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POLITECNICO DI TORINO

Road, railway & airport superstructures

Exercises solved during the course

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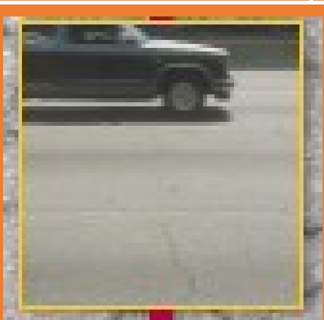
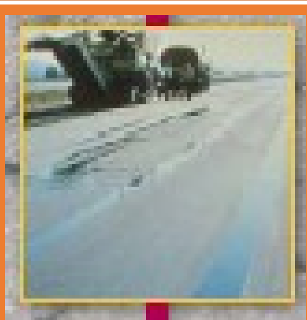




Exercises #1

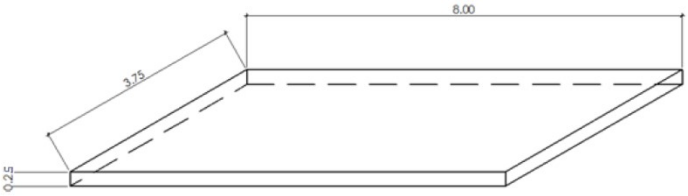
#1.1 Plain Concrete Slab

A plain concrete slab (geometrical and mechanical characteristics in Table(1) is supported by a subgrade whose modulus of subgrade reaction is 100 MN/m³.

Table 1.

E	[GPa]	30
v	[-]	0.15
γ	[kN/m³]	25
α	[1/°C]	9.E-06

h	[m]	0.25
L_x	[m]	8.00
L_y	[m]	3.75



#1.1.1 Stress and deflections – Westergaard's closed formulas

Given a circular load (P=30 kN, q=650 kPa), using closed-form formulas determine maximum stress and deflection in the different situations. The formulas given are:

Corner Loading

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{\ell}\right)^{0.6}\right]$$

$$\Delta_c = \frac{P}{k\ell^2}\left[1.1 - 0.88\left(\frac{a\sqrt{2}}{\ell}\right)\right]$$

Interior Loading

$$\sigma_i = \frac{3(1+\nu)P}{2\pi h^2}\left(\ln\frac{\ell}{b} + 0.6159\right)$$

$$b = a \qquad \text{when } a \geq 1.724h$$

$$b = \sqrt{1.6a^2 + h^2} - 0.675h \qquad \text{when } a < 1.724h$$

$$\Delta_i = \frac{P}{8k\ell^2}\left\{1 + \frac{1}{2\pi}\left[\ln\left(\frac{a}{2\ell}\right) - 0.673\right]\left(\frac{a}{\ell}\right)^2\right\}$$

Edge Loading

$$\underset{\text{(circle)}}{\sigma_e} = \frac{3(1+\nu)P}{\pi(3+\nu)h^2}\left[\ln\left(\frac{Eh^3}{100ka^4}\right) + 1.84 - \frac{4\nu}{3} + \frac{1-\nu}{2} + \frac{1.18(1+2\nu)a}{\ell}\right]$$

$$\underset{\text{(semicircle)}}{\sigma_e} = \frac{3(1+\nu)P}{\pi(3+\nu)h^2}\left[\ln\left(\frac{Eh^3}{100ka^4}\right) + 3.84 - \frac{4\nu}{3} + \frac{(1+2\nu)a}{2\ell}\right]$$

$$\underset{\text{(circle)}}{\Delta_e} = \frac{\sqrt{2+1.2\nu}\,P}{\sqrt{Eh^3k}}\left[1 - \frac{(0.76+0.4\nu)a}{\ell}\right]$$

$$\underset{\text{(semicircle)}}{\Delta_e} = \frac{\sqrt{2+1.2\nu}\,P}{\sqrt{Eh^3k}}\left[1 - \frac{(0.323+0.17\nu)a}{\ell}\right]$$

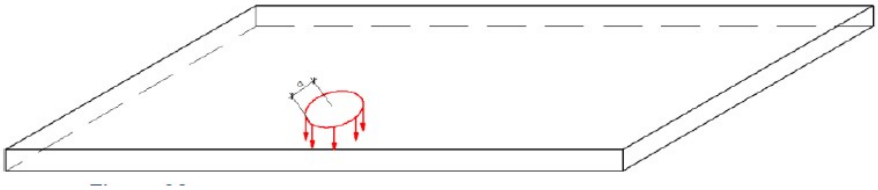
$$\sigma_i = \frac{3 \cdot (1+0.15) \cdot 30\,kN}{2 \cdot \pi \cdot 0.25^2\,m^2} \cdot \left[ln\left(\frac{79.5\,cm}{12.45\,cm}\right) + 0.6159\right] = 651\,kPa$$

$$\Delta_i = \frac{30\,kN}{8 \cdot 100 \cdot 10^3\,\frac{kN}{m^3} \cdot 0.795^2\,m^2} \cdot \left\{1 + \frac{1}{2\pi} \cdot \left[ln\left(\frac{12.12\,cm}{2 \cdot 79.5\,cm}\right) - 0.673\right] \cdot \left(\frac{12.12\,cm}{79.5\,cm}\right)^2\right\} = 0.059\,mm$$

- **Edge circular loading**

$$\sigma_e = \frac{3 \cdot (1+0.15) \cdot 30\,kN}{(3+0.15) \cdot \pi \cdot 0.25^2\,m^2} \cdot \left[ln\left(\frac{30 \cdot 10^3\,MPa \cdot 0.25^3\,m^3}{100 \cdot 100 \cdot 10^3\,\frac{kN}{m^3} \cdot 0.1212^4\,m^4}\right) + 1.84 - \frac{4 \cdot 0.15}{3} + \frac{1-0.15}{2} + \frac{1.18 \cdot (1+2 \cdot 0.15) \cdot 12.12\,cm}{79.5\,cm}\right] = 1285.5\,kPa$$

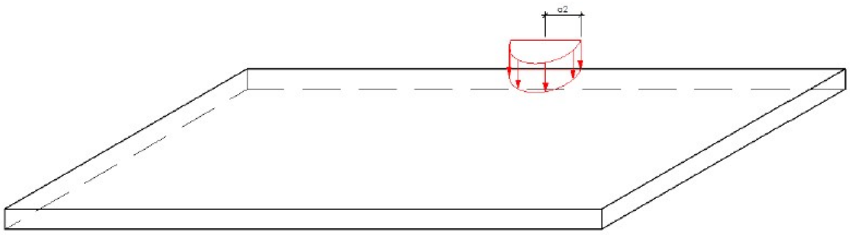
$$\Delta_e = \frac{\sqrt{2+1.2 \cdot 0.15} \cdot 30\,kN}{\sqrt{30 \cdot 10^3\,MPa \cdot 0.25^3\,m^3 \cdot 100\,\frac{MN}{m^3}}} \cdot \left\{1 - \frac{(0.76+0.4 \cdot 0.15) \cdot 12.12\,cm}{79.5\,cm}\right\} = 0.179\,mm$$



- **Edge semicircular loading**

$$\sigma_e = \frac{3 \cdot (1+0.15) \cdot 30\,kN}{(3+0.15) \cdot \pi \cdot 0.25^2\,m^2} \cdot \left[ln\left(\frac{30 \cdot 10^3\,MPa \cdot 0.25^3\,m^3}{100 \cdot 100 \cdot 10^3\,\frac{kN}{m^3} \cdot 0.1212^4\,m^4}\right) + 3.84 - \frac{4 \cdot 0.15}{3} + \frac{(1+2 \cdot 0.15) \cdot 12.12\,cm}{2 \cdot 79.5\,cm}\right] = 763.3\,kPa$$

$$\Delta_e = \frac{\sqrt{2+1.2 \cdot 0.15} \cdot 30\,kN}{\sqrt{30 \cdot 10^3\,MPa \cdot 0.25^3\,m^3 \cdot 100\,\frac{MN}{m^3}}} \cdot \left\{1 - \frac{(0.323+0.17 \cdot 0.15) \cdot 12.12\,cm}{79.5\,cm}\right\} = 0.097\,mm$$



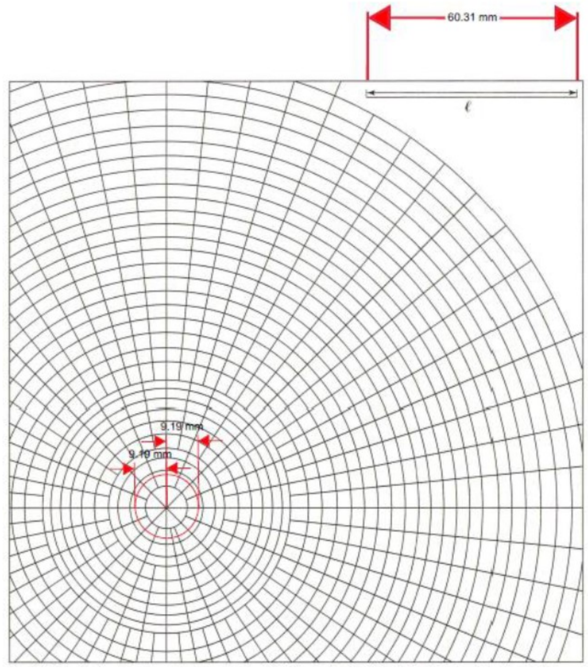
The next table summarizes the results:

Load case		Maximum stress σ [kPa]	Maximum deflection Δ [mm]
Corner loading		866.5	0.432
Interior loading		651.0	0.059
Edge loading	**Circular**	1285.5	0.179
	Semicircular	763.3	0.097

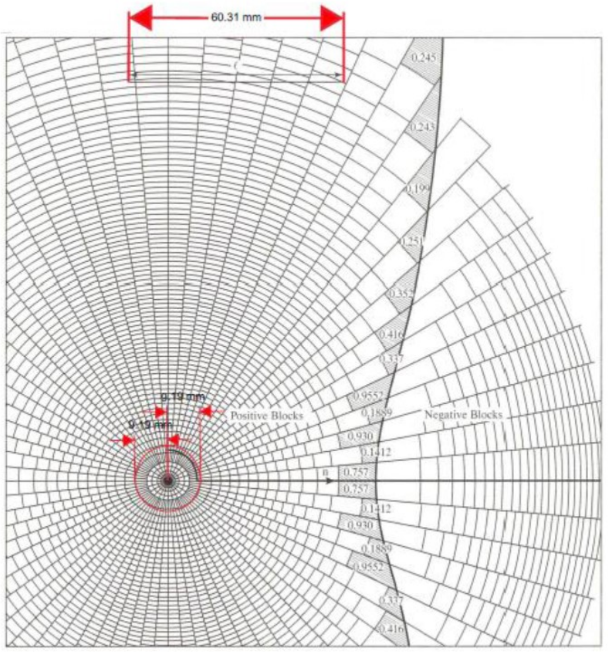
The charts are used as the following:

Having the values of (l) and (a), one can measure the length of (l) in the chart and find the scaled value of (a), knowing that it is the radius of the load, one draws the correspondent circle and counts the number of blocks.

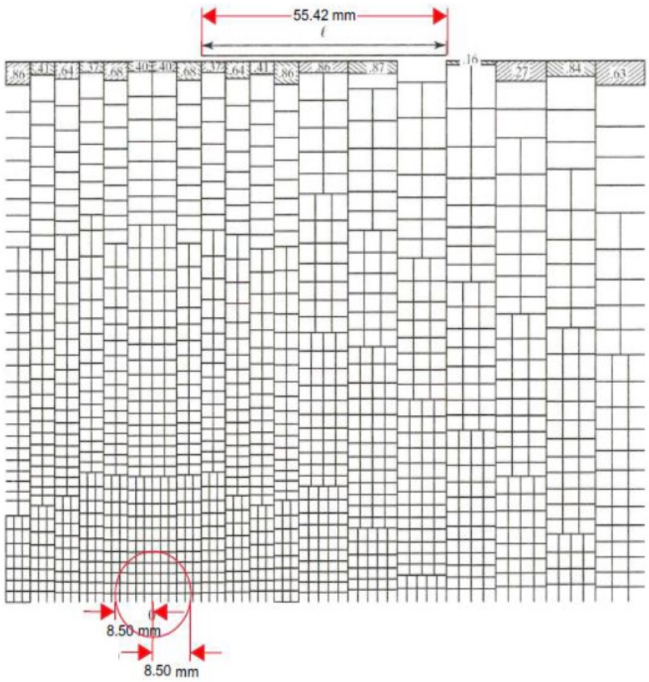
Influence chart for deflection due to interior loading



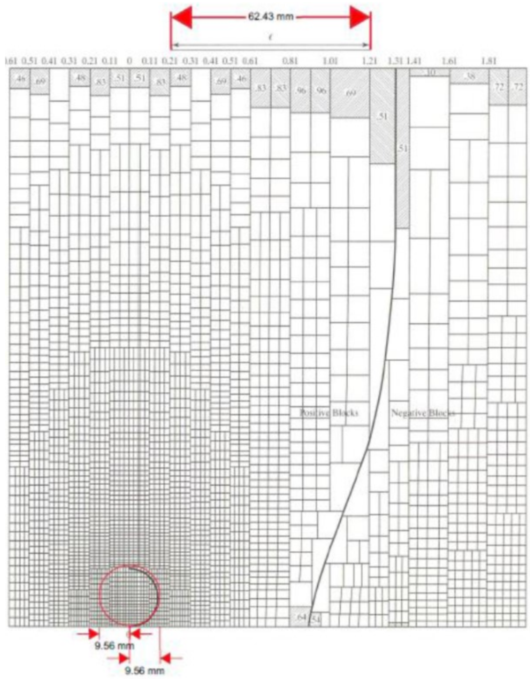
Influence chart for moment due to interior loading



Influence chart for deflection due to circular edge loading



Influence chart for moment due to circular edge loading



In the first case, $T_{top} > T_{bottom}$, there are springs which push up the pavement while in the second case, $T_{top} < T_{bottom}$ there are compressed springs.

- **Westergaard & Bradbury**

He gives an analytical solution for an infinite slab. The following equations are normal strains caused by stresses σx and σy, where the first term is a direct effect in the considered direction and the second term is an indirect effect. The total stress will be the sum of these 2 components.

$$\sigma = \frac{E \cdot \alpha_T}{2 \cdot (1 - \nu^2)} + \nu \cdot \frac{E \cdot \alpha_T}{2 \cdot (1 - \nu^2)}$$

Until now it has been considered infinite slabs, but in real cases they finite, so we correction factors must be considered, through which stresses at the center of the slab are calculated:

$$\sigma_{interior} = \frac{E \cdot \alpha_T}{2 \cdot (1 - \nu^2)} \cdot (C_x + \nu \cdot C_y)$$

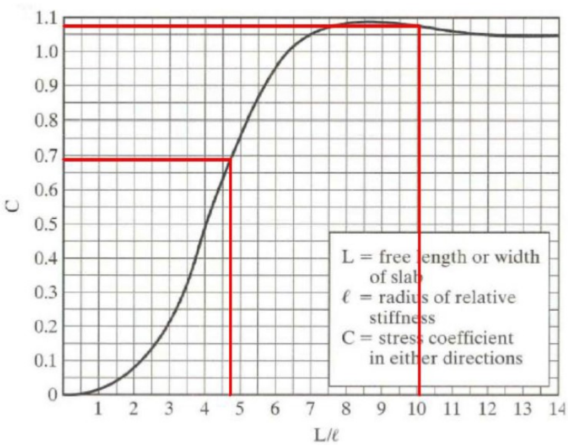
The edge stress at the midspan of the slab is calculated assuming no stresses in the perpendicular direction, considering ν = 0, no Poisson effect.

$$\sigma_{edge,midspan} = \frac{E \cdot \alpha_T}{2} \cdot C_x$$

Bradbury relates C_X and C_y to the length of the slab or width, depending on the direction that is considered and the radius of relative stiffness, previously calculated:

$$\frac{L_x}{l} = 10.062 \qquad ; \qquad \frac{L_y}{l} = 4.717$$

Entering with this values on the related chart, the factors C_X and C_y are obtained:



Load case	C_X	C_y	Curling stresses [kPa]	
			σ_x	σ_y
Interior loading	1.07	0.69	1940	1400
Edge loading			1700	1130

- **Eisenmann**

Eisenmann theory is based on field observations: he studied slabs with different lengths, calculating the upward deflection due to curling. He doesn't consider the negative temperature (upward curling) because the negative gradient is usually half of the positive one. That's why normally cracks are given by +ΔT on top.

In this approach, the concept of **critical length** is introduced, which affects the slab's thermal bending: deflection increases with slab length close to the critical length, beyond which, the upward deflection remains constant.

Since there is the condition **Lx/Ly > 1.2**, thus the plate is assumed rectangular and **β=20**, the critical length is calculated as follows:

As one can see, after the calculations made, in the x direction, the stresses are higher, because there is more curling effect on the most demanding direction (higher stress), that, in this case, is the longitudinal one.

In general terms, making a comparison between Westergaard & Bradbury and Eisenmann methods, one can note that the curling values have almost the same order of magnitude, being for the first approach bigger in the y direction; instead, for the x direction the values are similar, but the highest values are obtained using Eisenmann method.

