NUMERO: 2381A

ANNO: 2018

A P P U N T I

STUDENTE: Pirro Giulia

MATERIA: Geomatics Teoria - Prof. Piras

Objectives

- TO KNOW
- TO DO

Topics

- GNSS
- LIDAR
- Photogrammetry
- Inertial platform
- Data processing and statistics
- 3D network
- GIS

Contact

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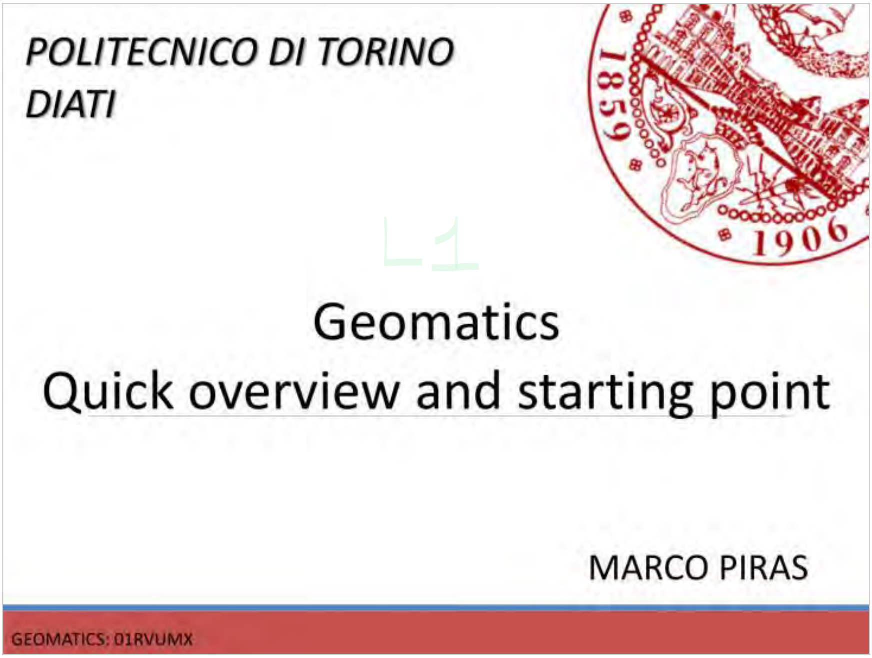
Educational Materials

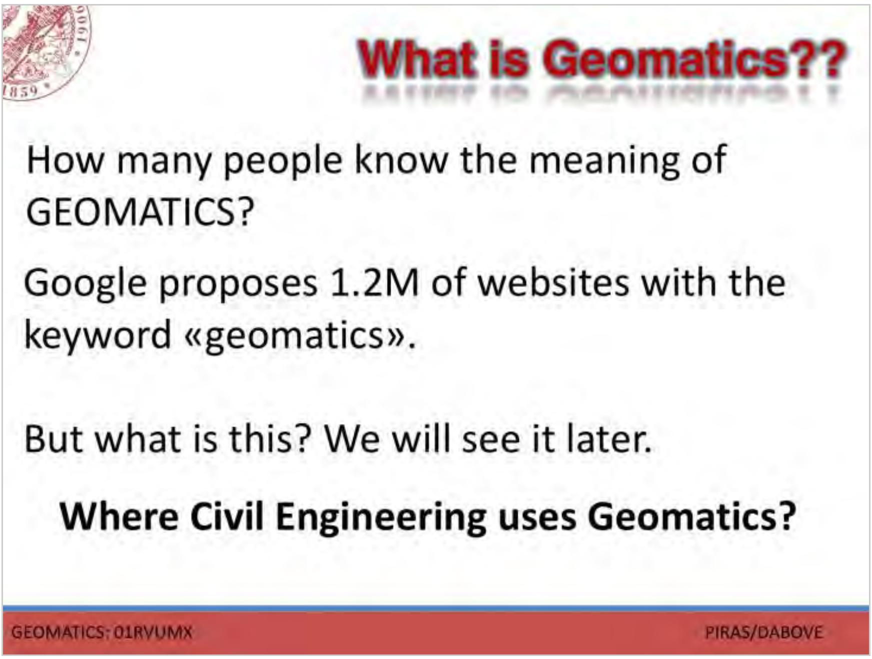
On the course web page:

- Lecture notes
- Exercises notes and dataset
- Additional material: papers, documents, websites, etc…

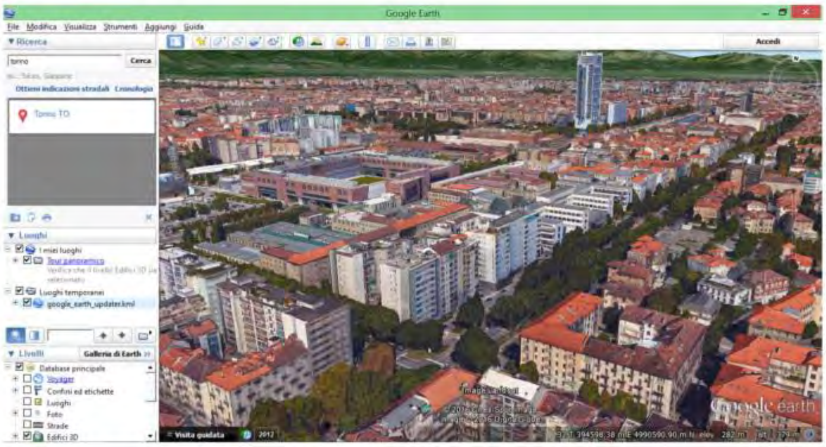
7.03.18

mercoledì 7 marzo 2018 08:36





What is Geomatics??



GEOMATICS: 01RVUMX PIRAS/DABOVE

Are you Geomatician??

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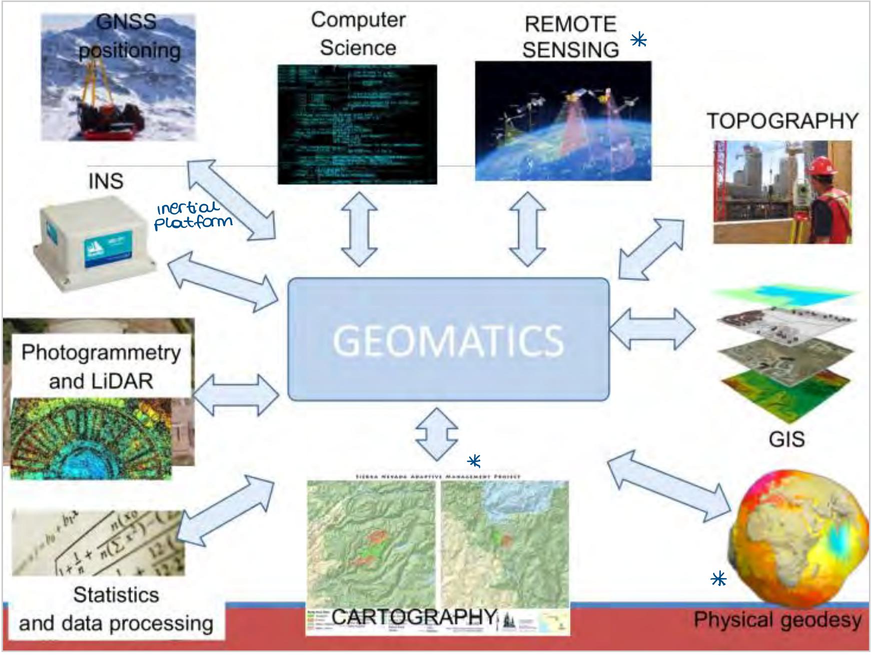
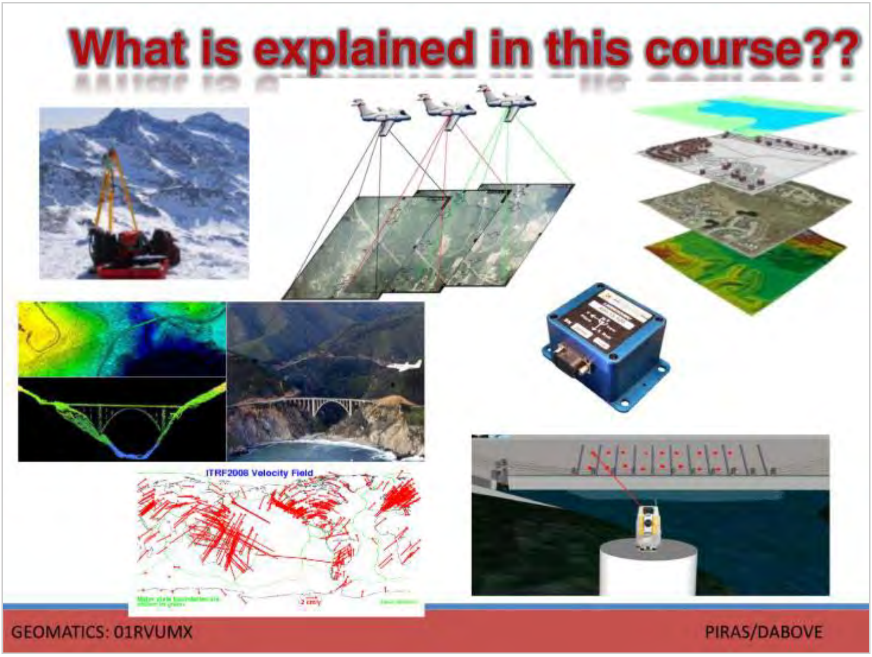

GNSS positioning
Computer Science
REMOTE SENSING *
TOPOGRAPHY
INS
inertial platform
GEOMATICS
Photogrammetry and LiDAR
GIS
*
*
Statistics and data processing
CARTOGRAPHY
Physical geodesy


What is explained in this course??
ITRF2008 Velocity Field
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Topography

from angle and distance we can define the position of a body

Topography is the oldest discipline of Geomatics and it is devoted to define the position and height of the points, making measurements of distance and angles.

Several years ago, only mechanics instruments were used, today the modern surveyor use Total Station, even with piezoelectric engine.

$$X_P = X_s + di \cdot sen\,\varphi \cdot sen(SP)$$
$$Y_P = Y_S + di \cdot sen\,\varphi \cdot cos(SP)$$
$$Z_P = Z_S + hs - hp + di \cdot cos\,\varphi$$

CIVIL ENGINEERING: Monitoring dam or other elements (historical part of cities, bridges, ...)

→ TOTAL STATION

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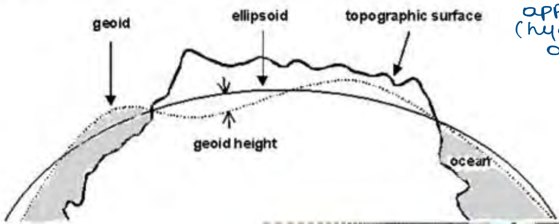
Topography

Topography allows also to estimated the height differences between the ground points, with high accuracy.

different application (hydrology or others)



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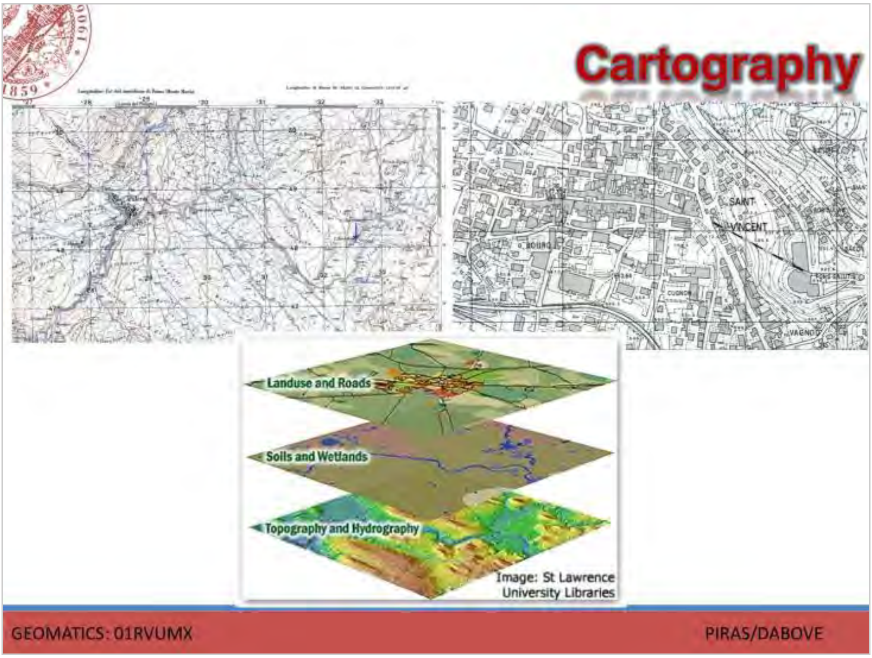
Cartography
Landuse and Roads
Soils and Wetlands
Topography and Hydrography
Image: St Lawrence University Libraries
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Digital Mapping
Cartography
OpenStreetMap
Il Geoportale Nazionale
Geoportale Nazionale
https://www.openstreetmap.org/#map=5/51.500/-0.100
http://www.pcn.minambiente.it/GN/
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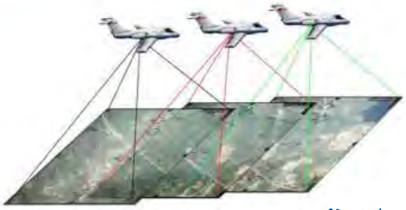
Photogrammetry

nowadays it's cheaper application

Drones or planes to collect images



Aerial → different for the distance and the dimension of the object ← Terrestrial

Photogrammetry is the science of obtaining reliable information about the properties of surfaces and objects without physical contact with the objects, and of measuring and interpreting this information.

↳ From image to 3D objects

(geometric, physical, semantic and temporal information)

http://www.isprs.org/

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↱ From image to 3D

Photogrammetric Workflow

IMAGE ACQUISITION

Canon EOS 5D
Ricoh GR

IMAGE ORIENTATION

C 'S

IMAGE MATCHING

feature extraction from images
Points Matching

TIN - DSM Generation

Constrained Triangulation
regularization

ORTOPHOTO Production

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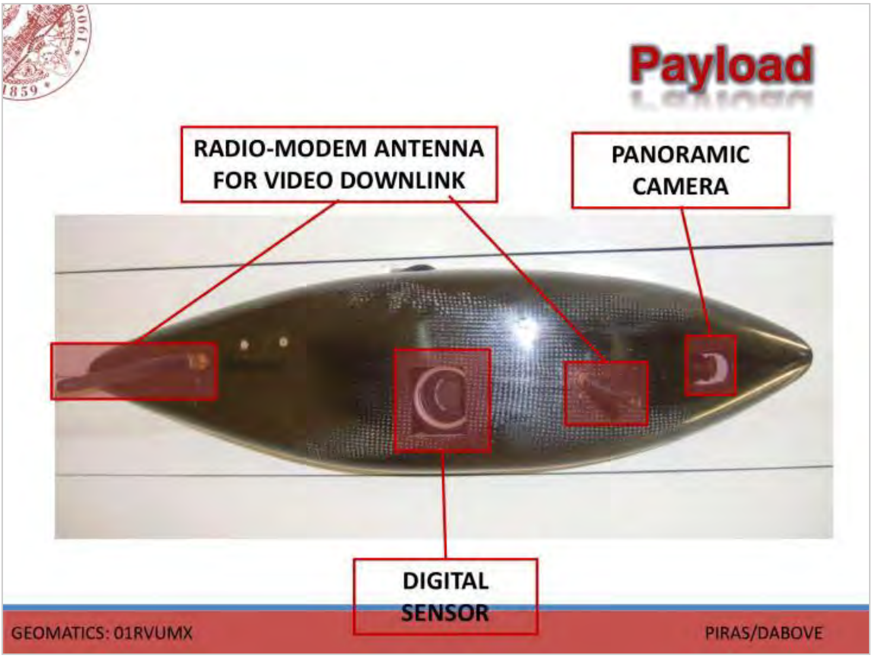
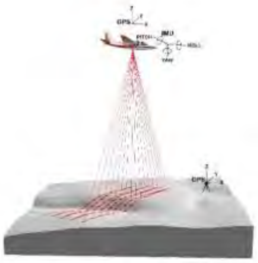
Aerial Platform
RADIO-MODEM
ANTENNA
POD
GROUND STATION
GEOMATICS: 01RVUMX
PIRAS/DABOVE


Payload
RADIO-MODEM ANTENNA
FOR VIDEO DOWNLINK
PANORAMIC
CAMERA
DIGITAL
SENSOR
GEOMATICS: 01RVUMX
PIRAS/DABOVE

LIDAR

LIDAR (Light Detection and Ranging o Laser Imaging Detection and Ranging)

Each measure is related to the center of the sensor (integrate informations)



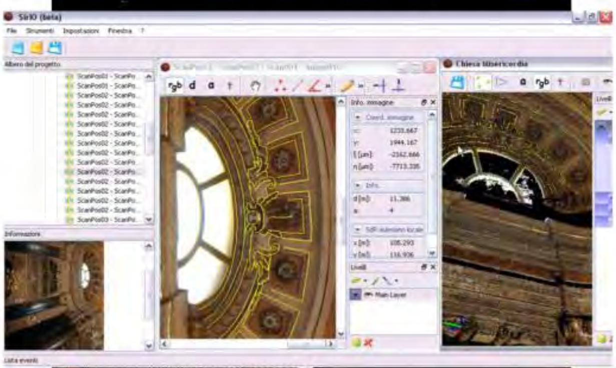
Aerial

Terrestrial

Photogrammetry – LIDAR Integration

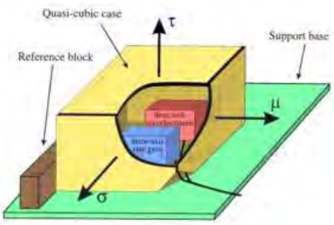
we can associate to each point a colour

Development of algorithms for LIDAR data processing and the integratrion between photogrammetry and LIDAR techniques



IMU (Inertial Measurement Unit)

An Inertial Measurement Unit (IMU) can determine position, speed and attitude of a body in motion by processing data measured by two orthogonal terns of sensors: one tern of accelerometers and one of gyroscopes.



A Distance Measurement Unit (DMI) allows to calculate the covered distance by measuring the turns of the car wheel.

IMU (Inertial Measurement Unit)

> 100k €	> 20k €	3k - 10k €	50 – 100 €
			

Low cost IMUs can be integrated with satellite sensors (GNSS) in order to reduce the long-term errors of the instrument.
The combined solution is composed of:

- ✓ Position, Speed and Attitude of the body in motion;
- ✓ High Frequency solution (over 100 Hz);
- ✓ Uninterrupted solution, also in presence of obstructions.

Geomatics and Civil engineering

In the main Civil engineering field (transport, geotechnics….) it is mandatory to have a CORRECT 3 model of the environment or object (dam, river, etc) which is considered as "state of the art". This model is adopted to design the new parts or planning some new engineering proposal.

SOME EXAMPLES…..

The origin of GNSS

4/10/1957: URSS has launched the first artificial satellite around the Earth. It was the Sputnik 1.

The satellite positioning is beginning: DoD (Department of Defence) USA considers the possibility to define the point positioning using the Doppler effect.

→ the old name of GPS

1958: TRANSIT program starts. It is a constellation based on 6 satellites,with height equal to 1100 km, which allows to define the positioning using the doppler effect using a unique frequency.

The accuracy was about 200-500 m. This have lead the DoD to move their attention on a new system, faster and more precise: **GPS system**.

GNSS segments

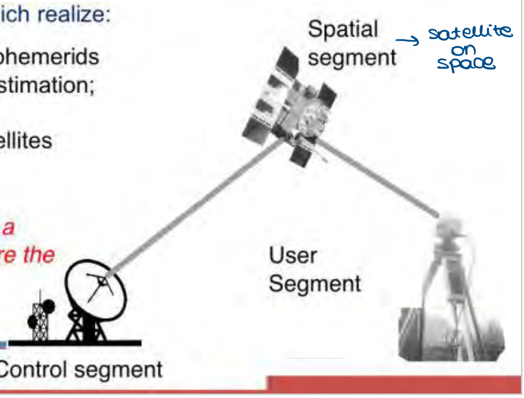
Spatial segment: MEO (*Medium Earth Orbit*) satellites, with an altitude between 19.100 km and 23.200 km

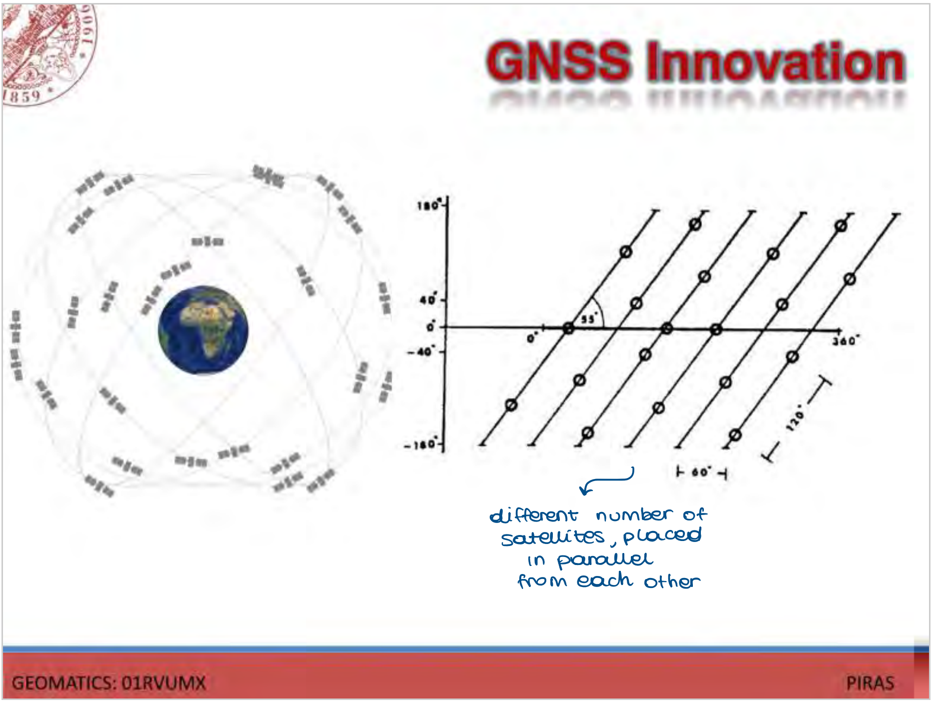
- Broadcasting (spreading data)
- maintenance of an accurated time reference scale
- receiving and storage the information about the control segment

Control segment: ground stations, which realize:

- Satellite tracking, to estimate the ephemerids
- Control of satellite clock and bias estimation;
- Orbit corrections
- Uploading new information into satellites

User segment: *each person who has a receiver and an antenna able to acquire the signal*




GNSS Innovation
180°
40°
0°
-40°
-180°
0°
55°
360°
120°
60°
different number of satellites, placed in parallel from each other
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PIRAS

GNSS Innovation

All the components

The signal is based on a fundamental frequency f0 =10.23 MHz
↳ it's generated by an atomic clock inside the satellite

IMPULSIVE COMPONENT: C/A and P code (pseudorange)

↱ for civil application (but used in any case)

- $f_{C/A} = f_0/10 = 1.023$ MHz ($\lambda = c/f_{C/A} \approx 300$ m) → wave length
- $f_P = f_0 = 10.23$ MHz ($\lambda = c/f_P \approx 30$ m)

⇒ the accuracy of our position it's linked to the wave lengths

SINUSOIDAL COMPONENT: L1 and L2 (carrier phase)

- $f_{L1} = 154\ f_0 = 1575.42$ MHz ($\lambda = c/f_{L1} \approx 19$ cm)
- $f_{L2} = 120\ f_0 = 1227.60$ MHz ($\lambda = c/f_{L2} \approx 24$ cm)

} very low wave length

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GNSS Innovation

-**Third frequency L5 (115 f_0): higher power and bandwidth. It is better to have less attack to noise**; this will allows to improve the estimation of ionospheric delay, with the combination L1/L5.
↳ we combine L5 with L1 or L2 to remove errors or to improve precision of the position

- **L5** with f (10.23 MHz) is ten times greater than C/A. Better correlation with a factor = 10

- **the new paseudoranges are based on** L2 and L5, they have a bigger chip and it will improve the quality of the correlation algorithm

- Better condition for fixing the ambiguity. Possible create new combination with L5:

L1/L5, with λ =75 cm, (ML),
L2/L5, λ = 5.87 m, (EWL).

⇒ not always it's possible to recieve all the signals: we have the position, but not the best result.

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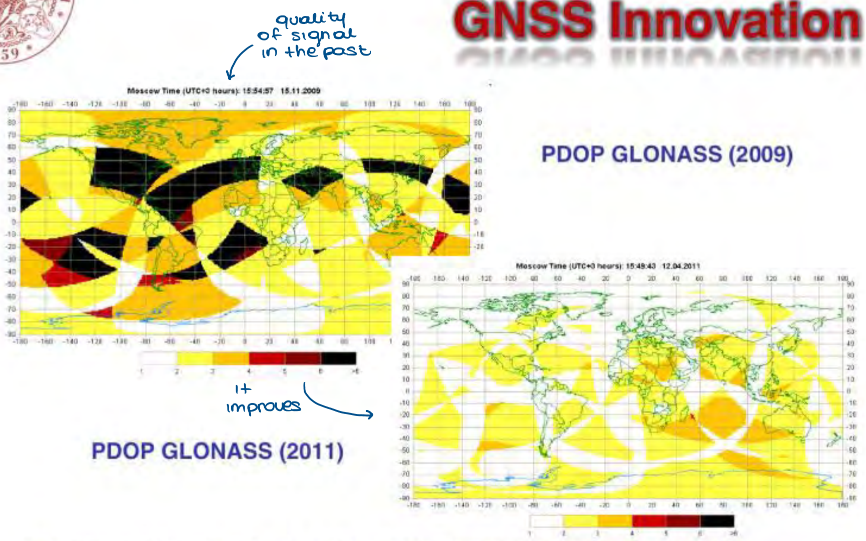
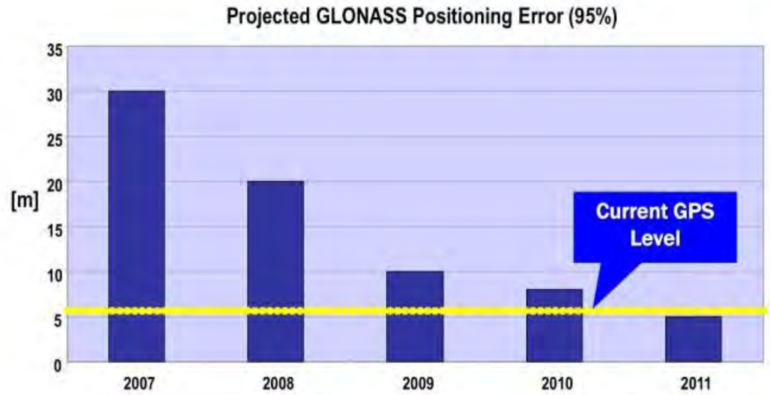
GNSS Innovation



http://www.glonass-ianc.rsa.ru/pls/htmldb/f?p=202:25:679198155068327

GEOMATICS: 01RVUMX PIRAS

GNSS Innovation



Improvement due to precision of ephemerids and time

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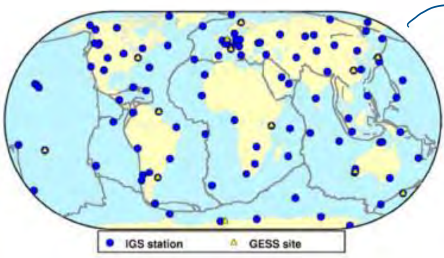
GNSS Innovation

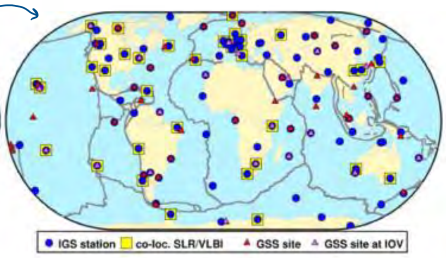
- we can use the same antenna for GPS and GALILEO (pro)
- if there are problems both sistems can't work (contro)

- GTRF is the GALILEO DATUM
- GGSP (Galileo Geodetic Service Provider) manages this RF
- 13 Galileo stations for GESS (*Galileo Experimental Sensor Station for the GIOVE mission*) and 131 IGS stations

number of stations increase in time



GTRF today



GTRF tomorrow…(maybe!)

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GNSS Innovation

Not possible to work only with it (increase number of satellites increase the accuracy) it helps GPS and GLONASS

- COMPASS/Beidou

BEidou 1 – GEO satellites (3 sats) (2000)
BEidou 2 (Compass) – Similar to GPS (35 sats) (2007)

2013: 5 GEO satellites, 6 MEO satellites and 7 IGSO (*Inclinated Geosynchronous Satellite Orbit*) satellites are available.

Interface Control Document (*ICD*) is now available on line

26 active satellites (35 full)

GEOMATICS: 01RVUMX PIRAS

GNSS Innovation

QZSS (*Quasi-Zenith Satellite System*)

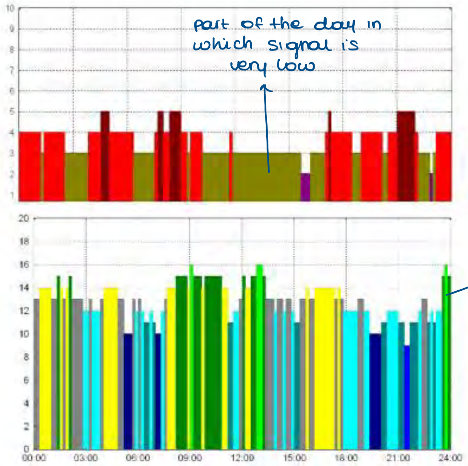
- → increase number of satellites in a specific area
- → that's way satellite has these orbits
- → it's again an helping constellation (not autonomos)

INRSS (*Indian Regional Navigation Satellite System*): 2006

http://www.mycoordinates.org/india-heads-for-regionalnavigation.november-2006.php

GEOMATICS: 01RVUMX PIRAS

GNSS: new and old scenario



GPS satellite visibility (1990)

10 times the before values → it's a system which works very well

GNSS satellite visibility (today) (GPS+GLONASS)

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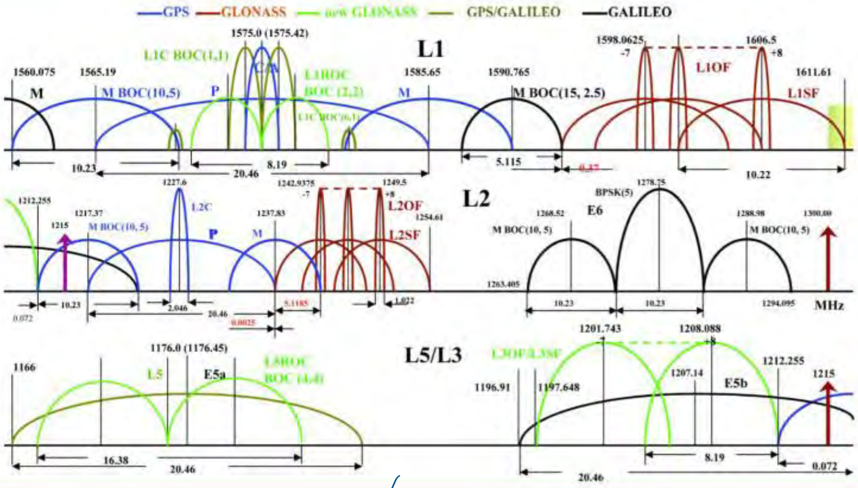
GNSS Innovation

- Different frequencies related to different satellites
- some problem to one constellation can lead to problems also in other constellations



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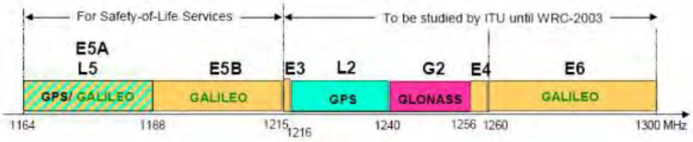
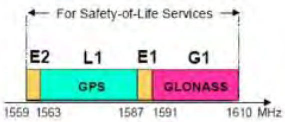
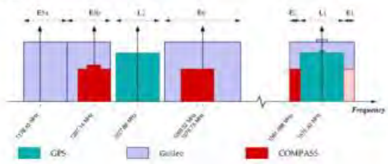
GNSS Innovation

may be we see satellites but we can't define the position







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9.03.18

venerdì 9 marzo 2018 09:08

POLITECNICO DI TORINO
DIATI

L3

GNSS positioning
Principles and strategies

MARCO PIRAS

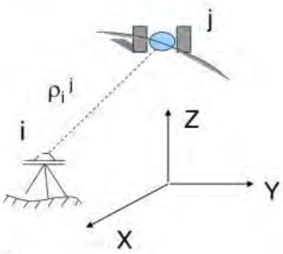
GEOMATICS: 01RVUMX

Stand alone positioning: an overview

↱ position starting from distance

The concept of point positioning is a trilateration on space with survey "range" of at least 4 satellites. We can write a range equation for each satellite j:

$$\rho_i^j(t) = \sqrt{(X^j(t) - X_i)^2 + (Y^j(t) - Y_i)^2 + (Z^j(t) - Z_i)^2}$$



$(XYZ)^j$: satellite coordinates (known by ephemeris)

$(XYZ)_i$: receiver coordinates (unknown)

in **WGS84** reference system. The relation with the geodetic coordinates (φ, λ, h) are:

$$X = (N+h)\cos\varphi\cos\lambda$$
$$Y = (N+h)\cos\varphi\,\mathrm{sen}\lambda$$
$$Z = [N(1-e2)+h]\,\mathrm{sen}\varphi$$

where

$$N = \frac{a}{W}$$
$$W = \left(1 - e^2\,\mathrm{sen}^2\varphi\right)^{\frac{1}{2}}$$

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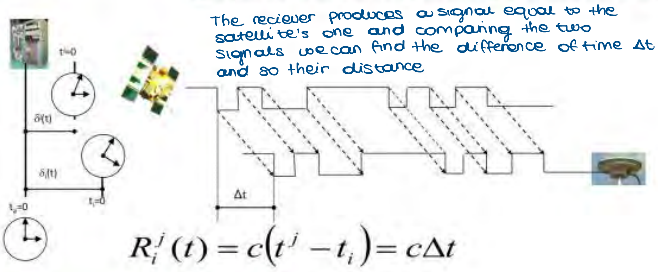
If we write RANGE EQUATION for each satellite the only unknowns are "my position" (x, y, z)

↳ we need to know how to compute position of satellites

⇒ 4 equation in 3 unknowns

Point positioning with code

The reciever produces a signal equal to the satellite's one and comparing the two signals we can find the difference of time Δt and so their distance



$$R_i^j(t) = c\left(t^j - t_i\right) = c\Delta t$$

but….considering clock offset: → offset of reciever is unknown

$R_i^j = c\,[\Delta t + \delta^j(t) - \delta_i(t)] = c\,[\Delta t + \delta_i^j(t)]$ → we always measure distance affected by errors

$\delta^j(t)$: satellite (known)

$c\,\Delta t$: it's the geometrical range + error

$\delta_i(t)$ = offset receiver clock (~10^{-3} s *$3\cdot10^8$ m/s= 300 km) UNKNOWN! → it's the 4 unknown in the system
↳ error in computation of distance

Point positioning with code

Pseudorange equation (rover-satellite) is:

$$R_i^j(t) = \rho_i^j(t) - c\left(\delta^j(t) + \delta_i(t)\right)$$

(error due to effects)

$$\rho_i^j = \sqrt{\left(X^j - X_i\right)^2 + \left(Y^j - Y_i\right)^2 + \left(Z^j - Z_i\right)^2}$$

Considering t^j as data streaming time from satellite to receiver, time needs to be corrected as :

$$t^j = t^j - \delta^j(t) \quad \text{where} \quad \delta^j(t) = a_0 + a_1(t - t_{oc}) + a_2(t - t_{oc})^2 + \Delta t_r - T_{GD}$$

→ rotation of satellite with respect to rotation of the Earth

Δt_r, T_{GD} Relativistic delay and group delay (other errors linked to phisical aspects) ⇒ we'd like to minimize them

Dividing unknowns and knowns terms: $R_i^j(t) + c\delta^j(t) = \rho_i^j(t) - c\delta_i(t)$

We have to linearize this equation around an approx values : $X_i^{(0)} Y_i^{(0)} Z_i^{(0)}$ (approximate value of position to start the iteration)

$$X_i = X_i^{(0)} + x_i; \quad Y_i = Y_i^{(0)} + y_i; \quad Z_i = Z_i^{(0)} + z_i;$$

x_i, y_i, z_i = corrections (they become the unknowns)

Oc means offset clock

➔ Positioning only N estimation (Initialization).

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Point positioning with carrier phase

Model is similar to code case, but phase ambiguities are included:

one additional column because we have also N as unknown

$$\begin{bmatrix} \frac{X_i^{(0)}-X^{(1)}}{\rho_i^{1(0)}} & \frac{Y_i^{(0)}-Y^{(1)}}{\rho_i^{1(0)}} & \frac{Z_i^{(0)}-Z^{(1)}}{\rho_i^{1(0)}} & \lambda_i^{(1)} & 0 & 0 & 0 & 0 & -1 \\ \frac{X_i^{(0)}-X^{(2)}}{\rho_i^{2(0)}} & \frac{Y_i^{(0)}-Y^{(2)}}{\rho_i^{2(0)}} & \frac{Z_i^{(0)}-Z^{(2)}}{\rho_i^{2(0)}} & 0 & \lambda_i^{(2)} & 0 & 0 & 0 & -1 \\ \frac{X_i^{(0)}-X^{(3)}}{\rho_i^{3(0)}} & \frac{Y_i^{(0)}-Y^{(3)}}{\rho_i^{3(0)}} & \frac{Z_i^{(0)}-Z^{(3)}}{\rho_i^{3(0)}} & 0 & 0 & \lambda_i^{(3)} & 0 & 0 & -1 \\ \frac{X_i^{(0)}-X^{(4)}}{\rho_i^{4(0)}} & \frac{Y_i^{(0)}-Y^{(4)}}{\rho_i^{4(0)}} & \frac{Z_i^{(0)}-Z^{(4)}}{\rho_i^{4(0)}} & 0 & 0 & 0 & \lambda_i^{(4)} & 0 & -1 \\ \frac{X_i^{(0)}-X^{(5)}}{\rho_i^{5(0)}} & \frac{Y_i^{(0)}-Y^{(5)}}{\rho_i^{5(0)}} & \frac{Z_i^{(0)}-Z^{(5)}}{\rho_i^{5(0)}} & 0 & 0 & 0 & 0 & \lambda_i^{(5)} & -1 \end{bmatrix} \begin{pmatrix} x_i \\ y_i \\ z_i \\ N_i^{(1)} \\ N_i^{(2)} \\ N_i^{(3)} \\ N_i^{(4)} \\ N_i^{(5)} \\ c\delta_i \end{pmatrix} - \begin{pmatrix} \phi_i^{(1)}+c\delta^{(1)}-\rho_i^{1(0)} \\ \phi_i^{(2)}+c\delta^{(2)}-\rho_i^{2(0)} \\ \phi_i^{(3)}+c\delta^{(3)}-\rho_i^{3(0)} \\ \phi_i^{(4)}+c\delta^{(4)}-\rho_i^{4(0)} \\ \phi_i^{(5)}+c\delta^{(5)}-\rho_i^{5(0)} \end{pmatrix} = \begin{pmatrix} v^{(1)} \\ v^{(2)} \\ v^{(3)} \\ v^{(4)} \\ v^{(5)} \end{pmatrix}$$

ambiguity is related to each satellite (not to constellation)

It is necessary to collect more epochs to estimate the ambiguities. When N are estimates, they are removed by unknowns. → reciever for professional application (this method needs more computations)

The procudure devoted to estimate N is called INITIALIZATION.

(when we need to have a mm accuracy we need to use carrier phase)

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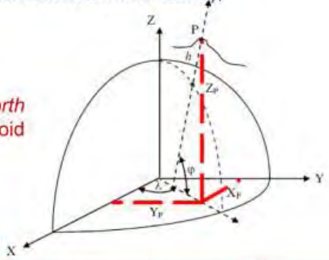
Point positioning with carrier phase

WGS84 (*World Geodetic System 1984*) is a geocentric system or ECEF (*Earth Centered Earth Fixed*) defined by:

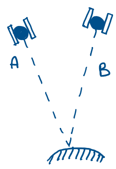
- **Origin in the mass centre** of the Earth;
- **Z** oriented along the Conventional Pole suggested by IERS (*International Earth Rotation Service*);
- **X** oriented along the intersection point between the equatorial plane and the «zero» meridian, defined by (*Bureau International de l'Heure*);
- **Y** completes the clockwise tern.

WGS84 was identical to DATUM NAD83 (*North American DATUM*, 1983) which uses the ellipsoid GRS80.



GEOMATICS: 01RVUMX

- range equation with satellite in the sistem A
- range equation with satellite in the sistem B



we need to know which are the parameters to these transformations

ERRORS
- time
- reference sistem

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Time reference scale

The GLONASS time (GLONASST) is kept by the clock of the Central Syncronizer Maser (CS) Time and is periodically corrected by an integer number of seconds (leap seconds) to align it with the UTC time. The time scale Soviet UTC (SU) = UTC + 3 hours (with a difference of 1 ms)

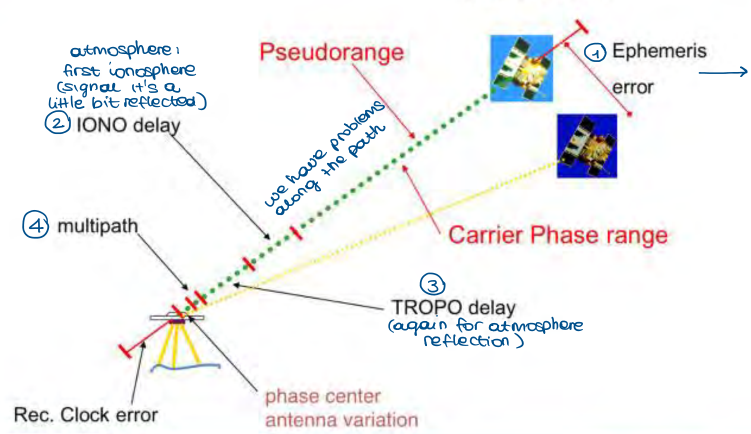
Galileo Time: GST (Galileo System Time): starts at 00:00 on 22 August 1999. The navigation message contains the coefficients used to calculate the offset between the GST and scale the UTC and GPS for interoperability between constellations.

time scale Beidou: BDT (Beidou navigation satellite system Time). The start time is 00:00:00 on January 1, 2006 UTC time. periodically checked to have an offset to UTC time less than 100 ns

ERRORS:

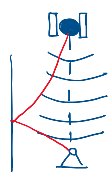
BIASES

they are called biases because they are sistematic



1 if the satellite's coordinates are wrong it leads to errors also in the position determined by reciever

4 the recieve can also recieve the waves reflected by several surfaces



$R = \rho + B$ → biases (R → range, ρ → pseudo-range)

Bias and correlations

Bias can be spatial or time correlated (spatial correlation is usually stronger) depending or not on signal frequency (different on L1 and L2).

Nature of bias	*Bias*	Spatial correlation
Not dispersive (frequency independent)	Ephemeris	High on big distance (>100 km)
	Troposphere	Regional (about 10 km)
Not dispersive (but frequency dep.)	Receiver clock	Identical on the same station
	Satellite clock	Identical on the same satellite
Dispersive (depending on f and they are different on L1 o L2)	Ionosphere	Regional (about 10 km)
	Multipath	No correlation. Site depending

Troposphere → the effect of this bias is similar if the distance is less than 10 km

Multipath → depends on the surfaces

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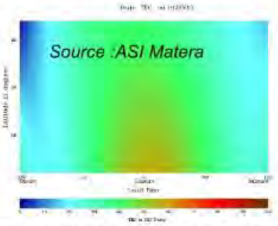
Dispersive BIAS: Ionospheric delay

(reflection linked to the activity of iones in the atmosphere)

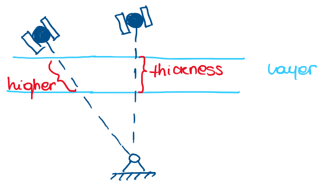
Ionospheric delay change with position, time and sun;
- it is correlable with lenght of baseline
- it change from few **mm/s** to **2 cm/s.**

⇒ Update correction: every 10s



activity is linked to
- time (in a day)
- position (more or less near to the Equator)

Ionospheric delay could be estimated with a model, for example **Klobuchar**:
- Only 50% of delay is removed!!
- It is based on 8 parameters: 4 *ION_ALPHA* and 4 *ION_BETA* ⟶ these parameters are inside the signal broad cast by the satellite
- They define the coefficients of a polynomial function (3° degree), which allows to estimate the zenith delay and model period (parameters depend on the position of satellite)
- *ION_ALPHA* and *ION_BETA* usually are in the Header of the navigation file (see later)



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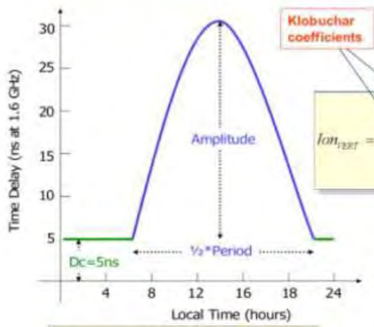
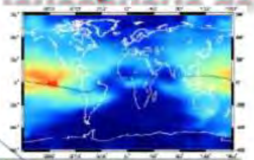
Dispersive BIAS: Ionospheric delay

Klobuchar model





$$Ion_{VERT} = \begin{cases} DC + A\cos\left[\frac{2\pi(t-\Phi)}{P}\right] & (day) \\ DC \quad ; \quad if \left|\frac{2\pi(t-\Phi)}{P}\right| > \frac{\pi}{2} & (night) \end{cases}$$

Bemg

$$A = \sum_{n=0}^{3} \alpha_n \varphi^n \quad ; \quad P = \sum_{n=0}^{3} \beta_n \varphi^n$$

φ = Geomagnetic Latitude

$$Ion_{SLANT} = Ion_{VERT} \cdot m(elev)$$

$$m(elev) = 1 + 16(0.53 - elev/\pi)^3$$

Where:
DC= 5ns
Φ= 14 (ctt. phase offset)
t = Local Time

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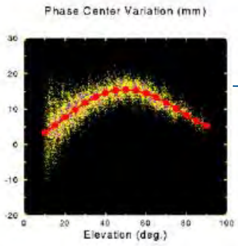
Dispersive BIAS: phase center variation (PCV)

Phase center variation depends on satellite elevation. It is usually considered with cylindrical symmetry.



→ variation of the center phase (where we collect the signal) with respect to elevation

CHOKE RING - L1

.0	1.4	2.4	3.2	3.8	4.2	4.4	4.6	4.7	**4.8**
4.7	4.5	4.2	3.7	3.0	2.0	.7	.0	.0	

Geodetic Antenna - L1

.0	4.6	8.9	12.6	15.8	18.3	20.0	20.9	**21.1**	20.6
19.5	18.1	16.3	14.5	13.0	12.0	11.8	.0	.0	

Calibration: it is made by producer or *NGS* (*www.ngs.noaa.com*) in comparison to a reference antenna. Sometimes it can be different, in particular if satellite has low elevation. => in the website there are the model for the most used antennas

Dispersive BIAS: phase center variation (PCV)

Wubbena et al, (2000) have proposed a calibration using an antenna movement, considering azimuth and elevation

```
TYPE= LEIAT503
NO OF FREQUENCIES=2
OFFSETS L1=-0.00055 0.00046 0.06083
OFFSETS L2=-0.00012 0.00019 0.08499
ELEVATION INCREMENT=5
AZIMUTH INCREMENT=5
VARIATIONS L1=
 -0.00799 -0.00513 -0.00260 -0.00026 0.00189 0.00374 0.00511 0.00588 .. ..
 -0.00790 -0.00502 -0.00247 -0.00012 0.00204 0.00388 0.00524 0.00598 .. ..
 -0.00782 -0.00492 -0.00236  0.00001 0.00217 0.00402 0.00536 0.00608 .. ..
 -0.00777 -0.00484 -0.00226  0.00011 0.00229 0.00413 0.00545 0.00615 .. ..
 -0.00774 -0.00477 -0.00218  0.00020 0.00238 0.00421 0.00553 0.00621 .. ..
 -0.00772 -0.00472 -0.00211  0.00027 0.00244 0.00427 0.00557 0.00624 .. ..
 -0.00772 -0.00467 -0.00206  0.00031 0.00247 0.00429 0.00558 0.00625 .. ..
 -0.00772 -0.00463 -0.00203  0.00033 0.00247 0.00427 0.00556 0.00623 .. ..
 -0.00774 -0.00461 -0.00202  0.00031 0.00243 0.00422 0.00552 0.00619 .. ..
 -0.00775 -0.00461 -0.00203  0.00027 0.00236 0.00415 0.00544 0.00613 .. ..
 -0.00777 -0.00463 -0.00208  0.00019 0.00227 0.00405 0.00535 0.00605 .. ..
........................
```

Not dispersive BIAS: Troposphere and ephemerids

Troposphere bias is related to atmosphere parameter so if we eliminate all the biases a part from this we can use itself to estimate meteleological aspects.

linked to atmosphere parameters

I can estimate a model (e.g. *Hopfield, Saastamoinen …*)

$$\delta R_{trop} = \frac{K}{\sqrt{\cos(z^2 + 6{,}25°)}} + \frac{L}{\sqrt{\cos(z^2 + 2{,}25°)}}$$

Dry component:

$$K = 155.28 * 10^{-7} \frac{P}{T_K}(40136 + 148.72T_K)$$

Wet component

$$L = (-0.028512T_K + 817.96)\frac{e_P}{T_K^2}$$

equations aren't important

$\delta R_{trop} = (R_0 - R)$ *tropospheric error*

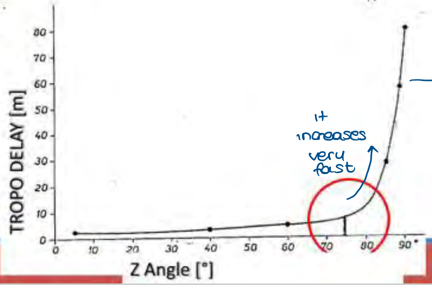
R = geometrical range

R_0 = *range measured*

P = pressure [millibar];

T_K = temperature [K]

e_P = partial vapour pressure [millibar]



we can stop to use the satellites (CUT OFF ANGLE or ELEVATION MASK) over the 70° angles (so the satellites with worse signals aren't used) → it's possible to set this command directly on the reciever or in the office (may be instead the number of satellites isn't enough when we cut of some of them)

z = 0° satellite is in the zenith position
z = 90° satellite is in the horizont

Not dispersive BIAS: Troposphere and ephemerids

available from different GNSS websites

different kind of ephemerids:	Orbita / Clock	Accuracy	latency	rate
Broadcast	*Orbit*	~ 100 cm	Real time	daily
	Clock sat.	RMS ~ 5 ns σ ~ 2.5 ns		
Ultrarapid (predicted)	*Orbit*	~ 5 cm	Real time	15 minutes
	Clock sat.	RMS ~ 35 ns σ ~ 1.5 ns		
Ultrarapid (observed)	*Orbit*	~ 3 cm	3 – 9 hours	15 minutes
	Clock sat.	RMS ~ 150 ps σ ~ 50 ps		
Rapid	*Orbit*	~ 2.5 cm	17 – 41 hours	15 minutes
	Clock sat.	RMS ~ 75 ps σ ~ 25 ps		5 minutes
Final	*Orbit*	~ 2.5 cm (very accurate)	12 – 18 days	15 minutes
	Clock sat.	RMS ~ 75 ps σ ~ 20 ps		30 seconds

very high quality — it's not possible to work in real time

this ephemerids can be used to precise engineering problems (we need to wait)

BIASES summary

- Not modelled error clock (receiver and satellite)
- ephemeris bias
- ionospheric delay
- tropospheric delay
- others (multipath, phase center antenna variation, receiver noise ...)

