NUMERO: 2309A ANNO: 2018

A P P U N T I

STUDENTE: Chiforeanu Loredana

MATERIA: Hydrology - Teoria + Esercitazioni - Prof. Tamea

2/10/17

Hydrology

Stefania Tamea Stefania.Tamea@polito.it USGS

→ Organise the assignments/exercises in a report: → groups of 2/3 people
- equations
- basic results / numbers
- theory

→ They'll be used in the exam (but with not much writing).

⊕ Final optional assignment: technical report on a problem of relevance of hydrology (design or verification)
↓
Will be presented to the students and solved. −1, 0, +1, +2 to add to the final exam.
Given in January

→ Written exam: 2h
- problems
- theoretical questions
- demonstrations
- quiz

(no calcolatrice)

Variables associated to a probability of an occurrence related to water:
- discharge or river flow $\left[\frac{m^3}{s}\right]$
- precipitation (intensity $\left[\frac{mm}{h}\right]$, volumes, ...) $\frac{m^3}{1\,m^2} = \frac{\text{volume}}{\text{unit surface}} \Rightarrow [mm]$
- temperature (in phase change)
- soil moisture $\left[\frac{m^3_{H_2O}}{m^3_{soil}}\right] \Leftrightarrow [/]$ non dimensional
- radiation (in energy balances). $\left[\frac{\text{Energy}}{\text{unit surface}}\right]$

If ∃ datas ⇒ use data-based approach

If ∄ datas ⇒ use physically based approach (quantitative modelling)

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- Simple Drainage Systems $P_{ex} = 0,1 = 10\%$

→ The Law imposes a higher protection depending on the consequences of the events. Pex is the probability of failure.

"La probabilità di collasso residua delle strutture deve essere uguale a quella imposta dalla legge, cioè uguale alla Pex, data per ogni struttura".

Descriptive Statistics

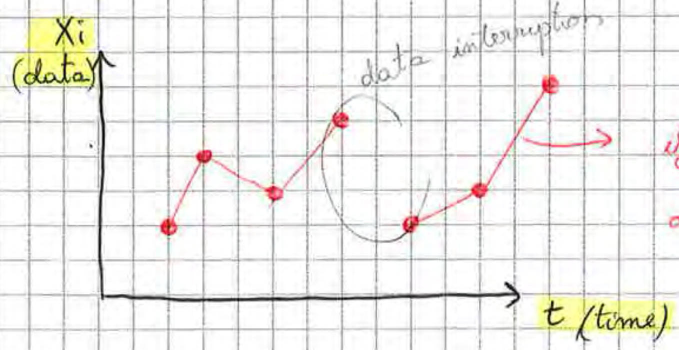
Starting from a SAMPLE OF DATA, whose elements are indicated as x, χ, x_i.

- The aim is to define the POPULATION OF THE RANDOM VARIABLE called X.
- The sample has a SIZE n.

We have to summarise and represent data.

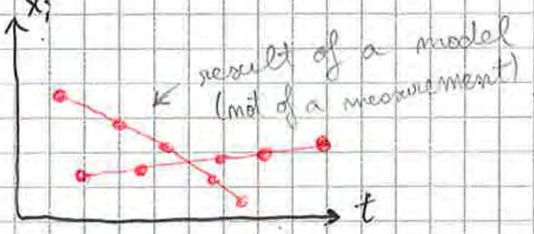
TIME SERIES

→ If I have data connected to time.



→ if there is temporal continuity, we represent it with a line, otherwise we interrupt the line.

Ex



This kind of data and lines follow a mathematical law, but we have to check if it is physically possible that the event follows that law.

3

Or we can use: $\boxed{K = 2 \cdot n^{0.4}}$

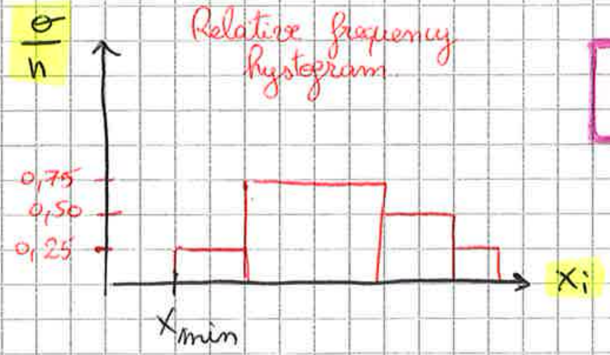
→ NORMALIZE THE AXIS OF THE OCCURENCES (σ)

because the hystogram depends on the size n, so it changes everytime I change the data:

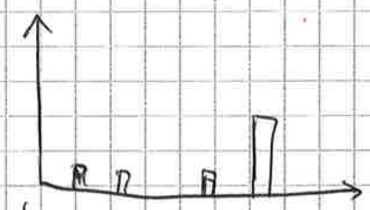
⇒ normalization: $\boxed{\frac{\sigma}{n}}$ → so if we sum all the frequencies we obtain 1.

Relative frequency Hystogram

$$\boxed{0 \leq \frac{\sigma}{n} \leq 1}$$

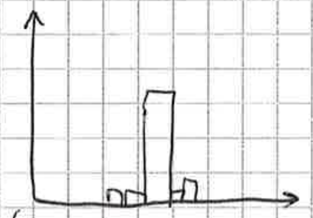


! Too many classes K



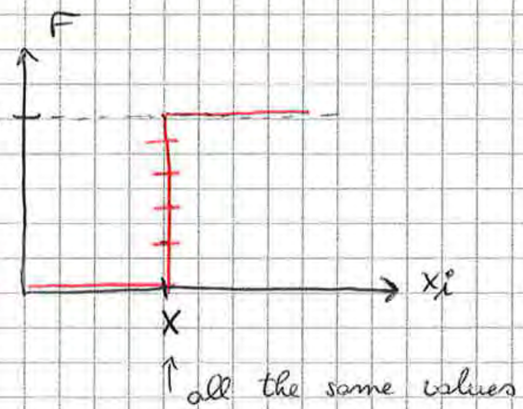
↳ dispersed

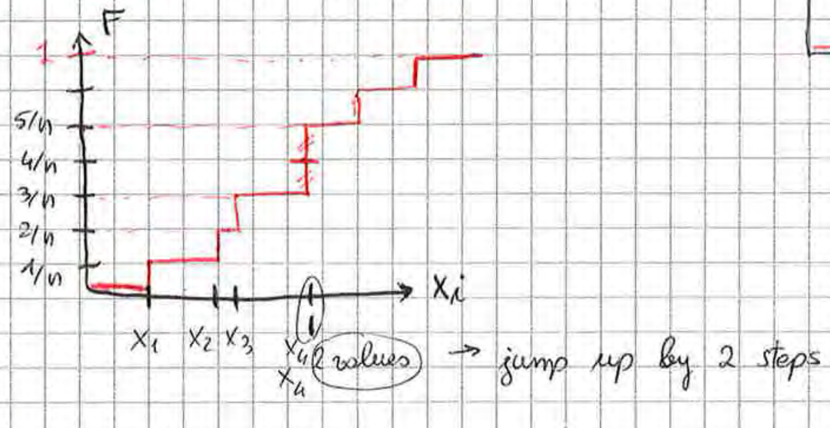
Too few classes K.



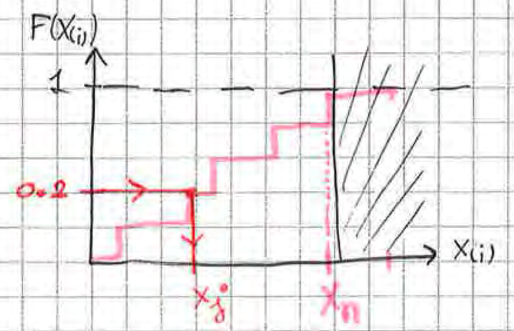
↳ not all data are rightly considered

" Se voglio che $P(X = X_j) = 0{,}02 = 2\%$, entro dalla $F(X_j) = 0{,}02$ fino alla curva e poi vado giù verso l'asse delle X_i; anche al contrario se voglio sapere la probabilità di x_j, vado dall'asse X_i alla $F(X_i)$. Oltre l'ultimo valore massimo X_n, non posso più dire né prevedere nulla!! (A1)





(A1) If want to know the value x_j corresponding to a given probability:
$P(X = X_j) = 0.02 = 2\%$; I have to entry from the value $F = 0{,}02$ on the ordinate axis and intersect the graph, then I have to go downwards and intersect the X_j value on the x axis. After the maximum value X_n I can't predict or suppose anything about the distribution.



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$$\bar{x}_w = \sum_{i=1}^{n} x_i \cdot \left(\frac{P_i}{P_{TOT}}\right)$$

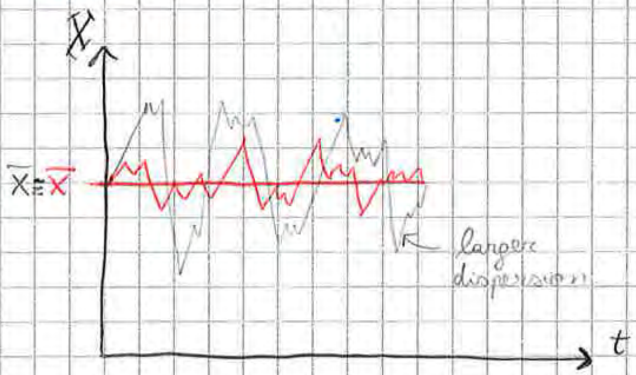
(River discharge)

P_i = population.

$P_{TOT} = \sum_{j=1}^{n} P_j$

$(x_i \cdot P_i)$ = number of undernourished people (only if I choose this case).
But if I choose a different type of weight (for ex. the size of the country S_i) it would be $(x_i \cdot S_i)$ which is $\neq$ from nr of undernourished

DISPERSION



- How to characterize two different samples, with same mean $\bar{x}$ but different dispersion?

because it's around the mean

2nd central moment / Standard deviation

$$\boxed{\sigma = s = \sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \frac{1}{n}}}$$

We want the dispersion around the mean
(I have this expression because I don't want that opposite values cancel so I loose information).

2nd (not central moment) $\boxed{\sqrt{\sum_{i=1}^{n} x_i^2 \cdot \frac{1}{n}}}$

→ If I want to compare 2 different objects with different scales, mean and dispersion, I can normalize in this way:
↓
Sample Coefficient of variation $\boxed{CV = \frac{s}{\bar{x}}}$

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We can quantify τ using probability.

If $p = P_{ex}(x \geq X_T)$ I can have success or failure.

$(\tau - 1)$ where the event didn't occure: $\begin{cases} (\tau-1) & (1-p) \quad \text{NOT OCCURENCE} \\ 1 & p \quad \text{OCCURENCE.} \end{cases}$

$$\boxed{(1-p)^{\tau-1}\, p^1}$$

PROBABILITY of an event that occures one time every τ years.

EXPECTED VALUE ≡ EXPECTED MEAN ≡ SAMPLE MEAN: $E(x)$:

$$\boxed{E(x) = \underbrace{\sum_{\tau=1}^{\infty} \tau \cdot \left[(1-p)^{\tau-1} \cdot p\right]}_{\text{weighted mean}}} =$$

$$= p \left[\underbrace{1 + 2(1-p) + 3\cdot(1-p)^2 + 4(1-p)^3 + \ldots}_{\text{POWER SERIES EXPANSION}}\right] =$$

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2}x^2 + \ldots$$

$$\Rightarrow E(x) = p\left[1-(1-p)\right]^{-2} \qquad x = -(1-p) \quad a = -2$$

$$\Rightarrow E(x) = p \cdot \frac{1}{p^2} = \frac{1}{p}$$

$\Rightarrow \boxed{E(x) = \frac{1}{p}}$ = RETURN PERIOD T, which is the inverse of the probability of the occurence of the event.

$$\Rightarrow \boxed{T = \frac{1}{P_{ex}(x \geq X_T)}}$$

- River banks: $P_{ex} = 0{,}01 \Rightarrow T = \frac{1}{0{,}01} = 100$ years
- Dams $P_{ex} = 0{,}0001 \Rightarrow T = \frac{1}{0{,}0001} = 10\,000$ years
- Drainage syst. $P_{ex} = 0{,}1 \Rightarrow T = \frac{1}{0{,}1} = 10$ years.

GOAL: FIND THE DESIGN VALUE, OR A VALUE CORRESPONDING TO A GIVEN PROBABILITY

TOOLS: SAMPLE (= limited information)

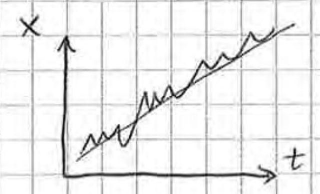
STEPS: From a sample to a whole population (= ∞ info)
Draw the info about the design value X_{DESIGN} (PROBABILITY)

$\Downarrow$

USE THE STATISTICAL INFERENCE

1° STEP: Description of data of the sample. (plot time series, frequency hystograms, particular points, outlayers ≡ ≡ spourious data, dispersion, mean, peculiar behaviour: if $\exists$ a trend., if $\exists$ deterministic parts, ...)

ex



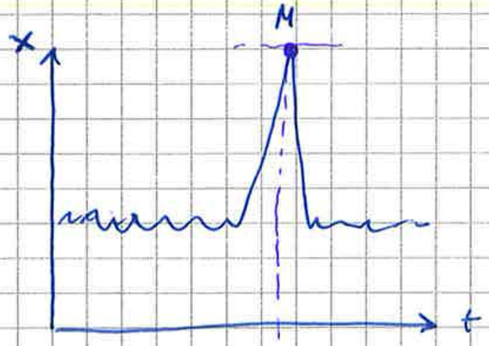
NON STATIONARY SERIES

2 5

The 2 parts, (1) and (2), are from different population. ⇒ what to do?

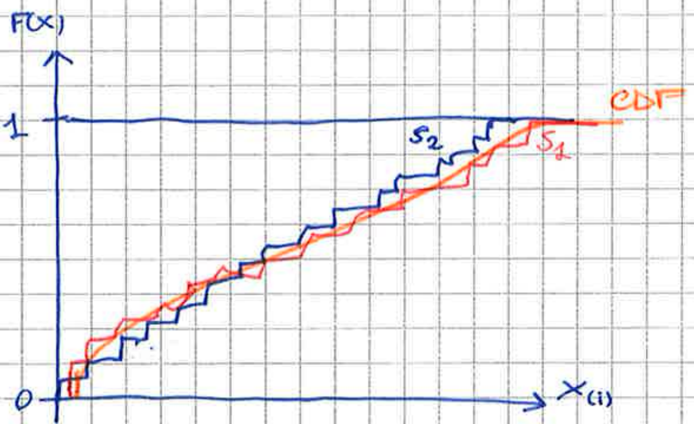
- If we have a longer part but older we choose the part which is the most recent data because it describes the current situation.

OUT LAYERS



M can be the most important point (maximum flow) or an error
↳ check rainfall data
↳ check also in other sections of the river

STEP (2) CHOICE OF THE PROBABILITY DISTRIBUTION



! when we have a SORTED SAMPLE the pedix is in brackets $x_{(i)}$

S_1 and S_2 can be from the same population or not.

If we increase the number of data, we have smaller and thinner steps ⇒ if $n \to \infty$ we will have a line called CUMULATIVE DISTRIBUTION FUNCTION (CDF)

$P(x) = \lim_{n\to\infty} F(x_i)$

sample	population
n	$n \to \infty$
$F(X_{(i)})$	$\lim_{n\to\infty} F(X_{(i)}) = P(X)$

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⇒ • Quantile function: $\boxed{x = [\ln(1-p)] \cdot (-\theta)}$

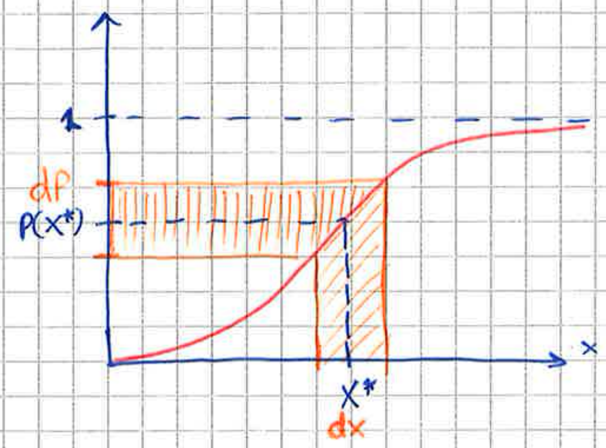
• Return period: 100 years

• $P_{ex} = \frac{1}{100} = 0{,}01$

• if $P = 0{,}99$

⇒ $x = -\theta \left[\ln(1 - 0{,}99) \right] = 4{,}6 \cdot \theta$ Design value

EX)



• Which is the probability of x*?
$P(x^*) = 0$ because if I have a CONTINUOUS SAMPLE, the probability of picking the exact value x^* is ≈ 0.
(but it's kind of paradox).

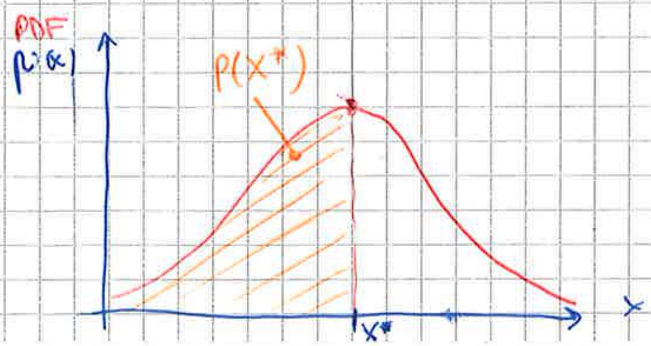
⇒ So it's a better-posed question to ask the probability of an INTERVAL AROUND that value

$$\boxed{\frac{dP(x)}{dx} = p(x)}$$ PROBABILITY DENSITY FUNCTION (PDF)

CDF = $\int$ PDF
(CDF)' = PDF

$p(x) = P'(x)$

$$\int_{x_{min}}^{x^*} dP(x) = \int_{x_{min}}^{x^*} p(x)\, dx \Rightarrow \boxed{P(x^*) = \int_{x_{min}}^{x^*} p(x)\, dx}$$

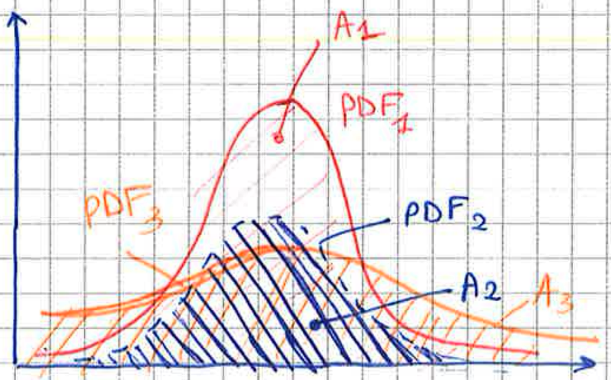


DIMENSIONS

- F, P NON DIMENSIONAL
- f, p $\frac{1}{[X]} = [X]^{-1}$ (look at its formula).

ex: X = river flow $[m^3/s]$

$\Rightarrow [p(x)] = [s/m^3]$



- If x has PDF_1 it can't have PDF_2 because the area under the curve must be = 1:

$A_2 < A_1 = 1$

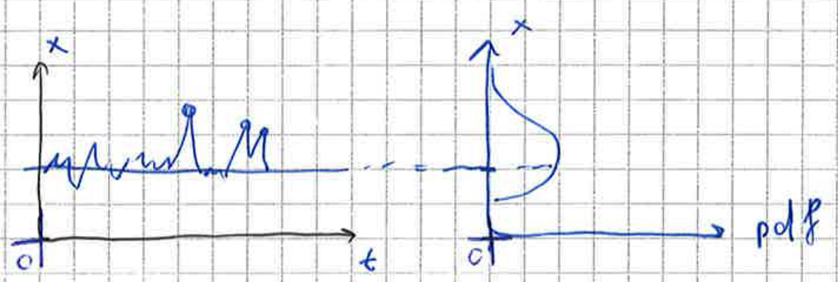
But if I have PDF_3 = it's ok because it has a different shape

→ Choosing a probability distribution needs 2 steps:

1) Identify the mathematical model (shape)
2) Estimate the parameters $P(x) = g(x, \sigma_1, \sigma_2, \sigma_3, ...)$

→ FOR HYDROLOGIC DATA:

- sample size is small ($20 \div 100 = n$) ⇒ FEW PARAMETERS.
- domain is POSITIVE
- samples are asymmetric



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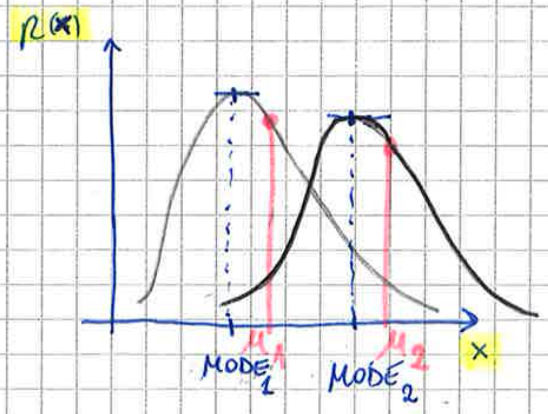
CHARACTERISTICS OF DISTRIBUTIONS

- DISTRIBUTION MEAN (EXPECTED VALUE) ≡ POPULATION MEAN.

$$\boxed{\mu = \int_{x_{min}}^{x_{max}} x \cdot \underbrace{p(x)}_{PDF}\, dx}$$

is the weight that we apply to any possible variable x



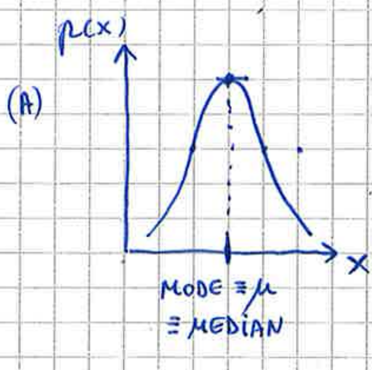
$\mu_2 > \mu_1$

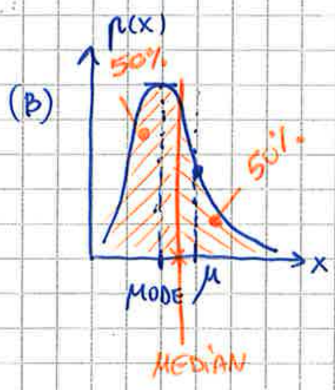
! x_{min} and x_{max} are the limits of the domain (that are for ex. 0 and ∞ or −∞ and +∞)

- MODE : value of x corresponding to the maximum value of PDF

(A) $\mu \equiv$ MODE for symmetrical distribution.

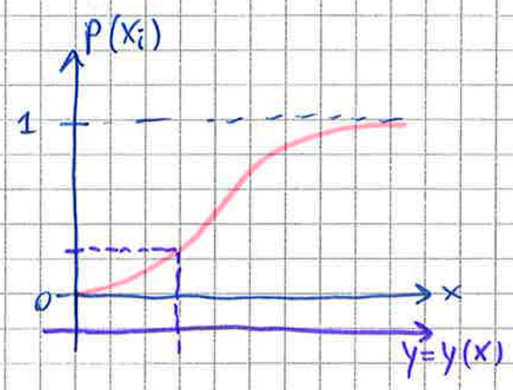
(B) $\mu >$ MODE for NON-symmetrical distrib. (positive)





- MEDIAN : value that is greater than 50% of values ≡ 50% quantile
  → Divide the area under the PDF into 2 equal parts

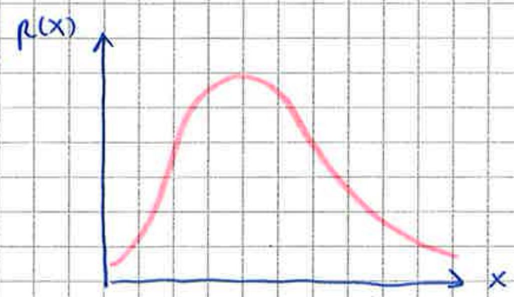
① Find CDF



⇒ Change variable $P(y) = P(x)\big|_{x=x(y)}$ it's easy for the CDF

② Find PDF

$(CDF)' = (P(x))' = [P(x(y))]'$



PDF $=(CDF)'$ ⇒ I have the derivative of a composition of functions

$$\boxed{p(y) = p(x)\Big|_{x=x(y)} \cdot \frac{dx(y)}{dy}}$$

derivative of the INVERSE of y:

$(y(x))^{-1} = x(y)$

$\equiv$

DERIVATIVE OF THE QUANTILE FUNCTION.

Ex EXPONENTIAL DISTRIBUTION

PDF $p(x) = \frac{1}{\theta} \cdot e^{-x/\theta}$

- Derived variable $y = a + bx$

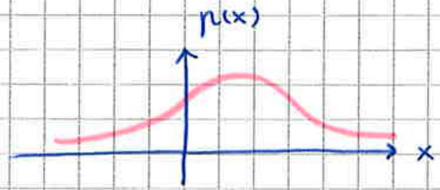
  ↳ inverse $x(y) = \frac{y-a}{b}$

⇒ $p(y) = \frac{1}{\theta} \cdot e^{-\frac{y-a}{b\theta}} \cdot \boxed{\frac{1}{b}} = \frac{dx(y)}{dy}$

$\frac{1}{\theta} \cdot e^{-\frac{y-a}{b\theta}} = p(x)\Big|_{x=x(y)}$ → substitution of $x(y)$ in the PDF

② GAUSSIAN DISTRIBUTION

PDF
$$\boxed{p(x) = \frac{1}{\vartheta_2 \sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\vartheta_1}{\vartheta_2}\right)^2}}$$



- Parameters = 2 → ϑ_1 = expresses position; → ϑ_2 = " dispersion (= scale)
- Mean = $\mu = \int_{-\infty}^{+\infty} x \cdot p(x)\, dx = \vartheta_1$
- Std deviation & Variance $\sigma^2 = \int_{-\infty}^{+\infty} (x-\vartheta_1)^2 p(x)\, dx = \vartheta_2^2$

$$\boxed{\begin{cases} \mu = \vartheta_1 \\ \sigma = \vartheta_2 \end{cases}}$$

! The Gaussian distrib is not usefull for hydrological data:

WHAT WE HAVE
- limited nr of data
- positive values → no need of the negative region
- no simmetry in precipitations, flows, ...

⇒ We use another distribution:

③ LOG - NORMAL DISTRIBUTION

Q = river flow

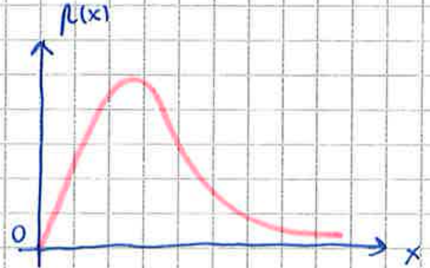
$\boxed{x = \ln(Q)}$ → impose this relationship from the beginning; → MONOTONIC

→ p(x) is a normal distribution if Q is log-normal

→ The log-normal is derived from the normal:

PDF →
$$\boxed{p(x) = p(\ln Q) = \frac{1}{\vartheta_2 \sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{\ln Q - \vartheta_1}{\vartheta_2}\right)^2} \cdot \frac{1}{Q}}$$

- Parameters = 2 → ϑ_1 and ϑ_2
- Domain = $[0; +\infty]$
- Non simmetric $\gamma = CA > 0$



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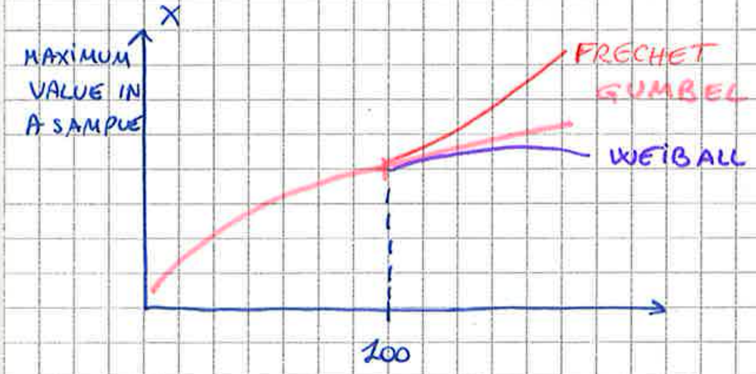
⑤ GEV DISTRIBUTION (Generalized Extreme Value)

CDF $$\boxed{P(x) = \exp\left[-\left(1-\frac{\sigma_3}{\sigma_2}(x-\sigma_1)\right)^{1/\sigma_3}\right]}$$

- parameters = 3 → σ_1, σ_2, σ_3

  - ! Don't overfit. → GEV is used only if the sample size is large at least 50 : $\boxed{n > 50}$

! Related to EXTREME EVENTS and the TIME THAT PASSES between extreme events.



PROBABILITY PLOT

Graphical tool to assess how well a family distribution fits a sample of data. It doesn't depend on the parameter values.

Q - Q PLOT, SPECIAL CASE
QUANTILE - QUANTILE

$F(X_i)_{max} = \frac{n}{n+1}$ The probability of the max value is very large but not = 1 = 100%

1) SORT THE SAMPLE $(X_{(i)})$

2) COMPUTE THE EMPIRICAL DISTRIBUTION FUNCTION $\boxed{F(X_i) = \frac{i}{n+1}}$ PP — IMPLIES THAT THE MAX VALUE OF F(X) IS NOT 1

   ! CDF $F(x) = \frac{i}{n}$ → $F(X_i)_{max} = 1$

3) COMPUTE REDUCED VARIATE U_i

4) PLOT POINTS $(X_{(i)}, U_i)$

$\Rightarrow$ $\boxed{U_i = \phi^{-1}(F) = \frac{\ln(x) - \sigma_1}{\sigma_2}}$ ϕ^{-1} = quantile function / inverse of std normal distrib.

! Std normal distrib $\begin{cases} \mu = 0 \\ \sigma = 1 \Leftrightarrow \sigma^2 = 1 \end{cases}$

$\Rightarrow$ If I see that the points aren't alligned $\Rightarrow$ I can discard that family of distributions

~~ END OF STEP 2 ~~~~~~~~~~~~

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EX EXP. DISTRIB: $\left[\mu = \sigma \right.$, $\bar{x} = \text{number}$ $\Rightarrow \sigma = \text{number}$.

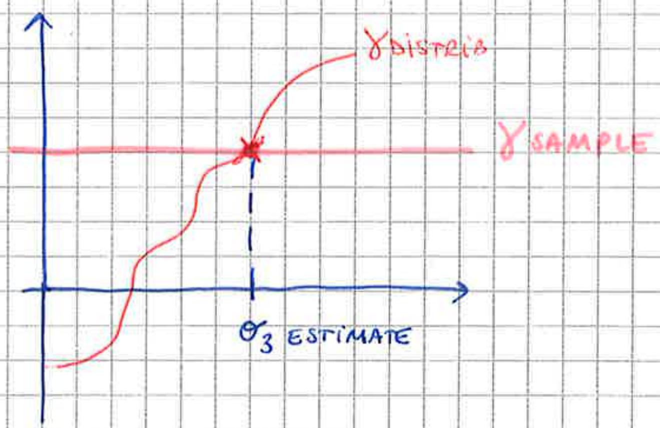
GUMBEL DISTRIB: $\boxed{\begin{aligned}\mu &= \theta_1 + 0.5772\,\theta_2 = \bar{x}\\ \sigma^2 &= \frac{\pi^2}{6}\cdot\theta_2^2 = s^2\end{aligned}}$

$$\Rightarrow \left[\begin{aligned} \theta_2^2 &= \frac{6s^2}{\pi^2} \rightarrow \theta_2 = \frac{\sqrt{6}\,s}{\pi} \\ \theta_1 &= \bar{x} - 0.5772\cdot\frac{\sqrt{6}}{\pi}\cdot s \end{aligned}\right.$$

GEV DISTRIB: $\left[\begin{aligned} &\mu(\theta_1, \theta_2, \theta_3) = \bar{x} \\ &\sigma^2(\theta_1, \theta_2, \theta_3) = s^2 \\ &\gamma_{DISTR}(\theta_3) = \gamma_{SAMPLE} \end{aligned}\right\}$ implicit $\rightarrow$ useless

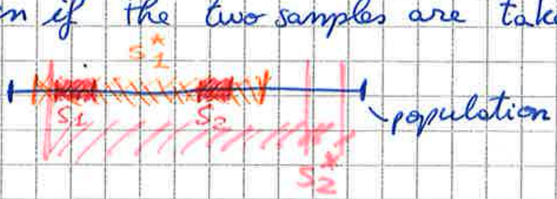
$\theta_1 = \dots$
$\theta_2 = \dots$
$\theta_3 = \dots$

$\Rightarrow$ I have to build a function



PRO & CONS

- The most limited is the size of the sample, the most variability I have between two samples even if the two samples are taken from the same population:



$S_1 \neq S_2$

$S_1^* \approx S_2^*$

$\Rightarrow$ $m \geq 20$.

$$\boxed{b_2 = \frac{1}{m} \cdot \sum_{i=1}^{m} \frac{(i-1)(i-2)}{(m-1)(m-2)} \cdot X(i)}$$

Giov 12/10/17
Assignment_1

$L_1 = \ell_1 \qquad \ell_1 = b_0$

$L_2 = \ell_2 \qquad \ell_2 = 2b_1 - b_0$

$L_3 = \ell_3 \qquad \ell_3 = 6b_2 - 6b_1 + b_0$

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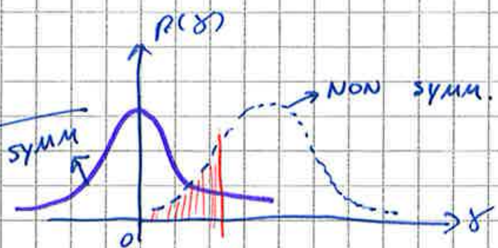
→ Rejection of H_0 doesn't (generally) imply the acceptance of H_1.

	H_0 TRUE	H_0 FALSE
POSITIVE TEST H_0 ACCEPTED	CORRECT ANSWER	ERROR OF TYPE 2
NEGATIVE TEST H_0 REJECTED	ERROR OF TYPE 1	CORRECT ANSWER



↳ I find CA ≡ γ that belongs to the symm and NON SYMM. DISTRIB.

→ STEP 4: of STATISTICAL INFERENCE

GOODNESS - OF - FIT TESTS

¿ H_0 : is THE SAMPLE DRAWN FROM THIS GIVEN DISTRIBUTION WITH THESE GIVEN PARAMETER VALUES ?

CHI - SQUARED TEST ≡ PEARSON TEST

Comparison between observed and theoretical frequences.

↳ ① SUBDIVIDE THE SAMPLE IN CLASSES. → K CLASSES

$$K = 2 \cdot m^{0.4} \quad \text{(approx to the nearest integer)}$$

↳ ② TEST STATISTICS

$$\boxed{X^2 = \sum_{\underset{\text{CLASS}=1}{i=1}}^{K} \frac{(O_i - E_i)^2}{E_i}}$$

O_i = number of observed elements in the i-TH class

E_i = number of the expeted frequency in the i-TH class if H_0 is true (means that the sample is originated from that distribution).

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- Can we discard values around 0? Because if $X^2 = 0$ means that the distribution is very closed to data, which is good and means that I have choosen the best type of distribution.

The LIMIT X^2_{LIM} is set by knowing 1 parameter and α:

DEGREE OF FREEDOM

$$\Rightarrow \quad X^2_{LIM} = X^2_{LIM}\left(\overbrace{k - m_p - 1}^{\text{DEGREE OF FREEDOM}} ; \alpha\right)$$

k = nr of classes

m_p = nr of parameters of the distribution that I'm fitting

$$\boxed{X^2_{LIM}(k - m_p - 1; \alpha)}$$

$\Rightarrow$ I find 2 cases:

- $X^2 \leq X^2_{LIM}(k - m_p - 1; \alpha)$ → TEST is POSITIVE ⇒ H_0 is ACCEPTET ⇒ SAMPLE IS DRAWN FROM THE SUPPOSED DISTRIB.
- $X^2 > X^2_{LIM}(k - m_p - 1; \alpha)$ → TEST IS NEGATIVE ⇒ H_0 is REJECTED AT THE α % OF SIGNIFICANCE

"Bisogna specificare sempre l'α% di significance in caso di test positivo o negativo, perché usando un α% diverso, lo stesso test può avere un altro risultato."

- What changes if we change the distribution to fit?

