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NUMERO: 2173A

ANNO: 2017

# **A P P U N T I**

STUDENTE: Cavallo Federico

MATERIA: Fisica II - Teoria+Esercizi - Prof. Kaniadakis

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**ATTENZIONE: QUESTI APPUNTI SONO FATTI DA STUDENTIE NON SONO STATI VISIONATI DAL DOCENTE.  
IL NOME DEL PROFESSORE, SERVE SOLO PER IDENTIFICARE IL CORSO.**

PARTE 1  
da 1 a 9 (1)

1) Legge di Coulomb

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

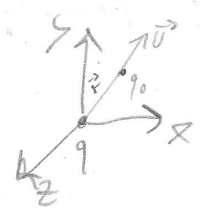
$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$\epsilon_0 = 8,85 \cdot 10^{-12} \text{ C}^2/\text{m}^2$$

$$= k = 9 \cdot 10^9$$

$$q_{e^-} = -1,6 \cdot 10^{-19} \text{ C}$$

2)  $F = k \frac{q_0 q}{r^2} \vec{u}$

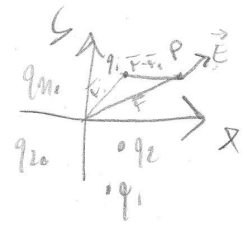


$$\vec{u} = \frac{\vec{r}}{r} = \frac{x\vec{i} + y\vec{j} + z\vec{k}}{\sqrt{x^2 + y^2 + z^2}}$$

$$E = k \frac{q_0}{r^2}$$

$$E = \frac{F}{q} \rightarrow F = E \cdot q$$

3) Sistema Discreto



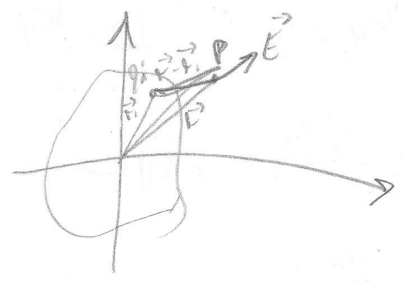
$$\vec{E}_i = \frac{k q_i}{|\vec{r} - \vec{r}_i|^3} |\vec{r} - \vec{r}_i|$$

$$\vec{E}(\vec{r}) = k \sum_{i=1}^n \frac{q_i}{|\vec{r} - \vec{r}_i|^2} (\vec{r} - \vec{r}_i)$$

$$\vec{E} = \sum \vec{E}_i$$

4) Sistema Continuo

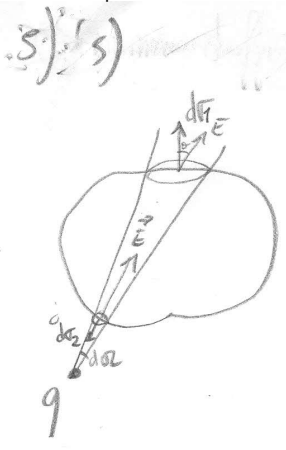
$$\frac{q}{V} = \rho(r)$$



$$E_i = \frac{k \rho(r)}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i)$$

$$E = \int_V \vec{E} dV = \int_V \frac{k \rho(r)}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i) dV$$

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$$d\phi_1 = kq d\Omega_1 \approx +kq d\Omega$$

$$d\phi_2 = kq d\Omega_2 \approx -kq d\Omega$$

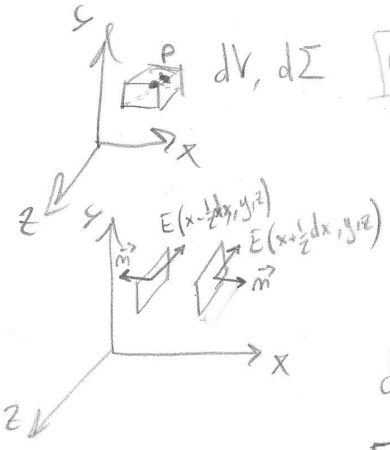
$$d\phi_1 + d\phi_2 = 0$$

$$\oint \vec{E} d\vec{\sigma} = 0$$

b) F. integrale:  $\oint_{\Sigma} \vec{E} d\vec{\sigma} = \frac{q}{\epsilon_0} = \frac{1}{\epsilon_0} \int_V \rho dV \rightarrow \oint \vec{E} d\vec{\sigma} = \frac{1}{\epsilon_0} \int_V \rho dV$

F. differenziale:  $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

Teorema di Gauss in forma differenziale



$\rho(x, y, z)$   
 $dV = dx dy dz$   
 $d\Sigma = 2 dx dy + 2 dy dz + 2 dx dz$

$d\phi = d\phi_x + d\phi_y + d\phi_z$ ;  $d\phi = \frac{\rho dV}{\epsilon_0}$  Gauss

$$d\phi_x = \vec{E} \cdot d\vec{\sigma} \Rightarrow d\phi_x = E_x(x + \frac{1}{2} dx, y, z) dy dz - E_x(x - \frac{1}{2} dx, y, z) dy dz$$

$$= \left[ \left( E_x(x, y, z) + \frac{\partial E_x(x, y, z)}{\partial x} \frac{1}{2} dx \right) - \left( E_x(x, y, z) - \frac{\partial E_x(x, y, z)}{\partial x} \frac{1}{2} dx \right) \right] dy dz =$$

$$= \frac{\partial E_x(x, y, z)}{\partial x} dx dy dz = \frac{\partial E_x}{\partial x} dV$$

idem per  $d\phi_y$  e  $d\phi_z$

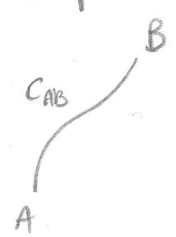
$$d\phi = \frac{\partial E_x}{\partial x} dV + \frac{\partial E_y}{\partial y} dV + \frac{\partial E_z}{\partial z} dV = \left( \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right) dV = \vec{\nabla} \cdot \vec{E} dV = \frac{\rho dV}{\epsilon_0}$$

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$



(5)

4) Campo Conservativo



$$\int_{CAB} \vec{E} \cdot d\vec{l} = V_A - V_B$$

V: potenziale Elettrostatico

$$\vec{E} = -\nabla V$$

In forma differenziale:  $\vec{E} \cdot d\vec{l} = -dV \rightarrow \vec{E} = -\frac{dV}{d\vec{l}} \rightarrow \vec{E} = -\nabla V$

ossia  $E = -\frac{\partial V}{\partial x} - \frac{\partial V}{\partial y} - \frac{\partial V}{\partial z}$

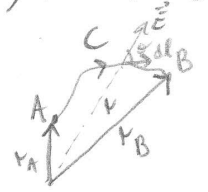
$\text{div } \vec{E} = \frac{\rho}{\epsilon_0} \rightarrow \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \rightarrow \nabla \cdot (\nabla V) = \frac{\rho}{\epsilon_0} \rightarrow \nabla^2 V = -\frac{\rho}{\epsilon_0}$  Eq. di Poisson

ossia  $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$\text{rot } \vec{E} = \nabla \wedge \vec{E} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = i \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + j \left( \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) + k \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right)$

$\nabla \wedge (\nabla f) = 0$  cioè  $\text{rot}(\text{grad}(f)) = 0 \rightarrow \text{rot } \vec{E} = 0$

5) Potenziale del Campo Elettrostatico Coulombiano



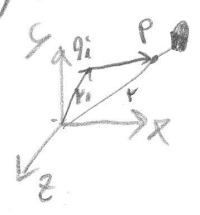
$$\int_{CAB} \vec{E} \cdot d\vec{l} = \int_C \frac{kq}{r^3} \vec{r} \cdot d\vec{l} = kq \int_C \frac{r \cdot dl \cos \theta}{r^3} = kq \int_C \frac{1}{r^2} dl \cos \theta = kq \int_{r_A}^{r_B} \frac{dr}{r^2} = kq \left[ -\frac{1}{r} \right]_{r_A}^{r_B}$$

$$= -\frac{kq}{r_B} + \frac{kq}{r_A} = \frac{kq}{r_A} - \frac{kq}{r_B} = \Delta V = V_A - V_B = V(r_A) - V(r_B)$$

$\rightarrow V(r) = \frac{kq}{r} + \text{cost}$ , sapendo che  $V(\infty) = 0 \rightarrow \text{cost} = 0 \rightarrow \boxed{V(r) = \frac{kq}{r}}$

6) Potenziale creato da cariche discrete e continue

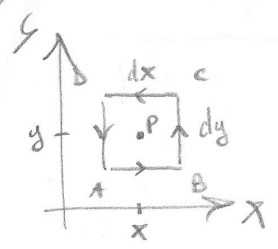
è conservativo V



$$V(r) = k \sum_{i=1}^n \frac{q_i}{|r - r_i|}$$

$$V(r) = k \int \frac{\rho(r')}{|r - r'|} dV'$$

10) Circuazione Elementare / Flusso Elementare



$P(x, y)$

$$d\varepsilon = E_x(x, y + \frac{1}{2}dy)dx + E_y(x + \frac{1}{2}dx, y)dy + E_x(x, y - \frac{1}{2}dy)dx + E_y(x - \frac{1}{2}dx, y)dy$$

$$= (E_x(x, y) - \frac{1}{2} \frac{\partial E_x}{\partial y} dy) dx + (E_y(x, y) + \frac{1}{2} \frac{\partial E_y}{\partial x} dx) + (E_x(x, y) + \frac{1}{2} \frac{\partial E_x}{\partial y} dy) dx + (E_y(x, y) - \frac{1}{2} \frac{\partial E_y}{\partial x} dx) dy =$$

$$= (\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}) dx dy = (\text{rot } \vec{E})_z dx dy = (\vec{\nabla} \wedge \vec{E})_z (d\sigma)_z \rightarrow$$

$d\varepsilon = \vec{\nabla} \wedge \vec{E} \cdot d\vec{\sigma}$

$d\varepsilon(\vec{E}) = d\phi(\vec{\nabla} \wedge \vec{E})$

$\varepsilon(\vec{E}) = \phi(\vec{\nabla} \wedge \vec{E}) \rightarrow \oint_C \vec{E} \cdot d\vec{l} = \int_\Sigma \nabla \wedge \vec{E} \cdot d\vec{\sigma}$  Teorema di Stokes

Applicazioni della legge di Gauss

A) Simmetria Sferica

A1) punto materiale carico

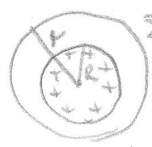
$\phi(E) = \frac{q}{\epsilon_0} = \oint_\Sigma \vec{E} \cdot d\vec{\sigma} \rightarrow \frac{q}{\epsilon_0} = E \cdot 4\pi r^2 \rightarrow E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \checkmark$



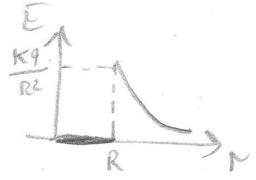
A2) Superficie Sferica carica uniformemente

$\Sigma$  densità di carica  $\sigma = \frac{q}{S} = \frac{q}{4\pi R^2} \rightarrow q = \sigma \cdot 4\pi R^2$

$r > R \oint_\Sigma \vec{E} \cdot d\vec{\sigma} = \frac{q}{\epsilon_0} \rightarrow E \cdot 4\pi r^2 = \frac{4\pi R^2 \sigma}{\epsilon_0} \rightarrow E = \frac{\sigma}{\epsilon_0} \frac{R^2}{r^2} \checkmark \quad E = \frac{\rho}{4\pi\epsilon_0} \checkmark$



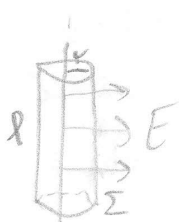
$r < R \oint_\Sigma \vec{E} \cdot d\vec{\sigma} = \frac{q}{\epsilon_0} \Rightarrow 0 \rightarrow E = 0$



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B) Simmetria Cilindrica

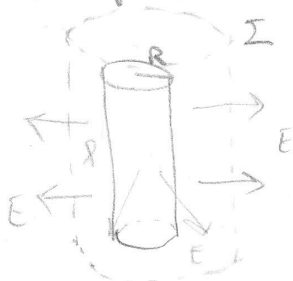
B1) filo rettilineo indefinito di densità lineare  $\lambda$



$$\phi(E) = \frac{q}{\epsilon_0} \quad \phi = \frac{\lambda l}{\epsilon_0}$$

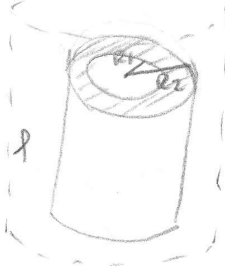
$$E(2\pi r l + 0 + 0) = \frac{\lambda l}{\epsilon_0} \rightarrow E = \frac{\lambda}{2\pi \epsilon_0 r}$$

B2) Superficie cilindrica di raggio R con densità di carica uniforme  $\sigma$



$$E \cdot 2\pi r l = \frac{\sigma \cdot 2\pi R l}{\epsilon_0} \rightarrow E = \frac{\sigma R}{\epsilon_0 r}$$

B3) guscio cilindrica di raggi  $R_1$  e  $R_2$ , con densità  $\rho$  uniforme



$$\rho = \frac{q}{\pi(R_2^2 - R_1^2)l}$$

$$r < R_1 \rightarrow E = 0$$

$$R_1 < r < R_2 \rightarrow E \cdot 2\pi r l = \frac{\rho \cdot \pi(r^2 - R_1^2)l}{\epsilon_0 \cdot \pi(R_2^2 - R_1^2)} \rightarrow E = \frac{\rho}{2\pi \epsilon_0 r} \frac{r^2 - R_1^2}{(R_2^2 - R_1^2)}$$

$$r > R_2 \rightarrow E \cdot 2\pi r l = \frac{q}{\epsilon_0} \rightarrow E = \frac{\rho \pi (R_2^2 - R_1^2) l}{2\pi r l \epsilon_0}$$

B4) Cilindro pieno carico uniformemente con densità  $\rho$



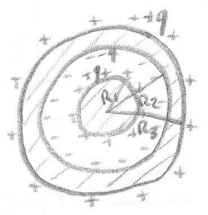
$$r < R \rightarrow E \cdot 2\pi r l = \frac{\rho \cdot \pi r^2 l}{\epsilon_0 \pi R^2 l} \quad \rho = \frac{q}{\pi R^2 l}$$

$$\rightarrow E = \frac{\rho \cdot r}{2\pi \epsilon_0 R^2}$$

$$r > R \rightarrow E \cdot 2\pi r l = \frac{q}{\epsilon_0} \rightarrow E = \frac{q}{2\pi r l \epsilon_0}$$

(11)

B) Sistemi di Conduttori, carica q



$$E = \begin{cases} v < R_1 & 0 \\ R_1 < v < R_2 & E \cdot 4\pi v^2 = \frac{q}{\epsilon_0} \rightarrow E = \frac{kq}{v^2} \\ R_2 < v < R_3 & E = 0 \\ v > R_3 & E \cdot 4\pi v^2 = \frac{q}{\epsilon_0} \rightarrow E = \frac{kq}{v^2} \end{cases}$$

$$V = \begin{cases} v < R_1 & c_1 \\ R_1 < v < R_2 & \frac{kq}{v} + c_2 \\ R_2 < v < R_3 & c_3 \\ v > R_3 & \frac{kq}{v} + c_4 \end{cases}$$

$$V(R_1) = V(R_1^+) \rightarrow c_1 = +\frac{kq}{R_1} - \frac{kq}{R_2} + \frac{kq}{R_3}$$

$$V(R_2) = V(R_2^+) \rightarrow c_2 = -\frac{kq}{R_2} + \frac{kq}{R_3}$$

$$V(R_3) = V(R_3^+) \rightarrow \frac{kq}{R_3} = c_3$$

$$V(\infty) = 0 \rightarrow c_4 = 0$$

$$\rightarrow \begin{cases} \frac{kq}{R_1} + \frac{kq}{R_2} + \frac{kq}{R_3} \\ \frac{kq}{v} - \frac{kq}{R_2} + \frac{kq}{R_3} \\ \frac{kq}{R_3} \\ \frac{kq}{v} \end{cases}$$

il - dipende dal segno della carica q

$V_1 = \frac{kq}{R_1} - \frac{kq}{R_2} + \frac{kq}{R_3}$  potenziale 1° conduttore

$V_2 = \frac{kq}{R_3}$  potenziale 2° conduttore

$$\Delta V = V_1 - V_2 = kq \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

