NUMERO: **2462A** ANNO: 2020

A P P U N T I

STUDENTE: Chiforeanu Loredana

MATERIA: Earthquake engineering - Part 2 - laboratories project hmw LMC - Prof. Ceravolo

POLITECNICO DI TORINO

Earthquake Engineering

Report of exercises solved during the laboratory lessons

GROUP 14

Students

Loredana Mihaela Chiforeanu

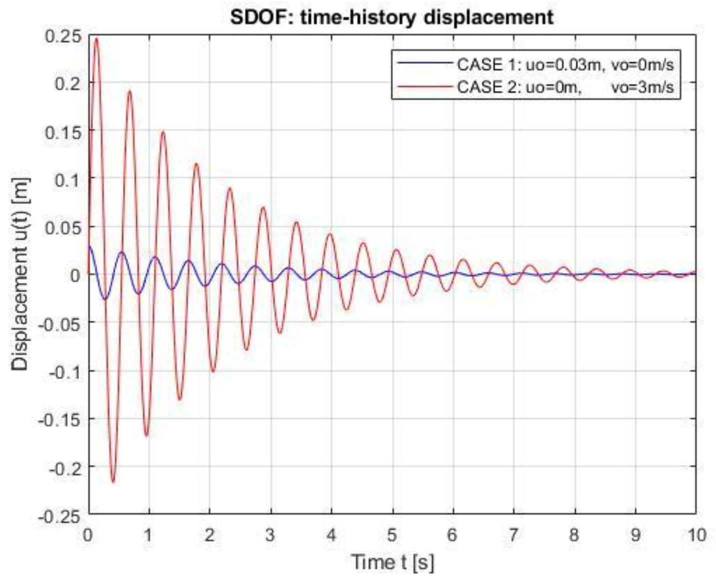
Professors

Rosario Ceravolo

2017/2018

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 5 di 174

Comparison	CASE 1	CASE 2
Stiffness [N/m]	2106996	2106996
Natural circular frequency [rad/s]	11.47551	11.47551
Damped frequency [rad/s]	11.46632	11.46632
Natural frequency [1/s]	1.82638	1.82638
Natural period [s]	0.547530	0.547530
Maximum displacements [m]	0.030000	0.245688

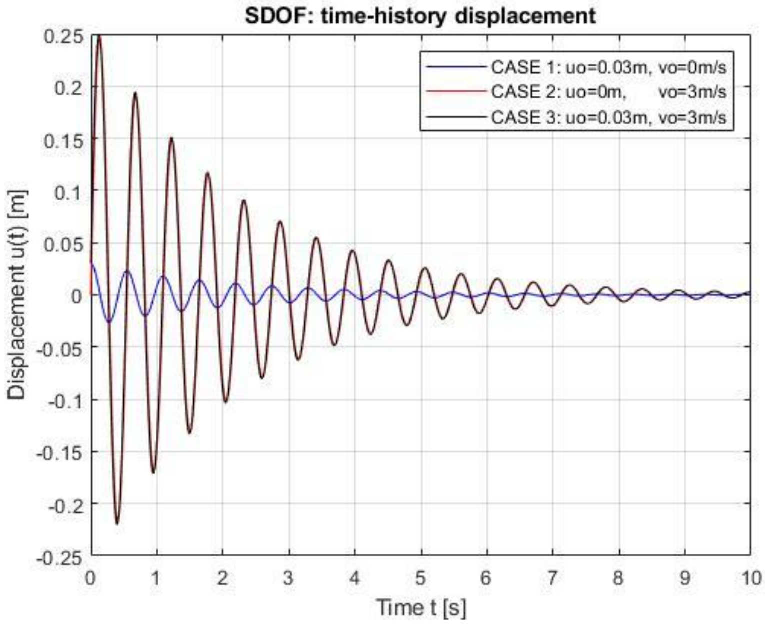


In case 1 (blue graph) it is applied an initial displacement of 3 cm and 0 velocity, so, since the structure is damped, its response vanishes quickly: after about 5 s the displacement u(t) is close to 0.

In case 2 (red graph) it is applied on the same structure an initial velocity of 3 m/s and 0 displacement, so the response initially is very high reaching a maximum displacement of 24.57 cm at t = 0.13 s, after which it tends to decrease due to damping, reaching zero values after 10 s.

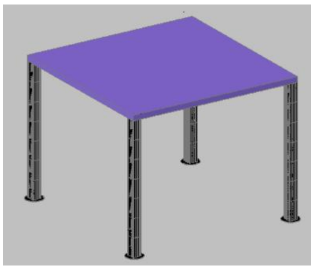
The structure is more affected by the application of an initial velocity.

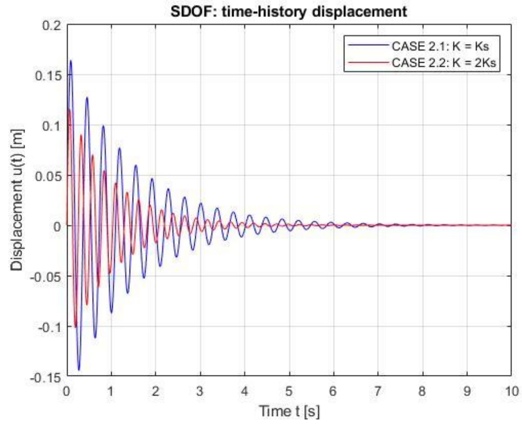
I analysed also a 3rd case (black graph) in which there are both initial displacement and velocity and the response is very close to case 2: the peak is of 24.95 cm, reached at t = 0.12 s.



Far more, we analysed the structure of case 2 having two different stiffnesses but same initial conditions.

CASE 2.1	CASE 2.2
l=4;	l=4;
rho=1000;	rho=1000;
hc=4.5;	hc=4.5;
Jc=3*10^-5;	Jc=3*10^-5;
E=3*10^11	E=3*10^11
nr_col=4;	nr_col=4;
zita=4/100;	zita=4/100
uo=0;	uo=0;
vo=3;	vo=3;
K=Ks	K=2Ks

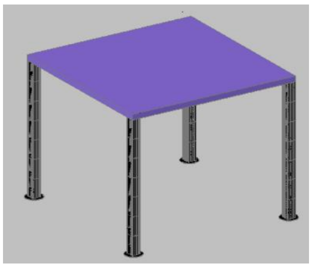


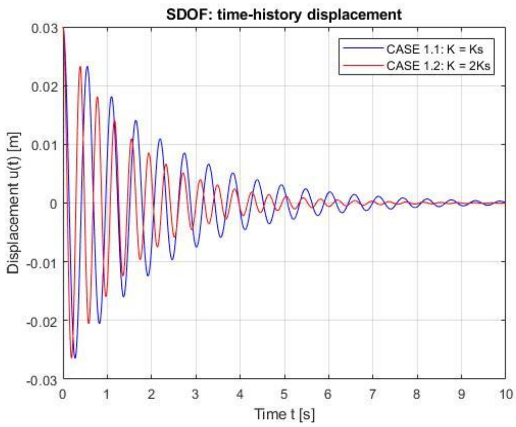


We can see that the stiffer structure (indicated as CASE2.2) has a smaller peak displacement (u=11,56 cm) and its vibration vanishes faster.

On the other hand, we confronted also the structure of case 1 having two different stiffnesses but same initial conditions.

CASE 1.1	CASE 1.2
l=4;	l=4;
rho=1000;	rho=1000;
hc=4.5;	hc=4.5;
Jc=2*10^-5;	Jc=2*10^-5;
E=2*10^11;	E=2*10^11;
nr_col=4;	nr_col=4;
zita=4/100;	zita=4/100;
uo=0.03;	uo=0.03;
vo=0;	vo=0;
K=Ks	K=2Ks





As expected the response vanishes quickly at a higher frequency, as in CASE1.2, due to the increase in stiffness.

The last frequency, much higher than the natural one, makes it possible to represent a well-defined sinusoidal trend that is very similar to the theoretical one.

Nyquist Criterion: It defines the minimum frequency necessary to sample a signal in order to reconstruct it without losing any information and avoiding aliasing. The signal x(t) must have null Fourier transform outside its band width, denoted as $2B_f$.

$$\left(f_s = \frac{1}{\Delta t_s}\right) \geq 2B_f$$

$$f_s = f_{NY} = f_{min}$$

Following the Nyquist principle, the maximum frequency of a measurable signal with a 100 Hz sampling will be equal to:

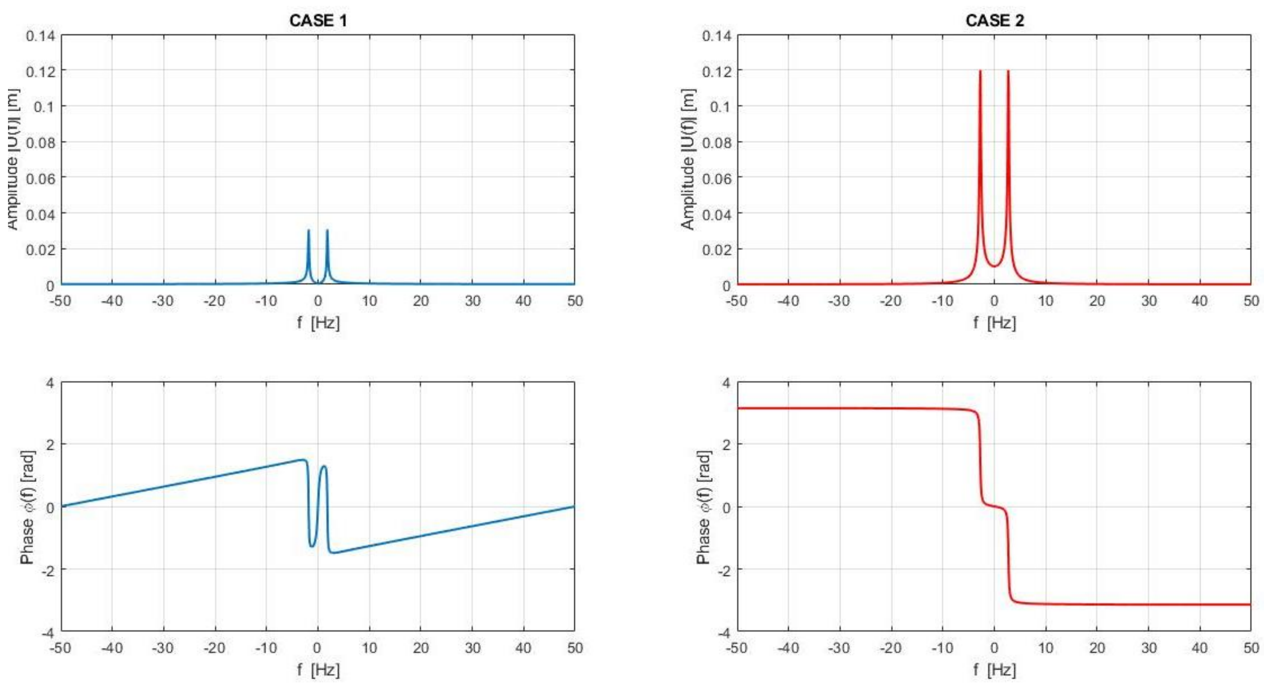
$$B_f = \frac{100}{2} = 50\ Hz$$

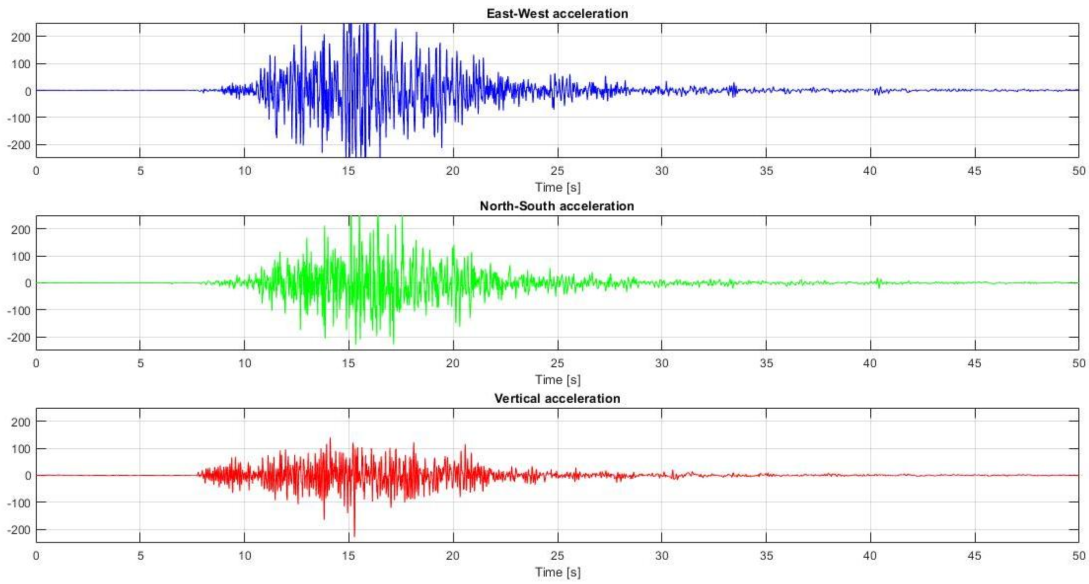
1.3 Fourier transform

The Fourier transform is now applied to the 100 Hz sampled signal related to the first oscillator. We then move on to the frequency domain, useful for determining the frequencies that most stress the structure.

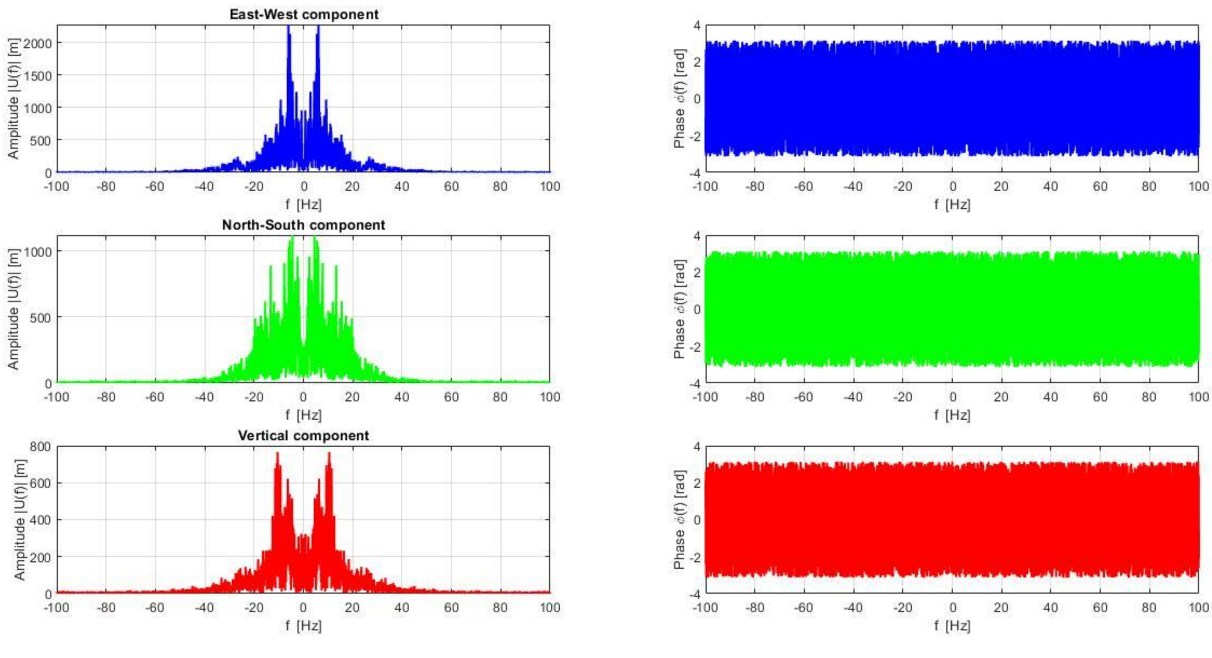
The Fourier transform of a signal is a complex signal in the frequency domain. Moreover the algorithm produces a Fourier transform composed of negative and positive frequency components, but the negative ones are shifted after the positive ones and mirrored, so we solved this inconvenient by applying the *fftshift* command. The FFT must be also scaled using the sampling time.

In this part we did the transform for both cases 1 and 2 (structure of point 1.1). We used a sampling frequency fs=100 Hz and we imposed a frequency resolution for the x-axis of f_{res} = 1/tend =0.1 Hz.



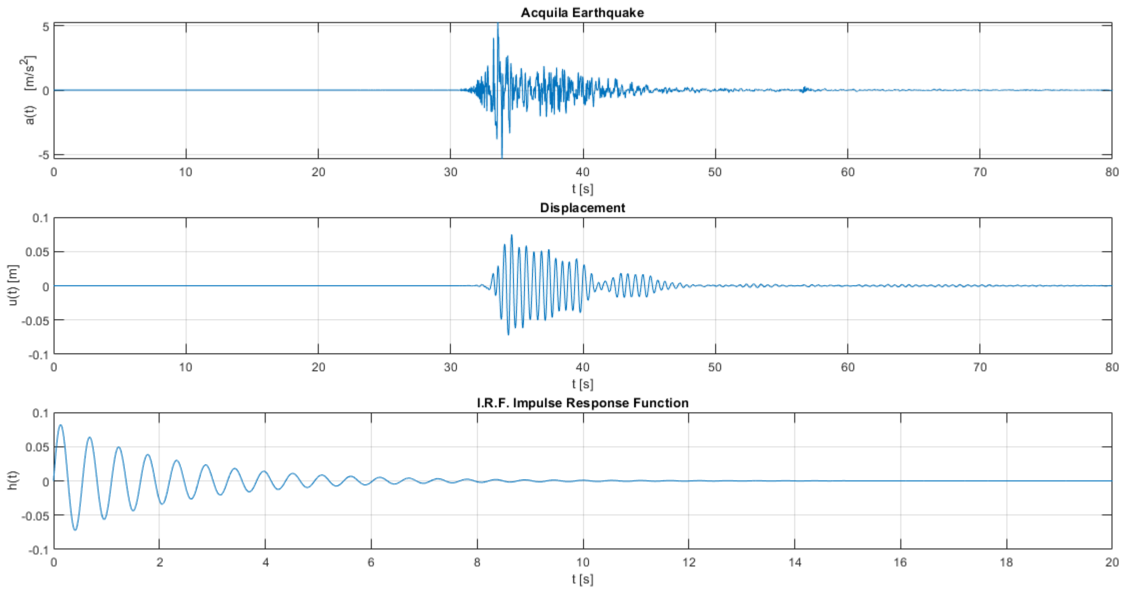


The Fourier transform (as described above) is applied to each component of the signal. Also in this case there is a maximum value for the natural frequency but there are several other peaks unlike what has been seen previously. This is because the signal is not sinusoidal with a single frequency, but aperiodic and composed by many frequencies.