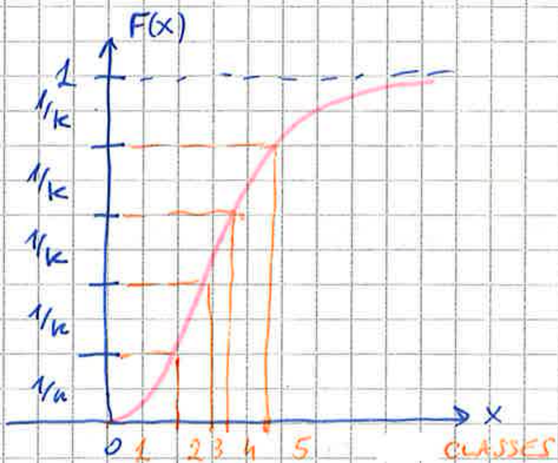
$$\boxed{q_i = \frac{1}{k}} \qquad q_i = \frac{i}{k} \quad i = 1, \dots, k$$

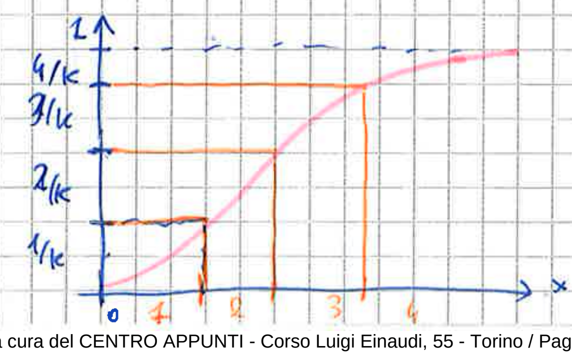
Different distributions lead to different CLASS LIMITS

↓

Defined through the QUANTILE FUNCTION

$x^{-1}(P)$





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4) Class limits and find x_i $(i = 1, 2, 3, 4)$

CLASS	LIMITS	O_i	E_i
1	$0 \div x_1$	0	1.6
2	$x_1 \div x_2$	4	1.6
3	$x_2 \div x_3$	2	1.6
4	$x_3 \div x_4$	1	1.6
5	$x_4 \div +\infty$	1	1.6
		m	

$x_1 = x^{-1}\left(\frac{1}{5}\right) = 2.23$

$x_2 = x^{-1}\left(\frac{2}{5}\right) = 5.11$

$x_3 = x^{-1}\left(\frac{3}{5}\right) = 9.16$

$x_4 = x^{-1}\left(\frac{4}{5}\right) = 16.1$

5) Count how many elements of the sample fall into each of these ranges.

↳ $\sum O_i = m$ "La somma di O_i deve essere uguale a m"

6) Determine E_i and χ^2

$$E_i = \frac{m}{k} = 1.6 \text{ in all classes}$$

$$\chi^2 = \frac{(0-1.6)^2}{1.6} + \frac{(4-1.6)^2}{1.6} + \frac{(2-1.6)^2}{1.6} + \frac{(1-1.6)^2}{1.6} + \frac{(1-1.6)^2}{1.6} = 5.75$$

7) Find χ^2_{LIM} and compare χ^2 with χ^2_{LIM}

$\chi^2_{LIM}(5-1-1\ ;\ \alpha = 5\%) = \chi^2_{LIM}(3; 0{,}05) = 7.81$ → Found from TABLES.

$\Rightarrow$ $\chi^2 \leq \chi^2_{LIM}$ TEST is POSITIVE

$5.75 < 7.81$ $\Rightarrow$ Ho is accepted at a 5% of significance

→ χ^2 is a test that is usually passed for many distrib. but we need a more selective test.

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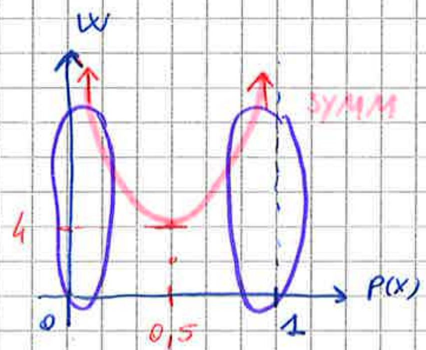
ANDERSON - DARLING

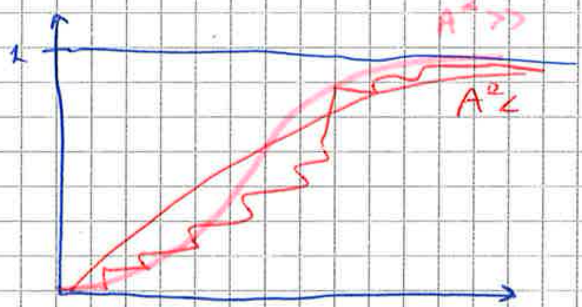
$$A^2 = m \int_{x_{min}}^{x_{max}} \frac{\overbrace{\left[P(x) - F(x)\right]^2}^{\text{measure of the distance squared}}}{P(x) \cdot \left[1 - P(x)\right]} P(x)\, dx$$

$\left[P(x) - F(x)\right]$ distance metric



$$w = \frac{1}{P(x)\left[1 - P(x)\right]} \quad \text{weight}$$

It's a weight function that amplifies the distance on the tails, because on the tails we can see how well the distribution captures the data.



→ FROM THE PREVIOUS FORMULA WE CAN OBTAIN A FINITE FUNCTION:

$$\boxed{A^2 = -m - \frac{1}{m} \cdot \sum_{i=1}^{m} \left\{ (2i - 1) \ln\left[P(x_i)\right] + (2m + 1 - 2i) \cdot \ln\left[1 - (P(x_i))\right] \right\}}$$

x_i = element in sorted sample

$P(x_i)$ = probability given by the tested distribution for x_i

$A^2 = 0 \div 4$

→ But find also A^2_{LIM} ⇒
- $A^2 \leq A^2_{LIM}$ { TEST POSITIVE, H_0 ACCEPTED
- $A^2 > A^2_{LIM}$ { TEST NEGATIVE, H_0 REJECTED

41

STEP 5) DEFINITION OF DESIGN VALUE.

CASE 0: NO DISTRIBUTIONS PASS BOTH TESTS

⇒ Change the distribution: maybe ∃ distrib that we didn't consider (ex: γ-distrib; Pierson-distrib; ....).

CASE 1: 1 DISTRIBUTION PASSING THE TESTS

⇒ Compute the value of x corresponding to a given return period ex. $T = 100$ years. ⇒ find the associated probability

$\underset{\text{NON EXEEDANCE}}{P(X)} = 1 - \frac{1}{T}$ ⇒ $\underset{\text{NON EXEED}}{X^{-1}(P)}$ using the model tested

$\underset{\text{OCCURENCE}}{P(X)} = \frac{1}{T}$

CASE 2: MORE DISTRIBUTIONS PASS THE TESTS

⇒ $X^*_{GUMBEL} = 1300\ m^3/s$

$X^*_{LOG-NORMAL} = 900\ m^3/s$ $\qquad X^* = X^{-1}(P)$

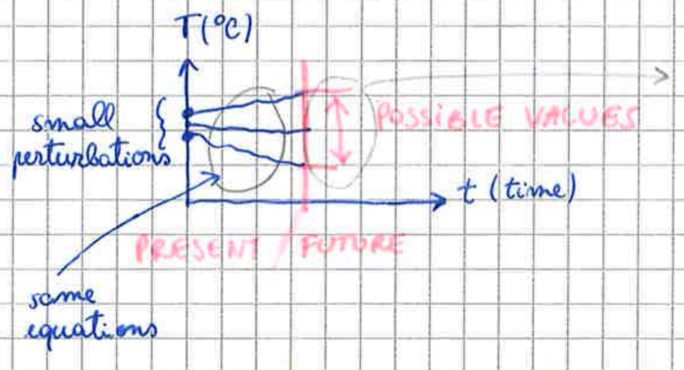
The values are extreme values because they're associated to a return period of $T = 100$ years.

⇒ Which one do we choose? ⇒ Take the mean:

$$X^*_{MEAN} = \frac{X_1^* + X_2^* + X_3^* + \ldots + X_m}{m_{values}}$$

Because we have to consider safety and costs

→ it's also a STATISTICAL TECHNIQUES: MODEL AVERAGING.



→ different values

⇒ Take the MEAN TO PREDICT FUTURE

⇒ The variability of X^*_{MEAN} must be given because for $X^*_{MEAN} = 1100\ \frac{m^3}{s}$: $1300 \div 900 \neq 1115 \div 1085$.

END OF STEP 5

i	Sample element sorted	$\frac{i-1}{n-1}$	$\frac{(i-1)(i-2)}{(n-1)(n-2)}$
$i=1$	x_1	0	0
$i=2$	x_2	$\frac{1}{n-1}$	0
$i=3$	x_3	$\frac{2}{n-1}$	$\frac{2\cdot 1}{(n-1)(n-2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

↓ growing weights

⇒ Finally I can calculate the sample L-moment

$$l_1 = b_0 = \bar{x} \qquad\qquad L_1(\theta_1, \theta_2, \theta_3) = l_1 \quad \text{] 1 param}$$

$$l_2 = 2b_1 - b_0 \quad \Rightarrow \quad L_2(\theta_1, \theta_2, \theta_3) = l_2 \quad \text{] 2 param}$$

$$l_3 = 6b_2 - 6b_1 + b_0 \qquad L_3(\theta_1, \theta_2, \theta_3) = l_3 \quad \text{] 3 param}$$

System of equations to work out the values $\theta_1, \theta_2, \theta_3$. The number of equations depend on how many parameters needs a chosen function.

$\boxed{\text{Ex}}$

$$P(x) = e^{-e^{-\frac{x-\theta_1}{\theta_2}}}$$

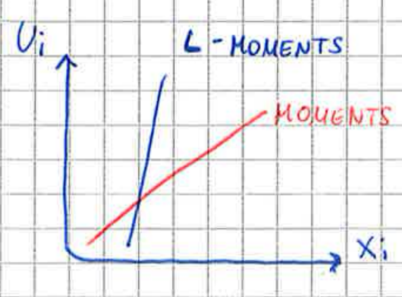
$$L_1 = \int_{x_{min}}^{x_{max}} x \cdot p(x)\, dx = \theta_1 + \gamma_E \theta_2 = \mu$$

$l_1 = \bar{x}$ mean of the sample

$$L_2 = 2\int_{x_{min}}^{x_{max}} x\, p(x)\, P(x)^1\, dx - \int_{x_{min}}^{x_{max}} x \cdot p(x)\, dx = \theta_2 \cdot \ln 2$$

$l_2 =$ number

$$\Rightarrow \text{SYSTEM} \begin{cases} \theta_1 + \gamma_E \theta_2 = \bar{x} \\ \theta_2 \cdot \ln 2 = l_2 \end{cases} \Rightarrow \begin{cases} \theta_2 = \frac{l_2}{\ln 2} \\ \theta_1 = \bar{x} - \gamma_E \theta_2 \end{cases} \qquad \gamma_E = 0.5772$$



23/10/2017

PART 2

DISTRIBUTION OF WATER ON EARTH

WATER IN THE ATMOSPHERE is IN VOLUME

DRY
- 78 % NITROGEN
- 21 % OXIGEN
- 0.9 % ARGON
- 0.04 % CARBON DIOXIDE

0.001% ÷ 5 % WATER VAPOUR very eterogeneous and 0.25% in MASS.

Temperature influences the water amount in the air.

1st PRINCIPLE OF THERMODYNAMICS.

$$\boxed{c_p \cdot dT = dq + v \cdot dP}$$

c_p = specific heat at constant pressure $\left[1000 \frac{J}{kg\,K}\right]$

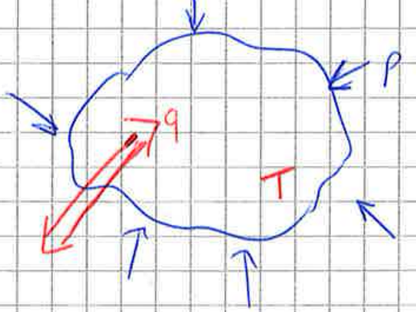
dT = temperature variation $[K]$

dq = amount of energy ≡ heat exchanged $\left[\frac{J}{kg}\right]$

v = specific volume $= \frac{1}{\rho}$ $\left[\frac{m^3}{kg}\right]$

dP = variation of pressure $[Pa] = \left[\frac{N}{m^2}\right]$

⇒ The eq is applied to a given volume of air (a system), submitted to pressure, exchanging heat (provided or outdrawn).



$c_p\, dT$ = internal energy

$V dP$ = work done by the system.

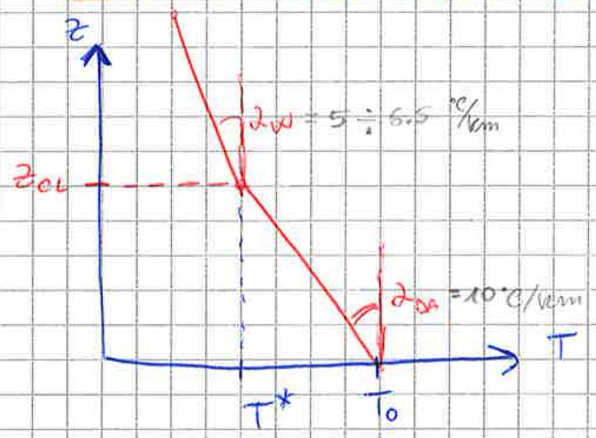
This result is for DRY CONDITIONS, water vapour is not considered.

Let's see what happens after the point of CONDENSATION. When condensation occures (phase passage from vapour to liquid). It's an EXOGENOUS transformation ⇒ it's not more adiabatic. ⇒ there is a release of heat :

$$c_p \, dT = dq - g\,dz$$

The value of dq depends on how much is the amount of vapour.

$$\boxed{\underbrace{\frac{dT}{dz} = -\frac{g}{c_p} + \text{POSITIVE QUANTITY}} \quad \text{saturated or wet lapse rate} \quad \alpha_w = -(5 \div 6.5)\ \frac{°C}{km} \quad |\alpha_w| < |\alpha_{DA}|}$$



∃ a first faster change of temperature with a rate of α_{DA}, but at a certain altitude, the change becomes slower with a rate of α_W.

Relative Humidity

It's a measure that quantifies the amount of water in the air

$$[/] = \frac{[Pa]}{[Pa]_{sat}}$$

$$\boxed{RH = \frac{e}{e_s} \qquad RH \in [0,1]}$$

e = water vapour pressure of air

e_s = saturated pressure of water vapour in the air

Saturated condition = the maximum capacity of air to hold water at that pressure and temperature, after which water condensates. $e_s = e_s(T)$

MIXING RATIO

$$w = \frac{\text{mass of water (vapour)}}{\text{mass of air}} \qquad \frac{[kg]_{H_2O}}{[kg]_{air}} = [/]$$

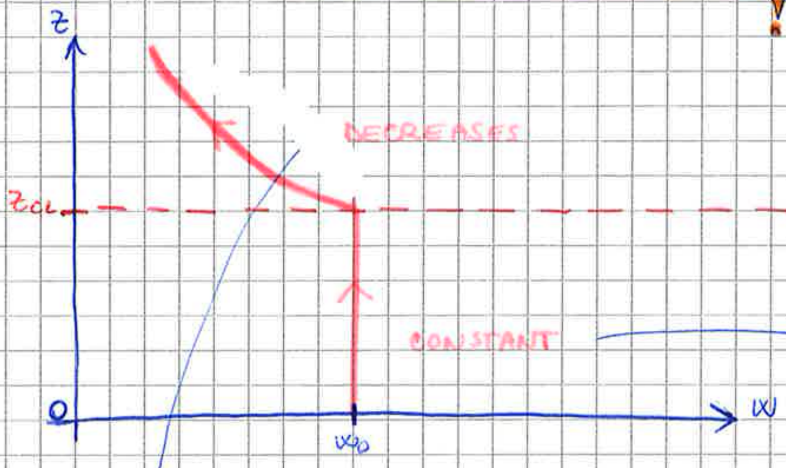
after applying the gas law

$$w = 0.622 \cdot \frac{e}{P_{atm}}$$

VAPOUR PRESSURE

e = vapour pressure ≡ partial pressure of the vapour.

! $\underbrace{P_{atm}}_{\text{mixed gas.}} = \sum_{i=1}^{m} P_i$ $\quad m$ = number of gases composing the air.



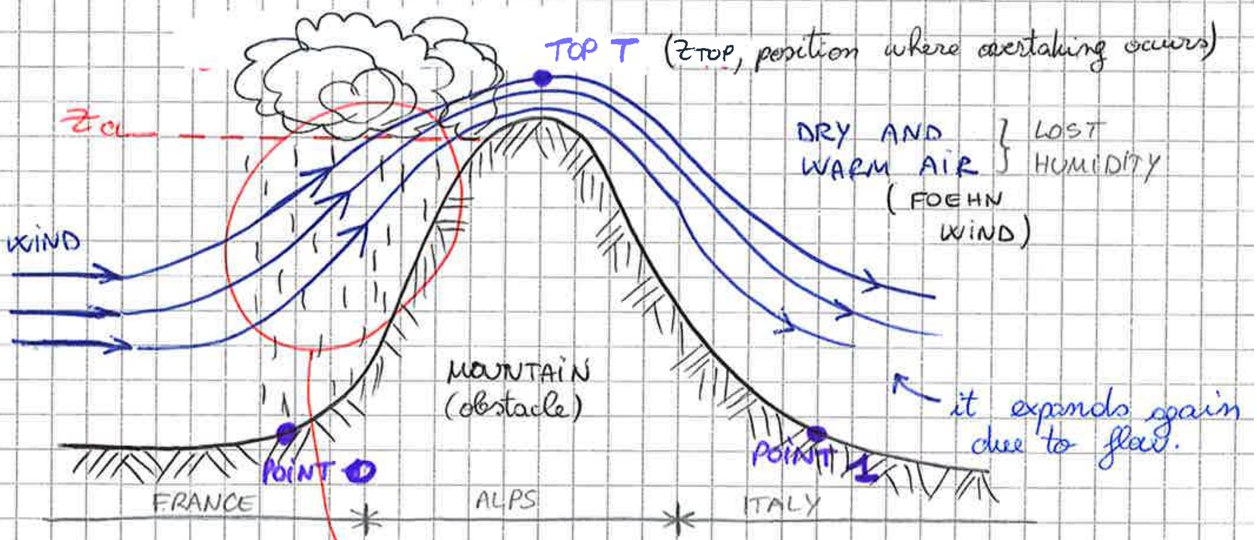
CONSTANT → Because ∄ condensation.
↓
w is about MASSES.
↓
Below z_{CL} is an adiabatic transformation.

Water condensates: becomes drops of liquid that start to move → loosing water

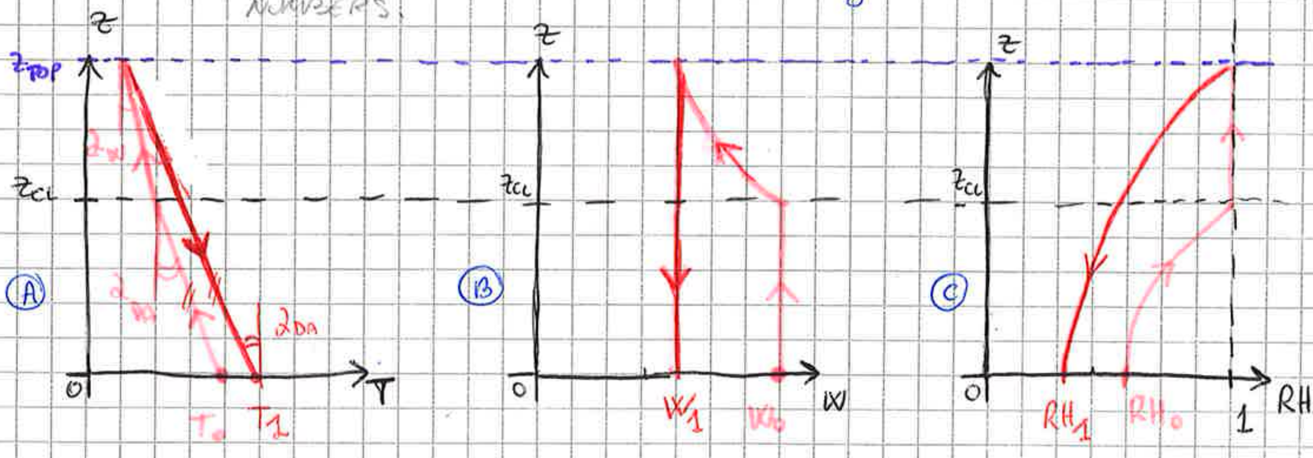
PRECIPITATION IS ALWAYS FORMED WHEN / WHERE AN UPWARD MOTION OF HUMID AIR OCCURS:

- OROGRAPHIC PRECIPITATION
- CONVECTIVE "
- CYCLONIC "

OROGRAPHIC PRECIPITATION



TRY TO DO DIAGRAMS WITH NUMBERS.

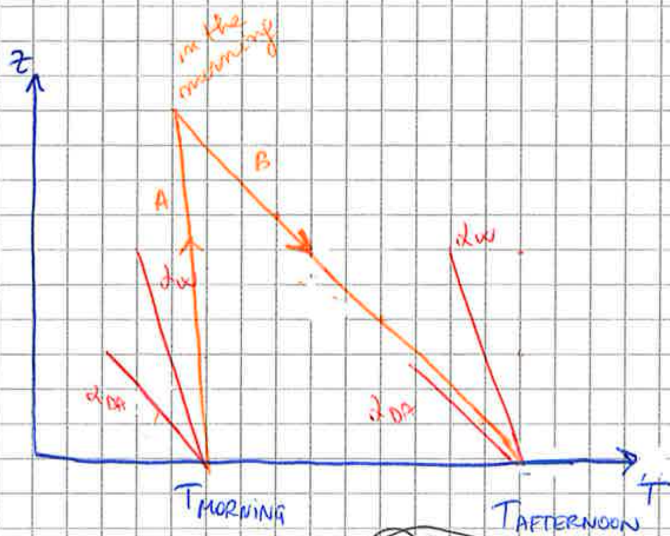


Ⓐ After the TOP T, a lot of water has been lost, it won't condense anymore, so we can say we're in an adiabatic condition again, so the temperature of air, which is going down again, increases following a line with α_{DA}.

Ⓑ After the top, the air still has a low water content, but the path is constant, so it's adiabatic.

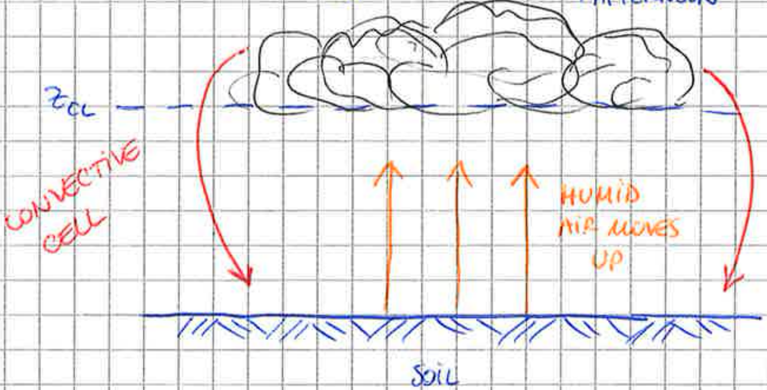
Ⓒ After the top, RH decreases while temperature increases.

INSTABILITY RELATED TO THE FORMATION OF STORMS (CONVECTIVE PRECIPITATIONS)



A: Morning stable

B Afternoon unstable



Since a lot of heat from the soil is released ⇒ a lot of air moves up and ∃ a lot of condensation ⇒ storms

Then colder air goes down

↓

After precipitation finishes and the humidity is not more sufficient for the condensation, the storm stops.

STORMS:

- INTENSITY IS VERY STRONG

  $> 20 \div 50 \ \frac{mm}{h}$

- DURATION depends on the time required by the convective cells to be exhausted

  $<$ FEW HOURS

- TOTAL VOLUMES (MAX) $100 \div 200 \ mm^3$

  In Torino annual volume is $\approx 700 \ mm^3$

- CRITICS FOR THE DRAINAGE SYSTEM.

Precipitation drops move in a turbolent way inside the cloud and hits other drops so they increase syze until the weight can overcome the power of the wind and then fall down.

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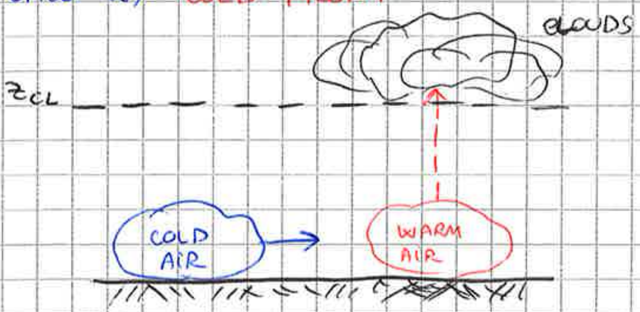
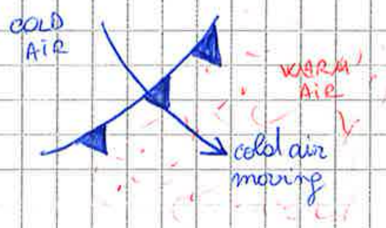
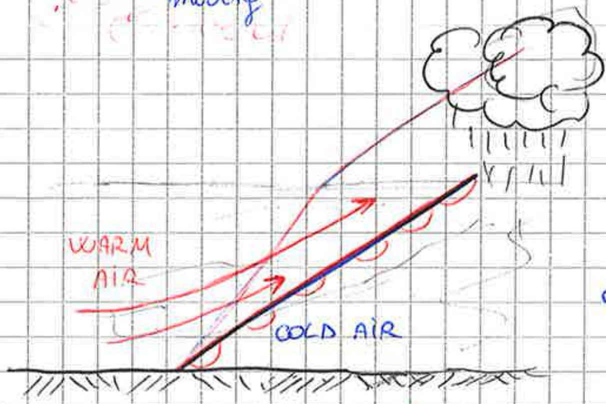
CASE 2) COLD FRONT

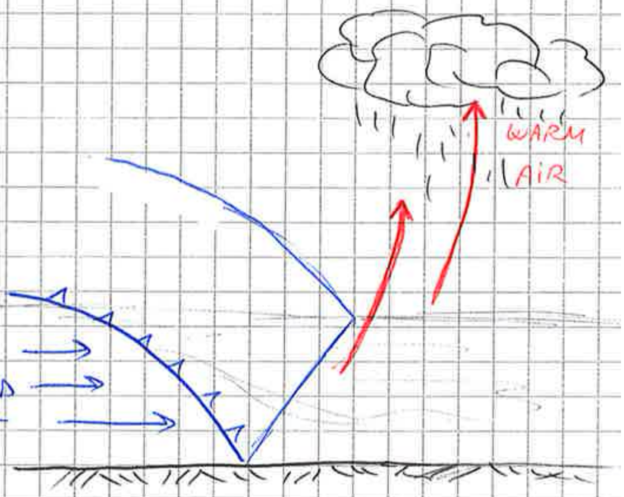


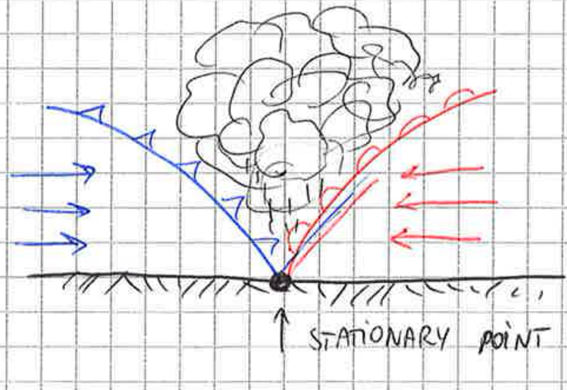
Cold front moves towards the warm air. The warm air moves up because it has less weight.

- INTERMEDIATE INTENSITY OF PRECIP $\left(5 \div 20 \frac{mm}{h}\right)$
- INTERMEDIATE DURATION (12 ÷ 24 H)









57

CLOUDS = formation of condensated water in the air. They form above a certain level (z_{CL}). It's formed by small droplets which diameter is $10 \div 30\ \mu m$. The density of droplets is $0.5 \div 1\ g/m^3$ for a maximum of $3\ g/m^3$.

$A_b = 1\ m^2$

AMOUNT OF WATER $\overbrace{\underbrace{3\ g/m^3 \cdot 4000\ m \cdot \underbrace{A_b}_{\text{per unit area}}}_{\text{MASS}} \cdot \frac{1}{1000\ kg/m^3}}^{\text{VOLUME}} = 12\ A_b\ g/Kg = 12 \cdot 10^{-3}\ m^3 = 12\ mm$ of RAINFALL DEPTH.

30/10/2017

3 PHASES OF PRECIPITATION FORMATION

1) CONDENSATION $T > 0°C$ FORMATION OF DROPLETS
$-40°C < T < 0°C$ " DROPLETS AND ICE CRYSTALS
$T < -40°C$ " ICE CRYSTALS

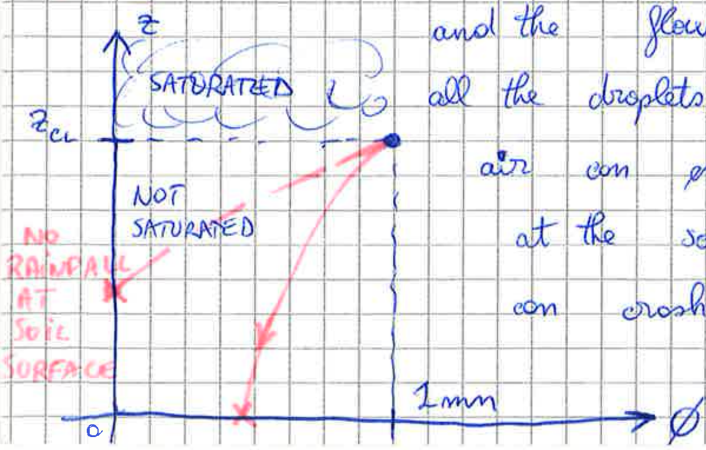
MAXIMUM DIMENSION OF DROPLETS : $100\ \mu m$.

Darker clouds are the tallest and absorb light by reflecting and refracting it much more than a white cloud, which is also less dense.

CONDENSATION NUCLEI (DUST, IONS, PARTICLES) WITH DIMENSION $10^{-3} \div 10\ \mu m$.
↳ without them condensation cannot start.

2) COALESCENCE : the droplets are small, and the turbolence brings them around the cloud so they meet each other and grow: DROPLETS GROWING BY HITTING OF DROPS. The air flow can be very slow: 0.5 cm/s but it can carry around the smallest droplets. The FINAL DIMENSION is $0.5 \div 2$ mm IN DIAMETER. The coalescence process can be very long.

3) FALL THROUGH THE CLOUD BASE : when the weight is sufficient and the flow can't carry them anymore. Not all the droplets reach the soil: the unsaturated air can evaporate them totally and ∄ rainfall at the soil. Also very turbolent air flow can crash the biggest droplets.

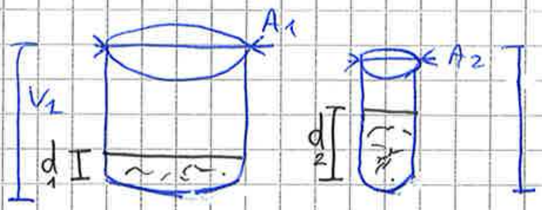


RAINFALL MEASUREMENTS

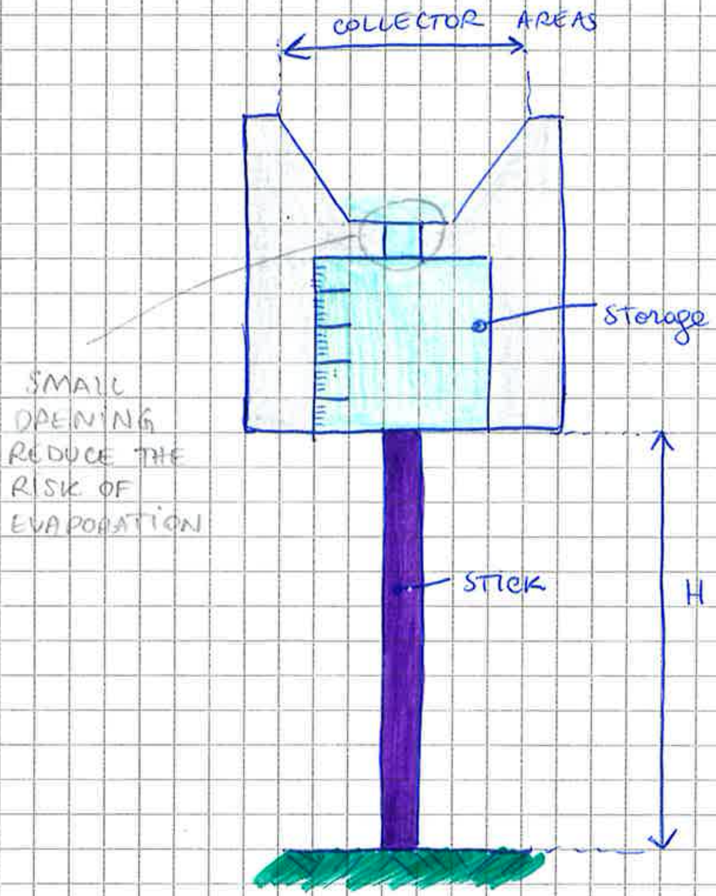
RAINFALL (precipitation) DEPTH = $\frac{\text{VOLUME}}{\text{AREA}}$

↓ $\boxed{d = \frac{V}{A}}$

PLUVIOMETER



$d_1 = d_2$ because of that ratio.



$\phi = 20$ cm WEATHER BUREAUX

$\phi = 37$ cm ITALY

H is important because if it's too low it can be disturbed by people/animals/vegetation, ... but it must be also near the soil:

$H = 170$ cm.

- NON - RECORDING GAUGES : someone must go and read it in time:
  - STANDARD : ONCE A DAY
  - STORAGE : ONCE A MONTH OR SEASON : in area difficult to reach
- RECORDING GAUGES
  - WEIGHTING TYPE (after weighting the bucket, empties it).
  - FLOAT TYPE (reads the DEPTH)
  - TIPPING BUCKET TYPE. (reads the tipping TIMES).

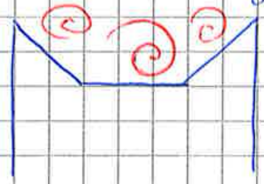
↳ One can draw a continuous curve of precipitation.

→ WIND can affect the measurements:

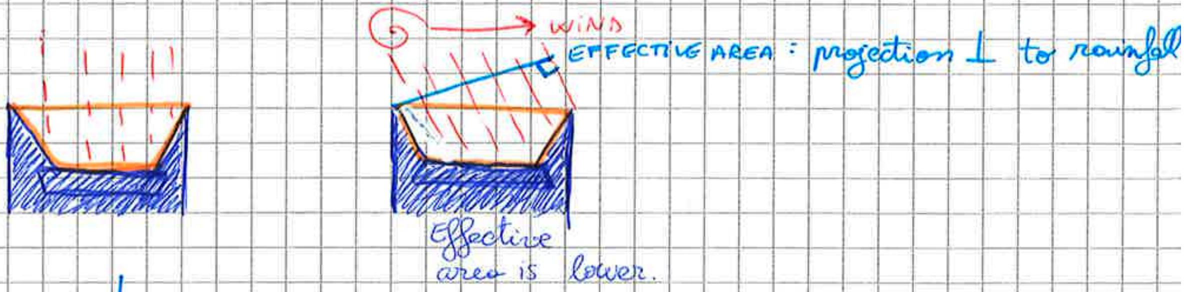
- creates turbolence → vortex inside the gauge → under measurement

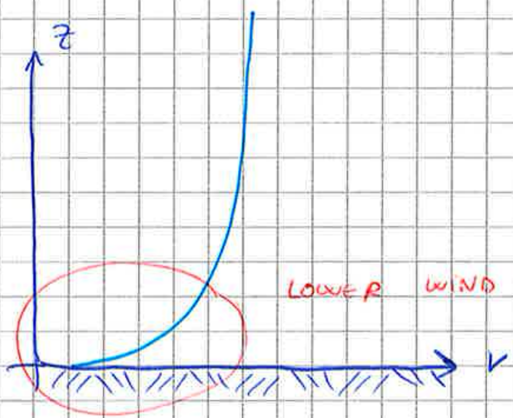
UNDER CATCHING

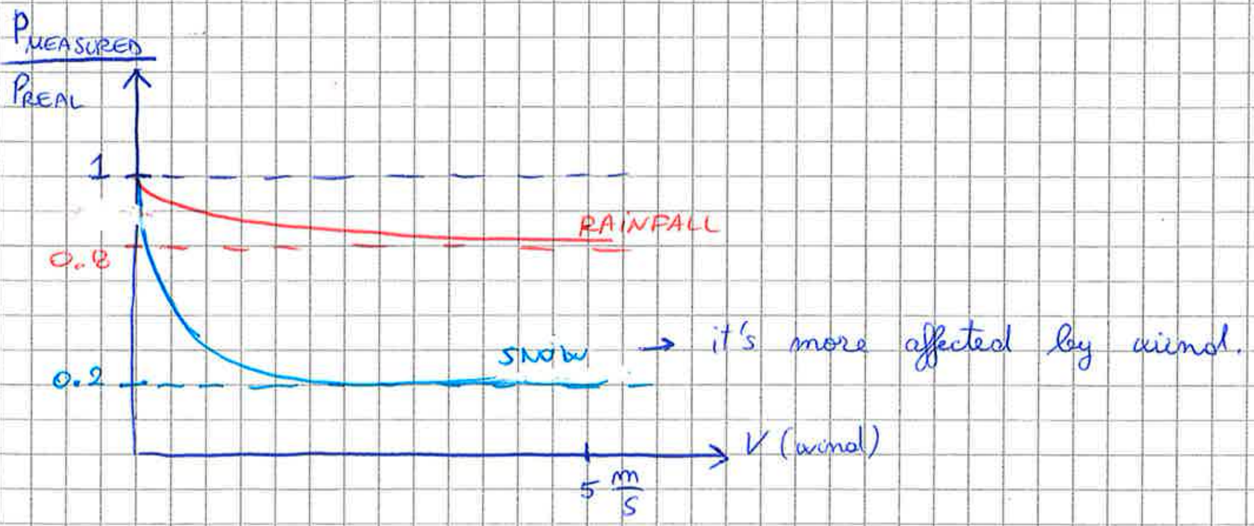
Deviation of droplets due to turbolence



- the collecting area changes according to the direction of wind







FOR THE SNOW WE USE THE HEATED PLUVIOMETER. that measures the snow water equivalent.

