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A P P U N T I

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MATERIA: Teoria e Progetto dei Ponti - Bertagnoli

TABLE OF CONTENTS

4. INFLUENCE FUNCTION

- metodo diretto
- metodo indiretto

5. GIRDER BRIDGES

- Slab design
- Courbon method
- Engesser method
- Girder bridges influence function
- Transversely rigid girder bridges

6. DESIGN OF REINFORCED CONCRETE SHELL ELEMENTS

- Internal action and sign convention
- Resisting mechanism definition
- Internal layer design
- External layer design
- Slab bridges - sandwich model: numerical example

7. LOCAL EFFECTS

- Shear lag in T beams
- Curved beams
- Design for local effects
- Slab design as a rectangular slab
- Local effects in steel orthotropic deck

8. STRUT & TIE

9. PRESTRESSED BOX GIRDER BRIDGE

Static scheme, section shapes and construction procedure

- Evolution of static scheme and section shapes
- Most common construction procedures
- Longitudinal design
- Transverse design
- Shear keys
- Precast sequental construction
- Incremental launching

Analysis in transverse direction
- Bleadt anomaly
- Operative steps of the analysis in transverse direction
- Procedure to take into account the transverse effects
- Transverse actions induced on the box girders with variable depth
- Dimensioning of diaphragms

Interaction between longitudinal shear and transverse bending in the design of the webs
- Sandwich model : actions
- Sandwich model : interaction domains
- Numerical example

Discontinuity regions details
- Transfer of the actions from the deck to pier
- Diffusion of prestressing in the extreme zone (of the box girder beams)
- Numerical example
- Flange: web connection

10. BEARINGS
- Joints

11. PIERS
- Description
- External actions
- Internal actions evaluation
- General method for 2nd order effect
- Details

12. ABUTMENTS
- Description
- Actions
- Verifications
- Details

13. FOUNDATIONS
- Description
- Preliminary dimensioning
- Jet-grouting
- Footings on piles

- Footings on walls
- Foundation on micropiles
- Shaft foundations
- Caisson foundations
- Tied foundations

14. IL PROGETTO STRUTTURALE

- Precompressione con cavi esterni
- Comportamento a taglio di elementi monolitici con armature ordinaria
- Comportamento a torsione di elementi monolitici con armature ordinaria
- Comportamento a taglio di elementi prefabbricati con giunti non armati
- Comportamento a torsione di elementi prefabbricati con giunti non armati

1. BASIS OF DESIGN

Input data in bridge design

• Planimetric configuration	Layout (Radius of curvature, span...) Obstacles to be crossed (rivers, roads, railways, valleys...)
• Altimetric configuration	Max and min depth, slope,... Curvature radius in the vertical plane
• Required transverse section (Geometry)	Live load (train, truck...) Gangway number and width Carriageway number (single, multiple) Sidewalk, services...
• Foundation	Geological and geotechnical investigations Hydraulic studies (max flood, score) → SCALZAMENTO Stratigraphies Landslides (old landslides, landslides in action...) → FRANE
• Local conditions for buildabiliy	Access roads Local availability of materials Local availability of common and specialized workers
• Environmental and metheorological conditions	Expected water level Expected tide level Expected drought period Expected temperatures during the construction
• Foundation	Geological and geotechnical investigations Hydraulic studies (max flood, score) Stratigraphies Landslides (old landslides, landslides in action...)

• Local conditions for buildabiliy	Access roads Local availability of materials Local availability of common and specialized workers
• Environmental and metheorological conditions	Expected water level Expected tide level Expected drought period Expected temperatures during the construction
• Environmental conditions in which the bridge has to be built	Open field Hill landscape Valley between close mountains Historical city (old and small depth buildings) Modern city (new and tall buildings)
• Environmental requirements	Aesthetic requirements Noise protection Wind protection Splash protection
• Functionality requirements	Deformability limitations (High speed lines) Limitation and control of vibration amplitude and frequency (High speed lines and pedestrian bridges)

Classification of bridges

- obstacle to be crossed
- service class
- material to be used
- statical scheme

(a) obstacle to be crossed

- **Bridges** — River / Channel / Gorge → Hydraulic restraints →
  - Maximum flood (during construction and lifetime)
  - Expected erosion
  - Maximum level of water
  - Clearance for navigation and/or objects
  - Expected score

spazio vuoto tra ponte e corso d'acqua

- **Viaducts** — Land cutting / Horographic constraints / Functional requirements → Restraints →
  - Existing infrastructures
  - Geological and geotechnical conditions
  - Landscape requirements

- **Overpasses and underpasses** (Overpassing structures) → Restraints →
  - Clearance
  - Hydraulic restraints

- **Flyover** (Overpassing of urban areas) → Restraints →
  - Urban constraints
  - Hindrances to the execution

(b) service class

- 1° category road bridges → più importanti per i carichi supportati
- 2° category road bridges
- Pedestrian bridges
- Railway bridges : Passenger traffic / Mixed traffic

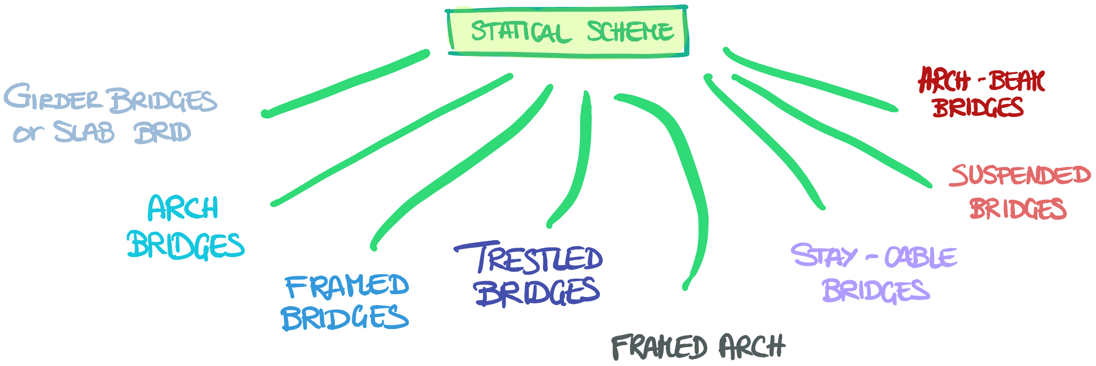
} Deck bridges and Trough bridges

- Channel bridges or installation-bearing bridge

(c) material to be used

- reinforced concrete
- prestressed concrete
- steel
- steel-concrete
- masonry
- wood

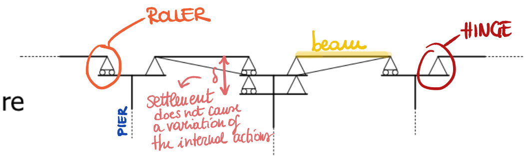
(d) static scheme



• Girder bridges or slab bridges (ponti a graticcio e a piastra)

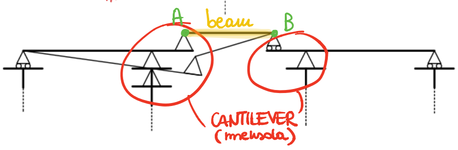
ponti fatti di travi

- Simply supported (ISOSTATIC) — Prefabrication; Settlements, temperature

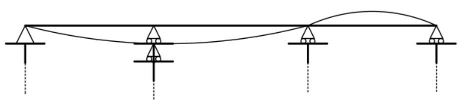


- Gerber (ISOSTATIC) — Internal actions distributions; Settlements, temperature

if I wanna solve a structure by hand calculation (small deformations)



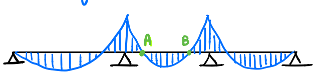
- Continuous Beam (IPERSTATIC) — Best use of material



↓ problem of maintenance
↓ I solve the issue using

I have to check foundation behaviour 'cause I don't want any settlements.
(settlements cause variation of internal actions)

it has the same bending moment of a continuous beam



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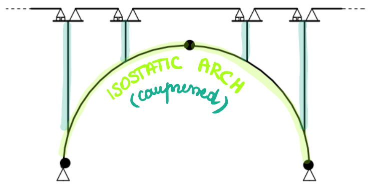
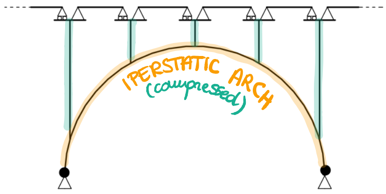
Arch bridges

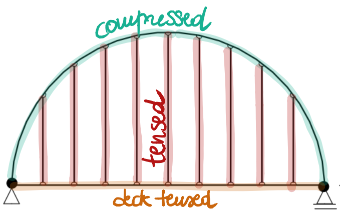
- Three hinges arches



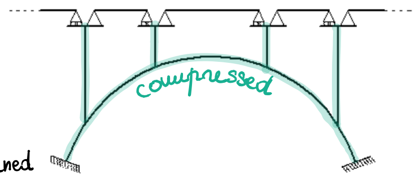
- Two hinges arches



- Bow string
  (bottom deck)



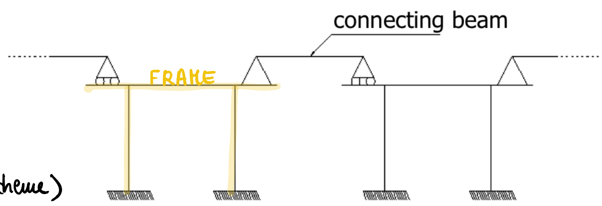
foundations work only for vertical forces ⇒ cheaper. Due to this horizontal forces (coming from the arch) goes on the deck and not in the foundations



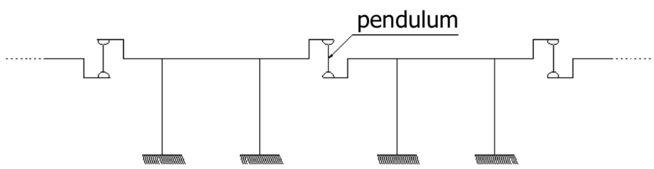
- Fully restrained arch
  used only on solid rocks due to the fully restrained supports (⊥)

Framed bridges (intelaiati)

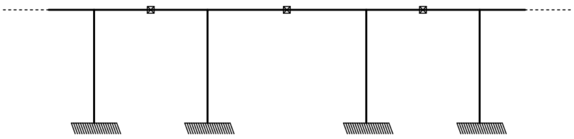
- Frames with connecting beam
  (not a nice statical scheme)



- Frames connected by pendula



- Continuous frames after mutual connection

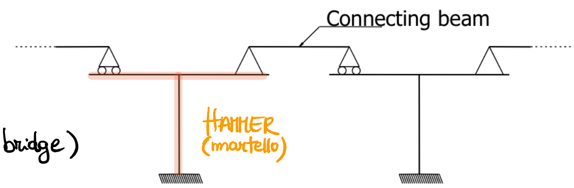


{ - Low cost bridges
- Good seismic dissipation

if there aren't joints
↓
INTEGRAL BRIDGES

Calculation for
– temperature
– settlement
are not easy
↳ and in general all imposed deformation (creep - shrinkage)

- Hammer with connecting beam



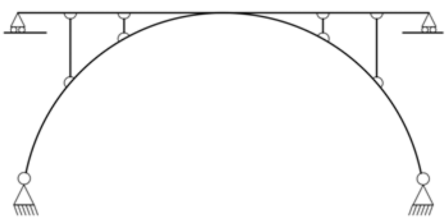
(Similar to Gerber bridge)

Trestle bridges (ponti a cavalletto)
↳ not used anymore

- Single trestle

- Tied trestle

Framed arch – Maillart like



–6–

Stay-cable bridges

FAN SOLUTION (soluzione a ventaglio, gli stralli convergono in un solo punto A) (VS HARP SOLUTION) → or mixed solution

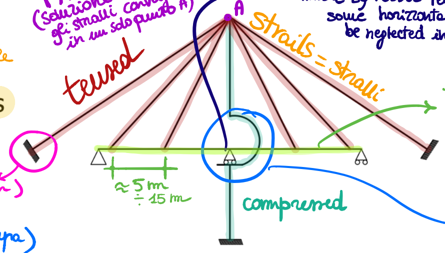
it is not a very roller 'cause the support is made by rubber restrain, so there is some horizontal restrain but it can be neglected in calculations.

- With ground-anchored stays

are deformable

non voglio vincoli rigidi perchè il deck è molto deformabile (non voglio che si rompa)

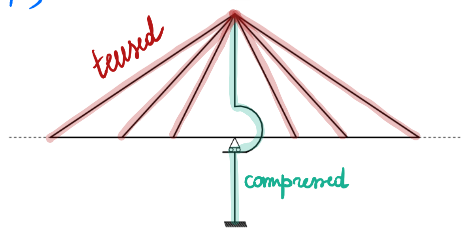
ANCHORING STAY (anchored on foundation)



thin deck ⇒ no stiffness

there is no connection between the deck and the pier (continuos pier from the bottom to the top)

- With stays anchored to the deck

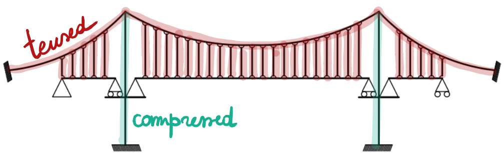


The FAN SOLUTION has the problem to have a lot of cable in a short point (•A) → too much internal action

Suspended bridges

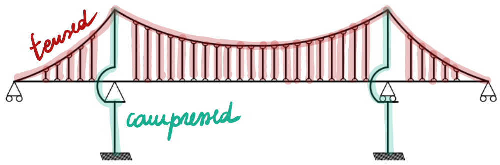
(if I double the span the difficulty increases on the square ($2^2=4$))

- Anchored to the ground



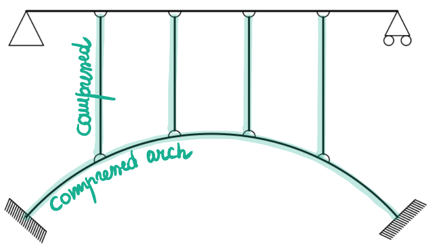
Wind is the real problem of this kind of bridges 'cause the deck is very thin ⇒ wind moves and lift the deck

- Self-anchored (to the deck)



Arch-beam bridges

- Inverted suspended bridge



CONSTRUCTION SYSTEM

- MOVING SCAFFOLDING
- FREE CANTILEVERING
- CANTILEVER WITH AUXILIARY TENDONS
- CANTILEVER WITH AUXILIARY TRUSS BEAM
- PRECAST SEGMENTAL CONSTRUCTION
- FULL SPAN PREFABBRICATION
- INCREMENTAL LAUNCHING

(a) Moving scaffolding (scaffolding = cassero)

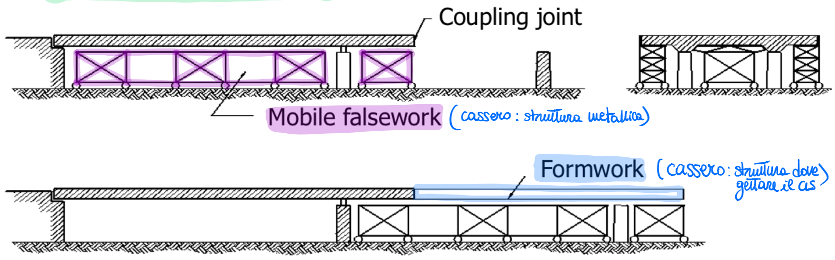
It is used to save money: I use the same scaffolding in many different positions

- Formwork on fixed falsework (o anche MOLD (cassero)) (SCAFFOLDING → Steel structure)
  - Falsework safety
  - Deformation during the casting
  - Removal of falsework
- Formwork on mobile falsework
  - Sliding beams of formwork
  - Foundations of formwork
  - Deformation during the casting

Formwork is made of panels and accessories that act as a mold to form a desired shape with concrete (for any purpose). Falsework is the temporary support structure for the forms.

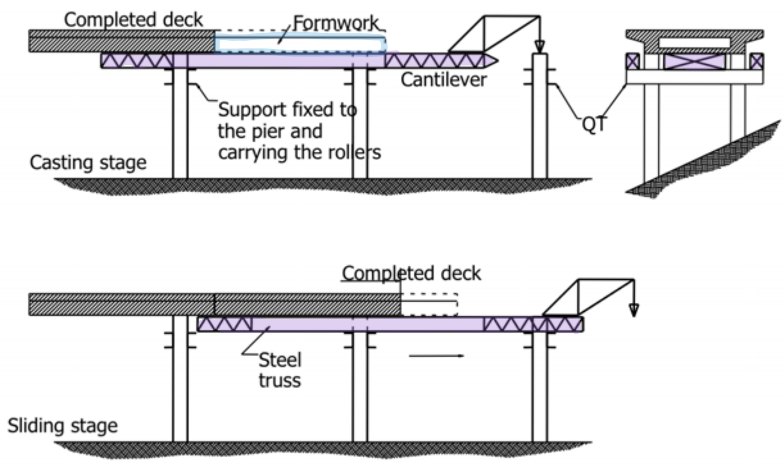
Falsework refers to temporary structures used in the construction to support arched structures and concrete forms (moulds) in order to hold the component in place until its construction is sufficiently far advanced to support itself.

a) Falsework on the ground (h ≤ ~ 10 m)



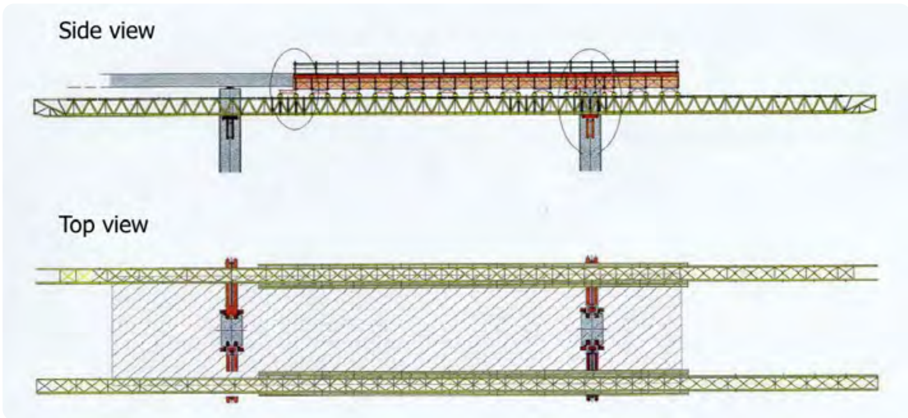
Like "Rivalta Bridge": I place the scaffolding on the first span and a little cantilever of the second one, and then I cast concrete. After harde-ning I move the scaffolding upwards and I go on.

b) Falsework sliding on rollers from pier to pier

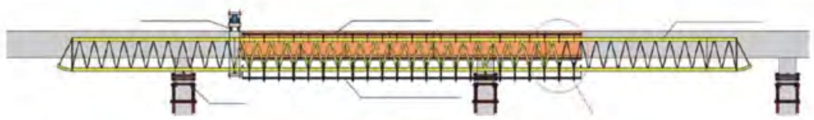


The scaffolding is sliding not connected to the ground but to the piers.

It cost more then the previous tecquinique, so it is used to build long spans.



← SCAFFOLDING OVER FALSEWORK

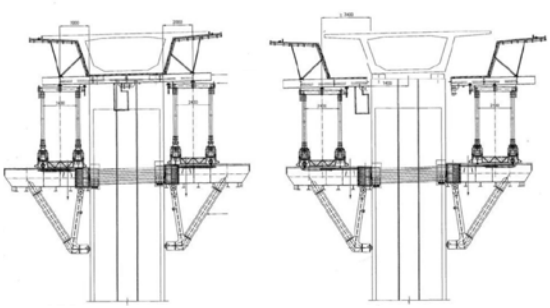


← SCAFFOLDING WITHIN FALSEWORK

Transverse view of scaffolding movement: transverse slide

Closed position: concreting

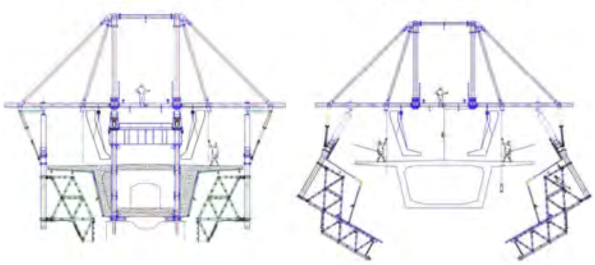
Open position: avancement



Transverse view of scaffolding movement: opening with rotation

Closed position: concreting

Open position: avancement

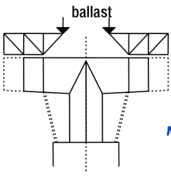


(b) FREE CANTELEVERING [Finsterwalder / dywidag]

The idea is: I build the pier and then from the top of it I start to build the deck in segments.

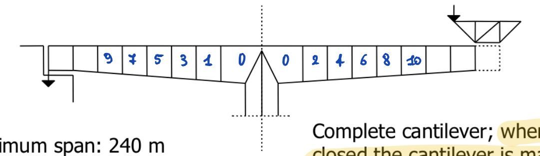
I use this tecquinique when the deck is far away to the ground ('cause I can't build a falsework so tall, or 'cause I have to overpass water)

a) Classical cantilever with two ballasted falsework and formwork



Equilibrium by means of full restrain to the pier or external ties

← Hammer tecquinique

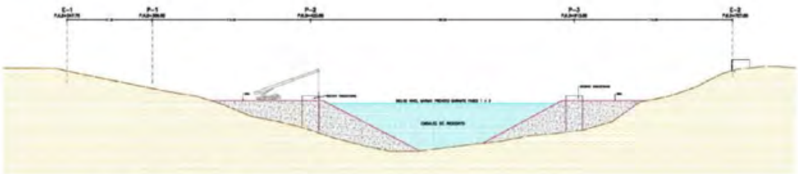


Maximum span: 240 m (Hamana/Giappone)

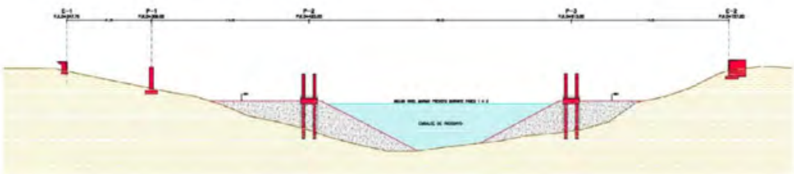
Complete cantilever; when the key is closed the cantilever is made continuous

CONSTRUCTION PHASES

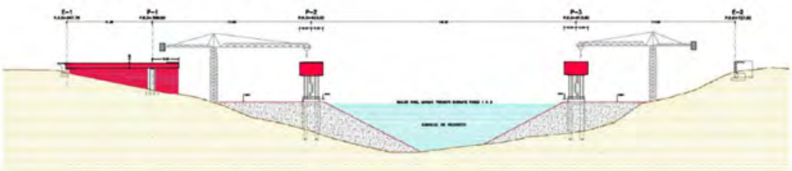
1) foundations



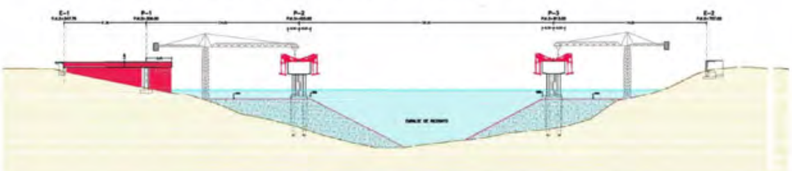
2) Piers and abutments



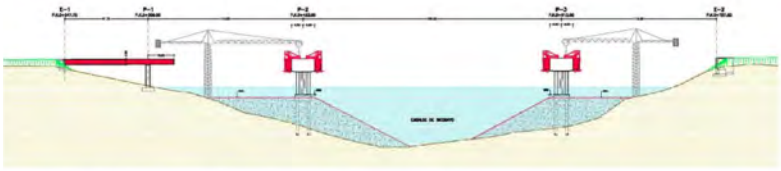
3) Pier segments



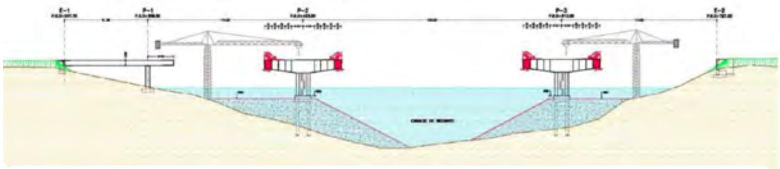
4) Positioning of cranes used for the scaffolding



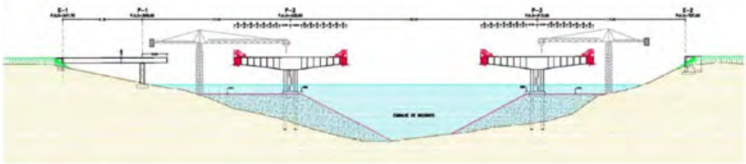
5) Concreting of segments 1



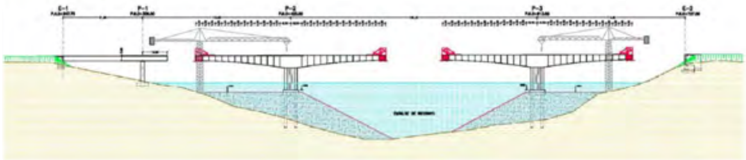
6) Concreting of following segments



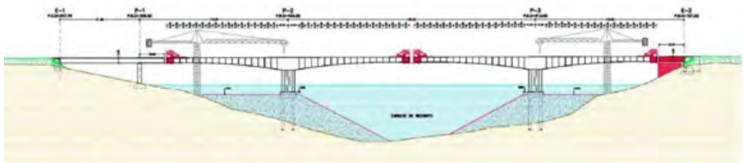
7)



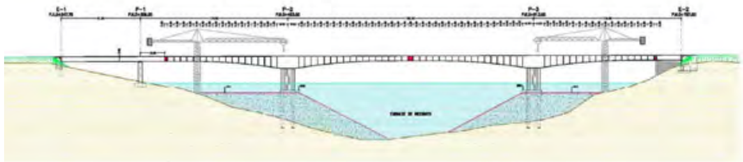
8)



9) Hammers completion



10) Concreting of key segments



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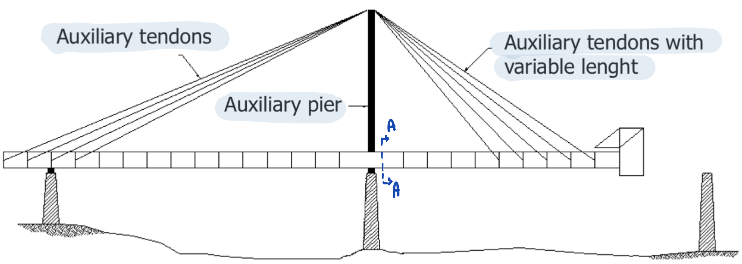
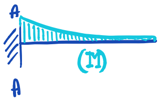
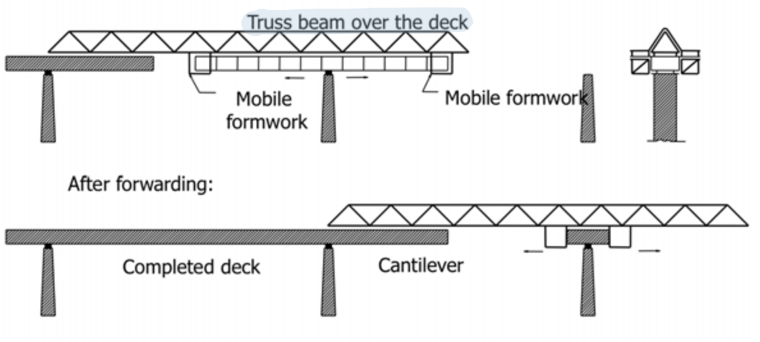
(c) CANTILEVERING WITH AUXILIARY TENDONS



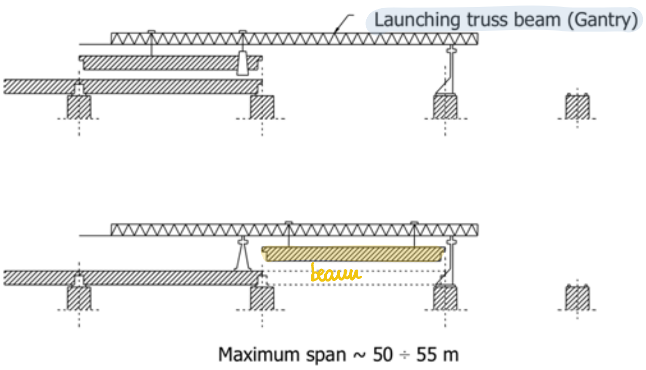
to reduce stress in section AA



(d) CANTILEVERING WITH AUXILIARY TRUSS BEAM



(e) FULL SPAN PREFABBRICATION (close to the construction site)



Both **overhead travelling** cranes and **gantry cranes** are types of Crane which lift objects by a **hoist** which is fitted in a trolley and can move horizontally on a rail or pair of rails fitted under a beam. An overhead travelling crane, also known as an overhead crane or as a suspended crane, has the ends of the supporting beam resting on wheels running on rails at high level, usually on the parallel side walls of a factory or similar large industrial building, so that the whole crane can move the length of the building while the hoist can be moved to and fro across the width of the building. A gantry crane has a similar mechanism supported by uprights, usually with wheels at the foot of the uprights allowing the whole crane to traverse.

A **hoist** is a device used for lifting or lowering a load by means of a drum or lift-wheel around which rope or chain wraps. It may be manually operated, electrically or pneumatically driven and may use chain, fiber or wire rope as its lifting medium.

(f) PRECAST SEGMENTAL CONSTRUCTION

No tensile stresses at the edges in serviceability conditions

Maximum span 135 ÷ 140 m

holes for prestressing tendons

shear keys = chiavi di taglio

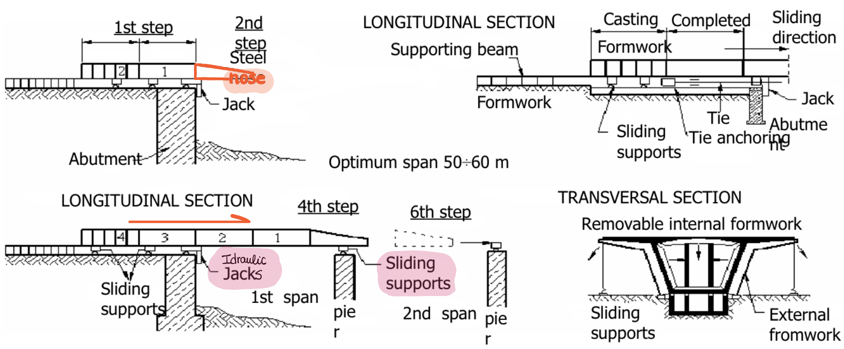
vertical tendons used to connect the zero element to the pier for safety reason during construction phases ⇐ not present during service life

diaphram (stronger element)

il collegamento con un'altra sezione avviene per mezzo di un gel

⇐ PIER SEGMENT (concio zero sulla pila)

(g) INCREMENTAL LAUNCHING



1st) I build piers
2nd) I build meanwhile the deck on one side already aligned with the abutment (in the direction of the bridge)
3rd) I start to push the deck

Advantages: I can start to build the deck before the end of the building of the pier ⇒ quicker

Workers work in a safe environment, no risk to fall from high decks.

The principle of incremental launching: casting in segment on the rear of the abutment and trusting from pier to pier.
Limit span: 600÷700 m

Limitations
- planimetric: R = cost = ∞
- altimetric: R very large

OTHER CRITERIA

✱ CRITERIA FOR SPAN CHOICE

$\Delta\sigma_{perm} \gg \Delta\sigma_{var}$

- Controlling parameter $\Rightarrow$ Performance of the bridge $\boxed{\mu = \dfrac{\Delta\sigma_{\underset{loads}{var}}}{\Delta\sigma_{\underset{loads}{var}} + \Delta\sigma_{\underset{loads}{perm}}}}$ $\Rightarrow$ CONDITION $\mu = 0$ $\rightarrow$ Limit span $\rightarrow$ this is a physical theoretical limit (not an economical one)
- Economical limit of performance $\begin{cases} STEEL: \mu_{lim} \cong 0{,}5 \\ CONCRETE: \mu_{lim} = 0{,}10 - 0{,}15 \end{cases}$

  ↳ peso 95% del carico permanente, non economico
- Comparison of different performances for bridge type and materials

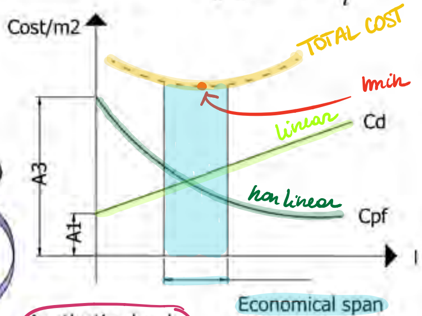
Type	Material	Economical limit span [m]	Maximum span realized [m]
Wall web and continuous beam	P.C.	~250	240 (Japan)
Wall web and continuous beam	STEEL	~350	345 (Jugoslavia)
Stay cable	P.C.	~500	400 (Thailand)
Stay cable	STEEL	~1800	404 (France)
Trussed gerber beam	STEEL	550÷600	549 (Canada)
Arch	STEEL	350÷400	366 (Canada)
Trussed arch	STEEL	~700	511 (U.s.a.)
Arch	R.C.	~400	~390 (Jugoslavia)
Suspended	STEEL	3500	~1900 (Japan)

Economical criteria for multispan bridges

$$\boxed{C = C_d + C_{pf}}$$

- C = Total cost
- C_d = Deck cost
- C_{pf} = Foundations and pier cost

From the back analysis of existing bridges $\begin{cases} C_d \cong A_1 + A_2 l \\ C_{pf} \cong A_3 + \dfrac{A_4}{l} \end{cases}$



Minimum cost

Deck cost $\cong$ Cost (pier + foundations)

Construction system
Need for prefabrication and transport

Quality of the soil
Expensive protections is needed

Aesthetic: l > h
↳ big span and small piers

I want to reduce C_{pf} $\Rightarrow$ ↑ span
↑ C_d

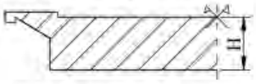
-14-

* BRIDGE TRANSVERSE SECTION SHAPE

- Influencing parameters:

- Span, with reference to statical scheme
- Depth or slenderness required (l/h)
- Available technology for execution
- Cost (slenderness implies increase of steel quantity)
- q/g ratio (live load/dead load) for dynamic behaviour

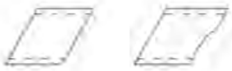
- Slab bridges cast in situ



Isostatic → Span ≤ 20 m
Continuous → Span ≤ 30 m

25 ≤ depth ≤ 70 cm

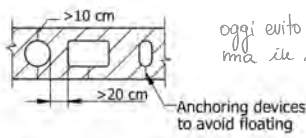
Good solution for skew crossing (obliquo) or irregular shapes



Slenderless { R.C. l/H = 15 ÷ 22
P.C. l/H = 18 ÷ 30

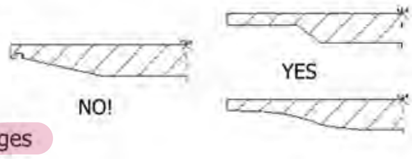
{ R.C. = reinforced concrete
P.C. = prestressed concrete

With depth > 70 cm ⇒ Voided slab (pay attention to durability!)



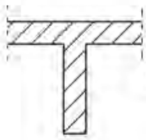
oggi evito di mettere i fori, ma in passato si faceva. Se lascio dei vuoti devono essere ispezionabili!

Variable thickness slabs to increase the aesthetic

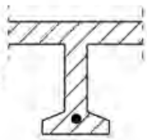


- T-beam cast in situ bridges

Very simple shape

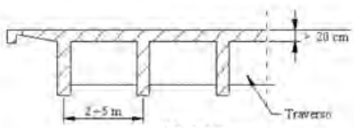


Best shape

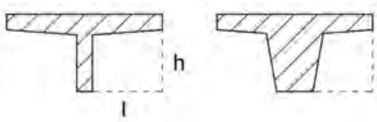


Typical configuration

Mono-beam



Multi-beam



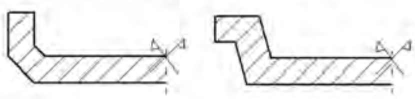
For both the sections:

$\frac{l}{h} \cong 20$

Slabs are more slender than beams

- Inverted T-beams (Channel bridges)

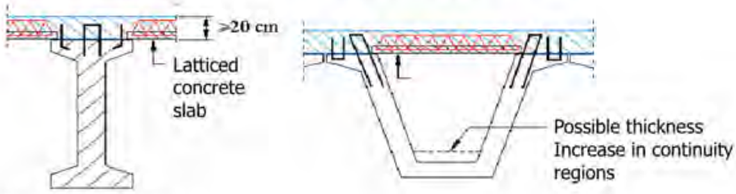
Typical section



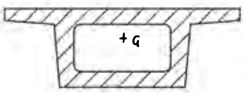
- Small depth
- Heavy appearance
- Bad for positive bending moment

-15-

- T or V precast beams, connected by casting to the slab



- Box girder beams



due to inertia properties (mass distant from centroid G)

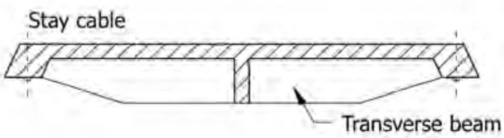
High performance! $\eta \cong 0.5$ $\frac{l}{h} \leq 30$ (continuity)

Depth { Constant → $l < 60\,m$; Variable → $l > 60\,m$ }

- Box girder with double deck

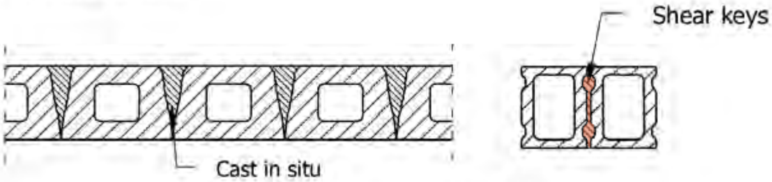
Internal → Railway
External → Road

- Suspended slabs

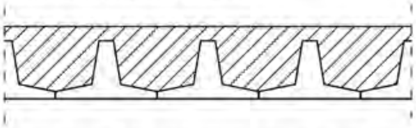


We want a deck thin and light weight

- Precast slabs (not so common today)



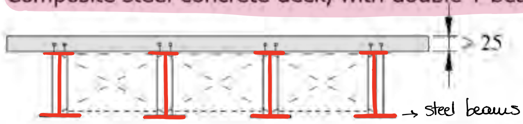
- Slabs with infilled beams



$l = 10 \div 15\,m$

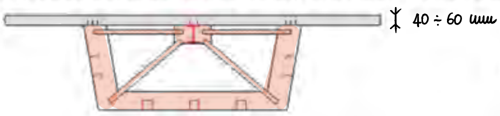
- Composite steel-concrete deck, with double T-beams → used both in isostatic scheme and in continuous beam even if the best use of construction materials is to use the isostatic solution (concrete compressed - steel tensed)

I use predalles to cast concrete



$l \leq 50 \div 90\,m$

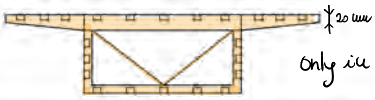
- Composite steel-concrete deck, with box girder beam



$l \geq 50\,m$

- Steel bridges with hortotropic deck

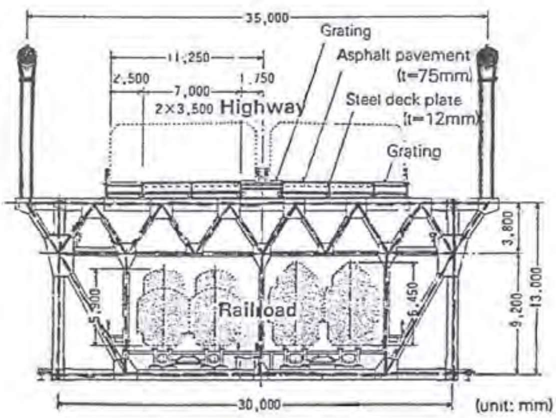
$l \geq 120 \div 150\,m$



only in steel

Solution for continuous beam	$100 \leq l \leq 200\,m$		**Deck bridge** Box girder with hortotropic deck
Solution for stay-cable bridges	$200 \leq l \leq 300\,m$		**Stay-cable bridge with flexural rigidity deck** Deck suspended in the central region
	$300 \leq l \leq 1500\,m$		**Stay-cable bridge with truss behaviour**

- Cross section of suspended bridge with mixed traffic



2. ACTIONS ON BRIDGES

Actions on road bridges

Permanent actions

- Dead load g_1 (structural weight)
- Carried permanent loads g_2 (not structural: kerbs, pavements, barriers)
- Other permanent loads g_3 (ground and hydraulic actions)

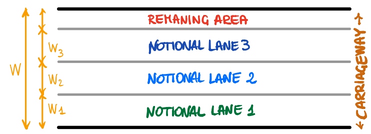
Traffic actions

Imposed deformations

- Design imposed deformations ε_1 (example: prestressing, displacement)
- Creep and shrinkage ε_2 (actions variable in time, only for concrete)
- Thermal actions ε_3
- Ground settlements ε_4

definition of NOTIONAL LANES

The width of the carriageway w has to be divided in notional lane in order to place traffic loads.



Inside the carriageway we find a set of notional lanes (W_1, W_2, W_3) and a remaining area. The transverse position of these lanes is up to you. The designer choses where to place them in order to maximixe internal actions. The width W_1, W_2, W_3, are function of the width w of the carriageway. They can be determined by the following table of EC:

Carriageway width w	Number of notional lanes	Width of a notional lane w_l	Width of the remaining area
$w < 5{,}4$ m	$n_l = 1$	3 m	w - 3 m
$5{,}4 \text{ m} \le w < 6 \text{ m}$	$n_l = 2$	$\frac{w}{2}$	0
$6 \text{ m} \le w$	$n_l = Int\left(\frac{w}{3}\right)$	3 m	$w - 3 \times n_l$
NOTE For example, for a carriageway width equal to 11m, $n_l = Int\left(\frac{w}{3}\right) = 3$, and the width of the remaining area is $11 - 3 \times 3 = 2$m.			

ROAD BRIDGE CATEGORY

To the value of concentrated loads Q_K and distributed one q_K we apply an adjustment factor α.

$\alpha_Q Q_K$ concentrated loads

$\alpha_q q_K$ distributed loads

The value of adjustment factors should be selected depending on the expected traffic. In absence of specification these factors should be taken equal to unity.

NATIONAL ANNEX: ITALY (NTC)

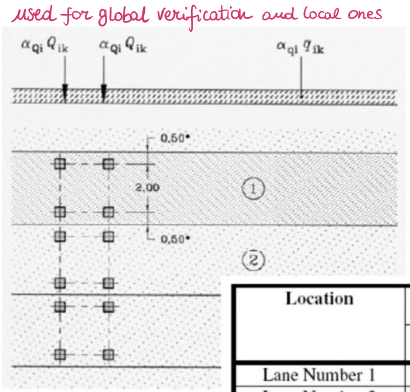
- 1° CATEGORY BRIDGES ⇒ $\alpha_Q = \alpha_q = 1.0$ (always today)
- 2° CATEGORY BRIDGES ⇒ $\alpha_Q = \alpha_q = 0.8$ (until 2008 old bridges)

(for new bridges, today there is no difference between 1° and 2° category)

VERTICAL TRAFFIC LOADS MODEL

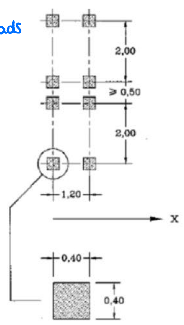
- **Load model 1 (LM1)** → made of distributed loads plus a tandem system

used for global verification and local ones



$\alpha_Q Q_k$

$\alpha_q q_k$

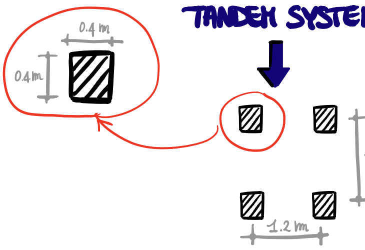


Location	Tandem system *TS*	UDL system
	Axle loads Q_{ik} (kN)	q_{ik} (or q_{rk}) (kN/m²)
Lane Number 1	300	9 ≈ 1 ton/m²
Lane Number 2	200	2,5
Lane Number 3	100	2,5
Other lanes	0	2,5
Remaining area (q_{rk})	0	2,5

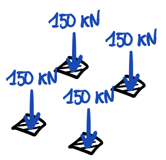
How can we use these loads? The distributed load can be divided in segments by the designer as many times as he wants in order to place it where it is needed to maximize the actions (inside the element for example q_k q_k)

While the tandem system can move, so I can place this load of 4 forces wherever I like but only one time on each lane (the three lanes are completely independent)

TANDEM SYSTEM = $\underbrace{1 \text{ axle}}_{2 \text{ wheels}} + \underbrace{1 \text{ axle}}_{2 \text{ wheels}}$



for instance, Lane 1
Axle load Q_{1k} = 300 kN



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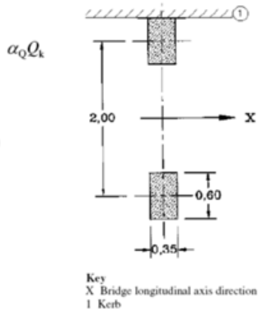
- **Load model 2 (LM2)**

Tandem or single tyre load used for global and local verification

400 kN axle load
For global and local verifications

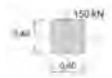
A single tyre (contact area)
Of 200 kN may be used if more conservative



→ only one axle

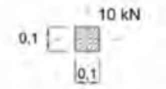
- **Load model 3 (LM3)**

Concentrated load for local verification (only for small details)
↳ 0.4 x 0.4 m



- **Load model 4 (LM4)**

Concentrated load for local verification
↳ 0.1 x 0.1 m



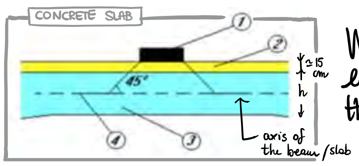
- **Load model 5 (LM5)**

Distributed Crowd load for general and local verification

Crowd loading, if relevant, should be represented by Load Model consisting of a uniformly distributed load (which includes dynamic amplification) equal to 5 kN/m²

N.B. combination value of crowd loading is 2.5 kN/m² → on kerbs!

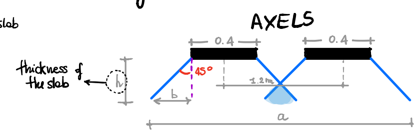
DISPERSAL OF CONCENTRATED LOADS



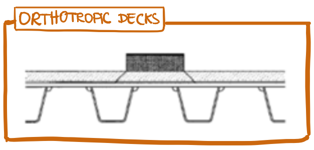
① Wheel contact pressure
② Pavement
③ Concrete slab
④ Middle surface of concrete slab

We can take into account the dispersal of concentrated loads through pavement and a concrete slab, only if the thickness of the slab allows to do that.

AXELS



In longitudinal direction the concentrated forces becomes a uniformed distributed load on a lenght

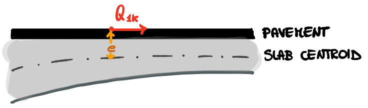


The stress variation generated by a load in a steel structure is much higher then the stress variation due to the same load 'cause different thicknen.
Moreover steel structures have big problems due to fatigue (while fatigue problems are almost null on cls structures)

HORIZONTAL TRAFFIC LOADS

• BREKING AND ACCELERATION FORCES

We consider an horizontal force placed at the level of the pavement (at the level of traffic).



There is a vertical eccentricity e (distance between pavement and slab centroid) that gives arise to an additional bending moment → $Q_{1k} \cdot e$. In our bridge model I have to take into account this eccentricity e.

The value of this force is a function of the uniformely distributed loads q_k and of the concentrated force Q_k :

$$\alpha_{Q1}\, 180\ \text{kN} \leq Q_{1k} = 0.6\, \alpha_{Q1} \left(2\, Q_{1k} \right) + 0.1\, \alpha_{q_1}\, q_{1k}\, w_1\, L \leq 900\ \text{kN}$$

L = length of the deck or of the part under consideration

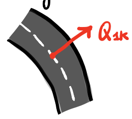
This force should be taken into account as located along the axis of any lane. However if the eccentricity effects are not significant, the force may be considered to be applied only along the carriageway axis, and uniformely distributed over the loaded length.

• CENTRIFUGAL FORCES (only in a curve)

The centrifugal force Q_{tk} should be taken as a transverse force acting at the finished carriageway level and radially to the axis of the carriageway.

$Q_{tk} = 0{,}2 Q_v$ (kN)	if $r < 200$ m
$Q_{tk} = 40 Q_v / r$ (kN)	if $200 \leq r \leq 1500$ m
$Q_{tk} = 0$	if $r > 1500$ m

The centrifugal force is a force pointing outside of the curve



where :

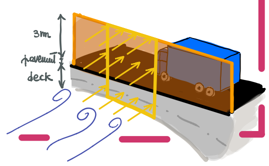
r is the horizontal radius of the carriageway centreline [m]

Q_v is the total maximum weight of vertical concentrated loads of the tandem systems of LM1, *i.e.* $\sum_i \alpha_{Q_i} (2 Q_{ik})$

N.B. WHEATHER AND WIND ACTIONS

- On SNOW ACTIONS we apply general rules. Snow loads can't be combined with traffic loads except from covered bridges. If I place on my bridge LM1, I'm not going to place snow and viceversa. LM1 >> snow. I use snow in verifications of the construction phases.

- WIND ACTION: general rules apply. The surface offered by the vehicles moving on the bridges is assumed as a rectangular continuous stripe 3 m high from pavement.



VALUES OF THE MULTI COMPONENT ACTIONS

	Loads on carriageway					Loads on footways
	Vertical			Horizontal		Vertical
Group of actions	Main action LM1-2-3-4-6	Special vehicles	Crowd	Braking Accel.	Centrifugal	Uniform
1	Characteristic value					2.5 kN/m²
2a	Frequent value			Characteristic value		
2b	Frequent value				Characteristic value	
3 (*)						5.0 kN/m²
4 (**)			5.0 kN/m²			5.0 kN/m²
5(***)	To be defined in design	Characteristic value				

(*) Only for footbridges
(**) Only for urban bridges
(***) Only if special vehicles are taken into account

I have to load the bridge not only with the single action we have seen untill now, but we have to load it with a set of actions:

- GROUP 1 (strong vertical stress)
- GROUP 2a (strong horizontal action in longitudinal direction + small vetrcal action)
- GROUP 2b (strong horizontal action in transverse direction + small vetrcal action)
- GROUP 3 (only on footbridges)
- GROUP 4 (crowd everywhere without traffic only for urban area)
- GROUP 5 (construction)

[Characteristic value means → 100% Q
Frequent value → $\psi \cdot Q$ (with $\psi < 1$)

Once I have the group of actions due to traffic, I have to combine it with permanent loads and imposed deformations.

FATIGUE LOAD MODELS

Load model 1

Load model 2

Load model 3

Load model 4

Load model 5

a) Fatigue Load Models 1, 2 and 3 are intended to be used to determine the maximum and minimum stresses resulting from the possible load arrangements on the bridge of any of these models ; in many cases, only the algebraic difference between these stresses is used in EN1992 to EN1999.

b) Fatigue Load Models 4 and 5 are intended to be used to determine stress range spectra resulting from the passage of lorries on the bridge.

c) Fatigue Load Models 1 and 2 are intended to be used to check whether the fatigue life may be considered as unlimited when a constant stress amplitude fatigue limit is given. Therefore, they are appropriate for steel constructions and may be inappropriate for other materials. Fatigue Load Model 1 is generally conservative and covers multi-lane effects automatically. Fatigue Load Model 2 is more accurate than Fatigue Load Model 1 when the simultaneous presence of several lorries on the bridge can be neglected for fatigue verifications. If that is not the case, it should be used only if it is supplemented by additional data. The National Annex may give the conditions of use of fatigue load models 1 and 2.

d) Fatigue Load Models 3, 4 and 5 are intended to be used for fatigue life assessment by reference to fatigue strength curves defined in EN1992 to EN1999. They should not be used to check whether fatigue life can be considered as unlimited. For this reason, they are not numerically comparable to Fatigue Load Models 1 and 2. Fatigue Load Model 3 may also be used for the direct verification of designs by simplified methods in which the influence of the annual traffic volume and of some bridge dimensions is taken into account by a material-dependent adjustment factor λ_e.

e) Fatigue Load Model 4 is more accurate than Fatigue Load Model 3 for a variety of bridges and of the traffic when the simultaneous presence of several lorries on the bridge can be neglected. If that is not the case, it should be used only if it is supplemented by additional data, specified or as defined in the National Annex.

f) Fatigue Load Model 5 is the most general model, using actual traffic data.

Indicative number of heavy vehicles expected per year and per slow lane

	Traffic categories	N_{obs} per year and per slow lane
1	Roads and motorways with 2 or more lanes per direction with high flow rates of lorries	$2{,}0 \times 10^6$
2	Roads and motorways with medium flow rates of lorries	$0{,}5 \times 10^6$
3	Main roads with low flow rates of lorries	$0{,}125 \times 10^6$
4	Local roads with low flow rates of lorries	$0{,}05 \times 10^6$

Only heavy loads generates fatigue (no cars, no vans but heavy lorries)

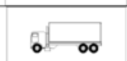
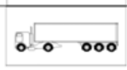
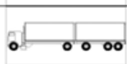
- **Fatigue load model 1** (similar to LM1 but the intensity of loads is different) — *same shape*

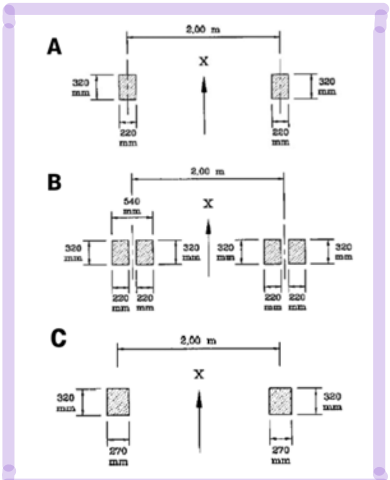
It has the configuration of the characteristic Load model 1, with the value of the axle loads equal to $0.7\ Q_{ik}$ and the values of the uniformely distributed loads equal to $0.3\ q_{ik}$ and (unless otherwise specified) $0.3\ q_{rk}$.

- **Fatigue load model 2**

It consists of a set of idealised lorries, called "frequent" lorries (they have the same shape of a common lorry but they are much hevier). Each "frequent lorry" is defined by:

- the number of axles and the axle spacing
- the frequent load of each axle
- the wheel contact area and the transverse distance between wheels

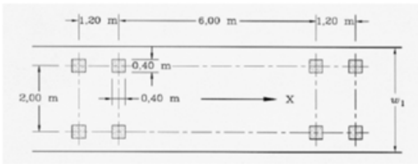
1 LORRY SILHOUETTE	2 Axle spacing (m)	3 Frequent axle loads (kN)	4 Wheel type (see Table 4.8)
	4,5	90 190	A B
	4,20 1,30	80 140 140	A B B
	3,20 5,20 1,30 1,30	90 180 120 120 120	A B C C C
	3,40 6,00 1,80	90 190 140 140	A B B B
	4,80 3,60 4,40 1,30	90 180 120 110 110	A B C C C



Definition of wheels and axes

- **Fatigue load model 3**

This model consist of four axles (2 tandem system). The weight of each axle is equal to 120 kN and the contact surface of each wheel is a square of 0.4 x 0.4 m. Dove è rilevante, bisogna prendere in considerazione due veicoli nella stessa corsia.



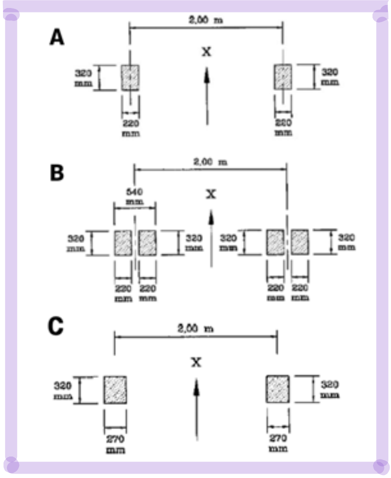
Key
w_l : Lane width
X : Bridge longitudinal axis

- **Fatigue load model 4**

It consists of sets of standard lorries which together produce effects equivalent to those of typical traffic on european roads.