➔ **The presence of bias decreases position accuracy. For high positioning accuracy we can follow two ways:**

1) **Relative positioning** The aim is to reduce common bias by differencing position of two receivers. (combining results of different recievers)
↳ not real time application
Differential positioning: The aim is to reduce common bias by differencing position of two receivers.
↳ real time application
2) **Precise Point Positioning (PPP):** The aim is to model bias in a combined phase and code observations in a single receiver. (advantage is using only 1 reciever)

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Receivers, data acquisition techniques and format

GEOMATICS: 01RVUMX MARCO PIRAS

GNSS and other sensors

GNSS receiver is able to manage or to support other sensors, such as Inertial platform, Laser scanner, digital camera, atomic clock, geophisics tools, navigation tool, etc...)

- shooting trigger to a digital camera

In some receiver, there is:
- 1PPS: pulse to an external sensors
- EVENT MARKER: pulse which comes from an external sensors. It is possible to store the time of a «specific» event.

Strategy for data acquisition

It is possible to realize several kind of positioning:

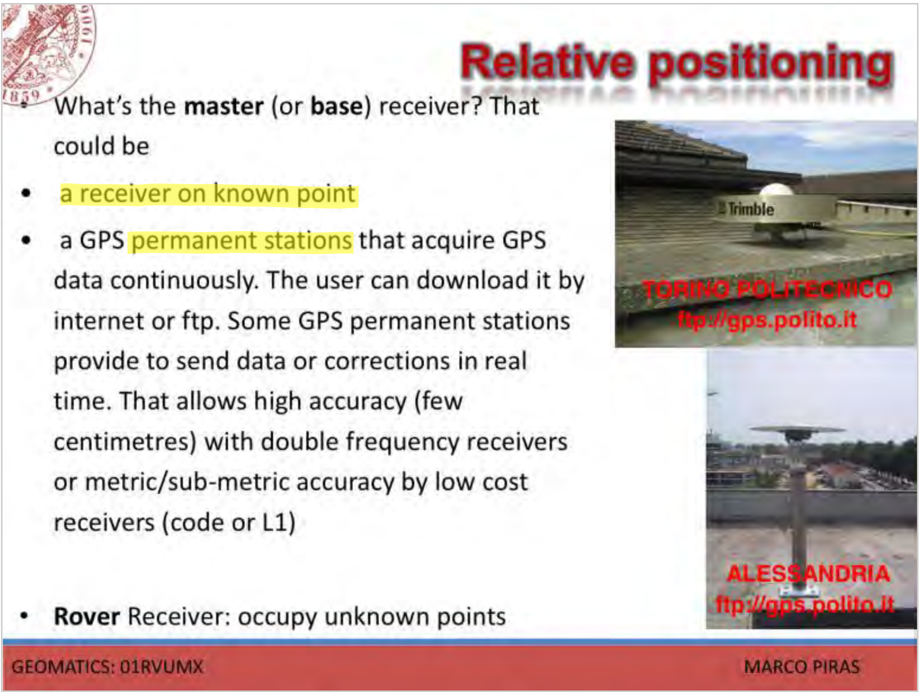
- Stand-alone: positioning with pseudorange only
Precision (95%): 5- 10 m without SA
It could be used for navigation but not for survey.

- DGPS: range observations are used with differential corrections, which are estimated by a master station. RTCM is used to streaming the corrections.

- Relative: baseline is estimated with a post-processing. There are many different strategies:

9.03.18

venerdì 9 marzo 2018 11:19



RINEX DATA (Read INdipendent Exchange format)

=> TO STORAGE DATAS (in different formats)

```
ssssdddf.yyt      ssss:   4-character station name designator
                   ddd:   day of the year of first record
                     f:   file sequence number within day
                          0: file contains all the existing
                             data of the current day
                    yy:   year
                     t:   file type:
                          O: Observation file
                          N: Navigation file
                          M: Meteorological data file
                          G: GLONASS Navigation file
                          H: Geostationary GPS payload nav mess file
                          B: Geostationary GPS payload broadcast data
                          C: Clock files (see separate documentation)
```

(file sequence number within day → if we have more files during one day)

(O, N, M, G → most common)

(H, B, C → other things)

ftp://igscb.jpl.nasa.gov/pub/data/format/

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RINEX DATA (Read INdipendent Exchange format)

* NAVIGATION FILE (*.YYN or *.NAV)

(.NAV → old format)

a_0, a_1, a_2 satellite clock errors

(1 → n° of satellite; 0 10 10 8 0 0.0 → time)

		a_0	a_1	a_2
1	0 10 10 8 0 0.0	.148013234138D-03	.170530256582D-11	.000000000000D+00
	.226000000000D+03	-.616250000000D+02	.466983737461D-08	.285302438067D+01
	-.329[illegible]88191414D-05	.491300632711D-02	.415742397308D-05	.515366060638D+04
	.201600000000D+06	.335276126862D-07	.156408352384D+01	-.242143869400D-07
	.961530[illegible]82141D+00	.299687500000D+03	-.174198327438D+01	-.821141346690D-08
	-.2471531[illegible]2064D-09	.100000000000D+01	.108300000000D+04	.000000000000D+00
	.200000000000D+01	.000000000000D+00	-.325962901115D-08	.482000000000D+03
	.194400000000D+06			

IODE (→ .226000000000D+03)

t_{oc} in GPS WEEK (→ .201600000000D+06)

The ephemerid are update each 4 hours about

when the orbit has been updated → when informations are updated, the number changes

GEOMATICS: 01RVUMX MARCO PIRAS

14.03.18

mercoledì 14 marzo 2018 08:30

POLITECNICO DI TORINO
DIATI

Relative and DGPS positioning

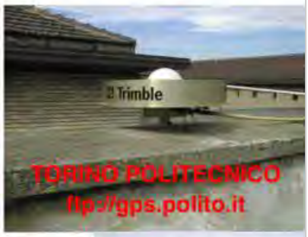
MARCO PIRAS

GEOMATICS: 01RVUMX

Relative positioning

What's the **master** (or **base**) receiver? That could be

- a receiver on known point
- a GPS permanent stations that acquire GPS data continuously. The user can download it by internet or ftp. Some GPS permanent stations provide to send data or corrections in real time. That allows high accuracy (few centimetres) with double frequency receivers or metric/sub-metric accuracy by low cost receivers (code or L1)



- **Rover** Receiver: occupy unknown points

GEOMATICS: 01RVUMX MARCO PIRAS

RINEX DATA
(Read INdipendent Exchange format)

↳ how to collect datas : owner format or RINEX format

```
ssssdddf.yyt        ssss:    4-character station name designator
                     ddd:    day of the year of first record
                       f:    file sequence number within day
                             0: file contains all the existing
                                data of the current day
                      yy:    year
                       t:    file type:
                             O: Observation file
                             N: Navigation file
                             M: Meteorological data file
                             G: GLONASS Navigation file
                             H: Geostationary GPS payload nav mess file
                             B: Geostationary GPS payload broadcast data
                             C: Clock files (see separate documentation)
```

ftp://igscb.jpl.nasa.gov/pub/data/format/

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RINEX DATA
(Read INdipendent Exchange format)

NAVIGATION FILE (*.YYN or *.NAV)

↳ GPS (we have parameters to estimate the position)

satellite atomic clocks (offsets)

```
                               a0                  a1                  a2
1  0 10 10  8  0  0.0  .148013234138D-03  .170530256582D-11  .000000000000D+00
    .226000000000D+03 -.616250000000D+02  .466983737461D-08  .285302438067D+01
   -.329688191414D-05  .491300632711D-02  .415742397308D-05  .515366060638D+04
    .201600000000D+06  .335276126862D-07  .156408352384D+01 -.242143869400D-07
    .961530482141D+00  .299687500000D+03 -.174198327438D+01 -.821141346690D-08
   -.247153152064D-09  .100000000000D+01  .108300000000D+04  .000000000000D+00
    .200000000000D+01  .000000000000D+00 -.325962901115D-08  .482000000000D+03
    .194400000000D+06
```

IODE

t_{oc} in GPS WEEK

The ephemerid are update each 4 hours about

GEOMATICS: 01RVUMX — MARCO PIRAS

RINEX DATA (Read INdipendent Exchange format)

- DATUM is provided in which the positions of the GLONASS satellites, through the ephemeris
- also called PE90 = Parameters of the Earth 1990
- as the WGS84 is a system ECEF
- with government decision of 20 June 2007, the system PZ90.02 is the new reference system GLONASS

reference system of GLONASS

PZ90 parameters

Ellipsoid		
Semi-major axis of the ellipse	a	$6\,378\,136.0\ m$
Flattening factor	f	$1/298.257839303$
Earth angular velocity	ω_E	$7\,292\,115.0 \cdot 10^{-11}\ rad/s$
Gravitational constant	μ	$3\,986\,004.4 \cdot 10^{8}\ m^3/s^2$
Speed of light in a vacuum	c	$2.99792458 \cdot 10^{8}\ m/s$
Second zonal harmonic coefficient	J_2^0	$1\,082\,625.75 \cdot 10^{-9}$

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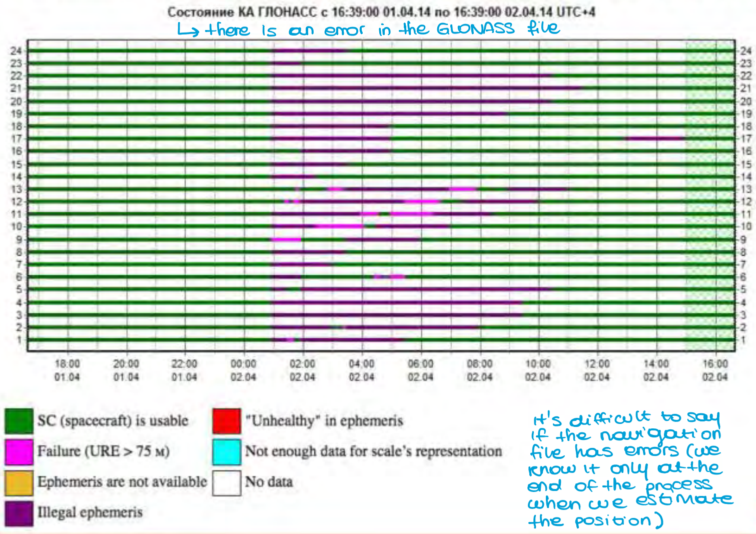
RINEX DATA (Read INdipendent Exchange format)



↳ there is an error in the GLONASS file

it's difficult to say if the navigation file has errors (we know it only at the end of the process when we estimate the position)

Ephemerids: SP3 format

XYZ coordinates and satellite clock error with rating = 900 s

it contains coordinates not the orbit (we need an interpolation)

```
•/* CENTER OF ORBIT DETERMINATION IN EUROPE (CODE)
•/* RAPID ORBIT PREDICTION (48 HOURS) FOR YEAR-DAY 00284
•/* THIS IS A PREDICTED ORBIT, USE WITH CARE.
•/* PREDICTED BROADCAST CLOCKS INCLUDED
•*  2000 10 10  0  0  0.00000000
•P  1    -442.181639 -26512.612214   -919.452911    147.954921
•P  2 -24953.146652  -9776.350787   -585.752971   -281.303666
•P  3  13475.970321 -11281.678068  19869.178689     47.399055
•P  4 -22691.533738   3553.815945 -13436.998794    483.903604
•P  .........Omissis.........
•P 29 -21074.725425  16022.321507   1287.893789    142.070527
•P 30 -15457.722946  -6208.426271  20515.285016    -41.428080
•P 31   8083.459492  13372.680515 -21701.701807     14.192701
•*  2000 10 10  8 15  0.00000000
•P  1  15487.291137   6543.545739  20727.275520    148.005569
•P  2  11431.870308 -13725.284069 -18920.302270   -281.462362
•...............................
```

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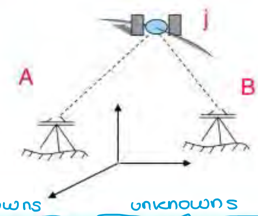
Single differences

RELATIVE POSITION

- Two points (A, B) and one satellite (j) are involved. We can write two raw equations, that have the unknown on the right site, and one single difference equation.
- Differencing it we obtain the final form of single differences.
- Note that satellite clock bias are deleted. For the same reason the tropospheric, ionospheric and ephemeris biases are reduced (eliminate for short baseline, about 15-20 km)
- The unknown are now the rover receiver coordinates, the combined clock offset and combined phase ambiguities. *for each epoch is always the same not a f(t)*



Rover — Master reciever now known position

Goal: find the base line (the line between rover and master reciever)

knowns | unknowns

$$\phi_A^j(t) - f^j\delta^j(t) = \frac{1}{\lambda}\rho_A^j(t) + N_A^j - f^j\delta_A(t)$$

$$\phi_B^j(t) - f^j\delta^j(t) = \frac{1}{\lambda}\rho_B^j(t) + N_B^j - f^j\delta_B(t)$$

it's the general solution without all the bias

$$\phi_{AB}^j(t) = \frac{1}{\lambda}\rho_{AB}^j(t) + N_{AB}^j - f^j\delta_{AB}(t)$$

→ it is called SINGLE DIFFERENCE
↓
for small base lines we can estimate troposphere and ionosphere bias (they are eliminated and they can be considered similar for rover and Master)

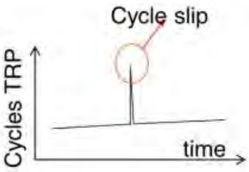
→ it's the difference between the two measurements
→ the satellite offset is completely removed

GEOMATICS: 01RVUMX — MARCO PIRAS

TD advantages:
1. First approximation position
2. Find cicle slips

Computational strategy

1) Approximate positioning with code pseudoranges (accuracy: few metres).
2) Single differences between 2 receivers and one satellite, for each satellite.
3) Triple differences solution (independent of ambiguity phase) ➔ $(\Delta X\ \Delta Y\ \Delta Z)_{TRP}$ (accuracy: few centimetres). We can also find cycle slips:
4) $(\Delta X\ \Delta Y\ \Delta Z)_{TRP}$ are known position; we can estimate (lest square) ambiguity phases *N* for each satellite by double differences. *N* is the floating number. We can estimate also a new solution: $(\Delta X\ \Delta Y\ \Delta Z)_{FLT}$, with more accuracy.
5) The ambiguity's nature is an integer number. We must to Fix it to integer: $N \in$ float ➔ $N \in$ Integer. If we are able to fix the value of N as integer we called it FIXED
6) Double difference solution with *N* FIX. The only unknown are the baseline's components. We can estimate $(\Delta X\ \Delta Y\ \Delta Z)_{FIX}$ as a final solution. components of our base line → find the rover position



$$N_{AB}^{jk} = \phi_{AB}^{jk}(t) - \frac{1}{\lambda}\rho_{AB}^{jk}(t)$$

$$\rho_{AB}^{jk}(t) = \lambda\left[\phi_{AB}^{jk}(t) - N_{AB}^{jk}\right]$$

GEOMATICS: 01RVUMX MARCO PIRAS

→ if we have problems in estimation of ambiguity these errors are reflected to the final position

Ambiguity estimation: ratio test

- The ratio is strongly indicator for correct ambiguity phase fixing.
- We can find ambiguities solution in to an research interval, when there are some values candidate.
- To fix phase ambiguity, need to resolve double difference solutions with many different integer combinations. We can analyze the "least square" reference variance $\sigma_0^{2\,(i)}$ of each "*i*" solution. We consider the first and the second smaller values.
- The test consist in ratio of the second on the first smaller values of reference variance:
  - $ratio = \sigma_0^{2\,(ii)} / \sigma_0^{2\,(i)}$ quality of the better solution we have found

1,5 in static conditions; dinamic / kinematic conditions

- Ratio should be greater than three, especially for short baseline (about 5 km). When the ratio is large number, the line is an accurate measurement.
- The high values of ratio, greater than 3, indicating that the chosen solution is al least 3 times better than the next most likely solution.

⇒ with a bad value of phase ambiguity may be we have a good precision, but the estimated value is far from the true one (accuracy) — LESS ACCURACY

- We start from a floating solution (several candidates and we need to find the correct one) ↳ possible integer values
- For each phase ambiguity we estimate the solution and the variance (= it is the quality as much as it is small, higher is the quality)
- We list the variances and we consider the ratio between the second and the first
- if tresholds are respected we have a high probability that phase ambiguity is correct and viceversa

GEOMATICS: 01RVUMX MARCO PIRAS

⇒ Benefits on relative positioning is higher with carrier phase than pseudo range (= less qualities)

Baseline solution: an example

```
• From Station    To Station       Solution      Slope          Ratio
• 3               903           Iono free fixed  23087.240      19.0

• Occupation Time      Meas. Interval (seconds): 03.54.00.00          15.00
• Solution Type:                 Iono free fixed double difference
• Solution Acceptability:        Passed ratio test
• Ephemeris:                     Broadcast
• Baseline Slope Distance):      23087.240                  0.000444
•
```

GNSS Permanent station

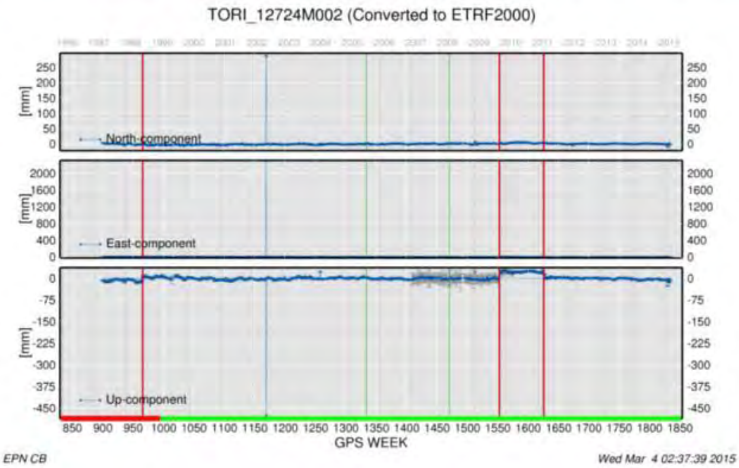
PERMANENT STATION

MAIN PROPERTIES:

- Dual frequency and GNSS
- good materialization and stability
- Connection with a topographic network
- daily raw data
- good atomic clock
- coordinates are periodically estimated
- differential corrections are availble

GEOMATICS: 01RVUMX MARCO PIRAS

TORI: POLITO permanent station



GEOMATICS: 01RVUMX MARCO PIRAS

RT without corrections (5 m)
- Stand alone

RT with corrections (1-2 m)
- DGPS (only with pseudoranges)
  ↳ PRC
- Carrier phase
  ↳ CPC

} single stations RTK

- NRTK
  - VRS } RTCM
  - MAC
  - FKP } FKP
  - NRT

NMEA (way to broadcast signal)

3) Interpolate bias of master stations with a plane

1) Not a real station
↳ reciever computes double differences baseline is small so errors are the same of the VS so we can fix the N
↳ even single frequencies recievers

2) for multi frequencies recievers

4) we use corrections of the master station which is nearer (NRT)
↳ single frequencies

POST PROCESSING
- differences
- single point positioning
- precise point positioning

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DGPS - pseudoranges

(Note: suppose base and master have the same biases)

- The base station code range A, at t_0 epoch, can be modelled by:

$$R_A^j(t_0) = \rho_A^j(t_0) + c\delta^j(t_0) - c\delta_A(t_0) + \\ +\Delta\rho_A^j(t_0)_{EPH} + \Delta\rho_A^j(t_0)_{IONO} + \Delta\rho_A^j(t_0)_{TROPO}$$

(Note: we need to consider the latency which is due to the passage of the signal between the base and the master)

- Pseudorange correction for satellite j at t_0 epoch is given by:

$$PRC^j(t_0) = -R_A^j(t_0) + \rho_A^j(t_0) = \\ -c\delta^j(t_0) + c\delta_A(t_0) - \Delta\rho_A^j(t_0)_{EPH} + \\ -\Delta\rho_A^j(t_0)_{IONO} - \Delta\rho_A^j(t_0)_{TROPO}$$

- To calculate, transmit and apply correction, require few seconds. This time is called "latency". The RRC can be evaluated by numeric differentiation

$$PRC^j(t) = PRC^j(t_0) + RRC(t - t_0) =$$

(Note: LATENCY — $(t - t_0)$; t: at which rover recieves the correction → so correction can be applied by the rover itself ↳ it increases the accuracy until 1.5/2 m (from 5/6 m we had before))

(Note: RRC variation of correction during the time (it's like a velocity); if master and rover are near RRC is similar to the one of the master)

$$R_B^j(t)_{correct} = R_B^j(t) + PRC(t) = \\ = \rho_B^j(t) + c\delta^j(t) - c\delta_B(t) - c\delta^j(t) + [\Delta\rho_B^j(t) - \Delta\rho_A^j(t)]_{EPH} \\ +c\delta_A(t) + [\Delta\rho_B^j(t) - \Delta\rho_A^j(t)]_{IONO} + [\Delta\rho_B^j(t) - \Delta\rho_A^j(t)]_{TROPO} \\ = \rho_B^j(t) - c\Delta\delta_{AB}(t)$$

(Note: ⇒ master and rover have to work with the same satellite (to use the same corrections) → so check the visibility of master and rover)

GEOMATICS: 01RVUMX — MARCO PIRAS

DGPS – carrier phase

(Note: we have a correction called CPC (we have another problem because we need a "small intelligent" inside which estimate phase ambiguity in the phase called INITIALISATION))

The base station carrier phase measure A, at t_0 epoch, can be modelled by:

$$\lambda\phi_A^j(t_0) = \rho_A^j(t_0) + c\delta^j(t_0) - c\delta_A(t_0) + \lambda N_A^j \\ +\Delta\rho_A^j(t_0)_{EPH} + \Delta\rho_A^j(t_0)_{IONO} + \Delta\rho_A^j(t_0)_{TROPO}$$

(Note: known; CPC)

Following the same procedure as before, the correct phase range at the receiver clock at epoch t are obtained by:

$$\lambda\phi_B^j(t)_{correct} = \rho_B^j(t) + c\delta_{AB}(t) + \lambda N_{AB}^j$$

DGPS is like single differences with zero latency. It's possible to difference one more time respect an other satellite K. The equation is:

$$\lambda\phi_B^k(t)_{correct} = \rho_B^k(t) + c\delta_{AB}(t) + \lambda N_{AB}^k$$

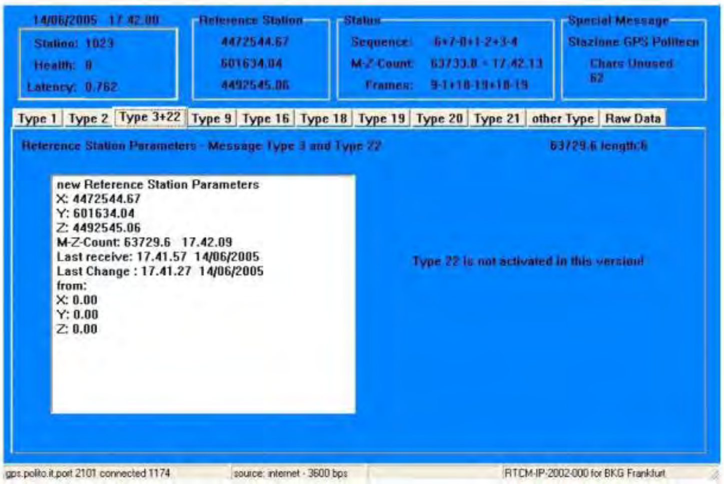
Finally, differencing the last two equations we obtain an equation without receivers clock bias. This approach is like the double differencing with zero latency. It's used in geodetic receivers for centimetres accuracy in real time.

$$\lambda\phi_B^{jk}(t) = \rho_B^{jk}(t) + \lambda N_{AB}^{jk}$$

This technique is called Real Time Kinematics RTK

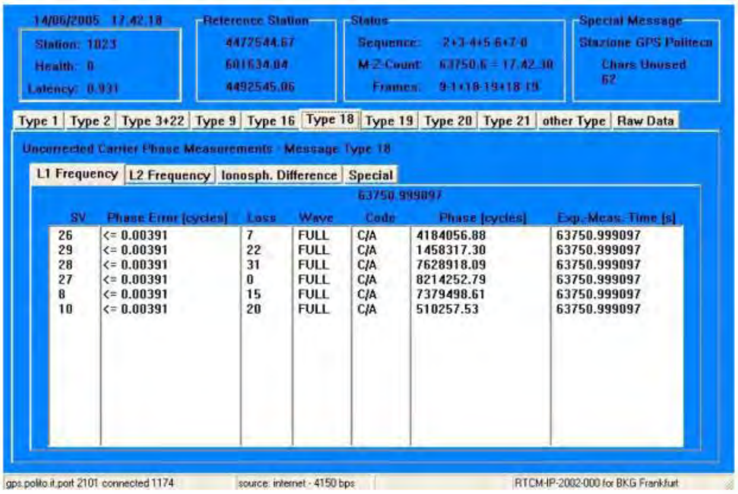
GEOMATICS: 01RVUMX — MARCO PIRAS

RTCM v2 – Type 3 - Station



GEOMATICS: 01RVUMX | MARCO PIRAS

RTCM v2 – Type 18 correction



GEOMATICS: 01RVUMX | MARCO PIRAS

RTCM v3 – Message group

Group	Subgroup	Type Message
Observations	GPS L1	1001
		1002
	GPS L1 / L2	1003
		1004
	GLONASS L1	1009
		1010
	GLONASS L1 / L2	1011
		1012
Station coordinates		1005
		1006
Antenna description		1007
		1008
Auxiliary information		1013

GEOMATICS: 01RVUMX MARCO PIRAS

RTCM v3 – service

Service	Group	Rover	Base station (Type)	
			min	full
Accuracy GPS L1	GPS obser.	1001-1004	1001	1002
	Antenna desc.	1005 & 1006	1005 or 1006	1005 or 1006
	Antenna desc.	1007 & 1008	1007 or 1008	1007 or 1008
	Auxiliary info			1013
Accuracy GPS RTK, L1 & L2	GPS obser.	1003-1004	1003	1004
	Antenna desc.	1005 & 1006	1005 or 1006	1005 or 1006
	Antenna desc.	1007 & 1008	1007 or 1008	1007 or 1008
	Auxiliary info			1013
GLONASS L1	IDEM	……..	……..	……..
GLONASS RTK	IDEM	……..	……..	……..
GPS+GLONASS L1	IDEM	……..	……..	……..
GPS+GLONASS RTK	IDEM	……..	……..	……..

GEOMATICS: 01RVUMX MARCO PIRAS

RTCM v3 - 1001 – 1002

- It containts all raw observations in single freq.

(DF)	Description
Satellite number	
GPS Code index on L1	0= C/A 1= P(Y)
Pseudoranges code GPS L1	resolution 2 cm
Carrier Phase GPS L1	resolution 0.5 mm
Time index L1	Time from last cycle slip. It becomes 0 every cycle slip.
Integer ambiguity on L1	
GPS L1 CNR	Carrier to Noise Ratio = noise in dB of satellite (0 = not evaluate)

GEOMATICS: 01RVUMX MARCO PIRAS

RTCM v3 - 1003 – 1004

(DF)	Description
Satellite number	
GPS Code index on L1	0= C/A 1= P(Y)
Pseudoranges code GPS L1	resolution 2 cm
Carrier Phase GPS L1	resolution 0.5 mm
Time index L1	Time from last cycle slip. It becomes 0 every cycle slip.
Integer ambiguity on L1	
GPS L1 CNR	Carrier to Noise Ratio = noise in dB of satellite (0 = not evaluate)
GPS Code index on L2	0=C/A o L2c, 1=P(Y) (no AS), 2=P(Y) with crosscorrelation, 3=owner method on L2
Pseudoranges code GPS L2	resolution 2 cm
Carrier Phase GPS L2	resolution 0.5 mm
Time index L2	Time from last cycle slip. It becomes 0 every cycle slip.
Integer ambiguity on L2	
GPS L2 CNR	Carrier to Noise Ratio = noise in dB of satellite (0 =

Datalink devices

we can share the results directly ↑ rover and master with the same frequency

1) Radio modem:

advantages: no costs

user's number: infinite

Disadvantages: short distance (few km) 2/3

communication: one way ↳ only master trasmittes to rover not viceversa

2) GSM modem:

advantages: long distance

(need GSM coverage) → it could be a big problem

communication: two way

Disadvantages: user's number: one each telephone line, costs

3) Internet IP: → the main technology used today

advantages: long distance (need GSM coverage)

communications: two way

International PORT : 2101 the IP we need to use to broadcast corrections

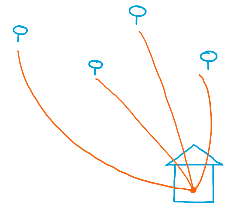
NTRIP PROTOCOL: one IP -port and different service

If we have permanent station we can distribute only one infor using the PORT 2101, instead the NTRIP protocol we are able to distribute different products with the same port.
↳ it is a dedicated protocol

Network architecture

A control centre receives data from all permanent stations and a model of corrections is made. This is valid only for bias spatial correlated.
Control centre is independent on single station.

several permanent stations distributed in the territory and a CONTROL STATION (not a GNSS reciever but it collects all the row data of the stations)



Communication:
1) Control centre and single permanent station
2) Control centre (and user)

it's better using several permanent stations in a region => cover an area (SPECIFIC AREA)

Network RTK implementation:
↳ Three main architectures

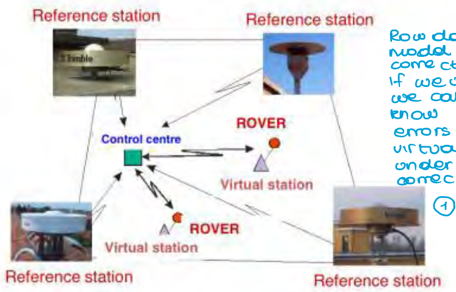
1) *Virtual Reference Station* (*VRS*);
2) *Multi Reference Station* (*MRS*) with corrections transmitting;
3) *Network RTK* with corrections transmitting by cell:
   Master Auxiliary (*MAX*).

GEOMATICS: 01RVUMX — MARCO PIRAS

Virtual reference station (VRS)

It uses the single row data to generate a global model

Differential corrections from network GPS are interpolated by control centre in a position close to rover. In this way a virtual station is made on the rover position.



Row data → global model → we need the correction in the exact point. If we use the pseudorange we can approximately know x,y,z and compute errors as this is a virtual station so to understand the corrections

(1) base lines are very precise

ROVER has a few meters a virtual station which realizes a baseline close to null. In this way, spatial correlated error can be eliminated.

(2) we use directly the RTK (traditional one even if the way to correct the position isn't based on one master stations but on a network)

GEOMATICS: 01RVUMX — MARCO PIRAS

(3) we can consider the influence of the bases in the region, not considering only one master station

Virtual reference station (VRS)

- Communication: 2 ways (internet or GSM) ➔ it needs approximate rover position
- Limited acces (computational and transmission load)
  ➔ n° user = n° different calculus of corrections
- If distance between rover and VRS pass a limit (i.e 3 km), a new VRS can be estimated. This is important to maintain a good network corrections quality → a useful way to avoid these limits is to use the same virtual station for differen users
- This method is applied with RTCM 2.x (*Type* messages18&19 or 20&21) then old receivers can use this kind of architecture. They must able to send their position by NMEA message.
  used to broadcast informations

=> the virtual station lasts only the time the user needs it (= after the user goes away from the network the VRS disappears)

=> we need to garantee a good quality of comunication

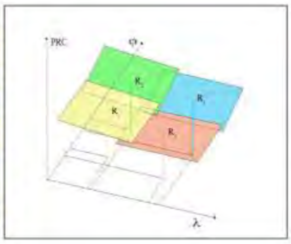
Several stations → row data → virtual rinex

Multi reference station (VRS)

It needs to distinguish between:

- Calculus of network solution and bias interpolation: this can be made by control centre with rigorous approach;
- Interpolative model transmission ➔ correction localized around a particular station. This is more easy, using the parameters belong to an interpolator plane.

FKP – Geo++ GNNET



FKP is equivalent to send the parameters of plane: each station has a particular FKP set.

"Transition area" are not important even though the planes are different. In fact they are very similar and corrections have extensive validity.

VRS and MRS

- Bias estimation model is evaluated using owner algorithm and it is send to *rover*. This not allows to use optimal interpolation algorithms or models
- RTCM 2.x is not optimal, overall to transmit network corrections
- VRS and FKP are not conform with RTCM philosophy, because they contain modeled data instead of raw data.
- FKP is not a RTCM standard, but nowadays it can be considered like that!
- VRS or (VRS generated by FKP) is a single base solution, with a short baseline. Control and advantages are limited, in respect to real time multibase solution

These are the reasons because RTCM v.3.0 has been created, which is particularly indicated for Master Auxiliary Stations (MAX) approach.

Master auxiliary: some considerations

Rover can use:

1) **predefined cell**, manually created, which send corrections divided in dispersive and not dispersive terms.

➔ Communication **1 way is enough. User** decides where take the corrections.

2) **AUTO MAX service**, where the cell is automatically created in respect to rover position.

➔ **2 ways communication**

Access problems are not present if internet communication is used, using also 2 ways communication.

This kind of approach needs that rover is able to **decode RTCM 3.0**

Individualized Master auxiliary (i-MAX)

This method is studied for receiver not able to decode RTCM 3.0:

- It is similar to **VRS approach**.
- It uses a real station to generate the network corrections.
- If rover position is known, master is defined by control centre that estimates the network corrections in respect to rover position. After that, control centre applies these corrections to master corrections, to send them to rover.

- 2 ways communication is necessary
- **RTCM 2.x is enough**

GEOMATICS: 01RVUMX MARCO PIRAS

BIAS INTERPOLATION - 1

Linear combination depended on α applied to residuals V of *pseudorange or carrier fase* or to double difference residuals. α are evaluated by LS

$$\hat{V}_u = \alpha\hat{V} = \alpha_1\hat{V}_{1n} + \alpha_2\hat{V}_{2n} + \dots + \alpha_{n-1}\hat{V}_{n-1,n}$$

Linear Combination Model (LCM): Linear combination depended on α . This method models the spatial errors. Main Hypothesis are:

$$\sum_{i=1}^{n-1}\alpha_i = 1$$

$$\sum_{i=1}^{n-1}\alpha_i\left(\hat{X}_u - \hat{X}_i\right) = 0$$

$$\sum_{i=1}^{n-1}\alpha_i^2 = \min$$

with u = *rover* and i = *master*

GEOMATICS: 01RVUMX MARCO PIRAS

BIAS INTERPOLATION - 2

Distance Interpolation Method (DIM): Weight average . $w_i = 1/d_i$ .It is used to estimate the iono effect:

$$\Delta\nabla I_u = \frac{\sum_{i=1}^{n-1} w_i \Delta\nabla I_i}{w}$$

Coeff. are: $\alpha = \left[\frac{w_1}{w}\ \frac{w_2}{w} \dots \frac{w_n}{w}\right]$

Linear Interpolation Method (LIM): It is used to evaluate iono residuals in DD.

$$V_{un} = a\Delta X_{un} + b\Delta Y_{un}$$

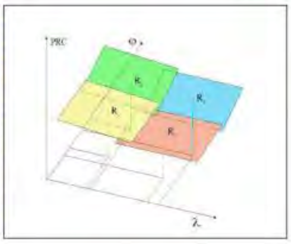
with Δ coordinate differences between master and rover

GEOMATICS: 01RVUMX MARCO PIRAS

→ problems related to linear interpolations

MRS: some considerations

FKP – Geo++ GNNET



Geometrical component:

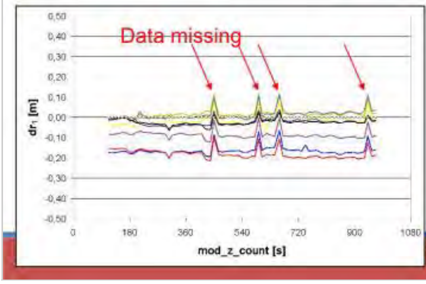
$$\delta r_0 = 6.37\left(N_0(\varphi-\varphi_R)+E_0(\lambda-\lambda_R)\cos\varphi_R\right)$$

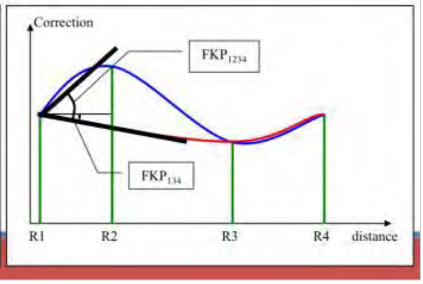
Dispersive component:

$$\delta r_I = 6.37H\left(N_I(\varphi-\varphi_R)+E_I(\lambda-\lambda_R)\cos\varphi_R\right)$$

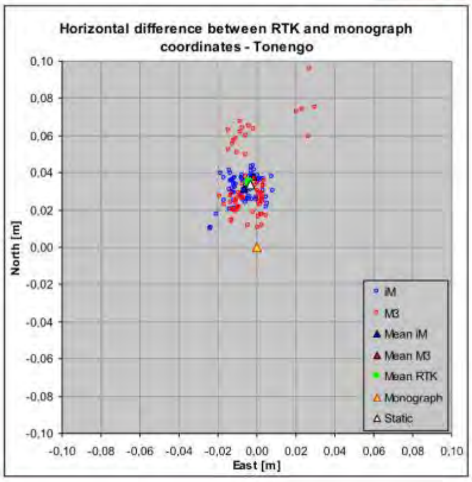
$$H = 1+16(0.53-\alpha/\pi)^3$$

$$\delta r_1 = \delta r_0 + (120/154)\delta r_I$$

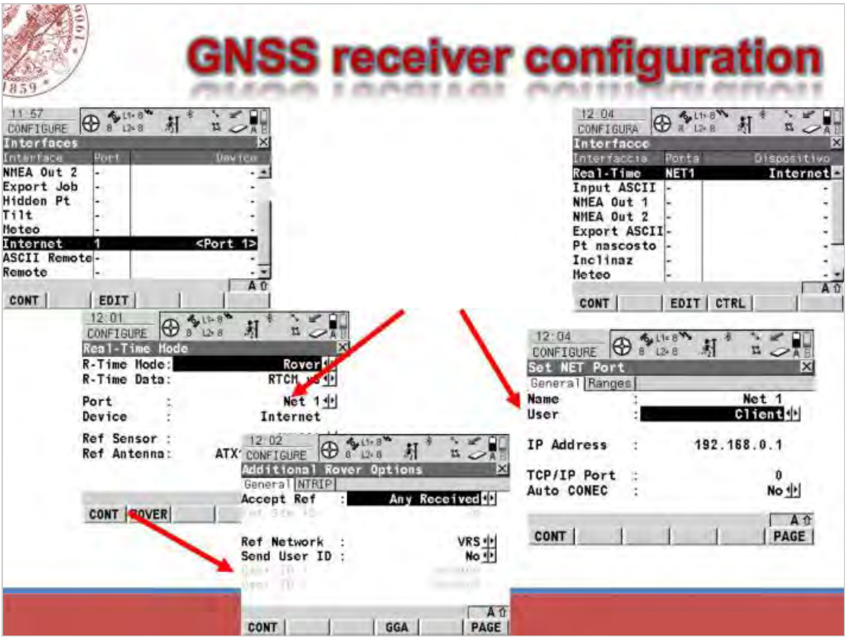


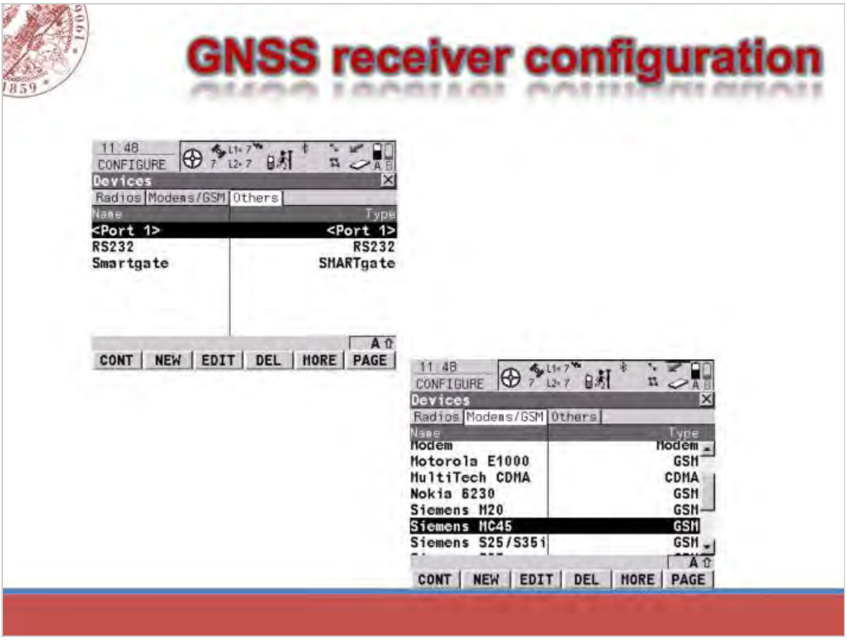


Performances



GEOMATICS: 01RVUMX MARCO PIRAS


GNSS receiver configuration
CONFIGURE
Interfaces
Interface Port Device
NMEA Out 2
Export Job
Hidden Pt
Tilt
Meteo
Internet 1 <Port 1>
ASCII Remote
Remote
CONT EDIT
CONFIGURA
Interfacce
Interfaccia Porta Dispositivo
Real-Time NET1 Internet
Input ASCII
NMEA Out 1
NMEA Out 2
Export ASCII
Pt nascosto
Inclinaz
Meteo
CONT EDIT CTRL
Real-Time Mode
R-Time Mode: Rover
R-Time Data: RTCM
Port : Net 1
Device : Internet
Ref Sensor :
Ref Antenna:
CONT ROVER
Additional Rover Options
General NTRIP
Accept Ref : Any Received
Ref Network : VRS
Send User ID : No
CONT GGA PAGE
Set NET Port
General Ranges
Name : Net 1
User : Client
IP Address : 192.168.0.1
TCP/IP Port : 0
Auto CONEC : No
CONT PAGE


GNSS receiver configuration
CONFIGURE
Devices
Radios Modems/GSM Others
Name Type
<Port 1> <Port 1>
RS232 RS232
Smartgate SMARTgate
CONT NEW EDIT DEL MORE PAGE
CONFIGURE
Devices
Radios Modems/GSM Others
Name Type
Modem Modem
Motorola E1000 GSM
MultiTech CDMA CDMA
Nokia 6230 GSM
Siemens M20 GSM
Siemens MC45 GSM
Siemens S25/S35i GSM
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venerdì 16 marzo 2018 10:22

POLITECNICO DI TORINO
DIATI

L6

Basic concepts of Reference systems in Geodesy

Paolo Dabove

GEOMATICS

The aim of positioning

To estimate positions of points by the available observations!

Needs
Constraints on the degrees of freedom intrinsic to this problem.

→ the materialization of the reference system
A reference frame is needed
Increasing complexity:
Case 1D, case 2D, case 3D.

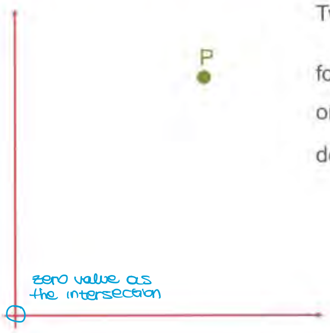
Case 1D

A different origin...

Case 1D

A different origin implies a different x'_P

Case 2D: the planimetric problem



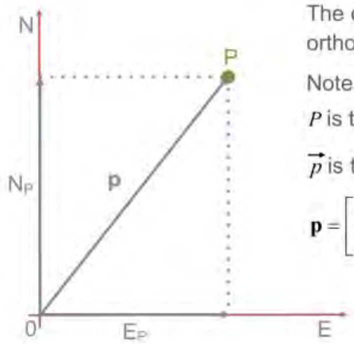
Two orthogonal axes,

for example $\vec{E}$ (East) e $\vec{N}$ (North),

one unit of lengths, → in general we could also use different ones, but not in geodesical problems

define the Reference System.

Case 2D: coordinates



The coordinates of a point are its orthogonal projections on the axes.

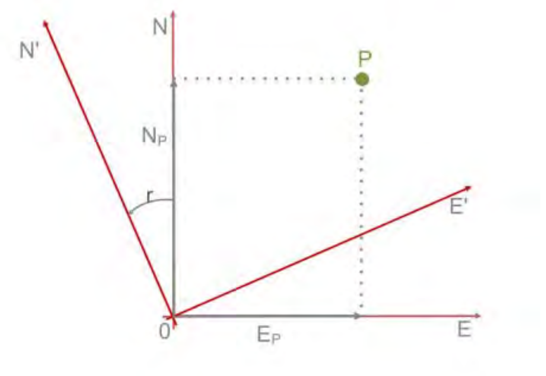
Note

P is the point, (expressed by a vector)

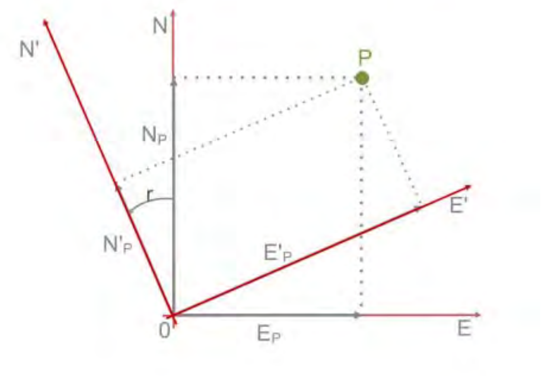
$\vec{p}$ is the vector between the origin and P,

$\mathbf{p} = \begin{bmatrix} E_P \\ N_P \end{bmatrix}$ is P position wrt the axes.

Case 2D: effect of a rotation of the axes



Case 2D: effect of a rotation of the axes



The lesson of the 2D case

Given the unit of lengths,

one origin and one direction are imposed.

3 parameters (two cohordinates and the angle)

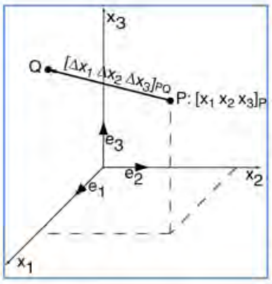
One alternative approach

One point is materialized, its position is assigned,

the azimuth angle to another point is assigned.

By properly combined observations

the positions of other points can be estimated.



3D Reference Systems

Three axes $\vec{x}_1, \vec{x}_2, \vec{x}_3$ with common origin,

reciprocally orthogonal, and the unit of lengths (unitary vectors $\vec{e}_1, \vec{e}_2, \vec{e}_3$).

The coordinates of a point are the lengths of its orthogonal projections on the three axes. The vector between two points is the oriented difference between their coordinates.

Given a point P, whose coordinates in I are $\mathbf{x}_{P_I} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{P,I}$

Its coordinates in II are given by the

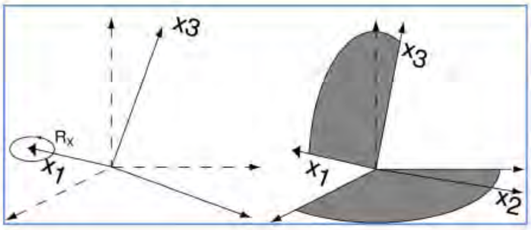
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{P,II} = \mathbf{x}_{P_{II}} = \mathbf{t} + \lambda \mathbf{R} \mathbf{x}_{P_I}$$

3D rotation: a matrix $[3 \times 3]$, with only 3 independent angles!

In geodetic framework, $\mathbf{R}$ is implemented by the composition of three planar rotations around the 3 axes.

$$\mathbf{R}_1(r_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos r_1 & \sin r_1 \\ 0 & -\sin r_1 & \cos r_1 \end{bmatrix}$$

these signs change in relation of the starting reference system



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The order of the three rotations is important:

we adopt the sequence

$\mathbf{R}_1 \Rightarrow \mathbf{R}_2 \Rightarrow \mathbf{R}_3$, i.e. $\mathbf{R}(r_1, r_2, r_3) = \mathbf{R}_3(r_3)\mathbf{R}_2(r_2)\mathbf{R}_1(r_1)$,

$\mathbf{R}$ depends on r_1, r_2, r_3:

the whole transformation depends on 7 parameters.

Helmert or similarity transformation.

The final rotation matrix

$$\mathbf{R} = \begin{bmatrix} c r_2\, c r_3 & c r_2\, s r_3 + s r_1\, s r_2\, c r_3 & s r_1\, s r_3 - c r_1\, s r_2\, c r_3 \\ -c r_2\, s r_3 & c r_1\, c r_3 - s r_1\, s r_2\, s r_3 & s r_1\, c r_3 + c r_1\, s r_2\, s r_3 \\ s r_2 & -s r_1\, c r_2 & c r_1\, c r_2 \end{bmatrix}$$

→ no need to remember it.

$cr_j = \cos(r_j)$, $sr_j = \sin(r_j)$

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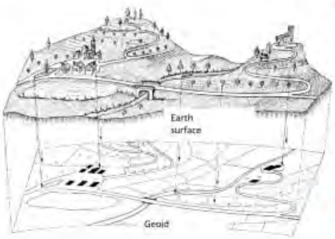
How is possible to represent the Earth surface?



The Earth Surface is very difficult to be described with analytical equation. It is very difficult (or practically impossible) to define angles, distances, areas, volumes, ect.

IT IS FUNDAMENTAL TO DEFINE A REFERENCE SURFACE!

to represent the 3D solid in 2D surface

First hypothesis is to project all "real" points with respect the vertical direction on a ideal surface. **This surface is called GEOID.**

We have to accept a more simple model and some initial conditions.

The Geoid

$\vec{g} = f(\rho, V)$ function of the density (continues to change) and of the volume

Geoid: Equipotential surface of gravity force, passing in a conventional point with known Geoid height.

$\vec{g} = g(XYZ)$ allows a potential = $dW = \vec{g} \bullet d\vec{P}$

$$\vec{g} = grad(W) \Rightarrow \left(g_X = \frac{\delta W}{\delta X}, g_Y = \frac{\delta W}{\delta Y}, g_Z = \frac{\delta W}{\delta Z} \right)$$

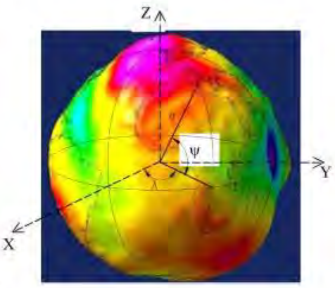
↳ not evaluate in each point so we pass from W to its components U and T

$W = \text{costante} \quad dW = 0 \quad \Rightarrow \quad g \bullet dP = 0$

P is orthogonal to gravity force g

$$V = \frac{GM}{r}\left(1 + \sum_{n=2}^{n_{max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \bar{P}_{nm}(\cos\phi)\left[\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right]\right)$$

(description with spherical harmonics)



split the problem into planimetric or altimetric problem (geoid)

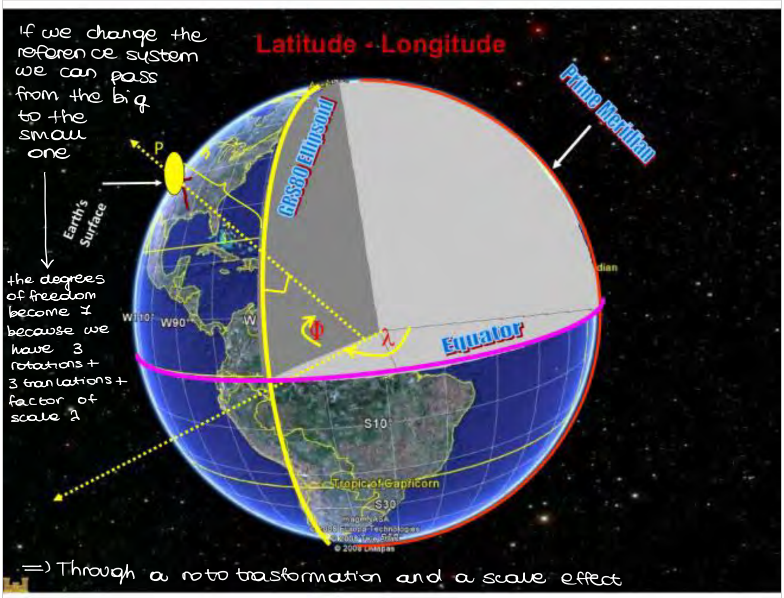
Geoid equation is not easily to be solved: a good approximation for horizontal solution is a ellipsoid of rotation!

