NUMERO: **2458A** ANNO: 2020

A P P U N T I

STUDENTE: Sobrero Giovanni

MATERIA: Machine design - Theory + schemes + exercises - Prof. Brusa

Machine Design (MD) (8 cfu)

Professor

MD: Prof. Eugenio Brusa

Giovanni Sobrero's Schemes

A.A. 2018 – 2019

2019 Program

1 – The Design Process
(3 hrs., lecture; application to the subject of the Technical Project, 1.5 hrs. tutorials)
methods, goals, activities. Needs, requirements, constraints, innovation targets. Tools and examples of design of systems and machines. Outlines of related standards. Concept, synthesis, verification and validation. Safety and reliability. Functional, operational and architectural requirements. Role of standards, best practices and modelling activity in design. Example of deployment of the whole design process.

2 – Fundamentals of Machine Design
(reading material provided; tutorial 3 hrs)
Review of applied criteria for static strength of isotropic materials.

3 – Design against failure: Fatigue and Fracture
(lectures 9 hrs, tutorials 9 hrs)
- Overview of fatigue problems (reading and self-instruction, material provided);
- Stress-life fatigue: basic material properties, specimen testing and specimen fatigue (ref. to FKM standards);
- Stress-life fatigue: component fatigue, finite and infinite life (ref. to FKM standards);
- Stress-life fatigue: thermal effects and thermomechanical fatigue behaviour, relation with creep and other effects;
- Crack propagation: linear fracture mechanics, basics, applications, Paris law;
- Tutorials: use of the main fatigue diagrams; application of FKM standards; notch effect; application to the Technical project; fracture mechanics: computation of crack propagation and path.

4 – Design of Assemblies: Supports and Bearings
(lectures 9 hrs, tutorials 9 hrs)
- Contact mechanics and damage (reading and self-instruction, material provided) ;
- Rolling bearings: static loading, fatigue conditions;
- Design of bearing assemblies, main solutions for bearing arrangements;
- Tutorials: application of Hertz theory on a selection of contact cases; angular contact bearings, preload diagram; load-life rating of the bearings for the Technical Project; bearing assemblies and related problems;

5 – Design of Power Transmission: Gears
(lectures 9 hrs, tutorials 9 hrs)
- Summary of motion transmission, tooth shape (reading and self-instruction, material provided);
- Spur and helical gears with parallel axes: kinematics, geometry, forces;
- Cutting techniques and profile displacement;
- Criteria for strength assessment of gears: fatigue, hertz contact, wear, scuffing;
- Tutorials: geometry and kinematics of gears, spur gears profile shift, design of the gears of the technical Project;

6 – Design of joining systems: Bolted Connections

(lectures 6 hrs, tutorials 6 hrs)

- Threaded fasteners and connections (reading and self-instruction, material provided);
- Prestressed single bolt connections (non-gasketed);
- Refinements and special problems;
- Elements of gasketed bolted connections;
- Tutorials: selection of overview exercises;
- Tutorials: application to a hydraulic piston or to a tie-rod connection;

7 – Advanced Topics

(Seminar lecture 3 hrs.)

Overview of some widely-used technologies based on the electromechanical energy conversion and Multiphysics modelling in designing systems and machines. Adaptive systems. Role of the product scale. Special, functional and new materials. Examples and current applications.

8 – Seminars, visits, unplanned teaching and student support

(lectures 1.5 hrs, tutorials 3 hrs)

Teaching organization

Credits 8, 81 classroom hours (40.5 lecture hours, 40.5 tutorial hours). The total study load for this subject is 200 to 240 total hours, i.e., 25 to 30 hours per credit. This includes classroom hours, self-study, completion of tutorials at home and reporting.
Class hours are equally shared between theoretical lectures and application tutorials, in order to achieve a balance between knowledge and skills. The subject is organised to allow students to progress incrementally in the development of their knowledge and skills under expert supervision.
All lecture materials will be made available on the subject unit website before the class activity. Students are urged to download or print them so to have them at hand to take notes.
Lectures on a section of the material will be followed by specific tutorials, where students are required to apply knowledge to working context problems. The tutor will provide organised materials and frames for solutions. However, the students will solve the proposed tasks themselves in small groups (max 3 students)
Moreover, there will be a semester-long project, the so – called "Technical project": in order to enhance problem solving capabilities, encourage independent thinking and develop professional reporting skills.
For each task, each group of students will produce a final report. The set of all reports will be examined during the final exam. Students are asked to work cooperatively in a small group.
The tutor will assist the groups during the tutorial class hours, supporting students in their learning progression and clarifying their doubts. Attendance to both lectures AND tutorials is strongly recommended, this being vital to achieve the expected learning outcomes. The teacher and the tutor are available weekly during the teaching period in order to meet students for consultation; please contact them by e-mail.
Tutorials may benefit from using EXCEL or MATLAB. Writing reports with editing software is not required. Although a clear and professional presentation is strictly required. Drawing tools (pencil, compass, scale rulers...) are necessary.

Required or recommended texts: readings, handouts, other educational material

Lectures: the subject is fully treated in the slides provided by the teacher. Reference textbooks of international standing are suggested (some herein at the end). Some additional slides cover several topics belonging the required background of students (mainly fundamentals of strength of materials and of machine design)
Tutorials: texts of problems, datasheets of materials, extracts or abstracts of standards and handbooks will be provided by the tutor. All lecture materials will be made available on the subject website before the lecture. Students should either download or print the files before the lecture and use the copy to facilitate taking notes.

Selection of references (just suggested readings):

- P.R.N. Childs, Mechanical design engineering handbook, Elsevier, 2014.
- R.C. Juvinall, K.M. Marshek, Fundamentals of machine component design, John Wiley & Sons, 2011
- R. Budynas, Shigley's mechanical engineering design, McGraw Hill, 2014.
- R.Stephens,A.Fatemi,R.Stephens,H.Fuchs, Metal fatigue in engineering, Wiley,2000.
- N. Recho, Fracture mechanics and crack growth, ISTE Wiley, 2012.
- S.S.Manson, Fatigue and durability of metals at high temperatures,ASM, 2009.
- V.Popov, Contact mechanics and friction, Springer, 2010.
- F.Litvin, Gear geometry and applied theory, Cambridge University Press, 2004.
- T. Harris, M. Kotzaias, Rolling bearing analysis – Essential concepts of bearing technology, CRC,2006.
- J. Bickford, Introduction to the design and behaviour of bolted joints, CRC Press, 2007
- J.E.Carryer, Introduction to mechatronic design, Pearson, 2010.

Criteria, rules and procedures for the exam

Achieved learning outcomes will be assessed by means of a final exam. This is based on an analytical assessment of student achievement of the "expected learning outcomes" described above.
In order to properly assess such achievement, the examination is composed of different sections:
a) Written session, day 1:
-) A test, duration 2 hrs, closed books, composed of three questions, two on chapters or sections of the lectures to assess knowledge, one problem to assess problem solving skills; each question scores max 8 points.
To be eligible to attend the oral exam the student must score in the written test a minimum total of 12 points with a minimum of 4 (50%) points for each question.
b) The oral session, day 2:
-) A review of the written output, in which examiners inform the student on grading criteria, and receive any student appeal supported by appropriate explanations;
-) A technical conversation with the lecturer to bring out the ability to deal responsibly with a mechanical design problem identifying an appropriate application of acquired knowledge (max additional 4 points);
-) A discussion with the tutor, to whom the student will submit the full set of tutorial reports; the tutor will investigate the effective personal achievement of skills and know how based on contents of reports (max additional 4 points).

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0.2) WHAT IS A TRADE-OFF ACTIVITY?

"TRADE-OFF" REFERS TO WHAT KIND OF ARCHITECTURE AND OF TECHNOLOGY CAN BE USED IN THE DESIGN PROCESS.

+ WHAT'S THE DESIGN ANALYSIS?

THE DESIGN ANALYSIS MUST BE PERFORMED TO EVALUATE THE REQUIREMENTS AND THE ARCHITECTURE IN ORDER TO PROVIDE A PROJECT SUMMARY.

AS SOON AS THE PROCESS IS APPLIED, SEVERAL ANALYSES ARE PERFORMED.

(THUS INVOLVING ALL THE TEACHING MODULES WE ARE STUDYING: BEO, MMI, MSD, THM, NM, AET, IMS)

THEREFORE, IT INCLUDES:

ANALYTICAL APPROACHES, NUMERICAL APPROACHES, GEOMETRICAL MODELLING, EMPIRICAL RULES, ECONOMIC ANALYSES, ... + *1

0.3) WHAT ARE THE MAIN ACTIVITIES OF DESIGN SYNTHESIS?

THE DESIGN SYNTHESIS BASICALLY PROVIDES A CONSOLIDATION OF MAIN DETAILS OF THE PRODUCT, THROUGH A PRELIMINARY VERIFICATION AND VALIDATION AND OF REQUIREMENTS ALLOCATION.

- **VERIFICATION**: PRODUCT DESIGNED WAS EXACTLY WHAT WE CONCEIVED AT THE BEGINNING OF DEVELOPMENT (IT'S WHAT WE THOUGHT).
- **VALIDATION**: PRODUCT MANUFACTURED WILL BE EXACTLY WHAT CUSTOMER ASKED (IT'S WHAT HE/SHE THOUGHT).
- **ALLOCATION**: FOR EACH FUNCTION OF THE MACHINE (+) THERE IS A COMPONENT OR A SET OF COMPONENTS PROVIDING THAT FUNCTION. + *2

+ WHAT IS A MACHINE?

A MACHINE IS AN ASSEMBLY OF MECHANICAL COMPONENTS ALL CONCURRING TO CONVERT ENERGY, TO PROVIDE A DEFINE FUNCTION IN OPERATION, USUALLY THROUGH THE WORK OF MOVEABLE INNER PARTS.

0.4) WHO/WHAT IS A STAKEHOLDER FOR PRODUCT DESIGN?

STAKEHOLDER IS EACH OF THE SUBJECTS (PROJECT COMPONENTS OR PEOPLE) DIRECTLY OR INDIRECTLY INVOLVED IN A PROJECT OR IN THE ACTIVITY OF A COMPANY.

(WHAT) CONSIDERING A KAPLAN TURBINE, STAKEHOLDERS CAN BE: THE WATER INJECT, THE ELECTRIC GENERATOR, THE PROCESS CONTROL, THE OPERATOR AND THE ENVIRONMENT.

(WHO) CONSIDERING THE PRODUCT DESIGN ACTIVITY, STAKEHOLDERS ARE ALL THOSE PEOPLE WHO HAVE A CERTAIN ROLE IN THE PRODUCT DESIGN. EVERY STAKEHOLDER HAS DIFFERENT NEEDS AND GOALS ACCORDING TO HIS ROLE. (BUSINESS REPRESENTATIVES, PROJECT MANAGER, TECHNICAL EXPERTS

*[1] THE **ANALYSIS** ALLOWS THE DESIGNER:
- **REFINING** THE REQUIREMENTS;
- **ASSESSING** THE MACHINE ARCHITECTURE;
- **COMPLETING** THE DESIGN STEP OF PRODUCT DEVELOPMENT;

*[2] ONCE THAT THIS STEP (ANALYSIS) IS COMPLETED, A **FINAL SYNTHESIS** IS REQUIRED TO:
- **DELIVER** A CONSOLIDATED PROJECT TO THE PRODUCTION LINE;
- **PERFORM** ALL THE CHECKS ABOUT RELIABILITY, SAFETY, MAINTAINABILITY AND COST.
- **INTEGRATING** THE MACHINE WITHIN A LARGER SYSTEM, IF PRESENT.

Q.7) WHAT'S VERIFICATION?

VERIFICATION: **PRODUCT DESIGNED** WAS EXACTLY WHAT WE CONCEIVED AT THE BEGINNING OF DEVELOPMENT (IT IS WHAT WE THOUGHT).

NOTE: VERIFICATION IS OFTEN PERFORMED ON A **VIRTUAL MOCK-UP**, I.E. A NUMERICAL MODEL OF THE MACHINE.

Q.8) WHAT'S VALIDATION?

VALIDATION: **PRODUCT MANUFACTURED** WILL BE EXACTLY WHAT CUSTOMERS ASKED (WHAT HE/SHE THOUGHT).

NOTE: VALIDATION ALWAYS REQUIRES AN **EXPERIMENTAL EVIDENCE** BASED ON A WHOLE PROTOTYPE OR/AND MACHINE COMPONENT.

Q.9) WHAT'S ALLOCATION?

ALLOCATION: **FOR EACH FUNCTION OF THE MACHINE** THERE IS A COMPONENT OR A SET OF ASSEMBLED COMPONENTS PROVIDING THAT FUNCTION.

Q.10) WHERE ALL CONCEPTS HERE ABOVE MENTIONED APPEAR IN YOUR TECHNICAL REPORT?

...

2. DAMAGE

2.1 DEFINE: YIELD, RUPTURE, BUCKING, CREEP, FATIGUE, IMPACT, CORROSION, WEAR, FRETTING;

STATIC LOAD (CONSTANT LOAD IN TIME):

1) YIELDING (YIELD STRESS / STRENGTH)

ELASTIC-PLASTIC BEHAVIOUR OF DUCTILE MATERIAL ABOVE A STRESS THRESHOLD LEADING TO HAVE A PLASTIC STRAIN, AT CONSTANT TEMPERATURE, WITH SLOWLY INCREASING LOADING CONDITION. [$\sigma_y = R_{eH} = R_{p0.2}$]

2) RUPTURE

MATERIAL BREAKING IN BOTH DUCTILE AND BRITTEL MATERIALS, REACHED AT CONSTANT TEMPERATURE, WITH SLOWLY INCREASING LOADING CONDITIONS [$R_m = \sigma_{UTS}$].

3) BUCKLING

COLLAPSE OF STRUCTURE (INDIPENDENTLY ON RUPTURE) UNDER COMPRESSION IN SLENDER ELEMENTS, DUE TO ELASTIC INSTABILITY.

4) CREEP

MATERIAL FLOW INDUCED BY HIGHER TEMPERATURE (ABOVE A THRESHOLD) EVEN WHEN LOADED BY A CONSTANT LOADING.

DYNAMIC DOMAIN (VARIABLE LOAD IN TIME):

1) FATIGUE

REDUCED OF STRENGTH OF MATERIAL UNDER EFFECT OF CYCLIC LOADING CONDITION.

2) IMPACT

RUPTURE OF MATERIAL UNDER AN IMPULSIVE LOAD INTRODUCING A LEVEL OF ENERGY LARGER THAN THE AMOUNT LEADING TO HAVE A PURE ELASTIC-PLASTIC WORK ON THE MATERIAL.

3) CORROSION

MATERIAL DEGRADATION DUE TO CORROSIVE AGENT; EVEN STRESS-CORROSION IF LOADED.

4) WEAR

MATERIAL DAMAGE TO FRICTION BETWEEN SURFACES.

5) FRETTING

WEAR UNDER LOADING CONDITION, TYPICALLY ASSOCIATED TO VIBRATION, EVEN TO CORROSION (FRETTING CORROSION).

6) OTHERS

(DELAMINATION IN COMPOSITES, SHORT-CIRCUIT IN PIEZOCERAMICS, ...)

EXAMPLES:

STRUCTURAL STEEL: S235 ($\sigma_y = 235$ [MPa]; A% = 26%)
S275 ($\sigma_y = 275$ [MPa]; A% = 22%)
S355 ($\sigma_y = 355$ [MPa]; A% = 22%)

ANNEALED STEEL: C30 ($\sigma_y = 400$ [MPa]; A% = 18%)
C60 ($\sigma_y = 580$ [MPa]; A% = 11%)
41CrG ($\sigma_y = 800$ [MPa]; A% = 11%)
36NiCrMo3 ($\sigma_y = 1050$ [MPa]; A% = 9%)

2.4) WHICH ARE THE DIFFERENCES BETWEEN BRITTLE AND DUCTILE MATERIALS?

BRITTLE MATERIAL BEHAVIOUR:

- ELASTIC DEFORMATION: UNDER A TENSILE LOAD, THE MATERIAL UNDERGOES A DEFORMATION ACCORDING TO $\sigma = \varepsilon E$. REMOVED THE LOAD, IT TURNS TO THE (ORIGINAL SHAPE) INITIAL STATE. HOWEVER, IF THE STRESS REACHES A LIMIT VALUE ($\sigma \geq R_m$ OR σ_{UTS}) THEN THE MATERIAL UNDERGOES A FRACTURE (BRITTLE MATERIAL).

$\varepsilon = \frac{\sigma}{E}$ (=> IF $\sigma \nearrow$ => $\varepsilon \nearrow$) (A% $= 100 \cdot \frac{L_u - L_o}{L_o}$; A% < 5% => BRITTLE MATERIAL)

E IS THE YOUNG MODULUS (ELASTIC PROPERTY) [MPa] (=> IF $E \nearrow$ => $\varepsilon \searrow$) DUCTILE M.

($E_{Fe} \cong 210$ [GPa]; $E_{STEEL} \cong 190 \div 215$ [GPa]; $E_{Al} \cong 64$ [GPa]; $E_{Al\ Alloys} \cong 70 \div 80$ [GPa])

EXAMPLE OF BRITTLE MATERIALS: GREY CAST IRON: G10, G20, G30.

DUCTILE MATERIAL BEHAVIOUR:

- YIELD POINT: IT IS THE POINT ON THE STRESS-STRAIN CURVE THAT INDICATES THE LIMIT OF ELASTIC BEHAVIOUR AND THE BEGINNING OF PLASTIC BEHAVIOUR.

YIELDING MEANS THE START OF BREAKING OF FIBRES.

YIELD STRENGTH OR YIELD STRESS (σ_y): IS THE MAT. PROPERTY DEFINED AS THE STRESS AT WHICH A MATERIAL BEGINS TO DEFORM PLASTICALLY, WHEREAS YIELD POINT IS THE POINT WHERE NON LINEAR (ELASTIC + PLASTIC) DEFORMATION BEGINS. (THE YIELD POINT DETERMINES THE LIMITS OF PERFORMANCE FOR MECHANICAL COMPONENTS, SINCE IT REPRESENTS THE UPPER LIMIT TO FORCES THAT CAN BE APPLIED WITHOUT PERMANENT DEFORMATION).

OFFSET YIELD POINT (OR PROOF STRESS) (R_{p02}): IS THE STRESS AT WHICH 0.2% PLASTIC DEFORMATION OCCURS. (THEORETICALLY IT SHOULD BE THE POINT WHERE THE SLOPE OF THE CURVE CHANGES WITH RESPECT TO THE STRETCH RELATIVE TO THE ELASTIC PHASE; BEING DIFFICULT TO IDENTIFY, BY CONVENTION WE CONSIDER AS σ_y THE POINT CORRESPONDING TO A PERMANENT DEFORMATION EQUAL TO 0.2%: $\sigma_y = R_{p02}$)

σ_{sup} OR R_{eH} (CONSIDERING DUCTILE MATERIAL WITH EVIDENT YIELD): IS THE LOAD NECESSARY TO START THE MOVEMENT OF THE DISLOCATIONS.

2.5) HOW THE MATERIAL CURVE (STRESS-STRAIN) DEPENDS ON THE STRAIN RATE ($\dot{\varepsilon}$)?

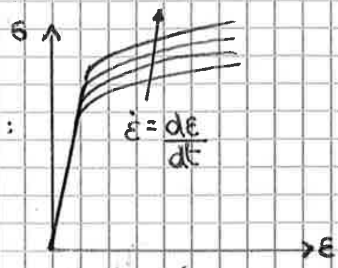
STRAIN RATE ($\dot{\varepsilon}$) - ELASTIC DEFORMATION

$\dot{\varepsilon} = \frac{d\varepsilon}{dt}\ [s^{-1}]$, $\left(\varepsilon = \frac{\sigma}{E}\ \text{STRAIN}\right)$

STEEL: $6 \leq \frac{\Delta\sigma}{\Delta t} \leq 30 \left[\frac{N/mm^2}{s}\right] \Rightarrow \dot{\varepsilon} = \frac{\Delta\varepsilon}{\Delta t} \left(= \frac{\Delta\sigma}{E\Delta t} = \frac{30\,[N/mm^2]}{210000\,[N/mm^2][s]}\right) \leq 1.43 \cdot 10^{-4}\ [s^{-1}]$

ALUMINIUM ALLOY: $2 \leq \frac{\Delta\sigma}{\Delta t} \leq 10 \left[\frac{N/mm^2}{s}\right] \Rightarrow \dot{\varepsilon} = \frac{\Delta\varepsilon}{\Delta t} \left(= \frac{\Delta\sigma}{E\Delta t} = \frac{10\,[N/mm^2]}{73000\,[N/mm^2][s]}\right) \leq 1.37 \cdot 10^{-4}\ [s^{-1}]$

STRAIN RATE ($\dot{\varepsilon}$) - YIELDING



YIELDING DEPENDS ON THE STRAIN RATE ($\dot{\varepsilon}$).

COWPER AND SYMONDS SUGGESTED THE CONSTITUTIVE EQUATION:

$\dot{\varepsilon} = D\left(\frac{\sigma_0'}{\sigma_0} - 1\right)^q$, $\sigma_0' \geq \sigma_0$

WITH:

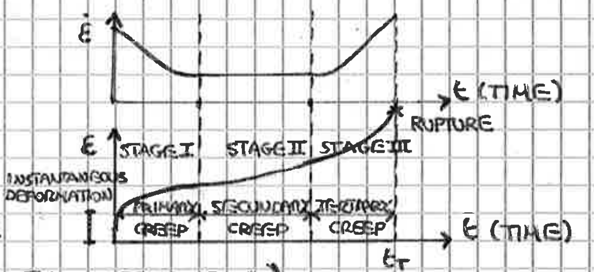
σ_0' = DYNAMIC FLOW STRESS, AT A UNIAXIAL PLASTIC STRAIN RATE $\dot{\varepsilon}$;

σ_0 = STATIC FLOW STRESS

D, q = CONSTANTS FOR A CERTAIN MATERIAL

STRAIN RATE ($\dot{\varepsilon}$) - CONSIDERING THE CREEP PHENOMENON

GENERAL CASE (VISCOUS FLOW DUE TO CREEP AS A FUNCTION OF TIME):



$\varepsilon = f(\sigma, T, t)$

$\varepsilon = kt^x$

$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = xkt^{(x-1)} = k't^{-m}$, $m = 1 - x$

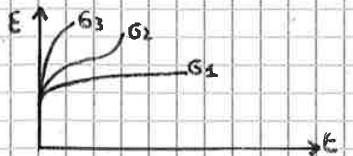
REFERRING TO STAGE I (PRIMARY CREEP):

$\dot{\varepsilon} = C_1 \sigma^{C_2} \varepsilon^{C_3} e^{-\frac{C_4}{T}}$ (VARIABLE: DECREASING WITH RESPECT TO t)

REFERRING TO STAGE II (SECONDARY CREEP):

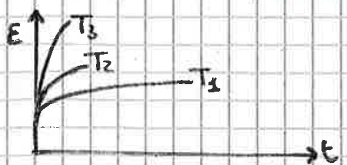
$\dot{\varepsilon} = C_5 \sigma^{C_6} e^{-\frac{C_7}{T}}$ (CONSTANT WITH RESPECT TO t) ($C_1, \dots, C_7$ CONSTANTS TYP. OF MATERIAL)

I. CASE: T = CONST, SEVERAL STRESS LEVELS ($\sigma_3 > \sigma_2 > \sigma_1$)



$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = C\sigma^m$; C, m = CONSTANTS OF MATERIAL (DEPENDING ON T)

II. CASE: σ = CONST, SEVERAL TEMPERATURES ($T_3 > T_2 > T_1$)



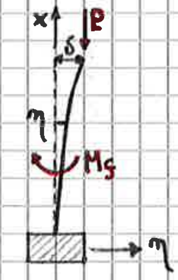
$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = Ae^{-\frac{Q}{RT}}$, A = CONSTANT OF MATERIAL; R = GAS CONST

Q = ACTIVATION ENERGY; T = ABS. TEMP.

III. CASE: σ = CONST, T = CONST

$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = C_5 \sigma^{C_6} e^{-\frac{C_7}{T}} = k\sigma^m e^{-\frac{Q}{RT}}$

2.7) DESCRIBE THE BUCKLING PROBLEM. WHAT ARE CRITICAL LOAD (P_{CR}) AND STRESS (σ_{CR})?



BUCKLING OF SLENDER BAR

δ IS WHAT LEADS TO INSTABILITY.

WE WANT TO FIND WHAT IS THE MINIMUM LOAD TO SEND THE BEAM INTO INSTABILITY ($\Rightarrow P_{CR}$)

$$M_f + P(\delta - \eta) = 0$$

$$\frac{d^2\eta}{dx^2} = -\frac{M_f}{EJ} \Rightarrow M_f = -\frac{d^2\eta}{dx^2}EJ$$

$$-\frac{d^2\eta}{dx^2}EJ + P\delta - P\eta = 0$$

$$\frac{d^2\eta}{dx^2} + \frac{P}{EJ}\eta = \frac{P}{EJ}\delta \quad , \left(\omega = \sqrt{\frac{P}{EJ}}\right)$$

GENERAL SOLUTION:

$$\eta = A\cdot\cos\sqrt{\frac{P}{EJ}}\cdot x + B\cdot\sin\sqrt{\frac{P}{EJ}}\cdot x + \delta$$

- WHEN $x = 0 \Rightarrow \eta = 0 \; ; \; \frac{d\eta}{dx} = 0 \Rightarrow 0 = A\cos(0) + \delta \Rightarrow \begin{cases} B = 0 \\ A = -\delta \end{cases}$

WE OBTAIN:

$$\eta = -\delta\cos\sqrt{\frac{P}{EJ}}x + \delta = \delta\left(1 - \cos\sqrt{\frac{P}{EJ}}x\right)$$

- WHEN $x = L \Rightarrow \eta = \delta$

$$\delta = -\delta\cos\sqrt{\frac{P}{EJ}}L + \delta = \delta\left(1 - \cos\sqrt{\frac{P}{EJ}}L\right)$$

THEREFORE:

$$\cos\sqrt{\frac{P}{EJ}}L = 0 \iff \sqrt{\frac{P}{EJ}}L = \frac{\pi}{2} + m\pi$$

SINCE WE ARE NOT INTERESTED IN ALL SOLUTIONS BUT ONLY THE LOWEST:

$$\sqrt{\frac{P_{CR}}{EJ}}L = \frac{\pi}{2} \Rightarrow \frac{P_{CR}}{EJ}L^2 = \frac{\pi^2}{4}$$, WE OBTAIN:

$$P_{CR} = \frac{\pi^2 EJ}{4L^2}$$ EULER'S CRITICAL LOAD

GENERALLY: $P_{CR} = \frac{\pi^2 EJ}{\ell_0^2}$, WITH ℓ_0 = FREE FLEXURAL LENGTH DEPENDING ON CONSTRAINS

$\begin{cases} \text{APP.-APP.}: \ell_0 = L \\ \text{INC.-LIB.}: \ell_0 = 2L \\ \text{INC.-INC.}: \ell_0 = L/2 \\ \text{INC.-APP.}: \ell_0 = 0.7L \end{cases}$

AND THE STRESS:

$$\sigma_{CR} = \frac{P_{CR}}{A} = \frac{\pi^2 EJ}{\ell_0^2 A}$$

CONSIDERING: $\rho_{min}^2 = \frac{J}{A}$; $\lambda = \frac{\ell_0}{\rho}$ $\left(\frac{J}{A} = \rho_{min}^2 \; ; \; \frac{1}{\ell_0^2} = \frac{1}{\lambda^2\rho^2}\right)$, WE OBTAIN:

$$\sigma_{CR} = \frac{\pi^2 E}{\lambda^2}$$ CRITICAL STRESS

THIS STRESS VALUE MIGHT BE LESS THAN YIELDING STRESS ($\sigma_y = R_{p02}$) OR RUPTURE (R_m).

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+ WHAT ARE DISLOCATIONS? DESCRIBE THEIR MECHANISM.

DISLOCATIONS ARE LINE DEFECTS THAT SLIP THROUGH A CRYSTAL LATTICE WHEN A MINIMUM SHEAR STRESS IS APPLIED. THEY INITIALLY SLIP ALONG THE CLOSEST PACKED PLANES SINCE THIS REQUIRES THE LEAST ENERGY OR APPLIED STRESS.

- EDGE DISLOCATIONS;
- SCREW DISLOCATIONS;

AN EDGE DISLOCATION CONSISTS OF AN UNFINISHED ATOMIC PLANE, THE EDGE OF THE PLANE BEING THE LINE OF THE DISLOCATION, AND IT IS CHARACTERISED BY A BURGERS VECTOR, AT 90° TO THE DISLOCATION LINE.

SCREW DISLOCATIONS HAVE A BURGERS VECTOR PARALLEL TO THE DISLOCATION LINE AND CAN SLIP ON ANY CLOSE PACKED PLANE CONTAINING BOTH LINE AND BURGERS VECTOR, WHEREAS EDGE DISLOCATIONS ONLY SLIP ON THE PLANE DEFINED BY THE LINE AND THE PERPENDICULAR BURGERS VECTOR.

WHEN A POLYCRISTALLINE METAL IS DEFORMED, MANY OF THE GRAINS WILL HAVE SLIP PLANES SUITABLY ALIGNED TO THE APPLIED STRESS, AND SLIP WILL OCCUR WHEN THE SHEAR STRESS EXCEEDS THE CRITICAL RESOLVED SHEAR STRESS. DISLOCATIONS MULTIPLY AT SOURCES, THUS INCREASING THE DISLOCATION DENSITY. MORE DISLOCATIONS INTERACTIONS OCCUR AND SOMETIMES THEY INHIBIT THE SLIP. PARTICULARLY, DISLOCATIONS ARRIVING AT GRAIN BOUNDARIES MAY NOT BE ABLE TO SLIP INTO THE NEXT GRAIN DUE TO A DIFFERENT ORIENTATION. THOSE MECHANISMS CONTRIBUTE TO WORK HARDENING OF METAL, I.E. AS THE METAL IS DEFORMED, PLASTIC FLOW BECOMES INCREASINGLY MORE DIFFICULT AND MORE STRESS HAS TO BE APPLIED TO ACCOMPLISH IT.

IN THE PRIMARY STAGE OF CREEP TEST, THE STRESS IS CONSTANT ($\sigma = const$), AND THESE MECHANISMS LEAT TO A DECREASING STRAIN RATE ($\dot{\varepsilon} \searrow$).

[STAGE I: $\sigma = const$; ($\varepsilon \nearrow$); $\dot{\varepsilon} \searrow$ $\Rightarrow$ APPARENT HARDENING]

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STAGE III (TERTIARY CREEP) [GRAIN BOUNDARY SLIDING]

THE START OF TERTIARY CREEP IS A SIGN THAT STRUCTURAL DAMAGE HAS OCCURRED IN AN ALLOY.

ROUNDED AND WEDGE-SHAPED VOIDS ARE SEEN.

THE MECHANISM OF VOID FORMATION INVOLVES GRAIN BOUNDARY SLIDING WHICH OCCURS UNDER THE ACTION OF SHEAR STRESSES ACTING ON THE BOUNDARIES.

EVIDENCE FOR GRAIN BOUNDARY SLIDING IS THE DISPLACEMENT OF SCRATCH LINES DURING A CREEP TEST.

THE GRAIN BOUNDARY SLIDING MAY ACCOUNT FOR 10% TO 65% OF THE TOTAL CREEP STRAIN; THE CONTRIBUTION INCREASES WITH INCREASING TEMPERATURE AND STRESS, AND REDUCES THE GRAIN SIZE.

ABOVE ABOUT $0.6 T_m$ THE GRAIN BOUNDARY REGION IS THOUGHT TO HAVE A LOWER SHEAR STRENGTH THAN THE GRAINS THEMSELVES. BOUNDARIES LYING AT ABOUT 45° TO THE APPLIED TENSILE STRESS EXPERIENCE THE LARGEST SHEAR STRESS AND WILL SLIDE THE MOST.

[DIFFUSION FLOW]

THE THIRD DISTINCT MECHANISM FOR CREEP IS SIGNIFICANT AT LOW STRESS AND HIGH TEMPERATURE.

UNDER THE DRIVING FORCE OF THE APPLIED STRESS, ATOMS DIFFUSE FROM THE SIDES OF THE GRAINS TO THE TOPS AND BOTTOMS.

THE GRAIN BECOMES LONGER AS THE APPLIED STRESS DOES WORK, AND THE PROCESS WILL BE FASTER AT HIGH TEMPERATURES AS THERE ARE MORE VACANCIES.

ATOMIC DIFFUSION IN ONE DIRECTION IS THE SAME AS VACANCY DIFFUSION IN THE OPPOSITE DIRECTION.

THROUGH THE GRAINS, THE ATOMS HAVE A SLOWER JUMP FREQUENCY, BUT MORE PATHS.

ALONG THE GRAIN BOUNDARIES, THE JUMP FREQUENCY IS HIGHER, BUT FEWER PATHS EXIST.

THE RATE CONTROLLING MECHANISM IS AGAIN VACANCY DIFFUSION, OR SELF-DIFFUSION.

THEREFORE, AT HIGHER TEMPERATURE, STRENGTH OF GRAINS IS LOWER, GRAIN BOUNDARIES SLIP, AND THERE IS ALSO A DIFFUSION FLOW DUE TO TEMPERATURE THUS LEADING TO MORE VACANCIES AND MORE CLIMB AND MOTIVATING THE RUPTURE OF MATERIAL.

3. FATIGUE

3.1) DEFINE FATIGUE PHENOMENON APPLIED TO A ROTATING SHAFT.

- **FATIGUE PHENOMENON** IS ASSOCIATED TO **CYCLIC LOADING CONDITIONS**.
- **STRENGTH IS REDUCED** AS LONG AS STRUCTURE IS REPEATEDLY LOADED.
- **RUPTURE SUDDENLY OCCURS** AND IS **BRITTLE EVEN IN DUCTILE MATERIAL**.
- IT IS AFFECTED BY SEVERAL EFFECTS ASSOCIATED TO THE **GEOMETRY**, THE **SURFACE TREATMENT**, THE **MANUFACTURING PROCESS** OF THE ANALYSED MECHANICAL COMPONENT.
- IT REQUIRES A DEEP INVESTIGATION TO IDENTIFY SOME SUITABLE **TOOLS** TO BE APPLIED WITHIN THE **DESIGN ACTIVITY**.
- **UNDER CYCLIC LOAD** AN INITIATION OF FAILURE OCCURS AT **SURFACE** OF MATERIAL AS A **LOCAL MICROCRACK**, THEN IT **GROWS UP AND PROPAGATES** INITIALLY **STABLY**, THEN **UNSTABLY** THUS LEADING TO A **FINAL BRITTLE FRACTURE**.
- **STRESS INDUCING THE FATIGUE RUPTURE** IS **LOWER THAN** TYPICAL REFERENCES OF **YIELDING AND ULTIMATE TENSILE STRESS** THUS MAKING THIS DAMAGE QUITE CRITICAL AND DANGEROUS. ($\sigma < \sigma_Y (\sigma_{P02})$ OR $R_m (\sigma_{UTS})$)
- **RUPTURE SUDDENLY OCCURS**, SINCE COMPONENT LOOKS WORKING REGULARLY UNTIL THAT CRACK PROPAGATES UNSTABLY AND IT IS BROKEN.
- **WÖHLER** DID CHARACTERIZE THIS PHENOMENON ON SOME ROTATING SHAFT UNDER BENDING THUS **DEFINING BOTH** THE **LOADING CONDITION** AND THE **MATERIAL BEHAVIOUR** THROUGH THE **EXPERIMENTAL RESULTS** OF THE WÖHLER CURVE.

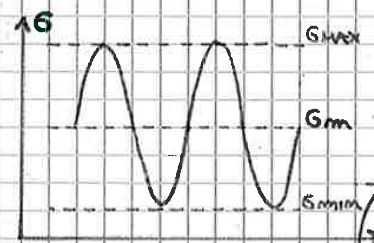
3.2) DEFINE RELEVANT STRESSES FOR FATIGUE DESIGN.

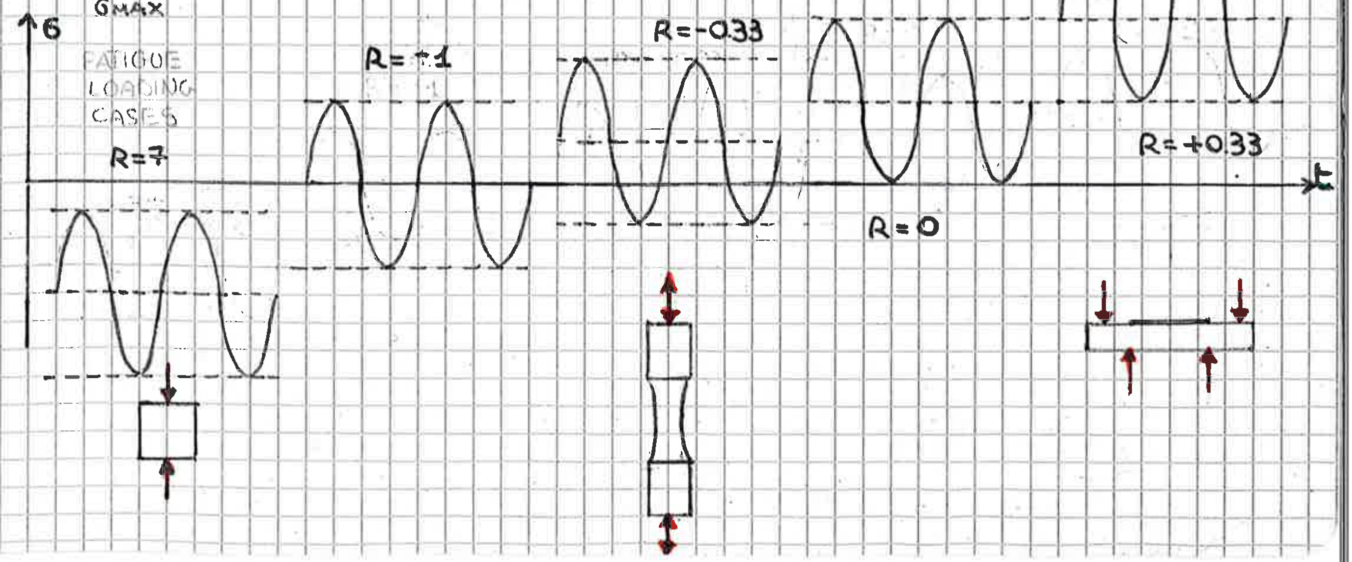
$\sigma_m = \dfrac{\sigma_{MAX} + \sigma_{min}}{2}$ **MEAN STRESS**

$\sigma_a = \dfrac{\sigma_{MAX} - \sigma_{min}}{2}$ **STRESS AMPLITUDE/** ALTERNATING STRESS

$\Delta\sigma = \sigma_{MAX} - \sigma_{min} = 2\sigma_a$ **STRESS RANGE**

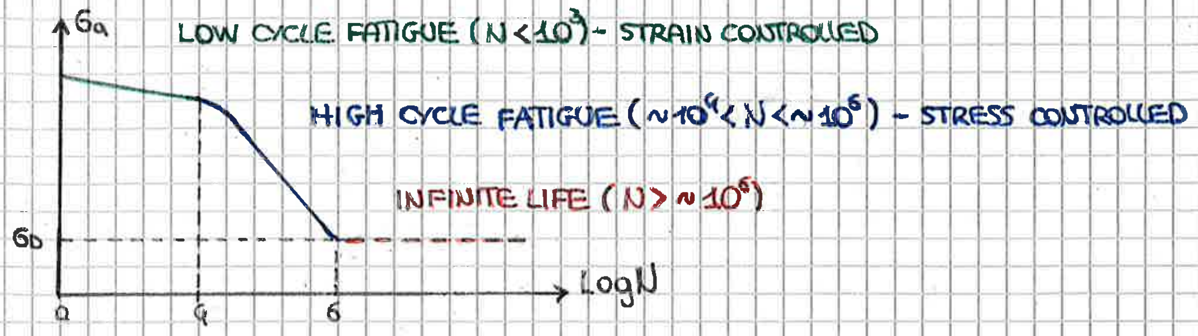
$R = \dfrac{\sigma_{min}}{\sigma_{MAX}}$ **STRESS RATIO**





3.4) WÖHLER CURVE

WÖHLER CURVE CAN BE DIVIDED IN THREE PARTS:



3.5) THREE STEPS OF FATIGUE DAMAGE (a), (b), (c). MATERIAL BEHAVIOUR UNDER F. LOADING:

" THE PROCESS OF **PROGRESSIVE (1), LOCALIZED (2), PERMANENT (3)** STRUCTURAL CHANGE OCCURRING IN A MATERIAL, SUBJECTED TO CONDITIONS THAT PRODUCE **FLUCTUATING** STRESSES AND STRAINS (4) AT SOME POINT OR POINTS AND THAT MAY CULMINATE IN CRACKS OR COMPLETE FRACTURE (5) AFTER A SUFFICIENT NUMBER OF FLUCTUATIONS".

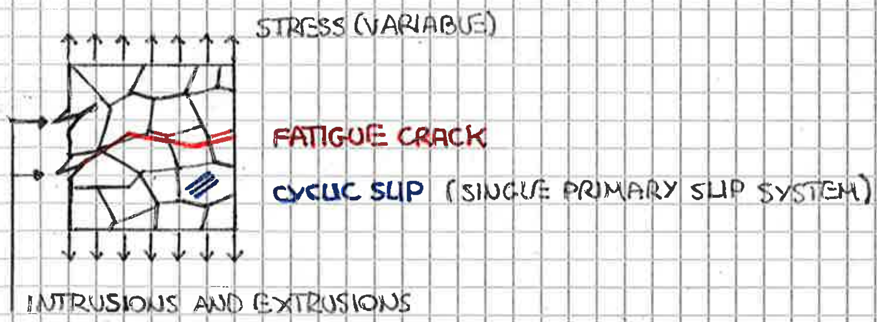
(1), (2) THERE IS A PROGRESSION IN DAMAGE:

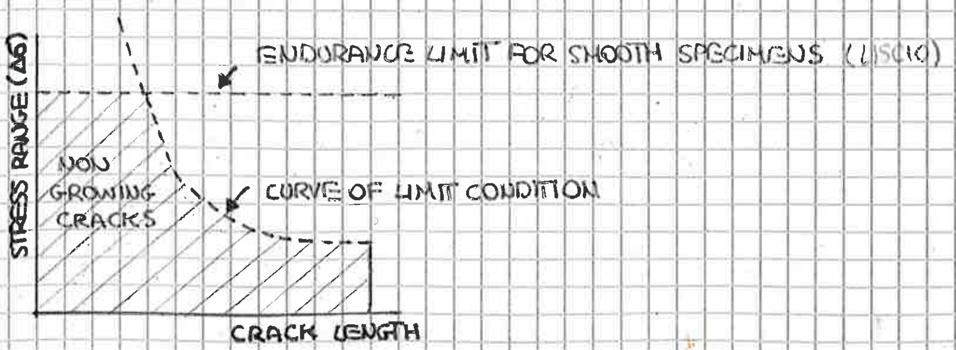
a) **NUCLEATION OF A CRACK**: NUCLEATION INSIDE GRAINS WITH LOCAL SLIP INTRUSIONS AND EXTRUSIONS ("MICROCRACK")

b) **CRACK GROWTH**: FIRST "SMALL CRACK" THEN CONVENTIONAL "LARGE CRACK" GOVERNED BY LINEAR ELASTIC OR PLASTIC FRACTURE MECHANISM.
("LARGE CRACK" MEANS MUCH LARGER THAN THE GRAIN SIZE)

b-c) CRACK PROPAGATION THROUGH THE MATERIAL AND ACROSS THE GRAINS.

c) **FINAL INSTABILITY: COMPLETE FRACTURE.**





3.7) DESCRIBE TYPICAL PROPERTIES OF A CROSS SECTION OF A COMPONENT COLLAPSED BECAUSE OF FATIGUE (MARKS, ROUGHNESS...)

3.8) COMPARE CROSS SECTION COLLAPSED BECAUSE OF FATIGUE WITH / WITHOUT NOTCH.

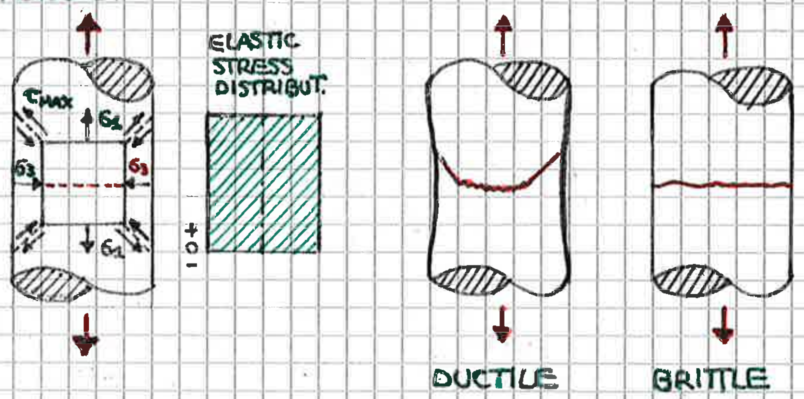
FRACTURE MECHANISM: ACCORDING TO MATERIAL AND PART GEOMETRY, THE TYPE OF FAILURE WILL BE SOMEWHERE BETWEEN TWO EXTREME SITUATIONS:

1) FULLY DUCTILE: THE REDUCED CROSS SECTION FAILS AT YIELD, EXACTLY AS IT WOULD HAPPEN IN A SMOOTH SPECIMEN UNDER TENSION WITH THE SAME CROSS SECTION AREA.

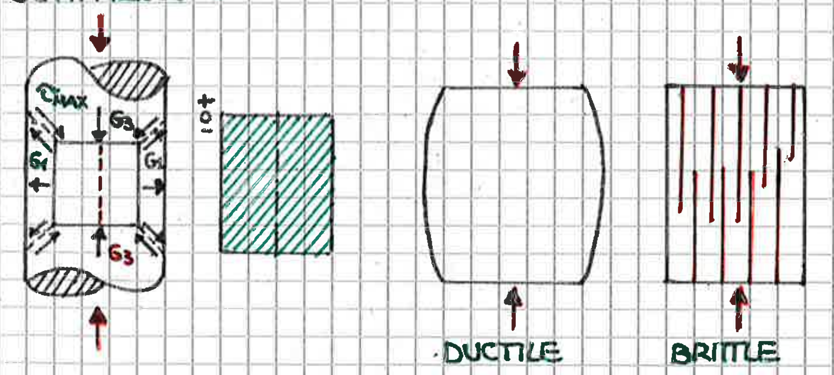
2) FULLY BRITTLE: THE REDUCED CROSS SECTION FAILS IN BRITTLE MANNER BECAUSE THE CRACK BECOMES UNSTABLE, I.E. IT PROPAGATES ALL OF A SUDDEN. (ROUGH SECTION)

FRACTURE PLANE: DEPENDS ON THE FATIGUE PHENOMENON AND ON MATERIAL.

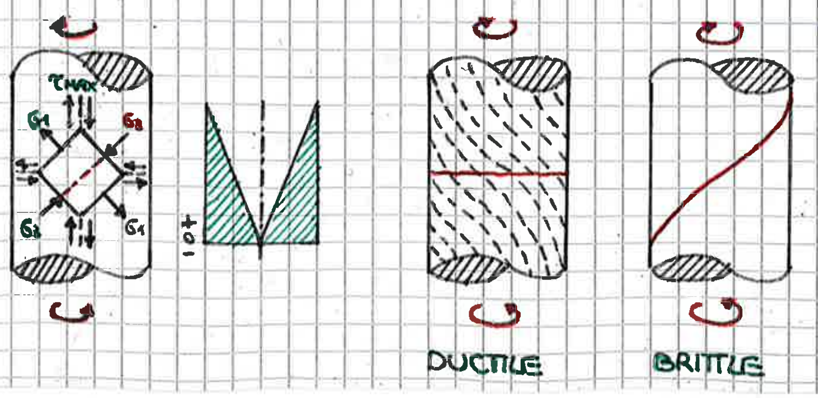
TENSION



COMPRESSION



TORSION



3.9) HOW WÖHLER CURVE IS EXPERIMENTALLY FOUND? (STAIRCASE METHOD)

3.10) DEFINE THE BASQUIN MODEL, 3.11) THE FATIGUE LIMIT?

DIAMETER OF ROUND SPECIMENS:

STEEL: $d = 6 \div 8$ [mm] FREQUENT USED VALUE

LIGHT ALLOYS: $d = 8 \div 13$ [mm]

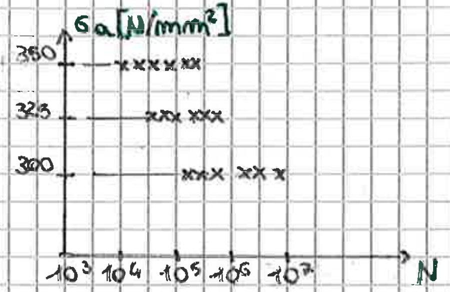
WIDTH OF RECTANGULAR SPECIMENS:

STEEL: $W = 2 \div 5$ [mm]

LIGHT ALLOYS: $W = 4 \div 8$ [mm]

EXPERIMENTAL DETERMINATION OF WÖHLER CURVES

A GROUP OF SPECIMENS IS TESTED AT EACH OF (AT LEAST) THREE STRESS LEVELS SPANNING THE EXPECTED RANGE OF FINITE LIFE STRESS AMPLITUDE.

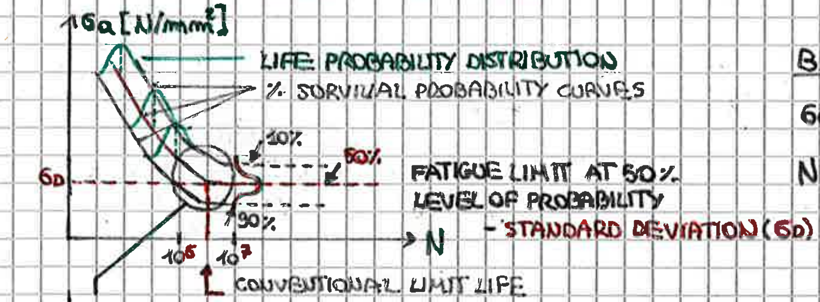


3 LEVELS

6 SPECIMENS EACH

X: BROKEN SPECIMEN

THE DATA ARE USED TO OBTAIN THE "LIFE PROBABILITY CURVES" AT EACH STRESS LEVEL, ON WHICH THE "STRESS-LIFE CURVES" ARE THEN CONSTRUCTED.



BASQUIN MODEL

$$\sigma_a = A N^B$$

$$N \sigma_a^k = B$$

X : BROKEN SPECIMENS
O : UN-BROKEN SPECIMENS

THE FATIGUE LIMIT CAN BE DETERMINED WITH THE "STAIRCASE" METHOD.

A SMALL NUMBER OF EQUALLY SPACED STRESS LEVELS IS SET AROUND THE EXPECTED FATIGUE LIMIT. THE FIRST SPECIMEN IS TESTED AT HIGHEST STRESS LEVEL.

IF FAILURE OCCURS, A FURTHER TEST IS CARRIED OUT AT A ONE-STEP DOWN STRESS.

IF FAILURE DOES NOT OCCUR (SAY, AT 10^7 CYCLES) THE TEST IS STOPPED, AND A FURTHER SPECIMEN IS TESTED AT ONE-STEP UP STRESS LEVEL.

UP TO 15-20 SPECIMENS MUST BE EMPLOYED.

THROUGH THE APPROPRIATE STATISTICAL TOOLS, THE FATIGUE LIMIT AT 50% LEVEL OF PROBABILITY IS EXTIMATED, WITH ITS STANDARD DEVIATION (σ_D).

⊕ DESCRIBE THE EFFECTS ON STRENGTH, ACCORDING TO FKM STANDARD

THE MAIN POINTS TO BE REMEMBERED ABOUT THE FKM-GUIDELINE NAMED "ANALYTICAL STRENGTH ASSESSMENT OF COMPONENTS IN MECHANICAL ENGINEERING" ARE:

VALID FOR COMPONENTS PRODUCED WITH OR WITHOUT MACHINING OR BY WELDING OF STEEL, OF IRON OR OF ALUMINIUM MATERIALS THAT ARE INTENDED FOR USE UNDER NORMAL OR ELEVATED TEMPERATURE CONDITIONS, AND IN DETAIL:

- FOR STATIC LOADING;
- FOR FATIGUE LOADING (WITH MORE THAN ~ 10^4 CYCLES (HCF) CONST OR VARIABLE AMPL);
- FOR COMPONENTS WITH GEOMETRICAL NOTCHES;
- FOR COMPONENTS WITH WELDED JOINTS;
- FOR MILLED OR FORGED STEEL, ALSO STAINLESS STEEL (SS), CAST IRON MATERIALS, AS WELL AS ALUMINIUM ALLOYS OR CAST ALUMINIUM ALLOYS.
- FOR COMPONENTS TEMPERATURES:
  FROM -40 [°C] TO 500 [°C] FOR STEEL
  FROM -25 [°C] TO 500 [°C] FOR CAST IRON
  FROM -25 [°C] TO 200 [°C] FOR ALUMINIUM.
- FOR A NON-CORROSIVE ENVIRONMENT.

⊕ HOW THE STANDARD DEFINES THE MECHANICAL PROPERTIES OF MATERIAL?

$R_{m,N}$: **STATIC FAILURE STRESS** : IS THE MAXIMUM STRESS EXPERIENCED IN A TENSILE TEST. (MINIMUM GUARANTEED VALUE), IN PARTICUR:

1. 97.5 % PROBABILITY; (N INDICATES THAT THE VALUES ARE IN THE PUBLISHED TABLES OF THE STANDARD (NORM))
2. TESTED AT ROOM TEMPERATURE (T_R);
3. TESTED ALONG THE MAIN DIRECTION OF MILLING AND FORGING;
4. IN CASE OF STEEL IT APPLIES TO THE SMALLEST DIMENSION OF A SEMI-FINISHED PRODUCT;
5. IN CASE OF CAST IRON AND CAST ALUMINIUM ALLOYS IT APPLIES TO THE TEST PIECE SIZE DEFINED IN STANDARDS.

