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A P P U N T I

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MATERIA: Foundations exercises - Prof. Dominijanni, Lancellotta

POLITECNICO DI TORINO

FOUNDATIONS

Homeworks

Department of Civil Engineering
Master Course of Foundations

2017/18

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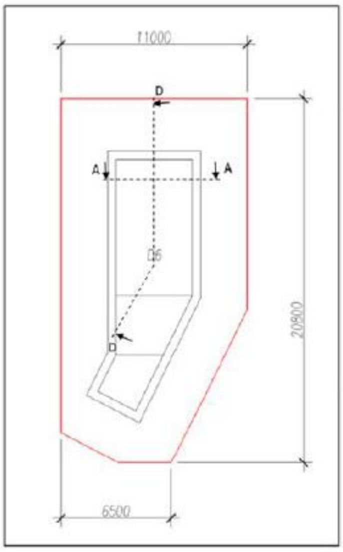
Homework 1
Soil investigation report

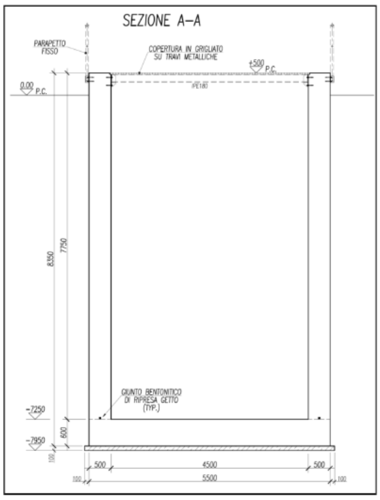
1. Introduction

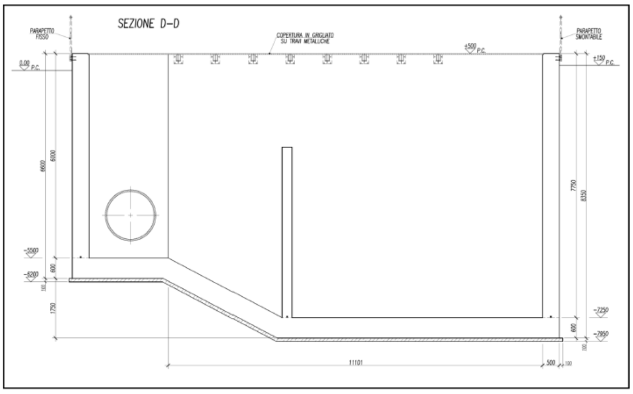
This report resumes the estimated values after an accurate analysis of the results of a geotechnical investigation of the soil present in the industrial plant of Mantova.
The motive of the investigation is the construction of a new underground tank within the area.
The customer is Versalis S.p.A.
The soil investigations took place between 08/08/2013 and 09/08/2013.

The following images show respectively the general layout of the tank, the vertical section A-A and the vertical section D-D.



TECNO IN
SERVIZI DI INGEGNERIA
LABORATORIO PROVE SUI TERRENI
Conc.Min.LL.PP. N° 53363
del 06-05-05
CARATTERISTICHE FISICHE GENERALI





The soil investigation was made by doing the following tests:

- 1 borehole S-01 drilled to a depth of 20 m
- 12 SPT tests within the borehole
- 6 Lefranc tests within the borehole
- 1 undisturbed sample was taken from the borehole
- 2 CPTU tests
- Laboratory tests: UU test, granulometry

<table>
<tr><th colspan="2">Relative Density of Sand</th><th colspan="3">Strength of Clay</th></tr>
<tr><th>Penetration Resistance N (blows/ft)</th><th>Relative Density</th><th>Penetration Resistance N (blows/ft)</th><th>Unconfined Compressive Strength (tons/ft²)</th><th>Consistency</th></tr>
<tr><td>0–4</td><td>Very loose</td><td><2</td><td><0.25</td><td>Very soft</td></tr>
<tr><td>4–10</td><td>Loose</td><td>2–4</td><td>0.25–0.50</td><td>Soft</td></tr>
<tr><td>10–30</td><td>Medium</td><td>4–8</td><td>0.50–1.00</td><td>Medium</td></tr>
<tr><td>30–50</td><td>Dense</td><td>8–15</td><td>1.00–2.00</td><td>Stiff</td></tr>
<tr><td>>50</td><td>Very dense</td><td>15–30</td><td>2.00–4.00</td><td>Very stiff</td></tr>
<tr><td></td><td></td><td>>30</td><td>>4.00</td><td>Hard</td></tr>
</table>

From Terzaghi and Peck, 1948.

As said before, also the description of the soil done by the operators and the following granulometric analysis in the lab is good to understand the profile. This considerations are reported in the tables.

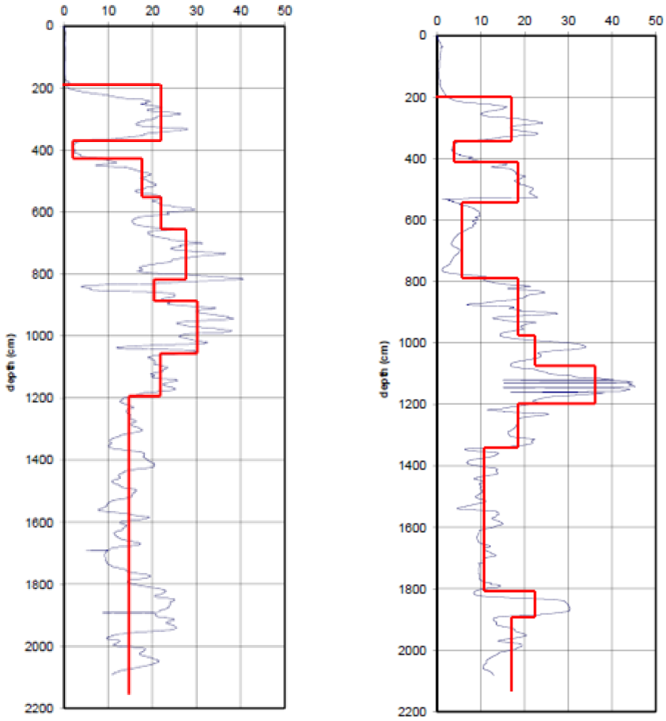
<table>
<tr><th colspan="2" rowspan="2">Depth range [m]</th><th rowspan="2">N° of blows</th><th rowspan="2">Consistency</th><th rowspan="2">Classification</th><th colspan="2">Observations</th></tr>
<tr><th>reaction with HCl</th><th>plasticity</th></tr>
<tr><td>3,00</td><td>3,45</td><td>24</td><td>medium</td><td>GP - gravel with low silty sand</td><td rowspan="3">low</td><td rowspan="3">non plastic</td></tr>
<tr><td>4,90</td><td>5,35</td><td>31</td><td>dense</td><td>SM, SC - silt with low clayey sand</td></tr>
<tr><td>6,00</td><td>6,45</td><td>44</td><td>dense</td><td rowspan="2">SM, SP - low silty sand</td></tr>
<tr><td>7,50</td><td>7,95</td><td>39</td><td>dense</td><td rowspan="9">no reaction</td><td rowspan="9">non plastic</td></tr>
<tr><td>8,50</td><td>8,95</td><td>41</td><td>dense</td><td>SM - silty sand</td></tr>
<tr><td>9,50</td><td>9,95</td><td>32</td><td>dense</td><td rowspan="3">SM, SP - low silty sand</td></tr>
<tr><td>10,50</td><td>10,95</td><td>33</td><td>dense</td></tr>
<tr><td>11,50</td><td>11,95</td><td>34</td><td>dense</td></tr>
<tr><td>12,50</td><td>12,95</td><td>27</td><td>medium</td><td>SW, SP - low silty sand</td></tr>
<tr><td>13,50</td><td>13,95</td><td>36</td><td>dense</td><td rowspan="2">SW, SP - low silty sand</td></tr>
<tr><td>16,50</td><td>16,95</td><td>40</td><td>dense</td></tr>
<tr><td>19,50</td><td>19,95</td><td>37</td><td>dense</td><td>SM, SP - low silty sand</td></tr>
</table>

<table>
<tr><th colspan="2" rowspan="2">Depth range [m]</th><th rowspan="2">N° of blows</th><th colspan="4">Granulometry [%]</th><th rowspan="2">Description</th></tr>
<tr><th>Silt</th><th>Clay</th><th>Sand</th><th>Gravel</th></tr>
<tr><td>3,00</td><td>3,45</td><td>24</td><td>3</td><td>8</td><td>44</td><td>46</td><td>Medium low silty sand – brown colour</td></tr>
<tr><td>4,90</td><td>5,35</td><td>31</td><td>7</td><td>51</td><td>42</td><td>0</td><td>Medium sand – brown colour</td></tr>
<tr><td>6,00</td><td>6,45</td><td>44</td><td>1</td><td>8</td><td>87</td><td>4</td><td>Medium sand – brown colour</td></tr>
<tr><td>7,50</td><td>7,95</td><td>39</td><td>1</td><td>10</td><td>89</td><td>0</td><td>Medium sand, with located low silt content – brown colour</td></tr>
<tr><td>8,50</td><td>8,95</td><td>41</td><td>3</td><td>14</td><td>80</td><td>3</td><td>Medium sand, with located low silt content – brown greyish colour</td></tr>
<tr><td>9,50</td><td>9,95</td><td>32</td><td>0</td><td>8</td><td>90</td><td>2</td><td>Medium sand, with located low silt content – brown greyish colour</td></tr>
<tr><td>10,50</td><td>10,95</td><td>33</td><td>0</td><td>8</td><td>92</td><td>0</td><td>Medium coarse low silty sand – brown colour</td></tr>
<tr><td>11,50</td><td>11,95</td><td>34</td><td>1</td><td>7</td><td>92</td><td>0</td><td>Medium coarse low silty sand – brown colour</td></tr>
<tr><td>12,50</td><td>12,95</td><td>27</td><td>1</td><td>9</td><td>89</td><td>1</td><td>Medium coarse low silty sand - grey colour</td></tr>
<tr><td>13,50</td><td>13,95</td><td>36</td><td>0</td><td>9</td><td>91</td><td>0</td><td>Medium coarse low silty sand - grey colour</td></tr>
<tr><td>16,50</td><td>16,95</td><td>40</td><td>0</td><td>7</td><td>93</td><td>0</td><td>Medium coarse low silty sand - grey colour</td></tr>
<tr><td>19,50</td><td>19,95</td><td>37</td><td>2</td><td>8</td><td>90</td><td>0</td><td>Medium coarse low silty sand - grey colour</td></tr>
</table>

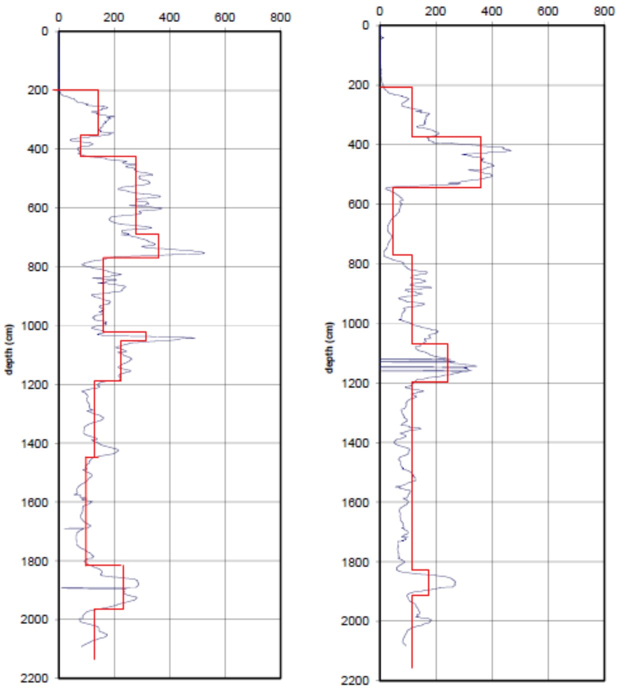
different peaks while advancing in depth. For this reason, the red line is defined by the engineer, who choose the criteria to estimate the parameters for the classification.

We estimate the needed values obtained from the two tests in order to get a final soil profile with averaged parameters.

- **Tip Stress resistance Qc [MPa]**



- **Sleeve friction Fs [MPa]**



The pore pressure graphs are very different, and, in the test number 1 (on the left), the behaviour is constant up to 8m depth, then it increases. That depth coincides with the water table and that means that the soil in this test is consolidated, even the clayey layers.
In the second test, at a level of 4m depth, we found a higher value of pressure: this could be explained due to the presence of the clayey soil in the form of lenses of clay or due to the consolidation process.

From the previous graphs we try to find suitable values to use for engineerical calculations, so in the following table, our estimations are collected.