By setting a vector of the times t in which the time step is equal to the sampling time of the given accelerogram ($\Delta t = t_S = 0.01$ s), the impulse response function $h(t_i)$ is calculated for each value t_i. Using the command *conv (ht, At) * ts*, where ht is the vector of the numerical values $h(t_i)$ obtained, and At is the vector of the measured accelerations of the earthquake, we calculate the response.

The graphs of the results obtained are shown below.



The provided accelerogram lasted 100 seconds, for a total of 20001 sampled instants. Applying the convolution, the vector of the displacements in response offers 40002 elements distributed over 200 seconds, part of which is of little interest as occupied by a null response, and for this reason not plotted.

2.2 Frequency response function FRF: numerical and analytical ways

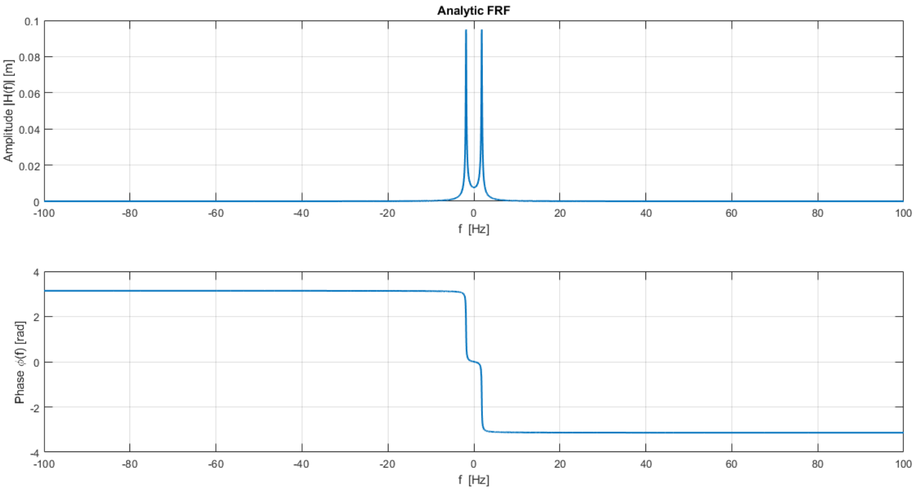
We now aim to calculate the displacements through analysis in the frequency domain, where the convolution becomes just a product:

$$U(f) = A(f) \cdot H(f)$$

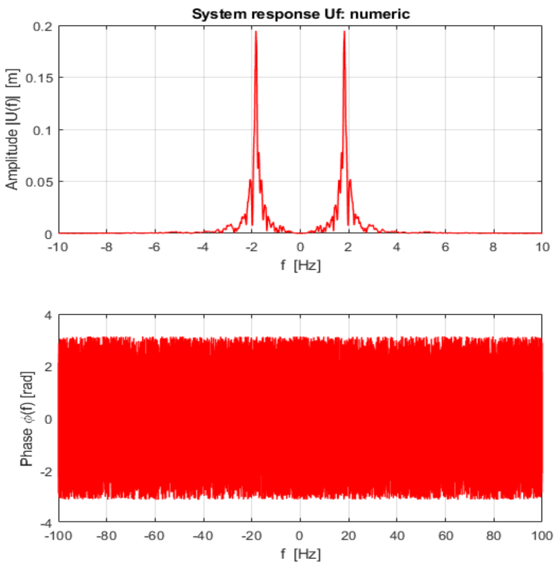
To do this it is necessary to calculate the Frequency Response Function (FRF) and obtain the accelerogram A(t) in the frequency domain by executing the Fourier transform.

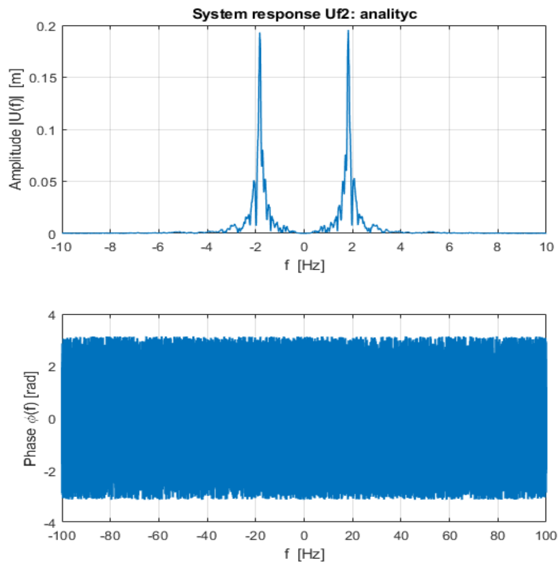
$$R_d\left(\frac{\omega}{\omega_n},\zeta\right)=\frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$\theta\left(\frac{\omega}{\omega_n},\zeta\right)=arctan\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$



The two Frequency Response Functions are practically equal.
To make a comparison between the two procedures, the two U(f) obtained in terms of module and phase were compared graphically. The two resulting graphs are shown below. On the left side there is the result obtained from the numerical procedure (red) and on the right side the one obtained with the analytical calculation (blue). It can be easily seen that the graphs are the same unless for negligible variations due to numerical calculations.
For a better visualization the negligible terms of the amplitude graphs have been not plotted.





the oscillator mass (i.e. on the first floor and not on the ground), accelerations of the ground must be added, element by element, in order to obtain an acceleration value referred to an absolute and non-relative reference system.

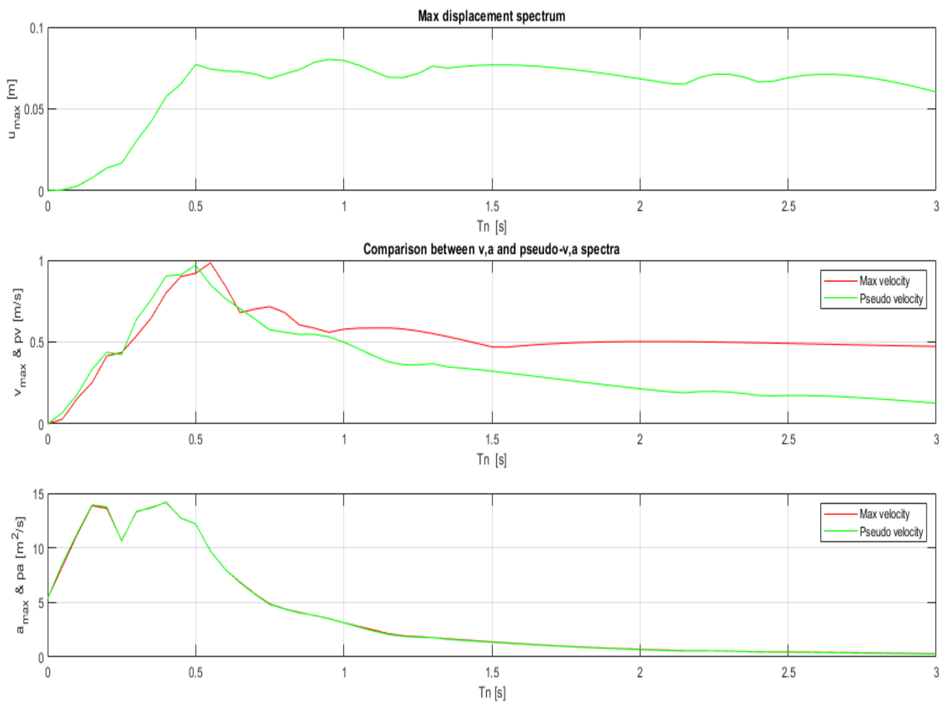
$$\ddot{u}_{absolute} = \ddot{u}_{relative} + \ddot{u}_{ground}$$

The actual values of the velocity and acceleration spectra are obtained by taking again the maximum in absolute value respectively of the velocity and acceleration vector, creating vectors long like Tn.
Then the i-values (where (ii) is the counter for the Tn vector) of the pseudo-velocity and of the pseudo-acceleration are obtained multiplying the i-th value of the displacement spectrum respectively for the natural pulsation and for the squared natural pulsation.

$$pV = \omega_n \cdot D_{max}$$
$$pA = {\omega_n}^2 \cdot D_{max}$$

At the end of the process the values relating to the null period were inserted, because they were not computable in the previous process because it would have involved numerical errors: for the spectrum of displacements, velocity and pseudo-velocities, null values were assumed, representing a situation of initial quietness, while for the spectrum of accelerations and pseudo-accelerations the initial value has been set equal to the maximum absolute value of At, being the maximum ground acceleration or PGA (Peak Ground Acceleration) corresponding to the maximum acceleration of an oscillator with null period.

The results can be seen in the following graphs: in the first one there is the response spectrum of the displacements, in the second the velocity response spectrum together with the pseudo-velocity and in the third the acceleration response spectrum together with the pseudo-acceleration.

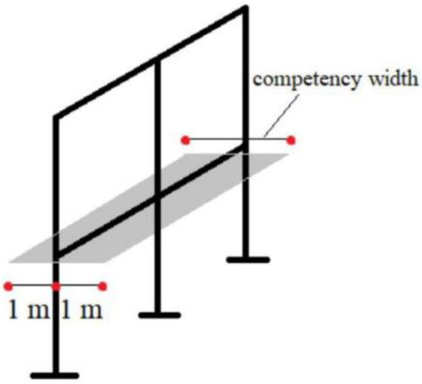


Laboratory 3

In this exercise we consider a more realistic case: we examine a 3D structure made of reinforced concrete with two floors above ground and an under roof.
The frame analysed here is only one and it is taken from the building that is shown in Laboratory 4.
We have to define the matrix of masses and of stiffness to solve the usual 3 DoF system using the Time Hisotry method and the Spectrum Response method.

3.1 Definition of the mass matrix [M]

The mass matrix is determined by calculating for each plane the mass that weighs on an area of influence of 2 m width on the plane perpendicular to the frame. Since the matrix in this Lab is considered diagonal, this means that the assumption of a shear-type frame is used, with the masses concentrated in the floors.



The combination of actions is carried out following the indications of NTC 2008:

$$M = G_1 + G_2 + \sum_j \psi_{2j} \cdot Q_{kj}$$

where G1 it is the permanent dead load, G2 the non structural permanent and Qk the variable loads, all expressed in [Kg]. ψ_{2j} is the combination coefficient for the quasi-permanent condition, indicated in Table 2.5.1 of the NTC and hereinafter reported:

Tabella 2.5.I – *Valori dei coefficienti di combinazione*

Categoria/Azione variabile	ψ_{0j}	ψ_{1j}	ψ_{2j}
Categoria A Ambienti ad uso residenziale	0,7	0,5	0,3
Categoria B Uffici	0,7	0,5	0,3
Categoria C Ambienti suscettibili di affollamento	0,7	0,7	0,6
Categoria D Ambienti ad uso commerciale	0,7	0,7	0,6
Categoria E Biblioteche, archivi, magazzini e ambienti ad uso industriale	1,0	0,9	0,8
Categoria F Rimesse e parcheggi (per autoveicoli di peso ≤ 30 kN)	0,7	0,7	0,6
Categoria G Rimesse e parcheggi (per autoveicoli di peso > 30 kN)	0,7	0,5	0,3
Categoria H Coperture	0,0	0,0	0,0
Vento	0,6	0,2	0,0
Neve (a quota ≤ 1000 m s.l.m.)	0,5	0,2	0,0
Neve (a quota > 1000 m s.l.m.)	0,7	0,5	0,2
Variazioni termiche	0,6	0,5	0,0

In Matlab the resolution is implemented by the *eig* command: it is sufficient to insert the matrices [M] and [K] to obtain the two matrices containing the eigenvalues (lam, square diagonal matrix) and the eigenvectors (sha, square matrix) related to the vibrating modes of the structure.
The elements of both matrices must be ordered in ascending order in order to make sure that for each eigenvalue (pulsation ω^2_k) the correct eigenvector corresponds (modal form ϕ_k).
The number of eigenvalues and eigenvectors is equal to the number of degrees of freedom of the structure.

$$\text{sha} = \begin{bmatrix} \{\phi_1\} & \{\phi_2\} & \{\phi_3\} \end{bmatrix} \qquad \text{lam} = \begin{bmatrix} \omega_1^2 & 0 & 0 \\ 0 & \omega_2^2 & 0 \\ 0 & 0 & \omega_3^2 \end{bmatrix}$$

This means that they can be normalized with the criterion that says that the normalization is chosen with respect to the mass matrix.
We can verify that the program returns the already normalized eigenvectors by checking the product $\{sha\}^T \cdot [m] \cdot \{sha\}$ results an identity matrix. If this does not happen it is necessary to divide each eigenvector by a coefficient like $c_k = \sqrt{\{\phi\}_k^T \cdot [m] \cdot \{\phi\}_k}$ obtaining normalized eigenvectors
$\{U\}_k = \frac{\{\phi\}_k}{c_k}$.

[check_sha]		
1	-1,166E-16	-1,391E-16
-7,069E-17	1	1,569E-16
-2,769E-17	1,503E-16	1

We can see that unless for negligible values the test gives already an identity matrix, but it is usual to write always the code for normalization to be sure every time; we get:

[U]		
1,392E-03	3,148E-03	-2,539E-03
2,921E-03	1,074E-03	2,934E-03
3,698E-03	-3,555E-03	-2,380E-03

[check_U]		
1	-1,166E-16	-1,391E-16
-7,069E-17	1	1,569E-16
-2,769E-17	1,503E-16	1

The eigenvectors can therefore be graphically represented, giving an idea of the 3 different ways of vibrating.

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 23 di 174

This factor describes the importance of each mass (or degree of freedom) of the structure in the global behaviour under excitation. In particular, we have the following factors:

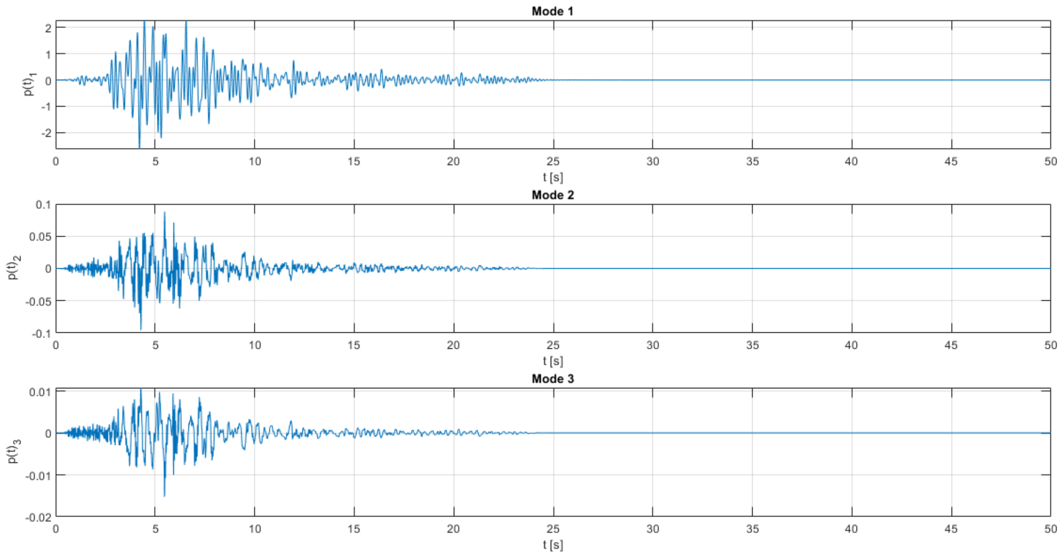
Modes	Γ_k	**T_k [s]**
Mode 1	351,399	0,184
Mode 2	119,592	0,063
Mode 3	-52,853	0,042

Far more also the diagonal damping matrix is calculated and it is:

$[2\varsigma\omega_k]$		
3,421E+00	0	0
0	1,001E+01	0
0	0	1,489E+01

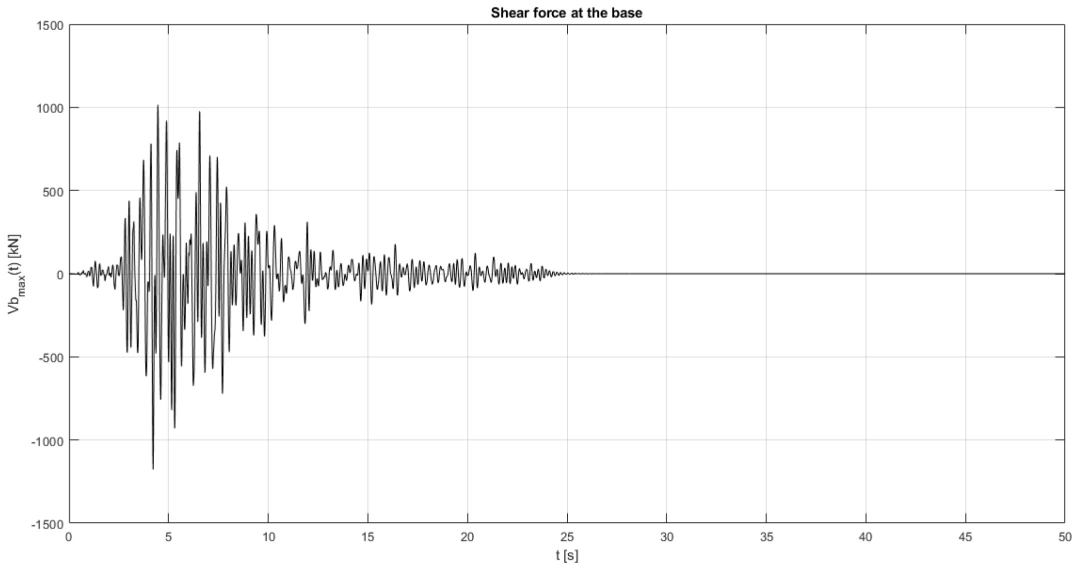
Then with the Duhamel integral (or convolution) the solution is determined (as in Laboratory 2):

$$p_k(t) = h_k(t) * (-\Gamma_k \cdot \ddot{u}_g(t))$$



From the modal coordinates, one can then return to the geometric coordinates (displacements) by pre-multiplying the normalized eigenvectors for the matrix:

$$\{u(t)\} = [U] \cdot \{p(t)\}$$



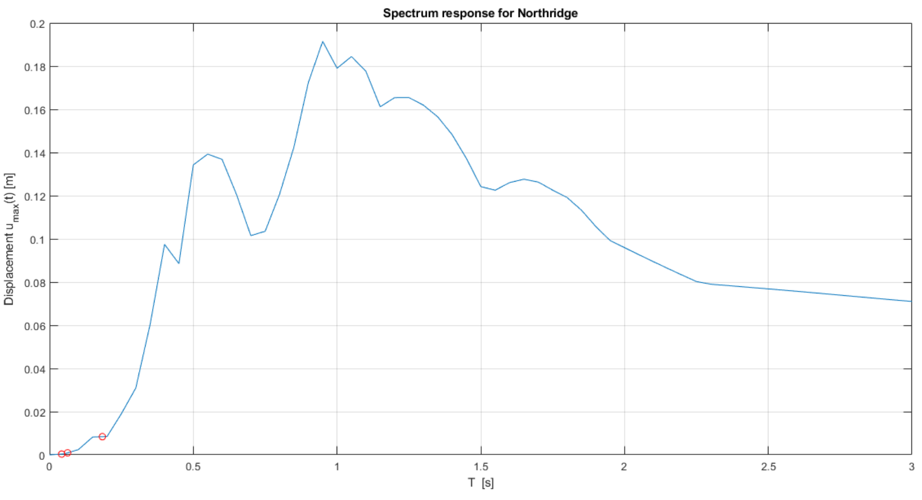
Its maximum absolute value is equal to 1177,05 kN.