⇒ we need to semplify this formula and move from g to W

$W = U + T$

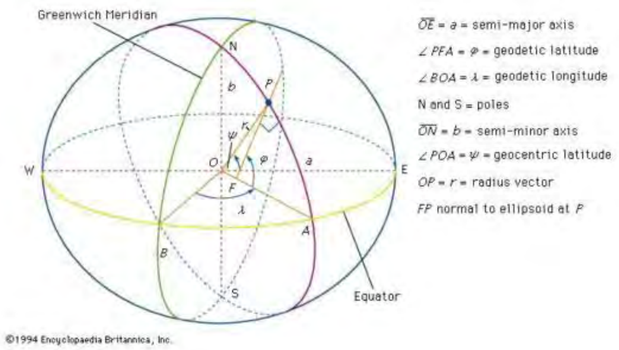
U : component of normal

T : anomaly trend (different density, mass attraction inside)



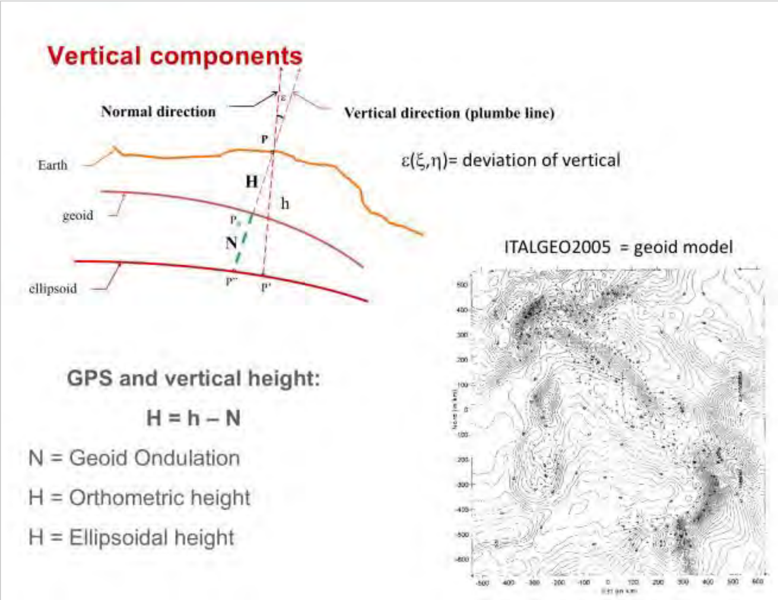
The ellipsoid

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -T_x \\ -T_y \\ -T_z \end{bmatrix} + \lambda R \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + \lambda R^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



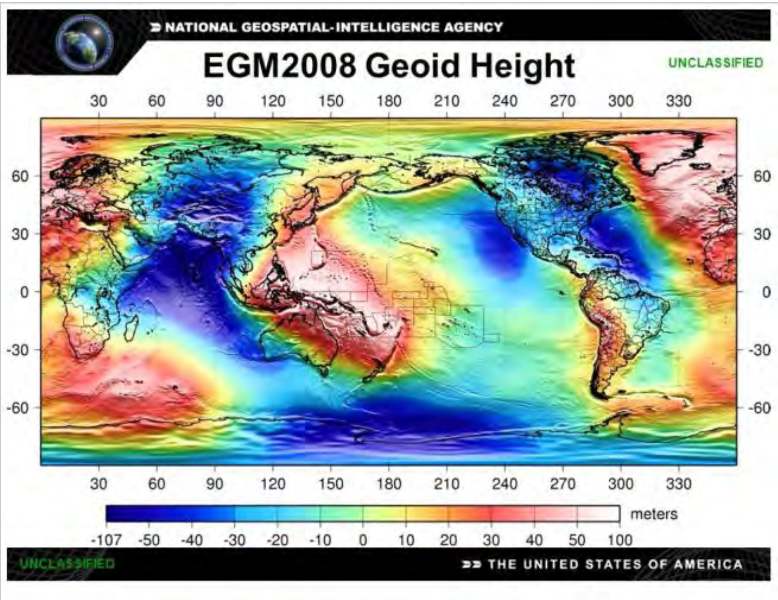
we can use the GNSS constellation because it is indipendent from the motion of the Earth

⇒ We need a frame (materialization of a reference sistem)



It gives us the possibility to move from the ellissoidal height that is measured by GNSS instruments to the orthometric height

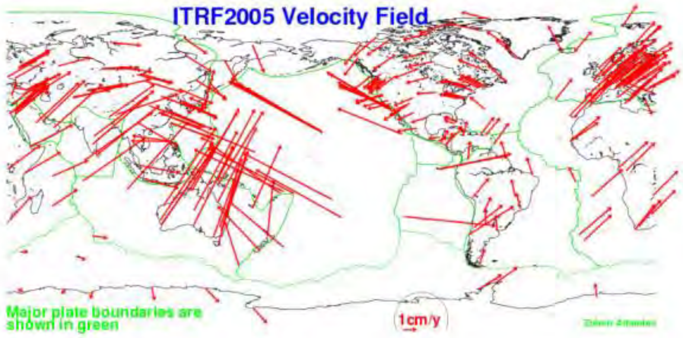
It's possible only if we know the Geoid ondulation. We have to estimate the distance between geoid and ellissoid; if we can't there are mathematical approches that interpolate measured values.
In this way we can find orthometric height.



There are cases in which the geoid is above the ellissoid: the mean value of N is between 27 and 53 m. So the geoid is under the ellipsoid in the ocean

REPRESENTATIONS OF THE EARTH

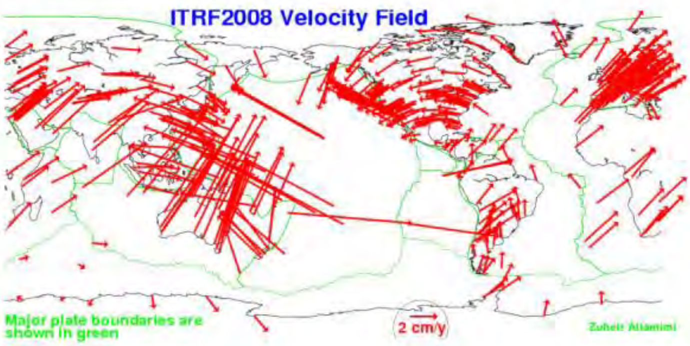
This has to take Tectonic Motions into account



When tectonic plates move, our benchmark locations change 30mm/yr equals 3 meters in 100 years

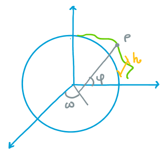
REPRESENTATIONS OF THE EARTH

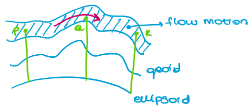
This has to take Tectonic Motions into account



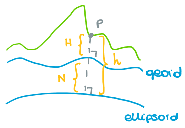
When tectonic plates move, our benchmark locations change 30mm/yr equals 3 meters in 100 years

GNSS
- static
  ↳ we need a post-processing for ONE materialization
- kinematic
  ↳ a set of constellations which broadcast directly the correction to a reference point
  {VRS, FKP, MAX o MAC}





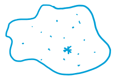
If we consider the ellipsoid it means that heigh of P is higher than Q which is less than R so it means that flow goes from P to Q and from R to P (it isn't true, it goes from P to R)



With a GNSS position we obtain h, but we would like to know H

$h = H + N$

We can compute N through gravimeters so create a grid with N values



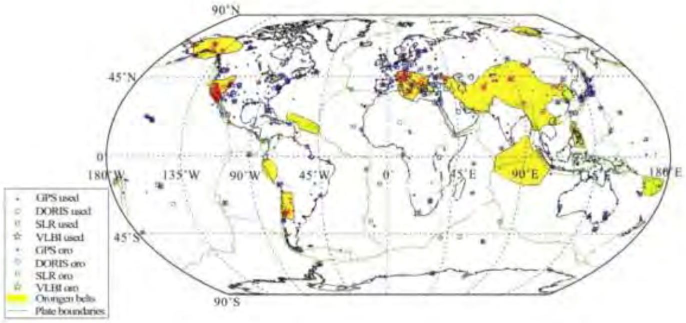
- in the kinematic approach we can't process anything so we need to pass from ellipsoide to the geoid earth
  we assume N = cost for small areas (< 200 m)
  Ivrea N changes 4 m for each 5 m²
  If we use the GARMIN we estimate a global geoid not local so we have less concentration of points

- we need a way to pass from angles φ and λ to meters

$$\begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix} \rightarrow \begin{bmatrix} E \\ N \\ H \end{bmatrix}$$ they depend on reference system

REPRESENTATIONS OF THE EARTH

This has to take Tectonic Motions into account

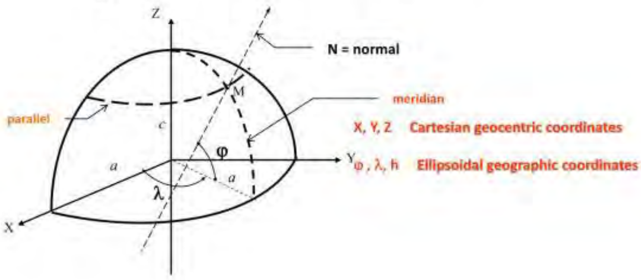


Reference systems and reference surfaces

For geodetic aims, the ECEF system is a based on a reference ellipsoid , which is usually **a** (semi-major axis) and **e^2** (First eccentricity).

$$e^2 = \frac{a^2 - c^2}{a^2}$$ First eccentricity

$$\alpha = \frac{a - c}{a}$$ flattening



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Coordinates transformation: geographic to ECEF

Starting from the coordinates of P in WGS84: φ=45° , λ=8° h=300 m, to estimate the coordinate in ECEF system. Considering also the inverse procedure.

If the direct equation is applied, considering WGS84 and N = 6388838,29 m, the coordinates are:

$$\begin{cases} X = (N+300)\cos 45° \cdot \cos 8° = 4473836{,}06\ 3\ \text{m} \\ Y = (N+300)\cos 45° \sin 8° = 628756{,}655\ \ \text{m} \\ Z = [N(1-e^2)+300] \cdot \sin 45° = 4487560{,}54\ 1\ \text{m} \end{cases}$$

Considering the inverse equation:

$$\lambda = \text{arctg}\ \frac{628756{,}655}{4473836{,}06\ 3} = 8°$$

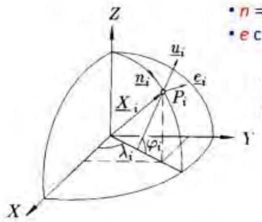
$$\text{r} = \sqrt{\text{X}^2 + \text{Y}^2} = 4517803{,}01\ 1$$

φ appr.	N	h	ϕ ris.
44,80758585	6388766,24	-20976,693	45,00064509
45,00064509	6388838,53	371,695	44,99999783
44,99999783	6388838,29	299,760	45,00000000
45.00000000	6388838,29	300,001	45,00000000
45.00000000	6388838,29	300,000	45,00000000

Coordinates transformation: ECEF to local plane (e,n,u)

We define a reference system with origin in Pi and the following axes:

- *u* = normal direction in *Pi*
- *n* = tangent of meridian in *Pi*
- *e* clockwise system



Given φ λ of Pi (origin) and X,Y,Z of the point P, to estimate e,n,u

1) Origin translation:

$$\begin{cases} X_i = N\cos\varphi_i \cos\lambda_i \\ Y_i = N\cos\varphi_i \sin\lambda_i \\ Z_i = N(1-e^2)\sin\varphi_i \end{cases} \qquad N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}}$$

2) Rotation of φ and λ:

local reference plane ⟶ not east north or up

$$\begin{pmatrix} e \\ n \\ u \end{pmatrix} = R(\varphi,\lambda)\begin{pmatrix} X-X_i \\ Y-Y_i \\ Z-Z_i \end{pmatrix} = \begin{bmatrix} -sen\lambda & \cos\lambda & 0 \\ -sen\varphi\cos\lambda & -sen\varphi sen\lambda & \cos\varphi \\ \cos\varphi\cos\lambda & \cos\varphi sen\lambda & sen\varphi \end{bmatrix}\begin{pmatrix} X-X_i \\ Y-Y_i \\ Z-Z_i \end{pmatrix}$$

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The ITR Frame

→ materializato through real points, stations

↱ to move from the system to the frame

Three spatial geodesy techniques:

VLBI: Very Long Baseline Interferometry

SLR: Satellite Laser Ranging

GNSS

The global GNSS network: International GNSS Service

A IAG service, established in 1993, with the following goals:

to contribute to the realization and distribution of ITRS,

to distribute GNSS products (ephemerides, EOP, …),

to define the standards for GNSS permanent networks,

to support GNSS research.

PS: about 350.

several Analysis centres, Working groups, Pilot projects, Services, a Central Bureau.

A ITRF consists of a catalogue of PSs contributing to the solution.

t_0 is the ITRFyy reference epoch. For each station,

its initial geocentric positions $\mathbf{x}(t_0)$ and related covariance $\mathbf{C}_{00}$

its velocity $\dot{\mathbf{x}}$ and related covariance $\mathbf{C}_{vv}$

ITRF2005 estimates for some italian GPS PS's

Nome	$X\,(m)$	$Y\,(m)$	$Z\,(m)$	$\dot X\,(m/y)$	$\dot Y\,(m/y)$	$\dot Z\,(m/y)$
Lampedusa	5073164.830	1134512.480	3683181.068	-.0147	.0168	.0153
Matera	4641949.648	1393045.330	4133287.386	-.0179	.0188	.0155
Cagliari	4893378.891	772649.688	4004182.102	-.0132	.0197	.0127
Bologna	4461400.834	919593.484	4449504.712	-.0182	.0190	.0110
Padova	4389531.226	923253.699	4519256.380	-.0172	.0177	.0113

Regional Networks: EPN (EUREF PN)

The main goal of EPN is the maintenance of ETRS.

For each ITRFyy, one ETRFyy.

About 245 PS's.

A hierarchy of analysis centers and governing boards similar to IGS.

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The parameters and their derivatives are given by EUREF

Table 5: Transformation parameters from ITRF2005 to ETRF2000 **at epoch 2000.0** and their rates/year

	T1 mm	T2 mm	T3 mm	D 10^{-9}	R1 mas	R2 mas	R3 mas
	54.1	50.2	-53.8	0.40	0.891	5.390	-8.712
Rates	-0.2	0.1	-1.8	0.08	0.081	0.490	-0.792

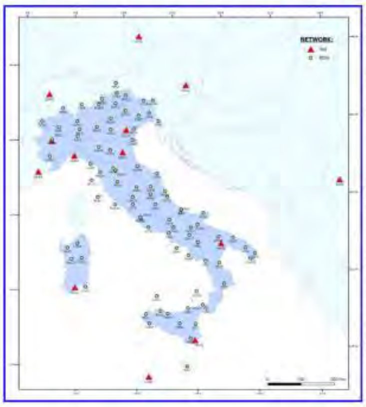
Table from official EUREF Memo (Boucher, Altamimi, 2008)

Then, the coordinates can be backward propagated to 1989.0

$$\mathbf{x}(1989.0)_{P,E00} = \mathbf{x}(t)_{P,E00} + (1989.0 - t)\dot{\mathbf{x}}_{P,E00}$$

$\dot{\mathbf{x}}$ is the velocity of the point in the European Reference Frame.

The Italian network: RDN (Dynamic National Network)



A set of PSs has been selected by Istituto Geografico Militare

to establish the new national zero order network:

RDN has been adjusted in ITRF2005 and

transformed in ETRF2000;

It materializes the European reference frame at the national scale.

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Geodetic coordinates in a 3D RF

In Earth positioning applications, a geometric reference surface that well approximates the Earth, wrt which a simple coordinates representation is possible, is required:

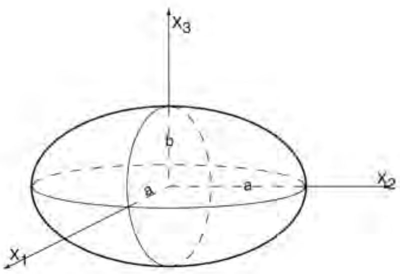
Rotational ellipsoid

The rotational ellipsoid is a geometric figure that satisfies the previous requirements.

A rotational ellipsoid in 3D is defined as the surface whose points satisfy the following condition

$$\frac{X^2+Y^2}{a^2}+\frac{Z^2}{b^2}=1$$

where
a major semiaxis;
b minor semiaxis.



Other a and b dependent parameters can be defined:

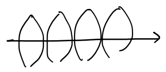
the eccentricity $e=\sqrt{1-\frac{b^2}{a^2}}$, the flattening $f=\frac{(a-b)}{a}$

are related by the $e^2=f(2-f)$

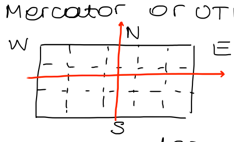
WGS84 → RS
→ ellipsoid

UTM32 → projection of ellipsoid in 2D
↳ with distorsions

Different projections
- Gauss Boaga



- Mercator or UTM



same angles

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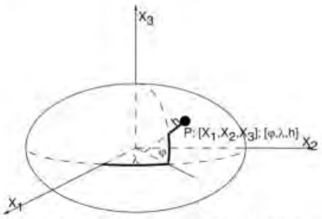
Geodetic coordinates of a point P



Given a RF and the related ellipsoid, the geodetic coordinates of a point P are defined as

φ: (geodetic latitude): angle between the normal to ellipsoid passing per P and the equatorial plane $[x_1,x_2]$;

λ: (geodetic longitude): angle between the meridian plane passing per P and the origin meridian plane $[x_1,x_3]$;

h: (geodetic height) distance between P and the ellipsoid surface.

COORDINATES

WGS84 → (x, y, z)
→ (φ, λ, h) ↳ ellipsoid height

UTM → cartographic (E, N, U)

h = H + N
in general, but it can be h

The relation between cartesian and geodetic coordinates is given by the

$$X_P = (N + h_P)\cos\varphi_P \cos\lambda_P$$
$$Y_P = (N + h_P)\cos\varphi_P \sin\lambda_P$$
$$Z_P = [N(1-e^2) + h_P]\sin\varphi_P$$

where $N = \dfrac{a}{\sqrt{(1 - e^2 \sin^2\varphi_P)}}$,

N is the East-West curvature radius (to be not confused with geoid undulation).

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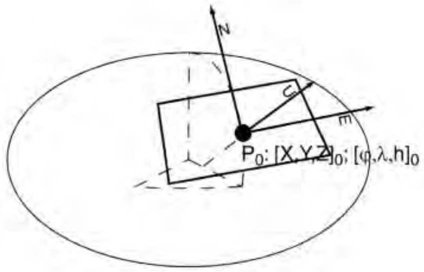
Local coordinates

Given the origin

$$\begin{bmatrix}\varphi_0\\ \lambda_0\\ h_0\end{bmatrix} \leftrightarrow \begin{bmatrix}X_0\\ Y_0\\ Z_0\end{bmatrix}$$



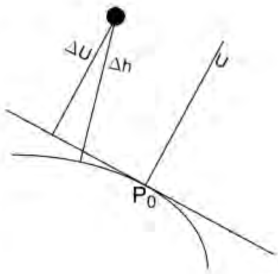
The other points have coordinates

$$\begin{bmatrix}E\\ N\\ U\end{bmatrix}_P = \begin{bmatrix}-\sin\lambda_0 & \cos\lambda_0 & 0\\ -\sin\varphi_0\cos\lambda_0 & -\sin\varphi_0\sin\lambda_0 & \cos\varphi_0\\ \cos\varphi_0\cos\lambda_0 & \cos\varphi_0\sin\lambda_0 & \sin\varphi_0\end{bmatrix}\begin{bmatrix}X_P - X_0\\ Y_P - Y_0\\ Z_P - Z_0\end{bmatrix}$$

Local coordinates provide a metrically well defined orthogonal coordinate system; Up component DOESN'T describe the height,

because it is the normal to the tangent plane in the origin:

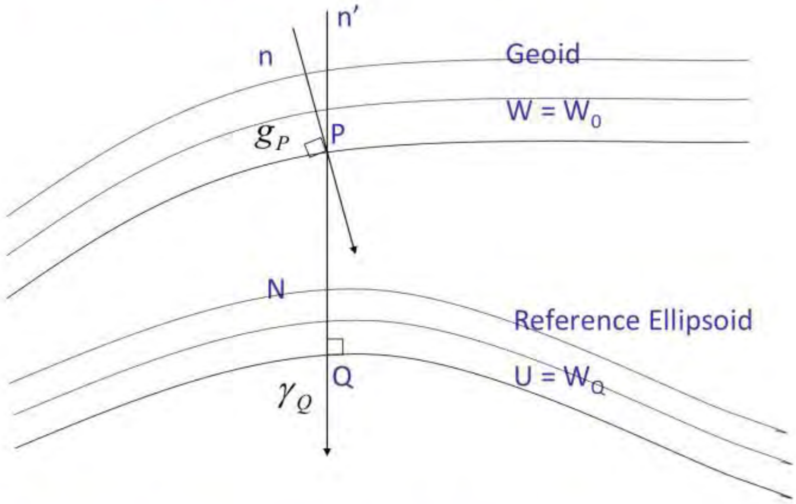
for points 500 m away difference between Up and height is of about 2 cm.



21.03.18

mercoledì 21 marzo 2018 09:36

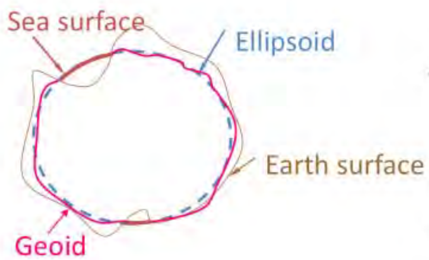
Definition of elevation

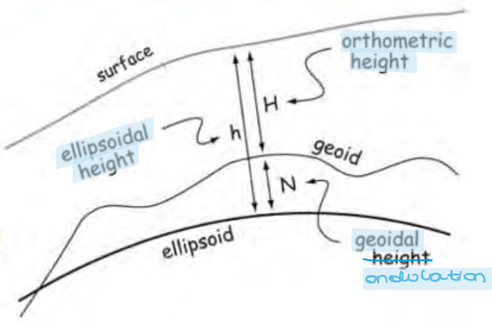


From Figure 2-12, p.83 of Heiskanen and Moritz, 1967, *Physical Geodesy*

Three systems for measuring elevation

What reference system (datum) is used?

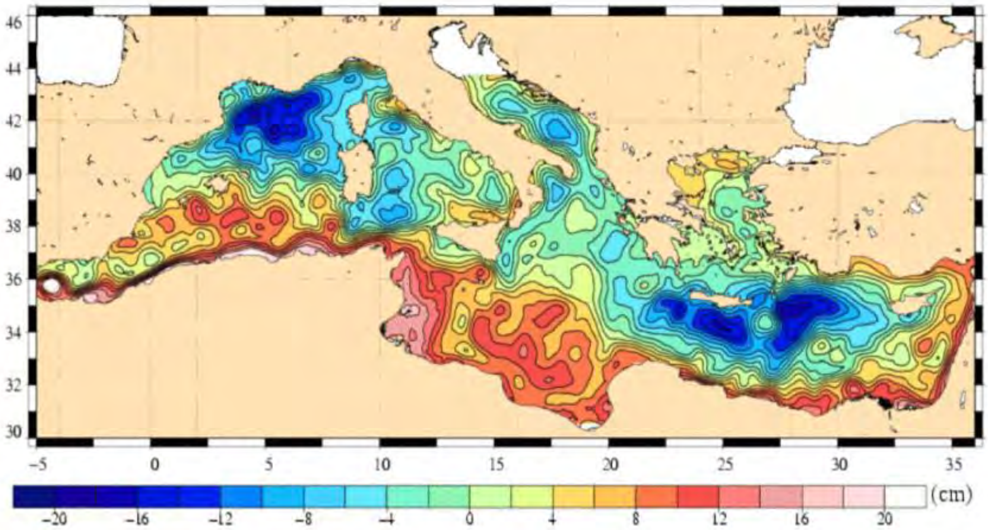




1. Orthometric heights (land surveys, geoid) through levels
2. Ellipsoidal heights (lidar, GPS) or GNSS systems
3. Tidal heights (Sea water level)
   ↳ To find the mean sea level we need the zero value (the zero is the sea level ~ it depends on the position)

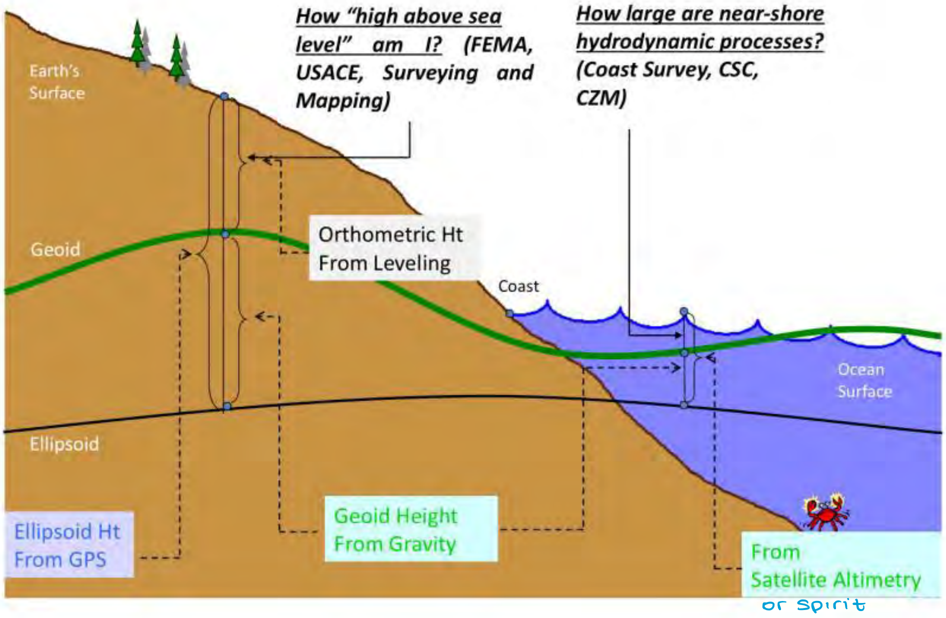
Geoid is mean sea level at a particular point and varies with gravitational potential

Sea Surface Topography on Mediterranean Sea



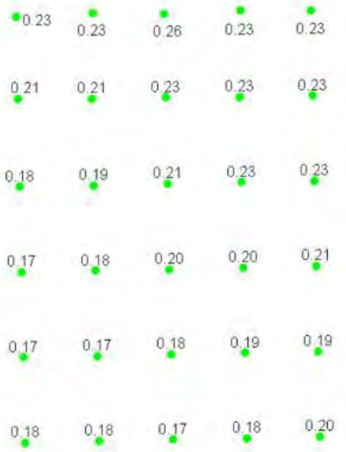
⟷ a lot of differences

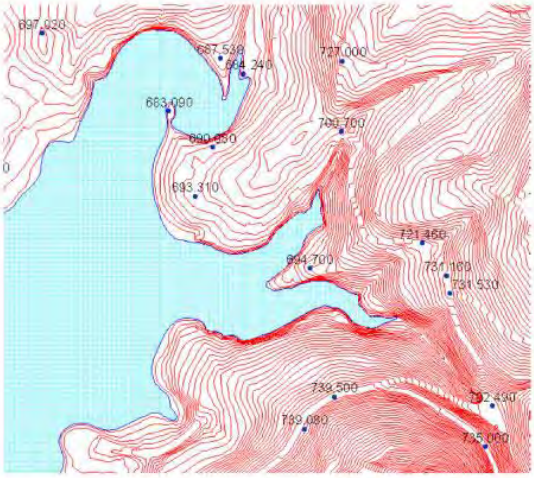
Gravity measurements help answer two big questions…



or spirit levelling

Converting vertical datums





Point file attributed with the elevation difference between Datum1 and Datum2

Datum1 terrain + adjustment = Datum2 terrain elevation

we have the sea level as f of g

=> Grid both for surface and for the sea

=> so TIDE GAGES (mareografi) can't be used → now we use GRAVIMETRY

Times systems

READ NOT STUDY ↕ all the time slides

Definition

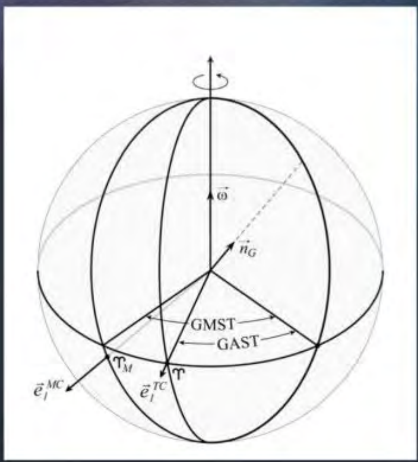
A time systems consists of (a) a time instant serving as the origin (zero time value)
(b) a time interval serving as the unit of time

Transformation between time systems: $\tilde{t} = at + b$

CLOCK	TIME SYSTEM
Rotating earth	Sidereal Time Universal Time
Solar system in motion	Dynamic Time
Atomic clocks (oscillators)	Atomic Time

Sidereal and Universal Time

Greenwich Apparent Sidereal Time (GAST / GST)



angle between

the Greenwich meridian

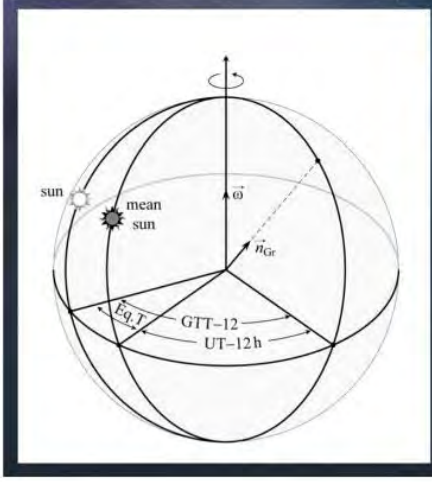
and the meridian of the true (instantaneous) vernal equinox ϒ.

Has been simplified to:

Greenwich Sidereal (GST)

Sidereal and Universal Time

Greenwich True Time (GTT)



angle between

the Greenwich meridian
and the meridian of the sun plus 12 h (in order to change date at midnight).
It is a "true solar time"

PROBLEM:
Variation of duration of true solar day due to varying apparent angular velocity of the rotation of the sun around the earth (true angular velocity of the earth in its orbit around the sun) – Kepler's 2nd law.

Sidereal and Universal Time

The UT day is larger than the sidereal day by
$3^m\ 55^s.909$ UT

one sidereal day lasts
$23^h\ 56^m\ 4^s.091 \pm 0.005$ UT

↳ earth rotation is not perfectly constant we need bisextil years

Difference δ between sidereal day and one revolution of the earth
δ = 0.0084 sec

$$\delta \approx \frac{1}{28500 \times 366.5} \text{ days}$$

Difference Δ between sidereal and (mean) solar day UT
Δ = 3 min 55.909 sec

$$\Delta \approx \frac{1}{366.25} \text{ days}$$

1 sidereal year = 366.5 days, precession period (1 cycle for ♈) = 28500 years

Sidereal and Universal Time

Instead of UT we refer to UT1 for historic reasons:

UT0: measured by astronomic observations
(referred to rotation axis - true terrestrial reference system)
UT1: value after UT0 was corrected for polar motion

Today GST (GAST) and UT (UT1) is connected to the earth rotation angle θ, determined by space geodetic techniques:

Relation of θ to UT1:

$$\theta = 2\pi(0.7790572732640 + 1.00273781191135448 T_u)$$

$$T_u = \text{JD} - 2451545.0$$

JD = Julian Date in UT

$$\frac{24 \cdot 60 \cdot 60}{24 \cdot 60 \cdot 60 - 235.909} = \frac{\text{sidereal day}}{\text{UT day}}$$

$$3^m\ 55^s.909 = 235^s.909$$

⟹ reference value for time is measured by ATOMIC CLOCKS

Dabove Pagina 11

Atomic Time

International Atomic Time (TAI): realized by a set of atomic clocks

Unit of time: SI second (International System of Units) defined as

the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the Cesium 133 atom

Origin of TAI: such that TAI = UT1 on 0^h UT1 January 1, 1958

1991: Terrestrial Time (TT) replaces TDT (TT≡TDT):

TT unit = TAI unit
TT origin: such that TT = TDT = TAI + $32^s.184$

2000: new definition of TT in relation to relativistic time coordinate
TT = TDT remains (instead of TT≡TDT)

Atomic Time

UTC

TAI = UT1 on 0^h UT1 January 1, 1958
Since then earth rotations decelerates → TAI is ahead of UT1

1972: Universal Time Coordinated (UTC) - a compromise between TAI & UT1
Unit of time = TAI second
Discontinuous time system with "leap seconds" (1 sec jumps)
to keep |UTC-UT1|< 0.9 sec

UTC = TAI – n · (1 sec)

without this correction we can't link atomic or local time

Date	TAI-UTC	Date	TAI-UTC	Date.	TAI-UTC	Date.	TAI-UTC
1-1-1972	10	1-1-1978	17	1-1-1988	24	1-6-1997	31
1-6-1972	11	1-1-1979	18	1-1-1990	25	1-1-1199	32
1-1-1973	12	1-1-1980	19	1-1-1191	26	1-1-2006	33
1-1-1974	13	1-6-1981	20	1-6-1992	27	1-1-2009	34
1-1-1975	14	1-6-1982	21	1-6-1993	28		
1-1-1976	15	1-6-1983	22	1-6-1994	29		
1-1-1977	16	1-6-1985	23	1-1-1196	30		

GPS Time

GPS Time: unit = TAI sec Different realization (set of atomic clocks)
Origin: GPS = UTC(USNO) at 0^h January 6, 1980
GPS Time = UTC + (n-19)s +C = TAI – 19s +C

=> LEAP SECOND : UTC time is realined with the universal one through this second

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venerdì 23 marzo 2018 08:40

POLITECNICO DI TORINO
DIATI

L8

Introduction to statistics and Least square method

Paolo Dabove

GEOMATICS

one sample is one estruction of all possible samples so in order to understand if it is reliable or not we need statistic parameters
→ mean
→ median
→ mode
→ st. dev.

Statistics

We use statistics for many reasons:

- To mathematically describe/depict our findings
- To draw conclusions from our results
- To test hypotheses
- To test for relationships among variables

Numerical representations of our data Can be:

- **Descriptive statistics** summarize data.
- **Inferential statistics** are tools that indicate how much confidence we can have when we generalize from a sample to a population.

Statistics: What's What?

Comparative objectives/ hypotheses

Descriptive objectives/ research questions:

Comparative objectives/ hypotheses

1) MEAN

$M = \sum_i^N \frac{\hat{x}_i}{n}$

x : is a representation of sample
E(x) is the expected value ($\bar{x}$)
$\hat{x}$ is one estimation

- not representative (if we have an error mean becomes not reliable!)

ES.
5, 6, -3, 4, 10, -8, -100 error

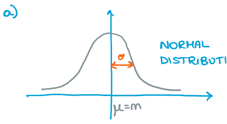
2) MEDIAN
- order from the lower to the upper
- find the value in the middle
  ↳ if numbers are even we take the mean of the two central numbers
- the gross error (if present) is automatically excluded

ES.
-8, 3, 4, 5, 6, 10
4,5

4) STANDARD DEVIATION
- we can consider the variance (s)
- its squared rooth (σ)
- or the difference with the values and the mean ($\hat{x}_i - \mu$)

If $N \to \infty$ we can represent the distribution of data

a)

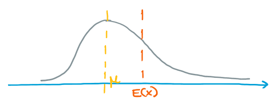


NORMAL DISTRIBUTION or GAUSSIAN DISTRIBUTION

mean = median = mode
mean = expected value

we can standardize and obtain $\mu = 0$ and $\sigma = 1$

b)



skewness: to understand if mean and expected value are different

σ: probability that one value can be obtained
↳ for flexural point the distance from the mean is equal to σ (standard deviation)

Useful because if we have a value very different from zero it means that there is an error.

If we know σ we can find also y

$y = f(\hat{x})$

$x = \hat{x}_i \pm \hat{\sigma}_i$

how spread is the distribution (positive or neg.)

Variance:
$\hat{\sigma}_i^2 = (\sigma_i)^2$

$\sigma_y^2 = ?$

$\sigma_y^2 = \left(\frac{\partial y}{\partial \hat{x}}\right)^2 \cdot \hat{\sigma}_{\hat{x}}^2$

↳ split the equation in two parts if eq. is linear or not

LINEAR

$\overline{AB} = a_1$
$\overline{BC} = b_1$

$\overline{AC} = \overline{AB} + \overline{BC} = a_1 + b_1$

$\hat{\sigma}_{AC}^2 = 1 \cdot \sigma_{AB}^2 + 1 \cdot \sigma_{BC}^2 = \sigma_{AB}^2 + \sigma_{BC}^2$

So MANDATORY is giving measurement + uncertanties (variance or standard deviation)

Median (Mid-point)

Example (6 scores)
3, 3, 7, 10, 12, 15

Even number of scores= Median is half-way between these scores
Sum the middle scores (7+10=17) and divide by 2
17/2= **8.5**

Please remember that: Insensitive to extremes

3, 3, 7, 10, 12, 15, 200

Mean: Arithmetic Average

Mean is half the sum of a set of values:
Scores: 5, 6, 7, 10, 12, 15
Sum: 55
Number of scores: 6
Computation of Mean: 55/6= **9.17**

Influenced by extremes
Only appropriate with interval or ration data

Is this four-point scale ordinal or interval?

1= Strongly Agree	3=Disagree
2=Agree	4=Strongly Disagree

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Mode: Frequency

Mode is the most frequently occurring value in a set.
Best used for nominal data.

Example:
Find the mode of the following set of scores.
14 11 15 9 11 15 11 7 13 12

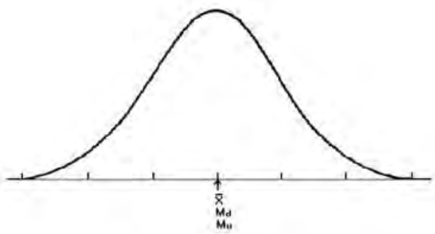
Solution:
The mode is 11 because 11 occurred more times than the other numbers.

If the observations are given in the form of a frequency table, the mode is the value that has the highest frequency.

Shapes of Distribution

Normal Curve (aka Bell Curve)
Repeated sampling of a population should result in a "normal" distribution- clustering of values around a central tendency.

<u>Remember</u>: In a symmetrical distribution, median, mode and mean all fall at the same point



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Skewness: positive

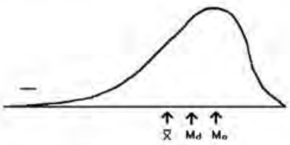
The mode is smaller than the median, which is smaller than the mean.

This relationship exists because the mode is the point on the x-axis corresponding to the highest point, that is the score with greatest value, or frequency.

The median is the point on the x-axis that cuts the distribution in half, such that 50% of the area falls on each side.

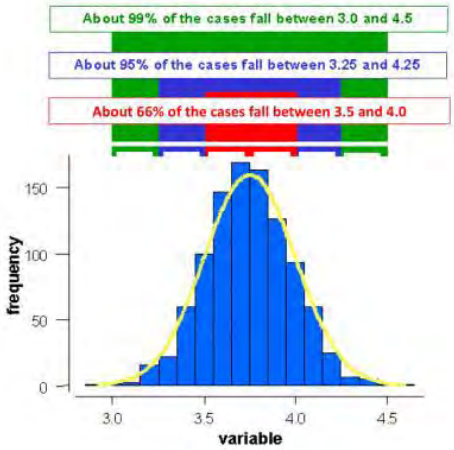
Skewness: Negative

An extremely easy test will result in a lot of high grades, and will skew to the left (negative)

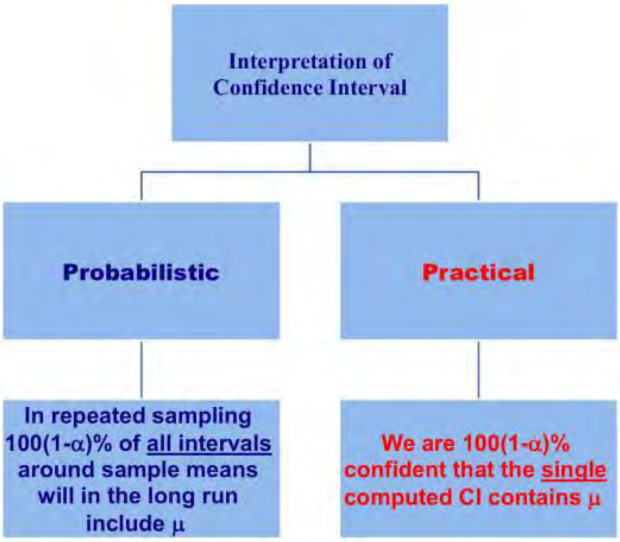


The order of the measures of central tendency would be the opposite of the positively skewed distribution, with the mean being smaller than the median, which is smaller than the mode.

Standard Deviation: 66, 95, 99%

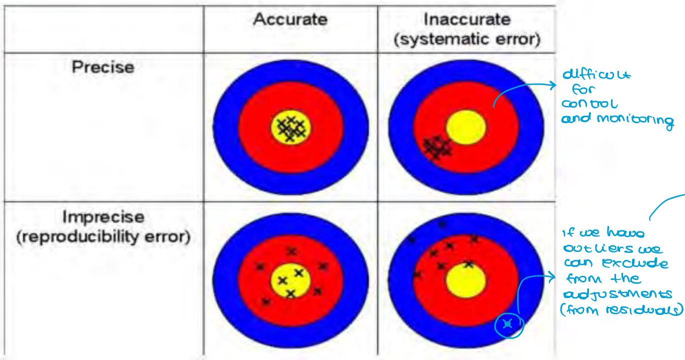


	Statistic		Parameter
Mean:	$\bar{X}$	estimates	_μ___
Standard deviation:	s	estimates	_σ___
Probability:	p	estimates	_π___
	from sample		from entire population



Precision and accuracy

- ✓ **Accuracy** – a description of how close a measurement is to the true value of the quantity measured
  expected value
- ✓ **Precision** – the exactness of a measurement



residual $v = y - \hat{y}$
- measurement estimated from solution
- measurement we have

→ outlier when all equations have same unit of measurement and one number is very different from the other ones

→ we can standardize residuals so to have PURE NUMBERS which can be compared without any problem

$w = \frac{v_i}{\sigma_{v_i}}$ (*) →

⇒ we know the expected value because in general we fix the origin of the local system there and so it's easy to measure distances from it

Inferential Statistics

- Allows for comparisons across variables
  i.e. is there a relation between one's occupation and their reason for using the public library?
- Hypothesis Testing

✓ H_0 Null Hypothesis states the Assumption to be tested e.g. SBP of participants = 120 (H0: m = 120).

✓ H_1 Alternative Hypothesis is the opposite of the null hypothesis (SBP of participants ≠ 120 (H1: m ≠ 120).

It may or may not be accepted and it is the hypothesis that is believed to be true by the researcher

Error Type

Table of error types		Null hypothesis (H_0) is	
		True	False
Decision About Null Hypothesis (H_0)	Accept	Correct inference (True Positive)	Type II error (False Negative)
	Reject	Type I error (False Positive)	Correct inference (True Negative)

Probability

By using inferential statistics to make decisions, we can report the probability that we have made a Type I error (indicated by the *p* value we report)

By reporting the p value, we alert readers to the odds that we were incorrect when we decided to reject the null hypothesis

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Propagation of errors

The error in the average ("error in the mean").

The average of several measurements each with the same uncertainty (σ) is given by:

$$\mu = \frac{1}{n}(x_1 + x_2 + \dots x_n)$$

$$\sigma_\mu^2 = \sigma_{x_1}^2\left(\frac{\partial\mu}{\partial x_1}\right)^2 + \sigma_{x_2}^2\left(\frac{\partial\mu}{\partial x_2}\right)^2 + \dots \sigma_{x_n}^2\left(\frac{\partial\mu}{\partial x_n}\right)^2 = \sigma^2\left(\frac{1}{n}\right)^2 + \sigma^2\left(\frac{1}{n}\right)^2 + \dots \sigma\left(\frac{1}{n}\right)^2 = n\sigma^2\left(\frac{1}{n}\right)^2$$

$$\sigma_\mu = \frac{\sigma}{\sqrt{n}}$$

We can determine the mean better by combining measurements. But the precision only increases as the square root of the number of measurements.

- Do not confuse σ_μ with σ!
- σ is related to the width of the *pdf* (e.g. Gaussian) that the measurements come from.
- σ does not get smaller as we combine measurements.

Least square method

if the variance is zero
$\hat{v} = \hat{x}_i - \mu = 0$
there aren't errors
in general we have
$v \neq 0$ so we need
to make minimum
the sum of the
squared values of
the residual

$$\sum_{i=1}^{N} (x_i - \mu)^2 = \min$$

↳ if m<n we can't invert the matrix that's why the system hasn't a solution

if m=n σ_0^2 isn't possible to be computed
$C_{\hat{x}\hat{x}} = \hat{\sigma}_0^2 N^{-1}$ (matrix)

we'd like to have variance $\sigma_0^2 = 0$ it's possible to have $\sigma_0^2 = 0$ when the point is fixed and very well known (no uncertanties of its position)

PROPERTIES
- All the values in the matrix have to be positive
- $\sigma_{xy} = \sigma_{yx}$

$$\begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1x_2} \\ \sigma_{x_2x_1} & \sigma_{x_2}^2 \end{bmatrix}$$

- it must be defined positive or semi positive det > 0

C_{xx} is given by a value σ_0^2 multiplied by a matrix N^{-1} so we can estimate C_{xx} a priori to have an idea of the quality of the shape of the environment
↳ it can be estimated before the survey

N is called normal matrix; two cases are possible:

- A is full rank (its columns are linearly independent one from the others)

$$\mathbf{Ax} = \mathbf{0} \Rightarrow \mathbf{x} = \mathbf{0}$$

in this case there is no rank deficiency and the normal matrix N is invertible

- A is not a full rank matrix, (some of its columns are linear combinations of some others)

$$\mathbf{Ax} = \mathbf{0} \quad \text{for some} \quad \mathbf{x} \neq \mathbf{0}$$

in this case the problem has a rank deficiency, N is not invertible.

$$y_0 = A \cdot \hat{x} \qquad y_0 = A \cdot \hat{x} + a$$

$$A^T \cdot y_0 = A^T \cdot A \cdot \hat{x} \qquad A^T \cdot (y_0 - a) = A^T \cdot A \cdot \hat{x}$$

Existence: has always a solution
Uniqueness: the solution is unique if the columns of A are linearly independent

$$A^T \cdot y_0 = N \cdot \hat{x} \qquad A^T \cdot l = A^T \cdot (y_0 - a) = N \cdot \hat{x}$$

The solution is: $\hat{x} = N^{-1} A^T l$

But we can also take into account the Weights (W or P) …

WEIGHT MATRIX

$N = A^T \cdot P \cdot A = A^T \cdot W \cdot A$
$P = W$ (weight matrix)
m×m
↳ related to number of equations
↳ ∝ of quality of measurements more is accurate more high is the value

$$W = \begin{bmatrix} w_1 & & & \\ & w_2 & & \\ & & w_3 & \\ \emptyset & & & \ddots \end{bmatrix}$$

zero values because there aren't correlations between one measurement and the others.

$w_1 = \frac{1}{\sigma_{y}^2}$ sometimes we have the standard deviation not variance

⇒ it's important to compare two different weight matrixes (focus on ratio between weights and not on single values)

$\hat{x} = N^{-1} A^T W (\hat{y} - b)$
↳ inside N we have the weight matrix so also inside C_{xx} and in v

$C_{xx} = \sigma_0^2 N^{-1}$
$\hat{v} = y - \hat{y}$

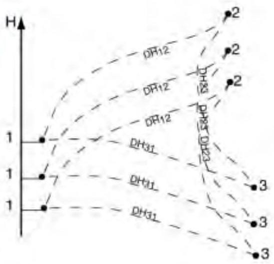
$\sigma_0^2 = \frac{v^T W v}{(m-n)}$ redundancy

The triangle levelling network

$$\mathbf{y}_O = \begin{bmatrix} dH_{12O} \\ dH_{23O} \\ dH_{31O} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} + \mathbf{v}$$

$$= \mathbf{Ax} + \mathbf{v}$$

$$\mathbf{C}_{yy} = \sigma_0^2 \mathbf{I}$$



A is not full rank; in particular

$$\mathcal{K}(\mathbf{A}) = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} H, \ \forall H \right\}, \ \mathbf{N} = \mathbf{A}^T\mathbf{A}, \ det(\mathbf{N}) = 0$$

the solution, can be written as follows

$$\mathbf{x}_r = \mathbf{N}_r^{-1}\mathbf{A}_r^T\mathbf{y}_o - \mathbf{D}\mathbf{x}_d,$$

where $\underset{r \times r}{\mathbf{N}_r} = \mathbf{A}_r^T\mathbf{A}_r$

The $\mathbf{x}_r$ component of the unknown parameter vector is a function of $\mathbf{x}_d$: there is no way to estimate the two components at the same time.

There are three possible alternatives to solve a rank deficient problem:

- ✓ Minimum constraints (either deterministic or stochastic) solution
- ✓ Minimum norm solution
- ✓ Tykonov's solution

Two definitions and a note

We define as intrinsic the rank deficiency of a network, made by a given type of observations, which cannot be reduced by modifying the network design; that is, by adding new observables of the same kind to the design itself.

In the geodetic network adjustment, solutions which eliminate the intrinsic rank deficiency must be adopted and define the reference system and frame.

