NUMERO: 2386A ANNO: 2018

A P P U N T I

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MATERIA: Foundations - Esercizi + Temi di Esame +Teoria - Prof. Lancellotta Dominija

BEARING CAPACITY

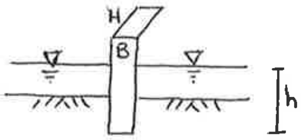
• SAND

$$q_{lim} = \frac{1}{2} \gamma' B_R N_\gamma s_\gamma i_\gamma b_\gamma g_\gamma + c' N_c s_c d_c i_c b_c g_c + q' N_q s_q d_q i_q b_q g_q$$

- q' se livelli di destra e sinistra sono diversi prendere il più basso
- $\frac{B}{L} = \frac{smallest}{higher} = \frac{B_R}{L_R}$
- If water above the soil

  $N' = N - \gamma_w h \cdot BH$

  ↳ use it in formula
  ↳ B and H not reduced



EURO CODE

1) N_c, N_γ, N_q, B_R
2) s, i, b, g, d
3) q_{lim}
4) $R_d = \frac{q_{lim}}{2{,}3} \cdot B_R \cdot L_R$

TRADITIONAL APPROACH

4) $q_{ad} = \frac{q_{lim}}{3}$
5) $N_{ad} = q_{ad}\, B_R \cdot L_R$

   ↳ EX. 29

• CLAY (UNDRAINED CONDITIONS) ~ drained as sand

NON - STRIP

$c_u = K + mz$
c_0 (z = base of foundation)
$q_{lim} = c_u N_c s_c d_c i_c b_c g_c + q$
↳ at the base

STRIP

$$q_{lim} = F\left[(2+\pi)c_0 + \frac{\rho b}{4}\right] + \gamma z$$

$\rho = m$ (of c_u)
b (side $\neq \infty$)
F (table, if < 1 take 1)

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Assessment Shallow footings ~ bearing capacity

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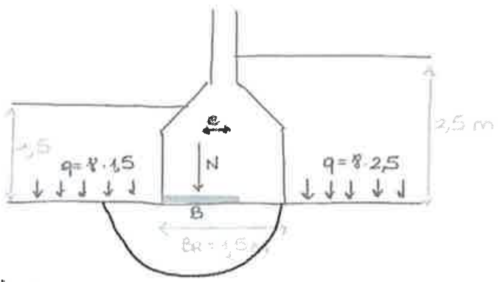
1) SAND

Strip footing in sand



1) e = 0
2) e = 0,25 m

γ = 18 kN/m³
Gk = 440 kN/m
Qk = 205 kN/m
ϕ = 36°

- check the foundation
- evaluate effect of eccentricy of vertical load

(1)

Strip foundation s = 1 | not-inclined ground g = 1 | not-inclined base b = 1 | not-inclined force i = 1

we need to consider separately left side and right side

LEFT (d = 1) ~ also on right because we haven't always D ≥ 2m ↳ we are on the safe side

$q_{lim} = \frac{1}{2}\gamma B N_\gamma + q' N_q$

$N_q = tg^2\left(45° + \frac{\phi}{2}\right)e^{\pi tg\phi} = 37,75$

$N_\gamma = 2(N_q + 1) tg\phi = 56,31$

$B = 1,5$

$q' = q = \gamma \cdot 1,5 = 27$ kN

$q_{lim} = 1779,435$ kPa

$R_d = \frac{q_{lim}}{2,3} \cdot B = 1160,50$ kN/m

$E_d = 1,3 \cdot G_k + 1,5 \cdot Q_k = 879,5$ kN/m

} $R_d > E_d$

(2)

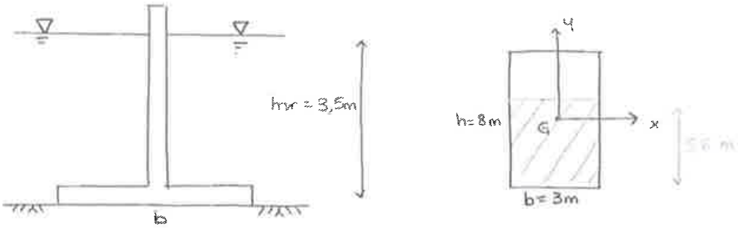
$B_1 = B - 2e = 1$

$$Rd = \frac{q_{lim}}{2{,}5} \cdot b h_1 = 2767{,}82 \text{ kN}$$

$$Ed = N' = 2030 \text{ kN}$$

$Rd > Ed$

3) SAND



$q' = 0$

$$q_{lim} = \frac{1}{2} \gamma' b \cdot N_\gamma \cdot s_\gamma \cdot i_\gamma = 203{,}59 \text{ kPa}$$

$$Rd = \frac{q_{lim}}{2{,}3} \cdot b h_1 = 1487{,}1 \text{ kN}$$

$$Ed = N' = 2030 \text{ kN}$$

$Rd < Ed$

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$A' = A - 2e_y = 2{,}3$ m
$B' = B - 2e_x = 1{,}6$ m

$s_q = s_\gamma = 1{,}246 \quad f\left(\frac{B'}{A'}\right)$

$q_{lim} = 1103{,}54$ kPa
$R_d = 1765{,}66$ kN
$E_d = 1200$ kN } $R_d > E_d$

4) $H = \sqrt{H_x^2 + H_y^2} = 100$ kN
$e_y = \frac{H_y}{N} = 0{,}28$
$A' = A - 2e = 2{,}44$ m

$s_q = s_\gamma = 1{,}29 \quad f\left(\frac{B}{A'}\right)$

$$m = \frac{2 + \frac{B}{A'}}{1 + \frac{B}{A'}} = 1{,}55$$

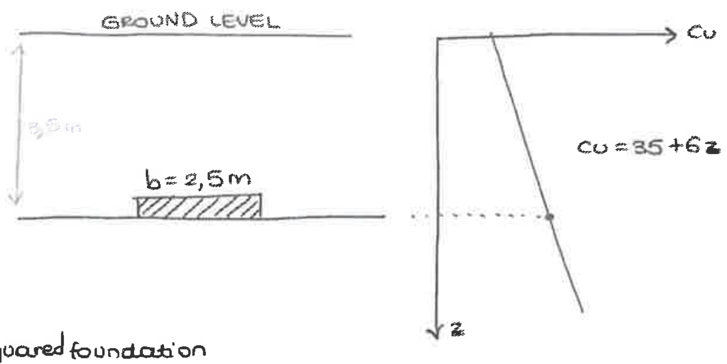
$i_\gamma = 0{,}78$
$i_q = 0{,}86$

$q_{lim} = \frac{1}{2} B \gamma N_\gamma s_\gamma \cdot i_\gamma + q N_q s_q i_q = 1070{,}27$ kPa

$R_d = 2270{,}83$
$N_d = 1100$ kN } $R_d > E_d$

=> All points are verified

5) CLAY



squared foundation
$\gamma = 18$ kN/m³

1) $N_d = 770$ kN $\quad e_x = e_y = 0$
2) $N_d = 770$ kN $\quad e_x = 0{,}32$ m $\quad e_y = 0$

1) $c_u = c_0 + p \cdot z$
$c_0 = 35 + 6 \cdot 3{,}5 = 56$ kPa

Settlements

Settlements

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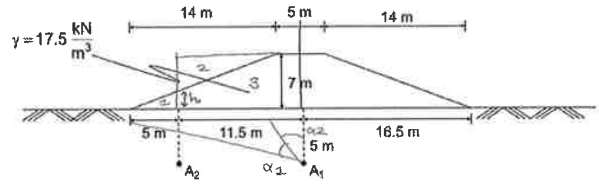
7)

Determine the vertical stress increase under the embankment at points A_1 and A_2.



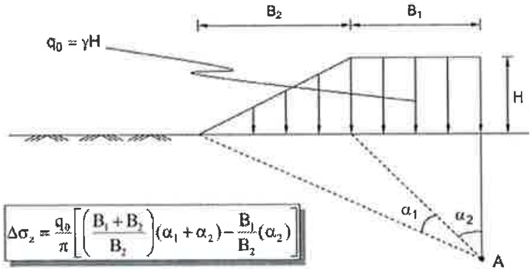
$\Delta\sigma_z = q_0 \cdot I$

$$I\left(\frac{B_1}{z},\frac{B_2}{z}\right)=\frac{1}{\pi}\left[\left(\frac{B_1/z}{B_2/z}+1\right)(\alpha_1+\alpha_2)-\frac{B_1/z}{B_2/z}(\alpha_2)\right]$$

$$\alpha_1(\text{radians})=\tan^{-1}\left(\frac{B_1+B_2}{z}\right)-\tan^{-1}\left(\frac{B_1}{z}\right)$$

$$\alpha_2=\tan^{-1}\left(\frac{B_1}{z}\right)$$

Vertical Stress Due to Embankment Loading



$$\Delta\sigma_z=\frac{q_0}{\pi}\left[\left(\frac{B_1+B_2}{B_2}\right)(\alpha_1+\alpha_2)-\frac{B_1}{B_2}(\alpha_2)\right]$$

$$\alpha_1(\text{radians})=\tan^{-1}\left(\frac{B_1+B_2}{z}\right)-\tan^{-1}\left(\frac{B_1}{z}\right)$$

$$\alpha_2=\tan^{-1}\left(\frac{B_1}{z}\right)$$

A_1) $B_1 = 2{,}5$ m
$B_2 = 14$ m

$\alpha_2 = \text{arctg}\ \frac{2{,}5}{5} = 0{,}464$ rad

$\alpha_1 = \text{arctg}\ \frac{14+2{,}5}{5} - \alpha_2 = 0{,}813$ rad

$q_0 = \gamma \cdot 7 = 122{,}5$ kPa

$\Delta\sigma_z = 2 \cdot \frac{q_0}{\pi}\left[\left(\frac{B_1+B_2}{B_2}\right)(\alpha_1+\alpha_2) - \frac{B_1}{B_2}(\alpha_2)\right] = 110{,}9$ kPa

A_2) • Triangolo (1)

$\beta = \text{arctg}\ \frac{7}{14} = 0{,}464$

$h = 5 \cdot \text{tg}\,\beta = 2{,}5$ m

$\alpha_2 = 0$ rad

$\alpha_1 = \text{arctg}\ \frac{5}{5} = 0{,}785$ rad

$q_0 = 17{,}5 \cdot h = 43{,}75$ kPa

$\Delta\sigma_z = \frac{q_0}{\pi}[\alpha_1] = 10{,}93$ kPa

• Trapezio (2+3)

$B_2 = 14$ m
$B_1 = 14$ m

$\alpha_2 = 1{,}23$ rad
$\alpha_1 = 0{,}164$ rad

$q_0 = 17{,}5 \cdot H = 122{,}5$ kPa

$\Delta\sigma_z = 60{,}75$ kPa

• Triangolo 2

$\bar{h} = 7 - 2{,}5 = 4{,}5$ m
$B_2 = 9$ m
$B_1 = 0$
$\alpha_2 = 0$
$\alpha_1 = 1{,}064$ rad

$q_0 = 17{,}5 \cdot \bar{h} = 78{,}75$ kPa

$\Delta\sigma_z = 26{,}67$ kPa

$\Rightarrow \Delta\sigma_v = 45$ kPa

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$\Delta q' = q' - \sigma'_{v0} = 200 - \sigma'_{v0} = 172{,}04$ kPa
$\sigma'_{v1} = \gamma'\left(D + \frac{B}{2}\right) = 51{,}26$ kPa   ! a B/2 sotto la base della fondazione !

$\Sigma \frac{I_{zi}}{E_i} = 0{,}0569$

$\Delta q\, \Sigma = 9{,}78$ kPa

$C_1 = 0{,}919$
$C_2 = 1{,}495$

$w = C_1 C_2 \Delta q \Sigma = 13{,}5$ mm

11) Berardi e Lancellotta ~ III method for the 9^{th} exercise

$D_R = \sqrt{\frac{C_N \cdot N_{SPT}}{60}}$

$\sigma'_{v0} = 18{,}64 \cdot 2{,}75 = 51{,}26$

at the medium point of the depth of influence

$C_N = 1{,}32$
$D_R = 0{,}81 = 81\%$
$K_E = 800$

$B_e = \sqrt{\frac{4BL}{\pi}} = 2{,}82$ m

$\Delta\sigma\left(z = \frac{B}{2}\right) = I\,\Delta q' = \left\{1 - \frac{1}{\left[\left(\frac{B_e}{2z}\right)^2 + 1\right]^{3/2}}\right\}\Delta q' = 121{,}21$ kPa

at B/2 from the foundation level

$E'_{0,1} = 84\,465$ kPa
$I = I_G\, I_F\, I_E$
$I_G = 0{,}86$
$I_F = 0{,}79$
$I_E = 0{,}9$

$I = 0{,}61$

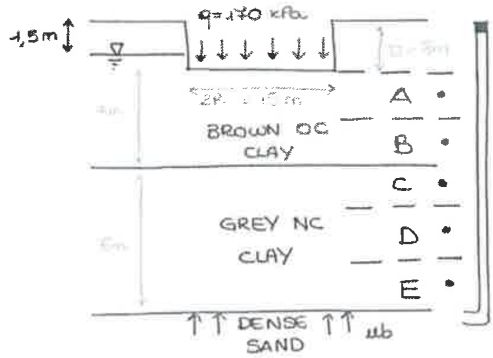
$\left(\frac{w}{B}\right)^{0,3} = \frac{125\, q\, I\,(1-\nu^2)}{E'_{0,1}} = 0{,}153$

$w = 4{,}7$ mm

12) Settlements on clay

$\gamma_1 = 19$ kN/m³
$OCR_1 = 5$
$CR_1 = 0{,}177$
$RR_1 = 0{,}03$

$\gamma_2 = 18$ kN/m³
$CR_2 = 0{,}124$
$RR_2 = 0{,}014$



- compute safety factor

$u = \gamma \cdot h = 10 \cdot 13 = 130$ kPa
$\sigma_v = \gamma_1 \cdot 4 + \gamma_2 \cdot 6 = 184$ kPa

$F_s = \frac{\sigma_v}{u} = 1{,}42$

- compute settlements

$T_v = 0,196$

$H_d = H/2$! perché sotto c'è uno strato permeabile !

$t = \frac{T_v H_d^2}{c_v} = 0,42$ yr

$c_v = \frac{k}{m_v \gamma_w} = 2,4 \cdot 10^{-7}$ m²/s

$m_v = \frac{1}{M} = \frac{1}{E} \frac{(1-\nu-2\nu^2)}{1-\nu} = 8,33 \cdot 10^{-4}$ 1/kPa

$U = 95\%$

$T_v = 1,129$

$t = 2,41$ yr

B) $t = 1$ month $= 2592000$ s

$T_v = \frac{c_v t}{H_d^2} = 0,03888$

we try to use the approximate solution

$U_s = 2\sqrt{\frac{T_v}{\pi}} = 0,22 < 0,6$ so it is correct

! qui c'è H totale $\neq H_d$

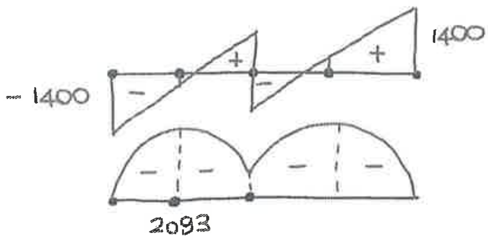
$w_c = m_v \cdot H \cdot q = 0,8$ m

$w = U_s \cdot w_c = 0,176$ m

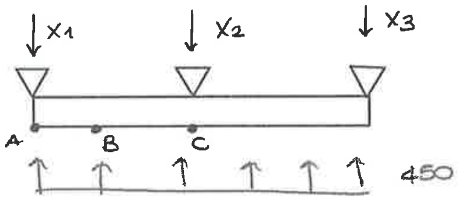
Contact linear load

$$r = \frac{P_1 + P_2 + P_3}{L} = 450 \text{ kN/m}$$

POINTS	V (kN)	M (kN·m)
A	-1400	0
B	-275	-2093,75
C	{ 850 -850	-1375



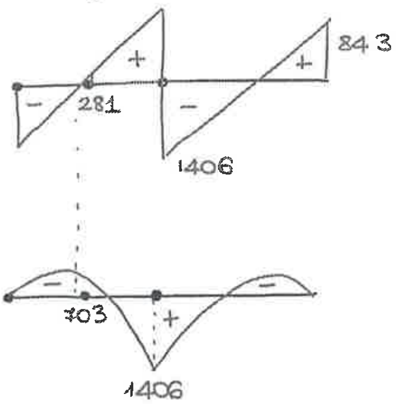
16)



$$x_1 = x_3 = \frac{3}{8} q \frac{L}{2} = 843,7 \text{ kN}$$

$$x_2 = \frac{10}{8} q \frac{L}{2} = 2812,5$$

POINTS	V (kN)	M (kN·m)
A	-843,7	0
B	281,3	-703
C	1406,3 -1406,3	1406,5



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$$r_{AA} = \frac{N_d}{L} + 12 \cdot \frac{N_d \cdot e_x}{L^3} \cdot \left(\frac{L}{2} - a\right) = 342 \text{ kN}$$

$$V_{AA} = (510 + 342) \cdot \frac{1{,}2}{2} = 511{,}2 \text{ kN}$$

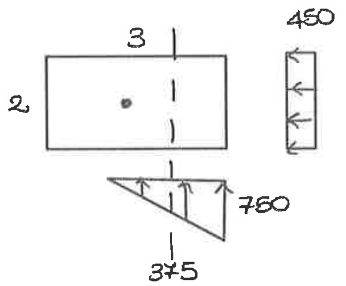
$$M_{AA} = 342 \cdot \frac{1{,}2^2}{2} + \frac{1}{2}(510 - 342) \cdot 1{,}2 \cdot \frac{2}{3} \cdot 1{,}2 = 326{,}9 \text{ kN m}$$

C) $N_d = 900$ kN
$M_x = 630$ kNm

$e_x = 0{,}7$ m $> L/6$ (a part of the section is tensed)

$$b = 3u = 3 \cdot \left(\frac{L}{2} - e_x\right) = 2{,}4 \text{ m}$$

$$r_{max} = \frac{2}{3} \frac{N_d}{u} = 750 \text{ kN/m}$$



$750 : 3u = r_A : (3u - a)$

$$r_A = \frac{750\,(3u - a)}{3u} = 375$$

$$V_A = (750 + 375) \cdot \frac{1{,}2}{2} = 675 \text{ kN}$$

$$M_A = 375 \cdot \frac{1{,}2^2}{2} + (750 - 375) \cdot \frac{1{,}2}{2} \cdot \frac{2}{3} \cdot 1{,}2 = 450 \text{ kN m}$$

D) $N_d = 720$ kN
$M_{xd} = 432$ kNm
$M_{yd} = 216$ kNm

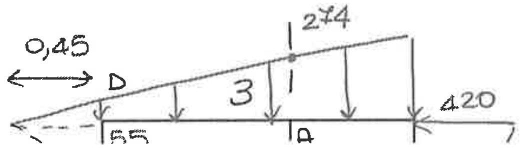
$$e_x = \frac{M_x}{N_d} = 0{,}6 \text{ m}$$

$$e_y = \frac{M_y}{N_d} = 0{,}3 \text{ m}$$

$$\frac{e_x}{3} = 0{,}2 \text{ m}$$

$$\frac{e_y}{2} = 0{,}15 \text{ m}$$

$k = 3{,}5$
$x = 0{,}6 \cdot 3 = 1{,}8$ m
$y = 0{,}8 \cdot 2 = 1{,}6$ m



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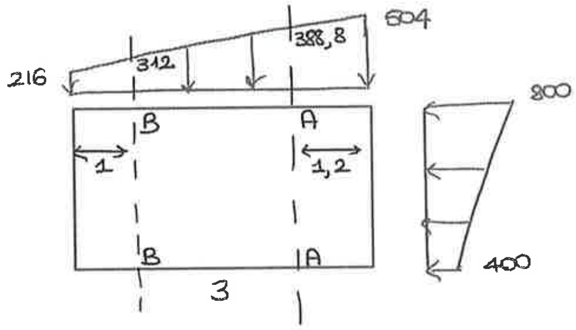
$$r_{max,x} = \frac{N}{L}\left(1 + \frac{6e_x}{L}\right) = 504 \quad kN/m$$

$$r_{min,x} = \frac{N}{L}\left(1 - \frac{6e_x}{L}\right) = 216 \quad kN/m$$

$$r_{max,y} = \frac{N}{B}\left(1 + \frac{6e_y}{B}\right) = 800 \quad kN/m$$

$$r_{max,y} = \frac{N}{B}\left(1 - \frac{6e_y}{B}\right) = 400 \quad kN/m$$



$$\frac{504-216}{3} = \frac{504-x}{1,2} \longrightarrow x = 388,8 \ kN/m$$

$$\frac{504-216}{3} = \frac{504-x}{2} \longrightarrow x = 312 \ kN/m$$

$$V_{AA} = \frac{388,8+504}{2} \cdot 1,2 = 535,7 \ kN$$

$$M_{AA} = 388,8 \cdot \frac{1,2^2}{2} + (504-388,8) \cdot \frac{1,2}{2} \cdot \frac{2}{3} \cdot 1,2 = 335,2 \ kN/m$$

$$V_{BB} = \frac{216+312}{2} \cdot 1 = 264 \ kN$$

$$M_{BB} = 216 \cdot \frac{1^2}{2} + (312-216) \cdot \frac{1}{2} \cdot \frac{1}{3} = 124 \ kN$$

$\ell_b = n\phi = 560$ mm

$\ell_{b,eff} = \alpha \cdot \frac{A_s}{A_{s eff}} \cdot \ell_b = 378$ mm

$\ell_{b,eff} < CD$

$\alpha = 0,7$

Hook bar

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$$T_d = \frac{R \cdot \left(x - \frac{a}{4}\right)}{0,85\,d} = 662 \text{ kN} = 662000 \text{ N}$$

$$A_s = \frac{T_d}{f_{yd}} = 1693 \text{ mm}^2$$

$\phi = 14$

$n = 11$

$$i = \frac{B - 2c}{n - 1} = 190 \text{ mm}$$

Vertical part

$$h = \max \begin{cases} \frac{1}{3} \cdot \ell_{b,net} = \frac{1}{3} n \phi = \\ 10\,\phi = 140 \text{ mm} \\ 200 \text{ mm} \end{cases}$$

$\frac{r_c}{3u - v} = \frac{r_{t\,max}}{3u}$

$r_c = 276,92$ kN/m

$M_{cc} = r_c \cdot \frac{v}{2} \cdot v + \frac{1}{2}(r_{t\,max} - r_c) \cdot v \cdot \frac{2}{3} \cdot v = 505,6$ kN m

$A_s = 2873,67$ mm²

$\phi = 18$ mm

$n = 12$

$i = \frac{B - 2c}{n - 1} = 0,218$ m

100 < i < 300

$e_{b,net} = n\,\phi = 36\ \phi = 648$ mm

$e_{available} = v - d - c = 975$ mm

$e_{b,eff} = 609,8$ mm (for straight bar)

$e_{b,eff} < e_{available}$

B) DRAINED CONDITIONS

ACTIVE

	z(m)	σv (kPa)	u (kPa)	σ'v (kPa)	σ'a (kPa)	σa (kPa)
• G	4	100	0	100	34,23	34,23
• B'	6	140	20	120	42,35	62,35
• B''	6	140	20	120	26,04	46,04
• C	9,8	208,4	58	150,4	32,64	90,64
• A	0	20	0	20	1,75	1,75

$\sigma'_a = k'_a \sigma'_v - 2c'\sqrt{k'_a}$ (clay) (sand)

$\sigma'_a = 0$

$z = \frac{2c\sqrt{K_a}}{\gamma K_a} - \frac{q}{\gamma} = -0,215$ m

we assume $z < H_w$ so $\gamma' = \gamma'$ and

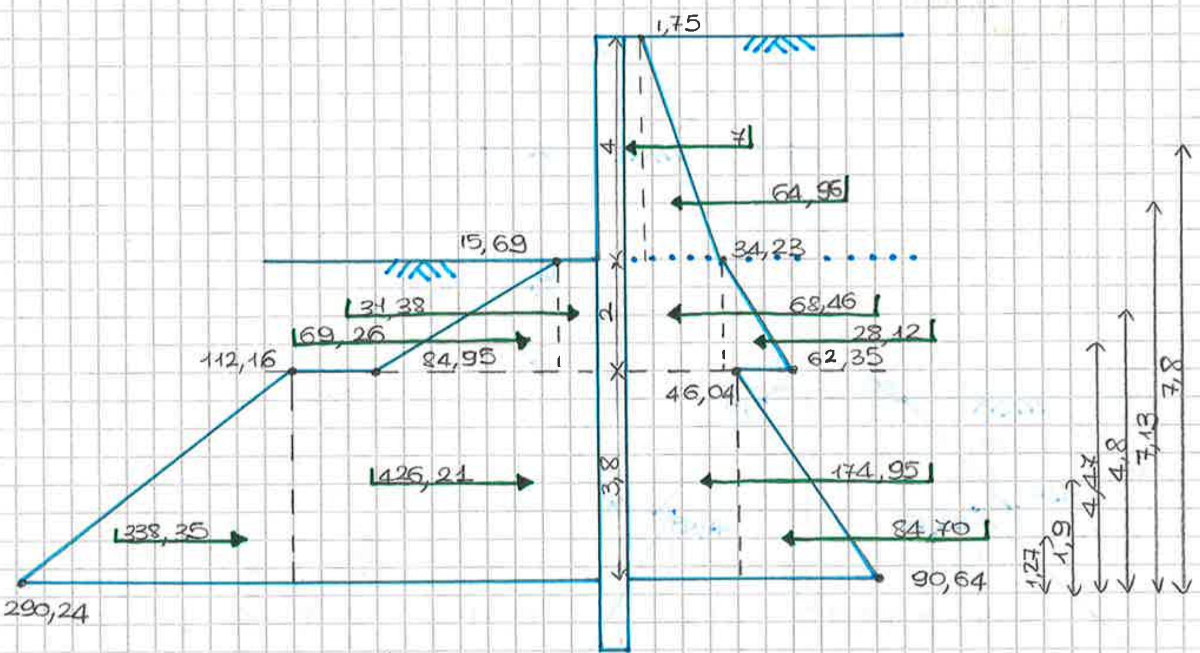
$k'_a = 0,406$ $k'_a = 0,217$

PASSIVE

	z(m)	σv (kPa)	u (kPa)	σ'v (kPa)	σ'p (kPa)	σp (kPa)
• D	0	0	0	0	15,69	15,69
• E'	2	40	20	20	64,95	84,95
• E''	2	40	20	20	92,16	112,16
• F	5,8	108,4	58	50,4	232,24	290,24

$k_p = \frac{1}{K_a} = 2,463$ $k_p = 4,608$

$\sigma_p = k_p \sigma'_v + 2c\sqrt{k_p}$ (clay) (sand)



M_A = 1412,04 kN m/m
M_P = 1699,72 kN·m/m

F_S = 1,2

Is lower than before so drained conditions are the worst (most critical)

$\ell_{b,net} = 36 \cdot \phi = 504$ mm

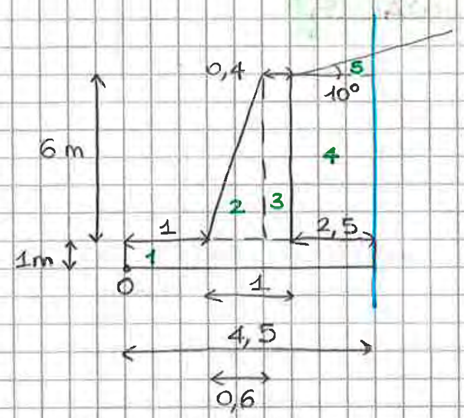
$\ell_1 = \vartheta - d - c = 0{,}55$ m $= 550$ mm

$\ell_{b,eff} = \ell_{b,net} \dfrac{A_{s,th}}{A_{s\,eff}} = 468{,}01$ mm

$\ell_{b,eff} < \ell_1$ so straight bar

25

$\varphi'_p = 32°$
$\varphi'_{cv} = 26°$
$\gamma = 18\ kN/m^3$



$z = 7 + 2{,}5\ tg\ 10° = 7{,}44\ m$

$\sigma'_v = \gamma z \cos i = 131{,}88\ kPa$

$\sigma'_a = k_a\ \sigma'_v = 42{,}99\ kPa$

$P_A = \sigma'_a \cdot \frac{z}{2} = 159{,}93$

$$K_a = \frac{\cos i - \sqrt{\cos^2 i - \cos^2\varphi}}{\cos i + \sqrt{\cos^2 i - \cos^2\varphi}} = 0{,}326$$

$P_{A,h} = P_A \cdot \cos i = 157{,}5\ kN/m$
$P_{A,v} = P_A \cdot \text{sen}\, i = 27{,}77\ kN/m$

$M_{UNS,O} = P_{A,h} \cdot \frac{z}{3} - P_{A,v} \cdot B = 265{,}63\ kN\,m/m$

$W_1 = 112{,}5\ kN$	$b_1 = 2{,}25\ m$
$W_2 = 45\ kN$	$b_2 = 1{,}4\ m$
$W_3 = 60\ kN$	$b_3 = 1{,}8\ m$
$W_4 = 270\ kN$	$b_4 = 3{,}25\ m$
$W_5 = 9{,}92\ kN$	$b_5 = 3{,}67\ m$

$M_{STA,O} = \Sigma W_i b_i = 1338{,}03\ kN\,m/m$

- ROTATION

$FS = \frac{M_{STA}}{M_{UNS}} = 5{,}04 \geq 1{,}8$

- TRASLATION

$\delta = \varphi_{cv} = 26°$ ⚠ Nelle altre formule sempre φ'_p ⚠

$FS = \frac{(W_{TOT} + P_{A,v})\, tg\,\delta}{P_{A,h}} = 1{,}63 \geq 1{,}5$

- BEARING CAPACITY

$V = W_{TOT} + P_{A,v} = 525{,}19\ kN/m$

$e = \frac{B}{2} - \frac{M_{STA} - M_{UNS}}{V} = 0{,}208\ m$

$B_R = B - 2e = 4{,}084\ m$

$q_{lim} = \frac{1}{2} \gamma N_\gamma B_R i_\gamma = 380{,}87\ kPa$

$N_\gamma = 30{,}21$

$i_\gamma = \left(1 - \frac{H}{N}\right)^{m+1} = 0{,}343$

$m = 2$ (strip foundations)

$N = V = 525{,}19\ kN/m$

$Q_{lim} = q_{lim} \cdot B = 380{,}87 \cdot 4{,}5 = 1555{,}47\ kN/m$

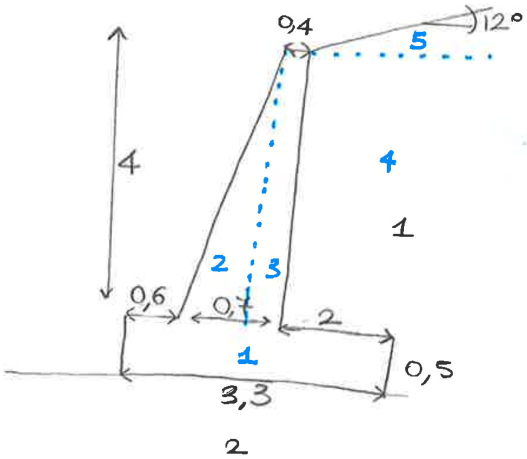
$FS = \frac{Q_{lim}}{N} = 2{,}96 \geq 2{,}5$

