NUMERO: 2261A

ANNO: 2017

A P P U N T I

STUDENTE: Sobrero Giovanni

MATERIA: Meccanica Applicata alle Macchine
Prof. Ferraresi-Raparelli.

MECCANICA APPLICATA

Programma

• Meccanica dei sistemi di corpi rigidi: Moto di un corpo rigido, vincoli e gradi di libertà in un sistema di corpi rigidi, moto di traslazione e di rotolamento, catene cinematiche, proprietà d'inerzia dei corpi, equilibrio dinamico dei sistemi di corpi rigidi - forze d'inerzia, energia nei sistemi di corpi rigidi.
• Forze agenti negli organi di macchine: Forze elastiche, fenomeni di aderenza e attrito, resistenza al rotolamento, forze viscose, analisi dinamica di comuni sistemi meccanici (meccanismi, trasmissioni, veicoli) sottoposti a un sistema di forze.
• Componenti dei sistemi di trasmissione della potenza meccanica: giunti, trasmissioni mediante flessibili, trasmissioni mediante ingranaggi, rotismi ordinari ed epicicloidali, trasmissioni a vite-madrevite, freni ad attrito, frizioni, camme e punterie, cuscinetti a rotolamento, a strisciamento, lubrificati.
• Caratteristiche generali di un sistema di trasmissione del moto: riduzione dell'inerzia e delle forze/coppie ad un dato asse, accoppiamento motore-utilizzatore, rendimento, macchine a regime periodico, equilibramento.
• Vibrazioni meccaniche: vibrazioni libere e forzate di un sistema a un grado di libertà, trasmissibilità.

Criteri, regole e procedure per l'esame

L'esame viene tenuto in forma scritta. E' richiesta la risoluzione di alcuni problemi (solitamente 3) relativi agli argomenti trattati nelle lezioni e nelle esercitazioni, è inoltre richiesta la risposta aperta ad una domanda relativa alla teoria.
A ciascuna domanda è attribuita una valutazione variabile secondo la difficoltà.

MECCANICA APPLICATA ALLE MACCHINE

APPUNTI VIDEOLEZIONI

PROF. CARLO FERRARESI (VIDEOLEZIONI ONLINE) PROF: LUIGI GARIBALDI

LIBRO: MECCANICA APPLICATA; FERRARESI, RAPARELLI (2007, CLUT)

FISICA — MECCANICA — TERMODINAMICA — ELETTROTECNICA — STRUTTURALE — FUNZIONALE

MECCANICA APPLICATA — ALLE MACCHINE — A...

MECCANICA — STATICA — CINEMATICA (STUDIA IL MOVIMENTO COSÌ COM'È, SENZA INTERROGARSI SULLE CAUSE) — DINAMICA (CORRELAZIONE, RELAZIONE CAUSALITÀ TRA MOVIMENTO E CAUSE CHE LO DETERMINANO)

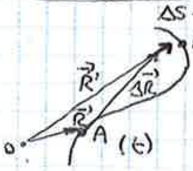
CINEMATICA — DEL PUNTO — DEL CORPO ESTESO RIGIDO — SISTEMI DI CORPI RIGIDI

RIPASSO; NOZIONI BASE

(LA POSIZIONE) È RAPPRESENTATA DA UN VETTORE (→ INDIVIDUATO DA UN ORIGINE E UN VERSO E DIREZ.)

$\vec{R} = x\vec{i} + y\vec{j}$, x, y COORD. CARTES. QUANT. SCALARI (t)

ΔS - VARIAZIONE DI POSIZIONE → TRAIETTORIA



$\Delta\vec{R} = \vec{R}' - \vec{R}$ VARIAZIONE (DIFFERENZA) DI POSIZIONE (È RAPPR. DAL TRIANGOLO)

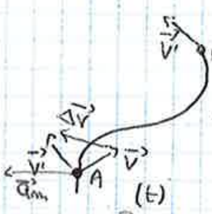
POSIZ. FINALE − POSIZ. INIZIALE

$$\vec{V}_m = \frac{\Delta\vec{R}}{\Delta t}\ ;\quad \vec{V}' = \lim_{\Delta t\to 0}\frac{\Delta\vec{R}}{\Delta t} = \frac{d\vec{R}}{dt}$$

(VELOCITÀ ISTANTANEA/REALE)

$\vec{V}$ È SEMPRE TANGENTE ALLA TRAIETTORIA (NB!)

($\vec{V}_m, \Delta\vec{R}$ VETTORE STESSA DIR. E VERSO)

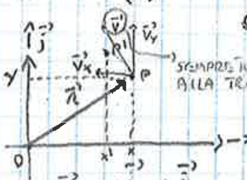


$\Delta\vec{V} = \vec{V}' - \vec{V}$

$\vec{V}' = \vec{V} + \Delta\vec{V}$

$$\vec{a}_m = \frac{\Delta\vec{V}}{\Delta t}\ ;\quad \vec{a} = \lim_{\Delta t\to 0}\frac{\Delta\vec{V}}{\Delta t} = \frac{d\vec{V}}{dt} = \frac{d^2\vec{R}}{dt^2}$$

$\vec{a}$: PUÒ AVERE COMPONENTE TANGENTE ALLA TRAIETTORIA E COMPONENTE NORMALE ALLA TRAIETTORIA.



SEMPRE TG ALLA TRAIETTORIA (DA P A P')

ORIGINE E (DUE ASSI DI RIFERIMENTO)

$\vec{R} = x\vec{i} + y\vec{j}$

$$\vec{V} = \frac{d\vec{R}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \underbrace{x\frac{d\vec{i}}{dt}}_{=0} + \underbrace{y\frac{d\vec{j}}{dt}}_{=0} = \vec{V}_x + \vec{V}_y$$

IN QUANTO I VERSORI SONO COSTANTI

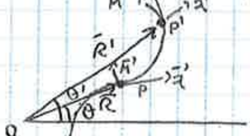
$$|\vec{V}| = \sqrt{V_x^2 + V_y^2}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d^2x}{dt^2}\vec{i} + \frac{d^2y}{dt^2}\vec{j} = \vec{a}_x + \vec{a}_y$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

(↑ SIST. DI RIF. CARTESIANO)

SIST. DI RIFERIMENTO POLARE (POLO E ASSE DI RIFERIMENTO)



(ORIGINE + ASSE DI RIF)

$$\begin{cases} R = R(t) & \text{RAGGIO} \\ \theta = \theta(t) & \text{ANOMALIA} \end{cases}$$

VELOCITÀ ANGOLARE $= \frac{d\theta}{dt} = \omega = (\dot{\theta})$

ACCELERAZIONE ANGOLARE $= \frac{d^2\theta}{dt^2} = \dot{\omega} = (\ddot{\theta})$

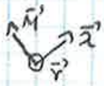
$\vec{\theta}$ = VETTORE LA CUI DIREZIONE È PERPENDICOLARE A QUELLA DEL MOTO (NB!)

POSITIVO SE VERSO USCENTE (ANTIORARIO)

NEGATIVO SE VERSO ENTRANTE (ORARIO)

$\vec{\lambda}$ = VERSORE LONGITUDINALE

$\vec{M}$ = VERSORE TRASVERSALE



$\Rightarrow \vec{R} = R\vec{\lambda}$

$\Rightarrow \vec{\theta} = \theta\vec{\gamma}$

$\vec{R} = R\vec{\lambda}$

$\vec{v} = \frac{dR}{dt}\vec{\lambda} + R\frac{d\vec{\lambda}}{dt} = \vec{v}_\lambda + \vec{v}_M = \dot{R}\vec{\lambda} + R\dot{\theta}\vec{M}$

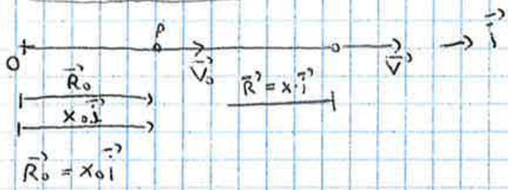
$\vec{a} = \frac{d^2R}{dt^2}\vec{\lambda} + \frac{dR}{dt}\frac{d\theta}{dt}\vec{\mu} + R\frac{d^2\theta}{dt^2}\vec{M} + R\frac{d\theta}{dt}\frac{d\theta}{dt}(-\vec{\lambda}) + \frac{dR}{dt}\frac{d\theta}{dt}\vec{\mu}$

$\vec{a} = \left(\frac{d^2R}{dt^2} - R\left(\frac{d\theta}{dt}\right)^2\right)\vec{\lambda} + \left(2\frac{dR}{dt}\frac{d\theta}{dt} + R\frac{d^2\theta}{dt^2}\right)\vec{\mu} = \vec{a}_\lambda + \vec{a}_M$

RICHIAMI DI CINEMATICA

STABILIRE IN OGNI ISTANTE POSIZIONE, VELOCITA' E ACCELERAZIONE (STUDIO CINEMATICO)

MOTO RETTILINEO



$$\begin{cases} \vec{v} = \frac{d\vec{R}}{dt} = \frac{dR}{dt}\vec{i} \\ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2R}{dt^2}\vec{i} \end{cases}$$

$\vec{R}_0 = x_0\vec{i}$

- $v = k \rightarrow$ MOTO RETTILINEO UNIFORME

$\boxed{v = \frac{dx}{dt}} = k$ EQUAZIONE DEL MOTO

$\int_{x_0}^{x} dx = v\int_{t_0}^{t} dt \quad x - x_0 = v(t-t_0) \Rightarrow x = x_0 + v(t-t_0) = (t_0 = 0) \Rightarrow$

$\boxed{x = x_0 + v \cdot t}$ LEGGE DEL MOTO

- $v \neq k$; $a = k \rightarrow$ MOTO RETTILINEO UNIFORMEMENTE ACCELERATO

$\boxed{a = \frac{dv}{dt}} = k$ EQUAZIONE DEL MOTO

$\int_{v_0}^{v} dv = a\int_{t_0}^{t} dt \quad v = v_0 + a(t-t_0) = (t_0 = 0) \Rightarrow$

$v = v_0 + at = \frac{dx}{dt}$

$x = \int dx = \int v\,dt = v_0\int_{t_0}^{t} dt + a\int_{t_0}^{t}(t-t_0)\,dt = v_0(t-t_0) + \frac{a}{2}(t-t_0)^2 = (t_0 = 0) \Rightarrow$

$\boxed{x = v_0 t + \frac{1}{2}at^2}$ LEGGE DEL MOTO

MOTO CIRCOLARE

$\boxed{R = k}$

$\vec{R} = R\vec{\lambda}$

$v = R\omega\vec{M}$ (IN QUANTO $\vec{v} = \underbrace{\cancel{\frac{dR}{dt}}}_{=0}\vec{\lambda} + R\frac{d\vec{\lambda}}{dt}$) $= v_T$ (V. TANGENZIALE)

$a = \vec{a}_\lambda + \vec{a}_M = \vec{a}_m + \vec{a}_t$

↳ ACC. TANGENZIALE

↳ ACC. NORMALE DETTA ANCHE CENTRIPETA (PERCHE' DIRETTA VERSO IL CENTRO)

$a_T = R\frac{d\omega}{dt}\vec{M}$; $a_N = R\omega^2(-\vec{\lambda})$

- $\omega = k \rightarrow$ MOTO CIRCOLARE UNIFORME

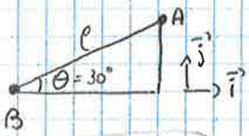
$\boxed{\omega = \frac{d\theta}{dt}} = k$ EQUAZIONE DEL MOTO

- $\boxed{\theta = \theta_0 + \omega t}$ LEGGE DEL MOTO

$\frac{d\omega}{dt} = k$ MOTO CIRCOLARE UNIFORMEMENTE ACCELERATO

$\boxed{\alpha = \frac{d\omega}{dt}}$ EQUAZIONE DEL MOTO $\quad \omega = \omega_0 + \underbrace{\frac{d\omega}{dt}}_{(=\alpha)} t \Rightarrow \boxed{\theta = \theta_0 + \omega t + \frac{1}{2}\alpha t^2}$ LEGGE DEL MOTO

ES 1.1



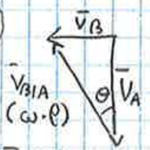
NOTI

- $V_A = 2\,m/s\ (-\vec{j}) = K\ (a_A = 0)$
- $\vec{V}_B = ?\ ;\ a_B = ?$
- $\ell = 200\,mm$
- $\theta = 30°$

$V_B = V_A + \vec{V}_{B/A}$

$\vec{V}_{B/A}$: V. DI B INTORNO AD A

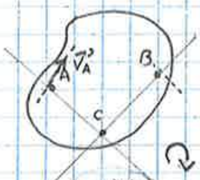
1° METODO



$V_B = V_A \operatorname{tg}\theta = 2 \operatorname{tg} 30° = 1{,}15\,m/s$

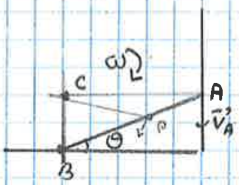
$V_{B/A} = \omega \ell = \dfrac{V_A}{\cos\theta} \Rightarrow \omega = \dfrac{V_A}{\ell \cos\theta} = 11{,}5\ rad/s$

CENTRO DI INSTANTANEA ROTAZIONE (CENTRO DELLE V) 2° METODO



ASSI PERPENDICOLARI ALLE DIREZIONE DEL MOTO DEI PUNTI

$\omega = \dfrac{V_A}{AC}$



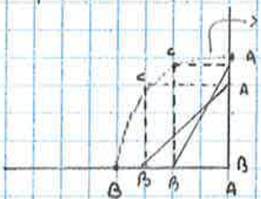
$V_C = 0$

$\omega = \dfrac{V_A}{AC} = \dfrac{2}{0{,}2 \cdot \cos 30°}$

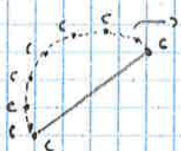
$V_B = \omega BC = \omega \ell \sin\theta$
$V_A = \omega AC = \omega \ell \cos\theta$
$\Rightarrow \dfrac{V_B}{V_A} = \operatorname{tg}\theta \Rightarrow V_B = V_A \operatorname{tg}\theta$

$V_B = \omega \cdot BC = \omega \ell \cdot \sin\theta$

$V_P = \omega \cdot PC$



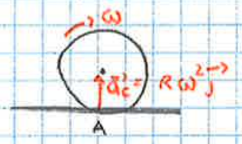
POLARE FISSA (RACCOGLIE TUTTI I POLI DI ROTAZIONE)



POLARE MOBILE

SISTEMA DI RIFERIMENTO SOLIDALE AL CORPO CHE SI MUOVE

NELLA POSIZIONE $\theta = 0 \Rightarrow \begin{cases} x_{C'} = 0 \\ \dot{x}_{C'} = 0 \\ \ddot{x}_{C'} = 0 \end{cases} \quad \begin{cases} y_{C'} = 0 \\ \dot{y}_{C'} = 0 \\ \ddot{y}_{C'} = R\omega^2 \end{cases}$



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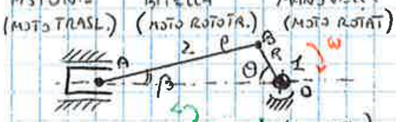


$\omega_2 = \dot{\beta}$ (NON NOTA)

[OGNI CORPO HA UN'ω DIVERSA]

*ANALIZZARE SEMPRE IL MECCANISMO

NOTI:

$\omega = 1500$ Rpm $= K$

$R = 125$ mm $\to 0{,}125$ m

$\rho = 350$ mm $\to 0{,}35$ m

$\theta = 60°$

$\vec{V}_A = ?$ $\vec{a}_A = ?$

MANOVELLA = MOVENTE

PISTONE = CEDENTE

(DUE METODI PER DET. LA VELOCITA')

PER $\vec{V}_A$: FORMULA FONDAMENTALE DELLA CINEMATICA

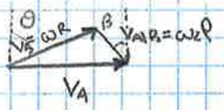
OPPURE

METODO DI INSTANTANEA ROTAZIONE

TROVO β: $\rho \sin\beta = R \sin\theta$

$$\frac{\rho}{\sin\theta} = \frac{R}{\sin\beta}$$

$$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B} = (R\omega) + (\omega_2 \rho)$$



$$\begin{cases} V_A = \omega R \sin\theta + \omega_2 \rho \sin\beta \\ \omega R \cos\theta = \omega_2 \rho \cos\beta \end{cases}$$

$$\omega_2 = \frac{\omega R \cos\theta}{\rho \cos\beta}$$

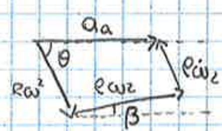
PER LE ACCELERAZIONI (→ T. DI RIVALS)

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

$$\vec{a}_B = \vec{a}_{B_n} + \cancel{\vec{a}_{B_t}}, \quad \vec{a}_{B_t} = 0$$

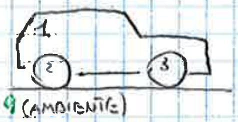
$$\vec{a}_A = \vec{a}_{B_n} + \vec{a}_{A/B_n} + \vec{a}_{A/B_t}$$

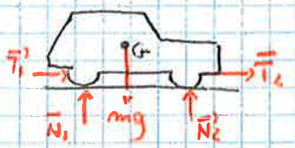
	$\vec{a}_A$	$\vec{a}_{B_n}$	$\vec{a}_{A/B_n}$	$\vec{a}_{A/B_t}$
(M)	?	$R\omega^2$	$\rho\omega_2^2$	$\rho\dot{\omega}_2$
(D/V)	—	θ	β	β

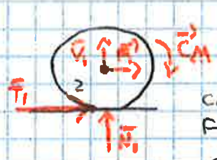


$$a_A + \rho\dot{\omega}_2 \sin\beta = R\omega^2 \cos\theta + \rho\omega_2^2 \cos\beta$$

$$R\omega^2 \sin\theta = \rho\omega_2^2 \sin\beta + \rho\dot{\omega}_2 \cos\beta$$







DIAGRAMMI DI CORPO LIBERO

CONSENTONO IL PASSAGGIO DAL MODELLO FISICO AL MODELLO MATEMATICO

$$\begin{cases} \Sigma\vec{F} = 0 \\ \Sigma\vec{M} = 0 \end{cases}$$

LEGGI DELLA DINAMICA

a) (I) $\vec{R} = \Sigma\vec{F}_e = 0$ $\begin{cases} \vec{V} = 0 & \text{CORPO RIMANE FERMO} \\ \vec{V} \neq 0 & \text{M.R.U. } (\vec{V} = k) \end{cases}$ [1ª LEGGE DELLA DINAMICA] (LEZIONE N° 13)

b) (II) $\vec{R} = \Sigma\vec{F}_e \neq 0$

$\Sigma\vec{F}_e = m\vec{a} = \vec{R}$ [SECONDA LEGGE DELLA DINAMICA (NEWTON)]

c) (III) PRINCIPIO DI AZIONE E REAZIONE

$\vec{F}_{21} = -\vec{F}_{12}$

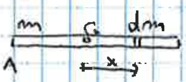
d) $\Sigma\vec{M}_{e_G} = b_1 F_1 \vec{k} + b_2 F_2 \vec{k} + b_3 F_3 \vec{k}$ $[N \cdot m] = \left[\frac{Kg \cdot m^2}{s^2}\right]$

e) $\Sigma\vec{M}e_G = I_G \dot{\vec{\omega}}$, I_G MOMENTO D'INERZIA BARICENTRICO $[Kg\, m^2]$

SECONDA LEGGE DELLA DINAMICA PER UN CORPO ESTESO (MOTO ROTATORIO)

MOMENTO D'INERZIA

$$I = \int_m R^2 dm$$



$I_G = \int x^2 dm$ $\quad m = \mu L \Rightarrow dm = \mu dx$

$$I_G = \mu \int_{-L/2}^{L/2} x^2 dx = \mu \frac{L^3}{12} = \frac{mL^2}{12}$$

$$I_A = \mu \int_0^L x^2 dx = \mu \frac{L^3}{3} = \frac{mL^2}{3}$$

PRINCIPIO DI HUGENS-STEYNER

$I_A = I_G + ma^2$, a = DISTANZA DALL'ASSE BARICENTRICO

NOTA: $I_G = m\rho_G^2 \rightarrow$ RAGGIO D'INERZIA (IN UN PROBLEMA POSSIAMO TROVARE ρ_G INVECE CHE LA DISTANZA DALL'ASSE G; PER CUI NOI LO AVREMO AL QUAD. E MOLT. X ?)

$\left(\rho_G = \frac{R}{\sqrt{2}}\right)$

(ESPRESSIONI VALIDE NELLO SPAZIO 3D)

$$\begin{cases} \Sigma\vec{F}_e = m\vec{a}_G \\ \Sigma\vec{M}_e = I_G\dot{\vec{\omega}} \end{cases} \quad \begin{cases} \Sigma\vec{F}_e - m\vec{a}_G = 0 \\ \Sigma\vec{M}e_G - I_G\dot{\vec{\omega}} = 0 \end{cases} \quad \begin{cases} \Sigma\vec{F}_e + \vec{F}_i = 0 \\ \Sigma\vec{M}e_G + \vec{M}_{iG} = 0 \end{cases}$$

$\vec{F}_i$ = RISULTANTE DELLE FORZE D'INERZIA

$\vec{M}_{iG}$ = MOMENTO RISULT. DELLE FORZE D'INERZIA

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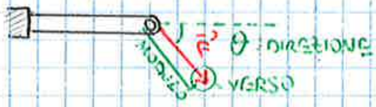
DINAMICA

(CORRELAZIONE TRA FORZE E MOTO) $\begin{cases} \text{ANALISI DIRETTA = DALLE FORZE AL MOTO} \\ \text{ANALISI INVERSA = DAL MOTO ALLE FORZE (CAUSE)} \end{cases}$

IN INGLESE: STATICS
DYNAMICS ┬ KINEMATICS (LA NOSTRA CINEMATICA)
└ KINETICS (LA NOSTRA DINAMICA)

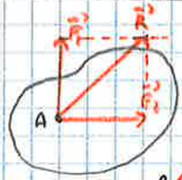
(LEZIONE N° 11)

LE FORZE SONO VETTORI APPLICATI CARATTERIZZATE DA:

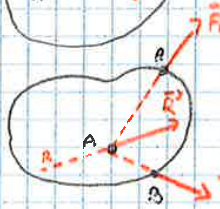


- MODULO
- DIREZIONE
- VERSO
- PUNTO DI APPLICAZIONE

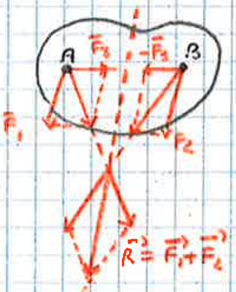
PRINCIPIO DI TRASMISSIBILITÀ : VETTORE = SLIDING VECTOR (SCORREVOLE)



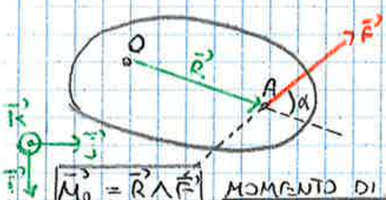
$\vec{R}' = \vec{F_1} + \vec{F_2}$ IMPORTANTE INDIVIDUARE LA RETTA D'AZIONE



$\vec{R} = \vec{F_1} + \vec{F_2}$ INDIVIDUO LA RETTA D'AZIONE TRAMITE L'APPLICAZIONE DEL PRINCIPIO DI TRASMISSIBILITÀ.



MOMENTO DI F



A PUNTO DI APPLICAZIONE

O POLO DI RIFERIMENTO

$\vec{R}$ POSIZIONE DI A

$\boxed{\vec{M_0} = \vec{R} \wedge \vec{F}}$ MOMENTO DI $\vec{F}$ RISP. A O

$M_0 = RF\sin\alpha\ \vec{k}$ $b = R\sin\alpha$ BRACCIO $\vec{M_0} = b \cdot F \cdot \vec{k}$

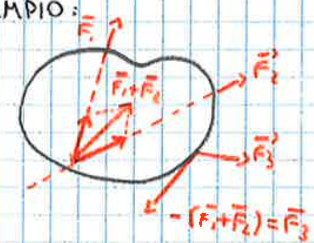
EQUILIBRIO DI UN SISTEMA DI FORZE (NEL PIANO)

$\boxed{\vec{R} = \Sigma \vec{F} = 0}$

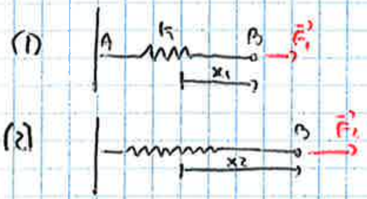
$\begin{cases} \vec{R}' = \vec{F} + (-\vec{F}) = 0 \\ \vec{M} = 0 \end{cases}$ ($\hat{=}$ STESSA RETTA D'AZIONE)

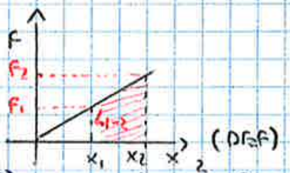
LE DUE FORZE DEVONO ESSERE UGUALI IN MODULO, OPPOSTE IN VERSO E ALLINEATE (STESSA RETTA D'AZIONE)

ESEMPIO:



$\rightarrow \begin{cases} \vec{R} = \Sigma_i \vec{F} = 0 \\ \vec{M} = 0 \end{cases}$ $\begin{cases} \vec{F_1} + \vec{F_2} + \vec{F_3} = 0 \\ \vec{F_3} = -(\vec{F_1} + \vec{F_2}) \end{cases}$



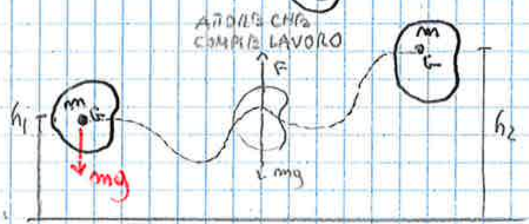


$dL = F \cdot dx = kx\,dx$

$L_{1-2} = k\int_{x_1}^{x_2} x\,dx = \frac{1}{2}k(x_2^2 - x_1^2)$, $\frac{1}{2}kx^2 = E_{P_{ELASTICA}}$

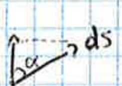
$L_{1-2} = \Delta E_{P_{ELASTICA}}$

ALTRO CASO: E_g



$F = mg$

$dL = \vec{F} \cdot d\vec{s} = F\,ds\cos\alpha$



$dL = mg\,dh$

$L_{1-2} = mg\int_{h_1}^{h_2} dh = mg(h_2 - h_1)$, $mgh = E_{P_{GRAVITAZIONALE}}$

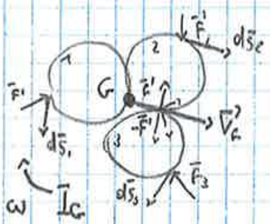
$L_{1-2} = \Delta E_{P_{GRAVITAZIONALE}}$

ENERGIA MECCANICA

$E_M = E_c + E_e + E_g$

$L = \Delta E_M = \Delta E_c + \Delta E_e + \Delta E_g$

SE $L = 0 \Rightarrow \Delta E_M = 0$ $E_M = k$ SISTEMA CONSERVATIVO



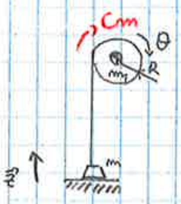
$L_i + L_e = \Delta E_c + \Delta E_e + \Delta E_g$ EQUAZIONE DELL'ENERGIA

$\Delta E_g = mg(h_f - h_i)$

$\Delta E_e = \frac{1}{2}k(x_f^2 - x_i^2)$

$\Delta E_c = \frac{1}{2}m(V_{G_f}^2 - V_{G_i}^2) + \frac{1}{2}I_G(\omega_f^2 - \omega_i^2)$

ESERCIZIO 2.4 (APPLICANDO L'EQUAZIONE DELL'ENERGIA CON TALE METODO NON POSSO CALCOLARE LE REAZ. VINC. IN ⊕ (PUNTO FISSO ⇒ NON COMPAIONO LE R NELL'EQ.)



$z = R\theta$; $I_G = m_T\frac{R^2}{2}$

DATI

$m_T = 100\,kg$; $m = 200\,kg$; $R = 15\,cm$

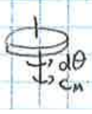
	(1)	(2)
t:	0	1
z:	0	h = 2m
ż:	0	V = 1m/s

$\Rightarrow C_m = ?$

$\Delta E_e = 0$; $\Delta E_c = \frac{1}{2}mV^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}mV^2 + \frac{1}{2}m_T\frac{R^2}{2}\frac{V^2}{R^2}$; $\Delta E_g = mgh$

$L_i + L_e = \Delta E_c + \underbrace{\cancel{\Delta E_e}}_{=0} + \Delta E_g$

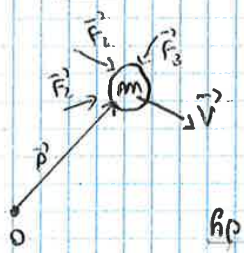
ORA ANALIZZIAMO IL LAVORO:



$dL = C_M \cdot d\theta \Rightarrow L_e = C_M \cdot \frac{z}{R}$

$L_i = T\,dz + (-T\,dz) = 0$ ⇒

TEOREMA DEL MOMENTO DELLA QUANTITÀ DI MOTO



$\vec{Q} = m\vec{V}$

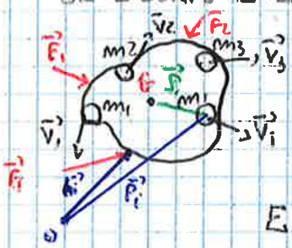
$\Sigma \vec{M}e_O = \vec{\rho} \wedge \vec{F}_1 + \vec{\rho} \wedge \vec{F}_2 + \rho \wedge \vec{F}_3 = \Sigma \vec{\rho} \wedge \vec{F}_i$

$\vec{\rho} \wedge \vec{Q} = \vec{\rho} \wedge m\vec{V} = \vec{K}_O$ MOMENTO DELLA Q.M. RISPETTO A O

hp: SE $\vec{V}_O = 0$

$\boxed{\Sigma \vec{M}e_O = \frac{d\vec{K}_O}{dt}}$ T. DEL MOMENTO DELLA Q.M.

SE IL CORPO È ESTESO:



$\vec{Q}_i = m_i \vec{V}_i$

$\Sigma_i \vec{\rho}_i \wedge m_i \vec{V}_i = \vec{K}_G$ MOMENTO RISULTANTE DELLA Q.M. RISP. AL BARIG. G

$\Sigma_i \vec{P}_i \wedge m_i \vec{V}_i = \vec{K}_O$ MOMENTO RISULTANTE DELLA Q.M. RISP A O

$\Sigma_i \vec{\rho}_i \wedge \vec{F}_i = \Sigma \vec{M}e_G$ RISULTANTE DEI MOMENTI DELLE F. ESTERNE RISP. A G.

$\Sigma_i \vec{P}_i \wedge \vec{F}_i = \Sigma \vec{M}e_O$ RISULTANTE DEI MOMENTI DELLE F ESTERNE RISP. A O

$\boxed{\Sigma \vec{M}e_G = \frac{d\vec{K}_G}{dt}}$ RISPETTO A G

T. DEL MOMENTO DELLA Q.M. PER UN CORPO ESTESO

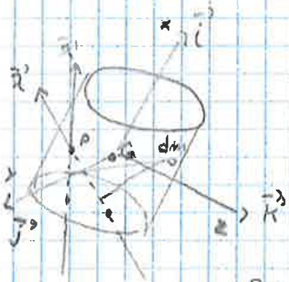
hp: $\vec{V}_O = 0$ NECESSARIA $\boxed{\Sigma \vec{M}e_O = \frac{d\vec{K}_O}{dt}}$ RISPETTO A O

$\begin{cases} \Sigma \vec{F}_e + \vec{F}_i = 0 \\ \Sigma \vec{M}e + \vec{M}_i = 0 \end{cases}$

$\vec{F}_i = -\Sigma \vec{F}_e = -\frac{d\vec{Q}}{dt} = -\frac{d(m\vec{V}_G)}{dt} = -m\frac{d\vec{V}_G}{dt} = -m\vec{a}_G$ $\Rightarrow$ $\begin{cases} \Sigma \vec{F}_e = m\vec{a}_G \\ \\ \end{cases}$

$\vec{M}_i = -\Sigma \vec{M}_e = -\frac{d\vec{K}}{dt}$, $\vec{K}_{(G/O)}$ G: BARICENTRO, O: SE $\vec{V}_O = 0$

$\vec{K}_G = ?$ $\vec{K}_O$ $(\vec{V}_O = 0)$



$\int_m R^2 dm = I_P$

$\int_m R'^2 dm = I_{P'}$

$[I_P(MAX) \quad I_P(min) \quad I_P(\perp)]$ MOMENTI PRINCIPALI D'INERZIA PER P

$[I_G(MAX) \quad I_G(min) \quad I_G(\perp)]$ MOMENTI CENTRALI D'INERZIA

$I_x \; I_y \; I_z$ MOMENTI CENTRALI D'INERZIA

$\vec{i} \; \vec{j} \; \vec{k}$ TERNA CENTRALE D'INERZIA

$\vec{\omega}$ VEL ANG. DEL CORPO

$\omega = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$

$\vec{K}_G = I_x \omega_x \vec{i} + I_y \omega_y \vec{j} + I_z \omega_z \vec{k}$ $\Rightarrow$ $\vec{M}_{iG} = -\frac{d\vec{K}_G}{dt}$ (↓ LEZIONE N° 19)

$$\frac{d\vec{K}_G}{dt} = I_x \omega_x \frac{d\vec{i}}{dt} + I_y \omega_y \frac{d\vec{j}}{dt} + I_z \omega_z \frac{d\vec{k}}{dt} + I_x \dot{\omega}_x \vec{i} + I_y \dot{\omega}_y \vec{j} + I_z \dot{\omega}_z \vec{k}$$

$\frac{d\vec{i}}{dt} = \vec{\omega}_T \wedge \vec{i}$ SE $\vec{i} \; \vec{j} \; \vec{k}$ È SOLIDALE AL CORPO: $\vec{\omega}_T = \vec{\omega}$ $\quad$ ω VEL ANG CORPO, ω_T VEL ANG TERNA

$\frac{d\vec{\lambda}}{dt} = \vec{\omega_T} \wedge \vec{\lambda}$; $\frac{d\vec{\mu}}{dt} = \vec{\omega_T} \wedge \vec{\mu}$

SE SOLIDO : GIROSCOPICO $\vec{\omega_T} = \vec{\omega_1} = \omega_1 \vec{k}$

(ω_T VEL ANGOL. DELLA TERNA A CUI APPARTIENE QUEL VETTORE)

$\frac{d\vec{\lambda}}{dt} = \omega_1 \vec{k} \wedge \vec{\lambda}' = \omega_1 \cos\alpha(-\vec{\nu})$; $\frac{d\vec{\mu}}{dt} = \omega_1 \vec{k} \wedge \vec{\mu} = \omega_1 \sin\alpha \, \vec{\nu}$

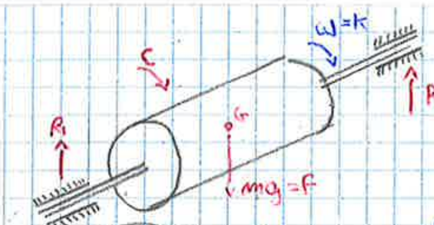
$$\frac{d\vec{K_g}}{dt} = I_\lambda(\omega_1 \sin\alpha - \omega_2)\omega_1 \cos\alpha(-\vec{\nu}) + I_\mu \omega_1^2 \cos\alpha \sin\alpha \, \vec{\nu} = -\vec{M}_{iG}$$

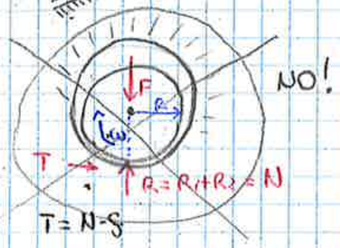
$L_i + L_e = \Delta E_M$ SE $L_i + L_e = 0 \Rightarrow E_M = E_c + E_e + E_g = k$

$\Sigma \vec{F_e} = \frac{d\vec{Q}}{dt}$ SE $\Sigma \vec{F_e} = 0 \Rightarrow \vec{Q} = k$

$\Sigma \vec{M_e} = \frac{d\vec{K}}{dt}$ SE $\Sigma \vec{M_e} = 0 \Rightarrow \vec{K} = k$

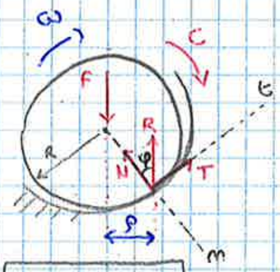
(LEZIONE N° 22)





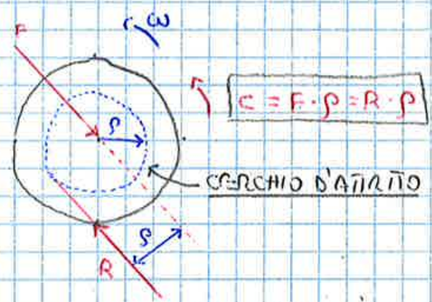
$T = N \cdot f$

$C = T \cdot R$

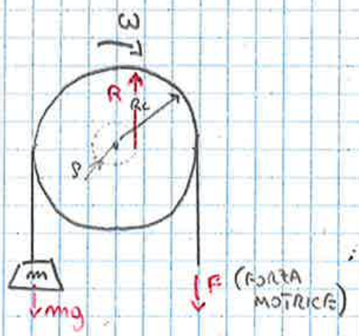


$\rightarrow \boxed{C = F \cdot \rho = R \cdot \rho}$

$\boxed{\rho = R \sin\varphi}$



$\boxed{C = F \cdot \rho = R \cdot \rho}$ $\quad \rho = R \sin\varphi$



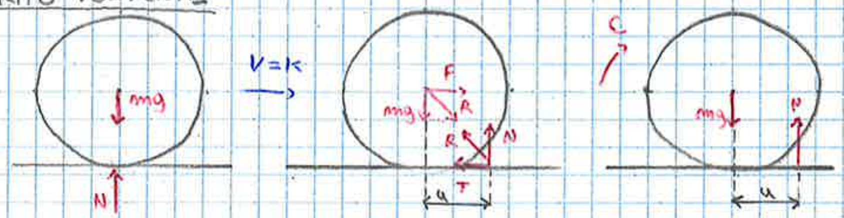
$\rho = R \cdot \sin\varphi \ , \ R \neq R_c$

$F(R_c - \rho) = mg(R_c + \rho)$

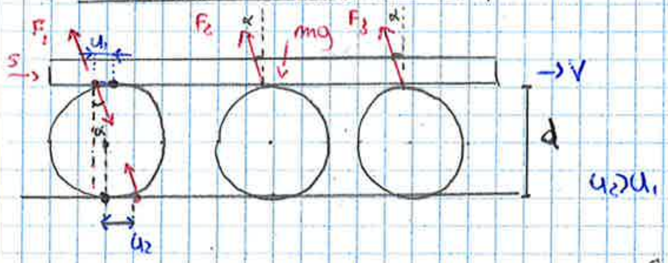
$F = mg\left(\frac{R_c + \rho}{R_c - \rho}\right) \qquad F > mg$

ATTRITO VOLVENTE

(LEZIONE N° 23)



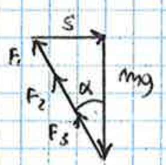
u : PARAMETRO DI ATTRITO VOLVENTE (O DI ROTOLAMENTO)



$\boxed{Tg\alpha = \frac{u_1 + u_2}{d}}$

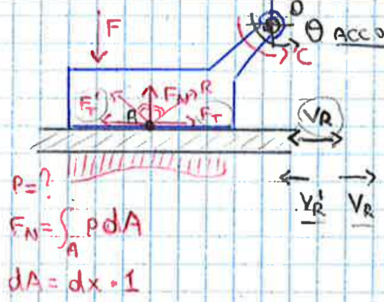
$\vec{S} + \vec{mg} + \vec{F_1} + \vec{F_2} + \vec{F_3} = 0$

$S = mg\, Tg\alpha = mg\, \frac{u_1 + u_2}{d}$



FRENI AD ACCOSTAMENTO RIGIDO = 1 GRADO DI LIBERTA'

(1)



θ ACCOSTAMENTO = UN GRADO DI LIBERTA' : θ

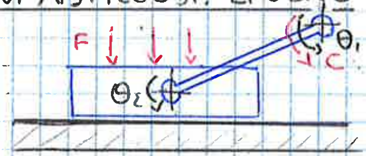
$p = ?$

$F_N = \int_A p\,dA$

$dA = dx \cdot 1$

FRENI AD ACCOST. LIBERO = 2 GRADI DI LIBERTA'

(2)

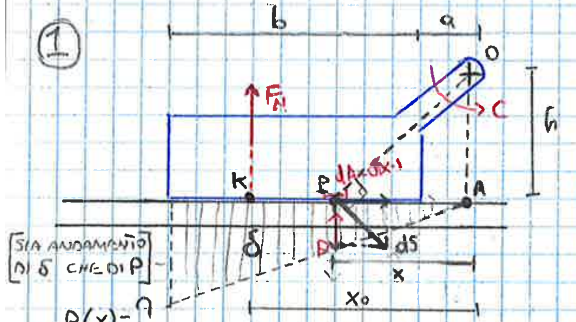


ACCOSTAMENTO = DUE GRADI DI LIBERTA' : θ_1 e θ_2

ANALISI

$F \;(C) \rightarrow$ [MODELLO FUNZIONALE] $\rightarrow F_T$

(1) ACCOSTAMENTO RIGIDO



[SIA ANDAMENTO DI δ CHE DI P]

$p(x) = ?$

$dF_N = p\,dA = p\,dx \qquad (a < x < a+b)$

$\boxed{\delta \cdot dA \propto f\,p \cdot dA\,V_R}$ $\qquad V_R = K \;;\; f = K \qquad$ (IPOTESI DELL'USURA)

$dF_T = f \cdot dF_N$

(ds) IN DIREZIONE ORIZZONTALE NON VIENE CONSUMATO NULLA

$\delta = K \cdot x = p(x)$

$F_N = \int dF_N = \int p\,dx = \int_a^{a+b} Kx\,dx = \frac{1}{2}Kx^2 \Big|_a^{a+b}$

$[-F_N x_0 + C = 0]$

$F_N \cdot x_0 = C = \int dF_N \cdot x = \int_a^{a+b} Kx^2\,dx = \frac{1}{3}Kx^3 \Big|_a^{a+b}$

$x_0 = \frac{C}{F_N} = \frac{2}{3}\frac{(a+b)^3 - a^3}{(a+b)^2 - a^2}$

$\sim$

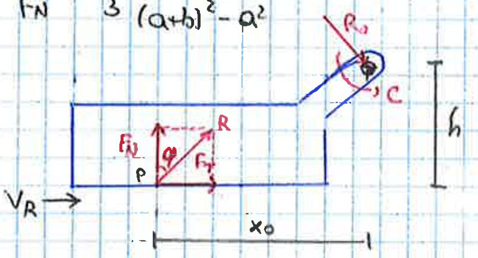
$dF_N = p\,dx$

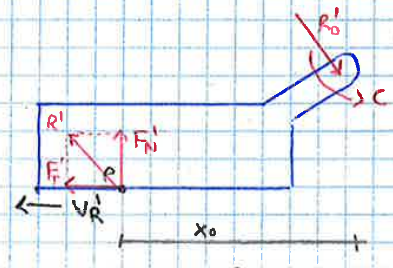
$F_N = \int p\,dx = \int_a^{a+b} Kx\,dx = \frac{1}{2}Kx^2 \Big|_a^{a+b}$

