Usually  $\eta=1$  in plastic design in order to stay on safety side.  $\gamma_{M0}$  is equal to 1 considering EC3 but if we are in Italy it is equal to 1,05.

As we can see in composite structures the contribution to vertical shear is given by the steel beam and the contribution of concrete is neglected.



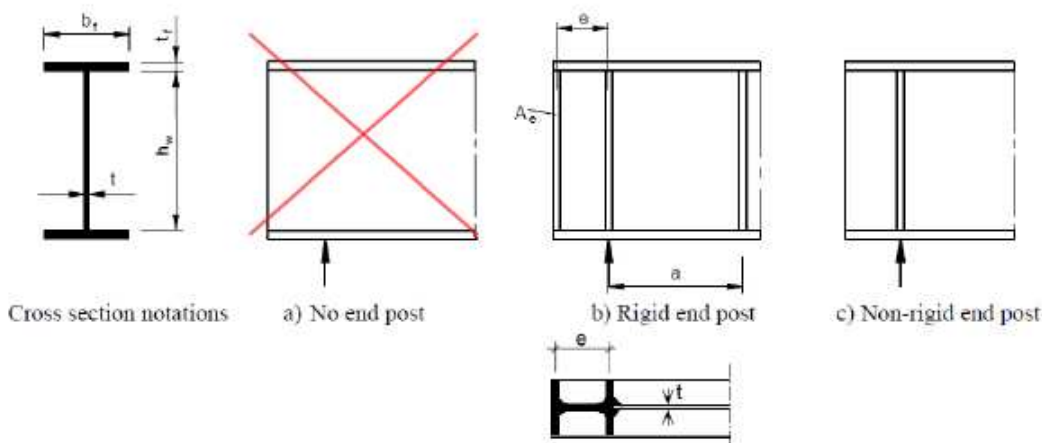
(6) In addition the shear buckling resistance for webs without intermediate stiffeners shall be according to section 5 of EN 1993-1-5, if

EC3-1-1

$$\frac{h_w}{t_w} > 72 \frac{\epsilon}{\eta}$$

Usually in bridge field this slenderness ratio is always greater than the second member and so I have to evaluate the shear buckling resistance.

About the shear buckling resistance we can see from the following picture that a tension field is developed in the post-buckling state. This tension field is anchored in the flanges following the picture on the right. Then the girder behaves quasi like a truss with the tension fields acting as tension diagonals.



EC3-1-5

- (2) A rigid end post should comprise of two double-sided transverse stiffeners that form the flanges of a short beam of length  $h_w$ , see Figure 5.1 (b). The strip of web plate between the stiffeners forms the web of the short beam. Alternatively, a rigid end post may be in the form of a rolled section, connected to the end of the web plate as shown in Figure 9.6.
- (3) Each double sided stiffener consisting of flats should have a cross sectional area of at least  $4h_w t^3 / e$ , where  $e$  is the centre to centre distance between the stiffeners and  $e > 0.1 h_w$ , see Figure 5.1 (b). Where a rolled section other than flats is used for the end-post its section modulus should be not less than  $4h_w t^2$  for bending around a horizontal axis perpendicular to the web.

EC3-1-5 gives this rules to be sure to have a rigi end-post.

NOTE 2 The slenderness parameter  $\bar{\lambda}_w$  may be taken as follows:

a) transverse stiffeners at supports only:

$$\bar{\lambda}_w = \frac{h_w}{86,4 t \varepsilon} \tag{5.5}$$

b) transverse stiffeners at supports and intermediate transverse or longitudinal stiffeners or both:

$$\bar{\lambda}_w = \frac{h_w}{37,4 t \varepsilon \sqrt{k_\tau}} \tag{5.6}$$

in which  $k_\tau$  is the minimum shear buckling coefficient for the web panel.

) For plates with rigid transverse stiffeners and without longitudinal stiffeners ~~or with more than two longitudinal stiffeners~~, the shear buckling coefficient  $k_\tau$  can be obtained as follows:

$$\begin{aligned} k_\tau &= 5,34 + 4,00 (h_w / a)^2 + k_{\tau u} & \text{when } a / h_w \geq 1 \\ k_\tau &= 4,00 + 5,34 (h_w / a)^2 + k_{\tau u} & \text{when } a / h_w < 1 \end{aligned} \tag{A.5}$$

here  ~~$k_{\tau u} = 9 \left( \frac{h_w}{a} \right)^2 \sqrt[4]{\left( \frac{I_{st}}{t^3 h_w} \right)^3}$  but not less than  $\frac{2,1}{t} \sqrt[3]{\frac{I_{st}}{h_w}}$~~

- a) is the distance between transverse stiffeners (see Figure 5.3);
- ~~$I_{st}$  is the second moment of area of the longitudinal stiffener about the z-axis, see Figure 5.3 (b). For webs with two or more longitudinal stiffeners, not necessarily equally spaced,  $I_{st}$  is the sum of the stiffness of the individual stiffeners.~~

NOTE No intermediate non-rigid transverse stiffeners are allowed for in equation (A.5).

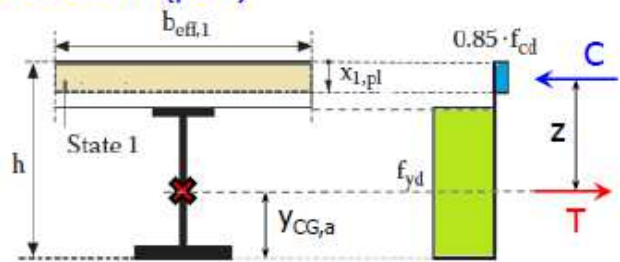
⇒ **Sagging areas, without reductions due to shear ( $\rho=0$ )**

$$C = b_{eff,1} \cdot x_{1,pl} \cdot 0.85 \cdot f_{cd}$$

$$T = A_a \cdot f_{yd}$$

$$x_{1,pl} = (A_a \cdot f_{yd}) / (b_{eff,1} \cdot 0.85 \cdot f_{cd})$$

$$M_{1,pl,Rd} = A_a \cdot f_{yd} \cdot (h - y_{CG,a} - 0.5 \cdot x_{1,pl})$$



So this is the case in which  $V_{Ed} < 0,5 V_{Rd}$ . In this case the moment is positive and then we have plastic neutral axis in the upper part of the section. We suppose that the plastic neutral axis is inside the concrete slab and in particular  $x_{1,pl}$  is the depth of the compressed part of the slab. Then using the stresses given in the previous table we have the distribution of the stresses that we can see; the compressive concrete works at a stress  $0,85f_{cd}$ , the tensile part of concrete works at 0 and the structural steel works at a stress  $f_{yd}$  because in this case I don't have any reduction of the strength induced by the shear.

If the applied axial force is 0 then it will be  $C=T$  and from this equality I can evaluate the depth of the plastic neutral axis  $x_{1,pl}$ .

The resisting plastic moment is given by one of these forces multiplied by the lever arm. ( $T \cdot z$ )

As the steel section is subjected to tensile stresses, any cross section is in class 1 or 2 in this situation, so in sagging area probably all the sections are in class 1 or 2.

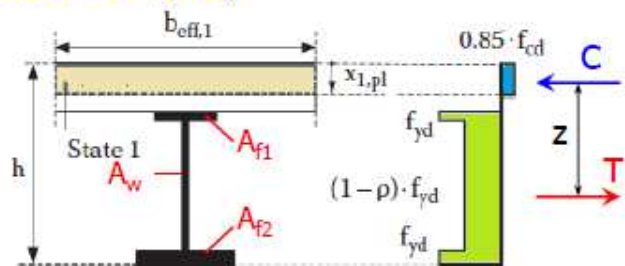
⇒ **Sagging areas with reduction due to shear ( $\rho \neq 0$ )**

$$C = b_{eff,1} \cdot x_{1,pl} \cdot 0.85 \cdot f_{cd}$$

$$T = (A_{f1} + A_{f2}) \cdot f_{yd} + A_w \cdot (1 - \rho) \cdot f_{yd}$$

$$x_{1,pl} = T / (b_{eff,1} \cdot 0.85 \cdot f_{cd})$$

$$M_{1,pl,V,Rd} = T \cdot z$$



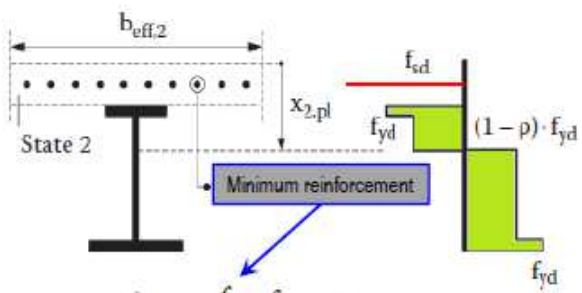
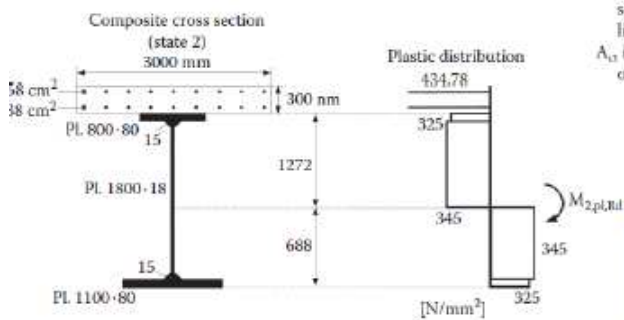
If the shear action applied is greater than 0,5 the shear capacity, then the steel web has to work at a tension different than  $f_{yd}$ . So I have to take into account a reduction of bending capacity due to shear.

This case can be evaluated as the previous one, but with a different T. In this case T is not applied at the center of gravity of the steel beam, so I have to evaluate the actual position of it.

➔ **Class 1 and 2 cross sections in hogging areas with reductions due to shear ( $\rho \neq 0$ )**

Evaluate the plastic neutral axis position by imposing  $C+T=N_{Ed}$  and calculate the plastic resisting moment  $M_{2,pl,V,Rd}$  with an equation of equilibrium at rotation about the center of gravity

**Example of application ( $N_{Ed}=0, \rho=0$ , flanges in class 1, web in class 2)**



$$\rho_s = \frac{A_s}{A_c} \geq \delta \cdot \frac{f_y}{235} \cdot \frac{f_{cm}}{f_{tk}} \cdot \sqrt{k_c}$$

$\delta$  is 1.0 or 1.1 for cross-sectional class 1 or correspondingly 2  
 $A_s$  is the area of tension reinforcement of ductility class B or C  
 $A_c$  is the area of the concrete flange within the effective width  $b_{eff,2}$   
 $f_y$  is the yield strength of structural steel in [N/mm<sup>2</sup>]  
 $f_{cm}$  is the mean tensile strength of concrete in [N/mm<sup>2</sup>]

$$k_c = \frac{1}{1 + h_c / (2 \cdot z_{1,0})} + 0.3 \leq 1.0$$

$z_{1,0}$  is the distance between centroids of the concrete flange and the composite section calculated with the short-term modular ratio  $n_s$

$\sigma_s$  is the maximum permissible stress in the reinforcement after cracking that may be set to  $f_{td}$  or to a lower value depending on the bar diameter to satisfy the crack width limits

$A_s$  is the area of the tension zone prior to cracking that may be set equal to  $A_c$ , the area of the concrete flange within the effective width  $b_{eff,2}$

### RESISTANCE TO BENDING: CLASS 3 CROSS SECTIONS

Opposite to class 1 or 2 sections, verifications for class 3 cross sections at ULSs are performed in the level of stresses rather in the level of internal forces and moments (refer to practical lessons)

**Stress design for class 3 cross sections (shear buckling not relevant)**

Material	Stresses	Verification
1 Concrete	Compression	$\sigma_{c,Ed} \leq \frac{f_{ck}}{\gamma_c} = \frac{f_{tk}}{1.5}$ <b>Note 1</b>
2 Structural steel	Direct stresses	$\sigma_{s,Ed} \leq f_{td} = \frac{f_{tk}}{\gamma_{M0}} = \frac{f_{tk}}{1.0}$ <b>Note 2</b>
3	Shear stresses	$\tau_{s,Ed} \leq \frac{f_{td}}{\sqrt{3} \cdot \gamma_{M0}} = \frac{f_{tk}}{\sqrt{3} \cdot 1.0}$
4	von Mises stresses	$\left(\frac{\sigma_{s,Ed}}{f_{tk}/\gamma_{M0}}\right)^2 + 3 \cdot \left(\frac{\tau_{s,Ed}}{f_{tk}/\gamma_{M0}}\right)^2 \leq 1.0$
5 Reinforcement	Tension (or compression)	$\sigma_{s,Ed} \leq f_{td} = \frac{f_{tk}}{\gamma_s} = \frac{f_{tk}}{1.15}$

Note: The von Mises stress refers to points where direct and shear stresses coexist, for example, in webs.

