NUMERO: 2152A ANNO: 2017

A P P U N T I

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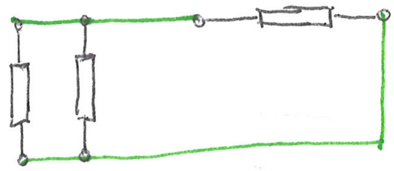
MATERIA: Introduction to electrical engineering - Electrical machines - Prof Freschi Ferraris

Introduction to electrical engineering

- We describe the electrical objects using two quantities
  - current $i(t)$
  - voltage $v(t)$

- two - terminal component

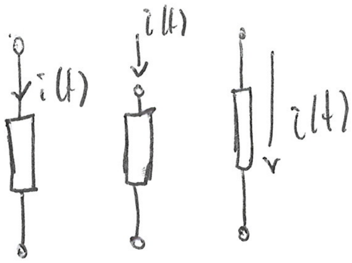
- In a circuit there are many components interconnected



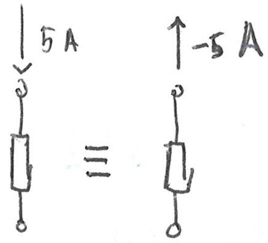
Connections are ideal

Current

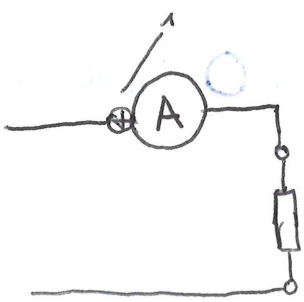
$i(t) \triangleq \frac{dq}{dt}$ [A], $q > 0$: Current is the flow of positive charges



Three different representations of current flowing through a component



The plus indicates the measurement direction (⟶)

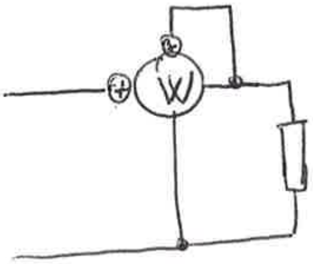


Ammeter is used to measure the current inside a circuit

Power

$$p(t) = \frac{dW}{dt} = \frac{dW}{dq} \cdot \frac{dq}{dt} = v(t) \cdot i(t) \quad [W]$$

Power is measured by using an ammeter and a voltmeter in combination or by using a wattmeter



The wattmeter has four terminals
- 2 for voltage
- 2 for current

(P) $P_{abs}(t) = v(t) \cdot i(t)$ - Power absorbed by the component

$P_{abs} \gtrless 0$

(A) $P_{del.}(t) = v(t) \cdot i(t)$ - Power delivered by the component

$P_{del} \gtrless 0$

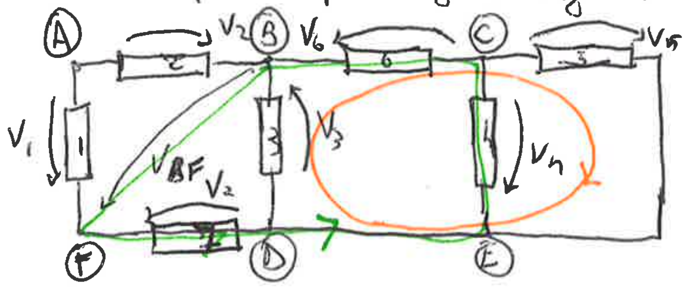
Energy

$$W(t_0, t_1) = \int_{t_0}^{t_1} p(t)\, dt = \int_{t_0}^{t_1} v(t) \cdot i(t) \cdot dt \qquad [J]$$

Power absorbed in an interval of time

- Kirckhoff's voltage law (KVL)

The algebraic sum of voltages along any oriented and closed path passing through nodes is always zero



- $+V_3(t) - V_6(t) + V_5(t) = 0$
- $+V_{BF}(t) - V_7(t) - V_4(t) + V_6(t) = 0$

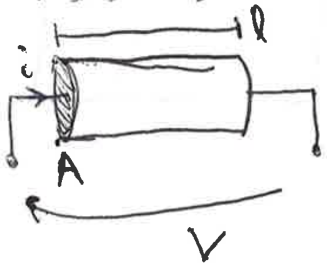
+ : V with the same orientation of the loop

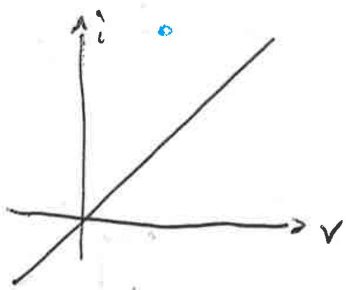
− : V with the opposite orientation of the loop

Both KCL and KVL are valid regardless the component.

Constitutive equations − Relationships V−i

- Resistors





$i \propto V$

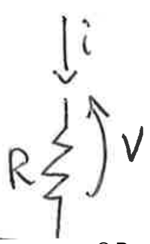
$V \propto i$

$V(t) = R \cdot i(t)$

R : resistance

$R = \rho \frac{l}{S}$ $\left[\frac{V}{A}\right] = [\Omega]$

ρ : resistivity $[\Omega m]$



Ohm's law

(P) $v(t) = R \cdot i(t)$; (A) $v(t) = -R \cdot i(t)$

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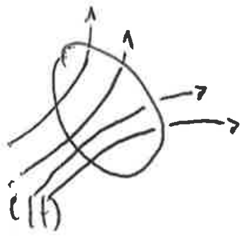
(P) $P_{abs} = v(t) \cdot i(t) = C\, v(t) \cdot \frac{dv(t)}{dt} \gtrless 0$

$W_E = W_{stored} = \int P_{abs}\, dt = \frac{1}{2} C\, v^2(t) \geq 0$

• Inductor



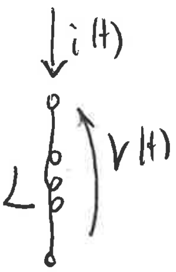
$\lambda = L \cdot i(t)$

λ : linked magnetic flux through the coil
L : inductance [H]

$\frac{d\lambda}{dt} = L \cdot \frac{di(t)}{dt} \Longrightarrow$ $\boxed{v(t) = L \cdot \frac{di(t)}{dt}}$ [$V = \frac{d\lambda}{dt}$ for Faraday's law]

$L = N^2 \cdot \mu \cdot \frac{A}{l}$

N → number of coils
μ → permeability [H/m]



$v(t) = L \cdot \frac{i(t)}{dt}$

$i(t) = I_0 + \frac{1}{L} \int_{t_0}^{t} v(t)\, dt$

In DC $i(t) = I_0$ and $v(t) = 0 \Longrightarrow$ short circuit

(P) $P_{abs} = v(t) \cdot i(t) = L \cdot i(t) \cdot \frac{di(t)}{dt} \gtrless 0$

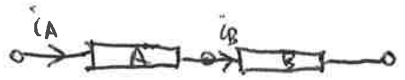
$W_M = W_{STORED} = \int P_{abs}\, dt = \frac{1}{2} L \cdot i^2(t) \geq 0$

Equivalence systems

- Series connections

  Two elements are connected in series when they share one mode and nothing else is connected to that mode

  If two elements are connected in series, they have the same current

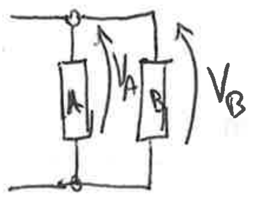


$i_A = i_B$

- Parallel connections

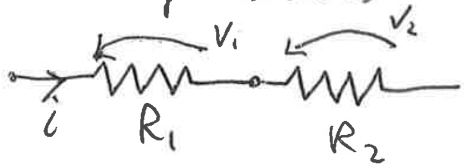
  Two elements are connected in parallel if one mode of the first element is connected to one mode of the second element and the other modes are connected toghether

  If two elements are connected in parallel, they have the same voltage



$V_A = V_B$

- Series of resistors



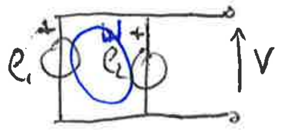
$R_{eq} = R_1 + R_2$

$R_{eq} = \sum_{k=1}^{N} R_k$

Series with a S.C $\Rightarrow$ ~~R_{eq}~~ $= R + R_{SC} = R$

Series with an O.C $\Rightarrow R_{eq} = R + R_{OC} = \infty \Rightarrow$ O.C.

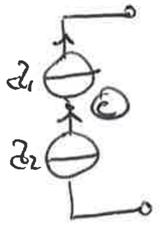
- Parallel of voltage generators



KVL : $e_1 - e_2 = 0 \Rightarrow e_1 = e_2$

We cannot put two voltage generators in parallel if they haven't the same imposed voltage.

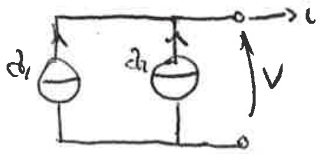
- Series of current generators



$KCL_{Ⓒ}$: $a_1 - a_2 = 0 \Rightarrow a_1 = a_2$

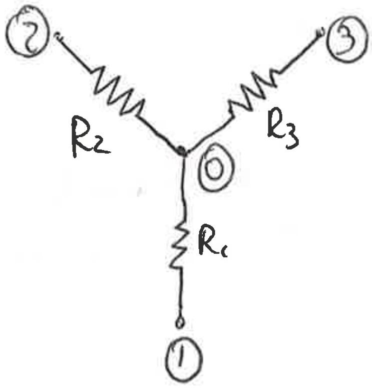
We cannot put two current generators in series if they haven't the same imposed current

- Parallel of current generator

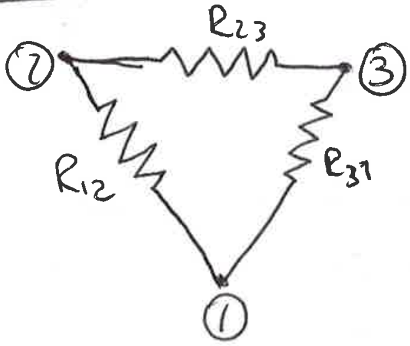


$i = a_1 + a_2 = a \quad , \forall v$

Star & Delta connections



Star/ Y / Wye



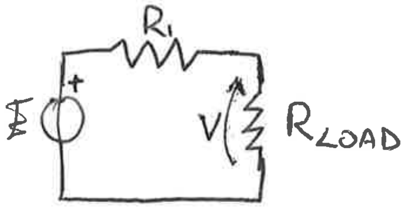
Delta/ triangle

Always a star connection can be transformed in a triangle one and viceversa

- From Δ to Y

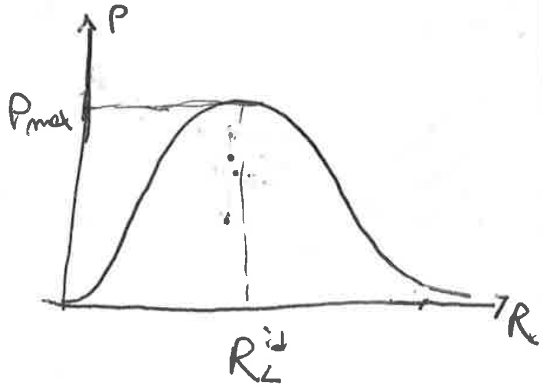
$$R_1 = \frac{R_{12} \cdot R_{31}}{R_{12} + R_{23} + R_{31}} \; ; \; R_2 = \frac{R_{23} \cdot R_{12}}{R_{12} + R_{23} + R_{31}} \; ; \; R_3 = \frac{R_{31} \cdot R_{23}}{R_{12} + R_{23} + R_{31}}$$

Maximum power - DC



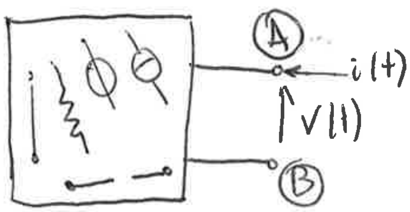
$$V = \frac{R_L}{R_1 + R_L} \cdot E$$

$$P = \frac{V^2}{R_L} = \frac{R_L \cdot E^2}{(R_1 + R_L)^2}$$



$$\frac{dP}{dR_L} = E^2 \frac{(R_1+R_L)^2 - 2(R_1+R_L) \cdot R_L}{(R_1+R_L)^4} = 0 \implies R_1 = R_L$$