BOUNDARY AND TESTING CONDITIONS

$R_{e,N}$: **YIELD STRENGTH** (MINIMUM GUARANTEED VALUE).
(R_p : YIELD STRESS, A GENERALIZATION OF R_{eH} AND $R_{p0.2}$; SAME CONDITIONS OF $R_{m,N}$)

σ_{D-1} : **FATIGUE LIMIT** FOR $R=-1$

NW : IT IS TESTED IN **TENSION-COMPRESSION** (FKM DOES NOT USE VALUES FROM ROTATING BENDING FOR ROUND SPECIMENS OR FROM ALTERNATING BENDING FOR FLAT SPECIMENS)

2 THE MATERIAL STRENGTH DEPENDS ON THE DIRECTION OF MILLING/FORGING

[ROLE OF PROCESS]

$R_m = K_{d,m} K_A R_{m,N}$

$K_{d,m}$: TECHNOLOGICAL SIZE FACTOR

K_A: ANISOTROPY FACTOR, DEPENDING ON R_m AND TYPE OF MATERIAL

EXAMPLES:

R_m [MPa]	UP TO 600	FROM 600 TO 900	FROM 900 TO 1200	ABOVE 1200
STEEL	$K_A = 0.90$	$K_A = 0.86$	$K_A = 0.83$	$K_A = 0.80$
R_m [MPa]	UP TO 200	FROM 200 TO 400	FROM 400 TO 600	
AL. ALLOYS	$K_A = 1.00$	$K_A = 0.95$	$K_A = 0.90$	

3 THE FATIGUE LIMIT IN FKM IS DEFINED AS A RESULT OF TENSION-COMPRESSION TEST (tc)

[ROLE OF LOAD]

$\sigma_{D-1}^{tc} = f_{W,\sigma} \cdot R_m$

WITH $f_{W,\sigma}$: DEPENDING ON TYPE OF MATERIAL

$\tau_{D-1}^{tc} = f_{W,\tau} \cdot \sigma_{D-1}^{tc} = f_{W,\tau} \cdot f_{W,\sigma} \cdot \sigma_{D-1}^{tc}$

WITH $f_{W,\tau}$: DEPENDING ON TYPE OF MATERIAL

CONSIDERATIONS:

- FOR A GIVEN $\sigma_{D-1,N}^{b}$, I.E. ROTATING BENDING, THE CORRESPONDING STRENGTH IN TENSION-COMPRESSION:

$\sigma_{D-1}^{tc} = 0.7 - 0.8 \cdot \sigma_{D-1,N}^{b}$

- MOREOVER, IF A GIVEN STRENGTH IS EXPRESSED IN TERMS OF NORMAL STRESS THE CORRESPONDING VALUE OF SHEAR STRESS, IN STATICS AND FOR A DUCTILE MATERIAL:

$\sigma_{EQ} = \sqrt{\sigma^2 + 3\tau^2}$ (VON MISES IN STATICS)

IF $\sigma = 0 \Rightarrow \sigma_{EQ} = \sqrt{3\tau^2} = \sqrt{3} \cdot \tau$, THEREFORE:

$\tau_{D-1} = \dfrac{\sigma_{D-1}}{\sqrt{3}} = 0.577 \cdot \sigma_{D-1}$

VALUES FOR $N = 10^6$ CYCLES

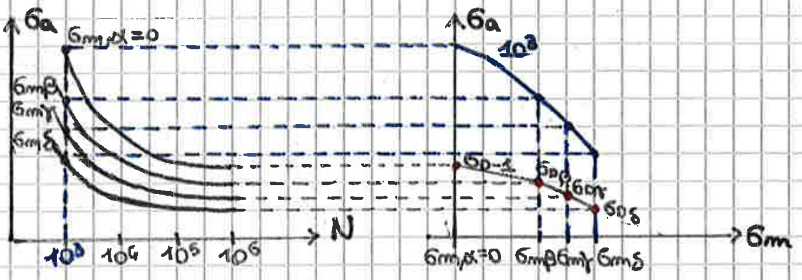
EXAMPLES:

TYPE OF MATERIAL	$f_{W,\sigma}$	$f_{W,\tau}$
CASE HARDENING STEEL	0.40	0.577
STAINLESS STEEL (SS)	0.40	0.577
FORGING STEEL	0.40	0.577
STEELS OTHER THAN THESE	0.45	0.577
GS - STEEL CASTINGS	0.34	0.577
GJS - SPHEROIDAL GRAPHITE CAST IRON	0.34	0.65
GJM - MALLEABLE CAST IRON	0.30	0.75
GJL - GREY CAST IRON	0.30	0.85

TYPE OF MATERIAL	$f_{W,\sigma}$	$f_{W,\tau}$
WROUGHT AL-ALLOYS	0.30	0.577
CAST AL-ALLOYS	0.30	0.577

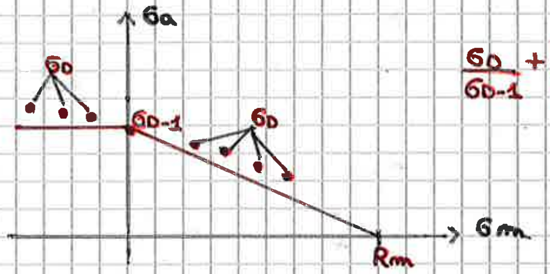
(5)
- SAME N AND DIFFERENT σ_m

FOR EACH VALUE OF NUMBER OF CYCLES (N), A NEW CURVE CAN BE OBTAINED BY COMPARING DIFFERENT MEAN STRESSES (σ_m).



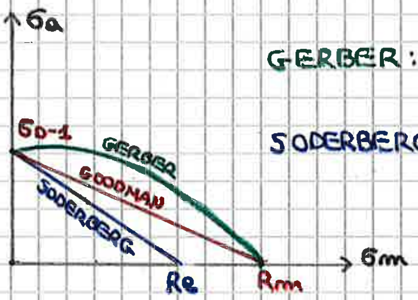
(5)
- APPROXIMATIONS OF THE EXPERIMENTAL RESULTS ON THE HAIGH DIAGRAM ($\sigma_a - \sigma_m$)

- GOODMAN ($\sigma_{m\,MAX} = R_m$)



$$\frac{\sigma_D}{\sigma_{D-1}} + \frac{\sigma_m}{R_m} = 1$$

- OTHER APPROXIMATIONS: GERBER ($\sigma_{m\,MAX} = R_m$) AND SODERBERG ($\sigma_{MAX} = R_e$)



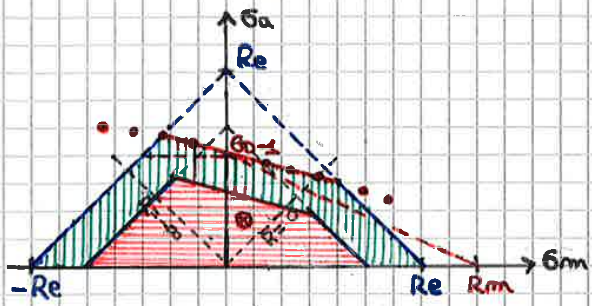
GERBER: $\frac{\sigma_D}{\sigma_{D-1}} + \left(\frac{\sigma_m}{R_m}\right)^2 = 1$

SODERBERG: $\frac{\sigma_D}{\sigma_{D-1}} + \frac{\sigma_m}{R_e} = 1$

3.17) SAFETY FACTOR AND AMMISSIBLE STRESS ?

ONCE THE LIMIT HAIGH THRESHOLDS AND AREA HAVE BEEN DECIDED ACCORDING TO A STANDARD, FOR A GIVEN MATERIAL AND FOR A CERTAIN COMPONENT, APPROPRIATE **SAFETY LIMITS** ARE APPLIED, PRODUCING THE ADMISSIBLE AREA HATCHED IN RED. ( )

APPLIED STRESS SATISFIES DESIGN REQUIREMENTS WHEN IT FALLS INSIDE THAT AREA (⊗).



NOTE FOR DRAWINGS (IN PARTICULAR FOR INTERS. R=0, FKM REF, Re LINES)

$\sigma_{D-1} = [4]$

$Re = [6.6]$

$R_m = [8.8]$

⊕ WHEN WE PASS FROM THE SPECIMEN TO THE COMPONENT, HOW MANY DETAILS MIGHT CHANGE? ($\sigma_{D-1} \rightarrow \sigma_{C,D-1}$)

WE HAVE TO CONSIDER:

1) SURFACE EFFECTS

1.1 - SURFACE FINISH (b_0 OR $K_{R,\sigma}$ IN FKM STANDARD): ROUGHNESS AND MACHINING

1.2 - SURFACE TREATMENT (K_V): THERMAL AND MECHANICAL

1.3 - SURFACE COATING (K_S)

2) (VOLUME) GRADIENT EFFECTS

2.1 - SIZE EFFECT IN BENDING AND TORSION ($\sigma_{D-1,T}$)

2.2 - (FATIGUE) NOTCH FACTOR FOR COMPONENT SHAPE (K_f)

3) COMPOSITE STRESSES, OTHER THAN THOSE IN SIMPLE STANDARD CASES.

3.1 - EFFECT OF COMBINED STRESS COMPONENTS

⊕ WHAT DOES FKM APPROACH EVALUATE?

THE FKM APPROACH BASICALLY EVALUATES:

- THE FATIGUE LIMIT OF COMPONENT (STARTING FROM THE MATERIAL)

$\sigma_{C,D-1} = \sigma_{D-1,T}\, b_0 K_V K_S$, WITH $\sigma_{D-1,T}$: TEST

- THE EFFECTIVE STRESS IN SERVICE

$\sigma_{EFF}^{x} = \sigma_{NOM}^{x} K_f^{x}$, WHERE THE INDEX X QUALIFIES: $\begin{cases} a: \text{AXIAL LOAD (TENS.-COMPR.)} \\ b: \text{BENDING} \\ t: \text{TORSION} \end{cases}$

- EQUIVALENT STRESS

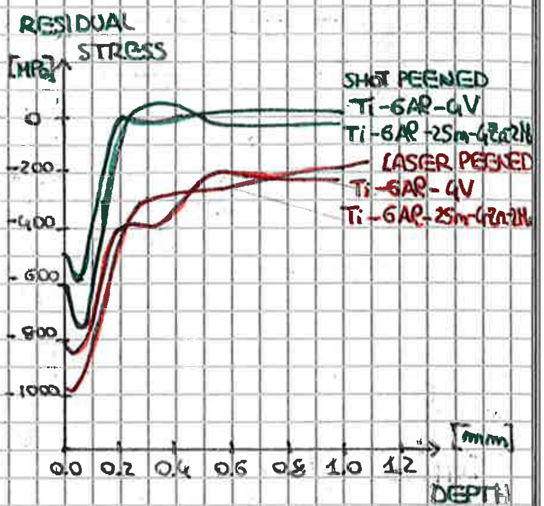
σ_{EQ}

$\sigma_{c,D-1} = \sigma_{D-1,T} \ldots K_V \ldots$

SURFACE HEAT TREATMENT ON STEEL	UN-NOTCHED COMPONENT FROM-TO	NOTCHED COMPONENT FROM-TO
NITRIDING	$1.1 \div 1.25$	$1.3 \div 3.0$
CASE HARDENING	$1.1 \div 2.0$	$1.2 \div 2.5$
CARBO-NITRIDING	1.80	

SURFACE MECH. TREATMENT ON STEEL	UN-NOTCHED COMPONENT FROM-TO	NOTCHED COMPONENT FROM-TO
COLD-ROLLING	$1.1 \div 1.4$	$1.3 \div 2.2$
SHOT PEENING	$1.1 \div 1.3$	$1.1 \div 2.5$
INDUCT/FLAME HARDENING	$1.2 \div 1.6$	$1.5 \div 2.8$

VALUES OF K_V FOR SPECIMENS: d FROM 8 TO 40 [mm]



1.3 - SURFACE COATING (K_S)

FOR STEEL AND CAST-IRON MATERIALS: $K_S = 1$

FOR ALUMINIUM ALLOYS WITHOUT COATING $K_S = 1$

FOR ALUMINIUM ALLOYS WITH AN ANODIC COATING OF THICKNESS X [μm]:

$K_S = 1 - 0.27 \log(x)$

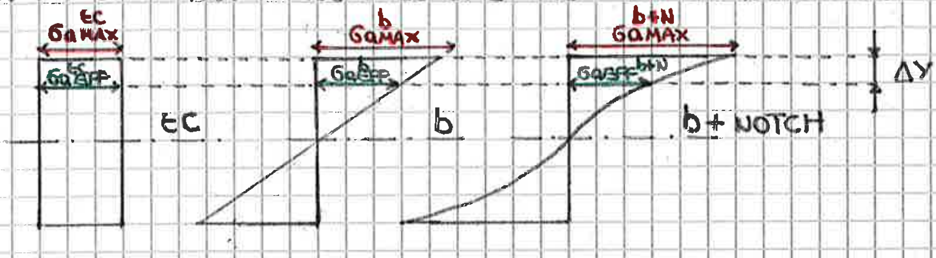
$\sigma_{c,D-1} = \sigma_{D-1,T} \ldots K_S$

3.22) NOTCH EFFECT AND INFLUENCE ON STATIC AND FATIGUE BEHAVIOURS?

2.2 (FATIGUE) NOTCH FACTOR FOR COMPONENT SHAPE (K_f)

- WHAT HAPPENS WHEN THE GRADIENT IS INDUCED BY THE NOTCH EFFECT?

=) COMPARING UNNOTCHED AND NOTCHED STRUCTURES UNDERGOING THE SAME EFFECTIVE STRESS ($\sigma_{a\,EFF}$): THE STRENGTH IS THE SAME IN AXIAL LOADING, IN BENDING AND IN NOTCHED SPECIMEN UNDER BENDING.



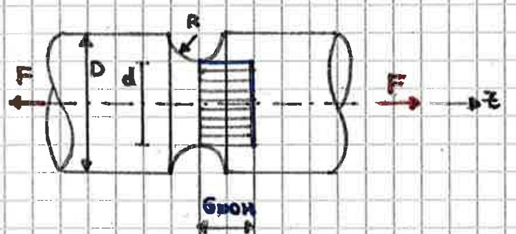
$$\sigma_{a\,EFF} = \sigma_a^{tc} < \sigma_{a\,MAX}^{b} < \sigma_{a\,MAX}^{b+N}$$

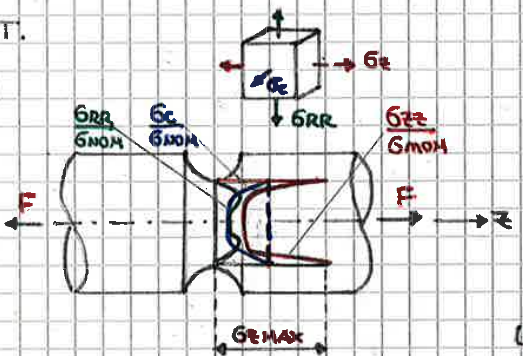
THAT RESULT IS CRUCIAL TO UNDERSTAND THE INTRINSIC MEANING OF THE STRESS CONCENTRATION FACTOR FOR NOTCHED STRUCTURE IN FATIGUE, K_F. LET'S RECALL THE:

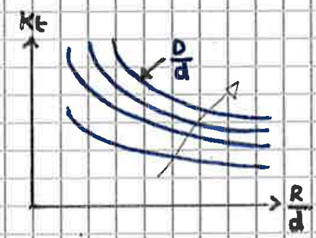
- CONCEPT OF NOTCH

AN ABRUPT REDUCTION OF CROSS SECTION INDUCES A STRESS CONCENTRATION (3D) THE STRESS AT NOTCH SECTION IS THREE DIMENSIONAL, AND NO LONGER CONSTANT; SHEAR STRESSES ARE PRESENT.

TENSION-COMPRESSION (tc)





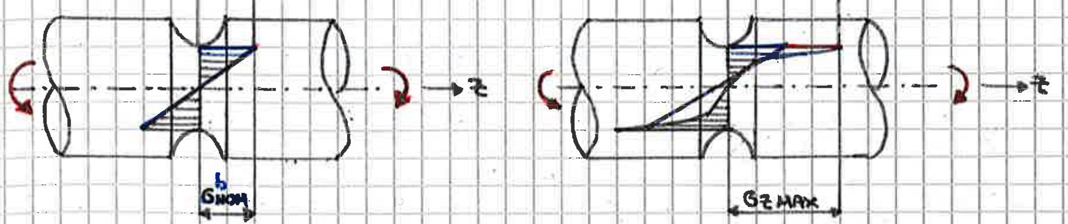


LOCAL STRESS DUE TO NOTCH

$$\sigma_{NOM} = \frac{4F}{\pi d^2} = \frac{1}{A}\int_A \sigma_z\, dA \quad \text{NOMINAL STRESS (BASED ON BEAM THEORY)}$$

$\sigma_{z\,MAX} = K_t\, \sigma_{NOM}$ WITH K_t = THEORETICAL STRESS CONCENTR. FACTOR

BENDING (b)



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FKM DEFINES $(1+\chi\Delta y)$ WITH THE SYMBOL m_σ $\quad [\sigma_{a\,EFF} = K_F \cdot \sigma_{a\,NOM} \leq \sigma_{D-1}^{tc}]$

$$K_F^{tc} = \frac{K_t^{tc}}{m_\sigma(R)}$$

$$K_F^{b} = \frac{K_t^{b}}{m_\sigma(R)\, m_\sigma(d)}$$

$$K_F^{t} = \frac{K_t^{t}}{m_\tau(R)\, m_\tau(d)}$$

WHERE THE DENOMINATORS MEAN:

$m_\sigma(R), m_\tau(R)$: FACTORS FOR σ, τ DUE TO GRADIENT AT NOTCH RADIUS

$m_\sigma(d), m_\tau(d)$: FACTORS FOR σ, τ DUE TO COMPONENT SIZE-RELATED GRADIENT (BENDING AND TORSION).

THE **NOMINAL STRESS** IS DEFINED AS THE MAXIMUM STRESS CALCULATED BY TRACTION, OR BENDING, OR TORSION FORMULAS, FOR THE SECTION OF LOWER AREA, WITHOUT NOTCH (BEAM FORMULA).

$$\sigma_{a\,NOM}^{tc} = \frac{4F}{\pi d^2}$$

$$\sigma_{a\,NOM}^{b} = \frac{M_b}{W_b}$$

$$\tau_{a\,NOM}^{t} = \frac{M_t}{W_t}$$

IF WE SUMMARIZE PREVIOUS CONTENTS, EFFECTIVE STRESSES ARE:

$$\sigma_{EFF}^{tc}(t) = \sigma_N^{tc}(t)\, K_F^{tc} \quad , \quad K_F^{tc} = \frac{K_t^{tc}}{m_\sigma(R)}$$

$$\sigma_{EFF}^{b}(t) = \sigma_N^{b}(t)\, K_F^{b} \quad , \quad K_F^{b} = \frac{K_t^{b}}{m_\sigma(R)\, m_\sigma(d)}$$

$$\tau_{EFF}^{t}(t) = \sigma_N^{t}(t)\, K_F^{t} \quad , \quad K_F^{t} = \frac{K_t^{t}}{m_\tau(R)\, m_\tau(d)}$$

FKM FIRST CALCULATES SEPARATELY RELATIVE GRADIENTS FOR NOTCH RADIUS (R) AND COMPONENT SIZE (d) EFFECTS : $\chi_\sigma(R)$; $\chi_\sigma(d)$.

THEN, $m_\sigma(R)$, $m_\tau(R)$ AND $m_\sigma(d)$, $m_\tau(d)$ ARE CALCULATED.

WE ANALYSE:

a) GROOVE ("SCANALATURA") ON SHAFT AND FLAT BAR, NORMAL STRESS σ $(\chi_\sigma(R); \chi_\sigma(d))$

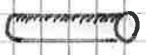
b) SHOULDER ON SHAFT AND FLAT BAR, NORMAL STRESS σ $(\chi_\sigma(R); \chi_\sigma(d))$

c) GROOVE AND SHOULDER ON SHAFT, SHEAR STRESS τ $(\chi_\tau(R); \chi_\tau(d))$

3.23) COMBINATION OF STRESSES IN FATIGUE ? (GOUGH AND POLLARD)

3.1 EFFECT OF COMBINED STRESS COMPONENTS

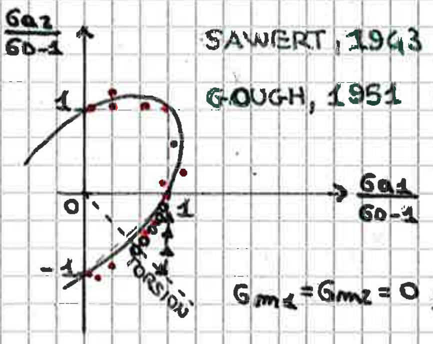
WHEN SEVERAL STRESS COMPONENTS ARE SUPERPOSED, THE MAIN PROBLEM IS FINDING A SUITABLE FORMULATION TO COMBINE ALL TOGETHER INTO AN EQUIVALENT STRESS WHICH MIGHT REPRESENT THE SAME CONTENT OF RISK ASSOCIATED TO DAMAGE NUCLEATION AS THE SET OF REAL STRESSES APPLIED.



$\overset{\sigma_a}{\longleftrightarrow}$ τ_a $\overset{\sigma_m}{\longrightarrow}$ τ_m $\Rightarrow$ $\sigma_{EQ_a} = ?$ $\sigma_{EQ_m} = ?$

GOUGH AND POLLARD'S WORKS ON FATIGUE (FROM THE 1920'S TO THE 1950'S) PRODUCED THEIR "RULE" FOR COMBINED BENDING AND TORSION, WHICH IS AN EMPIRICAL RELATION FOUND THROUGH LABORATORY EXPERIMENTS.
THIS WAS DONE COMBINING NORMAL STRESSES DUE TO BENDING WITH TANGENTIAL STRESSES DUE TO TORSION.

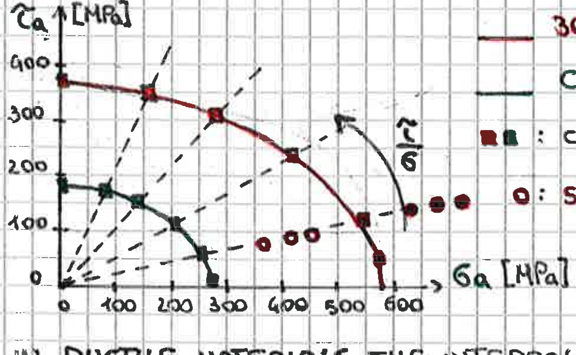
THEY PERFORMED SOME TESTS BY COMBINING THE BENDING LOAD WITH THE TORSIONAL BEHAVIOUR, NAMELY THE NORMAL AND THE SHEAR STRESSES.
TO SIMPLIFY, ONLY CYCLES IN-PHASE (IP) OR EXACTLY OUT-OF-PHASE (OP) WERE ANALYZED.



SAWERT, 1943 (• ANNEALED MILD STEEL)
GOUGH, 1951 (o Cr-V STEEL ; ▲ "SILAL" CAST IRON)

$\sigma_{m1} = \sigma_{m2} = 0$; 10^7 [CYCLES]

COMBINED FATIGUE LIMIT



30 Ni Cr Mo12 ; $R_m = 900$ [MPa]
C15 ; $R_m = 425$ [MPa]
■■ : COMBINED FATIGUE LIMIT POINTS
o : SAMPLE OF INFINITE LIFE ; • SAMPLE OF FINITE LIFE

IN DUCTILE MATERIALS THE INTERPOLATION OF DATA GIVES THIS KIND OF EXPRESSION:

$$\left(\frac{\sigma_a^b}{\sigma_{aD-1}^b}\right)^2 + \left(\frac{\tau_a^t}{\tau_{D-1}^t}\right)^2 \leq 1$$

IN CASE OF NOTCHED SECTION:

$$\sigma_{aEQ} = \sqrt{\left(K_\zeta^s \cdot \sigma_a^s + \frac{K_\zeta^t \sigma_a^t}{0.85}\right)^2 + 3\left(K_\zeta^\tau \cdot \tau_a\right)^2}$$

$$\sigma_{mEQ} = \sqrt{\left(K_\zeta^s \cdot \sigma_m^s + K_\zeta^t \sigma_m^t\right)^2 + 3\left(K_\zeta^\tau \cdot \tau_m\right)^2}$$

ABOUT THE MEAN STRESS (σ_{mEQ}):

1) DESPITE ITS NATURE OF STATIC LOAD, THE STRESS CONCENTRATION DUE TO NOTCH EFFECT IS FOUND BY MULTIPLYING THE NOMINAL STRESS BY K_ζ (INSTEAD OF $K_t = \sigma_{MAX\,EFF}/\sigma_{MAX\,NOM}$); THE MAXIMUM STRESS IS USED AND IT'S THE SUM OF MEAN STRESS AND ALTERNATE STRESS.

2) IN THIS CASE THE STATIC STRENGTH IS INVESTIGATED BY THE TENSILE TEST MACHINE AND SCALING (I.E. COEFFICIENT 0.85) IS NOT REQUIRED BECAUSE THE COMPARISON BETWEEN STRESS IN OPERATION AND STRENGTH IS APPLIED TO THE AXIAL LOAD.

SIMILITUDE WITH THE VON MISES' FORMULA IMMEDIATELY SUGGESTS THAT IN A PRINCIPAL REFERENCE FRAME FOR STRESS, THE EQUIVALENT STRESS BECOMES:

$$\sigma_{aEQ} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{a1}-\sigma_{a2})^2 + (\sigma_{a1}-\sigma_{a3})^2 + (\sigma_{a2}-\sigma_{a3})^2}$$

AND IN A NOTCHED SECTION IS:

$$\sigma_{aEQ} = \sqrt{\left(\sigma_{aNOM}^{tc} K_\zeta^{tc} + \sigma_{aNOM}^{b} K_\zeta^{b}\right)^2 + 3\left(\tau_{aNOM}^{t} K_\zeta^{t}\right)^2}$$

IN BRITTLE MATERIALS:

GOUGH AND POLLARD FOUND THAT IN CASE OF BRITTLE MATERIAL THE INTERPOLATING FUNCTION IS MORE COMPLEX.

$$\left(\frac{\tau_a^t}{\tau_{D-1}^t}\right)^2 + \left(\frac{\tau_a^t}{\sigma_a^b} - 1\right)\left(\frac{\sigma_a^b}{\sigma_{D-1}^b}\right)^2 + \left(2 - \frac{\sigma_{D-1}^b}{\tau_{D-1}^t}\right)\left(\frac{\sigma_a^b}{\sigma_{D-1}^b}\right) = 1$$

HOW LONG NEGLECTING THE CONTRIBUTION OF MEAN SHEAR STRESS MAKES SENSE IN DESIGN OF SHAFT AGAINST FATIGUE?

ACTUALLY, THE INTERPRETATION KNOWN AS SINES' THEORY SEEMS APPLICABLE TO A VERY SPECIFIC TEST AND MIGHT GENERATE SOME CONFUSION IN THE USERS. THINKING ABOUT THE DESIGN OF SHAFT:

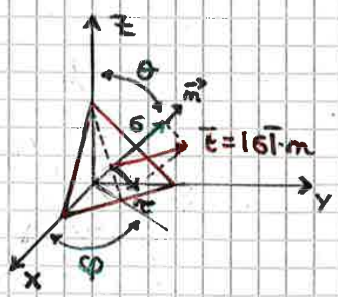
FIRST THE MEAN STRESS INDUCED BY TORQUE IS USED TO DEFINE THE DIAMETER OF CROSS SECTION; THEN IS NEGLECTED IN FATIGUE DESIGN. MOREOVER, USING A SINGLE VALUE OF K_f IS SOMEHOW WRONG, SINCE IN MULTIAXIAL FATIGUE AND NOTCHED COMPONENT EACH LOAD HAS ITS OWN STRESS CONCENTRATION FACTOR (K_f).

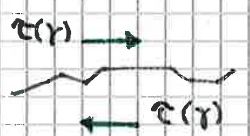
THOSE MOTIVATIONS SUGGEST THAT A **STANDARD APPROACH AS THE FKM STANDARD PROPOSES IS BETTER, CLOSER TO THE ACTUAL BEHAVIOUR OF MATERIAL** AND **MORE PRECISE**; ALTHOUGH FORMULAS MIGHT SEEM MORE DIFFICULT AND CALCULATIONS A LITTLE BIT LONGER.

REMARK:

GOUGH AND POLLARD APPROACH ALLOWS PREDICTING ROUGHLY THE **OPERATING CONDITION UNDER FATIGUE IN CASE OF COMBINED STRESSES IN OR OUT-OF-PHASE** (I.E. PEAKS OF SHEAR STRESS ARE PERFECTLY ALIGNED WITH THOSE OF NORMAL STRESS OR PERFECTLY OPPOSITE).

IN CASE OF MIXED PHASES, OTHER APPROACHES ARE AVAILABLE; IN PARTICULAR ONE IS QUITE POPULAR AND IS BASED ON **FINDING THE CRITICAL PLANE** WHERE ARE APPLIED THE SHEAR STRESSES RESPONSIBLE FOR THE SLIP MODES CAUSING THE NUCLEATION OF DEFECTS (THEORY OF CRITICAL PLANE).



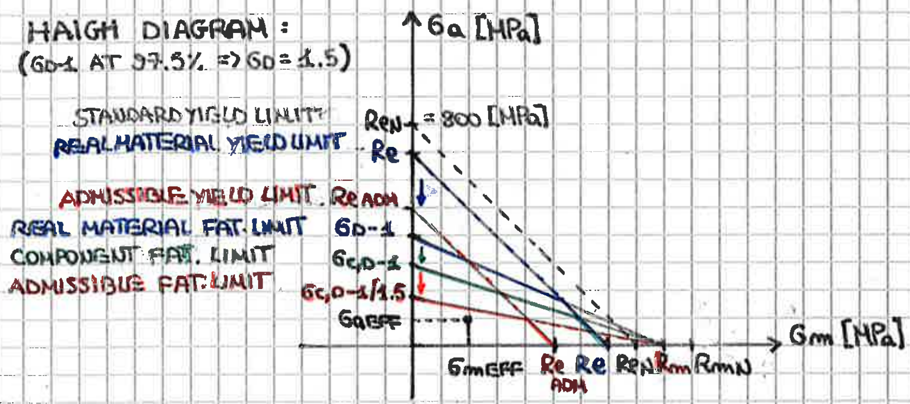


MAXIMUM SLIP EFFECT

- UNIAXIAL CASE:

IN THE SIMPLE UNIAXIAL CASE, σ_{aEFF} AND σ_{mEFF} ARE TO BE PUT RESPECTIVELY ON THE VERTICAL AXIS AND ON THE HORIZONTAL AXIS OF THE HAIGH DIAGRAM CONSTRUCTED ON THE BASIS OF FATIGUE DATA FOR TENSION-COMPRESSION (EC) (NO GRADIENT); THEY REPRESENT THE "EFFECTIVE" STRESSES TO BE COMPARED TO UNIAXIAL FATIGUE EC LIMITS.

HAIGH DIAGRAM: (σ_{D-1} AT 97.5% ⇒ $\sigma_D = 1.5$)



- BIAXIAL CASE:

ALSO IN THE BIAXAL CASE, σ_{aEFF} AND σ_{mEFF} ARE TO BE PUT ON THE VERTICAL AND ON THE HORIZONTAL AXIS OF THE SAME HAIGH DIAGRAM AS BEFORE; HOWEVER, NOW THEY REPRESENT THE "EQUIVALENT" AND "EFFECTIVE" STRESSES TO BE COMPARED TO UNIAXIAL FATIGUE EC LIMITS.

SAME HAIGH DIAGRAM: BUT, INSTEAD OF σ_{aEFF}, σ_{mEFF}, WE ENTER WITH σ_{aEQEFF}, σ_{mEQEFF}

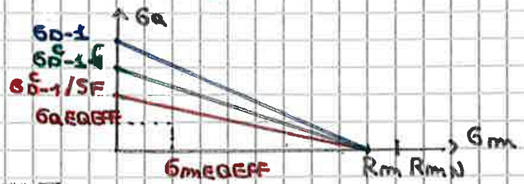
CONSIDERATION: WE COULD REPRESENT THE HAIGH DIAGRAM WITHOUT OF THE YIELD STRESS LINES (R_{eN}, R_e, R_{eADM}). IN PRINCIPLE, THE EQUIVALENT STRESS (σ_{EQ}) IS NEVER A REAL VALUE OF STRESS REACHED IN OPERATION, BUT JUST A CONVENTIONAL REFERENCE, USED TO COMPARE THE RISK OF DAMAGE WITH THE STRENGTH OF MATERIAL. THEREFORE IS SOMEHOW QUESTIONABLE COMPARING THE EQUIVALENT STRESS (σ_{EQ}) DIRECTLY TO YIELD.

⇒ THE FKM STANDARD FACES THIS POINT BY INTRODUCING SOME SUITABLE DEFINITIONS OF EQUIVALENT STRESS, TO BE CONSERVATIVE, AND ASSUMES THAT AS SOON THE EQUIVANT VALUE REACHES THE VALUE OF YIELD, THE LINEAR ELASTIC BEHAVIOUR IS NO LONGER ASSURED.

⊕ DESCRIBE ANOTHER INTERPRETATION FOR σ^{c}_{D-1}.

(WITH RESPECT TO $\sigma_{c,D-1} = b_0 K_V K_S \sigma_{D-1,T}$, FKM STANDARD)

$$\sigma^{c}_{D-1} = \frac{C_L \cdot C_S \cdot C_F}{K_f} \sigma_{D-1}$$



WITH:

σ^{c}_{D-1} : COMPONENT FATIGUE LIMIT

σ_{D-1} : BENDING SPECIMEN FATIGUE LIMIT ($d \approx 10$ [mm])

C_L : LOAD TYPE FACTOR : $C_L = 1$ FOR PLANE BENDING ; $C_L = 0.7$ FOR TENS.- COMPR.

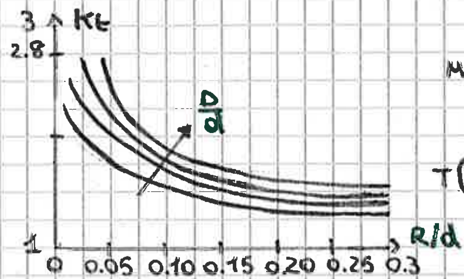
C_S : SIZE FACTOR (REPLACES C_L FOR BENDING ONLY IF $d < 10$ [mm]).

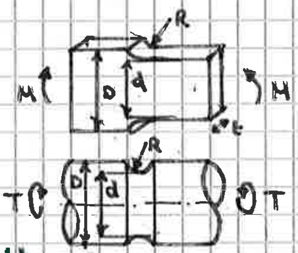
C_F : SURFACE FINISH FACTOR (ROUGHNESS, MACHINING, ...)

K_f : (FATIGUE) NOTCH EFFECT (BEING USUALLY SELECTED AMONG THE LOADING CASES AS THE HIGHEST ONE).

IN CASE OF **NOTCH**:

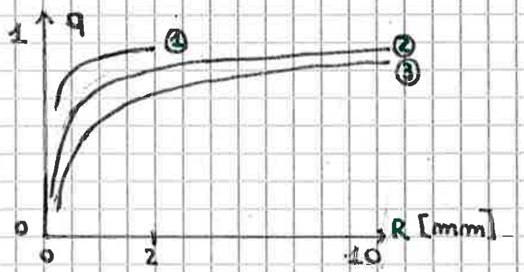
$$q = \frac{K_f - 1}{K_t - 1} \Rightarrow K_f = 1 + q(K_t - 1) \Rightarrow K_f = f(K_t, q) \begin{cases} K_t = f(D/d\,;\, R/d) \\ q = f(\text{TREATMENT}, R) \end{cases}$$





$$\sigma_{NOM} = \frac{Mc}{I} = \frac{6M}{td^2}$$

$$\tau_{NOM} = \frac{Tc}{J} = \frac{16T}{\pi d^3}$$



① QUENCHED AND TEMPERED STEEL

② ANNEALED OR NORMALIZED STEEL

③ AVERAGE Al- ALLOY (BARS AND SHEETS)

THIS INTERPRETATION ($\sigma^{c}_{D-1} = \frac{C_L C_S C_F}{K_f} \sigma_{D-1}$) **LOOKS SIMPLER, BUT IF A MULTIAXIAL FATIGUE CASE IS ANALYZED:**

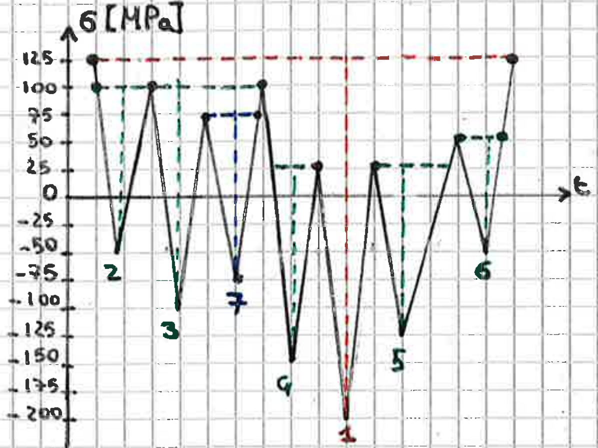
- WHICH LOAD DEFINES C_L?
- WHICH ONE DRIVES IN SELECTING C_S?
- WHICH LOAD IS ASSOCIATED TO K_f?

STEP 1: COUNTING OF CYCLES (BATHTUB METHOD)

1.1 - A SECTION OF THE TIME HISTORY MUST BE ISOLATED SO THAT IT BEGINS AND ENDS AT THE MAXIMUM VALUE (POINTS •)

1.2 - THE DIAGRAM IS INTERPRETED AS IF IT WERE A BASIN FULL OF LIQUID.

1.3 - THE BOTTOM POINT 1 IS WHERE LIQUID IS SPILLED; THE BASIN EMPTIES.

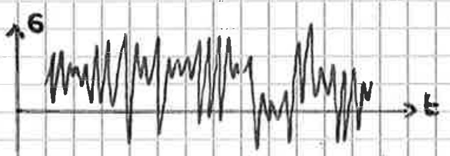


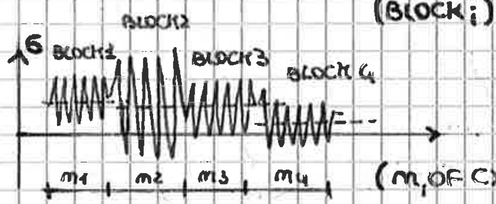
i	σ_{min_i}	σ_{MAX_i}	$\Delta\sigma_i$	σ_{a_i}	σ_{m_i}
1	-200	125	325	162.5	-37.5
2	-50	100	150	75	25
3	-100	100	200	100	0
4	-150	25	175	87.5	-62.5
5	-125	25	150	75	-50
6	-50	50	100	50	0
7	-75	75	150	75	0

$\Delta\sigma = \sigma_{MAX} - \sigma_{min}$; $\sigma_a = \frac{\sigma_{MAX} - \sigma_{min}}{2} = \frac{\Delta\sigma}{2}$

$\sigma_m = \frac{\sigma_{MAX} + \sigma_{min}}{2}$

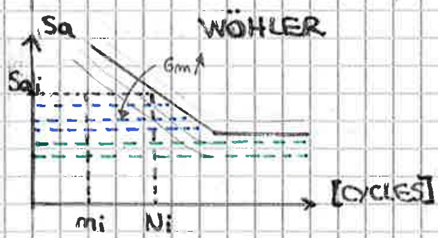
STEP 2: DAMAGE EVALUATION





($BLOCK_i$)

(n_i OF CYCLES OF $BLOCK_i$)



i	S_{mi}	S_{ai}	m_i	N_i	$D_i = \frac{m_i}{N_i}$
1	$\frac{\sigma_{mi}}{R_m} \cdot 100$	$\frac{S_{ai}}{R_m} \cdot 100$	...	...	...
*	...	...	...	...	...

m_i = n. OF CYCLES OF BLOCK (i) | * UNTIL N_i EXISTS! ⇒ ONLY BLUE ONES

N_i = n. OF CYCLES TO RUPTURE IF THERE WHERE ONLY BLOCK (i)

$D_i = \frac{m_i}{N_i}$ RELATIVE DAMAGE (BLOCK (i)); THE SYSTEM IS EXPLOITING A % OF ITS TOTAL LIFE.

STEP 3: DAMAGE CUMULATION

WHEN THE TOTAL DAMAGE EQUALS ONE: $D_{TOT} = \sum_i D_i = 1$ ⇒ FAILURE OCCURS!

(THEREFORE, IF $D_{TOT} = \sum_i D_i = \sum_i m_i/N_i = 0.93$ ⇒ NO FAILURE SHOULD OCCUR)

WE DEFINE:

m_{TOT} = TOTAL NUMBER OF VALID CYCLES ($m_{TOT} = \sum_i m_i$ (IF N_i EXISTS) ⇔ $N_i \leq 10^6$))

$\alpha_i = \frac{m_i}{m_{TOT}}$ = UTILIZATION FRACTION

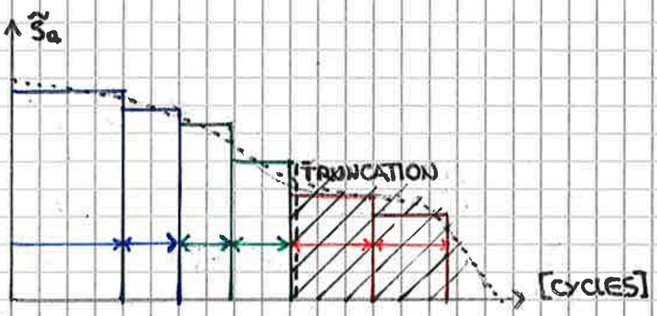
(A) $N_{TOT} = \frac{1}{\sum_i \frac{\alpha_i}{N_i}} \rightarrow \tilde{S}_{aEQ} = \left(\frac{10^6 S_{D-1}^k}{N_{TOT}}\right)^{\frac{1}{k}}$ | (B) $\tilde{S}_{aEQ} = \left(\sum_i \alpha_i \tilde{S}_{ai}^k\right)^{\frac{1}{k}} \rightarrow N_{TOT} = 10^6 \left(\frac{S_{D-1}}{\tilde{S}_{aEQ}}\right)^k$

DEFINITION OF EXCEEDANCE DIAGRAM

THIS DIAGRAM IS USED TO MONITOR FOR HOW MANY CYCLES (m) THE STRESS AMPLITUDE (Sa) EXCEEDS A GIVEN VALUE. THIS ALLOWS:

- DESCRIBING THE SYSTEM MISSION PROFILE;
- REALYZING HOW AN EQUIVALENT REFERENCE CYCLE COULD BE USED TO CHARACTERIZE THE SYSTEM FAILURE.

AS IN THE S-N CURVE THE AMPLITUDES WHICH ARE NOT CONTRIBUTING TO SYSTEM FAILURE ARE EXCLUDED BY TRUNCATION.

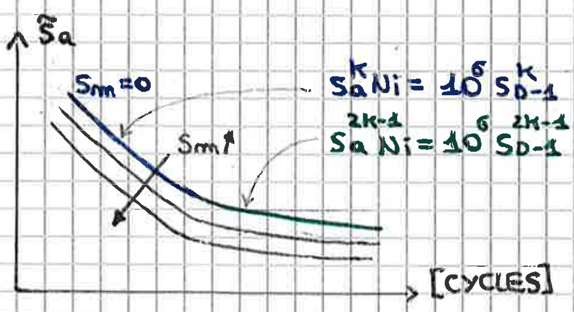


3.29) HAIBACH EXTENSION OF STRENGTH?

IN SOME CASES, RELATED TO SPECIFIC APPLICATIONS AND/OR MATERIALS, TO AVOID THAT TRUNCATION, A LOWER DEPENDENCE ON THE NUMBER OF CYCLES IS ASSUMED BY CHANGING THE EXPONENT OF THE BASQUIN EQUATION DESCRIBING THE CURVE.

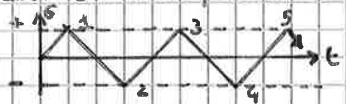
THIS APPROACH IS REFERRED TO A "HAIBACH EXTENSION".

THIS IS TYPICAL, FOR INSTANCE, IN TECHNICAL STANDARDS TO WELDED JOINTS.



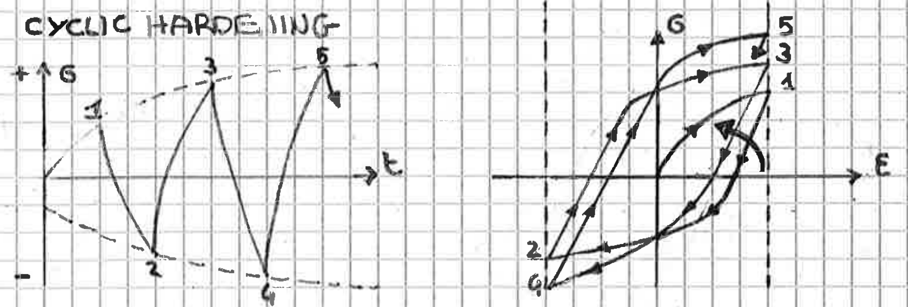
CHARACTERIZING THE PLASTIC STRAIN IS THEREFORE MANDATORY, BUT IT IS NEVER TRIVIAL BECAUSE OF MANY VARIABLE ISSUES:

A) THE HYSTERESIS LOOP MIGHT HAVE DIFFERENT SHAPES, ACCORDING TO LOADING CONDITIONS, IN CASE OF CYCLIC LOAD.

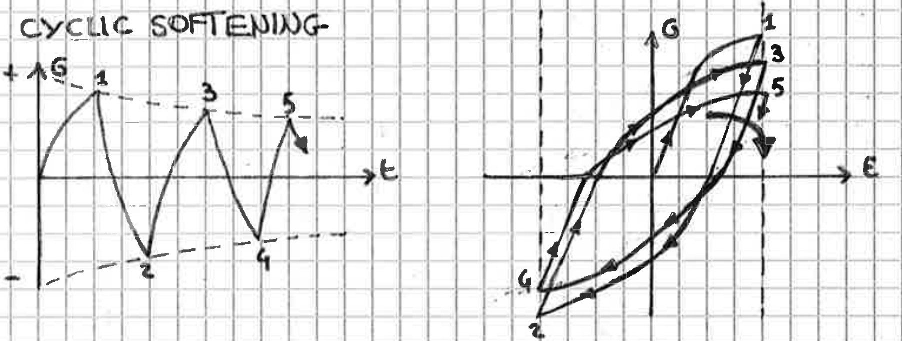


B) BOTH HARDENING AND SOFTENING RESPONSES COULD BE FOUND AND DEPEND UPON THE STABILITY OF MICROSTRUCTURE IN MATERIAL UNDER CYCLIC SLIP MODE.

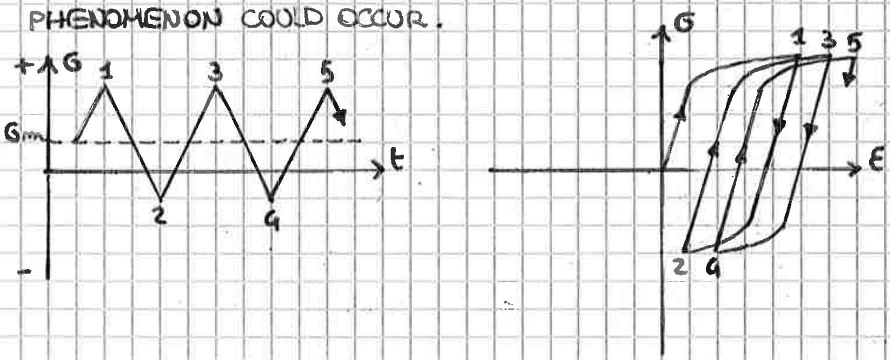
CYCLIC HARDENING



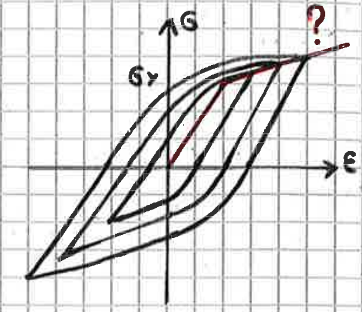
CYCLIC SOFTENING



C) IF THE MAXIMUM AND MINIMUM STRAINS ARE CONTROLLED, A RATCHETTING PHENOMENON COULD OCCUR.



3.32) DESCRIBE THE RAMBERG AND OSGOOD MODEL OF MATERIAL.



IN CASE OF PLASTIC BEHAVIOUR:

A CYCLIC HARDENING CURVE STRESS-STRAIN (σ-ε) IS USUALLY DESCRIBED BY AN ANALYTICAL EXPRESSION AS:

$\sigma = g^{-1}(\varepsilon_p) \iff \varepsilon_p = g(\sigma)$

THE CRUCIAL ISSUE IS DEFINING A SUITABLE FUNCTION g.

ACCORDING TO **RAMBERG AND OSGOOD**:

$\sigma = \sigma_y + K_y \varepsilon_p^{1/M_y}$

WITH K_y, M_y ARE CONSTANTS.

THUS LEADING TO HAVE:

$\varepsilon_p = \left(\frac{\sigma_s - \sigma_y}{K_y} \right)^{M_y}$

$\Delta\varepsilon_p = \left(\frac{\Delta\sigma}{K_c} \right)^{M_c}$

* ONCE THAT THESE PARAMETERS ARE KNOWN, A MODEL IS PROVIDED. (M_c, K_c)

WHILE ELASTIC STRAIN:

$\varepsilon_E = \frac{\sigma}{E}$

$\Delta\varepsilon_E = \frac{\Delta\sigma}{E}$

* FINDING THOSE PARAMETERS IS NEVER TRIVIAL.

SOME MODELS OF CYCLIC PLASTICITY ARE KNOWN:

- ISOTROPIC HARDENING
- KINEMATIC HARDENING

NONE OF THOSE PERFECTLY DESCRIBES THE ACTUAL BEHAVIOUR OF MATERIAL, BUT A COMBINATION OF THOSE TWO MODELS IS USED.

MOREOVER THE SHAPE OF HARDENING DEPENDS ON THE MATERIAL, EVEN ON THE TEMPERATURE.

WHICH ARE THE LIFE PREDICTION RELATIONS TO DESCRIBE THE STRAIN LIFE OF MATERIAL?

ONCE THAT AN ASSUMPTION TO DESCRIBE THE PLASTIC BEHAVIOUR OF MATERIAL IS MADE, A DESCRIPTION OF THE STRAIN LIFE OF MATERIAL CAN BE DONE IN CASE OF MECHANICAL, THERMAL AND THERMOMECHANICAL FATIGUE.

CRITERION	LIFE PREDICTION RELATION
BASQUIN-MANSON-COFFIN	$\varepsilon_a = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f (2N_f)^c$
MORROW	$\varepsilon_a = \frac{\sigma'_f}{E}\left(1-\frac{\sigma_m}{\sigma'_f}\right)(2N_f)^b + \varepsilon'_f\left(1-\frac{\sigma_m}{\sigma'_f}\right)^{\frac{c}{b}}(2N_f)^c$
WALKER	$\varepsilon_a = \frac{\sigma'_f}{E}\left(\frac{1-R}{2}\right)^{1-\gamma}(2N_f)^b + \varepsilon'_f\left(\frac{1-R}{2}\right)^{\frac{c(1-\gamma)}{b}}(2N_f)^c$
SMITH-WATSON-TOPPER	$\sigma_{MAX}\,\varepsilon_a = \frac{\sigma'^2_f}{E}(2N_f)^{2b} + \sigma'_f\varepsilon'_f(2N_f)^{b+c}$

PREVIOUS MODELS OF DAMAGE PREDICTION ASSUME THAT **TEMPERATURE REMAINS CONSTANT**. NEVERTHLESS IN **TF** AND **TMF** EVENT **T CHANGES**.

TO TAKE CARE OF THIS EFFECT **TWO MAIN PROBLEMS** HAVE TO BE FACED:

1) FINDING AN **EQUIVALENT CONSTANT TEMPERATURE** ALLOWING TO APPLY THE LCF METHODS AND INDUCING THE SAME FINAL DAMAGE;

2) CONSIDERING THE EVENTUAL OCCURRENCE OF **CREEP** (FLOW INDUCED BY HIGH T).

THOSE TWO ISSUES CAN BE MANAGED BY RESORTING TO **SOME THEORIES**, LIKE:

- THE **TAIRA'S APPROACH** FOR **EQUIVALENT TEMPERATURE**.
- THE **SEHITOGLU'S APPROACH** TO COMBINE THE DAMAGE INDUCED BY **FATIGUE**, **CREEP** AND **OXIDATION**.

3.36) DESCRIBE THE MAIN INNOVATION OF THE TAIRA'S MODEL IN FATIGUE.

TAIRA MODEL

THE APPROACH PROPOSED BY TAIRA INVESTIGATES THE CORRELATION BETWEEN **THERMAL FATIGUE (TF)** AND **ISOTHERMAL LCF** WITHIN THE SAME PLASTIC STRAIN RANGE AT ELEVATED TEMPERATURE, IN TERMS OF CHANGE OF THE MICROSTRUCTURE OF MATERIAL DURING THE FATIGUE PROCESS.

IN CASE OF VARIABLE T TAIRA ASSUMES THAT THE MANSON-COFFIN MODEL HAS TO BE MODIFIED AS FOLLOWS:

$$\Delta\varepsilon_p \cdot N_f^{\alpha} = C \implies \lambda(T)\cdot(\Delta\varepsilon_p)^m \cdot N_f = C_1$$

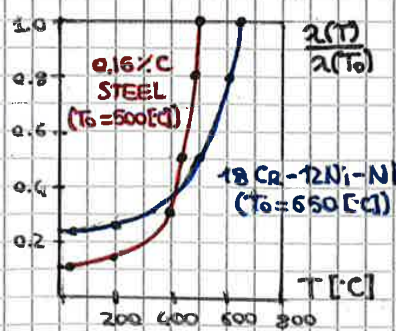
WHERE:

λ, m : CHARACTERISTICS OF MATERIAL TO BE EXPERIMENTALLY CHARACTERIZED.

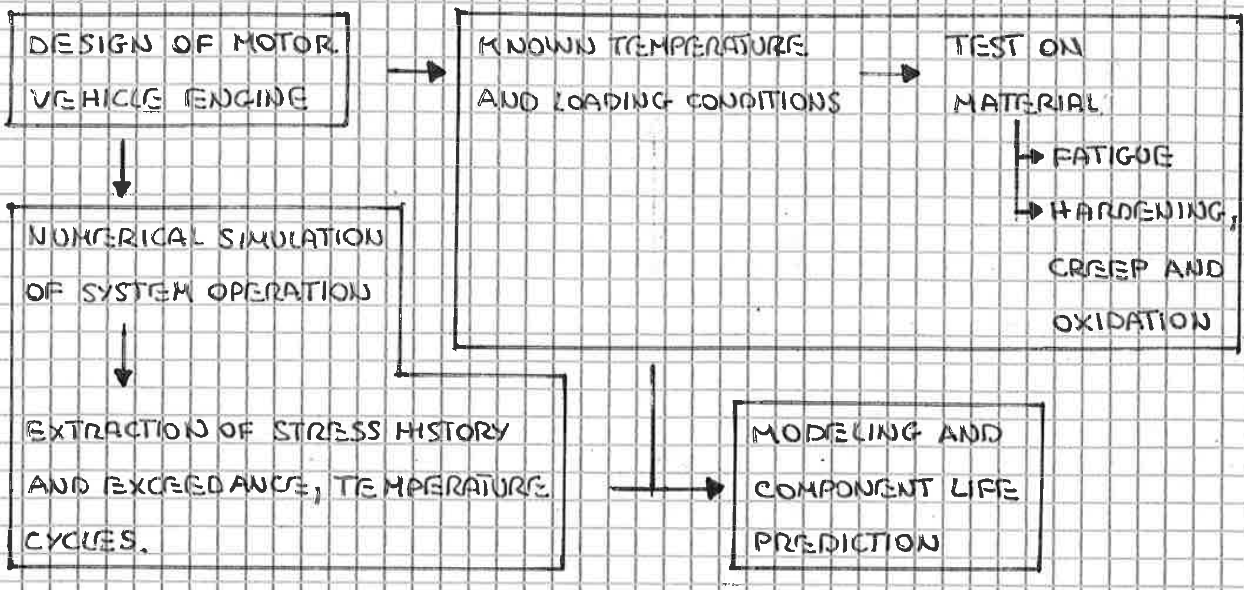
C_1: CONSTANT WITH T.

TESTS ARE PERFORMED TO FIND FUNCTION $\lambda(T)$ THUS LEADING TO HAVE:

$$\frac{\lambda(T)}{\lambda(T_0)} = \frac{N_f(T_0)}{N_f(T)}$$



GIVE AN EXAMPLE OF INDUSTRIAL PROBLEM WITH COMBINED EFFECTS.



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3.39) WHAT'S MATERIAL TOUGHNESS? HOW IT IS USED IN DESIGN?