**Note 1: in Italy consider in addition  $\alpha_{cc}=0.85$**

**Note 2: in Italy is  $\gamma_{M0}=1.05$**

**Stress design for class 3 cross sections (shear buckling is relevant)**

Material	Stresses	Verification
1 Concrete	Compression	$\sigma_{c,Ed} \leq \frac{f_{tk}}{\gamma_c} = \frac{f_{tk}}{1.5}$ <b>Note 1</b>
2 Reinforcement	Tension (or compression)	$\sigma_{s,Ed} \leq f_{td} = \frac{f_{tk}}{\gamma_s} = \frac{f_{tk}}{1.15}$
3 Structural steel	Direct stresses	$\sigma_{s,Ed} \leq f_{td} = \frac{f_{tk}}{\gamma_{M0}} = \frac{f_{tk}}{1.0}$ <b>Note 2</b>
4	Shear stresses	$\tau_{s,Ed} \leq \frac{\chi_w \cdot f_{tk}}{\sqrt{3} \cdot \gamma_{M1}} = \frac{\chi_w \cdot f_{tk}}{\sqrt{3} \cdot 1.1}$
5	Direct and shear stresses (von Mises)	$\left(\frac{\sigma_{s,Ed}}{f_{tk}/1.0}\right)^2 + 3 \cdot \left(\frac{\tau_{s,Ed}}{\chi_w \cdot f_{tk}/1.1}\right)^2 = \left(\frac{\sigma_{s,Ed}}{f_{tk}/1.0}\right)^2 + 3 \cdot \left(\frac{\tau_{s,Ed}}{\chi_w \cdot f_{tk}/1.1}\right)^2 \leq 1.0$

The same approach can be used for outstand compression elements and that are the flanges for I section

outstand compression elements:

$\rho = 1,0$  for  $\bar{\lambda}_p \leq 0,748$

$\rho = \frac{\bar{\lambda}_p - 0,188}{\bar{\lambda}_p} \leq 1,0$  for  $\bar{\lambda}_p > 0,748$

where  $\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28,4 \varepsilon \sqrt{k_{\sigma}}}$

Non-effective areas

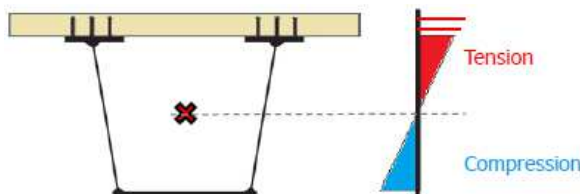
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Table 4.2: Outstand compression elements

Stress distribution (compression positive)		Effective <sup>a</sup> width $b_{eff}$			
		$1 > \psi \geq 0$ $b_{eff} = \rho c$			
		$\psi < 0$ $b_{eff} = \rho b_c = \rho c / (1 - \psi)$			
$\psi = \sigma_2 / \sigma_1$	1	0	-1	$1 \geq \psi \geq -3$	
Buckling factor $k_{\sigma}$	0,43	0,57	0,85	$0,57 - 0,21\psi + 0,07\psi^2$	
		$1 > \psi \geq 0$ $b_{eff} = \rho c$			
		$\psi < 0$ $b_{eff} = \rho b_c = \rho c / (1 - \psi)$			
$\psi = \sigma_2 / \sigma_1$	1	$1 > \psi > 0$	0	$0 > \psi > -1$	-1
Buckling factor $k_{\sigma}$	0,43	$0,578 / (\psi + 0,34)$	1,70	$1,7 - 5\psi + 17,1\psi^2$ 23,8	

In general in order to evaluate the properties of the effective cross section we can follow this procedure:

1. We evaluate the direct stresses on the basis of the cross section



For instance in this case (open box section without longitudinal stiffness) we have a position of G and if the section is subjected only to a negative bending moment I have a compression in the bottom part and that is the gross section.

2. Then considering this distribution of stresses related to the gross section I can evaluate the effective area of the compression flange and the corresponding direct stresses (note the variation of the gravity centre). So I will eliminate the central part for instance of the bottom flange and in this case I have a variation of the position of G and the same bending moment gives us a different distribution of stresses



## LATERAL TORSIONAL BUCKLING (LTB)

When a beam is subject to major axis bending, one of the flanges, (and an adjacent portion of web), is compressed and is, therefore, prone to buckling.

Buckling of the compression flange in the web plane is normally prevented by the web, because of the large rigidity of the latter in its own plane. Buckling of the flange in the flange plane is often critical because the web provides only limited restraint in this direction

When the web is very slender, it is likely to be distorted so that the restraint is negligible and lateral-torsional buckling is very close to flange buckling by bending about the weak axis of the cross-section. A stocky web, however, behaves roughly as a rigid plate element and lateral-torsional buckling causes the section to twist. Because of the rotation of the principal axes compared to their initial direction, the resulting deformation is a combination of torsion and lateral bending.

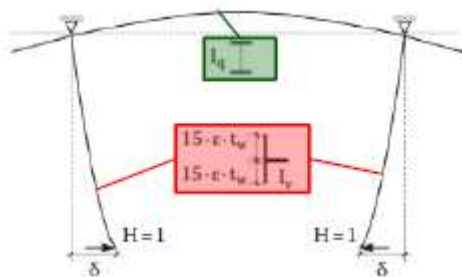


Usually we are in the first situation.

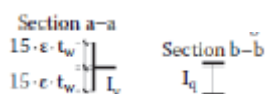
In steel-concrete composite bridge, LTB is an issue both in sagging and hogging areas in the launch phase and at concreting because in this cases the resisting section is the steel section alone and then we can have lateral torsional buckling both in positive and negative bending regions.

After the hardening of concrete, out-of-plane buckling of upper flange is prevented by the slab, whereas it is still possible for the bottom flange. So during the service of the structure we can have buckling only of the bottom flange of the steel girder.

Resistance LTB may be enhanced by addition of bracing that provides lateral support. Depending on the conditions, the lateral support may be rigid or flexible. Rigid support is generally provided by the connection of bracing elements directly to the compression flange (and that is we have a bracing that is connected to the compression flange), whereas elastic support provides cross frames or bracing elements in the tension flange or the web. In the following, we are going to show a simplified method for LTB verification; this method transforms the buckling verification of a member in bending in a buckling verification of a member in compression.



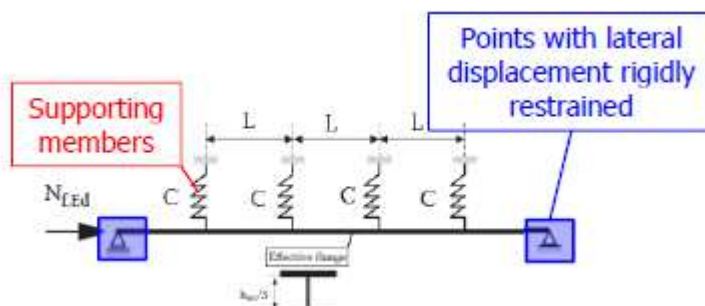
So I can evaluate the stiffness of the indirect support evaluating the stiffness of the U frames. The U frame is composed by the transverse cross beam, the steel cross beam, generally we disregard the contribution of the concrete slab, and by the vertical stiffener plus a portion of the web that we consider working with the vertical stiffener.



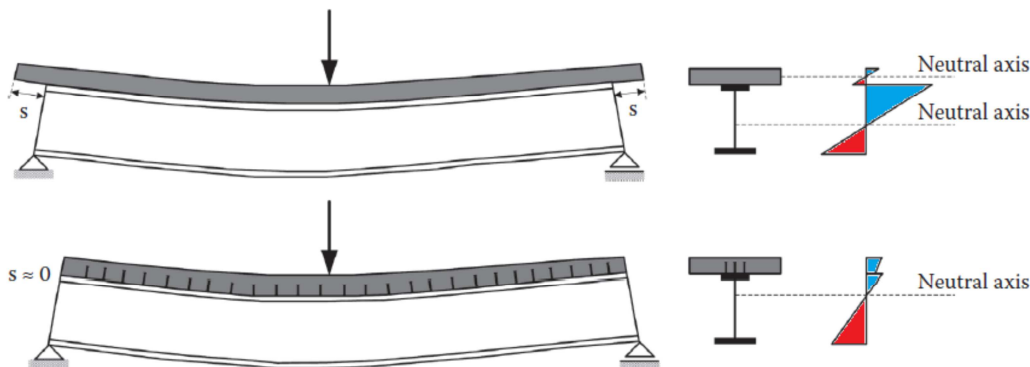
In particular we can see in the picture an horizontal section of the web so we have the vertical stiffener and the portion of the web that works with the stiffener; the portion has a length of  $15\epsilon t_w$ . so we have that these vertical members of the U frame have this section (so we can evaluate the area and the moment of inertia of the section) the top element of the U frame has an I section. I can evaluate the stiffness of this U frame applying two unit force at the level of the compressive flange and evaluate the lateral displacement. So the stiffness of this frame and that is the stiffness of the spring that reduces the prone of the compressive flange to buckling can be evaluated dividing the applied force H by the displacement  $\delta$ . In this case for this U frame we have a closed form solution for this displacement.

$$C = \frac{H}{\delta} = \frac{E_a \cdot I_v}{\frac{h_v^3}{3} + \frac{h^2 \cdot b_g \cdot I_v}{2 \cdot I_q}}$$

The total length of this beam is the length between two point where the later displacement is rigidly restrained.



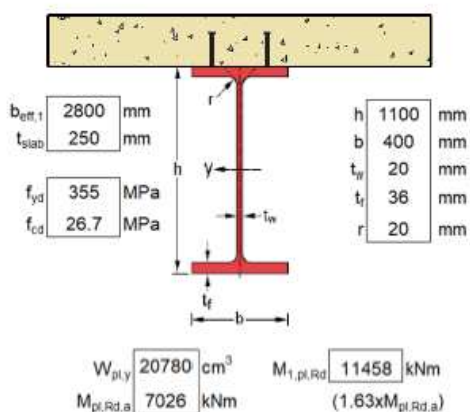
### SHEAR CONNECTION



When a concrete slab rests on a steel girder without any connection, it deflects like the girder but has its own neutral axis so that its top fibres shorten while its bottom fibres elongate. The fibres of the steel girder are also subject to similar displacements in longitudinal direction, so a differential displacement appears at the concrete–steel interface due to the fact that the bottom fibres of the slab elongates, while the top fibre of the girder shortens. So if we don't have any connection between the steel beam and the concrete slab we have this longitudinal differential displacement  $s$ .