3.5 Spectrum response method

Using the response spectrum determined in Laboratory 2, the maximum displacements expressed in the modal coordinates are obtained:

$$p_{k,max} = \Gamma_k \cdot D_k$$

where D_k is the spectral displacement corresponding to the period T_k, calculated from the pulsation ω_k.
The *interp1* command was used to determine D_k as a function of T_k on the spectrum.



As done in Time History, geometric displacements and equivalent static forces are determined. It is important to work independently to achieve a distinct structural response for each mode. Then the combination is carried out according to a statistical criterion: the NTC 2008, describes two methods for combining the effects of the different modes.

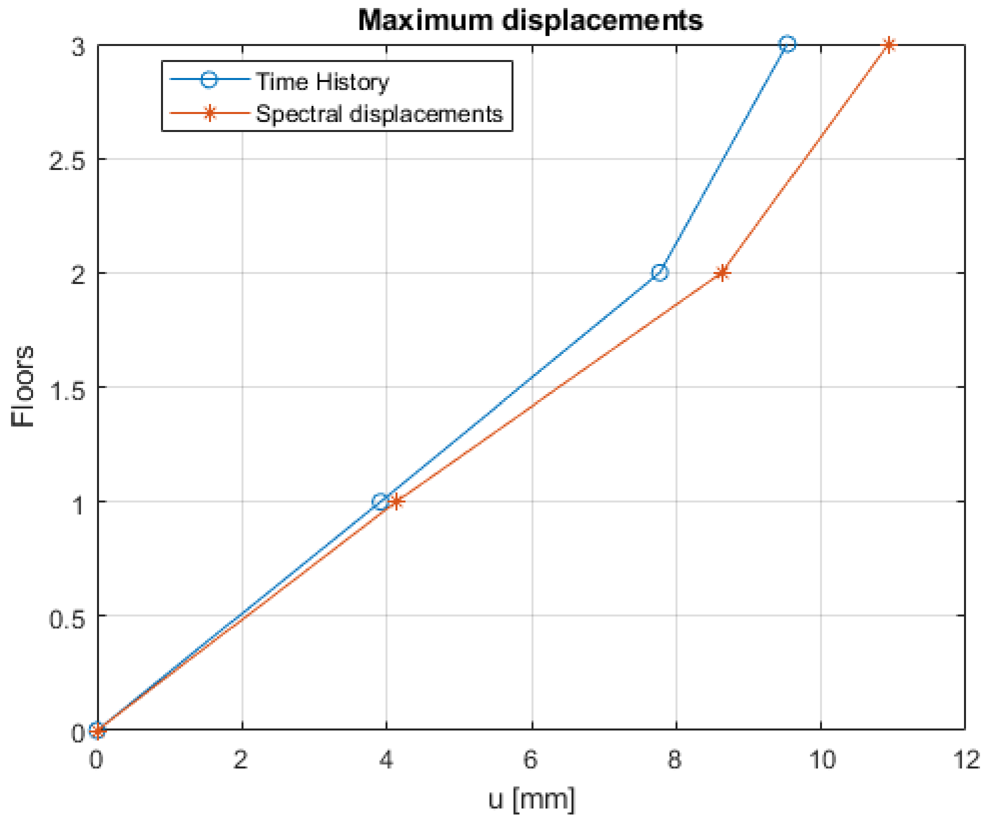
Floor	$\{p\}_{k,max}$ **[m]**	$\{f\}_{m1,max}$ **[N]**	$\{f\}_{m2,max}$ **[N]**	$\{f\}_{m3,max}$ **[N]**
1	2,954	2,632E+05	1,900E+05	5,728E+04
2	0,110	5,522E+05	6,484E+04	-6,618E+04
3	-0,019	3,999E+05	-1,228E+05	3,072E+04
TOT	$\{f\}_{mk,max}$ **[N]**	1,215E+06	1,321E+05	2,181E+04

The floor forces are added for each mode, obtaining the shear at the maximum base for the single mode. The combination gives:

$$V_{b,\max} = \sqrt{V_b^2(m_1) + V_b^2(m_2) + V_b^2(m_3)} = 1{,}223\text{E} + 06\ N \approx 1222{,}552\ KN$$

It can be noticed that the stress calculated with Time History (1177,05 KN) is smaller and therefore it is not in favour of safety.

Finally we can compare the maximum displacements obtained with the two methods:



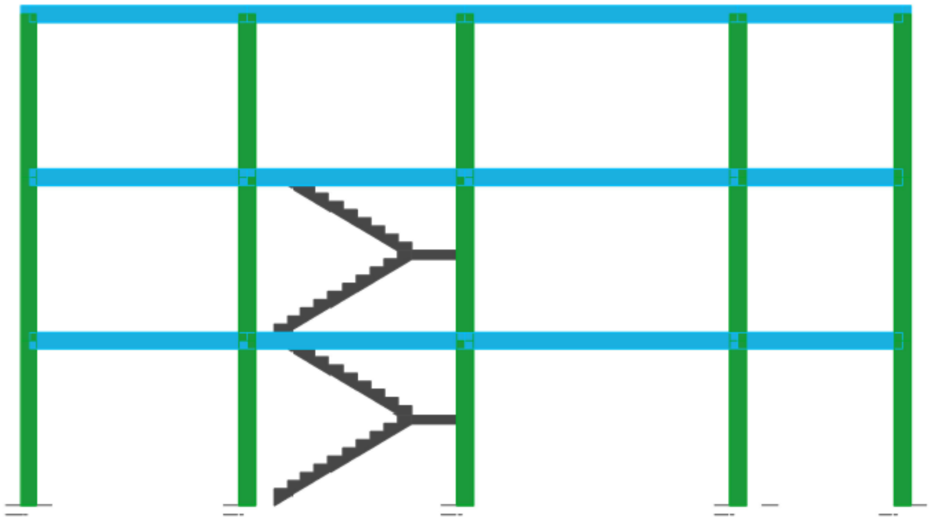


Figure 3 - Frame JJ'

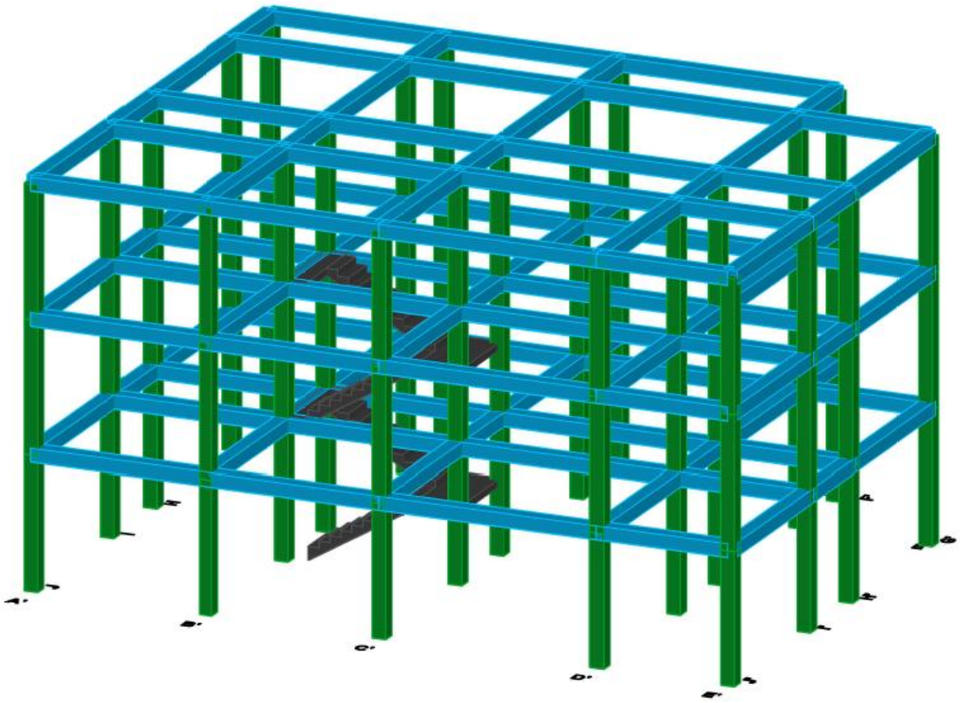


Figure 4 – 3D Frame

The seismic analysis of a building consists in determining the non-damped free oscillations of the structure by solving the system:

$$[M]\{\ddot{q}\} + [H]\{q\} = \{0\}$$

Where {q} represents the vector of displacements with length equal to the degrees of freedom, in fact, having indicated with u_i the transverse displacement in the x direction of the i-th floor, with v_i the transverse displacement in y direction of the i-th floor and with γ_i the rotation around a vertical axis passing through the pole O of the i-th floor, the vector {q} and the accelerations have the following form:

DIVIDERS		
Material	**thickness [m]**	**γ [kN/m³]**
hollow bricks	0.12	8
plaster	0.02	20
total thickness	0.14	
wall height	3	
Total linear weight	4.08 kN/m	
Total areal weight	1.20 kN/m²	

EXTERNAL WALLS		
Material	**thickness [m]**	**γ [kN/m³]**
bricks	0.28	21
plaster	0.02	20
total thickness	0.30	
wall height	3	
Total linear weight	18.84 kN/m	

We have also to consider also the masses coming from the structural elements such as beams, inside the floor, and columns both having section 30 x 30 cm.

BEAMS		**COLUMNS**	
b [m]	0.3	b [m]	0.3
h [m]	0.3	h [m]	0.3
		H [m]	3
γ [kN/m³]	25	γ [kN/m³]	25
Total linear weight	2.25 kN/m	Total weight (24 columns)	162 kN

The roofing of the building consists, for simplicity, of tiles.
The roof is symmetrical and plane, with dimensions equal to the previous Prèdalles slab. The non structural but permanent dead load G2 is equal to 0.6 kN/m².

Finally we consider the stairwell, which leads from the ground floor to the second floor.

STAIRS			
Material	**γ [kN/m³]**	**H [m]**	**G1 [kN/m²]**
reinforced concrete	25	0.1	2.5
TOT G1			**2.5**
			G2 [kN/m²]
plaster	20	0.015	0.3
steps of wood	12	0.20	2.4
TOT G2			**2.7**

Knowing the values of the loads, we moved to determine the mass matrix. The values presented here will be rescaled from [kN/m²] to measurements in [kg].

To evaluate the combination of the loads in a seismic region we referred to the NTC equation (section 3.2.4, Eq. 3.2.17).

$[M_{x\gamma}] = [M_{\gamma x}]$		
-2,5524E+06	0	0
0	-2,5524E+06	0
0	0	-1,1572E+06

$$[M_{y\gamma}] = [M_{\gamma y}] = \begin{bmatrix} m_1 \cdot X_{G,1} & 0 & 0 \\ 0 & m_2 \cdot X_{G,2} & 0 \\ 0 & 0 & m_3 \cdot X_{G,3} \end{bmatrix}$$

$[\mathrm{M}_{y\gamma}] = [\mathrm{M}_{\gamma y}]$		
-3,3067E+06	0	0
0	-3,3067E+06	0
0	0	-1,4943E+06

Finally $J_{O,i}$ is the polar moment of inertia of the i-th plane with respect to the pole O.

$$[M_{\gamma\gamma}] = \begin{bmatrix} J_{O,1} & 0 & 0 \\ 0 & J_{O,2} & 0 \\ 0 & 0 & J_{O,3} \end{bmatrix}$$

$[\mathrm{M}_{\gamma\gamma}]$		
5,9822E+07	0	0
0	5,9822E+07	0
0	0	2,6663E+07

It is therefore possible to proceed with assembly of the total mass matrix:

$$[M] = \begin{bmatrix} [M_{xx}] & [0] & [M_{x\gamma}] \\ [0] & [M_{yy}] & [M_{y\gamma}] \\ [M_{\gamma x}] & [M_{\gamma y}] & [M_{\gamma\gamma}] \end{bmatrix}$$

[M]								
3,8080E+05	0	0	0	0	0	-2,5524E+06	0	0
0	3,8080E+05	0	0	0	0	0	-2,5524E+06	0
0	0	1,7380E+05	0	0	0	0	0	-1,1572E+06
0	0	0	3,8080E+05	0	0	-3,3067E+06	0	0
0	0	0	0	3,8080E+05	0	0	-3,3067E+06	0
0	0	0	0	0	1,7380E+05	0	0	-1,4943E+06
-2,5524E+06	0	0	-3,3067E+06	0	0	5,9822E+07	0	0
0	-2,5524E+06	0	0	-3,3067E+06	0	0	5,9822E+07	0
0	0	-1,1572E+06	0	0	-1,4943E+06	0	0	2,6663E+07

Starting from these matrices the components of the global stiffness matrices are assembled following these steps:

$[H_{xx}] = \sum_i [H_{xx}]_i$

$[H_{xx}]$		
3,3144E+09	-1,7076E+09	2,3557E+08
-1,7076E+09	2,7737E+09	-1,3423E+09
2,3557E+08	-1,3423E+09	1,1226E+09

$[H_{yy}] = \sum_j [H_{yy}]_j$

$[H_{yy}]$		
3,5014E+09	-1,8175E+09	2,3538E+08
-1,8175E+09	3,0296E+09	-1,4710E+09
2,3538E+08	-1,4710E+09	1,2458E+09

$[H_{x\gamma}] = [H_{\gamma x}] = -\sum_i [H_{xx}]_i \cdot Y_i$

$[H_{x\gamma}]$		
-2,1243E+10	1,1013E+10	-1,5687E+09
1,1013E+10	-1,7732E+10	8,5658E+09
-1,5687E+09	8,5658E+09	-7,1144E+09

$[H_{y\gamma}] = [H_{\gamma y}] = \sum_j [H_{yy}]_j \cdot X_j$

$[H_{y\gamma}]$		
2,7622E+10	-1,4389E+10	1,9172E+09
-1,4389E+10	2,3766E+10	-1,1521E+10
1,9172E+09	-1,1521E+10	9,7102E+09

$[H_{\gamma\gamma}] = \left(\sum_j [H_{yy}]_j \cdot X_j^2\right) + \left(\sum_i [H_{xx}]_i \cdot Y_i^2\right)$

$[H_{\gamma\gamma}]$		
5,3182E+11	-2,7737E+11	3,8640E+10
-2,7737E+11	4,5112E+11	-2,1816E+11
3,8640E+10	-2,1816E+11	1,8221E+11

It is now possible to proceed with the assembly of the global stiffness matrix:

$$[H] = \begin{bmatrix} [H_{xx}] & [0] & [H_{x\gamma}] \\ [0] & [H_{yy}] & [H_{y\gamma}] \\ [H_{\gamma y}] & [H_{\gamma y}] & [H_{\gamma\gamma}] \end{bmatrix}$$

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 37 di 174

The normalization of the matrix [sha] was carried out automatically according to the following formula (normalization with respect to the mass matrix):

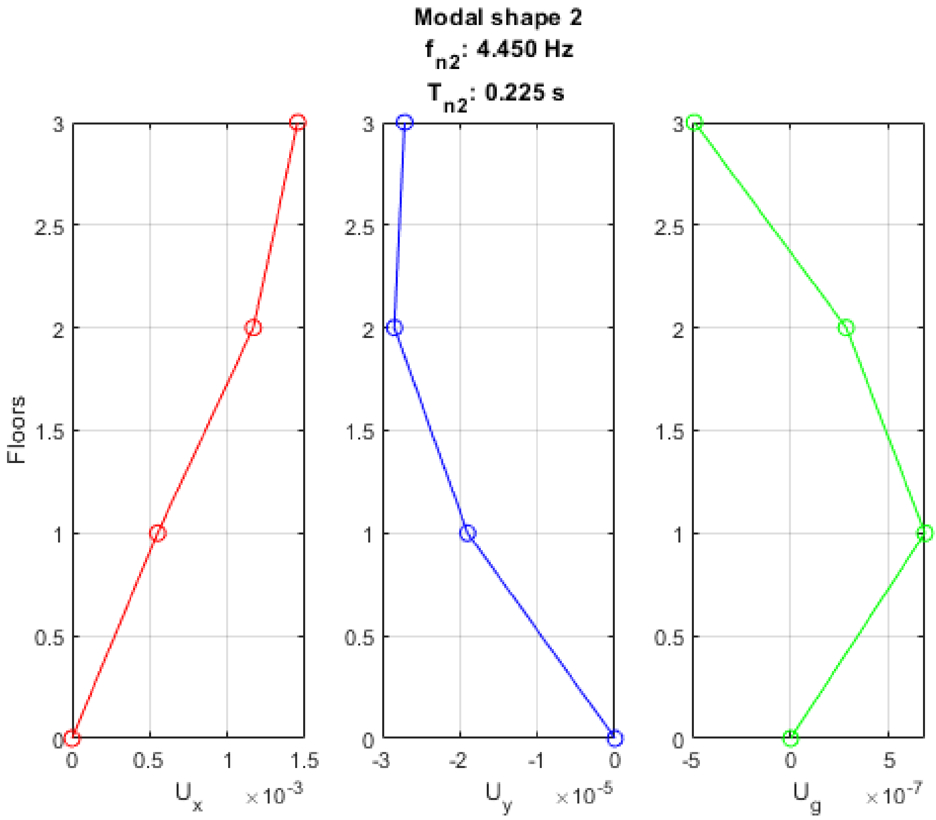
$$\{U\}_K = \frac{\{\phi\}_k}{\sqrt{\{\phi\}_k^T \, [M] \, \{\phi\}_k}}$$

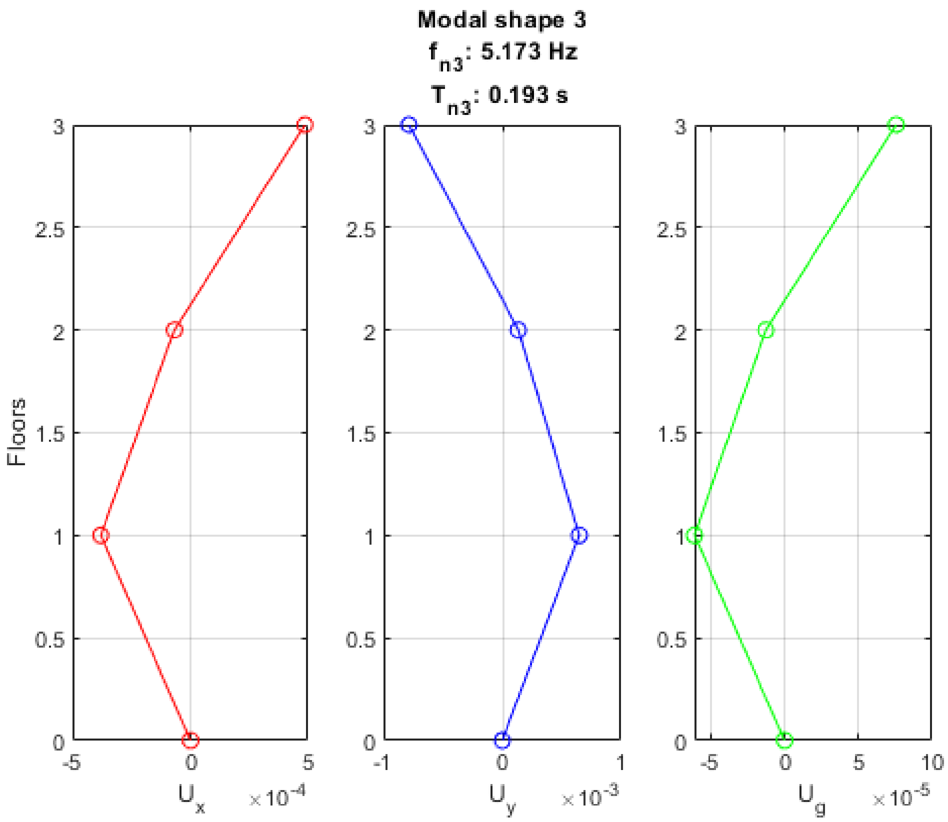
Where $\{U\}_K$ is the column vector coinciding with the k-th column of [sha] and is therefore the normalized eigenvector corresponding to the k-th mode of vibrating; $\{\phi\}_k$ is the k-th autovector of the non-normalized system.

