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A P P U N T I

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MATERIA: Theory and Design of Reinforced and Prestressed Concrete Structures - Teoria - Prof. Fantilli

02/10/2017

Reinforced Concrete Structures RC

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Why do we build?
- safe ⟶ Effect < Strength ($E < S$)
- sustainance
- sex

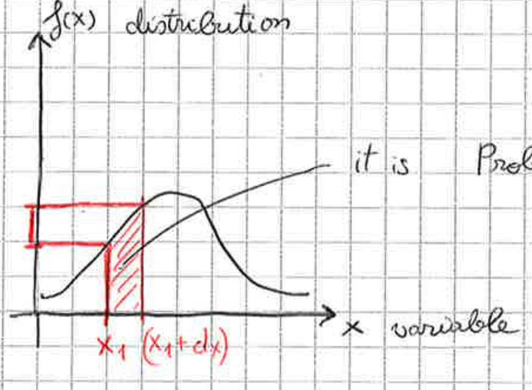
sollecitazione sulla struttura — collapse — serviceability — resistenza della struttura

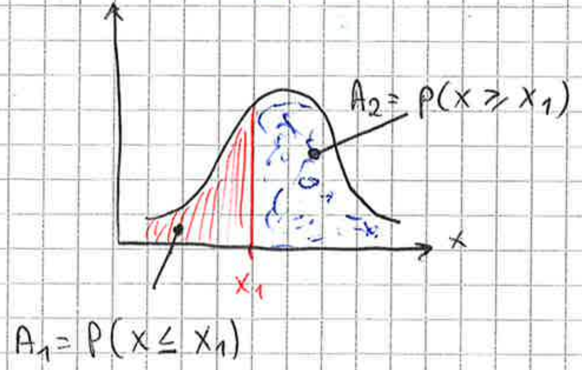
E and S are random variables
↓
NOT WELL DEFINED ELEMENTS.

Effect produced by loads: UNCERTAINTIES
- intensity
- position.
-

Strength: depends on the concrete

How can we represent a random variable? Using a diagram:





$A_1 + A_2 = 1$

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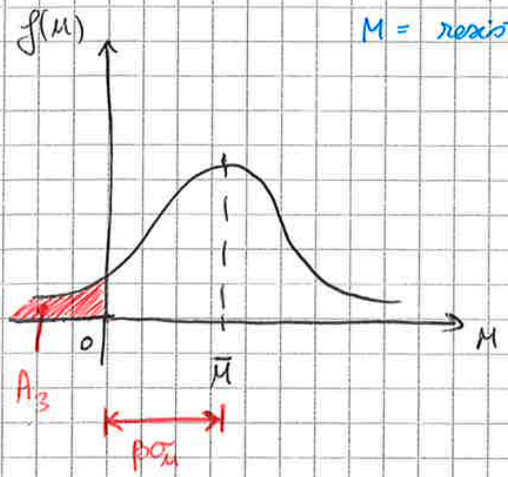
The equation $\boxed{M = S - E > 0}$ M = "safety Margin"

M = resistence − sollecitation > 0



A_3 is the probability of failure : $P_{FAILURE}(M < M_d)$

$\beta\sigma_\mu$ is the distance between 0 and $\bar{M}$

β reliability index.

σ_μ = standard deviation of M

We must garontee a sufficent distance between $\bar{M}$ and 0, using a correct β.

But it's very difficult to find the values Ed and Sd. So we use a **SEMI-PROBABILISTIC METHOD**, which is:

$$\boxed{E_d \leq R_d}$$

sollecitat. ≤ resistance

EC2 / ACI-318

ACI-318

$E_d = U$ = factored load

$R_d = \phi \cdot S_n$

ϕ = reduction factor.

S_n = nominal strength

EC2

$E_d = \gamma_q \cdot Q_i$

γ_q = partial safety factor for loads

Q_i = nominal value of loads

$R_d = \dfrac{R_k}{\gamma_m}$

R_k = characteristic value of strength

γ_m = partial safety factor for materials

! *1 "Valore caratteristico di un'azione è il valore frattile caratterizzato da una definita probabilità di essere superato".

$$P(z > z_k) = 5\% \div 95\%$$

$$\Rightarrow \begin{cases} R_k = R_{95\%} = P(R > R_k) = 95\% & \text{resistence} \\ E_k = E_{5\%} = P(E > E_k) = 5\% & \text{effect = sollecitation} \end{cases}$$

Fundamental Combination: (EUROCODE 0)

combination factor (coefficient)

VARIABLES

$$\underbrace{G_{1k} \cdot \gamma_{G1}}_{\text{DESIGN LOAD OF STRUCTURAL ELEMENTS}} + \underbrace{G_{2k} \cdot \gamma_{G2}}_{\text{SAME BUT NOT STRUCTURAL}} + \underbrace{\gamma_P \cdot G_P}_{\text{PRESTRESSED LOAD}} + \gamma_{q1} Q_{k1} + \underbrace{\sum_{i=2}^{m} \psi_{0i} \cdot Q_{ki} \cdot \gamma_{qi}}_{\text{OTHER VARIABLE LOADS}}$$

nominal value (G_{1k}) · partial coefficient (γ_{G1})

$\gamma_{q1} Q_{k1}$: FUNDAMENTAL VARIABLE LOAD THAT HAS THE BIGGEST EFFECT ON THE STRUCTURE

$\gamma_{G1} = \begin{cases} 1.3 & \text{where unfavorable (if it increases the stress)} \\ 1.0 & \text{" favorable (if it reduces the stress)} \end{cases}$

$\gamma_{G2} = \begin{cases} 1.5 & \text{unfavourable} \\ 0 & \text{favorable. (IT'S 0 because we can remove it)} \end{cases}$

$\gamma_P = \begin{cases} 1.3 & \text{unfav.} \\ 1 & \text{fav.} \end{cases}$

This coeffic γ_P can be lower in other countries (depending also on the economy of the country)

$\gamma_q = \begin{cases} 1.5 & \text{unfav.} \\ 0 & \text{fav.} \end{cases}$

$\psi_0 < 1$. (from tables) and depends on the type of construction/building

Ex Imagine an unfavorable condition of G_1 and Q_1:

ECO $\quad 1{,}3\ G_1 + 1{,}5\ Q_1$

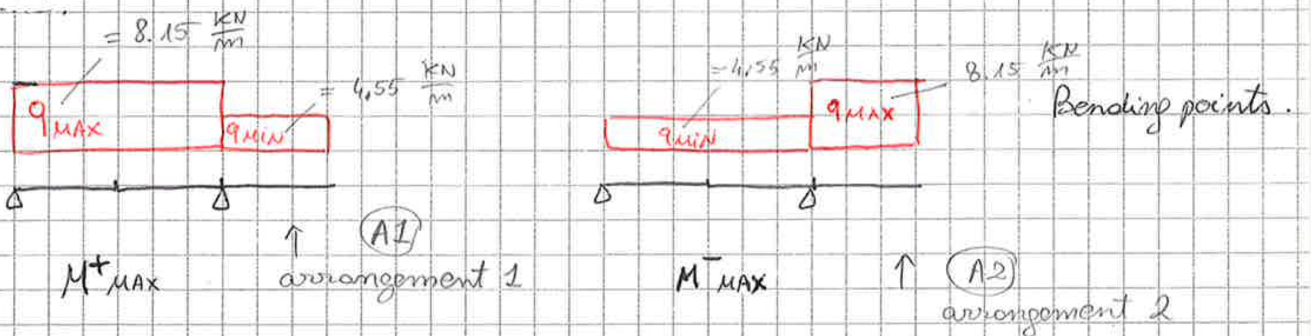
ACI-318 $\quad 1{,}2\ G_1 + 1{,}6 \cdot Q_1$

In general in ECO we have 6 combinations for ULS and SLU

ULS →
- FUNDAM. COMBIN.
- ACCIDENTAL COMBIN.
- SEISMIC COMBIN.

SLS →
- CHARACTERISTIC COMB.
- FREQUENT COMB
- QUASI – PERMANENT COMB.

Now if we want to maximise the positive moment, we have to maximise the load q_2 and minimise the negative one given by q_1; and viceversa.



Problem

Structure with different loads: (with positive sign if according to the gravity)

q_p = permanent (G_1) = 5 KN/m ↓

q_s = live loads = 0,4 KN/m ↓

q_n = snow = 1.1 KN/m ↓ → MAX variable load

q_v = wind = −0,3 KN/m ↑

ULS → fund comb. $G_{1k}\,\gamma_{G1} + G_{2k}\,\gamma_{G2} + Q_{k1}\,\gamma_{q_1} + \sum_{i=1}^{n} Q_{ki}\cdot\psi_{0i}\cdot\gamma_{qi}$

nomi ci sono in questo caso

γ_{G1}: 1.3 / 1.0 γ_{q_1}: 1.5 / 0 γ_{qi}: 1.5 / 0

ψ_{0i}: 0 live loads; 0,6 wind; 0,5 snow

$q_{max} = q_p \cdot \underset{fav}{1,3} + \boxed{q_n\ \underset{fav}{1,5}} + q_s\ \underset{fav}{1,5}\cdot 0 + q_v\cdot \underset{unfav}{0}\cdot 0,6 =$

$= 5\cdot 1,3 + 1,1\cdot 1,5 = 8.15\ KN/m$

⇒ We have found the maximum load that gives the maximum moment.

$q_{min} = q_p\cdot 1.0 + \underset{\text{minimum}}{q_v}\cdot\underset{\text{but not coefficient}}{1,5} + q_n\cdot 0\cdot 0,5 + q_s\cdot 0\cdot 0 = 5\cdot 1 + (-0,3)\cdot 1.5 =$

$= 4.55\ KN/m$

Then we put q_{max} and q_{min} on the beam.

→ (Represent the diagrams in Excel).
Need to know their functions

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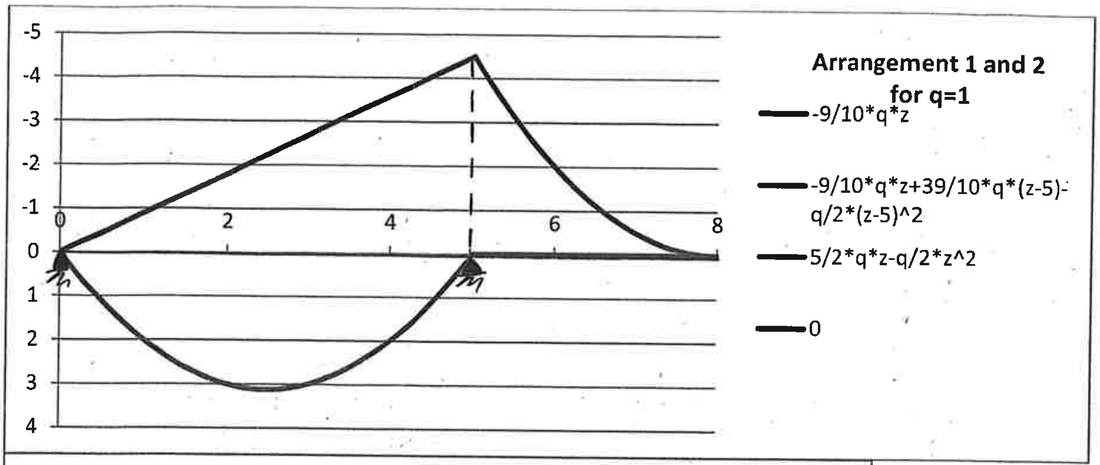
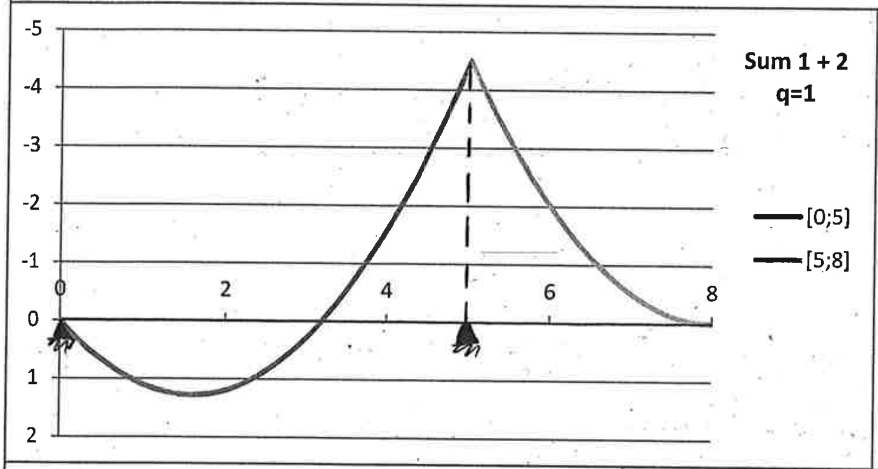
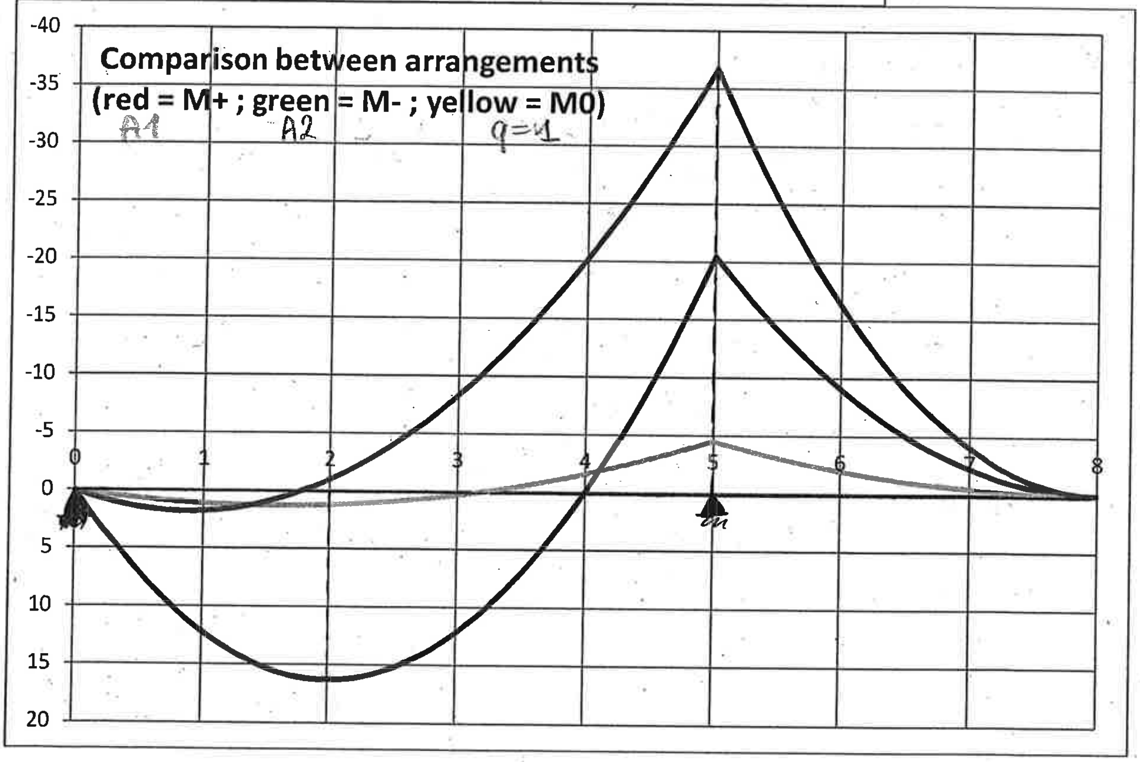
PART 3







FUNDAMENTAL COMBINATION

$$E_d = G_{1k} \cdot \gamma_{G1} + G_{2k} \cdot \gamma_{G2} + G_p \cdot \gamma_p + Q_{k1} \cdot \gamma_{q1} + \sum_{i=2}^{m} \psi_{0i} \cdot Q_{ki} \cdot \gamma_{qi}$$

SEISMIC ACTIONS

There are horizontal loads, depending on the mass, the stiffness and on the viscous damper (AMMORTIZZATORE) of the structure and on ground acceleration.