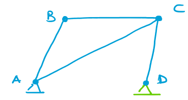
Levelling with respect to the sea level

$$\begin{cases} \Delta_{AB} = H_B - H_A \\ \Delta_{BC} = H_C - H_B \\ \Delta_{AC} = H_C - H_A \\ \Delta_{CD} = H_D - H_C \end{cases}$$

$m = n$ so not possible to solve the system with least square method

we fix the reference sistem (we consider only the variation of height with respect to a fixed point – sea level) → nº of parameter = or grater than nº of degree of freedom

↳ we have only a degree of freedom
DOF = 1 → minimum constrain approach



global redundancy = 1 (4-3)
r = 2 (we assume D as a known point)

If residual vectors are the same we obtain σ_0^2 first lower and then bigger

$$\hat{y} = \begin{bmatrix} \Delta_{AB} \\ \Delta_{BC} \\ \Delta_{CA} \\ \Delta_{CD} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} H_B \\ H_C \\ H_D \end{bmatrix} + \begin{bmatrix} -H_A \\ 0 \\ -H_A \\ 0 \end{bmatrix}$$

(columns: H_B, H_C, H_D; H_A)

vector of observations; design matrix 4x3 · unknowns

From the definition of Δ we perform partial derivaties and we find the design matrix (partial deriviaties with respect of unknowns)
⇒ we can solve the system, compute the normal matrix, the weight matrix (optional) and so the solution

$$\hat{x}_c = \begin{bmatrix} \hat{H}_B \\ \hat{H}_C \\ \hat{H}_D \end{bmatrix}$$

we can evaluate y considering the $\hat{x}$ estimated than we can find v and C_{xx}

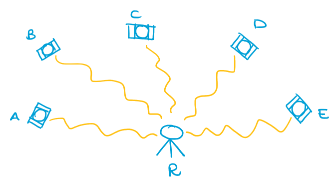
$$y = A\hat{x} + b$$
$$v = y - \hat{y}$$

$$C_{xx} = \begin{bmatrix} \sigma_{HB}^2 & \sigma_{HBHC} & \sigma_{HBHD} \\ // & \sigma_{HC}^2 & \sigma_{HCHD} \\ & & \sigma_{HD}^2 \end{bmatrix}$$ variance and covariance (simmetry)

Then correlation coefficients:

$$\rho = \frac{\sigma_{HBHC}}{\sigma_{HB}\sigma_{HC}}$$

If we have 200 equations as in GPS positioning we have complex sistems and complex equations



Receiver position: we can write the pseudoranges of each satellite (5 equations)

$$P_R^A = \underbrace{\sqrt{(x^A - x_R)^2 + (y^A - y_R)^2 + (z^A - z_R)^2}}_{d_R^A} + \Delta t$$

↳ we don't consider errors

P_R^B; P_R^C; P_R^D; P_R^E

we can compute coordinates in ITRF in the reference system of the satellites then we can convert into the reference system we like.

$$\begin{cases} \text{Five equations} \\ \text{4 unknowns} \end{cases}$$

$$\hat{y} = \begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 1 & 6 & \cdot & 16 \\ 2 & 7 & \cdot & \cdot \\ 3 & 8 & \cdot & \cdot \\ 4 & \vdots & \cdot & \cdot \\ 5 & \vdots & \cdot & \cdot \end{bmatrix}$$

(columns: x_R, y_R, z_R, ΔT)

Not linear, but it can be linearized. If A isn't from a linear system we call it A_J (Jacobian matrix)

$$g(x,y) = \begin{pmatrix} g_1(x,y) \\ g_2(x,y) \\ \vdots \\ g_l(x,y) \end{pmatrix} = 0 \qquad y \in R^m \qquad x \in R^n$$

Constraint: $(y_0 - \hat{y})^T P (y_0 - \hat{y}) = min$

Under the condition: $g(\hat{x}, \hat{y}) = 0$

We know the approximate value of x: $\hat{x} = \tilde{x} + \xi$

and g can be linearized: $y \cong g(\tilde{x}) + \left(\frac{\partial g}{\partial \tilde{x}}\right)\xi;$

$$\frac{\partial g}{\partial x} = \left[\frac{\partial g_i}{\partial x_j}\right] = A$$

$\hat{y} \cong a + A\hat{\xi};$ $\qquad \hat{v} = (\hat{y} - y_0) = A\hat{\xi} + (a - y_0)$

$\hat{v}^T P \hat{v} = \min$ $\qquad$ Under the condition: $\hat{y} \cong a + A\hat{\xi};$

And then iterate…

The local redundancy values

$$\hat{v} = (y_0 - \hat{y}) = l - A\hat{x} = R\, y_0 \qquad \Delta v = R \Delta y_0$$

An error on y_0 modifies the residuals according to R matrix: usually, a wrong measurement involves all residuals. Under the hypothesis that there is an error only in the k-th measurement:

$$\Delta y_k \neq 0 \qquad \Delta v_j(k) = r_j^k \Delta y_k \qquad \Delta v_k(k) = r_k^k \Delta y_k$$

Remember that $\underline{\underline{R}} = \frac{1}{\hat{\sigma}_0^2} P \cdot C_{\hat{v}\hat{v}} = I - PAN^{-1}A^T$

r_{ij} are pure numbers. R can be obtained before the survey. However, if pj and qj represent the weights before and after the adjustment

$p_j = \frac{\sigma_0^2}{\sigma_{yj}^2};$ $\qquad q_j = \frac{\hat{\sigma}_0^2}{\hat{\sigma}_{\hat{y}j}^2}$ $\qquad$ Their ratio is equal to $\qquad r_j = \frac{\sigma_0^2 \sigma_{\hat{v}j}^2}{\sigma_y^2 \hat{\sigma}_0^2}$

(*) residual $v = y - \hat{y}$
- measurement estimated from solution
- measurement we have

→ outlier when all equations have same unit of measurement and one number is very different from the other ones

→ we can standardize residuals so to have PURE NUMBERS which can be compared without any problem

$w = \frac{v_i}{\sigma_{v_i}}$

then we can exclude the outlayers and do the procedure again

we have also a test to define if a value is an out layer or not

$\hat{\tau}_v < \hat{\tau}_\alpha$ not an outlyer

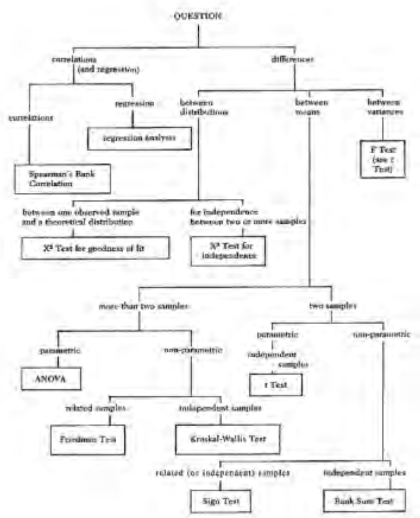
probability to accept an error even if it doesn't exist ($\alpha = 5\%$ in 95% of cases we are right!)
→ for application of people and goods (safe)
→ other applications $\alpha = 10\%$

The First Question

After examining your data, ask: does what you're testing seem to be a question of **relatedness** or a question of **difference**?

If **relatedness** (between your control and your experimental samples or between you dependent and independent variable), you will be using tests for correlation (positive or negative) or regression.

If **difference** (your control differs from your experimental), you will be testing for independence between distributions, means or variances. Different tests will be employed if your data show parametric or non-parametric properties.



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Chi-square test

It happens that some biases on "a" measurements exist:

$$\underline{y} = \left(\underline{\underline{A}} + \delta\underline{\underline{A}}\right)\underline{x} + \underline{a} \qquad \underline{y} = \underline{\underline{A}}\underline{x} + \underline{a} + \delta\underline{a}$$

The variance of the unit of weight increases

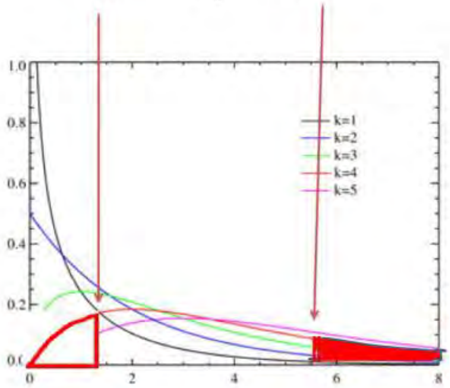
$$M\left[\hat{\sigma}_0^2(wrong)\right] = \hat{\sigma}_0^2 + \frac{\delta a' P \delta a}{m-n}$$

$$k = (m-n)\frac{\hat{\sigma}_0^2}{\sigma_0^2} \sim \chi^2_{m-n,\alpha} \qquad if \;\; k > \chi^2_{m-n,1-\frac{\alpha}{2}}$$

We reject the hypothesis: $\sigma_0^2 = \hat{\sigma}_0^2$

It may happens that an over parametrization occur: then we consider a TWO TAILED test

$$\chi^2_{\left(\frac{\alpha}{2}\right)} < r\frac{\hat{\sigma}_0^2}{\sigma_0^2} < \chi^2_{\left(1-\frac{\alpha}{2}\right)}$$



But you can consider only a subset of parameters

If **E** is the matrix that extracts *only k elements* from x that can be tested:

$$\xi = \begin{pmatrix} x_2 \\ x_5 \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = Ex$$

We can propagate the variance $C_{\xi\xi} = EC_{\hat{x}\hat{x}}E$

We want to test ξ values

$$\frac{(\hat{\xi}-\xi)'(EN^{-1}E)^{-1}(\hat{\xi}-\xi)}{\sigma_0^2} = \chi_k^2$$

Again, if σ_0^2 is unknown, it is possible to evaluate

$$\frac{(\hat{\xi}-\xi)'(EN^{-1}E)^{-1}(\hat{\xi}-\xi)}{r\hat{\sigma}_0^2} = F_0$$

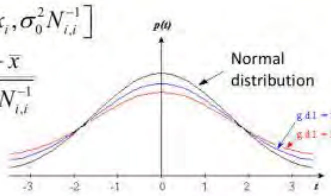
Ad it is possible to verify if $F_0 \sim F_{k,m-n;\alpha}$

But you can consider only ONE parameter

If the parameter is only one: $F_{1,r} = t_r$

If the parameter is only one: $\hat{x}_i \sim N\left[x_i, \sigma_0^2 N_{i,i}^{-1}\right]$

It is mandatory to standardize the measurement $z = \frac{\hat{x}_i - \bar{x}}{\sigma_0\sqrt{N_{i,i}^{-1}}}$



Very often I can't quantify σ_0 but I can estimate the RMS after the LS approach
Then …

$$t_r = \frac{z}{\sqrt{\chi_r^2/r}} = \frac{\sqrt{\sigma_0^2}z}{\sqrt{\hat{\sigma}_0^2}} = \frac{(\hat{x}_i - \bar{x})}{\sigma_{\hat{x}i}}$$

Test if the mean is equal to a hypothesized value or to create a graph varying the significant level of parameter

Useful for testing **over-parametrization**
(is this variable significant or not?) and for testing displacements

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Techniques to produce digital maps

↳ 3 methods to generate new maps

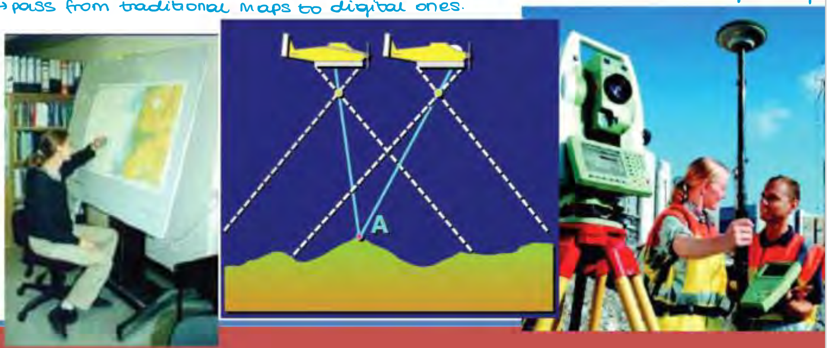
I generation: Direct surveying in the field, with traditional topographic tools (total station or levelling) or GNSS → collect in fields points and move to maps. Export to use tools. When we have a limited signal to be mapped.

II generation: Photogrammetry (also for very big scale)

III generation: Traditional maps are digitalized.

This classification doesn't consider the precision factors. → it isn't a scan (we have info like data base not only on image)

↳ pass from traditional maps to digital ones.



↳ we can use drones

Importance of the photogrammetry

Architecture and Engineering activities are working on portion of real world

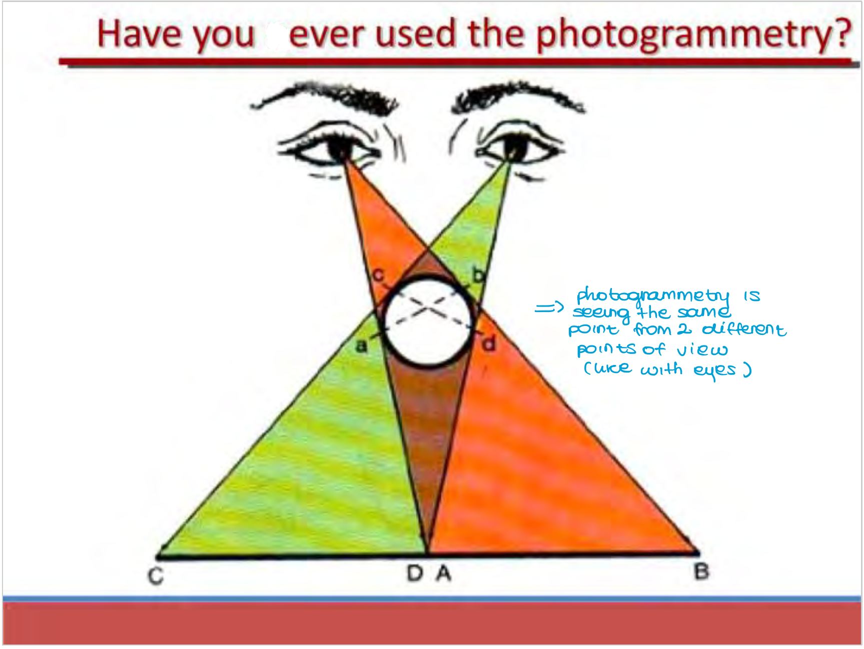
In order to make the correct decision, it is mandatory to know very well the object, considering:

- **correctness (precision and detail levels)**
- **completeness (geometry and shape)**
- **sustainable (working time, cost, performances)**

data acquisition and processing have costs

Geomatics offers tools and techniques, which are devoted to acquire these information, even in rapid, efficient and economic ways. In same case, some automatic solutions are available

Photogrammetry could be a good solution!

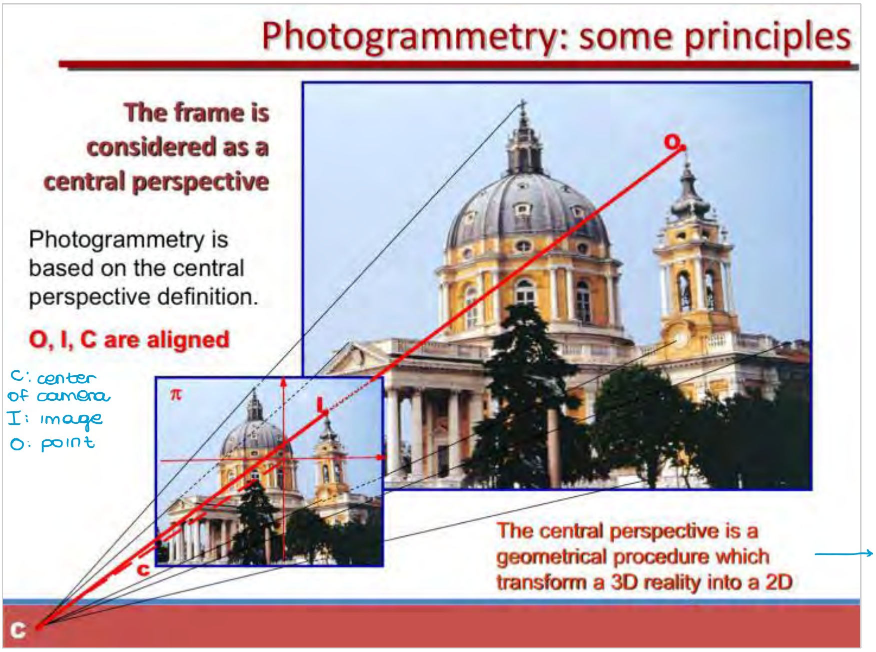


=> photogrammetry is seeing the same point from 2 different points of view (like with eyes)

=> The goal is to produce digital maps using images

it's important to control quality of our product (we can desig precision so:
- quality of camera
- distance between camera and objects)

know what is the final precision before collecting images

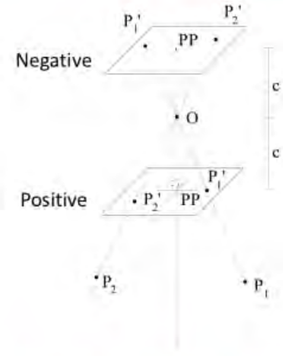


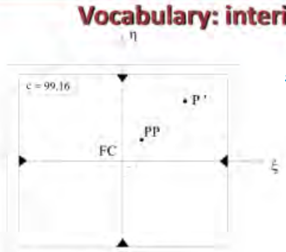
C: center of camera
I: image
O: point

→ we loose the tridimensional concept

Photogrammetry: some principles

Vocabulary: interior orientation





coordinate system related to the frame

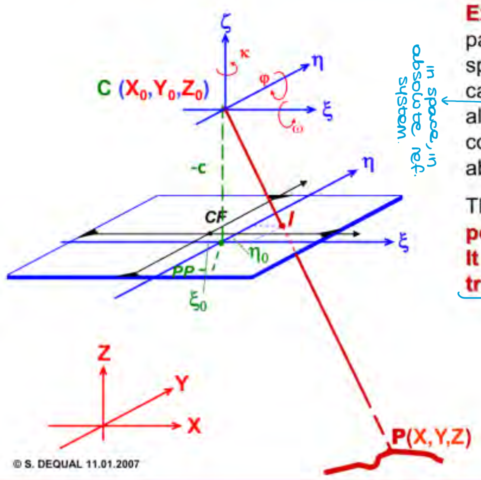
O = perspective center → where we collect data (camera)
PP = principal point
c = principal distance or focal length (from O to PP)
FC = center of inner coordinate system (or fiducial coordinate system)

Interior orientation is the set of parameters which allows to define the inner geometry of the camera. The parameters are:

ξ_0, η_0 coordinates of the principal point in Interior coordinate system
c principal distance

Photogrammetry: some principles

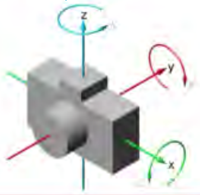
Vocabulary: exterior orientation



Exterior orientation is the set of parameters which allow to define the spatial position and attitude of the camera (frame and principal point already defined in the interior coordinate system) soncidering a 3D absolute coordinate system. (in space, in absolute ref. system)

This orientation defines **the spatial position of the camera in the space. It is defined by 6 parameters: 3 translations and 3 rotations**

C(x,y,z) — φ, ω, κ



Fotogrammetria Pagina 6

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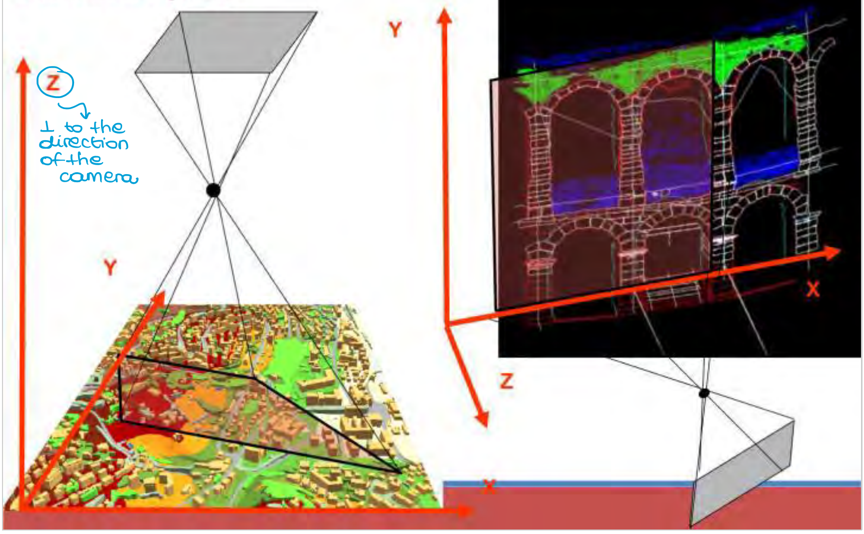
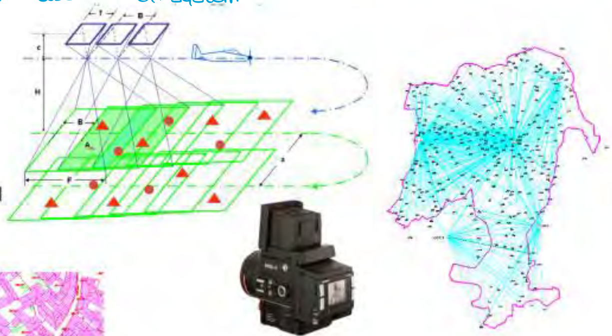
Photogrammetry: some principles

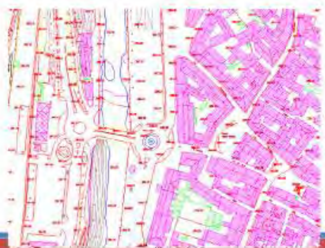
Coordinate system



Photogrammetry: the main steps

- Data collection
- Ground network and Control point (use info to convert in absolute ref. system)
- Orientation
- Plotting
- Integration and editing
- advanced editing and map export





Each step of the photogrammetric process requires a careful planning and verification.
↳ errors are repeated in the next steps
The procedure is sequential:
an error in one step brings to have some negative effects to all the subsequent phases
↳ the final quality depends on all steps

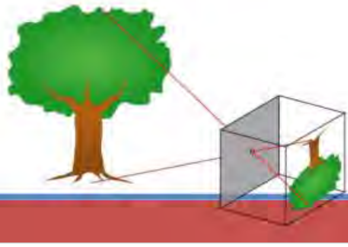
Background

Alhazen , or Abū Alī al-Hasan ibn al-Hasan ibn al-Haytham, doctor, philosopher, mathematician, physicist and Arab astronomer (Bassora, 965 - Cairo, 1039), He has studied light, the eye, the vision

The camera obscura was initially a real room with a small hole on a wall.

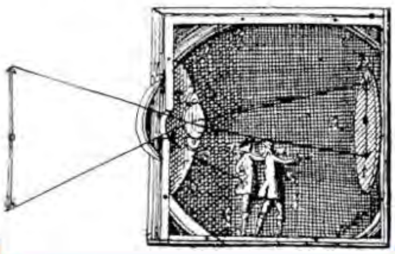
The light passing through this hole produced a perspective of the space outside the chamber on the opposite wall.

The modest amount of light and the diffraction effects at the edges of the hole generated blurred images.



Background

The possible "begin" of the **history of photogrammetry** can be considered coincident with the work of **Leonardo da Vinci** (1452-1519): his studies of geometry, optics, mechanics and his fervid intuition allowed to graphically demonstrate in **1492 the principles of optical projection**



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Background

The importance of binocular vision for distance perception was studied by the German astronomer Johannes Kepler (Weil der Stadt, 1571 - Regensburg, 1630). In two remarkable works, Astronomiae Pars Optica (The optical part of astronomy), published in 1604, and Dioptrice (Diottrica, the part of optics that deals with refraction), of 1611, Kepler provided a detailed description of the optics of eye, explained the operation of the lenses and developed a theory of stereoscopic vision

Background

the Florentine painter Jacopo Chimenti (1551-1640) produces what may be considered the first stereoscopic pair of drawings (**Museo Wicar - Lille - France**).

Background

Johann Heinrich Lambert (1728-1777)

in the treatise "Perspectiva Liber" developed the mathematical principles of a perspective image using the spatial intersection to identify a point in space from two different images

This describes the geometric fundamental of the technique that 100 years later will be called PHOTOGRAMMETRY

French admiral Beautemps-Beaupré has realized the coastal topographic maps from stereoscopic pairs of perspectives using these mathematical formulations.

The concept of stereoscopic drawings was used for a survey by **F. Kapeller** in 1726 that produced a topographic map of **Mount Pilatus on Lake Lucerne.**

Background

In the same period the **English Fox Talbot** managed to produce some negatives with his camera obscura, while Arago and Niepce announced the discovery of the method of heliographic reproduction.

In 1837 Louis-Jacques-Mandé Daguerre (1787-1851) presented the first positive images on a metal plate (daguerreotypes) at the French Academy of Arts and Sciences. His method was soon acquired, improved and spread throughout Europe.

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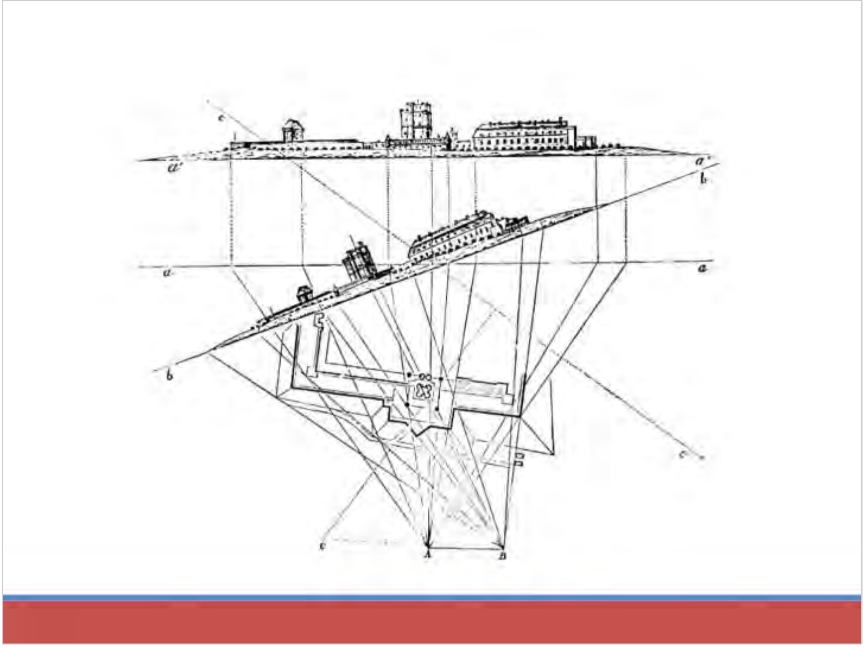

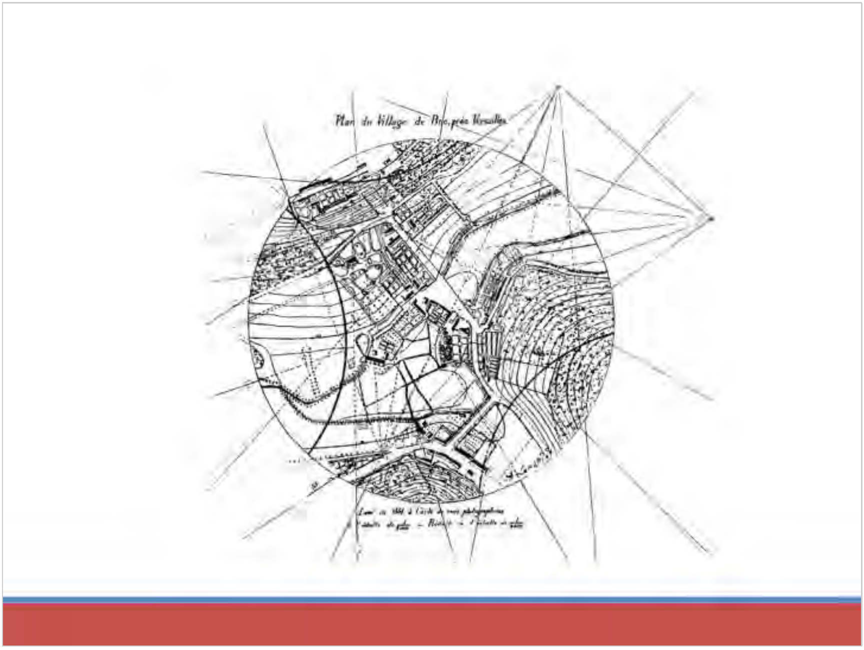

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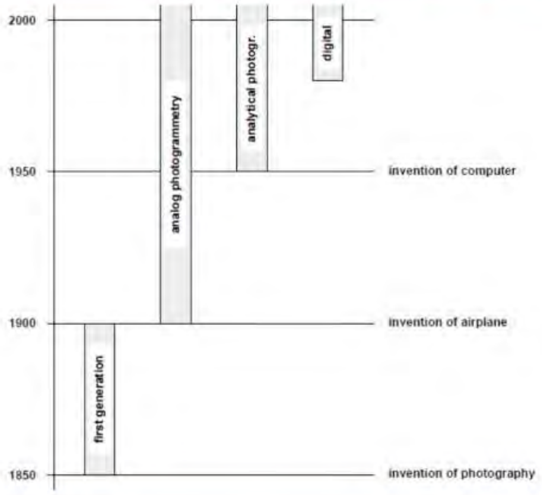
Background

In **1958**, **Helava** presents the prototype of an analytical photogrammetric plotter in which all the problems of orientation and plotting have been solved by a computer connected to an instrument (the stereo-comparator) for the identification and measurement of the coordinates of the homologous points.

In **1981** **Sarjakoski** demonstrated the possibility to produce and use digital images for photogrammetric purposes. Thus began the current period of evolution of the photogrammetric technique which sees the possibility of achieving complete automation of the entire production process.

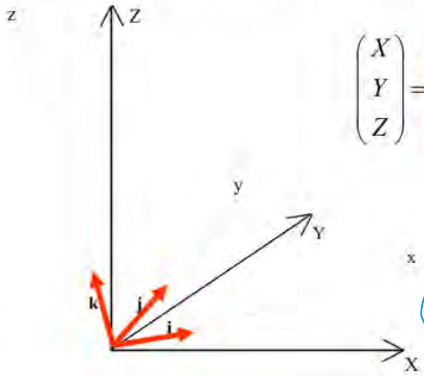
Photogrammetry evolution



Analytical principles

Mathematical model: rotation in the space

All mathematical concepts which are valid in the plane, could be also applied in the 3D space, to transform a point *P(x,y,z)* into a system *X, Y, Z*, using the directional cosines



$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos(xX) & \cos(yX) & \cos(zX) \\ \cos(xY) & \cos(yY) & \cos(zY) \\ \cos(xZ) & \cos(yZ) & \cos(zZ) \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

i j k

Considering the **i**, **j**, **k** as vectors in the coordinate system (*x, y, z*). → because they contain directional cosines → they describe rotation between two reference systems (also φ, ω and κ are related to directional cosines)

→ they are =1 because the are NORMALIZED

6/04/18

Analytical principles

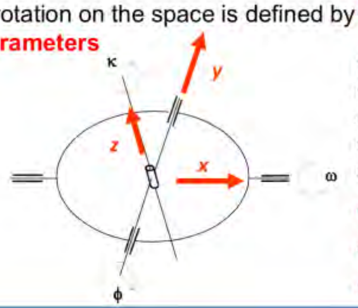
Mathematical model: rotation in the space

Three orthogonality conditions and three conditions of normalization could be defined to describe the nine elements of the rotation matrix ***R***.

$i^T i = j^T j = k^T k = 1$

$ij = ik = jk = 0$

A rotation on the space is defined by **three independent parameters**



In photogrammetry, three rotation angles are traditionally used: ω, φ, κ . These angles are considered with respect the three axis. It is necessary to know exactly the order of the rotation, as in the case of an universal joint:

ω = primary rotation around x axis

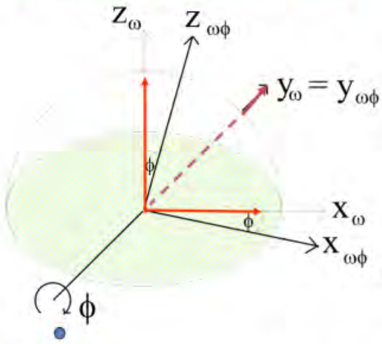
φ = secondary rotation around y axis

κ = tertiary rotation around z axis

Analytical principles

Mathematical model: rotation in the space

Secondary (second) rotation ϕ



$$x_\omega = x_{\omega\phi} \cos\phi + z_{\omega\phi} \text{ sen } \phi$$

$$y_\omega = y_{\omega\phi}$$ (rotation around y)

$$z_\omega = - x_{\omega\phi} \text{ sen } \phi + z_{\omega\phi} \cos$$

the components are around x and z because rotation is around y

$$\mathbf{x}_\omega = \begin{pmatrix} \cos\phi & 0 & \text{sen}\,\phi \\ 0 & 1 & 0 \\ -\text{sen}\,\phi & 0 & \cos\phi \end{pmatrix} \cdot \begin{pmatrix} x_{\omega\phi} \\ y_{\omega\phi} \\ z_{\omega\phi} \end{pmatrix} = \mathbf{R}_\phi \mathbf{x}_{\omega\phi}$$

Replacing this equation in the previous one:

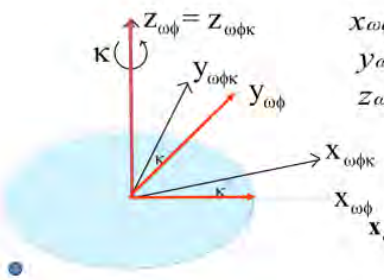
$$X = R_\omega \cdot R_\phi \cdot X_{\omega\phi}$$

(secondary rotation matrix; result of primary rotation)

Analytical principles

Mathematical model: rotation in the space

Tertiary (third) rotation κ



$$x_{\omega\phi} = x_{\omega\phi\kappa} \cos\kappa - y_{\omega\phi\kappa} \text{ sen } \kappa$$

$$y_{\omega\phi} = x_{\omega\phi\kappa} \text{ sen } \kappa + y_{\omega\phi\kappa} \cos\kappa$$

$$z_{\omega\phi} = z_{\omega\phi\kappa}$$

$$\mathbf{x}_{\omega\phi} = \begin{pmatrix} \cos\kappa & -\text{sen}\,\kappa & 0 \\ \text{sen}\,\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_{\omega\phi\kappa} \\ y_{\omega\phi\kappa} \\ z_{\omega\phi\kappa} \end{pmatrix} = \mathbf{R}_\kappa \mathbf{x}$$

Replacing this equation in the last one:

$$\mathbf{X} = \mathbf{R}_\omega \mathbf{R}_\varphi \mathbf{R}_\kappa \mathbf{x}$$

(primary rotation; secondary and third rotation)

→ It's important to maintain the position (1,2,3)

EXAMPLE: Given this rotation matrix

$$\begin{pmatrix} -0.034011 & 0.999407 & 0.004822 \\ -0.999419 & -0.034096 & 0.000621 \\ 0.000784 & -0.004798 & 0.999988 \end{pmatrix}$$

WARNING WHEN ϕ = 0!

GOAL: estimate the two series of rotations

$\text{sen}\phi = 0.004822 \rightarrow \phi_1 = 0.3070\ gon$ (complementary) $\phi_2 = 199.6930\ gon$

$0.000621/\cos\phi_1 = -\text{sen}\,\omega_1 = 0.0006210$
$0.999988/\cos\phi_1 = \cos\omega_1 = 0.9999996$ $\rightarrow \omega_1 = 399.9605\ gon$

$r_{23} = -\text{sen}\,\omega\cos\phi$
$r_{33} = \cos\omega\cos\phi$

$0.000621/\cos\phi_2 = -\text{sen}\,\omega_2 = -0.0006210$
$0.999988/\cos\phi_2 = \cos\omega_2 = -0.9999996$ $\rightarrow \omega_2 = 199.9605\ gon$

$0.999407/\cos\phi_1 = -\text{sen}\,\kappa_1 = 0.9994186$
$-0.034091/\cos\phi_1 = \cos\kappa_1 = -0.0340914$ $\rightarrow \kappa_1 = 297.8292\ gon$

$r_{12} = -\cos\phi\,\text{sen}\kappa$
$r_{11} = \cos\phi\cos\kappa$

$0.999407/\cos\phi_2 = -\text{sen}\,\kappa_2 = -0.9994186$
$-0.034091/\cos\phi_2 = \cos\kappa_2 = 0.0340914$ $\rightarrow \kappa_2 = 97.8292\ gon$

Analytical principles

Some consideration about the rotation matrix

The dependence of the rotation matrix on independent parameters is not a standard but can be expressed in many ways that depend on the conventional definition of:

- 3 parameters used to define the 9 directional cosines (3 angles, but also other definitions, e.g Rodriguez matrix);
- if they are angles of the positive direction of rotation (clockwise or anticlockwise);
- if they are angles from the order of rotations (primary, secondary and tertiary);

The photogrammetric plotters often use different conventions: the transition from one software to another must include a conversion in the parameters that describe the rotations.

9 directional cosines (the 9 numbers) which compose the rotation **matrix are NOT DEPENDENT ON the conventional definition**. When it is possible, it is better to use the complete rotation matrix instead to use 3 parameters only .

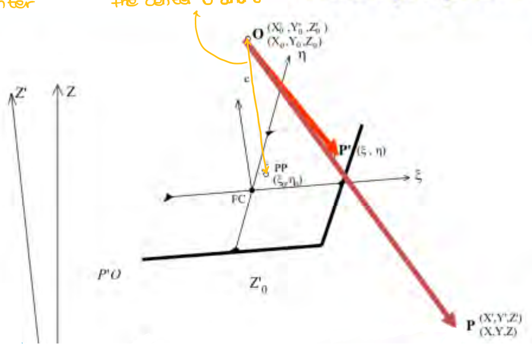
Analytical principles

→ defining position of a point, starting from the image.

Central perspective

When a frame is collected, **the object point P, the centre O and the image point P' are on the same straight line.**
We can introduce a new coordinate sytem called «Terrain coordinate system» defined by X',Y',Z', **which is parallel to the image CS** ξ,η,ζ (ζ = 0 for all points and ζ = c for the centre) but with

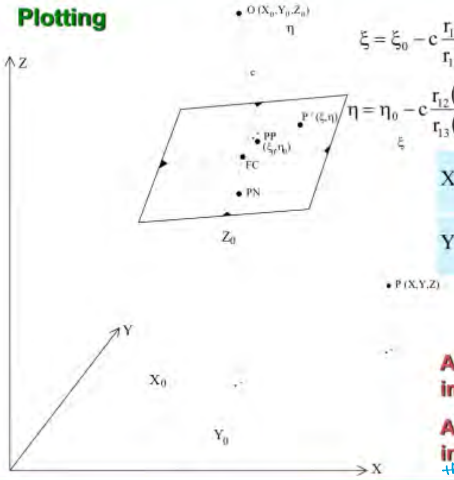
O is the principle center ← c is the principle distance between the center O and o



Analytical principles

Collinearity equation

Plotting



$$\xi = \xi_0 - c\frac{r_{11}(X-X_0)+r_{21}(Y-Y_0)+r_{31}(Z-Z_0)}{r_{13}(X-X_0)+r_{23}(Y-Y_0)+r_{33}(Z-Z_0)} = \xi_0 - c\frac{Z_x}{N}$$

$$\eta = \eta_0 - c\frac{r_{12}(X-X_0)+r_{22}(Y-Y_0)+r_{32}(Z-Z_0)}{r_{13}(X-X_0)+r_{23}(Y-Y_0)+r_{33}(Z-Z_0)} = \eta_0 - c\frac{Z_y}{N}$$

$$X = X_0 + (Z-Z_0)\frac{r_{11}(\xi-\xi_0)+r_{12}(\eta-\eta_0)-r_{13}c}{r_{31}(\xi-\xi_0)+r_{32}(\eta-\eta_0)-r_{33}c}$$

$$Y = Y_0 + (Z-Z_0)\frac{r_{21}(\xi-\xi_0)+r_{22}(\eta-\eta_0)-r_{23}c}{r_{31}(\xi-\xi_0)+r_{32}(\eta-\eta_0)-r_{33}c}$$

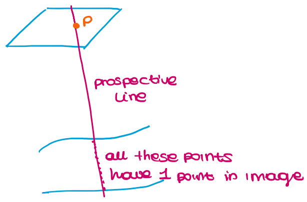
→ These equations allow to exstract also the coordinates of the object (PLOT)

At each object point corresponds an image point. (UNIQUE!)

At each image point can match infinite object points (because we have the problem that image is 2D and reality is 3D) → all the points in the prospective line belong to the same pixel of the image

Analytical principles

Linearization of Collinearity equation (ξ)



$$\xi = \xi_0 - c\frac{r_{11}(X-X_0)+r_{21}(Y-Y_0)+r_{31}(Z-Z_0)}{r_{13}(X-X_0)+r_{23}(Y-Y_0)+r_{33}(Z-Z_0)} = \xi_0 - c\frac{Z_x}{N}$$

because they aren't linear and so very difficult

$$-(\xi-\xi_0) - c\frac{r_{11}(X-X_0)+r_{21}(Y-Y_0)+r_{31}(Z-Z_0)}{r_{13}(X-X_0)+r_{23}(Y-Y_0)+r_{33}(Z-Z_0)} = 0$$

$$f(X_0, Y_0, Z_0, \omega, \varphi, \kappa, X, Y, Z / \bar{\xi}) = 0$$

$$X_0 = X_0^{\ 0} + dX_0 \quad Y_0 = Y_0^{\ 0} + dY_0 \quad Z_0 = Z_0^{\ 0} + dZ_0$$
$$\omega = \omega^0 + d\omega \quad \varphi = \varphi^0 + d\varphi \quad \kappa = \kappa^0 + d\kappa$$
$$X = X^0 + dX \quad Y = Y^0 + dY \quad Z = Z^0 + dZ$$

→ These are trial values from which start to find the solution: a possible way is to apply after that the minimum squared differences

$$-(\bar{\xi}-\xi_0) + \left(\frac{\partial\xi}{\partial X_0}\right)_{\bar{0}} dX_0 + \left(\frac{\partial\xi}{\partial Y_0}\right)_{\bar{0}} dY_0 + \left(\frac{\partial\xi}{\partial Z_0}\right)_{\bar{0}} dZ_0 +$$
$$+\left(\frac{\partial\xi}{\partial\omega}\right)_{\bar{0}} d\omega + \left(\frac{\partial\xi}{\partial\varphi}\right)_{\bar{0}} d\varphi + \left(\frac{\partial\xi}{\partial\kappa}\right)_{\bar{0}} d\kappa + \left(\frac{\partial\xi}{\partial X}\right)_{\bar{0}} dX + \left(\frac{\partial\xi}{\partial Y}\right)_{\bar{0}} dY + \left(\frac{\partial\xi}{\partial Z}\right)_{\bar{0}} dZ = 0$$

Analytical principles

Linearization of Collinearity equation (η)

$$\frac{\partial \eta}{\partial X_0} = -\frac{c}{N^2}\left(r_{13}Z_y - r_{12}N\right) = b_2 \qquad \frac{\partial \eta}{\partial X} = -\frac{c}{N^2}\left(Nr_{12} - Z_y r_{13}\right) = b_8$$

$$\frac{\partial \eta}{\partial Y_0} = -\frac{c}{N^2}\left(r_{23}Z_y - r_{22}N\right) = b_3 \qquad \frac{\partial \eta}{\partial Y} = -\frac{c}{N^2}\left(Nr_{22} - Z_y r_{23}\right) = b_9$$

$$\frac{\partial \eta}{\partial Z_0} = -\frac{c}{N^2}\left(r_{33}Z_y - r_{32}N\right) = b_4 \qquad \frac{\partial \eta}{\partial Z} = -\frac{c}{N^2}\left(Nr_{32} - Z_y r_{33}\right) = b_{10}$$

$$\frac{\partial \eta}{\partial \omega} = -\frac{c}{N}\left\{\left[(Y-Y_0)r_{33} - (Z-Z_0)r_{23}\right]\frac{Z_y}{N} - (Y-Y_0)r_{32} + (Z-Z_0)r_{22}\right\} = b_5$$

$$\frac{\partial \eta}{\partial \phi} = \frac{c}{N}\left[\left(Z_x \cos\kappa - Z_y sen\kappa\right)\frac{Z_y}{N} + Nsen\kappa\right] = b_6$$

$$\frac{\partial \eta}{\partial \kappa} = +\frac{c}{N}Z_x = b_7 \qquad t_\eta = \overline{\eta} - \eta_0$$

$$-t_\eta + b_2 dX_0 + b_3 dY_0 + b_4 dZ_0 + b_5 d\omega + b_6 d\varphi + b_7 d\kappa + b_8 dX + b_9 dY + b_{10} dZ = 0$$

Α α	alpha	Ν ν	nu
Β β	beta	Ξ ξ	ksi
Γ γ	gamma	Ο ο	omicron
Δ δ	delta	Π π	pi
Ε ε	epsilon	Ρ ρ	rho
Ζ ζ	zeta	Σ σς	sigma
Η η	eta	Τ τ	tau
Θ θ	theta	Υ υ	upsilon
Ι ι	iota	Φ φ	phi
Κ κ	kappa	Χ χ	chi
Λ λ	lambda	Ψ ψ	psi
Μ μ	mu	Ω ω	omega

Steps of photogrammetric process

Principles of Photogrammetry

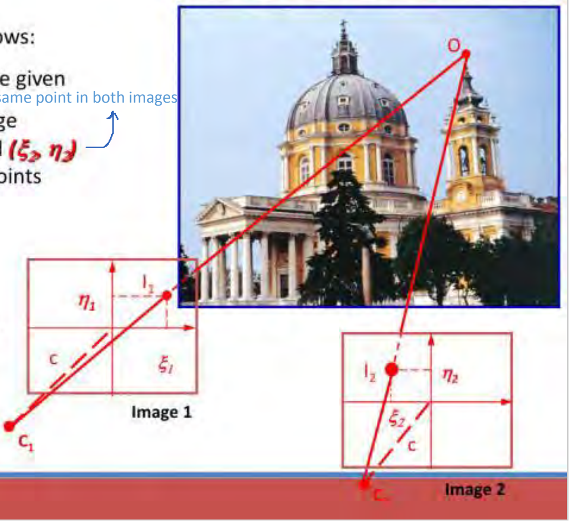
The Photogrammetry allows:

- If at least two frames are given

We have to be able to measure the same point in both images

- And measuring the image coordinates (ξ_1, η_1) and (ξ_2, η_2) about the homologous points

To estimate the coordinates of the object (X, Y, Z) of points in the object space



Interior orientation

Under these conditions:

- It is possible to measure the image coordinates:
  - Definition of the image reference system
  - To be able to measure the image coordinates (point on the frame)

- the position of the projection center is known (internal orientation)

- the position of the projection center and its attitude are known in the object reference system (exterior orientation)

Considering:

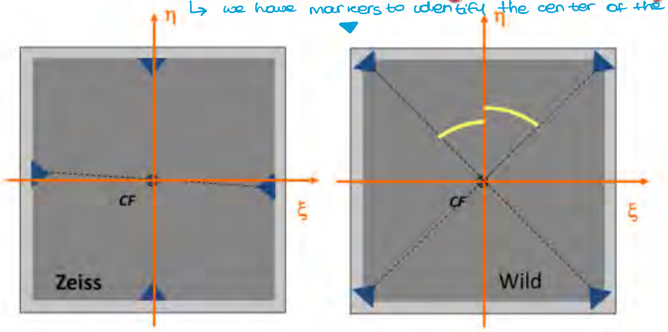
The «ideal» central perspective is not satisfied due to many operative conditions as errors and distortions

The camera are different, in particulare considering errors and distortions.

Interior orientation

Definition of the image coordinate system

↳ we have markers to identify the center of the frame



Interior orientation

Definition of the image coordinate system

Classification of the used camera in photogrammetry

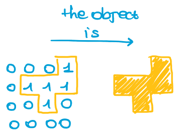
- **metric camera** : Interior orientation is **known** (by calibration) and **stable during the time**
  Specific applications (very expensive) → Pedestrian or aerial use (only by plane)
- **semi-metric camera**: Interior orientation is **known** (by calibration) and **it can change during the time**
- **professional camera (and mass market)**: Interior orientation is **unknown** and **variable during the time**

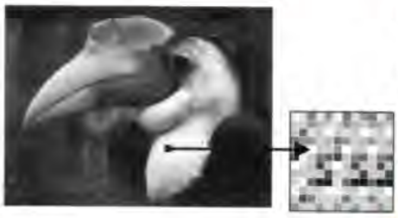
Digital images

Conventional radiometric definition (B/W)

If the digital image represent an object composed by two colors (a black-white portrait), the radiometric information is defined only by two integer numbers: **0 = white 1= black**

The data storage requires only 1 bit for each pixel.





If the digital image represents an object in gray scale (e.g black-and-white image), radiometry can be expressed with an integer variable between 0 (black) and 255 (white). Intermediate values at these two extremes represent the various gray scale. The black-white interval is divide into 256 parts for two main considerations:

- **the human eye can distinguish about 80 different grays**
- **an integer between 0 and 255 can be stored in 1 byte**

black ↑ white ↑ — so we have 256 values

→ so 1 pixel for 1 byte

resolution of image and required memory space

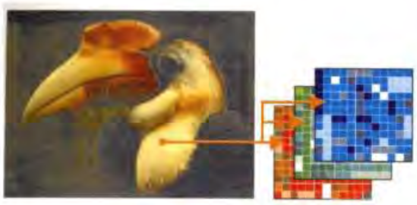
Digital images

Conventional radiometric definition (color)

If the image has to represent an coloured object, there are two possible approaches to define the radiometry:

True color Image (RGB)

Each color is described as the **sum of three bands corresponding to the main colors (red, green and blue)**
Each band is represented by 256 values from 0 (= no color) to 255 (= color saturation).
The radiometry of a pixel is represented by three integers which describe the saturations of the three main bands; this representation requires 3 bytes for each pixel

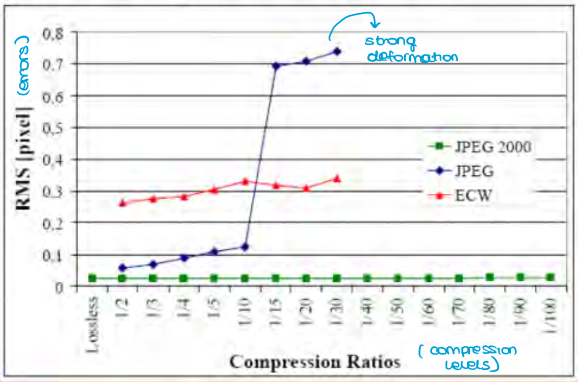


Digital images

Geometric degradation vs compression ratios

Geometric degradation can be represented by standard deviation differences in the markers positions between the compressed image and the uncompressed original image. These positions are expressed in image coordinates as pixel units.

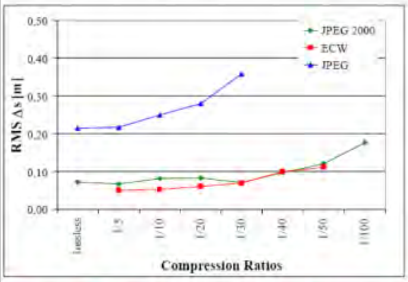
The trend is almost linear with low compression ratios, while it is very strong variation (that is not acceptable in photogrammetry) with compression values in the range 1/7 - 1/10.

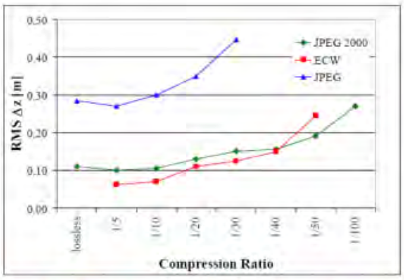


Digital images

Geometric degradation vs compression ratios

Geometric degradation has been also verified in photogrammetric plotting. The same points were plotted within a model based on uncompressed images (reference data) and in the various models composed by images with increasing compression ratios. The differences can be represented as standard deviation, in object coordinates.