↳ Non dividere per γ perché é verifica con FS

43

$\varphi'_2 = 28°$
$\gamma_2 = 18\ kN/m^3$
$\varphi'_1 = 34°$
$\gamma_1 = 17\ kN/m^3$
$\gamma_{cls} = 25\ kN/m^3$

- ribaltamento, scorrimento, bearing capacity
- armatura



1) $z = 4 + 0,5 + 2\,tg\,12 = 4,93\ m$
$\sigma_v = \gamma\, z = 83,73\ kPa$
$\sigma_{v,h} = \gamma\, z \cos 12 = 81,9\ kPa$

$$k_a = \frac{\cos i - \sqrt{\cos^2 i - \cos^2\varphi}}{\cos i + \sqrt{\cos^2 i - \cos^2\varphi}} = 0,307$$

$\sigma_a = \sigma_{v,h} \cdot k_a = 25,11\ kPa$

$P_A = \sigma_a \cdot \frac{z}{2} = 61,9\ kN$

$P_{A,h} = P_A \cos i = 60,54\ kN$
$P_{A,v} = P_A \sin i = 12,9\ kN$

$M_{INS} = P_{A,h} \cdot \frac{z}{3} - P_{A,v} \cdot B = 56,92\ kN \cdot m$

$W_1 = 41,25\ kN/m$	$b_1 = 1,65\ m$
$W_2 = 15\ kN/m$	$b_2 = 0,8\ m$
$W_3 = 40\ kN/m$	$b_3 = 1,1\ m$
$W_4 = 136\ kN/m$	$b_4 = 2,3\ m$
$W_5 = 7,23\ kN/m$	$b_5 = 2,63\ m$

$M_{STA} = \Sigma W_i\, b_i = 455,88\ kN \cdot m$

- ribaltamento

$FS = \frac{M_{STA}}{M_{INS}} = 8 \geq 1,8$ ✓

- scorrimento

$\delta = \varphi'_2 = 28°$

$FS = \frac{(W_{TOT} + P_{a,v})}{P_{a,h}} \cdot tg\,\delta = 2,22 \geq 1,5$ ✓

- Bearing capacity

$V = W_{TOT} + P_{a,v} = 252,38\ kN$

$e = \frac{B}{2} - \frac{M_{STA} - M_{INS}}{V} = 0,07\ m$

$B_R = B - 2e = 3,16\ m$

$q_{lim} = \frac{1}{2}\gamma_2\, N_\gamma\, B_R\, i_\gamma = 208,75\ kPa$

$N_q = tg^2\left(45° + \frac{\varphi_2}{2}\right) e^{\pi\, tg\varphi_2} = 14,72$

$N_\gamma = 2(N_q + 1)\, tg\varphi_2 = 16,72$

$i_\gamma = \left(1 - \frac{P_{a,h}}{V}\right)^2 = 0,439$

$Q_{lim} = q_{lim} \cdot B_R = 688,9\ kN/m$

$FS = \frac{Q_{lim}}{V} = 2,73 \geq 2,5$ ✓

(44)

$H = 60\ kN$ $\quad E_p = 25\,000$

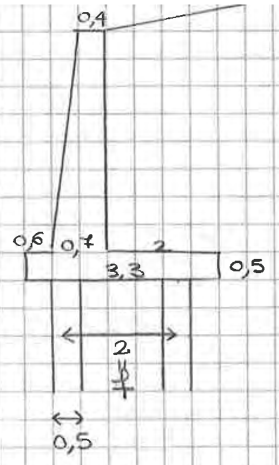
$V = 250\ kN$ $\quad d = 0,5\ m$

$i = 3\ m$

$e = 0,07\ m$

$G = 20\ MPa$

$\nu = 0,3$



$H_{TOT} = H i = 180\ kN$

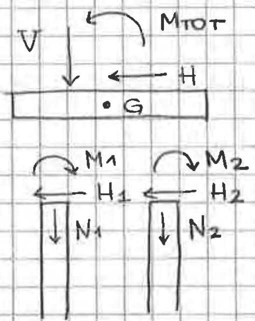
$V_{TOT} = V i = 750\ kN$

$M_{TOT} = V_{TOT} \cdot e = 52,5\ kN \cdot m$

$$L_c = d \left(\frac{E_p}{G_c(L_c)\left(1+\frac{3\nu}{4}\right)} \right)^{\frac{2}{7}} = 3,62\ m$$

$\rho_c = 1$

$$M_f = -\frac{1}{2} \frac{0,375}{\sqrt{\rho_c}} \cdot H_1 \cdot L_c = -61,09\ kN \cdot m$$



$$H_1 = H_2 = \frac{H_{TOT}}{2} = 90\ kN$$

$$\begin{cases} N_1 \cdot \frac{f}{2} - N_2 \cdot \frac{f}{2} - |M_{TOT}| - |2M_f| = 0 \\ N_1 + N_2 = V_{TOT} \end{cases}$$

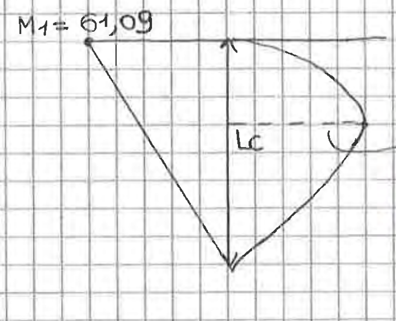
$M_1 = M_2 = -M_f = +61,09\ kN \cdot m$

$$\begin{cases} N_2 = V_{TOT} - N_1 \\ N_1 \cdot \frac{f}{2} - V_{TOT} \cdot \frac{f}{2} + N_1 \cdot \frac{f}{2} - |M_{TOT}| - |2M_f| = 0 \end{cases}$$

$$\begin{cases} N_1 = \frac{V_{TOT}}{2} + \frac{|M_{TOT}| + 2|M_f|}{f} = 462,34\ kN \\ N_2 = 287,66\ kN \end{cases}$$

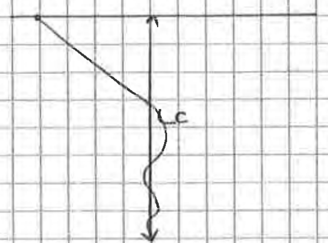
$$F = \frac{1}{\rho_c \cdot G} \sqrt{\frac{L_c}{d}} = 0,134$$

$$u = F \left(0,27 - \frac{0,11}{\sqrt{\rho_c}}\right) H_1 \left(\frac{L_c}{2}\right)^{-1} = 1,07\ m$$



$\frac{L_c}{4} = 0,905\ m$

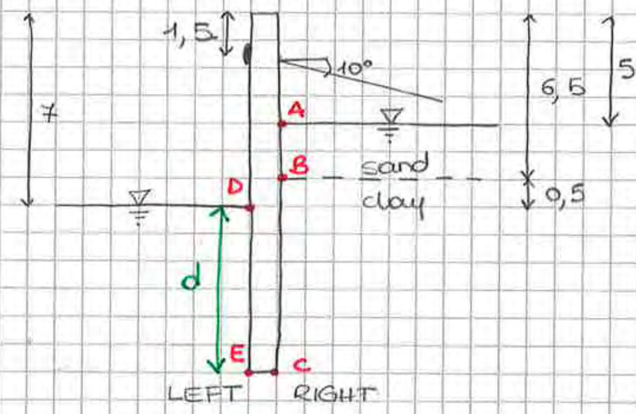
$0,1\ H \frac{L_c}{\rho_c}$ $\quad \begin{cases} z = L_c/4 & \rho_c = 1 \\ z = L_c/3 & \rho_c = 0,5 \end{cases}$



(27)

SAND
$\varphi = 36°$
$\gamma = 19\ kN/m^3$

CLAY
$\varphi = 33° = 0,524$ rad
$\gamma = 20\ kN/m^3$



$u = u_{STATIC} + u_{DINAMIC}$

Filtration (water table ≠)
↳ in clay

$u_s = \gamma_w \cdot (z)$ → dalla falda

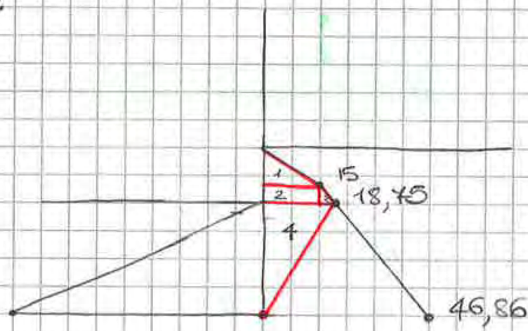
$u_d = \begin{cases} i\gamma_w \cdot z & (\uparrow) \\ -i\gamma_w \cdot (z) & (\downarrow) \end{cases}$
↳ dal moto di filtrazione

$i_c = \frac{\gamma - \gamma_w}{\gamma_w} = 1$

$\Delta h = 2$ m

$i = \frac{\Delta h}{L_f} = \frac{2}{2d + 0,5} = 0,25$

$F_s = \frac{i_c}{i} \sim 4$



$i \simeq \frac{i_c}{4}$

$\frac{2}{2d+0,5} = \frac{1}{4}$

$2d + 0,5 = 8$
$d = \frac{7,5}{2} = 3,75$

we verify this value

k_a, sand $= \frac{1 - \sin\varphi}{1 + \sin\varphi} = 0,26$

k_a, clay $= 0,295$

k_p, clay/h $= 5,37$

k_p, clay $= 4,52$

$\delta = \frac{2}{3}\varphi$ → k_p !

δ, clay $= 22° = 0,384$ rad

δ, sand $= 24°$

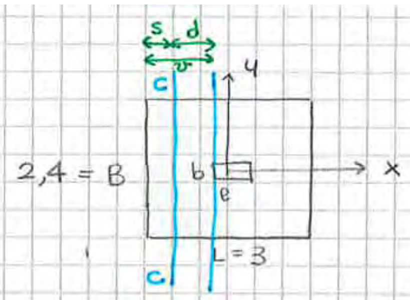
ACTIVE

	z (m)	σv (kPa)	u	σ'v (kPa)	σ'a	σa
A	5	95	0	95	24,7	24,7
B'	6,5	123,5	15	108,5	28,21	43,21
B''	6,5	123,5	15	108,5	32,01	47,01
C	10,75	208,5	46,86*	161,64	47,68	94,54

PASSIVE

E	3,75	75	46,86	28,14	151,11	197,97
D	0	0	0	0		

(28)



$b = 0{,}3$ m
$e = 0{,}45$ m
$N_d = 980$ kN
$M_{xd} = -150$ kN·m
$M_{yd} = 100$ kN·m

C 25/30

ALONG X

$e_x = \frac{M_{xd}}{N_d} = -0{,}15$ m

$e_y = \frac{M_{yd}}{N_d} = 0{,}102$ m

$\frac{e_x}{L} = 0{,}05$
$\frac{e_y}{B} = 0{,}04$
} so small that we are in case 1

$r_{t,(min\,max)} = \frac{N}{L}\left(1 \pm \frac{6e_x}{L}\right)$ → 424,67 kN/m ; → 226,67 kN/m

SHEAR ANALYSIS

$\frac{(424{,}62 - 226{,}67)}{3} = \frac{(424{,}67 - r_{cc})}{s}$

$d = 0{,}5$ m

$v = \frac{L}{2} - \frac{e}{2} = 1{,}275$ m

$s = v - d = 0{,}775$ m

$r_{cc} = 373{,}53$ kN/m

$V_{cc} = (424{,}67 + 373{,}53) \cdot \frac{s}{2} = 309{,}30$ kN

$k = 1 + \sqrt{\frac{200}{d}} = 1{,}63 \leq 2$

$\tau_{rd} = 0{,}035\, k^{3/2} f_{ck}^{1/2} = 364{,}18$

$V_{Rd} = \tau_{rd}\, d B = 437{,}02$ kN

BENDING ANALYSIS

$\frac{v}{h} = \frac{v}{d + 0{,}05} = 2{,}3 \geq 2$

$r_{AA} = 340{,}54$ kN/m

$M_{AA} = r_{AA} \cdot \frac{v^2}{2} + (r_{max} - r_A) \cdot \frac{v}{2} \cdot \frac{2}{3} \cdot v = 322{,}38$ kN m/m

$A_s = \frac{M_{AA}}{0{,}9\, d\, f_{yd}} = 1832{,}24$

$\phi = 14$ mm

$n = 12$

$s = \frac{L - 2c}{n - 1} = 264$ mm

$100 \text{ mm} < s < 300 \text{ mm}$

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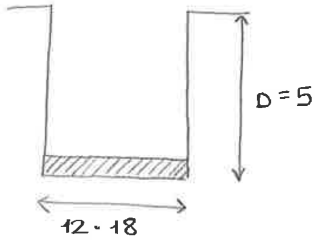
CLAY OC

$c_u = 45$ kPa

$\gamma = 19$ kN/m³



- compute bearing capacity and admissible load

Traditional

$q_{lim} = c_u \cdot N_c \cdot s_c \cdot d_c + q$

$N_c = 2 + \pi$

$s_c = 1 + 0,2 \cdot \frac{B}{L} = 1,13$

$d_c = 1 + 0,4 \cdot \frac{D}{B} = 1,17$

$q = D \cdot \gamma = 95$ kPa

$q_{lim} = 400,8$ kPa

$q_{ad} = \frac{q_{lim}}{FS}$ (3 = FS) $= 133,6$ kPa

$N_{ad} = q_{ad} \, B \cdot L = 28\,857,7$ kN

Eurocode

$R_d = \frac{q_{lim} \, BL}{\gamma_R} = 37\,640,35$ kN

- consider q^N

Traditional

$q^N_{lim} = q_{lim} - q$

$q^N_{adm} = q_{adm} - q$

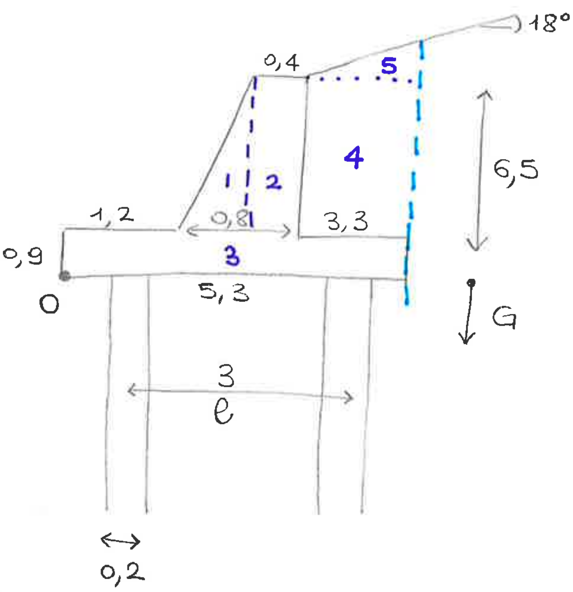
$FS = \frac{q^N_{lim}}{q^N_{adm}} = 3$

$q_{adm} = \frac{q^N_{lim}}{FS} + q = \frac{q_{lim} - q}{FS} + q = 196,93$ kPa

Eurocode

$R_d = \frac{q^N_{lim} \, BL}{\gamma_R} + qBL = 49\,238$ kN

(31)



$\gamma = 20\ kN/m^3$

φ 30°

OCR = 3

$G = 36 + 0{,}42\,z$

i piles = 2 m

$\gamma_{cls} = 24\ kN/m^3$

$P_1 = 31{,}2\ kN/m$ $b_1 = 1{,}47\ m$

$P_2 = 62{,}4$ $b_2 = 1{,}8\ m$

$P_3 = 114{,}48$ $b_3 = 2{,}65\ m$

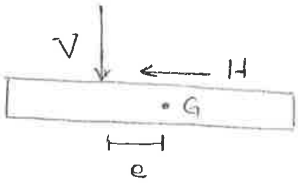
$P_4 = 429$ $b_4 = 3{,}65\ m$

$P_5 = 35{,}38$ $b_5 = 4{,}2\ m$

$M_A = 2176{,}01\ kNm/m$

↳ M_{STA}

$$k_a = \frac{\cos i - \sqrt{\cos^2 i - \cos^2\varphi}}{\cos i + \sqrt{\cos^2 i - \cos^2\varphi}} = 0{,}415$$



$z = 6{,}5 + 0{,}9 + 3{,}3\ \mathrm{tg}\ 18° = 8{,}47\ m$

$\sigma'_a = \gamma z\, k_a \cos i = 66{,}86\ kPa$

$P_a = \sigma'_a \cdot \frac{z}{2} = 283{,}15\ kN/m$

$P_{a,h} = P_a \cos i = 269{,}29\ kN/m$

$P_{a,v} = P_a \sin i = 87{,}5\ kN/m$

$V = \sum P_i + P_{a,v} = 760\ kN/m$

$H = P_{a,h} = -269{,}29\ kN/m$

$M_{UNS} = P_{a,h} \cdot \frac{z}{3} - P_{a,v} \cdot 5{,}3 = 296{,}54\ kNm/m$

$$e = \frac{B}{2} - \frac{M_{STA} - M_{UNS}}{V} = 0{,}18\ m$$

$H_{TOT} = H_i = -538{,}58\ kN$

$V_{TOT} = V_i = 1520\ kN$

$M_{TOT} = V_{TOT} \cdot e = -273{,}6\ kNm$

$L_c = 4{,}3\ m$

$$\rho_c = \frac{G(L_c/4)}{G(L_c/2)} = 0{,}988 \longrightarrow \text{should be } \ominus$$

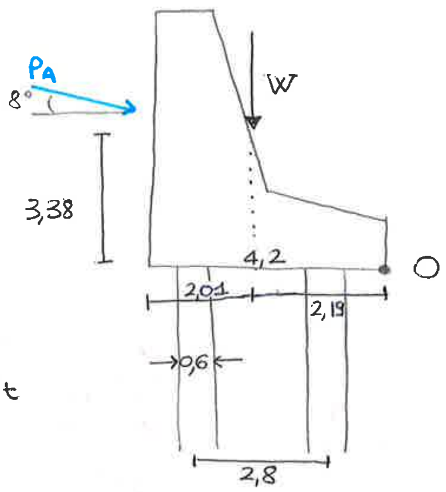
$$M_f = -\frac{1}{2} \cdot \frac{0{,}375}{\sqrt{\rho_c}}\ (H)\ L_c = -218{,}7\ kNm$$

$$\begin{cases} H_1 = H_2 = \dfrac{H_{TOT}}{2} = 269{,}52\ kN \\ N_1 + N_2 = V_{TOT} \\ N_1 \cdot \dfrac{e}{2} - N_2 \cdot \dfrac{e}{2} = M_{tot} + 2M_f \\ M_1 = M_2 = M_f \end{cases}$$

$$\begin{cases} N_1 = V_{TOT} - N_2 \\ V_{TOT} \cdot \dfrac{L}{2} - N_2 \cdot L = M_{TOT} + 2M_f \end{cases}$$

(32)

$W = 546$ kN/m
$P_A = 101$ kN/m
$c_u = 20$ MPa
$\nu = 0,15$
$E_p = 25'000$ MPa
$i_{long} = 2,5$ m



• azioni alla testa dei pali, diagrammi e u (horizontal displacement)

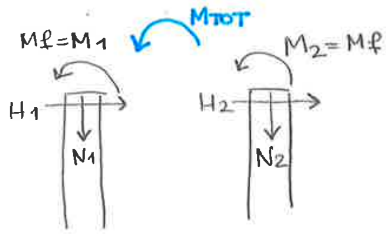
$P_{a,h} = P_a \cos 8° = 100,02$ kN/m
$P_{a,v} = P_a \sin 8° = 14,06$ kN/m

$V = W + P_{a,v} = 560,06$ kN/m
$H = P_{a,h} = 100,02$ kN/m

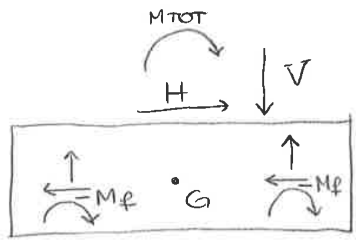
$M_{STA} = W \cdot 2,19 = 1195,74$ kN m/m
$M_{UNS} = P_{a,h} \cdot 3,38 - P_{a,v} \cdot 4,2 = 279,02$ kN m/m

$$e = \frac{B}{2} - \frac{M_{STA} - M_{UNS}}{V} = 0,46 \text{ m}$$

$H_{TOT} = H \cdot i = 250,05$ kN
$V_{TOT} = V \cdot i = 1400,15$ kN
$M_{TOT} = V_{TOT} \cdot e = 644,07$ kN · m



$$L_c = d \left(\frac{E_p}{G_{(Lc)} \left(1 + \frac{3}{4}\nu\right)} \right)^{\frac{2}{7}} = 4,46 \text{ m}$$



$\rho_c = 1$

$$M_f = -\frac{1}{2} \cdot \frac{0,375}{\sqrt{\rho_c}} \cdot H_1 L_c = -104,55 \text{ kNm}$$

$$H_1 = H_2 = \frac{H_{TOT}}{2} = 125,02 \text{ kN}$$

$$\begin{cases} N_1 \cdot \frac{2,8}{2} - N_2 \cdot \frac{2,8}{2} + |M_{TOT}| + |2 M_f| = 0 \\ N_1 + N_2 = V_{TOT} \end{cases}$$

$$\begin{cases} N_1 = 395,4 \text{ kN} \\ N_2 = 1004,75 \text{ kN} \\ M_1 = M_2 = M_f = -104,55 \text{ kN} \cdot \text{m} \end{cases}$$

$$F = \frac{1}{\rho_c \cdot G} \sqrt{\frac{L_c}{d}} = 0,136$$

$$u = F\left(0,27 - \frac{0,11}{\sqrt{\rho}}\right) H_1 \left(\frac{L_c}{2}\right)^{-1} = 1,22 \text{ m}$$

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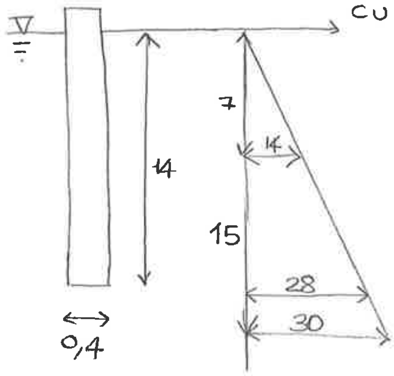
33 PILES

Driven pile in clay NC

$\gamma = 18$ kN/m³

$\gamma_w = 10$ kN/m³

$\gamma_{cls} = 24$ kN/m³



- calcolare Q_{LIM} e Q_{ADM} con approccio α e $\beta = 0,3$

Approccio α

$\frac{Su}{\sigma'_{vo}(L/2)} = 0,25 < 1 \rightarrow \alpha = \frac{0,5}{\left(\frac{Su}{\sigma'_{vo}}\right)^{0,5}} = 1$

$f_s = \alpha\, cu\left(\frac{L}{2}\right) = 14$ kPa

$Q_s = A_s \cdot f_s = \pi d \cdot L \cdot f_s = 246,3$ kN

$q_{lim} = Cu(L) \cdot N_c + \sigma'_{vo}(L)$

$N_c = (2+\pi)\, s_c \cdot d_c = 9$

$q_{lim} = 9 \cdot 28 + \gamma \cdot 14 = 504$ kPa

$Q_b = A_b \cdot q_{lim} = 63,33$ kN

$Q_{ADM} = Q_s + Q_b = 309,63$ kN

$Q_{lim} = Q_s + Q_b - W = 267,41$ kN

Approccio β

$\beta = 0,3$

$f_s = \beta \cdot \sigma'_{vo} = 16,8$ kPa

$Q_s = 295,56$ kN

$Q_{ADM} = 358,89$ kN

$Q_{lim} = 316,67$ kN

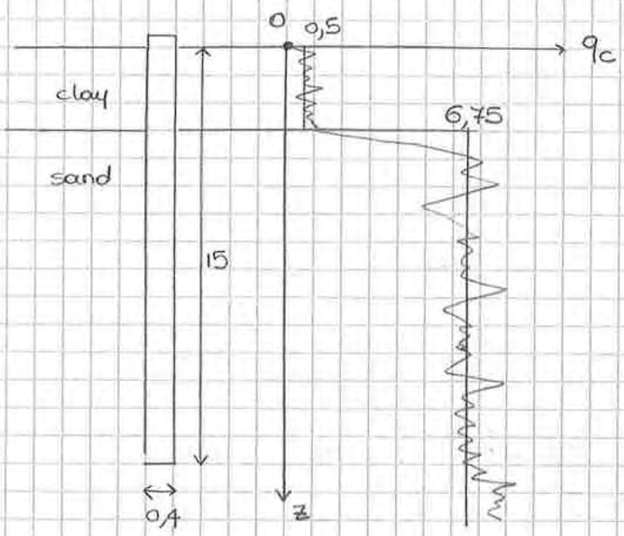
(35)

Driven pile

CLAY
$\gamma = 19\ kN/m^3$

SAND
$\gamma = 19\ kN/m^3$
$\varphi'_{cv} = 34°$

$\gamma_{cls} = 24\ kN/m^3$



- admissible and limiting capacity

clay
$\beta = 0,3$
$f_s = \beta\, \sigma'_{v0} = 8,55\ kPa$
$Q_s = 32,23\ kN$
$A_s = \pi d\, L_{clay} = 3,77$

sand
$f_s = \dfrac{q_c}{150} = 45\ kPa$
$Q_s = A_s f_s = 678,6\ kN$
$A_s = \pi d\, L_{sand} = 15,08\ m^2$

$Q_{s,TOT} = 710,8\ kN$

$q_{c,TIP} = 7000\ kPa$

$q_b = 0,4\ q_{c,TIP} = 2800\ kPa$

$Q_b = A \cdot q_b = \dfrac{\pi d^2}{4}\ q_b = 351,86\ kN$

$Q_{adm} = Q_s + Q_b = 1062,66\ kN$
$Q_{lim} = Q_{adm} - W = 1017,42\ kN$

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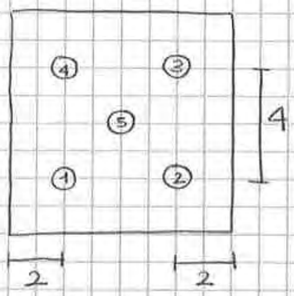


$k_v = 907{,}31 \ kN/mm$

$N = 5000 \ kN$

$$\alpha = \frac{\ln \frac{r_m}{s}}{\underbrace{\ln \frac{r_m}{r_0}}_{4}}$$

$\alpha_{11} = 1$

$\alpha_{12} = \alpha_{14} = \frac{1}{4} \ln \frac{r_m}{4} = 0{,}251$

$\alpha_{13} = \frac{1}{4} \ln \frac{r_m}{4\sqrt{2}} = 0{,}164$

$\alpha_{15} = \frac{1}{4} \ln \frac{r_m}{2\sqrt{2}} = 0{,}338$

$\ln \frac{r_m}{r_0} = 4$

$r_m = r_0 e^4 = 10{,}92 \ m$

$$\begin{cases} \omega_1 = 2\alpha_{12} \frac{P_2}{k_v} + \alpha_{13} \frac{P_3}{k_v} + \alpha_{15} \frac{P_5}{k_v} + \frac{P_1}{k_v} \\ \omega_2 = \omega_3 = \omega_4 \\ \omega_5 = 4\alpha_{15} \frac{P_1}{k_v} + \frac{P_5}{k_v} \\ P_1 + P_2 + P_3 + P_4 + P_5 = N \end{cases}$$

$P_1 = P_2 = P_3 = P_4 = P$

$$\begin{cases} \omega_1 = (2\alpha_{12} + \alpha_{13} + 1) \frac{P}{k_P} + \alpha_{15} \frac{P_5}{k_v} \\ \omega_5 = 4\alpha_{15} \frac{P}{k_v} + \frac{P_5}{k_v} \\ 4P + P_5 = N \\ \omega_1 = \omega_5 \end{cases}$$

$P_5 = N - 4P$

$\omega_1 = (2\alpha_{12} + \alpha_{13} + 1) \frac{P}{k_v} + \alpha_{15} \frac{(N - 4P)}{k_v} = 4\alpha_{15} \frac{P}{k_P} + \frac{N}{k_v} - \frac{4P}{k_v}$

$\hookrightarrow = (2\alpha_{12} + \alpha_{13} - 4\alpha_{15} + 1) \frac{P}{k_v} + \alpha_{15} \frac{N}{k_v} = 4\alpha_{15} \frac{P}{k_v} + \frac{N}{k_v} - \frac{4P}{k_v}$

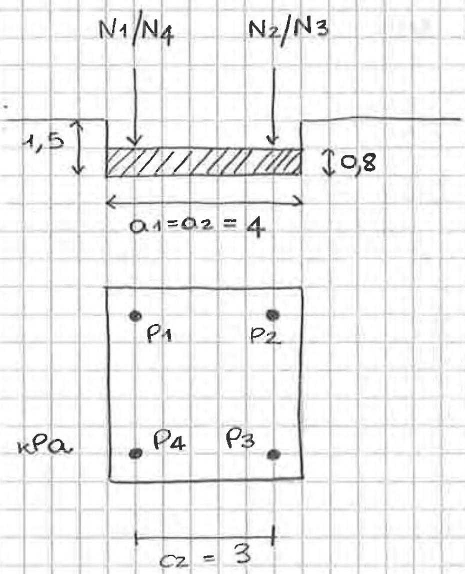
$(2\alpha_{12} + \alpha_{13} - 8\alpha_{15} + 5) \frac{P}{\cancel{k_v}} + (\alpha_{15} - 1) \frac{N}{\cancel{k_v}} = 0$

$$\begin{cases} P = \frac{(1 - \alpha_{15}) N}{2\alpha_{12} + \alpha_{13} - 8\alpha_{15} + 5} = 1117{,}5 \ kN \\ P_5 = 530 \ kN \\ \omega = 2{,}25 \ mm \end{cases}$$

