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A P P U N T I

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PRESTRESSED BOX GIRDER BRIDGES

1 It is the most used kind of bridge nowadays.

2 Static scheme

Up to middle 80s, the prevailing static scheme was the one of simply supported beam or with Gerber scheme. Continuous beam was used only in particular situations.

Then, in coincidence with an international conference, durability problems were discussed in detail and the designers and the owners of bridges understood that the simply supported static scheme of deck was not so convenient because

→ use of materials - it is designed from the mid span section

→ problems with durability.

So, they started to adopt the static scheme of continuous beam, with length up to 1500 m.

ADVANTAGES OF CONTINUOUS BEAM

→ more favorable distribution of internal actions, because maximum bending moment is $q\ell^2/16$ or less, instead of $q\ell^2/8$.

→ REDUCTION OF JOINTS, that implies a reduction of maintenance cost because joints are weak points, especially in zones where salt is used - it penetrates inside the structure through the joints and corrodes reinforcement.

→ smaller deformability with the same span with respect to simply supported beam - $1/3$ less.

→ better behaviour in seismic regions because the horizontal plane is a continuous deck and is able to distribute internal actions into the piers in a more suitable way.

DISADVANTAGES OF CONTINUOUS BEAM

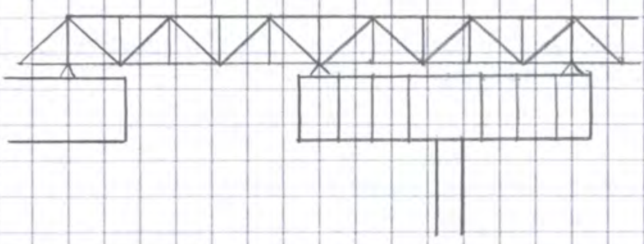
→ in presence of non-compatible imposed deformations - e.g. thermal variations, settlement of the foundation -, internal actions arise. So, we have to take into account of thermal effects and settlements.

this is a problem of the past because nowadays we use improved techniques to realize foundations, pile foundations and conservative methods.

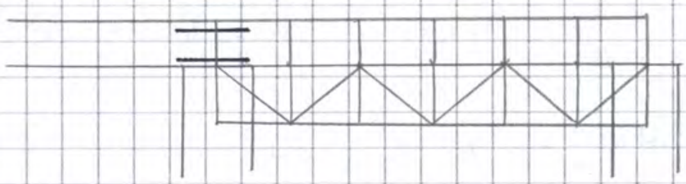
Construction procedures

1 MOST COMMON PROCEDURES

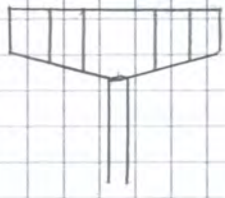
→ precast segments with match cast joints mounted with a launching girder.
The launching girder is supported by the already built part of the bridge and the building part and new segments are transported with a launching crane.
Every segment is cast against the subsequent one and it will be mounted with that order.

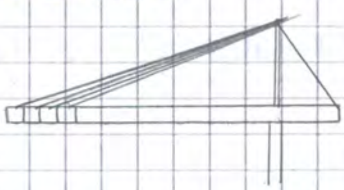


→ precast segments and mounting of full span, with an important truss beam able to support the weight of segments aligned on the beam.
Then, prestressing is applied to connect the span to previous built ones.