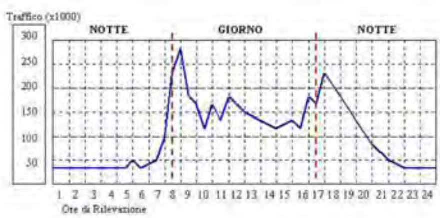
VEHICLE TYPE			TRAFFIC TYPE			
1	2	3	4	5	6	7
			Long distance	Medium distance	Local traffic	
LORRY	Axle spacing (m)	Equivalent axle loads (kN)	Lorry percentage	Lorry percentage	Lorry percentage	Wheel type
	4,5	70 130	20,0	40,0	80,0	A B
	4,20 1,30	70 120 120	5,0	10,0	5,0	A B B
	3,20 5,20 1,30 1,30	70 150 90 90 90	50,0	30,0	5,0	A B C C C
	3,40 6,00 1,80	70 140 90 90	15,0	15,0	5,0	A B B B
	4,80 3,60 4,40 1,30	70 130 90 80 80	10,0	5,0	5,0	A B C C C

The load of LM4 is different of the LM2
(LM4 → loads of real traffic, LM2 → loads much heavier)



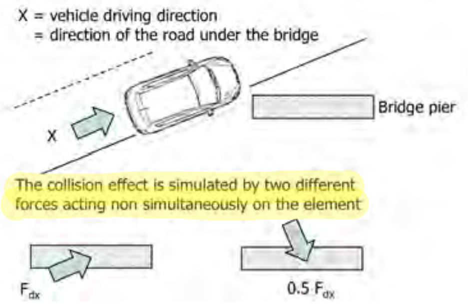
- **Fatigue load model 5**

It consists of the direct application of recorded traffic data, supplemented by statistical extrapolations.



ACTIONS FOR ACCIDENTAL DESIGN SITUATION

• Impact of vehicles on vertical elements (Collision forces from vehicles under the bridge)



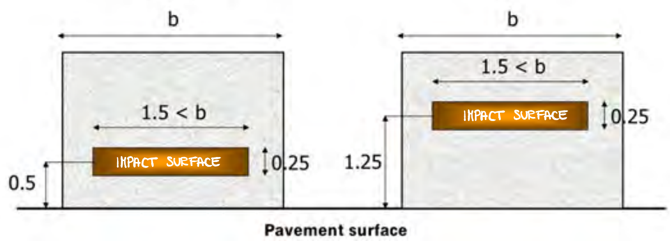
Collision forces from vehicles under the bridge

Values of F_{dx} → is function of [- road type - vehicle type

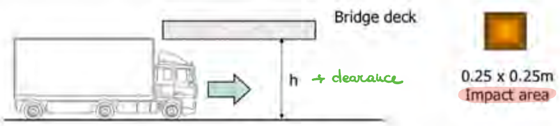
Road type	Vehicle type	Force F_{dx} [kN]
Motorways or important roads	-	1000
Standard roads	-	750
Urban streets	-	500
Parking facilities	Cars Vans > 3.5 t	50 150

Impact areas dimensions [m]

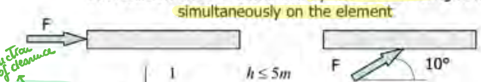
Car crash → cambia solo l'altezza ← Lorry crash



• Impact of vehicles on horizontal elements



The collision effect is simulated by two forces acting non simultaneously on the element



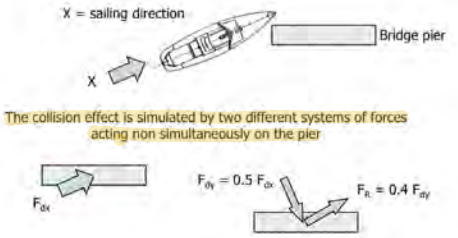
function of clearance

$F = r \cdot F_{dx}$

$$r = \begin{cases} 1 & h \le 5m \\ 1-(h-5) & 5m < h \le 6m \\ 0 & h > 6m \end{cases}$$

parameter function of clearance

• Impact of boat on piers



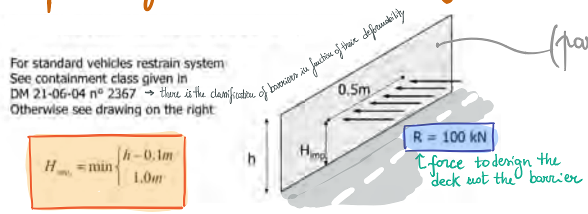
Values of F_{dx} → function of [- boat class - length - weigth

Boat class	Length [m]	Tonnage [t]	F_{dx} [kN]
Small	50	3000	30000
Medium	100	10000	80000
Large	200	40000	240000
Huge	300	100000	460000

-25-

- **Impact of vehicles on restraint systems** (Collision forces from vehicles ON the bridge) on the deck

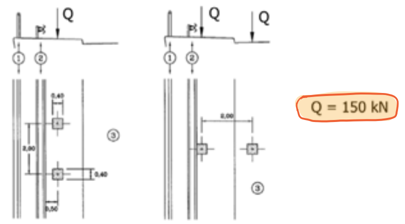
(parapets, safety barriers, new-jersy, ...)



they works in non linear field, it has to be strong and at the same time deformable to absorbe the impact energy.

The maximum force that can be transfered by the barrier to the bridge is a function of the only resistance of the barrier. The design is done with the aim to let the barrier work in plasticity field while the part of the deck work in elastic field.

- **Vehicles on footways and cycle tracks**



In this case we have a big load in a zone where it shouldn't be! (example: lorry on footways).

- **Other variable actions**
  - hydraulic actions
  - impact of ice on the pier
  - impact of flying vehicles

All the actions seen have to be combined at SLU and SLE (combination of actions) through safety factors:

SLU actions safety factors

		Coeff.	EQU[1]	A1 STR	A2 GEO
Dead load	FAV	γ_{G1}	0,90	1,00	1,00
	UNFAV		1,10	1,35	1,00
Permanent loads	FAV	γ_{G2}	0,00	0,00	0,00
	UNFAV		1,50	1,50	1,30
Traffic variable loads	FAV	γ_{Q}	0,00	0,00	0,00
	UNFAV		1,35	1,35	1,15
Other variable loads	FAV	γ_{Qi}	0,00	0,00	0,00
	UNFAV		1,50	1,50	1,30
Design imposed deformations	FAV	$\gamma_{\varepsilon 1}$	0,90	1,00	1,00
	UNFAV		1,00[3]	1,00[4]	1,00
Other imposed deformations (creep, temperature, settlements...)	FAV	$\gamma_{\varepsilon 2}, \gamma_{\varepsilon 3}, \gamma_{\varepsilon 4}$	0,00	0,00	0,00
	UNFAV		1,20	1,20	1,00

Actions	Description	Ψ_0 Charact.	Ψ_1 Freq.	Ψ_2 Q. Perm.
Traffic actions	LM1 Tandem	0,75	0,75	0,0
	LM1-5-6 Distributed	0,40	0,40	0,0
	LM3-4	0,40	0,40	0,0
	LM2	0,0	0,75	0,0
	2	0,0	0,0	0,0
	3	0,0	0,0	0,0
	4	----	0,75	0,0
	5	0,0	0,0	0,0
Wind	Wind – bridge unloaded SLU and SLE	0,6	0,2	0,0
	In construction phases	0,8	----	0,0
	Wind – loaded bridge	0,6		
Snow	SLU and SLE	0,0	0,0	0,0
	Execution	0,8	0,6	0,5
Temperature	T_k	0,6	0,6	0,5

N.B. **SLU and SLE verifications**

- SLU verifications are the same of building (as seen in "CEMENTO ARMATO")
- SLE verifications are generally different.

SLE deformation: deformation shouldn't cause trouble to traffic 'cause drivers are already subjected to movements and accelerations while they are driving. They don't feel the movement of the bridge 'cause they are moving too. It can be rilevant in pedestrian or in railway bridges.

SLE stress limitation:

Material	SLE combination	Stress limit
Concrete	Characteristic	$0.60\ f_{ck}$
Concrete	Quasi permanent	$0.45\ f_{ck}$
Steel	Characteristic	$0.80\ f_{yk}$

SLE cracking control:

Definition of cracking limit states

(a) Decompression: $\sigma_1 < 0$ → my structure has to work completely in compression. This goal is easy to be achived in longitudinal direction (thanks to prestressing), but not so easy in transverse direction. I could put some prestressing also in transverse direction.

(b) Crack formation: $\sigma_1 < \frac{f_{ctm}}{1.2}$

if (a) and (b) are simultaneously satisfied, the structure is completely in elastic field.

(c) Crack opening:

$w_1 = 0{,}2$ mm
$w_2 = 0{,}3$ mm
$w_3 = 0{,}4$ mm

Environmental class groups

Group	Environmental classes
Standard	X0, XC1, XC2, XC3
Aggressive	XC4, XD1, XS1
Very aggressive	XD2, XD3, XS2, XS3

Environmental class group	SLE combination	Reinforcement			
		Sensible (prestressing)		Less sensible (ordinary reinforcement)	
Standard	Frequent	Crack opening	w_2	Crack opening	w_3
	Quasi perman.	Crack opening	w_1	Crack opening	w_2
Aggressive	Frequent	Crack opening	w_1	Crack opening	w_2
	Quasi perman.	Decompression	-	Crack opening	w_1
Very aggressive	Frequent	Crack formation	-	Crack opening	w_1
	Quasi perman.	Decompression	-	Crack opening	w_1

Actions on rail bridges

Permanent actions

- Dead load g_1
  (structural weight)
- Carried permanent loads g_2
  (not structural: tracks, ballast, barriers, ducts, cable, parapets)
  - BALLAST → $\gamma = 18\ kN/m^3$ (straight track)
  - BALLAST → $\gamma = 20\ kN/m^3$ (curved)

  (in realtà γ è sempre uguale, ma poiché in curve la quantità di ballast utilizzato è superiore a quello utilizzato in rettilineo, si usa questo trick)
- Other permanent loads g_3
  (ground and hydraulic actions)

Traffic actions

Imposed deformations

- Design imposed deformations ε_1
  (example: prestressing, displacement)
- Creep and shrinkage ε_2
  (actions variable in time, only for concrete)
- Thermal actions ε_3
- Ground settlements ε_4

LOAD MODELS FOR RAILWAY TRAFFIC

Tracks in Italy

- NORMAL TRACKS (passengers + commercial)
- HIGH SPEED LINES (passengers)
- SMALLER LOCAL TRACKS (passengers)

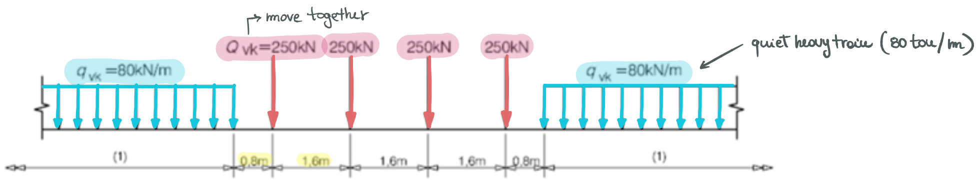
Rail traffic actions are defined by means of load models.
Five models of railway loading are given:

- **Load model 71** → to represent normal traffic on mainline railways (passenger traffic)
- **Load model SW/0** → (for continuous bridges) to represent normal traffic on mainline railways (passenger traffic)
- **Load model SW/2** → to represent heavy loads (commercial traffic)
- **Load model HSLM** → to represent the loading from passenger trains at speeds exceeding 200 km/h (high speed lines)
- **Load model "unloaded train"** → to represent the effect of unloaded trains

• Load model LM71

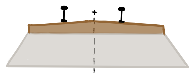
Load Model 71 represents the static effect of vertical loading due to normal rail traffic.

The uniform distributed load can be applied in segments in order to achieve the most unfavourable effect.



Key
(1) No limitation

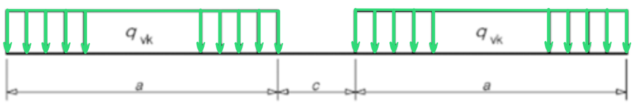
the load is applied in the mid of the track



• Load model SW

(1) Load Model SW/0 represents the static effect of vertical loading due to normal rail traffic on continuous beams.

(2) Load Model SW/2 represents the static effect of vertical loading due to heavy rail traffic.



→ they move together

Load Model	q_{vk} [kN/m]	a [m]	c [m]
SW/0	133	15,0	5,3
SW/2	150	25,0	7,0

• Load model "unloaded train"

(1) For some specific verifications (see EN 1990 A2, § 2.2.4(2)) a particular load model is used, called "unloaded train". The Load Model "unloaded train" consists of a vertical uniformly distributed load with a characteristic value of $\boxed{10{,}0 \text{ kN/m}}$.

UNIFORMLY DISTRIBUTED LOAD

N.B. Action for non public footpaths

↳ Uniform distributed load of 10 kN/m² without dynamic increment

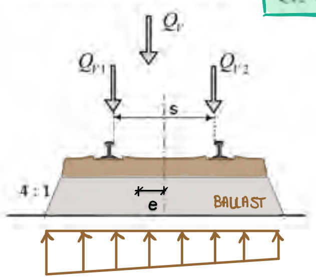
DISTRIBUTION OF LOADS

* Eccentricity of vertical loads

LM71 & SW0

The load may have a maximum transverse eccentricity *e* due to a difference in the vertical forces on the rail of:

$\boxed{Q_{V2}/Q_{V1}=1{,}25}$ → $Q_{V1}+Q_{V2}=Q_V$



$e \leq s/18$ distance between rails

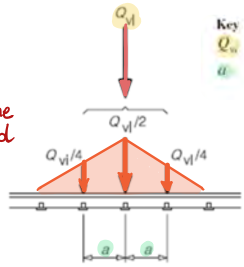
$s = 1435\ mm$ scartamento

at the base → distributed load (trapezoidal shape)

* Longitudinal distribution of wheel load by the rail

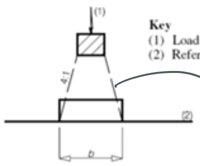
Key
Q_{vi} is the point force on each rail due to Load Model 71 or a wheel load of a Real Train in accordance with 6.3.5, Fatigue Train or HSLM (except for HSLM-B)
a is the distance between rail support points

This operation is done only on concentrated forces



triangular longitudinal distribution of load

* Longitudinal distribution of wheel load by the ballast



Key
(1) Load on sleeper
(2) Reference plane

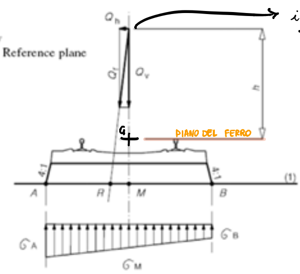
dispersion in the ballast

* Transverse distribution of load by sleepers and ballast

Transverse distribution of actions by the sleepers and ballast, track without cant (effect of eccentricity of vertical loads not shown)

STRAIGHT

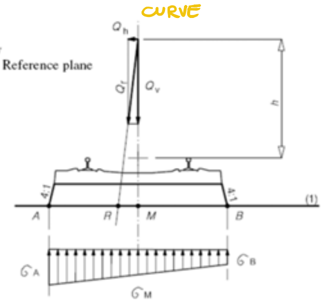
Key
(1) Reference plane



ideal position of the centroid of train's mass (G)

CURVE

Key
(1) Reference plane



DYNAMIC EFFECTS

dynamic enhancements
amplificazione dinamica

In road bridges dynamic effects are taken already by the codes. In railway bridges no. So I have to consider them.

Limits of bridge natural frequency n_0 [Hz] as a function of L [m]

The upper limit of n_0 is governed by dynamic enhancements due to track irregularities and is given by:

1st mode of vibration ← $n_0 = 94{,}76 L^{-0{,}748}$ (6.1)

non deve appartenere alla zona verde

The lower limit of n_0 is governed by dynamic impact criteria and is given by:

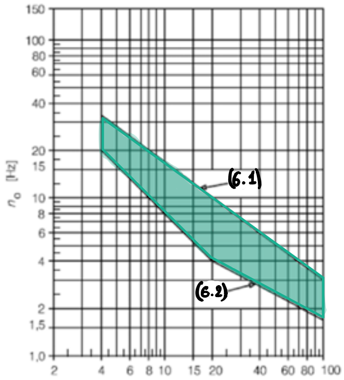
$n_0 = 80/L$ for $4\text{m} \leq L \leq 20\text{m}$

$n_0 = 23{,}58 L^{-0{,}592}$ for $20\text{m} < L \leq 100\text{m}$ (6.2)

where:

n_0 is the first natural frequency of the bridge taking account of mass due to permanent actions,

L is the span length for simply supported bridges or L_Φ for other bridge types.



L [m] (SPAN → lunghezza di 1 campata)

CASE 1

When all the following points are respected:

(a) • Usual bridge typology (e.g. simply supported beam, continuous beam...)

(b) • Maximum line speed at the site <200 km/h
↳ so never this case (case 1) in high speed lines

(c) • Natural frequency of the structure within the limits shown in the previous slide.

⬇

I use a STATIC EQUIVALENT ANALYSIS (simpler than a dynamic analysis), using the following DYNAMIC FACTORS ϕ (ϕ_2, ϕ_3) → they are function of the irregularity of the railways (↑ irregularity ⇒ ↑ dynamic factor)

L_ϕ = DETERMINANT LENGTH (length associated with ϕ)
↳ it is function of (static scheme, element designed)
↳ it is tabled

CASE 2

If one of (a), (b), (c), is NOT respected

⬇

I use a FULL DYNAMIC ANALYSIS

(a) For carefully maintained track:

$$\boxed{\Phi_2 = \frac{1{,}44}{\sqrt{L_\Phi} - 0{,}2} + 0{,}82}$$

with: $1{,}00 \leq \Phi_2 \leq 1{,}67$

(b) For track with standard maintenance:

$$\boxed{\Phi_3 = \frac{2{,}16}{\sqrt{L_\Phi} - 0{,}2} + 0{,}73}$$

the railway authority impose to use ϕ_3 (worst conditions)

with: $1{,}00 \leq \Phi_3 \leq 2{,}0$

The coefficient Φ (Φ_2, Φ_3) can NOT be used for:

- The Unloaded train
- Real trains
- Trains for fatigue analysis

} 'cause they are more complex

the ballast is a damper that absorbs vibrations

For steel deck without ballast (track directly connected to the deck) should be considered a coefficient β additional to Φ (Φ_2, Φ_3)
↳ increases dynamic effects

$\beta = 1{,}0$ for $L_\phi < 8m$ and $L_\phi > 90m$

$\beta = 1{,}1$ for $8m < L_\phi < 90m$

HORIZONTAL ACTIONS

* Centrifugal force

The centrifugal forces act outwards in a horizontal direction at a height of 1.8 m from the rail.

(3)P The centrifugal force shall always be combined with the vertical traffic load. The centrifugal force shall not be multiplied by the dynamic factor Φ_2 or Φ_3.

$$\boxed{Q_{tk} = \frac{v^2}{g \times r}(f \times Q_{vk}) = \frac{V^2}{127r}(f \times Q_{vk})}$$ → concentrated force

$$\boxed{q_{tk} = \frac{v^2}{g \times r}(f \times q_{vk}) = \frac{V^2}{127r}(f \times q_{vk})}$$ → distributed force

where:

- Q_{tk}, q_{tk} Characteristic values of the centrifugal forces [kN, kN/m]
- Q_{vk}, q_{vk} Characteristic values of the vertical loads specified in 6.3 (excluding any enhancement for dynamic effects) for Load Models 71, SW/0, SW/2 and "unloaded train". For load model HSLM the characteristic value of centrifugal force should be determined using Load Model 71.
- f Reduction factor (see below) → used in order to have not to big horizontal forces
- v Maximum speed in accordance with 6.5.1(5) [m/s]
- V Maximum speed in accordance with 6.5.1(5) [km/h]
- g Acceleration due to gravity [9,81 m/s²]
- r Radius of curvature [m]

For the load models SW/2 and "unloaded train" the value of the reduction factor f should be taken as 1,0.

(8) For Load Model 71 (and where required Load Model SW/0) the reduction factor f is given by:

→ function of the speed of the train

$$f = \left[1 - \frac{V-120}{1000}\left(\frac{814}{V} + 1{,}75\right)\left(1 - \sqrt{\frac{2{,}88}{L_f}}\right)\right] \quad (6.19)$$

PLOTTING (6.19)

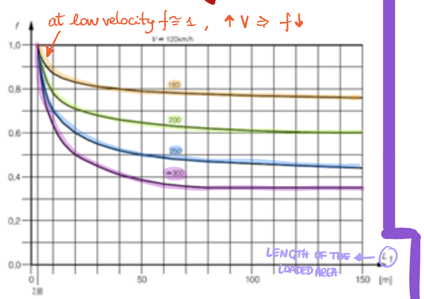
subject to a minimum value of 0,35 where:

- L_f is the influence length of the loaded part of curved track on the bridge, which is most unfavourable for the design of the structural element under consideration [m].
- V is the maximum speed in accordance with 6.5.1(5).

$f = 1$ for either $V \le 120$ km/h or $L_f \le 2{,}88$ m

$f < 1$ for 120 km/h $< V \le 300$ km/h (see Table 6.7 or Figure 6.16 or equation 6.19) and $L_f > 2{,}88$m

$f_{(V)} = f_{(300)}$ for $V > 300$ km/h.

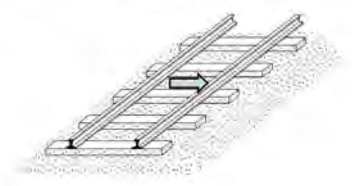


Factor f for Load Model 71 and SW/0

* Nosing force (Serpeggio)

(2)P The characteristic value of the nosing force shall be taken as Q_{sk} = 100 kN. It shall not be multiplied by the factor Φ (see 6.4.5) or by the factor f in 6.5.1(4).

(4)P The nosing force shall always be combined with a vertical traffic load.



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* Traction and breaking forces

Traction force: $Q_{lak} = 33$ [kN/m] $L_{a,b}$ [m] ≤ 1000 [kN]
for Load Models 71, SW/0, SW/2 and HSLM

is less intense than braking force

Braking force: $Q_{lbk} = 20$ [kN/m] $L_{a,b}$ [m] ≤ 6000 [kN]
for Load Models 71, SW/0 and Load Model HSLM

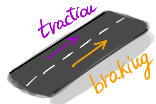
$Q_{lbk} = 35$ [kN/m] $L_{a,b}$ [m] (heavy trains)
for Load Model SW/2

The characteristic values of traction and braking forces shall not be multiplied by the factor Φ (see 6.4.5.2) or by the factor f in 6.5.1(6).

Traction and braking forces for decks with more than 1 track

I have to load the bridge

Bridges with 2 tracks:
- 1° track: braking
- 2° track: traction

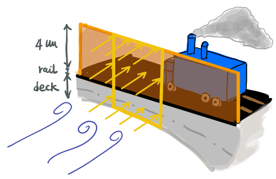


I have to load the bridge

Bridges with more than 2 tracks:
- 1° track: braking
- 2° track: traction
- 3° track: 50% braking
- 4° track and subsequent : Unloaded

WEATHER ACTIONS

Wind: general rules apply. The surface offered by the trains moving on the bridge is assumed as a rectangular continuous stripe 4m high from the rail.



Temperature: general rules apply.
the following thermal variations have to be taken into account

Uniform thermal variation

constant - no gradient

buona resistenza al calore

- Concrete deck (r.c. & p.c.) ±15°C
- Composite steel-concrete deck ±15°C
- Steel deck with ballast ±20°C
- Steel deck without ballast ±25°C
- Other concrete structures ±15°C

ballast → reduce the thermal stress on the deck

±20°C → higher than cs structure 'cause it is thinner and less resistance to temperature variations

For joints and bearings all theses values need to be raised by 50%

Non uniform thermal variation

gradient of temperature

- Between intrados and extrados of the deck ±5°C
- Between inside and outside of box section decks ±5°C
- Between concrete slab and steel beam ±5°C
- Between inside and outside of box section piers ±10°C
- Between the pier and its foundation ±5°C

più alta perchè lo spessore delle pareti è maggiore rispetto a quello dell'impalcato

Thermal variation on the rail

rail → binari (elementi saldati - welded) non sopportano bene gli spostamenti del deck

1) The thermal variation on the rail can be neglected if the rail has expansion joints.
2) Otherwise +30°C - 40°C from the rail regulation temperature.

- For STEEL decks the thermal variations of the rail should be applied with the thermal variation of the deck with the same sign.
- For CONCRETE or composite decks the thermal variations (positive and negative) of the rail should be coupled with nil thermal variation of deck and the thermal variation of the deck (positive or negative) should be coupled with nil thermal variation of the rail.

the movement of the bridge shouldn't tense the rail

INTERACTION EFFECTS
AMONG TRACK - BALLAST - DECK - PIERS - FOUNDATION

(1) Where the rails are continuous over discontinuities in the support to the track (*e.g.* between a bridge structure and an embankment) the structure of the bridge (bridge deck, bearings and substructure) and the track (rails, ballast etc.) jointly resist the longitudinal actions due to traction or braking. Longitudinal actions are transmitted partly by the rails to the embankment behind the abutment and partly by the bridge bearings and the substructure to the foundations.

(2) Where continuous rails restrain the free movement of the bridge deck, deformations of the bridge deck (*e.g.* due to thermal variations, vertical loading, creep and shrinkage) produce longitudinal forces in the rails and in the fixed bridge bearings.

(3)P The effects resulting from the combined response of the structure and the track to variable actions shall be taken into account for the design of the bridge superstructure, fixed bearings, the substructure and for checking load effects in the rails.

Parameters affecting the combined response of the structure and track

a) Configuration of the structure:
- simply supported beam, continuous beams or a series of beams,
- number of individual decks and length of each deck,
- number of spans and length of each span,
- position of fixed bearings,
- position of the thermal fixed point,
- expansion length L_T between the thermal fixed point and the end of the deck.

b) Configuration of the track:
- ballasted track or non-ballasted track systems,
- vertical distance between the upper surface of the deck and the neutral axis of the rails,
- location of rail expansion devices.

c) Properties of the structure:
- vertical stiffness of the deck,
- vertical distance between the neutral axis of the deck and the upper surface of the deck,
- vertical distance between the neutral axis of the deck and the axis of rotation of the bearing,
- structural configuration at bearings generating longitudinal displacement of the end of the deck from angular rotation of the deck,
- longitudinal stiffness of the structure defined as the total stiffness which can be mobilised by the substructure against actions in the longitudinal direction of the tracks taking into account the stiffness of the bearings, substructure and foundations.

d) Properties of the track:
- axial stiffness of the rail,
- resistance of the track or the rails against longitudinal displacement considering either:
  - resistance against displacement of the track (rails and sleepers) in the ballast relative to the underside of the ballast, or
  - resistance against displacement of the rails from rail fastenings and supports *e.g.* with frozen ballast or with directly fastened rails,

  where the resistance against displacement is the force per unit length of the track that acts against the displacement as a function of the relative displacement between rail and the supporting deck or embankment.

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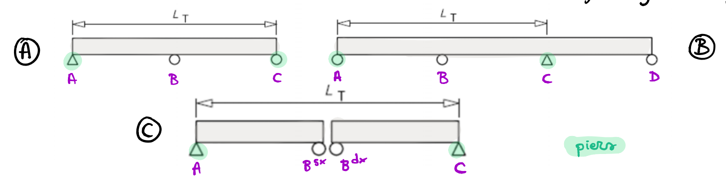
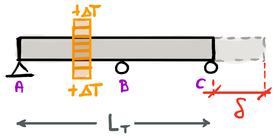
Let's consider the following structures: Ⓐ, Ⓑ, Ⓒ have all the same span (distance between the piers is the same even though they have different static scheme)

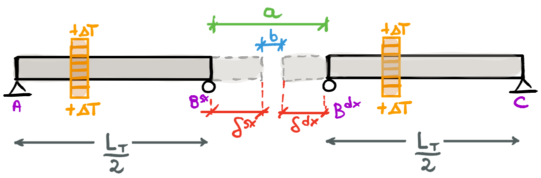


CASE Ⓐ : owing to thermal variation this bridge can increase/decrease its length. Point A is fixed, the max displacement occurrs in B, and this displacement is proportional to $2L_T$:



$\delta = \alpha \Delta T \cdot 2L_T$

CASE Ⓒ : the point on the left will move to the right because thermal effect $\delta^{sx} = 2 \cdot \alpha \Delta T \cdot \frac{L_T}{2}$; the point on the right because thermal expantion will move to the left: $\delta^{dx} = 2\alpha \Delta T \frac{L_T}{2}$. So the gap between the two bearings will reduce. How much? $\delta^{sx} + \delta^{dx} = \alpha \Delta T L_T + \alpha \Delta T L_T$



a = initial distance between the two bearings (B^{sx}, B^{dx})

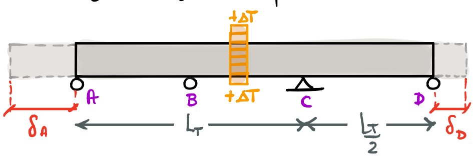
b = final distance between the two bearings (B^{sx}, B^{dx})

$b = a - \delta^{sx} - \delta^{dx}$

The gap between the two bearings will reduce, as already said, of a quantity: $2\alpha \Delta T L_T$. So the value of the displacement that the rail has to suffer in scheme Ⓒ is the same of the scheme Ⓐ.

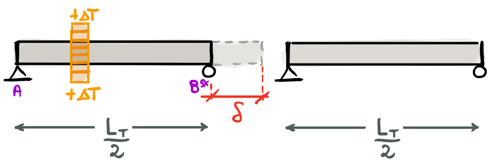
So from the point of view of the rail, scheme Ⓐ & Ⓒ are the same, 'cause they produce the same displacements.

CASE Ⓑ : Because of thermal expantion, A will move to the left of a quantity: $\delta_A = 2\alpha \Delta T \cdot L_T$; B will move to the right of a quantity: $\delta_D = 2\alpha \Delta T \cdot \frac{L_T}{2}$.



A is the worst point for the rail track 'cause occours the max displacement.

These 3 static schemes from the point of view of the stress in the rail because of thermal expansion are equivalent. But if we change static scheme mixing scheme Ⓑ with scheme Ⓒ, we can reduce the maximum displacement.

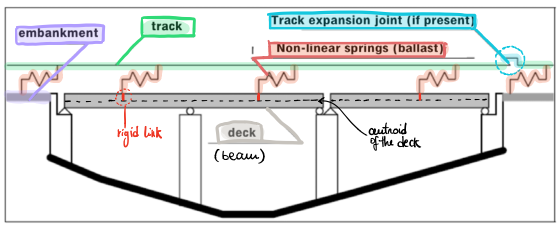


We get a total displacement of

$$\delta = 2\,\alpha\,\Delta T\,\frac{L_T}{2}$$

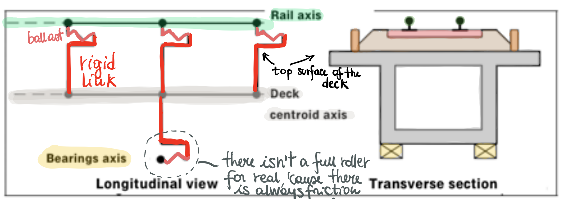
it's a better solution for the rail track

Scheme of structural model to evaluate the interactions



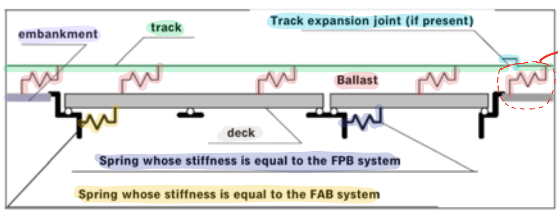
if we have horizontal forces, they go to the springs (ballast) and then to the deck trough rigid links. Then go down to the bearings, and then to the piers. So it's a very complex system that takes into account many different stiffness to transfer horizontal forces from track to foundation.

Scheme of track-deck-bearings connection



there isn't a full roller for real 'cause there is always friction and other stuff
⇒ -ʌʌʌ- I use spring (some horizontal stiffness)

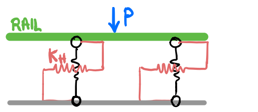
Simplified structural model



the stiffness of this spring (ballast) is hard to evaluate

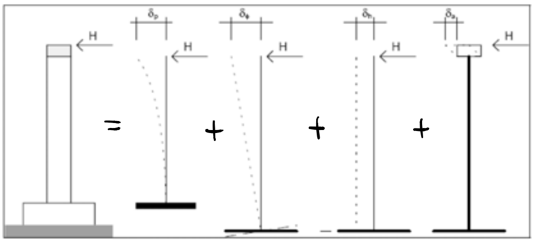
FPB = Foundation – Pier – Bearings
FAB = Foundation – Abutment - Bearings

Example: longitudinal total stiffness of FPB



ANALYSIS IN NON LINEAR FIELD

is a non linear elastoplastic horizontal spring, whose stiffness is a function of the vertical load.

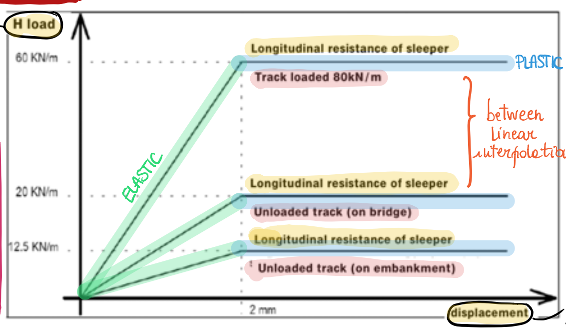


$K = H / \Sigma\delta_i$ $\quad \Sigma\delta_i = \delta_p + \delta_\varphi + \delta_h + \delta_a$

Ballast: variation of longitudinal shear force with vertical load for 1 track

horizontal load that acts on ballast

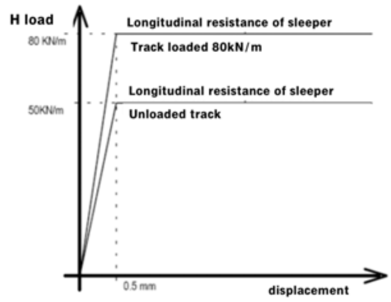
The stiffness of the spring (ballast) is not constant, but it is function of the vertical load



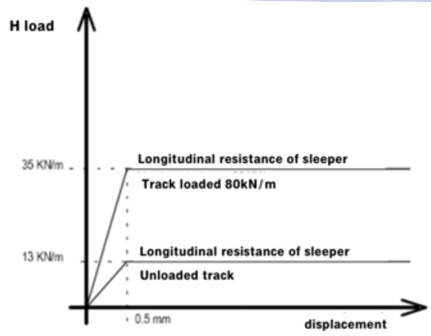
horizontal displacement that the ballast is going to allow.

As shown in the graph, the mechanical response of the ballast is NON LINEAR: there is an elastic part and then a plastic part.

K joint: variation of longitudinal shear force with vertical load for 1 track



Elastic joint:
variation of longitudinal shear force with vertical load for 1 track



AERODYNAMIC ACTIONS

function of the shape of the train

Simple vertical surfaces parallel to the track (*e.g.* noise barriers)

Simple horizontal surfaces above the track

Simple horizontal surfaces adjacent to the track

Multiple-surface structures alongside the track with vertical and horizontal or inclined surfaces

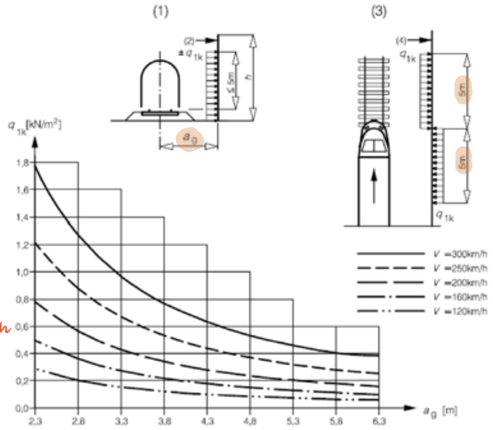
Surfaces enclosing the structure gauge of the tracks over a limited length

(2) The characteristic values apply to trains with an unfavourable aerodynamic shape and may be reduced by:

- a factor $k_1 = 0{,}85$ for trains with smooth sided rolling stock
- a factor $k_1 = 0{,}6$ for streamlined rolling stock (*e.g.* ETR, ICE, TGV, Eurostar or similar)

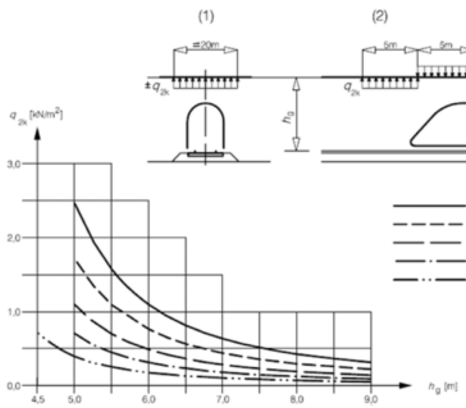
(3) If a small part of a wall with a height $\leq 1{,}00$ m and a length $\leq 2{,}50$ m is considered, *e.g.* an element of a noise protection wall, the actions q_{1k} should be increased by a factor $k_2 = 1{,}3$.

the load q is function of the speed V of the train and of a_g (distance between me and the centroid of the train)



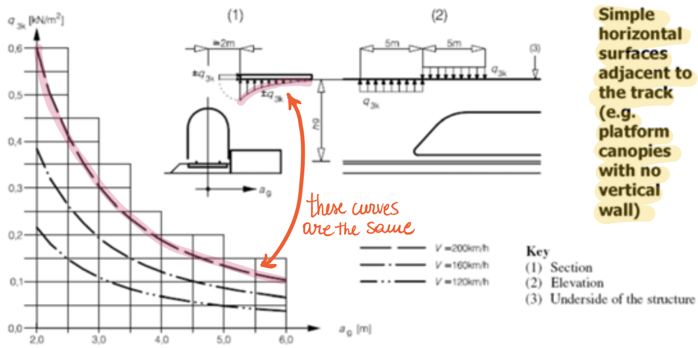
Simple vertical surfaces parallel to the track (e.g. noise barriers)

Key
(1) Section
(2) Surface of structure
(3) Plan view
(4) Surface of structure

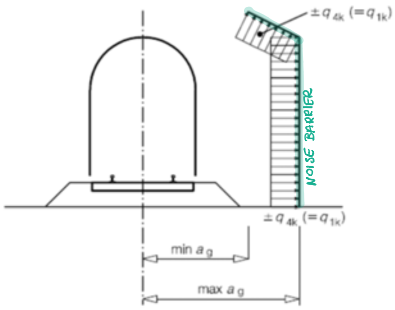


Simple horizontal surfaces above the track (e.g. overhead protective structures)

Key
(1) Section
(2) Elevation
(3) Underside of the structure



Simple horizontal surfaces adjacent to the track (e.g. platform canopies with no vertical wall)

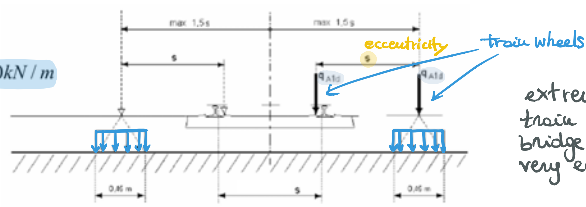


Multiple surfaces structures alongside the the track with vertical and horizontal or inclined surfaces (e.g. bent noise barriers)

DERAILMENT OVER THE BRIDGE

Case 1

$q_{A1d} = 60 kN/m$



extreme condition, train still on the bridge but in a very eccentric position

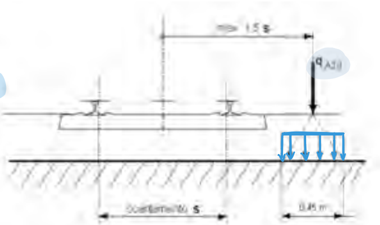
The load includes dynamic effect and may be placed transversally in every position within the field ± 1.5 s

Only small entity damage can be accepted in order to re-open the line after light maintenance

Case 2

bigger load than case 1 ← $q_{A2d} = 80 \cdot 1.4 \, kN/m$

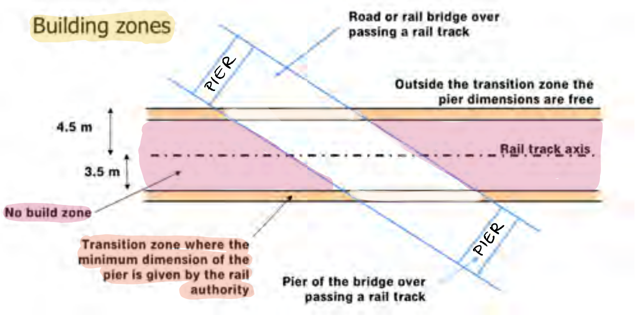
is the dynamic coefficient

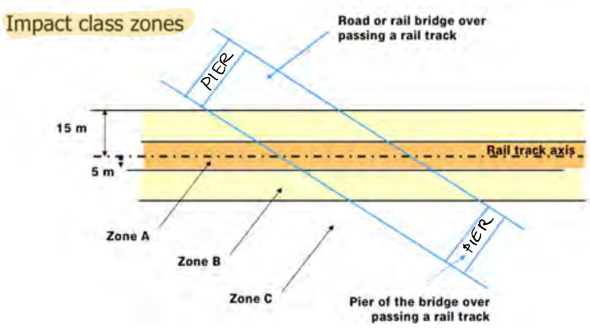


The load has to be placed for a maximum longitudinal extension of 20 m and in every position within the field ±1.5 s

For global verifications only (Equilibrium of the deck, collapse of the main structure). Severe damage accepted.

DERAILMENT UNDER THE BRIDGE





Impact forces

Zone A
- 4000 kN parallel to the track
- 1500 kN orthogonal to the track

Zone B
- 2000 kN parallel to the track
- 750 kN orthogonal to the track

Zone C
- Nothing

These forces should be considered at 1.80 m from the rail level and not acting simultaneously

All the actions seen have to be combined at SLU and SLE (combination of actions)

NUMBER OF TRACKS TO BE LOADED

N° of tracks	Loaded tracks	Normal traffic (passengers) Case a	Case b	Heavy Traffic (goods)
1	1st	1,0 (LM 71"+"SW/0")	-	1,0 SW/2
2	1st	1,0 (LM 71"+"SW/0")	-	1,0 SW/2
	2nd	1,0 (LM 71"+"SW/0")	-	1,0 (LM 71"+"SW/0")
≥ 3	1st	1,0 (LM 71"+"SW/0")	0,75 (LM 71"+"SW/0")	1,0 SW/2
	2nd	1,0 (LM 71"+"SW/0")	0,75 (LM 71"+"SW/0")	1,0 (LM 71"+"SW/0")
	Others	- envelope	0,75 (LM 71"+"SW/0")	-

LM71 "+" SW0 Means "which of the two that gives the most unfavourable effect"

Load Type	Vertical actions		Horizontal actions		
Group	Vertical load	Unloaded train	Braking Traction	Centrifugal	Nosing
1	1.00	-	0.5(0.0)	1.0(0.0)	1.0(0.0)
2	-	1.00	0.0	1.0(0.0)	1.0(0.0)
3	1.0(0.0)	-	1.0	0.5(0.0)	0.5(0.0)
4	0.8 (0.6,0.4)	-	0.8 (0.6,0.4)	0.8 (0.6,0.4)	0.8 (0.6,0.4)

100%

80%

Group	Comment
1	Maximum vertical and lateral actions
2	Lateral stability
3	Maximum longitudinal action
4	Crack control

Group 4 should be considered only for crack control

Value 0.6 should be used for 2 tracks loaded, while value 0.4 should be considered for more than 2 tracks

N.B. SLU and SLE verifications

- SLU verifications

SLU actions safety factors

		Coeff.	EQU[1]	A1 STR	A2 GEO	Accidental situation	Seismic situation
Permanent actions	Fav. Unfav.	γ_{G1}	0,90 1,10	1,00 1,35	1,00 1,00	1,00 1,00	1,00 1,00
Non struct. [2] permanent actions	Fav. Unfav.	γ_{G2}	0,00 1,50	0,00 1,50	0,00 1,30	1,00 1,00	1,00 1,00
Ballast [3]	Fav. Unfav.	γ_B	0,90 1,50	1,00 1,50			
Variable traffic loads [4]	Fav. Unfav.	γ_Q	0,00 1,45	0,00 1,45			
Other variable actions	Fav. Unfav.	γ_{Qi}	0,00 1,50	0,00 1,50			
Prestressing	Fav. Unfav.	γ_P	0,90 1,00[6]	1,00 1,00[7]			

(1) Equilibrium which is independent from the resistance and deformability characteristics of the ground; otherwise GEO values apply.
(2) If non structural loads are defined in detail, the same values as for permanent loads may be applied.
(3) If sensible variations of the ballast load are expected, they should be explicitly taken into account in the design.
(4) Traffic actions should be treated using the multi component actions shown in slide 112.
(5) Ratio of traffic load to be taken into account.
(6) 1.30 for instability in external prestressed structures.
(7) 1.20 for local effects.

- SLE verifications

SLS actions combination factors

Azioni		ψ_0	ψ_1	ψ_2
Single traffic actions	Load on the embankment	0,80	0,50	0,0
	Aerodynamic actions	0,80	0,50	0,0
Multi Comp. actions	gr_1	0,80[2]	0,80[1]	0,0
	gr_2	0,80[2]	0,80[1]	-
	gr_3	0,80[2]	[illegible]	0,0
	gr_4	1,00		
Wind	F_{Wk}	0,60		
Snow	In construction	0,80		
	SLU and SLE	0,0		
Temper.	T_k	0,60		

(1) 0.8 for only one track loaded, 0.6 for two tracks loaded, 0.4 if three or more tracks are loaded
(2) If the wind is taken as the base action, Ψ_0 coefficients for traffic loads should be taken equal to 0.

SLE Verifications for rail-traffic safety

- Vertical acceleration of the deck

comfort for passengers

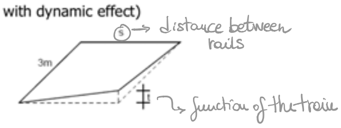
$a_v \leq 3.5 m/s^2$ if $0 < f < 20$ Hz

natural frequency of the deck

- Torsional deformation of the deck

(to be calculated with LM71 incremented with dynamic effect)

V≤ 120 km/h; t ≤ 4,5 mm/3m
120<V≤ 200 km/h; t ≤ 3,0 mm/3m
V > 200 km/h; t ≤ 1,5 mm/3m



If V>200 km/h ⇒ t≤1.2 mm/3m for real trains with dynamic effect

- Horizontal deflection of the deck

(to be calculated with LM71 incremented with dynamic effect, Wind, nosing force, centrifugal force, temperature variation)

The curvature radius for single span deck is given by $R = \dfrac{L^2}{8\,\delta_h}$

length of span

Where δ_h is the horizontal deflection

horizontal deflection

Speed [km/h]	Maximum angular variation	Minimum curvature radius	
		Single span	>1 span
V ≤ 120	0,0035 rd	1700 m	3500 m
120 < V ≤ 200	0,0020 rd	6000 m	9500 m
200 < V	0,0015 rd	14000 m	17500 m

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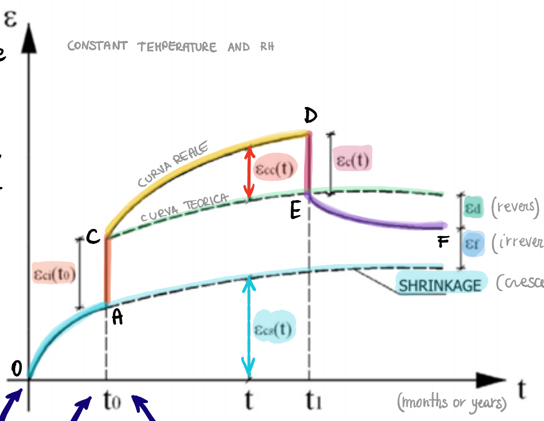
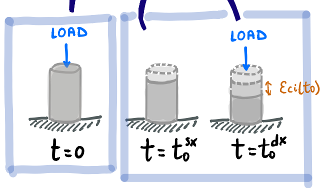
3. CREEP EFFECTS

CREEP AND RELAXATION FUNCTION

Imagine to have a concrete specimen loaded only in compression at time t=0. Then I remove the load. Owing to SHRINKAGE the specimen gets shorter following the curve from 0→A. At time t = t0 I load again my specimen, and it gets shorter immidiatly $\varepsilon_{ci}(t_0)$.





$\varepsilon_{ci}(t_0)$ = Elastic immediate deformation at time t_0

$\varepsilon_{cs}(t)$ = Shrinkage deformation at time t

$\varepsilon_{cc}(t)$ = Creep deformation at time t

$\varepsilon_{c}(t)$ = Elastic deformation at down loading at time t_1 ($\varepsilon_c(t) < \varepsilon_{ci}(t_0)$) by effect of elastic modulus increase with aging)

$\varepsilon_{d}(t)$ = Delayed elasticity

$\varepsilon_{f}(t)$ = Delayed plasticity

for $\sigma_c \leq 0.4\, f_{ckj}$ → $\varepsilon_{ci}, \varepsilon_f, \varepsilon_{cc}, \varepsilon_c, \varepsilon_d \propto \sigma$ ⇒ linear creep → so I can use the superposition principle

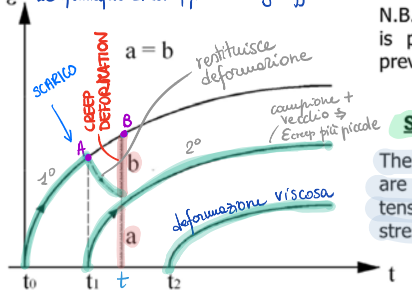
Now I'm in point C of the curve. From here it is supposed that the shortening of the specimen follows a curve parallel to the little blu dotted curve (the green one), but it isn't so. Infact the specimen shortening follows the yellow curve: the elongation is higher due to the CREEP effect (additional deformation due to load in time). At time t = t1 I remove the load (point D). What happens? The specimen will recover only a part of the elongation ($\varepsilon_c(t) < \varepsilon_{ci}(t_0)$) because at time t1 the modulus of elasticity E(t) is different than the E(t0) cause concrete is an aging material (E↑ in time ⇒ specimen becomes stiffer ⇒ it allows small deformations). After this immediate recover $\varepsilon_c(t)$, the specimen becomes longer and longer (it is relaxing), from E→F it is recovering the creep deformation but not completely but only a percentage ε_d of the total (elasticity), while it remains a plastic deformation ε_f.

This graph is important because of the relative value of the curves. The most important part of the total deformation is due to SHRINKAGE & CREEP EFFECTS, while the elastic one is small.

So during the construction phase and during the bridge life I have to check the stress limitation: $\sigma_c \leq 0.4\, f_{ck}$, in order to be in linear creep field. If it is so, I can use the superposition principle thanks to the MAC-HENRY PRINCIPLE.

MAC-HENRY PRINCIPLE

↳ supera i problemi della non linearità (grafico precedente) ⇒ rende applicabile il principio di sovrapposizione degli effetti



⊛ A stress variation $\Delta\sigma$ applied at time t_1 has the same effect, whichever is the age at loading and the sign of $\Delta\sigma$.
(trazione o compressione)

N.B. The effect of the aging at loading is pictured on the diagram of the previous slide .

APPLICABILITY OF SUPERPOSITION PRINCIPLE

The creep laws used for compression are supposed to be valid also for tension and for multi-axial state of stress.

+vecchio è il campione, minori saranno le deformazioni viscose

Prendiamo 2 campioni uguali: al tempo t_0 carico con carico costante il 1° campione. La deformazione elastica istantanea non è rappresentata nel diagramma per semplicità. Dopo di questa, segue una deformazione viscosa (in verde). A t_1 scarico il 1° provino, e carico il 2° con lo stesso carico. Quest'ultimo poiché più vecchio ha delle deformazioni di fluage più piccole (curva più bassa), questo perché la deformazione viscosa è funzione della età di messa in carico. Considerando un tempo t generico, successivo a t_1, la quantità di deformazione restituita dal 1° campione misurata non rispetto a quello che aveva accumulato (A), ma rispetto a quello che avrebbe accumulato se non l'avessi scaricato ⇒ da (B) la quantità di deformazione che mi viene restituita è uguale alla quantità di deformazione viscosa che accumula il 2° campione in compressione (a=b). ⇒ ⊛. Nel caso di una variazione della storia tensionale complessa, mi consente di analizzare l'effetto della storia di tensione come somma degli effetti delle variazioni prodotte da ciascun intervallo in cui la tensione è costante ⇒ principio di sovrapposizione effetti.

CONCRETE = AGEING MATERIAL, WITH LINEAR VISCOELASTIC BEHAVIOUR

Creep deformation in concrete (only creep) →

$$\varepsilon_{cc}(t,t_0)=\underbrace{\frac{\sigma_c(t_0)}{E_{ci}}}_{\text{deformazioni elastiche istantanee}}\phi(t,t_0)$$

t = time now
t_0 = time in wich the load was applied
ε function of the stren at time t_0 (deriving from the application of the load) and of the modulus of elasticity at 28 days → convention

$\Phi(t,t_0)$ = creep coefficient (non dimensional) ← misura l'entità della deformazione viscosa rispetto quella elastica per carico applicato a t_0 e valutato al tempo t ($\phi \simeq 2 \div 4.5$ dunque la ε_{fluage} è fino a 4 volte maggiore rispetto la $\varepsilon_{elastica\ istantanea}$)

$\sigma_c(t_0)/E_{ci}$ = elastic deformation at time t_0 (elastic modulus E_{ci} at 28 days)

The overall deformation at time t by effect of a constant stress σ_c, is

Strain in concrete due to load (stress) →

$$\varepsilon_{c\sigma}(t,t_0)=\underbrace{\sigma_c(t_0)}_{\text{tensione iniziale}}\left[\underbrace{\frac{1}{E_c(t_0)}}_{\text{elastic strain}}+\underbrace{\frac{\phi(t,t_0)}{E_{ci}}}_{\text{creep strain}}\right]=\sigma_c(t_0)J(t,t_0)$$

→ if we want to add the effect of the SHRINKAGE we have to add: $+\varepsilon_{shrinkage}(t,t_0)$ ↑ it is not function of the load, but only of time and environmental conditions

J = *creep function* $[F^{-1}L^2]$ → overall deformation at time t by effect of a unit stress

is an inverse modulus of elasticity

Disregarding the effect of ambient conditions (T[°], RH[%]) and considering only the effect of stress history by application of superposition principle and of linearity hypothesis, the evolution law of overall deformation may be expressed as follows (sum of a first part due to stresses and possible imposed deformation $\varepsilon_{cn}(t)$):

THE MOST GENERAL FORMULA

$$\varepsilon_c(t)=\varepsilon_{cn}(t)+\int_0^t J(t,\tau)\frac{\partial\sigma_c(\tau)}{\partial\tau}\partial\tau$$

t = today
τ = variable from $\tau \in [0;t]$
↳ imposed deformation (like thermal strain, shrinkage one) not depending on stren

τ = time to which the stress variation $\partial\sigma/\partial\tau$ is applied.

Assuming $t = t_0$ $\sigma(t) = \sigma(t_0)$ e $\varepsilon_{cn}(t_0)=0$ it results:

we can rewrite it

$$\varepsilon_c(t,t_0)=\sigma_c(t_0)J(t,t_0)+\int_{t_0}^{t}J(t,\tau)\frac{\partial\sigma_c(\tau)}{\partial\tau}\partial\tau+\varepsilon_{cn}(t) \qquad (1)$$

If the stress variation is applied by means of finite intervals, it results:

$$\varepsilon_c(t,t_0)=\sigma_c(t_0)J(t,t_0)+\sum_{i=1}^{n}J(t,t_i)\,\Delta\sigma(t_i)+\varepsilon_{cn}(t)$$

If on the opposite one works on the stresses, for an assigned deformation history, the relaxation integral is obtained.

Cos'è il concetto del rilassamento? È un modo diverso di vedere la stessa cosa: il creep. Se metto in tensione un provino fisso all'estremità, leggo inizialmente una certa tensione. Col tempo il provino non può subire accorciamenti perché vincolato; ma misurando la tensione si vede che essa sarà diminuita (rilassamento)

variation of the imposed deformation in time

$$\sigma_c(t) = \int_0^t R(t,\tau) \frac{\partial(\varepsilon_c(\tau) - \varepsilon_{cn}(\tau))}{\partial\tau} \partial\tau$$

this is the same equation of the page before, but that one gives me the strain when I apply a force (a stress), this one gives me the stress when I apply a displacement

R = *relaxation function* [FL^{-2}] → stress at time t caused by a unit imposed deformation applied at time τ (elastic modulus at time t)

By analogy with the previous expressions:

N.B. se applico ad un materiale viscoso
- $\varepsilon \Rightarrow \sigma \downarrow$ nel tempo
- $\sigma \Rightarrow \varepsilon \uparrow$ nel tempo

(se elastico stesso comportamento)

$$\sigma(t,t_0) = [\varepsilon_c(t_0) - \varepsilon_{cn}(t_0)]R(t,t_0) + \int_{t_0}^t R(t,\tau) \frac{\partial(\varepsilon_c(\tau) - \varepsilon_{cn}(\tau))}{\partial\tau} \partial\tau \qquad (2)$$

In the previous equations the direct solution is simple, the inverse one implies the use of Volterra integral equations, which are difficult to be solved.