(39) Schmertmann

$\gamma = 18{,}5 \ kN/m^3$
$\gamma_{cls} = 24 \ kN/m^3$

	N_1	N_2	N_3	N_4
1	1050	1097	693	652
2	850	798	712	850
3	900	1316	797	680
	933	1070	734	727



- Schmertmann

$\Delta q = q' - \sigma'_{v0}$

$q' = \frac{N_1+N_2+N_3+N_4}{a_1 \, a_2} + \gamma_{cls} \cdot d = 235{,}7 \ kPa$

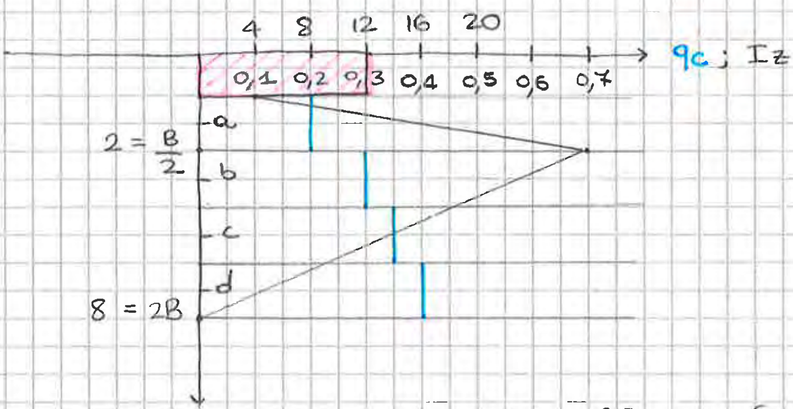
$\sigma'_{v0} = \gamma \cdot 1{,}5 = 27{,}75 \ kPa$

$\Delta q = 207{,}95 \ kPa$

$\frac{L}{B} = 1$

$\sigma'_{vi}\left(\frac{B}{2}\right) = \gamma\left(\frac{B}{2} + 1{,}5\right) = 64{,}75 \ kPa$

$I_{max} = 0{,}5 + 0{,}1 \left(\frac{\Delta q}{\sigma'_{vi}}\right)^{0,5} = 0{,}7$



	q_c	Δz	I_z	$E = q_c \cdot 2{,}5$	$\frac{\Delta z \, \Delta q}{E} \cdot I_z$ (mm)
a	8	2	0,4	20	8,3
b	12	2	0,55	30	7,6
c	14	2	0,35	35	4,2
d	16	2	0,15	40	1,6 // 21,7 mm

$C_1 = 1 - 0{,}5\left(\frac{\sigma'_{v0}}{\Delta q}\right) = 0{,}933$

$C_2 = 1 + 0{,}2 \log\left(\frac{t}{0{,}1}\right) = 1{,}495$

$W_t = C_1 \, C_2 \sum \frac{\Delta z \, \Delta q}{E} I_z = 30{,}27 \ mm$

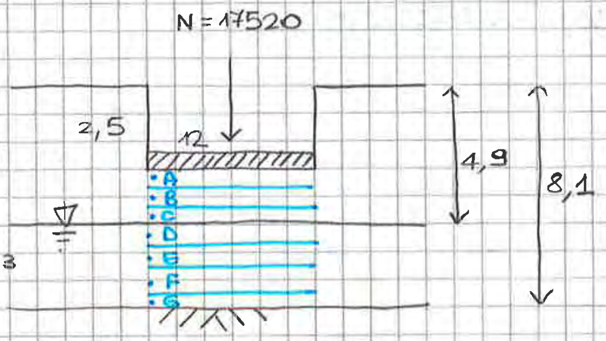
$W_0 = 20{,}24 \ mm$
$\quad \hookrightarrow C_2 = 1$

41

$q = \frac{N}{\pi \frac{d^2}{4}} = 154{,}9 \text{ kPa}$

$\sigma_{vo} = 2{,}5 \cdot \gamma = 45 \text{ kPa}$

OCR = 2
$\gamma_A = 18 \text{ kN/m}^3$
RR = 0,014
CR = 0,12
OCR = 4
$\gamma_B = 19 \text{ kN/m}^3$
RR = 0,028
CR = 0,18



	z	σ_{vo}	u	σ'_{vo}	$\Delta\sigma_v$	σ'_{vf}		σ'_p	
A	0,4	52,2	0	52,2	109,87	162,07	>	104,4	1
B	1,2	66,6	0	66,6	109,07	175,67	>	133,2	
C	2	81	0	81	106,42	187,42	>	162	
D	2,8	95,8	4	91,8	101,59	193,39	≤	367,2	2
E	3,6	111	12	99	94,93	193,93	≤	396	
F	4,4	126,2	20	106,2	87,17	193,37	≤	424,8	
G	5,2	141,4	28	113,4	79,03	192,43	≤	453,6	

$$\Delta\sigma_v = (q - \sigma_{vo}) \left\{ 1 - \frac{1}{\left[\left(\frac{R}{z}\right)^2 + 1\right]^{3/2}} \right\}$$

$$\sigma'_{vf} = \sigma'_{vo} + \Delta\sigma_v$$

$$\omega_1 = H_o \left[RR \log\left(\frac{\sigma'_p}{\sigma'_{vo}}\right) + CR \log\left(\frac{\sigma'_{vf}}{\sigma'_p}\right) \right]$$

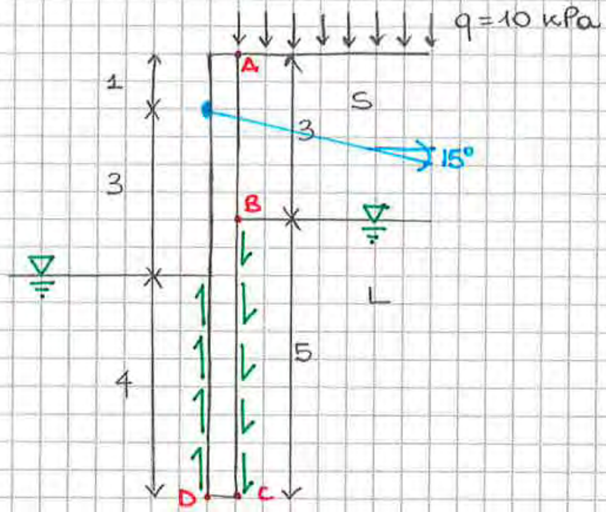
$$\omega_2 = H_o \, RR \log\left(\frac{\sigma'_{vf}}{\sigma'_{vo}}\right)$$

45

$i = 2\,m$
$\delta_p = \frac{2}{3}\varphi'$

S) $\gamma = 19\ kN/m^3$
$\alpha = 15°$
$c' = 0$
$\varphi' = 34°$
$k = 10^{-5}\ m/s$

L) $\gamma = 18\ kN/m^3$
$c' = 0$
$\varphi' = 32°$
$k = 10^{-7}\ m/s$



- calcolare f nel tirante

$i_c = \frac{\gamma_L - \gamma_w}{\gamma_w} = 0,8$

$i = \frac{\Delta h}{L_f} = \frac{1}{2d + \Delta h} = 0,11$

S) $K_a = \frac{1 - \operatorname{sen}\varphi_S}{1 + \operatorname{sen}\varphi_S} = 0,283$

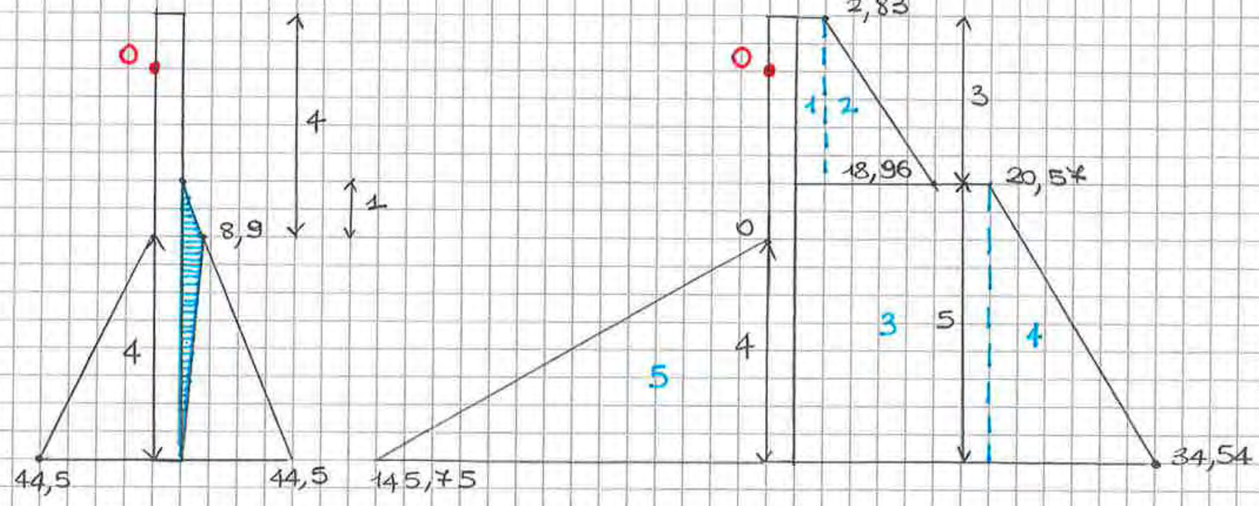
L) $K_a = \frac{1 - \operatorname{sen}\varphi_L}{1 + \operatorname{sen}\varphi_L} = 0,307$

$\delta_p = \frac{2}{3}\varphi_L = 21,33°$

$k_p = 5,3$

$$u = \begin{cases} \gamma_w \cdot z_w + i\,\gamma_w \cdot z_f & (\uparrow) \\ \gamma_w \cdot z_w - i\,\gamma_w \cdot z_f & (\downarrow) \end{cases}$$

ACTIVE		z(m)	σ_v(kPa)	u (kPa)	σ'_v(kPa)	σ'_a (kPa)
S)	A	0	10	0	10	2,83
	B'	3	67	0	67	18,96
L)	B''	3	67	0	67	20,57
	C	8	157	44,5	112,5	34,54
PASSIVE						σ'_p (kPa)
L)	D	4	72	44,5	27,5	145,75



6.03.18 (intro)

lunedì 5 marzo 2018 17:35

FOUNDATION ENGINEERING

01RVSMX
A.A. 2017-2018

Course Presentation

Teachers:
Prof. Renato Lancellotta
Dr. Andrea Dominijanni

How to contact me?

by e-mail: andrea.dominijanni@polito.it

I share the office with Prof. Mario Manassero at the 1st floor of the Department of Structural, Geotechnical and Building Engineering (entrance 2). My office phone number is 4820.

↳ 011-0904820

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Course Structure

First Part (teacher: Andrea Dominijanni)

- Course presentation (45 min)
- Historical notes on soil mechanics (2 hr)
- In situ investigations (6 hr)
- Reliability analysis and design methods (4.5 hr)
- Bearing capacity of shallow foundations (6 hr)
- Settlement predictions for shallow foundations (7.5 hr)
- Introduction to soil-structure analysis (3 hr)

end of this part of the course ~ middle April

Course Structure

Second Part

Shallow foundations and earth retaining structures
(teacher: Andrea Dominijanni)
Monday – *3 hr per week*

- Soil-structure interaction (6 hr)
- Shallow foundation design (6 hr)
- Flexible earth retaining structures (6 hr)
- Reinforced-soil retaining structures (3 hr)

Pile Foundations
(teacher: Prof. Renato Lancellotta)
Tuesday and Thursday – *3 hr per week*

Exam

The exam includes a **written test** and an **oral discussion** about all the topics that are dealt with during the course lessons (lectures and exercise classes).

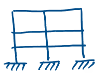
The written test consists of calculation exercises, that have to be solved within 2 hours. During the written test, the students can consult only the recommended textbooks (+ notes) and can use a calculator. It is not allowed the use of smartphones, tablets and laptops. The admission to the oral discussion is only obtained by a written test score not lower than 16/30 (maximum score equal to 30/30).

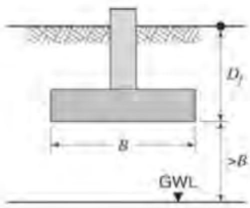
Homework is checked at the beginning of the oral discussion. The oral examination score can increase the written test score of no more than 8/30.

Structural Engineering | **Geotechnical Engineering**

the footing of the superstructure

(it's just a constrain)





the superstructure is the load on the soil

Soil-structure interaction

In this course, we will deal with uni-dimensional problems (mainly)

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6.03.18

lunedì 5 marzo 2018 17:37

FOUNDATION ENGINEERING

01RVSMX
A.A. 2017-2018

Historical notes on Soil Mechanics

1. Coulomb and the birth of Soil Mechanics

Charles Augustin De Coulomb (1736-1806)

studies of electricity and magnetism

Essai sur une application des règles de maximis et minimis à quelques problèmes de statique relatifs à l'Architecture (1773).

soil mechanics + structural elements

- shear strength of masonry and soils
- earth pressure
- stability of arches and the strength of beams

The stability analysis method (based on the mechanics of rigid bodies) developed by Coulomb is called "*limit equilibrium method*" and is still today commonly used, especially for the assessment of slope stability.

Coulomb starts from the study of RIGID BODIES

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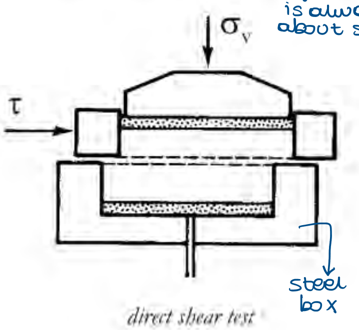
Soils are particulate materials

Soil (shear) strength depends on the confining stress

strength ↳ is always about shear

confining stress ↳ very low it behaves as a fluid, very high it behaves as a rigid rock



direct shear test

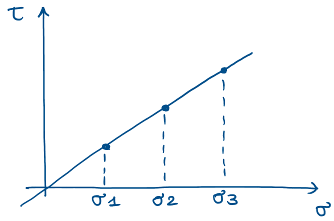
$$\tau = \sigma_v \cdot \tan(\varphi)$$

(failure criterion for dry soils)

$T = N \ tg\,\phi$

Dividing by the contact area A we have

$\tau = \sigma' \ tg\,\phi$

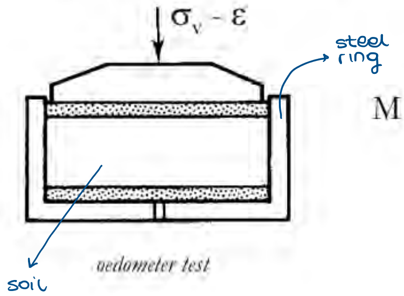


⇒ measure vertical and horizontal displacements when both vertical and horizontal forces are applied, util failure

Soils are particulate materials

↳ skeleton is made by grains

Soil (shear) stiffness depends on the confining stress



oedometer test

$$M = \frac{d\sigma_v}{d\varepsilon} = m \cdot \sigma_r \cdot \left(\frac{\sigma_v}{\sigma_r}\right)^{1-j}$$

(Janbu, 1963)

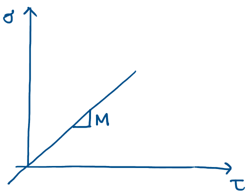
↳ under edometric conditions (radial strains aren't allowed) $\varepsilon_h = 0$

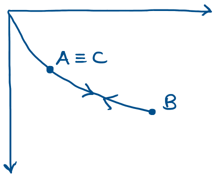
↓

only vertical strains are possible

If M is constant we have a straight line

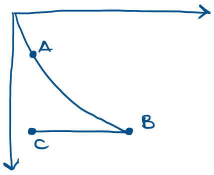
↳ in soils M isn't constant





ε is all elastic

2) If we haven't strain during unloading phase so we have a perfect plastic behaviour



ε is all irriversible
⇒ any swalling

Compression in grained soil produces a new arrangement and fractures and deformation
↳ if we have plastic strains there isn't the possibility to come back to the initial point
- deformation (a part is recoverable)
- fragmentation is irriversible

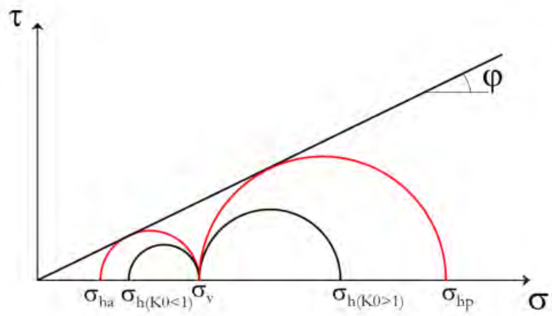
2. Rankine and the limiting states of stress

William John Macquorn Rankine
(1820-1872)

On the stability of loose earth (1857)

$$\sigma_{ha} = \underbrace{\frac{1-\sin(\varphi)}{1+\sin(\varphi)}}_{K_a} \cdot \sigma_v$$

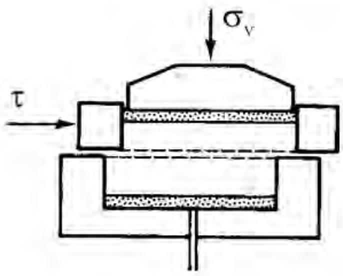
$$\sigma_{hp} = \underbrace{\frac{1+\sin(\varphi)}{1-\sin(\varphi)}}_{K_p} \cdot \sigma_v$$



Soils are particulate materials

Soil (shear) strength depends on the confining stress

↳ not a characteristic property



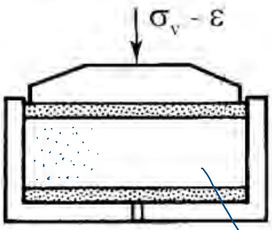
direct shear test

$$\tau = \sigma_v \cdot \tan(\varphi)$$

(failure criterion for dry soils)

Soils are particulate materials

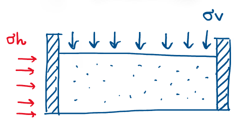
Soil (shear) stiffness depends on the confining stress



oedometer test

$$M = \frac{d\sigma_v}{d\varepsilon} = m \cdot \sigma_r \cdot \left(\frac{\sigma_v}{\sigma_r}\right)^{1-j}$$

(Janbu, 1963)



After compression we have two mechanisms:
- rearrangement in the more dense configuration
- fracture of grains

⟹ irreversible (both effects)
- increase of σh to maintain εh = 0

2. Rankine and the limiting states of stress

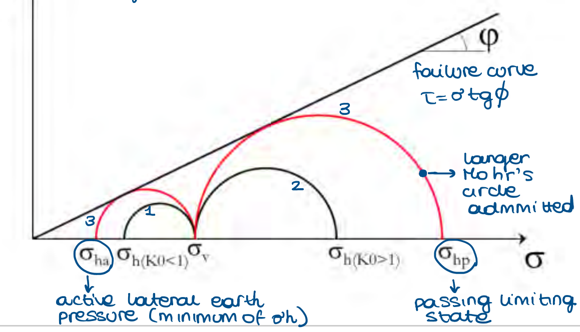
William John Macquorn Rankine
(1820-1872)

On the stability of loose earth (1857)

1) initial stress with $\sigma_v > \sigma_h$ (σ_h decreases)
2) initial stress with $\sigma_v < \sigma_h$ (σ_h increases)
3) limit condition when initial circle is tangent to the failure line

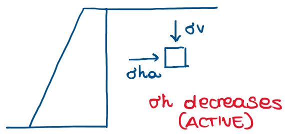
$$\sigma_{ha} = \underbrace{\frac{1-\sin(\varphi)}{1+\sin(\varphi)}}_{K_a} \cdot \sigma_v$$

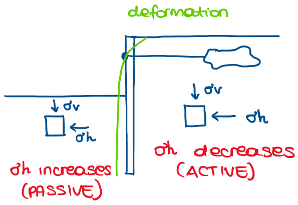
$$\sigma_{hp} = \underbrace{\frac{1+\sin(\varphi)}{1-\sin(\varphi)}}_{K_p} \cdot \sigma_v$$



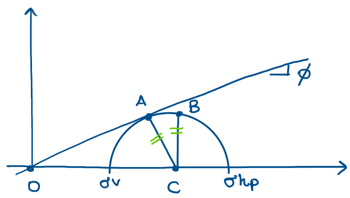
If σ_h is too high or to low we can have shear failure of soil
we can define the limits through Mohr's circles. (SLIDE ←)

we have an active limiting state in case of gravity walls





EX



$\overline{OC} = \sigma_{hB} + \frac{\sigma_v - \sigma_{hA}}{2} = \frac{\sigma_{hB} + \sigma_v}{2}$

$\overline{CB} = \frac{\sigma_v - \sigma_{hA}}{2}$

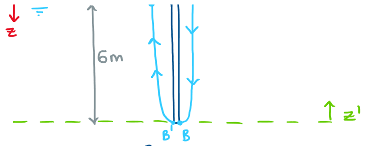
$\overline{AC} = \overline{CB} = \overline{OC}\ \text{sen}\phi$

$\frac{\sigma_v - \sigma_{hA}}{2} = \frac{\sigma_v + \sigma_{hA}}{2}\ \text{sen}\phi$

$\frac{\sigma_{hA}(1+\text{sen}\phi)}{2} = \frac{\sigma_v(1-\text{sen}\phi)}{2}$

$\frac{\sigma_{hA}}{\sigma_v} = \frac{1-\text{sen}\phi}{1+\text{sen}\phi} = K_a$

$\frac{\sigma_v}{\sigma_{hp}} \Rightarrow \frac{\sigma_{hp}}{\sigma_v} = \frac{1}{K_a} = K_p$



$\gamma = 20\ kN/m^3$
We have a water flow from the higher side to the lower

POINTS	z(m)	σ_v(MPa)	z'(m)	h(m)	u (kPa)	u_s
A	2.5	50	7,5	7,5	0	0
B	10	200	0	6,67	66,7 <	75
B'	6	120	0	6,67	66,7 >	60
C	0	0	6	6	60	0

$\sigma_v = \gamma \cdot z$

↓ static conditions (without flow)

z' is a reference plane
h_A and h_C is known because it coincides with water table

$h_B = h_A - \frac{\Delta h}{\Delta s} \cdot s = 7{,}5 - \frac{1{,}5}{13{,}5} \cdot 7{,}5 = 6{,}67$

13,5 = 7,5 + 6 (from A to B + from B' to C)

$u = \gamma_w (h - z')$
$\gamma_w = 10$

4. Terzaghi and the modern era of Soil Mechanics

Karl von Terzaghi
(1883-1963)

- First book on Soil Mechanics published in 1925
- Istanbul 1916-1925
- Boston (MIT) 1925-1929
- Vienna 1929-1937
- Boston (Harvard) 1938-1963

He was the first President of the International Society of Soil Mechanics and Geotechnical Engineering (ISSMGE)

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12.03.18

lunedì 12 marzo 2018 14:29

Effective stress in (saturated) soils

"All measurable effects of a change of stress, such as compression, distortion and a change of shearing resistance, are due exclusively to changes of effective stress. The effective stress σ' is related to the total stress and pore pressure by

$$\sigma' = \sigma - u."$$

The Terzaghi's principle of effective stress is given by:

1. the definition of the effective stress: $\sigma' = \sigma - u$
2. the basic postulate that the behavior of soil is related only to the effective stress

$$\tau = \sigma' \cdot \tan(\varphi) \qquad d\varepsilon = \frac{d\sigma'}{M(\sigma')} \qquad \sigma'_h = K \cdot \sigma'_v$$

K_0 K_a K_p

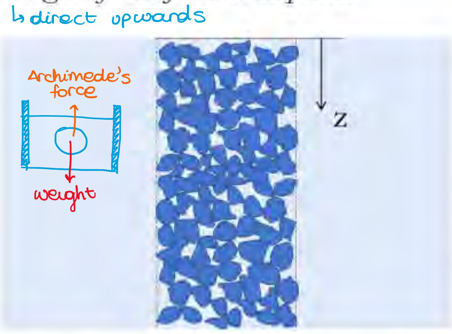
=> all quantities must be expressed using effective stress

Archimedes (287-212 B.C.) → phisical interpretation of the principle

A body immersed in a fluid experiences a buoyant force equal to the weight of the fluid it displaces

buoyant: galleggiante

↳ direct upwards



total stress:

$$(1-n)\cdot\gamma_s\cdot z + n\cdot\gamma_w\cdot z = \sigma$$

weight of grains (even outside water) — weight of water in the voids

pore pressure:

$$\gamma_w \cdot z = u$$

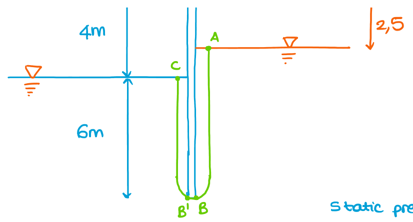
submerged grain stress:

$$(1-n)\cdot\gamma_s\cdot z - (1-n)\cdot\gamma_w\cdot z = \sigma - u = \sigma'$$

The effective stress is equal to the stress caused by the weight of the submerged grains; the physical principle that soil strain and strength are only related to ***this*** stress is called the "principle of effective stress".

$$(1-n)\gamma_s \cdot z - \gamma_w \cdot z + n\gamma_w \cdot z =$$

$$\underbrace{(1-n)\gamma_s z + n\gamma_w z}_{\sigma} - \underbrace{\gamma_w \cdot z}_{u} = \sigma'$$

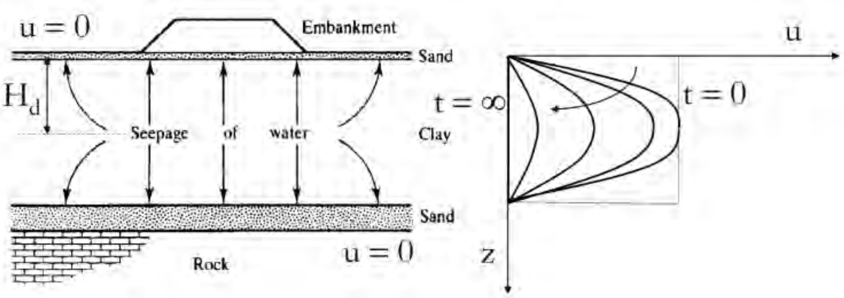


Static pressure of water

Points	σ_v (kPa)	u (kPa)	u_s (kPa)	σ'_v (kPa) $= \sigma' - u$	σ'_v (kPa) $= \sigma' - u$
A	30	0	0	50	50
B	200	66,7	75	200 − 66,7 = 133,3	125
B'	120	66,7	60	120 − 66,7 = 53,3	60
C	0	0	0	0	0

B : $u < u_s$ so $\sigma'_v > \sigma'_{v,s}$ (seepage is downwards)
B' : $u > u_s$ so $\sigma'_v < \sigma'_{v,s}$ (see page is upwards ⇒ SIFONAMENTO : PIPING)
↳ the effective stress is reduced and can make the soil behaves as a liquid

Consolidation



$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

c_v → both characteristics of soil and water → how much time it's necessary to dissipate the extra water pressure

$c_v = \frac{kM}{\gamma_w}$

$$U_s = \frac{s(t)}{s_\infty} = 1 - \sum_{k=0}^{\infty} \frac{2}{M^2} \exp\left(-M^2 \cdot T_v\right)$$

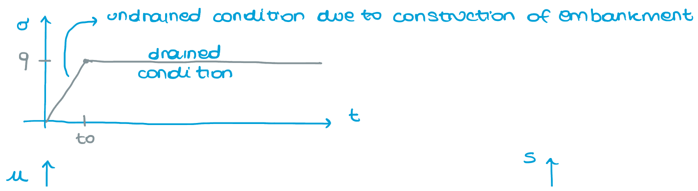
$$M = \frac{2k+1}{2}\pi \qquad T_v = \frac{c_v \cdot t}{H_d^2}$$

U_s ↓ average value of the settlement $0 < U_s < 1$

solution of this equation.

load can be considered as infinite

Same boundery conditions as sample in edometric test



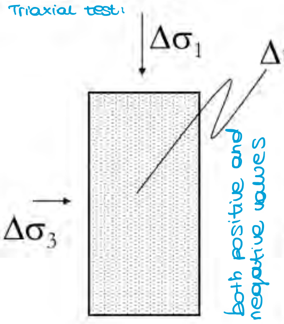
5. Skempton and the pore pressure parameters

Sir Alec Westley Skempton
(1914-2001)

The pore pressure parameters A and B (1954)
↳ under generic load

$$\Delta u = B\left[\Delta\sigma_1 + A(\Delta\sigma_1 - \Delta\sigma_3)\right]$$

Triaxial test:



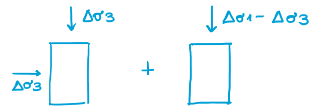
B = 1 for saturated clays

Clay Type	A = f(OCR)
High sensitivity clay	0.75 - 1.5
Normalconsolidated clay	0.5 - 1
Lightly overconsolidated clay	0 - 0.5
Heavely overconsolidated clay	−0.5 - 0

↳ as it is tensioned not compressed

clay tends to increase its volume so A has a negative value

SUPERPOSITION OF EFFECTS



$\Delta u = B\,\Delta\sigma_3 = \Delta\sigma_3$
$\Delta u = (A)\ (\Delta\sigma_1 - \Delta\sigma_3)$
function of over consolidation ratio

Undrained conditions: analysis in terms of total stress

↳ effective stress analysis to evaluate change of u ⟶ changing of pour pressure under undrained condition in field ⟶ perform a total stress analysis

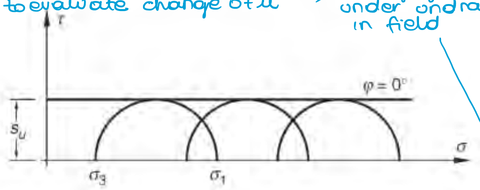
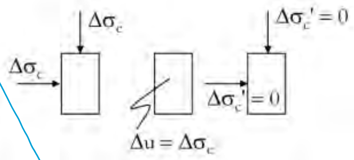
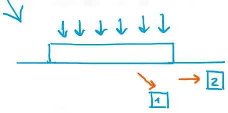


Figure 5.38 Failure criterion in terms of total stress.
from Lancellotta (2009)



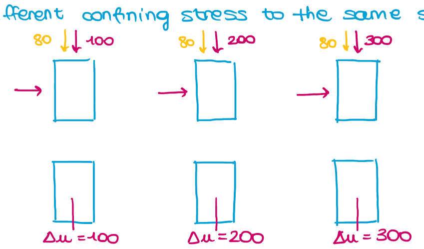


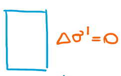
Two points have very different behaviours (we have only Skempton equation we need a solution)

Koutsoftas and Ladd (1985)

$$\frac{c_u}{\sigma'_{v0}} = (0.22 \pm 0.03)\cdot OCR^{0.8} \qquad OCR < 10$$

Different confining stress to the same samples





for all the samples

Δu = Δσ because we are in isotropic conditions

7. Soil dilatancy and dissipation work

Donald Wood Taylor
(1900-1955)

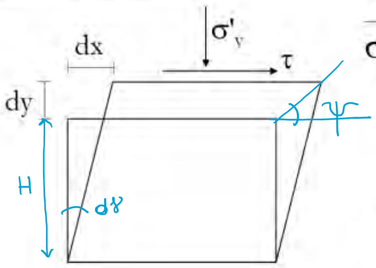
Fundamentals of soil mechanics (1948)

$$\tau \cdot dx - \sigma'_v \cdot dy = \tan(\varphi_{cv}) \cdot \sigma'_v$$

$$\frac{\tau}{\sigma'_v} = \tan(\varphi) = \tan(\varphi_{cv}) + \frac{dy}{dx}$$

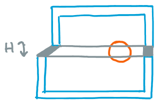
isn't only friction angle but is given by φ_{cv} + dilation

$$= -\frac{d\varepsilon_v}{d\gamma} = \tan(\psi)$$



ψ is the angle of dilation
(related to grain interlocking)

Direct shear stress box :

