NUMERO: 2450A

ANNO: 2019

A P P U N T I

STUDENTE: Zara Serena

MATERIA: Applied Geophysics - Prof. Godio

INTRODUCTION

Basics: spatial resolution
↗ seismic and ground penetrating radar → related to the wavelength
↘ electrical and electromagnetic methods → depends on the aquisition parameters of the adopted method and interpretation process

Problem
1D
2D
3D
4D → 3D + time

Natural methods

based on the natural field
- gravimetric m. = change of the gravitational acceleration
- magnetic m. = magnetic prop. of rocks

Artificial methods

are based on the analysis of the Earth behaviour to propagation of an artificially induced field
- Seismic = P and S wave
- Electrical = conductivity / resistivity
- Electromagnetic = σ at different frequency, electrical permettivity

magnetic: detect the presence of ferro-magnetic materials naturally or not

electrical and electromagnetic: environmental application

seismic: mechanical properties of soil and rock or exploration at great depth.

PLANE WAVE EQUATION

P wave propagation in a cartesian plane (x, t)

Generic function

$G = f(x, t) \rightarrow$ general form $\rightarrow G = f(at - bx)$

First and second derivative, respect to the time

$$G' = a f'(at - bx) = \frac{df}{dt}$$

$$G'' = a^2 f''(at - bx) = \frac{d^2 f}{dt^2} \longrightarrow f''(at-bx) = \frac{d^2 f}{dt^2} \cdot \frac{1}{a^2}$$

and respect to the space

$$\frac{df}{dx} = b f'(at - bx)$$

$$\frac{d^2 f}{dx^2} = b^2 f''(at - bx) \longrightarrow f''(at - bx) = \frac{d^2 f}{dx^2} \cdot \frac{1}{b^2}$$

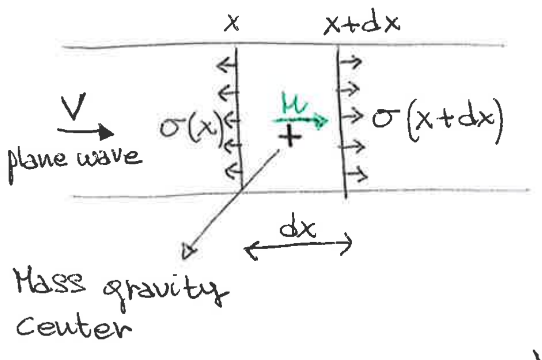
We pose equal the two terms

$$\frac{d^2 f}{dt^2} = \left(\frac{a}{b}\right)^2 \frac{d^2 f}{dx^2} \longrightarrow \boxed{\frac{d^2 f}{dt^2} = v^2 \frac{d^2 f}{dx^2}}$$

generic plane wave equation that propagates in
- homogeneous
- isotropic
- elastic

material

In an elastic medium



dx : thickness

σ : strain ⊥ to the surface S

u : displacement of the center of mass gravity (due to the plane wave)

$M = \rho dx S$

M : mass of the element of thickness dx

ρ : density

S : surface

dx : thickness

The $V = dx\, S$ is subjected to the stress $\sigma(x)$ and the stress $\sigma(x + dx)$ on the opposite surfaces.

MECHANICAL PARAMETERS

E [Pa] Young modulus

G [Pa] Shear modulus

k [Pa] Dynamic bulk modulus $k = \dfrac{\sigma^i}{\Delta V}$

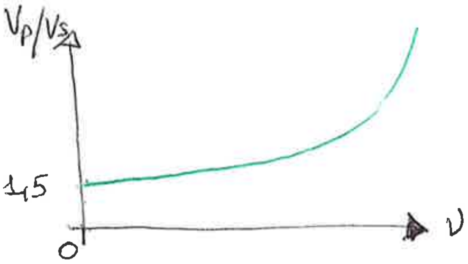
ν Poisson ratio

ρ [kg/m³] Density (of bulk)

$$\frac{V_P}{V_S} = \sqrt{\frac{2(1-\nu)}{1-2\nu}}$$

If we know the ratio V_P/V_S we can calculate the ν

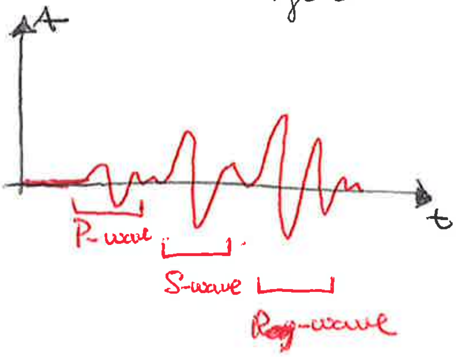
$V_P \uparrow$ if $k \uparrow$ $\rho \downarrow$ $\nu \downarrow$



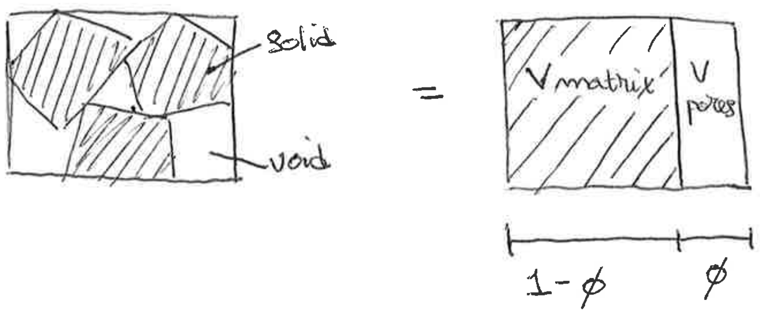
The ratio is related to some geological factors:
- litology and facies
- pore volume and fluid content
- mechanical prop. of components

Consolidation and water content $\uparrow$
$\Rightarrow V_P/V_S \uparrow$

The sensor that records the wavefront is the SEISMOGRAM.
P-waves arrives first, S-waves for second, surface waves (Rayleigh and Love) for third.
The amplitude of the soil vibration decades, with an exponential low, with distance from the source for two different phenomena:
- geometrical spreading (dispersion)
- intrinsic attenuation (dissipation)



ROCK DENSITY – EQUIVALENT POROSITY



ϕ : porosity

$1-\phi$: solid rock, matrix

- bulk density (d) : total density of the considered rock volume ~~con~~ including pore
- matrix density (d_m) : mean density of the matrix material without pore
- pore density (d_p) : density of the fluid present in the pore

So for a porous rock : coexist 2 ~~fasi~~ phases

$$d = (1-\phi)\, d_m + \phi\, d_p\, S_w$$

If is present a gas phase and a fluid phase

$$d = (1-\phi)\, d_m + \phi \cdot \left[S_w d_w + (1-S_w) d_g \right]$$

S = saturation, coexist 3 phases

If is also present a non miscible phase (oil)

$$d = (1-\phi)\, d_m + \phi \cdot \left[S_w d_w + (1 - S_w - S_o) d_g + S_o d_o \right]$$

coexist 4 phases

$S_w = \dfrac{V_w}{V_{voids}}$ water saturation

$S_o = \dfrac{V_o}{V_{voids}}$ oil saturation

$S_g = 1 - S_w - S_o$ gas saturation

WILLIE'S EQUATIONS OR TIME AVERAGE EQUATION

If the distance source-point is 10 times the wavelength, ~~ca~~ the wave can be approximate to a plane wave.

IN FLUID

$$G = 0$$

$$v_p = \sqrt{\frac{K_{fl}}{\rho}}$$

IN POROUS

$$V_s = \sqrt{\frac{G_{sat}}{\rho}} \qquad G_{sat} \approx (1 - k\phi) G_0$$

k = empired coeff 1,5 ÷ 2

G_0 = shear modulus of minerals

UNCOMPLETELY SATURATED

$$V_s = \sqrt{\frac{G_{sat}}{\rho}} \qquad G_{sat_{DRY}} \simeq G_{sat_{WET}}$$

MIXTURE OF FLUID

Different fluid phases = different bulk moduli

but if the phases are intimately mixed, ~~it's assumed~~ an average value is assumed.