Only for sake of completeness, if $\boldsymbol{L'} = (\boldsymbol{L_{crit}} = \mathbf{5.69}\ \boldsymbol{m})$, the slab has double curling curves with a contact point in the central area and the stress calculated is:

$$\sigma'_{\omega} = 1.2 \cdot \sigma_{\omega} = 2287\ kPa$$

- **Joint opening**

In order to do a complete analysis of the pavement, one can calculate also the joint opening value, that depends on the shrinkage characteristics of concrete rather than on the stress. In fact, longer joint spacings cause the joint to open wider and decrease the efficiency of load transfer. Using the approximated formula of **Darter & Barenberg**, the results are:

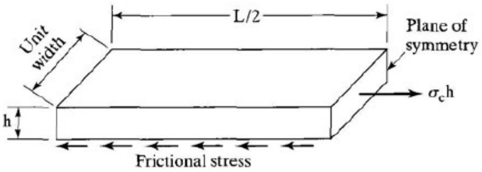
$$\Delta L = C \cdot L \cdot (\alpha_t \cdot \Delta T + \varepsilon) = 0.65 \cdot 8\ m \cdot \left(9 \cdot 10^{-6} \frac{1}{°C} \cdot 12\ °C + 2.5 \cdot 10^{-4}\right) = 1.86\ mm$$

Where:

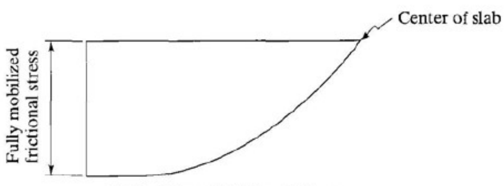
- C = correction coefficient due to slab-subbase friction: for stabilized base it's equal to 0.65 and 0.80 for granular base;
- L = slab longitudinal length(Lx) or joint spacing;
- α_t = coefficient of thermal expansion of concrete = 9e-6 1/°C;
- ΔT = temperature differential = 12°C
- ε = drying shrinkage coefficient of concrete that ranges between $[0.5 \cdot 10^{-4} \div 2.5 \cdot 10^{-4}]$ = chosen as $2.5 \cdot 10^{-4}$.

#1.1.4 Friction (tensile) stresses in concrete

Given a coefficient of friction of 1.5, determine stress in concrete due to friction.



(a) Free Body Diagram

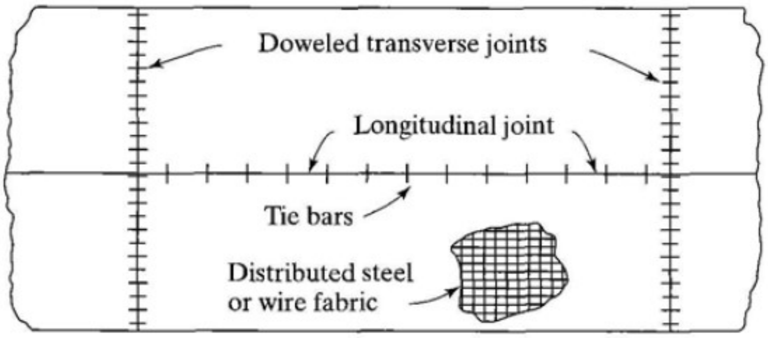


(b) Variation of Frictional Stress

#1.2 Jointed Reinforced Concrete Pavement

A reinforced slab (geometrical and mechanical characteristics in Table 2) is supported by a subgrade whose modulus of subgrade reaction is 100 MN/m3. The friction coefficient is 1.5.

E [GPa]	30	A_y [mm²/m]	100
ν	0.15	A_{tie} [mm²/m]	140
γ_c [kN/m³]	25	ϕ_{dowel} [mm]	30
h [m]	0.25	z [m]	0.005
L_x [m]	8.00	E_d [GPa]	200
L_y [m]	3.75	k_{ds} [GN/m³]	400
A_x [mm²/m]	280	s [m]	0.3



#1.2.1 Tensile stresses in the longitudinal and transverse reinforcement

Given a coefficient of friction of 1.5, determine stress due to friction in longitudinal and transverse reinforcement.

It is important to remember that all tensile stress, induced by friction, is assumed to be absorbed only by the steel reinforcement, which is a wire fabric/mesh. So, in this case, the slab equilibrium can be determined by:

$$\sigma_{steel} \cdot A_{steel} = \sigma_{friction} \cdot A_{friction}$$

$$\sigma_{steel} \cdot A_{steel} \cdot w = f_a \cdot \gamma_c \cdot \frac{L}{2} \cdot h \cdot w$$

$$(w = 1m) - -\rightarrow \ \sigma_{steel} = \frac{f_a \cdot \gamma_c \cdot h}{A_{steel}} \cdot \frac{L}{2}$$

Now the tensile stress depends also on the slab's thickness. The steel is usually placed at the middepth of the slab and discontinued at the joint. The stress calculated below refers to the centre of the slab.

The functions of the reinforcement are:

- Increase joint spacings (Lx);
- Tie cracked concrete together to allow load transfer through aggregate interlocking;
- No influence on the structural\bearing capacity of the slab.

So considering the both directions, these stresses appear in the reinforcement due to friction:

$$\sigma_{steel,\ x} = \frac{1.5 \cdot 25\ \frac{kN}{m^3} \cdot 0.25\ m}{280 \cdot \frac{10^{-6} m^2}{m}} \cdot \frac{8\ m}{2} = 133.9\ kPa$$

$$\sigma_{steel,\ y} = \frac{1.5 \cdot 25\ \frac{kN}{m^3} \cdot 0.25\ m}{100 \cdot \frac{10^{-6} m^2}{m}} \cdot \frac{3.75\ m}{2} = 175.8\ kPa$$

#1.2.3 Bearing stress between dowel and concrete under loading

Determine maximum bearing stress between dowel and concrete when two 40 kN loads are applied in correspondence of dowels A and B (Figure 1).

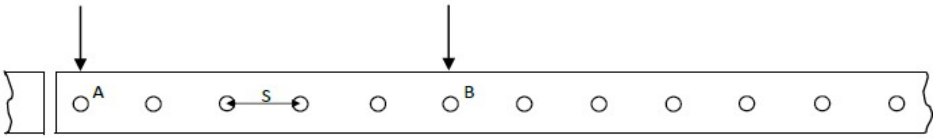
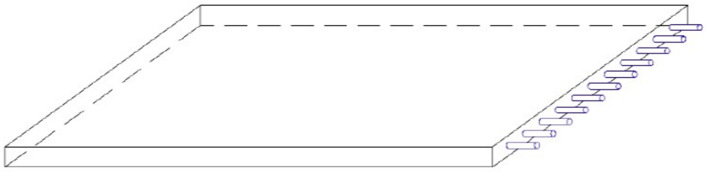


Figure 1.



The maximum bending moment will appear at a distance of 1.8 times the radius of relative stiffness from the point of application of the load (in the transverse direction). Other important constants are the inertia of the dowel and the stiffness β, that is the relative stiffness between dowel and concrete:

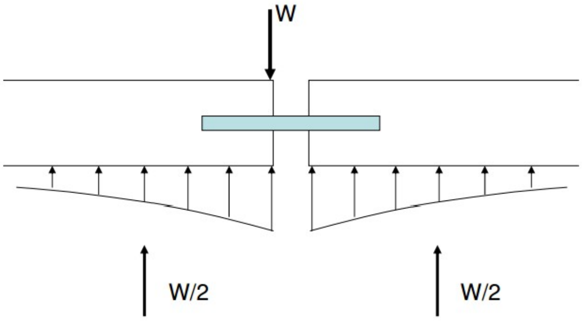
$$l = \sqrt[4]{\frac{E \cdot h^3}{12 \cdot (1 - \nu^2) \cdot K}} = \sqrt[4]{\frac{30 \cdot 10^3\ MPa \cdot 0.25^3\ m^3}{12 \cdot (1 - 0.15^2) \cdot 100\ \frac{MN}{m^3}}} = 0.795\ m$$

$$L_{max} = 1.8 \cdot l = 1.8 \cdot 0.795 = 1.43\ m$$

$$I_{dowel} = \frac{\pi \cdot \emptyset_d^4}{64} = 3.98 \cdot 10^{-8}\ m^4$$

$$\beta = \sqrt[4]{\frac{k_{ds} \cdot \emptyset_d}{4 \cdot E_d \cdot I_{dowel}}} = 24.8\ m^{-1}$$

The figure below shows the hypothesis made for shear distribution, for an edge load. The formula is used to calculate the distribution of the load if considering a dowel group 100% efficient:



Starting from **Timoshenko's equations** and considering (P_t) as the transferred load, **Friberg** obtained the following equations:

$$y_0 = \frac{P_t}{4 \cdot \beta^3 \cdot EI} \cdot (2 + \beta \cdot z) \qquad where \qquad P_t = \alpha_i \cdot \frac{\frac{W}{2}}{\sum \alpha_i} = \alpha_i \cdot \frac{20\ kN}{\sum \alpha_i}$$

Where, y_0 is the deflection of the dowel at the joint. When $z = 0$, there is $y_{0,max}$.
Then, σ_0 is the bearing pressure on concrete at the joint and k_{ds} is the Young's modulus of the dowel support (under the hypothesis of a Winkler foundation) that must be lower than the resisting one (f_b):

The following table shows the results:

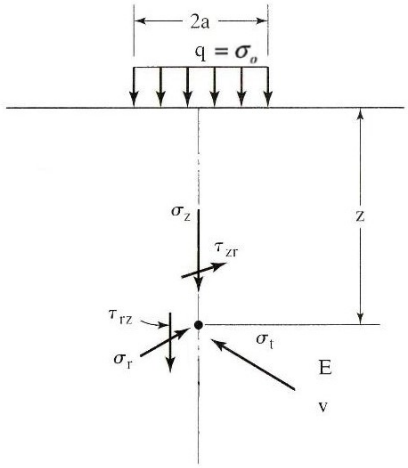
Joint	**Distance [m]**	**Case A**		**Case B**		**Total L [kN]**
		αi	**Loads Li [kN]**	**αi**	**Loads Li [kN]**	
1A	0	1,00	6,89	0	0	**6,89**
2	0,3	0,79	5,44	0,16	0,67	**6,12**
3	0,6	0,58	4,00	0,37	1,54	**5,54**
4	0,9	0,37	2,56	0,58	2,42	**4,97**
5	1,2	0,16	1,11	0,79	3,29	**4,40**
6B	1,5	0	0	1,00	4,16	**4,16**
7	1,8	0	0	0,79	3,29	**3,29**
8	2,1	0	0	0,58	2,42	**2,42**
9	2,4	0	0	0,37	1,54	**1,54**
10	2,7	0	0	0,16	0,67	**0,67**
11	3	0	0	0	0	**0**
12	3,3	0	0	0	0	**0**

As expected the dowel nearest to the pavement edge is the most critical and should be used for design purpose, in fact in the following table the deflections and the applied bearing stresses are compared; the applied stress is smaller than the allowable one (f’c assumed equal to 25 MPa) :

Point	y_0 **[mm]**	σ_0 **[MPa]**	f_b **[MPa]**
A	0.028	11.35	33.08
B	0.017	6.85	33.08

Boussinesq's equations for a circular uniformly distributed load. (Ullidtz and Forlag, 1998)

$$\sigma_z = \sigma_o\left(1 - \frac{1}{\left(\sqrt{1+\left(\frac{a}{z}\right)^2}\right)^3}\right)$$



$$\sigma_r = \sigma_t = \sigma_o\left[\frac{1+2\nu}{2} - \frac{1+\nu}{\sqrt{1+\left(\frac{a}{z}\right)^2}} + \frac{1}{2\left(\sqrt{1+\left(\frac{a}{z}\right)^2}\right)^3}\right]$$

$$\varepsilon_z = \frac{(1+\nu)\sigma_o}{E}\left[\frac{\frac{z}{a}}{\left(\sqrt{1+\left(\frac{z}{a}\right)^2}\right)^3} - (1-2\nu)\left(\frac{\frac{z}{a}}{\sqrt{1+\left(\frac{z}{a}\right)^2}} - 1\right)\right]$$

$$\varepsilon_r = \frac{(1+\nu)\sigma_o}{2E}\left[\frac{-\frac{z}{a}}{\left(\sqrt{1+\left(\frac{z}{a}\right)^2}\right)^3} - (1-2\nu)\left(\frac{\frac{z}{a}}{\sqrt{1+\left(\frac{z}{a}\right)^2}} - 1\right)\right]$$

$$d_z = \frac{(1+\nu)\sigma_o a}{E}\left[\frac{1}{\sqrt{1+\left(\frac{z}{a}\right)^2}} + (1-2\nu)\left(\sqrt{1+\left(\frac{z}{a}\right)^2} - \frac{z}{a}\right)\right]$$

The parameters needed are:

- $z = 60\ cm = 0.60\ m$; *distance of the point from the surface*
- $a = 15\ cm = 0.15\ m$; *radius of the circular area of loading*
- $q1 = 300\ kPa = \sigma_o$; *load value*
- $E = 100\ MPa$; *layer's elastic modulus*
- $\nu = 0.5$; *Poisson's value for incompressible layer*

- **Stresses:**

$$\sigma_z = 300\ kPa \cdot \left(1 - \frac{1}{\left(\sqrt{1+\left(\frac{0.15\ m}{0.60\ m}\right)^2}\right)^3}\right) = 26.077\ kPa$$

$$\sigma_r = \sigma_t = 300\ kPa \cdot \left(\frac{1+2\cdot 0.5}{2} - \frac{1+0.5}{\sqrt{1+\left(\frac{0.15\ m}{0.60\ m}\right)^2}} + \frac{1}{2\left(\sqrt{1+\left(\frac{0.15\ m}{0.60\ m}\right)^2}\right)^3}\right) = 0.397\ kPa$$

- **Strains:**

$$\varepsilon_z = \frac{(1+0.5)\cdot 300\ kPa}{100\cdot 10^3\ kPa}\cdot\left(\frac{\frac{0.60\ m}{0.15\ m}}{\left(\sqrt{1+\left(\frac{0.60\ m}{0.15\ m}\right)^2}\right)^3} - (1-2\cdot 0.5)\cdot\left(\frac{\frac{0.60\ m}{0.15\ m}}{\sqrt{1+\left(\frac{0.60\ m}{0.15\ m}\right)^2}} - 1\right)\right) = 256.802\ \mu strain$$

$$\varepsilon_r = \frac{(1+0.5)\cdot 300\ kPa}{2\cdot 100\cdot 10^3\ kPa}\cdot\left(\frac{-\frac{0.60\ m}{0.15\ m}}{\left(\sqrt{1+\left(\frac{0.60\ m}{0.15\ m}\right)^2}\right)^3} - (1-2\cdot 0.5)\cdot\left(\frac{\frac{0.60\ m}{0.15\ m}}{\sqrt{1+\left(\frac{0.60\ m}{0.15\ m}\right)^2}} - 1\right)\right) = -128.401\ \mu strain$$

- **Strains:**

$$\varepsilon_z = \frac{(1+0.5)\cdot 21\ kN}{2\cdot\pi\cdot 0.85^2\ m^2\cdot 100\cdot 10^3\ kPa}\cdot\left(3\cdot(\cos 45°)^3 - 2\cdot 0.5\cdot\cos 45°\right) = 24.859\ \mu strain$$

$$\varepsilon_r = \frac{(1+0.5)\cdot 21\ kN}{2\cdot\pi\cdot 0.85^2\ m^2\cdot 100\cdot 10^3\ kPa}\cdot\left(-3\cdot(\cos 45°)^3 + (3-2\cdot 0.5)\cdot\cos 45° - \frac{1-2\cdot 0.5}{1+\cos 45°}\right) = 24.859\ \mu strain$$

$$\varepsilon_t = \frac{(1+0.5)\cdot 21\ kN}{2\cdot\pi\cdot 0.85^2\ m^2\cdot 100\cdot 10^3\ kPa}\cdot\left(-\cos 45° + \frac{1-2\cdot 0.5}{1+\cos 45°}\right) = -49.718\ \mu strain$$

- **Displacements:**

$$d_z = \frac{(1+0.5)\cdot 21\ kN}{2\cdot\pi\cdot 0.85\ m\cdot 100\cdot 10^3\ kPa}\cdot\left(2\cdot(1-0.5)\cdot(\cos 45°)^3\right) = 0.089\ mm$$

$$d_r = \frac{(1+0.5)\cdot 21\ kN}{2\cdot\pi\cdot 0.85\ m\cdot 100\cdot 10^3\ kPa}\cdot\left(\cos 45°\cdot\sin 45° - \frac{1-2\cdot 0.5}{1+\cos 45°}\right) = 0.029\ mm$$

$$d_t = 0\ mm$$

#2.1.3 Total stresses, strains and displacements - Boussinesq

The final results in point A are given by the sum of those obtained before thanks to the application of the Superposition Principle.

Stresses [kPa]		Strains [μstrain]		Displacements [μm]	
σ_z	31.049	ε_z	281.662	d_z	253.205
σ_r	5.369	ε_r	-103.542	d_r	29.831
σ_t	0.400	ε_t	-178.120	d_t	0.000

#2.1.4 Stress and strains – Load q1 – Foster and Ahlvin charts

The solution can be achieved also by using the Foster and Ahlvin influence charts by treating again the two loads separately.