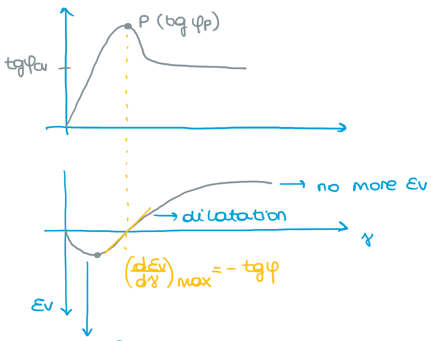


we consider an element in the shear surface

$d\gamma = \frac{dx}{H}$ $\quad d\varepsilon_v = \frac{dy}{H}$

Volumetric strain is positive for compression (decrease of volume)

If we consider a typical result of this test for dense sand



$$\left(\frac{d\varepsilon_v}{d\gamma}\right)_{max} = -tg\,\varphi$$

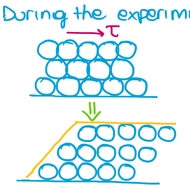
$$tg\,\psi = \frac{dy}{dx} = -\frac{d\varepsilon_v}{d\gamma}$$

The beaviour is explained by TAYLOR trough an energy equation

$$\tau dx - \sigma'_v dy = tg(\varphi_{cv}) \cdot \sigma'_v$$

external work — energy dissipated trough soil friction

During the experiment grains change



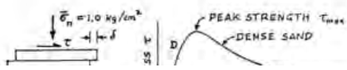
grains are in a very densified state so they move horizontally and the final configuration has a higher volume

8. Casagrande and the critical void ratio of sands

independent from initial condition

Arthur Casagrande
(1902-1981)

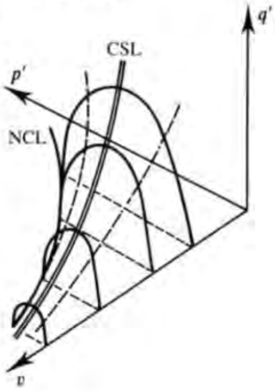
Characteristics of cohesionless soils affecting the stability of earth fills (1936)



9. Roscoe and the Cambridge model for disturbed soils

Kenneth Harry Roscoe
(1914-1970)

Schofield, A. and Wroth, P. (1968)
Critical State Soil Mechanics



State boundary surface $\frac{q}{p'} = M - \frac{d\varepsilon_v^p}{d\varepsilon_s^p}$

Critical state line (CSL) $q = M \cdot p'$

$v = \Gamma - \lambda \cdot \ln(p')$

10. International Society - ISSMFE (1936-1997) and ISSMGE (1997-present)

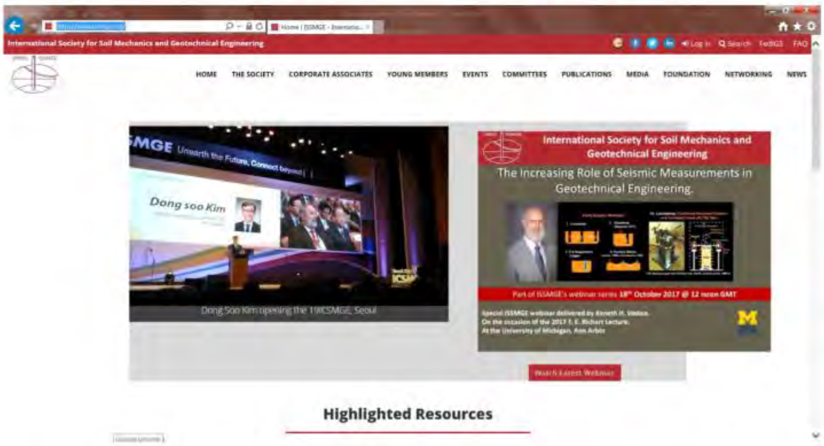
The International Society had its origins in the First International Conference on Soil Mechanics and Foundation Engineering held in **Harvard in 1936**. A total of 206 delegates attended from 20 countries. In order to ensure continuation of this very successful initiative, an Executive Committee was set up with Karl Terzaghi as President and Arthur Casagrande as Secretary; but war intervened and the Second ICSMFE was not held until **1948 in Rotterdam**. Again this proved to be a great success, with 596 delegates. By the time of the Third ICSMFE in Zurich in 1953, the International Society had become firmly established, with Terzaghi as President and Donald Taylor as Secretary. In 1957 A W Skempton became President, and the Secretariat moved to the UK.
In 1997, Council approved a change in name to the **International Society for Soil Mechanics and Geotechnical Engineering** to reflect more accurately the activities of the Society.
ISSMGE has experienced a rapid growth in membership, the 32 Member Societies and 2500 individual members in 1957 increasing to 50/11500 in 1977, 71/16500 in 1998, and 89/20800 in 2017. The growth in membership has been matched by increasing activities, largely through the establishment of many active Technical Committees and Task Forces.

ISSMGE Technical Committees

Italian Chairmen

Laboratory Stress Strain Strength Testing of Geomaterials	TC101
Ground Property Characterization from In-Situ Tests	TC102
Numerical Methods	TC103
Physical Modelling in Geotechnics	TC104
Geo-Mechanics from Micro to Macro	TC105
Unsaturated Soils	TC106
Laterites and Lateritic Soils	TC107
Geotechnical Aspects of Dykes and Levees and Shore Protection	TC201
Transportation Geotechnics	TC202
Earthquake Geotechnical Engineering and Associated Problems	TC203
Underground Construction in Soft Ground	TC204
Safety and Serviceability in Geotechnical Design	TC205
Interactive Geotechnical Design	TC206
Soil-Structure Interaction and Retaining Walls	TC207
Slope Stability in Engineering Practice	TC208
Offshore Geotechnics	TC209
Dams & Embankments	TC210
Ground Improvement	TC211
Deep Foundations	TC212
Scour and Erosion	TC213
Foundation Engineering for Difficult Soft Soil Conditions	TC214
Environmental Geotechnics	TC215
Frost Geotechnics	TC216
Land Reclamation	TC217
Reinforced Fill Structures	TC218
Preservation of Historic Sites	TC301
Forensic Geotechnical Engineering	TC302
Coastal and River Disaster Mitigation and Rehabilitation	TC303
Engineering Practice of Risk Assessment and Management	TC304
Geotechnical Infrastructure for Megacities and New Capitals	TC305
Geo-engineering Education	TC306
Sustainability in Geotechnical Engineering	TC307
Energy Geotechnics	TC308
Machine Learning and Big Data	TC309

ISSMGE Website: https://www.issmge.org/



- Information on conferences, symposia and other relevant events concerning the geotechnical community
- Reports from technical committees
- Free download of conference proceedings

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RIG – 50th Anniversary

Suggested readings

Lancellotta, R. (2017). Editorial – More than a birthday, Rivista Italiana di Geotecnica, supplement to n. 4/2017, 3-13.

Calabresi, G. (2017). 50th Anniversary of Rivista Italiana di Geotecnica (RIG). Thoughts and Memories, supplement to n. 4/2017, 14-36.

To review Soil Mechanics' basic notions:

Lancellotta R. (2009). Geotechnical Engineering 2nd Edition, Taylor & Francis, Abingdon (UK) and New York (USA):
Parts 4.1-4.2, 4.3-4.7, 5.1-5.2, 5.4-5.5, 5.8, 5.10, 6.2, 6.10

which finishes along the CLS

For each value we can built a line with a φ_p as angular coefficient

- depend on e_0
- depend on σ'_p

For example B $\varphi_{PB} = f(e_0, \sigma'_{PB})$

If we built a line passing through all the three points the angular coefficient is different

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2. Phases of site characterization

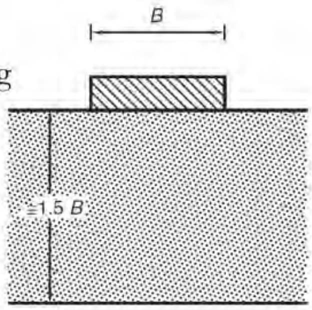
- Collection of available information such as maps, existing reports, site plans.
- Site visit ("*You have to see it to solve it*"*) to provide a general picture of the topography and geology of the site. * Lommler "Geothecnical Problem Solving"
- Detailed soil exploration: mapping the subsurface (stratigraphy) using nondestructive (geophysical methods) and destructive methods (boreholes); soil sampling for laboratory tests; *in situ* tests.
- Laboratory testing.
- Write a report (the report must contain a description of the soils at the site, methods of exploration, soil stratigraphy, *in situ* and laboratory test methods and results, and the location of the groundwater.

3. Exploration programme

- For ordinary buildings it is recommended to plan four borings, one at each corner of the structure.
- The footprint of a structure (plan area covered by the structure) should be divided using a grid approximately 20 m to 40 m for large areas and boreholes should be located at node points on the grid.
- If the locations of the loads on the footprint of the structure are known, you should consider drilling at least one borehole at the location of the heaviest load.

The depth of the borehole usually depends on the depth at which the soil is still affected by the stresses induced by the foundation loads. In general, it can be observed that at a depth of 1.5 times the foundation width the induced vertical stress is of the order of 20% of the unit load applied at the surface, so the depth of the borehole should be at least 1.5 times the foundation width.

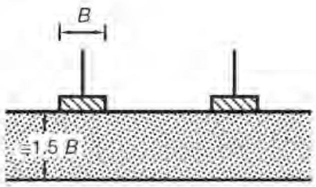
Spread footing



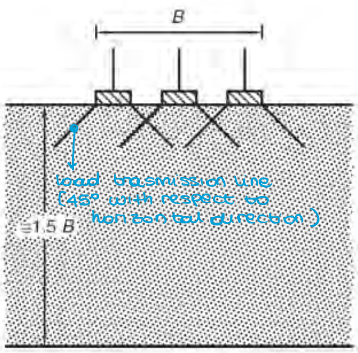
$\Delta\sigma_V \approx 20\% \cdot q$

$\Delta\sigma_V \approx 10\% \cdot \sigma'_{V0}$

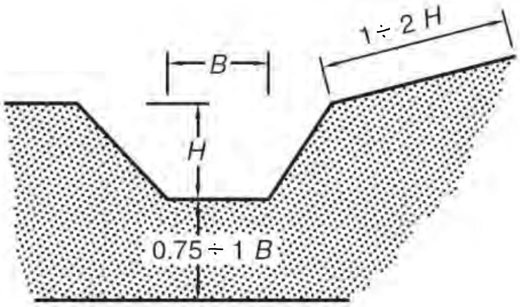
If the same footings are so closely spaced that there is overlap of the induced stresses, the depth of the borings should be compared to the overall area, as in the case of a raft foundation.



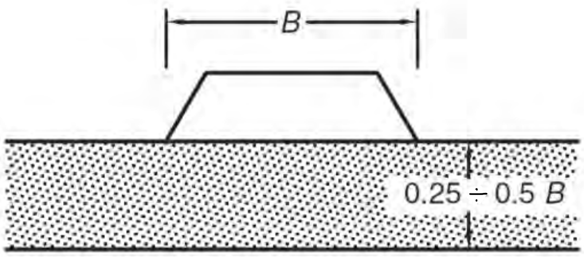
So we don't consider the 1,5 · B (=the single foundation) but 1,5 · Btot (= of the all serie of foundation)



For deep cuts, the depth of a sliding surface depends on the width of the cut, whereas the lateral extension is related to the depth of the cut.



For railroad embankments, the depth of borings should be higher than the thickness of the weathered strata, and sufficient to allow the analysis of stability, the drainage conditions and possible frost effects.



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4. Trial pits (TRINCEA)

Trial pits represent one of the simplest and most reliable methods of soil investigation.
It allows one to directly explore soil conditions, so that the sequence of strata as well as macrofabric features can be accurately detected. This procedure is a great advantage when dealing with stiff fissured clays, and in addition it enables one to obtain block samples, which can be cut from the bottom or from the sides of the pit. Obviously, the shortcoming of this procedure is the depth of the investigation, which is confined to a maximum of 5–6 m in stiff clays, and to shallow depth in coarse-grained soils, also depending on groundwater table.

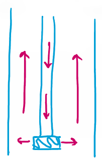
From Lefebvre and Poulin (1979)

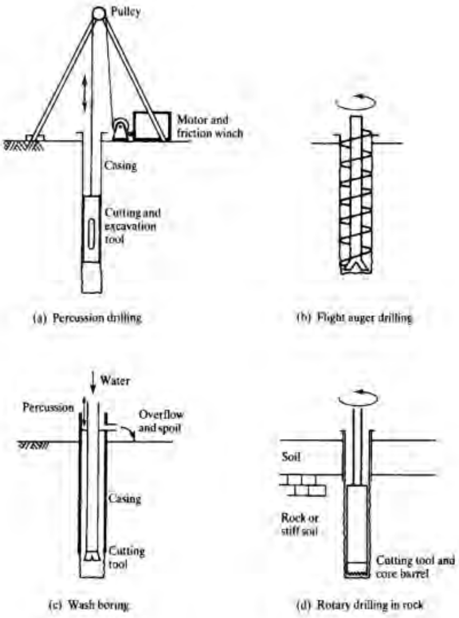
Auger boring.

The short-flight auger, used to advance the hole, is a helic of limited length attached to a Kelly bar. This is a drilling rod which is rotated and pushed into the soil to obtain the advancing of the hole.
The diameter of the auger is usually from 75 mm to 300 mm and this technique is mainly used in clays, because the hole does not require support. Bentonite slurry is introduced to support unstable holes, whereas casing is generally avoided because it requires the removal of the auger before introducing the casing. Difficulties with this procedure generally arise when cobbles and boulders are encountered and the main limitation comes from the fact that different soil strata are mixed, therefore it is difficult to recognize where stratigrafical changes occur.

Wash boring

This is a simple procedure to obtain relatively deep holes. In this case, water is pumped through a hollow rod and is forced through small slots in the drill bit. The soil fragments are carried to the surface through the annular space between the casing and the washing rod. The hole is generally encased, but mud can also be used as an alternative. The soil fragments carried to the surface do not allow an accurate identification of soil type, therefore the method has to be considered only as a borehole making procedure.





6. Stabilization of boreholes

15/03/18

The need to prevent the sides and bottom of the hole from collapse is common to all previously mentioned drilling methods. Dry boreholes can be stable above the water table and at relatively shallow depths, but as soon as the depth increases and the water table is encountered a stabilization procedure must be adopted.

7. Sampling techniques

A soil sample can be considered an *undisturbed soil sample* if it has been so carefully sampled to preserve both the structure and the water content of the soil in the field.

However, the process of sampling has some influence on the structure of the soil and the associated changes are called *disturbance*.

The quality of the sample is by far the most important requirement, since the disturbance of a soil sample tends to modify the original soil structure and all information related to this structure, implying that the soil partially loses its memory.

DISTURBANCE

1. change in stress condition
2. mechanical disturbance of soil structures
3. change water content and porosity (related but different!)
4. chemical change
5. mixing and segregation of soil consituents

GOAL

- Minimize the first three
- Eliminate the last two

$S \cdot e = G_s \cdot w$

degree of saturation — specific gravity

$S = \frac{V_w}{V_v}$ $\begin{cases} 0 & \text{dry soil} \\ 1 & \text{saturated soil} \end{cases}$

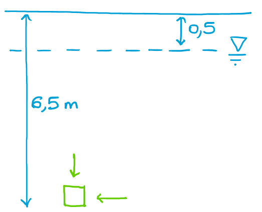
$e = \frac{V_v}{V_s} = \frac{n}{1-n}$ → n is the porosity → when $0<n<1$, $0<e<\infty$

$G_s = \frac{\gamma_s}{\gamma_w} \sim 2{,}70$

$w = \frac{W_w}{W_s}$

↳ water content contains two informations about soil density and about degree of saturation
↳ loose or not

EX



$\gamma_w = 10$ kN/m³
$\gamma = 19{,}15$ kN/m³
$K_0 = 0{,}5$

we can evaluate vertical and horizontal stresses in terms of total and effective terms.

$\sigma_{v0} = 19{,}15 \cdot 6{,}5 = 124{,}5$ kPa
$u_0 = 10\,(6{,}5 - 0{,}5) = 60$ kPa
$\sigma'_{v0} = \sigma_{v0} - u_0 = 64{,}5$ kPa
$\sigma'_{h0} = K_0\, \sigma'_{v0} = 32{,}25$ kPa
$\sigma_{h0} = \sigma'_{h0} + u_0 = 92{,}25$ kPa

we have to follow this order

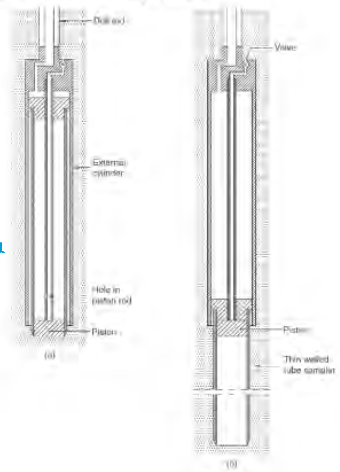
We can now evaluate the effective average effective stress (p'_0)
↳ in the horizontal direction we have two stresses equal to each other

$p'_0 = \frac{\sigma'_{v0} + 2\sigma'_{h0}}{3} = 43$ kPa

| z

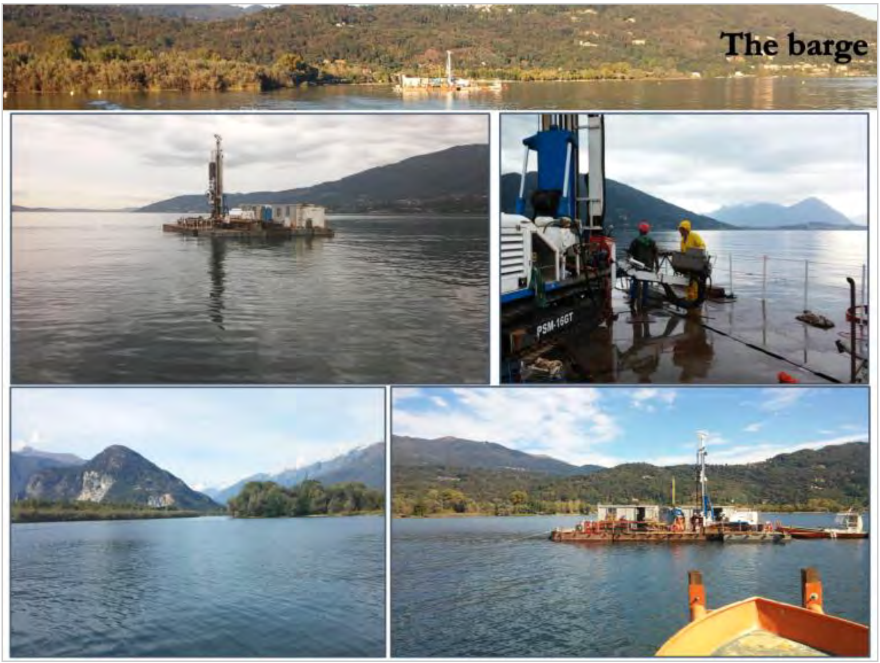
In clay soils, the open ended thin-wall tube samplers are usually pushed at a constant rate of the order of 150 to 300 mm/s (Hvorslev, 1949). Once the sample has been recovered, the ends of the tube are sealed in order to avoid moisture changes. Limitations of the open-ended samplers arise from the fact that remoulded material cannot be prevented from entering the tube from the sides and from the bottom of the hole.

The quality of the samples can be increased by using the **stationary piston sampler**; a thin-walled tube is fitted, in this case, with a piston which closes the end of the tube until the bottom of the borehole is reached. In such a way, the remoulded soil is prevented from entering the tube; the tube is then pushed past the piston to sample the soil. The available diameters usually range from 35 to 100 mm (but larger diameters have also been successfully used), and the length of the sample is of the order of 1 m.



→ first application in Italy

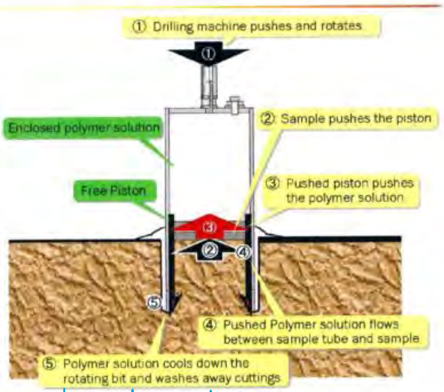
The use of the Gel-Push Sampler for the characterization of the Maggiore Lake sediments.



WHAT IS GP SAMPLING ?

"GP" means "Gel - Push".
All GP Samplers have a free piston or a fixed piston.
The piston encloses high density polymer solution in the sampler inside.
GP Samplers have structures to push the enclosed polymer solution to the tip of the samples.

①: Drilling or pushing
②: The piston of GP Sampler is relatively pushed from ground as the sampler is penetrating.
③: The piston pushes the polymer solution as the sample comes into the sample tube
④: The pushed polymer solution flows to sample
⑤: The polymer solution cools down the rotation bit and washes away cuttings

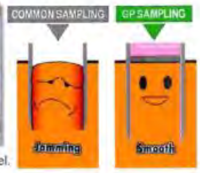


↳ in order to reduce friction during the sampling and during sample extrusion

POLYMER GEL REDUCES FRICTIONS DURING THE SAMPLING

It is not only the tip of the sampler (Bit, Cutting shoe) that the reason of disturbed sample. The friction of the sample tube gives stress to a sample, too.
Polymer solution reduces friction of sample tube with a sample.

Photo 1 : GP Sampler which is covered with polymer gel.



POLYMER GEL REDUCES FRICTIONS WHEN THE SAMPLE EXTRUSION

A minimum friction by polymer gel makes easy to extrude a sample from the sample tube.

Figure 1. Result of the laboratory experiments to check pressure and relations of the quantity of displacement necessary to push the sand in the thin-wall tube. The result compare a common thin-wall tube with thin-wall tube applied a polymer. A tube with polymer is easier to extrude sample than a tube with no-polymer.

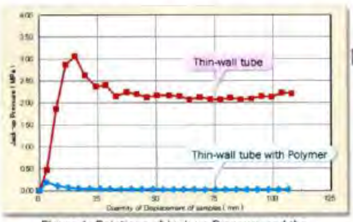
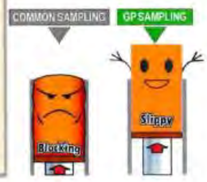
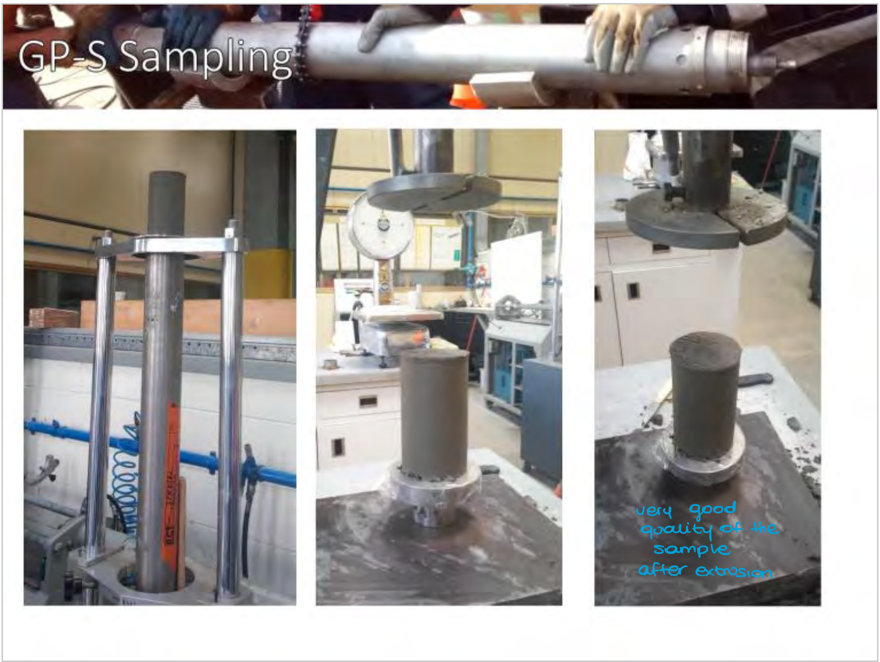
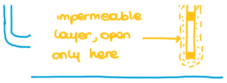


Figure 1. Relations of Jack-up Pressure and the quantity of displacement of sample in tube.




GP-S Sampling
very good quality of the sample after extrusion


GP-TR Sampling
(a)
(b)
(c)
(d)
inner tube in PVC
(a) (b) Sampler preparation
(c) (d) Sample extraction from the sampler



high hydraulic conductivity → the acquifer in this case is confined → the hydraulic head is higher than the top of the layer

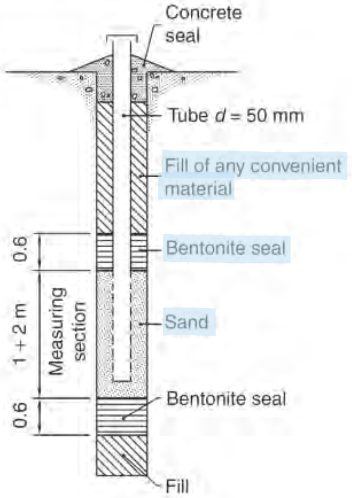
It's difficult to evaluate pore pressure
- static in acquifer but different values for the upper and the lower one
- dynamic in acquitard

used only in simple conditions when we have only an acquifer and not in cases when there are several layers crossed by the instrument itself.
↳ ex only for the upper freatic acquifer (!!)
↳ in more complex conditions we need to change the piezometer

If the perforated section is not sealed along its entire length, the open standpipe piezometer creates a vertical connection between the strata and the obtained measurements may be misleading. → each piezometer mesures only one layer without connecting water from different acquifers (!!)

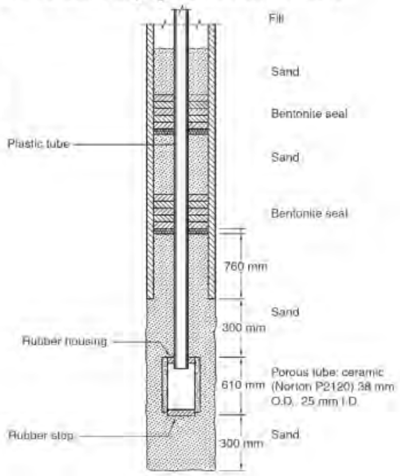
Therefore it should be installed only in homogeneous permeable strata, in which it is assumed that the pore pressure is hydrostatic.

For reliable measurements of local pore pressure, it is necessary to seal the perforated section with the surrounding filter.



In order to have the same pore pressure value in the pipe and in the surrounding soil, a long period of time is required to allow the water to flow into the piezometer, so that this type of piezometer is usually installed in relatively pervious soils.

In order to reduce the response time, *Casagrande* (1949) designed a standpipe piezometer, consisting of a cylindrical porous ceramic tube, 300 mm long and 38 mm in diameter, connected to a standpipe of 10 mm in diameter. → more accurate evaluation



Standard penetration test (SPT)

The SPT (Standard Penetration Test) was developed in USA in 1927, and is a test performed during boring in order to obtain a measure of soil resistance to the dynamic penetration and a disturbed soil sample.

↱ within the boreholes

The test is carried out by dropping a falling weight of 63.5 kg onto the drill rods from a height of 760 mm, and the number of blows (N_{SPT}) necessary for penetration of a predefinite distance of a standard sampler is the required measure of the state condition of the soil.

↳ the weight and the height have to be constant so the energy of the hammer is constant

When making such measurements, the sampler is driven a total of 450 mm, but this penetration is divided in three separate advances of 150 mm: the initial seating drive of the sampler of 150 mm is disregarded, because it reflects the influence of the disturbed soil at the bottom of the hole; then the operator counts the number of blows to advance the sampler each of the remaining 150 mm and adds these last two counts to obtain the number of blows (N_{SPT}) necessary for penetration of 300 mm, which represents the required measure.

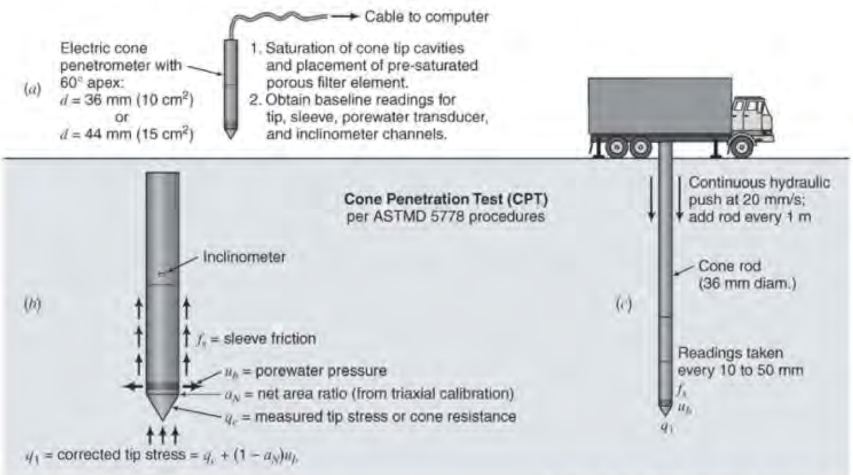
The test is usually performed at intervals ranging from 45 to 105 cm in a borehole, with a diameter ranging from 65 to 115 mm, as larger diameters can have a significant influence on the test results. The drilling operation must be interrupted each time the test is performed.

⇒ granulometric curve has to be controlled

The thrusts required to drive the cone and the sleeve into the ground are measured independently so that the end resistance (or cone resistance, q_c) and the side friction (or sleeve resistance, f_s) may be estimated separately.

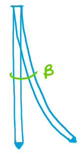
The piezocone (CPTU) has porous elements inserted into the cone or sleeve (guaina) to allow porewater pressure measurements (u).

The CPT test results are normally recorded electronically (electric cone) by running a cable carrying the signals from the load transducers and the pore water pressure transducer through the hollow drill rods and connecting it to a data acquisition system at the ground surface.



Source: Prof. Paul Mayne, Georgia Tech

If we have a very deep test the bending could be high



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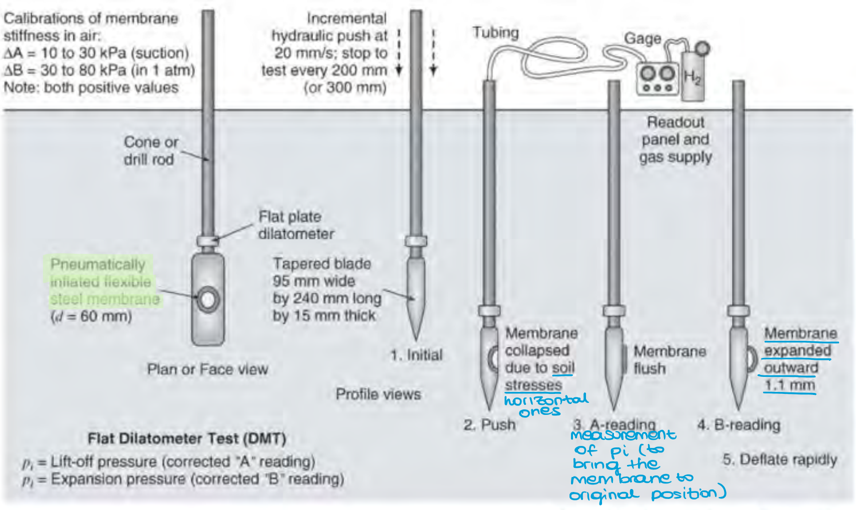
Flat dilatometer (DMT)

The flat dilatometer was developed in Italy by Marchetti in the late 1970s. A flat plate, 14 mm thick, hosts a flexible stainless steel membrane, 60 mm in diameter, located on one face. The plate is pushed down from the surface, by using the same technique used for CPT, and the test is performed at spaced intervals of 200 mm.
This consists of inflating the membrane by using nitrogen pressure, and the pressure required to just lift the membrane off the sensing disc (*lift-off* pressure, p_0) and the pressure required to cause a 1.1 mm deflection (p_1) are recorded.