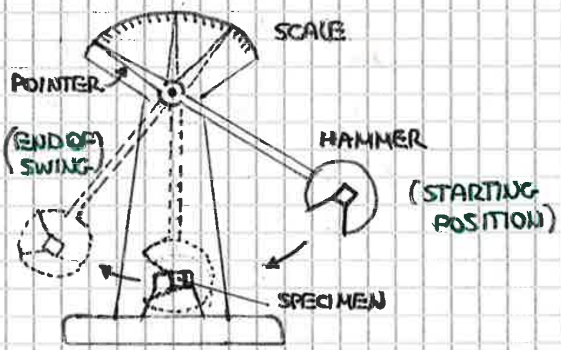
MATERIAL TOUGHNESS

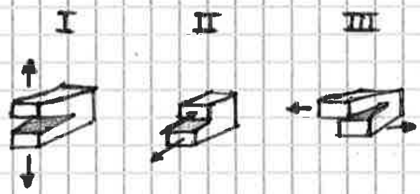
$$\sigma\sqrt{a} \geq \sqrt{\frac{2E\Gamma}{\pi}} = K_c \qquad \text{GRIFFITH LAW}$$

K_c = MATERIAL TOUGHNESS

IT POINTS THAT IN FRACTURE MECHANISM NOT ONLY THE STRESS (σ) EVEN THE CRACK LENGTH (a) HAS A ROLE.

K_c IS MEASURED THROUGH A SPECIFIC EXPERIMENTAL TEST: CHARPY TEST. THIS TEST IS BASED ON THE IMPACT OF A MOVING MASS WITH DEFINED GRAVITATIONAL POTENTIAL AGAINST A STANDARD SPECIMEN.
HOWEVER TOUGHNESS IS DIFFERENT DEPENDING ON THE FRACTURE MODE EXCITED. THREE ARE THOSE DEFINED (I, II, III). THEREFORE, TESTS ARE MADE TO IDENTIFY K_{Ic}, K_{IIc}, K_{IIIc}



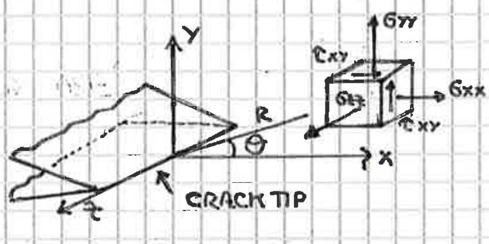


THE STRESS DISTRIBUTION AROUND THE CRACK TIP CAN BE FOUND THROUGH WESTERGAARD APPROACH.

$$\sigma_{ij}(r,\theta) = \frac{K_{I,II,III}}{\sqrt{2\pi r}} f_{ij}(\theta)$$, WITH: $K_{I,II,III}$: STRESS INTENSITY FACTOR

EX: MODE I

$K_I = Y\sigma\sqrt{a}$, Y DEFINED BY HANDBOOK CASE BY CASE

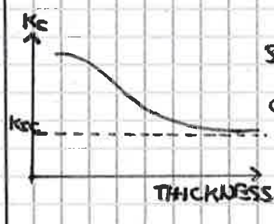


$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + \dots$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + \dots$$

$$\sigma_{zz} = 0$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} + \dots$$



STABLE PROPAGATION OCCURS WHEN:

$$K_I = Y\sigma\sqrt{a} < K_{Ic}$$

$K_I = Y\sigma\sqrt{a}$ RELATED TO THE OPERATION

$K_{Ic} = \sqrt{\frac{2E\Gamma}{\pi}}$ MAT. PROPERTY MEASURED BY TEST

4. BEARINGS

4.1 HERTZ THEORY AND APPLICATIONS

4.1.1) DESCRIBE THE MAIN ASSUMPTIONS OF THE HERTZ'S THEORY FOR ELASTIC CONTACT BETWEEN BODIES.

ASSUMPTIONS OF THE HERTZ'S THEORY

CONTACT CONDITIONS:

A) GEOMETRY

- TWO BODIES IN CONTACT.
- IN A NON-SINGOLAR POINT OF THEIR SURFACES (REGULAR AT LEAST TO THE SECOND DERIVATIVE); THEN (FIRST DERIVATIVES) THE COMMON TANGENT PLANE EXISTS.

B) MATERIAL

- ELASTIC, ISOTROPIC.
- NO FRICTION

C) HYPOTHESIS (EMPIRICALLY OBSERVED, POST MATHEMATICALLY PROVED)

- SMALL CONTACT SURFACE (LENGTH AND WIDTH SMALL COMPARED TO CURVATURE RADII OF BODIES IN CONTACT).

4.1.2) DEFINE ANALYTICALLY THE EXPRESSION OF SURFACE $z(x,y)$, CURVATURES (α, β, γ) AND DESCRIBE THE REFERENCE FRAME USED TO INVESTIGATE THE RELATIVE DISPLACEMENT BETWEEN ELASTIC BODIES.
WHAT'S THE MEANING OF CURVATURE SIGN?
IS CURVATURE RELATED TO RADIUS OR TO DIAMETER?

4.1.3) HOW THE CONTACT AREA LOOKS? HOW DIMENSIONS ARE CALCULATED?
(-> LOOK 4.1.4 AND 4.1.6)

LET'S NOW CONSIDER TWO BODIES, 1, 2, BOTH WITH CURVATURES

TWO BODIES WITH SURFACE 1 AND 2 IN CONTACT ARE TANGENT THROUGH THE SAME COMMON PLANE.

TO FACILITATE AN INTUITIVE PRACTICAL APPROACH TO SIGN DEFINITIONS, IT IS EXPEDIENT TO DEFINE AXIS z SO THAT CURVATURES ARE POSITIVE WHEN THE BODY IS CONVEX ( )(*); THIS IS ALREADY THE CASE OF AXIS z_1 FOR BODY 1.

(*) A SURFACE SECTION IS CONVEX AT A POINT OF A BODY WHEN IT IS SEEN BY AN OBSERVER, OUTSIDE THE BODY MATERIAL, ON THE OPPOSITE SIDE OF THE TANGENT PLANE TO THE BODY IN THAT POINT.

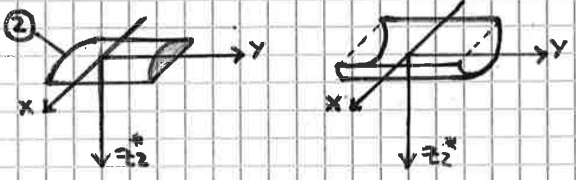
SURFACE PROPERTIES FOR BODY 2

FOR BODY 2 THE AXIS z_2 IS CHOSEN INSIDE (DOWNWARDS) SO THAT WITH POSITIVE CURVATURES THE SURFACE COORDINATES ARE POSITIVE (DOWNWARDS).

$$z_2^* = \{x \; y\} \begin{bmatrix} \beta_{xx} & \beta_{xy} \\ \beta_{yx} & \beta_{yy} \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix}$$

$$\beta_{xx} = \frac{1}{2} \cdot \frac{1}{R_{xx,2}}$$

$$\beta_{yy} = \frac{1}{2} \cdot \frac{1}{R_{yy,2}}$$



CONVEX CONCAVE

THE PRACTICAL RULE IS THAT FOR EACH BODY ITS OWN AXIS z IS CHOOSEN INSIDE THE MATERIAL IN ORDER TO HAVE POSITIVE CURVATURES (AND THIS OCCURS WHEN THE CONSIDERED BODY IS CONVEX).

WHEN THE CONTACT IS ANALYZED THE RELATIVE POSITION OF EACH POINT OF THE BODIES SURFACES IS DEFINED $\Rightarrow$ WE MUST EXPRESS BOTH BODIES IN THE SAME REFERENCE SYSTEM, THAT WE MAY CHOOSE TO BE (x, y, z) OF BODY 1, THEN:

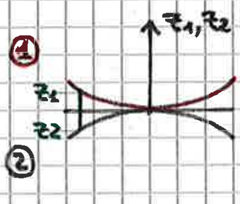
$$z_2 = -z_2^*$$

$$z_2 = \{x \; y\} \begin{bmatrix} -\beta_{xx} & -\beta_{xy} \\ -\beta_{yx} & -\beta_{yy} \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix}$$

A COMMON REFERENCE SYSTEM IS CONVENIENT TO CALCULATE THE VERTICAL DISTANCE ($z_1 - z_2 = z_1 + z_2^*$) BETWEEN THE TWO BODIES (1, 2).

$$z_1 - z_2 = z_1 + z_2^* = \{x \; y\} \begin{bmatrix} \alpha_{xx} + \beta_{xx} & \alpha_{xy} + \beta_{xy} \\ \alpha_{yx} + \beta_{yx} & \alpha_{yy} + \beta_{yy} \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix}$$

(RELATIVE DISPLACEMENT)



EACH SURFACE 1 AND 2 IS EXPRESSED, IN THE SIMPLEST WAY, IN IT'S OWN PRINCIPAL REFERENCE SYSTEM:

$$z_1 = \{X_1 Y_1\}\begin{bmatrix}\alpha_X & 0\\ 0 & \alpha_Y\end{bmatrix}\begin{Bmatrix}X_1\\ Y_1\end{Bmatrix} \iff z_1 = \alpha_X X_1^2 + \alpha_Y Y_1^2$$

$$z_2 = \{X_2 Y_2\}\begin{bmatrix}-\beta_X & 0\\ 0 & -\beta_Y\end{bmatrix}\begin{Bmatrix}X_2\\ Y_2\end{Bmatrix} \iff z_2 = -(\beta_X X_2^2 + \beta_Y Y_2^2)$$

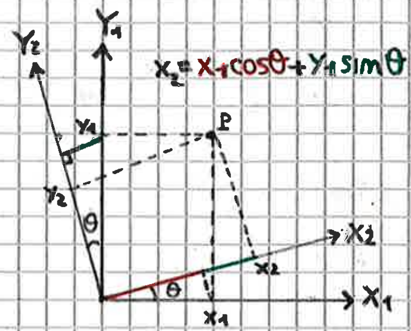
$$z_1 - z_2 = \{X\, Y\}\begin{bmatrix}\gamma_X & 0\\ 0 & \gamma_Y\end{bmatrix}\begin{Bmatrix}X\\ Y\end{Bmatrix} \iff z_1 - z_2 = h = \gamma_X X^2 + \gamma_Y Y^2$$

PHYSICAL ROTATION BETWEEN THE TWO BODIES:

$$\begin{Bmatrix}X_2\\ Y_2\end{Bmatrix} = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{Bmatrix}X_1\\ Y_1\end{Bmatrix}$$

THE INVERSE IS EQUAL TO THE TRANSPOSE:

$$\begin{Bmatrix}X_1\\ Y_1\end{Bmatrix} = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{Bmatrix}X_2\\ Y_2\end{Bmatrix}$$



ABOUT THE RELATIVE DISTANCE $(z_1 - z_2)$

$$\gamma_X + \gamma_Y = \alpha_X + \alpha_Y + \beta_X + \beta_Y$$

$$\gamma_X - \gamma_Y = \pm\sqrt{(\alpha_X-\alpha_Y)^2 + (\beta_X-\beta_Y)^2 + 2\cos 2\theta(\alpha_X-\alpha_Y)(\beta_X-\beta_Y)}$$

THE RELATIVE CURVATURES DEPEND UPON THE ANGLE BETWEEN THE TWO PLANES, BUT THIS ANGLE IS KNOWN AND CAN BE EASILY TAKEN INTO ACCOUNT BY INTRODUCING A SUITABLE PARAMETER, NAMELY τ.

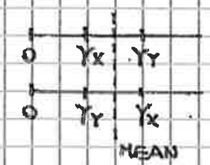
THE CONSEQUENCE OF:

$\gamma_X \geq 0$

$\gamma_Y \geq 0$

IS THAT $\gamma_X + \gamma_Y$ IS ALWAYS HIGHER THAN THE ABSOLUTE VALUE OF $\gamma_X - \gamma_Y$.

THERE ARE TWO POSSIBILITIES:



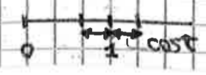
AND IN EITHER CASE:

$$0 \leq \frac{|\gamma_X - \gamma_Y|}{\gamma_X + \gamma_Y} \leq 1$$

WE GIVE TO THE FRACTION $\frac{|\gamma_X - \gamma_Y|}{\gamma_X + \gamma_Y}$ THE NAME "$\cos\tau$" DUE TO LIMITS $0 \div 1$;
IT IS USED AS A GOVERNING PARAMETER.

$\cos\tau = \frac{|\gamma_X - \gamma_Y|}{\gamma_X + \gamma_Y}$ RELATIVE DEVIATION (FROM THE MEAN)

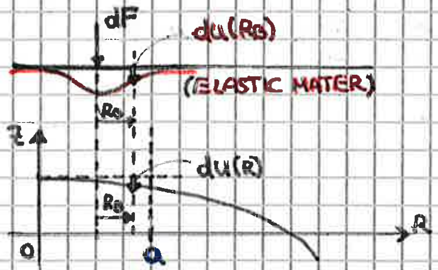
$$\lambda_1, \lambda_2 = \gamma_X, \gamma_Y = \frac{(\gamma_X+\gamma_Y) \pm |\gamma_X - \gamma_Y|}{2} = \frac{(\gamma_X+\gamma_Y)}{2}\left[1 \pm \frac{|\gamma_X-\gamma_Y|}{\gamma_X+\gamma_Y}\right] = \frac{(\gamma_X+\gamma_Y)}{2} + [1 \pm \cos\tau]$$



Q.1.5) DESCRIBE THE THEORY OF CONTACT DEFORMATION PROPOSED BY BOUSSINESQ.

RELATION BETWEEN DEFORMED SHAPE AND ELASTIC PROPERTIES

HERTZ STARTED FROM BOUSSINESQ SOLUTION OF THE "ELASTIC HALF-SPACE" LOADED BY A CONCENTRATED FORCE.



$$du(R_B) = \frac{1-\nu^2}{\pi E}\frac{dF}{R_B}$$

IN OUR CASE THE ELASTIC HALF SPACE IS LIMITED IN ANY SECTION PLANE BY A CURVE OF LOCAL RADIUS R; HOWEVER, AS $a \ll R$, THE CURVE CAN BE APPROXIMATED BY ITS TANGENT.

SO THE SOLUTION IS THE ONE VALID FOR THE PLANE HALF SPACE!

ON EACH AREA ELEMENT $dA = dx'dy'$ INSIDE THE CONTACT SURFACE AT A POINT (x', y'), AN INFINITESIMAL FORCE (dF) IS APPLIED:

$$dF = p(x', y')dx'dy'$$

WHICH PRODUCES AT ANY OTHER POINT OF COORDINATES (x,y) A CONTRIBUTION TO DISPLACEMENTS (du):

$$du(x,y) = \frac{1-\nu^2}{\pi E}\frac{1}{R_B}p(x',y')dx'dy' \quad , \text{ WITH } R_B = \sqrt{(x-x')^2+(y-y')^2}$$

$$u(x,y) = \frac{1-\nu^2}{\pi E}\iint_{\text{CONTACT AREA}} \frac{p(x',y')dx'dy'}{R'}$$

$$\frac{u_1(x,y)}{u_2(x,y)} = \frac{\frac{1-\nu_1^2}{E_1}}{\frac{1-\nu_2^2}{E_2}}$$

THEREFORE, IN THE DEFORMED ZONE IN CONTACT, $0 \leq R \leq a$, THE DISTANCE $u_1 + u_2 = \delta - (z_1 - z_2)$ BETWEEN THE TWO UNDEFORMED CURVES IS SPIT INTO TWO PARTS IN THE PROPORTION u_1/u_2.

⊕ DESCRIBE CONTACT BETWEEN CYLINDERS. (4.1.7)

CONTACT BETWEEN CYLINDERS

THE PREVIOUS FORMULAS AND THE RELATED DIAGRAMS CANNOT BE EXTENDED TO THE CASE OF CYLINDER / CYLINDER CONTACT WITH A LIMIT OPERATION SUCH AS $\cos\tau \to 1$.

IN FACT THIS EXTRAPOLATION:

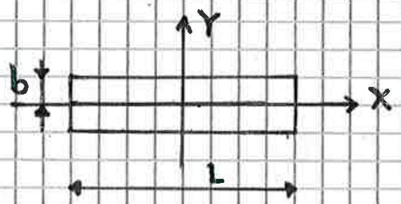
$\alpha_x = \beta_x \to 0 \qquad \theta = 0,\ \cos\tau = 1 \qquad a^* \to \infty\ ;\ b^* \to 0$

I.E. THE CONTACT AREA IS FINITE BECAUSE ONE SEMI AXIS TENDS TO INFINITE AND THE OTHER TO ZERO, BUT IT REQUIRES THAT THE MAJOR AXIS TO BE INFINITE WHILE IN CYLINDRICAL CONTACTS IT IS LIMITED.

FORMULAS FOR CYLINDRICAL CONTACT

HOWEVER, PRACTICAL CONTACTS HAVE A **FINITE LENGHT L**, AND A **FINITE SEMI-WIDTH b**, GIVEN BY THE FOLLOWING FORMULA:

$$P = \frac{2F}{\pi L b} \cdot \sqrt{1 - \left(\frac{Y}{b}\right)^2}$$



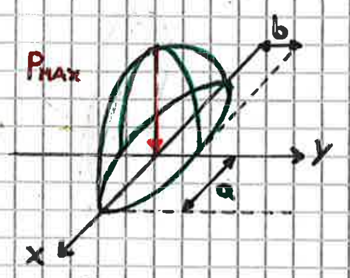
WITH $b = \sqrt{\frac{4}{\pi} \frac{F}{L} \frac{1}{2(\alpha_x + \beta_x)} \left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)}$

$P_{MAX} = \frac{2F}{\pi L b}$, $P_{MEAN} = \frac{F}{2Lb}$

$P_{MAX} = \frac{4}{\pi} P_{MEAN}$

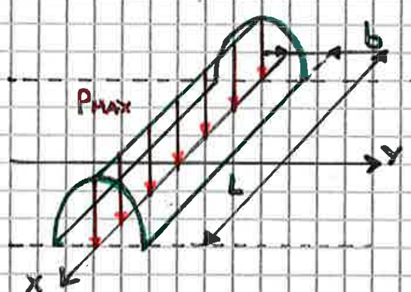
⊕ DRAW THE PRESSURE DISTRIBUTION: COMPARISON BETWEEN: GENERAL CONTACT AT A POINT & CYLINDER / CYLINDER CONTACT. (4.1.7)

GENERAL CONTACT IN A POINT



P: MAXIMUM IN THE MIDDLE

CYLINDER / CYLINDER CONTACT



P: UNIFORM DISTRIBUTION (LINE)

4.1.7 WHAT KIND OF DIFFERENCE OCCURS BETWEEN SPHERE-SPHERE CONTACT AND CYLINDER / CYLINDER? DRAW THE PRESSURE DISTRIBUTION IN BOTH THOSE CASES (→ ANSWERS = PREVIOUS ⊕, ⊕ & NEXT ⊕)

4.1.8) DEFINE THE MEANING OF REFERENCE PRESSURE (P_{REF}) AND ITS USE IN HANDBOOKS.

REFERENCE PRESSURE (P_{REF}) — SPHERE - SPHERE

$$P_{REF} = \frac{F}{\frac{\pi}{4} d^2}$$

- P_{REF} IS THE MEAN PRESSURE THROUGH THE CENTRAL, I.E. THE GREATEST, CROSS SECTION OF THE SPHERE OF DIAMETER d. ($A_{REF} = \pi d^2/4$)
- P_{REF} IS THE LOWEST POSSIBLE MEAN PRESSURE, GIVEN THE BODY DIMENSION d, AND THE APPLIED LOAD F.
- P_{REF} IS THE PARAMETER WHICH GOVERNS P_{MEAN} AND P_{MAX} ON IT, AND THE APPROACH δ.

(CONSIDERING STEEL ON STEEL)

THE MEAN PRESSURE (P_{MEAN}) ON THE CONTACT SURFACE IS:

$$P_{MEAN} = \frac{F}{\pi a b} = \frac{F\left(\frac{1}{d} \pm \frac{1}{D}\right)^{\frac{2}{3}}}{\pi (1.51 \cdot 10^{-2})^2 F^{2/3}} = \frac{\sqrt[3]{\pi/4}}{\pi (1.51 \cdot 10^{-2})^2} \sqrt[3]{P_{REF}} \sqrt[3]{\left(1 \pm \frac{d}{D}\right)^2}$$

... AND THE APPROACH (δ) IS:

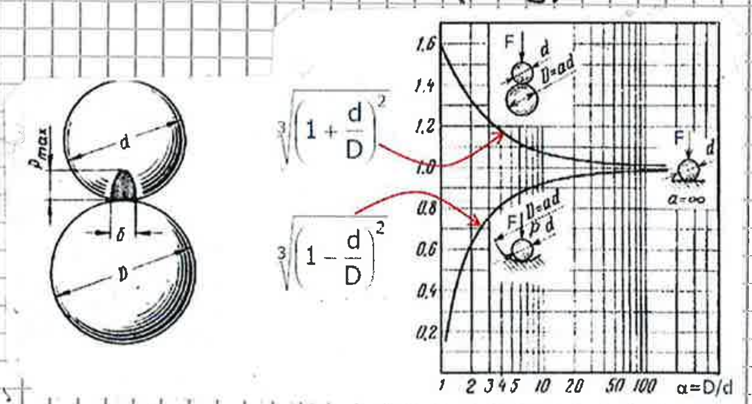
$$\delta = 3.60 \cdot 10^{-4} \sqrt[3]{F^2 \Sigma \frac{1}{d}} = 4.53 \cdot 10^{-4} \sqrt[3]{\frac{F^2}{d}\left(1 \pm \frac{d}{D}\right)}$$

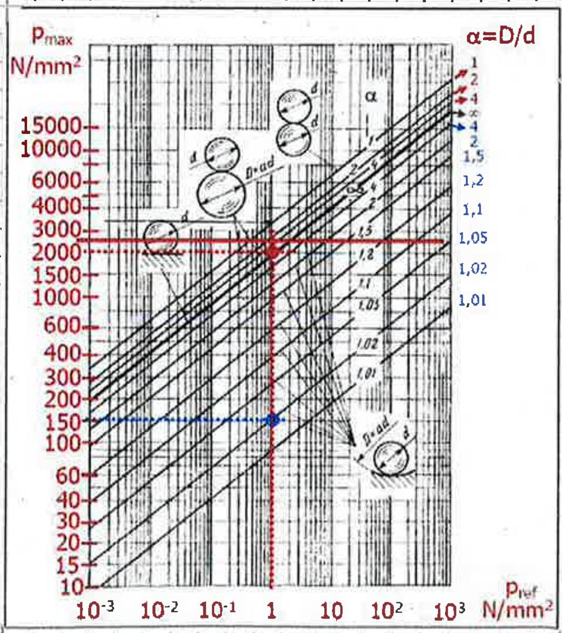
THEN:

$$P_{MAX} = \frac{3}{2} P_{MEAN} = 1930 \sqrt[3]{P_{REF}} \sqrt[3]{\left(1 \pm \frac{d}{D}\right)^2}$$

$$\frac{\delta}{d} = 3.86 \cdot 10^{-4} \sqrt[3]{P_{REF}^2} \sqrt[3]{\left(1 \pm \frac{d}{D}\right)}$$

GEOMETRICAL FACTOR $\sqrt[3]{\left(1 \pm \frac{d}{D}\right)^2}$ ($\alpha = D/d$ = GEOMETRICAL RATIO FACTOR)





THIS SHOWS THE EFFECT OF D/d FOR A GIVEN P_{REF}, I.E., F AND d.

NW: THE WORST CASE IS WHEN TWO SPHERES (CONV.-CONV. CONTACT) HAVE EQUAL DIAM: $D=d$

Q.1.9) IF WE COMPARE ROLLING ELEMENTS IN BEARINGS AS BALLS AND ROLLS, WHEN THEY ARE RESPECTIVELY SUITABLE FOR APPLICATIONS ?

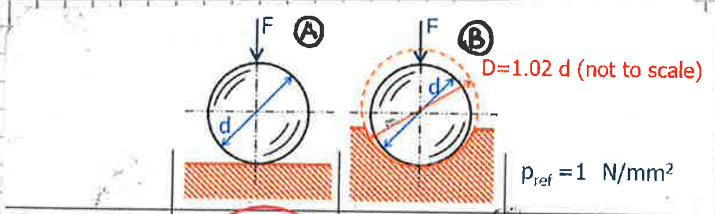
DIRECT COMPARISON

A VALUE $P_{REF} = 1\,[N/mm^2]$ IS QUITE REALISTIC, AS IT PRODUCES CONTACT STRESSES OF THE ORDER OF MAGNITUDE OF THOSE THAT A BEARING STEEL WILL TAKE IN OPERATING CONDITIONS.

TWO REFERENCE CASES ARE EXAMINED (AT THE THRESHOLD OF HERTZ TH. VALIDITY):

A) SPHERE ON PLANE

B) SPHERE IN A SPHERICAL CAVITY



	A	B	
p_{max}	≈ 2000	≈ 150	N/mm²
i.e., p_{max}	≈ 2000	≈ 150	times p_{ref}

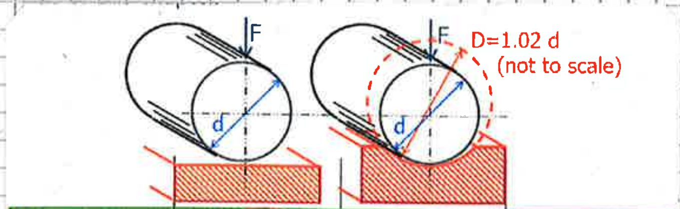
CONSIDERING CYL. CONTACT

SEE HERE CYLINDER P_{MAX}, FOR $P_{REF} = 1\,[N/mm^2]$ AND FOR $P_{REF} = 60\,[N/mm^2]$ (WHERE P_{MAX} IS ABOUT 2000 [MPa]) AND COMPARE WITH THE PREVIOUS CASE.

TWO REFERENCE CASES ARE EXAMINED:

A) CYLINDER ON PLANE

B) CYLINDER IN A CYLINDRICAL CAVITY



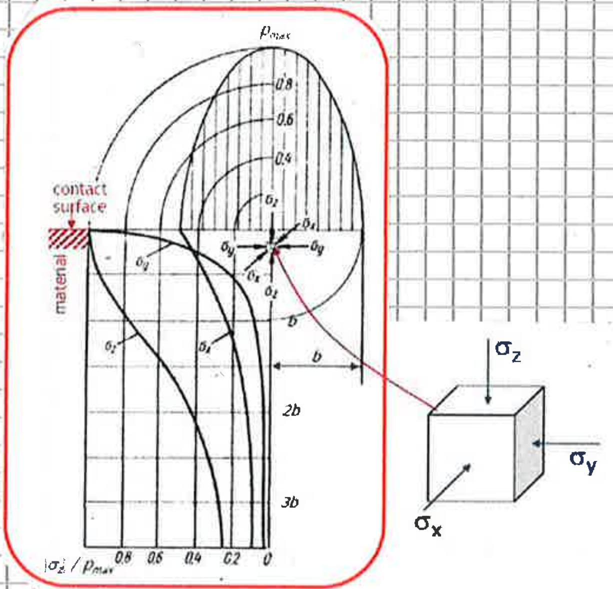
		A	B	
p_{ref}=1 MPa	p_{max}	≈ 285	≈ 40	N/mm²
	i.e., p_{max}	≈ 285	≈ 40	times p_{ref}
p_{ref}=60 MPa	p_{max}	≈ 2000	≈ 300	N/mm²
	i.e., p_{max}	≈ 33	≈ 5	times p_{ref}

Q.1.10 DRAW THE TYPICAL DISTRIBUTION OF STRESS COMPONENTS UNDER SURFACE AND FIND THE MOST CRITICAL QUOTE AT WHICH STRESS IS MAXIMUM.

WHERE IS LOCATED THE MAXIMUM STRESS AND DAMAGE OCCURS?

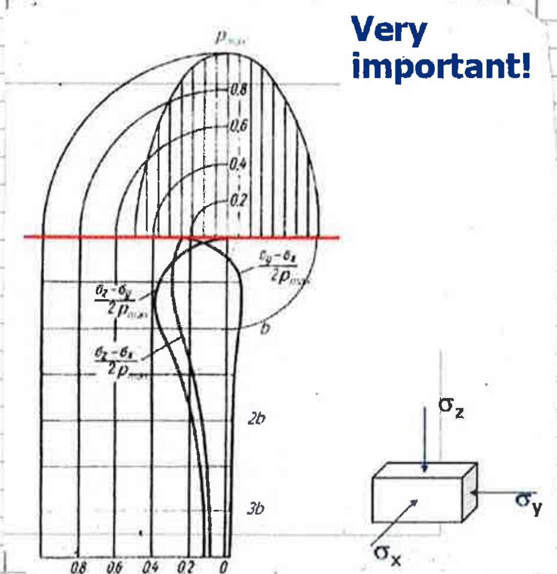
SURFACE AND SUB-SURFACE STRESSES FOR THE CYLINDER CONTACT

EXPERIMENTAL EVIDENCE SHOWS THAT FAILURE STARTS AT POINTS BELOW THE SURFACE. IT IS THEN OF INTEREST TO DETERMINE SUBSURFACE STRESSES, WHICH ARE HERE SHOWN FOR THE CASE OF THE CYLINDER CONTACT.



Axis z is on the centre of the contact; symmetry implies that along this axis shear stresses τ_{ij} (on any infinitesimal element) are zero. Then normal stresses are principal.

THE FOLLOWING FIGURE, NOT VERY ACCURATE, SHOWS MAXIMUM SHEAR STRESS ON THE THREE PRINCIPAL PLANES. THEIR DOUBLE ARE MOHR'S DIAMETERS ON THE PRINCIPAL PLANES.



THE HIGHEST DIAMETER IS, ACCORDING TO TRESCA THE EQUIVALENT STRESS; IT IS $(\sigma_z - \sigma_y)$, WHICH HAS A MAXIMUM AT ABOUT DEPTH $z \cong 0.65\,b$ WHERE b IS THE CONTACT SEMIWIDTH.

IT IS QUITE DIFFICULT TO STATE PRECISELY AT WHICH DEPTH THE MAXIMUM STRESS IS, AS THE CURVES ARE QUITE FLAT, MOREOVER IT IS NOT CLEAR WHETHER WE SHOULD TAKE TRESCA OR VON MISES. FORTUNATELY, THEY PRODUCE ALMOST EQUAL RESULTS:

TRESCA

- CYLINDER/CYLINDER CONTACT: $\sigma_{EQ} = \sigma_z - \sigma_y = 0.60\,P_{MAX}$, AT $\frac{z}{b} = 0.78$
- SPHERE/SPHERE CONTACT: $\sigma_{EQ} = \sigma_z - \sigma_y = 0.63\,P_{MAX}$, AT $\frac{z}{b} = 0.47$

VON MISES

- CYLINDER/CYLINDER CONTACT: $\sigma_{EQ} = 0.59\,P_{MAX}$, AT $\frac{z}{b} \cong 0.6$
- SPHERE/SPHERE CONTACT: $\sigma_{EQ} = 0.63\,P_{MAX}$, AT $\frac{z}{b} = 0.47$

G.1.11) DEFINE THE MOST USED CRITERIA TO COMPARE PRESSURE (P) AND YIELD (Re) IN CYLINDER/CYLINDER AND SPHERE/SPHERE CONTACTS.

WHAT KIND OF CRITERION IS USED TO FIND THE SAFETY FACTOR? (1,7 ; 1,6)

MAXIMUM CONTACT STRESS AT THE CENTRE OF CONTACT SURFACE, AT YIELD ONSET, IS OBTAINED AS FOLLOWS:

- CYLINDER / CYLINDER CONTACT: $\sigma_{EQ} = \sigma_z - \sigma_y = 0.60\,P_{MAX} = Re \Rightarrow P_{MAX} = \frac{1}{0.60} Re \cong 1.7\,Re$
- SPHERE/SPHERE CONTACT: $\sigma_{EQ} = \sigma_z - \sigma_y = 0.63\,P_{MAX} = Re \Rightarrow P_{MAX} = \frac{1}{0.63} Re \cong 1.6\,Re$

YIELD ONSET (Re) IN BEARING STEELS IS AROUND $Re = 1800 \div 2000\ [N/mm^2]$

THUS FOR A SPHERE ON SPHERE WITH $Re = 2000\ [N/mm^2]$ THE P_{MAX} WILL PRODUCE YIELD WHEN: $P_{MAX} \cong 1.6\,Re = 1.6 \cdot 2000 \cong 3200\ [MPa]$

4.2 ROLLING BEARINGS : STATIC LOADING

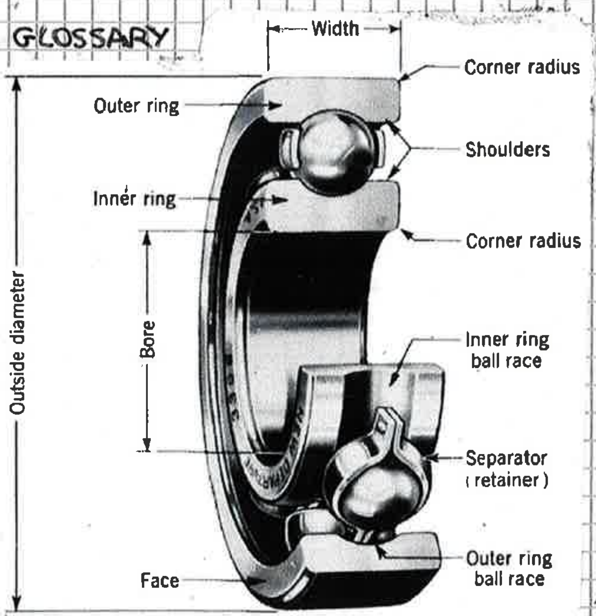
4.2.1) DESCRIBE THE MAIN FUNCTIONS OF BEARINGS

BEARINGS

1. THEY ARE KEY COMPONENTS OF THE MACHINE ASSEMBLY SINCE THEY ALLOW CONNECTING A ROTATING PART TO THE FIXED STATOR.
2. THEY ARE SMALL SYSTEMS WHERE SEVERAL INTERFACES HAVE TO BE CONSIDERED.
3. THEY ARE A CRUCIAL APPLICATION OF FATIGUE AND CONTACT THEORIES TO PREDICT THE SYSTEM LIFE.
4. THEY REQUIRE A DESIGN TO PERFORM A SUITABLE ASSEMBLING.
5. ABOUT ASSEMBLY AND LAYOUT: SEVERAL SOLUTIONS ARE PROPOSED TO ALLOW ONE BODY ROTATING, WHILE SUPPORT IS KEPT FIXED, AND REDUCING THE WEAR BY SUBSTITUTING THE SLIDING FRICTION BETWEEN MATERIALS WITH ROLLING FRICTION.

⊕ DESCRIBE BEARING'S STRUCTURE WITH ITS COMPONENTS AND ANALYSE THE DESIGN GOALS

GLOSSARY



COMPONENTS

1. INNER RING (USUALLY FIXED WITH ROT. PART)
2. OUTER RING (USUALLY FIXED WITH STATOR)
3. ONE OR MORE CROWNS OF ROLLING BODIES THAT ROLL ON THE RING (BALL) RACEWAYS.
4. SEPARATOR / RETAINER / CAGE. 5. SEAL

+. EVENTUAL GASKETS AND PROTECTIVE SCREENS FOR THE LUBRICANT AND AS PROTECTION AGAINST DIRT FROM THE EXTERNAL WORKING ENVIRONMENT.

ABOUT THE FACTORY ASSEMBLY (5 STEPS)

POSITIONING OF, FIRST, OUTER RING, SECOND, CROWN OF ROLLING ELEMENTS, THEN INNER RING; CENTERING OF INNER RING; RETAINER AND LUBRIC.

DESIGN GOALS

1. SELECT THE CONFIGURATION
2. SELECT THE ROLLING ELEMENTS
3. DEFINE THE SIZE ACCORDING TO THE LOADING CONDITIONS AND THE REQUIRED LIFE.
4. DEFINE THE LUBRIFICATION SYSTEM AND MAINTENANCE.
5. DEFINE THE SYSTEM OF BEARINGS AND THE MOUNTING LAYOUT.

4.2.4) DESCRIBE THE DIFFERENCES BETWEEN BALL AND ROLLER BEARINGS.

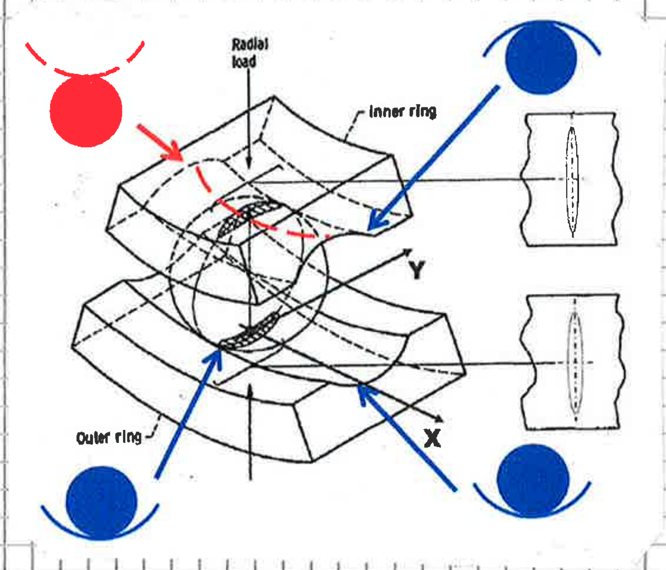
BALL: SMALL CONTACT AREA; HIGHER PRESSURE; LOWER FRICTION
$\Rightarrow$ LOWER LOAD; HIGHER SPEED

ROLLER: LARGER CONTACT AREA; LOWER PRESSURE; HIGHER FRICTION
$\Rightarrow$ HIGHER LOAD; LOWER SPEED.

4.2.5) DEFINE SURFACE CURVATURES ($\alpha_X, \alpha_Y, \beta_{Xi}, \beta_{Yi}, \beta_{Xo}, \beta_{Yo}$) IN BEARINGS.

CONTACT PROPERTIES

CURVATURES CONSIDERING BALL BEARING



BALL (SPHERE, d_s)

$\alpha_X = \alpha_Y = \frac{1}{d_s}$

INNER RING (D_i)

$\beta_{Xi} = +\frac{1}{D_i}$

$\beta_{Yi} = -\frac{1}{1.05\, d_s}$

OUTER RING (D_o)

$\beta_{Xo} = -\frac{1}{D_o}$

$\beta_{Yo} = -\frac{1}{1.05\, d_s}$

G.1.7) DERIVE THE MATHEMATICAL EXPRESSION OF THE RELATION BETWEEN RADIAL FORCE AND DISPLACEMENT IN RADIAL BEARING, INCLUDING THE CLEARANCE

G.1.8) DEFINE THE LOAD DISTRIBUTION ALONG THE RING OF A BALL RADIAL BEAR.

G.1.9) COMPUTE THE GLOBAL RADIAL FORCE APPLIED BY THE RADIAL BEARING (R)

- ANALYSIS OF LOAD DISTRIBUTION

SPHERE AND ROLLER CONTACTS HAVE THE SAME FORM FOR THE APPROACH - FORCE RELATION (HERE BELOW FOR STEEL ON STEEL).

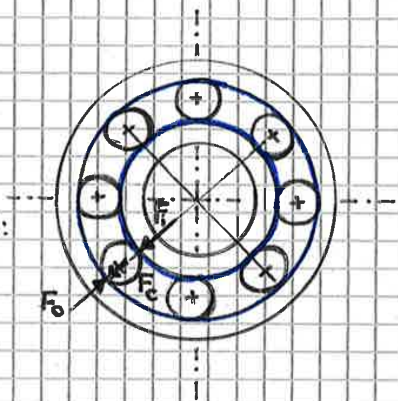
$$\begin{cases} \text{SPHERE}: \ \delta = \delta^* 3.60 \cdot 10^{-4} \sqrt[3]{\Sigma \frac{1}{\mathcal{R}}} \, F^{2/3} \\ \text{CYLINDER}: \ \delta = 3.84 \cdot 10^{-5} \frac{F^{0.9}}{L^{0.8}} \end{cases}$$

NOTE THAT IF CENTRIFUGAL FORCE (F_c) IS ABSENT OR NEGLIGIBLE:

$F_i = F_o = F$

OTHERWISE:

$F_o = F_i + F_c$



LATER WE SHALL NEED THE EQUATION IN THE FORM:

$F = K \delta^m$

$$\begin{cases} \text{SPHERE}: \ m = 1.5 \\ \text{CYLINDER}: \ m = 1.1 \end{cases}$$

THEN

$F_o = K_o \delta_o^m$

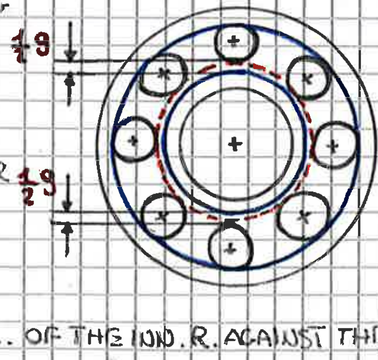
$F_i = K_i \delta_i^m$

$$\left.\begin{array}{l} F = K_{TOT} \delta_{TOT}^m \quad \text{OR} \quad \delta_{TOT} = \left(\frac{F}{K_{TOT}}\right)^{1/m} = F^{1/m} \left[\frac{1}{K_o^{1/m}} + \frac{1}{K_i^{1/m}}\right] \\ \text{WITH } K_{TOT} = \left[\frac{1}{K_o^{1/m}} + \frac{1}{K_i^{1/m}}\right]^{-m} \end{array}\right\} \text{IF } F_i = F_o$$

- CLEARANCE (g)

IN THE CASE OF RADIAL BEARINGS WE SHALL NOW START FROM THE BASIC CONF. WHERE ALL RACEWAYS ARE CONCENTRIC AND THE ROLLING ELEMENTS IN CONTACT WITH THE OUTER RACEWAY.

WE ASSUME BOTH RINGS ARE MOUNTED IN PERFECTLY RIGID SUPPORTS, I.E. THEY WILL REMAIN PERFECTLY CIRCULAR UNDER LOAD, WITH EXCEPTION OF LOCAL DEFORMATIONS DUE TO HERTZ CONTACT.



$g = \frac{1}{2}g + \frac{1}{2}g$ TOTAL RADIAL CLEARANCE (TOT POSSIBLE DISPL. OF THE INN. R. AGAINST THE O.R.)
($1/2\, g$ = RADIAL CLE.)

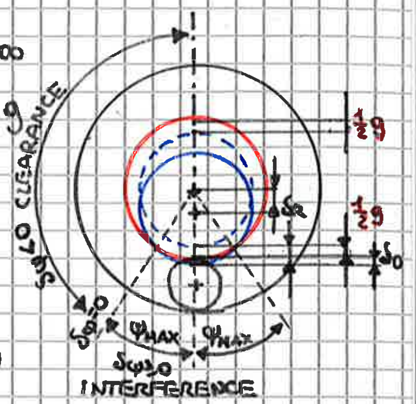
- AT WHICH ANGLE THE CONTACT VANISHES? → (4.2.8)

ψ_{MAX} IS THE ANGULAR HALF-OPENING OF THE LOADED PART OF THE BEARING, I.E. WHERE $\delta\psi \geq 0$; OUTSIDE THIS ANGLE THERE IS NO CONTACT.

WHERE $\delta\psi = 0$:

$$0 = \delta_R \cos\psi_{MAX} - \frac{1}{2}g \Rightarrow \delta_R \cos\psi_{MAX} = \frac{1}{2}g \Rightarrow \psi_{MAX} = \arccos\left(\frac{g}{2\delta_R}\right)$$

$$\begin{cases} \cos\psi_{MAX} = \frac{1}{2}\frac{g}{\delta_R} \\ \psi_{MAX} = \arccos\left(\frac{g}{2\delta_R}\right) \end{cases} \qquad \begin{cases} \text{FOR } g=0 \Rightarrow \psi_{MAX} = 90^\circ \\ \text{FOR } g>0 \Rightarrow \psi_{MAX} \to 90^\circ \text{ FOR } \delta_R \to \infty \\ \text{FOR } g>0 \Rightarrow \psi_{MAX} = 0 \text{ AT } \delta_R = \frac{1}{2}g \end{cases}$$



$$\frac{\delta\psi}{\delta_0} = \frac{\cos\psi - \frac{1}{2}\frac{g}{\delta_R}}{1 - \frac{1}{2}\frac{g}{\delta_R}} = \frac{\cos\psi - \cos\psi_{MAX}}{1 - \cos\psi_{MAX}}$$

SYNTHESIS

1) A RELATION BETWEEN LOAD (F) AND APPROACHING DISPL. (δ) IS DEFINED AS:

$F = K\delta^m$

2) IN THE CASE OF BEARING TWO CONTACTS OCCUR (INNER AND OUTER RINGS)

$F = K_{TOT}\,\delta_{TOT}^m$; $K_{TOT} = \left[\frac{1}{K_i^{1/m}} + \frac{1}{K_o^{1/m}}\right]^{-m}$

3) IF A RADIAL CLEARANCE EXISTS, IT CAN BE IDENTIFIED AS "g".

4) ALONG THE MAIN DIRECTION OF CLEARANCE IT SUPERPOSES TO THE RADIAL DISPLAC.

EX: MAIN DIRECT. OF CLEARANCE = VERTICAL ; FOR $\psi = 0 \Rightarrow \delta_0 = \delta_R - \frac{1}{2}g$

5) ALONG OTHER DIRECTIONS IT CONTRIBUTES WITH A LOCAL PROJECTION:

FOR $\psi \geq 0$ $\delta\psi = \delta_R\cos\psi - \frac{1}{2}g$ UNTIL $\delta\psi \geq 0 \Rightarrow$ INTERFERENCE.

6) HOW LARGE IS THE PORTION OF RACEWAY IN CONTACT?

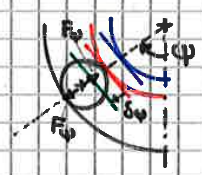
$\psi_{MAX} = \arccos\left(\frac{g}{2\delta_R}\right)$ (WHERE $\delta\psi = 0$)

- HOW THE LOAD (F) IS RELATED TO THE ANGULAR POSITION (ψ) OF ROLLING ELEMENTS?

$F = K_{TOT}\,\delta_{TOT}^m = K_{TOT}(\delta_i + \delta_o)^m$ → (4.2.8)

$F_\psi = K_{TOT}\,\delta_\psi^m$

$$\frac{F_\psi}{F_0} = \left(\frac{\delta_\psi}{\delta_0}\right)^m = \left(\frac{\cos\psi - \cos\psi_{MAX}}{1 - \cos\psi_{MAX}}\right)^m$$



SUMMARIZING ... (RADIAL BEARING)

CONTACT

$F_0 = K_0 \delta_0^m$

$F_i = K_i \delta_i^m$

APPROACHING

$\delta_\psi = \delta_R \cos\psi - \frac{1}{2} g$

$$\frac{\delta_\psi}{\delta_0} = \frac{\cos\psi - \frac{1}{2}\frac{g}{\delta_R}}{1 - \frac{1}{2}\frac{g}{\delta_R}} = \frac{\cos\psi - \cos\psi_{MAX}}{1 - \cos\psi_{MAX}}, \quad \psi_{MAX} = \arccos\left(\frac{g}{2\delta_R}\right)$$

$$\frac{F_\psi}{F_0} = \left(\frac{\delta_\psi}{\delta_0}\right)^m = \left(\frac{\cos\psi - \cos\psi_{MAX}}{1 - \cos\psi_{MAX}}\right)^m$$

SUM

$$R = \sum_{-\psi_{MAX}}^{+\psi_{MAX}} F_{\psi_i} \cdot \cos\psi_i$$

STRIBECK'S LAW

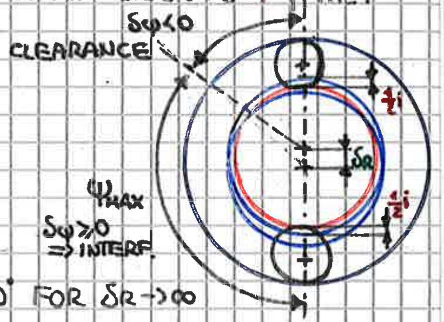
$$F_0 \cong \frac{5 \cdot R}{z}$$

Q.2.11) DESCRIBE THE DISTRIBUTION OF RADIAL DISPLACEMENT IN RADIAL BEARING WITH INTERFERENCE.

CASE 3: INTERFERENCE

IF INSTEAD OF HAVING A CLEARANCE g WE HAVE AN INTERFERENCE i, i.e.:

$i = -g$, THEN:



ANGLE ψ_{MAX} WHERE $\delta_\psi = 0$

$0 = \delta_R \cos\psi_{MAX} + \frac{1}{2} i$

$$\begin{cases} \cos\psi_{MAX} = -\frac{1}{2}\frac{i}{\delta_R} \\ \psi_{MAX} = \arccos\left(-\frac{i}{2\delta_R}\right) \end{cases}$$

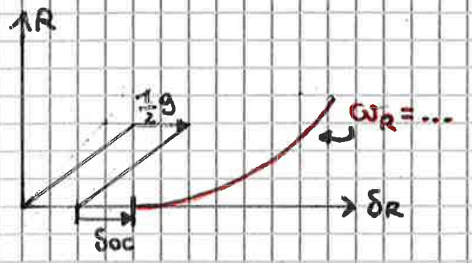
$$\begin{cases} \text{FOR } i = 0 \Rightarrow \psi_{MAX} = 90° \\ \text{FOR } i > 0 \Rightarrow \psi_{MAX} \to 90° \text{ FOR } \delta_R \to \infty \\ \text{FOR } i > 0 \Rightarrow \psi_{MAX} \to 180° \text{ FOR } \delta_R = \frac{1}{2} i \end{cases}$$

$$\frac{\delta_\psi}{\delta_0} = \frac{\delta_R \cdot \cos\psi + \frac{1}{2} i}{\delta_R + \frac{1}{2} i} = \frac{\cos\psi + \frac{1}{2}\frac{i}{\delta_R}}{1 + \frac{1}{2}\frac{i}{\delta_R}}$$

9. TOTAL RADIAL FORCE

$$R(\delta_R) = \sum_{-j_{MAX}}^{+j_{MAX}} F_{ij}(\delta_R)\cos\varphi_i$$

10. PLOT OF RADIAL FORCE

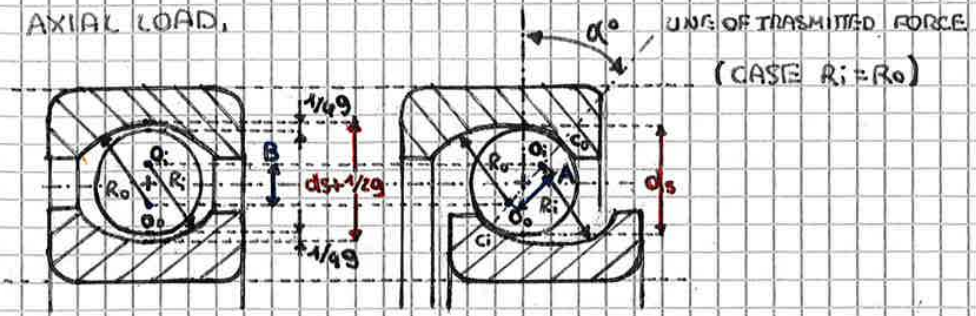


11. ...

ONCE THE DIAGRAM IS AVAILABLE, IT IS USED TO CHOOSE THE RADIAL FORCE (R) AND FIND THE CORRESPONDENT RADIAL APPROACH (δ_R); THEN, KNOWING δ_R, OTHER VALUES LIKE φ_{MAX}, F_{ij}, F_{oj}, AND IN PARTICULAR F_{io}, F_{oo}, HENCE ALSO THE MAXIMUM CONTACT PRESSURE ARE AT HAND (P_{MAX}).

- CHARACTERIZATION OF DEEP GROOVE BEARING GEOMETRY UNDER AXIAL LOAD

WHILE THERE IS RADIAL PLAY, UNDER THE ACTION OF AN AXIAL THRUST THE INNER RING DISPLACES AXIALLY, THE BALL TOUCHES INNER AND OUTER RACEWAYS RESPECTIVELY IN POINTS C_i AND C_o THROUGH WHICH THE LINE OF TRASMITTED FORCE PASSES; THEN, THE BEARING IS ENABLE TO TAKE AXIAL LOAD.



$$d_s + \frac{1}{2}g = R_i - B + R_o \qquad d_s = R_i - A + R_o$$

SUBTRACTING:

$$\frac{1}{2}g = A - B \Rightarrow B = A - \frac{1}{2}g$$

DEFINING:

$$\cos\alpha^\circ = \frac{B}{A} = \frac{A - \frac{1}{2}g}{A}$$

OR, CONSIDERING:

$$\begin{cases} B = R_i + R_o - d_s - \frac{1}{2}g \\ A = R_i + R_o - d_s \end{cases}$$

$\Rightarrow$

$$\cos\alpha^\circ = \frac{B}{A} = \frac{R_i + R_o - d_s - \frac{1}{2}g}{R_i + R_o - d_s} = 1 - \frac{1}{2}\frac{g}{R_i + R_o - d_s}$$

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Q.2.15 DEMONSTRATE HOW RADIAL AND AXIAL LOADS ARE COMBINED IN AN ANGULAR CONTACT BEARING AND DRAW RELATED VECTORS DIAGRAM.

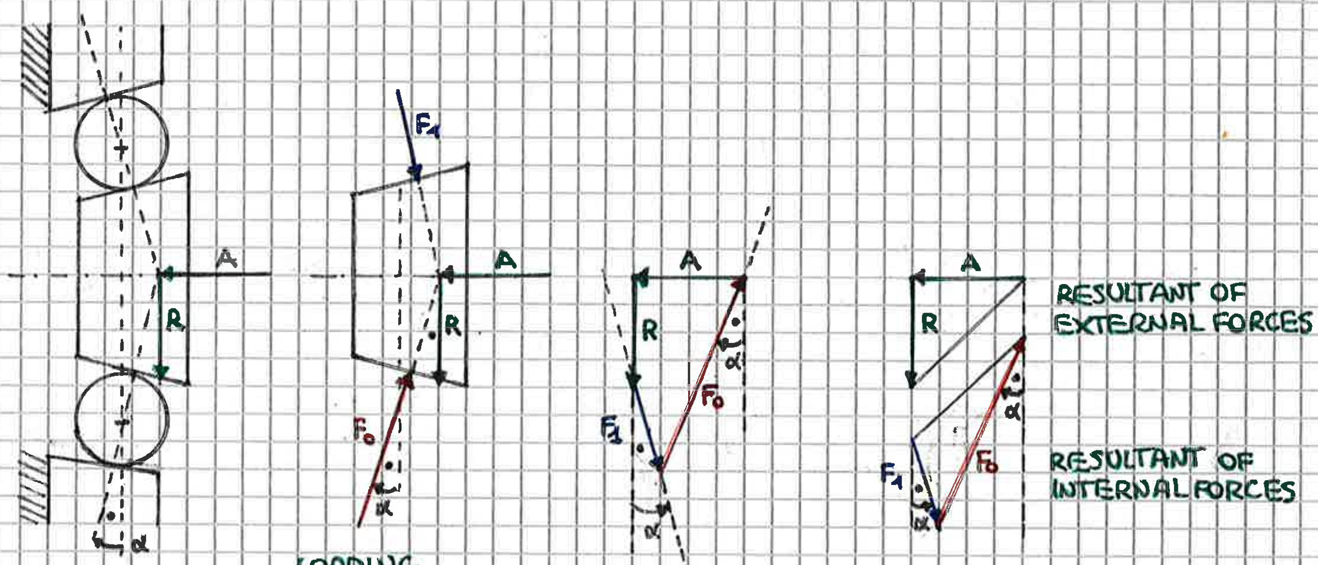
- DETERMINATION OF THE EQUIVALENT LOAD P APPLIED TO THE BEARING

FOR RADIAL "R" AND AXIAL "A" (COMBINED) LOADS.

