NUMERO: 2329A ANNO: 2018

A P P U N T I

STUDENTE: Carrasso Francesco

MATERIA: Fisica I - Teoria e Dimostrazioni - Prof. Andrianopoli

(1)

MOTO RETTILINEO UNIFORME

v costante

$$x(t) = x_0 + v\int_{t_0}^{t} dt$$

$$x(t) = x_0 + v(t-t_0) \quad \left[\text{con } t_0 = 0 \quad x(t) = x_0 + v(t-)\right.$$

MOTO RETTILINEO UNIFORMEMENTE ACCELERATO

a costante

$$v(t) = v_0 + \int_{t_0}^{t} a(t)dt$$

$$v(t) = v_0 + a(t-t_0)$$

$$x(t) = x_0 + \int_{t_0}^{t} v(t)dt$$

$$x(t) = x_0 + \int_{t_0}^{t}\left[v_0 + a(t-t_0)\right]dt = x_0 + v_0(t-t_0) + \frac{1}{2}a(t-t_0)^2$$

$$\left[\text{con } t_0 = 0 \quad x(t) = x_0 + v_0 t + \frac{1}{2}at^2\right.$$

Velocità istantanea

$$v = \frac{\partial x}{\partial t} \Rightarrow \partial x = v\,\partial t$$

$$\int_{x_0}^{x} dx = \int_{t_0}^{t} v(t)dt, \text{ attraverso cambi. di variabile}$$

$$x(t) - x_0 = \int_{t_0}^{t} v(t)dt$$

Accelerazione istantanea

$$a(t) = \frac{\partial v}{\partial t} = \frac{\partial^2 x}{\partial t^2}$$

$$a(t)\partial t = \partial v$$

$$\int_{v_0}^{v} dv = \int_{t_0}^{t} a(t)dt$$

$$v(t) - v_0 = \int_{t_0}^{t} a(t)dt$$

MOTO ARMONICO SEMPLICE

$$x(t) = A\cos(\omega t + \varphi)$$

$$v(t) = -A\omega\,\text{sen}(\omega t + \varphi) = \dot{x}$$

$$a(t) = -A\omega^2\cos(\omega t + \varphi) = \ddot{x} = -\omega^2 x(t)$$

$$\frac{\partial^2 x(t)}{\partial t^2} = -\omega^2 x(t) \Rightarrow \frac{\partial^2 x(t)}{\partial t^2} + \omega^2 x(t) = 0$$

se $x(0) = x_0$, $v(0) = v_0$ $\quad \begin{cases} x_0 = A\cos\varphi \\ v_0 = -A\omega\,\text{sen}\varphi \end{cases}$

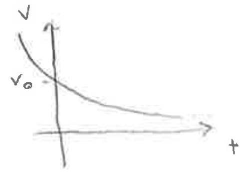
$$A = \pm\sqrt{x_0^2 + \frac{v_0^2}{\omega^2}} \qquad x_0^2 + \frac{v_0^2}{\omega^2} = A^2$$

$$-\frac{v_0}{x_0\omega} = \tan\varphi$$

MOTO RETTILINEO SMORZATO ESPONENZIALMENTE

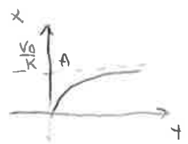
$$v(t) = v_0 e^{-kt}$$

$$t_0 = 0,\ v(0) = v_0$$



$$a(t) = -k v_0 e^{-kt} = -k v(t) = \ddot{x}$$

$$\frac{\partial v(t)}{\partial t} = -k v(t)$$



$$x(0) = x_0$$

$$\int_{x_0}^{x} dx = \int_{t_0}^{t} v(t)dt = v_0\int_{t_0}^{t} e^{-kt}dt = \frac{v_0}{-k}e^{-kt} + \frac{v_0}{k}$$

$$x(t) = x_0 + \frac{v_0}{k}\left(1 - e^{-kt}\right)$$

$$A = \frac{v_0}{k}$$

ACCELERAZIONE VETTORIALE

In maniera intrinseca:

$$\frac{d\vec r(s(t))}{dt} = \frac{d\vec r}{ds}\frac{ds}{dt} = v\frac{d\vec r}{ds} \quad , \quad \frac{d\vec r}{ds} \parallel \vec v \quad e \quad \frac{d\vec r}{ds} = \hat u_T$$

$$\left|\frac{d\vec r}{ds}\right| = \lim_{\Delta s\to 0}\left|\frac{\Delta \vec r}{\Delta s}\right| = \lim_{\substack{\Delta s\to 0\\ P'\to P}}\left|\frac{\overline{PP'}}{\widehat{PP'}}\right| = 1 \quad , \quad \vec v = v u_T$$

$\vec a \parallel d\vec v$, $\vec v = v\hat u_T$

[la derivata di un vettore costante e il vettore stesso, sono $\perp$]

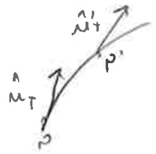
$$\vec a = \frac{d\, v\hat u_T}{dt} = \frac{dv}{dt}\hat u_T + v\frac{d\hat u_T}{dt}$$

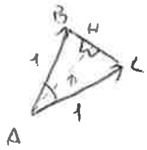
$\hat u_N$ = vettore perpendicolare a $\hat u_T$, e rivolto verso la concavità

$\hat u_N \perp \hat u_T$, $|\hat u_N| = 1$

$$\frac{d\hat u_T}{dt} \parallel \hat u_N$$

$$\frac{d\hat u_T}{dt} = \lim_{\Delta t\to 0}\frac{\Delta \hat u_T}{\Delta t}$$





$|\Delta \hat u_T| = BC = 2\overline{BH} = 2\overline{HC}$

$\overline{BH} = 1 \cdot \operatorname{sen}\left(\frac{\phi}{2}\right)$

$|\Delta \hat u_T| = 2\operatorname{sen}\frac{\phi}{2}$

$\hat u_T(\phi(t))$

$$\frac{\Delta \hat u_T}{\Delta t} = \left|\frac{\Delta \hat u_T}{\Delta \phi}\right|\left|\frac{\Delta\phi}{\Delta t}\right|$$

$$\frac{d\hat u_T}{dt} = \lim_{\Delta\phi\to 0}\left|\frac{\Delta \hat u_T}{\Delta\phi}\right| \lim_{\Delta t\to 0}\left|\frac{\Delta\phi}{\Delta t}\right|$$

$$\lim_{\Delta\phi\to 0}\left|\frac{\Delta \hat u_T}{\Delta\phi}\right| = \lim_{\Delta\phi\to 0}\frac{2\operatorname{sen}\frac{\phi}{2}}{\phi} = 1 \Rightarrow \left|\frac{d\hat u(\phi)}{d\phi}\right| = 1 \ , \perp \hat u_T \Rightarrow \frac{d\hat u_T}{d\phi} = \hat u_N$$

$$\vec a = \frac{d^2 s}{dt^2}\hat u_T + v\frac{d\phi}{dt}\hat u_N$$

$$PP' = \Delta s \simeq R_C \Delta\phi$$

$$\frac{d\phi}{dt} = \lim_{\Delta t\to 0}\frac{\Delta\phi}{\Delta t} = \lim_{\Delta t\to 0}\frac{1}{R_C}\frac{\Delta s}{\Delta t} = \frac{1}{R_C}\frac{ds}{dt} = \frac{v}{R_C}$$

$$\vec a = \frac{dv}{dt}\hat u_T + \frac{v^2}{R_C}\hat u_N$$

MOTO CIRCOLARE

3

$r(t) = R \qquad s(t) = \theta R$

$v = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega \quad , \vec{\omega} \perp$ al piano $, \vec{\omega} = \omega(\pm \hat{u}_z)$, reg. mano dx

$\vec{v} = R\omega \hat{u}_T = \vec{\omega} \times \vec{r} \quad , \perp \vec{r}$ e $\vec{\omega}$

$\alpha = \frac{d\omega}{dt} = \ddot{\theta} = \dot{\omega}$

$\vec{a}_T = \frac{d\vec{v}}{dt}\hat{u}_T = R\ddot{\theta}\hat{u}_T \quad , \hat{a}_N = \frac{v^2}{R}\hat{u}_N = \omega^2 R \hat{u}_N$