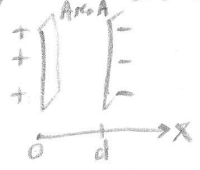
$$C = \frac{q}{\Delta V} = \frac{q}{kq \left( \frac{1}{R_1} - \frac{1}{R_2} \right)} = \frac{4\pi\epsilon_0}{\frac{1}{R_1} - \frac{1}{R_2}} = \frac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1}$$

c) Condensatore cilindrico

$$C = \frac{2\pi\epsilon_0 l}{\ln \frac{R_2}{R_1}}$$

capacità per cambi di lunghezza

d) Condensatore piano



$$E = \frac{\sigma}{\epsilon_0} \quad E = -\frac{dV}{dx} \quad V = -\int E dx + c = -\frac{\sigma}{\epsilon_0} x + c$$

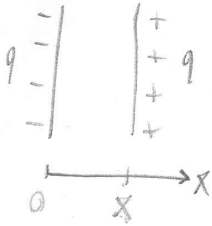
$$V_1 = V(0) = c$$

$$V_2 = V(d) = -\frac{\sigma d}{\epsilon_0} + c$$

$$\Delta V = V_1 - V_2 = \frac{\sigma d}{\epsilon_0}$$

$$C = \frac{q}{\Delta V} = \frac{q \epsilon_0}{\sigma d} = \frac{\sigma A \epsilon_0}{\sigma d} = \frac{\epsilon_0 A}{d}$$

6) Forza delle Armature di un Condensatore piano



$$U = \frac{q^2}{2C} = \frac{q^2 x}{2\epsilon_0 A}$$

Si sposta di dx l'armatura positiva

$$dU = \frac{q^2}{2\epsilon_0 A} dx \quad \text{energia spesa/formata}$$

$$dW = F dx \quad \text{Lavoro fatto dalle forze elettriche sulle armature durante lo spostamento dx}$$

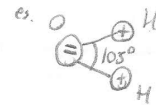
dato che  $dU = dW \Rightarrow F = \frac{q^2}{2\epsilon_0 A} \rightarrow F = \frac{\sigma^2}{2\epsilon_0} A \rightarrow \frac{F}{A} = \frac{\sigma^2}{2\epsilon_0} = \frac{1}{2} \epsilon_0 \left(\frac{\sigma}{\epsilon_0}\right)^2 = \frac{1}{2} \epsilon_0 E^2 = p$  Pressione Elettrica

MATERIALI DIELETRICI

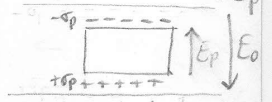
1) Due tipi di polarizzazione

→ polarizzazione elettronica degli atomi

→ polarizzazione per orientamento

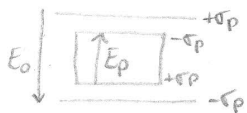


2) In presenza di un campo esterno  $E_0$  il materiale si polarizza creando un campo di polarizzazione  $E_p$



3)  $V_0 =$  ddp relativa a  $E_0$ , ossia quella tra le arm del condensatore in assenza di dielettrico.

$V_K =$  ddp relativa a  $E_K$ , " " " " " " in presenza del dielettrico.



$$E_K = E_0 - E_p$$

- definizioni:
- A)  $\epsilon_r = \frac{V_0}{V_K} > 1$  costante dielettrica relativa
  - B)  $\epsilon = \epsilon_r \epsilon_0$  costante dielettrica
  - C)  $\chi = \epsilon_r - 1$  suscettibilita' elettrica

d)  $V_0 = E_0 h$   
 $V_K = E_K h \rightarrow \epsilon_r = \frac{E_0}{E_K} \rightarrow E_K = \frac{E_0}{\epsilon_r}; V_K = \frac{V_0}{\epsilon_r}$

e)  $E_K = E_0 - E_p \Rightarrow \frac{E_0}{\epsilon_r} = E_0 - E_p \rightarrow E_p = E_0 \left(1 - \frac{1}{\epsilon_r}\right) = E_0 \left(\frac{\epsilon_r - 1}{\epsilon_r}\right) \Rightarrow E_p = \chi \frac{E_0}{\epsilon_r} \rightarrow E_p = \chi E_K$

f)  $E_p = \frac{\sigma_p}{\epsilon_0} \rightarrow \frac{E_p}{E_0} = \frac{\sigma_p}{\sigma_0} \rightarrow \frac{\chi}{\epsilon_r} = \frac{\sigma_p}{\sigma_0} \rightarrow \sigma_p = \frac{\chi \sigma_0}{\epsilon_r}$

g)  $C_K = \frac{q_0}{V_K} \rightarrow C_K = \frac{q_0}{E_K h} = \frac{q_0}{\frac{E_0 h}{\epsilon_r}} = \frac{q_0 \epsilon_r}{E_0 h} = \epsilon_r \frac{q_0}{V_0} = \epsilon_r C_0 = C_K$

4) Conduzione di Ohm

$\tau = \frac{l}{v}$   $\rightarrow$  cammino medio libero di urti  
 $\uparrow$   $v \rightarrow$  velocità dell'e-  
 Tempo medio tra due urti di due particelle

$\vec{a} = \frac{\vec{F}}{m} = -\frac{e\vec{E}}{m}$  accelerazione di ogni e- tra un urto e l'altro

$\vec{v}_d = \vec{a}\tau \rightarrow \vec{v}_d = -\frac{e\tau}{m}\vec{E}$  velocità di deriva

$\vec{J} = Ne\vec{v}_d = \frac{Ne^2\tau}{m}\vec{E} = \vec{J}$

$\sigma = \frac{Ne^2\tau}{m}$  conducibilità

$\vec{J} = \sigma\vec{E}$

$\rho = \frac{1}{\sigma}$  resistività

$\vec{E} = \rho\vec{J}$

Legge di Ohm



$\vec{J} = \frac{1}{\rho}\vec{E}$

$\vec{E} = \rho\vec{J} \rightarrow E = \frac{\rho i}{\Sigma}$

$V_A - V_B = V = Eh \Rightarrow E = \frac{V}{h}$

$V = \left(\frac{\rho h}{\Sigma}\right) i \rightarrow \boxed{V = Ri}$  Legge di Ohm

$R$  resistenza

$G = \frac{1}{R}$  conduttanza

5) Legge di Joule

$\vec{F} = e\vec{E}$  su ogni carica. Dunque lo spostamento di una carica  $dW = \vec{F} \cdot d\vec{l}$

Bisogna spendere energia per far circolare le cariche

$\rightarrow$  Potenza spesa per far circolare 1 elettrone:

$P_e = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{l}}{dt} = \vec{F} \cdot \vec{v} = e\vec{E} \cdot \vec{v} = P$

$\rightarrow$  Idem per N elettroni in un'unità di volume

$P_N = P_e \cdot N = Ne\vec{E} \cdot \vec{v} = Ne\vec{E} \cdot \frac{\vec{J}}{Ne} = \vec{J} \cdot \vec{E} = E\sigma E = \sigma E^2 = \rho J^2$

$\rightarrow$  Idem per far circolare gli e- in un volume  $dV = Sdh$  del conduttore

$dP = P_N dV = \rho J^2 S dh = \rho \frac{dh}{S} i^2$

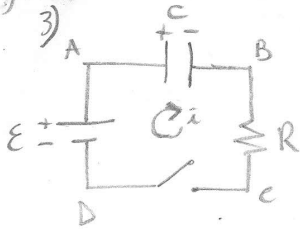
$\rightarrow$  Potenza spesa per far circolare gli e- nel conduttore di lunghezza h: e base S

$P = \int dP \rightarrow P = \rho \frac{h}{S} i^2 \rightarrow \boxed{P = Ri^2}$  legge di Joule

$\rightarrow$  Energia spesa nell'intervallo T per far circolare la corrente i:

$W = \int dW = \int \frac{dW}{dt} dt = \int_0^T P dt = \int_0^T Ri^2 dt$

3) Carica di un condensatore



$\begin{cases} \epsilon = \text{cost} \\ q(0) = 0 \end{cases}$

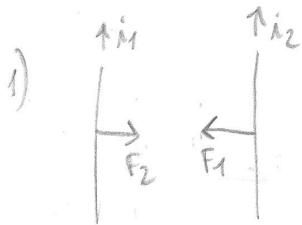
$$\sum V_i = 0 \rightarrow -\frac{q(t)}{C} - Ri + \epsilon = 0 \rightarrow \frac{q(t)}{C} + R \frac{dq(t)}{dt} = \epsilon$$

$$\rightarrow \frac{dq}{dt} = \frac{\epsilon}{R} - \frac{q(t)}{RC} \rightarrow \frac{dq}{\frac{\epsilon}{R} - \frac{q}{RC}} = dt \rightarrow \frac{dq}{q - C\epsilon} = -\frac{1}{RC} dt \rightarrow$$

$$\rightarrow \int_0^q \frac{dq}{q - C\epsilon} = \int_0^t -\frac{1}{RC} dt \rightarrow \ln \left| \frac{q - C\epsilon}{-C\epsilon} \right| = -\frac{t}{RC} \rightarrow \ln \left| 1 - \frac{q}{C\epsilon} \right| = -\frac{t}{RC} \rightarrow$$

$$\rightarrow 1 - \frac{q}{C\epsilon} = e^{-\frac{t}{RC}} \rightarrow \frac{q}{C\epsilon} = 1 - e^{-\frac{t}{RC}} \rightarrow q = C\epsilon \left( 1 - e^{-\frac{t}{RC}} \right)$$

FORZA MAGNETICA



$i_1$  crea  $\vec{B}_1$  (regola della vite)  
 $i_2$  immersa nel  $\vec{B}_1$  sente  $F_1$  (regola della mano destra)  
 e viceversa.

2) Forza Magnetica su una carica in moto in presenza di un campo  $\vec{B}$

$F = q\vec{v} \wedge \vec{B}$  Forza di Lorentz

a)  $F = qvB \sin \theta$  ( $F \perp v, B$ )

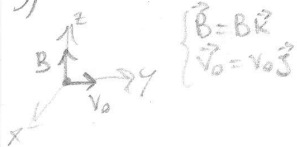
b)  $F$  è max se  $\theta = \frac{\pi}{2}$  c)  $F = 0$  se  $\theta = 0, \pi$   
 ( $\vec{v} \parallel \vec{B}$  o  $\vec{v} \perp \vec{B}$ )

d)  $F$  non compie lavoro  $dW = \vec{F} \cdot d\vec{l}$

$$W = \int_{AB} \vec{F} \cdot d\vec{l} = \int_{AB} q(\vec{v} \wedge \vec{B}) \cdot d\vec{l} = q \int_{AB} (\vec{v} \wedge \vec{B}) \cdot \frac{d\vec{l}}{dt} dt =$$

$$= q \int_{AB} (\vec{v} \wedge \vec{B}) \cdot \vec{v} dA = \int \vec{F} \cdot \vec{v} = 0 \quad (\vec{F} \perp \vec{v})$$

3) Moto di una carica in  $\vec{B}$



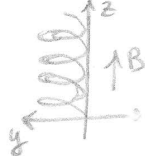
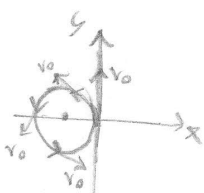
$\begin{cases} \vec{B} = B\vec{k} \\ \vec{v}_0 = v_0\vec{i} \end{cases}$

$F = m \cdot a$

in coordinate imbric.  $\vec{v} = v\vec{e}$ ;  $\vec{a} = a_z\vec{z} + a_m\vec{m}$  acc. tang. acc. centrip.  $\text{dove } \begin{cases} a_z = \frac{dv}{dt} \\ a_m = \frac{v^2}{R} \end{cases}$

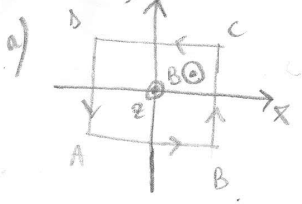
$\begin{cases} F_m = m \cdot a_m \\ F_z = m \cdot a_z \end{cases}$

$\rightarrow \begin{cases} qvB = m \frac{v^2}{R} \\ 0 = m \frac{dv}{dt} \end{cases} \rightarrow \begin{cases} v = \frac{qBR}{m} \\ v = \text{cost} \end{cases} \rightarrow \begin{cases} v = v_0 \\ R = \frac{mv_0}{qB} \end{cases}$



(19)

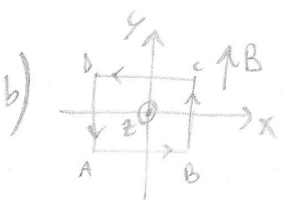
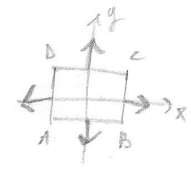
6) Spira rettangolare immersa in un campo  $\vec{B}$  uniforme



$$\vec{B} = B \vec{u}_z \quad \vec{F} = i \vec{I} \wedge \vec{B}$$

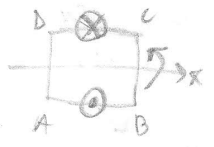
$$F_{AB} = -i l B \vec{u}_y ; F_{BC} = i l B \vec{u}_x ;$$

$$F_{CD} = i l B \vec{u}_y ; F_{DA} = -i l B \vec{u}_x$$



$$\vec{B} = B \vec{u}_y \quad F = i l B$$

$$F_{AB} = i l B \vec{u}_z ; F_{BC} = 0 ; F_{CD} = -i l B \vec{u}_z ; F_{DA} = 0$$



Si genera un momento meccanico attorno a x

$$\vec{M} = \int_{BC} \vec{r} \wedge \vec{F}_{CD} = l_{BC} \vec{u}_y \wedge (-i l_{CD} B) \vec{u}_z = -i l_{BC} l_{CD} B \vec{u}_z \wedge \vec{u}_y \Rightarrow$$

$$\rightarrow \vec{M} = i \vec{S} \wedge \vec{B} \quad \text{dove } \vec{m} = i \vec{S} \quad \text{Momento Magnetico della spira}$$

quindi  $\vec{M} = \vec{m} \wedge \vec{B}$

- $M = 0$  se  $m // B \rightarrow \theta = 0^\circ$  stabile  
 $\theta = \pi$  instabile
- Se  $B // x \rightarrow$  rotazione attorno a y

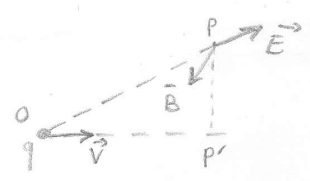
Spire in un campo elettrico  $\rightarrow$  spostamento

Energia potenziale  $U \rightarrow M = -\frac{dU}{d\theta} \quad (F = -\frac{dU}{dx}) \rightarrow \vec{M} = \vec{m} \wedge \vec{B} \rightarrow M = mB \sin\theta$   
 $U = -\vec{m} \cdot \vec{B} \rightarrow U = -mB \cos\theta$

CAMPO MAGNETICO CREATO DA UNA CORRENTE

1) B creato da una carica in moto

$$B = \frac{\mu_0}{4\pi} q \frac{\vec{v} \wedge \vec{r}}{r^3} \quad E = \frac{1}{4\pi\epsilon_0} q \frac{\vec{r}}{r^3}$$



$$a) \vec{B} = \mu_0 \epsilon_0 \vec{v} \wedge \left( \frac{1}{4\pi\epsilon_0} q \frac{\vec{r}}{r^3} \right) = \underbrace{\mu_0 \epsilon_0}_{= \frac{1}{c^2}} \vec{v} \wedge \vec{E} \rightarrow \vec{B} = \frac{\vec{v} \wedge \vec{E}}{c^2} \rightarrow \vec{B} \perp \vec{E}$$

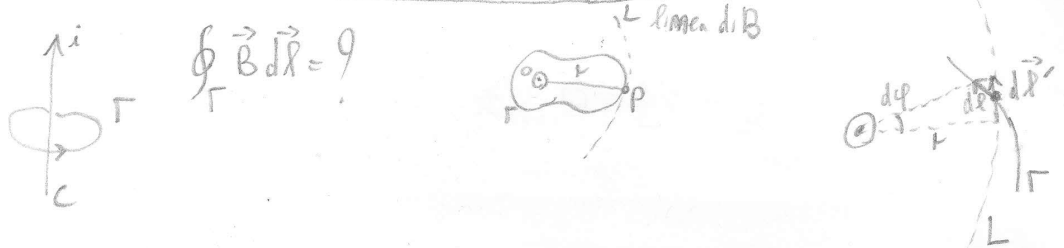
b) Sull'asse di moto di q, il  $\vec{B} = 0$



# LEGGI DI AMPERE - MAXWELL

(21)

