NUMERO: 2334A ANNO: 2018

A P P U N T I

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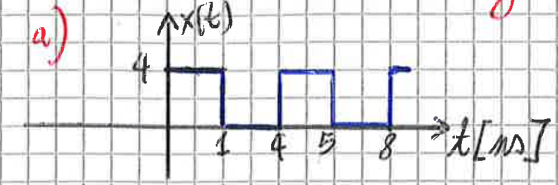
MATERIA: Advanced design for signal integrity and compliance - Esercizi - Prof. Canavero

3. SIGNAL SPECTRA EXERCISES

3.1. Periodic Signals

3.1.1. Determine the period and the fundamental frequency of the waveforms shown in figure. In addition determine the average value and hence c_0.

a)

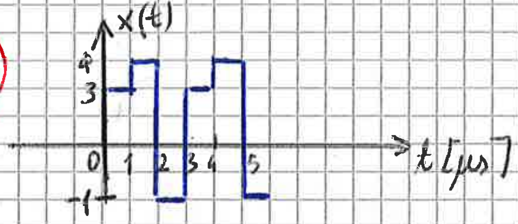


$T = 4\,ns \qquad \tau = 1\,ns \qquad A = 4$

$f_0 = \frac{1}{T} = \frac{1}{4\,ns} = 250\ MHz$

$c_0 = \frac{A\tau}{T} = \frac{4 \cdot 1\,\cancel{ns}}{4\,\cancel{ns}} = 1$

b)

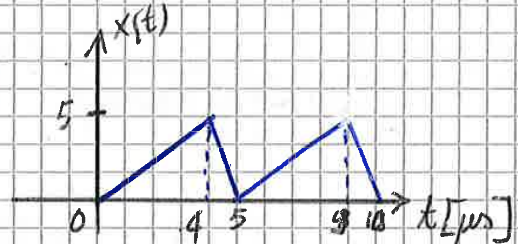


$T = 3\ \mu s$

$f_0 = \frac{1}{T} = \frac{1}{3\mu s} = 333{,}3\ kHz$

$c_0 = \frac{3 \cdot 1\,\cancel{\mu s} + 4(2-1)\cancel{\mu s} - 1\,(3-2)\cancel{\mu s}}{3\,\cancel{\mu s}} = 2$

c)



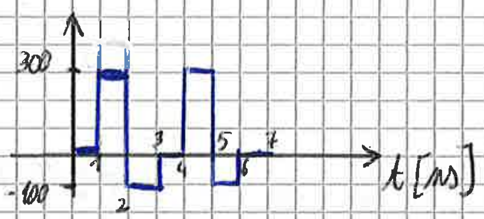
$T = 5\ \mu s$

$f_0 = \frac{1}{T} = \frac{1}{5\mu s} = 200\ kHz$

AREA OF TRIANGLE

$c_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{\cancel{5\mu s}} \cdot \frac{\cancel{5\mu s} \cdot 5}{2} = 2{,}5$

d)

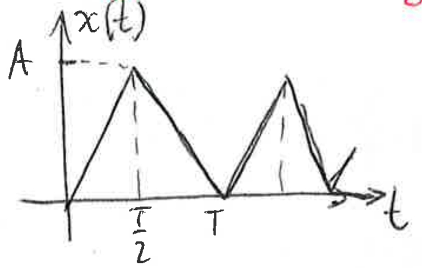


$T = 3\ ns$

$f_0 = \frac{1}{T} = \frac{1}{3\,ns} = 333{,}3\ MHz$

$c_0 = \frac{300 \cdot (2-1)\cancel{ns} - 100\,(3-2)\cancel{ns}}{3\,\cancel{ns}} = 66{,}67$

3.1.3 Determine the (one-sided) Fourier series expansion for the triangular wave



$$x(t) = \begin{cases} \frac{2A}{T}t & \text{for } 0 \le t < \frac{T}{2} \\ 2A\left(1 - \frac{t}{T}\right) & \text{for } \frac{T}{2} \le t \le T \end{cases}$$

$$C_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{\cancel{T}}\,\frac{A\cancel{T}}{2} = \frac{A}{2}$$

$$C_n = \frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}dt = \frac{1}{T}\int_0^{T/2}\frac{2A}{T}t\,e^{-jn\omega_0 t}dt + \frac{1}{T}\int_{\frac{T}{2}}^{T}2A\,e^{-jn\omega_0 t}dt +$$

$$+ \frac{1}{T}\int_{\frac{T}{2}}^{T}\frac{(-2A)}{T}t\,e^{-jn\omega_0 t}dt =$$

$$\boxed{\int t e^{\beta t}dt = \frac{e^{\beta t}}{\beta^2}(\beta t - 1)}$$

$$= \frac{1}{T}\frac{2A}{T}\int_0^{T/2}t e^{-jn\omega_0 t}dt + \frac{2A}{T}\int_{\frac{T}{2}}^{T}e^{-jn\omega_0 t}dt + \frac{(-2A)}{T^2}\int_{\frac{T}{2}}^{T}t e^{-jn\omega_0 t}dt =$$

$$= \frac{2A}{T^2}\left[\frac{e^{-jn\omega_0 t}}{(-jn\omega_0)^2}(-jn\omega_0 t - 1)\right]_0^{T/2} + \frac{2A}{T}\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{\frac{T}{2}}^{T} + \frac{(-2A)}{T^2}\left[\frac{e^{-jn\omega_0 t}}{(-jn\omega_0)^2}(-jn\omega_0 t - 1)\right]_{\frac{T}{2}}^{T}$$

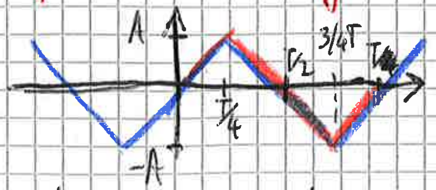
$$= \frac{2A}{T^2}\left\{\frac{e^{-jn\omega_0\frac{T}{2}}}{(n\omega_0)^2}\left(+jn\omega_0\frac{T}{2}+1\right) - \frac{1}{(n\omega_0)^2} + \frac{T e^{-jn\omega_0 T}}{-jn\omega_0} - \frac{T e^{-jn\omega_0 T/2}}{-jn\omega_0} + \right.$$

$$\left. - \left[\frac{e^{-jn\omega_0 T}}{(n\omega_0)^2}\left(+jn\omega_0 T + 1\right) - \frac{e^{-jn\omega_0\frac{T}{2}}}{(n\omega_0)^2}\left(+jn\omega_0\frac{T}{2}+1\right)\right]\right\}$$

$$= \frac{2A}{T^2(n\omega_0)^2}\left\{\cancel{e^{-jn\omega_0\frac{T}{2}}}\,jn\omega_0\frac{T}{2} + e^{-jn\omega_0\frac{T}{2}} - 1 + \cancel{Tjn\omega_0 e^{-jn\omega_0 T}} - \cancel{Tjn\omega_0 e^{-jn\omega_0\frac{T}{2}}} + \right.$$

$$\left. - \cancel{e^{-jn\omega_0 T}jn\omega_0 T} - e^{-jn\omega_0 T} + \cancel{e^{-jn\omega_0\frac{T}{2}}}\,jn\omega_0\frac{T}{2} + e^{-jn\omega_0\frac{T}{2}}\right\}$$

3.1.4. Determine the (one-sided) Fourier series expansion for the waveform in figura.



$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

- if $t \le \frac{T}{4}$ $\quad \frac{y-0}{A-0} = \frac{x-0}{T/4-0}$

$$y = \frac{4A}{T}x \rightarrow y = \frac{4A}{T}t$$

- if $\frac{T}{4} < t \le \frac{3T}{4}$ $\quad \frac{y-A}{-A-A} = \frac{x-T/4}{\frac{3}{4}T-\frac{T}{4}} \qquad \frac{y-A}{-2A} = \left(x-\frac{T}{4}\right)\frac{4}{2}\cdot\frac{1}{T}$

$$y = -\frac{4A}{T}\left(x-\frac{T}{4}\right)+A =$$

$$= -\frac{4A}{T}x + A + A = -\frac{4A}{T}x + 2A \rightarrow y = -\frac{4A}{T}t + 2A$$

- if $\frac{3}{4}T < t \le T$ $\quad \frac{y-(-A)}{0-(-A)} = \frac{x-\frac{3}{4}T}{T-\frac{3}{4}T}$

$$\frac{y+A}{A} = \left(x-\frac{3}{4}T\right)\frac{4}{T}$$

$$y = \frac{4A}{T}\left(x-\frac{3}{4}T\right) - A$$

$$y = \frac{4A}{T}t - 3A - A = \frac{4A}{T}t - 4A$$

$$y = \begin{cases} \frac{4A}{T}t & 0 \le t \le \frac{T}{4} \\ -\frac{4A}{T}t + 2A & \frac{T}{4} < t \le \frac{3}{4}T \\ \frac{4A}{T}t - 4A & \frac{3}{4}T < t \le T \end{cases}$$

$$C_0 = \frac{1}{T}\int_0^T x(t)\,dt = \cancel{\frac{1}{2}\frac{T}{2}A} + \cancel{\frac{1}{2}\frac{T}{2}(-A)} = 0$$

$$= \frac{A}{T^2 (n\omega_0)^2} \Big\{ 4e^{-jn\omega_0 \frac{T}{4}} \left(jn\omega_0 \frac{T}{4} + 1 \right) - 4 + 4 e^{-jn\omega_0 \frac{3}{4}T} \left(jn\omega_0 \frac{3}{4} T + 1 \right) +$$

$$+ 4e^{-jn\omega_0 \frac{T}{4}} \left(jn\omega_0 \frac{T}{4} + 1 \right) + 2(+jn\omega_0 T) e^{-jn\omega_0 \frac{3}{4}T} - 2jn\omega_0 T e^{-jn\omega_0 \frac{T}{4}} +$$

$$+ 4e^{-jn\omega_0 T} \left(jn\omega_0 T + 1 \right) - 4e^{-jn\omega_0 T\frac{3}{4}} \left(jn\omega_0 \frac{3}{4} T + 1 \right) +$$

$$- 4 (jn\omega_0 T) e^{-jn\omega_0 T} + 4 jn\omega_0 T e^{-jn\omega_0 T \frac{3}{4}} \Big\} =$$

$$= \frac{A}{(n\omega_0)^2 T^2} \Big\{ \cancel{e^{-jn\omega_0 \frac{T}{4}} jn\omega_0 T} + 4e^{-jn\omega_0 \frac{T}{4}} - 4 - \cancel{e^{-jn\omega_0 \frac{3}{4}T} jn\omega_0 T 3} - 4e^{-jn\omega_0 \frac{3}{4}T} +$$

$$+ \cancel{e^{-jn\omega_0 \frac{T}{4}} jn\omega_0 T} + 4e^{-jn\omega_0 \frac{T}{4}} + \cancel{2jn\omega_0 T e^{-jn\omega_0 \frac{3}{4}T}} - \cancel{2jn\omega_0 T e^{-jn\omega_0 \frac{T}{4}}} +$$

$$+ \cancel{4e^{-jn\omega_0 T} jn\omega_0 T} + 4e^{-jn\omega_0 T} - \cancel{e^{-jn\omega_0 T\frac{3}{4}} jn\omega_0 T 3} - 4e^{-jn\omega_0 \frac{3}{4}T} +$$

$$- \cancel{4jn\omega_0 T e^{-jn\omega_0 T}} + \cancel{4jn\omega_0 T e^{-jn\omega_0 \frac{3}{4}T}} \Big\} =$$

$$= \frac{A}{(n\omega_0)^2 T^2} \left[8e^{-jn\omega_0 \frac{T}{4}} - 8e^{-jn\omega_0 \frac{3}{4}T} + 4e^{-jn\omega_0 T} - 4 \right] =$$

$$= \frac{4A}{\frac{4\pi^2}{\cancel{T^2}} \cancel{T^2} n^2} \left[2e^{-jn\frac{2\pi}{T}\frac{T}{4_2}} - 2e^{-jn\frac{2\pi}{T}\frac{3}{4_2}T} + \cancel{1 e^{-jn\frac{2\pi}{T}T}}_{=1} - \cancel{1} \right] =$$

$$= \frac{A}{n^2\pi^2} \left[2e^{-jn\frac{\pi}{2}} - 2e^{-jn\frac{3}{2}\pi} \right] \longrightarrow n = \text{even (eg} = 2) \quad \frac{A}{n^2\pi^2} \left[\cancel{2e^{-j\frac{2\pi}{2}}} - \cancel{2e^{-j\frac{2}{2}3\pi}} \right] = 0$$

$n = \text{odd (eg} = 1)$ $\quad \frac{A}{n^2\pi^2} \left[\underset{\downarrow\, 2j}{2e^{-jn\frac{\pi}{2}}} - \underset{\downarrow\, +2j}{2e^{-jn\frac{3}{2}\pi}} \right] =$

$$\boxed{c_n = j\frac{4A}{n^2\pi^2}} \qquad \angle c_n = -n \cdot 90^\circ$$

$$x(t) = \frac{8A}{\pi^2} \cos(\omega_0 t - 90^\circ) + \frac{8A}{9\pi^2} \cos(3\omega_0 t + 90^\circ) + \frac{8A}{25\pi^2} \cos(5\omega_0 t + 90^\circ) + \dots$$

$$= \frac{1}{4(-jn\omega_0)}\left[e^{-jn\omega_0} - 1 + 2e^{-jn2\omega_0} - 2e^{-jn\omega_0} - 2e^{-jn3\omega_0} + 2e^{-jn2\omega_0} + - e^{-jn\omega_0 4} + e^{-jn\omega_0 3}\right] =$$

$$= \frac{j}{4n\omega_0}\left[4e^{-jn2\omega_0} - e^{-jn\omega_0} - 1 - e^{-jn4\omega_0} - e^{-jn3\omega_0}\right] =$$

$$= \frac{j}{\cancel{4}n\frac{2\pi}{\cancel{4}}}\left[4e^{-jn2\frac{2\pi}{4}} - e^{-jn\frac{2\pi}{4}} - 1 - \underbrace{e^{-jn4\frac{2\pi}{4}}}_{=1} - e^{-jn3\frac{2\pi}{4}}\right] =$$

$$= \frac{j}{2\pi n}\left[4e^{-jn\pi} - e^{-jn\frac{\pi}{2}} - 2 - e^{-jn\frac{3}{2}\pi}\right]$$

- $n = 4, 8, 12, \dots$ $\quad C_n = \frac{j}{n2\pi}\left[4 - 2 - 1 - 1\right] = \emptyset \quad |C_n| = 0 \quad \angle C_n = 0$

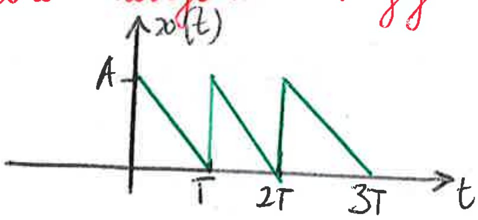
- $n = 2, 6, 10, \dots$ $\quad C_n = \frac{j}{n2\pi}\left[4 - \cancel{2} + \cancel{1} + \cancel{1}\right] = \frac{j\,\overset{2}{\cancel{4}}}{\cancel{2}n\pi} = j\frac{2}{n\pi}$

  $|C_n| = \frac{2}{n\pi} \quad \angle C_n = +90^\circ$

- $n = 1, 3, 5, \dots$ $\quad C_n = \frac{j}{n2\pi}\left[-4 - 2 - \cancel{1j} + \cancel{1j}\right] = \frac{-\overset{3}{\cancel{6}}}{\cancel{2}n\pi}j = -j\frac{3}{n\pi}$