$\dot{\vec{v}} = \vec{\omega}\times\vec{r} = \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\vec{r}}{dt} \quad , \vec{\alpha} = \frac{d\vec{\omega}}{dt}$

$\vec{a} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times(\underbrace{\vec{\omega}\times\vec{r}}_{\vec{v}})$

COORDINATE POLARI

$\vec{r}(t) = \begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad , \quad \begin{cases} \hat{u}_r = \frac{\vec{r}}{r} = \cos\theta\,\hat{u}_x + \sin\theta\,\hat{u}_y \\ \hat{u}_\theta = \frac{d\hat{u}_r}{d\theta} = -\sin\theta\,\hat{u}_x + \cos\theta\,\hat{u}_y \end{cases}$

$\vec{r} = r\hat{u}_r \iff \hat{u}_r = \frac{\vec{r}}{r}$

$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{u}_r + \frac{d\hat{u}_r}{dt}\cdot r = \boxed{v\hat{u}_r + r\cdot\dot{\theta}\,\hat{u}_\theta}$

$\frac{d\hat{u}_r(\theta(t))}{dt} = \frac{d\hat{u}_r}{d\theta}\frac{d\theta}{dt} = \hat{u}_\theta\dot{\theta}$

—o—

$\frac{d\hat{u}_\theta}{d\theta} = -\hat{u}_r \; ; \quad \frac{d\hat{u}_\theta}{dt} = -\dot{\theta}\hat{u}_r$

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_\theta$

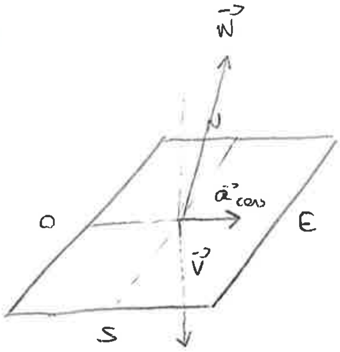
nel moto circolare

$\begin{cases} \hat{u}_T = \hat{u}_\theta \\ \hat{u}_N = -\hat{u}_r \end{cases}$

48

CORIOLIS

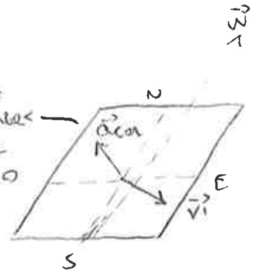
VERTICALE



$\vec{a}_{cor}$ contribuisce al moto verso est

ORIZZONTALE

2 componenti: quella orizzontale trascina il moto



MOTO ANGOLARE

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{L}}{dt} = \cancel{\vec{v} \times m\vec{v}}^{\,0} + \vec{r} \times \frac{d\vec{p}}{dt} = r \times \vec{F} \text{ , momento della forza}$$

Se in un campo agiscono forze centrali $\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L}$ si conserva $=$ cost.

$\vec{L} \perp$ al piano di $\vec{r}$ e $\vec{p}$. Ciò vuol dire perpendicolare sempre allo stesso piano $\Rightarrow$ si guarda moti: piani con velocità areale costante

Polarmente

$$\frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{u}_r + r\frac{d\theta}{dt}\hat{u}_\theta \quad , \vec{r} = r\hat{u}_r$$

$$|\vec{L}| = m\left(r\hat{u}_r \times \left(\cancel{\frac{dr}{dt}\hat{u}_r}^{\,0} + r\frac{d\theta}{dt}\hat{u}_\theta\right)\right) = m r^2 \frac{d\theta}{dt}\hat{u}_z$$

$$m r^2 \frac{d\theta}{dt} = \text{costante}$$

Se ho un ellisse lo approssimo ad un cerchio per Δt piccoli

$$\frac{\Delta A}{\Delta t} = \frac{\frac{1}{2} r^2 \Delta\theta}{\Delta t} + \frac{o(\Delta\theta)}{\Delta t} \qquad \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$

$$\frac{1}{2m} L = \frac{dA}{dt} \qquad \frac{dA}{dt} = K \Rightarrow dA = K\,dt$$

3 LEX

$$F = m w^2 r \qquad w = \frac{2\pi}{T}$$

$$G\frac{\cancel{m}M}{r^2} = \cancel{m} w^2 r = \left(\frac{2\pi}{T}\right)^2 r$$

$$\frac{GM}{4\pi^2} = \frac{r^3}{T^2} \qquad T^2 \propto r^3$$

TEOREMA DELLE FORZE VIVE

5B

$\vec F = \sum_i \vec F_i \quad , \quad \mathcal{L} = \int_{\gamma(A,B)} \vec F \cdot d\vec r = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2$

dove $\frac{1}{2} m v^2 = K$

$\sum_i \vec F_i = \frac{d\vec p}{dt} = m\vec a$

$w = F_{//}\, ds = m a_{//}\, ds = m \frac{dv}{dt} ds = m \frac{ds}{dt} dv = m v\, dv$

In coordinate intrinseche

$\vec a = \frac{dv}{dt}\hat u_T + \frac{v^2}{R_c}\hat u_N$

$d\vec r = ds\,\hat u_T = v\,dt\,\hat u_T$ $\qquad \hat u_T \cdot \hat u_N = 0$

$\mathcal{L} = \int_{\gamma(A,B)} \vec F d\vec r = \int_{\gamma(A,B)} \left(m\frac{dv}{dt}\hat u_T + m\frac{v^2}{R_c}\hat u_N \right) \cdot v\,dt\,\hat u_T = \int_{\gamma(A,B)} m \frac{dv}{dt} v\,dt =$

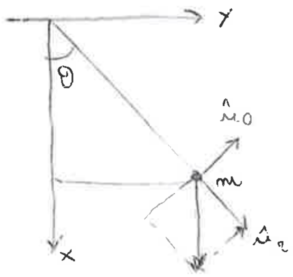
$v(t) \to v$

$\frac{dv}{dt} dt \to dv$

$= \int_{t(A)}^{t(B)} m \frac{dv}{dt} v\,dt$

$\mathcal{L} = \int_{v_A}^{v_B} m v\,dv = \frac{1}{2} m v^2 \Big|_{v_A}^{v_B} = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = K_B - K_A$

LAVORO DI FORZE SUL PENDOLO



$-\theta^0 \le \theta \le \theta^0$

$\vec F_p = mg\cos\theta\,\hat u_r - mg\sin\theta\,\hat u_\theta$

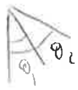
$\vec T = -N\hat u_r$

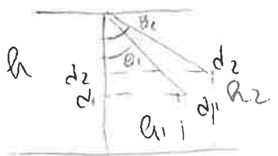
$d\vec r = l\,d\theta\,\hat u_\theta$

$\mathcal{L}_\gamma(T) = \int \vec T d\vec r = \int T\,l\,d\theta\,\hat u_r \hat u_\theta = 0$

$\mathcal{L}_\gamma(F_p) = \int_{\theta_1}^{\theta_2} -mg\sin\theta\,l\,d\theta\,|\hat u_\theta|^2 = -mg\int_{\theta_1}^{\theta_2} \sin\theta\,d\theta = mgl(\cos\theta_2 - \cos\theta_1)$

l'altra componente va a zero





$d_1 = l\cos\theta_1$

$d_2 = l\cos\theta_2$

$h_2 = h - d_2$

$h_1 = h - d_1$

$\mathcal{L}_{(\theta_1,\theta_2)} = mg(h_1 - h_2) = \frac{1}{2} m (v_2^2 - v_1^2)$

[dà segno al lavoro]