Depth [m]	Qc1 [MPa]	Qc2 [MPa]	Qc average Layer [MPa]	Qc average [MPa]
0,0	/	/	/	/
1,0	/	/	/	/
1,5	/	/	/	/
2,0	/	/	/	/
2,5	22	18	20	
3,0	22	18	20	15,83
3,5	11	4	7,5	
4,0	2	4	3	8,50
4,5	9	19	14	
5,0	18	19	18,5	
5,5	22	7	14,5	
6,0	22	7	14,5	
6,5	22	7	14,5	
7,0	28	7	17,5	17,61
7,5	28	7	17,5	
8,0	28	7	17,5	
8,5	20	19	19,5	
9,0	30	19	24,5	
9,5	30	19	24,5	
10,0	30	23	26,5	
10,5	30	23	26,5	27,30
11,0	22	37	29,5	
11,5	22	37	29,5	
12,0	15	19	17	
12,5	15	19	17	
13,0	15	19	17	
13,5	15	11	13	
14,0	15	11	13	
14,5	15	11	13	
15,0	15	11	13	
15,5	15	11	13	
16,0	15	11	13	14,71
16,5	15	11	13	
17,0	15	11	13	
17,5	15	11	13	
18,0	15	11	13	
18,5	15	22	18,5	
19,0	15	22	18,5	
19,5	15	17	16	
20,0	15	17	16	

The following chart shows how can be interpreted a CPTU test using the friction ratio and the cone bearing:

Soil type	k_z (cm/s)	Description	Drainage
Clean gravel (GW, GP)	>1.0	High	Very good
Clean sands, clean sand and gravel mixtures (SW, SP)	1.0 to 10^{-3}	Medium	Good
Fine sands, silts, mixtures comprising sands, silts, and clays (SM-SC)	10^{-3} to 10^{-5}	Low	Poor
Weathered and fissured clays			
Silt, silty clay (MH, ML)	10^{-5} to 10^{-7}	Very low	Poor
Homogeneous clays (CL, CH)	$<10^{-7}$	Practically impervious	Very poor

4. Estimation of the soil engineerical parameters

4.1. Undrained shear strength C_u From CPTU test

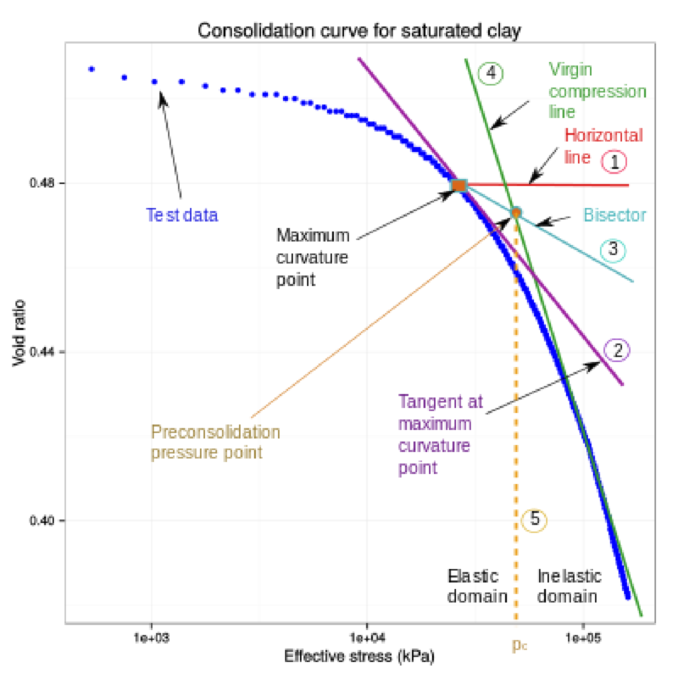
Since we have established that there is a clay layer, we can now calculate this parameter.

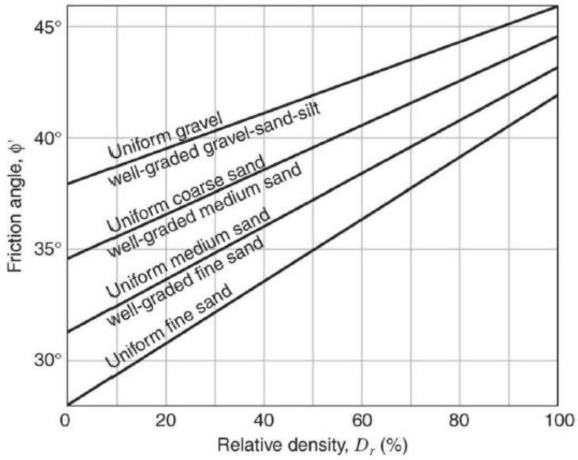
$$C_u = \frac{q_c - \sigma'_{vo}}{N_c}$$

Where:

- q_c = tip stress resistance [kPa]
- σ'_{vo} = vertical effective stress [kPa]
- $N_c = \begin{cases} 14 \text{ for NC clays} \\ 17 \pm 5 \text{ for OC clays} \\ 10 \div 30 \text{ for stiff fessured clays} \end{cases}$

To choose the N_c value, it is necessary to estimate the OCR so we need to know the pre-consolidation stress: it is employed the following graphical procedure using the curve obtained in the oedometric test.





The results are reported in the following table. Don't forget that after 7.63m depth we there is the water level, so we subtract the pressure from the vertical stress.

Depth [m]	γ [kN/m3]	Ϭ'vo [kPa]	Nspt	CN fine	CN coarse	CN max	DR [%]	soil type	φ°cv	φ°cv average layer
3	21	12,60	24	1,78	1,41	1,78	84,3	sand and gravel	44,70	45
4,4	19,79	40,31		/	/	/	/	/	/	/
6,45	18	77,21	44	1,13	1,08	1,13	91,0	well graded fine sand	42,00	40
7,95	18	89,49	39	1,06	1,04	1,06	82,8	uniform fine sand	39,60	
8,95	18	97,68	41	1,01	1,01	1,01	83,1	uniform fine sand	39,70	
9,95	18	105,87	32	0,97	0,98	0,98	72,3	well graded fine sand	39,80	
10,95	18	114,06	33	0,93	0,96	0,96	72,5	well graded fine sand	39,80	
11,95	18	122,25	34	0,90	0,93	0,93	72,6	well graded fine sand	39,90	
12,95	18	130,44	27	0,87	0,91	0,91	63,9	uniform fine sand	36,80	37
13,95	18	138,63	36	0,84	0,89	0,89	72,9	uniform fine sand	37,00	
16,95	18	163,20	40	0,76	0,83	0,83	74,2	uniform fine sand	38,40	
19,95	18	187,77	37	0,69	0,77	0,77	69,1	uniform fine sand	37,70	

4.3. Soil stiffness

4.3.1. Sands

It was estimated the effective elastic modulus E' for each sandy layer, using D'Apollonia's criterion. The sandy layers in the profiles were assumed as over consolidated layers. That is because its stress history must be similar to the clayey layer, from which was calculated the OCR, resulting in a value equal to 11.

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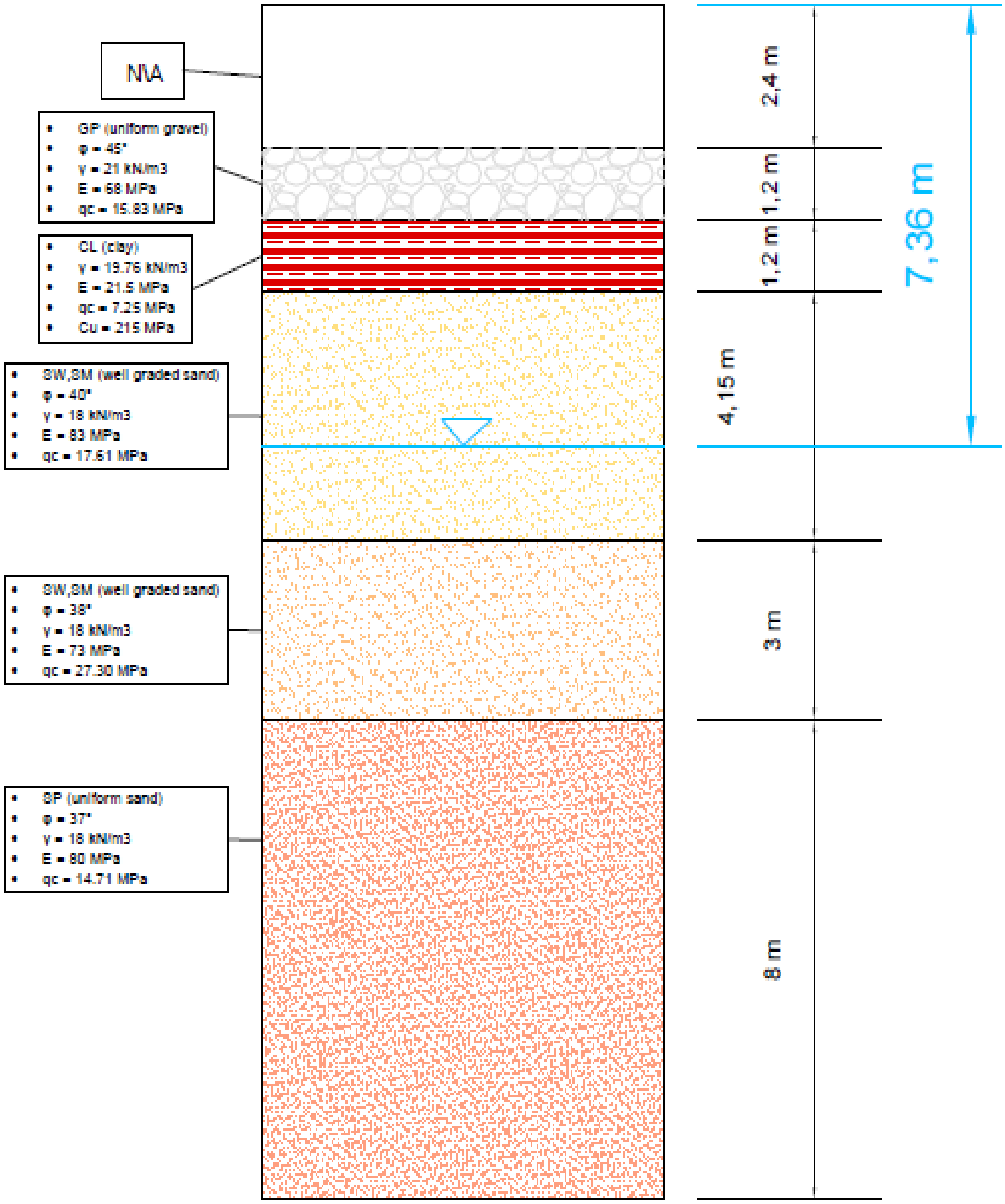
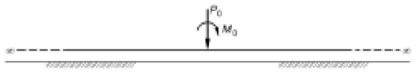
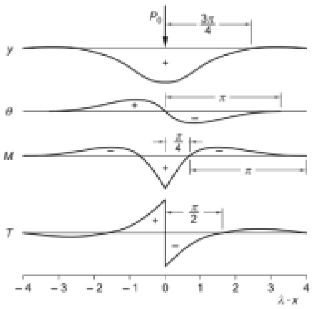
5. Definitive profile

Finally we can create a soil profile which is more useful for the engineerical purpose to construct a tank.



Infinite beam subjected to a concentrated load P_0

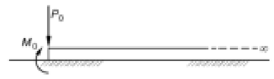


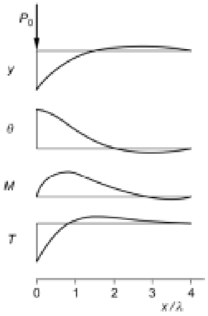


$$y(x) = A(x)\frac{\lambda P_0}{2K}$$
$$\vartheta(x) = \frac{dy}{dx} = -B(x)\frac{\lambda^2 P_0}{K}$$
$$M(x) = -EJ\frac{d^2y}{dx^2} = C(x)\frac{P_0}{4\lambda}$$
$$V(x) = -EJ\frac{d^3y}{dx^3} = -D(x)\frac{P_0}{2}$$

where $K = K_r \cdot B$, being B the breadth of the beam

Semi-infinite beam loaded by a vertical force P_0 at the free end

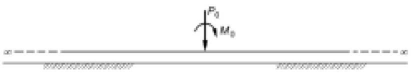


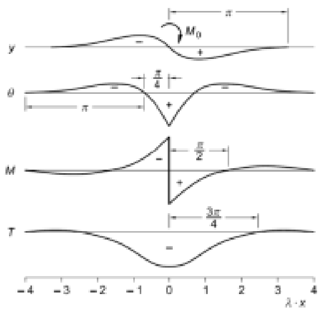


$$y(x) = D(x)\frac{2\lambda P_0}{K}$$
$$\vartheta(x) = \frac{dy}{dx} = -A(x)\frac{2\lambda^2 P_0}{K}$$
$$M(x) = -EJ\frac{d^2y}{dx^2} = -B(x)\frac{P_0}{\lambda}$$
$$V(x) = -EJ\frac{d^3y}{dx^3} = -C(x)P_0$$

where $K = K_r \cdot B$, being B the breadth of the beam

Infinite beam subjected to a concentrated external moment M_0

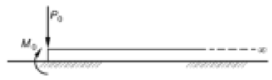


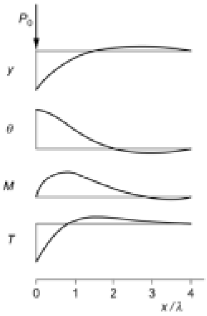


$$y(x) = B(x)\frac{\lambda^2 M_0}{K}$$
$$\vartheta(x) = \frac{dy}{dx} = C(x)\frac{\lambda^3 M_0}{K}$$
$$M(x) = -EJ\frac{d^2y}{dx^2} = D(x)\frac{M_0}{2}$$
$$V(x) = -EJ\frac{d^3y}{dx^3} = -A(x)\frac{\lambda M_0}{2}$$

where $K = K_r \cdot B$, being B the breadth of the beam

Semi-infinite beam loaded by a vertical force P_0 at the free end





$$y(x) = D(x)\frac{2\lambda P_0}{K}$$
$$\vartheta(x) = \frac{dy}{dx} = -A(x)\frac{2\lambda^2 P_0}{K}$$
$$M(x) = -EJ\frac{d^2y}{dx^2} = -B(x)\frac{P_0}{\lambda}$$
$$V(x) = -EJ\frac{d^3y}{dx^3} = -C(x)P_0$$