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31/10/17

TIME	h [mm]	max h_d = 1 hour	max h_d = 3 hours
7:15	1	–	–
7:30	2	–	–
7:45	2	–	–
8:00	4	9	–
8:15	3	11	–
8:30	5	14	–
8:45	6	18	–
9:0	4	18	–
9:15	5	20	–
9:30	7	22	–
9:45	5	21	–
10:00	3	20	47
10:15	4	19	50
10:30	1	13	49
10:45	1	9	48

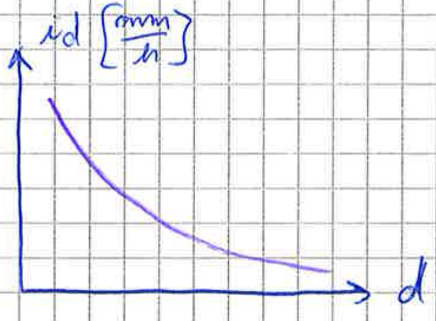
Duration of event : 3.75 h

h_d = 1 hour = 22 mm

h_d = 3 hours = 50 mm

i_d = 1 hour = 22 $\frac{mm}{h}$

i_d = 3 hours = $\frac{50}{3} \frac{mm}{h} = 16.7 \frac{mm}{h}$



$$\bar{i}_d = \frac{h_d}{d}$$

AVERAGE INTENSITY OVER A DURATION d

SERVIZIO IDROGRAFICO NAZIONALE (until 2002) → ANNALI IDROLOGICI (paper pubblication of data)

ARPA (after 2002).

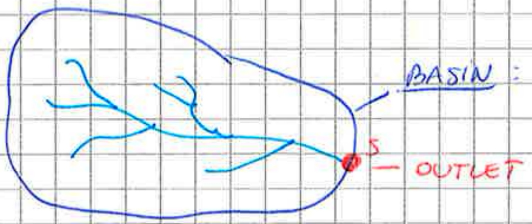
NOAA of USA Climate Production Center

Climate Research Unit (Univ of Anglia). – CRU

ECMWF for Europe – ERA section.

CRITICAL DURATION & CRITICAL INTENSITY

• CRITICAL DURATION

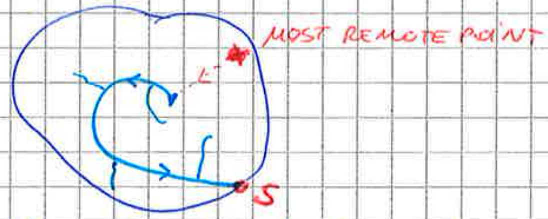


BASIN: portion of land containing a RIVER NETWORK (or also PIPE/ NETWORK) CHANNELS

→ Rainfall having
- CONSTANT INTENSITY i^*
- DURATION, d
- UNIFORMELY DISTRIBUTED IN SPACE (it rains in all points in the same way.

t_{cP} Concentration Time of a certain position in space: is the necessary time for a droplet that is in that point (on the soil) to arrive to the outlet S.

t_{cB} Concentration time of the basin is the concentr. time of the most hydraulically - remote point in the basin wrt S.
(≡ most distance in a hydraulic sense: is the longest path that the droplet follows).
It depends on the drainage characteristics of the basin (length, size, ...) and on the hydraulic characteristics. (roughness, speed, friction, ~~slopes~~, ...).



CASE 1 $d < t_{cB}$

All the points get wet at the same time but only the points near the S outlet will produce a flow which exits immediatly. For the others we can draw lines with equal concentration time.

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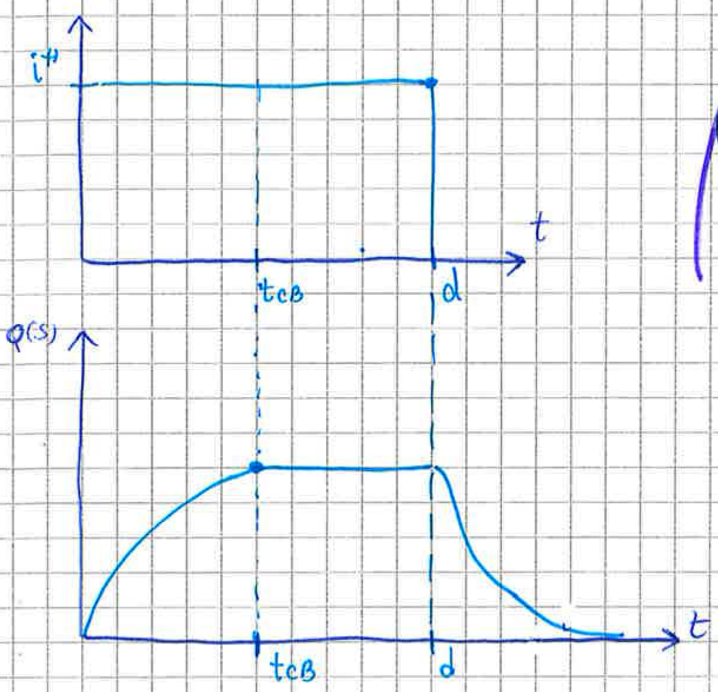
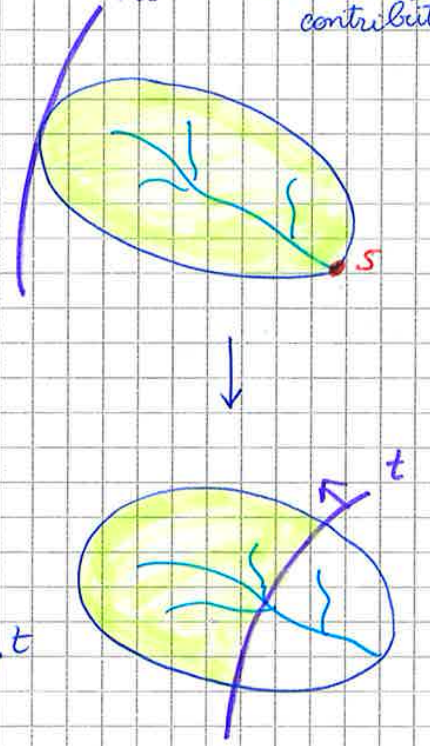
CASE 2 $\quad d > t_{cB}$

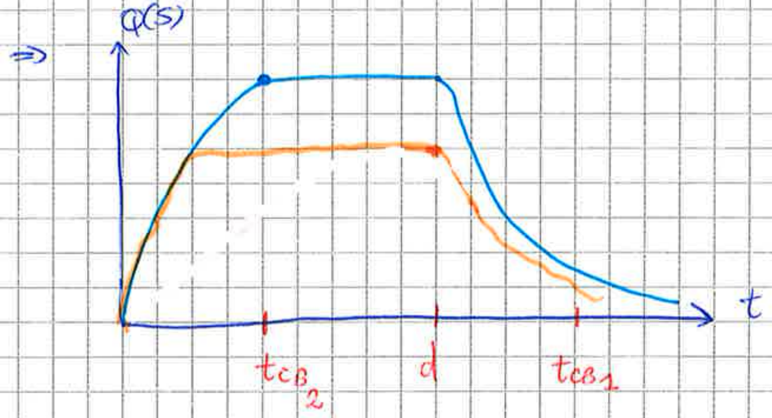


t_{cB} At t_{cB} all the basin is contributing



After t_{cB} the flow Q(S) is constant (no other area) and when precipitation stops at $t = d \Rightarrow$ it decreases.

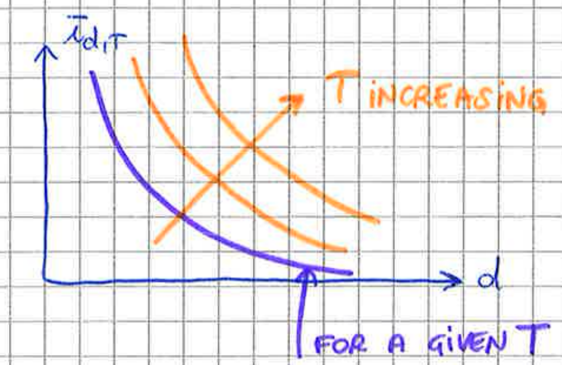
$\Rightarrow$

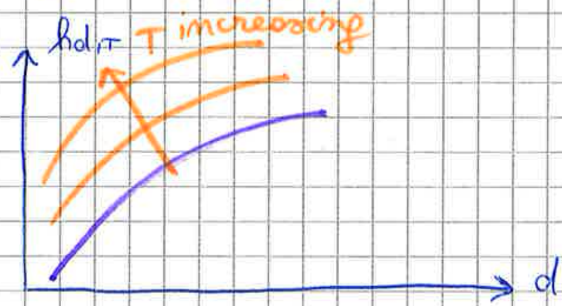


Return period in number of observations: $= T = \dfrac{n \text{ years}}{n \text{ observations}}$

Typical IDF FORM
$$\boxed{\bar{i}_{d,T} = \frac{A}{(d+c)^B}, \quad h_{d,T} = \frac{A \cdot d}{(d+c)^B}}$$

with A, B, C coefficients depending on T.





⇒ If we separate **d** and T we can use the GUMBEL MODEL FOR CURVES.
⇓
means using the Gumbel distribution (well suited for hydrological data)
⇓
It's also called EV1 (extreme value 1).

$$h_{d,T} = f(d) \cdot f(T)$$
$f(d)$ ↓ DETERMINISTIC PART; $f(T)$ ↓ PROBABILISTIC PART

→ $P(h_{d,T}) = e^{-e^{-\left(\frac{h-\sigma_1}{\sigma_2}\right)}}$

→ Parameters are defined with the method of moments:

$$\hat{\sigma}_{2,d} = \frac{\sqrt{6}}{\pi} S_{h,d}$$

$$\hat{\sigma}_{1,d} = \overline{h_d} - \gamma_E \frac{\sqrt{6}}{\pi} \cdot S_{h,d}$$

where $\overline{h_d}$, S_{hd} are the mean and the st deviation of the sample for duration d.

$$h_{d,T} = \hat{\sigma}_{1,d} - \hat{\sigma}_{2,d} \cdot \ln\left[-\ln\left(1 - \frac{1}{T}\right)\right]$$

$\frac{1}{T}$ ॥ p

$$h_{d,T} = \overline{h_d} - \gamma_E \frac{\sqrt{6}}{\pi} S_{h,d} - \frac{\sqrt{6}}{\pi} S_{h,d} \cdot \ln\left[-\ln\left(1 - \frac{1}{T}\right)\right] =$$

$$= \overline{h_d} \cdot \underbrace{\left\{1 - \frac{\sqrt{6}}{\pi} \cdot \underbrace{\frac{S_{h,d}}{\overline{h_d}}}_{cv_d} \left[\gamma_E + \ln\left[-\ln\left(1 - \frac{1}{T}\right)\right]\right]\right\}}_{\text{DEPENDS ONLY ON } T} =$$

$\dfrac{S_{hd}}{\overline{h_d}} = CV_d \Rightarrow \overline{CV}$ use a constant value not depending on d

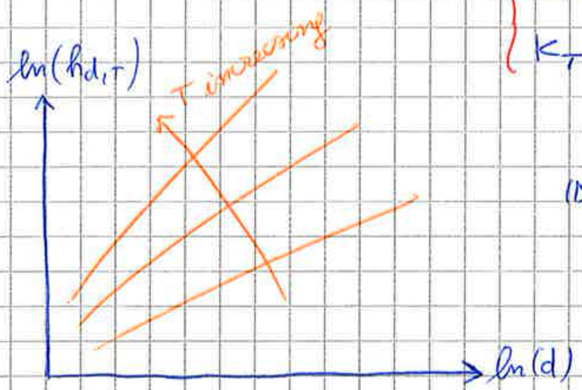
$$= \overline{h_d} \cdot f(T) = \overline{h_d} \cdot K_T$$

71

⇒ complete form : $\boxed{h_{d,T} = a \cdot d^n \, K_T}$

a, n derived from data (determ. part)

K_T derived from Gumbel model and T



IDF curves are straight lines.

! $[a] = [mm]$

$a \in [15; 50]$ mm how severe are extreme events

! $n \in [0.2; 0.6]$

! $K_T > 1$

~ END OF MATHEMATICAL MODEL & DEVELOPMENT OF IDF CURVES ~

$$\frac{m}{s} = 3600 \cdot 10^3 \frac{mm}{h}$$

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2) Find $\bar{a}$ and n:

- Compute $\ln(\bar{h}_{di})$ and $\ln(d)$ and plot
- Put the tendency line and show equation:

$$y = \underbrace{0,2491}_{n}\, x + \underbrace{3,3813}_{\ln(a)}$$

so find $a = \exp(3,3813) \cong 29,41\ mm$

$n = 0,2491$

3) Find $Q_{OUT}(y)$ and $i_T(d)$ = critical intensity:

Chezy formula

- $Q_{OUT}(y = 0.4\ m) = 0.397 \cdot 65 \cdot 0{,}4^{2.666} \sqrt{0{,}003} = 0.123\ \frac{m^3}{s}$
- From the formula $Q_{GROUND} = i \cdot Area \cdot C_{runoff}$, with $C_{runoff} = 1$, find i putting $Q_{GROUND} = Q_{OUT}$:

$$i = \frac{Q_{OUT}}{Area \cdot C_{runoff}} = \frac{0.123\ \frac{m^3}{s}}{10000\ m^2 \cdot 1} \cdot 3600 \cdot 10^3 \cong 44,22\ \frac{mm}{h}$$

4) Find the concentration time t_{cB} (parking lot ≡ basin)

- It's the time of the most distant particle to reach the pipe

$L = L_1 + L_2 + L_3 = 10\,m + 50\,m + 100\,m = 160\,m$

$$t_{cB} = \frac{L_1}{V_1} + \frac{L_2}{V_2} + \frac{L_3}{V_3} = \frac{10\ m \cdot s}{0.02\ m} + \frac{50}{0.1} + \frac{100}{0.1} = 2000\,s \simeq 0,555\ h$$

5) Find K_T:

$$\bullet\ K_T = \frac{i}{\bar{a} \cdot t_{cB}^{\,n-1}} = \frac{44,22\ \frac{mm}{h}}{29,41\ mm \cdot 0,555^{(0,2491-1)}\ h^{(0,2491-1)}} = 0,967 \cdot h^{0,2491}$$

6) Find $P_{NON\ EXCEED} = 1 - \frac{1}{T}$ and $P_{EXCEEDANCE} = \frac{1}{T}$

$$\bullet\ [\ldots] = \frac{1-K_T}{cv}\ ;\quad \ln\left[-\ln\left(1-\frac{1}{T}\right)\right] = \underbrace{\left(\frac{1-K_T}{cv} \cdot \frac{\pi}{\sqrt{6}} - \gamma_E\right)}_{z}\ ;$$

$$\Rightarrow\ 1 - \frac{1}{T} = \exp(-\exp(z)) \cong 0,53 = 53\%$$

$$\Rightarrow\ P_{EXCEED} = \frac{1}{T} \cong 47\%$$

⇒ $T = 2,12$ years return period. ⇒ Every **2** years (more or less) the probability that $Q_{RAINFALL} > Q_{PIPE}$ is about 47%. (it's a problem).

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ASSIGNMENT 1

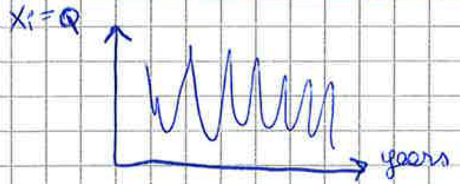
Design of the river banks' maximum height at a given river section

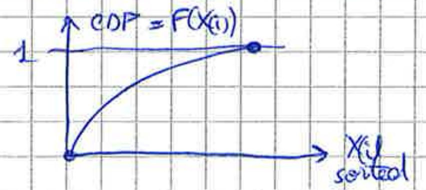
Measurements of the annual max discharge $\left(\frac{m^3}{s}\right)$ of the Dora Baltea River at Tavagnasco.

Relation between water level and discharge in presence of the embankments.

$Q = 142 \cdot (h + 0.05)^{1.81}$ $\quad Q$ = discharge; h = water level.

① Plot the time series and CDF and describe the sample





$F(X_{(i)}) = \frac{i}{N}$ $\quad i$ = position $1, \ldots, 74$

$N = 74$

- Mean $\bar{Q} = 834{,}74 \ \frac{m^3}{s}$ $\qquad$ Mean $(x = \ln Q) = 6{,}597$
- Variance $= 246\,474{,}9 \ \frac{m^6}{s^2}$ $\qquad$ Variance $(x = \ln Q) =$ ~~2~~
- Standard deviation $= 496{,}46 \ \frac{m^3}{s}$ $\qquad$ Std dev $(x = \ln Q) = 0{,}488$
- CV $= 0.59$
- CA $= 2.32$

② Probability plot test: find the $PP = \frac{i}{N+1}$ (empirical) and plot the reduced variate $U_i(PP)$ for exponential, normal, Gumbel and log-normal → ! you must use for the log-normal the $x = \ln(Q)$.

All the distributions pass this test

3) Estimate the parameters $\theta_1, \theta_2, \theta_3$ using the method of moments and the L-moments and plot the distributions with the parameters (for example using the θ found from the method of moments). For exponential, normal, Gumbel, GEV, log-normal.

	MOMENTS			L-MOMENTS		
	θ_1	θ_2	θ_3	θ_1	θ_2	θ_3
EXP	834.74	/	/	834.74	/	/
NORM	834.74	496.46	/	834.74	370.29	/
GUMBEL	611.20	387.29	/	638.44	340.10	/
GEV	606,94	311,901	-0,135	604,72	251,25	-0,258
LOG-NORM	6.58	0.55	/	1.88	0.07	/

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6/11/17

RECALL

- Gumbel Model for precipitation.

$$h_{d,T} = \bar{h}_d \cdot K_T$$

depends on duration ← ... → depends on return period

Other models can be used:

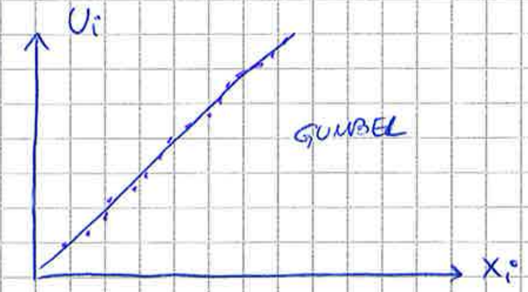
LOG - NORMAL MODEL

$$\boxed{P(h_{d,T}) = \phi\left(\frac{\ln(h_{d,T}) - \sigma_1}{\sigma_2}\right)}$$ → STANDARD NORMAL DISTRIBUTION

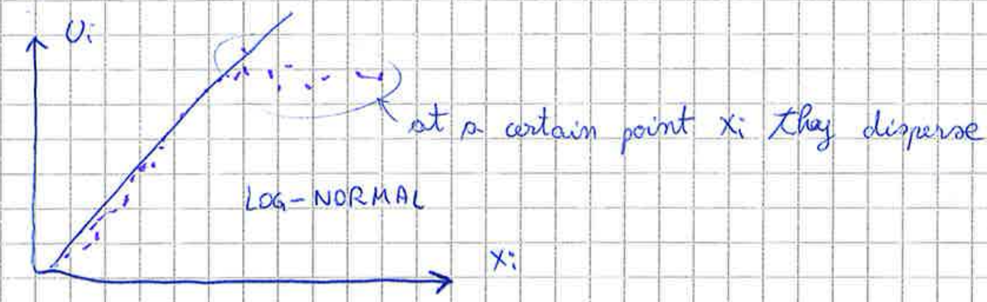
$$\boxed{\ln(h_{d,T}) = \sigma_{1,d} + \sigma_{2,d}\,\phi^{-1}\left(1 - \frac{1}{T}\right)}$$ → the parameters depend on the duration.

⇒ TRANSFORM DATA INTO $\ln(h_{d,T})$. ⇒ REPEATED FOR ALL $d = 1h, 3h, 6h, \dots$

⇒ In order to determine which model to use, we can study the probabilistic plots.



$$u_i\left(F(x_i)\right)$$



⇒ In this example we must choose Gumbel.

79

If we change the return period T from the beginning $\Rightarrow$ also the linear equation will change.

4) Generic IDF form: $h_{d,T} = a_T \cdot d^{n_T}$ (a and n are generic coefficients)

From the linear interpolation, one can have the $h_{d,T}$ for $d = 10\,h$

DEPTHS

$h = \bar{i} \cdot d$

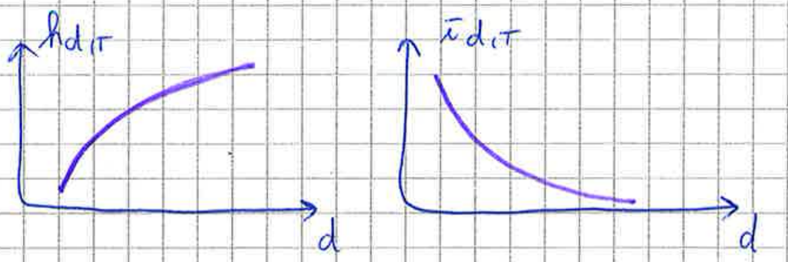
INTENSITY

$\bar{i} = \frac{h}{d}$

$$\boxed{h_{d,T} = a_T \cdot d^{n_T}} \rightarrow \boxed{\bar{i}_{d,T} = a_T \cdot d^{n_T - 1}}$$

↳ average intensity for a given duration d and a given return time T.

- $n_T \in 0.2 \div 0.6 < 1 \Rightarrow d^{(n_T - 1) < 0}$



$\lim_{d \to 0} i_{d,T} = \infty$ If duration goes to 0 intensity goes to ∞, but ∄ sense.

So ∃ IDF curves for short durations. ($d < 1\,h$)

h_{day} h_{24}

DAILY RAINFALL DEPTH — MAXIM. DEPTH OVER 24 HOURS

9 am – 9 am — 3:15 am – 3:15 am

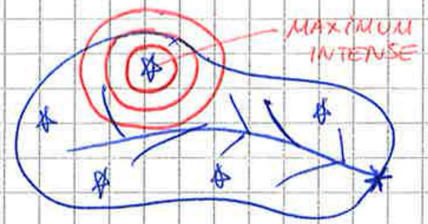
$\alpha = 1.1 \div 1.2$

POINT MEASUREMENTS

MEAN PRECIPITATION OVER AN AREA

↳ AREAL PRECIPITATION h_{AREA} [mm]

Very Intense precipit. are also local ones.



One tends to take the maximum h_d over a gauge where this storm happened, but this is also an overestimation, because it means that if we take that value we calculate for all the basin everything using that maximum value. → use ARF.

- COMPUTE AREAL PRECIPITATION
- INFER (h_{area}) FROM POINT MEASUREMENTS (h_{point})

AREAL REDUCTION FACTOR

$$\boxed{ARF(d,T,A) = \frac{h_a(d,T,A)}{h_p(d,T)}}$$

⇒ It's a reduction factor because the larger is the area and the smaller will become our value.

• if $T < 20 \div 30$ years : ARF DOES NOT DEPEND ON T.

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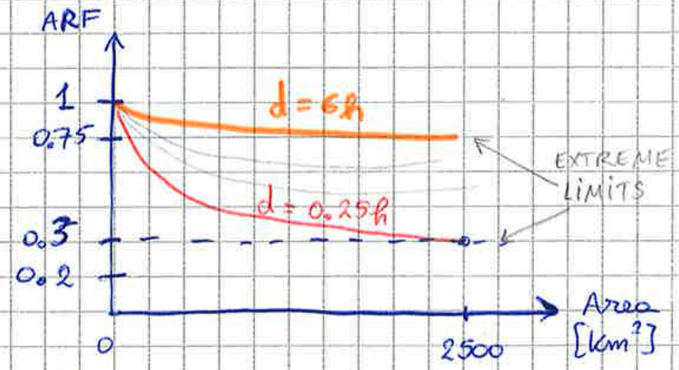
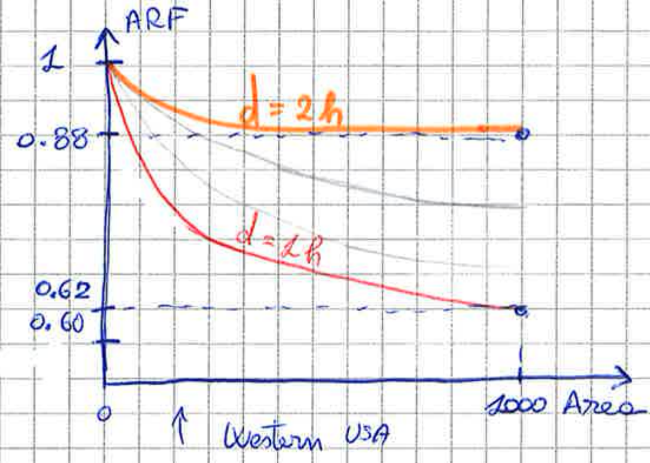
ARF can be defined in different ways: GRAPHICAL, TABULAR, EQUATION

EXPRESSIONS :



In Milan



UK : $ARF = 1 - a \cdot d^{b}$! [TO KNOW : – type of equation : powered – limits : 1]

$a = 0.0394 \cdot A^{0.354}$

$$b = \begin{cases} 0.40 - 0.0208 \cdot \ln(4.6 - \ln A) & \text{if } A < 20 \text{ Km}^2 \\ 0.40 - 0.00382 \, (4.6 - \ln A)^2 & \text{if } A \in 20 \div 100 \text{ Km}^2 \end{cases}$$

FRANCE : $ARF = A^{-\varepsilon}$ $\quad \varepsilon \approx 0.5$

$\varepsilon = (0.04 - 0.0332 \log(d)) \cdot (A^{0.105} - 0.73)$ for $A < 200$ hectars, $d < 1\,h$

⇒ Build IDF curves for areal precipitation.

$$\boxed{h_{area}(d, T, A) = a_T \cdot d^{n_T} \cdot ARF(d, A)}$$

GENERAL EXPRESSION OF IDF CURVE.

WHAT'S NEXT...

→ Measuring areal precipitation (h_{area})

→ Computation of spatially-averaged precipitation.

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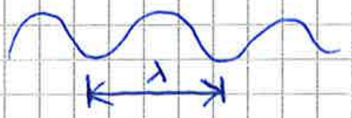
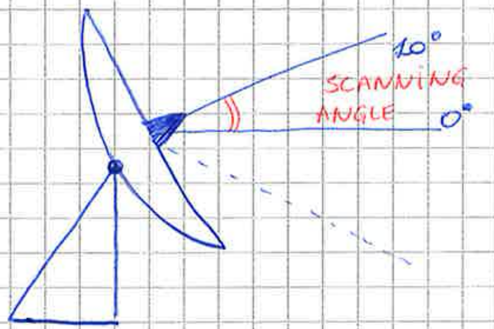
The power of the reflected signal depends on the number of droplets (volume of water of the precipitation)
Emission occurs in pulses so ∃ the TIME:
WAITING TIME between emission and reception is a function of distance. (The direction is also registered). The inclination of the emitter is between 0° and 10° (in case the antenna is on a mountain, it can go under the horizontal direction. They can measure up to 10000 km around them.





RADAR REFLECTIVITY FACTOR (Z) $[\mu m^6/m^3]$
depends on the integral of the drop size distribution.

$$dBZ \propto \log_{10}\left(\frac{Z}{Z_0}\right)$$

dBZ = decibel relative to Z
Z = radar reflectivity factor.

Z_0 = standard signal for 1 mm drop.

But it's difficult to convert dBZ into real precipitation at ground level, because also clouds are made by droplets.

$$i \approx a \cdot Z^b$$

One can make an accurate calibration by relating it to the rain gauges at the ground level.

COLOURED RADAR MAPS: red – violet are areas where dBZ is more intense but one have to calculate if it rains or not.

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SATELLITES

We can measure the TEMPERATURE / HUMIDITY PROFILE;

WATER AND WATER VAPOUR

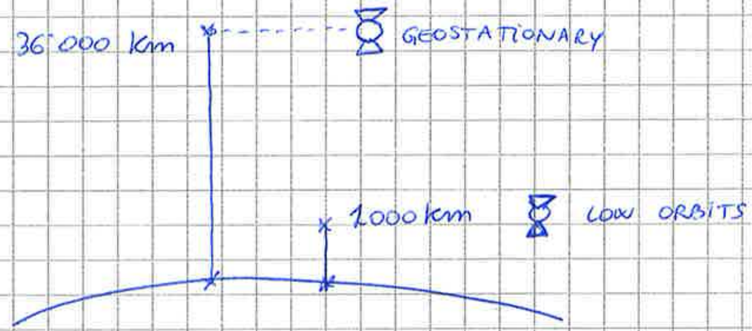
WIND RADIATION

LOW SPATIAL DISTRIBUTION

NEED A COMPLEX CALIBRATION

POST - PROCESSING OF DATA

∃ GEOSTATIONARY OR ORBITS SATELLITES



Project of low orbit satellite that carries X-radar around Earth

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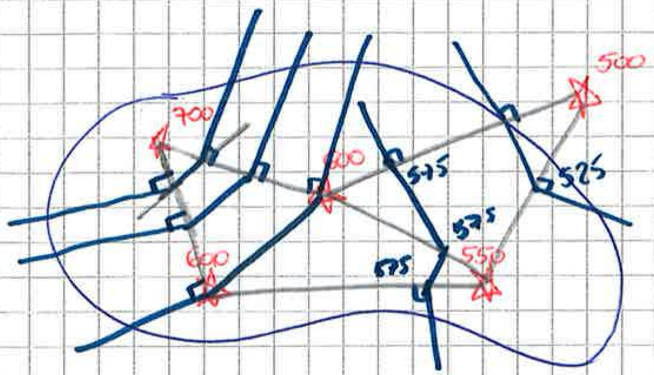
2 ISOHYETAL METHOD



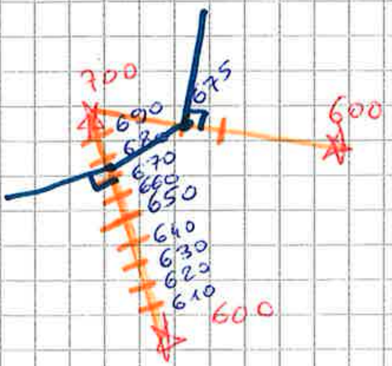
1) I have to know for ex. the mean annual [mm] of each gauge.

2) Draw the isohyetal lines that are lines connecting equal precipitation:

2.1) Connect nearby gauges

2.2) Perform divisions in equal parts of that line between the values.

2.3) Starting from the higher value 700 which is a point, connect 675 and 675 on 2 lines and draw the ⊥ to them.



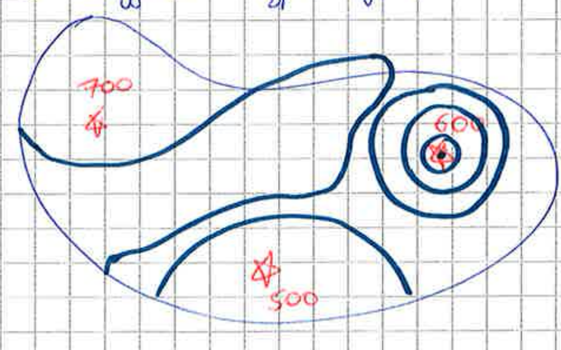
3) Compute the integral:

$$P_{AREA} = \frac{1}{A_b} \int_{A_b} P(x,y)\, dA$$

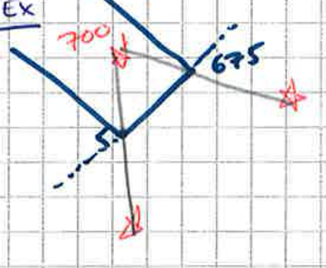
With this method ∃ a continuity between values. But it's more complex, and also the isohyetal lines can be non linear, in fact softwares are used.

! This lines can change in time because the annual measurements (and also after each precipitation event) data on gauges change

Ex: different types of areas.



Ex



We can choose to draw the ⊥ in this way. The choice is subjective. First we have to choose the rule then follow it for each application.

$\boxed{3}$ REC. DIST. METHOD

$$P = 150\left(\frac{\frac{1}{10}}{\frac{1}{10}+\frac{1}{20}}\right) + 100\left(\frac{\frac{1}{20}}{\frac{1}{10}+\frac{1}{20}}\right) =$$

$$= 150 \cdot \frac{20}{3 \cdot 10} + 100 \cdot \frac{20}{3 \cdot 20} =$$

$$= 133,4 \text{ mm}$$

$\boxed{4}$ REC. DIST. SQU. METHODS

$$P = 150\left(\frac{\frac{1}{10^2}}{\frac{1}{10^2}+\frac{1}{20^2}}\right) + 100\left(\frac{\frac{1}{20^2}}{\frac{1}{10^2}+\frac{1}{20^2}}\right) =$$

$$= 150\left(\frac{\frac{1}{100}}{\frac{1}{100}+\frac{1}{400}}\right) + 100\left(\frac{\frac{1}{400}}{\frac{1}{100}+\frac{1}{400}}\right) =$$

$$= 150 \cdot \frac{400}{5 \cdot 100} + 100 \cdot \frac{400}{5 \cdot 400}$$

$$= 140 \text{ mm}$$

$\boxed{5}$ KRIGING METHOD

It's a geostatistical method, complex, for large areas.
The assumption is that 2 points having distance h are related by a quantity called COVARIANCE.

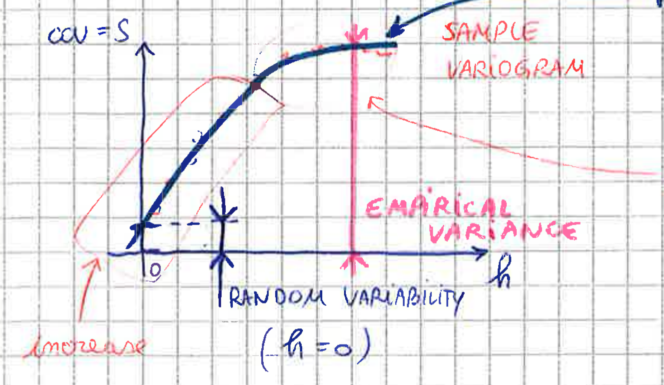
! Here we don't find P. directly.

1)

$$\text{cov} = S(h) = \frac{1}{N} \cdot \sum_{i=1}^{N} P(X_i) \cdot P(X_i + h) - \left[\frac{1}{N}\sum_{i=1}^{N} P(X_i) \cdot \frac{1}{N}\sum_{i=1}^{N} P(X_i + h)\right]$$

The $\text{cov}(h) = S(h)$ determines how similar the measurements $P(X_i)$ and $P(X_i + h)$ are.

INTERPOLATION FUNCTION (exp ; log).
↓
Know how the variance changes with the distance.



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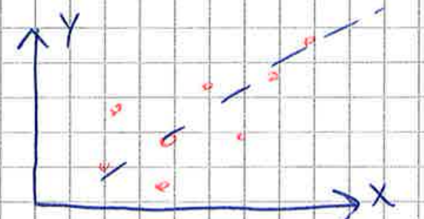
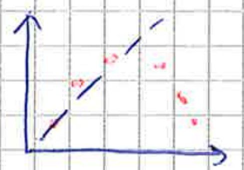
Linear Regression Models

It. Describes a deterministic (linear) relationship between x and y (+ NOISE). y = scalar dependent variable ; x = independent variables = regressor.
Could a linear relation describe the dependency of y on x ?

1) ASSESS A DEPENDENCY WITH A SCATTER PLOT. → "grafico di dispersione"



If there is a grouping around a direction → the relation can be linear.

EX 

this is no linear.