TEOREMA DI CONSERVAZIONE DELL'ENERGIA

6B

Per le forze conservative $\mathcal{L}_\gamma = E_p(\vec{r}_A) - E_p(\vec{r}_B)$

Per le non conservative $\mathcal{L}_\gamma = K_B - K_A$

Se conservative, valgono entrambe

$$\mathcal{L}_\gamma = E_p(A) - E_p(B) = K_B - K_A$$

$$E_p(A) + K_A = E_p(B) + K_B$$

ENERGIA MECCANICA

$$E = K(\vec{v}) + E_p(\vec{r})$$

$$\vec{F} = \vec{F}_c + \vec{F}_{nc}$$

$$\mathcal{L}_\gamma = K_B - K_A = \mathcal{L}_\gamma(\vec{F}_c) + \mathcal{L}_\gamma(\vec{F}_{nc}) = E_p(A) - E_p(B) + \mathcal{L}_\gamma(\vec{F}_{nc})$$

$$\mathcal{L}_\gamma(\vec{F}_{nc}) = K_B - K_A - E_p(A) + E_p(B) = E_B - E_A$$

, per una forza dissipativa l'energia meccanica del corpo varia nel tempo

FORZA PESO

$$\vec{F} = F_0 \hat{u}$$

$$\delta\mathcal{L}(\vec{F}) = F_0 \hat{u}\, \hat{u}_T ds \quad (\vec{F} d\vec{r})$$

$$\vec{F} = -mg\hat{u}_z \qquad \uparrow z$$

$$\mathcal{L}_\gamma(\vec{F}_p) = mg\int -\hat{u}_z\, dz\, \hat{u}_z$$

$$d\vec{r} = dz\,\hat{u}_z$$

$$\mathcal{L} = \int -mg\,\hat{u}_z\, dz\,\hat{u}_z = -mg\int_{h_A}^{h_B} dz = mg(h_A - h_B) = E_p(A) - E_p(B)$$

Per il pendolo

$$\mathcal{L}(\vec{F}_p) = mg(\cos\theta_2 - \cos\theta_1)$$

$h_1 = h - d_1$

$$\mathcal{L} = mg(d_2 - d_1) = mg(h - h_2 - h + h_1) = mg(h_1 - h_2) = E_p(h_1) - E_p(h_2)$$

FORZA ELASTICA

$$\vec{F} = -K(x - l_0) \qquad d\vec{r} = dx\,\hat{u}_x$$

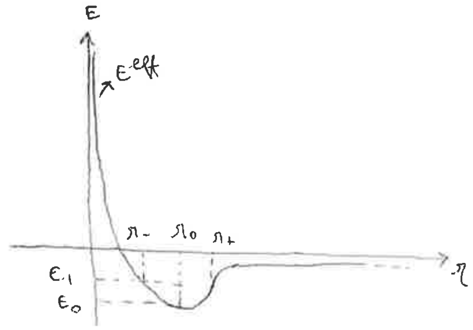
$$\delta\mathcal{L} = -K(x - l_0)\,\hat{u}_x\,(dx\,\hat{u}_x) = -K(x - l_0)dx$$

$$\mathcal{L} = \int_{x_A}^{x_B} -K(x - l_0)dx \qquad \begin{array}{l} z = x - l_0 \\ dz = dx \end{array} \quad = -\frac{1}{2}Kz^2\Big|_{z_A}^{z_B} = -\frac{1}{2}K(x_B - l_0)^2 + \frac{1}{2}K(x_A - l_0)^2$$

$$E_p(x) = \frac{1}{2}(x - l_0)^2$$

$$\vec{F} = -\vec{\nabla}E_p = -\frac{\partial E_p}{\partial x}\hat{u}_x - \frac{\partial E_p}{\partial z}\hat{u}_z = 0 - mg\hat{u}_z$$

$$\vec{F} = F(x)\,\hat{u}_x \qquad F(x) = -\frac{\partial E_p}{\partial x} = -K(x - l_0)$$



$$E^{cf}(r) = \frac{\alpha}{r^2}$$

$$E_p(r) = -\frac{\gamma}{r}$$

$$\left[\vec{F} = -\frac{d\vec{E}}{dx}\hat{u}_x\right]$$

se $E_0 = E_p^{eff}\Big|_{r_0 \text{ minimo}}$ $\quad \frac{dr}{dt} = 0$, il contributo polare, invece, $\frac{d\theta}{dt} = \frac{L}{m r_0^2} = \text{cost} = \omega_0$ (L costante, forza centrale; r_0 costante)

moto circolare uniforme

Se $E_0 < E_1 < 0$

Il moto è ellittico, il corpo è vincolato tra due raggi, afelio, perielio

Se $E = 0$, interseca il punto di E_p^{eff} e il corpo torna indietro descrivendo una parabola

MOTO CIRCOLARE

$\gamma > 0$, forza attrattiva

$$E_0 = \min\left(E_p^{eff}\right)$$

$E_p^{eff} = -\frac{\gamma}{r} + \frac{L^2}{2mr^2}$, per trovare il raggio associato al valore minimo $\frac{dE_p^{eff}}{dr} = \frac{\gamma}{r^2} - \frac{L^2}{mr^3} = 0$

$\boxed{r_0 = \frac{L^2}{m\gamma}}$, $r = r_0$, $\omega = \frac{L}{m r_0^2} = \frac{L m^2\gamma^2}{m L^4} = \frac{m\gamma^2}{L^3}$, moto costante

$\boxed{\text{se } L = 0 \Rightarrow \omega = 0}$

— o —

$E^{cf} = \frac{L^2}{2mr^2}$, $F^{cf} = -\frac{dE_p^{cf}}{dr} = \frac{L^2}{mr^3}\hat{u}_r$

$\frac{L^2}{mr^3} = \frac{(mr^2\omega)^2}{mr^3} = \frac{m^2 r^4 \omega^2}{mr^3} = m\omega^2 r$ ← forza centrifuga

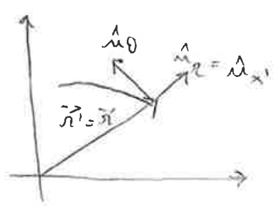
Dati due S.R.

S.I $\quad \vec{r} = r\hat{u}_r = \vec{r}\,'$

S.N.I $\quad \vec{r}\,' = x'\hat{u}_{x'}$ $\quad x' = r$, $\vec{r}\,' \equiv \vec{r}$ $\quad \hat{u}_r \equiv \hat{u}_{x'}$

posizione dell'ogg. lungo lo sbarra

$\vec{r}\,' = x'\hat{u}_{x'}$



$$\vec{v}\,' = \frac{dx'}{dt}\hat{u}_{x'} = \frac{dr}{dt}\hat{u}_r$$

$$\vec{v} = \vec{v}\,' + \omega \times \vec{r}\,'$$

$$\vec{a} = \vec{a}\,' + \vec{\alpha}\times\vec{r}\,' + 2\omega\times v' + \omega\times(\omega\times r')$$

$\vec{a}\,' = \vec{a} - \vec{\omega}\times\vec{\omega}\times\vec{r} \underbrace{- \vec{\alpha}\times\vec{r} - 2\vec{\omega}\times\vec{v}\,'}_{\text{vengono meno questi 2 fattori}}$. $E^{eff} = -\frac{\gamma}{r} + \frac{L^2}{2mr^2}$; $F = -\frac{dE_p^{eff}}{dr}\hat{u}_r = -\frac{\gamma}{r^2}\hat{u}_r + m\omega^2 r\hat{u}_r$

Ciò è valido se L è costante : $\alpha \times r = \frac{d\omega}{dt} r(\hat{u}_z\times\hat{u}_r)$, $2\omega\times v' = 2\omega\frac{dr}{dt}(\hat{u}_z\times\hat{u}_r)$

devo verificare che

$\frac{d\omega}{dt}r + 2\omega\frac{dr}{dt} = 0$. Se $L = \text{cost} = mr^2\omega$ $\quad \frac{dL}{dt} = 2mr\frac{dr}{dt}\omega + mr^2\frac{d\omega}{dt} = mr\left(2\omega\frac{dr}{dt} + \frac{d\omega}{dt}r\right) = 0$

$\varepsilon = \sqrt{1 + \frac{2EL^2}{m\gamma^2}}$, $\varepsilon^2 = 1 + \frac{2EL^2}{m\gamma^2} = 1 + \frac{2E r_0}{\gamma}$ $\qquad r_0 = \frac{L^2}{m\gamma}$ (88)