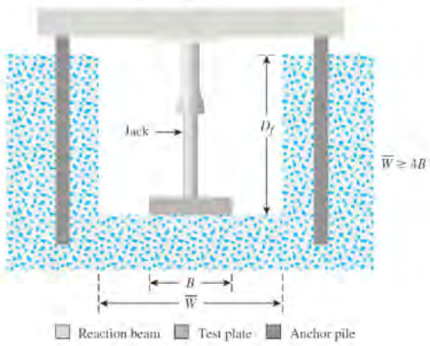
Source: Prof. Paul Mayne, Georgia Tech

Plate load test (PLT)

Plate load tests can be considered as one of the first examples of in situ tests performed in order to obtain soil stiffness. This test can be performed at the surface or at the base of a borehole.
The plates are made from steel with sizes varyng from 152 mm to 900 mm. Two common plates sizes used in practice ara a square plate of width 300 mm an a circular plate of diameter 300 mm. The test is carried out in a pit (buco) at the depth of the proposed footing.

Loads are applied in increments of not more than 95 kPa or 10% of the estimated allowable bearing capacity. Each load increment is held until settlement ceases. The final settlement at the end of each loading increment is recorded. Loading is continued until settlements are in excess of 10% of the plate diameter. The maximum load should be at least 1.5 times the estimated allowable bearing capacity.

common for evaluation of stiffness of soil (or bearing capacity of shallow foundation)



In situ investigation Pagina 32

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Originally, the cone penetrometer was used to measure just the tip resistance. Successively, the measurement of the sleeve friction was envisaged as an estimate of pile shaft resistance. Because in sands the tip resistance tends to be high and the friction ratio tends to be low, while the reverse is observed in clay soils, both these measurements have been used in recent times as indicators of the soil type (Robertson, 1990). However, there are limitations associated with the use of such empirical correlations. In particular it must be observed that, despite the good repeatability of the tip resistance values, the repeatability of the sleeve friction may be poor, as it depends on several factors such as maintenance of the instrument, mechanical compliance with the cone and inclusions in the seals.

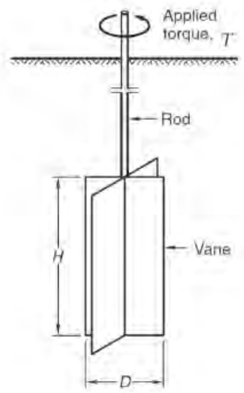
=> if we aren't sure of the quality of datas it's better considering the test only as a preliminary evaluation

Note that, since the resolution in terms of pore pressure is higher than that in terms of tip resistance, the CPTU is at present the most useful tool for detecting soil layering. However, it must be outlined that an important source of errors in the measured pore pressure may occur if the measuring system is not fully saturated. This usually leads to incorrected initial pore pressure values, lower capacity of detecting soil layering, and delayed recovery of the pore pressure level when restarting penetration phase after a dissipation test.

12. Undrained strength

The measured peak torque, T_{max}, from the vane shear device is converted to undrained shear strength using

$$c_u = \frac{2 \cdot T_{max}}{\pi D^3 \left(\frac{H}{D} + \frac{1}{3} \right)}$$



The ratio of the peak torque to the residual torque is the soil sensitivity, S_t

$$S_t = \frac{T_{max}}{T_r}$$

both th measured with the test SVT → so we can compute soil sensitivity

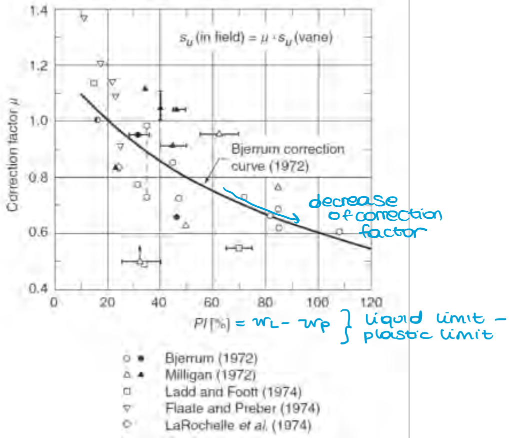
Sensitivity is a measure of the reduction of undrained shear strength due to soil disturbance.

The undrained strength is not a soil property, but it rather represents the soil response, so that it depends on the test used and may be different when considering different modes of failure of soil elements along a potential failure surface. For this reason, Bjerrum (1972, 1973) first suggested that the undrained strength from field vane should be corrected to obtain the strength mobilized at failure, using an empirical correction of the form:

$$c_u = \mu \cdot c_{u(SVT)}$$

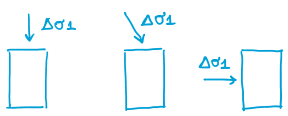
c_u: real value; μ → f(IP); $c_{u(SVT)}$ → measured value

it isn't a property of soil (response to total state of stress to soil)



OBSERVATIONS ABOUT cu

cu



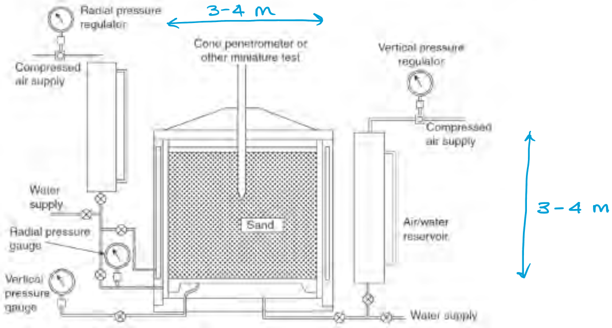
Assume that Δσ1 is always the same, but cu changes (different load pressure)
↳ depends on boundary conditions in terms of loads

Again real situation is different from the test for what concerns the over pressure.
↳ That's why there is μ as correction factor

13. Shear strength of coarse-grained soils (sands)

The interpretation of cone penetration tests in sand is a more difficult problem than interpretation in clay and the currently used approaches are mainly based on tests performed in calibration chamber, which is a double-walled cylinder, (huge dimensions) equipped with a base piston and a top lid. Sand specimens, to be tested within this cylinder, are usually obtained by pluvial deposition.

we use now the results performed in the past (only few times, not very common test)



The critical state angle φ_{cv} of sands depends on mineralogy, roughness and grading of particles, but it is independent of the initial conditions. In silica sands it can range from 30° to 38° (higher values apply to angular particles), and in feldspar or carbonatic sands can reach values up to 40°.

On the contrary, the peak value of the angle of shear resistance is linked to the rate of dilation, it depends on soil state and cannot be regarded as a soil property.

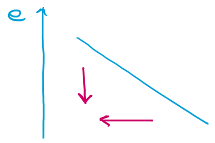
The rate of dilation depends on the specific volume and on the effective stress level, so that we reach the conclusion that peak value of the angle of shear resistance increases if the relative density increases, but decreases with higher level of effective stress.

$$tg\,\varphi = tg\,\varphi_c - \frac{dy}{dx}$$

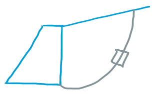
($\frac{dy}{dx}$ → $tg\,\psi$)

grain interlocking (soil dilation) ψ is the dilation angle

To evaluate ψ we refer to the foundamental graph e - lnp'



increase of the peak value of shear strength



$p'_f = \frac{2}{3}\sigma'_{v0}$

⇒ The only variable we need to estimate is D_R
we can evaluate it in two ways

→ standar penetration test

1) Skempton (1986) reviewed the SPT data from Japan, UK, China and USA, and has suggested the following correlation:

$$D_R^2 = \frac{N_1}{60}$$

$$N_1 = C_N \cdot N_{SPT}$$ → correction of Terzaghi

$$C_N = \begin{cases} \dfrac{2}{1+\dfrac{\sigma'_{v0}}{100}} & \text{for fine sand} \\ \dfrac{3}{2+\dfrac{\sigma'_{v0}}{100}} & \text{for coarse sand} \end{cases}$$

function of vertical effective stress (depth z)

For silty sands $N_{SPT} > 15$

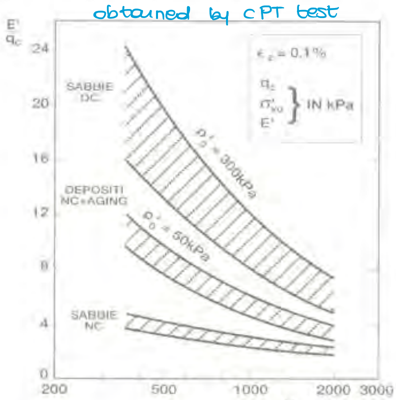
$N_{SPT} = 15 + \left(\frac{N_{SPT} - 15}{2}\right)$

where N_1 is the SPT value that refers to an average energy ratio of 60% and to a reference stress of 1 atmosphere. The overburden effective stress σ'_{v0} is in kPa.

14. Stiffness

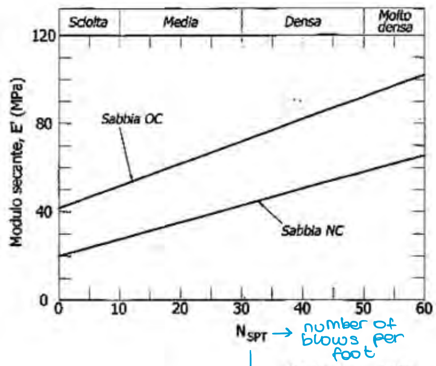
The Young modulus E for sands can be estimated from the following empirical correlations.

obtained by CPT test



we can also take an average value of each range

Berardi et al., 1991



N_{SPT} → number of blows per foot

standard penetration test

D'Apollonia, 1970

Correlation between E and in situ tests results.

The Young modulus for clays can be estimated from the following empirical correlations.

Undrained conditions

$$OCR = \left(\frac{q_c - \sigma_{v0}}{\alpha \cdot N_c \cdot \sigma'_{v0}} \right)^{1.25}$$

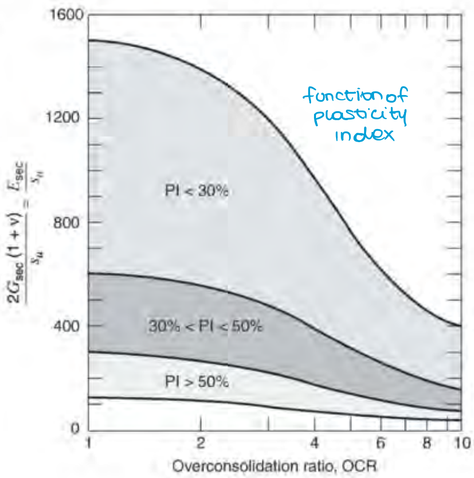
where α = 0.22-0.25.

N_c empirical coefficient

Normally consolidated clay = 14

Over cons. but intact clay = 17

Fessured clay = 10 ÷ 30



function of plasticity index

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FOUNDATION ENGINEERING

01RVSMX
A.A. 2017-2018

Reliability in geotechnical engineering

1. From laboratory to field applications

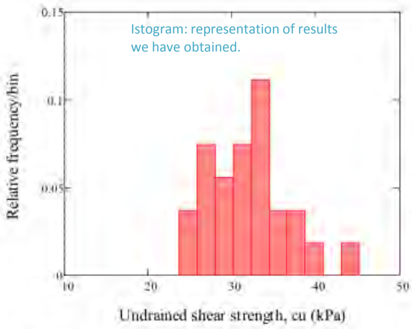
Soil Mechanics

Advanced theoretical models
- Inelastic behavior
- Peak and critical state shear strength
- Strain hardening and softening

Laboratory tests
- imposed stress or strain
- controlled boundary conditions
- accurate measurements

↳ advanced models to interpretate datas

Test number	Measured undrained shear strength, c_u (kPa)
1	39.2
2	23.9
3	28.3
4	34.8
5	31.9
6	28.9
7	33.7
8	32.7
9	30.7
10	36.6
11	26.5
12	31.7
13	45.2
14	27.4
15	34.9
16	33.1
17	37.3
18	28.8
19	27.3
20	32.7
21	26.6
22	30.6
23	34.2
24	33.2
25	23.7



Bin: 2.15 kPa

It's the width of the single column

No. of bins: 10

Mean: Most representative datum

$$\bar{c}_u = \frac{1}{N}\sum_{i=1}^{N} c_{u_i} = 31.75 \text{ kPa}$$

Dispersion of data

Standard deviation:

$$\sigma_{c_u} = \sqrt{\frac{\sum_{i=1}^{N}(c_{u_i} - \bar{c}_u)^2}{N-1}} = 4.9 \text{ kPa}$$

Coefficient of variation:

$$COV = \frac{\sigma_{c_u}}{\bar{c}_u} = \frac{4.9}{31.75} = 15\%$$

N=25 (number of total samples)

Function of cu

Probability density function: *normal distribution*

$$f(c_u) = \frac{1}{\sigma_{c_u}\cdot\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{c_u - \bar{c}_u}{\sigma_{c_u}}\right)^2\right]$$

We need to substitute the value of the mean and the standard deviation

Properties of the probability density function:

$$\int_{-\infty}^{\infty} f(c_u)\cdot dc_u = 1$$

Area under the curve which represent the probability distribution.

$$P[a < c_u < b] = \int_a^b f(c_u)\cdot dc_u$$

Probability to have numbers between a and b

Cumulative distribution function

$$P(c_u) = \int_{-\infty}^{c_u} f(x)\cdot dx = P[x \le c_u]$$

Probability to have numbers less than a fixed value of cu

For the margin of safety is possible to find an exact solution:

$$M = c_u - \frac{\gamma \cdot H}{4}$$

$$\bar{M} = \bar{c}_u - \frac{\bar{\gamma} \cdot H}{4} = 12 \text{ kPa}$$

they are the same results as before

$$\sigma_M^2 = \sigma_{c_u}^2 + \left(\frac{H}{4}\right)^2 \cdot \sigma_\gamma^2 = 4.8^2 + 2^2 = 27{,}04 \text{ kPa}^2$$

$$\sigma_M = \sqrt{27.04} = 5.2 \text{ kPa}$$

$c_u \rightarrow x_1$ $\quad a = \frac{H}{4}$
$\gamma \rightarrow x_2$
$F \rightarrow y$
$y = \frac{x_1}{x_2 \cdot a}$ — it isn't a linear function

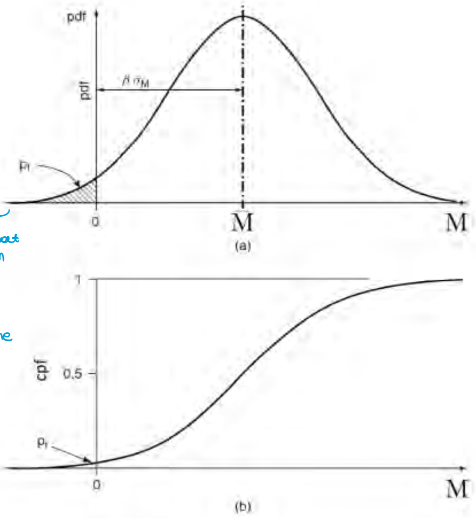
$M \rightarrow y$
$y = x_1 - a x_2$ — linear function

If R (i.e. c_u in this example) and Q (i.e. $\gamma \cdot H/4$) are normally distributed, M is normally distributed as well.

A reliability index β is defined as follows

$$\beta = \frac{\bar{M}}{\sigma_M}$$

which expresses the distance of the mean margin of safety from its critical value (M = 0) in units of standard deviation.



probability that safety margin is lower than zero → it represents the PROBABILITY OF FAILURE ↓ M is ≠ $\bar{M}$ (the expected one and it's <0)

A numerical approximation of first order derivatives is given by:

$$\frac{\partial y}{\partial x_i} = \frac{(y+\sigma_{x_i})-(y-\sigma_{x_i})}{2\cdot\sigma_{x_i}} = \frac{\Delta y_i}{2\cdot\sigma_{x_i}}$$

Using this approximation, the standard deviation can be calculated as follows:

$$\sigma_y^2 = \sum_{i=1}^{n}\left(\frac{\Delta y_i}{2}\right)^2.$$

If the FOSM method is applied to the margin of safety of the considered example, the obtained result is coincident with the exact solution, as the margin of safety is a *linear* function.
The method can be also applied to the factor of safety, which is not a linear function.

$$F = \frac{c_u}{\gamma\cdot H}$$

$$\bar{F} = \frac{\bar{c}_u}{\bar{\gamma}\cdot\bar{H}} = \frac{32}{20\cdot 4} = 1.6$$

it's also equal to the mean value of F

$$\sigma_F^2 = \sigma_{c_u}^2\cdot\left(\frac{4}{\bar{\gamma}\cdot\bar{H}}\right)^2 + \sigma_\gamma^2\left(-\frac{4\cdot\bar{c}_u}{\bar{\gamma}^2\cdot\bar{H}}\right)^2 =$$

$$= 4.8^2\cdot(0.05)^2 + 2^2\cdot(-0.08)^2 = 0.0832$$

$$\sigma_F = \sqrt{0.0832} = 0.29$$

In this case, the reliability index β is defined as follows

$$\beta = \frac{\bar{F}-1}{\sigma_F}$$

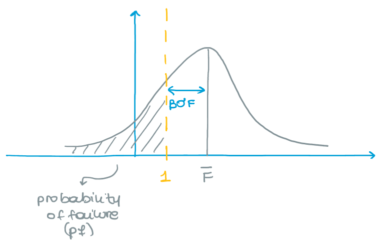
it's a distance between F̄ and 1

which expresses the distance of the mean factor of safety from its critical value (F = 1) in units of standard deviation.
If the factor of safety is assumed to be normally distributed, the probability of failure results equal to

$$p_f = \int_{-\infty}^{0} f(F)\cdot dF = 1.88\%$$

with an approximation error with respect to the exact value (1.05%).

because we truncate the Taylor serie at the first term



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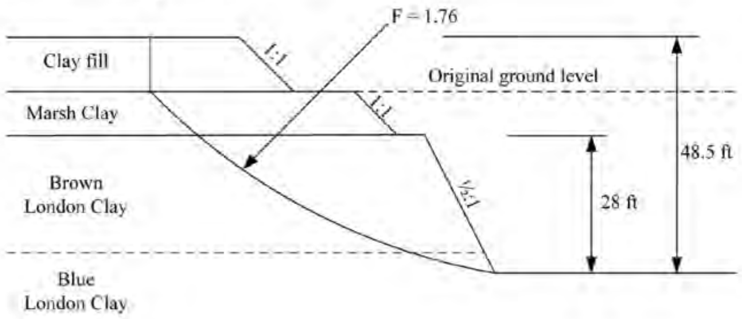
6. Judgement

Skempton and LaRochelle (1965) described failures in an excavated slope at the site of a nuclear power plant at Bradwell, England. The failures occurred within three weeks after excavation of the slopes for a nuclear reactor.
The failures were surprising because the calculated factor of safety of the slopes was 1.76.



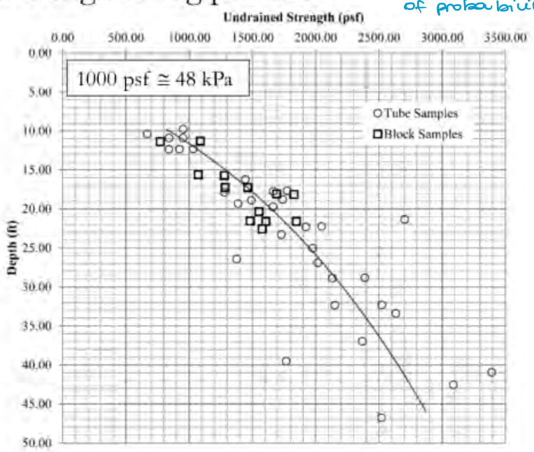
→ original ground soil

undrained and unconsolidated

Strengths measured in UU triaxial compression tests performed on tube and block samples of the London Clay from Bradwell had a COV of 16%. Using the same data, it is possible to calculate a probability of failure of 0.03–0.3%, values indicative of conservative engineering practice.

very low values of probability failure

it's difficult to explain the reason why



It is well recognized that *judgement* is essential in deterministic geotechnical engineering analyses, where sample disturbance can drastically alter laboratory test results, where soil properties may change with time, where the behaviour of soil deposits may differ from the behaviour of even high quality laboratory test specimens, and where site geology, which cannot be easily quantified, is of fundamental importance.

Computing probability of failure adds value to reliability studies because it provides an alternative to factor of safety.

before any mathematical computations

However, judgement is needed in probabilistic analyses for the same reasons judgement is needed in deterministic studies.

6. Limit State Design or Load and Resistance Factor Design

These methodologies represent attempts to apply probabilistically based methods to routine design procedures. They have been used successfully in structural engineering, but their application in geotechnical engineering, especially foundation engineering, has been controversial.
However, in Europe, after the introduction of Eurocode 7 (EN 1997), these methodologies are become the new standard for the design of foundations and geotechnical structures.

→ in Europe we use the semi probabilistic methods

1. From laboratory to field applications

Geotechnical Engineering

Simplified theoretical models
- Linear elasticity (serviceability limit states)
- Perfect plasticity (ultimate limit states)

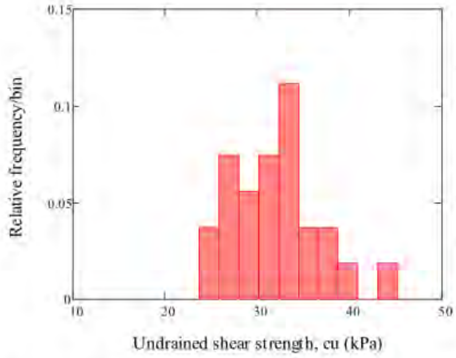
In situ tests
- Uncontrolled boundary conditions
- Undirect measurements
- Empirical relationships

2. How to deal with uncertainty?

- Ignore it
- Being conservative
- Using the observational method

  It involves *(1)* considering possible modes of unsatisfactory performance or other undesirable developments; *(2)* developing plans for dealing with each such development; *(3)* making field measurements during construction and operation to establish whether the developments are occurring; and *(4)* reacting to the observed behavior by changing the design or construction process.
- Quantifying uncertainty

  Reliability approach



Bin: 2.15 kPa

No. of bins: 10

Mean:

$$\bar{c}_u = \frac{1}{N}\sum_{i=1}^{N} c_{u_i} = 31.75 \text{ kPa}$$

Standard deviation:

$$\sigma_{c_u} = \sqrt{\frac{\sum_{i=1}^{N}(c_{u_i} - \bar{c}_u)^2}{N-1}} = 4.9 \text{ kPa}$$

Coefficient of variation:

$$COV = \frac{\sigma_{c_u}}{\bar{c}_u} = \frac{4.9}{31.75} = 15\%$$

Probability density function: *normal distribution*

$$f(c_u) = \frac{1}{\sigma_{c_u} \cdot \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{c_u - \bar{c}_u}{\sigma_{c_u}}\right)^2\right]$$

Properties of the probability density function:

$$\int_{-\infty}^{\infty} f(c_u) \cdot dc_u = 1$$

$$P[a < c_u < b] = \int_a^b f(c_u) \cdot dc_u$$

Cumulative distribution function

$$P(c_u) = \int_{-\infty}^{c_u} f(x) \cdot dx = P[x \le c_u]$$

For a random variable x:

$$\bar{x} = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$$

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \bar{x})^2 \cdot f(x) \cdot dx$$

If $y = a \cdot x + b$ then

$$\bar{y} = a \cdot \bar{x} + b$$

$$\sigma_y^2 = a^2 \cdot \sigma_x^2$$

If $y = \sum_{i=1}^{n} a_i \cdot x_i$ then

$$\bar{y} = \sum_{i=1}^{n} a_i \cdot \bar{x}_i$$

$$\sigma_y^2 = \sum_{i=1}^{n} a_i^2 \cdot \sigma_{x_i}^2$$

For the margin of safety is possible to find an exact solution:

$$M = c_u - \frac{\gamma \cdot H}{4}$$

$$\bar{M} = \bar{c}_u - \frac{\bar{\gamma} \cdot H}{4} = 12 \text{ kPa}$$

$$\sigma_M^2 = \sigma_{c_u}^2 + \left(\frac{H}{4}\right)^2 \cdot \sigma_\gamma^2 = 4.8^2 + 2^2 = 27.04 \text{ kPa}^2$$

$$\sigma_M = \sqrt{27.04} = 5.2 \text{ kPa}$$

$$p_f = \int_{-\infty}^{0} f(M) \cdot dM = 1.05\%$$

If the COV of c_u is assumed equal to 30%, the standard deviation of the margin of safety becomes:

$$\sigma_M^2 = \sigma_{c_u}^2 + \left(\frac{H}{4}\right)^2 \cdot \sigma_\gamma^2 = 9.6^2 + 2^2 = 96.16 \text{ kPa}^2$$

$$\sigma_M = \sqrt{96.16} = 9.8 \text{ kPa}$$

and the failure probability becomes:

$$p_f = \int_{-\infty}^{0} f(M) \cdot dM = 11.05\%$$

for the same expected value of the margin of safety.

4. First Order Second Moment (FOSM) Method

↳ Professor Duncan applied this method

The function that expresses the failure condition,

$$y = y(x_1, x_2, \ldots, x_n),$$

is expanded using the Taylor series about the mean values of the variables $x_1, x_2, \ldots, x_n$:

$$y = y(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) + \sum_{i=1}^{n} (x_i - \bar{x}_i) \frac{\partial y}{\partial x_i} +$$

$$+ \frac{1}{2!} \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - \bar{x}_i)(x_j - \bar{x}_j) \frac{\partial^2 y}{\partial x_i \partial x_j} +$$

$$+ \frac{1}{3!} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} (x_i - \bar{x}_i)(x_j - \bar{x}_j)(x_k - \bar{x}_k) \frac{\partial^3 y}{\partial x_i \partial x_j \partial x_k} + \ldots$$

$$\sigma_F^2 = \sigma_{c_u}^2 \cdot \left(\frac{4}{\bar{\gamma} \cdot H}\right)^2 + \sigma_\gamma^2 \left(-\frac{4 \cdot \bar{c}_u}{\bar{\gamma}^2 \cdot H}\right)^2 =$$

$$= 4.8^2 \cdot (0.05)^2 + 2^2 \cdot (-0.08)^2 = 0.0832$$

$$\sigma_F = \sqrt{0.0832} = 0.29$$

In this case, the reliability index β is defined as follows

$$\beta = \frac{\bar{F} - 1}{\sigma_F}$$

which expresses the distance of the mean factor of safety from its critical value (F = 1) in units of standard deviation.
If the factor of safety is assumed to be normally distributed, the probability of failure results equal to

$$p_f = \int_{-\infty}^{1} f(F) \cdot dF = 1.88\%$$

with an approximation error with respect to the exact value (1.05%).

5. Monte Carlo Method

Monte Carlo simulation enjoys a long history and a rich literature. Each continuous variable is replaced by a large number of discrete values generated from the underlying distribution; these values are used to compute a large number of values of function M or F and its distribution. The large numbers of computations once presented a constraint on the use of this method, but cheap modern computers have largely removed this obstacle.

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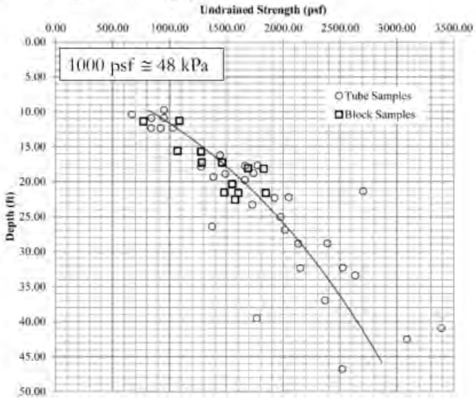
Strengths measured in UU triaxial compression tests performed on tube and block samples of the London Clay from Bradwell had a COV of 16%. Using the same data, it is possible to calculate a probability of failure of 0.03–0.3%, values indicative of conservative engineering practice.



From a deterministic point of view, the Bradwell failure can be understood by virtue of the fact that the London Clay is heavily overconsolidated "*stiff-fissured*" clay.

The fissures are oriented randomly, and developed when the deposit was unloaded due to erosion of a thick column of overlying sediments. As the erosion took place the vertical stress decreased, becoming smaller than the horizontal stress. Eventually, when the ratio of the vertical stress to the horizontal stress reached the passive stress extreme, the clay failed, resulting in development of ubiquitous failure planes (fissures) within the clay.

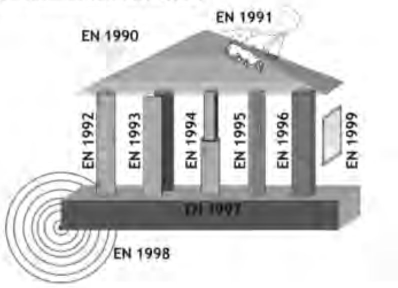
When a slope is excavated in this fissured clay, the horizontal stress in the ground decreases, fissures open, and the strength of the clay mass is reduced.

6. Limit State Design or Load and Resistance Factor Design

These methodologies represent attempts to apply probabilistically based methods to routine design procedures. They have been used successfully in structural engineering, but their application in geotechnical engineering, especially foundation engineering, has been controversial.
However, in Europe, after the introduction of Eurocode 7 (EN 1997), these methodologies are become the new standard for the design of foundations and geotechnical structures.

The Eurocode family of design standards comprises:

- o EN 1990, Basis of structural design
- o EN 1991, Actions on structures
- o EN 1992, Design of concrete structures
- o EN 1993, Design of steel structures
- o EN 1994, Design of composite concrete and steel structures
- o EN 1995, Design of timber structures
- o EN 1996, Design of masonry structures
- o EN 1997, Geotechnical design
- o EN 1998, Design of structures for earthquake resistance
- o EN 1999, Design of aluminium structures



In the case of ultimate limit states GEO and STR, where the strength of the structural material or the ground is significant, it will have to be verified that the design value of the effect of actions, E_d, never exceeds the design resistance, R_d:

$$E_d \le R_d$$

(→ available resistance; ↳ effect)

where:

(>1 we increase characteristics of loads)

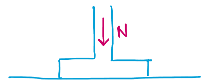
$$E_d = E\left[\gamma_F F_k ; \frac{X_k}{\gamma_M} ; a_d\right] \quad \text{or} \quad E_d = \gamma_E \cdot E\left[F_k ; \frac{X_k}{\gamma_M} ; a_d\right]$$

(we haven't partial factor)

$$R_d = \frac{1}{\gamma_R} R\left[\gamma_F F_k ; \frac{X_k}{\gamma_M} ; a_d\right]$$

where F_k = representative value of actions
X_k = characteristic value of geotechnical parameters
a_d = design value of geometrical data
γ_M = partial factors for geotechnical parameters (cu for example)
γ_R = partial factors for resistances
γ_E = partial factor for the effects of actions

If we apply a vertical load



- we apply γ_F to N
- compute bending moment

OR

- we use N (= characteristics value)
- compute bending moment
- apply partial factor directly to the moment

F_k = N
E = M

→ $\gamma_F \cdot N$
→ $\gamma_E \cdot M$

Eurocodes 7 offers three Design Approaches for verifying the GEO and STR ultimate limit states for persistent and transient situations.