But if the differential displacements at the interface are restraint as in the second picture, the slab and the girder behave as a composite girder with a single neutral axis. The restraint is provided by shear connectors, in this case by stud, while any natural bond between concrete and steel is ignored and we take into account only the resistance of the shear connectors. So we have to introduce the right number of shear connectors in order to make possible this behaviour and that is a composite behaviour.

#### ➔ Effect of composite action



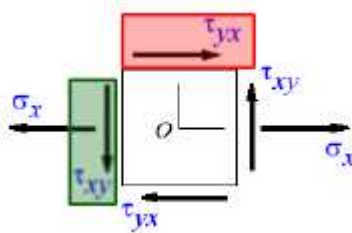
Here we can see the effect of the composite section on the capacity of the section. If we don't use any connections the resistance of the structure is equal to the capacity of the beam. In pure bending, as we can verify, the beam is in Class 1 and then the bending capacity of this steel beam alone is equal to the plastic moment of the section. But if I consider these two stud and that is I consider a composite behaviour of the section, then using the equations that we evaluated before for sagging regions and beam in class 1 or 2 we can evaluate the bending capacity that will be 1,63 times the one of the steel beam alone. So we have a great increase of the capacity of the section due to the presence of these two stud.

And that is if we don't design this shear connection in a correct way we have a great reduction of the bending capacity of the structure

The evaluation of the stud number can be performed with two different approaches considering an elastic behaviour or an inelastic behaviour

1. Elastic behaviour

In this case the longitudinal shear at the concrete-steel interface results from vertical shear forces  
 In fact if we have here a portion of the web we can see that we have the vertical shear stresses due to the shear force and in the longitudinal force we have to longitudinal shear stresses in the shear connector because these horizontal shear stresses should be hold by the shear connectors.  
 And for Cauchy we have  $\tau_{xy}=\tau_{yx}$ .



Then accordingly vertical and horizontal shear flow (shear force per unit length) is given by:

$$V_{L,Ed} = \frac{V_{Ed} \cdot S}{I}$$

- $V_{Ed}$  is the design vertical shear force
- $S=S_{1-2,L}$  is the first moment of area (static moment) of the concrete slab in respect to the center of gravity of the composite section for the load case and the time considered;
- $I=I_{1-2,L}$  is the second moment of area of the composite cross section for the load case and the time considered

LUCA GIORDANO – BRIDGE CONSTRUCTION AND DESIGN – LONGITUDINAL ANALYSIS

**POLITECNICO DI TORINO**

<b>Steel cross-section</b>
$A_s, I_s$ = Area and inertia moment
<b>Concrete cross-section</b>
$A_c, I_c$ = Area and inertia moment.
Transformed properties: $A_{c,1} = A_c/n_1, I_{c,1} = I_c/n_1$
<b>Reinforcement</b>
$A_{s,tot}$ = Total reinforcement within $b_{eff,2}$
<b>Fully cracked composite cross section (state II)</b>
Area: $A_{2,cr} = A_s + A_{c,cr}$
Center of gravity: $\bar{z}_{2,cr} = \frac{A_s \cdot \bar{z}_s}{A_{2,cr}}$
Moment of inertia: $I_{2,cr} = I_s + A_s(\bar{z}_{2,cr} - \bar{z}_s)^2 + A_{c,cr} \bar{z}_{c,cr}^2$
<b>Uncracked composite cross section (state I)</b>
Area: $A_{1,I} = A_s + \frac{A_c}{n_1}$
Center of gravity: $\bar{z}_{1,I} = \frac{A_s \cdot \bar{z}_s}{A_{1,I}}$
Static moment: $S_{1,I} = A_{c,1} \cdot \bar{z}_{c,1}$
Moment of inertia: $I_{1,I} = I_s + I_c + S_{1,I} \cdot \bar{z}_s$

The longitudinal shear resistance provided by shear connectors is

$$V_{L,Rd} = \frac{n \cdot P_{Rd}}{e_L} \geq V_{L,Ed}$$

- $n$  is the number of shear connectors at one cross section
- $e_L$  is the longitudinal spacing of connectors
- $P_{Rd}$  is the shear resistance

Born in mind that the longitudinal shear cannot be determined by combination of the vertical shear but has to be determined separately for each individual load case and then combined for the following reasons:

- The cross sections (and accordingly S and I), are not constant along the length of the bridge

■ Ultimate limit state of fatigue

See the local effect chapter

■ Serviceability limit states

⇒ Stress limitations

❖ Structural steel (characteristic combination of actions):

$$\sigma_a \leq f_y \quad \sqrt{\sigma_a^2 + 3 \cdot \tau^2} \leq f_y$$

❖ Reinforcement (characteristic combination of actions):

$$\sigma_s \leq k_3 \cdot f_{sk} \quad (k_3 = 0.8)$$

❖ Concrete:

characteristic combination of actions:  $\sigma_c \leq k_1 \cdot f_{ck} \quad (k_1 = 0.6)$

quasi-permanent combination of actions:  $\sigma_c \leq k_2 \cdot f_{ck} \quad (k_2 = 0.45)$

⇒ Cracking of concrete: concrete slab works in transverse direction to bear the loads to the girders and in the longitudinal direction through its effective width. Since concrete under tension inevitably cracks, cracking of concrete occurs both in transverse and longitudinal direction.

For the transverse direction we can use the dispersion of point loads and evaluate the crack width considering an equivalent beam in reinforced concrete (see the local effects chapter).

In longitudinal direction the crack width can be evaluated using the EC2 approach

$$w_k = s_{r,max} (\epsilon_{sm} - \epsilon_{cm})$$

$$s_{r,max} = 3.4c + 0.425k_1 k_2 \phi / \rho$$

$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_1 \frac{f_{ct,eff}}{\rho_{s,eff}} (1 + \alpha_s \rho_{p,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$$

(neglected on the side of safety)

- $\sigma_s$  is the stress in the tension reinforcement assuming a cracked section
- $\phi$  is the bar diameter
- $c$  is the cover to the reinforcement
- $k_1$  is a coefficient which takes account of the bond properties of the bonded reinforcement:  
= 0.8 for high bond bars
- = 1.6 for bars with an effectively plain surface
- $k_2$  is a coefficient which takes account of the distribution of strain:  
= 0.5 for bending  
= 1.0 for pure tension

According to EC2 it should be  $w_k \leq 0.3\text{mm}$  in the frequent combination of actions

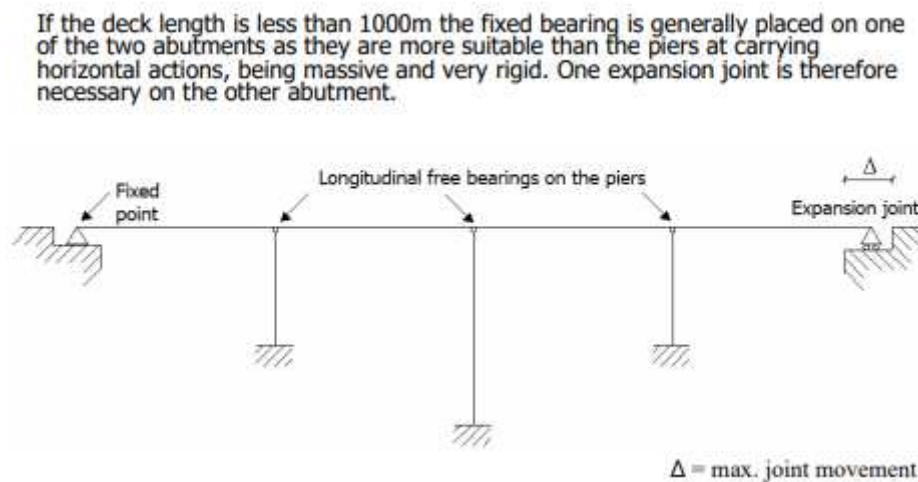
So in the frequent combination of action we have to evaluate the stress in the hogging zone.

considered rigid because they have some deformability related to many different phenomena: first of all the bending deformability of the deck it-self, if we place an horizontal force on the top of the pier it will bent and so the top of the pier will move horizontally and that's why we can see piers like elastic horizontal restraints.

Now that we have understood what these two pictures are talking about we can talk about bearings. The bearings transfer the loads both vertically and horizontally and horizontally loads can go both in longitudinal and transverse directions. So they transfer loads to the substructure that is made of abutments and piers.

Generally we have one fixed bearing in longitudinal direction on one abutment and one free bearing in longitudinal direction on the other abutment. On all the piers the bearings are free in longitudinal direction. it means that if the bridge wants to get longer it can move remaining fixed on the first point and will slide on the other bearings, without generating big forces on it. In fact we represent roller as pure roller but we know that this is a theoretical assumption because a pure roller doesn't exist and there will always be a minimum friction.

From the second picture we understand that we have two fixed points in the transverse direction and on the piers we have fixed bearing but the pier it-self is deformable and so the level of restraint is not fully fixed, but elastically fixed. In the vertical direction of course the pier's deformability is much lower and so we consider them as rigid bearings.



Here we see a typical arrangement for a bridge length that is small/medium. If the deck length is less than 1 km the fixed bearing is generally placed on one of the two abutments (in this case the left one) and a free bearing on the second abutment. Here we have the so called expansion joint that is a gap that allows the deck to become longer or shorter according to phenomena like thermal action: in summer the bridge is longer than in winter.

On the piers we have longitudinally free bearings it means that the deck can slide on the piers and transversally we already know that they are fixed.

Looking at the first picture we have two fixed bearings on the first abutment and bearings that are longitudinally free in all the piers and also on the second abutment. This layout is not very very good. First of all we have two bearings because on every pier we have at least two piers in transverse direction because if we have torque moments we need two vertical reactions to equilibrate torque. But the thing that may causes some problems in this layout is that each couple of bearings is fixed in transverse direction and the distance in transverse direction can't change. It means that if the weather becomes hotter and the bridge swells, becomes bigger because of thermal expansion, the bearings will try to keep the deck in the fixed position in the transverse direction and this distance cannot increase and so I will have big forces in the transverse direction in the bearings because they are working against a thermal swelling, and thermal swelling generates big forces because it is related to axial stiffness of the beams that is big. It is not very true because these bearings which are free in longitudinal direction and fixed in transverse direction are not perfect, so in transverse direction they have some residual movement that can be left free and so some gap is available. But if the bridge is large in transverse direction this tolerance is not enough to absorb the swelling.

So for broad bridges we have to move to the second solution that is the most typical bearing layout for bridges. We have a fully fixed bearing on one abutment which provides longitudinal and horizontal action and a completely free bearing; this one can be a full free bearing but can also be fixed in longitudinal direction and free in the transverse direction. On the piers all the bearings are free in longitudinal direction but in the transverse direction one is free to move whereas the other is restrained. The second abutment has the same layout of the piers.

If we have the necessity not to transfer torque to the piers we cannot place two bearings on the piers and then shift to the third solution where one bearing is placed on each pier. so for instance we can place two fixed bearings on the first abutment like in the first solution (but they can also be like in the second solution, or one fixed and the other free in longitudinal direction) then we have three piers on which we don't want any torque and then we have another pier on which we want to transfer torque again and so on it we will put two bearings, and finally on the abutment we have the last two bearings. Typically the distance between two points that are fixed in torsion should be about 100 m because if it is bigger, the torsional stiffness of the deck should be really big to transfer torsion and it becomes not so much economical.