WOOD'S LAW

$$\frac{1}{K_{fl}} = \sum_i \frac{S_i}{K_i}$$

K_{fl} = effective bulk modulus of fluid mix

K_i = single bulk modulus

S_i = saturation of each gas and liquid phase

es:

water and oil

$$\frac{1}{K} = \frac{S_w}{K_w} + \frac{S_o}{K_o} \qquad \text{where } S_o = 1 - S_w$$

water and gas

$$\frac{1}{K} = \frac{S_w}{K_w} + \frac{1 - S_w}{K_g}$$

water, oil and gas

$$\frac{1}{K} = \frac{S_w}{K_w} + \frac{S_{oil}}{K_{oil}} + \frac{(1 - S_w - S_{oil})}{K_g}$$

FRAME BULK MODULUS

$$K_n = \frac{K_{dry}}{K_0}$$

K_n : normalized modulus

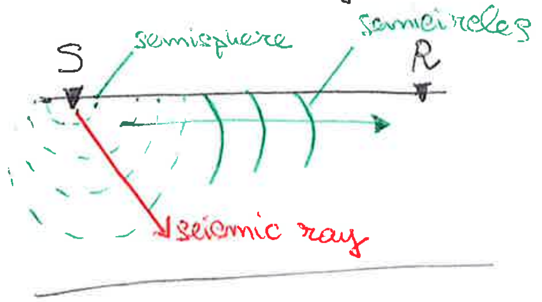
K_{dry} : bulk modulus (the entire skeleton) (grains interaction)

K_0 : mineral modulus

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 11 di 62

BASIC OF SEISMIC REFRACTION METHOD

Seismic survey

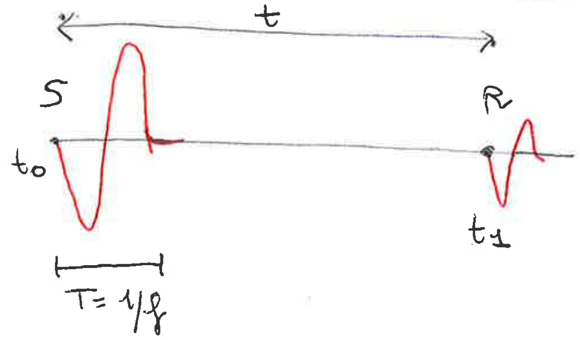


S: source, point of energization
Thanks to a shot, the seismic wave starts to propagate in a ELASTIC FIELD (basic assumption)

R: receiver

We consider the intersection between semisphere and vertical surface → semicircles

Seismic ray: ~~as semplific~~ semplification, indicates the wavefronts direction.
Is the ray that starts from the source and it's normal to the wavefront in every istant.



Peak attenuation

1) the time to arrive to the receiver depends on the distance between S-R
2) energetic attenuation → geometrical → dispersion: the energy is distributed on a surface bigger than the the previous one
   → intrinsic → dissipation: friction among particles

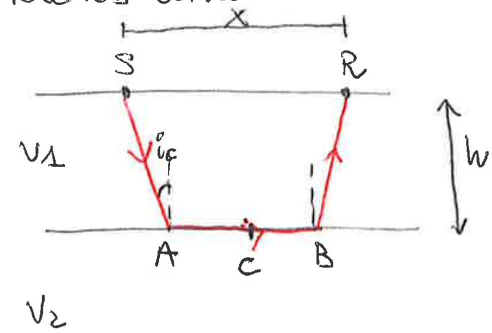
The energetic attenuation is linked to the amplitude
The amplitude of the wave in a generic istant t is

$$A(x,t) = \frac{A_0}{x} e^{-[\alpha x]}$$

A_0 : amplitude in the istant 0
x : term of geometrical spreading
α : attenuation coefficient $\left[\frac{1}{m}\right]$
αx : intrinsecal dispersion

$Q^{-1} \simeq 2\alpha$
quality factor: index of loss of energy in the system

REFRACTED WAVE



$$t_{refracted} = \frac{SA}{V_1} + \frac{AB}{V_2} + \frac{BR}{V_1}$$

$$SA = BR = \frac{h}{\cos(i_c)}$$

$$AB = x - 2h\,tg(i_c)$$

$$t_{refracted} = \frac{2h}{V_1 \cos(i_c)} + \frac{x - 2h\,tg(i_c)^{①}}{V_2 ②}$$

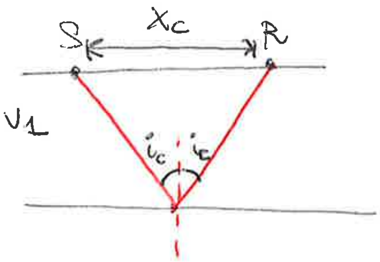
① $tg(i_c) = \frac{sen(i_c)}{\cos(i_c)}$; ② $sen\, i_c = \frac{v_1}{v_2} \Rightarrow v_2 = \frac{v_1}{sen\, i_c}$

$sen^2(i_c) = 1 - \cos^2(i_c)$

Application of Snell's law

$$t_{refracted} = \frac{x}{V_2} + \frac{2h}{V_1}\cos i_c$$

The minimum angle at wich we have the refraction phenomena is the critical angle i_c and the critical distance x_c

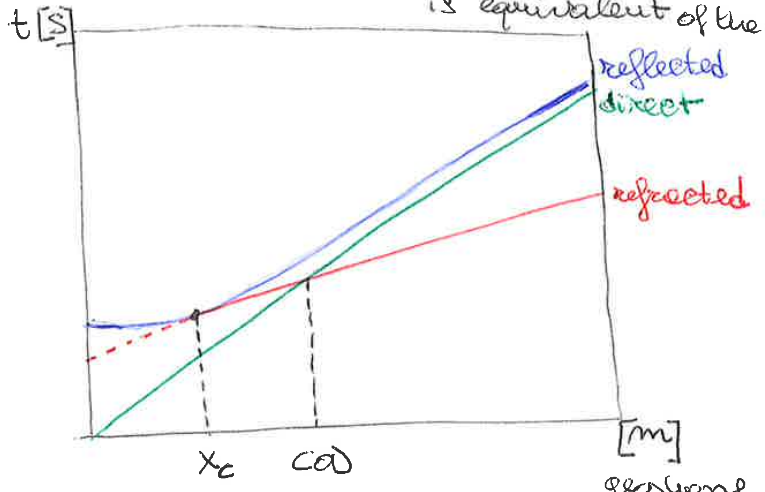


$$t_{x\,crit} = \frac{2\sqrt{\left(\frac{x_c}{2}\right)^2 + h^2}}{V_1}$$

for $x < x_c$ we don't have refraction but only reflection.

CROSS OVER DISTANCE

for $X <$ COD the direct waves are the 1^{st} to arrive to the receivitor. COD is the x at wich the triveltime of the refracted is equivalent of the direct



$x < x_c$: 1) direct
2) reflected
refracted doesn't exist

$x_c < x <$ COD 1) direct
2) refracted
3) reflected

$x >$ COD 1) refracted
2) direct
3) reflected

if x is very ~~big~~ large, the direct and reflected waves are confused

DETERMINATION OF THE CROSS OVER DISTANCE

Equalize the wave travel time equation

$$\underbrace{\frac{COD}{V_1}}_{\text{direct}} = \underbrace{\frac{COD}{V_2} + \frac{2h\cos i_c}{V_1}}_{\text{refracted}} \Rightarrow COD = 2h \cdot \frac{\sqrt{V_2+V_1}}{\sqrt{V_2-V_1}}$$

EXERCISE

$V_1 = 1000$ m/s $h = 10$ m

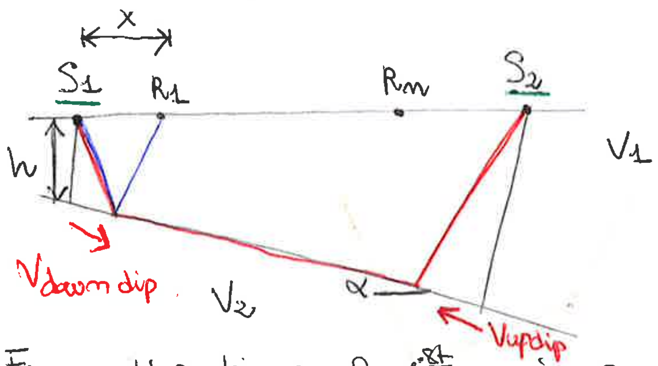
$V_2 = 2000$ m/s n° geophones = 24 → 5 for direct waves; → the other for the refracted

$$COD_t = 2 \cdot 10 \cdot \frac{\sqrt{2000+1000}}{\sqrt{2000-1000}} = 20 \cdot \sqrt{3} = 34,64 \text{ m}$$ ~~distance total~~ COD total

$$COD = \frac{COD_t}{\text{g direct}} = \frac{34,64}{5} \simeq 7 \text{ m}$$ distance between every geophone

$$X_{total} = 7 \cdot 24 = 168 \text{ m}$$ total survey length

INCLINED LAYER CASE

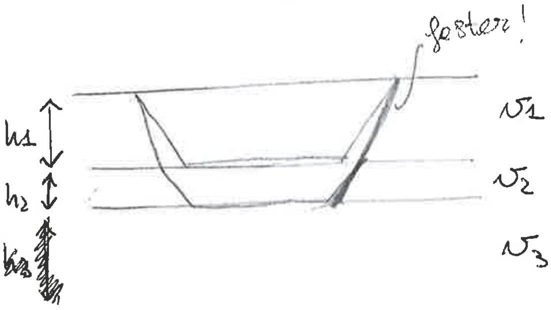


At least, 2 bursts, called reciprocal bursts, are needed at the beginning and at the end of the survey.