MASS
- is a property of a physical body
- it measures the body's resistance (or inertia) to acceleration when a force is applied:

$$\boxed{m = \frac{F_i}{a}} \qquad [kg] = \left[\frac{N}{\frac{m}{s^2}}\right]$$

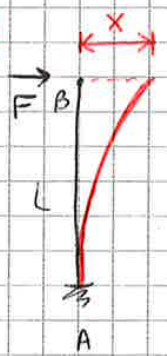
VISCOUS DAMPER

$$\boxed{c = \frac{F_d}{v}} \quad \text{FORZA VISCOSA} \qquad \left[\frac{N \cdot s}{m}\right] = \left[\frac{N}{\frac{m}{s}}\right] = \left[\frac{kg}{s}\right]$$

STIFFNESS

$$\boxed{K = \frac{F_e}{X}}$$ resistance to deformation $\left[\frac{N}{m}\right] = \left[\frac{N}{m}\right]$

Ex)



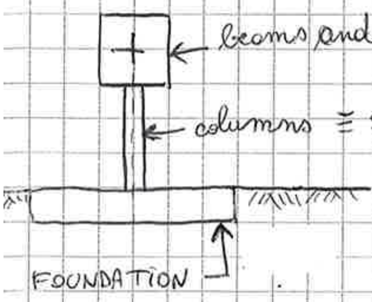
Due to F, there is a displacement in B, represented by X where $\boxed{X = \frac{FL^3}{3EI}}$

$$\Rightarrow \frac{F}{X} = K = \frac{3EI}{L^3} \quad \text{stiffness} \quad \left[\frac{N}{m}\right]$$

DAMPED LOAD = carico smorzato

The earthquake

Release of energy ~~released~~ in a point lungo una faglia.



An acceleration is applied to the foundation ⇒ oscillation of the mass, that can be larger than the one of the foundation. $X_g + X$

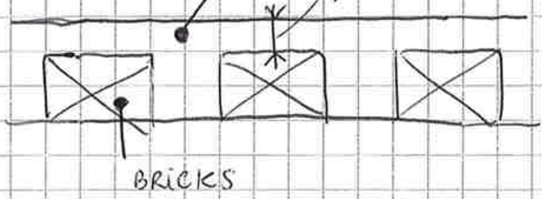
movement of foundation — GROUND — deformation of supports

M

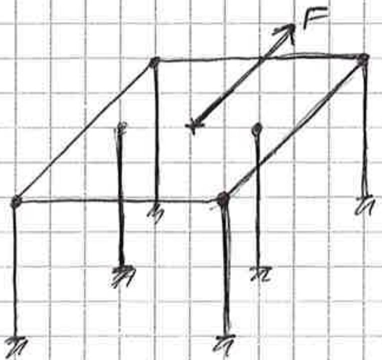
FLOOR SLAB:

RC = reinforced concrete

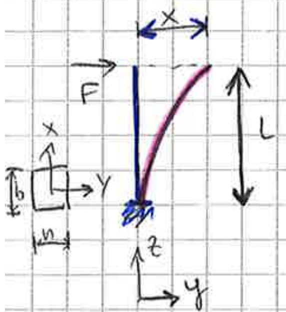
≥ 40 mm = 4 cm ⇒ the floor can be considered a rigid diagram.



DISTRIBUTION OF F ON THE COLUMNS



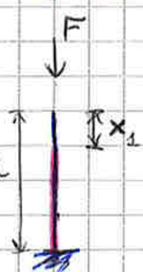
the horizontal diaphragm is statically indet.



$K = \frac{F}{x}$ stiffness $= \frac{3EI}{L^3}$ ⇒ x is dual wrt F

F and x have the same direction and orientation.

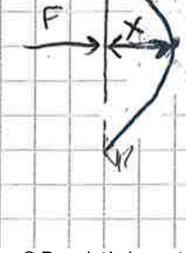
The stiffness of the column is not always the same:



$K_1 = \frac{F}{x_1}$ ⇒ $\boxed{x_1 = \frac{F}{EA} \cdot L}$

⇒ $K_1 = \frac{EA}{L}$

⇒ $K \neq K_1$ ⇒ the STIFFNESS DEPENDS ON THE TYPE OF LOAD, and also of the type of SUPPORT



$K_2 = \frac{F}{x_2} = \frac{\cancel{F}\, 48EI}{\cancel{F} L^3} = \frac{48EI}{L^3}$

$\boxed{x_2 = \frac{FL^3}{48EI}}$

⇒ $K \neq K_1 \neq K_2$

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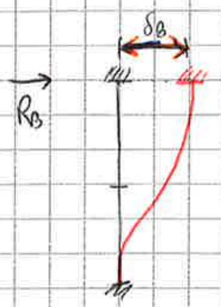
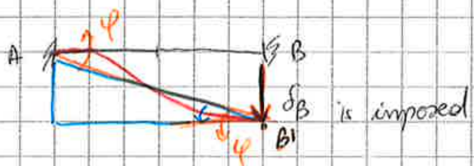
PROBLEM: How to find the stiffness in this case?



$$K = \frac{R_B}{\delta_B} = ?$$

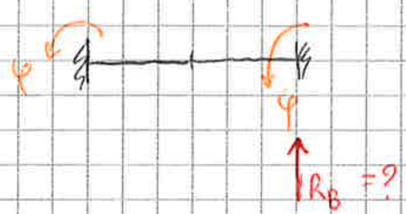
① Find reaction R_B, by applying an imposed displacement of B.



given $\delta_B \Rightarrow$ find the reaction R_B?

If we connect A and B' $\Rightarrow$ we can see that the real deformation is obtained by imposing 2 rotations φ: (small): $\boxed{\varphi = \frac{\delta_B}{L}}$

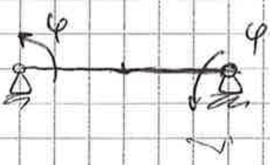
$\Rightarrow$ The system is equivalent to a double supported beam, subjected to a rotation.



→ We can solve with the Force Method
$\Rightarrow$ using the PLV.

1) STEP Define the PRINCIPAL STRUCTURAL SYST. SIMILAR TO THE REAL ONE ("cioè il sistema associato") "La sollecitazione esterna è la rotazione.

system 0



$g = 3 \rightarrow g = 2 \quad T, M$
$v = 2 \quad T, M$

$\Rightarrow$ It's statical for vertical forces.

$\Rightarrow M_0 = 0$

$T_0 = 0$

2) The VIRTUAL SYSTEM are 2, because the redudancy of constraints are 2 ("MOMENTO FLETTENTE NEI 2 vincoli").

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$$\Rightarrow \begin{cases} X_1 \cdot \int_L \frac{M_1^2}{EI} dz + X_2 \int_L \frac{M_1 M_2}{EI} dz = U\varphi \\ X_1 \cdot \int_L \frac{M_1 M_2}{EI} dz + X_2 \int_L \frac{M_2^2}{EI} dz = U\varphi \end{cases}$$

Since EI is the same all over the beam, EI = costant $\Rightarrow$ I can take it out of the integrals.

$$\Rightarrow \begin{cases} X_1 \cdot \int_L M_1^2 dz + X_2 \int_L M_1 M_2 dz = EI\, U\varphi \\ X_1 \int_L M_1 M_2 dz + X_2 \int_L M_2^2 dz = EI \cdot U\varphi \end{cases}$$

$$\int_L M_1^2 dz = U^2 \cdot \int_0^L \left(1 - \frac{z}{L}\right)^2 dz = U^2 \cdot \left(z + \frac{z^3}{3L^2} - \frac{2z^2}{2L} \Big|_0^L \right) = U^2 \left(L + \frac{L^3}{3L^2} - \frac{L^2}{L} \right) = + \frac{U^2 L}{3}$$

$\int_L M_2^2 dz = \frac{U^2 L}{3}$ because it's equal to the previous one because it's the integral (area) of the same shape.

$$\int_L M_1 M_2 dz = -U^2 \cdot \int_L \left(1 - \frac{z}{L}\right) \cdot \frac{z}{L} dz = -U^2 \int_L \left(\frac{z}{L} - \frac{z^2}{L^2} \right) dz = -U^2 \left(\frac{z^2}{2L} - \frac{z^3}{3L^2} \Big|_0^L \right) =$$

$$= -U^2 \left(\frac{L}{2} - \frac{L}{3} \right) = -\frac{U^2 L}{6}$$

$$\Rightarrow \begin{cases} \frac{U^2 L}{3} X_1 - \frac{U^2 L}{6} X_2 = EI\, U\varphi \\ -\frac{U^2 L}{6} X_1 + \frac{U^2 L}{3} X_2 = EI \cdot U \cdot \varphi \end{cases}$$

difference member by member: we get $X_1 = X_2$ $\Rightarrow$ then substitute in the 1st equation.

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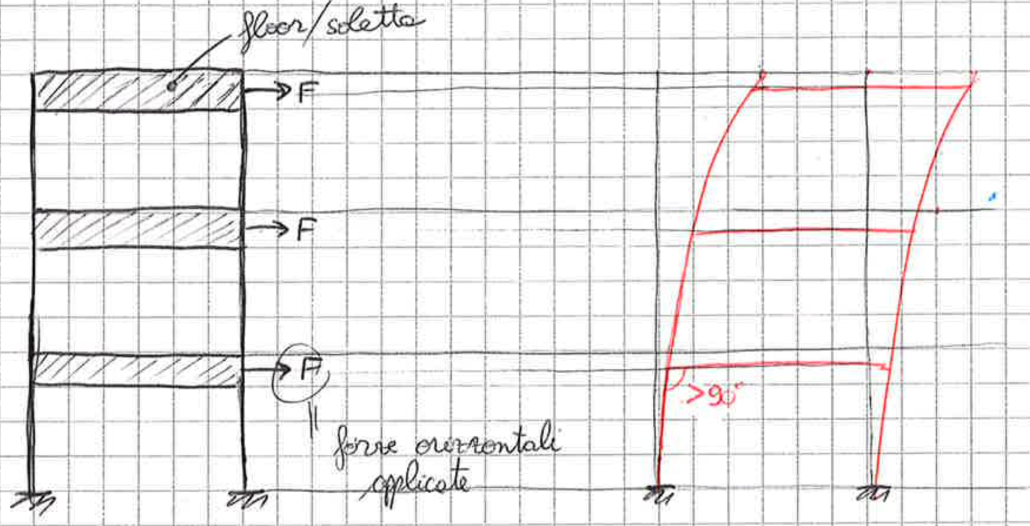
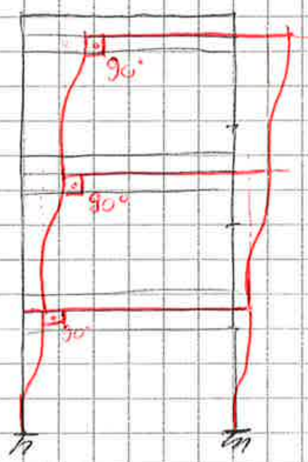
Ex FRAME

BENDING TYPE



① ↑ Angle bigger than 90°

SHEAR TYPE



← Alway 90° angle between column and floor

← RIGID DEFORMATION

②

⇒ The LOAD is applied to the floor's mass. In general if floor is a rectangle and the distribution is uniform → the force is applied ~~applied~~ in the CENTRE OF THE MASS.

$F_S = m \cdot a$

Then we have to distribute the force F_S to the columns below the floor: so we suppose that the column is a shear type deformation.

$$\Rightarrow \quad F_S = 4 \cdot \frac{12 EI_1}{L^3} \cdot \delta + 2 \cdot \frac{12 EI_2}{L^3} \delta$$

$$\delta = \frac{F_S \cdot L^3}{12 E} \cdot \frac{1}{(4 I_1 + 2 I_2)}$$

Now we substitute δ into the 2 equations:

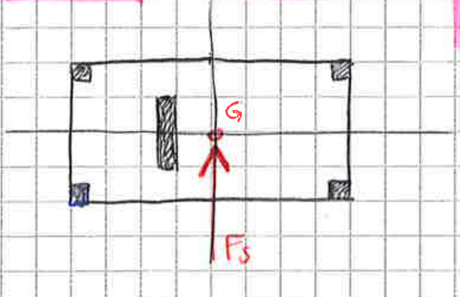
1° eq $$R_{B,1} = \frac{12 EI_1}{L^3} \cdot \frac{F_S L^3}{12 E} \cdot \frac{1}{4I_1 + 2I_2} = \frac{F_S I_1}{4I_1 + 2I_2}$$

2° eq $$R_{B2} = \frac{12 EI_2}{L^3} \cdot \frac{F_S L^3}{12 E} \cdot \frac{1}{4I_1 + 2I_2} = \frac{F_S I_2}{4I_1 + 2I_2}$$ rigidity of the element / moment of inertia

$\Rightarrow$ In GENERAL: (*1) $$\boxed{R_{B_i} = F_S \cdot \frac{I_i}{\sum_{j=1}^{m} I_j}}$$ → SOMMA DEI MOMENTI DI INERZIA DI TUTTI I TIPI DI ELEMENTI

N_j = number of columns of type j

3° case: COLUMNS WITH DIFFERENT GEOMETRY AND NO SYMMETRY WRT F_S.



(*1) The reaction $R_{B,i}$ is proportional to the applied (seismic) load and the constant of proportionality is: $\frac{I_i}{\sum I_j}$, which is the ratio of the moment of inertia of the column WRT the sum of all the moments of intertia.

Reactions $R_{B,i}$ are equal and opposite to the fraction of the (seismic) load applied to each column.

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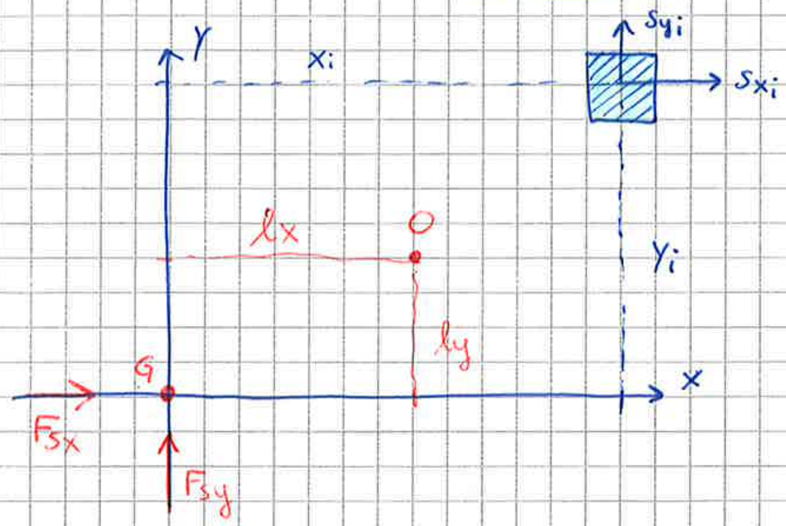
⇒ in GENERAL: THEORY BASED ON THE CENTRE OF TORSION

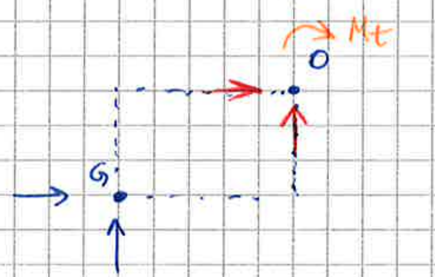


i-th column

S_{y_i} and S_{x_i} are the reactions' components on each column

G = centre of mass.
O = centre of torsion.