In both cases the equations require the availability of creep laws (derived from constant stress tests) and relaxation laws (derived from constant imposed deformation).

TYPE OF PROBLEM		CREEP FUNCTION J(t,τ)	RELAXATION FUNCTION R(t,τ)
	Problems with assigned stress history	$\varepsilon_c(t)$ = ? Simple integration	$\varepsilon_c(t)$ = ? Solution of Volterra integral equation
	Problems with assigned deformation history	$\sigma_c(t)$ = ? Solution of Volterra integral equation	$\sigma_c(t)$ = ? Simple integration

Type of problem in which the solution of Volterra integral equation is more frequent.

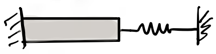
The rheological behaviour of concrete is characterized both by the availability of J(t,t) and of R(t,t).

I use table on codes with an error ±30%

For the practical applications, within the linear creep field, the structures can be divided into the following four groups:

I calculate it numerically

- **homogeneous structures with rigid (elastic) restraints** } 1
- **structures with constant restraint conditions**
- **heterogeneous structures with rigid (elastic) restraints** } 2
- **structures subjected to variation of the static scheme**



this structure is not OK 'cause spring is not a rigid restraint

The problems involving homogeneous structures may be easily solved by use of J and R functions. 1

The problems involving heterogeneous structures are controlled by one or more integral equations. 2

So we have some limits. The theory we gonna see on next pages is valid for: homogeneous, rigid restraints, eventually of static scheme.

2° PRINCIPLE OF LINEAR CREEP

(TEOREMA DELL'ISOMORFISMO)

A viscous-elastic body, homogeneous and with rigid restraints is subjected to imposed deformations irrespective of internal and external compatibility $\bar{\varepsilon}_A$.

Total deformation: $\bar{\varepsilon}_A + \varepsilon_A$ → internally and externally compatible (ELASTIC)

ε_A: complementary elastic deformation producing one self-equilibrated stress system σ_A. (ε_A born for internal and external congruence)

One add now a system of $\bar{\varepsilon}_B$ proportional to ε_A ($\bar{\varepsilon}_B = k\varepsilon_A$) than irrespective of internal and external compatibility; as a consequence a further system of complementary elastic deformation ε_B arises, so that $\bar{\varepsilon}_B + \varepsilon_B$ respect the internal and external compatibility.

ε_B involve the arising of self-equilibrated σ_B.

- Overall deformation (respective of internal and external compatibility)

$$\varepsilon = \bar{\varepsilon}_A + \varepsilon_A + \bar{\varepsilon}_B + \varepsilon_B$$ TOTAL DEFORMATION

- Total stress (self-equilibrated):

$$\sigma = \sigma_A + \sigma_B$$ TOTAL STRESS

To evaluate the effect of $\bar{\varepsilon}_B$ one suppose that:

$$\varepsilon_B = -\bar{\varepsilon}_B$$ hypothesis

Then: $\varepsilon = \bar{\varepsilon}_A + \varepsilon_A + \bar{\varepsilon}_B - \bar{\varepsilon}_B = \bar{\varepsilon}_A + \varepsilon_A$

$\bar{\varepsilon}_A + \varepsilon_A$ respective of internal and external compatibility. Moreover:

$$\varepsilon_B = -k\varepsilon_A \quad \Rightarrow \quad \sigma_B = -k\sigma_A$$

Then:

$$\boxed{\sigma = \sigma_A - k\sigma_A = \sigma_A(1-k)}$$ ⇒ the hypothesis was correct

k: coefficient of proportionality that relates the imposed B to the elastic A, in other words K is CREEP

with $\sigma_A(1-k)$ selfequilibrated.

As the proposed solution respects both internal and external compatibility and is self-equilibrated, it is the actual solution, because of the Kirchhoff theorem of the uniqueness of elastic equilibrium solution.

For instance, we can consider a beam prestressed by jacks and then restrained, in which creep takes place (proportional to the elastic deformation). The deformation state remains unchanged while the stress state varies remaining proportional to itself.

Then we can generalize as:

"The introduction in an elastic body, homogeneous and with rigid restraints and in state of eigen-stress of an imposed deformation proportional to the existing elastic deformation, leaves unchanged the deformation state, while the stress state changes proportionally to itself."

Creep is taking away stress. The introduction of an imposed deformation (proportional to the elastic one) doesn't change the total deformation but changes stress proportionally to the stress itself.

Il fluage è una deformazione impressa non congruente e non compatibile che è proporzionale alla deformazione elastica preesistente.

A such imposed deformation ($\bar{\varepsilon}_B$) is called "ISOMORFA". → because the imposed deformation introduced has the same shape of the elastic deformation

EXAMPLE

COLUMN IN CLS

I impose an imposed deformation $\bar{\varepsilon}_a$ (shortening of my column constant). What's appen inside my column? Imposing $\bar{\varepsilon}_a$ ⇒ stress arises. I have an elastic deformation $\varepsilon_a = -\bar{\varepsilon}_a$ and the stress $\sigma_a = E \cdot \varepsilon_a$. Arriva il creep (deformazione impressa) $\bar{\varepsilon}_b = k \cdot \sigma_a = k \cdot E \cdot \varepsilon_a$. E la sua ε elastica è $\varepsilon_b = -\bar{\varepsilon}_b$. Cosa succede? La faccia che ho spostato per accorciare il provino rimane lì (la deformazione totale non cambia), ma la σ nel provino diminuisce perché ho rilassato il provino → $\sigma = \sigma_a(1-k)$

1° PRINCIPLE OF LINEAR CREEP

(COROLLARIO DEL TEOREMA DELL'ISOMORFISMO)

Consider an elastic body, homogeneous with rigid restraints, in equilibrium under the action of a force system F.

There'll be equilibrated stresses σ_A and deformations ε_A which respects internal and external compatibility. (rispettano le Eq. di congruenza e le compatibilità coi vincoli)

A system of imposed deformations is added: $\bar{\varepsilon}_B = k'\varepsilon_A$.

$\bar{\varepsilon}_B$ respects the internal and external compatibility because it is proportional to ε_A. Then $\varepsilon_B = 0$ and $\sigma_B = 0$.

- Overall deformation: $\varepsilon = \varepsilon_A + \bar{\varepsilon}_B = \varepsilon_A(1+k')$ TOTAL DEFORMATION
- Overall stress: $\sigma = \sigma_A$ TOTAL STRESS (dovuto solo ai carichi perchè $\sigma_B = 0$)

For instance the creep changes the state of deformation proportionally to itself, whereas the state of stress remains unchanged.

"The introduction in an elastic body, homogeneous and with rigid restraints subjected to an equilibrated system of forces, of a deformation proportional to the existing one leaves unchanged the state of stress, whereas the state of deformation changes proportionally to itself. ".

From a quantitative point of view: $\bar{\varepsilon}_B \equiv linear\ creep$

$U_i^{el}(t)$ = deformation state of an elastic, homogeneous, rigidly restrained structure deriving by imposed deformations $\varepsilon_{cn}(t)$.

Following the application of an "isomorfa" deformation we get:

$U_i(t) = U_i^{el}(t)$

$$\boxed{\sigma_c(t) = \int_0^t R(t,\tau)\frac{\partial(\varepsilon_c(\tau)-\varepsilon_{cn}(\tau))}{\partial\tau}\partial\tau = \frac{1}{E_{co}}\int_0^t R(t,\tau)\frac{\partial\sigma_c}{\partial\tau}\partial\tau}$$

→ legge che esprime la variazione di tensione all'interno dell'elemento in cui, in presenza di deformazioni imposte, siamo andati ad aggiungere il fluage

If, on the opposite, $\sigma_c^{el}(t)$ is the stress state deriving by an equilibrated system of forces, it results:

$\sigma_c(t) = \sigma_c^{el}(t)$

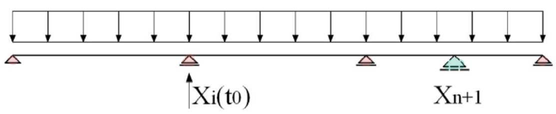
$$\boxed{U_i(t) = \int_0^t J(t,\tau)\frac{\partial\sigma(\tau)}{\partial\tau}\partial\tau = E_{co}\int_0^t J(t,\tau)dU_i^{el}(\tau)}$$

3° PRINCIPLE OF LINEAR CREEP

One change in static scheme exactly at time t_0 (time of loading)

(PRINCIPLE OF REINTRODUCTION OF DELAYED RESTRAINTS) → vincoli introdotti tutti al tempo t_0

The elastic body, homogeneous and with *n* rigid restraints is subjected to $X_i(t_0)$ reactions by effect of constant forces F applied at time t_0.



1ª configurazione → 4 appoggi
2ª configurazione → 5 appoggi

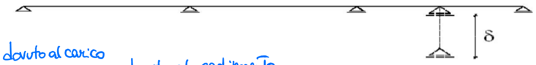
Immediately after the application of the load a further restraint is introduced, in which obviously at time t_0 the reaction is $X_{n+1}(t_0) = 0$.

Let's analyze the evolution of all the reactions by effect of creep.

Suppose to introduce the n+1 restraint before the application of the load; consequently a reaction will arise in it : $\mathbf{X_{n+1}(t_0)}$, and all other reactions will have variations $\Delta X_i(t_0)$.

To come back to the initial conditions, the restraints n+1 should be subjected to a settlement δ equal to the displacement of the same point deriving by the load.

As a consequence:



dovuto al carico — dovuto al cedimento

in n+1: $$\boxed{\underbrace{X_{n+1}(t_0)}_{A} - \underbrace{X_{n+1}(t_0)}_{B} = 0} \Rightarrow R_{TOT} = 0$$

quando tolgo il vincolo X_{n+1}

in i, in t_0: (in other beam(s)) $$\boxed{\underbrace{X_i(t_0)}_{C} + \underbrace{\Delta X_i(t_0)}_{D} - \underbrace{\Delta X_i(t_0)}_{E} = X_i(t_0)} \rightarrow \text{the reaction remains the same}$$

quando applico il carico (come se non ci fosse X_{n+1}) — derivano dalla presenza di X_{n+1} all'applicazione del carico

where:

A : effect of forces, than constant with time (reazione) (1° principio)

B : effect of imposed deformation, then variable in time with relaxation law (reazione dovuta al cedimento) (2° principio)

C : forces (reazione nel vecchio schema statico)

D : forces (variazione della reazione del vecchio schema dovuto all'introduzione del Δ)

E : imposed deformation (variazione della reazione del vecchio schema dovuto il cedimento del nuovo vincolo)

Considering the nature of different contributions, at time t, it will result:

evaluation in time of the reaction → follows the 2° principle of linear creep

$$X_{n+1}(t) = \underbrace{X_{n+1}(t_0)}_{\text{cost}} - \underbrace{X_{n+1}(t_0)\frac{R(t,t_0)}{E_c}}_{\text{varia con legge di rilassamento}} = X_{n+1}(t_0)\left[1-\frac{R(t,t_0)}{E_c}\right]$$

$$X_i(t) = \underbrace{X_i(t_0)+\Delta X_i(t_0)}_{\text{invarianti}} - \underbrace{\Delta X_i(t_0)\frac{R(t,t_0)}{E_c}}_{\text{variabili}} = X_i(t_0)+\Delta X_i(t_0)\left[1-\frac{R(t,t_0)}{E_c}\right]$$

As for t_0 = 28 days it results $R(t,t_0)/E_c$ = 0,15÷0,30, for $t = \infty$ it results:

$$X_{n+1}(t) = (0{,}70 \div 0{,}85)X_{n+1}(t_0)$$

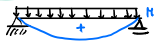
at the end of the life of the structure

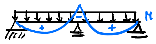
The final value of reaction in restraints n+1 is very close to the value that corresponds to the restraints introduced before the application of the load.

Several construction procedures imply variation of static scheme, but with time t_1 of introduction of new restraints sometime very different from t_0.

Then the previous principle should be generalized introducing the variable $t_1 > t_0$

EXAMPLE

Ⓐ  initial scheme (before variation)
↳ I solve this scheme ⇒ M,N,V,T,...

Ⓑ  statical scheme after variation
↳ I solve this scheme ⇒ M,N,V,T,...

Then we calculate the variation of internal actions:

$$\Delta X = Ⓑ - Ⓐ$$ (con ΔX che può essere ΔM, ΔN...)

and then:

↑t

$$\Delta X \cdot \left[1-\frac{R(t,t_0)}{E_c}\right]$$

at time $t=t_0$ $R(t_0,t_0)=E_c \Rightarrow 1-1 \Rightarrow \Delta X \cdot 0$ nothing happens ⇒ OK

$$Ⓐ + \Delta X\left[1-\frac{R(t,t_0)}{E_c}\right]$$

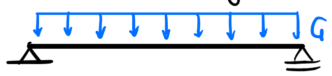
for $t \to \infty \Rightarrow R \cong 0.2\,E_c \Rightarrow Ⓐ + \Delta X\underbrace{\left[1-0.2\frac{\cancel{E_c}}{\cancel{E_c}}\right]}_{0.8}$

the yellow term is always < 1 ⇒ we have not scheme Ⓑ never (only if it was = 1) ← we have a variation of 80%

So the structure is slowly passing from scheme Ⓐ → to Ⓑ never reaching it

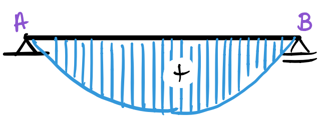
EXAMPLE

Let's consider a generic structure:



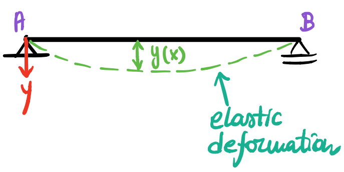
at time $t=t_0$ → the weight G starts to act on the structure.
produces internal actions M,N,V,T

just for example the bending moment plot is ←



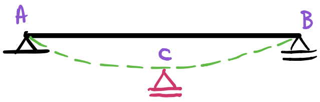
M (parabole function)

and of course I have a deformed structure:



deformed structure (cubic function)

Immediatly after the introduction of the load I change something in the static scheme of the structure (at time $t = t_0^+$)

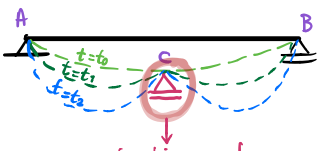


for example I add a roller in midspan.

What happens?

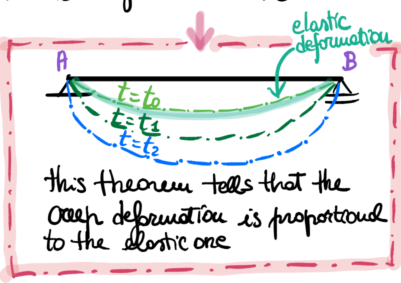
1 second after the introduction of the bearings nothing happens (same bending moment, same deformed shape exc.)

But because of creep the structure would like to deformate like this:



in time a force takes place inside this bearing

but the structure itself cannot do this because I have added a new bearing opposing to the deformation

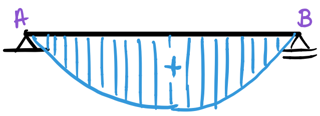


this theorem tells that the creep deformation is proportional to the elastic one

(at $t = t_0$ when I place the bearing $\rightarrow R_c(t_0) = 0$
at $t > t_0$ $\rightarrow$ $R_c \neq 0$ $(R_c(t_1) < R_c(t_2) < \ldots)$)

If a reaction takes place, the bending moment (as the other internal action) is gonna change. So we have:

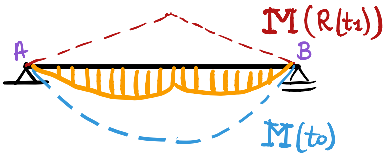
$M(t_0)$



$R_c(t_0)$

$\wedge$

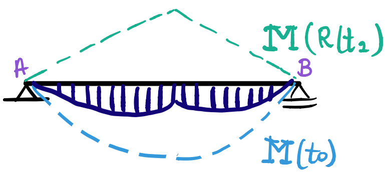
$M(t_1)$



$R_c(t_1)$

$\wedge$

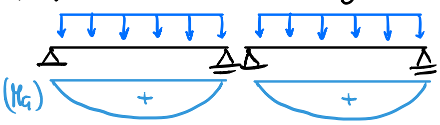
$M(t_2)$



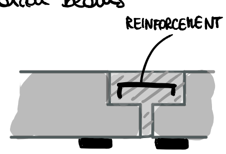
$R_c(t_2)$

$\rightarrow$ Variation of internal actions because of the introduction of $\triangle$.

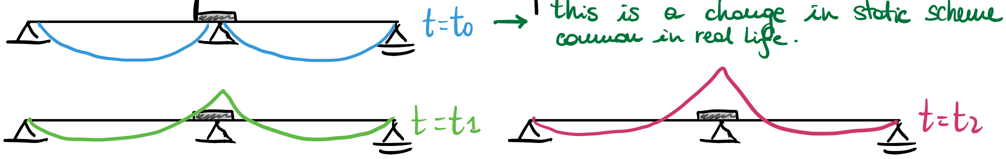
This is only a theoretic example, in real life noone add that $\triangle$. In reality in prefabrication something like this can occour: 2 set of prefabrication beams



When they are placed in position, I close the gap casting concret joining together the two beams



So I transform two isostatic span in a continuous beam:



→ this is a change in static scheme common in real life.

4° PRINCIPLE OF LINEAR CREEP

↳ it covers the same situation of the 3rd but the change in static scheme is done at time $t_1 > t_0$ (ovvero ad un tempo t_1 diverso da quello dell'applicazione dei carichi t_0)

One change in static scheme at time $t_1 > t_0$

Consider a modification of restraint conditions in a homogeneous structure with rigid restraints from *scheme 1* with *k* restraints to *scheme 2* with *m > k* restraints, obtained by the introduction of *m – k* additional restraints at time $t_1 > t_0$ (in t_0 permanent loads are applied).

Being:

$X_R(t)$: reactions at time $t > t_1$ of *k* existing restraints (R = 1,…, k)

$X_S(t)$: reactions at time $t > t_1$ of *m – k* additional restraints (S = k+1,…, m)

$X_R^{el,1}$: elastic reactions in static *scheme 1* (with *k* restraints)

ΔX_R^{el}, ΔX_S^{el}: corrections to be applied to the elastic solution in *scheme 1* to respect the (m – k) additional geometrical conditions imposed by addition restraints (m – k), supposed to be introduced before the introduction of loads (*scheme 2*).

By effect of first theorem of linear visco-elasticity, in presence of permanent actions the elastic values or reactions $X_R^{el,1}$ remain constant (scheme 1), whereas the elastic displacement $u^{el,1}$, evaluated with the E_c reference modulus, increase by means of the non-dimensional factor $E_cJ(t,t_0)$.

At time $t = t_1$ the displacement of points of application of (m – k) additional restraints (active for $t > t_1$) are:

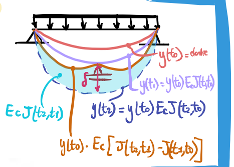
$$\boxed{u_S(t_1) = u_S^{el,1} E_c J(t_1,t_0)}$$

→ evolution from $t_0 \to t_1$

$y(t_0) \to$ elastic
$y(t_1) = y(t_0)\, E_cJ(t_1,t_0)$

The introduction of (m – k) restraints prevents the further deformability due to creep in the corresponding points, that is the restraints impose for $t > t_1$, (m – k) geometrical conditions corresponding to

$$\boxed{\bar{u}_S(t) = -u_S^{el,1} E_c\left[J(t,t_0) - J(t_1,t_0)\right]}$$



For the second principle of linear creep, the reply of reactions $\Delta X_R(t)$ and $\Delta X_S(t)$ to the system of imposed deformations $\bar{u}_S(t)$ for $t > t_1$ may be evaluated by integration from t_1 to t of the increments of elastic deformation, factorized by means of the relaxation factor $R(t,t)/E_c$.

$$\Delta X_R(t) = \Delta X_R^{el} \int_{t_1}^{t} R(t,\tau)\, dJ(\tau,t_0)$$

$$\Delta X_S(t) = \Delta X_S^{el} \int_{t_1}^{t} R(t,\tau)\, dJ(\tau,t_0)$$

Because the increment of reactions are:

$$d\Delta X_R^{el}\left[J(t,t_0) - J(t_1,t_0)\right]E_c = \Delta X_R^{el}\, dJ(\tau,t_0) E_c$$
$$d\Delta X_S^{el}\left[J(t,t_0) - J(t_1,t_0)\right]E_c = \Delta X_S^{el}\, dJ(\tau,t_0) E_c$$

It can be introduced now the function:

today

$$\boxed{\xi(t,t_1,t_0)=\int_{t_1}^{t} R(t,\tau)\,dJ(\tau,t_0)}$$

time of the changing of the static scheme

It results:

$$\boxed{\begin{aligned}\Delta X_R(t)&=\Delta X_R^{el}\,\xi(t,t_1,t_0)\\ \Delta X_S(t)&=\Delta X_S^{el}\,\xi(t,t_1,t_0)\end{aligned}}$$

The evolution in time of reactions for $t > t_1$ within the structure with modified restraints may be evaluated by application of superposition principle:

$$X_R(t)=X_R^{el,1}+\xi(t,t_1,t_0)\Delta X_R^{el}$$

$$X_S(t)=\xi(t,t_1,t_0)\Delta X_S^{el}$$

The function $\xi(t,t_1,t_0)$ measures the portion due to creep of difference between the reaction path corresponding to the application of load in scheme 2 and that one corresponding to scheme 1, for loads applied in t_0 in the scheme 1 and additional restraints introduced in t_1.

$$0 \le \xi \le 1$$

$\xi = 0$ per $t = t_1$

$\xi = 1$ per $t_1 = t_0^+$ (limit case)

for $t_1 = t_0^+$

$$\xi(t,t_0^+,t_0)=\int_{t_0}^{t} R(t,\tau)\,dJ(\tau,t_0)=1-\frac{R(t,t_0)}{E_c}$$

that corresponds to the previous case.

Introducing in (1) of pages 4-8:

$\sigma_c = 0$ per $t < t_0$ e $\sigma_c = 1$ per $t > t_0$ $\Rightarrow$ $\varepsilon_c(t,t_0) = J(t,t_0)$

and from (2)

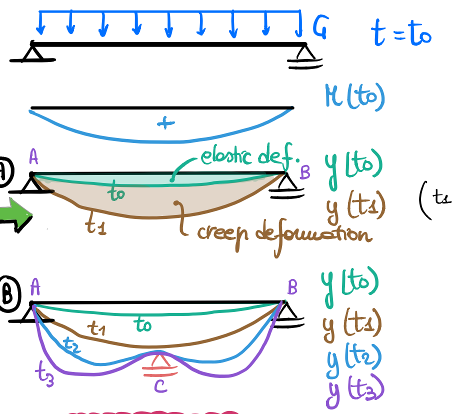
$$1=J(t_0,t_0)R(t,t_0)+\int_{t_0}^{t} R(t,\tau)\,dJ(\tau,t_0)$$

where $J(t_0,t_0) = 1/E_c$

EXAMPLE

I have a structure:

then I wait until time t_1. My structure meanwhile is deforming due creep.



(t_1 for istance could be after 1 month from t_0)

At time t_1 I change static scheme placing a new bearing

What is changing from 3rd principle? Now in scheme (A) we already have a creep deformation that was not present in 3rd principle

The variation ΔX from scheme A → B is smaller than the one computed with 3rd principle

$$R_c^{4^{th}\,pr.} < R_c^{3^{rd}\,pri.}$$

↳ because I lost the first deformation due to creep until time t_1

5° PRINCIPLE OF LINEAR CREEP

Multiple changes in static scheme at any time after the application of the loads

it is a generalization of the 3rd and the 4th principle. We're adding freedom. The 5th principle deals with multiple changes in static scheme, at any time after the introduction of the load.

The integral function $\xi(t,t_1,t_0)$ may also be used for homogeneous structures subjected to several variations of static scheme.
In such case the reaction in delayed restraints k introduced at time t_k in static scheme k *-1* is:

$$\boxed{X_k^{(k)}(t) = X_k^{el(k)}\xi(t,t_k,t_0)} \qquad t_k \le t \le t_{k+1}$$

Such relation produces a variation of reactions in restraints previously introduced, that can be "elastically" evaluated in agreement with first principle of linear creep.

Then the reactions in delayed restraints introduced in times $t_j \le t_k$ assume the expressions:

$$X_j^{(m)}(t) = X_j^{el(j)}\xi(t,t_j,t_0) + \sum_{k=j+1}^{m} a_{jk}^{(k-1)} X_k^k(t)$$

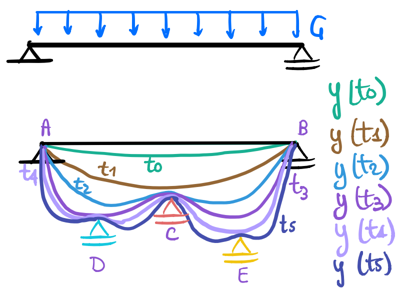
is the function that shows how much the reaction is changing because the introduction of another bearing.

Where $a_{jk}^{(k-1)}$ are the elastic reactions in delayed restraint j due to the application of $X_k = 1$ in static scheme *k-1*.

Finally in the restraint k a reaction arises corresponding to that one that should be present if the restraint had been introduced in the structure with scheme characterized by *k-1* restraints.

The preexisting restraints suffer, by effect of delayed restraint successively applied, variation of reactions that exclusively depend by the reaction $X_k^k(t)$ that arise in such restraints in the static scheme in which they are introduced.

EXAMPLE

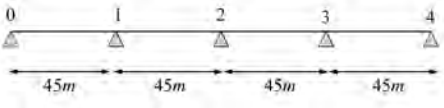


- $t = t_0$ → introducing of load
- $t = t_1$ → introducing of △ in C
- $t = t_3$ → introducing of △ in D
- $t = t_4$ → introducing of △ in E

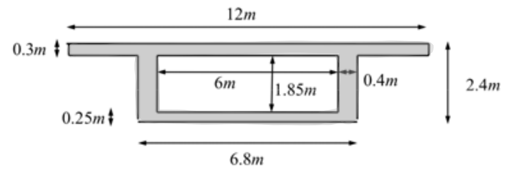
CREEP EFFECTS ON PHASED CONSTRUCTION
(example of 5th principle application)

The following exercise will show an application of the 5th principle of linear viscoelasticity to an homogeneous structure with rigid bearings built in several phases.

Final static scheme of the structure to build



CROSS-SECTION



$A = 0.3\cdot 12 + 2\cdot 0.4\cdot 1.85 + 0.25\cdot 6.8 = 3.6 + 1.48 + 1.7 = 6.78m^2$

exposed perimeter in box shaped cross-section bridges (perimetro che scambia umidità con l'ambiente)

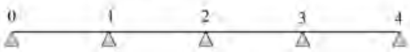
$u = 2\cdot\underbrace{(12+2.4)}_{\text{external perimeter}} + \underbrace{(1.85+6)}_{\text{metà perimetro interno}} = 28.8 + 7.85 = 36.7m$

$\frac{2A_c}{u} = \frac{2\cdot 6.78}{36.7} = 0.369m = 369mm$ $\quad g = \gamma\cdot A_c = 25\cdot 6.78 = 170kN/m$

IDRAULIC RADIUS

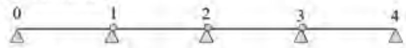
Construction phases

1. t=0d — Simultaneous concreting of the four spans
2. t=28d supported — Scaffolding removal – self weight action on simply beam scheme
3. t=30d — Concreting of key 1
4. t=45d — Concreting of key 2
5. t=60d — Concreting of key 3

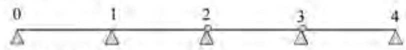


Static schemes corresponding to the different phases

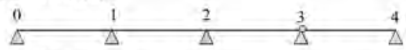
Scheme 0 28d<t<30d



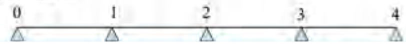
Scheme 1 30d<t<45d



Scheme 2 45d<t<60d



Scheme 3 60d<t

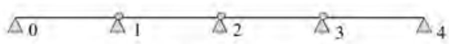


Calculation procedure

1. Solve each of the schemes seen above elastically and evaluate continuity bending moments.
2. Evaluate the functions $\alpha_j(z)$ that show the influence of the restraint j on the previous i:1,j-1.
3. Evaluate the functions $\xi(t,t_1,t_0)$ with respect to section dimensions, relative humidity and time (e.g. they are given in graphical form in CEB fib bulletins)
4. Apply 5th principle of linear viscoelasticity

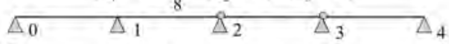
1. Linear elastic solutions

Scheme 0



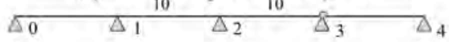
Scheme 1

$M_1^{el(1)} = -\frac{gl^2}{8} \quad M_2^{el(1)} = 0 \quad M_3^{el(1)} = 0$



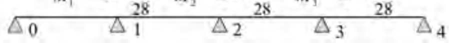
Scheme 2

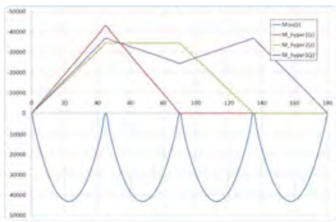
$M_1^{el(2)} = -\frac{gl^2}{10} \quad M_2^{el(2)} = -\frac{gl^2}{10} \quad M_3^{el(2)} = 0$

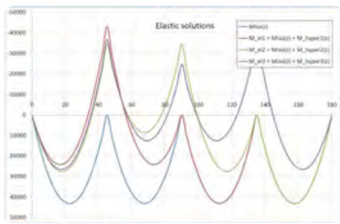


Scheme 3

$M_1^{el(3)} = -\frac{3gl^2}{28} \quad M_2^{el(3)} = -\frac{2gl^2}{28} \quad M_3^{el(3)} = -\frac{3gl^2}{28}$







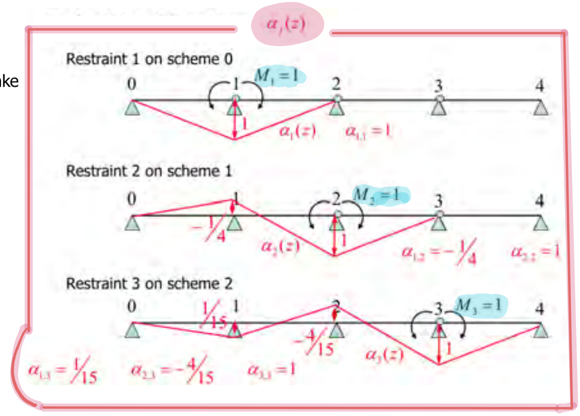
2. Evaluation of functions $\alpha_j(z)$

influence functions that measure the variation of the reaction in one bearing by the effect of the introduction of another bearing

The functions $\alpha_j(z)$ are the internal actions (for us only bending moment for sake of simplicity) calculated on the static scheme j-1 (the one previous to the introduction of the delayed restraint j) by the effect of the reaction given by the delayed restraint j set equal to unit.

Our restraint are hinges that becomes full restraints so the "reactions" are bending moments.

We have 3 delayed restraints so we will have 3 functions $\alpha_j(z)$.



3. Evaluation of functions $\xi(t,t_1,t_0)=\int_{t_1}^{t}R(t,\tau)dJ(\tau,t_0)$

tabled and plotted

$RH\%=50$ (for RH% = 80) $\quad f_{ck}=40MPa \quad t_0=28 \quad days$

1st restraint – key 1 $\quad t_1=30\cong 28 \quad days$

$$\left.\begin{array}{l}2A_c/u=200mm \Rightarrow \xi(45,30,28)=0.45\\ 2A_c/u=400mm \Rightarrow \xi(45,30,28)=0.39\end{array}\right\}\frac{2A_c}{u}=369mm\Rightarrow\xi(45,30,28)=0.40$$

$$\left.\begin{array}{l}2A_c/u=200mm \Rightarrow \xi(60,30,28)=0.50\\ 2A_c/u=400mm \Rightarrow \xi(60,30,28)=0.45\end{array}\right\}\frac{2A_c}{u}=369mm\Rightarrow\xi(60,30,28)=0.46$$

$$\left.\begin{array}{l}2A_c/u=200mm \Rightarrow \xi(\infty,30,28)=0.77\\ 2A_c/u=400mm \Rightarrow \xi(\infty,30,28)=0.76\end{array}\right\}\frac{2A_c}{u}=369mm\Rightarrow\xi(\infty,30,28)=0.76$$

2nd restraint – key 2 $\quad t_1=45 \quad days$

$$\left.\begin{array}{l}2A_c/u=200mm \Rightarrow \xi(60,45,28)=0.25\\ 2A_c/u=400mm \Rightarrow \xi(60,45,28)=0.22\end{array}\right\}\frac{2A_c}{u}=369mm\Rightarrow\xi(60,45,28)=0.22$$

$$\left.\begin{array}{l}2A_c/u=200mm \Rightarrow \xi(\infty,45,28)=0.67\\ 2A_c/u=400mm \Rightarrow \xi(\infty,45,28)=0.68\end{array}\right\}\frac{2A_c}{u}=369mm\Rightarrow\xi(\infty,45,28)=0.68$$

3rd restraint – key 3 $\quad t_1=60 \quad days$

$$\left.\begin{array}{l}2A_c/u=200mm \Rightarrow \xi(\infty,60,28)=0.64\\ 2A_c/u=400mm \Rightarrow \xi(\infty,60,28)=0.63\end{array}\right\}\frac{2A_c}{u}=369mm\Rightarrow\xi(\infty,60,28)=0.63$$

4. Apply 5th principle of linear viscoelasticity

Evaluation of the internal actions.

Phase 1 $\quad M=0 \quad \forall z \quad$ 'cause the scaffolding is supporting the structure

Phase 2 $\quad$ Simply supported beam moment $\quad M_1=M_2=M_3=0$

Phase 3 $\quad$ t=30 days Simply supported beam moment $\quad M_1=M_2=M_3=0$

t=45 days $\quad M_1=M_1^{el(1)}\cdot\xi(45,30,28)=-\frac{gl^2}{8}\cdot 0.40$

$M_2=M_3=0$

Phase 4 $\quad$ t=60 days $\quad M_2=M_2^{el(2)}\cdot\xi(60,45,28)=-\frac{gl^2}{10}\cdot 0.22$

$M_1=M_1^{el(1)}\cdot\xi(60,30,28)+\alpha_{1,2}\cdot M_2^{el(2)}\cdot\xi(60,45,28)=$

$=-\frac{gl^2}{8}\cdot 0.46+\frac{1}{4}\cdot\frac{gl^2}{10}\cdot 0.22$

Phase 5 $\quad$ t=∞ days

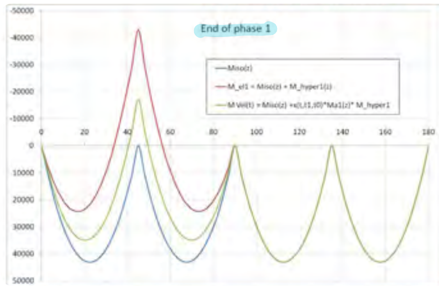
$$M_3=M_3^{el(3)}\cdot\xi(\infty,60,28)=-\frac{3gl^2}{28}\cdot 0.63$$

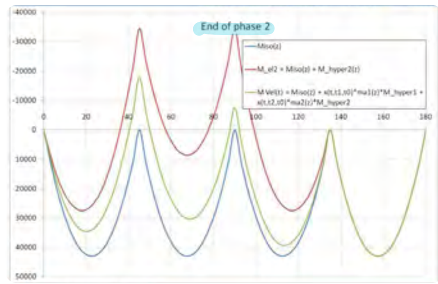
$$M_2=M_2^{el(2)}\cdot\xi(\infty,45,28)+\alpha_{2,3}\cdot M_3^{el(3)}\cdot\xi(\infty,60,28)=$$

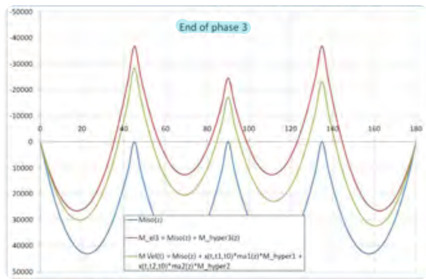
$$=-\frac{gl^2}{10}\cdot 0.68+\frac{4}{15}\cdot\frac{3gl^2}{28}\cdot 0.63$$

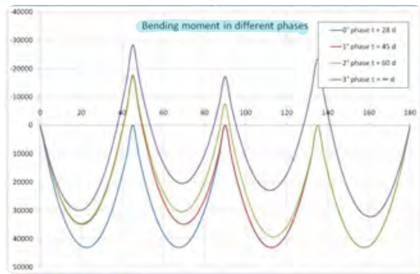
$$M_1=M_1^{el(1)}\cdot\xi(\infty,30,28)+\alpha_{1,2}\cdot M_2^{el(2)}\cdot\xi(\infty,45,28)+\alpha_{1,3}\cdot M_3^{el(3)}\cdot\xi(\infty,60,28)=$$

$$=-\frac{gl^2}{8}\cdot 0.76+\frac{1}{4}\cdot\frac{gl^2}{10}\cdot 0.68-\frac{1}{15}\cdot\frac{3gl^2}{28}\cdot 0.63$$





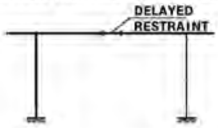




PRINCIPLE OF REINTRODUCTION OF DELAYED RESTRAINS

Construction procedures

In cases in which construction procedures imply a static scheme variation the effects induced by delayed restraints have to be taken into account.



i: initial static scheme
f: final static scheme

For $t_1 \cong t_0$

$$M_\infty = M_i + \Delta M_i\left(1-\frac{R(\infty,t_0)}{E_c}\right) = M_f - \Delta M_i\frac{R(\infty,t_0)}{E_c} =$$

$$= M_f\left(1-\frac{R(\infty,t_0)}{E_c}\right) + M_i\frac{R(\infty,t_0)}{E_c}$$

M_f: moment evaluated in the final static scheme
M_i: moment evaluated in the initial static scheme
DM_i: $M_f - M_i$

For $t_1 \neq t_0$ $$M_\infty = M_i + \xi(\infty,t_1,t_0)\Delta M_i$$

then $$M_\infty = M_f\xi(\infty,t_1,t_0) + M_i\left[1-\xi(\infty,t_1,t_0)\right]$$

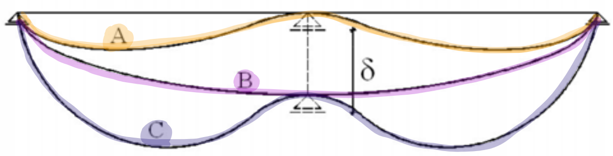
It's possible sensitive migration of actions that force to use an excess of resistant capacity compared to the necessary one if the structure was built from the beginning in the final static scheme.

IMPORTANT

The effect of delayed restraint doesn't interact with prestressing losses. According to the principle of acquisition of delayed restraints, the application of impressed deformation on the final static scheme, following the second principle of viscous-elasticity, doesn't change the deformations but only the stresses. It follows that the prestressing losses that happen for $t > t_1$ don't suffer the influence of delayed restraint effect, because they depend on deformation state.

we are using 2nd principle

Their evaluation should be done in the final static scheme (stress state subsequent to G + P). The deformations are going to be equal to the deformations that correspond of the scheme before the introduction of delayed restrains (in the demonstration).



A: deformation with restrain place at the beginning (due to loads)
B: deformation with settlement (due to impressed deformation)
C: evolution of the load-deformation due to creep, on which the presttressing losses have to be calculated. The load-deformation C is similar to the initial A.

Verification in longitudinal direction

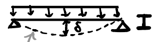
The cross-section is supposed to be not deformable, therefore we work with "BEAM" elements into the space.

The procedure of analysis and verification must go through the following steps (construction phases, progress from the piers):

• verification of every segment in presence of temporary and definitive prestressing.
• verification of the hammer (opened) in case of translation of launching girder
• evaluation of the prestressing losses when the key-segment is placed (delayed restraint)
• evaluation of the stresses due to selfweight, top prestressing (isostatic scheme - hammer) and bottom prestressing (hyperstatic scheme – completed deck)
• evaluation of the asymptotic value of the effect of delayed restraint and the consequent stress state
• evaluation of the effect prestressing losses (in the final static scheme)
• application of the permanents actions (possible inversion with the previous step)
• individuation of the load-cases due to the variable actions and correspondent analysis and verification

Deformation and cumbering

MONTE COSTRUTTIVO → it is not a deformation, but a particular shape that I give to the structure during construction → example: steel beam



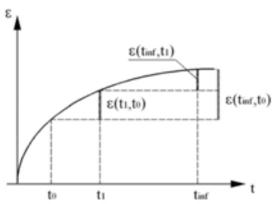
due to self weight

to avoid that I realize a beam:

so when I load it, it will be straight

Permanents actions:

- Selfweight, top prestressing } initial static schemes (e.g. Hammer construction)
- Bottom prestressing} intermediate schemes (e.g. closing of midspan key segments)
- Pavement, kerbs, barriers etc. } final static scheme
- Variable actions: final static scheme



δ': deformations in the initial static scheme ($t_0 < t < t_1$)
δ'': deformations in the final static scheme ($t_1 < t < \infty$)
It can be written:

$$\delta(t) = \delta'\varphi(t_1,t_0) + \delta''\varphi(t,t_1)$$
$$\delta_\infty = \delta'\varphi(t_1,t_0) + \delta''\varphi(t_\infty,t_1)$$

And as

$$\varphi(t_\infty,t_1) = \varphi(t_\infty,t_0) - \varphi(t_1,t_0)$$

We can write:

$$\delta_\infty = \delta'\varphi(t_\infty,t_0) + (\delta'' - \delta')[\varphi(t_\infty,t_0) - \varphi(t_1,t_0)]$$

The deformations induced by permanent actions at t_∞ are assigned as cumbering to obtain the desired profile.
It's better to overestimate slightly the cumbering to compensate possible mistakes in the evaluation of the creep parameters. An esthetically acceptable cambering is about equal to 1/2000 of the span, i.e. the same order of magnitude of the deformation induced by variables loads.

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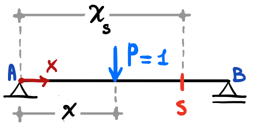
4. INFLUENCE FUNCTION

I ponti sono sollecitati da carichi accidentali mobili che possono assumere una qualsiasi posizione, per cui è necessario individuare quella situazione che rende massima una data caratteristica di sollecitazione o di deformazione nella sezione in cui si stanno eseguendo le verifiche. Per definire una linea di influenza (l.d.i in italiano, I.F. in inglese) si consideri una generica struttura comunque vincolata e si supponga che su di essa agisca un carico P in una sezione di ascissa x qualunque. Si ammette che il carico si sposti mantenendosi parallelo ad una data direzione che (verticale nel caso di carichi verticali). Se ora si considera una generica sezione S individuata dall'ascissa x_S, una qualsiasi grandezza G, sia essa una caratteristica di deformazione (χ, N, V, T, σ, τ, ecc.) o di spostamento (abbassamento, rotazione, ecc.) sarà descritta da una funzione del tipo:



$$G = G(x, x_S, P)$$

Mantenendo fissi alcuni parametri da cui dipende G e facendo variare gli altri si hanno i seguenti casi:

x_S VARIABILE	Si ottengono i diagrammi di stato, ovvero i diagrammi delle caratteristiche di sollecitazione che descrivono, per una data posizione fissa del carico, lo stato di sollecitazione o di deformazione nella struttura
x VARIABILE	Si ottengono le l.d.i., cioè un diagramma che, per una data sezione fissa S, indica con le sue ordinate generica $\mu(x)$ il valore che la grandezza in esame assume in S quando il carico P = 1 si trova nella sezione di ascissa x

Mediante le l.d.i. è possibile valutare l'effetto prodotto in una sezione da carichi mobili di vario tipo ed individuare le posizioni dei carichi per le quali si hanno i massimi ed i minimi valori della grandezza cercata.

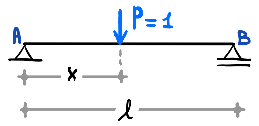
Per il tracciamento delle l.d.i. si può eseguire il calcolo con il metodo diretto o con il metodo indiretto.

METODO DIRETTO (explicit procedure)

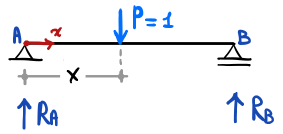
Il metodo diretto consiste nel costruire la linea di influenza per punti, calcolando G per diverse posizioni del carico. Più vicini sono i punti in cui si pone il carico e più approssimato sarà l'andamento della l.d.i.. Nel caso delle sollecitazioni è conveniente determinare, dapprima, le l.d.i. delle reazioni vincolari, essendo possibile da queste dedurre per una qualsiasi sezione le grandezze M, N, V.

Come si procede? Si pone il carico $P=1$ sull'ascissa x_1 e si calcola l'effetto che si desidera nella prefissata sezione S. Poi si sposta il carico sull'ascissa x_2 e si ripete nuovamente il calcolo in S. Si ripete il procedimento per altre posizioni x_i del carico ricavando i valori della grandezza G in S. Fatto ciò si riportano in un grafico i valori della grandezza G calcolata in S al variare della coordinata x.

ESEMPIO: trave semplicemente appoggiata



1) LINEA DI INFLUENZA DELLE REAZIONI VINCOLARI



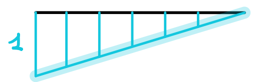
EQUAZIONI DI EQUILIBRIO

(↑) $R_A + R_B = 1$

A↻ $R_B \cdot \ell - \underset{1}{P}x = 0$ → $\boxed{R_B = \frac{x}{\ell} \qquad R_A = 1 - \frac{x}{\ell}}$

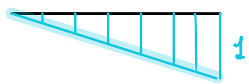
LINEE DI INFLUENZA DELLE REAZIONI

plottando



LINEA DI INFLUENZA DI RA

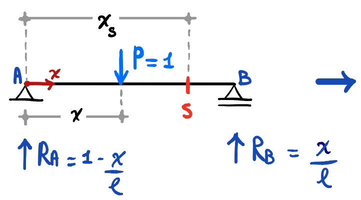
$R_A = 1 - \frac{x}{\ell} \rightarrow \begin{cases} \text{per } x=0 \rightarrow R_A = 1 \\ \text{per } x=\ell \rightarrow R_A = 0 \end{cases}$



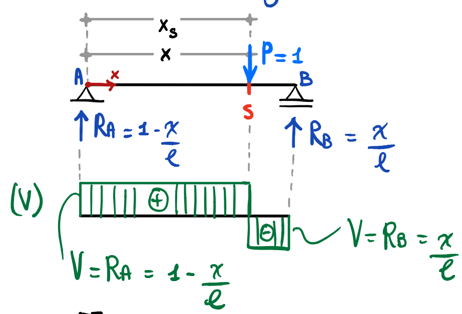
LINEA DI INFLUENZA DI RA

$R_B = \frac{x}{\ell} \rightarrow \begin{cases} \text{per } x=0 \rightarrow R_B = 0 \\ \text{per } x=\ell \rightarrow R_B = 1 \end{cases}$

2) LINEA DI INFLUENZA DEL TAGLIO (per la sezione S)



(a) considero prima come se P fosse applicato in S e calcolo il taglio



(b) definisco la linea di influenza del taglio in funzione della posizione x del carico

$$\begin{cases} x > x_S \Rightarrow V = R_A = 1 - \dfrac{x}{\ell} \\ x < x_S \Rightarrow V = R_B = \dfrac{x}{\ell} \end{cases}$$

plottando

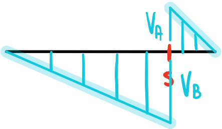
LINEA DI INFLUENZA DEL TAGLIO

per $x = 0 \rightarrow V = 0$

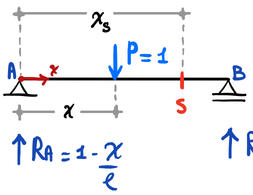
per $x = \ell \rightarrow V = 0$

per $x = x_s^{sx} \rightarrow V = \dfrac{x_s}{\ell} = V_B$

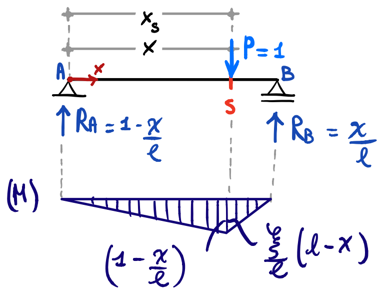
per $x = x_s^{dx} \rightarrow V = 1 - \dfrac{x_s}{\ell} = V_A$



3) **LINEA DI INFLUENZA DEL MOMENTO** (per la sezione S)



(a) considero prima come se P fosse applicato in S e calcolo il momento



(b) definisco la linea di influenza

$$\begin{cases} x > x_S \Rightarrow M = \left(1 - \dfrac{x_s}{\ell}\right) \\ x < x_S \Rightarrow M = \dfrac{x_s}{\ell}(\ell - x) \end{cases}$$

plottando

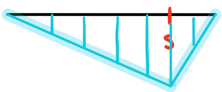
LINEA DI INFLUENZA DEL MOMENTO

per $x = 0 \rightarrow M = 0$

per $x = \ell \rightarrow M = 0$

per $x = x_s^{sx} \rightarrow M = \dfrac{x_s}{\ell}(\ell - x_s)$

per $x = x_s^{dx} \rightarrow M = \left(1 - \dfrac{x_s}{\ell}\right) x_s$



METODO INDIRETTO (implicit procedure)

Tutte le l.d.i. possono sempre ottenersi come particolari deformate della struttura immaginata caricata da un sistema di forze fittizie scelte opportunamente. Il metodo indiretto si basa sull'utilizzo dei principi di reciprocità (che sono validi nell'ipotesi di validità del principio di sovrapposizione degli effetti)!

Primo principio (teorema di Betti):

«Dati due insiemi di forze agenti separatamente sulla struttura, il lavoro compiuto dal primo insieme per gli spostamenti indotti dal secondo è uguale al lavoro compiuto dalle forze del secondo insieme per gli spostamenti indotti dal primo»

Secondo principio (teorema di Land-Colonnetti):

«Dati due insiemi di forze e distorsioni agenti separatamente sulla struttura, il lavoro mutuo generalizzato è nullo»

Terzo principio (teorema di Volterra):

«Dati due insiemi di distorsioni agenti separatamente sulla struttura, i due lavori mutui generalizzati sono uguali»

(teorema di Betti generalizzato):

«Dati due insiemi di forze e distorsioni agenti separatamente sulla struttura, il lavoro compiuto dalle forze e distorsioni del primo insieme per gli spostamenti e sollecitazioni indotti dal secondo insieme è uguale al lavoro compiuto dalle forze e distorsioni del secondo insieme per gli spostamenti indotti dal primo»

teorema del lavoro mutuo (teorema energetico del lavoro mutuo)

if I have two systems : system (a) , system (b) , applied on the same structure, I can write :

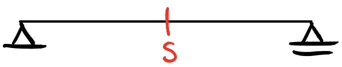
conservazione dell'energia ←

$$\boxed{\underbrace{A_a \cdot R_b}_{\text{ENERGY}} = \underbrace{A_b \cdot R_a}_{\text{ENERGY}}}$$

A = acting element
R = results

NOTATION

Influence function of the $\overbrace{\text{bending moment}}^{\text{unknowns (U)}}$ for $\underbrace{\text{vertical loods}}_{\text{known cause (KC)} \to \text{causa nota}}$ in $\overbrace{\text{midspan}}^{\text{Section}}$.



I.F. [U = M ; KC = V.L.↓ ; S = midspan

The idea is : I place in section S the item that is able to produce energy with my unknown (elemento duale al momento)

Gli enti duali

Dall' esame dei principi di reciprocita' emerge una relazione tra enti forza ed enti spostamento: ciascun elemento dell'insieme delle forze e' legato ad un elemento dell'insieme degli spostamenti, nel senso che ciascun ente forza compie lavoro per il corrispondente ente spostamento.

Ente Forza	Ente Spostamento
Forza	Spostamento
Coppia	Rotazione
Momento flettente	Distorsione di rotazione relativa
Taglio	Distorsione di scorrimento
Sforzo normale	Distorsione assiale

Intuitively let's have a look to this example :

$$\text{Energy} = \int_{x_1}^{x_2} M \cdot \chi \, dx$$

↑ For P.L.V. — M: bending moment — χ: curvature

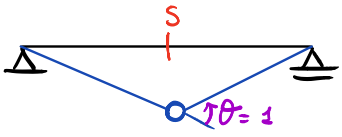
x_1, x_2 = 2 points of the beam

'Cause we're working for a single section, M is costant in the section, so I can put it out of the integral operation:

$$= M \underbrace{\boxed{\int_{x_1}^{x_2} \chi \, dx}}_{\text{LOCALIZED ROTATION } \theta}$$

→ we imagine to have a χ so big to get $\theta = 1$

So in our beam we place in our section S this rotation $\theta = 1$, it means that our beam has a concentrated rotation in midspan:



Taking again the general equation written before:

$$A_a \cdot R_b = A_b \cdot R_a$$

we're gonna apply it to our case:

- A_a = V.L. (= KC, the acting elements are V.L.)
- $R_a = M$ (= U, the results of system (a) is what we're looking for: bending moment generated by V.L.)
- $A_b = \theta = 1$ [-]

⇘ R_b from the point of view of dimension has to be:

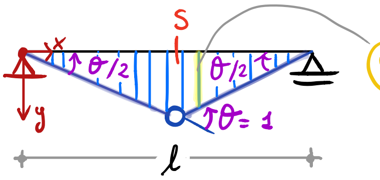
$$R_b = \frac{A_b \cdot R_a}{A_a} \rightarrow \frac{[-]\,[\cancel{kN}m]}{[\cancel{kN}]} \Rightarrow [m]$$

infact the item who makes energy with vertical load is a vertical displacement:

- $R_b = v_\theta$ (vertical displacement generated by θ)

⇒ we get so: $\quad V.L. \cdot v_\theta = \theta \cdot M_{V.L.}$

system (a) → REAL →	is the the system in which real vertical loads are applied to the structure, and real unkowns
system (b) → VIRTUAL → ('cause to get the deformation of system (b), the beam should be broken ⇒ but it isn't so ⇒ unreal)	we apply the complementary of the unknown and we calculate the complementary of the acting of the system (a)

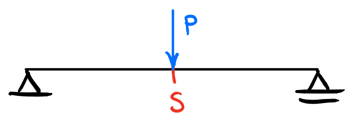


v_θ whose equation is

$$\begin{cases} y = \frac{\theta}{2} \cdot x & x \in [0; \frac{l}{2}] \\ y = \frac{\theta l}{4} - \frac{\theta}{2}\left(x - \frac{l}{2}\right) & x \in (\frac{l}{2}; l) \end{cases}$$

INFLUENCE FUNCTION OF DISPLACEMENT

using the I.F. if I wanna know the value of the bending moment due to P in midspan on section S:



$$V.L. \cdot v_\theta(S) = \theta \cdot M$$

$$\Downarrow$$

$$P \cdot \frac{\cancel{\theta}}{2} \frac{l}{2} = \cancel{\theta} \cdot M$$

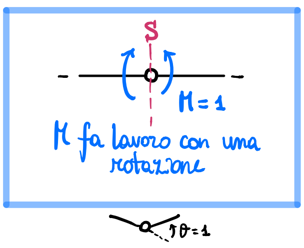
$$\Rightarrow \frac{Pl}{4} = M$$

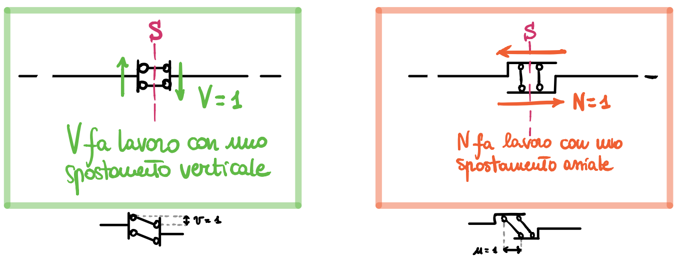
Quanto visto intuitivamente può essere schematizzato

LINEE DI INFLUENZA DELLE CARATTERISTICHE DI SOLLECITAZIONE (TEOREMA DI LAND - COLONNETTI)

Questo teorema, che deriva direttamente dal P.L.V., dice che le L.I. di M, N, V in una generica sezione S di una struttura si ottengono dalla deformata che risulta tagliando la struttura in S e applicando alle 2 facce del taglio 2 sistemi di forze fittizie eguali e contrarie. Queste vanno scelte in modo da produrre una distorsione unitaria corrispondente alla caratteristica di sollecitazione cercata e spostamenti relativi nulli rispetto alle altre due. Di ogni spostamento η va considerata la componente nella direzione della forza mobile. Una conseguenza del teorema di Land è che nelle strutture isostatiche le l.d.i. delle caratteristiche di sollecitazione sono sempre rettilinee. Infatti una volta effettuato il taglio nella generica sezione S la struttura diventa labile e quindi la distorsione voluta si può sempre ottenere con moti rigidi delle parti che risultano dal taglio. Invece le l.d.i. delle strutture iperstatiche, risultano sempre curve anche laddove l'applicazione del teorema di Land ne renda labili alcune parti.