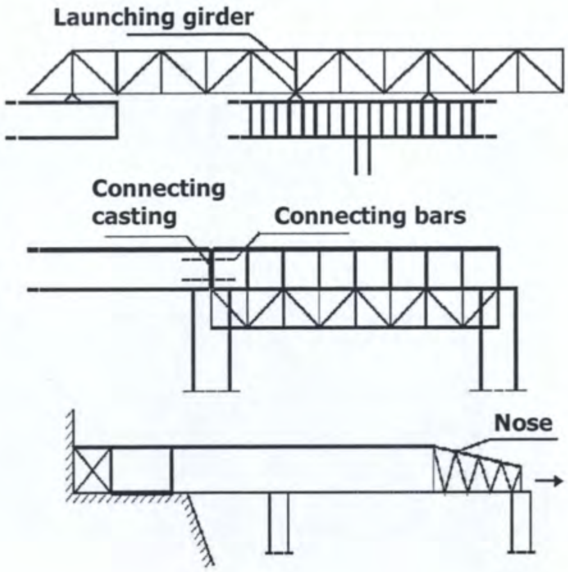
→ cast in situ classical cantilever, segment by segment.
We can use a classical system of cantilevering (BALANCED CANTILEVER), where two segments are cast at the same time on the opposite sides - a segment is about 5 m long.
Maybe, we can adapt temporary tendons to increase span and they will be removed at the end of construction.

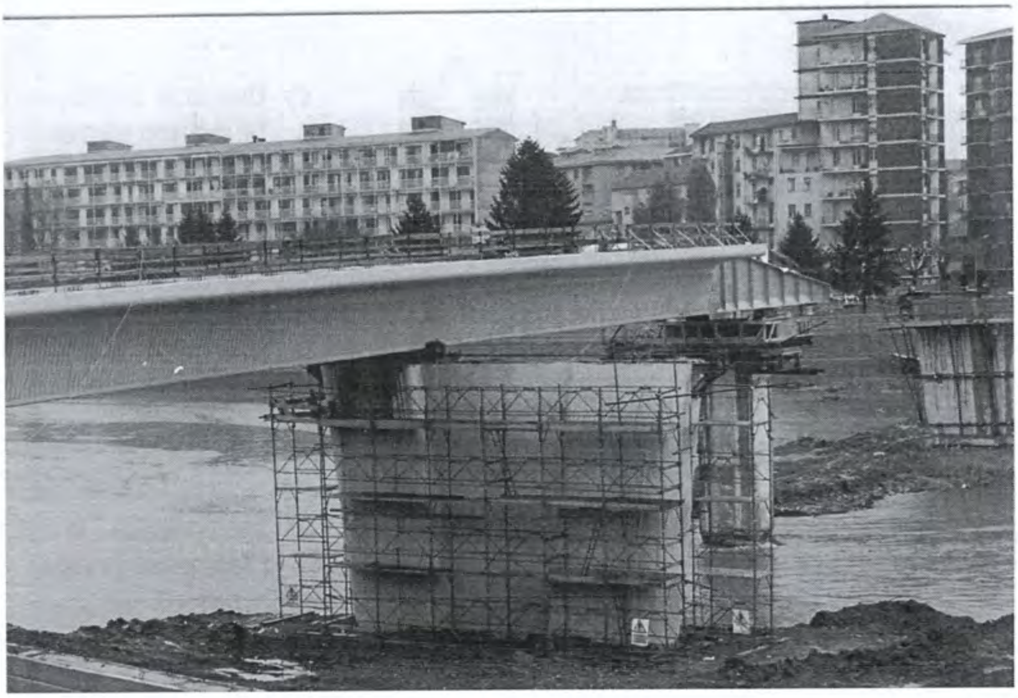




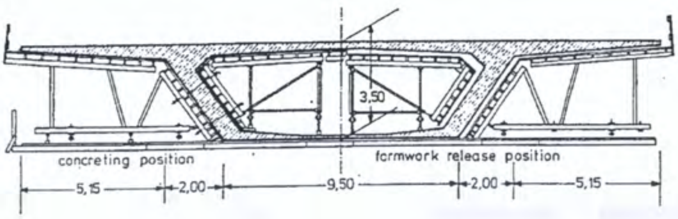
1.2 MOST COMMON CONSTRUCTION PROCEDURES

- Precast segments with match cast joints mounted with launching girder (eventually with temporary stays)
- Precast segments and mounting of full span, in a second time connected to previous built spans
- Cast in situ classical cantilever
- Incremental launching of elements with match cast joints