$r_0 = -\frac{\gamma}{2E}(1-\varepsilon^2)$ $\quad \rightarrow \quad -\frac{\gamma}{2E} = \frac{r_0}{1-\varepsilon^2}$ $\Rightarrow r_\pm = -\frac{\gamma}{2E}(1 \pm \varepsilon) = \frac{r_0}{(1-\varepsilon)(1+\varepsilon)}(1 \pm \varepsilon) = \frac{r_0}{(1 \mp \varepsilon)}$

$E = 0 \quad \varepsilon = 1 \qquad r(\theta) = \frac{r_0}{1+\cos\theta} \qquad 1+\cos\theta > 0 \quad -\pi < \theta < \pi$, parabola

$E > 0$, $\varepsilon > 1$

$r(\theta) = \frac{r}{1+\varepsilon\cos\theta} \qquad 1+\varepsilon\cos\theta > 0 \qquad \cos\theta > -\frac{1}{\varepsilon}$

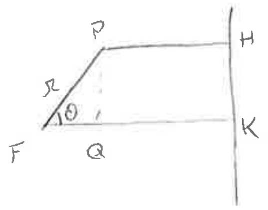
se $\varepsilon = 0 \qquad r_A = \frac{r_0}{2}$

Soluzione repulsiva $\gamma < 0$

$E = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{L^2}{2mr^2} - \frac{\gamma}{r}$ $\qquad r_\pm = -\frac{\gamma}{2E}(1+\varepsilon)$, Tolgo "$-$" gli overo st. ~~maggiore~~ minore

$E > 0$, $\gamma < 0$, $\varepsilon > 1$ $\qquad r(\theta) = \frac{r_0}{1 \pm \varepsilon\cos\theta}$, $r_0 < 0$

$r(\theta) = \frac{r_0}{1+\varepsilon\cos\theta} > 0$ poiché $r_0 < 0 \qquad 1+\varepsilon\cos\theta < 0 \qquad -\pi \leq \theta < \theta_0 \vee \theta_0 \leq \theta \leq \pi$



$\varepsilon = \frac{|PF|}{|PH|} \qquad FQ = r\cos\theta$

$FK = d$

$\varepsilon = \frac{r}{d - r\cos\theta}$

$\varepsilon d - \varepsilon r\cos\theta = r$

$r = \frac{\varepsilon d}{1+\varepsilon\cos\theta}$ $= r_0$

$L = \text{cost} \qquad A = \frac{1}{2}r^2 d\theta \qquad PP' \sim PH = r\,d\theta$

$dA = \frac{1}{2}r^2 \frac{d\theta}{dt}dt$

$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt}$, $L = mr^2\frac{d\theta}{dt}$, $\frac{dA}{dt} = \frac{L}{2m}$, cost.

$r_\pm = \frac{r_0}{1 \mp \varepsilon} \qquad a = \frac{1}{2}(r_+ + r_-) = \frac{r_0}{2}\left(\frac{1}{1-\varepsilon} + \frac{1}{1+\varepsilon}\right) = \frac{r_0}{1-\varepsilon^2}$ se $\frac{dA}{dt} = \frac{L}{2m} \Rightarrow dA = \frac{L}{2m}dt$

$\int_0^{A_{ell}} dA = \int_0^T \frac{L}{2m}dT \qquad A_{ell} = \frac{L}{2m}T = \pi ab = \pi\sqrt{r_0}\,a^{3/2}$

$a^3 = \boxed{\frac{L^2T^2}{4\pi^2 M^2 r_0}}$ cost

tutto ciò da

$$\begin{cases} \frac{dr}{dt} = \sqrt{\frac{2}{m}(E - E^{eff})} \\ \frac{d\theta}{dt} = \frac{L}{mr^2} \end{cases}$$

TEOREMA DI KÖNIG PER L'ENERGIA CINETICA

(38)

$$K = K^{int} + K^{CM} = K^{int} + \frac{1}{2} m V_{CM}^2$$

L'energia cinetica del sistema e' dovuta al moto del CM e al moto del sist. rispetto a CM.

$$K^{int} = \sum_i \frac{1}{2} m_i \vec{v}_i'^2 \quad \text{(nel sist. di r. del CM)}$$

$$K = \frac{1}{2} \sum_i m_i (\vec{v}_i \cdot \vec{v}_i) \qquad \vec{v}_i = \vec{v}_i' + \vec{V}_{CM}$$

$$= \frac{1}{2} \sum_i m_i v_i'^2 + \sum_i m_i \vec{v}_i' \vec{V}_{CM} + \frac{1}{2} \sum_i m_i V_{CM}^2$$

VINCOLI

$$m_1 v_1^{in} + m_2 v_2^{in} = m_1 v_1^{out} + m_2 v_2^{out}$$

$$\frac{1}{2} m_1 (v_1^{in})^2 + \frac{1}{2} m_2 (v_2^{in})^2 = \frac{1}{2} m_1 (v_1^{out})^2 + \frac{1}{2} m_2 (v_2^{out})^2$$

URTI ELASTICI

$$m_1 (v_1^{in} - v_1^{out}) = m_2 (v_2^{out} - v_2^{in}) \quad (1)$$

$$m_1 (v_1^{in\,2} - v_1^{out\,2}) = m_2 (v_2^{out\,2} - v_2^{in\,2})$$

$$m_1 [(v_1^{in} + v_1^{out})(v_1^{in} - v_1^{out})] = m_2 [(v_2^{out} - v_2^{in})(v_2^{out} - v_2^{in})] \quad (2)$$

Sost 1 in 2

$$m_2 \cancel{(v_2^{out} - v_2^{in})} (v_1^{in} + v_1^{out}) = m_2 \cancel{(v_2^{out} - v_2^{in})} (v_2^{out} + v_2^{in})$$

se $v_1^{out} \neq v_1^{in}$

$$\begin{cases} v_1^{in} + v_1^{out} = v_2^{out} + v_2^{in} & (a) \\ m_1 (v_1^{in} - v_1^{out}) = m_2 (v_2^{out} - v_2^{in}) & (b) \end{cases}$$

$m_1 (a) - (b)$:

$$; \; 2 m_1 v_1^{out} = \left(\frac{m_1 - m_2}{2 m_1}\right) v_2^{out} + \frac{(m_1 + m_2)}{2 m_1} v_2^{in}$$

Sost in (a)

$$v_1^{in} + \left(\frac{m_1 - m_2}{2 m_1}\right) v_2^{out} + \frac{m_1 + m_2}{2 m_1} v_2 = v_2^{in} + v_2^{out}$$

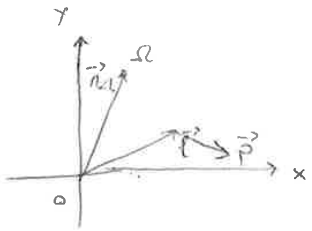
$$v_2^{out} = \left[\frac{m_1 - m_2}{2 m_1} - 1\right] = v_2^{in} \left[1 - \frac{m_1 + m_2}{2 m_1}\right] = v_1^{in}$$

$$v_2^{out} = \frac{2 m_1}{m_1 + m_2} v_1^{in} + \frac{m_2 - m_1}{m_1 + m_2} v_2^{in}$$

$$v_1^{out} = \frac{m_1 - m_2}{m_1 + m_2} v_1^{in} + \frac{2 m_2}{m_1 + m_2} v_2^{in}$$

L d pert. rispetto al polo Ω

10B



$$\vec{L} = (\vec{r} - \vec{r}_\Omega) \times \vec{p}$$

$$\frac{d\vec{L}}{dt} = \left(\frac{d\vec{r}}{dt} - \frac{d\vec{r}_\Omega}{dt}\right) \times \vec{p} + (\vec{r} - \vec{r}_\Omega) \times \frac{d\vec{p}}{dt}$$

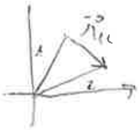
$$= \underbrace{(\vec{r} - \vec{r}_\Omega) \times \vec{F}}_{\tau_\Omega} - \vec{v}_\Omega \times \vec{p}$$