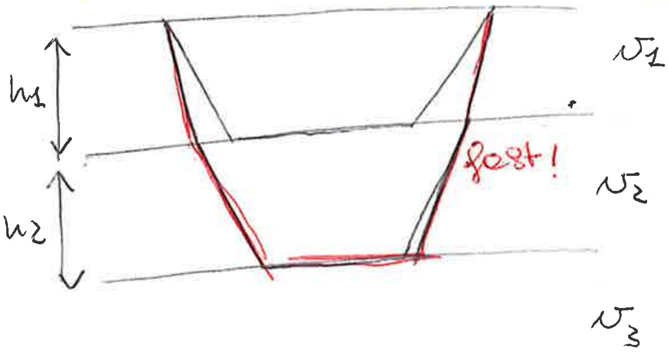
From the time of 1st arrivals of the direct and refracted waves, the direct and the conjugated dromochronous waves are constructed.
There are two intercept times; the inclination of the first stretches of straight line allows to obtain the value of speed of the direct wave.
The inclination of the two straight section relative to the arrivals of refracted waves allow us to estimate 2 apparent velocities V_{up}^{dip}: rising the refract horizon
$V_{down\,dip}$: descending " " "

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 17 di 62

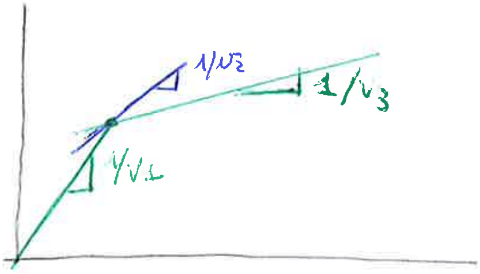
Hidden layer



The reflected waves from the 3rd layer arrives before than the 2nd layer waves! (they are faster!)
The wave spend little time to pass through the 2nd layer because is thin and arrives directly to the 3rd layer



$v_1 < v_2 < v_3$



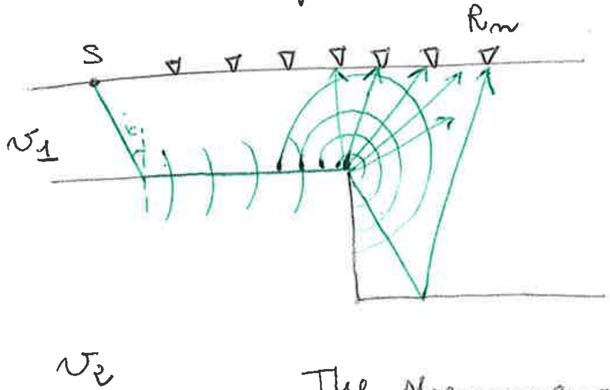
we observe the refraction of the second layer only for 1÷3 geophones → in the dromocrone is difficult notice the 2nd layer velocity.

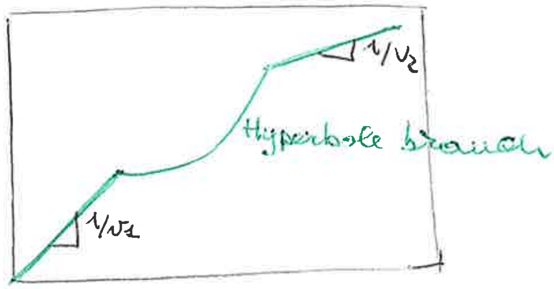
PITFALL CASE

$v_1 > v_2$ but $v_2 < v_3$

The resulting dromocrone is the same because the wave that travel with V_3 is faster than the wave with ~~velo~~ V_2 and arrives for first at the receiver

Vertical step

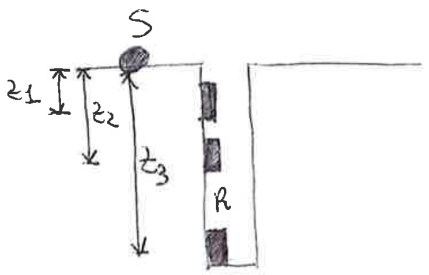


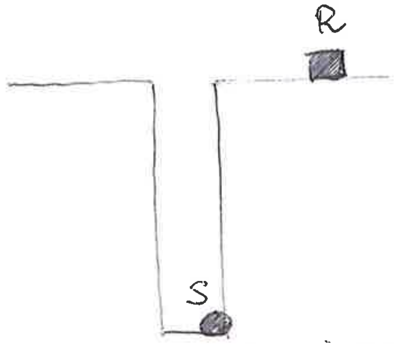


The phenomena of diffraction ~~occurs~~ occours where there are discontinuity in the mechanical properties like a vertical step.

The diffraction is based on the Huygens' principle: every point of a wavefront is a generator of new wavefront.
It happens on the ~~[illegible]~~ edge of the step.

DOWN HOLE TEST





- source on the surface
- receivers are moved along the borehole

↓

DOWN HOLE TEST

- a single receiver is close to the borehole's head
- the source is moved along the borehole

↓

UP-HOLE TEST

Source: shot of an hammer surface

Receivers: clamped mechanically or with pneumatic system

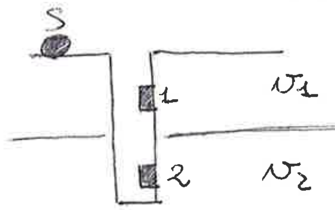
Each geophones record waves and the first arrival is picked

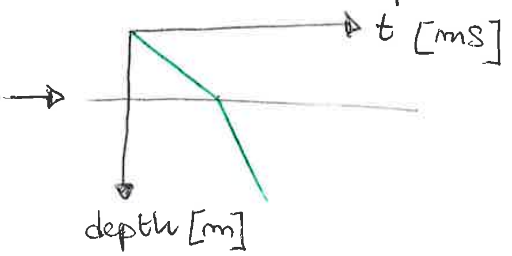
DATA ACQUISITION

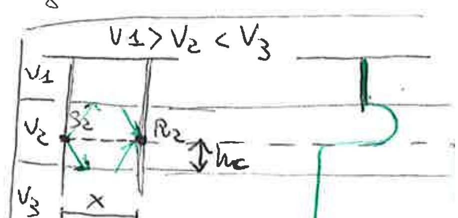
- a series of receivers along the borehole (only P waves)
- a single triaxial geophone (P and S waves)

RESULTS

The results are plot in a time-depth diagram







$$\frac{x}{v_2} = \frac{x}{v_3} + \frac{2h_c\cos(i_c)}{v_2}$$

LIMITS

- the result is the average wave velocity
- lower accuracy than cross hole test

→ we obtain h_c

* Pitfall case (inversion of velocity in the 2nd layer): when is detect the measure of velocity in the slowest layer, the seismic ray can be birefracted at the interface of layers 1-2 or 2-3. In this way can't be sure to measure a direct wave. h_c is the minimum distance below wich the birefracted waves arrives first

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 21 di 62

For that they are not affected by geom. attenuation. We can imagine that tube waves propagate along the interface water (in the hole) - ground. In the case of holes without casing, the tube waves are related to the shear waves of the geological unit.
At low frequency the c (tube wave velocity) can be determine by ↳ hole without case

$$c = \frac{V_f}{\sqrt{1 + \frac{\rho_f V_f^2}{\rho_s V_s^2}}}$$

V_f : velocity of elastic waves in the fluid (1500 m/s)
V_S : S-wave velocity in the formation
ρ_f : density of the fluid
ρ_s : density of the formation

$$V_f = \sqrt{\frac{k_f}{\rho_f}} = 1500\ m/s$$
$$V_S = \sqrt{\frac{G}{\rho_s}} \Rightarrow c = \frac{V_f}{\sqrt{1 + \frac{k_f}{G}}}$$

G : shear modulus of the formation
K_f : effective bulk modulus of fluid mix = 2,25 GPa

know c to calculate G

For hard rock with S wave velocity in the range 1200 - 3200 m/s the variation of tube wave velocity is very little → pay attention! ⇒ In hard rock it is difficult to estimate G.