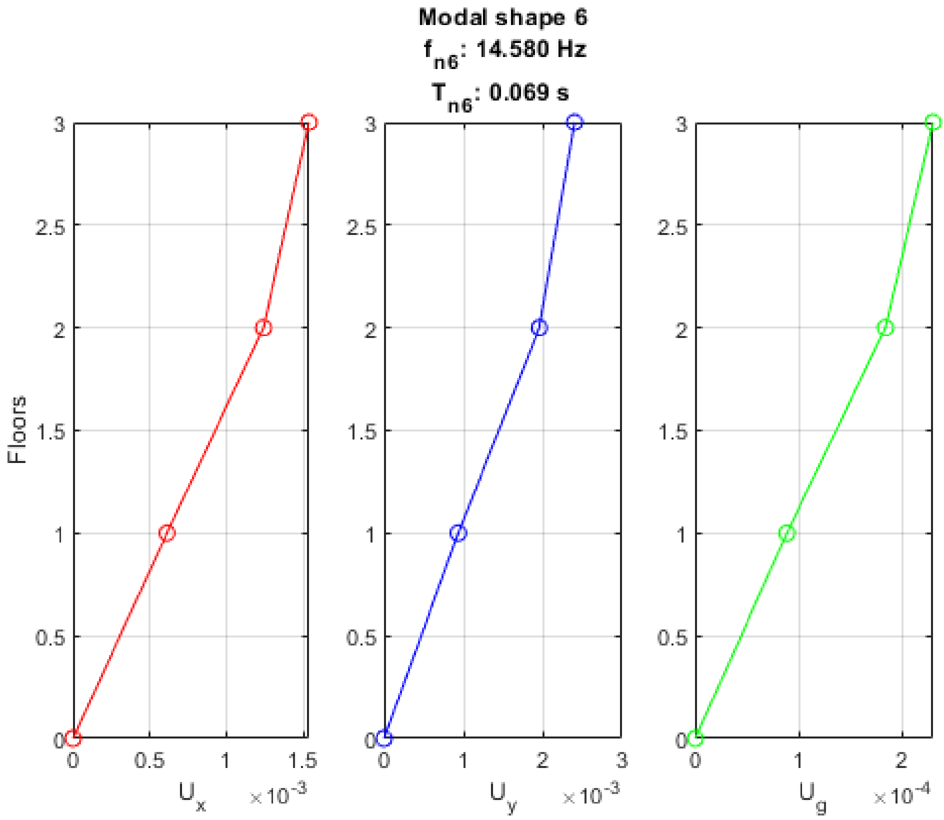
[U]								
U_1	**U_2**	**U_3**	**U_4**	**U_5**	**U_6**	**U_7**	**U_8**	**U_9**
1,7282E-04	5,5054E-04	-3,7993E-04	2,7776E-04	1,2160E-03	6,1207E-04	9,1302E-04	-1,3163E-03	9,2795E-04
3,6181E-04	1,1676E-03	-6,7509E-05	-3,5039E-04	2,8434E-04	1,2400E-03	-1,0924E-03	-2,5251E-04	-1,1567E-03
4,4880E-04	1,4525E-03	4,8595E-04	3,6414E-04	-1,5303E-03	1,5359E-03	1,1475E-03	1,6170E-03	1,3015E-03
-2,9080E-04	-1,8981E-05	6,4791E-04	-4,7258E-04	-3,0488E-05	9,3495E-04	-1,8393E-05	-2,0687E-03	1,4590E-03
-6,0758E-04	-2,8400E-05	1,3594E-04	5,6276E-04	-2,2763E-05	1,9587E-03	1,0125E-05	-3,9807E-04	-1,8195E-03
-7,5406E-04	-2,7082E-05	-7,8506E-04	-6,3521E-04	1,4050E-05	2,4019E-03	-2,9473E-05	2,5230E-03	2,0304E-03
2,7742E-05	6,8060E-07	-6,1132E-05	4,3179E-05	-6,2598E-07	8,8811E-05	-3,2910E-07	-1,9534E-04	1,3786E-04
5,8260E-05	2,8172E-07	-1,2574E-05	-5,5028E-05	-1,2355E-06	1,8419E-04	-9,0555E-07	-3,7449E-05	-1,7179E-04
7,2398E-05	-4,8790E-07	7,5627E-05	5,8326E-05	-2,0089E-06	2,3029E-04	-9,3317E-07	2,4155E-04	1,9460E-04

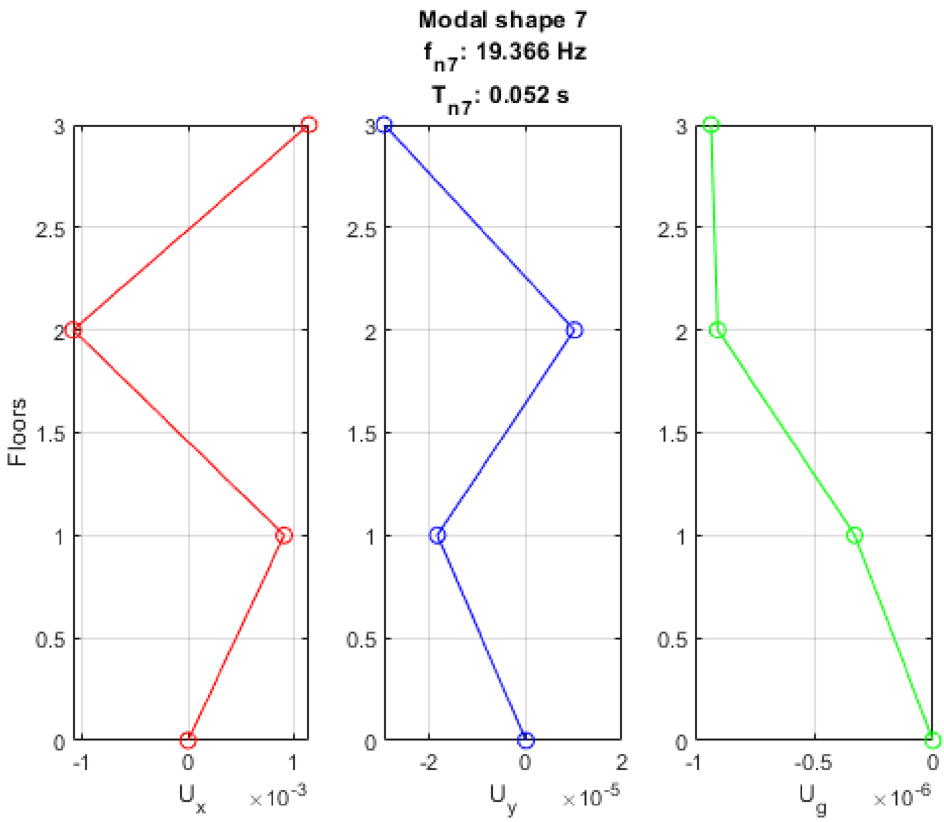
To verify the relation, it has been calculated that the product ($[sha]^T$ [M] [sha]) was equal to the identity matrix, except for negligible errors, as can be verified below.

[I]								
1	-5,2964E-18	4,7609E-17	-1,8132E-16	1,2103E-16	3,7360E-16	-1,3898E-16	-9,9189E-17	-1,7969E-17
-6,0232E-17	1	-4,0167E-18	9,5565E-17	1,1690E-16	-1,7823E-16	1,1442E-16	-9,2869E-18	3,2033E-17
6,6080E-17	-2,8600E-17	1	-1,3633E-16	-3,8273E-17	4,5571E-16	-2,8547E-17	-1,3469E-16	1,4382E-16
-1,4464E-16	6,1185E-17	-1,1702E-16	1	-1,1928E-16	4,0190E-16	5,6262E-17	5,0637E-17	9,1551E-17
1,2067E-16	1,2206E-16	-1,1070E-16	-1,0788E-16	1	-2,4405E-17	4,8353E-17	-1,8034E-16	8,2366E-17
4,6026E-16	-1,8510E-16	4,9197E-16	2,2915E-16	-4,3871E-17	1	2,6297E-16	-1,1995E-17	-1,5183E-16
-1,0126E-16	1,0116E-16	1,1432E-18	-5,7945E-17	9,3433E-18	4,1567E-16	1	-2,5311E-17	-2,6081E-16
-2,1110E-17	-2,6003E-17	-1,1983E-16	4,8522E-17	-2,4915E-17	-1,0062E-16	-5,9755E-17	1	-1,3318E-16
7,5438E-17	1,3291E-17	1,4281E-16	2,0839E-17	3,7627E-17	-3,4338E-16	-2,0783E-16	-4,3477E-17	1


Modal shape 2
f n2: 4.450 Hz
T n2: 0.225 s
Floors
U x ×10^-3
U y ×10^-5
U g ×10^-7

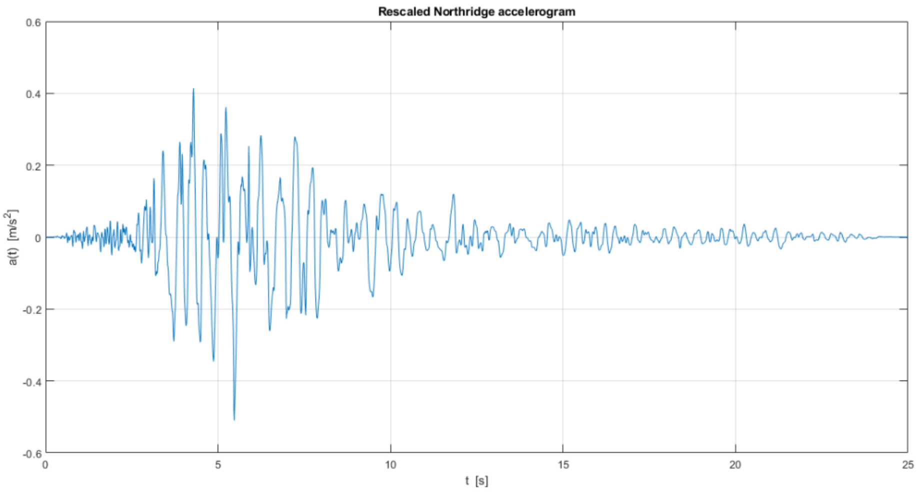

Modal shape 3
f n3: 5.173 Hz
T n3: 0.193 s
Floors
U x ×10^-4
U y ×10^-3
U g ×10^-5


Modal shape 6
f_n6: 14.580 Hz
T_n6: 0.069 s
Floors
U_x ×10^-3
U_y ×10^-3
U_g ×10^-4


Modal shape 7
f_n7: 19.366 Hz
T_n7: 0.052 s
Floors
U_x ×10^-3
U_y ×10^-5
U_g ×10^-6

4.3 Time-history analysis and structural response

The analysis is performed, with the Time History method, of the structure in question, subjected to the seismic action of the Northridge earthquake. To allow the comparison with the results from the analysis with elastic spectrum, the whole accelerogram was rescaled so that the maximum acceleration of the ground (PGA) was equal to that of the elastic spectrum obtained by the regulation (as explained in section 4.4).



For the purposes of the exercise it was also assumed that the earthquake acts 100% in the x direction, and 30% in the y direction, neglecting the fact that it would also be necessary to consider the opposite case, evaluating the most severe configuration.
To this end, the decoupled modal equations were used, introducing the modal coordinates {p}:

$$\ddot{p}_k + 2\zeta_k \omega_k \dot{p}_k + \omega_k^2 p_k = -\Gamma_k \ddot{u}_g(t)$$

Keeping in mind that here we analyse the response only along the x direction of the earthquake, we can define Γ_k , the participation factor:

$$\Gamma_k = \sum_i U_{ik} m_i$$

We define the vector of the scale factors:

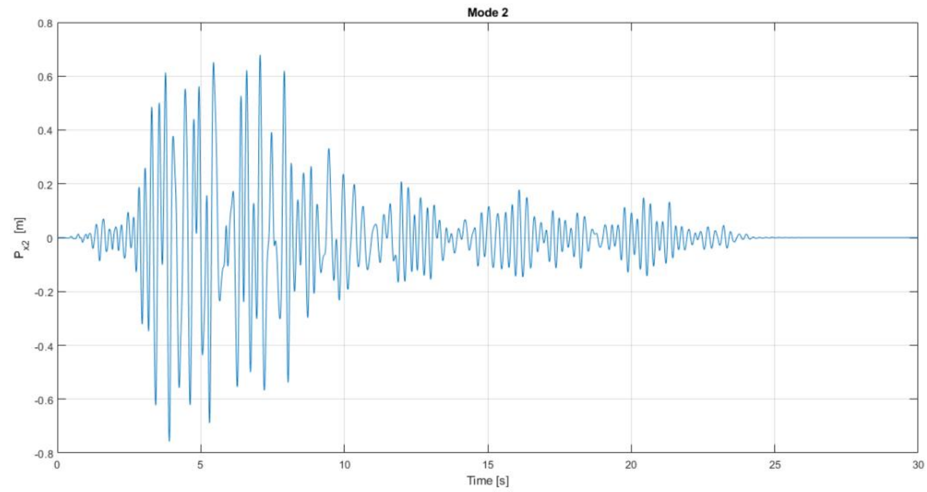
$$\{\Gamma_k\} = [U]^T [M]\{t_x\}$$

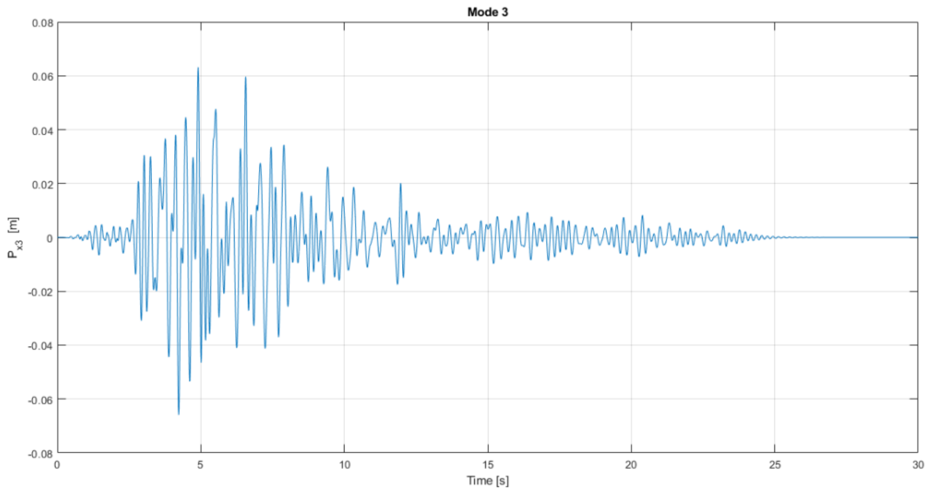
Then we define the drag vector:

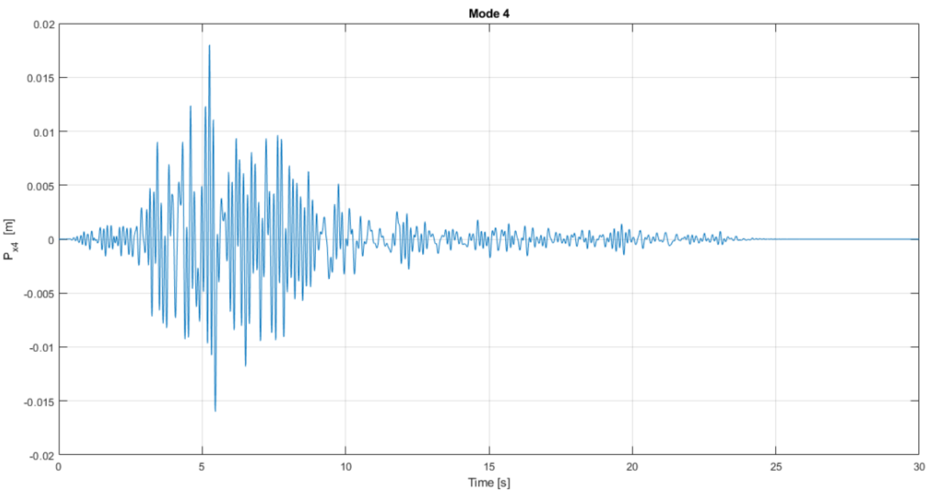
$$\{\mathrm{t_x}\} = [1 \quad 1 \quad 1 \quad 0.3 \quad 0.3 \quad 0.3 \quad 0 \quad 0 \quad 0]$$