$C = \int dF_N \cdot x = \int_a^{a+b} Kx^2\,dx = \frac{1}{3}Kx^3 \Big|_a^{a+b}$

INOLTRE SAPPIAMO:

CHE: $F_N x_0 = C \Rightarrow x_0 = \frac{C}{F_N} = \frac{2}{3}\frac{(a+b)^3 - a^3}{(a+b)^2 - a^2}$





$C = F_N x_0 - F_T h$, ESSENDO $F_N = F_T / f \Rightarrow$

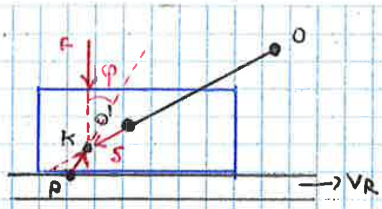
$\Rightarrow C = F_T\left(\frac{x_0}{f} - h\right) \Rightarrow$

$\Rightarrow \boxed{F_T = \frac{C}{\left(\frac{x_0}{f} - h\right)}}$

$C = F'_N x_0 + F'_T h$

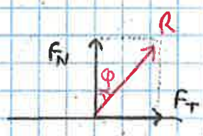
$\Rightarrow C = F'_T\left(\frac{x_0}{f} + h\right) \Rightarrow$

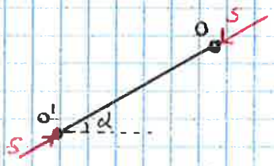
$\Rightarrow \boxed{F'_T = \frac{C}{\left(\frac{x_0}{f} + h\right)}} \qquad (< F_T)$

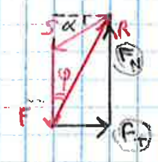


(I) $\sum \vec{F} = 0$

(II) CONCORRENTI NELLO STESSO PUNTO







$$F_N = \frac{F_T}{f} = F + S \cdot \sin\alpha \qquad S = \frac{F_T}{\cos\alpha}$$

$$\frac{F_T}{f} = F + F_T \operatorname{Tg}\alpha$$

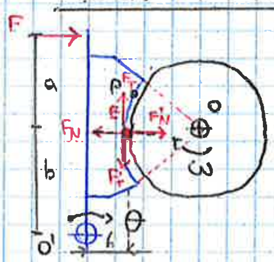
$$F_T\left(\frac{1}{f} - \operatorname{Tg}\alpha\right) = F$$

$$\boxed{F_T = \frac{F}{\frac{1}{f} - \operatorname{Tg}\alpha}}$$

FRENI A CEPPI

ACCOSTAMENTO RIGIDO

SISTEMA A UN GRADO DI LIBERTÀ



APPUCO hp DELL'USURA

OBIETTIVO TROVARE SEMPRE F_T (AZIONE FRENANTE)

SUPPONIAMO CHE IL PUNTO DI APPLICAZIONE SIA GIÀ DEFINITO : NELLA METÀ DELL'ARCO DEL TAMBURO CEPPO

$$M = F_T \cdot R$$

$$T_T = F_N \cdot f$$

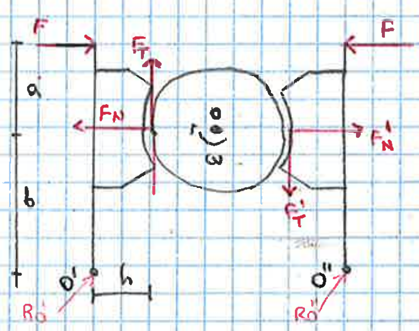
SX

$$F(a+b) - F_N b - F_T h = 0$$

$$F(a+b) = \frac{F_T}{f} b + F_T h =$$

$$F_T\left(\frac{b}{f} + h\right) = F(a+b)$$

$$\boxed{F_T = F \, \frac{a+b}{\frac{b}{f} + h}}$$



DX

$$F(a+b) - F_N' b + F_T' h = 0$$

$$F(a+b) = \frac{F_T'}{f} b - h F_T'$$

$$\boxed{F_T' = \frac{F(a+b)}{\frac{b}{f} - h}}$$

$$\boxed{F_T' > F_T}$$

SI CONSUMA DI PIÙ DI QUELLO DI SX

PUÒ TENDERE ANCHE A IMPUNTARSI

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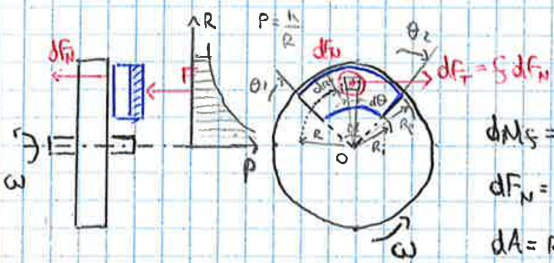
FRENO A DISCO



$dM_f = R \cdot dF_T$ $\qquad M_f$ MOMENTO FRENANTE

$dF_N = p \cdot dA$

$dA = R\,dR\,d\theta$

$F = \int_A dF_N = \int p\,dA$

$\delta\,dA \propto f\,p\,dA\,V_R$

$V_R = \omega R$

$\delta \propto f\,p\,\omega\,R \qquad \omega = k \quad f = k$

$p = \frac{k}{R}$

$F = \int \frac{k}{R}\,dA = \int \frac{k}{\cancel{R}}\,\cancel{R}\,dR\,d\theta$

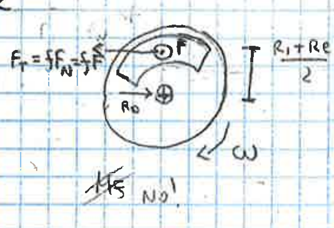
$F = \int k\,dR\,d\theta = \int_{\theta_1}^{\theta_2}\int_{R_i}^{R_e} k\,dR\,d\theta = k(R_e - R_i)(\theta_2 - \theta_1)$

$M_F = \int \cancel{R} f \frac{k}{\cancel{R}}\,R\,dR\,d\theta$

$M_F = \int f k R\,dR\,d\theta = \int_{\theta_1}^{\theta_2}\int_{R_i}^{R_e} f k R\,dR\,d\theta = f k(\theta_2 - \theta_1)\frac{R_e^2 - R_i^2}{2}$

$\frac{M_F}{F} = \frac{f \cancel{k}\cancel{(\theta_2-\theta_1)}\cancel{(R_e - R_i)}(R_e + R_i)}{2\cancel{k}\cancel{(R_e - R_i)}\cancel{(\theta_2 - \theta_1)}} = f \cdot \frac{R_e + R_i}{2}$

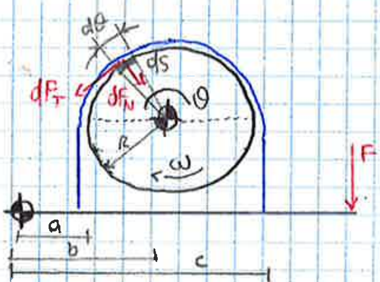
$\frac{M_F}{F} = f\,\frac{R_e + R_i}{2} \quad \Rightarrow \quad \boxed{M_F = f F\,\frac{R_i + R_e}{2}}$



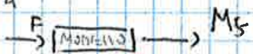
NB. α NON INFLUISCE SUL MOMENTO FRENANTE (↳ QUINDI LE DIMENSIONI DEL FRENO)

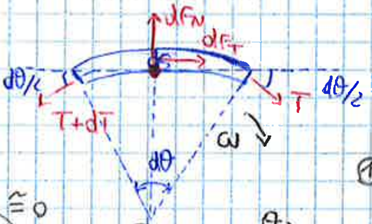
FRENO A NASTRO

(LEZIONE N° 29)



$\int_A dF_T \cdot R = M_f$





$$\begin{cases} dF_N = 2T \sin\frac{d\theta}{2} + \cancel{dT \sin\frac{d\theta}{2}} \cong 0 \\ dF_T = dT \cos\frac{d\theta}{2} \\ dF_T = f\,dF_N \end{cases}$$

$\sin\frac{d\theta}{2} = \frac{d\theta}{2} \qquad \cos\frac{d\theta}{2} = 1$

$\Rightarrow \begin{cases} dF_N = T\,d\theta \\ dF_T = dT \end{cases} \Rightarrow \frac{dT}{f} = T \cdot d\theta$

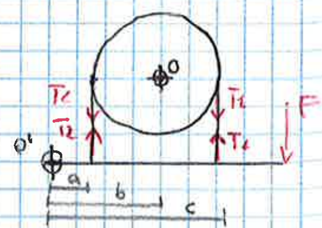
$\int_{T_1}^{T_2} \frac{dT}{T} = f \int_{\theta_1}^{\theta_2} d\theta$

$\ln\frac{T_2}{T_1} = f(\theta_2 - \theta_1) = f\theta^*$

① $\boxed{M_f = (T_2 - T_1) \cdot R}$

DEVO QUINDI TROVARE T_2 E T_1

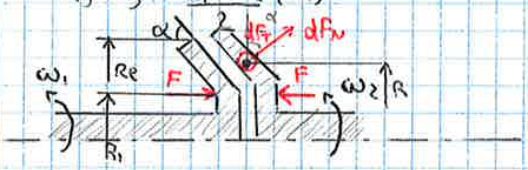
② $\boxed{\frac{T_2}{T_1} = e^{f\theta^*}} > 1 \Rightarrow T_2 > T_1$ $\quad$ [θ IN RADIANTI NON 180° MA π]



③ $\boxed{F \cdot c - T_1 \cdot b - T_2 \cdot a = 0}$

(LEZIONE N° 30)

$M_f = f F \frac{R_i+R_e}{2}$ (m)



FRIZIONE CONICA

$M_f = \int_A dF_T \cdot R$

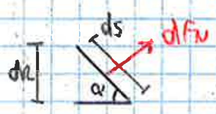
$dF_T = f\, dF_N$

$F = \int dF_N \sin\alpha$ $\qquad dF = dF_N \sin\alpha$

$\delta \cdot dA \propto f p\, dA\, V_R$, $V_R = \omega_R \cdot R$

$\delta \propto f p \omega_R R$ $\qquad \omega_R = k \quad f = k \qquad \delta = k \qquad p = \frac{k}{R}$

$dF_N = p \cdot dA$



$dR = ds \sin\alpha$; $dA = 2\pi R \cdot ds$

$dF_N = \frac{k}{R} 2\pi R \frac{dR}{\sin\alpha} = 2\pi k \frac{dR}{\sin\alpha}$

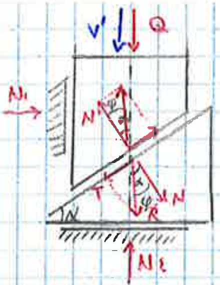
$F = \int_{R_i}^{R_e} \cancel{\sin}\alpha \frac{2\pi k}{\cancel{\sin}\alpha} dR = 2\pi k (R_e - R_i)$

$M_f = \int f\, 2\pi k \frac{dR}{\sin\alpha} \cdot R = \frac{\pi f k}{\sin\alpha}(R_e^2 - R_i^2)$

$\frac{M_f}{F} = \frac{\cancel{\pi} f \cancel{k} (R_e+R_i)\cancel{(R_e-R_i)}}{\sin\alpha \cdot 2\cancel{\pi}\cancel{k}\cancel{(R_e-R_i)}} = \frac{f}{\sin\alpha} \frac{R_e+R_i}{2}$

$\boxed{M_f = \frac{f}{\sin\alpha} F \frac{R_e+R_i}{2}}$; $\frac{f}{\sin\alpha} = f'$ COEFF. DI ATTRITO VIRTUALE $(>f)$

AL DI SOTTO DI UN CERTO VALORE DI α LA FRIZIONE SI IMPIANTA.



⊙ FUNZIONAMENTO INVERSO DEL SISTEMA (LEZIONE N 33)

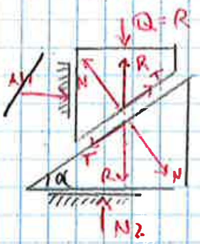
$F = \frac{C}{R}$

$V = \omega R$

$Q = R \cdot \cos(\alpha - \varphi)$

$F = R \sin(\alpha - \varphi)$

$\frac{F}{Q} = \tan(\alpha - \varphi)$, $\alpha > \varphi$



⊙ CONDIZIONE DI IRREVERSIBILITÀ (ES: IL CRICK)

$\frac{T}{N} = \tan\alpha$ $\quad \frac{T}{N} \leq f_a \Rightarrow \tan\alpha \leq f_a \Rightarrow \boxed{\alpha \leq \varphi_a}$ CONDIZ. DI IRREVERS.

φ_a ANGOLO DI ATTRITO STABILE

GIUNTI DI TRASMISSIONE (PAG 195)

GIUNTO → $\omega_1 = \omega_2$, ω_1, ω_2 V. ANGOLARI MEDIE

GIUNTO DI CARDANO / CARDANICO

$u = \frac{\omega_1}{\omega_2}$ $\quad \omega_1 = \frac{d\theta_1}{dt}$; $\omega_2 = \frac{d\theta_2}{dt}$

VEDI PAG 166 E 167

$\bar{\mu} = \cos\theta_1 \bar{j}_1 + \sin\theta_1 \bar{k}$

$\bar{v} = -\sin\theta_2 \bar{j}_2 + \cos\theta_2 \bar{k}$

$\bar{\mu} \cdot \bar{v} = 0$

$0 = -\cos\theta_1 \sin\theta_2 \bar{j}_1 \bar{j}_2 + \sin\theta_1 \cos\theta_2$

$\cos\theta_1 \sin\theta_2 \cdot \cos\alpha = \sin\theta_1 \cdot \cos\theta_2$

$\boxed{\tan\theta_1 = \tan\theta_2 \cdot \cos\alpha}$

; $\frac{d}{dt}(\tan\theta) = \frac{d(\tan\theta)}{dt}\frac{d\theta}{dt} = (1+\tan^2\theta)\omega$,

$(1+\tan^2\theta_1)\omega_1 = (1+\tan^2\theta_2)\omega_2 \cos\alpha$

$u = \frac{\omega_1}{\omega_2} = \frac{1+\tan^2\theta_2}{1+\tan^2_1} \cos\alpha$

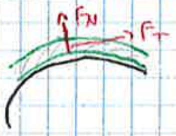
$\frac{\omega_1}{\omega_2} = \frac{1+\frac{\tan^2\theta_1}{\cos^2\alpha}}{1+\tan^2\theta_1}\cos\alpha = \frac{\cos^2\alpha + \tan^2\theta_1}{\cos\alpha(1+\tan^2\theta_1)} = \frac{\cos^2\theta_1 \cos^2\alpha + \sin^2\theta_1}{\cos\alpha(\underbrace{\cos^2\theta_1 + \sin^2\theta_1}_{=1})}$

$u = \frac{\omega_1}{\omega_2} = \frac{\cos^2\theta_1 \cos^2\alpha + 1 - \cos^2\theta_1}{\cos\alpha} = \frac{\cos^2\theta_1(\cos^2\alpha - 1) + 1}{\cos\alpha} = \frac{1 - \cos^2\theta_1 \sin^2\alpha}{\cos\alpha}$

$\boxed{u = \frac{\omega_1}{\omega_2} = \frac{1 - \cos^2\theta_1 \sin^2\alpha}{\cos\alpha}}$ RAPPORTO DI TRASMISSIONE

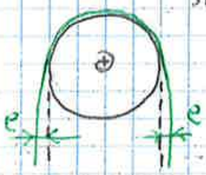
GIUNTO NON OMOCINETICO

FUNI



TREFOLO

2 FENOMENI:



→ SCOSTAMENTO ELASTICO

(e PARAMETRO DI SCOSTAMENTO ELAST.)

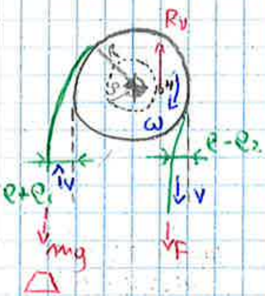
DOVUTO ALLA RIGIDEZZA ELASTICA DEI COMPONENTI DELLA FUNE

SE IL SISTEMA SI METTE IN MOTO SI VERIFICA ANCHE UN SECONDO EFFETTO:

DISCOSTAMENTO VERSO L'INTERNO (DOVUTO ALLE F. DI ATTRITO INTERNE)

→ SCOSTAMENTO ANELASTICO

(e_1 PARAM. SCOST. AN. IN INGRESSO
e_2 PARAM. DI SCOST. AN. IN USCITA)



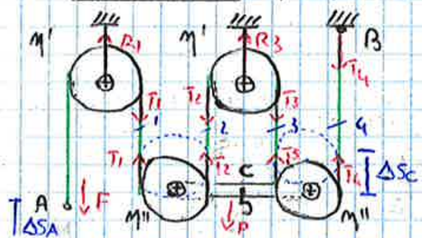
H) $F(R-\rho-e_2+e) = mg\,(R+\rho+e_1+e)$

$P_M = F\,V \qquad P_U = mg\,V$

$$\eta = \frac{mg}{F} = \frac{R-\rho-e_2+e}{R+\rho+e_1+e} < 1$$

PARANCO (PAG 105)

(LEZIONE N° 35)



2 PULEGGE FISSE
2 PULEGGE MOBILI (TRAVERSO MOBILE)

4 TRATTI DI FUNE PORTANTI (m) $\quad \Delta S_A = 4\Delta S_C$

$$\frac{\Delta S_A}{\Delta t} = \frac{4\Delta S_C}{\Delta t} \Rightarrow \boxed{V_A = 4V_C}$$

→ hp: FUNE INESTENSIBILE $AB = K$

→ hp: $\eta' = \eta'' = 1 \Rightarrow F = T_1 = T_2 = T_3 = T_4$

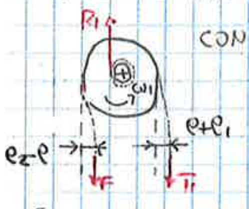
$$\boxed{P = \Sigma T_i = 4F = mF}$$

(IDEALE)

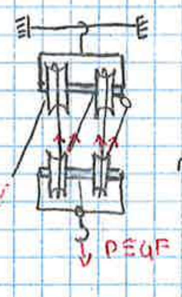
$$\eta_{TOT} = \frac{P_U}{P_M} = \frac{P \cdot V_C}{F \cdot V_A} = \frac{4}{4} = 1$$

CONSIDERANDO LE PULEGGE SINGOLARMENTE

(REALE)



$$\eta' = \frac{R+e-e_2-\rho}{R+e+e_1+\rho} < 1$$



BOZZELLO FISSO

BOZZELLO MOBILE

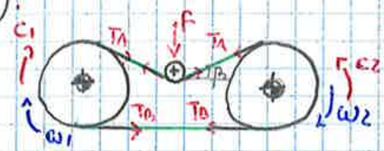
$$\begin{cases} T_1 = \eta' F \\ T_2 = \eta'' T_1 = \eta\eta' F \\ T_3 = \eta' T_2 = \eta'^2 \eta'' F \\ T_4 = \eta'' T_3 = \eta'^2 \eta''^2 F \end{cases}$$

$$P = \Sigma T_i = F\left(\eta' + \eta'\eta'' + \eta'^2\eta'' + \eta'^2\eta''^2\right)$$

$$\eta_{TOT} = \frac{P \cdot V_C}{F \cdot V_A} = \frac{\left(\eta' + \eta'\eta'' + \eta'^2\eta'' + \eta'^2\eta''^2\right)}{4} \quad (<1)$$

3 METODI PER RINFORZAMENTO NELLE CINGHIE CON LE PULEGGE:

1)

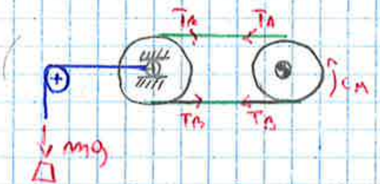


GALOPPINO

$F = 2T_A \sin\beta$

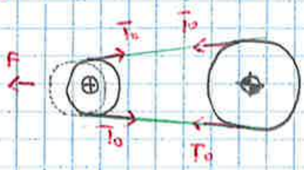
IL GALOPPINO FA' AUMENTARE ANCHE α $\Rightarrow$ + ATTRITO $\Rightarrow$ + COPPIA SI PUO' TRASMETTERE

2)



TENDITORE $mg = T_A + T_B$

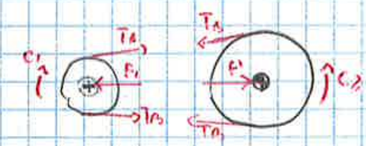
3)



TENSIONAMENTO (FORZAMENTO) INIZIALE

(ES: CATENA DELLA MOTO)

$T_0 \cong \dfrac{T_A + T_B}{2}$

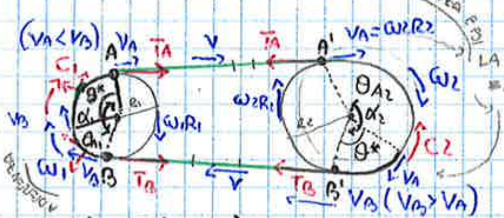


FLESSIBILE REALE

(LEZIONE N° 36)

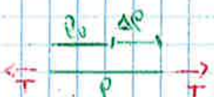
1) ELASTICITA'

2) MASSA $\boxed{q = \dfrac{dm}{d\ell}}$



LA TENSIONE AUMENTA ⇒ AUMENTA LA VELOCITA' (NON D'APPETITO) ⇒ CI DEV'ESSERE STRISCIAMENTO ⇒ θ^* IN CUI C'E' STRISCIAMENTO

$C_1 = (T_B - T_A) R_1$
$C_2 = (T_B - T_A) R_2$ $\Big\}$ $T_B > T_A$



$\Rightarrow \ell_B > \ell_A$ (DIPENDE DA T)

$\ell = \ell_0 + \Delta\ell$ $\quad \boxed{\varepsilon = \dfrac{\Delta\ell}{\ell_0} = \dfrac{\ell - \ell_0}{\ell}}$ $\quad \boxed{\sigma = \dfrac{T}{S\,(\text{SEZIONE})} = \varepsilon E}$ QUINDI: $\dfrac{T}{ES} = \dfrac{\ell - \ell_0}{\ell_0} \Rightarrow \ell = \ell_0\left(1 + \dfrac{T}{ES}\right)$ E = MODULO DI ELASTICITA' NORMALE

$\ell_B = \ell_0\left(1 + \dfrac{T_B}{ES}\right) = V_B \Delta t$

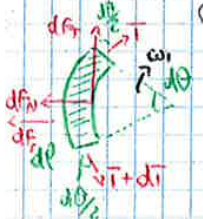
$\ell_A = \ell_0\left(1 + \dfrac{T_A}{ES}\right) = V_A \Delta t$

$\Rightarrow \boxed{\dfrac{V_B}{V_A} = \dfrac{1 + \frac{T_B}{ES}}{1 + \frac{T_A}{ES}}}$

θ^* STRISCIAMENTO, θ_{A1} ADERENZA, $\alpha_1 = \theta^* + \theta_{A1}$

SUI θ^* (ARCO DI SCORRIMENTO) $dF_T = f\,dF_N$ $\quad V_A \leq V \leq V_B$ $\quad$ POICHE' $T_A \leq T \leq T_B$

SU θ_{A1} (ARCO DI ADERENZA) $dF_T \leq f_a\,dF_N$ $\quad V = \omega_1 R_1$ $\quad$ POICHE' $T = T_B$



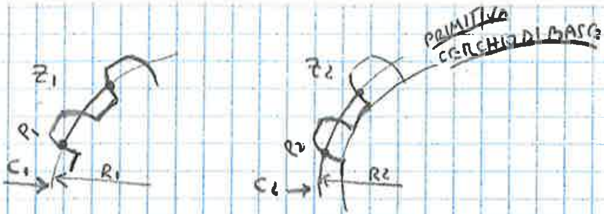
(FORZA D'INERZIA) $dF_i = dm\,R\omega^2 = q\,d\ell\,\dfrac{v^2}{R} = q\,R\,d\theta\,\dfrac{v^2}{R} = q v^2 d\theta$

$dF_T = dT \cdot \cos\dfrac{d\theta}{2}$ $\quad dF_N = 2T\sin\dfrac{d\theta}{2} + \underbrace{dT\sin\dfrac{d\theta}{2}}_{=0} - qv^2 d\theta$ $\quad$ (LEZIONE N° 37)

$\begin{cases} dF_T = dT \\ \dfrac{dF_T}{f} = T\,d\theta - qv^2 d\theta \end{cases} \Rightarrow \boxed{\dfrac{dT}{f} = (T - qv^2)\,d\theta} \Rightarrow \displaystyle\int_{T_A}^{T_B} \frac{dT}{T - qv^2} = f\int_0^{\theta^*} d\theta$

$\Rightarrow \ln\dfrac{T_B - qv^2}{T_A - qv^2} = f\theta^* \Rightarrow \dfrac{T_B - qv^2}{T_A - qv^2} = e^{f\theta^*}$ $\quad$ SE $q \cong 0 \rightarrow \dfrac{T_B}{T_A} = e^{f\theta^*} \Rightarrow C_1 = (T_B - T_A)R_1 = T_A\left(e^{f\theta^*} - 1\right)R_1$ $\quad C_1 \uparrow\ \theta^* \uparrow$

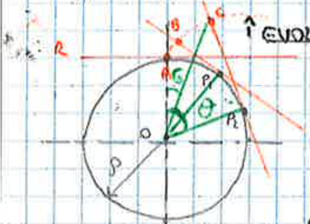
SE $\theta^* = \alpha_1$ AVREMO CHE: $V_B = \omega_1 R_1 \Rightarrow \boxed{C_{1MAX} = T_A\left(e^{f\alpha_1} - 1\right)R_1}$



$P_1 = \frac{2\pi R_1}{z_1}$ $\qquad$ $P_2 = \frac{2\pi R_2}{z_2}$ $\qquad$ P PASSO DELL'INGRANAGGIO

$\frac{P_1}{P_2} = \frac{R_1}{z_1}\frac{z_2}{R_2} = \frac{i}{i} = 1 \Rightarrow P = P_1 = P_2$

PROFILO DI UN DENTE



↑ EVOLVENTE DI CERCHIO

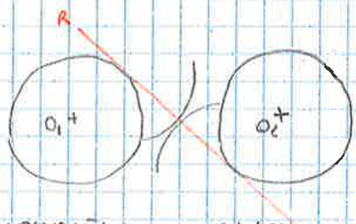
R RETTA GENERATRICE

ρ : RAGGIO DEL CERCHIO DI BASE

L'EVOLVENTE È SEMPRE ⊥ ALLA RETTA GENERATRICE

ROTOLAMENTO $\Rightarrow \widehat{AP_2} = \overline{CP_2}$

$(\sigma + \theta)\rho = \rho \, tg\theta \Rightarrow \boxed{\sigma = tg\theta - \theta}$ $\qquad$ ESPRESSIONE DELL'EVOLVENTE DI θ $\qquad$ $\sigma = ev(\theta)$

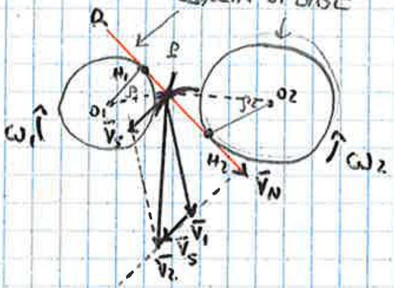


(GUARDA L'ANIMAZIONE CARICATA SUL PORTALE)

LE EVOLVENTI HANNO NEL LORO PUNTO DI CONTATTO LA STESSA NORMALE.

R È TANGENTE AI CERCHI DI BASE ; R RETTA DI PRESSIONE

(LEZIONE N° 40)



H_1, H_2 : LUOGO DEI CONTATTI

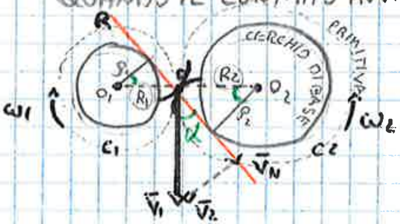
R : RETTA DI PRESSIONE

$V_1 = \omega_1 \overline{O_1 P}$

$V_2 = \omega_2 \overline{O_2 P}$

$\vec{V}_s = \vec{V}_2 - \vec{V}_1$ $\qquad$ VELOCITÀ RELATIVA (DI STRISCIAMENTO)

QUANDO IL CONTATTO AVVIENE COSÌ :

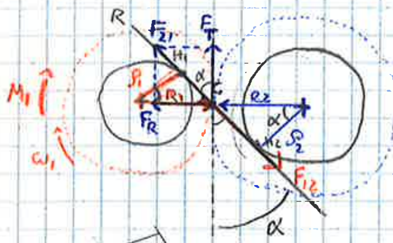


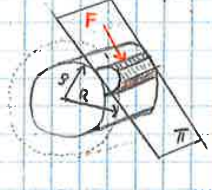
$R_1 = \overline{O_1 C}$ = RAGGIO PRIMITIVA 1 $\qquad$ C_1, C_2 : CIRCONF. PRIMITIVE

$R_2 = \overline{O_2 C}$ = RAGGIO PRIMITIVA 2 $\qquad$ C PUNTO DI TG DELLE DUE PRIMIT.

$\rho_1 = R_1 \cos\alpha$

$\rho_2 = R_2 \cos\alpha$





RUOTA CILINDRICA A DENTI DRITTI

IPOTESI:

1) IPOTIZZO F CONCENTRATA NELLA MEZZERIA DEL DENTE
2) UNA SOLA COPPIA DI DENTI IN PRESA, OVVERO CHE SI SCAMBIANO F
3) F E' SEMPRE SCAMBIATA NEL PUNTO C

$F_{12} = F_{21} = F$

$F_R = F \cdot \sin\alpha$

$F_T = F \cdot \cos\alpha$

(LEZIONE N° 41)

$$\begin{cases} M_1 = F\rho_1 = F_T R_1 \\ M_2 = F\rho_2 = F_T R_2 \end{cases}$$

$$\boxed{\eta = \frac{P_u}{P_e} = \frac{M_2\omega_2}{M_1\omega_1} = \frac{R_2}{R_1}\frac{R_1}{R_2} = 1}$$

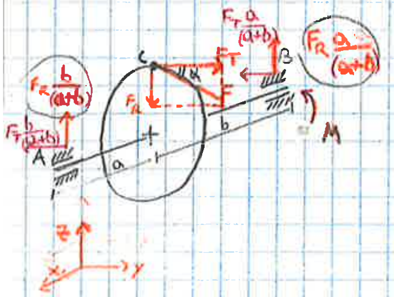
INGRANAGGIO IDEALE CON RENDIM. UNITARIO

$$\boxed{M_2 = M_1\frac{\omega_1}{\omega_2} = M_1 i}$$

(DATO DI FABB)

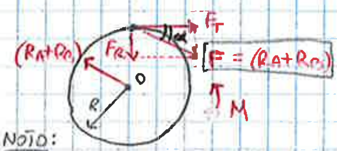
SE $\eta < 1$ $\quad \boxed{M_2 = M_1 \cdot i \cdot \eta}$

INGRANAGGIO CON RENDIM. NON UNITARIO



$F_R a = Z_B (a+b) \Rightarrow Z_B = F_R \frac{a}{(a+b)}$

NO FORZE ASSIALI

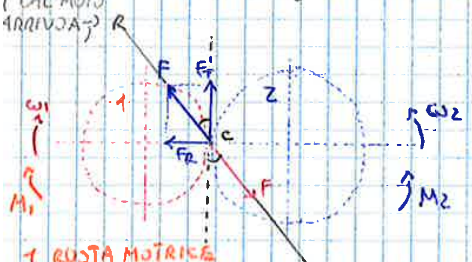


NOTO:

$M = F_T R \Rightarrow F_T = \frac{M}{R} \Rightarrow$

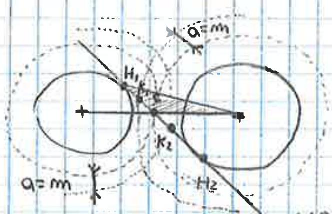
$\Rightarrow F_R = F_T \operatorname{tg}\alpha$

(DAL MOTO ARRIVA) R



1 RUOTA MOTRICE

COME FACCIO A DECIDERE QUANTI DENTI DEVONO AVERE LE RUOTE?



$\overline{H_1H_2}$ = LUOGO DEI CONTATTI (POSSIBILE)

$\overline{K_1K_2}$ = LUOGO REALE DEI CONTATTI

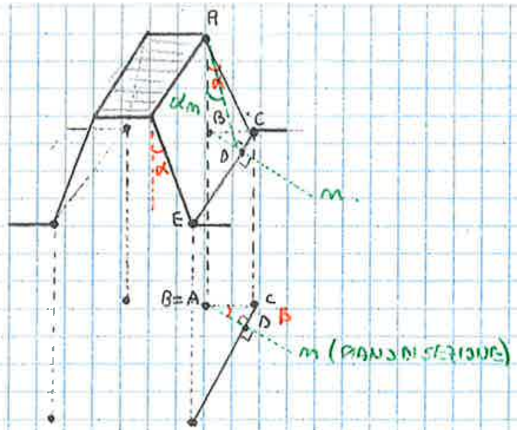
$K_1 \equiv H_1$ AL LIMITE

→ VOGLIAMO TUTTO IN FUNZ DELLA RUOTA PICCOLA

$$\begin{cases} \overline{O_2H_1} = R_2 + m \\ \overline{O_2C} = R_2 \\ \overline{H_1C} = R_1\sin\alpha \end{cases} \rightarrow \begin{cases} R_2 = iR_1 \\ m = \frac{2R_1}{z_1} \end{cases}$$

TRAMITE IL T. DI CARNOT: $H_1O_2 = \sqrt{\overline{H_1C}^2 + \overline{O_2C}^2 - 2\overline{H_1C}\,\overline{O_2C}\cos(\alpha + \pi/2)}$

$$\Rightarrow iR_1 + \frac{2R_1}{z_1} = \sqrt{R_1^2\sin^2\alpha + i^2R_1^2 + 2R_1^2 i\sin^2\alpha} \Rightarrow i + \frac{2}{z_1} = \sqrt{i^2 + \sin^2\alpha(1+2i)} \Rightarrow$$



$AB\, Tg\alpha = BC$

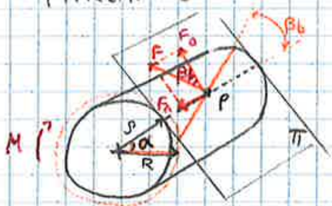
$AB\, Tg\alpha_N = BD = BC\cos\beta$

③ $\boxed{Tg\,\alpha_N = Tg\,\alpha\cos\beta}$

α ANGOLO DI P. FRONTALE

α_N ANGOLO DI P. NORMALE

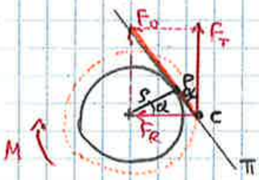
PARLANDO DI FORZE: (LEZIONE N° 43)



F SEMPRE Tg AL CILINDRO DI BASE (⇒ GIACENTE SU π)

hp:
1) UNA COPPIA DI DENTI IN PRESA
2) NO ATTRITI TRA I DENTI
3) F APPLICATA IN MEZZERIA

$\rho = R\cos\alpha$

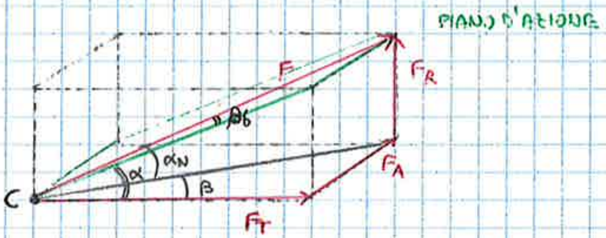


$F_0 = F\cos\beta b = \frac{M}{\rho}$

$F_A = F\sin\beta b$

$F_T = F_0\cos\alpha = F\cos\beta b\cos\alpha = \frac{M}{R}$

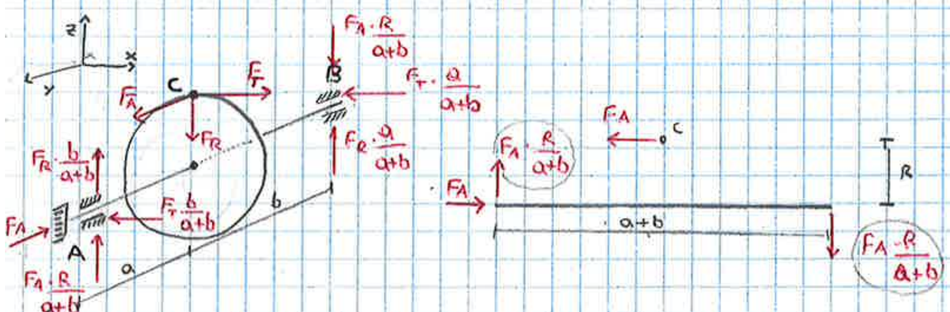
$F_R = F_0\sin\alpha = F\cos\beta b\sin\alpha$



$F_A = F\sin\beta b = F\cdot\cos\alpha_N\cdot\sin\beta$

$F_R = F\cos\beta b\cdot\sin\alpha = F\sin\alpha_N$

$F_T = F\cos\beta b\cdot\cos\alpha = F\cos\alpha_N\cos\beta$



PER EQUILIBRARE IL MOMENTO GENERATO DA $R\cdot F_A$

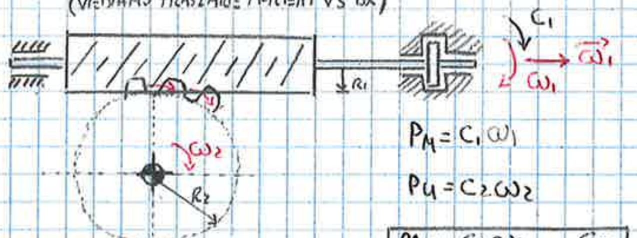
A: PUNTO PIÙ SOLLECITATO POICHÈ $F_R\frac{b}{a+b}$ E $F_A\frac{R}{a+b}$ HANNO LO STESSO VERSO ⇒ SI SOMMANO

⇒ IL CUSCINETTO IN A E' PIÙ CARICATO

$$R_A = \sqrt{\left(F_T\frac{b}{a+b}\right)^2 + \left(F_R\frac{b}{a+b} + F_A\cdot\frac{R}{a+b}\right)^2}$$

(LEZIONE N° 44)

(VEDIAMO TRASLARE I FILETTI VS DX)



$P_M = C_1 \omega_1$

$P_U = C_2 \omega_2$

$$\boxed{\eta = \frac{C_2 \omega_2}{C_1 \omega_1} = \frac{C_2}{C_1 i}}$$

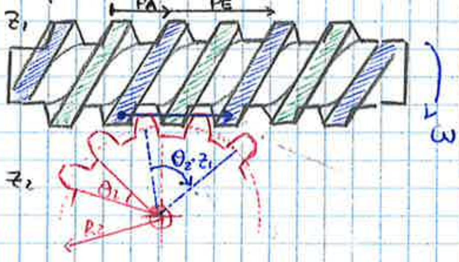
$$\boxed{C_2 = C_1 \cdot i \cdot \eta}$$

NOTA: VALE IN QUESTO CASO MA ANCHE IN GENERALE!

< 1 PER FENOMENI DI ATTRITO

$i = \frac{\omega_1}{\omega_2} = \frac{R_2}{R_1}$ NO!