Thevenin equivalent circuit



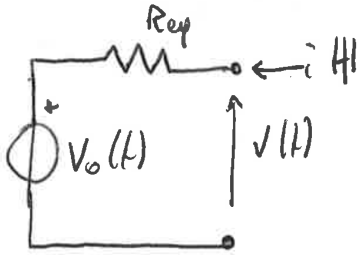
$i(t) = i_T(t)$

$$V(t) = \underbrace{\sum_k \alpha_K \cdot e_K + \sum_m \beta_m \cdot a_m}_{} + \beta_T i_T(t)$$

$$V(t) = V_0 + \beta_T \cdot i(t)$$

V_0 → open circuit voltage

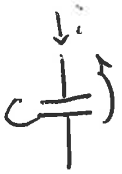
$\beta_T = \frac{V(t)}{i(t)}$ → R_{TH} : Equivalent resistance calculated with all the generators switched-off at the terminals



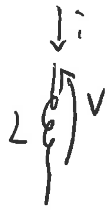
$$V(t) = V_0 + i(t) R_{eq}$$

Dynamic circuits

Capacitor and inductor

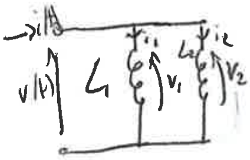


$$\begin{cases} i(t) = C \dfrac{dV(t)}{dt} \\ V(0) = V_0 \end{cases}$$



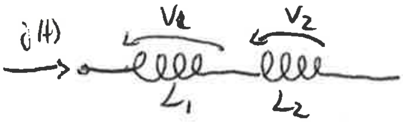
$$\begin{cases} V(t) = L \dfrac{di(t)}{dt} \\ i(0) = I_0 \end{cases}$$

- Parallel connections of inductors



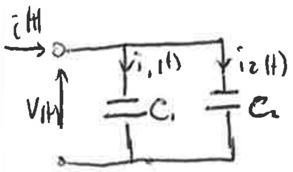
$$L_{eq} = L_1 \parallel L_2 = \frac{L_1 \cdot L_2}{L_1 + L_2}$$

- Series connections of inductors



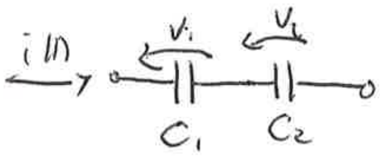
$$L_{eq} = L_1 + L_2$$

- Parallel connection of capacitors



$$C_{eq} = C_1 + C_2$$

- Series connection of capacitors



$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

- homogeneous equation

$$RC\frac{dV_c}{dt} + 1 V_c = 0$$

$$RC \cdot p^1 + 1 \cdot p^0 = 0 \implies p = -\frac{1}{RC}$$

$$V_{C,H} = K \cdot e^{-\frac{1}{RC} \cdot t}$$

- particular solution

$$V_{C,p}(t) = \text{const} \implies RC\underbrace{\frac{dV_{C,p}(t)}{dt}}_{=0} + 1 \cdot V_{Cp}(t) = E \implies V_{C,p} = E$$

- initial condition

$V_c(t) = K e^{-\frac{t}{RC}} + E \qquad t > 0$

$V_c(0^-) = V_0 \qquad t < 0$

$V_c(0^+) = ? = V_0$ → Current and voltage are continuous functions

$$V_c(t=0) = K e^0 + E = V_0$$

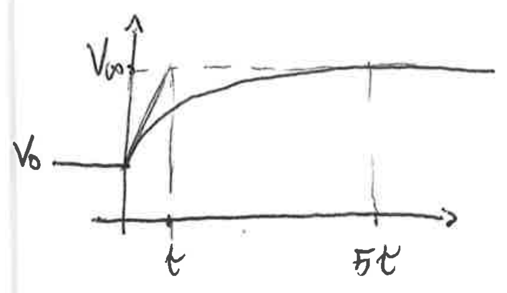
$$\implies K = V_0 - E$$

$$\longrightarrow V_c = (V_0 - E) e^{-\frac{t}{RC}} + E$$

$V_c(0) = V_0 - E + E = V_0$

$V_c(\infty) = E = V_\infty = V_p$

$RC = \tau$: time constant



The transient will extinguish after 5τ

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Sinusoidal steady state

The transient is negligible with respect to the steady state (few ms)

Sinusoidal waveform

$v(t) = \hat{V} \, sen(\omega t + \varphi)$

φ : phase shift

$\hat{V}$: peak value

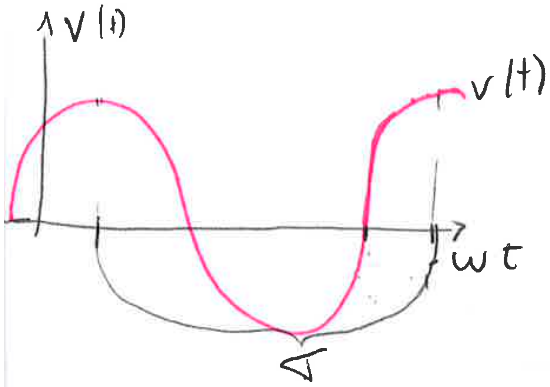
ω : angular frequency [rad/s]

T : period [s]

$T = \frac{2\pi}{\omega}$

f : frequency [s^{-1}]

$f = \frac{1}{T} = \frac{\omega}{2\pi}$



Phasors

$e^{j\theta} = \cos\theta + j \, sen\theta$

$\Rightarrow v(t) = \mathrm{Im}\left\{\hat{V} e^{j(\omega t + \varphi)}\right\} = \mathrm{Im}\left\{\hat{V} e^{j\varphi} \cdot e^{j\omega t}\right\} = \mathrm{Im}\left\{\underline{V} e^{j\omega t}\right\}$

($\hat{V} e^{j\varphi}$: constant complex number; $\underline{V}$: phasor)

If all generator show the same frequency, all electrical components have that frequency.

$\Rightarrow$ The relationship among the phasors is not affected by the frequency

$\Rightarrow$ $e^{j\omega t}$ can be neglected.

$v(t) \longleftrightarrow \underline{V} = \hat{V} e^{j\theta}$

time domain — phasor domain

• Rms and Peak value

$V = \sqrt{\frac{1}{T} \int_0^T \hat{V}^2 sen^2(\omega t + \varphi) \, dt} = \frac{\hat{V}}{\sqrt{2}} \Rightarrow v(t) = \sqrt{2} \, V sen(\omega t + \varphi) \longleftrightarrow \underline{V} = V e^{j\varphi}$

For resistors, capacitors and inductors, the relation is always the same $\longrightarrow$ $\underline{V} = Z \cdot \underline{I}$ (Generalized Ohm's law)

$$Z = \begin{cases} R & \text{resistors} \\ jX_C = j\frac{-1}{\omega C} & \text{capacitors} \\ jX_L = j\omega L & \text{inductors} \end{cases}$$

- Impedence

$\underline{Z}$ $[\Omega]$

$\underline{Z} = R + jX$, $X \begin{cases} >0 & \text{ohmic inductive} \\ <0 & \text{ohmic capacitive} \end{cases}$

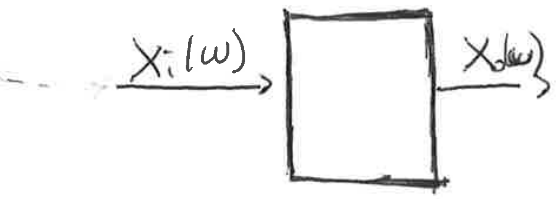
$R \rightarrow$ resistance, $X \rightarrow$ reactance

- Admittance

$\underline{Y} = \frac{1}{\underline{Z}}$ [S] $\quad \underline{Y} = G + jB$

$G = \frac{R}{R^2+X^2}$; conductance $\quad B = \frac{-X}{R^2+X^2}$; susceptance

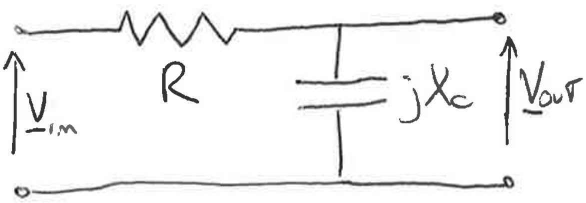
Filters



Transfer function

$G(\omega) = \frac{X_o(\omega)}{X_i(\omega)}$

- Low pass filter



$\underline{V}_o = \frac{jX_C}{R + jX_C} \cdot V_i$

$G(\omega) = \frac{\underline{V}_o}{\underline{V}_i} = \frac{jX_C}{R + jX_C} = \frac{j\frac{-1}{\omega C}}{R + j\frac{-1}{\omega C}} = \frac{j\frac{-1}{\omega C}}{\frac{-1}{\omega C}(-\omega CR + j)}$

$= \frac{-j}{\omega CR - j}$

Power in AC

$i(t) = I\sqrt{2}\ \text{sen}(\omega t + \varphi_I) \longleftrightarrow \underline{I}$

$v(t) = V\sqrt{2}\ \text{sen}(\omega t + \varphi_V) \longleftrightarrow \underline{V}$

- Instantaneous power

$$p(t) = v(t) \cdot i(t) = 2VI\ \text{sen}(\omega t + \varphi_I)\ \text{sen}(\omega t + \varphi_V)$$

$\vdots$

$$p(t) = \underbrace{VI \cos\varphi\,[1 - \cos(2\omega t + 2\varphi_I)]}_{P_{real}(t)} + \underbrace{VI\ \text{sen}\varphi\ \text{sen}(2\omega t + 2\varphi_I)}_{P_{reactive}(t)}$$

- $\langle P_{real}(t) \rangle = \langle p(t) \rangle = VI\cos\varphi = P$: real power [W]

$\cos\varphi$: power factor

$VI = S$: apparent power [VA]

$\Longrightarrow P = S\cos\varphi$

- $P_{reactive}(t)$ is an exchanged power (it has negative values) $\longrightarrow$ it generates losses

It must be registered $\longrightarrow$ peak value

$\max\{P_{reactive}(t)\} = VI\ \text{sen}\varphi = Q$: reactive power [Var]

- $P = VI\cos\varphi$, $Q = VI\ \text{sen}\varphi$ $\longrightarrow$ squaring and summing $\Longrightarrow P^2 + Q^2 = V^2 I^2 (\cos^2\varphi + \text{sen}^2\varphi) = S^2$

$\Longrightarrow S = \sqrt{P^2 + Q^2}$

- $\underline{V} \cdot \underline{I}^* = VI\cos\varphi + j\,VI\ \text{sen}\varphi = P + jQ = \underline{S}$: complex power

$|\underline{S}| = S = \sqrt{P^2 + Q^2} = VI$

Boucherot's theorem

$$\sum_{m=1}^{N_{gen}} \underline{S}_m = \sum_{m=1}^{N_{gen}} \underline{S}_m \implies \begin{cases} \sum_m P_m = \sum_m P_m \\ \sum_m Q_m = \sum_m Q_m \end{cases}$$

Power factor correction

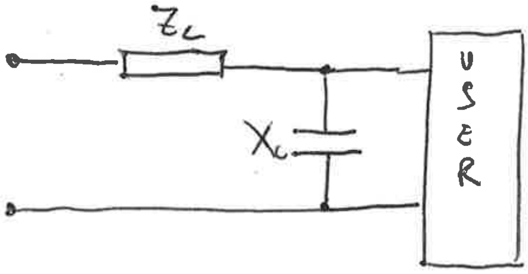
There are two ways for reducing losses in a line:

1. To reduce $R_L = \rho \frac{l}{S}$ → it is possible, but not always feasible from an economic p.o.v.

2. To decrease the current flowing in the line. For decreasing the current, the power factor has to be increased and, as a consequence, the reactive power decreases as well

$$Q\uparrow \implies \varphi\uparrow \implies \cos\varphi\downarrow \implies I\uparrow \implies P\uparrow \implies \eta\downarrow$$

A "Q generator" is a capacitor, put in parallel in the circuit.



In this way the overall reactive power seen by the circuit drops and the efficiency rises.