2 masse

$$\vec{L}_\Omega = \vec{L}_{1\Omega} + \vec{L}_{2\Omega} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$$

$$\frac{d\vec{L}_O}{dt} = \frac{d\vec{L}_{1,O}}{dt} + \frac{d\vec{L}_{2,O}}{dt}$$

$$\frac{dL_O}{dt} = \vec{r}_1 \times \frac{d\vec{p}_1}{dt} + \vec{r}_2 \times \frac{d\vec{p}_2}{dt}$$



$$\frac{d\vec{p}_1}{dt} = \vec{F}_1^{ext} + \vec{F}_{12} \; ; \; \frac{d\vec{p}_2}{dt} = \vec{F}_2^{ext} + \vec{F}_{21}$$

$$\frac{dL_O}{dt} = \vec{r}_1 \times \left(\vec{F}_1^{ext} + \vec{F}_{12}\right) + \vec{r}_2 \times \left(-\vec{F}_{12} + \vec{F}_2^{ext}\right) = \cancelto{0}{(\vec{r}_1 - \vec{r}_2) \times \vec{F}_{12}} + \vec{r}_1 \times \vec{F}_1^{ext} + \vec{r}_2 \times \vec{F}_2^{ext}$$

$$= \vec{\tau}_O^{ext}$$

$\vec{r}_{12} \parallel \vec{F}_{12}$

SISTEMA DI N PARTICELLE

$$\vec{L}_O = \sum_i \vec{r}_i \times \vec{p}_i \; ; \; \frac{d\vec{L}_O}{dt} = \sum_i \cancel{\frac{dr_i}{dt} \times p_i} + \sum_i r_i \times \frac{d\vec{p}_i}{dt}$$

$$\frac{d\vec{p}_i}{dt} = \vec{F}_i^{ext} + \sum_{i \neq j} \vec{F}_{ij}$$

$$\frac{d\vec{L}_O}{dt} = \cancel{\sum r_i \times \frac{dp_i}{dt}} = \underbrace{\sum_i \vec{r}_i \times \vec{F}_i^{ext}}_{\vec{\tau}_O^{ext}} + \sum_{i=1}^{n} r_i \times \sum_{j \neq i} \vec{F}_{ij} = \vec{\tau}_O^{ext} = \sum_i \vec{r}_i \times \vec{F}_i^{ext}$$

$$+ \sum_i \sum_j \cancelto{0}{(r_i - r_j) \times \vec{F}_{ij}}$$

$\Omega \neq 0$

$$\vec{L}_\Omega = \underbrace{\sum_i (\vec{r}_i - \vec{r}_\Omega) \times \vec{p}_i}_{\tau_\Omega^{ext}} \; ; \; \frac{d\vec{L}_\Omega}{dt} = \sum_i (\vec{r}_i - \vec{r}_\Omega) \times \frac{d\vec{p}_i}{dt} - \sum_i \vec{v}_\Omega \times \vec{p}_i =$$

$$= \sum_i (\vec{r}_i - \vec{r}_\Omega) \times \vec{F}_i^{ext} - v_\Omega \times \sum_i \vec{p}_i = \vec{\tau}_\Omega^{ext} - \vec{v}_{CM} M \times \vec{v}_\Omega$$

$[\, r_i - r_\Omega - (r_j - r_\Omega) = r_i - r_j \parallel F_{ij} \to 0 \,]$

Se Ω con CM.

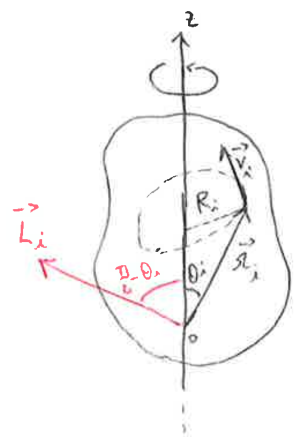
$$\vec{L}_C = \sum_i \underbrace{(\vec{r}_i - \vec{r}_{CM})}_{\vec{r}_i'} \times \vec{p}_i'$$

$r_i' = \vec{r}_i - r_{CM} \qquad r'_{CM} = 0$

$p_i' = m_i (v_i - v_{CM}) \Rightarrow \sum_i p_i' = P' = v'_{CM} M = 0$

MOMENTO ANGOLARE - MOMENTO DI INERZIA

113



$\vec{L}_{i,o} = \vec{r}_i \times m_i \vec{v}_i$ $\quad R_i = r_i \operatorname{sen}\theta_i$

$\vec{L} \perp$ al piano di $\vec{v}_i, \vec{r}_i$

con un angolo di $\frac{\pi}{2}-\theta_i$ $\quad r_i \perp v_i$

$L_i = m_i r_i v_i = m_i r_i \, w R_i$

Momento angolare assiale L_z

$L_{iz} = L_i \cos\left(\frac{\pi}{2}-\theta_i\right) = L_i \operatorname{sen}\theta_i = m_i r_i \operatorname{sen}\theta_i R_i w = m_i R_i^2 w \hat{u}_z$

$\vec{L}_z = \sum_i \vec{L}_{iz} = \left(\sum_i m_i R_i^2\right) w = I_z w$

$\vec{I}_z = \sum m_i R_i^2 = \sum_i m_i (x_i^2 + y_i^2)$

La componente assiale può variare solo in modulo. Quella perpendicolare può variare anche in direzione. Dipende dalla scelta del polo.

$\vec{L}_{i,\perp} = L_i \cos\theta_i = m_i r_i R_i w \cos\theta_i \hat{u}_\perp$ generalmente $\vec{L}_\perp$ è difficile da valutare

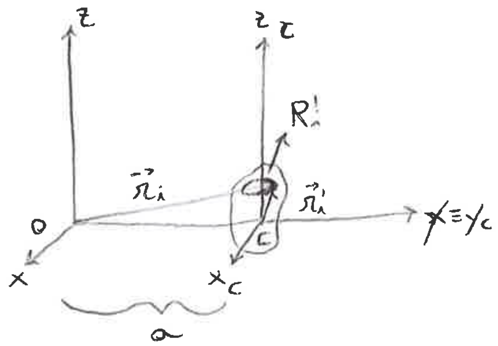
$$\boxed{\vec{L} = I_z \vec{w} + L_\perp}$$

Se ho un corpo continuo

$$I = \iiint_V R^2(x,y,z)\,\rho(x,y,z)\,\underbrace{dx\,dy\,dz}_{dV}$$

TEOREMA DI H-S

$I_z = I_c + Ma^2$ (a = distanza tra sistemi)



$$\begin{cases} y_i = y_i' + a \\ x_i' = x_i \\ z_i' = z_i \end{cases}$$

$I_z = \sum m_i R_i^2$

$R_i^2 = x_i^2 + y_i^2 = x_i'^2 + y_i'^2 + 2ay_i' + a^2 = R_i'^2 + 2ay_i' + a^2$

$$I_z = \underbrace{\sum m_i x_i'^2 + \sum m_i y_i'^2}_{R_i'^2} + \sum m_i \underbrace{a\, y_i'}_{0} + \sum m_i a^2 = \overbrace{\sum m_i R_i'^2}^{I_c} + Ma^2 + \sum m_i \underbrace{y_{cm}'}_{0}\, a$$

CASI PER CR

asse di rotazione $\hat{z}$ contiene un punto fisso in SR inerziale

$$\vec{\tau}_o^{\,ext} = \frac{d\vec{L}_o}{dt} \qquad \left(\text{generale: } \vec{L} = I_z\vec{\omega} + \vec{L}_\perp\right)$$

a_1) asse z è principale

$$\vec{L}_o = I_z\vec{\omega} \qquad \tau_o^{ext} = \frac{d(I_z\vec{\omega})}{dt}$$

$$\vec{\tau}_o^{\,ext} = I_z\frac{d\vec{\omega}}{dt} + \frac{dI_z}{dt}\vec{\omega}$$

se cambia l'asse di rotazione I_z varia

Per $\frac{dI_z}{dt} = 0 \Rightarrow \vec{\tau}_{ext} = I_z\vec{\alpha}$

Se $\tau^{ext} = 0 \qquad I\vec{\omega} = \text{cost.}$

a2) asse z non è principale

$L_z = I_z\omega$

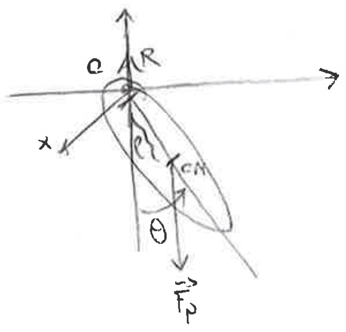
Possiamo fare considerazione L_z, $L_\perp$ più complicate e varia

in questo caso $\qquad \vec{\tau}^{\,ext}\cdot\hat{u}_z = \tau^{ext}$

$$\vec{\tau}_z^{\,ext} = \frac{dL_z}{dt} = \frac{d}{dt}(I_z\vec{\omega}) = \frac{dI_z}{dt}\vec{\omega} + I_z\frac{d\vec{\omega}}{dt}$$