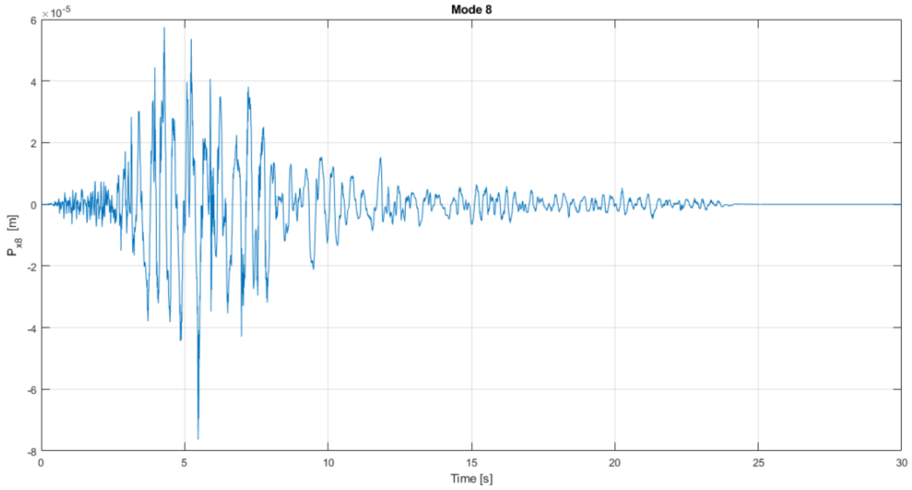
We have the following factors of participation of the masses:

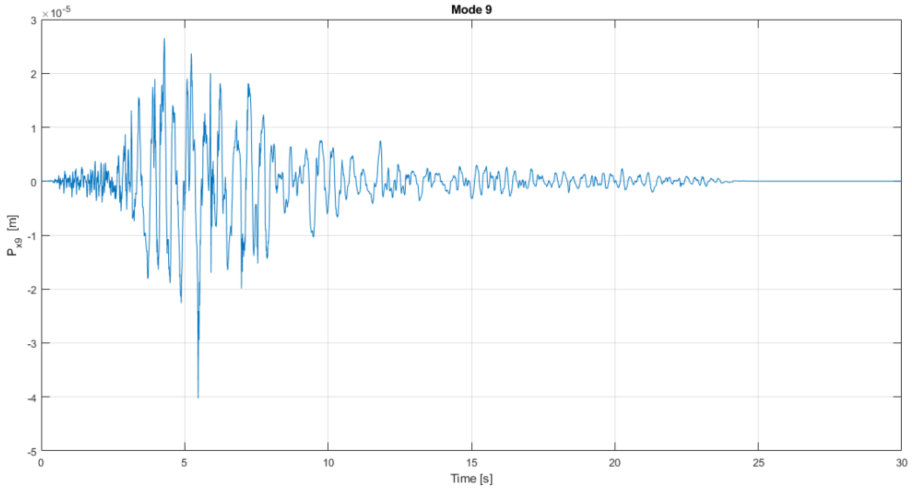
Modes	$\{\Gamma_x\}$
Mode 1	-281,4201
Mode 2	897,2647
Mode 3	102,5173
Mode 4	-38,8272
Mode 5	309,8079
Mode 6	90,6777


Mode 2
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
P x2 [m]
0
5
10
15
20
25
30
Time [s]


Mode 3
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
P x3 [m]
0
5
10
15
20
25
30
Time [s]


Mode 4
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
P x4 [m]
0
5
10
15
20
25
30
Time [s]





To obtain the displacements in geometric coordinates {q} it is sufficient to multiply the modal coordinates {p} for the normalized matrix of the eigenvectors [U], {q} is a 9x5001 matrix:

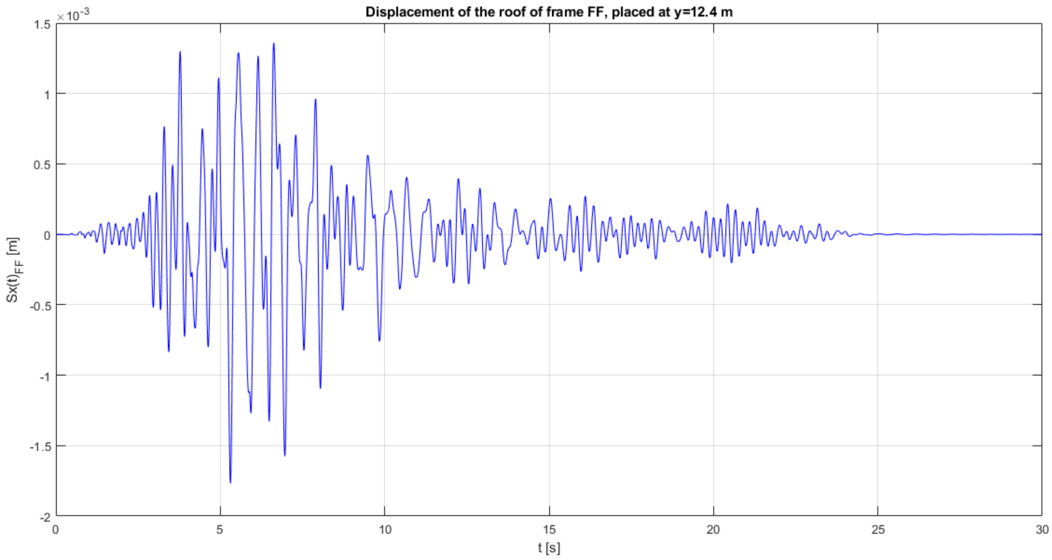
$$\{q\} = [U] \cdot \{p\}$$

The Time History of the displacements in geometric coordinates is shown below for each floor and for the three directions x,y,ϒ:

The displacements of each frame are calculated with the following formula:

$$\{S_x\}_{FF'} = \{u\} + \{\gamma\} \cdot 12.4\ m$$

The following image shows the Time History of the displacement parallel to the x axis of the last plane of the selected frame.



The maximum displacement is therefore:

$$S_{x(FF'),max} = 0.001767\ m \cong 1{,}8\ mm$$

4.4 Analysis with the elastic spectrum of the DM 2008 and structural response

The response analysis of the structure was repeated with the use of the elastic spectrum contained in the legislation of the DM 2008.

To derive this spectrum, the Excel program provided by the legislation, "Spectra-NTC", was used, containing the representative response spectra of the components (horizontal and vertical) of the seismic project actions for the chosen site of the national territory.

To derive the spectrum, the following data were considered: the building is located in the province of Rome, in the municipality of Rome, the ground below the building was considered a "bedrock" type, that is soil category A.

Having considered $k_R = 1$, to obtain the spectrum, a coefficient of use equal to 1 was also used, a nominal life of 50 years and the structure factor q_0 was determined according to the current Technical Construction, taking into account the fact that the structure is framed on several levels and it in category CD'B':

$$q_0 = 3{,}0 \cdot \frac{\alpha_u}{\alpha_1} \cdot k_R = 3{,}0 \cdot 1{,}3 = 3{,}9 > 1{,}5$$

As previously mentioned, the earthquake Northridge was rescaled to obtain the same PGA as the elastic spectrum, as can be seen in the following graph.

To do so the accelerogram has been divided by its maximum absolute value and remultiplied by the PGA of the elastic spectrum, which is the value corresponding to null period.

Obtaining the following values ($[P_{x,max}]$ is a 9x9 matrix containing the maximum modal coordinates for each mode):

Modes	D_k	$p_{x,k,max}$	T [s]
Mode 1	5,0255E-03	-1,4143E+00	0,5693
Mode 2	1,6439E-03	1,4750E+00	0,2247
Mode 3	1,2167E-03	1,2473E-01	0,1933
Mode 4	5,6758E-04	-2,2038E-02	0,1320
Mode 5	1,6961E-04	5,2548E-02	0,0759
Mode 6	1,3115E-04	1,1893E-02	0,0686
Mode 7	6,4484E-05	8,6749E-03	0,0516
Mode 8	9,7676E-06	-2,8608E-04	0,0233
Mode 9	4,1510E-06	4,8295E-05	0,0159

Multiplying each column $\{U\}_k$ of the array of the eigenvectors matrix [U] for the corresponding $p_{x,k,max}$, we obtain the maximum global displacements in geometric coordinates associated with the k-th mode $\{q\}_{k,max}$:

$$\{q_x\}_{k,max} = \begin{Bmatrix} \{u\}_{k,max} \\ \{v\}_{k,max} \\ \{\gamma\}_{k,max} \end{Bmatrix} = \{U\}_K \cdot p_{k,max}$$

$[\mathbf{q_x}]_{\max}$									
	Mode 1	**Mode 2**	**Mode 3**	**Mode 4**	**Mode 5**	**Mode 6**	**Mode 7**	**Mode 8**	**Mode 9**
u_{max} Floor 1	-2,4441E-04	8,1205E-04	-4,7389E-05	-6,1212E-06	6,3896E-05	7,2792E-06	7,9204E-06	3,7656E-07	4,4816E-08
u_{max} Floor 2	-5,1170E-04	1,7222E-03	-8,4205E-06	7,7217E-06	1,4941E-05	1,4747E-05	-9,4764E-06	7,2236E-08	-5,5863E-08
u_{max} Floor 3	-6,3472E-04	2,1425E-03	6,0613E-05	-8,0248E-06	-8,0416E-05	1,8266E-05	9,9544E-06	-4,6258E-07	6,2858E-08
v_{max} Floor 1	4,1128E-04	-2,7998E-05	8,0815E-05	1,0414E-05	-1,6021E-06	1,1119E-05	-1,5955E-07	5,9180E-07	7,0462E-08
v_{max} Floor 2	8,5928E-04	-4,1890E-05	1,6956E-05	-1,2402E-05	-1,1962E-06	2,3295E-05	8,7833E-08	1,1388E-07	-8,7875E-08
v_{max} Floor 3	1,0664E-03	-3,9946E-05	-9,7922E-05	1,3998E-05	7,3830E-07	2,8565E-05	-2,5568E-07	-7,2177E-07	9,8058E-08
γ_{max} Floor 1	-3,9235E-05	1,0039E-06	-7,6250E-06	-9,5156E-07	-3,2894E-08	1,0562E-06	-2,8549E-09	5,5883E-08	6,6580E-09
γ_{max} Floor 2	-8,2396E-05	4,1554E-07	-1,5684E-06	1,2127E-06	-6,4926E-08	2,1905E-06	-7,8556E-09	1,0713E-08	-8,2968E-09
γ_{max} Floor 3	-1,0239E-04	-7,1965E-07	9,4331E-06	-1,2854E-06	-1,0556E-07	2,7388E-06	-8,0952E-09	-6,9102E-08	9,3984E-09

Similarly to what it was done for the Time History analysis, we find the maximum displacements of each floor for the 9 ways of vibration of frame FF’, using the formula:

$$\{S_x\}_{FF',k,max} = \{u\}_{k,max} - \{\gamma\}_{k,max} \cdot 12.4 \text{ m}$$

Obtaining a matrix [3x9], whose lines represent the displacements of each plane in meters, for each of the 9 modes.

Attachments

A. Signal generation

In the first laboratory we learned how to generate a signal in Matlab.
Since all the steps have been developed later in the different laboratories, here is reported only the code.

```
close all;
clear;
clc;

%% INPUT
ts = 1/200;      %sampling time (200=sampling frequency [Hz])
wn(1) = 6.28;    %wn= row vector = natural pulsation
wn(2) = wn(1);
zita(1) = 0.02;  %two dampings
zita(2) = 0.06;

%% TIME
t = (0:ts:50-ts)';           %[s]; the length of the signal is 10000 so we put 50-ts; ts is the
                              step:   1/200,2/200,3/200...
tend = t(end);               %last value of vector t

wd = wn .* sqrt(1-zita.^2);   %damped pulsation (zit.^2=[0.02^2;0.06^2])

u = nan(size(t,1),2)         %nan = not a number; u is a matrix initialized here

%from this loop I get 2 vectors having 10000 rows and 1 column, that are in the matrix u with size
10000x2
for cc = 1:2                 %cc=is the column (1 and 2)
    u(:,cc) = exp(-zita(cc)*wn(cc)*t)/wd(cc) .* sin(wd(cc)*t);  %solution of a damped SDOF system
end

[umax,indmax_t] = max(abs(u)) %find the max values of matrix u in each column and their positions
tmax = t(indmax_t)           %find also the time corresponding to this positions

%% FREQUENCY
U=fft(u)*ts;                    %U = matrix 10000x2 = multiply fft with sampling time for Matlab
                                 reason (integration steps = 1), it has complex numbers
Us(:,1)=fftshift(U(:,1));       %shift of fft is used for plotting using positive values of the
                                 freq (modulus of complex nr)
Us(:,2)=fftshift(U(:,2));
absU = abs(Us);                 %the abs of a complex nr gives its modulus

fs = 1/ts;                      %sampling frequency
fres = 1/tend;                  %frequency resolution = invers of duration of the signal
f = (-fs/2:fres:fs/2)';         %vector  from -fs/2 with step of freq resolution up to fs/2

[~,indmax_f1] = max(absU(:,1));  % ~ = means I don't waant any value (we want only its position)
fmax(1) = abs(f(indmax_f1));

[~,indmax_f2] = max(absU(:,2));
fmax(2) = abs(f(indmax_f2));

%% TEXT
strtitle1 = sprintf('Signal u_1(t): zita = %.2f %%',100*zita(1));
strleget1 = sprintf('u_1(max) = %.3f m at %.3f s',umax(1),tmax(1));
strlegef1 = sprintf('f_m_a_x = %.1f Hz',fmax(1));

strtitle2 = sprintf('Signal u_2(t): zita = %.2f %%',100*zita(2));
strleget2 = sprintf('u_2(max) = %.3f m at %.3f s',umax(2),tmax(2));
strlegef2 = sprintf('f_m_a_x = %.1f Hz',fmax(2));

%% PLOT
figure;
subplot(2,2,1);
plot(t,u(:,1));
```

```
%CASE 2
uo2=0;                  %[m], initial displacement
vo2=3;                  %[m/s], initial velocity
%CASE 3
uo3=0.03;               %[m], initial displacement
vo3=3;                  %[m/s], initial velocity


%define the time vector
ts=0.01;                %[s], time step
tend=10;                %[s], time final value
t=0:ts:tend;

%displacement u
u1=exp(-zita*wn*t).*(uo*cos(wd*t)+(vo+zita*wn*uo)/wd*sin(wd*t));
u2=exp(-zita*wn*t).*(uo2*cos(wd*t)+(vo2+zita*wn*uo2)/wd*sin(wd*t));
u3=exp(-zita*wn*t).*(uo3*cos(wd*t)+(vo3+zita*wn*uo3)/wd*sin(wd*t));
u2max=max(abs(u2));
u2min=min(abs(u2));
u3max=max(abs(u3));

%plot
figure;
plot(t,u1,'b');
hold on
plot(t,u2,'r');
xlabel('Time t [s]');
ylabel('Displacement u(t) [m]');
legend('CASE 1: uo=0.03m, vo=0m/s','CASE 2: uo=0m,      vo=3m/s');
title('SDOF: time-history displacement');
grid on;
figure;
plot(t,u1,'b');
hold on
plot(t,u2,'r');
hold on
plot(t,u3,'k');
xlabel('Time t [s]');
ylabel('Displacement u(t) [m]');
legend('CASE 1: uo=0.03m, vo=0m/s','CASE 2: uo=0m,      vo=3m/s','CASE 3: uo=0.03m,    vo=3m/s');
title('SDOF: time-history displacement');
grid on;

%display
fprintf('Comparison:                      CASE 1           CASE 2\n');
fprintf('Stiffness [N/m]                  %.0f           %.0f\n',Ks,Ks);
fprintf('Natural circ. frequency [rad/s]  %.5f          %.5f\n',wn,wn);
fprintf('Natural frequency [1/s]           %.5f            %.5f\n',fn,fn);
fprintf('Damped frequency [rad/s]         %.5f          %.5f\n',wd,wd);
fprintf('Natural period [s]               %f          %f\n',Tn,Tn);
fprintf('Maximum displacements [m]        %f          %f\n',u1max,u2max);
fprintf('Maximum displacement 3 CASE [m]  %f',u3max);
```

Comparison 2

```
clc;
clear all;
close all;

%define the time vector
ts=0.01;                %[s], time step
tend=10;                %[s], time final value
t=0:ts:tend;
```

```
fprintf('Damped frequency [rad/s]          %.5f          %.5f\n',wd,wd2);
fprintf('Natural frequency [1/s]            %.5f          %.5f\n',fn,fn2);
fprintf('Natural period [s]                %f          %f\n',Tn,Tn2);
fprintf('Maximum displacements [m]         %f          %f\n',u1max,u2max);

%%%%% CASE 2.1 and 2.2
wn2_2=sqrt(2*Ks2/Ms2);
wd2_2=wn2_2*sqrt(1-zita2^2);
u2_1=u2;
u2_1max=u2max
u2_2=exp(-zita2*wn2_2*t).*(uo2*cos(wd2_2*t)+(vo2+zita2*wn2_2*uo2)/wd2_2*sin(wd2_2*t));
u2_2max=max(abs(u2_2))
%plot
figure;
plot(t,u2_1,'b');
hold on
plot(t,u2_2,'r');
xlabel('Time t [s]');
ylabel('Displacement u(t) [m]');
legend('CASE 2.1: K = Ks','CASE 2.2: K = 2Ks');
title('SDOF: time-history displacement');
grid on;

%%%%% CASE 1.1 and 1.2
wn1_2=sqrt(2*Ks/Ms);
wd1_2=wn1_2*sqrt(1-zita2^2);
u1_1=u1;
u1_1max=u1max
u1_2=exp(-zita*wn1_2*t).*(uo*cos(wd1_2*t)+(vo+zita*wn1_2*uo)/wd1_2*sin(wd1_2*t));
u1_2max=max(abs(u1_2))
%plot
figure;
plot(t,u1_1,'b');
hold on
plot(t,u1_2,'r');
xlabel('Time t [s]');
ylabel('Displacement u(t) [m]');
legend('CASE 1.1: K = Ks','CASE 1.2: K = 2Ks');
title('SDOF: time-history displacement');
grid on;
```

1.2 Nyquist criterion

```
fn=wn/(2*pi);
fs1=0.8*fn;
ts1=1/fs1;
t1=0:ts1:tend;
uc1=exp(-zita*wn*t1).*(uo*cos(wd*t1)+(vo+zita*wn*uo)/wd*sin(wd*t1));

fs2=2*fn;
ts2=1/fs2;
t2=0:ts2:tend;
uc2=exp(-zita*wn*t2).*(uo*cos(wd*t2)+(vo+zita*wn*uo)/wd*sin(wd*t2));

fs3=2.1*fn;
ts3=1/fs3;
t3=0:ts3:tend;
uc3=exp(-zita*wn*t3).*(uo*cos(wd*t3)+(vo+zita*wn*uo)/wd*sin(wd*t3));

fs4=100;
ts4=1/fs4;
t4=0:ts4:tend;
uc4=exp(-zita*wn*t4).*(uo*cos(wd*t4)+(vo+zita*wn*uo)/wd*sin(wd*t4));
```


© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 59 di 174

```
grid on;

subplot(2,2,2);
plot(f,amplitude2,'r','linewidth',1.5);
xlabel('f  [Hz]');
ylabel('Amplitude |U(f)| [m]');
title('CASE 2');
ylim([0 0.14]);
grid on;

subplot(2,2,4)
plot(f,phase2,'r','linewidth',1.5)
xlabel('f  [Hz]');
ylabel('Phase \phi(f) [rad]');
ylim([-4 4]);
grid on;
```

1.4 Loma Prieta

```
load quake

fs_loma=200;            %sampling frequency of Loma
ts_loma=1/fs_loma;      %sampling time of Loma
tend=50;                %duration of Loma
fres_loma=1/tend;
f_loma=[-fs_loma/2:fres_loma:fs_loma/2];
e=e';  n=n';  v=v';
t = ts_loma*(1:length(e))';

g = 0.0981;
e_resc=(1/(g*100))*e;
n_resc=(1/(g*100))*n;
v_resc=(1/(g*100))*v;

% acceleration-time plots
figure;
subplot(3,1,1), plot(t,e_resc,'b'), axis([0 50 -250 250]),xlabel('Time [s]'), title('East-West
acceleration'), grid on
subplot(3,1,2), plot(t,n_resc,'g'), axis([0 50 -250 250]),xlabel('Time [s]'), title('North-South
acceleration'),grid on
subplot(3,1,3), plot(t,v_resc,'r'), axis([0 50 -250 250]),xlabel('Time [s]'), title('Vertical
acceleration'), grid on

% FFT of e,n,v
FFT_e=fft(e)*ts_loma;
FFTs_e=fftshift(FFT_e);

FFT_n=fft(n)*ts_loma;
FFTs_n=fftshift(FFT_n);

FFT_v=fft(v)*ts_loma;
FFTs_v=fftshift(FFT_v);

figure;
subplot(3,2,1);
plot(f_loma,abs(FFTs_e),'b','linewidth',1.5);
xlabel('f  [Hz]');
ylabel('Amplitude |U(f)| [m]');
title('East-West component');
grid on;
subplot(3,2,2)
plot(f_loma,angle(FFTs_e),'b','linewidth',1.5)
xlabel('f  [Hz]');
ylabel('Phase \phi(f) [rad]');
grid on;
subplot(3,2,3);
plot(f_loma,abs(FFTs_n),'g','linewidth',1.5);
xlabel('f  [Hz]');
ylabel('Amplitude |U(f)| [m]');
title('North-South component');
grid on;
```

```
subplot(3,1,3)
plot(t,ht);
xlabel('t [s]');
xlim([0,20]);
ylabel('h(t)');
title('I.R.F. Impulse Response Function');
grid on;
```

2.2 Response function FRF

```
Hf=fft(ht)*ts;          %IRF in frequency domain
Hf=fftshift(Hf);

Af=fft(At)*ts;          %acceleration in frequency domain
Af=fftshift(Af);

%define the frequency axis
NFFT=length(ht);
Tf=NFFT/fs;
if mod(NFFT,2)==1
    f=[-fs/2:1/Tf:fs/2-1/Tf];  %even length
else
    f=[-fs/2:1/Tf:fs/2];       %odd length
end

%plot Af
figure;
subplot(2,1,1)
plot(f,abs(Af),'linewidth',1.5);
xlabel('f  [Hz]');
ylabel('Amplitude |A(f)| [m]');
title('FFT of the Earthquake');
grid on;
subplot(2,1,2)
plot(f,angle(Af),'linewidth',1.5)
xlabel('f  [Hz]');
ylabel('Phase \phi(f) [rad]');
grid on;

%plot Hf_numerical
figure;
subplot(2,1,1)
plot(f,abs(Hf),'r','linewidth',1.5);
xlabel('f  [Hz]');
ylabel('Amplitude |H(f)| [m]');
title('Numeric FRF');
grid on;
subplot(2,1,2)
plot(f,angle(Hf),'r','linewidth',1.5)
xlabel('f  [Hz]');
ylabel('Phase \phi(f) [rad]');
grid on;

%displacement response in freq. domain Uf
Uf=Hf.*(-Af);

%%%Analytically: Uf2
w=2*pi*f';                %circ. freq. of the quake
Rd=1./sqrt(((1-(w/wn).^2).^2)+(2*zita*(w/wn)).^2);
teta=atan2(2*zita*w/wn,1-(w/wn).^2);
Hf2=1/(wn^2)*Rd.*exp(-1i*teta);

%plot Hf_analytical
figure;
subplot(2,1,1)
plot(f,abs(Hf2),'linewidth',1.5);
xlabel('f  [Hz]');
ylabel('Amplitude |H(f)| [m]');
title('Analytic FRF');
grid on;
subplot(2,1,2)
plot(f,angle(Hf2),'linewidth',1.5)
xlabel('f  [Hz]');
```

```
    %Compute absolute acceleration
    a_ass=a(1:length(At))+At;

    %Compure response spectra
    maxv(ii)=max(abs(v));
    maxa(ii)=max(abs(a_ass));

    %Pseudo-velocity & Pseudo-acceleration
    pv(ii)=maxu(ii)*wN;
    pa(ii)=maxu(ii)*(wN^2);
end

Tn=[0 Tn];
maxu=[0 maxu];
maxv=[0 maxv];
pv=[0 pv];
maxa=[max(abs(At)) maxa];
pa=[max(abs(At)) pa];

figure;
subplot(3,1,1)
plot(Tn,maxu,'g')
xlabel('Tn  [s]');
ylabel('u_m_a_x [m]');
title('Max displacement spectrum');
grid on;
subplot(3,1,2)
plot(Tn,maxv,'r',Tn,pv,'g')
xlabel('Tn  [s]');
ylabel('v_m_a_x & pv [m/s]');
grid on;
title('Comparison between v,a and pseudo-v,a spectra');
legend('Max velocity','Pseudo velocity');
subplot(3,1,3)
plot(Tn,maxa,'r',Tn,pa,'g')
xlabel('Tn [s]');
ylabel('a_m_a_x & pa [m^2/s]');
grid on;
legend('Max velocity','Pseudo velocity');
```

3.1-2 Mass and stiffness matrices

```
%% THE SYSTEM
%Geometrical parameters and natural vibrations
g=9.81/1000;
zita=5/100;
L=12.4;                 %[m] frame's length
w=2;                    %[m] competence width

%Columns
b_col=0.3;              %[m] column's base of section
h_col=0.3;              %[m] column's height of section
H_col=3;                %[m] column's heigth
nr_col=5;               %number of columns in the frame
rho_col=2500;           %[kg/m^3] bricks weight
m_col=nr_col*(b_col*h_col*H_col)*rho_col; %[kN] column's weight

%External walls
b_pla=0.02;              %[m] plaster's width of section
rho_pla=2000;            %[kg/m^3] plaster's density 20kN/m3
b_wall=0.3-b_pla;        %[m] wall's width
l_wall=(L-(nr_col-1)*b_wall)+2*(w-0.15);%[m] wall's length = plaster's
H_wall=3;                %[m] wall's heigth
rho_wall=2100;           %[kg/m^3] wall's density
m_wall=H_wall*l_wall*((b_wall*rho_wall)+(b_pla*rho_pla)); %[kN] wall's weight

%Beams
b_beam=0.3;              %[m] column's base of section
h_beam=0.3;              %[m] column's height of section
l_beam=(L-(nr_col-1)*b_col)+5*(w-0.15);%[m] wall's length = plaster's
rho_beam=2500;           %[kg/m^3] bricks weight
m_beam=b_beam*h_beam*rho_beam*l_beam; %[kN] column's weight
```

3.4 Time History

```
%% TIME HYSTORY ANALYSIS of the frame under seismic excitation
%build the damping matrix [C]
C=sqrt(lam).*(2*zita);

%evaluate the participation factor
t_drag=ones(N,1);    %drag vector
gamma=U'*M*t_drag;       %T=capital gamma vector=partic. factor

%load earthquake
A=load('northridge.txt');
At=A(:,2);                 %[m/s^2] the 2 column has the accelerations
t=A(:,1);                  %[s] the 1 column has the time
ts=A(2);                   %the sampling time is 0.01 [s]
tend=ts*(length(A)-1);     %duration 25 [s]
fs=1/ts;                   %[Hz] sampling frequency

wn=sqrt(diag(lam));        %[rad/s], system's natural circular frequency
wd=wn*sqrt(1-zita^2);      %[rad/s], damped frequency
figure;
plot(t,At);
grid on;
title('Earthquake accelerations');
xlabel('t [s]');
ylabel('a [m^2/s]');

%apply convolution to calculate modal coordinates p(t)
for j=1:N
ht(j,:)=(1/wd(j))*exp(-zita*wn(j)*t').*sin(wd(j)*t'); %h(t)=IRF
F(j,:)=-gamma(j)*At';                                 %F(t)=excitation func.
P(j,:)=conv(ht(j,:),F(j,:))*ts;
end

%displacement of the 3 slabs
u=U*P;
umax_timehystory = max(abs(u.')).';      %get the 3 maximum displacements for the 3 slabs
tconv=0:ts:ts*(length(u)-1); %lenght of u(t) doubled after to convolution

for j=1:N
subplot(N,1,j)
    plot(tconv,P(j,:))
    str1=sprintf('Mode %d',j);
    title(str1);
    xlabel('t [s]');
    str2=sprintf('p(t)_%d',j);
    ylabel(str2);
    grid on;
end
figure;
for j=1:N
subplot(N,1,j)
    plot(tconv,u(j,:),'g')
    str1=sprintf('Floor %d',j);
    title(str1);
    xlabel('t [s]');
    str2=sprintf('u(t)_%d  [m]',j);
    ylabel(str2);
    grid on;
end

%equivalent static forces
f=K*U*P;
figure;
for j=1:N
subplot(N,1,j)
    plot(tconv,f(j,:),'r')
    str1=sprintf('Floor %d',j);
    title(str1);
    xlabel('t [s]');
    str2=sprintf('f(t)_%d  [N]',j);
    ylabel(str2);
    grid on;
end
```

4.1 System's geometry

```
%% GEOMETRICAL PARAMETERS in file "eeLab4_system_masses"
g=9.81/1000;            %constant to pass from [N] to [kg]
Lx=16;                  %plan length along x
Ly=12.4;                %plan length along y

%Columns
b_col=0.3;              %[m] column's base of section
h_col=0.3;              %[m] column's height of section
H_col=3;                %[m] column's heigth
nr_col=5;               %number of columns in one frame
rho_col=2500;           %[kg/m^3] bricks weight
m_col=(nr_col^2-1)*(b_col*h_col*H_col)*rho_col; %[kg] column's weight

%External walls
b_pla=0.02;                       %[m] plaster's width of section
rho_pla=2000;                     %[kg/m^3] plaster's density 20kN/m3
b_wall=0.3-b_pla;                 %[m] wall's width
l_wally=(Ly-(nr_col-1)*b_wall)*2; %[m] wall's length // y
l_wallx=(Lx-(nr_col-1)*b_wall)*2; %[m] wall's length // x
H_wall=3;                         %[m] wall's heigth
rho_wall=2100;                    %[kg/m^3] wall's density
m_wall=H_wall*(l_wally+l_wallx)*((b_wall*rho_wall)+(b_pla*rho_pla)); %[kN] wall's weight

%Beams
b_beam=0.3;                                              %[m] column's base of section
h_beam=0.3;                                              %[m] column's height of section
l_beamy=(Ly-(nr_col-1)*b_col)*4+((Ly-2)-(nr_col-2)*b_col); %[m] beams' length // y
l_beamx=(Lx-(nr_col-1)*b_col)*4+((Lx-3)-(nr_col-2)*b_col); %[m] beams' length // x
rho_beam=2500;                                           %[kg/m^3] bricks weight
m_beam=b_beam*h_beam*rho_beam*(l_beamx+l_beamx);         %[kN] column's weight

%Loads acting on the slabs
%roof
G1r=3.7;    %structural permanent for the floor
G2r=0.6;    %structural non-permanent for the floor
Qr=0.5;     %variable load
psir=0;     %combination factor

%1 & 2 floor
G1=3.7;     %structural permanent for the floor
G2=3;       %structural non-permanent for the floor
Q2=2;       %variable load
psi2=0.3;   %combination factor

%stairs
G1s=2.5;    %structural permanent load of stairs
G2s=2.7;     %structural non-permanent load of stairs

%The NTC rule (kN/m^2)
%roof's load
SLr=G1r+G2r+psir*Qr;

%1 and 2 floor load
SL2=G1+G1s+G2+G2s+psi2*Q2;
SL1=SL2;

%%MASSES in [Kg]
A_stairs = 2.2*4;                    %hollow area of atairs
A_gap    = 2*3;                      %hollow area of the gap
A_plain  = Lx*Ly;                    %total area of the slab
A_floor  = A_plain - A_gap - A_stairs;      %[m^2] real area of the floors
A_roof   = A_plain - A_gap;                 %[m^2] real area of the roof

M1=(A_floor*SL1/g)+ m_col + m_wall + m_beam;   %[kg] Floor 1 mass
M2=(A_floor*SL2/g)+ m_col + m_wall + m_beam;   %[kg] Floor 2 mass
Mr=(A_roof*SLr/g) + m_col/2 + m_wall/2 + m_beam;   %[kg] Roof's mass

%%%%WITH start i kN/m2
% M1=(A_floor*SL1/g+ m_col + m_wall + m_beam)/g;   %[kg] Floor 1 mass
% M2=(A_floor*SL2/g+ m_col + m_wall + m_beam)/g;   %[kg] Floor 2 mass
% Mr=(A_roof*SLr/g + m_col/2 + m_wall/2 + m_beam)/g;   %[kg] Roof's mass

%mass densities
```

```
KII=KJJ;
KHH=KJJ;
KGG=KJJ;
KFF=CFF^-1;
HXX=4*KJJ+KFF;

% stiffness matrix of the i-th frame parallel to the y axis:
CAA=[2.9400E-09   3.4763E-09    3.554E-09;
     3.4763E-09   7.7943E-09    8.532E-09 ;
     3.5540E-09   8.5320E-09    1.320E-08];
CBB=CAA;
CCC=CAA;
CDD=CAA;
CEE=[3.8100E-09   4.7054E-09    4.8080E-09 ;
     4.7054E-09   1.0647E-08    1.1787E-08 ;
     4.8080E-09   1.1787E-08    1.8210E-08];

KAA=CAA^-1;
KEE=CEE^-1;
KBB=KAA;
KCC=KAA;
KDD=KAA;
HYY=4*KAA+KEE;

HXG=-(KJJ*0+KII*4+KHH*6.2+KGG*10.4+KFF*12.4);
HGX=HXG;

HYG=(KAA*0+KEE*16+KBB*4+KCC*8+KDD*13);
HGY=HYG;

HGG=(KAA*0+KEE*16^2+KBB*4^2+KCC*8^2+KDD*13^2)+(KJJ*0+KII*4^2+KHH*6.2^2+KGG*10.4^2+KFF*12.4^2);

%ASSEMBLY OF THE GLOBAL STIFFNESS MATRIX OF THE STRUCTURE
H = [HXX  o   HXG ;
      o  HYY  HYG ;
     HGX HGY  HGG];

save('M_H_matrices.mat','M','H')
```

4.2 Eigenvalues problem

```
%% GEOMETRICAL PARAMETERS
g=9.81/1000;
zita=5/100;
N=9;       %DoFs

%load the matrices M & H from the file
load('M_H_matrices.mat')

%% EIGENVALUES AND EIGENVECTORS

[sha,lam]=eig(H,M);              % sha = eigenvectors, lam = eigenvalues
[lam,ind]=sort(diag(lam));       % ensure that the eigenvalues are stored in ascending order
lam=diag(lam);                   % put eigenvalues in a vector
sha=sha(:,ind');                 % exchange the columns of sha to maintain the same convention of lam
freq=sqrt(diag(lam))/(2*pi);     % evaluate the natural frequencies
Tn=1./freq;                      % natural periods of each mode
check_sha = sha'*M*sha;          % check:I=[sha]'*[M]*[sha]

sqrttt=sqrt(diag(lam));

% normalize if the check_sha3D is not identity
for k = 1:size(sha,1)            % normalize if check is not identity
    ck = sqrt((sha(:,k))' * M * sha(:,k));
    U(:,k) = sha (:,k) / ck;
end
check_U = U'*M*U;                % now check_U must be identity matrix

%plot the modal shapes
floors =(0:(N/3))';
for k = 1:size(U,1)
    figure(k);
    subplot(1,3,1);
```

```
%displacement of the 3 slabs and plot of vector [q] = [u, v, g] &  Px
qx=U*Px;


%qx geometric coord.  (X main direction)
figure; %u
for i=1:N/3
subplot(3,1,i)
plot(tconv,qx(i,:),'r');
stry=sprintf('u_%d  [m]',i);
ylabel(stry);
xlabel('Time [s]');
xlim([0,30]);
strtit=sprintf('Floor %d ',i);
title(strtit);
grid on;
end

figure; %v
for i=1:N/3
subplot(3,1,i)
plot(tconv,qx(i+3,:),'b');
stry=sprintf('v_%d  [m]',i);
ylabel(stry);
xlabel('Time [s]');
xlim([0,30]);
strtit=sprintf('Floor %d ',i);
title(strtit);
grid on;
end

figure; %rotation
for i=1:N/3
subplot(3,1,i)
plot(tconv,qx(i+6,:),'g');
stry=sprintf('rot_%d  [/]',i);
ylabel(stry);
xlabel('Time [s]');
xlim([0,30]);
strtit=sprintf('Floor %d ',i);
title(strtit);
grid on;
end

%Displacement of one choosen frame,along the prevalent direction X
%Frames // x-axis
y=[0 4 6.2 10.4 12.4];           %vector of distances of each frame from x-axis
Sx1=qx(1:3,:)-qx(7:9,:)*y(1);
Sx2=qx(1:3,:)-qx(7:9,:)*y(2);
Sx3=qx(1:3,:)-qx(7:9,:)*y(3);
Sx4=qx(1:3,:)-qx(7:9,:)*y(4);
Sx5=qx(1:3,:)-qx(7:9,:)*y(5);    %displ. of frame FF'(=Sx5) along x
Sxx=[Sx1;Sx2;Sx3;Sx4;Sx5];
for i=1:N/3
       Sx5_max_timehistory(i,:)=max(abs(Sx5(i,:))); %find max. displ. of each floor of frame Sx5
end
Sx5_max_roof_timehistory=Sx5_max_timehistory(3)  %[m] highlight max displ. at the roof of frame Sx5


%we choose only Sx5 and plot it
figure;
plot(tconv,Sx5(3,:),'b');
title('Displacement of the roof of frame FF, placed at y=12.4 m');
xlabel('t [s]');
xlim([0,30]);
ylabel('Sx(t)_F_F [m]');
grid on;

for i=1:N
qx_max(i,:)=max(abs(qx(i,:).')).';
end
```

```
plot([0;Sx5_max_timehistory],floors,'-o');
hold on
plot([0;Sx5_max_spectrum],floors,'-o');
xlabel('Sx [m]');
ylabel('Floors');
title('Maximum displacements of frame FF along x direction');
legend('Time History','Spectral analysis')
grid on;


figure;  %[mm]
plot([0;Sx5_max_timehistory*1000],floors,'-o');
hold on
plot([0;Sx5_max_spectrum*1000],floors,'-o');
xlabel('Sx [mm]');
ylabel('Floors');
title('Maximum displacements of frame FF along x direction');
legend('Time History','Spectral analysis')
grid on;
```