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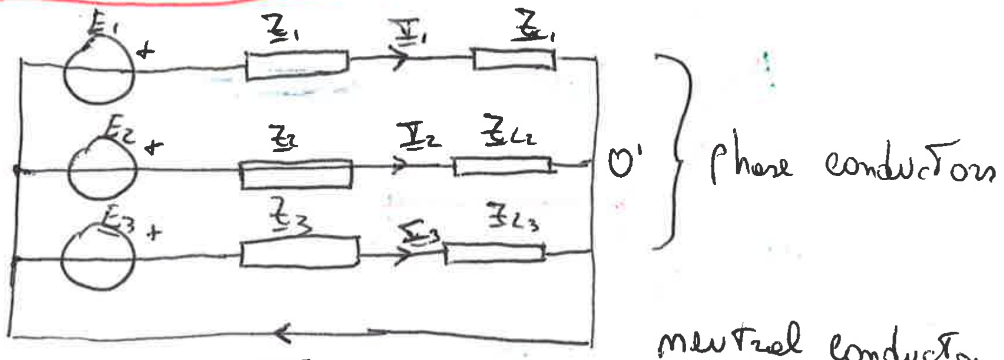
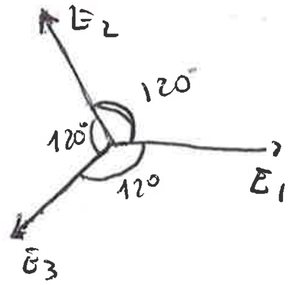
Three-phase circuit



$$\underline{I}_N = \underline{I}_1 + \underline{I}_2 + \underline{I}_3$$

- Symmetric triplet $(\underline{E}_1, \underline{E}_2, \underline{E}_3)$

$$\begin{cases} \underline{E}_1 = E_1 e^{j\varphi_1} \\ \underline{E}_2 = E_2 e^{j\varphi_2} \\ \underline{E}_3 = E_3 e^{j\varphi_3} \end{cases} \longrightarrow \begin{cases} E_1 = E_2 = E_3 = E \\ \varphi_1 = \varphi \\ \varphi_2 = \varphi + 120° \\ \varphi_3 = \varphi - 120° \end{cases}$$



$$\Longrightarrow \underline{E}_1 + \underline{E}_2 + \underline{E}_3 = 0$$

- Balanced three phase system

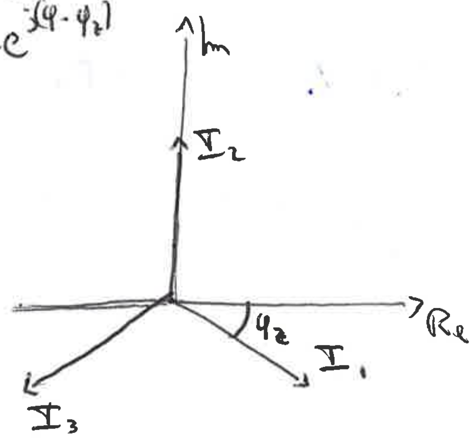
$$\begin{cases} \underline{Z}_{T1} = \underline{Z}_1 + \underline{Z}_{L1} \\ \underline{Z}_{T2} = \underline{Z}_2 + \underline{Z}_{L2} \\ \underline{Z}_{T3} = \underline{Z}_3 + \underline{Z}_{L3} \end{cases} \longrightarrow \underline{Z}_{T1} = \underline{Z}_{T2} = \underline{Z}_{T3} = \underline{Z}_T$$

- Symmetric and balanced three phase system

$$\underline{I}_1 = \frac{\underline{E}_1}{\underline{Z}_1 + \underline{Z}_2} \underset{\text{balanced}}{=} \frac{\underline{E}_1}{\underline{Z}_T} \underset{\text{symmetric}}{=} \frac{\underline{E}}{\underline{Z}_T} = \frac{E \cdot e^{j\varphi}}{Z_T e^{j\varphi_Z}} = \frac{E}{Z_T} \cdot e^{j(\varphi - \varphi_Z)}$$

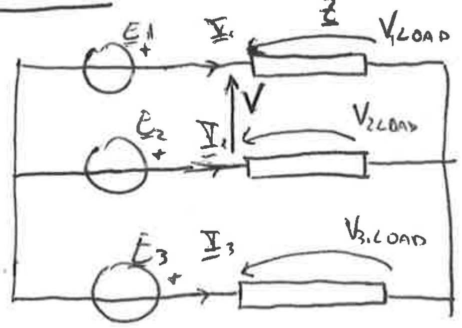
$$\underline{I}_2 = \dots = \frac{E}{Z_T} \cdot e^{j(\varphi + 120 - \varphi_Z)}$$

$$\underline{I}_3 = \dots = \frac{E}{Z_T} \cdot e^{j(\varphi - 120 - \varphi_Z)}$$



$$\Longrightarrow \underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0 = \underline{I}_N$$

Power



Symmetric & balanced

$$\underline{S} = \underline{V}_{1load} \cdot \underline{I}_1^* + \underline{V}_{2load} \cdot \underline{I}_2^* + \underline{V}_{3load} \cdot \underline{I}_3^* = 3 V_{load} \cdot I \, e^{j\varphi_z}$$

$$= \underbrace{3 V_{load} \cdot I \cos\varphi_z}_{P} + j \underbrace{3 V_{load} I \sin\varphi_z}_{Q}$$

$$\Longrightarrow \begin{cases} P = 3 V_{load} \cdot I \cos\varphi \\ Q = 3 V_{load} \cdot I \sin\varphi \end{cases} \xrightarrow[V_{load} = \frac{V}{\sqrt{3}}]{\text{with the line voltage}} \begin{cases} P = 3 \cdot \frac{V}{3} \cdot \sqrt{3} \, I \cos\varphi \\ Q = 3 \cdot \frac{V}{3} \cdot \sqrt{3} \, I \sin\varphi \end{cases}$$

$$\Longrightarrow \begin{cases} P = \sqrt{3} \, V \cdot I \cos\varphi \\ Q = \sqrt{3} \cdot V \cdot I \cdot \sin\varphi \end{cases} ; \quad S = \sqrt{3} \cdot V \cdot I \quad ; \cos\varphi = \frac{P}{S} \Longrightarrow Q = P \, tg\varphi$$

Benefits of three phase system

1. Simpler and more efficient for electrical machines
2. More efficient distribution (lower costs)
3. Constant power

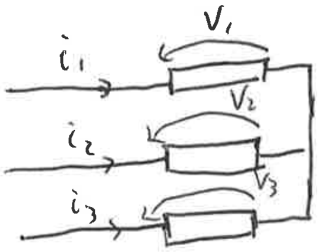
Let's demonstrate (2) and (3)

• (2)

DC, AC-1P and AC-3P are compared

The power delivered to the load (P), the voltage (V), the losses (P_P), the power factor $\cos\varphi$ and the value of each resistance ($R = \rho \frac{\ell}{S}$) are fixed

• (3)



$v_1 = \sqrt{2} \cdot V \, sen(\omega t)$
$i_1 = \sqrt{2} \cdot I \, sen(\omega t - \varphi)$

$V_2 = \sqrt{2} \cdot V sen(\omega t + 120°)$
$i_2 = \sqrt{2} \cdot I \, sen(\omega t + 120° - \varphi)$

$v_3 = \sqrt{2} \cdot V sen(\omega t - 120°)$
$i_3 = \sqrt{2} \cdot I \, sen(\omega t - 120° - \varphi)$

$p(t) = v_1 i_1 + v_2 i_2 + v_3 \cdot i_3$

$= 2VI \, sen(\omega t) sen(\omega t - \varphi) + 2VI \, sen(\omega t + 120°) sen(\omega t + 120° - \varphi) + 2VI \, sen(\omega t - 120°) \cdot sen(\omega t - 120° - \varphi)$

$= \ldots$

$= VI \cos\varphi - VI \cos(2\omega t - \varphi) + VI \cos\varphi - VI \cos(2\omega t - 120° - \varphi) + VI \cos\varphi - VI \cos(2\omega t + 120° - \varphi)$

$\Longrightarrow p(t) = 3 VI \cos\varphi + 0 = $ constant

3 sinusoidal waveforms with the same amplitude and a phase shift of 120°

Chapter 1

Resistive networks

Exercise 1

In the circuit of Fig. 1.1, calculate the power P_A supplied by the current generator A and the power absorbed by the resistor R_3.

Data

Only symbolic calculations are required.

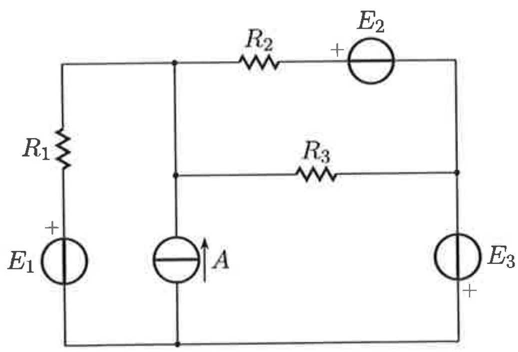


Figure 1.1: Circuit for Exercise 1.

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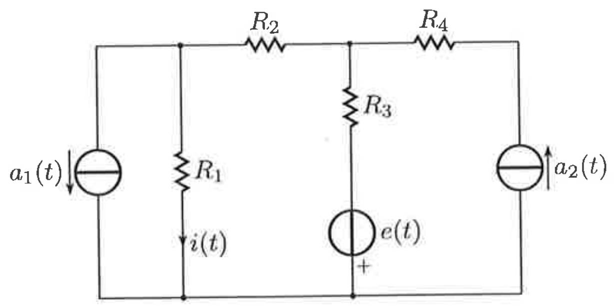

Figure 1.3: Circuit for Exercise 3.

Solution

$$i(t) = -\frac{R_2 + R_3}{R_1 + R_2 + R_3} a_1(t) - \frac{e(t)}{R_1 + R_2 + R_3} + \frac{R_3}{R_1 + R_2 + R_3} a_2(t)$$

Exercise 4

In the circuit of Fig. 1.4, calculate the voltage $v(t)$ applying superposition.

Data

Only symbolic calculations are required.

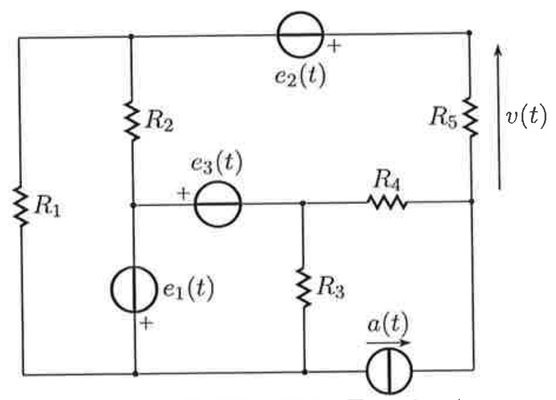

Figure 1.4: Circuit for Exercise 4.

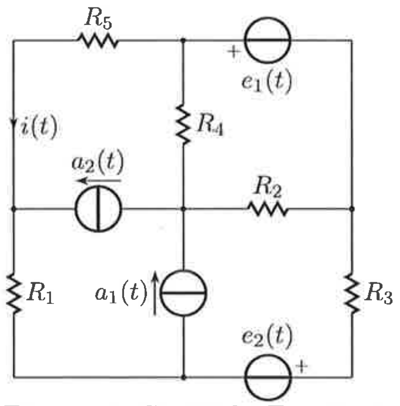

Figure 1.6: Circuit for Exercise 6.

Solution

$$i(t) = \frac{R_3}{R_1 + R_3 + R_5}a_1(t) - \frac{R_1 + R_3}{R_1 + R_3 + R_5}a_2(t) + \frac{e_1(t)}{R_1 + R_3 + R_5} + \frac{e_2(t)}{R_1 + R_3 + R_5}$$

Exercise 7

In the circuit of Fig. 1.8, calculate the voltage $v_1(t)$ across the resistor R_1 and the power supplied by the current generator A.

Data

Only symbolic calculations are required.

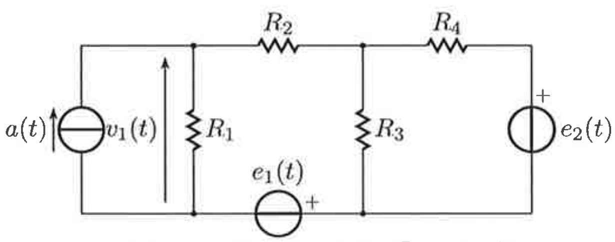

Figure 1.7: Circuit for Exercise 7

Solution

$$\begin{aligned} v_1(t) &= R_1\|(R_2 + R_3\|R_4)a(t) + \frac{R_1}{R_1 + R_2 + R_3\|R_4}e_1(t) \\ &+ \frac{(R_1 + R_2)\|R_3}{(R_1 + R_2)\|R_3 + R_4} \times \frac{R_1}{R_1 + R_2}e_2(t) \\ p_A(t) &= v_1(t)a(t) \end{aligned}$$

Solution

$$R_N = R_3 + (R_1 \| R_2)$$
$$i_N(t) = \frac{R_1 \| R_2}{R_1 \| R_2 + R_3} a(t) + \frac{R_2 \| R_3}{R_1 + R_2 \| R_3} \frac{e(t)}{R_3}$$

Exercise 10

For the circuit in Fig. 1.10, calculate

1. the Thévenin equivalent circuit of the two terminal component enclosed in the dashed box at $A - B$ terminals;
2. the current through the resistor R_5.

Data

Only symbolic calculations are required.