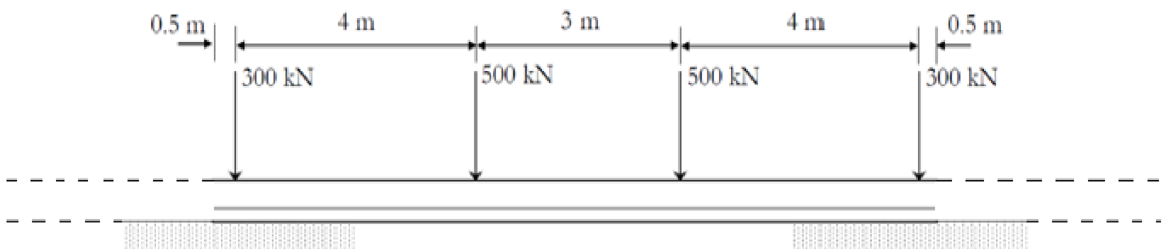
where $K = K_r \cdot B$, being B the breadth of the beam

Where:

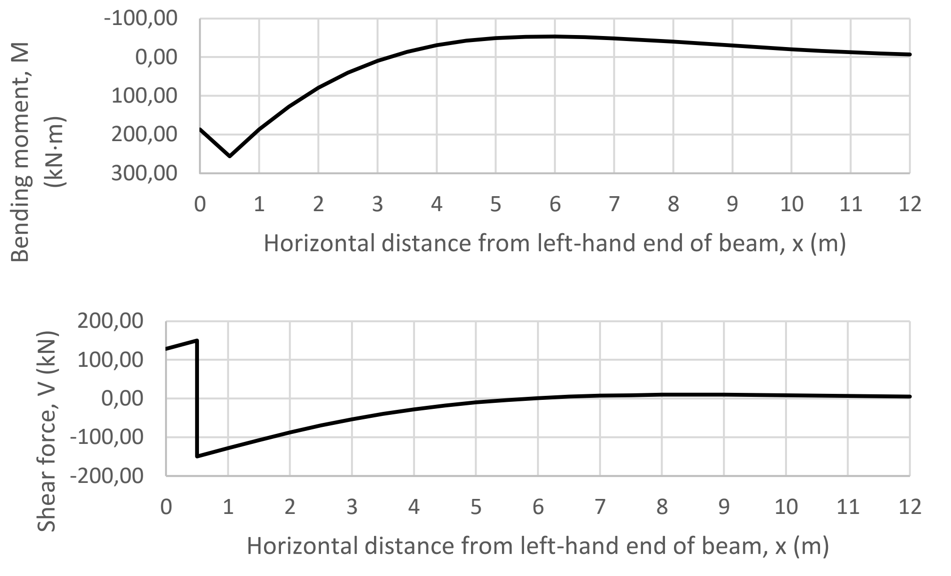
$$A(x) = e^{-\lambda x}(\cos\lambda x + \sin\lambda x)$$
$$B(x) = e^{-\lambda x}\sin\lambda x$$
$$C(x) = e^{-\lambda x}(\cos\lambda x - \sin\lambda x)$$
$$D(x) = e^{-\lambda x}\cos\lambda x$$

For the solution of this first case it was used the Excel file software, proposed by the professor.

Using the fundamental cases mentioned previously, one can obtain the solution for a finite beam subjected to a concentrated load in two ways: with an approximate solution or with an exact solution.
For both it must be evaluated the case of an infinite beam, subjected to a concentrated load, like in the figure:

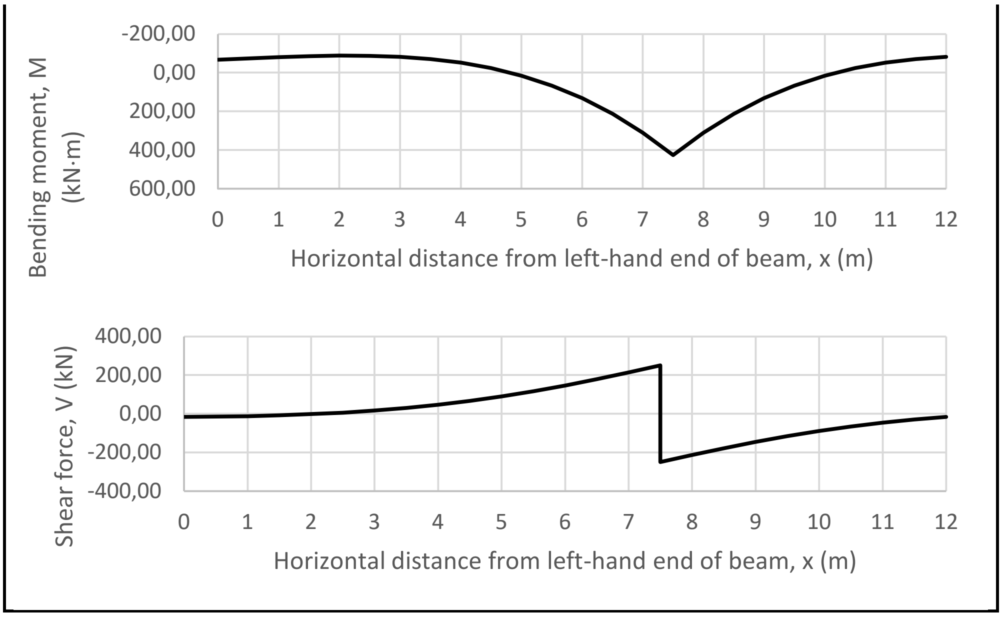


9,5	9	-0,03	0,03	-0,10	-0,06	-24,94	9,40
10	9,5	-0,04	0,02	-0,08	-0,06	-20,41	8,69
10,5	10	-0,04	0,01	-0,06	-0,05	-16,28	7,84
11	10,5	-0,04	0,00	-0,05	-0,05	-12,59	6,91
11,5	11	-0,04	0,00	-0,04	-0,04	-9,37	5,96
11,5	11	-0,04	0,00	-0,04	-0,04	-9,37	5,96
12	11,5	-0,04	-0,01	-0,03	-0,03	-6,62	5,04



P2 = 500 kN applied at x = 4.5 m

Load P	500	kN					
xP	4,5	m					
x	**x'**	**A**	**B**	**C**	**D**	**M (kN·m)**	**T (kN)**
0	4,5	0,33	0,26	-0,19	0,07	-82,03	16,74
0,5	4	0,41	0,29	-0,17	0,12	-70,44	30,12
0,5	4	0,41	0,29	-0,17	0,12	-70,44	30,12
1	3,5	0,49	0,31	-0,12	0,19	-51,40	46,56
1,5	3	0,58	0,32	-0,05	0,27	-23,34	66,27
2	2,5	0,68	0,32	0,04	0,36	15,44	89,41
2,5	2	0,77	0,31	0,16	0,46	66,64	115,97
3	1,5	0,86	0,27	0,31	0,58	131,96	145,82
3,5	1	0,93	0,22	0,50	0,71	212,95	178,59
4	0,5	0,98	0,13	0,73	0,85	310,93	213,63
4,5	0	1,00	0,00	1,00	1,00	426,81	250,00
4,5	0	1,00	0,00	1,00	1,00	426,81	-250,00
5	0,5	0,98	0,13	0,73	0,85	310,93	-213,63
5,5	1	0,93	0,22	0,50	0,71	212,95	-178,59
6	1,5	0,86	0,27	0,31	0,58	131,96	-145,82
6,5	2	0,77	0,31	0,16	0,46	66,64	-115,97
7	2,5	0,68	0,32	0,04	0,36	15,44	-89,41
7,5	3	0,58	0,32	-0,05	0,27	-23,34	-66,27
7,5	3	0,58	0,32	-0,05	0,27	-23,34	-66,27
8	3,5	0,49	0,31	-0,12	0,19	-51,40	-46,56
8,5	4	0,41	0,29	-0,17	0,12	-70,44	-30,12
9	4,5	0,33	0,26	-0,19	0,07	-82,03	-16,74
9,5	5	0,25	0,23	-0,21	0,02	-87,65	-6,14
10	5,5	0,19	0,20	-0,21	-0,01	-88,59	2,00
10,5	6	0,14	0,17	-0,20	-0,03	-86,01	7,99
11	6,5	0,09	0,14	-0,19	-0,05	-80,90	12,17
11,5	7	0,05	0,11	-0,17	-0,06	-74,09	14,84



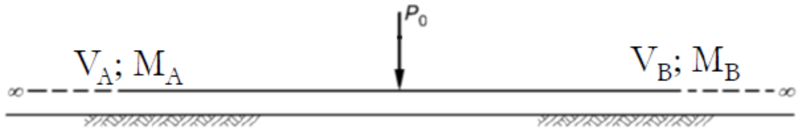
P4 = 300 kN applied at x = 11.5 m

Load P	**300**	**kN**					
xP	**11,5**	**m**					
x	**x'**	**A**	**B**	**C**	**D**	**M (kN·m)**	**T (kN)**
0	11,5	-0,04	-0,01	-0,03	-0,03	-6,62	-5,04
0,5	11	-0,04	0,00	-0,04	-0,04	-9,37	-5,96
0,5	11	-0,04	0,00	-0,04	-0,04	-9,37	-5,96
1	10,5	-0,04	0,00	-0,05	-0,05	-12,59	-6,91
1,5	10	-0,04	0,01	-0,06	-0,05	-16,28	-7,84
2	9,5	-0,04	0,02	-0,08	-0,06	-20,41	-8,69
2,5	9	-0,03	0,03	-0,10	-0,06	-24,94	-9,40
3	8,5	-0,02	0,05	-0,12	-0,07	-29,78	-9,89
3,5	8	0,00	0,07	-0,14	-0,07	-34,78	-10,05
4	7,5	0,02	0,09	-0,16	-0,07	-39,76	-9,77
4,5	7	0,05	0,11	-0,17	-0,06	-44,45	-8,90
4,5	7	0,05	0,11	-0,17	-0,06	-44,45	-8,90
5	6,5	0,09	0,14	-0,19	-0,05	-48,54	-7,30
5,5	6	0,14	0,17	-0,20	-0,03	-51,60	-4,80
6	5,5	0,19	0,20	-0,21	-0,01	-53,15	-1,20
6,5	5	0,25	0,23	-0,21	0,02	-52,59	3,68
7	4,5	0,33	0,26	-0,19	0,07	-49,22	10,05
7,5	4	0,41	0,29	-0,17	0,12	-42,26	18,07
7,5	4	0,41	0,29	-0,17	0,12	-42,26	18,07
8	3,5	0,49	0,31	-0,12	0,19	-30,84	27,93
8,5	3	0,58	0,32	-0,05	0,27	-14,00	39,76
9	2,5	0,68	0,32	0,04	0,36	9,26	53,64
9,5	2	0,77	0,31	0,16	0,46	39,99	69,58
10	1,5	0,86	0,27	0,31	0,58	79,18	87,49
10,5	1	0,93	0,22	0,50	0,71	127,77	107,15
11	0,5	0,98	0,13	0,73	0,85	186,56	128,18
11,5	0	1,00	0,00	1,00	1,00	256,09	150,00
11,5	0	1,00	0,00	1,00	1,00	256,09	-150,00
12	0,5	0,98	0,13	0,73	0,85	186,56	-128,18

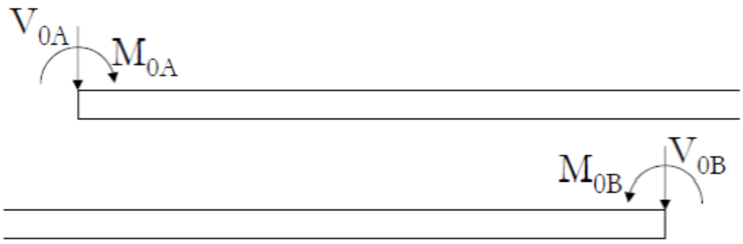
1.1.1. Approximate solution

These steps must be followed:

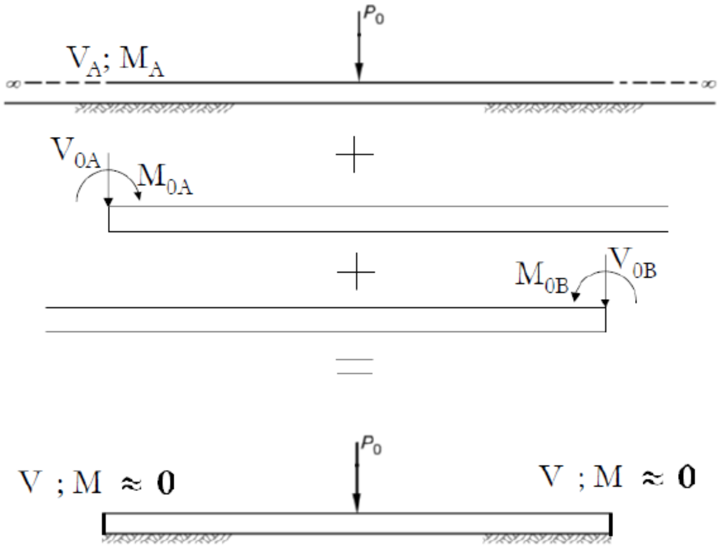
1) From the infinite beam solution obtained in the previous step must be extracted the values of moments and shears at the edges (12 m length beam):



2) The fundamental cases of semi-infinite beam subjected to a concentrated load in the edge must be solved using the values extracted in the previous step, but with the contrary sign.



3) Using the superposition principle, the solution is the sum of the 3 cases.