The needed parameters are:

- $z = 60\ cm = 0.60\ m$; *distance of the point from the surface*
- $a = 15\ cm = 0.15\ m$; *radius of the circular area of loading*
- $q1 = 300\ kPa$; *load value*
- $r = 0$; *distance of the point from the axis of symmetry*
- $\frac{r}{a} = 0$; *parameter of the graph*
- $\frac{z}{a} = \frac{0.6}{0.15} = 4$; *parameter of the graph*

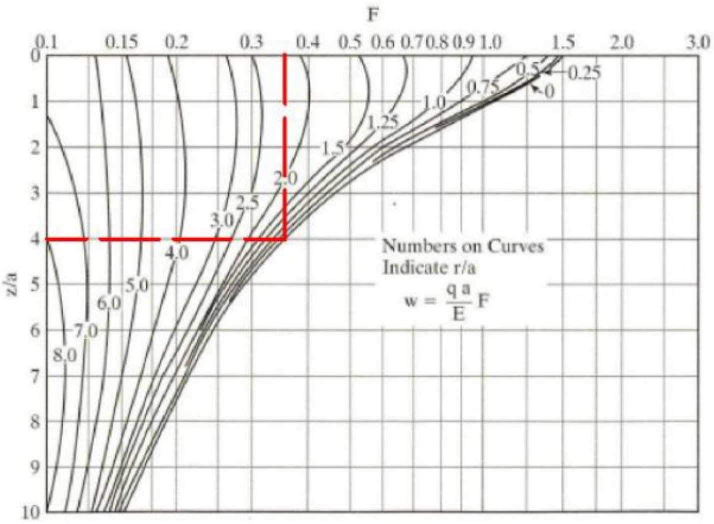
By interpolating the graphs, the following <u>stresses</u> are obtained:

$$\frac{\sigma_z}{q}\cdot 100\ [\%] = 8.4 \quad\rightarrow\quad \sigma_z\cdot q = \frac{8}{100}\cdot 300\ kPa = 25.2\ kPa$$

$$\varepsilon_t = \frac{\sigma_t}{E} - \nu\frac{\sigma_r}{E} - \nu\frac{\sigma_z}{E} = \frac{1}{100 \cdot 10^3\ kPa} \cdot (0.3 - 0.5 \cdot 0.3 - 0.5 \cdot 25.2)\ kPa = -124.50\ \mu strain$$

Far more, from the graphs it is possible to get the vertical displacement:

- $F = 0.36$



$$d_z = \frac{q \cdot a}{E} F = \frac{300\ kPa \cdot 15\ cm}{100 \cdot 10^3 kPa} \cdot 0.36 = 0.0162\ cm = 0.162\ mm$$

#2.1.5 Stress and strains – Load q2 – Foster and Ahlvin charts

The same procedure is done also for load q2, where the needed parameters are:

- $z = 60\ cm = 0.60\ m$; *distance of the point from the surface*
- $a = 15\ cm = 0.15\ m$; *radius of the circular area of loading*
- $q2 = 300\ kPa$; *load value*
- $r = 0.60\ m$; *distance of the point from the axis of symmetry*
- $\frac{r}{a} = 4$; *parameter of the graph*
- $\frac{z}{a} = \frac{0.6}{0.15} = 4$; *parameter of the graph*

By interpolating the graphs, the following **stresses** are obtained:

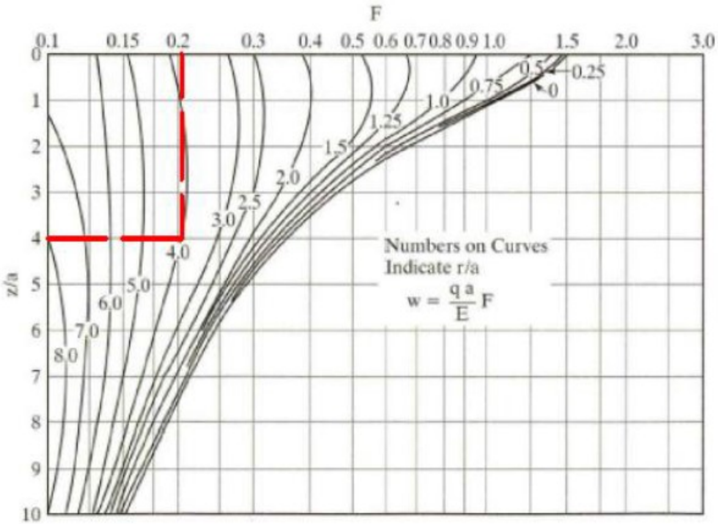
$$\frac{\sigma_z}{q} \cdot 100\ [\%] = 1.85 \quad \rightarrow \quad \sigma_z \cdot q = \frac{1.85}{100} \cdot 300\ kPa = 5.55\ kPa$$

$$\frac{\sigma_r}{q} \cdot 100\ [\%] = 1.7 \quad \rightarrow \quad \sigma_r \cdot q = \frac{1.7}{100} \cdot 300\ kPa = 5.1\ kPa$$

$$\frac{\sigma_t}{q} \cdot 100\ [\%] = 0 \quad \rightarrow \quad \sigma_t \cdot q = 0\ kPa$$

Far more, from the graphs it is possible to get the **vertical displacement**:

- $F = 0.21$



$$d_z = \frac{q \cdot a}{E} F = \frac{300\, kPa \cdot 15\, mm}{100 \cdot 10^3 kPa} \cdot 0.21 = 0.094\, mm$$

#2.1.6 Total stresses, strains and displacements - Foster and Ahlvin charts

The final results in point A are given by the sum of those obtained before thanks to the application of the Superposition Principle.

Stresses [kPa]		Strains [µstrain]		Displacements [µm]	
σ_z	30.75	ε_z	279.00	d_z	256.5
σ_r	5.4	ε_r	-101.25	d_r	/
σ_t	0.3	ε_t	-177.75	d_t	/

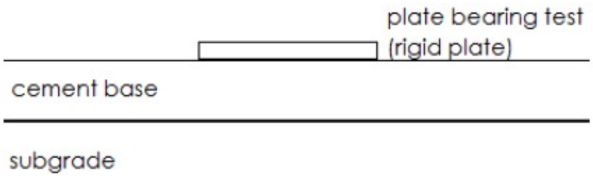
#2.1.7 Comparison between Boussinesq and Foster and Ahlvin charts

Foster and Ahlvin solution						Boussinesq solution					
Stresses [kPa]		Strains [µstrain]		Displacements [µm]		Stresses [kPa]		Strains [µstrain]		Displacements [µm]	
σ_z	30.75	ε_z	279.00	d_z	256.5	σ_z	31.049	ε_z	281.662	d_z	253.205
σ_r	5.4	ε_r	-101.25	d_r	/	σ_r	5.369	ε_r	-103.542	d_r	29.831
σ_t	0.3	ε_t	-177.75	d_t	/	σ_t	0.400	ε_t	-178.120	d_t	0.000

One can notice that the analytical and the graphical methods give quite the same results, as expected.
The graphical solution permits to estimate only the vertical displacement. It is important to acknowledge the limitation of the graphs, which can be used only for a value of Poisson ration of 0,5. In both cases, if the point is underneath the load, it will have a higher vertical stress rather than radial or tangential ones, whereas if the point is far away from the load, the radial and tangential components will be higher.

#2.2.2 Elastic modulus of the upper layer

Assuming that this deflection is too high, a cement treated base may be placed on top of the subgrade. To assess the mechanical parameters of the new material, a layer of 15 cm thick is laid over the subgrade and a new plate loading test is carried out. A deflection of 3.6 mm is measured. Assuming a Poisson ratio of 0.5, determine the elastic modulus of the treated base.



The elastic modulus E1 of the cement layer can be estimated by **Huang's charts and formula**: since the deflection is known, the factor F2 is calculated reversing the formula referring to a rigid plate and the ration E1/E2 is found from the chart.

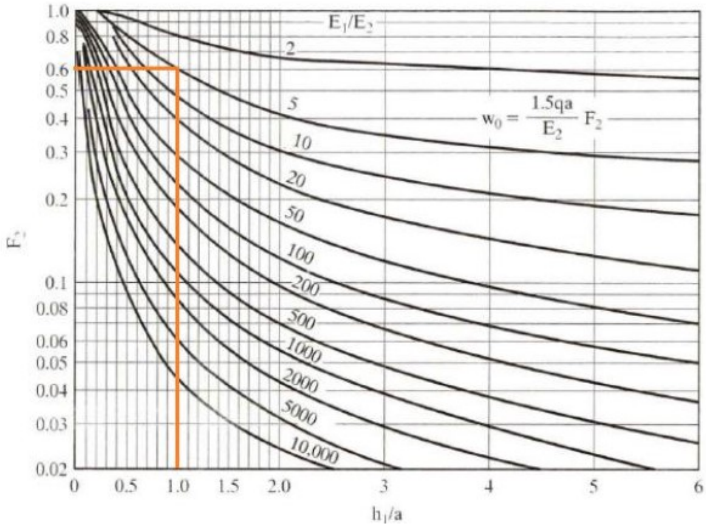
The parameters needed are:

- $a = 15\ cm$; *radius of the rigid plate*
- $P = 120\ kN$; *load value*
- $q = 1697.65\ kPa$; *distributed load, previously calculated*
- $w_0 = 3.6\ mm$; *deflection on the surface*
- $\nu = 0.5$; *Poisson's value for incompressible layer*
- $h_1 = 15\ cm$; *thickness of the cement base layer*
- $E_2 = 50\ MPa$; *elastic modulus of the subgrade*
- $\frac{h_1}{a} = \frac{15}{15} = 1$; *parameter of the graph*

So, the factor F2 is:

$$F_2 = \frac{w_0 \cdot E_2}{1.18 \cdot q \cdot a} = \frac{0.36\ cm \cdot 50 \cdot 10^3 kPa}{1.18 \cdot 1697.65\ kPa \cdot 15\ cm} = 0.599\ [/]$$

After interpolation, one finds:



$$\frac{E_1}{E_2} = 5 \rightarrow E_1 = 5 \cdot 50\ MPa = 250\ MPa$$

As is expected the cement base modulus is higher than the subgrade modulus. In this case, 5 times higher.

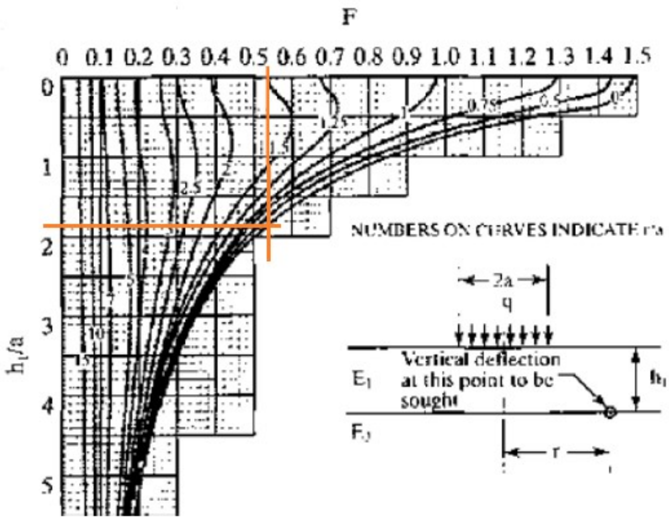
- w_1 = 1.0 mm → h_2
  The solution can be found from Huang's two-layer system diagram.
  The parameters needed are:
  - $q = 700\ kPa$; *distributed load*
  - $w_{1,max} = 1.0\ mm$; *deflection on the subgrade*
  - $E_2 = 50\ MPa$; *elastic modulus of the subgrade*
  - $a = \sqrt{\frac{P}{q \cdot \pi}} = 135\ cm$; *radius of the flexible tire*

  So, the factor F is:

$$F = \frac{w_1 \cdot E_2}{q \cdot a} = \frac{0.10\ cm \cdot 50 \cdot 10^3\ kPa}{700\ kPa \cdot 13.5\ cm} = 0.529\ [/]$$

  From the interpolation with the graph, the following ration and the thickness are obtained:

$$\frac{h_1}{a} = 1.9 \quad \rightarrow \quad h_1 = 1.9 \cdot a = 1.9 \cdot 13.5\ cm = 25.62\ cm$$



- σ_C = 245 kPa → h_3
  The solution can be found from Huang's two-layer system diagram, for a flexible plate.
  The parameters needed are:
  - $q = 700\ kPa$; *distributed load*
  - $\sigma_C = 245\ kPa$; *deflection on the subgrade*
  - $E_2 = 50\ MPa$; *elastic modulus of the subgrade*
  - $a = \sqrt{\frac{P}{q \cdot \pi}} = 135\ cm$; *radius of the flexible tire*

  So, the factor σ_C/q is:

$$\frac{\sigma_C}{q} = \frac{245\ kPa}{700\ kPa} = 0.35\ [/]$$

  From the interpolation with the graph, the following ration and the thickness are obtained:

$$\frac{a}{h_1} = 0.9 \quad \rightarrow \quad h_1 = 0.9 \cdot a = 0.9 \cdot 13.5\ cm = 14.98\ cm$$

#2.3 Three – Layer system

A flexible pavement composed of the following layers is considered:

- *a HMA course 15 cm thick with elastic modulus 3 GPa;*
- *a granular foundation 30 cm thick with elastic modulus 150 MPa;*
- *a subgrade with elastic modulus 75 MPa.*

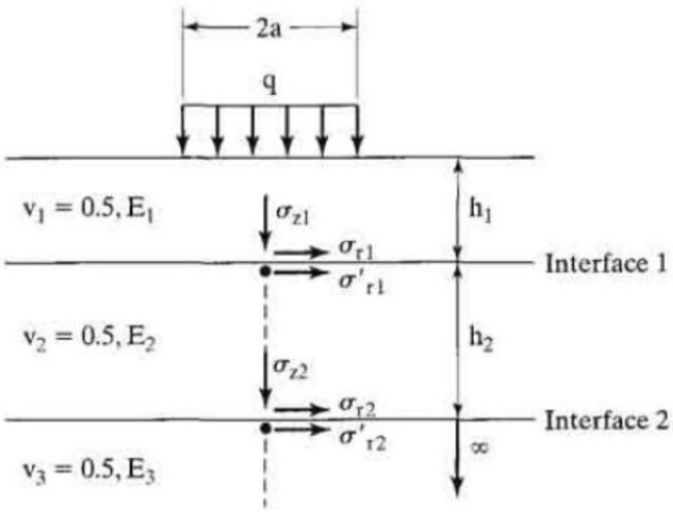
Hot mixed asphalt	h1 = 0.15m	E1 = 3000000kPa
Granular Foundation	h2 = 0.30m	E2 = 150000 kPa
Subgrade	E = 75000 kPa	

All layers are assumed to be incompressible. The asphalt pavement is subjected to a single-wheel load having contact radius 12 cm and contact pressure 700 kPa.
a. Determine all stresses and strains at the interfaces on the axis of symmetry using Jones' tables.
b. Determine stresses and strains at the top of the third layer using the Method of Equivalent Thickness.

#2.3.1 Stress and strains – Jone's tables

Jone's tables are very easy to be used, since by determining some quantities depending on the layers' properties, all the stresses and strains are determined.

At each interface the vertical stress is the same for the layer (i) and (i+1), but the radial stresses are different because the elastic moduli of the layers change while the radial deformations are the same due to the hypothesis of full adherence.



The parameters needed are:

- $a = 12\ cm$; $radius\ of\ the\ contact\ area$
- $q = 700\ kPa$; $pressure\ applied$
- $\nu = 0.5$; $Poisson's\ value\ for\ incompressible\ layer\ (1,2,3)$
- $h_1 = 15\ cm$; $thickness\ of\ the\ granular\ foundation\ layer\ (2)$
- $E_1 = 3\ GPa$; $elastic\ modulus\ of\ the\ HMA\ layer\ (1)$

Where, by convention, a positive sign of stress expresses compression.

The value for radial stress on the top of the first interface (σ_{r1}) is significantly bigger than the one calculated on its bottom (σ'_{r1}). This is due to the strong gap between the mechanical properties of the first two layers, that is observable in the elevated value of $k1 = E1/E2 = 20$.

#2.3.2 Stress and strains – Method of Equivalent Thicknesses

Another way to solve the problem is to homogenize the two upper layers by finding their thickness such that they will have the same stiffness as the subgrade layer. This is done by introducing the following proportion:

$$\frac{E_i \cdot h^2{}_i}{1-\nu^2{}_i} = \frac{E_{i+1} \cdot h^2{}_{i,equivalent}}{1-\nu^2{}_{i+1}} \quad \rightarrow \quad h_{i,equivalent} = f \cdot h_i \cdot \sqrt{\frac{E_i}{E_{i+1}} \cdot \frac{1-\nu^2{}_{i+1}}{1-\nu^2{}_i}}$$

Where, f is a constant that depends of the number of layers in the system: in a two-layer system, for the first interface f=1. For a multilayer system f=0.9. and, for a second interface, f=0.8 in general.

Since there are 2 layer to be homogenized, one proceeds by steps:

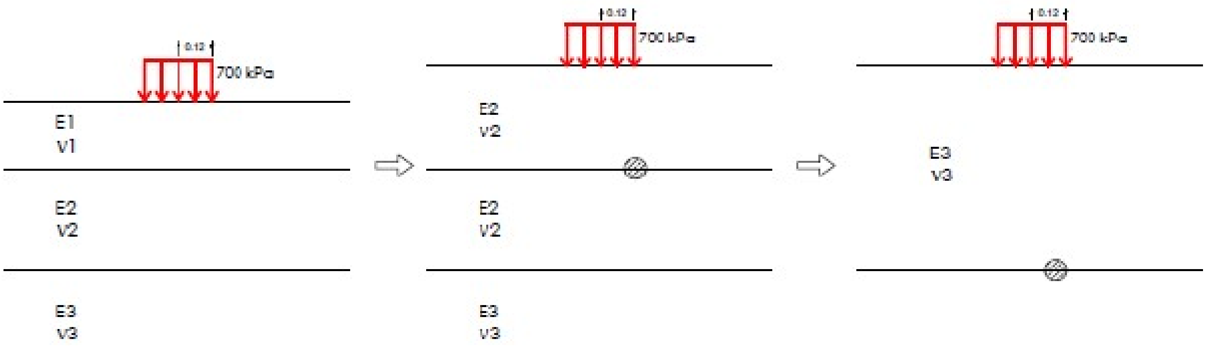
- Layer 1 – HMA (f=1)

$$h_{1,eq} = 1 \cdot h_1 \cdot \sqrt{\frac{E_1}{E_2} \cdot \frac{1-\nu^2{}_2}{1-\nu^2{}_1}} = 15\ cm \cdot \sqrt{\frac{3000\ MPa}{150\ MPa} \cdot \frac{1-0.5^2}{1-0.5^2}} = 40.72\ cm$$

- Layer 2 – granular foundation (f=0.8)

$$h_{2,eq} = 0.8 \cdot (h_2 + h_{1,eq}) \cdot \sqrt{\frac{E_2}{E_3} \cdot \frac{1-\nu^2{}_3}{1-\nu^2{}_2}} = 0.8 \cdot (30 + 40.72)\ cm \cdot \sqrt{\frac{150\ MPa}{75\ MPa} \cdot \frac{1-0.5^2}{1-0.5^2}} = 71.28\ cm$$

The process could be schematized as shown below:



Now it is possible to apply Boussinesq theory (as the previous exercises) in order to calculate stresses and strains at the top of the third layer under a circular uniformly distributed load.

The parameters needed are:

- $z = 71.28\ cm = h_{2,eq}$; *distance of the point from the surface*
- $a = 12\ cm$; *radius of the circular area of loading*
- $q = 700\ kPa = \sigma_o$; *load value*

Exercises #3

#3.1 Rail analysis

In this practice lesson, a railway track is analysed in order to find the stresses under which it acts.
The following loading and structure data are given:

Table 1.