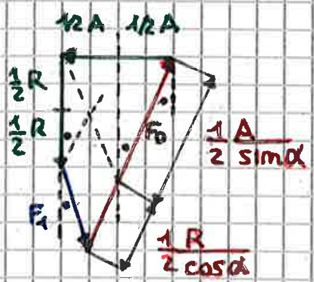
$$P_0 = X_0 R + Y_0 A$$

IF DEEP GROOVE RADIAL BEARINGS ARE SUBJECTED TO BOTH RADIAL AND AXIAL LOADS R AND A, AN "EQUIVALENT" STATIC RADIAL LOAD IS DEFINED WITH THE FOLLOWING (CATALOGUE).

$$\begin{cases} P_0 = 0.6R + 0.5A & (P_0 \leq C_0/S_F) \\ \text{HOWEVER} & \text{FOR SINGLE BEARINGS AND} \\ P_0 = R \quad \text{IF} \quad P_0 < R & \text{BEARING PAIRS ARRANGED IN TANDEM.} \end{cases}$$



GEOMETRY — LOADING CONDITION — ANALYSIS



$$F_0 = \frac{1}{2}\frac{R}{\cos\alpha} + \frac{1}{2}\frac{A}{\sin\alpha}$$

$$F_0 = \frac{1}{2}\frac{R}{\cos\alpha} + \frac{1}{2}\frac{A}{\sin\alpha} \leq F_{0,LIM} = \frac{1}{2}\frac{R_{LIM}}{\cos\alpha} + \frac{1}{2}\frac{A_{LIM}}{\sin\alpha}$$

$$\Downarrow$$

$$P_0 = \frac{1}{2}R + \frac{1}{2}\frac{A}{\tan\alpha} \leq C_0$$

EVALUATION OF TYPICAL PROPORTIONS OF VECTORS

FOR $\delta\psi = 0$: $\psi = \psi_{MAX}$

$$0 = \delta_R \cos\psi_{MAX} \cos\alpha + \delta_a \sin\alpha$$

THEN IF : $\frac{\delta_a}{\delta_R} \cdot Tg\alpha \leq 1$:

$$\cos\psi_{MAX} = -\frac{\delta_a}{\delta_R} \cdot Tg\alpha \quad \text{(NEGATIVE!)}$$

THEREFORE :

$$\frac{\delta\psi}{\delta_0} = \frac{\cos\psi + \frac{\delta_a}{\delta_R} Tg\alpha}{1 + \frac{\delta_a}{\delta_R} Tg\alpha} = \frac{\cos\psi - \cos\psi_{MAX}}{1 - \cos\psi_{MAX}}$$

NW : THIS FORMULA IS VALID ONLY WHEN $\delta_a Tg\alpha \leq \delta_R$

AND THE CONTACT EXTENDS OVER THE ANGLE : $\frac{\pi}{2} \leq \psi_{MAX} \leq \pi$

NOTE : WHEN $\psi_{MAX} = \frac{\pi}{2}$ FOR $\delta_a = 0$ ONLY BALLS IN THE LOWER-HALF ARE IN CONTACT.

- CASE I : WHEN $\frac{\pi}{2} \leq \psi_{MAX} \leq \pi$ ($\delta_a Tg\alpha \leq \delta_R$)

THEN $\cos\psi_{MAX}$ IS DEFINED, I.E., THE FORCE RATIO CAN BE WRITTEN TRIGONOMETRICALLY :

$$\frac{F\psi}{F_0} = \left(\frac{\delta\psi}{\delta_0}\right)^m = \left(\frac{\cos\psi - \cos\psi_{MAX}}{1 - \cos\psi_{MAX}}\right)^m$$

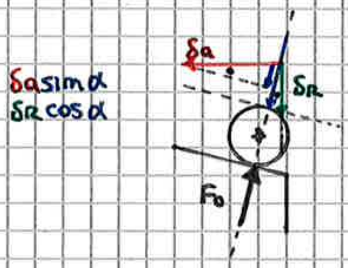
- CASE II : WHEN $\delta_a Tg\alpha > \delta_R$, WHICH IMPLIES $\psi_{MAX} = \pi$

THEN THE FORMULA MUST BE WRITTEN AS :

$$\frac{F\psi}{F_0} = \left(\frac{\delta\psi}{\delta_0}\right)^m = \left(\frac{\cos\psi + \frac{\delta_a}{\delta_R} Tg\alpha}{1 + \frac{\delta_a}{\delta_R} Tg\alpha}\right)^m$$

ON THE BOTTOM BALL , AT $\psi = 0$:

$$F_0 = K_{TOT} \cdot (\delta_a \sin\alpha + \delta_R \cos\alpha)^m$$



WITH:

$$\begin{cases} m = 1.5 & \text{: POINT CONTACT} \\ m \cong 1.1 & \text{: LINE CONTACT} \end{cases}$$

K_{TOT} = TOTAL STIFFNESS OF THE TWO CONTACTS.

SUMMARIZING...

CONTACT	APPROACHING
$F_0 = K_0 \delta_0^m$	$\delta_\psi = \delta_R \cos\psi \cos\alpha + \delta_a \sin\alpha$
$F_i = K_i \delta_i^m$	$\delta_0 = \delta_R \cos\alpha + \delta_a \sin\alpha$

ANGULAR CONTACT BEARING

$$\frac{\delta_\psi}{\delta_0} = \frac{\delta_R \cos\psi \cos\alpha + \delta_a \sin\alpha}{\delta \cos\alpha + \delta_a \sin\alpha} = \frac{\cos\psi + \frac{\delta_a}{\delta_R}\operatorname{Tg}\alpha}{1 + \frac{\delta_a}{\delta_R}\operatorname{Tg}\alpha} = \frac{\cos\psi - \cos\psi_{MAX}}{1 - \cos\psi_{MAX}}$$

- **CASE I:** $\delta_a \operatorname{Tg}\alpha \leq \delta_R$; $\frac{\pi}{2} \leq \psi_{MAX} \leq \pi$

$$\frac{F_\psi}{F_0} = \left(\frac{\delta_\psi}{\delta_0}\right)^m = \left(\frac{\cos\psi - \cos\psi_{MAX}}{1 - \cos\psi_{MAX}}\right)^m$$

$$\begin{cases} A = \sum_{-\psi_{MAX}}^{+\psi_{MAX}} F_{\psi_i} \sin\alpha = F_0 \sin\alpha \sum_{-\psi_{MAX}}^{+\psi_{MAX}} \left(\frac{\cos\psi_i - \cos\psi_{MAX}}{1 - \cos\psi_{MAX}}\right)^m \\ R = \sum_{-\psi_{MAX}}^{+\psi_{MAX}} F_{\psi_i} \cos\alpha \cdot \cos\psi_i = F_0 \cos\alpha \sum_{-\psi_{MAX}}^{+\psi_{MAX}} \left(\frac{\cos\psi_i - \cos\psi_{MAX}}{1 - \cos\psi_{MAX}}\right)^m \cos\psi_i \end{cases}$$

$$\psi_{MAX} = \arccos\left(-\frac{\delta_a}{\delta_R}\operatorname{Tg}\alpha\right)$$

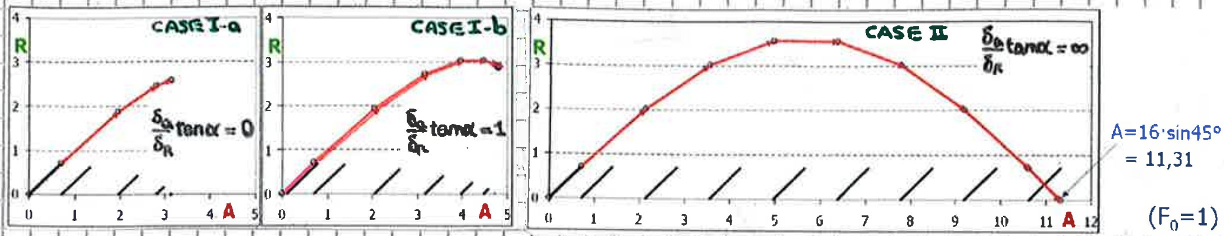
- **CASE II:** $\delta_a \operatorname{Tg}\alpha > \delta_R$ WHICH IMPLIES: $\psi_{MAX} = \pi$

$$\frac{F_\psi}{F_0} = \left(\frac{\delta_\psi}{\delta_0}\right)^m = \left(\frac{\cos\psi + \frac{\delta_a}{\delta_R}\operatorname{Tg}\alpha}{1 + \frac{\delta_a}{\delta_R}\operatorname{Tg}\alpha}\right)^m$$

$$\begin{cases} A = \sum_{-\pi}^{+\pi} F_{\psi_i} \sin\alpha = F_0 \sin\alpha \sum_{-\pi}^{+\pi} \left(\frac{\cos\psi_i + \frac{\delta_a}{\delta_R}\operatorname{Tg}\alpha}{1 + \frac{\delta_a}{\delta_R}\operatorname{Tg}\alpha}\right)^m \\ R = \sum_{-\pi}^{+\pi} F_{\psi_i} \cos\psi_i \cos\alpha = F_0 \cos\alpha \sum_{-\pi}^{+\pi} \left(\frac{\cos\psi_i + \frac{\delta_a}{\delta_R}\operatorname{Tg}\alpha}{1 + \frac{\delta_a}{\delta_R}\operatorname{Tg}\alpha}\right)^m \cos\psi_i \end{cases}$$

$$\psi_{MAX} = \pi$$

THREE CASES FOR $z = 16$, $\alpha = 45°$



LET'S SUMMARIZE THE **DESIGN ISSUES**:

1. IN CASE OF ANGULAR CONTACT A RADIAL LOAD EXCITES TWO REACTIONS IN THE **FRONTAL PLANE** OF BEARING (CONTRIBUTING TO **R**) BUT EVEN A **ORTHOGONAL ACTION** DUE TO THE ORIENTATION OF THE RING SURFACE (CONTRIBUTING TO **A**)

2. TO DESCRIBE THE DISTRIBUTION OF LOCAL CONTRIBUTION TO R AND A RESPECTIVELY, A **PLOT** WAS PROPOSED, THUS REALIZING THAT IF WE SUM ROLLING ELEMENT BY ROLLING ELEMENT THE TOTAL EFFECT IN TERMS OF R AND A WE FIND THEIR INTEGRAL VALUES.

3. SOME DIFFERENCES CAN BE FOUND WHEN COMPARING THE ANGULAR CONTACT TO THE PURE RADIAL BEARING, I.E. THE **MOST LOADED ROLLING ELEMENTS** ARE IN THE MIDDLE OF THE LOWER LOADED ARC, BUT A CONTRIBUTION TO AXIAL REACTION IS PROVIDED **EVEN BY THE UPPER ARC**.

THE QUESTION IS:

IN CASE OF RADIAL BEARING, STRIBECK SHOWS THAT A SIMPLE RELATION BETWEEN LOCAL FORCE (F) AND TOTAL REACTION (R) EXISTS : $F = \frac{5R}{z}$

IS IT POSSIBLE TO FIND A SIMILAR RELATION HERE BETWEEN R AND F, A AND F?

IN BOTH CASE I AND CASE II :

AXIAL (A) AND RADIAL (R) LOADS CAN BE WRITTEN:

$$\begin{cases} A = \sum_{-\psi_{MAX}}^{+\psi_{MAX}} F_\psi \cdot \sin\alpha = F_0 \cdot \sin\alpha \cdot z \cdot \tilde{a} \\ R = \sum_{-\psi_{MAX}}^{+\psi_{MAX}} F_\psi \cdot \cos\alpha \cdot \cos\psi = F_0 \cdot \cos\alpha \cdot z \cdot \tilde{R} \end{cases}$$

WITH:

$\tilde{a}, \tilde{R}$: TAKE INTO ACCOUNT ONLY THE PROPORTION OF LOAD SHARING AMONG ROLLING BODIES.

IT IS NOW INTERESTING TO TAKE $\frac{\delta_a}{\delta_R}\operatorname{Tg}\alpha$ AS A **PARAMETER** AND CALCULATE $\tilde{a}, \tilde{R}$.

(THE DIAGRAMS IN THE NEXT SHOW THE RESULT FOR FOUR REPRESENTATIVE CASES: 2/8/12/16 SPHERES)

MEANING OF C_0

THE REFERENCE CONDITION IS USED TO SET THE LIMIT.

WHEN F_0 REACHES ITS LIMIT $F_{0,LIM}$ VALUE, R_{LIM} REACHES ITS LIMIT VALUE WHICH WE CALL C_0.

$$F_{0,LIM} = \frac{C_0}{z}\left(\frac{2.26}{\cos\alpha} + \frac{A_{LIM}}{R_{LIM}}\frac{1}{\sin\alpha}\right) \qquad ; \qquad \frac{A_{LIM}}{R_{LIM}} = \tan\alpha \, \frac{\hat{a}_1}{\tilde{R}_1}$$

THEN:

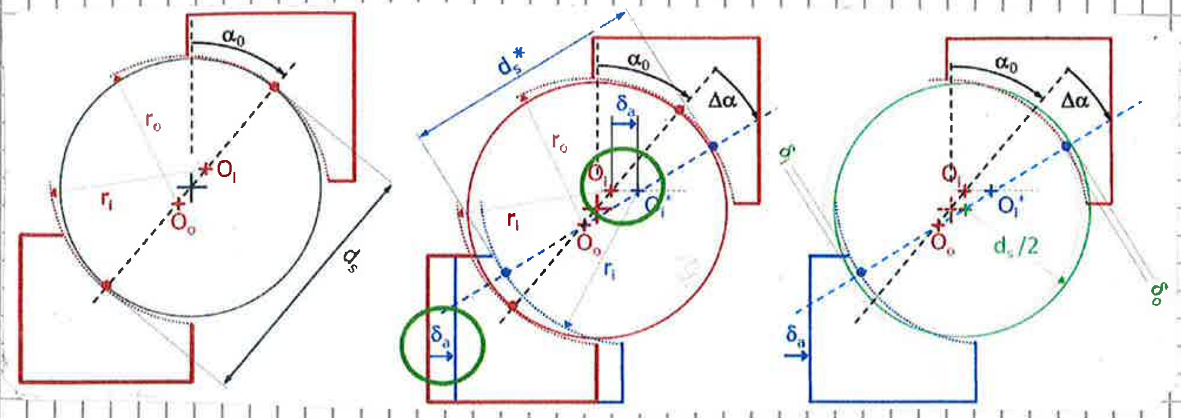
$$F_0 = \frac{R}{z}\left(\frac{2.26}{\cos\alpha} + \frac{A}{R}\frac{1}{\sin\alpha}\right) \leq F_{0,LIM} = \frac{C_0}{z}\left(\frac{2.26}{\cos\alpha} + \frac{\hat{a}_1}{\tilde{R}_1}\frac{1}{\cos\alpha}\right) = \frac{C_0}{z\cos\alpha}\frac{1}{\tilde{R}_1}$$

REMEMBER : $2.26 = \frac{(1-\hat{a}_1)}{\tilde{R}_1}$

THEN:

$$(1-\hat{a}_1)R + \frac{\tilde{R}_1}{\tan\alpha}A \leq C_0$$

BEFORE THE APPLICATION OF AN AXIAL FORCE:

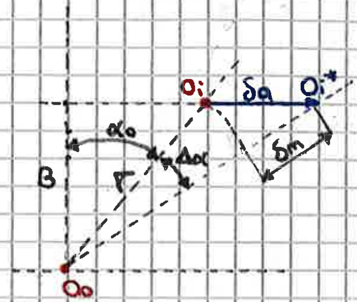


$$d_s = R_i - \overline{O_oO_i} + R_o$$

$$d_s^* = R_i - \overline{O_oO_i^*} + R_o$$

$$\delta_m = \delta_i + \delta_o = d_s - d_s^* = \overline{O_oO_i^*} - \overline{O_oO_i}$$

$$\overline{O_oO_i} = A \qquad \cos\alpha_o = \frac{B}{A}$$



$$\tan(\alpha_o + \Delta\alpha) = \left(\frac{B\tan\alpha_o + \delta_a}{B}\right) = \tan\alpha_o + \frac{\delta_a}{B}$$

$$\Delta\alpha = -\alpha_o + \arctan\left(\tan\alpha_o + \frac{\delta_a}{B}\right)$$

$$\delta_m = \delta_i + \delta_o = d_s - d_s^* = \overline{O_oO_i^*} - \overline{O_oO_i} \equiv \overline{O_oO_i^*} - A = \frac{B}{\cos(\alpha_o + \Delta\alpha)} - A$$

NORMALLY, BEARINGS ARE USED WITH A SLIGHT INTERNAL CLEARANCE UNDER OPERATING CONDITIONS. HOWEVER, IN SOME APPLICATIONS, BEARINGS ARE GIVEN AN INITIAL LOAD; THIS MEANS THAT THE BEARINGS' **INTERNAL CLEARANCE** IS NEGATIVE BEFORE OPERATION.

THIS IS CALLED **"PRELOAD"** AND IS COMMONLY APPLIED TO **ANGULAR BALL BEARINGS** AND **TAPERED ROLLER BEARINGS**, IN WHICH THE CLEARANCE (AND THE INTERF.) CAN BE ADJUSTED DURING ASSEMBLY.

THE **MAIN PURPOSES** AND SOME TYPICAL APPLICATIONS OF **PRELOADED BEARINGS** ARE:

1) TO MANTAIN THE BEARINGS IN EXACT POSITION BOTH RADIALLY AND AXIALLY AND TO SUPPRESS SHAFT RUNOUT.

2) TO INCREASE BEARING RIGIDITY.

3) TO MINIMIZE NOISE DUE TO AXIAL VIBRATION AND RESONANCE.

4) TO PREVENT SLIDING BETWEEN THE ROLLING ELEMENTS AND RACEWAYS DUE TO GYROSCOPIC MOMENTS.

5) TO MAINTAIN THE ROLLING ELEMENTS IN THEIR PROPER POSITION WITH THE BEARING RINGS.

Q.2.19 DRAW A BACK-TO-BACK AND FACE-TO-FACE ARRANGEMENTS

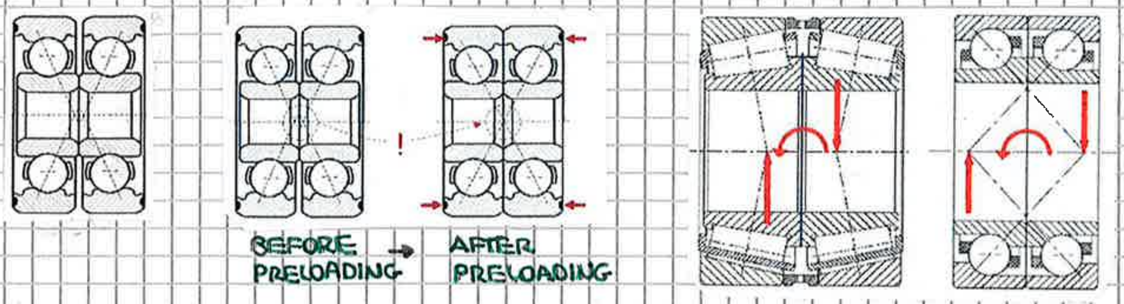
THE FACE IS WHERE THE CONTACT FORCES MEET;

THE BACK IS THE OPPOSITE.

DF ARRANGEMENTS : FACE-TO-FACE

THE TWO BEARINGS ARE PLACED SO THAT CONTACT ANGLE LINES OF THE BEARINGS CONVERGE INWARDLY.

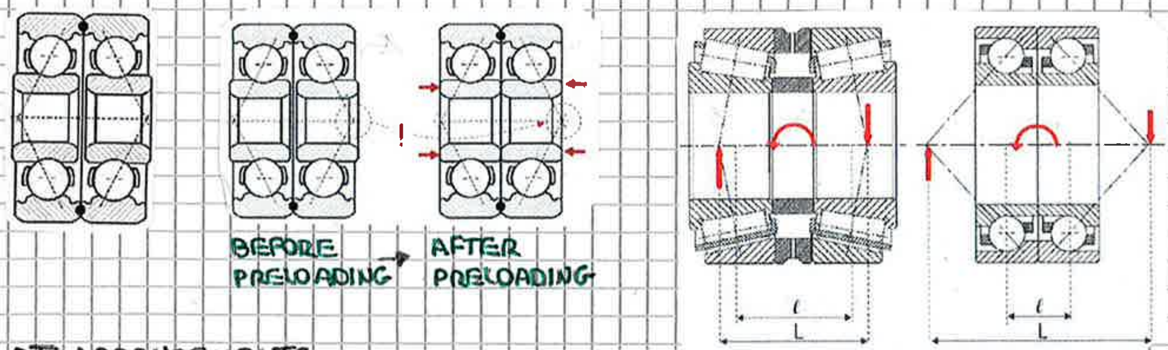
THESE CONTACT ANGLE LINES FORM "X" SHAPE, HENCE CALLED: "X ARRANGEMENT".



DB ARRANGEMENTS : BACK-TO-BACK

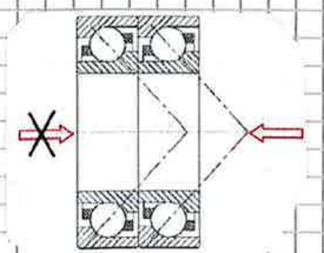
THE TWO BEARINGS ARE PLACED SO THAT CONTACT ANGLE LINES OF THE BEARING DIVERGE INWARDLY.

THESE CONTACT ANGLE LINES FORM "O" SHAPE, HENCE CALLED : "O ARRANGEMENT"

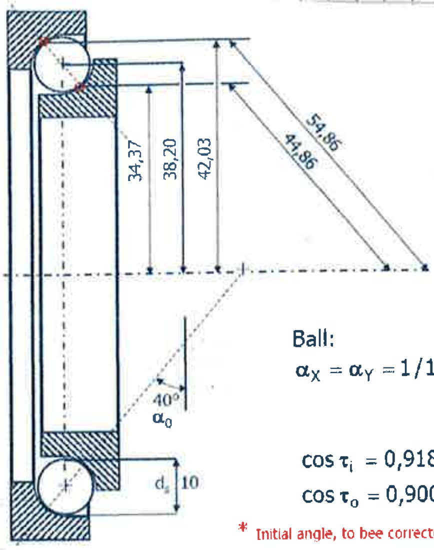


DT ARRANGEMENTS

THE TANDEM BEARING ARRANGEMENT :



EXAMPLE.



$Z=12$ number of balls

$\alpha_0=40°$ *

$E= 2\cdot10^5$ MPa

$\nu= 0,3$

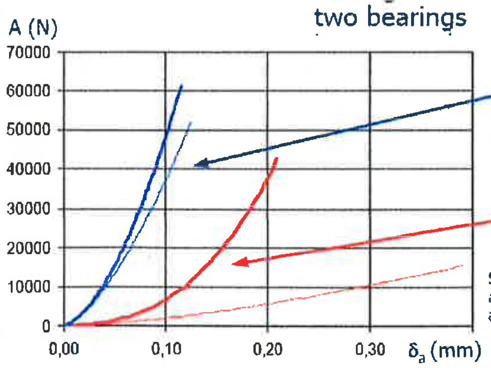
Ball:	Inner ring:	Outer ring:
$\alpha_X = \alpha_Y = 1/10$	$\beta_{Xi} = +1/89,73$	$\beta_{Xo} = -1/109,7$
	$\beta_{Yi} \doteq -1/10,50$	$\beta_{Yi} \doteq -1/10,50$

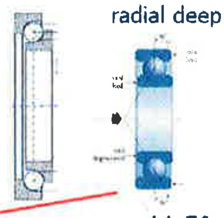
$\cos\tau_i = 0,918$	$a^* = 3,37$	$b^* = 0,440$	$\delta^* = 0,646$
$\cos\tau_o = 0,900$	$a^* = 3,10$	$b^* = 0,460$	$\delta^* = 0,678$

* Initial angle, to bee corrected with axial approach according to Sect.10 sl.6 of this Chapter

This figure shows the axial force A vs. the axial displacement of inner ring agaist the outer ring. It is clear that, particularly for radial deep groove bearings, the solution without angle correction lead to wrong results (thin lines).

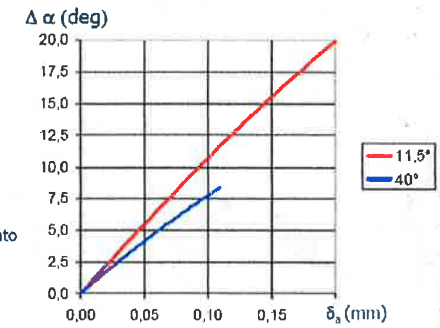
Once again, the thin lines are incorrect (no angle change), and are shown to indicate the error due to omission of change.



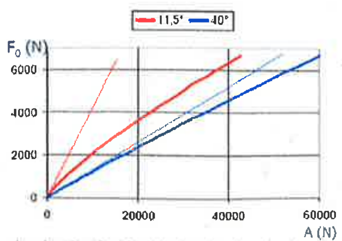


$\alpha_0=40°$ $\alpha_0=11,5°$

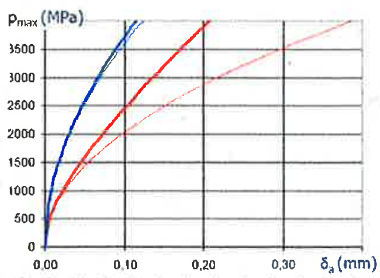
Solid lines are calculated taking into account the angle increase $\Delta\alpha$ according to Sect.10.



calculated up to a maximum contact pressure p_{max}=4000 MPa

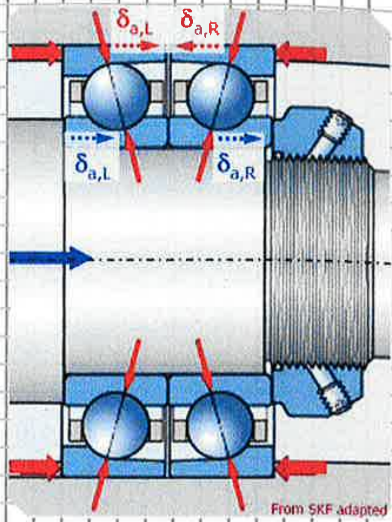


This figure shows the relation between the force F_0 at the contacts and the total axial force A.

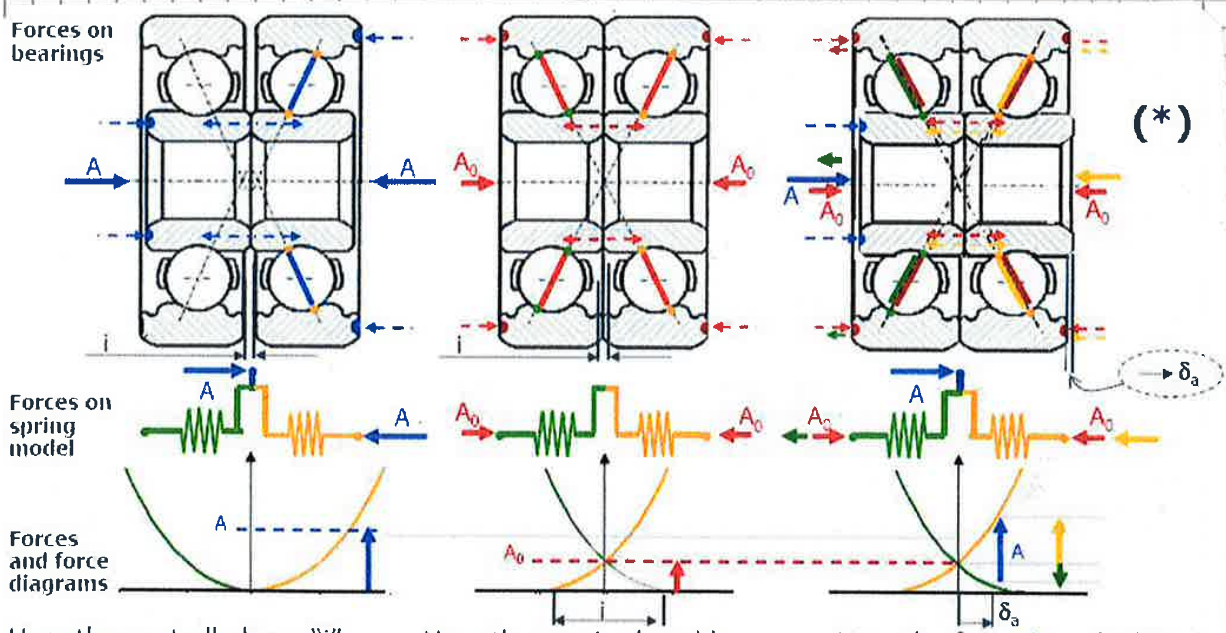


4.2.21) DRAW THE CHARACTERISTIC CURVE OF FACE-TO-FACE AND BACK-TO-BACK ARRANGEMENTS.

PRELOADING OF A BEARING COUPLE



IN THIS SECTION WE SHALL LEARN THE RATIONALE BEHIND PRELOADING ANGULAR BEARINGS, EITHER BALL OR TAPERED ROLLERS.
THE WORKED OUT EXAMPLE CONCERNS THE CASE OF "DF" OR "X" ASSEMBLY AND "FIXED POSITION" PRELOADING, WHERE THE PRELOAD A_0 IS APPLIED BY DRIVING INTO CONTACT THE OUTER RINGS THROUGH OPPOSITE DISPLACEMENTS $\delta a_L = \delta a_R \equiv \delta a$
THE SHAFT SHOULDER TRASMITS AN AXIAL FORCE A TO THE PAIR OF INNER RINGS, WHICH MOVE TOGETHER AXIALLY BY THE SAME AMOUNT $\delta a_L = \delta a_R \equiv \delta a$

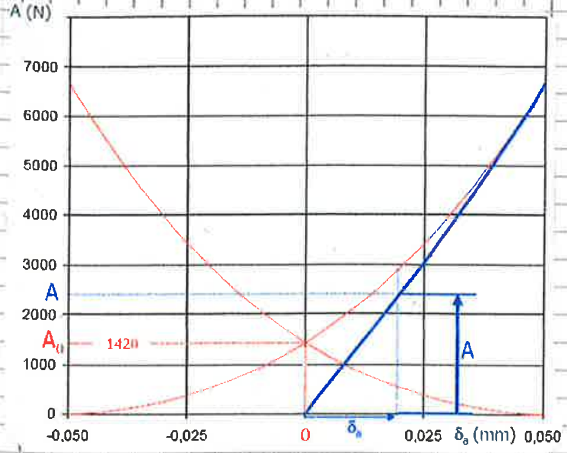


Here the controlled gap "i" between the outer rings will produce the interference when pressed. The force A on the inner rings loads only the right bearing balls.

Here the gap is closed by compressing the outer rings. The preload A_0 passes through the right and left balls and loads the inner rings one against the other.

Here the force A applied to the inner rings is taken by the right and left rows of balls. They take also the axial preload A_0. Notice the inner rings moving to the right.

...



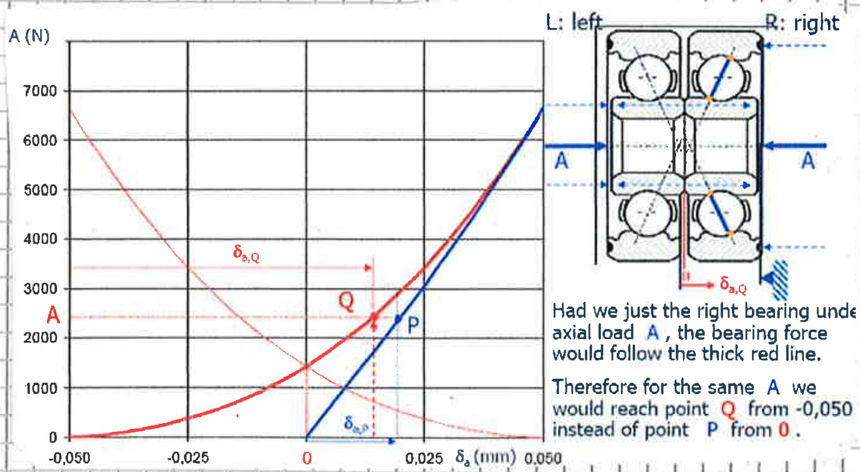
THE BLUE LINE REPRESENTS THE: **ADDITIONAL AXIAL FORCE A** APPLIED TO THE SHAFT AND TO THE INNER RINGS **VS.** THEIR COMMON **ADDITIONAL DISPLACEMENT δ_a**

(THE FACT THAT A-δ_a RELATION BECOMES "ALMOST" LINEAR IS ADDITIONAL BONUS)

IT IS CLEAR THAT THE STIFFNESS OF THE **PRELOADED PAIR OF BEARINGS PRODUCES A STIFFNESS** WHICH IS MUCH **HIGHER** FOR TWO REASONS:

1) THEY WORK FROM A POINT WHERE THEIR INDIVIDUAL STIFFNESS IS HIGHER THAN FROM ZERO.

2) THE SLOPE DEPENDS ON THE DIFFERENCE OF THE TWO CURVES, THEN IT IS HIGHER THAN EACH ONE SEPARATELY.



THE (SECANT) STIFFNESS SEEN FROM THE SHAFT IS, IN THIS EXAMPLE, AROUND 3,4 TIMES SMALLER FOR THE SINGLE BEARING (I.E., DOUBLE BUT NOT PRELOADED) COMPARED TO THE DOUBLE PRELOADED, AS IT IS EASY TO OBSERVE THAT δ_{aQ} IS ABOUT 3.4 TIMES $\delta_{a,P}$.

Q.3.1) DESCRIBE THE PITTING PHENOMENON AND THE SPALLING PHENOMENON.

ROLLING ELEMENT BEARING SURFACE FATIGUE IS CHARACTERIZED BY **PITTING** AND **SPALLING**, WHICH **LIBERATE PARTICLES** FROM THE SURFACE OF A RACEWAY OR OF A ROLLING ELEMENT IN THE LOAD ZONE, AND **LEAVES CRATERS** THAT ACT AS **STRESS CONCENTRATION SITES**. **SUBSEQUENT CONTACTS** AT THOSE SITES **CAUSE PROGRESSION OF THE SPALLING PROCESS**.

WE CAN TODAY SAY THAT THE TWO **MOST DOMINANT RCF MECHANISMS** ARE:

- **SURFACE ORIGINATED PITTING**
- **SUBSURFACE ORIGINATED SPALLING**.

THESE ARE OFTEN COMPETING MODES OF FAILURE, AND THE ULTIMATE MECHANISM THAT PREVAILS DEPENDS ON A NUMBER OF FACTORS; E.G.:

SURFACE QUALITY,
MATERIAL QUALITY,
LUBRICANT CLEANLINESS,
ETC.

- **SURFACE ORIGINATED PITTING**

IT OCCURS IN CASES WHERE **SURFACE DENTS** OR **SCRATCHES** ARE PRESENT. HERE, A **CRACK INITIATES** AT THE **SURFACE STRESS CONCENTRATOR** AND THEREAFTER **PROPAGATES** AT A **SHALLOW ANGLE** (15 ÷ 30°) TO THE SURFACE.

PITTING APPEARS AS **SHALLOW CRATERS** AT **CONTACT SURFACES** WITH A **DEPTH OF**, AT MOST, **THE THICKNESS OF THE WORK-HARDENED LAYER** (~10 [μm])

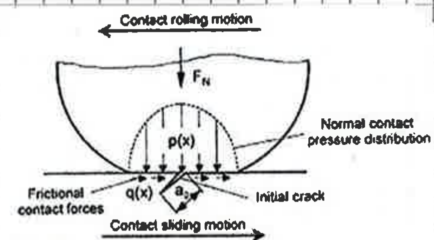


fig. a)

④ WHICH ARE THE MOST IMPORTANT DIFFERENCES BETWEEN CLASSICAL FATIGUE AND RCF?

1) THE STATE OF STRESS AT CONTACTS IS COMPLEX AND MULTIAXIAL, GOVERNED BY THE HERTZ CONTACT THEORY. CONTRARY TO MOST CLASSICAL FATIGUE PHENOMENA, ROLLING CONTACT FATIGUE (RCF) IS TYPICALLY A **MULTIAXIAL FATIGUE MECHANISM**.
$\Rightarrow$ A MULTIAXIAL FATIGUE CRITERION SHOULD BE APPLIED.

2) ROLLING CONTACT FATIGUE (RCF) HAS **NO ENDURANCE LIMIT**. IF ONE COMPARES THE FATIGUE **LIVES** OF CYCLIC TORSION OR BENDING WITH ROLLING CONTACT, THE LATTER ARE **7 ORDERS OF MAGNITUDE GREATER**. ROLLING CONTACT FATIGUE LIFE INVOLVES 10^7 TO 10^8 OF CYCLES.

3) THERE IS A **HIGH HYDROSTATIC STRESS COMPONENT** WHICH IS ABSENT IN CLASSICAL TENSION-COMPRESSION OR BENDING FATIGUE.

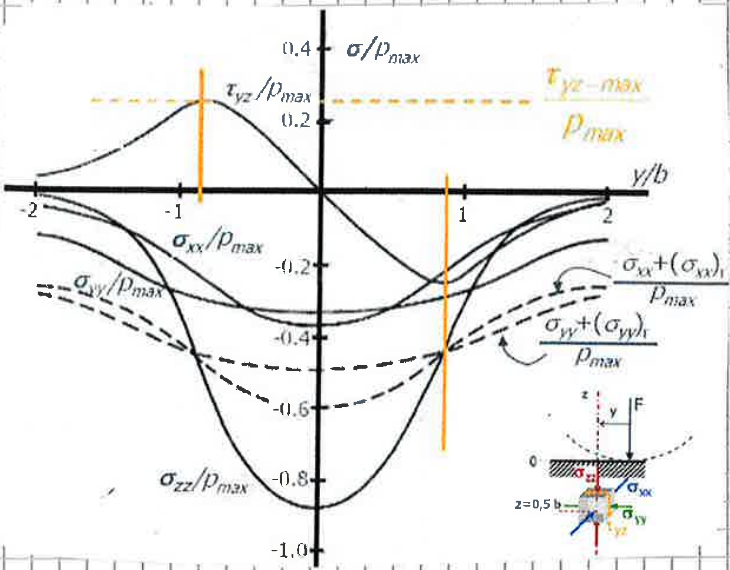
4) **FATIGUE OCCURS IN A VERY SMALL VOLUME** OF STRESSED MATERIAL. TYPICAL BEARING **CONTACT WIDTHS** ARE OF THE ORDER OF **200-1000 [μm]**

5) CONTRARY TO CLASSICAL FATIGUE, **THE LOADING HISTORY AT A POINT BELOW THE SURFACE IS NON-PROPORTIONAL** I.E., THE STRESS COMPONENTS DO NOT RISE AND FALL WITH TIME IN THE SAME PROPORTION TO EACH OTHER (THE PEAKS OF THE TWO NORMAL STRESSES (**σ**) DO NOT COINCIDE WITH THE PEAKS FOR THE SHEAR STRESS (**τ**)). **THERE IS A COMPLETE REVERSAL OF THE SHEAR STRESS WHILE THE NORMAL STRESSES ALWAYS REMAIN COMPRESSIVE.**

6) **THE PRINCIPAL AXES** IN NON-CONFORMAL CONTACTS **CONSTANTLY CHANGE** IN DIRECTION DURING A STRESS CYCLE **DUE TO WHICH THE PLANES OF MAXIMUM SHEAR STRESS ALSO KEEP CHANGING**. THUS, IT IS DIFFICULT TO IDENTIFY THE PLANES WHERE MAXIMUM FATIGUE DAMAGE OCCURS.

7) A FURTHER COMPLICATION ARE **RESIDUAL STRESSES** PRODUCED WHEN THE **ELASTIC LIMIT IS EXCEEDED IN THE FIRST APPLICATION OF THE LOAD** BUT ALL FURTHER CYCLES ARE WHITHIN THE ELASTIC LIMIT. (BROKEN LINES)

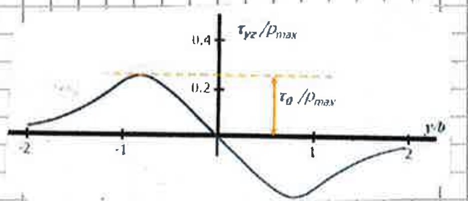
ROLLING CONTACT OF ELASTIC-PLASTIC CYLINDERS



—— : ELASTIC STRESSES AT DEPTH $z = 0.5b$

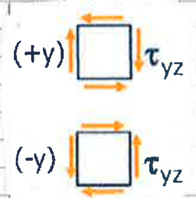
- - - : WITH ADDITION OF $(\sigma_y)_R$ AND $(\sigma_x)_R$ I.E. WITH RESIDUAL STRESSES (AFTER SHAKEDOWN)

ROLLING CONTACT: TANGENTIAL STRESSES AT $y=0$, $z=0.5b$



LATER WE SHALL EMPLOY THE SHORTER SYMBOL: τ_0 FOR THE **MAX TANGENTIAL STRESS AMPLITUDE AT DEPTH** z_0 INSTEAD OF $\tau_{yx\,max}$.

τ_0 = **MAXIMUM ALTERNATING SHEAR** (TANGENTIAL) **STRESS**



SHEAR STRESS **CHANGES SIGN** BECAUSE OF ROLLING EFFECT THUS INDUCING FATIGUE.

FOR EACH ELEMENTARY VOLUME:

$\Delta V_\psi = \xi z_0 a \frac{d_i}{2} \Delta\psi$

THE PROBABILITY OF SURVIVAL (I.E. NO CRACK INITIATED) IS WRITTEN BY **WEIBULL** AS:

$S_\psi = 1 - f(\tau_0, N, z_0) \cdot \Delta V_\psi$ **PROBABILITY OF SURVIVAL (WEIBULL)**

WHERE:

τ_0 = MAX TANGENTIAL STRESS AMPLITUDE AT DEPTH z_0

N = NUMBER OF LOADING-UNLOADING CYCLES

z_0 = DEPTH

IT IS SEEN THAT THE **FAILURE PROBABILITY** $f(\tau_0, N, z_0)\Delta V_\psi$ IS PROPORTIONAL TO THE VOLUME, I.E., THE GREATER THE VOLUME THE HIGHER THE PROBABILITY THAT A MATERIAL DEFECT (SUCH AS A NON-METALLIC INCLUSION) WILL FIND ITSELF NEAR THE LOCATION OF MAXIMUM STRESS.

FOR $i = 1$ TO k VOLUMES ΔV_i:

$S = S_1 \cdot S_2 \cdot \ldots S_i \cdot \ldots S_k = \prod_{i=1,k} [1 - f(\ldots)\Delta V_i]$ **TOTAL PROBABILITY OF SURVIVAL**

$\ln S = \sum_{i=1,k} \ln [1 - f(\ldots)\Delta V_i]$

$f(\ldots)\Delta V_\psi \ll 1 \Rightarrow \ln[1 - f(\ldots)\Delta V_\psi] \cong -f(\ldots)\Delta V_\psi$

THEREFORE:

$\ln S = \sum_{i=1,k} -f(\ldots)\Delta V_i$ OR $\ln S = -\int_V f(\ldots) dV$; $\ln \frac{1}{S} = \int_V f(\ldots) dV$

FOLLOWING WEIBULL, LUNDBERG AND PALMGREN INTRODUCED THE EMPIRICAL FUNCTION:

$f(\tau_0, N, z_0) = m \frac{\tau_0^c N^e}{z_0^h}$

WHERE:

m, c, e, h ARE **MATERIAL PARAMETERS** TO BE LATER DETERMINED THROUGH COMPARISONS BETWEEN THE PREDICTED TO THE OBSERVED BEARING FATIGUE LIFE.

THE EXPONENT e IS THE **WEIBULL SLOPE** FOR THE EXPERIMENTAL LIFE DATA PLOTTED ON A **WEIBULL PROBABILITY PAPER**.

WE FIND (MAGICALLY <=> WHERE IS ζ?) THAT:

$$\left(= \frac{m \, T \cdot P^c \cdot u_0^e \cdot d_0}{2 z_0^{h-1} A^{h-2}} \left(\frac{1}{7 J_s}\right)^{\frac{2c-h+2}{3}} \left(\frac{\cos\psi - \cos\psi_{MAX}}{1-\cos\psi_{MAX}}\right)^{\frac{2c-h+2}{3}} \cdot R^{\frac{2c-h+2}{3}} \cdot \tilde{L}^e \, d\psi \right)$$

THE SURVIVAL PROBABILITY FOR THE **OUTER RACEWAY** RESULTS TO BE:

$$\ell_n S_o = -\int_{-\psi_{MAX}}^{+\psi_{MAX}} m \frac{\tau_o^c N^e}{z_o^h} dV = -\int_{-\psi_{MAX}}^{+\psi_{MAX}} (\ldots) d\psi \cdot R^{\frac{2c-h+2}{3}} \cdot \tilde{L}^e = -J_o \cdot R^{\frac{2c-h+2}{3}} \cdot \tilde{L}^e$$

WHICH IS SHORTLY WRITTEN AS:

$$\ell_n S_o = -J_o R^R \cdot \tilde{L}^e \quad , \text{WITH } R = \frac{2c-h+2}{3}$$

THE **INNER RACEWAY** PRODUCES A FORMULA WITH THE SAME STRUCTURE AND EXPONENTS, HOWEVER WITH A DIFFERENT COEFFICIENT.

$$\ell_n S_i = -J_i R^R \cdot \tilde{L}^e \quad , \text{WITH } R = \frac{2c-h+2}{3}$$

USING "L" FOR "**MILLIONS OF REVOLUTIONS**", IN SUMMARY:

$$\ell_n S_o = -\tilde{J}_o R^R \cdot L^e$$

$$\ell_n S_i = -\tilde{J}_i \cdot R^R \cdot L^e$$

G.3.6) WRITE AND DESCRIBE THE LIFE EQUATION AND THE RELIABILITY EQUATION.

G.3.7) DESCRIBE THE DIFFERENCE BETWEEN THE LOAD-LIFE EQUATION IN NOMINAL BEHAVIOUR AND THE REAL EXPRESSION CORRECTED TO INCLUDE THE EFFECTS AFFECTING THE BEARING OPERATION. DEFINE ALL THOSE CORRECTIONS.

G.3.8) DESCRIBE THE RELATION BETWEEN THE LOAD-LIFE EQUATION AND THE MINER'S RULE.

$L_{10} = \left(\frac{C}{P}\right)^p$

LET'S NOW ANYLISE THE **REAL EXPRESSION** CORRECTED TO INCLUDE THE EFFECTS AFFECTING THE BEARING OPERATION.

$L_{ma} = a_1 \cdot a_2 \cdot a_3 \cdot L_{10} = a_1 \cdot a_2 \cdot a_3 \cdot \left(\frac{C}{P}\right)^p$ ALSO WRITTEN AS: $L_{ma} = a_1 \cdot a_{23} \left(\frac{C}{P}\right)^p$

OR: $L_{ma} = a_1 \cdot a_{SKF} \left(\frac{C}{P}\right)^p$

WITH:

L_{ma} = RATING LIFE ; $m = 100(1-S)$ = % FAILURE PROBABILITY ; "a" = "ADJUSTED"

$a_1 = \frac{L_{ma}}{L_{10}} = \left(\frac{\ln S}{\ln 0.9}\right)^{\frac{2}{3}}$ = LIFE ADJUSTMENT FACTOR FOR THE SELECTED RELIABILITY LEVEL

S	a_1	m

a_2 = ADJUSTMENT FACTOR FOR THE MATERIAL

a_3 = ADJUSTMENT FACTOR FOR OPERATING CONDITIONS (LUBRIFICATION)

THE EQUATION FOR SKF RATING LIFE IS IN ACCORDANCE WITH: ISO 281: 1990 / /AMENDMENT 2: 2000:

$L_{ma} = a_1 \cdot a_{SKF} \left(\frac{C}{P}\right)^p$

THE SKF LIFE MODIFICATION FACTOR a_{SKF} APPLIES THE CONCEPT OF FATIGUE LOAD LIMIT P_u ANALOGOUS TO THAT USED WHEN CALCULATING OTHER MACHINE COMPONENTS (IN BEARINGS, ASYMPTOTIC INFINITE LIFE EVALUATED AT 10^{11} CYCLES). IT IS UNDERLINED THAT WHILE a_{23} WAS INDIPENDENT OF LOAD P, THE NEWER a_{SKF} FACTOR DEPENDS ON THE RATIO $\frac{P_u}{P}$ THROUGH THE LOAD PARAMETER: $\frac{P - P_u}{C}$

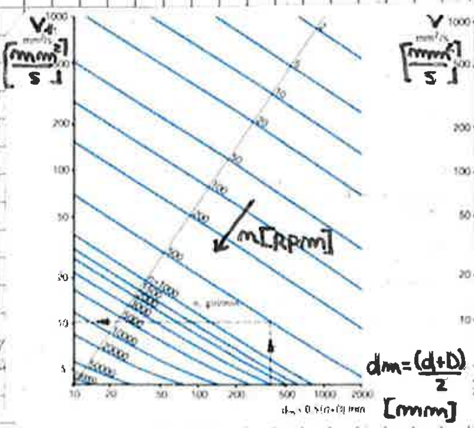
$a_{SKF} = a_{ISO} = a_{ISO}\left(K, \eta_c, \frac{P_u}{P}\right)$

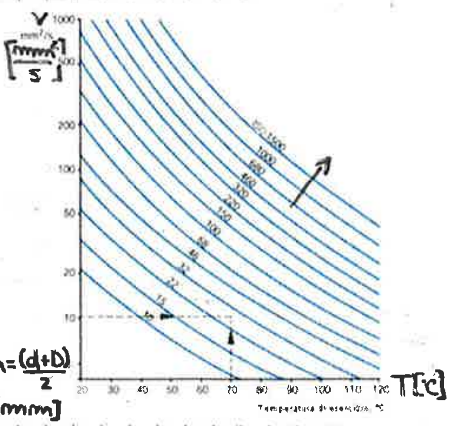
WITH:

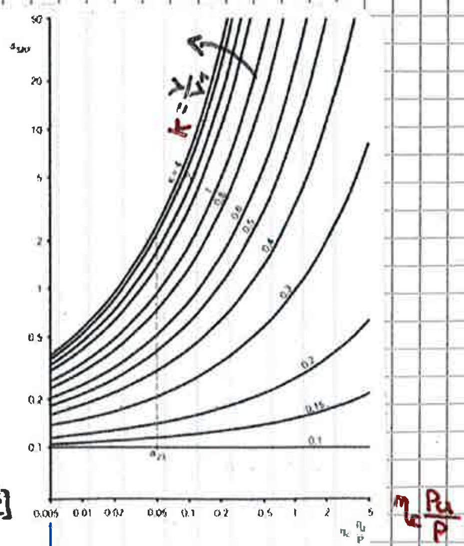
$K = \frac{V}{V_1}$ = VISCOSITY RATIO ; $V_1 = f(d_m = (d+D)/2 \,;\, m\,[RPm])$; $V = f(T[C], m[RPm])$

η_c = CONTAMINATION FACTOR (BY TABLE)

P_u = FATIGUE LIMIT OF THE ROLLING BEARING







* MINER'S RULE

MINER'S RULE ASSUMES THAT **FATIGUE FAILURE** IS PREDICTED WHEN THE **SUM** OF THE **DAMAGES** EQUATES TO **1**. THIS CAN BE DEFINED AS:

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_i}{N_i} = 1$$

WHEN $> 1 \Rightarrow$ FAILURE OCCURS

n_i = n. OF CYCLES AT i^{th} LOAD VALUE; (L_i)

N_i = n. OF CYCLES TO FAILURE AT i^{th} LOAD VALUE; (L_{ma_i})

$\frac{n_i}{N_i}$ = DAMAGE RATIO AT THE i^{th} LOAD VALUE;

IF THE FRACTION OF CYCLES AT EACH LOADING VALUE IS KNOWN AS PERCENTAGE RATHER THAN ACTUAL CYCLE NUMBER, THE NUMBER OF CYCLE CAN BE EXPRESSED AS:

$$n_i = \alpha_i N$$

α_i = CYCLE RATIO (FRACTION OF CYCLES) AT THE i^{th} LOAD VALUE;

N = RESULTANT FATIGUE LIFE (TOTAL CYCLES) (L_{TOT})

THUS THE RESULTANT FATIGUE LIFE (N) CAN BE EXPRESSED AS:

$$N = \frac{1}{\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \ldots + \frac{\alpha_i}{N_i}}$$

- CERAMIC MATERIALS FOR BEARINGS

CERAMICS, **FOR SPHERES ONLY**, HAVE BEEN INTRODUCED **FOR HIGH TEMPERATURE AND HIGH SPEED** APPLICATIONS.

COMPARISON (BETWEEN MATERIALS):

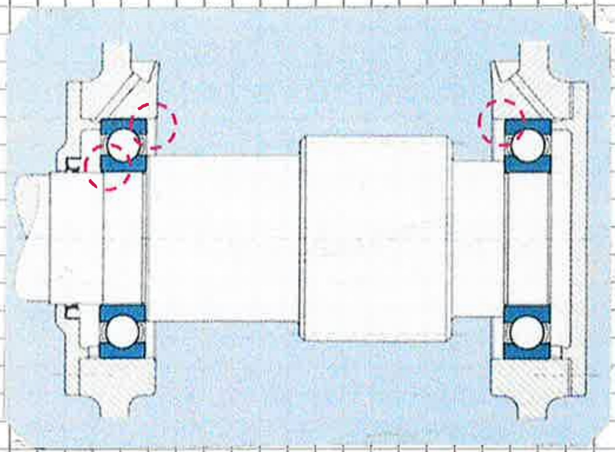
	HRC (20°)	T_{MAX} [°C]	ρ [kg/m³]	E [MPa]	ν	α [°C⁻¹]
M50	64	320	7.6	190'000	0.28	$12.3 \cdot 10^{-6}$
SILICON NITRIDE	78	1200	3.2	310'000	0.26	$2.9 \cdot 10^{-6}$

THE VERY GOOD **ELECTRICAL INSULATING PROPERTY** IS ONE OF THE ESSENTIAL FEATURES OF THE SILICON NITRIDE. THIS PROTECTS THE RINGS FROM ELECTRIC CURRENT DAMAGE AND SO-CALLED **WASHBOARDING**, AND THUS INCREASES BEARING SERVICE LIFE.

(LOWER FRICTION MEANS COOLER RUNNING AND LONGER LUBRICANT SERVICE LIFE).

ARRANGEMENTS

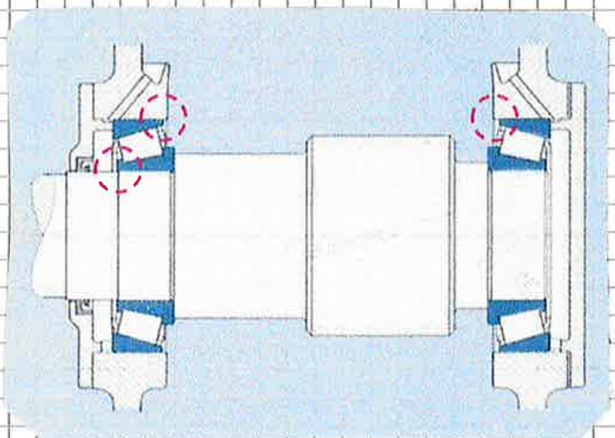
DEEP GROOVE BALL BEARINGS



- COST-FAVOURABLE
- MODERATE POWER
- HIGH SPEED OPERATION
- LOW FRICTION
- SMALL LUBRIFICATION
- AXIAL CLEARANCE BETWEEN OUTER RING AND THE COVER.