2) DEFINE THE LINEAR MODEL

$$\begin{cases} y = \hat{y} + \varepsilon \\ \hat{y} = b_0 + b_1 x \rightarrow \text{linear} \end{cases}$$

$\hat{y}$ = deterministic
ε = noise ≡ error

$$\Rightarrow y = (b_0 + b_1 x) + \varepsilon.$$

REGRESSOR $p = 1$: x

! A linear model can also be : $y = b_0 + b_1 \cdot \ln(x) + \varepsilon$ because it's linear WRT coefficents.

3) ESTIMATE THE PARAMETERS b_0, b_1 :

"LEAST SQUARE METHOD" "metodo dei minimi quadrati"

$$\boxed{S_\varepsilon = \sum_{i=1}^{N} \varepsilon_i^2 = \sum_{i=1}^{N} (y_i - b_0 - b_1 x_i)^2}$$

N = sample size : number of couples $(x, y)_i$

4) TEST: t - STUDENT TEST

To assess the dependency of y on x.

1° HP: $\hat{b}_1$ is significantly $\neq 0$. ($\hat{b}_1$ = value calculated.)

⇒ the linear relation between x and y is significant (level α)

b_0, b_1 are random variables because they come from $(x_i; y_i)$ which are variables. So b_0 and b_1 have a mean and a variance. Which is their variance?

$$\text{variance}(b_1) = \frac{\sigma_\varepsilon^2}{S_{xx}}$$

$$\sigma_\varepsilon^2 = \frac{S_\varepsilon}{N-2}$$

$(N-2)$ → is like having lost 2 degrees of freedom because we have estimated 2 parameters.

+ 2° HP: if residuals (errors) are normally distributed $\varepsilon \sim N(0, \sigma_\varepsilon^2)$

$$\Rightarrow b_1 \sim \mathcal{N}\left(b_{TH}, \frac{\sigma_\varepsilon^2}{S_{xx}}\right)$$

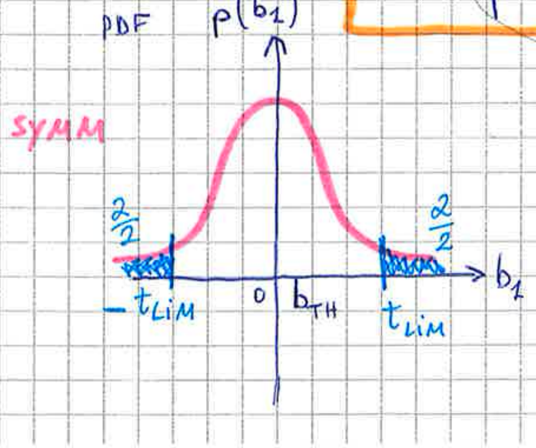
b_{TH}: theoretical mean of b_1

⇒ I can build the TEST VARIABLE T that has a t - student distribution with N - 2 degrees of freedom.

$$\boxed{T = \frac{\hat{b}_1 - b_{TH}}{\sqrt{\frac{\sigma_\varepsilon^2}{S_{xx}}}}}$$

$\sqrt{\frac{\sigma_\varepsilon^2}{S_{xx}}}$: dispersion of x and y

We want to know if $b_{TH} = 0$



If I am on the tails I can say that b_1 is signif different from 0.

• if $|T| > t_{LIM}\left(\frac{\alpha}{2}, N-2\right)$ ⇒ ↑

⇒ Dependency = TREND is SIGNIFICANT

97

MULTIVARIATE LINEAR REGRESSION MODELS

1) I want to establish the relation (to define)

$$V = b_0 + b_1 x + b_2 y + b_3 z + \varepsilon$$

(this is linear) in coeffic and variables.

REGRESSORS p : $p = 3$: x, y, z the variables that I want to connect.

2) Define the parameters for (ex with least square method)

$\Rightarrow (\hat{b}_0, \hat{b}_1, \hat{b}_2, \hat{b}_3)$ → equations are more.

3) Apply t-student test to each parameter (b_1, b_2, b_3) ⇒ to say which one is significant and which is not.

EX: b_1, b_2 NON SIGNIFICANT

b_3 SIGNIF.

⇒ The new model with only significant variables:

$$V = b_0 + b_3 z + \varepsilon$$

4) Recalculate again the parameters (point 2) because this is a different model.

! BIVARIATE $\sigma_\varepsilon^2 = \dfrac{S_\varepsilon}{N-2}$ → NEW DEGREES OF FREEDOM $N - (p+1) = N - 2$.

MULTIVARIATE : $\sigma_\varepsilon^2 = \dfrac{S}{N-p-1}$

⇒ t-student: $t_{LIM}\left(\dfrac{\alpha}{2}, N-p-1\right)$

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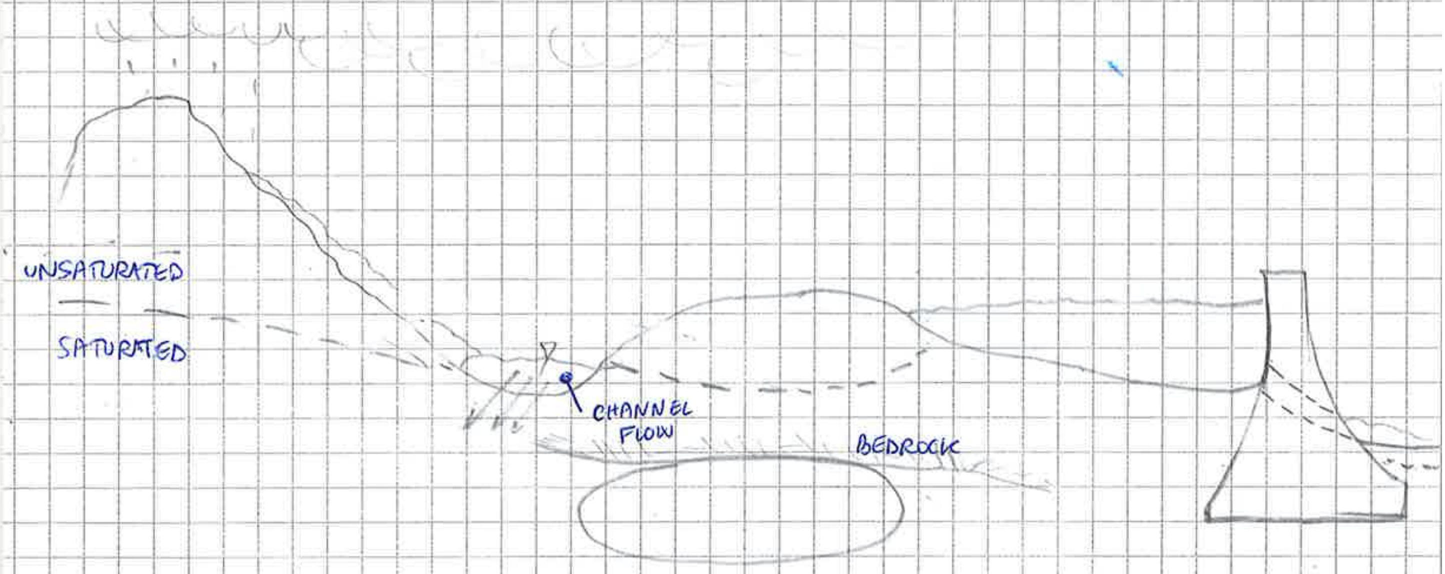
3ª parte

PRECIPITATION: WHAT HAPPENS WHEN IT REACHES THE SOIL?

- → SURFACE STORAGE
- → SUBSURFACE STORAGE

STORAGE:
- RETENTION (LONG. TERM, SLOW DYNAMICS, DEPLETED BY EVAPOTRANSPIRATION)
- DETENTION (SHORT TERM, FAST DYNAMICS, DEPLETED BY FLOWS



SURFACE STORAGE
- OVERLAND FLOW
- CHANNEL FLOW
- velocity $10 \div 100 \frac{cm}{s}$

SUBSURFACE STORAGE
- UNSATURATED FLOW
- SATURATED (GROUNDWATER) FLOW
- velocity $10^{-8} \div 10^{-1} \frac{cm}{s}$

Catchments are the medium by which an input (rainfall) is transformed into an output (discharge, runoff).

RAINFALL – RUNOFF MODELS

The size of a basin can be extremely variable: $km^2 \rightarrow 10'000\ km^2$

urban area — regions / countries

There are different types:

(URBAN):
- industrial / commercial areas
- residential
- rural areas
- natural areas

(from more waterproof ↓ to more infiltration)

1) Fix the outlet
2) Trace back the path followed by a droplet.

! There are some cases (Aral lake) where there is no outlet.

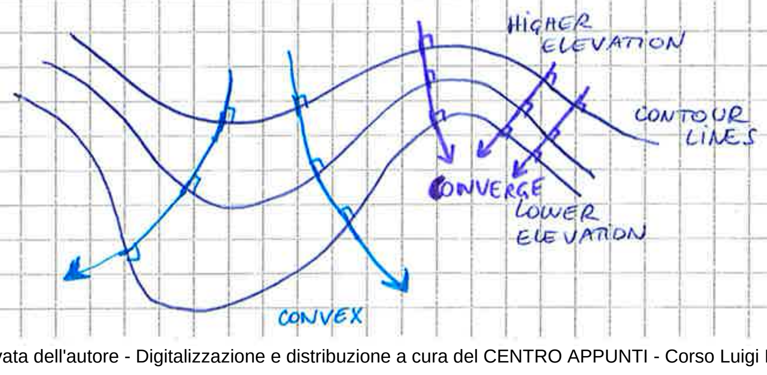
Drainage Network & Topography (DN.)

DN shapes topography (erosion, sediment, long time scale)

DN is shaped by topography

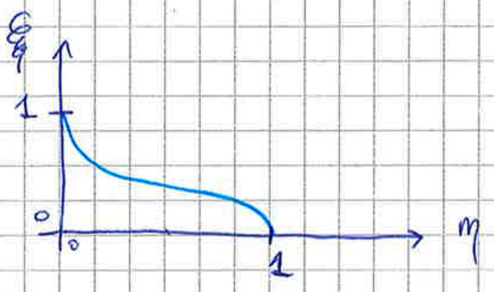
same z level = ISOIPSA

Topography is expressed by contour lines: ISOHYPES. Water flows by gravity, perpendicular to the contour lines from higher to lower z.

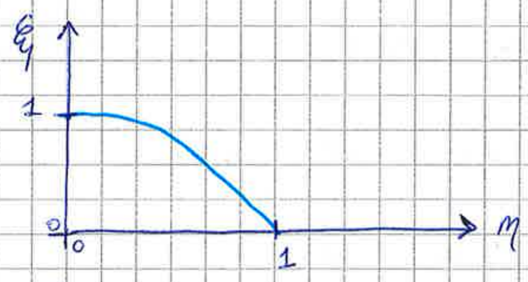


To enable comparisons between basins ⇒ use normalization.

$$\boxed{\xi = \frac{z - z_{min}}{z_{max} - z_{min}} \qquad \eta = \frac{A_i}{A_b}}$$



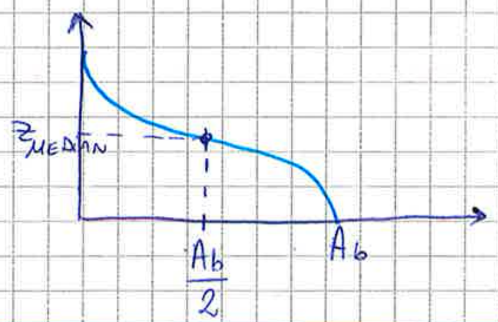
For higher elevation only a small area is in the basin
Drop of elevation near the outlet.
In the middle there is a large flatter area.

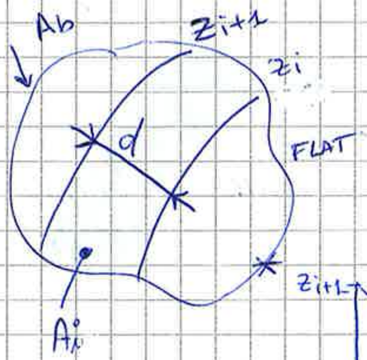


Higher elevations are present in a bigger area.
After this, there is a quite constant decrease of high.

MEDIAN ALTITUDE z_M

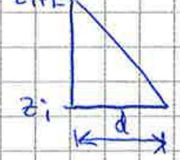
MEAN ALTITUDE z_a





$$\boxed{z_a = \sum_i \left(\frac{z_i + z_{(i+1)}}{2} \cdot \frac{A_i}{A_b} \right)}$$

or $\bar{z} = \frac{\sum z_i A_i}{A_{TOT}}$

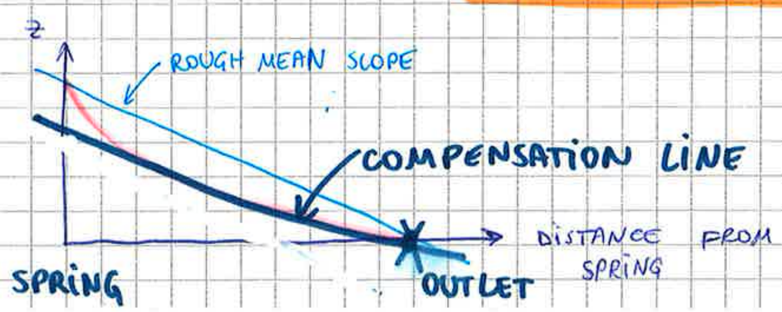


SLOPE OF CATCHMENT :

$$\boxed{SLOPE = \frac{z_{(i+1)} - z_i}{d} = \frac{z_{(i+1)} - z_i}{\left(\frac{A_b}{L_{\text{CONTOUR LINES}}} \right)}}$$

CONTOUR LINES = ISOHYPES

SLOPE OF STREAM



- **FORM FACTOR** ("fattore di forma") $FF = \dfrac{A_b}{L_{ms}^2}$ [/] $\dfrac{\text{basin area}}{(\text{length of the main stream})^2}$

- **ELONGATION FACTOR** ("rapporto di allungamento") $EF = \dfrac{L_{ms}}{\sqrt{A_b}}$ [/] $\dfrac{\text{length of the main stream}}{\sqrt{\text{basin area}}}$

- **CIRCULARITY FACTOR** ("rapporto di circolarità") $CF = \dfrac{A_b \cdot 4\pi}{P^2}$ [/] (*1) $\dfrac{4\pi \cdot \text{basin area}}{\text{perimeter of an equivalent circle}}$

- **UNIFORMITY FACTOR** (GRAVELIUS INDEX) ("coefficiente di uniformità") $UF = \dfrac{P_b}{2 \cdot \sqrt{\pi A}}$ [/] (*2) $\dfrac{\text{basin perimeter}}{2\sqrt{\pi}\sqrt{\text{area of an equivalent circle}}}$

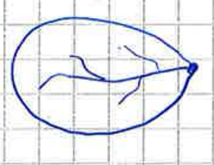
(*1) $\dfrac{A_b}{\pi R^2} = \dfrac{4\pi A_b}{4\pi^2 R^2} = \dfrac{4\pi A_b}{\underbrace{(2\pi R)^2}_{P^2}}$

(*2) $\dfrac{P_b}{2\pi R} = \dfrac{P_b}{2\sqrt{\pi^2 R^2}} = \dfrac{P_b}{2\sqrt{\pi A}}$

↳ MOST ELONGATED :

UF = 1.6

UF = 1.3

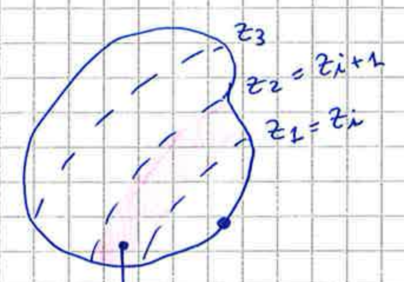


UF = 1.1

CLOSE TO A CIRCLE

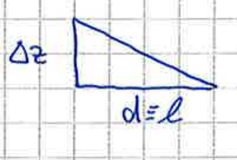
- **ELEVATION** (median or mean altitude, hypsographic curve ...)

- **SLOPE** : ALVARD – HORTON METHOD FOR SLOPE :



⇒ LOCAL SLOPE OF A_i :

$$s_i = \frac{z_{i+1} - z_i}{d_i} = \frac{\Delta z}{d=l}$$

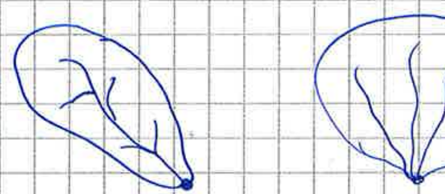


$$= \frac{\Delta_i}{\left(\frac{A_i}{l_i}\right)}$$

← because the distance between two contour lines changes so we need to take a sort of mean defined in that way.

(It would be better to take the $l_{i+\frac{1}{2}}$ (middle line) of the area, but if it si discretize very small it's good)

* STREAM NETWORK



"estensione della rete"
total length of rivers

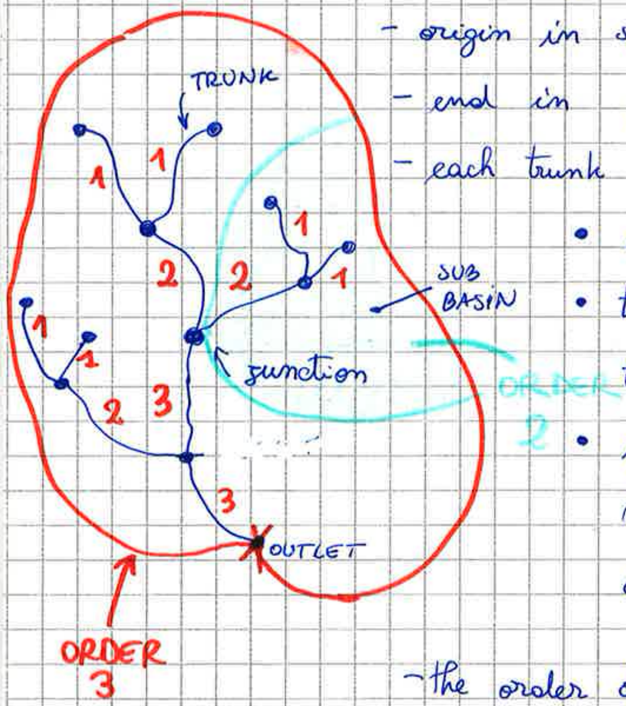
$$\left[\frac{Km}{Km^2}\right] \quad \boxed{\text{DRAINAGE DENSITY} = \frac{L_{TOT,\ STREAMS}}{A_b}}$$

as high as $0.93 - 1.24 \left[\frac{1}{Km}\right]$

* Denser network means a more rapid and complete response of the basin to the precipitations.

HORTON - STRAHLER THEORY

- Streams are formed by trunks
  - origin in springs or junctions
  - end in junctions or outlet
  - each trunk is characterize by an order:
    - all the head trunks : order 1
    - two trunks of same order i joining; they generate a trunk of order $i+1$
    - if two trunks meet of different order meet, the order will remain the same as the biggest $\quad i \oplus (i+1) \rightarrow (i+1)$



- the order of the catchment = order of outlet truck : the higher is the order, the most complicated is the shape and the network of the basin.

ex
- rivers in central Italy : order $7 \div 8$
- Mississipi order 10.
- Amazon river order 12.
- 80% of world's rivers are order $1 \div 3$

21/11/17

CONCENTRATION TIME OF THE CATCHMENT

- GIANDOTTI : • $t_{cB}[h] = \dfrac{4\sqrt{A_b\,[km^2]} + 1.5\,L_{ms}\,[km]}{0.8\sqrt{(z_m - z_{min})\,[m]}}$ (hours)

- KIRPICH • $t_{cB}\,[h] = 0.000325 \cdot L_{ms}\,[m]^{0.77} \cdot S^{-0.385}$

  S → slope of the catchment

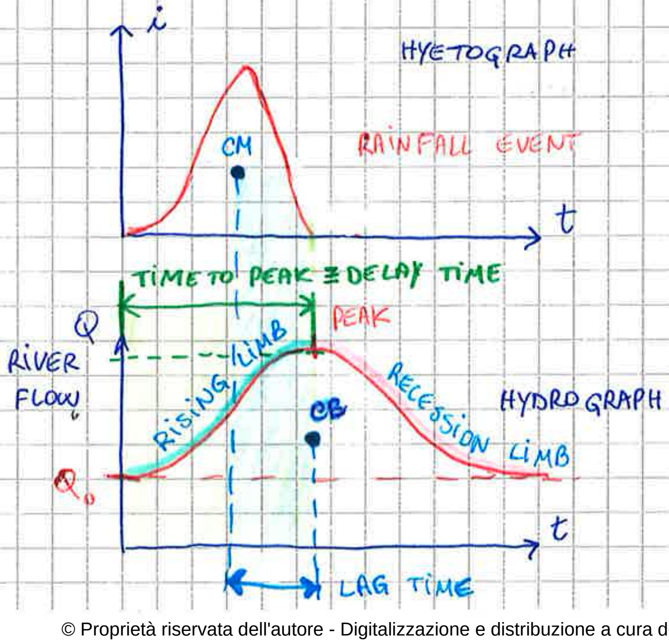
- VALID FOR ROUGH BEDS (SMALL VELOCITIES):
  - FOR CONCRETE SURFACES • t_{cB} (KIRPICH) · 0.4
  - FOR CONCRETE SURFACES + CHANNELS • t_{cB} (KIRPICH) · 0.2
  - FOR HIGHER VELOCITIES.

- SOIL CONSERVATION SERVICE (FOR AGRICULTURAL BASINS)

$$t_{cB}\,[h] = 0.00227 \cdot L_{ms}\,[m]^{0.8} \cdot \left[\frac{100}{CN} - 9\right]^{0.7} \cdot S^{-0.7}$$

CN = curve number : characterizes the type of soil and its usage.

LAG TIME ("tempo di attesa")



The event is concentrated in a small time interval. The intensity is very high.

After the event, the Q returns to its initial state Q_0

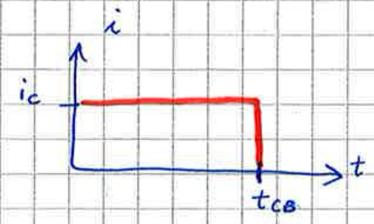
LAG TIME: is the distance (in time) between the 2 CMs.

A UNIFORM HYETOGRAPH

(Easiest) Design storm with constant rainfall intensity.

- Duration = Concentration time.
- Intensity (constant) derived from IDF curve

$$\boxed{i_{d,T} = a \cdot d^{n-1} \cdot K_T}$$ Average intensity



⇒ critical intensity i_c :

$$\boxed{i_c = a \cdot t_{CB}^{\,n-1} \cdot K_T}$$

→ Regular rectangular shape

⇒ But we have to consider time:

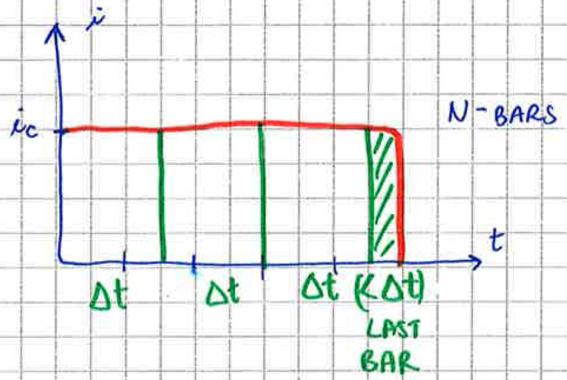
DEFINE COMPUTATION TIME STEP Δt and COMPUTE THE NUMBER OF STEPS $\boxed{\dfrac{t_{CB}}{\Delta t}}$ ex: $\dfrac{3 \text{ hours}}{10 \text{ min}} = 18$

but usually there are no integers: $\dfrac{2.3457 \text{ h}}{10 \text{ min}} = 17, \ldots$

⇒ so we have to take the upper integer number:

$$INT_{UP}\left(\frac{t_{CB}}{\Delta t}\right)$$

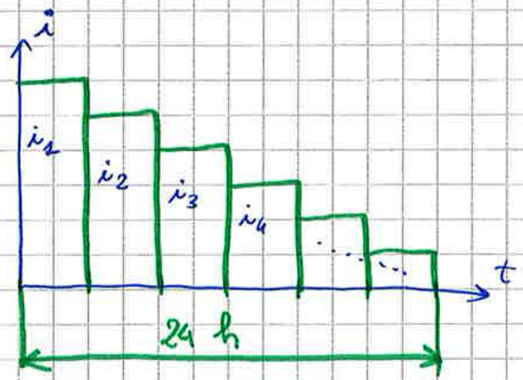
⇒ Now we have to discretize the hyetograph:



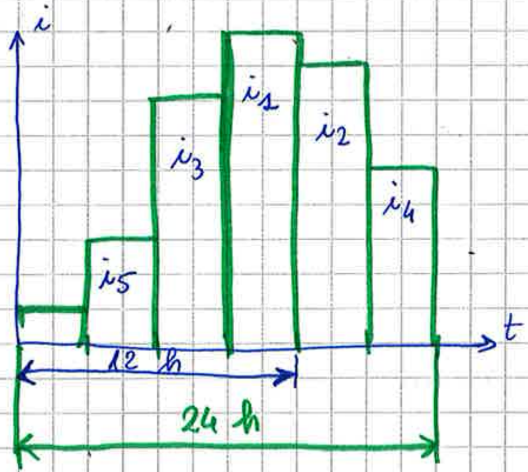
13

⇒ POSITIONING OF BLOCKS :
- INITIAL PEAK
- END PEAK (not much used)
- CENTRAL PEAK

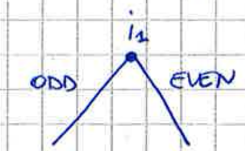
INITIAL PEAK :



CENTRAL PEAK



Put the maximum in the center at 12 hours; then the second on the right and the third on the left, and so on...

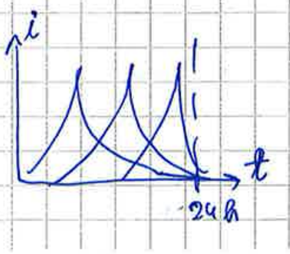


This representation is consistent with an IDF CURVE. The sequence $i_1, i_2, i_3, i_4, \ldots$ are all intensities referred to time steps Δt, so they are different from the values obtained with IDF curves.

$$i_{d,T} = a \cdot d^{n-1} \, K_T \rightarrow \begin{cases} a \cdot \Delta t^{n-1} \cdot K_T \\ a \cdot (2\Delta t)^{n-1} \cdot K_T \\ a \cdot (3\Delta t)^{n-1} \cdot K_T \\ \vdots \end{cases}$$

INCREASING TIME

But here we used the same time Δt.



WATER CONTENT $\boxed{\theta = \dfrac{V_L}{V_A + V_L + V_S} \qquad \theta \in [0 \div n]}$

↳ varies with depth $\theta(z)$

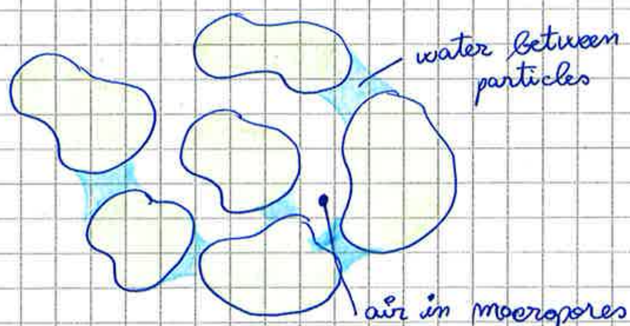
0 → COMPLETELY DRY; n → POROSITY

RELATIVE WATER CONTENT ≡ SOIL MOISTURE ("indice di saturazione").

$$\boxed{s = \frac{\theta}{n}} \qquad s \in [0 \le s \le 1]$$

0 → COMPLETELY DRY SOIL; 1 → SATURATED SOIL

CROSS SECTION OF A SOIL:



MOTION OF WATER IN SOIL (DARCY EQ)

$$\vec{q} = -K_x \frac{\partial h}{\partial x}\vec{i} - K_y \frac{\partial h}{\partial y}\vec{j} - K_z \cdot \frac{\partial h}{\partial z}\vec{k}$$

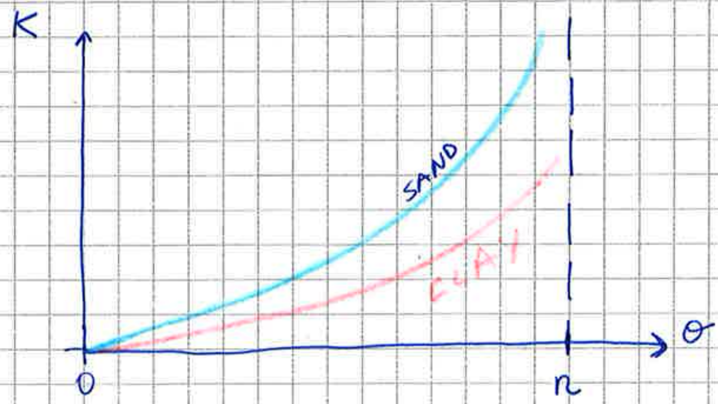
$\vec{q}$: FLOW PER UNIT AREA $\left[\frac{m}{s}\right]$

$$\Rightarrow \boxed{\vec{q} \propto K \frac{\partial h}{\partial \text{space}}}$$

K = hydraulic conductivity

h = piezometric head.

K depends on the water content in the soil: (already present):



K increases with θ.

117

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VERTICAL FLOW

$$\vec{q}_z = -k(\theta)\cdot\frac{dh}{dz}\vec{k} = -k(\theta)\frac{d}{dz}\left(z+\psi(\theta)\right)\cdot\vec{k} =$$

$$= -k(\theta)\left[1+\boxed{\frac{d\psi}{d\theta}}\cdot\boxed{\frac{d\theta}{dz}}\right]\cdot\vec{k}$$

if we know the relation between ψ and θ, this derivative is defined: USE THE EMPIRICAL RELATION (FOR EX).

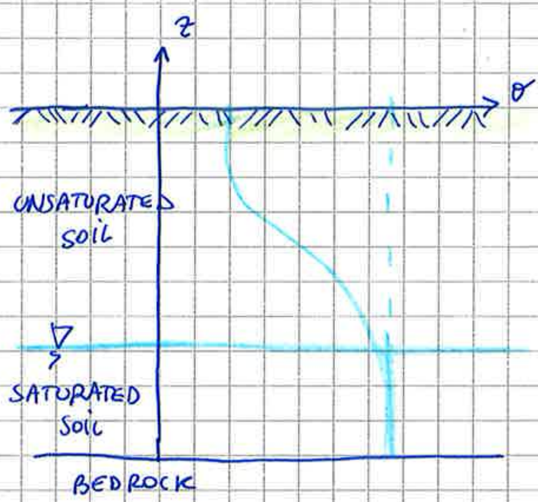
Vertical profile of flow given by the VARIATION OF THE GRAVITY FORCE & PRESSURE FORCE.

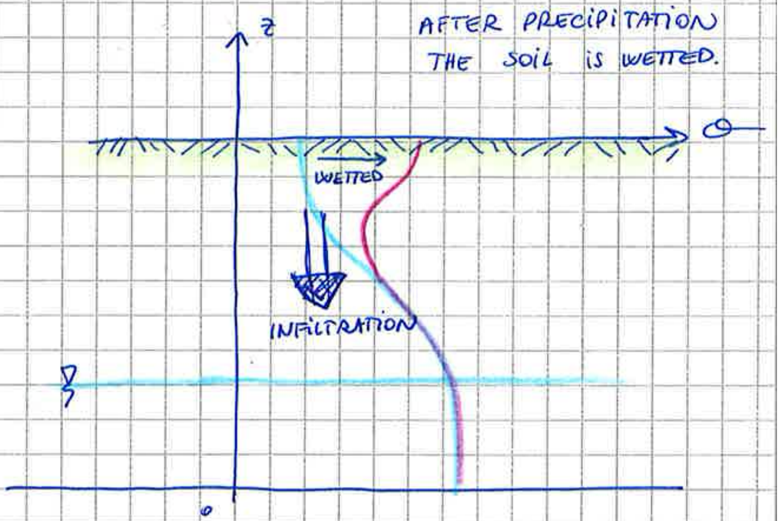
$\vec{F_g} = \vec{P} \Rightarrow$ no motion

$\vec{F_g} > \vec{P} \Rightarrow \downarrow$

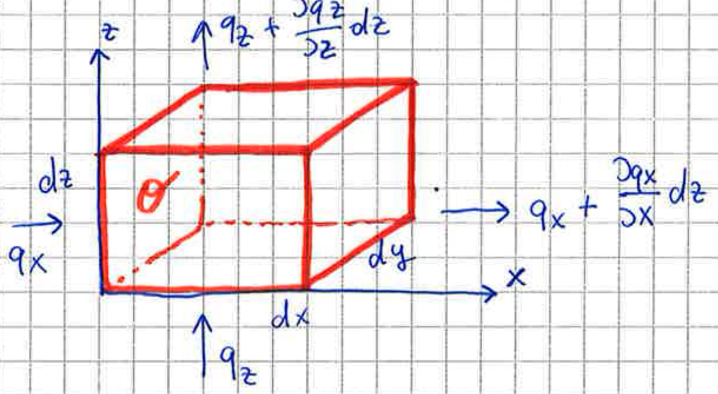
$\vec{F_g} < \vec{P} \Rightarrow \uparrow$.

VERTICAL PROFILE





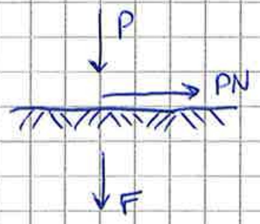
RICHARD'S EQUATION $\Rightarrow$ CONTINUITY EQUATION



The amount of water entering is equal to the amount of water exiting plus modifications that happened inside.

27/11/2017

GOAL: QUANTIFY THE NET INFILTRATION



PN = precipitation networks : contributes to the peak runoff (hydrograph)

F = infiltration.

Richard's equation in vertical direction:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K(\theta)\left(1+\frac{\partial \psi}{\partial \theta}\cdot\frac{\partial \theta}{\partial z}\right)\right] =$$

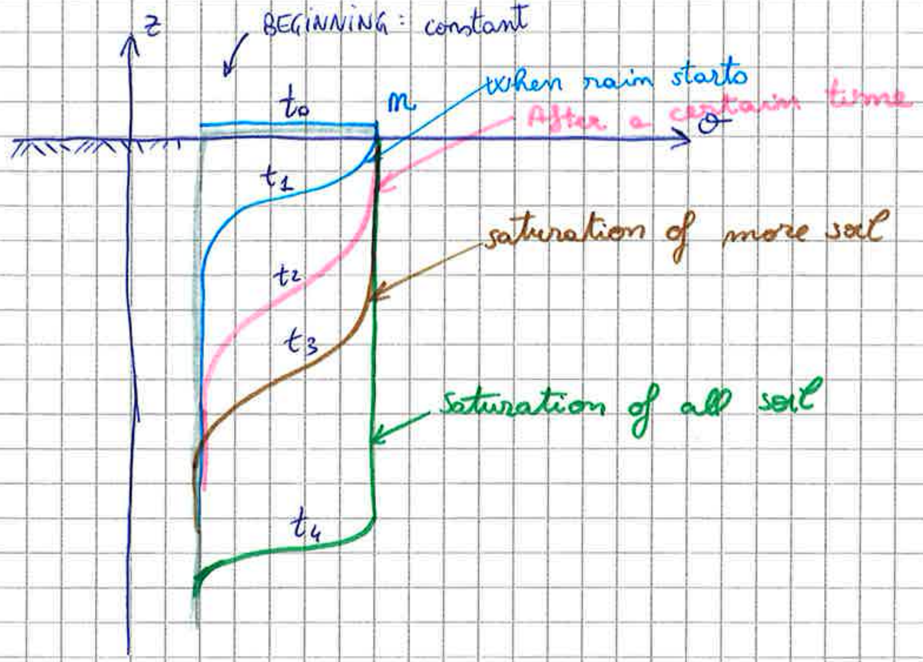
$$= \frac{\partial}{\partial z}\left[K(\theta) + \underbrace{\left(K(\theta)\frac{\partial \psi}{\partial \theta}\right)}_{\text{DIFFUSIVITY } D}\frac{\partial \theta}{\partial z}\right]$$

precise description but with uncertain info to put in.