(2 FILETTI = 2 PRINCIPI)



$z_1 = 2$ (FILETTI)

$P_E = z_1 P_A$

P_A = PASSO ASSIALE

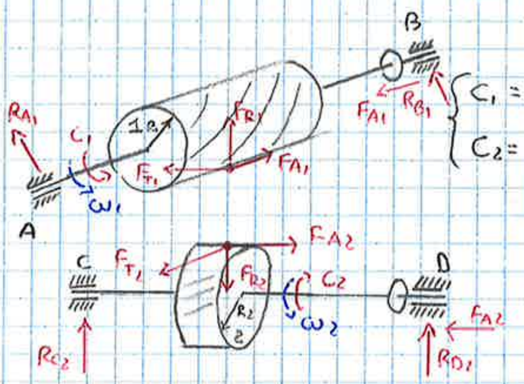
P_E = PASSO ELICOIDALE

$\theta_2 = \frac{2\pi}{z_2}$

$$\left.\begin{array}{l} 2\pi = \omega_1 \cdot \Delta t \\ \theta_2 \cdot z_1 = \omega_2 \Delta t \end{array}\right\}$$

$P = \frac{2\pi R_1}{z_1}$

$$i = \frac{\omega_1}{\omega_2} = \frac{2\pi}{\theta_2 \cdot z_1} = \frac{2\pi \cdot z_2}{z_1 \cdot 2\pi} = \frac{z_2}{z_1}$$



$$\left\{\begin{array}{l} C_1 = F_{T_1} \cdot R_1 \\ C_2 = F_{T_2} R_2 \end{array}\right.$$

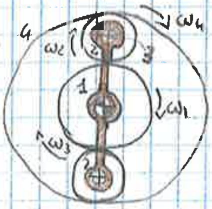
SE TRASCURO L'ATTRITO:

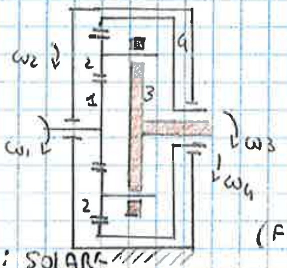
$$\left\{\begin{array}{l} F_{A2} = F_{T_1} \\ F_{R2} = F_{R_1} \\ F_{T_2} = F_{A_1} \end{array}\right.$$

ROTISMI EPICICLOIDALI =

(LEZIONE N 47)

ALCUNE R.O. HANNO L'ASSE DI ROTAZIONE MOBILE



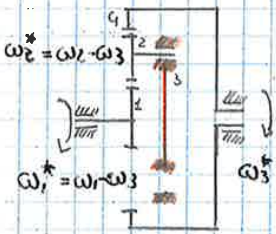


(FIG *)

1) RUOTA CENTRALE : SOLARE
2) RUOTE (2) : SATELLITI
3) ELEMENTO (3) : PORTATRENO / TRENO DI SATELLITI
4) ELEMENTO (4) : CORONA

SE BLOCCO IL PORTATRENO ⟹ ROTISMO ORDINARIO

IMMAGINIAMO DI BLOCCARE IL PORTATRENO E STUDIAMO LE VELOCITÀ ANGOLARI RELATIVE AL PORTATRENO



$\omega_2^* = \omega_2 - \omega_3$

$\omega_4^* = \omega_4 - \omega_3$

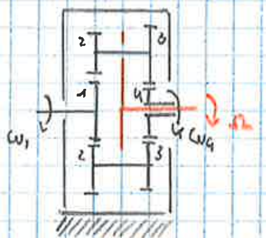
$\omega_1^* = \omega_1 - \omega_3$

$\omega_3^* = \omega_3 - \omega_3 = 0$

$$\frac{\omega_1^*}{\omega_4^*} = \frac{\omega_1}{\omega_2}\frac{\omega_2}{\omega_4} = -\frac{z_2}{z_1}\frac{z_4}{z_2} = -\frac{z_4}{z_1} = \frac{\omega_1 - \omega_3}{\omega_4 - \omega_3}$$

$$\frac{\omega_1^*}{\omega_4^*} = -\frac{z_2}{z_1}\frac{z_4}{z_2} = \boxed{-\frac{z_4}{z_1} = \frac{\omega_1 - \omega_3}{\omega_4 - \omega_3}}$$

FORMULA DI WILLIS [2 GRADI DI LIBERTÀ]



$$\frac{\omega_1^*}{\omega_4^*} = \frac{\omega_1 - \Omega}{\omega_4 - \Omega} = \frac{\omega_1^*}{\omega_2^*}\frac{\omega_2^*}{\omega_3^*}\cdot\frac{\omega_3^*}{\omega_4^*} = \left(-\frac{z_2}{z_1}\right)1\left(-\frac{z_4}{z_3}\right) = \frac{z_2 z_4}{z_1 z_3} = \frac{\omega_1 - \Omega}{\omega_4 - \Omega} = i^*$$

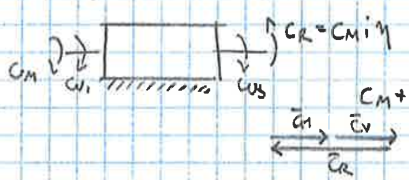
SE SALDO 4 CON IL TELAIO ($\omega_4 = 0$)

$$i^* = \frac{\omega_1 - \Omega}{-\Omega} = -\frac{\omega_1}{\Omega} + 1 \qquad \frac{\omega_1}{\Omega} = 1 - i^* \Rightarrow \frac{\Omega}{\omega_1} = \frac{1}{1 - i^*} = i_{TOT} \quad (\text{PER } i^* \to 1 \; i_{TOT} \to \infty) \; (\eta \to 0)$$

RELATIVO A (FIG *) : SE SALDO 4 CON IL TELAIO ($\omega_4 = 0$):

$$-\frac{z_4}{z_1} = \frac{\omega_1 - \omega_3}{-\omega_3} = -\frac{\omega_1}{\omega_3} + 1$$

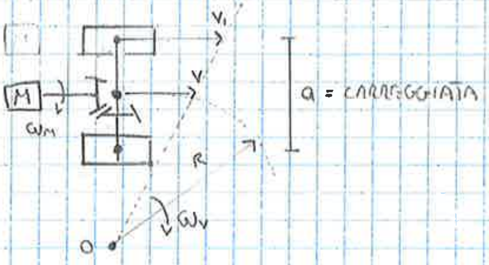
$$\frac{\omega_1}{\omega_3} = 1 + \frac{z_4}{z_1} \quad (>1) \qquad \text{RIDUTTORE DI VELOCITÀ}$$



$C_M + C_R + C_V = 0$ (A REGIME)

(LEZIONE N° 48)

DIFFERENZIALE AUTOMOBILISTICO



$V = \omega_V \cdot R$

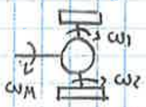
$V_1 = \omega_V \left(R + \frac{a}{2}\right)$

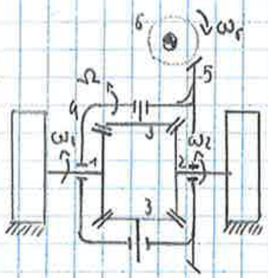
$V_2 = \omega_V \left(R - \frac{a}{2}\right)$

$\omega_1 = \frac{V_1}{R}$

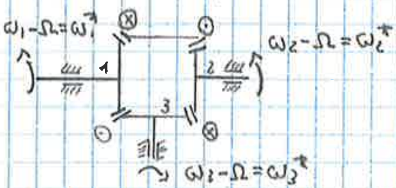
$\omega_2 = \frac{V_2}{R}$

$\boxed{\omega_1 \neq \omega_2}$





$\Omega = \omega_5$



$$\frac{\omega_1^*}{\omega_2^*} = -\frac{z_2}{z_1} = -1 = \frac{\omega_1 - \Omega}{\omega_2 - \Omega}$$

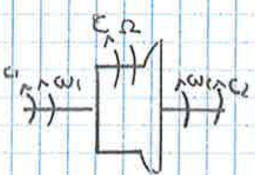
$$\omega_1 = \frac{V_1}{R} = \frac{\omega_V}{R}\left(R + \frac{a}{2}\right) = \frac{V}{RR}\left(R + \frac{a}{2}\right)$$

$$\omega_2 = \frac{\omega_V}{R}\left(R - \frac{a}{2}\right) = \frac{V}{RR}\left(R - \frac{a}{2}\right)$$

$$\omega_1 - \Omega = -\omega_2 + \Omega$$

$$\Omega = \frac{\omega_1 + \omega_2}{2}$$

$$\Omega = \frac{V}{2RR}\left(R + \cancel{\frac{a}{2}} + R - \cancel{\frac{a}{2}}\right) = \frac{V}{R}$$



$P_M = C\Omega$

$\boxed{C_1 + C_2 + C = 0}$

$C_1\omega_1 + C_2\omega_2 + C\Omega = 0$

$C = -(C_1 + C_2)$ $\quad \boxed{\Omega = \frac{\omega_1 + \omega_2}{2}}$

$$C_1\omega_1 + C_2\omega_2 - (C_1 + C_2)\frac{\omega_1 + \omega_2}{2} = 0$$

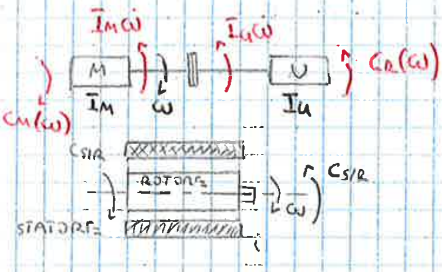
$$C_1\omega_1 + C_2\omega_2 - \frac{C_1\omega_1}{2} - \frac{C_1\omega_2}{2} - \frac{C_2\omega_1}{2} - \frac{C_2\omega_2}{2} = 0$$

$$\frac{C_1\omega_1}{2} + \frac{C_2\omega_2}{2} - \frac{C_1\omega_2}{2} - \frac{C_2\omega_1}{2} = 0$$

$$C_1(\cancel{\omega_1 - \omega_2}) - C_2(\cancel{\omega_1 - \omega_2}) = 0 \quad \Rightarrow \quad \boxed{C_1 = C_2 = -\frac{C}{2}}$$

(LEZIONE N° 50)

Ⓐ DIRETTO

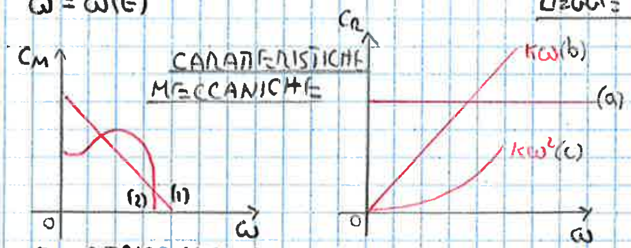


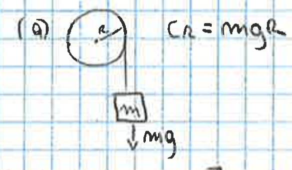
$$\boxed{C_M(\omega) - C_R(\omega) - (I_M + I_u)\frac{d\omega}{dt} = 0}$$

EQUAZIONE DEL MOTO INTEGRANDO QUESTA SI OTTIENE:

$\omega = \omega(t)$

LEGGE DEL MOTO

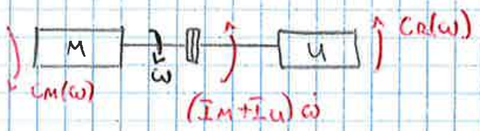




(1) MOT. ELETTRICO LINEARE (IN C.C.)

(2) MOT. ELETTRICO (IN C.A.)

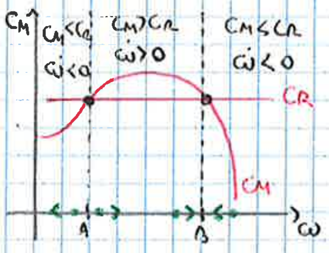
(b)-(c) [A FLUIDO]



$I = (I_M + I_u)$

$C_M - C_R - I\dfrac{d\omega}{dt} = 0$

SE $C_M = C_R \rightarrow \dot{\omega} = 0 \Rightarrow \omega = k$



Ⓐ EQUILIBRIO INSTABILE

Ⓑ EQUILIBRIO STABILE

B = PUNTO DI FUNZIONAMENTO

CONDIZIONE DI REGIME

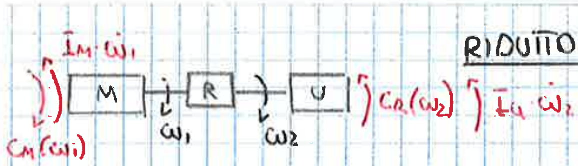
$$\boxed{\frac{d\omega}{dt} = 0}$$

PARLIAMO DI REGIME SOLO SE SIAMO IN CONDIZIONE DI EQUILIBRIO STABILE

$C_M(\omega_R) = C_R(\omega_R)$ $\quad\omega_R$ VELOCITA' DI REGIME

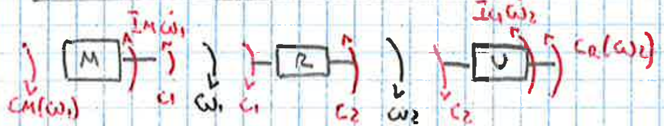
RIDUTTORE (B)

(LEZIONE N° 51)



$\eta = \dfrac{C_2 \omega_2}{C_1 \cdot \omega_1}$; $i = \dfrac{\omega_1}{\omega_2}$ CARATTERISTICHE DEL RIDUTTORE $C_2 = C_1 \cdot i \eta$

DIVIDIAMO IL SISTEMA IN 3 BLOCCHI:



$$\begin{cases} C_M - C_1 - I_M \dot{\omega}_1 = 0 \\ C_2 = C_1 \cdot i \cdot \eta \\ C_2 - C_R - I_U \dot{\omega}_2 = 0 \end{cases}$$

RIPORTIAMO TUTTO DAL PUNTO DI VISTA DEL MOTORE (IN FUNZIONE DI ω_1)

$$\begin{cases} C_M - C_1 = I_M \dot{\omega}_1 \\ C_2 = C_1 \cdot i \cdot \eta \\ C_2 = C_R + I_U \dot{\omega}_2 = C_R + \dfrac{I_U}{i} \dot{\omega}_1 = C_1 \cdot i \cdot \eta \end{cases} \qquad C_1 = \frac{C_R}{i\eta} + \frac{I_U}{i^2 \eta} \dot{\omega}_1$$

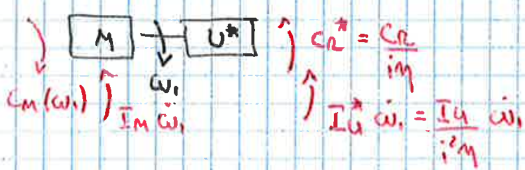
EQ. DEL MOTO

$$\Rightarrow \boxed{C_M - \frac{C_R}{i\eta} - \frac{I_U}{i^2\eta} \dot{\omega}_1 - I_M \dot{\omega}_1 = 0}$$

INTEGRANDO QUESTA $\rightarrow \omega_1(t)$ LEGGE DEL MOTO

$$C_M - \frac{C_R}{i\eta} - \left(I_M + \frac{I_U}{i^2\eta}\right)\dot{\omega}_1 = 0$$

$$\rightarrow C_M - C_R^* - (I_M + I_U^*)\dot{\omega}_1 = 0 \ , \ I_U^* = I_U / i^2\eta \ , \ C_R^* = \frac{C_R}{i\eta}$$



U^* UTILIZZATORE VIRTUALE [RIDUTTORE + UTILIZZATORE]

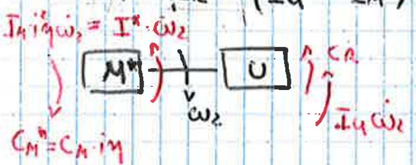
RIPORTIAMO TUTTO DAL PUNTO DI VISTA DELL'UTILIZZATORE (IN FUNZIONE DI ω_2)

$$\begin{cases} C_M - C_1 - I_M \dot{\omega}_1 = 0 \\ C_2 = C_1 \cdot i \cdot \eta \\ C_2 = C_R + I_U \dot{\omega}_2 \end{cases} \qquad \begin{aligned} C_1 &= C_M - I_M \dot{\omega}_2 \cdot i \\ C_2 &= C_M \cdot i\eta - I_M i^2 \eta \cdot \dot{\omega}_2 = C_R + I_U \dot{\omega}_2 \end{aligned}$$

EQ. DEL MOTO

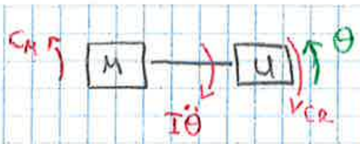
$$\Rightarrow \boxed{C_M \cdot i\eta - C_R - (I_U + I_M \cdot i^2 \eta)\dot{\omega}_2 = 0}$$

$$\rightarrow C_M^* - C_R - (I_U + I_M^*)\dot{\omega}_2 = 0 \ , \ I_M^* = I_M i^2 \eta \ , \ C_M^* = C_M \cdot i\eta$$

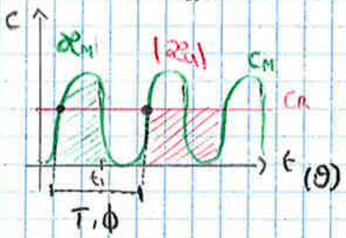


M^* MOTORE VIRTUALE [MOTORE + RIDUTTORE]

(LEZIONE N° 52)



$$C_M - C_R = I \frac{d\omega}{dt}$$



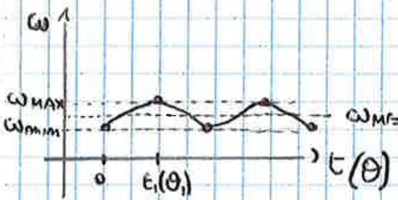
$0 < t < t_1$ $\quad C_M > C_R \quad \dot{\omega} > 0$ $\qquad t_1 < t < T \quad C_M < C_R \quad \dot{\omega} < 0$

$0 < \theta < \theta_1$

T = PERIODO (TEMPO)

Φ = PERIODO ANGOLARE

SISTEMA A REGIME PERIODICO



ω OSCILLA TRA UN VALORE MINIMO E UN VALORE MASSIMO (CHE SI RIPETONO NEL TEMPO)

$\Rightarrow \omega_{MED}$ (QUESTO VALORE SI PUÒ MANTENERE COSTANTE)

$$\omega_{MED} = \frac{1}{T}\int_0^T \omega\, dt = \frac{\Phi}{T}$$ (MEDIA INTEGRALE) VELOCITÀ MEDIA

$$\omega_{MED} \simeq \frac{\omega_{MAX} + \omega_{MIN}}{2}$$ (MEDIA ARITMETICA)

$$i = \frac{\omega_{MAX} - \omega_{MIN}}{\omega_{MED}}$$ GRADO DI IRREGOLARITÀ PERIODICA

E' IMPORTANTE MANTENERE PICCOLO QUESTO GRADO $\Rightarrow$ LA DIFF DI ω_M E ω_m $\Rightarrow$ TRAMITE IL VOLANO, GRANDE I

DAL PUNTO DI VISTA ENERGETICO, DINAMICO: LAVORO:

$$\mathcal{L}_M = \int C_M\, d\theta$$

$$\mathcal{L}_u = \int -C_R\, d\theta$$

$0 < t < T$

$$\mathcal{L}_M + \mathcal{L}_u = \int_0^T (C_M - C_R)\, d\theta = \Delta E_c \boxed{= 0}$$

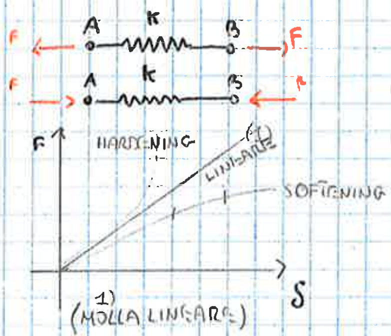
IN TERMINI DI POTENZA:

$$W_{MED} = \frac{\mathcal{L}_M}{T} = \frac{\int_0^\Phi C_M\, d\theta}{T} = \frac{|\mathcal{L}_u|}{T} = \frac{\int_0^\Phi C_R\, d\theta}{T} = C_R \frac{\Phi}{T} = C_R\, \omega_{MED}$$ POTENZA MEDIA

VIBRAZIONI IN SISTEMI MECCANICI

(LEZIONE N° 53)

PARLIAMO DI ELASTICITÀ:

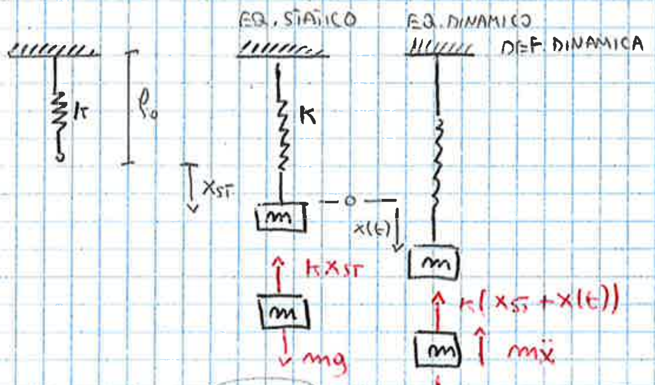


$F = k(x_B - x_A)$ ALLUNG. $\quad x_B - x_A$ SPOSTAMENTO RELATIVO TRA I DUE PUNTI

$F = k(x_A - x_B)$ ACCORC. $\quad$, k RIGIDEZZA ELASTICA

$F = k\delta$; δ : DEFORMAZIONE

NOI CONSIDEREMO SOLO SISTEMI ELASTICI LINEARI A UN GRADO DI LIBERTÀ

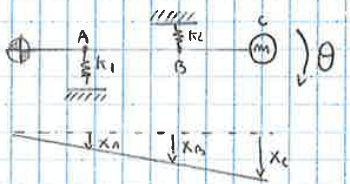


$kx_{ST} - mg = 0$

$$m\ddot{x} + kx(t) + \underbrace{kx_{ST} - mg}_{=0} = 0$$

$m\ddot{x} + kx = 0$ EQ DEL MOTO

$x(t) = \dots$ LEGGE DEL MOTO

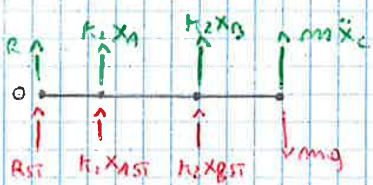


REGIME DI PICCOLE OSCILLAZIONI

$x_A = OA\,\theta \qquad \dot{x}_A = AO\,\dot{\theta} \qquad \ddot{x}_A = OA\,\ddot{\theta}$

$x_B = OB\,\theta$

$x_C = OC\,\theta$



$k_1 x_{AST}\,OA + k_2 x_{BST}\,OB - mg\,OC = 0$ STATICO

$k_1 x_A\,OA + k_2 x_B\,OB + m\ddot{x}_C\,OC = 0$ DINAMICO

SONO INDIPENDENTI → LEGGE DEL MOTO

$k_1 \cdot \theta_{ST} \cdot OA^2 + k_2\theta_{ST} OB^2 - mg\,OC = 0 \Rightarrow \theta_{ST}$

$k_1 OA^2\theta + k_2 OB^2\theta + m OC^2\ddot{\theta} = 0 \qquad\qquad m\ddot{x} + kx = 0$

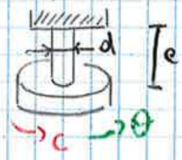
$\ddot{\theta} + \dfrac{k_1 OA^2 + k_2 OB^2}{m OC^2}\theta = 0$ IN FORMA CANONICA $\qquad \ddot{x} + \dfrac{k}{m}x = 0$

$\ddot{\theta} + \omega_m^2\theta = 0 \qquad \omega_m$ = PULSAZIONE NATURALE $\qquad \ddot{x} + \omega_m^2 x = 0$

$$\begin{cases} x = me^{\lambda t} \\ \dot{x} = m\lambda e^{\lambda t} \\ \ddot{x} = m\lambda^2 e^{\lambda t} \end{cases} \qquad m\lambda^2 e^{\lambda t} + \omega_m^2 m e^{\lambda t} = 0$$

$\lambda^2 + \omega_m^2 = 0$ EQ. CARATTER.

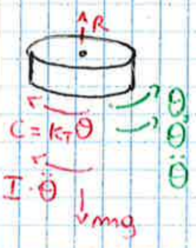
$\lambda_{1,2} = \pm i\omega_m \qquad x = ae^{\lambda_1 t} + be^{\lambda_2 t}$



NB!

$\boxed{C = \frac{G J}{\ell}\theta}$ $\qquad J = \frac{\pi}{32} d^4$

$\boxed{C = k_t \theta}$



$$I\ddot{\theta} + k_T\theta = 0$$

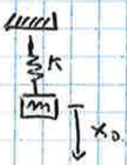
$$\ddot{\theta} + \frac{k_T}{I}\theta = 0$$

$$\ddot{\theta} + \omega_m^2\theta = 0$$

$$T = \frac{2\pi}{\omega_m}$$

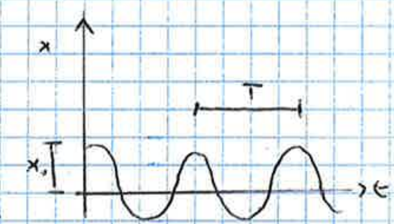
$k_{eq} = k_{1T} + k_{2T}$

MODELLO FISICO / MATEMATICO

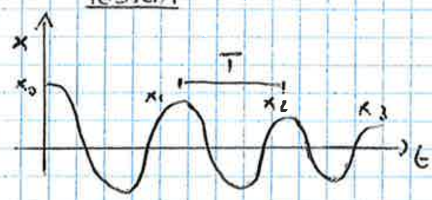


$\ddot{x} + \omega_m^2 x = 0$ EQ. DEL MOTO

$x = x_0 \sin(\omega_m t + \varphi_0)$ LEGGE DEL MOTO $\qquad T = \frac{2\pi}{\omega_m}$

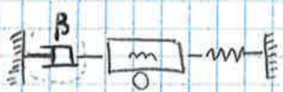


REALTÀ



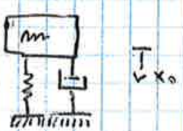
→ DEVO INTRODURRE UN ELEMENTO DISSIPATIVO: LO SMORZATORE VISCOSO (A FLUIDO)

E' PROPORZIONALE ALLA VELOCITÀ DI DEFORMAZIONE



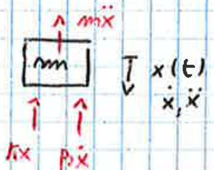
$$F = \beta(\dot{x}_B - \dot{x}_A)$$

$\boxed{K \text{ e } \beta}$



$$m\ddot{x} + \beta\dot{x} + kx = 0$$

$$m\,OC^2\ddot{\theta} + \beta OB^2\dot{\theta} + k OA^2\theta = 0$$



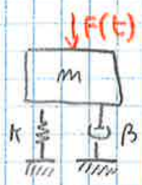
$$\begin{cases} \ddot{x} + \frac{\beta}{m}\dot{x} + \frac{k}{m}x = 0 \\ \ddot{\theta} + \frac{\beta OB^2}{m OC^2}\dot{\theta} + \frac{k OA^2}{m OC^2}\theta = 0 \end{cases}$$ FORMA CANONICA

$$\begin{cases} \ddot{x} + 2\zeta\omega_m\dot{x} + \omega_m^2 x = 0 \\ \ddot{\theta} + 2\zeta\omega_m\dot{\theta} + \omega_m^2\theta = 0 \end{cases}$$ FORMA GENERALIZZATA

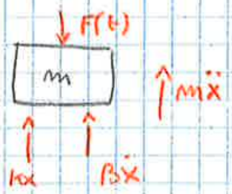
$\boxed{\zeta\,\omega_m}$ $\qquad \zeta$: FATTORE DI SMORZAMENTO

$\boxed{\frac{\beta}{m} = 2\zeta\omega_m}$ $\qquad [\zeta]$ ADIMENSIONATO (NUMERO PURO)

(LEZIONE N° 55)



$$m\ddot{x} + \beta\dot{x} + kx = F(t)$$



SERIE DI FOURIER

$$F(t) = F^* + \Sigma_m \left[a_m \cos(m\omega t) + b_m \sin(m\omega t) \right]$$

$\boxed{F(t) = F_0 \sin \Omega t}$ SUPPOSIZIONE : ARMONICA PURA (SEMPLICE SINUSOIDE)

$$m\ddot{x} + \beta\dot{x} + kx = F_0 \sin \Omega t$$

$$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m}\sin\Omega t$$

$$\boxed{\ddot{x} + 2\zeta\omega_m\dot{x} + \omega_m^2 x = A\sin\Omega t}$$

COM'È FATTO IL SIST. — COM'È SOLLECITATO

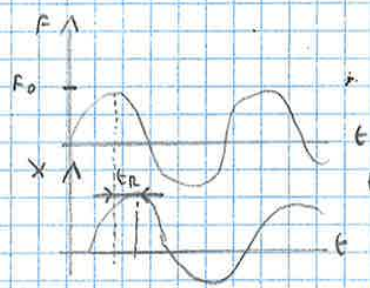
DOBBIAMO IMPARARE A RISOLVERE QUESTA : $x = x_G + x_P$ (OMOG., PART.)

OMOGENEA $(x = x_G)$

$$\ddot{x} + 2\zeta\omega_m\dot{x} + \omega_m^2 x = 0$$

$$\begin{cases} \zeta < 1 : & x_G = X^* \cdot e^{-\zeta\omega_m t}\sin(\omega_s t + \varphi_0) \\ \zeta \geq 1 : & x_G = a e^{\lambda_1 t} + b e^{\lambda_2 t} \end{cases}$$

$$\left(\lim_{t\to\infty} x_G = 0\right)$$



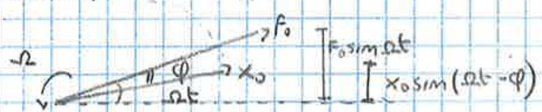
t_R TEMPO DI RISPOSTA

INTEGRALE PARTICOLARE $(x = x_P)$

$$\boxed{\begin{aligned} F &= F_0 \sin\Omega t \\ x &= x_0 \sin(\Omega t - \varphi) \end{aligned}}$$

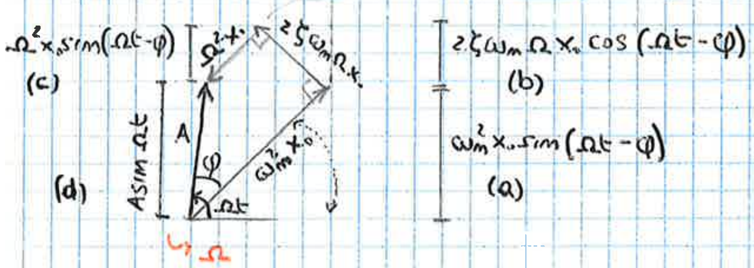
φ = SFASAMENTO IN RITARDO (INDICA CHE LA RISPOSTA (MAX) AVVIENE POCO DOPO LA FORZANTE (MAX))

$$\ddot{x} + 2\zeta\omega_m\dot{x} + \omega_m^2 x = A\sin\Omega t$$



$$\begin{cases} x = x_0 \sin(\Omega t - \varphi) \\ \dot{x} = x_0 \Omega \cos(\Omega t - \varphi) \\ \ddot{x} = -x_0\Omega^2 \sin(\Omega t - \varphi) \end{cases}$$

$$-x_0\Omega^2\sin(\Omega t - \varphi) + 2\zeta\omega_m x_0 \Omega\cos(\Omega t - \varphi) + \omega_m^2 x_0 \sin(\Omega t - \varphi) = A\sin\Omega t$$



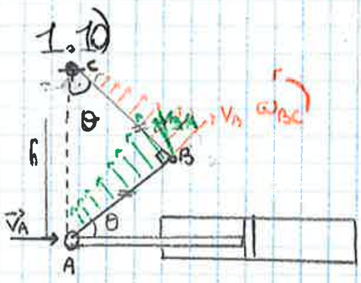
$$a + b + c - d = 0$$

ESERCITAZIONE

(PROF.SSA MATTIAZZO)

(LEZIONE N° 8)

1.10)



DATI

$V_A = 0,5\ m/s$

$a = BC = AB = 125\ mm$

$h = 175\ mm$

$\omega_{AB} = ?\quad \omega_{BC} = ?$

$h = BC\cos\theta + AB\sin\theta = a(\cos\theta + \sin\theta)$

$h^2 = a^2(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta) = a^2(1 + \sin 2\theta)$

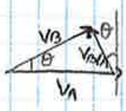
NOTA: $2\sin\theta\cos\theta = \sin(2\theta)$

$\frac{h^2}{a^2} - 1 = \sin 2\theta \qquad \theta = \frac{1}{2}\arcsin\left(\frac{h^2}{a^2} - 1\right) = 0,64\ rad = 36,9°$

1° METODO (FORM. FONDAMENTALE DELLA CINEMATICA)

$\vec{V}_B =$	$\vec{V}_A +$	$\vec{V}_{B/A}$	
	0,5		M
θ	→	θ	D/V

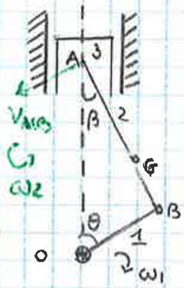
⇒



$V_{B/A} = V_A\sin\theta = \omega_{AB}a \Rightarrow \omega_{AB} = \frac{V_A}{a}\sin\theta = 2,4\ rad/s$

$V_B = V_A\cos\theta = \omega_{BC}a \Rightarrow \omega_{BC} = \frac{V_A}{a}\cos\theta = 3,2\ rad/s$

1.9)



DATI

$OB = 42,5\ mm \qquad \theta = 60°$

$AB = 107,5\ mm \qquad \omega_1 = 1500\ rpm$

$AG = 75\ mm \qquad \omega_2 = ? \qquad V_G\ (BARIC) = ?$

$a_A = ?$

ACC ANGOLARE BIELLA = ? ($\dot{\omega}$)

$\frac{AB}{\sin\theta} = \frac{OB}{\sin\beta}$

DETERMINIAMO:

$\beta = \arcsin\left(\frac{OB}{AB}\sin\theta\right) = 20,02°$

1° METODO

$\vec{V}_A =$	$\vec{V}_B +$	$\vec{V}_{A/B}$	
?	$\omega_1 OB$		M
|	θ	β	D/V

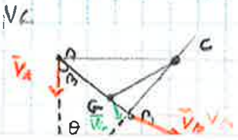
$V_B\cos\theta = V_{A/B}\cos\beta$

$V_{A/B} = \omega_1 OB\frac{\cos\theta}{\cos\beta}$ CHE È ANCHE UGUALE A:

$V_{A/B} = \omega_2 AB \Rightarrow \omega_2 = \omega_1\frac{OB}{AB}\frac{\cos\theta}{\cos\beta} = 33,05\ rad/s$

DETERMINIAMO V_G

2° METODO



$V_G = \omega_2\overline{CG}$

$OA = AB\cos\beta + OB\cos\theta$

$\Rightarrow V_G = \omega_2\overline{CG} = 6,58\ m/s$

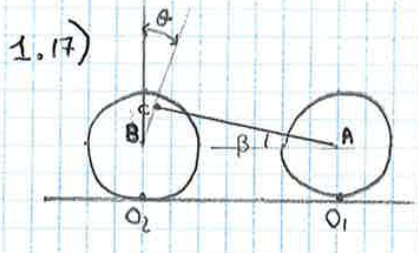
$AC = AO\,tg\theta = 211,74\ mm$

$CG = \sqrt{AC^2 + AG^2 - 2AC\,AG\sin\beta} = 198,96\ mm$

(T. DI CARNOT)

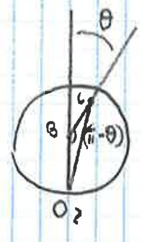
ESERCITAZIONE

1.17)



DATI

$V_A = 5\ m/s$
$AC = 800\ mm$
$R = 250\ mm$
$BC = 200\ mm$
SE $\theta = 0$ DET: ω_{AC}; V_C, V_B = ?
NO STRISCIAMENTO!



$$O_2C = \sqrt{R^2 + BC^2 - 2RBC\cos(\pi-\theta)}$$
$$O_2C = \sqrt{R^2 + BC^2 + 2RBC\cos\theta}$$

1ª RELAZIONE

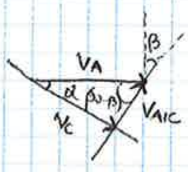
$$\frac{BC}{\sin\alpha} = \frac{O_2C}{\sin(\pi-\theta)} = \frac{O_2C}{\sin\theta} \qquad \sin\alpha = \frac{BC}{O_2C}\sin\theta$$

2ª RELAZIONE

$$\frac{BC}{\sin\beta} = \frac{AC}{\sin(\frac{\pi}{2}-\theta)} \qquad \sin\beta = \frac{BC}{AC}\cos\theta$$

3° RELAZIONE

$\vec{V_A}$ =	$\vec{V_C}$ +	$\vec{V_{A/C}}$	
5m/s	?	?	M
→	α	β	D/V



$$\begin{cases} V_C\sin\alpha = V_{A/C}\sin(90-\beta) \\ V_A = V_C\cos\alpha + V_{A/C}\cos(90-\beta) \end{cases}$$

$(\theta = 0)$

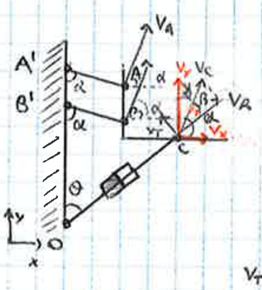
$O_2C = R + BC$

$\alpha = 0 \quad \sin\beta = \dfrac{BC}{AC}$

$$\Rightarrow \begin{cases} 0 = V_{A/C}\sin(90-\beta) \Rightarrow V_{A/C} = 0 \Rightarrow \omega_{AC} = 0 \\ V_A = V_C \end{cases}$$

1.22)

(LEZIONE N° 10)

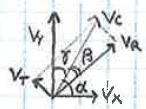


DATI

$V = 0{,}1\ m/s$
$\theta = 45°$
$\alpha = 60°$
$V_y = ?$

CON V_C VELOCITÀ ASSOLUTA
V_R VELOCITÀ RELATIVA
V_T VELOCITÀ TRAS

$$\vec{V_C} = \vec{V_R} + \vec{V_T}$$



PER COSTRUZIONE: $\gamma = 30°$; $\beta = 15°$

$$V_C = \frac{V_R}{\cos\beta}$$

$$V_y = V_C\cos\gamma = \frac{V_R\cos\gamma}{\cos\beta} = 0{,}09\ m/s$$

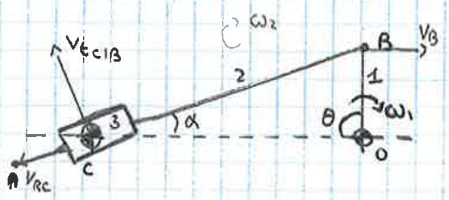
ESERCITAZIONE

1.25)

DATI

OB = 250 mm
OC = 600 mm
ω_1 = 5 rad/s

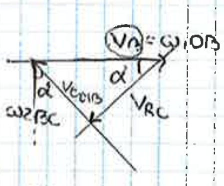
V_B = ? V_{Rc} = ? ω_2 = ?



$Tg\,\alpha = \frac{OB}{OC} \Rightarrow \alpha = 22{,}62°$

$V_C = 0 \qquad \vec{V}_C = \vec{V}_{Rc} + \vec{V}_{tc} \quad , \quad \vec{V}_{tc} = \vec{V}_{tB} + \vec{V}_{t\,C|B}$

	$\vec{V}_C =$	$\vec{V}_{Rc}$ +	$\vec{V}_{tB}$ +	$\vec{V}_{t\,C\|B}$
M	0		$\omega_1 OB$	$\omega_2 BC$
D/V	0	$\angle\alpha$	$\longrightarrow$	$\perp$



$V_{Rc} = V_B \cos\alpha = \omega_1 OB \cos\alpha = 1{,}15 \text{ m/s}$

$\omega_2 BC = V_B \sin\alpha$

$\omega_2 = 0{,}74 \text{ rad/s}$

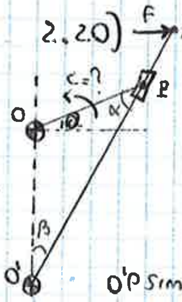
$\Sigma F_\beta = 0$

$F_p = R_c$

$p_c \frac{\pi d^2}{4} = R_c \Rightarrow p_c = \frac{R_c 4}{\pi d^2} = 2{,}24\ MPa$

ESERCITAZIONE

2.20)



DATI

$OP = 0{,}3\ m$

$O'A = 0{,}8\ m$

$O'O = 0{,}4\ m$

$\theta = 25°$

$F = 100\ N$

$C = ?$ (⇒ EQUIL.)

$R_{O} = ?$

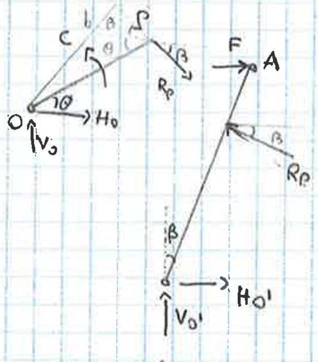
$$\left.\begin{aligned} & O'P \sin\beta = OP\cos\theta \Rightarrow O'P = OP\frac{\cos\theta}{\sin\beta} \\ & \alpha + \beta + \theta + \frac{\pi}{2} = \pi \\ & OP\sin\theta + OO' = O'P\cos\beta \quad O'P = \frac{OP\sin\theta + OO'}{\cos\beta} \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow OP\frac{\cos\theta}{\sin\beta} = \frac{OP\sin\theta + OO'}{\cos\beta}$$

$$\frac{\sin\beta}{\cos\beta} = \frac{OP\cos\theta}{OP\sin\theta + OO'} \Rightarrow \beta = 27{,}3° \Rightarrow O'P = 0{,}59\ m$$

$$\alpha = 37{,}7°$$

DIAGRAMMA DI CORPO LIBERO:



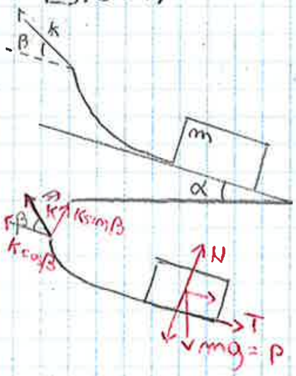
$b = OP\sin(\theta + \beta)$

$O')\ F\,O'A\cos\beta - R_P\,O'P = 0 \Rightarrow R_P = \frac{F\,O'A\cos\beta}{O'P} = \frac{100\cdot 0{,}8\cdot\cos 27{,}3}{0{,}59} = 119{,}9\ N \quad 120{,}5\ N$

$O)\ R_P b - C = 0 \Rightarrow C = R_P b = R_P\,OP\sin(\theta+\beta) = 28{,}4\ Nm$

ESERCITAZIONE
(LEZIONE N° 24)

ES. 3.17)



DATI

$30\% \Rightarrow \mathrm{Tg}\,\alpha = 0{,}3$

$m = 500\,kg \quad ; \quad f = 0{,}2$

$\beta = ? \ (\Rightarrow k = k_{min})$

$$\begin{cases} \uparrow)\ N + k\sin\beta - p\cos\alpha = 0 & \Rightarrow N = P\cos\alpha - k\sin\beta \\ \rightarrow)\ k\cos\beta - T - P\sin\alpha = 0 & \Rightarrow T = k\cos\beta - P\sin\alpha \\ T = fN & \Rightarrow k\cos\beta - P\sin\alpha = f\,P\cos\alpha - f\,k\sin\beta \end{cases}$$

$\Rightarrow k = \dfrac{P\sin\alpha + f\,P\cos\alpha}{\cos\beta + f\sin\beta}$

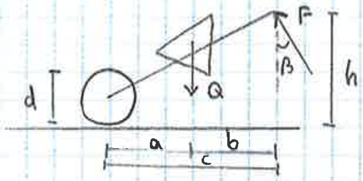
NB AFFINCHÈ $k = k_{min}$ IL DENOMINATORE DEVE ESSERE MASSIMO
$\Rightarrow$ DERIVO IL DEN. E PONGO $= 0$

$-\sin\beta + f\cos\beta = 0 \Rightarrow \mathrm{Tg}\,\beta = f \Rightarrow \beta = 11{,}31°$

PER ESSERE SICURI CHE CORRISPONDA A UN MAX E NON A UN MIN $\rightarrow$ DERIVATA SECONDA <= DEV'ESSERE NEGATIVA

$k = 2303\,N$

3.14) TRE NOTE IMPORTANTI!