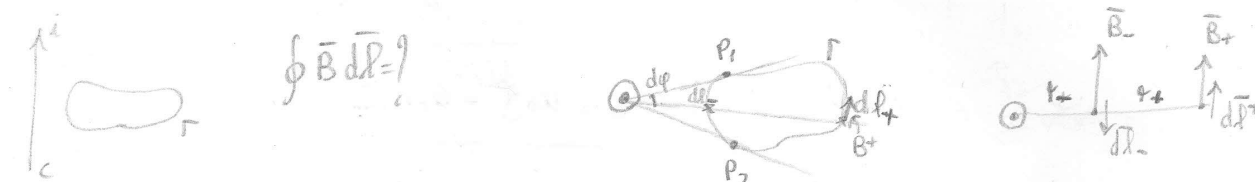
Im condizioni stazionaria, in forma integrale

a)   $\oint_{\Gamma} \vec{B} d\vec{l} = ?$

$dl = dl' + dl_{\perp}$   
 $dl' = r d\varphi$

$$\oint_{\Gamma} \vec{B} d\vec{l} = \oint_{\Gamma} \vec{B} (d\vec{l}' + d\vec{l}_{\perp}) = \oint_{\Gamma} \vec{B} d\vec{l}' = \oint_{\Gamma} \frac{\mu_0 i}{2\pi r} d\vec{l}' = \frac{\mu_0 i}{2\pi} \oint_{\Gamma} \frac{x d\varphi}{r} = \frac{\mu_0 i}{2\pi} \cdot 2\pi = \mu_0 i$$

$\rightarrow \oint_{\Gamma} \vec{B} d\vec{l} = \mu_0 i$  convene con la corrente

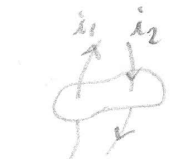
b)   $\oint_{\Gamma} \vec{B} d\vec{l} = ?$

$$\oint_{\Gamma} \vec{B} d\vec{l} = \int_{\Gamma'} \vec{B} (d\vec{l}_+ + d\vec{l}_-)$$


$$\int_{\Gamma'} \vec{B} (dl_+ + dl_-) = \int_{\Gamma'} \vec{B} (dl_+ + dl_+_{\perp} + dl_- + dl_-_{\perp}) =$$

$$= \int_{\Gamma'} (B_+ dl_+ - B_- dl_-) = \int_{\Gamma'} \left( \frac{\mu_0 i}{2\pi r_+} r_+ d\varphi - \frac{\mu_0 i}{2\pi r_-} r_- d\varphi \right) = \int_{\Gamma'} 0 d\varphi = 0$$

$B_+ \nearrow dl_+$   
 $B_- \searrow dl_-$

c)   $\oint_{\Gamma} \vec{B} d\vec{l} = \mu_0 i$  dove  $i = \sum i_i = i_1 - i_2$

Im condizioni stazionarie, in forma differenziale

  $\vec{A} d\vec{l} = \int_{\Sigma} \vec{\nabla} \wedge \vec{A} d\vec{\sigma}$  ; per def  $i = \int_{\Sigma} \vec{j} d\vec{\sigma}$  ; Ampere-Maxwell  $\oint_{\Gamma} \vec{B} d\vec{l} = \mu_0 i$  im f. integrale

$\downarrow$

$$\oint_{\Gamma} \vec{B} d\vec{l} = \mu_0 \int_{\Sigma} \vec{j} d\vec{\sigma}$$

$\downarrow$

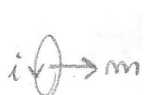
$$\int_{\Sigma} \vec{\nabla} \wedge \vec{B} d\vec{\sigma} = \mu_0 \int_{\Sigma} \vec{j} d\vec{\sigma} \rightarrow \int_{\Sigma} (\vec{\nabla} \wedge \vec{B} - \mu_0 \vec{j}) d\vec{\sigma} = 0 \rightarrow \vec{\nabla} \wedge \vec{B} - \mu_0 \vec{j} = 0$$

$\downarrow$

$$\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{j}$$

## DIPOLO MAGNETICO

23



- 1) Momento magnetico della spira  $\vec{m} = i \vec{S}$  ;  $\vec{S} = S \vec{m}$
- 2) Momento meccanico delle forze magnetiche  $\vec{M} = \vec{m} \wedge \vec{B}$
- 3) Energia potenziale magnetica  $\vec{U} = -\vec{m} \cdot \vec{B}$

Analogie tra dipolo elettrico e magnetico

$\begin{cases} \vec{p} \leftrightarrow \vec{m} \\ \vec{d} \leftrightarrow \vec{S} \\ q \leftrightarrow i \end{cases}$	$-q \quad \vec{p} \quad +q$ $\vec{p} = q \vec{d}$ $\vec{M} = \vec{p} \wedge \vec{E}$ $\vec{U} = -\vec{p} \cdot \vec{E}$	$i \quad \vec{m}$ $\vec{m} = i \vec{S}$ $\vec{M} = \vec{m} \wedge \vec{B}$ $\vec{U} = -\vec{m} \cdot \vec{B}$
---	---	---

Teorema di Gauss  $\begin{cases} \int_{\Sigma} E d\tau = 0 \\ \vec{\nabla} \cdot \vec{E} = 0 \end{cases}$  NO ma  $\begin{cases} \int_{\Sigma} \vec{B} d\tau = 0 \\ \vec{\nabla} \cdot \vec{B} = 0 \end{cases} \rightarrow$  campo solenoidale

Non esistono i mono poli magnetici

### LEGGI DI FARADAY

in forma differenziale

In condizioni stazionarie il campo  $\vec{E}$  è conservativo  $\vec{\nabla} \wedge \vec{E} = 0$ ,  
 in condizioni dinamiche (dipendenti dal tempo)  $\vec{\nabla} \wedge \vec{E} = -\dot{\vec{A}}$

Condizioni che  $\vec{A}$  deve soddisfare:

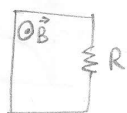
- 1)  $A=0$  in cond stazionarie
  - 2)  $\vec{\nabla} \cdot \vec{A} = 0$  in cond dinamiche
- $\vec{\nabla} \cdot (\vec{\nabla} \wedge \vec{A}) = \vec{\nabla} \cdot \dot{\vec{A}}$

Determinazione di  $\vec{A}$ :

T. di Gauss per  $\vec{B}$ :  $\vec{\nabla} \cdot \vec{B} = 0$ ,  $\frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{B} = 0 \rightarrow \vec{\nabla} \cdot \frac{\partial \vec{B}}{\partial t} = 0 \rightarrow \vec{\nabla} \cdot a \frac{\partial \vec{B}}{\partial t} = 0$   $\vec{A} = a \frac{\partial \vec{B}}{\partial t}$   
soddisfa le condizioni

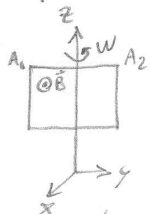
Legge di Faraday  $\vec{\nabla} \wedge \vec{E} = -a \frac{\partial \vec{B}}{\partial t}$   
 da determinare la costante  $a$

4) Circuito immerso in un  $\vec{B}(t)$

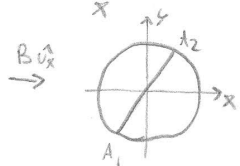


$$\mathcal{E} = -\frac{\partial \phi}{\partial t} = -\frac{\partial AB(t)}{\partial t} = -A \frac{\partial B(t)}{\partial t} \rightarrow i(t) = -\frac{A}{R} \frac{\partial B(t)}{\partial t}$$

5) Spirale rotante in presenza di  $\vec{B}$



$$\mathcal{E} = -\frac{\partial \phi}{\partial t} = -\frac{\partial BA(t)}{\partial t} = -B \frac{\partial A(t)}{\partial t} = i(t)$$



$$A(t) = A \sin(\omega t + \varphi_0) \quad \text{se per } t=0 \begin{cases} | & A(t) = A \cos \omega t \\ - & A(t) = A \sin(\omega t) = A \cos(\frac{\pi}{2} - \omega t) \end{cases}$$

$$\mathcal{E}(t) = -B \omega A \cos(\omega t + \varphi_0) \quad \text{corrente alternata}$$

AUTOINDUZIONE



$$i(t) = \frac{\mathcal{E}(t)}{R}$$

La corrente  $i(t)$  genera un campo  $B(t)$  che investe il circuito con flusso  $\phi(t)$ .

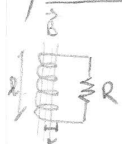
In ogni punto dello spazio  $B(t) \propto i(t)$ , in quanto  $B(t) = \frac{\mu_0 i(t)}{4\pi} \oint_C \frac{d\vec{l} \times \vec{r}}{r^3}$ . Inoltre  $\phi(t) = \int_S \vec{B} \cdot d\vec{\sigma}$ , quindi anche  $\phi(t) \propto i(t)$ .

$$\phi(t) = \int_S \left[ \frac{\mu_0 i(t)}{4\pi} \oint_C \frac{d\vec{l} \times \vec{r}}{r^3} \right] d\sigma = \left( \frac{\mu_0}{4\pi} \int_S d\sigma \oint_C \frac{d\vec{l} \times \vec{r}}{r^3} \right) i(t) = L \cdot i(t)$$

↑ induttanza o coeff. di autoinduzione

$$\mathcal{E}_L(t) = -\frac{d\phi(t)}{dt} \rightarrow \mathcal{E}_L(t) = -L \frac{di(t)}{dt} \quad ; \quad i_L(t) = \frac{\mathcal{E}_L(t)}{R} \rightarrow i_L(t) = -\frac{L}{R} \frac{di(t)}{dt}$$

1) Induttanza di un solenoide

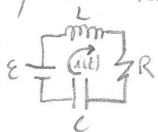


$$B(t) = \mu_0 n i(t) \quad n = \frac{N}{l} \quad \phi(t) = N S_i B(t) = N \pi r^2 \mu_0 \frac{N}{l} i(t) \quad \phi(t) = L i(t)$$

$$\mathcal{E}_L(t) = -L \frac{di(t)}{dt} \quad ; \quad i_L(t) = -\frac{L}{R} \frac{di(t)}{dt}$$

$$L = \mu_0 \pi r^2 \frac{N^2}{l}$$

2) Circuiti RLC



$$\sum V_i = 0 \rightarrow \mathcal{E} - L \frac{di(t)}{dt} - R i(t) - \frac{1}{C} q(t) = 0 \rightarrow \frac{d\mathcal{E}}{dt} - L \frac{d^2 i}{dt^2} - R \frac{di}{dt} - \frac{1}{C} i(t) = 0 \rightarrow$$

$$\rightarrow \frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i(t) = \frac{1}{L} \frac{d\mathcal{E}}{dt}$$

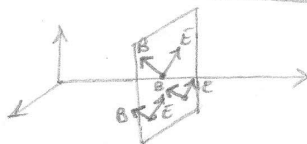
# ONDE ELETTROMAGNETICHE

PARTE 3

(27)

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = 0 & (1) \text{ Gauss } \vec{E} \\ \vec{\nabla} \cdot \vec{B} = 0 & (2) \text{ Gauss } \vec{B} \\ \vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t} & (3) \text{ Faraday} \\ \vec{\nabla} \wedge \vec{B} = \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} & (4) \text{ Ampere-Maxwell} \end{cases}$$

I) Soluzioni piane  $\vec{E}(x,t), \vec{B}(x,t)$



$$\begin{cases} E = E_x \vec{i} + E_y \vec{j} + E_z \vec{k} \\ B = B_x \vec{i} + B_y \vec{j} + B_z \vec{k} \end{cases}$$

6 incognite!

Dalle equazioni si può capire che:

$$\bullet \frac{\partial E_x}{\partial x} = 0 \quad (1) \quad \bullet \frac{\partial B_x}{\partial x} = 0 \quad (2)$$

$$\vec{\nabla} \wedge \vec{E} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & 0 & 0 \\ E_x & E_y & E_z \end{vmatrix} = -\frac{\partial E_z}{\partial x} \vec{j} + \frac{\partial E_y}{\partial x} \vec{k}$$

per Faraday

$$\rightarrow -\frac{\partial B}{\partial t} = \vec{\nabla} \wedge \vec{E} \rightarrow$$

$$\rightarrow -\frac{\partial E_z}{\partial x} \vec{j} + \frac{\partial E_y}{\partial x} \vec{k} = -\frac{\partial B_x}{\partial t} \vec{i} - \frac{\partial B_y}{\partial t} \vec{j} - \frac{\partial B_z}{\partial t} \vec{k} \quad (3) \rightarrow \begin{cases} \frac{\partial E_x}{\partial t} = 0 & (3.a) \\ \frac{\partial E_y}{\partial t} = -\frac{1}{\epsilon_0 \mu_0} \frac{\partial B_z}{\partial x} & (3.b) \\ \frac{\partial E_z}{\partial t} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial B_y}{\partial x} & (3.c) \end{cases}$$

$$\vec{\nabla} \wedge \vec{B} = -\frac{\partial B_z}{\partial x} \vec{j} + \frac{\partial B_y}{\partial x} \vec{k} \xrightarrow{\text{per Ampere-Maxwell}} -\frac{\partial B_z}{\partial x} \vec{j} + \frac{\partial B_y}{\partial x} \vec{k} = \epsilon_0 \mu_0 \frac{\partial E_x}{\partial t} \vec{i} + \epsilon_0 \mu_0 \frac{\partial E_y}{\partial t} \vec{j} + \epsilon_0 \mu_0 \frac{\partial E_z}{\partial t} \vec{k} \quad (4)$$

$$\begin{cases} \frac{\partial E_x}{\partial t} = 0 & (4.a) \end{cases}$$

$$\begin{cases} \frac{\partial E_y}{\partial t} = -\frac{1}{\epsilon_0 \mu_0} \frac{\partial B_z}{\partial x} & (4.b) \end{cases}$$

$$\begin{cases} \frac{\partial E_z}{\partial t} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial B_y}{\partial x} & (4.c) \end{cases}$$

II) Determinazione di  $E_x(x,t), B_x(x,t)$

$$(1) \left. \begin{cases} \frac{\partial E_x}{\partial x} = 0 \\ \frac{\partial E_x}{\partial t} = 0 \end{cases} \right\} \rightarrow E_x = \text{cost} \rightarrow \boxed{E_x = 0}$$

$$(2) \left. \begin{cases} \frac{\partial B_x}{\partial x} = 0 \\ \frac{\partial B_x}{\partial t} = 0 \end{cases} \right\} \rightarrow B_x = \text{cost} \rightarrow \boxed{B_x = 0}$$

III) Determinazione di  $E_y, E_z, B_y, B_z$

$$\begin{cases} \frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t} & (3.b) \end{cases}$$

$$\begin{cases} \frac{\partial E_z}{\partial t} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial B_y}{\partial x} & (4.c) \end{cases}$$

$$\begin{cases} \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} & (3.c) \end{cases}$$

$$\begin{cases} \frac{\partial E_y}{\partial t} = -\frac{1}{\epsilon_0 \mu_0} \frac{\partial B_z}{\partial x} & (4.b) \end{cases}$$

x) Campi E e B

(29)

$$\begin{cases} E(x,t) = E_y(x-ct) \vec{j} + E_z(x-ct) \vec{k} \\ B(x,t) = B_y(x-ct) \vec{j} + B_z(x-ct) \vec{k} \end{cases}$$

occorre conoscere  $\vec{E}$  e  $\vec{B}$  per  $t=0$ , ossia

$$\begin{cases} E(x,0) = E_y(x) \vec{j} + E_z(x) \vec{k} \\ B(x,0) = B_y(x) \vec{j} + B_z(x) \vec{k} \end{cases}$$

x1) Calcolo di B(x,t) a partire da E(x,t)

$$(3b) \quad \frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x} \quad / \quad \begin{cases} B_y(x,t) = B_y(w) \\ E_z(x,t) = E_z(w) \\ w = x-ct \end{cases}$$

$$B_y = \int \frac{\partial B_y}{\partial t} dt = \int \frac{\partial E_z}{\partial x} dt = \int \frac{\partial E_z}{\partial w} \frac{\partial w}{\partial x} dt = \int \frac{\partial E_z}{\partial w} \frac{\partial w}{\partial x} \frac{\partial t}{\partial w} dw = -\frac{1}{c} \int \frac{\partial E_z}{\partial w} dw = -\frac{1}{c} E_z$$

$$B_y = -\frac{E_z}{c}$$

$$(3c) \quad \frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}$$

analogo

$$B_z = \int \frac{\partial B_z}{\partial t} dt = \dots = \frac{1}{c} E_y$$

$$B_z = \frac{E_y}{c}$$

quindi

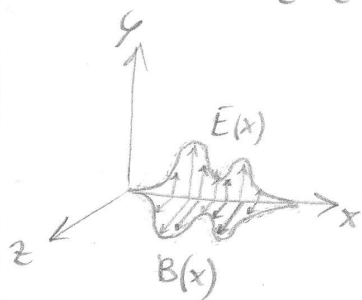
$$\begin{cases} E(x,t) = E_y(w) \vec{j} + E_z(w) \vec{k} \\ B(x,t) = -\frac{1}{c} E_z(w) \vec{j} + \frac{1}{c} E_y(w) \vec{k} \end{cases}$$