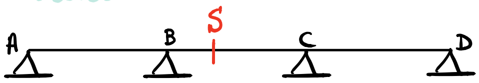
Utilizzando il metodo indiretto, nel caso si cerchino le l.d.i. delle caratteristiche di sollecitazione in una data sezione S, basterà introdurre ivi uno dei vincoli riportati di seguito con la relativa sollecitazione indicata:





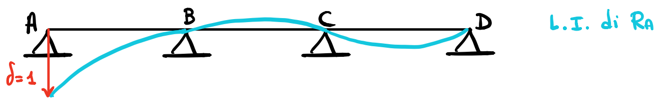
per il teorema di Land la deformata calcolata coinciderà con la l.d.i. cercata.

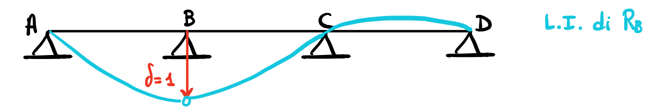
EXAMPLE



1) L.I. delle reazioni

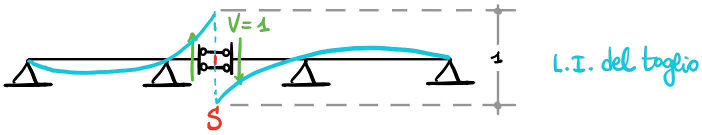
↳ devo imporre un cedimento e traccio la deformata qualitativa





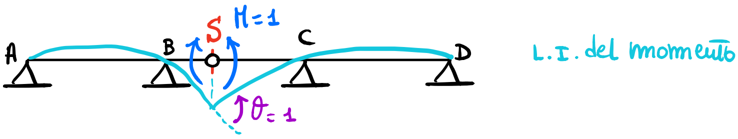
2) L.I. del taglio

↳ devo inserire nella sezione S la sconnessione che fa lavoro col taglio

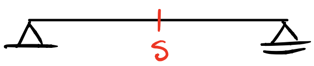


3) L.I. del momento

↳ devo inserire nella sezione S la sconnessione che fa lavoro col momento

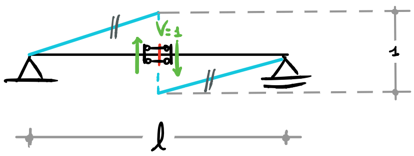


EXAMPLE



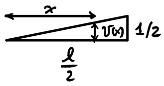
U = V
KC = V.L. ↓
S = midspan

$$E = \int_{x_1}^{x_2} V \underbrace{\eta}_{\text{shear shift}} dx \rightarrow = V \underbrace{\int_{x_1}^{x_2} \eta \, dx}_{v = 1 \ [m]} \longrightarrow V.L. \cdot \underbrace{v_b}_{\substack{\text{vertical displac.} \\ \text{generated by } v}} = \underbrace{v_a}_{\substack{\text{relative displacement} \\ \text{of the two sections}}} \cdot V$$



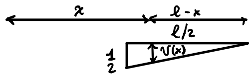
The two curves have to be parallel 'cause the double pendulum does not allow relative rotation !

↳ L.I. equation will be:



$v(x) : \frac{1}{2} = x : \frac{\ell}{2}$

$\boxed{v(x) = \frac{1}{\cancel{2}} \cdot x \cdot \frac{\cancel{2}}{\ell} = \frac{x}{\ell}}$ for $x \in [0; \ell/2]$

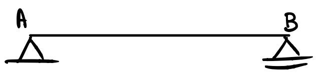


$v(x) : (\ell - x) = \frac{1}{2} : \frac{\ell}{2}$

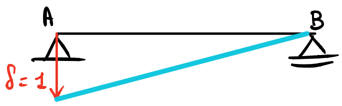
$\boxed{v(x) = \frac{1}{\cancel{2}}(\ell - x) \cdot \frac{\cancel{2}}{\ell} = 1 - \frac{x}{\ell}}$

→ L.I.
$$\begin{cases} y = \frac{x}{\ell} & x \in [0; \frac{\ell}{2}] \\ y = 1 - \frac{x}{\ell} & x \in [\frac{\ell}{2}; \ell] \end{cases}$$

EXAMPLE



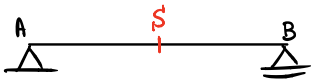
U = R
KC = V.L. ↓
S = A



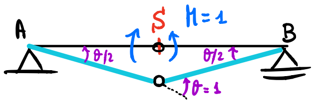
$v(x) : x = 1 : \ell$

$\boxed{v(x) = \frac{x}{\ell}}$ L.I.

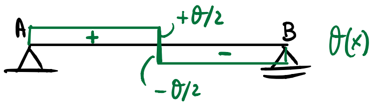
EXAMPLE



$U = M$ ↳ fa lavoro con una rotazione
$KC = \curvearrowright$ (moving coupler)
$S = S'$



↳ diagram of rotation (1st derivate of the little-blue curve)

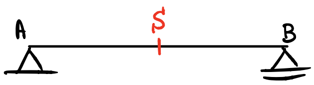


(+) sign convention

Couple · rotation = rotation · M

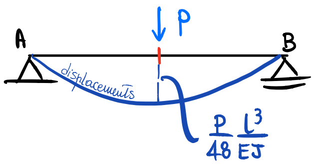
$$C \cdot \frac{\cancel{\theta}}{2} = \cancel{\theta} \cdot M \rightarrow M = \frac{C}{2}$$

EXAMPLE

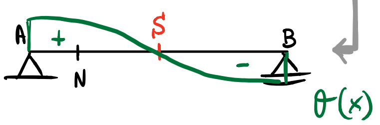


U = displacement
$KC = \curvearrowright$
$S = S'$

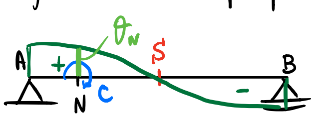
U makes work with displacement



calculating the 1st derivative of the displacements we get the rotation



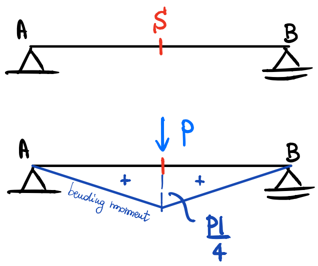
if I have a couple placed in N



→ the displacement that I have in midspan can be computed:

$$C \cdot \theta_N = P \cdot y_c \rightarrow y_c = \frac{P \cdot \theta_N}{C}$$

EXAMPLE

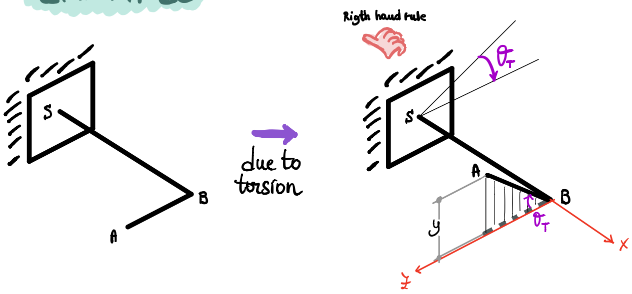


U = displacement
KC = temperature
S = S'

$\bar{\varepsilon} = \alpha \Delta T$ (imposed deformation)

$\bar{\chi}$ → imposed curvature (fa lavoro con M)

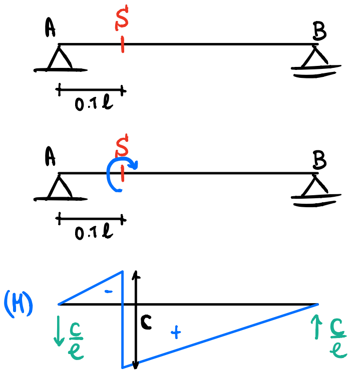
EXAMPLE



U = T ⟶ fa lavoro con un angolo θ_T di torsione
KC = vertical load
S = S'

$y = \theta_T \cdot x \Rightarrow V.L. \cdot V.D._{\theta_T} = \theta_T \cdot T_{V.L.}$

EXAMPLE



U = θ rotation ⟶ fa lavoro con una coppia C
KC = distributed load
S = S'

devo trovare spostamenti verticali (che fanno lavoro con ↓↓↓↓)

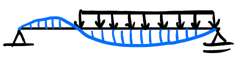
→ I go to the $\chi = \frac{M}{EJ}$ ⇒ diagram of χ has the same shape of M

$y(x) = \int \theta(x)\, dx \leftarrow \theta = \int \chi(x)\, dx$

$$q \cdot V.D._C = C \cdot \theta_q$$

$$\theta_q = \int q(x) \frac{y(x)}{C} dx$$

se voglio massimizzare lo spostamento in S devo piazzare q qui ⟶



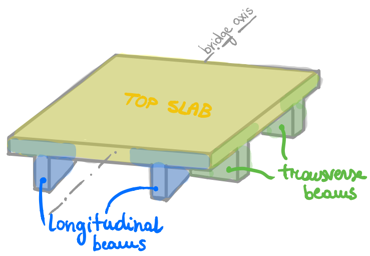
–65–

5. GIRDER BRIDGES

For span up to ≃ 35 m/50 m, girder bridges (ponti a graticcio) are the most economical solution.

Main advantages:

The top slab:

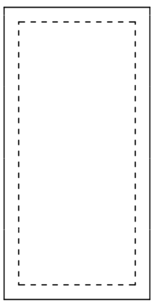


- transfers live load to beams
- is the chord (corrente) of beams
- contributes to distribute the concentrated actions to the beams
- works like a membrane in presence of horizontal actions
- increases the internal level arm, for M>0

Design criteria

- Few longitudinal beams if are cast in situ (not more than 4)
- Higher number longitudinal beams if precast elements are used (to reduce their weight).
- Generally for casting in situ i (interasse tra le travi longitudinali) = 5÷10m and with precast beams i= 2,5÷7m. → typicall 3÷4 m
- Transverse beam on the bearings to bear torque moments in beams (bending moment in transverse beams). Transverse beams can be omitted along the span and with thick slabs; a transverse beam in the mid-span produces about the same effect of two transverse beams in l/3. Further transverse beams are not useful. → typicall 3 or 5 trasverse beams
- Web thickness is decreasing with the increasing number of beams. High thickness produces a high torsional rigidity and then helps in the transversal distribution of actions. In the cracked stage this effect is strongly reduced because the torsional rigidity drops of 4÷5 times.

SLAB DESIGN



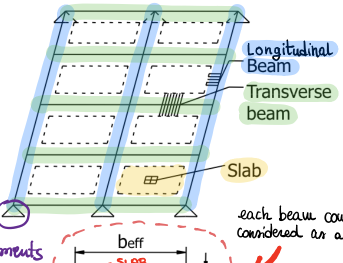
The static scheme is a "long" slab continuously supported in the longitudinal direction, and supported only on the extreme transverse beams in the transversal direction (in fact generally it is not in contact with intermediate transverse beams, if any, in order to uniform the internal actions).

To reduce the positive bending moment it is better to introduce some slab cantilevers externally to edge beams. In fact it is better to accept negative moments grater than the positive ones in order to use the positive effects of a possible hardening ribble in the web (that is an increase in the web thickness).

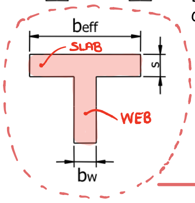
Naturally the restraint degree given to the slab by the beams depends on the ratio between flexural rigidity of the slab and torsional rigidity of the beam. The presence of transverse beams along the span improves the torsional behaviour of longitudinal beams and then improves the restrain degree between the slab and the beams.

Methods for the analysis of internal actions
Beams / Transverse beams / Slabs ⇒ Courbon / Massonnet / Computer → Program able to solve girders or 3D frames of mixed finite element

The typical problem related to the calculation of these bridges, is the transverse distribution of actions. The Courbon method allows us to calculate the amount of a concentrate force, placed in a general position on a longitudinal beam, that goes on the the longitudinal beams, and the amount that goes to the transverse ones.

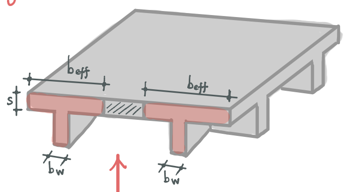


each beam could be considered as a T beam



Constitutive elements of a girder bridges ⇒ Beam / Transverse beams / Slab

→ Space system
↓
Plane system composed by
→ Beams with collaborator slab (longitudinal and transversal)
→ Slabs



The design implies the evaluation of internal actions corresponding to the combination load and to the load case the most unfavorable ones for the structural region in consideration.

⇓

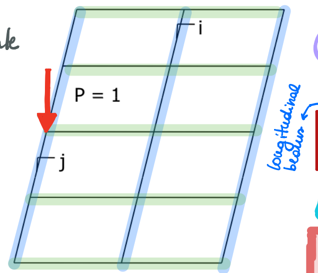
"Transversal distribution of actions"

thanks to

Evaluation of transversal distribution coefficient:

i = trave su cui valuto la percentuale di carico che va a finire

j = trave su cui è applicato il carico P



$\rho_{ij} \equiv$ Percentage of load P acting on beam i when the load is on the beam j.

ρ_{ij} = is the amount of the force P that remains on the beam

It should result:

$$\sum_{i=1}^{n} \rho_{ij} = 1$$ ← Vertical equilibrium condition (100% of the total load P)

$$\boxed{P_i = \rho_{ij} P}$$ For $P \neq 1$ the principle of superposition may be used

→ All this calculation are done in elastic field, so we can use the superposition principle

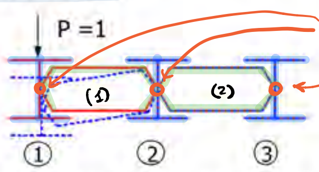
If the load is uniformly distributed:

$$\rho_{ij} = \frac{1}{n} \qquad n = \text{beam number}$$

Limit cases for deck behaviour

We have 3 longitudinal beams and transverse beams are hinged to the longitudinal ones

a) Transverse beams and slab without flexural rigidity or connected by means of hinges to the beams.



per la trave i=1 valuto ρ spostando P sulle altre travi (cambiando j)

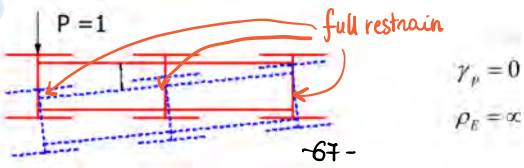
$\rho_{11} = 1$

$\rho_{12} = \rho_{13} = 0$

If I apply the load P on beam 1, 'cause the precence of the hinges, transverse beam (1) rotates. So the effect of P is only the vertical displacement of L.B. ①, while % P that goes on other L.B. is null.

transverse beams are full restrained to the longitudinal ones, and the stiffness of T.B. is very high if compared with the L.B. one.

b) Transverse beams with infinite flexural rigidity and beams with null torsional rigidity.



$\gamma_p = 0$

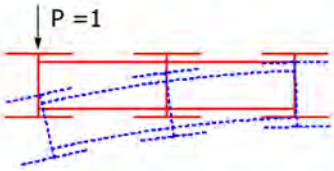
$\rho_E = \infty$

The torsional stiffness of longitudinal beams is zero

-67-

more realistic than the previous ones 'cause it introduces the bending deformability of the transverse beams

c) Transverse beams with finite flexural rigidity and beams with null torsional rigidity.



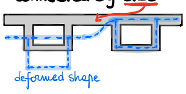
$\gamma_p = 0$

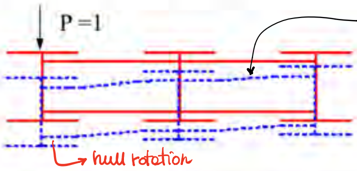
$\rho_E \neq 0$

uncommon case

d) Transverse beams with finite flexural rigidity and beams with infinite torsional rigidity.

this case can occur when we had two boxed section connected by slab





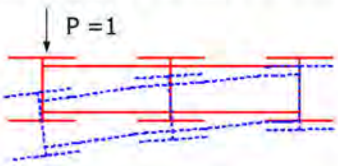
pure shear deformation (S deformation shape for T.B.)

$\gamma_p = \infty$

$\rho_E \neq 0$

real life

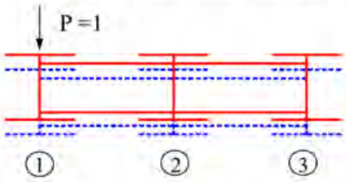
e) Transverse beams and beams with finite flexural and torsional rigidities.



$\gamma_p \neq 0$

$\rho_E \neq 0$

f) Transverse beams with infinite flexural rigidity and beams with finite torsional rigidity.



$\gamma_p = \infty$

$\rho_p = \infty$

Uniform distribution $\rho_{1,1} = \rho_{1,2} = \rho_{1,3} = 1/3$

- Main parameters influencing the girder behaviour
  - Flexural rigidity of transverse beams and beams
  - Torsional rigidity of beams

- Main analysis procedures
  - Finite element (combination of slab and beam elements)
  - Courbon method (valid for case b) → the COURBON METHOD is based on b) hypothesis, but it works also for bridges that satisfy the e) hypothesis.
  - Orthotropic slabs analysis: Massonnet
  - Beam girders (beam finite elements)

- Most used methods in the design
  - Courbon, Massonnet, Beam girders → Analytical calculation; Conceptual design and preliminary design (and detailed design in some cases) → Acceptable approximation in cases that respect the basic hypotheses
  - (f.e.) → Computer Design → Detailed design and slab drawings

COURBON METHOD

HYPOTHESIS:

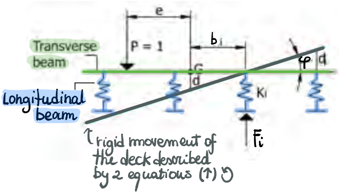
- the deck is transverse rigid with infinite bending stiffness

Why? The reason is: I have a sufficient number of trasnverse beam that is enough to connect the longitudinal beams. The stiffness of transverse beam is much higher than the longitudinal one because of the different length of the spans (L.B. works on ≈ 30m span, T.B. works on ≈ 3 m spans), if the number of transverse beam and the thickness of the slab is enough, we can imagine that the transverse connection is infinitely rigid. What does this mean? If I have a force applied transversaly, the deck will move down (rotating a little bit) but it will not bend ⇒ **RIGID MOVEMENT**

it can be seen as the sum of

only 2 d.o.f. { VERTICAL TRASLATION OF THE GRAVITY CENTER + ROTATION φ

STATIC SCHEME OF TRANSVERSE BEAM (continuous beam on elastic bearings)



each longitudinal beam is seen as a vertical spring. So longitudinal beams are able only to transfer a vertical force to the deck.

- the longitudinal beams have null torsional rigidity: there is no torsion between the spring and the deck. The longitudinal beams, as already said, transfer only vertical forces to the deck.

The force transfered from each beam to the deck is equal to the stiffness of each longitudinal beam (K of the spring) multiplied by the vertical displacement of each longitudinal beam (Hooke law: $F = K \cdot x$):

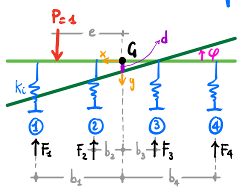
$$F_i = K_i \cdot d_i$$

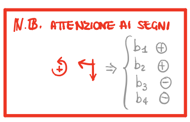
STIFFNESS OF THE SPRING (K_i) — DISPLACEMENT OF THE SPRING (d_i)

the displacement of each spring d_i can be written in function of the displacement of the centroid and the rotation φ of the deck:

$$d_i = d + \varphi \cdot b_i$$

← BRACCIO (level arm)





We get:

$$F_i = K_i \cdot d + K_i \cdot \varphi \cdot b_i = K_i (d + \varphi b_i)$$

What are my unknowns? d & φ ⇒ I need to write two equation of equilibrium: one in the vertical direction, and the other one of rotation aroun G.

(n → n° of the beam)

$$\begin{cases} (\uparrow) \quad \sum_{i=1}^{n} F_i = P \\ (\circlearrowleft) \quad \sum_{i=1}^{n} F_i \cdot b_i = P \cdot e \end{cases}$$

Finally in general case we get:

$$F_i = \rho_{ie} = \frac{K_i}{\sum K_i} \oplus \frac{K_i \cdot e \cdot d_i}{\sum K_i \cdot d_i^2}$$

+ for "e" and "d_i" with the same sign
- for "e" and "d_i" with different sign

But usually girder briges have same longitudinal beams ($\Rightarrow K_i = K$) and the spacing of beams is regular, so we can simplify:

AMOUNT OF LOAD ON BEAM "i" for a variable position of acting load P=1

$$\boxed{\rho_{i,e} = \frac{K}{nK} + \frac{K e d_i}{K \sum d_i^2} = \frac{1}{n} + \frac{e d_i}{\sum d_i^2}}$$

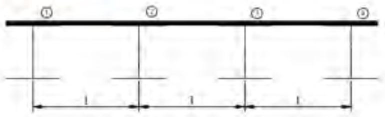
COURBON TRANSVERSAL DISTRIBUTION COEFFICIENT

esempio:

ρ_{11} = quantità di carico che va nella trave i=1 quando il carico è sulla trave e=1

ρ_{12} = quantità di carico che va nella trave i=2 quando il carico è sulla trave e=1

EXAMPLE



the most stressed beam is the most eccentric (both ones on the two sides)

Beams 1 e 4 $\quad \rho_{1,e} = \frac{1}{4} + \frac{1.5e}{2(1.5^2+0.5^2)} = 0.25 + 0.3e$

$$\left.\begin{aligned} \rho_{1,1} &= 0.25+0.3*1.5 = 0.70 \\ \rho_{1,2} &= 0.25+0.3*0.5 = 0.40 \\ \rho_{1,3} &= 0.25-0.3*0.5 = 0.10 \\ \rho_{1,4} &= 0.25-0.3*1.5 = -0.2 \end{aligned}\right\} \sum = 1 \xrightarrow[\text{Theorem}]{\text{Betti Maxwell}}$$

$\rho_{1,2} = \rho_{2,1}$

Obviously is linear because $J_T = \infty$

Beams 2 e 3 $\quad \rho_{2,e} = \frac{1}{4} + \frac{0.5*e}{2(1.5^2+0.5^2)} = 0.25 + 0.1e$

$$\left.\begin{aligned} \rho_{2,1} &= 0.25+0.1*1.5 = 0.40 \\ \rho_{2,2} &= 0.25+0.1*0.5 = 0.30 \\ \rho_{2,3} &= 0.25-0.1*0.5 = 0.20 \\ \rho_{2,4} &= 0.25-0.1*1.5 = 0.10 \end{aligned}\right\} \sum = 1 \xrightarrow[\text{Theorem}]{\text{Betti Maxwell}}$$

if the point of application of the force is near the centroid G (len tork) all the deck is shifted down.

Within the Courbon formula it can be introduced $e = e' b_0$ and $d_i = d_i' b_0$ then:

$$\rho_{i,e} = \frac{1}{n} + \frac{e' b_0 d_i' b_0}{\sum_{i=1}^{n} d_i'^2 b_0^2} = \underbrace{\frac{1}{n} + \frac{e' d_i'}{\sum_{i=1}^{n} d_i'^2}}$$

indipendent by b_0

For beams with the same mutual distance the coefficient ρ_{ij} is independent from that distance; then the influence lines of the coefficient of distribution may be drawn as a function of the beam number.

The value of coefficient ρ_{ij} for i = cost and $2 \leq n \leq 10$ are presented in the table

Pay attention! It is valid for identical beams with constant mutual distance

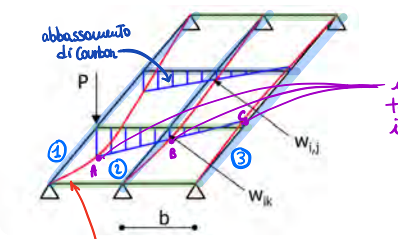
n° of L.B.

n° of beams	beam	Load in: 1	2	3	4	5	6	7	8	9	10
2	1	1	0								
3	1	0,833	0,333	-0,166							
	2	0,333	0,333	0,333							
4	1	0,7	0,4	0,1	-0,2						
	2	0,4	0,3	0,2	0,1						
5	1	0,6	0,4	0,2	0	-0,2					
	2	0,4	0,3	0,2	0,1	0					
	3	0,2	0,2	0,2	0,2	0,2					
6	1	0,524	0,381	0,238	0,095	-0,048	-0,19				
	2	0,382	0,296	0,21	0,124	0,037	-0,049				
	3	0,24	0,21	0,161	0,152	0,123	0,094				
7	1	0,463	0,357	0,25	0,143	0,36	-0,71	-0,178			
	2	0,355	0,285	0,214	0,143	0,072	-0,001	-0,07			
	3	0,25	0,215	0,179	0,143	0,107	0,071	0,035			
	4	0,143	0,143	0,143	0,143	0,143	0,143	0,143			
8	1	0,416	0,333	0,25	0,167	0,083	0	-0,083	-0,166		
	2	0,331	0,273	0,213	0,133	0,095	0,037	-0,022	-0,082		
	3	0,251	0,215	0,179	0,143	0,107	0,071	0,035	-0,001		
	4	0,167	0,155	0,143	0,131	0,119	0,107	0,095	0,083		
9	1	0,38	0,312	0,245	0,178	0,111	0,044	-0,023	-0,09	-0,157	
	2	0,31	0,261	0,211	0,161	0,111	0,061	0,011	-0,039	-0,089	
	3	0,243	0,21	0,177	0,144	0,111	0,078	0,045	0,012	-0,02	
	4	0,18	0,162	0,145	0,128	0,111	0,094	0,077	0,06	0,043	
	5	0,111	0,111	0,111	0,111	0,111	0,111	0,111	0,111	0,111	
10	1	0,343	0,289	0,235	0,181	0,129	0,073	0,019	-0,035	-0,089	-0,143
	2	0,289	0,249	0,205	0,163	0,121	0,079	0,037	-0,005	-0,047	-0,089
	3	0,235	0,205	0,175	0,145	0,115	0,085	0,055	0,025	-0,005	-0,035
	4	0,181	0,163	0,145	0,127	0,109	0,091	0,073	0,055	0,037	0,019
	5	0,127	0,121	0,115	0,109	0,103	0,097	0,091	0,085	0,079	0,073

N.B. se ↑ n° travi ⇓ ↓ ρ_{ij}

Due to the hypotheses that $I_{traverso} = \infty$ and $J_{t\ trave} = 0$ there is no mutual influence of transverse beams in the load distribution effect.

Let's consider this girder bridge:



in A, B, C in direzione trasversale e longitudinale i 2 abbassamenti

deformation of longitudinal beam ① due to the application of P

Using the Courbon method, the number of the transverse beams is completely useless, 'cause the Courbon theory works on the approximation of the infinitely rigid deck in transverse direction, so for 3 or 30 transverse beam I obtain the same result.

If I want to imagine the deformed shape of this deck, I should imagine the deformed shape of the only longitudinal beam loaded with the force P, and then we multiply the deformed shape for Courbon coefficient ρ obtaining the other longitudinal beam deformed shape.

For a general beam i:

$$\frac{w_{i,j}}{w_{i,k}} = \text{cost.}$$

As the loaded transverse beam is linear, the unloaded one is linear too.

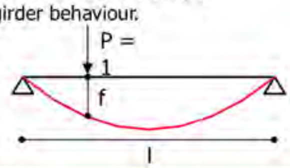
The transverse beam "j" follows the deformation of the "k" one, remaining linear.

On the opposite if $I_{ft} \neq \infty$, the deformed shape of transverse beam is curved like the one of "j" transverse beam.

In practice the spring deformability depends on the transverse beam position.

$$f = c\frac{l^3}{E I_{ft}} \quad (\text{for } z = l/2 \rightarrow c = 1/48)$$

Internal actions arise, which have an influence on the girder behaviour.



If we suppose that $I_{ft} \neq \infty$, the solution of continuous beam on elastic bearings shows that the distribution of internal actions is a function of following parameter:

$$\boxed{Z = \left(\frac{l}{b}\right)^3 \frac{I_{ft}}{I_{fb}} c}$$

length of the longitudinal beam

c → depends on the position of the force in longitudinal direction because the stiffness of the longitudinal beam is not constant

moment of inertia of longitudinal beams

transverse distance between longitudinal beams

$Z = \infty \Rightarrow$ infinitely rigid transverse beam

For $Z = \infty$, because $I_{ft}/I_{fb} \cong 1$, it is necessary that $l >> b$ (long and narrow decks). In practice, with $Z \geq 20$ one can consider $Z = \infty$.

Then with $I_{ft}/I_{fb} \cong 1$ it should result $(l / b) > 10$ so that the transverse beam can be considered like infinitely rigid.

Example:

Deck: $l = 30$ m $B = 10$ m 4 beams $b = 2.5$ m

$l/b = 30/2.5 = 12$ Mid span transverse beam $\Rightarrow c = 1/48$ $Z = 1/48 \cdot 12^3 \cdot 1 = 36$

it's a big number $\Rightarrow Z \approx \infty$

- For $Z \geq 20 \Rightarrow$ My bridge could be considered in light of the Courbon method → for $Z < 20$ Courbon hypothesis works fine too but with an higher level of approximation.
- For $Z \to \infty \Rightarrow$ I'm close to the Courbon method hypothesis → 2 ways to satisfy Courbon hypotesis:
  - lunghezza l grande e una b piccola
  - very "tuff" transverse beams and very "wick" longitudinal beams
    ↳ typical we're not in this condition 'cause $I_{ft} \approx I_{fb}$

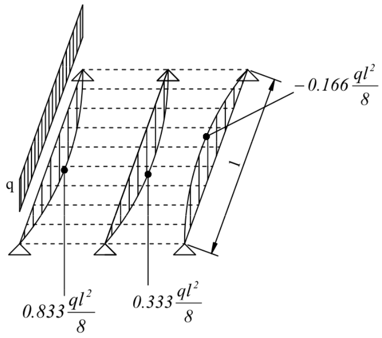
What we have seen for displacement also applies on internal action of the beam. So the bending moment of the 1st beam has the same shape of the 2nd one and the 3rd one and so on.

Despite what happens for displacement (in A the longitudinal deformed beams ≡ with the transverse) in internal action longitudinal and transverse actions ≠ more ⇒ there is a discontinuity ⇒ the effect of the transverse beam is neglected → that's true 'cause the hypotesis was that the stiffness of the transverse beams is smeared (spalmato) on the all width of the deck: la rigidezza del traverso è spalmata su tutta la lunghezza del ponte, quindi la sua rigidezza non è concentrata nel punto dove arriva.

So with Courbon I have no idea of the amount of internal action in transverse direction (I cannot get any kind of information with Courbon)

INTERNAL ACTIONS IN THE BEAMS

Case of infinite (or high number) of rigid transverse beam
COURBON



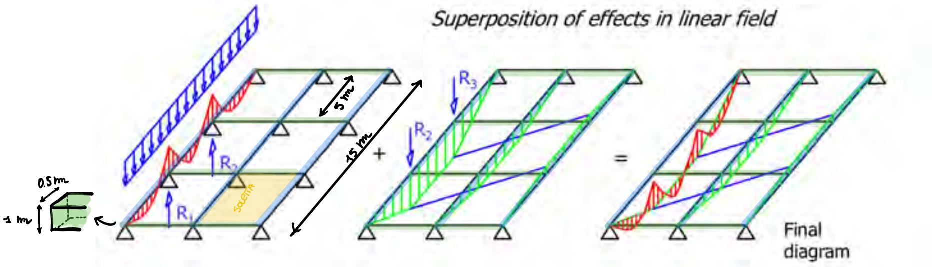
A load applied with a generic law on a beam is distributed with the same law on the other beams, proportionally to the transverse distribution coefficients.

(Rough hypothesis but good approximation in the result)

Case of finite number of transverse beams, infintely rigid
ENGESSER METHOD

ENGESSER METHOD

Engesser takes in consideration the real presence of transverse beams.



For istance let's imagine that the slab is 30 m thick $\Rightarrow J_{slab} = \frac{bh^3}{12} = \frac{bt^3}{12} = \frac{1 \cdot 0.3^3}{12} = 0.0025\ m^4$ (1 m of slab)

The inertia of the slab on 5 m of span is : $5 \cdot J_{slab} = 0.01125\ m^4$

Now compare this 5 m of slab inertia to the inertia of 1 transverse beam $\Rightarrow J_{trans.} = \frac{bh^3}{12} = \frac{0.5 \cdot 1^3}{12} = 0.04166\ m^4$

↳ for simplicity we consider only the web, neglecting the flange contribution

$$J_{T.B.} > J_{slab}$$

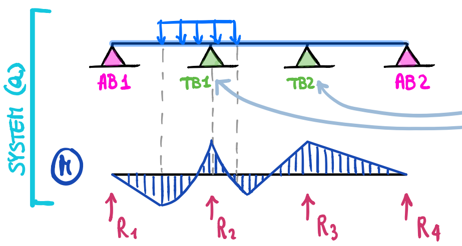
$$0.04166\ m^4 > 0.01125\ m^4$$

L'effetto di 5 m di slab è più piccolo rispetto l'effetto di 1 singola trave $\Rightarrow$ l'ipotesi di considerare tutto spalmato nel ponte non è poi così corretto. È meglio considerare tutta la rigidezza concentrata nelle travi trasversali; commettendo un errore più piccolo.

Il collegamento tra le travi longitudinali avviene attraverso 2 sistemi statici: la soletta, e le travi trasversali. La rigidezza della soletta sull'interasse delle travi trasversali è 0.01, la rigidezza di 1 trave trasversale (che opera sullo stesso campo) è 0.04 (considerando solo l'anima della trave trasversale e ignorando in realtà che è una trave a T per la presenza della soletta soprastante ⇒ in realtà sarebbe ancora maggiore). I 2 sistemi di collegamento tra le travi longitudinali, cioè la soletta e le travi trasversali non sono efficienti allo stesso modo. Il collegamento operato dalla soletta non funziona un cazzo, il collegamento operato coi traversi è 4÷5 volte più rigido e quindi funziona. Cioè il vero collegamento tra le travi longitudinali non è la soletta ma sono solo i traversi (sono loro ad avere tutta l'inerzia). Dunque l'ipotesi di considerare un collegamento spalmato e infinitamente rigido tra le travi longitudinali (ipotesi di Courbon) nella realtà non è vera. Può darsi che questa approssimazione (di Courbon) però mi porti a fare non un grosso errore se Z è alto, ma l'ipotesi fisica di base è palesemente falsa.

So Engesser modified the Courbon theory saying that all the stiffness of the connection between longitudinal beams is assigned to the transverse beams. Longitudinal beam n°1 will collaborate with longitudinal beam n°2 transfering forces not with a smired (spalmata) stiffness, but transfering forces only in correspondence of the transverse beams (no contribution of the slab).

• **STEP 1:** Consider the loaded longitudinal beam alone (no girder, simply beam)

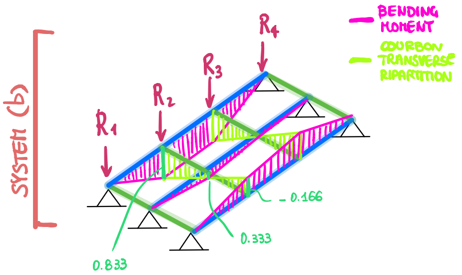


I consider transverse beam as rigid supports.

AB1 & AB2 = real supports (abutments)
TB1 & TB2 = fittitious supports (transverse beams)

Solving this structure we get internal action on the beam (for example it is drawn the bending moment) and the reactions.

• **STEP 2:** Consider the girder → load this static scheme with the reactions of the system (a) changed in sign. I could not draw R_1 and R_4 'cause they are influent: 100%. R_1 goes on the △ support below, the same happens with R_4 (they are forces applied on bearings ⇒ contribution to deformed shape = zero).



Once done this stuff, Engesser says: use Courbon method to transfer the concentrated forces transversally (Courbon Coefficients). So my girder is subjected to a bending moment which has a trapezoidal shape (because concentrated forces) on each longitudinal beam. The variation between M_1, M_2, M_3 is given by table of ρ in previous pages.

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- **STEP 3 :** - On the loaded beam superpose (a) and (b). So if I want to know the bending moment on beam n° 1:

$$M^{(1)} = M^{(1)}_a + M^{(2)}_b$$

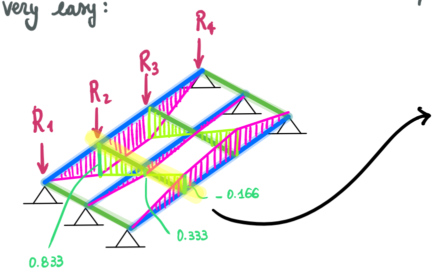
- On unloaded beams we will have only system (b).

N.B. If more than 1 beam is loaded we do this stuff for each beam

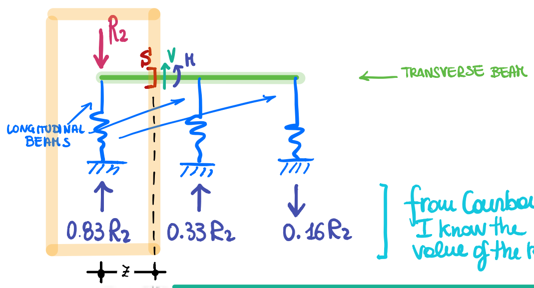
N.B. If the number of transverse beams is ≥ 3 the differences between the previous approaches are negligible. Any case the Engesser method is closer to the reality for the shear evaluation.

What is the idea of Engesser? Transversally forces can move going only to transverse beams. With this hypothesis we also can calculate the internal actions in transverse beams (this was not possible with Courbon method). How can we do it?

Once known the repartition coefficients p_{ij}, the evaluation of internal actions in transverse beams is very easy:



Let's image we want to know the internal action on this transverse beam:



from Courbon table I know the value of the reaction

If I want to calculate the internal action in S I have to write equilibrium equation of the rectangled part:

(↑) $V + 0.83 R_2 - R_2 = 0 \rightarrow V$

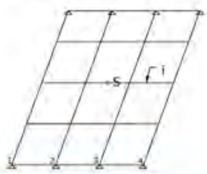
S) $-0.83 R_2 \cdot z + R_2 \cdot z + M = 0 \rightarrow M$

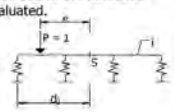
EXAMPLE

Internal actions in transverse beams

Once known the repartition coefficients p_{ij} , the evaluation of internal actions in transverse beams is very easy.

a) The load is moved transversely along the transverse beam: the influence line of bending moment in section S is to be evaluated.

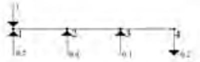




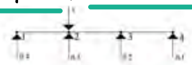
$M_s = \sum p_{ij} d_j - 1e$

$p_i = \begin{cases} 0.7 / 0.4 / 0.1 / -0.2 & \text{Beams 1/4} \\ 0.4 / 0.3 / 0.2 / -0.1 & \text{Beams 2/3} \end{cases}$

Those values have been calculated before



$M_s^1 = (0.7-1)*1.5 + 0.4*0.5 = -0.25$



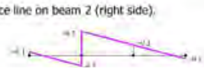
$M_s^2 = 0.4*1.5 + (0.3-1)*0.5 = +0.25$

Two points of the influence line are enough, because one deal with midspan section and the function is linear.

Now we can evaluate the shear influence line on beam 2 (right side).

$T^1_{S,r} = -1 + 0.7 + 0.4 = 0.1$

$T^2_{S,r} = +0.4 - 1 + 0.3 = -0.3$



b) The load is moved along the generic beam: Engesser procedure applies:

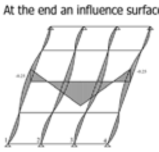
b₁) All the nodes are supposed to be restrained for vertical displacements and the reactions R_{ij} as a function of the position of load P=1 are evaluated.

b₂) The reactions R_{ij} , with opposite sign, are applied to the girder and their distribution between different beams is evaluated.

b₃) The bending moment in "S" is evaluated with the procedure discussed in "a".

b₄) All the procedure is repeated for other portions of load P=1.

At the end an influence surface is obtained:



In practice it's evaluated the influence line of reactions in a continuous beam on fixed bearings, and its ordinates are multiplied by the factor 0.25 (in this case).

The ordinates of influence line are obviously null when the load is located over the other transverse beams, because they are not able to influence mutually themselves.

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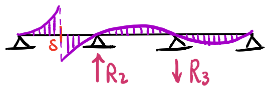
We can generate influence surfaces using the Engesser theory, 'cause with this theory we are able to calculate internal action both in longitudinal and transverse beams. So we can combine them together and get influence surfaces for moving loads (both on longitudinal and transverse beams).

I wanna know the influence lines of a longitudinal beam for a given internal action.

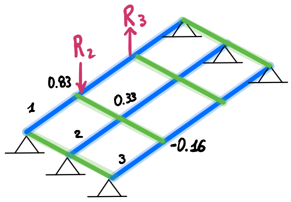
- V = Shear
- section: S
- l.i. vertical moving loads ↓P

→ I use SCHEME (a). I place the bearings corresponding to the transverse beams, and on this scheme I draw the influence line of shear.

SCHEME (a)

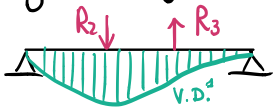


I calculate reaction and I put them on the girder.



SCHEME (b)

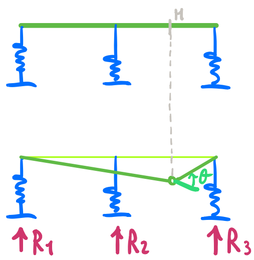
What should I calculate to draw lines of influence? The vertical displacement of this girder subjected to this load:



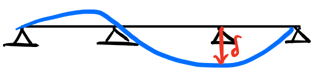
What about beam 2? I have to do the same stuff using Courbon. So the vertical displacement (V.D.) of beam n°1 is V.D.1 multiply by 0.83, the same for n° 2 multiply by 0.33 and the same for n°3 multiply by -0.16.

And then I have to superpose for beam n°1: scheme (a) + scheme (b) $(0.83 \cdot VD^{(1)})$, instead only scheme (b) for beams n°2 and n°3.

We can also calculate influence lines in transverse beams

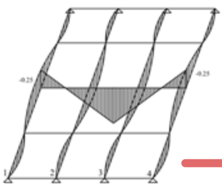


if for each beam I go back in scheme (a),



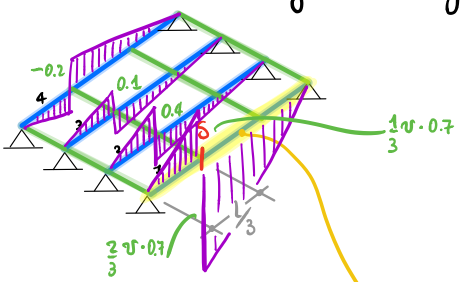
I'll have a force R1 which corresponds to a vertical displacement of beam 1 (located on a bearing 'cause scheme (a) is a continuous beam ⇒ so it corresponds to the reaction of a bearing subjected by a settlement)

What we get is this →

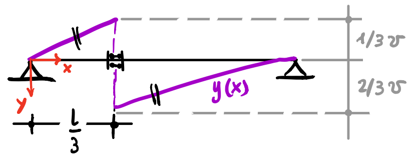


GIRDER BRIDGES - INFLUENCE FUNCTION

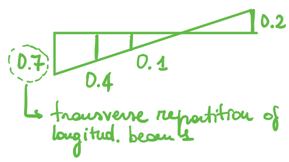
Let's consider a girder bridge. Using COURBON theory the calculation of influence lines is simple because we have the hypothesis that in trasverse direction the deck is infinitly rigid. Every section has the same ability to transfer load in transverse direction. COURBON HYPOTHESIS allows us only to calculate i.f. of the longitudinal beams. Transverse beam does not exist 'cause is smined.



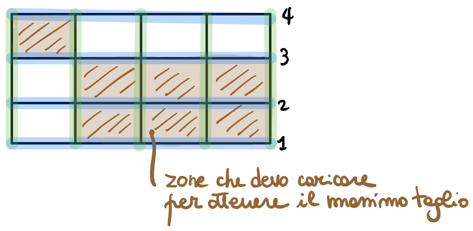
1) $U = V$
2) $kC = \downarrow\downarrow\downarrow\downarrow$
3) section S



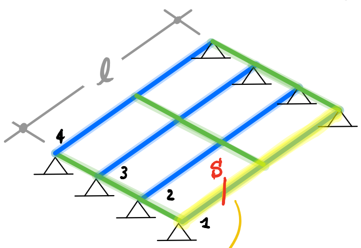
The transverse distribution of Courbon method is



TOP VIEW



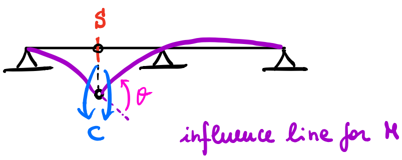
Now we are gonna see how to calculate l.i. with ENGESSER THEORY. Let's consider a girder bridge. With Engesser we can draw both influence lines (for longitudinal beams and for transverse beams).

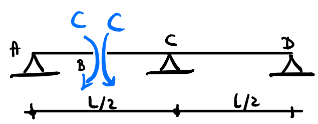


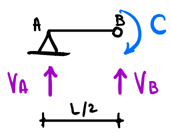
$$\begin{cases} U = M \\ kC = V.L. \\ S = \frac{l}{4} \end{cases}$$

1st step of Engesser theory was to consider the longitudinal beam alone. So let's consider beam 1.

BEAM 1





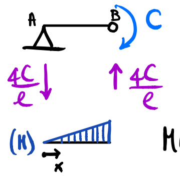


$(\uparrow)$ $V_A + V_B = 0$

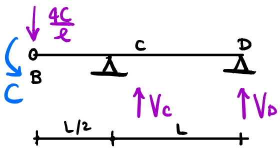
$A\circlearrowright$ $V_B \frac{L}{4} = C \quad \Rightarrow V_B = 4\frac{C}{\ell}$

$\Rightarrow V_A = -4\frac{C}{\ell}$

$\Rightarrow$



$M(x) = -\frac{4C}{\ell}\cdot x$



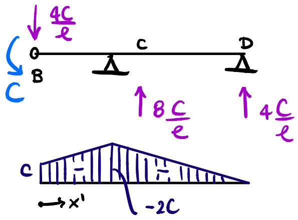
$(\uparrow)$ $V_C + V_D - V_B = 0$

$\circlearrowright$ $C + V_B \frac{\ell}{4} + V_D \frac{\ell}{2} = 0$

$\Rightarrow V_C = 8C/\ell$

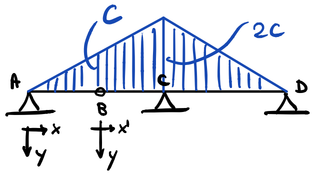
$\Rightarrow V_D = 4C/\ell$

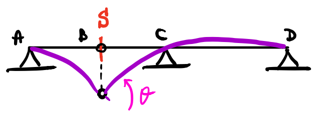
$\Rightarrow$



$M(x') = -C - \frac{C}{\ell}x'4 \quad \forall x' \in [B, C]$

$M(x') = -2C + \frac{2(x' - \frac{L}{4})2C}{L}$





$M_{AB}(x) = -4\frac{C}{\ell}\cdot x$ for $x \in [A; B]$

$M_{BC}(x') = -C - C x' \frac{4}{\ell}$ for $x \in [B; C]$

$M_{CD}(x') = -2C + \frac{4C(x' - \ell/4)}{\ell}$ for $x \in [C; D]$

To calculate the deformed shape I have to write χ, and then integrate 2 times:

$$\begin{aligned} \chi_{AB}(x) &= M_{AB}(x)/EJ \\ \chi_{BC}(x') &= M_{BC}(x')/EJ \\ \chi_{CD}(x') &= M_{CD}(x')/EJ \end{aligned} \rightarrow \begin{aligned} \theta_{AB}(x) &= \int \chi_{AB}(x)\,dx + k_1 \\ \theta_{BC}(x') &= \int \chi_{BC}(x)\,dx + k_3 \\ \theta_{CD}(x') &= \int \chi_{CD}(x)\,dx + k_5 \end{aligned} \rightarrow \begin{aligned} y_{AB}(x) &= \int \theta_{AB}(x)\,dx + k_2 \\ y_{BC}(x') &= \int \theta_{BC}(x')\,dx + k_4 \\ y_{CD}(x') &= \int \theta_{CD}(x')\,dx + k_6 \end{aligned}$$

We need 6 boundary conditions (1 for each k):

(1) $y_A = 0$

(2) $y_{AB}(B) = y_{BC}(B)$ → for the continuity of displacement

(3) $y_{BC}(C) = 0$ → 'cause we are on a bearing

(4) $\theta_{BC}(C) = \theta_{CD}(C)$ → 'cause the rotation in C is the same (continuity)

(5) $y_{CD}(C) = 0$

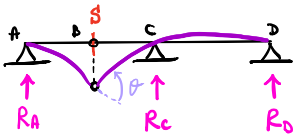
(6) $y_{CD}(D) = 0$

$\Rightarrow$ Now I have 6 equation and 6 boundary condition, I can solve the system

$\Rightarrow$ I'll find → 3 equations for displacement $y(x)$; → 3 equations for the rotation $\theta(x)$ → function of C

$\Rightarrow$ To calculate C I have to add a new equation: $\underbrace{\theta_{BC}(B) - \theta_{AB}(B) = \theta^*}_{\text{we find C to get the } \theta^* \text{ we wanted at start}}$

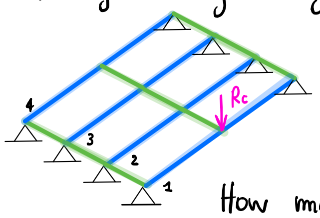
imposed by us



we found also the reactions:

$$R_A = R_A(\theta^*, EJ, L)$$
$$R_C = R_C(\theta^*, EJ, L)$$
$$R_D = R_D(\theta^*, EJ, L)$$

We are interested in R_C 'cause with the Engesser theory, R_C is the reaction that goes on the girder changed in sign:



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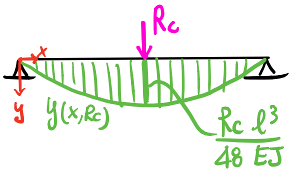
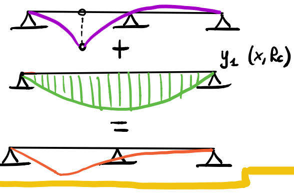


R_C fa lavoro con spostamenti verticali

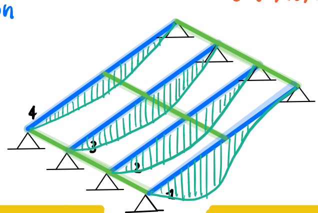
How many longitudinal beams we have? 4 → repartition of this deformed shape in transverse direction by Courbon method coefficient.

$$y_1(x, R_C) = 0.7\ y(x, R_C)$$
$$y_2(x, R_C) = 0.4\ y(x, R_C)$$
$$y_3(x, R_C) = 0.1\ y(x, R_C)$$
$$y_4(x, R_C) = -0.2\ y(x, R_C)$$

On beam n° 1 we have the superposition



On the other one, only y_2, y_3, y_4



EXAMPLE – application of Courbon method

Girder deck dimensions - Bottom view

Girder deck dimensions – Interaxis between main members

Girder deck dimensions – Cross section

The beams subjected to the highest bending moment are the external ones, so the other beams are designed as they were subjected to the same actions. This reduces design time and is a safe approximation.

We proceed calculating the internal actions (bending moment and shear) in the mid-span section of an external beam called beam 1.

Values of the multi component actions

In this exercise we will solve the structure only for the multi component action group n° 1. Needless to say that the other groups have to be taken into account too.

	Loads on carriageway					Loads on footways
	Vertical			Horizontal		Vertical
Group of actions	Main action LM1-2-3-4-6	Special vehicles	Crowd	Braking Accel.	Centrifugal	Uniform
1	Characteristic value					2.5 kN/m²

Load analysis

- **Dead load** g_1
  1. Longitudinal beams
  2. Transverse beams
  3. Slab

1. Longitudinal beam $g_{1lb} = b \cdot h \cdot l \cdot \gamma = 0.5m \cdot 1.20m \cdot 15m \cdot 25\frac{kN}{m^3} = 225kN$
2. Transverse beam $g_{1tb} = b \cdot h \cdot l \cdot \gamma = 0.3m \cdot 1.00m \cdot 2.5 \cdot 3m \cdot 25\frac{kN}{m^3} = 56kN$
3. Slab $g_{1s} = b \cdot l \cdot h \cdot \gamma = 12m \cdot 15m \cdot 0.25m \cdot 25\frac{kN}{m^3} = 1125kN$

- **Dead load** g_1

Total weight of the girder

$$g_{1tot} = 4 \cdot g_{1lb} + 4 \cdot g_{1tb} + g_{1s} = 4 \cdot (225 + 56) + 1125 = 2249kN$$

Dead load on the outermost beam

$$g_{1,1b} = g_{1tot}/4l = 2249kN/4 \cdot 15m = 38\frac{kN}{m}$$

Two simplifications:

a. Dead weight uniformly distributed among beams
b. Dead weight of transverse beams taken as uniformly distributed instead of 4 concentrated forces

- **Permanent loads** g_2
  1. Kerb
  2. Pavement
  3. Vehicle restraint system
  4. Pedestrian parapet

1. Kerb $\quad g_{2k} = b \cdot h \cdot l \cdot \gamma = 1.5m \cdot 0.23m \cdot 15m \cdot 25\frac{kN}{m^3} = 129kN$

2. Pavement $\quad g_{2p} = b \cdot l \cdot \gamma = 1.5m \cdot 15m \cdot 3\frac{kN}{m^2} = 67.5kN$

The load value for the pavement takes into account that several layers of asphalt may be placed one over another during maintenance of the road:

3. V. R. S. $\quad g_{2vrs} = l \cdot \gamma = 15m \cdot 2\frac{kN}{m} = 30kN$

4. Pedestr. parapet $\quad g_{2pp} = l \cdot \gamma = 15m \cdot 1.0\frac{kN}{m} = 15kN$

Permanent load on the outermost beam

$$g_{2,1b} = (g_{2,k} + g_{2,p} + g_{2,vrs} + g_{2,pp})/l =$$
$$= (129 + 68 + 30 + 15)kN/15m = 242/15 = 16kN/m \cong 44\% g_{1,1b}$$

One simplification:

a. The permanent load for the outermost beam is grater then for the other beams. In this example the load of kerb and barriers is fully given to the outermost beam, in reality it would distribute itself according to Courbon theory on the others beams resulting in a lesser weight for beam one.

Internal actions due to permanent loads

In the mid-span section of beam 1 we find the following internal actions:

Bending moment

$$Mg_{1b} = Mg_{1,1b} + Mg_{2,1b} = \frac{(g_{1,1b} + g_{2,1b}) \cdot l^2}{8}$$
$$= \frac{(38+16) \cdot 15^2}{8} = 1068 + 450\ kNm = 1518kNm$$

Shear

$$Vg_{1b} = Vg_{1,1b} + Vg_{2,1b} = 0 + 0 = 0kN$$

Load analysis

- **Variable traffic load** q_1

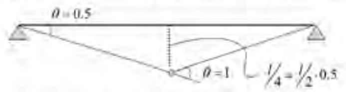
We need to trace the influence lines of bending moment and shear for the mid-span cross section of the beam for moving vertical loads.

We apply a disconnection dual to the desired internal action and we calculate the function of the entity dual to the known action (vertical force).