DATI

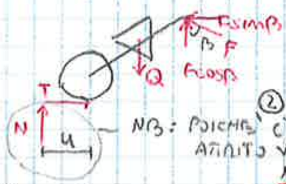
$F = ? \ (\Rightarrow V = k)$

$M = 30\,kg \ ; \ Q = Mg \ ; \ u = 10\,mm \ ; \ d_p = 30\,mm$

$f = 0{,}2 \ ; \ a = 0{,}7\,m \ ; \ b = 0{,}5\,m \ ; \ c = 1{,}2\,m \ ; \ d = 0{,}4\,m \ ; \ h = 0{,}9\,m$

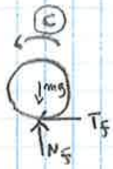
① NB: QUESTA DIFFERENZA:

DIAGRAMMA DI CORPO LIBERO:



MOTO TRAINATO
$\Rightarrow$ T_S È LA CAUSA DEL MOTO DELLA RUOTA

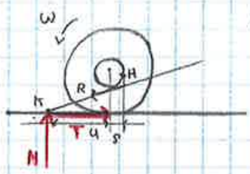
DIVERSO DA:



T_S SI OPPONE AL MOTO, ALLA COPPIA

② NB: POICHÈ C'È ATTRITO VOLVENTE
N IN AVANTI PERCHÈ OSTACOLA IL MOTO

$$\begin{cases} \uparrow)\ N - Q + F\cos\beta = 0 \\ \rightarrow)\ T = F\sin\beta \\ \overset{\curvearrowleft}{K})\ F\cos\beta\,(\rho + u) + F\sin\beta\,h - Q\,(a + u) = 0 \end{cases}$$



③ CERCHIO D'ATTRITO

$\boxed{\rho = R\sin\varphi} \ ; \ \boxed{\mathrm{Tg}\,\varphi = f} \ ; \ R = d/2$

$\overset{\curvearrowleft}{H})\ N(u + \rho) - TR = 0$

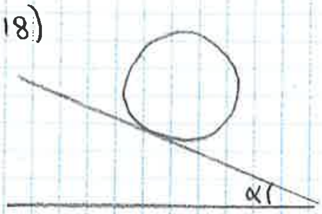
PROCEDENDO PER SOSTITUZIONE OTTENIAMO:

$\mathrm{Tg}\,\beta = \dfrac{(u+\rho)[(a+u) - (u+\rho)]}{h(u+\rho) - (a+u)R} \Rightarrow \beta = 2{,}86°$

$F = \dfrac{Q(u+\rho)}{R\sin\beta + (u+\rho)\cos\beta} = 444\,N$

ESERCITAZIONE

3.18)



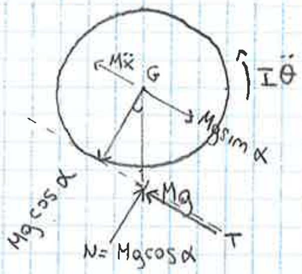
DATI

$v(t=0)=0$

$d = 1\,m$; $M = 10\,000\,kg$; $f_a = 0{,}2$; $f = 0{,}15$; $u = 2\,cm$

$t\,(x = 200\,m) = ?\begin{cases} - \alpha = 10° \\ - \alpha = 45° \end{cases}$

DIAGRAMMA DI CORPO LIBERO:



CON IL MIO METODO:

$$\begin{cases} \nearrow)\ N - Mg\cos\alpha = 0 \Rightarrow N = Mg\cos\alpha \\ \searrow)\ Mg\sin\alpha - T = Ma \\ \overset{\curvearrowright}{G})\ Nu - TR = -I\alpha \end{cases}$$

METODO PROF:

$$\begin{cases} \nearrow)\ N = Mg\cos\alpha \\ \searrow)\ Mg\sin\alpha = T + M\ddot{x} \\ \overset{\curvearrowright}{G})\ I\ddot{\theta} - TR + Nu = 0 \end{cases}$$

Hp: ROTOLA SENZA STRISCIARE: $\alpha = \frac{a}{R}$

DETERMINIAMO COSÌ: $\alpha = \frac{mg\,(R\sin\alpha - u\cos\alpha)}{I + mR^2}$ $\begin{cases} (\alpha = 10°)\ \alpha = 1{,}75\,\frac{rad}{s^2} \\ (\alpha = 45°)\ \alpha = 6{,}88\,\frac{rad}{s^2} \end{cases}$

$\ddot{\theta} = \frac{\ddot{x}}{R}$

$\ddot{\theta} = \frac{mg\,(R\sin\alpha - u\cos\alpha)}{I + mR^2}$

$N \begin{cases} \alpha = 10° & N = 96610\,N \\ \alpha = 45° & N = 69367\,N \end{cases}$ $\quad T \begin{cases} \alpha = 10° & T = 8284\,N \\ \alpha = 45° & T = 24967\,N \end{cases}$ $\quad N \langle \quad T \langle$

PERTANTO CONDIZIONE DOBBIAMO VERIFICARE:

$\frac{T}{N} \leq f_a$

PER $\alpha = 10°$ SI VERIFICA

PER $\alpha = 45°$ NON SI VERIFICA

$\Rightarrow$ PER $\alpha = 45°$ NON È VALIDA LA $\alpha = \frac{a}{R}$ (CONDIZIONE DI ROTOLAMENTO PURO) E AGGIUNGIAMO UN'ALTRA EQUAZIONE:

$T = fN$ E TROVEREMO CHE: $a = 5{,}89\ m/s^2$

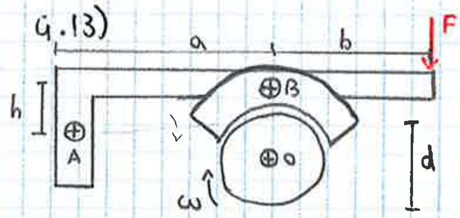
$\alpha = 3{,}05\ rad/s^2$

$x(t) = \underbrace{\cancel{x_0}}_{=0} + \underbrace{\cancel{v_0}}_{=0}t + \frac{1}{2}at^2$

$t = \sqrt{\frac{2x}{a}} = \begin{cases} \alpha = 10° & t = 21{,}3\,s \\ \alpha = 45° & t = 8{,}2\,s \end{cases}$

ESERCITAZIONE

4.13)

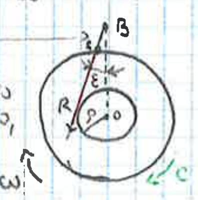


DATI

$a = 15\ cm$; $b = 30\ cm$; $h = 5\ cm$; $d = 22\ cm$

$F = 100N$; $f = 0{,}4$; $C = ?$ $(\omega = k)$

(DA QUESTA PARTE PERCHE' IL MOMENTO GENERATO SI OPPONE AL MOTO, CIOE' A ω)



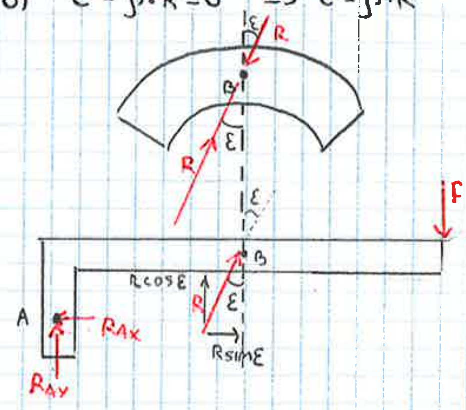
$OB = \frac{d}{2} + h$

$\rho = \frac{d}{2} \sin\varphi \qquad f = \tan\varphi \Rightarrow \varphi$

$\rho = 0{,}04\ m$

$\rho = \left(\frac{d}{2} + h\right) \sin\varepsilon \Rightarrow \varepsilon = 14{,}77°$

O) $C - \rho \cdot R = 0 \Rightarrow C = \rho \cdot R$



A) $R\cos\varepsilon \cdot a - R\sin\varepsilon \cdot h - F(a+b) = 0 \Rightarrow R = 340{,}14\ N$

$C = \rho \cdot R = 13{,}87\ Nm$

IN PIU' (PER COMPLETEZZA):

↑) $R_{Ay} + R\cos\varepsilon = F$

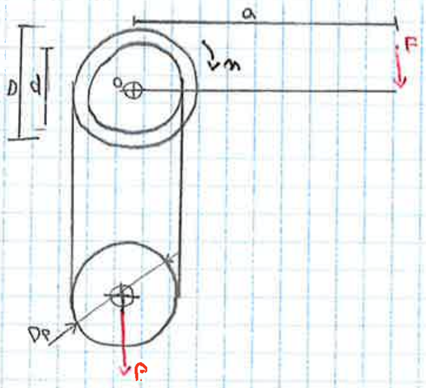
→) $R_{Ax} - R\sin\varepsilon = 0$

(FUNI)

ESERCITAZIONE

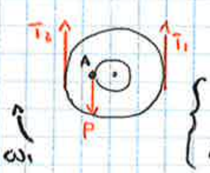
(LEZIONE N° 38)

5, 23) PARANCO DIFFERENZIALE



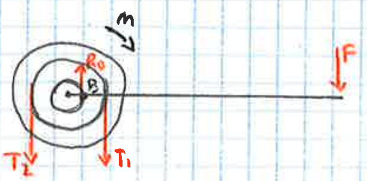
DATI

$d = 400$ mm
$D = 500$ mm
$D_p = 450$ mm
$a = 500$ mm
$R_p = 30$ mm
$f = 0,1$ SIAMO IN PRESENZA DI FENOMENI D'ATTRITO $\rightarrow \rho$
$P = 5000$N
$n = 30$ rpm
$F = ?$ $V_S = ?$
$\eta = ?$



$\rho = R_p \sin\varphi = 3$ mm $\quad (\mathrm{Tg}\,\varphi = f)$

$$\begin{cases} \uparrow) \; P = T_1 + T_2 \\ \hat{A}) \; T_1\left(\frac{D_p}{2} + \rho\right) = T_2\left(\frac{D_p}{2} - \rho\right) \end{cases} \Rightarrow \quad \begin{matrix} T_1 = 2466\,N \\ T_2 = 2533\,N \end{matrix}$$



$$\hat{B}) \; F(a - \rho) + T_1\left(\frac{d}{2} - \rho\right) - T_2\left(\frac{D}{2} + \rho\right) = 0$$

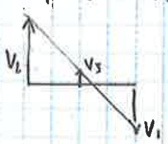
$F = 312\,N$

$V_F = \omega a$

$V_1 = \omega \frac{d}{2}$ $\quad$ CON $\omega = \frac{2\pi}{60} n$ $\quad$ VELOCITA' ANGOLARE DI INSTANTANEA ROTAZIONE

$V_2 = \omega \frac{D}{2}$

$V_F = 1,57$ m/s ; $V_1 = 0,628$ m/s ; $V_2 = 0,785$ m/s

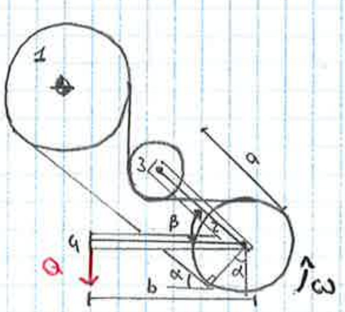


$$\boxed{V_S = \frac{V_2 - V_1}{2}} = 0,07 \text{ m/s}$$

$\eta = \frac{P_u}{P_e} = \frac{P V_S}{F V_F} = 0,8$ $\quad$ ($\neq 1$ DUE CERNIERE IN PRESENZA DI FENOM. DISSIPATIVI)

ESERCITAZIONE

5.22)



DATI

GIRI AL MINUTO

$n_2 = 360$ rpm V. ANGOLARE DI ROT DELLA PULEGGIA 2

$Q = 180$ N

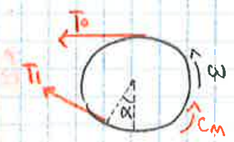
$D_2 = 300$ mm ; $a = 300$ mm ; $b = 405$ mm

$\alpha = 30°$; $\beta = 60°$; $f_a = 0,3$

$T_1 = ?$; $T_2 = ?$

$$\begin{cases} V = T_0 \\ H = T_0 \end{cases}$$

$$\circlearrowleft O) \; Qb - Va\cos\beta - Ha\sin\beta = 0 \Rightarrow T_0 = \frac{Qb}{a\cos\beta + a\sin\beta} = 178\ N$$



$\theta_{AW} = \pi + \alpha = 210°$

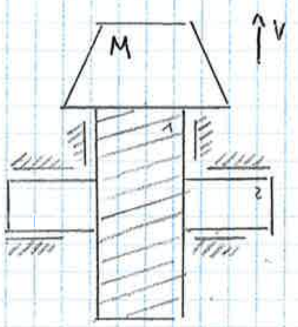
$$\boxed{\frac{T_1}{T_0} = e^{f\theta_{AW}}} \quad T_1 = T_0 e^{f\theta_{AW}} = 534\ N$$

CONDIZIONE LIMITE DI ADERENZA

$$W = (T_1 - T_0)\frac{d_2}{2}\omega_2 = 2015\ W$$

5.24)

(LEZIONE N° 39)



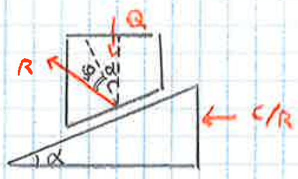
DATI

$M = 100$ kg

$d = 30$ mm ; $\alpha = 3°$; $f = 0,1$

$C = ?$ ($\dot{x} = k$)

$C' = 5$ Nm , $\ddot{x} = ?$



$Q = Mg$; $\tan\varphi = f$

$$\begin{cases} Q = R\cos(\alpha+\varphi) \\ \frac{C}{R} = R\sin(\alpha+\varphi) \end{cases} \qquad \begin{matrix} \uparrow) \; -Q + R\cos(\alpha+\varphi) = 0 \\ \rightarrow) \; -\frac{C}{R} + R\sin(\alpha+\varphi) = 0 \end{matrix}$$

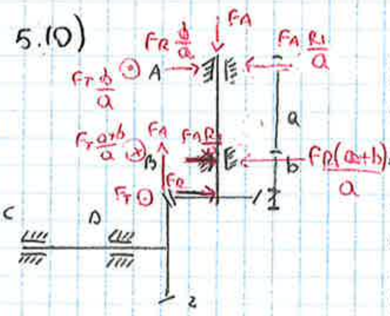
$$\frac{C}{R} = Q\tan(\alpha+\varphi) \Rightarrow C = RQ\tan(\alpha+\varphi) = 2,25\ Nm$$

$$\begin{cases} -Q + R'\cos(\alpha+\varphi) = M\ddot{x} \\ -\frac{C'}{R} + R'\sin(\alpha+\varphi) = 0 \end{cases}$$

$$\frac{C'}{R} = (Q + M\ddot{x})\tan(\alpha+\varphi) \Rightarrow \ddot{x} = 11,95\ m/s^2$$

ESERCITAZIONE

5.10)



DATI

$W = 20 kW$

$n_1 = 1500$ rpm

$i = 2$

$z_1 = 14$

$m = 5$ mm

$\alpha = 20°$

$a = 200$ mm

$b = 1000$ mm

$F_{R_A} = ?$ $F_{R_B} = ?$

$C_1 = \frac{W}{\omega_1} = 127,33 N$

$i = \frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} = \frac{z_2}{z_1} = \frac{\sin\delta_2}{\sin\delta_1}$

$z_2 = i z_1$

$m = \frac{p}{\pi} = \frac{2\pi R_1}{z_1 \pi} \Rightarrow m = \frac{2R_1}{z_1}$ $\quad R_1 = \frac{m z_1}{2} = 35$ mm

$R_2 = i R_1 = 70$ mm

$\delta_1 + \delta_2 = \frac{\pi}{2}$ $\quad \delta_2 = \frac{\pi}{2} - \delta_1$ $\quad \sin\delta_2 = \sin\left(\frac{\pi}{2} - \delta_1\right) = \cos\delta_1$

$i = \frac{\cos\delta_1}{\sin\delta_1} \Rightarrow \delta_1 = 26,6°$

$$\begin{cases} F_T = F\cos\alpha \\ F_R = F\sin\alpha\cos\delta_1 \\ F_A = F\sin\alpha\sin\delta_1 \end{cases}$$

$C_1 = F_T R_1 \Rightarrow F_T = \frac{C_1}{R_1} = 3638 N$

$F = \frac{F_T}{\cos\alpha} = 3871,5 N$

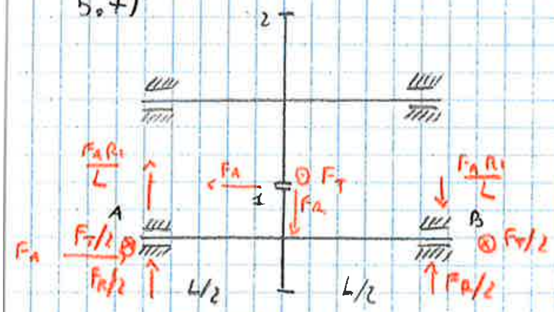
$F_R = 1184 N$

$F_A = 592,9 N$

5.7)

ESERCITAZIONE

(LEZIONE N° 46)



DATI

$i = 2$

$a = 156$ mm

$\alpha = 20°$

$m_n = 2,75$ mm

$z_1 = 37$

$L = 76$ mm

$P_e = 1$ CV

$n_1 = 720$ rpm

$\eta = 1$

$R_1, R_2 = ?$ $\beta = ?$ R (SUL CUSCINETTO PIÙ CARICATO)

↳ (ANGOLO DI INCLINAZIONE DEI DENTI)

$$\begin{cases} R_1 + R_2 = a \\ i = \frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} \end{cases} \qquad R_1 = 52 \text{ mm} \quad R_2 = 104 \text{ mm}$$

$$m_n = m \cos\beta \qquad m = \frac{m_n}{\cos\beta} = \frac{p}{\pi} = \frac{2\pi R_1}{z_1 \pi} \Rightarrow \cos\beta = \frac{m_n z_1}{2R_1} = 0,98 \Rightarrow \beta = 11,94°$$

$$P_e = C_1 \omega_1 \Rightarrow C_1 = \frac{P_e}{\omega_1} = \dots$$

$$C_1 = F_T R_1 \Rightarrow F_T = \frac{C_1}{R_1} = 187,5 \text{ N}$$

(BISOGNA SAPERE COME PASSARE α, α_n → β, β_b) →

$$\begin{cases} F_A = F \sin\beta_b = F\cos\alpha_n \sin\beta \\ F_T = F\cos\beta_b \cos\alpha = F\cos\alpha_n \cos\beta \\ F_R = F\cos\beta_b \sin\alpha = F\sin\alpha_n \end{cases}$$

$$\rightarrow \begin{cases} Tg\beta_b = Tg\beta\cos\alpha \\ Tg\alpha_n = Tg\alpha\cos\beta \end{cases}$$

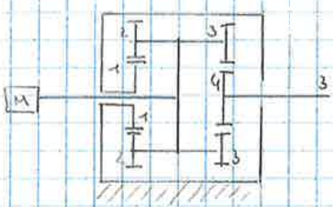
$$F = \frac{F_T}{\cos\beta\cos\alpha} = 203,94 \text{ N}$$

$F_R = 69,74$ N

$F_A = 39,65$ N

$$R_A = \sqrt{\left(\frac{F_T}{2}\right)^2 + \left(\frac{F_A R_1}{L} + \frac{F_R}{2}\right)^2} = 112,4 \text{ N}$$

5.15) ESERCITAZIONE



DATI

$z_1 = 97$ $\alpha = 20°$

$z_2 = 17$ $P = 1{,}2\ kW$

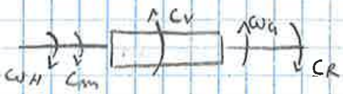
$z_3 = 18$ $n = 300\ rpm$

$m = 5\ mm$ $\omega_H = n\frac{2\pi}{60}$

$$i^* = \frac{\omega_1 - \omega_H}{\omega_4 - \omega_H} = \left(-\frac{z_2}{z_1}\right)\left(-\frac{z_4}{z_3}\right) = 0{,}93$$

$R_1 + R_2 = R_4 + R_3 \rightarrow z_4 = 96$

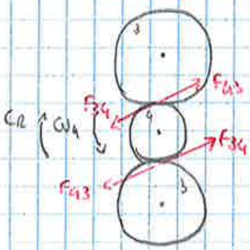
$$i^* = \frac{\cancel{\omega_1} - \omega_H}{\omega_4 - \omega_H} \quad \omega_1 = 0 \Rightarrow \frac{\omega_H}{\omega_4} = \frac{i^*}{i^* - 1} = -14{,}15$$



$$C_m = \frac{P}{\omega_H} = 38{,}2\ Nm$$

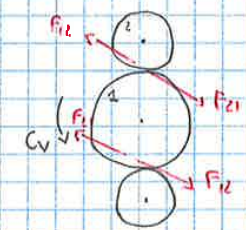
$$\eta = 1 = \frac{C_R \omega_4}{C_m \omega_H} \Rightarrow C_R = 540{,}5\ Nm$$

$C_m - C_v + C_R = 0 \qquad C_v = 585{,}17\ Nm$



$C_R = 2F_{34} R_4 \cos\alpha$

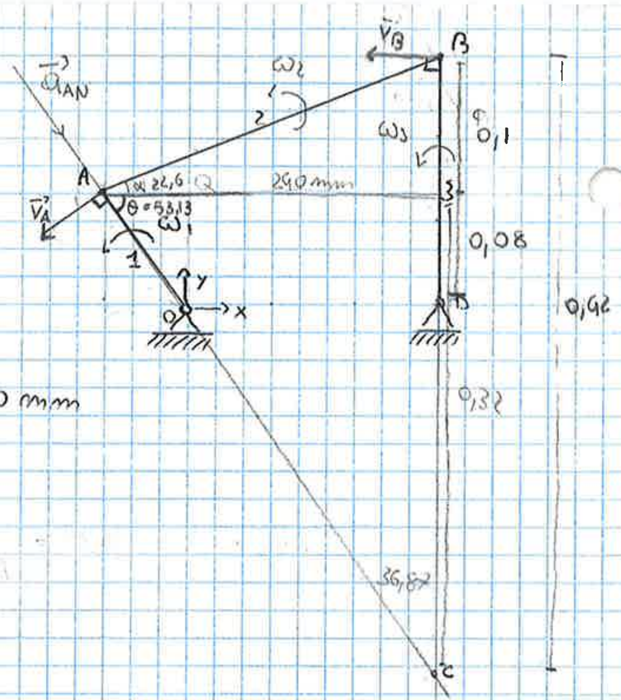
$$F_{34} = \frac{C_R}{2R_4 \cos\alpha} = 1212\ N$$



$C_v = 2F_{12} R_1 \cos\alpha$

$$F_{12} = \frac{C_v}{2R_1 \cos\alpha} = 1284\ N$$

1.2)



DATI

$\omega_1 = 10$ rad/s

$X_A = -60$ mm

$Y_A = 80$ mm

$\omega_2 = ?$ $\omega_3 = ?$ $\dot{\omega}_2 = ?$ $\dot{\omega}_3 = ?$

OA = 100 mm ; AB = 260 mm ; BD = 180 mm ; OD = 180 mm

$V_A = \omega_1 OA = 1$ m/s

$\overline{AC} = 400$ mm ; $BC = 420$ mm

$\omega_2 = \frac{V_A}{AC} = \frac{\omega_1 OA}{AC} = 2,5$ rad/s

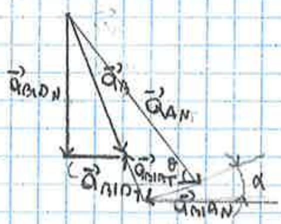
$\omega_3 = \frac{V_B}{BD}$, $V_B = \omega_2 BC$ =>

=> $\omega_3 = \frac{\omega_2 BC}{BD} = 5,83$ rad/s

$\vec{a}_B = \vec{a}_{A_0} + \vec{a}_{B|A}$, $\vec{a}_{B|A} = \vec{a}_{B|A_N} + \vec{a}_{B|A_T}$ =>

=> $\vec{a}_B = \vec{a}_{A_N} + \vec{a}_{B|A_N} + \vec{a}_{B|A_T}$

$\vec{a}_B = \vec{a}_{B|D_N} + \vec{a}_{B|D_T}$



$\vec{a}_A = \omega_1^2 \overline{OA} = 10$ m/s^2

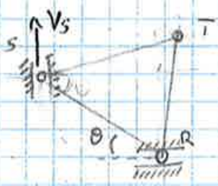
$\vec{a}_{B|A_N} = \omega_2^2 AB = 1,625$ m/s^2

$\vec{a}_{B|D_N} = \omega_3^2 BD = 6,12$ m/s^2

$\vec{a}_{B|D_T} = \dot{\omega}_3 BD$

$\vec{a}_{B|A_T} = \dot{\omega}_2 AB$

1.5)



DATI

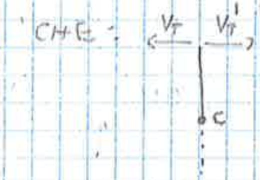
$l = 0,5\ m$ (EQUILATERO)

$V_S = 0,8\ m/s$

$\theta' = ?\ (V_{T\perp} = 0)$

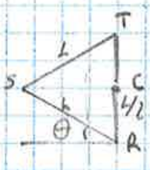
$\theta = \theta'\quad V_T = ?\quad a_R = ?$

AFFINCHÉ $V_{T\perp} = 0$ (QUINDI V_T SOLO COMPONENTE ORRIZ) C DEV'ESSERE SULLA $\perp$ IN MODO CHE:



AFFINCHÉ SI VERIFICHI TALE CONDIZIONE:

=>



DONQUE:

$\frac{L}{2} = L \sin\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

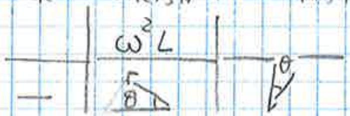
$\theta = 30°\qquad \overline{SC} = \sqrt{L^2 - \left(\frac{L}{2}\right)^2} = 0,43\ m$

$\omega = \frac{V_S}{\overline{SC}} = \frac{0,8}{0,43} = 1,86\ rad/s$

$V_T = \omega\, \overline{CT} = \omega \cdot \frac{L}{2} = 1,86 \cdot \frac{0,5}{2} = 0,46\ m/s\qquad \vec{V_T} = 0,46\ m/s$

$a_R = a_S + a_{RIS}\qquad a_S = 0\qquad a_{RIS} = a_{RIS_N} + a_{RIS_T}$

$a_R = a_{RIS_N} + a_{RIS_T}$



$a_{RIS_N} = a_R \cos\theta \Rightarrow a_R = \frac{a_{RIS_N}}{\cos\theta} = \frac{\omega^2 L}{\cos\theta} = \frac{(1,86)^2 \cdot 0,5}{\cos 30} = 1,997\ m/s^2\qquad \vec{a}_R = -1,997\ m/s^2$

1.8)

DATI

$V_A = 0,8\ m/s$

$AB = 1\ m$; $BD = 0,5\ m$; $OB = 500\ mm = 0,5\ m$

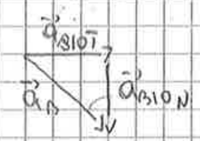
$V_D = ?$; $\omega_{OB} = ?$ $\omega_{AD} = ?$; $a_B = ?$; $\alpha_{AD} = ?$

$V_B = V_A/_A = 0,8\ m/s$ $\qquad$ $OB = AB \sin\alpha$

$\omega_{OB} = \dfrac{V_B}{OB} = \dfrac{0,8}{0,5} = 1,6\ m/s$ $\qquad$ $\alpha = \sin^{-1}\dfrac{OB}{AB} = 30°$

$V_D = 0,8\ m/s$

$\omega_{AD} = 0$ $\qquad$ $\vec{a}_B = \vec{a}_{B/O_N} + \vec{a}_{B/O_T}$



$a_{B/N} = \dfrac{V_B^2}{OB} = \dfrac{(0,8)^2}{0,5} = 1,28\ m/s^2$

$a_{B/T} = \omega\, OB = a_{B/O_N} \operatorname{tg}\alpha = 0,739\ m/s^2$

$a_B = \sqrt{a_{B_N}^2 + a_{B_T}^2} = 1,478\ m/s^2$

$\omega_{AD} = \dfrac{a_B}{AB} = 1,478\ rad/s^2$

NB

$\dfrac{GIRI}{MIN} = \dfrac{GIRI \cdot 2\pi}{60} = \dots \dfrac{rad}{sec}$

$\dfrac{GIRI}{SEC} = \dfrac{GIRI \cdot 2\pi}{SEC} = \dots \dfrac{rad}{sec}$

1.9)

DATI

$\omega_1 = 1500$ giri/minuto $\qquad 1500 \cdot 2\pi/60 \Rightarrow 157\ rad/s$

$OB = 42,5\ mm$; $AB = 107,5\ mm$; $AG = 75\ mm$; $\theta = 60°$

$\omega_2 = ?$ $V_G = ?$ $\alpha_2 = ?$ $a_A = ?$

$V_B = \omega_1 OB = 157 \cdot 0,0425 = 6,67\ m/s$

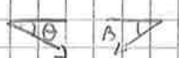
$V_G = V_B + V_{G/B}$

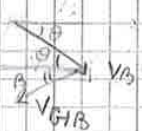
? $\quad$ 6,67

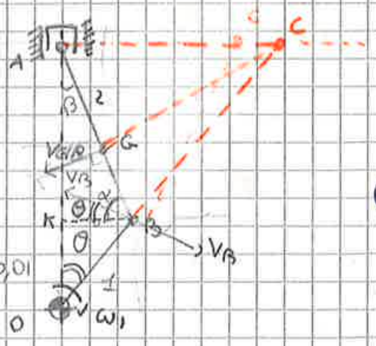
$KB = AB\cos\alpha \Rightarrow \alpha = \cos^{-1}\left(\dfrac{KB}{AB}\right) = 69,98$

$HB = OB\sin\theta = 0,0425\sin 60 = 0,0368$

$KB = AB\sin\beta \Rightarrow \beta = \sin^{-1}\left(\dfrac{KB}{AB}\right) = 20,01$

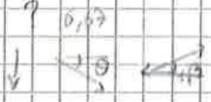


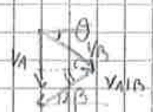




$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$

? $\quad$ 6,67





$V_B\cos\theta = V_{A/B}\cos\beta \Rightarrow V_{A/B} = \dfrac{V_B\cos\theta}{\cos\beta} = 3,55\ m/s$ $\qquad$ NOI SAPPIAMO CHE: $V_{A/B} = \omega_2 \overline{AB} \Rightarrow \omega_2 = \dfrac{V_{A/B}}{AB} =$

$3,55/0,1075 = 33,02\ rad/s$

$V_A = V_B\sin\theta + V_{A/B}\sin\beta = 6,67\sin 60 + 3,55\sin 20 = 6,99\ m/s$

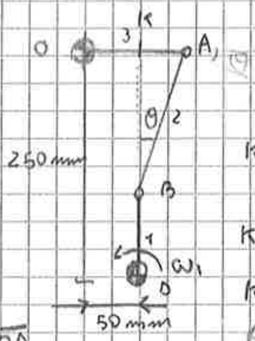
$AO = AB\cos\beta + OB\cos\theta = 0,1075 \cdot \cos 20,01 + 0,0425\cos 60 = 0,1223\ m$

$AC = AO\operatorname{tg}\theta = 0,1223 \cdot \operatorname{tg} 60 = 0,2118\ m$ $\qquad$ $CG = \sqrt{AC^2 + AG^2 - 2AC\,AG\cos(90-\beta)} = 0,1991\ m$

$V_G = \omega_2 CG = 33,02 \cdot 0,1991 = 6,57\ m/s$

1.11)

DATI



$\omega_1 = 2\ rad/s \qquad OA = 100\ mm\ ;\ BD = 75\ mm$

$\omega_2 = ?\ \ \omega_3 = ?\ \ \dot\omega_2 = ?\ \ \dot\omega_3 = ?\ \ V_A = ?\ \ a_A = ?$

$KA = \overline{OA} - 50 = 50\ mm$

$KB = 250 - 75 = 175\ mm$

$KA = KB\,tg\,\theta$

$\theta = tg^{-1}\left(\frac{KA}{KB}\right) = 15{,}94°$

$\overline{AB} = \sqrt{\overline{KA}^2 + \overline{KB}^2} = 0{,}182\ m$

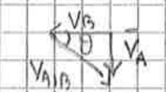
$V_B = \omega_1 \overline{BD} = 2 \cdot 0{,}075 = 0{,}15\ m/s$

$\vec V_A = \vec V_B + \vec V_{A|B}$

$\omega_3 \overline{OA} \qquad 0{,}15 \qquad \omega_2 \overline{AB}$

$\vec V_A = \vec V_O + \vec V_{A|O} \Rightarrow \vec V_A = \vec V_{A|O}$

$V_A = \omega_3 \overline{OA}$



$V_A = V_B\,tg\,\theta = 0{,}15\,tg\,15{,}94 = 0{,}0428\ m/s$

$V_B = V_{A|B}\cos\theta$

$\omega_3 = \frac{V_A}{\overline{OA}} = \frac{0{,}0428}{0{,}1} = 0{,}428\ rad/s$

$V_{A|B} = \frac{V_B}{\cos\theta} = \frac{0{,}15}{\cos 15{,}94} = 0{,}156\ m/s$

$\omega_2 = \frac{V_{A|B}}{\overline{AB}} = \frac{0{,}156}{0{,}182} = 0{,}857\ rad/s$

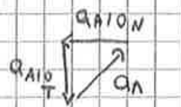
$\vec a_A = \vec a_B + \vec a_{A|B}$

$\vec a_A = \vec a_B + \vec a_{A|B_N} + \vec a_{A|B_T}$

$? \qquad \omega_1^2 \overline{BD} \qquad \omega_2^2 \overline{AB} \qquad \dot\omega_2 \overline{AB}$

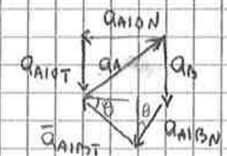
$\vec a_A = \vec a_{A|O_N} + \vec a_{A|O_T}$

$? \qquad \omega_3^2 \overline{OA} \qquad \dot\omega_3 \overline{OA}$



$a_{A|O_N} = a_{A|B_T}\cos\theta + a_{A|B_N}\sin\theta$

$\omega_3^2 \overline{OA} = \dot\omega_2 \overline{AB}\cos\theta + \omega_2^2 \overline{AB}\sin\theta$



AVENDO SCELTO COME VERSO DI $a_{A|B_T}$ ⇒ $\dot\omega_2$ È ORARIA

QUINDI

$\dot\omega_2 = \frac{\omega_3^2 \overline{OA} - \omega_2^2 \overline{AB}\sin\theta}{\overline{AB}\cos\theta} = \frac{0{,}428^2 \cdot 0{,}1 - 0{,}857^2 \cdot 0{,}182 \cdot \sin 15{,}94}{0{,}182\cos 15{,}94} = -0{,}105\ rad/s^2$

$a_{A|O_T} + a_{A|B_T}\sin\theta = a_B + a_{A|B_N}\cos\theta$

$\dot\omega_3 \overline{OA} + \dot\omega_2 \overline{AB}\sin\theta = \omega_1^2 \overline{BD} + \omega_2^2 \overline{AB}\cos\theta$

$\dot\omega_3 = \frac{\omega_1^2 \overline{BD} + \omega_2^2 \overline{AB}\cos\theta - \dot\omega_2 \overline{AB}\sin\theta}{\overline{OA}} = \frac{2^2 \cdot 0{,}075 + 0{,}857^2 \cdot 0{,}182 \cdot \cos(15{,}94) - (-0{,}105) \cdot 0{,}182 \cdot \sin(15{,}94)}{0{,}1}$

$= 4{,}34\ rad/s^2$ (ORARIA)

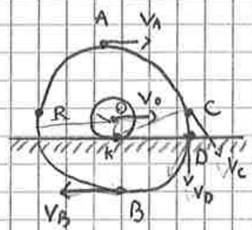
$a_A = \sqrt{a_{A|O_N}^2 + a_{A|O_T}^2} = \sqrt{(\omega_3^2 \overline{OA})^2 + (\dot\omega_3 \overline{OA})^2} = \sqrt{0{,}428^4 (0{,}1)^2 + (4{,}34)^2 (0{,}1)^2} = 0{,}43\ m/s^2$

1.13)

DATI

$r = 50\ mm$; $R = 250\ mm$; $\vec{V}_O = 0{,}8\ \vec{i}\ m/s$ $\quad \vec{a}_O = -1{,}4\ \vec{i}\ m/s^2$

$V_A, V_B, V_C, V_D = ?$ $\quad a_A, a_D = ?$



$\vec{j}$, $\vec{i}$

$\vec{V}_A = \vec{V}_O + \vec{V}_{A|O} = 0{,}8 + 16 \cdot 0{,}25 = 4{,}8\ m/s$

? $\quad 0{,}8 \quad \omega R$

$\omega = \dfrac{V_0}{r} = \dfrac{0{,}8}{0{,}05} = 16\ rad/s$

$KD = \sqrt{R^2 - r^2} = 0{,}2449\ m$

NB $KD \neq R$!!

$V_A = \omega(R+r) = 16 \cdot 0{,}3 = 4{,}8\ m/s$ $\qquad \vec{V}_A = 4{,}8\ \vec{i}\ m/s$

$V_B = \omega(R-r) = 16 \cdot 0{,}2 = 3{,}2\ m/s$ $\qquad \vec{V}_B = -3{,}2\ \vec{i}\ m/s$

$\vec{V}_B = \vec{V}_O + \vec{V}_{B|O} = 0{,}8 - 16 \cdot 0{,}25 = -3{,}2\ m/s$

$0{,}8 \quad \omega R$

NB

$V_C = \omega\sqrt{R^2 + r^2} = 4{,}08\ m/s$ $\quad \omega \cdot$ BRACCIO $\quad$ NO SCOMPOSIZIONE V !

$\vec{V}_C = \vec{V}_O + \vec{V}_{C|O} \Rightarrow V_C = \sqrt{V_O^2 + V_{C|O}^2} = \sqrt{0{,}8^2 + (16 \cdot 0{,}25)^2} = 4{,}08\ m/s$

$0{,}8 \quad \omega R$

$\vec{V}_C = 0{,}8\ \vec{i} - 4\ \vec{j}\ m/s$

$V_D = \omega R = 16 \times 0{,}25 = 4\ m/s$ $\qquad \vec{V}_D = -4\ \vec{j}\ m/s$

$\vec{V}_D = \vec{V}_O + \vec{V}_{D|O}$

$0{,}8 \quad \omega\sqrt{R^2 + r^2}$

$a_{T_{O_K}} : r = a_{T_{A_K}} : (R+r)$ $\qquad a_{TA_K} = a_O (R+r)/r = -1{,}4 \cdot 0{,}3 / 0{,}05 = -8{,}4\ m/s^2$

$a_{TA_K} = -8{,}4\ \vec{i}\ m/s^2$

(1) $\vec{a}_A = \vec{a}_O + \vec{a}_{A|O}$

RISP. AO. $\vec{a}_A = \vec{a}_O + \vec{a}_{A|O_N} + \vec{a}_{A|O_T}$ $\qquad \vec{a}_A = -8{,}4\ \vec{i} - 64\ \vec{j}\ m/s^2$

$-1{,}4 \quad \omega^2 R \quad \dot{\omega} R$

NB $\quad \vec{a}_K = \vec{a}_{K|O_N} = \omega^2 r\ \vec{j}$ (*)

(2) $\vec{a}_A = \vec{a}_K + \vec{a}_{A|K}$, $\cancel{\vec{a}_K = 0}$ $\qquad |\dot{\omega}| = \dfrac{a_O}{r} = \dfrac{1{,}4}{0{,}05} = 28\ rad/s^2$

RISP. AK $\quad \vec{a}_A = \vec{a}_{A|K_N} + \vec{a}_{A|K_T} + a_K$

$\omega^2(R+r) \quad \dot{\omega}(R+r) \quad \omega^2 r$ $\qquad a_A,\ a_{A|K_N},\ a_{A|K_T}$

$\Rightarrow \vec{a}_A = [\omega^2(R+r) - \omega^2 r]\ \vec{j} - 8{,}4\ \vec{i}\ m/s^2$

$\Rightarrow$ QUINDI COERENTE CON METODO (1)

$a_A = \sqrt{16^2 \cdot 0{,}3^2 + (-28 \cdot 0{,}3)^2} = \cancel{9{,}67}\ m/s^2$ $\qquad \vec{a}_A = -8{,}4\ \vec{i} - \cancel{76{,}8}\ \vec{j}\ m/s^2$

(*) $\vec{a}_K = \vec{a}_O + \vec{a}_{K|O_N} + \vec{a}_{K|O_T}$

$1{,}4 \quad \omega^2 r \quad \dot{\omega} r = 1{,}4$ $\qquad \vec{a}_K = 0\ \vec{i} + 12{,}8\ \vec{j}\ m/s^2$ $\qquad \Rightarrow \vec{a}_K = \vec{a}_{K|O_N}$

NON R! $\quad$ NON R!

$\vec{a}_D = \vec{a}_K + \vec{a}_{D|K_N} + \vec{a}_{D|K_T}$ $\qquad \vec{a}_D = \omega^2 KD\ \vec{i} + (12{,}8 + \dot{\omega} KD)\ \vec{j} = -62{,}7\ \vec{i} + 19{,}66\ \vec{j}$

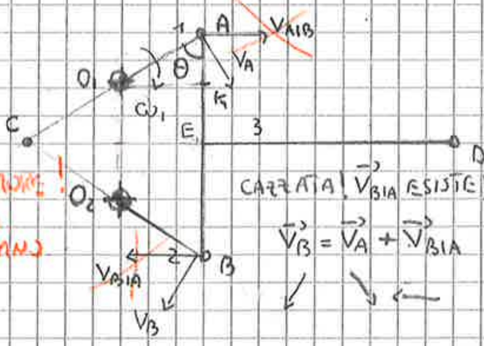
1.16)

DATI

$\overline{O_1A} = \overline{O_2B} = 0{,}2\ m$; $\overline{AE} = \overline{EB}$; $\overline{AB} = 0{,}5\ m$; $\overline{ED} = 0{,}5\ m$; $\overline{O_1O_2} = 0{,}3\ m$

$\omega_1 = 10\ rad/s$

$\omega_2 = ?$ $\omega_3 = ?$ $V_D = ?$



$V_A = \omega_1 \overline{O_1A} = 10 \cdot 0{,}2 = 2\ m/s$

$\vec{V}_B = \vec{V}_A + \cancel{\vec{V}_{B/A}}$

SECONDA VOLTA CHE FACCIO QUESTO ERRORE! B NON RUOTA INTORNO A A MA SOLO INTORNO A O_2!

CAZZATA! $\vec{V}_{B/A}$ ESISTE! GUARDA I VETTORI!

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

V_B È UGUALE A V_A PER QUESTO $\vec{V}_B \neq \vec{V}_A$!

(SECONDA VOLTA CHE DICO QUESTA CAZZATA!)

$V_B = V_A = 2\ m/s$

$\overline{AE} = 0{,}25\ m$; $\dfrac{\overline{O_1O_2}}{2} = 0{,}15$

$\omega_2 = \dfrac{V_B}{\overline{O_2B}} = 10\ rad/s$

$\overline{AK} = \overline{AE} - \dfrac{\overline{O_1O_2}}{2} = 0{,}10\ m$ $\quad \overline{AK} = \overline{O_1A}\cos\theta \Rightarrow$

① $\omega_3 = \dfrac{V_A}{\overline{CA}} = \dfrac{2}{0{,}5} = 4\ rad/s$ $\quad \theta = \cos^{-1}\left(\dfrac{\overline{AK}}{\overline{O_1A}}\right) = 60°$

$\overline{CA} = \dfrac{\overline{AE}}{\cos\theta} = 0{,}5\ m$

② $\vec{V}_E = \vec{V}_A + \vec{V}_{E/A}$
? 2 $\omega_3 \cdot \overline{AE}$

$\vec{V}_E = \vec{V}_B + \vec{V}_{E/B}$
? 2 $\omega_3 \cdot \overline{BE}$

$V_{E/A} = V_A \sin 30 = 1\ m/s$

$\omega_3 = \dfrac{V_{E/A}}{\overline{AE}} = \dfrac{1}{0{,}25} = 4\ rad/s$

$V_E = \dfrac{V_{E/A}}{Tg\,30} = \dfrac{1}{Tg\,30} = 1{,}732\ m/s$

$\vec{V}_D = \vec{V}_E + \vec{V}_{D/E}$
1,732 $\omega_3 \overline{ED}$

$V_D = 1{,}732 + (4 \cdot 0{,}5) = 3{,}73\ m/s$ $\quad \vec{V}_D = -3{,}73\,\vec{j}\ m/s$

1.20)

DATI

$R = 0,25\ m$

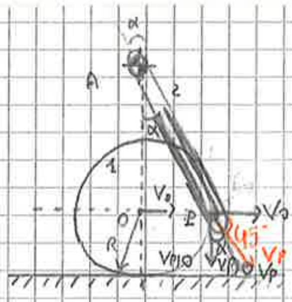
$V_0 = 2,5\ m/s$

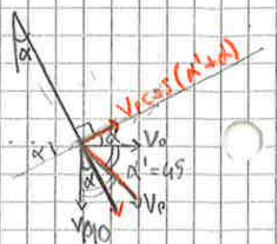
$\alpha = 30^\circ$

$\omega_1 = ?\quad V_P = ?\quad \omega_2 = ?$

$R = AP \sin\alpha$

$AP = \frac{R}{\sin\alpha} = \frac{0,25}{\sin 30} = 0,5\ m$





NB: $\boxed{\omega_1 = \frac{V_0}{R}} = \frac{2,5}{0,25} = 10\ rad/s$

NB: $\boxed{\vec{V}_P = \vec{V}_0 + \vec{V}_{P10}}$

$\beta \quad 2,5 \quad \omega_1 R$

NO AVENDO $\vec{V}_0$ E $\vec{V}_{P10}$ STESSO MODULO:

$\alpha' = 45^\circ$!!!