C. Project's Matlab Code

System's geometry

```
%% GEOMETRICAL PARAMETERS in file "eeLab4_system_masses"

g=9.81/1000;             %constant to pass from [N] to [kg]
Lx=16;                   %plan length along x
Ly=12.4;                 %plan length along y

%Columns
b_col=0.3;               %[m] column's base of section
h_col=0.3;               %[m] column's height of section
H_col=3;                 %[m] column's heigth
nr_col=5;                %number of columns in one frame
rho_col=2500;            %[kg/m^3] bricks weight
m_col=(nr_col^2-1)*(b_col*h_col*H_col)*rho_col; %[kg] column's weight

%External walls
b_pla=0.02;                         %[m] plaster's width of section
rho_pla=2000;                       %[kg/m^3] plaster's density 20kN/m3
b_wall=0.3-b_pla;                   %[m] wall's width
l_wally=(Ly-(nr_col-1)*b_wall)*2;   %[m] wall's length // y
l_wallx=(Lx-(nr_col-1)*b_wall)*2;   %[m] wall's length // x
H_wall=3;                           %[m] wall's heigth
rho_wall=2100;                      %[kg/m^3] wall's density
m_wall=H_wall*(l_wally+l_wallx)*((b_wall*rho_wall)+(b_pla*rho_pla)); %[kN] wall's weight

%Beams
b_beam=0.3;                                           %[m] column's base of section
h_beam=0.3;                                           %[m] column's height of section
l_beamy=(Ly-(nr_col-1)*b_col)*4+((Ly-2)-(nr_col-2)*b_col); %[m] beams' length // y
l_beamx=(Lx-(nr_col-1)*b_col)*4+((Lx-3)-(nr_col-2)*b_col); %[m] beams' length // x
rho_beam=2500;                                        %[kg/m^3] bricks weight
m_beam=b_beam*h_beam*rho_beam*(l_beamx+l_beamx);      %[kN] column's weight

%Loads acting on the slabs
%roof
G1r=3.7;     %structural permanent for the floor
G2r=0.6;     %structural non-permanent for the floor
Qr=0.5;      %variable load
psir=0;      %combination factor

%1 & 2 floor
```

```
%DISTANCES OD G FROM POLE O
dG1=sqrt(XG^2+YG^2);            %[m] of floor 1
dG2=dG1;                        %[m] of floor 2
dGr=sqrt(XGr^2+YGr^2);          %[m] of roof

%POLAR MOMENTS OF INERTIA
J01=mu1*(Jxx+Jyy+A_floor*dG1^2);
J02=J01;
J0r=mur*(Jxxr+Jyyr+A_roof*dGr^2);

%MATRIX OF POLAR MOMENTS of inertia with respect to the pole "O"
Mgg=[J01 0 0;0 J02 0; 0 0 J0r];

%ASSEMBLY OF THE GLOBAL MASS MATRIX OF THE 3D STRUCTURE:
o=zeros(3,3);
Mxx=[M1 0 0; 0 M2 0; 0 0 Mr];
Myy=[M1 0 0; 0 M2 0; 0 0 Mr];
Mxg=[-M1*YG 0 0; 0 -M2*YG 0; 0 0 -Mr*YGr];   %Mxg=Mgx
Myg=[-M1*XG 0 0; 0 -M2*XG 0; 0 0 -Mr*XGr];   %Myg=Mgy

M=[Mxx  o   Mxg ;
    o   Myy Myg ;
   Mxg  Myg Mgg];

%DETERMINATION OF MATRIX OF RIGIDITY:
%stiffness matrix of the i-th frame parallel to the x axis:
CJJ=[3.14E-09   3.799E-09   3.886E-09 ;
    3.799E-09   8.6972E-09  9.5699E-09;
    3.886E-09   9.5699E-09  1.485E-08];
CII=CJJ;
CHH=CJJ;
CGG=CJJ;
CFF= [3.9900E-09    5.0752E-09  5.2280E-09;
      5.0752E-09    1.1642E-08  1.3080E-08;
      5.2280E-09    1.3080E-08  2.0260E-08];

KJJ=CJJ^-1;
KII=KJJ;
KHH=KJJ;
KGG=KJJ;
KFF=CFF^-1;
HXX=4*KJJ+KFF;

% stiffness matrix of the i-th frame parallel to the y axis:
CAA=[2.9400E-09   3.4763E-09    3.554E-09;
     3.4763E-09   7.7943E-09    8.532E-09 ;
     3.5540E-09   8.5320E-09    1.320E-08];
CBB=CAA;
CCC=CAA;
CDD=CAA;
CEE=[3.8100E-09   4.7054E-09    4.8080E-09 ;
     4.7054E-09   1.0647E-08    1.1787E-08 ;
     4.8080E-09   1.1787E-08    1.8210E-08];

KAA=CAA^-1;
KEE=CEE^-1;
KBB=KAA;
KCC=KAA;
KDD=KAA;
HYY=4*KAA+KEE;

HXG=-(KJJ*0+KII*4+KHH*6.2+KGG*10.4+KFF*12.4);
HGX=HXG;
```

```
    xlabel(str2);
end

%Build the damping matrix [C]
C=sqrt(lam)*2*zita;
%Create the drag vectors
vtx=[1 1 1 0.3 0.3 0.3 0 0 0]';   %combination 1: maximize actions on frames // X
vty=[0.3 0.3 0.3 1 1 1 0 0 0]';   %combination 2: maximize actions on frames // Y
%Evaluate the participation factor
gammax=U'*M*vtx;
gammay=U'*M*vty;
```

Elastic spectrum

```
%% SOLUTION WITH RESPONSE SPECTRUM

%LOAD SPECTRA FROM NTC
Spectrum_SLD=load('SLD-SLE.txt');   %load the design elastic spectrum, SLD
Se_SLD=Spectrum_SLD(:,2).*9.81;                      %spectrum accelerations
Ts_SLD=Spectrum_SLD(:,1);                            %periods of the spectrum
Spectrum_SLV=load('SLV-SLU.txt');   %load the inelastic spectrum, ultimate state SLV
Se_SLV=Spectrum_SLV(:,2).*9.81;                      %spectrum accelerations
Ts_SLV=Spectrum_SLV(:,1);                            %periods of the spectrum

%interpolation of the elastic spectrum to find pseudo-accelerations for my periods
Sa_SLD=zeros(length(lam),1);                     %initialization
Sa_SLV=zeros(length(lam),1);
for k=1:length(lam)
    Sa_SLD(k)=interp1(Ts_SLD,Se_SLD,Tn(k));      %Pseudo-acceleration
    Sa_SLV(k)=interp1(Ts_SLV,Se_SLV,Tn(k));

    D_SLD(k)=Sa_SLD(k)/lam(k,k);                 %Maximum displacements
    D_SLV(k)=Sa_SLV(k)/lam(k,k);
end

%plot spectra & interpolation
figure;
plot(Ts_SLD,Se_SLD,'b',Tn,Sa_SLD,'ok')       %elastic spectrum from NTC
hold on
plot(Ts_SLV,Se_SLV,'r',Tn,Sa_SLV,'*k')
xlabel('Periods T [s]')
ylabel('Spectral accelerations [m/s^2]')
legend('Elastic spectrum SLD','Sa(SLE)','Inelastic spectrum SLV','Sa(SLU)')
title('Spectra of maximum response accelerations')
grid on;

%MAXIMUM MODAL COORDINATES Pxk (comb 1) & Pyk (comb 2)
Pxk_SLD=gammax.*D_SLD';   %slabs' max modal displacements
Pxk_SLV=gammax.*D_SLV';   %slabs' max modal displacements

Pyk_SLD=gammay.*D_SLD';   %slabs' max modal displacements
Pyk_SLV=gammay.*D_SLV';   %slabs' max modal displacements

format long;
vmax=0;
for i=1:length(lam)
    %evaluate the displacements q=vector=(u1,u2,u3,v1,v2,v3,g1,g2,g3) related to each mode
    qmx_SLD(:,i)=U(:,i)*Pxk_SLD(i);
    qmx_SLV(:,i)=U(:,i)*Pxk_SLV(i);

    qmy_SLD(:,i)=U(:,i)*Pyk_SLD(i);
    qmy_SLV(:,i)=U(:,i)*Pyk_SLV(i);
end

%In particular we have:
%combination 1 tx
umx_SLD=qmx_SLD(1:3,:);
vmx_SLD=qmx_SLD(4:6,:);
gmx_SLD=qmx_SLD(7:9,:);

umx_SLV=qmx_SLV(1:3,:);
```

```
plot([0;S_HHy_SLD_max*1000],floors,'--o')
xlabel('S_H_H [mm]')
ylabel('Floors')
title('Displacements of frame HH in X direction')
legend('vtx','vty')
grid on
subplot 122
plot([0;S_CCx_SLD_max*1000],floors,'-o')
hold on
plot([0;S_CCy_SLD_max*1000],floors,'--o')
xlabel('S_C_C [mm]')
ylabel('Floors')
title('Displacements of frame CC in Y direction')
legend('vtx','vty')
grid on
```

SLV Verifications

```
%Evluate equivalent static forces for each mode for each floor
F_HHx=KHH*S_HHx_SLV;    %Comb 1 tx
F_HHy=KHH*S_HHy_SLV;    %Comb 2 ty

F_CCx=KCC*S_CCx_SLV;    %Comb 1 tx
F_CCy=KCC*S_CCy_SLV;    %Comb 2 ty

%load N,V,M from excel
FHHtx=xlsread('Ordered N,V,M.xlsx',1,'C2:AL22');
FHHty=xlsread('Ordered N,V,M.xlsx',2,'C2:AL22');
FCCtx=xlsread('Ordered N,V,M.xlsx',3,'C2:AL22');
FCCty=xlsread('Ordered N,V,M.xlsx',4,'C2:AL22');

%initialization and division of data into N,V,M
M_HHtx=zeros(21,9); M_HHty=zeros(21,9); M_CCtx=zeros(21,9); M_CCty=zeros(21,9);
V_HHtx=zeros(21,9); V_HHty=zeros(21,9); V_CCtx=zeros(21,9); V_CCty=zeros(21,9);
N_HHtx=zeros(21,9); N_HHty=zeros(21,9); N_CCtx=zeros(21,9); N_CCty=zeros(21,9);

for k=1:N
    N_HHtx(:,k)=FHHtx(:,4*k-2); N_HHty(:,k)=FHHty(:,4*k-2); N_CCtx(:,k)=FCCtx(:,4*k-2);
N_CCty(:,k)=FCCty(:,4*k-2);
    V_HHtx(:,k)=FHHtx(:,4*k-1); V_HHty(:,k)=FHHty(:,4*k-1); V_CCtx(:,k)=FCCtx(:,4*k-1);
V_CCty(:,k)=FCCty(:,4*k-1);
    M_HHtx(:,k)=FHHtx(:,4*k);   M_HHty(:,k)=FHHty(:,4*k);   M_CCtx(:,k)=FCCtx(:,4*k);
M_CCty(:,k)=FCCty(:,4*k);
end

%initialization and calculation of CQC (combination of 9 modes)
N_HHtx_max=zeros(21,1); N_HHty_max=zeros(21,1); N_CCtx_max=zeros(21,1); N_CCty_max=zeros(21,1);
V_HHtx_max=zeros(21,1); V_HHty_max=zeros(21,1); V_CCtx_max=zeros(21,1); V_CCty_max=zeros(21,1);
M_HHtx_max=zeros(21,1); M_HHty_max=zeros(21,1); M_CCtx_max=zeros(21,1); M_CCty_max=zeros(21,1);

if CQC==1
    for z=1:21      % cycle for all the sections (21)
        n1=0;n2=0;n3=0;n4=0;
        v1=0;v2=0;v3=0;v4=0;
        m1=0;m2=0;m3=0;m4=0;
        for k=1:N   % k e j are i-th mmodes of combination
            for j=1:N
                beta=Tn(j)/Tn(k);
                rho(k,j)=(8*zita^2*beta^(3/2))/((1+beta)*((1-beta)^2+4*beta*zita^2));
                n1=n1+rho(k,j)*N_HHtx(z,k)*N_HHtx(z,j);
                n2=n2+rho(k,j)*N_HHty(z,k)*N_HHty(z,j);
                n3=n3+rho(k,j)*N_CCtx(z,k)*N_CCtx(z,j);
                n4=n4+rho(k,j)*N_CCty(z,k)*N_CCty(z,j);
                v1=v1+rho(k,j)*V_HHtx(z,k)*V_HHtx(z,j);
                v2=v2+rho(k,j)*V_HHty(z,k)*V_HHty(z,j);
                v3=v3+rho(k,j)*V_CCtx(z,k)*V_CCtx(z,j);
                v4=v4+rho(k,j)*V_CCty(z,k)*V_CCty(z,j);
                m1=m1+rho(k,j)*M_HHtx(z,k)*M_HHtx(z,j);
                m2=m2+rho(k,j)*M_HHty(z,k)*M_HHty(z,j);
                m3=m3+rho(k,j)*M_CCtx(z,k)*M_CCtx(z,j);
                m4=m4+rho(k,j)*M_CCty(z,k)*M_CCty(z,j);
            end
        end
```

POLITECNICO DI TORINO

Earthquake Engineering

Design and verification of a building subjected to seismic excitation

GROUP 14

Students

Loredana Mihaela Chiforeanu

Professor

Rosario Ceravolo

2017/2018

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 83 di 174

Introduction

This report illustrates a multi-story building that has been constructed considering the code rules related to seismic applied actions and structural response.
In particular the subject of this laboratory is the modal analysis of a 3D structure with multiple degrees of freedom (MDOF), which is the same that will be used in the project of an existing structure in reinforced concrete, plants and elevations are shown below.

The building is a reinforced concrete building, having 3 floors, just like in the following plan.

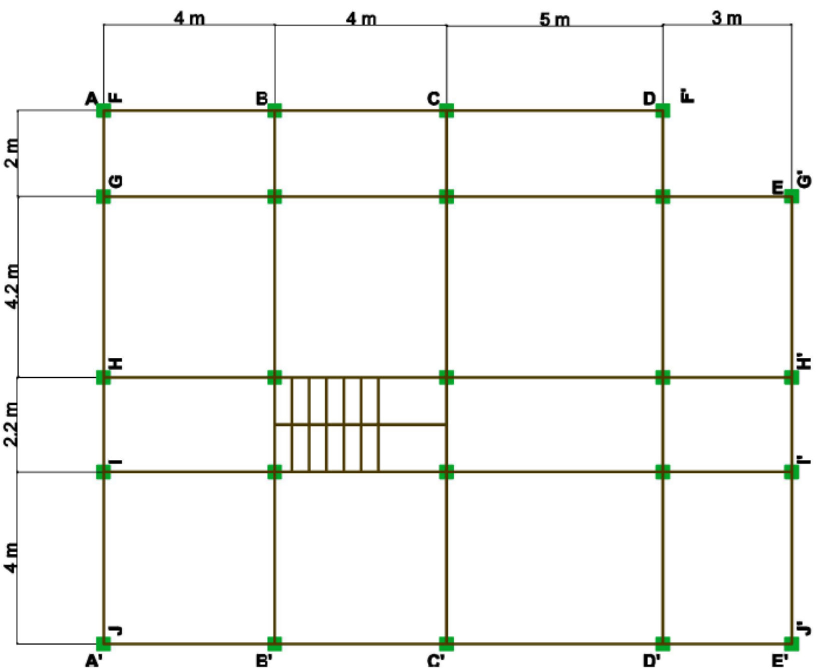


Figure 1 - Building plan

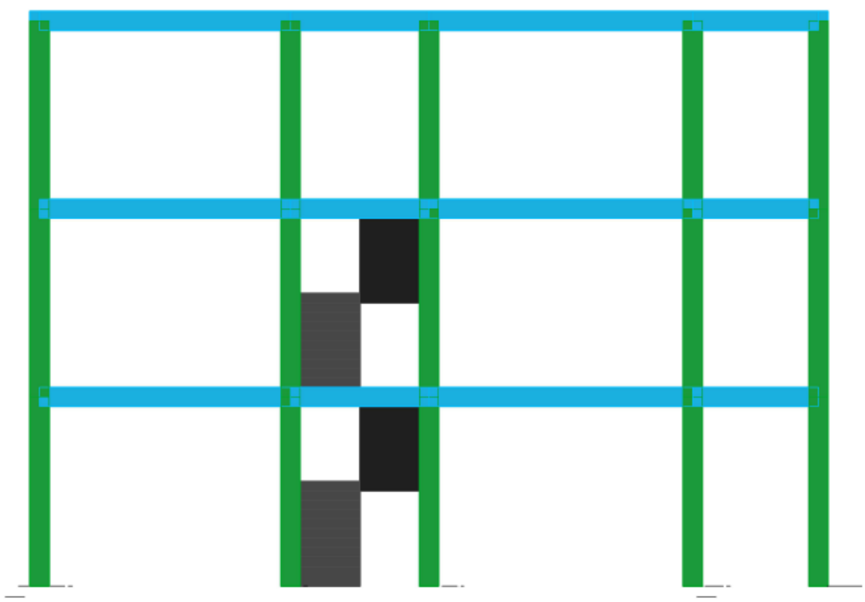
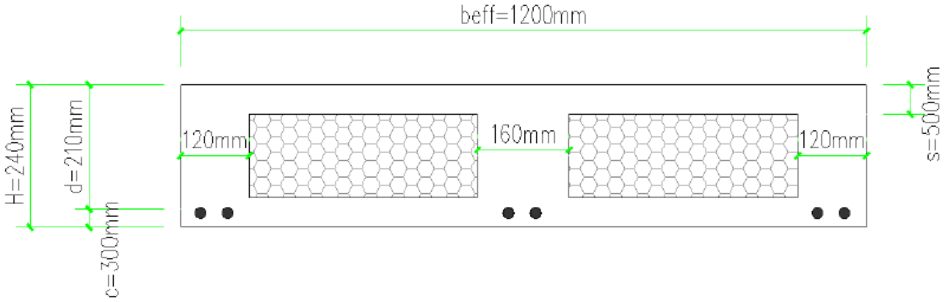


Figure 2 - Frame AA'

Geometry of the system: loads analysis

The analysis of the loads acting on the structure is now reported.
The slab appears to have the same composition on the 3 floors of the structure. We have chosen a Prèdalles slab, having the following parameters:



Dead load (G1) of the slabs:

SLAB				
Material	**γ [kN/m³]**	**H [m]**	**G1 [kN/m²]**
base of reinforced concrete	25	0.05	1.25
joists of reinf. concrete	25	0.05	1.17
polystyrene	0.3	0.09	0.028
cover of reinf. concrete	25	0.05	1.25
TOT G1			**3.7**

Slab's permanent non-structural load G2:

SLAB			
Material	**γ [kN/m³]**	**H [m]**	**G2 [kN/m²]**
plaster	20	0.015	0.3
massetto of cement mortar	21	0.05	1.05
pavement of ceramics	20	0.02	0.4
dividers			1.2
TOT G2			**3.0**

The internal walls (partitions) have been already considered in G2 of the slab: the partitions are considered, as usual as indicated by the NTC (upon condition, generally satisfied, of the capacity of the floor to distribute this load appropriately, wherever it is), as a uniformly distributed load on the entire floor.
Then the external walls and are introduced. In the creation of the mass matrix, it has been decided to leave the perimeter walls in their position, as they are not expected to be removed or significantly reduced during the lifetime of the structure.