P_{axle}	[kN]	176.6
V_{max}	[km/h]	110
R_{curve}	[m]	600
h	[cm]	15
s	[m]	1.5
R_w	[m]	0.625
y_G	[m]	1.29
A_{rail}	[mm^2]	7686
α_{rail}	[°C^{-1}]	1.2E-05
E_{rail}	[GPa]	210
I_{rail}	[cm^4]	3055
v_{rail}	[-]	0.3
c_{rail}	[m]	0.15
u	[MPa]	25

Determine vertical forces (with respect to the rolling plane) by using the Eisenmann and the ORE approaches in the following conditions: Level of reliability equal to 99.7% (t = 3); Track in standard conditions (φ = 0.2); Braking conditions (DAFORE = 1.11)
Determine lateral forces (with respect to the rolling plane) by taking into account: non-uniform distribution of the load among the connected axles (β = 1.1); abnormal movements.
Determine contact stresses at the wheel-rail interface.
Determine deflection and bending moment in the longitudinal axis according to both Zimmermann (γ = 0 m-2) and Pasternak (γ =3 m-2) solutions.
According to the Zimmermann solution, use Talbot's formulae to determine: maximum deflection in the rail; maximum bending moment in the rail.
Given a temperature increase with respect to the temperature of placement of 40°C, determine longitudinal forces by taking into account the effects of abnormal movements, breakings, crashes (ΔL = 15%L).

#3.1.1 Vertical forces acting on the rolling plane

Determine vertical forces (with respect to the rolling plane) by using the Eisenmann and the ORE approaches in the following conditions:

- Level of reliability equal to 99.7% (t = 3);
- Track in standard conditions (ϕ = 0.2);
- Braking conditions (DAFORE = 1.11).

The force of the axle (P_{axle}) is vertical with respect to an absolute reference system, but the rails are on a plane which is inclined by an angle α:

#3.1.2 Lateral force acting on the rolling plane

Determine lateral forces (with respect to the rolling plane) by taking into account:

- non-uniform distribution of the load among the connected axles (β = 1.1);
- abnormal movements.

The total lateral force is given by the previous one (H) plus the contribution of abnormal movements (H'):

$$H' = \frac{P_{axle} \cdot \mathrm{V}\left[\frac{\mathrm{km}}{\mathrm{h}}\right]}{1000} = 19.43\ kN$$

$$H_{tot} = \beta \cdot H + H' = 30.91\ kN$$

#3.1.3 Contact stress at the wheel-rail interface – practical formulae

To calculate the contact stress between the wheel and the rail, the practical formulas coming from the Eisenamann approach are used: he approximates the stress as a semi-elliptical distribution on a rectangular area, having axis:

$$b = 6\ mm$$

$$a = \sqrt{\frac{(1 - \nu^2{}_{rail})}{\pi \cdot E_{rail}} \cdot \frac{4 \cdot P_{ed}}{b} \cdot R_w} = 11\ mm$$

In the transverse direction of the rail there is a mean value of the stress given by:

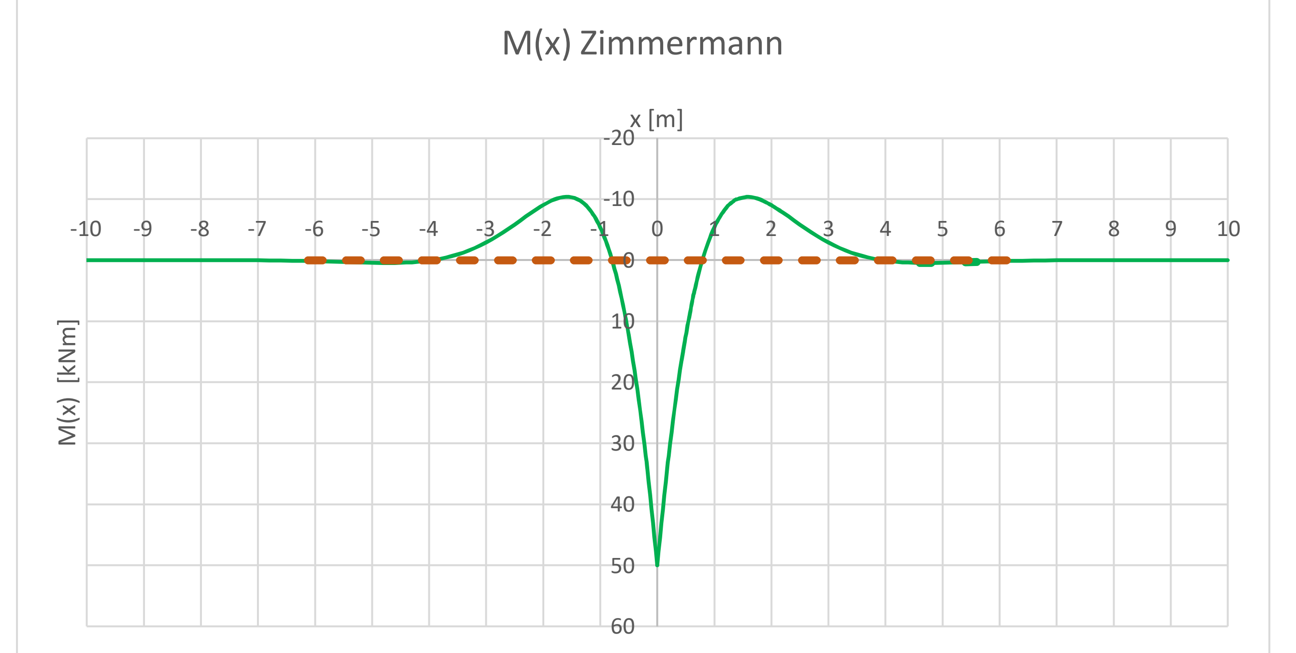
$$\sigma_{mean} = \sqrt{\frac{\pi \cdot E_{rail}}{64 \cdot (1 - \nu^2{}_{rail})} \cdot \frac{P_{ed}}{R_w \cdot b}} = 774.76\ MPa$$

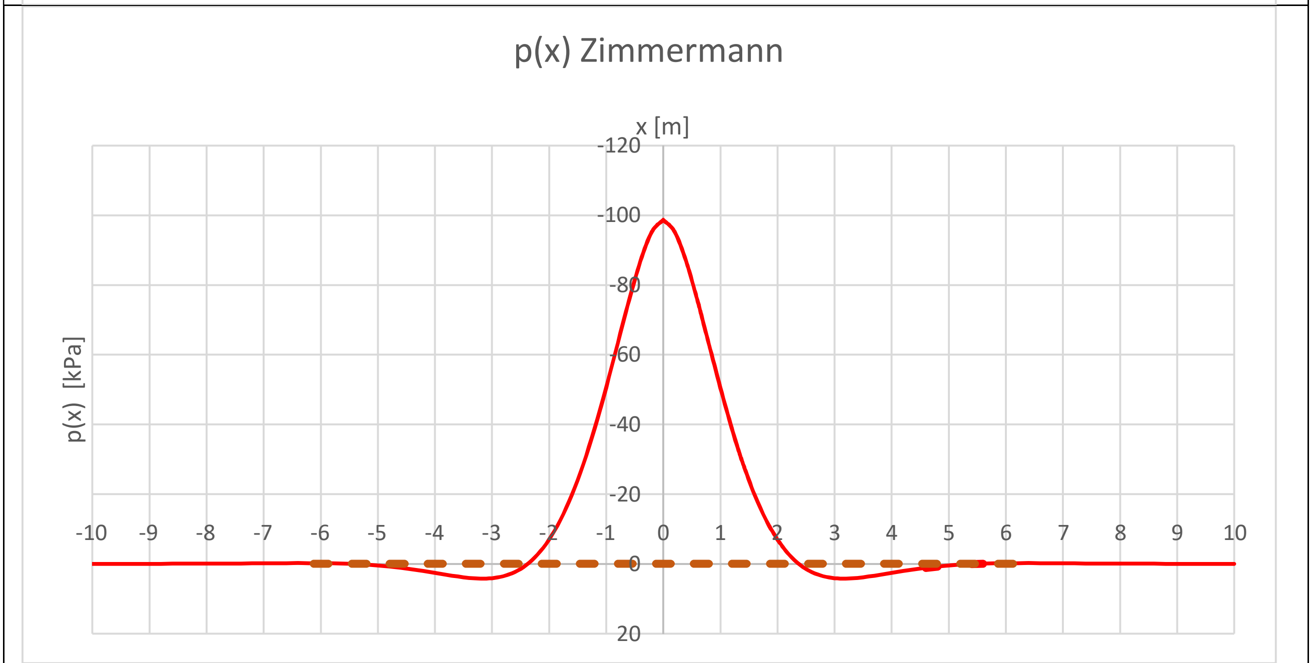
While along the longitudinal direction of the rail there is a maximum value, given by:

$$\sigma_{max} = \sqrt{\frac{\pi \cdot E_{rail}}{4 \cdot \pi^2 \cdot (1 - \nu^2{}_{rail})} \cdot \frac{P_{ed}}{R_w \cdot b}} = 986.46\ MPa$$

And the ratio between the two stresses is:

$$\frac{\sigma_{max}}{\sigma_{mean}} = 1.273$$





One can see that at about 2.4 m from the point of application of the load, the deflection changes sign, which means that the rails tend to detach from the support. The bigger amount of stress is supported by the first 4 sleepers.

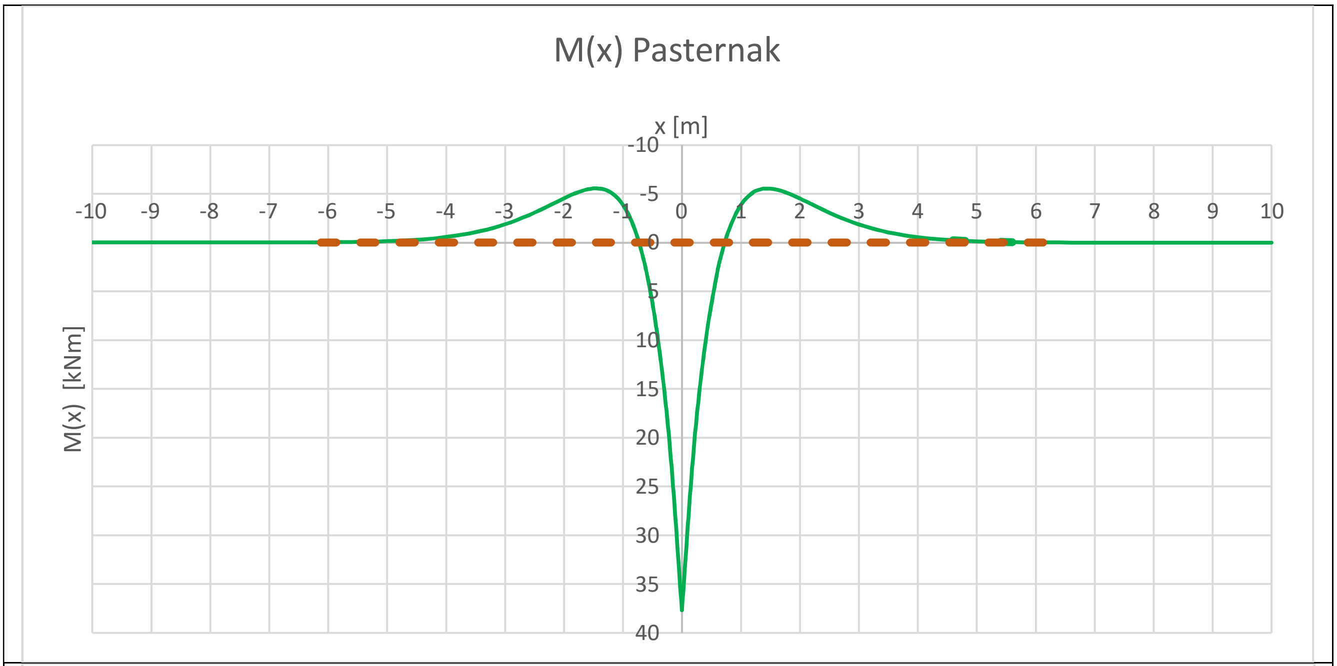
- **Pasternak approach**

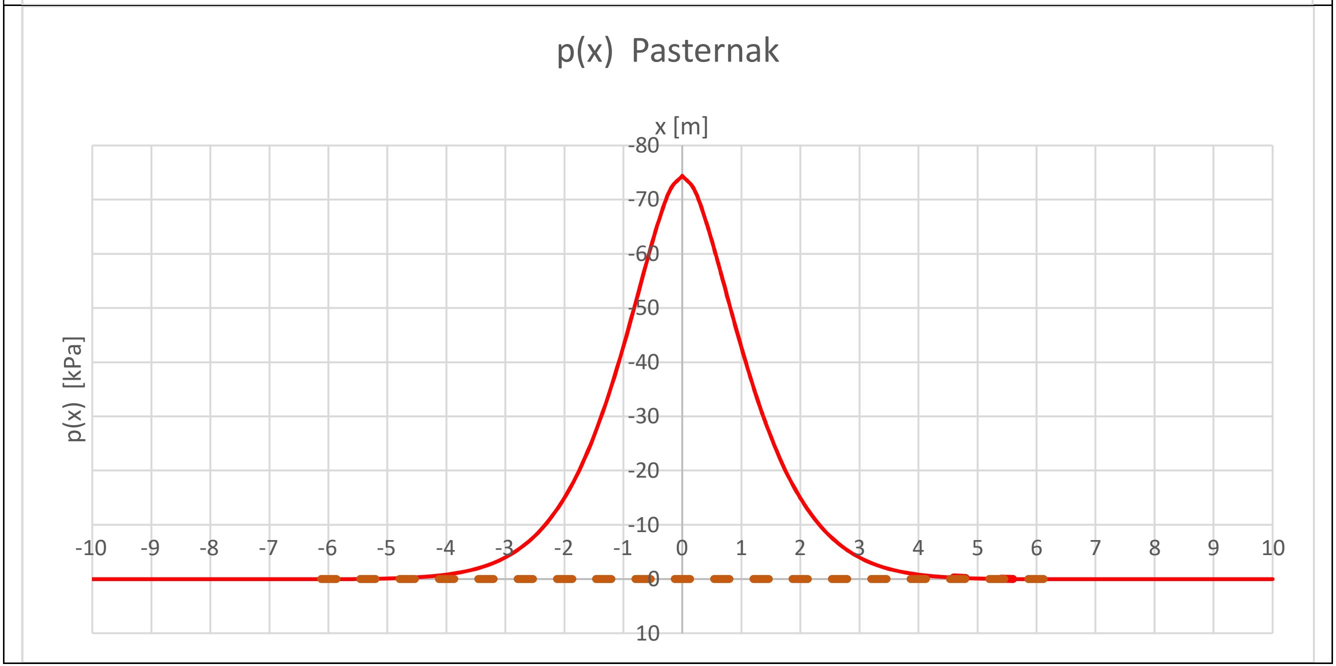
  This approach is as the previous one but more generalized and it takes into account the shear stiffness of the support.

  The equations are:

  - Deflection

$$y(x) = \frac{P_{ed}}{8 \cdot E_{rail} \cdot I_{rail} \cdot a \cdot b \cdot \beta^2} \cdot e^{-b \cdot x} \cdot (a \cdot \cos(ax) + \mathrm{b} \cdot \sin(ax))$$


M(x) Pasternak
x [m]
M(x) [kNm]
-10
-5
0
5
10
15
20
25
30
35
40
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10


p(x) Pasternak
x [m]
p(x) [kPa]
-80
-70
-60
-50
-40
-30
-20
-10
0
10
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

#3.2 Sleeper and ballast analysis

On the basis of results obtained from Exercise #3.1 and structure data given in Table 2:

Table 2.

L_{tie}	[cm]	260
L_{1tie}	[cm]	151
b_{tie}	[cm]	26
h_{tie}	[cm]	16
I_{tie}	[cm^4]	7475
s_{tie}	[m]	0.666
h_{ball}	[m]	0.3
α_{ball}	[°]	45
E_{ball}/E_{sub}	[-]	3
E_{sub}	[MPa]	100

Determine seating loads on ties. Determine bending moment in ties according to the following approaches: rigid body with full support; rigid body with end support; ORE empirical method (φ = 1.375; B = 0.5; χ = 1.35). Determine pressure on subgrade by means of the combined use of the Odemark's method and of the infinite strip theory.

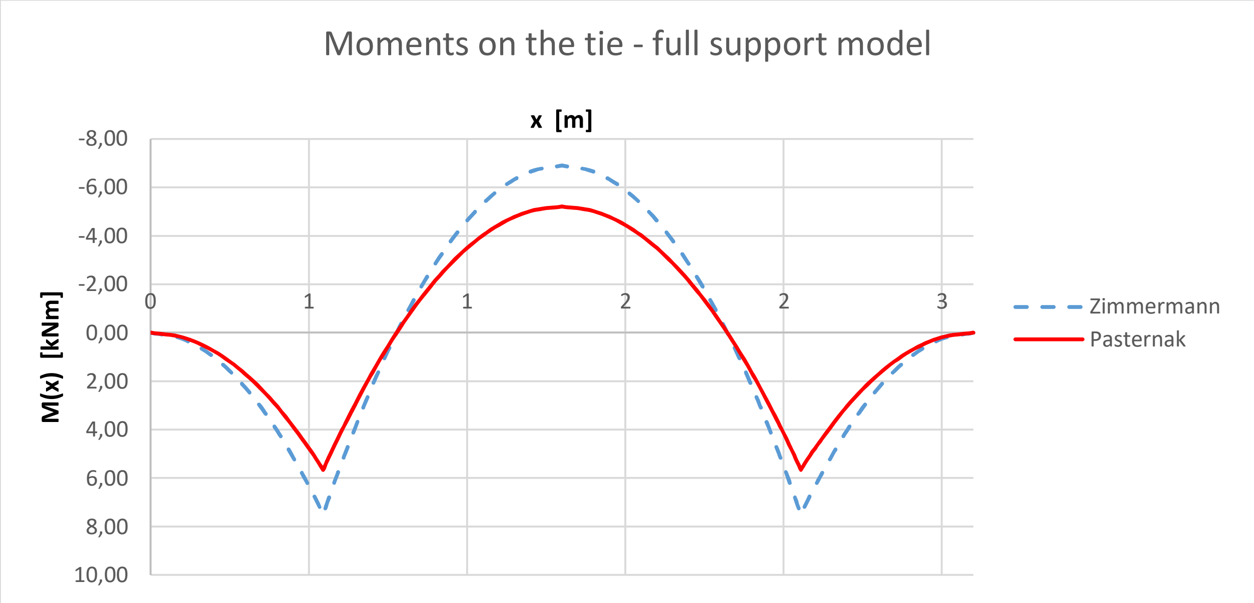
#3.2.1 Seating loads Q on ties

The seating loads are those transferred from the rails to the ties. They can be found integrating the pressure function p(x) founded previously with respect to the influence area equal to the spacing (s) between sleepers. A practical method is to approximate the area to a rectangle (base = s; height = p_i) and so to find easily the result:

$Q_{Zimmermann} = p_{max} \cdot s$ [kN]	$Q_{Pasternak} = p_{max} \cdot s$ [kN]
65,74	49,56
47,61	37,29
21,26	20,83
4,63	9,97

The seating load is calculated taking the maximum value of the pressure, exactly under the load application point.