θ = soil water content

ψ = soil matric potential

SOIL WATER CONTENT VERTICAL PROFILE



CONSTANT DIFFUSIVITY AND HYDRAULIC CONDUCTIVITY:

$$\frac{\partial \theta}{\partial t} = D\,\frac{\partial^2 \theta}{\partial z^2} + \text{constant} \qquad \text{DIFFUSION EQUATION}$$

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POTENTIAL INFILTRATION RATE f_p

(it occurs when f is max. $f \leq f_p$)

CUMULATIVE INFILTRATION F $\left[\frac{mm^3}{m^2}\right] = [mm]$

= volume of infiltrated water per unit area of soil surface

$$= \quad F(t) = \int_0^t f(\tau)\, d\tau$$

0 = time when rainfall starts

$$\Rightarrow \quad f(t) = \frac{dF(t)}{dt}$$

HORTON MODEL

FOR POTENTIAL INFILTRATION RATE

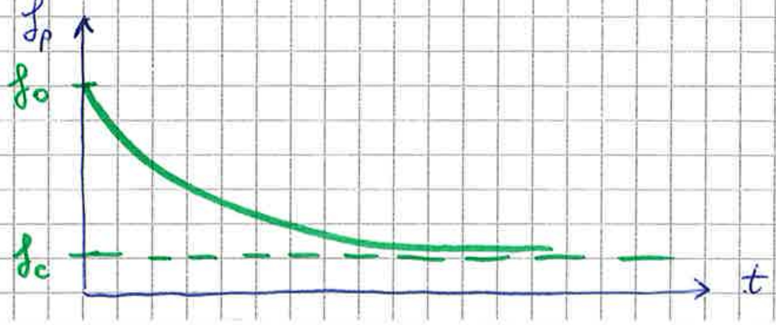
$\exists$ an initial maximum rate f_0 and ends with asymptote f_c

Rate of change is proportional to the difference with f_c

$$\frac{df_p}{dt} = -\alpha\left(f_p - f_c\right)$$ (semplification of the equation)

The solution of the eq is exponential: DECAY

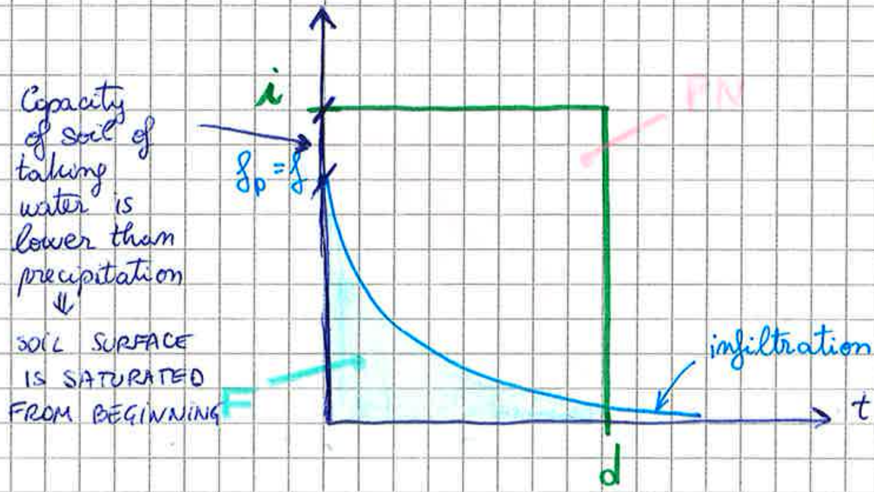
$$f_p(t) = f_c + \left(f_0 - f_c\right) e^{-\alpha t}$$



REAL INFILTRATION RATE

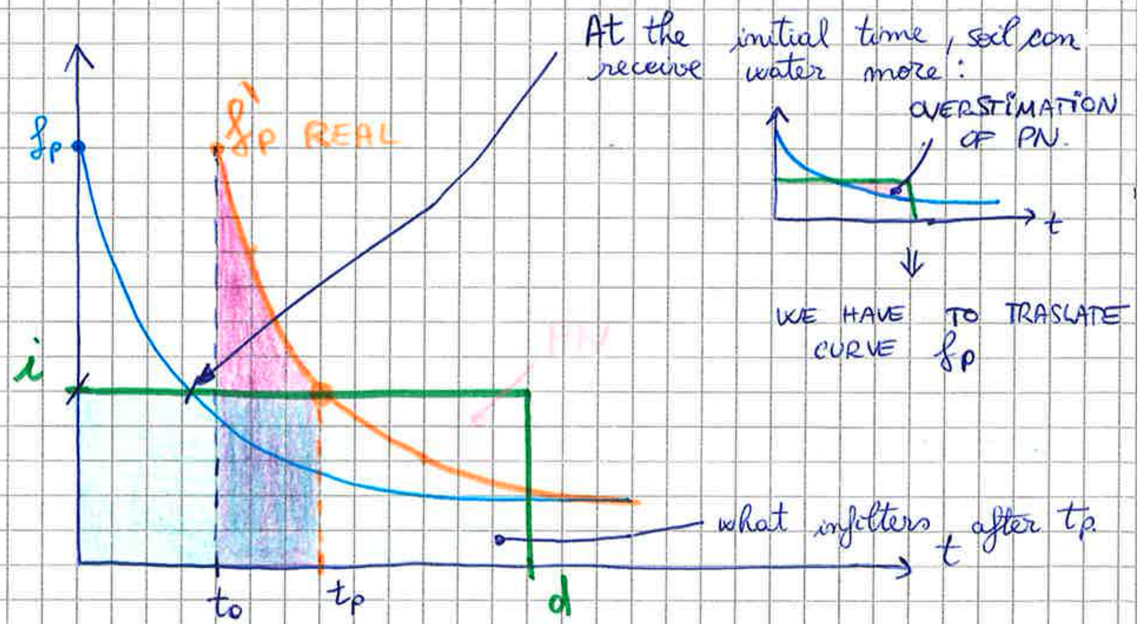
CASE 1: (Rainfall Rate) > (Potential infiltr. Rate)

⇒ (Potential) = (Real)



$$PN = \underbrace{i \cdot d}_{\text{area of rectangle}} - \int_0^d f(t)\,dt$$

CASE 2: (Rainfall Rate) < (Potential infiltr. Rate)



f'_p intersects i curve in the point of saturation at the time called PONDING TIME $= t_p$

t_0 = TRASLATION TIME

28/11/17

ASSIGNMENT (OPTIONAL) (MAX 3) (10-15 min) (Point) 0, +1, +2

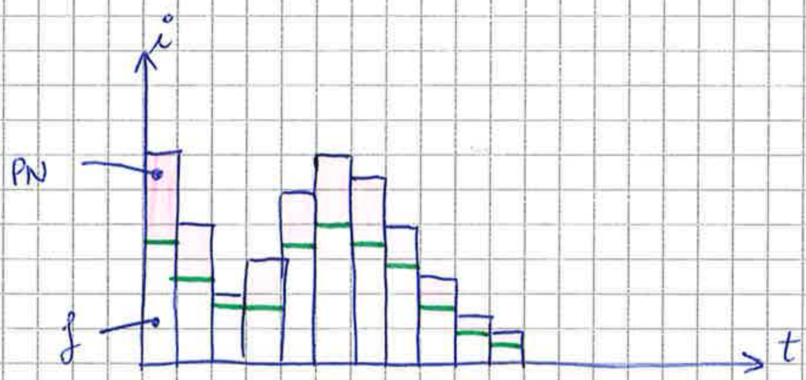
Technical report : INTRO : geomorphological charact

EVENT: flud, intense rainfall event, infrastructure related to water.

ANALITICAL : calculations

In the 2° threshold, the last bar is already the sum of PNs above 5 because it's cumulative. The subtraction between bars gives the values of bars in 1° threshold.

- PERCENTAGE (3) PROPORTIONAL METHOD

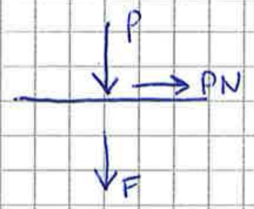


"in(t)"

$PN(t) = C \cdot i(t)$

C = RUN OFF COEFFICIENT

But rainfall water doesn't divide only into flow and infiltration: ∃ also other phenomenons that occur inside a catchment.



$PN = Q$ neglecting G_{NET}, ET, I, ΔV

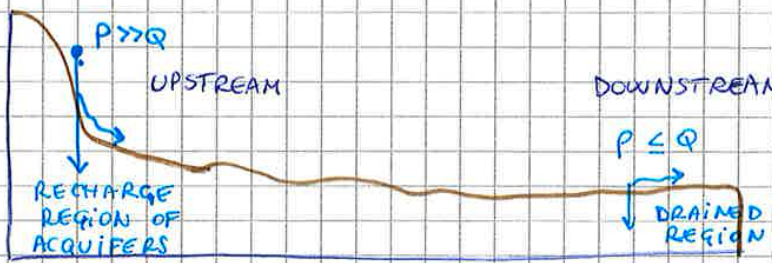
may be considered

<u>Runoff</u> coefficient includes $F, I, \Delta V$ (G_{NET}, ET) negligible contribution.

↳ $C = \frac{Q}{P} = 13\%$ (assigment 3).

⇒ 87% is LOST WITH THE OTHER FLOW TERMS

In this case the surface catchment coincides with underground catchment so there is no a significant G_{IN} but water infiltrates.



$\Rightarrow \quad F = P - PN - I$

$$\cdot S\left(\frac{P-PN-I}{S}\right) = \left(\frac{PN}{P-I}\right)\cdot S$$

$$P^2 - P \cdot PN - PI - PI + PNI + I^2 = PN \cdot S$$

$$PN(P - I + S) = P^2 - 2PI + I^2$$

$$\Rightarrow \quad PN = \frac{(P-I)^2}{P-I+S}$$

$$I = (0,1 \div \underset{\text{MORE OFTEN}}{\underline{0,2}})S$$

$$S = S_0\left(\frac{100}{CN} - 1\right)$$

S_0 = scale factor $= 254\,m$ [m]

CN = Curve Number [/]

if CN = 100 $\Rightarrow$ P = PN

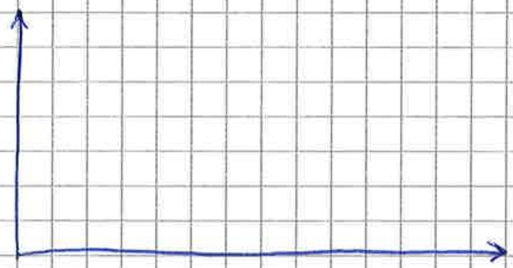
CN : depends on :

- TYPE OF SOIL A, B, C, D
  - A: Sandy soil: large infiltration.
  - D: clay or waterproof soils (rocks)
- TYPE OF LAND USE : WOOD, PASTURES, CROPLAND, URBAN ....

		A	B	C	D
• WOOD	THICK	62	68	74	79
	LOOS	34	39	45	50
• CROPLAND	...				
	...				

131

Ex



- RECTANGULAR HYETOGRAPH : constant precipitation with $i = 25 \frac{mm}{h}$ of duration $d = 3h$. and use a time step $\Delta t = 0,5\ h$
- CATCHMENT 30% LOOSE WOOD ON SAND
  70% CROPLAND ON SILT

→ COMPUTE NET RAINFALL HYETOGRAPH

1) DEFINITION OF CN : 30% woodland on soil A : CN = 66
70% cropland on soil C : CN = 83
↳ more infiltration

$$\Rightarrow CN = 66 \cdot \boxed{\frac{30}{100}} + 83 \cdot \frac{70}{100} = 77,9 = CN(II)$$

↓ IF GIVEN IN PERCENTAGE ⇒ DO AS THIS

! In DESIGN USE CN (III) that corresponds to more water in the soil corresponding to critical conditions.

2) TABLE

	$t\,[h]$	$i\,\left[\frac{mm}{h}\right]$	$P\,[mm]$	$PN\,[mm]$	$i_{NET}\,\left[\frac{mm}{h}\right]$
30 min =	0,5	25	12,5 = 25·0,5	0	0
	1	25	25	1.36	2.71
	1,5	25	37,5	5.60	8.48
	2	25	50	11.77	12.33
	2,5	25	62,5	19.25	14.96
	3	25	75	27.67	16.85
			cumulative		

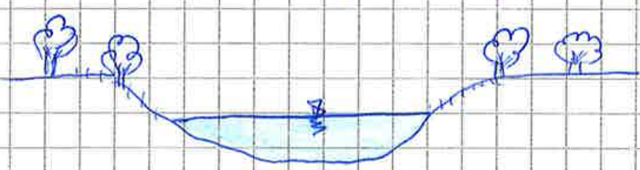
4/12/17

HOW TO DETERMINE A DESIGN STREAMFLOW WITHOUT STREAMFLOW DATA.

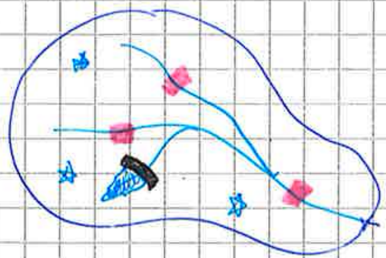
1) Definition of design storm.
2) Model the catchment response
3) Design streamflow { RAINFALL - RUNOFF TRANSFORMATION }

Measure river flow is more difficult because it costs, ∄ enough instruments, cross-sections are complex: geometry, change in time.

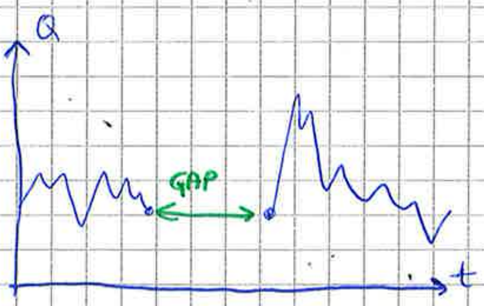
$Q = V \cdot A$



Also velocity is different in the river depths.

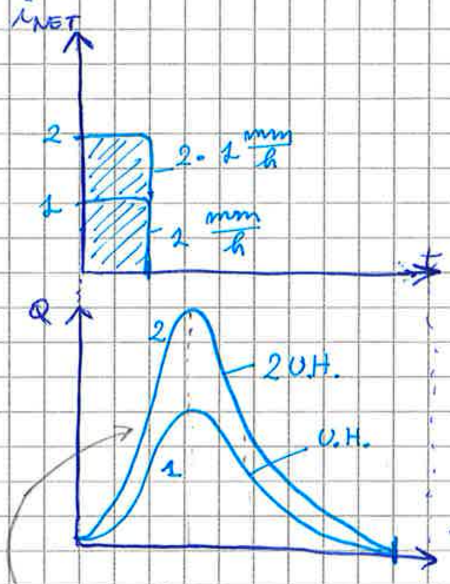


OTHER APPLICATIONS OF R-R TRANSFORMATION:

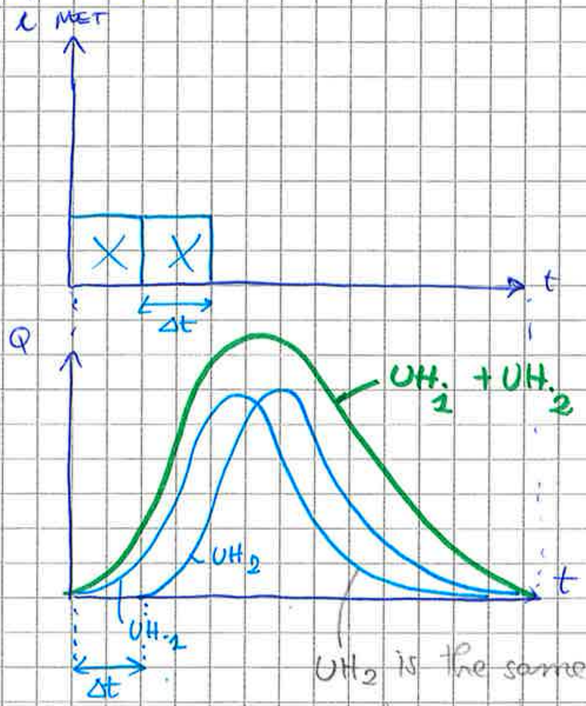


- FILLING GAPS in data of (Q - t)
- UNCERTAINTY IMPROVEMENT
- FLOOD ANTICIPATION
- ASSESS SCENARIOS: in the case of modifications in the catchment, ∃ will be modifications at the outlet, reservoirs

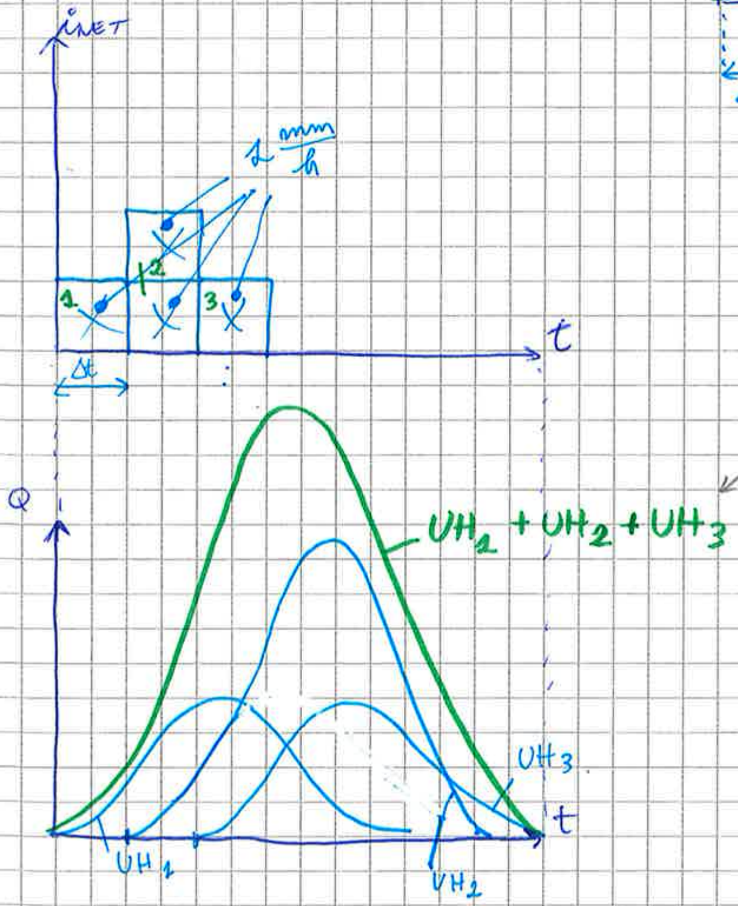
- U.H. depends on the characteristic of the catchment.
- assumption:
  - the shape of hydrograph is indipendent of time
  - duration is invariant with intensity.
- response of the catchment is linear:
  - H. $\propto$ INTENSITY (hydrograph)
  - final H results from superimposition of terms



values of 2·UH are those of UH but doubled.



UH_2 is the same as UH_1 but traslated by Δt.



SUM ALL UHs at the END.

137

5) 3^{rd} time step $2\Delta t \rightarrow 3\Delta t$: $Q_3 \propto i_3 \mu_1 + i_2 \mu_2 + i_1 \mu_3$

$$\Rightarrow \underbrace{Q_t}_{\text{RUNOFF AT OUTLET } [\frac{m^3}{s}]} = \underbrace{K}_{\text{CONVERSATION FACTOR } = \frac{1000}{3600}} \cdot \sum_{j=1}^{t} \left(\underbrace{i_j}_{\text{NET PRECIP INTENSITY } [\frac{mm}{h}]} \cdot \underbrace{\mu_{t-j+1}}_{\text{CONTRIBUTING AREAS } [km^2]} \right)$$

⇝ When precipitation stops or all the areas have contributed,

! If $\exists \neq$ gauges $\Rightarrow \exists \neq$ values of i $\Rightarrow$ so first compute the areal average because I want uniform precipitation.

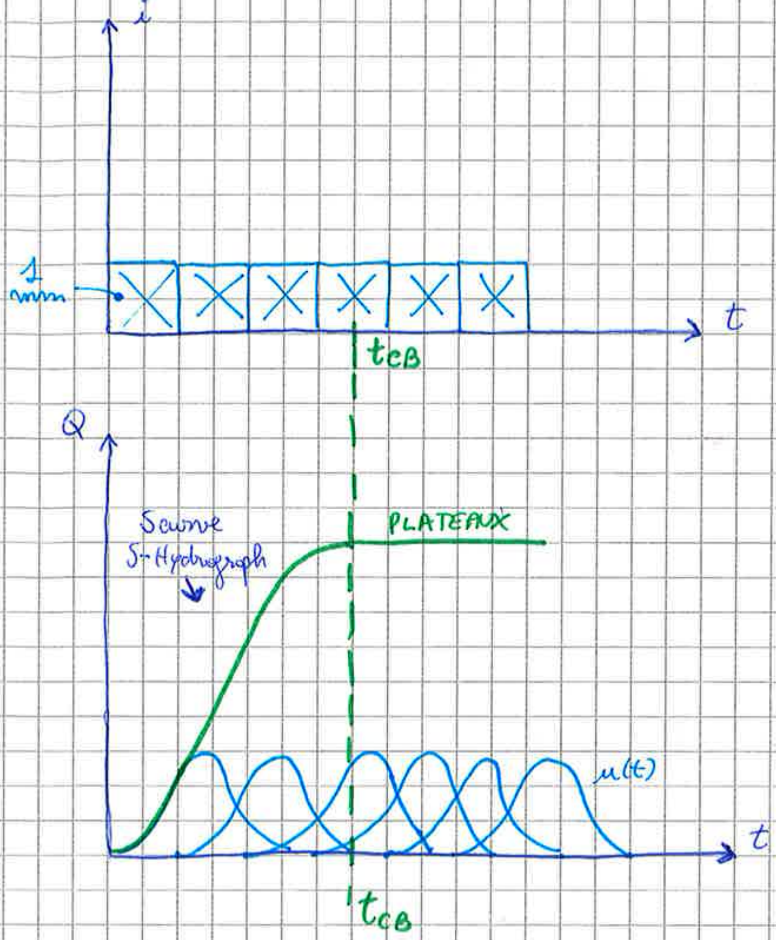
⇒ MAKE A TABLE

AREA / INTENSITY	μ_1	μ_2	μ_3	μ_4	Q = SUM BY ROWS decreasing
i_1	$\mu_1 i_1$				$Q_1 = \mu_1 i_1$
i_2	$\mu_1 i_2$	$\mu_2 i_1$			$Q_2 = \mu_1 i_2 + \mu_2 i_1$
i_3	$\mu_1 \cdot i_3$	$\mu_2 i_2$	$\mu_3 i_1$		$Q_3 = \dots$
i_4	$\mu_1 i_4$	$\mu_2 i_3$	$\mu_3 i_2$	$\mu_4 i_1$	$Q_4 = \dots$
i_5	$\mu_1 i_5$	$\mu_2 i_4$	$\mu_3 i_3$	$\mu_4 i_2$	$Q_5 = \dots$
i_6	$\mu_1 i_6$	$\mu_2 i_5$	$\mu_3 i_4$	$\mu_4 i_3$	$Q_6 = \dots$
		$\mu_2 i_6$	$\mu_3 i_5$	$\mu_4 i_4$	$Q_7 = \dots$
			$\mu_3 i_6$	$\mu_4 i_5$	$Q_8 = \dots$
				$\mu_4 i_6$	$Q_9 = \dots$

$t_{CB} = 4 \cdot \Delta t$

$d = 6 \cdot \Delta t$ — number of intensity

138



What if $d > t_{cB}$?

All the basin contributes

$$S = \int_0^t u(\tau)\, d\tau$$ integral of unit hydrograph

But what is the function $u(t)$? How to define it?

↓

∃ couples of methods

METHODS TO DERIVE U.H. (REQUIRE MEASUREMENTS)

DECONVOLUTION METHOD:

SET OF CORRESPONDING MEASURES OF RAINFALL AND FLOW RATE

FORWARD ↙ ↘ BACKWARD

starting from $\frac{Q_n}{i_n} = u_m$.

1

$$u_1 = \frac{Q_1}{i_1}$$

$$u_2 = \frac{Q_2 - i_2 u_1}{i_1}$$

$$\vdots$$

$$u_3 = \frac{Q_3 - i_2 \cdot u_2 - i_3 \cdot u_1}{i_1}$$

$$\vdots$$

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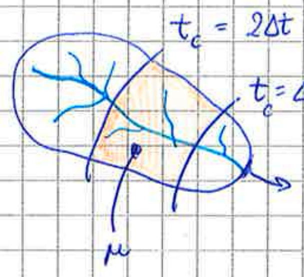
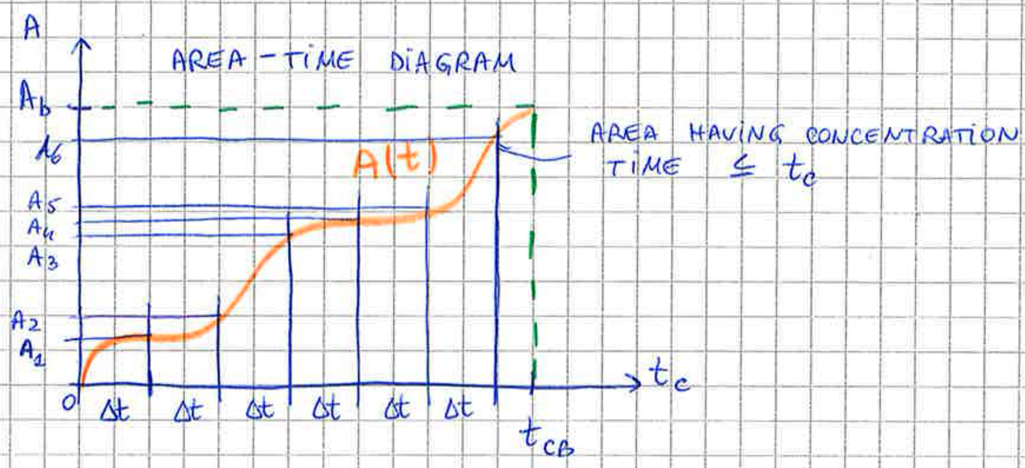
METHOD NOT REQUIRING MEASUREMENTS Q:

BASED ON THE AREA-TIME DIAGRAM

μ = area contributing, between two iso-CHRONE lines



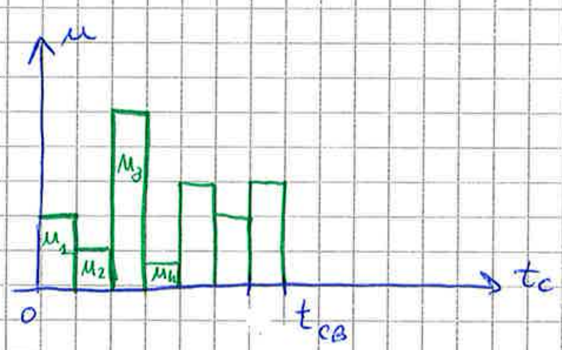
Since we have assumed an uniform distribution, properties over all catchment, (μ) can be found not considering the iso-chrones



Δt : $\quad \mu_1 = A_1$

$\Delta t \div 2\Delta t \quad \mu_2 = A_2 - A_1$

$2\Delta t \div 3\Delta t \quad \mu_3 = A_3 - A_2$



$$\Rightarrow \mu(t) = \frac{dA}{dt_c}$$

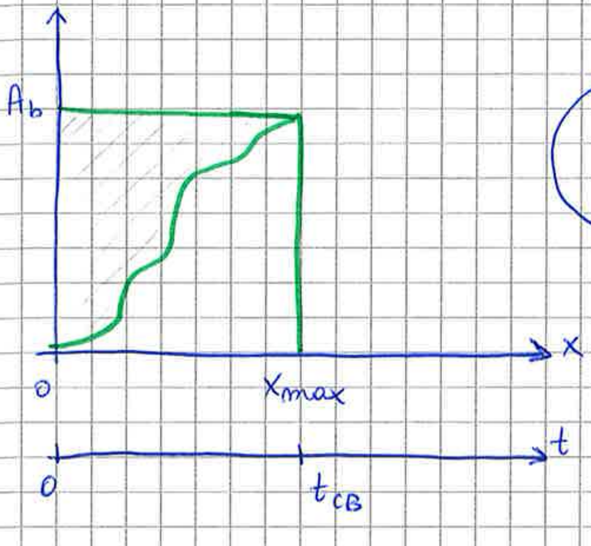
$\mu(t)$ is the derivative of the function $A(t)$, but how to define $A(t)$?

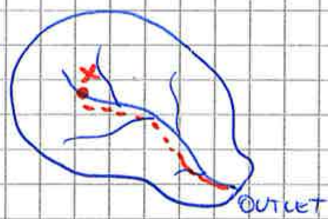
↓

$\exists \neq$ methods:

143

METHOD (B) FOR AREA - TIME CURVE





There is parallelism between area - time diagram point inside my basin.

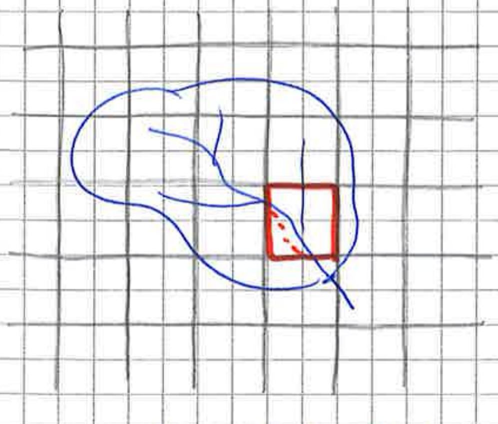
I will find the closest point on the network (stream flow network) and I measure the distance along the network untill the outlet.

PROCEDURE:
- area distance curve
- use time axis
- convert time steps put into t_{CB} to find x.
- enter x to the curve and find area
- differentiate the area

$$\frac{x}{X_{max}} = \frac{t_c}{t_{CB}}$$

x = distance of a point along the network
X_{max} = max dist.
t_c = concentration point of any point
t_{CB} = concentration time of the basin.

METHOD (C) FOR AREA - TIME CURVE

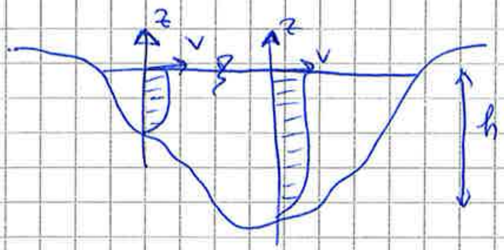


PROCEDURE:
- superpose a grid on the basin
- for any cell find:
  - channel path and length
  - inclination (averaged)
  - channel cross - section
  - velocity $v = \chi\sqrt{R \cdot i}$ (Chezy)
  - travel time $t = \frac{x}{v}$ (time from entering in the cell to exiting).
  - identify down stream path

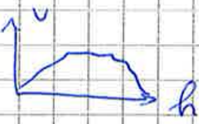
- quantify additional time t^* to reach the channel
- Travel time for droplets fallen in the cell

$$t_c = t^* + \sum_{\forall i} t_i$$

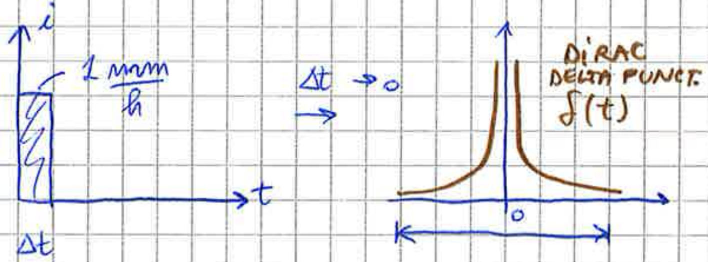
VELOCITY PROBES $\quad Q = \bar{V} \cdot A$



A device is immerged. It has an helix whose speed is function of the flow velocity. We have different velocity ⇒ take the mean $\bar{V}$ of each depth (h)



(4₁) ISTANTANEOUS UNIT HYDROGRAPH (IUH)



The integral is = 1 as $\Delta t \to 0$.

→ CONVOLUTION METHOD $\quad \phi(t) = \int_0^t i(\tau) \cdot \underbrace{u(t-\tau)d\tau}_{= dA}$

$\sum_{j=1}^{t} i_j \cdot u_{t-j+1}$, $\quad Q(t) = \int_0^t \delta(\tau)\, u(t-\tau)\, d\tau$

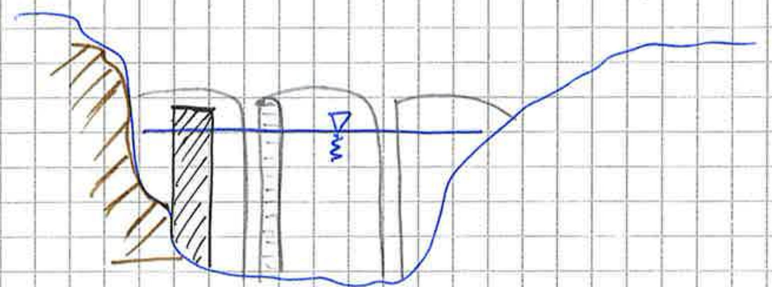
⇒ Property of δ-DIRAC : $\underbrace{Q(t)}_{\text{hydrograph}} = \underbrace{u(t)}_{\text{unit hydrograph}} \cdot K \qquad K$ = factor of proportionality

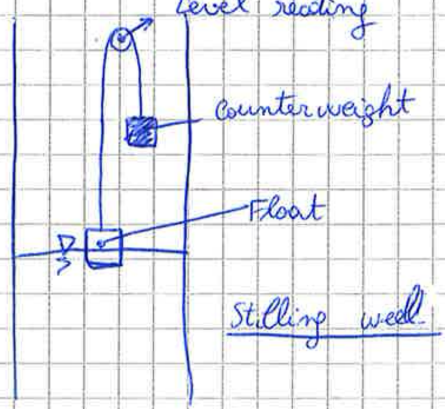
$\boxed{Ex}$ RAINFALL → RUNOFF TRANSFORMATION. ?

11/12/17

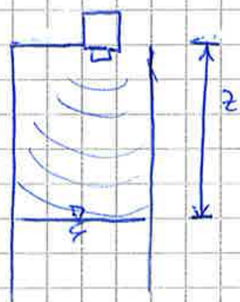
MEASUREMENTS

- Level Measurements
- Velocity Measurements
- Other systems.



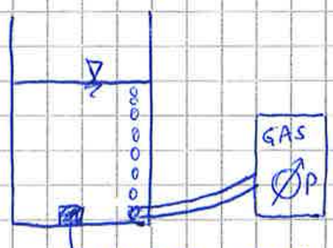


Ultrasound level gauges

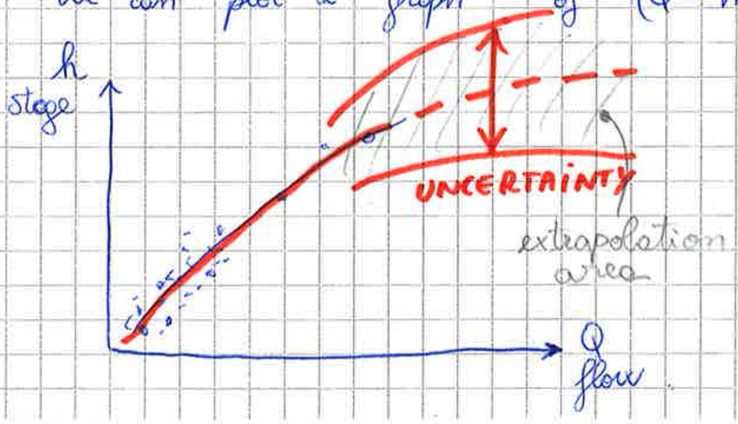


Pressure:

$$h = \frac{p}{\gamma}$$ specific weig



We can plot a graph of $(Q - h)$:

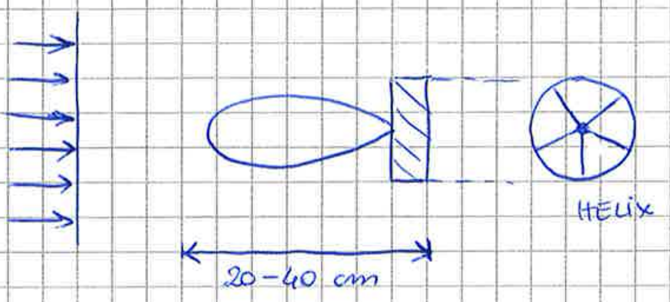


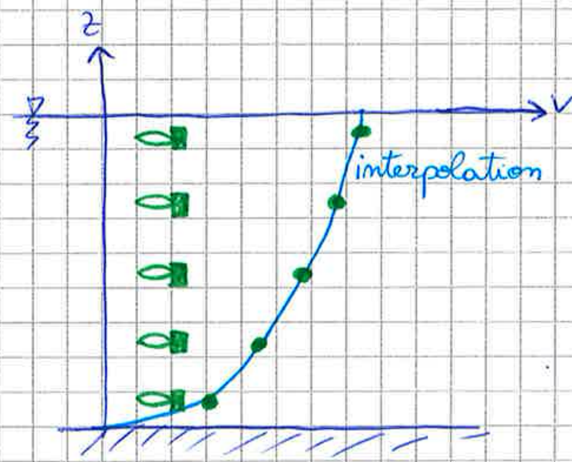
How to measure Q?