We can traslate the applied forces from G to the point O and add the torsional moment M_t:



$$\boxed{M_t = -F_{Sx} \cdot l_y + F_{Sy} \cdot l_x}$$

if ↻+ we have to pay attention to the sign of the components.

From point O, we find

$$\begin{cases} S_{x_i}^{TOT} = S_{x_i}^{(TRASL)} + S_{x_i}^{(M_t)} \\ S_{y_i}^{TOT} = S_{y_i}^{TRASL} + S_{y_i}^{(M_t)} \end{cases}$$

(TRASL) = (F)

1° CONTRIB 2° CONTRIB

1° CONTRIBUTION

$$\boxed{\begin{cases} S_{x_i}^{(F)} = F_{Sx} \cdot \dfrac{I_{y_i}}{\sum_{j=1}^{n} I_{y_j}} \\ \\ S_{y_i}^{(F)} = F_{Sy} \cdot \dfrac{I_{x_i}}{\sum_{j=1}^{n} I_{x_j}} \end{cases}}$$

LOAD

SOMMA DEI MOMENTI D'INERZIA NELLA DIREZIONE Y DI TUTTI I TIPI DI COLONNA

LOAD

But what is the POSITION OF O?

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a_x is the TRASLATION OF THE COLUMN IN THE X-DIRECTION DUE TO ROTATION

$$\boxed{a_x = - Y_{0i} \cdot \sin\alpha \approx - Y_{0i} \cdot \alpha}$$

because the verse is opposite to x

LOAD STIFFNESS DEFORMATION

$$\Rightarrow \begin{cases} S_{x_i}^{(M_t)} = -K_{y_i} \cdot a_{x_i} = -\dfrac{12\,E\,I_{y_i}}{L^3}\,\alpha\,Y_{0i} \\ S_{y_i}^{(M_t)} = K_{x_i} \cdot a_{y_i} = \dfrac{12\,E\,I_{x_i}}{L^3}\,\alpha\,X_{0i} \end{cases}$$

$$K = \frac{12EI}{L^3}$$

α can be defined by the equilibrium to the torsion :

$$M_t \underset{(\text{wrt } O)}{} = \sum_{i=1}^{m} S_{y_i}^{(M)} \cdot X_{0i} - \sum_{i=1}^{m} S_{x_i}^{(M)} \cdot Y_{0i}$$

substitute the previous formulas (of condition 2)

$$M_t = \frac{12 \cdot E\,\alpha}{L^3} \underbrace{\left[\sum_{i=1}^{m} I_{x_i} \cdot X_{0i}^2 + \sum_{i=1}^{m} I_{y_i} \cdot Y_{0i}^2 \right]}_{\psi}$$

$$\Rightarrow M_t = \frac{12\,E\,\psi}{L^3} \cdot \alpha$$

M_t is known $M_t = F_{sy} \cdot d$

$$\Rightarrow \boxed{\alpha = \frac{M_t \cdot L^3}{12\,E\,\psi}}$$

Now we put α:

$$\boxed{\begin{cases} S_{x_i} = -M_t \dfrac{I_{y_i} \cdot Y_{0i}}{\psi} \\ S_{y_i} = M_t \cdot \dfrac{I_{x_i} \cdot X_{0i}}{\psi} \end{cases}}$$

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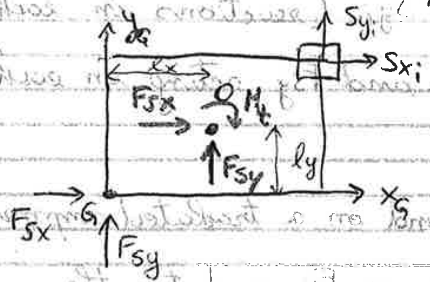
$\Rightarrow$ All the EQUATIONS TOGETHER:

$$\boxed{\begin{cases} S_{x_i}^{(TOT)} = F_{Sx} \cdot \dfrac{I_{y_i}}{\sum_{j=1}^{m} I_{y_j}} - \dfrac{M_t}{\psi} \cdot I_{y_i} \cdot Y_{0i} \\ S_{y_i}^{(TOT)} = F_{Sy} \cdot \dfrac{I_{x_i}}{\sum_{j=1}^{m} I_{x_j}} + \dfrac{M_t}{\psi} \cdot I_{x_i} \cdot X_{0i} \end{cases}}$$

VALID FROM $i \to m$ so we can distribute load on the columns. It's valid for all the distribution of the column

DEMONSTRATION OF EQUATIONS OF 3° CASE OF SEISMIC LOADS (STEPS)

1) Traslate the forces $F_{S,x}$ and $F_{S,y}$ from G to O: so a moment M_t appears.



2) S_{x_i} and S_{y_i} are forces (reactions) due to 2 contributions: traslated forces and torque

3) Traslated contribution:

$$\begin{cases} S_{x_i}^{(F)} = F_{Sx} \dfrac{I_{y_i}}{\sum I_{y_i}} \\ S_{y_i}^{(F)} = F_{Sy} \cdot \dfrac{I_{x_i}}{\sum I_{x_i}} \end{cases}$$

4) Position of center of rotation O? Do static moment equation of forces WRT $(X_G; Y_G)$ axis.

5) Find the value of $S_{x_i}^{(M_t)}$ and $S_{y_i}^{(M_t)}$ depending on angle α.

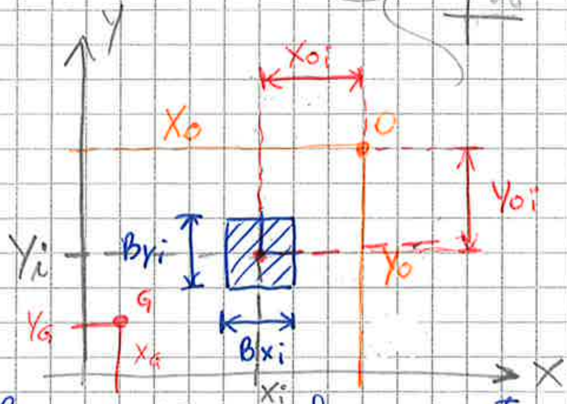
7) Find angle α by equl equation of rotation around O of the components $S_{x_i}^{(M_t)}$ and $S_{y_i}^{(M_t)}$

8) Put contributions together ⇒ GET EQUATIONS.

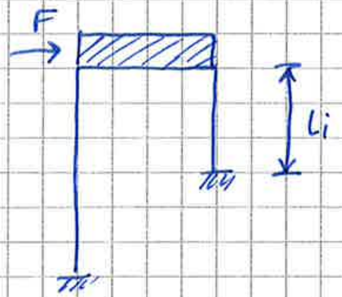
! We have to repeat all the formulas for every column ⇒ create an algorythm that computes the distribution of horizontal loads.

"mettetelo nell'outline"

ALGORYTHM



- INPUT DATA
  - → n° of columns
  - → B_{x_i}, B_{y_i}
  - → The position of all the columns WRT a reference system. Y_i, X_i
  - → L_i, the length of each column
  - → F_{sy}, F_{sx}
  - → X_G, Y_G



- OUTPUT DATA
  - → X_0, Y_0

$$X_0 = \frac{\sum_{i=1}^{m} \frac{I_{x_i}}{L_i^3} \cdot X_i}{\sum_{j=1}^{m} \frac{I_{x_j}}{L_j^3}} \; ; \quad Y_0 = \frac{\sum_{i=1}^{m} \frac{I_{y_i}}{L_i^3} \cdot Y_i}{\sum_{j=1}^{m} \frac{I_{y_j}}{L_j^3}}$$

WRT previous formula we have generalized

where:

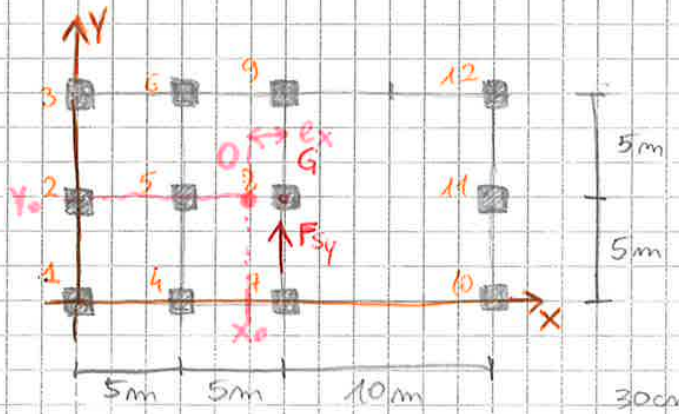
$$I_{x_i} = \frac{B_{x_i} \cdot B_{y_i}^3}{12}$$

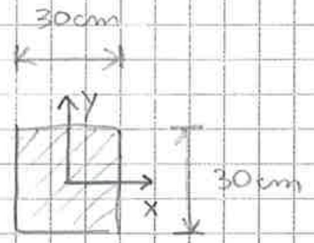
$$I_{y_i} = \frac{B_{y_i} \cdot B_{x_i}^3}{12}$$

and

$$e_y = Y_0 - Y_G$$
$$e_x = X_0 - X_G$$

Ex





- All the columns are the same
- $F_{Sy} = 50$ kN , $F_{Sx} = 0$
- 12 columns , $L = 3$ m
- Scheme :

geom. data :

column	Bx_i	By_i	X_i	Y_i	l_i
1	0,3	0,3	0	0	3
2	0,3	0,3	0	5	3
⋮ 8	0,3	0,3	10	5	3
m = 12	0,3	0,3	20	10	3

! Use the same units for every length [m]

! Define (x,y) and so the number of column and its corresponding (x_i, y_i)

loads and (x_G, y_G)

Fx	0
Fy	50
X_G	10
Y_G	5

- Output :

centre of torsion

$X_o = 8.75$
$Y_o = 5$
$l_x = e_x = 1.25$
$l_y = e_y \cong 0$ → it's approximated to 0.

O is closed to the most rigid zone

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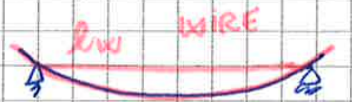
11/10/2017

THEORY OF ELASTIC COACTIONS

Elastic Coactions are a STATE OF STRESS IN A BODY, in which a constraint is generated between some parts of the body, i.e. a LIMITATION OF MOVEMENT. Used to study also non-linear behaviour (cioè / id est)

deflection due to loads

$l_{wire} < L$



→ state of stress without the application of any external loads → if I want to remove the state of stress I have to remove the wire.

⇒ ∃ many situations where ∃ state of stress ⇔ imposed strains without the application of any external loads:

- EFFECT OF TEMPERATURE (especially if not uniform)
- SHRINKAGE (= ritiro)
- PRE-STRESS

GUSTAVO COLONNETTI (1896 - 1968)

If you impose a strain on a body $\{\bar{\varepsilon}\}$ = IMPOSED STRAIN, it's INCOMPATIBLE. ⇒ then a COMPLEMENTARY ELASTIC STRAIN is GENERATED $\{\varepsilon\}_{el}$ WITHIN THE BODY and THE TOTAL STRAIN $\boxed{\{\varepsilon_{TOT}\} = \{\bar{\varepsilon}\} + \{\varepsilon\}_{el}}$ is COMPATIBLE.

In general $\boxed{\{\bar{\varepsilon}\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}}$ and $\boxed{\begin{aligned} \{\sigma\} &= [D] \cdot \{\varepsilon\}_{el} = \\ &= [D] \cdot \left(\{\varepsilon\}_{TOT} - \{\bar{\varepsilon}\}\right) \end{aligned}}$

ANGULAR DISTORSION

STATE OF STRESS IN THE ELASTIC BODY

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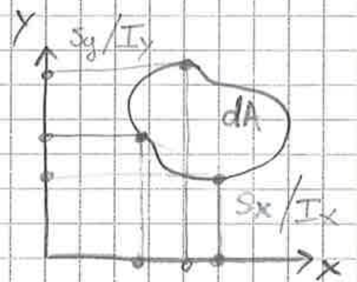
$$\varepsilon_{zTOT} = \bar{\varepsilon}_z + \varepsilon_{zel} = \lambda + \mu_x \cdot y + \mu_y \cdot x$$

$$\varepsilon_{zel} = \lambda + \mu_x y + \mu_y x - \bar{\varepsilon}_z$$ eq of a plane



$$\sigma_z = E \cdot \varepsilon_{el} = E(\lambda + \mu_x y + \mu_y \cdot x - \bar{\varepsilon}_z)$$

If you substitute σ_z into N, M_x, M_y ⇒ :

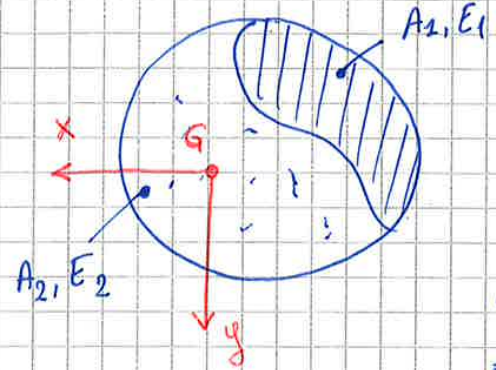
$$N = \int \sigma_z \, dA = \int E(\lambda + \mu_x y + \mu_y x - \bar{\varepsilon}_z)\, dA =$$

$$= \int E\lambda \, dA + \int E\mu_x \overset{S_x}{y\,dA} + \int E\mu_y \overset{S_y}{x \cdot dA} - \int E\bar{\varepsilon}_z \, dA$$

$$M_x = \int \sigma_z y \, dA = \int E\lambda \overset{S_x}{y\,dA} + \int E\mu_x \overset{I_x}{y^2 dA} + \int E\mu_y \overset{I_{xy}}{xy\,dA} - \int E\cdot\bar{\varepsilon}_z \overset{S_x}{y\,dA}$$

$$M_y = \int \sigma_z x \, dA = \int E\lambda x \, dA + \int E\mu_x xy\,dA + \int E\mu_y x^2 dA - \int E\,\bar{\varepsilon}_z x\, dA$$

Ex IN GENERAL : MORE MATERIALS



⊕ and $\bar{\varepsilon}_z$ on A_1. (the imposed strain is considered only on area A_1)

⊕ (x,y) axis on the CENTROID of the section.

⇒ we have to HOMOGENIZE THE VARIABLES

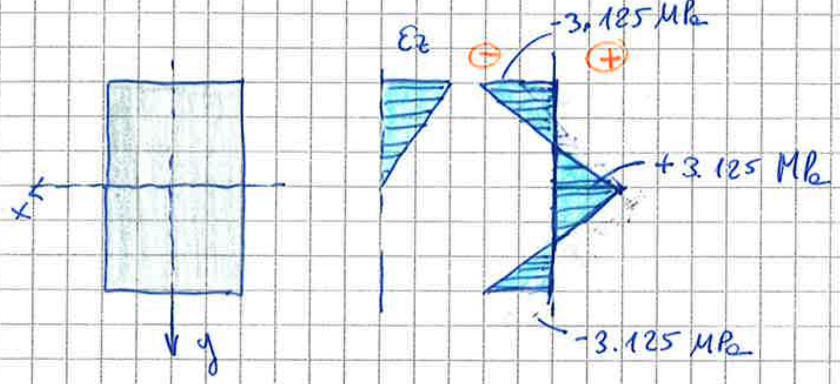
⇒ so we have to choose an E_0 and divide them by E_0.