Offset source - head of borehole : with zero offset different phenomena can be superimposed.
~~With~~ hydrophones are able to detect only P waves and tube waves, NOT S WAVE, but ~~w~~ they can detect some shear mechanical properties. For that the plot P wave velocity - depth has only P waves; the covered area is just a triangle due to the disposition source - receiver.

DATA PROCESSIN TOMOGRAPHY

1) each source send a signal to all the receivers → traveltimes' picking.

2) Build a mesh ~~of~~ in the interested area (area between the two boreholes). THE NUMBER OF CELLS MUST BE LOWER THAN THE EXPERIMENTAL DATA TO HAVE A GOOD ~~MATHEMATICALLY~~ ELABORATION (MATHEMATICALLY ROBUST)

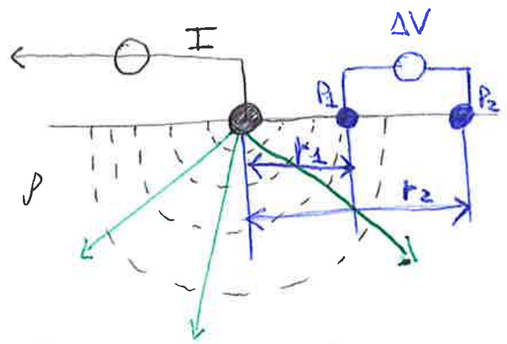
3) A STRAIGHT RAY PROPAGATION IS ASSUMED. (simplified approach, in the reality the distribution is curvilinear) THE VELOCITY IS CONSTANT IN EACH CELL.

4) Solve the forward modeling: supposed a velocity distribution, it compute the ray tracing and the theoretical traveltimes (given a discretization in cells of the region) ~~suppo~~ It's a sintetic model

5) Solve the inverse problem: starting from the collected data (traveltime), it computes the velocity distribution.

4) Is given a discretization in cells of the region. A velocity distribution is supposed the model computes the ray tracing and the theoretical travel time.

Electrical field with a single current electrode



→ current flow
---- equipotential surface
● current electrode

The second electrode is an infinity distance far away from the 1st one. The 2nd electrode closes the circuit.

$V = RI$ $\qquad R = \rho \frac{L}{A}$

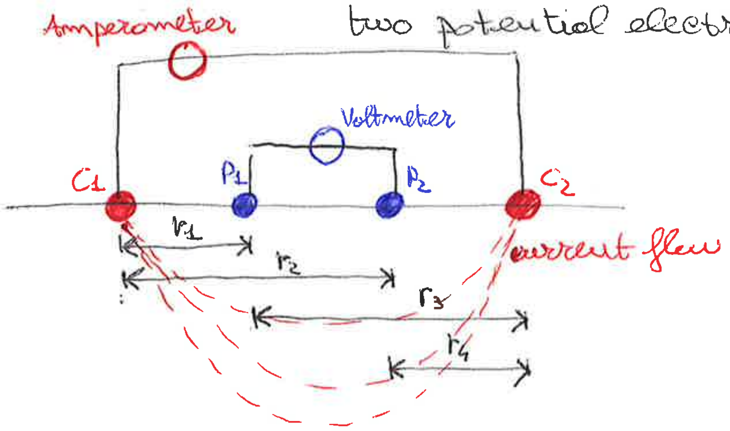
$\rho = R \frac{A}{L}$ $\qquad V = \frac{\rho I}{2\pi r}$ → the potential is related to the current by the constant R

● potential electrode
○ voltmeter

$$\Delta V = V_1 - V_2 = \frac{\rho I}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Electrical field : two current electrodes (current injection) C_1, C_2

two potential electrodes (voltage measurement) P_1, P_2



Generic formula

$$\rho = K \frac{\Delta V}{I}$$

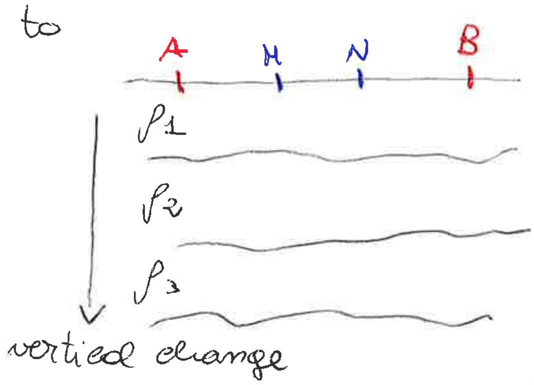
→ geometrical factor depending on the array (distance between the 4 electrodes)

$$\Delta V = \Delta V_1 + \Delta V_2 = \frac{\rho I}{2\pi}\left[\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - \left(\frac{1}{r_4} - \frac{1}{r_3}\right)\right]$$

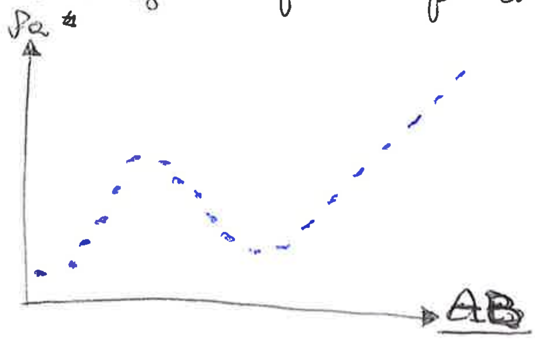
ELECTRICAL ARRAYS

1) WENNER - SCHLUMBERGER
2) DIPOLE - DIPOLE

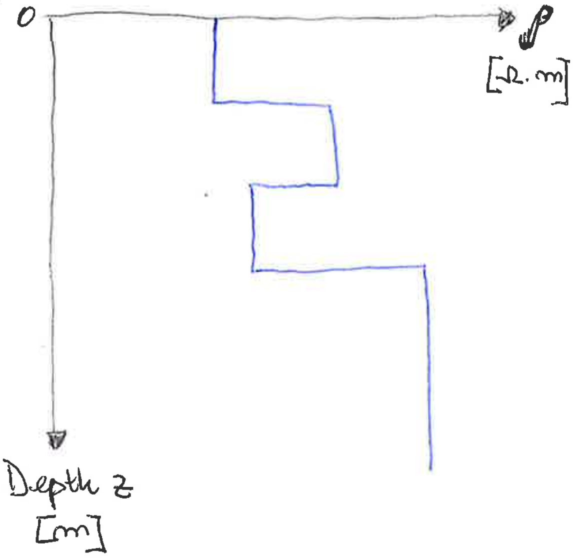
1) the potential dipole (MN) are internal to the current one (AB)
- is more sensitivity with depth → ρ vertical changes
- is less sensitive to the noise



To transform ρ_a in ρ true a inversion procedure is used.



inversion
$\longleftrightarrow$



4 LAYER CASE

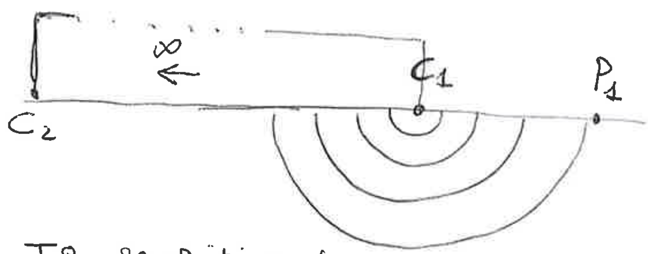
$$\rho_a = f \begin{cases} \rho_1, \rho_2, \rho_3, \rho_4 \\ h_1, h_2, h_3 \end{cases}$$

The inversion procedure provides 'infinite' solutions
Some known parameters are required to ~~solve~~ reduce the possible number of solutions.

In conclusion

- wenner: electrodes are equally spaced (distance = a)
  so $\rho = k \frac{V}{I}$ where $k = 2\pi a$
- schlumberger: $\frac{AB}{2} \geq 4 \frac{MN}{2}$ ~~to be sure~~ to do a good survey

3) Pole - Pole: only one electrode of current and one electrode of potential. The 2nd current electrode is an ∞ distance as well as the potential one.
The electrical current can be approximated by emispherical surfaces



$\rightarrow P_2 \rightarrow \infty$

$L_{C_1P_1} \ll L_{C_1C_2}$

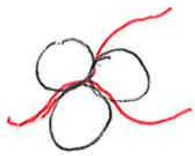
Is sensitive to the vertical and longitudinal variation of ρ

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 29 di 62

One example is the clay. For it the conductivity is mainly affected by ~~the~~ surface phenomena.