In Design Approach 1 (DA1), factors are applied to actions alone (in Combination 1) and mainly to material factors (in Combination 2). → separate uncertanties of actions and material factors

In Design Approach 2 (DA2), factors are applied to actions (or effects of actions) and to resistances simultaneously. (NOT γ_M)
↳ load and resistance factors of USA

In Design Approach 3 (DA3), factors are applied to structural actions (but not to geotechnical actions) and to material properties simultaneously. → mixture between design approach 1 and 2

⇒ so four possibilities because the first approach has 2 combinations.

Tab. 6.2.I – *Coefficienti parziali per le azioni o per l'effetto delle azioni*

	Effetto	Coefficiente Parziale γ_F (o γ_E)	EQU	(A1)	(A2)
Carichi permanenti G_1	Favorevole	γ_{G1}	0,9	1,0	1,0
	Sfavorevole		1,1	1,3	1,0
Carichi permanenti G_2[(1)]	Favorevole	γ_{G2}	0,8	0,8	0,8
	Sfavorevole		1,5	1,5	1,3
Azioni variabili Q	Favorevole	γ_{Qi}	0,0	0,0	0,0
	Sfavorevole		1,5	1,5	1,3

[(1)] Per i carichi permanenti G_2 si applica quanto indicato alla Tabella 2.6.I. Per la spinta delle terre si fa riferimento ai coefficienti γ_{G1}

Tab. 6.2.II – *Coefficienti parziali per i parametri geotecnici del terreno*

Parametro	Grandezza alla quale applicare il coefficiente parziale	Coefficiente parziale γ_M	(M1)	(M2)
Tangente dell'angolo di resistenza al taglio	$\tan \varphi'_k$	$\gamma_{\varphi'}$	1,0	1,25
Coesione efficace	c'_k	$\gamma_{c'}$	1,0	1,25
Resistenza non drenata	c_{uk}	γ_{cu}	1,0	1,4
Peso dell'unità di volume	γ_γ	γ_γ	1,0	1,0

Tab. 6.4.I – *Coefficienti parziali γ_R per le verifiche agli stati limite ultimi di fondazioni superficiali*

Verifica	Coefficiente parziale (R3)
Carico limite	$\gamma_R = 2,3$
Scorrimento	$\gamma_R = 1,1$

Tab. 6.8.I - *Coefficienti parziali per le verifiche di sicurezza di opere di materiali sciolti e di fronti di scavo*

COEFFICIENTE	R2
γ_R	1,1

- bearing resistance failure] DA2
- sliding]
- loss of overall stability – DA1, combination 2
- structural failure – DA2

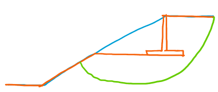
DA2

γ_F or γ_E — permanent loads (1,3); variable loads (1,5)

$\gamma_M = 1$

γ_R — bearing resistance (2,3); sliding (1,1)

DA1, combination 2 ~ it's used for overall stability (cantilever wall on a slope)



modify the slope (a) to build a road so introduce the wall
↳ check the stability of the new arrangement

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FOUNDATION ENGINEERING

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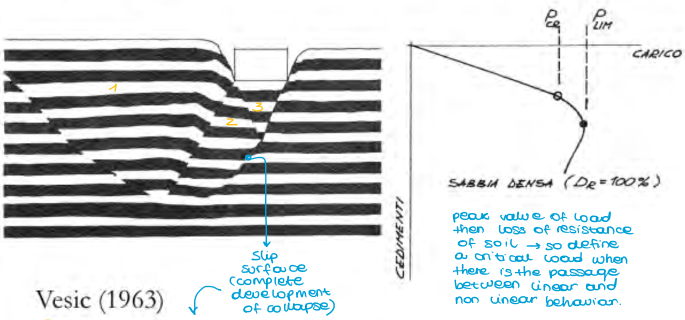
Bearing capacity of shallow footings

1. Modes of failure

General shear



Vesic (1963)

③ blocks with relative movement

⇒ peak value of resistance is linked to a SANDY SOIL

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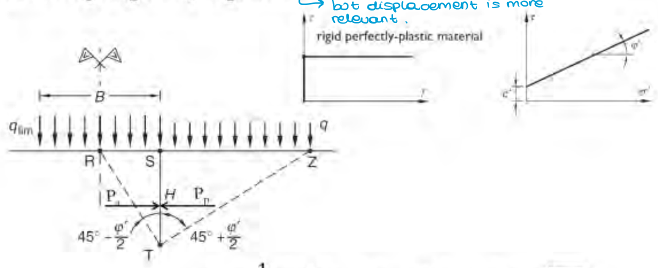
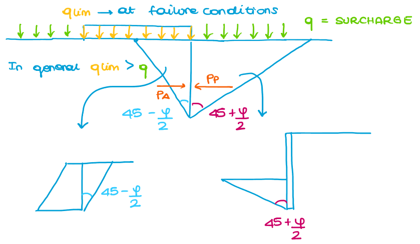
2. Bearing capacity equation

it isn't the limit state that governs the failure state → but displacement is more relevant.

→ we have a strip foundation (∞ in the orthogonal direction)





$$P_a = \frac{1}{2}\gamma H^2 K_a + q_{lim} H K_a - 2cH\sqrt{K_a}$$

$$P_p = \frac{1}{2}\gamma H^2 K_p + q H K_p + 2cH\sqrt{K_p}$$

kp instead of ka

$$P_a = P_p$$

$$H = \frac{B}{2}\cdot\frac{1}{tg\left(45-\frac{\varphi}{2}\right)}$$

$$K_a = \frac{1-\text{sen}\varphi}{1+\text{sen}\varphi} = tg^2\left(45-\frac{\varphi}{2}\right)$$

$$H = \frac{B}{2}\cdot\frac{1}{\sqrt{K_a}}$$

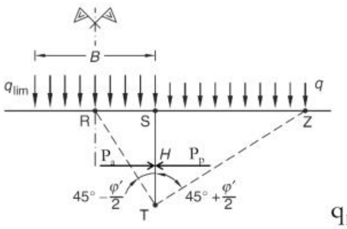
$$P_a = \frac{1}{2}\gamma\frac{B^2}{4}\frac{1}{K_a}\cdot K_a + q_{lim}\frac{B}{2}\frac{1}{\sqrt{K_a}}K_a - 2c\frac{B}{2}\frac{1}{\sqrt{K_a}}\sqrt{K_a}$$

$$P_p = \frac{1}{2}\gamma\frac{B^2}{4}\frac{1}{K_a}\cdot K_p + q\frac{B}{2}\frac{1}{\sqrt{K_a}}K_p + 2c\frac{B}{2}\frac{1}{\sqrt{K_a}}\sqrt{K_p}$$

$$q_{lim}\left(\frac{B}{2}\sqrt{K_a}\right) = \frac{1}{8}\gamma B^2\left(\frac{K_p}{K_a}-1\right) + q\frac{B}{2}\frac{1}{\sqrt{K_a}}K_p + cB\left(\frac{\sqrt{K_p}}{K_a}+1\right)$$

↳ $q_{lim} = \frac{1}{2}\gamma B\cdot N_\gamma + qN_q + cN_c$

27/03/18



$$q_{lim} = \frac{1}{2}\gamma B\cdot N_\gamma + q\cdot N_q + c\cdot N_c$$

- $\frac{1}{2}\gamma B \cdot N_\gamma$: strength due to the own weight of the soil
- $q \cdot N_q$: due to lateral load/stress acting on the sides of the foundations
- $c \cdot N_c$: cohesion

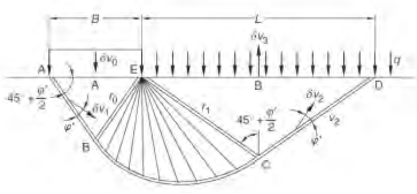
$$N_\gamma = \frac{1}{2\sqrt{K_a}}\left(\frac{K_p}{K_a}-1\right)$$

$$N_q = \frac{K_p}{K_a}$$

$$N_c = \frac{2(\sqrt{K_p}+\sqrt{K_a})}{K_a}$$

=> very simplified case

=> more realistic mechanism of failure: we obtain the same equation, but different values for the coefficients N_q, N_γ and N_c (PRATICAL APPLICATION)



$$N_q = tg^2\left(45° + \frac{\varphi}{2}\right)\cdot e^{\pi \cdot tg\varphi}$$

$$N_c = (N_q - 1)\,\text{cotg}\,\varphi$$

$$N_\gamma = 2(N_q + 1)\,tg\,\varphi$$

difficult case to achieve the result with easy computations as before (but concepts are always the same)

SHALLOW FOUNDATIONS

$q_{adm} \le q_{lim}$ → admissible vertical stress → it must be lower than bearing capacity

→ when we build a shallow foundation
- excavation

$$q_{\lim} = \frac{1}{2}\gamma B N_\gamma s_\gamma i_\gamma b_\gamma g_\gamma + c' N_c s_c d_c i_c b_c g_c + q' N_q s_q d_q i_q b_q g_q$$

we already have the contribution of the lateral pressure but we need another contribution

Depth factors:

↳ it increases the bearing capacity

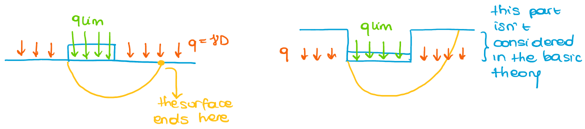
$$d_q = 1 + 2\tan\varphi' (1 - \sin\varphi')^2 \frac{D}{B} \qquad (D \le B)$$

$$d_q = 1 + 2\tan\varphi' (1 - \sin\varphi')^2 \tan^{-1}\frac{D}{B} \qquad (D > B).$$

$$d_c = d_q - \frac{1 - d_q}{N_c \tan\varphi'}$$

to be used for depths D ≥ 2 m

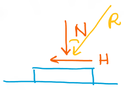
=> so both the equations can be used (are correct!!)



That's why we have the depth correction factor. Moreover, the top soil is not good as the foundation soil so in general it's better considering it only if depth is significant (> 2m)

$$q_{\lim} = \frac{1}{2}\gamma B N_\gamma s_\gamma i_\gamma b_\gamma g_\gamma + c' N_c s_c d_c i_c b_c g_c + q' N_q s_q d_q i_q b_q g_q$$

Inclined force factors:



$$i_\gamma = \left(1 - \frac{H}{N + BLc'\cot\varphi'}\right)^{m+1}; \qquad i_q = \left(1 - \frac{H}{N + BLc'\cot\varphi'}\right)^{m}$$

$$i_c = i_q - \frac{1 - i_q}{N_c \tan\varphi'}; \qquad m = \frac{2 + \frac{B}{L}}{1 + \frac{B}{L}}$$

for strip foundation m = 2 because L → ∞

$$q_{\lim} = \frac{1}{2}\gamma B N_\gamma s_\gamma i_\gamma b_\gamma g_\gamma + c' N_c s_c d_c i_c b_c g_c + q' N_q s_q d_q i_q b_q g_q$$

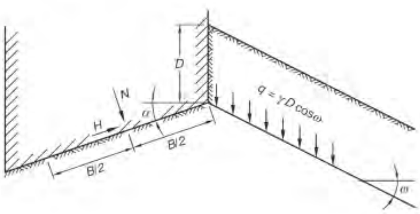
Inclined founding base factors:

↳ base of foundation

$$b_q = (1 - \alpha \tan\varphi')^2$$

$$b_\gamma = b_q$$

$$b_c = b_q - \frac{1 - b_q}{N_c \tan\varphi'}$$



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4. Shear resistance angle for sands

It is possible to make reference to the peak strength, provided that the stress level is properly taken into account. In dense sands, this can be done by referring to the relation introduced by Bolton (1986):

$$\varphi - \varphi_{cv} = m \cdot DI \quad < 12°$$ peak friction angle

$$DI = D_R \cdot (Q - \ln p'_f) - 1$$

where:
- p'_f is mean effective stress at failure (expressed in kPa);
- m depends on deformation constraints (3 for triaxial strain conditions and 5 for plane strain conditions);
- Q depends on grain crushing strength and ranges from 10, for quartz and feldspar sands, to 8 for limestone and 5.5 for chalk.

The mean effective stress at failure can be linked to the applied bearing stress, as suggested by De Beer (1970):

$$p'_f = \frac{1}{4}(q_{lim} + 3 \cdot q)(1 + \sin(\varphi))$$

↳ function of bearing capacity

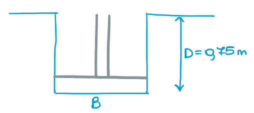
through an iteration procedure, since the operational strength depends on the computed bearing stress.

⇒ ITERATIVE APPROACH

$\varphi^{(1)} \to q_{lim} \to p'_f \to DI \to \varphi^{(2)}$

it stops when variation of φ is very low

EX



B = 2 m
L = 3 m
Sand
D_R = 70%
γ = 20 kN/m³
φ_{cv} = 32°

q_{lim} = ?

$q_{lim} = \frac{1}{2}\gamma B N_\gamma s_\gamma + q N_q s_q d_q$

↳ All the other corrective factors are = 1
d_q = 1 because D < 2 m (only 0,75 m)
$q = \gamma D = 20 \cdot 0,75 = 15$ kPa
$\varphi_p^{(1)} = 35°$ → TRIAL VALUE (1)

N_q = 33,3
N_γ = 48,03
m = 3
↳ we can use Mayeroff formula

$s_\gamma = s_q = 1 + 0,1 \cdot \frac{1 + \text{sen } 35°}{1 - \text{sen } 35°} \cdot \frac{2}{3} = 1,25$

$q_{lim} = \frac{1}{2} \cdot 20 \cdot 2 \cdot 48,03 \cdot 1,25 + 15 \cdot 33,3 \cdot 1,25 = 1819$ kPa

$p_f = \frac{1}{4}(1819 + 3 \cdot 15)(1 + \text{sen } 35) = 199$ kPa

$DI = [0,7 (10 - \ln 199)] - 1 = 2,296$

$\varphi_p^{(2)} = 32° + 3 \cdot 2,296 = 38,9°$

→ then N_γ, N_q, s_γ, s_q, q_{lim}, p_f, DI

→ $\varphi_p^{(3)} = 37,9$
$\varphi_p^{(4)} = 38,1$ } they are equal
$\varphi_p^{(5)} = 38,1$ } so the exact value is this one

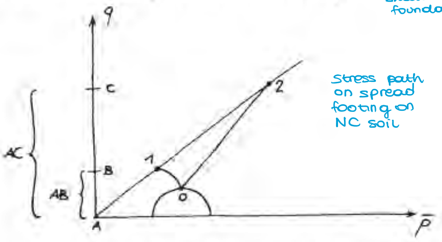
$q_{lim}(\varphi_p = 38,1) = 2976$ kPa

it's a very high number (so it's difficult to have a problem in bearing capacity the main problems are linked to settlements)

5/04/18

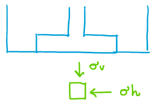
5. Bearing capacity in clays

Stress path for a normally consolidated clay → a point under shallow foundation



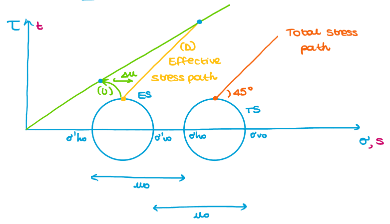
0-1: effective stress path under undrained conditions
0-2: effective stress path under drained conditions

AB: short-term undrained shear strength (c_u)
AC: long term drained shear strength



$s = \frac{\sigma'_v + \sigma'_h}{2}$ mean stress

$t = \frac{\sigma'_v - \sigma'_h}{2}$ shear stress



⇒ both for drained conditions

Δu > 0 for NC clays

- undrained conditions most critical
- drained conditions better because shear strenght is higher

EX
circular foundation

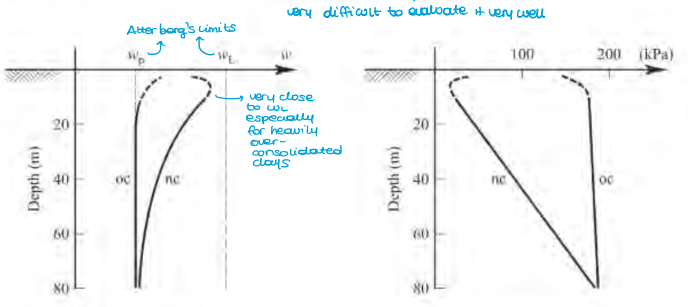
Stress path for a overconsolidated clay

⇒ These are the effective stress paths, we can add the total ones (u)

$\frac{1}{2}(\sigma_v - \sigma_h)$ $\frac{1}{2}(\sigma_v - \sigma_h)$

5. Assessment of the undrained shear strength of clays

↳ it's not a soil property (= result of a calculation)
very difficult to evaluate it very well



(a) Water content

physical parameter which controls shear strength

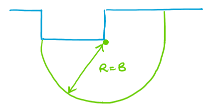
(b) Undrained strength

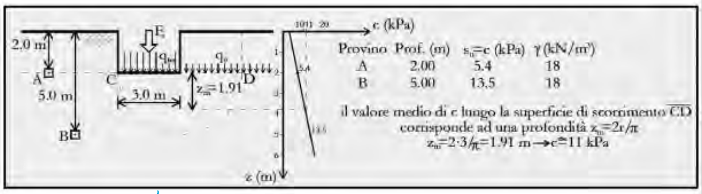
relationships between shear strength and depth

Effect of non-homogeneity

The undrained shear strength of soft clays increases with depth, the ratio c_u/σ'_{v0} being constant for a given deposit. This gives the soil a character of inhomogeneity in the vertical direction and, in such a situation, a common practice is to rely on the use of a slip circle analysis, with the introduction of an averaged value of the undrained strength.

↳ its radius is equal to B

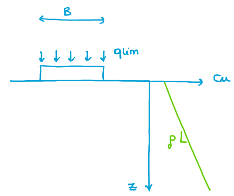




This approach isn't correct

Davis and Booker (1973) provided a rigorous treatment of the problem, where the undrained strength increases linearly with depth:

the theory says →



$$c_u = c_0 + \rho \cdot z$$

and by using the method of stress characteristics have shown that the limit load Q of a rigid strip footing is expressed in the form:

$c_u = c_0 + \rho \cdot z$
If c_u = cost
$q_{lim} = N_c \cdot c_u$
$N_c = 2+\pi$

$$Q/b = F \cdot [(2+\pi) \cdot c_0 + \rho b / 4]$$

(Q/b = q_{lim})

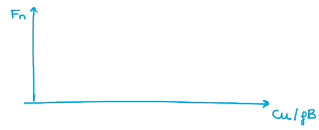
where b is the breadth of the footing, c_0 is the strength at the founding level and F is a dimensionless factor depending on the ratio $\rho b/c_0$ and on the roughness of the footing:

Table 8.1 Dimensionless factor F for rough footings

$\rho b/s_o$	1	2	3	4	6	8
F	1.20	1.36	1.45	1.50	1.57	1.62

Source: Deduced from Davis and Booker (1973)

Graphical representation →

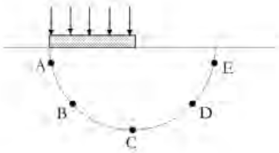


This solution proves that the usual practice of introducing an averaged value of the shear strength provides reliable results if

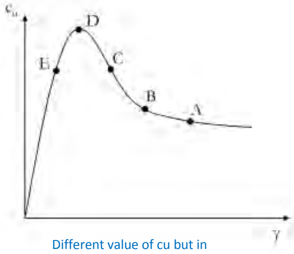
$F\,[(2+\pi)c_0 + \frac{\rho B}{4}] = (2+\pi)c_u z_{eq} = (2+\pi)(c_0 + \rho z_{eq})$

Furthermore, the development of strains along a failure surface forces a large number of soil elements well past the peak strength, leading to a *progressive failure* characterized by an operational strength lower than the peak.

Circular slip surface



We have a peak and then a softening decrease



Different value of cu but in A,B and C it isn't possible to achieve the peak value anymore. In E and D is still possible.

In order to account for both these aspects, Koutsoftas and Ladd (1985) have suggested to evaluate the operational strength by means of the expression:

We don't need to consider 0,03

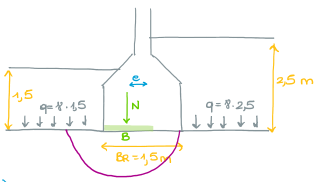
$$\frac{c_u}{\sigma'_{v0}} = (0.22 \pm 0.03) \cdot OCR^{0.8} \qquad OCR < 10$$

If soil is NC we have the ratio just equal to 0,22

Note that the above relation is supported by experimental evidences related to values of OCR up to 10, and a lot of care must be taken when analysing the behaviour of *stiff fissured clays*. Moreover, it must be observed that it is at least doubtful if the behaviour of fissured clays can be defined fully undrained.

EX 1

Strip footing in sand



1) e=0
2) e = 0,25 m

γ = 18 kN/m³
G_k = 440 kN/m
Q_k = 205 kN/m
ϕ = 36°

- check the foundation
- evaluate effect of eccentricity of vertical load

N_d = 1,3 · 440 + 1,5 · 205 = 880 kN/m
↳ design value of vertical load (with partial safety factors)

$$q_{lim} = \frac{1}{2}\gamma B N_\gamma s_\gamma i_\gamma b_\gamma g_\gamma + c' N_c s_c d_c i_c b_c g_c + q N_q s_q d_q i_q b_q g_q$$

(s_γ → 1 strip foundation; $i_\gamma, b_\gamma, g_\gamma$ → 1; $c' N_c s_c d_c i_c b_c g_c$ crossed out because c=0; s_q, i_q, b_q, g_q → 1: soil, ground and base are horizontal)

So bearing capacity is given by a reduced equation:

$$q_{lim} = \frac{1}{2}\gamma B N_\gamma + q N_q d_q$$

1) e = 0
so B = BR
↳ the load is in the middle of the base
we haven't a symmetrical problem so the most probable slip surface goes from the right to the left (ш)

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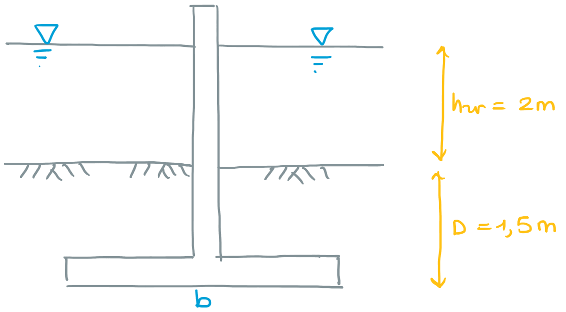
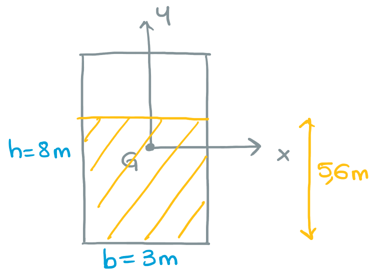
EX 2



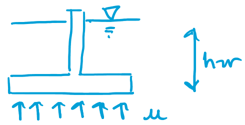


SILTY SAND

$\gamma = 18$ kN/m^2
$\varphi = 32°$
$N_d = 2870$ kN
$M_{xd} = 0$
$H_{xd} = 0$
$H_{yd} = 487$ kN
$M_{yd} = 2436$
$\gamma_w = 10$ kN/m3

- point out the horizontal stress
- verify the foundation

Uplift pressure force



$U = \gamma_w \cdot h_w \cdot A = 10 \cdot 3{,}5 \cdot 8 \cdot 3 = 840$ kN
$U = u \cdot$ Area
$N_d' = N_d - U = 2870 - 840 = 2030$ kN
↳ effective load acting on the foundation

$e_x = 0$

$$e_y = \frac{M_y}{N'} = \frac{2436}{2030} = 1{,}2 \text{ m}$$

$B = b = 3m$
$L = a - 2e_y = 8 - 2 \cdot 1{,}2 = 5{,}6$ m
↳ we have a reduced side of the foundation (u)

CORRECTION FACTORS

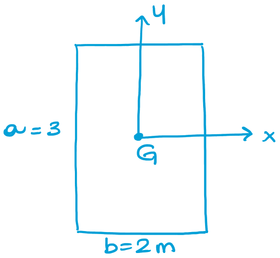
$$s_\gamma = s_q = 1 + 0{,}1 \cdot \frac{1+\sin\varphi}{1-\sin\varphi} \cdot \frac{B}{L} = 1 + 0{,}1 \cdot \frac{1+\sin 32}{1-\sin 32} \cdot \frac{3}{5{,}6} = 1{,}17$$

$$m = \frac{2 + B/L}{1 + B/L} = \frac{2 + 3/5{,}6}{1 + 3/5{,}6} = 1{,}65$$

$$i_\gamma = \left(1 - \frac{H_d}{N_d'}\right)^{m+1} = \left(1 - \frac{487}{2030}\right)^{2{,}65} = 0{,}48$$

EX 4

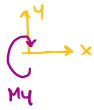
SAND



γ = 18 kN/m^3
φ = 34° (TX)
q = 10 kPa

- check the foundation

My is aroud x



1) Nd = 1100 kN, Hx = Hy = 0 kN, Mx = My = 0 kN m
2) Nd = 1100 kN, Hx = Hy = 0 kN, Mx = 0 kNm, My = 660 kNm
3) Nd = 1200 kN, Hx = Hy = 0 kN, Mx = - 240 kNm, My = 420 kNm
4) Nd = 1100 kN Hx = 60 kN Hy = - 80 kN, Mx = 0 kNm, My = 310 kNm

N_γ = 41,06
N_q = 29,44

1) B = b = 2m
L = a = 3m

$s_\gamma = s_q = 1 + 0,1 \cdot \frac{1 + \text{sen } 34}{1 - \text{sen } 34} \cdot \frac{2}{3} = 1,24$

d_q = 1 because lateral load is very small (small lateral embankement)

$q_{lim} = \frac{1}{2} \cdot 18 \cdot 2 \cdot 41,06 \cdot 1,24 + 10 \cdot 29,44 \cdot 1,24 = 1281$ kPa

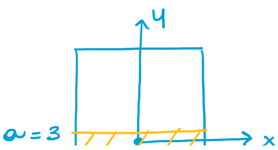
$R_d = \frac{q_{lim} \cdot A}{\gamma_R} = \frac{7686}{2,3} = 3341$ kN

$R_d > N_d$ ✓

2) $e_y = \frac{M_y}{N} = \frac{660}{1100} = 0,6$ m

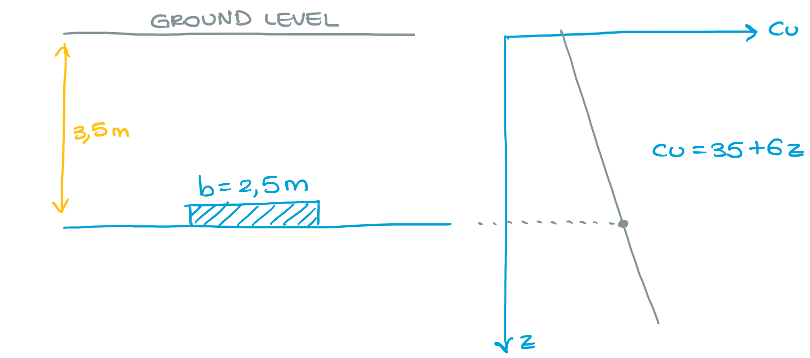
$B = a - 2e_y = 3 - 2 \cdot 0,6 = 1,8$ m
$L = b = 2$m

because it becomes the lowest side



$R_d > N_d$ ✓

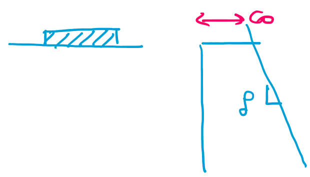
EX 5



$\gamma = 18$ kN/m³

1) $N_d = 770$ kN $e_x = e_y = 0$

2) $N_d = 770$ kN $e_x = 0,32$ m $e_y = 0$



$c_u = 35 + 6 \cdot 3,5 = 63$ kPa

$c_o = 56$ kPa

$\rho = 6$ kPa/m

$F = \frac{\rho b}{c_u} = \frac{6 \cdot 2,5}{56} = 0,27$ => we assume 1

↳ it is valid for strip foundation, but also useful to evaluate Cu

$q_{lim} = \underbrace{(2+\pi) c_o}_{288} + \underbrace{\frac{\rho \cdot b}{4}}_{4} =$

we can neglect increase of Cu with depth but relay only on the other factor which depend only on Co so we assume $c_u = C_o$

1) $B = L = 2,5$ m

$s_c^o = 1 + 0,1 \frac{B}{L} = 1,2$

$q_{lim} = (2+\pi) 56 \cdot 1,2 = 345$ kPa

$R_d = \frac{q_{lim} \cdot A}{\gamma_R} = 937$ kN

$R_d > N_d$ ✓

2) $B = b - 2e_y = 25 - 2 \cdot 0,32 = 1,86$ m

$L = 2,5$ m

$s_c^o = 1 + 0,2 \cdot \frac{1,86}{2,5} = 1,15$

$q_{lim} = 56 \cdot (2+\pi)\, 1,15 = 331$ kPa

$R_d = \frac{q_{lim} \cdot A}{\gamma_R} = \frac{331 \cdot 2,5 \cdot 1,86}{2,3} = 669$ kN

$R_d < N_d$ ✗

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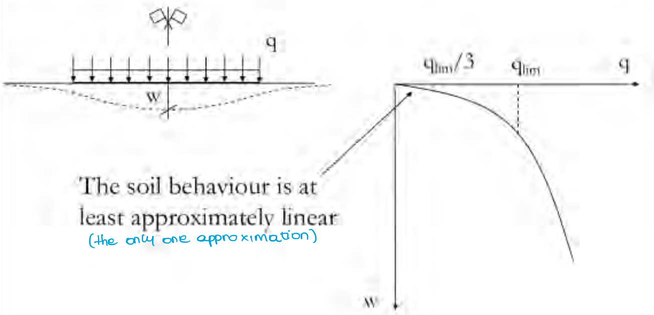
FOUNDATION ENGINEERING

01RVSMX
A.A. 2017-2018

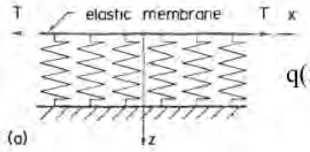
Settlements of shallow footings

1. Settlement prediction



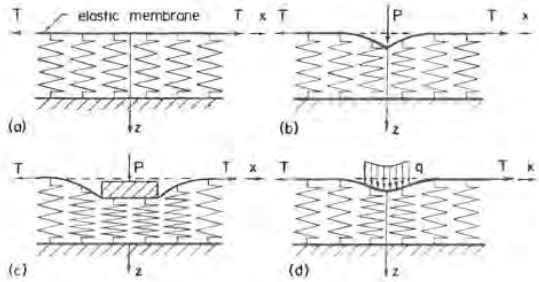
$w = \frac{q}{K_1}$

Two-parameters elastic model



$$q(x,y) = K_1 \cdot w(x,y) - T \cdot \nabla^2 w(x,y)$$

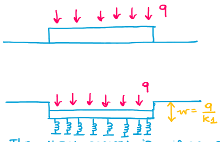
The model proposed by Filonenko-Borodich (1940, 1945) acquires continuity between the individual spring elements in the Winkler model by connecting them to a thin elastic membrane under a constant tension T. The model, with different interpretations of its parameters, has been derived also by Pasternak (1954), Reissner (1958) and Vlasov and Leontiev (1966).