EQUATIONS TO REPEAT FOR EACH MATERIAL

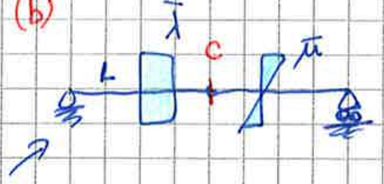
$$\begin{cases} N = E_0 \lambda A_0 - E_0 \int_{A_1} m_1 \bar{\varepsilon}_z \, dA \\ M_x = E_0 \mu_x I_{x_0} + E_0 \mu_y \cdot I_{xy,0} - E_0 \int_{A_1} m_1 \cdot \bar{\varepsilon}_z Y \, dA \\ M_y = E_0 \mu_y I_{y_0} + E_0 \mu_x \cdot I_{xy,0} - E_0 \int_{A_1} m_1 \cdot \bar{\varepsilon}_z X \, dA \\ m_1 = \dfrac{E_1}{E_0} \\ m_2 = \dfrac{E_2}{E_0} \\ A_0 = m_1 A_1 + m_2 A_2 \end{cases}$$

20/10/17

From last exercise



(b)



imposed $\bar{\lambda} = 12.5 \cdot 10^{-5}$

$\bar{\mu}_x = 1.25 \cdot 10^{-6} \frac{1}{mm}$

$L = 4m.$

Statically det beam.

$\Rightarrow$ Find $\begin{cases} M_b = 0 \\ N_b = 0 \\ T_b = 0 \end{cases}$ because only imposed strain is applied in the cross section. But the deformation is $\neq 0$.

"FRECCIA"

Compute DEFLECTION IN C. $f_c = ?$

We apply the equation of virtual work.

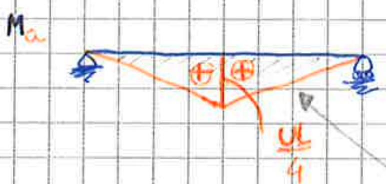
$$\boxed{L^{a,b}_{EXT} = L^{a,b}_{INT}}$$

a = virtual
b = real

(a)

$N_a = 0$

M_a



$\Rightarrow L^{a,b}_{EXT} = \underbrace{U}_{VIRT} \cdot \underbrace{f_c}_{REAL}$

$L^{a,b}_{INT} = \int_L \underbrace{M_a}_{VIRT} \underbrace{\mu_b}_{REAL} dz = \bar{\mu} \int_L M_a \cdot dz$

total curvature $\mu = \mu_{el} + \bar{\mu}$

AREA OF

$\Rightarrow U \cdot f_c = \bar{\mu} \int_L M_a dz = \bar{\mu} \frac{UL}{4} \cdot \frac{L}{2} = \bar{\mu} \frac{UL^2}{8} \Rightarrow f_c = \frac{\bar{\mu} L^2}{8} =$ DEFLECTION DUE ONLY TO THE IMPOSED STRAIN.

$= \frac{4000^2}{8} \cdot 1.25 \cdot 10^{-6} = 2.5 mm$

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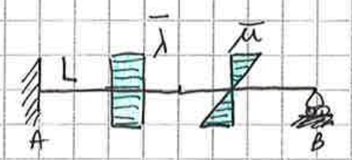
		$\mathcal{E}$ MODIFIES	
		KINEMATICAL CONDITIONS	STATICAL CONDITIONS
TYPE OF STRUCTURE	STATICALLY DETERMINATED	YES	NO
	STATICALLY INDETERMINATED	YES	YES

- CASE IN WHICH THE STRUCTURE IS STATICALLY INDETERMINATED

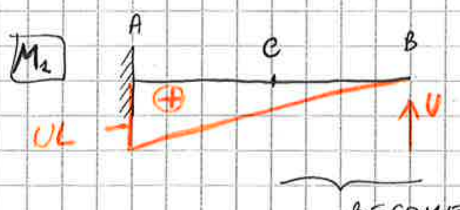


(1) Compute the reactions on the supports: $n = 1$ redundant support

→ Transform into a MAIN SYSTEM $^{(0)}$ which is statically determined.

$M_0 = 0$, $N_0 = 0$, $T_0 = 0$ } because the imposed $\bar{\mathcal{E}}_z$ don't produce sollecitations.

(2) SECONDARY SYSTEM → (1) ⇒ Find freccia in B : f_B ?



$$M_b = M_0 + X\,M_1$$

"è il momento reale = somma del momento della struttura associata e di quella con la x"

⇒ (3) $L_{EXT}^{a,b} = U \cdot \underbrace{0}_{f_B} = 0$ in general the ext work is $=0$ unless there are settlements of the supports.

↳ perchè è la condizione $v_B = 0$.

$$L_{INT}^{a,b} = \int_L M_1 \cdot \mu \, dz = \int_L M_1 \cdot \left(\frac{X M_1}{EI} + \bar{\mu} \right) dz =$$

$\mu \to \mu_{el} + \bar{\mu}$

$\mu_{el} \to \frac{M_b}{EI} = \frac{M_0 + X M_1}{EI}$

$$= \frac{X}{EI} \int_L M_1^2 \, dz + \bar{\mu} \int_L M_1 \, dz \Rightarrow$$

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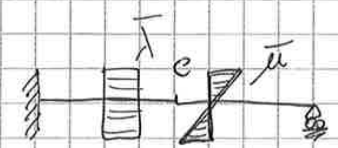
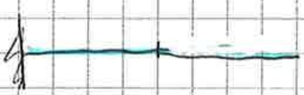
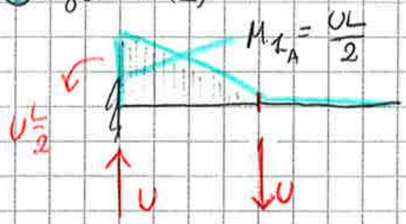
→ Find $f_c = ?$



1) System (0) : No ext forces : $M_0 = 0$, $T_0 = 0$, $N_0 = 0$



2) System (1)



$$M_1 = \begin{cases} -\frac{UL}{2} + Uz & \text{per } z \in [0, \frac{L}{2}] \\ 0 & \text{per } z > \frac{L}{2} \end{cases}$$

$$\Rightarrow M^{reale} = \cancel{M_0} + X M_1 = X M_1$$

3) PLV $\Rightarrow$ $f_c = 0 = v_c$:

$$U v_c = \int M_1 \, \mu^{reale} \, dz$$

$$0 = \int M_1 \left(\frac{X M_1}{EI} + \bar{\mu} \right) dz$$

$$\Rightarrow \frac{X}{EI} \int M_1^2 dz + \bar{\mu} \int M_1 dz = 0$$

$$X \frac{U^2 L^3}{16} + \bar{\mu} EI \cdot \frac{UL^2}{8} = 0 \quad \Rightarrow \quad X = -\frac{\bar{\mu} EI \, UL^2}{8} \cdot \frac{16}{U^2 L^3} = -2\bar{\mu} \frac{EI}{UL}$$

$\Rightarrow$ X depends on $\bar{\mu}$

⟶ END OF 1ST PART $E_d \leq R_d$

⇒ finished the computation of E_d.

CO-ACTIONS: IMPRESSED DEFORMATIONS

These are: - effect of temperature
- shrinkage
- pre-stress.

$$\{\varepsilon_{TOT}\} = \{\bar{\varepsilon}\} + \{\varepsilon_{el}\} \qquad \{\bar{\varepsilon}\} = \begin{Bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_y \\ \bar{\varepsilon}_z \\ \bar{\gamma}_{xy} \\ \bar{\gamma}_{xz} \\ \bar{\gamma}_{yz} \end{Bmatrix}$$

$$\{\sigma\} = [D]\cdot\{\varepsilon_{el}\} = [D]\left[\{\varepsilon_{TOT}\} - \{\bar{\varepsilon}\}\right]$$

$$\{\varepsilon_{TOT}\}_z = \lambda + \mu_x \cdot y + \mu_y \cdot x$$

INSERT IN:

Equations: $N = \int \sigma_z \, dA$

$M_x = \int \sigma_z \cdot y \, dA$

$M_y = \int \sigma_z \cdot x \cdot dA$

GET THIS:

39A

Matrix

$$\begin{Bmatrix} N \\ M_x \\ M_y \end{Bmatrix} = E_0 \begin{bmatrix} A_0 & 0 & 0 \\ 0 & I_{x_0} & I_{xy_0} \\ 0 & I_{xy_0} & I_{y_0} \end{bmatrix} \begin{Bmatrix} \lambda \\ \mu_x \\ \mu_y \end{Bmatrix} - E_0 \begin{Bmatrix} \int_{A_1} m_1 \bar{\varepsilon}_z \, dA \\ \int_{A_1} m_1 \bar{\varepsilon}_z \, dA \\ \int_{A_1} m_1 \bar{\varepsilon}_z \, dA \end{Bmatrix}$$

$$\begin{Bmatrix} \lambda \\ \mu_x \\ \mu_y \end{Bmatrix} = \frac{1}{E_0} \begin{bmatrix} 1/A_0 & 0 & 0 \\ 0 & \frac{I_{y_0}}{D} & \frac{-I_{xy_0}}{D} \\ 0 & \frac{-I_{xy_0}}{D} & \frac{I_{x_0}}{D} \end{bmatrix} \begin{Bmatrix} N \\ M_x \\ M_y \end{Bmatrix} + \begin{bmatrix} \frac{1}{A_0} & 0 & 0 \\ 0 & \frac{I_{y_0}}{D} & \frac{-I_{xy_0}}{D} \\ 0 & \frac{-I_{xy_0}}{D} & \frac{I_{x_0}}{D} \end{bmatrix} \cdot \begin{Bmatrix} \int m_1 \bar{\varepsilon}_z \, dA \\ \int m_1 \bar{\varepsilon}_z^{\,y} \, dA \\ \int m_1 \bar{\varepsilon}_z^{\,x} \, dA \end{Bmatrix}$$

$$D = I_{x_0} I_{y_0} - I_{xy_0}^2 \qquad \| \qquad A_0 = n_1 A_1 + n_2 A_2$$

1

⇒ HOMOGENIZE THE CROSS SECTION

$E_0 = E$ we have only 1 material

$A_0 = \underset{1}{m_1} BH = 200 \cdot 400 = 80\,000 \text{ mm}^2$

$I_{x_0} = \dfrac{BH^3}{12} = \dfrac{200 \cdot 400^3}{12} = 1{,}07 \cdot 10^9 \text{ mm}^4$

$I_{y_0} = \dfrac{HB^3}{12} = \dfrac{400 \cdot 200^3}{12} = 2{,}67 \cdot 10^8 \text{ mm}^4$

$I_{x_0 y_0} = 0$ because central axis

2

$\Rightarrow \boxed{\lambda} = \dfrac{1}{A_0} \int_{A_1} m_1 \bar{\varepsilon}_z \, dA = \dfrac{1}{A_0} B \cdot \int_{-\frac{H}{2}}^{0} \bar{\varepsilon}_z \, dy =$ $\quad dA = dx \cdot dy$, $dx = B$ constant ⇒ la $\bar{\varepsilon}_z$ non varia lungo B

$= -\dfrac{B}{A_0} \cdot \dfrac{2{,}5 \cdot 10^{-6}}{\text{mm}} \int_{-\frac{H}{2}}^{0} y \, dy = -\dfrac{B}{A_0} \dfrac{2{,}5 \cdot 10^{-6}}{\text{mm}} \left(-\dfrac{H^2}{8}\right) = 12{,}5 \cdot 10^{-5}$

"È come se forzassi una deformaz nella direz ⊥ alla lavagna, e ciò porta a un lavoro di trazione pari a quel $\bar{\lambda}$."

3

$\Rightarrow \boxed{\bar{\mu}_x} = \dfrac{1}{I_{x_0}} \int_{A_1} m_1 \bar{\varepsilon} y \, dA = \dfrac{1}{I_{x_0}} B \cdot \int_{-\frac{H}{2}}^{0} y \cdot \bar{\varepsilon}_z \, dy = \dfrac{-B}{I_{x_0}} \cdot \dfrac{2{,}5 \cdot 10^{-6}}{\text{mm}} \int_{-\frac{H}{2}}^{0} y^2 dy = -\dfrac{2{,}5}{2} 10^{-6} =$

$= -1{,}25 \cdot 10^{-6} \dfrac{1}{\text{mm}}$

4

$\Rightarrow \boxed{\bar{\mu}_y} = \dfrac{1}{I_{y_0}} \int_{A_1} \overset{1}{m_1} \bar{\varepsilon} x \, \overset{dx\,dy}{dA} = \dfrac{1}{I_{y_0}} \int_{-\frac{H}{2}}^{0} \bar{\varepsilon} \, dy \cdot \int_{-\frac{B}{2}}^{\frac{B}{2}} x \, dx =$

$= -\dfrac{1}{I_{y_0}} \cdot \dfrac{2{,}5 \cdot 10^{-6}}{\text{mm}} \int_{-\frac{H}{2}}^{0} y \, dy \cdot \underbrace{\left.\dfrac{x^2}{2}\right|_{-\frac{B}{2}}^{\frac{B}{2}}}_{=0} = 0$

⇒ ∄ curvature on y-axis because of symmetry there can't be any rotation around y axis, but only around x axis.

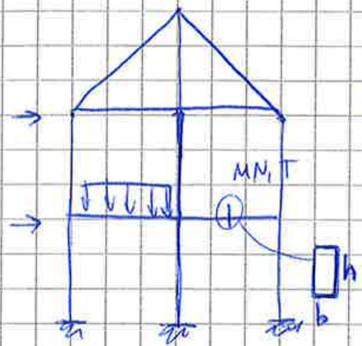
$\Rightarrow \sigma_z = E\left(\bar{\lambda} + \bar{\mu}_x y + \bar{\mu}_y x - \bar{\varepsilon}_z\right)$ ➤ No external forces ⇒ no elastic deformations ⇒ only imposed strains.

$\Rightarrow \sigma_z = \sigma_z(y) = 25\,000 \left(12{,}5 \cdot 10^{-5} - 1{,}25 \cdot \dfrac{10^{-6}}{\text{mm}} y - \bar{\varepsilon}_z\right)$

↳ is a sum of different linear ~~equations~~ functions

23/10/17

THE PROJECT



Statically indet. → when I know the loads → so find state of stress → define cross sections. But I need to know in advance the cross section!
If it was static determ ⇒ the state of stress won't depending on the cross section.

⇒ We have to predict the cross section and verify if it's suitable or not to carry the loads. If it's not ok, we change the shape.
↓
There are some rules given by codes. In general $h \geq 2L$

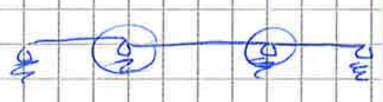
$\frac{L}{16} \leq h \leq \frac{L}{11}$ SIMPLY SUPPORTED

for ex. where is not important deflection.