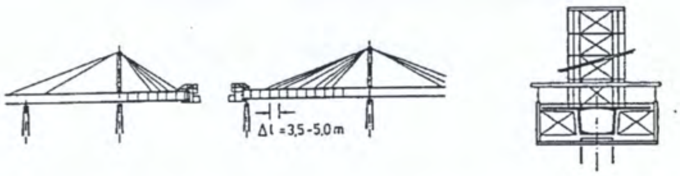


From T. Schlaich and Scheef



Formwork for box girder bridges

a) Classical balanced cantilever (Rhine Bridge)



b) Classical cantilever with auxiliary stays (Lahntal Bridge)

Launch of scaffolding	30 ÷ 60 m	500 ÷ 1000 m	15 ÷ 40 m/week
Incremental launching	40 ÷ 80 m	200 ÷ 600 m (over friction forces too much big	15 ÷ 30 m/week

2 Extimation of material quantities

When we define this type of bridge, we have to do a first extimation of the quantities in order to have an idea of costs

→ CONCRETE:

it is defined as an equivalent thickness of the slab, i.e. all section is smelled in one plate

$$t_m\,[m^3/m^2] = 0{,}35 + \frac{0{,}45 \cdot \ell\,[m]}{100}$$

↓

m^3 to realize $1\ m^2$ of deck

→ PRESTRESSING STEEL

$$A_p\,[kg\,m^{-2}] = 4{,}5 + 0{,}5 \cdot \ell\,[m]$$

→ ORDINARY REINFORCEMENT, which is assumed with a constant value in every case.

$$A_s = 150\ kg\,m^{-3}$$

2 Longitudinal shape of the section

The deck depth can remain uniform along the span up to length of 90 m, adopting the ratio

$$\frac{\ell}{d} \leq 20 \div 25 \quad , \quad \ell \leq 90\,m$$

Generally, near the piers we have to increase the depth of the bottom slab because there negative bending moment is significant and the width of the bottom slab is smaller than the one of the top slab along the span. This aspect should be considered for spans beyond 60 m.

In case of VARIABLE DEPTH, the ratio between maximum depth d_s - in the piers - and minimum depth d_m - in the midspan - should vary within the range

$$\frac{d_s}{d_m} = 1,5 \div 3$$

→ $d_s/d_m \geq 1,5$ due to aestetychs

→ $d_s/d_m \leq 3$ because it is not a cantilever.

How can we choose correctly the variation of depth?

Firstly, we notice that webs thickness is constant. The reason is that, during construction, the internal formwork should be able to be closed and then extracted. To avoid too much complication, at least one dimension should remain constant and, to have variable depth, we are obliged to have large excursion in the vertical plane and small excursion in the horizontal plane - about 1 cm -, in order to extract the formwork.
It means that <u>webs should be designed with constant thickness.</u>

So, if webs have constant thickness, how can we choose the variable shape?

In a constant depth deck, the engagement for shear is constant if shear is constant and bending moment varies linearly.
The same concept is applied to variable depth:

bending moment is a couple of forces in the compressed and tensed chord and the shape of section should be chosen so that the variation of these forces due to bending moment is linear, taking into account that lever-arm is increasing or decreasing. This is equivalent to have CONSTANT ENGAGEMENT FOR SHEAR.

Applying this concept, it results that

$$\frac{\ell}{d_s} = 12 \div 24 \qquad \frac{\ell}{d_m} = 18 \div 72$$

→ top slab

$$\underset{\downarrow}{20\text{ cm}} < t_3 \quad , \quad \frac{\ell_3}{30} \leq t_3$$