• QUESTIONABLE CONSTRAINTS APPLIED TO RINGS

TWO TAPER ROLLER FACE-TO-FACE



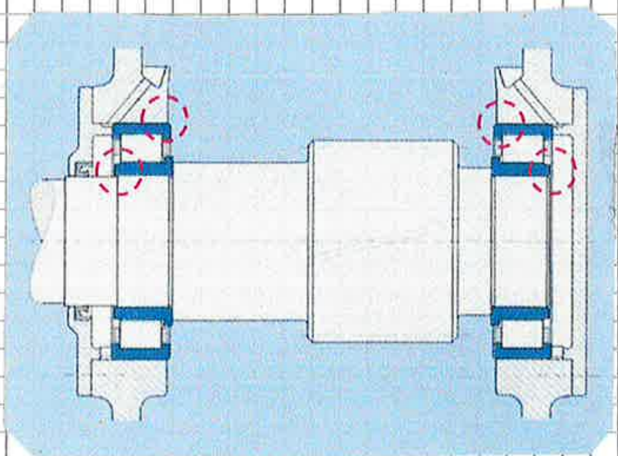
- HIGHER POWER
- LOWER SPEED OPERATION
- HIGHER FRICTION
- ZERO CLEARANCE (*)
- SLIGHT PRE-LOAD

• RISK OF AXIAL CLAMPING

(NO RISK OF BENDING CLAMPING)

(NOT SURE BECAUSE OF (*))

CYLINDRICAL ROLLER BEARINGS

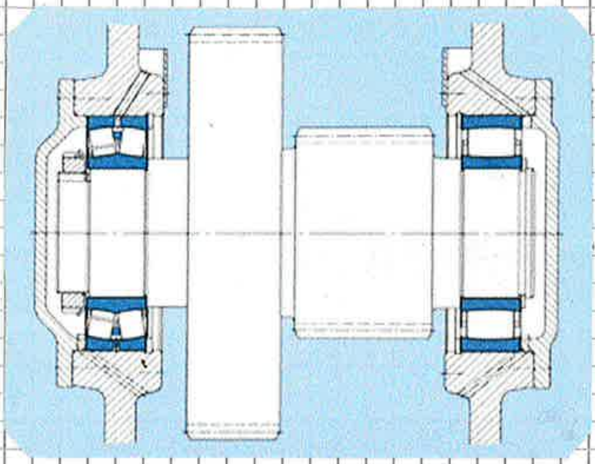


- HIGHER POWER
- LOWER SPEED OPERATION
- HIGHER FRICTION
- HIGH RADIAL STIFFNESS
- COOLING PARTICULARLY GOOD
- AXIAL FORCES TRASMITTED VIA THE FLANGES AND ROLLER ENDS.

• RISK OF BENDING CLAMPING.

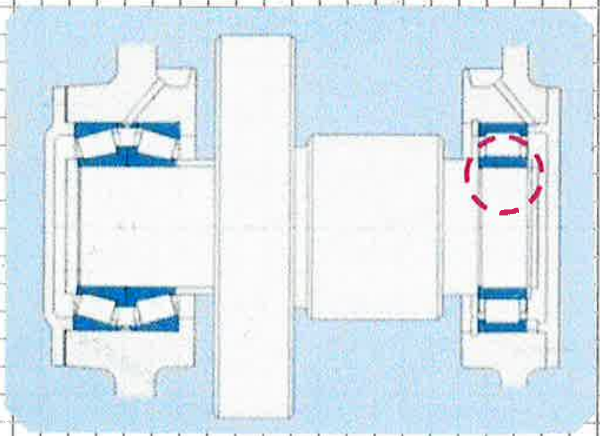
(NO RISK OF AXIAL CLAMPING)

COMBINED ROLLER BEARING + SPHERICAL ROLLER BEARING (CARB)



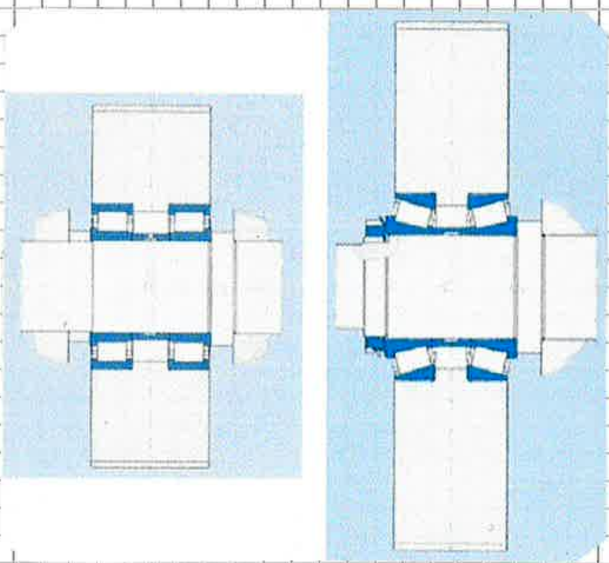
- HEAVY LOADS: HIGH LOAD CARRYING CAPACITY OF THE CARB
- LOWER SPEED OPERATION
- HIGHER FRICTION
- LIMITS SET BY THE DISTANCE BETWEEN THE SHAFTS.
- RISK OF BEARING OVER LOADING
- NO RISK OF HOUSING OVER LOADING
- (NO RISK OF BENDING CLAMPING)

BENDING MOMENT BEARINGS (TANDEM) + CYLINDRICAL ROLLER BEARING



- HEAVY LOADS: RADIAL AS WELL AXIAL LOA S
- LOWER SPEED OPERATION
- HIGHER FRICTION
- LIMITS SET BY THE DISTANCE BETWEEN THE SHAFTS.
- (RISK OF BEARING OVER LOADING)
- (NO RISK OF HOUSING OVER LOADING)
- (NO RISK OF BENDING CLAMPING)
- (NO RISK OF AXIAL CLAMPING)

SPACER AND SYMMETRICAL ROLLER BEARINGS



- LARGE GEARS USE OF SPACER AND SYMMETRIC BEARINGS.

5. GEARS

5.1 POWER TRANSMISSION AND GEARS GEOMETRY

5.1.1 WRITE THE TYPICAL EXPRESSIONS OF POWER.

CONCEPT OF POWER TRANSMISSION

- IT OCCURS BETWEEN TWO MOVEABLE MACHINE COMPONENTS TO PRODUCE WORK
- MOTION IS INDUCED BY ONE BODY UPON THE OTHER ONE BY PROVIDING ENERGY.
- IT IS BASED ON SOME MUTUAL ACTIONS EXCHANGED BY THE TWO PARTS.
- IT EXPLOITS THE TRANSFER OF POWER BETWEEN ELEMENTS.
- AS IS VERY WELL KNOWN, POWER RESULTS FROM A TORQUE APPLIED AT A CERTAIN SPIN SPEED OR A FORCE APPLIED AT A GIVEN SPEED.

$P = F \cdot V = C \cdot \omega$ POWER, F = FORCE, V = SPEED ($P = C \cdot \omega = M \cdot m$)

$C = F \cdot b$ MOMENT, b = ARM (DISTANCE) ($b = R$)

$\omega = V/R$ ANGULAR SPEED, V = SPEED, R = RADIUS (DISTANCE)

5.1.2 DESCRIBE IN SPUR GEARS HOW THE POWER IS TRASMITTED FROM PINION TO GEAR

POWER TRANSMISSION IN CASE OF SPUR GEARS

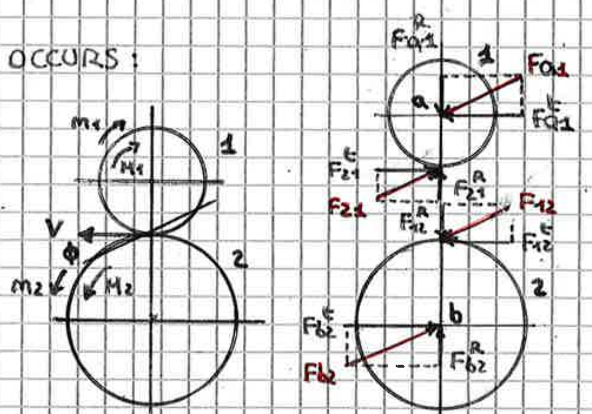
ASSUMING THAT NO DISSIPATION OCCURS:

$P_1 = P_2$

$P_1 = M_1 m_1 = F_{t1} V_1$

$P_2 = M_2 m_2 = F_{t2} V_2$

(WITH $M = C = F_t \cdot R$; $m = V/R$)

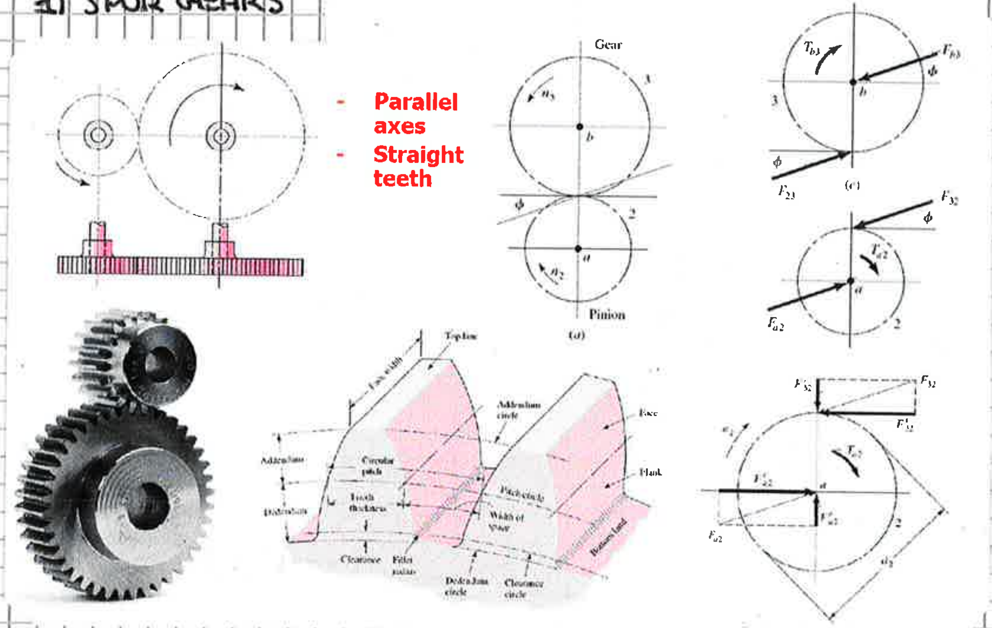


5.1.Q) DRAW A SYSTEM OF TWO: SPUR, HELICAL, BEVEL, WORM, EPICYCLIC GEARS

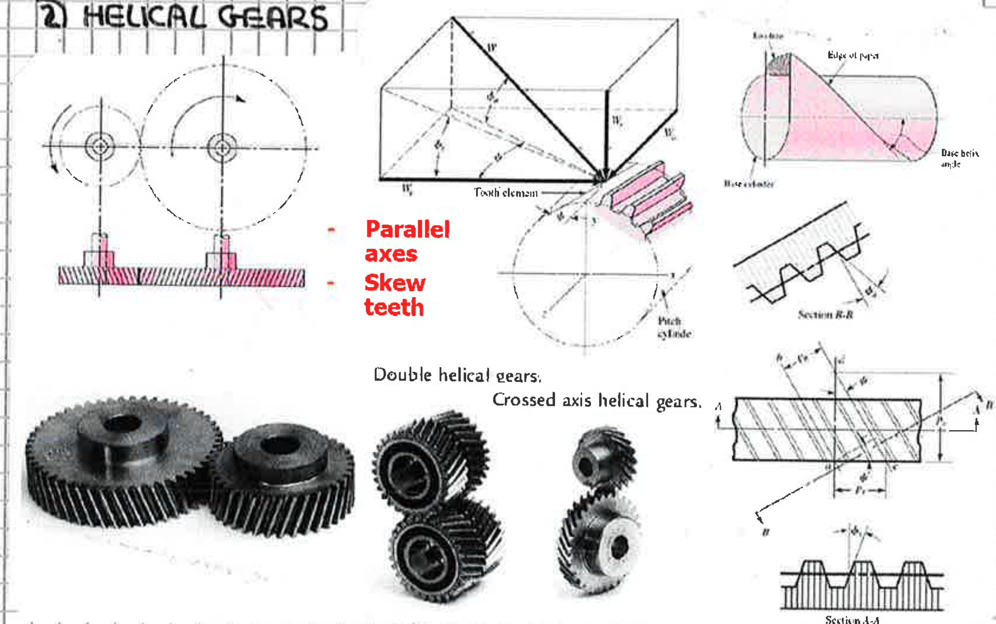
GEAR TYPOLOGIES:

1) SPUR GEARS
2) HELICAL GEARS
3) BEVEL GEARS
4) WORM GEARS
5) EPICYCLIC GEARS

1) SPUR GEARS



2) HELICAL GEARS



5.1.5 DESCRIBE: THE MATERIAL PROCESSING; THE METHODS TO MANUFACTURE THE GEARS AND THE DESIGN OF GEARS.

TRASMISSION OF POWER, MOTION AND ACTIONS IS A CRUCIAL ISSUE OF MACHINE DESIGN TO ALLOW SYSTEMS WORKING.

IN CASE OF GEARS IT IS WORTH NOTICING THAT **DESIGN MAINLY FOCUSES ON**:
- THE **GEOMETRY OF GEAR**, TO BE COMPATIBLE WITH A CONTINUOUS REGULAR AND NEVER NOISY CONTACT BETWEEN GEARS;
- THE **TOOTH PROFILE** AND **STRENGTH**, TO ASSURE PRECISION AT HIGH SPEED AND AVOID PROBLEMS OF WEAR AND FATIGUE.

- **MATERIAL PROCESSING**

THE **GEOMETRY OF GEARS** IS RATHER DIFFICULT TO BE UNDERSTOOD, BECAUSE THERE ARE TWO RELEVANT ISSUES:
- THE **PROFILE OF TOOTH**
- THE **OVERALL SHAPE OF GEAR**

THE **KINEMATIC BEHAVIOUR** OF GEARS IN CONTACT DEPENDS ON THOSE **TWO ARCHITECTURAL DETAILS** AS WELL AS THE **WEAR CONDITION** AND THE **ACTIONS EXCHANGED** BETWEEN GEARS.

A GOOD PRACTICE STARTS FROM UNDERSTANDING:
- HOW THE GEAR IS **MANUFACTURED** TO CREATE THE PROFILE OF TOOTH
- WHICH ARE THE **MAIN DESIGN PARAMETERS** TO BE DESIGNED.

LET'S HAVE AN IMMEDIATE IMPRESSION ... TO UNDERSTAND THE DETAILS OF GEAR GEOMETRY, THE **MOTION OF RACK TOOL** DURING THE MANUFACTURING PROCESS MUST BE REMINDED.

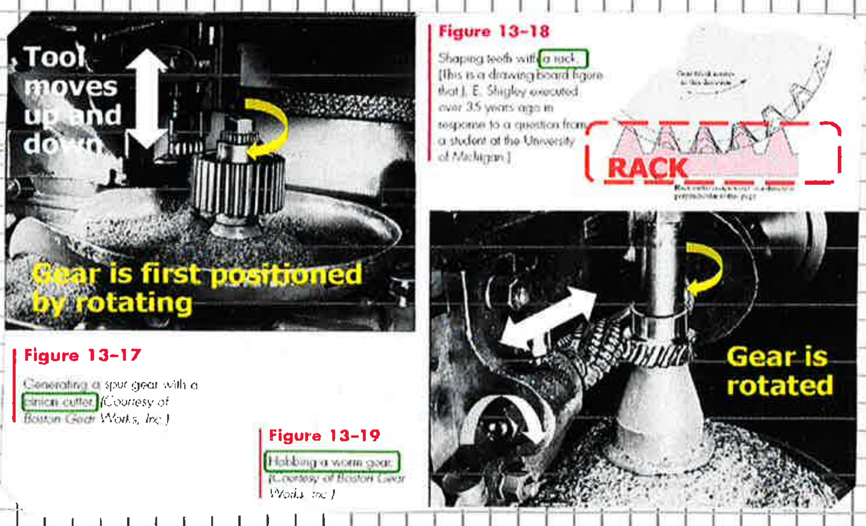


Figure 13-17
Generating a spur gear with a pinion cutter. (Courtesy of Boston Gear Works, Inc.)

Figure 13-18
Shaping teeth with a rack. [This is a drawing board figure that J. E. Shigley executed over 35 years ago in response to a question from a student at the University of Michigan.]

Figure 13-19
Hobbing a worm gear. (Courtesy of Boston Gear Works, Inc.)

4) HOBBING

The hobbing process is illustrated in Fig. 13–19. The hob is simply a cutting tool that is shaped like a worm. The teeth have straight sides, as in a rack, but the hob axis must be turned through the lead angle in order to cut spur-gear teeth. For this reason, the teeth generated by a hob have a slightly different shape from those generated by a rack cutter. Both the hob and the blank must be rotated at the proper angular-velocity ratio. The hob is then fed slowly across the face of the blank until all the teeth have been cut.

5) FINISHING

Gears that run at high speeds and transmit large forces may be subjected to additional dynamic forces if there are errors in tooth profiles. Errors may be diminished somewhat by finishing the tooth profiles. The teeth may be finished, after cutting, by either shaving or burnishing. Several shaving machines are available that cut off a minute amount of metal, bringing the accuracy of the tooth profile within the limits of 250 μin.

Burnishing, like shaving, is used with gears that have been cut but not heat-treated. In burnishing, hardened gears with slightly oversize teeth are run in mesh with the gear until the surfaces become smooth.

Grinding and lapping are used for hardened gear teeth after heat treatment. The grinding operation employs the generating principle and produces very accurate teeth. In lapping, the teeth of the gear and lap slide axially so that the whole surface of the teeth is abraded equally.

- **DESIGN OF GEARS**

THE DESIGN OF GEARS INCLUDES:
- THE SELECTION OF **SPEED AND TORQUE RATIOS** BETWEEN THE TWO GEARS IN CONTACT
- THE DEFINITION OF THE **NUMBER**, **PROFILE**, **DIMENSIONS**, **MATERIAL OF TEETH**
- A **STRESS ANALYSIS**, OF TOOTH AND CHECK ON THE GEAR BODY.
- A **CLEAR DEFINITION OF THE KINEMATICS** OF THE GEARS COUPLED BY THE POWER TRANSMISSION.

TO PERFORM THE DESIGN ACTIVITY IT IS REQUIRED:
- KNOWING THE TYPICAL GEOMETRY OF GEARS.
- HANDLING THE KINEMATICS OF GEARS.
- INVESTIGATING THE DAMAGE OF MATERIALS, MAINLY AT TOOTH.
- APPLYING A TECHNICAL STANDARD DRIVING ALL OF DESIGN STEPS.

5.2 GEOMETRICAL DESIGN AND GEAR KINEMATICS : PART 1

5.2.1) DESCRIBE THE PROPERTIES OF CONJUGATE PROFILES. DEMONSTRATE HOW RATIOS BETWEEN SPEEDS OF GEAR AND PINION ARE FOUND (FOR DIFFERENT CIRCUMFERENCES).

- CONJUGATE ACTION

WHAT KIND OF PROFILE ASSURES A CONSTANT RATIO BETWEEN THE ANGULAR SPEEDS OF GEARS?

THIS IS A TYPICAL PROPERTY OF THE SO-CALLED CONJUGATE ACTION BETWEEN THE TWO GEARS, TYPICAL OF CONJUGATE PROFILES.

A CONJUGATE ACTION OCCURS WHEN:

1) THE TWO SURFACES IN CONTACT HAVE A COMMON TANGENT PLANE (t-t)

2) CONTACT OCCURS IN CORRESPONDENCE OF A GIVEN POINT P (PITCH POINT)

3) CONTACT FORCES ARE APPLIED ALONG THE DIRECTION NORMAL TO THE TANGENT PLANE

4) THE LINE CONNECTING THE TWO CENTERS O_1 AND O_2 OF THE PROFILES IN CONTACT MEET THE LINE ALONG WHICH THAT ACTION IS APPLIED AT POINT C AND THIS POINT DOES NOT CHANGE.

 4.1) O_1C IS THE RADIUS OF PRIMITIVE CIRCUMFERENCE OF ONE WHEEL.

 4.2) O_2C IS THE RADIUS OF PRIMITIVE CIRCUMFERENCE OF THE OTHER WHEEL.

 4.3) RATIO BETWEEN THE TWO ANGULAR SPEEDS Ω_1 AND Ω_2 IS INVERSELY PROPORTIONAL TO THE RATIO BETWEEN RADII OF PRIMITIVE CIRCUMFERENCES, BUT EVEN RADII OF BASE CIRCUMFERENCES.

 CALLING:

 $R_1 = O_1C$; $R_2 = O_2C$; $\Omega_1 = \frac{V}{R_1}$; $\Omega_2 = \frac{V}{R_2}$ $V = \Omega_1 R_1 = \Omega_2 R_2$

 $$\frac{\Omega_1}{\Omega_2} = \frac{R_{2p}}{R_{1p}} = \frac{R_{2b}}{R_{1b}}$$

 SPEED (V) OF POINT P WHEN IT CORRESPONDS TO POINT C
 - ITS DIRECTION IS UNIQUELY DEFINED.
 - IT IS EQUAL FOR WHEEL 1 AND WHEEL 2
 - NO OTHER COMPONENTS ARE ACTIVE.

5) THIS RATIO IS CONSTANT IF POINT C DOES NOT CHANGE.

MAIN RESULTS:

1) THE ACTION LINE ALONG WHICH FORCES ARE APPLIED IS KNOWN: (m-m)
2) TANGENTIAL AND RADIAL COMPONENTS ARE EASILY FOUND.
3) SPEEDS RATIO IS KNOWN, CAN BE RELATED TO RADII AND IS CONSTANT.
4) ALL THOSE PROPERTIES ARE TRUE BECAUSE PROFILES ARE JUST CONJUGATE.
5) TWO CIRCLES ARE SIGNIFICANT: THE PRIMITIVE (EVEN REFERRED TO AS PITCH CIRCLE) AND THE BASE CIRCLE.

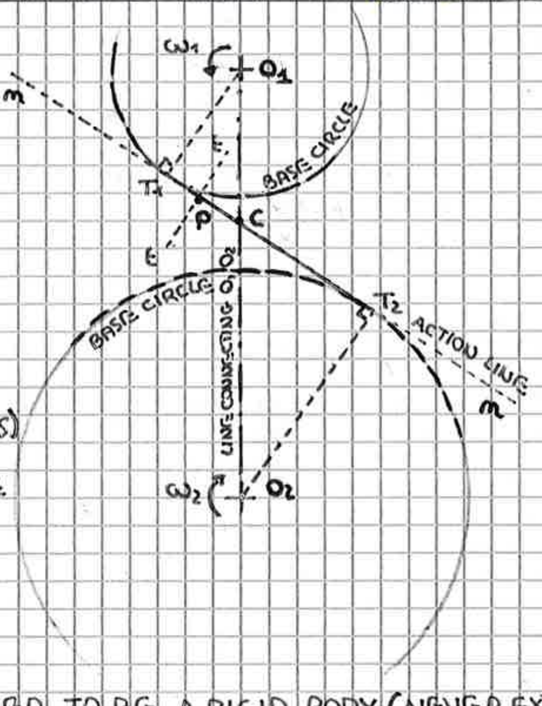
$\frac{\Omega_1}{\Omega_2} = \frac{R_2}{R_1}$ (PRIMITIVE CIRCUMFERENCES)

(BECAUSE OF... SIMILAR TRIANGLES...)

$\frac{\Omega_1}{\Omega_2} = \frac{R_{2b}}{R_{1b}}$ (BASE CIRCUMFERENCES)

52.2) DEFINE THE ACTION LINE, THE CONTACT PATH / SEGMENT AND DRAW A SKETCH.

CONTACT POINT P MOVES ALONG THE ACTION LINE (m-m), BETWEEN THE TWO POSITIONS T_1 AND T_2 WHERE THE LINE IS TANGENT TO THE BASE CIRCLES, BY DEF.

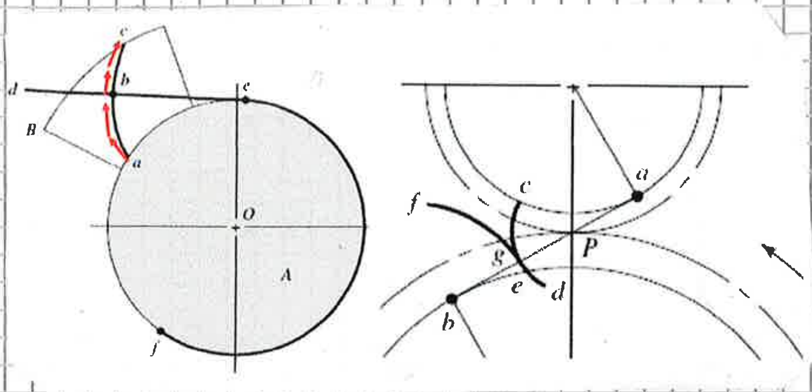


GEARS LOOK LIKE TWO PULLEYS, ON WHICH THE ACTION LINE IS APPLIED LIKE A BELT.
(THE ACTION LINE IS TANGENT TO BASE CIRCLES)
THE CONTACT POINT P ALWAYS LIES ON THE BELT, MOVING ALONG THE LINE, WHILE POINT C IS FIXED.
THE SPEED OF BELT IS THE SAME OF THE TWO PULLEYS, PROVIDED THAT IT IS ASSUMED TO BE A RIGID BODY (NEVER EXTENSIBLE!)

B) GRAPHICAL INTERPRETATION

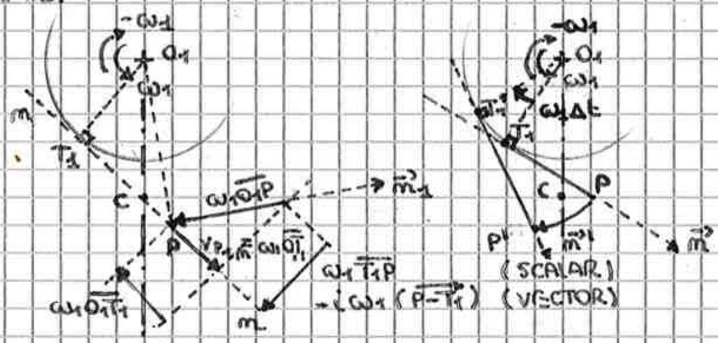
PRATICALLY SPEAKING, THE INVOLUTE OF CIRCLE IS DRAWN BY A PENCIL FIXED ON RIGID SEGMENT $\overline{de}$ WHEN CIRCLE ROTATES AND THE TIP MOVES FROM a TO b TO c...

THE INVOLUTE OF CIRCLE WILL BE THE SURFACE OF CONTACT BETWEEN GEARS.



THE OBSERVER "SEAT" ON WHEEL 1 SEES P MOVING FROM THE BASE CIRCLE WITH A SPEED EXACTY OPPOSITE TO V_{P1}, I.E IS SOMETHING LIKE APPLYING AN OPPOSITE ROTATION TO P, $-\Omega_1$. $(-\omega_1)$

MOTION CAN BE DETECTED BY THINKING **SEGMENT T_1P BOUNDED ON THE BASE CIRCLE, WHILE ROTATING, THUS MAKING P (MOVING TO P') DESCRIBING AN INVOLUTE OF CIRCLE WHICH IS EXACTLY THE PROFILE OF THE TOOTH ALLOWING TO REALIZE THE MOTION OF P ALONG THE ACTION LINE SEEN** BY A FIXED REFERENCE FRAME.



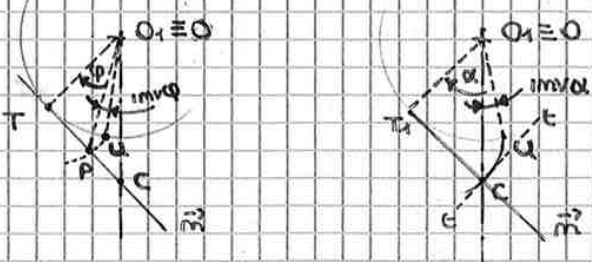
C) EQUATION OF THE INVOLUTE

LET'S FOLLOW POINT P IN ITS MOTION: WHEN THE CABLE IS COMPLETELY ROLLED ON THE WHEEL IS U, THEN IT GOES TO P. THE ARC DESCRIBED IS EQUAL TO SEGMENT TP.

$$\begin{cases} T\hat{O}P = \varphi \\ \overline{TP} = \widehat{TU} \end{cases}$$

$$\overline{TO} \cdot Tg\,\varphi = \overline{TO}\,(\varphi + inv\,\varphi)$$

$$\begin{cases} inv\,\varphi = Tg\,\varphi - \varphi \\ \overline{OP} = \overline{TO} \cdot \dfrac{1}{\cos\varphi} \end{cases}$$



WHEN POINT P IS SUPERPOSED TO C: φ CORRESPONDS TO α, WHICH DEFINES THE INCLINATION OF THE ACTION LINE.

5.2.Q) DEFINE THE CONTACT ANGLE AND DESCRIBE HOW IS SELECTED. IS THERE ANY LIMITATION TO ITS VALUE?

CONTACT/PRESSURE ANGLE (α)

1) ANGLE α CAN BE **ARBITRARY CHOSEN**, SINCE THE SPEED RATIO IS ONLY RELATED TO THE DIMENSIONS OF GEARS.

2) THIS ANGLE IS USUALLY DEFINED BASING ON **SOME MANUFACTURING ISSUES**, TO AVOID INTERFERENCES BETWEEN BODIES.

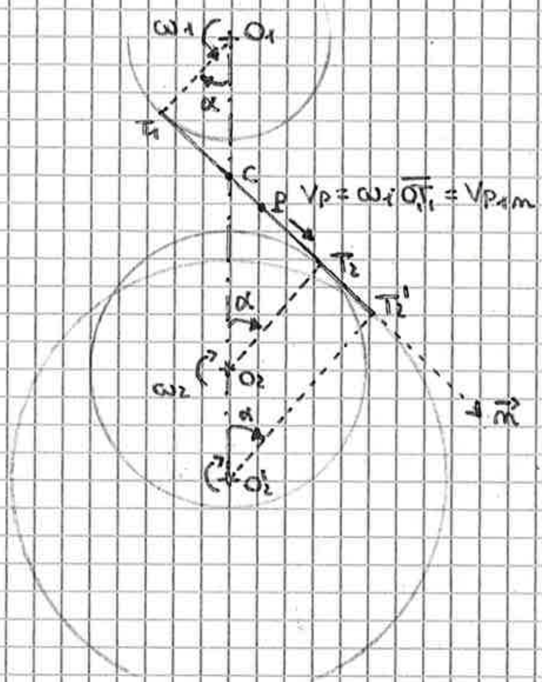
3) IF THE SIZE OF WHEELS CHANGES, THE SAME ANGLE CAN BE APPLIED.

$$\omega_2 = \frac{V_{P2m}}{O_2T_2} = \frac{V_{P1m}}{O_2T_2}$$

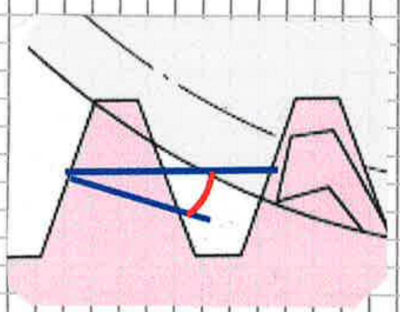
$$\omega_1 = \frac{V_{P1m}}{O_1T_1}$$

THEN:

$$\frac{\omega_1}{\omega_2} = \frac{O_2T_2}{O_1T_1}$$



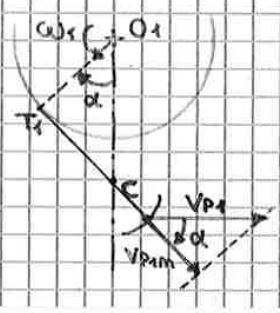
4) PRESSURE ANGLE (α) DEFINES EVEN THE **INCLINATION OF THE RACK**.



5) THE COMPONENT OF SPEED ALONG THE **DIRECTION NORMAL TO THE TOOTH PROFILE** IS THE **TANGENTIAL SPEED OF BASE CIRCLE**, WHILE ALONG THE **HORIZONTAL DIRECTION** IS THE **TANGENTIAL SPEED OF PITCH CIRCLE**.

$$V_{P1m} = \Omega_1 \cdot O_1T_1$$

$$V_{P1} = \Omega_1 \frac{O_1T_1}{\cos\alpha} = \Omega_1 \cdot O_1C$$



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SYNTHESIS:

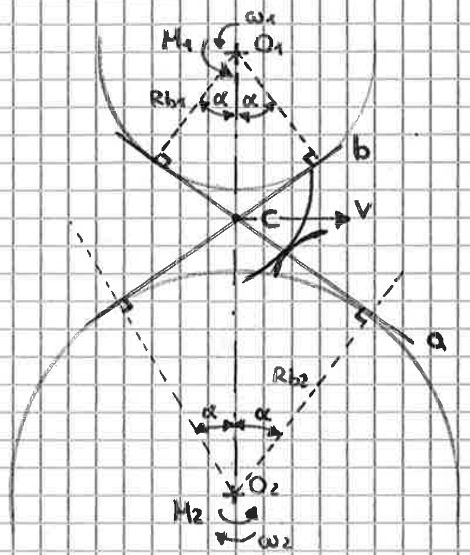
1) GIVEN CENTERS O_1 AND O_2, THE DISTANCE IS KNOWN AND FIXED JUST BY CONNECTING THOSE.

2) FOR A GIVEN CONTACT ANGLE α, DRAWING O_1T_1 AND ITS SYMMETRIC POINT (BASE RADII) THE ACTION LINES a AND b CAN BE FOUND.

3) INTERSECTION BETWEEN a AND b AND THE CONNECTING LINE BETWEEN CENTERS IS C, WHERE PRIMITIVE CIRCUMFERENCES MEET.

4) TRASMISSION RATIO IS CONSTANT:

$$\left|-\frac{\omega_1}{\omega_2}\right| = \frac{O_2T_2}{O_1T_1}\left(=\frac{R_{b2}}{R_{b1}}\right) = \frac{O_2C}{O_1C}\left(=\frac{R_2}{R_1}\right)$$



5) POWER IS THE SAME:

$$P = M_1\Omega_1 = M_2\Omega_2$$

6) TANGENTIAL SPEED IN C IS THE SAME:

$$V = R_1\Omega_1 = R_2\Omega_2$$

7) ACTIONS ARE:

$$F_{t1} = \frac{M_1}{R_1} \; ; \; F_{t2} = \frac{M_2}{R_2}$$

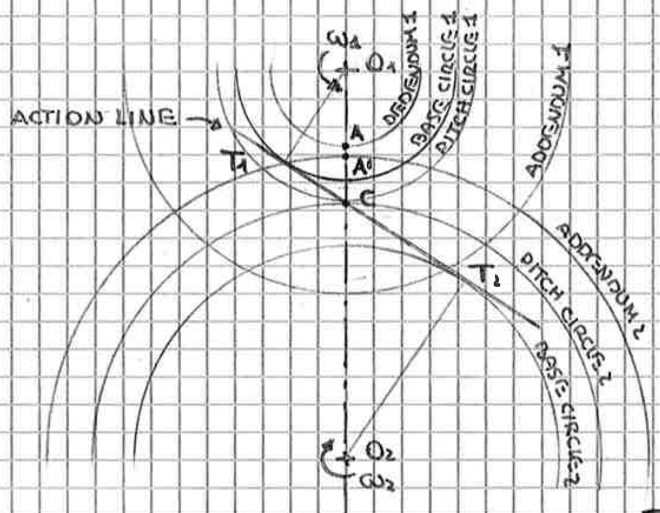
$$F_{R1} = F_{t1}\,Tg\,\alpha \; ; \; F_{R2} = F_{t2}\,Tg\,\alpha$$

5.2.7) DESCRIBE THE PROPERTIES OF ADDENDUM, DEDENDUM, PITCH AND MODULUS OF GEARS

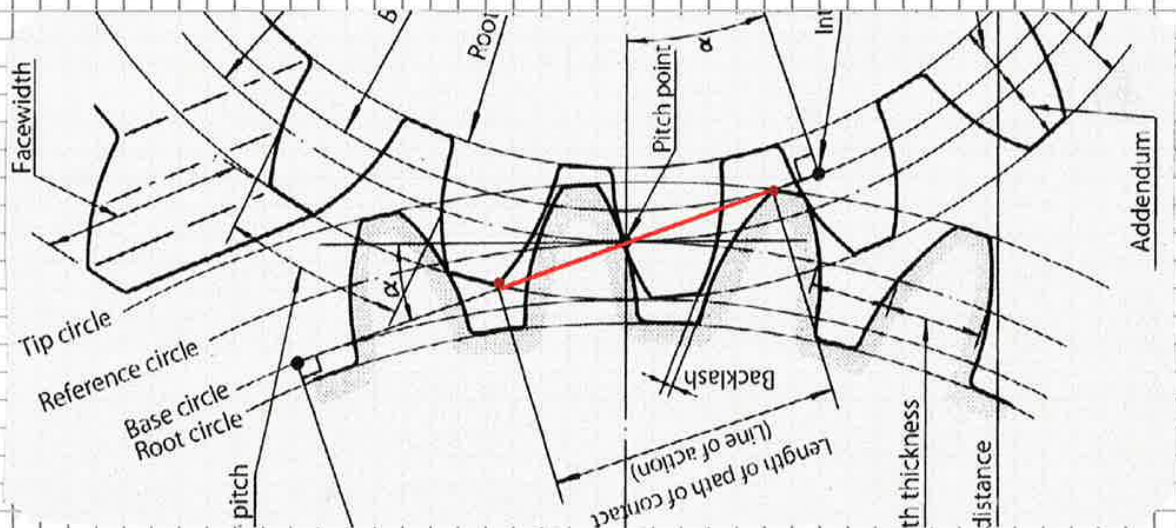
⦿ GEOMETRY PROPERTIES OF TEETH

1) **CONTACT** OCCURS ALONG THE SEGMENT OF CONTACT **ONLY BETWEEN T_1 AND T_2.**
2) THE PORTION OF TOOTH EXCEEDING THE PITCH CIRCLE IS THE **ADDENDUM** ("TO BE ADDED") WHILE THE PORTION BELOW IS THE **DEDENDUM** ("TO BE SUBTRACTED", IN LATIN).
3) CONTACT COULD EXPLOIT THE WHOLE LATERAL SURFACE OF TOOTH, BUT IN PRACTICE IT COULDN'T, **TO AVOID AN INTERFERENCE BETWEEN GEARS.**
4) **THE TOOTH IS TRUNCATED AT ITS TIP**, TO AVOID CONTACT OUTSIDE THE SEGMENT.

DEDENDUM CIRCLE 1 IS OUT OF ADDENDUM CIRCLE 2 ($A \neq A'$)



5) THE REAL LENGTH OF THE **SEGMENT OF CONTACTS** IS **SHORTER THAN $\overline{T_1T_2}$.**



5.2.8 DEFINE HOW THE THICKNESS OF TOOTH IS DESIGNED. WHICH IS THE LAW FOR A RIGHT CONTACT BETWEEN THE TWO GEARS? HOW IT VARIES ALONG THE RADIUS OF GEAR?

...

- THICKNESS OF TEETH (...7)

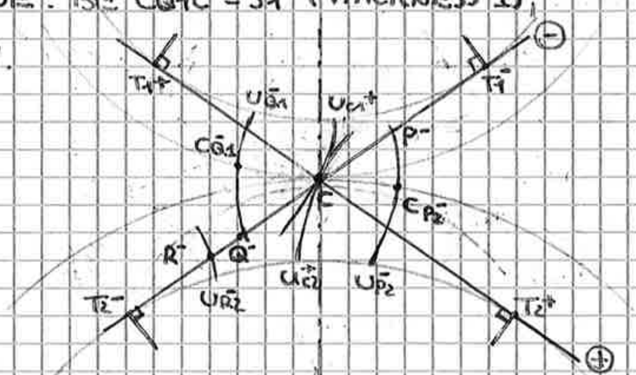
- THICKNESS OF TEETH IS THE SAME ON THE TWO GEARS?

7.1) DRAW PRIMITIVE AND BASE CIRCLES, AND ACTION LINES "+" AND "−" (ONE PER VERSUS OF ROTATION)

7.2) FIND THE SEGMENT OF CONTACTS ($T_1T_2^+$, $T_1T_2^-$)

7.3) DRAW THE INVOLUTES OF CIRCLE IN C, U_{C1} AND U_{C2}.

7.4) CHOOSE A THICKNESS OF TOOTH AND DRAW THE RELATED INVOLUTE TO FIND U_{Q1} AND Q ON THE ACTION LINE. BE $\widehat{C_{Q1}C} = S_1$ (THICKNESS 1);



7.5) NOW SIMILARLY CHOOSE A THICKNESS FOR THE TOOTH OF 2 AND FIND C_{P2}, U_{P2}, P.

7.6) SET THE PITCH ON THE ACTION LINE AS $PR = P_b$ $\qquad \widehat{C_{P2}C} = S_2$

7.7) THICKNESSES WERE CHOSEN ARBITRARILY, THEREFORE R AND Q MIGHT NOT SUPERPOSE. THIS DIFFERENCE MIGHT LEAD TO A GAP OR AN INTERFERENCE $g_b = RQ$

- MAY WE APPLY A RULE TO ASSURE THE COMPATIBILITY OF TEETH THICKNESSES?

7.8) THE RULE IS THAT, FOR GIVEN GAP g_b ($=RQ$) ON THE BASE CIRCLE, THE PITCH MUST BE THE SUM OF THICKNESSES OF THE TEETH, MEASURED ON THE PITCH CIRCLE, IN ABSENCE OF GAP.

$$\widehat{U_{R2}^-U_{P2}^-} = P_b$$

$$P_b = g_b + \overline{Q^-C} + \overline{CP^-}$$

$$\widehat{C_{Q1}^-C} = \widehat{U_{Q1}^-U_{C1}^-} \cdot \frac{R_1}{R_{b1}} = \overline{Q^-C} \cdot \frac{R_1}{R_{b1}} \rightarrow \frac{1}{\cos\alpha}$$

$$\Rightarrow \overline{Q^-C} = S_1\cos\alpha \qquad \overline{CP^-} = S_2\cos\alpha$$

$$\Rightarrow P_b = g_b + S_1\cos\alpha + S_2\cos\alpha$$

$$\frac{P_b}{\cos\alpha} = \frac{g_b}{\cos\alpha} + (S_1+S_2) \Rightarrow S_1 + S_2 = P$$

④ DEFINE CONTACT SEGMENT AND CONTACT RATIO

- CONTACT SEGMENT AND RATIO (...9)

9.1) THE HEIGHT OF TOOTH IS PROPORTIONAL TO THE MODULUS.
(ADDENDUM + DEDENDUM = $m + 1.25\,m = 2.25\,m$)

9.2) ON THE ACTION LINE THE SEGMENT OF CONTACTS IS SHORTER AND SPANS FROM A TO E.

9.3) CONSEQUENTLY WE CAN DEFINE:

$$\varepsilon_a = \frac{AE}{p_b} \quad \text{CONTACT RATIO}$$

9.4) CONDITION FOR A REGULAR AND CONTINUOUS CONTACT BETWEEN GEARS IS:

$$\varepsilon_a = \frac{AE}{p_b} > 1$$

(THIS MEANS THAT AS SOON AS A COUPLE OF TEETH LEAVES THE CONTACT THERE IS ALREADY THE NEXT ONE IN CONTACT)

$$\overline{AC} = \overline{AT_2} - \overline{CT_2}$$

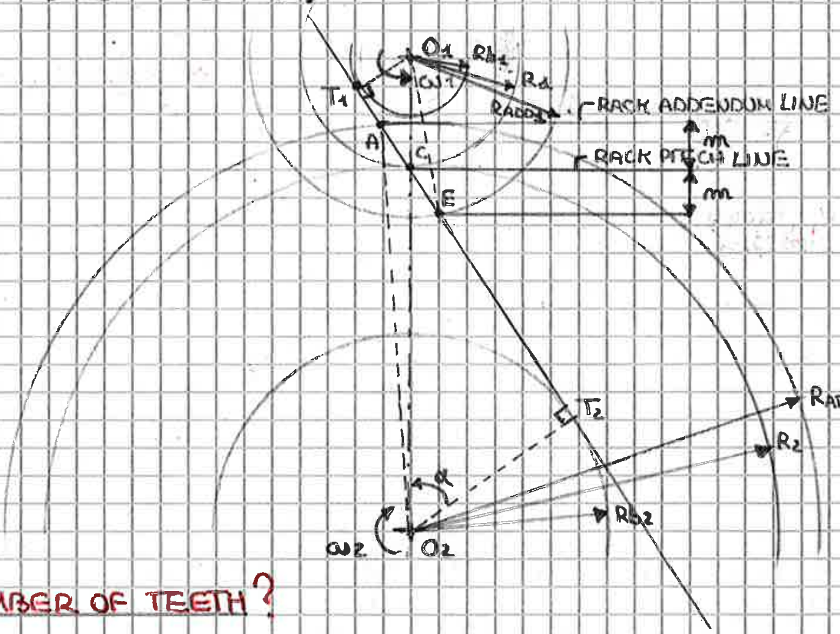
$$\begin{cases} \overline{AT_2} = \sqrt{(R_2+m)^2 - R_{b2}^2} \\ \overline{CT_2} = R_2 \cdot \sin\alpha \end{cases}$$

$$\overline{AC} = \sqrt{(R_2+m)^2 - R_{b2}^2} - R_2 \sin\alpha$$

$$\overline{EC} = \overline{ET_1} - \overline{CT_1}$$

$$\begin{cases} \overline{ET_1} = \sqrt{(R_1+m)^2 - R_{b1}^2} \\ \overline{CT_1} = R_1 \cdot \sin\alpha \end{cases}$$

$$\overline{EC} = \sqrt{(R_1+m)^2 - R_{b1}^2} - R_1 \sin\alpha$$



④ IS THERE A MINIMUM NUMBER OF TEETH?

- MINIMUM NUMBER OF TEETH (...10)

10.1) THE MAXIMUM HEIGHT OF TOOTH IS GIVEN BY AC (ON THE PITCH CIRCLE) AS IT LOOKS ON THE RACK.

10.2) BY THE FOLLOWING DISCUSSION IT CAN BE REALIZED THAT A RELATION BETWEEN Z AND α HOLDS.

$$\overline{AC} = \frac{m}{\sin\alpha} \leq \overline{CT_1} = \overline{O_1C} \cdot \sin\alpha$$

$$\frac{2m}{\sin\alpha} \leq 2R_1 \sin\alpha \Rightarrow \frac{2}{\sin^2\alpha} \leq \frac{2R_1}{m} = z_1$$

$$\Rightarrow z_1 \geq \frac{2}{\sin^2\alpha} \qquad (\text{EX}: \alpha = 20^\circ : z_1 \geq 17.1)$$

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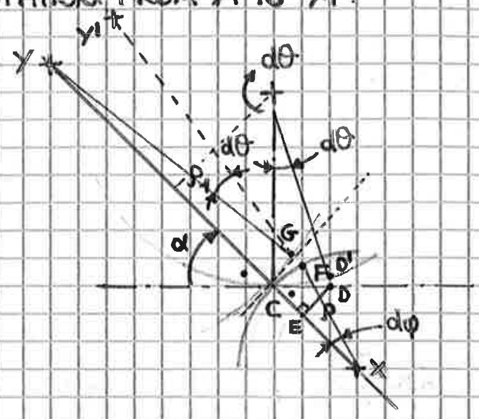
STEP 3

1) THE GEAR ROTATES OF $d\theta$

2) POINT G (GEAR) CORRESPONDS TO F (RACK).

3) THEREFORE, THE GEAR ROTATES ABOUT G OF ANGLE $d\theta$,

AND BRINGS THE CENTER OF INSTANTANEOUS ROTATION FROM Y_1 TO Y_1':



$$C\hat{Y_1}G = d\theta - d\varphi$$

$$\overline{DE} = R_1 d\theta \sin\alpha = (\rho - R_1 d\theta \cos\alpha) Tg\, d\varphi$$

$$Tg\, d\varphi \cong d\varphi = \frac{R_1 d\theta \sin\alpha}{(\rho - R_1 d\theta \cos\alpha)}$$

BEING $\rho - R_1 d\theta \cos\alpha \cong \rho$

WE OBTAIN THAT:

$$d\varphi = d\theta \frac{R_1 \sin\alpha}{\rho}$$

(A PART FROM THE DEMONSTRATION, IT IS RELEVANT THAT A RELATION HOLDS BETWEEN THE TWO ANGLES. THIS ALLOWS FINDING A RELATION BETWEEN CURVATURE OF RACK SURFACE AND OF GEAR TOOTH.)

$$d\theta - d\varphi = d\theta \left(1 - \frac{R_1 \sin\alpha}{\rho}\right)$$

$$\rho_1 (d\theta - d\varphi) = \rho\, d\varphi$$

$$\frac{1}{\rho_1} = \frac{1}{\rho} \cdot \frac{d\theta - d\varphi}{d\varphi} = -\frac{1}{\rho} + \frac{1}{R_1 \sin\alpha}$$

THEREFORE:

$$\frac{1}{\rho_1} + \frac{1}{\rho} = \frac{1}{R_1 \sin\alpha}$$

$$\frac{1}{\rho_2} = \frac{1}{\rho} + \frac{1}{R_2 \sin\alpha}$$

THE RELATIVE CURVATURES BETWEEN RACK AND GEAR DEPENDS ON THE PRIMITIVE RADIUS (R_1), ON THE CONTACT ANGLE (α), BUT NEVER ON THE SPECIFIC SHAPE OF THE PROFILE, PROVIDED THAT CONTACT IS ASSURED DURING THE RELATIVE MOTION.

5.2.10) SUMMARIZE BRIEFLY ALL THE RELEVANT PROPERTIES OF A SPUR GEAR

[NEXT REVIEWING MOMENT]

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REMARK:

a) $P_0 = \pi m_0$: PITCH (DISTANCE BETWEEN TEETH MEASURED AT PRIMITIVE CIRCLE)

b) $R = z m_0$: PITCH/PRIMITIVE RADIUS

c) $R_b = R \cos\alpha_0$: BASE RADIUS

d) m = ADDENDUM ; 1.25 = DEDENDUM

e) $S_0 = P_0/2$: TOOTH THICKNESS (MEASURED AT THE PITCH CIRCLE)

f) THE PITCH LINE OF RACK IS TANGENT TO THE PITCH CIRCLE OF WHEEL IN C.

g) $a_0 = R_1 + R_2$: DISTANCE BETWEEN CENTRES OF GEARS.

WHAT WE NEED?

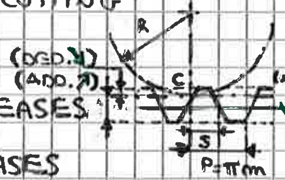
A) $a \to a'$: SOMETIMES WE WANT TO CHANGE SLIGHTLY THE DISTANCE BETWEEN CENTERS OF GEARS [$\boxed{A_1}$ $a' > a$; $\boxed{A_2}$ $a' < a$]

B) $s \to s'$: SOMETIMES THE STRENGTH OF TOOTH IS NOT SUFFICIENT, THEREFORE ITS THICKNESS MUST BE INCREASED. [A ($a \to a'$) INVOLVES B ($s \to s'$)] (?)

=> THIS IS POSSIBLE JUST BY MODIFYING THE GEARS ALREADY BUILT.

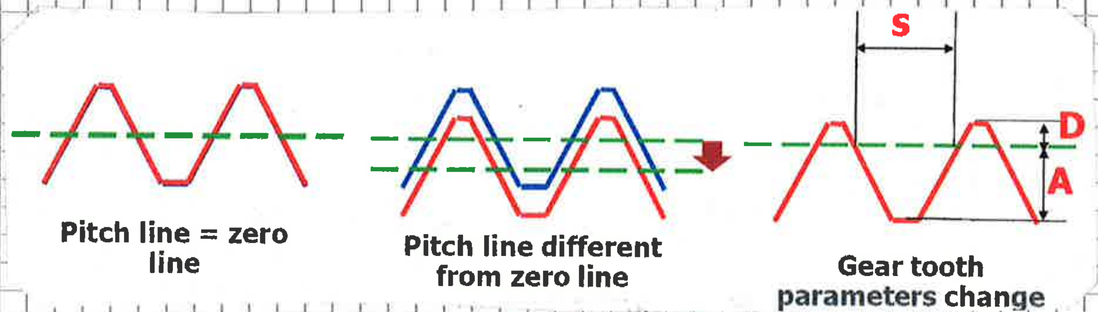
PROFILE SHIFTING

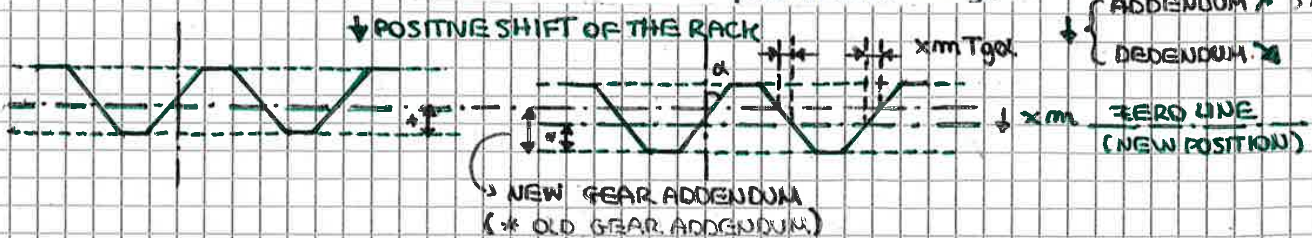
THE PITCH LINE (ALWAYS REFERENCE TO MEASURE ALL THE PARAMETERS) IN THE NEW CONFIGURATION IS LOCATED EITHER ABOVE OR BELOW THE ZERO LINE (CUTTING TOOL REFERENCE), WHICH WAS MOVED.



↓ POSITIVE SHIFT : ZERO LINE MOVES DOWN => ADDENDUM OF GEAR INCREASES

↑ NEGATIVE SHIFT : ZERO LINE MOVES UP => ADDENDUM OF GEAR DECREASES





THE RACK MOVES DOWN.

THE PITCH LINE REMAINS THE REFERENCE TO MEASURE THE GEAR PARAMETERS BUT THE PROFILE OF RACK MOVES DOWN.

NEW PARAMETERS:

THE ORIGINAL PARAMETERS OF WHEEL 1 ARE:

R_1 = PITCH RADIUS | $R_{a1} = R_1 + m$ = ADDENDUM RADIUS

$R_{b1} = R_1 \cos\alpha$ = BASE RADIUS | $R_{d1} = R_1 - 1.25m$ = DEDENDUM RADIUS

$P = \pi m$ = PITCH | $S_1 = P/2$ = THICKNESS OF TOOTH

AFTER POSITIVE SHIFTING (↓) (THE RACK MOVES DOWN).

$R_1' = R_1 + xm$ = POSITION OF THE NEW ZERO LINE OF RACK (WITH RESP. TO THE GEAR CENT.)

$R_{a1}' = R_{a1} + xm = (R_1 + m) + xm = R_1 + (1+x)m$ = NEW ADDENDUM RADIUS (LONGER)

$R_{d1}' = R_{d1} - xm = (R_1 - 1.25m) - xm = R_1 - (1.25 + x)m$ = NEW DEDENDUM RADIUS

$S_1' = S_1 + 2xm\,Tg\,\alpha = P/2 + 2xm\,Tg\,\alpha$ = NEW THICKNESS OF TOOTH (MEASURED AT PL!)

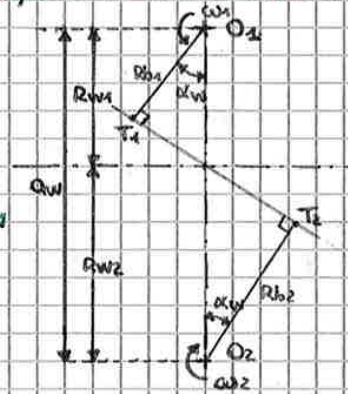
⇒ POSITIVE SHIFTING ⇒ LONGER ADDENDUM ($R_{a1}' > R_{a1}$) AND LARGER THICKNESS ($S_1' > S_1$)

SOME GENERAL RELATIONS CAN BE WRITTEN

PARTICULARLY, THE NEW POSITION OF RACK WILL CUT A DIFFERENT WHEEL (!)