Se non ho punti fissi di asse con SRI non uso $\tau = \frac{dL}{dt} \qquad \tau_c^{ext} = \frac{dL_c}{dt}$

PENDOLO FISICO



$$\vec{\tau}_R + \vec{\tau}_{F_p} = 0 + \vec{r}\times\vec{F}_p = -Mgl\sin\theta$$

asse di rotazione non è principale

$$\vec{\tau}_x^{\,ext}\cdot\hat{u}_x = \tau_x = I\alpha$$

$$-Mgl\sin\theta = I\ddot{\theta} \qquad \text{per } \theta \to 0$$

$$-\frac{Mgl}{I}\theta = \cancel{I}\ddot{\theta} \qquad \omega_0 = \sqrt{\frac{Mgl}{I}}$$

$$\sqrt{\frac{Mgl}{I}} = \sqrt{\frac{g}{l'}} \Rightarrow \text{l'equivalente di un pendolo} \quad T = 2\pi\sqrt{\frac{I}{Mgl}}$$

ENERGIA ASSOCIATA AD UN CORPO RIGIDO

(13B)

$$E_K = \sum_i^n m_i v_i^2 - \frac{1}{2} M V_{CM}^2 + E_K^{int}$$

ad esempio di rotazione attorno a CM

$$\Rightarrow \quad \vec{v}_i = \vec{\omega} \times R_i$$

$$v_i = \omega r_i \sin\theta_i$$

$$E_K^{int} = \frac{1}{2} \sum m_i (\omega^2 R_i^2) = \frac{1}{2} \left(\sum_i m_i R_i^2 \right) \omega^2 = \frac{1}{2} I_C \omega^2$$

se l'asse di rotazione è principale

$$L = I\omega \qquad \omega = \frac{L}{I} \qquad K_K^{int} = \frac{1}{2} \frac{L^2}{I}$$

Energia potenziale

Costante in quanto le posizioni sono fisse che interazioni si elidono

$$U = E^{int} = E_K^{int} + E_p^{int}$$

$$E^f - E^i = \mathcal{L}^{ext}$$

$$\mathcal{L}^{ext} = \Delta \frac{1}{2} M V_{CM}^2 + \Delta E_K^{int};$$

Per ogni pto del CR

$$\vec{v}_i = \vec{V}_{CM} + \underbrace{(\vec{v}_i - \vec{V}_{CM})}_{\vec{v}_i'}$$

$$\vec{v}_i' = \vec{\omega} \times \vec{r}_i'$$

$$\vec{v}_i = v_{CM} + \omega \times (\vec{r}_i - \vec{r}_{CM})$$

$$E_K = \frac{1}{2} M V_{CM}^2 + \frac{1}{2} I_C \omega^2$$

R-TR

$$E_K = \frac{1}{2} M V_{CM}^2 + \frac{1}{2} I_C \omega^2$$

R-PU

(Steiner)

$$E_K = \frac{1}{2} I_O \omega^2 = \frac{1}{2} I_C \omega^2 + \underbrace{\frac{1}{2} M R^2 \omega^2}_{V_{CM}^2}$$

velocità fondamentale

$$\vec{V}_P = V_{CM} + \omega \times (\vec{r}_P - \vec{r}_{CM}) = \cancel{V_{CM}} + \omega \times r_P - \cancel{\omega \times r_{CM}} = \vec{\omega} \times \vec{r}_P$$

Potenza motrice a CR

$$W = \frac{d\mathcal{L}^{est}}{dt} \qquad \mathcal{L}^{est} = \int \vec{F}^{ext} \cdot d\vec{r} = \int \vec{F} (\vec{\omega} \times \vec{r}) dt = \int \vec{\omega} \cdot (\vec{r} \times \vec{F}) dt = \int \vec{\omega}\, \vec{\tau}^{ext} dt$$

$$\frac{d\vec{r}}{dt} = \vec{v} = \vec{\omega} \times r \qquad d\vec{r} = (\vec{\omega} \times \vec{r}) dt$$

$$W = \frac{d\mathcal{L}}{dt} = \vec{\omega} \cdot \tau^{ext} = \tau^{ext} \frac{d\theta}{dt} dt \Rightarrow \mathcal{L} = \int \tau^{ext} d\theta$$

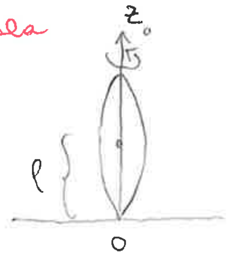
MOTO GIROSCOPICO

(19B)

È il caso in cui

$$\frac{dL_0}{dt}=0 \quad \text{ma} \quad \frac{d\vec{L}}{dt}\neq 0$$

Trottola

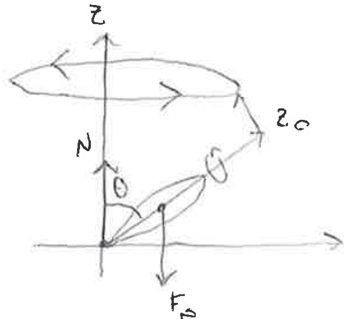


$\vec{\omega}=\omega\,\hat{u}_{z_0}$

se z_0 è verticale

$\vec{\tau}_0 = \vec{OC}\times\vec{F_p} \rightarrow \vec{L}_0 = \text{costante}$

Situazione iniziale



z_0 asse di simmetria $\quad \ell = OC$

N, F_p sono forze che tendono a far cadere la trottola

$$\vec{\tau}_0 = \vec{OC}\times\vec{F_p} = -\ell Mg(\hat{u}_{z_0}\times\hat{u}_z)$$

$$\vec{\tau}_0 \perp \hat{u}_{z_0}\,,\quad \vec{L}_0 = I\vec{\omega} = I\omega\,\hat{u}_{z_0}$$

Se L è suff. grande, la trottola descrive un cono con origine O attorno all'asse z.

Moto di precessione, l'asse di ~~simmetria~~ rotazione si sposta nel tempo

$$\frac{d\vec{L}}{dt}=\vec{\tau}\neq 0 \qquad L\perp\frac{d\vec{L}}{dt} \Rightarrow \frac{d\vec{L}}{dt}=\vec{\Omega}\times\vec{L}\ (*)$$

vel. angolare attorno a z

$$\vec{\tau}=\vec{\Omega}\times\vec{L}$$

$$\vec{\tau}_0 = -\ell Mg\,\hat{u}_{z_0}\times\hat{u}_z = \ell Mg(\hat{u}_z\times\hat{u}_{z_0})$$

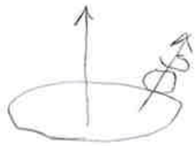
$$\vec{L}_0 = I\omega\,\hat{u}_{z_0}$$

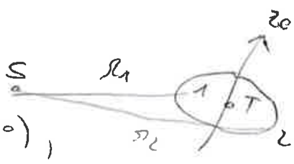
$$\vec{\Omega}=\Omega\,\hat{u}_z \qquad \ell Mg(\hat{u}_z\times\hat{u}_{z_0}) = \Omega I\omega(\hat{u}_z\times\hat{u}_{z_0})\ (*)$$

$$\Omega = \frac{\ell Mg}{I\omega}$$

Se $\Omega \sim \omega$ parliamo di nutazione, ruotando descrive dei merletti.