In porous aquifer the electrical conductivity depends on
- porosity of medium
- water saturation
- salinity
- ionic mobility
- temperature

In fractured rocks the electrical conductivity depends on
- density of fractures
- tortuosity of the electrical paths inside the fractures.



THEORIES AND MODELS DESCRIBING ELECTRICAL PROPERTIES OF ROCKS
1) Model of Archie for saturated rock
2) Model for unsaturated rock
3) Effect of clay particles

1) The model of Archie is an empirical relationship for sandy soil (non clayey soil). The model assums that the electrical resistivity of the soil skeleton is negligible (trascurabile)

$\sigma_m = a\phi^m \cdot \sigma_f$

SATURATION CONDITION

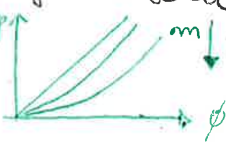
σ_m : el. cond. of the sand

σ_f : el. cond. of the pore water

ϕ : porosity

a, m : experimental parameters depending on the shape of the solid particles

m is the cementation exponent.



For • unconsolidated sand $m = 1{,}3$
• consolidated sandstone $1{,}8 < m < 2{,}0$

m keeps into account the grains disposition that is the tortuosity of porosity channel of the sample → is a index of E.C. !

$m \uparrow$ if grain sphericity $\downarrow$ ➡

Model of Waxman - Smith → unsaturated porous media with surface conductivity

$$\sigma_m = \phi^m S_w^n \left(\sigma_f + \frac{BQ_v}{S_w} \right)$$

surface conductivity → CEC Cation Exchange Capacity

ELECTRICAL PERMITTIVITY

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

ε_r = relative electrical permittivity
ε = absolute value of permittivity
ε_0 = permittivity in vacuum $\left(8{,}85 \cdot 10^{-12} \frac{F}{m}\right)$

ε_r varies from 1 = vacuum
to 81 = water

So high water content, high ε_r value!

For $f \gtrsim 1$ MHz we can write

$$v = \frac{1}{\sqrt{\varepsilon \cdot \mu}}$$

(in seismic $V_P = \sqrt{\frac{E}{\rho}}$)

μ = magnetic permeability
$\mu_0 = 4\pi \cdot 10^{-7}$ Henry/m

If I rewrite v equations in terms of relative values:

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \cdot \mu_0 \mu_r}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}\sqrt{\varepsilon_r \mu_r}}$$

$$= \underset{c}{\underset{\shortparallel}{v_{vacuum}}} \cdot \frac{1}{\sqrt{\varepsilon_r \mu_r}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$$

where $c = 3 \cdot 10^8$ m/s speed of light

In most of rocks $\mu \simeq 1$ so

$$v = \frac{c}{\sqrt{\varepsilon_r}}$$

CRIM : COMPLEX REFRACTIVE INDEX MODEL

Check!

$$\sqrt{\varepsilon_r} = (1-\phi)\sqrt{\varepsilon_{soil}} + \phi\sqrt{\varepsilon_{fluid}}\, S_w$$

(Same for density in seismic)

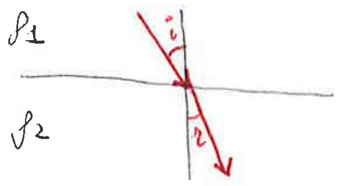
So I can rewrite in

$$\frac{1}{v_{bulk}} = (1-\phi)\frac{1}{v_{soil}} + \frac{\phi\, S_w}{v_{fluid}}$$

(Similar to the Willie's law)

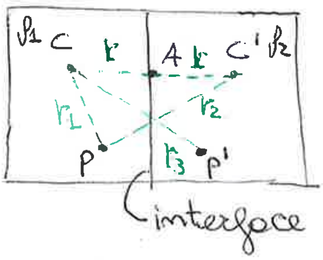
At the interface between two different media, the current lines are refracted according to:

$$\frac{\tan(i)}{\tan(r)} = \frac{\rho_1}{\rho_2}$$



if $\rho_1 > \rho_2$ the current flow tends to diverge with respect to the normale behaviour.
il $\rho_2 > \rho_1$ the current flow tends to converge.

ELECTRICAL POTENTIAL OF 2-HALF SPACES



The interface is considered as a semi-reflecting surface with a reflection coeff. $= k$

C = intensity current source I

C' = image point

$$V_P = \frac{I\rho_1}{4\pi}\left(\frac{1}{r_1} + \frac{k}{r_2}\right)$$

$$V_{P'} = \frac{I\rho_2}{4\pi}\left(\frac{1-k}{r_3}\right)$$

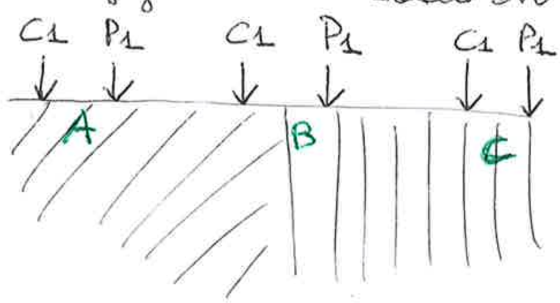
At the point A $V_P = V_{P'}$ and $r = r_1 = r_2 = r_3$

→ we calculate k from $V_P = \frac{I\rho_1}{4\pi}\left(\frac{1+k}{r}\right) = \frac{I\rho_2}{4\pi}\left(\frac{1-k}{r}\right) = V_{P'}$ a solution in function of ρ_1, ρ_2

Now we know ρ_1 and ρ_2 and we can calculate the potential trend in each point.

The image source theory can be performed with

- a ~~combination~~ pole-pole array configuration in boreholes
- a semi-Wenner array (pole-pole array) to study discontinuity both vertical and horizontal. 3 different configuration based on the interface position



A $\frac{\rho_a}{\rho_1} = 1 + \left[\frac{ka}{(2s-a)}\right]$

B $\frac{\rho_a}{\rho_1} = 1 + k$

C $\frac{\rho_a}{\rho_1} \simeq k^*\left[1 - \frac{ka}{(2s+a)}\right]$

$$k^* = \frac{1+k}{1-k}$$

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 35 di 62

At the interface layer 1 - layer 2 a new reflection - transmission phenomena occurs.

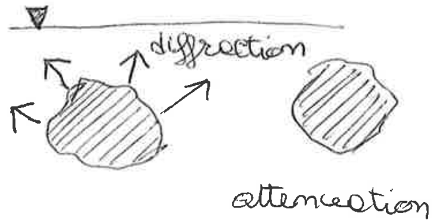
→ For each interface there is a scattering phenomenon, that is a loss of energy.

e.m. impedence : $z = \rho v \rightarrow z = f(\varepsilon, \mu)$

$R = \dfrac{z_2 - z_1}{z_2 + z_1}$ REFLECTION COEFFICIENT

So we have to keep into account:

- loss of energy for attenuation that is an intrinsic parameter
- loss of energy for diffraction of the different materials



ELECTROMAGNETIC METHODS: FREQUENCY DOMAIN INVESTIGATION

The e.m. techniques can be classified in

FEM: frequency domain, use one or more frequencies

$i(t) = i_0 \, sen(\omega t)$
$\quad \hookrightarrow f$

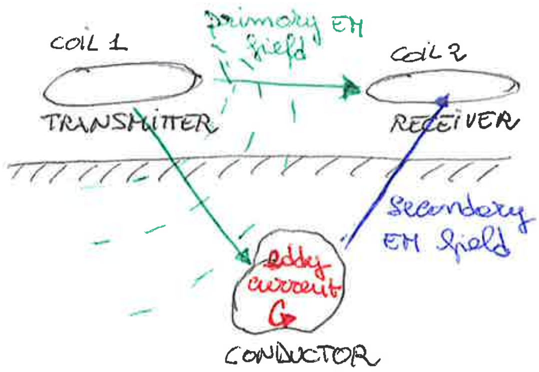
TEM: time domain, the measurements are a time function. IS observed the signal behaviour during the time.

Methods are divided in:

- passive methods: utilizes the natural groun signal
- active methods: an artificial transmitter is used. Can be in the near field or in the far field

The aim is to map the conductivity/resistivity of the subsoil.