One can observe that at x=0m, there is a big difference between the results of Zimmermann and Pasternak, which tends to be less evident as moving away from this point.



- **Rigid body with end support**

  This approach also considers the sleeper as a rigid finite beam, but lying on a support that responds only in the edges, while in the midspan, after a distance $L=L_{end}$, moments are equal to 0.

  It can be calculated the linear pressure (not depending on the tie's width b) and the one per unit area (depending on the tie's width b):

$$p'_{ballast} = \frac{Q}{4 \cdot L_{end}}$$

$$p_{ballast} = \frac{Q}{4 \cdot L_{end} \cdot b_{tie}}$$

  The moments are calculated as following:

$$M_R = \frac{Q}{4} \cdot L_{end}$$

$$L_{midspan} = L_{tie} - 4 \cdot \frac{(L_{tie} - L_{1,tie})}{2} = 0.42\ m$$

$$M_C = 0$$

  So, the results are:

Moments on ties		Zimmermann	Pasternak
$p'_{ballast}$	kN/m	60.31	45.46
$p_{ballast}$	kN/m^2	231.97	174.86
M_R	kNm	8.96	13.50
M_C	kNm	0	0

$$M_C = -1.2 \cdot M_R \cdot \frac{(E \cdot I)_{center}}{(E \cdot I)_{rail}}$$

So, the results are:

Moments on ties		
$p'_{midspan}$	kN/m	264.37
$p_{midspan}$	kN/m^2	1016.79
$p'_{ballast}$	kN/m	75.19
$p_{ballast}$	kN/m^2	289.18
M_R	kNm	12.78
M_C	kNm	-15.34

- **Comparison**
  The results can be compared:

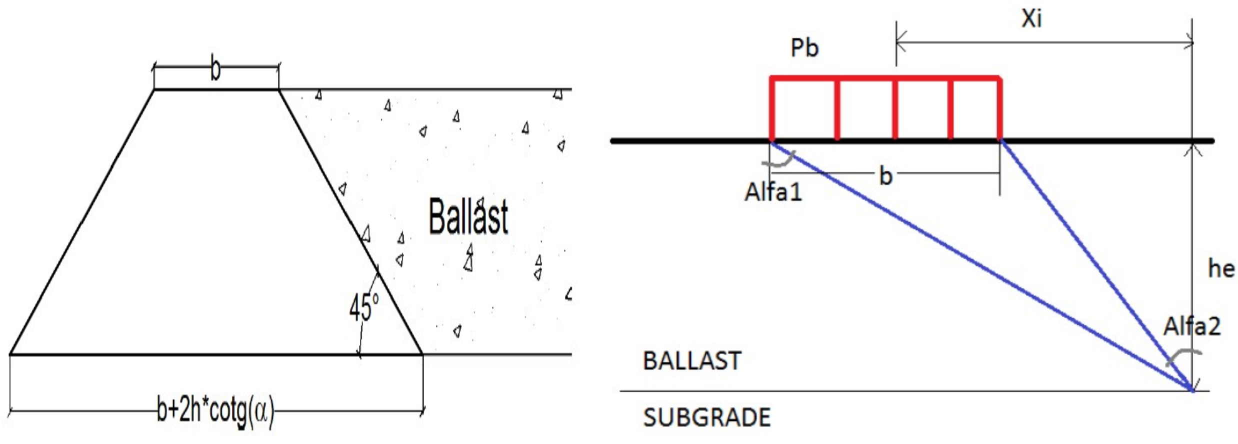
Moments on ties		ORE	Rigid body with full support		Rigid body with end support	
			Zimmermann	Pasternak	Zimmermann	Pasternak
M_R	kNm	12.78	7.51	5.66	8.96	13.50
M_C	kNm	-15.34	-6.90	-5.20	0	0

The highest moment under the rail is given by the Rigid body – end support approach, while the highest in the middle is given by Rigid body – full support approach.
The ORE method, which is empirical, gives even higher values.

#3.2.3 Multi-load configuration

The stresses on the ballast and on the subgrade layers induced by the forces Q, that are supported also by a certain number of adjacent ties, that apply a pressure themselves on the ballast and on the subgrade, can be calculated as:

$$\sigma_i = \frac{2 \cdot Q_i}{A_{tie}} = \frac{2 \cdot p_i \cdot s}{A_{tie}}$$



n° tie	x [m]	α1 [°]	α2 [°]	f(x)	p(x) [kN/m]	Q [kN]	σi [KPa]	σz [kPa]		σz,tot ballast [kPa]
1	**0,00**	**18,46**	**-18,46**	**0,3963**	**98,71**	**65,74**	**194,50**	**77,09**	**x1**	**85.74**
2	**0,67**	**63,93**	**54,00**	**0,0295**	**71,48**	**47,61**	**140,84**	**4,15**	**x2**	
3	**1,33**	**75,09**	**72,05**	**0,0027**	**31,92**	**21,26**	**62,89**	**0,17**	**x2**	
4	**2,00**	**79,63**	**78,22**	**0,0006**	**6,96**	**4,63**	**13,71**	**0,01**	**x2**	
5	2,66	82,07	81,26	1,874E-04	-2,83	-1,89	-5,58	-1,047E-03	**x2**	\\\
6	3,33	83,58	83,06	7,774E-05	-4,16	-2,77	-8,20	-6,372E-04	**x2**	
7	4,00	84,61	84,25	3,774E-05	-2,63	-1,75	-5,19	-1,958E-04	**x2**	
8	4,66	85,35	85,09	2,046E-05	-1,04	-0,69	-2,04	-4,175E-05	**x2**	
9	5,33	85,92	85,72	1,202E-05	-0,14	-0,09	-0,28	-3,375E-06	**x2**	
10	5,99	86,36	86,20	7,520E-06	0,16	0,11	0,31	2,367E-06	**x2**	
11	6,66	86,72	86,59	4,940E-06	0,17	0,11	0,33	1,636E-06	**x2**	
12	7,33	87,01	86,90	3,378E-06	0,09	0,06	0,19	6,273E-07	**x2**	
13	7,99	87,26	87,16	2,387E-06	0,03	0,02	0,06	1,505E-07	**x2**	
14	8,66	87,46	87,39	1,734E-06	0,00	0,00	0,00	3,309E-09	**x2**	
15	9,32	87,64	87,57	1,290E-06	-0,01	-0,01	-0,02	-1,965E-08	**x2**	
16	9,99	87,80	87,74	9,789E-07	-0,01	0,00	-0,01	-1,264E-08	**x2**	
17	10,66	87,93	87,88	7,564E-07	0,00	0,00	-0,01	-4,886E-09	**x2**	

Comparing this last stress at the subgrade (which takes into consideration that each tie generates a bubble of stress that influences the adjacent sleepers) with the calculation of a rigid body - full support, made taking into account a linear projection, one can notice that the last approach gives a higher stress, because it is more accurate.

TABLE 6.4 Asphalt Institute's Equivalent Axle Load Factors

Axle load (lb)	Equivalent axle load factor: Single axles	Tandem axles	Tridem axles	Axle load (lb)	Equivalent axle load factor: Single axles	Tandem axles	Tridem axles
1000	0.00002			41,000	23.27	2.29	0.540
2000	0.00018			42,000	25.64	2.51	0.597
3000	0.00072			43,000	28.22	2.76	0.658
4000	0.00209			44,000	31.00	3.00	0.723
5000	0.00500			45,000	34.00	3.27	0.793
6000	0.01043			46,000	37.24	3.55	0.868
7000	0.0196			47,000	40.74	3.85	0.948
8000	0.0343			48,000	44.50	4.17	1.033
9000	0.0562			49,000	48.54	4.51	1.12
10,000	0.0877	0.00688	0.002	50,000	52.88	4.86	1.22
11,000	0.1311	0.01008	0.002	51,000		5.23	1.32
12,000	0.189	0.0144	0.003	52,000		5.63	1.43
13,000	0.264	0.0199	0.005	53,000		6.04	1.54
14,000	0.360	0.0270	0.006	54,000		6.47	1.66
15,000	0.478	0.0360	0.008	55,000		6.93	1.78
16,000	0.623	0.0472	0.011	56,000		7.41	1.91
17,000	0.796	0.0608	0.014	57,000		7.92	2.05
18,000	1.000	0.0773	0.017	58,000		8.45	2.20
19,000	1.24	0.0971	0.022	59,000		9.01	2.35
20,000	1.51	0.1206	0.027	60,000		9.59	2.51
21,000	1.83	0.148	0.033	61,000		10.20	2.67
22,000	2.18	0.180	0.040	62,000		10.84	2.85
23,000	2.58	0.217	0.048	63,000		11.52	3.03
24,000	3.03	0.260	0.057	64,000		12.22	3.22
25,000	3.53	0.308	0.067	65,000		12.96	3.41
26,000	4.09	0.364	0.080	66,000		13.73	3.62
27,000	4.71	0.426	0.093	67,000		14.54	3.83
28,000	5.39	0.495	0.109	68,000		15.38	4.05
29,000	6.14	0.572	0.126	69,000		16.26	4.28
30,000	6.97	0.658	0.145	70,000		17.19	4.52
31,000	7.88	0.753	0.167	71,000		18.15	4.77
32,000	8.88	0.857	0.191	72,000		19.16	5.03
33,000	9.98	0.971	0.217	73,000		20.22	5.29
34,000	11.18	1.095	0.246	74,000		21.32	5.57
35,000	12.50	1.23	0.278	75,000		22.47	5.86
36,000	13.93	1.38	0.313	76,000		23.66	6.15
37,000	15.50	1.53	0.352	77,000		24.91	6.46
38,000	17.20	1.70	0.393	78,000		26.22	6.78
39,000	19.06	1.89	0.438	79,000		27.58	7.11
40,000	21.08	2.08	0.487	80,000		28.99	7.45

Note. 1 lb = 4.45 N.

Now, the total truck factor must be calculated, considering the EALF and the percentage of each truck type:

Vehicle types	[%]	TFi	TTFi
2-axle single unit trucks	0,5	0,00518	0,0026
3-axle single unit trucks	0,2	1,4362	0,2872
5-axle multiple unit trucks	0,3	4,2962	1,2889

$$TF_i = \sum_{j=axle\ type}^{nr\ of\ axles\ of\ a\ vehicle\ (3)} (axle\ type)_j \cdot EALF_j$$

$$TTF_i = TF_i \cdot (\%)_i$$

So the total truck factor TTF is:

$$TTF = \sum TTF_i = 1.57869$$

It is now possible to obtain the ESAL for an annual growth rate of 2.5% and to plot the growth of ESAL's during the analysis period (30 years):

$$\mathrm{ESAL_{year\ n}} = 365 \cdot \mathrm{AADT} \cdot \mathrm{D_D} \cdot \mathrm{D_L} \cdot \%\mathrm{HV} \cdot \mathrm{TTF} \cdot \mathrm{G(Y)}$$

#4.2 Effective Roadbed resilient modulus M_R

Figure 2.4 shows the monthly subgrade resilient moduli (M_{Ri}) estimated by means of a laboratory relationship between resilient modulus and moisture content of soil beneath the pavement. Determine the effective roadbed resilient modulus (M_R).

It's possible to determine the relative damage calculated for each month with the following formula:

$$u_{f,i} = 1.18 \cdot 10^8 \cdot M_{R,i}^{-2.32}$$

Month	M_R [psi]	Relative Damage $u_{f,i}$
Jan	20000	0,012
Feb	20000	0,012
Mar	2500	1,544
Apr	4000	0,519
May	4000	0,519
June	7000	0,142
July	7000	0,142
Aug	7000	0,142
Sept	7000	0,142
Oct	7000	0,142
Nov	4000	0,519
Dec	20000	0,012
	TOT u_f	**3,846**

Then the values are averaged with respect to the period n = 12 months.

$$\overline{u_f} = \frac{1}{n} \cdot \sum u_{f,i} = 0.321$$

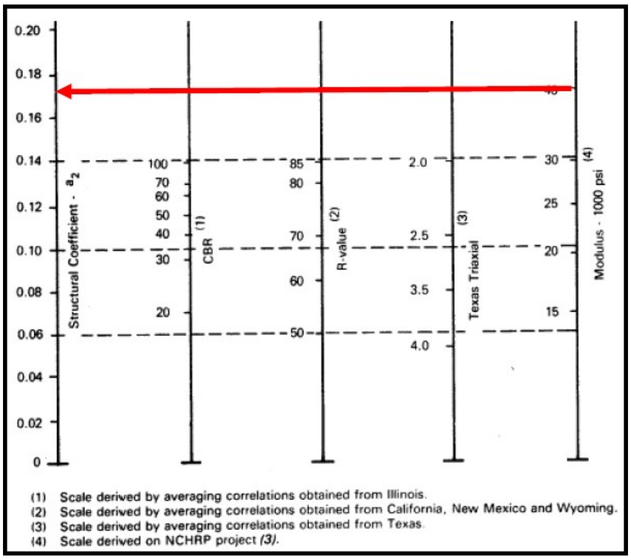
Finally, the resilient modulus can be obtained with the following equation:

$$M_R = 10^{\left(-\frac{1}{2.32}\log\left(\frac{\overline{u_f}}{1.18 \cdot 10^8}\right)\right)} = 4923\ psi$$

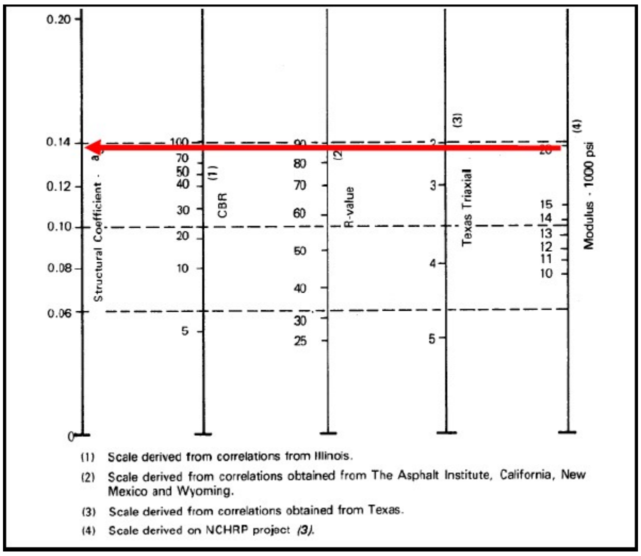
#4.3 Layer thickness

Calculate the required layer thicknesses for a new flexible pavement consisting of an asphalt concrete surface, a crushed-stone base, and a granular subbase. The subbase has a resilient modulus (E3) of 20 ksi, resilient modulus of the base (E2) is 40 ksi, and resilient modulus of asphalt concrete (E1) is 400 ksi. It is estimated that water drains out of the pavement within a period of one day and the pavement structure will be exposed to moisture levels approaching saturation for 20 percent of the time. As cumulative ESAL in the design lane, consider the traffic versus time relationship developed in example #1 and a performance period of 15 years. As effective roadbed soil consider the result obtained in example #2. Assume a reliability level of 95 percent and an overall standard deviation of 0.45. The initial serviceability index (pi) is 4.2 and the terminal serviceability index (pt) is 2.5.