WHOSE WORKING PARAMETERS (W) WILL BE DIFFERENT

$a_W = a' = R_{W1} + R_{W2}$ = NEW DISTANCE BETWEEN CENTRES, SUM OF NEW WORKING RADII (R' ON THE TWO GEARS)



$R_{W1} = R_{b1}/\cos\alpha_W$

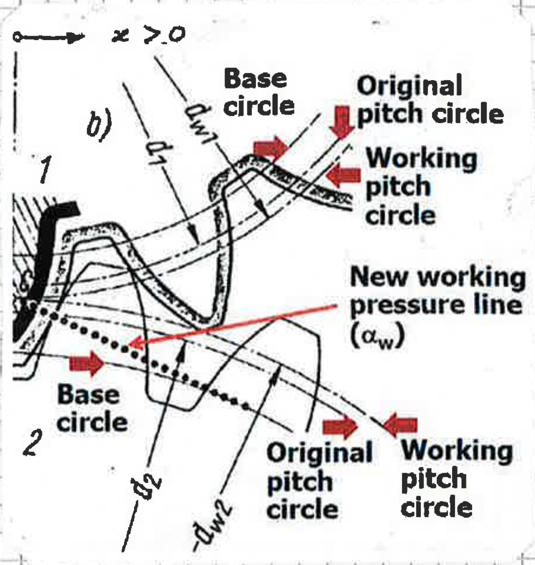
$R_{W2} = R_{b2}/\cos\alpha_W$

$$\frac{\omega_1}{\omega_2} = \frac{z_2}{z_1} = \frac{R_{W2}}{R_{W1}} = \frac{R_{b2}}{R_{b1}}$$

ASSUMING THAT BASE CIRCLES (R_{b1}, R_{b2}) DO NOT CHANGE, THE PRESSURE ANGLE (α) MUST CHANGE.

(WE WILL REFER TO PINION WITH 1, TO WHEEL WITH 2)

IN PRACTICE... (EXAMPLE: POSITIVE SHIFTING ($x>0$))



1. IN OUR EXAMPLE $X>0$ LIKE IN THIS SKETCH
2. THE ADDENDUM OF THE GEAR TEETH IS BIGGER
3. WE CAN SEE THAT IN ORIGIN THE TWO GEARS WERE TANGENT AT THE PITCH CIRCLE (d_1, d_2).
4. THE PRESSURE LINE WAS IDENTIFIED BY ANGLE α.
5. AS WE APPLY THE PROFILE SHIFTING THE SHAPE OF EACH TOOTH CHANGES, THIS REQUIRES THAT THE TWO GEARS WORK BEING NOW TANGENT AT THE SO-CALLED WORKING PITCH CIRCLE (d_{w1}, d_{w2})
6. BASE CIRCLES REMAIN THE SAME, THEREFORE TO MAKE COMPATIBLE THE WORKING PITCH CIRCLES AND THE BASE CIRCLES, THE REAL PRESSURE LINE OF THE SHIFTED PROFILES IS NOW IDENTIFIED BY THE NEW ANGLE α_w.
7. ALL THE REQUIRED RELATIONS TO FIND WORKING CIRCLES (R_{w1}, R_{w2}) AND ANGLE α_w ($\mathrm{inv}\,\alpha_w$) ARE:

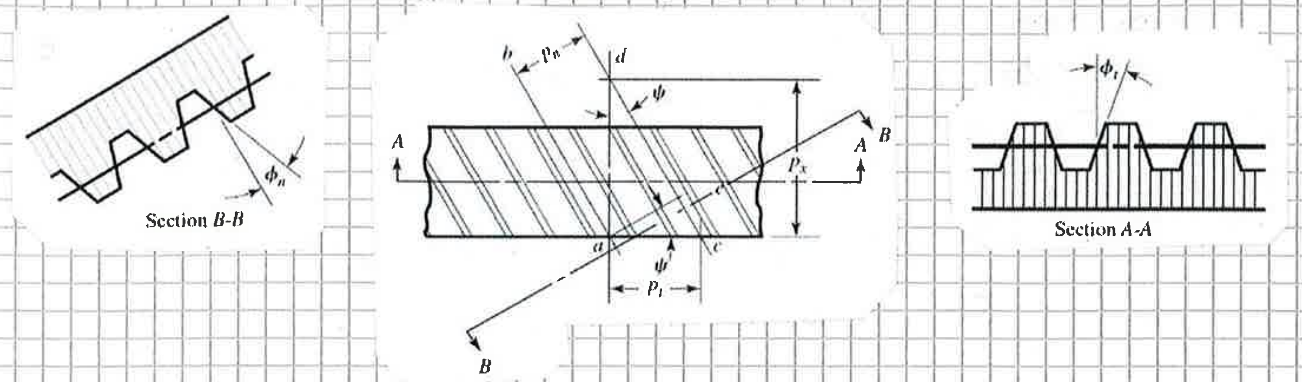
$$R_{w1} = R_{b1} / \cos\alpha_w$$

$$R_{w2} = R_{b2} / \cos\alpha_w$$

$$\mathrm{inv}\,\alpha_w = \mathrm{inv}\,\alpha + \frac{2(x_1+x_2)\,\mathrm{Tg}\,\alpha}{z_1+z_2}$$

HELICAL GEARS

1. THIS GEOMETRY IS STUDIED IN THE PLANE NORMAL TO THE TOOTH AS IN THE SPUR GEARS (WHOSE TRACE IS B-B).
2. THE COMPLETE DESIGN REPORT DELIVERED TO MANUFACTURER DESCRIBES ALL THE PARAMETERS IN THE FRONTAL PLANE (A-A) EVEN REFERRED TO AS TANGENTIAL.
3. RULES TO "TRANSLATE" INFORMATION BETWEEN PLANES MUST BE DEFINED.

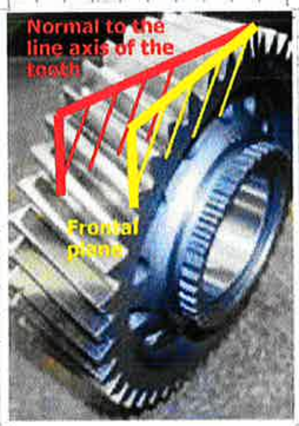


TO DEFINE THE RELATIONS BETWEEN THE TWO PLANES A RATIONALE MUST BE DEVELOPED:

→ WE NEED TO UNDERSTAND HOW THE GEOMETRY OF THE HELICAL TOOTH IS DEVELOPED AND HOW IT WORKS. (IN B-B)

→ THEN WE'LL EXAMINE THE CONTACT BETWEEN TEETH OF HELICAL GEARS. (IN A-A)

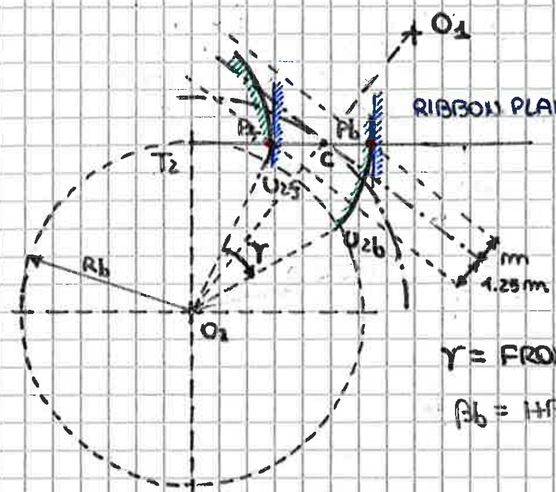
→ FINALLY WE'LL WRITE THE RELATIONS BETWEEN PARAMETERS DESCRIBED IN THE TWO PLANES.



5.3.5) DESCRIBE HOW THE PROFILE OF TEETH CHANGE IN A HELICAL GEAR AND WHY.

PROFILE OF TEETH (γ) (βb)

PROFILES LOOK ROTATED FROM THE FRONT TO BACK, LIKE IN ROTATION OF SPUR GEARS. THE CONTACT POINT MOVES UP ALONG THE PROFILE OF THE INVOLUTE LIKE IN THE SPUR GEARS, BUT IT REMAINS ON THE RIBBON PLANE. MOREOVER, IT MOVES FROM THE FRONT TO THE BACK OF THE CYLINDER.



RELATION BETWEEN FRONT ANGLE (γ) - HELICAL A. (βb)

$(\overline{Q_b P_b} = b\,Tg\beta_b = \overline{P_5 Q_b}\,Tg\beta_b)$

$\widehat{U_{25}U_{2b}} = \overline{Q_b P_b} = b\,Tg\beta_b = \overline{P_5 Q_b}\,Tg\beta_b$

$\widehat{U_{25}U_{2b}} = Rb\cdot\gamma$

$$\frac{d\gamma}{db} = \frac{\gamma}{b} = \frac{\widehat{U_{25}U_{2b}}}{Rb}\,\frac{Tg\beta b}{\widehat{U_{25}U_{2b}}} = \frac{1}{Rb}\,Tg\beta b$$

γ = FRONT ANGLE

β_b = HELICAL ANGLE

5.3.6) DEFINE THE TRAJECTORY FOLLOWED BY THE CONTACT POINT IN HELICAL GEAR IN OPERATION AND DRAW A SKETCH

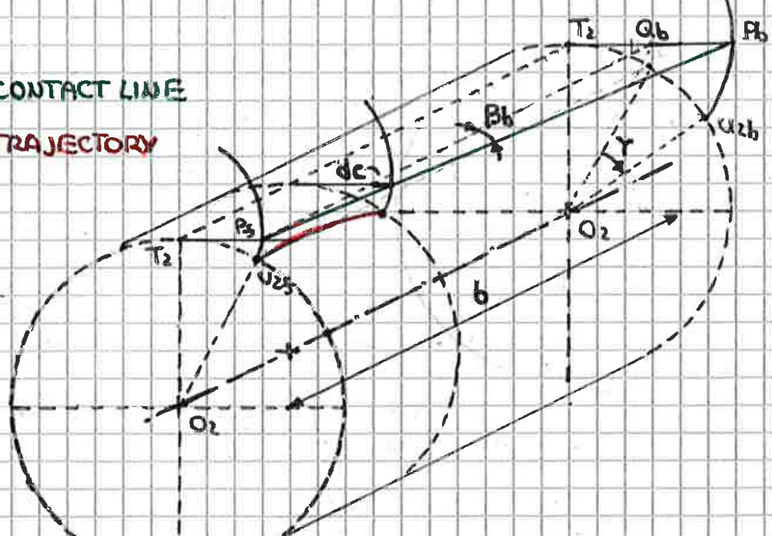
TRAJECTORY OF THE CONTACT POINT

IF THE TRAJECTORY OF THE CONTACT POINT ACTUALLY IS STRAIGHT ON THE UNROLLING RIBBON, THE CORRESPONDING DISPLACEMENT OF THE PROFILE TO ALLOW THAT TRAJECTORY COMES OUT FROM THE COMBINATION OF A MOTION ALONG THE LINE AXIS OF THE CYLINDER AND A ROTATION OF GEAR (HELICAL).

REMARK: ANGLE βb IS USED TO DEFINE THE HELICAL BECAUSE THE IDEAL UNROLLING RIBBON IS ACTUALLY THE PLANE IN WHICH THE CONTACT OCCURS!

CONTACT LINE

TRAJECTORY



$(\overline{Q_b P_b} = b\cdot Tg\beta b)$ $(\overline{Q_b P_b} = \widehat{U_{25}U_{2b}} = Rb\gamma)$

$dc = db\cdot Tg\beta b$

$dc = Rb\cdot d\gamma$

$$\frac{d\gamma}{db} = \frac{1}{Rb}\cdot Tg\beta b$$

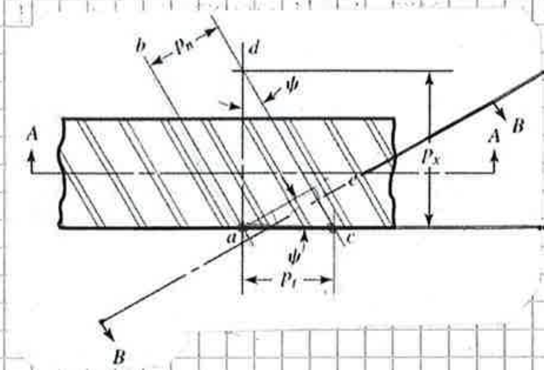
⊕ DESCRIBE THE CUTTING OPERATION.

5.3.8) DESCRIBE THE CONTACT PATH IN HELICAL GEAR.

CUTTING OPERATION

THE MAIN GOAL HERE IS DEMONSTRATING THAT ALL THE DESIGN CRITERIA LEARNT ON THE SPUR GEARS APPLY TO THE HELICAL GEARS, IF WE CONSIDER A PLANE NORMAL TO THE HELICAL (B-B).

NEVERTHELESS, TO ALLOW DRAWING THE GEARS WE MUST TRANSFER THE WHOLE INFORMATION FROM THE PLANE NORMAL TO THE HELICAL (B-B) TO THE PLANE NORMAL TO THE LINE AXIS OF THE CYLINDER, I.E. THE FRONTAL PLANE (A-A).



PLANE NORMAL TO THE HELICAL (B-B)

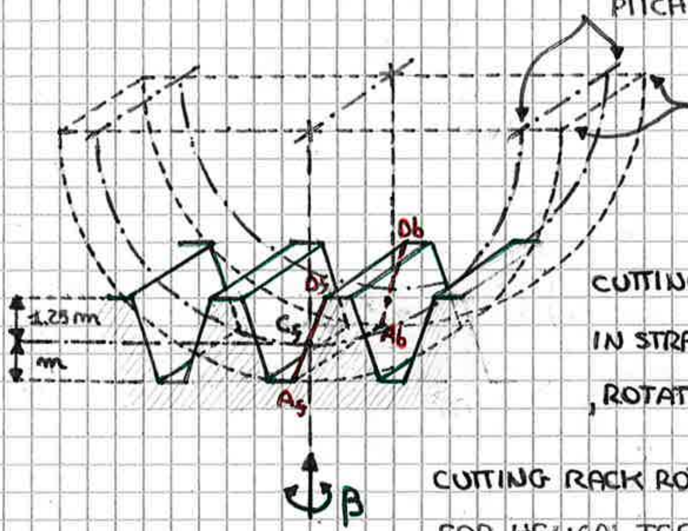
FRONTAL PLANE (A-A) (NORMAL TO CYL. LINE AXIS)

STEP 1

A FIRST DIFFERENCE BETWEEN SPUR AND HELICAL GEAR IS THAT THE RACK IN CUTTING IS ROTATED JUST BY THE ANGLE β.

PITCH CYLINDER

ADDENDUM CYLINDER



CUTTING RACK:

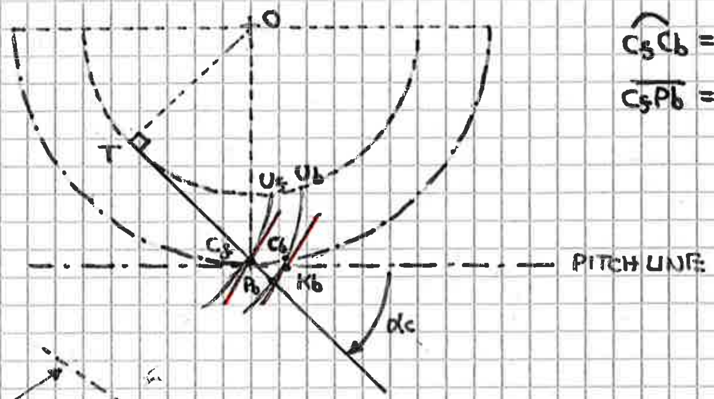
IN STRAIGHT TEETH CUTTING → SPUR GEAR

, ROTATING BY β → HELICAL GEAR

CUTTING RACK ROTATION ANGLE β FOR HELICAL TEETH CUTTING (ROTATION ABOUT C_5)

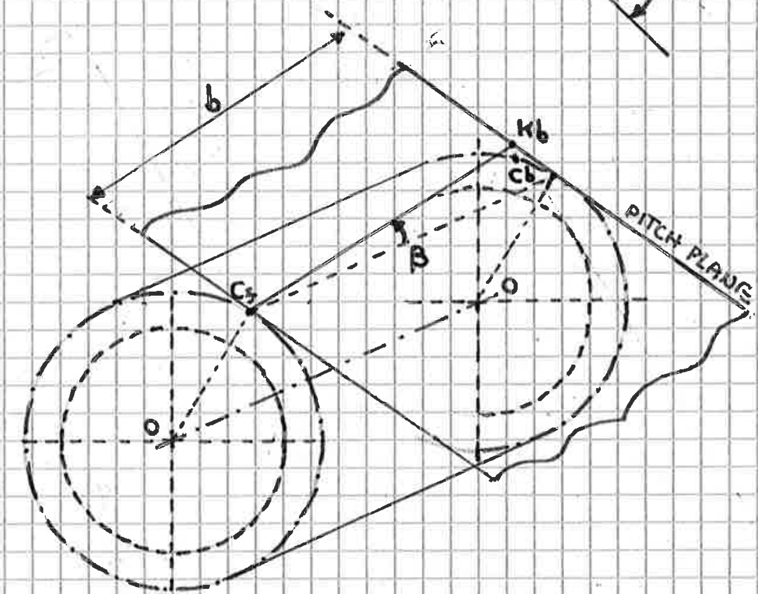
STEP 3

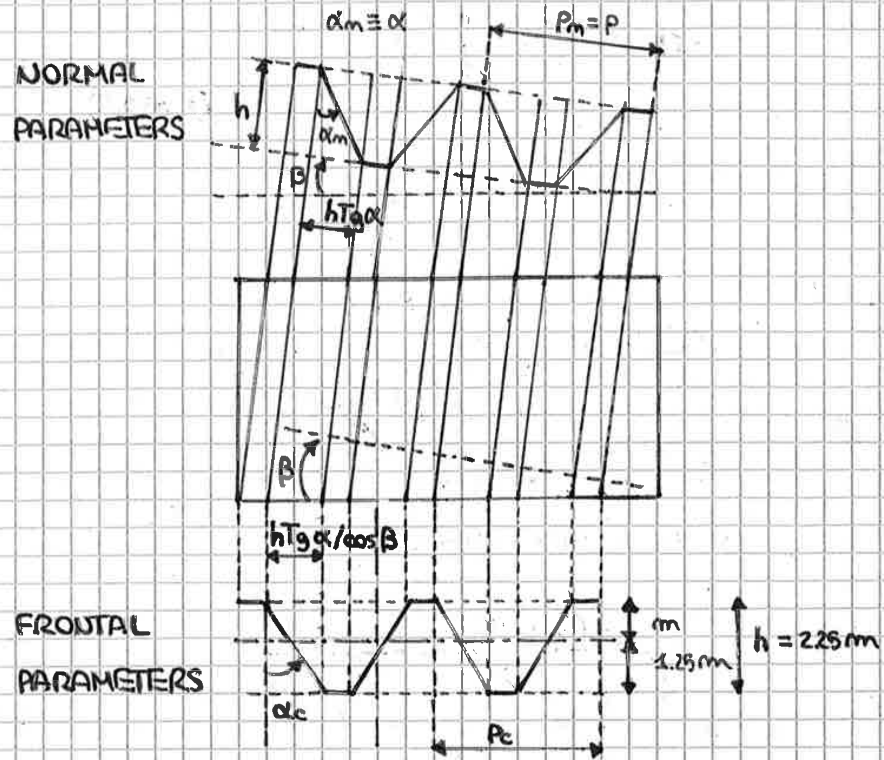
IF WE LOOK AT THE PITCH LINE OF THE RACK IT SEEMS ROTATING BY ANGLE β IN OPPOSITE VERSUS (COUNTER CLOCKWISE) WITH RESPECT TO THE PRIMITIVE CIRCLE OF THE GEAR.



$$\widehat{C_S C_b} = \overline{C_S K_b} = b \operatorname{Tg} \beta$$

$$\overline{C_S P_b} = \widehat{U_S U_b} = \overline{C_S K_b} \cos \alpha_c$$

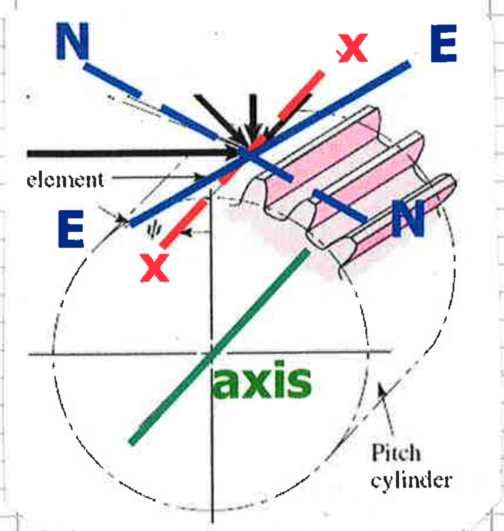


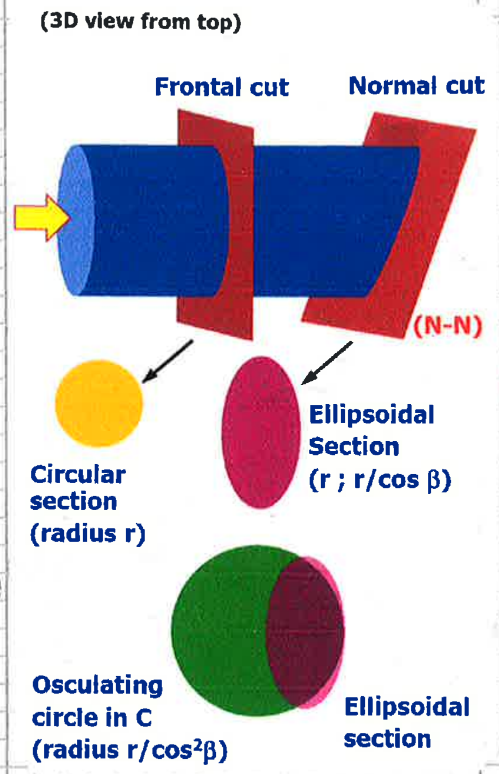


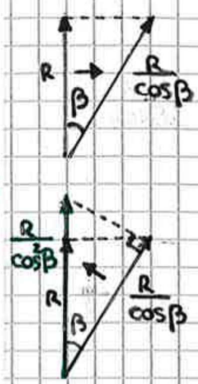
$$Tg\alpha_c = \frac{h \cdot Tg\alpha}{\cos\beta} \cdot \frac{1}{h} = \frac{Tg\alpha}{\cos\beta}$$

5.3.12) DESCRIBE THE SHAPE OF THE GEAR SECTION IN FRONT AND NORMAL CUTS AND HOW THIS IS USED TO RELATE THE PARAMETERS BETWEEN PLANES.

RELEVANT PARAMETERS IN THE TWO MAIN PLANES



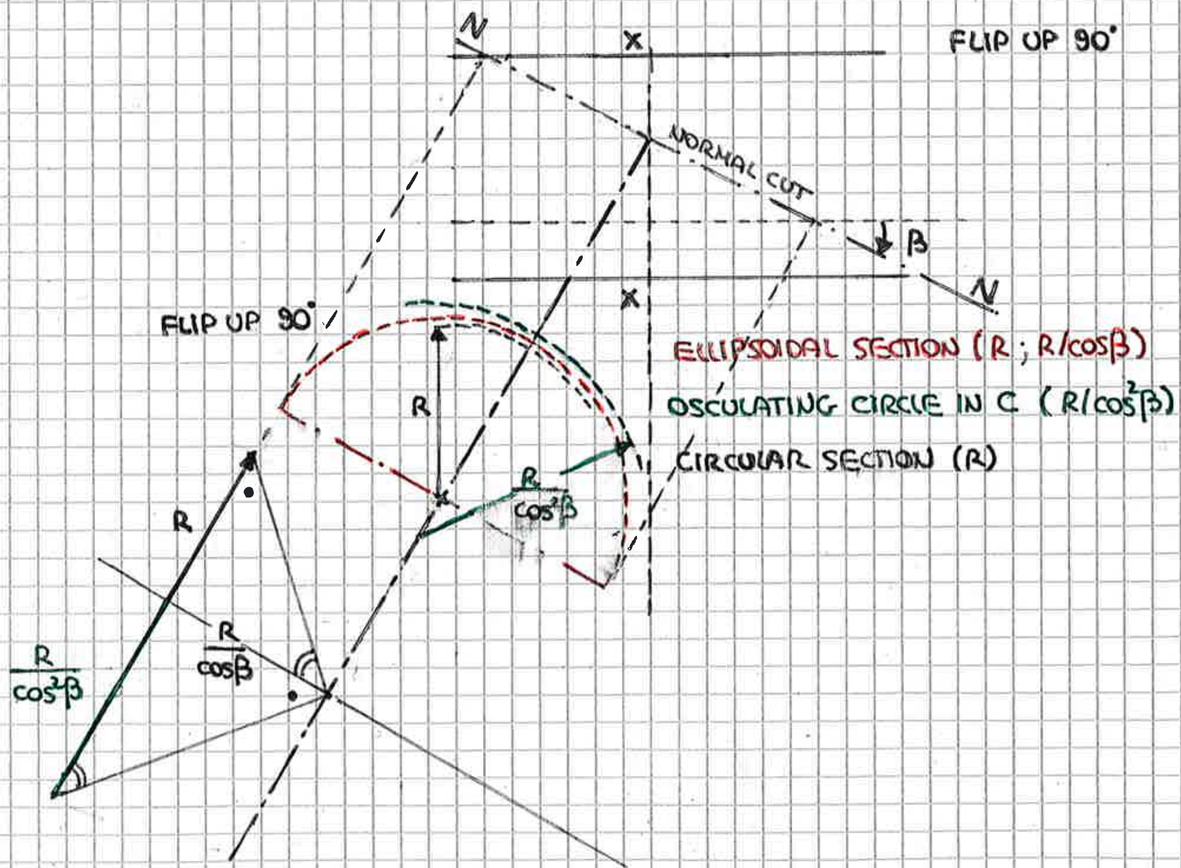




X-X PITCH CYLINDER (// TO CYL.AXIS)

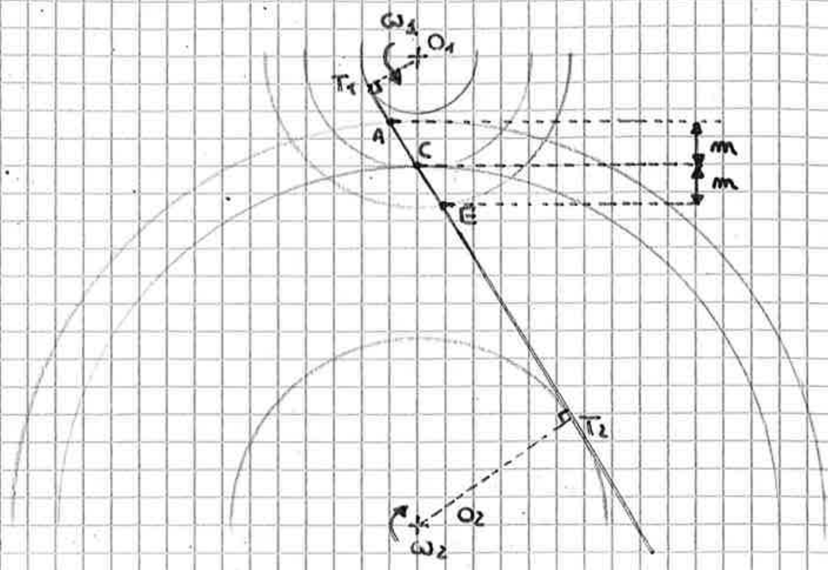
E-E DIRECTION OF THE HELICAL

N-N IS ⊥ TO E-E



5.3.13) DEFINE THE CONTACT RATIOS IN HELICAL GEAR AND SHOW THE MEANING ON A SKETCH

CONTACT RATIO



IN CASE OF SPUR GEARS:

$\varepsilon_\alpha = \frac{\overline{AE}}{P_b}$ CONTACT RATIO

IN CASE OF HELICAL GEARS:

$A_S F$, DE_b : FULL-LENGTH CONTACT LINES

BA_b, $A_S F$: PARTIAL-LENGTH CONTACT LINES

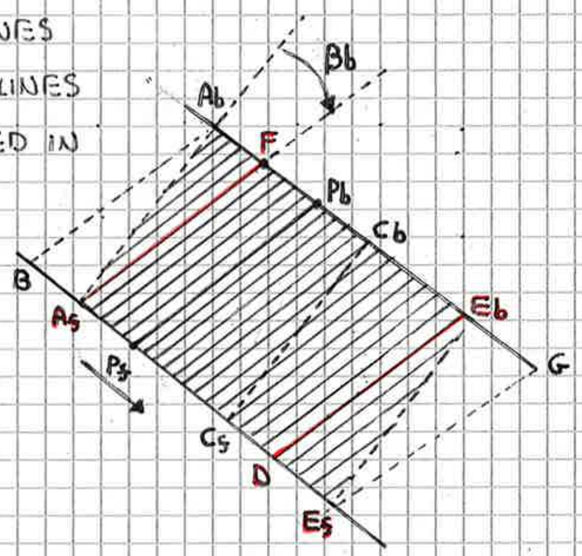
THE WHOLE REGION B TO E_S IS INVOLVED IN THE CONTACT, THEREFORE:

$\varepsilon_{TOT} = \varepsilon_\alpha + \varepsilon_\beta$ CONTACT RATIO

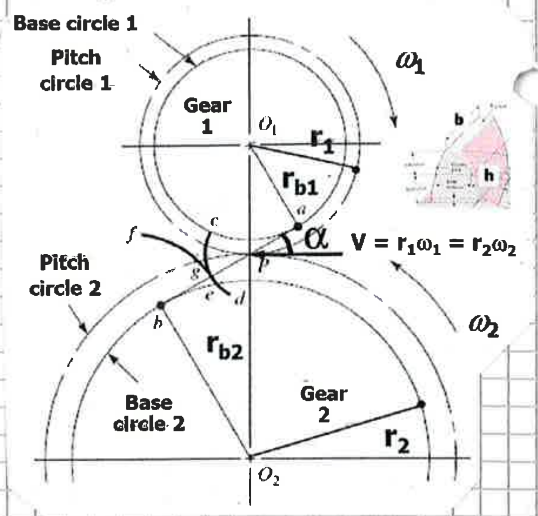
WITH:

$\varepsilon_\alpha = \frac{\overline{AE}}{P_c \cos\alpha_c}$

$\varepsilon_\beta = \frac{\overline{BA_S}}{P_c \cos\alpha_c} = \frac{b \, Tg \beta_b}{P_c \cdot \cos\alpha_c}$



BE COMPARED TO SPUR GEARS ....



$$\frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} = \frac{R_{b2}}{R_{b1}} = \frac{M_2}{M_1} = \frac{\omega_{PINION}}{\omega_{WHEEL}} = i = K = \frac{1}{\tau}$$

$$R_b = R\cos\alpha$$

$$\tau = \frac{\omega_{WHEEL}}{\omega_{PINION}} = \frac{d_{PINION}}{d_{WHEEL}} = \frac{z_{PINION}}{z_{WHEEL}} = \frac{1}{K}$$

$$P = \frac{2\pi R}{z}$$

$$m = \frac{d}{z} = \frac{P}{\pi}$$

$$h = R_a + R_d = m + 1.25m$$

$$\lambda = \frac{b}{m}$$

$$z_{min} = \frac{2}{(1+2K)\sin^2\alpha}\left(K + \sqrt{K^2 + (1+2K)\sin^2\alpha}\right), \quad K = i = \frac{\omega_1}{\omega_2} = \frac{\omega_{PINION}}{\omega_{WHEEL}}$$

Table 21.9: Reference values for **Dynamic Factor (K_v)** of Gear wheel pairs

Dynamikfaktor

$$K_V \approx 1 + \left(1 + \frac{k_c \cdot f_{pe}}{w \cdot K_1} 0{,}65\,\text{N/mm}\right) C_V \cdot C_\beta \cdot v$$

k_c in μm^{-1}	Paarungsbeiwert nach Tab. 21.10, Seite 470,
f_{pe} in μm	zulässige Eingriffsteilungs-Abweichung für das Rad 2 nach Tab. A 21.6,
w in N/mm	spezifische Nennumfangskraft $= F_t/b$,
K_1	Betriebsfaktor nach Tab. 21.3 Seite 445,
C_V in s/m	Belastungsbeiwert (untenstehend),
C_β	Schrägungsbeiwert (untenstehend), abhängig von der Sprungüberdeckung ε_β (Gl. 21.41),
v in m/s	Umfangsgeschwindigkeit der Teilkreise.

$w \cdot K_1$ N/mm	C_V in s/m bei v in m/s: 1	2	4	6	8	10	12	14	16	18	20
25	0,057	0,056	0,056	0,046	0,034	0,029	0,026	0,024	0,021	0,019	0,017
50	0,050	0,049	0,048	0,041	0,034	0,028	0,025	0,023	0,021	0,019	0,017
100	0,038	0,037	0,032	0,030	0,027	0,025	0,024	0,023	0,021	0,019	0,017
200	0,033	0,030	0,025	0,024	0,023	0,023	0,023	0,022	0,021	0,019	0,017
300	0,031	0,029	0,025	0,023	0,023	0,023	0,023	0,022	0,021	0,019	0,017
400	0,030	0,029	0,025	0,023	0,023	0,023	0,023	0,022	0,021	0,019	0,017
500	0,030	0,029	0,024	0,023	0,023	0,023	0,023	0,022	0,021	0,019	0,017
1000	0,029	0,028	0,024	0,023	0,023	0,023	0,023	0,022	0,021	0,019	0,017
2000	0,028	0,028	0,024	0,023	0,023	0,023	0,023	0,022	0,021	0,019	0,017

ε_β	$\leq 0{,}4$	0,5	0,6	0,7	0,8	0,9	≥ 1
C_β	1	0,95	0,9	0,85	0,8	0,75	0,7

Table A21.11: Form Factor (Y_F) in function of the **profile shift factor (x)** and of the **virtual teeth number (z_n or z_{vn})** (compiled from DIN 3990)

z_n z_{vn}	Zahnformfaktor Y_F bei Profilverschiebungsfaktor x = −0,6	−0,5	−0,4	−0,3	−0,2	−0,1	0	+0,1	+0,2	+0,3	+0,4	+0,5	+0,6	+0,7	+0,8	+0,9	+1,0	+1,1	+1,2	+1,3	+1,4
7												2,84									
8											2,98	2,69	2,47								
9											2,84	2,60	2,40	2,22							
10										2,99	2,73	2,52	2,34	2,18							
11									3,15	2,87	2,65	2,46	2,30	2,16	2,05						
12									3,03	2,79	2,58	2,41	2,27	2,14	2,04						
13									2,93	2,72	2,53	2,38	2,24	2,12	2,03	1,96					
14							3,36	3,10	2,86	2,66	2,48	2,34	2,22	2,11	2,03	1,95					
15							3,25	3,01	2,79	2,60	2,44	2,31	2,20	2,10	2,02	1,95	1,89				
16						3,45	3,16	2,95	2,74	2,56	2,42	2,29	2,18	2,09	2,02	1,95	1,89				
17						3,35	3,09	2,88	2,69	2,53	2,39	2,27	2,17	2,08	2,01	1,95	1,89	1,85			
18					3,53	3,26	3,02	2,82	2,65	2,50	2,37	2,26	2,16	2,08	2,01	1,95	1,90	1,86			
19				3,72	3,44	3,20	2,96	2,78	2,61	2,47	2,35	2,24	2,15	2,07	2,01	1,95	1,90	1,87	1,83		
20				3,62	3,35	3,12	2,91	2,74	2,58	2,45	2,33	2,23	2,14	2,07	2,01	1,95	1,90	1,87	1,84		
21				3,53	3,28	3,07	2,87	2,70	2,55	2,43	2,32	2,22	2,14	2,06	2,01	1,95	1,91	1,87	1,84	1,82	
22				3,45	3,20	3,01	2,83	2,67	2,52	2,41	2,30	2,21	2,13	2,06	2,00	1,95	1,91	1,88	1,85	1,83	
23			3,64	3,38	3,15	2,96	2,80	2,64	2,50	2,39	2,29	2,20	2,12	2,06	2,00	1,95	1,91	1,88	1,85	1,83	1,82
24			3,55	3,30	3,10	2,92	2,75	2,61	2,48	2,37	2,28	2,19	2,12	2,06	2,00	1,95	1,91	1,88	1,86	1,84	1,83
25		3,73	3,45	3,25	3,05	2,88	2,72	2,58	2,46	2,36	2,27	2,19	2,12	2,05	2,00	1,95	1,92	1,88	1,86	1,84	1,83
30	3,61	3,35	3,18	3,01	2,85	2,72	2,60	2,48	2,38	2,30	2,22	2,16	2,10	2,04	2,00	1,96	1,93	1,90	1,88	1,86	1,85
40	3,15	3,00	2,86	2,75	2,63	2,54	2,45	2,37	2,30	2,24	2,18	2,13	2,08	2,04	2,01	1,97	1,95	1,93	1,91	1,90	1,89
50	2,90	2,78	2,68	2,59	2,50	2,43	2,36	2,31	2,25	2,20	2,15	2,11	2,07	2,03	2,02	1,98	1,97	1,94	1,93	1,92	1,91
60	2,75	2,65	2,57	2,50	2,42	2,37	2,32	2,25	2,22	2,17	2,13	2,10	2,08	2,04	2,02	1,99	1,98	1,96	1,94	1,94	1,93
100	2,46	2,40	2,35	2,32	2,26	2,24	2,21	2,17	2,15	2,12	2,10	2,08	2,06	2,04	2,03	2,01	2,00	1,99	1,98	1,98	1,97
200	2,27	2,24	2,21	2,19	2,17	2,15	2,14	2,12	2,10	2,10	2,08	2,07	2,05	2,04	2,04	2,02	2,02	2,01	1,98	2,00	2,00
400	2,17	2,15	2,14	2,13	2,12	2,11	2,10	2,09	2,08	2,08	2,08	2,07	2,06	2,06	2,05	2,04	2,04	2,04	2,03	2,03	2,03
∞	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07	2,07

Table A21.13: Material factors (Z_M) (according to DIN 3990)

Rad			Gegenrad			Werkstoffaktor Z_M
Werkstoff	Kurzzeichen	Elastizitätsmodul E N/mm²	Werkstoff	Kurzzeichen	Elastizitätsmodul E N/mm²	$\sqrt{N/mm^2}$
Stahl	St	210000	Stahl	St	210000	272
			Stahlguß	GS-60	205000	270
				GS-52	205000	270
			Gußeisen mit Kugelgraphit	GGG-60	176000	259
				GGG-40	175000	258
			Guß-Zinnbronze	G-CuSn	105000	222
			Kupfer-Zinn (Zinnbronze)	CuSn8	115000	228
			Gußeisen mit Lamellengraphit (Grauguß)	GG-25	128000	237
				GG-20	120000	232
Stahlguß	GS-60	205000	Stahlguß	GS-52	205000	268
			Gußeisen mit Kugelgraphit	GGG-60	176000	257
			Gußeisen mit Lamellengraphit (Grauguß)	GG-20	120000	231
Gußeisen mit Kugelgraphit	GGG-60	176000	Gußeisen mit Kugelgraphit	GGG-40	175000	248
			Gußeisen mit Lamellengraphit (Grauguß)	GG-20	120000	224
Gußeisen mit Lamellengraphit (Grauguß)	GG-25	128000	Gußeisen mit Lamellengraphit (Grauguß)	GG-20	120000	209
	GG-20	120000				205
Stahl	St	210000	Duroplast-Schichtstoff (Hartgewebe)		8000	80,5

5.3.2) DESCRIBE THE LEWIS APPROACH TO PREDICT THE BENDING STRESS IN TOOTH: ASSUMPTIONS, PROCEDURE AND RESULTS. DRAW A SKETCH.

BENDING STRENGTH OF A GEAR TOOTH

THE DOMINANT PHENOMENON TO BE TAKEN INTO ACCOUNT WHEN CALCULATING THE STRENGHT OF A GEAR TOOTH IS THE **FATIGUE**.

THE SO-CALLED "LEWIS SIMPLIFIED METHOD" HERE SHOWN IN DETAIL ONLY FOR SPUR GEARS, IS A CANTILEVER BENDING CALCULATION BASED ON THE FOLLOWING **ASSUMPTIONS**.

A1: THE NORMAL LOAD (F_m) IS APPLIED AT THE TIP OF THE TOOTH.

A2: ONE COUPLE ONLY OF TEETH IN CONTACT (*).

A3: ONLY THE BENDING COMPONENT (W) OF THE FORCE (F_m) IS TAKEN INTO ACCOUNT. THIS COMPONENT, IN THE CASE OF SPUR GEARS, HAS AN EFFECTIVE VALUE:

$W = F_m \cos\phi$, BEING $F_t = F_m \cos\alpha$

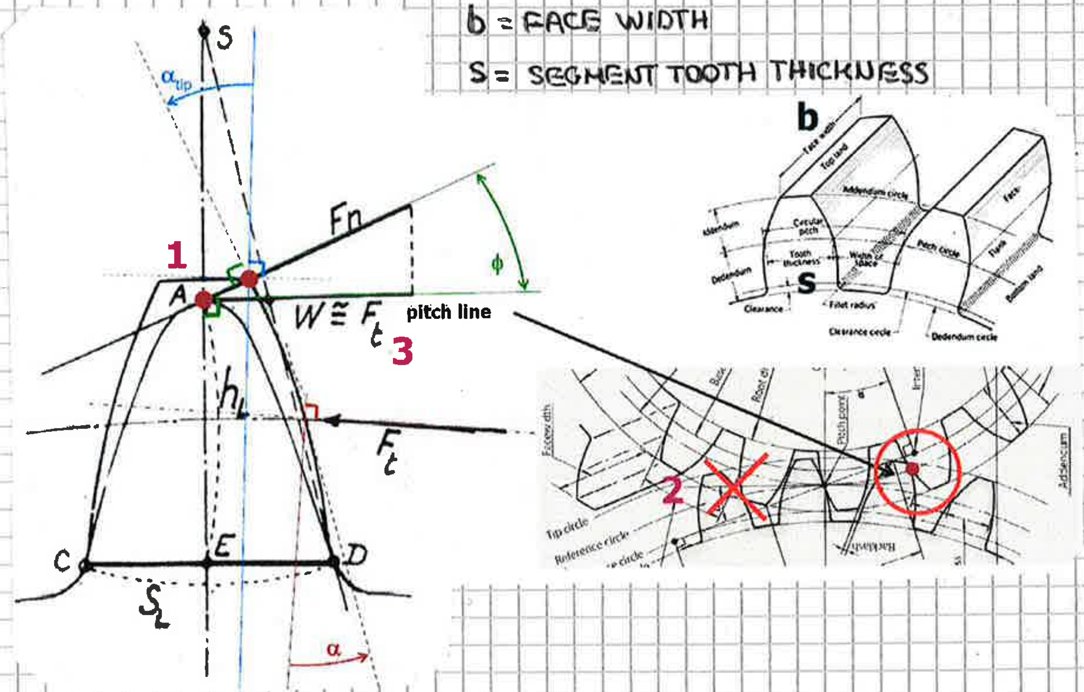
$W = F_t \frac{\cos\phi}{\cos\alpha} \cong F_t$

HOWEVER, W IS TAKEN EQUAL TO THE PITCH LINE FORCE : $W \cong F_t$ $\left(\frac{\cos\phi}{\cos\alpha} \approx 1\right)$

(*) THE APPROACH IS CONSERVATIVE, THE ACTION APPLIED TO THE TIP GIVES THE MAXIMUM BENDING MOMENT. IT IS TRUE THAT WHEN IT IS ACTUALLY APPLIED TO THE HEAD OF TEETH AT LEAST ANOTHER COUPLE OF TEETH IS ALREADY IN CONTACT, THEREFORE THE LOAD IS SHARED BETWEEN THE TWO COUPLES.

b = FACE WIDTH

S = SEGMENT TOOTH THICKNESS



AFTER TRACING THE TOOTH PROFILE, ROOT AND FILLET INCLUDED, THE PARABOLA IS TRACED THROUGH POINT A (POINT OF APPLICATION OF THE FORCE ON THE CANTILEVER AXIS) AND TANGENT TO THE ROOT PROFILE.

SUCH PARABOLA IS THE CONSTANT STRESS BEAM IN BENDING ACCORDING TO:

$$\sigma = \frac{M}{bs^2/6} = \frac{Wh}{bs^2/6} = \sigma_{ROOT} \quad \text{THAT IS CONSTANT}$$

$$h = \left(\frac{\sigma_{ROOT}\, b}{6W}\right) s^2 = \left(\frac{4\sigma_{ROOT}\, b}{6W}\right) x^2 \quad , \text{WITH } x = \frac{s}{2}$$

WHICH IS THE EQUATION OF A PARABOLA WITH ABSCISSA x AND ORDINATE $y = h$, HAVING $y = h = 0$ IN POINT A, POSITIVE "DOWN".

IT IS SEEN THAT AT ALL $h < h_L$ (WHERE THE SUFFIX L STANDS FOR "LEWIS") THE WIDTH S OF THE PARABOLA IS LOWER THAN THE WIDTH OF THE TOOTH, THIS MEANING THAT THE STRESS IN THE TOOTH IS LOWER THAN IN THE PARABOLIC SHAPED CANTILEVER.

ONLY AT POSITION: $h = h_L = AE$, WHERE $s = s_L = CD$, THEN $\sigma_{ROOT} = \sigma_{MAX}$.

THE DETERMINATION OF THE TANGENCY POINTS C, D IS GREATLY FACILITATED BY THE PROPERTY OF THE PARABOLA THAT SA = AE. IN FACT:

$$\frac{dh}{dx} = \left(\frac{4\sigma_{ROOT} \cdot b}{6W}\right) 2x$$

$$SE = \frac{s_L}{2}\left(\frac{dh}{dx}\right)_L = \frac{s_L}{2}\left(\frac{4\sigma_{ROOT} \cdot b}{6W}\right) 2x_L = 2\left(\frac{4\sigma_{ROOT}\, b}{6W}\right) x_L^2 = 2h_L = 2AE$$

AT POSITION h_L, WITH $W = F_t$:

$$\sigma_{ROOT} = \frac{6F_t h}{bs_L^2} = \frac{F_t}{bm} Y_L \quad , \text{WITH: } Y_L = \frac{6\,\frac{h_L}{m}}{\left(\frac{s_L}{m}\right)^2} \quad \text{LEWIS FACTOR}$$

THE LEWIS APPROACH ILLUSTRATED ABOVE DOES NOT TAKE INTO ACCOUNT LOAD SHARING BETWEEN MORE THAN ONE COUPLE OF TEETH, AND FOR GEARS OF SUFFICIENT ACCURACY IS TOO SEVERE, NOT REPRESENTING THE REAL WORKING CONDITIONS. (= LIMITATIONS OF LEWIS APPROACH)

5.4.3) DESCRIBE THE PROBLEM OF FATIGUE APPLIED TO THE GEAR TOOTH.

FATIGUE - SHOCK OVERLOADING - FOREIGN HARD BODY

- THE DOMINANT FACTOR IN **BENDING FAILURE** IS FATIGUE.
TOOTH BREAKAGE FROM FATIGUE FAILURES UNDER THE CYCLIC BENDING LOADS APPLIED TO TEETH IN NORMAL GEAR OPERATION, IS ONE OF THE MAIN FAILURE CRITERIA USED IN GEAR DESIGN.
THE FAILURE STARTS AS A CRACK WHICH IS USUALLY AT THE ROOT OF THE TOOTH AND PROCEEDS ACROSS THE BASE OF THE TOOTH UNTIL THE COMPLETE TOOTH BREAKS AWAY FROM THE GEAR.
WHEN FAILURE ARISES FROM THIS CAUSE THERE ARE OFTEN OTHER ADJACENT TEETH SHOWING CRAKS AT AN EARLIER STAGE OF DEVELOPMENT.
- **SHOCK OVERLOADING** CAN ARISE FROM SOME MAJOR TORSIONAL SHOCK IN THE DRIVE SYSTEM, OFTEN ASSOCIATED WITH THE FAILURE OF A MACHINE DRIVEN BY THE GEAR BOX. IN THESE CASES THE FRACTURED SURFACES SHOW A SINGLE BRITTLE BREAK WITH NO SIGN OF FATIGUE MARKING, AND USUALLY ONE OR TWO TEETH ARE AFFECTED. APART FROM THE BROKEN TEETH, THE OTHERS ARE USUALLY IN GOOD CONDITIONS WITH NO SIGNS OF CRACKS.
- SIMILAR EFFECTS CAN ALSO OCCUR IF A **FOREIGN HARD BODY** ENTERS THE GEAR MESH. THIS IS GENERALLY OBVIOUS FROM THE NATURE OF THE IMPRESSION IN THE TEETH AND THE GEAR BLANK, AND THE BODY WHICH HAS CAUSED THE PROBLEM CAN GENERALLY BE FOUND IN THE FAILURE DEBRIS IN THE BOTTOM OF THE GEAR BOX.

4 $K_{F\beta}$: LONGITUDINAL LOAD DISTRIBUTION FACTOR

IT REQUIRES TO SPECIALLY DETERMINED CASE BY CASE (TAKING INTO ACCOUNT TOOTH STRAIGHTNESS OR INTENTIONALLY ALTERED LONGITUDINAL TOOTH PROFILES, TOOTH LONGITUDINAL DEFORMATION, SHAFT OR GEARBOX OR GEAR-BODY DEFORMATION).

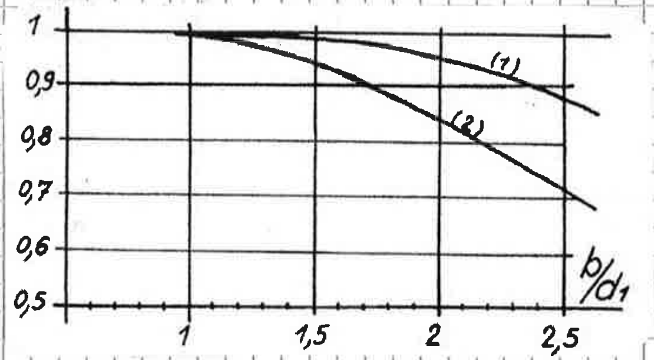
IN ORDINARY CONSTRUCTION IT IS ASSUMED:

$K_{F\beta} = 1$ FOR SPUR GEARS

$K_{F\beta} = 1.2$ FOR CONICAL GEARS

HOWEVER, NEXT FIGURE SHOWS $K_{F\beta}$ WHICH DEPENDS MAINLY ON:

- MISALIGNMENT OF TEETH AND SHAFTS, IN TORSION AND IN BENDING, DUE TO ALL POSSIBLE CAUSES.



CURVE 1 HOLDS FOR TEETH WITH LONGITUDINALLY ALTERED PROFILES (SEE APPENDIX II)

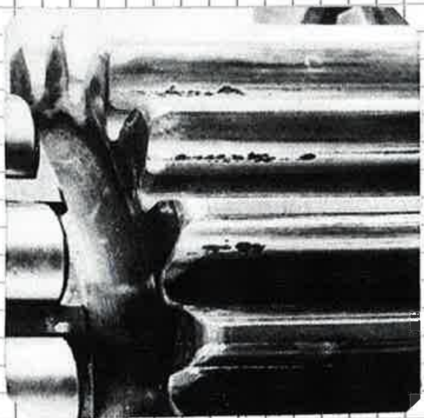
CURVE 2 HOLDS FOR NON ALTERED PROFILES.

THE DIAGRAM COMES FROM BUREAU VERITAS, AS QUOTED BY HENRIOT.

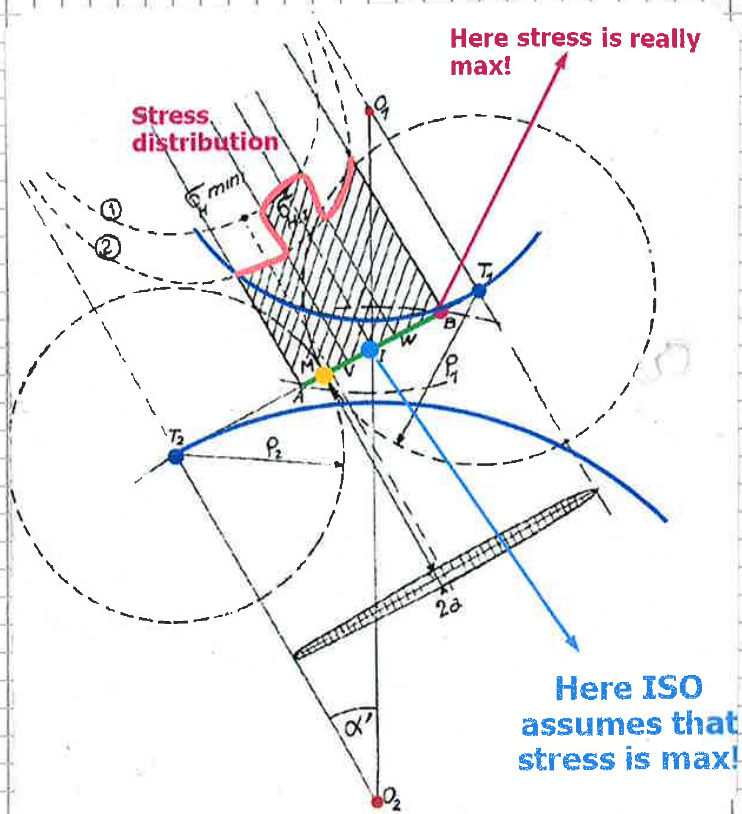
IT IS SEEN THAT FOR $\frac{b}{d_1} < 1$ WE CAN TAKE $K_{F\beta} = 1$, I.E., THIS FACTOR COUNTS ONLY FOR RELATIVELY LONG GEARS.

5.4.6) FIND THE MAXIMUM STRESS IN GEAR CONTACT AND DESCRIBE HOW IT IS CALCULATED

SURFACE STRENGTH OF A GEAR TOOTH - MAXIMUM STRESS



← PITCH LINE PITTING ON A CASE HARDENED PINION WHICH HAS OPERATED AT HIGH LOAD AND LOW SPEED.



THE DRAWING ON THE RIGHT SHOWS TWO WHEELS WITH CENTERS O_1, O_2 AND BASE CIRCLES TANGENT TO THE PRESSURE LINE AT POINTS T_1, T_2.

CONTACT SEGMENT IS $\overline{AB}$.

WHEN THE CONTACT POINT IS AT M, THE PROFILES OF THE TEETH PAIR IN CONTACT ARE APPROXIMATED TO THE SECOND ORDER BY TWO CIRCLES OF RADII ρ_1, ρ_2;

OF COURSE IT IS ALWAYS: $\rho_1 + \rho_2 = \overline{T_1T_2}$

IN CYLINDRICAL HERTZ CONTACTS THE MAX CONTACT STRESS (σ_{MAX}) IS PROPORTIONAL TO $\sqrt{F_n\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)}$

IN: $\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right) = \frac{\overline{T_1T_2}}{\rho_1(\overline{T_1T_2}-\rho_1)}$ THE VALUE OF THE DENOMINATOR IS A PARABOLIC FUNCTION OF ρ_1, ZERO IN T_1 AND T_2 AND MAXIMUM AT THE MEAN POINT OF SEGMENT $\overline{T_1T_2}$. THEREFORE, σ_{MAX} IS INFINITE AT POINTS T_1 AND T_2, IS MINIMUM IN THE MIDPOINT M OF THE SEGMENT $\overline{T_1T_2}$ AND SYMMETRIC ABOUT M.

MOREOVER, DIAGRAM SHOWS THE VALUE OF MAX CONTACT STRESS IF ONLY ONE TEETH PAIR TAKES THE FULL LOAD F_n. THE HIGHEST STRESS OCCURS AT THE POINT (IN THIS CASE POINT B) WHICH IS MORE FAR FROM MIDPOINT; MIDPOINT POSITION IS INDICATED BY THE DASHED LINE CROSSING $\overline{T_1T_2}$ VERY NEAR TO POINT M.