XII) Proprietà dei vettori  $\vec{E}$  e  $\vec{B}$  (1)

1)  $B^2 = \frac{E^2}{c^2} \rightarrow B = \frac{1}{c} E$

2)  $\vec{E} \cdot \vec{B} = 0 \rightarrow \vec{B} \perp \vec{E}$

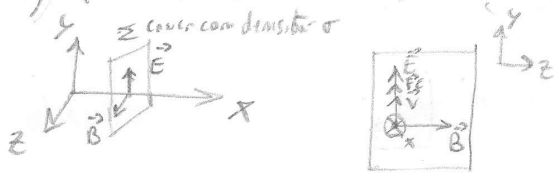
3)  $\vec{E} \wedge \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & E_y & E_z \\ 0 & -\frac{E_z}{c} & \frac{E_y}{c} \end{vmatrix} = \begin{vmatrix} E_y & E_z \\ -\frac{E_z}{c} & \frac{E_y}{c} \end{vmatrix} \vec{i} = \left( \frac{E_y^2}{c} + \frac{E_z^2}{c} \right) \vec{i} = \frac{E^2}{c} \vec{i} = c B^2 \vec{i} = E B \vec{i}$



Scala degli assi x, y, z

$$t=0 \Rightarrow \begin{cases} \vec{E} = E(x) \vec{j} \\ \vec{B} = \frac{1}{c} E(x) \vec{k} \end{cases}$$

XVI) Quantità di moto dell'onda armonica e pressione di radiazione em (31)



$$\vec{F}_E = q\vec{E} ; m\vec{a} = \vec{F}_E \rightarrow \vec{v} \parallel \vec{E}$$

$$\vec{F}_B = q\vec{v} \wedge \vec{B} = qv \hat{u}_y \wedge B \hat{u}_z = qvB \hat{u}_x$$

q carica nell'arco A

$F_B$  per unità di area  $\frac{F_B}{A} = \frac{q}{A} vB \hat{u}_x = \sigma vB \hat{u}_x = P_r$  pressione dell'onda o radiazione em

$\frac{dP}{dt} = F \rightarrow dP = F dt \rightarrow dP = P_r dA dt$   $P = \sigma vB$  quantità di moto ceduta dalla radiazione alla superficie per unità di tempo e superficie

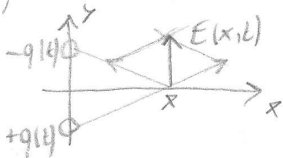
Il campo E trasporta una potenza (energia per unità di tempo)  $P = F_E \cdot v = qEv$

$\frac{P}{A} = \frac{qEv}{A} = \sigma Ev$  potenza trasportata per unità di superficie

$I = \sigma Ev = \sigma cBv = \boxed{cP = I}$  potenza media per unità di superficie (intensità d'onda)

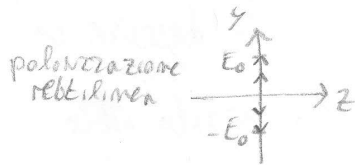
XVII) Polarizzazione lineare delle onde armoniche

a) Creazione di un'onda armonica



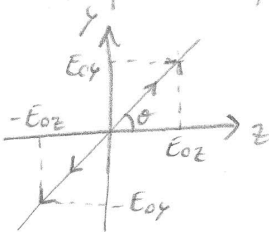
$E(x,t) = E_0 \sin(kx - \omega t)$

Il campo  $E(x,t)$  sta oscillando sul piano xy

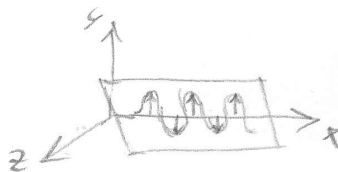


Sul piano yz si trova sempre un segmento rettilineo

b) Il campo  $E(x,t) = E_{0y} \sin(kx - \omega t) \hat{u}_y \pm E_{0z} \sin(kx - \omega t) \hat{u}_z$  è ancora polarizzato rettilineamente



$\frac{E_y(x,t)}{E_z(x,t)} = \pm \frac{E_{0y}}{E_{0z}} = \tan \theta \rightarrow E_{0y} = \tan \theta \cdot E_{0z}$



# Onde Sferiche

PARTE 4

93

da (33) a (37)

Onda piana lungo l'asse x:  $E = E_0 \text{sem}(kx - \omega t)$      $\vec{E}_0 \perp \hat{u}_x$

Onda piana nella direzione  $\frac{\vec{k}}{k}$ :  $E = E_0 \text{sem}(\vec{k}\vec{r} - \omega t)$      $\vec{E}_0 \perp \frac{\vec{k}}{k}$  ;  $\vec{r} = x\hat{u}_x + y\hat{u}_y + z\hat{u}_z$

Onda sferica emessa dall'origine  $E = E_0(r) \text{sem}(kr - \omega t)$      $\vec{E}_0(r) \perp \vec{r}$

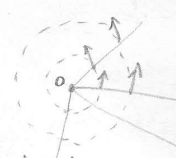
$$I = v_m c = \langle E E^2 \rangle c = \epsilon_0 E_0^2(r) c \langle \text{sem}^2(kr - \omega t) \rangle = \frac{1}{2} \epsilon_0 E_0^2(r) c$$

La potenza che attraversa una superficie sferica  $\Sigma$  deve essere indipendente da  $r$

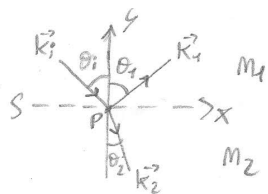
$$4\pi r^2 \frac{1}{2} \epsilon_0 E_0^2(r) c = a \rightarrow E_0(r) = \sqrt{\frac{a}{2\pi \epsilon_0 c}} \frac{1}{r}$$

$$E_0(r) = \frac{E_0}{r}$$

$$\vec{E} = \frac{\vec{E}_0}{r} \text{sem}(kr - \omega t) ; I = \frac{1}{2} \epsilon_0 c E_0^2 \frac{1}{r^2} \Rightarrow I = \frac{I_0}{r^2}$$



## Riflessione e rifrazione di onde piane



$$n = \frac{c}{v} > 1$$

- Il piano di separazione dei due mezzi è // al piano xz
- $\omega$  dipende solo dalla sorgente  $\rightarrow$  è uguale nei due mezzi
- $k = \frac{\omega}{v}$  è diverso nei due mezzi  $k_1, k_2$

Onda incidente  $E_i = E_{0i} \cos(k_i r - \omega t)$

Onda riflessa  $E_r = E_{0r} \cos(k_r r - \omega t)$

Onda rifratta  $E_2 = E_{02} \cos(k_2 r - \omega t)$

In P le fasi delle tre onde sono uguali in ogni istante  $\rightarrow \vec{k}_i \vec{r} = \vec{k}_r \vec{r} = \vec{k}_2 \vec{r}$

$$\vec{k}_i \vec{r} = \vec{k}_r \vec{r} = \vec{k}_2 \vec{r}$$

$$\downarrow$$

$$k_{ix} x = k_{rx} x + k_{r2z} z = k_{2x} x + k_{2z} z$$

$$\vec{k}_i = k_{ix} \hat{u}_x + k_{iy} \hat{u}_y \quad (k_i \in xy)$$

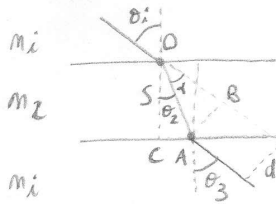
$$\vec{k}_r = k_{rx} \hat{u}_x + k_{ry} \hat{u}_y + k_{r2z} \hat{u}_z$$

$$\vec{k}_2 = k_{2x} \hat{u}_x + k_{2y} \hat{u}_y + k_{2z} \hat{u}_z$$

$$\vec{r} = x \hat{u}_x + z \hat{u}_z \quad (S \in xz)$$

$$\rightarrow \begin{cases} k_{rx} x + k_{r2z} z = k_{ix} x \\ k_{2x} x + k_{2z} z = k_{ix} x \end{cases} \rightarrow \begin{cases} (k_{rx} - k_{ix}) x + k_{r2z} z = 0 \\ (k_{2x} - k_{ix}) x + k_{2z} z = 0 \end{cases} \rightarrow \begin{cases} k_{rx} = k_{ix} \\ k_{r2z} = 0 \\ k_{2x} = k_{ix} \\ k_{2z} = 0 \end{cases}$$

Aberrazione di un raggio di una lamina



OBA:  $\text{sen } d = \frac{d}{OA} \rightarrow OA = \frac{d}{\text{sen } \alpha}$

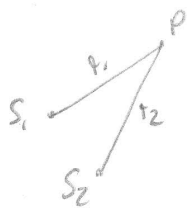
O:  $\theta_i = \theta_2 + \alpha \rightarrow \alpha = \theta_i - \theta_2$

OCA:  $\text{cos } \theta_2 = \frac{S}{OA} \rightarrow OA = \frac{S}{\text{cos } \theta_2}$

$OA = OA \rightarrow \frac{d}{\text{sen}(\theta_i - \theta_2)} = \frac{S}{\text{cos } \theta_2} \rightarrow d = S \frac{\text{sen}(\theta_i - \theta_2)}{\text{cos } \theta_2} = S \frac{\text{sen } \theta_i \text{cos } \theta_2 - \text{sen } \theta_2 \text{cos } \theta_i}{\text{cos } \theta_2} =$

$= S \left( \text{sen } \theta_i - \frac{\text{sen } \theta_2}{\sqrt{1 - \text{sen}^2 \theta_2}} \text{cos } \theta_i \right)$   $\text{sen } \theta_2 = \frac{m_1}{m_2} \text{sen } \theta_i$   $\text{Legge di Snell}$   $\rightarrow d = S \text{sen } \theta_i \left( 1 - \frac{\text{cos } \theta_i}{\sqrt{\left(\frac{m_2}{m_1}\right)^2 - \text{sen}^2 \theta_i}} \right)$

Interferenza di due onde sferiche



$E_1 = A_1 \cos(\omega t - k r_1)$ ;  $A_1 = \frac{\epsilon_0}{r_1}$ ;  $I = b A_1^2$

$E_2 = A_2 \cos(\omega t - k r_2)$ ;  $A_2 = \frac{\epsilon_0}{r_2}$ ;  $I = b A_2^2$

$E = E_1 + E_2$

$E = A \cos(\omega t + \alpha)$   $A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \text{cos } k(r_1 - r_2)}$   
 $\text{tg } \alpha = \frac{A_1 \text{sen } k r_1 + A_2 \text{sen } k r_2}{A_2 \text{cos } k r_2 + A_1 \text{cos } k r_1}$

$I = b A^2 = I_1 + I_2 + 2\sqrt{I_1 I_2} \text{cos } \delta$  dove  $\delta = k(r_1 - r_2)$

Interferenza costruttiva  $\rightarrow \text{cos } \delta = 1 \rightarrow \delta = 2m\pi$ ;  $m = 0, \pm 1, \pm 2, \dots$

$r_1 - r_2 = m\lambda$

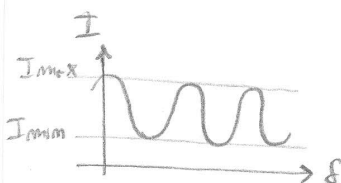
$I_{\text{max}} = I_1 + I_2 + 2\sqrt{I_1 I_2} > I_1 + I_2$

Interferenza distruttiva  $\rightarrow \text{cos } \delta = -1 \rightarrow \delta = (2m+1)\pi$ ;  $m = 0, \pm 1, \pm 2, \dots$

$r_1 - r_2 = (2m+1)\frac{\lambda}{2}$

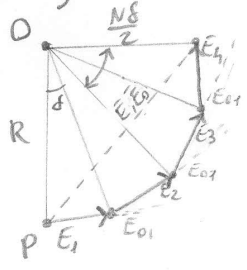
$I_{\text{min}} = I_1 + I_2 - 2\sqrt{I_1 I_2} < I_1 + I_2$

se  $I_1 = I_2 \rightarrow I_{\text{min}} = 0$





N. generico)



$$\begin{cases} \text{sem} \frac{\delta}{2} = \frac{E_{01}}{2R} \\ \text{sem} \frac{N\delta}{2} = \frac{E_0}{2R} \end{cases} \rightarrow \frac{\text{sem} \frac{N\delta}{2}}{\text{sem} \frac{\delta}{2}} = \frac{E_0}{E_{01}} \rightarrow E_0 = E_{01} \frac{\text{sem} \frac{N\delta}{2}}{\text{sem} \frac{\delta}{2}}$$

$$E_0^2 = E_{01}^2 \left( \frac{\text{sem} \frac{N\delta}{2}}{\text{sem} \frac{\delta}{2}} \right)^2$$

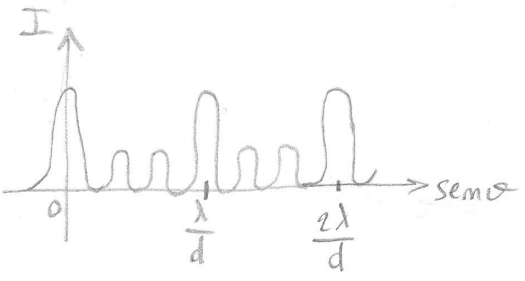
$$I = I_1 \left( \frac{\text{sem} \frac{N\delta}{2}}{\text{sem} \frac{\delta}{2}} \right)^2$$

$$\delta = k(r_1 - r_2)$$

$$r_1 - r_2 \approx d \text{sem} \theta \quad \text{se } d \ll L$$

$$\rightarrow \delta = kd \text{sem} \theta \quad \downarrow \quad \frac{2\pi}{\lambda}$$

$$I = I_1 \left( \frac{\text{sem} \frac{N\pi d \text{sem} \theta}{\lambda}}{\text{sem} \frac{\pi d \text{sem} \theta}{\lambda}} \right)^2$$

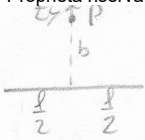


Massimi principali:  $\frac{\pi d \text{sem} \theta}{\lambda} = m\pi \rightarrow I = I_1 \left( \frac{\text{sem} (Nm\pi)}{\text{sem} (m\pi)} \right)^2 = I_1 N$

Minimi:  $\frac{N\pi d \text{sem} \theta}{\lambda} = m'\pi$

Minimi secondari tra i minimi.