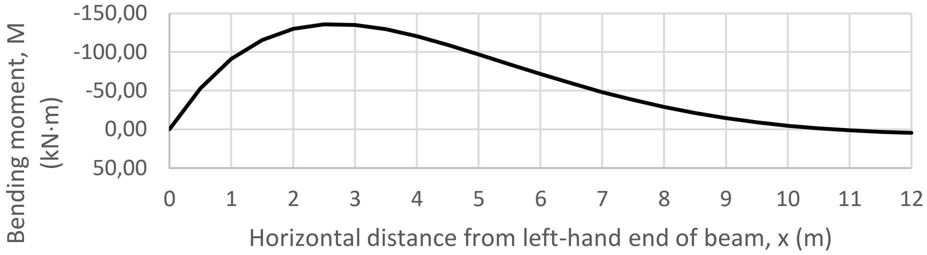


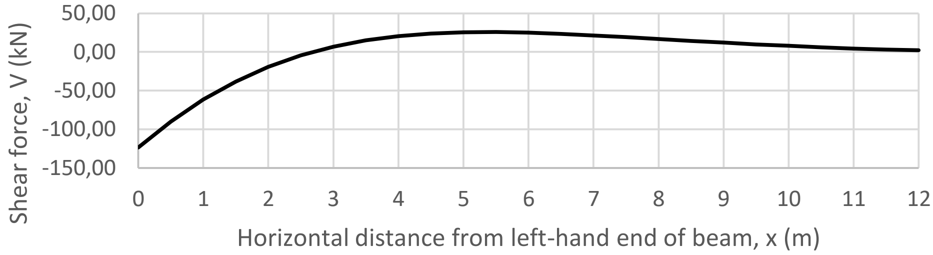
4) The solution is not exact because there are residual values of moments and shear in the edges that come from the semi-infinite beams.

Moment M_{OA}	-31,6464	kN·m					
x_{MOA}	0	m					
x	x'	A	B	C	D	M (kN·m)	V (kN)
0	0	1,00	0,00	1,00	1,00	-31,65	0,00
0,5	0,5	0,98	0,13	0,73	0,85	-31,03	2,34
0,5	0,5	0,98	0,13	0,73	0,85	-31,03	2,34
1	1	0,93	0,22	0,50	0,71	-29,42	3,99
1,5	1,5	0,86	0,27	0,31	0,58	-27,13	5,08
2	2	0,77	0,31	0,16	0,46	-24,42	5,70

Load V_{0A}	123,61	kN
x_{PA}	0	m

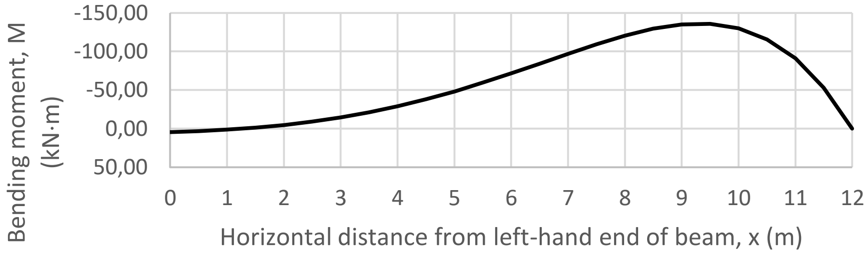
x	x'	A	B	C	D	M (kN·m)	V (kN)
0	0	1,00	0,00	1,00	1,00	0,00	-123,61
0,5	0,5	0,98	0,13	0,73	0,85	-53,19	-90,05
0,5	0,5	0,98	0,13	0,73	0,85	-53,19	-90,05
1	1	0,93	0,22	0,50	0,71	-90,91	-61,67
1,5	1,5	0,86	0,27	0,31	0,58	-115,69	-38,22
2	2	0,77	0,31	0,16	0,46	-129,89	-19,30
2,5	2,5	0,68	0,32	0,04	0,36	-135,67	-4,47
3	3	0,58	0,32	-0,05	0,27	-134,96	6,76
3,5	3,5	0,49	0,31	-0,12	0,19	-129,43	14,89
4	4	0,41	0,29	-0,17	0,12	-120,51	20,40
4,5	4,5	0,33	0,26	-0,19	0,07	-109,39	23,76
4,5	4,5	0,33	0,26	-0,19	0,07	-109,39	23,76
5	5	0,25	0,23	-0,21	0,02	-97,04	25,38
5,5	5,5	0,19	0,20	-0,21	-0,01	-84,23	25,66
6	6	0,14	0,17	-0,20	-0,03	-71,55	24,91
6,5	6,5	0,09	0,14	-0,19	-0,05	-59,44	23,43
7	7	0,05	0,11	-0,17	-0,06	-48,21	21,46
7,5	7,5	0,02	0,09	-0,16	-0,07	-38,04	19,19
7,5	7,5	0,02	0,09	-0,16	-0,07	-38,04	19,19
8	8	0,00	0,07	-0,14	-0,07	-29,04	16,79
8,5	8,5	-0,02	0,05	-0,12	-0,07	-21,25	14,37
9	9	-0,03	0,03	-0,10	-0,06	-14,65	12,04
9,5	9,5	-0,04	0,02	-0,08	-0,06	-9,19	9,85
10	10	-0,04	0,01	-0,06	-0,05	-4,77	7,86
10,5	10,5	-0,04	0,00	-0,05	-0,05	-1,29	6,07
11	11	-0,04	0,00	-0,04	-0,04	1,35	4,52
11,5	11,5	-0,04	-0,01	-0,03	-0,03	3,26	3,19
11,5	11,5	-0,04	-0,01	-0,03	-0,03	3,26	3,19
12	12	-0,04	-0,01	-0,02	-0,03	4,58	2,09

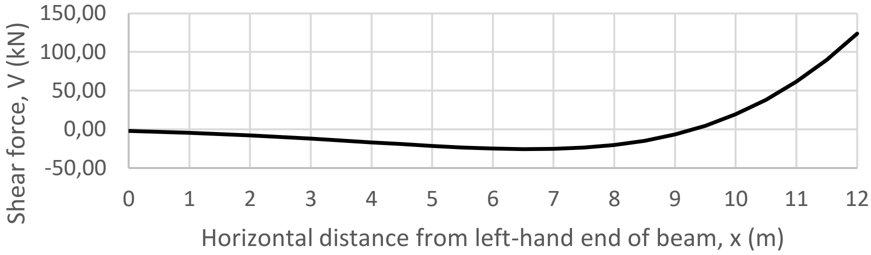




Load V_{0B}	123,61	kN
x_{PB}	12	m

x	x'	A	B	C	D	M (kN·m)	V (kN)
0	12	-0,04	-0,01	-0,02	-0,03	4,58	-2,09
0,5	11,5	-0,04	-0,01	-0,03	-0,03	3,26	-3,19
0,5	11,5	-0,04	-0,01	-0,03	-0,03	3,26	-3,19
1	11	-0,04	0,00	-0,04	-0,04	1,35	-4,52
1,5	10,5	-0,04	0,00	-0,05	-0,05	-1,29	-6,07
2	10	-0,04	0,01	-0,06	-0,05	-4,77	-7,86
2,5	9,5	-0,04	0,02	-0,08	-0,06	-9,19	-9,85
3	9	-0,03	0,03	-0,10	-0,06	-14,65	-12,04
3,5	8,5	-0,02	0,05	-0,12	-0,07	-21,25	-14,37
4	8	0,00	0,07	-0,14	-0,07	-29,04	-16,79
4,5	7,5	0,02	0,09	-0,16	-0,07	-38,04	-19,19
4,5	7,5	0,02	0,09	-0,16	-0,07	-38,04	-19,19
5	7	0,05	0,11	-0,17	-0,06	-48,21	-21,46
5,5	6,5	0,09	0,14	-0,19	-0,05	-59,44	-23,43
6	6	0,14	0,17	-0,20	-0,03	-71,55	-24,91
6,5	5,5	0,19	0,20	-0,21	-0,01	-84,23	-25,66
7	5	0,25	0,23	-0,21	0,02	-97,04	-25,38
7,5	4,5	0,33	0,26	-0,19	0,07	-109,39	-23,76
7,5	4,5	0,33	0,26	-0,19	0,07	-109,39	-23,76
8	4	0,41	0,29	-0,17	0,12	-120,51	-20,40
8,5	3,5	0,49	0,31	-0,12	0,19	-129,43	-14,89
9	3	0,58	0,32	-0,05	0,27	-134,96	-6,76
9,5	2,5	0,68	0,32	0,04	0,36	-135,67	4,47
10	2	0,77	0,31	0,16	0,46	-129,89	19,30
10,5	1,5	0,86	0,27	0,31	0,58	-115,69	38,22
11	1	0,93	0,22	0,50	0,71	-90,91	61,67
11,5	0,5	0,98	0,13	0,73	0,85	-53,19	90,05
11,5	0,5	0,98	0,13	0,73	0,85	-53,19	90,05
12	0	1,00	0,00	1,00	1,00	0,00	123,61



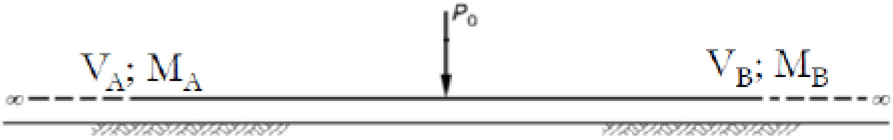


Then, summing all the cases (the 4 semi-infinite beams and the infinite one) it is obtained:

1.1.2. Exact solution

In order to get the exact solution, these steps must be followed:

1) From the infinite beam solution obtained in the previous step must be extracted the values of moments and shears at the edges (12 m length beam) :



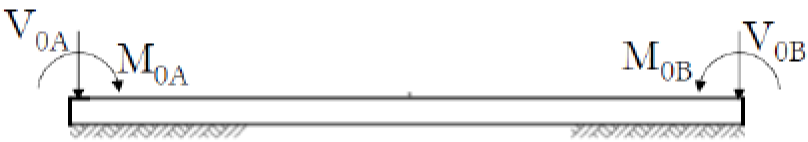
2) The general equation must be solved with the new boundary conditions:

$$EJ\frac{d^2y}{dx^2}(0) = M(0) = -Ma$$
$$EJ\frac{d^3y}{dx^3}(0) = V(0) = -Va$$
$$EJ\frac{d^2y}{dx^2}(l) = M(l) = -Mb$$
$$EJ\frac{d^3y}{dx^3}(l) = V(l) = -Vb$$

3) The constants are obtained solving the system of equations:

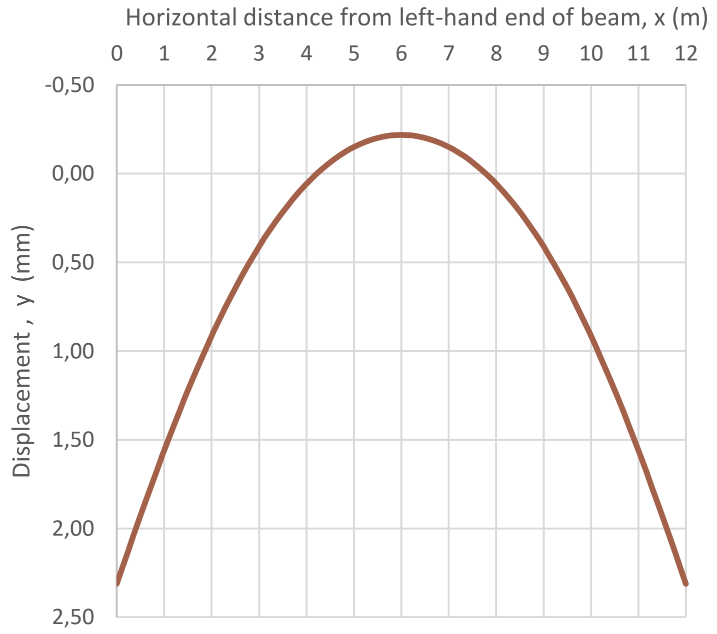
Linear system to be solved to take the finite length into account	Matrix				constant term vector	
	0	1	0	-1	0,165812	
	-1	1	1	1	2,2113505	
	12,238919	-31,28902	-0,010842	0,027719	0,165812	
	43,527941	-19,0501	-0,016877	-0,038561	-2,21135	
	Inverse matrix				**solution for arbitrary constants**	
	-0,001254	0,0002847	-0,016919	0,0277374	-0,063721	**c1**
	-0,001725	-0,000236	-0,038633	0,0108572	-0,031222	**c2**
	1,0021965	1,000756	0,0603475	0,006023	2,3758858	**c3**
	-1,001725	-0,000236	-0,038633	0,0108572	-0,197034	**c4**

4) Using them, it can be obtained the exact solution for a finite beam subjected to moments and loads at its edges by putting the coeffcients into the system's equations.

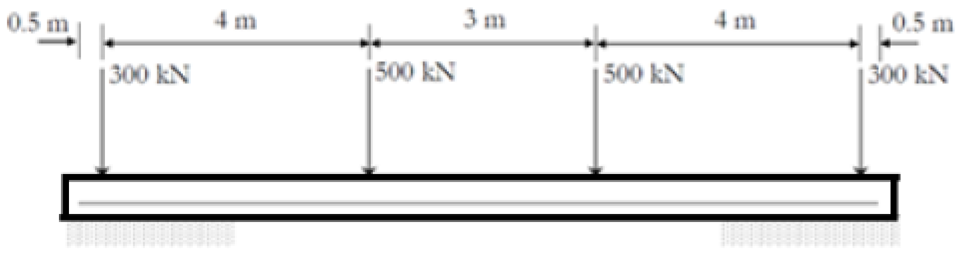


5) Using the superposition principle, summing the infinite beam subjected to a concentrated load to the case of opposite loads at the edges, the final moments and shears at the edges are going to be eliminated giving in this way an exact solution.