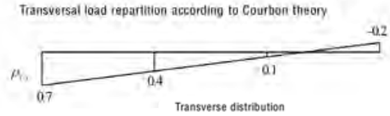
Bending moment in mid-span

- **Drawing influence surface**

One dimensional influence line for longitudinal simply supported beam



Transversal load repartition according to Courbon theory



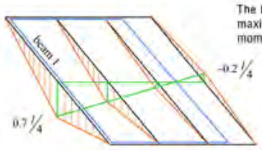
Transverse distribution

$\rho_{1,j}$ Is the amount of the load P=1 applied on the beam 1 that goes on the beam j (j=1÷4)

Or:

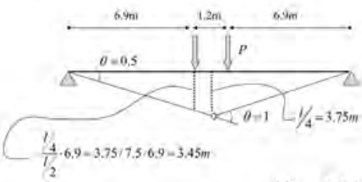
$\rho_{j,1}$ Is the amount of the load P=1 applied on the beam j (j=1÷4) that goes on the beam 1

If we modulate the two graph seen before we obtain



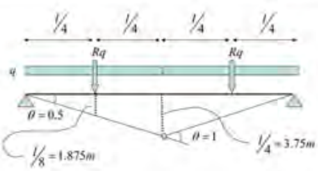
The blue area has to be loaded to maximize the mid-span bending moment in beam 1

Longitudinal distribution (concentrated loads)



$$\Rightarrow M_{s,P} = 2 \cdot 3.45 \cdot P = 6.9P$$

Longitudinal distribution (uniformly distributed loads)



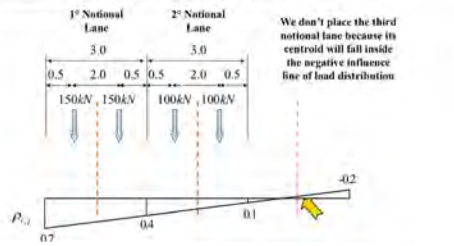
$$\Rightarrow M_{s,q} = 2 \cdot 1.875 \cdot Rq = 3.7 \cdot l/2\, q = 3.7 \cdot 7.5q = 27.75q$$

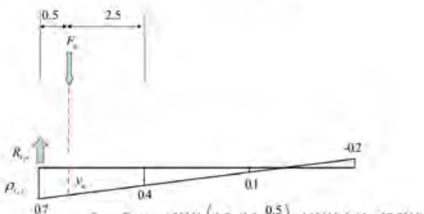
Transverse distribution

Carriageway width = 9m
Width of each notional lane = 3m
Number of notional lanes = 3

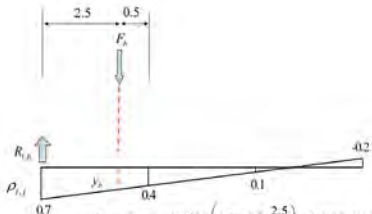
Location	Tandem system TS	UDL system
	Axle loads Q_{ik} (kN)	q_{ik} (or q_{rk}) (kN/m²)
Lane Number 1	300	9
Lane Number 2	200	2.5
Lane Number 3	100	2.5
Other lanes	0	2.5
Remaining area (q_{rk})	0	2.5

Transverse distribution (concentrated loads)

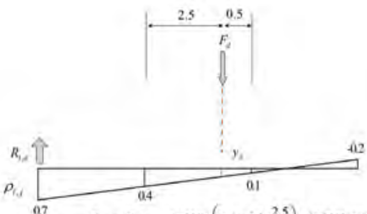




$$R_{1,a} = F_a \cdot y_a = 150kN \cdot \left(0.7 - 0.3 \cdot \frac{0.5}{3}\right) = 150kN \cdot 0.65 = 97.5kN$$



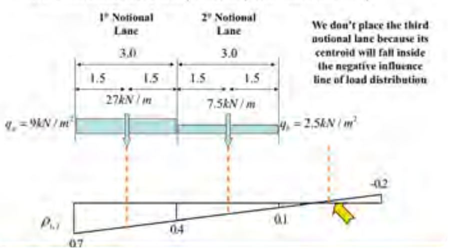
$$R_{1,b} = F_b \cdot y_b = 150kN \cdot \left(0.7 - 0.3 \cdot \frac{2.5}{3}\right) = 150kN \cdot 0.45 = 67.5kN$$

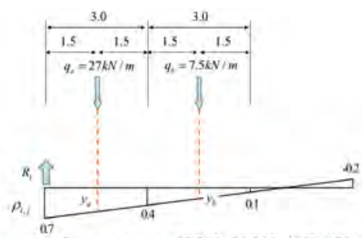


$$R_{1,d} = F_d \cdot y_d = 100kN \cdot \left(0.4 - 0.3 \cdot \frac{2.5}{3}\right) = 100kN \cdot 0.15 = 15.0kN$$

$$R_{1,concentrated} = R_{1,a} + R_{1,b} + R_{1,c} + R_{1,d} = \\ = 97.5 + 67.5 + 35 + 15 = 215kN$$

Transverse distribution (uniformly distributed loads)

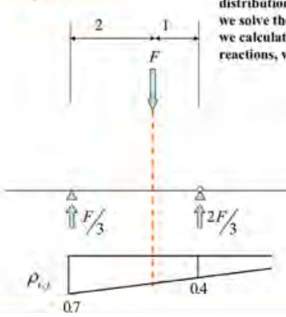




$$R_{1,uniform} = R_{1,a} + R_{1,b} = q_a \cdot y_a + q_b \cdot y_b = 27 \cdot 0.55 + 7.5 \cdot 0.25 = 14.85 + 1.87 = 16.7kN/m$$

Pay attention!

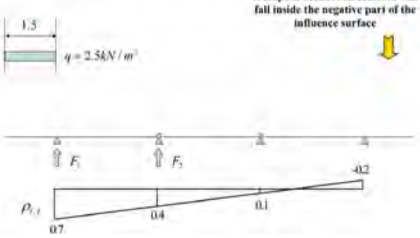
If we consider a force F and we calculate its distribution using directly the influence line, or we solve the static scheme shown below and then we calculate the force in the beam 1 with the reactions, we obtain the same result.

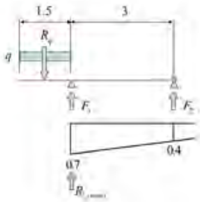


$$R_1 = F \cdot 0.5 = F/3 \cdot 0.7 + 2F/3 \cdot 0.4$$

Transverse distribution (crowd)

We don't place crowd on this footpath because its centroid will fall inside the negative part of the influence surface





$q = 2.5kN/m^2$

$R_q = q \cdot 1.5m = 3.75kN/m$

$F_2 = -3.75 \cdot 0.75/3 = -0.94kN/m$

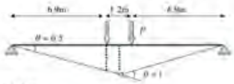
$F_1 = 3.75 + 0.94 = 4.69kN/m$

$R_{1,crowd} = F_1 \cdot 0.7 + F_2 \cdot 0.4 = 2.9kN/m$

Same result if we extrapolate the Curbon transverse line outside beam n1

Bending moment in mid-span

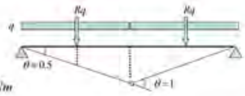
Concentrated tandem system



$R_{1,concentrated} = P = 215kN$

$M_{1,concentrated} = 6.9R_{1,concentrated} = 1484kNm$

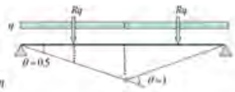
Uniformly distributed



$R_{1,u.distr.} = q = 16.7kN/m$

$M_{1,u.distr.} = 27.75R_{1,u.distr.} = 463kNm$

Crowd



$R_{1,crowd} = q = 2.9kN/m$

$M_{1,crowd} = 27.75R_{1,crowd} = 81kNm$

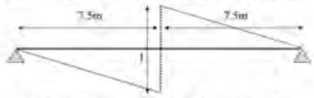
Total bending moment from vertical traffic actions

$M_{traffic} = M_{concentrated} + M_{u.distr.} + M_{crowd} = 1484 + 463 + 81 = 2028kNm$

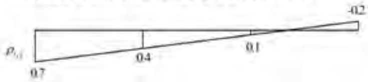
Shear in mid-span

- **Drawing influence surface**

One dimensional influence line for longitudinal simply supported beam

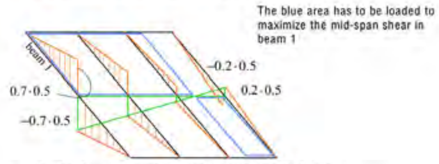


Transversal load repartition according to Curbon theory

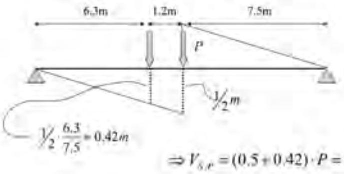


If we modulate the two graph seen before we obtain

The blue area has to be loaded to maximize the mid-span shear in beam 1



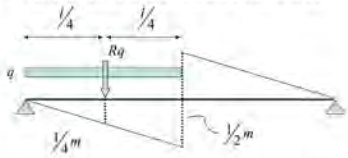
- **Variable concentrated traffic load**



$\frac{1}{2}\cdot\frac{6.3}{7.5} = 0.42m$

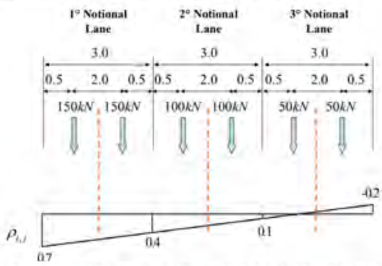
$\Rightarrow V_{S,P} = (0.5 + 0.42) \cdot P = 0.92P$

- **Variable uniformly distributed traffic load**



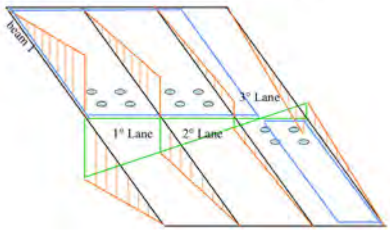
$\Rightarrow V_{S,q} = q\frac{l}{2}\cdot\frac{1}{4} = q\cdot\frac{15}{8} = 1.875q$

Transverse distribution (concentrated loads)

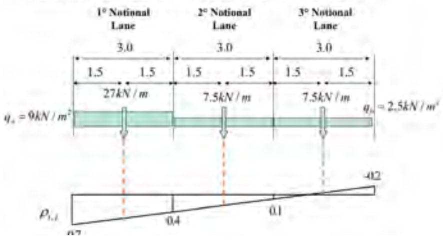


$R_{1,concentrated} = 150 \cdot (0.65 + 0.45) + 100 \cdot (0.35 + 0.15) + 50 \cdot (-0.05 + 0.15) =$
$= 165 + 50 + 5 = 220kN$

Longitudinal location of previously seen concentrated loads

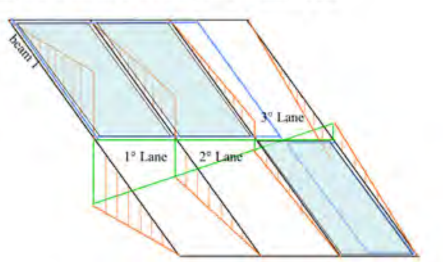


Transverse distribution (uniformly distributed loads)

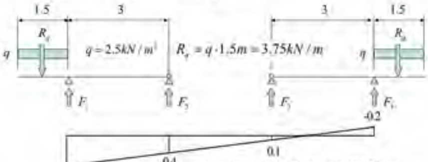


$$R_1, u.distr. = 27 \cdot 0.55 + 7.5 \cdot 0.25 + 7.5 \cdot 0.05 = 17.1 kN/m$$

Longitudinal location of previously seen distributed loads



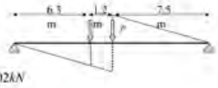
Transverse distribution (crowd)



$$F_1 = F_4 = 3.75 + 0.94 = 4.69 kN/m$$
$$F_2 = F_3 = -3.75 \cdot 0.75/3 = -0.94 kN/m$$
$$R_{1,crowd} = F_1 \cdot 0.7 + F_2 \cdot 0.4 + F_3 \cdot -0.1 + F_4 \cdot 0.2 =$$
$$= 4.69 \cdot 0.9 - 0.94 \cdot 0.3 = 3.94 kN/m$$

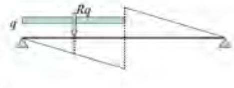
Shear in mid-span

Concentrated tandem system



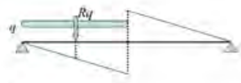
$$R_{1,concentrated} = P = 220kN$$
$$V_{1,concentrated} = 0.92 R_{1,concentrated} = 202kN$$

Uniformly distributed



$$R_{1,u.distr.} = q = 17.1kN/m$$
$$V_{1,u.distr.} = 1.875 R_{1,u.distr.} = 32kN$$

Crowd



$$R_{1,crowd} = q = 2.9kN/m$$
$$V_{1,crowd} = 1.875 R_{1,crowd} = 5.4kN$$

Total shear from vertical traffic actions

$$V_{1,traffic} = V_{1,concentrated} + V_{1,u.distr.} + V_{1,crowd} = 202 + 32 + 5 = 239kN$$

Non traffic actions: WIND

Location: Piemonte 250m o.s.l.

Wind referring speed $v_b = v_{b,0} = 25\frac{m}{s}$

Kinetic referring pressure $q_b = \frac{1}{2}\rho v_b^2 = \frac{1}{2} 1.25 \cdot 25^2 = 391\frac{N}{m^2}$

Geografic zone 1
Terrain roughness class D (open land without obstacles)
Site exposition category II $\quad k_r = 0.19$
$z_0 = 0.05m$
$z_{min} = 4$ m

Maximum height of the structure z=3m

Exposure coefficient $c_e(z) = c_e(z_{min}) = k_r^2 \cdot \ln\left(\frac{z_{min}}{z_0}\right)\left[7 + \ln\left(\frac{z_{min}}{z_0}\right)\right]$

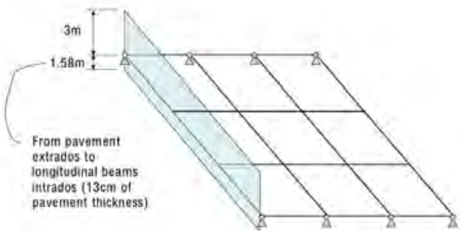
$$c_e(z) = 0.19^2 \cdot \ln\left(\frac{4}{0.05}\right)\left[7 + \ln\left(\frac{4}{0.05}\right)\right] = 1.8$$

Dynamic coefficient = 1

Shape coefficient = 1

Wind pressure $p = q_b \cdot c_e \cdot c_p \cdot c_d = 391 \cdot 1.8 \cdot 1 \cdot 1 = 0.74\frac{kN}{m^2}$

Surface exposed to the wind



Vertical position of the centroid of the deck

Total mass of the bridge

1. Longitudinal beams $Mg_{1lb} = 225kN \cdot 4 = 900kN$
2. Transverse beams $Mg_{1tb} = 56kN \cdot 4 = 224kN$
3. Slab $Mg_{1s} = 1125kN$

Total $Mg = 900 + 224 + 1125 = 2249kN$

Static moment of bridge masses with respect to the intrados

1. Longitudinal beams $Sg_{1lb} = 225kN \cdot 4 \cdot 0.6m = 540kNm$
2. Transverse beams $Sg_{1tb} = 56kN \cdot 4 \cdot 0.5m = 112kNm$
3. Slab $Sg_{1s} = 1125kN \cdot 1.325m = 1491kNm$

Total $Sg = 540 + 112 + 1491 = 2143kNm$

Vertical position of the centroid $y_g = \frac{S_g}{M_g} = \frac{2143}{2249} = 0.95m$

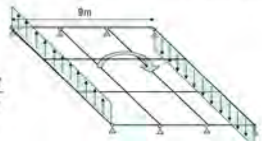
Wind resultant $q_{wind} = p \cdot h = 0.74 \frac{kN}{m^2} \cdot 4.58m = 3.39 \frac{kN}{m}$

Torque moment $Mt = q_{wind} \cdot e = 3.39 \frac{kN}{m}(4.58/2 - 0.95) = 4.54 \frac{kN}{m} m$

Equivalent vertical load acting on beams 1 and 4

$q_{Vert,wind} = Mt/9m =$

$4.54/9 = 0.50 \frac{kN}{m}$



Bending moment in mid-span of beam 1 due to wind action

$$M_{S,wind} = \frac{q_{Vert,wind} \cdot l^2}{8} = \frac{0.50 \cdot 15^2}{8} = 14.1kNm$$

Shear in mid-span of beam 1 due to wind action

$$V_{S,wind} = 0kN$$

Pay attention:

It's not possible to evaluate the internal actions in the transverse beams using **Courbon**, because Courbon hypothesis doesn't locate transverse beams in a specific position but smears them in the whole length of the deck.

If we want to know the internal actions in the transverse beam we have to use the **Engesser** model.

ULS combination

Bending moment in mid-span of beam 1

$$M_{S,tot} = 1.35M_{S,perm} + 1.35M_{S,traffic} + 1.50M_{S,wind} =$$
$$= 1.35 \cdot 1518 + 1.35 \cdot 2028 + 1.50 \cdot 14 = 4808kNm$$

Shear in mid-span of beam 1

$$V_{S,tot} = 1.35V_{S,perm} + 1.35V_{S,traffic} + 1.50V_{S,wind} =$$
$$= 1.35 \cdot 0 + 1.35 \cdot 239 + 1.50 \cdot 0 = 323kN$$

EXAMPLE – application of Engesser method

We will analyze the same deck seen with the Courbon approach with Engesser theory.

We will calculate bending moment and shear in the mid-span of beam 1 exactly as we have done with Courbon for the same multi component actions.

For sake of simplicity we will assume for dead load and permanent actions the same values seen in Courbon example (there's very little difference as the deformation due to these loads is cylindrical).

We will then focus only on variable traffic loads.

$$y_k(z,x) = \left(0.7 - 0.9\frac{x}{3b}\right) \begin{cases} \frac{1}{10l^2}\left(-z^3 + 6l^2 z\right) & for\ 0 \le z \le l \\ \frac{1}{10l^2}\left(-z^3 + (z-l)^3 + 6l^2 z\right) & for\ l \le z \le 2l \\ \frac{1}{10l^2}\left(-z^3 + (z-l)^3 + (z-2l)^3 + 6l^2 z\right) & for\ 2l \le z \le 3l \end{cases}$$

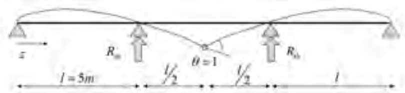
Transverse direction

Longitudinal direction

Bending moment in mid-span

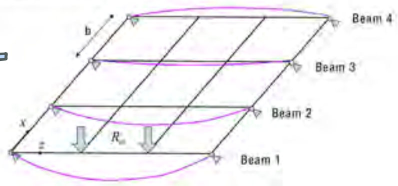
- **Drawing influence surface**

One dimensional influence line for longitudinal beam (continuous on transverse beams)



$$y_{el}(z) = \begin{cases} \frac{z}{10}\left(\frac{z^2}{l^2} - 1\right) & for\ 0 \le z \le l \\ \frac{1}{5}\left(\frac{3z^2}{2l} - 2z + \frac{l}{2}\right) & for\ l \le z \le \frac{3}{2}l \end{cases} \qquad R_{th} = \frac{3EI_y}{5l^3}$$

We apply the virtual reactions R_{th} on the girder and we calculate with Courbon theory the global deformation of the deck.



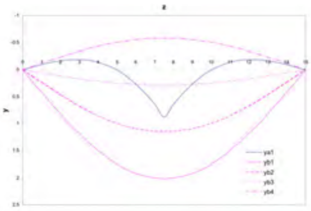
That can become for the single beams

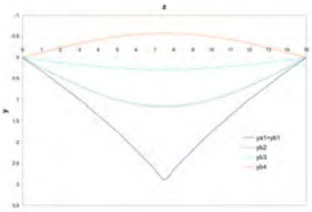
$$y_{b,j}(z) = \rho_{i,j} \cdot \begin{cases} \frac{1}{10l^3}\left(-z^3+6l^2z\right) & for\ 0 \le z \le l \\ \frac{1}{10l^3}\left(-z^3+(z-l)^3+6l^2z\right) & for\ l \le z \le 2l \\ \frac{1}{10l^3}\left(-z^3+(z-l)^3+(z-2l)^3+6l^2z\right) & for\ 2l \le z \le 3l \end{cases}$$

Transverse direction

Longitudinal direction

$$\rho_{ij} = \begin{matrix} 0.7 & \textbf{Beam 1} \\ 0.4 & \textbf{Beam 2} \\ 0.1 & \textbf{Beam 3} \\ -0.2 & \textbf{Beam 4} \end{matrix}$$

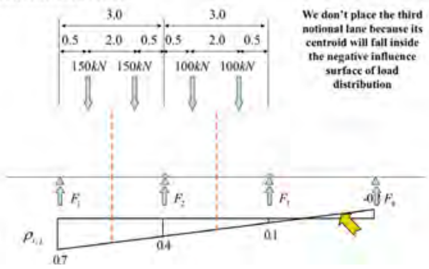




- **Procedure**

1. We have to distribute on the longitudinal beams the vertical loads acting on the slab using the simply supported schemes seen before
2. Once the loads are on the beams we can use the influence lines shown in the previous slide to calculate the bending moment in mid-span.

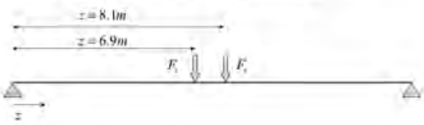
Concentrated loads



1. We have to distribute on the longitudinal beams the vertical loads acting on the slab using the simply supported schemes seen before

$$F_1 = 150kN$$
$$F_2 = 150+100 = 250kN$$
$$F_3 = 100kN$$

2. Once the loads are on the beams we can use the influence lines shown in slide 61 to calculate the bending moment in mid-span.



$$y_{a,1}(6.9)+y_{b,1}(6.9) = y_{a,1}(8.1)+y_{b,1}(8.1) = 0.6+2.0 = 2.60$$
$$y_{b,2}(6.9) = y_{b,2}(8.1) = 1.14$$
$$y_{b,3}(6.9) = y_{b,3}(8.1) = 0.285$$

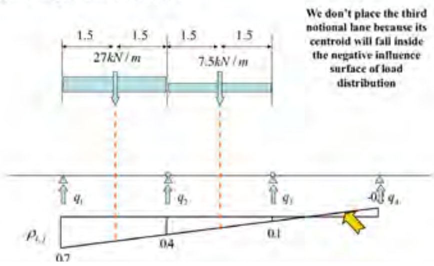
$$M_{s,F1} = 2\cdot\left[y_{a,1}(6.9)+y_{b,1}(6.9)\right]\cdot F_1 = 2\cdot 2.60\cdot 150 = 780kNm$$
$$M_{s,F2} = 2\cdot y_{b,2}(6.9)\cdot F_2 = 2\cdot 1.14\cdot 250 = 570kNm$$
$$M_{s,F3} = 2\cdot y_{b,3}(6.9)\cdot F_3 = 2\cdot 0.285\cdot 100 = 57kNm$$

$$M_{S,concentrated} = M_{S,F1}+M_{S,F2}+M_{S,F3} = 780+570+57 = 1407kNm$$

With Courbon model it was

$M_{S,concentrated} = 1484kNm$ 5% difference

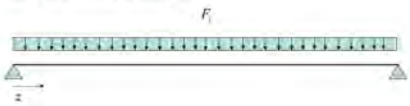
Uniformly distributed loads



1. We have to distribute on the longitudinal beams the vertical loads acting on the slab using the simply supported schemes seen before

$$q_1 = 13.5kN/m$$
$$q_2 = 13.5+3.75 = 17.25kN/m$$
$$q_3 = 3.75kN/m$$

2. Once the loads are on the beams we can integrate the influence lines shown in slides 56 and 59 for a uniformed distributed load to calculate the bending moment in mid-span.



"a" system

$$y_{a1}(z)=\begin{cases}\dfrac{z}{10}\left(\dfrac{z^2}{l^2}-1\right) & for\ 0\le z\le l\\ \dfrac{1}{5}\left(\dfrac{3z^2}{2l}-2z+\dfrac{l}{2}\right) & for\ l\le z\le \dfrac{3}{2}l\end{cases}$$

$$\int_0^{3l} q\cdot y_{a1}(z)dz=2\left(\int_0^l q\cdot\frac{z}{10}\left(\frac{z^2}{l^2}-1\right)dz+\int_l^{3l/2} q\cdot\frac{1}{5}\left(\frac{3z^2}{2l}-2z+\frac{l}{2}\right)dz\right)$$

$$=2q\left(\int_0^l \frac{z}{10}\left(\frac{z^2}{l^2}-1\right)dz+\int_l^{3l/2}\frac{1}{5}\left(\frac{3z^2}{2l}-2z+\frac{l}{2}\right)dz+\right)$$

$$=2q\left(-\frac{l^2}{40}+\frac{3l^2}{80}\right)=\frac{1}{40}q\cdot l^2=0.625q$$

"b" system

$$y_{b,j}(z)=\rho_{1,j}\cdot\begin{cases}\dfrac{1}{10l^2}\left(-z^3+6l^2z\right) & for\ 0\le z\le l\\ \dfrac{1}{10l^2}\left(-z^3+(z-l)^3+6l^2z\right) & for\ l\le z\le 2l\\ \dfrac{1}{10l^2}\left(-z^3+(z-l)^3+(z-2l)^3+6l^2z\right) & for\ 2l\le z\le 3l\end{cases}$$

$$\int_0^{3l} q\cdot y_{b,j}(z)dz=\rho_{1,j}\left(2\int_0^l q\cdot\frac{1}{10l^2}\left(-z^3+6l^2z\right)dz+\int_l^{2l} q\cdot\frac{1}{10l^2}\left(-z^3+(z-l)^3+6l^2z\right)dz+\right)$$

$$=\rho_{1,j}\frac{q}{10l^2}\left(2\int_0^l\left(-z^3+6l^2z\right)dz+\int_l^{2l}\left(-z^3+(z-l)^3+6l^2z\right)dz+\right)$$

$$=\rho_{1,j}\frac{q}{10l^2}\left(\frac{11}{2}l^4+\frac{11}{2}l^4\right)=\rho_{1,j}q\frac{11}{10}l^2$$

"a" + "b" systems

Beam 1 $\left(\frac{1}{40}+\frac{11}{10}\cdot 0.7\right)q\cdot l^2=\frac{41\cdot 25}{100}q=\frac{41}{4}q$

Beam 2 $\left(\frac{11}{10}\cdot 0.4\right)q\cdot l^2=\frac{11}{25}q=0.44q$

Beam 3 $\left(\frac{11}{10}\cdot 0.1\right)q\cdot l^2=\frac{11}{100}q=0.11q$

Total $M_{S,a.distr.}\ \frac{41}{4}q_1+0.44q_2+0.11q_3=\frac{41}{4}13.5+0.44\cdot 17.25+0.11\cdot 3.75=146kNm$

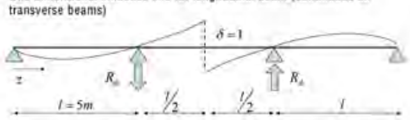
With Courbon model it was

$M_{S,a.distr.}=463kNm$ 69% difference

Shear in mid-span

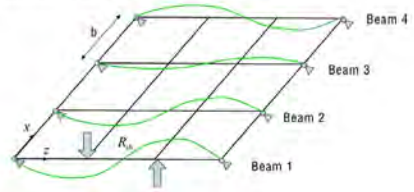
- **Drawing influence surface**

One dimensional influence line for longitudinal beam (continuous on transverse beams)



$$y_{a}(z)=\begin{cases}\dfrac{z}{3l}\left(1-\dfrac{z^2}{l^2}\right) & for\ 0\le z\le l\\ \dfrac{z}{3l}\left(1-\dfrac{z^2}{l^2}\right)+\left(\dfrac{z}{l}-1\right)^3 & for\ l\le z\le \dfrac{3}{2}l\end{cases}\qquad R_{ib}=\frac{6EI_b}{l^3}$$

We apply the virtual reactions R_{ib} on the girder and we calculate with Courbon theory the global deformation of the deck.



$$y_b(z,x)=\left(0.7-0.9\frac{x}{3b}\right)\cdot\begin{cases}\dfrac{2}{l^3}\left(\dfrac{z^3}{6}-\dfrac{l^2z}{3}\right) & for\ 0\le z\le l\\ \dfrac{2}{l^3}\left(\dfrac{z^3}{6}-\dfrac{(z-l)^3}{2}-\dfrac{l^2z}{3}\right) & for\ l\le z\le 2l\\ \dfrac{2}{l^3}\left(\dfrac{z^3}{6}-\dfrac{(z-l)^3}{2}-\dfrac{(z-2l)^3}{2}-\dfrac{l^2z}{3}\right) & for\ 2l\le z\le 3l\end{cases}$$

Transverse direction

Longitudinal direction

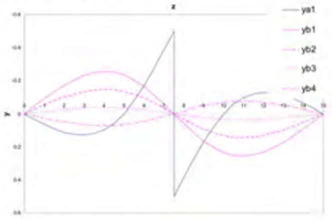
That can become for the single beams

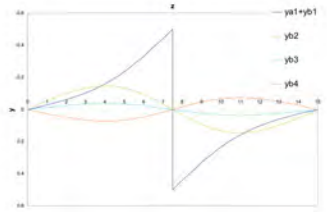
$$y_b(z,x)=\rho_{1,j}\cdot\begin{cases}\dfrac{2}{l^3}\left(\dfrac{z^3}{6}-\dfrac{l^2z}{3}\right) & for\ 0\le z\le l\\ \dfrac{2}{l^3}\left(\dfrac{z^3}{6}-\dfrac{(z-l)^3}{2}-\dfrac{l^2z}{3}\right) & for\ l\le z\le 2l\\ \dfrac{2}{l^3}\left(\dfrac{z^3}{6}-\dfrac{(z-l)^3}{2}-\dfrac{(z-2l)^3}{2}-\dfrac{l^2z}{3}\right) & for\ 2l\le z\le 3l\end{cases}$$

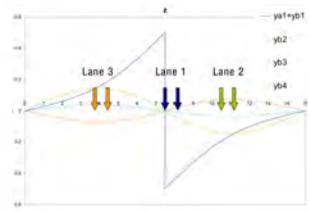
Transverse direction

Longitudinal direction

$$\rho_1=\begin{array}{ll}0.7 & \textbf{Beam 1}\\ 0.4 & \textbf{Beam 2}\\ 0.1 & \textbf{Beam 3}\\ -0.2 & \textbf{Beam 4}\end{array}$$

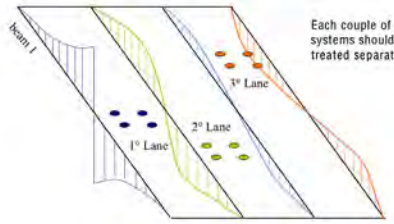






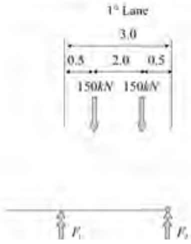
Concentrated loads

Longitudinal location of previously seen concentrated loads



Each couple of tandem systems should be treated separately

Concentrated loads : 1° lane



$F_1 = F_2 = 150kN$

$y_{a,1}(7.5) + y_{b,1}(7.5) = 0.5 + 0.0 = 0.5$

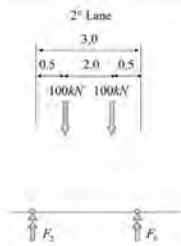
$y_{a,1}(8.7) + y_{b,1}(8.7) = 0.27 + 0.13 = 0.40$

$y_{b,2}(7.5) = 0$

$y_{b,2}(8.7) = 0.08$

$V_{c,1} = F_1(0.50 + 0.40 + 0 + 0.08)$
$= 0.98 \cdot 150 = 147kN$

Concentrated loads : 2° lane



The z corresponding to the maximum value of $y_{b,2}(z)$ has to be calculated. For sake of simplicity it is done for 0<z<7.5 and then used for the symmetric points with z>7.5m.

$$y_{b,2}(z) = 0.4\frac{2}{l^3}\left(\frac{z^3}{6} - \frac{l^2 z}{3}\right)$$

$$\frac{\partial y_{b,2}(z)}{\partial z} = 0 \Rightarrow \frac{\partial}{\partial z}\left[0.4\frac{2}{l^3}\left(\frac{z^3}{6} - \frac{l^2 z}{3}\right)\right] = 0$$

$$z = \sqrt{\frac{2}{3}}l = 4.08m$$

$z_{f1} = 4.08 - 0.6 = 3.48m$

$z_{f2} = 4.08 + 0.6 = 4.68m$

$y_{b,2}(11.52) = -y_{b,2}(3.48) = 0.140$

$y_{b,2}(10.32) = -y_{b,2}(4.68) = 0.140$

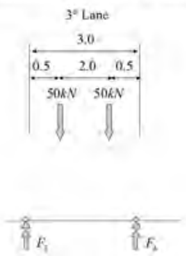
$y_{b,3}(11.52) = -y_{b,3}(3.48) = 0.035$

$y_{b,3}(10.32) = -y_{b,3}(4.68) = 0.035$

$F_2 = F_3 = 100kN$

$V_{c,2} = 2 \cdot F_2(0.140 + 0.035)$
$= 0.350 \cdot 100 = 35kN$

Concentrated loads : 3° lane



The tandem loads are placed symmetrically to the ones of the 2° lane with respect to the mid-span of the bridge because of the anti-symmetry of the influence line of beams 3 and 4

$y_{b,3}(3.48) = -0.035$

$y_{b,3}(4.68) = -0.035$

$y_{b,4}(3.48) = 0.070$

$y_{b,4}(4.68) = 0.070$

$F_3 = F_4 = 50kN$

$V_{c,3} = 2 \cdot F_3(0.070 - 0.035)$
$= 0.070 \cdot 50 = 3.5kN$

Concentrated loads : total shear in mid-span

We add the contribution of the three lanes

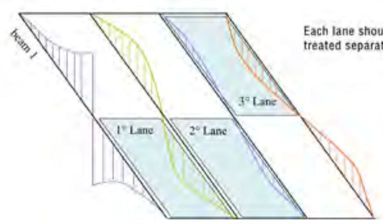
$V_{S,concentrated} = V_{c,1} + V_{c,2} + V_{c,3} = 147 + 35 + 3.5 = 185.5kN$

With Courbon model it was

$V_{S,concentrated} = 202kN$ 8.9% difference

Uniformly distributed loads

Location of uniformly distributed loads



Each lane should be treated separately

"a" system

N.B. For sake of simplicity the following calculations are done for 0<z<7.5 and then used for the symmetric values with z>7.5m.

$$y_{a1}(z) = \begin{cases} \frac{z}{3l}\left(1 - \frac{z^2}{l^2}\right) & for\ 0 \le z \le l \\ \frac{z}{3l}\left(1 - \frac{z^2}{l^2}\right) + \left(\frac{z}{l} - 1\right)^3 & for\ l \le z \le \frac{3}{2}l \end{cases}$$

$$\int_0^{3l/2} q \cdot y_{a1}(z)dz = \int_0^l q \cdot \frac{z}{3l}\left(1 - \frac{z^2}{l^2}\right)dz + \int_l^{3l/2} q \cdot \left[\frac{z}{3l}\left(1 - \frac{z^2}{l^2}\right) + \left(\frac{z}{l} - 1\right)^3\right]dz$$
$$= q\left(\int_0^l \frac{1}{3l}\left(z - \frac{z^3}{l^2}\right)dz + \int_l^{3l/2} \frac{1}{3l}\left(z - \frac{z^3}{l^2}\right) + \left(\frac{z}{l} - 1\right)^3 dz\right)$$
$$= ql\left(\frac{1}{12} - \frac{25}{192} + \frac{1}{64}\right) = -\frac{1}{32}ql = -\frac{1}{32}q5 = -0.156q$$

"b" system

$$y_{b1}(z) = \rho_{1,i} \cdot \begin{cases} \frac{2}{l^3}\left(\frac{z^3}{6} - \frac{l^2 z}{3}\right) & for\ 0 \le z \le l \\ \frac{2}{l^3}\left(\frac{z^3}{6} - \frac{(z-l)^3}{2} - \frac{l^2 z}{3}\right) & for\ l \le z \le 2l \end{cases}$$

$$\int_0^{3l/2} q \cdot y_{b1}(z)dz = \rho_{1,i}\left\{\int_0^l q \cdot \frac{2}{l^3}\left(\frac{z^3}{6} - \frac{l^2 z}{3}\right)dz + \int_l^{3l/2} q \cdot \left[\frac{2}{l^3}\left(\frac{z^3}{6} - \frac{(z-l)^3}{2} - \frac{l^2 z}{3}\right)\right]dz\right\}$$
$$= \rho_{1,i}\left\{\frac{2q}{3l^3}\int_0^l \left(\frac{z^3}{2} - l^2 z\right)dz + \frac{2q}{l^3}\int_l^{3l/2}\left(\frac{z^3}{6} - \frac{(z-l)^3}{2} - \frac{l^2 z}{3}\right)dz\right\}$$
$$= \rho_{1,i}\left\{-ql\frac{1}{4} + ql\left(\frac{65}{192} - \frac{1}{64} - \frac{5}{12}\right)\right\} = -\rho_{1,i}\frac{11}{32}ql = -\rho_{1,i}\frac{11}{32}q5 = -\rho_{1,i}1.72q$$

Then $\int_{3l/2}^{3l} q \cdot y_{b1}(z)dz = -\int_0^{3l/2} q \cdot y_{b1}(z)dz$ is:

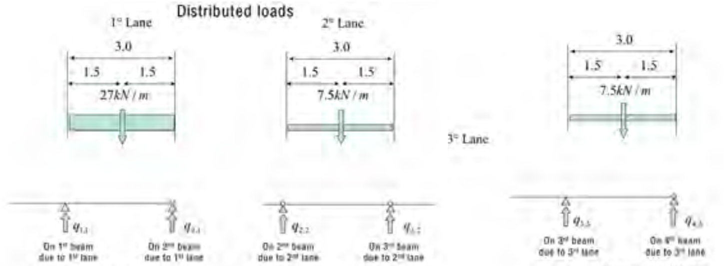
For beam 1: $\rho_{1,1}1.72q = 0.7 \cdot 1.72q + 0.156q = 1.36q$

For beam 2: $\rho_{1,2}1.72q = 0.4 \cdot 1.72q = 0.688q$

For beam 3: $\rho_{1,3}1.72q = 0.1 \cdot 1.72q = 0.172q$

For beam 4: $\rho_{1,4}1.72q = -0.2 \cdot 1.72q = -0.344q$

Distributed loads



$q_{1,1} = q_{2,1} = 27/2 = 13.5kN/m$ $\quad q_{2,2} = q_{3,2} = 7.5/2 = 3.75kN/m$ $\quad q_{3,3} = q_{4,3} = 7.5/2 = 3.75kN/m$

The shear due to distributed loads is then:

Beam 1 - lane 1: $V_{d,1,1} = 1.36 \cdot q_{1,1} = 1.36 \cdot 13.5 = 18.36kN$

Beam 2 - lane 1: $V_{d,2,1} = 0.688 \cdot q_{2,1} = 0.688 \cdot 13.5 = 9.29kN$

Beam 2 - lane 2: $V_{d,2,2} = 0.688 \cdot q_{2,2} = 0.688 \cdot 3.75 = 2.58kN$

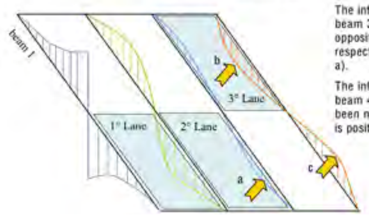
Beam 3 - lane 2: $V_{d,3,2} = 0.172 \cdot q_{3,2} = 0.172 \cdot 3.75 = 0.65kN$

Beam 3 - lane 3: $V_{d,3,3} = -0.172 \cdot q_{3,3} = -0.172 \cdot 3.75 = -0.65kN$

Beam 4 - lane 3: $V_{d,4,3} = +0.344 \cdot q_{4,3} = +0.344 \cdot 3.75 = 1.29kN$

Pay attention to the signs !
See next slide

Location of uniformly distributed loads



The distributed load on lane 3 is on the opposite side with respect to lane 1 and 2.

The influence line on beam 3 has for lane 3 opposite sign with respect to lane 2 (point a).

The influence line on beam 4 would have been negative in c but is positive in b

Distributed loads : total shear in mid-span

We add the contribution of the three lanes

$$V_{S,distributed} = V_{d,1,1} + V_{d,2,1} + V_{d,2,2} + V_{d,3,2} + V_{d,3,3} + V_{d,4,3} =$$
$$= 18.35 + 9.29 + 2.58 - 0.65 + 0.65 + 1.29 = 31.5kN$$

With Courbon model it was

$V_{S,concentrated} = 32kN$ 0% difference

TRANSVERSELY RIGID GIRDER BRIDGES

The transverse section may be open or closed but cannot change its shape due to the presence of a diaphragm system.

- $\frac{\partial w}{\partial x} = \theta(x)$ ROTATION
- $\frac{\partial^2 w}{\partial x^2} = \frac{\partial \theta}{\partial x} = \chi(x)$ CURVATURE
- $M(x) = -EJ\chi(x)$ BENDING MOM.
- $\frac{\partial M}{\partial x} = -EJ\frac{\partial^3 w}{\partial x^3} = V$ SHEAR

$\downarrow$

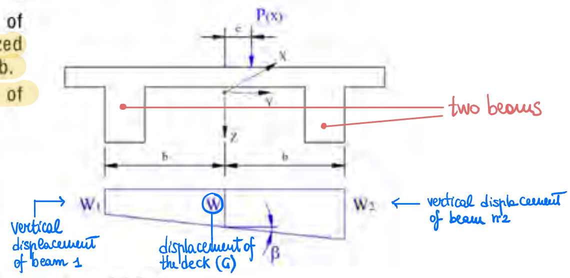
- $-EJ\frac{\partial^4 w}{\partial x^4} = \frac{\partial V}{\partial x} = q(x)$ LINEAR ELASTIC EQUATION

We may suppose, in sake of simplicity, that the section is realized with two beams connected by a slab.

Due to the shape indeformability of the section it results:

ricordando che

$$w_1 = w - b\beta$$
$$w_2 = w + b\beta$$
$$w_1 + w_2 = 2w$$



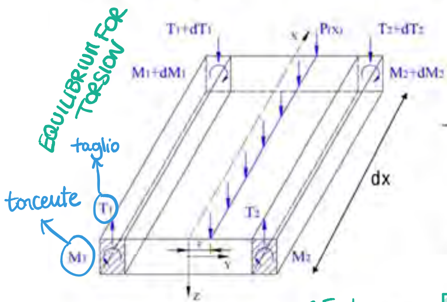
Furthermore:

$$EI\frac{\partial^4 w_1}{\partial x^4} = EI\left(\frac{\partial^4 w}{\partial x^4} - b\frac{\partial^4 \beta}{\partial x^4}\right) = -P_1$$
$$EI\frac{\partial^4 w_2}{\partial x^4} = EI\left(\frac{\partial^4 w}{\partial x^4} + b\frac{\partial^4 \beta}{\partial x^4}\right) = -P_2$$

Adding:

$$\boxed{2EI\frac{\partial^4 w}{\partial x^4} = -(P_1 + P_2) = -P} \quad \text{(b)}$$

Let's analyze in detail the equilibrium of an elementary beam segment, having length dx.



Equilibrium to the rotation around longitudinal axis X:

$$(T_1 - T_2)b + \left[-(T_1 - dT_1) + (T_2 - dT_2)\right]b +$$
$$-M_1 - M_2 + M_1 + dM_1 + M_2 + dM_2 + m\,dx = 0$$

X

m = p(x) e

Simplifying:

$$-dT_1 b + dT_2 b + dM_1 + dM_2 + m\,dx = 0$$

ci sarebbe pure il momento flettente attorno Y, ma poichè non dà contributo di torcente, lo ignoro.

$$\boxed{\left(-\frac{dT_1}{dx} + \frac{dT_2}{dx}\right)b + \frac{dM_1}{dx} + \frac{dM_2}{dx} + m = 0} \quad \text{(a)}$$

EQUILIBRIUM EQUATION

Remembering that:

$$T_1 = -EI\frac{\partial^3 w_1}{\partial x^3} = -EI\left(\frac{\partial^3 w}{\partial x^3} - b\frac{\partial^3 \beta}{\partial x^3}\right); \qquad T_2 = -EI\frac{\partial^3 w_2}{\partial x^3} = -EI\left(\frac{\partial^3 w}{\partial x^3} + b\frac{\partial^3 \beta}{\partial x^3}\right)$$

It follows:

$$\frac{dT_1}{dx} = -EI\left(\frac{\partial^4 w}{\partial x^4} - b\frac{\partial^4 \beta}{\partial x^4}\right); \qquad \frac{dT_2}{dx} = -EI\left(\frac{\partial^4 w}{\partial x^4} + b\frac{\partial^4 \beta}{\partial x^4}\right)$$

And because:

$$M_1 = GI_t\frac{\partial \beta}{\partial x} = M_2 \quad \Rightarrow \quad \frac{dM_1}{dx} + \frac{dM_2}{dx} = 2GJ_t\frac{\partial^2 \beta}{\partial x^2}$$

Variation of torsion beam 1 — Variation of torsion beam 2

EQUILIBRIUM EQUATION

By substitution in (a) it can be obtained:

$$-2EIb^2\frac{\partial^4 \beta}{\partial x^4} + 2GJ_t\frac{\partial^2 \beta}{\partial x^2} + m = 0$$

TORK ANGLE

TORK DUE TO LOADS

TORK STIFFNESS

The equation may be re-written by using the term $\overline{J}_t = 2J_t$:

it relate a torsion equilibrium to a bending stiffness. The torsional stiffness is helped by bending stiffness. For open section T profiles, the red number is big and the green number is small

$$\boxed{2EIb^2 \frac{\partial^4 \beta}{\partial x^4} - G\overline{J}_t \frac{\partial^2 \beta}{\partial x^2} = m(x)} \qquad (c)$$

4th order differential equation with a big contribution of the term in red (related to the 4th derivative of displacement) and a small contribution of the tork part

Depending by I_{tot} (2I) of the deck with respect to Y and by the distance of the beams from the deck axis (b).

Global torsional rigidity of the deck, evaluated in agreement to De Saint Venant.

May be generalized considering this term like a secondary torsional rigidity, function of the flexural contribution of beams.

With n beams: $I_{yz} = \sum_{i=1}^{n} I_i b_i^2$ and $2EIb^2 = EI_{yz}$

With beams with the same cross section: $I_{yz} = I \sum_{i=1}^{n} b_i^2$

By substitution in (b) and (c) we obtain:

$$\begin{cases} E\sum_{i=1}^{n} I_i \dfrac{\partial^4 w}{\partial x^4} = -p(x) \\ \underbrace{EI_{yz} \dfrac{\partial^4 \beta}{\partial x^4}}_{[a]} - \overbrace{G\overline{J}_t \dfrac{\partial^2 \beta}{\partial x^2}}^{[b]} = m(x) \end{cases}$$

primary torsional stiffness of the section

Limit cases: $\begin{cases} \overline{J}_t \cong 0 & \text{Solution without torsional contribution} \\ I_{yz} \cong 0 & \text{De Saint Venant solution} \end{cases}$

(⇒ ricordo in Courbon) → [b] = 0

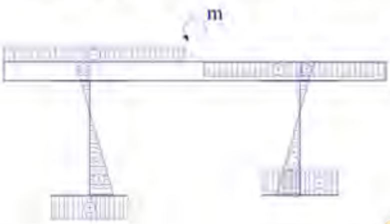
(no warping) → [a] = 0

Looking at the second equation we can appreciate that the acting torque moment m(x) do not corresponding to the resisting one $\left(GJ_t \frac{\partial^2 \beta}{\partial x^2}\right)$ but an additional flexural contribution has to be taken into account.

The differential equation of fourth order (linear, with constant coefficients) can be easily integrated.

- For a deck with 2 beams the torque moment m(x) may be substituted with two equal and opposite forces applied to the beams, which are then obliged to deflect in opposite directions (bi-moment). The secondary torsional behaviour may be considered as a flexural secondary one, as can be read in the following expression:

$$M^* = -EI \frac{\partial^2 (w_2 - w)}{\partial x^2} = -EI \frac{\partial^2 (w - w_1)}{\partial x^2} = -EI \frac{\partial^2 \beta}{\partial x^2} b$$



Longitudinal stresses σ_x diagram

By substitution in the fourth order differential equation we get (d)

$$(d) \qquad -\frac{\partial^2 M^*}{\partial x^2} + \frac{G}{E} \frac{2J_t}{2Ib^2} M^* = \frac{m}{2b}$$

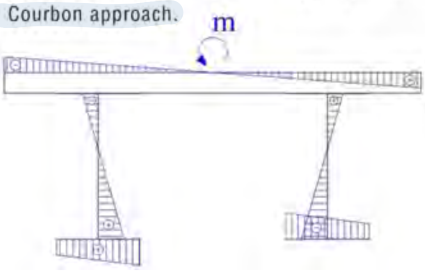
Remark 1: for $J_t \to \infty$ it results: $M^* \to 0$

In closed (box) sections the secondary torsional effect may be disregarded.

Remark 2: for $J_t \to 0$ it results: $\frac{\partial^2 M^*}{\partial x^2} = \frac{m}{2b}$

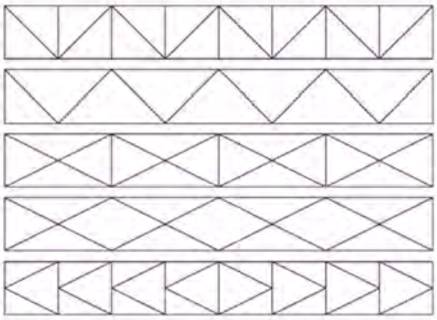
And by integration: $\frac{\partial M^*}{\partial x} = T^* = -\frac{1}{2b}\int m dx = -\frac{M}{2b}$

Because $q = \frac{pe}{2b}$ is the acting load applied on the two beams (supposed to be alike) one comes back, with $J_t = 0$ and transversely rigid section, at the Courbon approach.



As a matter of fact the correct solution of the problem implies some differences in the stress diagram, which play in practice a minor role.

- In case of a deck made of two beams if we close the bottom (i.e. by means of a truss beam), we may change its behaviour in a torsional one. In such a case the analysis should be performed like a box girder section having a fictitious thickness "s" of the bottom wall, which may be evaluated imposing the equality of internal actions and displacements between the actual and the fictitious wall.



Most usual geometry of bottom stiffening stresses.

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6. DESIGN OF REINFORCED CONCRETE SHELL ELEMENTS

The method we'll see now (SANDWICH MODEL) is a method to verify a shell at ULS (there is nothing (no methods) for SLS) 'cause it is based on the static theorem of plasticity. So the best element to use for our calculations is the beam (shell is complicated to use).

SANDWICH MODEL
- INTERNAL ACTIONS AND SIGN CONVENTIONS
- RESISTING MECHANISM DEFINITION
- INTERNAL LAYER DESIGN
- EXTERNAL LAYER DESIGN
  - X & Y reinforcement
  - α & β reinforcement

INTERNAL ACTIONS AND SIGN CONVENTION

Structural analysis of 2 dimension elements

(from the actions → to the internal actions)

Structural analysis of 2 dimension elements can be done in several way:

- closed form solutions (simple structures under simple loads conditions)
- tabular methods (simple structures under simple loads conditions)
- approximate methods (stripe methods)
- numerical methods (finite differences, finite elements)

Finite element methods is nowaday the most popular as it allows to analyse generic structures under generic load and restraint conditions.

The complete solution of the most generic 2D structure, a shell, gives as output:

shear inside a plane

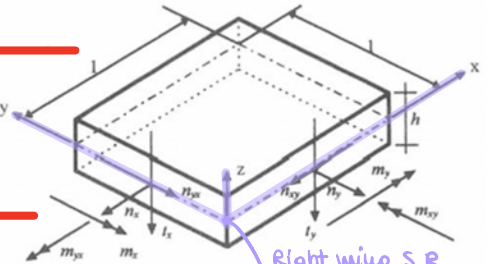
- **3 membrane components** n_x, n_y, $\overbrace{n_{xy} = n_{yx}}$
  EQUAL for CAUCHY HYPOTHESIS
- **5 plate components**
  - **2 bending moments and a torque moment** m_x, m_y, $m_{xy} = m_{yx}$
  - **2 out of the plane shears** t_x, t_y

Pay attention

All these internal actions are for unit lenght, for instance:

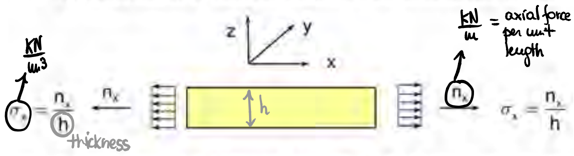
$n_x, n_y, n_{xy}, t_x, t_y \rightarrow$ [kN/m]

$m_x, m_y, m_{xy} \rightarrow$ [kNm/m]



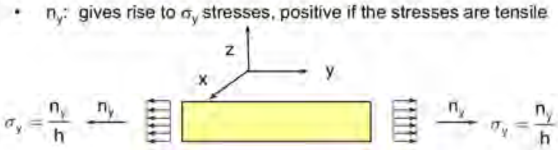
Sign convention for membrane internal actions:

- n_x: gives rise to σ_x stresses, positive if the stresses are tensile



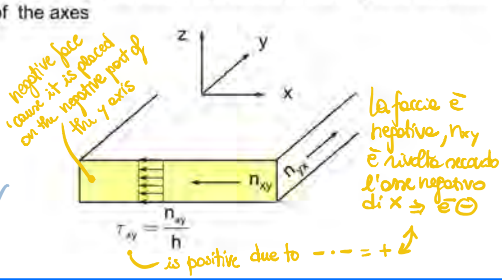
$$\sigma_x = \frac{n_x}{h}$$

- n_y: gives rise to σ_y stresses, positive if the stresses are tensile



$$\sigma_y = \frac{n_y}{h}$$

- n_{xy}: gives rise to τ_{xy} stresses, positive if:
  - on the positive face it is oriented in the positive direction of the axes
  - on the negative face it is oriented in the negative direction of the axes

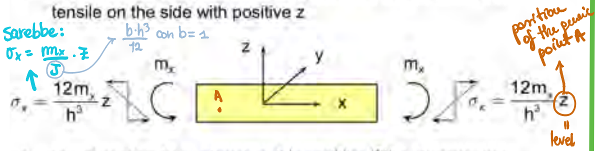


$$\tau_{xy} = \frac{n_{xy}}{h}$$

Sign convenction for plate internal actions

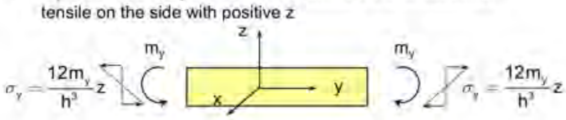
- m_x: gives rise to σ_x stresses, it is positive if the stresses are tensile on the side with positive z

Sarebbe: $\sigma_x = \frac{m_x}{J} \cdot z$, $J = \frac{b \cdot h^3}{12}$ con b = 1



$$\sigma_x = \frac{12 m_x}{h^3} z$$

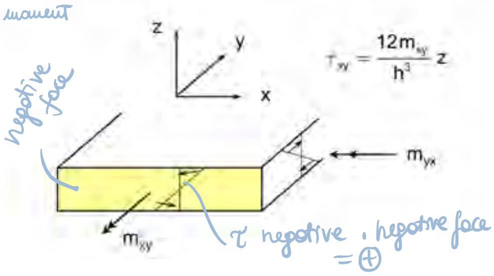
- m_y: gives rise to σ_y stresses, it is positive if the stresses are tensile on the side with positive z



$$\sigma_y = \frac{12 m_y}{h^3} z$$

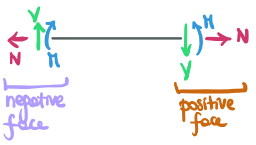
- m_{xy}: gives rise to τ_{xy} stresses, it is positive as shown in the figure

torque moment



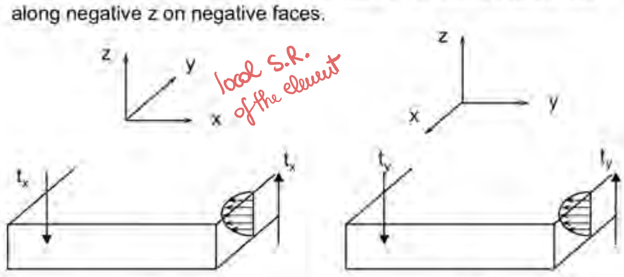
$$\tau_{xy} = \frac{12 m_{xy}}{h^3} z$$

Stessa cosa che avviene nelle travi:



Sign convention for out of the plane shears:

They are positive if directed along positive z on positive faces and along negative z on negative faces.

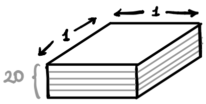


$$(\tau_{xz})_{max} = \frac{3}{2}\frac{t_x}{h} \qquad (\tau_{yz})_{max} = \frac{3}{2}\frac{t_y}{h}$$

t_x ↑ ⇒ ↑ z ⇒ ⊕

t_y ↓ ⇒ ↑ z ⇒ ⊖

RESISTING MECHANISM DEFINITION



high number of layer in order to get good accuracy (linked to the position of the neutral axis at ULS)

1° step: divide the thickness of the slab into several layer (i.e. 20) and check if concrete is cracked in each layer. That is to say calculate the principal tensile stress and check if it is bigger than the resisting tensile strength.

The stresses can be derived from the internal actions as follows.

$\rightarrow \frac{N_x}{A} + \frac{M_x}{J} z \rightarrow$ con $b=1$!

come out from JOURASWKY

1) $$\sigma_x = \frac{n_x}{h} + \frac{12 m_x}{h^3} z$$

2) $$\sigma_y = \frac{n_y}{h} + \frac{12 m_y}{h^3} z$$

3) 4) $$\underbrace{\tau_{xy} = \tau_{yx} = -\frac{n_{xy}}{h} + \frac{12 m_{xy}}{h^3} z}$$

Same structure of the 1) & 2) formula (pay attention to the sign)

5) $$\tau_{yz,\max} = \frac{3}{2}\frac{t_y}{h}$$

6) $$\tau_{xz,\max} = \frac{3}{2}\frac{t_x}{h}$$

for each layer we calculate these stresses in the elastic field imagining that our element is not cracked (elastic prediction of stresses).