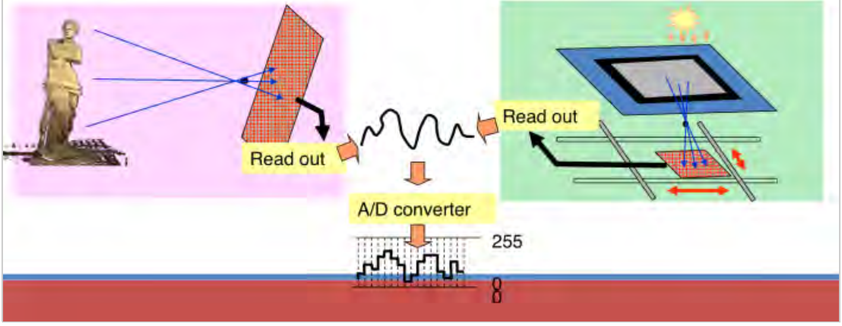
Using JPEG, the maximum level of compression is = 7. ECW or JPEG2000 format can be adopted up to a compression level = 30.

Acquisition of digital images

Digital images can be acquired in two different methods:

Direct acquisition: the image is created by digital camera on the digital support. The camera could be metric or not-metric.

Indirect acquisition: Image is created making a scan of a traditional image (hardcopy). In this case, it is necessary to use special scanner (photogrammetric or DTP), in order to reduce the distortions.



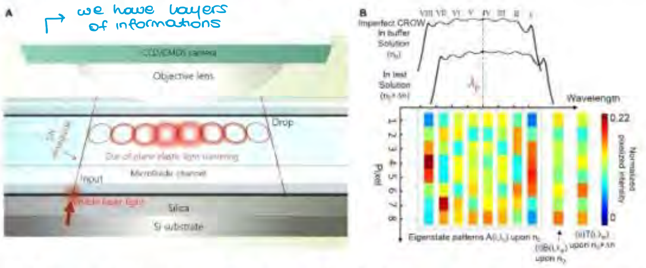
Acquisition of digital images

Direct acquisition

How does a digital camera works ?

The image storage is based on **Electronic principles**. The support where the image is stored is called «digital sensor»

we have layers of informations



The digital sensor is composed by a matrix of elements (pixels), where each of which is sensitive to a primary color. Each pixel consists of a capacitor able to record the amount of energy (light intensity) incident and to convert it into a integer value

↳ informations are strongly related to light intensity

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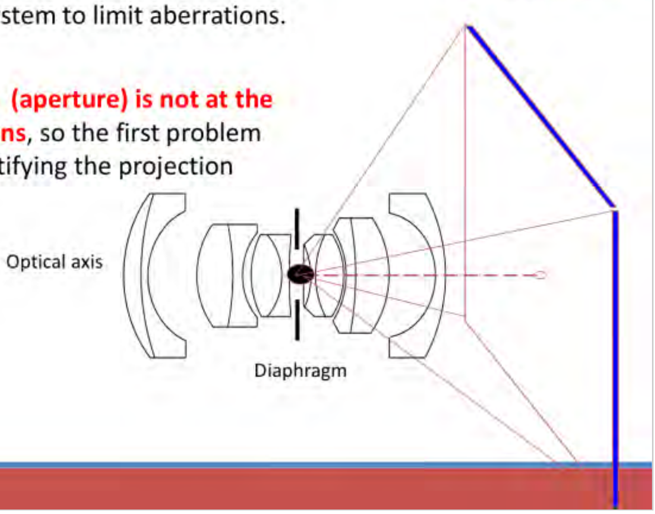
Interior orientation

Metric camera

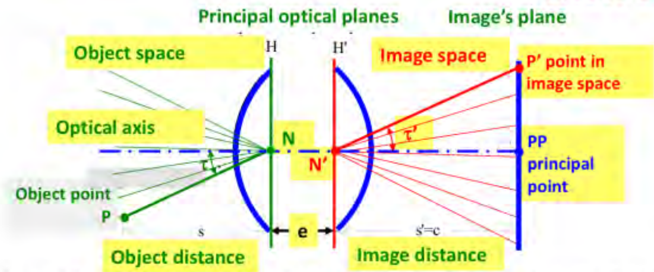
In photogrammetric cameras, the lens is a complex lens system to limit aberrations.

The diaphragm (aperture) is not at the center of the lens, so the first problem consists in identifying the projection center.



Calibration

Metric camera



In each optical system there are two principal planes that intersect the optical axis in N and N ', called nodal points (external and internal)

N can be considered the center of projection in the object space and N ' in the image space

With respect to the simplified schema (lens as point), **the two stars of directions, external and internal, are separated by the segment "e", but the parallelism ensures the maintenance of directions**

Acquisition of digital images

The geometry of the film support

The film support has several deformations:
- no planarity of the film
- mechanical stress (stretching)
- chemicals stress (development).

The sensor could be considered geometrically stable during the time

In the digital sensors

The sensor is produced on a silicon support and it is quite stable during the time in geometric terms. It does not present any significant planarity problems for structural reasons or chemical stresses.

It can have some mechanical stress (e.g. sensor cleaning devices). If this camera is used for photogrammetric purposes this function must be disabled.

it modifies the mechanical behavior

There are some effects due to the sensor geometry.

1) Square pixel
2) Lines perpendicular to the columns

Not suqared pixel
Lines not perpendicular to the columns

we have to estimate the skew angle and the real size of pixels

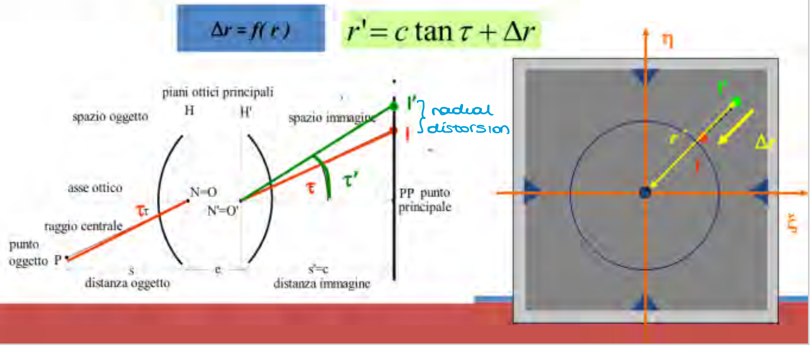
Calibration of metric camera

Lens distortions

The incident ray passes through the lens with an incident angle equal to τ and it has a final angle equal to $\tau' \neq \tau$

Lens of the metric camera have a spherical symmetry perfectly defined: in this case, the distortion has mainly a radial component.

$\Delta r = f(\tau)$ $\quad r' = c \tan \tau + \Delta r$



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Calibration of «commercial» camera

$$\Delta\xi_1 = \Delta r \frac{\xi - \xi_0}{r} = (\xi - \xi_0).(k_1 r^2 + k_2 r^4 + k_3 r^6 + ...)$$

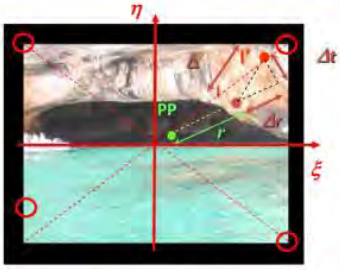
$$\Delta\eta_1 = \Delta r \frac{\eta - \eta_0}{r} = (\eta - \eta_0).(k_1 r^2 + k_2 r^4 + k_3 r^6 + ...)$$

In addition to radial distortions, there are:

Tangential distortions

$$\Delta\xi_2 = \left[p_1(r^2 + 2\xi^2) + 2p_2\xi\eta\right](1 + p_3 r^2)$$

$$\Delta\eta_2 = \left[p_2(r^2 + 2\eta^2) + 2p_1\xi\eta\right](1 + p_3 r^2)$$



Autocalibration will be discussed later.
↳ for professional cameras

Modified collinearity equations

$$\xi = \xi_0 + \Delta\xi_1 + \Delta\xi_2 - c\frac{r_{11}(X - X_0) + r_{21}(Y - Y_0) + r_{31}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)} = \xi_0 + \Delta\xi_1 + \Delta\xi_2 - c\frac{Z_x}{N}$$

$$\eta = \eta_0 + \Delta\eta_1 + \Delta\eta_2 - c\frac{r_{12}(X - X_0) + r_{22}(Y - Y_0) + r_{32}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)} = \eta_0 + \Delta\eta_1 + \Delta\eta_2 - c\frac{Z_y}{N}$$

Calibration of digital camera

Radial distortions

$$\Delta\xi_1 = \Delta r \frac{\xi - \xi_0}{r} = (\xi - \xi_0).(k_1 r^2 + k_2 r^4 + k_3 r^6 + ...)$$

$$\Delta\eta_1 = \Delta r \frac{\eta - \eta_0}{r} = (\eta - \eta_0).(k_1 r^2 + k_2 r^4 + k_3 r^6 + ...)$$

Tangential distortions

$$\Delta\xi_2 = \left[p_1(r^2 + 2\xi^2) + 2p_2\xi\eta\right](1 + p_3 r^2)$$

$$\Delta\eta_2 = \left[p_2(r^2 + 2\eta^2) + 2p_1\xi\eta\right](1 + p_3 r^2)$$

Distortion of the sensors

$$\Delta\xi_3 = c_1(\xi - \xi_0) + c_2(\eta - \eta_0)$$

sensors distortion because pixel aren't squared and lines aren't perpendicular (we need to add CORRECTION FACTORS)

Modified collinearity equations

$$\xi = \xi_0 + \Delta\xi_1 + \Delta\xi_2 + \Delta\xi_3 - c\frac{r_{11}(X - X_0) + r_{21}(Y - Y_0) + r_{31}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)} = \xi_0 + \Delta\xi_1 + \Delta\xi_2 + \Delta\xi_3 - c\frac{Z_x}{N}$$

$$\eta = \eta_0 + \Delta\eta_1 + \Delta\eta_2 - c\frac{r_{12}(X - X_0) + r_{22}(Y - Y_0) + r_{32}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)} = \eta_0 + \Delta\eta_1 + \Delta\eta_2 - c\frac{Z_y}{N}$$

Digital camera classification

Professional Camera

They have an interior orientation that is unknown and variable during the time. Given the stability of the sensor, these parameters depend only on continuous focusing: in the case of constant focusing they are constant over time. **Using analytical techniques so called "autocalibration", it is possible to estimate the inner orientation parameters, which can be considered characteristic of the camera for a defined focal length. Radial, tangential and sensor-related distortions are used.**

Mass Market camera

They have an inner orientation that is unknown and variable. They have a system that retract the optics in a protected position when the camera is turned off. **This movement modifies the optical definition and inner orientation parameters and the distortions changes every "switch on". If the camera does not switch off and the focus is not changed, these parameters can be considered constant: it is possible to estimate them using autocalibration.**
Radial, tangential and sensor-related distortions are used.

calibration changes as soon as we switch on or off the camera
↳ we change the focal length (c)

Aerial digital camera

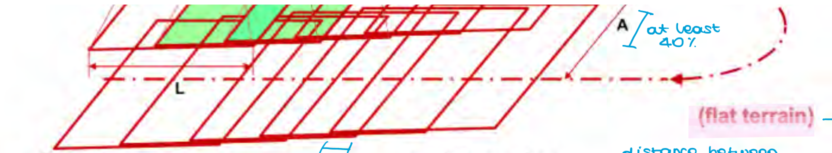
↓ aerial photogrammetry

General properties

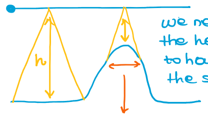
The **geometry is quite different** (principal distance, sensor size, et.) therefore it is better to consider the **GSD** (Ground Sample Distance) — it's a parameter better than resolution
Some sensors are able to work in **INFRARED band**
Image shifting is continously corrected **TDI (Time Delay and Integration)**

Frame
RECTANGURAL MULTIPLE SENSORS
DMC (*Digital Modular Camera*) di Z/I;
Ultracam D and X by Microsoft Vexcel;
DiMAC (*Digital Modular Aerial Camera*) by DiMAC System;

Pushbroom → data are collected line by line and not like a matrix (it's a merger of lines)
LINEAR MULTIPLE SENSORS
ADS 40 (*Aerial Digital Scanner*, I and II generation) e ADS 80 by Leica Geosystem;
3-DAS-1 and 3DAS-2 (*Digital Aerial Scanner*) by Wehrli & Associates/Geosystem;
JAS 150 by Jena Optronic AG.



→ if terrain isn't regular we have problems



we need to increase the height of flight to have always the same h

the covered surface is less than before

B = air base (at least 60%)
H = relative altitude (elevation)
L = size of the area (->%overlap)

I = image size
A = strips inter-distance (-> %sidelap) (distance between two data collections)
A_m = area of the model

⇒ we need to garantee 60% of overlapping in horizontal (between two images along same trajectory) and 40% for two following trajectories.

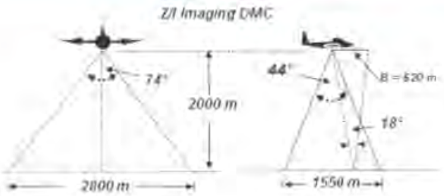
higher is the height higher is the dimension of the pixels and so lower is resolution

Considering problems about
- weather
- birds
- eagle attack

Aerial digital camera

DMC by Z/I

Flight planning with a relative altitude = 2000 m



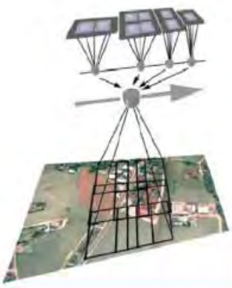
H [m]	Sm (1:)	GSD [m]	LT [m]	LL [m]	B (40 %LL) [m]
500	4167	0,05	691	384	154
1000	8333	0,10	1382	768	307
2000	16667	0,20	2765	1536	614
3000	25000	0,30	4147	2304	922
4000	33333	0,40	5530	3072	1229
5000	41667	0,50	6912	3840	1536
6000	50000	0,60	8294	4608	1843

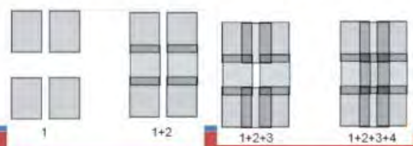
Parameters with different altitudes

Aerial digital camera

UltraCam by Vexcel/Microsoft

4 panchromatic sensors 4992 x 3328 (9 µm) and a focal length = 100 mm. Final image 14430 x 9420 where all distortions are corrected.

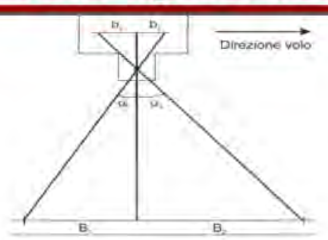




Aerial digital camera

ADS40 by Leica Geosystems

16° backward
26° foreward



H [m]	Sm (1:)	GSD [m]	LT [m]
500	7966	0,05	621
1000	15931	0,10	1243
2000	31862	0,21	2485
3000	47794	0,31	3728
4000	63725	0,41	4971
5000	79656	0,52	6213
6000	95587	0,62	7456

↳ higher zone but also worse quality

Semi-metric digital camera

ROLLEI d30 Metric

One of the first calibrated digital camera.
↳ we know internal calibration (we need to estimate other parameters) → ex lens

sensor: 2552 x 1920 pixel
size 2/3"

Focal length: 10-30 mm (such as a 40-120 mm for camera standard)

Focus with 2 fixed calibrated steps

Professional digital camera

Sony Nex 5K => little size and not heavy, so suitable for drones

CMOS (APS-C)
14.2 Mpx *(4592 x 3056)*
Size 23,5 x 15,6 *mm*
Interchangeable lens

Panasonic DMC-GF3

Sensor Live MOS
12.1 Mpx
Size *17.3 x 13 mm*.
Interchangeable lens

Mass market digital camera

Ricoh GX200

CCD with 12 Mpixel *(4000x3000)*
size 1/1.7"
Focal length 5,1 - 15,3 mm
(equivalent focal length at 35 mm: from 24 to 72 mm).

Canon IXUS 980 IS

CCD with 14.7 Mpixel *(4416 x 3312)*
Size 1/1.7"
Focal length = 6,1 - 29.0 mm
(equivalent focal length at 35 mm: from 36 to 133 mm).

Nokia N86 8M

CMOS with 8 Mpixel *(3280 x 2464)*
Focal length = 4,61 mm
└→ inside we have digital camera, GPS inertial platform (very useful tools)

Exterior orientation

Analytical procedures

The unknown are the exterior orientation parameters:

angles with respect to x, y, z

$(X_0, Y_0, Z_0, \omega, \phi, \kappa)_j$ j=1…num. fot,

position and orientation of the camera in absolute reference system

There are several approaches:

Independent orientation with 1 frame (j=1)

Simultaneaus orientation with 2 frames and 1 step (j=1-2)

Simultaneaus orientation with 2 frames and 2 steps(j=1-2) :

Relative orientation

Absolute orientation

Photogrammetric triangulation (**Simultaneaus orientation with all frames** which compose the block, j=1-n)

Independent orientation with 1 frame

→ only one image

Unknown parameters are 6:

we use collinearity equation to find the solution.

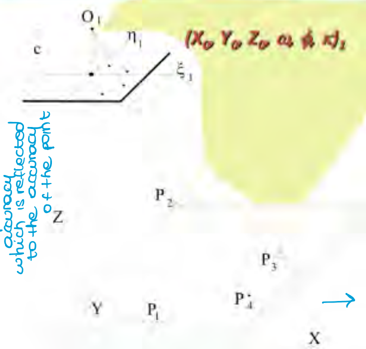
$$\xi = \xi_0 - c\frac{r_{11}(X - X_0) + r_{21}(Y - Y_0) + r_{31}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$

$$\eta = \eta_0 - c\frac{r_{12}(X - X_0) + r_{22}(Y - Y_0) + r_{32}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$

maybe we are using GPS or a map or satellite data

specific accuracy which is reflected to the accuracy of the point

Some ground control points (points with known coordinates) (X_i, Y_i, Z_i), are defined in the image coodinates(ξ_i, η_i)



Starting from the ground we obtain position of the camera (in photogrammetry in general we do the opposite thing)

For a generic point *i*, it is possible to write the following collinearity equations:

Two equations for each control point

$$\xi_i = \xi_0 - c\frac{r_{11}(X_i - X_0) + r_{21}(Y_i - Y_0) + r_{31}(Z_i - Z_0)}{r_{13}(X_i - X_0) + r_{23}(Y_i - Y_0) + r_{33}(Z_i - Z_0)} \quad \xi_i = f(\xi_0, c, \underline{X_0}, \underline{Y_0}, \underline{Z_0}, \underline{\omega}, \underline{\phi}, \underline{\kappa}, X_i, Y_i, Z_i)$$

$$\eta_i = \eta_0 - c\frac{r_{12}(X_i - X_0) + r_{22}(Y_i - Y_0) + r_{32}(Z_i - Z_0)}{r_{13}(X_i - X_0) + r_{23}(Y_i - Y_0) + r_{33}(Z_i - Z_0)} \quad \eta_i = f(\eta_0, c, \underline{X_0}, \underline{Y_0}, \underline{Z_0}, \underline{\omega}, \underline{\phi}, \underline{\kappa}, X_i, Y_i, Z_i)$$

solving the system we can find all the unknowns → all the ground control points are in one image

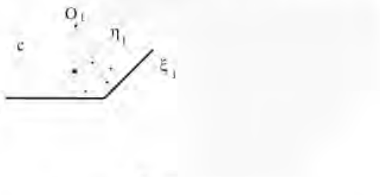
Independent orientation with 1 frame

It is necessary to have at least 3 ground control points (GCPs). It is possible to write 6 equations, with 6 unknown (underlined parameters). Usually, 4-6 GCPs are used, which are well distributed in the image.

to have a certain redundancy

$$\xi_i = f(\xi_0, c, \underline{X_0}, \underline{Y_0}, \underline{Z_0}, \underline{\omega}, \underline{\phi}, \underline{\kappa}, X_i, Y_i, Z_i)$$

$$\eta_i = f(\eta_0, c, \underline{X_0}, \underline{Y_0}, \underline{Z_0}, \underline{\omega}, \underline{\phi}, \underline{\kappa}, X_i, Y_i, Z_i)$$



Collinearity equation are not linear (in the unknowns) and therefore a linearization around approximate exterior orientation parameters is required.

The approximate values can be obtained:

- from the flight planning (aerial image)

Independent orientation with 1 frame

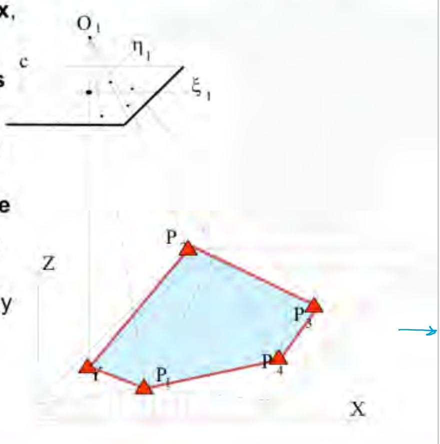
This procedure, called **pyramid vertex**, has the following disadvantages:

1. The concept of ray intersection is not considered

2. At least 3 GCPs are required, but usually more GCPs are used, especially on the board of the image

A good solution is only obtained in the area described by the GCPs. The extrapolation could be realized but only with a limited area (5-10%).

we are able to use only the points inside the chosen region (not OUTSIDE)



bases of the pyramid is made by points, vertex by the position of the cameras

=> limitation related to the board of the image

Independent orientation with 2 frames and 1 step

6 for the 1st frame
6 for the 2nd one

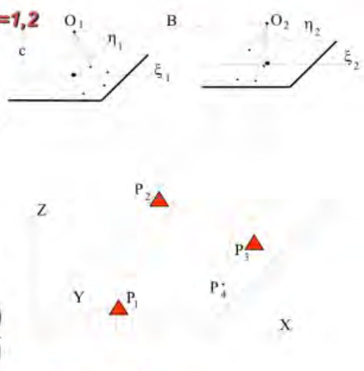
more roboustness of our system

There are 12 unknowns: ***$(X_0, Y_0, Z_0, \omega, \phi, \kappa)_j$*** ***j=1,2***

GCPs and tie point are used.

constrain, but not known position

For each GCP, it is possible to write 4 collinearity equations (2 for each frame)



$$\xi_{ij} = \xi_0 - c\frac{r_{11j}(X_i - X_{0j}) + r_{21j}(Y_i - Y_{0j}) + r_{31j}(Z_i - Z_{0j})}{r_{13j}(X_i - X_{0j}) + r_{23j}(Y_i - Y_{0j}) + r_{33j}(Z_i - Z_{0j})}$$

$$\eta_{ij} = \eta_0 - c\frac{r_{12j}(X_i - X_{0j}) + r_{22j}(Y_i - Y_{0j}) + r_{32j}(Z_i - Z_{0j})}{r_{13j}(X_i - X_{0j}) + r_{23j}(Y_i - Y_{0j}) + r_{33j}(Z_i - Z_{0j})}$$

$$\xi_{i1} = f(\xi_0, c, \mathbf{X_{01}}, \mathbf{Y_{01}}, \mathbf{Z_{01}}, \boldsymbol{\omega_1}, \boldsymbol{\varphi_1}, \mathbf{k_1}, X_i, Y_i, Z_i)$$
$$\eta_{i1} = f(\eta_0, c, \mathbf{X_{01}}, \mathbf{Y_{01}}, \mathbf{Z_{01}}, \boldsymbol{\omega_1}, \boldsymbol{\varphi_1}, \mathbf{k_1}, X_i, Y_i, Z_i)$$
$$\xi_{i2} = f(\xi_0, c, \mathbf{X_{02}}, \mathbf{Y_{02}}, \mathbf{Z_{02}}, \boldsymbol{\omega_2}, \boldsymbol{\varphi_2}, \mathbf{k_2}, X_i, Y_i, Z_i)$$
$$\eta_{i2} = f(\eta_0, c, \mathbf{X_{02}}, \mathbf{Y_{02}}, \mathbf{Z_{02}}, \boldsymbol{\omega_2}, \boldsymbol{\varphi_2}, \mathbf{k_2}, X_i, Y_i, Z_i)$$

for each point we can write 4 equations

all unknowns

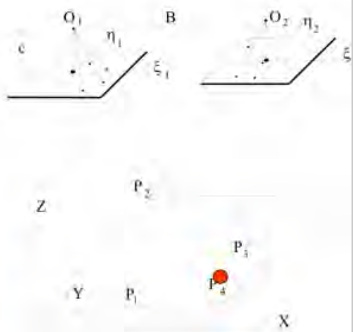
Independent orientation with 2 frames and 1 step

It is possible to use some tie points, which are points without known coordinates, but well detectable in the image.

For each TIE POINT, it is possible to define the unknown obkect coordinates ***(X_i, Y_i, Z_i)***, and to measure the image coordinates ***(ξ_{ij}, η_{ij})*** ***j=1,2***

↳ we recognise the same point in the 2 frames

For each tie point, it is possible to write 4 collinearity equations, adding 3 new unknowns (cyan boxes):



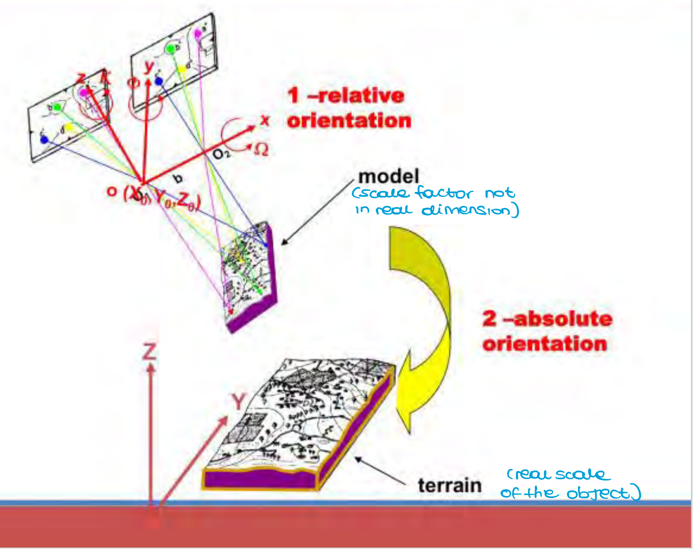
TIE POINTS depends on accuracy of orientation, operators, other factors

CONTROL POINTS
↳ used to control the results

- (x,y,z) estimated by plotting

$$\xi_{i1} = f(\xi_0, c, \mathbf{X_{01}}, \mathbf{Y_{01}}, \mathbf{Z_{01}}, \boldsymbol{\omega_1}, \boldsymbol{\varphi_1}, \mathbf{k_1}, \mathbf{X_i}, \mathbf{Y_i}, \mathbf{Z_i})$$
$$\eta_{i1} = f(\eta_0, c, \mathbf{X_{01}}, \mathbf{Y_{01}}, \mathbf{Z_{01}}, \boldsymbol{\omega_1}, \boldsymbol{\varphi_1}, \mathbf{k_1}, \mathbf{X_i}, \mathbf{Y_i}, \mathbf{Z_i})$$
$$\xi_{i2} = f(\xi_0, c, \mathbf{X_{02}}, \mathbf{Y_{02}}, \mathbf{Z_{02}}, \boldsymbol{\omega_2}, \boldsymbol{\varphi_2}, \mathbf{k_2}, \mathbf{X_i}, \mathbf{Y_i}, \mathbf{Z_i})$$

Independent orientation with 2 frames and 2 steps



Independent orientation with 2 frames and 2 steps

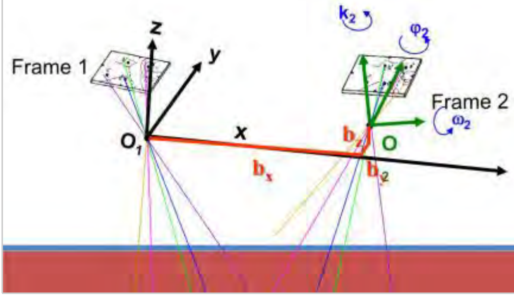
=> NOT VERY USED

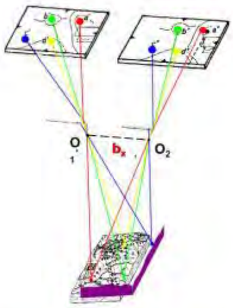
The aim is create a model, allowing the intersection between the infinite couples of rays.

It is possible to realize this condition:
- Blocking 1 frame and moving the frame 2

There are 6 DOF (degree of freedom):
- 3 translations b_x, b_y, b_z
- 3 rotations ω_2, φ_2, κ_2

- But the value b_x defines only the scale of the model





Relative orientation is based on 5 parameters (unknowns) : $b_y, b_z, \omega_2, \varphi_2, \kappa_2$

→ we fix the base between the frames (the distance between the two images)

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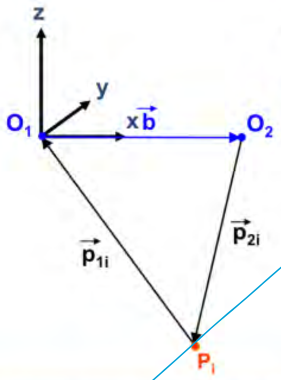
Independent orientation with 2 frames and 2 steps

Analytical solution

=> NOT IMPORTANT



Two straight lines have intersection only if they have the complanarity conditions

This condition could be described as this multiplication

b, p_1i, p_2i

$$\vec{b} \wedge \vec{p_{2i}} \times \vec{p_{1i}} = 0$$

i = 1 ... ≥5

$$p_\eta = \frac{c}{h}db_y + \frac{\eta_2}{h}db_z + \frac{\xi_1\eta_1}{c}d\phi_1 - \left(c+\frac{\eta_1^2}{c}\right)d\omega_1 - \xi_1 d\kappa_1 - \frac{\xi_2\eta_2}{c}d\phi_2 + \left(c+\frac{\eta_2^2}{c}\right)d\omega_2 + \xi_2 d\kappa_2$$

L'orientamento di due fotogrammi

Asymmetric relative orientation

Starting from the collinearity eq., it is possible to define **the parallax equation of height**.

$$p_\eta = \frac{c}{h}db_y + \frac{\eta_2}{h}db_z - \frac{\xi_2\eta_2}{c}d\phi_2 + \left(c+\frac{\eta_2^2}{c}\right)d\omega_2 + \xi_2 d\kappa_2$$

With $p_\eta = 0$ and writing at least 5 of these equations for 5 homologous points, it is possible to estimates ***(b_y, b_z, ω_2, ϕ_2, κ_2)***

Symmetric relative orientation

$$p_\eta = -\xi_1 d\kappa_1 + \xi_2 d\kappa_2 + \frac{\xi_1\eta_1}{c}d\phi_1 - \frac{\xi_2\eta_2}{c}d\phi_2 + \left(c+\frac{\eta_2^2}{c}\right)d\omega_2$$

With $p_\eta = 0$ and writing at least 5 of these equations for 5 homologous points, it is possible to estimates ***(ϕ_1, κ_1, ω_2, ϕ_2, κ_2)***

If the equations are more than 5, the solution is estimated by LS. The residuals are the «residual parallaxes», which describe the quality of the orientation. **In the relative orientation, GCPs are not required.** only absolute one

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Independent orientation with 2 frames and 2 steps

Analytical solution

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X_u \\ Y_u \\ Z_u \end{pmatrix} + m \cdot \mathbf{R} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\boldsymbol{X} \approx \boldsymbol{x} = \boldsymbol{X}^0 \Rightarrow \Omega = d\Omega,\ \Phi = d\Phi,\ K = dK,\ m = 1 + dm,\ \boldsymbol{X}_u = d\boldsymbol{X}_u$$

$$\begin{pmatrix} \cos\phi\cos\kappa & -\cos\phi sen\kappa & sen\phi \\ \cos\omega sen\kappa + sen\omega sen\phi\cos\kappa & \cos\omega\cos\kappa - sen\omega sen\phi sen\kappa & -sen\omega\cos\phi \\ sen\omega sen\kappa - \cos\omega sen\phi\cos\kappa & sen\omega\cos\kappa + \cos\omega sen\phi sen\kappa & \cos\omega\cos\phi \end{pmatrix}$$

$$d\boldsymbol{R} = \begin{pmatrix} 1 & -dK & d\Phi \\ dK & 1 & -d\Omega \\ -d\Phi & d\Omega & 1 \end{pmatrix} \qquad (1 + dm)(-dK) = -dK - dKdm \sim -dK$$

$$m \cdot \boldsymbol{R} = (1 + dm)d\boldsymbol{R} = \begin{pmatrix} 1+dm & -dK & d\Phi \\ dK & 1+dm & -d\Omega \\ -d\Phi & d\Omega & 1+dm \end{pmatrix} = \boldsymbol{I} + \begin{pmatrix} dm & -dK & d\Phi \\ dK & dm & -d\Omega \\ -d\Phi & d\Omega & dm \end{pmatrix}$$

Independent orientation with 2 frames and 2 steps

Analytical solution

$$X = dX_u + X^0 dm + Z^0 d\Phi - Y^0 dK + X^0$$

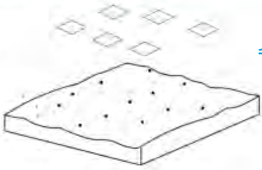
$$Y = dY_u + Y^0 dm - Z^0 d\Omega + X^0 dK + Y^0$$

$$Z = dZ_u + Z^0 dm + Y^0 d\Omega - X^0 d\Phi + Z^0$$

Photogrammetric triangulation

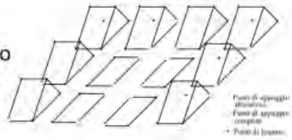
Photogrammetric triangulation can be made using two different approaches:

1) **Bundle block adjustment** based on the direct use of the equations of collinearity: this method allows to obtain the maximum possible precision and is the only one that allows to use frames of different nature (metric, semi-metric and non-metric) allowing the determination of the parameters of interior and exterior orientation unknown ;



=> working with all frames

2) **Independent model**, which is based on the analytical absolute orientation, considering all stereo models which compose the block, already realized.
↳ pair of the process
→ not very used



In this way, it is possible to reduce the number of Ground control points
→ we can use Tie points as equations and have additional info using all the frames together
→ high number of equations and unknowns

Stereoscopic plotting

from image to objects space

The plotting requires that the **geometry of the two frames is defined by means an analytical projection.** The problem is easier if the parameters of **interior and exterior orientation of the frames are known**.

Image 1 (L)

Image 2 (R)

The plotting of a stereoscopic model: **consists in the determination of the coordinates object of a number of points, which are sufficient to describe the geometry and dimensions of this object**.

It is implemented, as all orientation steps, **by means of instruments called analytical plotting (optical-mechanical) or digital plotting**

computers

Stereoscopic plotting

Geometrical solution

Knowing the interior and exterior orientation parameters, the image coordinates of the homologous points P1 and P2 will be measured and using the equations of collinearity, the object coordinates (X, Y, Z) of the point P are measured

collinearity eq. on the object point of view (opposite than usual)

$$X = X_0 + (Z - Z_0)\frac{r_{11}(\xi-\xi_0)+r_{12}(\eta-\eta_0)-r_{13}c}{r_{31}(\xi-\xi_0)+r_{32}(\eta-\eta_0)-r_{33}c}$$

$$Y = Y_0 + (Z - Z_0)\frac{r_{21}(\xi-\xi_0)+r_{22}(\eta-\eta_0)-r_{23}c}{r_{31}(\xi-\xi_0)+r_{32}(\eta-\eta_0)-r_{33}c}$$

$$X = X_{01} + (Z - Z_{01})\, k_{x1}$$
$$Y = Y_{01} + (Z - Z_{01})\, k_{y1}$$
$$X = X_{02} + (Z - Z_{02})\, k_{x2}$$
$$Y = Y_{02} + (Z - Z_{02})\, k_{y2}$$

linear equation from which compute X, Y, Z

coordinates of points in object reference system

There are **4 linear equations and three unknowns *X, Y, Z***. Considering the first and third equations, ***Z*** could be estimated as:

$$Z = \frac{X_{02} - Z_{02}k_{x2} + Z_{01}k_{x1} - X_{01}}{k_{x1} - k_{x2}}$$

→ replace this value over there

Moreover, using the first or third equation, X is estimated and Y is estimated by the other equations (2nd or 4th). If the X or Y values are slighly different, an average value is considered.

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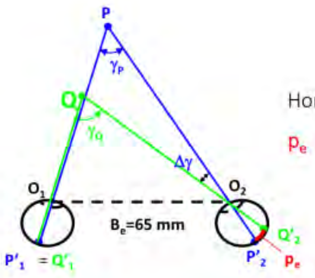
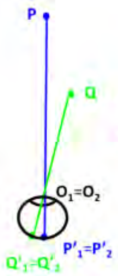
Stereoscopic vision

Natural stereoscopic vision



Horizontal parallax

p_e



Vertical parallax

$p_v=0$

HORIZONTAL VISUAL PLANE

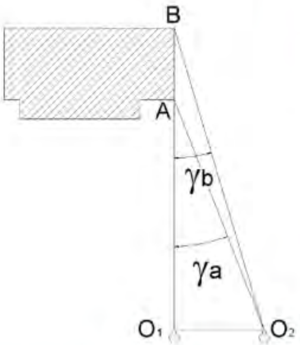
VERTICAL VISUAL PLANE

in photogrammetry we use only horizontal parallax (not vertical)

The fusion of the two images which are focused on the retinas in a unique 3D image is called stereoscopic vision. The images on the retinas have an horizontal parallax but no vertical parallax.

Stereoscopic vision

Natural stereoscopic vision: benefits



We define **stereoscopic acuity or stereoscopic power $d\gamma_{px}$** the minimum value of parallax angle which allow to distinguish ~~to~~ 2 different points and to measure them. → between our sight and the object

$$\gamma_b - \gamma_a = d\gamma_{ps} \quad \Longrightarrow \quad A \equiv B$$

Under normal «eyes condition» , the stereoscopic acuity range is 5" - 10". The monocular acuity is 1'.

Su una distanza standard di focamento l'acuità stereoscopica corrisponde a 6÷12 µm e quella monoculare a 60÷100 µm

A first conclusion is that in a plotting process, it is possible to collimate the homologous points with a better sensibility and precision, working with a stereoscopic vision.

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Stereoscopic vision

Artificial stereoscopic vision

In photogrammetry, the plotting process is possible ONLY IF you have two frames collected by two different points of view

Observing the two images of the same object with the eyes (left eye → left image, right eye → right image) it is possible to artificially create the stereoscopic vision.

The image fusion (3D model) in a «virtual space» will be possible only if:

- **no vertical parallax condition during the shooting step**
- **the scale between of the images is lower than 14%** → size of the object remains more or less constant
- **horizonal parallax is lower than 1.3 gon**

Stereoscopic vision

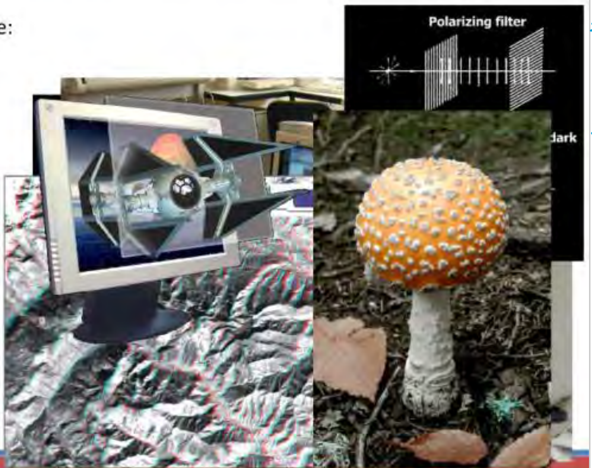
Artificial stereoscopic vision

It is necessary to see separately the single images. The brain will merge the images into a unique 3D model.

The subtractive techniques are:

1) **Geometrical separation** (stereoscope);
2) **Physical separation** (anaglyph or Polarization
3) **Temporal separation** (different image emission).

The additive techniques are realized using some particular image processing, which produce lenticular images



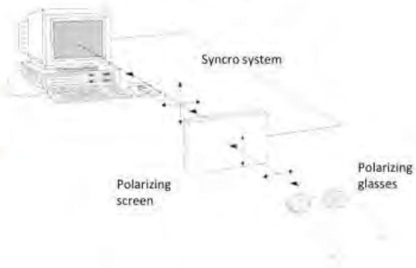
1) Two images in movement to generate sterescopic vision
2) with particular supports (gloss) we split two images
3) we separate temporary images for right and left eye (1 second for the left, 1 second for the right and so one)...

Stereoscopic vision

Artificial stereoscopic vision: polarized filter

Polarized filters use the physical phenomenon of the light polarization. These filters are placed in front of the PC monitor on which the stereoscopic images are projected. The images are alternately projected with a very high frequency (100 Hz). The filter is appropriately synchronized with the screen and is able to polarize the light in a horizontal and vertical direction. Stereoscopic vision can be perceived using glasses with polarized lenses in both directions. Nowadays, it is possible to have the same "effect", but using "active" glasses and normal screen. The process is directly made in the glasses (glasses for 3D gaming)

we distinguish between vertical and horizontal polarization
↳ before we distinguish between colours (red and blue)



→ we don't modify colour of image

Stereoscopic vision

Artificial stereoscopic vision: stereoscopic room

In this case, we need:

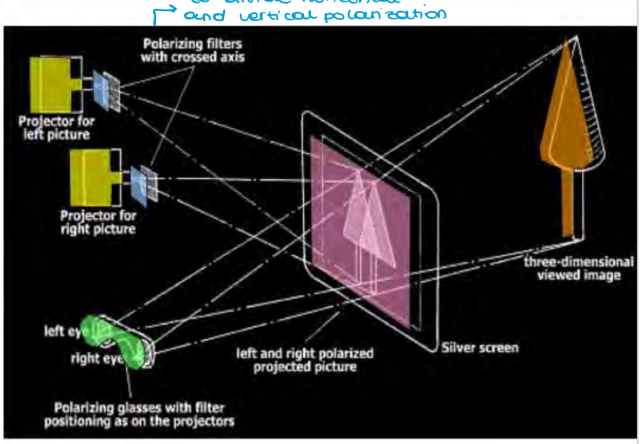
Two Projectors;

Polarizing filters over each projection lens aligned at right angles;

A **Silver screen,** in order to avoid the lost the light polarization

Polarizing glasses with filter (same polarization of the projector)

↳ to divide horizontal and vertical polarization



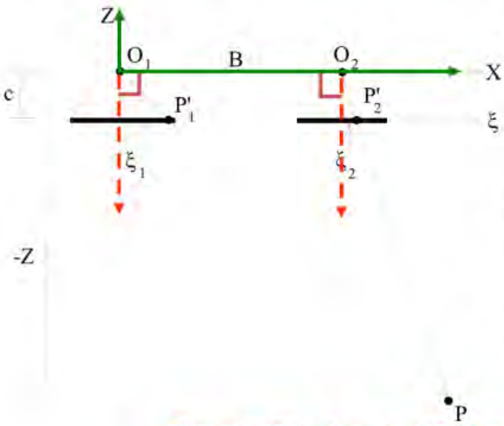
The precision of photogrammetry

camera

distance between camera and object

design the better conditions

The case with «normal» axis



Photogrammetry is used to plot the spatial objects starting from two frames using stereoscopic vision (human or simulated by the PC). If the axes of the two camera (normal direction of the shooting) are normal to the base (a segment that joins the two camera centers) and are parallel to each other, the plotting is particularly easy.

$$X_{01} = Y_{01} = Z_{01} = 0$$
$$X_{02} = B \qquad Y_{02} = Z_{02} = 0$$
$$\xi_{01} = \eta_{01} = \xi_{02} = \eta_{02} = 0$$
$$\omega_1 = \omega_2 = \phi_1 = \phi_2 = \kappa_1 = \kappa_2 = 0$$

we don't have rotations (inner orientation is very semplified ~ difficult to have this condition in reality)

This condition is very difficult to be verified in the field, but the aim is to be quite close to this condition.

The precision of photogrammetry

The case with «normal» axis

used in terrestrial not easy in aerial condition!

The rotation matrix becomes the **Identity matrix of third order**. The collinearity equation will be simplified as:

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

→ only an identity matrix → no rotation between the two frames

$$X = X_0 + (Z - Z_0)\frac{r_{11}(\xi-\xi_0)+r_{12}(\eta-\eta_0)-r_{13}c}{r_{31}(\xi-\xi_0)+r_{32}(\eta-\eta_0)-r_{33}c} \qquad Y = Y_0 + (Z - Z_0)\frac{r_{21}(\xi-\xi_0)+r_{22}(\eta-\eta_0)-r_{23}c}{r_{31}(\xi-\xi_0)+r_{32}(\eta-\eta_0)-r_{33}c}$$

(X_0 → B)

In particular:

Image 1 $X = -Z\dfrac{\xi_1}{c} \qquad Y = -Z\dfrac{\eta_1}{c}$

→ so we have the relation between ground position (x, y) and image one (ξ, η)

Image 2 $X = B - Z\dfrac{\xi_2}{c} \qquad Y = -Z\dfrac{\eta_2}{c}$

Starting from the equations with Y, it is possible to define:

$$\eta_1 = \eta_2 \qquad p_\eta = \eta_2 - \eta_1$$

(Vertical parallax is null)

The precision of the photogrammetry

The theory of errors

Working in the same way on the other two equations, we finally get the following expressions that express the standard deviation of the object coordinates in the "normal" case :

$$\sigma_X = \sqrt{\left(\frac{\xi_1}{c} m_b \frac{Z}{B} \sigma_{P_\xi}\right)^2 + \left(m_b \cdot \sigma_\xi\right)^2}$$

$$\sigma_Y = \sqrt{\left(\frac{\eta_1}{c} m_b \frac{Z}{B} \sigma_{P_\xi}\right)^2 + \left(m_b \cdot \sigma_\eta\right)^2}$$

$$\sigma_Z = \frac{Z^2}{c \cdot B} \sigma_{P_\xi}$$

It's possible to estimate "a priori" the precision of the final results

The precision of the photogrammetry

The theory of errors

Image coordinates $\xi_1 = \eta_1 = 50\ mm\ \pm 7\ \mu m$
Parallax precision $\sigma_{P\xi} = \pm 5\ \mu m$
Principal distance $c = 150\ mm$

$$\sigma_Z = \frac{Z^2}{c \cdot B} \sigma_{P_\xi}$$

B=1, Z=3

		B/Z = 1:1		B/Z = 1:3		B/Z = 1:10		B/Z = 1:20	
Z [m]	m_b	σ_{XY}	σ_Z	σ_{XY}	σ_Z	σ_{XY}	σ_Z	σ_{XY}	σ_Z
7500	50000	0.36	0.25	0.43	0.75	0.90	2.50	1.70	5.00 *m*
1500	10000	0.72	0.50	0.86	1.50	1.81	5.00	3.41	10.00 *dm*
150	1000	0.72	0.50	0.86	1.50	1.81	5.00	3.41	10.00 *cm*
15	100	0.72	0.50	0.86	1.50	1.81	5.00	3.41	10.00 *mm*
3.75	25	0.18	0.13	0.22	0.38	0.45	1.25	0.85	2.50 *mm*

- With the same B/Z ratio, **standard deviations of the coordiantes are directly proportional to the scale factor** of the image. T**hen, it is possible to obtain all level of precision, but it is FUNDAMENTL to define a correct scale of the frame!!**
- with the same scale of frame, **the errors in Z are inversely proportional to the B/Z ratio. In fact, the errors on X and Y are lightly influenced by B/Z ratio variation**. If B/Z ratio is a little bit lower than 1:1, the threes stadard deviations have the same values.
- with a specific base B, **Z errors increase with the square of the distance between camerea and object (shooting distance).** $\sigma_Z \propto Z^2$

mb: changing scale factor, resolution changes too so with a B/Z = n we can improve quality decreasing mb (Z is fixed so we need to change the focal length)

error in Z is strongly correlated with B/Z, its values are in inverse proportion to B/Z (higher is B/Z less is mb)

=> distance Z can change so steps of data process are complicated to design.

The precision of the photogrammetry

Empirical rules

These previous values could be used also for **NATURAL POINTS**, but it is important to consider the uncertainity of point definition

we have to add a term

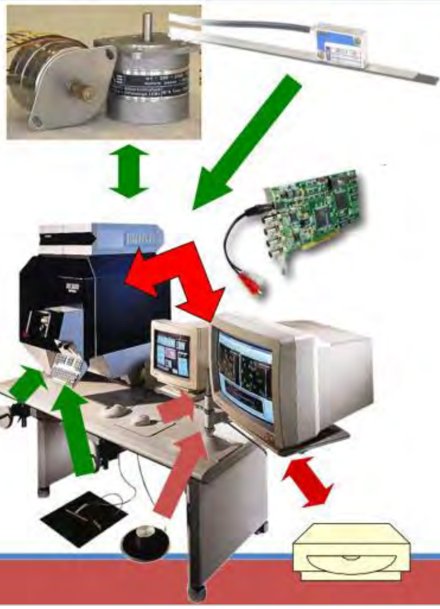
$$\sigma_{XY(nat)} = \sqrt{\sigma^2_{XY(seg)} + \sigma^2_{XY(def)}} \qquad \sigma_{Z(nat)} = \sqrt{\sigma^2_{Z(seg)} + \sigma^2_{Z(def)}}$$

theoretical — accuracy of the points we choose

points	SD – horizontal [cm]	SD – Vertical [cm]
Edge of building or fence	7-12	8-15
Manhole egde or center	4-6	1-3
Farming edge	20-100	10-20
Bushes or trees	20-100	20-100

very low quality of precision

Analytical plotter



Digital photogrammetry

Automation of the photogrammetric process

The digital nature of the digital data allows automatic processing on a computer using special algorithms and techniques.

The images can be automatically pre-processed in order to adapt both the radiometric content (image enhancement and image restoration techniques) and the geometry (image resampling) as the photogrammetry needs.

Measurements of characteristic points (fiducial marks, homologous points, ground control points) can be done in semi-automatic mode, with approximate collimation of the user and a better refinement of the collimation with automatic techniques (image matching). The same operations can be performed in a **fully automatic mode, combining image matching techniques with automatic methods for extracting characteristic points (feature extraction).**

The relative orientation operations and aerial triangulation use robust estimation techniques (e.g. LMS) to automatically eliminate some possible gross errors in the direct measurements (image coordinates).

Cartographic products in vector format (geometric entities) or raster (DTM, orthophotos) can be extracted semi-automatically with numerous digital techniques (automatic DSM extraction, edge extraction, digital orthoprojection)

Digital photogrammetry

PRO vs CONS

These factors have several **PRO** and **CONS**:

+
- User has not to be a very skilled and expert user
- Number of potential user is widely increased → increase potential users because PC isn't so difficult
- Photogrammetry is more diffused
- Cost are lower than the «classical» photogrammetry
- New products can be generated

−
- Software solutions are more complex
- It is important to use more rigorous statistical algorithms (robust techniques)
- Not-expert user cannot use the digital photogrammetry in a correct way

(→ we need procedures to control results; → to remove errors; → softwares in general do everything alone — someone gives directly the result without infos about statistical analysis and so precision)

However, critical operations (shooting, ground point and orientation) are always steps to be reserved for expert operators. **The possible automatic steps not preserve from a bad planning and execution of these three fundamental stages.**

27/04/18

DIGITAL IMAGE: can be planned and modify a little bit, change its quality, extract from it different info ←

Image processing

→ computer vision: a lot of algoritm of photogrammetry used to have a computer vision itself

Classification

↱ to improve quality of image

The image-preprocessing techniques are used in several disciplines (Fotogrammetria, *Computer Vision, Remote Sensing*) , in order to improve the **radiometric quality adiometrica** of an image, considering the aim of the application. extract info from live image point from I/E orientation

In photogrammetry, these techniques are used to reduce the noise (degradation) of the image, generated during the acquisition or to improve the quality. This step is called ***image restoration o image enhancement.***

The filtering operator

Image processing

Convolution

applied to original matrix to modify original info multiplying image itself and convolution matrix.