5.G.7) DESCRIBE HOW THE DESIGN AGAINST WEAR IS PERFORMED AND THE MEANING OF $K_I \cdot K_V \cdot K_{H\alpha} \cdot K_{H\beta}$ AND HOW IS DEFINED THE SAFETY FACTOR.

TOOTH SURFACE FATIGUE STRENGTH ACCORDING TO DIN STANDARDS

$W_{Ht} = W \cdot K_I \cdot K_V \cdot K_{H\alpha} \cdot K_{H\beta}$ SPECIFIC PITCH-LINE FORCE

W, K_I, K_V ARE THE SAME AS THE GEAR TOOTH BENDING STRENGTH CALCULATION

$K_{H\alpha}$: SPUR GEAR LOAD DISTRIBUTION FACTOR

$$K_{H\alpha} = 1 + 2 \cdot (q_L - 0.5)\left(\frac{1}{Z_\varepsilon^2} - 1\right)$$

THAT FOR SPUR GEARS TAKES INTO ACCOUNT THE NON UNIFORM DISTRIBUTION OF TEETH PAIRS IN SIMULTANEOUS CONTACT ; IT DEPENDS ON :

$$Z_\varepsilon = \sqrt{\left[\frac{4 - \varepsilon_\alpha}{3}(1 - \varepsilon_\beta) + \frac{\varepsilon_\beta}{\varepsilon_\alpha}\right]\cos\beta_b} \quad \text{CONTACT RATIO FACTOR}$$

β_b : HELIX ANGLE ON THE BASE CYLINDER , CALCULATED AS: $\sin\beta_b = \sin\beta \cdot \cos\alpha_n$

q_L : IS THE SAME AS THE GEAR TOOTH BENDING STRENGTH CALCULATION.

$K_{H\beta}$: LONGITUDINAL LOAD DISTRIBUTION FACTOR ($K_{H\beta} = 1$ FOR SPUR GEARS)

THE HERTZ STRESS (σ_H) IS CALCULATED AS:

$$\sigma_H = \sqrt{\frac{W_{Ht}}{d_1} \frac{u+1}{u}} \cdot Z_H \cdot Z_M \cdot Z_\varepsilon$$

d_1 : PINION PITCH DIAMETER

$u = R_2/R_1$: GEAR RATIO

$$Z_H = \frac{1}{\cos\alpha_t}\sqrt{\frac{\cos\beta_b}{\tan\alpha_{wt}}} \quad : \text{FLANK FORM FACTOR}$$

α_t : STANDARD PRESSURE ANGLE

α_{wt} : WORKING PRESSURE ANGLE

Z_M : MATERIAL FACTOR THAT CAN BE OBTAINED BY TABLE A21.13

THE SAFETY FACTOR (S_H) IS CALCULATED AS:

$$S_H = \frac{\sigma_{HD}}{\sigma_H}$$

σ_{HD} : ALLOWABLE FATIGUE STRESS OF THE TOOTH MATERIAL FROM TABLE A21.12

$S_H \geq 1.25$ FOR INFINITE LIFE CALCULATION AND $Z_1 > 20$

$S_H \geq 1.6$ FOR INFINITE LIFE CALCULATION AND $Z_1 \leq 20$

5.4.9) DESCRIBE THE SCUFFING PHENOMENON

DESIGN TO SCUFFING (ADHESIVE WEAR) AND ITS BALANCING

(SCUFFING = SCORING = ADHESIVE WEAR = "GRIPPAGGIO")

"SCUFFING", ALSO TERMED "SCORING" (INCORRECT ACCORDING TO GEAR STANDARDS), IS A SEVERE TYPE OF ADHESIVE WEAR WHICH INSTANTLY DAMAGES TOOTH SURFACES THAT ARE IN RELATIVE MOTION. IN FACT, A SINGLE OVERLOAD CAN LEAD TO CATASTROPHIC FAILURE.

SCUFFING WELDS TOGETHER UNPROTECTED SURFACES IN METAL-TO-METAL CONTACT. METAL PARTICLES DETACH AND TRANSFER FROM ONE OR BOTH MESHING TEETH. DURING SUCCESSIVE ROTATIONS, THESE PARTICLES CAN SCRATCH TEETH FLANKS IN THE SLIDING DIRECTION. THIS TYPE OF DAMAGE GENERALLY HAPPENS IN AREAS OF HIGH CONTACT PRESSURE AND SLIDING VELOCITY (V_g), FAR FROM THE PITCH SURFACE. CONDITIONS, THERE, ARE LESS FAVOURABLE TO FORM A PROTECTIVE LUBRICANT LAYER THAT WOULD PREVENT DIRECT METAL-TO-METAL CONTACT. THIS PROTECTIVE LAYER COULD BE A THICK OIL FILM (RELATIVE TO SURFACE ROUGHNESS) OR AN ABSORBED OR CHEMICALLY DEPOSITED LAYER ESTABLISHED BY LUBRICANT ADDITIVES.

THEREFORE, GEAR SCUFFING IS CHARACTERIZED BY MATERIAL TRANSFER BETWEEN SLIDING TOOTH SURFACES. GENERALLY THIS CONDITION OCCURS WHEN INADEGUATE LUBRICATION FILM THICKNESS PERMITS METAL-TO-METAL CONTACT BETWEEN GEAR TEETH. WITHOUT LUBRIFICATION, DIRECT METAL CONTACT REMOVE THE PROTECTIVE OXIDE LAYER ON THE GEAR METAL, AND THE EXCESSIVE HEAT GENERATED BY FRICTION WELDS THE SURFACES AT THE CONTACT POINTS.
AS THE GEARS SEPARATE, METAL IS TORN AND TRANSFERRED BETWEEN THE TEETH. SCUFFING IS MOST LIKELY TO OCCUR IN NEW GEAR SETS DURING THE RUNNING-IN PERIOD BECAUSE THE GEAR TEETH HAVE NOT SUFFICIENT OPERATING TIME TO DEVELOP SMOOTH SURFACES.

SCUFFING IS CONTROLLED BY APPROPRIATE LUBRICATION CONDITIONS.
AN OLD CRITERION, CREDITED TO ALMEN, IS KNOWN AS THE : (BALANCING)

PV (PRESSURE-VELOCITY) CRITERION

- THE PRODUCT $\sigma_H V_g$, NAMED ALMEN PARAMETER, I.E. THE MAX HERTZ STRESS TIMES THE SLIDING SPEED, IS CALCULATED AND COMPARED WITH A LIMIT VALUE WHICH IS (EXPERIMENTALLY) CHARACTERISTIC OF THE LUBRICANT EMPLOYED.
- ALMEN PARAMETER ($\sigma_H V_g$) IS PROPORTIONAL TO A SPECIFIC POWER: INFACT, NAMING THE HERTZ CONTACT AREA A_H, ASSUMING THE FRICTION COEFF. μ, THE LOST POWER IS: $\sigma_H A_H \mu V_g$

5.9.10 DEFINE THE CROWNING OF PROFILES AND ITS USE.

APPENDIX II : CROWNING OF PROFILES

IN ORDER TO AVOID OVERLOADS AT THE ENDS OF TEETH DUE TO A GREAT VARIETY OF UNFAVOURABLE EVENTS:

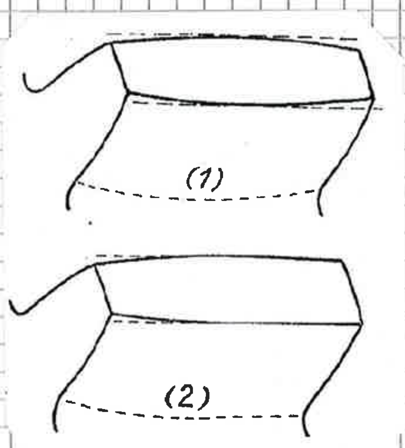
- TOLERANCES IN MACHINING GEARBOXES;
- GEARBOX DEFORMATIONS UNDER LOADS;
- ERRORS OF GEARS ALIGNMENT;
- TORSION AND BENDING DEFORMATIONS OF PINION;

IT IS NOT CONVENIENT TO MACHINE A "PERFECT" TOOTH.

RATHER, IT IS CONVENIENT TO MODIFY THE TOOTH FORM IN THE LONGITUDINAL DIRECTION, AS INDICATED QUALITATIVELY IN NEXT FIGURE, CASE (1). THIS IS DONE SYMMETRICALLY BECAUSE IT IS DIFFICULT TO FORESEE ON WHICH SIDE THE GREATER DEFLECTIONS WILL OCCUR.
IN THE CASE OF CAR-SHIFT GEARBOXES A CROWNING MAX OF 10 TO 15 [μm] IS USUAL.

ON THE CONTRARY, IF WE CAN FORESEE THAT LOAD WILL CONCENTRATE AT A SPECIFIC END, CROWNING WILL TAKE THE FORM OF CASE (2), I.E., A LIGHT TAPER OF THE TOOTH.
THIS IS THE TYPICAL CASE OF CANTILEVERED PINIONS OF ELECTRIC MOTORS, WHERE THE LOAD CONCENTRATES ON THE TOOTH SIDE WHICH IS NEARER TO THE MOTOR.
HENRIOT REPORTS A TAPER OF 10 MINUTES OF DEGREE (1 MINUTE = 1/60 OF A DEGREE) ON THE HALF OF THE PINION WIDTH.

THE GENERAL IDEA IS THAT THESE CORRECTIONS WILL BRING THE TOOTH IN UNIFORM CONTACT WHEN THE LOAD IS APPLIED; ITERATIVE COMPUTING IS NEEDED TO DETERMINE THE DEFLECTIONS UNDER LOAD ON WHICH THE CONTACT DEPENDS.



PRESTRESSING

WE SHALL DEAL HERE ONLY WITH "PRESTRESSED BOLT CONNECTIONS", I.E., THESE CONNECTIONS ARE LOADED WITH A HIGH AXIAL FORCE (MOUNTING PRESTRESSING) DURING ASSEMBLY.
THIS PRESTRESSING PROVIDES THE NECESSARY FORCE BOND OF CONTACT SURFACES OF THE CONNECTED MATERIALS AND IS NECESSARY TO INSURE A CORRECT LOAD SHARING BETWEEN BOLT AND PART DURING THE APPLICATION OF VARIABLE EXTERNAL LOADS.
PRE-STRESSED BOLT CONNECTIONS FORM THE MAJORITY OF BOLT AND THREADED CONNECTIONS USED IN PRACTICE.

TECHNICAL STANDARDS

OUR MAIN REFERENCE WILL BE TO A SPECIFIC STANDARD:
VDI RICHTLINIEN - VDI 2230 BLATT 1 / 2 / 3 (PART 1, PART 2, PART 3) - 2003:
PART 1: DEALS WITH THE CALCULATION OF THE SINGLE-BOLTED JOINT.
IT IS BASED ON THE ELASTIC BEHAVIOUR OF THE JOINT IN THE IMMEDIATE SOURROUNDING OF THE BOLT AXIS. DURING ASSEMBLY AND IN THE SERVICE CASE, THIS REGION HAS A CONSIDERABLE EFFECT ON THE DEFORMATION AND THUS ON THE LOADING OF THE BOLT. (RULES).
PART 2: DEALS WITH JOINTS COUPLED BY MANY BOLTS: MULTI-BOLTED JOINTS; IN THIS CASE THE PROBLEM IS THE IDENTIFICATION OF THE MOST LOADED BOLT AND RELATED LOADS, EITHER THROUGH BEAR OR PLATE THEORY, OR THROUGH FINITE ELEMENTS; THIS BEING DONE, THE CALCULATION THEN PROCEEDS MUCH IN THE WAY OF PART 1 FOR EACH BOLT OR THE MOST STRESSED ONE. (ANALYSIS).
PART 3: PROVIDES DETAILED RECOMMENDATIONS ON BOLT ASSEMBLY PROBLEMS AND TECHNIQUES, AND RELATED REFINEMENTS IN CALCULATIONS. (TECHNOLOGY).

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6.1.3) DESCRIBE SOME PROCESS TO MANUFACTURE BOLTS AND DESCRIBE ITS MAIN CHARACTERISTICS.

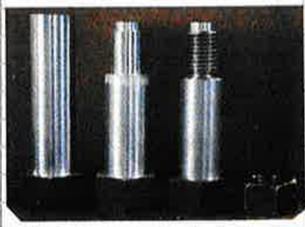
MANUFACTURING PROCESS (TO READ)

THE MANUFACTURING PROCESSES ARE HEREIN SYNTHESIZED JUST TO:
- → KNOW THEM
- → UNDERSTAND RELATION BETWEEN STRENGTH OF MATERIAL AND PROCESS
- → IDENTIFY THE MAIN ISSUES OF DESIGN
- → DETECT TYPICAL DEFECTS AND DAMAGES.

REMARKS:

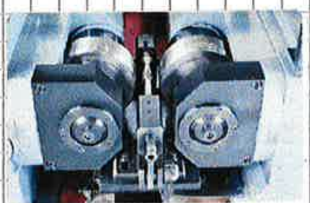
1) THE **SIZE OF BOLT** SUGGESTS TO APPLY DIFFERENT MATERIAL PROCESSING.
2) THE **FATIGUE STRENGTH** IS DEEPLY AFFECTED BY THE PROCESS (COLD ROLLING IS BENEFICIAL, HOT ROLLING DECREASES THE STRENGTH).
3) THE PROCESS MIGHT INTRODUCE SOME **VERY SMALL DEFECTS**, DEPENDING ON THE ACTION AND ON THE MOTION OF APPLIED TOOLS.

MACHINING

IT IS ONLY USED **ON UNIQUE DESIGNS** OR WITH SCREWS EITHER **TOO SMALL** OR **TOO LARGE** TO BE MADE IN ANY OTHER WAY.
THE MACHINING PROCESS IS **EXACT**, BUT **TOO TIME CONSUMING**, **WASTEFUL** AND **EXPENSIVE**.
DEFECTS (MICROCRACKS AND GRAIN BOUNDARIES) WHICH FORM AT THE SURFACE DURING THIS PROCESS ARE PREFERRED SITES FOR **FATIGUE CRACK INITIATION**. THUS, **MACHINED THREADS** OFTEN EXHIBIT ONLY **LIMITED FATIGUE PROPERTIES**.
MACHINING IS ECONOMICALLY ADVANTAGEOUS FOR SMALL QUANTITIES AND NON-STANDARD GEOMETRIES.

ROLLING

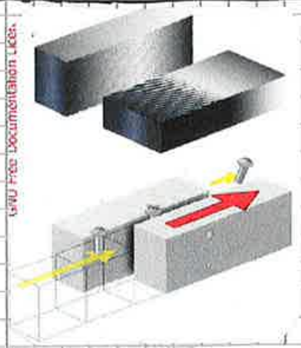


FATIGUE PROPERTIES ARE MARKEDLY ENHANCED **BY ROLLING THE THREADS AFTER* HEAT TREATMENTS**.
DURING THE PROCESS, **GRAINS ARE BEING ALIGNED IN THE ROLLING DIRECTION (MECHANICAL FIBERING)** AND **BENEFICIAL COMPRESSIVE RESIDUAL STRESSES** ARE INTRODUCED INTO THE MATERIAL.

*(⊗ **ERROR IN SLIDE 11 (PP 6/27)**: "FATIGUE STRENGTH IS MARKEDLY ENHANCED BY ROLLING THREADS **BEFORE** HT" **WRONG!** CHECKED)

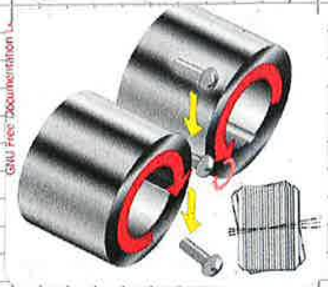
3.1 RECIPROCATING DIE ROLLING

TWO FLAT DIES ARE USED TO ROLL THE SCREW THREAD; ONE DIE IS **STATIONARY**, WHILE THE OTHER MOVES IN A RECIPROCATING MANNER, AND THE SCREW BLANK IS ROLLED BETWEEN THE TWO.



3.2 CYLINDRICAL DIE ROLLING

THE SCREW BLANK IS ROLLED BETWEEN TWO OR THREE ROUND DIES, IN ORDER TO CREATE THE FINISHED THREAD.



3.3 PLANETARY ROTARY DIE ROLLING

IT USES ONE STATIONARY SEGMENT/CIRCULAR DIE (1) AND ONE ROTARY CIRCULAR DIE (2)

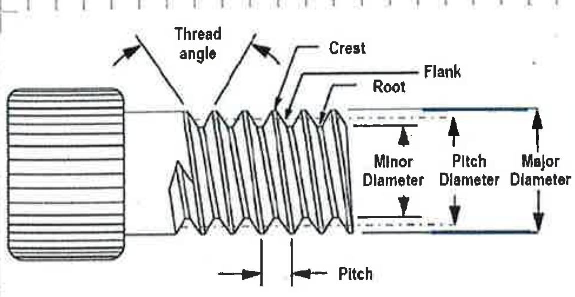
ALL THREE ROLLING METHODS CREATE **HIGHER STRENGTH SCREWS** THAN THE MACHINE-CUT VARIETY. THIS IS BECAUSE THE **THREAD IS NOT LITERALLY CUT INTO THE BLANK** DURING THE THREAD-ROLLING PROCESS, RATHER IT IS **IMPRESSED INTO THE BLANK**. THUS, **NO METAL MATERIAL IS LOST**, AND **WEAKNESS IN THE MATERIAL IS AVOIDED** (RESIDUAL STRESSES AND METAL STRUCTURE ARE MORE FAVOURABLE, IN PARTICULAR TO FATIGUE).

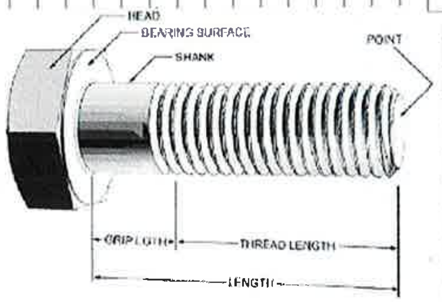
ON AVERAGE, THE **COLD HARDENING MACHINE** PRODUCES **100 TO 550** SCREW BLANKS **PER MINUTE**.

THE MORE PRODUCTIVE OF THE THREAD-ROLLING TECHNIQUES IS BY FAR THE **PLANETARY ROTARY DIE**, WHICH CREATES SCREWS AT A SPEED UP TO **2000 PARTS PER MINUTE**, HOWEVER IN THE CASE OF **SMALLER BOLTS UP TO 1/8 [inch], 3 [mm]**; DOWN TO **100÷400 PARTS PER MINUTE** FOR **BOLTS UP TO 1/2 [inch]**.

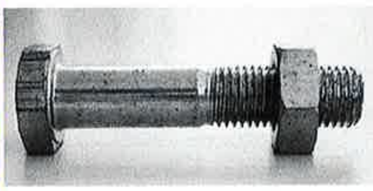
6.1.G) DRAW A SKETCH OF A SCREW AND INDICATE ALL OF ITS PARAMETERS

DESCRIPTION OF THE GEOMETRY

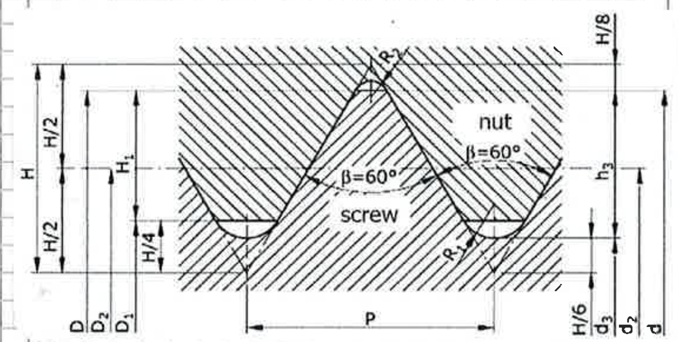




DESIGNATION EXAMPLES OF VARIOUS THREADS			
ORIGIN	COARSE THREAD	FINE THREAD	EXTRA FINE THREAD
METRIC	M 8	M 8 X 1.0	M 8 X 0.75
U.S.A.	3/8 - 16 UNC	3/8 - 24 UNF	3/8 - 32 UNEF
BRITISH	1/2 BSW	1/2 BSF	NONE
ORIGIN	MINIATURE THREAD	PIPE TAPERED THREAD	PIPE PARALLEL THREAD
METRIC	M 1.0	M 12 X 1.5T (TAPER)	M12 X 1.5
U.S.A.	1.0 UNM	1/8 - 27 NPT	1/8 - 27 NPS
BRITISH	10 BA	1/2 BSPT	1/2 BSPF



METRIC THREAD DIMENSIONS:



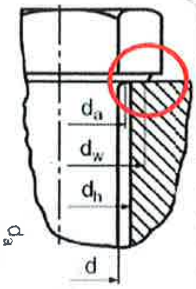
d : NOMINAL (BOLT CREST) / MAJOR DIAMETER

d_2 : PITCH (MEAN) DIAMETER (OF THE IMAGINARY CYLINDER PASSING THROUGH THE THREAD IN A MANNER AS TO EQUALIZE THE WIDTH OF THREAD RIDGE AND GROOVE).

d_3 : MINOR DIAMETER

SURFACE PRESSURE [N/mm²] UNDER THE HEAD OF A HEXAGON SCREW – COARSE THREADS

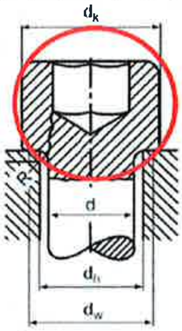
Nominal thread Ø d	Width across flats s_{max}	Ø of the bearing surface $d_{w\,min.}$	Through hole (ISO 273) d_h	Bearing surface A_p	Stressed cross-section A_s	Surface pressure under the head [1] $\left[\frac{N}{mm^2}\right]$		
	mm	mm	mm	mm²	mm²	8.8	10.9	12.9
M 4	7	5,9	4,5	11,4	8,78	385	568	665
M 5	8	6,9	5,5	13,6	14,2	528	777	909
M 6	10	8,9	6,6	28	20,1	364	532	625
M 8	13	11,6	9	42,1	36,6	442	649	761
M10	16	14,63	11	73,1	58	405	594	695
M10	17	15,6	11	96,1	58	308	452	529
M12	18	16,63	13,5	74,1	84,3	580	853	999
M12	19	17,4	13,5	94,6	84,3	454	668	782
M14	21	19,64	15,5	114,3	115	517	759	888
M14	22	20,5	15,5	141,4	115	418	613	718
M16	24	22,5	17,5	157,1	157	515	756	885
M18	27	25,3	20	188,6	192	541	769	901
M20	30	28,2	22	244,4	245	532	761	888
M22	34	31,71	24	337,3	303	480	685	803
M22	32	30	24	254,5	303	637	908	1065
M24	36	33,6	26	355,8	353	528	750	880
M27	41	38	30	427,3	459	576	821	960
M30	46	42,7	33	576,7	561	520	740	865



← HEXAGON SCREW

SURFACE PRESSURE [N/mm²] UNDER THE HEAD OF A SOCKED HEAD CAP SCREW

Nominal thread Ø d	Ø of head d_k	Ø of the bearing surface $d_{w\,min.}$	Through hole (ISO 273) d_h	Bearing surface A_p	Stressed cross-section A_s	Surface pressure under the head [1] $\left[\frac{N}{mm^2}\right]$		
	mm	mm	mm	mm²	mm²	8.8	10.9	12.9
M 4	7	6,53	4,5	17,6	8,79	250	370	432
M 5	8,5	8,03	5,5	26,9	14,2	268	394	461
M 6	10	9,38	6,6	34,9	20,1	292	427	502
M 8	13	12,33	9	55,8	36,6	333	489	574
M10	16	15,33	11	89,5	58	331	485	567
M12	18	17,23	13,5	90	84,3	478	702	822
M14	21	20,17	15,5	130,8	115	452	663	776
M16	24	23,17	17,5	181,1	157	447	656	767
M18	27	25,87	20	211,5	192	482	686	804
M20	30	28,87	22	274,5	245	474	678	791
M22	33	31,81	24	342,3	303	473	675	792
M24	36	34,81	26	420,8	353	447	635	744
M27	40	38,61	30	464	459	530	756	884
M30	45	43,61	33	638,4	561	470	869	782



← SOCKED HEAD CAP SCREW

6.17) WHICH PARAMETERS AFFECT THE SELECTION OF BOLT MATERIAL?

TYPICAL MATERIALS

PARAMETERS AFFECTING THE SELECTION:

- WORKING ENVIRONMENT
- APPLICATION
- ADDITIONAL REQUIREMENTS (LIKE THOSE RELATED TO THE ELECTRIC BEHAVIOUR)

CARBON STEELS AND CORROSION-RESISTANT MATERIALS - 1

CARBON STEELS, COATED

LOW CARBON, MEDIUM CARBON AND LOW-ALLOY STEELS CAN BE MADE MORE RESISTANT TO ATMOSPHERIC CORROSION BY COATING OR BY PLATING THEM.

STAINLESS STEELS

DESCRIPTION BY A COMBINATION OF LETTERS AND FIGURES, STAMPED ON BOLT HEAD OR NUT: A2-70 → ABBREVIATION OF PROPERTY CLASS: 50, 70, 80 (1/10 OF TENSILE STRENGTH) [MPa]

A → COMPOSITION GROUP (A, C, F)

2 → ABBREVIATION OF CHEMICAL COMPOSITION

A = AUSTENITIC STAINLESS STEELS

- MOST COMMON OF SS AND MORE CORROSION-RESISTANT THAN THE OTHER SS.
- NON MAGNETIC
- CANNOT BE HEAT-TREATED BUT CAN BE COLD-WORKED.
- GOOD HIGH AND LOW TEMPERATURE PROPERTIES (CHEMICAL COMP.: 1, 2, 3, 4, 5)

C = MARTENSITIC STAINLESS STEELS (UTS 70-180 [KSI])

- MAGNETIC
- HEAT TREATABLE
- CAN EXPERIENCE STRESS CORROSION IF NOT PROPERLY TREATED.

F = FERRITIC STAINLESS STEELS

- MAGNETIC
- CANNOT BE HEAT-TREATED OR COLD WORKED

CARBON STEELS AND CORROSION-RESISTANT MATERIALS - 3

NIMONIC

- A FAMILY OF Ni-Cr ALLOYS
- HIGH FATIGUE STRENGTH
- HIGH CORROSION RESISTANCE
- HIGH RESISTANCE TO HIGH MECHANICAL STRESSES AT HIGH T, UP TO 1000 [°C]
- APPLICATIONS: ROTATING COMPONENTS AT HIGH T, SPRINGS, FASTENERS, COMBUSTION CHAMBER COMPONENTS, BLADES...

TITANIUM GRADES 1, 2, 3, 4

- HIGH STRENGTH IN RELATION TO LOW DENSITY.
- EXCELLENT RESISTANCE TO CORROSION UNDER CHLORIDE.
- APPLICATIONS: COMPONENTS WITH WEIGHT SAVING AND HIGH STRENGTH, SUBJECTED TO STRONG OXIDIZING CONDITIONS SPECIALLY CHLORIDES, CHEMICAL INDUSTRY, SEAWATER DESALINATION, POWER STATION TECHNOLOGY, MEDICAL TECHNOLOGY...

TITANIUM GRADE 5

- HIGH SPECIFIC STRENGTH
- APPLICATIONS: COMPONENTS FOR AEROSPACE INDUSTRY, CHEMICAL PROCESSING TECHNOLOGY, ROTATING COMPONENTS, FASTENERS, ...

TITANIUM GRADE 7, 11

- Ti-Pd ALLOY
- HIGH RESISTANCE TO CORROSION IN MOIST MEDIA CONTAINING CHLORINE.
- GRADE 11 HAS INCREASED DEFORMATION PROPERTIES
- APPLICATIONS: CHEMICAL AND PETROCHEMICAL PLANTS, ...

④ DEFINE TENSILE AND YIELD STRENGTHS

TENSILE STRENGTH IS THE AXIAL STRESS IN THE BOLT OR SCREW AT FRACTURE, I.E. WHEN REACHING THE MINIMUM VALUE OF THE "ULTIMATE STRESS" OR "TENSILE STRESS" R_m

THE BREAKING LOAD OF A FASTENER IS DETERMINED BY: $F = A_s R_m$ (A_s = STRESS AREA) $[mm^2] \rightarrow R_m [MPa]$

YIELD STRENGTH IS THE AXIAL STRESS IN THE BOLT OR SCREW AT YIELD, I.E. WHEN REACHING THE MINIMUM VALUE OF THE "YIELD STRESS" R_{p02}.

ONLY AXIAL LOAD IS APPLIED, I.E. WITH NO TORSIONAL STRESS DUE TO A TORQUE.

THE YIELD LOAD OF A FASTENER IS DETERMINED BY: $F_{0.2} = A_s R_{p02}$

YIELD STRENGTH SHOULD NOT BE CONFUSED WITH THE YIELD STRENGTH IN OPERATION (EITHER IN ASSEMBLY TIGHTENING OR IN OPERATION) WHERE THE AXIAL STRESS DUE TO AXIAL LOAD PRODUCES A YIELD STRESS IN THE BOLT IN COMBINATION WITH SHEAR STRESSES DUE TO TORQUE.

Property Class for Nuts

Property class of nut	Mating bolts: Property class	Mating bolts: Thread range	Nuts Style 1: Thread ranges	Nuts Style 2: Thread ranges
4	3.6, 4.6, 4.8	> M16	> M16	—
5	3.6, 4.6, 4.8	≤ M16	≤ M39	—
	5.6, 5.8	≤ M39		
6	6.8	≤ M39	≤ M39	—
8	8.8	≤ M39	≤ M39	> M16 ≤ M39
9	9.8	≤ M16	—	≤ M16
10	10.9	≤ M39	≤ M39	—
12	12.9	≤ M39	≤ M16	≤ M39

ISO 898-2:1992

This Table shows nut proof stress values for property classes 8 to 12.

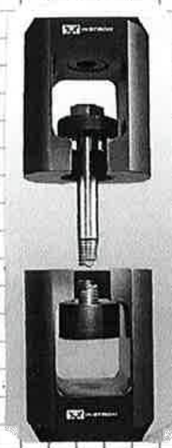
Thread		Property class 8										10				
		Stress under proof load Sp	Vickers hardness HV		Nut		Stress under proof load Sp	Vickers hardness HV		Nut		Stress under proof load Sp	Vickers hardness HV		Nut	
greater than	less than or equal to	N/mm²	min	max	state	style	N/mm²	min.	max.	state	style	N/mm²	min.	max	state	style
—	M4	800	180	302	NQT	1	—	—	—	—	—	1 040	272	353	QT	1
M4	M7	855	200									1 040				
M7	M10	870										1 040				
M10	M16	880										1 050				
M16	M39	920	233	353	QT		890	180	302	NQT	2	1 060				

ISO 898-2:2019

This (more recent) Table gives only the prof loads.

Thread		Property class 12									
		Stress under proof load Sp	Vickers hardness HV		Nut		Stress under proof load Sp	Vickers hardness HV		Nut	
greater than	less than or equal to	N/mm²	min.	max.	state	style	N/mm²	min.	max.	state	style
—	M4	1 140	295	353	QT	1	1 150	272	353	QT	2
M4	M7	1 140					1 150				
M7	M10	1 140					1 160				
M10	M16	1 170					1 190				
M16	M39	—	—	—	—	—	1 200				

TENSILE TESTING OF A SCREW



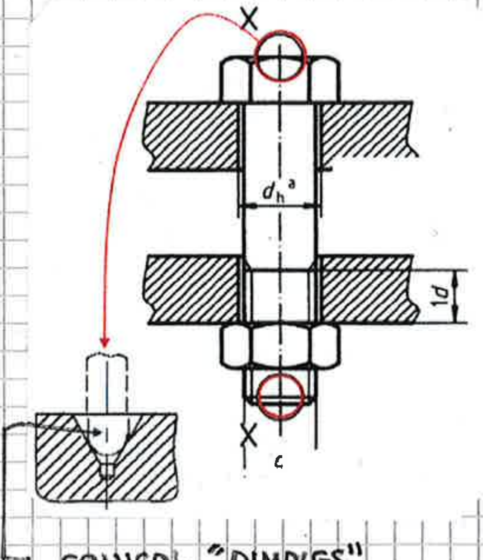
IN ORDER TO CARRY OUT A VALID TEST, THE FRACTURE MUST OCCUR IN THE SHANK OR THE FREE THREADED LENGTH OF THE BOLT AND NOT AT THE JUNCTION OF THE HEAD AND THE SHANK.
WHERE BOLTS AND NUTS CANNOT BE TESTED FULL SIZE, TESTS ARE CONDUCTED USING TEST SPECIMENS MACHINED FROM THEM.

PROOF LOAD (F_p) / STRESS (S_p) TESTING OF A SCREW

IT CONSISTS OF TWO STEPS:
1) APPLYING A TENSILE LOAD
2) MEASURING THE EVENTUAL PERMANENT ELONGATION DUE TO THAT TENSILE LOAD.

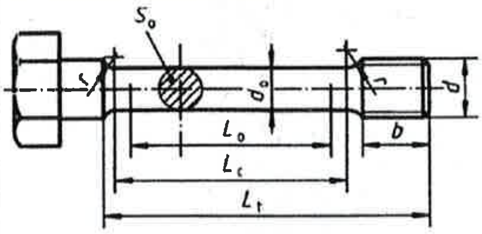
THE SCREW LENGTH MUST BE MEASURED BEFORE AND AFTER THE TEST BY MEANS OF AN INSTRUMENT ON TWO SPHERICAL CONTACTS SET IN CONICAL "DIMPLES" AT THE BOLT ENDS.

PROOF LOAD IS THE MAXIMUM AXIAL FORCE FOR WHICH THE SCREW LENGTH IS THE SAME BEFORE AND AFTER THE TEST WITH A TOLERANCE OF ± 12.5 [μm] TO ALLOW FOR MEASUREMENT ERRORS.



CONICAL "DIMPLES"

Yield stress testing of screw materials



Key

d = nominal diameter
d_o = diameter of test piece (d_o < minor diameter of thread)
b = threaded length ($b \geq d$)
$L_o = 5\,d_o$ or ($5{,}65\sqrt{S_o}$): original gauge length for determination of elongation
$L_o \geq 3\,d_o$: original gauge length for determination of reduction of area
L_c = length of straight portion ($L_o + d_o$)
L_t = total length of test piece ($L_c + 2r + b$)
L_u = final gauge length (see ISO 6892:1998)
S_o = cross-sectional area before tensile test
S_u = cross-sectional area after fracture
r = fillet radius ($r \geq 4$ mm)

NUT PROOF LOAD - TEST METHOD

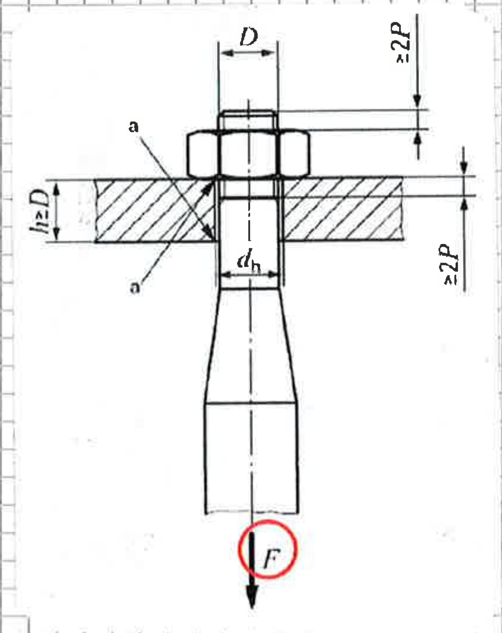
THE NUT IS ASSEMBLED ON A HARDENED AND THREADED TEST MANDREL AS SHOWN IN THE FIGURE.
THE HARDNESS OF THE TEST MANDREL MUST BE 45 [HRC] MINIMUM.

THE SPECIFIED PROOF LOAD IS APPLIED AGAINST THE NUT IN AN AXIAL DIRECTION, AND SHALL BE HELD FOR 15 [S].

THE NUT MUST RESIST THE LOAD WITHOUT FAILURE BY STRIPPING OR RUPTURE, AND MUST BE REMOVABLE BY THE FINGERS AFTER THE LOAD IS RELEASED.

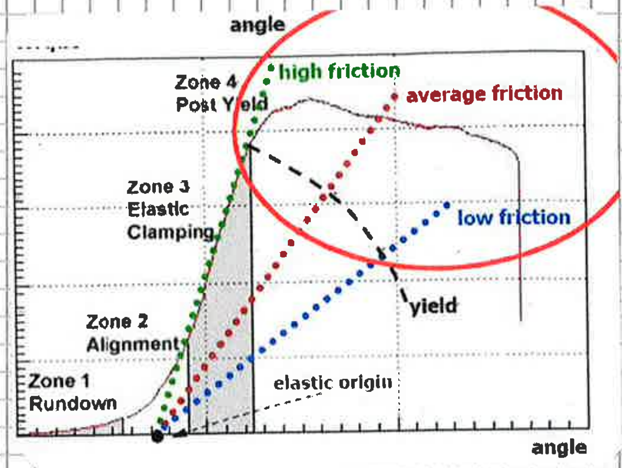
IF THE THREAD OF THE MANDREL IS DAMAGED DURING THE TEST, THE TEST IS DISCHARGED.

IT MAY BE NECESSARY TO USE A MANUAL WRENCH TO START THE NUT IN MOTION. SUCH WRENCHING IS PERMISSIBLE PROVIDED THAT IT IS RESTRICTED TO ONE HALF TURN AND THAT THE NUT IS THEN REMOVABLE BY FINGERS.



ZONE 4 (POST YIELD)

YIELDING IN THE JOINT, DUE TO YIELD OF THE THREADS IN THE NUT FOR ISTANCE.

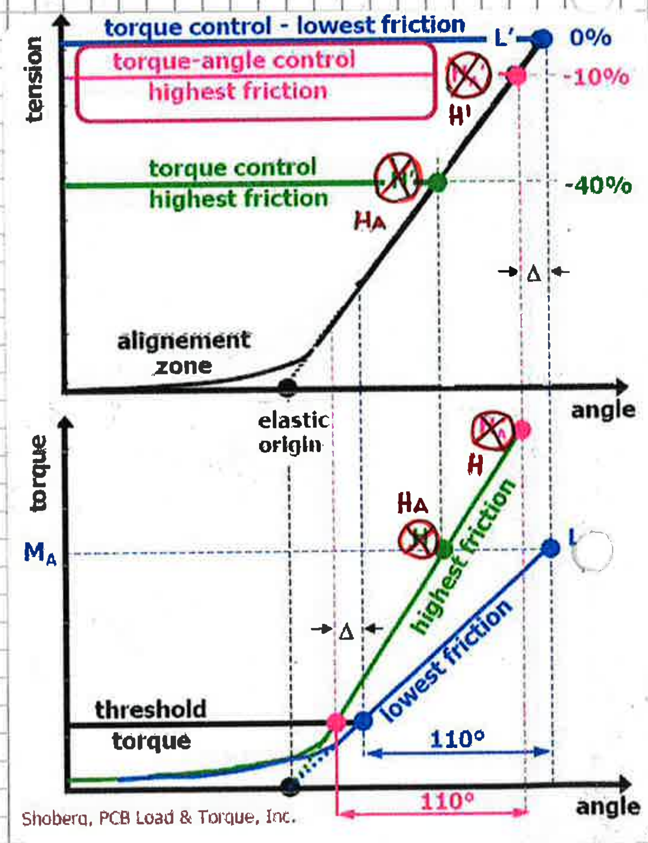


A DECREASE IN FRICTION, IN EITHER THE THREAD OR UNDERHEAD REGIONS, RESULTS IN A PROPORTIONAL DECREASE IN THE SLOPE OF THE TORQUE-ANGLE SIGNATURE.

AS FRICTION INCREASES, THE BOLT CLAMP FORCE AT THE YIELD POINT IS REDUCED, WHILE THE TORQUE REQUIRED TO REACH THE YIELD POINT INCREASES.

• ANGLE CONTROL

THE TENSION-ANGLE CURVE IS UNIQUE, WHILE THE TORQUE-ANGLE CURVE DEPENDS ON FRICTION COEFFICIENT (FRICT. ↑ TORQUE ↑)



IN TIGHTENING WITH TORQUE CONTROL ONLY, THE RESULTING BOLT TENSION CAN VARY BETWEEN A MAXIMUM (L' LOWEST FRICTION) AND A MINIMUM (H', HIGHEST FRICTION), WHICH CAN BE MORE OR LESS 40% LOWER THAN THE MAXIMUM.

~~im~~

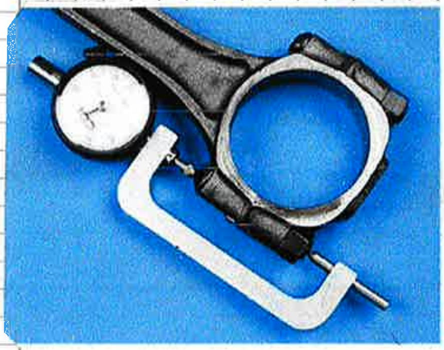
THE ALTERNATIVE TORQUE-ANGLE-TENSION CONTROL:

- TORQUE IS APPLIED UNTIL A SPECIFIED "THRESHOLD TORQUE" LEVEL IS ATTAINED
- AN ADDITIONAL ANGLE-OF-TURN IS APPLIED TO COMPLETE ASSEMBLY TIGHTENING.

WHEN THE THRESHOLD TORQUE IS REACHED, AND THE FRICTION IS THE LOWEST, AN ADDITIONAL ANGLE THAT HERE IS CHOOSEN, BY WAY OF EXAMPLE, AT 110°, TAKES TO POINT L WHILE IF FRICTION IS HIGHEST TAKES TO POINT H. THIS (H) CORRESPONDS TO POINT H' IN THE TENSION-ANGLE DIAGRAM, WHICH IS A TENSION ONLY ABOUT 10% LOWER THAN THE MAXIMUM, THEN THE SCATTER ACHIEVED HERE IS MUCH LESS THAN THE SCATTER OBTAINED WITH TORQUE-ONLY CONTROL.

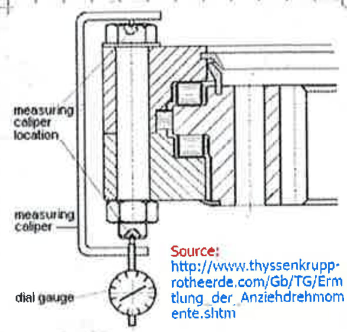
BOLT-ELONGATION (STRETCH) CONTROLLED TIGHTENING

IN ORDER TO MEASURE **BOLT STRETCH**, FIRST THE **TOTAL ROD BOLT LENGTH** IS **MEASURED** (FROM THE HEAD SURFACE TO THE TIP OF THE SHANK) IN THE RELAXED STATE.
THEN BOLT LENGTH IS MONITORED DURING TIGHTENING UNTIL THE SPECIFIED AMOUNT OF BOLT STRETCH HAS BEEN ACHIEVED.
THIS CAN BE DONE EITHER VIA ULTRASOUND PROBES OR VIA MECHANICAL GAUGE.
THE ASSEMBLY FORCE IS INDIPENDENT OF FRICTION.



An example of a simple rod bolt stretch gauge.
High-performance rod bolts feature a dimple at each end.
This provides a convenient location for a stretch gauge.

Photos Mike Mavrigian - http://www.precisionenginetech.com/tech-explained/2009/06/03/measuring-connecting-rod-bolt-stretch-part-1/



Source: http://www.thyssenkrupp-rotheerde.com/Gb/TG/Ermtlung_der_Anziehdrehmomente.shtm

BOLT ELONGATION MEASUREMENT CAN BE USED TO CALIBRATE THE TIGHTENING TORQUE, HOWEVER FOR EACH SPECIFIC GEOMETRY AND MATERIAL.

TESTS AND PRACTICAL EXPERIENCE HAVE SHOWN TIME AND AGAIN THAT THE CALCULATED TORQUES FOR BOLTS > M30 (OR 1/4") ARE NOT COINCIDING WITH THE ACTUAL VALUES WITH ADEGUATE PRECISION.
THE MAIN INFLUENTIAL FACTOR FOR THESE DIFFERENCES IS THREAD FRICTION IN THE BOLT AND NUT CONTACT AREA, FOR WHICH TO A LARGE EXTENT ONLY EMPIRICAL OR ESTIMATED VALUES ARE AVAILABLE.

THE ELASTIC LONGITUDINAL ELONGATION AT 70% PRESTRESS OF THE YIELD POINT IS DETERMINED THEORETICALLY VIA THE ELASTIC RESILIENCE OF THE BOLT.

THE BOLT IS LOADED (AND TORQUED) THROUGH A SPANNER UNTIL THE PREVIOUSLY DETERMINED BOLT ELONGATION IS DISPLAYED ON THE DIAL GAUGE. THIS TORQUE IS THEN READ OFF THE TORQUE SPANNER.

TO ACCOUNT FOR ANY VARIATIONS, AN AVERAGE VALUE FROM SEVERAL MEASUREMENTS SHOULD BE DETERMINED. CAUTION: AFTER A CERTAIN OPERATING TIME THE BOLT CONNECTION MUST BE RECHECKED FOR PRE-STRESS AND RETIGHTENED, AS REQUIRED TO COMPENSATE FOR ANY SETTLING PHENOMENA REDUCING BOLT PRE-STRESS, WHICH RESULTS IN A LOSS IN CLAMP LOAD.

- TORQUE-CONTROLLED TIGHTENING

A PRELOAD FORCE:

F_M : TENSILE ASSEMBLY (/MONTAGE) FORCE IN THE BOLT SHAFT IS TO BE GENERATED

ALL TIGHTENING TECHNIQUES DO NOT SENSE THE PRELOAD PRODUCED IN THE BOLT DIRECTLY BUT INDIRECTLY.

THE BOLT PRELOAD FORCE F_M IS A FUNCTION OF:

M_G : TORSION MOMENT ON THE BOLT SHAFT (TRASMITTED THROUGH THE THREADS, IN GERMAN "GEWINDE"). THIS IS PRODUCED BY:

M_A : ASSEMBLY TORSION MOMENT (IN GERMAN "ANZIEHDREHMOMENT")

THE ASSEMBLY TORSIONAL MOMENT M_A APPLIED TO NUT OR BOLT IS:

$$M_A = M_K + M_{Kzu} + \underbrace{M_G + M_{\ddot{u}}}_{M_S}$$

M_S: MOMENT ON THE BOLT SHAFT

M_K : FRICTION MOMENT IN THE HEAD (KOPF) OR NUT BEARING AREA

M_{Kzu}: ADDITIONAL HEAD MOMENT, WHEN USING ELEMENTS WHICH PREVENT SLACKENING (EX. IN GRIP NUTS ①)

M_G: TORQUE ACTING IN THE THREAD ("GEWINDE")

$M_{\ddot{u}}$: OVERBOLTING MOMENT, WHEN USING ELEMENTS WHICH PREVENT THE BOLT FROM ROTATING LOOSE (EX. IN SELF LOCKING NUTS/LOCKNUTS ②)

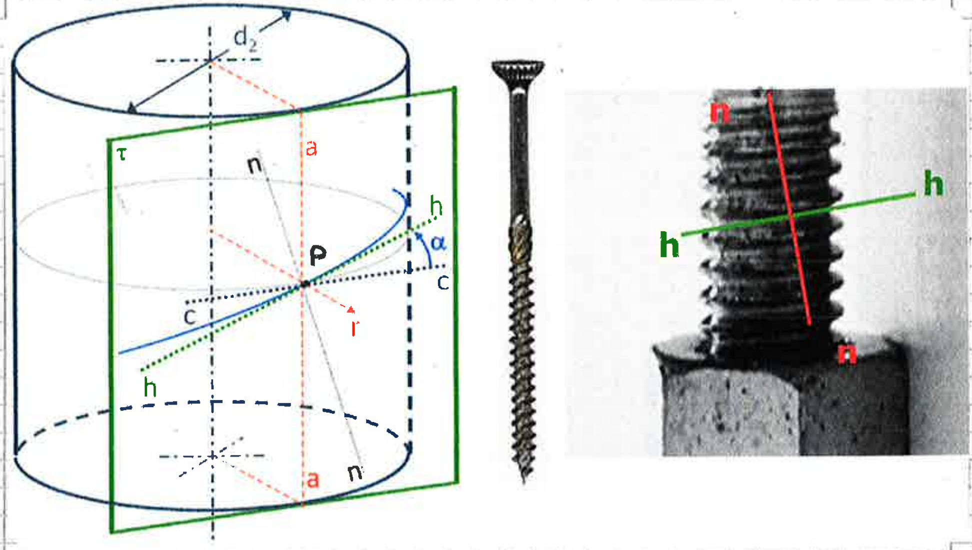
M_{Kzu} AND $M_{\ddot{u}}$ CAN BE IGNORED IN HIGHLY PRELOADED JOINTS.

①


②


6.2.2) DESCRIBE THE MAIN PLANES USED TO ANALYSE THE BEHAVIOUR OF BOLT.

- ACTIONS ON THREAD: MAIN PLANES



d_2 : BOLT THREAD PITCH DIAMETER

PLANE τ : TANGENT TO THE CYLINDER ALONG THE LINE a-a (PARALLEL TO CYL. AXIS).

A PART OF THE HELIX IS SHOWN; ONE OF ITS POINTS IS CROSSED BY RADIUS R. (P)

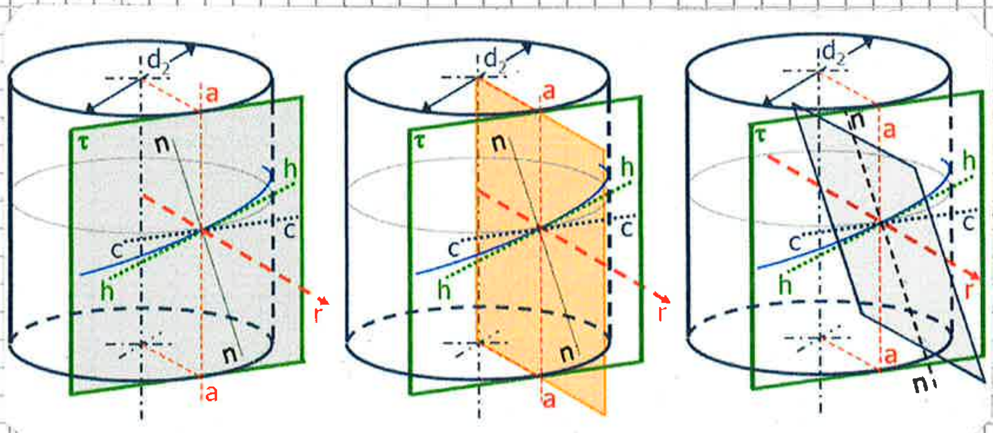
LINE h-h : TANGENT TO THE HELIX, PASSANT THROUGH POINT (P)

LINE c-c : CIRCUMFERENTIAL LINE ($\perp$ TO RADIUS R AND TO AXIS a-a)

α : HELIX ANGLE OF THREAD = INCLINATION OF LINE h-h WITH RESPECT TO LINE c-c.

LINE n-n : ON PLANE τ, NORMAL TO LINE h-h IN POINT P.

THREE IMPORTANT PLANES ARE DEFINED:



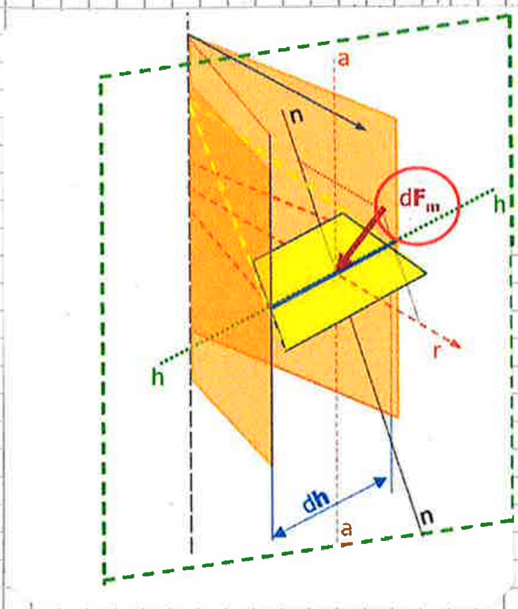
PLANE τ = TANGENT PLANE (CONTAINING LINE a-a AND $\perp$ TO RADIUS R)

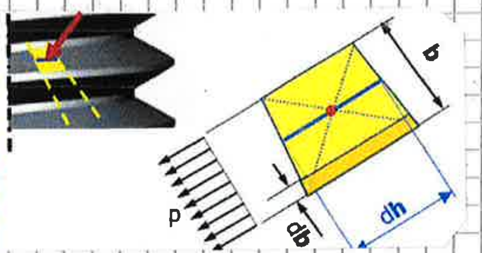
RADIAL PLANE (CONTAINING LINE a-a AND RADIUS R) (= a-a/R PLANE)

PLANE NORMAL TO THE HELIX (CONTAINING RADIUS R AND LINE n-n)(= n-n/R)

(↳ START WITH THE RADIAL PLANE AND ROTATE ABOUT RADIUS R)

- PRESSION RESULTANT FORCE (F_m) (NORMAL TO THE FLANK SURFACE)



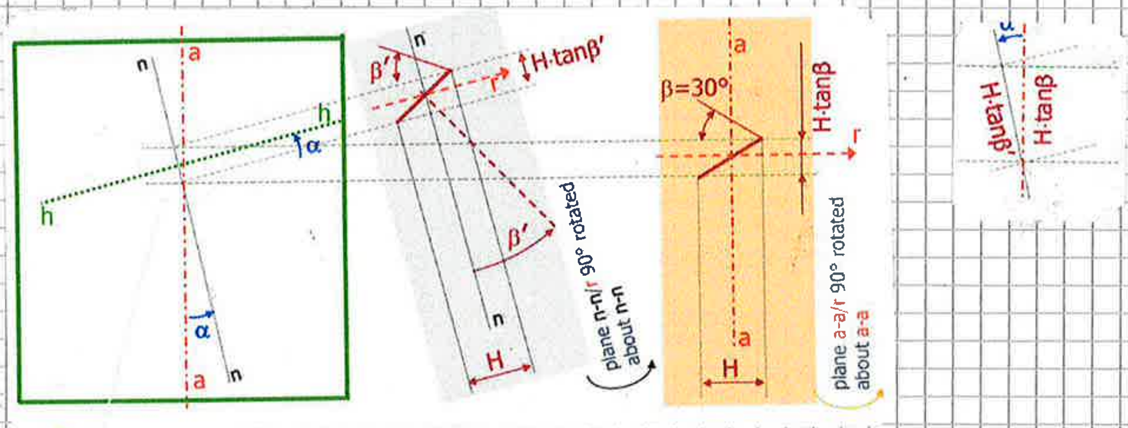


WE ASSUME THAT NORMAL PRESSURE p IS CONSTANT ALONG THE THREAD FLANK WIDTH b

$dF_m = p \cdot b\,dh$ NORMAL FORCE COMP. ON THE FLANK.

RESULTANT FORCE DUE TO PRESSURE p APPLIED TO THE FLANK'S INFINITESIMAL HELIX ARC dh, NORMAL TO THE FLANK SURFACE AND APPLIED IN ITS AREA CENTROID, APPROXIMATELY AT THE INTERSECTION OF RADIUS r AND LINE h-h, I.E. ON THE TRACED HELIX ON THE PITCH CYLINDER.