In the following slide we can see a very common operation that can be done for long bridges which is changing the fixed point during construction. Sometimes during construction procedures we start building the bridge from one abutment and then we move to another abutment. So let's suppose to realize the first part of the bridge from the first abutment to the first pier. In order to do this operation we need the deck to be fixed in a point and then we fix it in the first abutment and of course we keep it free longitudinally on the first pier. then we move and realize the second pier and the second part of the pier always maintaining the deck fixed on the first abutment.

During construction we have prestressing action that means negative axial force and compression in longitudinal direction, and so shortening 1. Then we have shrinkage and this means that deck is shortening in longitudinal direction and so we have the displacement 2. Then we have creep, that is something that is related to the state of stress; if we have mainly compression state it means that compression is going from 2 to 3. As we can see now we are analysing only horizontal displacements. And we are analysing it at the level of the bearing, not at the level of the deck. That's different because it is a combination of the horizontal displacement at the axis of the beam plus a contribution due to the rotation of the section around the axis. Then we have thermal action during construction that can be positive or negative. Wind can also be in one or in the opposite direction. Remember that temperature and wind action during construction are related to a return period related to the construction time.

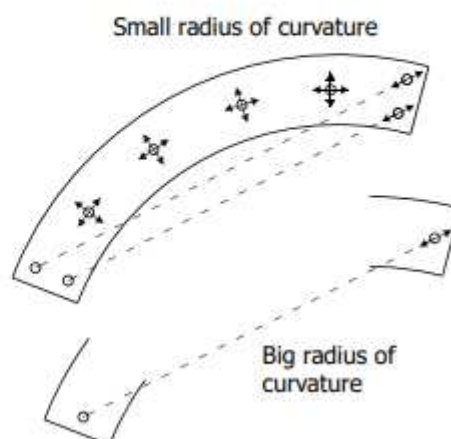
At the end of construction we move the fixed bearing so the direction of shortening changes and we will have shrinkage, creep and actions in the other direction. we have already considered shrinkage and creep with 2 and 3 deformations. Now we reconsider them from the end of construction to the end of service life. During construction time is short but both shrinkage and creep are fast because concrete is new. From the end of construction to the end of service life time is bigger but creep and shrinkage are slower because concrete is not fresh. The new have thermal action that can cause shortening or lengthening. Again now we consider thermal action in service life and so  $\delta$  is always bigger than  $\delta_4$  from the point of view of intensity because  $\delta_4$  was related to few months. Finally we have shortening or lengthening due to live loads and they may be not be the same depending on the actions. The last is wind during service life.

By summing these lengths we will obtain the total one.

Generally, in bridges with straight axis, torque moment are caused by variable loads (traffic actions), but in bridge decks with curved axis the bearings should carry torque moments coming from permanent loads also.

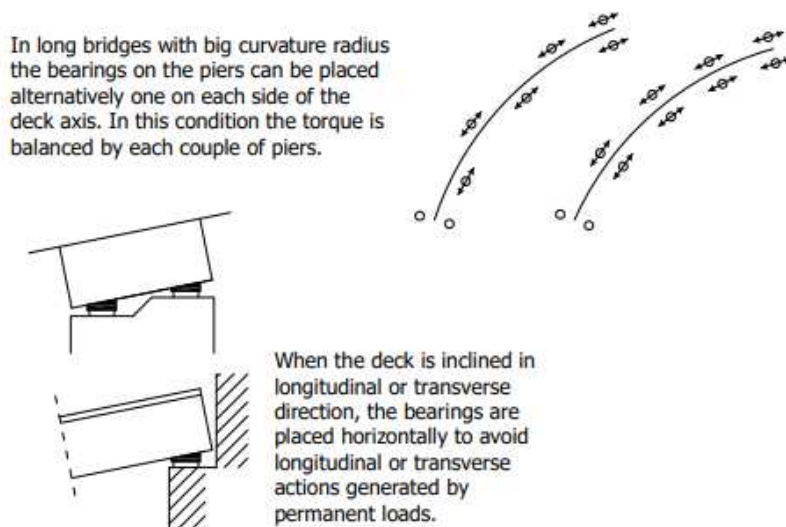
If the radius of curvature and the spans are small, the piers can be supplied with a single bearing placed under the bridge axis and therefore unable to carry torque. Torque is therefore equilibrated only on the abutments.

If the radius of curvature is big, centroid bearings can be placed also on the abutments, but they should be fixed in the direction orthogonal to the line that joins them.



Curvature bridges are different than straight one as they have torque moment caused by permanent actions like self-weight and dead loads whereas in straight bridges torque is only caused by variable loads.

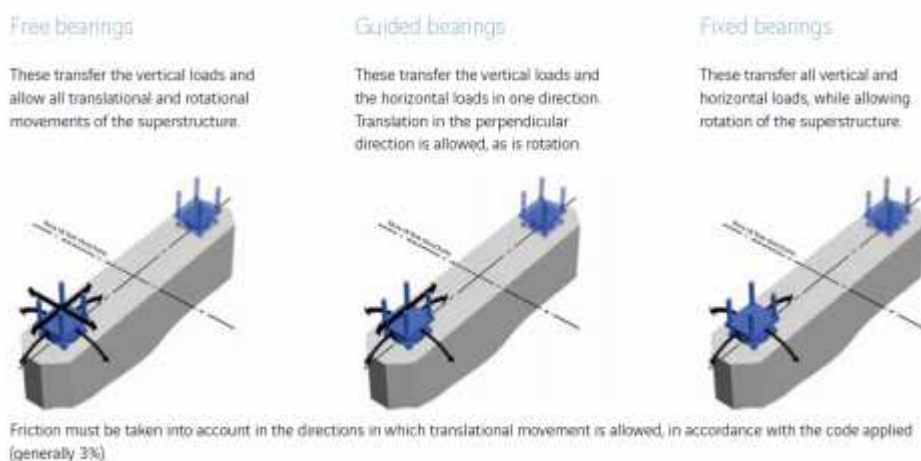




Here we can see two alternative solutions for counterbalancing the torque moment due to permanent loads in curved bridges in two different ways: the right solution is the most used one having two torques on the piers that counterbalance torque moment. The left solution is more unusual but it works.

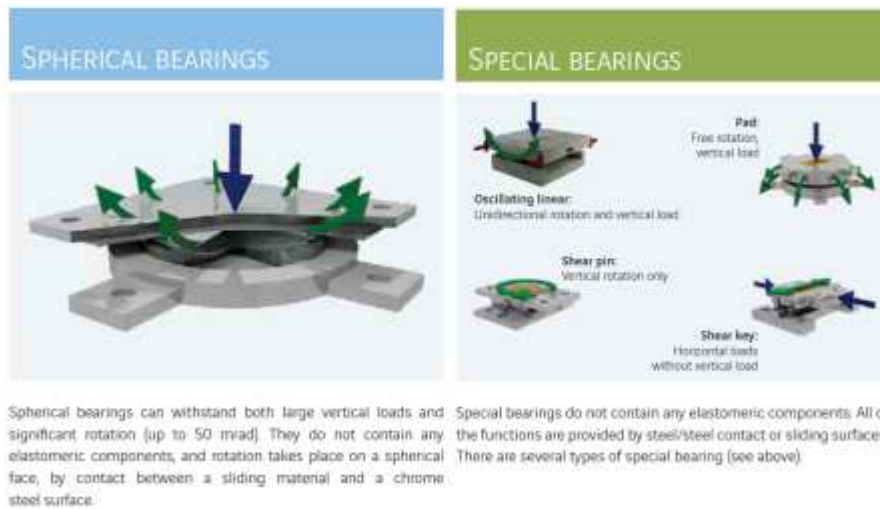
## 2. BEARINGS

We will see now different types of bearings in the detail.



Here we can see three different families of bearings: the first type is a free bearing, the one that allow movement in transverse and longitudinal direction and rotations. The picture represents the situation described before of a pier with two bearings on it. This bearing is able to transfer only vertical actions from the deck to the pier.

Then we have guided bearings that allow rotation around two axis but they allow movement only in one direction (in the picture only transverse movement is allowed).

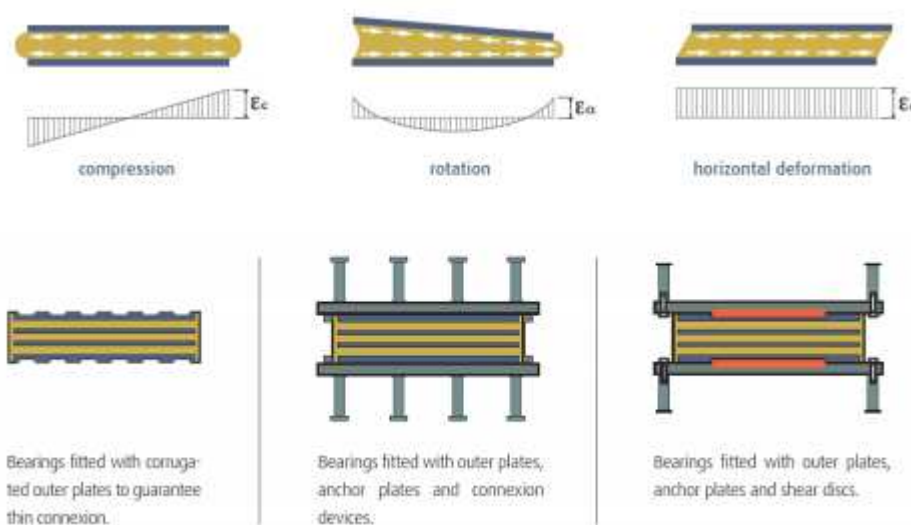


## 2.1 ELASTOMERIC BEARINGS

An elastomeric bearing is made of rubber between two steel plates. Then a set of these simple element are placed one on the other. When we apply uniform compression the rubber is squeezed between the steel plates and wants to go out, so in the centre we will have nil deformation because of symmetry and when we go to the exterior we find an increasing deformation. So the horizontal deformations are linear as we can see.

When we apply a rotation to the steel plates a quite complicate mechanism is rising because we have compression on one side and tension on the other side and so when we have tension we have exactly the opposite of before and so the rubber is coming in.

Horizontal deformation has a simple layout because the steel plates are moving horizontally one with respect to the other.