The number of points are more frequent for lower flows and usually ∄ data for big flows: we interpolate and provide a range of uncertainty

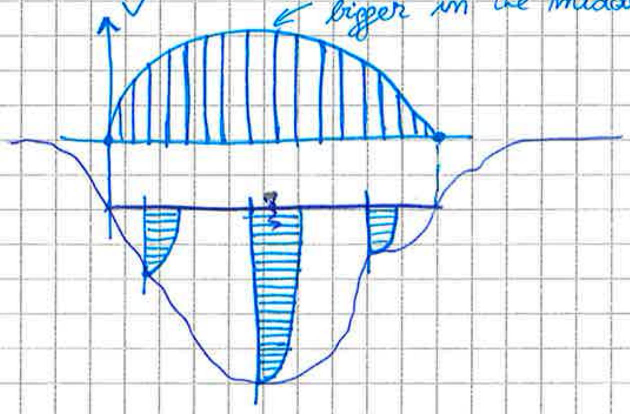
VELOCITY PROBES FOR POINT MEASUREMENTS.

Small device that measures point velocities, but it takes time





Velocity changes also along the horizontal profile

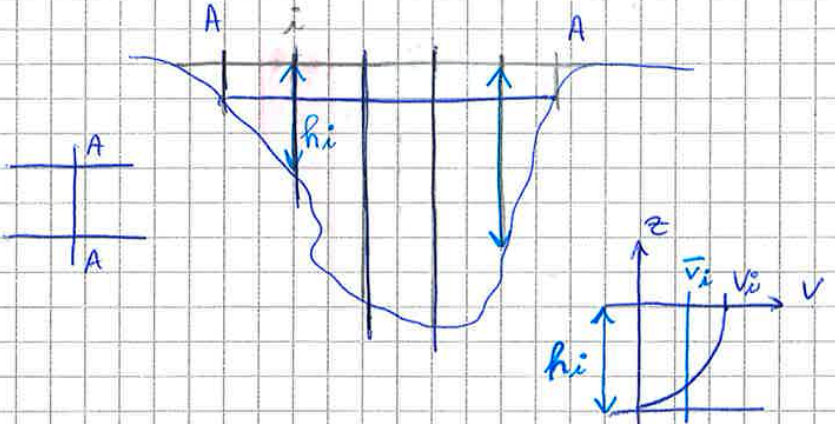


- Take cross - section
- Divide into sections
- Operator stays outside the water
- Immerge the device till the bottom in each section.
- Obtain N profiles
- Assume the same velocity V_i in the area ( ) and take its mean
- Calculate flow Q_i:

$$Q_i = \overline{V}_i \cdot d_i \cdot h_i$$

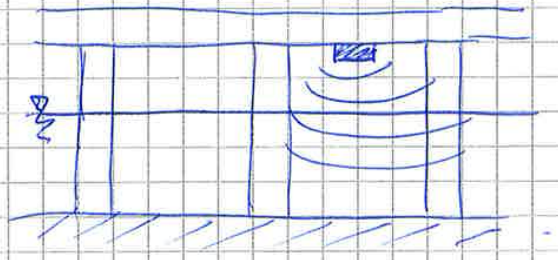
- Calculate Q_{TOT}:

$$Q_{TOT} = \sum_i \cdot Q_i$$



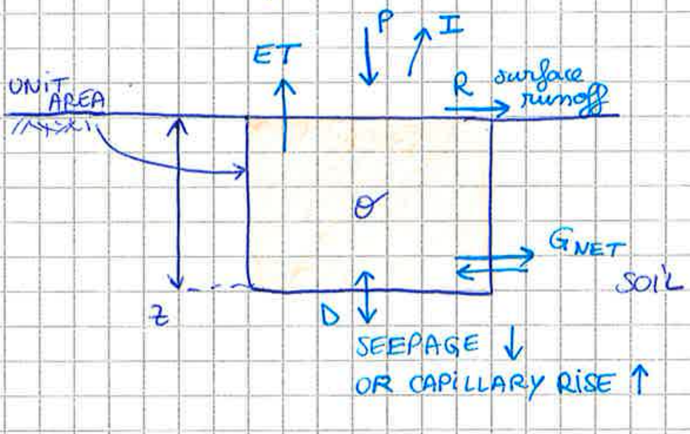
⇒ This procedure is long and during flud events is difficult, but it gives good results.

ULTRASOUND



SOIL WATER BALANCE AT A POINT

∃ ≠ terms in this equation, that we can simplify making some assumptions.



What is the storage of water in this unit area of soil? It's quantified by θ = water content in soil:

$$P - I - R - (ET + D + G_{NET}) = \underbrace{\frac{d}{dt}}_{\text{temporal}} \Big(\theta \cdot \underbrace{\overset{\text{constant}}{z} \cdot 1}_{\text{area}}\Big) = z \cdot \frac{d\theta}{dt}$$

variation

Radiations

Every body with $T > 0°K$ emits electromagnetic radiations, whose amount depends on the body's temperature following some laws:

BLACK BODIES: that don't reflect light

Stefan-Boltzmann Law $J = \sigma \cdot T^4$

J = radiance or energy per unit area and time $\left[\frac{W}{m^2}\right]$

σ = Stefan - Boltzmann constant $= 5.76 \cdot 10^{-8} \left[\frac{W}{m^2 K^4}\right]$

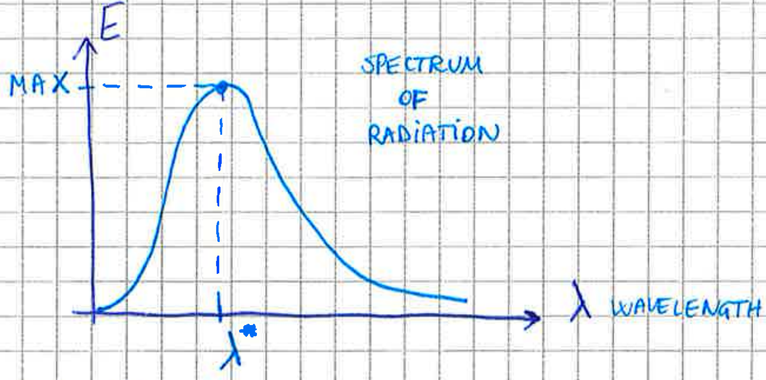
T = body temperature $[K]$

NON BLACK BODIES (general): $J = \varepsilon \sigma T^4$ ε = emissivity

$\varepsilon = 1$ for black bodies

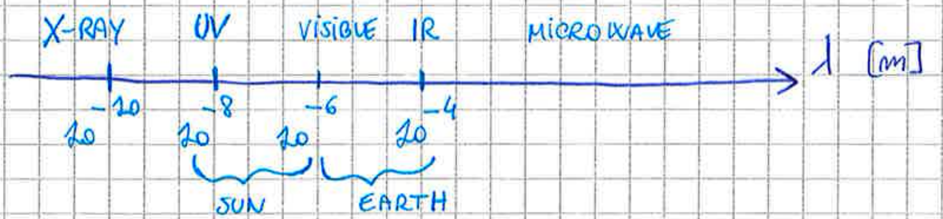
$\varepsilon \in [0,9 ; 1]$ for soil

$\varepsilon \in [0,7 ; 0,8]$ for vapour / clouds.



Wien's Law

$$\lambda^* = \frac{2900 \cdot 10^{-6}}{T} \qquad [T] = [K] \;,\; [\lambda^*] = [m]$$



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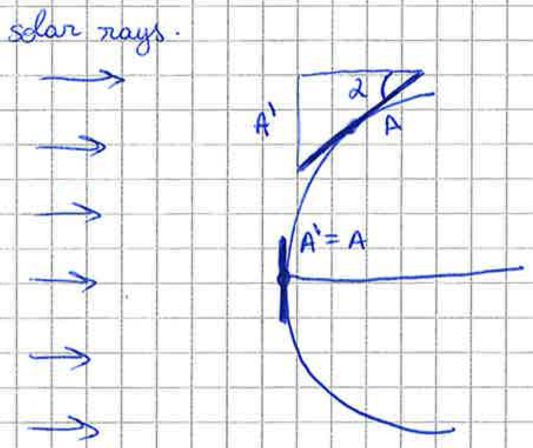
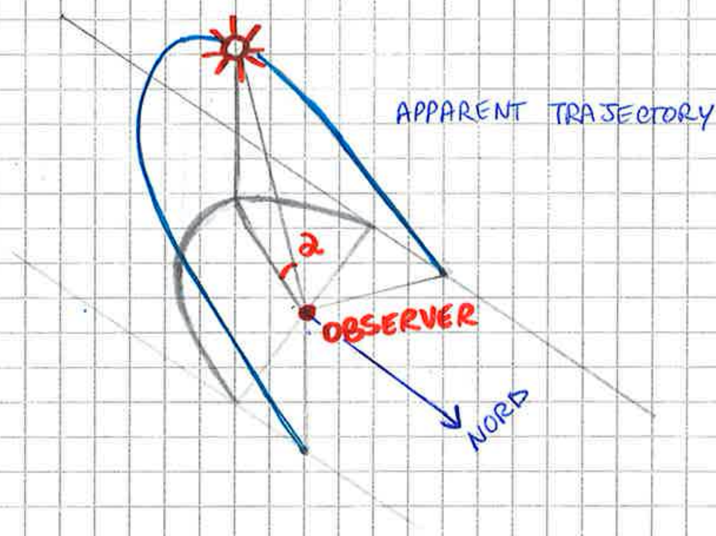


$$\sin\alpha = \frac{A'}{A}$$

A = unit surface

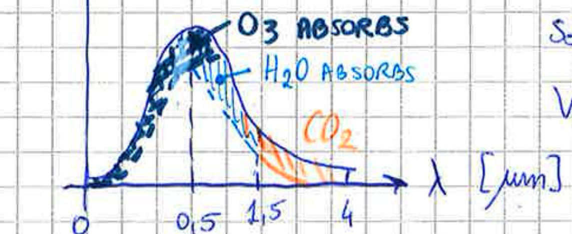
$$\Rightarrow \quad S' = S_0 \cdot d_r \cdot \sin\alpha$$

α DEPENDS ON:
- latitude
- inclination of earth axis (declination) depends on day of year
- time of day.



$S' = S_0 \cdot d_r \cdot \sin(\alpha)$ time-specific and place-specific extra-atmospheric radiation, wich has a given spectrum.

Energy emitted by Sun: S



Solar radiation: $0{,}17 \div 4\ \mu m$ (λ)

Visible radiation $0{,}4 \div 0{,}7\ \mu m$

Different wave lengths are absorbed by different components of the atmosphere: O_3 OZONE shorter waves.

H_2O WATER IN ATMOSPHERE:

CO_2 ABSORBS

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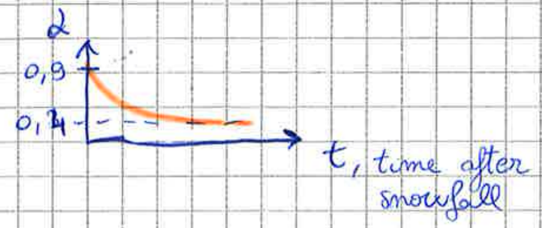


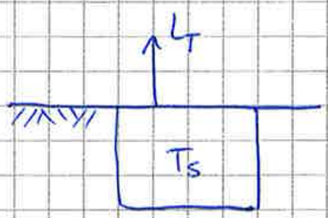
FEEDBACK : cases in which a dynamic is influenced by some variables, but the variables depend on feedback.

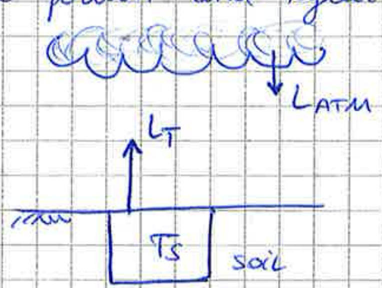
ex → CLIMATE CHANGE : increase of temperature ⇒ increase of ice - melting ⇒ more surface with lower albido → less reflection (outgoing radiation) ⇒ increase of temperature

LONG WAVE RADIATION :

$T_{SOIL} = 15°C = 288°K$ ⇒ longer wave length : range of infrared

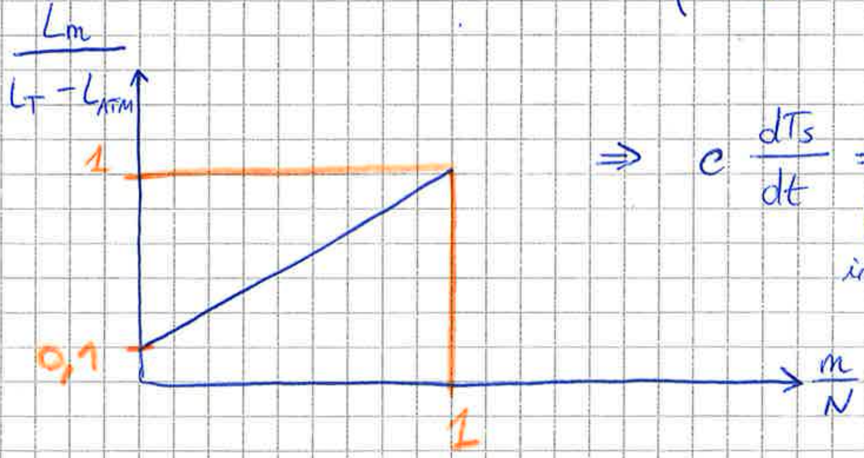
$L_T \cong \sigma T_{SOIL}^4$ Clouds absorb a portion and reflect it back





$L_{ATM} = 0,75\ \sigma\ T_{AIR}^4$

$$\Rightarrow L_{net} = (L_T - L_{ATM}) = \left(0,9 \frac{m}{N} + 0,1\right)$$



$$\Rightarrow c \frac{dT_s}{dt} = \overbrace{S_n}^{S_T - \alpha S_T} - L_n - ET \rho \lambda$$

(+°C) (−°C)
incoming outgoing

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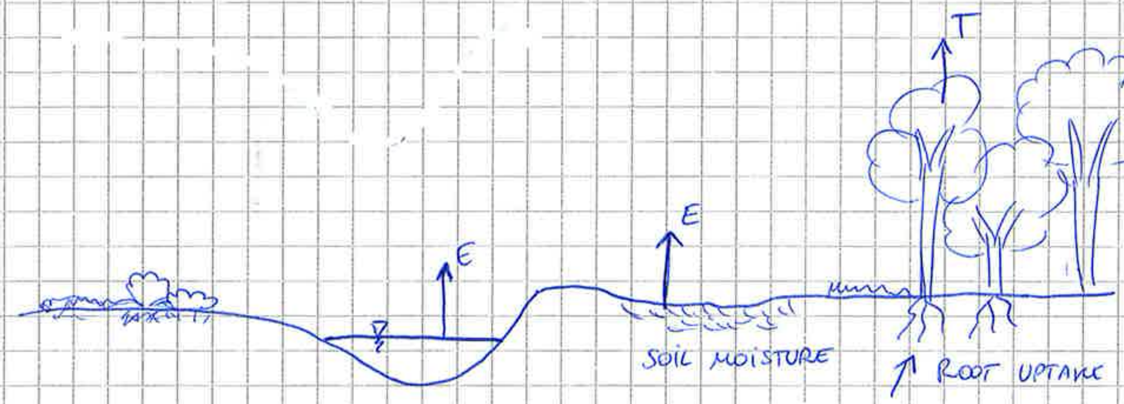
EVAPOTRANSPIRATION

• EVAPORATION



HRTW

Endogenous process : - radiation
- humidity of air (vapour pressure)
- wind (that moves humid air).
- temperature

⇒ Use the energy balance equation to find the POTENTIAL EVAPOTRANSPIRATION that gives an upper bound to ET: it's the maximum rate allowed by energetic condition.

$$c\,\frac{dT_s}{dt} = S_n - L_n - ET\cdot\rho_w\lambda$$

We consider long time scale because it's a long process ⇒

$$\frac{dT_s}{dt} = 0 \quad \Rightarrow \quad \boxed{ET_{POTENTIAL} = \frac{S_n - L_n}{\rho_w\cdot\underbrace{\lambda}_{\text{latent heat}}}}$$

ex summer day (Piemonte):

$$S_n > L_n \quad \Rightarrow \quad S_n - L_n = 150\ \frac{W}{m^2} \qquad \rho_w = 1000\ \frac{kg}{m^3}$$

$$\Rightarrow ET_p = \frac{150\ \frac{W}{m^2}}{1000\ \frac{kg}{m^3}\cdot 2{,}25\cdot 10^{-6}\ \frac{J}{kg}} \simeq 6{,}6\cdot 10^{-8}\ \frac{m}{s} = 5{,}7\ \frac{mm}{day}$$ MAX ALLOWED BY ATMOSPH

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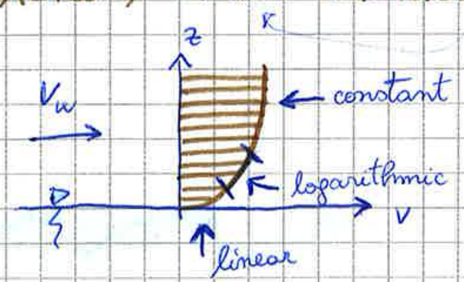
Energy Balance Method

$$ET_p = \frac{S_n - L_n}{\rho \cdot \lambda} = E_{p,r} \rightarrow \text{radiation}$$

AERODYNAMIC METHOD



$$E_{p,a} = B\,(e_s - e) = B \cdot e_s (1 - RH)$$

saturated vapour press. (e_s); vapour pressure (e)

⇒ MIXED METHOD (Penmann) → to compute potential evaporation

$$E_p = \frac{\Delta}{\Delta + \gamma} \cdot E_{p,r} + \frac{\Delta}{\Delta + \gamma} \cdot E_{p,a} \rightarrow \text{average the 2 components}$$

γ = PSYCROMETRIC CONSTANT

$$\gamma = \frac{P_{air} c}{0{,}622 \cdot \lambda}$$

P_{air} = pressure of air
c = specific heat
λ = latent heat

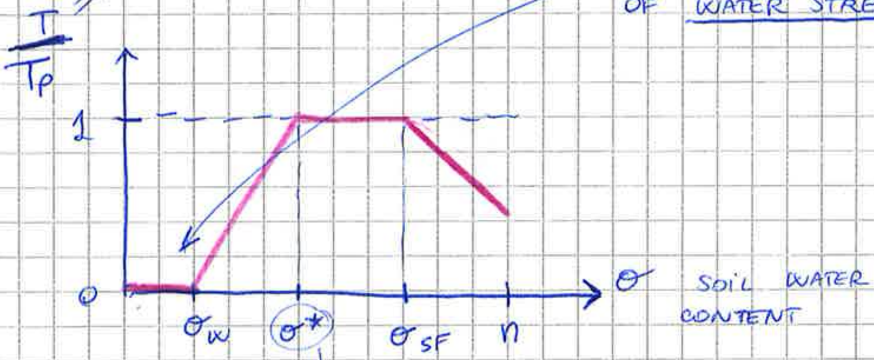
Δ = GRADIENT OF SATURATED VAPOUR PRESSURE

$$\Delta = \left.\frac{d e_s(T)}{dT}\right|_{T = T_{air}}$$

→ ! This rules apply both for evap from free liquid surface and for soil surface at $\theta = m$ (saturated soil, so ∃ water on the surface).

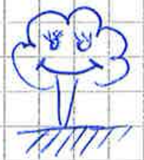
T = transpiration

PLANTS SUFFER OF WATER STRESS → stomata closed



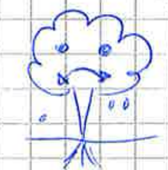
θ_W WILTING POINT

θ_{SF} FIELD CAPACITY
↓
water content at which gravity forces equal the capillarity rised = capillarity exist. If F_g > capillary ⇒ ∃ drainage and drynage.
↓
MIX OF SOIL, WATER, OXIGEN
⇒



θ^* → FULL OPENING OF STOMATA

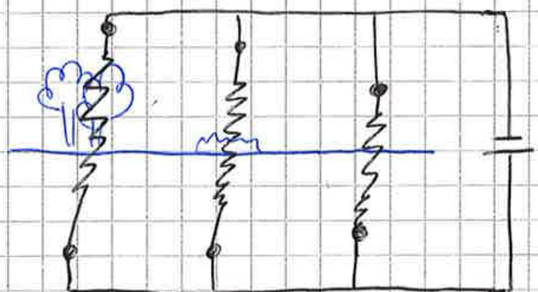
For $\theta > \theta_{SF}$ we go toward saturation
⇓
MIX OF SOIL, WATER BUT NO OXIGEN: roots die.



Unless ∃ plants adapted to an ANOXIS STRESS

ET FROM EMPIRICAL MODEL

PHYSICALLY BASED METHOD : PENMANN – MONTEITH FORMULA



It assimilates the different resistances to an electrical circuit in parallel.

THORNTHWAITE FORMULA

$$ET_p = 16\left(10 \cdot \frac{T_m}{I}\right)^{\alpha} \cdot L_m$$

T_m = mean mounthly temperature [°C]

I = thermal index that includes the temperatures throughout the year

19/12/17

EVAPOTRANSPIRATION FROM CROP FIELDS (FAO)

REFERENCE EVAPOTRANSPIRATION ET_0

↳ UNIFORM AND STANDARD VEGETATION ⇝ grass field

The field has enough water for the grass to live well.

CROP EVAPOTRANSPIRATION $ET_C = K_c \cdot ET_0$ K_c = crop coefficient.

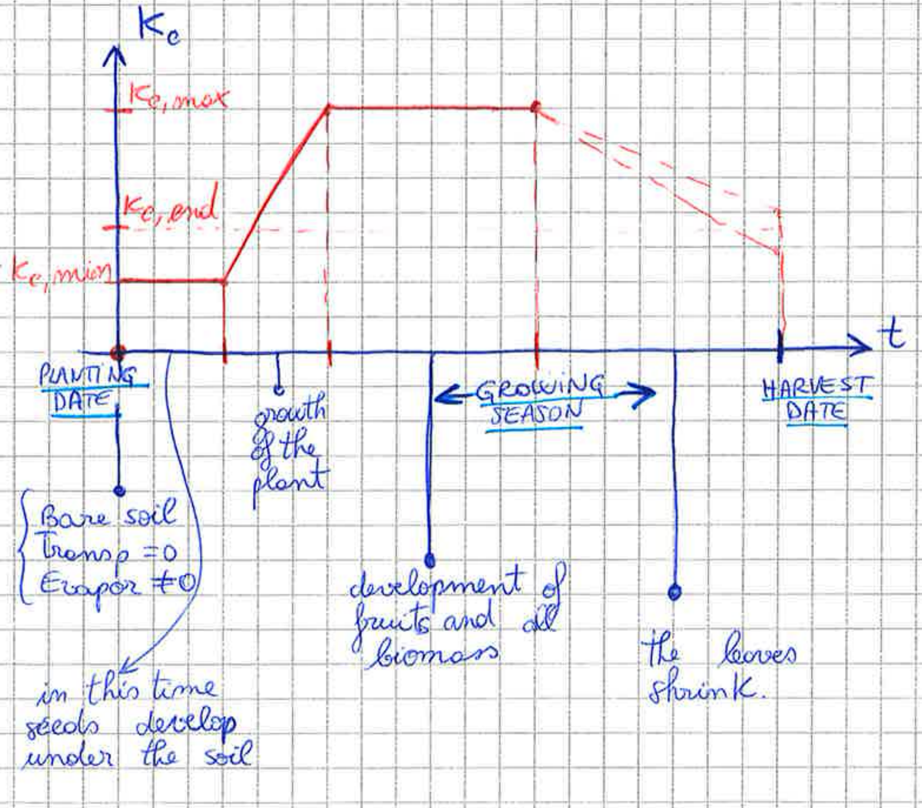
↳ SPECIFIC CROP (fruit trees, wood plantation, ...)

The field has enough water and nutrients.

ACTUAL EVAPOTRANSPIRATION $ET_A = K_s \cdot ET_C$, K_s = water stress coeffic.

↳ SPECIFIC CROP $ET_A = K_s \cdot K_c \cdot ET_0$

Real water conditions



($K_{c,max}$) depends on the crop type

($K_{c,min}$) depends on the soil (wetness)

($K_{c,end}$) depends on the type of crop and agricultural methods

$K_{c,max}$ can be > 1 but usually is < 1, i.e. a reduction coefficent

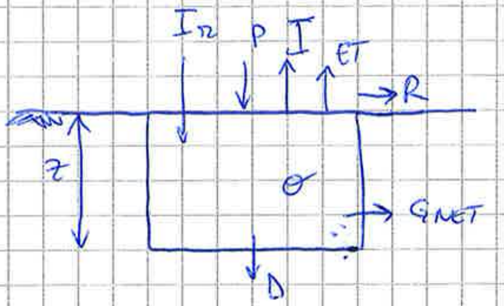
To know how much water to provide, use soil water balance:

$$z \cdot \frac{d\theta}{dt} = \underbrace{(P - I - R)}_{\text{that infiltrates}} + \underbrace{I_r}_{\text{irrigation}} - ET - \underbrace{D}_{\text{pass over the root zone and recharge ground water}} - G_{NET}$$

AVAILABLE

EXITING FROM CONTROL AREA / VOLUME

depth of active soil $\equiv$ depth of root zone.



$P - I - R = P_{eff}$ Effective precipit.

D is mantained ≈ 0

$G_{NET} = 0$ because surface is flat

⇒ Simplified equation: $P_{eff} + I_r - ET = 0$

$I_r = ET - P_{eff}$

$ET_a = ET_c$ → this is what we want to achieve

if negative ⇒ no need of I_r

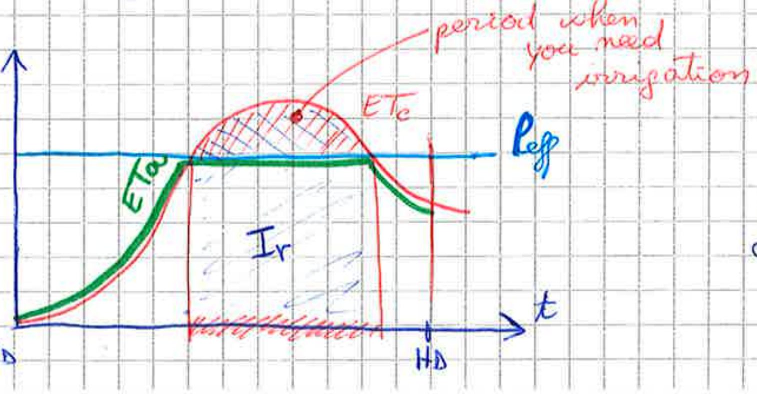
↳ $ET_c - ET_a < 0$

Bulk Assessment $I_r = \int_{PD}^{HD} (ET_c - ET_a)\, dt$ → ! maybe with this expression we see that we don't need irrigat.

HD = Harvest date, PD = Planting date

But if we consider temporal variability:

$$\begin{cases} I_{r,i} = ET_{c,i} - ET_{a,i} & \text{if } I_{r,i} > 0 \\ i = \text{DAY / WEEK / MOUNTH} & \\ I_{r,i} = 0 & \text{if no need of irrigation.} \end{cases}$$



$$z \frac{d\theta}{dt} = P_{eff} - ET_a - D - G_{net}$$

$$P_{eff} = P - I - R$$

Irrigated soil water content is described by $z \frac{d\theta}{dt} = P_{eff} + I_r - ET_c - D - G_{net}$

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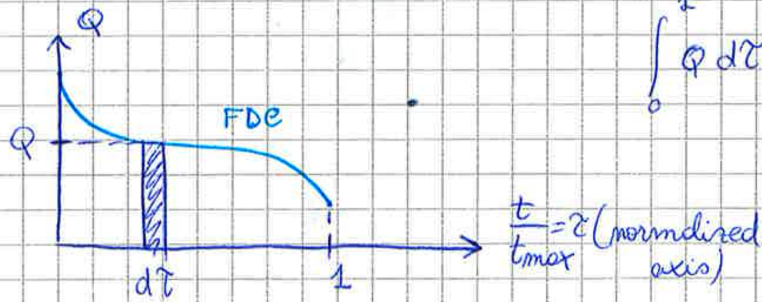
Q = mean daily flow rate

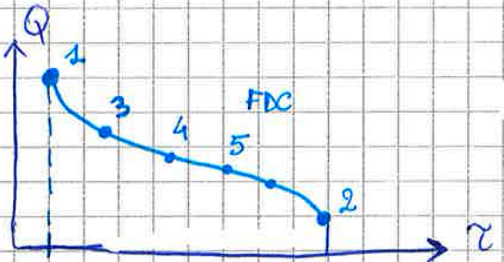
t_{max} = 1 year.



$$\int_0^1 Q \, d\tau = \frac{|W|}{t_{max}} = \overline{Q}$$

$|W|$ → water volume.

$\overline{Q}$ = mean annual flow rate



① $t = 1$ day   max annual daily flow rate
$\tau = \frac{1}{365}$

② $t = 365$ days, $\tau = 1$ min annual daily flow rate

③ $t = 91$ days, $\tau = 0,25$ ordinary high flow

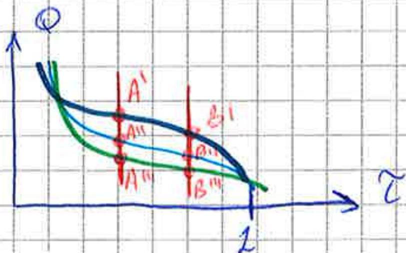
④ $t = 182$ days, $\tau = 0.5$ semi-permanent flow

⑤ $t = 274$ days, $\tau = 0.75$ ordinary low flow.

Multi annual time series of Q

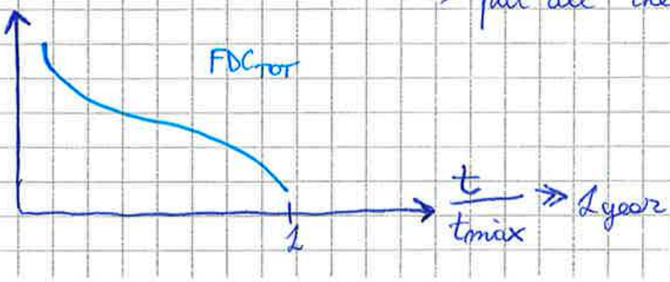
- Mean Annual FDC → build the FDC for each year.
  ⇒ take the vertical mean in each point

$$A = \frac{A' + A'' + A'''}{3} \; ; \; B = \frac{B' + B'' + B'''}{3} \; ; \ldots$$



⇒ plot $(A, B, C, \ldots)$ to have the mean curve.
→ we can see the intra annual variability

OR   TOTAL DC → we can see the inter-annual variability
⇒ put all the years together.



Q	i	$F(i)$
Q_{max}	1	$1/366$
Q_{364}	2	$2/366$
Q_{363}	3	$3/366$
Q_{362}	4	$4/366$
	⋮	⋮
Q_1	365	$365/366$ → a low probabil. that the amount is not exceeded

HYDROPOWER PLANTS

1) Design a flow rate from this curve: evaluate cost and benefits

2) The design flow rate cannot be the maximum because you take out all water and this kills the environment
⇓
∃ limitations: ∃ a min flow in the river to sustain environment.

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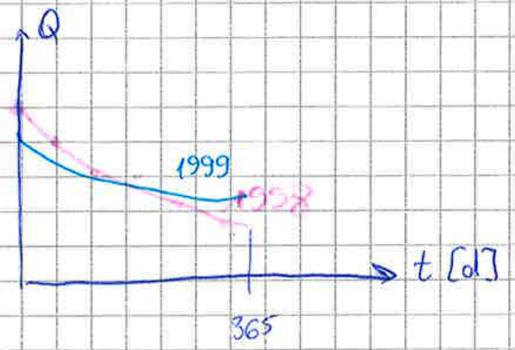
3. Plot the sorted sequence

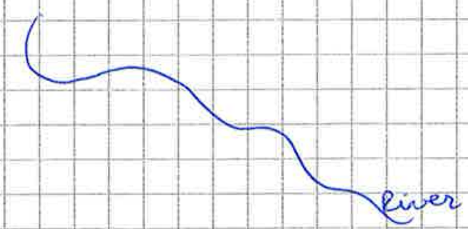


4. Repeat for all years

5. Take the mean : you have to take the mean of each row of the table

6. Plot the mean values.

FDC is specific for a given cross sections.
This serves to know how much water is in the river.



- domestic use (rural areas)
- industrial use (cooling, pollution, ..)
- agricultural use (irrigation) volume
- hydropower use volume + economical

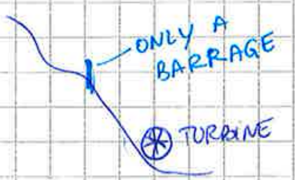
Or they're classified by <u>head</u>:

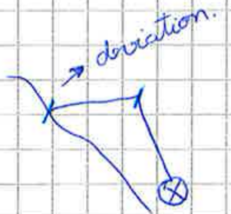
LOW	HEAD	$H < 50\ m$
MEDIUM	"	
HIGH	"	$H > 200\ m$

Or for <u>facility type</u>

① IN - STREAM
② RUN - ON - RIVER
③ RESERVOIR
④ PUMPED STORAGE

SIMPLE ↓ COMPLEX

①


②


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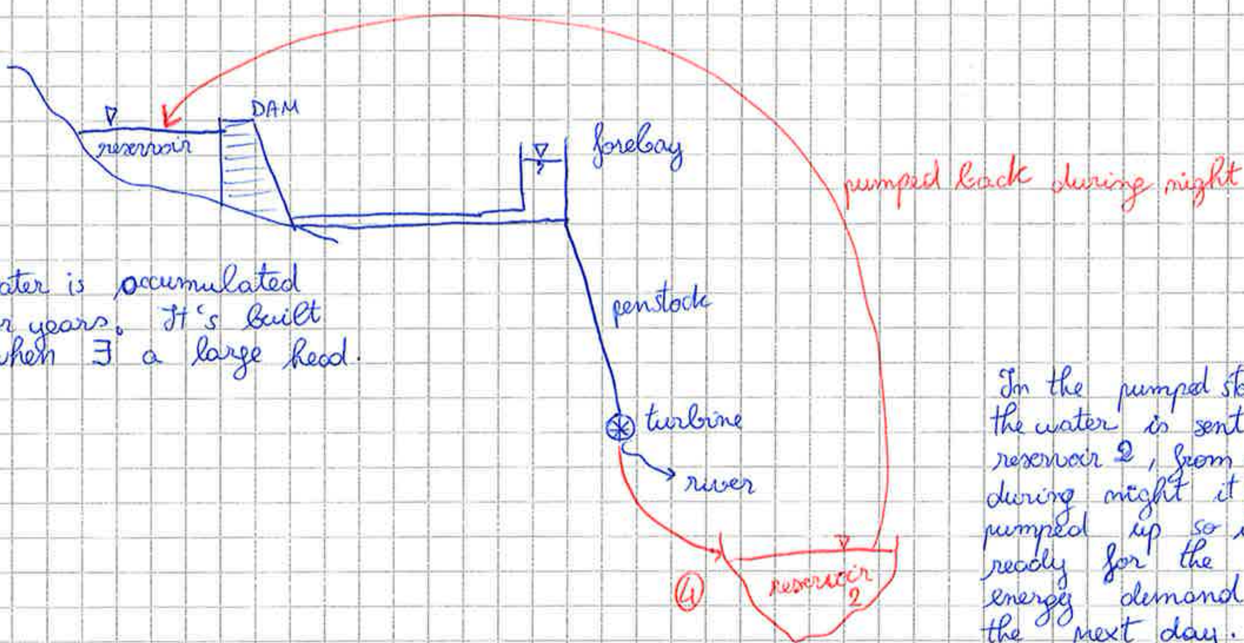
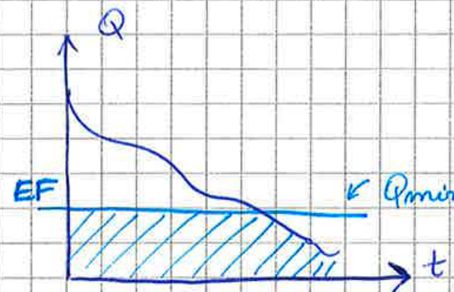


Water is accumulated for years. It's built when ∃ a large head.

In the pumped storage, the water is sent to reservoir 2, from which during night it's pumped up so it's ready for the energy demand of the next day.

Env. flow Q (EF) : ∃ a minimum istantaneous flow to be guaranted downstream a withdrawal



$$\boxed{EF = K\,\bar{Q}\cdot M\cdot A}$$

K = fraction given over homogeneous areas based on the low flows of the river

$K \in [0{,}1 \, ; \, 0{,}15]$

$\bar{Q}$ = mean annual natural flow in the cross-section of interest.