MAX MIN

$\frac{L}{12}$ $\frac{L}{20}$ (ITA) $\leq h \leq \frac{L}{18,5}$ $\frac{L}{14}$ ONE END CONTINUOUS

$\frac{L}{14} \leq h \leq \frac{L}{21}$ BOTH END CONTINUOUS



$\frac{L}{5} \leq h \leq \frac{L}{8}$ CANTILEVER

→ The Italian is lower because of money

$h \leq 2L$

$b \geq 200$ mm → $b_{min} = 250$ mm
$b \leq \frac{L}{4}$

25/10/17

Rd : CAPACITY OF ELEMENTS OF STRUCTURES

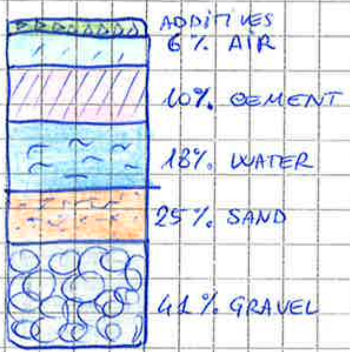
REINFORCED CONCRETE (RC) → want to know the mechanical properties of concrete & rebars.

$$8\,\frac{kN}{m^3} = 800\,\frac{kg}{m^3}$$

$$8000\,\frac{N}{m^3} = \underbrace{800\,\frac{kg}{m^3}\cdot 10\,\frac{m}{s^2}}_{8000\,\frac{N}{m^3}}$$

CONCRETE

Artificial rock made by
- → WATER
- → SAND
- → AGGREGATES : COARSE / FINE (Sand)
- → AIR
- → ADDITIVES



- DENSITY
  ↓
- WEIGHT → $\frac{kN}{m^3}$ usual used

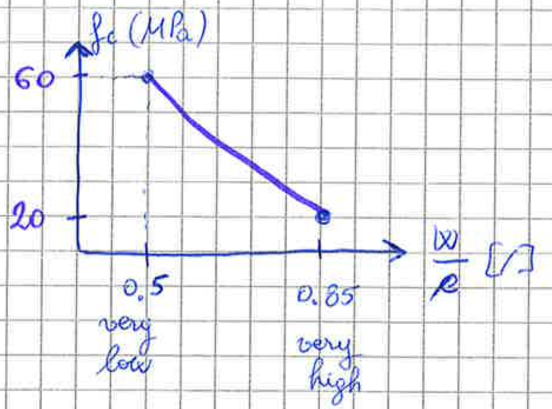
MC 2010 :
- LIGHT WEIGHT CONCRETE : $8 \div 20\ \frac{kN}{m^3} = 800 \div 2000\ \frac{kg}{m^3}$
- NORMAL " " : $20 \div 26\ \frac{kN}{m^3}$
- HEAVY " " : $\geq 26\ \frac{kN}{m^3}$

ex. Light concrete in floors in seismic zones
Heavy " " nuclear plants, shield, hospitals] → obtained by changing the type of aggregates.
Normal is made by silicate carbonate minerals.

The strenght f_c depends on many factors:

1) water / cement ratio. $\frac{w}{c} = \frac{Kg_{H_2O}}{Kg_{cement}}$

↳ if $w \uparrow \Rightarrow f_c \downarrow$



2) type of cement:

Normal → TYPE I
Modified → TYPE II
Hearly Strength → TYPE III
Low heat → TYPE IV
Sulfate Resisting TYPE V

3) Supplementary materials:

→ FLY ASH
→ SILICA FUME

→ pozzolanic effect → it reduces the $\frac{w}{c}$ ratio increasing the strength.

4) Type of aggregate

→ STRONG → QUARZITE
→ NORMAL → LIME STONE
→ WEAK → SANDSTONE

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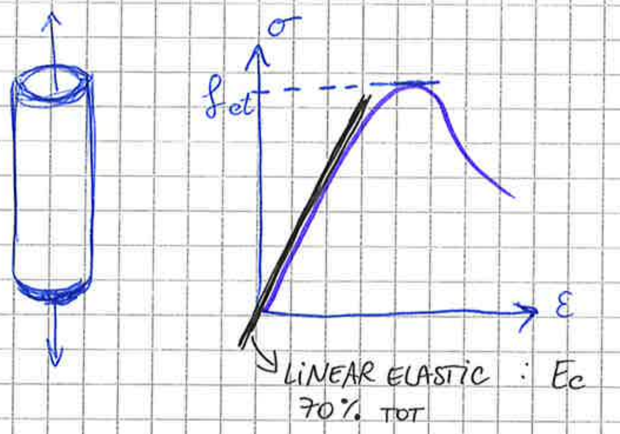
BEHAVIOUR IN TENSION → TEST on CYLINDER



$$\boxed{8\% f_c \le f_{ct} \le 15\% f_c}$$

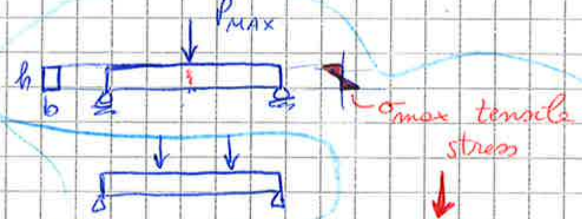
So f_{ct} for traction is lower usually that the one of compression.

→ NORMAL STRENGTH CONCRETE

$$\boxed{\begin{aligned} f_c &\cong 30\ MPa \\ f_{ct} &\approx 3\ MPa \\ E_c &\cong 22000 \cdot \left(\frac{30}{10}\right)^{0.3} = 30000\ MPa = 30\ GPa \end{aligned}}$$

Generally the direct test is not performed because it's difficult, so we use different test:

- 3-POINT BENDING TEST
- 4-POINT BENDING TEST



MODULUS OF FRACTURE $f_{ct,f} = \frac{3}{2}\frac{PL}{bh^2} = \frac{PL6}{4bh^2} = \sigma_{max} = \frac{M_{MAX}\,6}{bh^2}$

$f_{ct,f} > f_{c,t}$

3 points test ↳ direct test

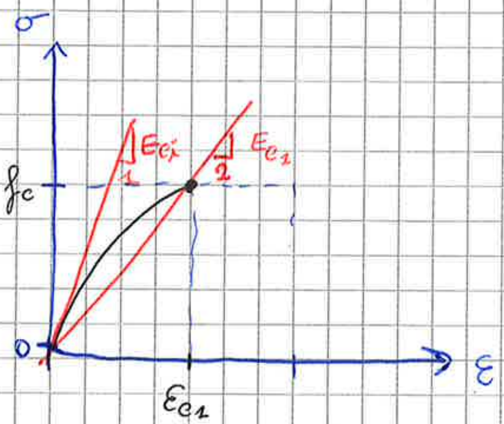
- SPLITTING TEST / BRASILIAN TEST

FLEXURAL TEST

30/10/17

STRESS STRAIN RELATIONSHIP

↳ COMPRESSION → PRE-PEAK STAGE → SARGIN'S PARABOLA



Equation of the parabola:

$$\boxed{\sigma = f_c(-)\frac{k\eta - \eta^2}{1+(k-2)\eta}}$$

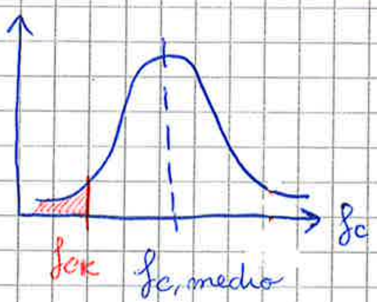
f_c is the average value of the compressive strength.

- $\eta = \dfrac{\varepsilon}{\varepsilon_{c1}}$ relative strain.
- $k = \dfrac{E_{ci}}{E_{c1}}$ plasticity number (E_{ci} in the origin, E_{c1} at the peak of stress.)

We use this equation to describe the behaviour

The parameters η and k can be defined by EC2 if there is no test available.

Also the model code 2010 defines the parameters from the class of concrete: class of concrete are called C30, where 30 is the characteristic strength which is the 5% percentile (the value of the compressive stress is overcomed in 95% of the cases).



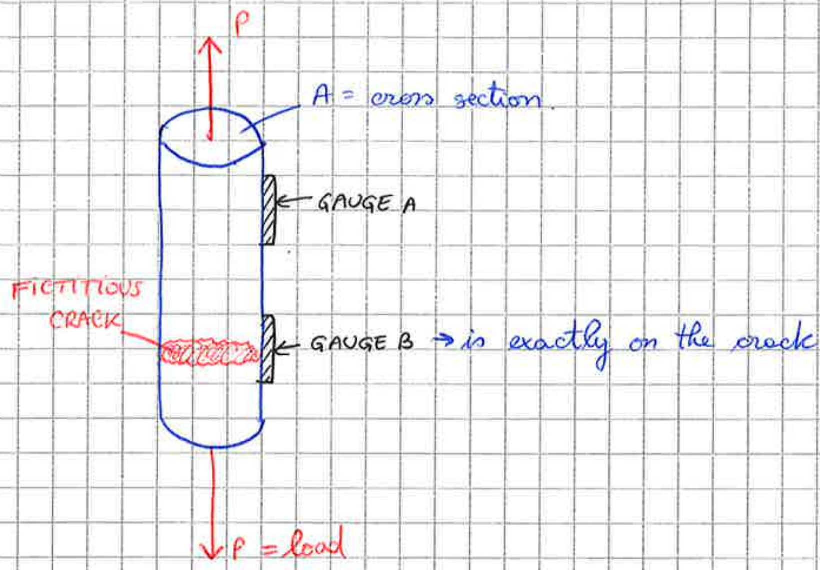
In Italy C30/35 → cube R_{ck}

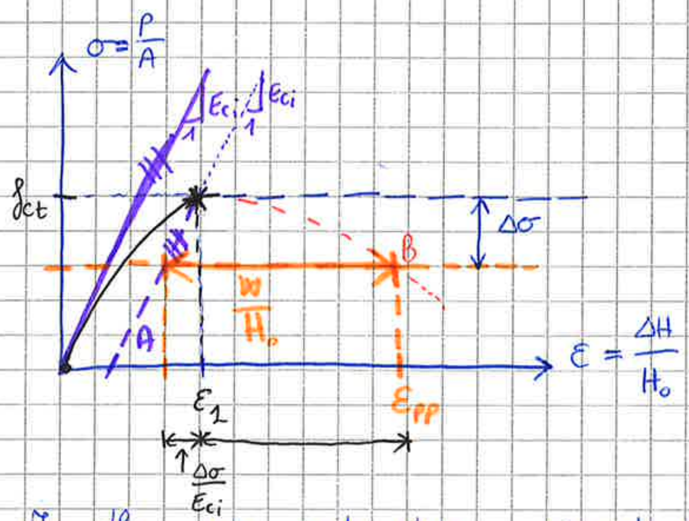
C30 ↓ cylindrical f_{ck}

and $\dfrac{f_{ck}}{R_{ck}} \cong 0.83$

$$\boxed{\begin{cases} f_{c,m} = f_{ck} + \Delta f \\ \Delta f = 8\ \text{MPa} \end{cases}}$$

↳ TENSILE TEST → HILLERBORG TEST 1976 → POST-PEAK

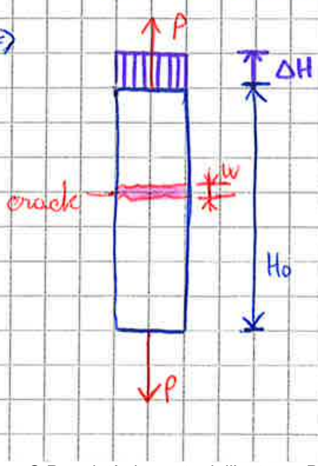




In the pre-peak stage, A and B have the same stress-strain relationship. After the peak, the branches are different:

- B shows an increment of deformation ε.
- A goes back, showing an unloading path, with the slope equal to the initial E_{ci} (ELASTIC).

⇒



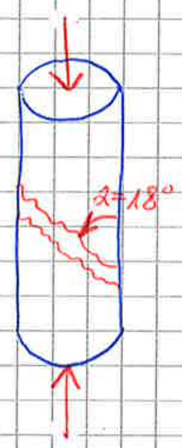
We have an increment of crack width w.

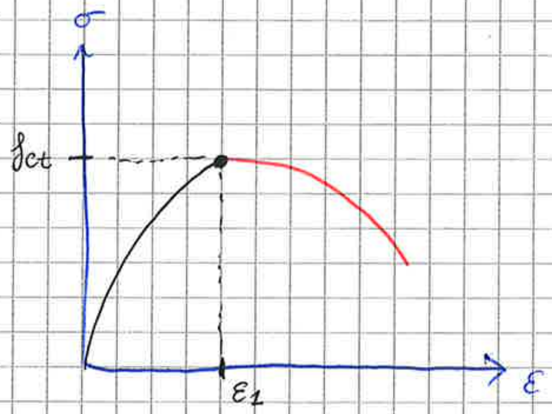
⇒ The deformation ε in the post peak is

$$\boxed{\varepsilon_{\substack{POST\\PEAK}} = \varepsilon_1 + \frac{w}{H_0} - \frac{\Delta\sigma}{E_{ci}}}$$

$\frac{w}{H_0}$ is the increment of width of the crack

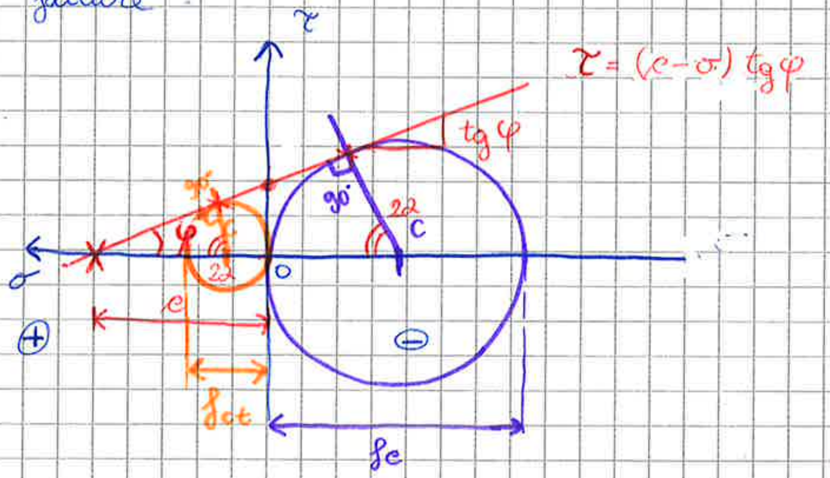
↳ COMPRESSIVE TEST → POST PEAK STAGE





DEMONSTRATION OF 18°

The crack in compression is inclined by an angle of $\alpha = 18°$ (subvertical). We are in the failure situation: we can assume the Mohr–Coulomb failure:



FAILURE CRITERIUM called COLOUMB FAILURE MODEL

$2\alpha = 180° - 90° - \varphi = 90° - \varphi$

If f_c and f_{ct} are known, you can obtain α (first to know φ and c)

$$\begin{cases} \sin\varphi = \dfrac{\frac{f_c}{2}}{c + \frac{f_c}{2}} \\ \sin\varphi = \dfrac{f_{ct}}{c - \frac{f_{ct}}{2}} \end{cases}$$

develop this system: and find, using this previous Hp)

$\Rightarrow c, \varphi$; when $f_c = 10 \cdot f_{ct}$

$\Rightarrow \varphi = 54.1°$

$f_{ct} = \frac{9}{10} c$

$\Downarrow$

$\sin\varphi = \frac{9}{11}$

$$\Rightarrow \alpha = \frac{90° - \varphi}{2} = \frac{90° - 54.1}{2} \simeq 18°$$

- m, ε_{c2}, ε_{cu2} → are the parameters, functions of the CLASS OF CONCRETE
   - if class of concrete $< C50/60$: $\boxed{m = 2,\ \varepsilon_{c2} = 0.2\%,\ \varepsilon_{cu} = 0.85\%}$ [reduction of the test ③]

without any other tests, we need to know only the class of concrete in order to know these parameters and so $\sigma - \varepsilon$ relation.