We can arrange these $\underbrace{\text{6 components}}_{\hookrightarrow \text{for each layer}}$ of stress into a tensor. The tensor is a matrix 3x3

a uno dei termini σ_p

QUADRATIC MATRIX
SYMMETRIC MATRIX

Principal stresses are the eigenvalues of the stress tensor that is to say the solutions of the cubic equation that we get setting the determinant of the following matrix equal to zero.

principal stress

$$\begin{vmatrix} \sigma_x - \sigma_p & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y - \sigma_p & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_p \end{vmatrix} = 0$$

$\Rightarrow \det|A| = 0 \rightarrow$ trovo $\sigma_{p1}, \sigma_{p2}, \sigma_{p3}$

instead in a simple beam if I wanna check the crack state I do:
- $\sigma_{sollec} \leq \sigma_{trazione}$
oppure se ho compresenza di σ e τ calcolo la σ_{eq} e confronto:
- $\sigma_{eq,soll} \leq \sigma_{traz}$
[Scienze I]

in a shell → 6 components of stress

According to Model Code 90, $\boxed{\text{concrete is cracked if } \Phi>0}$, where:

$$\boxed{\Phi = \alpha \frac{J_2}{f_{cm}^{\,2}} + \lambda \frac{\sqrt{J_2}}{f_{cm}} + \beta \frac{I_1}{f_{cm}} - 1}$$

→ this formula tell us if the element is cracked or not and where ('cause I calculate Φ ∀ layer)

Where →

$I_1 = \sigma_1 + \sigma_2 + \sigma_3$ First invariant of stresses

$J_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$

$J_3 = (\sigma_1 - \sigma_m)(\sigma_2 - \sigma_m)(\sigma_3 - \sigma_m)$

Second and third invariant of deviatoric stresses, where:

$\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3)/3$

average stress

$$\alpha = \frac{1}{9k^{1.4}} \qquad \beta = \frac{1}{3.7k^{1.1}}$$

$$c_1 = \frac{1}{0.7k^{0.9}} \qquad c_2 = 1 - 6.8(k - 0.07)^2$$

$$\lambda = c_1 \cos[1/3 \arccos(c_2 \cos 3\theta)] \quad \text{for } \cos 3\theta \geq 0$$

$$\lambda = c_1 \cos[\pi/3 - 1/3 \arccos(-c_2 \cos 3\theta)] \quad \text{for } \cos 3\theta < 0$$

Where k is the ratio between tensile and compressive strength ≅ 0.1 of concrete

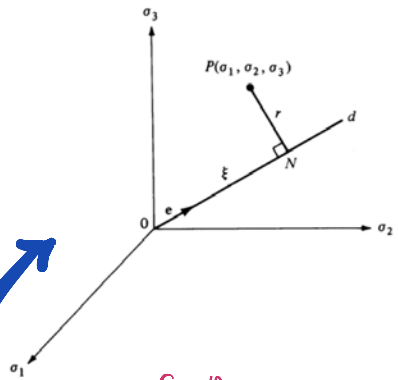
$$k = f_{ctm}/f_{cm}$$

Where θ is the lode angle on the octahedral plane:

$$\cos 3\theta = \frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}$$

We've just seen the matematical expression, but what is the physical meaning of this shit? Let's consider a 3D space where we have 3 orthogonal axis in which we plot the principal stresses $(\sigma_1, \sigma_2, \sigma_3)$

Principal stresses referring system
↳ $(\sigma_1, \sigma_2, \sigma_3)$



Why are useful principal stresses? Remember our problem is based on 6 components of stresses:

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yx} \\ \tau_{yz,max} \\ \tau_{xz,max} \end{Bmatrix} \Rightarrow \mathbb{R}^6$$

(non possiamo rappresentarli perché gli umani sono capaci di immaginare uno spazio 3D)

While principal stresses components are:

$$\{\sigma_P\} = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} \Rightarrow \mathbb{R}^3$$

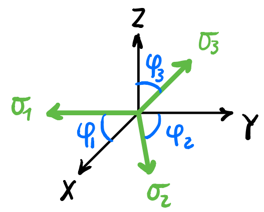
So the principal stresses are the stuff to translate an $\mathbb{R}^6$ vector to an $\mathbb{R}^3$ vector

Quindi: calcolo $\{\sigma\}$ → calcolo da queste $\{\sigma_P\}$

$\mathbb{R}^6$ $\xrightarrow{\text{passo a}}$ $\mathbb{R}^3$

domain: all 6 components are stresses

Codomain: 3 components are stresses, 3 components are angle



I love this 'cause I can plot the stresses in a 3D space ← $\{\sigma_P\} = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix}$, $\{\varphi\} = \begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{Bmatrix}$

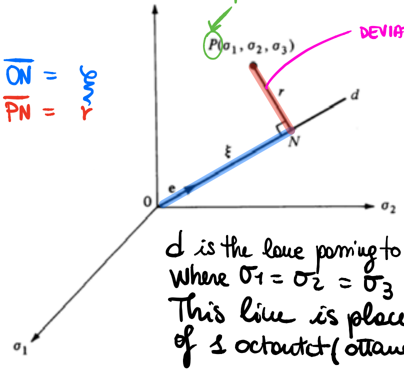
Let's consider a point P

it represent the stress in my structure → the position of this point can be represented in many different ways:

- cartesian coordinates: $P(\sigma_1, \sigma_2, \sigma_3)$
- using another S.R. based on line (d) ← HYDROSTATIC LINE

$\overline{ON} = \xi$

$\overline{PN} = r$



defining first a point N which is the point a line ⊥ to d passing by P itself cross the line d (∃ only 1 single line (r)). In this way we have defined a plane ⊥ to d and passing by P (c'è 1 e 1 solo piano ⊥ ad 1 retta che passa per un punto ∉ alla retta)

d is the line passing to the origin and is the line where $\sigma_1 = \sigma_2 = \sigma_3$ (hydrostatic state of stress). This line is placed exactly in the middle of 1 octant (ottano)

the planes ⊥ to d are called all DEVIATORIC PLANES: the definition of a deviatoric plane is a plane that in the principal stress system is ⊥ to the hydrostatic line d.

To define the point P in a 3D space we need 3 coordinates (like in the cartesian S.R. we had $(\sigma_1, \sigma_2, \sigma_3)$). In this new S.R. using d we have found till now only two coordinates: d & r. We miss one. To identify the third coordinate we change our point of view and we look at this picture

Why we do this? let's go back to this formula:

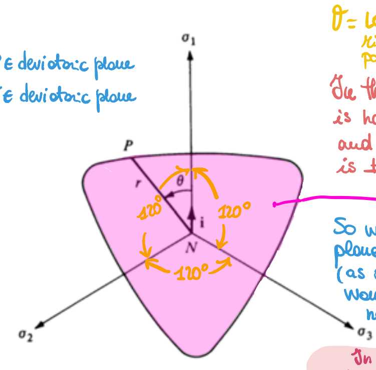
$$\boxed{J_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}$$

SECOND INVARIANT OF DEVIATORIC STRESS

this is a part of the Von Misses failure criteria for ductile materials (yelding criteria for metals). The Von Misses yielding criteria says that the yielding of a metal is reached when J_2 was close to a limit value: the yielding stress. According to Von Misses theory: failure of a material is reached when the loss of shape of that material is big enough (failure is not related to volume variation but in shape variation).

Octahedral plane and lode angle

P ∈ deviatoric plane
θ ∈ deviatoric plane



θ = LODE ANGLE preso per convenzione rispetto all'asse σ_1 individua la posizione di P

In this picture the d line is hortogonal to the paper, and the deviatoric plane is ⊥ to d.

So we are on the deviatoric plane, in the hydrostatic line (as shown in figure). If I wanna reach the P point I need two values: r, and θ (POLAR COORDINATES);

In such a way we have 3 coordinates: ξ, r, θ which are defining the position of P in the space 3D.

So we change the R.S. from a cartesian one to a polar one

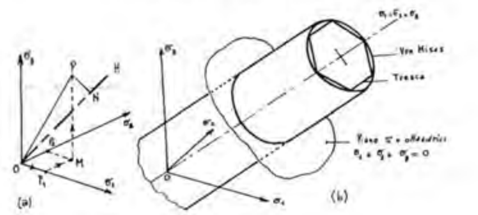
Moving along the hydrostatic line: σ in all direction are the same (uniform hydrostatic compression of the object) and it will never break 'cause is subjected to hydrostatic stress (never changes the shape but only the volume ⇒ Von Misses says → NO FAILURE: esempio sfere di metallo che immergiamo in acqua a profondità sempre maggiori → si riduce il volume man mano che si scende, ma la forma rimane sempre la stessa ⇒ NO ROTTURA).

When we start to move from the hydrostatic line on the deviatoric plane, we start to feel pain 'cause $\sigma_1 \neq \sigma_2 \neq \sigma_3$ ⇒ change in shape! ⇒ deformed shape (loss of shape). The radius r measure the loss of shape (a sort of pain measure): più una struttura si allontana dalla condizione idrostatica e più quel materiale è stressato per perdita di forma. Man mano che mi allontano da d (retta idrostatica), e quindi ↑ r, rischio di rompere il mio materiale. Il massimo valore di r produce un dominio di rottura. Esistono diverse formulazioni per individuare il criterio di rottura:

DOMANDA: Se mi sposto in una direzione o in un'altra cambia la sofferenza del materiale? DIPENDE → vediamo alcuni criteri di rottura per rispondere

- TRESCA FAILURE CRITERIA
- VON MISES FAILURE CRITERIA
- MOHR FAILURE CRITERION (friction criteria)
- (OTTOSEN)/KOTSOVOS/PAV_LOVIC (for concrete) FAILURE CRITERIA

Tresca and Von Mises failure criteria



Tresca

$$\sigma_3 - \sigma_2 = \pm\sigma_{cu} \quad \sigma_2 - \sigma_1 = \pm\sigma_{cu} \quad \sigma_1 - \sigma_3 = \pm\sigma_{cu}$$

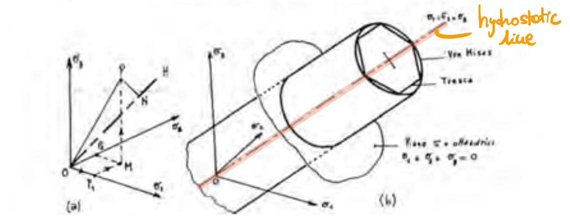
$$F(J_2', J_3') = 4J_2'^3 - 27J_3'^2 - 9\sigma_y^2 J_2'^2 + 6\sigma_y^4 J_2'^2 - \sigma_y^6 = 0$$

Von Mises

$$f(\sigma_{cu}) = \tfrac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] - \sigma_{cu} = 0$$

$$f(\sigma_{cu}) = \sqrt{6J_2} - \sigma_{cu} = 0$$

Tresca and Von Mises failure criteria



Tresca

$$\sigma_3-\sigma_2=\pm\sigma_{cu}\quad \sigma_2-\sigma_1=\pm\sigma_{cu}\quad \sigma_1-\sigma_3=\pm\sigma_{cu}$$

$$F(J_2',J_3')=4J_2'^2-27J_3'^2-9\sigma_y^2J_2'^2+6\sigma_y^4J_2'^2-\sigma_y^6=0$$

Von Mises

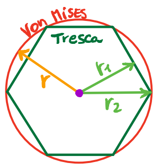
$$f(\sigma_{cu})=\tfrac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]-\sigma_{cu}=0$$

yielding stress

$$f(\sigma_{cu})=\sqrt{6J_2}-\sigma_{cu}=0$$

- Von Mises criterie (metals) → cylindric surface, the radius r is the yielding stress. No matter the direction we walk → same failure on the deviatoric plane (r è costante lungo tutta la retta idrostatica)

- Tresca criterie (special soils) → dice che si ha la rottura quando la differenza tra le tensioni principali raggiunge un certo limite ($\sigma_3-\sigma_2$; $\sigma_2-\sigma_1$; $\sigma_1-\sigma_3$)
  ↳ se risolvo questo criterio in funzione di J_2, J_3 ottengo un esagono centrato sulla retta idrostatica (inscritto nel cerchio di Von Mises). According to Tresca theory the direction where we walk on the deviatoric plane is important ('cause $r_1 < r_2$ ⇒ I reach failure before !!) ⇒ the properties of the material are not perfectly isotropic.

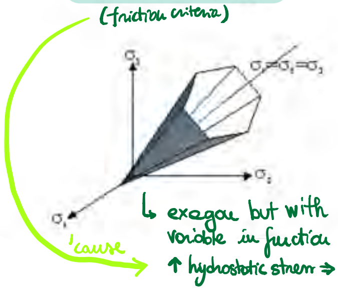


Mohr failure criterion (soil)

(friction criteria)

$$f(J_1,J_2,\theta)=\frac{J_1}{3}sen\varphi+\sqrt{J'_2}\,sen\left(\theta+\frac{\pi}{3}\right)+\frac{\sqrt{J'_2}}{\sqrt{3}}cos\left(\theta+\frac{\pi}{3}\right)sen\varphi-C\cos\varphi=0$$

$$f(\xi,r,\theta)=\sqrt{2}\xi sen\varphi+\sqrt{3}r\cdot sen\left(\theta+\frac{\pi}{3}\right)+r\cdot cos\left(\theta+\frac{\pi}{3}\right)sen\varphi-\sqrt{6}\cdot C\cos\varphi=0$$

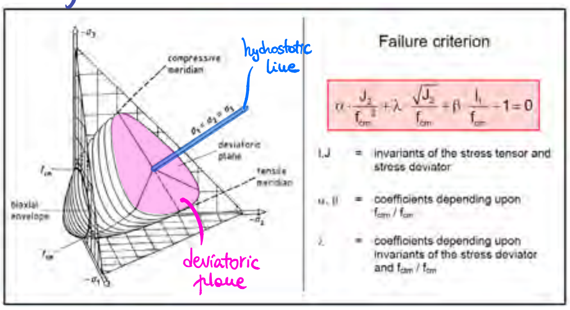


↳ exagon but with the dimension variable in function of the hydrostatic stress
↑ hydrostatic stress ⇒ ↑ dimension of the exagon ⇒ ↑ resistance of the material

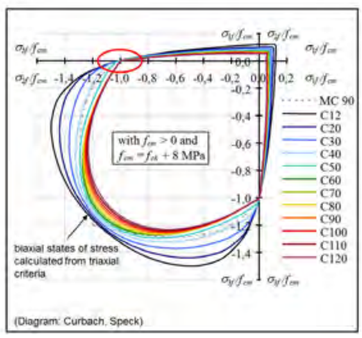
We have seen the soil and steel cases. But what about concrete? What is its behaviour? Concrete is a brittle material (no ductile as the steel): so it has not the same resistance in tensile and in compression, the domain of failure it will not be a cyrcle (but it will depends on θ = load angle), so it will not be like Von Mises. Moreover if I add confinement (compression), the concrete becomes more resistance (like the Mohr failure criterion).

RESISTENZA IN PRESENZA DI STATI DI TENSIONE MULTIASSIALI

formulazione di OTTOSEN

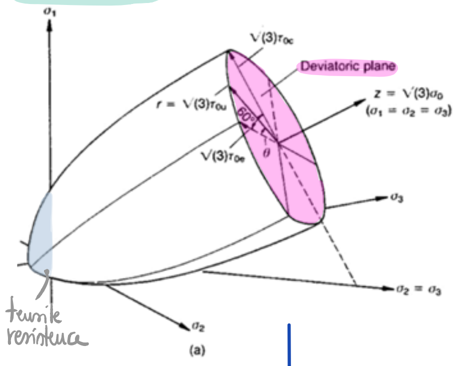


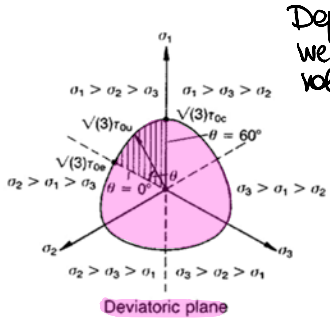
$$\alpha\cdot\frac{J_2}{f_{cm}^2}+\lambda\cdot\frac{\sqrt{J_2}}{f_{cm}}+\beta\cdot\frac{I_1}{f_{cm}}-1=0$$



Kotsovos, Pavlovic failure criteria

⊕ axis → compression
⊖ axis → tension

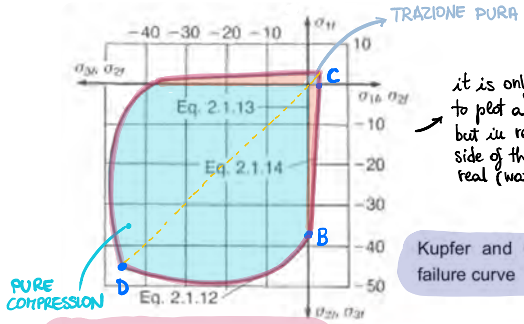




Depending on the lode angle θ we reach failure at different values (direction is important)

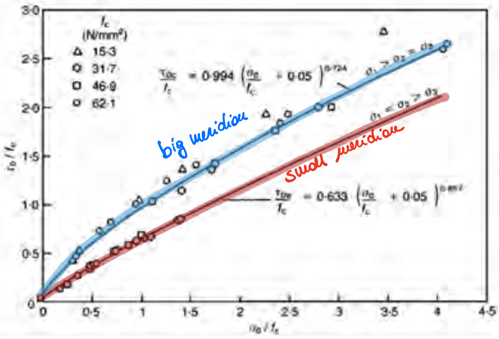
$$r=\sqrt{3}\tau_{0u}=\sqrt{3}\,\frac{2\tau_{0c}\left(\tau_{0c}^2-\tau_{0e}^2\right)\cos\theta+\tau_{0c}\left(2\tau_{0e}-\tau_{0c}\right)\sqrt{4\left(\tau_{0c}^2-\tau_{0e}^2\right)\cos^2\theta+5\tau_{0e}^2-4\tau_{0c}\tau_{0e}}}{4\left(\tau_{0c}^2-\tau_{0e}^2\right)\cos^2\theta+\left(\tau_{0c}-2\tau_{0e}\right)^2}$$

If we cut the surface with a plane (σ_1,σ_2) we get this:



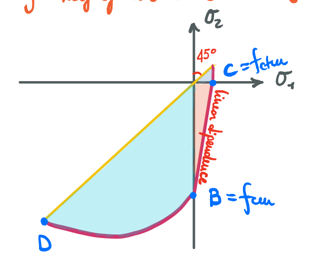
it is only a way to plot a close curve, but in reality only one side of this curve is real (watch down)

Kupfer and Gerstle failure curve



N.B. According to the definition of principal stresses: $\sigma_1>\sigma_2$ always! 'Cause σ_1 is the most positive and σ_2 is the most negative.

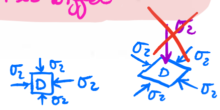
only 1 half of this curve is real:



B = uniaxial cylindric compressive strength of CLS → fcm
(B = monoaxial compression) no tension

C = tensile strength of CLS
↳ fctm

D = compression in all direction
(no hydrostatic situation 'cause there is nothing (σ_3) out of the plane ⇒ it is a uniform compression in a plane)



If concrete is not cracked (Φ≤0) the principal stresses are inside the resistance dominion, which can be evaluated, according to Model Code 90, with the following relations.

Biaxial compression and tension-compression for $\sigma_{3f}<-0.96f_{cm}$

$$\sigma_{3f}=-\frac{1+3.80\alpha}{(1+\alpha)^2}f_{cm}\qquad(2.1\text{–}12)$$

where $\alpha=\sigma_{2f}/\sigma_{3f}$.

Biaxial tension

$$\sigma_{1f}=f_{ctm}=\text{const.}\qquad(2.1\text{–}13)$$

Biaxial tension-compression for $\sigma_{3f}>-0.96f_{cm}$

in BIAXIAL COMPRESSION the resistance of concrete is bigger than in UNIAXIAL COMPR.

$$\sigma_{1f}=\left(1+0.8\frac{\sigma_{3f}}{f_{cm}}\right)f_{ctm}\qquad(2.1\text{–}14)$$

thanks to confinement the resistance of concrete is not fcm but is bigger

where

σ_{1f} is the largest principal stress at failure
σ_{2f} is the intermediate principal stress at failure
σ_{3f} is the smallest principal stress at failure.

If $\Phi>0$ we have to work in cracked state using a sandwich model.

The basic idea derives from the analogy with the beam element.

Three different layers are individuated inside the shell element.

this division has nothing in common with the previous one division (20 layers) (this is a second step). The value of the thicknesses t_s, t_c, t_i is up to me (I chose them with an iterative calculation). VARIABLE PARAMETERS

The two external ones bear membrane actions coming both from membrane and the plate external actions.

The inner one, working as a beam web bears the out of the plane shears.

Each layer has a constant thickness and the following quantities are introduced:

N.B.

Usually these second quantities are set equal to the respective ones presented in the previous slide.

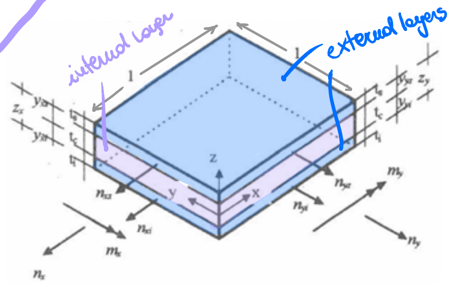
They can be set to different values in order to place different layers of reinforcement for the different actions in the same concrete layer.

scomposte in componente layer superiore e inferiore (n_{xs}, n_{xi})

s = superior, i = inferior, c = central

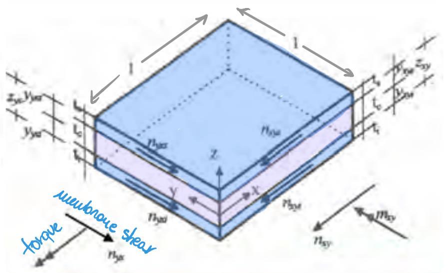
- z_x, z_y internal lever arms for m_x, n_x and m_y, n_y
- y_{xs}, y_{xi} distances of the reinforcement that bears m_x and n_x from the centroid of the shell ($z_x = y_{xs} + y_{xi}$)
- y_{ys}, y_{yi} distances of the reinforcement that bears m_x and n_x from the centroid of the shell ($z_y = y_{ys} + y_{yi}$)

AXIAL FORCE & BENDING

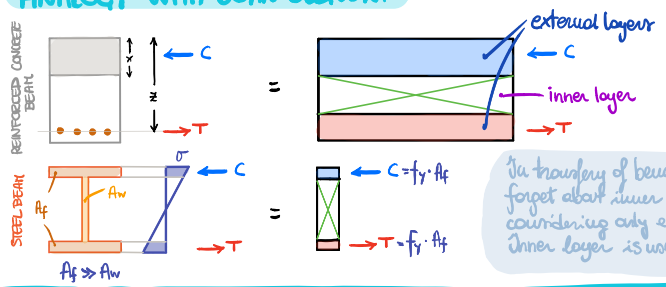


TORQUE & SHEAR

- z_{xy}, z_{yx} internal lever arms for m_{xy} and n_{xy}
- y_{xys}, y_{xyi} distances of the reinforcement that bears m_{xy} and n_{xy} from the centroid of the shell ($z_{xy} = y_{xys} + y_{xyi}$)
- y_{yxs}, y_{yxi} distances of the reinforcement that bears m_{xy} and n_{xy} from the centroid of the shell ($z_{yx} = y_{yxs} + y_{yxi}$)



ANALOGY WITH BEAM ELEMENT



In transfering of bending we forget about inner layer considering only external layers. Inner layer is used to transfer shear. (torsion)

N.B. Actions that give rise to normal stresses are assigned to the external layers (N, M), while actions that give rise to tangential stresses are assigned to the internal layer

The membrane actions that interest the external layer can then be calculated by writing equilibrium equations as follows

level arm; dist (G_{layer}; $G_{element}$) → se aumento $\uparrow t_s \Rightarrow y_{xs}\downarrow \Rightarrow n_{xs}\uparrow$

$$n_{xs} = n_x \frac{z_x - y_{xs}}{z_x} + \frac{m_x}{z_x} \qquad n_{yxs} = n_{yx} \frac{z_{yx} - y_{yxs}}{z_{yx}} - \frac{m_{yx}}{z_{yx}}$$

$$n_{xi} = n_x \frac{z_x - y_{xi}}{z_x} - \frac{m_x}{z_x} \qquad n_{yxi} = n_{yx} \frac{z_{yx} - y_{yxi}}{z_{yx}} + \frac{m_{yx}}{z_{yx}}$$

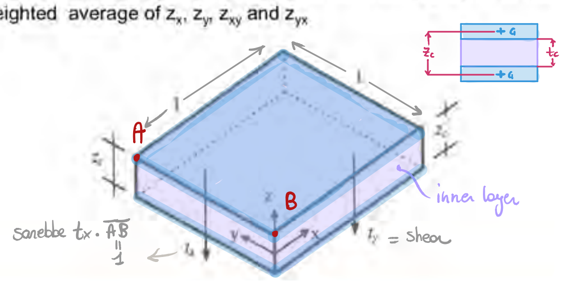
$$n_{ys} = n_y \frac{z_y - y_{ys}}{z_y} + \frac{m_y}{z_y} \qquad \underset{n_{yxs}}{\overset{}{n_{xys}}} = n_{xy} \frac{z_{xy} - y_{xys}}{z_{xy}} - \frac{m_{xy}}{z_{xy}}$$

$$n_{yi} = n_y \frac{z_y - y_{yi}}{z_y} - \frac{m_y}{z_y} \qquad \underset{n_{yxi}}{\overset{}{n_{xyi}}} = n_{xy} \frac{z_{xy} - y_{xyi}}{z_{xy}} + \frac{m_{xy}}{z_{xy}}$$

($n_{yxs} = n_{xys}$, $n_{yxi} = n_{xyi}$)

Out of the plane shears are carried by the internal layer of thickness t_c, that works in collaboration with the external ones.

Therefore we will consider an effective thickness z_c equal to the weighted average of z_x, z_y, z_{xy} and z_{yx}

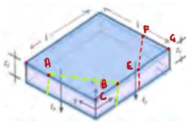


INTERNAL LAYER DESIGN

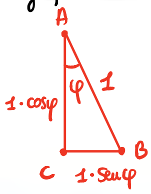
We start dimensioning the shell element from the internal layer and then we move to the external ones.

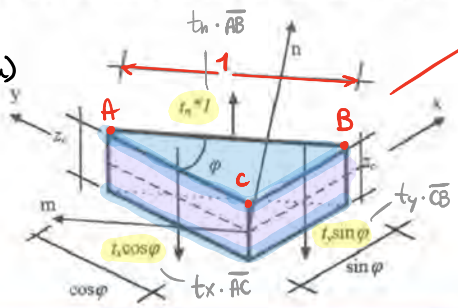
Calculation of the principal out of the plane shear direction

If we analyze the equilibrium to the vertical translation (z direction) of a prismatic triangular element obtained by sectioning the internal layer with a plane parallel to the z axis and orthogonal to a general n axis belonging to the [xy] plane, and inclined of the angle φ whit respect to the x axis, we get:



(↑) $t_n = t_x \cos\varphi + t_y \sin\varphi$
(equation of equilibrium in z direction)



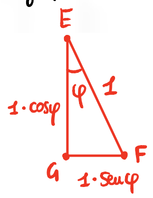


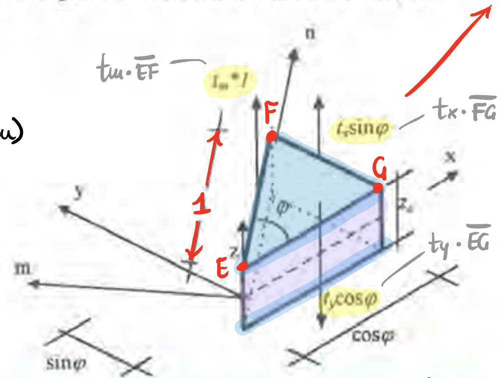
we imagine to do a cut to the element in order to have a lenght of the cut (AB) equal to 1.

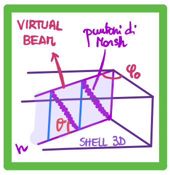
If now we analyze the equilibrium to the vertical translation (z direction) of another prismatic triangular element obtained by sectioning the internal layer with a plane parallel to the z axis and orthogonal to a general m axis belonging to the [xy] plane, and orthogonal to the n axis defined before, we get:

now we make another cut on the element ($\overline{EF} = 1$)

(↑) $t_m = -t_x \sin\varphi + t_y \cos\varphi$
(equation of equilibrium in z direction)







Perchè faccio tutto questo casino? Questo meccanismo serve per andare ad applicare ciò che più sappiamo del taglio da C.A. per le travi ad un elemento (2D) shell che una trave non è. Quindi in sostanza trovo la direzione del taglio principale n, e creo delle travi virtuali (tante travi //) e creo l'elemento come se fossero travi

It can be observed that the value of the principal shear

$t_0^2 = t_x^2 + t_y^2 = t_n^2 + t_m^2$ → invariant whatever φ I chose!

Is constant with respect to the angle φ.

Moreover there's a angle φ_0 that makes $t_n = t_0$ and $t_m = 0$.

This is the principal direction of shear and can be calculated as:

$\tan\varphi_0 = t_y / t_x$ → direction of principal shear

In this direction the shell element behaves like a beam as it is subjected to shear in only one direction. ⇒ I dimension my element like I do with a beam.

n = direzione del taglio principale

come fatto in cemento armato

if $t_0 > V_{Rd1}$ shear reinforcement should be provided. The resisting mechanism is equal to a beam, a unit length wide, and oriented along the principal shear direction. According to Model Code 90 we have:

- verification of compressed struts.

$$F_{Scw} = \frac{t_0}{\sin\vartheta} \le F_{Rcw} = f_{cd2} z_c \cos\vartheta$$

θ non è più l'angolo tra asse trave e biella compressa (puntone come C.A.) perchè adesso abbiamo un elemento 2D allora è l'inclinazione tra l'asse principale di taglio n e z (asse fuori-piano della shell)

- verification of tensed reinforcement (vertical stirrups)

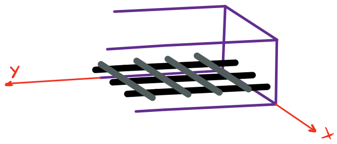
$$F_{Stw} = t_0 \le F_{Rtw} = \frac{A_{sw} f_{ywd}}{s} z_c \cot\vartheta$$

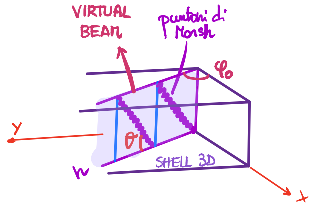
- force variation in longitudinal chords (tensed and compressed)

$$\Delta F_{st} = \Delta F_{sc} = \frac{t_0}{2} \cot\vartheta$$ (traslazione del momento flettente)

The angle θ is subjected to the same limitations seen for beams (26°<θ<45°). Whereas the forces $\Delta F_{st} = \Delta F_{sc}$ are directed in the direction φ_0 and should be decomposed along x and y.

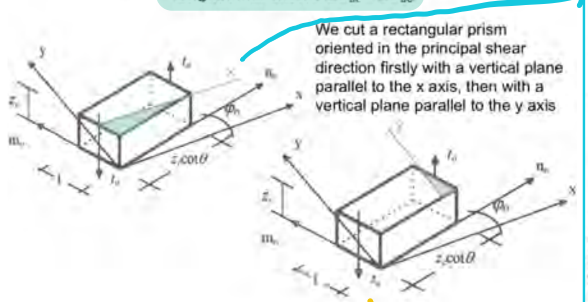
Traslazione del diagramma di momento flettente: la presenza del taglio porta ad una aggiunta di armatura longitudinale. Con quanto abbiamo visto l'aggiunta che otteniamo è in direzione n. Ma nello shell non dispongo armatura longitudinale in direzione n ma al 55.5% la dispongo // x e // y (vedi figura). Quindi devo capire adesso quanta armatura è necessario aggiungere in direzione x e quanta in direzione y.

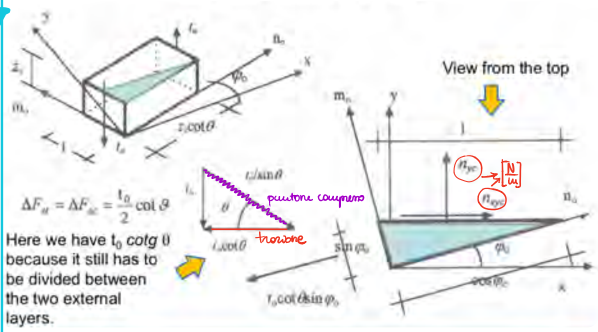




Components in x and y directions of force variation in longitudinal chords $\Delta F_{st} = \Delta F_{sc}$

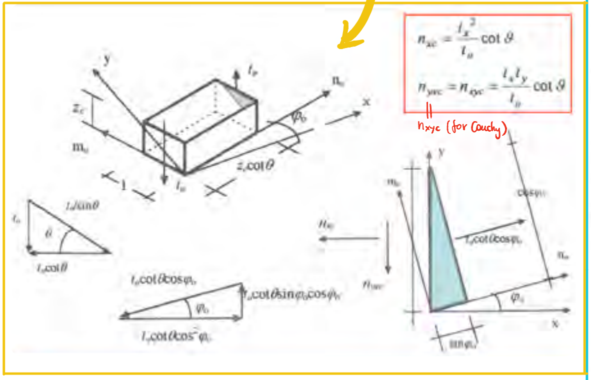
We cut a rectangular prism oriented in the principal shear direction firstly with a vertical plane parallel to the x axis, then with a vertical plane parallel to the y axis





$$\Delta F_{st} = \Delta F_{sc} = \frac{t_0}{2}\cot\vartheta$$

Here we have t_0 *cotg* θ because it still has to be divided between the two external layers.



$$n_{xc} = \frac{t_x^2}{t_0}\cot\vartheta$$

$$n_{yxc} = n_{xyc} = \frac{t_x t_y}{t_0}\cot\vartheta$$

n_{xyc} (for Cauchy)

$$n_{yc} = t_0 \cot\vartheta \sin^2\varphi_0 = t_0 \cot\vartheta \frac{tg^2\varphi_0}{1+tg^2\varphi_0} =$$

$$= t_0 \cot\vartheta \frac{t_y^2/t_x^2}{1+t_y^2/t_x^2} = t_0 \cot\vartheta \frac{t_y^2}{t_x^2+t_y^2}$$

$$n_{xyc} = t_0 \cot\vartheta \sin\varphi_0 \cos\varphi_0 = t_0 \cot\vartheta \frac{tg\varphi_0}{1+tg^2\varphi_0} =$$

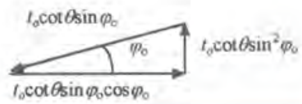
$$= t_0 \cot\vartheta \frac{t_y/t_x}{1+t_y^2/t_x^2} = t_0 \cot\vartheta \frac{t_x t_y}{t_x^2+t_y^2}$$

$$n_{yc} = \frac{t_y^2}{t_0}\cot\vartheta$$

n_{yxc} (for Cauchy) =

$$n_{xyc} = \frac{t_x t_y}{t_0}\cot\vartheta$$

membrane shear



n_{xc}, n_{yc}, n_{xyc} are the decomposition of the $\Delta F_{st} = \Delta F_{sc}$ and have to be added to the external actions n_x, n_y, n_{xy} and then divided between the two external layers.

EXTERNAL LAYERS DESIGN

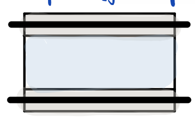
For seek of simplicity we refer to the mean level of reinforcement in each of the two layers:

$y_{xs}=y_{ys}=y_{xys}=y_{yxs}=y_s$

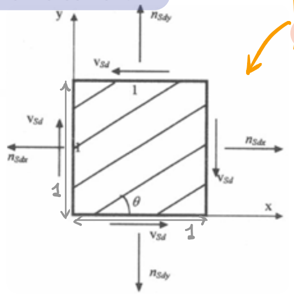
$y_{xi}=y_{yi}=y_{xyi}=y_{yxi}=y_i$

$z=y_s+y_i$

assumption (for simplicity):



reinforcement placed in the middle of each layer



Each layer is subjected to membrane actions (see figure).

The design can be done according to plasticity theory with a lower bound solution.

The values of the three internal actions shown beside for each layer is given by the equations presented in the next slide.

no stirraps ←

No shear reinforcement needed

$$n_{Sdx,s} = n_x \frac{z - y_s}{z} + \frac{m_x}{z}$$

$$n_{Sdx,i} = n_x \frac{z - y_i}{z} - \frac{m_x}{z}$$

$$n_{Sdy,s} = n_y \frac{z - y_s}{z} + \frac{m_y}{z}$$

$$n_{Sdy,i} = n_y \frac{z - y_i}{z} - \frac{m_y}{z}$$

$$v_{Sd,s} = n_{xy} \frac{z - y_s}{z} - \frac{m_{xy}}{z}$$

$$v_{Sd,i} = n_{xy} \frac{z - y_i}{z} + \frac{m_{xy}}{z}$$

Shear reinforcement needed

$$n_{Sdx,s} = n_x \frac{z - y_s}{z} + \frac{m_x}{z} + \frac{1}{2}\frac{t_x^2}{t_o}\cot\vartheta$$

$$n_{Sdx,i} = n_x \frac{z - y_i}{z} - \frac{m_x}{z} + \frac{1}{2}\frac{t_x^2}{t_o}\cot\vartheta$$

$$n_{Sdy,s} = n_y \frac{z - y_s}{z} + \frac{m_y}{z} + \frac{1}{2}\frac{t_y^2}{t_o}\cot\vartheta$$

$$n_{Sdy,i} = n_y \frac{z - y_i}{z} - \frac{m_y}{z} + \frac{1}{2}\frac{t_y^2}{t_o}\cot\vartheta$$

$$v_{Sd,s} = n_{xy} \frac{z - y_s}{z} - \frac{m_{xy}}{z} + \frac{1}{2}\frac{t_x t_y}{t_o}\cot\vartheta$$

$$v_{Sd,i} = n_{xy} \frac{z - y_i}{z} + \frac{m_{xy}}{z} + \frac{1}{2}\frac{t_x t_y}{t_o}\cot\vartheta$$

addition due to the traslation of the bending moment (or in the beams) due to the presence of the shear force

Two different approaches.

regular mesh ← **1**

Reinforcement is arranged along the two orthogonal directions x and y

local axes of the element

2 → irregular mesh

Reinforcement is arranged along two general directions respectively inclined of the angles α with the x axis and β with the y axis

In both cases the equilibrium in the directions x and y is granted by a compressive field inclined of an angle θ with respect to the x axis and by two tension fields directed along the directions of reinforcement.

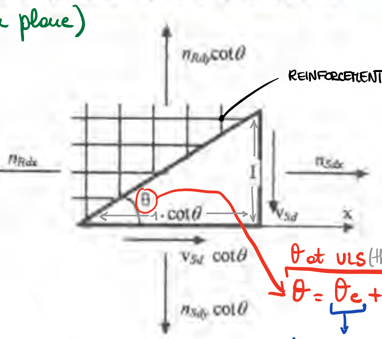
CASE 1: EXTERNAL LAYERS DESIGN – x & y REINFORCEMENT

We cut the layer with a plane parallel to the direction of compressions at failure. ($\theta \rightarrow$ in plane)

How do we chose θ? It's our choice, but we have some limits taken from the principal direction of compression

This direction can be chosen by the designer starting from the value of the inclination of principal stress σ_2 at cracking and changing it of $-20° < \Delta\theta < 20°$.

Remember that a big $\Delta\theta$ leads to wider cracks in service.

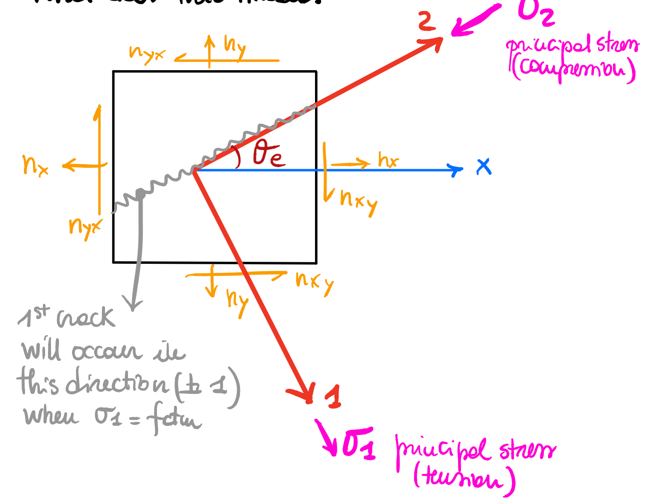


θ at ULS (the one we use)

$\theta = \theta_c + \Delta\theta$

elastic coming from the solution on FEM (elastic analysis)

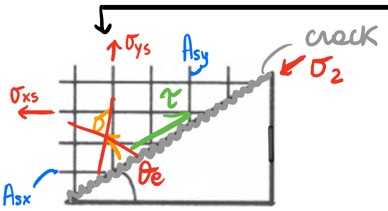
What does this mean?



We have a finite element subjected to membrane forces. Starting from this set of action we can calculate the position of axis (1,2) (principal axes) thanks to θ_e → angle between (2, x)

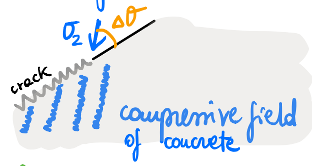
θ_e: indication of the compression fields respect x axis (elastic prediction)

After 1st crack occurs, if we increase the load keeping n_x, n_y, n_{xy} in the same proportion, what happens? The elastic stresses σ_1, σ_2 will increase proportionally with θ_e = cost ('cause we're in elastic field). But we are in reinforced concrete



- Typically $A_{sx} \neq A_{sy} \Rightarrow \sigma_{yx} \neq \sigma_{ys}$
- τ are due to: interlock, friction, dowel effect
- σ tensile resistance of concrete (post cracking) → small → neglected (by safe side)

What happens? At 1st cracking σ_2 is perfectly alligned to the crack. After the 1st crack takes place, σ_2 can be disalligned with the crack



{ inclinazione dei campi di compressione come C.A. dovuto alle τ }

Typically this is what happens:

1) 1st crack occours → load increases → reinforcement in elastic field ⇒ $\Delta\theta \cong 0$ (molto piccolo)

2) When one of the two reinforcements yields, the stiffness in one of the two direction (x or y) become very low because of the yielding of the rebars while in the other direction high stiffness ⇒ ≠ stiffness ⇒ suddently very big τ appears 'cause they have to counterbalance the loss of stiffness of one set of the rebars (for equilibrium) ⇒ $\Delta\theta \uparrow$

3) Yelding of the other set of rebars (at different loads and time)

What is the maximum variation that we can admit? → laboratory test: → BEAMS $\theta_{max} = 45°$, $\theta_{min} \cong 22°$ ⇒ $\Delta\theta \approx 23°$

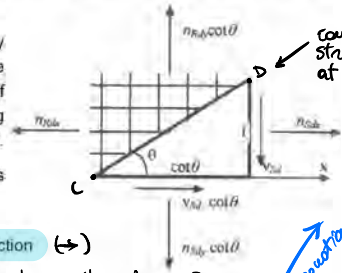
→ PANNEL $\Delta\theta_{max} = 20°$

Riprendiamo adesso da dove eravamo rimasti.

We cut the layer with a plane parallel to the direction of compressions at failure.

This direction can be chosen by the designer starting from the value of the inclination of principal stress σ_2 at cracking and changing it of $-20° < \Delta\theta < 20°$.

Remember that a big $\Delta\theta$ leads to wider cracks in service.



compression stress direction at failure

AB ⊥ CD !

Equilibrium to traslation in x direction (→)

$$n_{Rdx} = n_{Sdx} + V_{sd}\cot\theta = A_{sx}\sigma_{sx}$$ – stress inside reinforcement

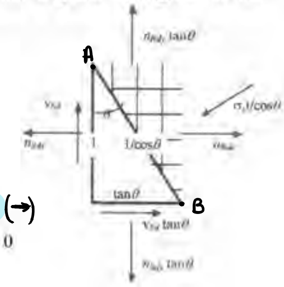
Equilibrium to traslation in y direction (↑)

$$n_{Rdy}\cot\theta = n_{Sdy}\cot\theta + V_{sd} \Rightarrow n_{Rdy} = n_{Sdy} + \frac{V_{sd}}{\cot\theta} = A_{sy}\sigma_{sy}$$

$\theta = f(\Delta\theta)$ – variable parameter (by our choice)

Asx → unknown; Asy → } 3 unknown in 2 equations ⇒ 3 equations i.e.

Then we cut the same layer with a plane orthogonal to the direction of compressions



Equilibrium to traslation in x direction (→)

$$-\frac{\sigma_c t}{\cos\theta}\cos\theta + n_{Rdx} - n_{Sdx} + V_{sd}\,\mathrm{tg}\theta = 0$$

(t → thickness)

And substituting the value of n_{Rdx} calculated in the previous slide we get

$$\sigma_c t = +\cancel{n_{Sdx}} + V_{sd}\cot\theta - \cancel{n_{Sdx}} + V_{sd}\,\mathrm{tg}\theta$$

The verification on concrete side is:

$$\sigma_c = V_{sd}\cdot(\cot\theta + \mathrm{tg}\theta)/t \qquad \sigma_c \le f_{cd2}$$

resistance of concrete in two dimensional beam

$f_{cd2} < \underbrace{f_{cd}}_{\text{monoaxial for beam}}$

Where f_{cd2} is the concrete compressive strength in cracked state, that can be calculated according to MC90 as:

(a) $$f_{cd2MC90} = 0.6\left(1 - \frac{f_{ck}}{250}\right)f_{cd}$$ (for beam)

Or according to a proposal of the authors:

If at least one reinforcement is yielded

(b) $$f_{cd2} = f_{cd2MC90}\cdot\boxed{(1 - 0.032\,|\Delta\vartheta|)}$$ → reduction $f(\Delta\theta)$

Where $\Delta\theta$ is the variation of the inclination of compressed struts from first cracking to failure.

Or if no reinforcement is yielded

(c) $$\sigma_c \le f_{cd2MC90}\left(0.85\frac{f_{cd}}{f_{cd2MC90}} - \frac{\sigma_c}{f_{cd}}\left(0.85\frac{f_{cd}}{f_{cd2MC90}} - 1\right)\right)$$

But remember that if NO reinforcement is yielded there CAN NOT be $\Delta\theta$ variation. → uso (c)

A comparison of the result obtained with this procedure with Marti and Kaufmann model on experimental results is presented in the next slide.

→ if we have no cracks we use fcd
→ if we have crack with NO yielding of the reinforcement we use fcd2MC90
→ if we have crack with yieded reinforcement we use fcd2

Something about layers thickness

The choice of the thickness of the layers is up to the designers as far as:

- $t_s \geq 2 \times b'_s$ where b'_s is the distance between the upper reinforcement centroid and the extrados of the slab
- $t_i \geq 2 \times b'_i$ where b'_i is the distance between the lower reinforcement centroid and the intrados of the slab
- $t_s + t_i < h$ ← total thickness of the slab

The reinforcement A_{sx} and A_{sy} calculated in the previous slides are supposed in the middle of the thickness of their layer. but

If reinforcement A_{sx} and A_{sy} are not placed in the middle of their layer but somewhere else like outer wards (i.e. with the minimum cover) leads to internal redistributions to respect the equilibrium.

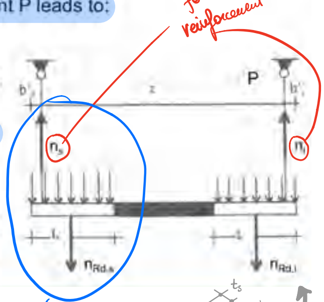
Equilibrium to rotation around point P leads to:

$$n_i = \frac{n_{Rd,x}\left(h - \frac{t_s}{2} - b'_s\right) + n_{Rd,y}\left(\frac{t_i}{2} - b'_i\right)}{z}$$

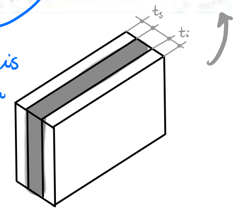
And equilibrium to translation to:

$$n_i = n_{Rd,x} + n_{Rd,y} - n_s$$

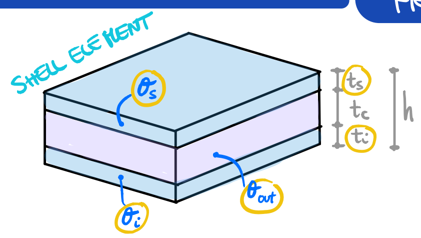
In this case redistributions inside concrete compression fields take also place but are generally neglected.



APPROXIMATION: the single layer is not in equilibrium but we neglecte this



PROCEDURA RIASSUNTIVA



N.B. i parametri cerchiati in giallo sono i parametri scelti dal progettista, quelli in viola sono gli OUTPUT

(1) This structure works in cracked field or not?
↳ I calculate Φ → >0 CRACKED; <0 NOT CRACKED

(2) Let's imagine $\Phi > 0$ (concrete cracked)

(3) We choose 3 layers: t_s = thickness of top layer; t_i = thickness of bottom layer
automatically $t_c = h - t_s - t_i$

(4) Shear design of the central layer: we choose a $22° < \theta_{out} < 45°$
↳ inclination of the compressive strut in shear out of the plane

(5) design of the top and bottom layers → we choose:

$$\theta_{se} - 20° < \theta_s < \theta_{se} + \underbrace{20°}_{\Delta\theta}$$
$$\theta_{ie} - 20° < \theta_i < \theta_{ie} + 20°$$

(6) At the end of the design I have to verify that:

(a) CENTRAL LAYER: $\sigma_c < f_{cd2}$ (resistance of compressive strut)
(b) TOP LAYER: $\sigma_{c,s} < f_{cd2,top}$
(c) BOTTOM LAYER: $\sigma_{c,i} < f_{cd2,bottom}$
↳ verification of 3 compression in concrete

(7) I got:
- Asw → amount of stirrups in central layer
- Asx, Asy → amount of reinforcement in top layer
- Aix, Aiy → amount of reinforcement in bottom layer

CRITERIA OF DIMENSIONING: I want to minimize the weight (W) of the reinforcement in order to save money

min (W) → but respecting always (a),(b),(c)

function of (Asw, Asx, Asy, Aix, Aiy)

{MODEL} → $\underbrace{\text{5 input parameter}}_{\mathbb{R}^5}$ → goal $\underbrace{\min(W)}_{\mathbb{R}^1}$

$\mathbb{R}^5 \Rightarrow \mathbb{R}^1$

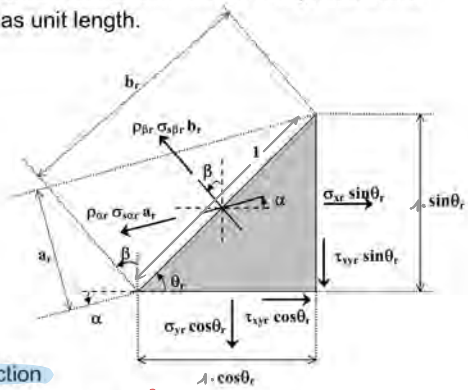
EXTERNAL LAYER DESIGN: α & β REINFORCEMENT (general case)

We cut the layer with a plane parallel to the direction of compressions at failure so that the cutting edge has unit length.

As before this direction is

$\theta_u = \theta_{cr} \pm \Delta\theta$ with

$-20° < \Delta\theta < 20°$.



(→) Equilibrium to traslation in x direction

$$\sigma_x \sin\theta_r + \tau_{xy}\cos\theta - \rho_\alpha \sigma_{s\alpha} a \cos\alpha - \rho_\beta \sigma_{s\beta} b \sin\beta = 0$$

Equilibrium to traslation in y direction

$$-\sigma_y \cos\theta - \tau_{xy}\sin\theta - \rho_\alpha \sigma_{s\alpha} a \sin\alpha + \rho_\beta \sigma_{s\beta} b \cos\beta = 0$$

Where:
ρ_α and ρ_β are the geometrical reinforcement ratios respectively in directions α and β.

a and b are the projections fo the unit length cut respectively in directions α and β.

$a = 1 \cdot \sin(\theta - \alpha)$ $\qquad b = 1 \cdot \cos(\theta - \beta)$

Solving the system of two equations in the two unknowns →

$\rho_\alpha \sigma_{s\alpha}$ and $\rho_\beta \sigma_{s\beta}$

And remembering that

$$\cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta = \cos(\alpha - \beta)$$

$$\cos\theta \cdot \cos\beta - \sin\theta \cdot \sin\beta = \cos(\theta + \beta)$$

$$\cos\theta \cdot \sin\alpha + \sin\theta \cdot \cos\alpha = \sin(\theta + \alpha)$$

We get

$$\rho_\alpha \sigma_{s\alpha} = \frac{\sigma_x \sin\theta \cos\beta - \sigma_y \cos\theta \sin\beta + \tau_{xy}\cos(\theta + \beta)}{\sin(\theta - \alpha)\cos(\alpha - \beta)}$$

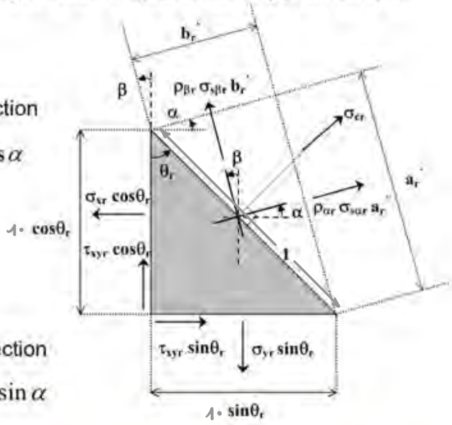
% armatura meccanica

$$\rho_\beta \sigma_{s\beta} = \frac{\sigma_x \sin\theta \sin\alpha + \sigma_y \cos\theta \cos\alpha + \tau_{xy}\sin(\theta + \alpha)}{\cos(\theta - \beta)\cos(\alpha - \beta)}$$

From which we can calculate the reinforcement ratios by choosing the reinforcement stress, remembering:

$\sigma_{s\,\alpha,\beta} \le f_{yd}$

Then we cut the same r layer with a plane orthogonal to the direction of compressions



Equilibrium to traslation in x direction

$$-\sigma_x \cos\theta + \tau_{xy} \sin\theta + \rho_\alpha \sigma_{s\alpha} a' \cos\alpha$$
$$-\rho_\beta \sigma_{s\beta} b' \sin\beta + \sigma_c \cos\theta = 0$$

Equilibrium to traslation in y direction

$$-\sigma_y \sin\theta + \tau_{xy} \cos\theta + \rho_\alpha \sigma_{s\alpha} a' \sin\alpha$$
$$-\rho_\beta \sigma_{s\beta} b' \cos\beta + \sigma_c \sin\theta = 0$$

Being the four equilibrium equations linearly correlated, only three of them are independent , and σ_c can be derived from both the previous two equations as:

From equilibrium to traslation in x direction

$$\sigma_c = \sigma_x - \tau_{xy} \tan\theta - \rho_\alpha \sigma_{s\alpha} \cos(\theta-\alpha)\frac{\cos\alpha}{\cos\theta} + \rho_\beta \sigma_{s\beta} \sin(\theta-\beta)\frac{\sin\beta}{\cos\theta}$$

From equilibrium to traslation in y direction

$$\sigma_c = \sigma_y - \frac{\tau_{xyr}}{\tan\theta} - \rho_\alpha \sigma_{s\alpha} \cos(\theta-\alpha)\frac{\sin\alpha}{\sin\theta} - \rho_\beta \sigma_{s\beta} \sin(\theta-\beta)\frac{\cos\beta}{\sin\theta}$$

Where:
a' and b' are the projections fo the unit length cut respectively in directions α and β.

$a' = 1 \cdot \cos(\theta-\alpha) \qquad b' = 1 \cdot \sin(\theta-\beta)$

The verification on concrete side is as seen for reinforcement along x and y: $\sigma_c \leq f_{cd2}$

N.B.

1. The values of α, β, ρ_α , ρ_β and θ can be different for the two external layers.
2. The θ angle used for external layers has NO relation with the θ angle used for out of the plane shear in the internal layer.
3. In the previous equations θ should be included in the same quadrant of θ_{cr} (angle on x axis of principal tension direction at cracking) and the solutions with nil denominator in equations of slide 148 correspond to cases in which, in disagreement with the code provisions, only one order of reinforcement is available (*$\cos(\alpha - \beta) = 0$*) or the equilibrium is not possible (*$\sin(\theta_r - \alpha) = 0$* and *$\cos(\theta_r - \beta) = 0$*).