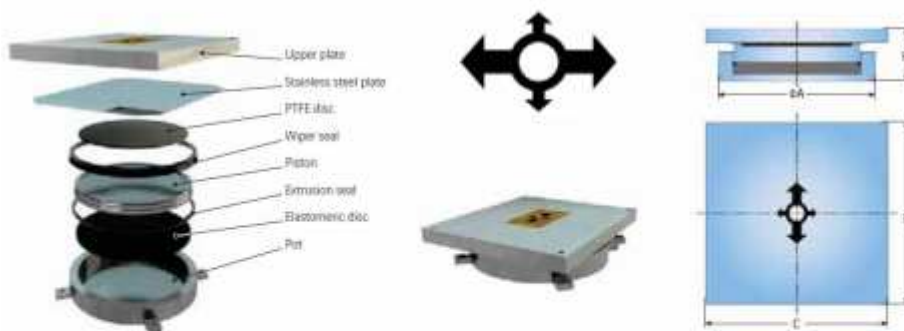


## 2.2 POT BEARINGS

### POT BEARINGS

Type	Free sliding bearing	Guided sliding bearing	Fixed bearing
	GL	GGL/GGT	FX
Symbol			
Vertical load			
Rotation	 Up to 30 mrad	 Up to 30 mrad	 Up to 30 mrad
Movement Horizontal	 Multidirectional	 Unidirectional	 Blocked

### Free Sliding POT Bearing



Bearings with  $\pm 200\text{mm}$  longitudinal and  $\pm 20\text{mm}$  transverse movement

	EM				SE				AAGITO				AL						
	DA	B	C	H	DA	B	C	H	DA	B	C	H	DA	B	C	H			
GL	300	400	40	180	430	280	90	150	425	270	80	180	430	280	90	150	430	280	90
GL	1,000	400	40	210	650	320	94	220	620	320	90	240	710	320	90	230	620	320	90
GL	1,500	400	40	240	670	330	104	260	640	330	94	280	750	330	90	280	670	330	90
GL	2,000	400	40	270	700	340	104	300	690	340	94	320	790	340	90	340	700	340	90
GL	3,000	400	40	400	750	410	120	400	740	400	104	400	800	400	100	420	750	400	100
GL	4,000	400	40	460	790	450	130	460	790	450	110	480	850	450	110	480	800	450	110

## 2.2 SPHERICAL BEARINGS

### SPHERICAL BEARINGS

Type	Free sliding bearing	Guided sliding bearing	Fixed bearing
	<b>GL</b>	<b>GGL/GGT</b>	<b>FX</b>
Symbol			
Vertical load			
Rotation	 Up to 50 mrad	 Up to 50 mrad	 Up to 50 mrad
Movement Horizontal	 Multidirectional	 Unidirectional	 Blocked

### Free Sliding Spherical Bearing

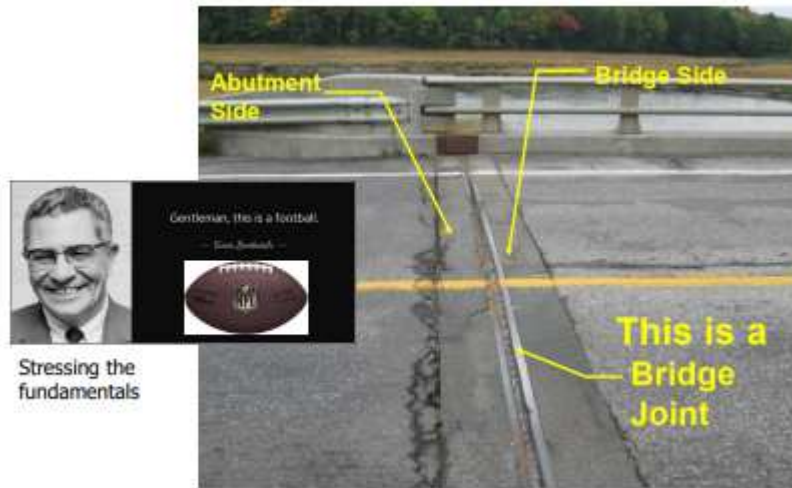


Bearings with  $\pm 200$ mm longitudinal and  $\pm 20$ mm transverse movement.

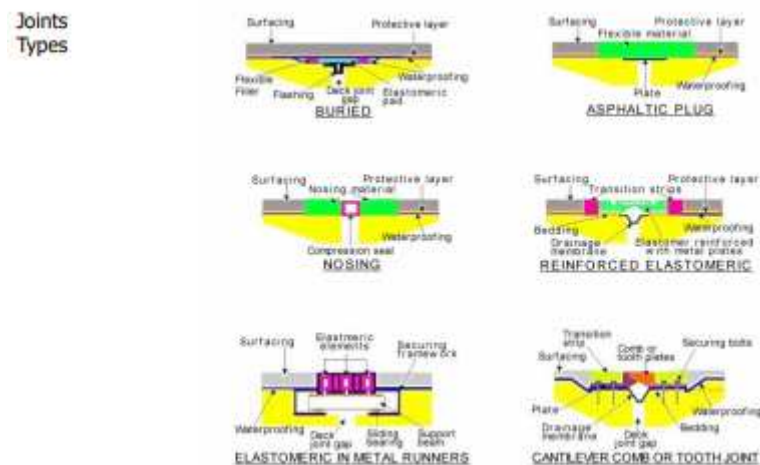
	PTFE								Incolloy®							
	EA	B	E	H	VA	B	C	H	EA	B	C	H	VA	B	C	H
GL 630 - 400 - 40	215	630	200	40	190	620	225	75	205	580	180	35	185	590	200	35
GL 1.030 - 400 - 40	220	630	290	35	200	630	240	85	210	590	200	50	195	590	200	35
GL 1.530 - 400 - 40	230	630	380	35	210	730	330	90	220	620	220	50	205	620	220	35
GL 2.030 - 400 - 40	240	730	300	35	220	730	370	95	230	640	240	50	215	640	240	35
GL 2.530 - 400 - 40	250	740	340	35	230	830	410	100	240	670	270	50	225	670	270	35
GL 3.030 - 400 - 40	260	770	370	35	240	830	450	110	250	680	300	50	235	680	300	35
GL 4.030 - 400 - 40	270	820	430	35	250	830	490	115	260	730	330	50	245	730	330	35

### 3. JOINTS

A joint is a very important element in bridges and we start with a joke as we can see: (?)



In the picture we have an abutment on the left side and the bridge on the right side and we need something to be placed between them to allow the movement of the deck with respect to the abutment which is moving too but moving with the ground, so we consider it as fixed. The joint is the device that allows this movement but allows also the traffic to pass by.



There are many different joint types and here we see some schematic of them.

The joint can be buried and in this case we don't see it because the bitumen is on it and when we walk on it we don't see it at all; of course in this case the joint should have a little movement otherwise the surface will crack or break. It is important to understand that the gap is small but the joint as we can see is wider because the interface between the joint and the deck is wider in order to redistribute actions on a wide part of the surfaces in order to reduce the stresses that are transferred to the surface.

**Joints  
Types**

JOINT TYPE	TOTAL ACCEPTABLE LONGITUDINAL MOVEMENT		MAXIMUM ACCEPTABLE VERTICAL MOVEMENT BETWEEN TWO SIDES OF JOINT (mm)
	Min (mm)	Max (mm)	
1. Balled joint under continuous surfacing	0	26	1.1
2. Asphaltic flag joint	0	40	3
3. Sliding joint with planed surface	0	12	0
4. Sliding with preformed compression seal	0	40	0
5. Reinforced elastomers	0	*	0
6. Elastomers in steel frames	0	*	0
7. Cantilever rubber tooth joint	25	*	0

The minimum of the range is given to indicate when the type of joint may not be economical.  
 \* Maximum value varies according to manufacturer or type.




This table shows us which joint in the one we have to use in function of the total acceptable longitudinal movement of the joint and the maximum acceptable vertical movement between the two sides of joint.



Here we have two pictures that show two very common joints. Typically elastomeric joints are better for small gaps while finger joints are better for bigger gaps. Finger joints are made by these two steel teethes that we can see and if they are triangular like in the picture is called finger joint, but we will see another type of joint which is similar to this and it is called comb joint in which the singular tooth is not triangular.

### 3.2 ELASTOMERIC JOINTS

Elastomeric joints

Model	Movement [mm]	Dimensions AxHxL [mm]	P [mm]	Weight [kg/m]
XJ 30	± 15	269 X 32 X 2000	200	17
XJ 50	± 25	272 X 41 X 2000	200	25
XJ 80	± 40	358 X 46 X 2000	250	32
XJ 100	± 50	388 X 53 X 2000	250	37
XJ 120	± 60	427 X 69 X 1000	250	59
XJ 140	± 70	465 X 80 X 2000	250	81
XJ 160	± 80	498 X 84 X 2000	250	88
XJ 200	± 100	802 X 71 X 2000	250	135
XJ 250	± 125	882 X 78 X 2000	250	170
XJ 330	± 165	1108 X 100 X 1000	250	285

Elastomeric joints are used a lot in bridge construction; looking at the picture we may recon this kind of joint that is very common to be found on joints.

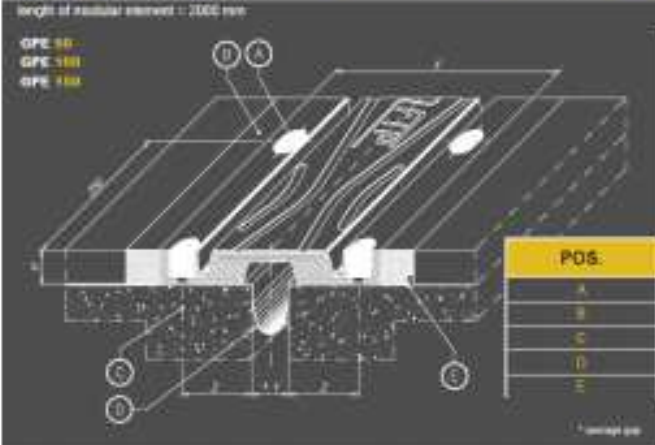
This joint is fastened to the two elements by using these two bolts like we can see in the cross section. The joint is made with elastomeric material and it is shapes with waves, because the gaps can become wider or longer and here there is the reason of the movement of the joint.

Elastomeric joints

JOINT TYPE	TOTAL MOVEMENT (mm)	HEIGHT	WIDTH	GAP	ANCHORS
		H (mm)	X (max)	Y (mm)	Z (mm)
GPE 30	30	40	300	20	70
GPE 50	50	60	300	20	110
GPE 100	100	80	300	20	150

length of modular element = 2000 mm

GPE 30  
GPE 50  
GPE 100



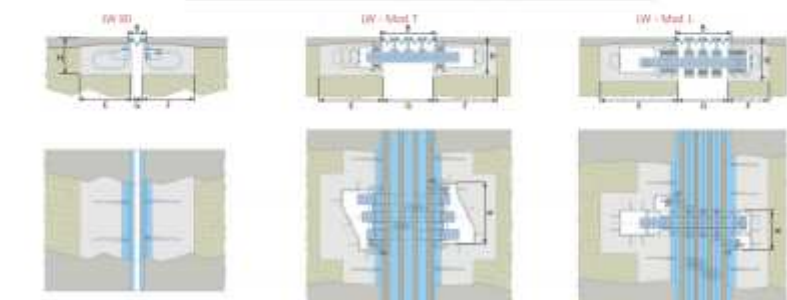
POS.	DESCRIPTION
A	Anchor
B	Transfer slip
C	MTB Anchoring system
D	Filter
E	5° tapered drainage profile

### 3.4 MODULAR JOINTS



The movement of the joint is in function of the number of the elements that are placed so we can have this joint without support bars or with multiple support bars.

Type	Range	Movement	R		G		E		F		H		K	
			min.	max.	min.	max.	min.	max.	min.	max.	min.	max.		
EW30	1	+40	80	160	30	110	350	250	250	-	-	-	-	-
EW160	2	+80	160	320	210	270	350	350	300	300	310	310	310	310
EW240	3	+120	240	480	330	430	590	590	570	570	570	570	570	570
EW320	4	+160	320	640	420	560	670	670	670	670	670	670	670	670
EW400	5	+200	400	800	510	750	750	750	750	750	750	750	750	750
EW480	6	+240	480	960	600	810	800	800	800	800	800	800	800	800
EW560	7	+280	560	1,120	710	1,070	880	880	880	880	880	880	880	880
EW640	8	+320	640	1,280	800	1,200	960	960	960	960	960	960	960	960
EW720	9	+360	720	1,440	870	1,350	1,040	950	950	950	950	950	950	950
EW800	10	+400	800	1,600	930	1,500	1,140	980	980	980	980	980	980	980
EW880	11	+440	880	1,760	990	1,710	1,200	990	990	990	990	990	990	990
EW960	12	+480	960	1,920	1,050	1,870	1,260	1,000	1,000	1,000	1,000	1,000	1,000	1,000



This table shows us some parameters for modular joints so we may see how it works.