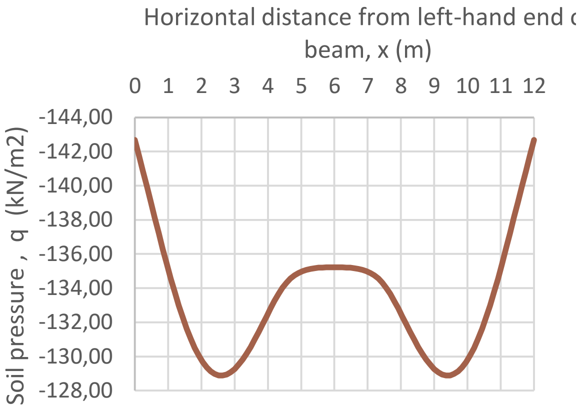
-0,20	-152,16	-3,36
-0,19	-152,61	-4,14
-0,18	-153,15	-4,88
-0,17	-153,78	-5,58
-0,16	-154,49	-6,22
-0,14	-155,27	-6,80
-0,12	-156,12	-7,31
-0,10	-157,02	-7,74
-0,08	-157,97	-8,09
-0,05	-158,96	-8,34
-0,02	-159,97	-8,49
0,00	-160,99	-8,53
0,04	-162,01	-8,45
0,07	-163,02	-8,25
0,10	-163,99	-7,91
0,14	-164,91	-7,43
0,18	-165,76	-6,80
0,22	-166,53	-6,01
0,27	-167,20	-5,05
0,31	-167,74	-3,91
0,36	-168,13	-2,59
0,41	-168,35	-1,07
0,46	-168,38	0,64
0,52	-168,19	2,57
0,58	-167,75	4,72
0,63	-167,05	7,09
0,70	-166,04	9,71
0,76	-164,71	12,56
0,82	-163,02	15,67
0,89	-160,94	19,04
0,96	-158,44	22,69
1,03	-155,48	26,60
1,11	-152,04	30,81
1,18	-148,08	35,31
1,26	-143,55	40,11
1,34	-138,44	45,22
1,42	-132,69	50,64
1,50	-126,27	56,39
1,59	-119,14	62,47
1,68	-111,26	68,88
1,76	-102,60	75,64
1,85	-93,10	82,74
1,94	-82,73	90,19
2,03	-71,44	98,00
2,13	-59,19	106,17
2,22	-45,94	114,71
2,31	-31,65	123,61



- Finite beam with concentrated loads

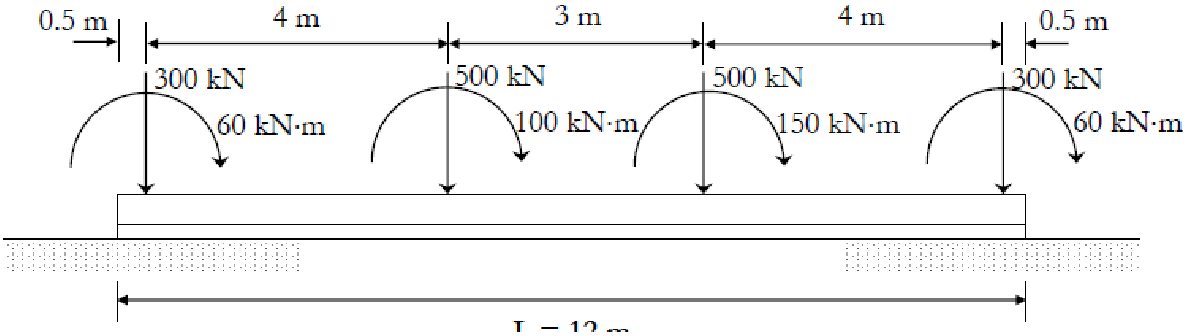


4,10	146,39	194,42	-134,32
4,09	140,69	-289,47	-134,02
4,08	106,91	-273,41	-133,66
4,07	75,07	-257,40	-133,25
4,06	45,14	-241,43	-132,81
4,04	17,12	-225,52	-132,36
4,03	-8,99	-209,67	-131,89
4,01	-33,20	-193,87	-131,43
4,00	-55,52	-178,13	-130,98
3,99	-75,96	-162,43	-130,56
3,98	-94,51	-146,79	-130,17
3,96	-111,19	-131,19	-129,82
3,96	-126,00	-115,63	-129,51
3,95	-138,94	-100,11	-129,26
3,94	-150,02	-84,61	-129,07
3,94	-159,25	-69,13	-128,94
3,94	-166,61	-53,66	-128,88
3,94	-172,13	-38,19	-128,89
3,94	-175,78	-22,72	-128,97
3,94	-177,58	-7,24	-129,13
3,95	-177,52	8,27	-129,36
3,96	-175,59	23,81	-129,67
3,97	-171,80	39,40	-130,05
3,99	-166,13	55,03	-130,50
4,00	-158,59	70,72	-131,03
4,02	-149,16	86,48	-131,62
4,04	-137,83	102,31	-132,27
4,06	-124,60	118,22	-132,99
4,09	-109,46	134,23	-133,75
4,11	-92,38	150,33	-134,57
4,14	-73,37	166,53	-135,42
4,16	-52,41	182,83	-136,30
4,19	-29,49	199,24	-137,20
4,22	-4,59	215,76	-138,12
4,25	16,30	-67,61	-139,04
4,27	9,19	-50,87	-139,95
4,30	4,09	-34,02	-140,86
4,33	1,03	-17,07	-141,77
4,36	0,00	0,00	-142,68
Max	146,39	289,47	-128,88
Min	-177,58	-289,47	-142,68



2. Case 2

Using the numerical approach, obtain shear, moment and displacement diagram of the following beam on an elastic foundation.



2.1. Numerical solution

In this case we add the external moments applied on the beam in the same sections defined before. We do the calculations only numerically, because analytically it would be very long and we should write the moments are pairs of concentrated vertical loads, then compute the variables (y,V,M) for each case and then sum all the found values.

- Input data

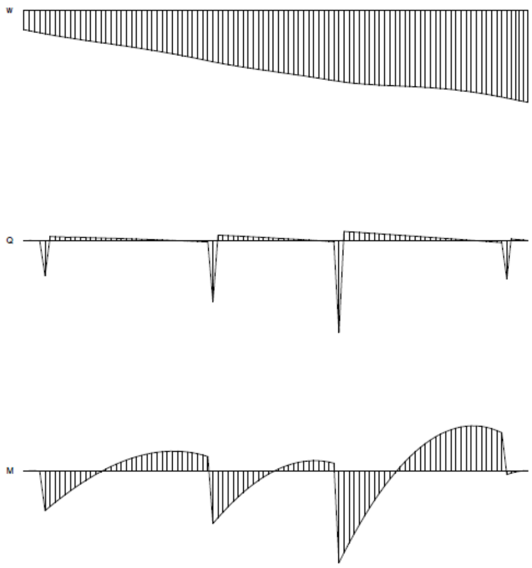
Sections

Layer	Length m	Load kN/m²	Spring kN/m³
1	0.500	0.000	29764.000
2	4.000	0.000	29764.000
3	3.000	0.000	29764.000
4	4.000	0.000	29764.000
5	0.500	0.000	29764.000

Forces

Point	x m	F kN	M kNm
0	0.000	0.000	0.000
1	0.500	60.000	60.000
2	4.500	100.000	100.000
3	7.500	150.000	150.000
4	11.500	60.000	60.000
5	12.000	0.000	0.000

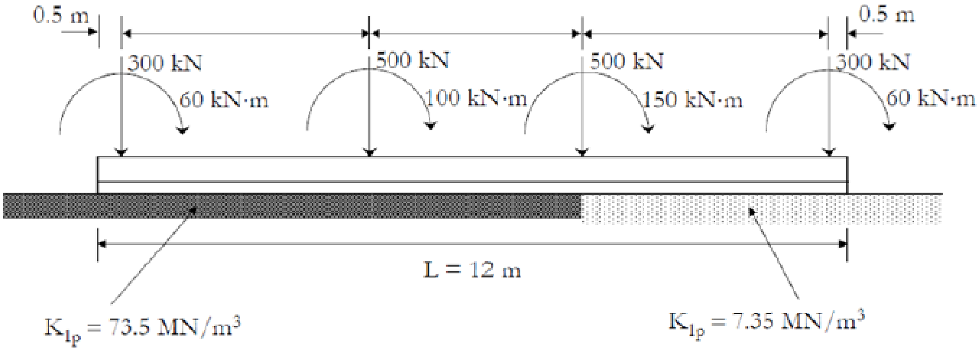
- Output data



Minimum displacement : 0.000 m
Maximum displacement : 0.002 m
Minimum shear force : -124.138 kN
Maximum shear force : 1271.282 kN
Minimum moment : -68.458 kNm
Maximum moment : 140.909 kNm

3. Case 3

Now it is interesting to see what happens in the following cases:



Foundation soil: variable

1) $K_{1p} = 73.5\ MN/m^3$ (from plate load test of side b = 0.305 m)
2) $K_{1p} = 7.35\ MN/m^3$ (from plate load test of side b = 0.305 m)
3) $K_{1p} = 73.5\ MN/m$ and $K_{1p} = 7.35\ MN/m^3$ as shown in the figure

3.1. K1p=73.5 MN/m³

This case is exactly the same as in section *2. Case 2.*

3.2. K1p=7.35 MN/m³

- Input data

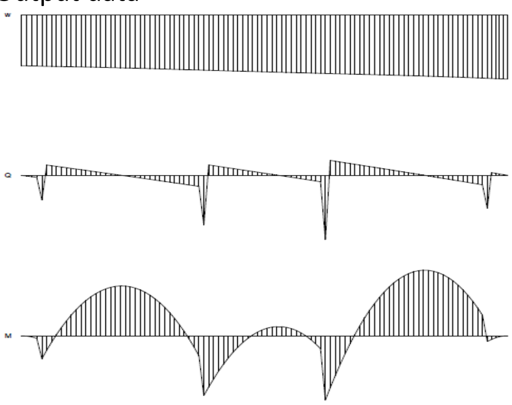
Sections

Layer	Length m	Load kN/m²	Spring kN/m³
1	0.500	0.000	2976.000
2	4.000	0.000	2976.000
3	3.000	0.000	2976.000
4	4.000	0.000	2976.000
5	0.500	0.000	2976.000

Forces

Point	x m	F kN	M kNm
0	0.000	0.000	0.000
1	0.500	300.000	60.000
2	4.500	500.000	100.000
3	7.500	500.000	150.000
4	11.500	300.000	60.000
5	12.000	0.000	0.000

- Output data



Minimum displacement : 0.040 m
Maximum displacement : 0.050 m
Minimum shear force : -326.851 kN
Maximum shear force : 1406.521 kN
Minimum moment : -214.467 kNm
Maximum moment : 209.849 kNm

3.3. K1p=73.5 MN/m³ on the right & K1p=7.35 MN/m³ on the left

Now we want to see the behaviour of the beam when it's located on a soil having two different stiffnesses.

- Input data

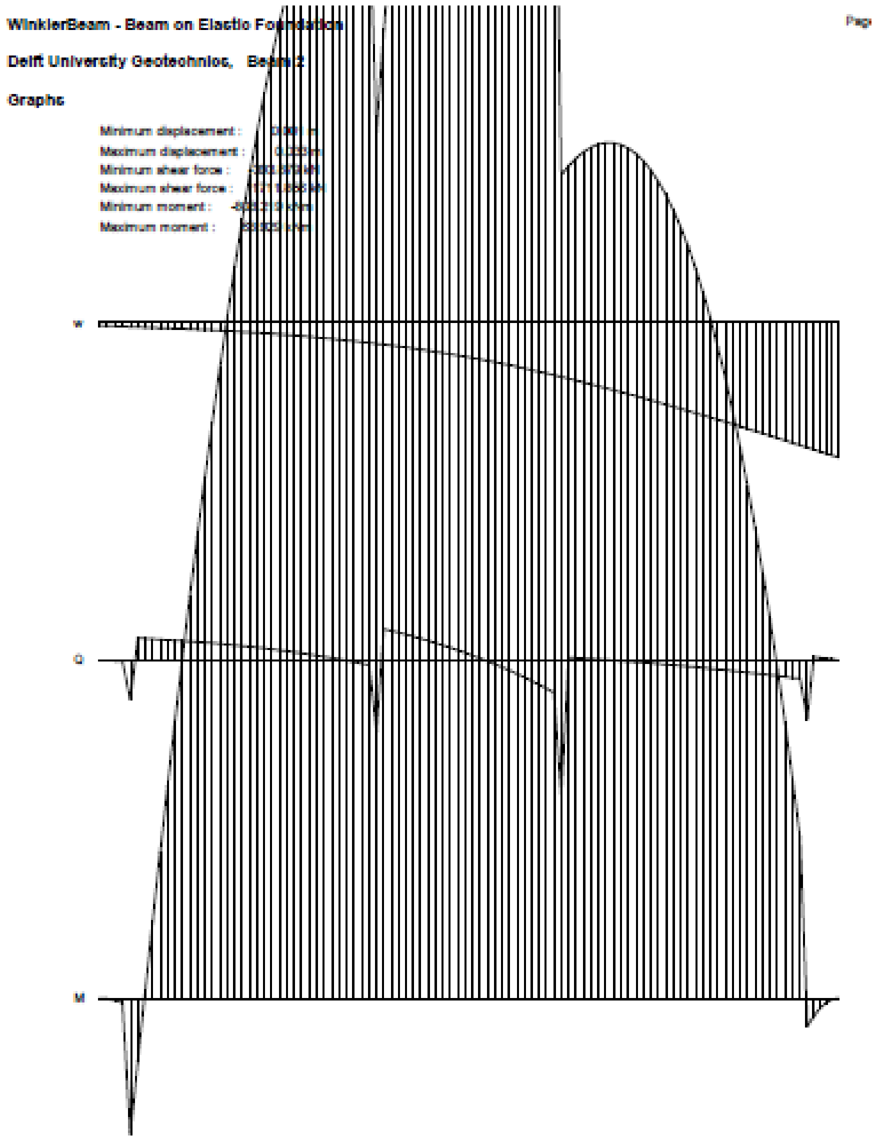
Sections

Layer	Length m	Load kN/m²	Spring kN/m³
1	0.500	0.000	29764.000
2	4.000	0.000	29764.000
3	3.000	0.000	29764.000
4	4.000	0.000	2976.000
5	0.500	0.000	2976.000

Forces

Point	x m	F kN	M kNm
0	0.000	0.000	0.000
1	0.500	300.000	60.000
2	4.500	500.000	100.000
3	7.500	500.000	150.000
4	11.500	300.000	60.000
5	12.000	0.000	0.000

- Output data



3.4. Remarks

- The concentrated moments cause jumps in the bending moment diagram. In the Verruijt's software this causes an error in the shear force diagram, showing a very high and no sense value. This is because the shear force is the derivative of the moment, and the slope of a vertical line (the jumps) is infinite, so a very high value of the shear is calculated wrongly.