  $|C_n| = \frac{3}{n\pi} \quad \angle C_n = -90^\circ$

3.1.7. Determine the Fourier series expansion coefficients for the waveform in figura



$$x(t) = -\frac{A}{T}t + A$$

$$c_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{\cancel{T}}\,\frac{A\cancel{T}}{2} = \frac{A}{2}$$

$$c_n = \frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}dt = \frac{1}{T}\int_0^T\left(-\frac{At}{T}+A\right)e^{-jn\omega_0 t}dt =$$

$$= -\frac{A}{T^2}\int_0^T t\,e^{-jn\omega_0 t}dt + \frac{A}{T}\int_0^T e^{-jn\omega_0 t}dt =$$

$$\int t e^{\beta t}dt = \frac{e^{\beta t}}{\beta^2}(\beta t - 1)$$

$$= -\frac{A}{T^2}\left[\frac{e^{-jn\omega_0 t}}{(-jn\omega_0)^2}(-jn\omega_0 t - 1)\right]_0^T + \frac{A}{T}\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_0^T =$$

$$= \frac{A}{T^2}\left[-\frac{e^{-jn\omega_0 t}}{(n\omega_0)^2}(+jn\omega_0 t + 1)\right]_0^T + \frac{A}{T}\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_0^T =$$

$$= \frac{A}{T^2(n\omega_0)^2}\left[-e^{-jn\omega_0 T}(jn\omega_0 T + 1) + 1 + e^{-jn\omega_0 T}(+jn\omega_0)T - jn\omega_0 T\right] =$$

$$= \frac{A}{T^2(n\omega_0)^2}\left[-\cancel{e^{-jn\omega_0 T}(jn\omega_0 T)} - \underbrace{\cancel{e^{-jn\omega_0 T}}}_{=1} + \cancel{1} + \cancel{e^{-jn\omega_0 T}(jn\omega_0 T)} - jn\omega_0 T\right]$$

$$= \frac{A}{T^2(n\omega_0)^2}\left[-jn\,\omega_0 T\right] = \frac{A}{\cancel{T^2}n^2\frac{4\pi^2}{\cancel{T^2}}}\left(-j\cancel{n}\frac{2\pi}{\cancel{T}}\cancel{T}\right) = -j\frac{A}{2\pi n}$$

$$\boxed{\angle c_n = -90^\circ \qquad |c_n| = \frac{A}{2\pi n}}$$

$$x(t) = c_0 + \sum_{n=1}^{\infty} 2|c_n|\cos(n\omega_0 t + \angle c_n) = \frac{A}{2} + \frac{2A}{2\pi}\cos(\omega_0 t - 90^\circ) + \frac{2A}{2\pi 2}\cos(2\omega_0 t - 90^\circ)$$

$$= \frac{A}{2} + \frac{A}{\pi}\cos(\omega_0 t - 90^\circ) + \frac{A}{2\pi}\cos(2\omega_0 t - 90^\circ) + \frac{A}{3\pi}\cos(3\omega_0 t - 90^\circ) + \dots$$

$$= \frac{A}{\tau T \, n^2 \frac{4\pi^2}{T^2}} \left[1 - e^{-j \frac{2\pi}{T} n \tau/2} \frac{\sin\left(n \frac{2\pi}{T} \frac{\tau}{2}\right)}{n \frac{2\pi}{T} \frac{\tau}{2}} \right] \cdot (-j n \omega_0 \tau) =$$

$$= \frac{A}{\cancel{\tau} n^{\cancel{2}} \frac{4\pi^{\cancel{2}}}{\cancel{T}}} \left(-j \cancel{n} \frac{2\cancel{\pi}}{\cancel{T}} \cancel{\tau}\right) \left[1 - e^{-j\pi n \tau/T} \frac{\sin(n\pi \tau/T)}{n\pi \tau/T} \right] =$$

$$\boxed{c_n = -j \frac{A}{2n\pi} \left[1 - e^{-j n\pi\tau/T} \frac{\sin(n\pi\tau/T)}{n\pi\tau/T} \right]}$$

$$= \frac{A}{T\sigma(n\omega_0)^2}\, j n\omega_0\sigma \left[e^{-jn\omega_0\tau} + \frac{e^{-jn\omega_0\sigma} - 1}{jn\omega_0\sigma} \right] =$$

$$= \frac{A}{T\sigma(n\omega_0)^2}\, j n\omega_0\sigma \left[e^{-jn\omega_0\tau} - e^{-jn\omega_0\frac{\sigma}{2}} \frac{\left(e^{+jn\omega_0\frac{\sigma}{2}} - e^{-jn\omega_0\frac{\sigma}{2}}\right)}{2jn\omega_0\frac{\sigma}{2}} \right] =$$

$$= \frac{A}{\cancel{T}\cancel{\sigma}\frac{4\pi^2}{\cancel{T^2}}n^2}\, j\cancel{n}\frac{2\pi}{\cancel{T}}\cancel{\sigma} \left[e^{-jn\omega_0\tau} - e^{-jn\omega_0\frac{\sigma}{2}} \frac{\sin\left(n\omega_0\frac{\sigma}{2}\right)}{n\omega_0\frac{\sigma}{2}} \right] =$$

$$= j\frac{A}{2\pi n} \left[e^{-jn2\pi\tau/T} - e^{-jn\frac{2\pi}{T}\frac{\sigma}{2}} \frac{\sin\left(n\frac{2\pi}{T}\frac{\sigma}{2}\right)}{n\frac{2\pi}{T}\frac{\sigma}{2}} \right] =$$

$$\boxed{c_n = j\frac{A}{2\pi n} \left[e^{-jn2\pi\tau/T} - e^{-jn\pi\sigma/T} \frac{\sin(n\pi\sigma/T)}{n\pi\sigma/T} \right]}$$

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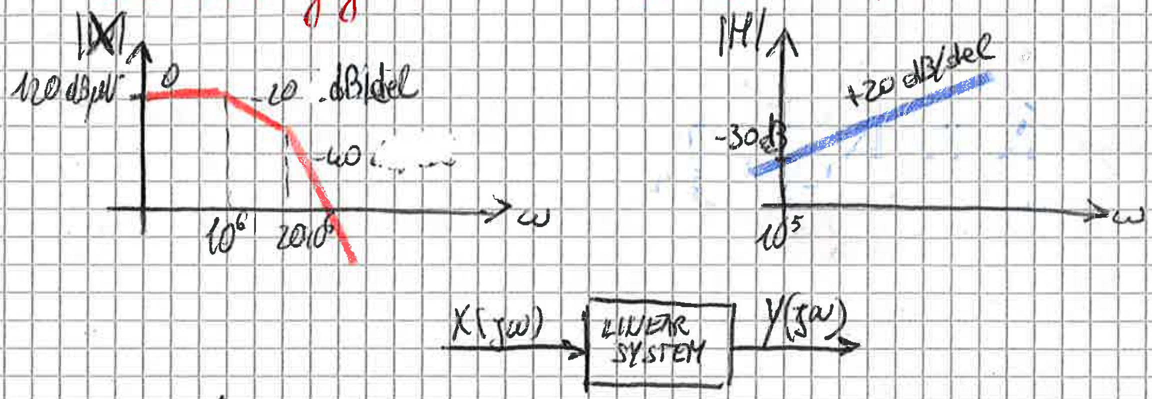
3.2. Spectra of Digital Waveforms

3.2.2. Determine the magnitude of the output of the system shown in figure at $\omega = 50 \cdot 10^6$ rad/s.

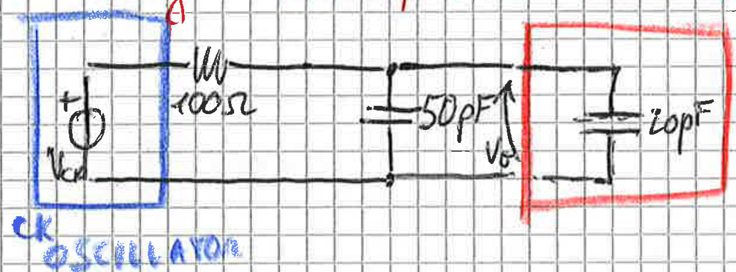


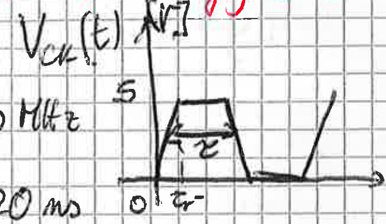
$$|X(j\,50\cdot 10^6)| = 120\ \text{dB}\mu\text{V} - 20\log\left(\frac{20\cdot 10^6}{10^6}\right) - 40\log\left(\frac{50\cdot 10^6}{20\cdot 10^6}\right) = 78{,}06\ \text{dB}\mu\text{V}$$

$$|H(j\,50\cdot 10^6)| = -30\ \text{dB}\mu\text{V} + 20\log\left(\frac{50\cdot 10^6}{10^5}\right) = +23{,}98\ \text{dB}\mu\text{V}$$

$$|Y(j\,50\cdot 10^6)| = 78{,}06\ \text{dB}\mu\text{V} + 23{,}98\ \text{dB}\mu\text{V} = 102{,}04\ \text{dB}\mu\text{V}$$

3.2.1. A 10-MHz clock oscillator transitioning from 0 to 5 V with rise/falltimes of 20 ns and a 50% duty cycle is connected to a gate as shown in figure. A filter is connected as shown. Determine the level of the 11th harmonic at the gate terminals. Obtain these by using the exact expression and by using the spectral bounds. Use PSPICE to verify this.





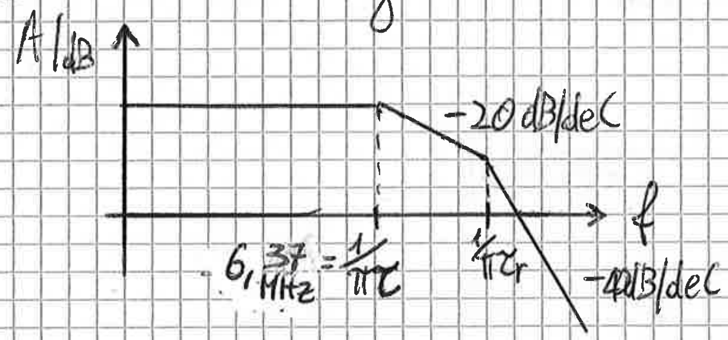
$f_0 = 10$ MHz

$\tau_r = \tau_f = 20$ ns

DC = 50%

$|Y(j \cdot 11\omega_0)| = ?$

Since a clock signal is considered:



$\omega_0 = 2\pi f_0 = 62,83 \cdot 10^6$ rad/s

$\tau = \frac{1}{2} T = \frac{1}{2 f_0} = 50$ ns

1st breakpoint $= \frac{1}{\pi\tau} = 6,37$ MHz

2nd breakpoint $= \frac{1}{\pi\tau_r} = \frac{1}{\pi \, 20ns} = 15,9$ MHz

$11 f_0 = 11 \cdot 10$ MHz $= 110$ MHz   11th harmonic

$|X(11f_0)| = 20\log(5 \cdot 10^6 \mu V) - 20\log\left(\frac{15,9 \text{ MHz}}{6,37 \text{ MHz}}\right) - 40\log\left(\frac{110 \text{ MHz}}{15,9 \text{ MHz}}\right)$

by interpolation with bounds

2nd breakpoint with respect to first

11th harm. with respect to 2nd break

$= 82,43$ dBμV

$|H(j\omega)| = \frac{V_G(j\omega)}{V_S(j\omega)} = \frac{Z_C}{Z_C + R_S}$, $Z_C = \frac{1}{j\omega(C_1+C_2)}$

$|H(j\omega)| = \left| \frac{\frac{1}{j\omega(C_1+C_2)}}{R + \frac{1}{j\omega(C_1+C_2)}} \right| = \left| \frac{1}{1 + j\omega R(C_1+C_2)} \right|$

$\omega \to 0 : H(j\omega) = 1$

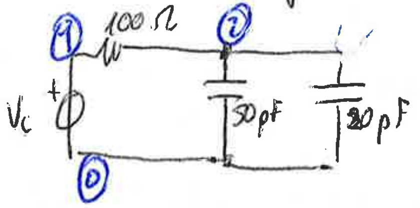
$\omega \to \infty : |H(j\omega)| = -20\log(\omega RC)$

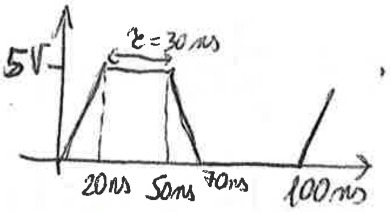
$\omega_c = \frac{1}{(C_1+C_2)R} = 142,85 \cdot 10^6$ rad/s $\to f_c = 22,74$ MHz

$|H(11f_0)| = -20\log\left(\frac{110 \text{ MHz}}{22,74 \text{ MHz}}\right) = -13,69$ dBμV

$|Y(11f_0)| = |X(11f_0)| + |H(11f_0)| = 72,74$ dB μV

• PSPICE Verification





PSPICE Netlist: VS 1 0 PULSE(0 5 0.01N 20N 20N 30N 100N)

R 1 2 100

C1 2 0 50p

C2 2 0 20p

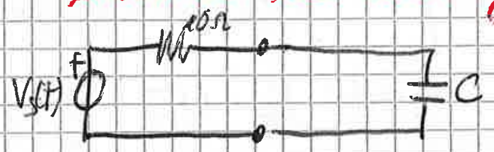
.TRAN 0.001N 100N

.FOUR 10MEG 11 V(2) ⇒ output Fourier component 4,[illegible] mV

f_0 # harmonic values   output Node

3.2.5. A 10-MHz clock oscillator having a 10 Ω internal resistance has an open-circuit voltage waveform as a trapezoidal pulse train with a 50% duty-cycle and rise- and falltimes of 2 ns. Determine the value of a capacitor (an ideal one) that when placed across the oscillator output will reduce the output voltage of the fifth harmonic by 10 dB. Given that the open circuit voltage transitions from 0 to 5 V, estimate the level of this fifth harmonic.

$\tau = 0,5 \cdot \frac{1}{10\,MHz} = 50\ ns$



$$|H(j\omega)| = \left| \frac{\frac{1}{j\omega C}}{R_S + \frac{1}{j\omega C}} \right| = \left| \frac{1}{1 + j\omega C R_S} \right|$$

$5f_0 = 50\ MHz$

$$|H(5f_0)| = -10\ dB \rightarrow 20\log\left(\frac{1}{\sqrt{1+(2\pi 5 f_0 R_S C)^2}}\right) = -10\ dB$$

$$\frac{1}{\sqrt{1+(2\pi 5 f_0 R_S C)^2}} = 10^{-\frac{10}{20}} \rightarrow (10^{\frac{1}{2}})^2 = \left(\sqrt{1+(2\pi 5 f_0 R_S C)^2}\right)^2$$

$$(2\pi 5 f_0 R_S C)^2 = 10 - 1 \rightarrow C = \frac{\sqrt{9}}{2\pi 5 f_0 R_S} = \frac{3}{2\pi \cdot 5 \cdot 10\,MHz \cdot 10\,\Omega}$$

$$\rightarrow C = 955\ pF$$

- bg approximation

$$f_1 = \frac{1}{\pi\tau} = 6,37\ MHz$$

$$f_2 = \frac{1}{\pi\tau_r} = 159,15\ MHz$$

$$|X(5f_0)| = 20\log(5 \cdot 10^{+6}\,\mu V) - 20\log\left(\frac{50}{6,37}\right)$$

$$= 116,08\ dB\mu V$$

$$|Y(5f_0)| = |X(5f_0)| + |H(5f_0)| = 116,08 - 10 = 106,08\ dB\mu V$$

$\angle I_{ns} = \pm \frac{n\pi\tau}{T}$

$\angle I_{1s} = -\frac{\pi}{2} = -90°$

$H(j\omega_0) = \frac{1}{1+j\omega L/R} \cdot \frac{1-j\omega L/R}{1-j\omega L/R} = \frac{1-j\omega L/R}{1+\omega^2(L/R)^2}$

$\angle H(j\omega_0) = \text{arctg}\left(\frac{-2\pi f_0 L/R}{1}\right) = -80{,}96°$

$= \text{arctg}\left(\frac{-2\pi\left(\frac{1}{25}\right)\cdot\frac{2H}{1\Omega}}{1}\right) = -80{,}36°$

$\angle I_{2s} = \phi$

$\angle I_{3s} = -90°$

$\angle H(3j\omega_0) = \text{arctg}\left(-\frac{3 \times 2\pi f_0 \cdot L/R}{1}\right) = -86{,}36°$

$\angle I_{4s} = 0$

$\angle I_{5s} = -90°$

$\angle H(5j\omega_0) = \text{arctg}\left(\frac{-5 \cdot 2\pi f_0\, L/R}{1}\right) = -88{,}18°$

$\angle I_{6s} = \phi$

$\angle I_{7s} = -90°$

$\angle H(7j\omega_0) = \text{arctg}\left(-\frac{7 \cdot 2\pi f_0\, L/R}{1}\right) = -88{,}7°$

Since the output is in the form $i(t) = \sum_{n=1}^{7} I_n \sin(n\omega_0 t + \theta_n)$

$\Downarrow$

$$\sum_{n=1}^{7} I_n \sin(n\omega_0 t) = \sum_{n=1}^{7} I_n \cos(n\omega_0 t - 90°)$$

$\Downarrow$

So $\theta_n = \angle H(jn\omega_0)$

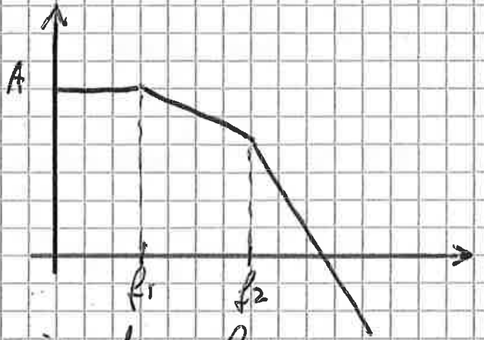
$\theta_1 = -80{,}36°$

$\theta_3 = -86{,}36°$

$\theta_5 = -88{,}18°$

$\theta_7 = -88{,}7°$

3.2.7. A 5-MHz, 5-V, 30% duty-cycle trapezoidal waveform having rise/fall times of 15 ns has the (one-sided) spectral bounds given in figure. Determine A, f_1 and f_2. Determine the levels of the first five harmonics.