La Terra nella sua rotazione su se stessa si comporta come una trottola → precessione degli equinozi





F_1 tenderebbe a raddrizzare z_0 a z

Ruotando grazie a ω e all'angolazione ($\sim 23°$), la Terra ruota attorno al Sole e non si raddrizza

$$F_{TS}\sim\frac{1}{R_{TS}^2}$$

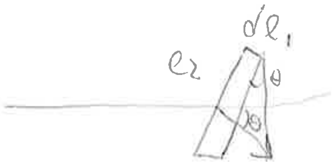
$T_{precessione} \sim 26\,000$ anni

$T_{nutazione} \sim 19$ anni

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 29 di 44

Regola di Faraday per linee di campo (15 B)

$$dN = \alpha |\vec{K}| dS_\perp$$



$$dS = dl_1 \cdot l_2$$

$$dS \cos\theta = dl_1 \, l_2 \cos\theta = dS_\perp$$

$$|d\phi| = \frac{1}{\alpha} dN$$

$$d\phi = \begin{cases} \frac{1}{\alpha} dN_{out} = |\vec{K}| dS_\perp \\ -\frac{1}{\alpha} dN_{in} = -|\vec{K}| dS_\perp \end{cases}$$

$$d\phi = |\vec{K}| \underbrace{\cos\theta \, dS}_{\pm dS_\perp}$$

$$\phi_S(\vec{K}) = \sum_i d\Phi_i = \frac{1}{\alpha}(N_{out} - N_{in})$$

Flusso di $\vec{E}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{u}_r$$

$$d\phi = \vec{E} \cdot \vec{n} \, dS = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{u}_r \hat{u}_n dS = \frac{Q}{4\pi\varepsilon_0} \frac{\pm dS_\perp}{r^2}$$

$$\Phi_{chiusa}(\vec{E}) = \frac{Q}{4\pi\varepsilon_0} \int_\Sigma \frac{dS_\perp}{r^2}$$

$\frac{dS_\perp}{r^2}$ adimensionale, $d\Omega$ angolo solido

$$dS_\perp = r^2 d\Omega$$

$$\Omega_{max} = \frac{dS_\perp}{r^2} = \frac{4\pi r^2}{r^2}$$

$$d\Omega = \frac{dS_\perp(r_0)}{r_0^2} = \frac{dS_\perp(r_1)}{r_1^2}$$

$$\Phi_\Sigma(\vec{E}) = \frac{Q}{4\pi\varepsilon_0} \int d\Omega = \frac{Q 4\pi}{4\pi\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

Se ho N cariche $\phi = \int \sum_i \vec{E}_i \cdot \vec{n} \, dS = \sum \phi = \frac{1}{\varepsilon_0} \sum Q$

Se la sorgente è esterna, ogni ~~vettore~~ componente di flusso annulla la sua simmetrica

Potenziale gravitazionale generato da M omogenea (sfera raggio R) (16B)

$$\vec{G} = -\vec{\nabla} V_G$$

$$\vec{G}(\vec{r}) = G(r)\hat{u}_r = -\hat{u}_r \frac{\partial V_G}{\partial r}$$

$$G(r) = -\frac{d}{dr}\left(V_{G(r)}\right)$$

$r > R$: $G(r) = -G_N \frac{M}{r^2} = -\frac{dV_G}{dr}$ $\qquad V_G = -\int G(r)dr + c = \int G_N \frac{M}{r^2} dr + c$

$$V = -\frac{G_N M}{r} + c \qquad r > R$$

Per $r < R$

$$G(r) = -\frac{G_N M}{R^3} r = -\frac{dV_G}{dr}$$

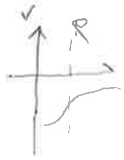
$$V_G = -\int G(r)dr + c' = \int \frac{GM}{R^3} r + c' = \frac{GM}{2R^3} r^2 + c'$$

V_G continua e differenziabile in R $\qquad V_G(R^-) = V_G(R^+)$

$$\frac{GM}{2R^3} R^2 + c' = -\frac{GM}{R^2} R + c \qquad c = 0 \text{ per } V \to 0, \; r \to +\infty$$

$$c' = -\frac{GM}{R}\left(\frac{1}{2} + 1\right) = -\frac{3}{2}\frac{GM}{R}$$

$$V(r) = \begin{cases} \frac{GM}{R}\left(\frac{r^2}{2R^2} - \frac{3}{2}\right), & r < R \\ -\frac{GM}{r}, & r > R \end{cases}$$



FLUIDI

Lavoro compiuto da $\Delta \vec{F}_\perp$ $\qquad \Delta F_\perp = p \Delta S \vec{n}$

$$\mathcal{L} = \int d\vec{F}_\perp d\vec{r} = \int p dS \vec{n} d\vec{r} = \int p dS dh = \iiint p dV = \int_{V_A}^{V_B} p dV$$

$$d\vec{r} = dr_{\parallel}\hat{u}_r + dh\vec{n}$$

17B

Per gas

$\rho \neq cost$

$\rho = \rho(P) = KP$

$PV = mRT$

$\frac{m}{V} \propto P$

$\rho \propto \frac{m}{V} = \frac{P}{RT}$

se T cost

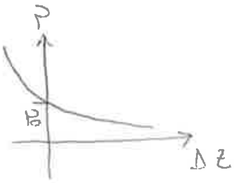
$\rho = KP$

$K = \frac{\rho}{P} = \frac{\rho_0}{P_0}$

$$\frac{dP}{dz} = -\rho(P)\, g = -KPg$$

$$\int_{P_0}^{P} \frac{dP}{P} = -\int_{z_0}^{z} gK\,dz \Rightarrow \ln \frac{P}{P_0} = -g\,K(z - z_0)$$

$$P = P_0\, e^{-gK(\Delta z)}$$



$$P(z) = P_0\, e^{-gK\Delta z} = P_0 e^{-\frac{\Delta z}{a}} \quad , \quad a \equiv \frac{1}{Kg}$$

$K = \frac{\rho_0}{P_0}$, $\rho_0^{aria} = 1{,}2\, Kg/m^3$

$P_0 = 10^5\, Pa$, $K = 1{,}2 \cdot 10^{-5} \frac{s^2}{m^2}$, $a \sim 10^4 m$

Se $\Delta z \sim a$

$$P = \frac{P_0}{e} \sim \frac{P_0}{2{,}7} \quad \text{in } 10\,Km$$

Se $\Delta z \ll a \sim 10\,Km$

$$P \simeq P_0\left(1 - \frac{\Delta z}{a}\right) = P_0 - gP_0 K \Delta z \simeq P_0 - g\rho_0 \Delta z$$

Per $\Delta z \ll a$ e $P = cost$

$\rho_0 \simeq cost$, $\vec{\nabla} P = \rho \vec{a} \simeq 0$

Barometro di Torricelli

$h = 76\,cm$

P_a in A

$$P - P_0 = -\rho_{Hg}\, g\, \Delta z$$

$z_A = z_0$

$P(z_A) = P_0$

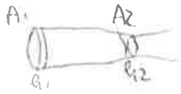
$$P_B - P_A = -\rho_{Hg}\, g(z_B - z_A)$$

$$P_A = \rho_{Hg}\, g(h)$$

CONSERVAZIONE DELLA MASSA - EQUAZIONE DI CONTINUITÀ

18B

In dt, fluido in A_1 si sposta di $d\vec{l_1} = \vec{v_1}\,dt$



$dm_1 = \rho\, dV_1 \qquad dV_1 = A\, dl_1 = A_1 v_1 dt$

$d\vec{l_2} = \vec{v_2}\,dt$

$dm_2 = \rho\, dV_2 = \rho A_2 v_2 dt$

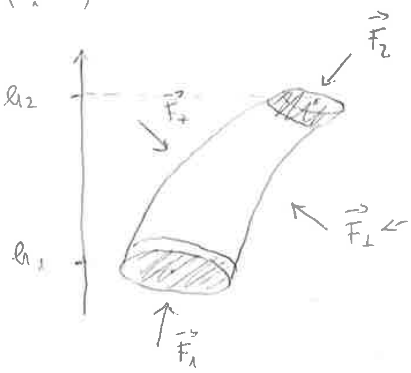
Poiché $dm_1 = dm_2$, abbiamo la stessa massa di fluido, in quanto non può distruggersi