better 25 ÷ 26 cm due to durability

→ web

$t_4 \geq 30$ cm → space for tendon

→ bottom slab

$t_5 \geq 15$ cm → 20 ÷ 25 cm for durability

4 Shear keys

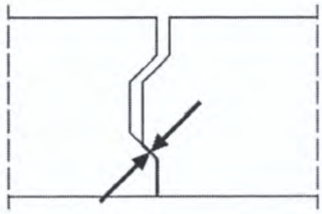
A typical construction procedure is cantilever with cast in situ elements, where segments are cast one against the other.
The external formwork is opened ~~and~~ laterally and we put in position the reinforcement cage on the internal formwork.
Then, the external formwork is closed, we insert the tubes for tendons and we close all with two fronts, typically made in steel plates.
Once we have cast the segment, we can open it after about 16 h, because it needs a compressive strength of about 100 ÷ 120 kg cm^{-2} to self-support and avoid cracking.
At this point, this segment is translated and a new segment is cast, using the ~~first~~ previous one as formwork on the frontal face. Thus, the second segment will be coupled with the first one and so on.
Segments will be numbered and mounted following the casting order, in order to have the best contact between them.

The contact surface is not smooth but presents shear keys, realized in the slabs and in the web:

before putting segments in contact, we put a layer of epoxide resine along the contact surface, with a thickness of about 0,5 cm, then segments are coupled to each other by means of temporary prestressing - bars that exert a pressure of about 2 kg cm^{-2}.
Shear keys ensure firstly the correct positioning of segments and epoxide resin works like a lubrificant in order to avoid direct coupling between segments because they have differences - even if cast together. Moreover, epoxide resine grants the sealing of the joint, protecting it from water, and makes the contact not localized and without pressure concentration, avoiding cracks.

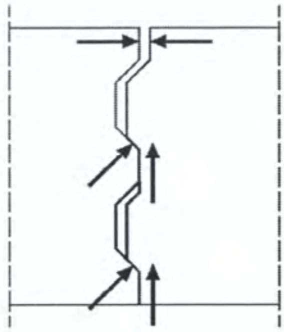
At the ultimate limit state:

- Ensure the continuity of the top slab, even when the joints are open



- Ensure the equilibrium for shear and torsion by means of a SHEAR FRICTION mechanism, in regions in which the joints (crossed by bonded tendons) are open

- Ensure the equilibrium for shear and torsion in regions in which the joints are closed but the friction is not enough



1.6 PRECAST SEGMENTAL CONSTRUCTION

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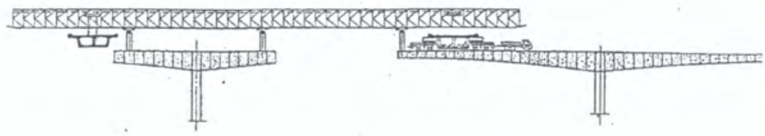
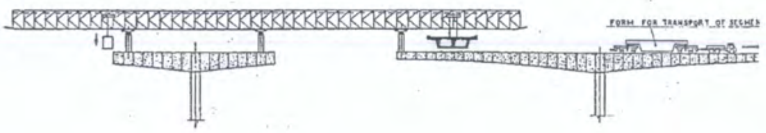
So, joints are firstly sealed, then groutin is performed at the end.

Focusing on the machinery, the girder is supported by two subsequent piers.
The segment, brought by a truck, is uplifted through a mobile crane mounted on the launching girder, rotated, transferred and then put in position on the pier.
When it is put in position, the girder is moved a bit in order to put its leg on the segment and stabilize it.
The process continues with alternate installation of segments on the two sides.
At a certain point, we are too far with the launching girder to arrive to mount other segments. So, the girder is moved longitudinally and it is partially carried by the already built hammer.
Finally, after the alignment, the span is closed with the cast in situ key segment.