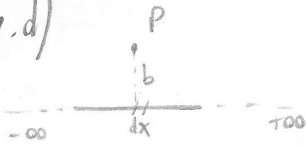
1.c)



$$\begin{cases} E_x = 0 \\ E_y = \frac{k\lambda}{b} \left( \frac{\frac{l}{2}}{\sqrt{\frac{l^2}{4} + b^2}} + \frac{\frac{l}{2}}{\sqrt{\frac{l^2}{4} + b^2}} \right) = \frac{2kq}{b} \frac{l}{\sqrt{l^2 + 4b^2}} \end{cases}$$

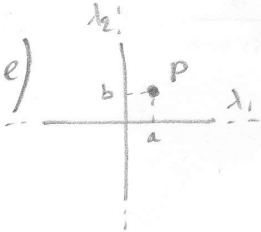
(2)

1.d)



$$\begin{cases} E_x = 0 \\ E_y = \frac{2k\lambda}{b} \lim_{l \rightarrow \infty} \frac{l}{\sqrt{l^2 + 4b^2}} = \frac{2k\lambda}{b} \end{cases}$$

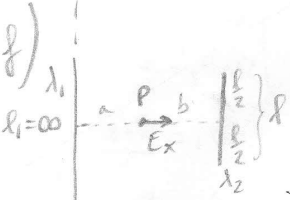
1.e)



Strette infinite condensabili  $\lambda_1, \lambda_2$

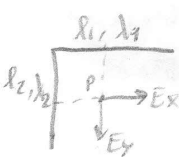
$$\begin{cases} E_x = \frac{2k\lambda_1}{b} \\ E_y = \frac{2k\lambda_2}{b} \end{cases}$$

1.f)



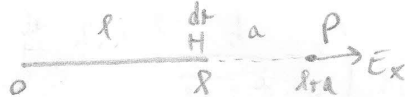
$$\begin{cases} E_y = 0 \\ E_x = E_{x_1} - E_{x_2} = \frac{2k\lambda_1}{b} - \frac{2k\lambda_2}{b} \frac{l}{\sqrt{4b^2 + l^2}} \end{cases}$$

1.g)



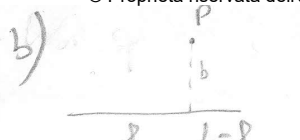
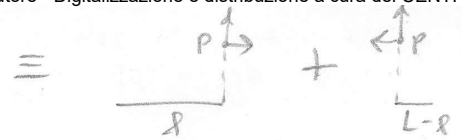
$$\begin{cases} \frac{4k\lambda_2 l_2}{l_2 \sqrt{l_1^2 + l_2^2}} = E_x \\ \frac{4k\lambda_1 l_1}{l_2 \sqrt{l_1^2 + l_2^2}} = E_y \end{cases}$$

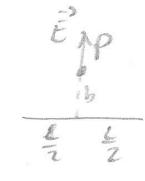
1.h)




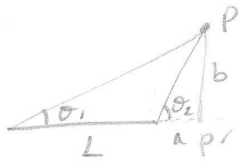
$b=0$

$$dE = k \frac{\lambda dx}{x^2} \rightarrow E = k\lambda \int_a^{l+a} \frac{1}{x^2} dx = k\lambda \left[ -\frac{1}{x} \right]_a^{l+a} = k\lambda \left( \frac{1}{a} - \frac{1}{l+a} \right)$$

b)   $\equiv$   (vedi caso a) (4)

in particolare   $\vec{E} = 4k\sigma \left( \frac{\pi}{2} - \arcsin \frac{b}{\sqrt{L^2 + 4b^2}} \right) \vec{j}$

c)  caso degenere, la variabile  $\theta$  non va bene. Si rifà il calcolo per  $b \rightarrow 0$



$$\theta_1 = \theta_1(b)$$

$$\theta_2 = \theta_2(b)$$

$$\frac{\sin \theta_2}{\sin \theta_1} = \dots$$

$$\tan \theta_1 = \frac{b}{L+a} \rightarrow \tan \theta = \frac{\sin \theta}{\cos \theta} \rightarrow \sin \theta = \frac{\tan \theta}{\frac{1}{\cos \theta} \sqrt{\sin^2 \theta + \cos^2 \theta}} = \frac{\tan \theta}{\sqrt{\tan^2 \theta + 1}}$$

$$\tan \theta_2 = \frac{b}{a}$$

$$\sin \theta_1 = \frac{\frac{b}{L+a}}{\sqrt{\left(\frac{b}{L+a}\right)^2 + 1}} \quad ; \quad \sin \theta_2 = \frac{\frac{b}{a}}{\sqrt{\left(\frac{b}{a}\right)^2 + 1}} \rightarrow \frac{\sin \theta_2}{\sin \theta_1} = \frac{\frac{b}{a} \sqrt{\left(\frac{b}{a}\right)^2 + 1}}{\frac{b}{L+a} \sqrt{\left(\frac{b}{L+a}\right)^2 + 1}} = \frac{L+a}{a}$$

$$\lim_{b \rightarrow 0} \frac{L+a}{a} \frac{\sqrt{\left(\frac{b}{a}\right)^2 + 1}}{\sqrt{\left(\frac{b}{L+a}\right)^2 + 1}} = \frac{L+a}{a}$$

$$E_x = 2k\sigma \lim_{b \rightarrow 0} \frac{L+a}{a}$$

Calcolo diretto:



$$E = \int d\vec{E} = \int \frac{2k\sigma dx}{r} = \left( \int_0^L \frac{2k\sigma dx}{L+a-x} \right) = \int_a^{L+a} \frac{2k\sigma dr}{r} = 2k\sigma \ln \frac{L+a}{a}$$

Se:  $L \rightarrow 0$  sbarra infinitesima  $E_x = \frac{2k\sigma L}{a} = \frac{2k\lambda}{a}$

$a \rightarrow \infty$   $E_x = \frac{2k\sigma L}{a} = \frac{2k\lambda}{a}$

4) Arco di circonferenza. Calcolare  $E$  al centro.

(6)



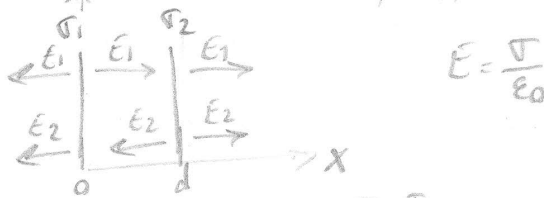
$$dq = \lambda dl = \frac{q}{L} dl$$

$$dE = dE_x = k \frac{dq}{r^2} = \frac{kq}{Lr^2} dl$$

$$dE = dE_{-x} + dE_{+x} = \frac{2kq}{Lr^2} dl \cos\sigma = \frac{2kq}{Lr^2} r d\sigma \cos\sigma$$

$$E = \int dE = \frac{2kq}{Lr} \int_0^\phi \cos\sigma d\sigma = \frac{2kq}{Lr} \sin\phi$$

5) Andamento di  $E(x)$  e  $V(x)$  tra due piani paralleli infiniti, arenati  $\sigma_1$  e  $\sigma_2$ , posti a distanza  $d$ .



$$E = \frac{\sigma}{\epsilon_0}$$

$$E(x) = \begin{cases} x < 0 & \rightarrow -\frac{\sigma_1 + \sigma_2}{2\epsilon_0} \\ 0 < x < d & \rightarrow \frac{\sigma_1 - \sigma_2}{2\epsilon_0} \\ x > d & \rightarrow +\frac{\sigma_1 + \sigma_2}{2\epsilon_0} \end{cases}$$

$$E = -\frac{dV}{dx} \rightarrow dV = -E dx \rightarrow V = -\int E dx$$

$$\rightarrow V = -Ex + C$$

Condizioni al contorno:

$$V(d^-) = V(d^+) \rightarrow \frac{\sigma_2 d}{2\epsilon_0} - \frac{\sigma_1 d}{2\epsilon_0} + C_2 = -\frac{\sigma_1 d}{2\epsilon_0} + C_3$$

$$\rightarrow C_3 - C_2 = \frac{\sigma_2 d}{\epsilon_0}$$

$$V(0^-) = V(0^+) \rightarrow 0 + 0 + C_1 = 0 + 0 + C_2 \rightarrow C_1 = C_2$$

$$C_1 = C_2$$

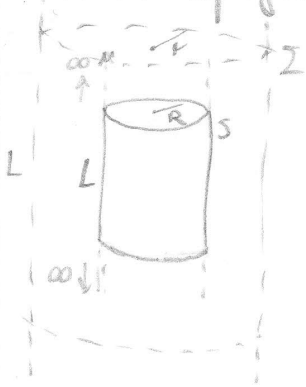
$$\text{Se } C_2 = 0 \rightarrow C_1 = 0 ; C_3 = \frac{\sigma_2 d}{\epsilon_0}$$

Caso speciale:  $\sigma_2 = -\sigma_1$  (Armature di un condensatore):

$$E(x) = \begin{cases} x < 0 & 0 \\ 0 < x < d & \frac{\sigma}{\epsilon_0} \\ x > d & 0 \end{cases}$$

$$V(x) = \begin{cases} x < 0 & 0 \\ 0 < x < d & -\frac{\sigma}{\epsilon_0} x \\ x > d & -\frac{\sigma}{\epsilon_0} d \end{cases} \quad \Delta V = \frac{\sigma d}{\epsilon_0}$$

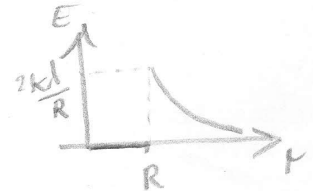
7) Determinare il campo  $\vec{E}$  creato da una carica distribuita uniformemente su una superficie cilindrica indefinita di raggio  $R$ , di densità lineare (omica)  $\lambda$ .  $\forall r \in \mathbb{R}$  (8)



$$\phi(E) = \oint \vec{E} \cdot d\vec{\sigma} = \frac{q}{\epsilon_0} \quad q = \lambda \cdot L$$

$$r > R \quad E \cdot 2\pi r L = \frac{\lambda L}{\epsilon_0} \rightarrow E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} = 2k \frac{\lambda}{r}$$

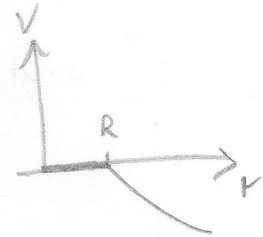
$$r < R \quad E \cdot 2\pi r L = \frac{0}{\epsilon_0} \rightarrow E = 0$$



$$V(r) = -\int E(r) dr + C \rightarrow V(r) = \begin{cases} r > R & -2k\lambda \ln r + C_1 \\ r < R & C_2 \end{cases}$$

$$V(R^-) = V(R^+) \rightarrow -2k\lambda \ln R + C_1 = C_2 \quad \text{poniamo } C_2 = 0$$

$$\rightarrow V(r) = \begin{cases} r < R & 0 \\ r > R & 2k\lambda \ln R - 2k\lambda \ln r \\ & = 2k\lambda \ln \frac{R}{r} \end{cases}$$



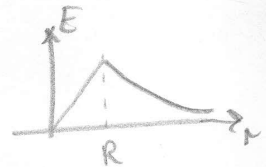
8) Cilindro pieno carico uniformemente con densità di carica  $\rho$ , raggio  $R$



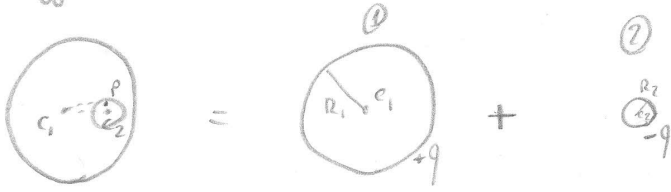
$$\phi(E) = \oint \vec{E} \cdot d\vec{\sigma} = \frac{q}{\epsilon_0} \quad \rho = \frac{q}{\pi R^2 L}$$

$$r > R \quad E \cdot 2\pi r L = \frac{\rho \pi R^2 L}{\epsilon_0} \rightarrow E = \frac{\rho R^2}{2\epsilon_0 r}$$

$$r < R \quad E \cdot 2\pi r L = \frac{\rho \pi r^2 L}{\epsilon_0} \rightarrow E = \frac{\rho r}{2\epsilon_0}$$



11) Sfera di raggio  $R_1$  e carica uniforme  $q$ , combinata con una cavità sferica di raggio  $R_2$ .  $\vec{E}$  in  $P$ , fuori  $\vec{C}_1P$  e  $\vec{C}_2P$  (10)



$$\rho = \frac{q}{\frac{4}{3}\pi(R_1^3 - R_2^3)}$$

$$q_1 = \rho \cdot \frac{4}{3}\pi R_1^3$$

$$q_2 = \rho \cdot \frac{4}{3}\pi R_2^3$$

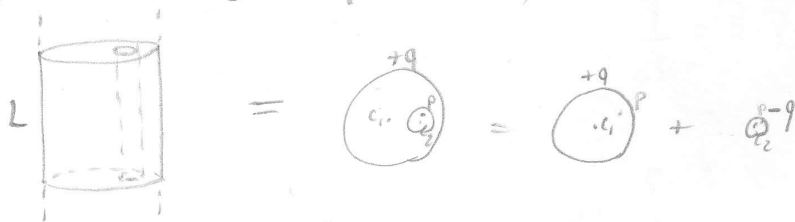
$$E = \frac{q}{4\pi\epsilon_0 r^2} = \frac{\rho \cdot \frac{4}{3}\pi r^3}{\epsilon_0} \rightarrow E = \frac{\rho r}{3\epsilon_0}$$

in P distribuita

$$\vec{E}_1 = \frac{\rho}{3\epsilon_0} \vec{C}_1P \quad ; \quad \vec{E}_2 = -\frac{\rho}{3\epsilon_0} \vec{C}_2P = \frac{\rho}{3\epsilon_0} \vec{PC}_2$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\epsilon_0} (\vec{C}_1P + \vec{PC}_2) = \frac{\rho}{3\epsilon_0} \vec{C}_1C_2$$

12) Caso analogo al precedente, ma con un cilindro pieno uniformemente caricato.  $E(P)?$  indefinito



$$\rho = \frac{q}{\pi R^2 L}$$

$$E \cdot 2\pi r L = \frac{\rho \cdot \pi r^2 L}{\epsilon_0} \rightarrow E = \frac{\rho r}{2\epsilon_0} \quad \text{dipendente}$$

in P distribuita

Il procedimento è analogo prima  $\rightarrow$  soluzione  $E = \frac{\rho}{2\epsilon_0} \vec{C}_1C_2$

12) Due superfici cilindriche indefinite di raggi  $R_1$  e  $R_2$  concambue  
 passiamo cariche distribuite uniformemente condensabili lineare  $\lambda_1, \lambda_2$  - E? V?



f. imb.  $\oint \vec{E} \cdot d\vec{\sigma} = \frac{1}{\epsilon_0} \int \rho dV$   
 f. diff.  $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

$q = \lambda \cdot L$

$$\begin{cases} r < R_1 & E = 0 \\ R_1 < r < R_2 & E \cdot 2\pi r L = \frac{\lambda_1 L}{\epsilon_0} \rightarrow E = 2k \frac{\lambda_1}{r} \\ r > R_2 & E \cdot 2\pi r L = \frac{\lambda_1 L + \lambda_2 L}{\epsilon_0} \rightarrow E = 2k \frac{\lambda_1 + \lambda_2}{r} \end{cases}$$

$V = -\int E dr + c$

$$V = \begin{cases} r < R_1 & c_1 \\ R_1 < r < R_2 & -2k \lambda_1 \ln r + c_2 \\ r > R_2 & -2k (\lambda_1 + \lambda_2) \ln r + c_3 \end{cases}$$

c.c.:  $V(R_1^-) = V(R_1^+)$

$c_1 - c_2 = -2k \lambda_1 \ln R_1$

$V(R_2^-) = V(R_2^+)$

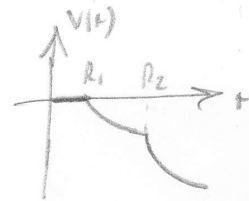
$-2k \lambda_1 \ln R_2 + c_2 = -2k (\lambda_1 + \lambda_2) \ln R_2 + c_3$

$c_3 = c_2 - 2k \lambda_1 \ln R_2 + 2k (\lambda_1 + \lambda_2) \ln R_2$

• se  $c_1 = 0 \rightarrow c_2 = +2k \lambda_1 \ln R_1$

$c_3 = 2k (\lambda_1 \ln R_1 - \lambda_1 \ln R_2 + (\lambda_1 + \lambda_2) \ln R_2)$

$$V = \begin{cases} r < R_1 & 0 \\ R_1 < r < R_2 & 2k \lambda_1 \ln \frac{R_1}{r} \\ r > R_2 & 2k \lambda_1 \ln \frac{R_1}{R_2} + 2k (\lambda_1 + \lambda_2) \ln \frac{R_2}{r} \end{cases}$$



Caso speciale  $\lambda_1 = -\lambda_2 = -\lambda$

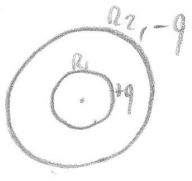
$$E = \begin{cases} r < R_1 & 0 \\ R_1 < r < R_2 & 2k \frac{\lambda}{r} \\ r > R_2 & 0 \end{cases} \quad V = \begin{cases} r < R_1 & 0 \\ R_1 < r < R_2 & 2k \lambda \ln \frac{R_1}{r} \\ r > R_2 & 2k \lambda \ln \frac{R_2}{R_1} \end{cases}$$

$\Delta V = V(R_1) - V(R_2) = 2k \lambda \ln \left( \frac{R_1}{R_1} \cdot \frac{R_2}{R_1} \right) = 2k \lambda \ln \frac{R_2}{R_1}$

(c.c. V(r))  $\begin{cases} c_1 \\ -2k \lambda \ln r + c_2 \\ c_3 \end{cases}$   
 • se  $c_1 = 0$   
 •  $c_2 = +2k \lambda \ln R_1$   
 •  $c_3 = -2k \lambda \ln R_2 + c_2$

16) Calcolare la capacità di un sistema di due superfici sferiche concentriche conduttrici di raggi  $R_1$  e  $R_2$  e di carica  $+q$  e  $-q$