Deterioration problems:

3 Water leakage and damage to deck and/or substructure



Joints are a weakness point of all bridges.

Their design life is generally around 15 years, so they need to be replaced several times during bridge life.

Maintenance on joints request partial or total traffic closure, therefore it is expensive.

Damage to a joint can cause additional damage to the structure that is generally much more expensive

Joints should be:

1. Very tough, as are they are directly stressed by the traffic;
2. Resistant to fatigue
3. Resistant to impacts
4. Perfectly aligned with the road pavement surface
5. Durable and resistant to harsh chemical attack (dust, oil, debris, water, salts...)
6. Simple to be replaced.
7. Installed with care by skilled professional.
8. Controlled and maintained periodically.

### Scuppers

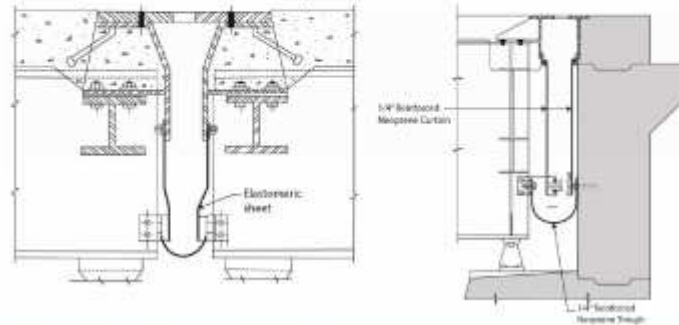
A scupper is defined typically as a vertical hole through a bridge deck for the purpose of deck drainage. However, a horizontal opening in the curb or barrier is also considered a scupper as it serves the same purpose of deck drainage. A scupper can be as simple as a circular penetration in the bridge deck, with or without an insert pipe in the bridge deck. Or it can be a combination of a small steel-grated casting typically with a steel pipe extended below the superstructure.



### Troughs

A drainage trough is typically used under open deck joints, such as finger joints, to divert runoff and associated debris away from underlying superstructure, bearings and substructure members. Drainage troughs on older bridges were made of steel and required significant maintenance based on flat slopes which allowed for the easy collection of debris and replacement or repair due to corrosion.

Drainage troughs on new bridges are typically made of elastomeric sheet



Clogged elastomeric through on Gerber joint in steel girder.



Also piers, which are the substructures that are between the abutment, can be also different from each other. From the point of view of aesthetic we prefer to have piers that have the same configuration but in the foundations sometimes they are different.

So we have for the abutment the foundation and in this case is called spread footing; the first abutment has no piles whereas the second abutment has piles under it, and then we have the breast wall which is also called front wall and the wind wall on the side.

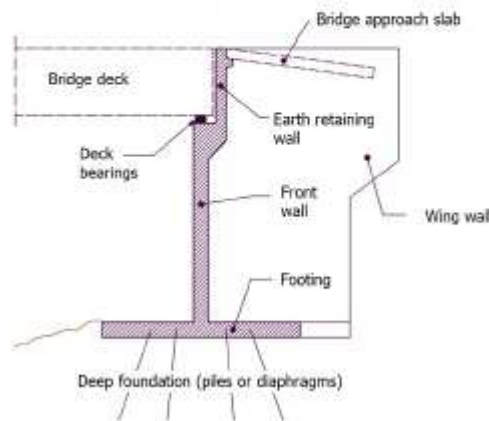
The pier has a cup on the top which is sometimes also called capital; then we have on the second pier two column and the footing at the base that can be above the piles or it can be a direct footing; sometimes we can have pier piles as we can see in the first pier on the left, so the same pile is coming up and becomes a vertical element of the pier.

They're the transition element between the bridge and the embankment

They're built almost essentially in concrete (masonry was also used in the past), sometimes in P.C. (using prefabricated elements assembled in situ by prestressing devices).

The backfill inside the abutment should be done with proper material whose grading has to be chosen. It should be laid in layers of 50+60 cm, which should undergo proper compaction. In such a way an internal friction angle of  $34^\circ \pm 38^\circ$  and a nil cohesion can be assumed

The transition slab should be modelled as a slab on elastic foundation, free on three sides and hinged on the embankment retaining wall

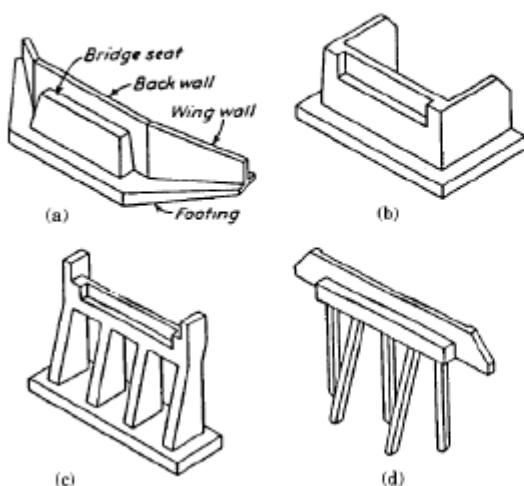


Now let's see a detailed situation of abutment. In this picture we have a vertical cross section parallel to the longitudinal axis of the deck. The bridge deck is placed over bearings, these bearing are placed on the top of the front wall where the front wall is the same we have seen in the last slide, it can be very thick and it is placed over a footing; the footing is also a foundation raft (rafts means slab) and this footing can be placed over piles or diaphragm. The difference between piles and diaphragms is that piles are generally smaller and circular while diaphragms are bigger and rectangular most of the times. A pile can be half a meter to 2 m diameter. On the top of the front wall generally there is a zone which is thicker to have room for the bearings and then we have another vertical element which is called heart retained wall which goes from the level of the bearings to the top surface of the deck. So the earth retained wall is as tall as the deck is deep. The function of this structure is keeping the ground away from going inside the deck. If we think of this element as a 3D element we have other two wall which are orthogonal to the earth retained wall and they are the wing walls. The wing walls start from the two side of the front wall and they go backwards. If we place together the front wall, the two wing walls, the footing we have a kind of a box without one face that is the face

Down on the right of the picture we have another kind of abutment and this abutment is called “crossing through” that is the embankment in going through the slab so we don’t have a front wall but we have vertical diaphragms and on the top of this wall we have a cup beam and it is where the deck is placed.

This abutment is not so strong as the previous but from the aesthetic point of view is better of course in urban environments solutions.

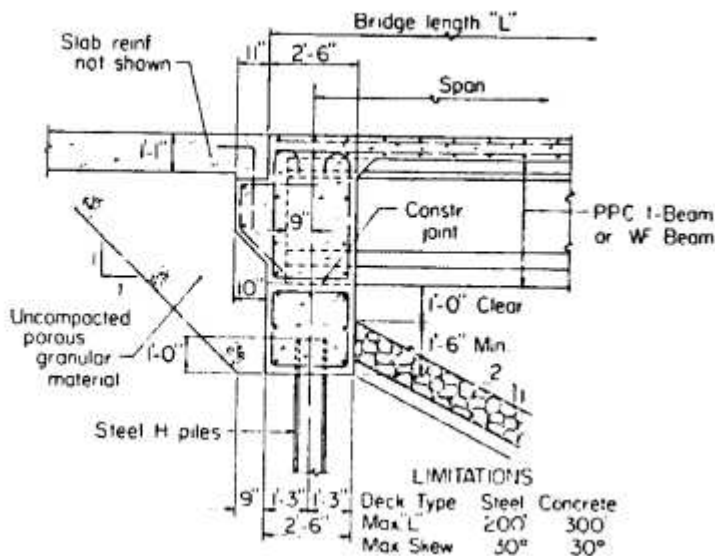
**Typologies**



**Figure 1-7** Various types of abutments; (a) typical gravity abutment with wing walls; (b) U-abutment; (c) spill-through abutment; (d) pile bent abutment with stub wings.

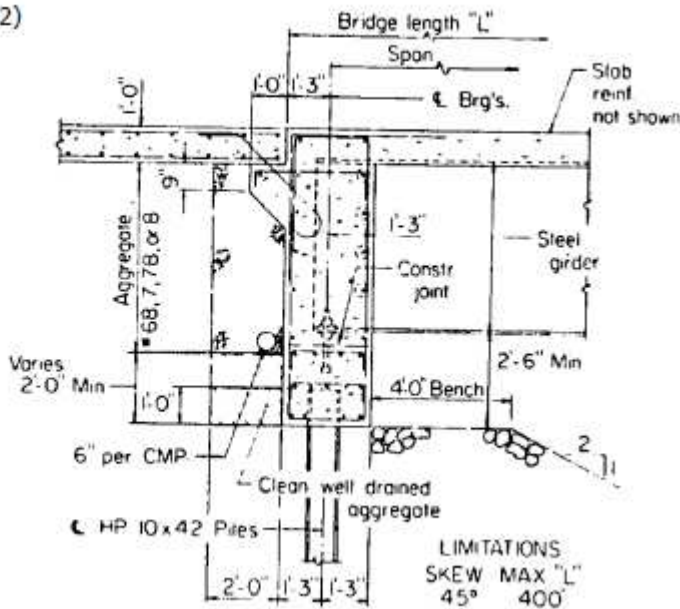
Here we can see different typologies of abutments and we have a which is a gravity abutment with wing walls that are not orthogonal to the front wall (that can be also called back wall in order to where we look). This abutment works because of its self-weight like and earth retained wall. Then we have b that is the most common abutment, it is like a but wing wall are at 90°. This is called C or U abutment because of the shape. Then we have c abutment which is open to the embankment to go through. Finally we have the solution for light weight deck that is the solution d and it is called pile bent abutment with stub wings.

Integral abutments (1)



Here we see an example of an abutment that is used for an integral bridge.

Integral abutments (2)



## SEISMIC DESIGN OF NEW BRIDGES

### Lezioni XXX-XXXI-XXXII (Luca Giordano)

The subject of today lesson is the seismic design of bridges and in particular the seismic design of new bridges.

We will refer to EC8. Because seismic action is more conventional with respect to other types of actions and we should find differences referring to a code or another.

From a general point of view:

- ➡ A bridge must be designed, constructed and maintained so that it adequately and in an economically reasonable way performs in earthquakes that may take place during its construction and service. More specifically, the bridge must:
  - ❖ remain fit for the use for which it has been designed
  - ❖ withstand **extreme**, **occasional** and **frequent** seismic actions likely to occur during its anticipated use and avoid damage by an **exceptional** earthquake to an extent disproportionate to the triggering event

These two points are the main performances that we ask to a bridge subjected to an earthquake.

- ➡ In order to meet these performances, we can define two serviceability limit states and two ultimate limit states and associated representative seismic action, with a prescribed probability of not being exceeded during the design service life

So we could define a lot of seismic actions that are correlated with the probability to be exceeded.