Precast segmental construction 6-7

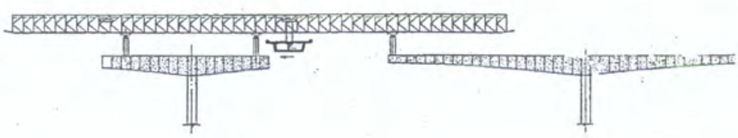


5) Left segment before the rotation
6) Positioning for the translation of right segment

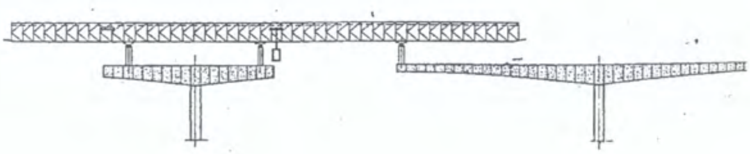


7) Rotation and launch of left segment
8) Uplift of right segment

Precast segmental construction 7-7



9) Assembling of left segment
10) Launching of right segment



11) Rotation and launch of right segment

OBSERVATION: In the incremental launching, friction plays an important role.
If we use steel plates with teflon inside to reduce friction, the friction coefficient is 0,03 at the first movement, then decreases to 0,02.
Actually, cast in situ segments present an irregular surface and, during the movement, we should assume a coefficient

$$\delta = 0{,}08 \div 0{,}09$$

It means that 10% of the weight is applied as a horizontal force.

2 GEOMETRICAL LIMITATIONS

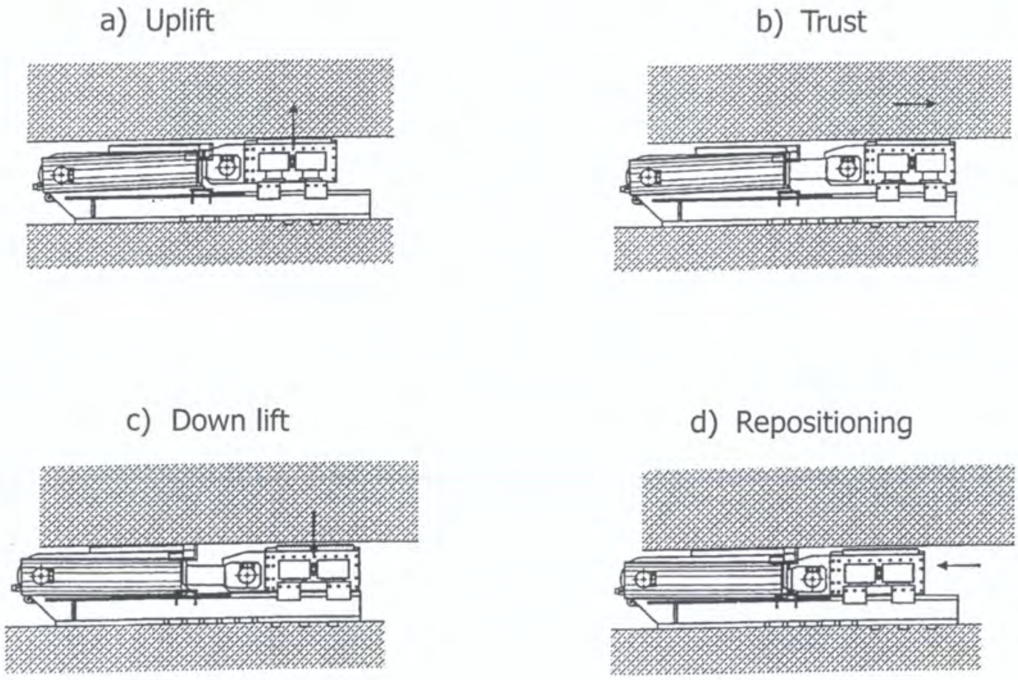
→ in the vertical plane, the deck should be horizontal or stay in a circular line or have a linear inclination, i.e. assume a constant slope.

→ in the horizontal plane, the deck should be straight or circular

These limitations are not independent

In vertical plane	In horizontal plane
Horizontal	Straight or circular
Circular	Straight
Circular	Circular
Linear inclination	Circular

If the deck is circular in the vertical plane, ~~it~~ it can be straight in the ~~verti~~ horizontal plane in order to cross the piers with the same section.
In the last two cases, the projections on the horizontal plane are ellypses and so the same segment will not cross the piers in the same position, but there is a lateral displacement and we need more space in piers to support the bridge.
For this reason, the circular + circular combination is avoided.