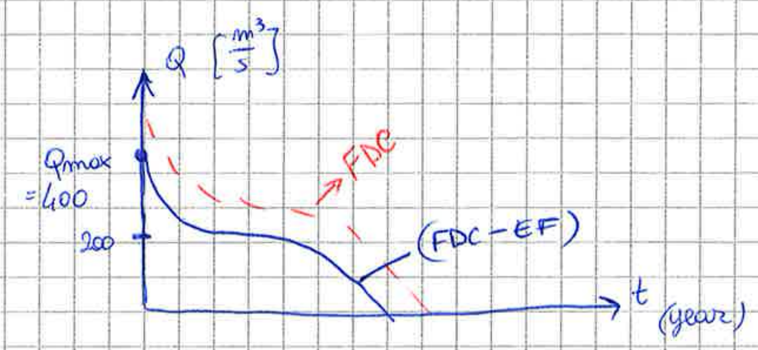
M = parameter that quantifies river bed morphology

$M \in [0{,}9 \, ; \, 1{,}3]$

A = parameter for groundwater exchange

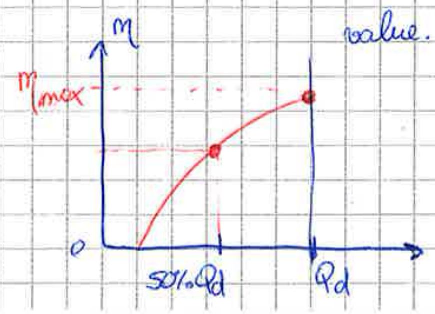
$A \in [0{,}7 \, ; \, 1{,}5]$

→ Build a NEW FDC that shows the real availability:

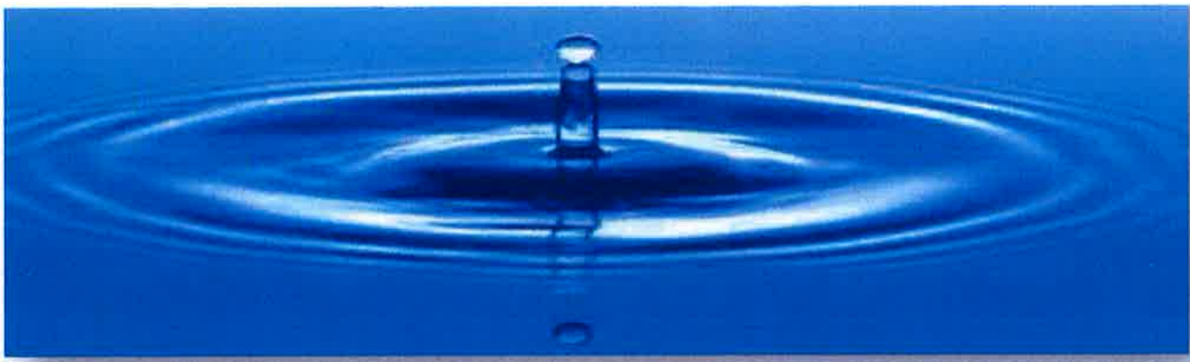


→ For the design of a hydroplant we know Q, H, γ_{g}. We focus on Q. and we look for a design value Qd. How to choose it?

- $Q_{max} = Q_d$ is an overestimated value → big expensive machine. and also the efficiency η decreases below the design value. → lost of energy.



→ Find η_{max} and $€_{min}$.



POLITECNICO DI TORINO

Master in Civil Engineering

Hydrology Course: Report

Collection of assignments

Student:

Loredana Mihaela Chiforeanu

Professor:

Stefania Tamea

2017/2018

ASSIGNMENT 1

Design of the river banks' maximum height at a given river section

The design of river embankments is sought for protecting from flooding the industrial area near the river Dora Baltea at Tavagnasco. In particular, the height of the embankments needs to be determined to prevent, in the area of interest, flood events with return periods of 100 years.

The time series of the annual maximum discharge (m^3/s) of the Dora Baltea river at Tavagnasco is provided in file dorabaltea_tavagnasco.txt.
Assuming that the relationship between water level and discharge in presence of the embankments (obtained via a hydraulic simulation software) is the following:

$$Q = 142 * (h + 0.05)^{1.81}$$

where Q is the discharge in m^3/s and *h* is the water level (height).

You are asked to provide the following:

1. Graphics and metrics of data analysis, including the time series plot, in order to identify potential non-stationarities, and the cumulative distribution function.

2. The probability plots of empirical data for the exponential, normal, log-normal and Gumbel probability distributions, assessing how well data aligns to the distributions.

3. Estimate distribution parameters for those distributions that successfully comply with the preliminary assessments plus the GEV distribution using moments and L-moments methods (use equations provided in the file: formule-distribuzioni.pdf). Where possible, plot the distributions with the estimated parameters into the probability plots.

4. Run the Pearson and Anderson-Darling statistical tests with a significance level of 5% on the distributions with estimated parameters.

5. On the basis of the distributions passing the statistical tests, provide an estimate of the design flow corresponding to the return period *T* = 100 years, and determine the corresponding embankments height (without considering safety factors).

There are several maximum points identifying maxima annual discharges: the first one is in 1920, when there has been a Q=2670 m^3/s. From 1948 to 1993 there have been some big discharges with values between 1646 and 2300 m^3/s.

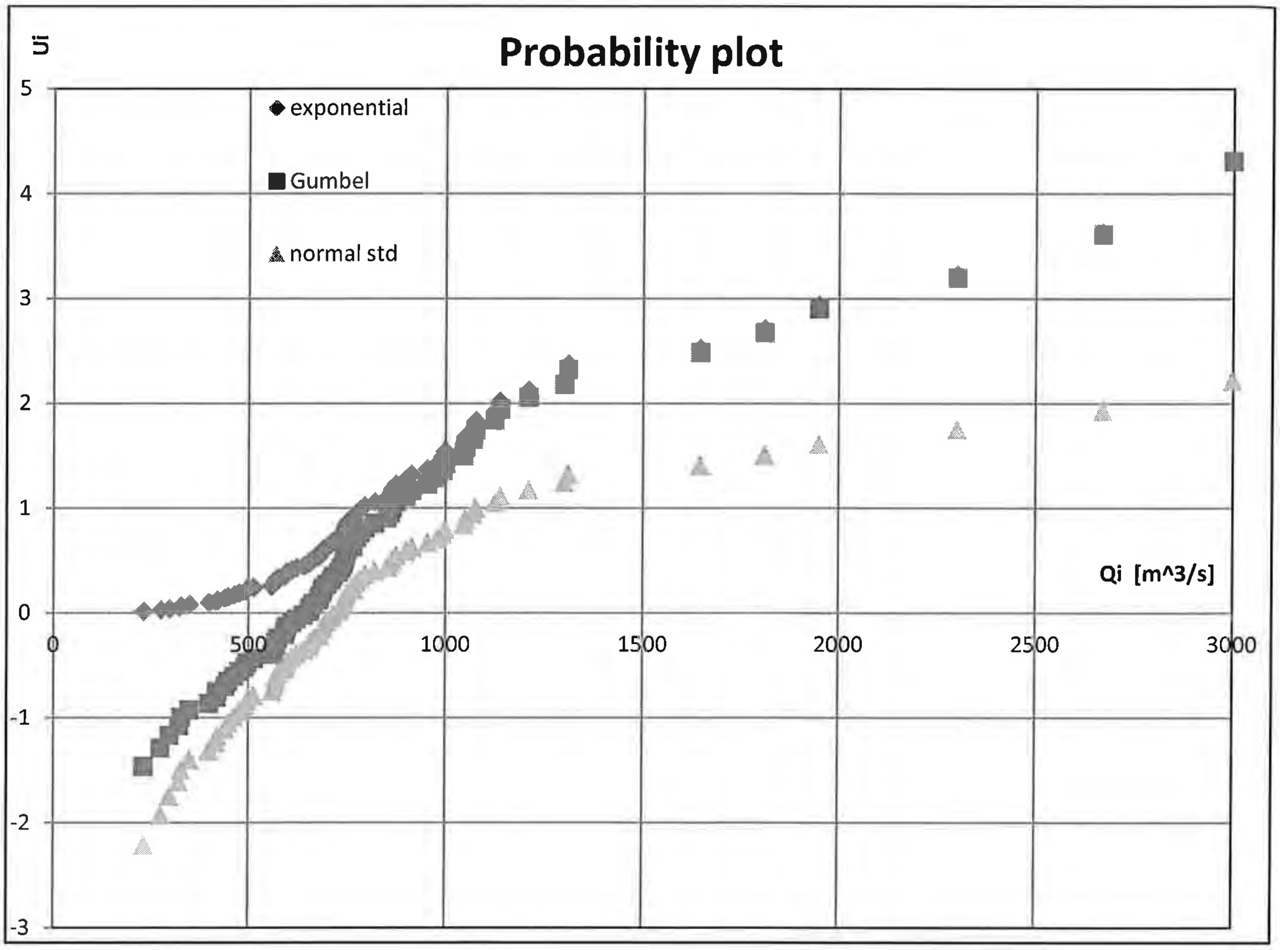
- Zero values:
  There are some values equal to 0 between 1920 and 1928 because there is no data information. The same is for year 2001.
- Other values:
  All the other discharge values are included in a range between 232 and 1310 m^3/s.

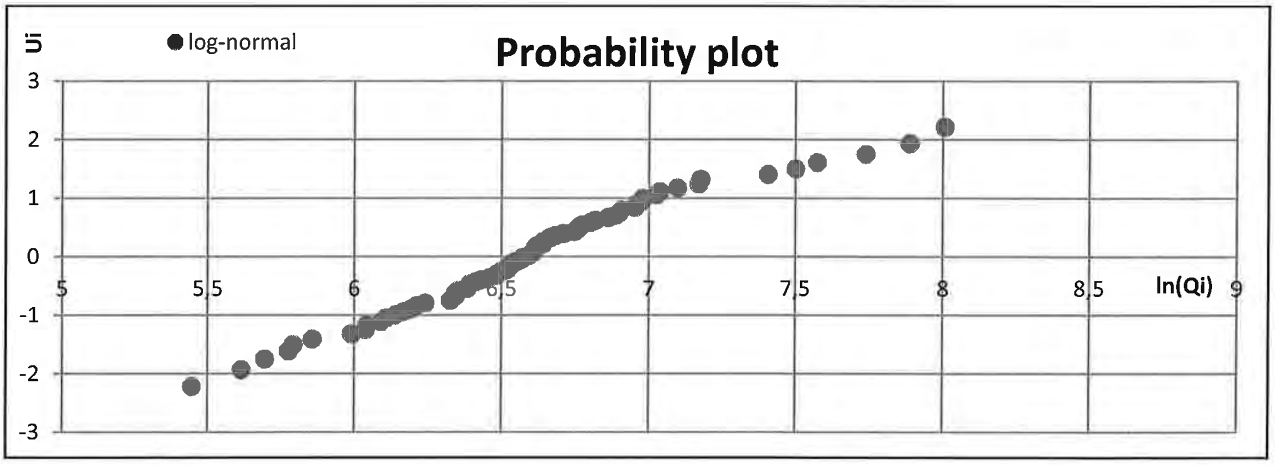
- Characterization and description of the sample:

	Mathematical formula	Value
Sample size	$N = count\ number\ of\ x_i$	74
Arithmetical mean	$Q_{mean} = \frac{1}{N}\sum_{i=1}^{N} Q_i$	834.74
Variance	$S^2 = \frac{1}{N}\sum_{i=1}^{N}(Q_i - Q_{mean})^2$	246474.79
Standard deviation	$S = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(Q_i - Q_{mean})^2}$	496.46
Coefficient of variation	$CV = \frac{S}{Q_{mean}}$	0.59
Coefficient of asymmetry	$CA = \frac{1}{S^3}\frac{1}{N}\sum_{i=1}^{N}(Q_i - Q_{mean})^3$	2.32

The sample is not symmetric since CA is different from 0.
For the Log_Normal distribution one must first remember to use as a variable y=ln(Q) in order to compute all the calculations. The formulas used are the same and results are the following:

Arithmetical mean	6.60
Standard deviation	0.49
Coefficient of variation	0.07
Coefficient of asymmetry	0.43





For values smaller than Q=1000 m^3/s all distributions are good because the points are on a quite straight line, but for bigger values the plots are more dispersed. We are looking for linear plots and, from this first representations, we can see that Gumbel and Log-Normal respect the criterion.

- <u>Method of L- moments</u>

First we have to define some quantities:

B_i = (sample) probability-weighted moments

$$B_0 = Q_{mean}$$

$$B_1 = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{i-1}{N-1}x_i\right)$$

$$B_2 = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{(i-1)(i-2)}{(N-1)(N-2)}x_i\right)$$

$$B_r = \frac{1}{N}\sum_{N=1}^{N}\left[\frac{(i-1)(i-2)\dots(i-r)}{(N-1)(N-2)\dots(N-r)}x_i\right]$$

$l_1 = B_0$	Position parameter	$l_1 = L_1$
$l_2 = 2B_1 - B_0$	Dispersion parameter	$l_2 = L_2$
$l_3 = 6B_2 - 6B_1 + B_0$	Asymmetry parameter	$l_3 = L_3$

Type	L_1	L_2	θ_1	θ_2
Exponential	θ	$\frac{\theta}{2}$	l_1	/
Normal	θ_1	$\pi^{-\frac{1}{2}}\theta_2$	l_1	$\pi^{\frac{1}{2}} l_2$
Log-Normal (Use formulas of normal)	θ_1	$\pi^{-\frac{1}{2}}\theta_2$	l_1	$\pi^{\frac{1}{2}} l_2$
Gumbel	$\theta_1 + 0.5772\theta_2$	$\theta_2\ln(2)$	$l_1 - 0.5772\, l_2$	$\frac{l_2}{\ln(2)}$

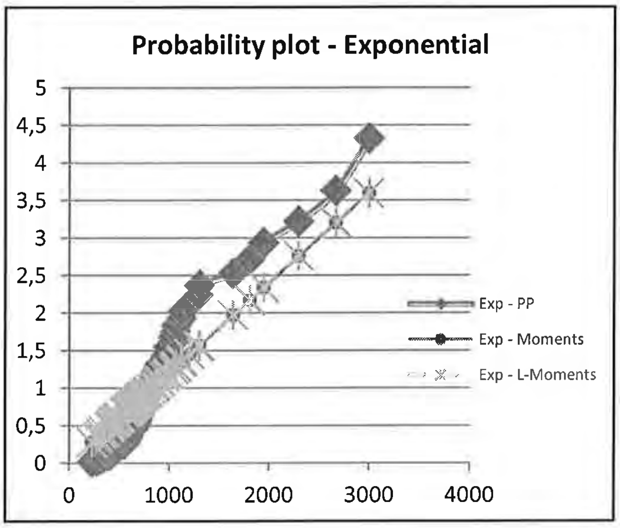
Type	l_1	l_2	θ_1	θ_2
Exponential	834.74	235.74	**834.74**	**/**
Normal	834.74	235.74	**834.74**	**370.29**
Log-Normal (Use formulas of normal)	6.60	0.27	**6.60**	**0.48**
Gumbel	834.74	235.74	**638.44**	**340.10**

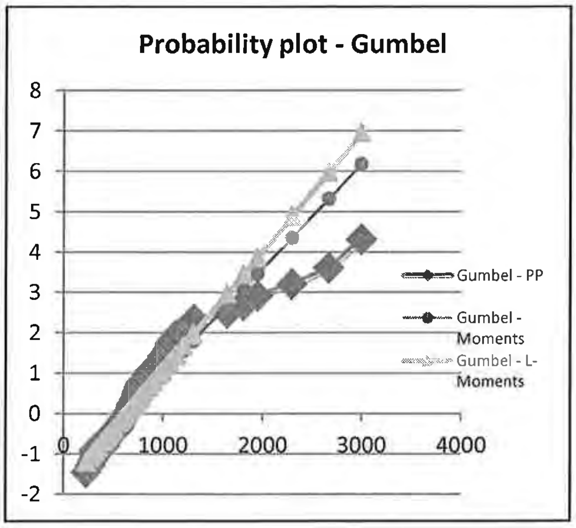
B_0	**834.74**
B_1	**535.24**
B_2	**409.72**
$B_{0\,log}$	**6.60**
$B_{1\,log}$	**3.43**

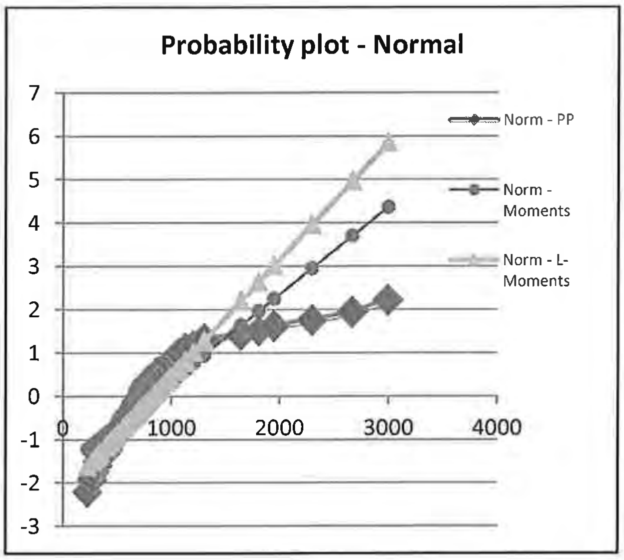
There is no need to calculate L_1 and L_2 because we can use the formulary in which we have the law of each teta. Remember that for all the calculations of the Log-Normal distribution we always use $y_i = \ln(Q_i)$.

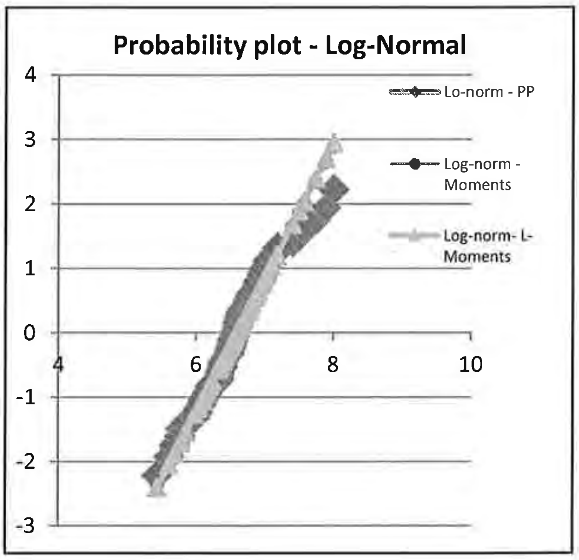
We can see that the best distribution approximating the sample is the Log-Normal because all the 3 series coincide.

It looks like that the Exponential may be good to use but it doesn't fit very well as the Gumbel distribution; it seems the Normal distribution isn't good for our data.









4. Now the **Pearson (Chi-Squared) test** must be performed and then, for those distributions that pass it, the Anderson Darling test is run.

- <u>Pearson test</u>

$$\chi^2 = \sum_{j=1}^{k} \frac{(O_j - E_j)^2}{E_j}$$

For this test we must define the number of classes, k, in which our data are divided. We assume that every class has the same probability q_j.

$$k = 2N^{0.4} = 11.187 \cong 11 \; classes$$

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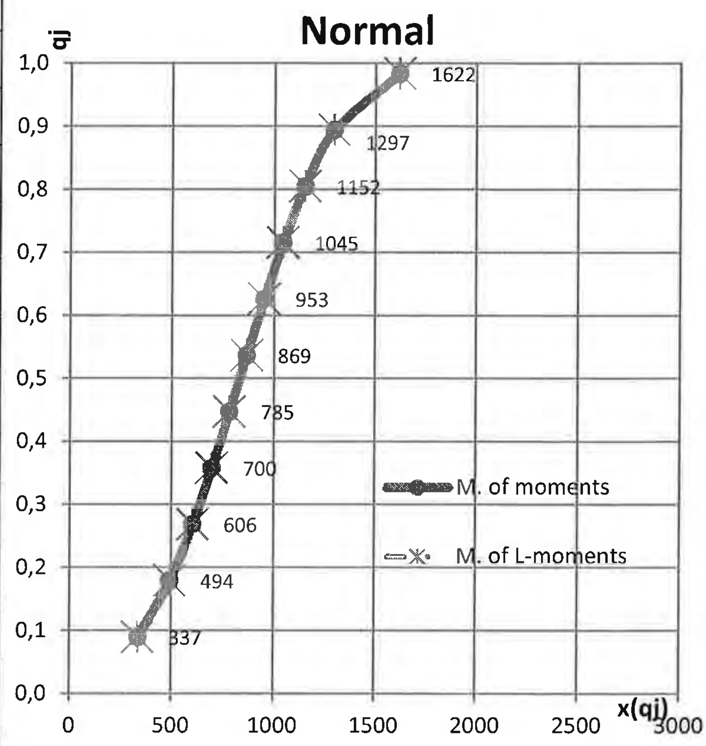
Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	26.30	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	27.59	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	28.87	34.80
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	30.14	36.19
20	8.260	10.851	12.443	15.452	19.337	23.83	28.41	31.41	37.57
22	9.542	12.338	14.041	17.240	21.337	26.04	30.81	33.92	40.29
24	10.856	13.848	15.659	19.037	23.337	28.24	33.20	36.42	42.98
26	12.198	15.379	17.292	20.843	25.336	30.43	35.56	38.89	45.64
28	13.565	16.928	18.939	22.657	27.336	32.62	37.92	41.34	48.28
30	14.953	18.493	20.599	24.478	29.336	34.80	40.26	43.77	50.89
40	22.164	26.509	29.051	33.660	39.335	45.62	51.80	55.76	63.69
50	27.707	34.764	37.689	42.942	49.335	56.33	63.17	67.50	76.15
60	37.485	43.188	46.459	52.294	59.335	66.98	74.40	79.08	88.38

		M. of moments		M. of L-moments	
q_j	j	$x(q_j)$	O_j	$x(q_j)$	O_j
0,089	1	337	5	337	5
0,179	2	494	9	494	9
0,268	3	606	10	606	10
0,358	4	700	11	700	11
0,447	5	785	12	785	12
0,536	6	869	5	869	5
0,626	7	953	3	953	3
0,715	8	1045	4	1045	4
0,804	9	1152	6	1152	6
0,894	10	1297	1	1297	1
0,983	11	1622	8	1622	8
		χ^2	18,79	χ^2	18,79
		χ^2_{LIM}	15,51	χ^2_{LIM}	15,51
		Test passed	**NO**	**Test passed**	**NO**

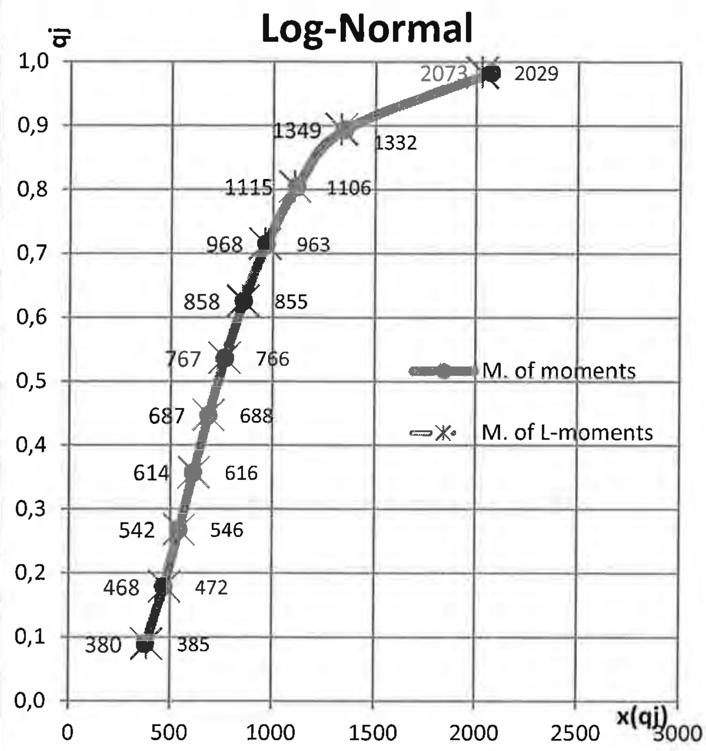


- Gumbel

This distribution passes the Pierson's test only for teta coming from the L-moment method. Its degree of freedom is v=8.

The division in classes now is different if we use the parameters coming from the method of moments or the method of L-moments.

		M. of moments		M. of L-moments	
q_j	j	$x(q_j)$	O_j	$x(q_j)$	O_j
0,089	1	380	6	385	6
0,179	2	468	6	472	6
0,268	3	542	4	546	4
0,358	4	614	9	616	9
0,447	5	687	7	688	8
0,536	6	767	13	766	12
0,626	7	858	5	855	5
0,715	8	968	6	963	6
0,804	9	1115	7	1106	7
0,894	10	1349	5	1332	5
0,983	11	2073	6	2029	6
		χ^2	9,12	χ^2	7,61
		χ^2_{LIM}	15,51	χ^2_{LIM}	15,51
		Test passed	YES	Test passed	YES



- GEV
  This distribution passes the Pierson's test for teta coming from both methods. Its degree of freedom is v=7.

  The division in classes now is also different.

- <u>Anderson-Darling test</u>

We perform this test only on the distributions that passed the previous one.

$$A^2 = -N - \frac{1}{N}\sum_{i=2}^{N}\{(2i-1)\cdot ln[P(x_{(i)})] + (2N+1-2i)\cdot ln[1-P(x_{(i)})]\}$$

$$\begin{cases} A^2 = 0 \div 4 \\ A^2 < A^2{}_{LIM} \quad Test\ positive \\ A^2 < A^2{}_{LIM} \quad Test\ negative \end{cases}$$

Where $P(x_{(i)})$ is the cumulative probability (CDF) of the function tested and N is the sample size. For practical reasons we have to transform the variable A^2 into ω using the following relations and find the coefficients ε, β, η from the table.

$$\begin{cases} \omega = 0.0403 + 0.116\left(\frac{A^2-\varepsilon}{\beta}\right)^{\frac{\eta}{0.861}} & if\ 1.2\varepsilon \le A^2 \\ \omega = \left[0.0403 + 0.116\left(\frac{0.2\,\varepsilon}{\beta}\right)^{\frac{\eta}{0.861}}\right]\frac{A^2-0.2\,\varepsilon}{\varepsilon} & if\ 1.2\varepsilon > A^2 \end{cases}$$

Type	ε	β	η
Gumbel, EV1	0.169	0.229	1.141
Norm, Log-norm	0.167	0.229	1.147
GEV	$0.147(1+0.13\,\theta_3+0.21\,\theta_3{}^2+0.09\,\theta_3{}^3)$	$0.189(1+0.20\,\theta_3+0.37\,\theta_3{}^2+0.17\,\theta_3{}^3)$	$0.186(1-0.04\,\theta_3-0.04\,\theta_3{}^2-0.01\,\theta_3{}^3)$

The limits are:

ω < 0.347	α = 0.10
ω < 0.461	α = 0.05
ω < 0.743	α = 0.01

In our case, since α=5%, if $\omega < \omega_{LIM}$ = 0.461 the test is positive.

- <u>Gumbel L-moments</u>

This distribution doesn't pass the test.

A^2	εp	1,2*εp	βp	ηp	ω
1,431	0,169	0,203	0,229	1,141	1,154
		$<A^2$			no

5. Embankment height

Finally, to calculate the height, we have to find the mean value of the maximum discharges given by the 3 distributions that have passed Anderson's test, for a return period of T=100 years.
The probability of non exceedance is:

$$P_{non\ exceed} = 1 - \frac{1}{T} = 0.99$$

So the values of discharge are given by the quintile functions $x^{-1}(P_{non\ exceed})$:

Type & method	**Q [m^3/s]**
Log-norm Mom	2286
Log-norm L-Mom	2233
GEV L-Mom	2822

Now we compute the mean and the deviation:

$$Q_{d,mean} = \frac{1}{3}\sum_{i=1}^{3} Q_i = 2447\ \frac{m^3}{s}$$

$$S_d = \sqrt{\frac{1}{3}\sum_{i=1}^{3}(Q_i - Q_{d,mean})^2} = 1201\ \frac{m^3}{s}$$

The discharge design value is Q_d = (2447 ± 1201) m^3/s.

The height is found by reversing the given formula:

$$h = -0.05 + \sqrt[1.81]{\frac{Q_{d,mean}}{142}} = 4.77\ m$$

Distribuzione Lognormale a 2 parametri

$$P(x) = \frac{1}{\theta_2\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{1}{2}\left(\frac{\ln(x)-\theta_1}{\theta_2}\right)^2\right] dx$$

$$p(x) = \frac{1}{x\theta_2\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln(x)-\theta_1}{\theta_2}\right)^2\right] \quad x > 0$$

$$x(F) = \exp\left[\theta_1 + \theta_2\Phi^{-1}(F)\right] \quad \theta_2 > 0$$

Momenti	*L Momenti*
$\mu = \exp(\theta_1 + \theta_2^2/2)$	$L_1 = \exp(\theta_1 + \theta_2^2/2)$
$\sigma^2 = [\exp(\theta_2^2) - 1]\exp(2\theta_1 + \theta_2^2)$	$L_2 = e^{\theta_1+\theta_2^2/2}\left[2\Phi(\theta_2/\sqrt{2}) - 1\right]$

Parametri(M)	*Parametri(L_M)*
$\widehat{\theta_1} = \ln\overline{x} - 1/2\ln(1 + s^2/\overline{x}^2)$	$\widehat{\theta_1} = \ln l_1 - \theta_2^2/2$
$\widehat{\theta_2} = \sqrt{\ln(1 + s^2/\overline{x}^2)}$	$\widehat{\theta_2} = \sqrt{2}\Phi^{-1}\left(\frac{1+l_2/l_1}{2}\right)$

0.4.1 Relazione con la distribuzione Normale

La distribuzione Lognormale è caratterizzata dal fatto che il logaritmo naturale della variabile, x, segue una distribuzione normale; quindi è possibile applicare le formule della distribuzione normale alla variabile modificata $\ln(x)$, ovvero

$$P(x) = \Phi\left(\frac{\ln x - \theta_1}{\theta_2}\right)$$

Distribuzione di Gumbel

$$P(x) = e^{-e^{-\frac{x-\theta_1}{\theta_2}}}$$

$$p(x) = \frac{1}{\theta_2} e^{-\frac{x-\theta_1}{\theta_2}} e^{-e^{-\frac{x-\theta_1}{\theta_2}}}$$

$$x(F) = \theta_1 - \theta_2 \ln[-\ln(F)]$$

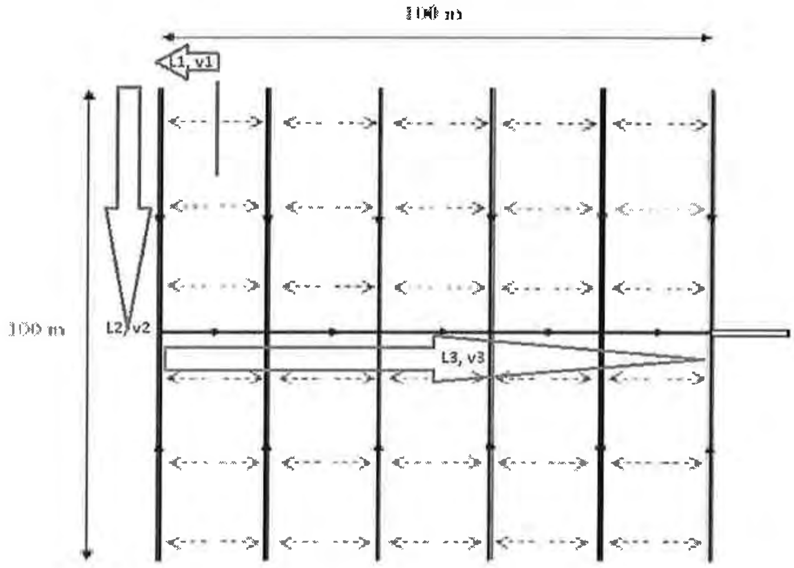
Momenti	*L Momenti*
$\mu = \theta_1 + 0.5772\theta_2$	$L_1 = \theta_1 + 0.5772\theta_2$
$\sigma^2 = \pi^2(\theta_2^2/6)$	$L_2 = \theta_2 \ln 2$
$\gamma = 1.1396$	$\tau_3 = 0.1699$
$\kappa = 5 + 2/5$	$\tau_4 = 0.1504$

Parametri(M)	*Parametri(L_M)*
$\widehat{\theta_1} = \overline{x} - 0,5772s(\sqrt{6}/\pi)$	$\widehat{\theta_1} = l_1 - 0.5772\theta_2$
$\widehat{\theta_2} = s(\sqrt{6}/\pi)$	$\widehat{\theta_2} = l_2/\ln 2$

ASSIGNMENT 2

Design and verification of a stormwater drainage system using IDF curves

Consider the asphalt parking lot outlined below: thick lines indicates the drainage gutters (arrows indicate the flow direction), dotted arrows indicate the slopes of asphalt pavement, while the rectangular element on the right indicates the stormwater outlet pipe, made of concrete and having a square section of dimension *y*.



Consider the annual maximum rainfall depth in the study area for durations 1, 3, 6, 12 and 24 hours given in the file 'dati.txt' (in mm) and the Intensity-Duration-Frequency (IDF) curves determined with the precipitation index method, thus based on the formulas

$$i_T(d) = K_T \cdot \bar{a} \cdot d^{n-1};$$

$$K_T = 1 - CV\left[\gamma_E \frac{\sqrt{6}}{\pi} + \frac{\sqrt{6}}{\pi}\ln\left(-\ln\left(1-\frac{1}{T}\right)\right)\right].$$

The critical duration of rainfall equals the concentration time of the parking lot area, for which you may consider a flow velocity over the asphalt pavement equal to 2 cm/s, and a flow velocity in the gutters equal to 10 cm/s. The stormwater discharge flowing out of the parking lot must be equal to the rainfall reaching the ground in the same unit of time.

The stormwater outlet pipe has a slope of j = 0.3% and a Strickler roughness coefficient for concrete of ks = 65 $m^{0.33}$/s, thus the maximum flow through the pipe can be determined by:

$$Q_{out}(y) = 0.397 \cdot k_s \cdot y^{2.666} \cdot \sqrt{j},$$

where y is the side length of the square section of the outlet pipe.

You should:

A. Determine the probability that, in a given year, the drainage system receives more stormwater than it is designed to handle and the existing outlet pipe is inadequate to fully drain the flow, considering an outlet square section with y = 40 cm.

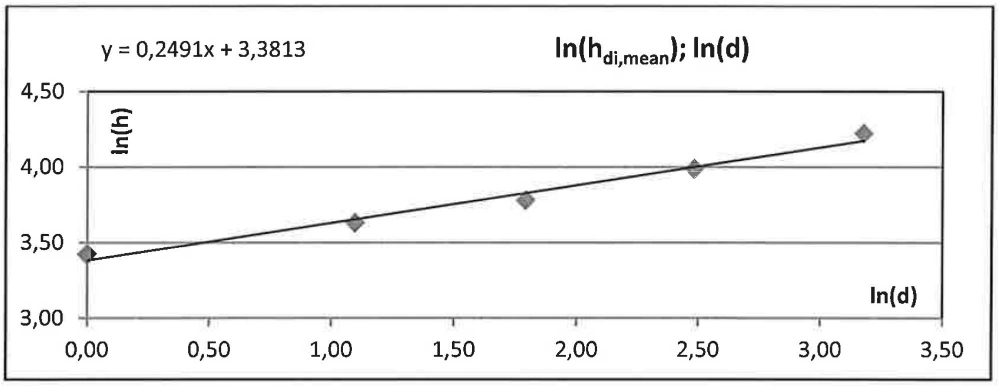
B. Design the stormwater outlet (square section) that properly drains the flow generated by a rainfall event corresponding to a return period of 10 years.

2. Now, we can find the parameters a and n by plotting ln(d) and ln($h_{di,mean}$) and ask for the tendency linear equation.

$$\overline{h_d} = a \cdot d^n \quad \leftrightarrow \quad \ln(\overline{h_d}) = \ln(a) + n \cdot \ln(d)$$

Results: ln(a) = 3.3813 ⇒ a = exp(3.3813) = 29.41 mm (intercept)

n = 0.2491 (slope)



3. The maximum flow through the pipe Q_{out} can be calculated and it is assumed equal to the rainfall reaching the ground in the same unit time:

$$Q_{out}(y = 0.4) = 0.397 \cdot K_s \cdot y^{2.666} \cdot \sqrt{j} = 0.123 \frac{m^3}{s}$$

$K_s = 65\ m^{0.33}/s$

$j = 0.003$

$Q_{ground} = i \cdot Area \cdot C_{runoff}$

$C_{runoff} = 1$

$Area = 10000\ m^2$

$Q_{out} = Q_{ground}$

So the critical intensity $i_T(d)$ is equal to the intensity i reaching the ground, that can be found by reversing the formula of Q_{ground}:

$$i = \frac{Q_{out}}{Area \cdot C_{runoff}} 3600 \cdot 10^3 = 44.22 \frac{mm}{h}$$

4. Now we need to find the critical duration d_c that is equal to the concentration time t_{cb} (the time that the most distant particle needs to reach the pipe).