$f_0 = 5$ MHz

$DC = 30\%$

$V_{MAX} = 5$ V

$\tau = 0{,}3 \cdot \frac{1}{5\text{ MHz}} = 60$ ns

$\tau_r = \tau_f = 15$ ns

- approximate values

$A_{(V)} = \frac{5 \cdot 60\text{ ns}}{90\text{ ns}} = 3{,}33 \text{ V} \cong 3 \text{ V} \leftarrow \frac{A\tau}{T}$

$A_{dB\mu V} = 20\log(3 \cdot 10^6 \mu V) = 129{,}54 \text{ dB}\mu V = |X(f_0)|$

$f_1 = \frac{1}{\pi\tau} = 5{,}305$ MHz

$f_2 = \frac{1}{\pi\tau_r} = 21{,}221$ MHz

$|X(2f_0)| = |X(10 \text{ MHz})| =$ ↙ since $2f_0 < f_2$

$= A_{dB\mu V} - 20\log\left(\frac{2f_0}{f_1}\right) = 129{,}54 - 20\log\left(\frac{10}{5{,}305}\right) = 124{,}04 \text{ dB}\mu V$

$|X(3f_0)| = |X(15 \text{ MHz})| = A_{dB\mu V} - 20\log\left(\frac{3f_0}{f_1}\right) = 129{,}54 - 20\log\left(\frac{15}{5{,}305}\right) = 120{,}51 \text{ dB}\mu V$

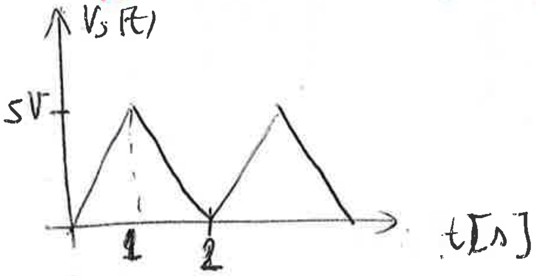
$|X(4f_0)| = |X(20 \text{ MHz})| = A_{dB\mu V} - 20\log\left(\frac{4f_0}{f_1}\right) = 129{,}54 - 20\log\left(\frac{20}{5{,}305}\right) = 118{,}02 \text{ dB}\mu V$

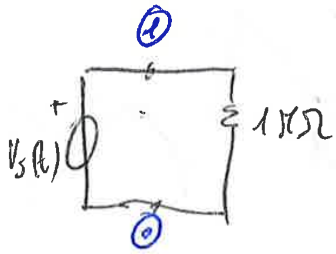
$|X(5f_0)| = |X(25 \text{ MHz})| = A_{dB\mu V} - 20\log\left(\frac{f_2}{f_1}\right) - 40\log\left(\frac{5f_0}{f_2}\right) =$

$= 129{,}54 - 20\log\left(\frac{21{,}221}{5{,}305}\right) - 40\log\left(\frac{25}{21{,}221}\right) = 114{,}65 \text{ dB}\mu V$

3.6. Use of SPICE in Fourier Analysis

3.6.1. Use the .FOUR function in PSPICE to verify expansion coefficient obtained for problem 3.1.3.





PSPICE Netlist:

```
VS 1 0 PWL (0 0 1 5, 2 0 3 5 4 0 5 5 ..)
R 1 0 1MEG
.TRAN. 0.001 10
.FOUR 0.5 V(1)
```

Fourier components $\left(2C_n = 2 \cdot \frac{2A}{(n\pi)^2}\right)$

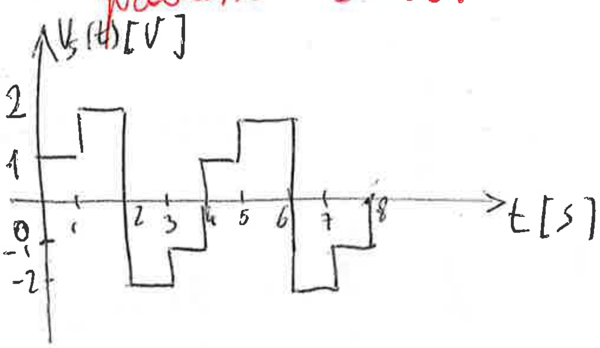
$|C_n^+| = 2C_n \rightarrow$

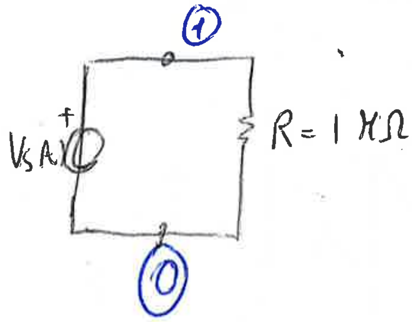
C_1^+ 2,026 V $\quad 2 \cdot \frac{2A}{\pi^2} = 2 \cdot \frac{2 \cdot 5}{\pi^2} = 2{,}026\ V$

C_2^+ $\approx 3{,}832 \cdot 10^{-6}\ V \rightarrow \emptyset$

C_3^+ $2{,}252 \cdot 10^{-6}\ V \leftrightarrow 225\ mV = 2 \cdot \frac{2 \cdot 5}{\pi^2 9}$ ...

3.6.3. Use the .FOUR function in PSPICE to verify problem 3.1.5.





PSPICE Netlist:

VS 1 0 PWL(0 0 0.001 1 1 1 1.001 2 2 2 2.001 -2
3 -2 3.001 -1 4 -1 4.001 1
5 1 5.001 2 6 2 6.001 -2
7 -2 7.001 -1 8 -1 8.001 0)

R 1 0 1MEG

.TRAN 0.001 8

.FOUR V(1)

.END

4n : ∅

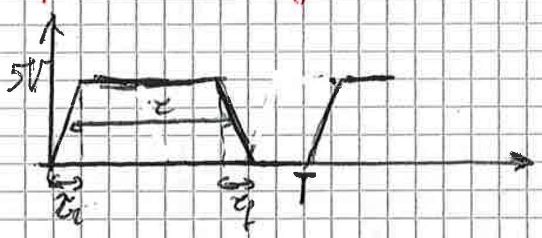
n even: 2, 6, 10 … $\left|\frac{2}{n\pi}\right| = |C_n|$

n odd : $\frac{3}{n\pi} = |C_n|$

	PSPICE out	theory $|C_n^+| = 2|C_n|$
$|C_n^+| = 2C_n \rightarrow C_1$	1,91 V	$2 \cdot \frac{3}{\pi} = 1,91$ V
$|C_2^+|$	$6,366 \cdot 10^{-1}$ V	$2 \cdot \frac{2}{2\pi} = 636,6$ mV
$|C_3^+|$	$6,366 \cdot 10^{-1}$ V	$2 \frac{3}{3\pi} = 636,6$ mV
$|C_4^+|$	$4,014 \cdot 10^{-8}$ V	∅
$|C_5^+|$	388 mV	$2 \frac{3}{5\pi} = 382$ mV

SAMPLE EXAM PROBLEMS

1) For a trapezoidal signal, whose amplitude is 5V, fundamental frequency is 30 MHz, duty-cycle is 75% and rising and falling times are 5 ns, compute all the relevant parameters of the spectrum.



$f_0 = 30$ MHz

$A = 5$ V

$\tau = \frac{75}{100} \cdot \frac{1}{30\,\text{MHz}} = 25$ ns

$\tau_r = \tau_f = 5$ ns

$$x(t) = C_0 + \sum_{n=1}^{\infty} 2|C_n| \cos\left(2\pi \frac{nt}{T} + \angle C_n\right)$$

$$C_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{A\,\tau}{T} = 3,75$$

$$C_n = \frac{1}{T}\int_0^T x(t) e^{-jn\omega_0 t} dt = \frac{1}{T}\int_0^{\tau_r} \frac{A}{\tau_r} e^{-jn\omega_0 t} dt + \frac{1}{T}\int_{\tau+\left(\frac{\tau_r+\tau_f}{2}\right)}^{\tau-\left(\frac{\tau_r+\tau_f}{2}\right)} \frac{A}{\tau_F} e^{-jn\omega_0 t} dt =$$

$$= \frac{A}{\tau_R T} \left.\frac{e^{-jn\omega_0 t}}{(-jn\omega_0)}\right|_0^{\tau_r} + \frac{A}{\tau_F T} \left.\frac{e^{-jn\omega_0 t}}{(-jn\omega_0)}\right|_{\tau+\left(\frac{\tau_r+\tau_F}{2}\right)}^{\tau-\left(\frac{\tau_R+\tau_F}{2}\right)} \overset{\tau_r=\tau_f}{=}$$

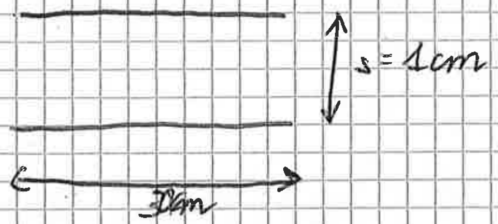
$$= \frac{A}{\tau_R T} \left.\frac{e^{-jn\omega_0 t}}{(-jn\omega_0)}\right|_0^{\tau_r} + \frac{A}{\tau_R T} \left.\frac{e^{-jn\omega_0 t}}{(-jn\omega_0)}\right|_{\tau}^{\tau+\tau_r}$$

$$= \frac{A}{\tau_R T(-jn\omega_0)} \left[e^{-\tau_r jn\omega_0} - 1 + e^{-jn\omega_0(\tau+\tau_r)} - e^{-jn\omega_0 \tau} \right] =$$

$$= \frac{2A}{2\tau_R T(-jn\omega_0)} \left[e^{-\frac{\tau_r}{2} jn\omega_0} \left(e^{-jn\omega_0 \frac{\tau_r}{2}} - e^{jn\omega_0 \frac{\tau_r}{2}} \right) + \right.$$

$$\left. + e^{-jn\omega_0 \tau} e^{-jn\omega_0 \frac{\tau_r}{2}} \left(e^{-jn\omega_0 \frac{\tau_r}{2}} - e^{jn\omega_0 \frac{\tau_r}{2}} \right) \right] =$$

2) Compute the maximum electric field radiated at a distance of 3 m from a short transmission line (length = 30 cm and wire separation = 1 cm); the current flowing in the line is a common-mode current, whose amplitude is 10 mA at 50 MHz frequency.



$s = 1\ cm$

$\mathcal{L} = 30\ cm$

$d = 3\ m$

$|\hat{I}_C| = 10\ mA$ @ $f = 50\ MHz$

$|\hat{E}_{CMAX}| > |\hat{E}_{DMAX}|$ in most of cases

$$|\hat{E}_{CMAX}| = 1{,}257 \cdot 10^{-6} \, \frac{|\hat{I}_C| f \mathcal{L}}{d} =$$

$$= 1{,}257 \cdot 10^{-6} \cdot \frac{10mA \cdot 50MHz \cdot 30cm}{3\ m} = 62{,}85\ mV/m$$

$$|\hat{E}_{CMAX}|_{dB} = 20 \log \left(\frac{62{,}85\ mV/m}{1\ \mu V} \right) = 95{,}97\ dB\mu V$$

4) Compute the amplitude of the crosstalk voltage V_{FE} for the frequency of 500 kHz, if you know that $|V_{FE}| = 50\,mV$ at 5 MHz. The line length is 5 m.

$|V_{FE}|_{500}$? @ 500 kHz

$|V_{FE}|_{5} = 50\ mV$ @ $f = 5$ MHz

$\mathcal{L} = 5\ m$ → 1st verify limit $\mathcal{L} \ll \frac{c}{f}$

500 kHz

$\mathcal{L} \leq \frac{10}{10}\frac{c}{f} = 6\ km$

$\mathcal{L} \leq 10\frac{c}{f} = 600\ m$

Model is valid!

$$\begin{cases} \left|\dfrac{\hat{V}_{FE5}}{\cancel{\hat{V}_S}}\right| = \left|\cancel{j}\omega_5\left(M_{FE}^{IND} \cancel{+ M_{FE}^{CAP}}\right)\right| \\ \left|\dfrac{\hat{V}_{FE500}}{\cancel{\hat{V}_S}}\right| = \left|\cancel{j}\omega_{500}\left(M_{FE}^{IND} \cancel{+ M_{FE}^{CAP}}\right)\right| \end{cases}$$

$$\frac{\hat{V}_{FE5}}{\cancel{2\pi} f_5} = \frac{\hat{V}_{FE500}}{\cancel{2\pi} f_{500}}$$

$$|\hat{V}_{FE}|_{500} = |\hat{V}_{FE5}| \cdot \frac{f_{500}}{f_5} = 50\ mV \cdot \frac{500\ kHz}{5\ MHz} = 5\ mV$$

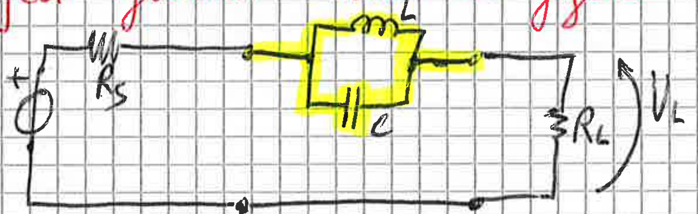
2) Two bare wires (radius =16 mils) are separated center-to-center by 50 mils. Determine the value of the p.u.l. capacitance and inductance.

$r_w = 16$ mils $\qquad s = 50$ mils

$$l = \frac{\mu_0}{\pi} \ln\left[\frac{s}{2r_w} + \sqrt{\left(\frac{s}{2r_w}\right)^2 - 1}\right] = 0{,}4065 \ \mu H/m$$

$$c = \frac{\mu_0 \varepsilon_0}{l} = 27{,}37 \ pF/m \quad \left(= \frac{\pi \varepsilon_0}{\cosh^{-1}(s/2r_w)} \right)$$

4) Determine an equation for the insertion loss of the bandreject filter shown in figure.



$$V_{L\omega_0} = V_S \frac{R_L}{R_S + R_L}$$

$$V_{L\omega} = V_S \frac{R_L}{R_S + R_L + Z_L // Z_C} = V_S \frac{R_L}{R_S + R_L + \left(\frac{L/C}{\frac{1}{j\omega C} + j\omega L}\right)} = V_S \frac{R_L}{R_S + R_L + \left(\frac{L}{C} \cdot \frac{j\omega C}{(1-\omega^2 LC)}\right)}$$

$$|IL|_{dB} = 20 \log\left(\left|\frac{V_{L\omega_0}}{V_{L\omega}}\right|\right) = 20 \log\left(\left|\cancel{V_S} \cdot \frac{\cancel{R_L}}{\cancel{R_S + R_L}} \cdot \frac{1}{\cancel{V_S}} \frac{(\cancel{R_S + R_L})}{\cancel{R_L}} \cdot \left(1 + \frac{j\omega L}{(1-\omega^2 LC)(R_S + R_L)}\right)\right|\right)$$

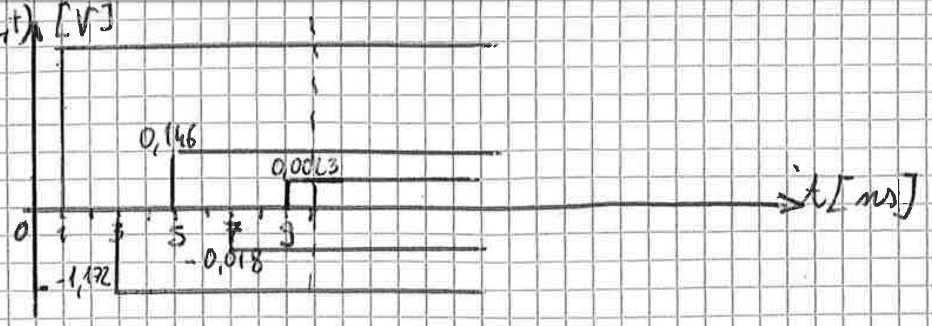
$$= 20 \log\left(\left| 1 + \frac{j\omega L}{(1-\omega^2 LC)(R_S + R_L)}\right|\right) = 20 \log\left(\left|\frac{1 - \omega^2 \overbrace{LC}^{\to 1/\omega_0^2} + j\omega \overbrace{L/(R_S + R_L)}^{\to \tau}}{1 - \omega^2 LC}\right|\right) =$$

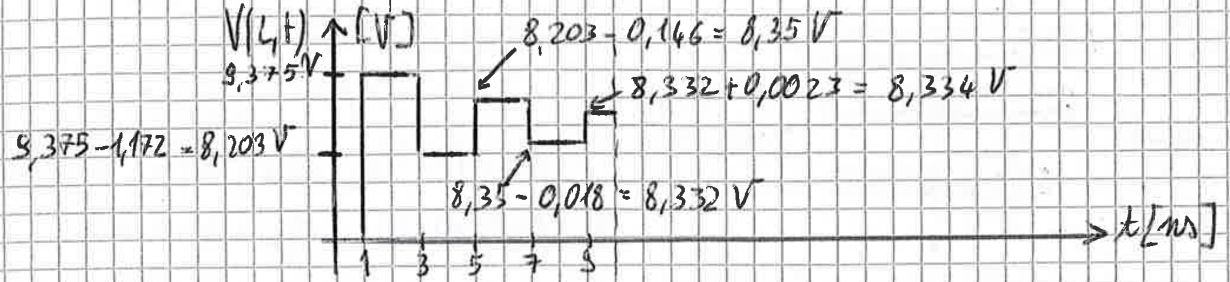
$$= 20 \log\left(\left|\frac{1 - (\omega/\omega_0)^2 + j\omega\tau}{1 - (\omega/\omega_0)^2}\right|\right) = 20 \log\left(\left| 1 + \frac{j\omega\tau}{1 - \left(\frac{\omega}{\omega_0}\right)^2}\right|\right) =$$

$$= 10 \log \cancel{\sqrt{1 + \frac{(\omega\tau)^2}{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2}}}^{\,2} = 10 \log\left(1 + \frac{(\omega\tau)^2}{\left(1 - (\omega/\omega_0)^2\right)^2}\right),$$