- $a_2 = 0.249 \cdot logE_2 - 0.977 = 0.16891$



- $a_3 = 0.227 \cdot logE_3 - 0.839 = 0.13733$



In the following table are shown the formulas used to define the thickness of each layer and that was compared to the thickness derived from the traffic calculation.

$$D^*_1 \geq \frac{SN_1}{a_1}$$

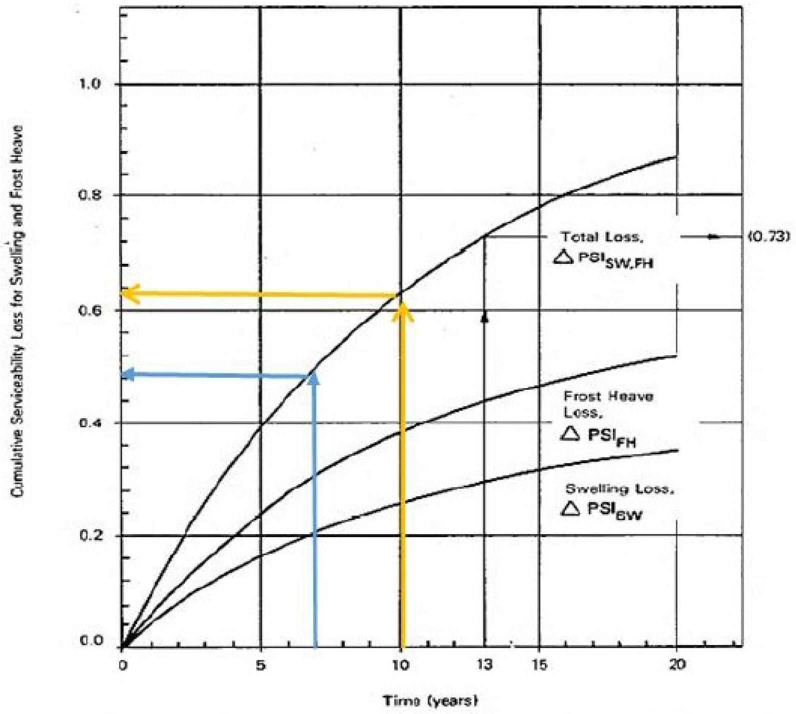
$$SN^*_1 = a_1 D^*_1 \geq SN_1$$

$$D^*_2 \geq \frac{SN_2 - SN^*_1}{a_2 m_2}$$

$$SN^*_1 + SN^*_2 \geq SN_2$$

$$D^*_3 \geq \frac{SN_3 - (SN^*_1 + SN^*_2)}{a_3 m_3}$$

	Minimum Thickness (inches)	
Traffic, ESAL's	Asphalt Concrete	Aggregate Base
Less than 50,000	1.0 (or surface treatment)	4
50,001–150,000	2.0	4
150,001–500,000	2.5	4
500,001–2,000,000	3.0	6
2,000,001–7,000,000	3.5	6
Greater than 7,000,000	4.0	6



Finally, to find the performance period, the following steps are iterative:

- Step1: suppose a trial performance period.
- Step 2: enter to the figure above and find the value for $\Delta PSI_{SW,FH}$.
- Step 3: use the following equation to determine the ΔPSI_{TRIAL} :

$$\Delta PSI_{TRIAL} = \Delta PSI - \Delta PSI_{SW,FH}$$

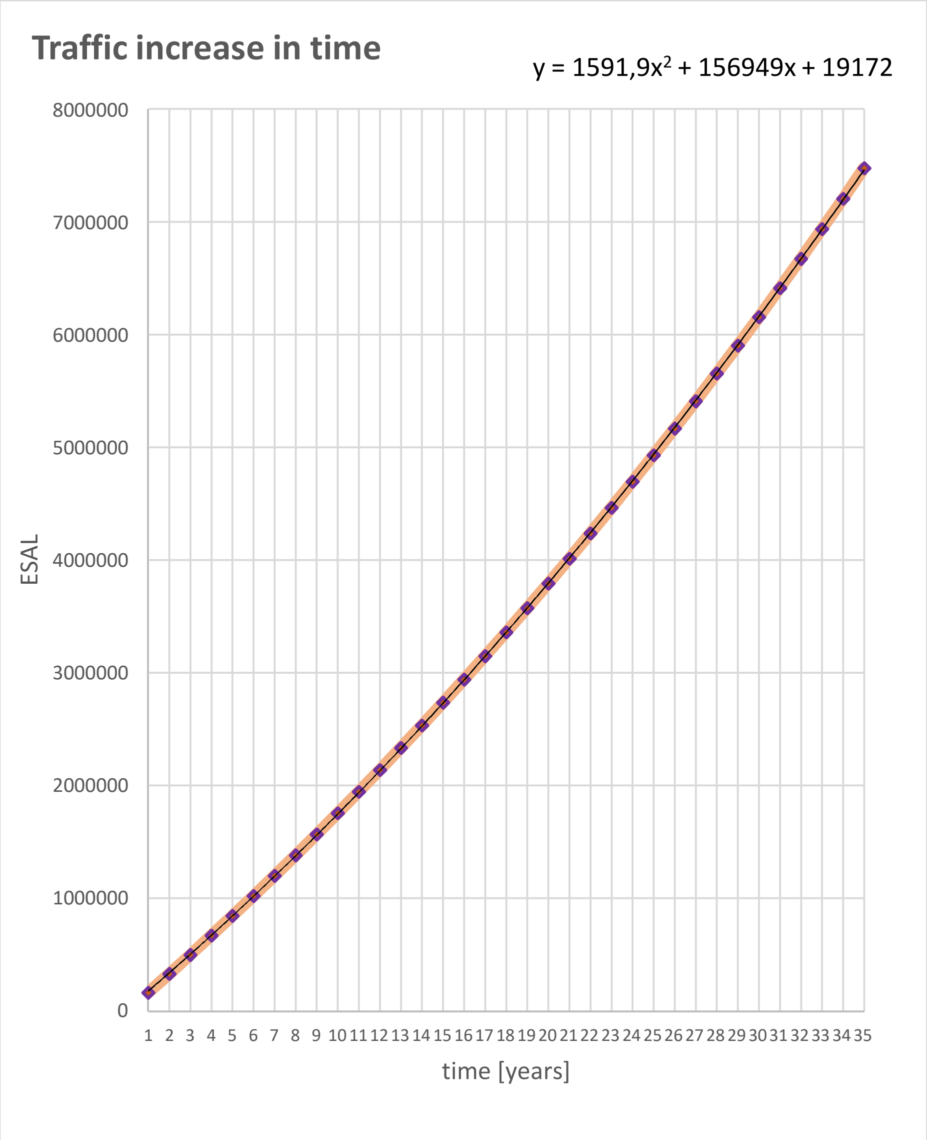
- Step 4: with the ΔPSI_{TRIAL} and $SN_{SUBGRADE}$ calculate a new W_{18}.
- Step 5: find on the graph above "Traffic increase in time", the time corresponding with that W_{18}.
- Step 6: if the value is equal to that assumed the process is ended, otherwise it is recommended to use the mean between the value assumed and the one read in the graph to start a new iteration.

The results are reported in the following table:

N° iter.	PP trial [years]	$\Delta PSI_{SW,FH}$	$\Delta PSI_{TRAFFIC}$	ΔPSI	ESAL	PP [years]	<1 year?	
1	13	0,73	0,97	1.7	5807593	5,38	7,62	NO
2	10	0,63	1,07	1.7	7298568	6,67	3,33	NO
3	7,5	0,54	1,16	1.7	8808945	7,92	0,42	OK

So the performance period of the pavement is about 7.5 years.

Year	G(Y)	ESAL
1	1	164000
2	2,015	330460
3	3,045225	499417
4	4,090903	670908
5	5,152267	844972
6	6,229551	1021646
7	7,322994	1200971
8	8,432839	1382986
9	9,559332	1567730
10	10,70272	1755246
11	11,86326	1945575
12	13,04121	2138759
13	14,23683	2334840
14	15,45038	2533863
15	16,68214	2735871
16	17,93237	2940909
17	19,20136	3149022
18	20,48938	3360258
19	21,79672	3574661
20	23,12367	3792281
21	24,47052	4013166
22	25,83758	4237363
23	27,22514	4464924
24	28,63352	4695897
25	**30,06302**	**4930336**
26	31,51397	5168291
27	32,98668	5409815
28	34,48148	5654963
29	35,9987	5903787
30	37,53868	6156344
31	39,10176	6412689
32	40,68829	6672879
33	42,29861	6936972
34	43,93309	7205027
35	45,59209	7477102



Consequently, the value of Z_R (standard normal deviate) is related to the value of reliability R_1, and its value is found as an interpolation from the above table: Z_R **=** -1.627.

Finally the reliability parameter is:

$$Z_R \cdot S_O = -1.627 \cdot 0.3 = -0.488$$

#5.3 Drainage coefficient Cd

Determine drainage coefficient Cd by considering a percent of time the pavement is exposed to moisture levels approaching saturation from 5 to 25% of the year, and a drainage system capable of removing excessive moisture in 1 day or less (Tables 2 and 2.5).

The drainage coefficient Cd depends on the quality of drainage and on the amount of time a pavement is exposed close to saturation conditions.

After looking at the table below, it is possible to deduce a drainage quality equivalent to "good". As a consequence, Cd can be obtained.

Table 2

Drainage Quality	Water Removed Within
Excellent	½ day
Good	1 day
Fair	1 week
Poor	1 month
Very Poor	(never removed)

Table 2.5. Recommended Values of Drainage Coefficient, C_d, for Rigid Pavement Design

Quality of Drainage	Percent of Time Pavement Structure is Exposed to Moisture Levels Approaching Saturation: Less Than 1%	1–5%	5–25%	Greater Than 25%
Excellent	1.25–1.20	1.20–1.15	1.15–1.10	1.10
Good	1.20–1.15	1.15–1.10	1.10–1.00	1.00
Fair	1.15–1.10	1.10–1.00	1.00–0.90	0.90
Poor	1.10–1.00	1.00–0.90	0.90–0.80	0.80
Very poor	1.00–0.90	0.90–0.80	0.80–0.70	0.70

So it is assumed: **Cd = 1.05**.

#5.4 Loss of support LS

Determine loss of support factor LS, indicative of the potential for voids to form beneath the slab, in the case of slabs directly placed on a granular subbase characterized by a modulus of 15,000 and 25,000 psi in wet and dry conditions, respectively (Table 2.7).

This coefficient translates the potential void formation underneath the slab due to erosion on the subbase. The loss of support factor can be determined with the table below, which defines the typical ranges of loss of support factors (LS) for various types of materials.

Given the composite K∞, it is possible to reach the value of relative damage (u_f), entering in the chart below:

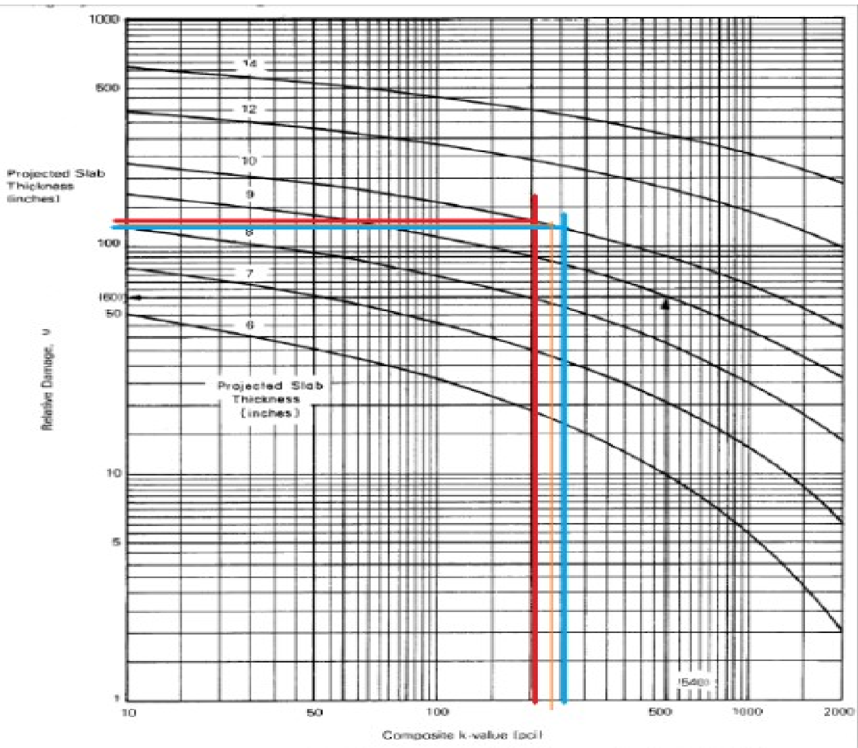


Figure 3.5. Chart for Estimating Relative Damage to Rigid Pavements Based on Slab Thickness and Underlying Support

The rigid foundation is not taken into account (the subgrade is considered to be semi-infinite, meaning it can be assumed as a half space). This premise is based upon the significant depth on which the rigid foundation lays (30 ft >> 10 ft). The **slab** has been hypothesized to be **10 in. Thick**.

A representative value for the relative damage can be pursued by calculating an average value; then the average k-value is calculated, using the previous chart (orange line). The final corrected value is found from the chart considering the loss of support and it is equal to **90 pci**.

The values are resumed in the following table:

Season	M_R [psi]	E_{SB} [psi]	K∞ [pci]	u_f
1	5000	15000	310	115
2	5000	15000	310	115
3	5000	15000	310	115
4	5000	15000	310	115
5	5000	15000	310	115
6	6500	25000	410	100
7	6500	25000	410	100
8	6500	25000	410	100
9	6500	25000	410	100
10	6500	25000	410	100
11	5000	15000	310	115
12	5000	15000	310	115

TOT u_f	1305,000
n° periods	12
u_f average	109
K∞ average [pci]	370
K∞ corrected of design [pci]	**90**

By using the following formula, it is possible to determine the thickness of the slab (D) by using the solver of Excel. The thickness is equal to 9.02 inches and can be rounded to 9.5 inches or 24 cm:

$$log_{10} w_{18} = z_R \cdot s_0 + 7.35 \cdot \log_{10}(D+1) - 0.06 + \frac{\log_{10}\left[\frac{\Delta PSI}{4.2 - 1 \cdot 5}\right]}{1 + \frac{1.624 * 10^7}{[D+1]^{8.46}}} + (4.22 - 0.32 p_t) \cdot \log_{10}\left[\frac{s'_c * C_d * [D^{0.75} - 1.132]}{215.63 * J * \left[D^{0.75} - \frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]}\right]$$

#5.7.2 Performance period

The performance period can be defined by making different hypothesis till the following condition is respected:

$$\Delta PSI_{TRIAL} = \Delta PSI - \Delta PSI_{SW,FH}$$

In which the difference between the trial performance period and the final performance period must be lower than 1. To do this, it is considered the chart below to obtain the environmental serviceability loss according to the time for swelling conditions. There have been made 3 hypotheses, as resumed in the table below:

N° iter.	PP trial [years]	ΔPSI_{SW}	$\Delta PSI_{TRAFFIC}$	ESAL	PP [years]	<1 year?	
1	20	0.24	1.76	6091750	29.73	9.73	NO
2	28	0.281	1.719	5954880	29,18	1,18	NO
3	29	0.285	1.715	5941521	29,13	0,13	OK

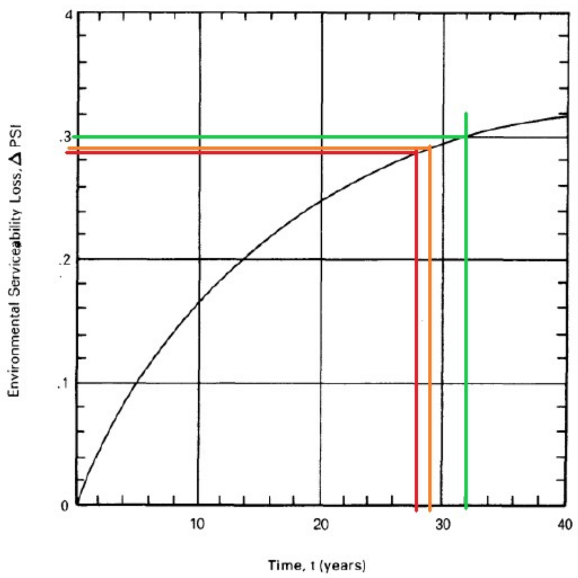
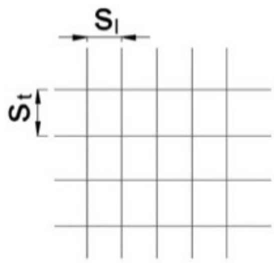


Figure I.2. Plot of Environmental Serviceability Loss Versus Time for Swelling Conditions Considered

After the iteration, a performance period of about 29 years have been found, but this is bigger than the requested one of 25 years: the problem comes from the initial overestimation of the subbase thickness of 10 inches and also of the slab's thickness. By overestimating them, the effects of swelling are already covered and the pavement is more resistant. In a real design all the procedure should be done again. For sake of exercise only, the iteration is not repeated and the results are used in the next steps.

- **Spacing:**



$s_l \in [4 \div 12\ in.]$

$s_t \in [4 \div 24\ in.]$

#5.9 Tie bars design

Design tie bars along the three longitudinal joints by using grade 40 steel (Fig 3.13 and 3.14).

- **Spacing**

To define the maximum tie bar spacing, it is normally used a chart, where the value is chosen according to the diameter of the bars, by considering a diameter of 1/2 inches and also of 5/8 in., entering to the charts with the distance to the closest free edge and knowing the slab thickness (9.5 in.), the following results are obtained:

Steel grade		40	**fy**	psi	40000	**F**	\	1,5
			Φ bar	**in.**	**0,5**	Φ bar	in.	0,625
L J1	ft	12	**s1**	in.	34	**s1**	in.	48
L J2	ft	22	**s2**	in.	19	**s2**	in.	30
L J3	ft	10	**s3**	in.	41	**s3**	in.	48

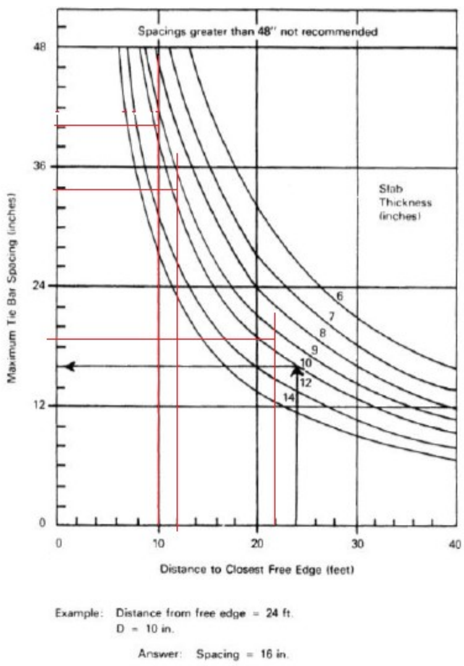


Figure 3.13. Recommended Maximum Tie Bar Spacings for PCC Pavements Assuming ½-inch Diameter Tie Bars, Grade 40 Steel, and Subgrade Friction Factor of 1.5

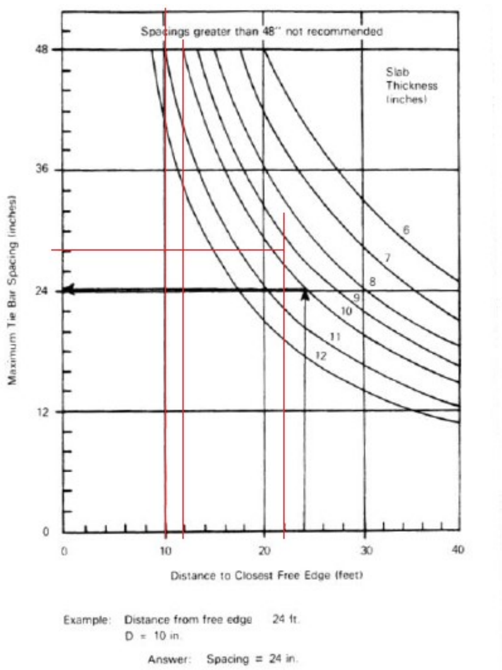


Figure 3.14. Recommended Maximum Tie Bar Spacings for PCC Pavements Assuming ⅝-inch Diameter Tie Bars, Grade 40 Steel, and Subgrade Friction of 1.5

$$f_b' = \frac{f_c'}{3} \cdot (4 - \Phi_d) = \frac{5429\ psi}{3} \cdot (4 - 1.19\ in) = 5085\ psi$$

It is possible now to calculate the maximum displacement following Timoshenko's solution:

$$y_0 = \frac{P_t}{4 \cdot \beta^3 \cdot EI} \cdot (2 + \beta \cdot z) \qquad where \qquad P_t = \alpha_i \cdot \frac{\frac{W}{2}}{\sum \alpha_i} = \alpha_i \cdot \frac{20\ kN}{\sum \alpha_i}$$

Displacement y_0 allows to calculate the bearing stress by knowing the modulus of dowel support Kds:

$$\sigma_0 = k_{ds} \cdot y_0 \qquad < \qquad f_b = \frac{4 - \phi_d}{3} \cdot f'_c$$

In this exercise there are multiple loads and superposition can be applied, but this is not done because it would be a long procedure.