$V_P = \sqrt{2,5^2 + 2,5^2} = 3,53\ m/s$

$\omega_2 = \frac{V_P \cos(\alpha'+\alpha)}{AP} = \frac{3,53 \cos(45+30)}{0,5} = \frac{0,914}{0,5} = 1,83\ rad/s$

1.21)

DATI

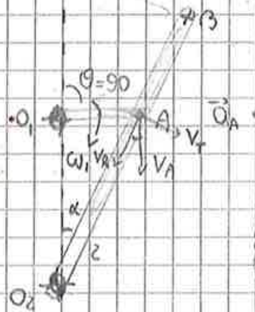
$\omega_1 = 1\ rad/s$

$\overline{O_1A} = 0,1\ m \qquad \overline{O_2B} = 0,4\ m$

$\overline{O_1O_2} = 0,2\ m$

$\omega_2 = ?\quad V_B = ?\quad \dot{\omega}_2 = ?\quad (\theta = 90)$

$\theta = ?\ (\omega_2 = 0)$



$V_A = \omega_1 \overline{O_1A} = 0,1\ m/s$

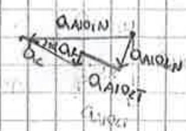
$\overline{O_1A} = \overline{O_1O_2}\ tg\,\alpha \qquad \alpha = \operatorname{cotg}\left(\frac{\overline{O_1A}}{\overline{O_1O_2}}\right) = 26,56 \qquad \overline{O_2A} = \sqrt{\overline{O_1A}^2 + \overline{O_1O_2}^2} = 0,2236\ m$

$V_{A_T} = V_A \sin\alpha = 0,1 \cdot \sin 26,56 = 0,0447\ m/s \qquad V_{R_A} = V_A \cos\alpha = 0,0894\ m/s$

$\omega_2 = \frac{V_{A_T}}{\overline{O_2A}} = 0,2\ rad/s$

$V_B = \omega_2 \cdot \overline{O_2B} = 0,2 \cdot 0,4 = 0,08\ m/s$

$\boxed{\vec{a}_{C_A} = 2\vec{\omega} \wedge \vec{V}_R}$ ACC. DI CORIOLIS



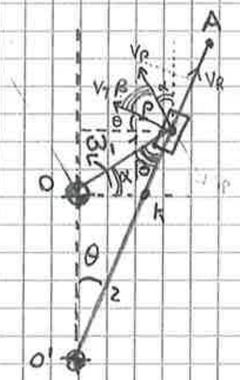
$\vec{a}_{A10_1} = \underset{\omega_1^2 \overline{O_1A}}{\vec{a}_{A10_1N}} + \underset{=0}{\vec{a}_{A10_1T}} \qquad \vec{a}_{A10_2} = \underset{\omega_2^2 \overline{O_2A}}{\vec{a}_{A10_2N}} + \underset{\dot{\omega}_2 \overline{O_2A}}{\vec{a}_{A10_2T}} + \underset{\text{INCOGNITA}}{\vec{a}_{A10_2R}} + \underset{2\omega_2 V_R}{\vec{a}_{A10_2C}}$

$\vec{a}_{A10_1} = \omega_1^2 \overline{O_1A} = 0,1\ m/s^2 \qquad a_{A10_1N} \cos\alpha = a_C + a_{A10_2T} \Rightarrow a_{A10_2T} = a_{A10_1N} \cos\alpha - a_C \Rightarrow$

$\Rightarrow \dot{\omega}_2 \overline{O_2A} = \omega_1^2 \overline{O_1A} \cos\alpha - 2\omega_R V_R \Rightarrow \dot{\omega}_2 = \frac{0,1^2 \cos(26,56) - 2 \cdot 0,2 \cdot 0,0894}{0,2236} = 0,24\ rad/s^2$

1.24)

DATI



$\omega_1 = 100$ rad/s ; $\theta = 25°$

$\overline{OP} = 0{,}3$ m ; $\overline{O'A} = 0{,}8$ m ; $\overline{O'O} = 0{,}4$ m

$\omega_2 = ?$ $\dot\omega_2 = ?$ $V_A = ?$ $a_A = ?$

$V_P = \omega_1 \overline{OP} = 100 \cdot 0{,}3 = 30$ m/s

$\overline{O'P} \sin\theta = \overline{OP} \sin(90-\alpha)$

$\overline{O'P} = \sqrt{\overline{O'O}^2 + \overline{OP}^2 - 2\overline{O'O}\,\overline{OP}\cos(90+\alpha)}$

$(\overline{O'O}^2 + \overline{OP}^2 - 2\overline{OO'}\,\overline{OP}\sin\alpha)\sin^2\theta = \overline{OP}^2\cos^2\alpha$

$\overline{OP}^2\cos^2\alpha + 2\overline{OO'}\,\overline{OP}\sin\alpha\sin^2\theta = \overline{OO'}^2\sin^2\theta + \overline{OP}^2\sin^2\theta$

$-\overline{OP}^2\sin^2\alpha + 2\overline{OO'}\,\overline{OP}\sin\alpha\sin^2\theta = -\overline{OP}^2 + \overline{OO'}^2\sin^2\theta + \overline{OP}^2\sin^2\theta$

$0{,}3^2\sin^2\alpha - 2\cdot 0{,}4\cdot 0{,}3\cdot 0{,}1786\sin\alpha = +0{,}3^2 - 0{,}4^2\cdot 0{,}1786 - 0{,}3^2\cdot 0{,}1786$

$0{,}09\sin^2\alpha - 0{,}0429\sin\alpha = 0{,}0454$

$\sin^2\alpha - 0{,}4767\sin\alpha - 0{,}5039 = 0$

$\alpha = \dfrac{+0{,}4767 \pm \sqrt{0{,}4767^2 - 4\cdot 0{,}5039}}{2}$

$\overline{OK} = \overline{OO'}\,\mathrm{tg}\,\theta = 0{,}4\cdot \mathrm{tg}\,25 = 0{,}1865$ m

$\overline{O'K} = \sqrt{\overline{OO'}^2 + \overline{OK}^2} = 0{,}4414$ m

$\overline{KP} =$ STESSA STORIA

$$\boxed{\frac{\overline{OP}}{\sin\theta} = \frac{\overline{O'P}}{\sin(90+\alpha)} = \frac{\overline{OO'}}{\sin\beta}}$$

POICHÉ $\boxed{\overline{OP} : \sin\theta = \overline{O'P} : \sin(90+\alpha) = \overline{OO'} : \sin\beta}$

$\beta = \sin^{-1}\left(\dfrac{\overline{OO'}}{\overline{OP}}\sin\theta\right) = 34{,}3°$

$90 + \alpha + \beta + \theta = 180° \Rightarrow \alpha = 30{,}7°$

$\overline{O'P} = \dfrac{\overline{OP}\sin(90+\alpha)}{\sin\theta} = 0{,}61$ m

$V_T = V_P\cos\beta = 30\cdot\cos 34{,}3 = 24{,}78$ m/s

$V_R = V_P\sin\beta = 30\sin 34{,}3 = 16{,}9$ m/s

$\omega_2 = \dfrac{V_T}{\overline{O'P}} = \dfrac{24{,}78}{0{,}61} = 40{,}63$ rad/s

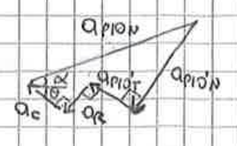
$\vec{a}_c = 2\vec\omega_2 \wedge \vec V_R$

$V_A = \omega_2\cdot\overline{O'A} = 40{,}63\cdot 0{,}8 = 32{,}5$ m/s

$\vec a_{P1O} = \vec a_{P1O_N} = \omega_1^2\cdot\overline{OP} = 3000$ m/s²

$$\underset{a_{t_N}}{} \quad \underset{a_{t_T}}{}$$

$\vec a_{P1O'} = \underbrace{\vec a_{P1O_N}}_{\omega_2^2\overline{O'P}} + \underbrace{\vec a_{P1O_T}}_{\dot\omega_2\overline{O'P}} + \underbrace{\vec a_R}_{?} + \underbrace{\vec a_c}_{2\omega_2 V_R}$



$a_{P1O_N}\cos(\alpha+\theta) = a_c + a_{P1O_T} \Rightarrow a_{P1O_T} = a_{P1O_N}\cos(\alpha+\theta) - a_c \Rightarrow \dot\omega_2\overline{O'P} = \omega_1^2\cdot\overline{OP}\cos(\alpha+\theta) - 2\omega_2 V_R$

NB!

$\Rightarrow \dot\omega_2 = 519$ rad/s²

$a_{A_T} = \dot\omega_2\cdot\overline{O'A} = 519\cdot 0{,}8 = 415{,}2$ m/s²

$a_{A_N} = \omega_2^2\,\overline{O'A} = 40{,}63^2\cdot 0{,}8 = 1320{,}64$ m/s²

$a_A = \sqrt{a_T^2 + a_N^2} = 1384$ m/s²

[CAPITOLO 2]

2.1)

DATI

$T_B = 9 daN$ $daN = 10N$

$R_A = 37 daN$

$4T_B - T_A = 0$ $T_A = 4T_B$

$P = T_A + T_B + R_A = 4T_B + T_B + R_A = 82 daN$

2.2)

$(T_A = 2T_B)$ $a_t = ?$

$T_A = 2T_B$

$-ma - T_B \cos 60 + T_A \cos 60 = 0$ NO

$-ma - T_B \cos 60 + 2T_B \cos 60 = 0$

$-ma + T_B \cos 60 = 0$

$a = \frac{T_B \cos 60}{m}$

↑) $T_A \sin 60 + T_B \sin 60 - mg = 0$

$(2T_B + T_B) \sin 60 - mg = 0$

$3T_B \sin 60 = mg$ $T_B = \frac{mg}{3 \sin 60}$

→) $-ma - T_B \cos 60 + T_A \cos 60 = 0$

$ma = T_B \cos 60 \Rightarrow a = \frac{T_B \cos 60}{m} = \frac{g \cos 60}{3 \sin 60} = \frac{9,8 \cdot \cos 60}{3 \cdot \sin 60} = 1,89\ m/s^2$

2.3)

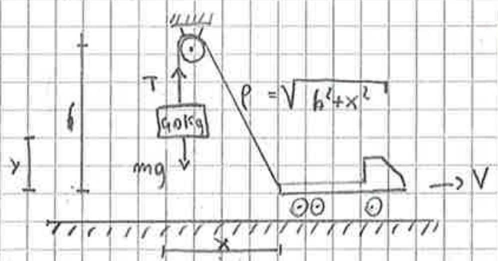
DATI

$V = 5\ m/s$

$m = 40\ kg$

$x = 12\ m$; $h = 16\ m$

$T = ?$



$+T - mg = ma$

$T = m(g + a)$

$L = h - y + \ell = h - y + \sqrt{h^2 + x^2}$

$y = \sqrt{f(x)}$: $y' = \frac{f'(x)}{2\sqrt{f(x)}}$

NB: FUNE INESTENSIBILE = $\frac{dL}{dt} = 0$

$\frac{dL}{dt} = -\dot{y} + (h^2 + x^2)^{-1/2} x \cdot \dot{x}$

$\dot{y} = (h^2 + x^2)^{-1/2} x \cdot \dot{x}$

DERIVANDO ANCORA:

$\ddot{y} = \dot{x}\dot{x}(h^2 + x^2)^{-1/2} + x\ddot{x}(h^2 + x^2)^{-1/2} - x\dot{x}(h^2 + x^2)^{-3/2} x\dot{x} = \dot{x}^2 (h^2 + x^2)^{-1/2} - x^2 \dot{x}^2 (h^2 + x^2)^{-3/2} = 0,8\ m/s^2$

$T = m(g + a) = 424\ N$

2.6)

DATI

$m = 30\,000$ kg

$\theta = 15°$; $v = k$

$R = 10 \cdot 1000\,N = 10\,000\,N$

$(a = 9\,m,\ b = 0{,}2\,m\ ;\ c = 0{,}18\,m)$

$T = ?$ $\quad P = ?$ $\quad S = ?$

$\theta)\ +T - R - mg\sin\theta = 0 \Rightarrow T = R + mg\sin\theta = 10000 + 30000 \cdot 9{,}8\sin(15) = 86170\ N$

$\varphi)\ +P + S - mg\cos\theta = 0$

$\circlearrowleft)\ Rc + bP - Tc - Sa = 0 \qquad P = \dfrac{T \cdot c + S \cdot a - R \cdot c}{b}$

$\dfrac{T \cdot c + S \cdot a - R \cdot c}{b} + S - mg\cos\theta = 0$

$S \cdot a + T \cdot c - R \cdot c + S \cdot b - mgb\cos\theta = 0$

$S(a + b) = R \cdot c + mgb\cos\theta - Tc$

$S\ \dfrac{R \cdot c + mgb\cos\theta - T \cdot c}{(a+b)} = \dfrac{10000 \cdot 0{,}18 + 30000 \cdot 9{,}8 \cdot 0{,}2 \cdot \cos 15 - 86170 \cdot 0{,}18}{(9 + 0{,}2)} = 4690\ N$

$P = \dfrac{TC + SA + RC}{b} = \dfrac{86170 \cdot 0{,}18 + 4690 \cdot 9 + 10000 \cdot 0{,}18}{0{,}2} = 279600\ N$

2.7)

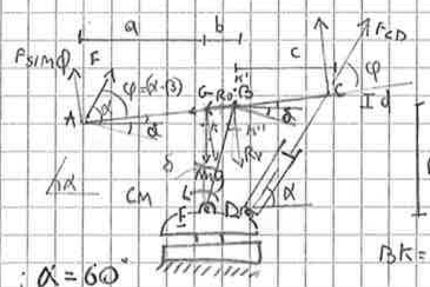
DATI

$F = 100\,N$

$m_{AC} = 25$ kg

$a = 0{,}75\,m$; $b = 0{,}25\,m$; $c = 0{,}5\,m$; $d = 0{,}1\,m$; $\alpha = 60°$

$F_{CD} = ?$ $\quad R_B = ?$ $\quad C_M = ?$



$\overline{EK'} = BE + BK = 1$

$\overline{EK} = \overline{BE} - \overline{BK} = 0{,}79 - 0{,}2067 = 0{,}5833\ m$

$a = \overline{BE}\cos\delta$

$\overline{BE} = \dfrac{a}{\cos\delta} = 0{,}79\,m$

$BK'' = \overline{AB}\sin\beta = 0{,}196\,m = BK\cos\delta \Rightarrow$

$BK = \dfrac{BK''}{\cos\delta} = 0{,}2067\,m$

$d = c\,Tg\beta \Rightarrow \beta = Tg^{-1}\left(\dfrac{d}{c}\right) = Tg^{-1}\left(\left(\dfrac{0{,}1}{0{,}5}\right)\right) = 11{,}31°$

$\varphi = (\alpha - \beta) = 60 - 11{,}31 = 48{,}69°$

$F\sin\varphi \cdot \overline{AB} - mg\cos\beta \cdot \overline{GB} - F_{CD}\sin\varphi \cdot \overline{BC} = 0$

$(a + b) = \overline{AB}\cos\beta \Rightarrow \overline{AB} = \dfrac{(a+b)}{\cos\beta} = \dfrac{1}{\cos(11{,}31)} = 1{,}02\ m$

$b = \overline{GB}\cos\beta \Rightarrow \overline{GB} = \dfrac{b}{\cos\beta} = \dfrac{0{,}25}{\cos(11{,}31)} = 0{,}255\ m$

$c = \overline{BC}\cos\beta \Rightarrow \overline{BC} = \dfrac{c}{\cos\beta} = \dfrac{0{,}5}{\cos(11{,}31)} = 0{,}51\ m$

$F_{CD} = \dfrac{F\sin\varphi \cdot \overline{AB} - mg\cos\beta \cdot \overline{GB}}{\sin\varphi\ \overline{BC}} = \dfrac{100 \cdot \sin 48{,}69 \cdot 1{,}02 - 25 \cdot 9{,}8\cos 11{,}31 \cdot 0{,}255}{\sin 48{,}69 \cdot 0{,}51} = 40{,}1\,N$

$b = a\,Tg\delta \Rightarrow \delta = Tg^{-1}\left(\dfrac{b}{a}\right) = 18{,}43°$

$\overline{EK}$ $\ F\cos(\alpha+\delta) + F_{CD}\cos(\alpha+\delta) \cdot \overline{EI} +$

$\left(\overline{EB} - \dfrac{b\,Tg\beta}{\cos\delta}\right) mg\sin\delta = C_M$

$C_M = 0{,}5833 \cdot 100 \cdot \cos 78{,}43 + 40 \cdot 1 \cdot \cos 78$

$- 25 \cdot 9{,}8 \cdot \sin 18{,}43 \cdot \left(\overline{EB} - \dfrac{b\,Tg\beta}{\cos\delta}\right)$

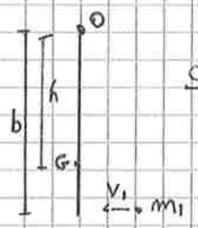
$= 37{,}79$

$R_V = F\sin\varphi + F_{CD}\sin\varphi - mg\cos\beta = (100 + 40)\sin 48{,}69 - 25 \cdot 9{,}8 \cdot \cos 11{,}31 = -135{,}08$

$R_O = F\cos\varphi - mg\sin\beta + F_{CD}\cos\varphi = (100 + 40)\cos 48{,}69 - 25 \cdot 9{,}8\sin 11{,}31 = 44{,}37$

$R_B = \sqrt{R_V^2 + R_O^2} = 142{,}2\ N$

2.15)



CONSER. DEL MOM. DELLA QUANT. MOTO

DATI:

$M = 25\ kg$

$\rho_G = 304\ mm = 0{,}304\ m$

$m = 30\ g$

$v = 500\ m/s$

$h = 0{,}9\ m \quad b = 1{,}1\ m$

$\omega = ?$

$I_O = I_G + Mh^2 = 22{,}5604\ kg\,m^2$ $\quad \boxed{I_G = M\rho_G^2} = 2{,}3\ kg\,m^2$

NB: USO LA CONSERVAZIONE DEL MOMENTO DELLA QUANTITA' DI MOTO

NO!!

$\cancel{m_1 V_1 + m_2 V_2 = (m_1 + m_2) V_{TOT}}$

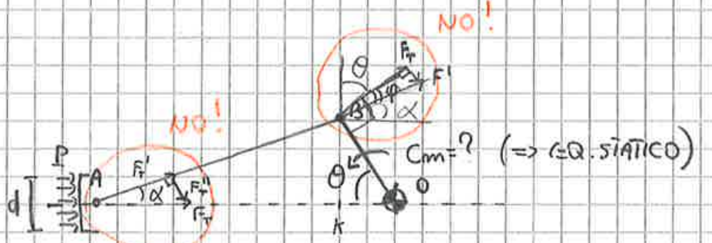
$\cancel{V_{TOT} = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2}}$

$b \cdot m_1 V_1 = I_O \omega + m_1 V_2 \cdot b$

$b \cdot m_1 V_1 = I_O \omega + m_1 \omega b^2$

$$\Rightarrow \omega = \frac{m_1 V_1 \cdot b}{I_O + m_1 b^2} = \frac{0{,}03 \cdot 500 \cdot 1{,}1}{22{,}5604 + 0{,}03\,(1{,}1)^2} = 0{,}73\ rad/s$$

2.16)



DATI:

$P = 100\ kPa$

$d = 40\ mm$

$\overline{OB} = 42{,}5\ mm$; $\overline{AB} = 107{,}5\ mm$; $\theta = 60°$

$C_m = ?$ $\quad R_O, R_A, R_B = ?$ $\quad$ AREA CERCHIO: $\pi R^2 = 3{,}14\,(0{,}02)^2 = 0{,}0013\ m^2$

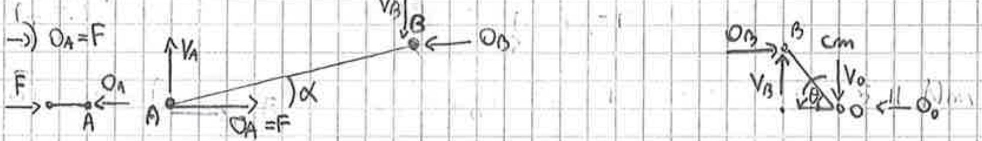
NB:

$P = \frac{F_\perp}{S} \Rightarrow F_\perp = P \cdot S = 100\,000 \cdot 0{,}001256 = 125{,}6\ N$ $\quad (dF = P\,dS)$

$\overline{BK} = \overline{OB} \sin\theta = 42{,}5 \cdot \sin 60 = 36{,}81\ mm = 0{,}03681\ m$

$\overline{BK} = \overline{AB} \sin\alpha \Rightarrow \alpha = \sin^{-1}\left(\frac{\overline{BK}}{\overline{AB}}\right) = \sin^{-1}\left(\frac{36{,}81}{107{,}5}\right) = 20{,}02°$

NB: NO SCOMPOSIZIONI DELLE FORZE (CENTRIAMO IN MECCANICA I)



$$\begin{cases} \uparrow)\ V_A - V_B = 0 & V_A = V_B = 45{,}76\ N \\ \rightarrow)\ F - O_B = 0 \Rightarrow O_B = F = 125{,}6\ N \\ \curvearrowleft_A)\ O_B \cdot \overline{AB}\sin\alpha - V_B \overline{AB}\cos\alpha = 0 \Rightarrow V_B = \frac{O_B \overline{AB}\sin\alpha}{\overline{AB}\cos\alpha} = O_B \operatorname{Tg}\alpha = 45{,}76\ N \end{cases}$$

$R_A = \sqrt{45{,}76^2 + 125{,}6^2} = 133{,}7\ N$

$\uparrow)\ V_B - V_O = 0 \qquad V_O = V_B = 45{,}76\ N$

$\rightarrow)\ O_B - O_O = 0 \Rightarrow O_O = O_B = 125{,}6\ N$

$R_O = R_B = \sqrt{45{,}76^2 + 125{,}6^2} = 133{,}7\ N$

$\curvearrowleft_O)\ C_m - O_B \overline{OB}\sin\theta - V_B \overline{OB}\cos\theta = 0 \Rightarrow C_m = O_B \overline{OB}\sin\theta + V_B \overline{OB}\cos\theta = 5{,}6\ Nm$

2.11)

DATI

NB: $Km/h \xrightarrow{(:3,6)} m/s$

$m = 1500\ Kg$

$V = 8\ Km/h = 8 \cdot \frac{1000}{3600} \frac{m}{s} = 2{,}22\ m/s \qquad \frac{8}{3{,}6} = 2{,}22\ m/s$

$K = ?$ (DEF $h = 150\ mm$)

CONSERVAZIONE DELL'ENERGIA

$E_{Mi} = E_{Mf}$

$E_{Ki} = \frac{1}{2} m V^2$; $E_{Pi} = 0$; $E_{Kf} = 0$; $E_{Pf} = \frac{1}{2} K x^2$

$\frac{1}{2} m V^2 = \frac{1}{2} K x^2$

$K_{TOT} = \frac{m V^2}{x^2} = \frac{1500 \cdot (2{,}22)^2}{(0{,}15)^2} = 322\,666{,}6$

OPPURE (INVECE DI TROVARE K_{TOT}):

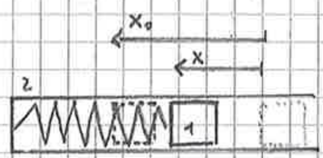
$\frac{1}{2} m V^2 = \frac{1}{2} K x^2 + \frac{1}{2} K x^2$

$\frac{1}{2} m V^2 = K x^2 \Rightarrow K = \frac{m V^2}{2 x^2} = 161\,333\ N/m$

$\frac{K_{TOT}}{2} = K$ (POICHE' ABBIAMO DUE MOLLE UGUALI)

$K = 161\,333\ N/m$ (PIU' PRECISAMENTE: USANDO $\frac{8}{3{,}6} = 2{,}2222$ AL POSTO DI $2{,}22$ IL RISULTATO SARA' $K = 164\,605\ N/m$)

2.12)



$L = \int F \cdot ds$

$F = Kx$

$dL_e = Kx\,dx \Rightarrow L_e = \int_{x}^{x_0} Kx\,dx = \frac{1}{2} K (x_0^2 - x^2)$

$P = -\frac{dL}{dt}$ IL $\ominus$ POICHE' IL LAVORO APPENA CALCOLATO E' DI COMPRESSIONE ($\sim$ EN. POTENZ.)

ESSENDO $V = \frac{dx}{dt} \Rightarrow \frac{1}{dt} = \frac{V}{dx} \Rightarrow$

$P = -\frac{dL}{dt} = -\frac{dL}{dx} V = -\frac{1}{2} K (2 x_0 V_0 - 2 x V) = K x \cdot V$ POICHE' $V_0 = 0$

$L_e - E_K = 0$

$\frac{1}{2} K (x_0^2 - x^2) = \frac{1}{2} m V^2 \Rightarrow V = \sqrt{\frac{K}{m}(x_0^2 - x^2)}$

$P = Kx \cdot V = Kx \cdot \sqrt{\frac{K}{m}(x_0^2 - x^2)}$

$\boxed{\frac{dP}{dx}} = K\sqrt{\frac{K}{m}(x_0^2 - x^2)} + Kx\sqrt{\frac{K}{m}} \cdot \frac{1}{2} \frac{-2x}{\sqrt{x_0^2 - x^2}} = K\sqrt{\frac{K}{m}}\left(\sqrt{x_0^2 - x^2} - \frac{x^2}{\sqrt{x_0^2 - x^2}}\right) = K\sqrt{\frac{K}{m}}\left(\frac{x_0^2 - x^2 - x^2}{\sqrt{x_0^2 - x^2}}\right) =$

$= K\sqrt{\frac{K}{m}}\left(\frac{x_0^2 - 2x^2}{\sqrt{x_0^2 - x^2}}\right) = \boxed{0}$

$\Rightarrow x_0^2 - 2x^2 = 0 \Rightarrow x = \frac{x_0}{\sqrt{2}}$

$P_{MAX} = K \frac{x_0}{\sqrt{2}} \sqrt{\frac{K}{m} \frac{x_0^2}{2}} = K \frac{x_0^2}{2} \sqrt{\frac{K}{m}}$

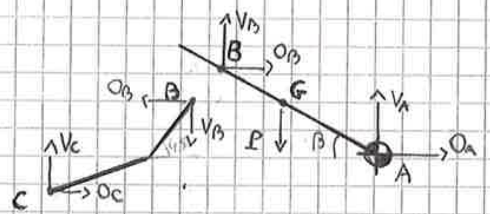
2.19)

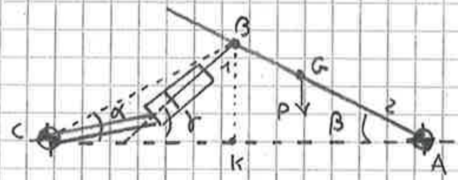
DATI

$P = 20\,kN$

$\alpha = \beta = 30°$; $\gamma = 45°$; $\overline{AB} = 4\,m$ $\overline{AG} = 2{,}5\,m$ $\varnothing = 0{,}2\,m$

$R_B = ?$ P_m (=> F.O. St.) = ?





$\overline{BK} = \overline{AB}\sin\beta$

$\overline{BK} = \overline{CK}\,\mathrm{Tg}\,\alpha \Rightarrow \overline{CK} = \dfrac{\overline{BK}}{\mathrm{Tg}\,\alpha}$

$$\begin{cases} \uparrow)\ V_B + V_A - P = 0 \\ \rightarrow)\ O_B + O_A = 0 \\ \overset{\curvearrowleft}{A})\ \overline{AG}\cos\beta \cdot P - \overline{AB}\cos\beta\, V_B - \overline{AB}\sin\beta\, O_B = 0 \end{cases}$$

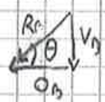
$$\begin{cases} \uparrow)\ V_C - V_B = 0 \\ \rightarrow)\ O_C - O_B = 0 \\ \overset{\curvearrowleft}{C})\ \overline{AB}\sin\beta \cdot O_B - \dfrac{\overline{AB}\sin\beta}{\mathrm{Tg}\,\alpha} \cdot V_B = 0 \end{cases} \Rightarrow V_B = \frac{\cancel{\overline{AB}\sin\beta}}{\cancel{\overline{AB}\sin\beta}}\,\mathrm{Tg}\,\alpha \cdot O_B$$

$\overset{\curvearrowleft}{A})\ \overline{AG}\cos\beta \cdot P - \overline{AB}\cos\beta\,\mathrm{Tg}\,\alpha\, O_B - \overline{AB}\sin\beta\, O_B = 0 \Rightarrow \overline{AG}\cos\beta \cdot P = O_B(\overline{AB}\cos\beta\,\mathrm{Tg}\,\alpha + \overline{AB}\sin\beta)$

$$\Rightarrow O_B = \frac{\overline{AG}\cos\beta \cdot P}{(\overline{AB}\cos\beta\,\mathrm{Tg}\,\alpha + \overline{AB}\sin\beta)} = \frac{2{,}5\cos 30° \cdot 20\,000}{(4\cdot\cos 30\cdot \mathrm{Tg}\,30 + 4\sin 30)} = 10\,825{,}3\ N$$

$V_B = O_B\,\mathrm{Tg}\,\alpha = 6\,250\ N$

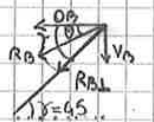
$R_B = \sqrt{V_B^2 + O_B^2} = 12\,500\ N$



$V_B = R_B \sin\theta$ $\theta = \sin^{-1}\left(\dfrac{V_B}{R_B}\right) = 30$

NB:

$P = \dfrac{F_\perp}{S}$, $S = \pi \dfrac{d^2}{4}$



$R_{B\perp} = R_B\cos(\gamma - \theta) = 12\,074{,}1\ N$

NB!

$P = \dfrac{R_{B\perp}}{S} = 384\,525{,}5\ \dfrac{N}{m^2} = 3{,}84\ bar$ POICHÉ NB: $1\ bar = 100\,000\ Pa = 10^5\ Pa$

2.21)

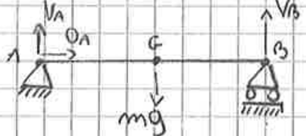
DATI

m, L

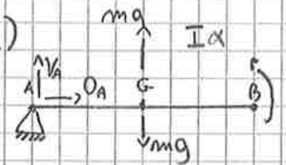
$R_A, R_B = ?$ (EQ. STAT.)

$R'_A = ?$ (SENZA APPOGGIO IN B)

1)



2)



$\alpha = \frac{a}{(L/2)}$ $\quad I = \frac{1}{12} mL^2$

1)
$$\begin{cases} \uparrow)\ V_A + V_B - mg = 0 \\ \rightarrow)\ O_A = 0 \\ \overset{\curvearrowleft}{A})\ -mg\frac{L}{2} + V_B \cdot L = 0 \end{cases}$$

$V_A = mg - V_B = mg - \frac{mg}{2} = \frac{mg}{2} = R_A$

$\Rightarrow V_B = \frac{mg}{2} = R_B$

2)
$$\begin{cases} \uparrow)\ V_A + ma - mg = 0 \\ \rightarrow)\ O_A = 0 \\ \overset{\curvearrowleft}{A})\ \frac{L}{2} \cdot ma - \frac{L}{2} mg + I\alpha = 0 \end{cases}$$

$\overset{\curvearrowleft}{A})\ \frac{L}{2} ma - \frac{L}{2} mg + \frac{1}{12} mL^2 \frac{a \cdot 2}{L} = 0 \Rightarrow a = \frac{3}{4} g \Rightarrow \uparrow)\ V_A = m(g-a) = mg\left(1 - \frac{3}{4}\right) =$

$= \frac{mg}{4} = R_A$

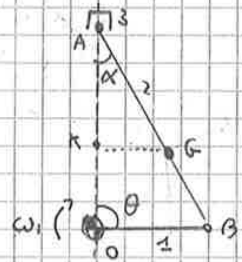
2.22)

DATI

$\overline{OB} = 42{,}5\ mm$

$\overline{AB} = 107{,}5\ mm$

$\overline{AG} = 75\ mm$



$\theta = 90°$; $m_2 = 0{,}6\ kg$; $\rho_2 = 28\ mm$; $m_3 = 0{,}82\ kg$; $\omega_1 = 3000$ GIRI/min

$F_A = ?$

$F_A = m_3 a_A$

$\overline{OB} = \overline{AB} \sin\alpha \Rightarrow \alpha = \sin^{-1}\left(\frac{\overline{OB}}{\overline{AB}}\right) = 23{,}29°$

$\overline{KG} = \overline{AG} \sin\alpha = 0{,}0297\ m$

$\overline{AK} = \overline{AG} \cos\alpha = 0{,}0689\ m$

$\overline{AO} = \overline{AB} \cos\alpha = 0{,}0987\ m$

$\overline{KO} = \overline{AO} - \overline{AK} = 0{,}0298\ m$; $\overline{KO} = \overline{BG}\cos\alpha$

$\Rightarrow \overline{BG} = \frac{\overline{KO}}{\cos\alpha} = 0{,}0324$

$F = [1522]$ $\quad a_A = 1856{,}09$

$\omega_1 = \frac{3000 \cdot 2\pi}{60} = 314\ rad/s$

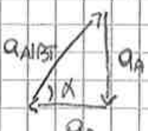
$V_B = \omega_1 \overline{OB} = 314 \cdot 0{,}0425 = 13{,}34\ m/s$

$\vec{V}_A = \vec{V}_B \Rightarrow \bar{V}_{AB} = \omega_2 \overline{AB} = 0$

13,34

$\vec{a}_A = \vec{a}_B + \vec{a}_{A|B\,N} + \vec{a}_{A|B\,T}$

$\quad \omega_1^2 \overline{OB} \quad \omega_2^2 \overline{AB} = 0 \quad \dot{\omega}_2 \overline{AB}$



$a_A = a_B \operatorname{tg}\alpha = \omega_1^2 \overline{OB} \cdot \operatorname{tg}\alpha = 1804\ m/s^2$

$a_B = a_{B|A\,T} \cos\alpha \Rightarrow a_{B|A\,T} = \frac{a_B}{\cos\alpha} = \frac{\omega_1^2 \overline{OB}}{\cos\alpha} = 4567$

$a_{B|A\,T} = \dot{\omega}_2 \overline{AB} \Rightarrow \dot{\omega}_2 = \frac{a_{B|A\,T}}{\overline{AB}} = 42481\ rad/s^2$

$\vec{a}_G = \vec{a}_A + \vec{a}_{G|A\,N} + \vec{a}_{G|A\,T}$

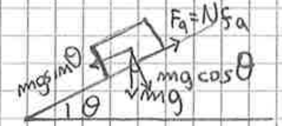
$\quad 1804\frac{m}{s^2} \quad \omega_2^2 \overline{AG} = 0 \quad \dot{\omega}_2 \overline{AG}$

[CAPITOLO 3]

3.1)

DATI

$\theta = ?$ (⇒ ADERENZA LIMITE)



$F_A = N f_a$

$$\begin{cases} \uparrow) \; N - mg\cos\theta = 0 \qquad N = mg\cos\theta \\ \rightarrow) \; -mg\sin\theta + N f_a = 0 \end{cases}$$

$$\rightarrow) \; \cancel{m}\cancel{g}\cos\theta \, f_a = \cancel{m}\cancel{g}\sin\theta \quad \Rightarrow f_a = \operatorname{Tg}\theta \Rightarrow \theta_{LIM} = \operatorname{Tg}^{-1}(f_a)$$

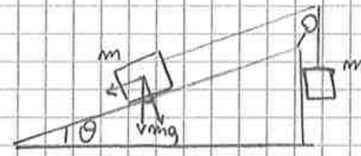
3.2)

DATI

$m = 100\,kg$

$\theta = 20°$

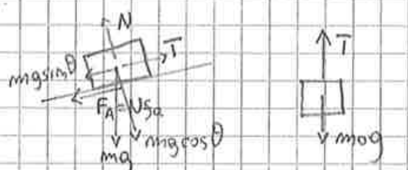
$f_a = 0{,}3$



A) $m_0 = ?$ (EQ. STAT. PRIMA CHE m VENGA TRASCINATA A DX)

B) $m_0 = ?$ (EQ. STAT. PRIMA CHE m ~~ve~~ SI SPOSTI VERSO SX)

A)

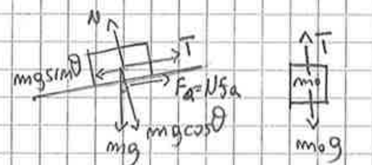


$m_0)\; +T - m_0 g = 0 \qquad T = m_0 g$

$$m)\begin{cases} \uparrow) \; N - mg\cos\theta = 0 \quad \Rightarrow N = mg\cos\theta \\ \rightarrow) \; -mg\sin\theta + T - N f_a = 0 \end{cases}$$

$$\rightarrow) \; -m\cancel{g}\sin\theta + m_0\cancel{g} - m\cancel{g}\cos\theta \cdot f_a = 0 \quad \Rightarrow m_0 = m(\sin\theta + f_a\cos\theta) = 62{,}4\,kg$$

B)



$m_0)\; +T - m_0 g = 0 \Rightarrow T = m_0 g$

$$m)\begin{cases} \uparrow) \; N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta \\ \rightarrow) \; T + f_a N - mg\sin\theta = 0 \end{cases}$$

$$\rightarrow) \; m_0\cancel{g} + f_a m\cancel{g}\cos\theta - m\cancel{g}\sin\theta = 0 \quad \Rightarrow m_0 = m(\sin\theta - f_a\cos\theta) = 6{,}01\,kg$$

3.4)

DATI

$d_1 = 0{,}18\ m$

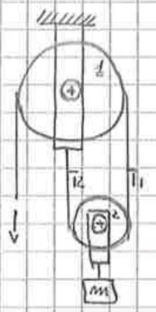
$d_2 = 0{,}090\ m$

$R_1 = 0{,}010\ m$

$R_2 = 0{,}006\ m$

$f_a = 0{,}25$

$m = 200\ kg$



A) CARICO SOLLEVATO V=K

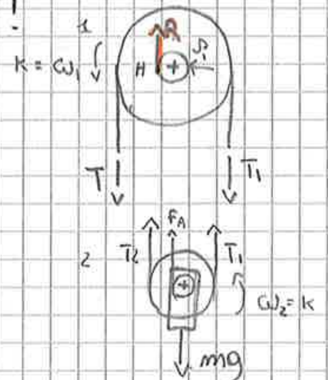
B) CARICO FATTO SCENDERE A V=K

$\Big\}\ T, T_1, T_2 = ?$

$\boxed{f = Tg\,\varphi \cong \sin\varphi \Rightarrow \rho = R\sin\varphi = R\cdot f}$

$\rho_1 = R_1 \cdot f = 0{,}0025\ m$

$\rho_2 = R_2 \cdot f = 0{,}0015\ m$



A) 2) $\begin{cases} (\uparrow)\ +T_1 + T_2 - mg = 0 \\ (K\curvearrowleft)\ -\left(\frac{d_2}{2}+\rho_2\right)mg + d_2 T_1 = 0 \end{cases}$

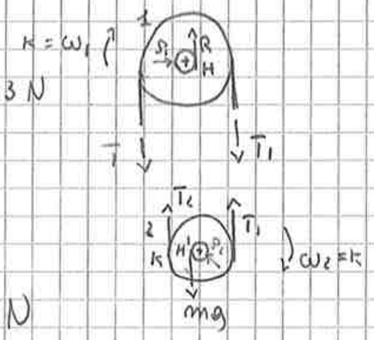
$\begin{cases} T_2 = mg - T_1 = 200\cdot 9{,}8 - 1013 = 947\ N \\ T_1 = \left(\frac{d_2}{2}+\rho_2\right)mg/d_2 = \left(\frac{0{,}090}{2}+0{,}0015\right)\cdot 200\cdot 9{,}8 / 0{,}090 = 1013\ N \end{cases}$

1) $\begin{cases} (\uparrow)\ -T - T_1 + R = 0 \\ (H\curvearrowleft)\ \left(\frac{d_1}{2}-\rho_1\right)T - \left(\frac{d_1}{2}+\rho_1\right)T_1 = 0 \end{cases} \Rightarrow T = T_1\left(\frac{d_1}{2}+\rho_1\right)\Big/\left(\frac{d_1}{2}-\rho_1\right) = 1013\left(\frac{0{,}18}{2}+0{,}0025\right)\Big/\left(\frac{0{,}18}{2}-0{,}0025\right)$

$= 1071\ N$

B) 2) $\begin{cases} (\uparrow)\ +T_2 + T_1 - mg = 0 \\ (K\curvearrowleft)\ -\left(\frac{d_2}{2}-\rho_2\right)mg + d_2 T_1 = 0 \end{cases}$

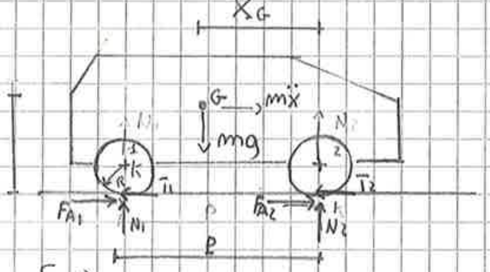
$\begin{cases} T_2 = mg - T_1 = 200\cdot 9{,}8 - 947 = 1013\ N \\ T_1 = \left(\frac{d_2}{2}-\rho_2\right)mg/d_2 = 947\ N \end{cases}$



1) $\begin{cases} (\uparrow)\ -T - T_1 + R = 0 \\ (H\curvearrowleft)\ T\left(\frac{d_1}{2}+\rho_1\right) - T_1\left(\frac{d_1}{2}-\rho_1\right) = 0 \end{cases}$

$T = T_1\left(\frac{d_1}{2}-\rho_1\right)\Big/\left(\frac{d_1}{2}+\rho_1\right) = 896\ N$

3.7)



DATI

$p = 2,5$ m (PASSO)

$X_G = 1,4$ m ; $Y_G = 0,8$ m

$R = 0,32$ m

$m = 1000$ kg

$f_A = 0,75$

$V_0 = 100$ km/h $= 27,8$ m/s

S (FRENATA) $= ?$

$C_1 = ?$ $C_2 = ?$ (COPPIE FRENANTI)

$$\left\{\begin{array}{l} \uparrow)\ N_1 + N_2 - mg = 0 \\ \curvearrowleft_K)\ -N_1 p + mg X_G - m a Y_G = 0 \end{array}\right.$$

$$\left\{\begin{array}{l} \uparrow)\ N_2 = mg - N_1 = 1000 \cdot 9,8 - 3136 = 6664\ N \\ \curvearrowleft_K)\ N_1 = \dfrac{m(g X_G - a Y_G)}{p} = 3136\ N \end{array}\right.$$

$$\underbrace{E_{K_i}}_{} = \overbrace{E_{K_F}}^{=0} + L_{F_A}$$

$$\frac{1}{2} m V_0^2 = L_{F_A} = \frac{1}{2} 1000 \cdot (27,8)^2 = 386\,420\ J$$

$$L_{F_A} = F_{A_{TOT}} \cdot S_R \Rightarrow S_R = \frac{L_{F_A}}{F_{A_{TOT}}}$$

$$F_{A_{TOT}} = F_{A_1} + F_{A_2} = N_1 f_a + N_2 f_a = f_a (N_1 + N_2) = 0,75 \cdot 9800 = 7350\ N$$

$$\boxed{S_R = \frac{L_{F_A}}{F_{A_{TOT}}}} = \frac{386\,420}{7350} = 52,6\ m$$

$$\cancel{S = \frac{S_R}{2} = \frac{52,6}{2}}$$ NO!

NB $S = S_R = \dfrac{L_{F_A}}{F_{A_{TOT}}}$ NON DIVIDERE PER 2 !!