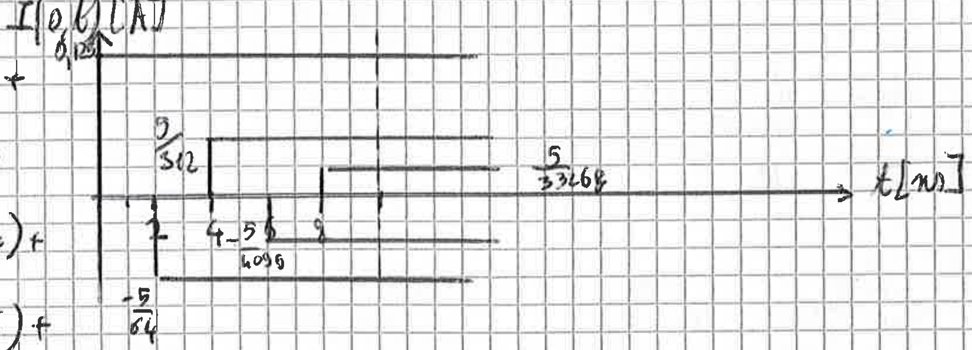
$$\tau = \frac{L}{R_S + R_L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

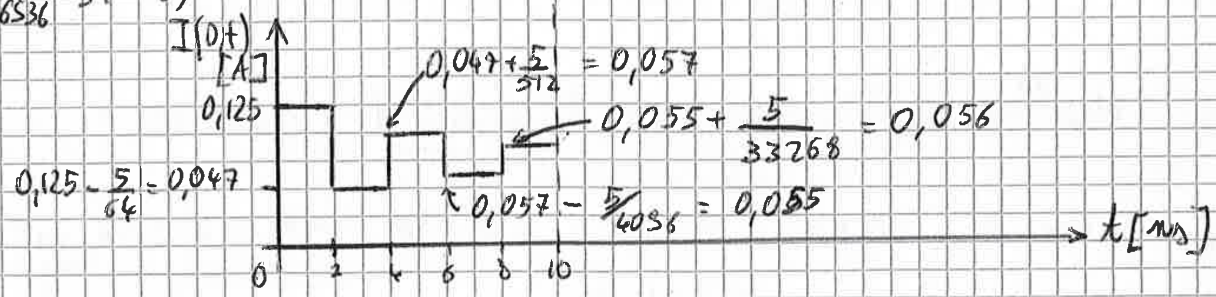
$$V_s(2,t) = \frac{15}{16} V_s(t-1) + - \frac{15}{128} V_s(t-3) + + \frac{15}{16}\cdot\frac{1}{64} V_s(t-5) + - \frac{15}{16}\cdot\frac{1}{512} V_s(t-7) + \frac{15}{16}\cdot\frac{1}{4096} V_s(t-9)$$





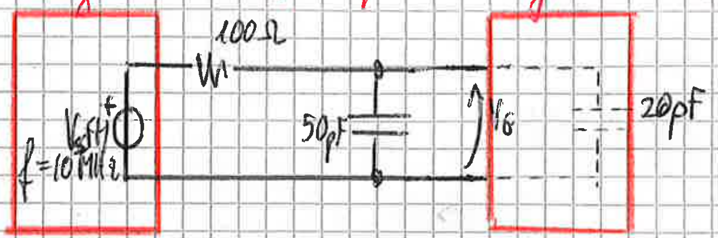
$$I(0,t) = \frac{1}{20} V_s(t) + - \frac{1}{128} V_s(t-2) + + \frac{1}{128}\cdot\frac{1}{8} V_s(t-4) + - \frac{1}{8192} V_s(t-6) + + \frac{1}{65536} V_s(t-8)$$





WRITTEN EXAM 04/07/2017 - B

1) A 10-MHz clock oscillator transitioning from ϕ to 5 V with rise/falltimes of 20 ns and a 50% dutycycle is connected to a gate as shown in figure.
Determine the first three coefficients V_{G0}, V_{G1}, V_{G2} of the voltage $v_G(t)$ expressed by $v_G(t) = \sum_n V_{Gn} \sin(n\omega_0 t + \theta_n)$



$\tau_r = 20\ ns = \tau_f$

$T = \frac{1}{f} = \frac{1}{10MHz} = 100\ ns$

$x(t) = c_0 + \sum_{n=1}^{\infty} 2|c_n| \cos\left(\frac{2\pi n}{T} t + \angle c_n\right)$, $c_0 = \frac{A\tau}{T}$

$|c_n^+| = 2|c_n| = 2\frac{A\tau}{T} \left|\frac{\sin(n\omega_0 \tau/2)}{n\omega_0 \tau/2}\right| \left|\frac{\sin(n\omega_0 \tau_r/2)}{n\omega_0 \tau_r/2}\right|$, $n \neq 0$ $\angle c_n = \pm n\omega_0 (\tau + \tau_r)/2$

Since D.C. is 50% → only the odd harmonics are present:

$|V_{2S}| = \phi \qquad \angle V_{2S} = \phi$

$\tau = \frac{50}{100} \cdot \frac{1}{10MHz} = 50\ ns$

$|V_{0S}| = \frac{A\tau}{T} = 5V \cdot 50\ ns \cdot 10\ MHz = 2,5\ V$

$|V_{1S}| = \cancel{2} \cdot \frac{A}{\cancel{2}} \left|\frac{1}{\frac{\pi}{2}}\right| \left|\frac{\sin(1 \cdot 2\pi \cdot 10MHz \cdot 20ns/2)}{2\pi\ 10MHz \cdot 20ns/2}\right| = 2,978\ V$ A

$\angle V_{1S} = \frac{7}{10}\pi$

$|H_G(j\omega)| = \left|\frac{\frac{1}{j\omega(C_1+C_2)}}{R_s + \frac{1}{j\omega(C_1+C_2)}}\right| = \left|\frac{1}{1 + j\omega(C_1+C_2)R_s}\right|$

$|V_{G0}| = |V_{G0}| \cdot 1 = 2,5\ V \rightarrow V_{G0dB\mu V} = 20\log\left(\frac{2,5V}{10^{-6}V}\right) = 127,96\ dB\mu V$

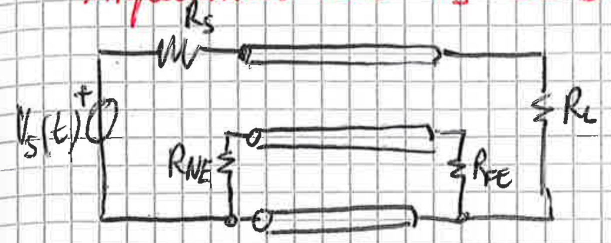
$|V_{G1}| = |V_{S1}| \cdot |H_1(j\omega)| = 2,978\ V \cdot \frac{1}{\sqrt{1 + (2\pi \cdot 1f(C_1+C_2)R_s)^2}} =$

$= 2,978\ V \cdot \frac{1}{\sqrt{1 + (2\pi \cdot 10MHz \cdot (20+50)pF \cdot 100\Omega)^2}} = 2,726\ V$

$|V_{G1}|_{dB\mu V} = 20\log\left(\frac{|V_{G1}|}{10^{-6}V}\right) = 128,71\ dB\mu V$

$\theta_1 = -\frac{n\pi\tau_r}{T}\frac{180}{\pi} + \text{arctg}(-2\pi f(C_1+C_2)R_s) = -59,74°$

5) For the ribbon cable shown in figure, assume the total mutual inductance and the total mutual capacitance to be $L_m = 0{,}4\ \mu H$ and $C_m = 400\ pF$.
If $V_s(t)$ is a 1 MHz sinusoid of magnitude 1 V, calculate the magnitude of the far-end cross-talk if the termination impedances are $R_S = 50\ \Omega$, $R_L = 50\ \Omega$, $R_{NE} = 50\ \Omega$, $R_{FE} = 50\ \Omega$.



$L_m = 0{,}4\ \mu H$

$C_m = 400\ pF$

$V_S(t) = V_{PK_S} \cos(2\pi f t)$, $V_{PK_S} = 1\ V$, $f = 1\ MHz$

$R_L = R_S = R_L = R_{NE} = 50\ \Omega$

$$V_{FE} = M_{FE} \frac{dV_S}{dt} = M_{NE}\, 2\pi f t\, V_{PK} \left(-\sin(2\pi f t)\right)$$

$$M_{FE}^{IND} = \frac{-R_{FE}}{R_{NE}+R_{FE}} \frac{L_m}{R_S+R_L} = \frac{-\cancel{R}}{2\cancel{R}} \cdot \frac{L_m}{2R} = \frac{-L_m}{4R} = -2 \cdot 10^{-9}$$

$$M_{FE}^{CAP} = \frac{R_{FE}R_{NE}}{R_{NE}+R_{FE}} \frac{R_L C_m}{R_S+R_L} = \frac{\cancel{R^2}}{2\cancel{R}} \cdot \frac{R C_m}{2\cancel{R}} = \frac{R C_m}{4} = 5 \cdot 10^{-9}$$

$$M_{FE} = M_{FE}^{IND} + M_{NE}^{CAP} = (5 \cdot 10^{-9} - 2 \cdot 10^{-9}) = 3 \cdot 10^{-9}$$

$$V_{FE_{PK}} = M_{FE}\, 2\pi f\, V_{PK_S} = 3 \cdot 10^{-9} \cdot 2\pi \cdot 1\ MHz \cdot 1\ V = 18{,}85\ mV$$

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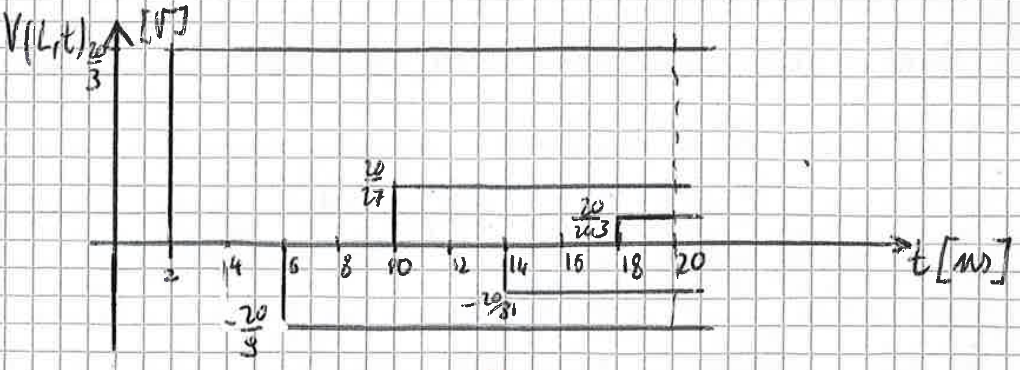
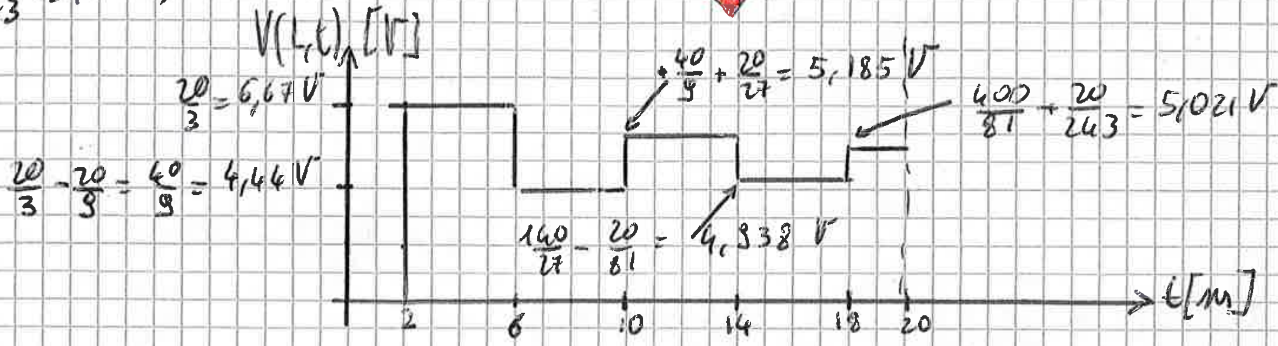
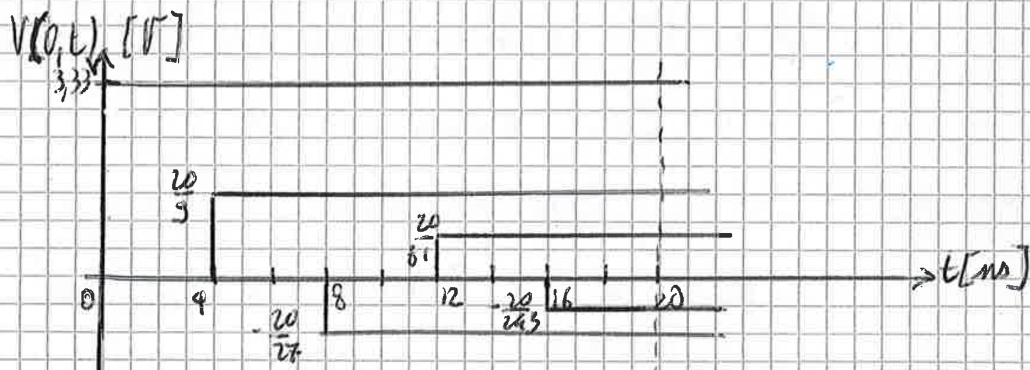
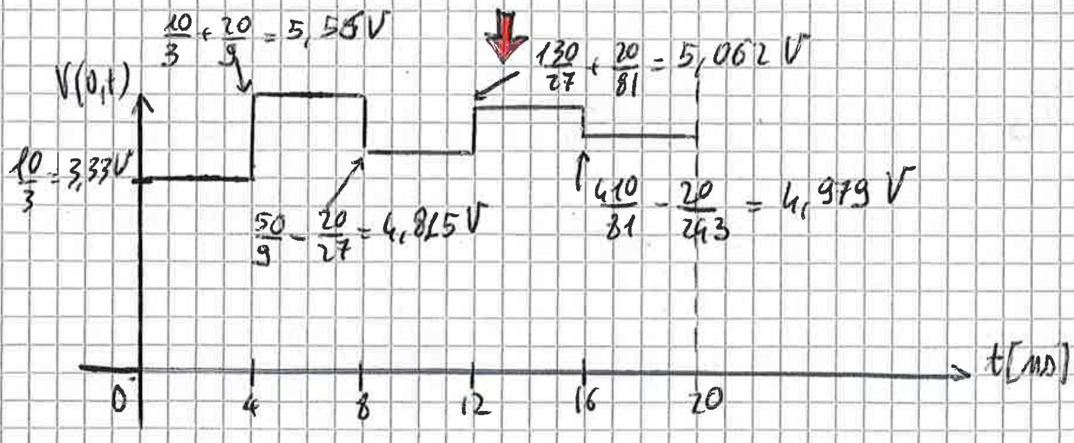
$$V(l,t) = \frac{4}{3} V_S(t-2) + -\frac{4}{9} V_S(t-6) + \frac{4}{27} V_S(t-10) - \frac{4}{81} V_S(t-14) + \frac{4}{243} V_S(t-18)$$





$$V(0,t) = \frac{2}{3} V_S(t) + \frac{4}{9} V_S(t-4) - \frac{4}{27} V_S(t-8) + \frac{4}{81} V_S(t-12) - \frac{4}{243}$$





2) A microstrip line is constructed on a FR-4 board having a relative permittivity of 4,7. The board thickness is 64 mils and the land width is 10 mils.
Determine the values of p.u.l. capacitance and inductance.

$\varepsilon_r = 4,7$ $\qquad h = 64$ mils $\qquad w = 10$ mils

$l\,?\quad c\,?$

$$\varepsilon_r' = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\,\frac{1}{\sqrt{1+10\frac{h}{w}}} = 3,075 \qquad (A)$$

$$\frac{w}{h} = \frac{10}{64} = 0,156 < 1$$

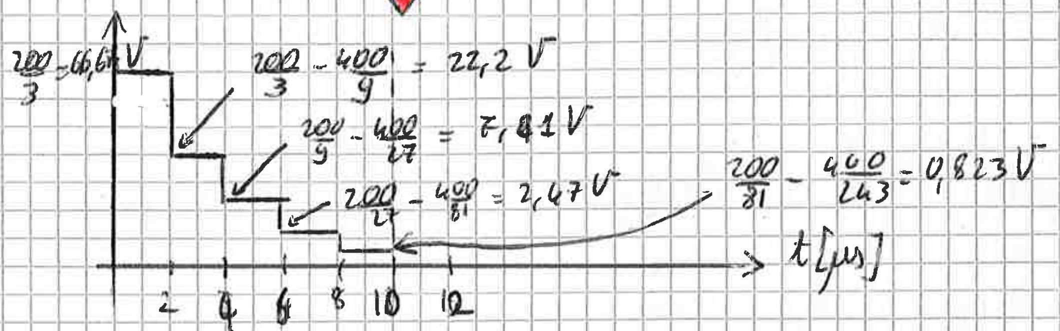
$$Z_c = \frac{60}{\sqrt{\varepsilon_r'}}\,\ln\left(\frac{8h}{w} + \frac{w}{4h}\right) = 134,6\,\Omega \qquad (B)$$

$$v = \frac{v_0}{\sqrt{\varepsilon_r'}} = \frac{3\cdot 10^8\,m/s}{\sqrt{3,075}} = 170,86\cdot 10^6\,m/s \qquad (C)$$

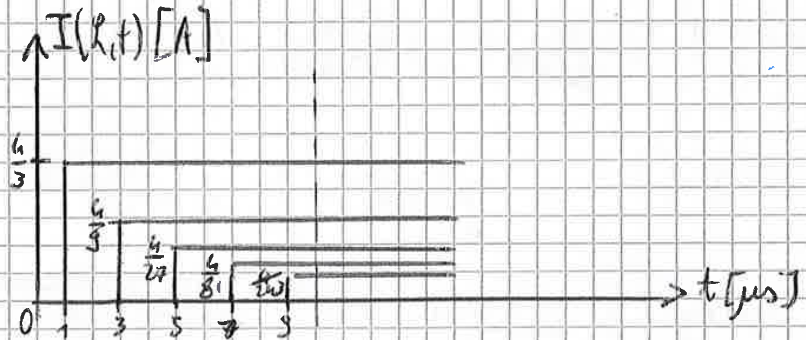
$$l = \frac{Z_c}{v} = \frac{134,6\,\Omega}{170,86\cdot 10^6\,m/s} = 0,787\,\mu H/m$$

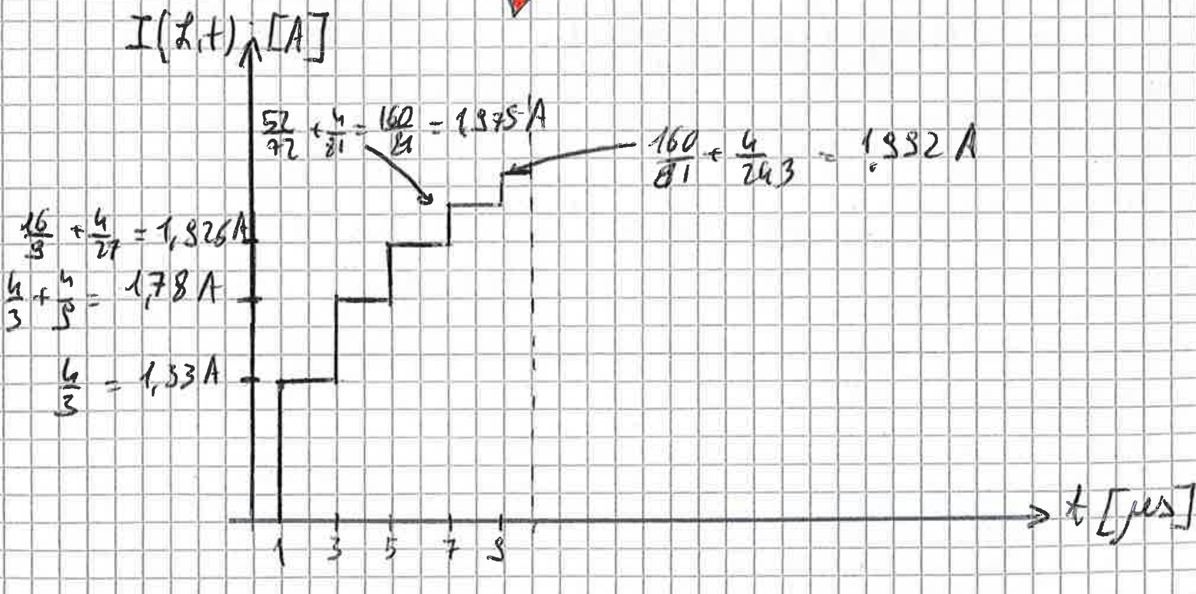
$$c = \frac{1}{Z_c\,v} = \frac{1}{134,6\,\Omega\cdot 170,86\cdot 10^6\,m/s} = 43,46\,pF/m$$

$$V(0,t) = \frac{2}{3} V_s(t) +$$
$$- \frac{4}{9} V_s(t-2) +$$
$$- \frac{4}{27} V_s(t-4) +$$
$$- \frac{4}{81} V_s(t-6) +$$
$$- \frac{4}{243} V_s(t-8)$$

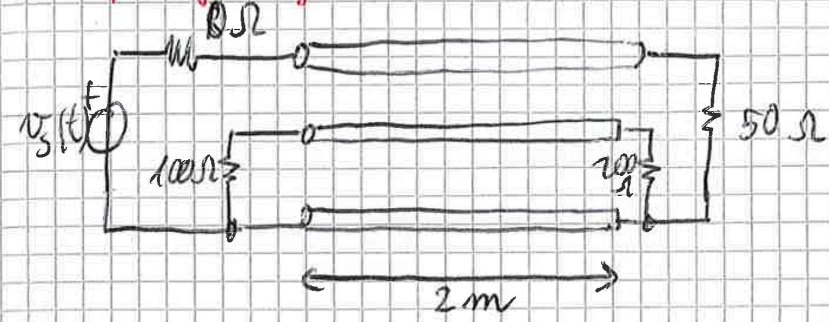


$$I(l,t) = \frac{2}{150} V_s(t-1) +$$
$$+ \frac{2}{150} \frac{1}{3} V_s(t-3) +$$
$$+ \frac{2}{150} \frac{1}{9} V_s(t-5) +$$
$$+ \frac{2}{150} \frac{1}{27} V_s(t-7) + \frac{2}{150} \frac{1}{81} V_s(t-9)$$





5) Consider the case of two wires above a ground plane. Suppose $l_m = 2$ nH/m, $c_m = 0{,}6$ pF/m, $L = 2$ m, $R_S = 0\,\Omega$, $R_L = 50\,\Omega$, $R_{NE} = 100\,\Omega$ and $R_{FE} = 200\,\Omega$. $V_S(t) = 1\cos(2\pi f t)$, where $f = 1$ MHz. Determine the near-end inductive and capacitive coupling coefficients.



$l_m = 2$ nH/m
$c_m = 0{,}6$ pF/m
$v_S(t) = 1\cdot\cos(2\pi f t)$, $f = 1$ MHz

$$V_S(t) = V_{PR}\cos(2\pi f t)$$

$$M_{FE} = M_{FE}^{IND} + M_{FE}^{CAP}, \qquad M_{NE} = M_{NE}^{IND} + M_{NE}^{CAP}$$

$$M_{NE}^{IND} = \frac{R_{NE}}{R_{NE}+R_{FE}} \cdot \frac{l_m \mathcal{L}}{R_L + \underbrace{R_S}_{=0}} = 2{,}67\cdot 10^{-11}$$

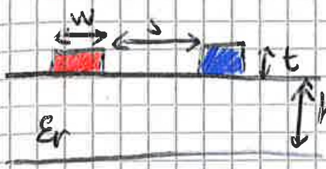
$$M_{NE}^{CAP} = \frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}} \cdot \frac{R_L c_m \mathcal{L}}{R_L+R_S} = 8\cdot 10^{-11} = M_{FE}^{CAP}$$

$$M_{FE}^{IND} = \frac{-R_{FE}}{R_{NE}+R_{FE}} \; \frac{l_m \mathcal{L}}{R_L+R_S} = -5{,}33\cdot 10^{-11}$$

$$M_{NE} = 2{,}67\cdot 10^{-11} + 8\cdot 10^{-11} = 106{,}7\cdot 10^{-12} = 10{,}67\cdot 10^{-11}$$

$$M_{FE} = 8\cdot 10^{-11} - 5{,}33\cdot 10^{-11} = 2{,}67\cdot 10^{-11}$$

2) A PCB shown in figure has land widths of 5 mils and an edge-to-edge separation of 5 mils. The board is glass epoxy having a relative permittivity of 4,7 and a thickness of 47 mils.
Determine the p.u.l. capacitance and inductance



$w = 5$ mils $\quad s = 5$ mils

$\varepsilon_r = 4{,}7 \quad h = 47$ mils

c? ℓ?

$$k = \frac{s}{s+2w} = 0{,}\overline{3}$$

$$\varepsilon_r' = \frac{\varepsilon_r+1}{2}\left\{\tanh\left[0{,}775\ln\left(\frac{h}{w}\right)+1{,}75\right]+\frac{kw}{h}\left[0{,}04-0{,}7k+0{,}01(1-0{,}1\varepsilon_r)(0{,}25+k)\right]\right\} =$$

$$= 2{,}825$$

$$k' = \sqrt{1-k^2} = 0{,}943$$

since $0 \le k < \frac{1}{\sqrt{2}} \Rightarrow Z_C = \dfrac{377\pi}{\sqrt{\varepsilon_r'}\ln\left(2\,\dfrac{1+\sqrt{k'}}{1-\sqrt{k'}}\right)} = 143{,}46\ \Omega$

$$v = \frac{v_0}{\sqrt{\varepsilon_r'}} = \frac{3\cdot 10^8\ m/s}{\sqrt{2{,}825}} = 178{,}49\cdot 10^6\ m/s$$

$$\ell = \frac{Z_C}{v} = 0{,}8037\ \mu H/m$$

$$c = \frac{1}{Z_C v} = 39{,}05\ pF/m$$

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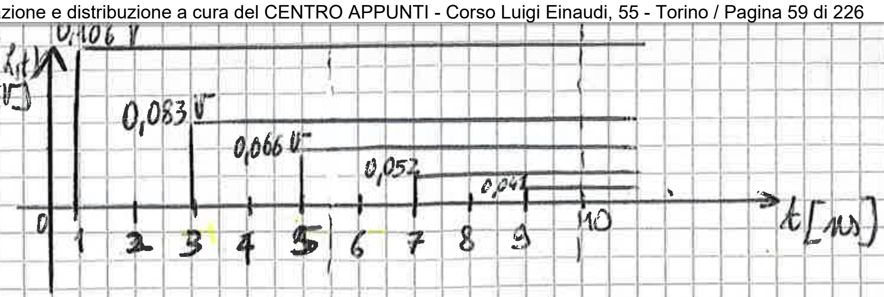
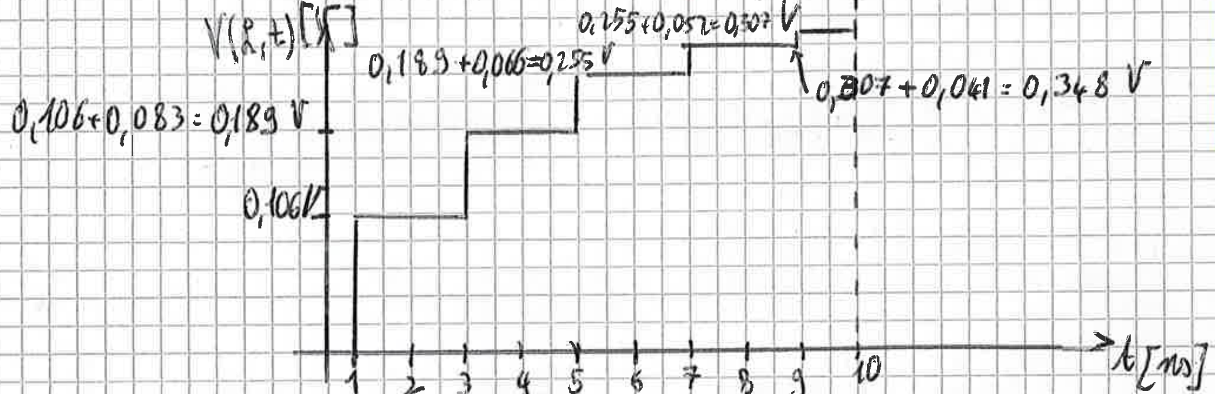
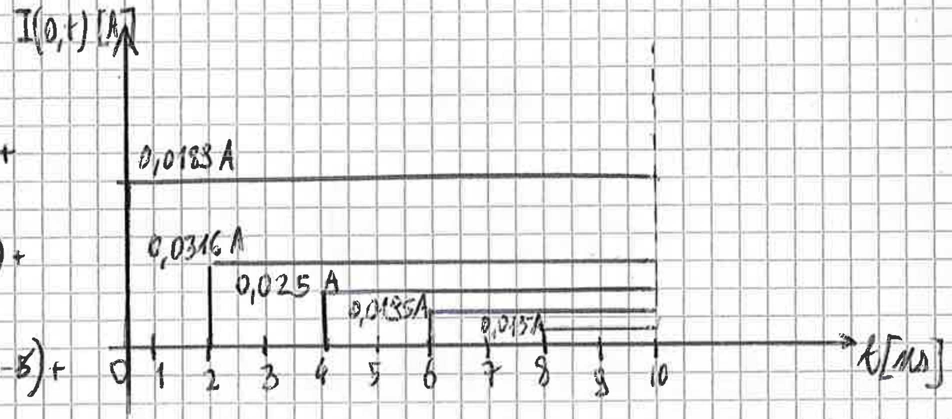
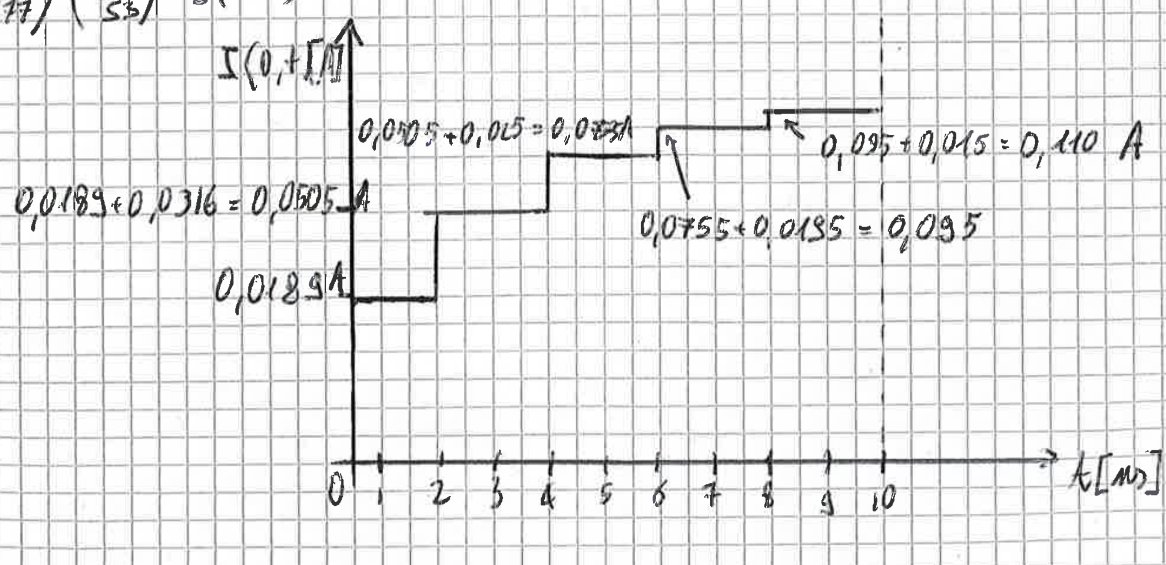
$$V(\ell,t) = \frac{50}{53}\,\frac{6}{53}\,V_s(t-1) + \frac{50}{53}\,\frac{6}{53}\cdot\left(-\frac{47}{53}\right)^2 V_s(t-3) + \frac{300}{2809}\cdot\left(\frac{-47}{53}\right)^4 V_s(t-5) + \frac{300}{2809}\left(-\frac{47}{53}\right)^6 V_s(t-7) + \frac{300}{2809}\left(-\frac{47}{53}\right)^8 V_s(t-9)$$





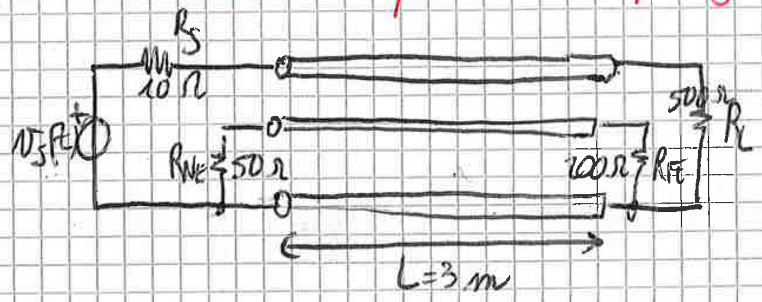
$$I(0,t) = \frac{1}{53}V_s(t) + \frac{1}{53}\,\frac{100}{53}\,\frac{47}{53}\,V_s(t-2) + \left(\frac{4700}{148877}\right)\left(\frac{47}{53}\right)^2 V_s(t-4) + \left(\frac{4700}{148877}\right)\left(\frac{47}{53}\right)^4 V_s(t-6) + \left(\frac{4700}{148877}\right)\left(\frac{47}{53}\right)^6 V_s(t-8)$$





5) Consider the case of two wires above a ground plane shown to the right.
Suppose $l_m = 3$ nH/m, $c_m = 0,8$ pF/m, $L = 3$ m,
$R_S = 10\ \Omega$, $R_L = 500\ \Omega$, $R_{NE} = 50\ \Omega$, $R_{FE} = 200\ \Omega$

$v_S(t) = 10 \cos(2\pi f t)$, $f = 100$ kHz. Determine the far-end inductive and capacitive coupling coefficients



$l_m = 3$ nH/m
$c_m = 0,8$ pF/m
M_{NE}^{IND}, M_{NE}^{CAP}, M_{FE}^{IND}, M_{FE}^{CAP}?

$$\mathcal{L} < \frac{c}{10 f} \rightarrow \mathcal{L} < \frac{3 \cdot 10^8 \text{ m/s}}{10 \cdot 100 \text{ kHz}} = 300 \text{ m} \rightarrow \mathcal{L} = 3 \text{ m } \underline{\text{OK}}$$

$$M_{NE}^{IND} = \frac{R_{NE}}{R_{NE}+R_{FE}} \cdot \frac{l_m \mathcal{L}}{R_S + R_L} = \frac{50}{50+200} \cdot \frac{3 \cdot 10^{-9} \cdot 3}{(10+500)} = 3,53 \cdot 10^{-12}$$

$$M_{NE}^{CAP} = M_{FE}^{CAP} = \frac{R_{FE} R_{NE}}{R_{NE}+R_{FE}} \cdot \frac{c_m \mathcal{L}}{R_S + R_L} = \frac{50 \cdot 200}{50+200} \cdot \frac{0,8 \cdot 10^{-12} \cdot 3}{10+500} = 0,188 \cdot 10^{-12}$$

$$M_{FE}^{IND} = \frac{-R_{FE}}{R_{NE}+R_{FE}} \cdot \frac{l_m \mathcal{L}}{R_S + R_L} = \frac{-200}{250} \cdot \frac{3 \cdot 10^{-9} \cdot 3}{(10+500)} = -0,048 \cdot 10^{-12}$$

$$= \frac{A}{\cancel{\pi} n \frac{2\pi}{\cancel{\pi}}} (+j) \left\{ 1 - e^{-jn\frac{2\pi}{T}\frac{\tau}{2}} \left(\frac{\sin\left(n\frac{2\pi}{T}\frac{\tau}{2}\right)}{n\frac{2\pi}{T}\frac{\tau}{2}} \right) \right\} =$$

$$= +\frac{jA}{2\pi n} \left\{ 1 - e^{-jn\pi(\tau/T)} \frac{\sin(n\pi\tau/T)}{n\pi\tau/T} \right\}$$

if $n = \text{even}$: $c_n = \frac{A}{2\pi n} j \overset{A=2}{=} j \frac{1}{\pi n}$

$n = \text{odd}$: $c_n = j\frac{A}{2\pi n} - \frac{A}{2\pi n} \frac{1}{n\pi\tau/T} \overset{A=2,\ \tau=2,\ T=4}{=} j\frac{1}{\pi n} - \frac{2}{2\pi n}\frac{1}{n\pi\frac{1}{2}} =$

$$= j\frac{1}{\pi n} - \frac{2}{(n\pi)^2}$$

first five harmonics:

$c_1 = \underset{A}{j\frac{1}{\pi}} - \underset{B}{\frac{2}{(\pi)^2}} \rightarrow |c_1| = \sqrt{\left(\frac{1}{\pi}\right)^2 + \left(\frac{2}{\pi^2}\right)^2} = 0{,}377$

$\angle c_1 = \operatorname{arctg}\left(\frac{1/\pi}{-\frac{2}{(\pi)^2}}\right) + 180 = 122{,}5°$ DEG

$c_2 = j\frac{1}{2\pi} \rightarrow |c_2| = \frac{1}{2\pi} = 0{,}1592$

$\angle c_2 = \frac{\pi}{2} = 90°$

$c_3 = j\frac{1}{3\pi} - \frac{2}{(3\pi)^2} \rightarrow |c_3| = \sqrt{\left(\frac{1}{3\pi}\right)^2 + \left(\frac{-2}{(3\pi)^2}\right)^2} = 0{,}1085$

$\angle c_3 = \operatorname{arctg}\left(\frac{1/3\pi}{-2/(3\pi)^2}\right) + 180 = 101{,}88°$

$c_4 = j\frac{1}{4\pi} \quad |c_2| = \frac{1}{4\pi} = 0{,}0796 \quad \angle c_2 = \frac{\pi}{2} = 90°$

$c_5 = j\frac{1}{5\pi} - \frac{2}{(5\pi)^2} \rightarrow |c_5| = \sqrt{\left(\frac{1}{5\pi}\right)^2 + \left(\frac{-2}{(5\pi)^2}\right)^2} = 0{,}0642$

$\angle c_5 = \operatorname{arctg}\left(\frac{1/5\pi}{-2/(5\pi)^2}\right) + 180° = 97{,}26°$

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$$v_0(t) = \frac{1}{2}\cdot 1 + 2\cdot\frac{1}{\pi}\cdot 0{,}3033 \cos(\pi t - 90^\circ - 72{,}34^\circ) +$$

$$+ 2\,\frac{1}{3\pi}\cdot 0{,}1055 \cos(3\pi t - 90^\circ - 83{,}94^\circ) + \ldots =$$

$$= \frac{1}{2} + 0{,}1931 \cos(\pi t - 162{,}34^\circ) + 0{,}0224 \cos(3\pi - 173{,}94^\circ) + \ldots$$

- PSPICE simulation

NODES

```
VS  1 0  PULSE (0  1  0.001  0.001  0.001  0.999  2)
R   1 2  1
C   2 0  1
.TRAN .0.001  10
.PROBE
.END
```

A; delay; tr; tf; TIME UP τ; TOT TIME T

VALUE

```
.TRAN print_step end_time
.FOUR fo [output_variable]
```

→ to see the Fourier analysis MUST BE ANTICIPATED BY TRANSIENT ANALYSIS

V [#Node]

EXAMPLE:

```
.TRAN  0.1N  20N
.FOUR  500MEG  V(4)
```

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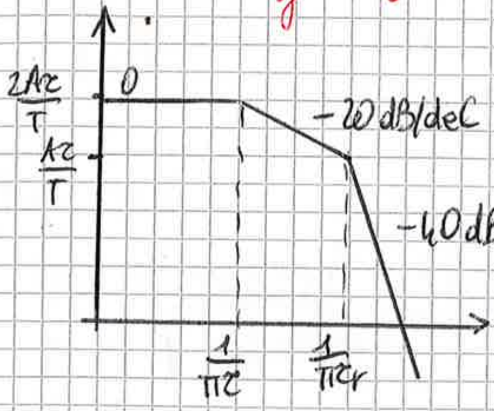
EXAMPLE 3.5.

For the 1 V, 10-MHz, 50% DC trapezoidal waveform whose measured spectra are shown in figure, determine the level at 110 MHz for the 20 ns rise/falltime and 5 ns rise/falltime. In addition reduce the fundamental frequency to 1 MHz and determine the level at 110 MHz for 20 ns rise/fall time



- $A = 1\,V$ $\quad \tau = \frac{1}{2}\frac{1}{10\,MHz} = 50\,ns$

$f = 10\,MHz \quad \tau_r = \tau_f = 20\,ns$

$\frac{1}{\pi\tau} = \frac{1}{\pi 50\,ns} = 6{,}37\,MHz$ (A)

$\frac{1}{\pi\tau_r} = \frac{1}{\pi 20\,ns} = 15{,}9\,MHz$ (B)

$$V|_{110\,MHz} = 20\log\left(\frac{2A\tau/T}{1\mu V}\right) - 20\log\left(\frac{\frac{1}{\pi\tau_r}}{\frac{1}{\pi\tau}}\right) - 40\log\left(\frac{\overset{110\,MHz}{f_1}}{\frac{1}{\pi\tau_r}}\right) =$$

$$= 20\log(10^6\mu V) - 20\log\left(\frac{15{,}9\,MHz}{6{,}37\,MHz}\right) - 40\log\left(\frac{110\,MHz}{15{,}9\,MHz}\right) = 78{,}45\,dB\mu V$$

- $\tau_r = \tau_f = 5\,ns \rightarrow \frac{1}{\pi\tau_r} = 63{,}66\,MHz$ (C)

$$V|_{110\,MHz} = 20\log(10^6\mu V) - 20\log\left(\frac{63{,}66\,MHz}{6{,}37\,MHz}\right) - 40\log\left(\frac{110\,MHz}{63{,}66\,MHz}\right) = 90{,}5\,dB\mu V$$

- $f = 1\,MHz \rightarrow \tau = \frac{1}{2}\frac{1}{1\,MHz} = 500\,ns \rightarrow \frac{1}{\pi\tau} = 637\,kHz$ (D)

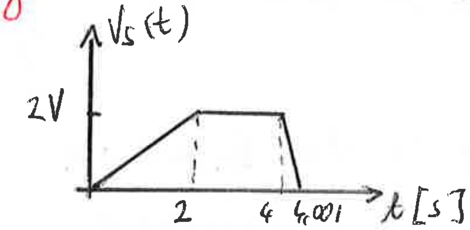
$\tau_r = \tau_f = 20\,ns \quad \frac{1}{\pi\tau_r} = 15{,}9\,MHz$

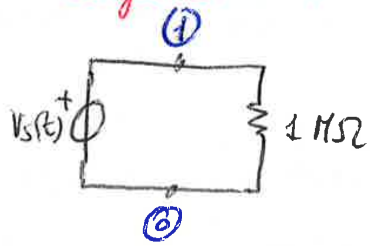
$$V|_{110\,MHz} = 20\log(10^6\mu V) - 20\log\left(\frac{15{,}9\,MHz}{637\,kHz}\right) - 40\log\left(\frac{110\,MHz}{15{,}9\,MHz}\right) =$$

$$= 58{,}46\,dB\mu V$$

EXAMPLE 3.6.

Determine the Fourier expansion coefficients for the wabeform in figure with PSPICE. (The waveform is the one of example 3.1)





A = 2

$\tau = 2$ s

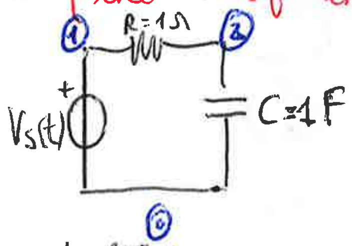
$T = 4$ s $\rightarrow f_0 = \frac{1}{T} = 0,25$ Hz

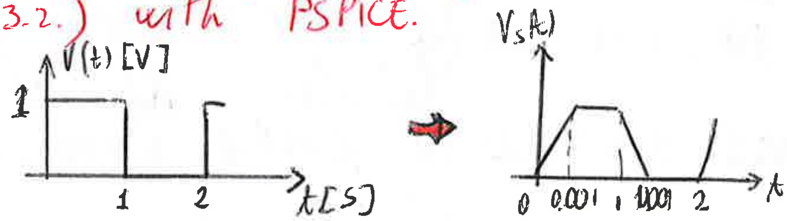
PSPICE Netlist:

```
VS  1  0  PWM(0  0  2  2  4  2  4.001  0)
              T0    T1    T2     T3
R   1  0  1E6
.TRAN  0,0001  4
.FOUR  0.25  V(1)  ← NODE
        ↑ fo
.PROBE
.END
```

EXAMPLE 3.7.

Determine the Fourier expansion coefficients for the waveform in figure (the one of example 3.2.) with PSPICE.





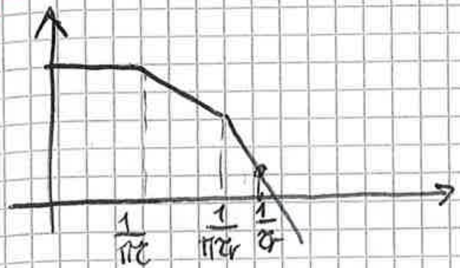
$T = 1$ s $\rightarrow f_0 = \frac{1}{2} = 0,5$ Hz

PSPICE Netlist:

```
VS   1  0   PWM(0  0  0.001  1   1   1   1.001  0   2   0)
R    1  2   1
C    2  0   1
.TRAN  0,0001   10
.FOUR  0.5   V(1)  V(2)
.PROBE        ↑      OUTPUT NODE
.END       INPUT NODE
```

REVIEW EXERCISE 3.3.

For a trapezoidal waveform having equal rise and falltimes, determine how much the bound is down at $f = \frac{1}{\tau_r}$ from the level at the second breakpoint of $\frac{1}{\pi\tau_r}$.

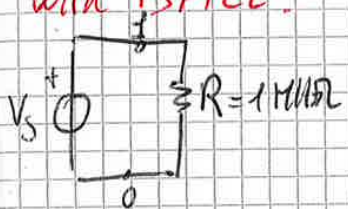


$$\text{Bound} = 40\log\left(\frac{\frac{1}{\tau_r}}{\frac{1}{\pi\tau_r}}\right) = 40\log(\pi) =$$

$$= 19,88 \text{ dB}$$

EXAMPLE 3.8.

Determine the Fourier expansion coefficients for a trapezoidal waveform of $f = 100$ MHz, $A = 5$ V, 50% DC, rise/falltimes 1ns with PSPICE.



$A = 5$ V DC = 50%

$f_0 = 100$ MHz $\tau_r = \tau_f = 1$ ns

```
VS 1 0 PULSE(0 5 0 1N 1N 4N 10N)
R 1 0 1E6
.TRAN 0.0001N 10N
.PROBE
.END
```

EXAMPLE 9.3

Determine the p.u.l. parameters for a shielded pair of wires consisting of 28-gauge stranded wires. It is reasonable to assume that $\theta_{GR} = 180°$. We have $d_G = d_R = 2r_w$, $r_w = 7{,}5$ mils and $r_{SH} = 4r_w$.

$d_G = d_R = 2r_w$

$r_w = 7{,}5$ mils (A)

$\theta_{GR} = 180° = \pi$ rad

$r_{SH} = 4r_w$

$$l_G = \frac{\mu_0}{2\pi} \ln\left(\frac{r_{SH}^2 - d_G^2}{r_{SH}\, r_{wG}}\right) = \frac{\mu_0}{2\pi} \ln\left(\frac{(4r_w)^2 - (2r_w)^2}{4r_w \cdot r_w}\right) = 220 \text{ nH/m} \quad (B)$$

$$l_R = \frac{\mu_0}{2\pi} \ln\left(\frac{r_{SH}^2 - d_R^2}{r_{SH}\, r_{wG}}\right) = 220 \text{ nH/m}$$

$$l_m = \frac{\mu_0}{2\pi} \ln\left[\frac{d_R}{r_{SH}} \sqrt{\frac{(d_G d_R)^2 + r_{SH}^4 - 2 d_G d_R r_{SH}^2 \cos\theta_{GR}}{(d_G d_R)^2 + d_R^4 - 2 d_G d_R^3 \cos\theta_{GR}}}\right]$$

-1, -1

$$= \frac{\mu_0}{2\pi} \ln\left[\frac{d_R}{r_{SH}} \sqrt{\frac{((d_G d_R) + (r_{SH})^2)^2}{((d_G d_R) + (d_R)^2)^2}}\right] =$$

$$= \frac{\mu_0}{2\pi} \ln\left[\frac{2r_w}{4r_w} \sqrt{\frac{((2r_w)^2 + (4r_w)^2)^2}{((2r_w)^2 + (2r_w)^2)^2}}\right] = 44{,}6 \text{ nH/m} \quad (C)$$

$$C_m = \frac{l_m}{v^2(l_G l_R - l_m^2)} = 10{,}7 \text{ pF/m} \quad (D)$$

$$C_G = \frac{l_R}{v^2(l_G l_R - l_m^2)} - C_m = 42{,}03 \text{ pF/m} \quad (E)$$

$$C_R = \frac{l_G}{v^2(l_G l_R - l_m^2)} - C_m = C_G = 42{,}03 \text{ pF/m}$$

$$Z_C = v_0 l_G = v_0 l_R = 65{,}9\ \Omega \qquad \text{insulation of wire}$$

$$Z_C = \sqrt{\frac{l_G}{C_G + C_m}} = 64{,}5\ \Omega$$

EXPERIMENTAL RESULTS

Consider a ribbon cable. The source voltage $V_S(t)$ will be 2,5V, 20 kHz trapezoidal pulse train having a 50% duty-cycle and rise/falltimes of 400ns. Consider $R_S=0$, $R_L=R_{NE}=R_{FE}=50\,\Omega$. Compute near-end cross-talk voltage.