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To analyze these two pavements, the Asphalt Institute Method is applied. This method considers two main criteria as critical for the design of asphalt pavement:

1. The horizontal tensile strain (ε_t) at the bottom of the asphalt layer which leads to fatigue cracking: ***Fatigue failure criterion***.

Since the volumetric characteristics are not provided, in this first step, the allowable number of load repetitions (N_f) will be calculated using standard values. Thus, for a bitumen volume of 11% and a void volume of 5% there will be:

$$N_f = 0.0796 \cdot \varepsilon_t^{-3.291} \cdot |E^*|^{-0.854}$$

2. The vertical compressive strain (ε_c), which results in permanent deformation or rutting: ***Rutting failure criterion.***

Considering ε_c as the vertical compressive strain on the top of the subgrade, it is possible to obtain the allowable number of load repetitions:

$$N_d = 1.365 \cdot 10^{-9} \cdot |\varepsilon_c|^{-4.477}$$

Knowing that:

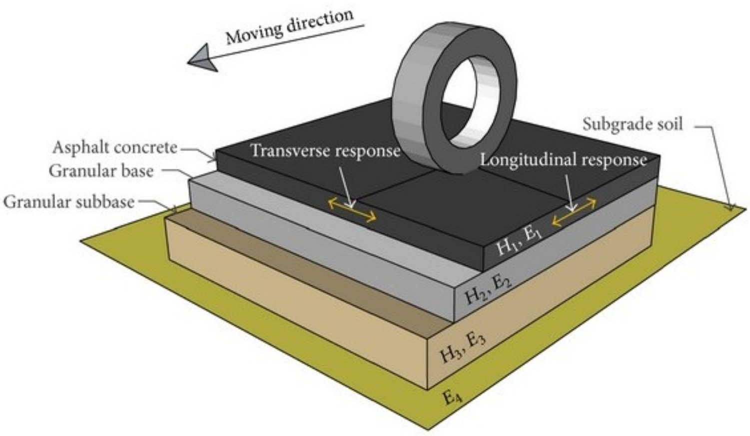
- *Principle horizontal tensile stress:*

$$\varepsilon_t = \frac{\varepsilon_x + \varepsilon_y}{2} + \sqrt{\left(\frac{\varepsilon_x + \varepsilon_y}{2}\right)^2 + \gamma_{xy}^2}$$

- *Shear strain (x plain, y direction)*

$$\gamma_{xy} = \frac{2 \cdot (1 + v)}{E} \cdot \tau_{xy}$$

Where τ_{xy} is the shear stress.



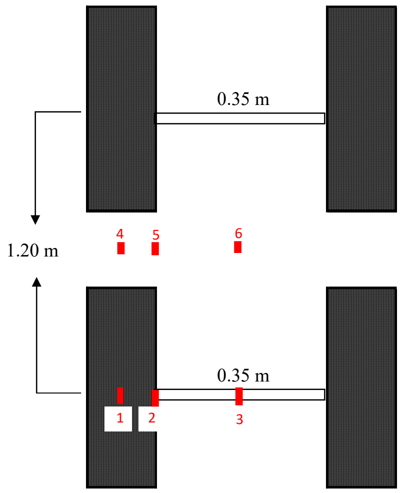
It's possible to calculate the damage ratio by using the following formulas:

$$D_{axle} = \frac{1}{N_f} \qquad D_{axle} = \frac{1}{N_d}$$

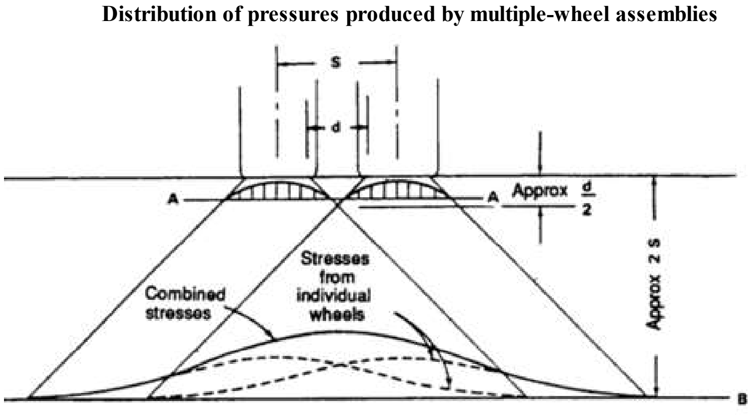
The Equivalent Axle Load Factor (EALF) will be calculated as:

$$EALF = \frac{D_x}{D_{std}}$$

As the goal is to compute the maximum amount of passages for each failure criteria, it is indispensable that ε_t and ε_c are considered as the sum of the effects caused with the passage of more tires in the case of the tridem and tandem axles.

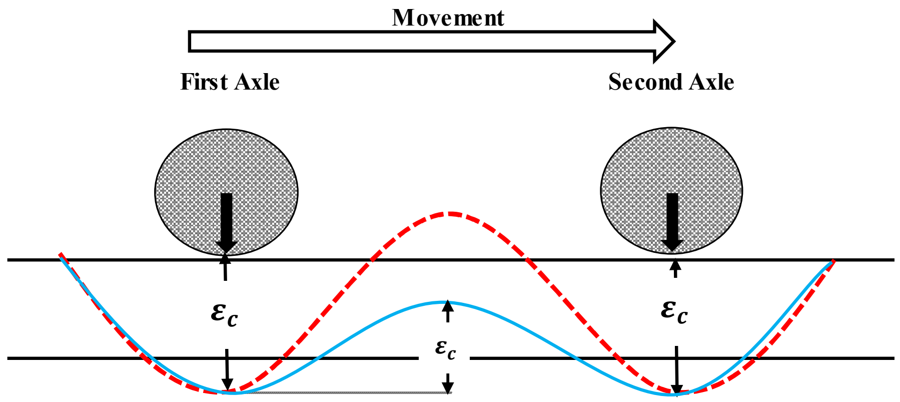


Plan View

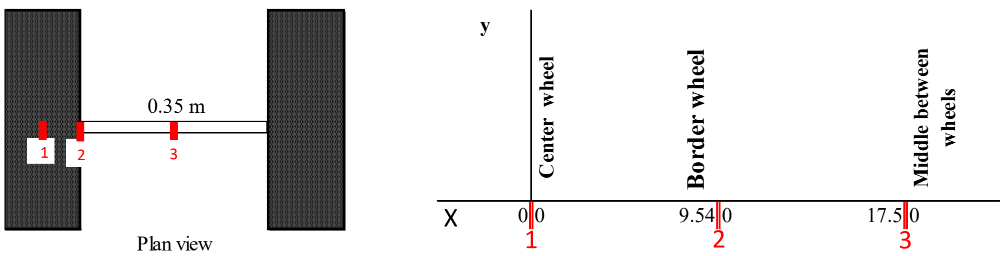


In this case for points that are not below of the first axle it is considered:

- As it is shown in the case of the red line, the vertical compressive strains are changing longitudinally: ε_c passes from compression to traction and back to compression, then for this specific case the rutting failure criterion considers two times the value of ε_c.
- On the other hand, the blue line represents a regular variation of ε_c, only in compression; in this case for calculating the rutting failure criterion it is needed to subtract to the value of ε_c of the first axle, the value of ε_c of the point in evaluation.

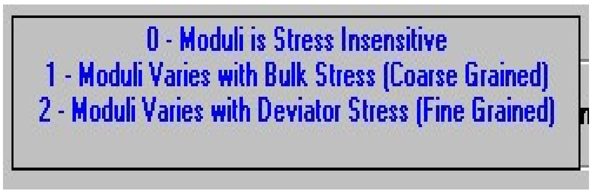


- The standard axle is an axle with dual tires whose maximum weight is 80KN, and the points evaluated for the relative damage due to rutting and cracking, are plot below.

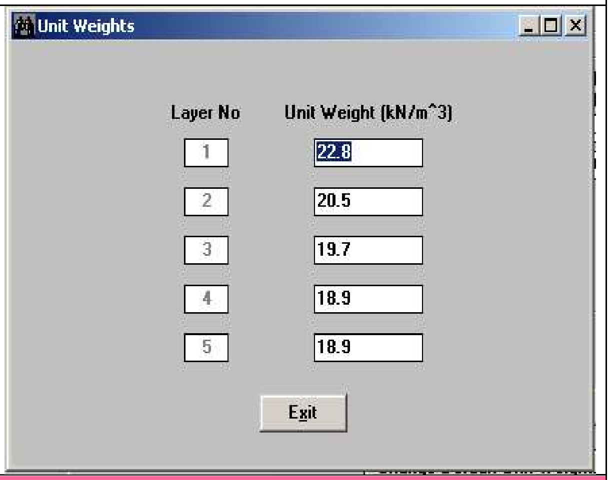


Where:

Layer ID	indication that assumes that the moduli of the layers depends on the type of material (own weight) of which it is composed	
0	insensitive	
1	E=k1*teta^(k2)	k1 = is the bulk stress
2	E = k3*sigma_deviatoric^(k4)	

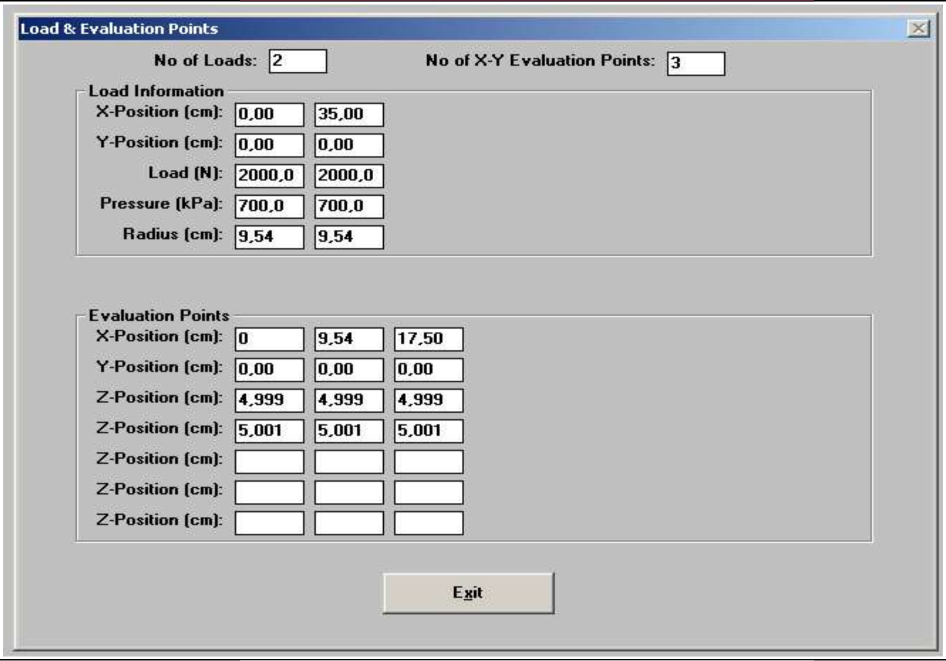


Or to define the weights of the materials:



Interface contact	
0	full slip
1	full adhesion
2-1000	calibrate partial slip

Definition of load and evaluation points



In the next table the values that have been obtained from the output data are presented.

Layer	Type	h	h	E	ν
		[cm]	[in.]	[Mpa]	[\]
1	HMA	21	8,27	Franken & Vanelstraete	0,35
2	Granular foundation	25	9,84	Shell	0,45
3	Subgrade	∞	∞	50	0,45

#6.2.1 ESAL after 20 years

To determine design ESAL, the following data on traffic are given:

AADT	**[\]**	3800	**TTF**	**[\]**	2,5
r (1-5 years)	**[%]**	4	**Dd**	**[\]**	0,5
r (6-20 years)	**[%]**	2	**Dl**	**[%]**	80
%HV	**[%]**	65	**PP**	**[years]**	20

Which, by applying the same formulas as the previous practical applications contained in this book, allows to obtain the ESAL for the following 20 years:

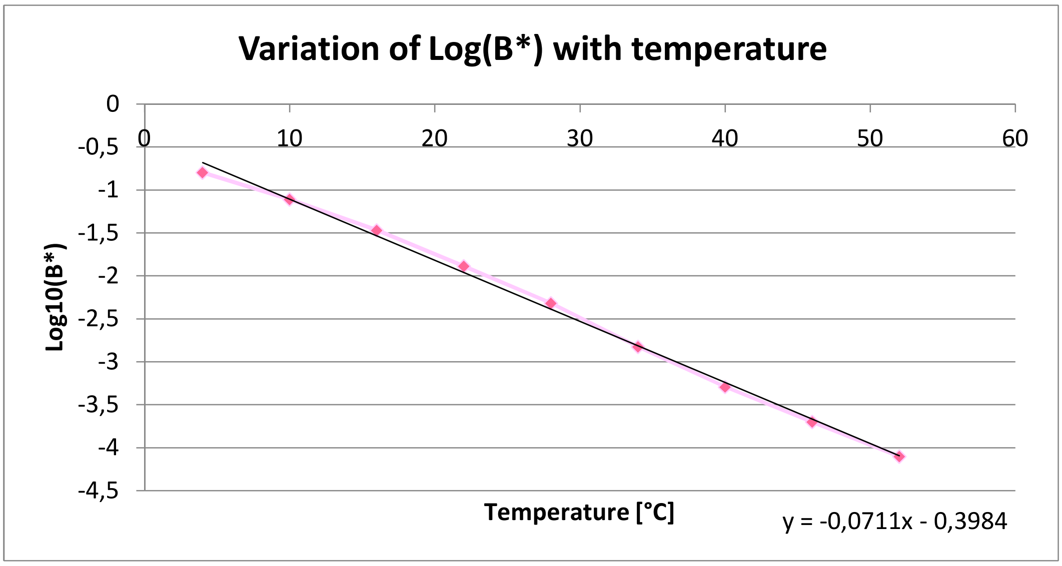
Year	G(Y)	ESAL	Year	G(Y)	ESAL
1	1,000	901550	**11**	12,169	10970705
2	2,040	1839162	**12**	13,412	12091669
3	3,122	2814278	**13**	14,680	13235053
4	4,246	3828400	**14**	15,974	14401304
5	5,416	4883086	**15**	17,293	15590880
6	6,308	5687086	**16**	18,639	16804248
7	7,434	6702378	**17**	20,012	18041883
8	8,583	7737976	**18**	21,412	19304270
9	9,755	8794285	**19**	22,841	20591906
10	10,950	9871721	**20**	24,297	21905294

$$G(Y) = \frac{(1+r)^n - 1}{r}$$

$$ESAL_{year\ n} = 365 \cdot AADT \cdot D_D \cdot D_L \cdot \%HV \cdot TTF$$

The traffic value after 20 years is needed because it will be multiplied by the damage of one axle (the one used to study the response of the pavement), in order to get the damage due to all the traffic. Since 4 seasons are considered, the ESAL at 20 years is divided by 4 (assuming to have an equal distribution of traffic during the seasons, which means 25% of the traffic in every season):

Season	% Traffic	ESAL
Winter	25%	5476323
Spring	25%	5476323
Summer	25%	5476323
Autumn	25%	5476323
TOT	**100%**	**21905294**



Now, knowing that:

Va	**[%]**	84,5
Vb	**[%]**	10,7
Va/Vb	**[\]**	7,897
Voids = 100-Va-Vb	**[%]**	4,8

It is possible to calculate the logarithm of the **reduced modulus** $Log(R^*)$ for each season as:

$$Log(R^*) = Log(B^*) \cdot \left\{1 - 1.35 \cdot \left[1 - exp\left(-0.13 \cdot \frac{Va}{Vb}\right)\right] \cdot [1 + 0.11 \cdot Log(B^*)]\right\}$$

The next step is the calculation of the **glassy elastic modulus** (E_∞), which for mixes containing pure bitumen, only depends on the volumetric composition of the mix:

$$E_\infty[MPa] = C \cdot \left(\frac{Va}{Vb}\right)^{0.55} \cdot \exp(-0.0584 \cdot v) = 33808.28 \text{ MPa}$$

Where:

- $v =$ is the void content [%]
- Va = the aggregate volume content [%]
- Vb = the binder content (Va + Vb + v = 100) [%]
- C = is equal to 14360 MPa for mixtures bound with pure bitumen

Finally, by using the relationship to estimate the effective resilient modulus of the HMA layer given by **Franken and Vanelstraete** in 1996, defined as:

$$|E^*|(T, Fr) = E_\infty \cdot R^*(T, Fr)$$

The following results were obtained for each season:

- *Fatigue failure criterion:*

$$Nf = 18.4 \cdot C \cdot 4.325 \cdot 10^{-3} \cdot {\varepsilon_t}^{-3.291} \cdot |E^*|^{-0.854}$$

$$M = 4.84 \cdot \left(\frac{V_b}{V_v + V_b} - 0.69\right) = 0.001561$$

$$C = 10^M = 1.003601$$

- For a standard mix with Vb of 11% and Vv of 5%, M=0 and:

$$Nf = 0.0796 \cdot {\varepsilon_t}^{-3.291} \cdot |E^*|^{-0.854}$$

- *Rutting failure criterion:*

$$Nd = 1.365 \cdot 10^{-9} \cdot |\varepsilon_c|^{-4.477}$$

Finally, considering the traffic that was previously calculated for each season, it can be determined the total damage by fatigue and rutting. The following table shows all the results obtained:

x [cm]	εt [µstrain]	εc [10^-6]	Nf	Nd	ESAL	Df	Dd
Winter							
0	0,00003679	-112,31					
9,537	**0,00003755**	-116,51	8,227E+07		5476323	0,096569297	
17,5	0,0000373	**-117,56**		535279391,7			0,010230776
Spring							
0	0,00005384	-149,34					
9,537	**0,00005499**	-155,64	3,867E+07		5476323	0,179631363	
17,5	0,00005457	**-157,22**		145669042,9			0,037594284
Summer							
0	0,000106	-242,99					
9,537	**0,000108**	-255,75	9,169E+06		5476323	0,687289048	
17,5	0,000107	**-258,9**		15614945,1			0,350710387
Autumn							
0	5,59E-05	-153,47					
9,537	**5,71E-05**	-160,02	3,563E+07		5476323	0,15369941809	
17,5	5,66E-05	**-161,66**		128593472,9			0,042586325
TOTAL damages:						**1,117189126**	**0,44112177210**