- f_{cd} design compressive strength

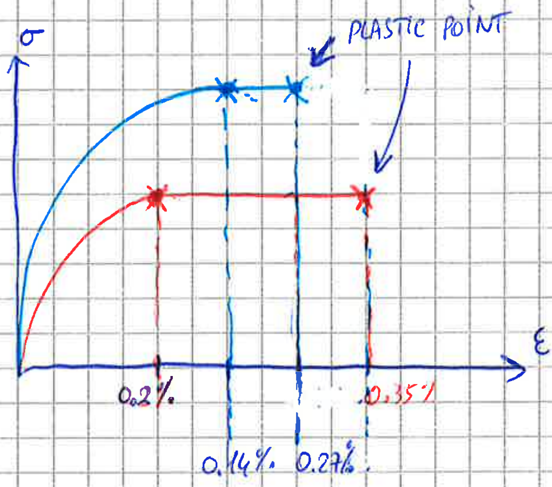
↳ $\boxed{\sigma_{cd} = \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_c} \quad \begin{cases} \alpha_{cc} = 0{,}85 \\ \gamma_c = 1.5 \end{cases}}$ [MPa] [include safety ②]

is the charact. value reduced by a coefficient.

↳ f_{ck} characteristic value (5% percentile)

- if $\alpha_{cc} = 0.85$ ⇒ effects of LONG TERM LOADS

Ex)



class $\leq C50/60$

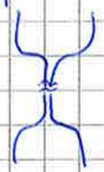
class $C70$

The strength increases but the plastic point reduces (so the ductility) so the deformation capacity ↓.
C70 is ductile than C50/60 but stronger.

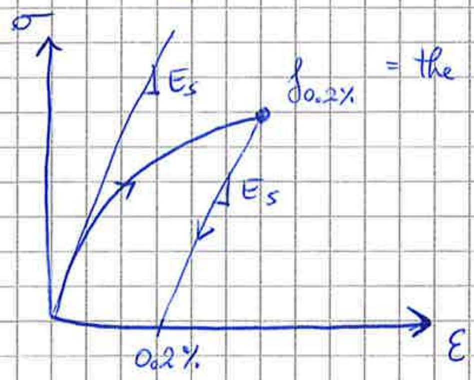
The LONGER is THE HORIZONTAL PART, the MORE DUCTILE IS THE MATERIAL

After the horizontal part you have plateaux f_t = ultimate stress = tensile strength.

When $\sigma = f_t$ there is a crack



(B) ∄ the constant stress and so it is difficult to define yielding stress, so it is defined as this:



$f_{0.2\%}$ = the stress in which you have a residual strain equal to 0,2% in the conventional yielding stress

After this, the behaviour is similar to case (A). The failure occurs at f_t

$f_t = f_{0.2\%} \cdot k$

⇒ In both case we have 3 parameters

f_y, $f_{0.2}$ yielding stress

f_t strength

$E_s = 200'000$ MPa

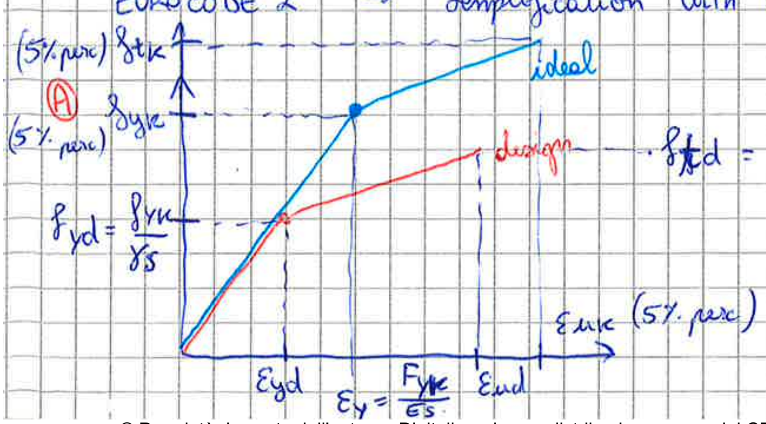
ε_u = strain at max stress

$k = \dfrac{f_t}{f_{y_{0,2}}}$

} properties regarding the ductility of steel

Also this is converted from real to ideal relationships for practical reasons.

EUROCODE 2 → Semplification with a bilinear relationship which is ideal



$f_{td} = \dfrac{f_{tk}}{\gamma_s}$

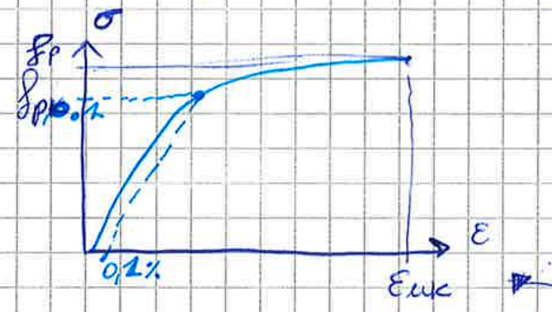
$\gamma_s = 1{,}15$

6/11/17

PRESTRESSING - STEEL

- BARS
- WIRES fili
- STRANDS "trefoli"

Tensile strength is 4 times larger than rebar



f_p = tensile strength

$f_{p0.1}$ = " yielding stress (residual strain = 0.1 %)

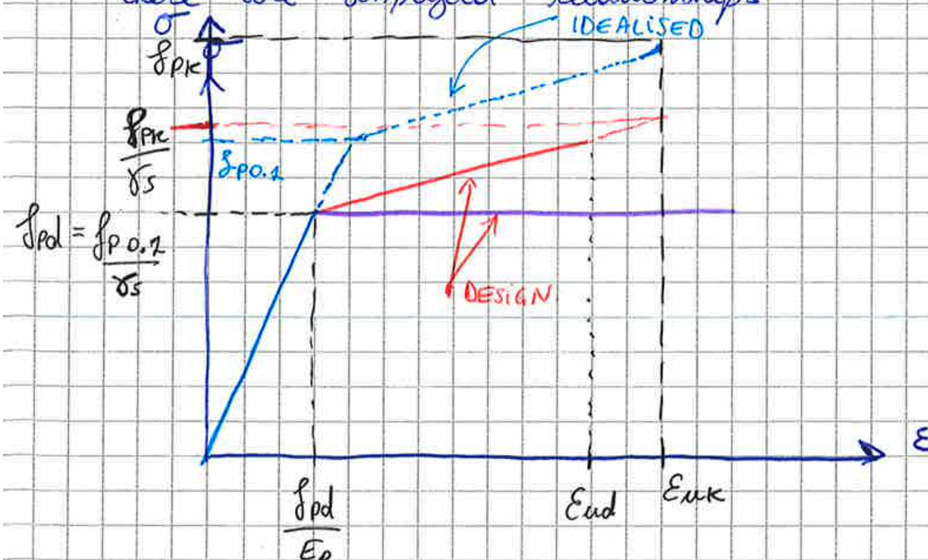
ε_u = strain at peak of stress.

$$K = \frac{f_p}{f_{p0.1}} = 1.1 \text{ recommended value}$$

$f_p = K f_{p0.1\%}$

Classification and parameters are about
- CLASS
- SIZE
- SURFACE PROPERTIES.

(*) Relationship found after testing directly the sample, but as usual, there are simplified relationships:



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These are strains that occur without applying any loads.

EC2 defines two types of shrinkage.

TOTAL = DRYING + AUTOGENOUS

$$\boxed{\begin{aligned} \varepsilon_{cs} &= \varepsilon_{cd} + \varepsilon_{ca} \\ \varepsilon_{cd} &= \beta_{ds}(t, t_s) \cdot K_h \cdot \varepsilon_{cd0} \end{aligned}}$$

ε_{cd0} = drying shrinkage at time $t = 0$ (immediatly after casting)

→ Table 3.2. EC2

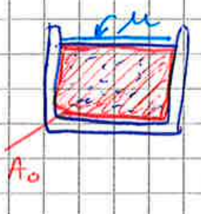
→ depends of the class of concrete and on the relative humidity: the strength is inversely proportional to relat. humidity.

$\beta_{ds}(t, t_s)$ is a coefficient that depends on t = at which we want to calculate β and t_s = time of setting/casting

→ if $t \to \infty \Rightarrow \beta_{ds} = 1$

K_h is a function of h_0 :

$$\boxed{h_0 = \frac{2A_0}{\mu} = \frac{2 \cdot \text{area of cross section}}{\text{perimeter exposed to the environment}}}$$



MAX SHRINKAGE DRY. → $\varepsilon_{cd_\infty} = K_h \, \varepsilon_{cd,0}$

$$\varepsilon_{ca} = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty)$$

MAX SHRINKAGE AUTOG. → $\varepsilon_{ca}(\infty) = 2.5 (f_{ck} - 10) \cdot 10^{-6}$ // (f_{ck} in MPa)

$$\beta_{as}(t) = 1 - e^{-0.2 t^{0.5}} \quad (t \text{ in days})$$

→ if $t \to \infty \Rightarrow \beta_{as} = 1$

MAX TOT $\Rightarrow \varepsilon_{cs,\infty} = K_h \cdot \varepsilon_{cd,0} + 2.5 \cdot (f_{ck} - 10) \cdot 10^{-6}$ It's not an exact formula

The variability from this result is ± 50%

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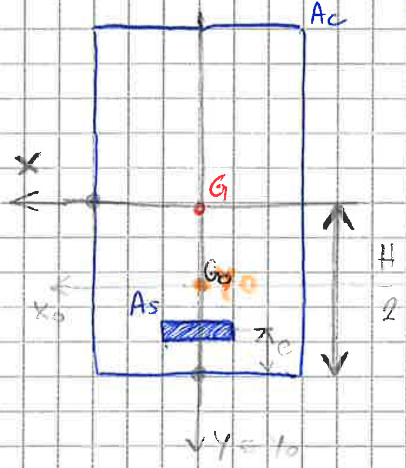
⇒ **3** Homogenize the cross section WRT concrete

$$E_0 = E_c$$

$$m_1 = \frac{E_c}{E_0} = 1$$

$$m_2 = \frac{E_s}{E_c} = \frac{210'000}{30'000} = 7$$

$$A_0 = m_1 A_c + m_2 A_s = 1 \cdot (BH - A_s) + 7 A_s = 79500 + 7 \cdot 500 = 83000 \text{ mm}^2$$



4 Using static moment find G_0 (baricenter of homogenized cross section)

$$y_0 = \frac{S_{x0}}{A_0} \quad \text{homogeneous}$$

$$S_{x0} = m_1 \underbrace{BH}_{\text{area}} \underbrace{\frac{H}{2}}_{\text{distance}} - m_1 A_s \cdot c + m_2 \cdot A_s \cdot c =$$

$$= 1 \cdot \frac{200 \cdot 400^2}{2} - 1 \cdot 500 \cdot 40 + 7 \cdot 500 \cdot 40 =$$

$$= 1{,}61 \cdot 10^7 \text{ mm}^3$$

$$\Rightarrow \quad y_0 = \frac{1.61 \cdot 10^7}{83'000} = 194{,}2 \text{ mm}$$

5 Compute

$$I_{x0} = \left[\frac{BH^3}{12} + \underbrace{BH\left(\frac{H}{2} - y_0\right)^2}_{\text{transportation inertia}}\right] m_1 - m_1 A_s (y_0 - c)^2 + m_2 A_s (y_0 - c)^2 =$$

$$= \left[\frac{200 \cdot 400^3}{12} + 400 \cdot 200 (200 - 194{,}2)^2\right] 1 - 1 \cdot 500 (194{,}2 - 40)^2 + 7 \cdot 500 (194{,}2 - 40)^2 =$$

$$= 1.14 \cdot 10^9 \text{ mm}^4$$

6 How to define the state of strain in the cross section.

$$\begin{Bmatrix} \lambda \\ \mu_x \\ \mu_y \end{Bmatrix}_{TOT} = \begin{Bmatrix} \bar{\lambda} \\ \bar{\mu}_x \\ \bar{\mu}_y \end{Bmatrix}_{IMPOSED} \qquad \mu_y = 0, \quad \bar{\mu}_y = 0$$

$$\bar{\lambda} = \frac{1}{A_0} \int_{A_c} \underbrace{m_1}_{=1} \underbrace{\bar{\varepsilon}_2}_{\text{CONSTANT}} dA = \frac{\bar{\varepsilon}_2 A_c}{A_0} = \frac{-6 \cdot 10^{-4} (400 \cdot 200 - 500)}{83'000} = -5{,}8 \cdot 10^{-4}$$

and it occurs after $\gg 20$ days after setting. If you leave these cracks in the atmosphere, the steel will corrode (expansive phenomenon) $\Rightarrow$ deterioration of the work

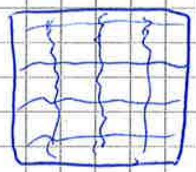
$\Downarrow$

How to reduce the shrinkage?

$\Downarrow$

Having the curing in water ($-30 \div 40\%$)

cracks without reinforcement



cracks with reinforcement

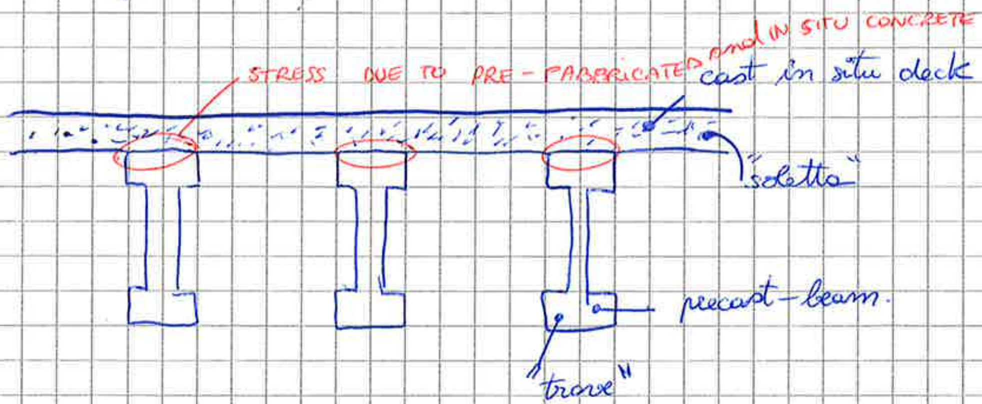
(8) How to check if the calculation is correct?

$$N = \int \sigma_z \, dA = 0$$

$$M_x = \int \sigma_z \, y \, dA = 0$$

Because $\nexists$ external load applied.

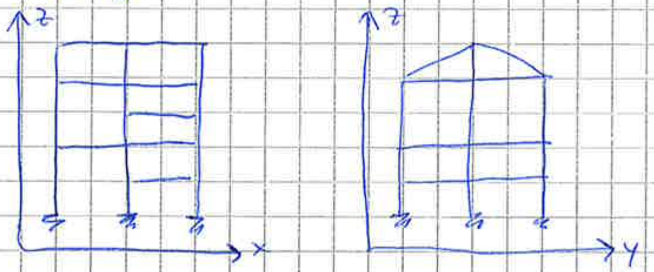
EX Bridge



8/11/17

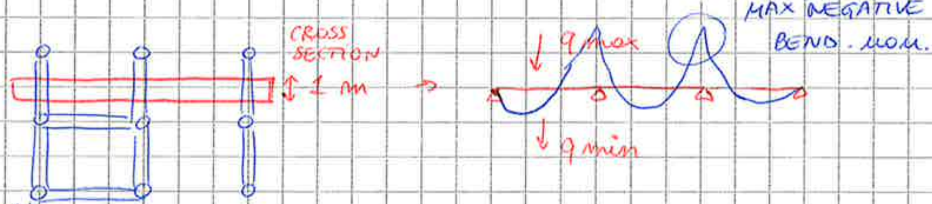
PROJECT

Simplify from 3D to 2D: and calculate separately by defining the vertical loads (neglecting horizontals now) that must be combined using the formula.