SLAB BRIDGES – SANDWICH MODEL: NUMERICAL EXAMPLE

Finite element model

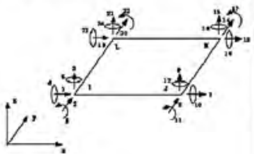
Mesh

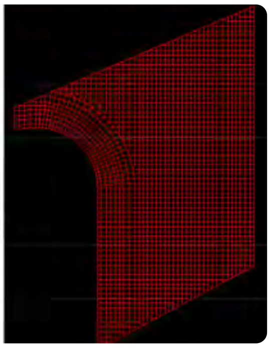
2215 shell elements

2285 nodes

6 D.o.F. per node

13710 D.o.F. in total





Guidelines to choose the mesh layout

1. Fit properly the geometry of the structure
   - 1.1 Avoid skew irregular elements
   - 1.2 Follow thickness changes and possibly main reinforcement distribution
2. Place a number of element able to describe properly the deformation of the structure
3. Choose a regular spacing between the nodes in order to be able to place live loads and prestressing equivalent loads as nodal loads (if you don't dispose of an advanced pre-processor)

Introduction of prestressing in the model

Equivalent loads method

Prestressing is seen as a system of equivalent external actions which is globally self equilibrated.

Such actions are applied to the structure both at the anchorages (or deviators) as concentrated loads and along the curved path of the tendons as distributed loads.

In isostatic structures no external reactions take place, whereas globally equilibrated reactions arise in hyperstatic ones.

With shell elements things become more complicated….

With rectangular elements we can have 3 different situations:

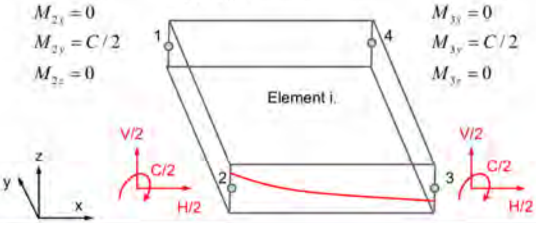
1. Tendon passing trough two nodes of the same side (this is the situation of the 28 longitudinal tendons of our slab)
2. Tendon passing trough two opposite nodes of the same element
3. Tendon crossing the element in a general position.

1. Tendon passing trough two nodes of the same side

$F_{2x} = H/2$
$F_{2y} = 0$
$F_{2z} = V/2$
$M_{2x} = 0$
$M_{2y} = C/2$
$M_{2z} = 0$

N.B These are the forces in the nodes 2 and 3 deriving only from the contribution of the part of the tendon in element i.

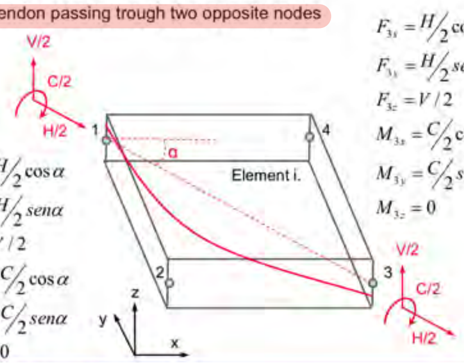
$F_{3x} = H/2$
$F_{3y} = 0$
$F_{3z} = V/2$
$M_{3x} = 0$
$M_{3y} = C/2$
$M_{3z} = 0$



2. Tendon passing trough two opposite nodes

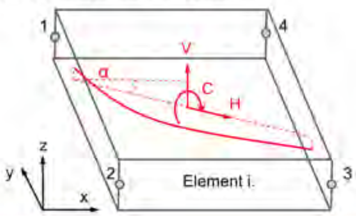
$F_{1x} = \frac{H}{2}\cos\alpha$
$F_{1y} = \frac{H}{2} sen\alpha$
$F_{1z} = V/2$
$M_{1x} = \frac{C}{2}\cos\alpha$
$M_{1y} = \frac{C}{2} sen\alpha$
$M_{1z} = 0$

$F_{3x} = \frac{H}{2}\cos\alpha$
$F_{3y} = \frac{H}{2} sen\alpha$
$F_{3z} = V/2$
$M_{3x} = \frac{C}{2}\cos\alpha$
$M_{3y} = \frac{C}{2} sen\alpha$
$M_{3z} = 0$



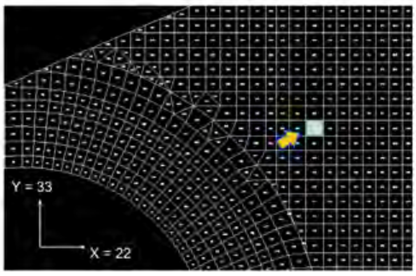
3. Tendon crossing the element in a general position

The actions H, V and C placed in the middle of the projection of the tendon segment on the centroid surface of the element should be firstly decomposed in their x y and z component and then distributed on the nodes according to the shape functions of the element, or, by simplification, as a function of the distance from the nodes.



Sandwich model:
Numerical example

Element chosen: n°682



Symbols, conventions
and general data

$\alpha = 0 \Rightarrow$ transverse reinforcement, Asx, direction 22

$\beta = 0 \Rightarrow$ longitudinal reinforcement, Asy, direction 33

Concrete properties f_{ctd} = 20.75 MPa

f_{ctm} = 3.16 MPa

f_{ctd} = 1.38 MPa

Steel properties f_{yd} = 373.9 MPa

Dimensioning of α reinforcement (transverse) in the inferior layer

	Layers thicknesses	
H sez.	**tsup**	**tinf**
(m)	(m)	(m)
1.0000	0.23	0.18

Distance of reinforcement from the outer surface = 6 cm

Combination type	Nsd22 (KN/m)	Nsd33 (KN/m)	Nsd23 (KN/m)	Msd22 (KNm/m)	Msd33 (KNm/m)	Msd23 (KNm/m)	Vsd12 (KN/m)	Vsd13 (KN/m)
Max M33	277	-5134	-230	616	1121	-476	95	-212

Load combination that maximizes this reinforcement

Increment of internal actions due to shear (for the single layer)		
nsd22	**nsd33**	**nsd23**
(KN/m)	(KN/m)	(KN/m)
0	0	0

Upper layer verification

Internal actions on the layer			Cracked ?	Concrete parameters			Actions at $t_{sup}/2$		Reinforcement calculated at $c+\phi/2$	
nsd22	**nsd33**	**nsd23**	case	θ	ν fcd	σ_c(f)	$n_{R1(x)}$	$n_{R2(y)}$	**A_s(x)nec**	**A_s(y)nec**
(KN/m)	(KN/m)	(KN/m)	(-)	(°)	(N/mm²)		(kN/m)	(kN/m)	(cm²/m)	(cm²/m)
-633	-4070	481	no.	65.0	17.6	17.6	0.0	0.0	15.7	15.7

Lower layer verification

Internal actions on the layer			Cracked ?	Concrete parameters			Actions at $t_{sup}/2$		Reinforcement calculated at $c+\phi/2$	
nsd22	**nsd33**	**nsd23**	case	θ	ν fcd	σ_c(f)	$n_{R1(x)}$	$n_{R2(y)}$	**A_s(x)nec**	**A_s(y)nec**
(KN/m)	(KN/m)	(KN/m)	(-)	(°)	(N/mm²)		(kN/m)	(kN/m)	(cm²/m)	(cm²/m)
909	-1064	-711	yes	23.1	11.1	11.1	1212.0	607.2	31.3	15.7

Dimensioning of β reinforcement (longitudinal) in the inferior layer

	Layers thicknesses	
H sez.	**tsup**	**tinf**
(m)	(m)	(m)
1.0000	0.23	0.19

Distance of reinforcement from the outer surface = 6 cm

Combination type	Nsd22 (KN/m)	Nsd33 (KN/m)	Nsd23 (KN/m)	Msd22 (KNm/m)	Msd33 (KNm/m)	Msd23 (KNm/m)	Vsd12 (KN/m)	Vsd13 (KN/m)
Max M22	261	-5134	-219	657	1014	-464	79	-197

Load combination that maximizes this reinforcement

Increment of internal actions due to shear (for the single layer)		
nsd22	**nsd33**	**nsd23**
(KN/m)	(KN/m)	(KN/m)
0	0	0

Upper layer verification

Internal actions on the layer			Cracked ?	Concrete parameters			Actions at $t_{sup}/2$		Reinforcement calculated at $c+\phi/2$	
nsd22	**nsd33**	**nsd23**	case	θ	ν fcd	σ_c(f)	$n_{R1(x)}$	$n_{R2(y)}$	**A_s(x)nec**	**A_s(y)nec**
(KN/m)	(KN/m)	(KN/m)	(-)	(°)	(N/mm²)		(kN/m)	(kN/m)	(cm²/m)	(cm²/m)
-695	-3904	474	no	45.0	17.6	17.6	0.0	0.0	15.7	15.7

Lower layer verification

Internal actions on the layer			Cracked ?	Concrete parameters			Actions at $t_{sup}/2$		Reinforcement calculated at $c+\phi/2$	
nsd22	**nsd33**	**nsd23**	case	θ	ν fcd	σ_c(f)	$n_{R1(x)}$	$n_{R2(y)}$	**A_s(x)nec**	**A_s(y)nec**
(KN/m)	(KN/m)	(KN/m)	(-)	(°)	(N/mm²)		(kN/m)	(kN/m)	(cm²/m)	(cm²/m)
956	-1229	-693	yes	20.6	11.1	11.1	1216.7	611.5	31.3	15.7

Dimensioning of α reinforcement (transverse) in the superior layer

	Layers thicknesses	
H sez.	**tsup**	**tinf**
(m)	(m)	(m)
1.0000	0.23	0.19

Distance of reinforcement from the outer surface = 6 cm

Combination type	Nsd22 (KN/m)	Nsd33 (KN/m)	Nsd23 (KN/m)	Msd22 (KNm/m)	Msd33 (KNm/m)	Msd23 (KNm/m)	Vsd12 (KN/m)	Vsd13 (KN/m)
Max M22	261	-5134	-219	657	1014	-464	79	-197

Load combination that maximizes this reinforcement

Increment of internal actions due to shear (for the single layer)		
nsd22	**nsd33**	**nsd23**
(KN/m)	(KN/m)	(KN/m)
0	0	0

Upper layer verification

Internal actions on the layer			Cracked ?	Concrete parameters			Actions at $t_{sup}/2$		Reinforcement calculated at $c+\phi/2$	
nsd22	**nsd33**	**nsd23**	case	θ	ν fcd	$\sigma_c(f)$	$n_{R1(x)}$	$n_{R2(y)}$	**A_s(x)nec**	**A_s(y)nec**
(KN/m)	(KN/m)	(KN/m)	(-)	(°)	(N/mm²)		(kN/m)	(kN/m)	(cm²/m)	(cm²/m)
-695	-3904	474	no	45.0	17.6	17.6	0.0	0.0	15.7	15.7

Lower layer verification

Internal actions on the layer			Cracked ?	Concrete parameters			Actions at $t_{sup}/2$		Reinforcement calculated at $c+\phi/2$	
nsd22	**nsd33**	**nsd23**	case	θ	ν fcd	$\sigma_c(f)$	$n_{R1(x)}$	$n_{R2(y)}$	**A_s(x)nec**	**A_s(y)nec**
(KN/m)	(KN/m)	(KN/m)	(-)	(°)	(N/mm²)		(kN/m)	(kN/m)	(cm²/m)	(cm²/m)
956	-1229	-693	yes	20.6	11.1	11.1	1216.7	611.5	31.3	15.7

Dimensioning of β reinforcement (longitudinal) in the superior layer

	Layers thicknesses	
H sez.	**tsup**	**tinf**
(m)	(m)	(m)
1.0000	0.23	0.19

Distance of reinforcement from the outer surface = 6 cm

Combination type	Nsd22	Nsd33	Nsd23	Msd22	Msd33	Msd23	Vsd12	Vsd13
	(KN/m)	(KN/m)	(KN/m)	(KNm/m)	(KNm/m)	(KNm/m)	(KN/m)	(KN/m)
Max M22	261	-5134	-219	657	1014	-464	79	-197

Load combination that maximizes this reinforcement

Increment of internal actions due to shear (for the single layer)		
nsd22	**nsd33**	**nsd23**
(KN/m)	(KN/m)	(KN/m)
0	0	0

Upper layer verification

Internal actions on the layer			Cracked ?	Concrete parameters			Actions at $t_{sup}/2$		Reinforcement calculated at $c+\phi/2$	
nsd22	**nsd33**	**nsd23**	case	θ	ν fcd	$\sigma_c(f)$	$n_{R1(x)}$	$n_{R2(y)}$	**A_s(x)nec**	**A_s(y)nec**
(KN/m)	(KN/m)	(KN/m)	(-)	(°)	(N/mm²)		(kN/m)	(kN/m)	(cm²/m)	(cm²/m)
-695	-3904	474	no	45.0	17.6	17.6	0.0	0.0	15.7	15.7

Lower layer verification

Internal actions on the layer			Cracked ?	Concrete parameters			Actions at $t_{sup}/2$		Reinforcement calculated at $c+\phi/2$	
nsd22	**nsd33**	**nsd23**	case	θ	ν fcd	$\sigma_c(f)$	$n_{R1(x)}$	$n_{R2(y)}$	**A_s(x)nec**	**A_s(y)nec**
(KN/m)	(KN/m)	(KN/m)	(-)	(°)	(N/mm²)		(kN/m)	(kN/m)	(cm²/m)	(cm²/m)
956	-1229	-693	yes	20.6	11.1	11.1	1216.7	611.5	31.3	15.7

Verification of maximum compressive stresses in concrete
Inferior layer

Combination type	Nsd22 (KN/m)	Nsd33 (KN/m)	Nsd23 (KN/m)	Msd22 (KNm/m)	Msd33 (KNm/m)	Msd23 (KNm/m)	Vsd12 (KN/m)	Vsd13 (KN/m)
Min V12	188	-5215	-148	-88	143	-245	-43	-48

Load combination that maximizes the stress

	Superior layer		Inferior layer		Centoid	
Thicnness [m]	$\sigma_{1,sup}$ (N/mm²)	$\sigma_{3,sup}$ (N/mm²)	$\sigma_{1,inf}$ (N/mm²)	$\sigma_{3,inf}$ (N/mm²)	$\sigma_{1,g}$ (N/mm²)	$\sigma_{3,g}$ (N/mm²)
1.0000	0.97	-6.32	0.23	-4.93	0.21	-5.22

Verification of maximum compressive stresses in concrete
Superior layer

Combination type	Nsd22 (KN/m)	Nsd33 (KN/m)	Nsd23 (KN/m)	Msd22 (KNm/m)	Msd33 (KNm/m)	Msd23 (KNm/m)	Vsd12 (KN/m)	Vsd13 (KN/m)
Min V12	188	-5215	-148	-88	143	-245	-43	-48

Load combination that maximizes the stress

	Superior layer		Inferior layer		Centoid	
Thicnness [m]	$\sigma_{1,sup}$ (N/mm²)	$\sigma_{3,sup}$ (N/mm²)	$\sigma_{1,inf}$ (N/mm²)	$\sigma_{3,inf}$ (N/mm²)	$\sigma_{1,g}$ (N/mm²)	$\sigma_{3,g}$ (N/mm²)
1.0000	0.97	-6.32	0.23	-4.93	0.21	-5.22

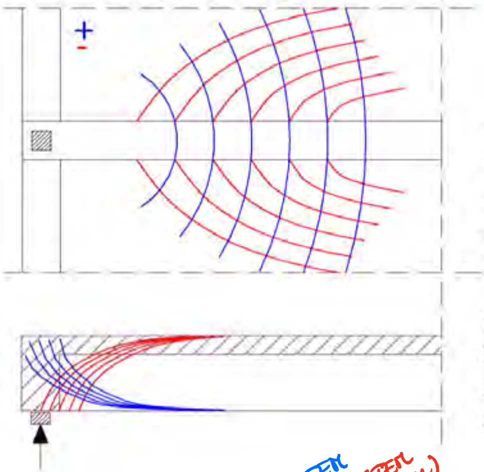
7. LOCAL EFFECTS

PARAGRAFI

SHEAR LAG IN T BEAMS (or double T beams)

ritardo

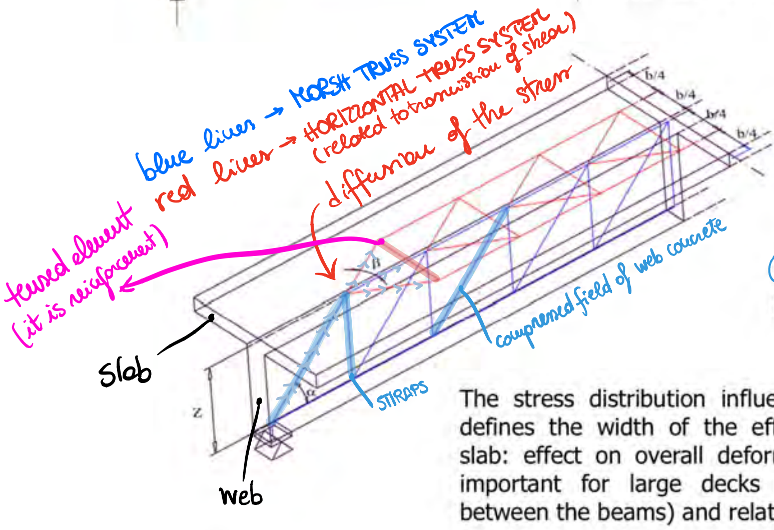
only

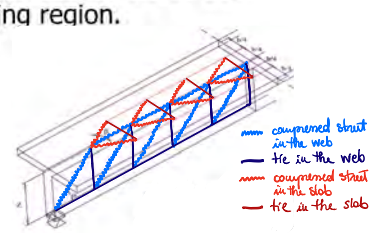


Shear lag: shear influence on distribution of longitudinal stresses

In the horizontal plane (slab): transverse tensile stresses by effect of stress distribution within the compressed chord of concrete truss generated by shear.

In the vertical plane (web): skew tensile stresses orthogonal to the direction of compression field in the web in the bearing region.



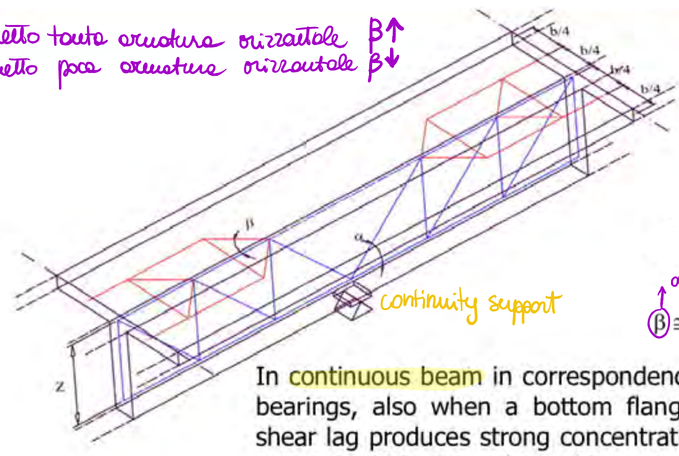


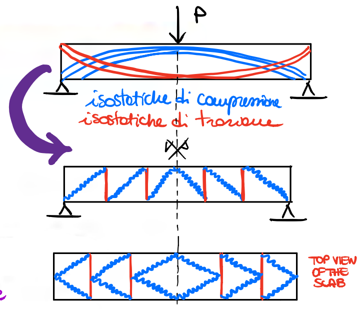
$\alpha \cong 40° \div 45°$

$\beta \cong 30° \div 35°$

The stress distribution influenced by the shear defines the width of the effective collaborating slab: effect on overall deformability, particularly important for large decks (or large distance between the beams) and relatively small spans.

- Se metto tanta armatura orizzontale β↑
- Se metto poca armatura orizzontale β↓





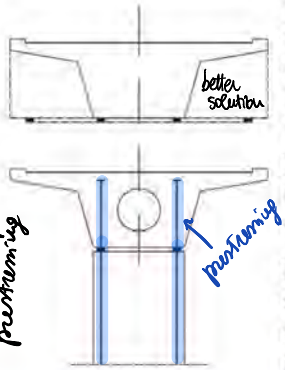
our choice
β ≅ 40° ÷ 45°

In continuous beam in correspondence of intermediate bearings, also when a bottom flange is present, the shear lag produces strong concentration of longitudinal stresses within the webs and increment of creep effects in serviceability conditions. In some cases also the bearing capacity may be reduced.

Single beam deck → used for b ≤ 6 ÷ 7 m. (esempio rampa autostrada)

The web should be designed to carry the torsion coming from live loads with their eccentricity (primary torsion, governing the equilibrium!)

typically used with 1 single notional lane



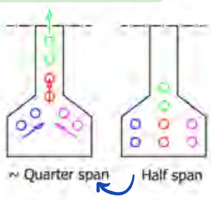
In the support regions the torsion should be equilibrated by means of two bearings with enough distance; if necessary, use a transverse beam to increase the lever arm of support reactions.

If it is impossible the equilibrium with dead and live loads (light beams), the deck shall be connected rigidly to the pier.

If necessary, use tensioned prestressing bars, crossing the bearings. Those bars should be extended within the pier so that they are anchored in a section on which the dead load of the upper part of the pier is enough.

To carry Torsion I have to put prestressing
↳ 'cause all the bearings usually work in compression
I want all bearings have to be compressed

Prestressed beams



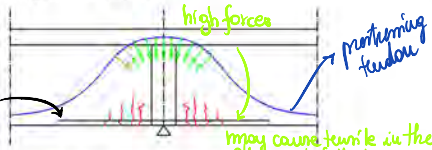
si annulla la forza orizzontale

Take into account the transverse forces induced by tendons when their layout (in proximity of support regions) implies deviation in the horizontal plane. When possible, design for symmetrical transverse tendon displacements.

to move the blue and pink tendon up I need horizontal displacements ⇒ horizontal curvature of the tendon ⇒ transverse bending moment (horizontal load due to tendon)

If coupled tendons are used in continuous beams, the couplers should be located in the regions where very low bending moments are expected. (~0.2 ÷ 0.25 l)

In case of continuous beams the continuity tendons should be curved downward at a short distance from the bearing to transfer the deviation forces directly to the support.



In such a way it is possible to obtain an hyperstatic bending moment of opposite sign with respect to that deriving by acting loads. With small deviation there is the possibility to generate tensile stresses in bottom chord. In such a case ordinary reinforcement should provided to control the crack opening.

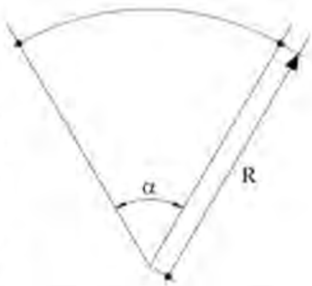
CURVED BEAMS

In this case I have torsion also for permanents loads.

It is possible to built curved decks for small α values; R plays a less important role.

For large α values it is better to use small spans and straight beams.

Using curved beams, $\alpha \leq 20°$ (simply supported), $\alpha \leq 40°$ (continuous). The end transverse beams should be very strong. Intermediate transverse beams may also be necessary to limit the torque moments in longitudinal beams.



In general it should be noted that curved beams are not an efficient solution. For small span it is better to use slab decks; for medium or high spans, box girder beams.

DESIGN FOR LOCAL EFFECTS

Global-Local is a formal distinction deriving by the necessity to simplify the design procedures.

Slab ⟶ elastically restrained by beams and transverse beams

⇓

Difficult definition of restraint conditions

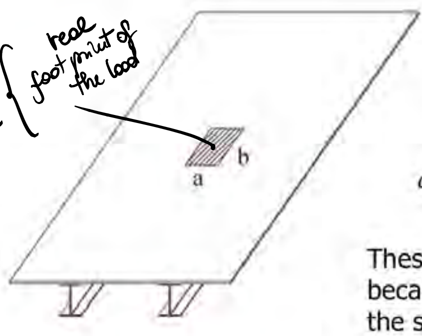
↙ Analysis with Courbon → Slab on rigid restraints (continuous)

↘ Analysis with Massonet → Slab on elastic restraints (maybe including the torsional connection)

- Simplification: portal restrain on the beams
- Further simplification → degeneration from slab to strip of "equivalent" slab
- Imprint of concentrated load and its repartition

it is taken into account the diffusion

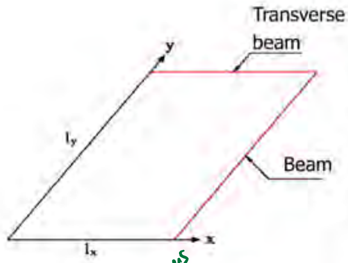
real foot print of the load



a_0 = actual contact area width
b_0 = actual contact area length
S_1 = paving thickness
S = slab thickness

$$a = a_0 + 2S_1 + S \qquad b = b_0 + 2S_1 + S$$

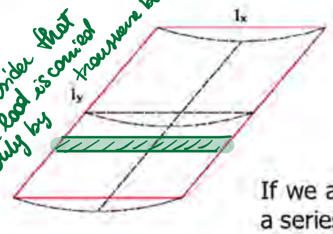
These expression is on the safety side because the imprint could spread as far as the slab reinforcements.



Design as slab with infinite length

If l_y / l_x > 2 one can assume $l_y \cong \infty$ (transverse beams largely spaced).

- With uniformly distributed load along y the deformed shape is a cylinder.
- If the load is acting only partially along ly the deformed surface has a double curvature.



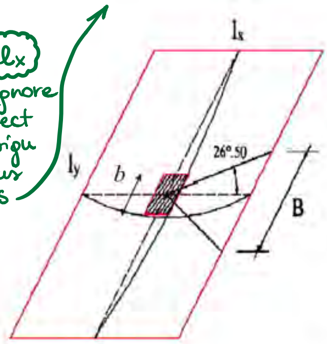
I consider that the load is carried only by transverse beams

In case of uniformly distributed load along y the bending moment My derives by the restrained transverse deformation.

$$M_y = -\nu \frac{\partial^2 w}{\partial x^2} \cdot \frac{Es^3}{12(1-\nu^2)}$$

If we assume ν=0 ⇒ M_y=0 ⇒ the slab degenerates in a series of adjacent independent beams.

Se $\boxed{l_y > 3\, l_x}$ ⇒ I can ignore slab effect and design as a series of beams

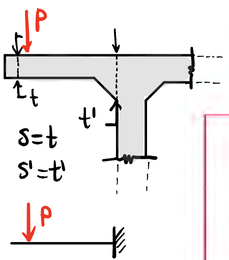


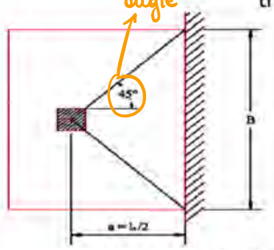
In case of partially extended loads bending moments $M_y < M_x$ exist.

As an approximation one can design the slab like a beam with an equivalent width:

$$B = b + \frac{l_x}{2}$$

That is spreading the load towards the beams with an angle of 26,5°.





diffusion angle

(come in acciaio)

In case of cantilevering slab, the load can be spread with 45° (uniform thickness of the slab), then

$$B = b + l_x$$

In this case with a point load it results:

$$m_x = \frac{Pa}{2a} = \frac{P}{2} = 0.5P \quad (\text{if } t = \text{cost})$$

If thickness increases towards the restrained edge also m increases:

$$\begin{cases} s/s' = 2 \quad m_x = 0{,}576P \quad m_x \uparrow \\ s/s' = 3 \quad m_x = 0{,}631P \quad m_x \uparrow \end{cases}$$

Pay attention that near longitudinal joints only B/2 is available for the spreading.

With an equivalent beam design the secondary bending moments are disregarded, then in that direction the slab should be design for a bending moment equivalent to 25% of principal bending moment.

In proximity of deck joint, due to the geometrical irregularities, a strong dynamic effect should be expected. Conventionally a dynamic coefficient ∅ = 3 is used and its application is extended at least to l_x /3 or l_{cantil} , any case not less than 1 meter.

SLAB DESIGN AS A RECTANGULAR SLAB

In such case $l_y/l_x \leq 2$.

References are available for different geometrical ratios, load cases and restraint conditions (Bittner, Bares, Pucher, Homberg); in addition analytical solution are available (Westergaard).

Generally the design aids are available as influence surfaces.

Extension in 2D of influence lines

Influence surface: surface giving the variation law of a parameter in one single point in function of the variation of the position of a unit load moving along the slab. Influence surfaces are available in literature for deformation components and internal actions. They are drawn by means of contour lines.

They are calculated in linear-elastic field, then within the field of validity of superposition principle.

Having defined G (x_o, y_o) the parameter under consideration, it results:

- for several concentrated loads

$$G(x_o, y_o) = \sum_i P_i(x_i, y_i)\,\Delta(x_o, y_o; x_i, y_i)$$

Effect produced in x_o, y_o by a unit load in x_i, y_i

- for a linear load (along "s")

$$G(x_o, y_o) = \int_s p(s)\,\Delta(x_o, y_o; x, y)\,ds$$

In case of uniform linear "p" load:

$$G(x_o, y_o) = p\int_s \Delta(x_o, y_o; x, y)\,ds = p\Omega$$

Area intercepted by a plane or a cylinder crossing by "s" the influence surface

- for a load distributed over an area A

$$G(x_o, y_o) = \iint_A p(x, y)\,\Delta(x_o, y_o; x, y)\,dxdy$$

In case of uniformly distributed load

$$G(x_o, y_o) = pV$$

Volume of influence surface intercepted by a cylinder having the base coincident with the imprint of the load.

- the practical use of influence surfaces is made easy by the invariability of influence surfaces for slabs having the same edge ratio.

Once known on the surface $\overline{l}_x, \overline{l}_y$

While the actual dimensions are l_x, l_y

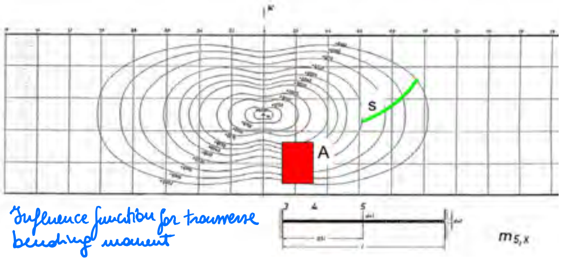
$$\frac{\overline{l}_x}{l_x} = \frac{\overline{l}_y}{l_y} = k$$

Size ratio

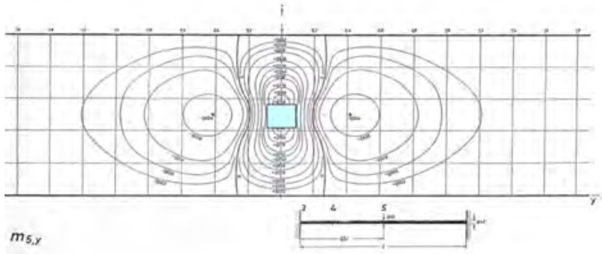
The load should be reduced in the same ratio: S/k (linear); A/k² (surface)

Having evaluated the $\overline{G}$ parameters from the surfce, it results :

- $G = \overline{G}$ For concentrated loads
- $G = k \cdot \overline{G}$ For linear loads
- $G = k^2 \cdot \overline{G}$ For surface loads



Influence function for transverse bending moment



Influence function for longitudinal bending moment

Drawing procedures of influence surfaces

All the influence surfaces of "G" parameters are obtained by the derivatives of displacement equation. It is necessary the integration of Lagrange equation:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = -\frac{p}{B}$$

The most known solution method is the Pucher one (Singularities method)

$$w = w_1 + w_2$$

Actual deformed shape

Deformed shape in proximity to the load of a circular plate loaded in the center, particular solution of differential equation.

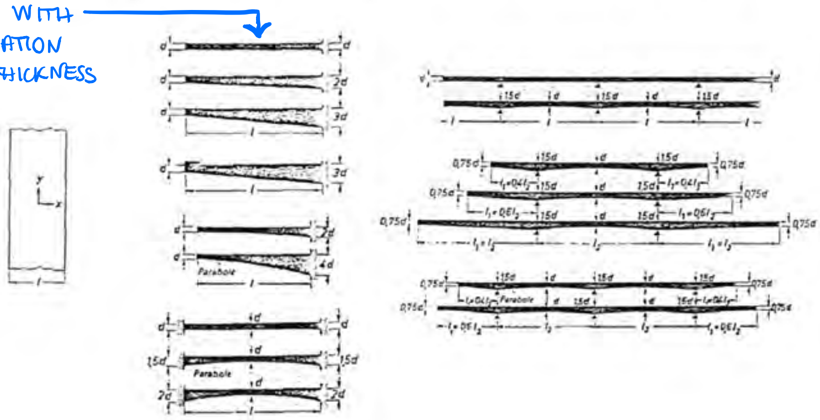
$$w = \frac{r^2}{8\pi B}\ln\left(\frac{r}{a}\right)$$

a = diameter

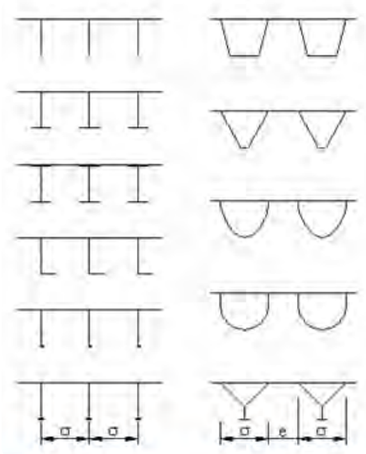
Deformed shape of slab loaded along the edges, so that to respect the restrain conditions (solution of homogeneous equation)

Most important cases for which influence surfaces are available.

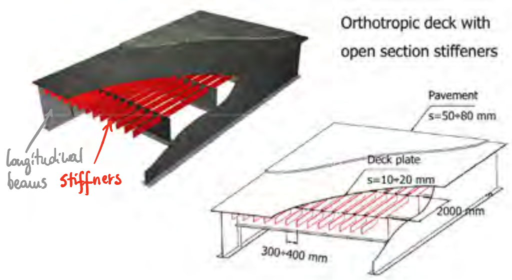
SLAB WITH VARIATION OF THICKNESS

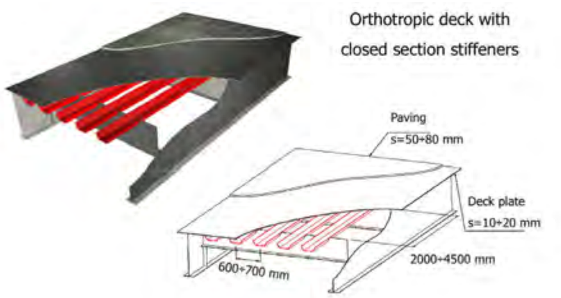


LOCAL EFFECTS IN STEEL ORTHOTROPIC DECK



- The steel plate transfers the action to the girder (first stress system)
- Stiffeners and plate transfers the action on the beam system with the help of transverse beams (second stress system)
- Overall behaviour of the section against the internal action coming by the overall analysis (third stress system)





5.1 Plate behaviour (Klöppel)

he osserved on load test on Orthotropic deck in laboratory

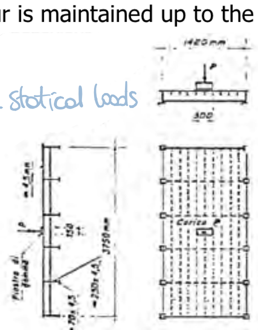
Four steps are outlined following the load increase up to the failure:

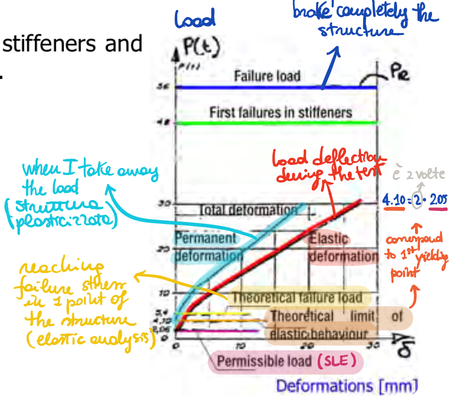
- 1st step: serviceability loads ⟶ plate within the elastic field; it behaves like an isotropic slab on several bearings. Flexural stresses are prevailing on membrane ones.
- 2nd step: important membrane stresses take place for increasing load and the structure remains in the elastic field also for load levels considerably greater than the ones that can be predicted by means of thin slab theory.
- 3rd step: membrane stresses continue to increase, assuming the same order of magnitude of flexural ones. Elastic-plastic field is reached.
- 4th step: plastic hinges appear along the longitudinal stiffeners and the membrane behaviour is maintained up to the failure.

What does it mean? Most of the resources of this structure are beyond elastic field. We have a safety factor from serviceability load ($\simeq P=2$) to failure load ($\simeq P=50$) $\Rightarrow \frac{50}{2} \simeq 25$!
So the elastic analysis I'm performing is extremely on the safe side

per $P=56$ broke completely the structure

- Very high safety margin respect to the failure. → never collapse for statical loads
- Deck behaviour variable as a function of the load level, particularly in 4th step.

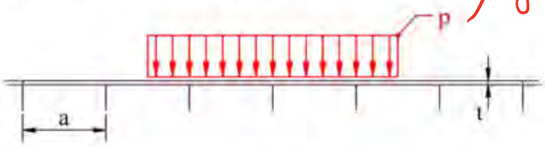




In the design the limit state of deformation is prevailing, in order to grant the durability of the pavement.

footprint of the wheel after dispersion (or tandem)

σ_I



It is allowed a limit displacement of:

$$w = \frac{a}{300}$$

Boobnov (experimental check by Klöppel):

$$w = \frac{a}{300} = \frac{1}{6}\frac{5}{384}\frac{pa^4}{EI} \Rightarrow t = 1.98\,a\sqrt[3]{\frac{p}{E}} \Rightarrow t = 0.015\,a\sqrt[3]{p}$$

thickness of the plate function of the load

$$I = \frac{t^3}{12} \qquad E = 2.1\cdot 10^6\,\frac{kg}{cm^2}$$

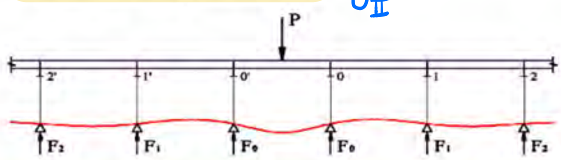
Where: a [cm]
p [kg/cm²]

5.2 Behaviour of stiffened plate

- Following the most used procedure, the stiffened plate is considered as an orthotropic plate (plate and longitudinal stiffeners) continuously supported on elastic restraints (transverse beams); this procedure is known as Pelikan-Esslinger method. (for the calculation)
- An alternative hypothesis is to consider (Huber) the deck like a girder with infinitesimal mesh (Guyon-Massonnet method).
- A third procedure is to use influence surfaces of orthotropic slabs.
- Having accepted the orthotropic behaviour, numerical algorithms can be used (finite differences or variations methods).
- Finally semi-empirical methods based on the experimental analysis of structural models may be used.

Pelikan-Esslinger Method

σ_{II}

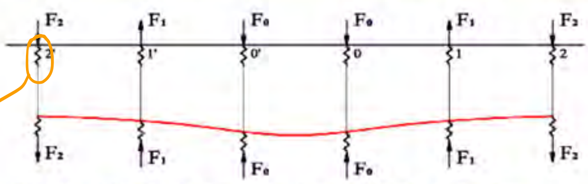


Transverse beams are supposed to be infinitely rigid.

carico la plate e calcolo le reazioni nei vincoli

4

Step 1 - Orthotropic slab continuously supported on infinitely rigid transverse beams.



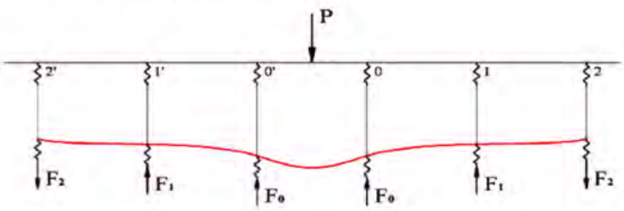
bending stiffness of the transverse beam

Reaction of 1st step are applied as actions to the slab now considered as supported on elastic restraints.

queste reazioni le cambio di segno e le applico ad un modello con le molle

Step 2 - Effect of transverse beams elastic behaviour.

Superposition of two previous steps.

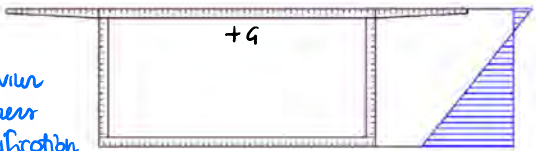


Step 3 – Orthotropic slab continuously supported on infinitely elastic transverse beams.

with this procedure I can design the stiffners

5.3 Overall behaviour of the section

σ_{III} come out from longitudinal global analysis (moving loads ext..)



σ_I plate behaviour
σ_{II} plate + stiffners
σ_{III} global verification

Small stresses arise.

Pay attention to the problem of local instability!

- Superposition of different stress states
- Reduction criterion: $\rho_I \sigma_I + \rho_{II} \sigma_{II} + \sigma_{III} \leq \sigma_{amm}$ longitudinal stresses $\rho_I, \rho_{II} < 1$ → to reduce stresses

In practice : $\rho_I = 0$ consideration taken to the high level of available safety

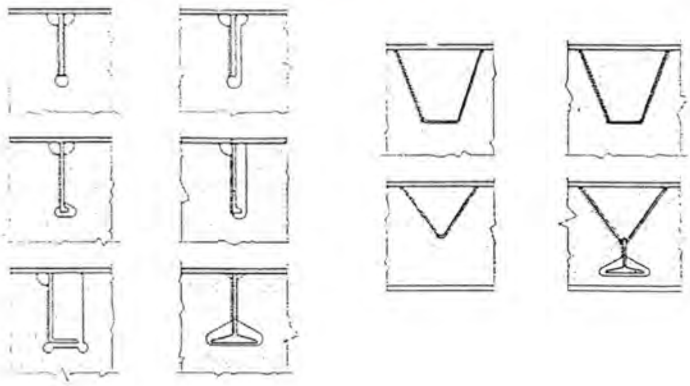
$$\rho_{II} = \frac{P_{adm}(teoric)}{P_{adm}(experimental)}$$

serviceability loads
The most used value is 0,45

- Strength verification: $0.45\sigma_{II} + \sigma_{III} \leq \sigma_{adm}$

↳ for example σ_y

Details for different stiffeners geometry.

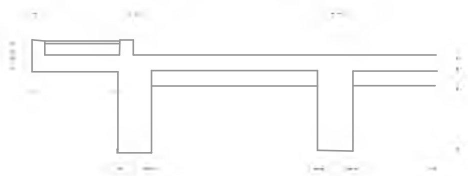


EXAMPLE OF LOCAL EFFECTS ON SLAB

1.1 Differences between free spans and axis spans

Axis spans   Free spans

In the bridge studied in the previous lessons the slab is separated from the transverse beams so we have:
•3 inner fields of 14.7x3m of axis spans
•2 external cantilevers of 14.7x1.5m of axis spans



1.2 Aspect ratios

Called b the longest side of a slab
Called a the shorter one

If $\frac{b}{a} > 2.5$ The slab can be dimensioned as equivalent beam

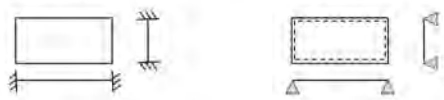
If $\frac{b}{a} < 2.5$ The slab has to be dimensioned as a real slab

1.3 Level of restraint

In both cases (real slab or equivalent beam) the real level of restraint provided by longitudinal beams and transverse beams is difficult to be correctly evaluated.

For a simplified analysis the slab can be dimensioned for the envelope of the two limit situations of:

rigid restraint on the edges   simple support on the edges



The rigid restraint condition is used for evaluating the maximum negative moments at the edges (i.e. points A and B), whereas the simple support condition is used for the bending moments in mid-span (i.e. point M)

In case of automatic design by use of f.e.m. both conditions are used for every point of the slab and the results are enveloped.

2. Actions on the slab

- **Dead load** g_1
- **Carried permanent loads** g_2
  Kerbs, pavement, barriers, parapets, etc.

- **Creep and shrinkage** ε_2
- **Thermal actions** ε_3

- **Load Model 1 (LM1)**
  Tandem and distributed loads (use lane 1)
- **Load Model 2 (LM2)**
  Tandem or single tyre load 200kN (0.35x0.60m)
- **Load Model 3 (LM3)**
  Concentrated load for local verification 150kN (0.4x0.4m)

2.1 Permanent loads

- **Dead load** $g_1 = h_s \cdot \gamma_s = 0.25m \cdot 25\frac{kN}{m^3} = 6.25kN/m^2$

- **Carried permanent loads**
  pavement $g_{2,p} = 3.00kN/m^2$
  kerb $g_{2,k} = h_k \cdot \gamma_k = 0.23m \cdot 25\frac{kN}{m^3} = 5.75kN/m^2$
  pedestrian barrier $g_{2,pb} = 1.00kN/m$

2.2 Imposed deformations

No imposed deformations will be taken into account for sake of simplicity

3. Equivalent beam dimensioning

3.1 Inner field of slab

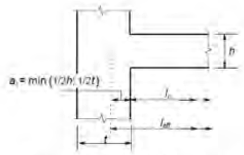
In our case $\frac{b}{a} = \frac{14.7}{3} = 4.9 > 2.5$ We can use the equivalent beam simplification on the transverse span

3.1.1 Effective span calculation

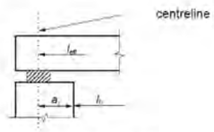
(1) The effective span, l_{eff}, of a member should be calculated as follows:

$$l_{eff} = l_n + a_1 + a_2 \qquad (5.8)$$

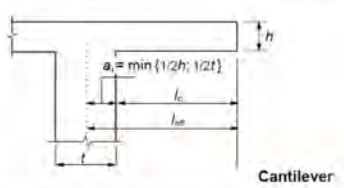
where:
l_n is the clear distance between the faces of the supports;
values for a_1 and a_2, at each end of the span, may be determined from the appropriate a_i values in Figure 5.4 where t is the width of the supporting element as shown.



Supports considered fully restrained



Bearing provided



Cantilever

Effective span calculation

Cantilever $l_{eff} = l_n + a_1 = 1.25 + \min\{0.125, 0.25\} = 1.38m$

Inner span
Fully restrained
$l_{eff} = l_n + a_1 + a_2 =$
$2.5 + \min\{0.125, 0.25\} + \min\{0.125, 0.25\} = 2.75m$

Inner span
Simply supported
$l_{eff} = l_n + a_1 + a_2 =$
$2.5 + 0.125 + 0.125 = 2.75m$

3.1.2 Bending moment due to permanent loads

Cantilever
$$M_{cant,g} = (6.25 + 5.75) \cdot \frac{1.38^2}{2} + 1.0 \cdot 1.38 =$$
$$11.4 + 1.4 = 12.8kN/m \cdot m$$

Inner span
Fully restrained
At the edge
$$M_{in\_edge,g} = (6.25 + 3.00) \cdot \frac{2.75^2}{12} = 5.83kN/m \cdot m$$

Inner span
Simply supported
In mid-span
$$M_{in\_mid,g} = (6.25 + 3.00) \cdot \frac{2.75^2}{8} = 8.74kN/m \cdot m$$

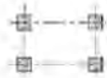
N.B.

If we had considered the load without dispersion we would have found a bending moment of:

$$M = Q\frac{l}{4} = 150\frac{2.75}{4} = 103kNm/m$$

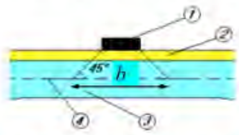
That is 21% greater than the value obtained before.

3.1.3 Bending moment due variable loads

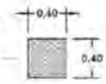


3.1.3.1 Load Model 1 (LM1)

Dispersal of concentrated loads



1 wheel dimension



2 pavement thickness = 0.13m

3 half slab thickness = 0.125m

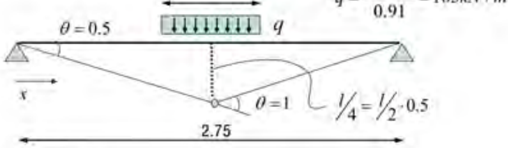
Width of the dispersed load area

$b = 0.40 + 2 \cdot (0.13 + 0.125) = 0.91m$

Maximum positive in mid span $\Rightarrow$ Scheme of simply supported beam.

Influence line of bending moment due to vertical forces (in transverse direction)

$b = 0.91m$ $\qquad q = \frac{150kN}{0.91} = 165kN/m$

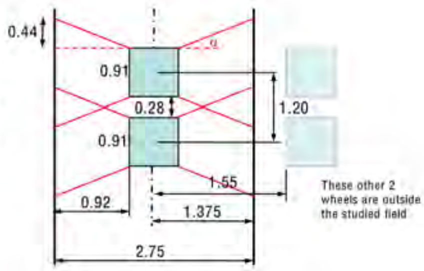


$$M = 2 \cdot \int_{(l-.91)/2}^{l/2} q \cdot y(x)dx = 2 \cdot \int_{0.92}^{1.375} q \cdot y(x)dx = 2 \cdot 0.455 \cdot q \cdot y_{ave} =$$
$$= 0.91 \cdot q \cdot \frac{1}{2}\left(\frac{2.75}{4} + \frac{0.92}{2}\right) = 0.52 \cdot q = 85.8KNm/m$$

What is the effective resisting cross section that bears the bending moment just calculated ?

In longitudinal direction we place 2 single wheels of each tandem along the mid-span longitudinal axis of the slab as the other two of each tandem fall on the other field of the slab.

The dispersion angle α is 25.6°



The dispersion length is: $l_{disp} = 0.91 + 2 \cdot 0.92 \cdot tg(25.6°) = 1.79m$

The overlapping length (painted in orange) spreads for

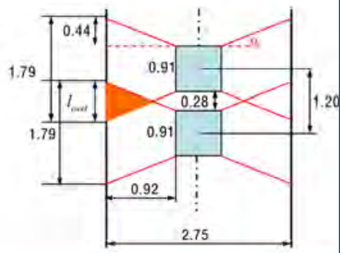
$l_{ovrl} = 0.44 \cdot 2 - 0.28 = 0.60m$

The acting bending moment is then

$M = M_{conc,trasv} / l_{disp} =$
$85.8 / 1.79 = 47.9kNm/m$

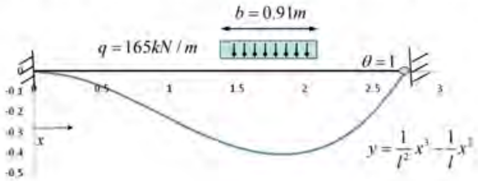
This value has to be doubled as the overlapping length is >0 so

$M_{conc,LM1} = 95.8kNm/m$



Maximum negative at the edges ⇒ Scheme of fully restrained beam.

Influence line of bending moment due to vertical forces (in transverse direction)



The bending moment is then

$$M(x) = \int_x^{x+0.91} q \cdot \left(\frac{1}{l^2}x^3 - \frac{1}{l}x^2\right)dx = \frac{q}{l}\int_x^{x+0.91} \frac{1}{l}x^3 - x^2 dx$$

$$M(x) = \frac{q}{l}\left[\frac{1}{4l}x^4 - \frac{1}{3}x^3\right]_x^{x+0.91}$$

To find the maximum value of M(x) we set equal to 0 its derivative and we find

$$M_{max} = M(x = 1.375) = -0.347q = -57.2kNm/m$$

What is the effective resisting cross section that bears the bending moment just calculated ?

In longitudinal direction we place 2 single wheels of each tandem in the position given us by the influence line just calculated.

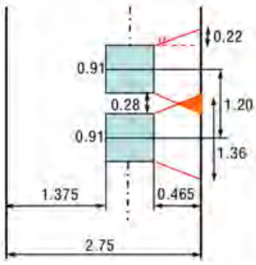
The dispersion angle α is 25.6°

The dispersion length is:

$$l_{disp} = 0.91 + 2 \cdot 0.465 \cdot tg(25.6°) = 1.36m$$

The overlapping length (painted in orange) is

$$l_{ovrl} = 0.22 \cdot 2 - 0.28 = 0.16m$$



The acting bending moment is then

$$M = M_{conc,trasv} / l_{disp} = 57.2 / 1.36 = 42.1kNm/m$$

This value has to be doubled as the overlapping length is >0 so

$$M_{conc,LM1} = 84.2kNm/m$$

For the distributed live load we get

Maximum positive in mid span ⇒ Scheme of simply supported beam.

$$M_{distr,LM1} = \frac{ql^2}{8} = \frac{9 \cdot 2.75^2}{8} = 8.51kNm/m$$

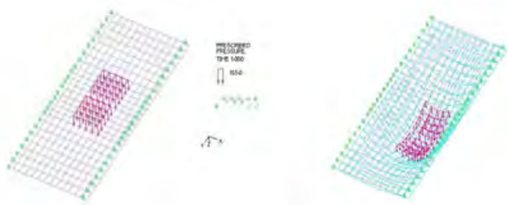
Maximum negative at the edges ⇒ Scheme of fully restrained beam.

$$M_{distr,LM1} = -\frac{ql^2}{12} = -\frac{9 \cdot 2.75^2}{12} = -5.67kNm/m$$

4. Finite element analysis

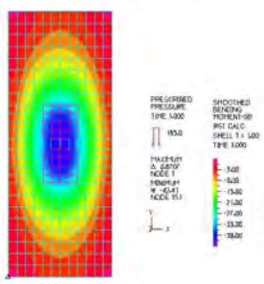
The same slab has been analyzed by F.E.M.

Maximum positive in mid span ⇒ hinge supports on the two sides.



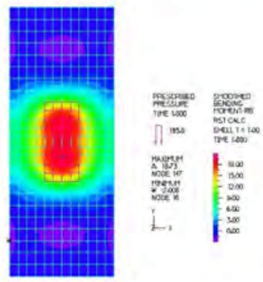
M_{yy} – transverse bending moment

$M_{yy,max}$ = 42.4 kNm



M_{xx} – longitudinal bending moment

$M_{xx,max}$ = 19.7 kNm



The transverse bending moment Myy = 42.4 kNm is similar to the acting bending moment calculated in slide 18 before overlapping .

$$M = M_{conc,trasv} / l_{disp} = 85.8 / 1.79 = 47.9kNm/m$$

F.E.M. analysis seems then to give smaller transverse bending moment but provides a better estimation of the longitudinal ones .

8. STRUT & TIE

I modelli tirante-puntone (S&T – *Strut and Tie*) sono utilizzati per la progettazione delle membrature in c.a. che non possono essere schematizzate come solidi snelli o "travi" alla Saint Venant quali ad esempio i plinti tozzi di fondazione, le travi parete, le mensole tozze, ma anche quelle regioni di travi snelle soggette a carichi concentrati o caratterizzate da brusche variazioni di sezione.

Il modello consiste nel ricondurre ad un traliccio reticolare, con tiranti e puntoni, ovvero con aste tese e compresse, il meccanismo strutturale resistente.

Le aste compresse (puntoni) sono materializzate dal calcestruzzo, mentre le aste tese (tiranti) sono costituite dalle armature.

Secondo l'EC2 i modelli tirante-puntone possono essere utilizzati per il progetto delle strutture allo stato limite ultimo (SLU) sia di zone di "continuità" sia di zone di "discontinuità".

Le regioni di continuità sono indicate come regioni "B" (da "Bernoulli" o dall'inglese "beam") e sono costituite da quelle zone di travi e piastre dove l'ipotesi di Saint Venant è soddisfatta.

Le regioni di discontinuità sono invece caratterizzate dalla presenza di discontinuità di tipo statico o geometrico (regioni tipo "D", dall'inglese "discontinuity"), dove l'ipotesi di Bernoulli non è soddisfatta. Le discontinuità di tipo statico comprendono la presenza di carichi concentrati, zone di appoggio di estremità, zone di ancoraggio di cavi di precompressione, ecc., mentre quelle di tipo geometrico includono brusche variazioni di sezione o di direzione dell'asse, presenza di aperture, elementi tozzi (mensole, travi parete, selle Gerber, ecc.).

I modelli tirante-puntone possono essere utilizzati anche per alcune verifiche agli stati limite di esercizio (SLE). A questo scopo l'EC2 suggerisce di orientare puntoni e tiranti lungo le linee isostatiche ricavate dall'analisi della struttura in fase non fessurata. In realtà è importante seguire questa strada anche per la verifica allo SLU

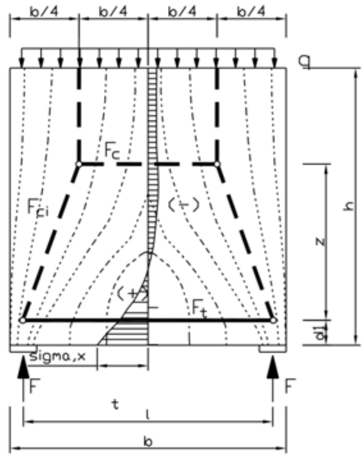


Figura 10.1 Modello S&T di una trave parete.

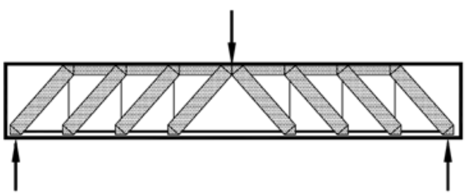


Figura 10.2 Traliccio di Ritter-Mörsch.

Una volta sostituiti eventuali carichi distribuiti con i carichi concentrati equivalenti, il progetto di una membratura in c.a. con il metodo S&T consiste nella schematizzazione del campo di sforzi presente nell'elemento strutturale mediante un traliccio reticolare di aste rettilinee in equilibrio con i carichi esterni.