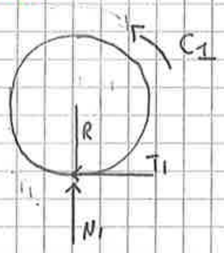
OPPURE PIÙ SEMPLICEMENTE:

$$\rightarrow)\ -T_1 - T_2 = -ma \Rightarrow a = \frac{T_1 + T_2}{m} = \frac{f_a (N_1 + N_2)}{m} = \frac{f_a \cancel{m} g}{\cancel{m}} = 7,35\ m/s^2$$

$$a \cdot S = \frac{1}{2} V^2 \Rightarrow S = \frac{1}{2} \frac{V^2}{a} = 52,6\ m$$

$$\cancel{-RT_1 + C_1} = I\alpha$$

NB: PER LE COPPIE FRENANTI



MA:

$$-RT_1 + C_1 = 0 \Rightarrow C_1 = RT_1 = R f_a N_1 = 753\ Nm$$

$$-RT_2 + C_2 = 0 \Rightarrow C_2 = RT_2 = R f_a N_2 = 1600\ Nm$$

3.10)

DATI

$M = 1000\,kg$ $\qquad V = K$

$b = 2{,}6\,m$

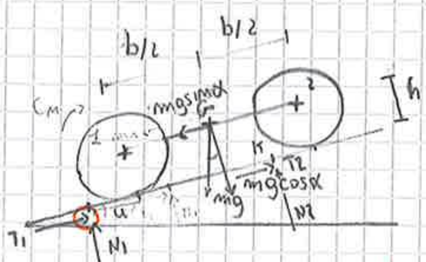
$h = 0{,}25\,m$

$u = 10\,cm = 0{,}1\,m$

$\alpha = 30°$

$N_1, T_1, N_2, T_2 = ?$

$C_M = ?$



HA QUESTO VERSO PERCHÉ DET LA CM

$C_M = hT_1 - uN_1$

$\uparrow)\ N_1 + N_2 - mg\cos\alpha = 0 \quad \Rightarrow N_2 = mg\cos\alpha - N_1 = 4099\,N$

$K)\ -N_1 b + mg\cos\alpha\left(\frac{b}{2} - u\right) + mg\sin\alpha\, h = 0 \quad \Rightarrow N_1 = \frac{mg\left[\left(\frac{b}{2} - u\right)\cos\alpha + h\sin\alpha\right]}{b} = 4388\,N$

$T_1 = f_a N_1$

$T_2 = f_a N_2$

$O_2)\ hT_2 - uN_2 = 0 \qquad T_2 = \frac{u}{h} N_2 = 1641\,N$

$\rightarrow)\ T_1 + T_2 - mg\sin\alpha = 0 \quad \Rightarrow T_1 = mg\sin\alpha - T_2 = 3264\,N$

$C_M = hT_1 - uN_1 = 377\,Nm$

3.11)

DATI $\qquad$ NB: IL PROBLEMA NON PARLA DI u, PARAM. DI ATTR. VOLV. QUINDI QUESTO NON VA BENE !!

$d = 50\,m$

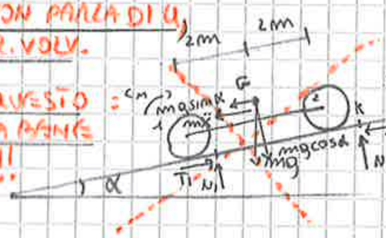
$V_F = 40\,km/h = 11{,}11\,m/s$

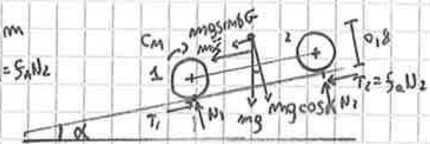
$M = 5000\,kg$

PENDENZA = 15%

$N_1, T_1, N_2, T_2 = ?$

$f_a = ?$ (⇒ NO STRISC.)





$a \cdot d = \frac{1}{2} v^2 \quad \Rightarrow a = \frac{v^2}{2d} = \frac{11{,}11^2}{2 \cdot 50} = 1{,}235\,m/s^2$

NB

$\boxed{\alpha = Tg^{-1}\left(\frac{15}{100}\right)}$ $\qquad 15 = 100\,Tg\,\alpha \Rightarrow \alpha = Tg^{-1}\left(\frac{15}{100}\right) = 8{,}5°$

$C_M = RT_1 + \cancel{uN_1}$

$\uparrow)\ N_1 + N_2 - mg\cos\alpha = 0 \quad \Rightarrow N_2 = mg\cos\alpha - N_1 = 21526\,N$

$K)\ -N_1(4m) + mg\cos\alpha \cdot (2m \cancel{+u}) + (mg\sin\alpha + ma)(0{,}8) = 0 \Rightarrow N_1 = \frac{mg\cos\alpha(2[m] \cancel{+u}) + (mg\sin\alpha + ma)(0{,}8)}{4[m]}$

$= 26936\,N$

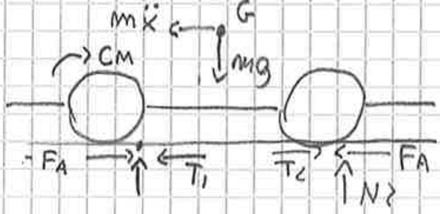
$\rightarrow)\ T_1 - T_2 - mg\sin\alpha = ma$

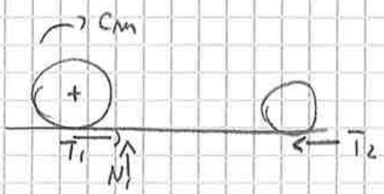
$O_2)\ RT_2 = 0 \quad \Rightarrow T_2 = 0$

$\rightarrow)\ T_1 = m(g\sin\alpha + a) = 13418\,N$

$T_1 = f_a N_1 \qquad f_a = \frac{T_1}{N_1} = 0{,}5 \qquad$ VERIFICHIAMO:

$T_2 = f_a N_2 \qquad \frac{T_2}{N_2} = 0 \leq f_a (= 0{,}5)$ VERIFICATA!





$-T_1 + T_2 = m\ddot{x}$ $\qquad$ $-T_1 = -T_2 + m\ddot{x}$

$T_1 = +T_2 - m\ddot{x}$

$+T_1 - T_2 + m\ddot{x} = 0$

$(2\bar{I}_1 + 2\bar{I})\ddot{\theta}$

$(2 \cdot 53{,}3 + 2 \cdot 0{,}4) = 107{,}4$

$-N_1 b + mg\cos\alpha\left(\frac{b}{2} - u\right) + mg\sin\alpha\, h = 0$

$$N_1 = \frac{mg\left[\cos\alpha\left(\frac{b}{2} - u\right) + \sin\alpha \cdot h\right]}{b} =$$

$$= \frac{1000 \cdot 9{,}8 \left[\cos 30\left(\overset{1{,}2}{1{,}3} - 0{,}1\right) + \sin 30 \cdot 0{,}25\right]}{2{,}6}$$

$= 4388{,}3\ N = 4{,}39\,kN$

3.13)

DATI

$M = 200\,kg$

$d = 0,2\,m$

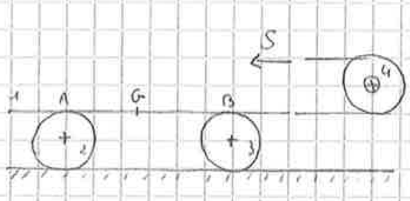
$u_1 = 10\,mm = 0,01\,m$

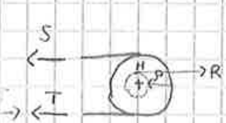
$u_2 = 5\,mm = 0,005\,m$

$D = 0,3\,m$

$R = 0,04\,m \quad \varphi = 30°$

$S = ?\ (\Rightarrow v = k)$



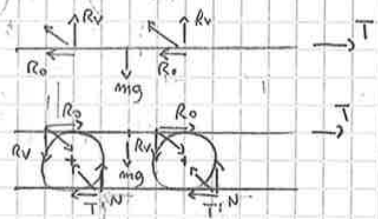


$\rho = R \sin\varphi = 0,04 \sin 30 = 0,02\,m$

4) $\rightarrow)\ -S + R - T = 0$

$H)\ S\left(\frac{D}{2} - \rho\right) - T\left(\frac{D}{2} + \rho\right) = 0$

$S = T\left(\frac{D}{2} + \rho\right) \Big/ \left(\frac{D}{2} - \rho\right)$



1) $\rightarrow)\ +T - R_{o1} - R_{o2} = 0 \qquad R_{o1} = R_{o2} \qquad 2R_o = T \qquad R_o = \frac{T}{2}$

$\uparrow)\ R_{v1} + R_{v2} - Mg = 0 \qquad R_{v1} = R_{v2} \qquad 2R_v = Mg \qquad R_v = \frac{Mg}{2}$

2-3)

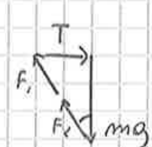
$\uparrow)\ +R_v + R_v + N + N = 0 \Rightarrow N = R_v$

$\rightarrow)\ R_o + R_o - T' - T' = 0 \Rightarrow T' = R_o$

$O)\ 2N u_1 + 2R_v u_2 - 2T' \frac{d}{2} - 2R_o \frac{d}{2} = 0 \Rightarrow \cancel{2}R_v(u_1 + u_2) = \cancel{4}^{2} R_o \frac{d}{2}$

$\Rightarrow \frac{Mg}{2}(u_1 + u_2) = T \frac{d}{2} \qquad T = Mg \frac{(u_1 + u_2)}{d}$

$\Rightarrow S = T\left(\frac{D}{2} + \rho\right) \Big/ \left(\frac{D}{2} - \rho\right) = Mg \frac{u_1 + u_2}{d} \cdot \frac{D + 2\rho}{D - 2\rho} = 192,4\,N$



$T = mg \,Tg\,\alpha$

$d = \frac{u_1 + u_2}{Tg\alpha}$

$Tg\alpha = \frac{u_1 + u_2}{d}$

$\Rightarrow T = mg \frac{(u_1 + u_2)}{d}$

3.(5)

DATI

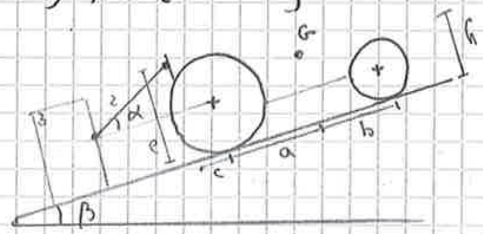
$V_F = 4\ m/s$

$t_F = 3s$

$a = 1m$; $b = 0{,}8m$; $c = 0{,}2m$; $h = 0{,}7m$; $e = 1m$; $\alpha = 10°$; $\beta = 20°$; $R = 0{,}65m$

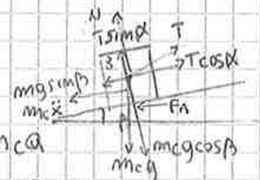
$I = 25\ kg\,m^2$; $f = 0{,}3$; $m_T = 2500\ kg$; $m_c = 600\ kg$

$N_1, T_1, N_2, T_2 = ?$



$a = \frac{V}{t} = \frac{4}{3} = 1{,}33\ m/s^2$

3) $\begin{cases} \uparrow) + T\sin\alpha + N - m_c g\cos\beta = 0 \\ \rightarrow) - m_c g\sin\beta + T\cos\alpha - fN = m_c a \end{cases}$

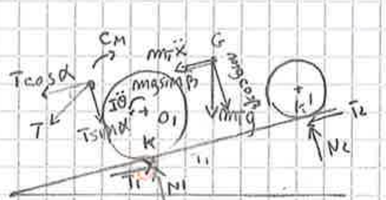


$\begin{cases} \uparrow) N = m_c g\cos\beta - T\sin\alpha \\ \rightarrow) - m_c g\sin\beta + T\cos\alpha - f m_c g\cos\beta + Tf\sin\alpha = m_c a \end{cases}$

$\Rightarrow \rightarrow)\ T(\cos\alpha + f\sin\alpha) = m_c a + m_c g\sin\beta + f m_c g\cos\beta$

$\Rightarrow T = \frac{m_c(a + g\sin\beta + f g\cos\beta)}{(\cos\alpha + f\sin\alpha)} = \frac{600(1{,}33 + 9{,}8\sin 20 + 0{,}3\cdot 9{,}8\cos 20)}{(\cos 10 + \sin 10\cdot 0{,}3)} = 4294\,N$

$\ddot\alpha = \frac{a}{R} = \frac{1{,}3}{0{,}65} = 2\ rad/s^2$



SUL TRATTORE:

$\begin{cases} \uparrow) N_1 + N_2 - m_T g\cos\beta - T\sin\alpha = 0 \\ \rightarrow) - T\cos\alpha - m_T g\sin\beta + T_1 - T_2 = m a \\ \overset{\frown}{O_1}) h - RT_1 + cT\sin\alpha + (e-R)T\cos\alpha + RT_2 - N_2(c+a+b) - m_T a(h-R) + a\cdot m_T g\cos\beta + \\ - (h-R) m g\sin\alpha = I\ddot\alpha \end{cases}$ TROPPE VARIABILI $\Rightarrow$

$\Rightarrow \overset{\frown}{K})\ I\ddot\theta + cT\sin\alpha + eT\cos\alpha + m_T a h - m_T g\cos\beta\cdot a + N_2(a+b) + m_T g\sin\beta\, h = 0$

$N_2 = \frac{a m_T g\cos\beta - I\ddot\theta - cT\sin\alpha - eT\cos\alpha - m_T a\cdot h - m_T g\sin\beta\cdot h}{(a+b)}$ $- 2500\cdot 1{,}3\cdot 0{,}7$

$= \frac{1\cdot 2500\cdot 9{,}8\cos 20 - 25\cdot 2 - 0{,}2\cdot 4294\cdot\sin 10 - 1\cdot 4294\cdot\cos 10 - 2500\cdot 9{,}8\sin 20\cdot 0{,}7}{(1+0{,}8)}$

È LA STESSA COSA ? NO !

$= 5808\,N$ * $\Rightarrow N_1 = m_T g\cos\beta + T\sin\alpha - N_2 = 17648\,N$ *

PERCHÉ * MI VENGONO DIVERSI ?!

OPPURE:

$\overset{\frown}{K'})\ I\ddot\theta + T\sin\alpha\cdot(a+b+c) + T\cos\alpha\cdot e + m_T a h + m_T g\sin\beta\, h + m_T g\cos\beta\cdot b$
$- N_1(a+b) = 0$

$\Rightarrow N_1 = \frac{I\ddot\theta + b\, m_T g\cos\beta + h m_T g\sin\beta + h m_T a + eT\cos\alpha + (a+b+c)T\sin\alpha}{(a+b)} =$

$= \frac{25\cdot 2 + 0{,}8\cdot 2500\cdot 9{,}8\cos 20 + 0{,}7\cdot 2500\cdot 9{,}8\sin 20 + 0{,}7\cdot 2500\cdot 1{,}3 + 1\cdot 4294\cdot\cos 10 + 4294\sin\alpha(1+0{,}8+0{,}2)}{(1+0{,}8)}$

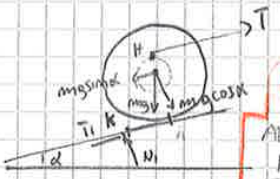
$= 17960\,N$ * $N_2 = m_T g\cos\beta + T\sin\alpha - N_1 = 5800\,N$ *

(A)

$$\begin{cases} \uparrow)\ N_1 - m_1 g \cos\alpha = 0 \\ \rightarrow)\ -m_1 g \sin\alpha + T_1 + T = 0 \\ \overset{\curvearrowleft}{K})\ m_1 g \sin\alpha \cdot R + (R+\rho)T - m_1 g \cos\alpha\, u = 0 \end{cases}$$

$$T = \frac{m_1 g(\sin\alpha\, R - \cos\alpha \cdot u)}{(R+\rho)} = \frac{60 \cdot 9{,}8\,(\sin 20 \cdot 0{,}2 - \cos 20 \cdot 0{,}02)}{0{,}2 + 0{,}016} = 135{,}05\ N$$

$$\rightarrow m_2 = 22{,}1\ kg \quad (\text{CORRETTO})$$



NB!!!

COME FACCIO A CAPIRE CHE DEVO DISEGNARE T_1 ?

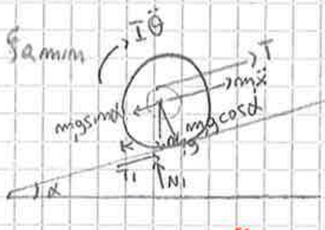
NB: PERCHE' (IN H) OSTACOLA IL MOTO

ALTERNATIV. POTEVA ESSERE IN H' MA IN TAL MODO NON OSTACOLEREBBE IL MOTO, ANZI LO ACCELEREREBBE!

SIGNIFICATO DI ATTR. NEL PERNO

$$\boxed{NB:\ f_{a\,min} = \frac{T_1}{N_1}}$$

(B) $m_2 = 20\ kg$ (NUOVO DATO) ORA CALCOLA $\ddot{x}$ E $f_{a\,min}$



$$\alpha = \frac{a}{R} \Rightarrow a = \alpha R$$

1) $\uparrow)\ N_1 - m g \cos\alpha = 0 \qquad N_1 = m g \cos\alpha$

$\rightarrow)\ -m g \sin\alpha + T + T_1 = -m_1 a$

$\overset{\curvearrowleft}{K})\ + m_1 g \sin\alpha \cdot R - u\, m_1 g \cos\alpha +$

$- T(R+\rho) - R \cdot m\ddot{x} - I\ddot{\theta} = 0$ CONSIDERANDO $\ddot{x} = \ddot{\theta} R$

$\overset{\curvearrowleft}{K})\ m_1 g \sin\alpha \cdot R - u \cdot m_1 g \cos\alpha - T(R+\rho) = R \cdot m\ddot{x} + I_0 \ddot{\theta}$

$\overset{\curvearrowleft}{K})\ m_1 g \sin\alpha \cdot R - u \cdot m_1 g \cos\alpha - T(R+\rho) = m_1 \ddot{\theta} R^2 + I \ddot{\theta}$

$\overset{\curvearrowleft}{K})\ m_1 g \sin\alpha \cdot R - u \cdot m_1 g \cos\alpha - T(R+\rho) = (I_0 + m_1 R^2)\ddot{\theta}$

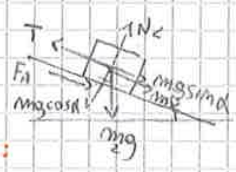
2) $\uparrow)\ N_2 - m_2 g \cos\alpha = 0 \Rightarrow N_2 = m_2 g \cos\alpha$

$\rightarrow)\ -T + F_A + m g \sin\alpha + m\ddot{x} = 0$

$\rightarrow)\ -T + f N_2 + m g \sin\alpha = -m\ddot{x}$

$\rightarrow)\ -T + f m_2 g \cos\alpha + m g \sin\alpha = -m\ddot{x}$ CONSIDERANDO:

$\rightarrow)\ T = f m_2 g \cos\alpha + m_2 g \sin\alpha + m_2 \ddot{\theta} R$ $\quad \ddot{x} = \ddot{\theta} R$



$\overset{\curvearrowleft}{K})\ m_1 g \sin\alpha \cdot R - u\, m_1 g \cos\alpha - (R+\rho)[f m_2 g \cos\alpha + m_2 g \sin\alpha] - (R+\rho) m_2 \ddot{\theta} R = (I_0 + m_1 R^2)\ddot{\theta}$

$$\ddot{\theta} = \frac{m_1 g \sin\alpha \cdot R - u\, m_1 g \cos\alpha - (R+\rho)(f m_2 g \cos\alpha + m_2 g \sin\alpha)}{I_0 + m_1 R^2 + m_2 R^2 + m_2 R\rho} = 0{,}65\ rad/s^2$$

$$\ddot{x} = \ddot{\theta} R = 0{,}65 \cdot 0{,}2 = 0{,}13\ m/s^2$$

$$\boxed{f_{a\,min} = \frac{T_1}{N_1}} = \frac{m_1 g \sin\alpha - T - m_1 \ddot{x}}{m_1 g \cos\alpha} = \frac{m_1 g \sin\alpha - f m_2 g \cos\alpha - m_2 g \sin\alpha - m_2\ddot{x} - m_1\ddot{x}}{m_1 g \cos\alpha}$$

$$= 0{,}124$$

3.18)

DATI

$d = 1\,m$

$M = 10\,000\ kg$

$f_a = 0{,}20$ COEFF. ADER.

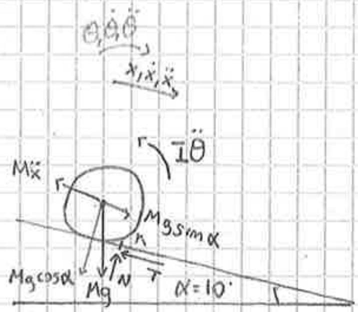
$f = 0{,}15$ COEFF. DI ATTR.

$u = 2\,cm = 0{,}02\,m$

A) $\alpha = 10°$ $t = ?$

B) $\alpha = 45°$ $t = ?$

$\Rightarrow S = 200\,m$



$\alpha = \frac{a}{R}$

$\ddot\theta = \frac{\ddot x}{R}$ $\ddot x = \ddot\theta R$

$\frac{T}{N} = f_a$ $T = f_a N$

A) E IL NUMERO DI GIRI

$$\begin{cases} \uparrow)\ N - Mg\cos\alpha = 0 \Rightarrow N = Mg\cos\alpha \\ \rightarrow)\ Mg\sin\alpha - T = Ma \quad T = F_A = fN = f\,Mg\cos\alpha \\ \curvearrowright)\ \frac{d}{2}M\ddot x + I\ddot\theta - \frac{d}{2}Mg\sin\alpha + u\,Mg\cos\alpha = 0 \end{cases}$$

$, \ddot x = \ddot\theta\frac{d}{2}$

$I_0 = \frac{1}{2}MR^2$

$, I_0 = \frac{1}{2}M\frac{d^2}{4} = \frac{Md^2}{8}$

$\curvearrowright)\ \frac{d}{2}Mg\sin\alpha - u\,Mg\cos\alpha = \left(I_0 + \frac{d^2}{4}M\right)\ddot\theta$

$I = Md^2\left(\frac{1}{8} + \frac{1}{4}\right) = Md^2\frac{3}{8}$

$\curvearrowright)\ \ddot\theta = \dfrac{\frac{d}{2}Mg\sin\alpha - u\,Mg\cos\alpha}{Md^2\frac{3}{8}} = \dfrac{\cancel{10\,000}\cdot 9{,}8\,(0{,}5\sin 10 - 0{,}02\cos 10)}{\cancel{10\,000}(1)^2\cdot 0{,}375} = 1{,}75\ rad/s^2$

$\Rightarrow \ddot x = \ddot\theta R = 1{,}75\cdot 0{,}5 = 0{,}88\,m/s^2$

$N = Mg\cos\alpha = 10\,000\cdot 9{,}8\cdot\cos 10° = 96\,511{,}2\ N$

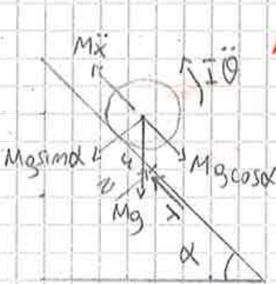
$T = Mg\sin\alpha - Ma = M(g\sin\alpha - a) = 10\,000\,(9{,}8\cdot\sin 10° - 0{,}88) = 8217{,}5\ N$

ADERENZA $\frac{T}{N} \le f_a$, $f_a = 0{,}20$

$\frac{T}{N} = 0{,}0851 < 0{,}20 \Rightarrow$ ADERENZA VERIFICATA $x = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2S}{a}} = 21{,}3\ s$

$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\,1{,}75\cdot 15^2 = 397{,}7°$

NB: $\boxed{n°\,GIRI = \frac{\theta}{2\pi}} = \frac{397{,}7°}{(2\cdot 3{,}14)} = 63{,}3$ GIRI



B) $\alpha = 45°$

$\ddot\theta = 8{,}87\ rad/s^2$

$\ddot x = \ddot\theta\cdot R = 8{,}87\cdot 0{,}5 = 4{,}44\ m/s^2$

$N = Mg\cos\alpha = 10\,000\cdot 9{,}8\cdot\cos 45° = 69\,297\ N$

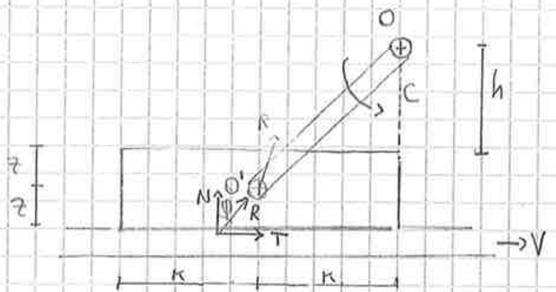
$T = Mg\sin\alpha - Ma = M(g\sin\alpha - a) = 10\,000\cdot(9{,}8\sin 45 - 4{,}44) = 24\,897\ N$

ADERENZA $\frac{T}{N} \le f_a$, $f_a = 0{,}20$

$\frac{T}{N} = 0{,}36 > f_a$ ADERENZA NON VERIFICATA $\Rightarrow$ $\boxed{\alpha \ne \frac{a}{R}}$ $\boxed{\ddot\theta \ne \frac{\ddot x}{R}}$ $\Rightarrow$

$a = g(\sin\alpha - f\cos\alpha) = 5{,}9\ m/s^2$; $t = \sqrt{\frac{2S}{a}} = 8{,}23\ s$ $\theta = \frac{1}{2}\alpha t^2$ PER TROVARE α FACCIO DI NUOVO $\curvearrowright$) CON NUOVO $\ddot x \rightarrow$

9.1) [CAPITOLO 9]



DATI

$h = 0{,}1\,m$; $k = 0{,}07\,m$; $z = 0{,}025\,m$

$f = 0{,}15$

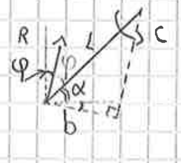
$m = 7\,kg$

$V = 5\,m/s$

$C = 40\,Nm$

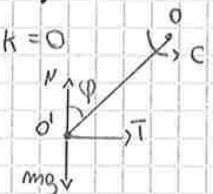
$T = ?$ (FORZA FRENANTE)

$W_d = ?$ (POTENZA DISSIPATA)



$C = bR = L\cos(\varphi+\alpha)R$

$C - Nk + T(h+z) + mgk = 0$



$T = fN \Rightarrow N = \frac{T}{f}$

$C - \frac{T}{f}k + T(h+z) + mgk = 0$

$T\left(h+z - \frac{1}{f}k\right) = -C - mgk$

$T = -\frac{(C+mgk)}{\left(h+z-\frac{1}{f}k\right)} = 131{,}1\,N$

INFO. NON USATE

$\varphi = \sin^{-1} f = 8{,}63°$

$(h+z) = k\,tg\alpha \Rightarrow \alpha = tg^{-1}\left(\frac{h+z}{k}\right) = 60{,}75°$

$k = L\cos\alpha \Rightarrow L = \frac{k}{\cos\alpha} = 0{,}143\,m$

POTENZA DISSIPATA

$P = \frac{dW}{dt} = F\frac{dS}{dt} = F\cdot V$

$W_{DISS} = T\cdot V = 131{,}1 \cdot 5 = 655{,}5\,W$

4.3) DATI

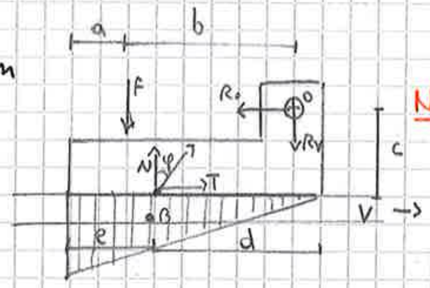
$a = 0{,}05\ m$; $b = 0{,}175\ m$; $c = 0{,}075\ m$

$f = 0{,}1$

$F = 500\ N$

$T = ?$ (FORZA FRENANTE)

$R_O, R_V = ?$ (REAZIONI AL PERNO)



NB: B BARICENTRO DELLA DISTRIBUZIONE

DETERMINIAMO e, d : IL PUNTO B SI TROVA A $\frac{1}{3}$ DELLA DISTRIBUZIONE TOTALE $(a+b)$ DA SX E A $\frac{2}{3}(a+b)$ DA DX $\Rightarrow$ $d = (a+b)\cdot\frac{2}{3}$; $e = (a+b)\cdot\frac{1}{3}$

$= 0{,}15\ m$; $= 0{,}075\ m$

$T = fN$

$$\begin{cases} \uparrow)\ N - F - R_V = 0 \\ \rightarrow)\ +T - R_O = 0 \\ \circlearrowleft)\ bF - N(b+a-e) + Tc = 0 \end{cases}$$

$\circlearrowleft)\ bF - \frac{T}{f}(b+a-e) + Tc = 0 \Rightarrow T\left(\frac{1}{f}(b+a-e) - c\right) = bF$

$$\Rightarrow T = \frac{b \cdot F}{\left[\frac{1}{f}(b+a-e) - c\right]} = \frac{0{,}175 \cdot 500}{\left[\frac{1}{0{,}1}(0{,}15) - 0{,}075\right]} = 61{,}4\ N$$

$R_O = T = 61{,}4\ N$

$R_V = N - F = \frac{T}{f} - F = \frac{61{,}4}{0{,}1} - 500 = 114\ N$

4.4)

DATI

$W = 5\ kN = 5000\ N$

$d = 40\ cm = 0{,}4\ m$ $R = 0{,}2$

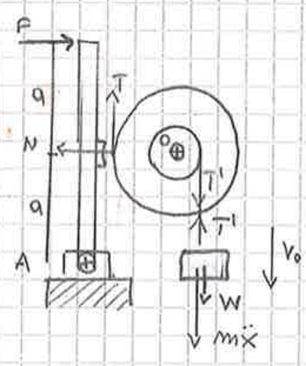
$D = 50\ cm = 0{,}5\ m$ $\mathcal{R} = 0{,}25$

$a = 30\ cm = 0{,}3\ m$

$V_0 = 0{,}2\ m/s$

$f = 0{,}5$

$F = ?$ ($\Rightarrow$ $V_F = 0$, $x = 1\ m$)



$T = fN$

$W = mg \Rightarrow m = \frac{W}{g} = \frac{5000}{9{,}8} = 510{,}2\ kg$

$ax = \frac{1}{2}V_F^2 - \frac{1}{2}V_0^2$

$a = -\frac{1}{2}\frac{V_0^2}{x} = -\frac{0{,}2^2}{2} = -0{,}02\ m/s^2$ $\qquad \ddot{x} = 0{,}02\ m/s^2$

$\uparrow)\ -W + T' = +m\ddot{x}$

$T' = W + m\ddot{x} = 5000 + 510{,}2 \cdot 0{,}02 = 5010\ N$

$\circlearrowleft)\ RT - \mathcal{R}T' = 0 \Rightarrow T = T'\frac{\mathcal{R}}{R} = 4008\ N$

$N = \frac{T}{f} = 8016\ N$

$\hat{A})\ -F \cdot 2a + N \cdot a = 0$ $\quad F = \frac{N}{2} = 4008\ N$

4.6)

DATI

$M = 150\,Nm$

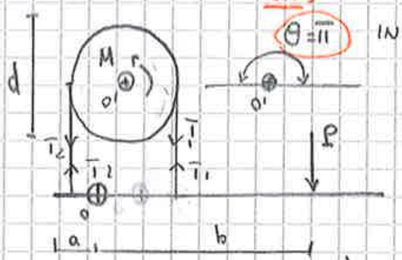
$a = 0{,}125\,m$

$b = 0{,}650\,m$

$d = 0{,}450\,m$

$f_a = 0{,}2$

$P_{MIN}\ (\Rightarrow \omega = 0) = ?$



NB! $\theta = \pi$ IN RADIANTI!

NB: $\boxed{\frac{T_1}{T_2} = e^{f\theta}}$ θ IN π (RAD)

POICHÈ: T_1, T_2: QUELLA CHE SI OPPONE AL MOMENTO DEVE ESSERE QUELLA PIÙ GRANDE $\Rightarrow T_2 = T_1 e^{f\theta}$

$O)\ T_1(d-a) - T_2 a - Pb = 0 \qquad P = \frac{T_1(d-a) - T_2 a}{b}$

$O')\ M - T_1\frac{d}{2} + T_2\frac{d}{2} = 0 \Rightarrow M = (T_1 - T_2)\frac{d}{2} \qquad T_1 > T_2 \qquad \frac{T_1}{T_2} = e^{f\theta} \qquad T_1 = T_2 e^{f\theta}$

$M = T_2(e^{f\theta} - 1)\frac{d}{2} \Rightarrow T_2 = \frac{M \cdot 2}{(e^{f\theta} - 1)d} = \frac{150 \cdot 2}{(\underbrace{e^{0{,}2 \cdot \pi} - 1}_{0{,}3932})0{,}45} = 763\,N$

$T_1 = T_2 e^{f\theta} = 763 e^{0{,}2 \cdot 3{,}14} = 1430\,N$

$P = \frac{1430(0{,}450 - 0{,}125) - 763 \cdot 0{,}125}{0{,}65} = 568{,}1\,N$

4.7)

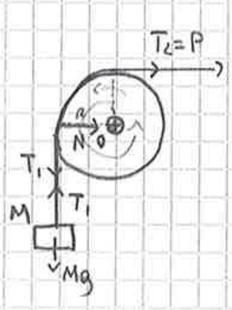
DATI

$M = 100\,kg$

$f_a = 0{,}3$

A) $\theta = 0 \quad P = ?$ ($\omega = 0$ NÈ $\uparrow$ NÈ $\downarrow$)

B) $P = 500\,N \quad \theta = ?$ (NO SCIV. DEL CARICO)



A) $T_1 = Mg = 980\,N$

⊙ SE IL CARICO TENDE A SCENDERE (ω ANTIOR.):

$\Rightarrow T_2$ SI OPPONE ALLA ROTAZIONE $\Rightarrow \left[\frac{T_2}{T_1} = e^{f\beta}\right]$

$T_2 = T_1 e^{f\beta} \quad , \beta = \frac{\pi}{2}$

$T_2 = 980 e^{0{,}3\pi/2} = 1570\,N = P_{MAX}$

⊙ SE IL CARICO TENDE A SALIRE (ω ORARIA):

$\Rightarrow T_1$ SI OPPONE ALLA ROTAZIONE $\Rightarrow \left[\frac{T_1}{T_2} = e^{f\beta}\right]$ PER GARANTIRE $\omega = 0$

$T_2 = \frac{T_1}{e^{f\beta}} \quad , \beta = \frac{\pi}{2}$

$T_2 = \frac{980}{e^{0{,}3 \cdot \pi/2}} = 612\,N = P_{min}$

4.9)

DATI

$M = 100\,kg$

$\rho = 0,3\,m$

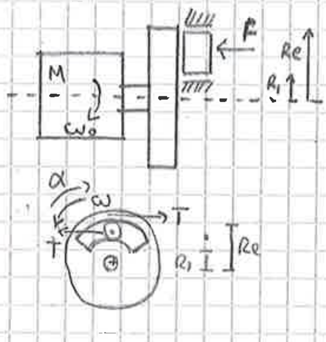
$\omega_0 = 1500$ GIRI/MIN $= 157\,rad/s$

$R_e = 20\,cm = 0,2\,m$

$R_i = 15\,cm = 0,15\,m$

$f = 0,3$

$F = ?$ ($\Rightarrow t_{ARRESTO} = 10\,s$)



$$\frac{(R_e - R_i)}{2} + R_i = \frac{R_e - R_i + 2R_i}{2} = \frac{R_e + R_i}{2}$$

$F_T = f F_N = f F$

$T = f N = f F$

$F = \frac{T}{f}$

$\cancel{I = \frac{1}{2} M \rho^2} = \frac{1}{2} 100 \cdot 0,3^2 = 4,5\,kg\,m^2$

$I = M\rho^2$!! NB: QUANDO CI DANNO ρ I È SEMPRE (!) UGUALE A $M\rho^2$

$= 9\,kg\,m^2$

$\omega = \omega_0 + \alpha t$

$0 = \omega_0 + \alpha t \Rightarrow \alpha = -\frac{\omega_0}{t} = -\frac{157}{10} = -15,7\,rad/s^2$

$T \frac{R_e + R_i}{2} = I\alpha$

$T = \frac{I\alpha \cdot 2}{R_e + R_i} = \frac{9 \cdot 15,7 \cdot 2}{0,2 + 0,15} = 807,4\,N$

$F = \frac{T}{f} = 2691\,N = 2,69\,kN$

4.11)

DATI

$D = 0,4\,m$

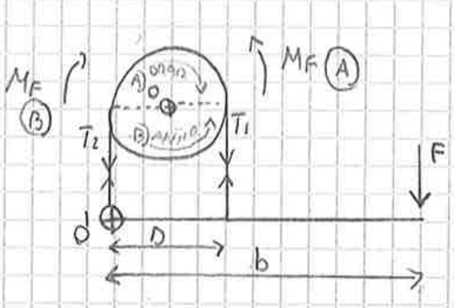
$b = 1\,m$

$\theta^* = \pi\,rad$

$f = 0,25$

$F = 200\,N$

$M_F = ?$ A) ORARIO
B) ANTI ORARIO



REAZIONI IN O:

A) $R_V - T_2 - T_1 = 0$

$R_V = T_2 + T_1 = 1596\,N$

B) $R_V - T_1 - T_2 = 0$

$R_V = T_1 + T_2 = 728,1\,N$

A) [ω ORARIA]

$\frac{T_2}{T_1} = e^{f\theta}$

$\circlearrowleft O)$ $-Fb + T_1 D = 0 \Rightarrow T_1 = \frac{Fb}{D} = \frac{200 \cdot 1}{0,4} = 500\,N$

$T_2 = T_1 e^{f\theta} = 500 e^{0,25 \cdot 3,14} = 1096\,N$

$\circlearrowleft O)$ $+ T_2 \frac{D}{2} - T_1 \frac{D}{2} = M_F \Rightarrow$

$\Rightarrow M_F = \frac{D}{2}(T_2 - T_1) = 0,2(1096 - 500) = 119,2\,Nm$

B) [ω ANTIORARIA]

$\frac{T_1}{T_2} = e^{f\theta}$ $\quad T_1 = 500\,N$

$T_2 = \frac{T_1}{e^{f\theta}} = \frac{500}{e^{0,25 \cdot 3,14}} = 228,1\,N$

$\circlearrowleft O)$ $- T_2 \frac{D}{2} + T_1 \frac{D}{2} = M_F$

$\Rightarrow M_F = \frac{D}{2}(T_1 - T_2) = 0,2(500 - 228,1) =$

$= 54,4\,Nm$

4.13)

DATI

$a = 15\ cm = 0,15\ m$

$b = 30\ cm = 0,3\ m$

$h = 5\ cm = 0,05\ m$

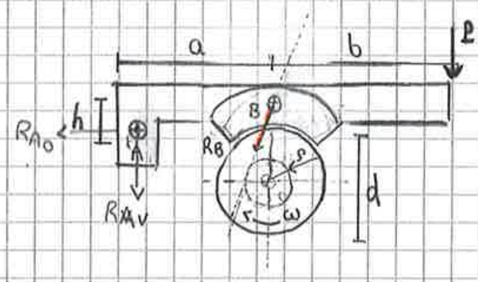
$d = 22\ cm = 0,22\ m$

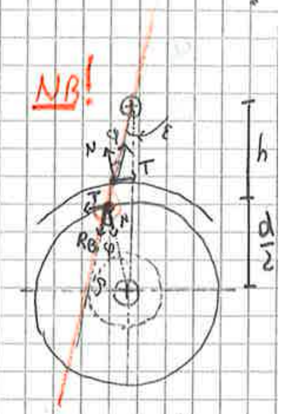
$P = 100\ N$

$f = 0,4$ $\Rightarrow$ LA FORZA ESERCITATA DAL CARRO NON GLI E' ESERCITATA SU O MA SARA' TANGENTE AL CERCHIO D'ATTRITO $\rho = \frac{d}{2} \cdot \sin\varphi$, $\varphi = tg^{-1} f$ (POICHE' $f = tg\varphi$)

$C = ?$ $(\omega = k)$

$R_A = ?$





$$\begin{cases} \uparrow) & -R_{A_V} - P + R_{B_V} = 0 \\ \rightarrow) & -R_{A_O} + R_{B_O} = 0 \\ \hat{A}) & +P(a+b) - R_{B_O}(h) + R_{B_V}(a) = 0 \end{cases}$$

$f = tg\varphi \Rightarrow \varphi = tg^{-1} f = 21,8°$

$\rho = \frac{d}{2}\sin\varphi = 0,04\ m$

$\rho = \left(h + \frac{d}{2}\right)\sin\varepsilon \Rightarrow \varepsilon = \sin^{-1}\left(\frac{\rho}{h + \frac{d}{2}}\right) = 14,5°$

$T = R_B \sin\varphi \Rightarrow R_B = \frac{T}{\sin\varphi}$

$R_{B_V} = R_B \cos\varepsilon = \frac{T}{\sin\varphi}\cos\varepsilon$

$R_{B_O} = R_B \sin\varepsilon = \frac{T}{\sin\varphi}\sin\varepsilon$

$\hat{A})\ -P(a+b) - \frac{T}{\sin\varphi}\sin\varepsilon(h) + \frac{T}{\sin\varphi}\cos\varepsilon(a) = 0$

$$\Rightarrow T = \frac{P(a+b)}{a\frac{\cos\varepsilon}{\sin\varphi} - h\frac{\sin\varepsilon}{\sin\varphi}} = 125,6\ N$$

$C = T\frac{d}{2} = 13,8\ Nm$

$\uparrow)\ R_{A_V} = \frac{T}{\sin\varphi}\cos\varepsilon - P = 227,7\ N$

4.15)

DATI

$\omega_0 = 32$ giri/min

$C_S = ?$

$t_S = ?$

$D_1 = 0{,}35\ m$; $D_2 = 0{,}8\ m$; $P = 800\ N$; $m = 420\ kg$; $I = 52\ kg\cdot m^2$; $f = 0{,}22$

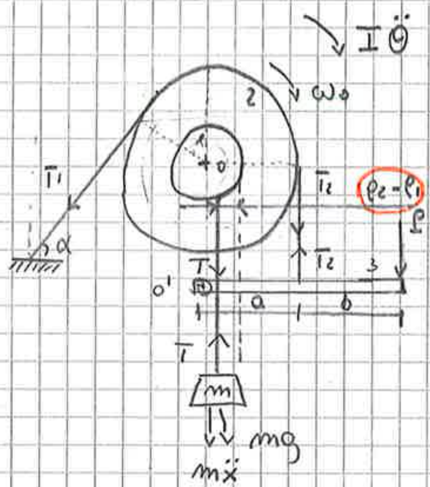
$e_1 = 0{,}02\ m$ RIGIDEZZA ELASTICA

$e_2 = 0{,}05\ m$ RIGIDEZZA ANELASTICA

$a = 0{,}3\ m$

$b = 0{,}65\ m$

$\alpha = 60°$



- $T - mg - m\ddot{x} = 0 \Rightarrow T = mg + m\ddot{x}$
- $(a+b)P - aT_2 = 0 \Rightarrow T_2 = \dfrac{(a+b)P}{a}$
- $\circlearrowleft O \quad I\ddot{\theta} + \dfrac{D}{2}T_2 + \left(\dfrac{D}{2} - (e_2 - e_1)\right)T - T_1\dfrac{D}{2} = 0$
- $\dfrac{T_1}{T_2} = e^{f\theta}$
- $\theta = 90 + \alpha = 90 + 60 = 150° = \dfrac{5}{6}\pi$ (IN QUANTO $180 : \pi = 150 : x \Rightarrow x = \dfrac{150}{180}\pi = \dfrac{5}{6}\pi$)

$\boxed{\circlearrowleft O)\ T_1\dfrac{D}{2} - T_2\dfrac{D}{2}} = C_F \Rightarrow C_F = (T_1 - T_2)\dfrac{D}{2}$

$T_2 = \dfrac{(a+b)P}{a} = 2533{,}3\ N$

$T_1 = T_2 e^{f\theta} = 4506{,}3\ N$

$\boxed{C_S = (T_1 - T_2)\dfrac{D}{2}} = 789{,}2\ Nm$ NB!