So, since the total damage due to fatigue is:

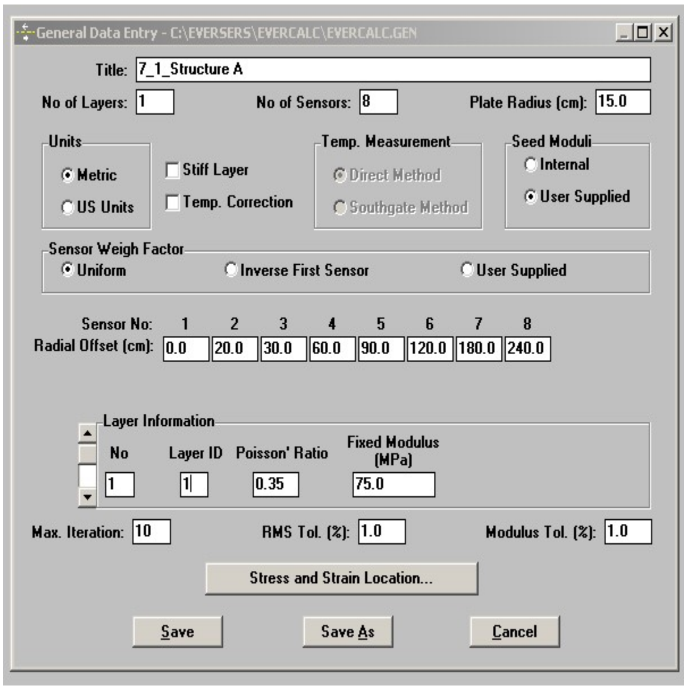
$$D_{f,tot} = 1.12 > 1$$

One can conclude that the pavement structure will fail due to fatigue.
For rutting the pavement is verified:

$$D_{d,tot} = 0.4412 < 1$$

In the Software Evercalc the parameters are set as shown in the following pictures:

- Setting Layer ID = 1 means that the modulus of the layer is known (ex. from laboratory tests), while if Layer ID = 0 means that it's unknown and it is needed an initial one and a range of variation, the program will calculate it by iteration.
- The tolerance is set to 0.1%.
- First are decided the General Data Entry files of structures A and B, then the Deflection Data files: the program calculates the iterations.



BACKCALCULATION by Evercalc© 5.0 - Detail Output

Route: 7_1_Structure A
Plate Radius (cm): 15.0
No of Sensors: 8
Offsets (cm): .0 20.0 30.0 60.0 90.0 120.0 180.0 240.0

No of Layers: 1
Stiff Layer: No
P-Ratio: .350

Station: 1
Thickness (cm):
No of Drops: 1
Average RMS Error(%): 55.56

Drop No: 1
Convergence: Error (76)
Load (N): 45000.0
No of Iterations: 0
RMS Error (%): 55.56

Sensor No:	1	2	3	4	5	6	7	8
Measured Deflections (microns):	914.000	742.000	639.000	426.000	285.000	197.000	.000	.000
Calculated Deflection (microns):	2235.037	915.209	578.109	281.615	186.907	139.964	93.208	69.879
Difference (%):	-144.53	-23.34	9.53	33.89	34.42	28.95	.00	.00

Layer No:	1
Seed Moduli (MPa):	75.00
Calculated Moduli (MPa):	75.00

Layer No:	1
Radial Distance (cm):	.00
Position:	Top
Vertical Stress (kPa):	-636.76
Radial Stress (kPa):	-541.25
Bulk Stress (kPa):	-1719.26
Deviator Stress (kPa):	-95.51
Vertical Strain (10^-6):	-3438.52
Radial Strain (10^-6):	-1719.26

One can see that in the case of the half-space the deflection basin is much higher than in the case of the 3 layer system. In both cases under the load there is the maximum deflection depending on the surface modulus in that point (Structure A → E = 75 MPa; Structure B → E = 8000 MPa), while far away from the load, at about 1 m, the values tend to be the same because the deflection here depends on the modulus of the subgrade which is the same (Structure A → E = 75 MPa; Structure B → E = 75 MPa).

#7.1.2 Surface moduli E_o

Using the surface deflections calculated above as input data, calculate the surface moduli by means of Boussinesq equations at different radial distances from the load. Plot and compare the obtained results.

To calculate the surface moduli, the following formulas are used:

- Point under the load at r = 0 m:

$$E_0 = \frac{(1-\nu^2)\cdot 2\cdot P}{\pi\cdot\delta\cdot a}$$

- Points at distance r from the load:

$$E_0 = \frac{(1-\nu^2)\cdot P}{\pi\cdot\delta\cdot r}$$

Where:

- $\nu = Poisson's\ ratio$
- $P = applied\ load = 45000\ N$
- $\delta = surface\ deflection\ [mm]$
- $a = plate's\ radius = 150\ mm$
- $r = distance\ from\ the\ load\ [mm]$

#7.2 Stiff Layer

The presence of a shallow stiff layer of 7500 MPa is considered in the case of Structure B presented in Exercise #7.1. The following deflections are obtained from a FWD test (Table 1).

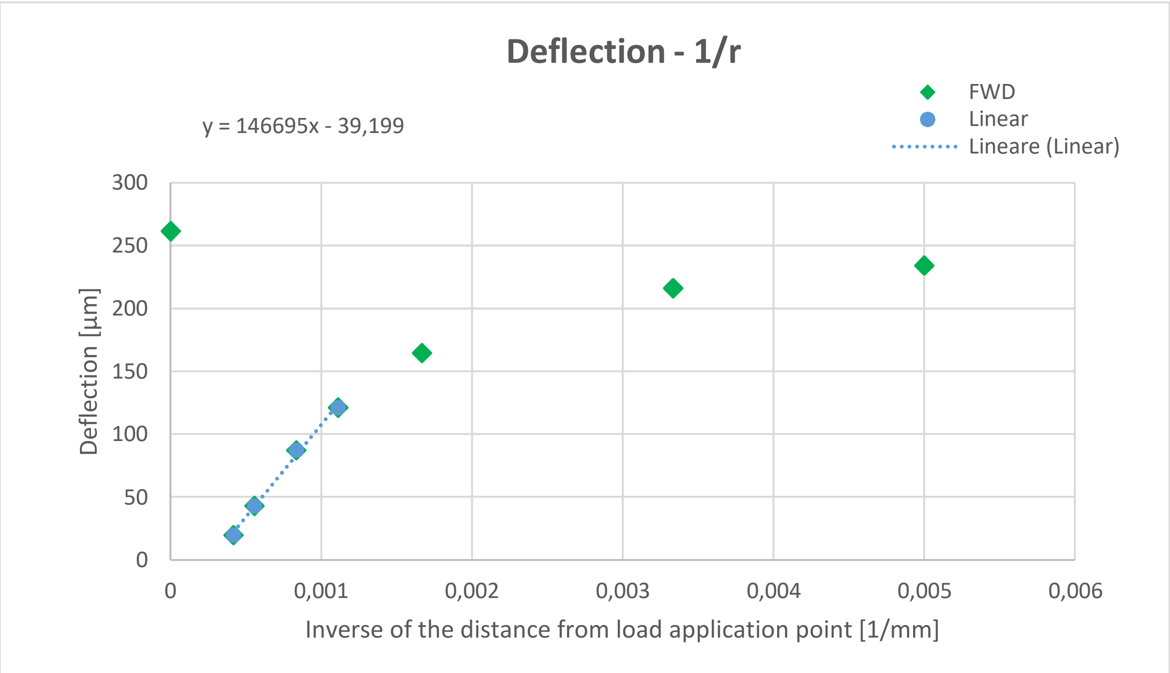
Estimate the depth of the bedrock.

Table 1.

Radial distance (mm)	0	200	300	600	900	1200	1800	2400
Surface deflection (microns)	261.630	234.212	216.194	164.632	121.157	87.219	42.975	19.714

To estimate the depth of the bedrock the first thing to do is to define the use of ratio (1/r) and then plot all this ratios according to the surface deflection.

Sensors	**Distance r [mm]**	**1/r [1/mm]**	**Deflection [µm]**
1	0	0	261,63
2	200	0,005	234,212
3	300	0,00333333	216,194
4	600	0,00166667	164,632
5	900	0,00111111	121,157
6	1200	0,00083333	87,219
7	1800	0,00055556	42,975
8	2400	0,00041667	19,714

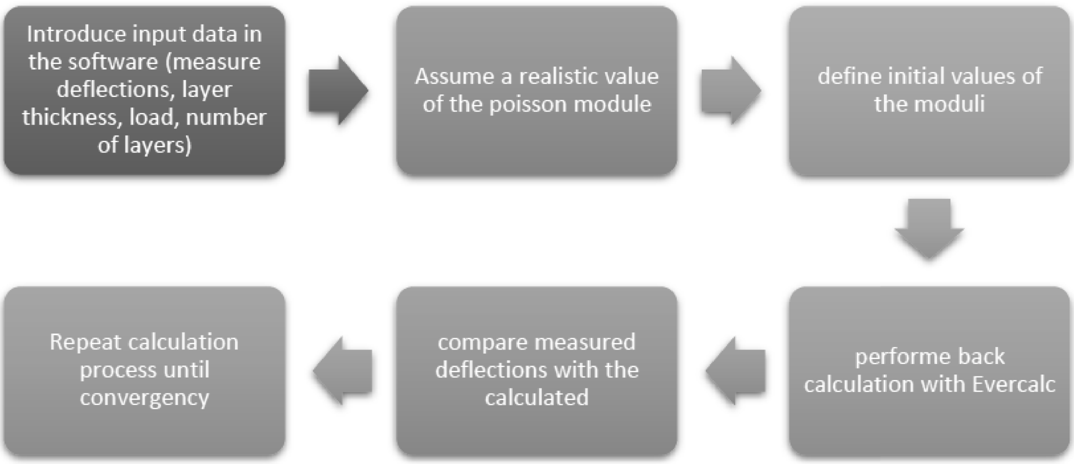


#7.3 Backcalculation

#7.3.1 Layer moduli backcalculation

Using the surface deflections determined in Exercise #7.1 for Structure B, backcalculate the layer moduli by means of the Evercalc software.

The procedure for back calculation can be resume in the following scheme:



The inputs are set as specified in the previous sections and the results are:

BACKCALCULATION by Evercalc© 5.0 - Detail Output

Route: 7_3_Structure B
Plate Radius (cm): 15.0 — No of Layers: 3
No of Sensors: 8 — Stiff Layer: No
Offsets (cm): .0 20.0 30.0 60.0 90.0 120.0 180.0 240.0 — P-Ratio: .350 .350 .350

Station: 1 — No of Drops: 1 — Average RMS Error(%): .05
Thickness (cm): 20.00 25.00 — Pavement Temperature (C): N/A

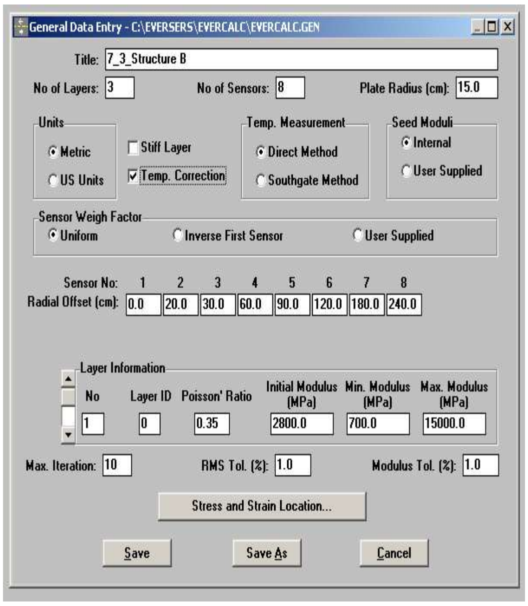
Drop No: 1 — Load (N): 45000.0 — No of Iterations: 3
Convergence: RMS Error Tolerance Satisfied — RMS Error (%): .05

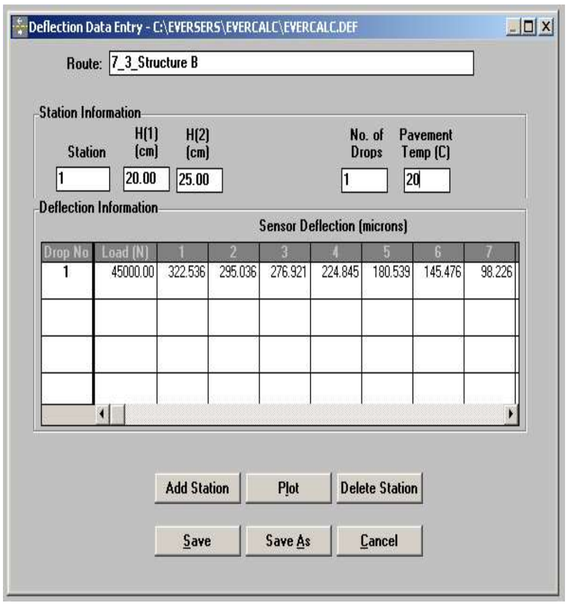
Sensor No:	1	2	3	4	5	6	7	8
Measured Deflections (microns):	322.536	295.036	276.921	224.845	180.539	145.476	98.226	71.201
Calculated Deflection (microns):	322.296	294.813	276.719	224.722	180.485	145.467	98.252	71.228
Difference (%):	.07	.08	.07	.05	.03	.01	-.03	-.04

Layer No:	1	2	3
Seed Moduli (MPa):	2800.00	180.00	100.00
Calculated Moduli (MPa):	7997.29	302.44	74.99

Layer No:	1
Radial Distance (cm):	.00
Position:	Bottom
Vertical Stress (kPa):	-61.28
Radial Stress (kPa):	1058.22
Bulk Stress (kPa):	2055.17
Deviator Stress (kPa):	-1119.50
Vertical Strain (10^-6):	-100.29
Radial Strain (10^-6):	88.69

Then, the same has been done using the command “Seed moduli: Internal”, from which the outputs are:





Again the calculation has been done comparing both commands, "User supplied" and "Internal", and the results are:

BACKCALCULATION by Evercalc© 5.0 - Detail Output

Route: 7_3_Structure B
Plate Radius (cm): 15.0
No of Sensors: 8
Offsets (cm): .0 20.0 30.0 60.0 90.0 120.0 180.0 240.0

No of Layers: 3
Stiff Layer: No
P-Ratio: .350 .350 .350

Station: 1
Thickness (cm): 20.00 25.00
No of Drops: 1
Average RMS Error(%): .05
Pavement Temperature (C): 20.0

Drop No: 1
Convergence: RMS Error Tolerance Satisfied
Load (N): 45000.0
No of Iterations: 3
RMS Error (%): .05

Sensor No:	1	2	3	4	5	6	7	8
Measured Deflections (microns):	322.536	295.036	276.921	224.845	180.539	145.476	98.226	71.201
Calculated Deflection (microns):	322.296	294.813	276.719	224.722	180.485	145.467	98.252	71.228
Difference (%):	.07	.08	.07	.05	.03	.01	-.03	-.04

Layer No:	1	2	3	1-(adj)
Seed Moduli (MPa):	2800.00	180.00	100.00	N/A
Calculated Moduli (MPa):	7997.29	302.44	74.99	5136.124

Layer No:	1
Radial Distance (cm):	.00
Position:	Bottom
Vertical Stress (kPa):	-61.28
Radial Stress (kPa):	1058.22
Bulk Stress (kPa):	2055.17
Deviator Stress (kPa):	-1119.50
Vertical Strain (10^-6):	-100.29
Radial Strain (10^-6):	88.69

BACKCALCULATION by Evercalc© 5.0 - Detail Output

Route: 7_3_Structure B
Plate Radius (cm): 15.0
No of Sensors: 8
Offsets (cm): .0 20.0 30.0 60.0 90.0 120.0 180.0 240.0

No of Layers: 3
Stiff Layer: No
P-Ratio: .350 .350 .350

Station: 1
Thickness (cm): 20.00 25.00
No of Drops: 1
Average RMS Error(%): .40
Pavement Temperature (C): 20.0

Drop No: 1
Convergence: RMS Error Tolerance Satisfied
Load (N): 45000.0
No of Iterations: 1
RMS Error (%): .40

Sensor No:	1	2	3	4	5	6	7	8
Measured Deflections (microns):	322.536	295.036	276.921	224.845	180.539	145.476	98.226	71.201
Calculated Deflection (microns):	322.734	295.161	277.070	225.201	181.092	146.134	98.872	71.720
Difference (%):	-.06	-.04	-.05	-.16	-.31	-.45	-.66	-.73

Layer No:	1	2	3	1-(adj)
Seed Moduli (MPa):	6896.55	344.83	83.24	N/A
Calculated Moduli (MPa):	7925.76	314.60	74.54	5090.185

Layer No:	1
Radial Distance (cm):	.00
Position:	Bottom
Vertical Stress (kPa):	-62.85
Radial Stress (kPa):	1046.05
Bulk Stress (kPa):	2029.25
Deviator Stress (kPa):	-1108.89
Vertical Strain (10^-6):	-100.32
Radial Strain (10^-6):	88.56

Layer	**With stiff layer (#7.2) E [MPa]**			**With stiff layer E [MPa]**		
1	7997.29	RMS Error: 0.05%	N° iterations: 3	7915.26	RMS Error: 0.06%	N° iterations: 3
2	302.44			306.88		
3	74.99			74.98		
4	/			7500.00		

One can see that the error is small and the values are quite the same as calculated in the previous point.