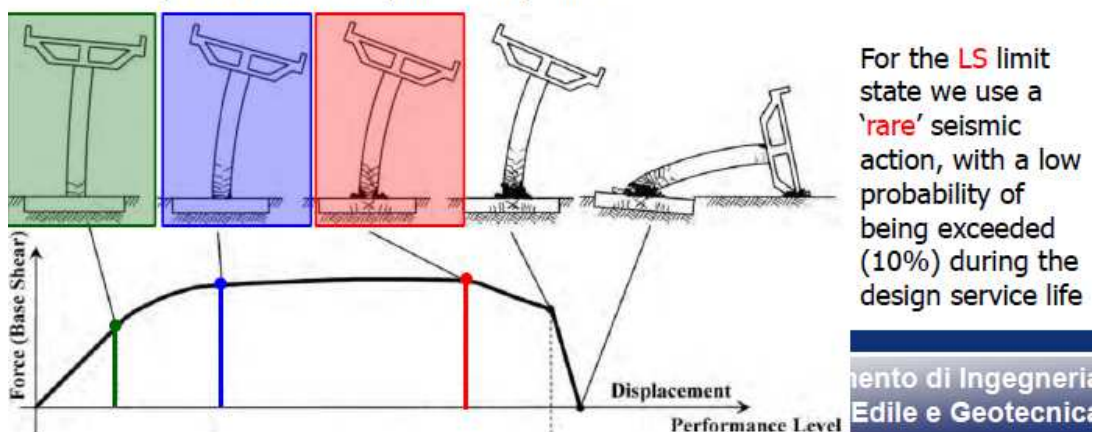
The performances of the bridge can be illustrated referring to the following graph in which we have on the y axis the total base shear due to the seismic action; in the picture we can see in the transversal direction a pier connected to a deck and we can see its performances. In the x axis we have the lateral displacement of the pier.

The first serviceability limit state is the **operational limit state**.

In the chart we are at the end of the linear elastic behaviour of the structure.

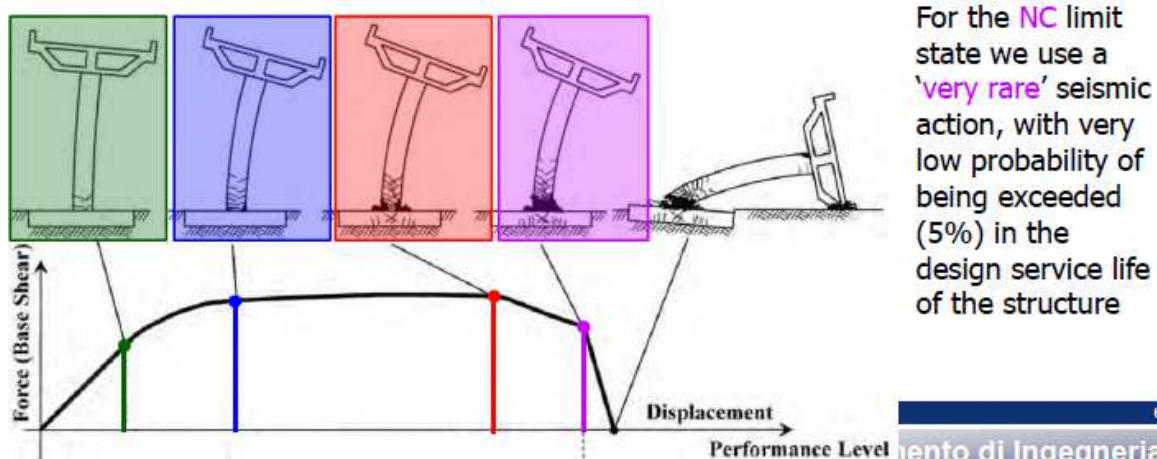
➔ Ultimate limit states:

- ❖ The **life safety (LS)** limit state. It is reached if any of the following conditions are met (but not surpassed): the structure is significantly damaged, but does not collapse; the structure does not provide sufficient safety for normal use, although it is safe enough for temporary use; non-structural components are seriously damaged, but do not obstruct emergency use; the structure is on the verge of losing capacity, although it retains sufficient load-bearing capacity and sufficient residual strength and stiffness to protect life for the period until the repair is completed.



We are in this case in the plastic range and so we have a lot of damage in the more stressed area and irreversible displacements. But we haven't had yet any decrease of the shear base as we can see.

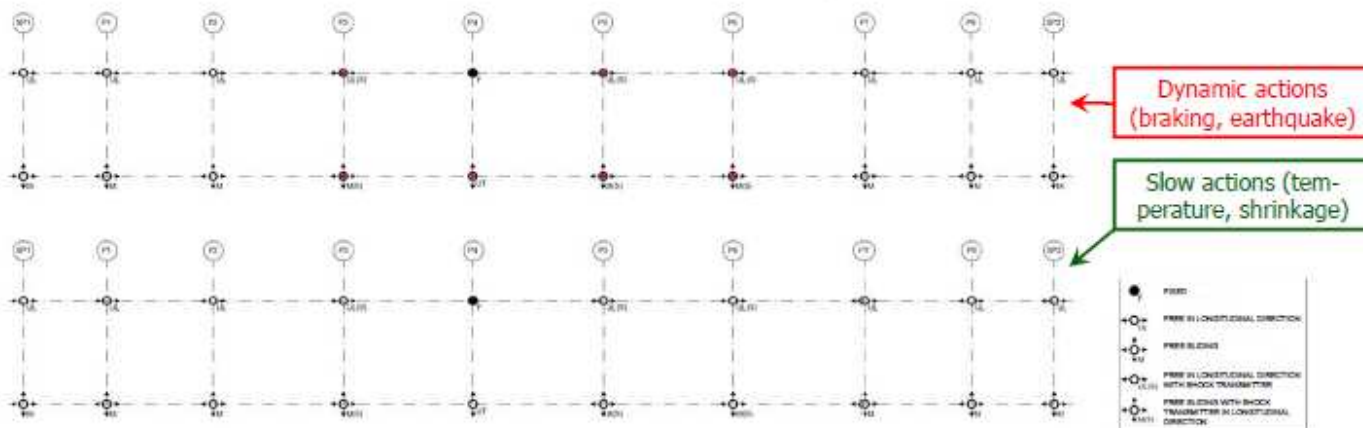
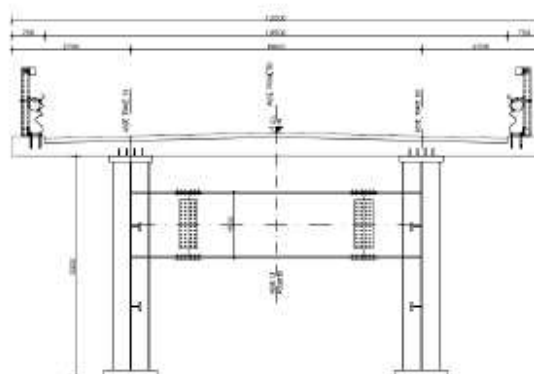
- ❖ The **near-collapse (NC)** limit state. It is reached if any of the following conditions are met: the structure is heavily damaged and is at the verge of collapse; although life safety is mostly ensured during the loading event, it is not fully guaranteed as there may be life-threatening injury situations due to falling debris; the structure is unsafe even for emergency use, and would probably not survive additional loading; the structure presents low residual strength and stiffness but is still able to support the quasi-permanent loads



The correct way to perform a correct dimensioning in seismic is considering the lateral displacements of the structure, and so a displacement base design; EC8 use a force base design approach.

➔ The prime aim of seismic design is to **accommodate the dynamic horizontal displacements** imposed by earthquake with controlled damage. In bridge structures this can be achieved in three main ways:

A. To fix or rigidly connect horizontally the deck to the top of at least one pier, but let it slide or move on flexible bearings at all other supports (including the abutments)



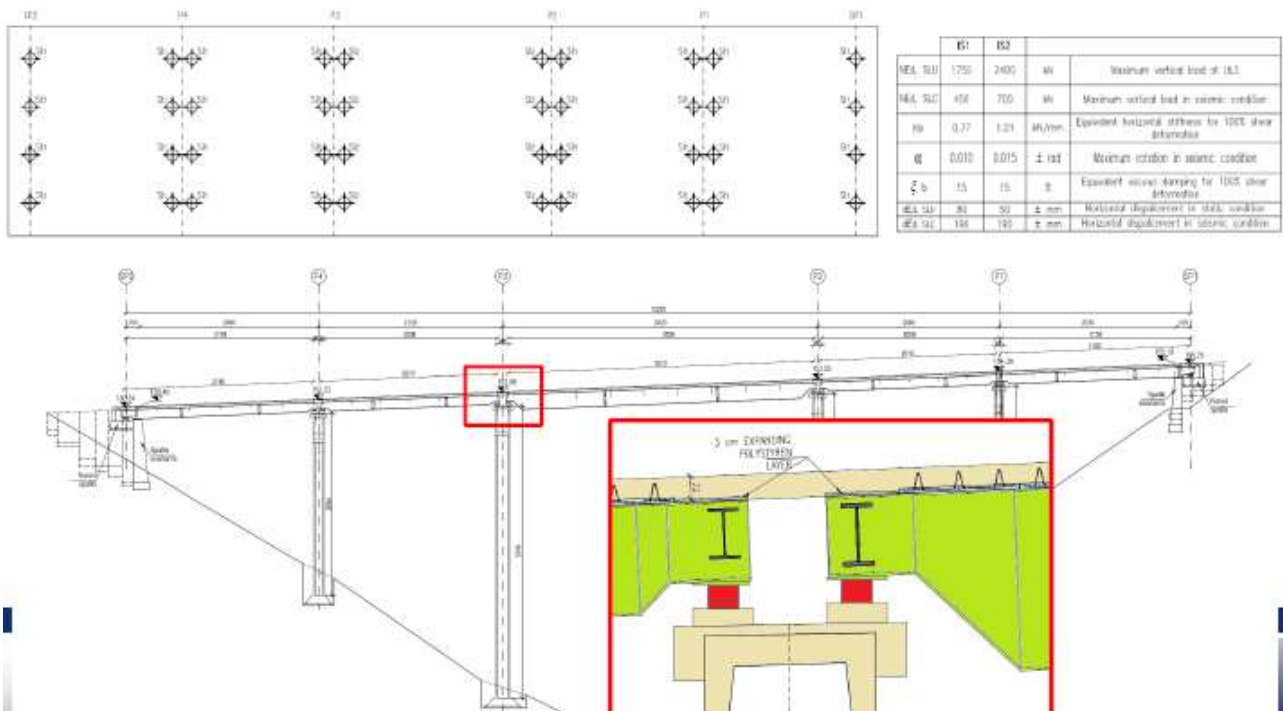
We can see here an example of this approach with the use also of a particular device that we are going to illustrate. We can see here a multi-span bridge from abutment 1 to abutment 2 through intermediate piers. It is composite steel concrete deck.

We can see the typology of bearings that we use in the design for each beam and for each support alignment. We have a lot of different bearings typologies. The first one is on the pier 4 and it is the fixed bearing, so in this point we connect the deck and the pier in this point in the longitudinal direction and also in transversal direction. In the other piers we have two types of other bearings and in particular on abutment 1,2,7,8 we have a bearing that allow the longitudinal displacement of the deck but not the transverse one. On piers 6,5,3 we have the same bearings but with an additional **shock transmitter** that is a device that has a different behaviour depending on the type of action that we apply on the bridge. If the deck has a slow velocity for instance a temperature variation or a shrinkage of concrete than the displacement is free and the shock transmitter doesn't transmit any force. But if the force is a dynamic action (braking, earthquake) the shock transmitter transmits the force and avoid an horizontal differential displacement from deck to pier.



This is an example in which the deck is connected to the abutment 2 and we can see that longitudinal actions are beard from abutment 2 while piers and abutment 1 are free in longitudinal direction and then the longitudinal force is carried by abutment 2.

- C. To place the deck on a system of sliding or horizontally flexible bearings (or bearing-type devices) at the top of the substructure (the abutments and all piers) and accommodate the horizontal displacements at this interface (seismic isolation); piers are designed to remain elastic during the design seismic action



In this example we can see a work of retrofitting of an existing bridge in which retrofitting is realized through a strengthening of the piers and of the abutments and with a complete replacement of the deck. In this case we have four beams and four bearings for each alignment in particular we have 4 high dumping elastomeric bearings that are isolation devices.

supported deck for vertical actions and we can analyse the structure with a gird model. But for seismic analysis we use a single line model at the centreline of the deck and we can analyse how to use the correct mass of the deck, the correct stiffness of the deck and a single line model simplify the analysis and so we have less number to control.