$\omega = \omega_0 - \ddot{\theta}t \Rightarrow \ddot{\theta} = \dfrac{\omega_0}{t}$

$v = v_0 - \ddot{x}t \Rightarrow \ddot{x} = \dfrac{v_0}{t}$ ESSENDO $v_0 = \dfrac{\omega_0 D_1}{2} \Rightarrow \ddot{x} = \dfrac{\omega_0 D_1}{2t}$

$T = m\left(g + \dfrac{\omega_0 D_1}{2t}\right)$

$\circlearrowleft O \quad I\dfrac{\omega_0}{t} + T_2\dfrac{D}{2} + \left(mg + m\dfrac{\omega_0 D_1}{2t}\right)\left(\dfrac{D_1}{2} - e_2 + e_1\right) - T_1\dfrac{D}{2} = 0$

$$t = \frac{I\omega_0 + \dfrac{m\omega_0 D_1}{2}\left(\dfrac{D_1}{2} + e_1 - e_2\right)}{T_1\dfrac{D_2}{2} - T_2\dfrac{D_2}{2} - mg\left(\dfrac{D_1}{2} + e_1 - e_2\right)} = 1{,}1\ s$$

5.3)

DATI — SI VUOLE FARE SCENDERE Q A V=K (Q SU VITE)

ATTRITO SOLO TRA I FILETTI = φ

ANGOLO DI INCL. FILETTI = α

ANALIZZARE LA MODALITÀ DI DISCESA DI Q = ? (A) $\varphi > \alpha$ B) $\varphi < \alpha$)

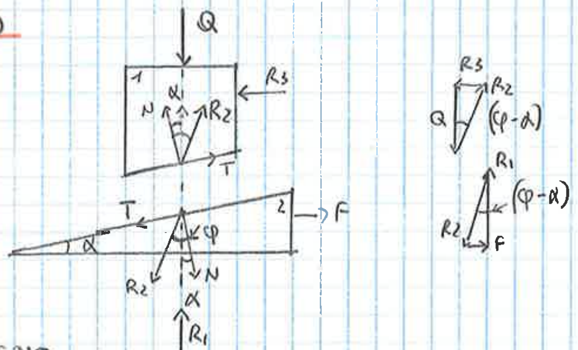
A) $\varphi > \alpha$ — NON SCENDE DA SOLO

VITE: $\uparrow)$ $-Q + R_2 \cos(\varphi - \alpha) = 0$

MADREVITE: $\rightarrow)$ $F - R_2 \sin(\varphi - \alpha) = 0$

$F = Q \tan(\varphi - \alpha)$

=> E' NECESSARIO TIRARE IL CUNEO 2



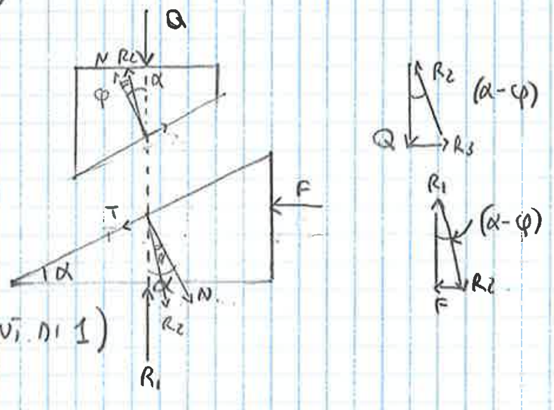
B) $\varphi < \alpha$ — IL CARICO SCENDE DA SOLO

VITE: $\uparrow)$ $-Q + R_2 \cos(\alpha - \varphi) = 0$

MADREVITE: $\rightarrow)$ $-F + R_2 \sin(\alpha - \varphi) = 0$

$F = Q \tan(\alpha - \varphi)$

=> SE SI VUOLE V=K OCCORRE FRENARE IL CUNEO 2 (E QUINDI LA DISCESA SPONT. DI 1)



5.4)

DATI

$i = 3$

RUOTA PICCOLA = MOTRICE (1)

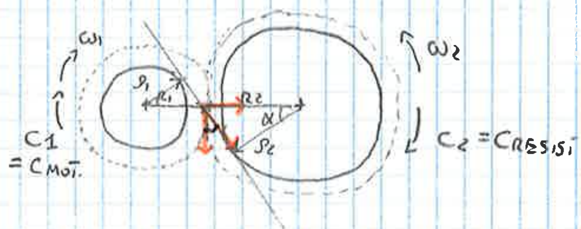
$z_1 = 30$

$de_1 = 0{,}128\ m$

$p = 0{,}01257\ m$

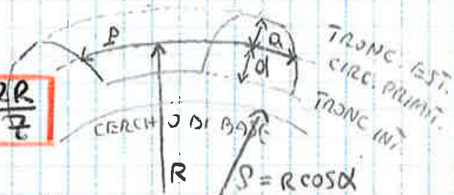
MODULO = ? DIAM. PRIMITIVI = ? DIAMETRI DI TRONCATURA INTERNA = ?

$de_2 = ?$ $z_2 = ?$



$\boxed{p = \frac{2\pi R}{z}}$

$\boxed{m = \frac{p}{\pi} = \frac{2R}{z}}$



$\boxed{\rho = R\cos\alpha}$

a = m ADDENDUM

d = 1,25 m DEDENDUM

$dp_1 = 2R_1 = 0{,}12\ m$

$dp_2 = 2R_2 = 0{,}36\ m$

$D_{TE_2} = 2(R_1 + a) = 0{,}368\ m$ $\quad D_{Ti_2} = 2(R_2 - 1{,}25m) = 0{,}35\ m$

INTERASSE = $R_1 + R_2$

$\boxed{i = \frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} = \frac{z_2}{z_1}}$

$a = m$

$d = 1{,}25\ m$

$z_2 = i z_1 = 90$

$m = \frac{p}{\pi} = 4 \cdot 10^{-3}\ m$

$D_{TE_1} = 2(R_1 + a) \Rightarrow R_1 = \frac{D_{TE_1}}{2} - a = 0{,}06\ m$

$D_{Ti_1} = 2(R_1 - 1{,}25m) = 0{,}11\ m$

$R_2 = i \cdot R_1 = 0{,}18\ m$

5.7)

DATI

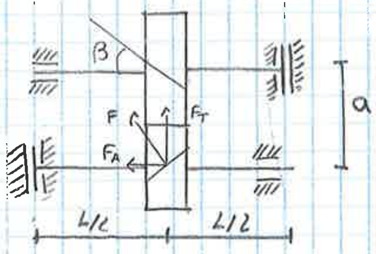
$i=2$; $a=0{,}156\,m$; $\alpha_m=20°$; $m_n=0{,}00275\,m$; $z_1=37$; $L=0{,}76\,m$ $P_{E_{NTRATA}}=1\,CV$

$\omega_1=720$ giri/min $=\frac{720\cdot 2\pi}{60}=75{,}36$ rad/s

$R_1=?$

$R_2=?$

$\beta=?$



NB!

$\boxed{m_n=m\cos\beta}$

$\boxed{tg\,\alpha_n=tg\,\alpha\cos\beta}$

$$\begin{cases} i=\frac{R_2}{R_1} \Rightarrow R_2=iR_1 \qquad i=\frac{R_2}{R_1}=\frac{\omega_1}{\omega_2} \\ R_1+R_2=a \Rightarrow R_1(1+i)=a \Rightarrow R_1=\frac{a}{1+i}=\frac{0{,}156}{1+2}=0{,}052\,m \end{cases}$$

$R_2=iR_1=0{,}104\,m$

$P_M=C_1\omega_1 \Rightarrow C_1=\frac{P_M}{\omega_1}=9{,}7\,Nm$

$\eta=1=\frac{C_2\omega_2}{C_1\omega_1}=\frac{C_2}{C_1 i} \Rightarrow C_2=1\cdot C_1 i=19{,}5\,Nm$

$\omega_2=\frac{\omega_1}{i}=37{,}7$ rad/s

$m=\frac{p}{\pi}=\frac{2\pi R_1}{\pi z_1}=\frac{2R_1}{z_1}=0{,}0028\,m$

$\beta=\cos^{-1}\left(\frac{m_n}{m}\right)=11{,}94°$

5.4)

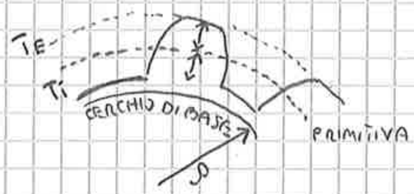
DATI

$i = 3$

$z_1 = 30$

$D_{T_1} = 0,128\ m$

$p_1 = 0,01257\ m$



$m = ?\ d_1, d_2$, INTERASSE, D_{T_2}, d_{T_1}, d_{T_2}, $z_2 = ?$

$i = \frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} = \frac{z_2}{z_1}$

$z_2 = i z_1 = 90$

$m = \frac{p}{\pi} = 0,004\ m \qquad d = 1,25 m = 0,005\ m$

$R_1 = \frac{D_{T_1}}{2} - m = 0,06\ m \qquad d_1 = 0,12\ m$

$R_2 = i R_1 = 0,18\ m \qquad d_2 = 0,36\ m$

INTERASSE $= R_1 + R_2 = 0,24\ m$

$d_{T_1} = 2(R_1 - d) = 2(0,06 - 0,005) = 0,11\ m$

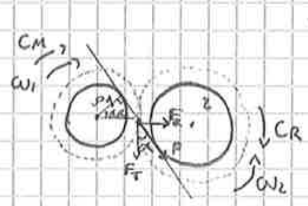
$d_{T_2} = 2(R_2 - d) = 2(0,18 - 0,005) = 0,35\ m$

$D_{T_2} = 2(R_2 + a) = 2(0,18 + 0,004) = 0,368\ m$

5.5)

DATI

$C_M = 150\ Nm$



$\alpha = 20°$ ANGOLO DI PRESSIONE

$C_M = R_1 F_T \Rightarrow F_T = \frac{C_M}{R_1} = \frac{150}{0,06} = 2500 N$

$F_T = F \cos\alpha \Rightarrow F = \frac{F_T}{\cos\alpha} = 2660,4\ N$

5.7)

DATI

$i = 2$

INTERASSE $(R_1 + R_2) = 0{,}156\ m$

$\alpha_m = 20°$; $m_m = 0{,}00275\ m$;

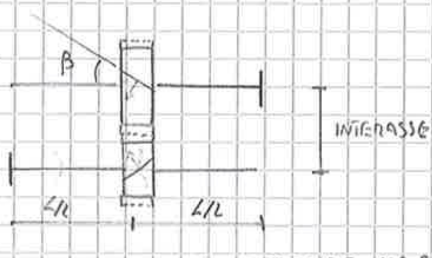
$z_1 = 37$

$L = 0{,}076\ m$

$P_e = 1\ CV = 735{,}5\ W$ $= 75{,}36\ rad/s$

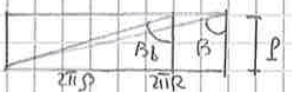
$\omega_1 = 720\ giri/min =$

$R_1 = ?\ R_2 = ?\ \beta = ?$ (SU CIL. PRIM.) F_R (CUSC. + CARICATO) = ?

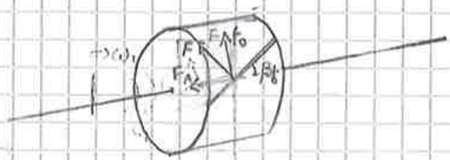


ESSENDO MESSI IN TAL MODO I CUSCINETTI ⇒

$$\begin{cases} i = \dfrac{R_2}{R_1} & R_2 = R_1 i \\ R_1 + R_2 = 0{,}156 & R_1(1+i) = 0{,}156 \end{cases} \Rightarrow R_1 = \frac{0{,}156}{(1+2)} = 0{,}052\ m \qquad R_2 = 0{,}104\ m$$



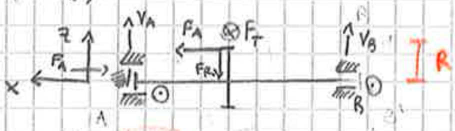
$p = \dfrac{2\pi R_1}{z_1} = 0{,}0088\ m$



$2\pi\rho = p\, Tg\beta_b = 2\pi R\cos\alpha$ $\Rightarrow \boxed{Tg\beta_b = Tg\beta\cos\alpha}$

$2\pi R = p\, Tg\beta$

$\boxed{Tg\,\alpha_m = Tg\alpha\cos\beta}$

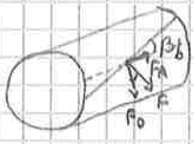


$\hat{A})\ +F_A R_1 + V_B L - F_R \dfrac{L}{2} = 0 \Rightarrow V_B = F_R\dfrac{L}{2} - F_A R_1$

$\hat{B})\ -V_A L + F_A R_1 + F_R \dfrac{L}{2} = 0 \Rightarrow V_A = F_R\dfrac{L}{2} \oplus F_A R_1$

⇒ (A) CUSCINETTO PIÙ CARICATO

$P_e = C_1\omega_1 \Rightarrow C_1 = \dfrac{P_e}{\omega_1} = \dfrac{735{,}5}{75{,}36} = 9{,}76\ Nm$



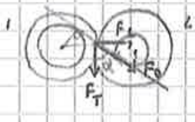
$F_0 = F\cos\beta_b$

$F_A = F\sin\beta_b$

$C_1 = \cancel{R_1 F_0 \Rightarrow F_0 = \dfrac{C_1}{R_1} = \dfrac{9{,}76}{0{,}052} = 187{,}7\ N}$ NO! $C_1 = R_1 F_T$

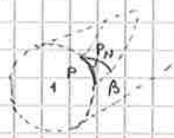
NB:

$C_1 = R_1 F_T \Rightarrow F_T = \dfrac{C_1}{R_1} = \dfrac{9{,}76}{0{,}052} = 187{,}7\ N$



$F_T = F_0\cos\alpha$

$F_R = F_0\sin\alpha$

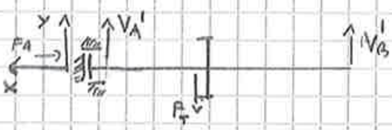


$m = \dfrac{p}{\pi} = \dfrac{2\pi R_1}{\pi z_1} = \dfrac{2R_1}{z_1} = \dfrac{2\cdot 0{,}052}{37} = 0{,}0028\ m$

$p_N = p\cdot\cos\beta$

$m_N = \dfrac{p_N}{\pi} = \dfrac{p\cos\beta}{\pi} = \dfrac{2R_1}{z_1}\cos\beta = m\cos\beta \Rightarrow \cos\beta = \dfrac{m_m}{m}$ $\quad \beta = \cos^{-1}\left(\dfrac{m_m}{m}\right) = 10{,}84°$

$Tg\alpha = \dfrac{Tg\,\alpha_m}{\cos\beta} \Rightarrow \alpha = Tg^{-1}\left(\dfrac{Tg\,\alpha_m}{\cos\beta}\right) = 20{,}33°$



$R_A = \sqrt{V_A'^2 + V_A^2} = 111{,}5\ N$

$V_A' + V_B' - F_T = 0 \Rightarrow V_A' = F_T - \dfrac{F_T}{2} = \dfrac{F_T}{2}$

$\hat{A})\ -F_T\dfrac{L}{2} + L V_B' = 0 \Rightarrow V_B' = \dfrac{F_T}{2}$

5.10)

DATI

$W_t = 20\,kW$ POTENZA TRASMESSA

$\omega_1 = 1500$ giri/min $= 157$ rad/s

$i = 2$

$z_1 = 14$

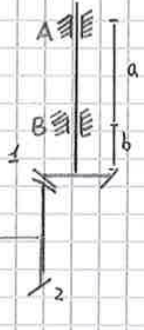
$m = 0{,}005\,m$ MODULO AL RAGGIO MEDIO

$\alpha = 20°$ ANGOLO DI PRESSIONE

$a = 0{,}2\,m$

$b = 0{,}1\,m$

$R_A = ?$ $R_B = ?$



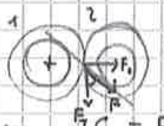
$\delta_1 + \delta_2 = 90$

$i = \frac{\sin\delta_2}{\sin\delta_1}$ ESSENDO $\sin\delta_1 = \cos\delta_2 \Rightarrow i = tg\,\delta_2$

$\Rightarrow \delta_2 = tg^{-1}(i) = 63{,}4°$

$\delta_1 = 26{,}6°$

$i = \frac{\omega_1}{\omega_2} \Rightarrow \omega_2 = \frac{\omega_1}{i} = \frac{157}{2} = 78{,}5$ rad/s



$P_E = C_1\omega_1 \Rightarrow C_1 = \frac{P_E}{\omega_1} = \frac{20\,000}{157} = 127{,}4\,Nm$

$C_1 = F_T R_1 \Rightarrow F_T = \frac{C_1}{R_1} = \frac{127{,}4}{0{,}035} = 3640\,N$

$i = \frac{z_2}{z_1} \Rightarrow z_2 = i z_1 = 28$

$F_0 = \cancel{F_R} = F_T\,tg\,\alpha = 3640 \cdot tg\,20 = 1324{,}8\,N$

NB! (RIPASSA RUOTE CONICHE)

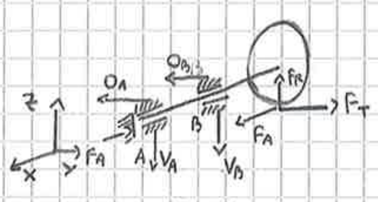
$F_R = F_0 \cos\delta_1 = 1184{,}5\,N$

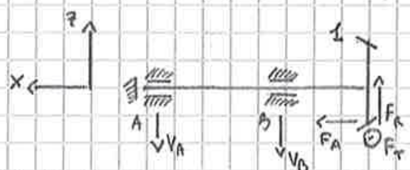
$F_A = F_0 \sin\delta_1 = 593{,}1\,N$

$p = \frac{2\pi R}{z}$ $\quad R = \frac{pz}{2\pi}$

$m = \frac{p}{\pi}$ $\quad p = m \cdot \pi = 0{,}005 \cdot 3{,}14 = 0{,}0157\,m$

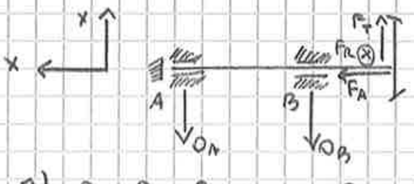
$R_1 = \frac{p z_1}{2\pi} = 0{,}035\,m$; $R_2 = \frac{p z_2}{2\pi} = 0{,}07\,m$ $\quad \rho_1 = R_1 \cos\alpha = 0{,}0329\,m$





$\uparrow)\ -V_A - V_B + F_R = 0 \Rightarrow V_A = F_R - V_B = 488{,}5\,N$

$\hat{A})\ -V_B a + F_R(a+b) - \rho_1 F_A = 0 \Rightarrow V_B = \frac{F_R(a+b) - \cancel{\rho_1} R_1 F_A}{a} = 1673\,N$



$\uparrow)\ -O_A - O_B + F_T = 0 \quad O_A = F_T - O_B = -1820\,N$

$\hat{A})\ -O_B a + F_T(a+b) = 0 \Rightarrow O_B = \frac{F_T(a+b)}{a} = 5460\,N$

$R_A = \sqrt{V_A^2 + O_A^2} = 1884\,N$

$R_B = \sqrt{V_B^2 + O_B^2} = 5710\,N$

5.13)

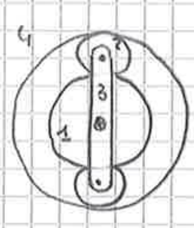
DATI

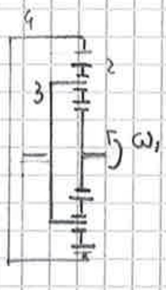
$z_1 = 30$; $z_2 = 18$

$\omega_1 = 30$ rad/s

$\omega_3 = ?$

TUTTE LE RUOTE HANNO LO STESSO MODULO!





$$\frac{\omega_1^*}{\omega_4^*} = \frac{\omega_1^*}{\omega_2^*}\frac{\omega_2^*}{\omega_4^*} = -\frac{z_2}{z_1}\cdot\frac{z_4}{z_2} = -\frac{z_4}{z_1} = \frac{\omega_1 - \omega_3}{\omega_4 - \omega_3}$$

$\omega_4 = 0$ FORMULA DI WILLIS

$$-\frac{z_4}{z_1} = -\frac{\omega_1}{\omega_3} + 1 \Rightarrow \frac{\omega_1}{\omega_3} = 1 + \frac{z_4}{z_1} = \frac{z_1 + z_4}{z_1}$$

$$\Rightarrow \omega_3 = \frac{z_1}{(z_1 + z_4)}\omega_1$$

STESSO m

$$m = \frac{2R}{z}$$

$$\begin{cases} \frac{2R_1}{z_1} = \frac{2R_2}{z_2} = \frac{2R_4}{z_4} \\ R_4 = (R_1 + 2R_2) \end{cases}$$

$$z_4 = \frac{z_2}{\cancel{2}R_2}\cdot\cancel{2}R_4 = \frac{z_2}{R_2}(R_1 + 2R_2) = z_2\left(\frac{R_1}{R_2} + 2\right) \qquad \frac{R_1}{R_2} = \frac{z_1}{z_2} \Rightarrow$$

$$z_4 = z_2\left(\frac{z_1}{z_2} + 2\right) = z_1 + 2z_2 = 66$$

$$\omega_3 = \frac{z_1}{z_1 + z_4}\omega_1 = 9{,}375 \text{ rad/s}$$

CONCORDE CON ω_1

5.14)

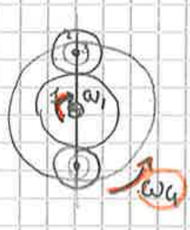
DATI

$\omega_1 = 400$ giri/min $= 41{,}8667$ rad/s

$\omega_4 = 50$ giri/min $= 5{,}23$ rad/s

$z_1 = 15$; $z_2 = 25$; $z_3 = 15$; $z_4 = 55$

$\omega_5 = ?$ $\omega_2 = ?$ $\frac{\omega_1}{\omega_5} = ?$



$$\frac{\omega_1^*}{-\omega_4^*} = \frac{\omega_1^*}{\omega_2^*}\frac{\omega_2^*}{\omega_3^*}\frac{\omega_3^*}{\omega_4^*} = -\frac{z_2}{z_1}(1)\frac{z_4}{z_3} = -\frac{z_2 z_4}{z_1 z_3} = \frac{\omega_1 - \omega_5}{\omega_4 - \omega_5}$$

$$\omega_1 - \omega_5 = -\frac{z_2 z_4}{z_1 z_3}(\omega_4 - \omega_5)$$

$$\omega_1 - \omega_5 = -\frac{z_2 z_4}{z_1 z_3}\omega_4 + \omega_5\frac{z_2 z_4}{z_1 z_3}$$

$$\omega_5\left(1 + \frac{z_2 z_4}{z_1 z_3}\right) = \omega_1 + \omega_4\frac{z_2 z_4}{z_1 z_3}$$

$$\omega_5 \cdot \left(\frac{z_1 z_3 + z_2 z_4}{z_1 z_3}\right) = \frac{\omega_1 z_1 z_3 + \omega_4 z_2 z_4}{z_1 z_3}$$

$$\omega_5 = \frac{\omega_1 z_2 z_3 + \omega_4 z_2 z_4}{z_1 z_3 + z_2 z_4} = 14{,}31 \text{ rad/s}$$

NO PERCHE' ω_4 E' DISCORDE CON ω_1!

$-\Omega = \omega_5$

$$\frac{\omega_1^*}{-\omega_4} = \frac{\omega_1^*}{\omega_2^*}\frac{\omega_2^*}{\omega_3^*}\frac{\omega_3^*}{\omega_4^*} = -\frac{z_2}{z_1}(1)\frac{z_4}{z_3} = -\frac{z_2 z_4}{z_1 z_3} = \frac{\omega_1 - \Omega}{-\omega_4 - \Omega}$$

$$-\frac{\omega_1 - \Omega}{\omega_4 + \Omega} = -\frac{z_2 z_4}{z_1 z_3}$$

$$\omega_1 - \Omega = \frac{z_2 z_4}{z_1 z_3}(\omega_4 + \Omega)$$

$$\Omega = \left(\omega_1 - \frac{z_2 z_4}{z_1 z_3}\omega_4\right)\frac{z_1 z_3}{z_1 z_3 + z_2 z_4} = 13{,}3 \text{ rad/s}$$

$$\frac{\omega_1^*}{-\omega_2^*} = -\frac{z_2}{z_1} = \frac{\omega_1 - \Omega}{-\omega_2 - \Omega}$$

$$-\frac{\omega_1 - \Omega}{\omega_2 + \Omega} = -\frac{z_2}{z_1} \Rightarrow \omega_2 + \Omega = (\omega_1 - \Omega)\frac{z_1}{z_2}$$

$$\omega_2 = \omega_1\frac{z_1}{z_2} - \Omega\frac{z_1}{z_2} - \Omega = \omega_1\frac{z_1}{z_2} - \Omega\left(1 + \frac{z_1}{z_2}\right) = 21{,}8 \text{ rad/s}$$

$$i = \frac{\omega_1}{\omega_5} = \frac{\omega_1}{\Omega} = 30{,}1$$

5.15)

DATI

$z_1 = 97$; $z_2 = 17$, $z_3 = 18$

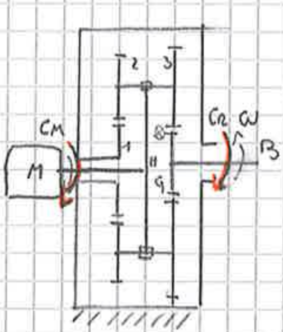
$m = 0{,}005$; $\alpha = 20°$

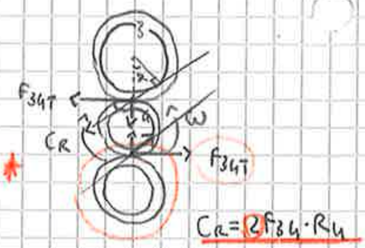
$P_E = 1{,}2\ kW$; $\omega_M = 300$ giri/min $= 31{,}4$ rad/s

$i = \frac{\omega_M}{\omega_B} = ?$

COPPIA DI REAZIONE DELLA STRUTTURA? (C_V)

F_T, F_R DI 1-2, 3-4 = ?





$C_R = 2F_{34} \cdot R_4$

$$\frac{\omega_1^*}{\omega_4^*} = \frac{\omega_1^*}{\omega_2^*}\frac{\omega_2^*}{\omega_3^*}\frac{\omega_3^*}{\omega_4^*} = -\frac{z_2}{z_1}(1)\left(-\frac{z_4}{z_3}\right) = \frac{z_2 z_4}{z_1 z_3} = \frac{\omega_1 - \Omega}{\omega_4 - \Omega}$$

$P_E = C_M \omega_M$

$C_M = \frac{P_E}{\omega_M} = \frac{1200}{31{,}4} = 38{,}2\ Nm$

$$\omega_4 - \Omega = (\omega_1 - \Omega)\left(\frac{z_1 z_3}{z_2 z_4}\right) \Rightarrow \omega_4 = (\omega_1 - \Omega)\frac{z_1 z_3}{z_2 z_4} + \Omega$$

$$\begin{cases} m = \frac{2R}{z} = \frac{2R_1}{z_1} = \frac{2R_2}{z_2} = \frac{2R_3}{z_3} = \frac{2R_4}{z_4} \\ R_1 + R_2 = R_3 + R_4 \end{cases}$$

$R_4 = R_1 + R_2 - R_3$ $\quad R_1 = \frac{R_2 z_1}{z_2}$ $\quad R_3 = \frac{R_2 z_3}{z_2}$ $\quad R_4 = \frac{R_2 z_1}{z_2} + R_2 - \frac{R_2 z_3}{z_2} = R_2\left(\frac{z_1}{z_2} + 1 - \frac{z_3}{z_2}\right) =$

$m = \frac{2R_2}{z_2}$ $\quad R_2 = \frac{m z_2}{2} = 0{,}0425$ $\quad = \left(\frac{z_1 + z_2 - z_3}{z_2}\right)R_2$

$\frac{2R_4}{z_4} = \frac{2R_2}{z_2}$ $\quad \cancel{2}\left(\frac{z_1 + z_2 - z_3}{z_2}\right)\frac{\cancel{R_2}}{z_4} = \frac{\cancel{2R_2}}{z_2}$ $\quad R_1 = \frac{m z_1}{2} = 0{,}2425$ $\quad R_4 = 0{,}24$

$z_4 = \frac{\cancel{z_2}(z_1 + z_2 - z_3)}{\cancel{z_2}} = 96$ $\quad R_3 = \frac{m z_3}{2} = 0{,}045$

$\omega_1 = 0$

$$\omega_4 = \Omega\left(1 - \frac{z_1 z_3}{z_2 z_4}\right) = 31{,}4\left(1 - \frac{97 \cdot 18}{17 \cdot 96}\right) = -2{,}19\ rad/s$$

$i = \frac{\Omega}{\omega_4} = -14{,}3$ $\quad C_R = F_{T_{34}} R_4$ $\quad \eta = 1 = \frac{C_R \omega_4}{C_M \omega_M} \Rightarrow C_R = \frac{C_M \omega_M}{\omega_4} = \frac{38{,}2 \cdot 31{,}4}{(-)2{,}19} = (-)547{,}94\ Nm$

$\vec{C}_M \longrightarrow$ $\quad C_V = C_M + C_R = 586\ Nm$

$\vec{C}_R \longrightarrow$

$F_{34T} = \frac{C_R}{R_4} = \frac{547{,}94}{0{,}24} = 2283{,}1\ N$ $\quad F_{34T} = F_{34}\cos\alpha \Rightarrow F_{34} = \frac{F_{34T}}{\cos\alpha} = 2429{,}6\ N$

$F_{34R} = F_{34T}\,tg\,\alpha = 831\ N$ $\quad$ DIMENTICHI CHE CI SONO DUE RUOTE 3!

$C_R = 2F_{34T} R_4 = 2F_{34}\cos\alpha\, R_4 \Rightarrow F_{34} = \frac{C_R}{2R_4\cos\alpha} = 1214{,}8\ N$

$C_3 = F_{43T} \cdot R_3 = 51{,}37\ Nm$

$C_3 = C_2 = F_{12T} R_2 = F_{12}\cos\alpha\, R_2 \Rightarrow F_{12} = \frac{C_2}{R_2 \cos\alpha} = 1286{,}3\ N$

5.17)

<u>DATI</u>

$R_1 = 0{,}2\,m$

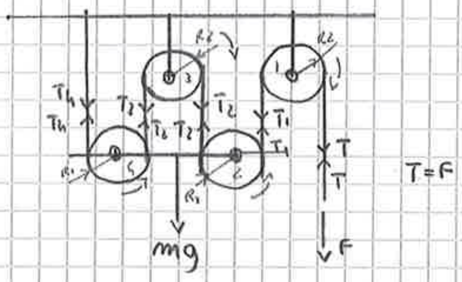
$R_2 = 0{,}15\,m$

$m = 800\,kg$

A) $F = ?$

B) $F' = ?$

$f = 0{,}3 \quad R_p = 0{,}02\,m$



A)

1) $\uparrow)\ R_{V_1} - T_1 - F = 0$

$\overset{\curvearrowleft}{O_1})\ +T_1 R_2 - F R_2 = 0 \Rightarrow T_1 = F$

3) $R_V - T_3 - T_2 = 0$

$\overset{\curvearrowleft}{O_3})\ T_3 R_2 - T_2 R_2 = 0 \Rightarrow T_3 = T_2$

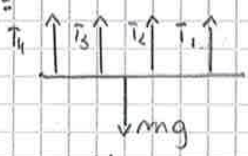
BOZZELLO: $R_4 + R_{V_3} = mg \Rightarrow R_4 = R_{V_3} = \frac{mg}{2}$

2) $T_2 + T_1 - \frac{mg}{2} = 0$

$\overset{\curvearrowleft}{O_2})\ -T_2 R_1 + T_1 R_1 = 0 \Rightarrow T_1 = T_2$

$\Rightarrow 2T_1 = \frac{mg}{2} \Rightarrow T_1 = \frac{mg}{4} = F = 1960\,N$

<u>PIÙ SEMPLICEMENTE:</u>



$T_1 = T_2 = T_3 = T_4 = T$

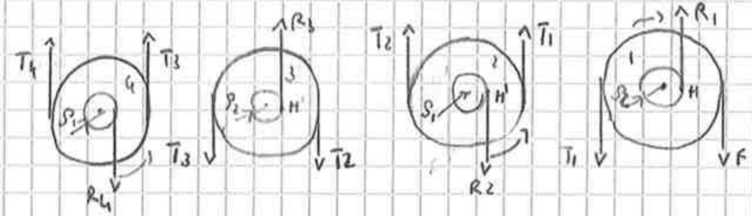
$$\begin{cases} mg = 4T \Rightarrow T = \frac{mg}{4} \\ T = F \Rightarrow F = \frac{mg}{4} = 1960\,N \end{cases}$$

B) $f = \tan\varphi \cong \sin\varphi$;

$\rho = R\sin\varphi \cong Rf$; $\varphi = \tan^{-1} f$

$\rho_1 = R_1 \sin\varphi = 0{,}057\,m$

$\rho_2 = R_2 \sin\varphi = 0{,}043\,m$



$T_1 + T_2 + T_3 + T_4 = mg$

$F'(R_2 - \rho_2) = T_1 (R_2 + \rho_2)$

$T_1(R_1 - \rho_1) = T_2(R_1 + \rho_1)$

$T_2(R_2 - \rho_2) = T_3(R_2 + \rho_2)$

$T_3(R_1 - \rho_1) = T_4(R_1 + \rho_1)$

$$T_3 = T_4 \frac{(R_1+\rho_1)}{(R_1-\rho_1)}$$

$$T_2 = T_4 \frac{(R_1+\rho_1)}{(R_1-\rho_1)} \frac{(R_2+\rho_2)}{(R_2-\rho_2)}$$

$$T_1 = T_4 \frac{(R_1+\rho_1)}{(R_1-\rho_1)} \frac{(R_2+\rho_2)}{(R_2-\rho_2)} \frac{(R_1+\rho_1)}{(R_1-\rho_1)}$$

$$F' = T_4 \left(\frac{R_1+\rho_1}{R_1-\rho_1}\right)^2 \left(\frac{R_2+\rho_2}{R_2-\rho_2}\right)^2$$

$$mg = T_4\left(1 + \frac{(R_1+\rho_1)}{(R_1-\rho_1)} + \frac{(R_1+\rho_1)}{(R_1-\rho_1)}\frac{(R_2+\rho_2)}{(R_2-\rho_2)} + \left(\frac{R_1+\rho_1}{R_1-\rho_1}\right)^2\left(\frac{R_2+\rho_2}{R_2-\rho_2}\right)\right)$$

$$F' = \frac{mg\left(\frac{R_1+\rho_1}{R_1-\rho_1}\right)^2\left(\frac{R_2+\rho_2}{R_2-\rho_2}\right)^2}{1 + \left(\frac{R_1+\rho_1}{R_1-\rho_1}\right) + \left(\frac{R_1+\rho_1}{R_1-\rho_1}\right)\left(\frac{R_2+\rho_2}{R_2-\rho_2}\right) + \left(\frac{R_1+\rho_1}{R_1-\rho_1}\right)^2\left(\frac{R_2+\rho_2}{R_2-\rho_2}\right)} = 2322\,N$$

$$\begin{cases} F' \cdot m\eta = mg \\ m = 4 \end{cases} \qquad \eta = \frac{mg}{4F'} = 0{,}8441$$

5.19)

DATI

$C_M =$

5.21)

DATI

$D = 0{,}08\ m$

$R = 0{,}3\ m$

$m_2 = 200\ kg$

$m_4 = 500\ kg$

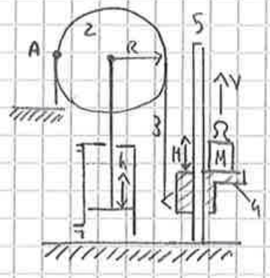
$M = 3000\ kg$

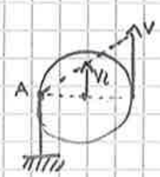
$\eta = 1$

$h = 3\ m$

$V = k$

$H = ?\quad p = ?$

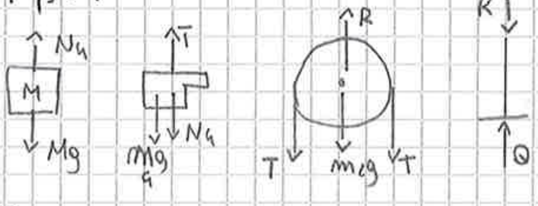




$V = \omega \cdot 2R \qquad V = \frac{H}{t} \qquad t = \frac{H}{V} \qquad V = 2V_2 \qquad t = \frac{H}{2V_2}$

$V_2 = \omega R \qquad V_2 = \frac{h}{t} \qquad t = \frac{h}{V_2} \qquad t = \frac{h}{V_2}$

$\Rightarrow \frac{H}{2V_2} = \frac{h}{V_2} \Rightarrow H = 2h = 6\ m$



$N_4 = Mg$

$T = m_4 g + N_4 = m_4 g + Mg$

$R = m_2 g + 2T = m_2 g + 2(m_4 g + Mg)$

$Q = R = m_2 g + 2(m_4 g + Mg)$

$p \; \frac{Q}{\frac{\pi D^2}{4}} = \frac{4m_2 g + 8(m_4 g + Mg)}{\pi D^2} = 14{,}05 \cdot 10^6\ PA$

5.22)

DATI

$\omega_2 = 37{,}7\ rad/s$

$Q = 180\ N$

$D_2 = 0{,}3\ m$

$a = 0{,}3\ m$

$b = 0{,}405\ m$

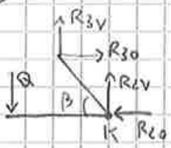
$\alpha = 30°$

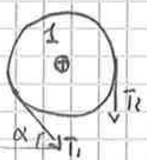
$\beta = 60°$

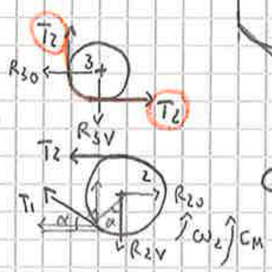
$f_a = 0{,}3$

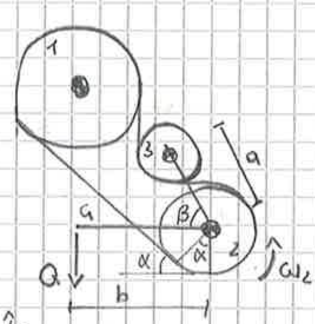
$T_1, T_2 = ?$

$P_{MAX} = ?$ (DA MOTA 2)









$R_{3V} = T_2$

$R_{3O} = T_2$

$C_M = (T_1 - T_2)\frac{D}{2}$

$\overset{\curvearrowleft}{K}$) $Qb - R_{3V} \cdot a\cos\beta - R_{3O}\, a\sin\beta = 0 \Rightarrow Qb - T_2 a\cos\beta - T_2 a\sin\beta = 0$

$T_2 = \frac{Qb}{a(\cos\beta + \sin\beta)} = 178\ N \qquad \frac{T_1}{T_2} = e^{f\theta},\ \theta = \frac{\pi}{6} + \pi = \frac{7}{6}\pi \Rightarrow T_1 = 534\ N$

$C_M = (T_1 - T_2)\frac{D}{2} = 53{,}4\ Nm \qquad P_{MAX} = C_M \cdot \omega_2 = 2014\ W$

5.24)

DATI

$M = 100\,kg$

$d = 0{,}03\,m$

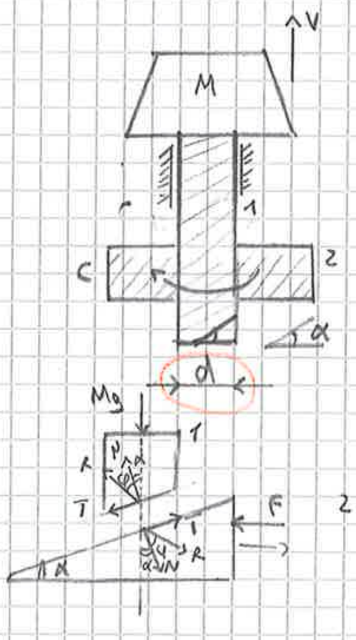
$\alpha = 3°$

$f = 0{,}1 \quad Tg^{-1} f = \varphi$

$v = k$

$C = ?$

$C' = 5\,Nm \rightarrow \ddot{x} = ?$



1) $\uparrow) \; -Mg + R\cos(\varphi+\alpha) = 0$

2) $\rightarrow) \; R\sin(\varphi+\alpha) - F = 0$

$$R = \frac{Mg}{\cos(\varphi+\alpha)}$$

$$F = R\sin(\varphi+\alpha) = Mg\,Tg(\varphi+\alpha) = 150{,}1\,N$$

$$C = F\frac{d}{2} = 2{,}26\,Nm$$

~ · ~

$C' = 5\,Nm \quad \ddot{x} = ?$

$$F = \frac{C}{d}2 = 333{,}3\,N$$

2) $R\sin(\varphi+\alpha) - F = 0$

$$R = \frac{F}{\sin(\varphi+\alpha)} = \frac{333{,}3}{\sin(5{,}71+3)} = 2201\,N$$

1) $-Mg + R\cos(\varphi+\alpha) = Ma$

$$a = \frac{R\cos(\varphi+\alpha)}{M} - \frac{\cancel{M}g}{\cancel{M}} = \frac{2201\cos 8{,}71}{100} - 9{,}8 = 11{,}95\,m/s^2$$

5.26)

DATI:

$a = 0{,}5\ m$

$b = 0{,}35\ m$

$F = 700\ N$

$d_m = 0{,}012\ m$

$p = 0{,}005\ m$

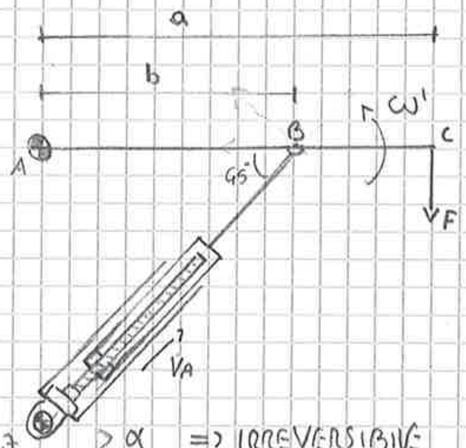
$f = 0{,}2 \quad \varphi = Tg^{-1} f = 11{,}3$

$f_a = 0{,}3 \quad \varphi_A = Tg^{-1} f_a = 16{,}7 > \alpha \Rightarrow$ IRREVERSIBILE

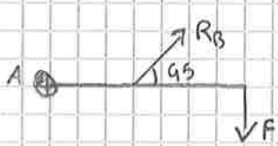
$P = 500\ W$

$C = ?$

$V_A, V_C = ?$ (V ATTUATORE ; V DEL PUNTO C)



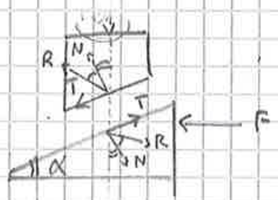
$p = \pi d_m Tg\alpha \quad \alpha = Tg^{-1}\left(\frac{p}{\pi d_m}\right) = 7{,}5°$



PER TROVARE R_B: NO TRAMITE EQ VERTICALE: $\uparrow)\ R_B \sin 45 - F = 0$

POICHE' MANCHEREBBE V_A $\Rightarrow$

$\Rightarrow A\curvearrowleft) + R_B \sin 45 \cdot b - F a = 0 \Rightarrow R_B = \frac{F\,a}{b \sin 45} = 1414{,}2\ N$

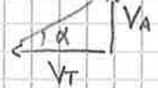


$\uparrow)\ -R_B + R\cos(\varphi+\alpha) = 0 \qquad R = \frac{R_B}{\cos(\varphi+\alpha)}$

$\rightarrow)\ -F + R\sin(\varphi+\alpha) = 0 \qquad F = R\sin(\varphi+\alpha) = R_B Tg(\varphi+\alpha) = 487{,}4\ N$