Geometrical limitations:

In vertical plane	**In horizontal plane**
horizontal	straight or circular
circular	straight
linear inclination	circular
circular	circular

In the last two cases the projections on the horizontal plane are ellipse circles

Incremental launching - Tiziano Bridge – Alessandria 1-5

Incremental launching - Tiziano Bridge – Alessandria 2-5

9-1

Incremental launching - Tiziano Bridge – Alessandria 5-5

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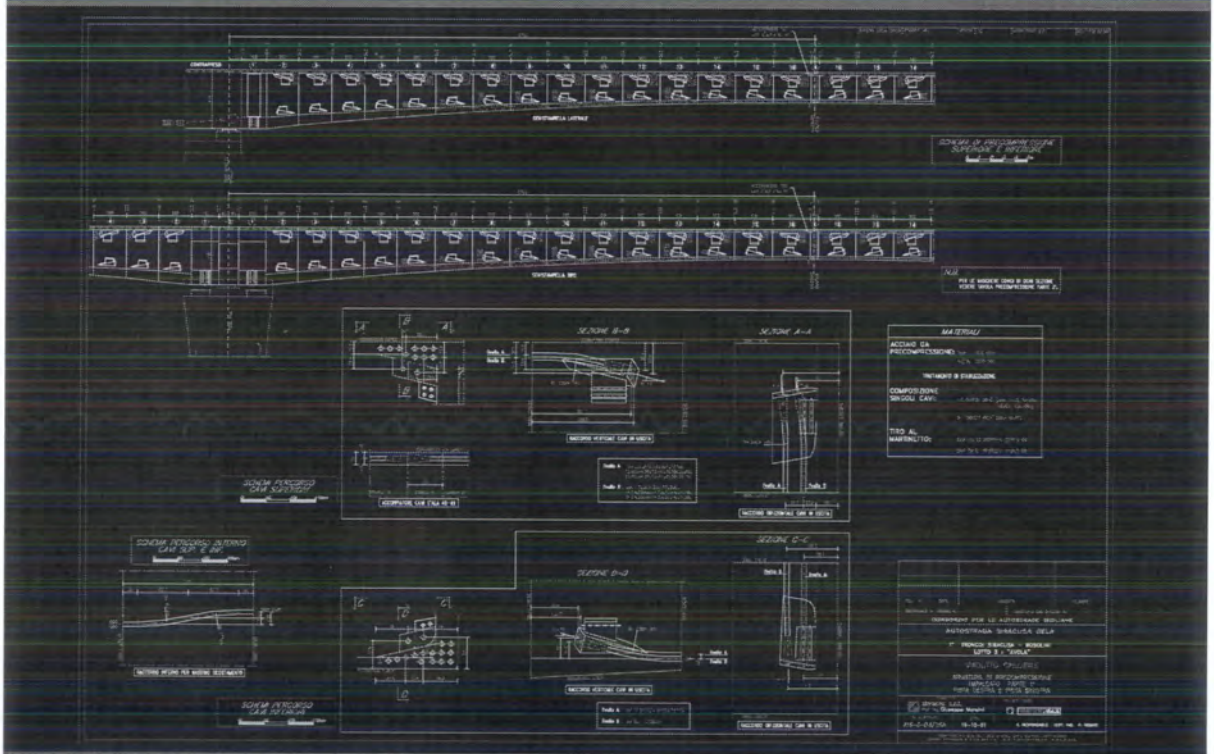
9-1

Executives 1-13



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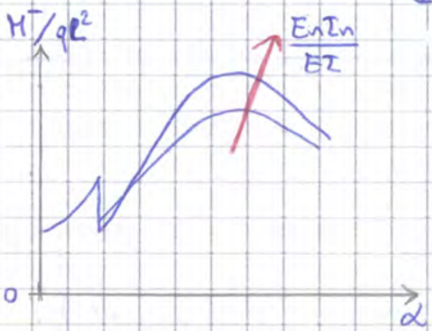
and we can distinguish 2 situations