- Comparing the 1st and 2nd problem, it can be seen the following aspects:

  - The values of the moments and shears are very much higher in the 2nd problem. That is because the soil is very much compliant in the second case, causing that the beam has to carry more loads. In the other hand in the 1st case the soil is stiffer and collaborates more with the beam.

  - The vertical displacements are very much higher in the 2nd problem. This happens because of the compliance of the soil that allows having much more displacement in this case.

- In the 3rd case can be observed the following:

  - It can be observed the loads are pretty much higher than in the 1st case, but lower than the second. This happens because the change in soil's subgrade reaction. The difference between the 2 parts of the beam causes that a very high negative moment value exists near the limit zone.

  - Since the difference in the soil subgrade reaction causes that the right part of the beam suffers more vertical displacements, the beam is rotating. This rotation causes the dangerous relative displacements in structures.

2. Stability check

We need firstly to check if a depth of embedment of 6 m is sufficient to guarantee the stability with a suitable margin of safety (make reference to the collapse of the structure for a rigid rotation around a given point – Design approach 1 – combination 2: A2+M2+R2).
First of all, the Design coefficients corresponding to the design approach mentioned previously are mentioned in order to develop the following calculation.

A2 + M2 + R2 γ coefficients	
$\gamma_{\text{Variable load q}}$	1.3
γ_M (tanφ)	1.25
γ_M (c'_k)	1.25
γ_M (c_{uk})	1.4
γ_M (γ_γ)	1
R_2	1.1

Then the Design soil mechanical properties are used, for each layer, to estimate the different parameters that will be needed to the evaluation of the stability.
The following equations were used:

$$\gamma_M := 1.25 \qquad \gamma_Q := 1.3$$

$$\varphi_d := \operatorname{atan}\left(\frac{\tan(\varphi)}{\gamma_M}\right) = \quad \text{deg}$$

$$\delta_{ad} := \frac{\varphi_d}{2} = \quad \text{deg} \qquad \delta_{pd} := \frac{2}{3}\cdot\varphi_d = \quad \text{deg}$$

$$q_d := \gamma_Q \cdot q = \quad \text{kPa}$$

$$\theta_a := \frac{1}{2}\cdot\left(\operatorname{asin}\left(\frac{\sin(\delta_{ad})}{\sin(\varphi_d)}\right) - \delta_{ad}\right) = \quad \text{deg}$$

$$\theta_p := \frac{1}{2}\cdot\left(\operatorname{asin}\left(\frac{\sin(\delta_{pd})}{\sin(\varphi_d)}\right) + \delta_{pd}\right) = \quad \text{deg}$$

$$K_a := \left[\frac{\cos(\delta_{ad})}{1+\sin(\varphi_d)}\cdot\left[\cos(\delta_{ad}) - \sqrt{(\sin(\varphi_d))^2 - (\sin(\delta_{ad}))^2}\right]\right]\cdot\exp(-2\cdot\theta_a\cdot\tan(\varphi_d)) =$$

$$K_{aRankine} := \frac{1-\sin(\varphi_d)}{1+\sin(\varphi_d)} =$$

$$K_p := \left[\frac{\cos(\delta_{pd})}{1-\sin(\varphi_d)}\cdot\left[\cos(\delta_{pd}) + \sqrt{(\sin(\varphi_d))^2 - (\sin(\delta_{pd}))^2}\right]\right]\cdot\exp(2\cdot\theta_p\cdot\tan(\varphi_d)) =$$

$$K_{pRankine} := \frac{1+\sin(\varphi_d)}{1-\sin(\varphi_d)} =$$

$$Ko = 1 - \sin(\varphi d) \rightarrow for\ sands$$

$$Ko = (1 - \sin(\varphi d)).OCR^{0.5} \rightarrow for\ clays$$

- Where the used parameters are:

γw	10	kPa	
d	6	m	(-) on the right side of the wall
deltah	1,5	m	
L flux	13,5	m	(+) on the left side of the wall
i	0,111		

$$u_{tot} = u_{static} + u_{dynamic}$$

$$u_{resultant} = u_{tot,\ right} - u_{tot,\ left}$$

$$\sigma v = \sum \sigma v_{i-th\ layer} = \sum \gamma_{i-th\ layer} \cdot Z$$

$$\sigma' v_{i-th\ layer} = (\sigma v - u_{tot})_{i-th\ layer}$$

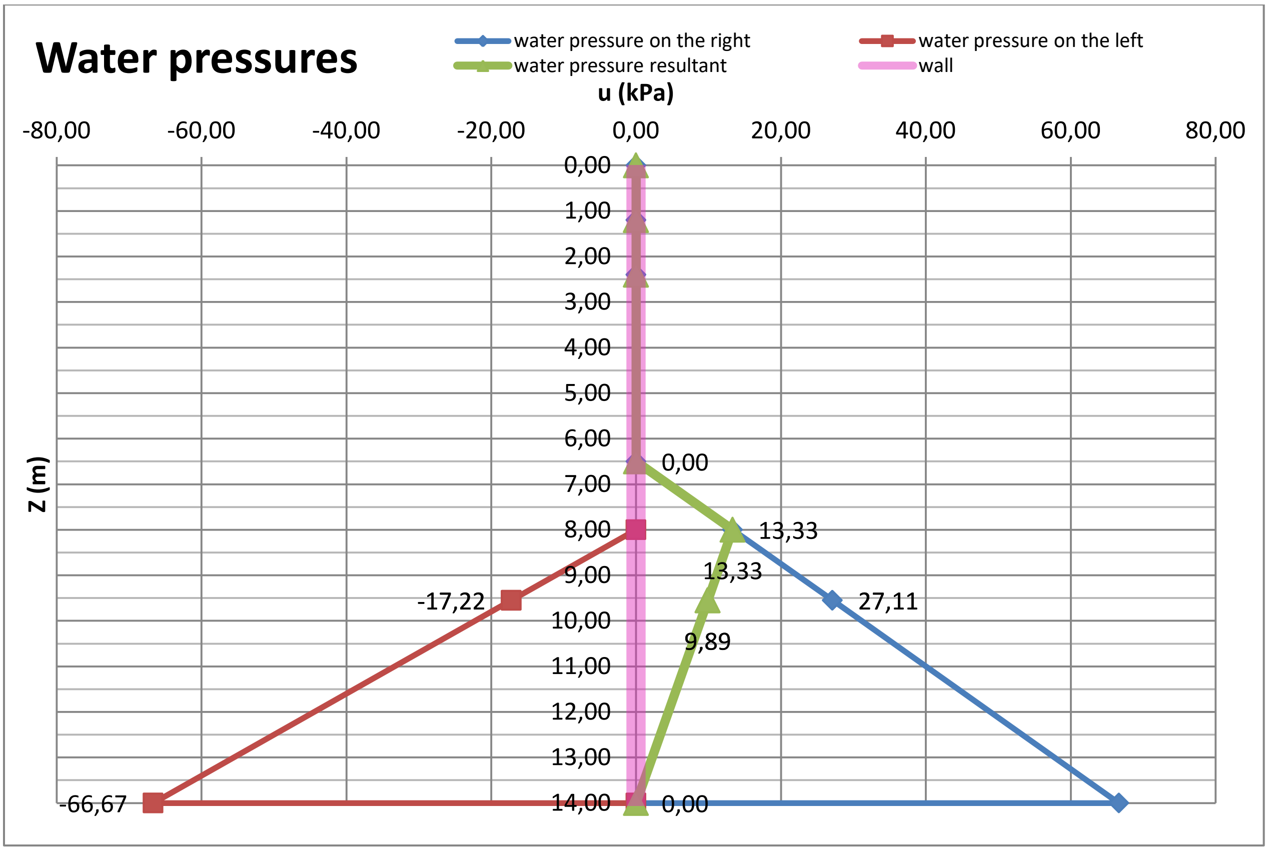
$$\sigma' a_{i-th\ layer} = \sigma q_{i-th\ layer} + Ka_{i-th\ layer} \cdot \sigma' v_{i-th\ layer}$$

$$\sigma' p_{i-th\ layer} = Kp_{i-th\ layer} \cdot \sigma' v_{i-th\ layer}$$

2.1. Loads and arms calculations

Diagrams with the resultant pressures, that have been obtained, are showed below and estimation of the different forces associated to the area that these represents, will be calculated.

- **Water pressures**



$$Q_3 = \sigma q_{of\ the\ \square} \cdot (Z_{\text{"end }\square\text{"}} - Z_{\text{"start }\square\text{"}}) = 5.19\ kPa \cdot (9.55\ m - 2.4\ m) = 37.108 \frac{kN}{m}$$

$$Q_4 = \sigma q_{of\ the\ \square} \cdot (Z_{\text{"}end\ \square\text{"}} - Z_{\text{"}start\ \square\text{"}}) = 5.62\ kPa \cdot (14\ m - 9.55\ m) = 25.010 \frac{kN}{m}$$

- **Surcharge arms with respect to the prop level**

$$Z_{prop} = 1\ m$$

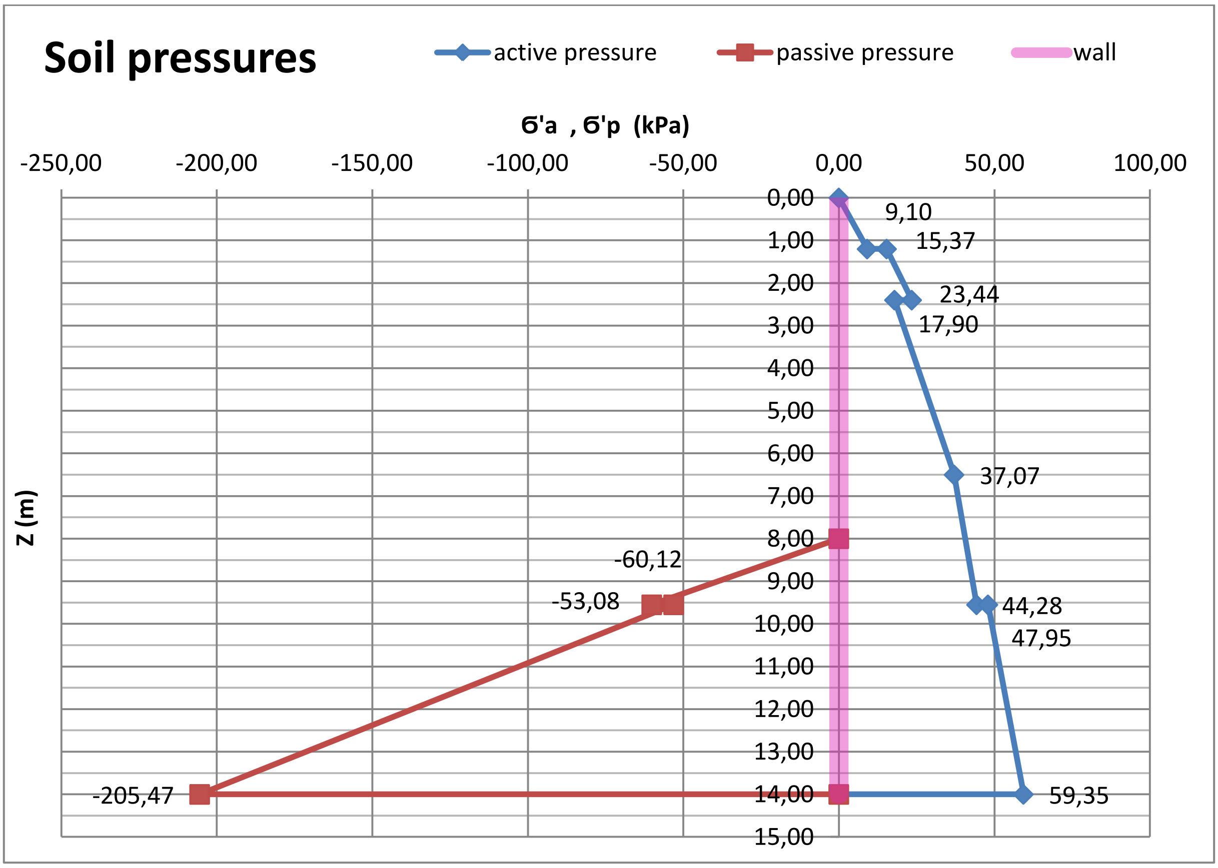
$$b_{Q1} = \frac{1}{2} \cdot (Z_{\text{"}end\ \square\text{"}} - Z_{\text{"}start\ \square\text{"}}) - (Z_{\text{start }\square} - Z_{prop}) = \frac{1}{2} \cdot (1.2 - 0)m + (0 - 1)m = -0.4\ m\ (over\ the\ prop)$$

$$b_{Q2} = \frac{1}{2} \cdot (Z_{\text{"}end\ \square\text{"}} - Z_{\text{"}start\ \square\text{"}}) - (Z_{\text{start }\square} - Z_{prop}) = \frac{1}{2} \cdot (2.4 - 1.2)m + (1.2 - 1)m = 0.8\ m$$

$$b_{Q3} = \frac{1}{2} \cdot (Z_{\text{"}end\ \square\text{"}} - Z_{\text{"}start\ \square\text{"}}) - (Z_{\text{start }\square} - Z_{prop}) = \frac{1}{2} \cdot (9.55 - 2.4)m + (2.4 - 1)m = 4.97\ m$$

$$b_{Q4} = \frac{1}{2} \cdot (Z_{\text{"}end\ \square\text{"}} - Z_{\text{"}start\ \square\text{"}}) - (Z_{\text{start }\square} - Z_{prop}) = \frac{1}{2} \cdot (14 - 9.55)m + (9.55 - 1)m = 10.77\ m$$