INTERACTION E.M. FIELD - GROUND ; E.M. INDUCTION

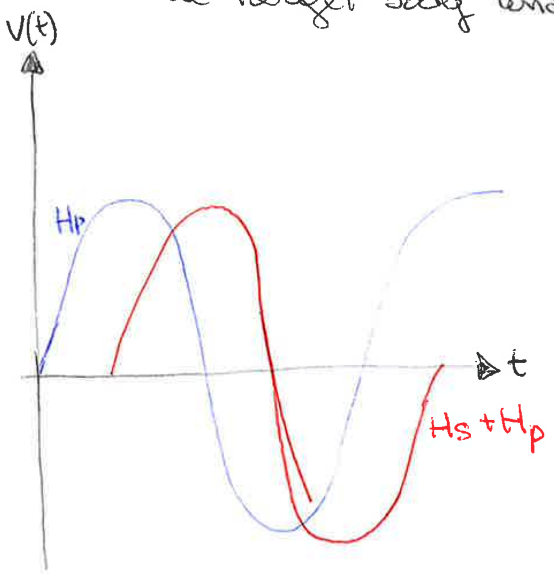


TRANSMITTER COIL: generates the primary e.m. field that propagates in the ground and in the atm.

If the subsurface is homogeneous there is no difference between the field propagated both in atmosphere and in the ground (only slight reduction in amplitude)

CONDUCTIVE ANOMALY: when the primary e.m. field meets a conductor in the ground, it is induced an alternating current (called Eddy current) within the conductor. The eddy current generates their own SECONDARY EM FIELD wich propagate to the coil 2 - THE RECEIVER

So to the RECEIVER arrives:
- the primary magnetic field induced by the transmitter
- the secondary e.m. field induced by the conductor
  → this is what I want to study because it comes from the target body underground.

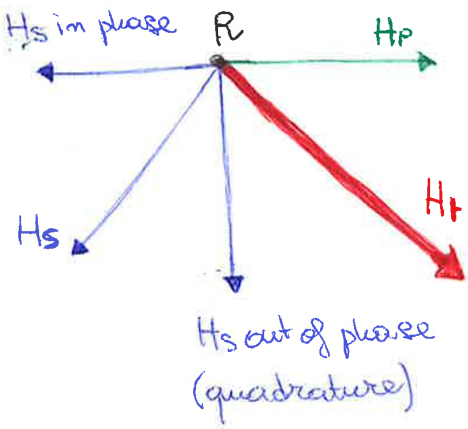


- primary e.m. field Hp
- secondary e.m. field + primary e.m. field Hs + Hp

The measured response at the receiver will differ in phase and in amplitude relative to the unmodulated Hp.
This differences reveals the presence of the conductor.

From the Hs it is possible to know the
- shape
- conductivity   of the conductor
- depth

At the receiver the situation is these



H_p : primary m. field

H_s : secondary m. field in 2 forms:

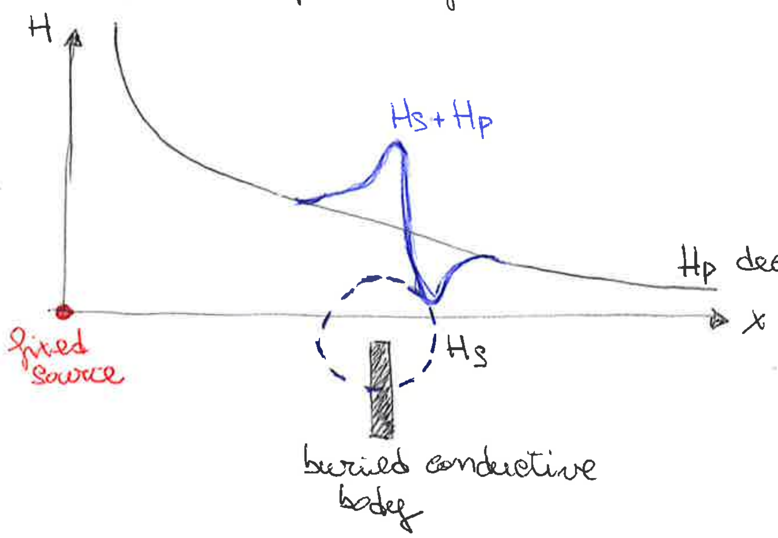
- H_s in phase, shifted of 180° respect to H_p
- H_s out of phase, shifted of 90° respect to H_s

H_r : resultant of the primary and secondary m. field.

H_s in phase : more sensitive to $\mu \to$ inductance

H_s out of phase : more sensitive to $\sigma \to$ resistivity.

Example : response of a vertical conductive body



The receiver (coil) is moved

H_p decrease with the distance

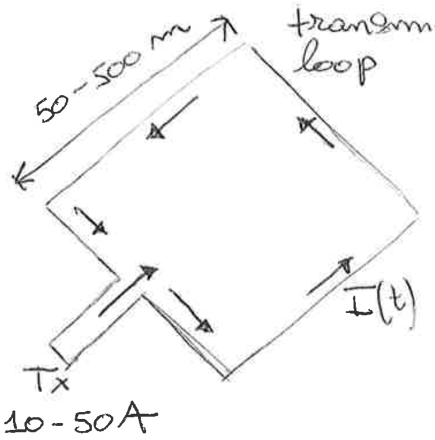
EQUIPMENT USING SMALL LOCAL SOURCE

Small distance source - receiver ; they are portable and are moved along a transect. The distance can vary from 1 m up to 20 - 40 m.

- Small depth 1-4 m , f = 20 kHz
- intermediate depth 4-6 m, f = 10 kHz
- high depth penetration 20-50 m, f = 300 Hz

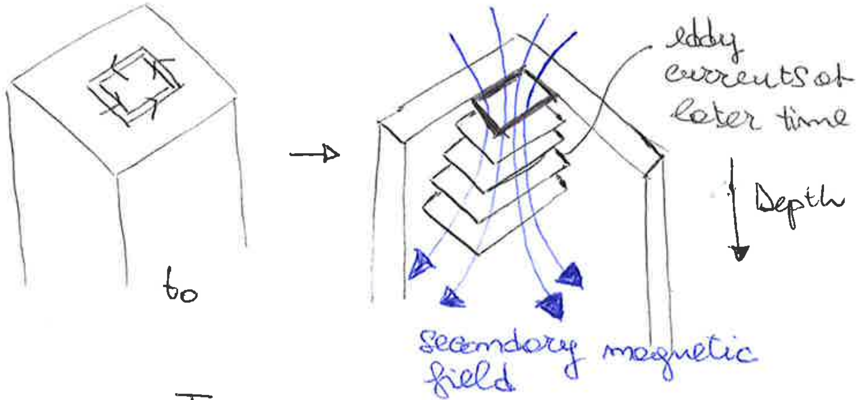
↑ depth ↓ f

→ concept of skin depth

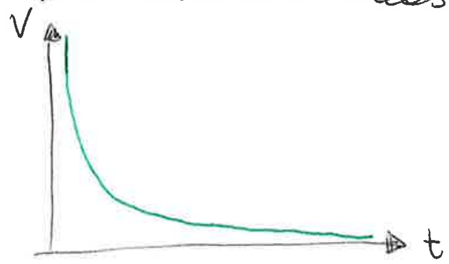


Max current injected 50 A

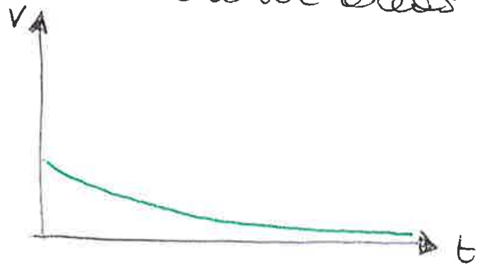
The bigger the loop, the greater the investigation depth.



In resistive areas



In conductive areas



Decay in time means decay of depth of investigation

The diffusion velocity is $\quad \mu$: magnetic permeability

$$v = 2\left(\frac{1}{\pi\sigma\mu t}\right)^{0,5}$$

and the distance of propagation is

$$d = \left(\frac{2t}{\sigma\mu}\right)^{0,5} \rightarrow$$

connection between time and depth. If I can observe for longer, I can go deeper!

and the skin depth is

$$\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{0,5} \rightarrow$$

if I express $\omega = 2\pi f$ in function of period T, $\omega = \frac{2\pi}{T}$, I can express δ in function of time

$$\Rightarrow \delta = \left(\frac{T}{\pi\mu\sigma}\right)^{0,5}$$

$\delta \downarrow$ if $\sigma \uparrow$

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 43 di 62

- TDEM can be used in topographically rugged areas

EM FIELD : in TDEM there aren't interference in the Hp because measurements ~~are~~ take place during the absence of energization → accurate vertical surveys also lateral surveys.