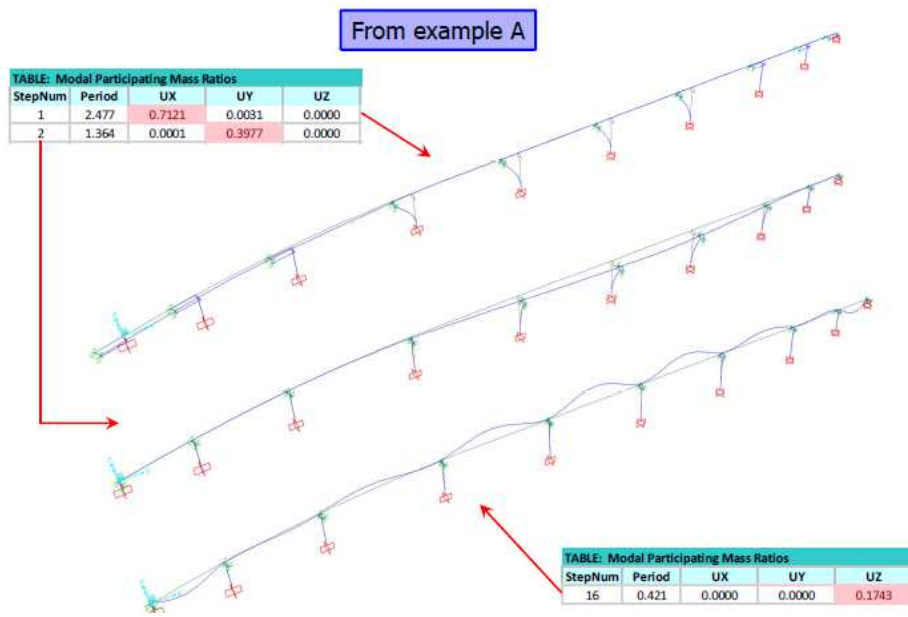
- ➡ Modal response spectrum analysis should take into account all modes contributing significantly to any response quantity of interest
- ➡ EC8 considers the total force resultant in the direction of each seismic action component as the prime response quantity of interest, and sets as a goal of the eigenvalue analysis the capture of at least 90% of its full value.

This is translated into a criterion for the participating modal mass: EC8 requires the N modes that are taken into account to provide together a total participating modal mass along any individual direction of the seismic action components considered in the design, at least 90% of the total mass

From example A

StepNum	Period	UX	UY	UZ	SumUX	SumUY	SumUZ
1	2.477	0.7121	0.0031	0.0000	0.712	0.003	0.000
2	1.364	0.0001	0.3977	0.0000	0.712	0.401	0.000
3	1.208	0.0001	0.0172	0.0000	0.712	0.418	0.000
4	0.967	0.0012	0.1532	0.0000	0.713	0.571	0.000
9	0.630	0.0012	0.0784	0.0000	0.716	0.655	0.015
15	0.431	0.0011	0.0553	0.0000	0.738	0.712	0.019
16	0.421	0.0000	0.0000	0.1743	0.738	0.712	0.194
17	0.416	0.0000	0.0007	0.0008	0.738	0.713	0.195
18	0.371	0.0000	0.0000	0.1538	0.738	0.713	0.348
19	0.352	0.0005	0.0000	0.0000	0.738	0.713	0.348
20	0.350	0.0000	0.0000	0.1437	0.738	0.713	0.492
50	0.100	0.0000	0.0000	0.0653	0.808	0.770	0.566
51	0.099	0.0000	0.0000	0.0588	0.808	0.770	0.625
52	0.099	0.0000	0.0000	0.0018	0.808	0.770	0.627

So for example from the A example we have the table of the modal participating mass ration and we can see the eigen shape and the period of each eigen shape.



This is the eigen shape of mode 1,2 and 16 (the first one with a value of mass in z direction)

- Previous equation can be applied at the end of the design process as the amount of vertical bars and their layout in the pier section should be known
- One way to bypass the problem is to use the pier reinforcement from the design of the bridge for the non-seismic actions (the analysis for which uses the uncracked stiffness of the pier), also taking into account the minimum reinforcement. Another way is the use of empirical equations

$$\frac{(EI)_{eff}}{(EI)_c} = a \left( 0.8 + \ln \frac{L_n}{h} \right) \left( 1 + 0.048 \frac{N}{A_c} [\text{MPa}] \right)$$

where  $A_c$  and  $h$  are the area and the depth of the pier section, respectively, and

for circular or rectangular piers:  $a = 0.081$

for hollow rectangular ones:  $a = 0.09$

- For circular piers: shear span ratio,  $L_n/D$ , from 1.0 and 8.5 (mean value 3.25), concrete strength,  $f_c$ , from 19 MPa to 90 MPa (mean value about 35 MPa), axial load ratio,  $N/A_c f_c$ , from -0.1 to 0.7 (mean value about 0.15), vertical reinforcement ratio from 0.5 to 5.7% (average 2.5%).
- For rectangular piers: shear span ratio,  $L_n/h$ , from 1.0 to 13 (mean value 4),  $N/A_c f_c$  from -0.05 to 0.9 (average about 0.125), section slenderness,  $h/b_w$ , from 0.2 to 4 (mean value 1.3), vertical reinforcement ratio between 0.11 and 8.5% (average 1.97%),  $f_c$  from 9.6 MPa to 118 MPa (mean value 37.2 MPa).
- For hollow rectangular piers: shear span ratio,  $L_n/h$ , from 0.6 to 8.3 (mean value 2.6),  $N/A_c f_c$  from 0 to 0.5 (average about 0.075), wall thickness,  $b_w$ , between 50 mm and 500 mm (average 120 mm), wall slenderness,  $h/b_w$ , from 2.5 to 36 (mean value 12.5), vertical reinforcement ratio between 0.34 and 6.2% (average 1.35%),  $f_c$  from 20 MPa to 102 MPa (mean value 43 MPa).

Range of applicability

- In any case, it should be borne in mind that, in general, in a forced-based seismic design context, internal forces arising from a linear seismic analysis are on the safe side, if stiffness values are overestimated (and hence periods of normal modes underpredicted). By contrast, the seismic displacements and deformations from such an analysis are underpredicted. The reverse is the case if the analysis is based on underestimated values of stiffness.

For this reason, EC8 suggests **checking a posteriori whether the stiffness values initially assumed for the members are consistent with the level of moments predicted for them**. If they are found to be significantly lower, they should be updated, and the analysis repeated

- ➔ If elastomeric bearings (a block of vulcanised elastomer reinforced internally with steel plates, chemically bonded to the elastomer during vulcanization) are provided at some support alignments, they have to be considered in the structural model with their stiffness (refer to EN1337 or to the manufacturer's catalogue)

## CONCEPTUAL DESIGN FOR EARTHQUAKE RESISTANCE

The design in general for seismic actions on bridges mainly concerns the piers and the way they are connected or supported to the deck. An important role is played by the mass of the deck and that is reducing the mass of the deck is of primary importance for bridge seismic design for displacements and forces demands.

⇒ Although the conceptual (and also the detailed) seismic design of bridges concerns mainly the piers and the way they are connected to or support the deck, **reducing the deck's self-weight** is of prime importance for the bridge seismic design as well: both seismic force and displacement demands increase with the deck mass (they are normally approximately proportional to its square root)

⇒ Seismic behavior of bridges is mainly affected by:

- ❖ the **continuity of the deck** over the piers
- ❖ whether the **deck is monolithic with the piers**



⇒ In fact, one of the most common damage in multi span bridges is local **loss of support of the deck**, due to unseating from a pier



⇒ The best way to prevent drop of a part of a multi-span deck from one or more piers is by providing **continuity of the spans over all piers**: a deck continuous from abutment to abutment. **Exceptions** are:

- ❖ **very long bridges** (several hundred or over a thousand)
- ❖ if it is considered likely that a strong earthquake may induce **significantly different movement at the base of adjacent piers** (the bridge straddles a potentially active tectonic fault or crosses non-homogeneous soil formations)

⇒ Intermediate movement joints are placed between two spans whose ends are supported through separate bearings on the same pier. In such a layout, the movement joint should be **wide enough to prevent pounding** between the ends of the two spans, in addition to providing sufficient support length against unseating

⇒ Deck spans composed of precast girders are normally simply supported on the piers. In this case, continuity of adjacent spans may be get through a **cast-in-situ topping slab** continuous over the joint between two girder ends

In Italy we must refer to the National code Nuove Norme Tecniche per le Costruzioni NTC18 and his explanatory document (Circolare 21-01-19) in order to design the bridge.

The Italian code in most of its parts descends from Eurocodes and in particular if we refer to traffic load models, they directly descend from Eurocode 1 because it is an Italian traducing of what is written in the Eurocode.

A general observation is that all the topics that aren't fully clear in Italian Code can be read in the Eurocodes.

## **GLOSSARY: BRIDGE TYPES**

Bridges can be classified by the types of loads on them :

### I Footbridges

bridges that are intended to be used only on foot or by bike; in general this kind of bridges is provided by physical barriers at the beginning and at the end of them. We need to consider the possibility that special-vehicles could pass on it, for example emergency vehicles or maintenance vehicles;

### II Road bridges

Bridges that are used to carry road vehicles like cars, lorries and also pedestrian and cycles tracks can be present on the same deck;

### III Rail bridges

bridges that are intended to carry the rail traffic.

## **GLOSSARY: PARTS OF A BRIDGE**

- Abutment (*spalla*)

it is the structure that bears the deck and the extreme edges. Generally it is the earth retaining wall and it could be a massive concrete structure with strong foundations that have to transfer all the load of the bridge to the ground;

- Deck (*impalcato*)

is the part of the bridge that carries the traffic loads. The deck can be located directly between two abutments or can be beard between the abutments by piers (piers can be of different types but also walls can be used to bear the deck);

- Moveable inspection platform (*generally called by-bridge*)

## GLOSSARY: ROAD BRIDGES

- Carriageway (*carreggiata*)  
is the part of the deck where vehicles can move; later we will see a stronger definition of carriageway in order to define design parameters;
- Central reservation (*fascia centrale*)  
it separates the physical traffic lanes and in this part vehicles in general can't pass;
- Hard shoulder (*corsia di emergenza*)  
it is a traffic lane in which vehicles can move only for emergency reasons but should be free during the ordinary movement of the traffic;
- Hard strip  
It has dimension of less than 2m wide; it is in addition to the hard shoulder and also could be used only for emergency situation, for vehicles that have to stop on the track;
- Abnormal load(*carico eccezionale*)  
It is a particular kind of load that, in order to pass on the infrastructure, has to be allowed from authorities. In general we can have 3 types of abnormal loads:
  - 1) bigger than ordinary loads for examples lorries that should have a permission from authority to pass on the bridge and has a signal on the vehicle that shows it is an abnormal load;
  - 2) a second case is loads that need a permission to pass but need also a car in front and on the back in order to pass on the specific infrastructure;
  - 3) the third is represented by lorries that are particularly high and have to be accompanied also from police's car to move on the structure;
- Notional lane  
It is a design tool. First of all we need to perform a distinction: physical lanes are the real lanes where the traffic passes every day, and with notional lane we denote a theoretical lane (not a physical lane) that we use to locate traffic actions on the bridges just for design requirements. It is not real on the track but is something that we can use it as designers to locate traffic loads on the deck;