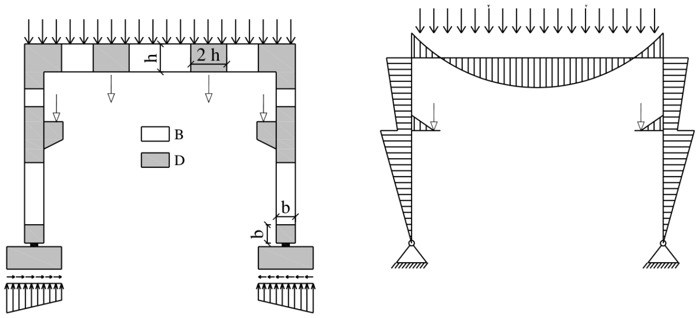


Figura 10.3 Suddivisione di un portale in zone "D" e "B" (ogni zona "D" si estende su ciascun lato della discontinuità per un tratto pari all'altezza dell'elemento strutturale).

Le curvature delle linee isostatiche vengono concentrate in corrispondenza di punti detti nodi, che rappresentano le intersezioni degli assi delle aste con quelli di altre aste, con i carichi applicati o con le reazioni vincolari. Una volta definita la geometria del traliccio, si calcolano gli sforzi normali nelle aste, si progettano le armature metalliche e si esegue la verifica di resistenza dei puntoni e dei nodi. !

La tecnica S&T rientra tra i metodi di analisi plastica delle strutture in c.a. ed in particolare può essere vista come un'applicazione del primo teorema dell'analisi limite (teorema del limite inferiore o teorema statico). Questo teorema può enunciarsi nel seguente modo: se la distribuzione degli sforzi all'interno di una struttura soddisfa tutte le condizioni di equilibrio (interne ed esterne) e non viola la condizione di resistenza dei materiali (*condizione di plasticità*), allora il carico associato a tale distribuzione non supera quello di collasso.

la geometria di un modello S&T deve essere scelta in modo tale che tutte le aste del traliccio possano attingere la loro resistenza di progetto e non si verifichi la rottura anticipata di un'asta o di un nodo.

è fortemente consigliabile definire la geometria del traliccio a partire dai campi tensionali ricavati nella fase elastica non fessurata, altrimenti la duttilità della membratura può non essere sufficiente ad attivare le resistenze di progetto di tutte le aste. In altre parole il campo discreto di sforzi normali del modello S&T può non instaurarsi a causa della rottura prematura di qualche elemento (asta o nodo).

PASSI PER L'APPLICAZIONE DEGLI S&T

1	Individuazione delle regioni di discontinuità ("D") in corrispondenza di carichi concentrati e/o di discontinuità geometriche
2	Definizione della estensione delle regioni di discontinuità mediante l'applicazione del postulato del Saint-Venant e conseguente suddivisione della struttura in regioni di continuità ("B") e regioni di discontinuità ("D")
3	Determinazione dello stato di sforzo e progetto delle armature delle regioni "B"
4	Calcolo delle forze agenti sul contorno delle regioni "D"
5	Definizione della geometria del traliccio S&T per ciascuna regione "D"
6	Calcolo degli sforzi nelle aste del traliccio di ogni regione "D"
7	Progetto delle armature (aree resistenti e disposizione geometrica), eventuale raffinamento del modello (per esempio per semplificare la disposizione delle armature), ricalcolo degli sforzi nelle aste e riprogetto delle armature
8	Verifica dei puntoni e dei nodi, eventuale ridimensionamento dei puntoni e dei nodi sulla base di considerazioni geometriche, raffinamento del modello, ricalcolo degli sforzi nelle aste e riverifica dei puntoni e dei nodi
9	Progetto dell'ancoraggio delle armature e delle armature diffuse per controllare la fessurazione

Identificazione della geometria del modello S&T

L'identificazione del modello tirante-puntone all'interno di una struttura in c.a. richiede innanzitutto l'individuazione delle regioni "D" di discontinuità e la definizione della loro estensione. Successivamente si suddivide la struttura in regioni "B" di continuità e regioni "D" di discontinuità

10.2.1 Posizione ed estensione delle regioni "D"

Le regioni "D" si collocano in corrispondenza di discontinuità statiche (carichi concentrati) e/o di discontinuità geometriche (ad es. brusche variazioni della linea d'asse). La loro e-stensione può essere determinata applicando il postulato del Saint Venant, secondo il quale a sufficiente distanza dall'area su cui sono applicati i carichi esterni, lo stato di tensione non dipende dalla particolare distribuzione di questi carichi, ma solo dalla risultante e dal momento risultante. La distanza alla quale questa condizione può ritenersi soddisfatta è all'incirca uguale alla maggiore delle dimensioni dell'area caricata (fig. 10.3, 10.4, 10.5).

Questa regola è approssimata; del resto essa serve solo come uno strumento qualitativo per l'identificazione delle regioni "D" e per il successivo sviluppo del modello tirante-puntone. Se l'elemento in esame è tozzo (es. trave parete con luce non superiore a due volte l'altezza, fig. 10.4a) la regione di discontinuità coincide con l'elemento stesso.

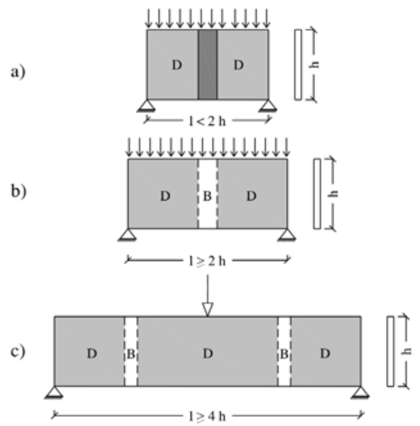


Figura 10.4 Suddivisione in zone "D" e "B" di travi appoggiate di diversa snellezza (ogni zona "D" si estende su ogni lato della discontinuità - rappresentata dal carico concentrato e dalla reazione vincolare - per un tratto pari all'altezza *h* dell'elemento strutturale).

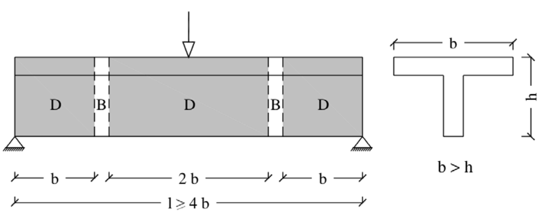


Figura 10.5 Suddivisione in zone "D" e "B" di una trave con sezione a T di larghezza *b* maggiore dell'altezza *h*.

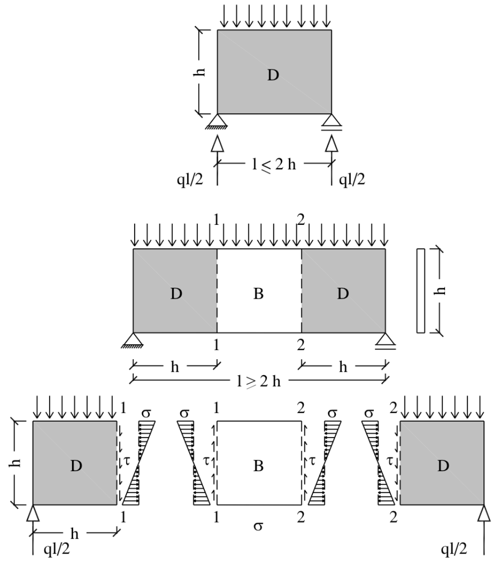


Figura 10.6 Determinazione delle forze al contorno di una regione "D": per la trave in alto ($l \leq 2h$) le forze al contorno coincidono con i carichi esterni e con le reazioni vincolari, mentre per la trave in basso ($l > 2h$) le forze sul bordo delle regioni "D" all'interfaccia con la regione "B" sono date dal momento risultante e dal taglio risultante nelle sezioni di estremità 1-1 e 2-2 della regione "B".

10.2.2 Calcolo degli sforzi e progetto delle armature nelle regioni "B"

Una volta suddivisa la struttura in regioni "B" e "D", le regioni "B" possono essere analizzate con i modelli validi e codificati per le travi snelle. Si possono così determinare le caratteristiche della sollecitazione in tutte le sezioni, in particolare in quelle che segnano il confine con le regioni "D", e progettare le armature.

10.2.3 Forze agenti sulle regioni "D"

Le forze al contorno delle regioni "D" sono date dai carichi e dalle reazioni vincolari direttamente applicati su di esse e dalle caratteristiche della sollecitazione agenti nelle sezioni terminali delle zone "B" adiacenti (vedi par. 10.2.2). Per gli elementi tozzi formati solo da regioni tipo "D" (ad esempio travi parete, plinti di fondazione, mensole) le forze al contorno coincidono con i carichi applicati e le reazioni vincolari.

Una volta determinate tutte le forze agenti su un'assegnata regione "D", prima di passare all'individuazione del modello S&T, occorre verificarne l'equilibrio di corpo rigido.

Eventuali forze distribuite sui bordi sono schematizzate con forze concentrate equivalenti: per esempio nelle travi della figura 10.7 il carico uniforme e quello trapezoidale sono suddivisi ciascuno in due parti di intensità pari alle corrispondenti reazioni vincolari. Infine eventuali forze distribuite agenti all'interno delle regioni "D", come per esempio il peso proprio, possono essere sostituite da forze distribuite sui bordi, che a loro volta possono essere ricondotte a forze concentrate equivalenti (figura 10.8).

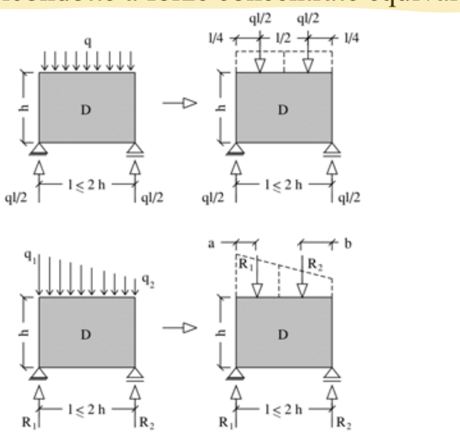


Figura 10.7 Schematizzazione di carichi distribuiti sui bordi (a sinistra) con forze concentrate equivalenti (a destra).

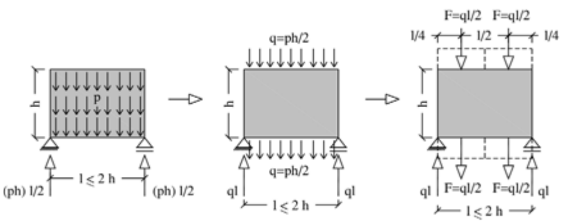


Figura 10.8 Schematizzazione di un carico distribuito per unità di superficie all'interno di una trave parete (carico p a sinistra) con carichi distribuiti sui bordi (carichi q al centro) e forze concentrate equivalenti (carichi F a destra).

10.3 Scelta del modello S&T di progetto

All'interno di un'assegnata regione "D" di discontinuità è possibile identificare più di un modello S&T in equilibrio con le forze applicate sul suo contorno e che soddisfa la condizione di resistenza dei materiali (figura 10.9).

In sintesi il modello S&T va scelto in modo tale che in nessun elemento del traliccio sia superata la capacità di deformazione del materiale, prima che gli sforzi normali di tutte le aste abbiano attinto i valori di progetto.

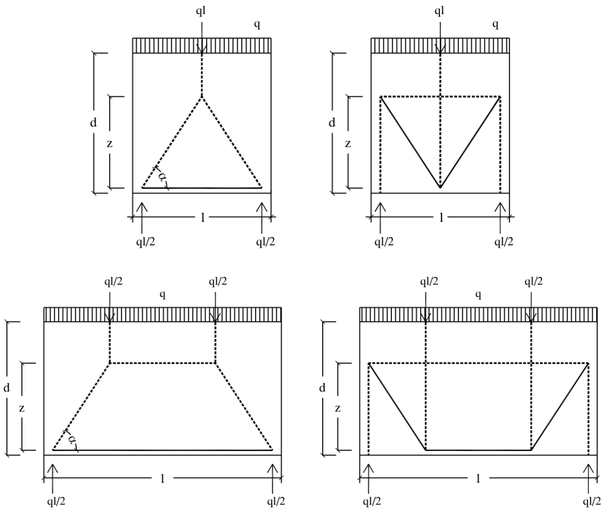


Figura 10.9 Possibili modelli S&T per una trave parete (linee tratteggiate = puntoni, linee continue = tiranti).

Nel selezionare tra tutti i possibili tralicci quello da utilizzare per il progetto di una regione "D" occorre tenere presente che i carichi tendono a seguire i percorsi ai quali corrispondono le minime tensioni e le minime deformazioni. Poiché i tiranti sono molto più deformabili dei puntoni di calcestruzzo, il modello con il minore sviluppo complessivo di tiranti (meno tiranti e più corti) è quello che funziona meglio.

Metodo dei percorsi di carico (load path method)

Con questo metodo sviluppato da Schlaich si uniscono le forze applicate sul contorno della regione "D" con "percorsi di carico" ad andamento curvilineo. A questo scopo il diagramma degli sforzi viene suddiviso in modo tale che ogni carico agente su un lato della regione trovino la sua controparte sul lato opposto. Ogni percorso fa capo a due forze situate su lati opposti della regione e ha nei punti di applicazione delle forze la loro stessa direzione. Possono inoltre esistere percorsi che iniziano e terminano sullo stesso lato e hanno una forma ad "U", come ad esempio il percorso B-B della trave parete in figura 10.11. Nel disegnare i percorsi di carico occorre evitare intersezioni ed è opportuno seguire la via più breve. I percorsi di carico ad andamento curvilineo sono poi sostituiti da percorsi poligonali, i quali sono infine integrati con altre aste per assicurare l'equilibrio dei nodi. Si ottiene così il modello S&T cercato.

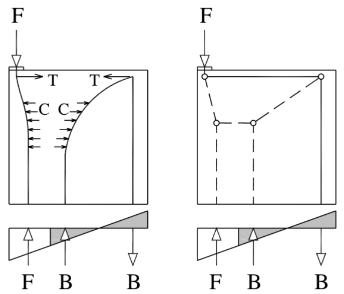


Figura 10.11 Percorsi di carico di una trave parete (puntoni = linee tratteggiate, tiranti = linee continue).

Metodo delle linee isostatiche

Con il metodo delle linee isostatiche le aste del modello tirante-puntone vengono collocate nella posizione e direzione delle linee isostatiche delle tensioni calcolate in campo elastico lineare. A questo scopo si esegue un'analisi elastica lineare agli elementi finiti della membratura in c.a. e si raggruppano i nodi dove gli sforzi principali hanno all'incirca la stessa intensità e direzione, separando gli sforzi positivi da quelli negativi. Ogni gruppo viene poi suddiviso in sottogruppi formati da nodi vicini ed ognuno di questi viene infine sostituito da una linea retta che passa per il centro di gravità degli sforzi principali del sottogruppo e ha direzione uguale a quella media di tali sforzi (Harisis e Fardis, 1991).

10.5 Regole pratiche per l'identificazione del traliccio

Per l'identificazione del modello a traliccio il Model Code 1990 (CEB/FIP, 1991) suggerisce una serie di regole pratiche, che vengono richiamate di seguito.

Inizialmente conviene adottare modelli S&T semplici, con poche aste, che possono essere perfezionati successivamente (fig. 10.15).

Le aste del traliccio vanno orientate nella direzione delle linee isostatiche ricavate in fase elastica non fessurata, in modo da riprodurre con sufficiente approssimazione il flusso medio degli sforzi in campo elastico. Tuttavia conviene disporre i tiranti in modo da semplificare la distribuzione delle armature, utilizzando armature parallele oppure ortogonali ai bordi dell'elemento strutturale (come per esempio nella figura 10.15c). Nelle zone meno sollecitate infatti la struttura è in grado di adattarsi alla geometria del traliccio scelto, anche se questo si discosta un po' dal campo di sforzi in fase I.

Gli angoli tra i puntoni ed i tiranti devono preferibilmente essere almeno pari a 45°, fatta eccezione per quei nodi dove un puntone interseca due tiranti ortogonali, caso per il quale l'EC2 fornisce una riduzione del 25% della resistenza di progetto del calcestruzzo rispetto ad un nodo tutto compresso (vedi par. 10.10.5); in particolare bisogna evitare angoli inferiori a 30° (fig. 10.16).

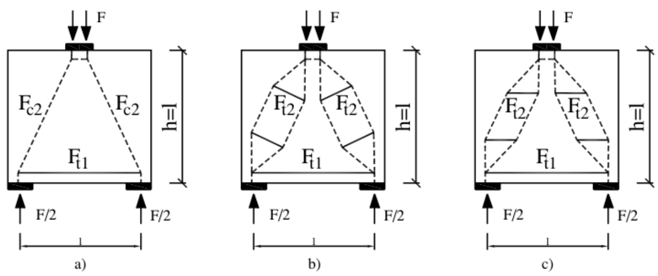


Figura 10.15 Trave parete soggetta a carico concentrato: a) modello S&T di base, b) modello raffinato con tiranti inclinati, c) modello raffinato con tiranti orizzontali (puntoni = linee tratteggiate, tiranti = linee continue).

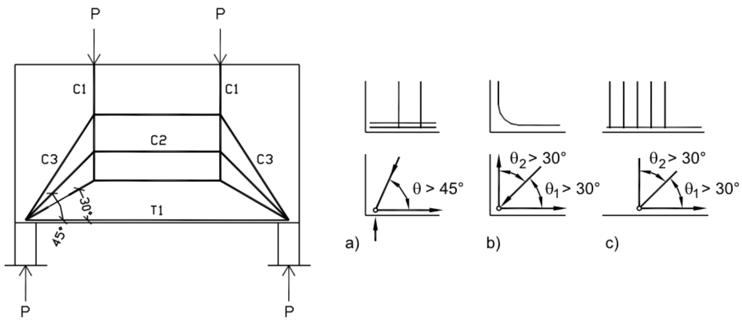


Figura 10.16 Variazione del modello S&T di una trave parete al variare dell'angolo tra puntoni inclinati e tirante orizzontale da 30° (valore minimo) a valori maggiori di 45°: intersezione di un puntone con uno (a) o con due tiranti ortogonali (b e c).

La limitazione sull'angolo tra puntoni e tiranti confluenti in un nodo serve a limitare la fessurazione ed evitare che l'accorciamento dei puntoni e l'allungamento dei tiranti avvengano all'incirca nella stessa direzione. Questa limitazione sull'angolo si ritrova anche nel traliccio ad inclinazione variabile utilizzato per il progetto a taglio delle travi snelle, dove l'angolo θ tra le bielle compresse e le armature longitudinali non può assumere valori inferiori a 21,8° (cot $\theta \leq 2{,}5$).

Le forze concentrate applicate sul bordo di un elemento strutturale tendono a diffondersi secondo un angolo di circa 32,5°, come si deduce dalla teoria dell'elasticità in un semispazio infinito (fig. 10.17).

In funzione della forma e delle condizioni al contorno della regione "D" l'angolo di diffusione di un carico concentrato varia ed il modello S&T deve essere adattato di conseguenza. Con riferimento alla trave parete mostrata nella figura 10.18, al crescere del rapporto L/h, l'angolo δ di diffusione del carico aumenta e l'angolo γ tra il puntone inclinato e il tirante orizzontale diminuisce. In particolare per $L = h$ (figura 10.18a) si ha $\delta \cong 26{,}5°$ e $\gamma \cong 63{,}5° > 45°$, mentre per $L = 3h$ (figura 10.18b) risulta $\delta \cong 56{,}5°$ e $\gamma \cong 33{,}5° < 45°$.

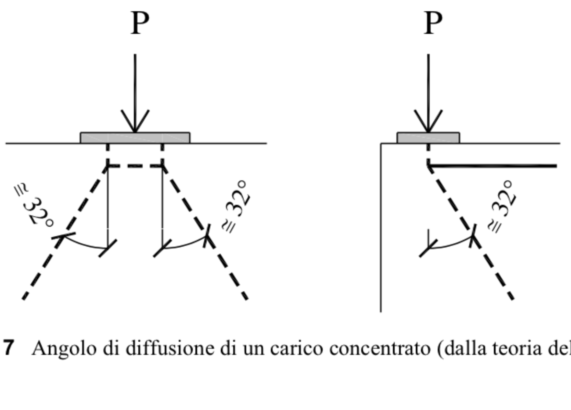


Figura 10.17 Angolo di diffusione di un carico concentrato (dalla teoria dell'elasticità).

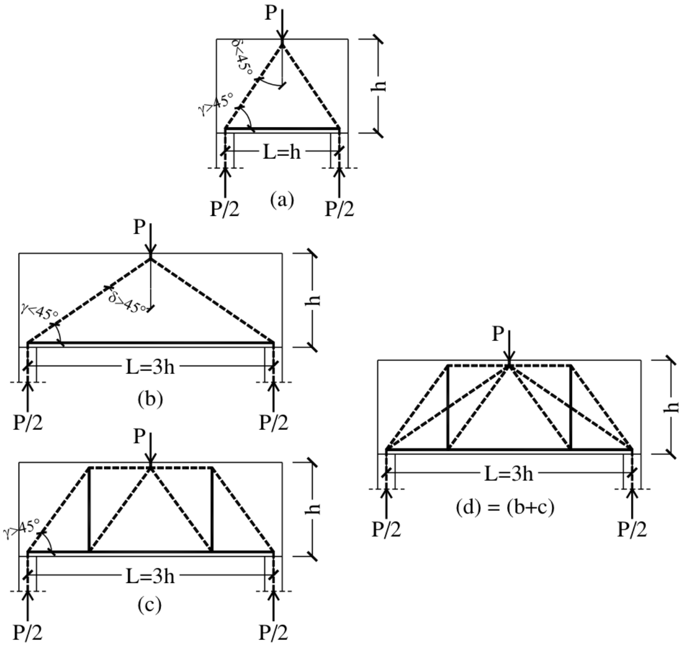


Figura 10.18 Modelli tirante-puntone di una trave parete al variare della snellezza.

Nel caso (b), affinché l'angolo γ non risulti inferiore a 45°, è opportuno modificare il modello S&T inserendo su ogni lato un tirante verticale posto a metà strada tra l'asse dell'appoggio ed il carico applicato. Si ottiene così il traliccio mostrato nella figura 10.18c, che può anche essere combinato con il modello (b) a formare un traliccio iperstatico. Per il calcolo degli sforzi nelle aste di quest'ultimo modello S&T occorre pertanto considerare la rigidezza delle aste.

Gli assi dei puntoni devono essere posizionati a sufficiente distanza dai bordi dell'elemento strutturale per tenere conto dell'ingombro trasversale dei puntoni stessi (figura 10.19); lo stesso discorso vale anche per i tiranti formati da armature distribuite su più strati e per i nodi.

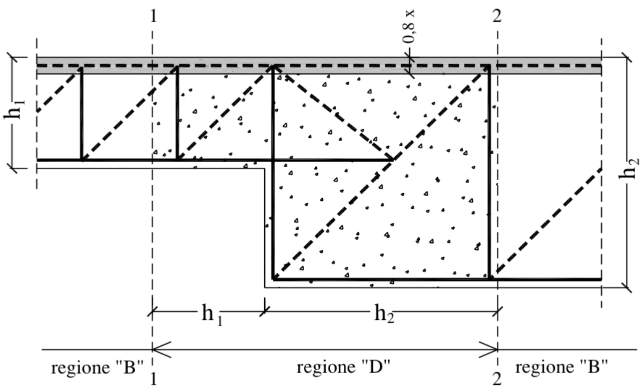


Figura 10.19 Distanza minima delle aste dai bordi: il corrente superiore avrà una distanza dal bordo non inferiore a 0,4 x, dove x è la profondità dell'asse neutro calcolata nella regioni "B" adiacenti (si può assumere $x = \max(x_1; x_2)$ essendo x_1 e x_2 le profondità dell'asse neutro nella sezione 1-1 a sinistra della regione "D" e nella sezione 2-2 a destra).

10.7 Verifica del traliccio

La rottura di un traliccio S&T può essere causata da:
- snervamento di uno o più tiranti;
- schiacciamento di un puntone di calcestruzzo;
- schiacciamento di un nodo;
- sfilamento di uno o più tiranti in corrispondenza di un nodo.

Se un elemento strutturale in c.a. è stato progettato correttamente utilizzando un traliccio tirante-puntone, la rottura si verifica secondo la prima delle quattro modalità sopra elencate [2]. → Rottura duttile

[2] Le Norme Tecniche per le Costruzioni (D.M. 14.1.2008) al p.to 4.1.2.1.5 prescrivono che nella verifica dei modelli S&T la resistenza associata allo snervamento delle armature sia inferiore a quella associata agli altri meccanismi di collasso, per garantire una rottura di tipo duttile dell'elemento strutturale (gerarchia delle resistenze).

Le tensioni limite nel calcestruzzo sono scelte in modo da scongiurare lo schiacciamento localizzato del calcestruzzo o la fessurazione longitudinale ("splitting") dei puntoni e dei nodi e sono generalmente basate sul grado di confinamento del calcestruzzo. Così nei nodi in cui convergono tre o più puntoni si possono accettare livelli tensionali maggiori, grazie all'elevato grado di confinamento del materiale, mentre nei nodi dove confluiscono uno o più tiranti il livello tensionale nel calcestruzzo deve essere minore.

10.8 Progetto delle armature

Le armature metalliche sono utilizzate sia come tiranti del modello tirante-puntone (armature principali) sia come elementi atti a resistere alle forze di trazione dovute alla diffusione del carico, che si instaurano in direzione ortogonale ai campi di compressione (armatura di frettaggio, par. 10.6.1). L'area A della sezione di ciascun tirante si ottiene dividendo lo sforzo normale di progetto N_{Ed} per la resistenza di progetto dell'acciaio: $A \geq N_{\mathrm{Ed}}/f_{yd}$.

L'armatura principale va distribuita sull'altezza del nodo nel quale è ancorata; i tiranti formati da più barre vanno disposti su più strati per evitare la congestione delle barre d'armatura e per migliorare la forma del nodo. Nel nodo le armature possono essere ancorate

Si assume che la lunghezza di ancoraggio inizi in corrispondenza della sezione dove le traiettorie degli sforzi di compressione nel puntone intercettano la barra o le barre di armatura e sono deviate da queste (figura 10.30). Le armature devono essere poi prolungate almeno fino alla faccia opposta del nodo, ossia devono attraversarlo completamente.

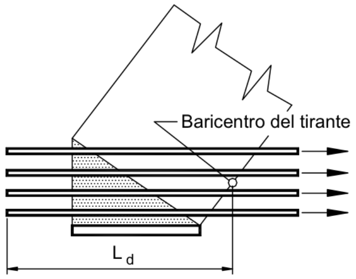


Figura 10.30 Ancoraggio in rettilineo delle armature in un nodo.

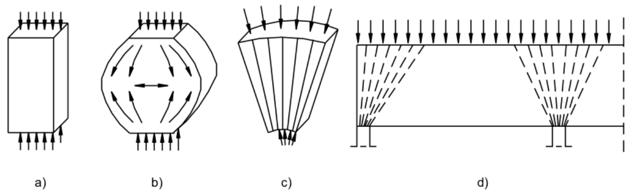


Figura 10.31 Campi tensionali di compressione: (a) “prismatico”, (b) a “collo di bottiglia”, (c, d) a “ventaglio”.

10.9 Verifica dei puntoni

I campi di compressione, ovvero i puntoni, sono sostanzialmente di tre tipi (figura 10.31): paralleli o prismatici, a “collo di bottiglia”, diffusivi a ventaglio.

I campi prismatici a) sono tipici delle regioni “B” e possono essere trattati alla stregua dei pilastri soggetti a sforzo normale centrato oppure del corrente compresso nel traliccio resistente delle travi inflesse.

I campi tensionali b), c) e d) riproducono invece la diffusione dello stato di sforzo: quelli a collo di bottiglia rappresentano stati di sforzo biassiali o triassiali nelle zone di applicazione di carichi concentrati, mentre il ventaglio rappresenta campi di compressione prevalentemente rettilinei (curvatura trascurabile) e pertanto con sforzi trasversali trascurabili. Quest’ultimo è il caso per esempio dei campi di compressione che in una trave parete soggetta a carico uniformemente distribuito si sviluppano a partire dalla superficie di applicazione del carico fino agli appoggi (figura 10.31d).

La verifica dei puntoni consiste nel controllare che la massima compressione del calcestruzzo sia inferiore alla resistenza di progetto. Inoltre per campi di compressione a collo di bottiglia occorre progettare un’apposita armatura in grado di assorbire le trazioni trasversali, calcolate con gli schemi riportati nel par. 10.6.1.

In assenza di campi di tensione trasversali o in presenza di campi trasversali di compressione, la resistenza di progetto di un puntone di calcestruzzo coincide con la resistenza di progetto del calcestruzzo:

$$\sigma_{Rd,\max} = f_{cd} \qquad [(6.55)]$$

mentre se sono presenti tensioni trasversali di trazione, la resistenza di progetto è inferiore a quella del calcestruzzo ed è data dalla seguente espressione:

$$\sigma_{Rd,max} = 0{,}60\ \upsilon'\ f_{cd} \qquad [(6.56)]$$

dove: $\upsilon' = 1 - f_{ck}/250 = (5/3)\,\upsilon$ [(6.57N)]

con $\upsilon = 0{,}6\,(1 - f_{ck}/250)$ (f_{ck} in N/mm^2). [(6.6N)]

Secondo le NTC 2008, per calcestruzzi di classe non superiore alla C70/85 il coefficiente υ è pari a 0,5 [3]; essendo poi υ' e υ legati dalla relazione $\upsilon' = (5/3)\upsilon$, si ha: $\upsilon' = 5/6 = 0{,}83$. Per le classi di resistenza C80/95 e C90/105 si utilizza invece il valore suggerito dall'EC2: $\upsilon' = 1 - f_{ck}/250$.

Per $f_{ck} \leq 70$ N/mm^2 la resistenza di progetto dei nodi in presenza di tensioni trasversali di trazione assume pertanto il seguente valore:

$$\sigma_{Rd,\max} = 0{,}60 \cdot 0{,}83 \cdot f_{cd} = 0{,}5\, f_{cd}$$ [4]

Il valore ridotto della resistenza dei puntoni in presenza di campi trasversali di trazione tiene conto del fatto che le fessure indotte dalle trazioni trasversali, interrompendo la compattezza del puntone, riducono la resistenza del calcestruzzo, anche quando esse sono parallele al puntone. La riduzione di resistenza è poi più accentuata per fessure inclinate rispetto alla direzione del puntone.

Il prospetto 10.8 elenca le espressioni della resistenza di progetto dei puntoni a seconda della presenza e del tipo di tensioni trasversali e della classe di resistenza del calcestruzzo.

Prospetto 10.8 Resistenze di progetto dei puntoni.

Tipo di tensioni trasversali	Resistenza di progetto	
nulle o di compressione	f_{cd}	per tutte le classi di resistenza del calcestruzzo
di trazione	$0{,}5\, f_{cd}$	se $f_{ck} \leq 70$ N/mm^2
	$0{,}41\, f_{cd}$	se $f_{ck} = 80$ N/mm^2
	$0{,}38\, f_{cd}$	se $f_{ck} = 90$ N/mm^2

[3] La resistenza a compressione del calcestruzzo fessurato per taglio è espressa come $0{,}5\, f_{cd}$ nelle NTC 2008 (formula 4.1.19) e come $\upsilon_1 f_{cd}$ nell'EC2 (formula 6.14), ne discende che secondo le NTC risulta $\upsilon_1 = 0{,}5$. Inoltre la Nota 1 al p.to 6.2.3(3) dell'EC2 raccomanda di assumere $\upsilon = \upsilon_1$, pertanto risulta anche $\upsilon = 0{,}5$.

[4] Si fa osservare che, in presenza di trazioni trasversali, la resistenza del calcestruzzo dei puntoni è pari a quella del calcestruzzo fessurato per taglio ($f'_{cd} = 0{,}5\, f_{cd}$), che compare nel calcolo della resistenza a taglio-compressione della travi dotate di armature trasversali (4.1.19-NTC).

10.9.1 Armature trasversali

Generalmente non è necessario verificare gli sforzi nei puntoni se le verifiche nei nodi sono soddisfatte e se viene adottata un'apposita armatura trasversale rispetto all'asse dei puntoni. La forza trasversale totale tra un'estremità ed il bulbo centrale di un puntone può essere assunta al massimo pari al 25 % dello sforzo di compressione nel puntone, come si evince dal modello S&T "D1" per la diffusione di un carico concentrato (vedi par. 10.6.1).

10.10 Verifica dei nodi

Un nodo di un modello S&T è definito come un volume di calcestruzzo contenuto all'interno delle intersezioni tra i campi di compressione dei puntoni e tra questi e le barre di armatura e/o le forze esterne.

I nodi sono "zone critiche" perché sede di un brusco cambiamento di direzione delle forze con conseguente concentrazione degli sforzi. In base a geometria ed estensione i nodi si classificano in "concentrati" e "diffusi": nei primi gli sforzi sono deviati in una zona molto ristretta rispetto alla lunghezza delle aste che vi confluiscono (figura 10.33a, b), nei secondi questa zona è più estesa (figura 10.33c, d). Normalmente i nodi "diffusi" sono meno critici e per essi non è necessaria la verifica degli sforzi di compressione nel calcestruzzo, viceversa in quelli "concentrati" occorre verificare sia lo sforzo massimo di compressione nel conglomerato sia l'ancoraggio delle armature.

Esempi di nodi concentrati sono: i punti di applicazione di carichi concentrati, gli appoggi, le zone di ancoraggio dove si ha concentrazione di armature ordinarie o da precompressione, le piegature delle armature, gli angoli dei portali.

Le forze che agiscono sui nodi devono essere equilibrate; inoltre per nodi tridimensionali occorre considerare la diffusione degli sforzi su due piani ortogonali (par. 10.10.4.1).

La capacità resistente dei nodi è strettamente connessa al dimensionamento e alla disposizione delle armature, in particolare al loro ancoraggio.

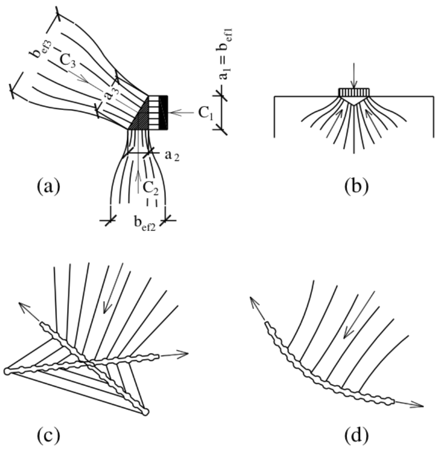


Figura 10.33 Nodi concentrati (a,b) e nodi diffusi (c,d).

10.10.1 Tipi di nodi

In funzione del tipo di aste che vi confluiscono, i nodi possono essere suddivisi in quattro tipologie (fig. 10.34):

- CCC: tre puntoni,
- CCT: due puntoni ed un tirante,
- CTT: un puntone e due tiranti,
- TTT: tre tiranti.

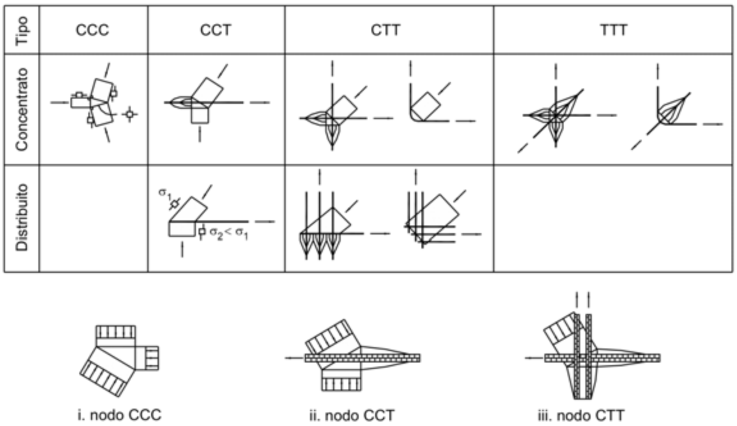


Figura 10.34 Tipi di nodo.

10.10.2 Resistenza dei nodi

Una volta che la geometria e le dimensioni del nodo sono state definite occorre eseguire la verifica a schiacciamento del calcestruzzo. La resistenza di progetto dei nodi è un'aliquota ν della resistenza a compressione del calcestruzzo, dove il coefficiente ν è detto *fattore di efficienza*.

Il fattore di efficienza assume valori diversi per i nodi CCC, CCT e CTT: i nodi CCT e CTT hanno fattori di efficienza più piccoli dei nodi CCC, perché le tensioni di trazione trasmesse per aderenza dalle barre d'armatura ne diminuiscono la resistenza.

Ne derivano tre diversi valori della resistenza: $\sigma_{1Rd,max}$ per i nodi CCC, $\sigma_{2Rd,max}$ per i nodi CCT e $\sigma_{3Rd,max}$ per i nodi CTT (prospetto 10.9). Il prospetto 10.10 riporta i valori di $\sigma_{1Rd,max}$, $\sigma_{2Rd,max}$, $\sigma_{3Rd,max}$ al variare della classe di resistenza del calcestruzzo.

Di seguito si descrivono i vari tipi di nodo.

Prospetto 10.9 Resistenze di progetto dei nodi.

Tipo di nodo	Fattore di efficienza	Resistenza di progetto [1]	
CCC	$k_1 = 1,0$	$\sigma_{1Rd,max}$	$1,0\ (\nu' f_{cd}) = 0,83\ f_{cd}$
CCT	$k_2 = 0,85$	$\sigma_{2Rd,max}$	$0,85\ (\nu' f_{cd}) = 0,705\ f_{cd}$
CTT	$k_3 = 0,75$	$\sigma_{3Rd,max}$	$0,75\ (\nu' f_{cd}) = 0,622\ f_{cd}$
TTT	–	–	$\min (f_{yd}; f_{y,ader})$
Compressione triassiale (3D)		$\sigma_{Rd,max}$	$3,00\ (\nu' f_{cd}) = 2,49\ f_{cd}$

[1] Valori validi per calcestruzzi di classe ≤ C70/85.

Prospetto 10.10 Valori delle tensioni $\sigma_{1Rd,max}$, $\sigma_{2Rd,max}$, $\sigma_{3Rd,max}$ in N/mm^2 ($\sigma_{1Rd,max} > \sigma_{2Rd,max} > \sigma_{3Rd,max}$).

f_{ck} (N/mm^2)	f_{cd} (N/mm^2)	$\sigma_{1Rd,max}$ (N/mm^2)	$\sigma_{2Rd,max}$ (N/mm^2)	$\sigma_{3Rd,max}$ (N/mm^2)
12	6,80	5,64	4,80	4,23
16	9,07	7,53	6,40	5,64
20	11,33	9,41	8,00	7,06
25	14,17	11,76	10,00	8,82
28	15,87	13,17	11,19	9,88
30	17,00	14,11	12,00	10,58
35	19,83	16,46	14,00	12,35
40	22,67	18,81	16,00	14,11
45	25,50	21,17	18,00	15,87
50	28,33	23,52	20,00	17,64
55	31,17	25,87	22,00	19,40
60	34,00	28,22	24,00	21,17
70	39,67	33,60	28,56	25,20

10.10.3 Nodi compressi (CCC)

I nodi tutti compressi si trovano in corrispondenza di carichi concentrati e sopra gli appoggi intermedi di travi snelle e travi parete, così come negli angoli rientranti di telai, nelle mensole tozze ed in corrispondenza di aperture.

Questi nodi sono caratterizzati dalla presenza di uno stato di compressione biassiale; il loro contorno viene schematizzato con superfici piane, che individuano regioni triangolari o poligonali.

La massima tensione che può essere applicata ai bordi di un nodo compresso ossia la tensione di progetto, è data dalla seguente espressione:

$$\sigma_{1\text{Rd,max}} = k_1 \, \upsilon' f_{cd} = 1{,}0 \cdot 0{,}83 \cdot f_{cd} = 0{,}83 \, f_{cd} \text{ (per } f_{ck} \leq 70 \text{ N/mm}^2\text{)} \qquad [(6.60)]$$

La figura 10.36 mostra due diversi tipi di nodi compressi con tre puntoni complanari. Il primo nodo è relativo all'appoggio di continuità di una trave parete ed il secondo è tipico degli angoli rientranti dei portali soggetti a momento flettente negativo.

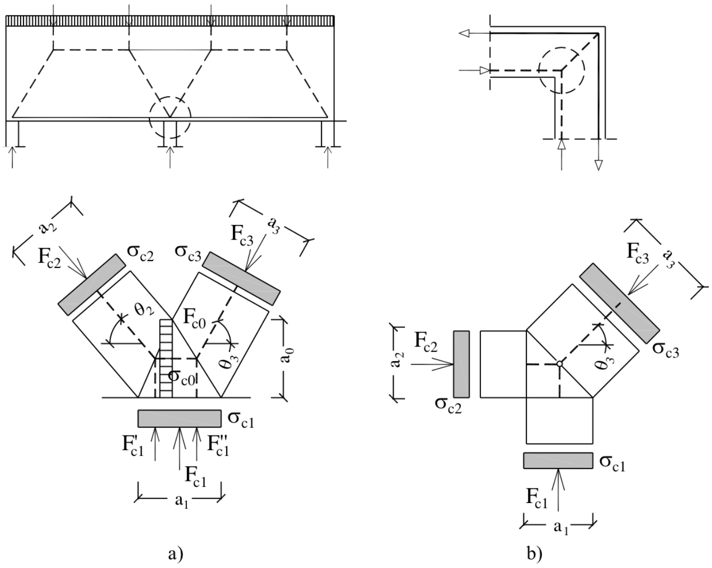


Figura 10.36 Nodi compressi con tre puntoni complanari: a) nodo sull'appoggio di continuità di una trave parete, b) nodo nell'angolo di un portale soggetto a momento negativo (che tende a chiudere il nodo).

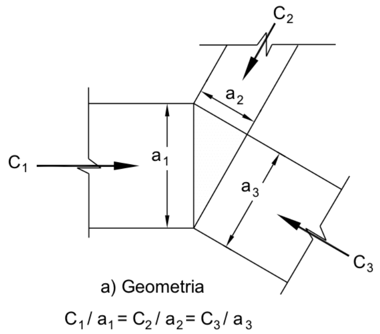


Figura 10.37 Nodo idrostatico.

10.10.3.1 *Nodi compressi idrostatici*

Se un nodo compresso è delimitato da facce ortogonali ai puntoni e le dimensioni delle sezioni trasversali dei puntoni sono proporzionali agli sforzi normali di compressione, le tensioni normali su tutte e tre le facce nodali sono uguali. In questo caso lo stato di sforzo nel nodo è idrostatico, ossia la tensione normale è la stessa su qualsiasi giacitura e d'ora in poi un nodo così fatto sarà denominato "nodo idrostatico" (figura 10.37).

A rigore il termine idrostatico implicherebbe l'uguaglianza di tutte e tre le tensioni principali; convenzionalmente nei modelli S&T piani questo termine è utilizzato per indicare lo stato di sforzo di un nodo dove sono uguali solo le due tensioni principali nel piano del modello. Se nel piano ortogonale al modello S&T si ha una significativa diffusione degli sforzi, lo stato tensionale fuori dal piano va studiato con un altro modello S&T (vedi par. 10.10.4.1).

In un nodo idrostatico non ci sono tensioni tangenziali, tuttavia il calcestruzzo della regione nodale è in grado di sopportarle, cosicché si possono accettare anche nodi non idrostatici. Per i nodi non idrostatici Schlaich (1987) raccomanda che il rapporto tra le tensioni di compressione massima e minima non sia superiore a due.

Peraltro non è facile avere nodi idrostatici, perché la geometria del nodo è dettata dalla disposizione delle armature e/o dalle dimensioni delle piastre di appoggio, dalle dimensioni trasversali dei pilastri oltre che dalla geometria del traliccio. Per esempio per il nodo in alto a destra della trave parete di spessore b mostrata nella figura 10.38, si può fissare la larghezza a_1 del puntone verticale e la larghezza a_2 di quello orizzontale affinché la tensione normale σ sulle due facce nodali 1-2 e 1-3 sia la stessa; se si pone $\sigma = \sigma_{1\mathrm{Rd,max}}$ (ossia pari alla resistenza del calcestruzzo in un nodo CCC) si ricavano i seguenti valori di a_1 e a_2:

$$a_1 = C_1 / (\sigma_{1\mathrm{Rd,max}} \cdot \mathrm{b})$$

$$a_2 = C_2 / (\sigma_{1\mathrm{Rd,max}} \cdot \mathrm{b})$$

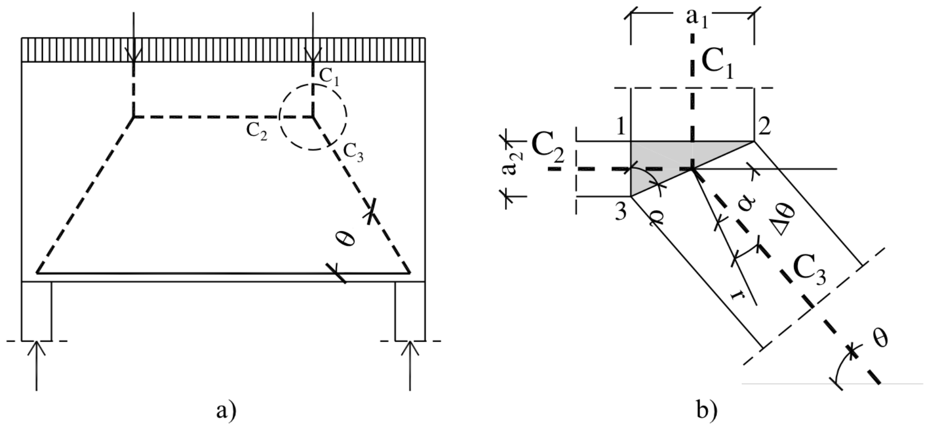


Figura 10.38 Esempio di nodo non idrostatico.

La larghezza a_3 della terza faccia del nodo (faccia 2-3) vale

$$a_3 = \sqrt{a_1^2 + a_2^2}$$

ed è inclinata dell'angolo $\alpha = \arctan\,(a_1 / a_2)$ rispetto all'orizzontale.

Si evince pertanto che il nodo è idrostatico solo se il terzo puntone è orientato lungo la direzione r, ortogonale alla faccia 2-3, mostrata nella figura 10.38b, ossia se la sua inclinazione θ con l'orizzontale coincide con α. In realtà la direzione del terzo puntone è già definita dalla geometria del traliccio ed in generale è ruotata di un angolo $\Delta\theta$ rispetto alla direzione r.

10.10.3.2 *Nodi compressi non idrostatici*

Nei nodi non idrostatici le facce nodali non sono ortogonali agli assi dei puntoni e le tensioni di compressione nei puntoni sono diverse. La verifica di un nodo non idrostatico può essere ricondotta a quella del nodo idrostatico con la stessa base, secondo la procedura descritta di seguito (fig. 10.39).

Si consideri il nodo con tre puntoni complanari, che potrebbe rappresentare per esempio il nodo di continuità di una trave parete. Se le facce a_2 e a_3 dei due puntoni inclinati formano un angolo diverso da 90° con le direzioni dei due puntoni diagonali, solo sulla faccia a_1 la pressione σ_{c1} risulta ortogonale e quindi coincide con una delle due tensioni principali nella regione nodale. Circa il valore dell'altra tensione principale σ_{c0}, agente sulla faccia a_0, normale alla faccia a_1 di base, si dimostra che:

$$\sigma_{c0} \le \sigma_{c1} \quad \text{se } a_0 \ge a_{0,\mathrm{idr}}$$

$$\sigma_{c0} > \sigma_{c1} \quad \text{se } a_0 < a_{0,\mathrm{idr}}$$

dove a_0 è l'altezza del nodo e $a_{0,\mathrm{idr}}$ è l'altezza del nodo idrostatico con la stessa base a_1.

La dimostrazione delle espressioni appena scritte è immediata se si fa riferimento alla figura 10.39, dove sono stati disegnati sia il nodo non idrostatico ABC sia il nodo idrostatico DBC con la stessa base.

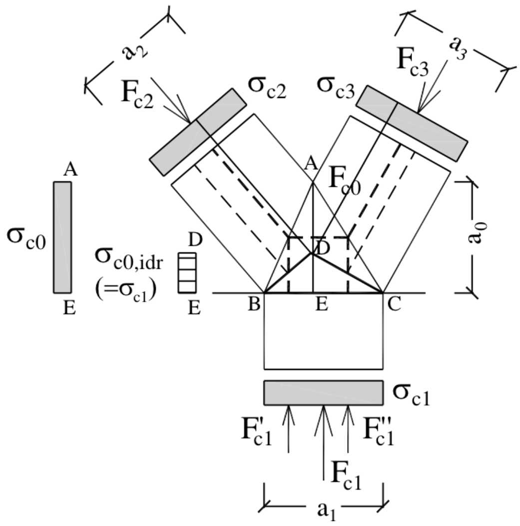


Figura 10.39 Nodo non idrostatico (ABC) e nodo idrostatico (DBC): i due nodi hanno la stessa base, ma altezze diverse (il nodo idrostatico è di altezza minore).

Per il nodo idrostatico DBC la tensione sulla faccia DE ($a_{0,\mathrm{idr}}$) è uguale alla tensione idrostatica σ_{c1}, mentre per il nodo non idrostatico ABC la tensione σ_{c0} sulla faccia AE (a_0), essendo AE ≥ DE ($a_0 \ge a_{0,\mathrm{idr}}$), è certamente inferiore a σ_{c1}. In questo caso è sufficiente verificare la sola tensione σ_{c1}.

In modo analogo si dimostra che se l'altezza del nodo è minore di quella del nodo idrostatico con la stessa base a_1, allora la tensione principale σ_{c0} sulla faccia AE è maggiore di quella sulla base BC ($\sigma_{c0} > \sigma_{c1}$) e quindi occorre verificare la tensione σ_{c0}.

Più in generale lo stato di sforzo in un nodo non idrostatico può essere determinato secondo la seguente procedura (Marti, 1985) (fig. 10.40):

1. si tracciano le circonferenze di Mohr dello stato di sforzo in ciascun puntone (per esempio la circonferenza di Mohr del puntone C_2 passa per l'origine degli assi e per il punto di ascissa $\sigma_{c2} = C_2/\,(AD \cdot s)$, dove σ_{c2} è la tensione di compressione nel puntone e s è lo spessore del nodo);
2. si determina il polo di ciascuna delle circonferenze dei tre puntoni (Q_A, Q_B e Q_C)[5];
3. si tracciano le rette parallele ai lati BC, CA ed AB del nodo passanti per i poli Q_A, Q_B e Q_C delle circonferenze di Mohr dei singoli puntoni; i punti di intersezione A, B e C di queste rette con le corrispondenti circonferenze di Mohr definiscono la circonferenza di Mohr dello stato di sforzo di compressione biassiale nel nodo ABC; il centro di questa circonferenza giace sull'asse delle tensioni normali e le rette Q_AA, Q_BB e Q_CC si intersecano nello stesso polo Q.

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[5] Il polo P della circonferenza di Mohr è quel particolare punto appartenente alla circonferenza tale che la retta passante per P, parallela ad una giacitura prefissata π, interseca la circonferenza nel punto rappresentativo dello stato di sollecitazione sulla giacitura π.

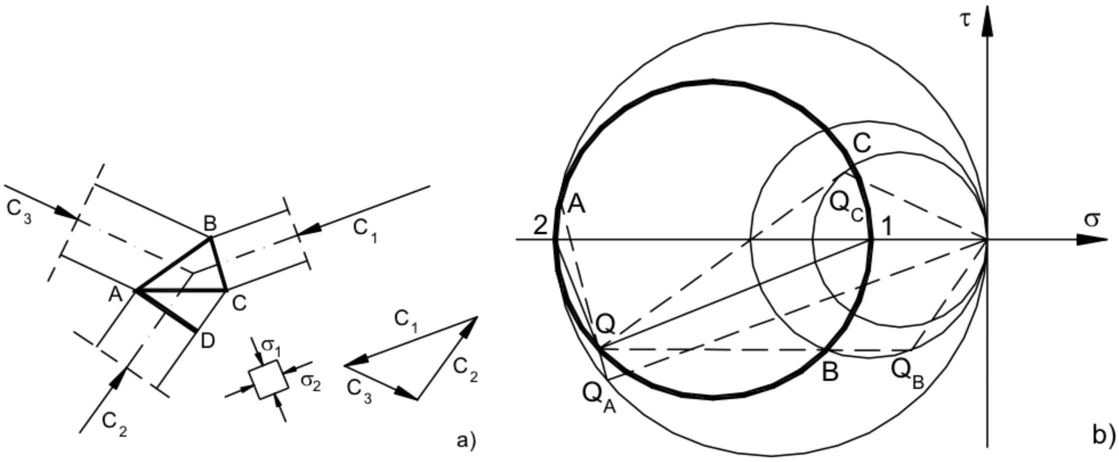

Figura 10.40 Costruzione grafica di Mohr per la determinazione dello stato di sforzo in un nodo CCC (Marti, 1985).

10.10.3.3 *Resistenza di un nodo CCC*

Ai fini pratici il calcolo della resistenza di un nodo può essere condotto come suggerito nelle ACI (2008). Fatta eccezione per i nodi idrostatici, dove la resistenza del nodo può essere calcolata con riferimento ad una giacitura qualunque, per i nodi non idrostatici la resistenza si ottiene moltiplicando la resistenza del calcestruzzo (che a sua volta è funzione del tipo di nodo - CCC, CCT, CTT) per la più piccola delle seguenti dimensioni:

– l'area della faccia nodale sulla quale agisce lo sforzo di compressione F_c, presa perpendicolarmente alla retta di azione di F_c; con riferimento alla figura 10.41*a*, poiché i due puntoni inclinati non sono ortogonali alle corrispondenti facce nodali, si prende per ognuna di esse la proiezione ortogonale all'asse del puntone (a_2 per il puntone di sinistra e a_3 per quello di destra), oppure
– l'area di una sezione che divide il nodo in due; nelle figure 10.41*b,c,d* sono indicate le sezioni passanti per i tre vertici del nodo ed ortogonali alle facce nodali; per esempio per la sezione AH_1 la risultante coincide con la proiezione orizzontale dello sforzo nel puntone 3 (o in modo equivalente di quello nel puntone 2).

La resistenza σ_R del nodo mostrato nella figura 10.41 è data da:

$$\sigma_R = \min\left(\frac{F_{c1}}{a_1 \cdot b};\frac{F_{c2}}{a_2 \cdot b};\frac{F_{c3}}{a_3 \cdot b};\frac{F_{AH1}}{AH_1 \cdot b};\frac{F_{CH2}}{CH_2 \cdot b};\frac{F_{BH3}}{BH_3 \cdot b}\right)$$

dove b è lo spessore dell'elemento strutturale nella direzione normale al piano del nodo.

10.10.3.4 *Suddivisione delle regioni nodali*

In molti casi per semplificare i calcoli è utile suddividere un nodo in più parti; ciascuna porzione in cui il nodo viene suddiviso trasferisce una parte delle forze agenti su di esso. Per esempio nel nodo mostrato nella figura 10.41b la reazione F_{c1} è suddivisa in una componente F'_{c1}, che equilibra la componente verticale di F_{c2}, ed in una componente F''_{c1}, che equilibra la componente verticale di F_{c3}.

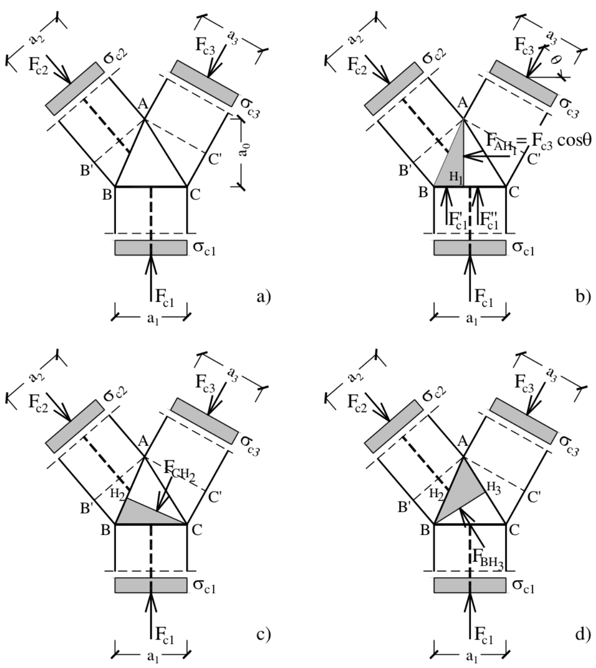

Figura 10.41 Nodo CCC: a) verifica sulle facce ortogonali agli assi dei puntoni (AB', AC', BC); b), c), d) verifica sulle sezioni del nodo perpendicolari alle facce nodali.