DEPTH : In TDEM the investigation depth is not linked to the skin depth but only to the power delivered by the transmitter and to the electrical noise of the receiver.

SURVEY : in TDEM is not necessary to move electrodes to go deeper. The depth depends only on the coil configuration.

BASIC EQUATIONS AND PARAMETERS

$\vec{D}$ electric displacement $[C/m^2]$
q charge volumic density $[C/m^3]$
$\vec{E}$ electric field intensity $[V/m]$
I electric current intensity $[A]$
$\vec{J}$ electric current density $[A/m^2]$
$\vec{H}$ magnetic field intensity $[A/m]$
$\vec{B}$ magnetic field flux $[T$ or $Wb/m^2]$

$$\left.\begin{aligned} \operatorname{div}\vec{D} &= q \\ \operatorname{div}\vec{B} &= \phi^{\text{zero}} \\ \operatorname{curl}\vec{H} &= \vec{J} + \frac{d\vec{D}}{dt} \\ \operatorname{curl}\vec{E} &= -\frac{d\vec{B}}{dt} \end{aligned}\right\}$$

Maxwell's equations

this equations clarly show that the fields $\vec{E}$ and $\vec{H}$ are linked and ~~that~~ when exists a time-varyin- E field there will be also a time varying H field.

+ curl = rotore

$$\begin{aligned} \vec{J} &= \sigma\vec{E} \\ \vec{D} &= \varepsilon\vec{E} \\ \vec{B} &= \mu\vec{H} \\ \operatorname{div}\vec{D} &= \phi \quad (\sigma > \phi) \end{aligned}$$

$$= \quad \left.\begin{aligned} \nabla^2\vec{H} &= \mu\sigma\frac{\partial\vec{H}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{H}}{\partial t} \\ \nabla^2\vec{E} &= \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t} \end{aligned}\right]$$

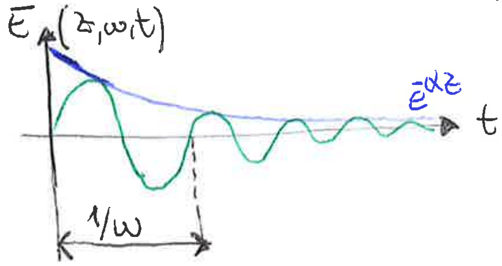
the 2 fields are $\perp$

can be $\mu\varepsilon << \mu\sigma \rightarrow 10^{-17} << 10^{-12}$

$$\Rightarrow \nabla^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t}$$

The two equations have the same solution in form of:

$$E = E_0 e^{-\alpha z} e^{i(\omega t - \beta z)}$$



α : dissipation factor $[1/m]$
β : propagation factor $[rad/m]$
$\frac{2\pi}{\beta} = \lambda \rightarrow$ wave length $[m]$
$\frac{\omega}{\beta} = v$ velocity $[m/s]$
z = depth

45

DATA PROCESSING

COMPONENTS	FUNCTION
2) move start - time :	main bang (interection of refllection with the ~~signal~~ receiver → the signal enters in the ground.) → eliminate useless time before the main bang
4) gain setting :	recover information by multipling amplitude for a linear growing function.
1) de - wow :	is eliminated the noise with low frequency thanks to a ~~lo~~ high-pass filter.
3) time cut :	a lot of data from a certain point on are useless because are only noise; they are cut to make the process faster.
5) Band pass filtering :	an antenna of 100 MHz will emitts a signal in the range of $100 - \frac{100}{2} \leq \text{signal} \leq 100 + \frac{100}{2}$ the central frequencies are usefull, all that's out of the band is cut and it's cut with a band pass filter.
6) Background removal :	average ~~of~~ traces on rows traces, from every row is substracted the average of the signal. The main bang is always the same on every column due to eletronics and physics.

7) Assembling the required GPR profiles, according to the GPS coordinates

On most of medium $\mu_r \simeq 1$ so the velocity can be expressed as

$v = \frac{c}{\sqrt{\mu\varepsilon}}$ is a approximated, simplified velocity

Also in GPR method the EM waves are attenuated ~~follo~~ for
- geometrical spreading → spherical divergence
- intrinsic attenuation → ~~dissipation~~

Now it's demonstrated that if I know ε_1 and R I can calculate ε_2.

I define $e_i = \sqrt{\varepsilon_i}$

$$R = \frac{e_1 - e_2}{e_2 + e_1} \rightarrow Re_1 + Re_2 = e_1 - e_2$$

$$Re_1 - e_1 = -(Re_2 + e_2)$$

$$(R-1)\,e_1 = -e_2(R+1)$$

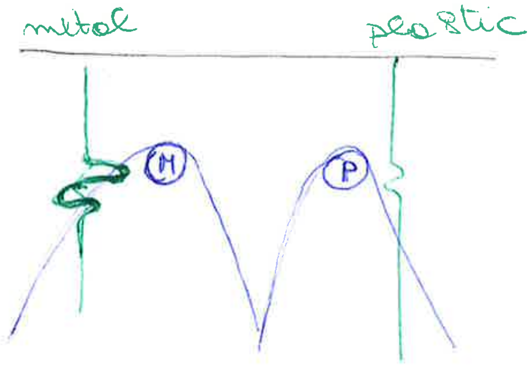
$$\rightarrow e_2 = -\frac{R-1}{R+1}\,e_1$$

When R = 1 the material is like a MIRROR and all the signals are reflected.

ex: metal $\sigma = 10^{-4}$ $\varepsilon = 1$ → MIRROR

plastic $\sigma = 10^{4}$ $\varepsilon = 3$

In the radargram



Interpolating the arrival times is obtained an hyperbole called DIFFRACTION HYPERBOLA

Is the response of the object to pulses.

Plastic is more transparents to EM pulse.

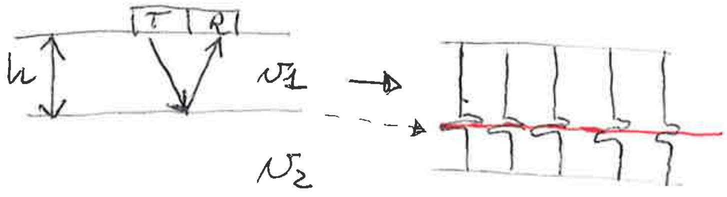
R - T rules are true if

- $\theta_{incident} \leq 20°$
- $\lambda_2 \ll h_2$ (thickness of the 2nd layer) → if not THIN LAYER THEORY

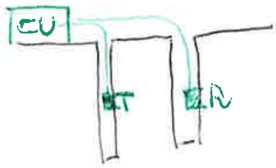
Reflector parallel to the ground surface

You can see in a radargram the reflected pulse on a horizontal line at the time (two way travel time)

$$twt = \frac{2h}{v_1}$$

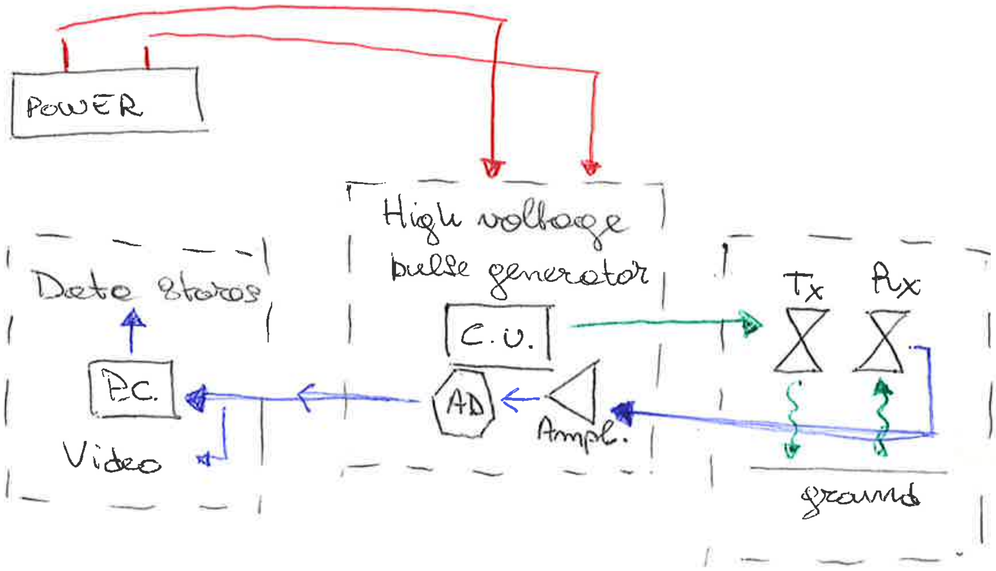


two boreholes: moving T and R at the same time in 2 differents boreholes.
moving only R to do a GPR TOMOGRAPHY