$L_1 = 10$ m $v_1 = 0.02$ m/s

$L_2 = 50$ m $v_2 = 0.1$ m/s

$L_3 = 100$ m $v_3 = 0.1$ m/s

$$t_{cb} = \frac{L_1}{v_1} + \frac{L_2}{v_2} + \frac{L_3}{v_3} = 2000\ s = 0.555\ h$$

5. Finally the coefficient K_T can be calculating by reversing the IDF formula ($d = t_{CB}$):

$$K_T = \frac{i}{a \cdot {t_{cb}}^{n-1}} = 0.967$$

6. To find the probability of non exceedance, we must reverse the K_T formula in order to find 1-1/T:

$$\ln\left(-\ln\left(1 - \frac{1}{T}\right)\right) = \frac{1 - K_T}{CV} \frac{\pi}{\sqrt{6}} - \gamma_E = Z \quad (called\ Z\ for\ simplicity)$$

$$P_{non\ exceed} = 1 - \frac{1}{T} = \exp(-\exp(Z)) = 0.53 = 53\%$$

ASSIGNMENT 3

An introduction to GIS in Hydrology - Catchment characterization

Given a digital elevation model (DTM) of the Arezzo province in central Italy, we are going to use the Q-GIS* functions to answer the following:

1. Identify the streamflow network over the whole DTM

2. Delineate the catchment that contributes to the following outlet point along the Arno river:
   E 11.8730646727, N 43.5866988302

3. Evaluate the catchment area and perimeter. Get the minimum, mean, and maximum catchment height. Determine the hypsometric curve of the catchment.

4. Add the geo-referenced annual rainfall data (gauges.csv) and via the Thyessen method find the spatial distribution of rainfall over the catchment.

5. Find the areal average precipitation over the whole catchment, and the corresponding volume.

6. Compare the annual volumes of precipitation and of water flown at the catchment outlet (streamflow gauge).

*QuantumGIS is an open source software that we can download and use on our own computer. It can run on Windows, MacOS, and Linux systems. We may download it here:
http://www.qgis.org/en/site/forusers/download.html#

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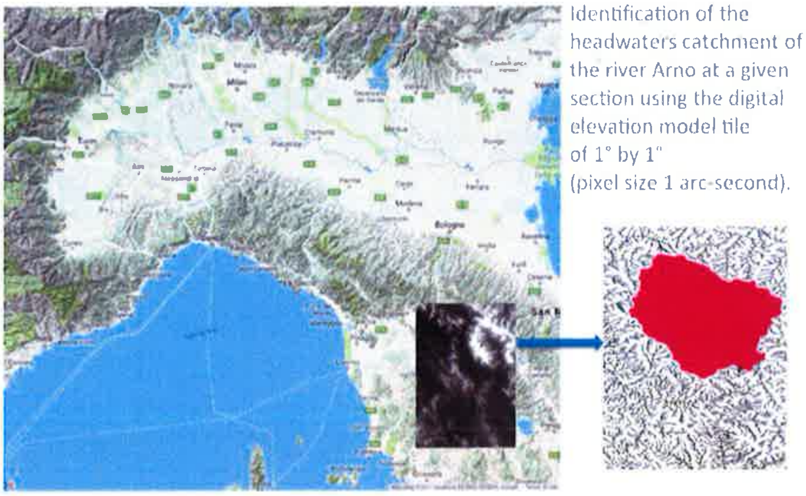
2. The studied region:

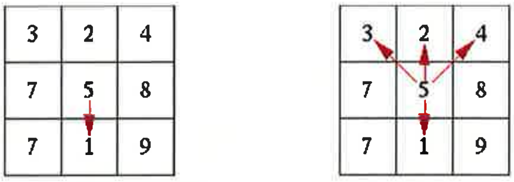
Our DEM: a tile from Central Italy



Using the function **r.watersheds** that allows to obtain, from a DEM file, the drainage direction map, the drainage density map and the stream network raster.

This function can use two different algorithms:

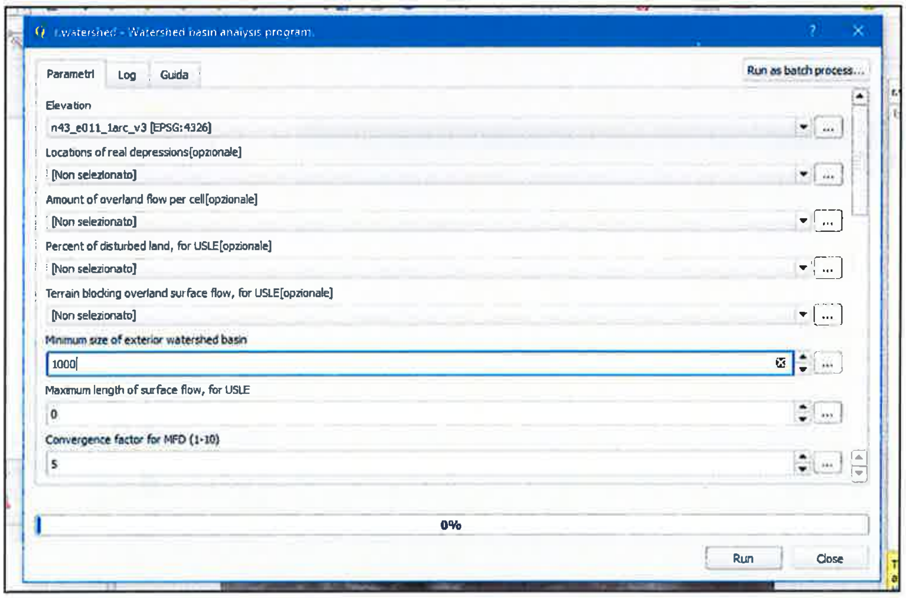
- Single flow direction (**SFD**): assumes that subsurface flow occurs only in the steepest down-slope neighbour;
- Multiple flow direction (**MFD**): assumes that subsurface flow occurs in all down-slope neighbours.



Flow direction to steepest downslope neighbor (SFD). **Flow direction to all downslope neighbors (MFD).**

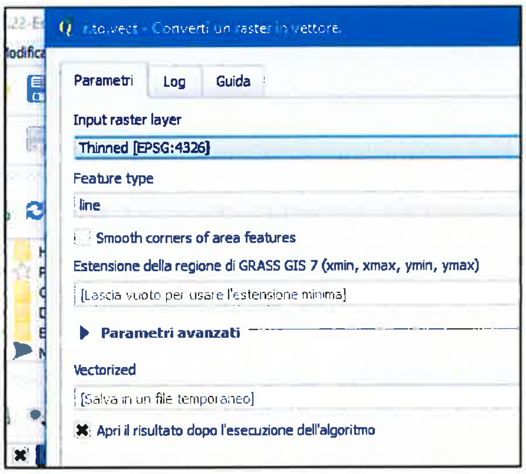
Figure 3: Difference between SFD and MFD algorithms

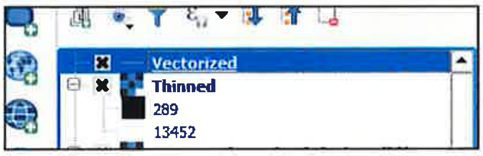
Set the following information and then run the function:



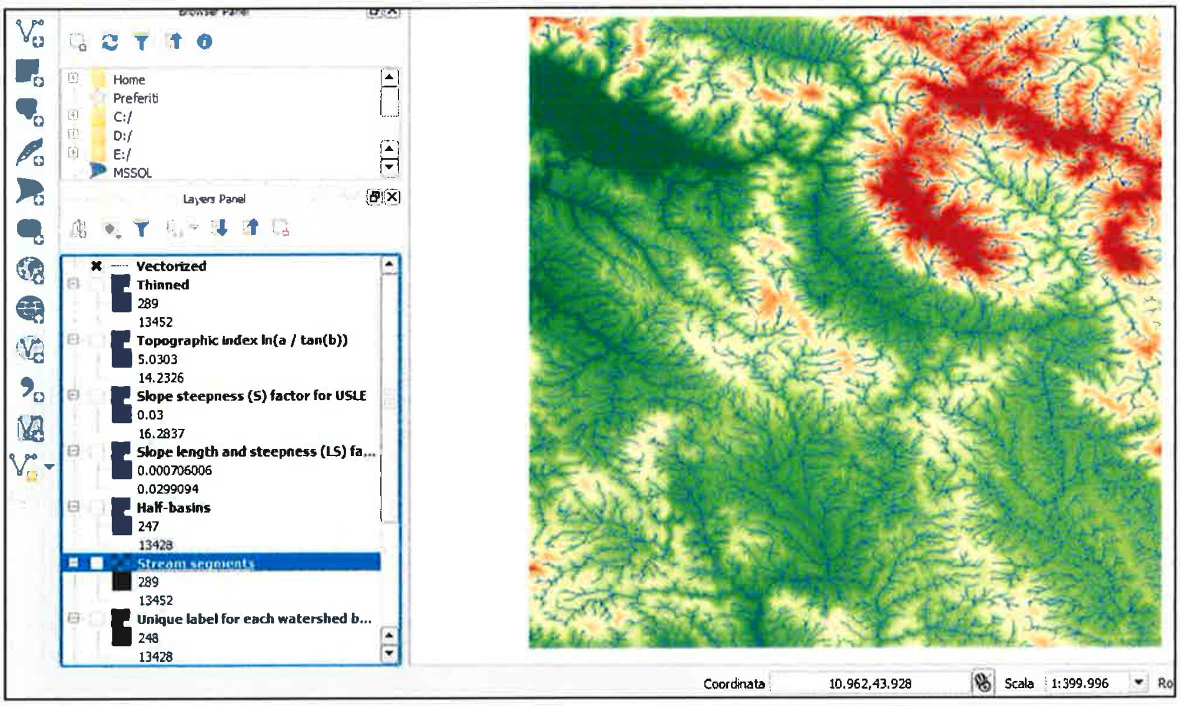
From **Processing** Tab > **Toolbox** > **Grass Gis** 7 **commands** > **Raster** > double-click on **r.to.vect**

As result we have a file named "Vectorized".





Result after setting a blue colour to the vectors and "banda singola falso colore" to the map:



2. Catchment delineation:

A catchment area is otherwise considered a drainage basin. The basin is where water flows over topographic terrain and consists of water runoff into surrounding rivers, streams and lakes. (Wagener et al. 2007).

To delineate the catchment we have to use the function **r.water.outlet** that generates a watershed basin from a drainage direction map and a set of coordinates representing the outlet point of watershed.

From **Processing** Tab > **Toolbox** > **Grass Gis 7 commands** > **Raster** > double-click on **r.water.outlet**

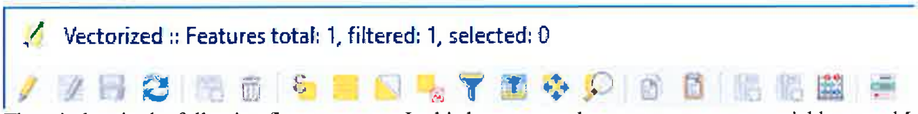
The coordinates of the outlet point are:

x=11.8730646727

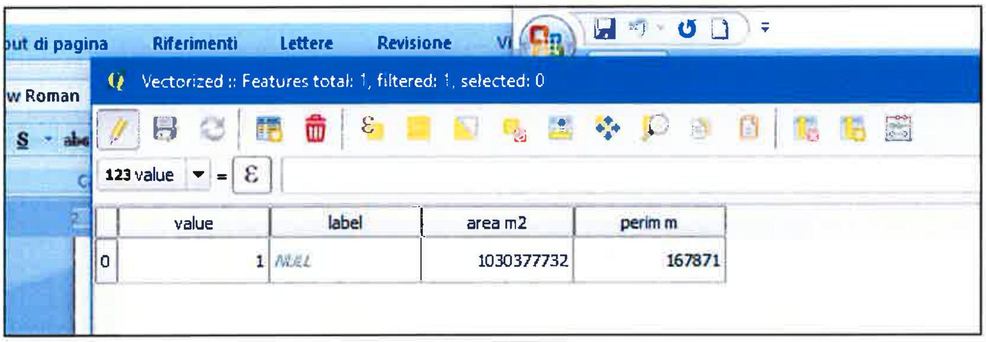
y=43.5866988302

3. Catchment stats:

The first step is to calculate the perimeter and the area of the catchment. To do this, we have to open the attribute tables of the last file "vectorized" that represents the vector basin. Firstly click on the "toggle editing mode" icon and then click on the "open field calculator" icon .

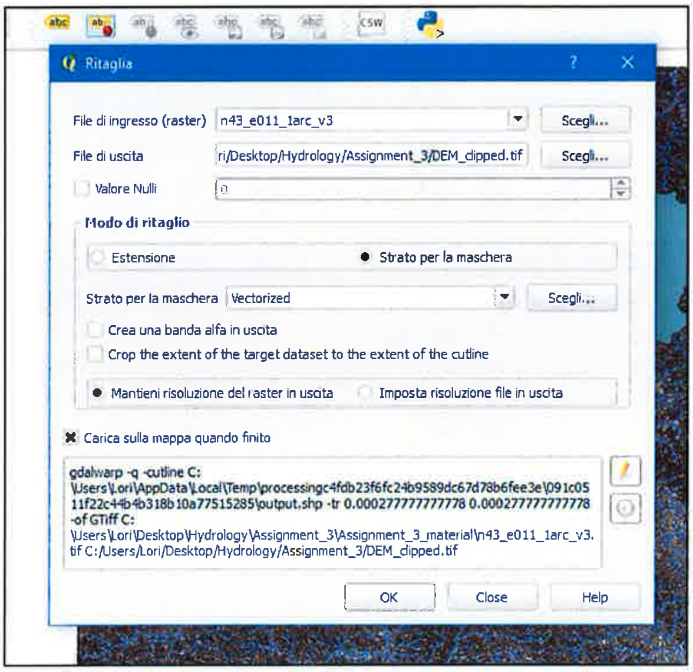


The window in the following figure appears. In this last one, we have to create a new variable named **[area] m2** in "output field name", then double-click on **Geometry** >"**$area**" and finally click **OK**. In this way we have created a new column in the table representing the area of basin.
We have to do the same thing with the perimeter: create a new variables named **[perim] m** > double-click on **Geometry** >"**$perimeter**".



The next step is to use the layer "vectorized" to cut the DEM with the command "clipper" from the **menu bar>raster>extraction>clipper.**

The input file is the initial raster "n43_e011_1arc_v3" while for the output file, we have to choose a folder destination and the file name "DEM_clipped". Then we select the option "mask layer" (after selection we don't change anything in the windows that appears) and choose "vectorized" as mask layer.



In the next step you want to calculate Z_{min} ,Z_{max},and Z_{mean}. Firstly you have to activate the "zonal statistical plugins" from the **menu bar> plugins.**

The output of the SAGA hypsometry function is a table. We copy the content of this table to obtain the graph (the hypsographic curve) in Excel.

In the "Relative Area" column we have the portion of the catchment area below a corresponding "Absolute Height". For example at 1652 m we have that the area of catchment below this height is 100%.

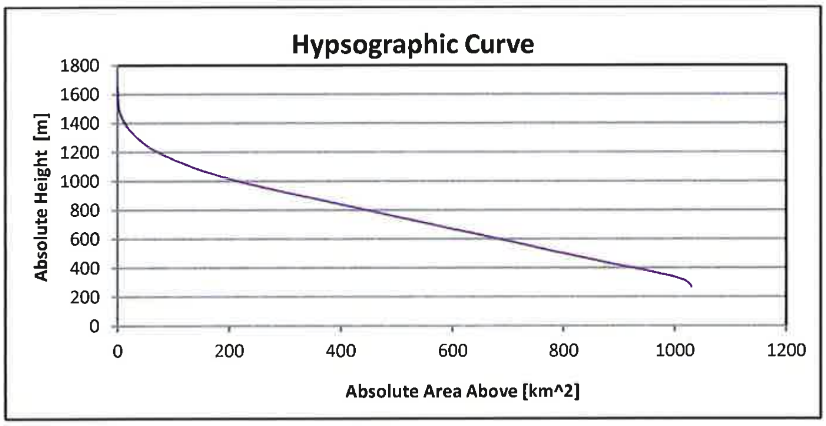
In order to make the hypsographic curve we want the area above a given absolute height.

Therefore we create a new column that is equal to 100 – relative area. Later, in a new column, we obtain the absolute area by multiplying the "relative area above" by the catchment area (**1030.38 Km2** calculated earlier).

Nr. Points	Relative Height	Relative Area Below	Relative Area Above	Absolute Area Above	Absolute Height	Absolute Area
	[%]	[%]	[%]	[km^2]	[m]	[km^2]
0	100	100	0	0	1652	0,08317
1	99	99,99295	0,00705	0,07264163	1638,1	0,083162
2	98	99,98358	0,01642	0,169188024	1624,2	0,083155
3	97	99,975324	0,024676	0,254256009	1610,3	0,083146
4	96	99,964192	0,035808	0,368957658	1596,4	0,083136
5	95	99,951946	0,048054	0,495137715	1582,5	0,083124
6	94	99,937474	0,062526	0,644253981	1568,6	0,083106
7	93	99,916694	0,083306	0,858366473	1554,7	0,083089
8	92	99,896007	0,103993	1,071520715	1540,8	0,083063
9	91	99,865022	0,134978	1,390783255	1526,9	0,083031
10	90	99,825874	0,174126	1,79415553	1513	0,082983
11	89	99,768265	0,231735	2,387745837	1499,1	0,082913
12	88	99,684681	0,315319	3,248976761	1485,2	0,082821
13	87	99,574009	0,425991	4,389316404	1471,3	0,082714
14	86	99,444783	0,555217	5,720832332	1457,4	0,082591
15	85	99,297096	0,702904	7,242566293	1443,5	0,082446
16	84	99,1226	0,8774	9,040534221	1429,6	0,082284
17	83	98,928158	1,071842	11,04402129	1415,7	0,082104
18	82	98,712101	1,287899	13,27022451	1401,8	0,081917
19	81	98,48686	1,51314	15,59105761	1387,9	0,08172
20	80	98,250116	1,749884	18,03041507	1374	0,081499
21	79	97,984428	2,015572	20,76800506	1360,1	0,081251
22	78	97,685529	2,314471	23,8477938	1346,2	0,080979
23	77	97,359357	2,640643	27,20859745	1332,3	0,080693
24	76	97,015187	2,984813	30,75484849	1318,4	0,080394
25	75	96,65534	3,34466	34,46263185	1304,5	0,080073
26	74	96,270074	3,729926	38,43232692	1290,6	0,079738
27	73	95,866625	4,133375	42,58937558	1276,7	0,079381
28	72	95,43813	4,56187	47,00449264	1262,8	0,078982
29	71	94,958426	5,041574	51,94725584	1248,9	0,078576
30	70	94,469817	5,530183	56,98177417	1235	0,078077
31	69	93,870164	6,129836	63,16046515	1221,1	0,07756
32	68	93,247875	6,752125	69,57239244	1207,2	0,077012
33	67	92,589222	7,410778	76,35900628	1193,3	0,076427

Nr. Points	Relative Height [%]	Relative Area Below [%]	Relative Area Above [%]	Absolute Area Above [km^2]	Absolute Height [m]	Absolute Area [km^2]
94	6	3,717866	96,282134	992,0696686	345,4	0,002041
95	5	2,45353	97,54647	1005,097105	331,5	0,001141
96	4	1,371947	98,628053	1016,241496	317,6	0,000663
97	3	0,796599	99,203401	1022,169753	303,7	0,000279
98	2	0,335078	99,664922	1026,925163	289,8	0,000056
99	1	0,066979	99,933021	1029,687595	275,9	0
100	0	0,000186	99,999814	1030,375815	262	0

To create the hypsographic curve plot on the y-axis the "**Absolute Height**" and on the x-axis the "**Absolute area above**".

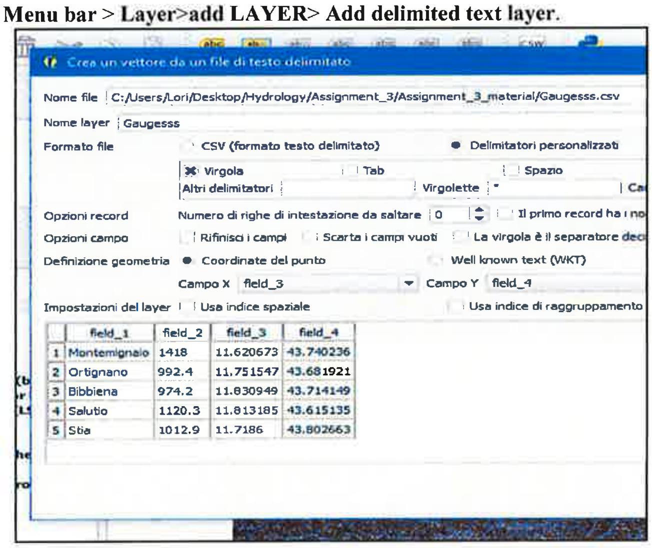


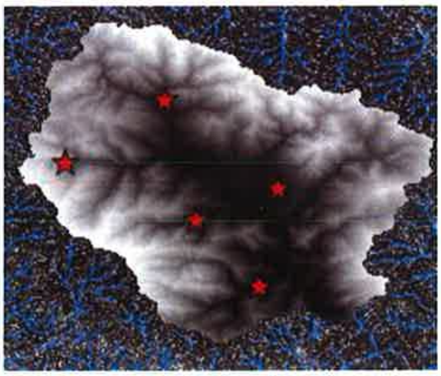
4. **Add rainfall gauges:**

From the file "rainfall_monthly_data" compute (excel or similar) mean annual rainfall totals for the gauges, and save them into **.csv**.

Import gauges coordinates into QGis via:

Menu bar > Layer>add LAYER> Add delimited text layer.





5. **First method for the catchment precipitation estimation: Thiessen polygons:**

The next step is to extract Voronoi/Thiessen polygon in order to identify the area of influence for each rainfall-gauge. We may use the function **v.voronoi** in the GRASS toolbox:

From **menu bar> Processing** Tab > **Toolbox** > **Grass Gis 7 commands** > **vector** > double-click on **v.voronoi**

Specify the region extent (xmin,xmax,ymin,ymax).

6. Method for catchment precipitation estimate: IDW inverse distance weighting:

You use the file “five_annual_rainfall_mm_e_n_svd2nd_titles” that contains the five rainfall gauges.

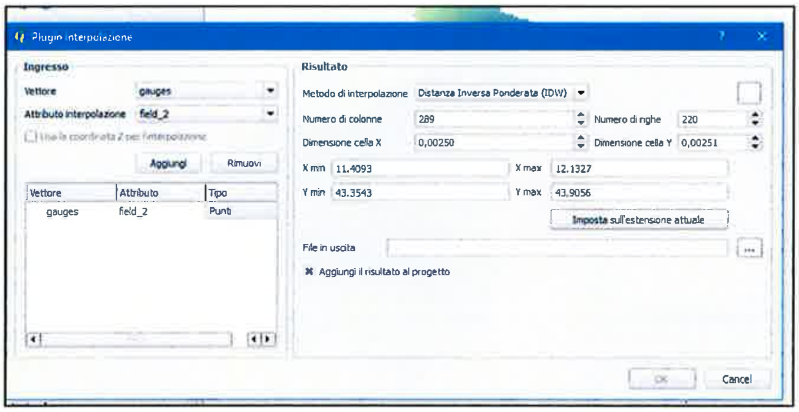
Right click on file > click **Set Porject CRS from Layer** .

Then we use the interpolation plugin via:

menu bar> Raster > **Interpolation** > **Interpolation.** (This plug-in must be installed).

On the left side of the dialog (**Input**), choose *five_annual_rainfall_mm_e_n_svd2nd_titles* as vector layers and *field_2* as interpolation attribute and then press “Add”.

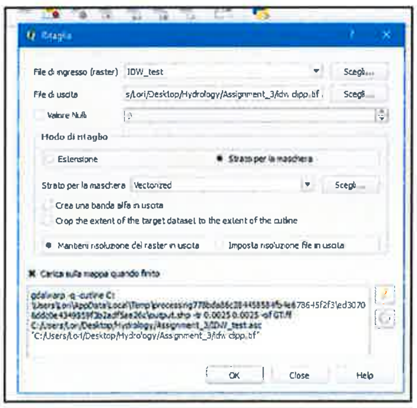
On the left side of the dialog (**Output**) press on “Set to current extent” and then modify “**Cellsize x” and “cellsize y” values into 0.0025**. Finally choose a name for the output file (example “IDW_test”) and select OK.

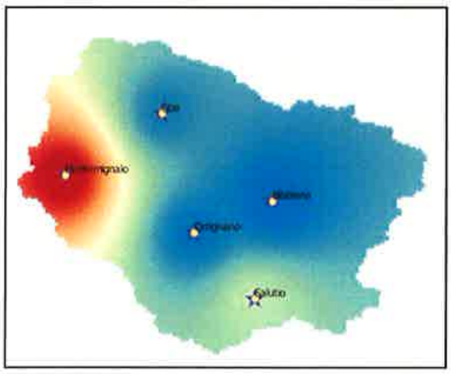


Now you cut the IDW raster just created (“IDW_test”) by using the catchment polygon via:

menu bar> Raster > **Extraction** > **Clipper.**

Choose IDW raster as *Input file* and a name for the *output file* (example “IDW_clipped”). As “Clipping Mode” select “Mask Layer” and choose “vectorized” as *mask layer*.





So we have a new file named “IDW_clipped”. We modify its symbology via:

Layer Panel > **right click on file** “DEM_clipped” > **Properties> Style**

Select “Singleband pseudocolor” as *Render type*, then tick the box “Invert” and press “Apply”.

8. Annual water volumes comparison (rainfall vs. streamflow):

				precipitation mm^3	x	y	Ai m^2	precip*Ai		mean precip mm^3
	a_cat	a_cat_	a_str_1	a_dbl_1	a_dbl_2	a_dbl_3	area polig			1076,18
	1	2	Ortignano	992,4	11,751547	43,681921	169070345,6	167,79		
thiessen	4	5	Stia	1012,9	11,7186	43,802663	221798428,6	224,66		
	5	3	Bibbiena	974,2	11,830949	43,714149	315978971,4	307,83		
	2	4	Salutio	1120,3	11,813185	43,615135	168823776,2	189,13		
	3	1	Montemignaio	1418	11,620673	43,740236	154974895	219,75		
							sum tot	1109,16		
	value	label	area basin	perimeter	annual precip mm^3	mean precip m^3				
IDW	1		1030646417	174230,83	1084,96	1,08496E-06				
				rainfall on the basin Bm^3						
	thiessen polygons		1109159430827,12	1,11						
	inverse squared		1,11821E+12	1,12						

To compare stream flow total annual volume with rainfall total annual volume, we need to transform the precipitation (mm) into m^3.

We obtain:

1° CASE (Thiessen Polygons) 1076.36 mm * catchment area [m^2]/ (1000 mm/m) = **1.11 Bm^3**.

2° CASE (IDW: Inverse Distance Interpolation) 1084.96 mm * catchment area [m^2]/ (1000 mm/m) = **1.12 Bm^3**.

Between 2011 and 2016 the mean annual volume of water precipitated over the catchment is approximately one billion m^3, while the amount of water flown at the catchment outlet is 140 million m^3.

Therefore approximately 13% of rainfall volume can be found in the river Arno at the catchment outlet.

streamflow		139,90	Mm^3	x10^6	ratio streamflow/rainfall	
rainfall	thiesson	1,11	Bm^3	x10^9	12,61301	%
	IDW	1,12	Bm^3	x10^9	12,51092	%
					13	%

ASSIGNMENT 4 – PART 1

Determine a design hydrograph in the absence of streamflow data

For the design of a road bridge, the design flow with a return period of 100 years is needed for the river Stura at the Lanzo cross-section (corresponding catchment outlined below). Suppose streamflow data are not available near the section of interest, so that you will use the convolution method of unit hydrographs.

PART 1: Determination of the design storm and the corresponding net rainfall hyetograph

1. Find the cacthment's time of concentration using the empirical formula proposed by Giandotti,

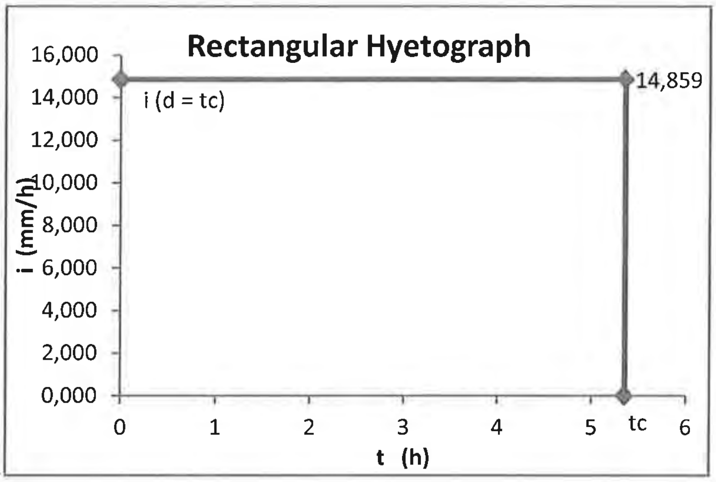
$$t_c = \frac{4\sqrt{S} + 1.5L}{0.8\sqrt{H'}}$$

where L = main channel length (in km); S = catchment area (in km^2); H' = catchment mean height minus catchment minimum height (in m); t_c concentration time (in h). You may use the following morphologic parameters: $L = 39$ km, $S = 582$ km^2, $z_{med} = 1773$ m, $z_{min} = 464$ m.

2. Use the monomial depth-duration-frequency curve (IDF) as derived with the Gumbel method:

$$h_{d,T} = a \cdot d^n \cdot K_T$$

with parameters $a = 24.91$ mm, $n = 0.2491$, $K_T = 2.103$ for a return period of 100 years. The curve is assumed to describe the mean areal precipitation over the catchment. Determine the design hydrographs with a 30-minute temporal resolution:



- <u>Chicago hyetograph</u>

Assuming a time step $\Delta t = 0.5h$ (half an hour) and $d = 24h$, we find the precipitation depth for each time step. The we calculate the actual depth in each half an hour and divide this value by 0.5h in order to find the intensity. At the end we order the values in a "central peak way".

Time steps Δt (h)	**Precipitation depth (mm)**	**Intensity in each Δt (mm/h)**
0 to Δt	h1	h1/ Δt
Δt to $2\Delta t$	h2-h1	(h2-h1)/ Δt
$2\Delta t$ to $3\Delta t$	h3-h2	(h3-h2)/ Δt
…..	…..	…..

Δt (h)	**hd,t (mm)**	**h(i) - h(i-1)**	**id,t (mm/h)**	**Decreasing order**	**Central peak order**
0	0,00	0,00	0,00	88,16	0
0,5	44,08	44,08	88,16	16,61	1,23
1	52,39	8,31	16,61	11,13	1,27
1,5	57,95	5,57	11,13	8,61	1,31
2	62,26	4,31	8,61	7,12	1,36
2,5	65,82	3,56	7,12	6,12	1,42
3	68,88	3,06	6,12	5,39	1,47
3,5	71,57	2,70	5,39	4,84	1,54
4	73,99	2,42	4,84	4,41	1,61
4,5	76,20	2,20	4,41	4,05	1,69
5	78,22	2,03	4,05	3,76	1,78
5,5	80,10	1,88	3,76	3,51	1,87
6	81,86	1,76	3,51	3,30	1,99
6,5	83,50	1,65	3,30	3,11	2,12
7	85,06	1,56	3,11	2,95	2,27
7,5	86,53	1,47	2,95	2,80	2,46
8	87,94	1,40	2,80	2,68	2,68
8,5	89,28	1,34	2,68	2,56	2,95
9	90,56	1,28	2,56	2,46	3,30
9,5	91,78	1,23	2,46	2,36	3,76

3. Net precipitation with Curve Number Method

First of all we have to estimate the Curve Number CN, that depends on the characteristics of the catchment. Using the following tables, we find the CN (II) that must be corrected into CN (III), because our soil is humid so it's class III.

Table 1 Hydrologic soil group

Group	Minimum Infiltration Rate (in/hr)	Hydrologic Soil Group
A	0.3 – 0.45	High infiltration rates. Deep, well drained sands and gravels
B	0.15 – 0.30	Moderate infiltration rates. Moderately deep, moderately well drained soils with moderately coarse textures (silt, silt loam)
C	0.05 – 0.15	Slow infiltration rates. Soils with layers, or soils with moderately fine textures (clay loams)
D	0.00 – 0.05	Very slow infiltration rates. Clayey soils, high water table, or shallow impervious layer

Table 2 Antecedent moisture conditions

Antecedent moisture conditions	Dormant season	Growing season
I	<12.7 mm	<35.5 mm
II	12.7-28.0 mm	35.5-53.3 mm
III	>28.0 mm	>53.3 mm

In the table there are $CN(II)_{min}$ and $CN(II)_{max}$ and we take the mean. The we transform $CN(II)_{mean,i}$ into $CN(III)_{mean,i}$ and take the mean weighted by the area.

$$CN(II)_{mean,i} = \frac{CN(II)_{max} + CN(II)_{min}}{2}$$

$$CN(III)_i = \frac{CN(II)_{mean,i}}{0.43 + 0.0057\, CN(II)_{mean,i}}$$

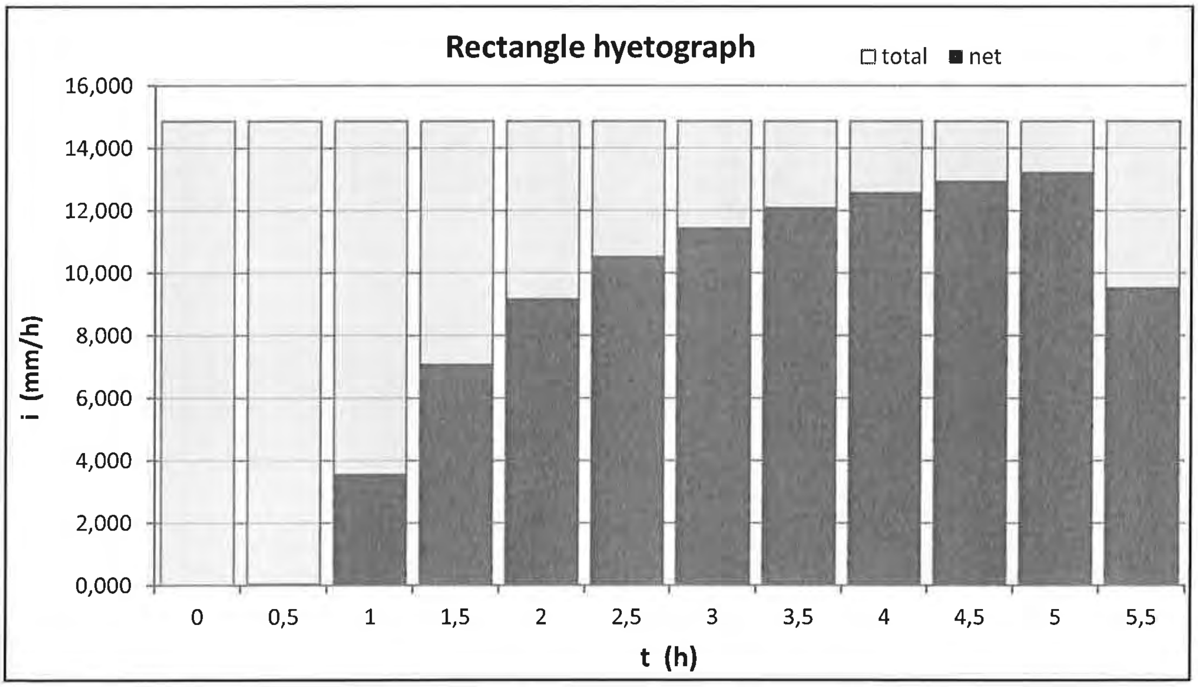
$$CN(III) = \sum_{i=1}^{3} CN(III)_i \frac{A_i}{A_b}$$

- Rectangular hyetograph

In order to calculate the net intensity we go through the following steps:
- take intensity values found before
- compute cumulative precipitation $P_{cum,i} = i * \Delta t_i$
- compute PN using the previous formula (previous page) ($P=P_{cum}$)
- calculate $i_{net}= (PN_i - PN_{i-1})/ \Delta t$ for Δt=0.5h

Δt (h)	i (mm/h)	Pcum (mm)	PN (mm)	i net (mm/h)
0,5	14,859	7,429352	0,038344	0,07668738
1	14,859	14,8587	1,822439	3,5681899
1,5	14,859	22,28806	5,371393	7,09790957
2	14,859	29,71741	9,969473	9,19615877
2,5	14,859	37,14676	15,24187	10,5448012
3	14,859	44,57611	20,97333	11,4629067
3,5	14,859	52,00546	27,03138	12,1161079
4	14,859	59,43481	33,33007	12,5973748
4,5	14,859	66,86417	39,81116	12,9621817
5	14,859	74,29352	46,43381	13,2452996
5,5	14,859	79,56997	51,20678	9,54594103
! Last value: Pcum= i*tc = 14.86*5.36				

So the hyetographs can be plotted:



So the hyetographs can be plotted: