



Appunti universitari

Tesi di laurea

Cartoleria e cancelleria

Stampa file e fotocopie

Print on demand

Rilegature

NUMERO: 2204A

ANNO: 2017

A P P U N T I

STUDENTE: Sciotto Miriam

**MATERIA: Esercitazioni di Elettrotecnica + Simulazione
d'Esame - Prof. Repetto**

Il presente lavoro nasce dall'impegno dell'autore ed è distribuito in accordo con il Centro Appunti.

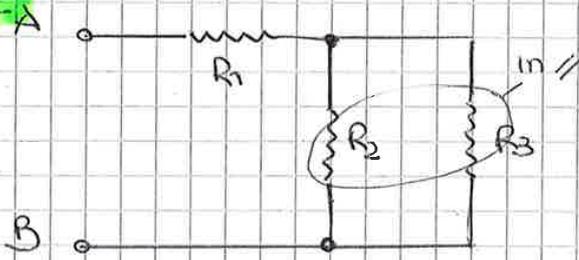
Tutti i diritti sono riservati. È vietata qualsiasi riproduzione, copia totale o parziale, dei contenuti inseriti nel presente volume, ivi inclusa la memorizzazione, rielaborazione, diffusione o distribuzione dei contenuti stessi mediante qualunque supporto magnetico o cartaceo, piattaforma tecnologica o rete telematica, senza previa autorizzazione scritta dell'autore.

**ATTENZIONE: QUESTI APPUNTI SONO FATTI DA STUDENTIE NON SONO STATI VISIONATI DAL DOCENTE.
IL NOME DEL PROFESSORE, SERVE SOLO PER IDENTIFICARE IL CORSO.**

10.10.16

ESERCIZIO 1

ES 1

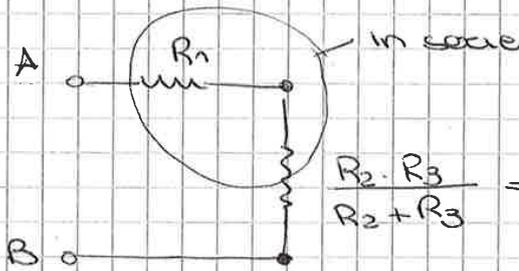


$R_1 = 10 \Omega$

$R_2 = 1 \Omega$

$R_3 = 9 \Omega$

$R_{AB} ?$

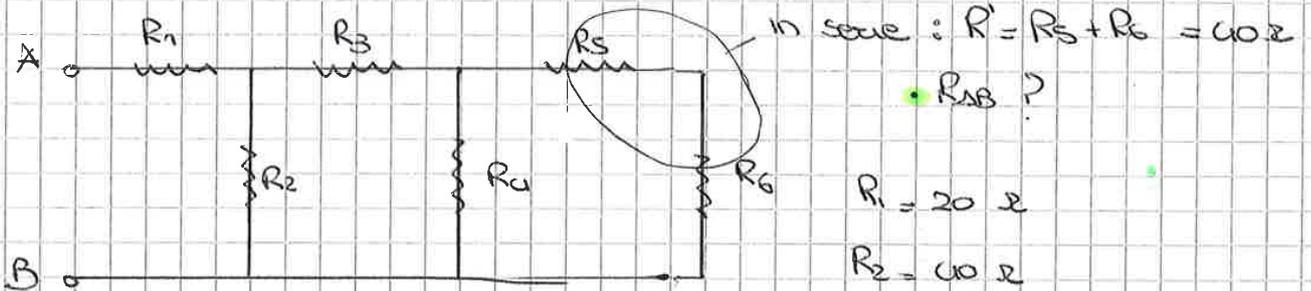


$$\frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{1 \cdot 9}{1 + 9} = \frac{9}{10} = 0,9 \Omega$$

$R_{AB} = R_1 + 0,9 = 10 + 0,9 = 10,9 \Omega$



ES 2



in serie : $R' = R_5 + R_6 = 60 \Omega$

$R_{AB} ?$

$R_1 = 20 \Omega$

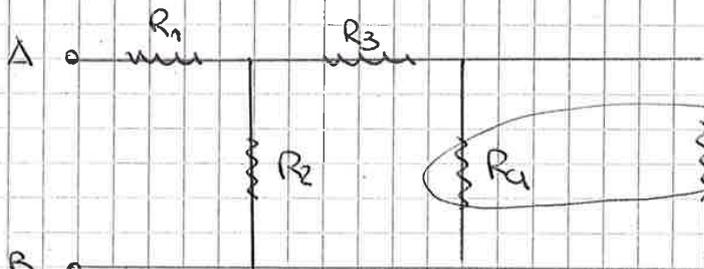
$R_2 = 60 \Omega$

$R_3 = 20 \Omega$

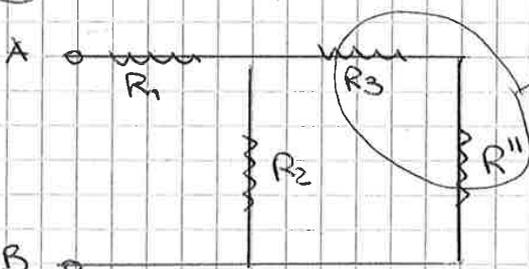
$R_4 = 60 \Omega$

$R_5 = 20 \Omega$

$R_6 = 20 \Omega$



in || : $R'' = \frac{R_4 \cdot R'}{R_4 + R'} = 20 \Omega$



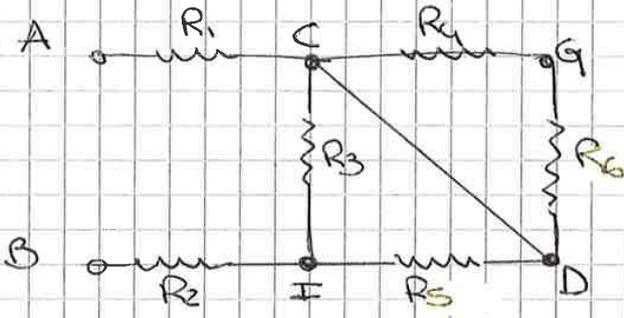
in serie : $R''' = R'' + R_3 = 20 + 20 = 40 \Omega = R'''$

Nota

Due resistenze uguali in ||
 → Req vale la metà
 (se 3 uguali ⇒ 1/3 ecc)

$R^{IV} = (R''' \cdot R_2) / (R''' + R_2) = 20 \Omega$

ES 4



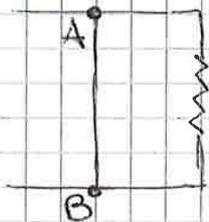
$R_{AB}?$

Ma se i nodi C-D non sono elettricamente distinti!

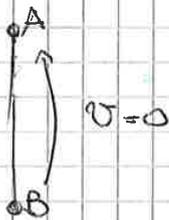
↑
C-D corto circuito

- $R_1 = 10 \ \Omega$
- $R_2 = 10 \ \Omega$
- $R_3 = 20 \ \Omega$
- $R_4 = 20 \ \Omega$
- $R_5 = 20 \ \Omega$
- $R_6 = 20 \ \Omega$

OSS

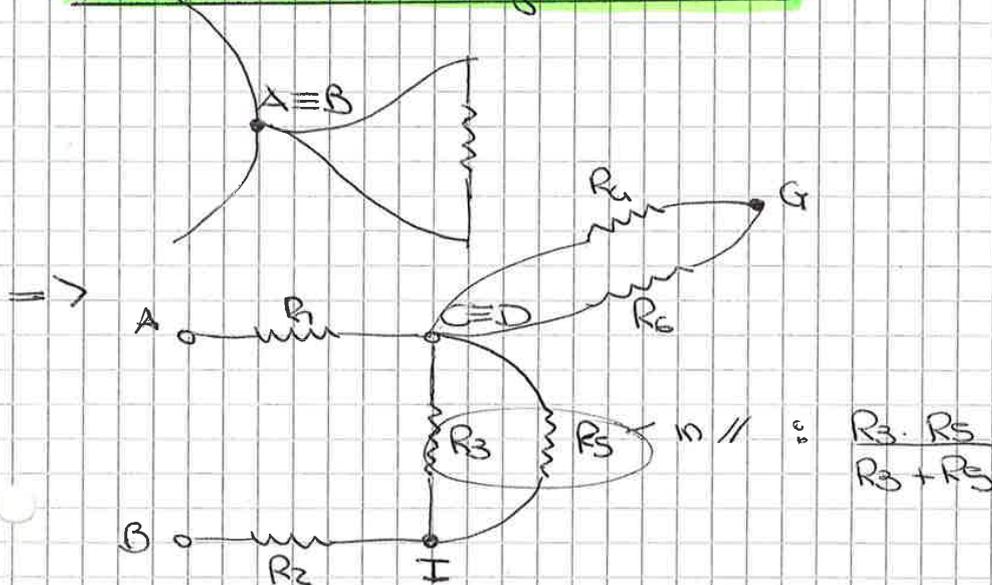


$R_{eq} = 0$



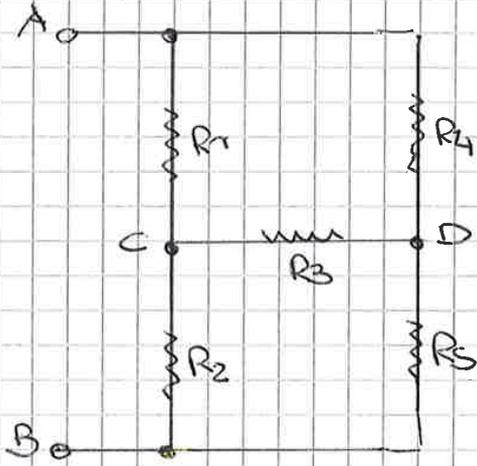
corto circuito

è equivalente disegnare il caso:



$$R_{AB} = R_1 + R_2 + \frac{R_3 \cdot R_5}{R_3 + R_5} = 10 + 10 + \frac{20 \cdot 20}{20 + 20} = 30$$

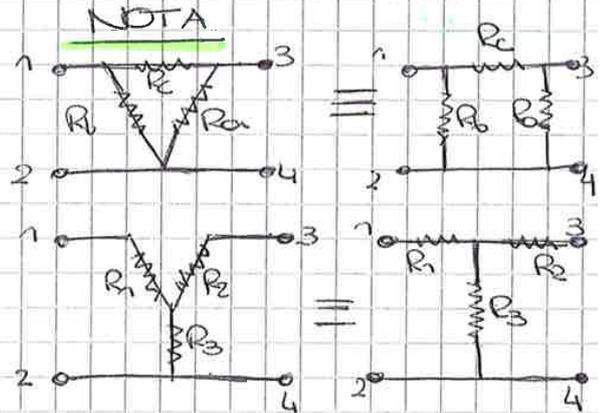
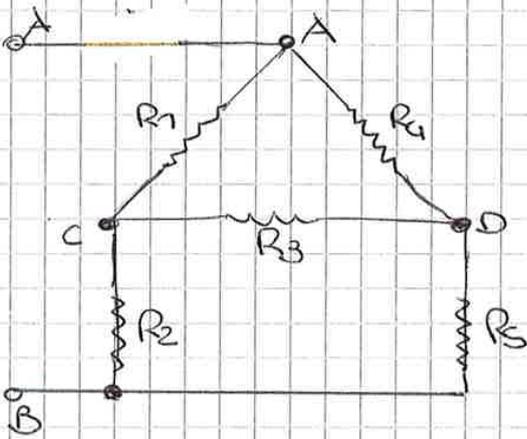
ES6



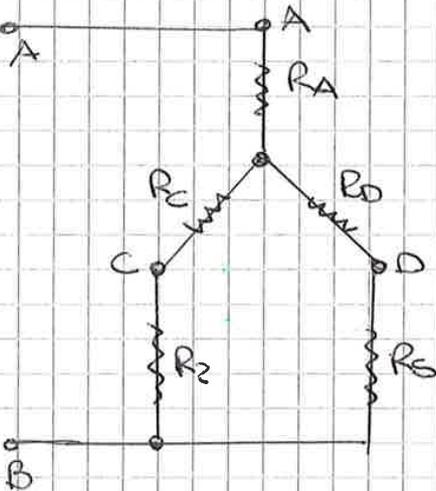
• R_{AB} ?

$R_1 = R_2 = R_3 = R_4 = R_5 = 30 \Omega$

↓ equivalente a

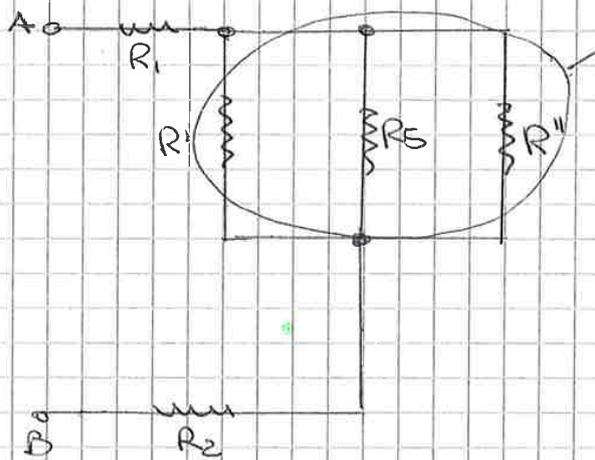


Ora posso trasformare il Δ in Y



$R_Y = \frac{R_{\Delta}}{3} = \frac{30}{3} = 10 \Omega$

$R_A = R_C = R_D = 10 \Omega$



$$R_{eq} = \frac{1}{\frac{1}{30}} = 10 \Omega$$

$$G_{eq} = G' + G_5 + G''$$

$$= \frac{1}{R} + \frac{1}{R_5} + \frac{1}{R''}$$

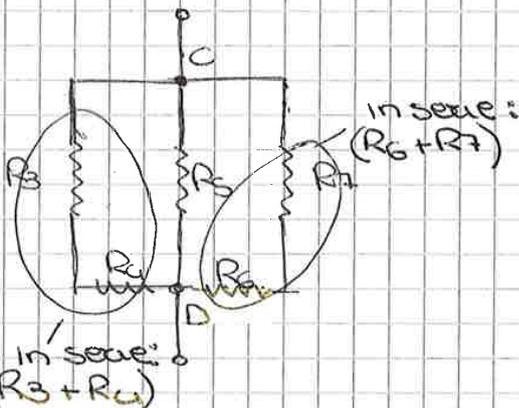
$$= \frac{1}{30} + \frac{1}{30} + \frac{1}{30}$$

$$= \frac{3}{30} = \frac{1}{10} \text{ S}$$

$$R_{eq} = \frac{1}{G_{eq}} = 10 \Omega$$

$$R_{AB} = R_1 + R_2 + R_{eq} = 10 + 10 + 10 = 30 \Omega$$

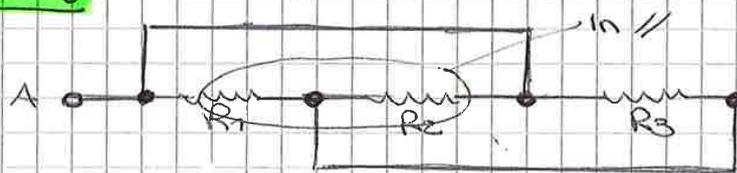
R_{CD}



in serie: $(R_6 + R_7)$ $(R_3 + R_4)$
 (non si considerano R_1 e R_2 perché A e B aperti)

$$R_{CD} = (10 + 20) \parallel R_5 \parallel (10 + 20) = \left(\frac{30 \cdot 30}{30 + 30} \right) \parallel 30 = \frac{30 \cdot 15}{30 + 15}$$

ES 8



$$\frac{30 \cdot 30}{30 + 30} = 15$$

$$= \frac{450}{45} = 10 \Omega$$

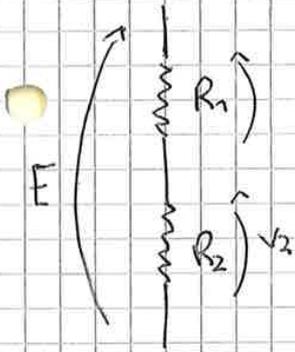
$$R_1 = R_2 = R_3 = 30 \Omega$$

R_{AB} ?

$$R' = R_1 \parallel R_2 = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{30 \cdot 30}{30 + 30} = 15 \Omega$$

$$R_{AB} = R' \parallel R_3 = \frac{15 \cdot 30}{15 + 30} = 10 \Omega$$

Per trovare di tensione per trovare V_2 :



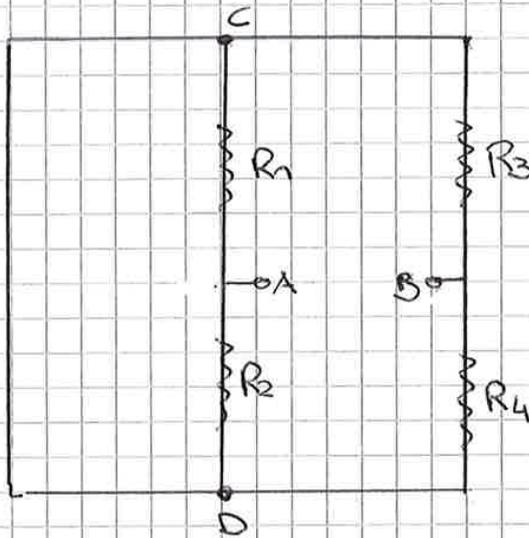
$$V_2 = E \cdot \frac{R_2}{R_1 + R_2} = 200 \cdot \frac{60}{10 + 60} = 200 \cdot \frac{4}{5} = 160 \text{ V}$$

Per trovare di tensione per trovare V_4 :

$$V_4 = E \cdot \frac{R_4}{R_3 + R_4} = 200 \cdot \frac{5}{20 + 5} = 200 \cdot \frac{1}{5} = 40 \text{ V}$$

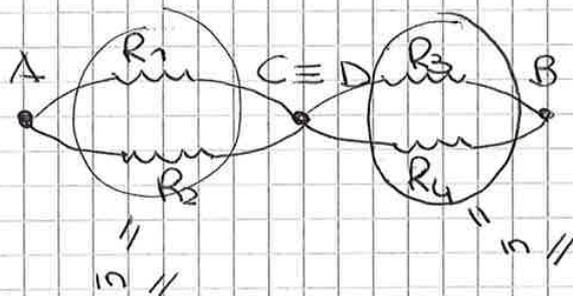
$$E_{eq} = 160 - 40 = 120 \text{ V}$$

Per trovare R_{eq} , passiamo la rete:



NOTA

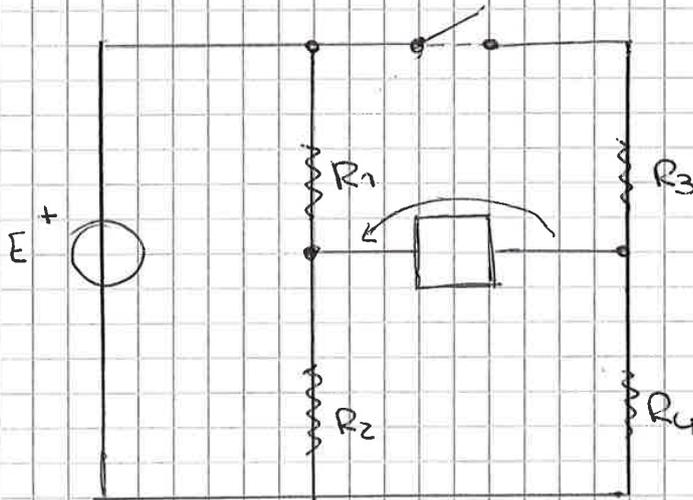
C e D non sono elettricamente distinti perché c'è un c.c. \Rightarrow è come se nodi cancellassero



$$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2} + \frac{R_3 \cdot R_4}{R_3 + R_4} = \frac{10 \cdot 60}{10 + 60} + \frac{20 \cdot 5}{20 + 5} = 8 + 4 = 12 \Omega$$

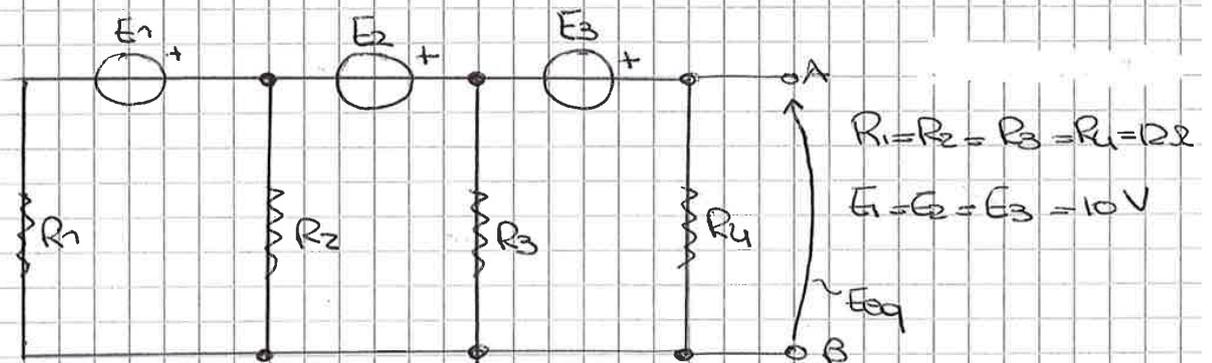
PONTE DI WHEATSTONE :

Consente di mettere in addece qualcosa che non funziona nel sistema. Δ AD



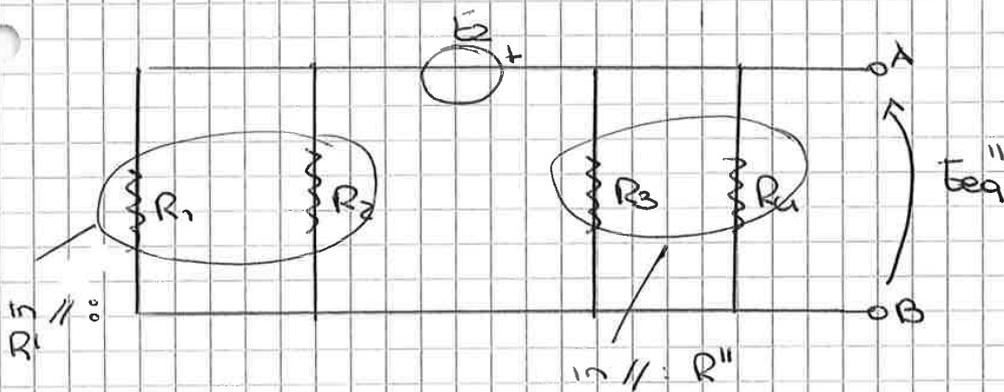
- Interruttore chiuso \Rightarrow Ponte equilibrato (tensione nulla)
- Se interruttore viene aperto \Rightarrow il Ponte non \bar{e} piú equilibrato

ES 2



- Circuito equivalente ai morsetti AB ?

Effetto 2 (E₂) :

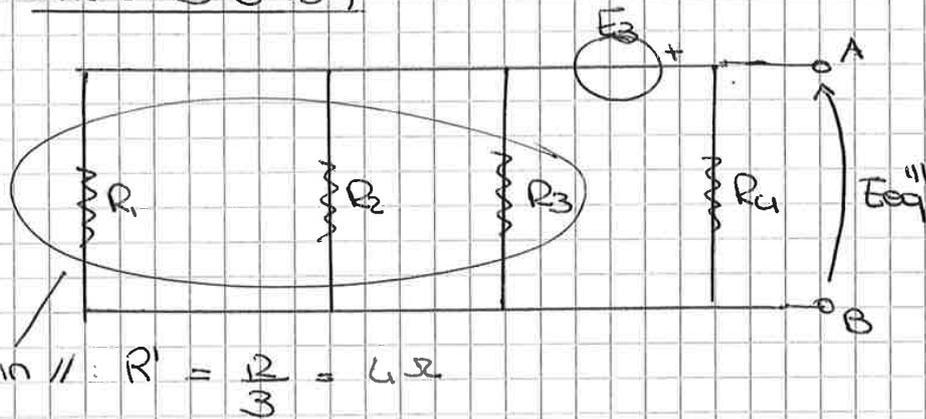


$$R = \frac{R_1 \cdot R_2}{R_1 + R_2} = 6 \Omega$$

$$R'' = \frac{R_3 \cdot R_4}{R_3 + R_4} = 6 \Omega$$

$$E_{eq}'' = E_2 \cdot \frac{R''}{R + R''} = 10 \cdot \frac{6}{6+6} = 10 \cdot \frac{6}{12} = \frac{10}{2} = 5 \text{ V}$$

Effetto 3 (E₃) :



$$R' = \frac{R}{3} = 4 \Omega$$

$$E_{eq}''' = E_3 \cdot \frac{R_4}{R_4 + R'} = 10 \cdot \frac{12}{12+4} = 10 \cdot \frac{12}{16} = \frac{30}{4} = \frac{15}{2} = 7,5 \text{ V}$$

$$E_{eq} = 2,5 + 5 + 7,5 = 15 \text{ V}$$

Nb. (per casa)

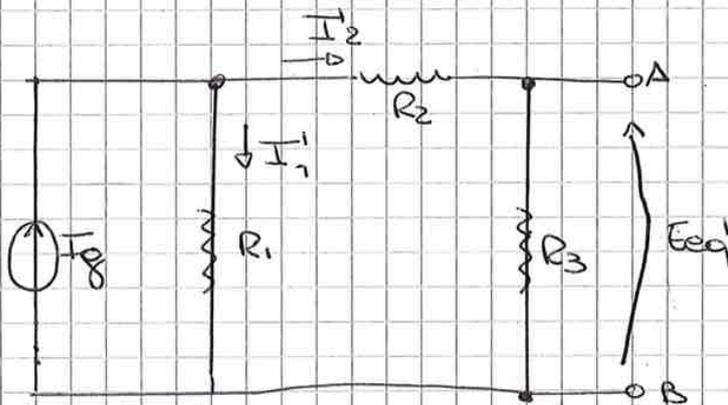
Calcolo con Millman V_{CB} e po. trova E_{eq}

Ora lo risolviamo con la sovrapposizione:

$$E_{eq} = E'_{eq} + E''_{eq}$$

\uparrow \uparrow
 I_g E

Effetto 1 (I_g):

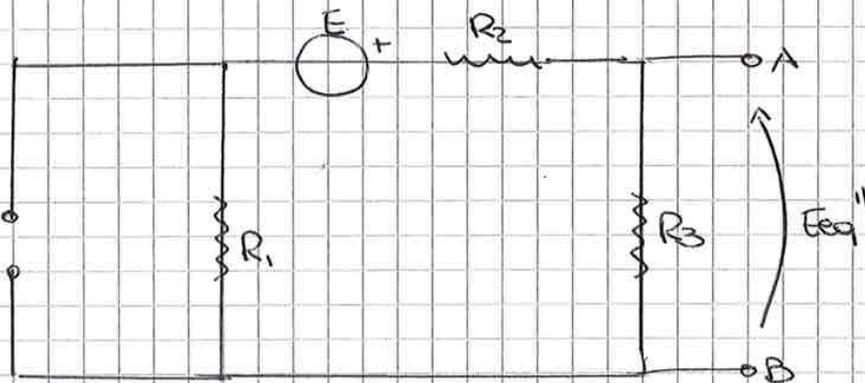


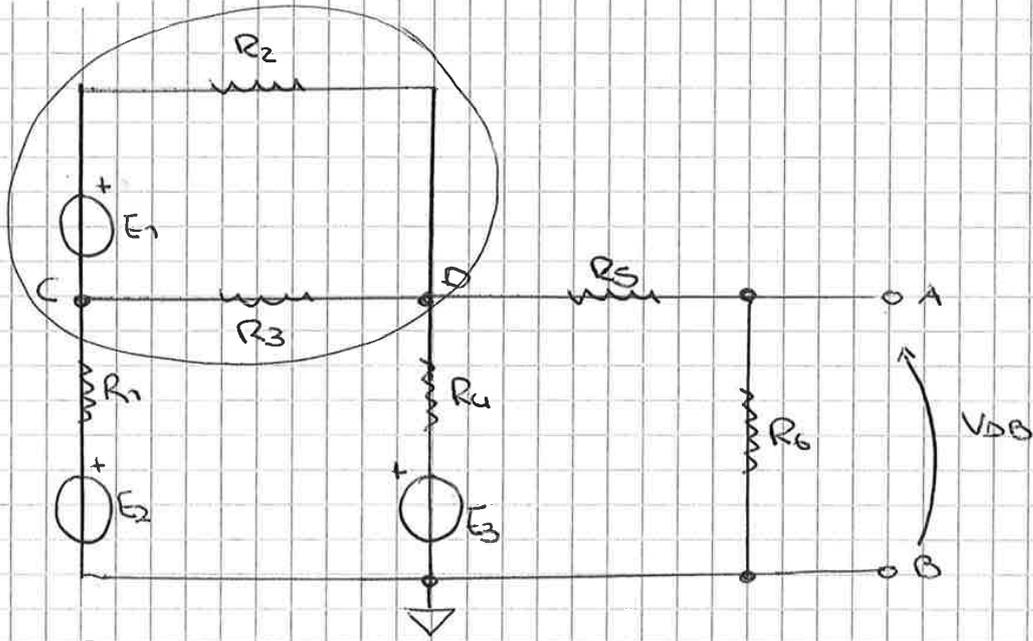
Partitore di corrente $I'_2 = I_g \frac{R_1}{R_1 + (R_2 + R_3)}$

$$= 10 \frac{10}{10 + 5 + 5} = 5 \text{ A}$$

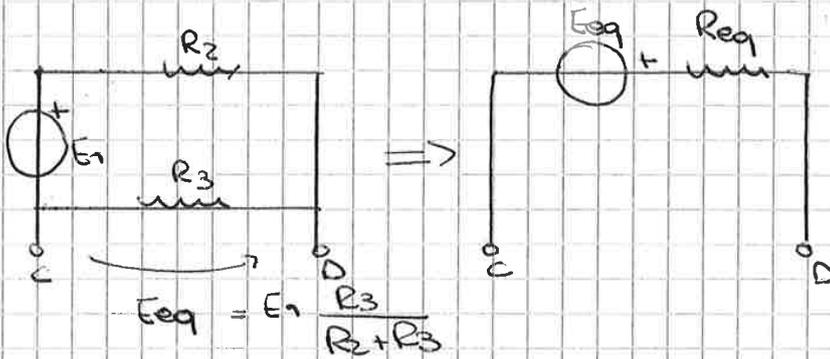
$$E'_{eq} = R_3 \cdot I'_2 = 5 \cdot 5 = 25 \text{ V}$$

Effetto 2 (E):





• $V_{AB} ?$

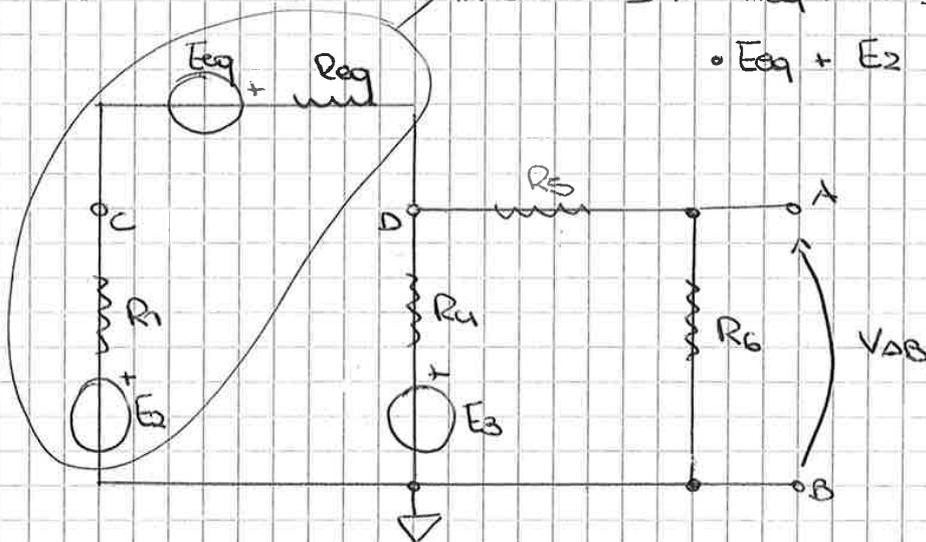


$$E_{eq} = E_1 \frac{R_3}{R_2 + R_3}$$

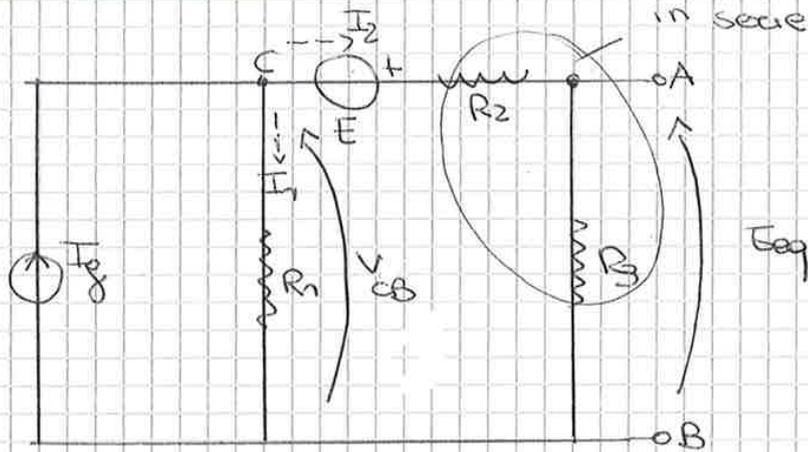
$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3}$$

In serie \Rightarrow

- $R_{eq} + R_1 = R'$
- $E_{eq} + E_2 = E'$



ESERCITAZIONE 2 - ES. 3 con MILMAN



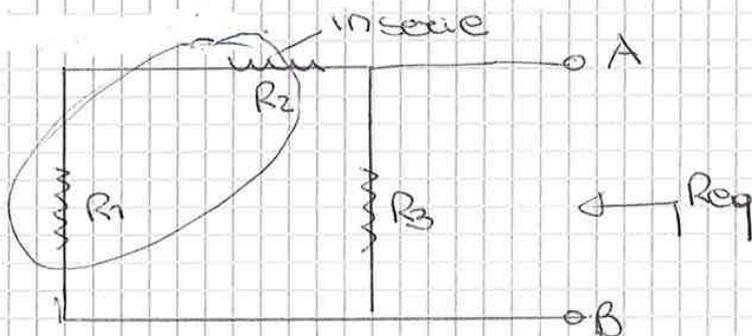
$$V_{CB} = \frac{I_g + (-E)/R_2 + R_3}{\frac{1}{R_1} + \frac{1}{R_2 + R_3}}$$

$$= \frac{10 - \frac{40}{10}}{\frac{1}{10} + \frac{1}{10}} = \frac{10 - 4}{\frac{2}{10}} = \frac{6}{\frac{1}{5}} = 30 \text{ V}$$

$$I_1 = \frac{V_{CB}}{R_1} = \frac{30}{10} = 3 \text{ A}$$

$$I_2 = I_g - I_1 = 10 - 3 = 7 \text{ A} \Rightarrow V_3 = E_{eq} = R_3 \cdot I_2 = 5 \cdot 7 = 35 \text{ V}$$

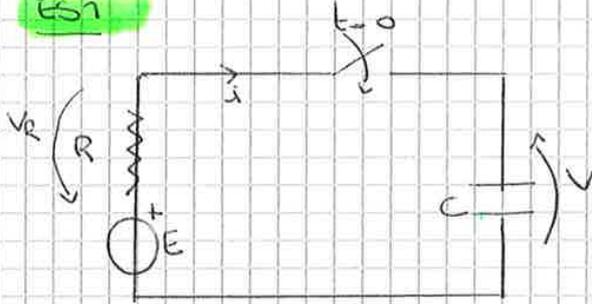
Req (sempre passivando la rete)



$$Req = (R_1 + R_2) \parallel R_3 = \frac{15 \cdot 5}{15 + 5} = \frac{75}{20} = \frac{15}{4} \Omega$$

ESERCITAZIONE 3

ES1



$$R = 100 \Omega$$

$$C = 100 \mu F$$

$$E = 10 V$$

$$Q = CV$$

$$\text{LKT: } E - V_R - V = 0$$

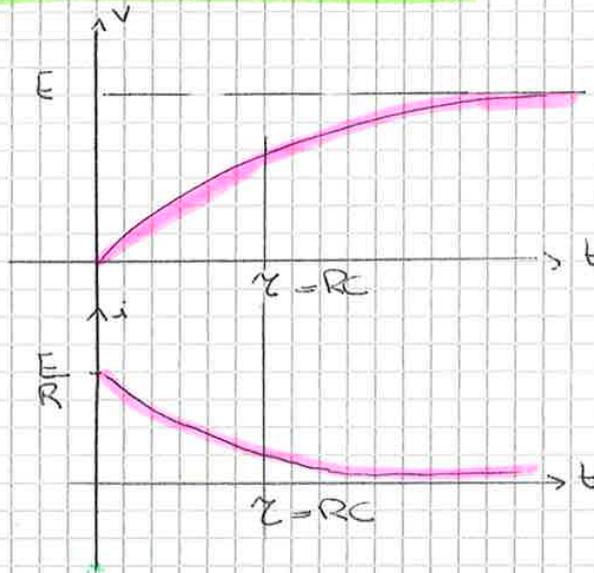
$$\text{Eq. COSTITUTIVA: } V_R = R \cdot i$$

$$\Rightarrow i = \frac{E - V}{R}$$

$$\text{Derivando si ottiene: } \frac{dQ}{dt} = C \frac{dV}{dt}$$

$$i = C \frac{dV}{dt}$$

Andamento di $V(t)$ e $i(t)$



$$V(0) = 0$$

$$V(\infty) = E$$

$$i(0) = \frac{E - V(0)}{R}$$

$$i(\infty) = 0$$

$$E - V - R \cdot i = 0$$

$$E - V - R \cdot C \frac{dV}{dt} = 0 \rightarrow \text{eq. differenziale di I ordine} *$$

$$\frac{dV}{dt} = \frac{(E - V)}{RC}$$

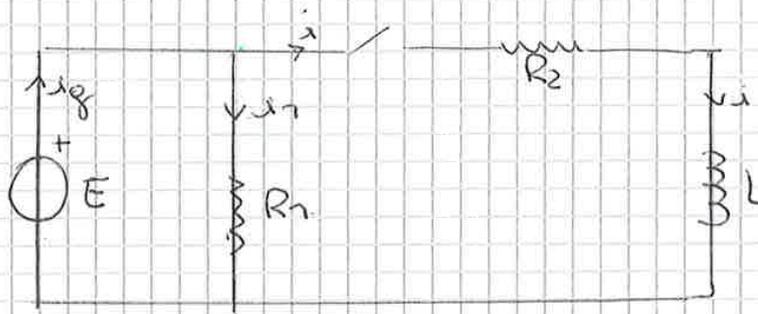
*

• Omogenea associata

$$E = 0$$

$$V + RC \frac{dV}{dt} = 0$$

ES 2



$E = 12V$
 $R_1 = R_2 = 10 \Omega$
 $L = 0,5 H$

Tasto aperto: $i_2 = i_1 = \frac{E}{R_1} = 1,2 A$

NOTA appena chiuso il tasto non si può avere corrente sull'induttanza perché i è continua $W_L = \frac{1}{2} Li^2$
 $\Rightarrow i(0^-) = i(0^+) = 0$
 Dunque la tensione è nulla di conseguenza.

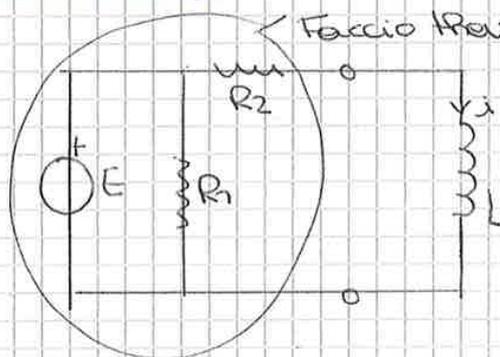
A regime: $\frac{di}{dt} = 0$ corrente stazionaria

$\Rightarrow V_L = L \frac{di}{dt} = 0$

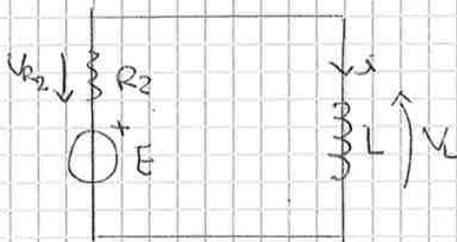
$i = \frac{E}{R_2} = 1,2 A$

$i_{ges} = i_1 + i_2 = 2,4 A$

$P_R(t) = E \cdot i_2 = E (i_1 + i)$



Faccio Thévenin: $E_{eq} = E$



$E - R_2 i - V_L = 0$
 $V_L = L \frac{di}{dt}$

$$j_g = j_n + j = \frac{E}{R_1} + \left[-\frac{E}{R_2} e^{-t/L/R_2} + \frac{E}{R_2} \right]$$

$$P_g = E \cdot j_g = E \cdot j_n + E \cdot j = \frac{E^2}{R_1} + \left[-\frac{E^2}{R_2} e^{-t/L/R_2} + \frac{E^2}{R_2} \right]$$

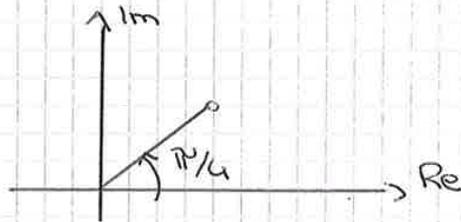
ES 2

$$\bar{y} = \frac{1}{10 + j10} \quad f = 100 \text{ Hz}$$

• $y(t)$?

$$|10 + j10| = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$$

$$\angle \bar{d} = \arctg \frac{10}{10} = \frac{\pi}{4}$$



$$\bar{y} = \frac{1}{10\sqrt{2} e^{j\pi/4}} = \frac{\sqrt{2}}{20} e^{-j\pi/4}$$

$$\omega = 2\pi f = 628 \frac{\text{rad}}{\text{s}}$$

$$y(t) = \frac{\sqrt{2}}{20} \cdot \frac{\sqrt{2}}{20} \sin\left(628t - \frac{\pi}{4}\right)$$

ES 3

$$\bar{z} = \frac{10 e^{j\pi/4}}{10 + j10} = \frac{10 e^{j\pi/4}}{10\sqrt{2} e^{j\pi/4}}$$

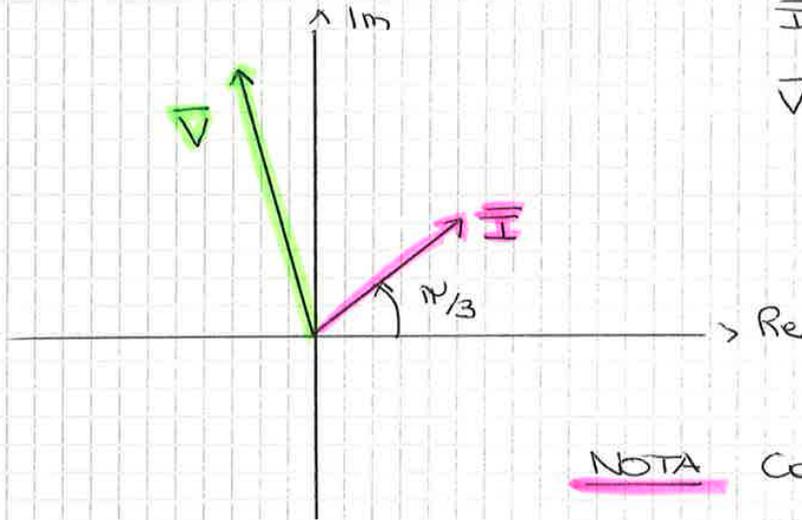
• Ridurre l'espressione

$$\bar{z} = \frac{10 e^{j\pi/4}}{10\sqrt{2} e^{j\pi/4}} = \frac{1}{\sqrt{2}} \quad \text{e}$$

$$\text{Modulo} = \frac{1}{\sqrt{2}} = 0,7$$

$$\text{fase} = 0$$

L'impedenza è un RESISTORE
perché ha solo parte reale



$$\bar{I} = b e^{j \frac{\pi}{3}} \text{ A}$$

$$\bar{V} = 10\sqrt{2} e^{j \frac{7}{12} \pi} \text{ V}$$

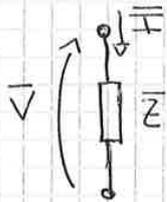
NOTA Corrente in ritardo rispetto \bar{V}

ES5

$\bar{Z} = (1 - j) \Omega$ — Impedenza di tipo OHMICO - CAPACITIVA

$\bar{V} = 100 \text{ V} = 100 \cdot e^{j0} \text{ V}$

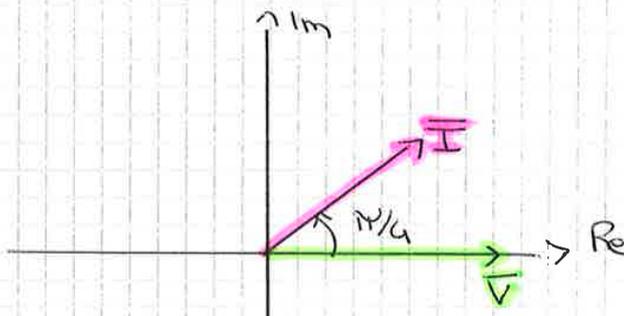
• $\bar{I} ?$



$$\begin{cases} |\bar{Z}| = \sqrt{2} \\ \angle \bar{Z} = -45^\circ = -\frac{\pi}{4} \end{cases}$$

$\Rightarrow \bar{Z} = \sqrt{2} e^{-j \frac{\pi}{4}}$

$$\bar{V} = \bar{Z} \cdot \bar{I} \Rightarrow \bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{100}{\sqrt{2} e^{-j \frac{\pi}{4}}} = \frac{100}{\sqrt{2}} e^{j \frac{\pi}{4}} = (70,7 e^{j \frac{\pi}{4}}) \text{ A}$$

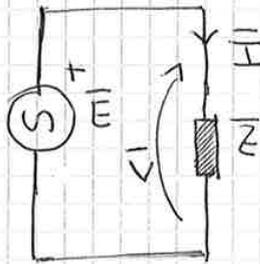


NOTA Corrente in anticipo rispetto tensione

31. 10. 16

ESERCITAZIONE 5

ES1



$$\bar{Z} = (1 + j) \Omega$$

$$\bar{I} = 10 e^{j\pi/3}$$

• $e(t)$?

$$v(t) = e(t)$$

$$\bar{I} \rightarrow i(t)$$

$$i(t) = \sqrt{2} |\bar{I}| \sin(\omega t + \arg \bar{I})$$

$$|\bar{I}| = 10 \text{ A} \quad \arg(\bar{I}) = \frac{\pi}{3}$$

$$i(t) = \sqrt{2} \cdot 10 \sin\left(\omega t + \frac{\pi}{3}\right)$$

$$\bar{Z} \stackrel{\Delta}{=} \frac{\bar{V}}{\bar{I}}$$

per definizione

$$|\bar{Z}| = \frac{V}{I}; \quad \arg(\bar{Z}) = \arg(\bar{V}) - \arg(\bar{I})$$

$$V = |\bar{Z}| \cdot I = \sqrt{2} \cdot 10 = (10\sqrt{2}) \text{ V}$$

$$\arg(\bar{Z}) = \arctan \frac{\text{Im}(\bar{Z})}{\text{Re}(\bar{Z})} = \arctan 1 = \frac{\pi}{4}$$

$$\arg(\bar{V}) = \arg(\bar{Z}) + \arg(\bar{I}) = \frac{\pi}{4} + \frac{\pi}{3} = \frac{7}{12} \pi$$

$$\bar{V} = \bar{Z} \cdot \bar{I} = (1 + j) 10 e^{j\pi/3} = \sqrt{2} \cdot e^{j\pi/4} \cdot 10 e^{j\pi/3}$$

$$= \underbrace{10\sqrt{2}}_V \cdot e^{j\frac{7}{12}\pi}$$

$$v(t) = \sqrt{2} \cdot V \sin(\omega t + \arg \bar{V})$$

$$= 20 \sin\left(\omega t + \frac{7}{12}\pi\right)$$

$$\bar{Z} = 10 \frac{(2\pi)^2}{100 + (2\pi)^2} + j 2\pi \cdot \frac{100}{100 + (2\pi)^2}$$

$$= (2.83 + j 4.50) \Omega$$

$f = 5 \text{ KHz}$

$$X_L = 2\pi \cdot 100 \Omega$$

$$\bar{Z} = 10 \frac{(2\pi \cdot 100)^2}{100 + (2\pi \cdot 100)^2} + j 2\pi \cdot 100 \cdot \frac{100}{100 + (2\pi \cdot 100)^2}$$

$$\bar{Z} = 10 + j 0.16$$

NOTA

f piccola

- Se R e L in serie, prevale la resistenza (prevale quello più grande)
- Se R e L in parallelo, prevale il comportamento induttivo

$$\bar{I}_L = \frac{\bar{U}}{j\omega L}$$

per ω piccola \Rightarrow per \bar{I}_L aumenta, mentre \bar{I}_R è sempre la stessa

\Rightarrow Se ω molto piccola, $\bar{I} = \bar{I}_L$

segue che R la faccio diventare c.a. essendo trascurabile

(Due elementi in parallelo prevale quello più piccolo)

$$I_n = |27,6 - j36,8| = 46 \text{ A}$$

$$\begin{aligned} \overline{Z_{eq2}} &= \frac{R \cdot 4j}{R + 4j} = \frac{3 \cdot 4j}{3 + 4j} = \frac{12j}{3 + 4j} \cdot \frac{(3 - 4j)}{(3 - 4j)} = \\ &= (1,92 + 1,44j) \Omega \end{aligned}$$

$$\overline{I_2} = \frac{230}{1,92 + 1,44j} = (76,67 - j57,5) \text{ A}$$

$$I_2 = |76,67 - j57,5| = 96 \text{ A}$$

d)

$$\bar{S} = P + jQ = (2101 + j 4202) \text{ VA}$$

$$|\bar{S}| = S = \sqrt{2101^2 + 4202^2} = 4700 \text{ VA}$$

POTENZA
APPARENTE

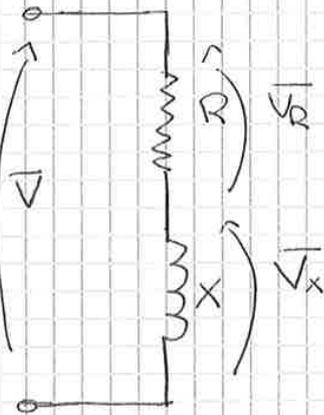
OPPURE

$$S = \sqrt{P^2 + Q^2} = \sqrt{(VI \cos \theta)^2 + (VI \sin \theta)^2} = VI$$

$$= 230 \cdot 20,5$$

$$= 4715 \text{ VA}$$

OPPURE



$$P = \frac{V_R^2}{R} = \frac{102,5^2}{5} = 2101 \text{ W}$$

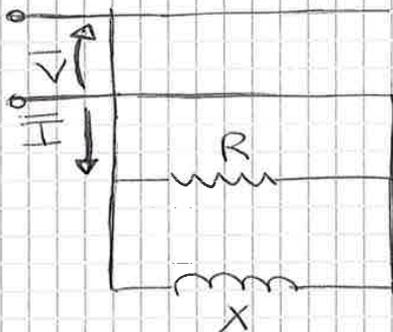
$$Q = \frac{V_X^2}{X} = \frac{205^2}{10} = 4202 \text{ VAR}$$

OPPURE

$$P = \frac{V^2}{Z} \cos \theta = \frac{230^2}{11,2} \cdot \cos(63,4^\circ) = 2115 \text{ W}$$

$$Q = \frac{V^2}{Z} \sin \theta = \frac{230^2}{11,2} \sin(63,4^\circ) = 4223 \text{ VAR}$$

ES 2



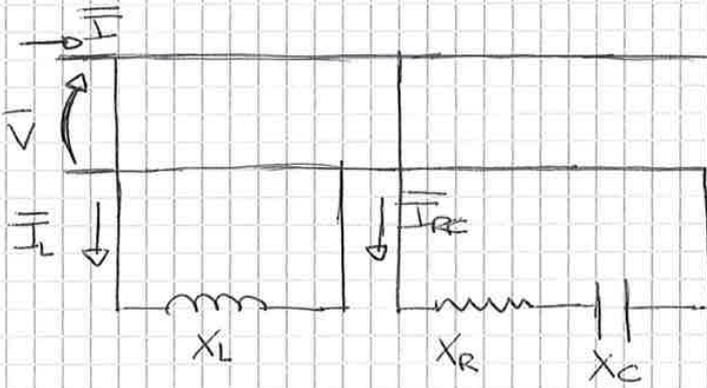
$$V = 230 \text{ V}$$

$$R = 5 \Omega$$

$$X = 10 \Omega$$

1. I ?
2. S ?

ES 3



$V = 230 \text{ V}$
 $X_L = 10 \ \Omega$
 $X_R = R = 10 \ \Omega$
 $X_C = 7,5 \ \Omega$

- a) \vec{I}_L ? b) \vec{I}_{RC} ?
 c) Q_L ? d) \vec{S}_{TOT} ?
 e) \vec{I} ?

a) $\vec{I}_L = \frac{\vec{V}}{\vec{Z}_L} = \frac{230 e^{j0}}{10 e^{j\pi/2}} = 23 e^{-j\pi/2} \text{ A} = 23 e^{-j90^\circ} \text{ A}$

b) $\vec{I}_{RC} = \frac{\vec{V}}{\vec{Z}_{RC}} = \frac{\vec{V}}{R - jX_C} = \frac{230}{12,5 \cdot e^{j37^\circ}} = 18,4 e^{j37^\circ} \text{ A}$
 moduli = 12,5 Ω
 fase = -37°

c) $Q_L = X_L I_L^2 = 10 \cdot 23^2 = 5290 \text{ VAR}$

d) $\vec{S}_{TOT} = \sum_k \vec{S}_k$

$\vec{S}_L = j Q_L \text{ VA}$

$\vec{S}_{RC} = \vec{Z}_{RC} \cdot \vec{I}_{RC}^2 = (10 - j7,5) \cdot 18,4^2 = 3385 - j2539 \text{ VA}$

$\vec{S}_{TOT} = \vec{S}_{RC} + j Q_L = 3385 - j2539 + j5290$
 $= (3385 + j2751) \text{ VA}$

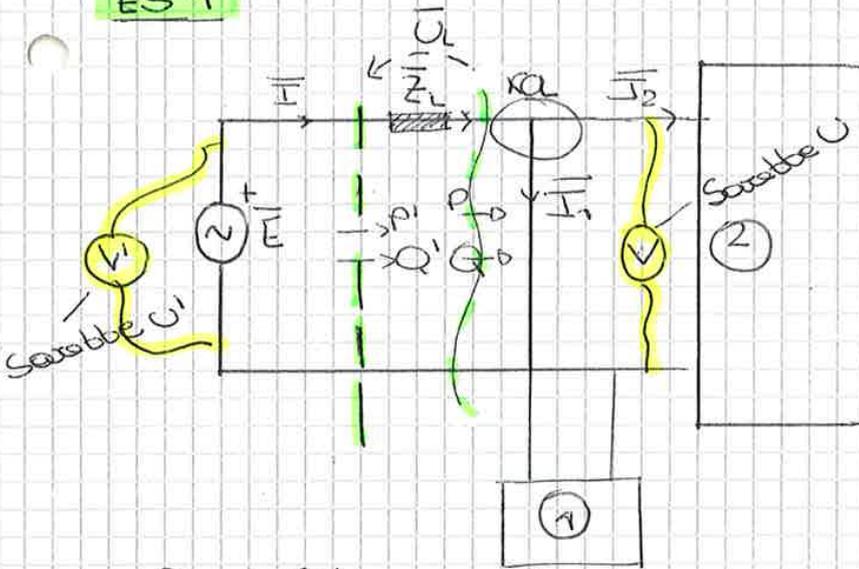
$S_{TOT} = \sqrt{P^2 + Q^2} = 4362 \text{ VA}$

e) $S_{TOT} = V \cdot I \Rightarrow I = \frac{S_{TOT}}{V} = \frac{4362}{230} = 18,9 \text{ A}$

Esercitazione 7

14.11.16

ES 1



$P_1 = 10 \text{ kW}$

$Q_1 = 7,5 \text{ KVAR}$

$P_2 = 20 \text{ kW}$

$Q_2 = 7,5 \text{ KVAR}$

$\bar{z} = 50 + j100 \text{ (m}\Omega\text{)}$

$U = 230 \text{ V}$ Nota è un valore efficace!

$$\begin{cases} \bar{S}_1 = P_1 + jQ_1 \\ \bar{S}_1 = U \cdot \bar{I}_1^* = U \left(\frac{U}{\bar{z}_1} \right)^* = U \cdot \frac{U^*}{\bar{z}_1^*} = \frac{U^2}{\bar{z}_1^*} \Rightarrow \\ \bar{z}_1 = \frac{U^2}{\bar{S}_1^*} = \frac{230^2}{P_1 - jQ_1} \end{cases}$$

$S = U \cdot I$ potenza apparente (per questa non vale il co. di conservazione di Boucherot)

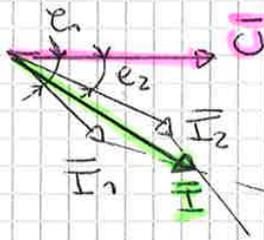
$$|S| = \sqrt{P^2 + Q^2} = 15\sqrt{5} = 33,54 \text{ KVA}$$

$P = P_1 + P_2 \quad Q = Q_1 + Q_2$

$I = \frac{S}{U} = \frac{33,54 \cdot 10^3}{230} = 145,82 \text{ A}$

$S' = U' \cdot I$ potenza apparente del generatore

$$\sqrt{P'^2 + Q'^2} \quad P' = P_1 + P \quad Q' = Q_1 + Q$$



$$\vec{I} = (86,95 + j43,47) - j(32,60 + 32,60)$$

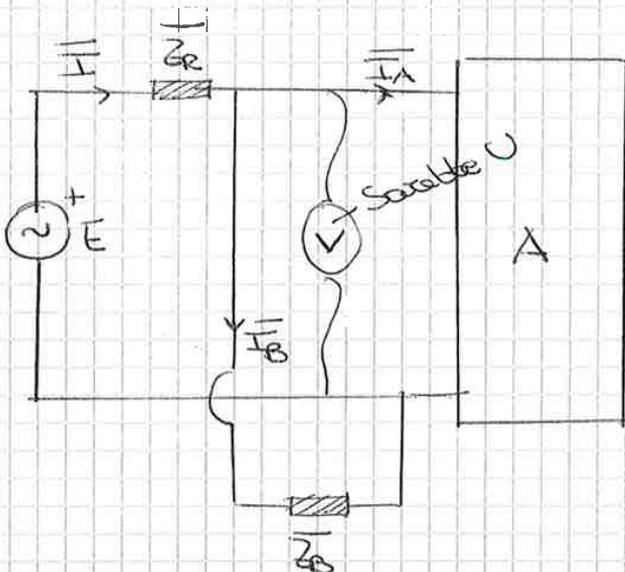
$$= 130,42 - j65,2 \text{ A}$$

Facendo il modulo di \vec{I} otterrei otteneva di nuovo 145,82 A

$$I = |\vec{I}| = 145,82 \text{ A}$$

$$\vec{E} = \vec{Z}_L \cdot \vec{I} + \vec{U} = (50 + j100) \cdot 10^{-3} \cdot (130,42 - j65,2) + 230$$

ES 2



$P_A = 20 \text{ kW}$
 $Q_A = 20 \text{ KVAR}$
 $\vec{S}_A = \vec{U} \cdot \vec{I}_A^*$
 $Z_B = 10 + j5 \text{ } \Omega$
 $Z_L = 75 + j100 \text{ (m}\Omega\text{)}$
 $V = 230 \text{ V}$
 $\vec{I} ?$
 $\vec{E} ?$

$$\vec{I}_A = \frac{\vec{S}_A^*}{\vec{U}} = \frac{(20 - j20) \cdot 10^3}{230} = 86,95 - j86,95 \text{ A}$$

$$\vec{I}_B = \frac{\vec{U}}{Z_B} = \frac{230}{10 + j5} = 18,4 - j9,2 \text{ A}$$

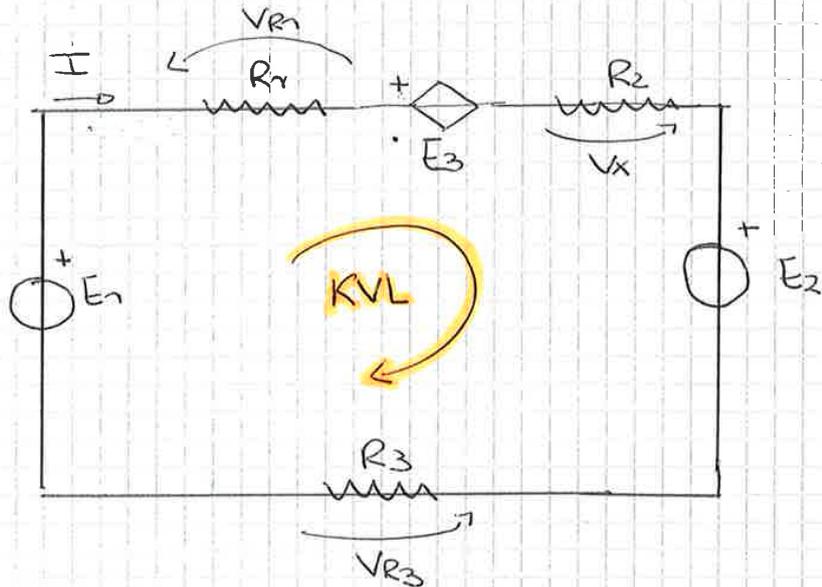
$$\vec{I} = \vec{I}_A + \vec{I}_B = 105,35 - j96,15 \text{ A}$$

$$I = |\vec{I}| \quad \vec{E} = \vec{Z}_R \vec{I} + \vec{U}$$

ESERCITAZIONE 8

16. 11.16

ES 1



- $R_1 = 4 \Omega$
- $R_2 = 2 \Omega$
- $E_2 = 120 \text{ V}$
- $E_3 = \alpha V_x \quad \alpha = 3$
- $R_3 = 6 \Omega$
- $E_1 = 12 \text{ V}$

KVL

$$E_1 - V_{R1} - E_3 + V_x - E_2 - V_{R3} = 0$$

$$E_1 - R_1 I - \alpha V_x + V_x - E_2 - V_{R3} = 0$$

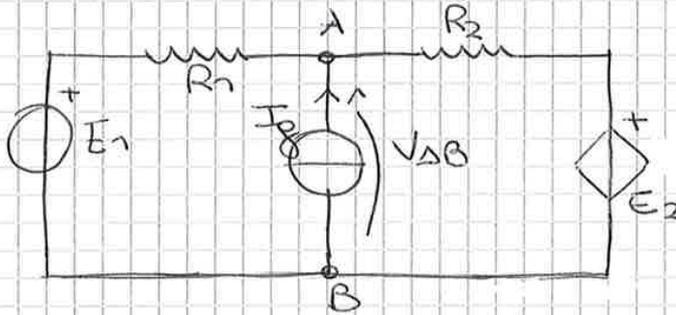
$$E_1 - \underline{R_1 I} - \underline{\alpha (-R_2 I)} + \underline{(-R_2 I)} - E_2 - \underline{R_3 I} = 0$$

$$I (-R_1 + \alpha R_2 - R_2 - R_3) = E_2 - E_1$$

$$I = \frac{E_2 - E_1}{-R_1 + \alpha R_2 - R_2 - R_3} = \frac{120 - 12}{-4 + 3 \cdot 2 - 2 - 6} = \frac{108}{-6} = -18 \text{ A}$$

$$I_2 = \frac{V_{AB}}{R_2} = \frac{48}{7} \cdot \frac{1}{8} = \frac{6}{7} \text{ A}$$

ES 3



$$\begin{aligned} R_1 &= 2 \Omega \\ R_2 &= 4 \Omega \\ E_1 &= 2 \text{ V} \\ I_0 &= 1 \text{ A} \\ E_2 &= \alpha V_{AB} \\ \alpha &= 2 \end{aligned}$$

MILLMAN

• $V_{AB}?$

$$\begin{aligned} V_{AB} &= \frac{\frac{E_1}{R_1} + I_0 + \frac{E_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} \\ &= \frac{R_2 E_1 + R_1 R_2 I_0 + R_1 E_2}{\frac{R_1 + R_2}{R_1 R_2}} \end{aligned}$$

$$V_{AB} = \frac{R_2 E_1 + R_1 R_2 I_0 + \alpha V_{AB} R_1}{R_1 + R_2}$$

$$V_{AB} ((1 - \alpha) R_1 + R_2) = R_2 E_1 + R_1 R_2 I_0$$

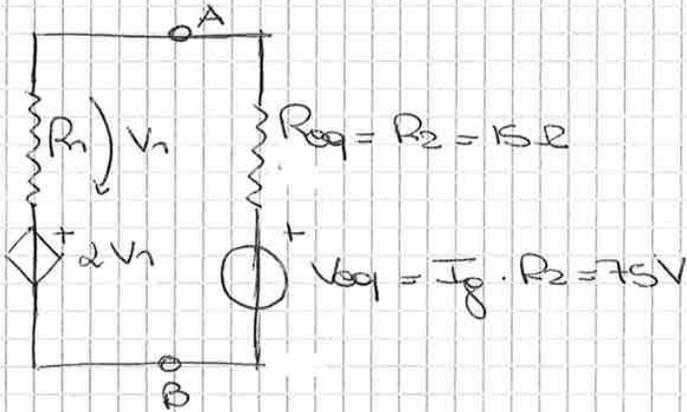
$$V_{AB} = 8 \text{ V}$$

Quindi:

$$V_{AB} = 6V_1 + 30 = 6 \cdot 10 + 30 = 90V$$

* NOTA

Posso anche fare:



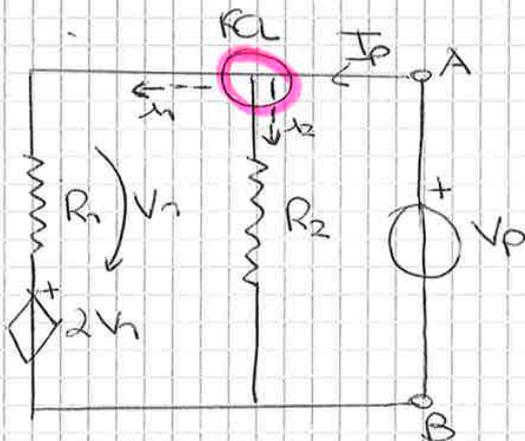
$$V_{AB} = \frac{\frac{2V_1}{R_1} + \frac{V_{eq}}{R_{eq}}}{\frac{1}{R_1} + \frac{1}{R_{eq}}} = \frac{V_1 + 5}{1} = 6V_1 + 30$$

Inoltre:

$$V_{AB} = 2V_1 - V_1 = 9V_1$$

Uguagliando $\Rightarrow 9V_1 = 6V_1 + 30 \Rightarrow 3V_1 = 30 \Rightarrow V_1 = 10V$
 $\Rightarrow V_{AB} = 90V$

R_{TH}



$$R_{TH} = \frac{V_p}{I_p}$$

$$I_p = i_1 + i_2$$

$$i_2 = \frac{V_p}{R_2}$$

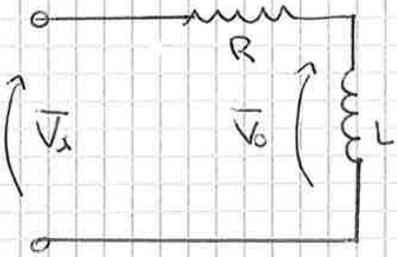
$$i_1 = -\frac{V_1}{R_1} = -\frac{V_p}{9R_1}$$

$$V_p = 2V_1 - V_1 = 9V_1$$

ESERCITAZIONE 9

17.11.16

ES 1



$R = 10 \text{ K}\Omega \quad L = 1 \text{ H}$

- A separare Bode moduli semplificati

$H(\omega) = \frac{V_o}{V_i} = \frac{j\omega L}{R + j\omega L}$ ~ pendenza di tens.

$[L/R] = S$

$$\begin{aligned} \bar{H}(\omega) &= \frac{j\omega L}{R(1 + j\omega \frac{L}{R})} \\ &= \frac{j\omega L/R}{1 + j\omega L/R} \end{aligned}$$

$\frac{L}{R} = \tau$

$\omega_0 = \frac{1}{L/R} = \frac{R}{L} = 10^4 \text{ rad/s}$

$$\bar{H}(\omega) = \frac{j\omega \frac{R}{L}}{1 + j\omega \frac{R}{L}} = \frac{j\omega/\omega_0}{1 + j\omega/\omega_0} = \frac{N(\omega)}{D(\omega)}$$

- N(ω)**
- Ha zero 0 nell'origine (pendenza +20 dB per decade)
 - per $\omega = \omega_0 \Rightarrow 0 \text{ dB}$

- D(ω)**
- per $\omega \rightarrow 0 \Rightarrow 1$ ovvero 0 dB
 - $\omega = \omega_0 \Rightarrow \frac{1}{\sqrt{2}}$ ovvero -3 dB
 - $\omega \rightarrow \infty \Rightarrow \dots \dots \dots -20 \text{ dB/dec}$

$|1 + j\omega/\omega_0|_{\omega=\omega_0} = |1 + j| = \frac{1}{\sqrt{2}} \Rightarrow -3 \text{ dB}$

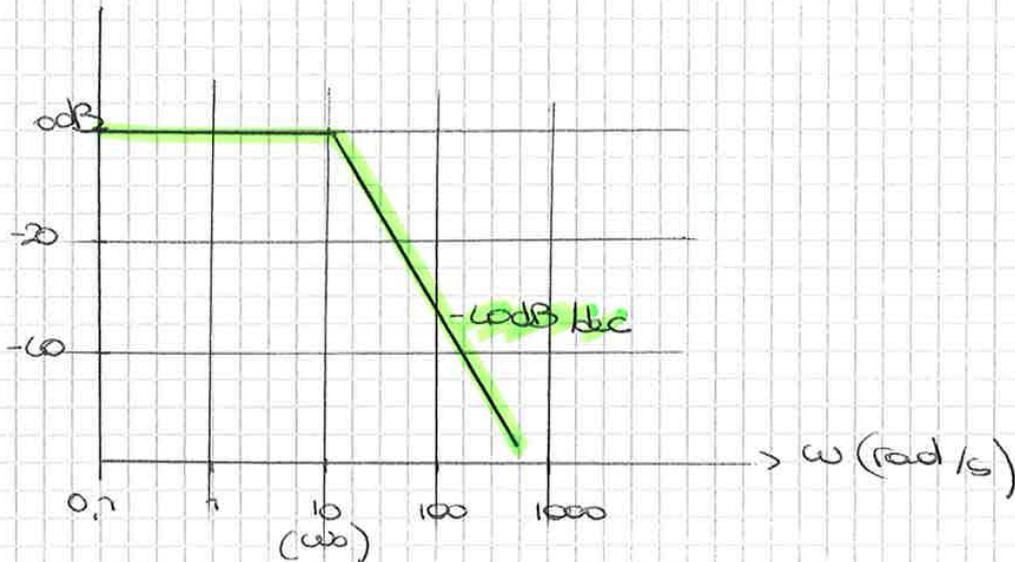
$$\bar{H} = \frac{\frac{R}{1 + j\omega RC}}{j\omega L + \frac{R}{1 + j\omega RC}} = \frac{\frac{R}{1 + j\omega RC}}{\frac{j\omega L(1 + j\omega RC) + R}{1 + j\omega RC}}$$

$$= \frac{R}{j\omega L - \omega^2 RLC + R}$$

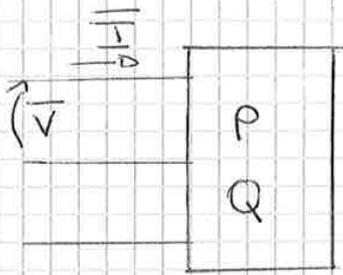
$$\bar{H}(\omega) = \frac{1}{1 + j\omega \frac{L}{R} - \omega^2 LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-1} \cdot 10^{-1}}} = 10 \text{ rad/s}$$

Denominatore : $\omega \rightarrow 0 \Rightarrow 1$ ovvero 0 dB
 $\omega \rightarrow \infty \Rightarrow -40 \text{ dB/dec}$



ES 41: TRIFASE



- $V = 400 \text{ V}$
- $P = 10 \text{ kW}$
- $Q = 5 \text{ KVAR}$
- $\cos \varphi$?
- I ?

$$P = \sqrt{3} V I \cos \varphi$$

$$Q = \sqrt{3} V I \sin \varphi$$

$$\frac{Q}{P} = \tan \varphi = \frac{\sqrt{3} V I \sin \varphi}{\sqrt{3} V I \cos \varphi}$$

$$S = P + jQ$$

$$|S| = \sqrt{P^2 + Q^2} = \sqrt{(\sqrt{3} V I \cos \varphi)^2 + (\sqrt{3} V I \sin \varphi)^2} = \sqrt{3} V I$$

$$\tan \varphi = \frac{Q}{P} = \frac{5}{10} = 0,5 \Rightarrow \cos \varphi \approx 0,9$$

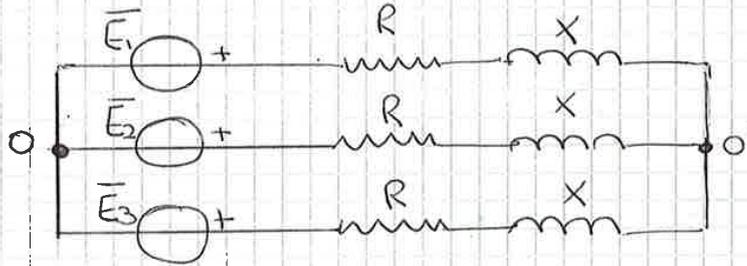
$$\tan \varphi = 0,484 \Leftrightarrow \cos \varphi = 0,9$$

$$P = \sqrt{3} V I \cos \varphi$$

\downarrow
 incognita

$$I = \frac{P}{\sqrt{3} V \cos \varphi} = \frac{10^4 \text{ W}}{\sqrt{3} \cdot 400 \cdot 0,9} = 6 \text{ A}$$

Esempio:

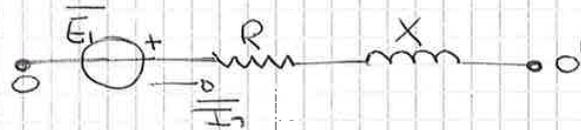


$$|\bar{E}_1| = |\bar{E}_2| = |\bar{E}_3| = 230 \text{ V}$$

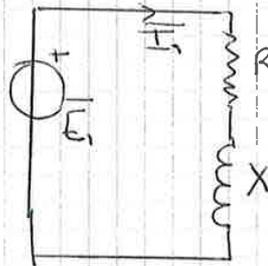
$$R = 1 \Omega$$

$$X = 1 \Omega$$

Se $400 = 0 \text{ V} = 0$



Overo



Tensione di linea = $V = \sqrt{3} \cdot E = 400 \text{ V}$

$$\bar{E}_1 = \bar{Z} \cdot \bar{I}_1 \Rightarrow \bar{I}_1 = \frac{\bar{E}_1}{\bar{Z}}$$

$$\bar{Z} = R + jX$$

$$|\bar{Z}| = \sqrt{R^2 + X^2} = \sqrt{2} \Omega$$

$$\angle \bar{Z} = \arctan \frac{X}{R} = \frac{\pi}{4}$$

$$\bar{I}_1 = \frac{E \cdot e^{j0}}{\bar{Z} \cdot e^{j\theta}} = \frac{230 e^{j0}}{\sqrt{2} \cdot e^{j\pi/4}} = 162,6 e^{-j\frac{\pi}{4}} \text{ A}$$

$$\bar{E}_2 = \bar{Z} \cdot \bar{I}_2 \Rightarrow \bar{I}_2 = \frac{\bar{E}_2}{\bar{Z}}$$

$$\bar{I}_2 = \frac{230 e^{-j\frac{2\pi}{3}}}{\sqrt{2} \cdot e^{j\pi/4}} = 162,6 e^{j(-\frac{2}{3}\pi - \frac{\pi}{4})} = 162,6 e^{-j\frac{11}{12}\pi} \text{ A}$$

ESAME ELETTROTECNICA

18.11.16

- 4 esercizi (ognuno 4 quesiti)
- 1) DC (analisi nodale, pot- enera usate qui ma non e' molto implementate)
 - 2) transistorio
 - 3) AC
 - 4) risposta in frequenza
- + domande aperte, risposta multiple

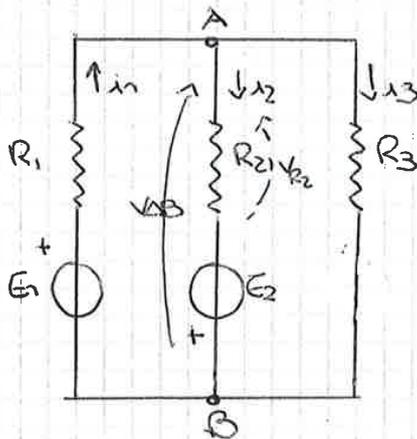
NOTA

Unità di misura SEMPRE!

2 R

SIMULAZIONE ESAME

ES1



$E_1 = 100 \text{ V}$
 $E_2 = 400 \text{ V}$
 $R_1 = R_2 = R_3 = 1 \Omega$

- a) i_1 ?
- b) i_2 ?
- c) i_3 ?
- d) potenza generata ?

Millman (pot- eno usate anche sovrapposizione effetti)

$$V_{AB} = \frac{E_1}{R_1} - \frac{E_2}{R_2} \cdot \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$= \frac{100 - 400}{1 + 1 + 1} = -\frac{300}{3} = -100 \text{ V}$$

$$V_{R2} = V_{AB} + E_2 = -100 + 400 = 300 \Rightarrow V_{R2} = R_2 \cdot i_2$$

$$\textcircled{i_2} = \frac{V_{R2}}{R_2} = 300 \text{ A}$$

$$V_{AB} + R_1 \cdot i_1 - E_1 = 0 \Rightarrow \textcircled{i_1} = \frac{E_1 - V_{AB}}{R_1}$$

$$= \frac{100 - (-100)}{1}$$

$$= 200 \text{ A}$$

$$i(0) = 0 \Rightarrow K e^{-t/\tau} + \frac{E}{R} = 0$$

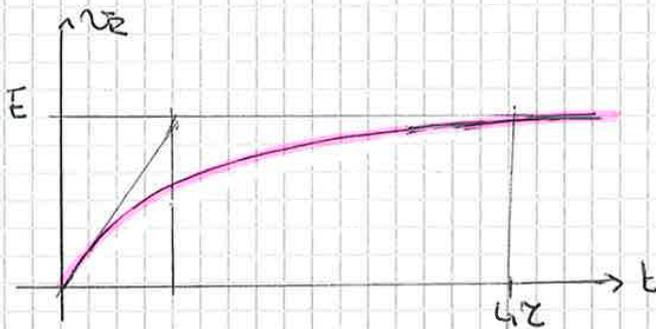
$$K = -\frac{E}{R}$$

$$i(t) = \frac{E}{R} (1 - e^{-t/\tau})$$



b)

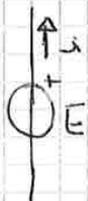
$$v_R(t) = R i(t) = R \cdot \frac{E}{R} (1 - e^{-t/\tau}) = E (1 - e^{-t/\tau})$$



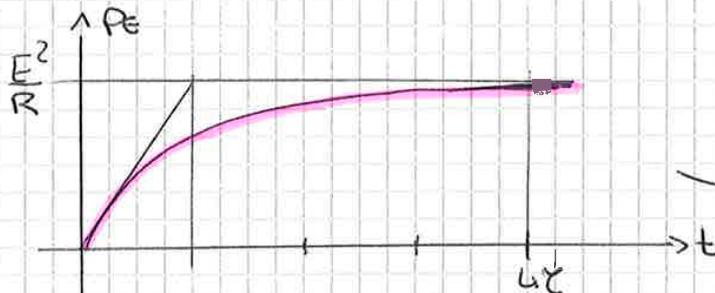
c) Energia in L per $t \rightarrow \infty$

$$W_L = \frac{1}{2} L I_{\infty}^2 = \frac{1}{2} L \left(\frac{E}{R} \right)^2 = \frac{1}{2} \cdot 10^3 \cdot \left(\frac{12}{1} \right)^2 = 72 \text{ mJ}$$

d)



$$P_R(t) = E \cdot i(t) = E \cdot \frac{E}{R} (1 - e^{-t/\tau}) = \frac{E^2}{R} (1 - e^{-t/\tau})$$



Lo stesso andamento corrente

Potenza max: all'infinito ($> 4\tau$)

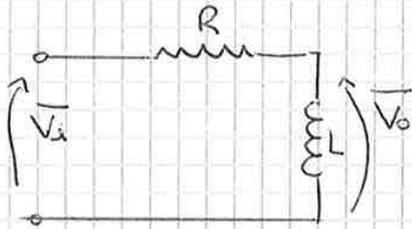
dipolo R-parco-induttivo.

$$|\bar{S}_{tot}| = \sqrt{4,2^2 + 1,6^2} = 4,4 \text{ kVA}$$

$$|\bar{S}_{tot}| = \underset{\substack{\uparrow \\ \text{valori} \\ \text{eff. cacci}}}{V} \cdot \underset{\downarrow}{\bar{I}} \Rightarrow \bar{I} = \frac{|\bar{S}_{tot}|}{V} = \frac{4,4 \cdot 10^3}{230} = 19,1 \text{ A}$$

è minore della somma
dei moduli di \bar{I}_{ec} ed \bar{I}_L !!!
 $I \neq I_{ec} + I_L$

ES4



$$R = 10 \text{ k}\Omega$$

$$L = 1 \text{ H} \Rightarrow X_L = \omega L$$

a) $\bar{Z}_{eq}(\omega)$

b) $\bar{H} = \frac{\bar{V}_o}{\bar{V}_i}$

c) ω_0

d) Bode modulus (semplificato)

a)

$$\bar{Z}_{eq}(\omega) = R + jX_L = R + j\omega L = (10^4 + j\omega) \Omega$$

b)

$$\bar{H}(\omega) = \frac{\bar{Z}_L}{\bar{Z}_{eq}} = \frac{j\omega L}{R + j\omega L} = \frac{j\omega L}{R(1 + j\omega \frac{L}{R})} = \frac{j\omega \frac{L}{R}}{1 + j\omega \frac{L}{R}}$$

c)

$$\omega_0 = \frac{1}{L/R} = \frac{R}{L} = \frac{10^4}{1} = 10^4 \text{ rad/s}$$

$$\bar{H}(\omega) = \frac{j \frac{\omega}{\omega_0}}{1 + j \frac{\omega}{\omega_0}}$$