Surface displacements: *(a)* basic model, *(b)* concentrated load, *(c)* rigid load, *(d)* uniform flexible load.

⇒ we eliminate the first limitation of the Winkler model and also the second one

If we have a beam subjected to a uniform load



we would use to evaluate M and S

The displacement is uniform so M=0 and T=0

2. Available solutions for a linear elastic isotropic continuum

NO NEED TO STUDY

we can use for exercise

The Boussinesq's problem

valid solution

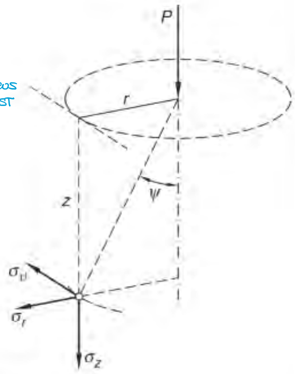
For isotropic and homogeneous elastic medium (so E = cost and ν = cost)

$$\sigma_z = \frac{3P}{2\pi}\frac{z^3}{R^5}$$

$$\tau_{rz} = \frac{3P}{2\pi}\frac{z^2 r}{R^5}$$

$$\sigma_r = \frac{P}{2\pi}\left[\frac{3zr^2}{R^5} - \frac{1-2\upsilon}{R(R+z)}\right]$$

$$\sigma_\vartheta = \frac{P}{2\pi}(1-2\upsilon)\left[\frac{1}{R(R+z)} - \frac{z}{R^3}\right]$$



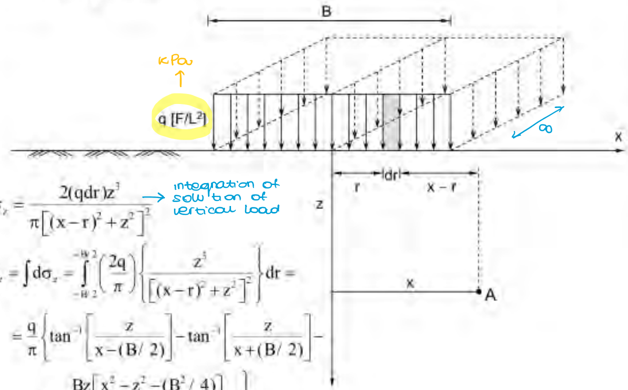
where $R^2 = z^2 + r^2$ and υ is the Poisson's ratio.

⇒ No elasticity parameters (no parameter of Elastic medium E, ν, G

tents to ∞ for z→0

2)

Vertical strip load



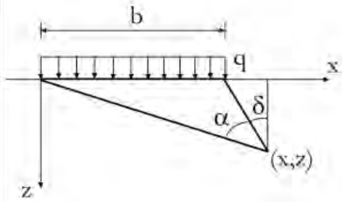
$$d\sigma_z = \frac{2(q\,dr)z^3}{\pi\left[(x-r)^2+z^2\right]^2}$$

→ integration of solution of vertical load

$$\Delta\sigma_z = \int d\sigma_z = \int_{-B/2}^{+B/2}\left(\frac{2q}{\pi}\right)\left\{\frac{z^3}{\left[(x-r)^2+z^2\right]^2}\right\}dr =$$

$$= \frac{q}{\pi}\left\{\tan^{-1}\left[\frac{z}{x-(B/2)}\right]-\tan^{-1}\left[\frac{z}{x+(B/2)}\right]-\frac{Bz\left[x^2-z^2-(B^2/4)\right]}{\left[x^2+z^2-(B^2/4)\right]^2+B^2z^2}\right\}$$

Vertical strip load



$$\sigma_z = \frac{q}{\pi}[\alpha + \sin\alpha\cdot\cos(\alpha+2\delta)]$$

$$\sigma_x = \frac{q}{\pi}[\alpha - \sin\alpha\cdot\cos(\alpha+2\delta)]$$

$$\sigma_y = \nu\cdot(\sigma_x+\sigma_z)$$

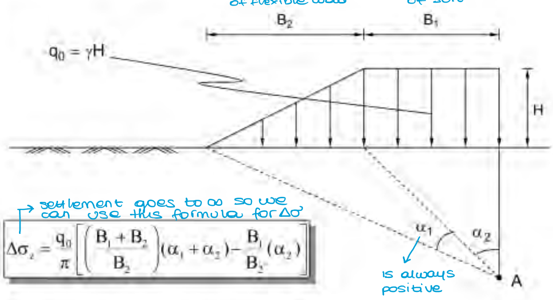
$$\tau_{xz} = \frac{q}{\pi}\sin\alpha\cdot\cos(\alpha+2\delta)$$

$$p = \frac{\sigma_x+\sigma_y+\sigma_z}{3} = \frac{2(1+\nu)q}{3\pi}\alpha$$

$$\alpha = \arctan\left(\frac{x}{z}\right)+\arctan\left(\frac{b-x}{z}\right)$$

Vertical Stress Due to Embankment Loading

typical example of flexible load → generally made of soil

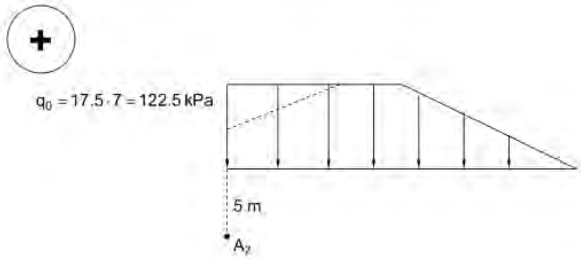


settlement goes to ∞ so we can use this formula for Δσ

$$\Delta\sigma_z = \frac{q_0}{\pi}\left[\left(\frac{B_1+B_2}{B_2}\right)(\alpha_1+\alpha_2)-\frac{B_1}{B_2}(\alpha_2)\right]$$

$$\alpha_1(\text{radians}) = \tan^{-1}\left(\frac{B_1+B_2}{z}\right)-\tan^{-1}\left(\frac{B_1}{z}\right)$$

$$\alpha_2 = \tan^{-1}\left(\frac{B_1}{z}\right)$$



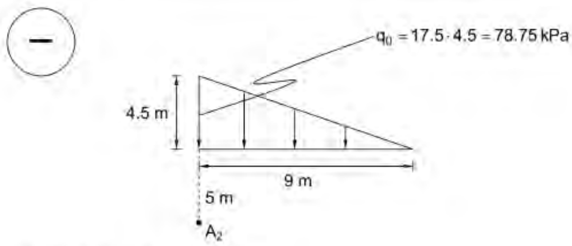
$B_2 / z = 14 / 5 = 2.8$

$B_1 / z = 14 / 5 = 2.8$

$I\left(\frac{B_1}{z}, \frac{B_2}{z}\right) = 0.497$

$\Delta\sigma_{z(2)} = 122.5 \cdot 0.497 = 60.88 \text{ kPa}$

−



$B_2 / z = 9 / 5 = 1.8$

$B_1 / z = 0 / 5 = 0$

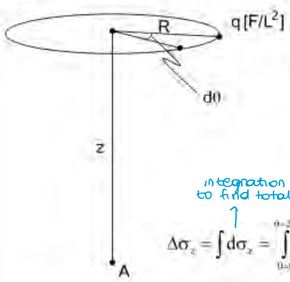
$I\left(\frac{B_1}{z}, \frac{B_2}{z}\right) = 0.339$

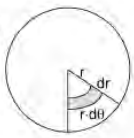
$\Delta\sigma_{z(3)} = 78.75 \cdot 0.339 = 26.7 \text{ kPa}$

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} - \Delta\sigma_{z(3)} = 10.94 + 60.88 - 26.7 = 45.12 \text{ kPa}$$

10/04/18

3) Vertical stress below the center of a uniformly loaded circular area





R = radius of the loaded area

it is exactly the load of the previous formula $\sigma_z = \frac{3P}{2\pi} \cdot \frac{z^3}{R^5}$

$$d\sigma_z = \frac{3(q \cdot dr \cdot r \cdot d\theta)}{2\pi} \frac{z^3}{(r^2+z^2)^{5/2}}$$

P (over $q \cdot dr \cdot r \cdot d\theta$); R^5 because $R = \sqrt{r^2+z^2}$

integration to find total σ

$$\Delta\sigma_z = \int d\sigma_z = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=R} \frac{3q}{2\pi} \frac{z^3 r}{(r^2+z^2)^{5/2}} dr d\theta = q \left\{ 1 - \frac{1}{\left[(R/z)^2+1\right]^{3/2}} \right\}$$

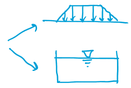
The solution is referred to the points under the centreline of the loaded area

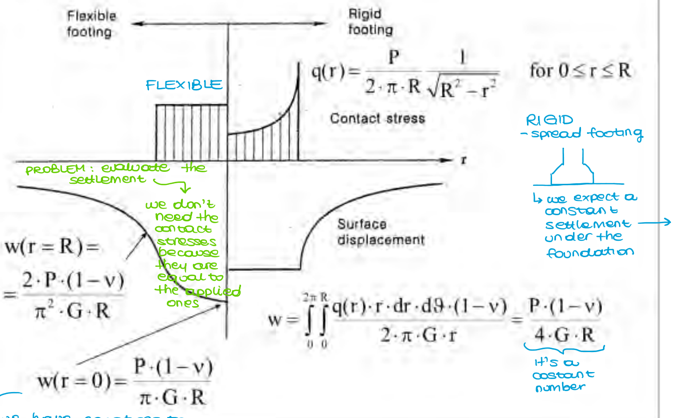
$$\Delta\sigma_z = q \left\{ 1 - \frac{1}{\left[(R/z)^2+1\right]^{3/2}} \right\}$$

→ increase of vertical stress is independent of parameters of elastic medium (E, ν)

Rigid circular foundation

↳ ≠ FLEXIBLE (embankment which can be consider as a direct load applied on the soil or liquid tank in which the load is just the weight of water)





$$q(r)=\frac{P}{2\cdot\pi\cdot R}\frac{1}{\sqrt{R^2-r^2}}\quad \text{for } 0\le r\le R$$

↳ we expect a constant settlement under the foundation → PROBLEM difficult to evaluate the contact stresses between soil and foundation

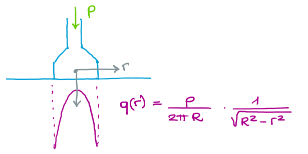
$$w(r=R)=\frac{2\cdot P\cdot(1-\nu)}{\pi^2\cdot G\cdot R}$$

$$w=\int_0^{2\pi}\int_0^{R}\frac{q(r)\cdot r\cdot dr\cdot d\vartheta\cdot(1-\nu)}{2\cdot\pi\cdot G\cdot r}=\frac{P\cdot(1-\nu)}{4\cdot G\cdot R}$$

(it's a costant number)

$$w(r=0)=\frac{P\cdot(1-\nu)}{\pi\cdot G\cdot R}$$

↳ we have equations to compute displacements everywhere

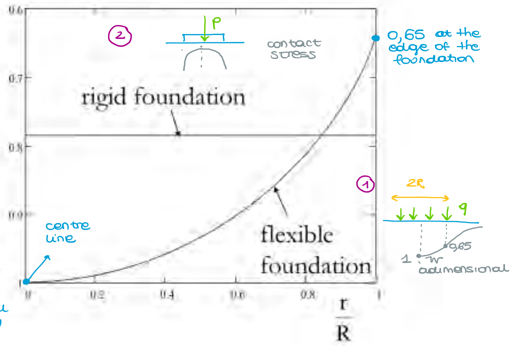


$$q(r)=\frac{P}{2\pi R}\cdot\frac{1}{\sqrt{R^2-r^2}}$$

For the edge of foundation so r=R we have an infinite stress
- elastic medium possible because we don't achieve failure
- elasto-plastic material we achive collapse for certain values of

Rigid circular foundation

↳ comparison between rigid and flexible foundation → two similar conditions ① ②



q is for the flexible load, for the rigid ones $q=\frac{P}{\pi R^2}$

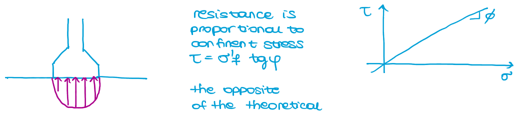
$$\frac{w\cdot G}{q\cdot(1-\nu)\cdot R}$$

plot of nondimensional displacement / settlement

$$\frac{r}{R}$$

SAND

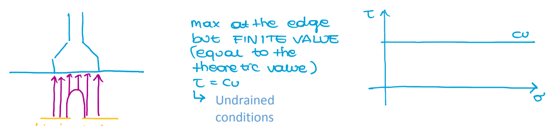
↳ because of low confinement we have this distribution of contact stress



resistance is proportional to confinent stress $\tau=\sigma'\tan\varphi$

the opposite of the theoretical elastic stress

CLAY

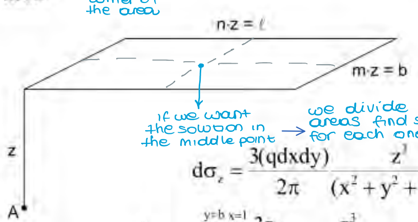


Limit value up to collapse

max at the edge but FINITE VALUE (equal to the theoretic value) $\tau=c_u$

↳ Undrained conditions

4) Vertical stress below the center of a uniformly loaded rectangular area → under the corner of the area



if we want the solution in the middle point → we divide in four areas find solution for each one and sum → they are equal so ·4

$$d\sigma_z=\frac{3(qdxdy)}{2\pi}\frac{z^3}{(x^2+y^2+z^2)^{5/2}}$$

$$\Delta\sigma_z=\int d\sigma_z=\int_{y=0}^{y=b}\int_{x=0}^{x=l}\frac{3q}{2\pi}\frac{z^3}{(x^2+y^2+z^2)^{5/2}}dxdy=q\cdot I_{rec}$$

$$I_{rec}=\frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\cdot\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right)+\tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right]$$

if $m^2n^2 > m^2+n^2+1$ it is necessary to add π

EX 1

2R = 4m

P = 2000 kN

$q=\frac{2000}{\pi 2^2}$

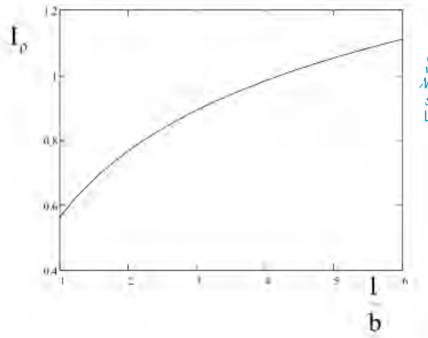
$w=\frac{2000\text{ kN }(1-\nu)}{4\cdot 6\cdot 2}$ (ν = 0,25)

G = 20 000 kPa

$w=\frac{P(1-\nu)}{4GR}=\frac{q(1-\nu)\pi R^2}{4GR}=\frac{q(1-\nu)\pi R}{4G}$

↳ settlement for rigid foundation is constant the smallest dimension of the area

$$w = \frac{q \cdot (1-\nu^2) \cdot b}{E} \cdot I_\rho \qquad I_\rho = \frac{1}{\pi} \cdot \frac{1}{b} \cdot \left[\sinh^{-1}\left(\frac{b}{l}\right) + \frac{b}{l} \sinh^{-1}\left(\frac{l}{b}\right) \right]$$



we can use it when we have the strip foundation ↳ q = ∞ so settlement should be ∞ too but it is also valid to use this diagram with a high value of $\frac{l}{b}$ (=6 is very high)

12/04/18

3. Mayne and Poulos (1999) method

Mayne and Poulos (1999) presented an improved formula for calculating the elastic settlement of foundations. The formula takes into account:

a) the rigidity of the foundation,
b) the depth of embedment of the foundation,
c) the increase in the modulus of elasticity of the soil with depth,
d) and the location of rigid layers at a limited depth.

To use Mayne and Poulos' equation, one needs to determine the equivalent diameter B_e as follows → we need an equivalent circular foundation

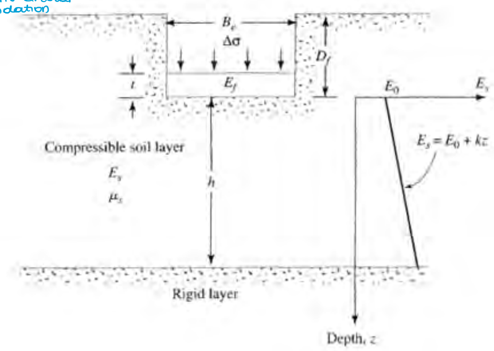
↱ it's the equivalent width

$B_e = D$ for circular foundations (D is the diameter)

$B_e = \sqrt{\frac{4 \cdot B \cdot L}{\pi}}$ for rectangular foundations of area B × L

$\frac{\pi B_e^2}{4} = B \cdot L$ → real foundation ⇒ from equation we can find the equivalent width

equivalent circular foundation



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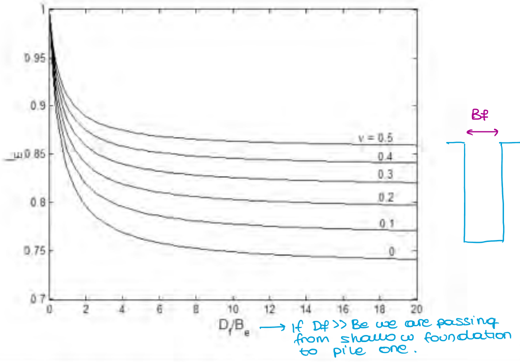
$$I_E = 1 - \frac{1}{3.5\exp(1.22\nu - 0.4)\left(\frac{B_e}{D_f} + 1.6\right)}$$

effect of foundation embedment



For a shallow foundation supported by silty clay, the following data are given:

Length	L = 1.5 m
Width	B = 1 m
Depth of foundation	D_f = 1 m
Thickness of foundation	t = 0.23 m
Load per unit area	$\Delta\sigma$ = 190 kPa
Foundation stiffness	E_f = 15·106 kPa.

The silty clay soil has the following properties:

h = 2 m

$\nu = 0.5$

E_0 = 9000 kPa

k = 500 kPa/m

$$B_e = \sqrt{\frac{4BL}{\pi}} = \sqrt{\frac{4 \cdot 1.5 \cdot 1}{\pi}} = 1.38 \text{ m}$$

$\Delta\sigma = 190$ kPa

$$\beta = \frac{E_0}{kB_e} = \frac{9000}{500 \cdot 1.38} = 13.04$$

$$\frac{h}{B_e} = \frac{2}{1.38} = 1.45$$

For $\beta = 13.04$ e $h/B_e = 1.45$, $I_G \cong 0.74$.

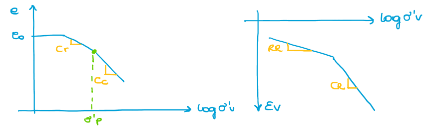
$I_F = 0.787$

$I_E = 0.922$

$$w = \frac{\Delta\sigma B_e I_G I_F I_E}{E_0}(1 - \nu^2) = 0.012 \text{ m} = 12 \text{ mm}$$

major steps:

1. Definition of a design profile, usually a layered soil profile, properly characterized in terms of stress history (σ'_p, σ'_{v0}) and compressibility (C_c and C_r). *they are refered to a specific graph*
2. For the layered soil profile the vertical stresses are computed assuming that the soil is isotropic, linearly elastic and homogeneous, ignoring the differences in soil properties of the different layers. *=> we can use Boussinesq*
3. The vertical strains of each layer are computed using the one-dimensional (oedometric) approach, and the reductions in thickness of the layers are added to obtain the settlement of the soil surface.

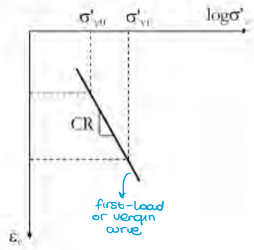


Cc: refered to the over consolidation part
Cr: vergin part

RR: recompression ratio
CR: compression ratio

$CR = \frac{Cc}{1+e_0}$ and $RR = \frac{Cr}{1+e_0}$ (e_0: initial void ratio)

Normally consolidated clay



$$\varepsilon_v = \frac{w}{H_0} \qquad \sigma'_{vf} = \sigma'_{v0} + \Delta\sigma'_v$$

$$\log\sigma'_{vf} - \log\sigma'_{v0} = \log\left(\frac{\sigma'_{vf}}{\sigma'_{v0}}\right)$$

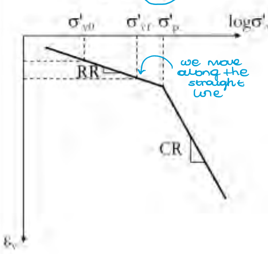
$$CR = \frac{C_c}{1+e_0}$$

$$w = H_0 \cdot CR \cdot \log\left(\frac{\sigma'_{vf}}{\sigma'_{v0}}\right)$$

H_0: initial height of the layer; σ'_{vf}, σ'_{v0}: final and initial effective stresses

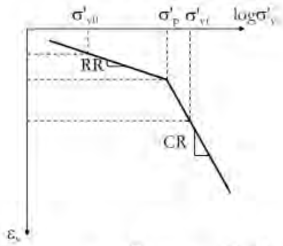
Over consolidated clay

$\sigma'_{vf} \leq \sigma'_p$ (PRE-CONSOLIDATION PRESSURE)



$$w = H_0 \cdot RR \cdot \log\left(\frac{\sigma'_{vf}}{\sigma'_{v0}}\right)$$

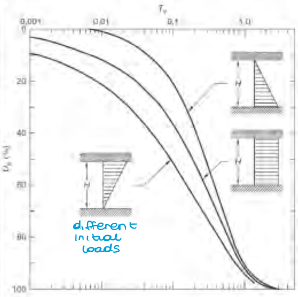
$\sigma'_{vf} > \sigma'_p$



$$w = H_0 \cdot \left[RR \cdot \log\left(\frac{\sigma'_p}{\sigma'_{v0}}\right) + CR \cdot \log\left(\frac{\sigma'_{vf}}{\sigma'_p}\right)\right]$$

$$U_s = \frac{s(t)}{s_\infty} = U_p = \frac{\int_0^H (u_0 - u)\cdot dz}{\int_0^H u_0 \cdot dz} = 1 - \sum_{k=0}^{\infty} \frac{2}{M^2} \exp\left(-M^2 \cdot T_v\right)$$

U_s	T_v
0	0
5	0.0017
10	0.0077
15	0.0177
20	0.0314
25	0.0491
30	0.0707
35	0.0962
40	0.126
45	0.159
50	0.196
55	0.238
60	0.286
65	0.342
70	0.403
75	0.477
80	0.567
85	0.684
90	0.848
95	1.129
100	∞



Approximate solutions:

$$U_s \cong \frac{2}{\sqrt{\pi}}\sqrt{T_v} \quad U_s \le 0.6$$

$$U_s = \frac{\left(4\frac{T_v}{\pi}\right)^{0.5}}{\left[1+\left(4\frac{T_v}{\pi}\right)^{2.8}\right]^{0.179}};$$

$$T_v = \frac{\frac{\pi}{4}U_s^2}{\left[1-U_s^{5.6}\right]^{0.357}}.$$

=> 1D : infinite load , different from real problems

Three-dimensional consolidation: circular footing of radius a

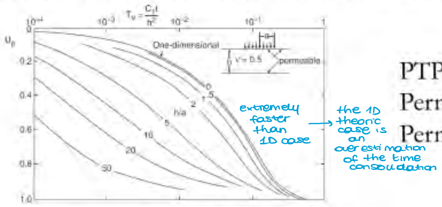


Figure 6.30 Rate of consolidation of a circular footing for drainage boundaries PTPB (reproduced from Davis and Poulos (1972) with permission).

PTPB:
Permeable top
Permeable bottom

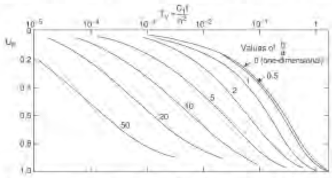


Figure 6.31 Rate of consolidation of a circular footing for drainage boundaries PTIB (reproduced from Davis and Poulos (1972) with permission).

PTIB:
Permeable top
Impermeable bottom

Three-dimensional consolidation: strip footing of width 2b

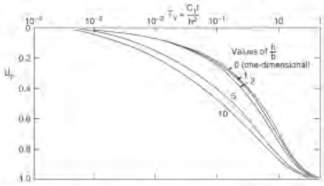


Figure 6.32 Rate of consolidation of strip footing for drainage boundaries PTPB (reproduced with permission from Davis and Poulos, 1972).

PTPB:
Permeable top
Permeable bottom

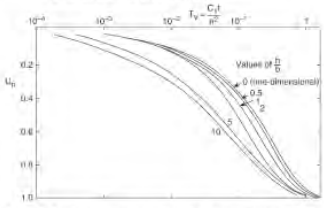


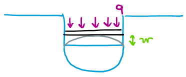
Figure 6.33 Rate of consolidation of strip footing for drainage boundaries PTIB (reproduced with permission from Davis and Poulos, 1972).

PTIB:
Permeable top
Impermeable bottom

19/04/18

6. Settlement of foundations on sands

Despite the knowledge of the main factors affecting the soil behaviour, due to the difficulties of sampling coarse-grained soils the prediction of settlements of foundations on sand still



The same expression applies to *overconsolidated sand*: where $q' \leq \sigma'_p$, the relevant contribution is only given by $q' \cdot B^{0.7} I_c/3$; while, if $q' > \sigma'_p$ the last expression must be used, in which σ'_{v0} is substituted by σ'_p.

Further corrections have also been suggested to take into account factors like the *shape of the foundation*, the *thickness of the compressible layer* and the *time dependent fraction* of the overall settlement.

The correction factor to be applied when the ratio of length to breadth of the foundation is greater than unity, i.e. L/B > 1, is the following:

$$f_s = \left[\frac{\frac{1.25L}{B}}{\frac{L}{B}+0.25}\right]^2 > 1$$

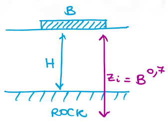
if $\frac{L}{B} = 1$ we have $f_s = 1$

→ it's just a function of L/B

When the thickness H of the sand or gravel layer is less than the depth of influence, then a correction f_H should be applied such that

$$f_H = \frac{H}{Z_I}\left(2 - \frac{H}{Z_I}\right) < 1$$

H → thickness of compressible layer; Z_I → depth of influence



Finally, the case records collected by Burland and Burbidge prove that footings on sand and gravels exhibit time dependent settlements. Therefore, the settlement at any time *t*, greater than *three years*, will be increased by a factor equal to:

$$f_t = \left(1 + R_3 + R \log\frac{t}{3}\right)$$

t: time in years

Values of R_3 and R depend on the nature of the imposed loads, so that values of 0.3 and 0.2 are suggested when dealing with static loads, while values of 0.7 and 0.8 are suggested when dealing with cyclic (for dynamic) loading. As an example, by considering the expect settlement the foundation can experience in a time interval of *30 years* (reference time for life of building), the value of f_t is equal to 1.5 for static loads, and to 2.5 in presence of cyclic loading (this could be the case of special structures such as chimneys).

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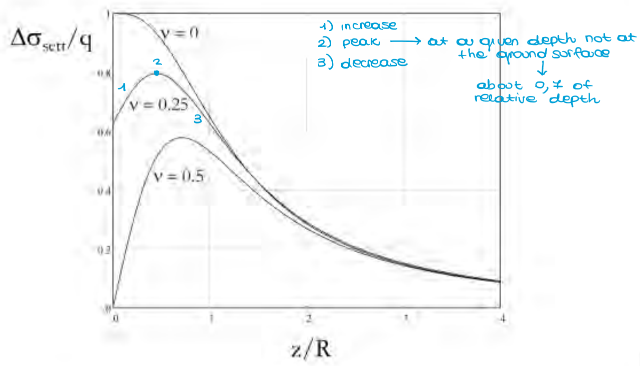
$$\frac{\Delta\sigma_z}{q} = 1 - \frac{1}{\left[\left(\frac{R}{z}\right)^2 + 1\right]^{3/2}}$$

$$\frac{\Delta\sigma_r}{q} = \frac{1}{2} + \nu - \frac{(1+\nu)}{\left[\left(\frac{R}{z}\right)^2 + 1\right]^{1/2}} + \frac{1}{2\cdot\left[\left(\frac{R}{z}\right)^2 + 1\right]^{3/2}}$$

$$\Delta\sigma_{sett} = (1-2\nu)\cdot\Delta\sigma_z + 2\nu\cdot(\Delta\sigma_z - \Delta\sigma_r)$$



⇒ so settlement is given by both vertical and deviatoric stress also for load propagation in the soil (that's why we can apply elastic theory, if we don't consider deviatoric stress we won't be able to apply elasticity theory)

Even though the soil is unlikely to be elastic, the figure show clearly the contribution that shearing is likely to make to settlement: it is not enough to consider only the change in vertical stress.
Moreover, the maximum value of $\Delta\sigma_{sett}$ does not occur at the surface. Indeed for high values of Poisson's ratio ($\nu \rightarrow 0.5$), the peak shear stress and hence peak value of $\Delta\sigma_{sett}$ occur at a depth of about $0.7\cdot R$.
Schmertmann (1970) takes inspiration from this to devise a procedure which can be applied more generally. If we write:

$$\Delta\sigma_{sett} = I_z \cdot q$$

where I_z is a dimensionless influence factor that varies with depth, then

$$w = q \cdot \int_0^\infty \frac{I_z}{E} \cdot dz$$

(I_z → function of z; E → under integral symbol because it isn't costant it varies with depth)

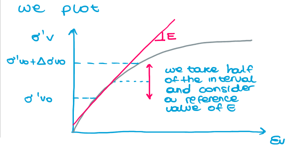
Berardi and Lancellotta (1991) method

The empirical relation suggested by Burland and Burbidge (1985) implicitly assumes that the compressibility index I_c does not depend on stress and strain level. Since one expects that the soil stiffness must depend on stress and strain level, Berardi and Lancellotta (1991) have reviewed the case histories collected by Burland and Burbidge (1985) and have suggested the following approach in order to account for *soil non linearity*. Consider the settlement as predicted by the theory of elasticity:

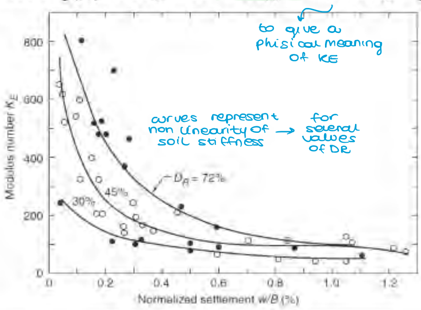
$$w = \frac{q}{E'} B (1 - \nu^2) \cdot I$$

and give the soil stiffness the following expression, in order to account for the current stress level:

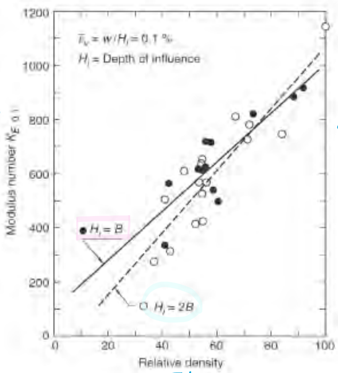
$$E' = K_E \cdot p_a \left(\frac{\sigma'_{vo} + 0.5 \Delta\sigma'_{vo}}{p_a} \right)^{0.5}$$

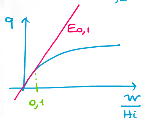


In the previous equation, the overburden effective stress and the increment of the vertical stress are supposed to be evaluated at a depth corresponding to half of the depth of influence H_i.
The modulus number K_E may be thought as the value the soil modulus assumes when the current vertical stress is equal to the reference stress p_a (i.e. $K_E = E'$ if $\sigma'_{v0} + 0.5 \cdot \Delta\sigma'_z = p_a$).