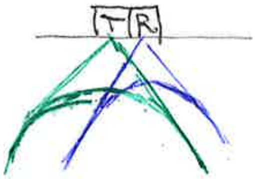


↳ CRIM RULE $\sqrt{\varepsilon} = (1-\phi)\sqrt{\varepsilon_s} + \phi\sqrt{\varepsilon_f}$

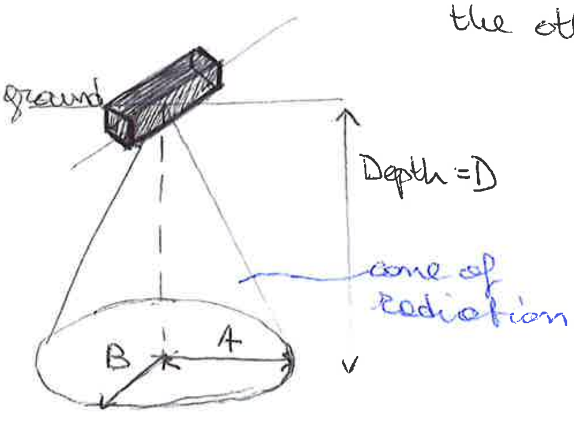
OUTILINES OF THE INSTRUMENTS



Antenna: emits EM energy in a cone of radiation; the receiving antenna gathers the return signal in a similar cone. This is the reason why the antenna can "see" out of it's vertical.



The antenna can be
- monostatic configuration: 1 dipole to transmitting and receiving
- bistatic conf.: 2 dipoles, one transmits the other receives



the footprint becomes bigger with the increasing depth and depends on characteristics of the soil and f

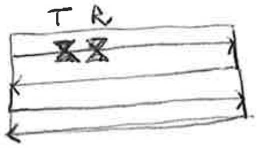
$A = \frac{\lambda}{4} + \frac{D}{\sqrt{\varepsilon_r + 1}}$ $\lambda = \frac{c}{f} = \frac{c}{\sqrt{\varepsilon_r}\, f}$

$B = \frac{A}{2}$

$\Delta t \rightarrow$ according to the Shannon's law $\Delta t \leq \frac{1}{2 f_{max}} \xrightarrow{better} \Delta t \leq \frac{1}{6 f_{max}}$

$T \rightarrow$ releated to the depth $T \geq 4 \frac{D}{v_{max}}$

The survey procedes with parallel lines and eventually orthogonal lines. Don't turn the antenna to not lose the geoposition references.



ELECTRICAL METHODS

$R = \frac{V}{I}$ [kom] ; $R = \rho \frac{L}{A} \Rightarrow \rho = 2\pi r \frac{V}{I} \Rightarrow \rho = k \frac{V}{I}$

$\Delta V = \Delta V_1 - \Delta V_2 = \frac{I\rho}{2\pi}\left[\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - \left(\frac{1}{r_3} - \frac{1}{r_4}\right)\right]$

Archie's model - saturated soil $\quad \sigma_{bulk} = a\phi^m \sigma_f$

- unsaturated " $\quad \sigma_{bulk} = a\phi^m \sigma_f S_w^n$

Uns. soil + clay presence $\quad \sigma_{bulk} = \sigma_f \theta(a\theta + b) + \sigma_s$

Waxman and Smith

uns. soil + surface phen. $\quad \sigma_{bulk} = \phi^m S_w^n \left(\sigma_f + \frac{BQ}{S_w}\right)$

Formation factor $F = \frac{\rho_o}{\rho_w} = \frac{1}{\phi^m} = \frac{\sigma_f}{\sigma_{bulk}}$

$\sigma_{surf} = \frac{BQ}{F}$

$\frac{\varepsilon}{\varepsilon_r} \rightarrow v = \frac{1}{\sqrt{\mu\varepsilon}} \Rightarrow v = \frac{c}{\sqrt{\varepsilon_r}}$

CRIM Complex refractive index model

$\sqrt{\varepsilon_r} = (1-\phi)\sqrt{\varepsilon_{soil}} + \phi S_w \sqrt{\varepsilon_w}$

$\rightarrow \frac{1}{v} = \cancel{\sqrt{\varepsilon_r}} \frac{1}{v_{soil}}(1-\phi) + \phi S_w \frac{1}{v_w}$

REFLECTION COEFF.F. $\quad k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$

IMAGE SOURCE THEORY $\quad \Delta V = \frac{I\rho_1}{4\pi}\left(\frac{1}{r_1} + \frac{k}{r_2}\right) \quad \Delta V' = \frac{I\rho_2}{4\pi}\left(\frac{1-k}{r_3}\right)$

ELECTROMAGNETISM

SKIN DEPTH $\quad \delta = \left(\frac{2}{\mu\sigma\omega}\right)^{0.5}$

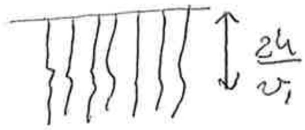
REFLECTION COEFF. $\quad RC = \frac{I_2 - I_1}{I_1 + I_2}$

MAXWELL'S EQ.

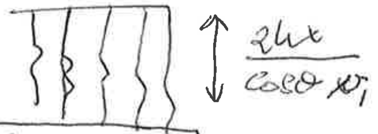
$$\begin{cases} \nabla \times E = -\mu \frac{\partial H}{\partial t} \\ \nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t} \end{cases} \longrightarrow \begin{matrix} E = E_0 e^{j\omega t} \\ H = H_0 e^{j\omega t} \end{matrix} \rightarrow \begin{matrix} \nabla^2 E = \mu\sigma j\omega E - \mu\omega^2 \varepsilon E \\ \nabla^2 H = \mu\sigma j\omega H - \mu\omega^2 \varepsilon H \end{matrix}$$

$|\sigma| \Leftrightarrow |\mu\omega|$

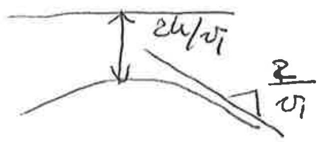
REFLECTOR PARALLEL $t_{tot} = \frac{2h}{v_1}$



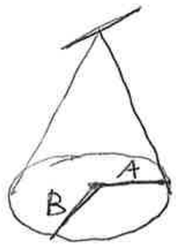
INCLINATED REFLECTOR $t_{tot} = \frac{2h\ x}{\cos\theta_i\, v_1}$



POINT LIKE OBJECT $t_{tot} = \frac{2h\sqrt{(x-x_0)^2+h^2}}{v_1}$



FRESNELL'S ELLIPSE



$A = \frac{\lambda}{4} + \frac{D}{\sqrt{\varepsilon_r+1}}$

$B = \frac{A}{4}$

minimum resolutions

$\Delta x = \sqrt{\frac{\lambda D}{2}}$ $\Delta z = \frac{\lambda}{4}$

N° OF SAMPLE $N = \frac{T}{\Delta t} + 1$

LABWORK 2

seismic_raypath.m

① Exercise of ray tracing in a heterogeneous medium

The code implement equation for ray-path in a medium were velocity gradually changes with depth according to the function $V = v_0 + Cz$

v_0 = surface v

C = constant

z = depth

The code ~~it~~ models 20 rays: each ray is continously refracted along the raypath

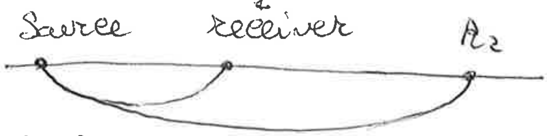
1^{st} figure = ray tracing → in the figure it's recreate the coordinates traveltime - distance from the source

2^{nd} " = wavefronts

~~labwork 2.m~~
~~plot the traveltimes of direct, reflected, refracted waves~~
~~① 2 layers~~

The wavefront are not longer spherical surfaces and traveltimes (T) depend on the curvature of the ray path

$$T(l) = \int_0^l \frac{dl}{v(l)} \sim \text{curvature}$$



↳ we can reconstruct traveltime from the curvature

② labwork2.m

plot the traveltimes of ~~direct~~, reflected and refracted waves

3 layers

$v_1 < v_2 < v_3$