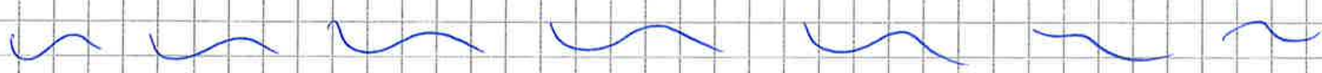


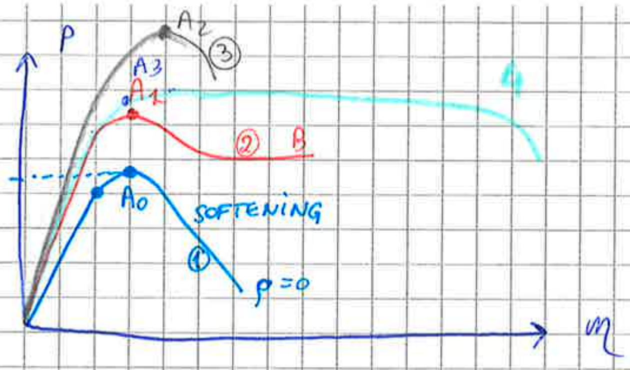
After computing the loads, you must combined them for each floor to find the max and min. values to find bending moment and shear on slabs:

EX 1ST floor



↝ "Use MACRO OF CORNETTI".

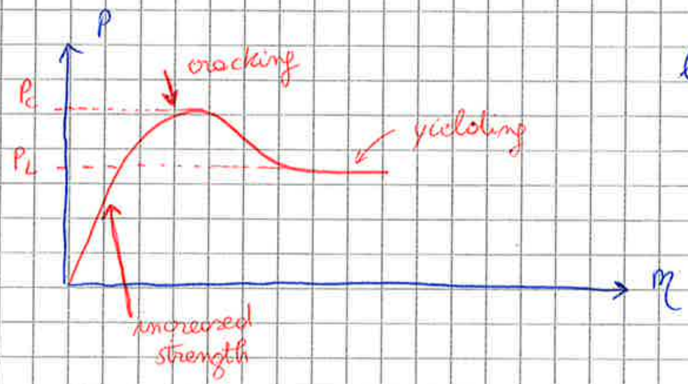




fixed by the code rules

② A LITTLE AMOUNT OF REINFORCE MATERIAL $A_s \neq 0$, $\rho < \rho_{MIN}$:

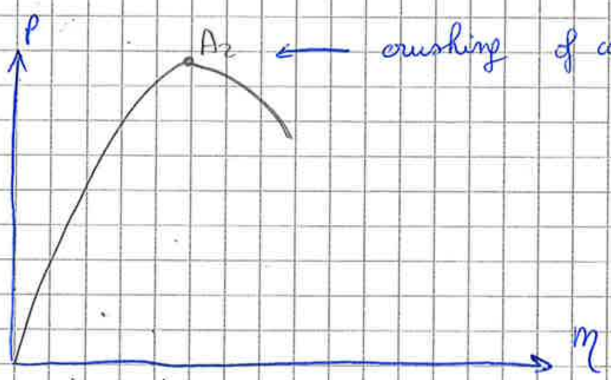
UNDER-REINFORCED CONCRETE BEAM.



loading at cracking > loading yielding

③ A LARGE QUANTITY OF REINFORCEMENT $A_s \neq 0$, $\rho > \rho_{MAX}$:

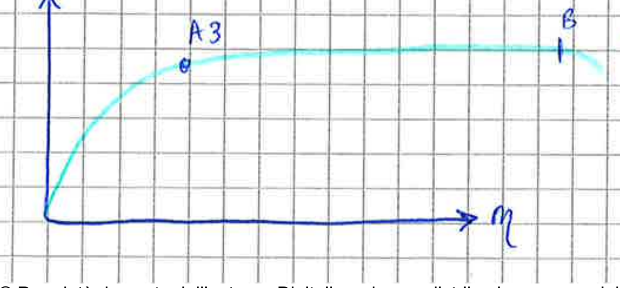
OVER-REINFORCED BEAM



← crushing of concrete in compression at point A_2

④ REINFORCEMENT INCLUDED BETWEEN THE BOUNDS OF THE CODE:

$A_s \neq 0$ $\rho_{MIN} \leq \rho \leq \rho_{MAX}$ → DUCTILE BEHAVIOUR



← It's lower than A_3 but it still high. In A_3 ∃ the yelding of steel. We have large deflection. The maximum load remains for large deformations.

For the cross section we consider the moment curvature relationship:
$M - \mu$ (function of N). This relation will be linear

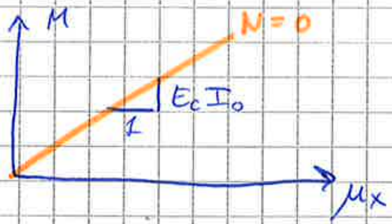
$\boxed{EX}$ If $O \equiv G$ (centroid G coincident with origin of axis O).
$O \equiv C$ (C point of application of load)

$\Rightarrow e = 0$

$S_0 = 0$

$\Rightarrow M = E_c \cdot \mu_x \cdot I_0$

LINEAR.

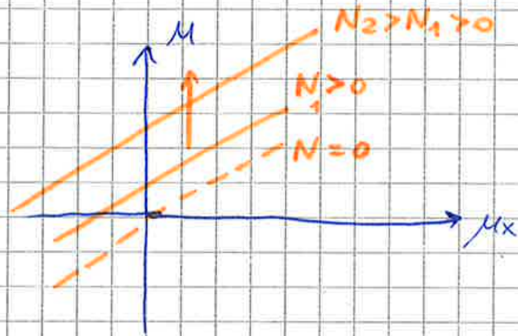


$\Rightarrow$ $E_c I_0$ = FLEXURAL STIFFNESS

If $C \not\equiv O \equiv G \Rightarrow S_0 = 0$

$\Rightarrow M + Ne = E_c \mu_x I_0$

$M = E_c I_0 \mu_x - Ne$



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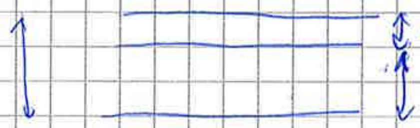
ex REI 90 → metodo tabellare geometrico

EC2 1-2

H=24cm

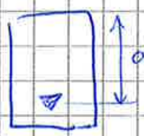


10 cm → 5cm should be made of concrete

14cm CONCRETE SLAB

REI 90 floor should resist to 90 min to a fire

Effective depth of a cross section



d

C 25/30

$R_{ck} = 30 \frac{N}{mm^2}$ $f_{ck} = 24,9 \frac{N}{mm^2}$ $f_{cd} = 14,1 \frac{N}{mm^2}$

Steel B450C

$f_{yk} = 450 \frac{N}{mm^2}$ $f_{yd} = 391 \frac{N}{mm^2}$

Calcolo del dead load = peso proprio della struttura

↓

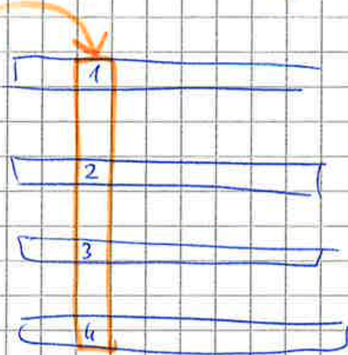
Calcolo carichi permanenti casa e outdoor (indoor are bigger).

↓

Calcolo carichi variabili

Nelle condiz di ULS, SLS (Rare), SLE (Frequent), SLE (Quasi-permanent) in cui cambiano i coefficienti delle equazioni. (for outdoor & indoor).

1 2 3 4



1 2 3 4

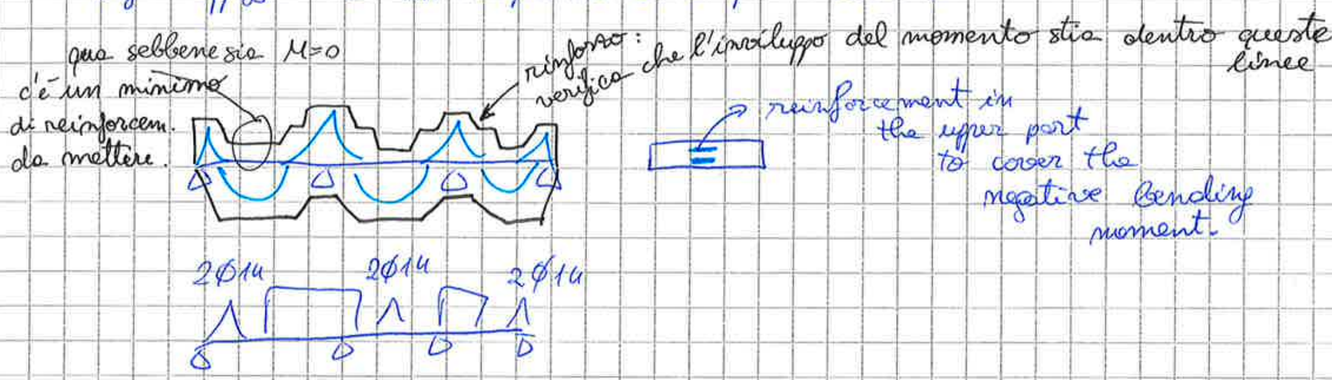
↓

ex M^+_{max} & M^-_{max}

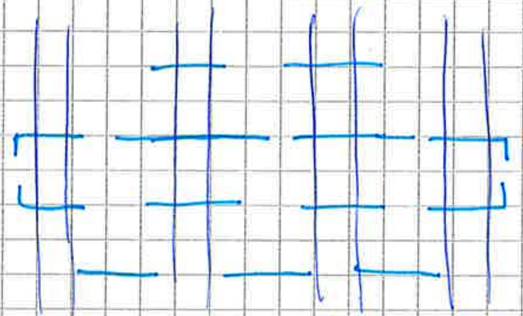
One can estimate the maximum $-\frac{QL^2}{8}$ or $+\frac{QL^2}{12}$ if there are no other values available.

75

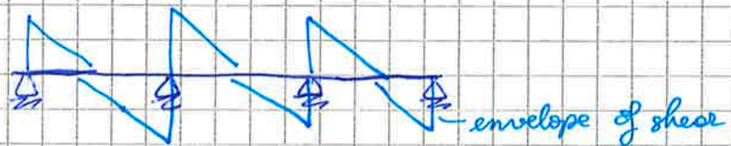
Sugli appoggi va in compressione la parte bassa.



⇒

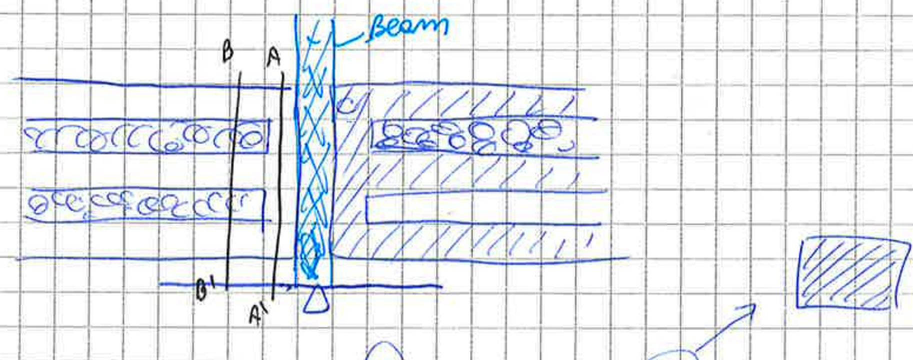


Principalmente il cls resiste a taglio, without reinforcement.



→ Si calcola anche per T: ULS, SLE,

TOP VIEW

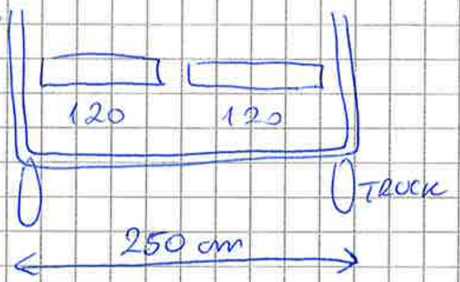


resistance of cross section (BB) is ≠ from (AA)

Si fa la verifica su BB.

Se la verifica non è soddisfatta: → si può ridurre la quantità di polistirolo.

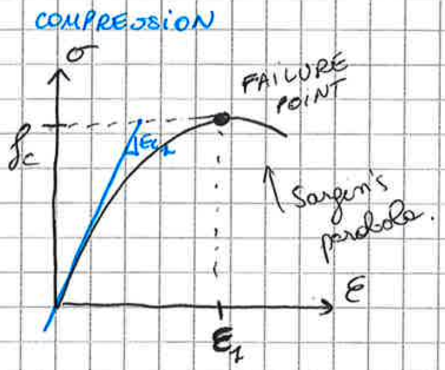
77



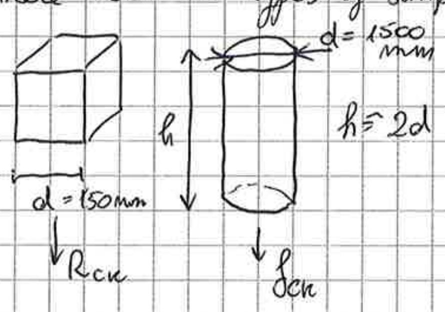
CALCESTRUZZO - CONCRETE

The concrete is a mixture of materials : 40% aggregates, 25% sand, 20% water, 10% cement, 5% air, additives.

It's density is usually for normal : $20 \div 26 \frac{kN}{m^3} = 2000 \div 2600 \frac{kg}{m^3}$.



There are 2 types of samples : cubic and cylindrical

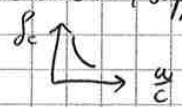


The resistance is bigger in R_{ck}:

$f_{ck} \cong 0,83\ R_{ck}$

Normal concrete = type I.

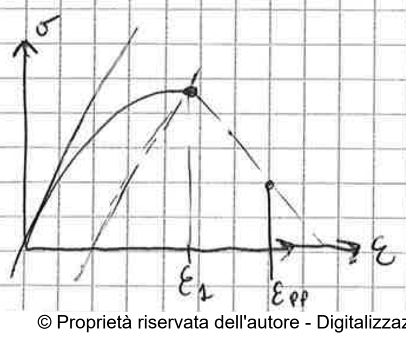
f_c = { type of aggregates
" " additives / supplem. materials
$\frac{w}{c}$ ratio
type of cement
rate of loading : creep rate $= 10^{-8}\frac{1}{s}$; static rate $= 10^{-5}\frac{1}{s}$; blast rate $= 10^{3}\frac{1}{s}$



E_c depends on the aggregates and composition. $E_c = \alpha f_c$

Normal := $E_c \simeq 30$ GPa

f_c (ITALY) $< 20 \div 70$ MPa, but can be > 150 MPa



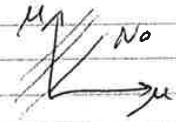
$$\varepsilon_{pp} = \varepsilon_1 + \frac{w}{H_0} - \frac{\Delta\sigma}{E_{ci}}$$

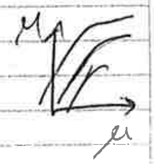
Diagramma M–μ equil eq + costit eq

$N = \int \sigma_z(\varepsilon)\, dz$

$Ne + M = \int \sigma_z(\varepsilon)\, dz$

Mette in relazione la curvatura della sez. al variare del momento flettente M (esterno) e serve a capire il comportamento meccanico di una sez. inflessa di cls. armato

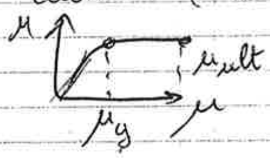
↳ Saint-Venant: relaz lineare 

↳ General: cls: parab - rect
steel elastic - perfect plastic 

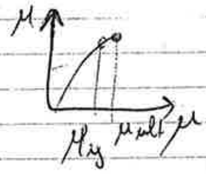
⇒ Failure avviene nel cls nella parte in tensione

↳ field 3 ⟶ steel in snervamento

↳ sez più duttile (ok x sismic zone)



↳ field 4 ⟶ steel in elastic zone:

↳ sez più fragile 

ρ = ductility = $\dfrac{\mu_{ult}}{\mu_y}$

After the first cracking if you increase the bending moment, other cross sections crack: we are moving from stage 1 to stage 2. We arrive on stage 2 line when we have the yielding of traction and we are in the ULS.