Convolution is a matrix operator which allow to elaborate an image $G=g(x,y)$ using a convolution matrix W, obtaining another imag $\overline{G} = \overline{g}(x,y)$. This image is filtered. This trasformation can be described as: :

$$\overline{g}_{i,j} = \sum_{k=-n}^{n} \left(\sum_{l=-n}^{l=n} g_{i-k,j-l} \cdot w_{n+1+k,n+1+l} \right)$$

FINAL IMAGE

$$\overline{G} = G * W$$

Where w is a generic element of the convolution matrix W, with size = (2n+1).

Changing the size and the values of W, it is possible to obtain **low-pass convolution filter or high-pass convolution filter**. (2 classes of matrixes)

The **low-pass convolution filter (LP)** are used for ***image-restoration.*** (reach a noise)

The high-pass convolution filter (HP) are used in ***image enhancement.*** → stress the radiometry difference to highlight the h of the object

Filter image + low frequency, we can remove noise in image

Image processing

LAPLACE operator

in our image we apply this convolution matrix W because we can remove noise and obtain the final image →

1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	2	1	1	1	1	1
1	1	2	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	3	1	1	1	1	1
1	1	1	1	1	1	8	2	1	1	1	1
1	1	1	1	2	8	8	9	4	2	1	1
1	1	1	1	1	2	5	1	1	1	1	1
1	1	2	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1

12x12

It is a LP filter, to extract the radiometric edge

$$W = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\overline{g}_{i,j} = \sum_{k=-n}^{n} \left(\sum_{l=-n}^{l=n} g_{i-k,j-l} \cdot w_{n+1+k,n+1+l} \right)$$

0	0	0	0	0	1	0	0	0	0
0	1	0	0	1	-4	1	0	0	0
1	-4	1	0	0	1	0	0	0	0
0	1	0	0	0	2	0	0	0	0
0	0	0	0	2	1	3	0	0	0
0	0	0	1	14	-18	11	4	1	0
0	0	1	3	-19	-2	-21	-3	1	1
0	1	0	2	7	-8	12	3	1	0
1	-4	1	0	1	4	0	0	0	0
0	1	0	0	0	0	0	0	0	0

10x10

It is the second derivatives of the radiometric values between the adiacent pixels.

I can hilight element and remove noise. I use laplace operator which works by second derivaty. It stresses the side of image when we have different values of radiometric informations, we can extract values so we can see h of all elements.

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Image processing

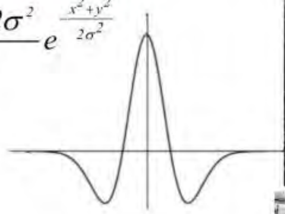
LoG operator

we can combine Laplace and Gauss to extract h of single elements. Applying filter we can see better the part of trees.

Laplacian of Gaussian (LoG) operator is a low-pass filter which allows to realizea LP filtering and to identify the NULL points, on the same time. It is based on a double difference of the GAUSS distribution.

$$LoG = \frac{x^2 + y^2 - 2\sigma^2}{2\pi\sigma^6} e^{\frac{x^2+y^2}{2\sigma^2}}$$

LoG plot is like a reverse sombrero. The unique variable is σ



without filters it's difficult to extract elements in dark regions.

Where σ is small, there are a lot of NULL points.

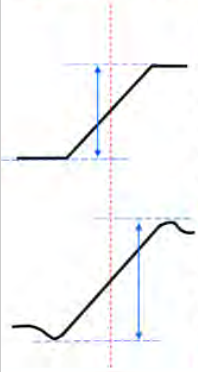
A big value of σ allows to exclude the nearest edges and extract only the raw information

Image enhancement

WALLIS filter

→ low pass filter

Wallis filter (Wallis, 1976) is widely used in photogrammetry for *image enhancement*. This filter allows to ***increase the dynamic range and to improve the radiometric range in the region with high contrast*** (radiometric edge and interesting point) e si ***aumenta la differenza agli estremi dei bordi radiometrici, senza modificarne la posizione geometrica.***



The aim of the Wallis filter is to force the average and standard deviation of the radiometric value of an image to predefined values (Baltsavias, 1991). Since the standard deviation is related to the contrast of an image, it can be said that the Wallis filter can improve or reduce the contrast, favoring the suppression or improvement of the radiometric edges. **The filter can therefore have a LP effect.** → to extract feature (element) from the image

The "forcing" operation of mean and standard deviation takes place locally, allowing to increase the contrast and to make visible radiometric details. This operation is very important because it optimizes **the performance of the feature extraction and matching techniques.**

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Image Matching

Classification

Image matching functions allow to ***automatize the operation of coordinates measurement of the homologous points.*** They are based on the radiometric index, which is defined as digital number in a digital image.

Image matching **can be defined as the whole of techniques devoted to automatically define a relation between the points which are extracted by two or more images.**

These techniques are classified as following:

- ***Feature Based Matching (FBM)***
- ***Area Based Matching (ABM)***
- ***Relational Based Matching (RBM)***

Image Matching

Area Based Matching

In **ABM techniques**, also called autocorrelation techniques, the automatic relation between homologous points is done by comparing the radiometric tones of the pixels around the points. Usually they are considered square areas, **called "correlation windows".**

The matching is made by evaluating the degree of similarity between the correlation windows, considering that homologous points must have identical correlation windows in terms of radiometric tone values.

This condition is guaranteed only if the object represented is not subject to perspective deformations and if the images are not subject to radiometric distortions (changes in lighting, noise, etc).
As known, images represent a central perspective of the object, it can be said that ABM techniques provide automatic couplings only if the object represented in the correlation windows can be approximated to a plane.

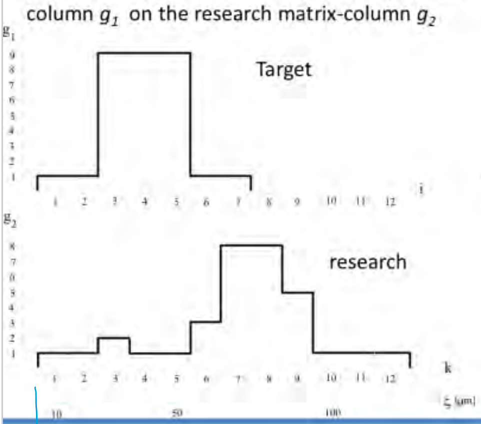
The most famous ***ABM*** techniques adopted in photogrammetry are:

- ***Cross-Correlation (CC)* or autocorrelation with «entire pixel»**
- ***Least Square Matching (LSM)* or sub-pixel autocorrelation**

Image Matching

Area Baser Matching: cross-correlation

The similarity between two images are calculated by means a correlation coefficient *(Normalized Cross-Correlation, NCC or r)* concerning all position of the target matrix-column g_1 on the research matrix-column g_2



we have a correlation coefficient made by pixel target and original image. The content of target is similar to the original image. We find where our reference is our original image.

$$r = \frac{\sigma_{12}}{\sigma_1 \cdot \sigma_2} = \frac{\sum (g_1 - \bar{g}_1) \cdot (g_2 - \bar{g}_2)}{\sqrt{\sum (g_1 - \bar{g}_1)^2 \cdot \sum (g_2 - \bar{g}_2)^2}}$$

r = coefficient of correlation

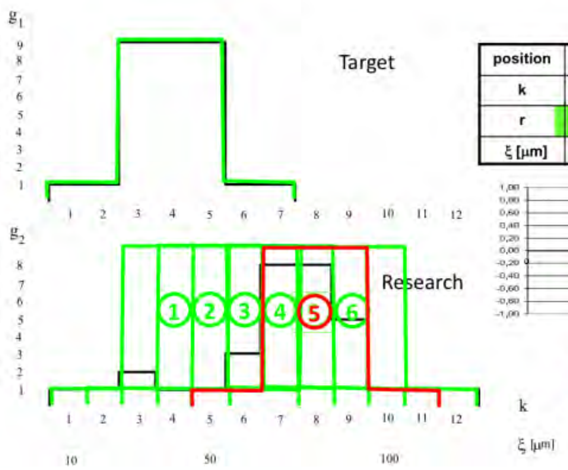
reference is a known geometry so matching means to find a known object in our image. Realize close correlation gives errors.

(pixel size = 10 mm)

diagram gives correlation but we need to pay attention if it's correct because not all points are real and correct.

Image Matching

Area Baser Matching: cross-correlation



r

position	1.	2.	3.	4.	5.	6.
k	1(4)7	2(5)8	3(6)9	4(7)10	5(8)11	6(9)12
r	-0.40	-0.51	0.00	0.73	0.92	0.24
ξ [µm]	40	50	60	70	80	90

The right position is where r is MAX, such as 5, with the coordinate x = 80 mm

Image Matching

ABM: Least Square Matching

(statistical solution)

Apply target matrix and look for solution by geometrical transformation

Under "theoretical" conditions, in a bi-dimensional image space x, y, the relation should be:

$$g_1(x,y) = g_2(x,y)$$

Where $g_1(x,y)$ is the radiometric value of one pixel in the target matrix with a size equal to $n.n$ and $g_2(x,y)$ is the radiometric value of one pixel in the research matrix, with the same size $n.n$

By generalizing the problem to a real case, with a geometric and radiometric distortions, known the approximate position of the target matrix with respect to the research matrix, it is possible to define:

$$g_1(x,y) = c \cdot g_2(u,v) + b$$

$u=u(x,y)$ $v=v(x,y)$ are the **two-dimensional geometric transformation** between the research matrix and the target matrix → look for best solution by LSM

c: **radiometric gain,** a multiplicative factor of the radiometric values.

b: **radiometric shift,** it is a shift in the gray scale values

Image Matching

ABM: Least Square Matching

If a geometric transformation between the research and target matrices is defined and writing the equation:

$$g_1(x,y) = c \cdot g_2(u,v) + b$$

for each pixel inside the target and research matrix, we can write a system of nxn equations in function of the unknowns *u, v, c, b* which can be solver by least squares (also called LSM). In this way it is possible to estimate the correct subpixel position of the research matrix with respect to the target matrix.

X TRANSLATION

Assuming that the target and research matrix are aligned along one direction (y) and there is no scale factor, **it is possible to considered a transformation with only one parameter which is a translation in the x direction**

$$u = x + t_x$$
$$v = y$$

$$g_1(x,y) = c \cdot g_2(x + t_x, y) + b$$

Image Matching

ABM: Least Square Matching, Tx

The approximate values of the unknowns are updated at each iteration by geometrically resampling the research matrix g_2 (x, y) and the radiometry in the target matrix g_1 (x, y).

For each iteration the correlation coefficient between the two matrices is calculated and the calculation using least squares is stopped when the correlation coefficient tends to 1, or when the values of correction of the unknowns have negligible values (lower than 1/10 of pixels in the translation component).

The least squares technique allows to estimate the subpixel tx translation of the research matrix with respect to the template matrix, which will be:

$$X_r = X_s + t_x$$

It also allows to estimate the **Variance and Covariance matrix of the unknowns**, thus providing information on the precision of the estimated parameters, in particular for the geometric ones (e.g σ_{tx}).

Image Matching

ABM: Least Square Matching, affinity

Under the hypothesis that the geometric relation between research and target matrix is an affinity, the equations of LSM are:

$$u = ax + by + c \qquad g_1(x,y) = c \cdot g_2(ax + by + c, dx + ey + f) + b$$
$$v = dx + ey + f$$

Equations are linearized and the corrections to unknowns are estimated at least squares with an iterative procedure. At each iteration, the parameters of the approximate unknowns are updated by geometrically resampling of the search matrix and radiometrically on the target matrix. The calculation is stopped when the corrections of the unknowns are negligible and the correlation coefficient tends to 1.

The final displacement of a pixel of the research matrix with respect to the target matrix will be:

$$X_r = aX_s + bY_s + Y_r = dX_s + eY_s + c$$

Feature Extraction

Classification

↳ To extract feature objects

Feature extraction techniques are used in photogrammtry to extract some «interesting region», as **points, lines or area which have radiometric discontinuity or *texture, with pupose to identify homologous points in a better, precise and faster manner***

This is a possible classification:

- ***Interest Operators***
- ***Edge detectors***
- ***Region detectors***

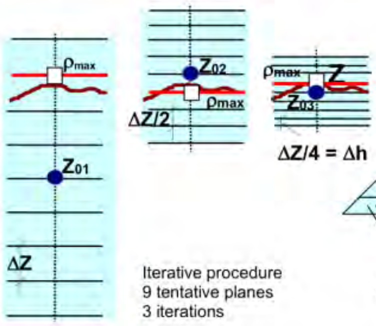
The operators of interest are used in photogrammetry for the extraction of points (pixels) on a digital image that can provide good results in image matching operations.

Edge detection techniques are used for the extraction of radiometric edges.

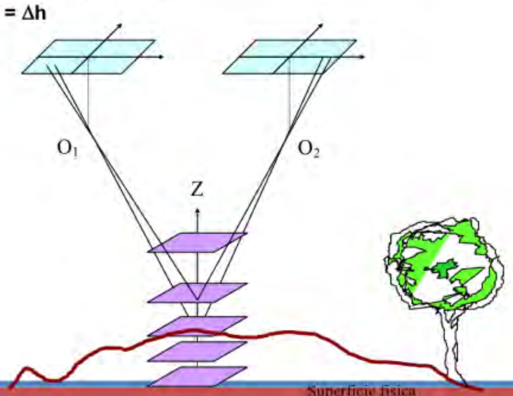
The region detectors are algorithms developed in Computer Vision (CV) for the extraction of portions of images (region) that guarantee good results in image matching operations on stereoscopic images with strong geometrical variations (scale variation, rotations, perspective deformations) and radiometric ones. (even if radiometry changes a lot)

Numerical elevation models

MA generation using VLL



VLL (Vertical Line Locus) define the planimetric coordinates and working for logical tentative, the corresponding height is determined



Plotting aiding

Geometrical primitive extraction

It is a sector that is not yet completely developed and it is still investigated.

The ambition is to allow the automatic extraction of some standard objects (roads, areas with homogeneous radiometry such as water, fields, etc., buildings).

They widely use the operators of interest and special filters for the detection of radiometric edges.

The results are interesting, but it is necessary to make other study and research

The algorithms are like "artificial intelligence" and they use very complex logical functions.

Processing time is still very high even on workstations.

we can extract element that impose terrain and environment, we can improve our intelligence with artificial one because we extract roads, field, buildings and then we say if results are correct or not. The process is very long also when we have computational capability because we use radiometric function to extract informations.

Plotting aiding

Geometrical primitive extraction

Road limit extraction

The operator detects some significant component of the road (in black). Using the radiometric and directional information of the plotted elements, it reconstructs the road axes.

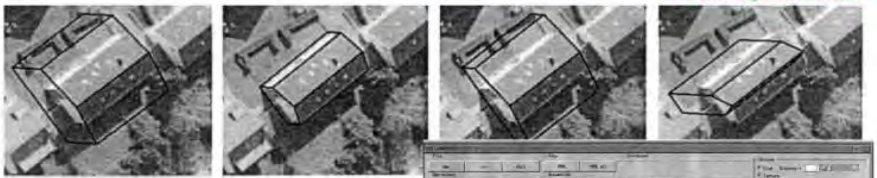
from images we extract limit of roads and also of buildings, of rivers and so one

In a second step, the implemented procedures can define geometry of the board of the road.

Plotting aiding

Geometrical primitive extraction

Building extraction



The geometric model is defined as perspective on the images. The operator moves one or more points which are very close, deforming the geometry of the model in order to adapt the solution to the building and the software completes the best adaptation operation.

Another automation includes automatic edge extraction, automatic search for a similar target model and automatic adaptation to real geometry.

⇒ not always model is coincident with reality

Digital photogrammetry

Conclusion and future works

Modern digital plattin systems are starting to introduce the automatic procedures proposed by the research community, producing tools which can be managed by non-expert users.

Photogrammetric operators have only to know and to realize the planning of flight, terrain point measurements and triangulation. The images will be automatically loaded with the exterior orientation parameters, with purpose to plotting some elements

Digital aerial cameras and metric terrestrial cameras will transform the traditional photographic cameras as old camera and direct photogrammetric techniques will be more used.

The digital photogrammetric system allows a complete integration with the GIS (numerical mapping)

PRO : new products

All new tools are born to do better operations already performed with older tools, but finally involve the emergence of new products:

1) traditional and precise digital orthophotos

2) Stereophotomap

3) Precise solid orthophoto

useful because we can define quality of objects, knowing only distance between camera - object + position of the camera

we can have a final product different from laser scanning because we can't check final position, we have to plan the plot

photogrammetry better tool to realize 3D models

↳ laser scanning can be fast and easy but also weak due to control of precision

⇒ we have lots of instruments which allow to create digital images:
- camera
- model with photogrammetry
  ↳ it's becoming current.

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Feature Extraction

Forstner operator

useful to extract shape of buildings or border of other objects.

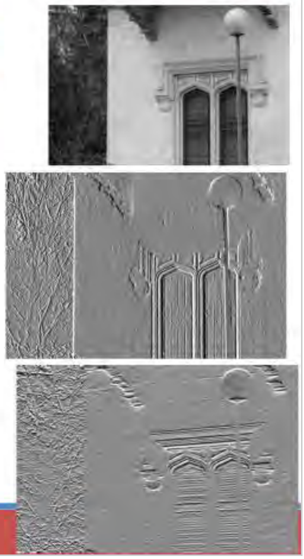
The Forstner operator selects points of interest by analyzing the size and shape of the standard error ellipses provided by the Cxx variance and covariance matrix of the LSM equation system

The size of the ellipses is defined by w. The shape of the ellipses is defined by q, so called circularity parameter:

$$q = \frac{4\det(\mathbf{N}_{x,y})}{tr(\mathbf{N}_{x,y})^2}$$

If $q=0$ N is singular. **The point is a radiometric edge.**

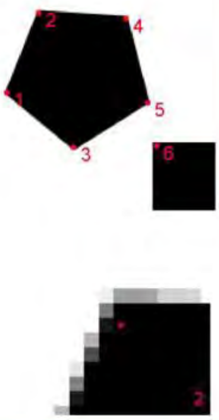
If $q=1$ the ellipse is a circle. This is a point of interest, which has a high radiometric contrast with respect the closest points

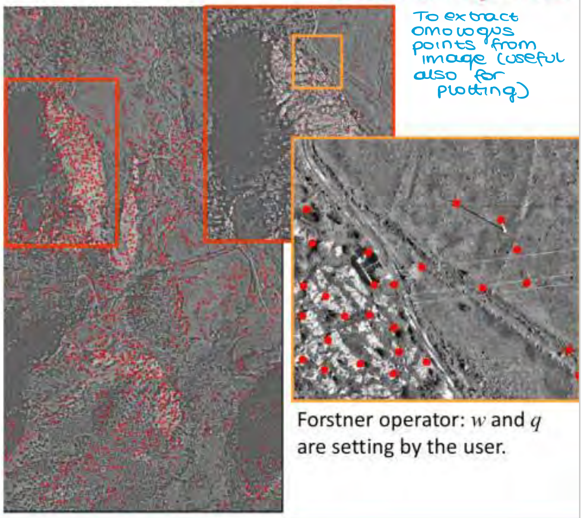


Feature Extraction

Forstner's operator

examples





To extract omologous points from image (useful also for plotting)

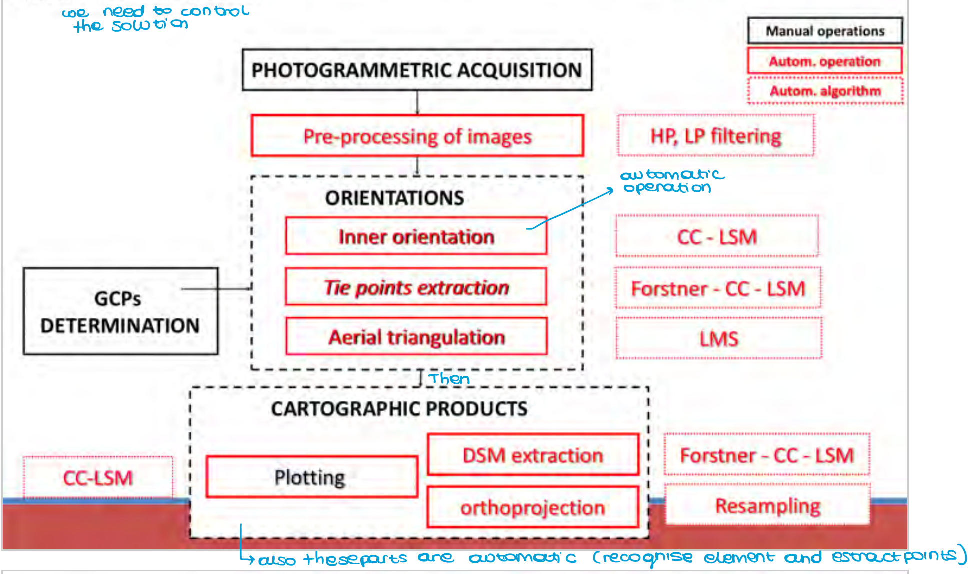
Forstner operator: w and q are setting by the user.

1300 points, camera DMC, GSD= 34 cm, 600 dpi) speed power ←

Automatic photogrammetric procedures

Combining the different algorithms, it is possible to obtain automatic or semi-automatic photogrammetric process.

we need to control the solution



Automatic photogrammetric procedures

The automation in digital photogrammetry needs to learn new concepts:

- **The determination of approximate parameters for a non-linear systems becomes a more important problem**, because direct solutions are often required (linear)
- In order to avoid the manual activities, robust estimators have a fundamental role due to the presence in the data automatically acquired (autocorrelation) of a large number of accidental, systematic and coarse errors

only LSM is dangerous because some errors aren't eliminated, we need to use robust approach to remove errors which we don't want to be influenced by.

The automatic photogrammetric procedures must have two important properties:

- **Robustness as the final solution is not influenced (or the influence is very very low) by model errors**
- **Diagnostics as the ability to identify the model errors**

The photomapping

Definition

In an aerial case, the photomappping is quite similar to a frame (photogram)

Frame?? **Photomapping??**

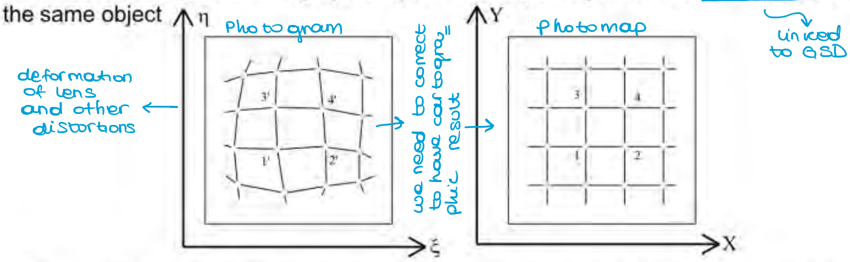
The photomapping

Photogram deformation

The difference between photogram and the photomapping is mainly a geometric difference:

- PHOTOGRAM is a perspective representation (central perspective) of the object (the terrain in aerial photogrammetry)

- PHOTOMAPPING is a cartographic representation, on a predefined scale factor, of the same object



The central perspective of a regular grid produces an irregular and deformed grid.

The example shown in the figure represents schematically the geometric deformation effect for each frame.

Rectification

Definition

we can measure, directly on the image, distances between points or dimensions of objects

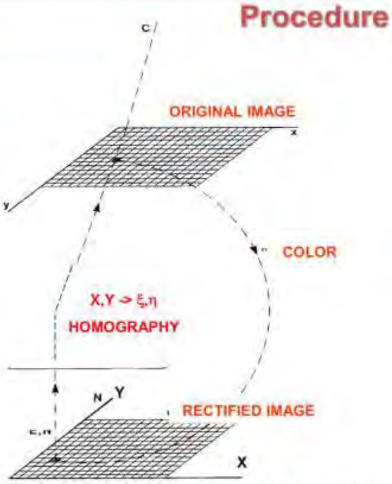
=> FAST AND EASY

Rectification

Procedure

Giving the image of the object and a series of known points (X,Y) on the object:

- **homography parameters are estimated** (at least 4 known points)
- **A rectified image is considered (at the begin it is empty matrix)**, where each pixel is defined with the object coordinates X, Y
- For each pixel in the image, homography equation is applied, with purpose to project the object point into image space.
- radiometric value is resampling (***RESAMPLING***) and it is assigned to this pixel
- this procedure is repeated for all pixels, in **order to obtain the RECTIFIED IMAGE**



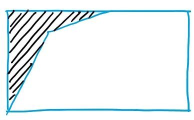
$$X = X_0 + d*(c-1)$$
$$Y = Y_0 - d*(r-1)$$

1) we create an empty space (image) and decide size of the pixel (we work with terrain coordinates)

2) we start from the first pixel, we apply homography transformation and find ξ and η

3) reading the radiography we find the pixel's coulor

=> it's possible that pixels have homologous points or not (if not the colour of the point is BLACK)



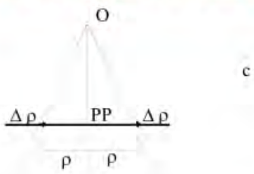
Rectification

The systematic error in the height

what happens to points which are in another surfaces?

← HEIGHT ERRORS

What are the height variation which can be considered as negligible in the rectification? Which parameters do they depend on?



The image points that do not exactly belong to the object plane are translated in the direction of the principal point.

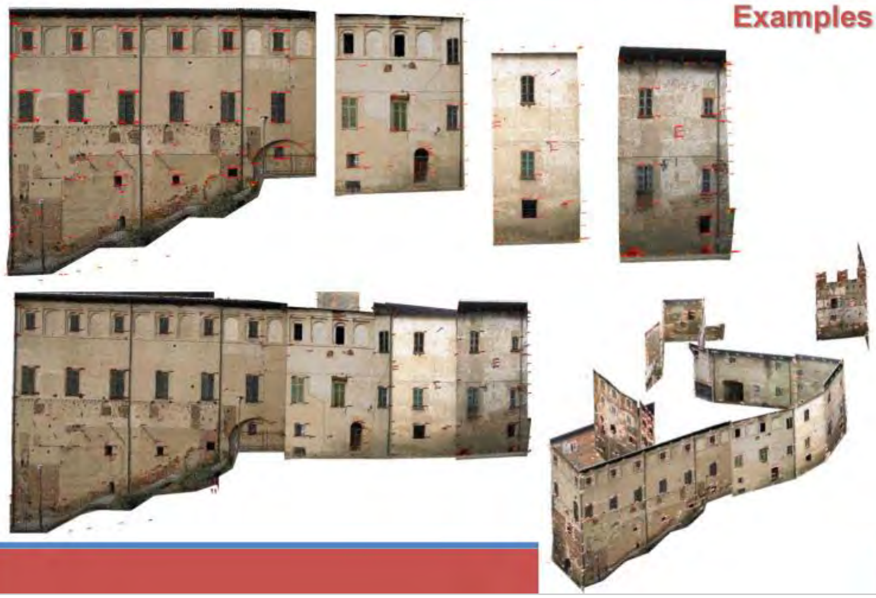
Rectification

Scale map	Scale frame	$c = 45$	$c = 100$	$c = 200$ *mm*
1:50	1:200	3	6	12 *cm*
	1:400	5	11	24 *cm*
1:100	1:400	5	11	24 *cm*
	1:800	10	23	48 *cm*
1:250	1:1.000	13	28	59 *cm*
	1:2.000	25	57	119 *cm*

Scale map	Scale frame	$c = 90$	$c = 150$	$c = 210$	$c = 300$ *mm*
1:1.000	1:3.000	1	1	2	3 *m*
	1:6.000	2	3	4	5 *m*
1:2.500	1:7.500	2	3	4	6 *m*
	1:15.000	4	6	9	13 *m*
1:5.000	1:15.000	4	6	9	13 *m*
	1:30.000	8	13	18	26 *m*
1:10.000	1:30.000	8	13	18	26 *m*
	1:60.000	15	26	36	51 *m*

Rectification

Examples



Orthoprojection

Classification

There are three different types of orthophotos:

- ***Rapid OP***
- ***Normal OP***
- ***Precise OP***

} different procedures

Rapid orthophoto is a photographic representation, which is produced with rapid and cheaper techniques. **The graphical precision is 0.3 mm of the nominal scale of the orthophoto for the objects on the terrain, and 0.9 mm of the nominal scale for all objects with a elevation (building, bridge, viaduct).** It is generally used for environmental application or thematic map (emergency management or early mapping). → low accuracy, but ready after 2-3 h

The normal orthophoto is a photographic representation, but with **0.2 nominal scale of the orthophoto for the objects on the terrain, and 0.6 mm of the nominal scale for all objects with a elevation.**

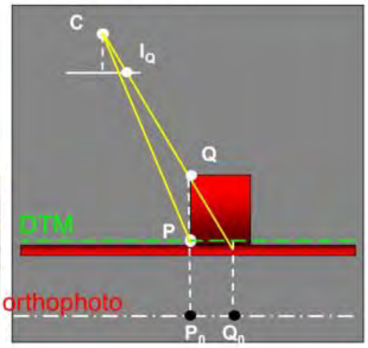
terrain is the plane where we apply the transformation

points in the terrain are less effected by deformations

Orthoprojection

Normal orthophoto

The orthoprojection is realized using a digital terrain model (DTM).



This product has some residual perspective deformations, due to a not correct geometry model.

we don't have a central, but lateral prospective (we can see facade)
↳ it's wrong we have low quality

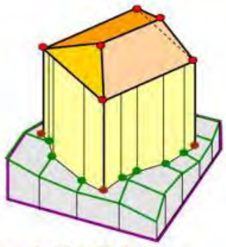
when there is a height we have prospective deformations

Precise orthophoto

Geometry (shape) model

In order to produce a precise ORTHOPHOTO, it is important to use a correct geometry (shape) model. This model could be created by:

1. ***A digital surface model (DSM) , starting by a digital terrain model (DTM).*** **MANUAL PROCEDURE and SHAPE FORMAT**
=> shape format (lines, points, areas, geometrical elements)



2. ***Dense digital surface model (DDSM)*** in **RASTER FORMAT. It is a model with an high density of points**

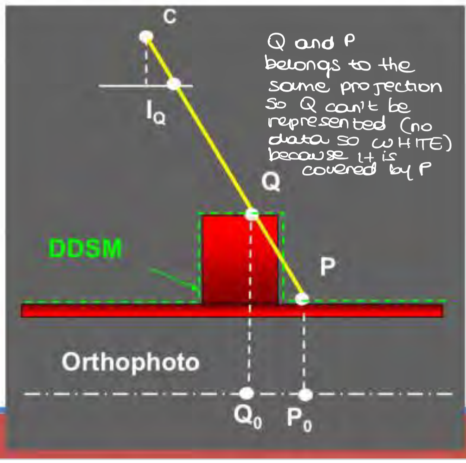
AUTOMATIC PROCEDURE.

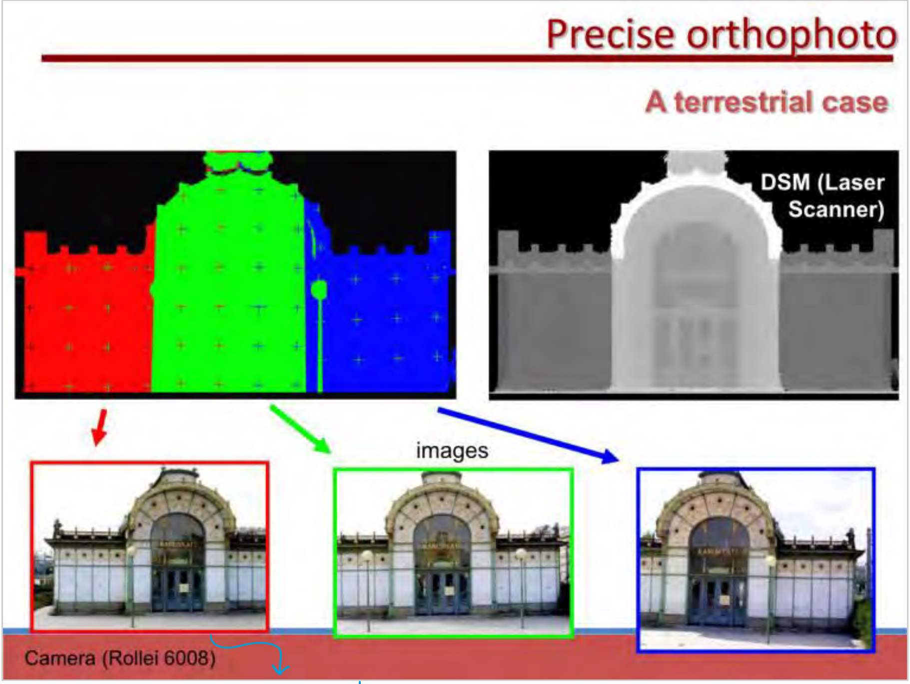
It is generated usign numerical map, LIDAR or photogrammetry
↳ a raster file (like a matrix!)

Precise orthophoto

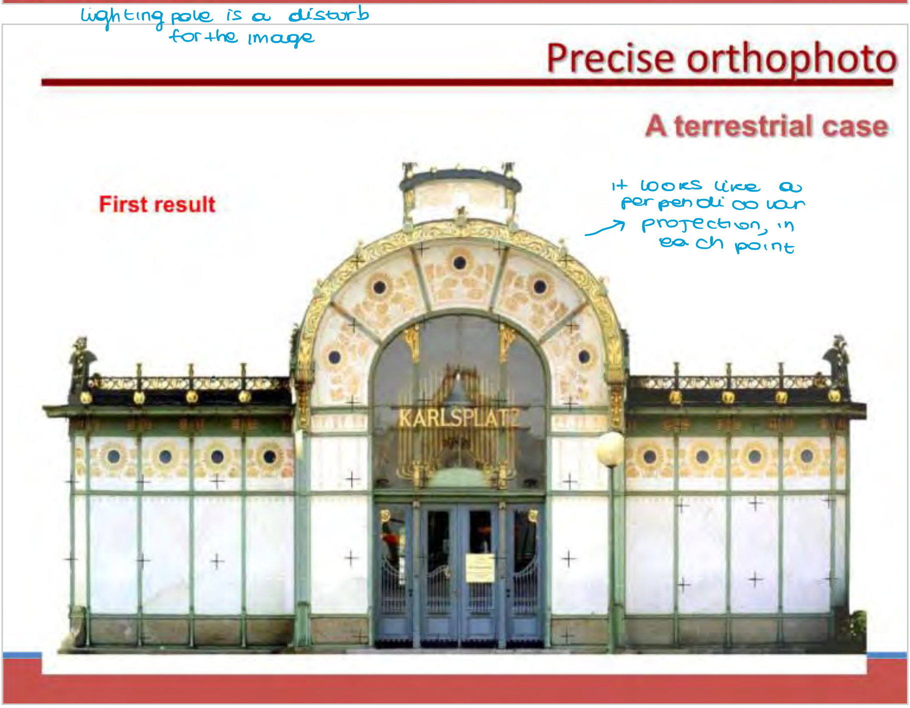
Images duplication

The correct shape model (DDSM) is not sufficient to eliminate the perspective deformations. In fact, using a correct description of the shape of the object and a single image, some visible points are duplicated on the orthophotos





lighting pole is a disturb for the image



it looks like a perpendicular projection, in each point

Precise orthophoto

An Aerial case

Orthophoto of Turin municipality

INPUT

Numerical map 1:2000

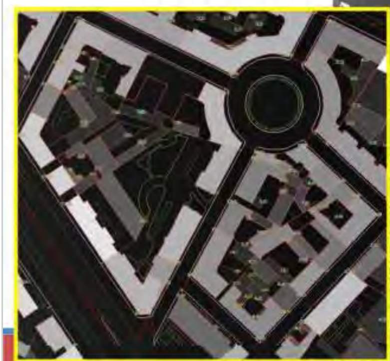
Photogrammetric flight frame scale 1:5000.

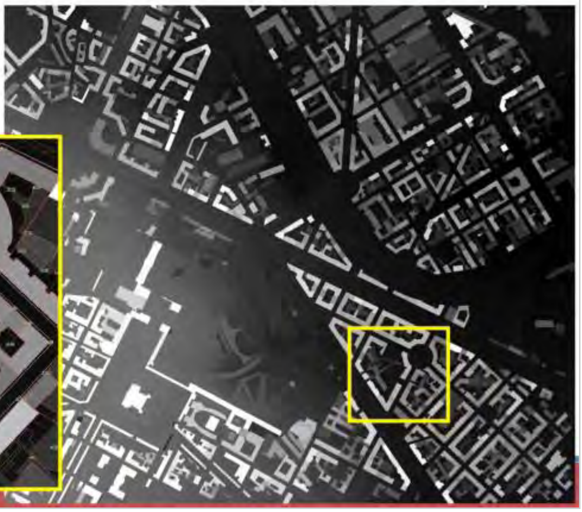
L'ortofoto di precisione

An aerial case

Generation of a DDSM from a numerical map

resolution: 20 cm





Orthophoto in large scale map

CISIS: italian guideline

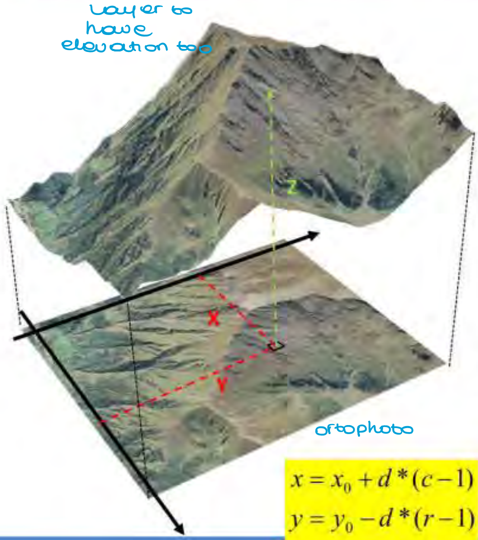
→ how to produce ortho photos or digital images in large scale

Precise solid orthophoto

+ additional layer to have elevation too

Definition

Red, Green, Blue } layer + Elevation (additional layer)



PSO is the fusion of the two-dimensional and chromatic content of the precision orthophoto (OP) with the three-dimensional information of a DDSM (Dense Digital Surface Model) of object.

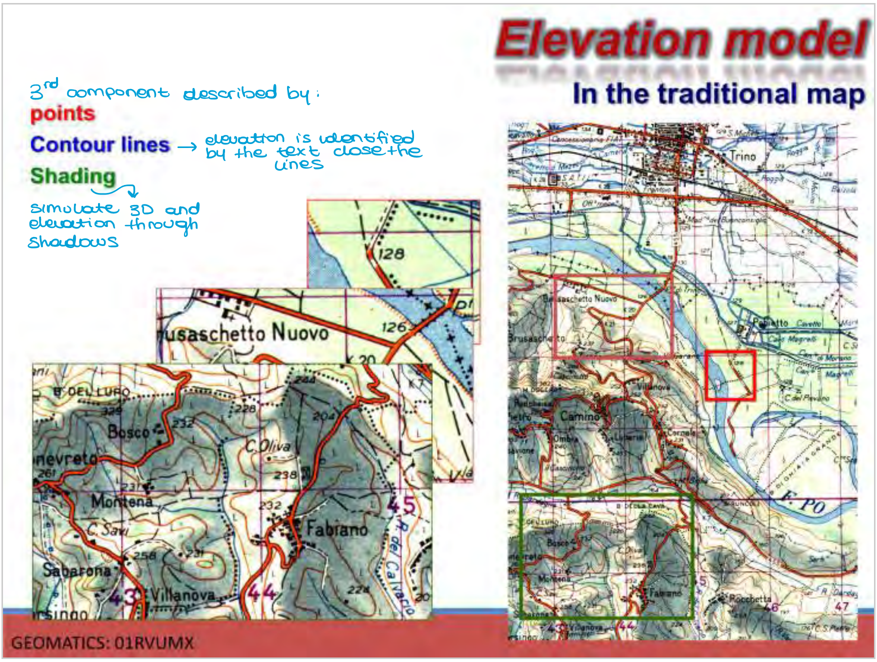
The position of the selected orthophoto pixel gives the horizontal coordinates while the altimetric model contains the corresponding height.

$$x = x_0 + d*(c-1)$$
$$y = y_0 - d*(r-1)$$
$$z = q(c,r)$$

→ so we have x, y and z (the distance is the slope one) → we can compute Δ volumes

27.04.18

sabato 28 aprile 2018 10:05



=> in traditional 2D map we can't see vertical component as in digital maps

Digital Elevation model

Dense Model

=> These 5 elements are foundamental both for DTM and for DSM
we need them for a digital model

If the number of points contained in the model is very high, it is not mandatory to describe correctly and completely the surface. Breaklines and Chorographic elements are not more mandatory to describe the details.

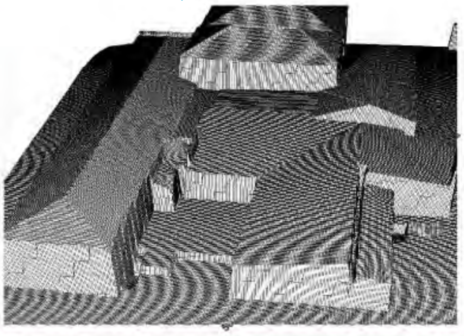
↳ because variation of slope can be described by points themselves

↳ used for laser scanning or photogrammetry

- **Dense Digital Elevation Model, DDEM or Dense Digital Terrain Model, DDTM);**
- **Dense Digital Surface Model, DDSM**

It is fundamental (MANDATORY) to include always:

- **Dead zone**
- **Limits of the area**



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Digital Elevation model

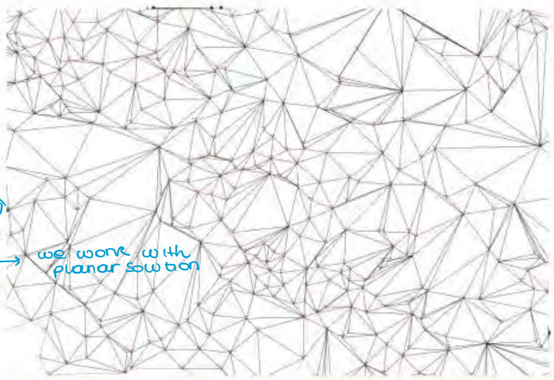
Point distributions

The points are distributed following a «not regular» distribution:

↳ the regular one should have same, costant distance
↳ random
↳ we collect them using laser scanning or photogrammetry
↳ we create a model by them

- ***point cloud,*** it is a whole of points, which are defined by 3 coordinates (E, N, Q) → or (x,y,z) without ant information on the surface between the points (*software interpolation*)

- ***Triangulated Irregular Network*** (TIN), it is a point clouds, where the points are connected using triangles (Delaunay): CONTRO
  - ***It is not «univocal»***, therefore it is necessary to describe clearly the triangles (*)
  - In each triangle, the surface is described as a plane → we work with planar solution
  - The triangles ***don't have to crossing*** the characteristic lines or breaklines.



PRO
- it is very easy and we use real points to create the model
- vertexes of triangles are made by real points

(*) we can fix different criteria for ex. max distance between points to realize the triangle. In this case we have to store (x,y,z) of each point and with them generate the model

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Digital Elevation model

guidelines

It is necessary to have an international and national congruence and uniformity of the data.

Digital Elevation model

Tollerance

=> There are different accuracies linked to DTM, DSM ecc.
If we consider DDTM or DDSM the density of point is higher than 1m per m2 so resolution = 1m2

level	type	resolution (m)	terrain: $T_{H(a)}$ (m)	With trees: $T_{H(b)}$ (DEM) (m)	building: $T_{H(c)}$ (DSM) (m)	T_{EN} (m)
0	DEM, DSM	40-100	30	30	30	20
1	DEM, DSM	20	10	20	10	10
2	DEM, DSM	20	4	½ al.m.al.	5	4
3	DEM, DSM	10	2	½ al.m.al.	3	2
4	DEM, DSM	5	0.60	1.20	0.80	0.60
5	DEM, DSM	2	0.40	0.80	0.54	0.40
6	DDEM, DDSM	1	0.60	1.20	0.80	0.60
7	DDEM, DDSM	0.50	0.30	0.60	0.40	0.30
8	DDEM, DDSM	0.10-0.20	0.20	0.30	0.26	0.20

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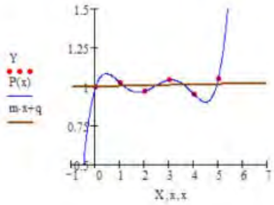
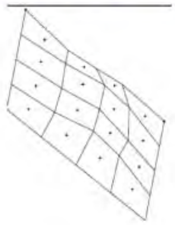
Digital Elevation model

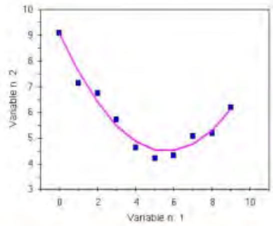
Interpolation and extrapolation

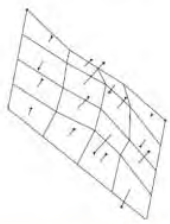
Interpolation
To select a correct model





approximation
To minimize the difference with respect known points





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Digital Elevation model

Classification

→ we have two big classifications related to approach or to number of points

1) Approach

Deterministic
The interpolation is made considering physical parameters
↳ usually the used one because we have height of points

Stochastic
The interpolation is made considering a statistical correlation (co-variance), which can be not correlated with physical parameters.
↳ or distribution

2) N° of points

Global method
All information are used to create the model; they are useful to identify general trend but they aren't suggested to model local phenomena.

Local method
- only neighbours points are used to create the model: nearest neighbour, moving average, inverse distance weight (IDW), spline, kriging.

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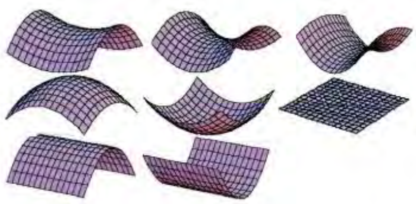
MODELS used for interpolation:

Digital Elevation model

Polynomial function

It is easy to be implemented
↳ we use distribution of points to interpolate them → different degrees → in general LESS THAN 3



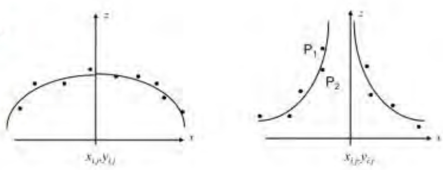


image made by pixel = point. For each pixel we have values of height so we can store regular model like an image because inside each pixel of image we have RGB (so in this case info related to height)

In regular distribution we define Δx, Δy but we also have two kinds of definition for resolution:

1) regular distribution defined in GEOGRAPHIC COORDINATES

Digital Elevation model

Spline

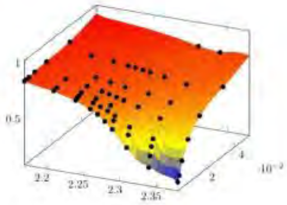
↱ we can have strange behaviour

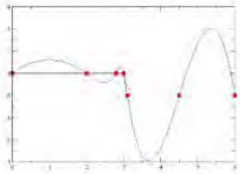
The interpolating function **is more smoothed** than the other methods (for example with the polynomial function).

However, if the data to be interpolated have a particular distribution (e.g. they create a step), the interpolant spline can suffer to the **Gibbs phenomenon**, with wide variation close to the step.

To avoid this problem, it is better to use **the smoothing spline or spline tension.**

⇒ so we should use this when we have a flat, regular distribution





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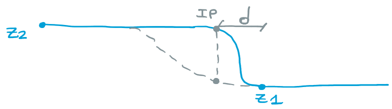
we can decide a criteria to use when we interpolate the number of points we have (ex. we fix the distance d = FIX or the number of points N without considering distances)

↳ we chose one criterion or the other considering the distribution of points: if we have a big number of points we can work with them, if not it's better working with number of points

It's most rigorous, but if terrain has a slope variation we can have problems

Ex. terrain and two points at z1 and z2 (real) and Ip which belongs to the grid



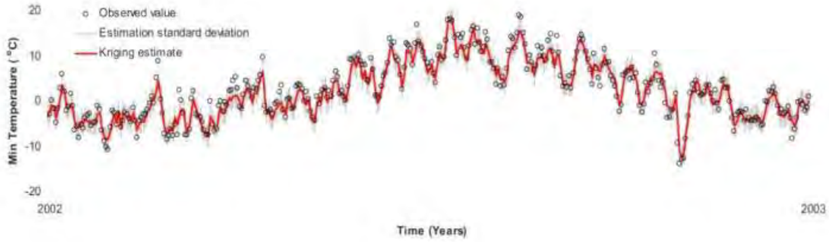
The correct value of IP could be z2, but we have to apply also weight of z1 so we apply a higher contribute to z1, just because it's closer even z2 is more correct. So final value of IP is closer to z1 and so the model is more distant from reality

Digital Elevation model

Kriking

It is a statistical approach, where it is possible to define the accuracy of the final model.

↳ we use it for environmental application, using elevation and standard deviation
↳ we use statistical info
↳ it is a stocastic method



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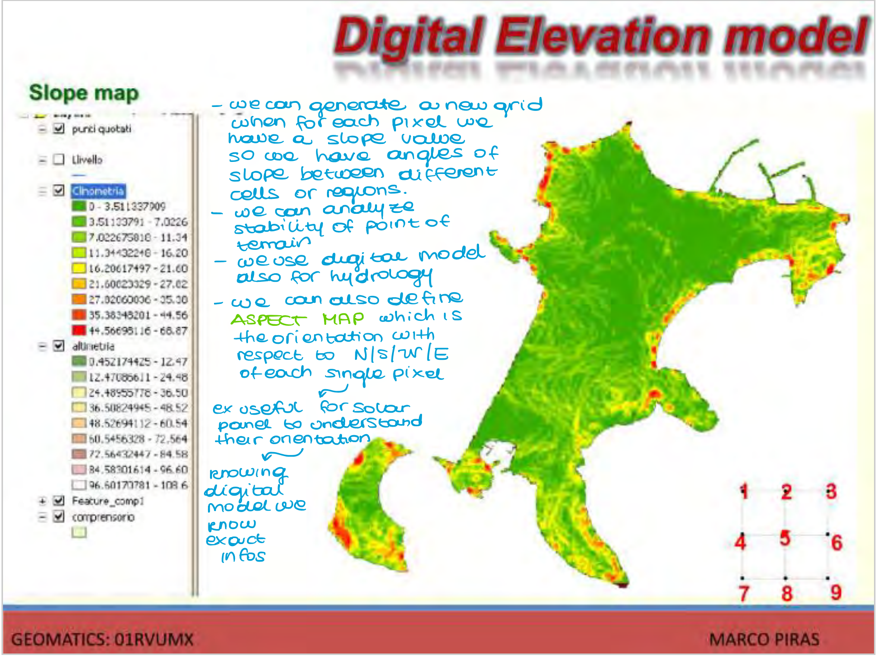

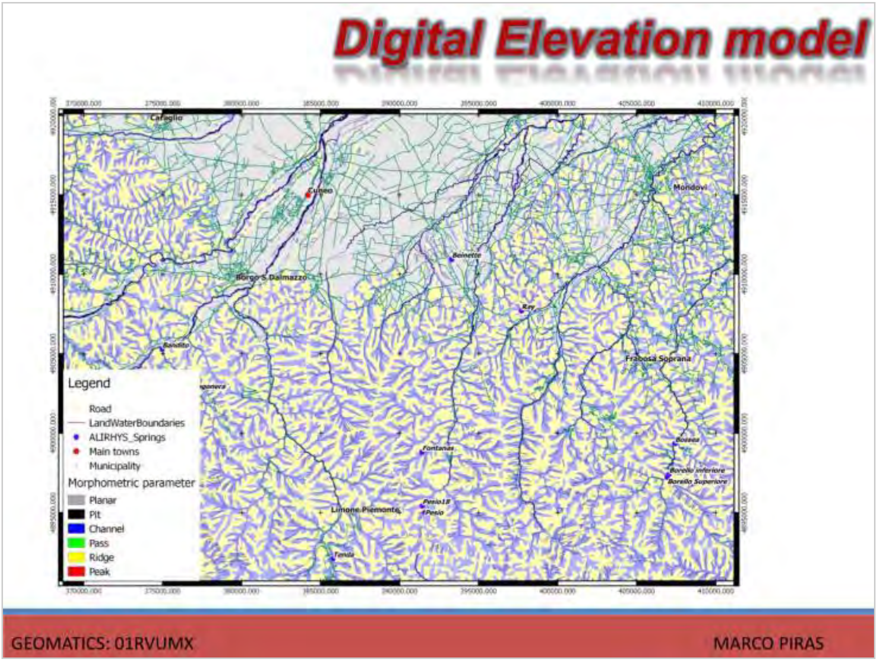

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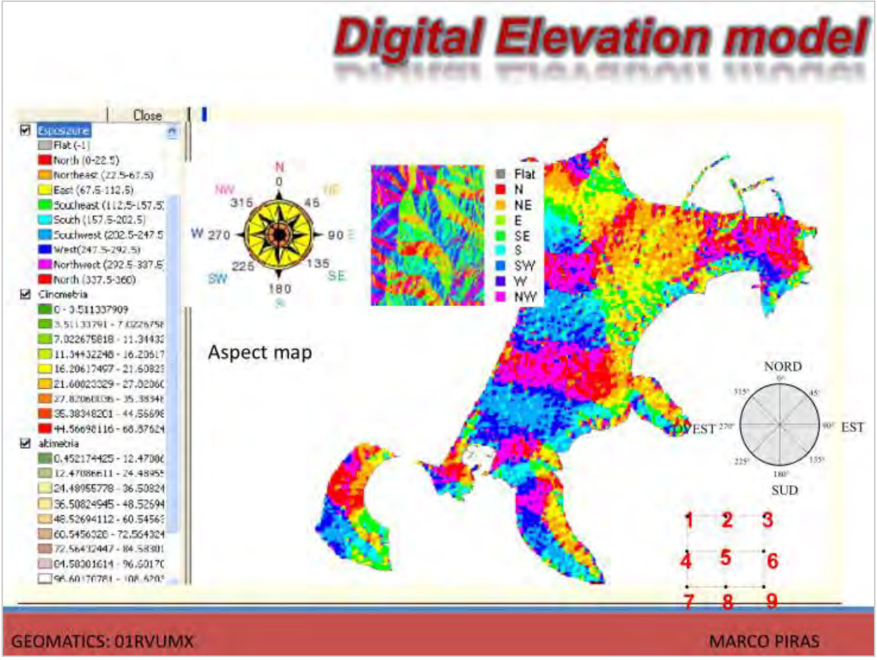
Digital Elevation model
Aspect map
Flat
N
NE
E
SE
S
SW
W
NW
NORD
EST
SUD
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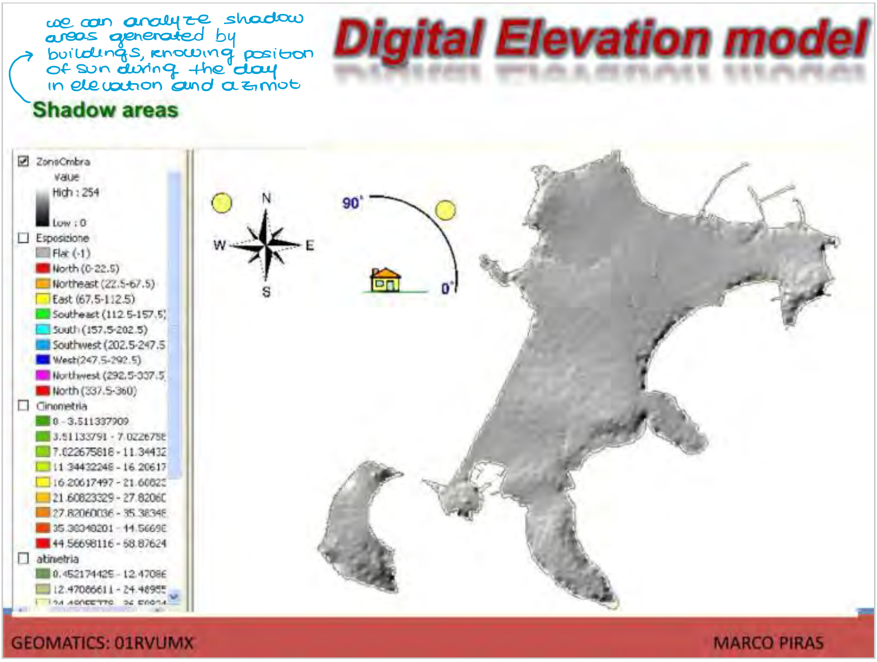
we can analyze shadow areas generated by buildings, knowing position of sun during the day in elevation and azimot
Digital Elevation model
Shadow areas
90°
0°
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2.05.18

mercoledì 2 maggio 2018 08:37

POLITECNICO DI TORINO
DIATI

L14.1

LiDAR (Laser Scanning)
Data acquisition, sensors, planning, data processing

MARCO PIRAS

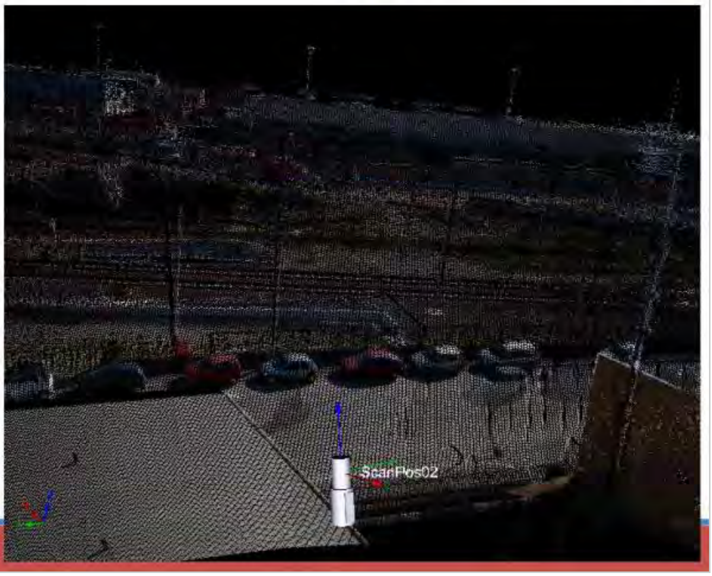
GEOMATICS: 01RVUMX

Introduction

Light Detection and Ranging

Laser scanning is a techniques which allows to acquire milions of 3D point in automatic way.

The final result is called **point clouds, where for each point is known the position (in local CS) and color (if a digital camera is included)**



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Both in local
And in absolute
Coordinate System

Techniques for 3D measurements

Light radiation

Light can be considered as a sequence of photons (set of electromagnetic energy) where is valid the following equation:

$$\lambda = c / f$$

where:

- λ is the wavelength
- f is the frequency of wave
- c is the light speed in the object

Frequency and wavelength Are inverse proportionally.

This relation defines that if the frequency and the material of the object are known, the wavelength is automatically determined.

Low frequency → high wavelength

GSM works with GigaHz: 900 Mhz -> λ=33 cm – 1,8GHz -> λ=17 cm)

In the case of visible light, the wavelength are smaller than GSM, therefore the frequencies are more bigger. (TeraHz)

We have a dipendency between accuracy and wavelength. Small wavelength to have high accuracy.

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Techniques for 3D measurements

Speed of light radiation

Speed light is 299.792.458 km/s, but it is generally considered as standard value in the vacuum equal to **c = 300.000 km/s**

If the light is in a different medium of the vacuum, the speed decreases.

The ratio between the speed of light in a vacuum and the speed of light in a medium (air, water, etc.) is called the refractive index:

$$n = c_{vacuum} / c_{medium}$$

Standard air	n= 1.00025
Water	n = 1.33

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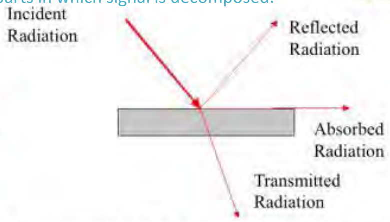
Techniques for 3D measurements

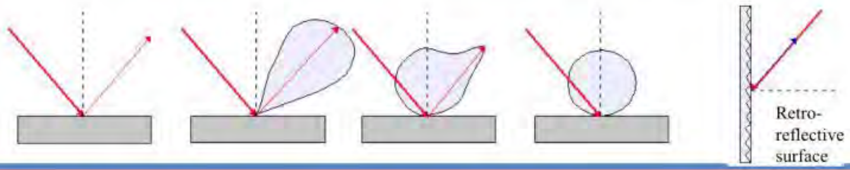
LiDAR

Each surface has different values to the three parts in which signal is decomposed.

When a radiation hits a real body surface (so called grey body), the radiation is partially absorbed (**$\alpha(\lambda)$**) partially is reflected (**Reflectivity $\rho(\lambda)$**) and partially is transmitted (**Transmissivity $\tau(\lambda)$**), consireding the wavelength λ. Starting from the principle of conservation of energy : **$\alpha+\rho+\tau=1$.** Principal of conservation of energy



Considering the reflective part only, it is possible to distinguish the surface of the object. The surfaces completely smoothed reflect as a mirror. A wrinkled surface **works like a** lambertial surface: the reflection direction is independent on the incident angle. Moreover, there are the retro-reflective surface, where the incident direction is equal to reflection direction.



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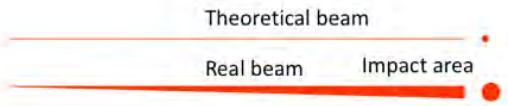
Reflection in not only on one direction, but it creates also a spectrum around itself (the spectrum is spread). Lambertial surface. Or winkler surface in power of reflection is very low.

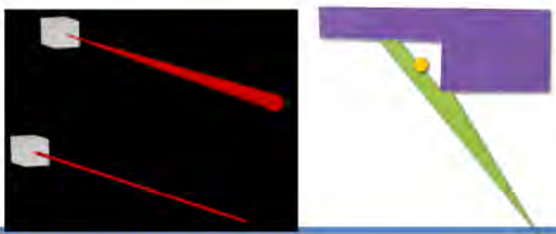
Techniques for 3D measurements

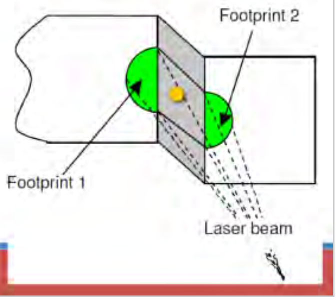
LiDAR

Beams are "practically" parallel, therefore they are affected by a slight divergence (working with limited range, it is absolutely negligible).



In the acquisition if the impact area meets two surfaces at different distances, a non-existent average point is generated (**scan artefacts**)





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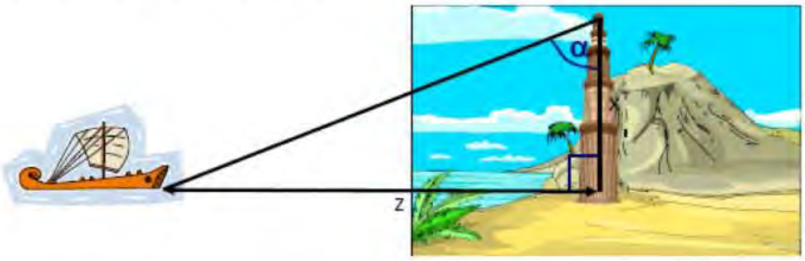
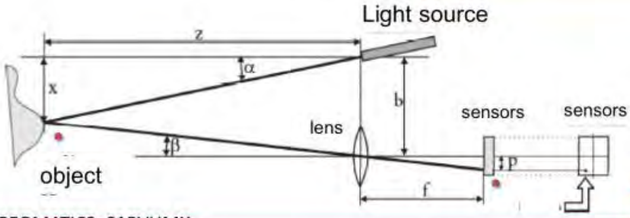
Increasing the distance the beam becomes a region (divergence of rail). So the distance is from a virtual point (CENTRAL POINT) - average points not real ones. It's difficult to find errors in a point cloud (ex. We apply a filter to remove some kind of errors).

Techniques for 3D measurements

LiDAR based on triangulation

This method was developed in VI century a.C. by Talete). Knowing a base x and two angles, it is possible to solve a triangles and to calculate z: $z = x \cdot \tan\alpha$





On a plane

$y=0$

$\tan\beta = \frac{p}{f}$

$z = \frac{b}{\tan\alpha + \tan\beta}$

$x = \frac{z}{\tan\alpha}$

always possible to find z only with geometrical consideration

=> it has a very HIGH PRECISION but we have limited distances 60-80 m (not more than 100m)

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Techniques for 3D measurements

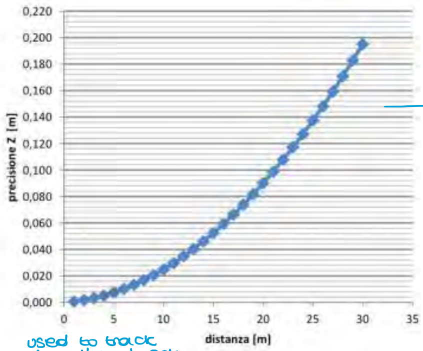
Precision

→ change with the square of the distance (if high the standar deviation increases a lot)

$$\sigma_X \cong \sigma_Y = \sqrt{\left(\frac{1}{\tan\alpha}\right)^2 \sigma_z^2 + \left(\frac{z}{\cos^2\alpha}\right)^2 \sigma_\alpha^2}$$

$$\sigma_Z = \sqrt{\left(\frac{z^2}{fb}\right)^2 \sigma_p^2 + \left(\frac{z}{\cos^2\alpha}\right)^2 \sigma_\alpha^2}$$

The precision (σ) changes with the **square of the distance.** at 5-10 m , σ is greater than 1 cm. For small distance, the precisions are close to 0.1 mm.

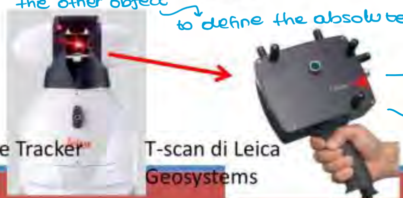


best precision (low values) if we are very very near to the object

Nowadays, for improving the ditance, the system is composed by two parts: the laser scanning (close to the object) and a tracker (which defines the laser position)

used to track the other object

to define the absolute position of the scanner



Absolute Tracker

T-scan di Leica Geosystems

located close to the object

same quality because the principal scanner is close to the point

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Techniques for 3D measurements

The ToF LIDAR system based on distance measurements

↳ like the pseudorange in GPS and for the phase the analogy is the career phase

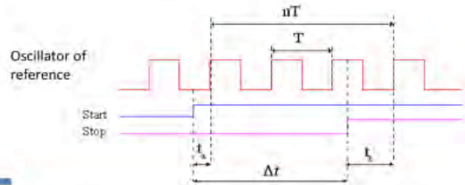
The principle is very easy. This approach is based on the measure of the time of flight (TOF) Δt which is required by a laser beam to cover the distance between the instrument and the object and viceversa. ↳ signal starts, is reflected and comes back

$$2D = c\Delta t$$

The distance measurement unit has an high precision clock p=10^{-8} and λ=20 m

After a time = Δ*t*, signal is back to receiver. Using this time difference, the distance is estimated with a SD = 9 m.

the quality of traditional clocks is not enough to a high value of precision



$$\Delta t = nT + t_a - t_b$$

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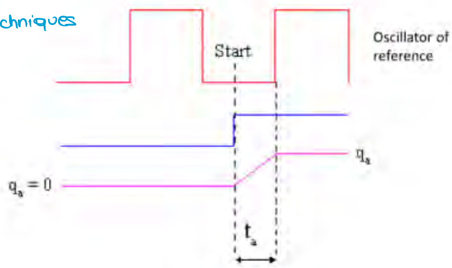
Techniques for 3D measurements

The ToF LIDAR system based on distance measurements

$$\Delta t = nT + t_a - t_b$$

↱ we can use an atomic clock or other techniques

In order to measure t_a and t_b with high accuracy, a time-voltage converter is used. It is composed by a capacitor which is charged with a constant current, for the time t_a and t_b.



we measure the difference to the voltage from the sent signal and the recieved one
↳ we can find the time delay and so what we need.

Knowing the voltage reached for a charge time corresponding to an oscillation period (T), it is easy, with a simple proportion, to obtain the required residual times as a function of the voltage reached by the capacitor

$$\frac{t}{q} = \frac{T}{Q}$$

⟹ range is 2-3 km

GEOMATICS: 01RVUMX

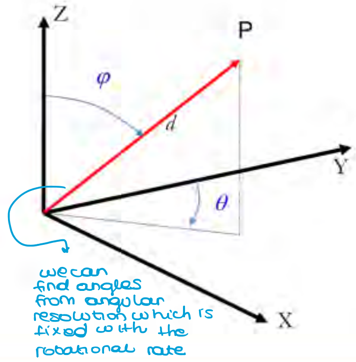
Techniques for 3D measurements

TOF system

The position (3D) of the single point (→ of the point clouds) is determined as the traditional «topographic» measurments, using distance and angles (→ as the total station).

$$X_P = X_s + d \cdot sen\varphi \cdot sen\vartheta$$
$$Y_P = Y_S + d \cdot sen\varphi \cdot \cos\vartheta$$
$$Z_P = Z_S + hs - hp + d \cdot \cos\vartheta$$

The source is able to rotate around a primary axis with respect to a rotation rate, but this axis rotates also around the orthogonal position to cover all the space. The laser collect points only in one line then we need to rotate it to obtain different lines



we can find angles from angular resolution which is fixed with the rotational rate

Techniques for 3D measurements

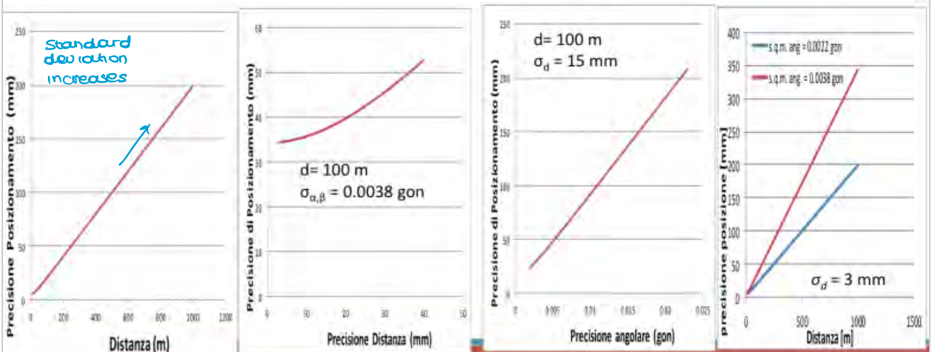
PRECISION

⇒ single points have different precisions → σ = k + ppm (so it depends on a fixed quantity and a f(distance))

$$X = d \cdot sen\alpha \cdot \cos\beta \qquad \sigma_X = \sqrt{(sen\alpha \cdot \cos\beta)^2 \cdot \sigma_d^2 + (d \cdot \cos\alpha \cdot \cos\beta)^2 \cdot \sigma_\alpha^2 + (-d \cdot sen\alpha \cdot sen\beta)^2 \cdot \sigma_\beta^2}$$
$$Y = d \cdot sen\alpha \cdot sen\beta \qquad \sigma_Y = \sqrt{(sen\alpha \cdot sen\beta)^2 \cdot \sigma_d^2 + (d \cdot \cos\alpha \cdot sen\beta)^2 \cdot \sigma_\alpha^2 + (d \cdot sen\alpha \cdot \cos\beta)^2 \cdot \sigma_\beta^2}$$
$$Z = d \cdot \cos\alpha \qquad \sigma_Z = \sqrt{\cos^2\alpha \cdot \sigma_d^2 + (-d \cdot sen\alpha)^2 \cdot \sigma_\alpha^2}$$

σ_d = 3 mm
$\sigma_{\alpha,\beta}$ = 0.0022 gon

$$\sigma_P = \sqrt{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2}$$



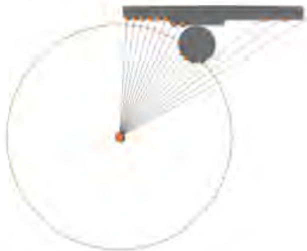
Techniques for 3D measurements

Characteristics

Acquisition speed → how many points we are able to collect in a second

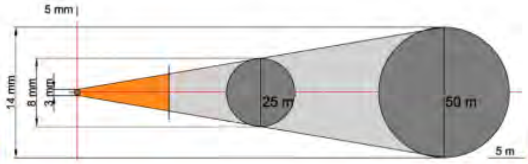
It is possible to acquire the points witha speed = 500000pts/s (→ half billion). The greater part of the data acquisition is due to the instrument setting, data storage and data processing.

⇒ more important aspects are data processing (80%), storing, ecc.



Resolution and divergency of the ray (divergency → distribution and density of points)

The scan resolution is theoretically equal to the value of the rotation angle between two laser beam. But in the real case, we have to consider the divergency of the ray. In some case, this value is bigger than the resolution.



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Techniques for 3D measurements

Characteristics

Real Range

The range depends on the medium where the laser beam passes..

Moreover, it depends on the material and the type of surface.

Moreover, it depends also by other external radiation noise (reflected sunlight, artificial light).

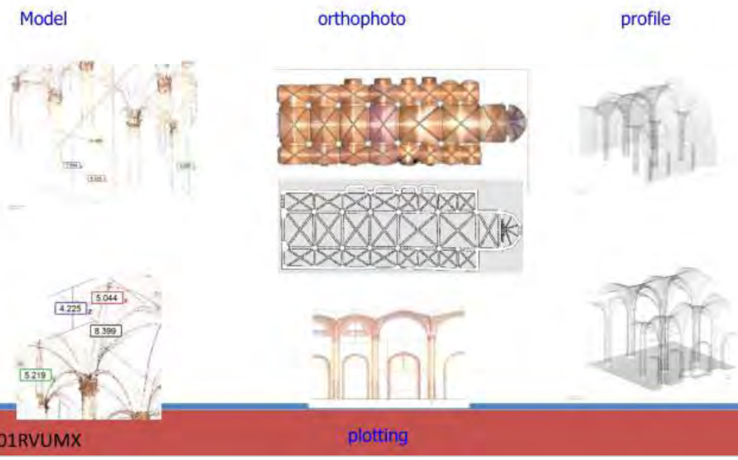
Field of work

Laser scanning can be fixed (no rotation of the body) or moving, with a rotation around 1 or 2 axes.

GEOMATICS: 01RVUMX

Some preliminary conclusions

- Processing of raw data
- Generation of final products



Data acquisition

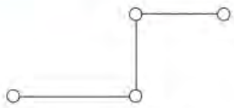
(very important)

Laser scanner is a instrument able to acquire automatically a lot of points (millions)

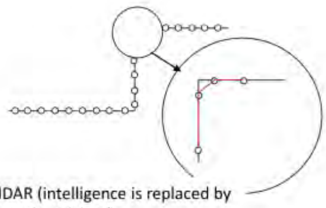
↳ we don't have any logical function, we substitute human intelligence with the number of points

Acquisition is made without any logical criteria. Usually, we have a lot of points, where we don't need and viceversa (Murphy's law)

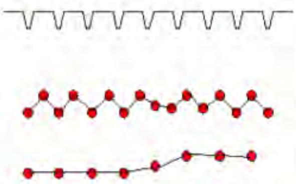
(ex. in corners, in focal points, ecc. → we need to extract the shape of the object)



Smart survey (human brain)



LIDAR (intelligence is replaced by quantity of data)



There are other problems: the discontinuities (break-lines) and the economic factor are fundamental. The dimensions of the objects and the required precisions vary in a very large range (from a few m to mm).

we don't have a regular distribution of points (= different precisions)
↳ each one has a noise so we need to smooth the surface to make it homogeneous

4.05.18

venerdì 4 maggio 2018 08:31

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L14.2

LiDAR (Laser Scanning)
Data acquisition, sensors, planning, data processing
Part 2

how to organize data position

MARCO PIRAS

GEOMATICS: 01RVUMX

Planning

Laser scanning is quite similar to a total staion, but without a telescope to collimate the points. **The quality of the «human brain» is replaced by the quantity of points and rapidity of data acquisition.**. million of points → because we don't need points so the only way is to have a huge number

If the planning is not correctly realized, the final result could be different to the «foreseen» model, therefore it is necessary to repeat the survey, with additional cost and time

In order to realize a good «laser scanning» acquistion, it is important to consider:

preformance

1) **Characteristic of the Laser: precision, range, FOV, resolution,** field of view
2) **Object (shape, size, etc)**
3) **Environment**
   - other people
   - natural obstacles } interferences
   - minimum level of light if we need to collect a digital image → we can work also in the night but if we merge it with the image the constrains are the ones of the image (so light)

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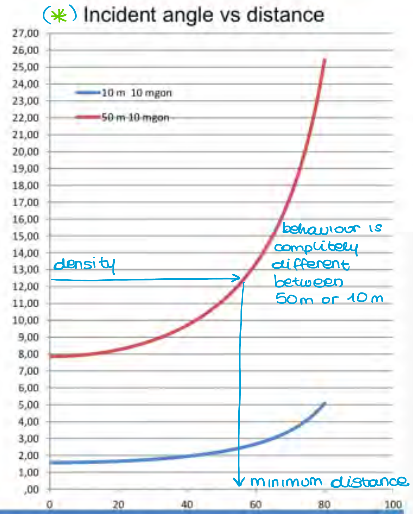
Planning

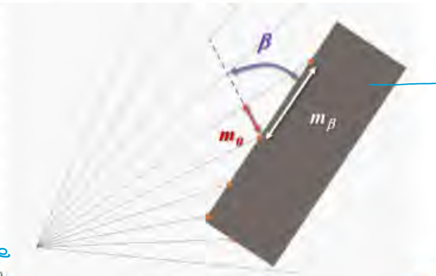
Density and inclination

(✳) Incident angle vs distance



The density is not constant everywhere (excluding a sphere surface).

Density depends on the distance, slope of the surface and direction of the incident laser beam. → we need to fix the min. density of the cloud of points



→ slope surface

$m_\beta = m_0/\cos(\beta)$

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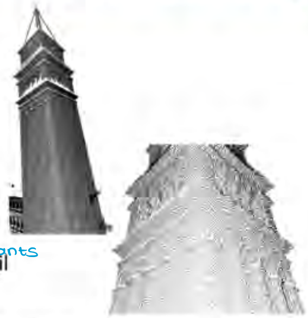
Planning

Definition of the density

It is not possible to have a constant density, but it is necessary to have a most regular density → very difficult to be controlled

The main «requirement» is the minimun value of the density



$$Q = 1 - \frac{m}{\lambda}$$

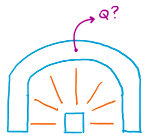
Q data quality index (%)

m points density } minimum distance between points

λ Size of the minimum detail ← it depends on the goal

we can fix the max distance allowed for a certain density, using the diagram (✳), then with m, we know λ and we can compute Q

If distance isn't good for the correct density we can use:
- mobile device
- photogrammetry
- total station (we need more number of points, but the quality is very high for sure)
- drones



For example:

- If the density is 10 mm and it is necessary to detect the brick's separation (interstices) (5 mm) : working with this density we aren't able to represent this level of detail
  Q=1-10/5=-1
- Using a density m =5 mm: Q= 0 → density = dimension of the element → very low probability to collect the element
- Using m= 2 mm → Q = 0.60. It means that 60 % of the interstices are correctly detected.

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Planning

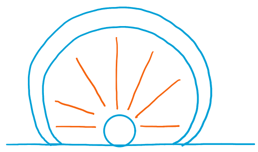
Density definition

The selection of the resolution is not a very «easy» and «rapid» ste, because there is not a «unique» solution.

It is important to consider the «natural» and «morphological» texture of the object.

we lost geometrical part of the object

Ex. TUNNELS
- cheap
- short
- no need to close tunnel for a long time
- mobile devices
- geo reference with IMU platform (inertial platform)



Data acquisition is not intelligent so it can happen that when we move, the points of the same details aren't exactly in the same position:
- high density of points in a small region
- points aren't coincident (some of them need to be eliminated)

Planning

Site inspection

Using a **sketch,** it is important to define:

- code or name of the single part
- position and name of each scan
  ↳ manage precisely the informations
- position, name or code, of each target
  → especially when the quantity of points is huge

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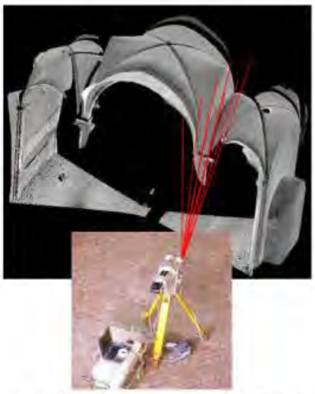
Planning

Site inspection

GEOMATICS: 01RVUMX

Planning

The influence of the geometrical distribution of the scans



One scan is not sufficient to acquire an object, even with a simple geometry

If many scans are realized, it is necessary to be able to join the single scans into a unique reference systems. Therefore, it is important to organize the network of targets

→ global or local

We have a mandatory condition: reference network is fundamental (topographic survey), and a series of points with purpose to realize the roto-translation» between the single scan.

position of target has to be in a unique reference system → if local, each one has a personal one so for merging info, we need to put them togheter

it requires a lot of time to merge different point clouds

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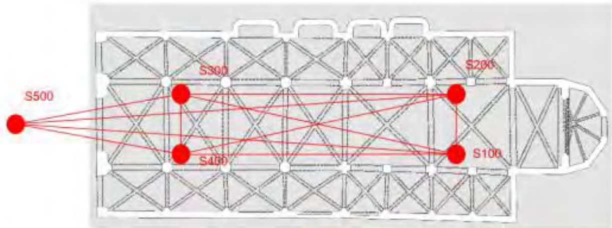
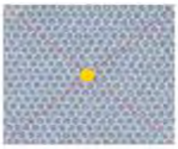
Planning

The influence of the geometrical distribution of the scans





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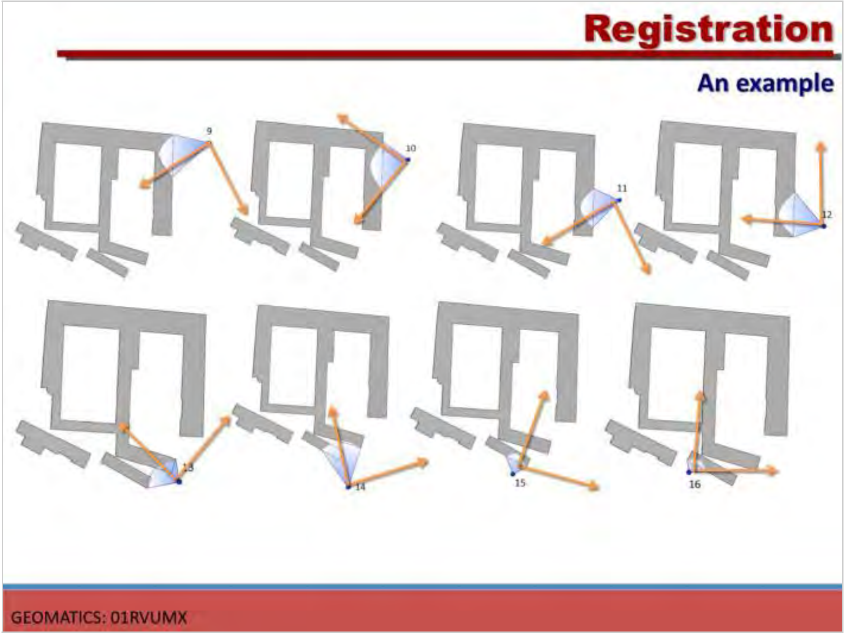
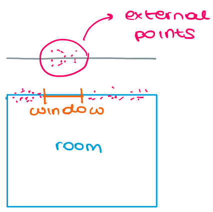
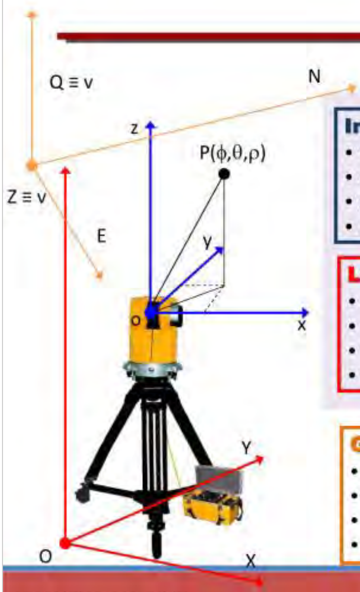

Registration
An example
9
10
11
12
13
14
15
16
GEOMATICS: 01RVUMX


Registration
An example
there are a lot of points outside the model object
ex. window which allows to collect no needed points
after registration we need to eliminate them


external points
window
room

Coordinates system



Instrinsic coordinates system (o,x,y,z)
- Origin (O) is in the instrumental centre
- z is coincident with rotation axis
- x is perpendicular to z and coincident with transverse axis
- y completes the Cartesian system.

LOCAL coordinates system (O,X,Y,Z)
- Origin (O) as user defined ↳ defined by the user
- Z coincident with vertical direction in O
- X perpendicular to Z and with orientation as user defines
- Y completes the cartesian system

GLOBAL Coordinates system (E,N,Q)
- Using the traditional reference system
- UTM WGS 84 or ED50 → cartographic or geographic
- Gauss-Boaga
- Orthometric height or ellipsoidal height.

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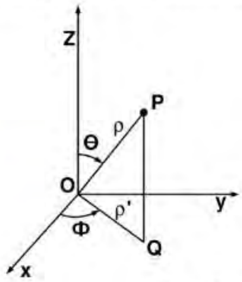
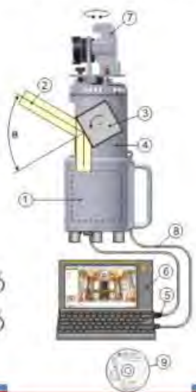
=> we need to combine several techniques for a better result

Coordinates sytems

Intrinsic coordinates system

Laser scanner measures the position of each point using the distance ρ and the angles (horizontal and vertical) (ϕ and θ).
The Intrinsic coordinates system is a conventional system, but is stable for each instrument.





From spherical coordinates to cartesian coordinates in intrinsic coordinates system

$$x = \rho \sin\theta \cos\phi$$
$$y = \rho \sin\theta \sin\phi$$
$$z = \rho \cos\theta$$

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Coordinates system

Estimation of parameters

The rototranslation:

- involves always **two references system only.**
- Define the relation between homologous points, which have different coordinates but belong to the same «REAL» points;

Estimation of R and T requires to know at least a certain number of homologous points in both references system (tie points, common points, correspondences, etc).

It is possible to find some correspondences if there is an overlapping between the scans at least of 30%.

How many common points?

The least number is 2….but it is better to have more points (>3), in order to have a system with redundancy (equations- unknowns)

Coordinates system

Selection of the points

The selection of «common» points could be made in several ways:

- **markers** (artificial target with retro-reflective surface), which can be automatically detected; usually we add natural points to markers.
- **significative elements,** which are directly identified in the scan, considering the shape and morphology of the object, using some **shape descriptor**
- automatically minimize the distance between the homologous points, in a overlapping region: **Iterative Closest Point** and possible variations

Alignment techniques

Iterative Closest Point (ICP)

→ automatically extract the markers

- It is the first solution about the automatical marker identifications
- Proposed on 1990 from Besl and McKay (1992), it has been used in "computer vision" and "reverse engineering".
- No tie points are required
- Iterative computation

If the dataset (A and B) are quite well aligned, the algorithms:

The common points are the closest in the clouds.

The transformation is made, minimizing the residuals.

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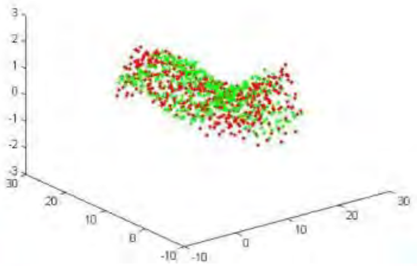
Alignment techniques

Iterative Closest Point (ICP)

For each point $\{v_i\}$ of V, the nearest poing of U is determined;
- transformation is estimated (R,t), in order to minimize the distances between the clouds;
- the positions of the points {vi} of V are updated with the parameters;
- the proedure is iterated up to the convergence (threshold is the minimum distance between the clouds).

⇒ reduce value of residual
quite complex if the surface has a huge number of discontinuities



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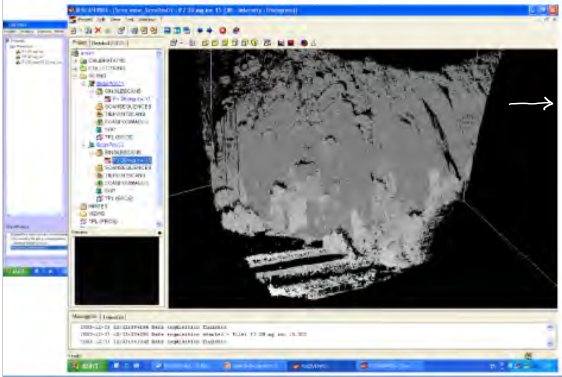
Post processing of the terrestrial LiDAR

Data smoothing or filtering

Smoothing of the acquired data allows to eliminate the noise component which is always include in laser data (automatic procedure)



→ raw data are affected by a huge noise (terrible data)

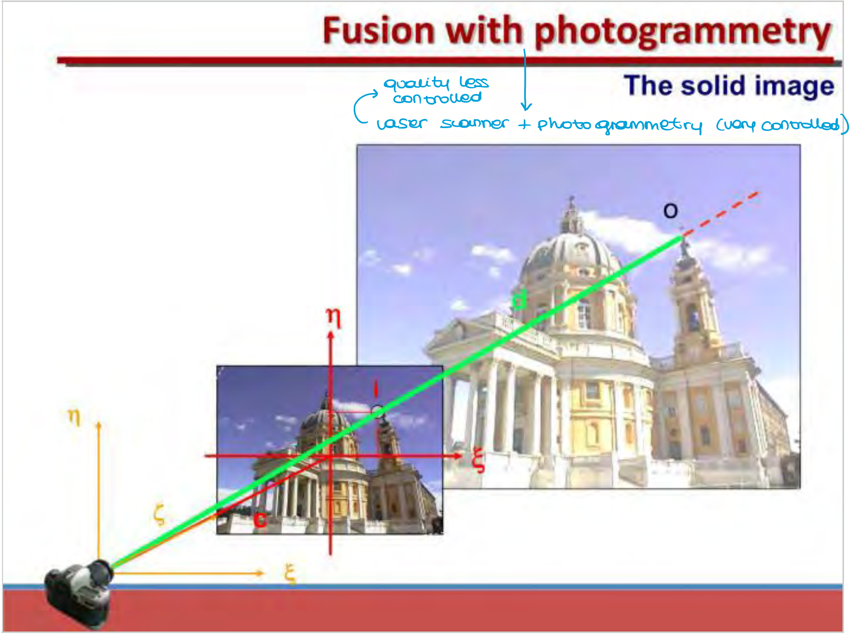
Post processing of the terrestrial LiDAR

Generation of 3D surface

↳ registration, georeference then 3D surface

→ clean model → without noise

→ we spend a lot of time removing points we don't need to identify surfaces


Fusion with photogrammetry
The solid image
quality less controlled
laser scanner + photogrammetry (very controlled)
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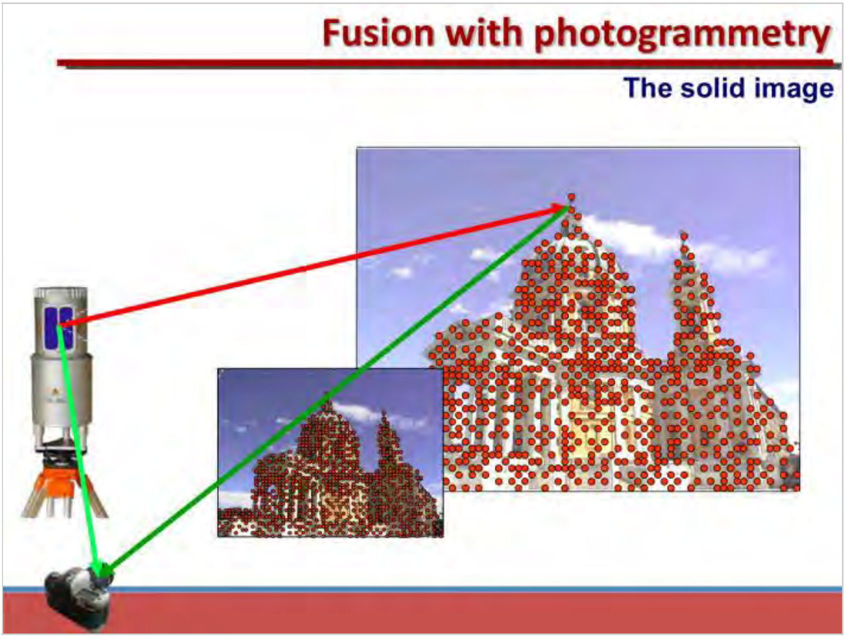
Fusion with photogrammetry
The solid image

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L14.3

LiDAR (Laser Scanning)
Aerial platform
Part 3

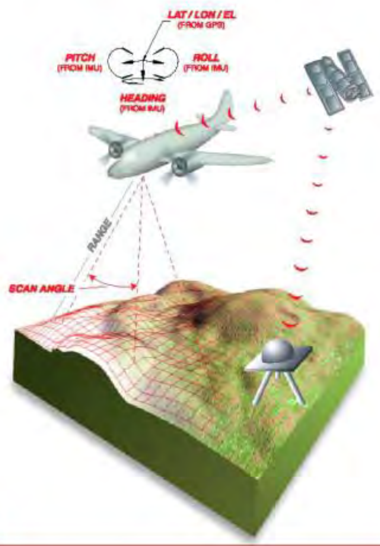
MARCO PIRAS

GEOMATICS: 01RVUMX

Aerial LiDAR

→ very high cost with respect to photogrammetry

Principles



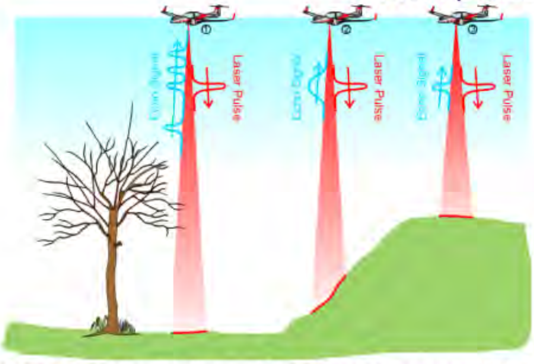
- system: airplane, helicopter, drone
- GPS defines the position (XYZ)
- Inertial platform(IMU) estimates the orientations (omega, phi, k)
- Laser source
- For each ground point, the coordinates (3D) are estimated

combination of angle of laser mirror and the angle of the plane where the laser is installed

photogrammetry: ground control points, here not

Aerial LiDAR

Multi-pulses (echo) registration



At the moment of impact with the surface the laser pulse will then be characterized by an impact region and not by a single point.
When the impact region of the laser pulse meets an obstacle (leaves, suspended lines), this is reflected at different times. so we have different pulses for each obstacle → the stored signal is only the first or the last (we need to choose which one, but in general only one pulse)
The backward signals are therefore more than one, which gives rise to the possibility or the need to measure a specific one or a series. it stores a huge number of info

Aerial LiDAR

Multi-pulses (echo) registration

The greater part of the laser system are able to collect only «one pulse», usually the first pulse. In some case, the last pulse is stored.

In other cases, the system is able to acquire up to 4 different pulses.

In order to distinguish the different pulses, it is necessary to have a discontinuity between the pulse (at least 1 m, it is related to the wavelength).

=> so when we have vegetation lidar is better, because photogrammetry even if it has better quality, can collect only the top of the trees

Aerial LiDAR

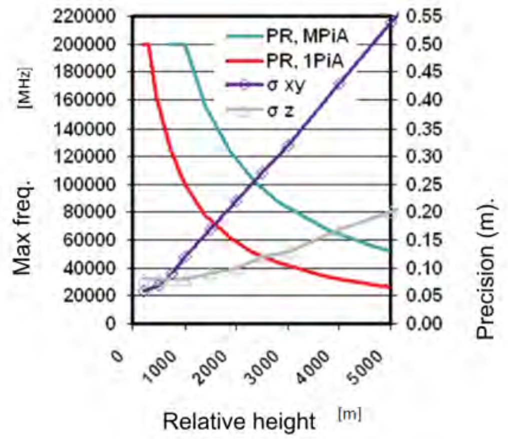
Errors

Systematic errors

Calibration errors
Time errors
Angular resolution errors
Inertial platform errors

Random errors

GNSS system
Distance measurements
Angular resolution errors
↳ Quality of the sensor and quality of laser scanner



9/05/18

Aerial LiDAR

Some examples

Precision: dm
Resolution: it depends on the height of flight, but up to 8-10 points/m^2)

Riegl LMS-Q560

Geometrical and radiometric measurements;

we flight over a very wide region when we use a lidar laser scanner
↓
if we need a small portion it's better using terrestrial or drones
↓
use photogrammetry when resolution isn't needed so high

Aerial LiDAR

Calibration of the aerial system

→ we use reference points (non points) to control if the sistem measures well
→ so the lever arm of each instrument with respect to the central point

A calibration field is required. It is important to verify the correct «geometry definition» between the sensors

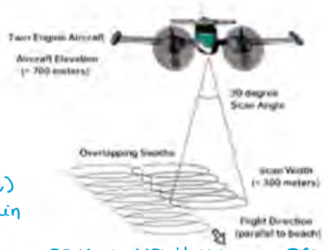
Aerial LiDAR

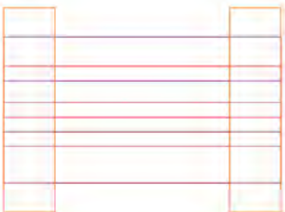
Design of the flight

WATER
→ it adsorbs all the signals
→ we don't have any point

The flights have to be designed, considering:

1. Number of pulses → only top of trees or also the terrain
2. FOV of the laser
3. Number of points (density) → height of the plane (if h ↑↑ d ↓↓) → kind of terrain
4. Terrain morphology
   - vegetation
   - building
   - water





we obtain a not-regular distribution so after that we need to perform an interpolation

In order to avoid some «gap» in the point clouds, it is important to have at least the 20% of overlapping between trasverse strips, but we have to considering the slope of the terrain. → magnitude of region common when we use aerial L.

It is a good practice, every 10 km, a trasverse acquisition has to be realized. In particular, it is fundamental to acquire a trasverse line at the begin and at the end of the lines. → make acquisition more robust

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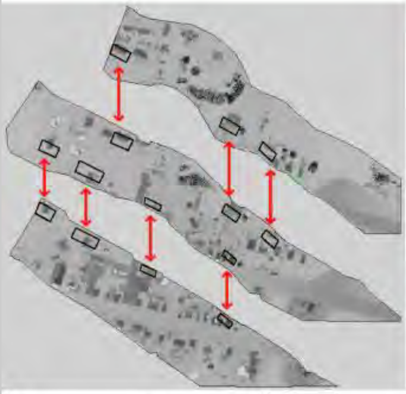
Aerial LiDAR

Georeferencing and alignment

to merge different lines
↳ we have noise so probably lines are wave, not easy alignment
↳ we use statistical approach



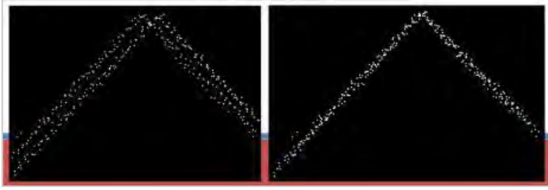
IMU and GNSS

alignment → we need to interpolate with the profiles given by inertial platform and GPS

Statistical approach, to minimize the residuals.

$$\Delta H = a_s + b_s\left(X - X_s^c\right) + c_s\left(Y - Y_s^c\right)$$

$$\Delta H = a_s + b_s\left(X - X_s^c\right) + c_s\left(Y - Y_s^c\right) - a_t - b_t\left(X - X_t^c\right) - c_t\left(Y - Y_t^c\right)$$



LiDAR: Aerial solution

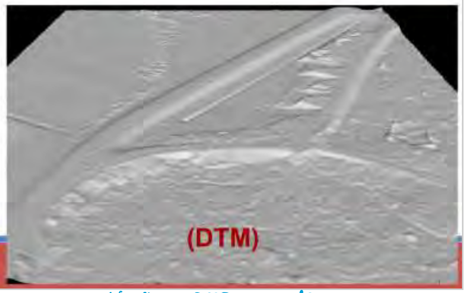
Filtering

It is important to remove the points which are not belong to the terrain (DTM)

↳ From RAW data we can find only DSM then after filtering we have DTM

we use different algoritms

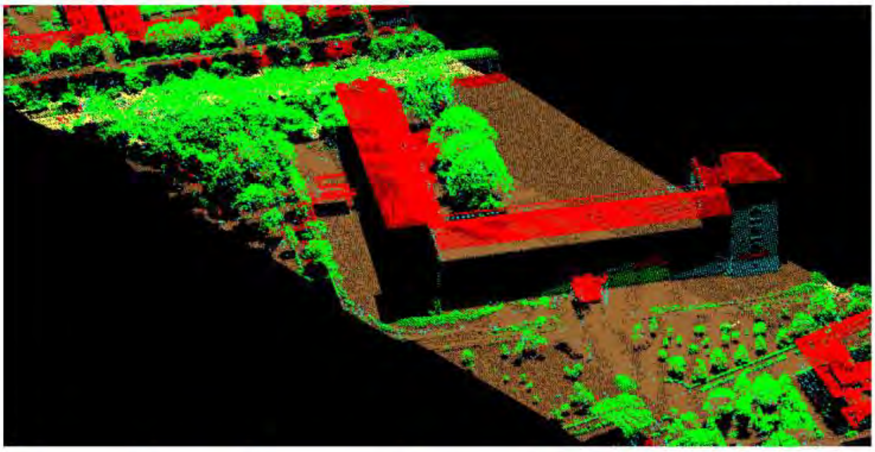
digital surface model



digital terrain model

LiDAR: Aerial solution

Classification

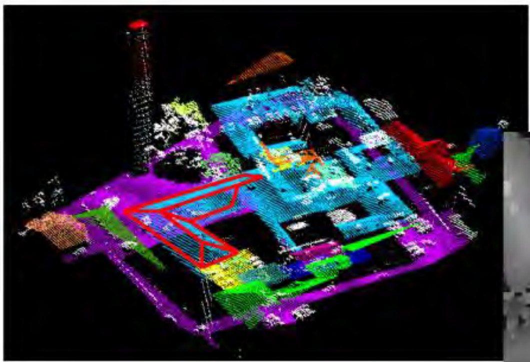


	terrain	Irregul. terrain	vegetation	building	Outlier	total
points[n.°]	414.858	34.808	226.092	270.976	12988	959.722
Error [%]	0,43	3,30	0,12	12,25	0,00	3,79

LiDAR: Aerial solution

Segmentation

Analytical procedure that combines points with common characteristics defining regions and objects



Region Growing by means cluster analisys

↳ we need more data / points

POLITECNICO DI TORINO
DIATI

Introduction to tracking

Paolo Dabove

GEOMATICS

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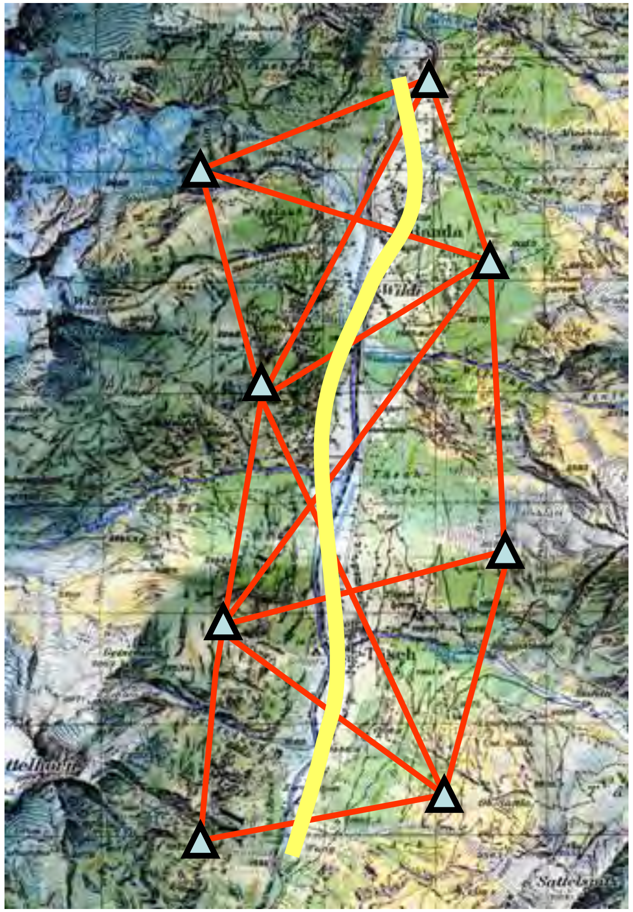
REFERENCE SYSTEMS

Materialization of the reference system on the ground using a **geodetic network** (often specifically made)



Vertex: → **Station** / **Orientamento** | Tunnel, viaducts, etc.

Survey: → **GNSS** / **classic**

DETAILS

Survey: → **Tracking with Total stations** / **GNSS-RTK** / **Photogrammetry**

The final product of these surveys constitutes the starting proint of the project design.

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FROM THE ANALYTICAL PROJECT TO THE REALIZATION

Problems in the transition from project to ground:

1) Loss of decimal numbers (measurement approximation)
2) The geometry of the measurement schemes affects the propagation of measurement errors in the tracking activities
3) relationship between design reference systems and tracking

DESIGN OF TOPOGRAPHIC MEASUREMENTS

Steps:

0) Reference systems definition

1) Measurements schema definition

2) Measurements accuracy and precision (weights)

3) Optimization of the schema

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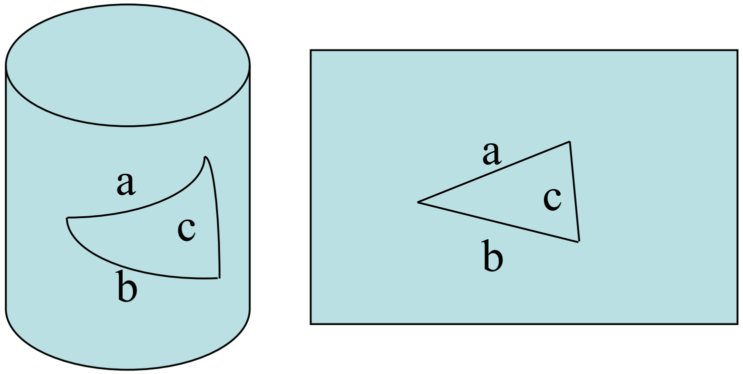
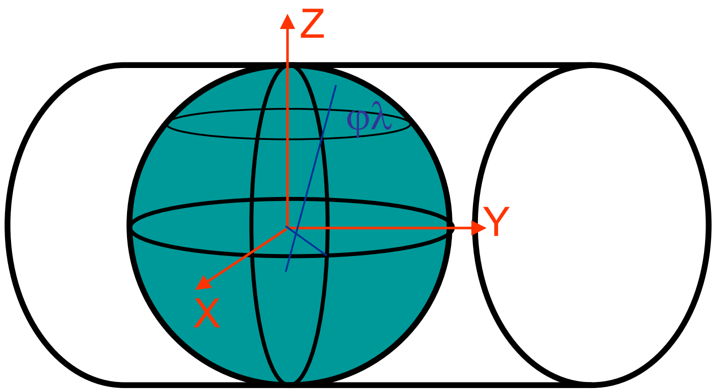
REFERENCE SYSTEMS
Cartographic representation





Cartographic projections "move" coordinates from an ellipsoid to a cartographic plane:
What we can do is:
- to preserve angles (conformal maps)
- to preserve surfaces (equivalent maps)
- to minimize the deformation, without having none of them actually null (compromise maps)

Geocentric (X,Y,Z) or geographic (φ,λ,h) coordinate systems
⇒ no deformations but ….

•) **not easy for design and tracking activities**
•) **valid only on local planes** (less than 15 km)

Cartographic representation: the official national cartography is a Gauss projection. There are two fuses of 6°30' each

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REFERENCE SYSTEMS
Isometric coordinates

E.g.: a linear element ***Lp=5 km*** in the middle of the fuse (m_L=***0.9996***) must be identified on the ground at ***h= 600 m***:

$$Lt = \frac{5000\,m}{0.9996\ (1 - 600\,m/R)} = 5002.47\,m$$

Their lenght on the map is ≠ with respect to the tracking one

1) Changing some project parameters during tracking phase(?)

2) consider the reductions in the project: *Lp=4997.53m* ⇒ *Lt=5000m*

3) ISOMETRIC systems ⇒ immediate use in tracking

Similarity between cartographic and tracking dimensions

It is not possible to obtain a null value of the linear deformation in the whole plane of representation ⇒ RS is isometric if it creates negligible residual deformations (e.g. 10 ppm?)

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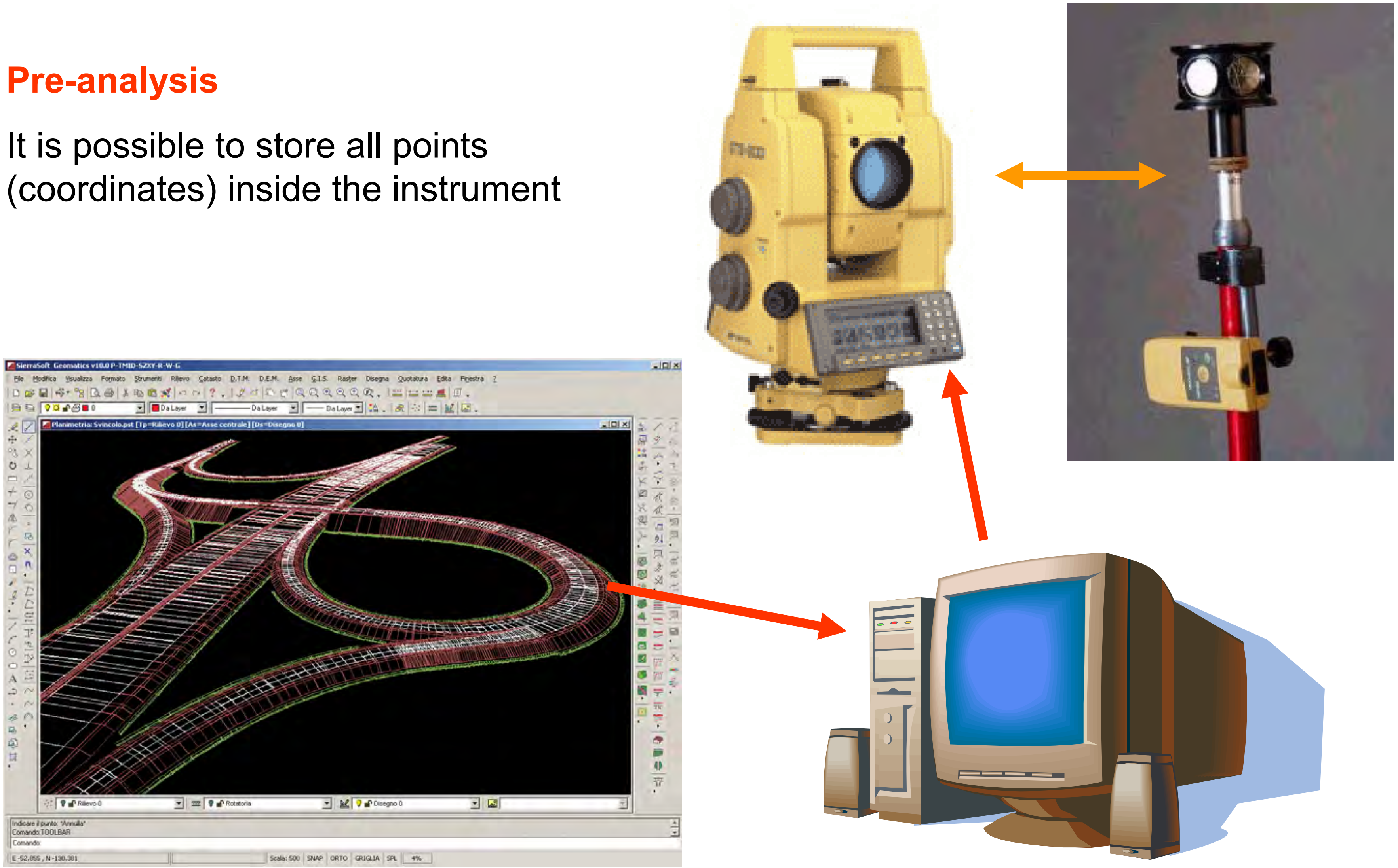
New instruments and methods

Tracking with total station

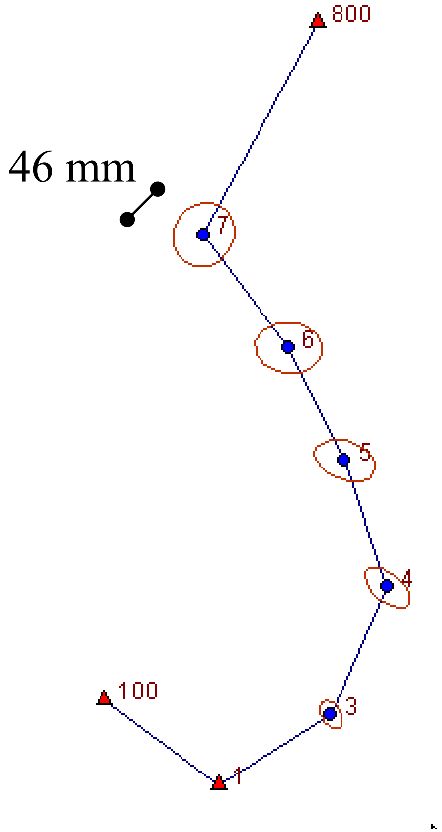
Pre-analysis

It is possible to store all points (coordinates) inside the instrument



New instruments and methods

Tracking with total station



Simulation of axis polygonal:
considering directions and distances, with steps of about 1km:

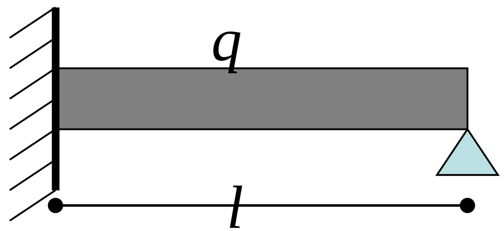
$$\sigma_\alpha = \pm 0.5 mgon$$

$$\sigma_d = \pm(3mm + 2\,ppm * d[km])$$

Ellipses at 95%

Static analogy:

$$f = \frac{ql^4}{185\,EJ}$$



But this only happens at the end of the excavation: before "closing" ...

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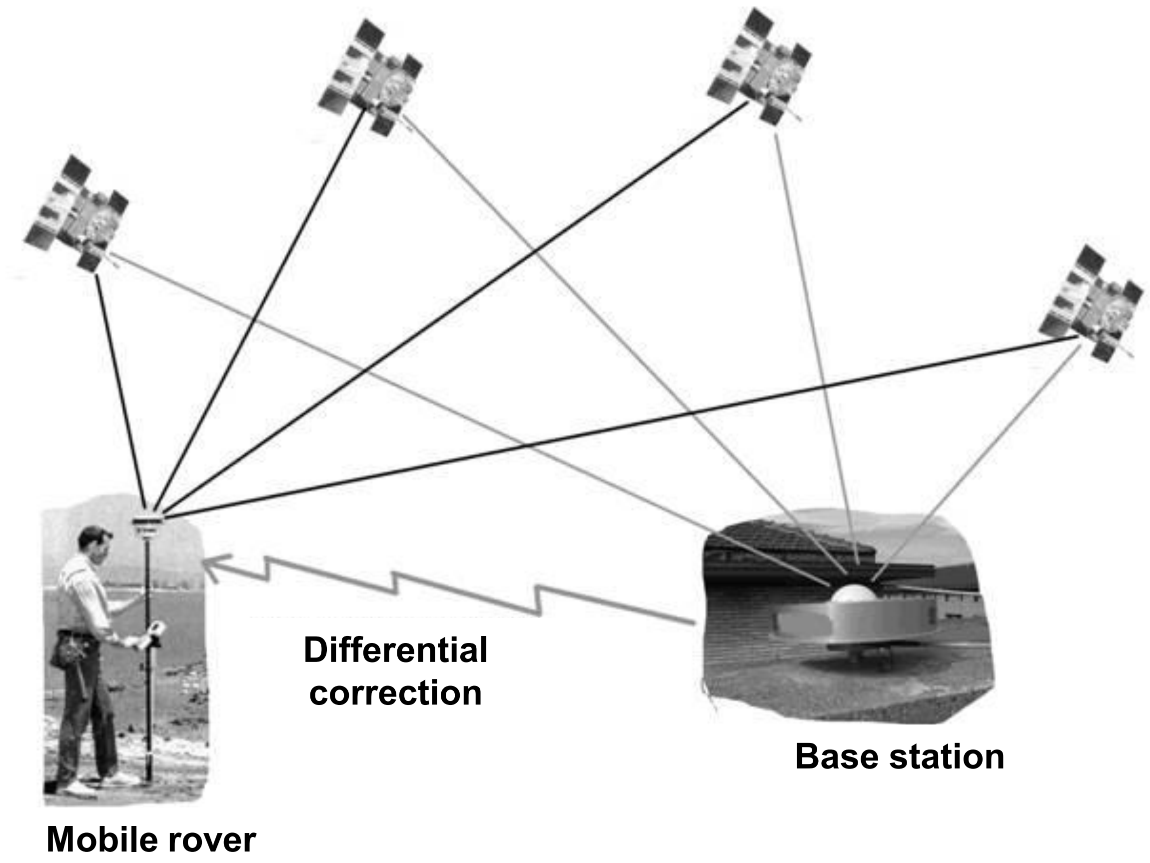
New instruments and methods

Tracking with GNSS-RTK



Real-time (RT) mode, centimetric accuracy, two receivers (base and *rover*), satellites visibility required (no tunnels)

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New instruments and methods

Tracking with GNSS-RTK

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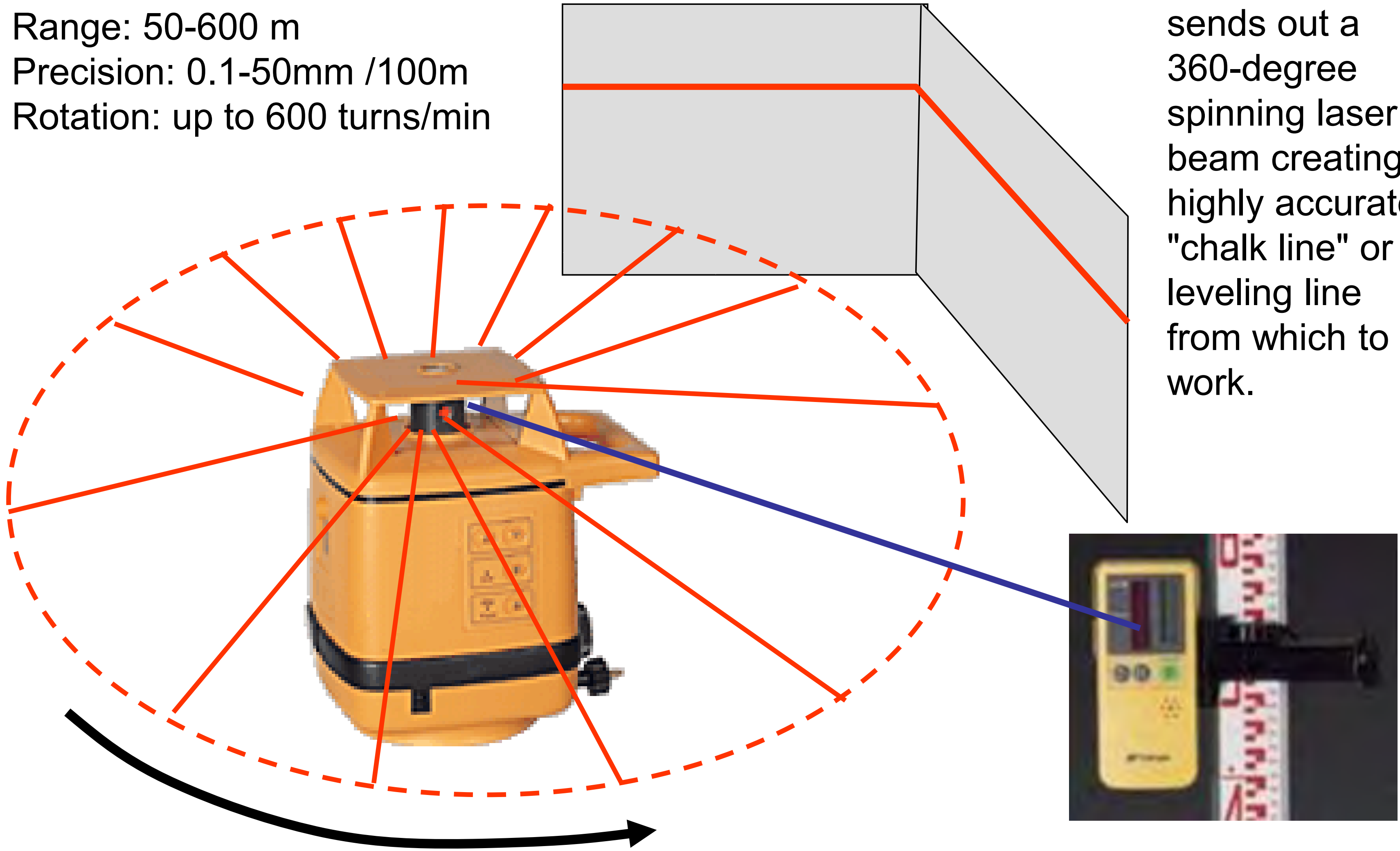
New instruments and methods

Rotary Lasers

Range: 50-600 m
Precision: 0.1-50mm /100m
Rotation: up to 600 turns/min

A rotary laser sends out a 360-degree spinning laser beam creating a highly accurate "chalk line" or leveling line from which to work.



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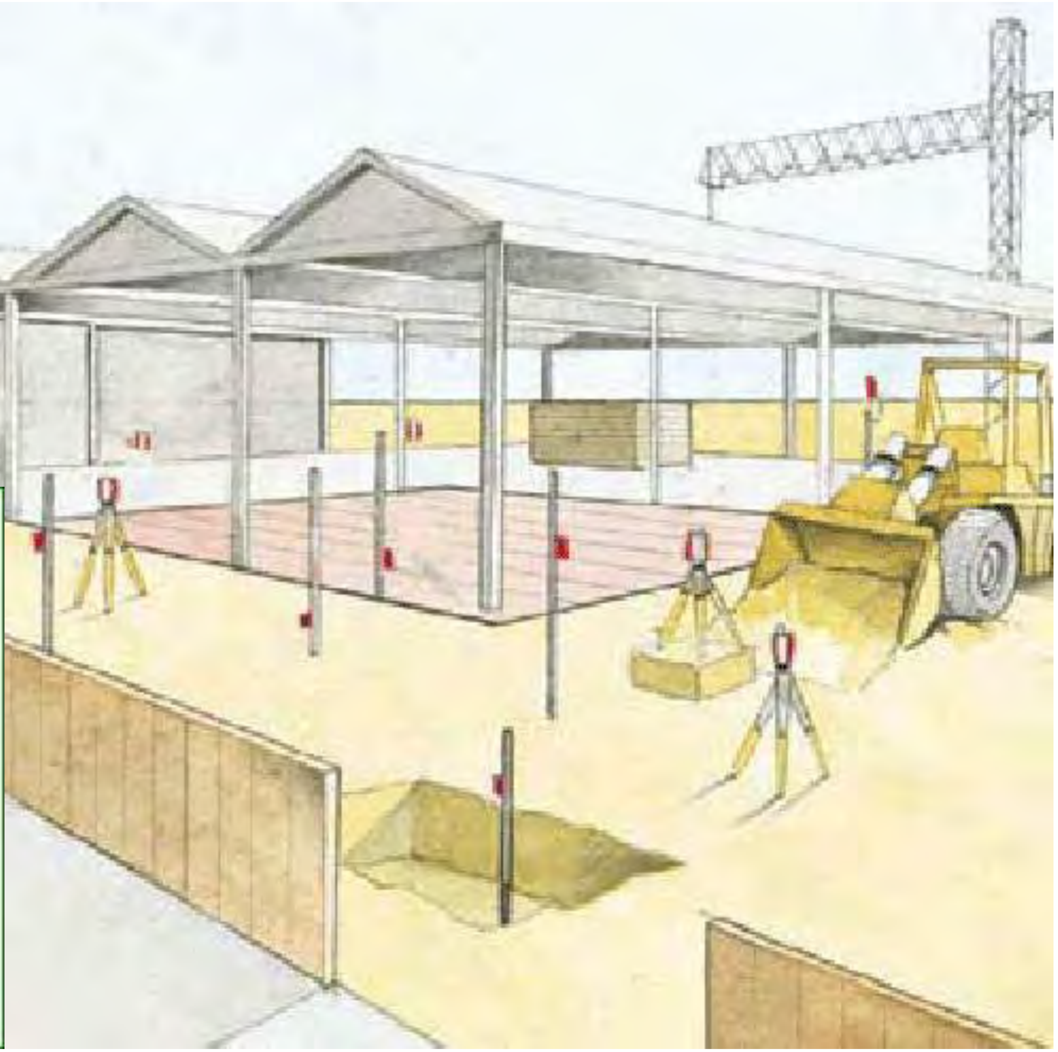
New instruments and methods

Applications for horizontal surfaces (outdoor)



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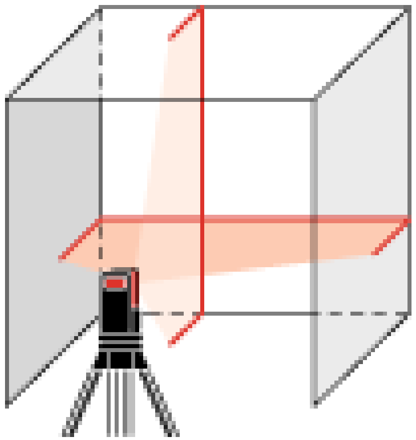
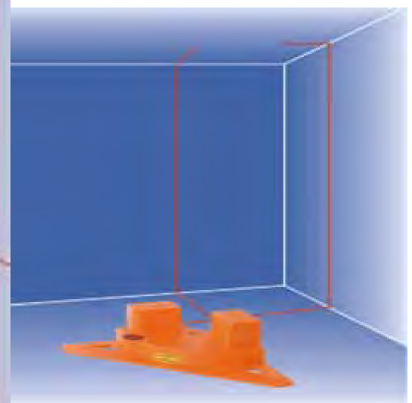
New instruments and methods

Line Lasers

Line lasers allow the user to establish a horizontal or vertical plane by projecting a beam, or line, of light usually around 180 degrees horizontally and vertically.

However, there are some line laser levels that can project a beam 360 degrees.





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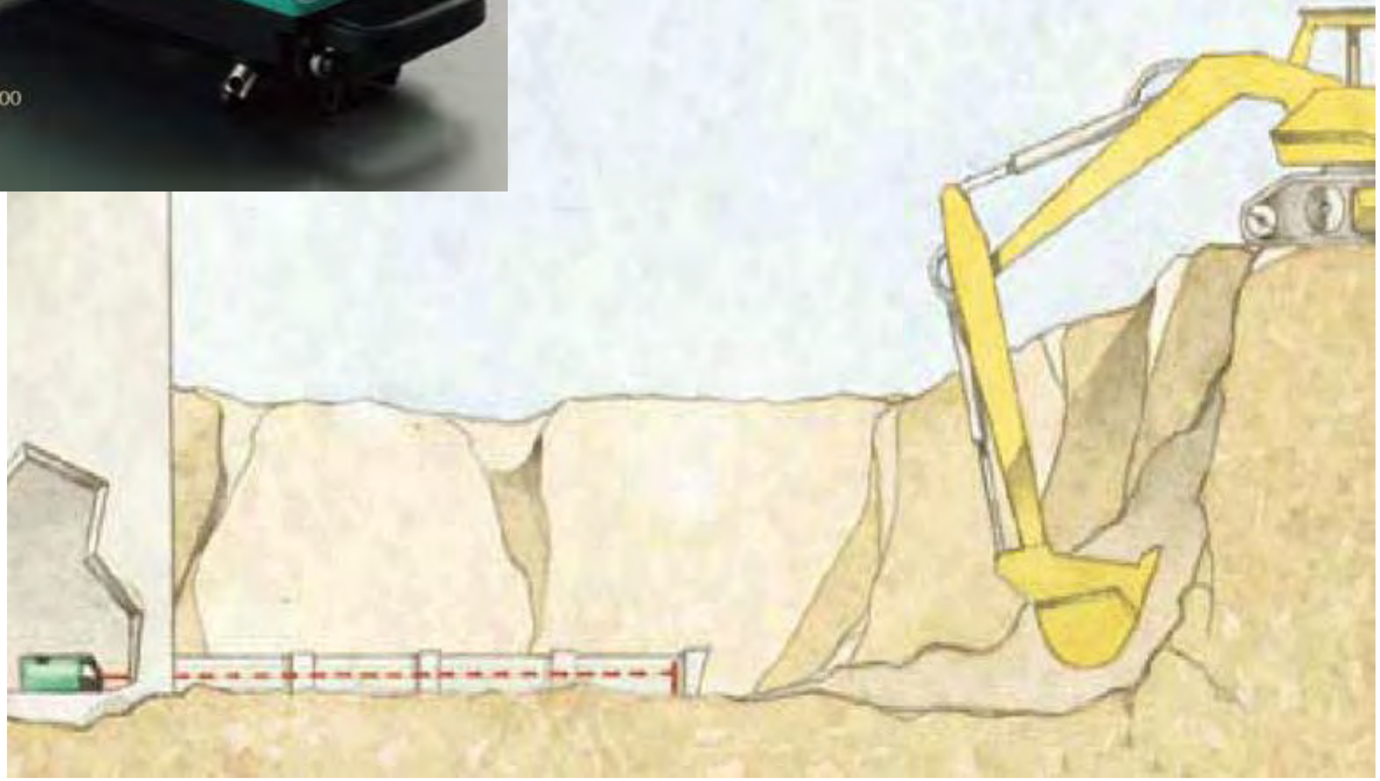
New instruments and methods

Pipe lasers

The clue is in the name, pipe lasers (also known as utility construction lasers) are used for work in pipes.

This can be above the ground, in manholes or in the pipes themselves.



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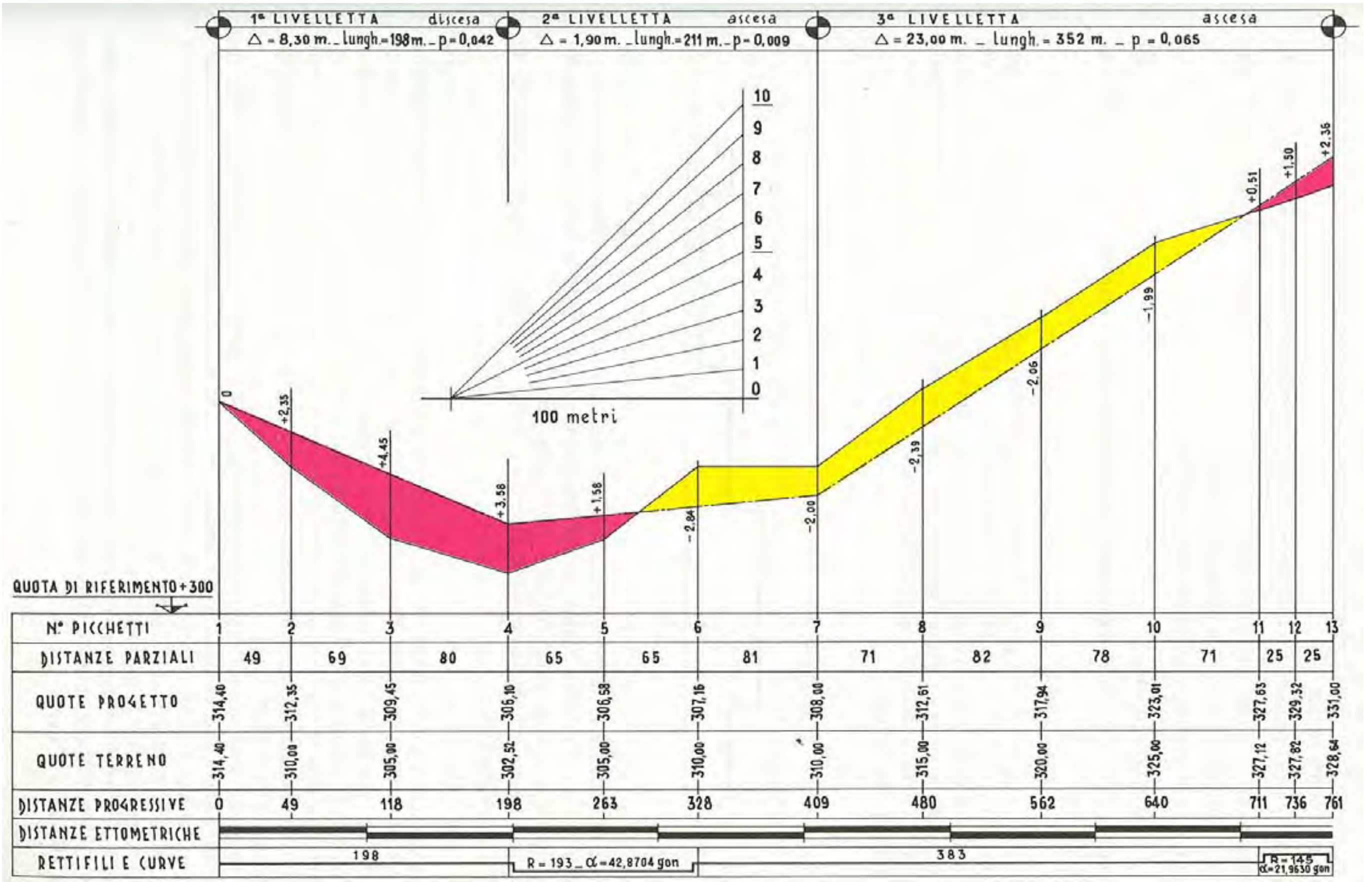
THE TRACKING ACTIVITIES OF ROAD INSFRASTRUCTURES

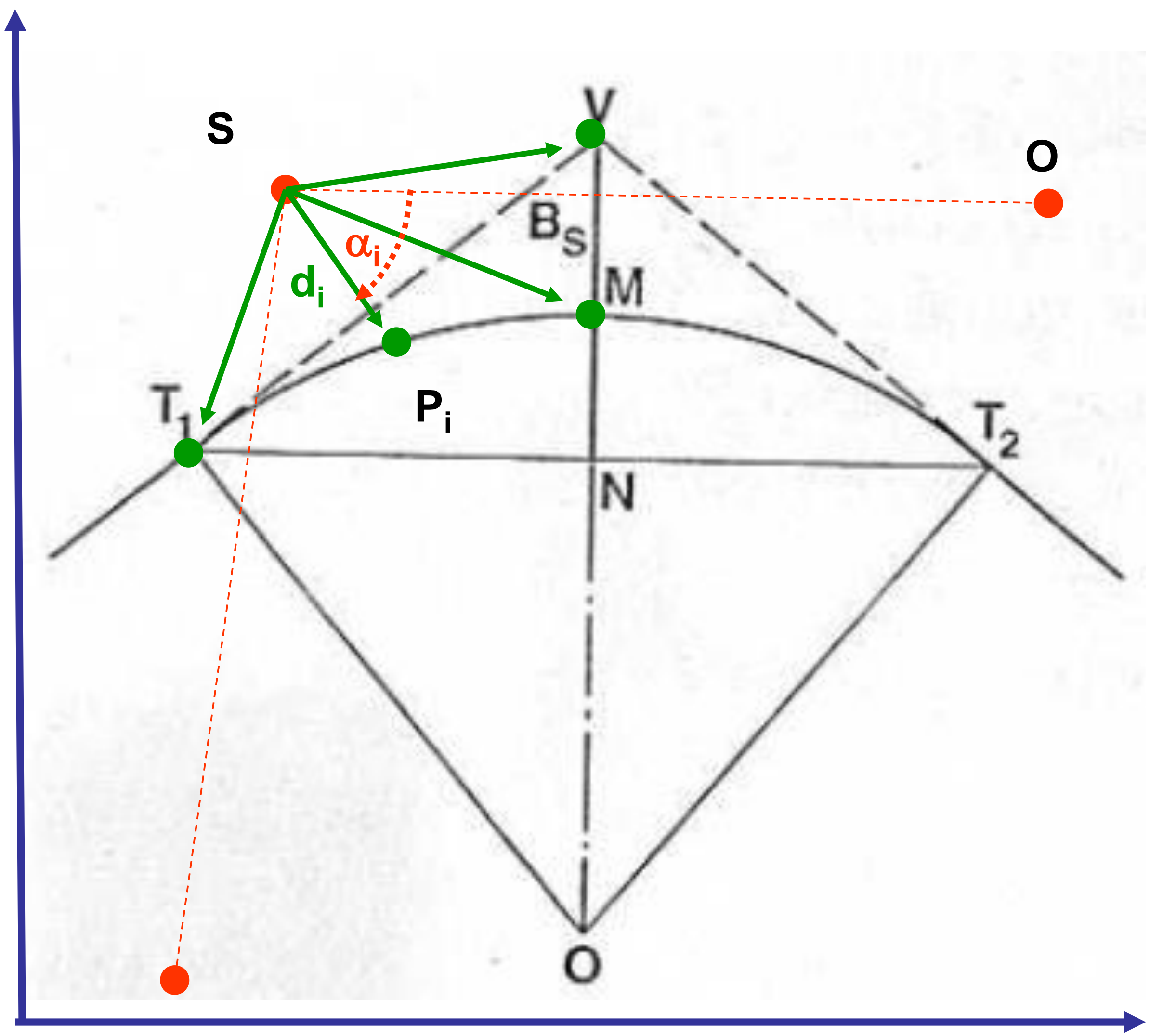
The road project - The longitudinal profile

Centerline stakes should be set at even 25 - and 50 - meter stations when practicable and intermediate stakes set at significant breaks in topography and at other points, such as breaks where excavation goes from cut to fill, locations of culverts, or significant obstructions.



THE TRACKING ACTIVITIES OF ROAD INSFRASTRUCTURES

Curve Widening



Relations

$$d_i = \sqrt{(x_i - x_S)^2 + (y_i - y_S)^2}$$

$$\alpha_i = atn\frac{x_i - x_S}{y_i - y_S} - atn\frac{x_O - x_S}{y_O - y_S}$$

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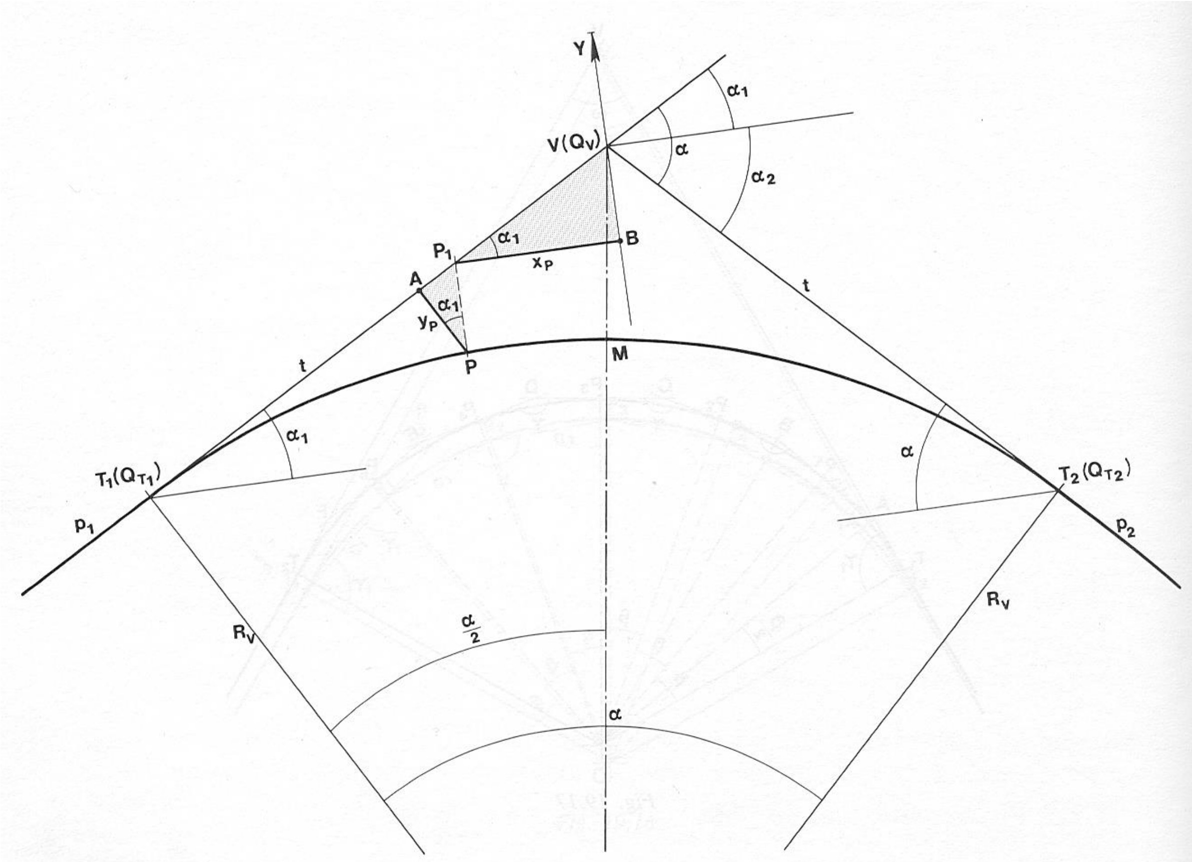
THE TRACKING ACTIVITIES OF ROAD INSFRASTRUCTURES

Altimetric definition – Vertical connections

The problem consists in the heights definition that will be used to stake out the vertical connection (circular or parabolic).

In case of **circular connection** (x_P y_P from profile)



$$t_{1(2)} = R \tan \frac{\alpha_{1(2)}}{2}$$

$$VM = R \frac{1 - \cos \alpha_{1(2)}/2}{\cos \alpha_{1(2)}/2}$$

$$Q_P = Q_V \pm x_P \pm \frac{y_P}{\cos p_{1(2)}}$$

+ convex crest curve

- concave sag curve

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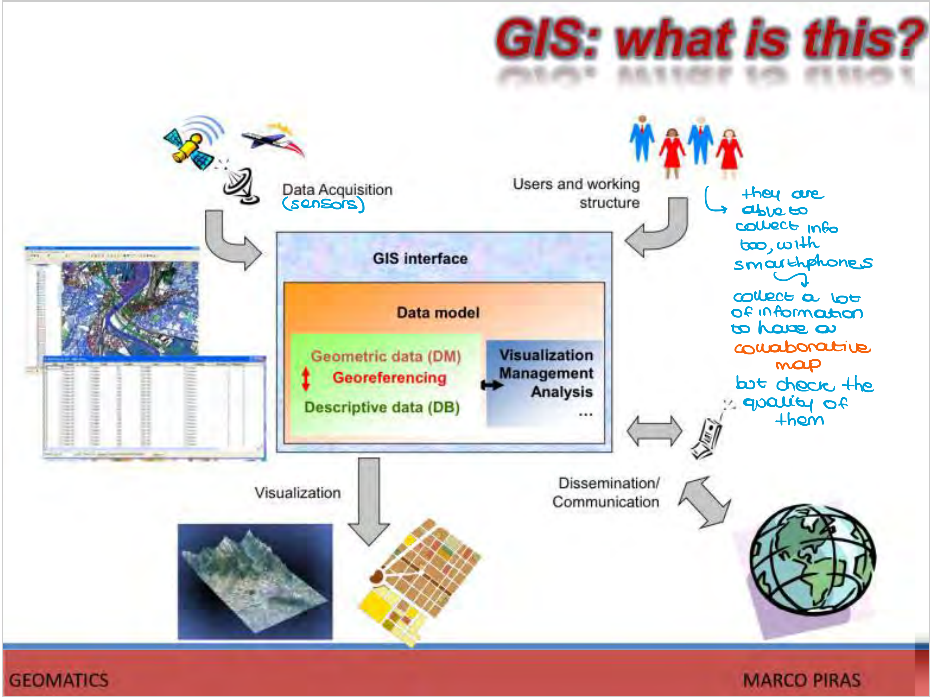


map → traditional
→ digital
↳ possible to pass to tematic maps very easily

GIS: what is this?

The main components

A GIS is composed by:

- a cartographic support / map (raster/vector)
- a database (where we collect data)
- a GIS management *software*
- some hardware tools for input/output
- an organization framework

(each item is called objects → in GIS objects are entities)

The **georeferenced data** describe real objects in terms of:

- **position** in a specific known reference system (local or global)
- **attributes** which describe the characteristics of the object
- **relationships** between objects: spatial or descriptive (↳ use attributes)

GEOMATICS MARCO PIRAS

A common reference language

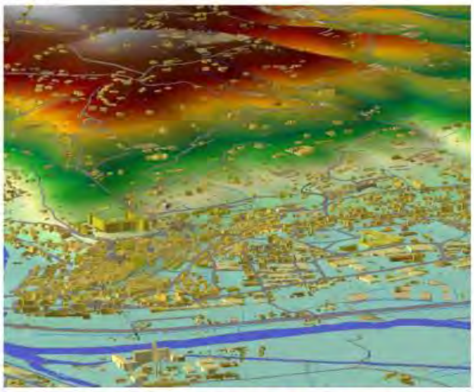
when we pass from reality to digital map we need to select data (not all of them can be stored – select info –)

The most common object of surveying is the terrestrial surface and man-made objects (infrastructure, buildings ...). You have to know the land and define models of representation metrically consistent to:

- Georeferenced data recording
- Information production and processing
- Modelling and simulation of different scenarios
- Updating data
- Visualization and presentation of information and analisys

restart to collect new info, new model, ecc.

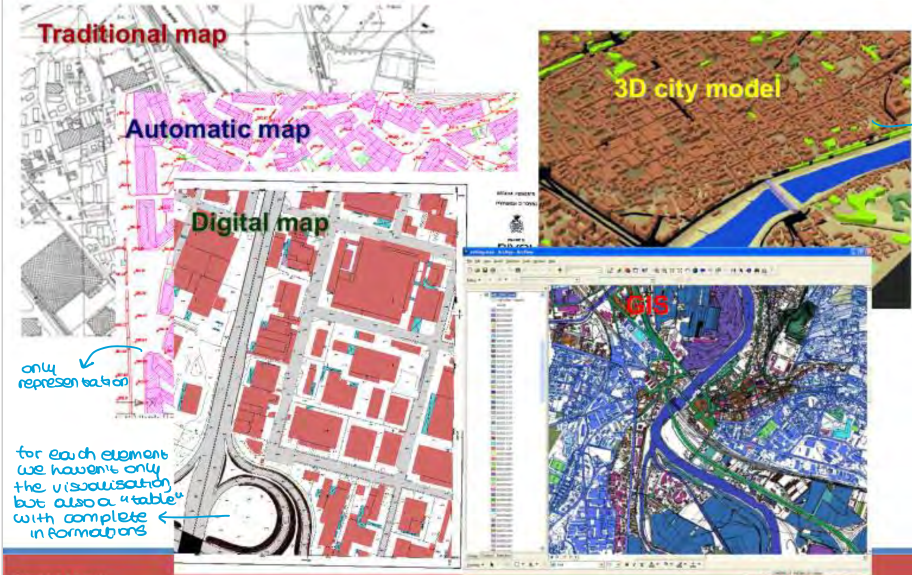
↳ procedure to produce 3D or digital data



Using a multidisciplinar/integrated approach which is based on a common reference language, the digital map.

GEOMATICS — MARCO PIRAS

Evolution of the map

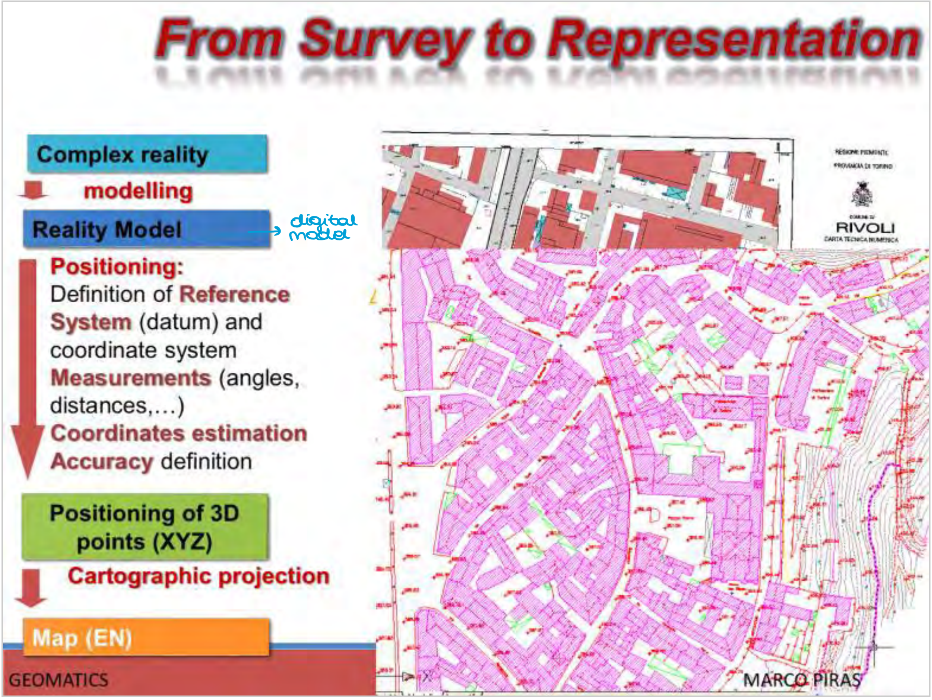


virtual scenario

only representation

for each element we haven't only the visualisation but also a "table" with complete informations

GEOMATICS — MARCO PIRAS



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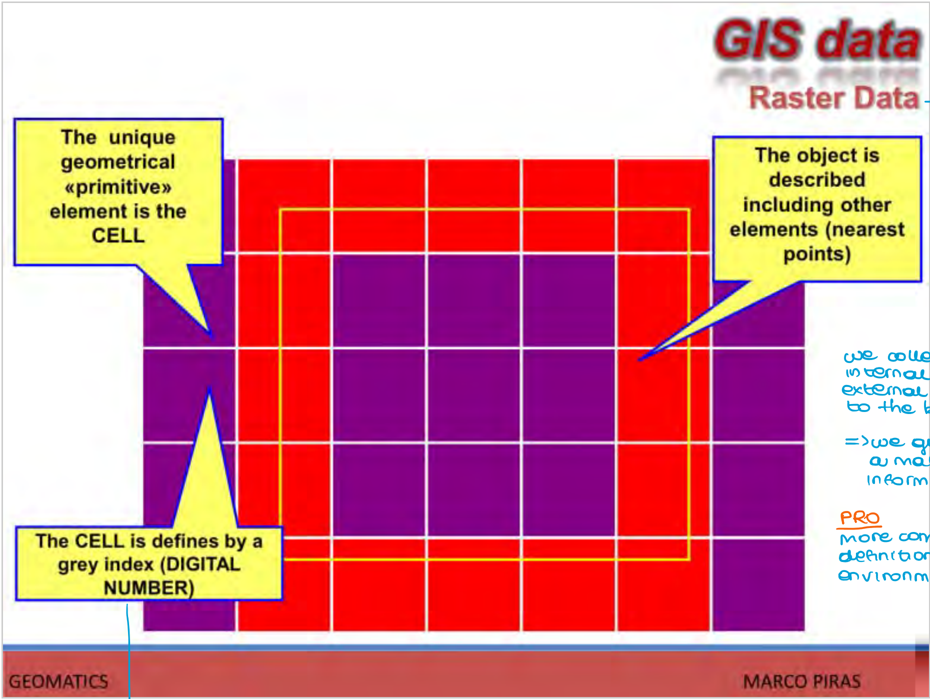


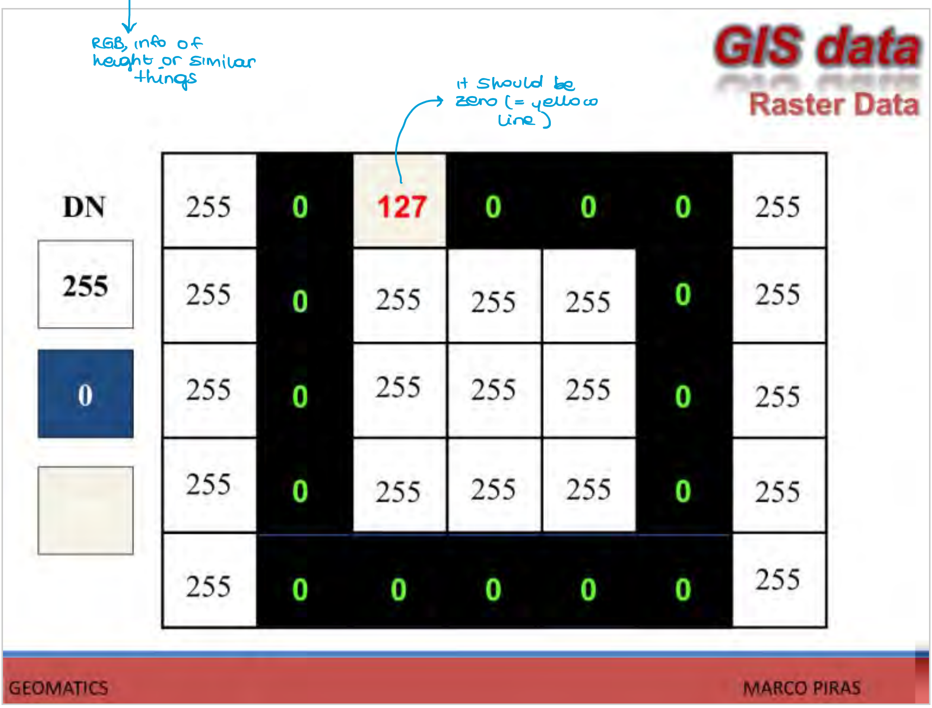
→ image

we collect both internal and external parts to the borders
=>we generate a matrix of informations

PRO
more complete definition of environment

CONTRO
size of data because we need all pixels to define data

RGB, info of height or similar things



255	0	127	0	0	0	255
255	0	255	255	255	0	255
255	0	255	255	255	0	255
255	0	255	255	255	0	255
255	0	0	0	0	0	255

it should be zero (= yellow line)