- RELATION BETWEEN β, β', α



H = THREAD V-SHAPE HEIGHT

IT IS SEEN BOTH IN THE n-n/r PLANE AND IN THE a-a/r PLANE

$H_{n-n} = H \cdot \tan\beta'$ PROJECTION ALONG n-n

$H_{a-a} = H \cdot \tan\beta$ PROJECTION ALONG a-a WITH $\beta = 30°$ FOR METRIC THREAD

SINCE: $H_{n-n} = H_{a-a} \cdot \cos\alpha \iff H \cdot \tan\beta = H \cdot \tan\beta' \cdot \cos\alpha$

WE HAVE THAT:

(⊗ ERROR IN THE SLIDE 15 (pp. 8/31) Bolts_2_2019: $H\tan\beta' = H\tan\beta\cos\alpha$: WRONG!)

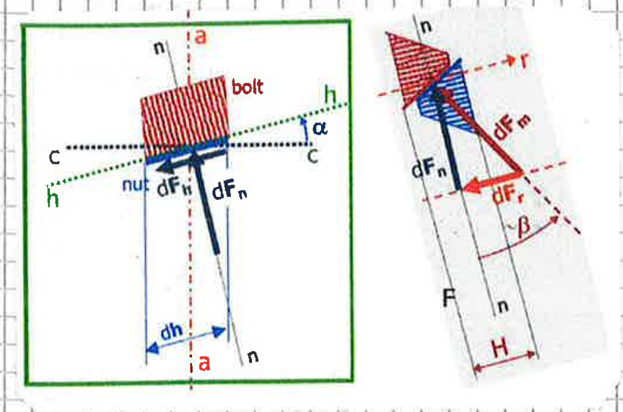
$\tan\beta = \tan\beta' \cdot \cos\alpha$

WITH α IN THE RANGE $1 \div 3$ DEGREES, THIS CORRECTION CAN BE NEGLECTED.

WE SHALL THEN ASSUME:

$\tan\beta' \cong \tan\beta \implies \beta' \cong \beta = 30°$

- FORCE COMPONENTS dF_m, dF_h, dF_R



$dF_m = p \cdot b \cdot dh$

$dF_n = dF_m \cdot \cos\beta' = dF_m \cdot \cos\beta$

$dF_h = dF_n \cdot \tan\varphi_G$

WITH: φ_G = THREAD FRICTION ANGLE

$dF_h = dF_m \dfrac{\tan\varphi_G}{\cos\beta}$ (*)

THE NUT-TO-BOLT RADIAL COMPONENT dF_R IS USUALLY NEGLECTED, BEING A RADIAL COMPRESSIVE FORCE THAT PRODUCES RELATIVELY LOW COMPRESSIVE STRESSES, MOREOVER OF A BENIGN NATURE TO FATIGUE. THE TWO REMAINING CONTACT FORCE COMPONENTS dF_m AND dF_h, SEEN ON PLANE τ, ARE LINKED BY THE EQUATION (*)

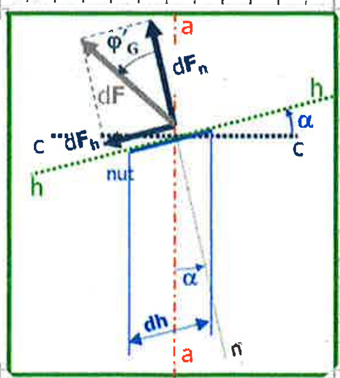
THE TANGENTIAL FRICTION FORCE dF_h IS LINKED TO dF_m BY

$dF_h = \mu'_G \cdot dF_m$

WITH:

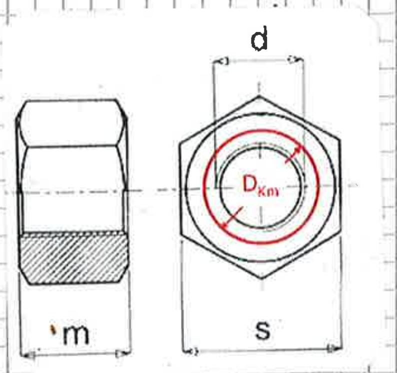
$\mu'_G = \tan\varphi'_G = \dfrac{\tan\varphi_G}{\cos\beta} = \dfrac{\mu_G}{\cos\beta}$ FICTITIOUS FRICTION COEFFICIENT

- DECOMPOSITION OF THE TOTAL IN PLANE FORCE (dF) ALONG THE HELIX (dF_h) AND ITS NORMAL (dF_m)



6.2.G) WRITE THE EXPRESSION OF TIGHTENING TORQUE (M_A)

• FRICTION TORSION MOMENT (M_K) ABOUT THE BOLT AXIS



D_{Km} = EFFECTIVE DIAMETER FOR THE FRICTION MOMENT AT THE BOLT HEAD OR NUT BEARING AREA.

THE CALCULATION MODEL ASSUMES THAT THE HEAD OR NUT AXIAL LOAD (F_a) IS DISTRIBUTED ON A CIRCLE AT DIAMETER D_{Km}.

$$M_K = F_a \frac{D_{Km}}{2} \mu_K$$ FRICTION TORSION MOMENT

WITH: $\mu_K = \tan \varphi_K$ FRICTION COEFF. (HEAD OR NUT)

NOTE D_{Km} IS DEFINED AS THE ARITHMETIC MEAN OF INNER AND OUTER EFFECTIVE CONTACT CIRCLES ON HEAD OR NUT BEARING AREA.

MATERIALS: COEFFICIENTS μ_G AND μ_K.
VDI 2230 BLATT 1 / PART 1 - 2003 - FRICTION COEFF. CLASSES WITH GUIDE VALUES FOR DIFFERENT MATERIALS / SURFACES AND LUBRIFICATION STATES IN BOLTED JOINTS.

Friction coefficient class	Range for μ_G and μ_K	Selection of typical examples for materials	Selection of typical examples for lubricants
A	0,04 to 0,10	metallically bright black oxide phosphated galvanic coatings such as Zn, Zn/Fe, Zn/Ni Zinc laminated coatings	solid lubricants, such as MoS2, graphite, PTFE, PA, PE, PI in lubricating varnishes, as top coats or in pastes; liquefied wax wax dispersions
B	0,08 to 0,16	all the above plus Al and Mg alloys	all the above plus greases; oils; delivery state
		hot-galvanized	MoS2; graphite; wax dispersions
		organic coatings	with integrated solid lubricant or wax dispersion
		austenitic steel	

VALUES APPLY AT ROOM TEMPERATURE.
FOR CLASSES C, D, E SEE THE ORIGINAL OF THE STANDARD.
THE AIM IS TO ACHIEVE COEFFICIENTS OF FRICTION WHICH FIT INTO THE FRICTION COEFFICIENT CLASS B IN ORDER TO APPLY AS HIGH A PRELOAD AS POSSIBLE WITH LOW SCATTER.

$$M_A = 78400\left[0.16 \cdot P + 0.58\, d_2 \mu_G + \frac{D_{Km}}{2}\mu_K\right] = 202000\left[0.12 + 0.48 + 0.40\right]$$

CONSIDERATION:

EVIDENTLY, A FUNDAMENTAL PROBLEM WITH TORQUE TIGHTENING IS THAT BECAUSE THE MAJORITY OF THE TORQUE IS USED TO OVERCOME FRICTION (USUALLY BETWEEN 85% AND 95% OF THE APPLIED TORQUE), SLIGHT VARIATIONS IN THE FRICTIONAL CONDITIONS CAN LEAD TO LARGE CHANGES IN THE BOLT PRELOAD.

⊕ DEFINE THE TIGHTENING FACTOR (TIGHTENING OPERATION)

TIGHTENING OPERATION

TIGHTENING DEPENDS ON THE OPERATOR AND ON THE WRENCH USED.
IT IS UNCERTAIN.
IT IS POSSIBLE DEFINING A RANGE OF VALUES, BY INTRODUCING AN INDEX I (IN ITALIAN STANDARDS) : $I = \alpha_A$ (α_A IN OTHER STANDARDS)

$\alpha_A = \frac{F_{M\,max}}{F_{M\,min}}$ TIGHTENING FACTOR

$\frac{1}{2}\frac{|\Delta F_M|}{F_{Mm}} = \frac{\alpha_A - 1}{\alpha_A + 1}$ SCATTER OF THE ASSEMBLY PRELOAD

WITH:

$$\begin{cases} \Delta F_M = F_{M\,max} - F_{M\,min} \\ F_{Mm} = (F_{M\,max} + F_{M\,min})/2 \end{cases}$$

Tightening factor α_A	Scatter $\frac{1}{2}\frac{\lvert\Delta F_M\rvert}{F_{Mm}} = \frac{\alpha_A - 1}{\alpha_A + 1}$	Tightening technique	Adjusting technique
1,05 to 1,2	2% to 10%	Elongation-controlled tightening with ultrasound measurement	Echo time
1,1 to 1,5	5% to 20%	Elongation-controlled tightening with mech. measurement	Adjustment via longitudinal measurement
1,2 to 1,4	9% to 17%	Yield-controlled tightening, motor or manually operated	Input of the relative torque/rotation-angle coefficient
1,2 to 1,4	9% to 17%	Angle-controlled tightening, motor or manually operated	Experimental determ. of pre-tightening torque and angle of rotation
1,6 to 2,0 (friction coeff. class B) 1,7 to 2,5 (friction coeff. class A)	23% to 33% 26% to 43%	Torque-controlled tightening with torque wrench, indicating wrench, or precision tightening spindle with dynamic torque measurement	Determination of the required tightening torque by estimating the friction coefficient (surface and lubricating conditions)

(REFERRING TO THE PREVIOUS FIGURES...)
THEESE FIGURES SHOW THE PROGRESSION OF CLAMPING FROM a) THE FIRST CONTACT BETWEEN HEAD / NUT UNDERFACES AND PART, AND THEREFORE PUT INTO EVIDENCE HOW CLAMPING IS GENERATED DURING THE TURNING OF HEAD OR NUT.
(MOREOVER THEY LINK THE LINEAR APPROACH TO CLAMPING OF THIS SECTION TO NON-LINEARITIES SHOWN IN SECTION 5).

THE PART CURVE HAS THE LINEAR TRACK (DESCRIBED IN SECT. 10 "PART RESILIENCE") THE ALIGNMENT ZONE (NON-LINEAR) AND THE ELASTIC ORIGIN (DESCRIBED IN SECT. 5)
THE BOLT CURVE HAS THE LINEAR TRACK (DESCRIBED IN SECT. 9 "BOLT RESILIENCE") AND THE NON-LINEAR TERMINAL TRACK DUE TO BOLT YIELDING.

Fig. b) SHOWS THE RELATIVE POSITION OF PART AND BOLT CURVES WHEN, FOR ISTANCE, ONE FULL TURN HAS BEEN GIVEN TO THE NUT AND THEREFORE THE BOLT CLAMPED LENGTH HAS BEEN REDUCED BY ONE PITCH (P).
(IT IS PREFERRED HERE TO REPRESENT THE BOLT AS FIXED)

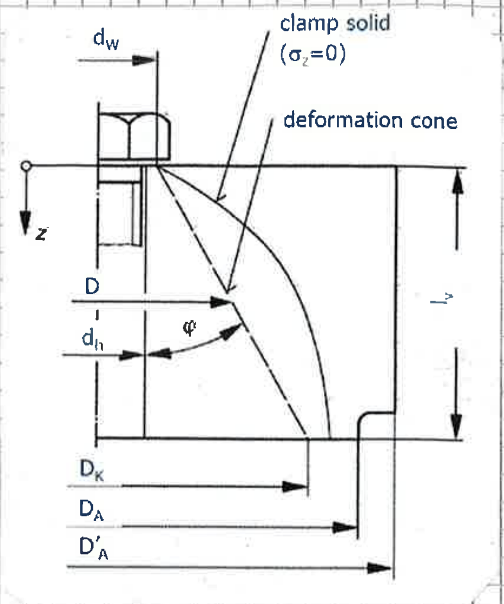
Fig c) SHOWS BOLT AND PART CURVES AFTER TWO TURNS.
THE DISTANCE BETWEEN PART AND BOLT ELASTIC ORIGINS IS THE ELASTIC INTERFERENCE i.

$i = m_i P = \frac{\alpha_i}{2\pi} P$ ELASTIC INTERFERENCE i

WITH:

$$\begin{cases} m_i : \text{NUMBER OF NUT/HEAD TURNS} \\ \alpha_i : \text{NUT/HEAD ROTATION ANGLE} \\ P : \text{THREAD PITCH} \end{cases}$$

PARTS RESILIENCE



STRESSES IN CLAMPED PART AFFECT A VOLUME WHICH WIDENS MOVING AWAY FROM THE BOLT HEAD OR NUT. CALCULATION OF PART RESILIENCE IS DONE ON A "SUBSTITUTION" CONE HAVING THE SAME RESILIENCE (δ_P)

THE CONTACT SURFACE RESILIENCE IS NOT TAKEN INTO ACCOUNT; FOR HEAD/NUT/WASHER CONTACT IT IS RECOVERED WHILE TURNING THE HEAD/NUT.

IF THE CLAMPED PARTS ARE A LARGE NUMBER OF TIN SHEETS, WHICH ARE NEVER FULLY FLAT, SURFACE EFFECTS MUST BE DETERMINED EXPERIMENTALLY.

$D(z) = d_w + 2 \cdot z \, \mathrm{Tan}\varphi$: DIAMETER OF CONE

$A(z) = \frac{\pi}{4}\left[(d_w + 2 \cdot z\,\mathrm{Tan}\varphi)^2 - d_h^2\right]$: CONE CROSS-SECTION AREA

$$\delta_P = \frac{1}{E_P}\int_{z=0}^{z=l_v} \frac{dz}{A(z)} = \frac{4}{\pi E_P}\int_{z=0}^{z=l_v} \frac{dz}{\left[(d_w + 2z\,\mathrm{Tan}\varphi)^2 - d_h^2\right]}$$

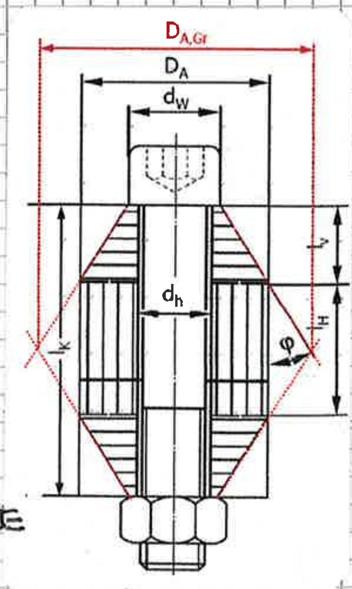
$$\delta_P = \frac{4}{\pi E_P}\int_{x=d_w}^{x=d_w+2l_v\mathrm{Tg}\varphi} \frac{dx}{\left[x^2 - d_h^2\right]}$$

(⊗ ERROR IN THE SLIDE 37 (PP. 19/31) BOLTS_2_2019)
← CORRECT

KNOWING THAT: $\int \frac{dy}{1-y^2} = \frac{1}{2}\ln\left|\frac{1+y}{1-y}\right| + C$

THEN:

$$\delta_P = \frac{1}{\pi E_P d_h \mathrm{Tan}\varphi}\left[\ln\left(\frac{(d_w+d_h)(d_w+2l_v\mathrm{Tan}\varphi - d_h)}{(d_w-d_h)(d_w+2l_v\mathrm{Tan}\varphi + d_h)}\right)\right]$$ CONE RESILIENCE



IF THE CYLINDRICAL PART HAS A DIAMETER (D_A) THAT:

$D_A \leq D_{A,GR}$, WITH:

$D_{A,GR}$: LIMIT (GRENZE) DIAMETER OF CONE INTERSECTION

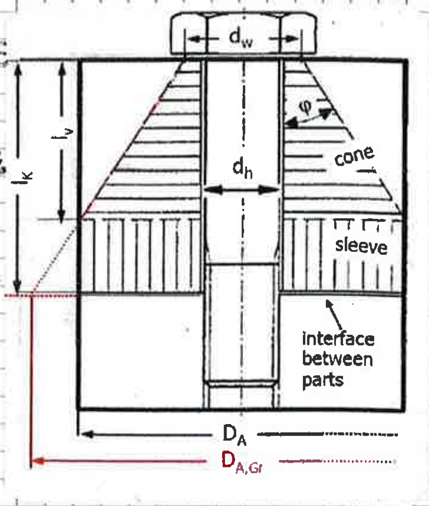
THEN, THE PART IS COMPOSED OF TWO CONES AND A CENTRAL CYLINDER SLEEVE (HÜLSE) OF LENGTH l_H.

(NORMALLY THE PARTS ARE MADE OF THE SAME MATERIAL WITH ONE E_P; HOWEVER ATTENTION MUST BE PAID TO SPECIAL CASES)

IF $D_A \geq D_{A,GR}$, THEN THE FOLLOWING HOLDS:

$$\delta_P = \frac{2\ln\left(\frac{(d_w+d_h)(d_w+l_K\mathrm{Tan}\varphi - d_h)}{(d_w-d_h)(d_w+l_K\mathrm{Tan}\varphi + d_h)}\right)}{\pi E_P d_h \mathrm{Tan}\varphi}$$ PARTS RESILIENCE

IN THE CASE OF TAPPED THREAD JOINT:



TO SIMPLIFY THE CALCULATION OF THE PLATE RESILIENCE THE TOP CONE AND THE BOTTOM TRUNCATED CONE ARE REPLACED BY ONE SUBSTITUTION DEFORMATION CONE OF THE SAME RESILIENCE, WHICH CAN BE FOLLOWED BY A SLEEVE.

THE CORRESPONDING FORMULA FOR RESILIENCE ARE:

IF $D_A \geq D_{A,GR}$:

$$\delta_P = \frac{\ln\left(\frac{(d_w+d_h)(d_w+2l_k\tan\varphi-d_h)}{(d_w-d_h)(d_w+2l_k\tan\varphi+d_h)}\right)}{\pi E_P d_h \tan\varphi}$$

IN THE CASE OF TAPPED THREAD JOINT THE SUBSTITUTION CONE ANGLE WHICH PRODUCES THE SAME RESILIENCE AS THE REAL BODY IS INTERPOLATED BY THE FORMULA:

$$\tan\varphi = 0.348 + 0.013 \ln\frac{l_k}{2d_w} + 0.193 \ln\frac{D'_A}{d_w}$$

IF $d_w < D_A \leq D_{A,GR}$:

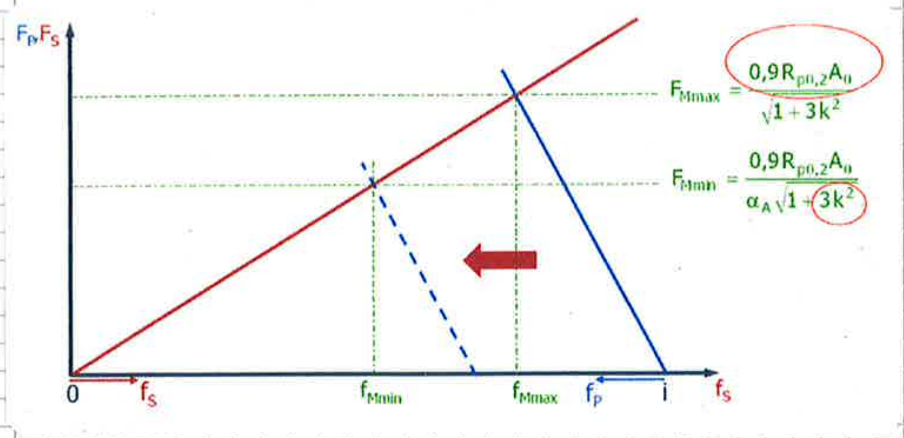
$$\delta_P = \frac{1}{\pi E_P}\cdot\left\{\frac{1}{d_h\tan\varphi}\ln\left(\frac{(d_w+d_h)(D_A-d_h)}{(d_w-d_h)(D_A+d_h)}\right) + \frac{4}{D_A^2-d_h^2}\left[l_k - \frac{(D_A-d_w)}{2\tan\varphi}\right]\right\}$$

6.2.7) DEFINE THE TIGHTENING UNCERTAINTY

• UNCERTAINTY OF TIGHTENING FORCE (DUE TO THE ASSEMBLY PROCESS)

TAKING INTO ACCOUNT THE "TIGHTENING FACTOR" (α_A), ALSO NAMED "TIGHTENING UNCERTAINTY FACTOR": $\alpha_A = \frac{F_{Mmax}}{F_{Mmin}}$

F_{Mmin} IS ALSO FOUND AND REPRESENTED.



RECALLING:

$\sigma = \frac{F}{\frac{\pi d^2}{4}}$ = AXIAL LOAD / SECTION DEFINED TO CALCULATE STRESS ; $\tau = \frac{m_t}{\frac{\pi d^3}{16}}$ ← TORQUE: $m_t = \frac{F}{2}\left(\frac{p}{\pi} + d_m \frac{tg\alpha}{\cos\beta}\right)$

MEAN DIAMETER (d_m)

($m_t = M_t$, $d_m = d_2$)

$\sigma_{id} = \sqrt{\sigma^2 + 3\tau^2} = \sigma\sqrt{1+3\left(\frac{\tau}{\sigma}\right)^2}$

$\frac{\tau}{\sigma} = \frac{16 m_t}{\pi d^3}\frac{\pi d^2}{4F} = \frac{4 m_t}{dF} = \frac{4}{dF}\frac{F}{2}\left(\frac{p}{\pi} + d_m\frac{tg\alpha}{\cos\beta}\right) = \frac{2}{d}\left(\frac{p}{\pi} + \frac{d_m Tg\alpha}{\cos\beta}\right) = K$ ($Tg\alpha$ OR $Tg\varphi_c$?) → ASK TO PROF.

$\sigma_{id} = \sigma\sqrt{1+3K^2}$

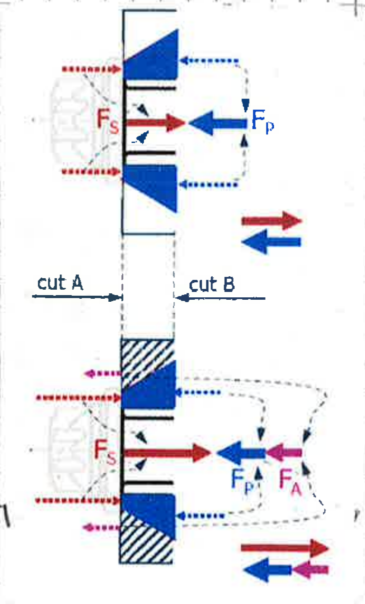
$\sigma = \frac{\sigma_{id}}{\sqrt{1+3K^2}} \iff F = \frac{\sigma_{id} A_0}{\sqrt{1+3K^2}}$

$$\begin{cases} F_{Mmax} = \frac{0.9 R_{p0.2} \cdot A_0}{\sqrt{1+3K^2}} \\ F_{Mmin} = \frac{0.9 R_{p0.2} A_0}{\alpha_A\sqrt{1+3K^2}} \end{cases}$$, WITH $K = \frac{2}{d}\left(\frac{p}{\pi} + \frac{d_m Tg\alpha}{\cos\beta}\right)$

(+) DESCRIBE THE LOADING

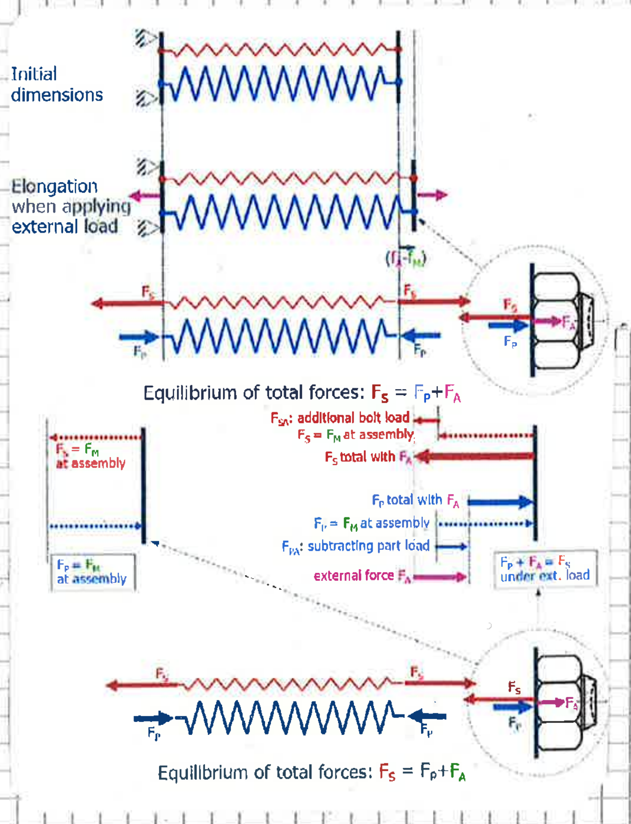
JOINT ASSEMBLY

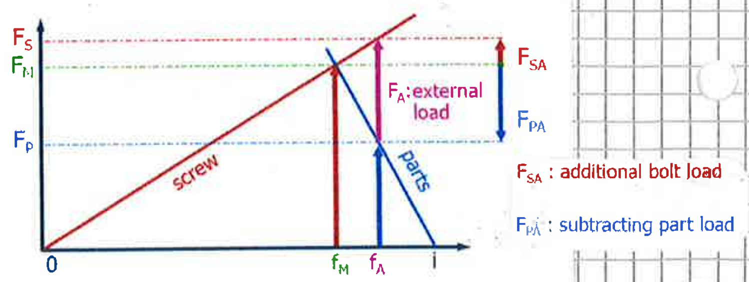
THIS FIGURE REPRESENTS ONE CUTAWAY END OF A BOLTED JOINT. CUT PLANES A AND B SEPARATE A PART SECTION, WHICH IS EQUILIBRIUM UNDER THE APPLIED AXIAL FORCES, WHOSE RESULTANTS ARE SHOWN IN SOLID LINES.
THE RESULTANT FORCE ON THE BOLT SECTION IS F_S.
A DISTRIBUTED FORCE UNDER THE HEAD TRANSMITS HEAD COMPRESSION TO THE CLAMPED PART AT CUT A.
TOTAL VALUE OF COMPRESSION IS $F_S = F_P$.
ON CUT B ANOTHER DISTRIBUTION PROVIDES A TOTAL RESULTANT FORCE F_P ACTING ALONG THE BOLT/PART AXIS.
AT ASSEMBLY F_S AND $F_P \Rightarrow F_M$.



JOINT LOADING

NOW AN EXTERNAL FORCE F_A IS APPLIED TO THE PART ON (OR VERY NEAR TO) ITS OUTER SURFACE.
THE PART RECEIVES FROM THE HEAD/PART CONTACT SURFACE, CUT A, THE FORCE F_S; FROM THE AJACENT PART MATERIAL THROUGH, CUT B, THE RESULTANT FORCE F_P. EQUILIBRIUM OF THE PART FRAGMENT REQUIRES THAT : $F_S = F_P + F_A$





$$\begin{cases} F_S = F_P + F_A \\ (F_M + F_{SA}) = (F_M - F_{PA}) + F_A \end{cases} \Rightarrow F_A = F_{SA} + F_{PA}$$

$$\begin{cases} f_A - f_M = \delta_S F_{SA} \\ f_A - f_M = \delta_P F_{PA} \end{cases} \Rightarrow F_{PA} = \frac{\delta_S}{\delta_P} F_{SA}$$

THEREFORE:

$$F_A = F_{SA} + \frac{\delta_S}{\delta_P} F_{SA} = F_{SA}\left(\frac{\delta_P + \delta_S}{\delta_P}\right)$$

THEN:

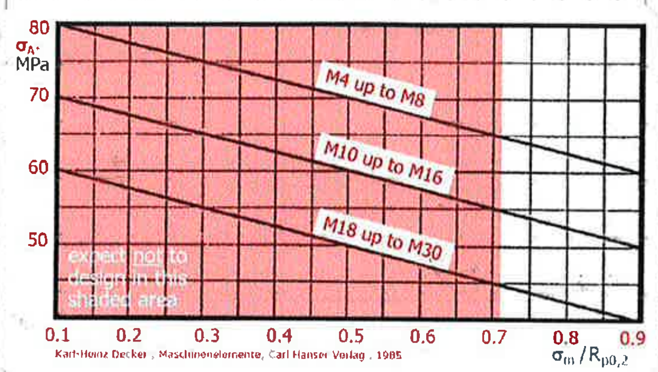
$$F_{SA} = F_A \frac{\delta_P}{\delta_S + \delta_P}$$

ACTING ON BOLT:

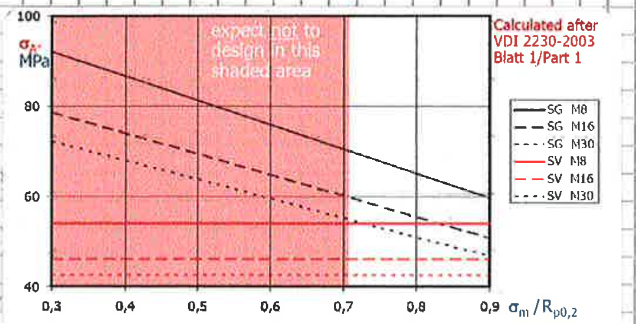
$$\sigma_m = \frac{1}{A_S}\left(F_{M\,MAX} + \frac{F_{SA\,MAX} + F_{SA\,min}}{2}\right)$$ MEAN STRESS

$$\sigma_a = \frac{F_{SA\,MAX} - F_{SA\,min}}{2A_S}$$ ALTERNATING STRESS (σ_a, σ_A)

FATIGUE LIMIT DIAGRAM



- STRESS AMPLITUDE OF THE FATIGUE LIMIT IS SET AT $\sigma_a = 0.9\sigma_a^*$
- THIS FATIGUE LIMIT DIAGRAM INCLUDES THE NOTCH EFFECT DUE TO THREAD.



- $\sigma_{A,SV}$: STRESS AMPLITUDE OF THE ENDURANCE LIMIT OF SV BOLTS (ROLLED BEFORE HT)
- $\sigma_{A,SG}$: STRESS AMPLITUDE OF THE ENDURANCE LIMIT OF SG BOLTS (ROLLED AFTER HT)

ACCORDING TO VDI 2230-2003:

- FOR BOLT THREADS ROLLED BEFORE HT:

  $\sigma_{A,SV} = 0.85\,(150/d + 45)$ $(\sigma\,[MPa],\ d\,[mm])$

  (WITHIN THE VALIDITY RANGE OF $0.3 \leq \sigma_{Sm}/R_{p02} < 1$)

- FOR BOLT THREADS ROLLED AFTER HT:

  $\sigma_{A,SG} = (2 - \sigma_{Sm}/R_{p02})\,\sigma_{A,SV}$

  (WITHIN THE VALIDITY RANGE OF $0.3 \leq \sigma_{Sm}/R_{p02} < 1$)

THIS HIGHER FATIGUE STRENGTH IS A FUNCTION OF STEADY STRESSES ON ACCOUNT OF THE INDUCED RESIDUAL COMPRESSIVE STRESSES.

6.2.8) DESCRIBE HOW THE BOLT DIAGRAM IS USED TO DESIGN IT IN STATIC LOADING.

6.2.9) DEFINE THE UPPER AND LOWER LIMITS OF DIAGRAM.

BOLT AND JOINT : STATIC LOADING

IN THE THREADED SECTION:

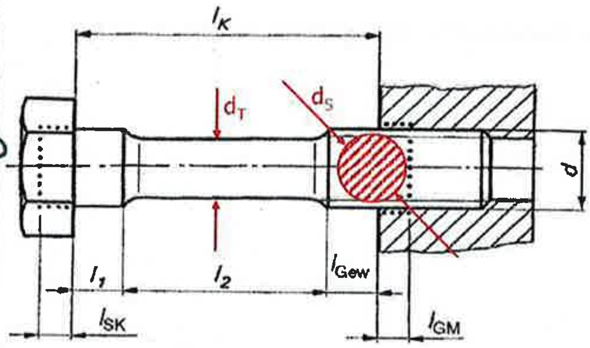
d_3 : MINIMUM BOLT CROSS SECT. DIAMETER

FOR STRENGTH CALCULATION :

d_T : SHANK DIAMETER

$d_s = (d_2 + d_3)/2$: STRESS DIAMETER

THE LOWEST OF d_T, d_s SHALL BE EMPLOYED AND IT WILL CALLED d_0 (DIAMETER AT THE SMALLEST CROSS SECT. OF THE BOLT EITHER d_T OR d_s).



$A_0 = \pi d_0^2/4$: APPROPRIATE MIN. CROSS SECT. A.

$W_P = \pi d_0^3/16 \cong 0.2 d_0^3$: MIN. POLAR RESISTANCE MODULUS

THE AIM IS TO UTILIZE THE BOLT STRENGTH TO THE GREATEST POSSIBLE EXTENT. NORMALLY THE EQUIVALENT STRESS IN THE BOLT (NOMINAL STRESS) AT ASSEMBLY IS 90% OF $R_{P0.2}$ (MINIMUM GUARANTEED).

THE THREAD TORQUE M_G PRODUCES TANGENTIAL STRESSES τ;

THE ASSEMBLY BOLT LOAD F_M PRODUCES (MEAN OR NOMINAL) TENSILE STRESS σ

$$\begin{cases} \sigma_{EQ} = 0.9 R_{P0.2} \\ \sigma_{EQ} = \sqrt{\sigma^2 + 3\tau^2} = \sqrt{\left(\frac{F_{MMAX}}{A_0}\right)^2 + 3\left(\frac{M_G}{W_P}\right)^2} = \frac{F_{MMAX}}{A_0}\sqrt{1+3K^2} \end{cases}$$

$$0.9 R_{P0.2} = \frac{F_{MMAX}}{A_0}\sqrt{1+3K^2}$$

$$F_{MMAX} = \frac{0.9 R_{P0.2} A_0}{\sqrt{1+3K^2}} \quad \text{, WITH : } K = \frac{\tau}{\sigma} = \frac{M_G}{W_P}\frac{A_0}{F_{MMAX}} = \frac{4}{d_0}\left[0.16P + 0.58 d_2 \mu_G\right]$$

WHEN THE YIELD POINT OF THE MATERIAL, I.E. THE FULLY PLASTIC STATE, IS REACHED IN THE CROSS SECTION A_0, THEN THERE IS A CONSTANT TORSIONAL STRESS τ OVER THE CROSS SECTION.

THIS CONDITION IS SATISFIED BY A CORRECTION TO THE TORSION SECTION MODULUS :

$W_P' = \pi d_0^3/12$: "CORRECTED" MINIMUM TORSION SECTION MODULUS

$$\text{THEN : } K = \frac{3}{d_0}\left[0.16P + 0.58 d_2 \mu_G\right]$$

PART 3

(+) DESCRIBE THE SITUATION WHEN THE LOAD IS APPLIED UNDER THE HEAD.

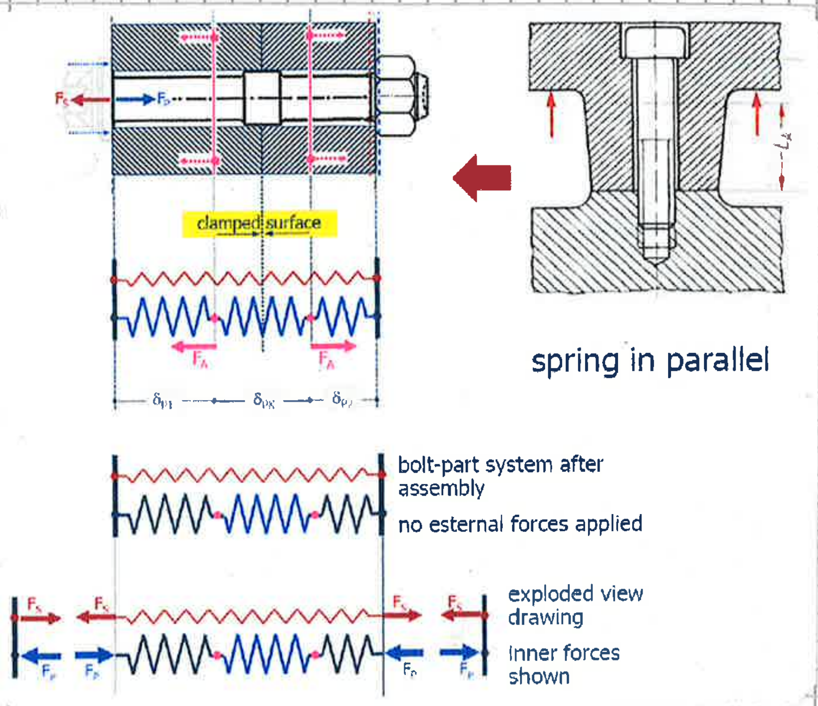
LOAD APPLIED UNDER THE HEAD

THE EXTERNAL FORCE F_A IS APPLIED TO THE PARTS AT A LOCATION WHICH IS NO LONGER (AS IT WAS IN THE ELEMENTARY APPROACH) AT THE LEVEL OF THE NUT/PART AND HEAD/PART CONTACT SURFACE.

THE SPRING MODEL COLLECTS FORCE POSITIONS AND STIFFNESS OF THE THREE SECTIONS IN WHICH PARTS ARE NOW DIVIDED

RESILIENCES: $\delta_{P1}, \delta_{PK}, \delta_{P2}$

ASSEMBLY FORCE EQ: $|F_S| = |F_P|$



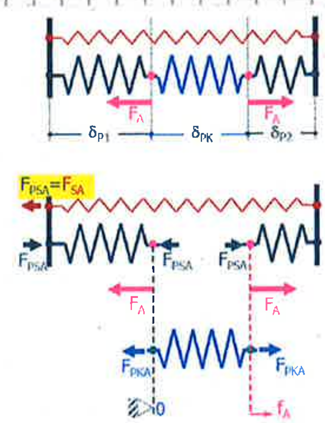
THE BODY LOADED BY THE EXTERNAL FORCE F_A IS SPIT INTO TWO SPRING SECTIONS:

- THE INNER SPRING (K)

  $\delta_{PK} < \delta_P$

- THE SERIES OF SPRINGS WITH RESILIENCES $\delta_{P1}, \delta_{P2}, \delta_S$

$\Rightarrow \delta_{PS} = \delta_{P1} + \delta_{P2} + \delta_S$ TOTAL RESILIENCE



THESE TWO SPRING SECTIONS ARE IN PARALLEL AND SHARE THE EXTERNAL LOAD F_A WITH THE SAME MECHANISM ALREADY DEMONSTRATED BEFORE.

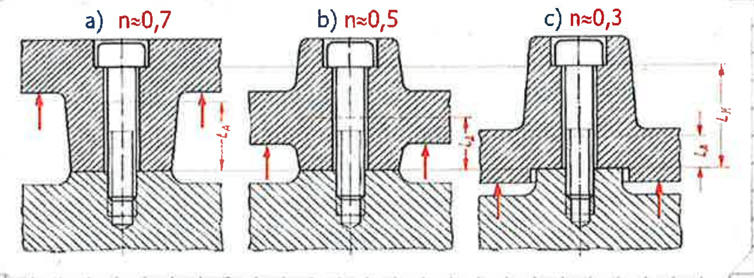
EQUILIBRIUM:

$F_A = F_{PKA} + F_{PSA}$

NOTE THAT: $F_{PSA} = F_{SA}$

f_A IS THE EXTENSION OF INNER SPRING

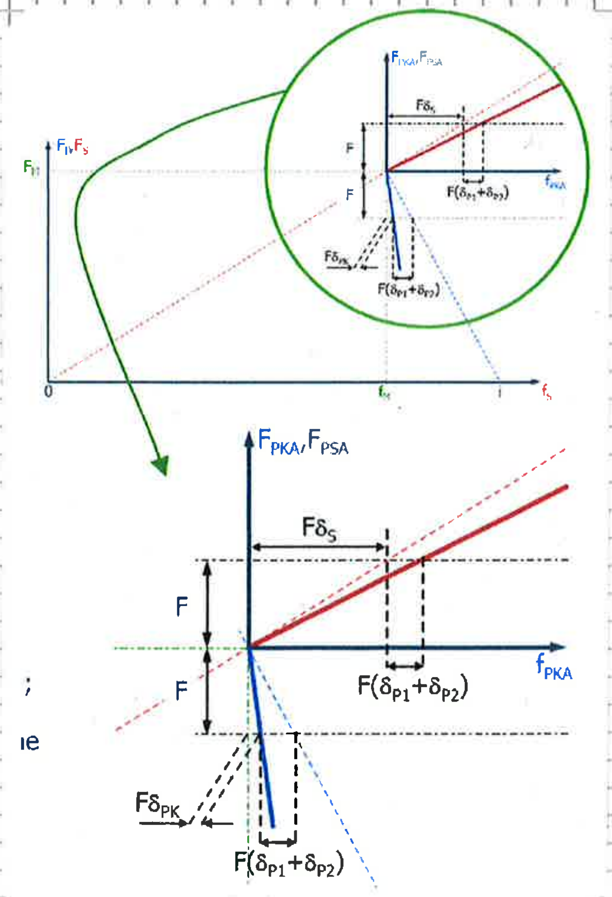
LOAD INTRODUCTION FACTOR : APPROXIMATE VALUES ($m = \delta_{PK}/\delta_P$)



THE VDI 2230-2003 DEFINES THE LOAD INTRODUCTION FACTOR WITH QUITE COMPLEX FORMULAS WHICH ARE BEYOND THE SCOPE OF THESE NOTES.

- A SIMPLE FIRST-APPROXIMATION APPROACH CAN BE GIVEN BY THE FIGURE.
- WHEN m IS NOT AVAILABLE, A SAFE CHOICE IS : $m = 1$.
- A FIRTS APPROXIMATION VALUE COULD BE : $m = \frac{L_A}{L_K}$ (HOWEVER IT IS REASONABLE ONLY IF CROSS SECTION OF THE PART IS NEARLY CONSTANT)

EXTERNAL LOADING JOINT DIAGRAM : CORRECT CONSTRUCTION



THE CORRECTED CONSTRUCTION REQUIRES THAT THE GRAPHICAL CONDITION BE SATISFIED; IN FACT THE (SCREW + PART P1 + PART P2) SERIES GAINS A DEFORMATION EQUAL TO THAT LOST BY THE PARTS.

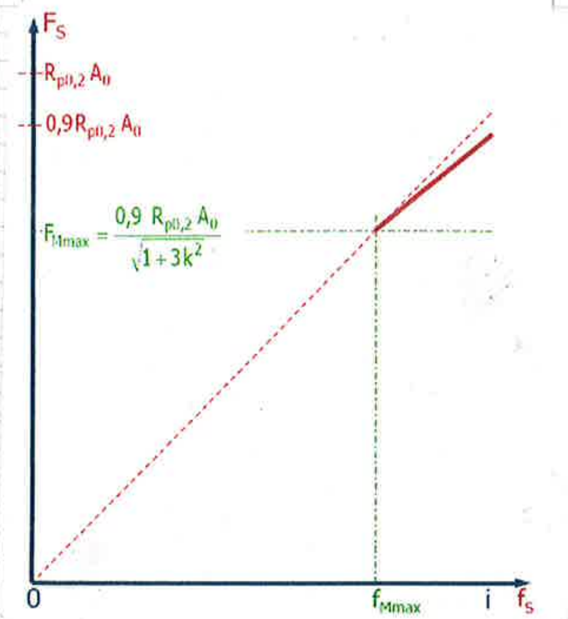
⊕ DESCRIBE THE SPRING BACK EFFECT

SPRING BACK EFFECT

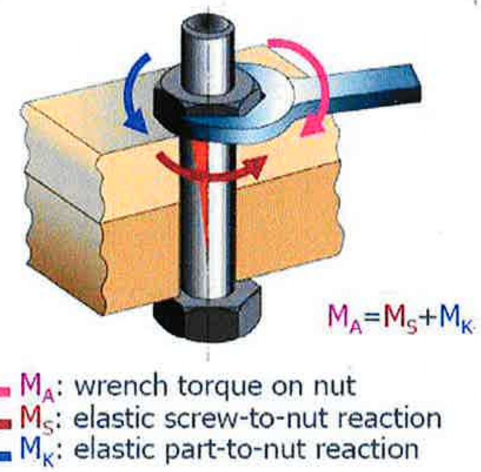
TIGHTENING TORQUE AND STRESS WERE CALCULATED BY NEGLECTING THE **ELASTIC REPLAY OF MATERIAL** WHICH **TENDS TO REDUCE THE REAL ROTATION** APPLIED TO THE SCREW.

THIS MOTIVATES THE NON LINEAR LOOK OF THE SCREW LINE AS FOR THE PART.

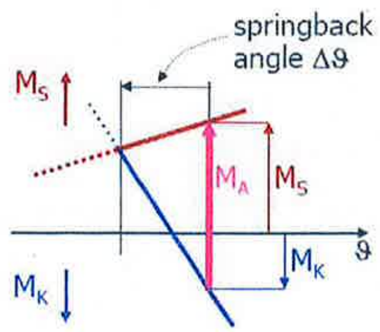
THIS EFFECT IS USUALLY QUITE CONTROLLED AND MORE EVIDENT IN THE PART OF LINE NOT EXPLORED.



THE WHOLE THING IS COMPLEX AND QUITE CONTROVERSIAL. **ACCORDING TO BICKFORD:** " MANY PEOPLE, IN FACT, WILL INSIST THAT TORSIONAL STRESS DISAPPEARS IMMEDIATELY AND COMPLETELY WHEN THE WRENCH IS REMOVED FROM THE FASTENER. OTHERS FIND THAT IT DOESN'T DISAPPEAR UNTIL A BREAKAWAY TORQUE IS APPLIED".



HOWEVER **CALCULATION AND CONTROL OF THE AMOUNT OF SPRINGBACK IS DIFFICULT.** SO, UNLESS PRECISE DATA ARE AVAILABLE, IT IS **SAFER TO ASSUME** THAT **SHEAR STRESS IN THE BOLT IS AT ITS UPPER LIMIT**, CALCULATED ACCORDING TO SLIDING HEAD-TO-PART AND NUT-TO-SCREW CONTACTS WITH THE FRICTION MODEL.



(THE LAST FIGURE ON THE RIGHT ASSUMES THAT **DURING SPRINGBACK THERE IS NO SLIDING** BETWEEN NUT AND PART, NUT AND BOLT.)

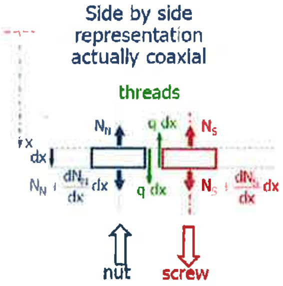
(TO READ)

Neglecting the fact that the thread is wound helicoidally around the pitch cylinder, a purely axial model is studied where an infinitesimal slice dx of the bolt is in contact with the corresponding slice of nut through the thread.

q : load per unit length (axial) exchanged between threads

N_S: load (axial) on the screw cross section

N_N: load (axial) on the nut cross section

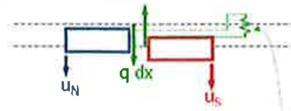


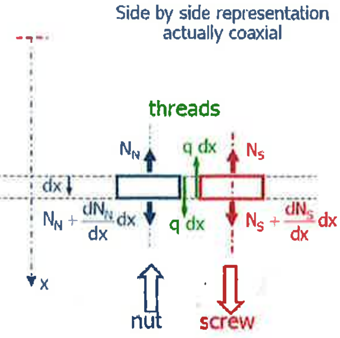
infinitesimal slice of nut and screw in contact at the thread mean diameter

Bolt equilibrium: $\frac{dN_S}{dx} = q$

Nut equilibrium: $\frac{dN_N}{dx} = -q$

Displacements u_N, u_S under load:





Thread distributed load q is due to the elastic reaction between bolt and nut threads, which deform as short plane strain beams in bending and shear, plus local contact elasticity:

$$q = k(u_S - u_N)$$

$$\frac{du_S}{dx} = \varepsilon_S = \frac{N_S}{E_S A_S} \qquad \frac{du_N}{dx} = \varepsilon_N = \frac{N_N}{E_N A_N}$$

Bolt and nut stress-strain relations

$q = k(u_S - u_N)$: thread load-displacement relation

$$\frac{dq}{dx} = k\left(\frac{du_S}{dx} - \frac{du_N}{dx}\right) = k\left(\frac{N_S}{E_S A_S} - \frac{N_N}{E_N A_N}\right)$$

$$\frac{d^2q}{dx^2} = k\left(\frac{1}{E_S A_S}\frac{dN_S}{dx} - \frac{1}{E_N A_N}\frac{dN_N}{dx}\right) = q\left(\frac{k}{E_S A_S} + \frac{k}{E_N A_N}\right)$$

Equilibrium, stress-strain and load displacement equations are combined in order to produce one differential equation of the second order with variable q

$$\left.\begin{array}{l}\frac{d^2q}{dx^2} - \chi^2 q = 0 \\ \text{with: } \chi^2 = \left(\frac{k}{E_S A_S} + \frac{k}{E_N A_N}\right)\end{array}\right\} \Rightarrow q = A_1 e^{\chi x} + A_2 e^{-\chi x}$$

(TO READ)

Boundary conditions for the "compressed" nut:

$$\frac{dq}{dx} = \chi\left(A_1 e^{\chi x} - A_2 e^{-\chi x}\right) = k\left(\frac{N_S}{E_S A_S} - \frac{N_N}{E_N A_N}\right)$$

a) $x=0 \Rightarrow N_N=0,\ N_S=0$

b) $x=h \Rightarrow N_N=-F_S,\ N_S=F_S$

Condition a) produces:

$$\chi(A_1 - A_2) = 0 \Rightarrow A_1 = A_2 \Rightarrow A$$

Then: $q = A\left(e^{\chi x} + e^{-\chi x}\right)$

Condition b) produces: $\chi A\left(e^{\chi h} - e^{-\chi h}\right) = F_S\left(\frac{k}{E_S A_S} + \frac{k}{E_N A_N}\right) = F_S \chi^2$

It then follows: $A = \frac{F_S \chi}{e^{\chi h} - e^{-\chi h}}$ hence : $q = F_S \chi \frac{e^{\chi x} + e^{-\chi x}}{e^{\chi h} - e^{-\chi h}}$

This is also written: $q = F_S \chi \frac{ch(\chi x)}{sh(\chi h)}$

The integral of q dx over the height of one thread turn P has the meaning of "fraction of screw load taken by one turn of thread".

$$Q(x) = F_S \int_X^{X+P} \chi \frac{ch(\chi x)}{sh(\chi h)} dx = \frac{F_S}{sh(\chi h)} \int_X^{X+P} [\chi\, ch(\chi x)]\, dx$$

$$= \frac{F_S}{sh(\chi h)} \int_X^{X+P} d[sh(\chi x)] = \frac{F_S}{sh(\chi h)} [sh(\chi(X+P)) - sh(\chi X)]$$

This is normally given for:
Thread 1: from X=h to X=h-P
Thread 2: from X=h-P to X=h-2P
etc., up to the last thread at the top of the nut

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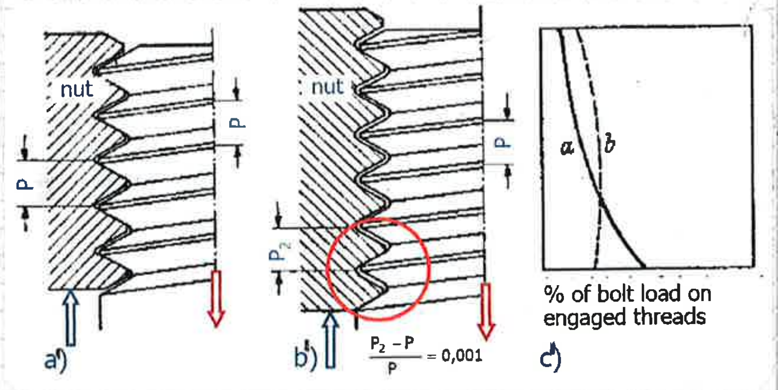
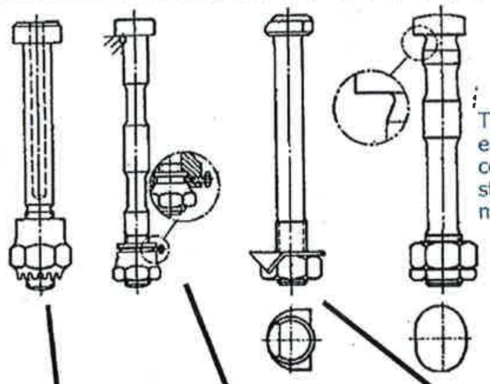
CONSIDERATIONS



a) THIS FIGURE SHOWS THE NORMAL ENGAGEMENT BETWEEN A SCREW AND A "COMPRESSED" NUT.

b) THERE IS HERE A SMALL DIFFERENCE BETWEEN BOLT AND NUT PITCH; THE FIRST THREAD TAKES LOAD ONLY AFTER SOME BOLT ELONGATION.

c) THIS FIGURE SHOWS THE BENEFIT OF SOLUTION B) AGAINST a). HOWEVER, THE SHAPE OF CURVE B) DEPENDS ON LOAD.



This has an eccentric circular-arc head to prevent rotation during tightening, easier to manufacture than in c); the shank is mostly waisted, i.e. more compliant both axially and in bending; a larger fillet under the head reduces peak stresses; contrary to case b), the thread ends at shank diameter reduction, thus minimizing local stress peaks

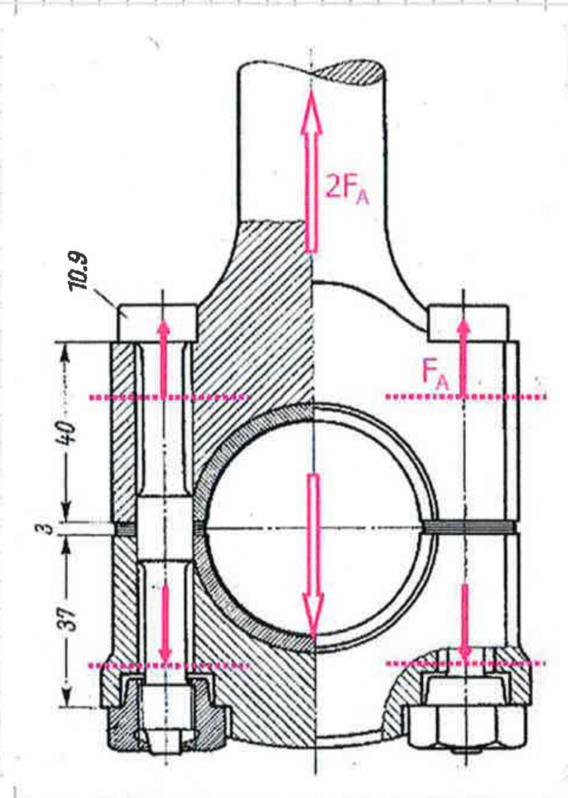
This shows a flattened bolt head (against a matching surface on the conrod) to prevent bolt rotation during tightening; this simple bolt is not waisted

example of "waisted" shank, even though the nominal diameter fitting cylinders take the most of the shaft length; inconveniently, the thread ends into the fitting part without a relief groove; bolt rotation during tightening is prevented by a dowel (pin) under the head

a bore along the bolt shaft increases its axial deformability, without using a "waisted " or "reduced shank" solution; compared to the latter, this approach has the shortcoming of a higher bending stiffness and higher stress peaks at the thread runout; a splined nut is a safety against unlocking, however it is doubtful whether this is effective against loss of pre-load; the high nut is due to the use of a fine thread; bolt rotation during tightening is prevented by a wrench applied to the head; the thread correctly ends in a relief groove

MODERN APPLICATION OF THE ADVANCED BOLT ON A CONROD

- FLAT HEAD AGAINST ROTATION DURING TIGHTENING.
- WAISTED SHANK
- PARTIALLY "SUSPENDED" NUT
- FIRST BOLT THREAD ONE PITCH INSIDE THE BOLT.



VIOLET FORCES APPLIED TO THE ROD AND TO ITS CAP INDICATE QUALITATIVELY THE NEED TO EVALUATE THE POSITION OF THE PART FORCE AT EACH BOLT LOCATION.