→ nose cantilevering : the nose has not reached the pier yet and it is in an isostatic situation (cantilever)

$$0 \leq \alpha \leq 1 - \frac{L_n}{L}$$

→ nose on the pier and the situation is the one of continuous beam

$$1 - \frac{L_n}{L} \leq \alpha \leq 1$$



If we evaluate bending moment in bearing B for different positions of the nose, i.e. in function of α, bending moment increases in its value at the beginning, as we are in a cantilevering situation. The increase follows a parabolic law.
Then, there is a discontinuity when the next pier is reached and then bending moment increases again.

This diagram is defined for

$$\frac{L_n}{L} = 0,80 \text{ (not } 0,65) \qquad \frac{q_n}{q} = 0,10$$

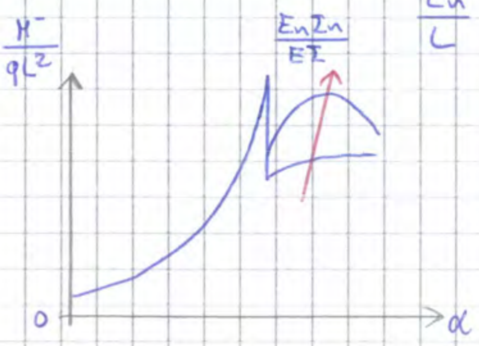
and it presents curves with different ratios of rigidity. The upper curve corresponds to a nose with lower rigidity.

We can notice that, after the launching, there is a strong increase of negative bending moment and bending moment in B is bigger during the stage in which the deck works as a continuous beam than in the one in which it is cantilevering.
We have to design for the maximum and, to make a good design, we should reduce one value and increase the other one