15  
Condensatore



$$C = \frac{q}{\Delta V}$$

$$E = \begin{cases} 0 & r < R_1 \\ k \frac{q}{r^2} & R_1 < r < R_2 \\ 0 & r > R_2 \end{cases} \rightarrow V = \begin{cases} C_1 & r < R_1 \\ k \frac{q}{r} + C_2 & R_1 < r < R_2 \\ C_3 & r > R_2 \end{cases}$$

$$C_3 = 0$$

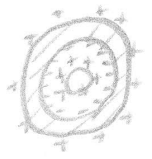
$$C_2 = -k \frac{q}{R_2}$$

$$C_1 = \frac{kq}{R_1} - \frac{kq}{R_2}$$

$$\Delta V = V_1 - V_2 = \frac{kq}{R_1} - \frac{kq}{R_2} - \frac{kq}{R_2} + \frac{kq}{R_2} = \frac{kq}{R_1} - \frac{kq}{R_2} = kq \frac{R_2 - R_1}{R_1 R_2}$$

$$C = q \cdot \frac{R_2 - R_1}{kq R_1 R_2} = 4\pi\epsilon_0 \frac{R_2 - R_1}{R_1 R_2}$$

17) Calcolare la capacità di un condensatore sferico formato da una sfera carica positivamente di raggio  $R_1$  e da un guscio sferico concentrico di raggi  $R_2$  e  $R_3$  interno ed esterno.



$$E = \begin{cases} 0 & r < R_1 \\ k \frac{q}{r^2} & R_1 < r < R_2 \\ 0 & R_2 < r < R_3 \\ k \frac{q}{r^2} & r > R_3 \end{cases}$$

$$V = \begin{cases} C_1 & r < R_1 \\ \frac{kq}{r} + C_2 & R_1 < r < R_2 \\ C_3 & R_2 < r < R_3 \\ \frac{kq}{r} + C_4 & r > R_3 \end{cases}$$

$$C_4 = 0$$

$$C_3 = \frac{kq}{R_3}$$

$$C_2 = \frac{kq}{R_3} - \frac{kq}{R_2}$$

$$C_1 = \frac{kq}{R_3} - \frac{kq}{R_2} + \frac{kq}{R_1}$$

$$V(r) = \begin{cases} kq \left( \frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3} \right) & \leftarrow V_1 \\ \frac{kq}{r} + kq \left( \frac{1}{R_3} - \frac{1}{R_2} \right) & \\ \frac{kq}{R_3} & \leftarrow V_2 \\ \frac{kq}{r} & \end{cases}$$

$$\Delta V = V_1 - V_2 = kq \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$C = \frac{q}{\Delta V} = \frac{4\pi\epsilon_0}{\frac{1}{R_1} - \frac{1}{R_2}}$$

18) In un condensatore piano si allungano le due armature da  $d_1$  a  $d_2$ . Calcolare la variazione di energia accumulata nei due casi:



Durante la variazione: ①  $V = \text{cost}$ ; ②  $Q = \text{cost}$

$$dW_{\text{carica spostata da un'armatura all'altra}} = V \cdot dq \rightarrow W = \int \frac{q}{C} dq = \frac{q^2}{2C} = U \rightarrow U = \frac{1}{2} CV^2$$

$$C = \frac{\epsilon_0 A}{d} \rightarrow U = \frac{1}{2} \epsilon_0 A V^2 \left( \frac{1}{d_1} - \frac{1}{d_2} \right)$$

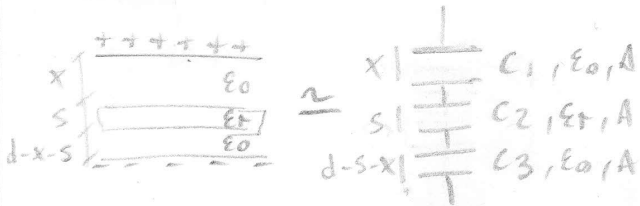
$$\text{① } \Delta U = U_2 - U_1 = \frac{1}{2} C_1 V^2 - \frac{1}{2} C_2 V^2 \rightarrow = \frac{1}{2} \epsilon_0 A V^2 \left( \frac{1}{d_1} - \frac{1}{d_2} \right)$$

$$\text{② } \Delta U = U_2 - U_1 = \frac{1}{2} \frac{Q^2}{C_1} - \frac{1}{2} \frac{Q^2}{C_2} \rightarrow = \frac{1}{2} \frac{Q^2}{\epsilon_0 A} \left( d_2 - d_1 \right)$$



20) Inserimento di un dielettrico tra le armature di un condensatore. Conoscendo  $Q, A, d, \epsilon_r, \epsilon_0, s, x$  calcolare  $\Delta U$  tra prima e dopo l'inserimento del dielettrico. ( $Q = \text{cost}$ )

16  
Dielettrico



$$C_1 = \frac{A\epsilon_0}{x}$$

$$C_2 = \frac{A\epsilon_r}{s}$$

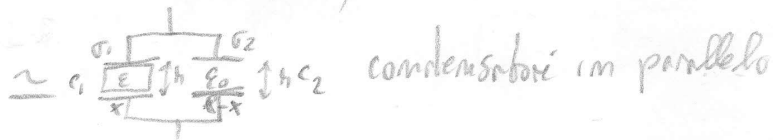
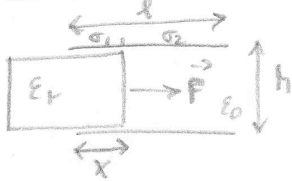
$$C_3 = \frac{A\epsilon_0}{d-s-x}$$

$$\Delta U = \frac{1}{2} \frac{Q^2}{C_{eq}} - \frac{1}{2} \frac{Q^2}{C_0} = \frac{1}{2} \frac{Q^2}{C_{eq}} - \frac{1}{2} \frac{Q^2}{\frac{A\epsilon_0}{d}}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{x}{A\epsilon_0} + \frac{s}{A\epsilon_r} + \frac{d-s}{A\epsilon_0} = \frac{x}{A\epsilon_0} + \frac{s}{A\epsilon_r} + \frac{d-s}{A\epsilon_0} - \frac{x}{A\epsilon_0}$$

$$\Delta U = \frac{1}{2} Q^2 \left( \frac{s}{A\epsilon_r} + \frac{d-s}{A\epsilon_0} - \frac{s}{A\epsilon_0} - \frac{x}{A\epsilon_0} \right) = \frac{1}{2} \frac{Q^2 s}{A(\epsilon_r - \epsilon_0)}$$

21) Inserimento di un dielettrico tra le armature di un condensatore. Si conoscono  $h, L, \epsilon_r, x, \epsilon_0, \epsilon_r, Q$  iniziale. Calcolare la  $\vec{F}$  di richiamo del dielettrico all'interno del condensatore ( $V = \text{cost}$ )



$$A = L \cdot L$$

$$C_1 = \frac{A_1 \epsilon_r}{h} = \frac{xL\epsilon_r}{h} ; C_2 = \frac{A_2 \epsilon_0}{h} = \frac{(L-x)L\epsilon_0}{h}$$

$$F = - \frac{dU}{dx} \rightarrow \Delta U \rightarrow U = \frac{1}{2} C V^2$$

$$\begin{cases} q_1 = VC_1 \\ q_2 = VC_2 \end{cases} \rightarrow V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{q_{eq}}{C_{eq}}$$

$$U = \frac{1}{2} C \left( \frac{q}{C} \right)^2 = \frac{1}{2} \frac{q^2}{C}$$

$$C_{eq} = \frac{xL}{h} \epsilon_r + \frac{(L-x)L}{h} \epsilon_0 = \frac{L}{h} [x(\epsilon_r - \epsilon_0) + L\epsilon_0]$$

$$V_1 = V_2 \rightarrow E_1 = E_2 \rightarrow \frac{\sigma_1}{\epsilon_r} = \frac{\sigma_2}{\epsilon_0} ; Q = \sigma \cdot A \rightarrow Q_1 = \sigma_1 \cdot xL ; Q_2 = \sigma_2 (L-x)L$$


$$\begin{cases} \sigma_1 \epsilon_0 - \sigma_2 \epsilon_r = 0 \\ Q = \sigma_1 \cdot xL + \sigma_2 (L-x)L \end{cases} \rightarrow \begin{cases} \sigma_1 = \frac{\sigma_2 \epsilon_r}{\epsilon_0} \\ \frac{Q}{L} = \frac{\sigma_2 \epsilon_r}{\epsilon_0} x + \sigma_2 (L-x) \end{cases} \rightarrow \begin{cases} \sigma_1 = \frac{Q}{L} \frac{\epsilon_r}{\epsilon_r x + \epsilon_0 (L-x)} \\ \sigma_2 = \frac{Q}{L} \frac{\epsilon_0}{\epsilon_r x + \epsilon_0 (L-x)} \end{cases}$$

combinava  
→

23) Una sfera metallica di raggio  $R_1$  è coperta da un guscio di spessore  $R_2 - R_1$  di dielettrico con costante dielettrica  $\epsilon_r$ . Calcolare la capacità del condensatore.

18

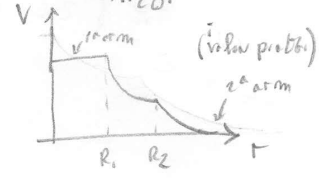
Corrente Elettrica

  $C = \frac{q}{\Delta V}$

$$E = \begin{cases} r < R_1 & 0 \\ R_1 < r < R_2 & \frac{q}{4\pi\epsilon_r r^2} \\ r > R_2 & \frac{q}{4\pi\epsilon_0 r^2} \end{cases} \rightarrow V = \int \frac{q}{4\pi\epsilon_r r^2} + C_2 + \int \frac{q}{4\pi\epsilon_0 r^2} + C_3$$

$C_3 = 0$   
 $C_2 = \frac{q}{4\pi\epsilon_0 R_2} - \frac{q}{4\pi\epsilon_r R_2}$   
 $C_1 = \frac{q}{4\pi\epsilon_r R_1} + \frac{q}{4\pi\epsilon_0 R_2} - \frac{q}{4\pi\epsilon_r R_2}$

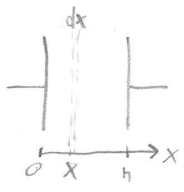
$$\rightarrow V = \frac{q}{4\pi\epsilon_0 r} \left( \frac{1}{\epsilon_r R_1} - \frac{1}{\epsilon_r R_2} + \frac{1}{\epsilon_0 R_2} \right)$$



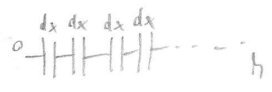
$$\Delta V = V(R_1) - V(\infty) = V(R_1) - 0 = \frac{q}{4\pi} \left( \frac{1}{\epsilon_r R_1} - \frac{1}{\epsilon_r R_2} + \frac{1}{\epsilon_0 R_2} \right)$$

$$C = q \cdot \frac{4\pi}{q} \cdot \left( \frac{1}{\epsilon_r} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{1}{\epsilon_0 R_2} \right)^{-1} = \frac{4\pi}{\frac{1}{\epsilon_r R_1} + \frac{\epsilon_r - \epsilon_0}{R_2 \epsilon_0 \epsilon_r}} = \frac{4\pi}{\frac{1}{\epsilon_r R_1} \left( 1 + \frac{\epsilon_r - \epsilon_0}{\epsilon_0} \frac{R_1}{R_2} \right)} = \frac{4\pi \epsilon_r R_1}{1 + \frac{\epsilon_r - \epsilon_0}{\epsilon_0} \frac{R_1}{R_2}}$$

24) Tra le armature di un condensatore piano esiste un mezzo dielettrico con  $\epsilon_r(x)$ . Calcolare la capacità del condensatore.



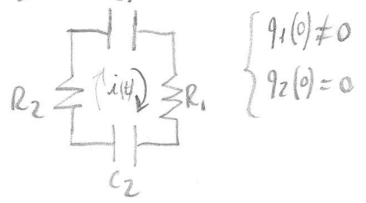
$$C = \frac{A \cdot \epsilon}{h} \rightarrow dC = \frac{A \cdot \epsilon(x)}{h} dx$$



$$\frac{1}{C} = \sum \frac{1}{C(x)} = \int \frac{1}{C(x)}$$

$$\frac{1}{C} = \int_0^h \frac{dx}{A \epsilon(x)} = \frac{1}{A} \int_0^h \frac{1}{\epsilon(x)} dx \rightarrow C = \frac{A}{\int_0^h \frac{1}{\epsilon(x)} dx}$$

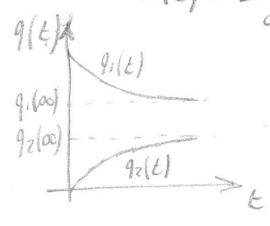
25) Carica di un condensatore  $C_2$  da un altro condensatore  $C_1$



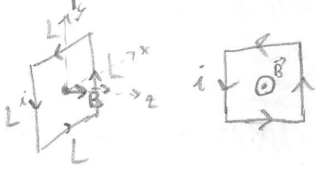
$$\sum V_i = 0 \rightarrow -\frac{1}{C_1} q_1(t) - R_2 i(t) - \frac{1}{C_2} q_2(t) - R_1 i(t) = 0$$

$$i(t) = \frac{dq_1(t)}{dt} = \frac{dq_2(t)}{dt}$$

$$\begin{cases} q_1(\infty) + q_2(\infty) = q_1(0) \\ \frac{q_1(\infty)}{C_1} = \frac{q_2(\infty)}{C_2} \end{cases}$$



28) Campo  $\vec{B}$  creato al centro di una spirale rettangolare di lato  $L$  percorsa da  $i$ . (20)



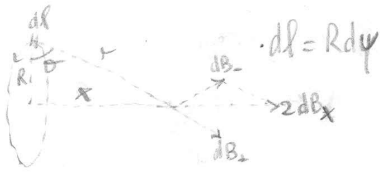
$$B_{\text{filo}} = \frac{\mu_0 i}{4\pi R} \frac{L}{\sqrt{L^2 + 4R^2}} \hat{u}_z \quad R = \frac{L}{2} \rightarrow B = \frac{\mu_0 i}{\pi L} \frac{L - \hat{u}_z}{\sqrt{\frac{L^2 + 4L^2}{4}}} = \frac{\mu_0 i}{\pi L} \frac{1}{\sqrt{2}} \hat{u}_z$$

$$\vec{B}_{\text{tot}} = 4B_{\text{filo}} = 2\sqrt{2} \frac{\mu_0 i}{\pi L} \hat{u}_z$$

29) Campo  $B$  creato sull'asse di una spirale circolare di raggio  $R$  percorsa da corrente  $i$ .