- ## Active and passive pressures



This graph refers to the active pressures considering the surcharge and calculated as:

$$\sigma' a_{\,i-th\ layer} = \sigma q_{\,i-th\ layer} + Ka_{i-th\ layer} \cdot \sigma' v_{\,i-th\ layer}$$

$$b_{P7} = \frac{2}{3} \cdot (Z_{"\Delta\, ends"} - Z_{"\Delta\, starts"}) + (Z_{"\Delta\, starts"} - Z_{prop}) = \frac{2}{3} \cdot (9.55 - 6.5)m + (6.5 - 1)m = 7.533\ m$$

$$b_{Pa8} = \frac{1}{2} \cdot (Z_{"end\,\square"} - Z_{"start\,\square"}) + (Z_{\text{start}\,\square} - Z_{prop}) = \frac{1}{2} \cdot (14 - 9.55)m + (9.55 - 1)m = 10.775\ m$$

$$b_{Pa9} = \frac{2}{3} \cdot (Z_{"\Delta\, ends"} - Z_{"\Delta\, starts"}) + (Z_{"\Delta\, starts"} - Z_{prop}) = \frac{2}{3} \cdot (14 - 9.55)m + (9.55 - 1)m = 11.516\ m$$

- **Passive loads**

$$P_{p1} = \frac{1}{2} \cdot (\sigma'_{a,earth})(at\ Z_{base\ of\ \Delta}) \cdot (Z_{"end\ \Delta"} - Z_{"start\ \Delta"}) = \frac{1}{2} \cdot 60.12\ kPa \cdot (1.55 - 0)m = 46.593 \frac{kN}{m}$$

$$P_{p2} = (\sigma'_{a,earth})(at\ Z_{start\ of\ \square}) \cdot (Z_{"end\,\square"} - Z_{"start\,\square"}) = 53.08\ kPa \cdot (6 - 1.55)m = 236.206 \frac{kN}{m}$$

$$P_{p3} = \frac{1}{2} \cdot (\sigma'_{a,earth,\ at\ end} - \sigma'_{a,earth,\ at\ start}) \cdot (Z_{"end\ \Delta"} - Z_{"start\ \Delta"}) = \frac{1}{2} \cdot (205{,}47 - 53.08)kPa \cdot (6 - 1.55)m = 339.067 \frac{kN}{m}$$

- **Passive arms with respect to the prop level**

$$Z_{prop} = 1\ m$$

$$b_{Pp1} = \frac{2}{3} \cdot (Z_{"\Delta\, ends"} - Z_{"\Delta\, starts"}) + (Z_{"\Delta\, starts"} - Z_{prop}) = \frac{2}{3} \cdot (1.55 - 0)m + (8 - 1)m = 8.033\ m$$

$$b_{Pp2} = \frac{1}{2} \cdot (Z_{"end\,\square"} - Z_{"start\,\square"}) + (Z_{\text{start}\,\square} - Z_{prop}) = \frac{1}{2} \cdot (6 - 1.55)m + (9.55 - 1)m = 10.775\ m$$

$$b_{Pp3} = \frac{2}{3} \cdot (Z_{"\Delta\, ends"} - Z_{"\Delta\, starts"}) + (Z_{"\Delta\, starts"} - Z_{prop}) = \frac{2}{3} \cdot (6 - 1.55)m + (9.55 - 1)m = 11.516\ m$$

2.2. Moments

With these forces and arms the Un-stabilizing moments can be calculated making the equilibrium in the prop point.

Type	Name	Load (kN/m)	Arm b (m)	Moment (kNm)	Factor γ	Moment design unstabilizing (kNm)
Active pressure	Pa1	3,04	-0,20	-0,61	1,0	0,00
	Pa2	10,28	0,80	8,23	1,0	8,23
	Pa3	4,84	1,00	4,84	1,0	4,84
	Pa4	52,11	3,45	179,78	1,0	179,78
	Pa5	78,56	4,13	324,67	1,0	324,67
	Pa6	97,20	7,03	682,85	1,0	682,85
	Pa7	22,02	7,53	165,88	1,0	165,88
	Pa8	188,37	10,78	2029,67	1,0	2029,67
	Pa9	50,73	11,52	584,21	1,0	584,21
Water pressure	U1	10,00	6,50	64,99	1,0	64,99
	U2	39,99	9,00	359,91	1,0	359,91
Surcharge pressure	Q1	4,84	-0,40	-1,93	1,3	0,00
	Q2	8,16	0,80	6,53	1,3	8,49
	Q3	37,11	4,97	184,43	1,3	239,75
	Q4	25,01	10,77	269,36	1,3	350,17
	Moment design unstabilizing total (kNm)					5003,43

Taking the values calculated in the previous step:

Point	σ_q (kPa)	Z (m)	u_{static} (kPa)	$u_{dynamic}$ (kPa)	u_{tot} (kPa)	$u_{resultant}$ (kPa)
A	4,03	0,00	0,00	0,00	0,00	0,00
B	4,03	1,20	0,00	0,00	0,00	0,00
B'	6,80	1,20	0,00	0,00	0,00	0,00
C	6,80	2,40	0,00	0,00	0,00	0,00
C'	5,19	2,40	0,00	0,00	0,00	0,00
D	5,19	6,50	0,00	0,00	0,00	0,00
E	5,19	9,55	30,50	-3,39	27,11	9,89
E'	5,62	9,55	30,50	-3,39	27,11	9,89
F	5,62	14,00	75,00	-8,33	66,67	0,00
I	0,00	0,00	0,00	0,00	0,00	0,00
H	0,00	1,55	15,50	1,72	17,22	0,00
H'	0,00	1,55	15,50	1,72	17,22	0,00
G	0,00	6,00	60,00	6,67	66,67	0,00

3.1. Total vertical stress

At the base of the grouting the total pressure is 66.67 kPa.
The total vertical stress is:

$$\sigma v = \gamma_{(un)treated} \cdot Z = 21 \frac{kN}{m^3} \cdot 6m = 126\ kPa$$

The design total vertical stress is:

$$\sigma v_d = \frac{\sigma v}{\gamma_R} = \frac{126\ kPa}{1.45} = 86.896\ kPa$$

The condition that must be satisfied is:

$$\sigma v_d > u_{tot}$$

Which is verified.

Total vertical stress (86.896 kPa) **>** *Total pressure (66.67 kPa) → Total water pressure verified*

3.2. Dynamic water pressure

At the base of the grouting the total pressure is 6.67 kPa.
The effective vertical stress is:

$$\sigma' v = \left(\gamma_{(un)treated} - \gamma_w\right) \cdot Z = (21 - 10) \frac{kN}{m^3} \cdot 6m = 66\ kPa$$

The design total vertical stress is:

$$\sigma' v_d = \frac{\sigma' v}{\gamma_R} = \frac{66\ kPa}{2} = 33\ kPa$$

The condition that must be satisfied is:

$$\sigma v_d > u_{tot}$$

Which is verified.

Total vertical stress (6.67 kPa) **>** *Total pressure (33 kPa) →Dynamic water pressure verified*

Properties of Soil Layers

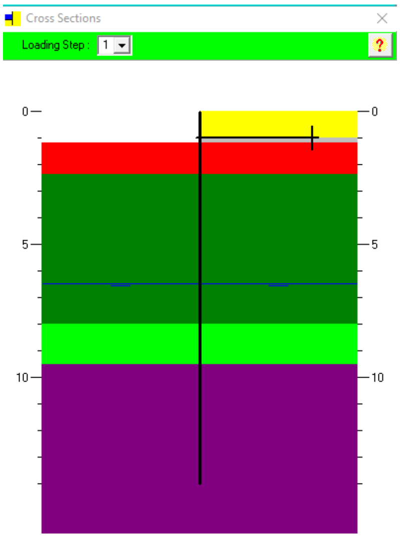
Loading Step : 1 Right Side

No.	Soil Name	H	Wd	Ws	Zw	Cap	q	c	Ka	Kp	Kn	Dw
		m	kN/m³	kN/m³	m	m	kN/m²	kN/m²	--	--	--	m
1	Soil 1	1.000	21.000	21.000	6.500	0.000	26.000	0.000	0.202	8.588	0.376	0.061
2	Soil 2	0.200	21.000	21.000	6.500	0.000	26.000	0.000	0.202	8.588	0.376	0.061
3	Soil 3	1.200	19.790	19.790	6.500	0.000	26.000	0.000	0.340	3.743	1.834	0.061
4	Soil 4	5.600	18.000	18.000	6.500	0.000	26.000	0.000	0.260	5.631	0.457	0.061
5	Soil 5	1.550	18.000	18.000	6.500	0.000	26.000	0.000	0.260	5.631	0.457	0.061
6	Soil 6	8.000	18.000	18.000	6.500	0.000	26.000	0.000	0.282	4.710	0.484	0.061

Properties of Soil Layers

Loading Step : 1 Left Side

No.	Soil Name	H	Wd	Ws	Zw	Cap	q	c	Ka	Kp	Kn	Dw
		m	kN/m³	kN/m³	m	m	kN/m²	kN/m²	--	--	--	m
1	Soil 1	1.000	0.000	10.000	6.500	0.000	0.000	0.000	1.000	1.000	1.000	1.000
2	Soil 2	0.200	0.000	10.000	6.500	0.000	0.000	0.000	1.000	1.000	1.000	1.000
3	Soil 3	1.200	19.790	19.790	6.500	0.000	26.000	0.000	0.340	3.743	1.834	0.061
4	Soil 4	5.600	18.000	18.000	6.500	0.000	26.000	0.000	0.260	5.631	0.457	0.061
5	Soil 5	1.550	18.000	18.000	6.500	0.000	26.000	0.000	0.260	5.631	0.457	0.061
6	Soil 6	8.000	18.000	18.000	6.500	0.000	26.000	0.000	0.282	4.710	0.484	0.061



And finally with the prop located and the excavation in the design depth:

Properties of Soil Layers

Loading Step : 2 Right Side

No.	Soil Name	H	Wd	Ws	Zw	Cap	q	c	Ka	Kp	Kn	Dw
		m	kN/m³	kN/m³	m	m	kN/m²	kN/m²	--	--	--	m
1	Soil 1	1.000	21.000	21.000	6.500	0.000	26.000	0.000	0.202	8.588	0.376	0.061
2	Soil 2	0.200	21.000	21.000	6.500	0.000	26.000	0.000	0.202	8.588	0.376	0.061
3	Soil 3	1.200	19.790	19.790	6.500	0.000	26.000	0.000	0.340	3.743	1.834	0.061
4	Soil 4	5.600	18.000	18.000	6.500	0.000	26.000	0.000	0.260	5.631	0.457	0.061
5	Soil 5	1.550	18.000	18.000	6.500	0.000	26.000	0.000	0.260	5.631	0.457	0.061
6	Soil 6	8.000	18.000	18.000	6.500	0.000	26.000	0.000	0.282	4.710	0.484	0.061

$$K = 939150 \frac{kN}{m}$$

$$Dw = \frac{200000 \frac{kN}{m} \cdot 4\, m}{939150 \frac{kN}{m}} = 0.852\, m$$

The results obtained are:

Output Tables

Loading Step : 1

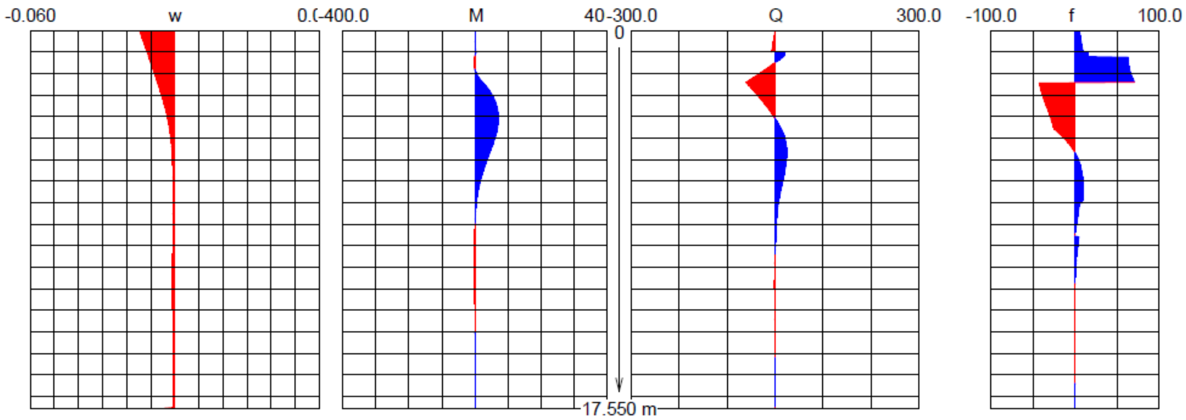
i	z	w	M	Q-	Q+	F	f
	m	m	kNm/m	kN/m	kN/m	kN/m	kN/m²
0	0.000000	-0.014516	0.000000	0.000000	0.000000	0.000000	0.000000
1	0.035714	-0.014406	0.003398	-0.190277	-0.190277	0.000000	-5.327799
2	0.071429	-0.014296	0.013688	-0.385964	-0.385964	0.000000	-5.479126
3	0.107143	-0.014187	0.031063	-0.587063	-0.587063	0.000000	-5.630817
4	0.142857	-0.014077	0.055717	-0.793571	-0.793571	0.000000	-5.782270
5	0.178571	-0.013967	0.087844	-1.005491	-1.005491	0.000000	-5.933807
6	0.214286	-0.013857	0.127635	-1.222821	-1.222821	0.000000	-6.085118
7	0.250000	-0.013747	0.175285	-1.445562	-1.445562	0.000000	-6.236798
8	0.285714	-0.013637	0.230986	-1.673714	-1.673714	0.000000	-6.388307
9	0.321429	-0.013528	0.294932	-1.907277	-1.907277	0.000000	-6.539633
10	0.357143	-0.013418	0.367317	-2.146250	-2.146250	0.000000	-6.691298
11	0.392857	-0.013308	0.448332	-2.390634	-2.390634	0.000000	-6.842807
12	0.428571	-0.013198	0.538173	-2.640429	-2.640429	0.000000	-6.994316
13	0.464286	-0.013089	0.637031	-2.895634	-2.895634	0.000000	-7.145597
14	0.500000	-0.012979	0.745100	-3.156250	-3.156250	0.000000	-7.297306
15	0.535714	-0.012869	0.862574	-3.422277	-3.422277	0.000000	-7.448816
16	0.571429	-0.012759	0.989645	-3.693714	-3.693714	0.000000	-7.600084
17	0.607143	-0.012650	1.126507	-3.970562	-3.970562	0.000000	-7.751806
18	0.642857	-0.012540	1.273354	-4.252821	-4.252821	0.000000	-7.903315
19	0.678571	-0.012430	1.430377	-4.540491	-4.540491	0.000000	-8.054824
20	0.714286	-0.012320	1.597771	-4.833571	-4.833571	0.000000	-8.206076