This real behaviour is different from the theoretical one.

EQUILIBRIUM EQUATIONS:

1) $N = \int_A \sigma_z \, dA$

2) $M + Ne = \int_A \sigma_z y \, dA$

COMPATIBILITY EQUATION

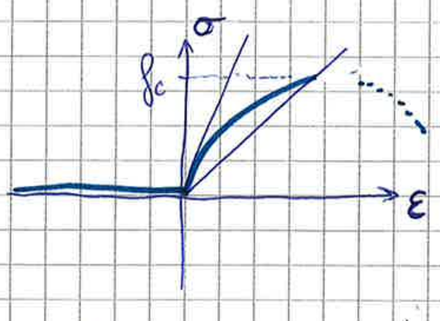
3) $\varepsilon = \lambda + \mu_x y$

COSTITUTIVE RELATION (NON LINEAR)

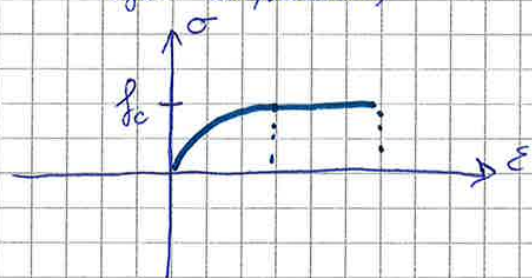
4) $\sigma_z \neq E\varepsilon \Rightarrow \sigma = \sigma(\varepsilon)$

! ④ In fact the relation is not linear: $\sigma - \varepsilon$ FOR CONCRETE

SARGIN RELATION

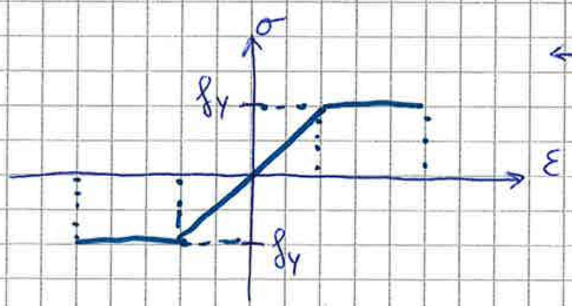


PARABOLA - RECTANGLE RELATION (for compression)



These relations are not linear.

$\sigma - \varepsilon$ STEEL

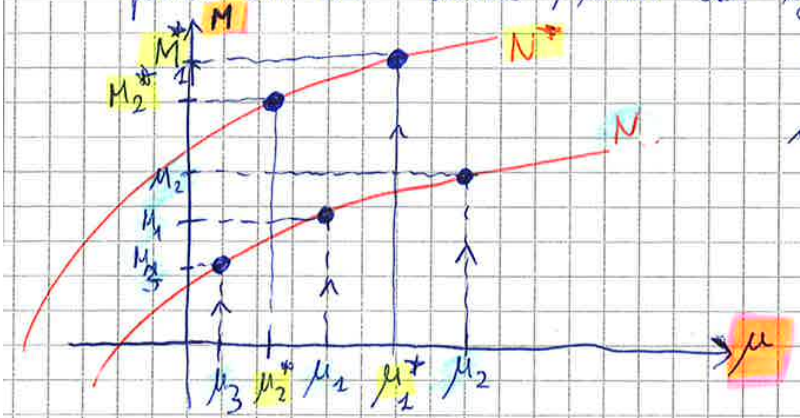


← the linear part is up to $\pm f_y$ (yielding) after which $\sigma \simeq$ constant.

8) If $N = N_1$ $\Rightarrow$ we can compute the bending moment.

$$M = \int_A \sigma_z \, y \, dA - Ne$$

9) Now I can change the curvature μ to find another point on the curve, and do again the calculations.

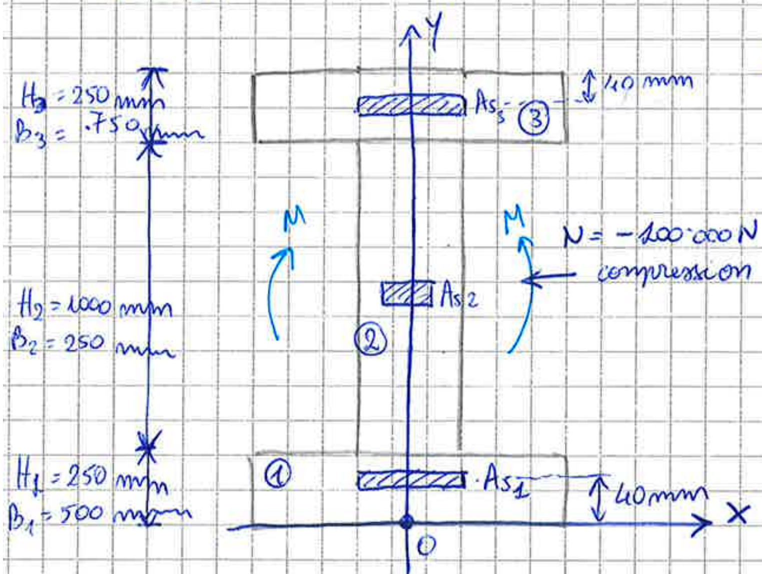


I also can change N into N^* and repeat again the procedure

↓

Of course this is solved by a computer:

MACRO: cross-section should be symmetric



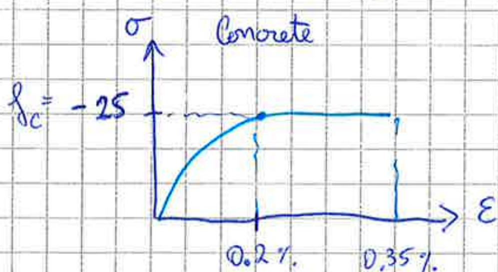
We have 3 rectangles, so we put at least 2 stripes of reinforcement.

$A_{s3} = 0$ if we don't have it.

$A_{s2} = 0$

$A_{s1} = 10\,000$ mm^2

$y_{A_{s1}} = 40$ mm from $y = 0$.

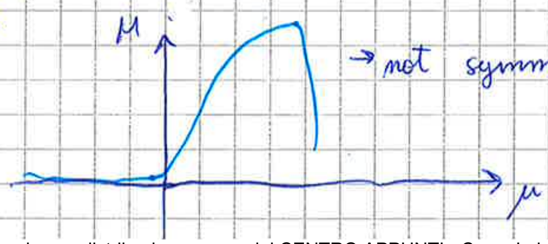


Steel

$F_y = 450$ $\quad \varepsilon_{psy} = 0.00225$

$F_u = 500$ $\quad \varepsilon = 0.065$

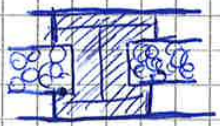
$\Rightarrow$ Result



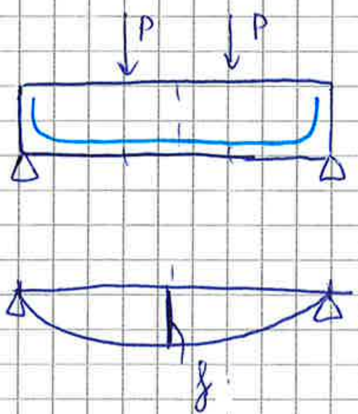
→ not symmetric

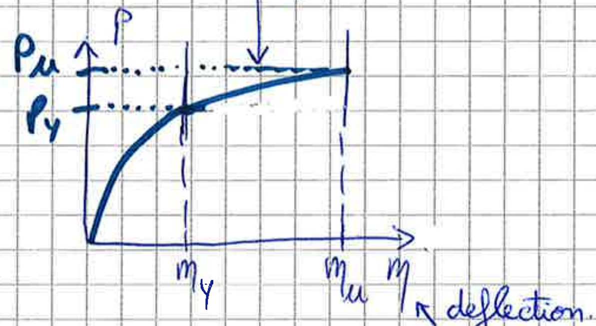
We need not only the strength but also a large ductility:
for ex we have the same M_{max} for and but the curvature (⇒ ductility) is bigger for . (example on slides), but it's more expensive and difficult to do in situ so the is used, but if it's pre-cast it can be used mixed with .

! Resisting joist in slab ≈ I

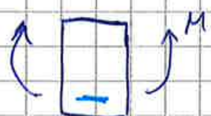


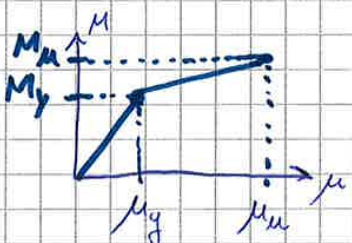
STRUCTURAL DUCTILITY





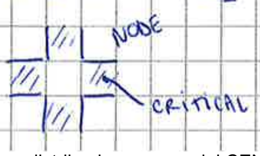
DUCTILITY RATIO FOR STRUCTURE $= \boxed{\dfrac{\eta_u}{\eta_y}}$





CROSS-SECTIONAL DUCTILITY $= \boxed{\dfrac{\mu_u}{\mu_y}}$

EC 8: for seismic zones:
- in critical regions 50% of As_2 (steel in tension) should be located in compression. $= As_1$



When does the failure occurs? $\exists$ 2 points:

CONCR $\varepsilon = 0.35\ \%$

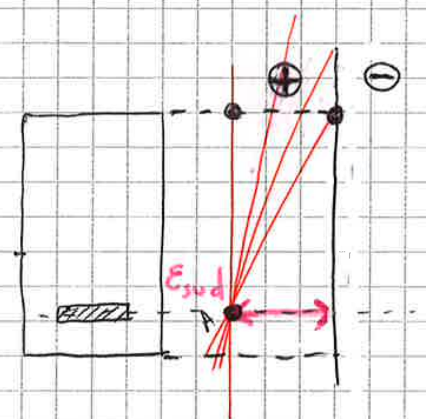
STEEL $\varepsilon = \varepsilon_{sud}$

→ 3° HP : ULS conditions occur when

$$\begin{cases} \text{Max strain in concrete} = 0.35\% \\ \text{and / or} \\ \text{Max strain is steel} = \varepsilon_{sud} \end{cases}$$

The slope of the strain profile variates in an ∞ range because $\exists$ ∞ combinations of having $\varepsilon_{c\,max}$ and $\varepsilon_{s\,max}$. We classify them into 5 Fields of ultimate limit strain profiles

1° FIELD : failure of only steel : cross is only in TENSION ⊕ (no compression)

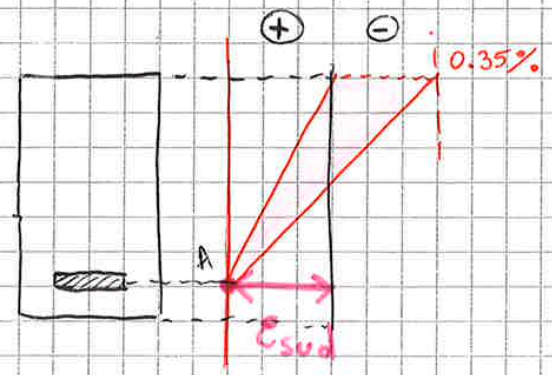


$\varepsilon_s = \varepsilon_{sud}$

$0 \leq \varepsilon_c \leq \varepsilon_{sud}$

Rotation around A.

2° FIELD

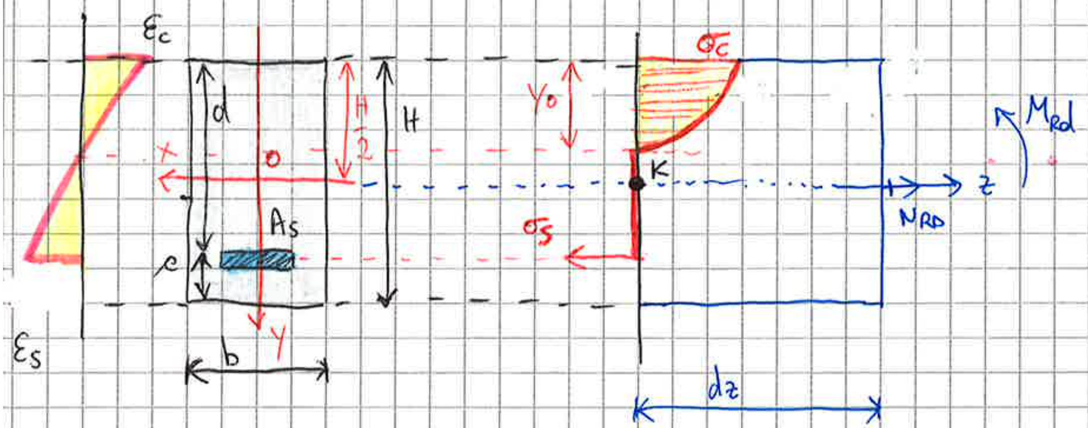


$\varepsilon_s = \varepsilon_{sud}$

$-0{,}35\% \leq \varepsilon_c \leq 0.$

4° HP For a given strain profile, the state of stress should be in equilibrium with M, N

ULS



d = depth effective

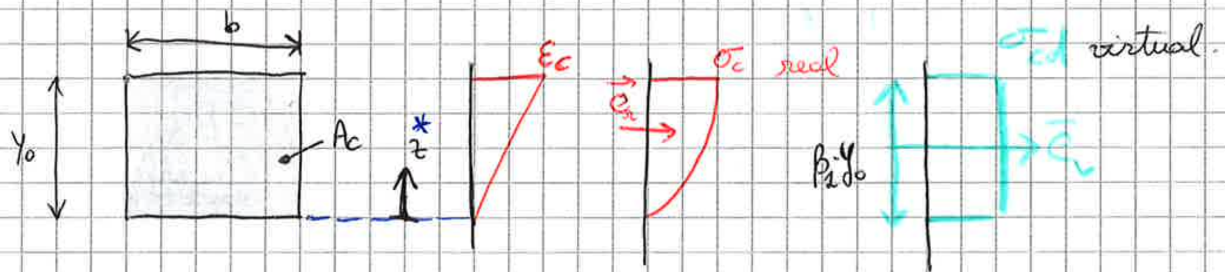
M_{Rd} = resistance bending moment " " "
N_{Rd} = resistance of normal force of the cross-section.

⇒ If equil is satisfied

⇒ Equil around K

$$\boxed{\begin{aligned} &(1)\quad N_{RD} - \sigma_s A_s + \int_{A_c} \sigma \cdot dA = 0 \qquad \xrightarrow{+} z \\ &(2):\ \overset{\curvearrowleft}{+})\quad M_{RD} - \sigma_s A_s \left(\frac{H}{2} - c\right) - \int_{A_c} \sigma \cdot y \cdot dA = 0 \end{aligned}}$$

We have to eliminate the integrals:



$\vec{C}$ = resultant of state of stress in Ac $\quad \vec{C} = \int_{A_c} \sigma \, dA$

We can consider a virtual σ_{cd}, uniform, with value that is the maximum. $\vec{C}_{real} = \vec{C}_{virtual}$: resultants must be equal.

$$\boxed{\vec{C} = \int_{A_c} \sigma \cdot dA = \sigma_{cd} \, \beta_1 \, Y_0 \, b}$$

reduces the real depth

$\beta_1 < 1$