Ampere-Laplace  $d\vec{B} = \frac{\mu_0}{4\pi} i \frac{d\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} i \frac{dl}{r^2} \sin \frac{\pi}{2} \hat{u}_z$



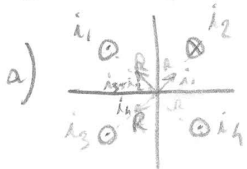
$$dB_x = \frac{\mu_0 i R dy}{4\pi r^2} \cos \theta \rightarrow B_x = \int_0^\pi dy \rightarrow$$

$$B = 2B_x = \frac{\mu_0 i R \pi}{4\pi r^2} \cos \theta \quad \cos \theta = \frac{R}{r} \rightarrow B = \frac{\mu_0 i R^2}{2r^3}$$

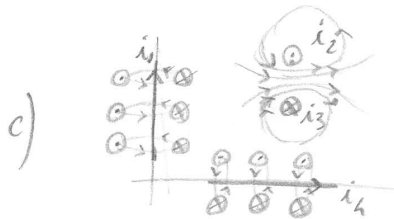
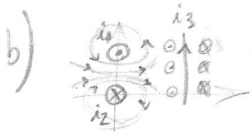
$$r = \sqrt{x^2 + R^2}$$

$$\vec{B} = \frac{\mu_0 i}{2} \frac{R^2}{(x^2 + R^2)^{3/2}} \hat{u}_x \quad ; \quad \text{Nel centro } x=0 \rightarrow B = \frac{\mu_0 i R}{2}$$

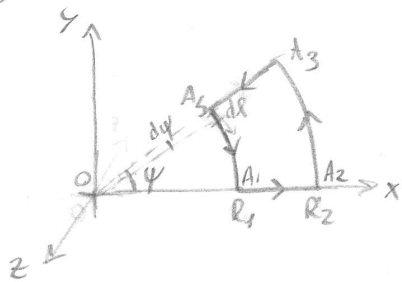
30) Calcolare il campo  $\vec{B}$  in un punto del piano, creato da, conduttori:



$$\text{in } O: B = \frac{\mu_0}{2\sqrt{2}\pi R} [(i_1 - i_2 - i_3 - i_4) \hat{u}_x + (i_1 + i_2 + i_3 - i_4) \hat{u}_y]$$



31) Calcolare  $\vec{B}$  in  $O$  creato dal circuito percorsa da una corrente  $i$  antioraria.



$$B_{A1A2} = B_{A3A4} = 0 \quad (d\vec{l} \parallel \vec{r} = 0 \text{ perché } d\vec{l} \parallel \vec{r})$$

$$dB_{A4A1} = \frac{\mu_0 i}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0 i}{4\pi} \frac{d\psi R_1}{R_1^2} (-\hat{u}_z) \rightarrow B_{A4A1} = \frac{\mu_0 i}{4\pi R_1} \hat{u}_z$$

analogamente  $B_{A2A3} = \frac{\mu_0 i}{4\pi R_2} \psi \hat{u}_z$

$$\text{quindi in } O: \vec{B} = \frac{\mu_0 i}{4\pi} \left( \frac{1}{R_2} - \frac{1}{R_1} \right) \hat{u}_z$$

34) Calcolare il campo  $B$  creato da un cilindro indefinito di raggio  $R$  carico uniformemente con densità di carica  $\rho$  che ruota con velocità angolare  $\omega$ . (22)



sup. cilindrica indefinita:  $B_{int} = \mu_0 i_m$   
 $B_{ext} = 0$

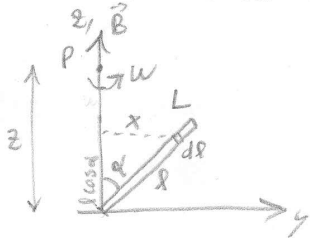
$$di = \frac{dq}{T}; T = \frac{2\pi}{\omega}; dq = \rho \cdot 2\pi r L dr$$

$$di = \frac{\rho \cdot 2\pi r L dr \omega}{2\pi} \rightarrow di_m = \frac{di}{L} = \rho \omega r dr$$

corrente per un'area di lunghezza

$$dB = \mu_0 di_m = \mu_0 \rho \omega r dr \rightarrow B = \mu_0 \rho \omega \int_0^R r dr \rightarrow B = \frac{1}{2} \mu_0 \rho \omega R^2$$

35) L'asta di lunghezza  $L$  possiede una carica  $q$  e forma un angolo  $\alpha$  con l'asse  $z$ . Ad una sua estremità coincide con  $O$  e ruota attorno a  $z$  con una velocità  $\omega$ . Calcolare  $\vec{B}$  sull'asse  $z$ .



$$\lambda = \frac{q}{L}; dq = \lambda dl = \frac{q}{L} dl; T = \frac{2\pi}{\omega}; z - l \cos \alpha = w$$

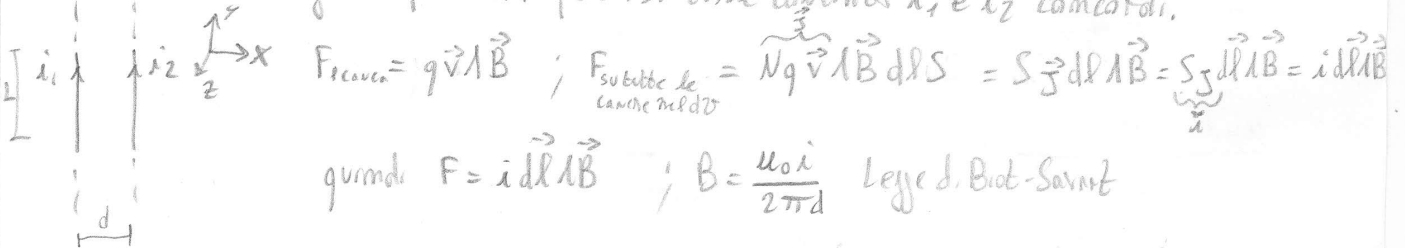
$$di = \frac{dq}{T} = \frac{q dl \omega}{2\pi L}$$

è equivalente a una spirale

$$\rightarrow dB_{\text{spirale}} = \frac{\mu_0}{2} di \frac{x^2}{(x^2 + w^2)^{3/2}}$$

$$dB = \frac{\mu_0}{2} \frac{q \omega dl}{2\pi L} \frac{x^2}{(x^2 + w^2)^{3/2}} \quad \begin{matrix} x = l \sin \alpha \\ w = z - l \cos \alpha \end{matrix} \quad B = \frac{\mu_0}{2} \frac{q \omega}{2\pi L} \sin^2 \alpha \int_0^L \frac{l^2}{(l^2 \sin^2 \alpha + (z - l \cos \alpha)^2)^{3/2}} dl$$

36) Calcolare la forza di interazione magnetica per unità di lunghezza di due conduttori rettilinei indefiniti paralleli percorsi dalle correnti  $i_1$  e  $i_2$  come in figura.



$$F_{\text{cavo}} = q \vec{v} \wedge \vec{B}; F_{\text{subtile de}} = N q \vec{v} \wedge \vec{B} dS = \int \vec{j} \wedge \vec{B} dS = \int \vec{j} \wedge \vec{B} = i d \vec{l} \wedge \vec{B}$$

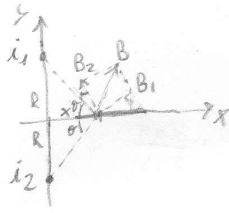
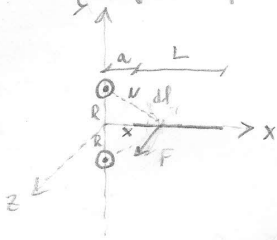
quindi  $F = i d \vec{l} \wedge \vec{B}; B = \frac{\mu_0 i}{2\pi d}$  Legge di Biot-Savart

$$F_{1s2} = i_2 L \vec{v}_1 \wedge \frac{\mu_0 i_2}{2\pi d} (-\vec{v}_2) = \frac{\mu_0 i_1 i_2}{2\pi d} L (-\vec{v}_x) \rightarrow \frac{F_{1s2}}{L} = -\frac{\mu_0 i_1 i_2}{2\pi d} \vec{v}_x$$

$$F_{2s1} = i_1 L \vec{v}_2 \wedge \frac{\mu_0 i_1}{2\pi d} \vec{v}_1 = \frac{\mu_0 i_1 i_2}{2\pi d} L \vec{v}_x \rightarrow \frac{F_{2s1}}{L} = \frac{\mu_0 i_1 i_2}{2\pi d} \vec{v}_x$$

40) Calcolare la  $F$  sul conduttore  $L$  orenbe  $i_2$  immerso nel campo  $B$  creato dai due conduttori indefinibili paralleli  $i_1$  e  $i_2$

(24)



$$\vec{B}_1 = \frac{\mu_0 i_1}{2\pi r} (\sin\theta \hat{U}_x + \cos\theta \hat{U}_y)$$

$$\vec{B}_2 = \frac{\mu_0 i_2}{2\pi r} (-\sin\theta \hat{U}_x + \cos\theta \hat{U}_y)$$

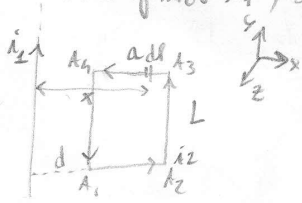
$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 \cos\theta}{2\pi r} (i_1 + i_2) \hat{U}_y$$

$$dF_{i_2 L} = i_2 dx \frac{\mu_0 \cos\theta (i_1 + i_2)}{2\pi r} (\hat{U}_x) \wedge (\hat{U}_y) = \frac{\mu_0 (i_1 + i_2) i_2 \cos\theta}{2\pi} \frac{dx}{r} \hat{U}_z$$

$$F_{i_2 L} = \frac{\mu_0 (i_1 + i_2) i_2}{2\pi} \int_a^{a+L} \frac{2x}{x^2 + R^2} dx = \frac{\mu_0 (i_1 + i_2) i_2}{4\pi} \ln\left(\frac{R^2 + (L+a)^2}{R^2 + a^2}\right)$$

$\cos\theta = \frac{x}{\sqrt{x^2 + R^2}}$

41) Filo indefinibile  $i_1$ , spira rettangolare  $i_2$ . Calcolare le forze agenti sulla spira.



$$F = i d\vec{l} \wedge \vec{B}$$

$$F_{A_4 A_1} = i_2 L \frac{\mu_0 i_1}{2\pi d} (-\hat{U}_y) \wedge (-\hat{U}_z) = \frac{\mu_0 i_1 i_2 L}{2\pi d} \hat{U}_x$$

$$F_{A_2 A_3} = i_2 L \frac{\mu_0 i_1}{2\pi(d+a)} (\hat{U}_y) \wedge (-\hat{U}_z) = \frac{\mu_0 i_1 i_2 L}{2\pi(d+a)} (-\hat{U}_x)$$

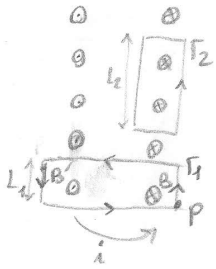
$$B_{A_3 A_4} = B_{A_1 A_2} = \frac{\mu_0 i_1}{2\pi x} \hat{U}_z \rightarrow dF_{A_3 A_4} = i_2 dx \frac{\mu_0 i_1}{2\pi x} (-\hat{U}_x) \wedge (-\hat{U}_z) = \frac{\mu_0 i_1 i_2 dx}{2\pi x} (-\hat{U}_y) \rightarrow F_{A_3 A_4} = \frac{\mu_0 i_1 i_2}{2\pi} \ln\left(1 + \frac{d}{a}\right) \hat{U}_y$$

$$dF_{A_1 A_2} = i_2 dx \frac{\mu_0 i_1}{2\pi x} (\hat{U}_x) \wedge (-\hat{U}_z) = \frac{\mu_0 i_1 i_2 dx}{2\pi x} \hat{U}_y \rightarrow F_{A_1 A_2} = \frac{\mu_0 i_1 i_2}{2\pi} \ln\left(1 + \frac{d}{a}\right) \hat{U}_y$$

$$F = \sum F_i = (F_{A_4 A_1} - F_{A_2 A_3}) \hat{U}_x + (F_{A_1 A_2} - F_{A_3 A_4}) \hat{U}_y = \frac{\mu_0 i_1 i_2 L}{2\pi} \left(\frac{1}{d} - \frac{1}{d+a}\right) \hat{U}_x = \frac{\mu_0 i_1 i_2 L}{2\pi d} \frac{a}{d+a} \hat{U}_x$$

45) Calcolare il campo  $\vec{B}$  creato da un solenoide indefinito contenente  $n$  spire per unità di lunghezza e percorso dalla corrente  $i$

(26)



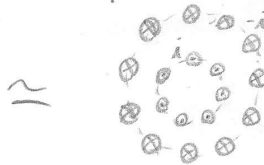
1)  $\Gamma_1: L_1 B - L_1 B' = \mu_0 (n L_1 i - n L_1 i) \rightarrow B = B'$

se prendo  $P$  all'infinito  $\rightarrow B = 0 \rightarrow B' = 0$

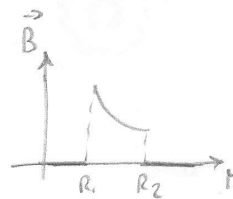
2) circolazione lungo  $\Gamma_2$ : contribuisce solo il tratto interno verticale  
 $(\vec{B} \cdot d\vec{l})$

$-B L_2 = \mu_0 (-L_2 n i) \rightarrow B = \mu_0 n i$

46) Calcolare il campo magnetico  $\vec{B}$  creato da un solenoide toroidale di raggi interno ed esterno  $R_1$  e  $R_2$  rispettivamente, aventi  $N$  spire e percorso dalla corrente  $i$ .



f. imb:  $\oint \vec{B} \cdot d\vec{l} = \mu_0 i$   
 f. diff:  $\nabla \times \vec{B} = \mu_0 \vec{j}$  } c. statiche

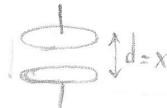


$r < R_1 \quad B \cdot 2\pi r = \mu_0 0 \rightarrow B = 0$

$R_1 < r < R_2 \quad B \cdot 2\pi r = \mu_0 N i \rightarrow B = \frac{\mu_0 N i}{2\pi r}$

$r > R_2 \quad B \cdot 2\pi r = \mu_0 (N i - N i) \rightarrow B = 0$

47) Un condensatore piano con armature a disco di raggio  $R$  è alimentato da una tensione variabile nel tempo secondo la legge  $V(t) = V_0 \sin(\omega t)$ . Calcolare il campo  $\vec{B}$  e  $\vec{E}$ .



$\vec{E} = -\frac{dV}{dx} \rightarrow E = -\frac{V}{d} = -\frac{V_0}{d} \sin(\omega t)$

$r < R$

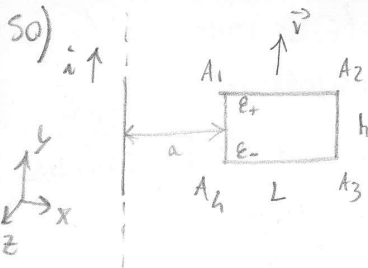
2) f. diff:  $\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$   
 f. imb:  $\oint \vec{B} \cdot d\vec{l} = \mu_0 i + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \oint \vec{E} \cdot d\vec{s}$  } c. dinamiche

$0 < r < R \quad B \cdot 2\pi r = \mu_0 \cdot 0 + \mu_0 \epsilon_0 \frac{\partial \phi(E)}{\partial t} \quad \phi(E) = \int_{\text{ip}} \vec{E} \rightarrow \frac{\partial \phi(E)}{\partial t} = \pi r^2 \left( -\frac{V_0}{d} \cos(\omega t) \cdot \omega \right)$

$B = -\frac{\mu_0 \epsilon_0 \omega V_0 r}{2d} \cos(\omega t)$

$r > R \quad B \cdot 2\pi r = \mu_0 \cdot 0 + \mu_0 \epsilon_0 \frac{\partial \phi(E)}{\partial t} \quad \phi(E) = \pi R^2 \cdot E \rightarrow \frac{\partial \phi(E)}{\partial t} = -\frac{V_0 \pi R^2 \omega}{d} \cos(\omega t)$

$B = -\frac{\mu_0 \epsilon_0 \omega V_0 R^2}{2d r} \cos(\omega t)$



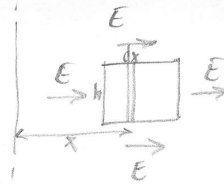
Calcolare la  $\mathcal{E}$  indotta.

$$\mathcal{E} = \mathcal{E}_+ + \mathcal{E}_- = \mathcal{E} - \mathcal{E} = 0$$

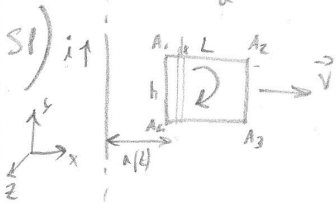
Usando Faraday:

$$d\phi = B(x) dA = \frac{\mu_0 i}{2\pi x} h dx = \frac{\mu_0 i h}{2\pi} \frac{dx}{x}$$

$$\phi = \int d\phi = \frac{\mu_0 i h}{2\pi} \int_a^{a+L} \frac{dx}{x} = \frac{\mu_0 i h}{2\pi} \ln\left(1 + \frac{L}{a}\right) ; \mathcal{E} = -\frac{\partial\phi}{\partial t} = -\frac{\partial}{\partial t} \left( \frac{\mu_0 i h}{2\pi} \ln\left(1 + \frac{L}{a}\right) \right) = 0$$



(28)



Calcolare la  $\mathcal{E}$  indotta.