Output Tables

Loading Step : 2

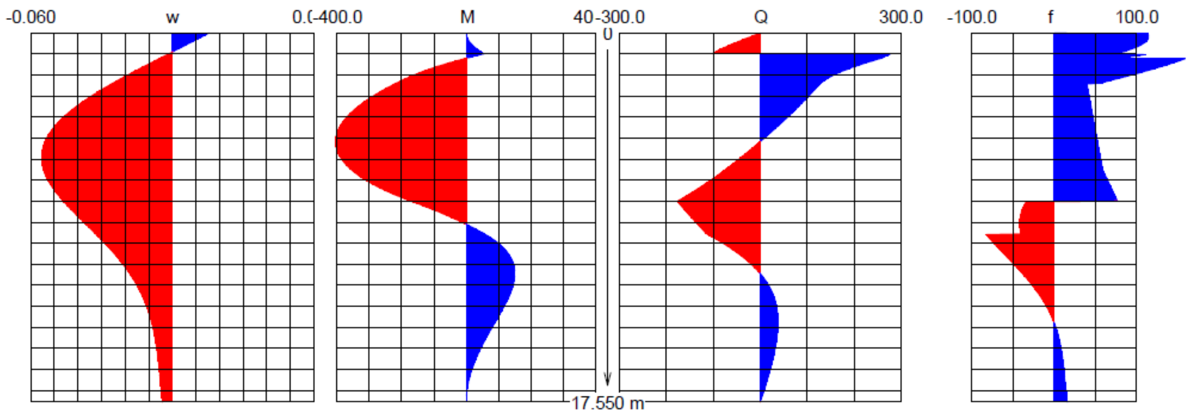
i	z	w	M	Q-	Q+	F	f
	m	m	kNm/m	kN/m	kN/m	kN/m	kN/m²
0	0.000000	0.015670	0.000000	0.000000	0.000000	0.000000	0.000000
1	0.035714	0.015054	0.072363	-4.052312	-4.052312	0.000000	-113.465644
2	0.071429	0.014439	0.289783	-8.123248	-8.123248	0.000000	-113.983928
3	0.107143	0.013823	0.652832	-12.207465	-12.207465	0.000000	-114.358991
4	0.142857	0.013208	1.161887	-16.299620	-16.299620	0.000000	-114.581257
5	0.178571	0.012592	1.817137	-20.394368	-20.394368	0.000000	-114.653861
6	0.214286	0.011976	2.618578	-24.486364	-24.486364	0.000000	-114.573597
7	0.250000	0.011361	3.566018	-28.570259	-28.570259	0.000000	-114.349975
8	0.285714	0.010745	4.659071	-32.640703	-32.640703	0.000000	-113.973344
9	0.321429	0.010129	5.897161	-36.692343	-36.692343	0.000000	-113.443651
10	0.357143	0.009513	7.279521	-40.719821	-40.719821	0.000000	-112.770286
11	0.392857	0.008897	8.805193	-44.717777	-44.717777	0.000000	-111.943664
12	0.428571	0.008281	10.473025	-48.680844	-48.680844	0.000000	-110.966764
13	0.464286	0.007665	12.281677	-52.603651	-52.603651	0.000000	-109.836399
14	0.500000	0.007048	14.229614	-56.480820	-56.480820	0.000000	-108.561600
15	0.535714	0.006431	16.315110	-60.306966	-60.306966	0.000000	-107.132945
16	0.571429	0.005814	18.536247	-64.076695	-64.076695	0.000000	-105.550301
17	0.607143	0.005197	20.890913	-67.784606	-67.784606	0.000000	-103.822339
18	0.642857	0.004579	23.376804	-71.425289	-71.425289	0.000000	-101.939940
19	0.678571	0.003961	25.991422	-74.993320	-74.993320	0.000000	-99.905667
20	0.714286	0.003343	28.732075	-78.483268	-78.483268	0.000000	-97.716590

Results after Step 1 :



Maxima : -0.014516 m 71.381983 kNm/m -60.593949 kN/m 70.762934 kN/m²

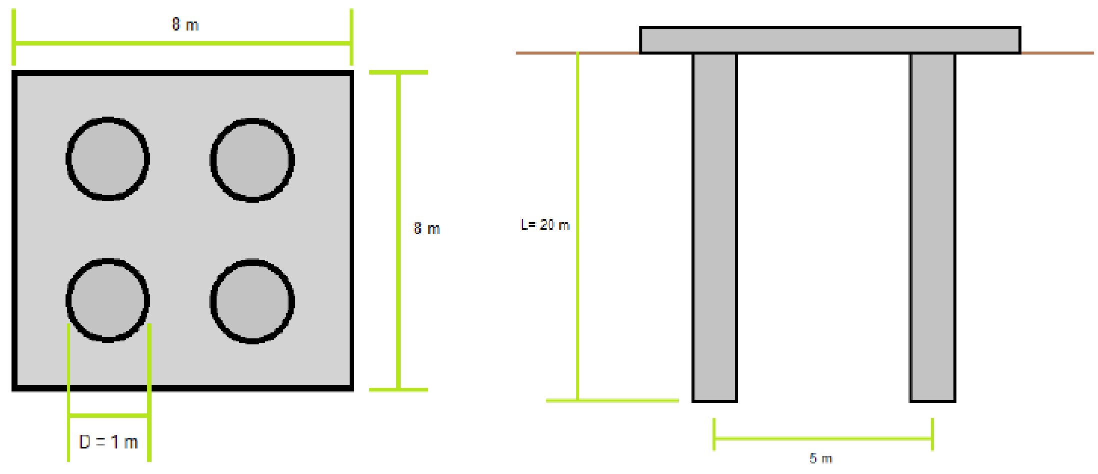
Results after Step 2 :



Maxima : -0.055331 m -404.277094 kNm/m 278.628958 kN/m 158.493823 kN/m²

Homework 4
Piles: settlement and bearing capacity

From the following configuration of a group of piles, calculate the bearing capacity and the settlement as individual one and as the group.



1. Bearing capacity

1.1. Single pile

For the estimation of the solution we use the in situ test CPT (showed in Homework 1), in order to use the values of the tip resistance qc. The final results are in the two tables.
The input data for the calculation is:

Bored piles	
D (m)	1
L (m) pile length	20
nr piles	5
S (m) spacing	5
Lr (m) raft length	8
Ap (m^2) pile area	0,785
Ar (m^2) raft area	64
Su (kPa) undrained shear strength in clay layer	214,852

The α coefficient, for calculating the friction value in clays, was obtained from two authors, but the used value was the S. and Kulhavy:

pa (kPa) atmospheric pressure	101,325
α (S. and Kulhavy)	0,333
α (Skempton)	0,45

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Nr of soil profiles	1	2	3	5
$\xi 3$	1,7	1,65	1,6	**1,5**
$\xi 4$	1,7	1,55	1,48	**1,34**

$R_{c,k}$	In situ test			Analytical
Q (kN)	Average	Minimum	Selected	
Qs	1390,13	1414,055	1390,13	4356,30
Qb	1540,00	1689,399	1540,00	14747,42
Q tot	2930,13	3103,45	**2930,13**	**19103,72**

Where:

$$R_{c,k} = min\left\{\frac{R_{c,k,average}}{\xi 3}; \frac{R_{c,k,minimum}}{\xi 4}\right\}$$

And finally the design resistance is:

Rd (kN)	In situ test	Analytical
base resistance	2170,47	14150,90
shaft resistance	2547,94	16611,93
overall resistance	2253,94	14695,17

Where:

$$R_d = \frac{R_{c,k,tot}}{\gamma_R}$$

1.2. Group of piles

For the estimation of the bearing capacity of the group, we can assume the following equation:

$$P_{group} = n \cdot P_{single,lim} = \begin{cases} P_{g,\ analytical} = n \cdot Q_{single,\ tot} \\ P_{g,\ in\ situ,\ i-th\ test} = n \cdot Q_{single,\ tot,\ in\ situ,\ i-th\ test} \end{cases}$$

Then, as we have 4 piles the final result will be (for both methods):

P_{group} (kN)		
Analytical		76414,881
In situ test	Pg1	18527,026
	Pg2	16634,514

$$E = \frac{68\ MPa + 21.5\ MPa + 79MPa}{3} = 56.20\ MPa$$

$$\bar{G} = \frac{E}{2(1+\nu)} = \frac{56.20\ MPa}{2(1+0.2)} = 23.42\ MPa$$

Then for the contribution of the tip, the G can be obtained directly:

$$G_{base} = \frac{E}{2(1+\nu)} = \frac{79\ MPa}{2(1+0.2)} = 32.92\ MPa$$

Then the Kv is equal to:

$$K_\nu = \frac{P_{single}}{w} = \frac{1}{2}\pi \cdot 20\ m \cdot 23.42\ kPa \cdot 10^3 + \frac{2 \cdot 1\ m \cdot 32.92\ kPa \cdot 10^3}{1-0.2} = 818061 \frac{kN}{m}$$

From the bearing capacity analysis, we can get the values of the overall forces, Rd:

- Analytical approach = 14695,17 kN
- In situ test method = 2253,94 kN
- γ_R = 2.5

Now using:

$$Rd = P_{single} \cdot \gamma_R$$

One can find:

$$P_{single,\ analytical} = \frac{Rd}{\gamma_R} = \frac{14695.17\ \text{kN}}{2.5} = 5878.10\ kN$$

$$P_{single,\ in\ situ\ test} = \frac{Rd}{\gamma_R} = \frac{2253.94\ \text{kN}}{2.5} = 901.57\ kN$$

Now it's possible to obtain the maximum settlement because, assuming that the load comes from the bearing capacity, the pile is mobilizing its maximum capacity.

$$w_{analytical} = \frac{P_{single,\ analytical}}{K_\nu} = \frac{5878.10\ \text{kN}}{818061 \frac{kN}{m}} = 0.00718\ m = 7.18\ mm$$

$$w_{in\ situ\ test} = \frac{P_{single,\ in\ situ\ test}}{K_\nu} = \frac{901.57\ \text{kN}}{818061 \frac{kN}{m}} = 0.00110\ m = 1.10\ mm$$

Then we can say that:

$$\begin{cases} \alpha_{11} \\ \alpha_{22} \\ \alpha_{33} \\ \alpha_{44} \end{cases} = 1$$

$$\alpha_{12} = \alpha_{13} = \alpha_{21} = \alpha_{24} = \alpha_{31} = \alpha_{34} = \alpha_{43} = \alpha_{42}$$
$$\alpha_{14} = \alpha_{23} = \alpha_{41} = \alpha_{32}$$

Then the results are:

$$\alpha_{12} = \frac{Ln\left(\frac{27.3\ m}{5\ m}\right)}{4} = 0.424$$

$$\alpha_{14} = \frac{Ln\left(\frac{27.3\ m}{5\sqrt{2}\ m}\right)}{4} = 0.337$$

The equation of the rigidity for all the piles are:

$$K_v = \frac{n°_{piles} \cdot K_v}{n°_{piles} \cdot (\alpha_{11} + \alpha_{12} + \alpha_{13} + \alpha_{14})} = \frac{4 \cdot 818061\ \frac{kN}{m}}{4 \cdot (1 + 0.424 + 0.424 + 0.337)} = 374398{,}7\ \frac{kN}{m}$$

Finally, the settlement of the group of piles is:

$$w_{analytical} = \frac{n°_{piles} \cdot P_{single,\ \ analytical}}{K_v} = \frac{4 \cdot 5878.10\ \text{kN}}{374398{,}7\ \frac{kN}{m}} = 0.0628\ m = 62.8\ mm$$

$$w_{\ in\ situ\ test} = \frac{n°_{piles} \cdot P_{single,\ \ in\ situ\ test}}{K_v} = \frac{4 \cdot 901.57\ \text{kN}}{374398{,}7\ \frac{kN}{m}} = \ 0.00963\ m = 9.63\ mm$$