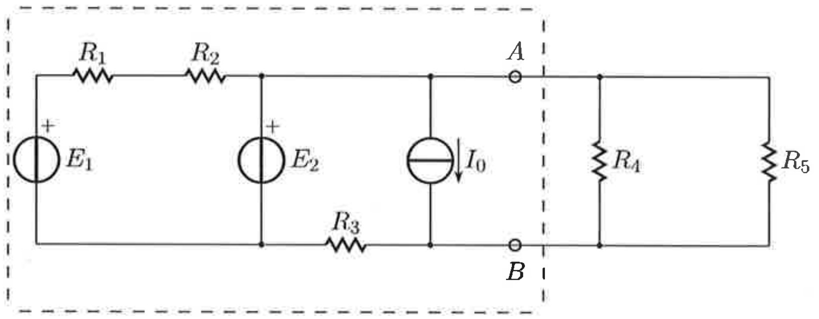

Figure 1.10: Circuit for Exercise 10

Solution

$$R_{TH} = R_3$$
$$V_{TH} = E_2 - R_3 I_0$$
$$I_5 = \frac{1}{R_5} \times \frac{R_4 \| R_5}{R_4 \| R_5 + R_{TH}} V_{TH}$$

Exercise 11

For the circuit in Fig. 1.11, calculate

1. the Thévenin equivalent circuit of the two terminal component enclosed in the dashed box at $1 - 3$ terminals;

Data

Only symbolic calculations are required.

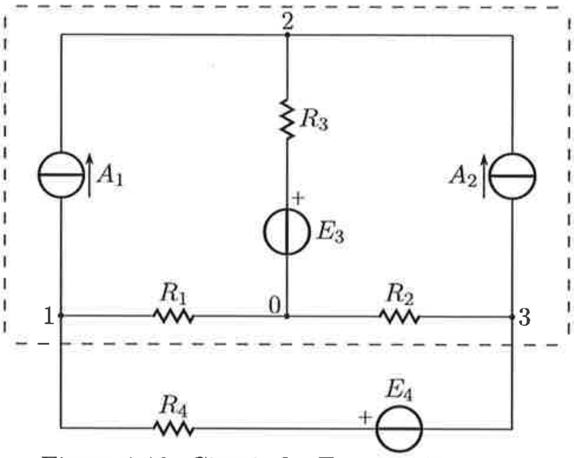

Figure 1.12: Circuit for Exercise 12

Solution

$$R_{TH} = R_1 + R_2$$
$$V_{TH} = V_{13} = -A_1 R_1 + A_2 R_2$$
$$P_4 = E_4 \frac{E_4 - V_{TH}}{R_4 + R_{TH}}$$

$$\begin{bmatrix} G_1 + G_4 & 0 & -G_4 \\ 0 & G_3 & 0 \\ -G_4 & 0 & G_2 + G_4 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix} = \begin{bmatrix} -A_1 + G_4 E_4 \\ A_1 + A_2 + G_3 E_3 \\ -A_2 - G_4 E_4 \end{bmatrix}$$

Exercise 27

In the circuit of Fig. 4.2, the switch T is initially open and the circuit is in steady state condition. At $t = t_0$ the switch is closed and the re-opened at $t = t_1$. Calculate:

1. the analytic expression of the voltage $v_C(t)$ when $t_0 < t < t_1$ and qualitatively plot it;

2. the energy absorbed by the resistor R_3 when $t > t_1$.

Data

$E = 6$ V, $R_1 = 100$ Ω, $R_2 = 300$ Ω, $R_3 = 200$ Ω, $C = 100$ μF, $t_0 = 0$ ms, $t_1 = 15$ ms.

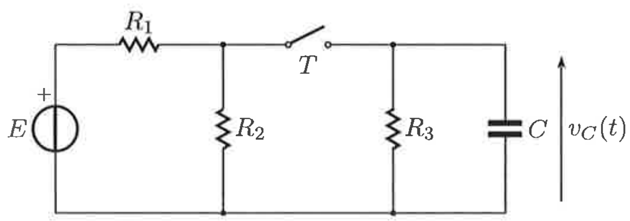

Figure 4.2: Circuit for Exercise 27

Solution

$$v_C(t) = 3.27\left(1 - e^{-\frac{t}{5.45\times10^{-3}}}\right) \text{ V}$$
$$W_{R_3} = 0.469 \text{ mJ}$$

Exercise 28

In the circuit of Fig. 4.3, the switch T is initially open and the circuit is in steady state condition. At $t = t_0$ the switch is closed and the re-opened at $t = t_1$. Calculate:

1. the analytic expression of the voltage $v_C(t)$ when $t_0 < t < t_1$ and qualitatively plot it;

2. the energy absorbed by the resistors R_2 and R_3 when $t > t_1$.

Exercise 33

In the circuit of Fig. 4.8, the switches T_1 e T_2 are initially open. The switch T_1 closes at $t = 0$, while T_2 closes at $t = t_0$. Calculate:

1. the charging and discharging time constants of the inductor;
2. the value of the current through the inductor when $t = t_0$;
3. the analytic expression of the current through the inductor when $t \leq 0$, $0 \leq t \leq t_0$ and $t \geq t_0$

Data

$E = 10$ V, $R_1 = 10\ \Omega$, $R_2 = 30\ \Omega$, $R_3 = 20\ \Omega$, $L = 50$ mH, $t_0 = 3$ ms.

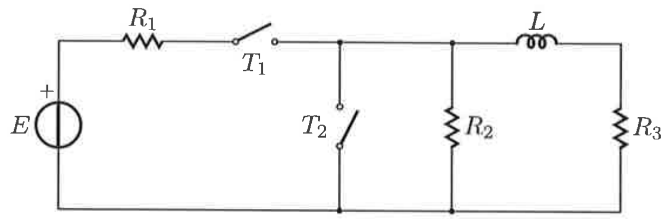

Figure 4.8: Circuit for Exercise 33

Solution

$$\tau_1 = 1.82 \text{ ms}$$
$$\tau_2 = 2.5 \text{ ms}$$
$$i_L(t_0) = 218 \text{ mA}$$
$$i_L(t) = \begin{cases} 0 \text{ A} & \text{when } t \leq 0 \text{ s} \\ 270\left(1 - e^{-\frac{t}{1.82\times10^{-3}}}\right) \text{ mA} & \text{when } 0 \text{ s} \leq t \leq t_0 \\ 218 e^{-\frac{t-t_0}{2.5\times10^{-3}}} \text{ mA} & \text{when } 0 \text{ s} t \geq t_0 \end{cases}$$

Exercise 34

In the circuit of Fig. 4.9, the switches T_1 e T_2 are initially open. The switch T_1 closes at $t = 0$, while T_2 closes at $t = t_0$. Calculate:

1. the charging and discharging time constants of the inductor;
2. the value of the voltage across the capacitor when $t = t_0$;
3. the analytic expression of the voltage across the capacitor when $t \leq 0$, $0 \leq t \leq t_0$ and $t \geq t_0$

Solution

$$W_L = 0.11 \text{ J}$$
$$i_L(t) = 20 - 13.33e^{-\frac{t}{1\times10^{-3}}} \text{ A}$$
$$v_L(t) = 66.66e^{-\frac{t}{1\times10^{-3}}} \text{ V}$$

Exercise 36

In the circuit of Fig. 4.11 the switch T is open and the circuit is in steady state. At $t = t_0$, the switch is closed and then re-opened at $t = t_1$. Calculate the analytic expression of the current $i_L(t)$ when $t_0 < t < t_1$ and plot it on a graph. Calculate the energy absorbed by the resistor R_3 when $t > t_1$.

Data

$E = 10$ V, $R_1 = 1\ \Omega$, $R_2 = 3\ \Omega$, $R_3 = 2\ \Omega$, $L = 100$ mH, $t_0 = 0$ s, $t_1 = 0.3$ s.

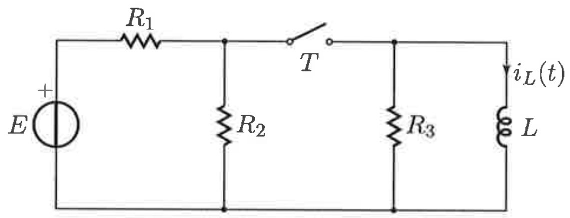

Figure 4.11: Circuit for Exercise 36

Solution

$$i_L(t) = 10\left(1 - e^{-\frac{t}{183\times10^{-3}}}\right) \text{ A}$$
$$W_{R_3} = 3.24 \text{ J}$$

Exercise 37

In the circuit of Fig. 4.12, the switch T is initially open. At $t = t_0$ the switch is closed when the initial current in the inductor is I_0. Calculate:

1. the initial energy W_0 stored in the inductor;
2. the analytic expression of the current $i_L(t)$ when $t \geq t_0$ and qualitatively plot it;
3. the maximum energy W_L stored in the inductor;
4. the current i_{R_1} through the resistor R_1 when $t < t_0$ and $t > t_0$.

Chapter 5

Sinusoidal steady state

Exercise 38

In the circuit of Fig. 5.1, calculate the power absorbed by the resistor R_2.

Data

$R_1 = 2\ \Omega$, $R_2 = 5\ \Omega$, $X_L = 2\ \Omega$, $X_C = -3\ \Omega$, $\underline{E}_1 = 3$ V, $\underline{E}_2 = \text{j}4$ V, $\underline{A} = -\text{j}2$ A.

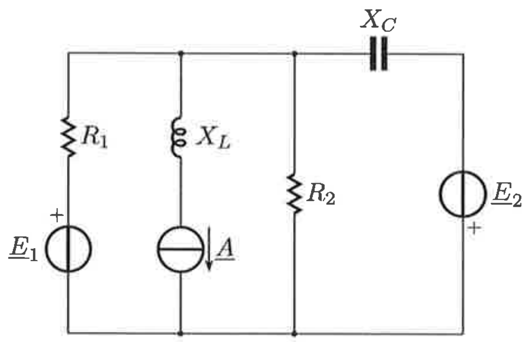

Figure 5.1: Circuit for Exercise 38

Solution

$P_{R_2} = 4.014$ W

Exercise 39

In the circuit of Fig. 5.2, calculate the reactive power absorbed by the inductor of reactance X_{L_2}.

Exercise 41

In the circuit of Fig. 5.4 calculate and draw the Thévenin equivalent circuit at the terminals $A - B$.

Data

Only symbolic calculations are required.

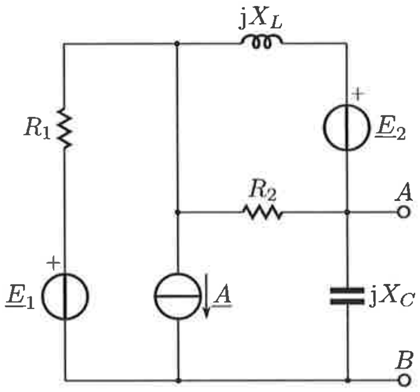

Figure 5.4: Circuit for Exercise 41

Solution

$$\dot{Z}_{TH} = jX_C \| \left[R_1 + (R_2 \| jX_L)\right]$$

$$\underline{V}_{TH} = -\frac{jX_C R_1}{R_1 + jX_C + R_2 \| jX_L}\underline{A} + \frac{jX_C}{jX_C + R_1 + R_2 \| jX_L}\underline{E}_1 - \frac{jX_C R_2}{R_2 + R_1 + jX_C} \times \frac{1}{jX_L + R_2 \| (R_1 + jX_C)}\underline{E}_2$$

Exercise 42

In the circuit of Fig. 5.5 calculate and draw the Thévenin equivalent circuit at the terminals $A - B$.

Data

Only symbolic calculations are required.

Solution

$$\dot{Z}_{\text{eq}} = R_1 + \mathrm{j}X_{C_2} \| \left[R_2 + \mathrm{j}\left(X_{L_1} + X_{L_2}\right)\right]$$
$$\underline{V} = -\frac{E}{\dot{Z}_{\text{eq}}} \times \left[\mathrm{j}X_{C_2} \| \left[R_2 + \mathrm{j}\left(X_{L_1} + X_{L_2}\right)\right]\right]$$

Exercise 49

In the circuit of Fig. 7.2, calculate:

1. the capacitor C_1 to be connected at terminals $A-B$ to obtain a power factor of 0.9;
2. the magnitude of the current I (with C_1 inserted);
3. the capacitor C_2 to be connected at terminals $M-N$ to obtain a power factor of 0.9 (with C_1 inserted).

Data

$P_1 = 10$ kW, $\cos\varphi_1 = 0.5$, $V_1 = 380$ V, $\dot{Z}_L = 1 + j5\ \Omega$, $f = 50$ Hz.