$V_S(t) \rightarrow V_{pk}=2{,}5\,V$ $\quad R_S=0$ $\quad \ell=4{,}737\,m$

$f=20\,kHz$ $\quad R_L=R_{NE}=R_{FE}=50\,\Omega$ $\quad R_O=0{,}921\,\Omega$

$DC=50\%$ $\quad l_m=0{,}24\,\mu H/m$

$\tau_r=\tau_f=400\,ns$ $\quad c_m=5{,}27\,pF/m$

$$M_{NE}^{IND}=\frac{R_{NE}}{R_{NE}+R_{FE}}\cdot\frac{L_m}{R_L+R_S}=11{,}37\cdot10^{-9} \quad (A)$$

$$M_{NE}^{CAP}=\frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}}\cdot\frac{c_m\,\ell\,R_L}{R_S+R_L}=742{,}52\cdot10^{-12}=0{,}743\cdot10^{-9} \quad (B)$$

$$M_{NE}^{CI}=\frac{R_{NE}}{R_{FE}+R_{NE}}\,\frac{R_O}{R_S+R_L}=9{,}21\cdot10^{-3} \quad (C)$$

$$V_{NE}(t)=\left(M_{NE}^{IND}+M_{NE}^{CAP}\right)\frac{dV_S(t)}{dt}+M_{NE}^{CI}\,V_S(t)=$$

$$=12{,}1\cdot10^{-9}\,\frac{dV_S(t)}{dt}+9{,}31\cdot10^{-3}\,V_S(t)$$

SLEW-RATE OF PULSE TRAIN: $\boxed{\frac{dV_S(t)}{dt}=\frac{A}{\tau_r}}=\frac{2{,}5V}{400ns}=6{,}25\cdot10^{6}\,V/s$

$$V_{NE}(t)=\underbrace{12{,}1\cdot10^{-9}\cdot6{,}25\cdot10^{6}\,V}_{\text{ignoring losses}}+9{,}31\cdot10^{-3}\cdot2{,}5\,V=$$

$$=75{,}7\,mV+23{,}02\,mV=98{,}72\,mV$$

$\downarrow M_{NE}^{CI}$

$\uparrow R_L=R_{NE}=R_{FE}$

$\uparrow M_{NE}^{IND}+M_{NE}^{CAP}$

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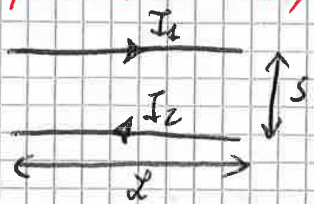
RADIATED EMISSION AND SUSCEPTIBILITY

EXAMPLE 8.1.

Consider the case of a ribbon cable constructed of 28-gauge wires separated a distance of 50 mils. Suppose the length of the wires is 1 m and they're carrying a 30 MHz differential-mode current. Compute the level of the differential-mode current that will give a radiated emission in the plane of the wires that just equals the FCC Class B limit (40 dBμV or 100 μV/m at 30 MHz).



$d = 3$ m (FCC Class B)

$|\hat{E}_D| = 100\ \mu V/m \quad @ f = 30$ MHz

$\ell = 1$ m $\quad s = 50$ mils $= 50 \cdot \frac{2{,}54\ cm}{1000} = 1{,}27$ mm

$$|\hat{E}_D| = 1{,}316 \cdot 10^{-14}\ \frac{|\hat{I}_D| f^2 \ell s}{d} =$$

$$|\hat{I}_D| = \frac{|\hat{E}_D|}{1{,}316 \cdot 10^{-14}} \frac{d}{f^2 \ell s} = \frac{100\ \mu V/m}{1{,}316 \cdot 10^{-14}} \cdot \frac{3\ m}{(30\ MHz)^2\ 1\ m \cdot 1{,}27\ mm} =$$

$$= 19{,}94\ mA.$$

EXAMPLE 8.2.

Consider the case of the ribbon cable of 28-gauge wires separated at a distance of 50 mils as previous problem. The length of wires is 1 m and they're carrying a 30 MHz common mode current. Find the value of the common-mode current that will give a radiated emission broadside to the cable that just equals the FCC Class B limit (100 μV/m @ 30 MHz)

$$|\hat{E}_C| = 1{,}257 \cdot 10^{-6}\ \frac{|\hat{I}_C| f \ell}{d}$$

$$|\hat{I}_C| = \frac{|\hat{E}_C|}{1{,}257 \cdot 10^{-6}} \frac{d}{f \ell} = \frac{100\ \mu V/m}{1{,}257 \cdot 10^{-6}} \frac{3\ m}{30\ MHz \cdot 1\ m} = 7{,}96\ \mu A$$

EXAMPLE 8.14.

Consider a 1 m cable and a current probe with a transfer impedance of 15 dBΩ. Compute the voltage from common current measurement in order to comply with the FCC Class B limit ($d = 3$ m) at 30 MHz of 40 dBμV/m.

$\mathcal{L} = 1\ m \qquad |Z_T| = 15\ dB\Omega$

$f = 30\ MHz \qquad |\hat{E}_{CMAX}| = 40\ dB\mu V/m \qquad d = 3\ m$

$|V_{SA}|?$

$$\begin{cases} |\hat{E}_{C_{MAX}}| = 1{,}257 \cdot 10^{-6} \dfrac{|\hat{I}_C| f \mathcal{L}}{d} \\ |I_{probe}| = 2 I_C \rightarrow |I_C| = \dfrac{|I_{probe}|}{2} = \dfrac{|V_{SA}|}{2 Z_T} \\ |V_{SA}| = |I_{probe}| Z_T \end{cases}$$

$$\rightarrow |\hat{E}_{CMAX}| = 1{,}257 \cdot 10^{-6} \frac{|V_{SA}| f \mathcal{L}}{2 |Z_T| d} =$$

$$\boxed{|\hat{E}_{CMAX}| = 6{,}28 \cdot 10^{-7} \frac{|V_{SA}| f \mathcal{L}}{|Z_T| d}}$$

$$|V_{SA}| = \frac{|Z_T| d |\hat{E}_{CMAX}|}{6{,}28 \cdot 10^{-7} \cdot f \cdot \mathcal{L}}$$

$$|V_{SA}|_{dB\mu V} = |\hat{E}_{CMAX}|_{dB\mu V} + 20\log d + |Z_T|_{dB\Omega} - 20\log(6{,}28 \cdot 10^{-7}) +$$

$$- 20\log(f) - 20\log(\mathcal{L}) =$$

$$= 40\ dB\mu V/m + 20\log(3) + 15\ dB\Omega - 20\log(6{,}28 \cdot 10^{-7}) +$$

$$- 20\log(30\ MHz) - 20\log(1\ m) = 39{,}04\ dB\mu V$$

EXAMPLE 8.6.

Consider a half-wave dipole having a gain in the main Beam of 2.15 dB (1.64 absolute), transmitting a 1 kW radiated power at 100 MHz. If the line is located at a distance of 3000 m from the antenna, compute the maximum electric and magnetic field.

$G = 1{,}68 \qquad (G\ dB = 2{,}15\ dB) \leftarrow 10^{\frac{2{,}15}{10}} \qquad d = 3000\ m$

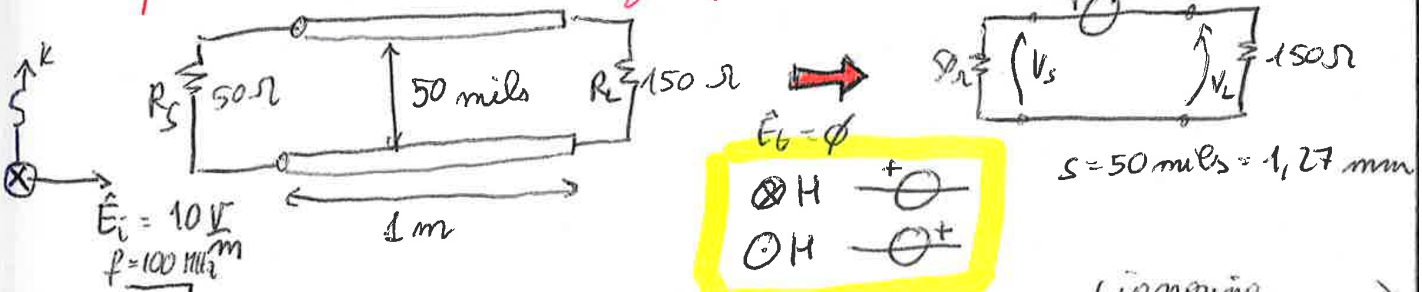
$P_T = 1\ kW \quad @\ f = 100\ MHz \qquad |E_{MAX}|?\quad |H_{MAX}|?$

$$|E_{MAX}| = \frac{\sqrt{60 P_T G}}{d} = \frac{\sqrt{60 \cdot 1\,kW \cdot 1{,}64}}{3000\ m} = 104{,}56\ mV/m$$

$$|H_{MAX}| = \frac{|E_{MAX}|}{Z_0} = \frac{104{,}56\ mV/m}{377\ \Omega} = 0{,}277\ mA/m$$

EXAMPLE 8.7.

Consider a 1 m ribbon cable, as shown in figure. The wires are 28-gauge 7x36 ($r_W = 7{,}5$ mils) and are separated by 50 mils. The termination impedances are $R_S = 50\,\Omega$ and $R_L = 150\,\Omega$. Compute the induced voltage of the cable.



$$Z_C = \sqrt{\frac{l}{c}} = \frac{1}{\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\ln\left(\frac{s}{r_W}\right) = 120\ln\left(\frac{s}{r_W}\right) = 228\ \Omega \qquad \left(\begin{array}{c}\text{ignoring wire dielectric insulation,}\\ \varepsilon_r = 1\end{array}\right)$$

$$\hat{H}_i = \frac{|E_i|}{Z_0} = \frac{10\ V/m}{377\ \Omega} = 26{,}52\ mA/m$$

$$|\hat{V}_S| = \frac{R_S}{R_L+R_S} j 2\pi f \mu_0 \mathcal{L} s |\hat{H}| = j\frac{50}{50+150} \cdot 2\pi \cdot 100 \cdot 10^6 \cdot 4\pi \cdot 10^{-7} \cdot 1\,m \cdot 1{,}27\,mm \cdot 26{,}52\,\frac{mA}{m}$$

$$= j\,6{,}65\ mV$$

$$|V_L| = \frac{-R_L}{R_S+R_L} j 2\pi f \mu_0 \mathcal{L} s |\hat{H}| = -j\ 19{,}95\ mV$$

TRANSMISSION LINES

REVIEW EXERCISE 4.1.

Determine the exact and approximate values for the p.u.l. inductance and capacitance of a two-wire ribbon cable consisting of two 28-gauge 7x36 wires ($r_w = 7,5$ mils) separated by 50 mils. Determine the ratio of wire separation to wire radius.

$s = 50$ mils
$r_w = 7,5$ mils

$$l_{APPROX} = \frac{\mu_0}{\pi} \ln\left(\frac{s}{r_w}\right) = 0,759 \ \mu H/m$$

$$= 10,16 \ln\left(\frac{s}{r_w}\right) = 19,27 \ nH/in$$

(for widely separated wires)

$$l_{EXACT} = \frac{\mu_0}{\pi} \cosh^{-1}\left(\frac{s}{2 r_w}\right) = 0,7495 \ \mu H/m$$

$$C_{EXACT} = \frac{\pi \varepsilon_0}{\cosh^{-1}\left(\frac{s}{2 r_w}\right)} = 14,84 \ pF/m$$

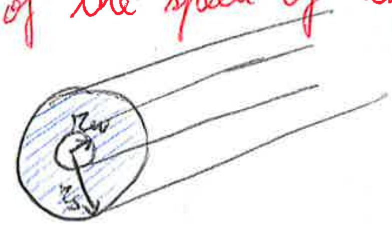
$$C_{APPROX} = \frac{\pi \varepsilon_0}{\ln\left(\frac{s}{r_w}\right)} = 14,66 \ pF/m$$

(for widely separated wires)

$$\frac{s}{r_w} = \frac{50 \text{ mils}}{7,5 \text{ mils}} = 6,67$$

REVIEW EXERCISE 4.3.

Consider a typical coaxial cable, RG58U, which consists of an inner 20-gauge solid wire ($r_W = 16$ mils) and a shield (braided) having an inner radius of 58 mils. The interior dielectric is polyethylene ($\varepsilon_r = 2{,}3$). Determine the p.u.l capacitance and inductance as well as the velocity of propagation as a percentage of the speed of light.



$r_W = 16$ mils

$r_{SH} = 58$ mils

$\varepsilon_r = 2{,}3$

$$l_{EXACT} = \frac{\mu_0}{2\pi} \ln\left(\frac{r_{SH}}{r_W}\right) = 0{,}258\ \mu H/m$$

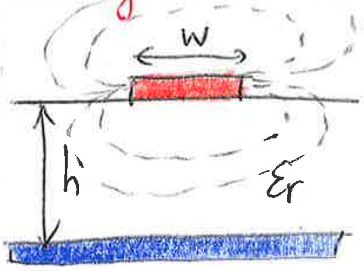
$$C_{EXACT} = \frac{2\pi\varepsilon}{\ln(r_S/r_W)} = \frac{55{,}56\,\varepsilon_r}{\ln(r_{SH}/r_W)}\ pF/m = 99{,}23\ pF/m$$

$$v = \frac{v_0}{\sqrt{\varepsilon_r}} = \frac{3 \cdot 10^8\ m/s}{\sqrt{2{,}3}} = 197{,}81 \cdot 10^6\ m/s$$

$$\frac{v}{v_0} 100 = \frac{197{,}81 \cdot 10^6}{3 \cdot 10^8} \cdot 100 = 66\%$$

REVIEW EXERCISE 4.5.

Determine the p.u.l. capacitance and inductance of a microstrip having dimensions of $h = 50$ mils, $w = 5$ mils, $\varepsilon_r = 4{,}7$.



$w = 5$ mils
$h = 50$ mils
$\varepsilon_r = 4{,}7$

l? c?

$$l = \frac{Z_c}{v}, \quad c = \frac{1}{Z_c v}$$

$$v = \frac{v_0}{\sqrt{\varepsilon_r'}} = \frac{3 \cdot 10^8}{\sqrt{3{,}034}} = 172{,}23 \cdot 10^6 \ m/s \quad (A)$$

$$\varepsilon_r' = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \frac{1}{\sqrt{1 + 10h/W}} = 3{,}034 \quad (B)$$

(non-homogeneous material)

$$\frac{W}{h} = 0{,}1 < 1 \rightarrow Z_c = \frac{60}{\sqrt{\varepsilon_r'}} \ln\left(\frac{8h}{W} + \frac{W}{h}\right) = 150{,}99 \ \Omega \quad (C)$$

$$l = \frac{Z_c}{v} = \frac{150{,}99 \ \Omega}{172{,}23 \cdot 10^6 \ m/s} = 0{,}877 \ \mu H/m$$

$$c = \frac{1}{Z_c v} = \frac{1}{150{,}99 \ \Omega \cdot 172{,}23 \cdot 10^6 \ m/s} = 38{,}46 \ pF/m$$

REVIEW EXERCISE 4.7.

Determine the characteristic impedances and the velocities of propagation for the wire-type lines in EXERCISES 4.1 - 4.3

4.1) $l = 0,7495\ \mu H/m$ $\quad c = 14,84\ pF/m$

$$v = \frac{1}{\sqrt{lc}} = 3 \cdot 10^8\ m/s$$

$$Z_C = \sqrt{\frac{l}{c}} = 224,73\ \Omega = 225\ \Omega$$

4.2) $l = 0,779\ \mu H/m$ $\quad c = 14,28\ pF/m$

$$v = \frac{1}{\sqrt{lc}} = 3 \cdot 10^8\ m/s \qquad Z_c = \sqrt{\frac{l}{c}} = 234\ \Omega$$

4.3) $l = 0,258\ \mu H/m$ $\quad c = 99,23\ pF/m$

$$v = \frac{1}{\sqrt{lc}} = 1,98 \cdot 10^8\ m/s \qquad Z_c = \sqrt{\frac{l}{c}} = 51\ \Omega$$

REVIEW EXERCISE 4.8.

Determine the characteristic impedances and velocities of propagation for the wire-type lines in EXERCISES 4.4 - 4.6.

4.4) $l = 0,461\ \mu H/m$ $\quad c = 113,2\ pF/m$

$$v = \frac{1}{\sqrt{lc}} = 1,38 \cdot 10^8\ m/s \qquad Z_c = \sqrt{\frac{l}{c}} = 63,8\ \Omega$$

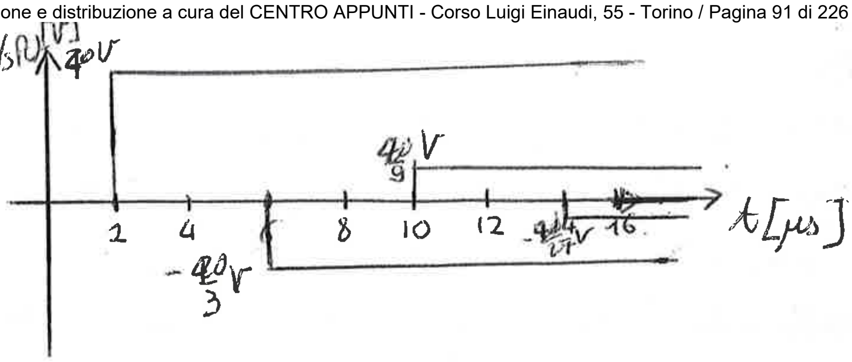
4.5) $l = 0,877\ \mu H/m$ $\quad c = 38,46\ pF/m$

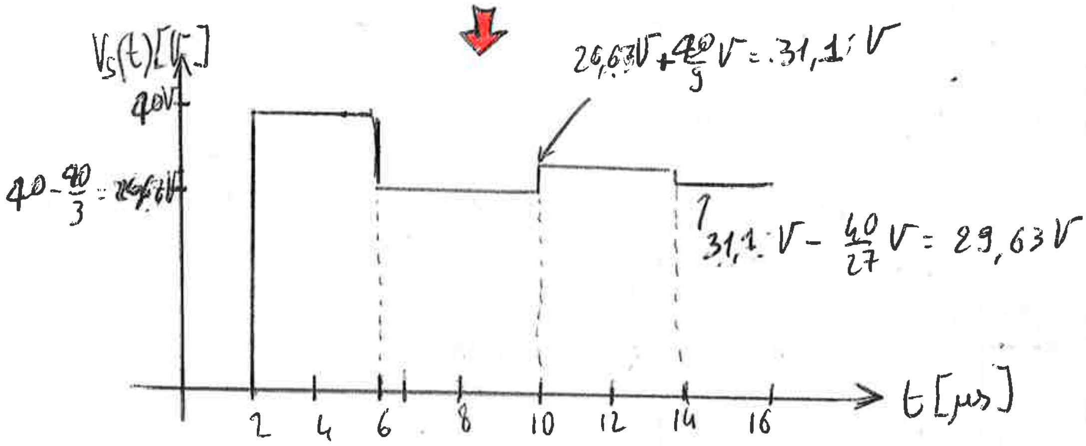
$$v = \frac{1}{\sqrt{lc}} = 1,72 \cdot 10^8\ m/s \qquad Z_c = \sqrt{\frac{l}{c}} = 151\ \Omega$$

4.6) $l = 0,804\ \mu H/m$ $\quad c = 38,5\ pF/m$

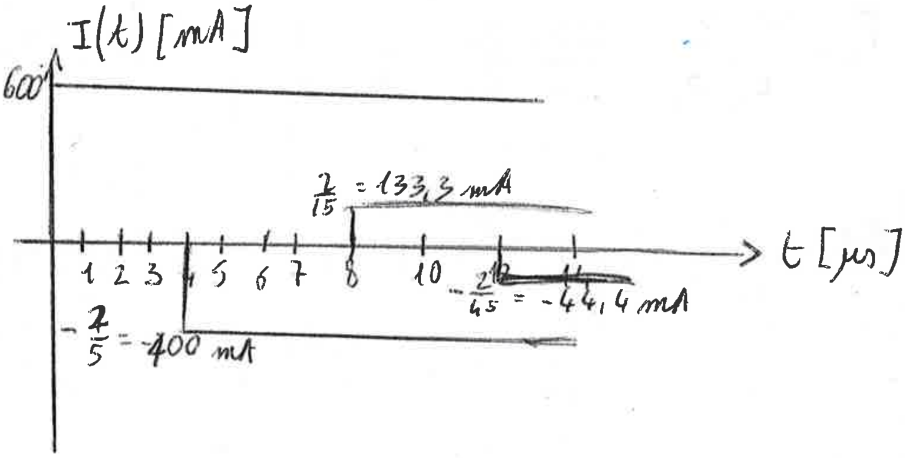
$$v = \frac{1}{\sqrt{lc}} = 1,8 \cdot 10^8\ m/s \qquad Z_c = \sqrt{\frac{l}{c}} = 144,5\ \Omega$$

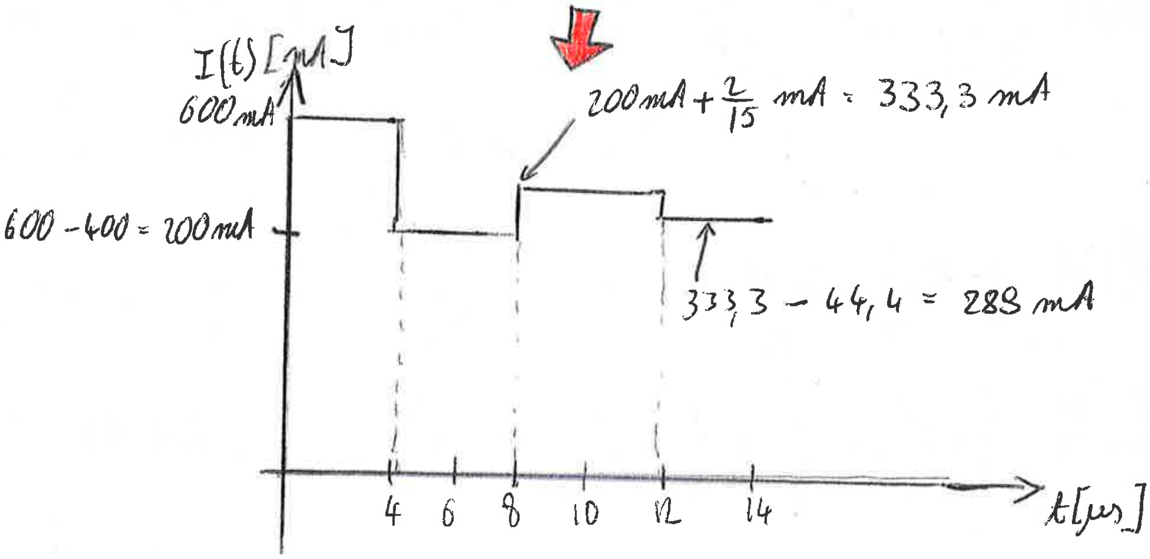
$$V(l,t) = \frac{2}{3}V_S(t-2) + -\frac{2}{9}V_S(t-6) + +\frac{2}{27}V_S(t-10) + -\frac{2}{81}V_S(t-14)$$



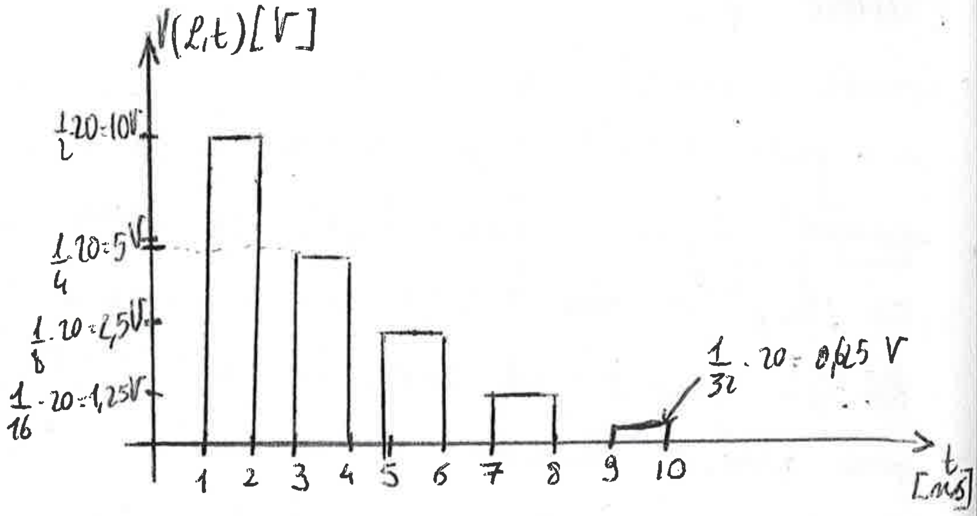


$$I(0,t) = \frac{1}{50}V_S(t) + -\frac{2}{150}V_S(t-4) + +\frac{1}{150}\cdot\frac{2}{3}V_S(t-8) + -\frac{2}{150}\,\frac{1}{9}V_S(t-12)$$

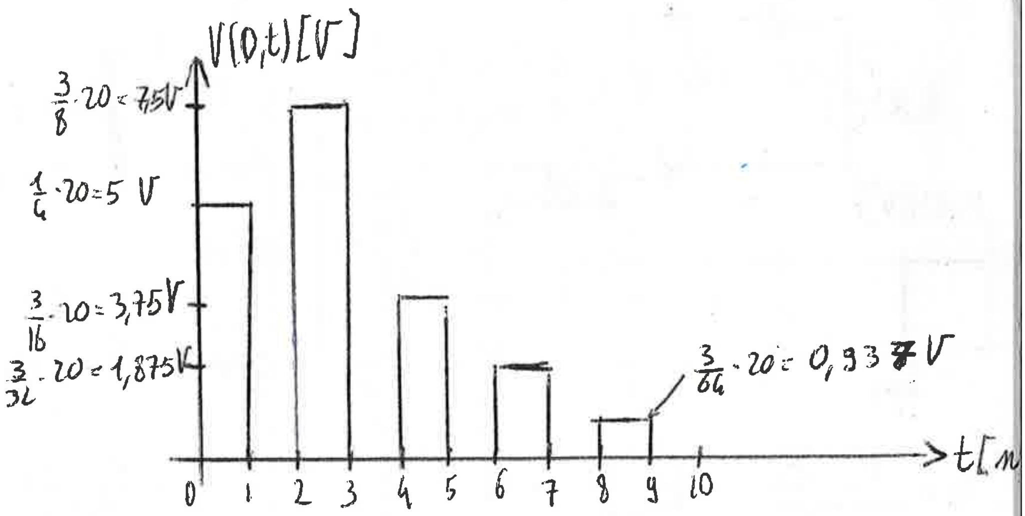




$$V(l,t) = \frac{1}{2} V_s(t-1) + \frac{1}{4} V_s(t-3) + \frac{1}{8} V_s(t-5) + \frac{1}{16} V_s(t-7) + \frac{1}{32} V_s(t-9)$$



$$V(0,t) = \frac{1}{4} V_s(t) + \frac{3}{8} V_s(t-2) + \frac{3}{16} V_s(t-4) + \frac{3}{32} V_s(t-6) + \frac{3}{64} V_s(t-8)$$



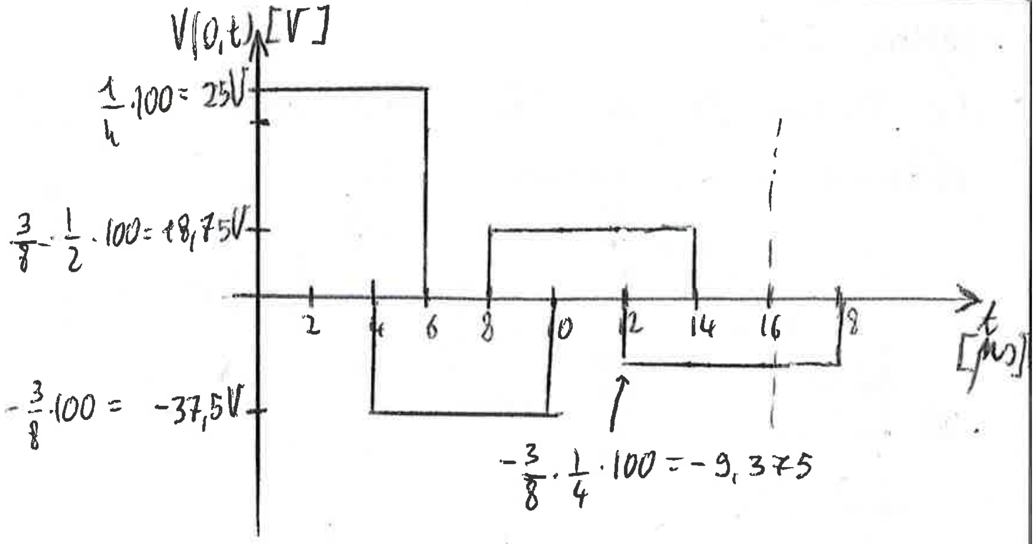
$$V(0,t) = \frac{1}{4} V_s(t) +$$

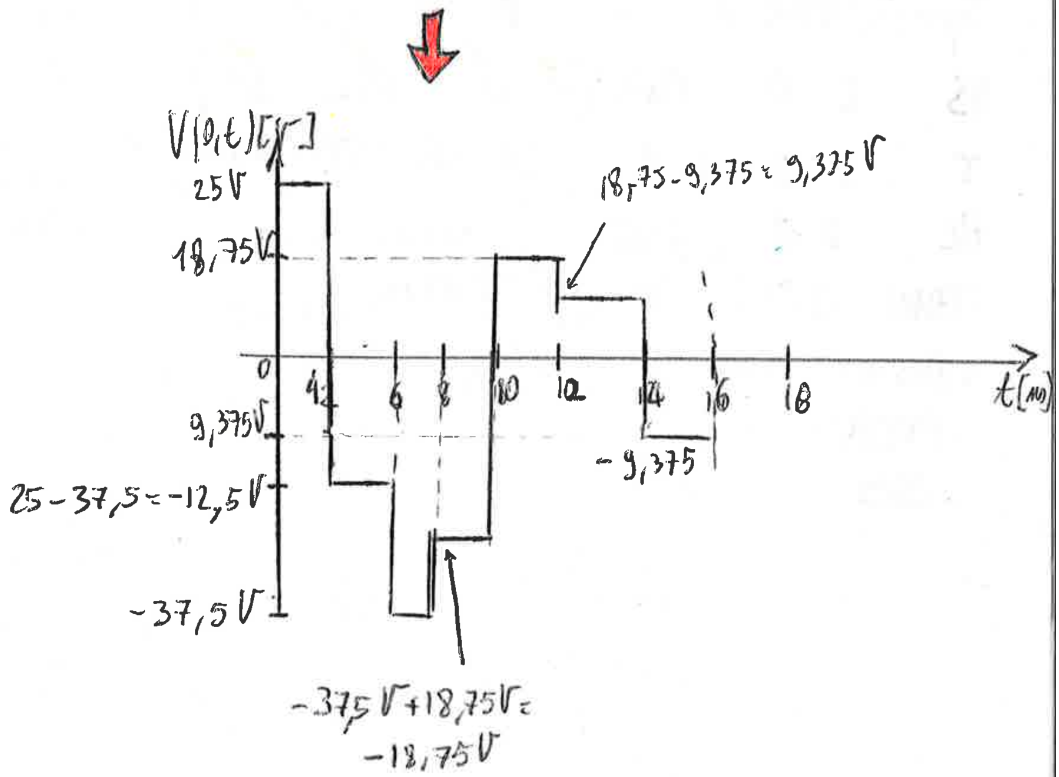
$$- \frac{1}{4} \cdot \frac{3}{2} \cdot V_s(t-4) +$$

$$+ \frac{3}{8} \cdot \frac{1}{2} V_s(t-8) +$$

$$- \frac{3}{8} \cdot \frac{1}{4} V_s(t-12) +$$

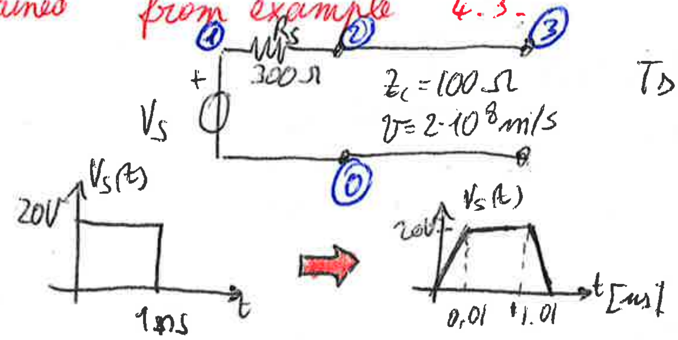
$$\ldots$$





EXAMPLE 4.6.

Use PSPICE to solve the problem shown in figure that was obtained from example 4.3.



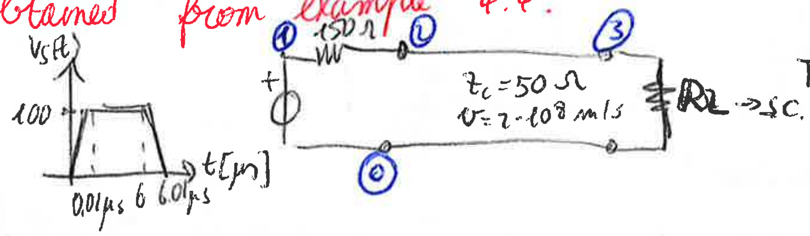
$T_D = 1$ ns

PSPICE NETLIST:

```
VS    1  0   PWL (0 0  0.01n  20  1n 20  1.01n 0)
RS    1  2   300
T     2  0   3 0   Z0=100  TD= 1n
RL    3  0   1E8  (simulate O.C.)
.TRAN  .01n  10n  0  0.01n
.PROBE
.END
```

EXAMPLE 4.7.

Use PSPICE to solve the problem shown in figure that was obtained from example 4.4.



$T_D = 2\ \mu s$

PSPICE NETLIST:

```
VS    1  0   PWL (0  0  0.01μ  6   1μ  6   1.01μ  0)
RS    1  2   150
T     2  0   3  0   Z0= 50   TD = 2u
RL    3  0   1E-6
.TRAN  .01u  20u   0  .01u
.PROBE
.END
```