$C = F \cdot \frac{d}{2} = 2{,}9\ Nm$

$P = C\omega \Rightarrow \omega = \frac{P}{C} = 172{,}4\ rad/s$



$V_A = V_T Tg\alpha = \omega \frac{d_m}{2} Tg\alpha = 0{,}14\ m/s$

$V_C = \omega' \cdot a$

NB $V_B = V_A + V_{B/A}$

$V_B = \omega' b = 2 V_A \cos 45$

$\Rightarrow \omega' = \frac{2 V_A \cos 45}{b} = 0{,}56\ rad/s$

$V_C = \omega' a = 0{,}28\ m/s$

6.2)

DATI

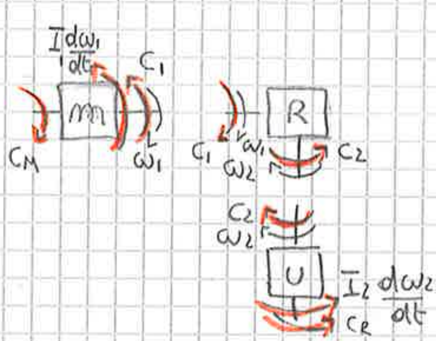
$C_M = 700\ Nm$

$i = \frac{\omega_1}{\omega_2} = 10 \quad ; \quad \eta = 0{,}9$

$C_R = C_U = k\omega_2 \quad , k = 80\ Nms$

$I_m = 12\ kgm^2 \ ; \ I_u = 25\ kgm^2$

$t = ?$ (DA $\omega = 0$ A $90\%\ \omega_R$)



IN TALE CONDIZIONE CALCOLARE LA COPPIA CHE IL RIDUTTORE ESERCITA SUL TELAIO = ? ($\Rightarrow C_V = ?$)

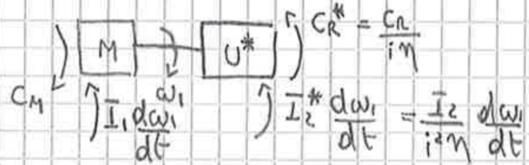
$$\begin{cases} \eta = \frac{C_2\omega_2}{C_1\omega_1} = \frac{C_2\omega_2}{C_1 i \omega_2} \Rightarrow C_2 = C_1 i \eta \\ i = \frac{\omega_1}{\omega_2} \Rightarrow \omega_1 = i\omega_2 \qquad i = \frac{\dot\omega_1}{\dot\omega_2} \Rightarrow \dot\omega_2 = \frac{\dot\omega_1}{i} \Rightarrow \frac{d\omega_2}{dt} = \frac{d\omega_1}{i\,dt} \end{cases}$$

DAL PUNTO DI VISTA DEL MOTORE

$$\begin{cases} C_M - C_1 - I_1\frac{d\omega_1}{dt} = 0 \\ C_2 = C_1 i\eta \Rightarrow C_1 = \frac{C_2}{i\eta} \Rightarrow C_1 = \frac{C_R}{i\eta} + \frac{I_2\, d\omega_1}{i^2\eta\, dt} \\ C_2 - C_R - I_2\frac{d\omega_2}{dt} = 0 \Rightarrow C_2 = C_R + I_2\frac{d\omega_2}{dt} = C_R + I_2\frac{d\omega_1}{i\,dt} \end{cases}$$

$$\Rightarrow C_M - \frac{C_R}{i\eta} - \frac{I_2\, d\omega_1}{i^2\eta\, dt} - I_1\frac{d\omega_1}{dt} = 0 \Rightarrow C_M - \frac{C_R}{i\eta} - \left(\frac{I_2}{i^2\eta} + I_1\right)\frac{d\omega_1}{dt} = 0$$

$$\to C_M - C_R^* - \left(I_2^* + I_1\right)\frac{d\omega_1}{dt} = 0 \quad , C_R^* = \frac{C_R}{i\eta} \quad , I_2^* = \frac{I_2}{i^2\eta}$$

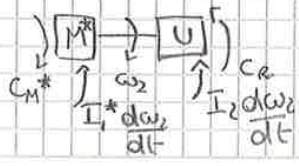


$\longrightarrow$ DAL PUNTO DI VISTA DELL'UTILIZZATORE

$$\begin{cases} C_M - C_1 - I_1\frac{d\omega_1}{dt} = 0 \Rightarrow C_1 = C_M - I_1\frac{d\omega_1}{dt} = C_M - I_1 i\frac{d\omega_2}{dt} \\ C_2 = C_1 i\eta \qquad C_2 = C_M i\eta - I_1 i^2\eta\frac{d\omega_2}{dt} \\ C_2 - C_R - I_2\frac{d\omega_2}{dt} = 0 \end{cases}$$

$$\Rightarrow C_M i\eta - I_1 i^2\eta\frac{d\omega_2}{dt} - C_R - I_2\frac{d\omega_2}{dt} = 0 \Rightarrow C_M i\eta - C_R - \left(I_1 i^2\eta + I_2\right)\frac{d\omega_2}{dt} = 0$$

$$\to C_M^* - C_R - \left(I_1^* + I_2\right)\frac{d\omega_2}{dt} = 0 \quad , C_M^* = C_M i\eta \quad , I_1^* = I_1 i^2\eta$$



6.3)

DATI

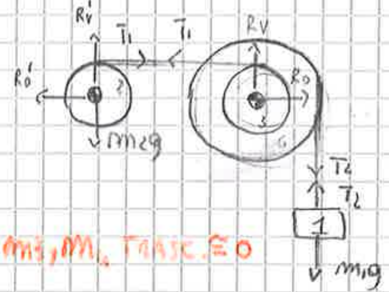
$R_2 = R_3 = 0,15\ m$

$R_4 = 0,6\ m$

$m_1 = 15\ kg$

$m_2 = 300\ kg$

$\alpha_2 = ?$



$\alpha = \frac{a_T}{R}$ $\quad a_T = \alpha R$

$I_2 = \frac{1}{2} m_2 R_2^2$

IN QUANTO m_3, m_4 TRASC. = 0

1) $T)\ +T_2 - m_1 g = -m_1 a \qquad T_2 = m_1(g-a)$

3-4) $O_3)\ +T_1 R_3 - T_2 \cdot R_4 = 0 \qquad T_1 = T_2 \cdot R_4 / R_3 = \frac{m_1(g-a)R_4}{R_3} = \frac{m_1(g-\alpha R_4)R_4}{R_3}$

2) $O_2)\ -T_1 R_2 = -I_2 \alpha_2 \qquad \Rightarrow \alpha_2 = \frac{T_1 R_2}{I_2} = \frac{m_1(g-\alpha R_4)R_4 R_2}{I_2 R_3}$

$\alpha_4 = \alpha_3 = \alpha_2$

$\alpha = \frac{m_1 g R_4 R_2 - m_1 \alpha R_4^2 R_2}{I_2 R_3} \Rightarrow \alpha I_2 R_3 + m_1 \alpha R_4^2 R_2 = m_1 g R_4 R_2$

$\alpha = \frac{m_1 R_4 R_2 g}{I_2 R_3 + m_1 R_4^2 R_2} = \frac{m_1 R_4 \cancel{R_2} g}{\cancel{R_2}(\frac{1}{2} m_2 R_2 R_3 + R_4^2 m_1)} = 10\ rad/s^2$

6.4)

DATI

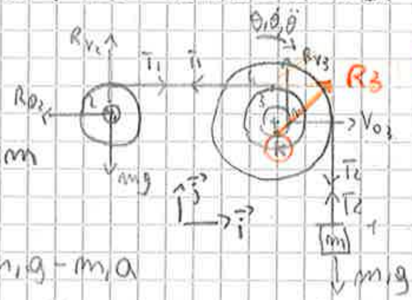
ATTRITO NEL PERNO DELLE PULEGGE 3-4 DELL'ES PRECEDENTE:

$R_p = 0,04\ m$

$f = 0,5 \qquad \varphi = Tg^{-1} f = 26,5651$

$\alpha = ? \qquad \rho = R_p \sin\varphi = 0,0179\ m$

$R_2 = ?\ R_3 = ?$



NB! $R_3 \neq \vec{R_{V_3}}$

1) $-m_1 g + T_2 = -m_1 a \Rightarrow T_2 = m_1 g - m_1 a$

3-4) $K) - T_2 R_4 + T_1(R_3 + \rho) = 0 \qquad T_1 = \frac{T_2 R_4}{(R_3+\rho)} = \frac{m_1 g R_4 - m_1 a R_4}{(R_3+\rho)}$

2) $O_2) - T_1 R_2 = -I_2 \alpha_2$

$\Rightarrow I_2 \alpha_2 = \frac{m_1 g R_4 R_2 - m_1 a R_4 R_2}{(R_3+\rho)} \qquad \alpha_3 = \alpha_4 = \alpha_2$

$I\alpha(R_3+\rho) + m_1 \alpha R_4^2 R_2 = m_1 g R_4 R_2$

$\alpha = \frac{m_1 g R_4 R_2}{I_2(R_3+\rho) + m_1 R_4^2 R_2} = \frac{15 \cdot 9,8 \cdot 0,6 \cdot 0,15}{\frac{300}{2} \cdot 0,15^2 \cdot \underbrace{(0,15+0,0179)}_{0,1679} + 15 \cdot 0,6^2 \cdot 0,15} = 9,6\ rad/s^2$

$R_{V_3} = T_2 \qquad R_3 = \sqrt{R_{V_3}^2 + R_{O_3}^2} = 224,4\ N \qquad R_{V_2} = m_2 g \qquad R_2 = \sqrt{R_{V_2}^2 + R_{O_2}^2} = 2951\ N$

$R_{O_3} = T_1 \qquad R_{O_2} = T_1$

6.6)

DATI

$d_1 = 0{,}5\ m$

$a = 0{,}3\ m$

$d_2 = 0{,}4\ m$

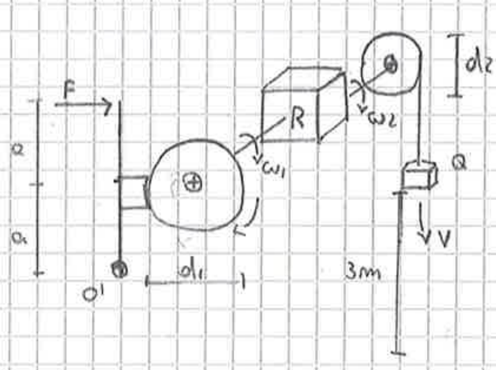
$i = \frac{\omega_1}{\omega_2} = 2$

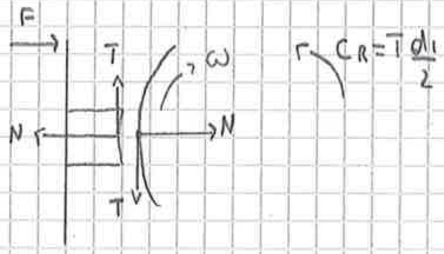
$Q = 10\,000\ N$

$f = 0{,}5$

$V_0 = 2\ m/s$

$F = ?\ (s = 3m \quad V = 0)$





$T = fN \Rightarrow N = \frac{T}{f}$

$O')\ +Na - F2a = 0$

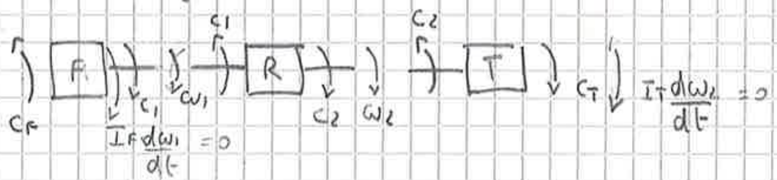
a) $\uparrow)\ T - Q = m a \qquad m = \frac{Q}{g} = 1020{,}4\ kg$

$a x = \frac{1}{2}v_f^2 - \frac{1}{2}v_0^2$

$a = \frac{1}{2}\frac{v_0^2}{x} = \frac{2^2}{2 \cdot 3} = 0{,}667\ m/s^2$

$C_2 = T\frac{d_2}{2} = (Q + ma)\frac{d}{2} = (10000 + 1020{,}4 \cdot 0{,}667)\frac{0{,}4}{2} = 2136{,}1\ Nm$

$\frac{d\omega_2}{dt} = \frac{a \cdot 2}{d_2} = 3{,}33\ rad/s^2$



SE NON CI DANNO $I \Rightarrow$
$\Rightarrow$ NO FORZE D'INERZIA.

$i = \frac{\omega_1}{\omega_2} = 2 \Rightarrow \omega_1 = 2\omega_2 \qquad \frac{d\omega_1}{dt} = \frac{d\omega_2}{dt} \cdot 2$

$\eta = 1 = \frac{C_1\omega_1}{C_2\omega_1} \Rightarrow C_1\omega_1 = C_2\omega_2 \qquad C_1 = C_2\frac{\omega_2}{\omega_1} = \frac{C_2}{2} = 1068{,}1\ Nm$

$C_1 = T\frac{d_1}{2} \Rightarrow T = \frac{C_1 \cdot 2}{d_1} = 4272\ N$

$N = \frac{T}{f} = 8544{,}5\ N$

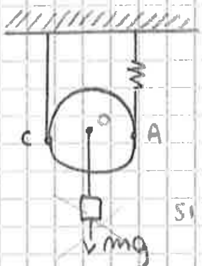
$F = \frac{N}{2} = 4272{,}2\ N$

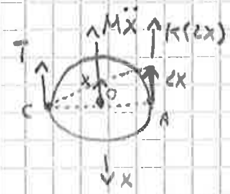
[CAPITOLO 7]

7.1)

DATI

$T = ?$





SI TRASCURINO LE FORZE COSTANTI

SE x È LO SPOSTAMENTO DI O => LO SPOSTAMENTO DI A SARÀ 2x!

$$\circlearrowleft C)\ +M\ddot{x}R + (k\,2x)\,2R = 0$$

$$\ddot{x} + \frac{4k}{M}x = 0$$

$$\ddot{x} + \omega_m^2 x = 0$$

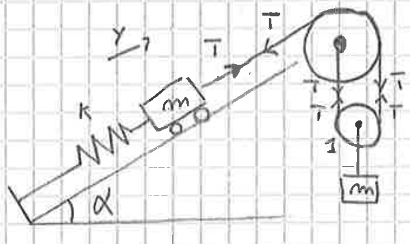
$$\omega_m = \sqrt{\frac{4k}{M}} = 2\sqrt{\frac{k}{M}}$$

$$T = \frac{2\pi}{\omega_m} = \frac{2\pi}{2}\sqrt{\frac{M}{k}} = \pi\sqrt{\frac{M}{k}}$$

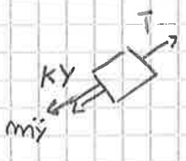
7.2)

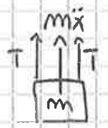
DATI

$\omega_m = ?$



SI TRASCURINO LE FORZE COSTANTI





$$m\ddot{x} + 2T = 0$$

$$m\ddot{y} + ky - T = 0$$

$$y = 2x \Rightarrow x = \frac{y}{2} \qquad \ddot{x} = \frac{\ddot{y}}{2}$$

$$T = -\frac{m\ddot{x}}{2} = -\frac{m\ddot{y}}{4}$$

$$m\ddot{y} + ky + \frac{m\ddot{y}}{4} = 0$$

$$\frac{5}{4}m\ \left(m + \frac{m}{4}\right)\ddot{y} + ky = 0$$

$$\ddot{y} + \frac{4ky}{5m} = 0$$

$$\ddot{y} + \omega_m^2 y = 0$$

$$\omega_m = \sqrt{\frac{4k}{5m}}$$

7.5)

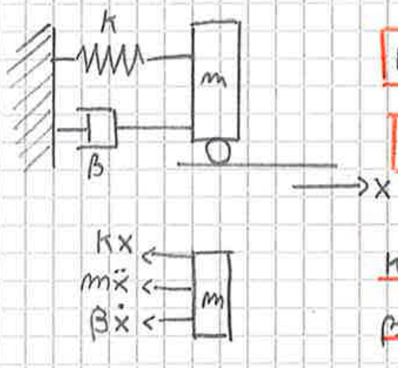
DATI

$k = 98\ N/m$

$m = 2kg$

$\beta = 42\ Ns/m$

$\zeta = ?$



$\boxed{F_{ELASTICA} = kx}$

$\boxed{F_{SMORZATORE} = \beta \dot{x}}$

k COEFF. ELASTICO

β COEFF. DI SMORZ.

ω_m PULSAZIONE NATURALE

ζ FATTORE DI SMORZAMENTO

$$m\ddot{x} + \beta\dot{x} + kx = 0$$

$$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\Downarrow$$

$$\boxed{\ddot{x} + 2\zeta\omega_m\dot{x} + \omega_m^2 x = 0}$$

$$\Downarrow$$

$$\omega_m^2 = \frac{k}{m} \Rightarrow \omega_m = \sqrt{\frac{k}{m}}$$

$$2\zeta\omega_m = \frac{\beta}{m} \Rightarrow \zeta = \frac{\beta}{2m\omega_m} = \frac{\beta}{2m}\sqrt{\frac{m}{k}} = \frac{\beta}{2\sqrt{km}} = 1,5$$

$\boxed{T = \frac{2\pi}{\omega_m}}$

$\boxed{T_s = \frac{2\pi}{\omega_s}}$

$\boxed{\omega_s = \omega_m\sqrt{1-\zeta^2}}$

7.6)

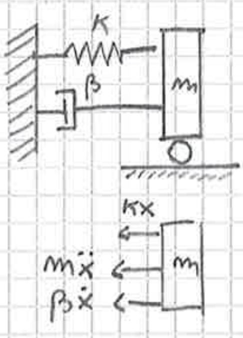
DATI

$T_s = 0,32\ s$

$m = 1\ kg$

$k = 850\ N/m$

$\beta = ?$



$$m\ddot{x} + \beta\dot{x} + kx = 0$$

$$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\Downarrow$$

$$\ddot{x} + 2\zeta\omega_m\dot{x} + \omega_m^2 x = 0$$

$$\omega_m^2 = \frac{k}{m} \Rightarrow \omega_m = \sqrt{\frac{k}{m}} = 29,15\ rad/s$$

$$T_s = \frac{2\pi}{\omega_s} \Rightarrow \omega_s = \frac{2\pi}{T_s} = 19,62\ rad/s$$

$$\omega_s = \omega_m\sqrt{1-\zeta^2}$$

$$\sqrt{1-\zeta^2} = \frac{\omega_s}{\omega_m}$$

$$1-\zeta^2 = \frac{\omega_s^2}{\omega_m^2} \Rightarrow \zeta = \sqrt{1-\left(\frac{\omega_s}{\omega_m}\right)^2} = 0,74$$

7.9) VAFFANCULO!

DATI

$m = 500\,kg$

$V = 25\,km/h$

$b = 0{,}025\,m$; $S = 1{,}2\,m$

DURANTE IL CARICAMENTO OGNI 75 Kg AGGIUNTI SUL CARRELLO NE CAUSANO UN ABBASSAMENTO DI 0,003 m

SMORZAMENTO NULLO

V_c (VELOCITA' CRITICA) (OSCILLAZ. MAX) = ?

AMPIEZZA DELL'OSCILLAZIONE VERTICALE = ?

$$\omega_m = \sqrt{\frac{k}{m}}$$

$$k = \frac{F}{\Delta l} = 245 \cdot 10^3\ N/m \quad ?$$

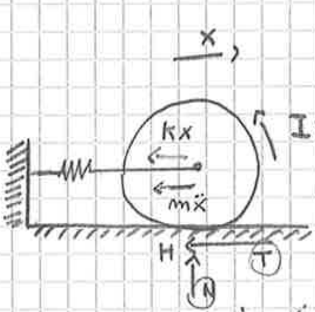
$$V = \frac{S}{T} \Rightarrow \cancel{T = \frac{S}{V} \Rightarrow \omega_m = \frac{2\pi}{T} = 2\pi\frac{V}{S}} \qquad T = \frac{\omega_m}{2\pi}\ ,\ \omega_m = \sqrt{\frac{k}{m}} = 22\ rad/s$$

$$V_c = \frac{\omega_m S}{2\pi} = 4{,}2\ m/s$$

7.10)

DATI

EQUAZIONE DEL MOTO IN TERMINI DI x = ?



$$I\ddot{\theta} = I\frac{\ddot{x}}{R}$$

$$I = \frac{1}{2}mR^2$$

$\rightarrow$) $-kx - m\ddot{x} - T = 0$

$$\overset{\curvearrowleft}{H}) \quad m\ddot{x}R + kxR + I_0\frac{\ddot{x}}{R} = 0$$

(NB: NO: $m\ddot{x}R + kxR + (I_0 + \cancel{mR^2})\frac{\ddot{x}}{R} = 0$ POICHE' HO GIA MESSO $m\ddot{x}R$!!)

$$m\ddot{x}\cancel{R} + kx\cancel{R} + \frac{1}{2}mR^{\cancel{2}}\frac{\ddot{x}}{\cancel{R}} = 0$$

$$\frac{3}{2}m\ddot{x} + kx = 0$$

$$\ddot{x} + \frac{2k}{3m}x = 0$$

$$\boxed{\ddot{x} + \omega_m^2 x = 0}$$

$$\omega_m^2 = \frac{2}{3}\frac{k}{m} \Rightarrow \boxed{\omega_m = \sqrt{\frac{2}{3}\frac{k}{m}}}$$

7.13)

DATI

$m_1 = 10\ kg$

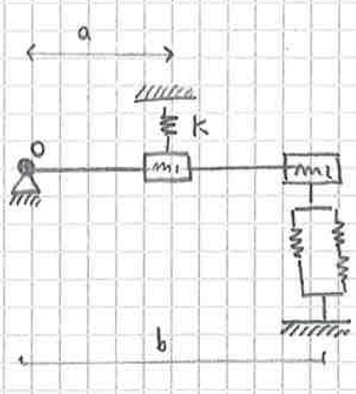
$m_2 = 20\ kg$

$k = 1000\ N/m$

$a = 1\ m$

$b = 2\ m$

$k_e = ?\ T = ?$

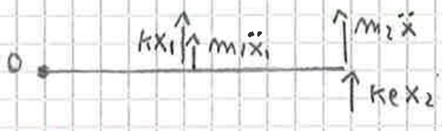


$\frac{1}{k'} = \left(\frac{1}{k} + \frac{1}{k}\right)$

$k' = \left(\frac{1}{k} + \frac{1}{k}\right)^{-1}$

$$k_e = k + \left(\frac{1}{k} + \frac{1}{k}\right)^{-1} = k + \frac{k}{2} = \frac{3}{2}k = 1500\ N/m$$

NO $I\ddot{\theta}$ POICHÈ IL PROBLEMA CI DICE DI TRASCURARE LA MASSA DEL BRACCIO



NB $\boxed{\frac{\ddot{x}_2}{b} = \frac{\ddot{x}_1}{a}} \Rightarrow \ddot{x}_2 = \frac{b}{a}\ddot{x}_1$ $\quad \left(\alpha = \frac{a_{T_2}}{b} = \frac{a_{T_1}}{a}\right)$ $\quad \boxed{\frac{x_2}{b} = \frac{x_1}{a}} \Rightarrow x_2 = \frac{b}{a}x_1$

$$\circlearrowleft O)\ m_2\ddot{x}_2 b + k_e x_2 b + k x_1 a + m_1\ddot{x}_1 a = 0$$

$$\circlearrowleft O)\ m_2\frac{b^2}{a}\ddot{x}_1 + k_e\frac{b^2}{a}x_1 + k x_1 a + m_1\ddot{x}_1 a = 0$$

$$\circlearrowleft O)\ \left(m_2\frac{b^2}{a} + m_1 a\right)\ddot{x}_1 + \left(k_e\frac{b^2}{a} + k a\right)x_1 = 0$$

$$\ddot{x}_1 + \frac{\left(k_e\frac{b^2}{a} + k a\right)}{\left(m_2\frac{b^2}{a} + m_1 a\right)}x = 0$$

$$\omega_n = \sqrt{\frac{k_e b^2 + k a^2}{m_2 b^2 + m_1 a^2}}$$

$$T = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m_2 b^2 + m_1 a^2}{k_e b^2 + k a^2}} = 0{,}71\ s$$

7.15)

DATI

$m_2 = 24\ kg$

$h = 0{,}1\ m$

$a = 0{,}2\ m$

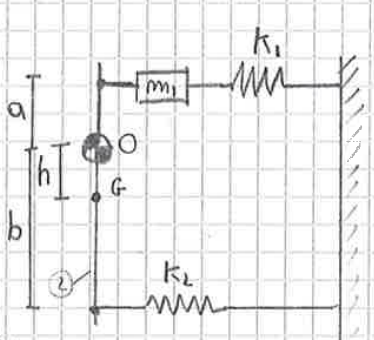
$b = 0{,}4\ m$

$m_1 = 40\ kg$

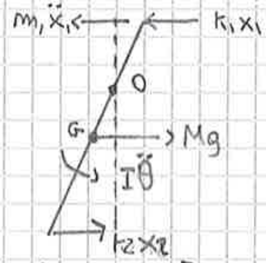
$k_1 = 500\ N/m$

$k_2 = 2000\ N/m$

$f = ?$



ULTERIORE DATO:
QUESTO SISTEMA GIACE SU UN PIANO ORIZZONTALE



$x_1 = a \sin\theta \Rightarrow \ddot{x}_1 = a \sin\ddot{\theta}$

$x_2 = b \sin\theta$

$I = m_2 \frac{(a+b)^2}{12} + m_2 h^2$ [POICHÉ NON VOGLIO CONSIDERARE $m_1 \ddot{x}$]

$\overset{\curvearrowleft}{O})\ k_2 x_2 b + I\ddot{\theta} + m_2 g \cos\theta \cdot h + m_1 \ddot{x}_1 a + k_1 x_1 a = 0$

$k_2 b^2 \sin\theta + \left(m_2 \frac{(a+b)^2}{12} + m_2 h^2\right)\ddot{\theta} + m_2 g \cos\theta\, h + m_1 a^2 \sin\ddot{\theta} + k_1 a^2 \sin\theta = 0$

PER $\theta \simeq 0 \Rightarrow \begin{cases} \sin\theta \simeq \theta \\ \cos\theta \simeq 1 \end{cases}$

FORZA PESO STATICA

$k_2 b^2 \theta + \left(m_2 \frac{(a+b)^2}{12} + m_2 h^2\right)\ddot{\theta} + [m_2 g h] + m_1 a^2 \ddot{\theta} + k_1 a^2 \theta = 0$

INDIPENDENTE DA θ; PRENDO E BUTTO VIA!

$\left(\frac{m_2 (a+b)^2}{12} + m_2 h^2 + m_1 a^2\right)\ddot{\theta} + k_1 a^2 \theta + k b^2 \theta = 0$

$f = \frac{1}{T} \quad T = \frac{2\pi}{\omega_n} \Rightarrow f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1 a^2 + k_2 b^2}{m_2\left(\frac{(a+b)^2}{12} + h^2\right) + m_1 a^2}} = 1{,}83\ Hz$

7.18)

DATI

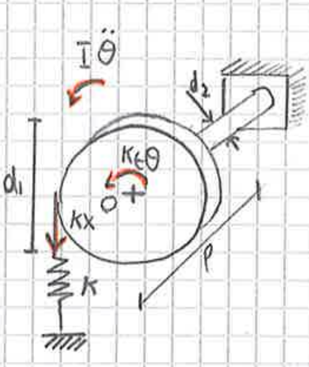
$d_1 = 0{,}18\ m$

$m = 1\ kg$

$d_2 = 0{,}004\ m$

$\ell = 0{,}1\ m$

$k = 10^3\ N/m$ $\rightarrow \boxed{k_t\theta}$



$k_t = G J_p/\ell$ RIGIDEZZA TORSIONALE DELL'ALBERO

$G = 8\cdot 10^{10}\ Pa$ MODULO DI ELASTICITÀ DEL MATERIALE

$J_p = \pi d^4/32$ MOMENTO D'INERZIA POLARE DI AREA

$\omega_n = ?$

$$\circlearrowleft)\ k_t\theta + kx\frac{d_1}{2} + I\ddot{\theta} = 0$$

$$\begin{cases} x = \dfrac{d_1}{2}\sin\theta \\ I = \dfrac{1}{2}m\left(\dfrac{d_1}{2}\right)^2 = m\dfrac{d_1^2}{8} \end{cases}$$

$$\circlearrowleft)\ k_t\theta + k\frac{d_1^2}{4}\sin\theta + m\frac{d_1^2}{8}\ddot{\theta} = 0$$

PER $\theta \simeq 0 \Rightarrow \begin{cases} \sin\theta \simeq \theta \\ \cos\theta \simeq 1 \end{cases}$

$$\circlearrowleft)\ k_t\theta + k\frac{d_1^2}{4}\theta + m\frac{d_1^2}{8}\ddot{\theta} = 0$$

$$\ddot{\theta} + \frac{\left(k_t + k\frac{d_1^2}{4}\right)}{\left(m\frac{d_1^2}{8}\right)}\theta = 0$$

$$\omega_n = \sqrt{\frac{8\left(k_t + \frac{k d_1^2}{4}\right)}{(m d_1^2)}} = 83{,}5\ rad/s$$

7.22)

DATI

$\ell = 2m$

$a = 1{,}2\ m$

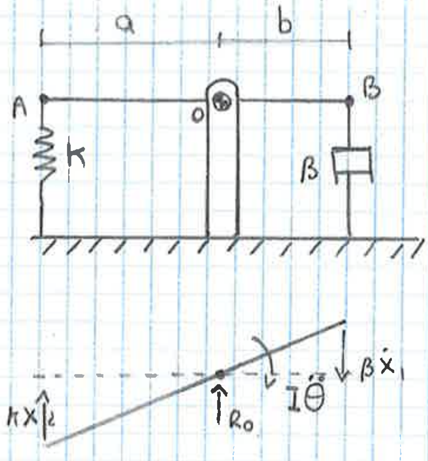
$b = 0{,}8\ m$

$m = 80\ kg$

$k = 50 \cdot 10^3\ N/m$

$\zeta = 0{,}5$

$\beta = ?$



$I = I_c + m\left(a - \frac{\ell}{2}\right)^2 = \frac{m}{12}\ell^2 + m\left(a - \frac{\ell}{2}\right)^2 = 30\ kg\,m^2$ [IN TAL MODO NON CONSIDERO $m\ddot{x}$]

$\circlearrowleft)\ -I\ddot{\theta} - \beta\dot{x}_1 b - kx_2 a = 0$ (NON CONSIDERIAMO LA FORZA PESO)

$\begin{cases} x_1 = b\sin\theta & \Rightarrow \dot{x}_1 = b\sin\dot{\theta} \\ x_2 = a\sin\theta \end{cases}$

$\circlearrowleft)\ +I\ddot{\theta} + \beta b^2 \sin\dot{\theta} + ka^2\sin\theta = 0$

PER $\theta \simeq 0 \Rightarrow \begin{cases} \sin\theta \simeq \theta \\ \cos\theta \simeq 1 \end{cases}$

$\circlearrowleft)\ I\ddot{\theta} + \beta b^2\dot{\theta} + ka^2\theta = 0$

$\ddot{\theta} + \frac{\beta b^2}{I}\dot{\theta} + \frac{ka^2}{I}\theta = 0$

$\omega_n = \sqrt{\frac{ka^2}{I}} = 49\ rad/s$

$2\zeta\omega_n = \frac{\beta b^2}{I} \Rightarrow \beta = \frac{2\zeta\omega_n I}{b^2} = 2291\ Ns/m$

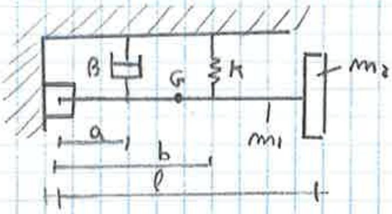
7.23)

7.27)

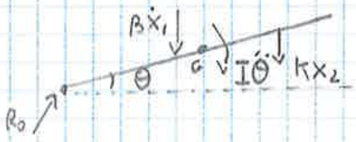
DATI

$\omega_m = ?$

$\zeta = ?$



$$I = m_1 \frac{\ell^2}{12} + m_1 \frac{\ell^2}{4} + m_2 \frac{R^2}{4} + m_2 \ell^2 = \frac{m_1 \ell^2}{3} + m_2 \left(\frac{R^2}{4} + \ell^2 \right)$$



$\circlearrowleft)\ -I\ddot{\theta} - \beta a \dot{x}_1 + k b x_2 = 0$ NON CONSIDERO LE FORZE PESO!

$$\begin{cases} x_1 = a \sin\theta \Rightarrow \dot{x}_1 = a \sin\dot{\theta} \\ x_2 = b \sin\theta \end{cases}$$

$\circlearrowleft)\ I\ddot{\theta} + \beta a^2 \sin\dot{\theta} + k b^2 \sin\theta = 0$

PER $\theta \simeq 0 \Rightarrow \begin{cases} \sin\theta \simeq \theta \\ \cos\theta \simeq 1 \end{cases}$

$\circlearrowleft)\ I\ddot{\theta} + \beta a^2 \dot{\theta} + k b^2 \theta = 0$

$$\ddot{\theta} + \frac{\beta a^2}{I} \dot{\theta} + \frac{k b^2}{I} \theta = 0$$

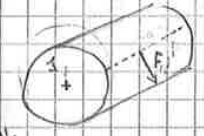
$$\omega_m = \sqrt{\frac{k b^2}{I}} = \sqrt{\frac{k b^2}{m_1 \frac{\ell^2}{3} + m_2 \left(\frac{R^2}{4} + \ell^2 \right)}}$$

$$2 \zeta \omega_m = \frac{\beta a^2}{I}$$

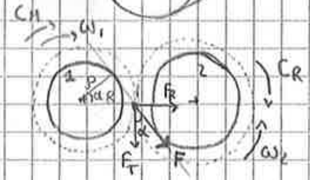
$$\zeta = \frac{\beta a^2}{2 I \omega_m}$$

22) RUOTE DENTATE A DENTI DRITTI

- $\rho = R\cos\alpha \qquad p = \frac{2\pi R}{z}$
- a = addendum = $m = \frac{p}{\pi} = \frac{2R}{z}$
- d = dedendum = $1.25\,m$



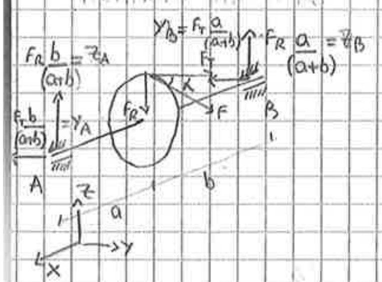
$F = F_{12}$ = FORZA ESERCITATA DA 1 SU 2

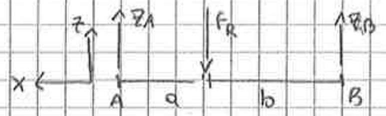


- $\boxed{F_T = F\cos\alpha}$
- $\boxed{F_R = F\sin\alpha}$
- $i = \frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} = \frac{z_2}{z_1}$
- $\eta = \frac{C_R\,\omega_2}{C_M\,\omega_1}$

NOTA: $\omega_R = \omega_1 - \omega_2$

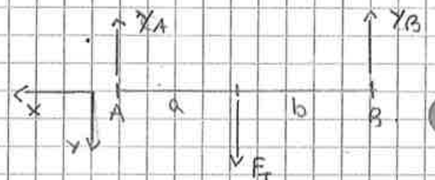
REAZIONI VINCOLARI:





$\hat{A})\ V_B(a+b) - F_R a = 0 \Rightarrow V_B = \frac{F_R a}{(a+b)}$

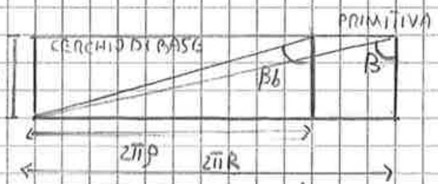
$\hat{B})\ F_R b - V_A(a+b) = 0 \Rightarrow V_A = \frac{F_R b}{(a+b)}$



$\hat{A})\ Y_B(a+b) - F_T a = 0 \Rightarrow Y_B = \frac{F_T a}{(a+b)}$

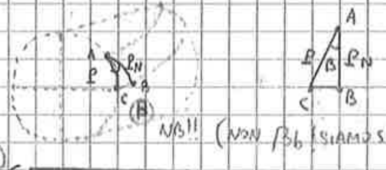
$\hat{B})\ F_T b - Y_A(a+b) = 0 \Rightarrow Y_A = \frac{F_T b}{(a+b)}$

23) RUOTE CILINDRICHE ELICOIDALI



$$\left.\begin{aligned} 2\pi\rho &= P\,\mathrm{tg}\,\beta_b = 2\pi R\cos\alpha \\ 2\pi R &= P\,\mathrm{tg}\,\beta \end{aligned}\right\} \Rightarrow$$

(1) $\Rightarrow \boxed{\mathrm{tg}\,\beta_b = \mathrm{tg}\,\beta\cos\alpha}$

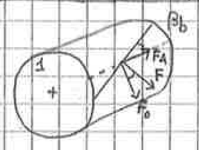


$p = \frac{2\pi R}{z}$

NB!! (NON β_b! SIAMO SULLA PRIMITIVA!)

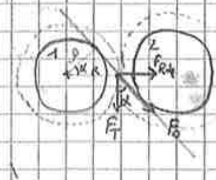
(2)
$$\begin{cases} \boxed{p_N = p\cos\beta} \\ m_n = \frac{p_N}{\pi} = \frac{p\cos\beta}{\pi} = \frac{2\pi R\cos\beta}{\pi z} = \frac{2R\cos\beta}{z} = m\cos\beta \\ \boxed{m_n = m\cos\beta} \end{cases}$$

(3) $\boxed{\mathrm{tg}\,\alpha_N = \mathrm{tg}\,\alpha\cos\beta}$ (A MEMORIA)



$\boxed{F_0 = F\cos\beta_b}$

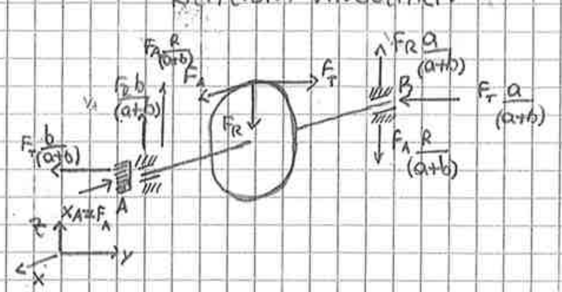
$\boxed{F_A = F\sin\beta_b}$



$\boxed{F_T = F_0\cos\alpha = F\cos\beta_b\cos\alpha}$

$\boxed{F_R = F_0\sin\alpha = F\cos\beta_b\sin\alpha}$

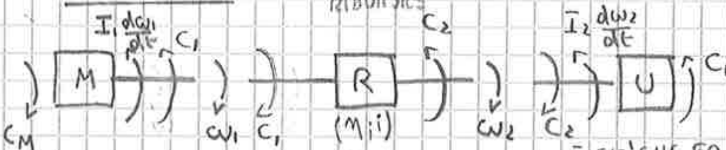
REAZIONI VINCOLARI:



TRANSITORI

NOTE: DISEGNARE PRIMA ω, POI C_M CONCORDE A ω_1, POI C_1 (PRESENZA DEL RIDUTTORE)

27) RIDUTTORE



CARATTERISTICHE DEL RIDUTTORE:

$$\eta = \frac{C_2\omega_2}{C_1\omega_1} \qquad i = \frac{\omega_1}{\omega_2} = \frac{\dot\omega_1}{\dot\omega_2}$$

$$\begin{cases} C_M - C_1 - I_1\frac{d\omega_1}{dt} = 0 \\ C_2 = C_1 i\eta \\ C_2 - C_R - I_2\frac{d\omega_2}{dt} = 0 \end{cases}$$

PUNTO DI VISTA DEL MOTORE

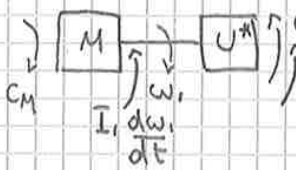
$$C_2 = C_R + I_2\frac{d\omega_2}{dt} = C_R + I_2\frac{d\omega_1}{i\,dt}$$

$$C_1 = \frac{C_2}{i\eta} = \frac{C_R}{i\eta} + \frac{I_2\,d\omega_1}{i^2\eta\,dt}$$

$$C_M - \frac{C_R}{i\eta} - \frac{I_2\,d\omega_1}{i^2\eta\,dt} - I_1\frac{d\omega_1}{dt} = 0$$

$$C_M - \frac{C_R}{i\eta} - \left(\frac{I_2}{i^2\eta} + I_1\right)\frac{d\omega_1}{dt} = 0$$

$$C_M - C_R^* - \left(I_2^* + I_1\right)\frac{d\omega_1}{dt} = 0$$



$$C_R^* = \frac{C_R}{i\eta} \qquad I_2^*\frac{d\omega_1}{dt} = \frac{I_2}{i^2\eta}\frac{d\omega_1}{dt}$$

PUNTO DI VISTA DELL'UTILIZZATORE

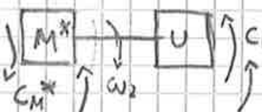
$$C_1 = C_M - I_1\frac{d\omega_1}{dt} = C_M - I_1 i\frac{d\omega_2}{dt}$$

$$C_2 = C_M i\eta - I_1 i^2\eta\frac{d\omega_2}{dt}$$

$$C_M i\eta - I_1 i^2\eta\frac{d\omega_2}{dt} - C_R - I_2\frac{d\omega_2}{dt} = 0$$

$$C_M i\eta - C_R - \left(I_1 i^2\eta + I_2\right)\frac{d\omega_2}{dt} = 0$$

$$C_M^* - C_R - \left(I_1^* + I_2\right)\frac{d\omega_2}{dt} = 0$$



$$I_1^*\frac{d\omega_2}{dt} = I_1 i^2\eta\frac{d\omega_2}{dt}$$

PER CONOSCERE LA VELOCITA' DI REGIME PONGO: $\frac{d\omega_1}{dt} = 0 \,/\, \frac{d\omega_2}{dt} = 0$

PER CALCOLARE LA COPPIA C_V CHE IL RIDUTTORE ESERCITA SUL TELAIO:

USANDO $\frac{d\omega_1}{dt} = i\frac{d\omega_2}{dt}$ MI SBARAZZO DI QUESTI

$$\begin{cases} C_2 = C_1 i\eta \\ C_V = \sqrt{C_1^2 + C_2^2} \end{cases}$$

28) FRIZIONE

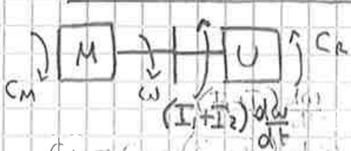
CARATTERISTICHE DELLA FRIZIONE:

$$C_f = M_f = f F \frac{R_i + R_e}{2} \quad ; \quad f' = \frac{f}{\sin\alpha} \text{ FRIZ. CONICA} \quad ; \quad f = mf \text{ FRIZ. A DISCHI MULTIPLI}$$

$$C_M - C_f - I_1\frac{d\omega_1}{dt} = 0 \;\rightarrow\; \omega_1(t)$$

$$C_f - C_R - I_2\frac{d\omega_2}{dt} = 0 \;\rightarrow\; \omega_2(t)$$

PER $t \geq t^*$ FRIZIONE INNESTATA $\omega_1 = \omega_2 = \omega^*$; SUCCESSIVAMENTE VALE:



$$C_M - C_R - (I_1 + I_2)\frac{d\omega}{dt} = 0$$

$$M_f = C_M - I_M\frac{d\omega}{dt} \;/\; M_f = C_R - I_U\frac{d\omega}{dt}$$