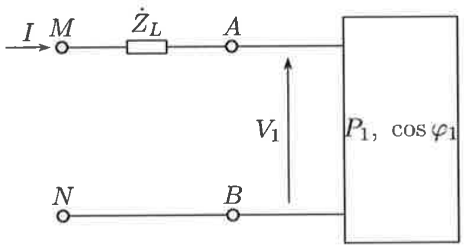

Figure 7.2: Circuit for Exercise 49

$$C_1 = 275\ \mu\text{F}$$
$$I = 29.24\ \text{A}$$
$$C_2 = 52.3\ \mu\text{F}$$

Exercise 50

In the single-phase system of Fig. 7.3, calculate:

1. the magnitude of the current I_A;
2. the magnitude of the current I_B;
3. the current at the beginning of the line I;
4. the voltage at the beginning of the line V.

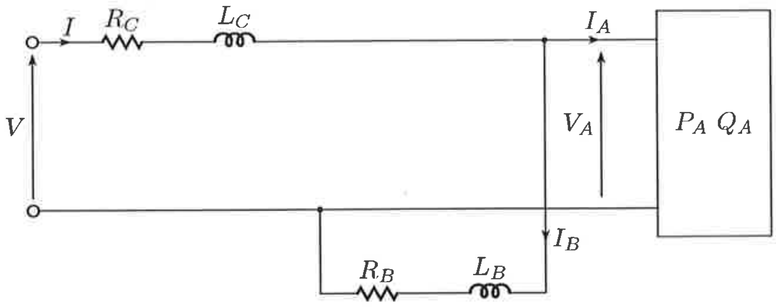

Figure 7.4: Circuit for Exercise 51

$$I_A = 4.18 \text{ A}$$
$$I_B = 2.59 \text{ A}$$
$$I = 4.92 \text{ A}$$
$$V = 222.87 \text{ V}$$

Exercise 52

In the single-phase system of Fig. 7.5, calculate:

1. the magnitude of the current I_3;
2. the magnitude of the current I_A;
3. the voltage at the beginning of the line V_1;
4. the current at the beginning of the line I_1.

Data

$\dot{Z}_L = 1 + \text{j}1\ \Omega$, $V_3 = 600$ V, $P_3 = 7$ kW, $Q_3 = 5$ kvar, $P_A = 5$ kW, $\cos\varphi_A = 0.707$ (inductive).

Figure 7.6: Circuit for Exercise 53

$$I_1 = 138.37 \text{ A}$$
$$P_2 = 48.086 \text{ kW}$$
$$Q_2 = 21.787 \text{ kvar}$$
$$V_2 = 381.52 \text{ V}$$

Exercise 54

In the single phase system of Fig. 7.7, calculate:

1. the magnitude of the current I_A;
2. the real power P_3;
3. the magnitude of the current I_3;
4. the power factor $\cos\varphi_3$.

Data

$P_1 = 10$ kW, $V_1 = 400$ V, $I_1 = 35$ A, $\dot{Z}_L = 0.25 + \text{j}0.3\ \Omega$, $P_A = 5$ kW, $Q_A = 3$ kvar.

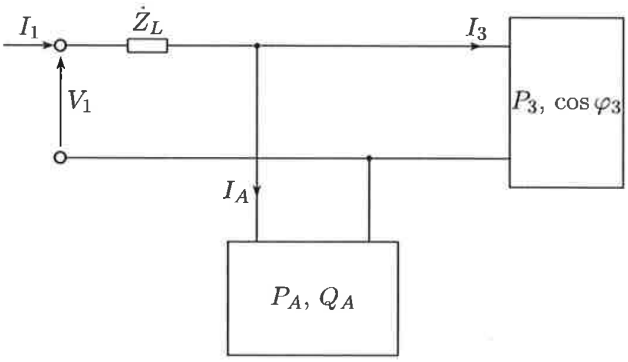

Figure 7.7: Circuit for Exercise 54

Solution

$$I_A = 15.1 \text{ A}$$
$$I_3 = 20.6 \text{ A}$$
$$P_3 = 4.694 \text{ kW}$$
$$\cos\varphi_3 = 0.59$$

Chapter 8

Three phase systems

Exercise 56

In the three phase system of Fig. 8.1, calculate:

1. the magnitude of the line voltage V_3;
2. the magnitude of the current I_2;
3. the magnitude of the current I_3.

Data

$P_1 = 25$ kW, $Q_1 = 15$ kvar, $I_1 = 30$ A, $R_2 = 2\ \Omega$, $L_2 = 10$ mH, $f = 60$ Hz.

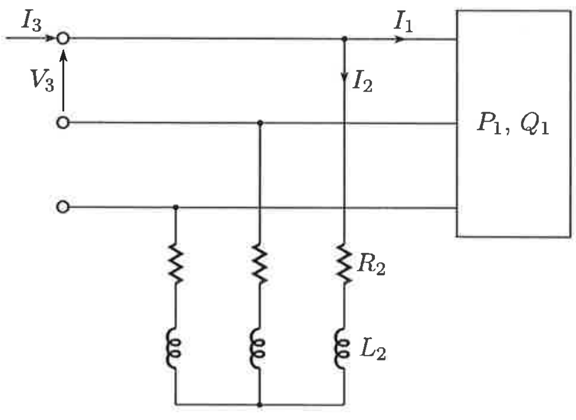

Figure 8.1: Circuit for Exercise 56

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Exercise 58

In the three phase system of Fig. 8.3, calculate:

1. the magnitude of current in the load 1;
2. the supply voltage V_{in};
3. the magnitude of current in the load 2;
4. the supply current I_{in};

Using the value of V_{in}, calculate the current I_{1sc} when the load 1 is replaced by a short circuit.

Data

$V_1 = 400$ V, $P_1 = 30$ kW, $\cos\varphi_1 = 0.6$, $\dot{Z}_L = 0.1 + j0.2\ \Omega$, $P_2 = 10$ kW, $Q_2 = 20$ kvar.

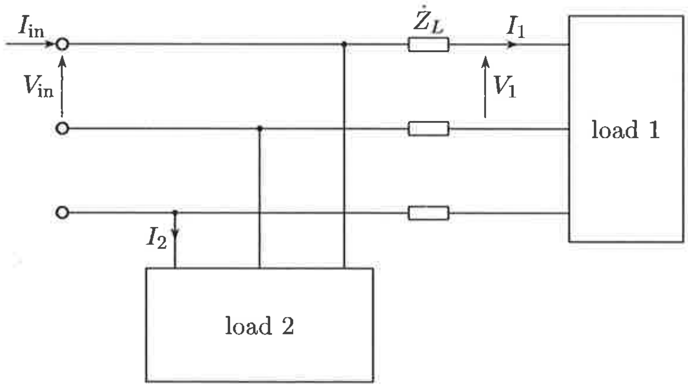


Figure 8.3: Circuit for Exercise 58

Solution

$$I_1 = 72.2 \text{ A}$$
$$V_{in} = 427,35 \text{ V}$$
$$I_2 = 30,22 \text{ A}$$
$$I_{in} = 102,1 \text{ A}$$
$$I_{1sc} = 1,103 \text{ kA}$$

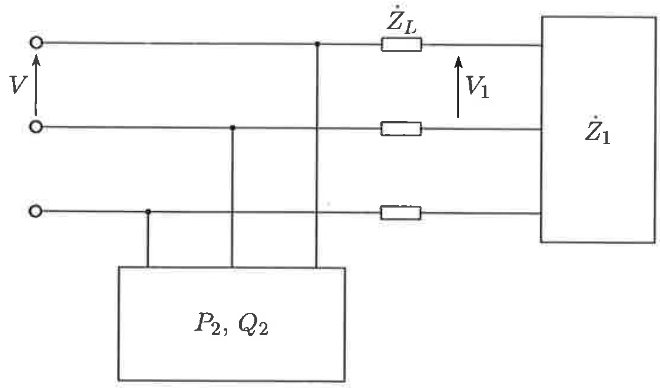


Figure 8.5: Circuit for Exercise 60

Solution

$$V = 396.97 \text{ V}$$
$$\cos\varphi = 0.651$$

Exercise 61

In the symmetric and balanced three phase system of Fig. 8.6, calculate:

1. the magnitude of the current I_A;
2. the magnitude of the current I_B;
3. the voltage at the line input terminals V.

Data

$R_A = 50\ \Omega$, $C_A = 63.7\ \mu\text{F}$, $V_A = 400$ V, $P_B = 1$ kW, $\cos\varphi_B = 0.8$, $R_C = 1\ \Omega$, $L_C = 3.185$ mH, $f = 50$ Hz

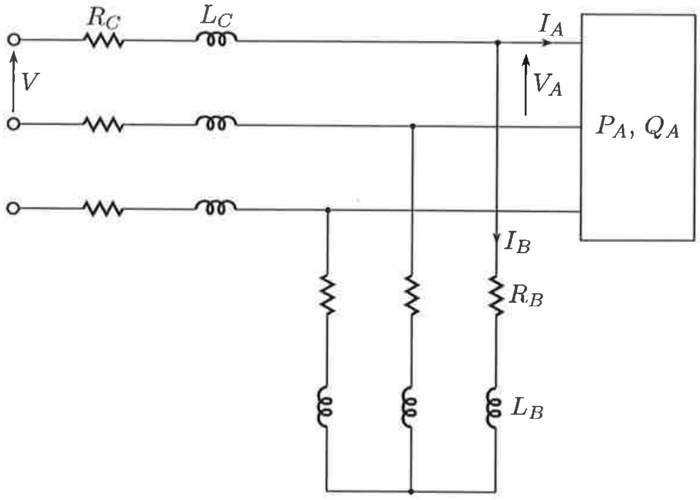


Figure 8.7: Circuit for Exercise 62

Solution

$$I_A = 3.246 \text{ A}$$
$$I_B = 1.796 \text{ A}$$
$$V = 404.4 \text{ V}$$

Exercise 63

In the symmetric and balanced three phase system of Fig. 8.8:

1. draw the equivalent single-phase system;
2. calculate the three line currents absorbed by the three-phase load $\dot{Z}_A$;
3. calculate the three line currents supplied by the voltage generators;
4. the total real power through the section ①.

Data

$\underline{E}_1 = 230$ V, $\underline{E}_2 = 230\angle{-120°}$ V, $\underline{E}_3 = 230\angle{+120°}$ V, $\dot{Z}_{L1} = \text{j}2\ \Omega$, $\dot{Z}_{L2} = 1.5\ \Omega$, $\dot{Z}_A = 20 + \text{j}20\ \Omega$, $\dot{Z}_B = 75 + \text{j}60\ \Omega$.

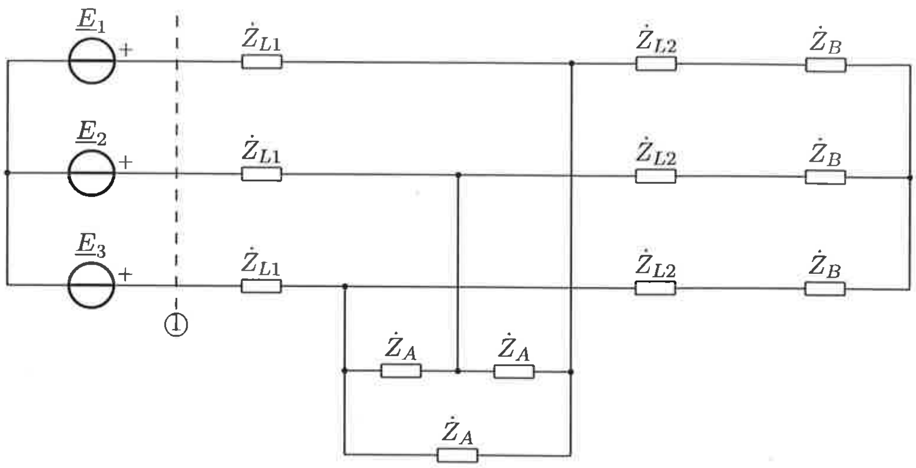


Figure 8.9: Circuit for Exercise 64

Solution

$$I_{1A} = 5.85\angle-49.12° \text{ A}, \quad I_{2A} = 5.85\angle-169.12° \text{ A}, \quad I_{3A} = 5.85\angle70.88° \text{ A}$$
$$I_1 = 13.41\angle-43° \text{ A}, \quad I_2 = 13.41\angle-163° \text{ A}, \quad I_3 = 13.41\angle77° \text{ A}$$
$$P_1 = 6.468 \text{ kW}$$

Exercise 65

In the symmetric and balanced three phase system of Fig. 8.10:

1. draw the equivalent single-phase system;
2. calculate the real power absorbed by the three-phase load $\dot{Z}_A$;
3. calculate the three line currents supplied by the voltage generators;

Data

$\underline{E}_1 = 230$ V, $\underline{E}_2 = 230\angle{-120°}$ V, $\underline{E}_3 = 230\angle{+120°}$ V, $\dot{Z}_L = \text{j}1.3\ \Omega$, $\dot{Z}_A = 51 + \text{j}51\ \Omega$, $\dot{Z}_B = 30 + \text{j}15\ \Omega$.

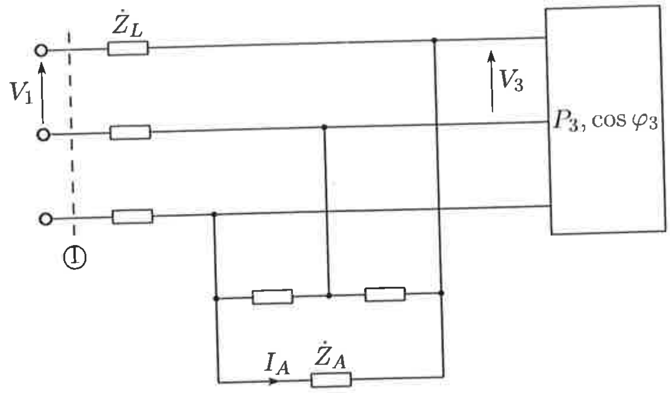


Figure 8.11: Circuit for Exercise 66

$V_3 = 367.95$ V
$I_A = 25.51$ A
$P_3 = 9.440$ kW $\quad \cos\varphi_3 = 0.332$

Exercise 67

Fig. 8.12 represents a symmetric and balanced three phase system. Calculate:

1. the voltage at the beginning of the line;
2. the rms value of the current absorbed by the load A;
3. the rms value of the current absorbed by the load B;
4. the value of the impedance (magnitude and phase) of the load B.

Data

$P_{\text{in}} = 20$ kW, $\cos\varphi_{\text{in}} = 0.57$, $I_{\text{in}} = 45$ A, $P_A = 10$ kW, $Q_A = 15$ kvar.

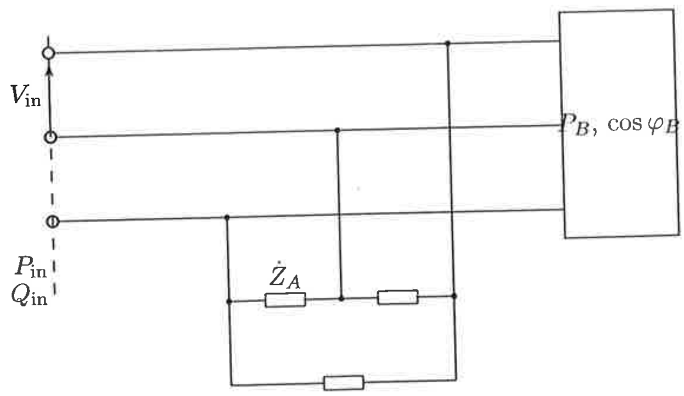


Figure 8.13: Circuit for Exercise 68

$I_{in} = 142.89$ A
$I_A = 80.54$ A
$I_B = 64.81$ A
$P_B = 37.566$ kW $\cos\varphi_B = 0.837$

Exercise 69

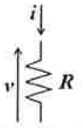
In the three-phase system of Fig. 8.14, calculate:

1. the current at the beginning of the line I_1;
2. the real power absorbed by the load P_2;
3. the supply voltage of the load V_2;
4. the power factor $\cos\varphi_2$.

Data

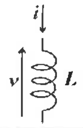
Dati: $V_1 = 406$ V, $P_1 = 50$ kW, $\cos\varphi_1 = 0.89$, $\dot{Z}_l = 0.1 + j0.2$ Ω

CONSTITUTIVE EQUATIONS



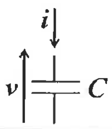
$v = Ri$

$\bar{Z} = R$



$v = L\dfrac{di}{dt}$

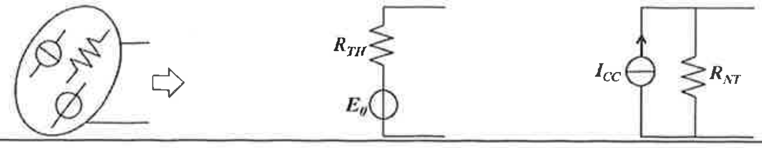
$\bar{Z} = jX_L = j\omega L$



$i = C\dfrac{dv}{dt}$

$\bar{Z} = jX_c = -j\dfrac{1}{\omega C}$

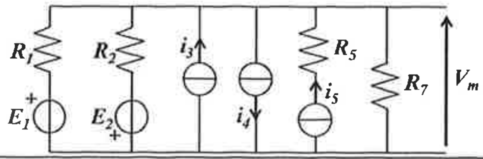
THEVENIN'S AND NORTON'S EQUIVALENT CIRCUIT



$E_0 = R_{TH} I_{CC}$

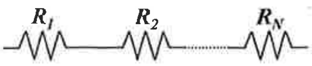
$R_{NT} = R_{TH}$

MILLMAN'S THEOREM

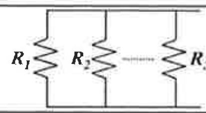


$$V_m = \frac{\frac{E_1}{R_1} - \frac{E_2}{R_2} + i_3 - i_4 + i_5}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_7}}$$

SERIES AND PARALLEL CONNECTION

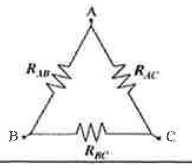


$$R_{eq} = \sum_{k=1}^{N} R_k$$



$$R_{eq} = \frac{1}{\sum_{k=1}^{N} \frac{1}{R_k}}$$

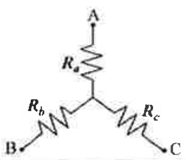
DELTA



STAR-DELTA TRANSFORMATION

$$R_{AB} = \frac{R_aR_b + R_bR_c + R_bR_c}{R_c} \qquad R_{AC} = \frac{R_aR_b + R_bR_c + R_bR_c}{R_b} \qquad R_{BC} = \frac{R_aR_b + R_bR_c + R_bR_c}{R_a}$$

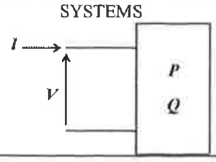
STAR



DELTA-STAR TRANSFORMATION

$$R_a = \frac{R_{AB}R_{AC}}{R_{AB} + R_{AC} + R_{BC}} \qquad R_b = \frac{R_{AB}R_{BC}}{R_{AB} + R_{AC} + R_{BC}} \qquad R_c = \frac{R_{AC}R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

POWER IN SINGLE PHASE SYSTEMS



$P = VI\cos\varphi$

$Q = VI\sin\varphi$

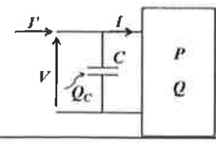
$S = VI = \sqrt{P^2 + Q^2}$

$tg\varphi = \dfrac{Q}{P} \rightarrow \varphi$

$\cos\varphi = \dfrac{P}{S}$

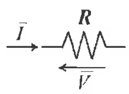
$\sin\varphi = \dfrac{Q}{S}$

POWER FACTOR CORRECTION TO *cosφ'*

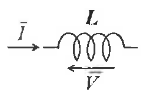


$Q_C = P(tg\varphi' - tg\varphi)$

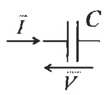
POWER ABSORBED BY TWO TERMINAL COMPONENTS



$P = RI^2 = \dfrac{V^2}{R}$

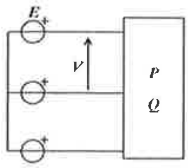


$Q_L = X_L I^2 = \dfrac{V^2}{X_L}$



$Q_C = X_C I^2 = \dfrac{V^2}{X_C} \leq 0$

SYMMETRIC AND BALANCED THREE-PHASE SYSTEMS



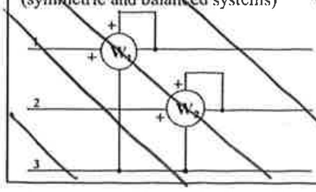
$P = \sqrt{3}VI\cos\varphi = 3EI\cos\varphi$

$Q = \sqrt{3}VI\sin\varphi = 3EI\sin\varphi$

$S = \sqrt{3}VI = \sqrt{P^2 + Q^2}$

$tg\varphi = \dfrac{Q}{P} \rightarrow \varphi$

ARON INSERTION (symmetric and balanced systems)



1st law

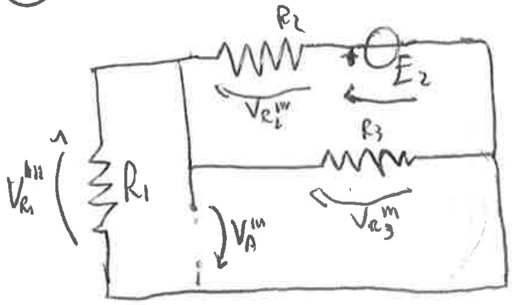
$$\begin{cases} W_1 + W_2 = P \\ \sqrt{3}(W_1 - W_2) = Q \end{cases} \rightarrow \begin{cases} W_1 = \frac{1}{2}\left(P + \frac{Q}{\sqrt{3}}\right) \\ W_2 = \frac{1}{2}\left(P - \frac{Q}{\sqrt{3}}\right) \end{cases}$$

2nd law

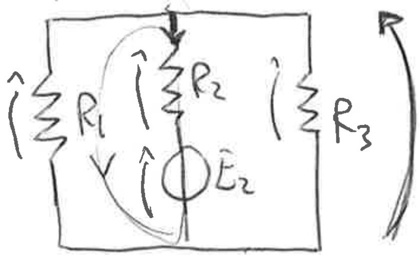
$W = -\dfrac{Q}{\sqrt{3}}$

Relation between phase and line voltages (symmetric systems)	Relation between phase and line currents (balanced loads)
$V = \sqrt{3}E$	$I_{PHASE} = \dfrac{I_{LINE}}{\sqrt{3}}$

(3) $E_1 = 0,\ A = 0,\ E_2 \neq 0,\ E_3 = 0$

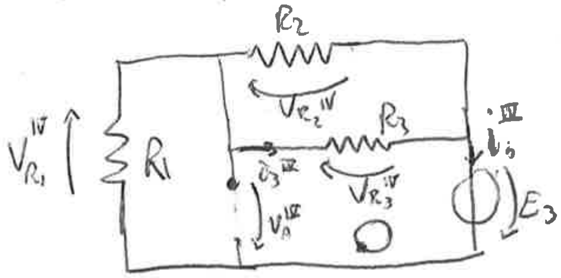


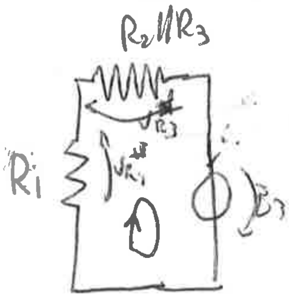
$V_A''' = V_{R_3}'''$



$$V = V_{R_3}''' = \frac{\frac{E_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

(4) $E_1 = 0,\ A = 0,\ E_2 = 0,\ E_3 \neq 0$





$$V_{R_3}^{IV} = + \frac{R_2 // R_3}{R_1 + R_2 // R_3} \cdot E_3$$

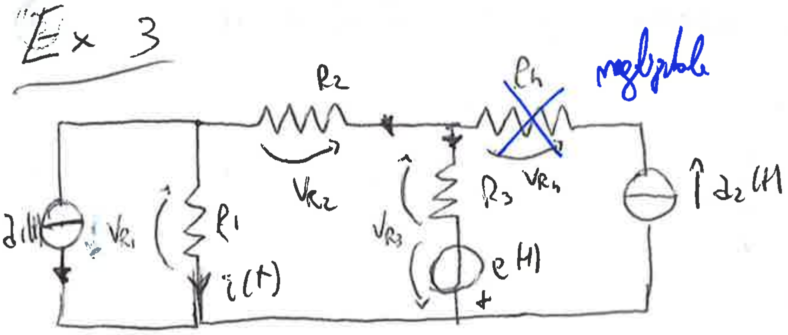
$$V_A^{IV} = V_{R_1}^{IV} = -E + V_{R_3}^{IV} = \frac{R_2 // R_3}{R_1 + R_2 // R_3} E_3 - E_3 = -E_3 \cdot \frac{R_1}{R_1 + R_2 // R_3}$$

$$\Longrightarrow V_A = \frac{R_2 // R_3}{R_2 // R_3 + R_1} \cdot E_1 + R_1 // R_2 // R_3 \cdot A + \frac{R_1 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2} E_2 - \frac{R_1}{R_1 + R_2 // R_3} E_3$$

$$V_3 = \frac{R_2 // R_3}{R_2 // R_3 + R_1} \cdot E_1 + R_1 // R_2 // R_3 \cdot A + \frac{R_1 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2} E_2 + \frac{R_2 // R_3}{R_1 + R_2 // R_3} E_3$$

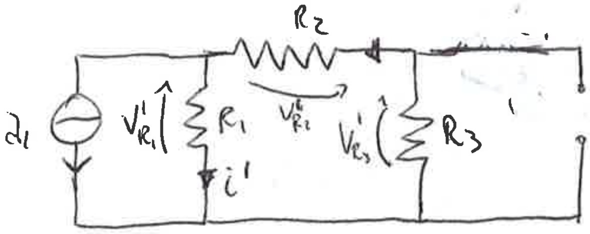
$$\Longrightarrow P_A = V_A \cdot A \qquad P_3 = V_3 \cdot i_3 = V_3 \cdot \frac{V_3}{R_3} = \frac{V_3^2}{R_3}$$

Ex 3



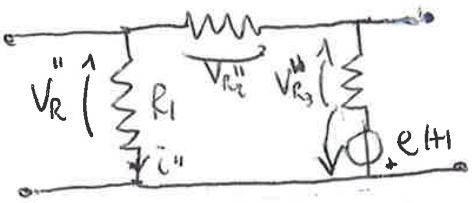
$i(t)$?

① $a_1 \neq 0$, $e = 0$, $a_2 = 0$



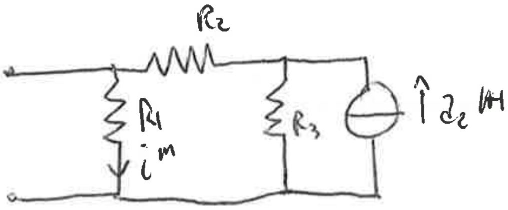
$$i' = -\frac{R_2 + R_3}{R_1 + R_2 + R_3} \cdot a_1$$

② $a_1 = 0$, $e \neq 0$, $a_2 = 0$



$$i'' = -\frac{e}{R_1 + R_2 + R_3}$$

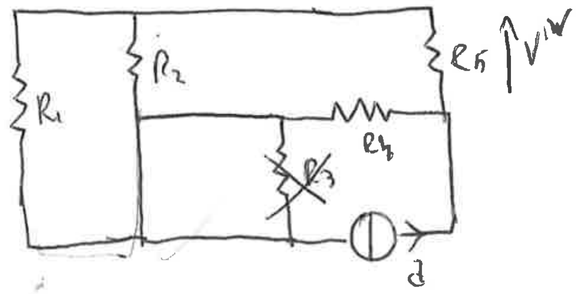
③ $a_1 = 0$, $e = 0$, $a_2 \neq 0$

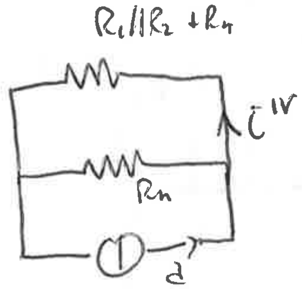


$$i''' = \frac{R_3}{R_1 + R_2 + R_3} \cdot a_2$$

$$i(t) = -\frac{R_2 + R_3}{R_1 + R_2 + R_3} a_1 - \frac{e}{R_1 + R_2 + R_3} + \frac{R_3}{R_1 + R_2 + R_3} \cdot a_2$$

(5) $e_1 = 0,\ e_2 = 0,\ e_3 = 0,\ d \neq 0$

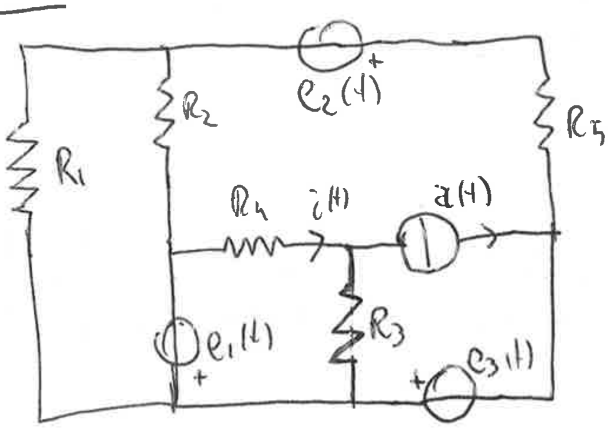




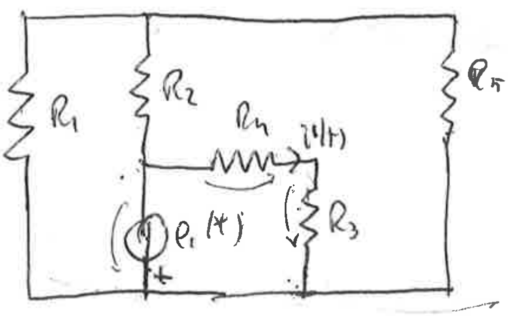
$$i^{IV} = \frac{R_4}{R_4 + R_5 + R_1//R_2} \cdot d$$

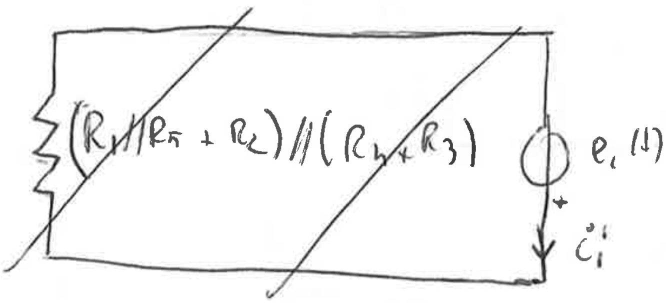
$$V^{IV} = R_5 \cdot i^{IV} = -\frac{R_5 R_4}{R_4 + R_5 + R_1//R_2} \cdot d$$

Ex 5



(1) $e_1 \neq 0,\ e_2 = 0,\ e_3 = 0,\ d = 0$

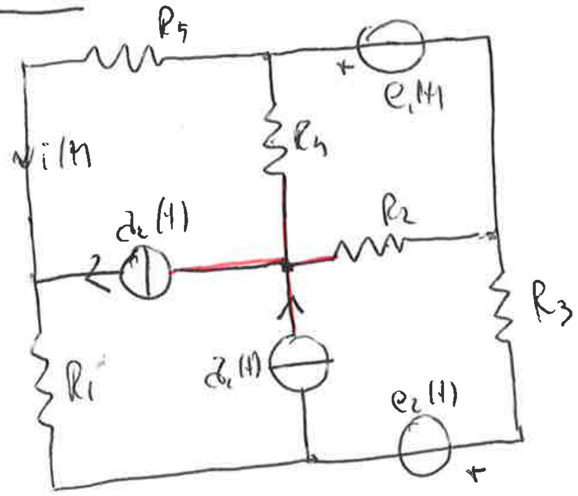




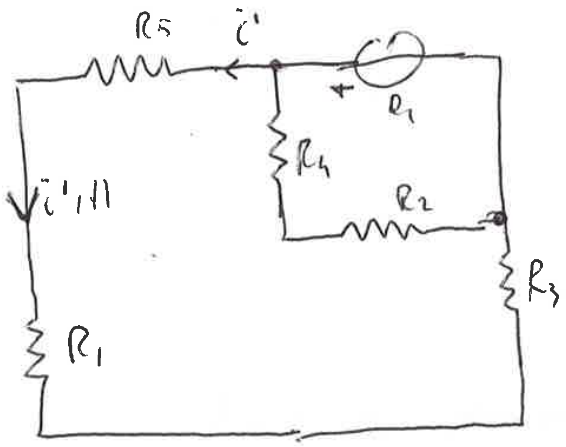
~~$$i_1'(t) = \frac{e_1}{(R_1//R_5 + R_2)//(R_4 + R_3)}$$~~

$$i'(t) = -\frac{e_1}{R_4 + R_3}$$

Ex 6

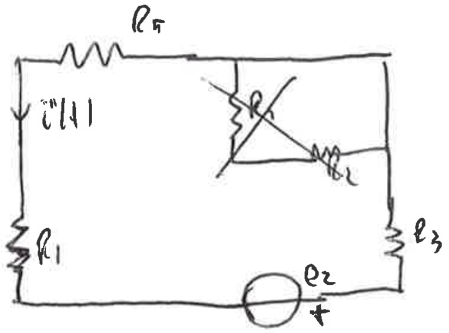


① $e_1 \neq 0$, $e_2 = 0$, $a_1 = 0$, $a_2 = 0$



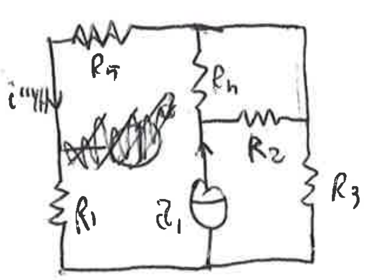
$$i' = \frac{e_1}{(R_5 + R_1 + R_3)}$$

② $e_1 = 0$, $e_2 \neq 0$, $a_1 = 0$, $a_2 = 0$



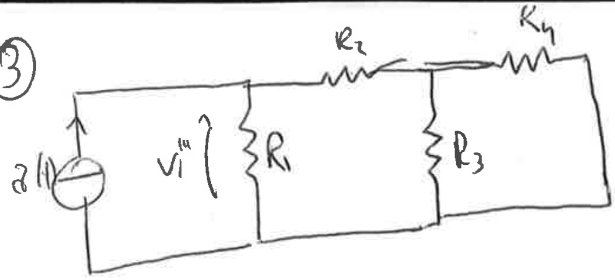
$$i'' = \frac{e_2}{(R_1 + R_3 + R_5)}$$

③ $e_1 = 0$, $e_2 = 0$, $a_1 \neq 0$, $a_2 = 0$



$$i''' = \frac{R_3}{(R_1 + R_5) + R_3} a_1$$

③

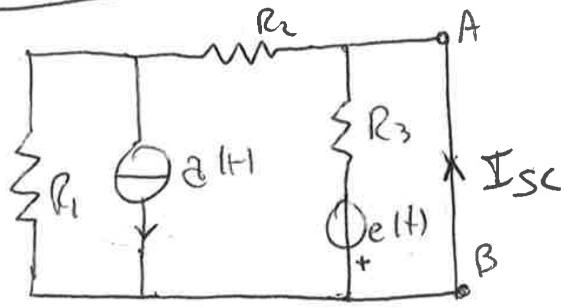


$$V_1''' = a\left(R_1 + R_2 + R_3 // R_4\right)$$

$$V_1 = \frac{R_1}{R_1+(R_2+R_4//R_3)} e_1 + \frac{R_3//(R_1+R_2)}{R_4+R_3//(R_1+R_2)} \cdot \frac{R_1}{R_1+R_2} e_2 + a\left(R_1 // (R_2 + R_3 // R_4)\right)$$

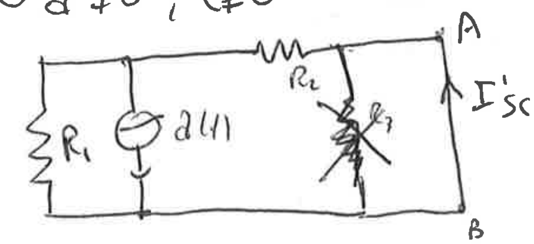
$$P_A = a \cdot V_1$$

Ex 8



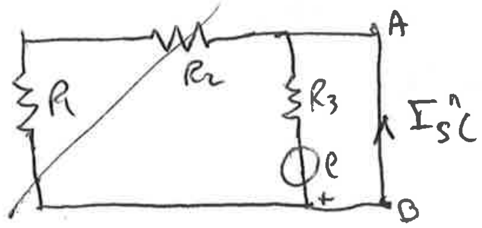
$$R_N = (R_2 + R_1) // R_3$$

① $a \neq 0$, $e = 0$



$$I'_{sc} = \frac{R_1}{R_1+R_2} \cdot a$$

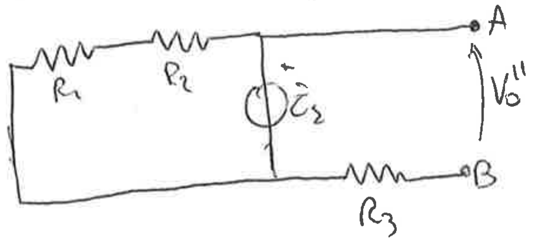
② $a = 0$, $e \neq 0$



$$I''_{sc} = \frac{e}{R_3}$$

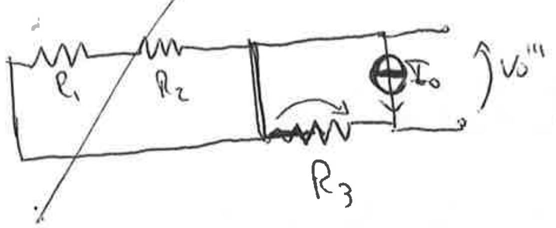
$$I_{sc} = \frac{R_1}{R_1+R_2} \cdot a + \frac{e}{R_3}$$

(2) $E_1 = 0$, $E_2 \neq 0$, $I_0 = 0$



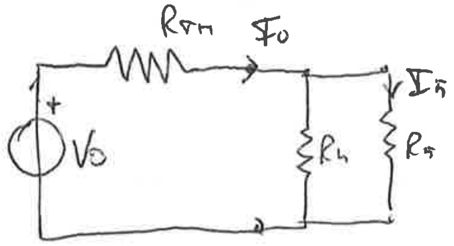
$$V_0'' = E_2$$

(3) $E_1 = 0$, $E_2 = 0$, $I_0 \neq 0$



$$V_0''' = -I_0 R_3$$

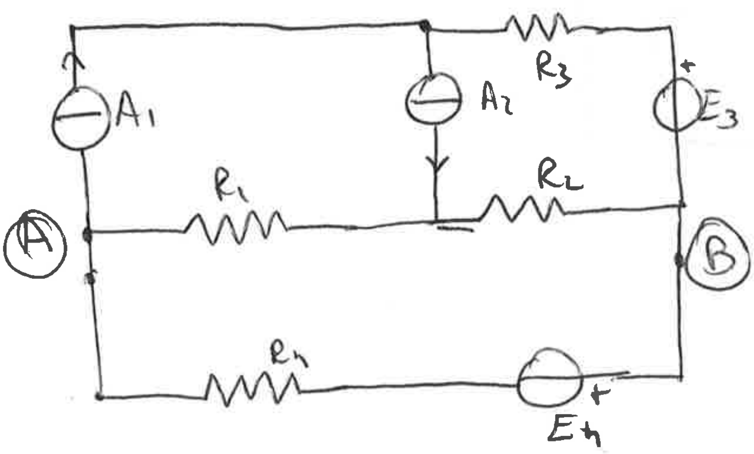
$$V_0 = E_2 - I_0 R_3$$

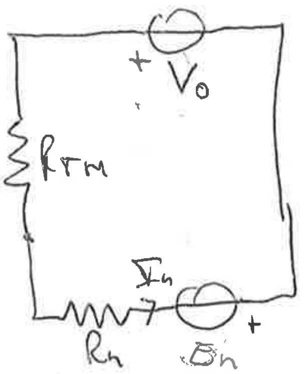


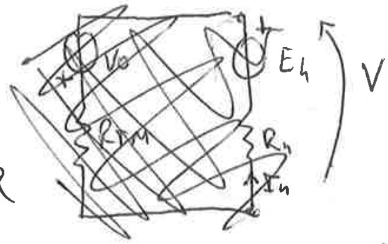
$$I_0 = (R_{TH} + R_4 // R_5)^{-1} \cdot V_0$$

$$I_5 = \frac{R_4}{R_4 + R_5} \cdot I_0 = \frac{R_4}{R_4 + R_5} \cdot (R_{TH} + R_4 // R_5)^{-1} \cdot V_0$$

Ex 11





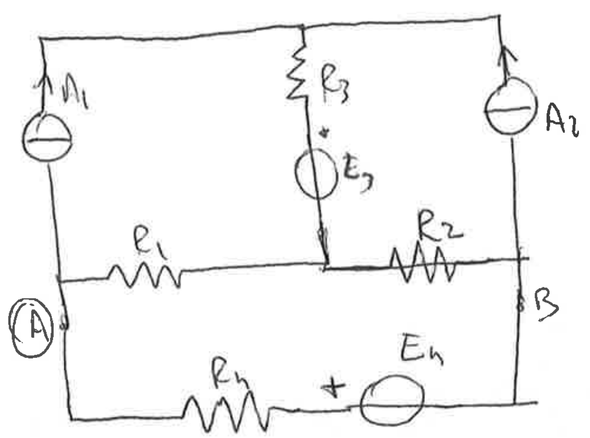


By superposition

$$I_h = \frac{E_h}{R_{TM}+R_h} + \frac{V_0}{R_{TM}+R_h} = \frac{E_h+V_0}{R_{TM}+R_h}$$

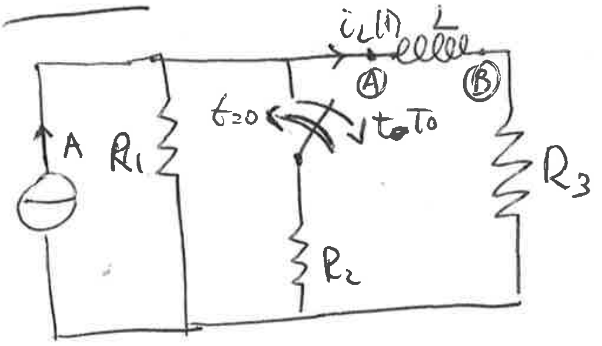
$$P_{abs,E_h} = -E_h \cdot I = -E_h \cdot \frac{E_h+V_0}{R_{TM}+R_h}$$

Ex 12



Single time constant transient

Ex 26



$A = 10\ mA$

$R_1 = R_2 = 1\ k\Omega$

$R_3 = 500\ \Omega$

$L = 5\ H$

$t_0 = 30\ ms$

$t \le 0$

$$i_L(t=0) = \frac{R_1}{R_1 + R_3} \cdot A = 6.67\ mA = I'_0$$

$0 < t \le 30\ ms$

Applying Norton's equivalent circuit

$$R_N = (R_1 // R_2) + R_3 = 1000\ \Omega$$

$$I_{SC} = \frac{R_1 // R_2}{R_1 // R_2 + R_3} A = I'_\infty = 5\ mA$$

$$\tau = \frac{L}{R_N} = 2 \cdot 10^{-6}\ s = 5\ ms$$

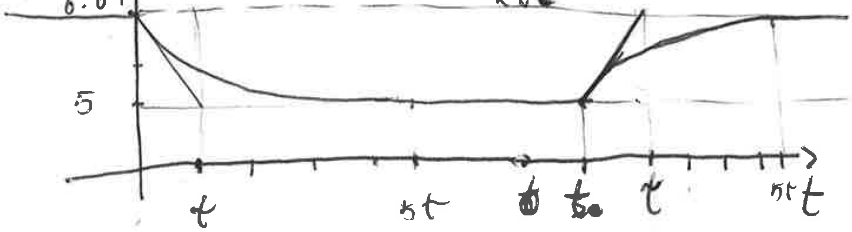
$$i'(t) = (I'_0 - I'_\infty)\, e^{-t/\tau} + I'_\infty = 1.67 \cdot e^{-\frac{t}{5 (ms)}}\ mA + 5\ mA$$

$t > 30\ ms$ (We have already passed $5\tau \rightarrow I''_0 = I'_\infty$) $\quad t^* = t - t_0$

$$I''_\infty = I'_0 \ ; \quad i(t^*) = (I'_\infty - I'_0)\, e^{-\frac{t^*}{\tau'}} + I'_0$$

$$R'_N = R_1 + R_3 = 1.5\ k\Omega$$

$$\tau' = \frac{L}{R'_N} = 3.333 \cdot 10^{-3}\ s$$



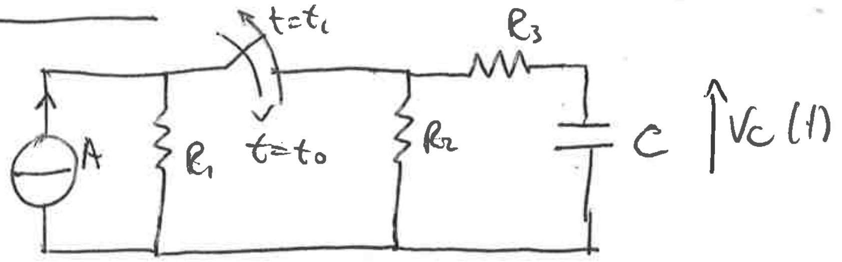
~~$i(t) = \begin{cases} 6.67\ mA & t < 0 \\ 1.67\, e^{-\frac{t}{0.2 (ms)}}\ mA + 5\ mA & 0 < t \\ -1.67\, e^{-\frac{t}{0 (ms)}}\ mA + 6.67\ mA & t > \end{cases}$~~

$V'_\infty = V_{C0} = 0$

(2) $E = \frac{1}{2} C \cdot V_0^2$ The energy absorbed by the resistor R_3 is equal to the energy delivered by the capacitor

$= 4.694 \cdot 10^{-4}$ J

Ex 28



A = 50 mA
$R_1 = 100\ \Omega$
$R_2 = 300\ \Omega$
$R_3 = 200\ \Omega$
C = 100 μF
$t_0 = 0$ ms
$t_1 = 80$ ms

(1) $t \leq 0$

$V_{C0} = 0$

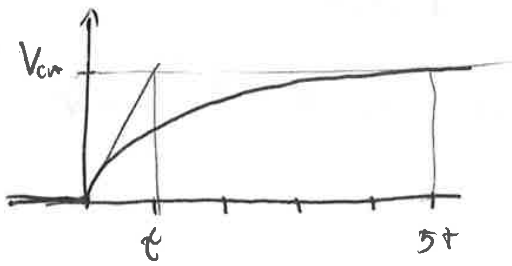
$0 < t \leq t_1$

$R_{TH} = R_3 + R_1 // R_2 = 275\ \Omega$

$\tau = C R_{TH} = 0.0275$ s

$V_\infty = R_1 // R_2 \cdot A = 3.75$ V

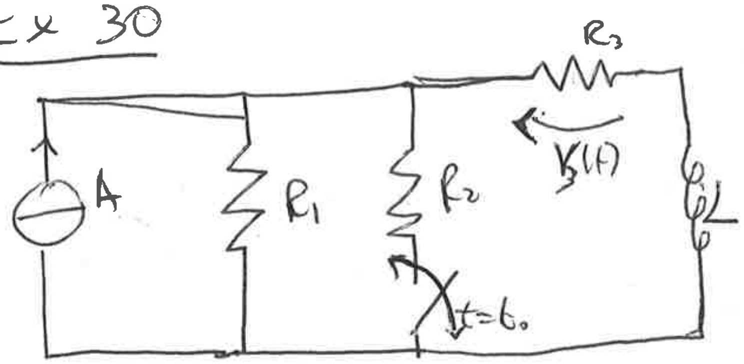
$V(t) = 3.75 \left(1 - e^{-\frac{t}{0.0275 s}}\right)$ V



(2) $V'_{0c} = V(80\ ms) = 3.545$ V

$E_{w_{R_2,R_3}} = \frac{1}{2} C V'^2_{0c} = 0.628$ mJ

Ex 30



$A = 50\ mA$

$R_1 = 300\ \Omega$

$R_2 = 200\ \Omega$

$R_3 = 500\ \Omega$

$L = 5\ mH$

$t_0 = 0\ s$

$t \le t_0$

$V_{30} = R_1 // R_3 // R_2 \cdot A = 4.834$

$t > t_0$

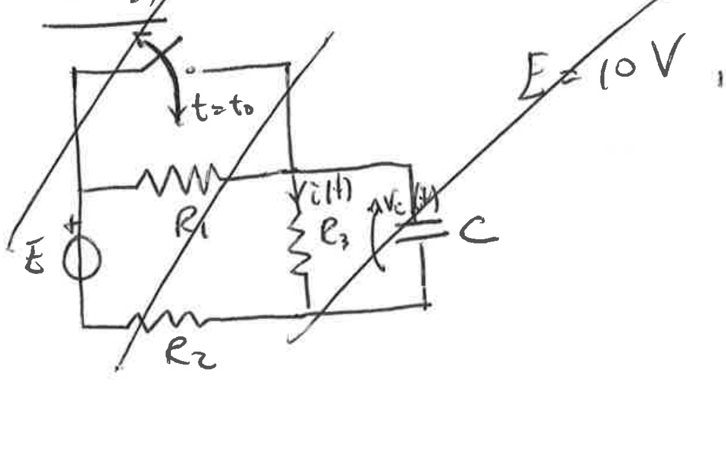
$R_{eq} = R_3 + R_1 = 800\ \Omega$

$\tau = 6.25 \cdot 10^{-6}\ s$

$V_{3\infty} = R_1 // R_3 \cdot A = 9.375\ V$

$v(t) = -4.536\, e^{-\frac{t}{6.25 \cdot 10^{-6}}}\ V + 9.375\ V$

Ex 31



$E = 10\ V$,

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