DIVIDERS		
Material	**thickness [m]**	**γ [kN/m³]**
hollow bricks	0.12	8
plaster	0.02	20
total thickness	0.14	

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 87 di 174

$$G1 + G2 + \sum_{j}^{n} \gamma_{2j} \cdot Q_{kj}$$

Where G1 are the structural masses and G2 are the non structural but permanent masses, that the structure must bear. Q_{kj} is the j-th variable action (in terms of mass) that acts on the structure (refer to the regulation, NTC, for their evaluation). Υ_{2j} is the combination coefficient, as shown in the following table.

Tabella 2.5.I – *Valori dei coefficienti di combinazione*

Categoria/Azione variabile	ψ_{0j}	ψ_{1j}	ψ_{2j}
Categoria A Ambienti ad uso residenziale	0,7	0,5	0,3
Categoria B Uffici	0,7	0,5	0,3
Categoria C Ambienti suscettibili di affollamento	0,7	0,7	0,6
Categoria D Ambienti ad uso commerciale	0,7	0,7	0,6
Categoria E Biblioteche, archivi, magazzini e ambienti ad uso industriale	1,0	0,9	0,8
Categoria F Rimesse e parcheggi (per autoveicoli di peso ≤ 30 kN)	0,7	0,7	0,6
Categoria G Rimesse e parcheggi (per autoveicoli di peso > 30 kN)	0,7	0,5	0,3
Categoria H Coperture	0,0	0,0	0,0
Vento	0,6	0,2	0,0
Neve (a quota ≤ 1000 m s.l.m.)	0,5	0,2	0,0
Neve (a quota > 1000 m s.l.m.)	0,7	0,5	0,2
Variazioni termiche	0,6	0,5	0,0

The building is designed for use as civil dwelling, therefore the categories of use is A. The roof is not accessible except for normal maintenance and repair, the categories of use therefore is H, according to Eurocode 1, Actions on Structures Part 1-1:

- The variable loads on the floors are: 2.0 kN/m^2.
- The variable loads on the roof are: 0.5 kN/m^2.

Determination of the mass matrix [M]

We now move on to determining the global mass matrix. The following are the mass matrices, the static moments and the moments of inertia of the structure:

$$[M_{xx}] = [M_{yy}] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

$[M_{xx}] = [M_{yy}]$		
3,8080E+05	0	0
0	3,8080E+05	0
0	0	1,7380E+05

Imposing $(X_{G,i}; Y_{G,i})$ as the coordinates of the centre of mass of the i-th floor, taking into account the 5% eccentricity with respect to the centre O of the reference system, we have:

$$[M_{x\gamma}] = [M_{\gamma x}] = \begin{bmatrix} -m_1 \cdot Y_{G,1} & 0 & 0 \\ 0 & -m_2 \cdot Y_{G,2} & 0 \\ 0 & 0 & -m_3 \cdot Y_{G,3} \end{bmatrix}$$

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 89 di 174

The stiffness matrices of the frames parallel to the direction x $[H_{xx}]_i$ and of those parallel to the direction y $[H_{yy}]_i$ are shown below. Since there are two types of frames both along x and y directions, there will be two types of matrices.

$[H_{xx}]_{JJ'=II'=HH'=GG'}$		
6,8468E+08	-3,5038E+08	4,6630E+07
-3,5038E+08	5,7456E+08	-2,7858E+08
4,6630E+07	-2,7858E+08	2,3467E+08

$[H_{xx}]_{FF'}$		
5,7566E+08	-3,0606E+08	4,9046E+07
-3,0606E+08	4,7547E+08	-2,2799E+08
4,9046E+07	-2,2799E+08	1,8389E+08

$[H_{yy}]_{AA'=BB'=CC'=DD'}$		
7,2823E+08	-3,7669E+08	4,7408E+07
-3,7669E+08	6,3354E+08	-3,0807E+08
4,7408E+07	-3,0807E+08	2,6212E+08

$[H_{yy}]_{EE'}$		
5,8849E+08	-3,1073E+08	4,5750E+07
-3,1073E+08	4,9547E+08	-2,3867E+08
4,5750E+07	-2,3867E+08	1,9732E+08

Starting from these matrices the components of the global stiffness matrices are assembled following these steps:

$[H_{xx}] = \sum_i [H_{xx}]_i$

$[H_{xx}]$		
3,3144E+09	-1,7076E+09	2,3557E+08
-1,7076E+09	2,7737E+09	-1,3423E+09
2,3557E+08	-1,3423E+09	1,1226E+09

$[H_{yy}] = \sum_j [H_{yy}]_j$

$[H_{yy}]$		
3,5014E+09	-1,8175E+09	2,3538E+08
-1,8175E+09	3,0296E+09	-1,4710E+09
2,3538E+08	-1,4710E+09	1,2458E+09

$[H_{x\gamma}] = [H_{\gamma x}] = -\sum_i [H_{xx}]_i \cdot Y_i$

Eigenvalues solution

The previously written equation can then be rewritten as:

$$\begin{bmatrix} [M_{xx}] & [0] & [M_{x\gamma}] \\ [0] & [M_{yy}] & [M_{y\gamma}] \\ [M_{\gamma x}] & [M_{\gamma y}] & [M_{\gamma\gamma}] \end{bmatrix} \{\ddot{q}\} + \begin{bmatrix} [H_{xx}] & [0] & [H_{x\gamma}] \\ [0] & [H_{yy}] & [H_{y\gamma}] \\ [H_{\gamma y}] & [H_{\gamma y}] & [H_{\gamma\gamma}] \end{bmatrix} \{q\} = \{0\}$$

Being an eigenvalue problem, the "eig" function of Matlab was used. The program returns two matrices ([sha] and [lam]) using as input [H] and [M].
Specifically, [sha] is a matrix that has, in columns, the eigenvectors of the problem that correspond to the eigenvalues returned on the diagonal of [lam] (diagonal matrix with the eigenvalues on the diagonal itself).

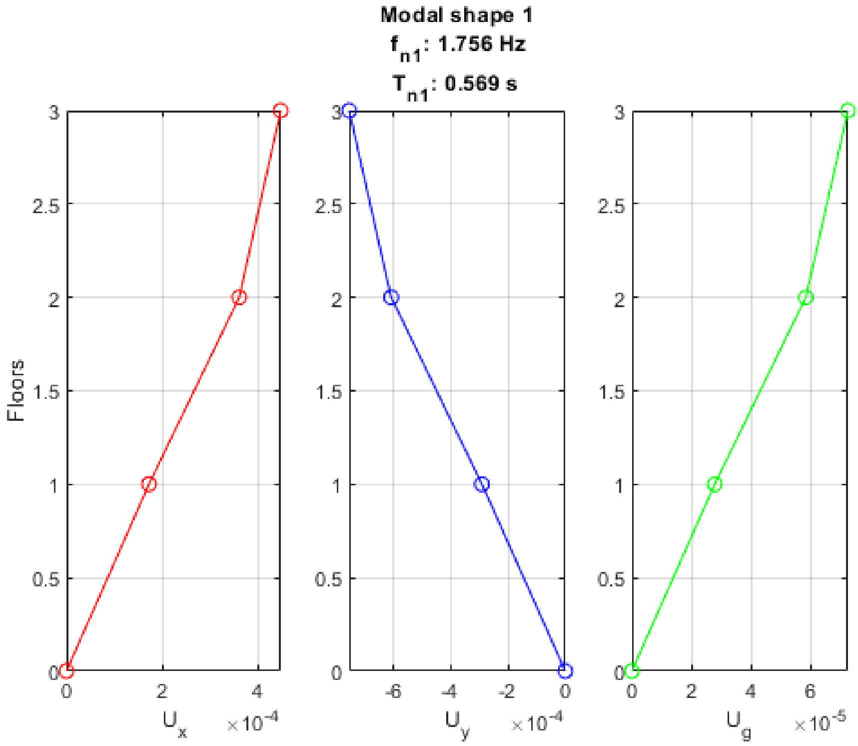
[lam]								
ω^2_1	ω^2_2	ω^2_3	ω^2_4	ω^2_5	ω^2_6	ω^2_7	ω^2_8	ω^2_9
1,2179E+02	0	0	0	0	0	0	0	0
0	7,8175E+02	0	0	0	0	0	0	0
0	0	1,0562E+03	0	0	0	0	0	0
0	0	0	2,2642E+03	0	0	0	0	0
0	0	0	0	6,8591E+03	0	0	0	0
0	0	0	0	0	8,3925E+03	0	0	0
0	0	0	0	0	0	1,4806E+04	0	0
0	0	0	0	0	0	0	7,2761E+04	0
0	0	0	0	0	0	0	0	1,5590E+05

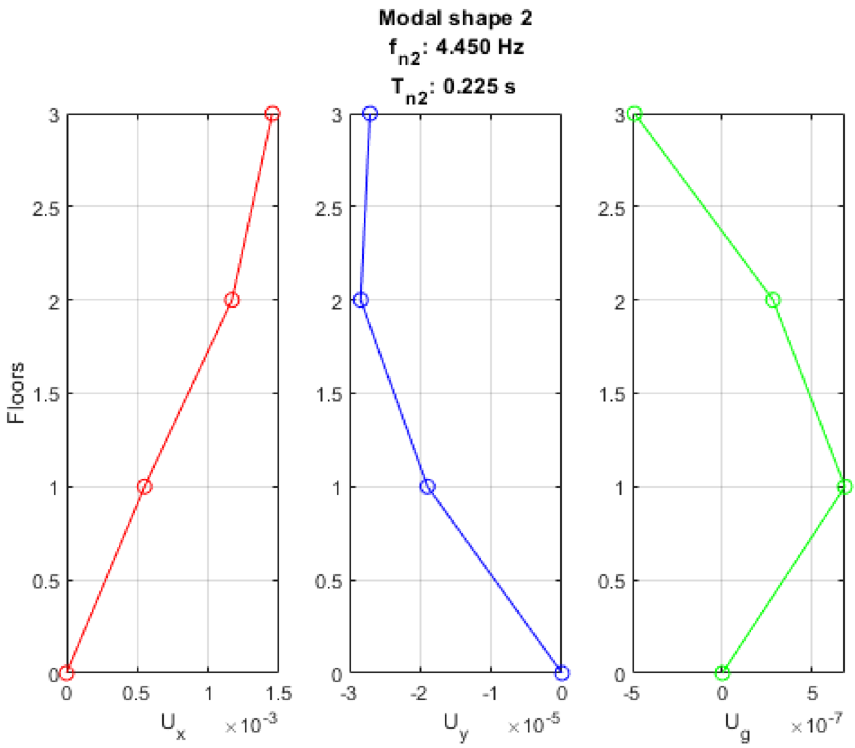
[sha]								
Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	Φ_7	Φ_8	Φ_9
1,7282E-04	5,5054E-04	-3,7993E-04	2,7776E-04	1,2160E-03	6,1207E-04	9,1302E-04	-1,3163E-03	9,2795E-04
3,6181E-04	1,1676E-03	-6,7509E-05	-3,5039E-04	2,8434E-04	1,2400E-03	-1,0924E-03	-2,5251E-04	-1,1567E-03
4,4880E-04	1,4525E-03	4,8595E-04	3,6414E-04	-1,5303E-03	1,5359E-03	1,1475E-03	1,6170E-03	1,3015E-03
-2,9080E-04	-1,8981E-05	6,4791E-04	-4,7258E-04	-3,0488E-05	9,3495E-04	-1,8393E-05	-2,0687E-03	1,4590E-03
-6,0758E-04	-2,8400E-05	1,3594E-04	5,6276E-04	-2,2763E-05	1,9587E-03	1,0125E-05	-3,9807E-04	-1,8195E-03
-7,5406E-04	-2,7082E-05	-7,8506E-04	-6,3521E-04	1,4050E-05	2,4019E-03	-2,9473E-05	2,5230E-03	2,0304E-03
2,7742E-05	6,8060E-07	-6,1132E-05	4,3179E-05	-6,2598E-07	8,8811E-05	-3,2910E-07	-1,9534E-04	1,3786E-04
5,8260E-05	2,8172E-07	-1,2574E-05	-5,5028E-05	-1,2355E-06	1,8419E-04	-9,0555E-07	-3,7449E-05	-1,7179E-04
7,2398E-05	-4,8790E-07	7,5627E-05	5,8326E-05	-2,0089E-06	2,3029E-04	-9,3317E-07	2,4155E-04	1,9460E-04

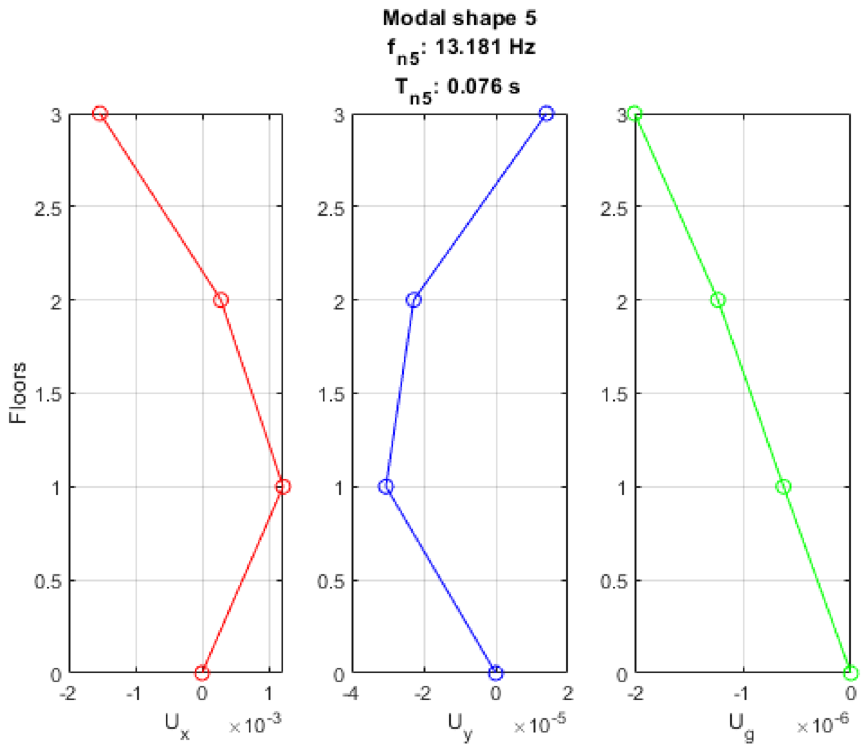
The normalization of the matrix [sha] was carried out automatically according to the following formula (normalization with respect to the mass matrix):

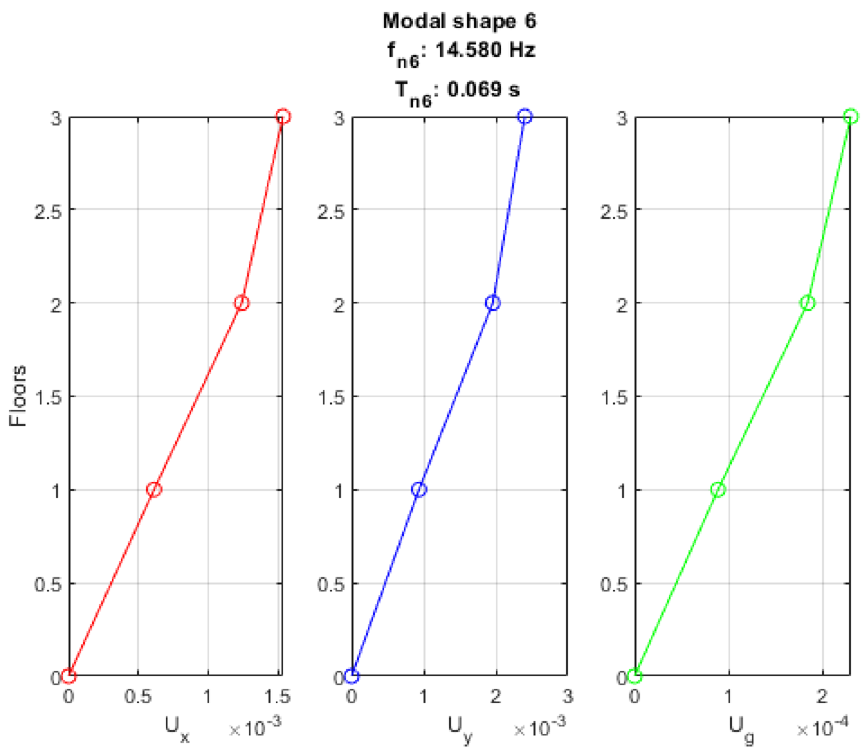
Mode 7	121,6808	19,3661	0,0516
Mode 8	269,7425	42,9309	0,0233
Mode 9	394,8431	62,8412	0,0159

The three modes of vibrating, two translational and one torsional are shown in the following 9 images (one image for each modal shape).






Modal shape 5
f n5: 13.181 Hz
T n5: 0.076 s
Floors
U x
×10-3
U y
×10-5
U g
×10-6


Modal shape 6
f n6: 14.580 Hz
T n6: 0.069 s
Floors
U x
×10-3
U y
×10-3
U g
×10-4