$\rho A_1 v_1 dt = \rho A_2 v_2 dt$

$A_1 v_1 = A_2 v_2 = Q =$ costante PORTATA

LEGGE DI BERNOULLI

$\mathcal{L}\left(\sum_i F_i\right) = \Delta K$



$\vec{F_\perp}$ ← non compiono lavoro perché non effettuano spostamenti attraverso il tubo di flusso

$\vec{F_1} = P_1 A_1 \hat{u}_T \qquad \vec{F_2} = P_2 A_2 \hat{u}_T$

$d\mathcal{L}(\vec{F_p}) = P_1 A_1 \hat{u}_{T_1} d\vec{l_1} - P_2 A_2 \hat{u}_{T_2} d\vec{l_2} = P_1 A_1 dl_1 - P_2 A_2 dl_2 = (P_1 - P_2)\, dV$

$d\mathcal{L}(\vec{F_P}) = dm\, g(h_1 - h_2) = \rho g\, dV (h_1 - h_2)$, poiché $\vec{F_P}$ conservativo

$dK = \frac{1}{2} m (v_2^2 - v_1^2) = \frac{1}{2}\rho\, dV (v_2^2 - v_1^2)$

$d\mathcal{L}(F_p) + d\mathcal{L}(F_P) = dK$

$(P_1 - P_2)\cancel{dV} + \rho g \cancel{dV}(h_1 - h_2) = \frac{1}{2}\rho \cancel{dV} (v_2^2 - v_1^2)$

$$\boxed{P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2}$$

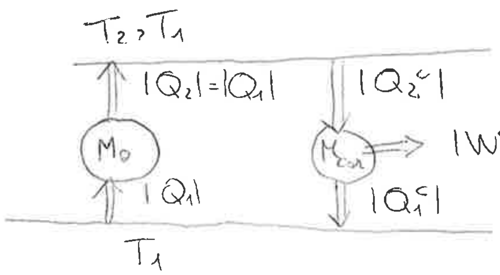
$$\underbrace{h}_{\text{altezza del fluido}} + \underbrace{\frac{P}{\rho g}}_{\text{altezza piezometrica}} + \underbrace{\frac{v^2}{2g}}_{\text{altezza cinetica}} = \text{cost}$$

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 39 di 44

$\not{C} \Rightarrow \not{K}$

21

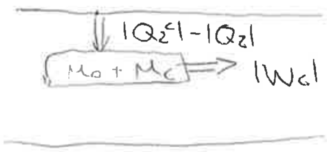
$\not{C}$



$W^c = Q_2^c + Q_1^c = |Q_2^c| - |Q_1^c| > 0$

Scelgo $|Q_1^c| = |Q_1|$ e considero la macchina complessiva

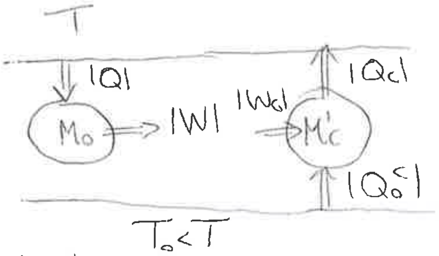
$M_o + M_c$



$W_c = |Q_2^c| - |Q_1^c|$

$|W_c| = |Q_2^{TOT}| = |Q_2^c| - |Q_1^c|$

$\not{K} \Rightarrow \not{C}$



M_c^{-1} frigor.

$|W| = |Q|$

$W > 0$

$W_c < 0$

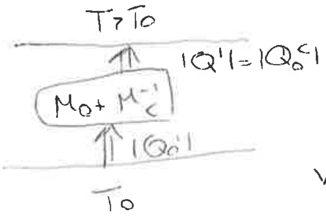
$|W_c| = |Q_c| - |Q_c^o|$

Scelgo $|W_c| = |W|$

Considero $M_o + M_c^{-1} = M'$

$|W_c| = |Q_c| - |Q_c^o|$

$|W| = |Q|$



$|Q| - |Q_c| = -|Q_o^c| < 0$

$|Q_o'| = |Q_o^c|$

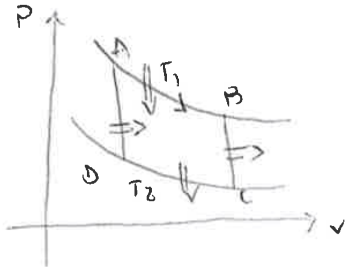
Violato Clausius

CICLO STIRLING

22

$r = \frac{V_2}{V_1} > 1$

2 isoterme
2 isocore



AB: T=cost, $\Delta U = 0$

$W_{AB} = Q_{AB} = mRT_2 \ln\left(\frac{V_2}{V_1}\right) > 0$

CD: T=cost.

$W_{CD} = Q_{AB} = mRT_2 \ln\left(\frac{V_1}{V_2}\right) < 0$

BC: $W_{BC} = 0$

$Q_{BC} = \Delta U_{BC} = mc_v(T_1 - T_2) < 0$

DA V=cost

$W_{DA} = 0$

$Q_{DA} = \Delta U_{DA} = mc_v(T_2 - T_1) > 0 = -Q_{BC}$

$Q_{ASS} = Q_{AB} + Q_{DA} = mRT_2 \ln r + mc_v(T_2 - T_1)$

$W_{TOT} = W_{AB} + W_{CD} = mR(T_2 - T_1)\ln r$

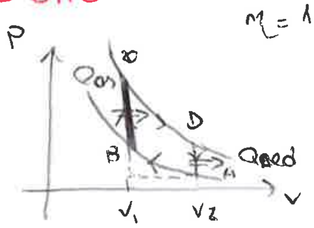
$\eta = \frac{|W_{TOT}|}{|Q_{ASS}|} = \frac{mR\ln r\,(T_2 - T_1)}{mRT_2 \ln r + mc_v(T_2 - T_1)} < \eta_c$

$Q_{medio\ scamb.} = Q_{AB} \Rightarrow \eta = \eta_c$ Rigeneratore

CICLO OTTO

2 adiabatiche
2 isocore

$\eta = 1 + \frac{Q_{ced}}{Q_{ass}}$



AB: adiabatica, compressione, $\delta Q = 0$

$\Delta U = -W$

BC: isocora, aumento di pressione

$\Delta U = Q = mc_v(T_C - T_B)$

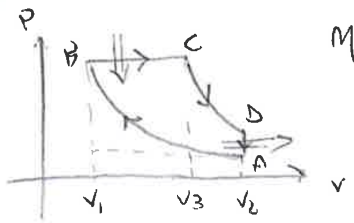
AD: isocora, dim. pressione

$\Delta U = Q = mc_v(T_A - T_D)$

$\eta = 1 + \frac{\cancel{mc_v}(T_A - T_D)}{\cancel{mc_v}(T_C - T_B)} = 1 - \frac{T_D - T_A}{T_C - T_B}$

CICLO DIESEL

[1 isobara progressiva]
2 adiabatiche rev
1 isocora rev.
1 isobara rev.



$\eta = 1 + \frac{Q_{ced}}{Q_{ass}} = \frac{|W|}{|Q_{ass}|}$

BC: isobara

$\Delta U = Q - W$

$mc_v(T_C - T_B) = Q - P(V_3 - V_1)$

$Q = P(V_3 - V_1) + mc_v(T_C - T_B) = \boxed{mc_p(T_C - T_B)}$

DA:

$\Delta U = Q = mc_v(T_A - T_D)$

$\eta = 1 - \frac{mc_v(T_A - T_D)}{mc_v(T_C - T_B) + P_{BC}(V_3 - V_1)} = \longrightarrow 1 + \frac{\cancel{m}c_v(T_A - T_D)}{\cancel{m}c_p(T_C - T_B)} = \boxed{1 - \frac{T_D - T_A}{\gamma(T_C - T_B)}}$

$r_C = \frac{V_2}{V_1}$, $r_E = \frac{V_2}{V_3}$

$T_D V_2^{\gamma-1} = T_C V_3^{\gamma-1}$, $T_A V_2^{\gamma-1} = T_B V_1^{\gamma-1}$

$\eta = 1 - \frac{1/r_E^{\gamma} - 1/r_C^{\gamma}}{\gamma(1/r_E - 1/r_C)}$