In order to highlight the influence of the relative density, it is convenient to select a reference value of soil modulus, such as corresponding to a normalized settlement equal to 0.1%.





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σ'_{v0} (z = 2,75) = 1,9 · 9,81 · 2,75 = 51,26 kPa

$\Delta\sigma'_{v0} = I_\sigma \cdot q^{NET} = \left\{1 - \frac{1}{\left[\left(\frac{B_r}{2z}\right)^2 + 1\right]^{3/2}}\right\} q^{NET} = 0,7 \cdot 172 = 120$ kPa

↳ formula of circular foundation

$I_\sigma = f$(depth)

$B_r = \sqrt{\frac{4B^2}{\pi}} = 2,82$ m

↳ equivalent diameter

$$\left(\frac{w}{B}\right)^{0.3} = \frac{q}{E'_{0.1}} \cdot \frac{125 \cdot 1 \cdot (1-\nu^2)}{1} = \frac{172}{84.000} \cdot \frac{125 \cdot 0.609 \cdot (1-0.15^2)}{1} = 0.152$$

w = 4.7 mm

→ because w is unknown so better to consider w/B

We need to have an estimation of I and ν

Drained soil (0,15 ÷ 0,3)

$I = I_G I_F I_E$ – HAYNE AND POULOS

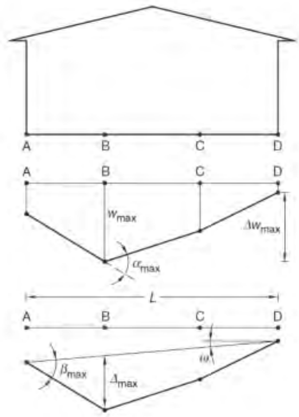
- I_G: presence of rigid layer and increase of soil stiffness with depth
  $\frac{h}{B} \approx 3$ so $I_G = 0,86$
- I_F: rigidity of foundation
  $I_F = 0,79$ for rigid foundation
- $I_E = 0,9$ (embedment of foundation)
  ↳ $\frac{B_r}{D} = \frac{2,82}{1,5}$

⇒ we have diagrams to estimate each I_G, I_F and I_E

EXAM on the topic of settlement
→ settlement on clay
→ settlement on sand
  → B&B
  → Schmertmann
  → Berardi and Lancellotta

7. Damage criteria and limiting values of settlement

different parameters to quantify settlements



w = Settlement
ω = Tilt
Δw = Relative settlement — or differential
β = Relative rotation
α = Angular strain
Δ = Relative deflection
Δ/L = Deflection ratio

Relative rotation and deflection ratio: very similar quantities → two ways to compute the same thing

Δ is max differential settlement after the tilt of foundation (rigid rotation)

L is the total length of the foundation

β is again Δ over only a portion of the building (of the foundation beam)

a) Deviation from the vertical in excess of about 1/250 can be distinguishable to the casual eye and may produce an unpleasant and sometimes alarming feeling. A deflection ratio of more than 1/250 will also be visible for horizontal members.

b) The most comprehensive analysis of limiting settlements of structures is that of Skempton and MacDonald (1956), based on the performance of 98 buildings. According to this analysis, limiting values of the relative rotation β should be of the order of 1/500 to avoid cracking in walls and partitions and of the order of 1/150 to avoid structural damage.

Polshin and Tokar (1957) quoted values of 1/500 and 1/250 and similar conclusions have also been reached by Meyerhof (1956).

Two different levels of damage
- if $\beta > \frac{1}{500}$ we have cracking in walls and partitions
- if $\beta > \frac{1}{150}$ structural damage
  ↳ or 250

Data collected by Bjerrum (1963) showed, in addition, that no damage to building on rafts on clay has been recorded for differential settlements of less than 125 mm and for total settlements of less than 250 mm. Damage has instead been reported for buildings on isolated footings on clay for differential settlements in excess of 50 mm and total settlements higher than 150 mm.

⇒ we need limited values of displacement because relative settlements play an important role in soil-foundation interaction

8. Recommended readings

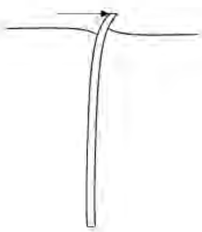
Lancellotta R. (2009). Geotechnical Engineering 2nd Edition, Taylor & Francis, Abingdon (UK) and New York (USA): **Chapter 9** and **Parts 6.10, 6.14**.

1. Examples → they are all one dimentional

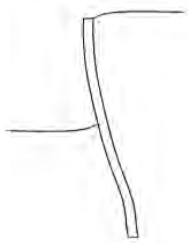
Pile under axial loading

Bending of a beam on an elastic foundation

Pile under lateral loading



Embedded wall



⇒ geometry level analysis (1D) is in common of all the 4 examples. For 2D or 3D we need final element differences

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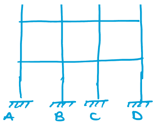
tial settlements with their effects on the super structure
↳ so geotechnical and structural engineering

rigid so without differential settlement we have no problem, but if not, we need to consider a change on inner forces of the structure

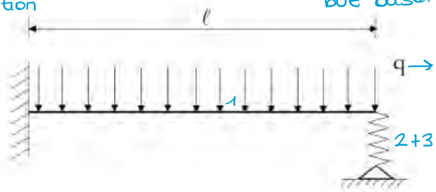


We can compare complex structures with this semplification

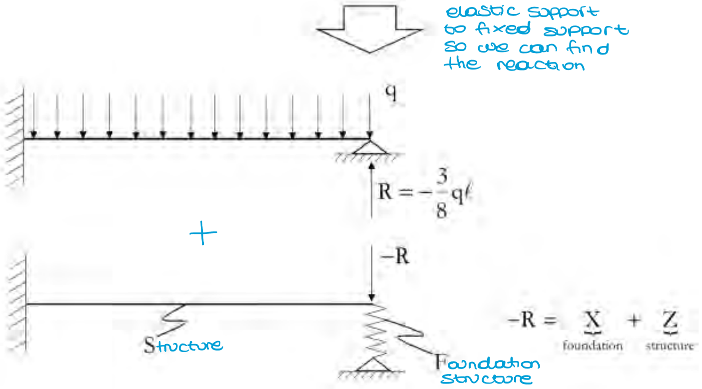
Same examples, but distributed load:

1) beam is the superstructure

2+3) elastic support represents the foundation structure and soil.

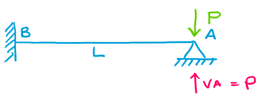


elastic support to fixed support so we can find the reaction



We can use even a SIMPLER RESPONCE

(1) cantilever beam with a vertical load on its free edge



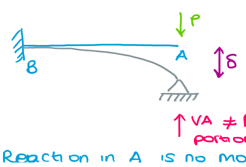
we suppose to put a rigid support on the free edge (rigid because without possible settlements)

we have a reaction VA so beam isn't stressed at all because T = M = 0

Beam: superstructure
Support: foundation

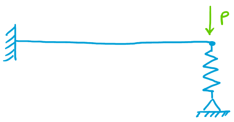
- any stress of the beam
- no settlements

If we have a DISPLACEMENT



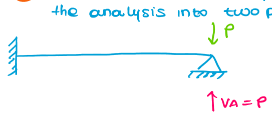
Reaction in A is no more equal to P

(2)

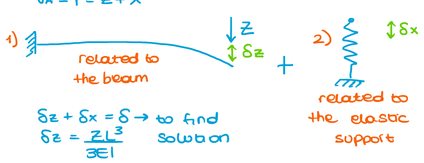


if we have an elastic support so displacement is not given

(HP) from elastic to fixed support and we divide the analysis into two parts:



$V_A = P = Z + X$



$\delta_z + \delta_x = \delta$ → to find solution

$\delta_z = \frac{ZL^3}{3EI}$

$\delta_x = \frac{X}{k}$

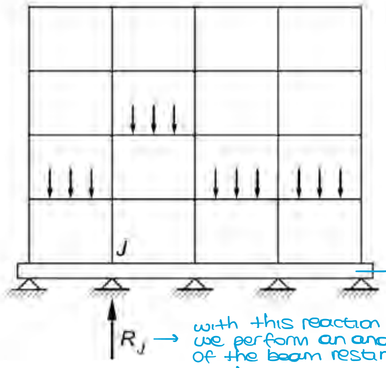
Combining

$\frac{3EI}{L^3} \cdot \delta + k\delta = P$

$\delta = \frac{P}{k + \frac{3EI}{L^3}}$

$X = k\delta = k \cdot \frac{P}{\ldots} = \frac{P}{\ldots}$

In the case of a frame structure, additional restrain can be introduced below the column bases and the reaction R_j (referred to the j-th column) can be computed.

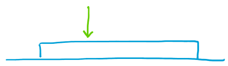


we can consider the foundation as more rigid as the super structure

with this reaction we perform an analysis of the beam resting on the elastic soil

frame is flexible with respect to the beam or viceversa

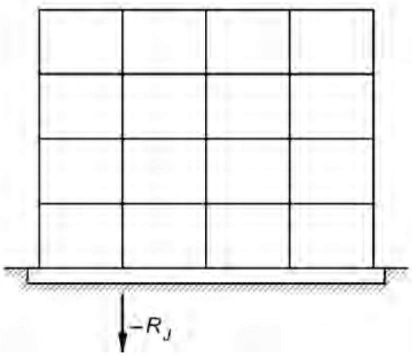
we apply the whole reaction of the superstructure we suppose that the beam is very RIGID



it leads to a settlement so we need to evaluate consequences of this displacement in the nodes of the superstructure

=> analizing all these aspects we can choose the kind of foundation to be used (spread footing, beam, plate, pile foundation)

The actual situation corresponds to the superposition of this first stage with that in which the reaction R_j are applied to the whole system given by the structure and the foundation beam.



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1) FLEXIBLE STRUCTURE: if the structure is considered flexible, then the transmitted loads do not depend on the relative settlements, because the structure has no rigidity. In this case, the loads are no longer unknowns and the problem is reduced to the analysis of a beam on an elastic soil.

2) RIGID STRUCTURE: if the structure is considered rigid, then the unknown loads can be obtained by the condition that settlements must lie on a straight line. Then, for each column, the equation holds:

$$y_i = y_0 + \alpha_0 x_i$$

y_0 being the settlement of the first column, where the origin of the x axis is taken and α_0 being the rigid tilt.

Two further equations must be added:

$$\sum_i P_i = Q \qquad \sum_i P_i \cdot x_i = Q \cdot e$$

where Q is the resultant of vertical loads transmitted by the superstructure and e is its eccentricity.
The settlements can be related to the loads P_j acting on the beam as follows:

$$y_i = \sum_i \eta_{ij} P_j$$

where the coefficients η_{ij} depend on the relative rigidity of the beam with respect to the soil.
By doing so, (n + 2) equations are obtained that allow one to compute the loads P_i transmitted to the foundation beam and the values of y_0 and α_0.

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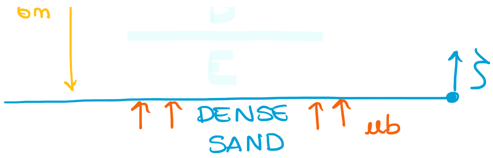


thickness of layer

LAYER	ΔH (m)	σ_{vo} (kPa)	ζ (m)	h (m)	u (kPa)	σ'_{vo} (kPa)	σ'_p = OCR σ'_{vo}
A	2	4 · 19 = 76	9	11,85	28,3	47,7	238,7
B	2	6 · 19 = 114	7	12,09	50,9	63,1	315,7
C	2	7 · 19 + 1 · 18 = 151	5	12,35	73,5	77,5	
D	2	7 · 19 + 3 · 18 = 187	3	12,61	96,1	90,9	
E	2	7 · 19 + 5 · 18 = 223	1	12,87	118,7	104,3	

we consider the layer from the top because we need to compute settlement of soil instead of before in which we were considering the base of foundation to verify its resistance to floating.

$$h(\zeta) = hb - \frac{\Delta h}{L} \cdot \zeta$$

↳ it is a linear combination of ζ

$\Delta h = 1,5$ m

$L = 11,5$ m

From Bernoulli equation we have:

$$h = u + \frac{\zeta}{\gamma_w}$$

$$u = \gamma_w (h - \zeta)$$

- we need to compute increment of vertical stress due to the foundation
  ↳ through Bousinnesq

$$\Delta\sigma_v = q^{net} \left\{ 1 - \frac{1}{\left[\left(\frac{R}{z_f}\right)^2 + 1\right]^{1,5}} \right\}$$

$$q^{net} = q - \sigma_{vo}(z = D) = 170 - 3 \cdot 19 = 113 \text{ kPa}$$

LAYER	z_f (m)	$\Delta\sigma_v$ (kPa)	σ'_{vf}	w (m)	w_i (m)	w_c (m)	w_{TOT} (m)
A	1	11,3	160,5	w_{oc} = 0,032	0,016	0,016	0,032
B	3	10,7	170,3	w_{oc} = 0,026	0,013	0,013	0,026
C	5	93,7	171,2	w_{NC} = 0,085	0,009	0,085	0,094
D	7	77,1	168	w_{NC} = 0,066	0,007	0,066	0,073
E	9	61,8	166,1	w_{NC} = 0,050	0,005	0,05	0,055

$\sigma'_{vf} = \sigma'_{vo} + (\Delta\sigma_v = \Delta\sigma'_v)$ (kPa)

- For A and B we need to compare σ'_{vf} with σ'_p

because $\sigma v = \sigma' 1$ and $\sigma 2 = \sigma' 3 = \sigma' h$

$\varepsilon h = \frac{1}{E}\left[\sigma' h - \nu(\sigma' v + \sigma' h)\right]$

=> constitutive equations

we impose $\varepsilon h = 0$ so

$\sigma' h = \nu(\sigma' v + \sigma' h)$

$\sigma' h (1-\nu) = \nu \sigma' v$

$\sigma' h = \frac{\nu}{1-\nu} \sigma' v$

coeff. of pressure at rest

$\varepsilon v = \frac{1}{E}\left(\sigma' v - 2\nu \cdot \frac{\nu}{1-\nu} \cdot \sigma' v\right) = \frac{1}{E}\left(\frac{1-\nu-2\nu^2}{1-\nu}\right)\sigma' v$

$\frac{1}{E_{edom}}$ = edometric compressibility $= \frac{1}{M} = m_v$

EDOMETRIC MODULUS

$M = \frac{1}{m_v} = \frac{1-\nu}{1-\nu-2\nu^2} \cdot E = 1{,}2 \cdot 10^3$ kPa

$m_v = 8{,}33 \cdot 10^{-6}$ kPa^{-1}

Compressibility coefficient:

$c_v = \frac{k}{m_v \cdot \gamma_w} = 2{,}4 \cdot 10^{-7}$ m^2/s $= 7{,}574$ m^2/yr

$1s = 3 \cdot 171 \cdot 10^{-8}$ yr

better passing from seconds to years

- consolidation settlement computed by linear elasticity:

$w_c = m_v \cdot H \cdot q = 0{,}8$ m

$U_s = 50\%$

$T_v = 0{,}196$

$T_v = \frac{c_v \cdot t}{H_d^2}$

length of drainage (half of height of layer) (= 4 m)

If we have clay instead of rock we have $H_d = H$ ⇓ REPEAT THE EXERCISE

$t_{50} = \frac{T_v \cdot H_d^2}{c_v} = 0{,}414$ yr

$U_s = 85\%$

$T_v = 1{,}129$

$t_{95} = \frac{T_v \cdot H_d^2}{c_v} = 2{,}385$ yr

$w\left(\frac{1}{12} yr\right)$

Approximate equation

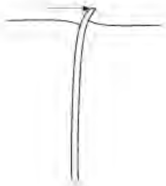
$U_s = \frac{2}{\sqrt{\pi}} \cdot \sqrt{T_v}$

$T_v = \frac{c_v \cdot t}{H_d^2} = 0{,}039$

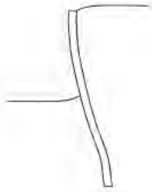
$U_s = 0{,}224 < 0{,}6$

$w\left(\frac{1}{12} yr\right) = U_s \cdot w_c = 0{,}179$ m

Pile under lateral loading

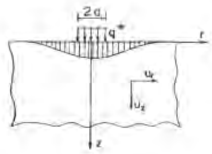


Embedded wall

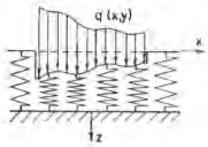


2. Soil models

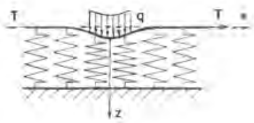
Elastic continuum

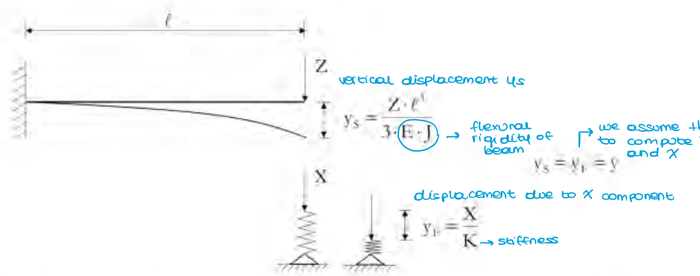


Winkler model



Two-parameter model





$$X + Z = -R \quad \Rightarrow \quad K \cdot y + \frac{3 \cdot E \cdot J}{\ell^3} \cdot y = \frac{3}{8} q\ell \quad \Rightarrow \quad y = \frac{\frac{3}{8} q\ell}{K + \frac{3 \cdot E \cdot J}{\ell^3}}$$

↳ value of settlement

elastic support:

$$X = K \cdot \left(\frac{\frac{3}{8} q\ell}{K + \frac{3 \cdot E \cdot J}{\ell^3}} \right)$$

cantilever beam:

$$Z = \frac{3 \cdot E \cdot J}{\ell^3} \cdot \left(\frac{\frac{3}{8} q\ell}{K + \frac{3 \cdot E \cdot J}{\ell^3}} \right)$$

→ Analizing X: we have ratio between flexural rigidity EJ of cantilever beam and stiffness K of elastic support

- if K increases the term goes to zero and X is considered with Z's variation that we find for fixed support. No vertical displacements of the edge of cantilever beam and load is taken by edge support, that is equivalent to say foundation.
- if K decreases with respect to EJ, the ratio goes to ∞ and X to 0. So Z goes to reaction R which means that cantilever isn't supported at the edge like if we have a simple cantilever beam without extra supports

Negative case from structural point of view:
↳ in the beam we represent the elastic support as a reaction given by:

R − Z = X

The reaction is due to the effects of settlements

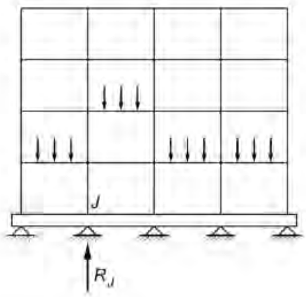
We generate this reaction when we know vertical force so we can evaluate V and bending moment in the beam.

For geotechnical engineering we need to evaluate settlements and bearing capacity qb, this value is X

We can evaluate rigidity of cantilever beam and K of elastic support. So for bending we consider rigidity and stiffness.

In the case of a frame structure, additional restrain can be introduced below the column bases and the reaction R_j (referred to the j-th column) can be computed.

we obtain a foundation system, we consider foundation more rigid than superstructure (with this assumption calculations are easier)



we introduce fixed supports and evaluate reaction RJ when J is for a certain column. We substitute elastic supports with fixed ones to evaluate the reaction itself.

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are taken into account.

After solving the equation ({4}) we can evaluate z and x using previous equations
↳ difficult to know D and F

1) FLEXIBLE STRUCTURE: if the structure is considered flexible, then the transmitted loads do not depend on the relative settlements, because the structure has no rigidity. In this case, the loads are no longer unknowns and the problem is reduced to the analysis of a beam on an elastic soil.

2) RIGID STRUCTURE: if the structure is considered rigid, then the unknown loads can be obtained by the condition that settlements must lie on a straight line. Then, for each column, the equation holds:

$$y_i = y_0 + \alpha_0 x_i$$

y_0 being the settlement of the first column, where the origin of the x axis is taken and α_0 being the rigid tilt.

Two further equations must be added:

$$\sum_i P_i = Q \qquad \sum_i P_i \cdot x_i = Q \cdot e$$

where Q is the resultant of vertical loads transmitted by the superstructure and e is its eccentricity.
The settlements can be related to the loads P_i acting on the beam as follows:

$$y_i = \sum_j \eta_{ij} P_j$$

we use inverse of stiffness matrix to relate load to settlement
↳ system of (n) equation (one for each column)

where the coefficients η_{ij} depend on the relative rigidity of the beam with respect to the soil.
By doing so, (n + 2) equations are obtained that allow one to compute the loads P_i transmitted to the foundation beam and the values of y_0 and α_0.

So we can obtain the solution for this limit case, because we have a relation similar to the previous equation that we use for foundation.

Relating position of load to the settlement, using K matrix, here we use inverse of matrix K to relate settlement to load at column J, inverse relation of previous equation.

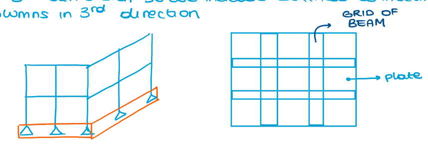
Now we have a system of N equation, one for each column, but we have n+2 unknowns
- loads P
- settlements y_0
- rigid tilt α_0

We need two more equations
- vertical force equilibrium $\Sigma P_i = Q$
- momentum equation $\Sigma P_i x_i = Qe$

If we have RIGID FOUNDATION: here R2 is higher than R1 and R3. we have higher load because we have a strip foundation for each column, we have uniform soil so we expect higher settlement for column 2, so different settlements for each column of the frame.

If settlement is significant we don't have flexible structure. If we have uniform settlements no problem, but higher load column 2.
↳ DIFFERENTIAL SETTLEMENTS (high effect on frame so we make connection between columns to avoid it.

If we have a 3D structure we have similar frames in 3rd dimension so we increase stiffness connecting columns in 3rd direction



when empty areas between beams are below a given ratio it's convinient to substitute the beam with a PLATE. In this case we have increase of rigidity of the foundation, from the spread foundation to grid of beams and plate.
we can have an unsatisfactory result if we have a compressible shallow layer so better using pile foundation to reduce settlements, produced in shallow layer

- spread footing
- beams
- grid of beams
- plate
- piles

we make foundation sufficiently rigid to limit settlements and in similar way there is a limiting condition of flexible superstructure. This isn't typical condition that is designed in practise, because the aim of foundation is to have a restriction of differential settlements to a value so low that it can be neglected in the analysis of the support structure.

2) **RIGID STRUCTURE**

↳ we have Q as vertical load and e as its eccentricity
Position of column has to be in straight line, so we have a settlement and a rotation of the structure that is due to structure rigidity.

All the settlements are located in straight line, the general equation of a generic settlement of a column is:

$y_i = y_0 + \alpha_0 x_i$

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FOUNDATION ENGINEERING

01RVSMX
A.A. 2017-2018

Beam on a Winkler medium

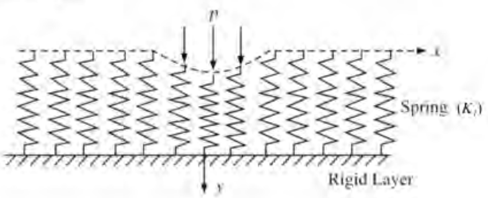
1. Winkler medium

The idealized model of soil media proposed by Winkler (1867) assumes that the deflection, y, of the soil medium at any point on the surface is directly proportional to the stress, p, applied at that point and independent of stresses applied at other locations, i.e.

settlement ← $$y = \frac{p}{K_1}$$

↱ unit weight

where K_l (F/L³) is termed the modulus of subgrade reaction with units of stress per unit length.



bed of independent springs, vertical displacement of single spring is generated by a single load applied to the single spring itself

From the previous equations, the governing equation of bending for a beam supported on a Winkler medium results to be:

$$EJ\frac{d^4y}{dx^4} = q(x) - K_1 B \cdot y(x)$$

the reaction is ∝ to Kd

modulus of subgrade reaction

whose general solution is given by:

$$y(x) = e^{\lambda x}\left(C_1 \cos\lambda x + C_2 \sin\lambda x\right) + e^{-\lambda x}\left(C_3 \cos\lambda x + C_4 \sin\lambda x\right)$$

where C_1, C_2, C_3, and C_4 must be determined from the boundary conditions of a particular problem, and the parameter λ (L^{-1}) is equal to:

relative stiffness coefficient

$$\lambda = \sqrt[4]{\frac{K_1 B}{4EJ}}$$

→ rigidity of soil

→ rigidity of beam

The *relative stiffness* of the beam-soil foundation system depends on the ratio between the beam length, L, and the characteristic length given by $1/\lambda$.

used to evaluate propagation of load applied at a certain point

The *relative stiffness* of the beam-soil foundation system depends on the ratio between the beam length, L, and the characteristic length given by $1/\lambda$ (Vesic, 1961):

$$\varepsilon = \frac{L}{(1/\lambda)} = L \cdot \sqrt[4]{\frac{k_s B}{4EJ}}$$

Vesic (1961) defined the flexibility of a footing as follows:

- the footing is classified as *rigid* when $\varepsilon < \pi/4$ (i.e. about 0.8)
- the footing is classified as *flexible* when $\varepsilon > \pi$ (i.e. about 3)
- the footing is classified as *intermediate* when $\pi/4 < \varepsilon < \pi$

it speaks about Winkler's method

From Haberfield, C. (2017), "*Practical application of soil structure interaction analysis. First Gregory Tschebotarioff Lecture on Soil-structure interaction*", Proceedings of 19th ICSMGE, 17-22 September 2017, Seoul, Korea:

"*The author has also been asked many times by both structural and geotechnical engineers what value of subgrade reaction should be adopted as the reference books all give different values. My reply has always been – none of them.*

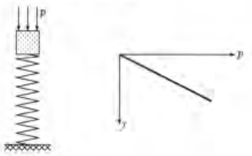
It is clear from these experiences that there are many structural and geotechnical engineers who do not understand the limitations of subgrade reaction theory, do not understand the difference between modulus of subgrade reaction (which applies to a 300 mm square plate) and spring stiffness (which applies to a footing of any type or size) and who have the misconception that spring constants are a soil property. Spring constants for a pad footing or pile for example, defined as load divided by displacement, depend upon many factors including geometry, scale, load magnitude and duration and soil properties. Spring stiffness values will also vary for the same footing with location across the foundation system (for example) and with each load case."

WINKLER MODEL

3. Modulus of subgrade reaction

Consider a circular rigid footing, under the uniform load p (F/L²).

Winkler medium → settlement is given by:



$$y = \frac{p}{K_1}$$

compare with

Linear elastic half-space

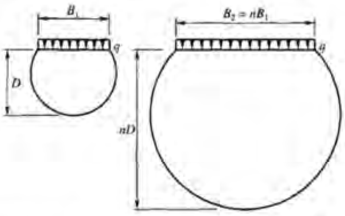
$$y = \frac{\pi(1-\nu^2)pD}{4E}$$

settlement is function of load, stiffness of soil and diameter of footing

where ν is the Poisson's coefficient and D is the footing diameter

Consider two foundation beams, of width B_1 and B_2 (such that $B_2 = n \cdot B_1$) resting on a compressible subgrade and each loaded so that the subgrade reactive pressure is uniform and equal to q for both beams.

we have two plates with two settlements which correspond to different values of K1 because the influence depth is different so settlements aren't equal too.



$$y_{(1)} = \frac{q}{K_{1(1)}}$$

$$y_{(2)} = \frac{q}{K_{1(2)}}$$

The differences of K1 depend not only on soil properties, but also on foundations

If the beams are resting on a subgrade whose deformation properties are more or less *independent of depth* (such as a stiff clay) then it can be assumed that the settlement increases in simple proportion to the depth of the pressure bulb:

$$y_{(2)} = n \cdot y_{(1)} \quad \Longrightarrow \quad K_{1(2)} = \frac{q}{n \cdot y_{(1)}} = \frac{q}{y_{(1)}} \cdot \frac{B_1}{B_2} = K_{1(1)} \cdot \frac{B_1}{B_2}$$

→ direct application of equation of linear elasticity

so a plate with B2 = 0 has n factors ≠ from B1, so n is a factor related also to settlement y and to K1

The modulus of subgrade reaction can be estimated from the results of plate loading tests (generally, a square plate with a side of *0.305 m* is used).

Terzaghi (1955) provided the following indications for the determination of the modulus of subgrade reaction:

- Stiff clay (stiffness constant with depth):

→ valued measured at plate test

$$K_1 = K_{1p} \cdot \left(\frac{\ell + 0.152}{1.5 \cdot \ell} \right) \cdot \frac{b}{B} \qquad K_1 = K_{1p} \cdot \frac{b}{1.5 \cdot B} \quad \text{for } \ell \to \infty$$

ℓ: length of the beam B: width of the beam

- Sand (stiffness that increases with depth): → not constant, we have solution given by Terzaghi

$$K_1 = K_{1p} \left(\frac{B + b}{2 \cdot B} \right)^2$$

where K_{1p} is the modulus obtained from a plate bearing test using a plate of width b, B is the width of the beam and ℓ is the length of the beam.

⇒ we calculate values after knowing real ones from tests.