1) Usando la legge di Lorentz

$$\vec{B} = -\frac{\mu_0 i}{2\pi x} \hat{u}_z$$

$$\vec{F} = q\vec{v} \wedge \vec{B}$$

$$F_{A_1 A_2} = -qv \frac{\mu_0 i}{2\pi x} (\hat{u}_x) \wedge (-\hat{u}_z) = \frac{\mu_0 i qv}{2\pi x} \hat{u}_y$$

non c'è corrente indotta nel tratto di conduttore. (le cariche non possono spostarsi in  $\hat{u}_F$ )

$$F_{A_3 A_4} = \frac{\mu_0 i qv}{2\pi x} (-\hat{u}_x) \wedge (-\hat{u}_z) = \frac{\mu_0 i qv}{2\pi x} \hat{u}_y$$

$$F_{A_1 A_4} = qv \frac{\mu_0 i}{2\pi a(t)} \hat{u}_x \wedge (-\hat{u}_z) = \frac{qv \mu_0 i}{2\pi a(t)} \hat{u}_y \quad \text{c'è corrente indotta (la F fa muovere le cariche dal basso verso l'alto)}$$

$$\vec{E}_{A_1 A_4} = \frac{F}{q} = \frac{\mu_0 i v}{2\pi a(t)} \hat{u}_y$$

$$\mathcal{E}_{A_1 A_4} = \int_{A_1 A_4} E dx = E \int_{A_1 A_4} dx = E \cdot h = \frac{\mu_0 i v h}{2\pi a(t)}$$

$$F_{A_2 A_3} = qv \frac{\mu_0 i}{2\pi (L+a(t))} (\hat{u}_x) \wedge (-\hat{u}_z) = \frac{qv \mu_0 i}{2\pi (L+a(t))} \hat{u}_y \quad \text{c'è corrente indotta}$$

$$\vec{E}_{A_2 A_3} = \frac{\mu_0 i v}{2\pi (L+a(t))} \hat{u}_y$$

$$\mathcal{E}_{A_2 A_3} = \int_{A_2 A_3} E dx = E \int_{A_2 A_3} dx = E \cdot h = \frac{\mu_0 i v h}{2\pi (L+a(t))}$$

$$E_{A_1 A_4} \uparrow \quad \uparrow \quad E_{A_2 A_3} = \uparrow \quad \mathcal{E} = \mathcal{E}_{A_1 A_4} - \mathcal{E}_{A_2 A_3}$$

$$\mathcal{E} = \mathcal{E}_{A_1 A_4} - \mathcal{E}_{A_2 A_3} = \frac{\mu_0 i v h}{2\pi} \left( \frac{1}{a(t)} - \frac{1}{L+a(t)} \right) = \frac{\mu_0 i v h}{2\pi a(t)} \frac{L}{L+a(t)}$$

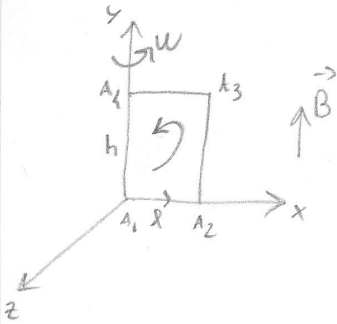
2) Usando la legge di Faraday

$$\mathcal{E} = -\frac{\partial\phi}{\partial t} ; \phi(t) = \frac{\mu_0 i h}{2\pi} \ln\left(1 + \frac{L}{a(t)}\right) = \frac{\mu_0 i h}{2\pi} \left( \ln(a(t)) + L + \ln(a(t)) \right)$$

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$$\frac{\partial\phi}{\partial t} = \frac{\mu_0 i h}{2\pi} \left( \frac{d\phi}{dt} \stackrel{V}{=} -\frac{d a(t)}{dt} - \frac{d a(t)}{dt} \right) = \frac{\mu_0 i v h}{2\pi} \left( \frac{1}{L+a(t)} - \frac{1}{a(t)} \right) \rightarrow \mathcal{E} = -\frac{\partial\phi}{\partial t} = \frac{\mu_0 i v h}{2\pi} \left( \frac{1}{a(t)} - \frac{1}{L+a(t)} \right)$$

53) Il circuito  $A_1A_2A_3A_4$  si trova sul piano  $xy$  come in figura. Il piano  $xy$  ruota attorno a  $z$ . Il campo  $\vec{B} = B\hat{y}$ . Determina la  $\mathcal{E}$  che nasce nel circuito. (30)



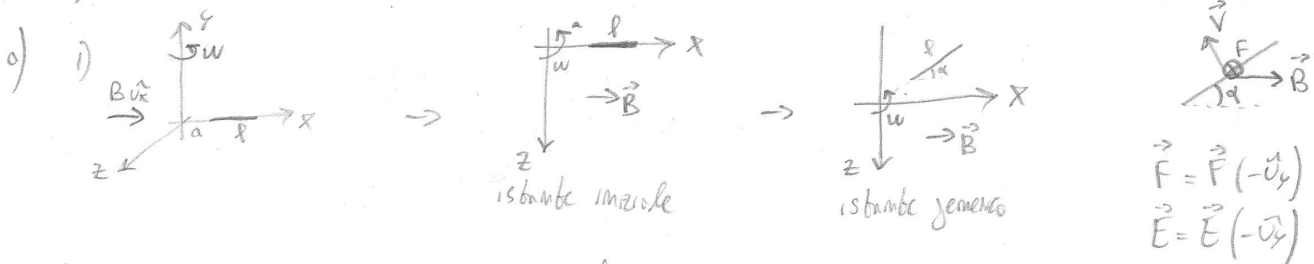
$$\begin{aligned}
 A_1A_2) \quad \vec{F} &= q\vec{v}\wedge\vec{B} = q\omega a B \hat{y} \times \hat{x} \rightarrow \vec{E} = \omega a B \hat{y} \times \hat{x} \rightarrow \mathcal{E} = \frac{1}{2}\omega B a^2 \\
 A_2A_3) \quad \vec{F} &= q\vec{v}\wedge\vec{B} = q\omega h B \hat{y} \times \hat{y} \rightarrow \text{NON ATTIVO (per la componente indotta)} \\
 A_3A_4) \quad \vec{F} &= q\vec{v}\wedge\vec{B} = q\omega a B \hat{y} \times \hat{x} \rightarrow \mathcal{E} = \omega a B \hat{y} \times \hat{x} \rightarrow \mathcal{E} = \frac{1}{2}\omega B a^2 \\
 A_4A_1) \quad \vec{F} &= q\vec{v}\wedge\vec{B} = 0 \quad (v_y=0) \rightarrow \text{NON ATTIVO}
 \end{aligned}$$



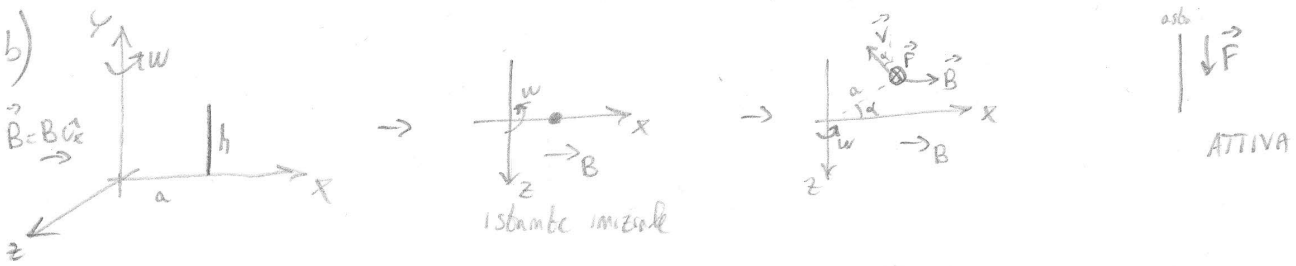
$\mathcal{E} = 0$

Con Faraday:  $\mathcal{E} = \frac{\partial \Phi}{\partial t} = \frac{\partial}{\partial t} 0 = 0$  l'area è uguale a zero; il  $B$  rimane costante nel tempo durante la rotazione

54) Calcolare  $\mathcal{E}_i$  sull'asta che ruota all'asse  $z$ :  
 a) mantenendo  $\perp$  ad  $z$  in presenza di  $\vec{B} \perp z$   
 b) mantenendo  $\parallel$  a  $z$  in presenza di  $\vec{B} \perp z$



$$\mathcal{E} = \int \vec{E} \cdot d\vec{l} = \int E(-v_y) dl (\cos\alpha \hat{u}_x + \sin\alpha \hat{u}_z) = \int E dl \cos\frac{\pi}{2} = 0$$



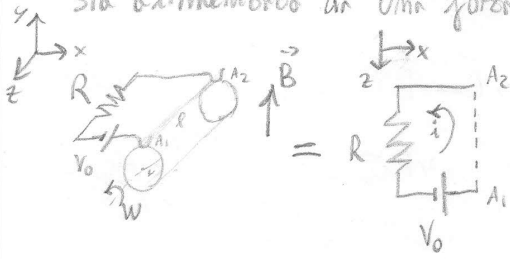
$$\begin{aligned}
 \mathcal{E} &= \int_{asb} \vec{E} \cdot d\vec{l} = \int_{asb} \frac{\vec{F}}{q} \cdot d\vec{l} = \int_{asb} \vec{v}\wedge\vec{B} \cdot d\vec{l} = \int_{asb} [v \sin\alpha (-\hat{u}_x) + \cos\alpha (\hat{u}_z)] \wedge B \hat{y} \cdot d\vec{l} = \int_{asb} B v \sin\alpha (-v_y) dl (v_y) = \\
 &= - \int_{asb} B \omega a \sin\alpha dl = -B \omega a \sin\alpha \int_0^h dl = -B \omega a h \sin\alpha
 \end{aligned}$$

se  $\omega = \cos t \rightarrow \mathcal{E} = -B \omega a h \sin(\omega t)$



57) Determina  $w(t)$  e  $i(t)$  dell'esercizio precedente nel caso che il circuito sia alimentato da una forza elettromotrice esterna  $V_0$  costante nel tempo.

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$$A_{1A2} \vec{F} = q \vec{v} \wedge \vec{B} = qvB (-\hat{u}_2) \rightarrow \vec{E} = \frac{\vec{F}}{q} = -w \wedge B \hat{u}_2$$

$$\mathcal{E} = \int \vec{E} d\vec{l} = -w \wedge l B$$

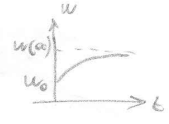
$$V_0 - Ri - w \wedge l B = 0 \rightarrow i = \frac{V_0 - B \wedge l w}{R}$$

$$F_{su A_1 A_2} = i l B$$

$$M_{meccanico} = r \cdot F_{su A_1 A_2} = r i l B = \frac{V_0 - B \wedge l w}{R} r l B$$

Eq. di moto

$$I \frac{dw}{dt} = \frac{V_0 - B \wedge l w}{R} r l B \rightarrow \frac{dw}{dt} = \frac{V_0 r l B}{R I} - \frac{B^2 r l^2}{R I} w \rightarrow \frac{dw}{dt} = c_1 - c_2 w \rightarrow \text{soluzione}$$

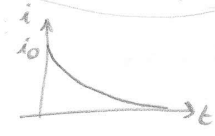


$$t \rightarrow \infty \quad \frac{dw}{dt} = 0 = c_1 - c_2 w \rightarrow w(\infty) = \frac{c_1}{c_2}$$

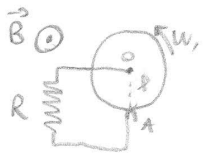
$$i(\infty) = \frac{V_0}{R} - \frac{B \wedge l}{R} \cdot \frac{V_0 r l B}{B^2 r l^2} = 0$$

$$t=0 \quad i(0) = \frac{V_0}{R} - \frac{B \wedge l}{R} w_0$$

$$i(t) = \frac{V_0}{R} - \frac{B \wedge l}{R} w(t)$$



58) In presenza di un campo B, un disco metallico di raggio l ruota con velocità angolare  $w$ . Il suo centro è in contatto ohmico tramite una resistenza R con la sua periferia. Determinare  $i$  che attraversa la resistenza R



OA si comporta come un'asta che ruota a velocità w



$$\vec{E} = \vec{v} \wedge \vec{B} = w r B (-\hat{u}_2) ; d\vec{l} = dr (\hat{u}_2) \quad \mathcal{E} = \int_0^l E d\vec{l} = - \int_0^l w B r dr = - \frac{1}{2} w B l^2$$

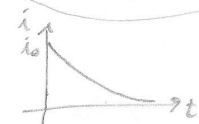
$$i = \frac{V}{R} = \frac{\mathcal{E}}{R} = - \frac{1}{2} \frac{w B l^2}{R}$$

forza su OA:  $dF = i B dr = i B dr = - \frac{w B^2 l^2}{2R} dr$  forza frenante

Momento meccanico:  $dM = r dF = - \frac{w B^2 l^2}{2R} r dr \rightarrow M = - \frac{w B^2 l^2}{2R} \int_0^l r dr = - \frac{w B^2 l^4}{4R}$  Momento frenante

Eq. di moto

$$I \frac{dw}{dt} = - |M| \rightarrow I \frac{dw}{dt} + \frac{B^2 l^4}{4R} w = 0 \rightarrow \text{soluzione} \rightarrow w(t) = w_0 e^{-\frac{B^2 l^4}{4RI} t} \rightarrow i = \frac{B l^2}{2R} w \rightarrow i(t) = \frac{B l^2 w_0}{2R} e^{-\frac{B^2 l^4}{4RI} t}$$





62) Le armature del condensatore C sono collegate attraverso un circuito  $\Gamma$  che compie un solenoidale percorso da  $i(t)$ . Il circuito è piano e  $\perp$  all'asse del solenoide. Calcolare la  $q(t)$  sulle armature del condensatore.

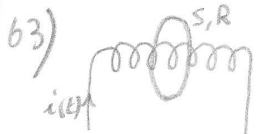
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Emergia  
Maggiore  
Imultanza

Il flusso di  $\vec{B}(t)$  del solenoide attraverso  $\Gamma$  è:  $\phi(t) = \pi r^2 B(t) = \pi r^2 \mu_0 m i(t)$

Sul  $\Gamma$  nasce una  $\mathcal{E}_i$  che alimenta il condensatore:  $\mathcal{E}_i(t) = -\frac{\partial \phi}{\partial t} = -\pi r^2 \mu_0 m \frac{\partial i(t)}{\partial t}$

La  $q(t)$  su ogni armatura è:  $q(t) = C \mathcal{E}_i(t) = -\mu_0 m \pi r^2 C \frac{\partial i(t)}{\partial t}$



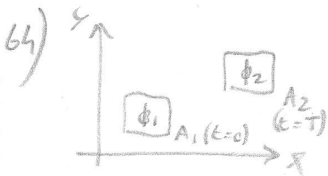
63) La spira piana S di resistenza R è attraversata perpendicolarmente da un solenoide indefinito di n spire per unità di lunghezza, di raggio a e percorso dalla corrente  $i(t)$ . Determinare la corrente I(t) che circola nella spira.



Flusso di B creato dal solenoide attraverso S:  $\phi(t) = \pi a^2 B(t) = \pi a^2 \mu_0 n i(t)$

$\mathcal{E}_i$  generata sulla spira:  $\mathcal{E}_i(t) = -\frac{\partial \phi}{\partial t} = -\pi a^2 \mu_0 n \frac{\partial i(t)}{\partial t}$

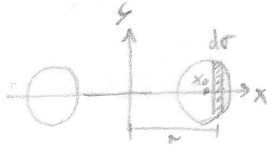
Corrente indotta che circola nella spira:  $I = \frac{\mathcal{E}_i(t)}{R} = -\mu_0 \frac{\pi a^2 n}{R} \frac{\partial i(t)}{\partial t}$



64) Calcolare la carica Q spostata nel circuito durante il suo spostamento dalla configurazione iniziale  $A_1$  a quella finale  $A_2$  in presenza di un campo B.

Durante lo spostamento da  $A_1$  a  $A_2$  cambia  $\phi$  e quindi circola corrente:  $i(t) = -\frac{1}{R} \frac{\partial \phi(t)}{\partial t}$   
 d'altra parte si ha che  $i(t) = \frac{dq(t)}{dt}$ , quindi  $\frac{dq}{dt} = -\frac{1}{R} \frac{\partial \phi(t)}{\partial t} \rightarrow \int_0^T \frac{dq}{dt} dt = -\frac{1}{R} \int_0^T \frac{\partial \phi(t)}{\partial t} dt \rightarrow$   
 $\rightarrow Q = -\frac{1}{R} (\phi(T) - \phi(0)) \rightarrow Q = \frac{\phi_1 - \phi_2}{R}$

65) Calcolo della L di un solenoide toroidale

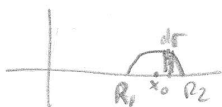


$R = \frac{R_2 + R_1}{2}$ ;  $x_0 = \frac{R_2 - R_1}{2}$ ;  $B = \frac{\mu_0 N i}{2\pi r}$

Immagi punto dentro il cerchio B è costante

$\phi = Li \rightarrow L = \frac{\phi}{i}$

$L = \frac{1}{i} N \int_{S_1} B dr = \frac{N}{i} \int_{S_1} \frac{\mu_0 N i}{2\pi r} dr = \frac{\mu_0 N^2}{2\pi} \int_{S_1} \frac{dr}{r}$   
 considera solo mezza circonferenza, quella superiore, e moltiplica per 2



$(x - \frac{R_2 + R_1}{2})^2 + y^2 = (\frac{R_2 - R_1}{2})^2 \rightarrow y = \sqrt{(\frac{R_2 - R_1}{2})^2 - (x - \frac{R_2 + R_1}{2})^2}$

$dr = y dx$ ;  $r = x$

$L = \frac{\mu_0 N^2}{\pi} \int_{R_1}^{R_2} \frac{1}{x} \sqrt{(\frac{R_2 - R_1}{2})^2 - (x - \frac{R_2 + R_1}{2})^2} dx$