⇒ we modify the ratio L_n/L

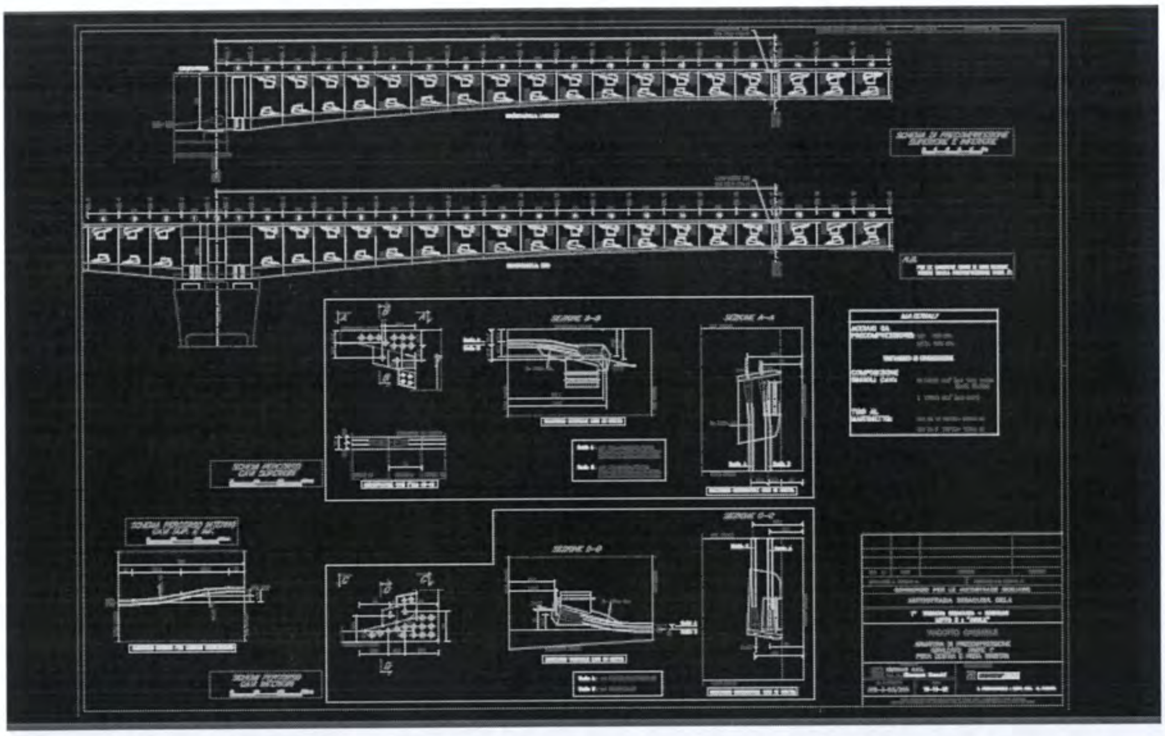
If we assume a ratio

$$\frac{L_n}{L} = 0,50 \quad \rightsquigarrow \text{ 50\% of bridge is cantilevering (short nose)}$$

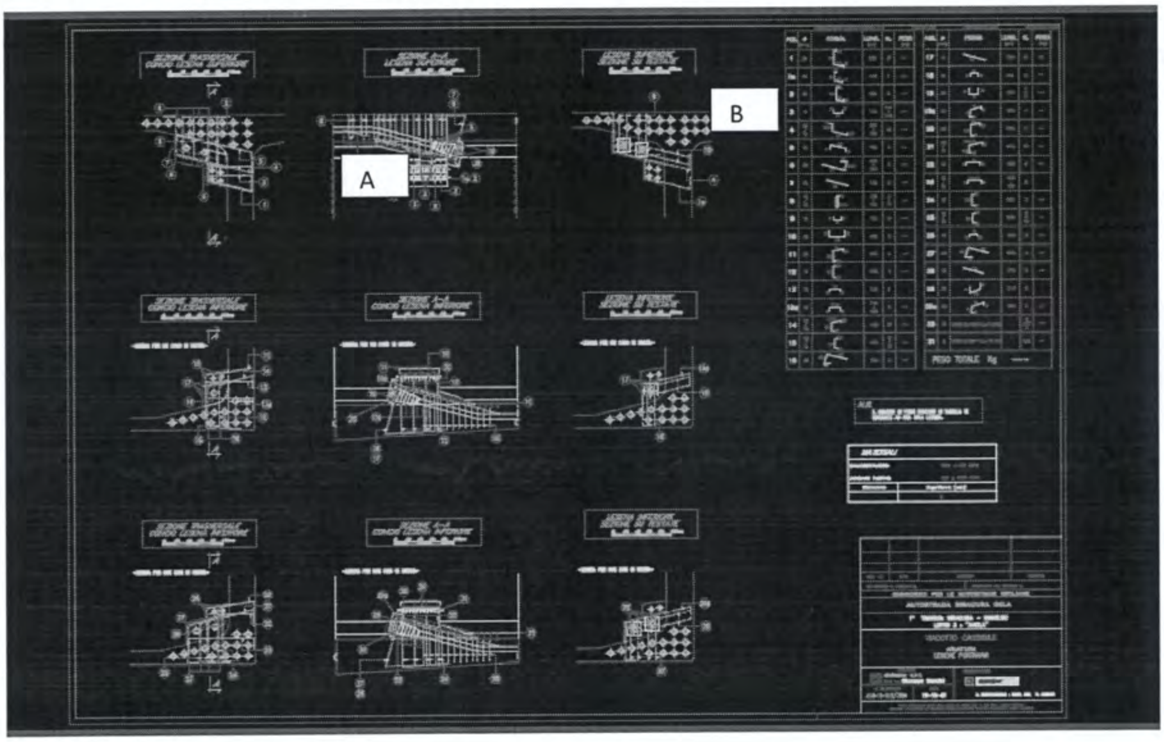


In this case, during cantilevering, negative bending moment is very high and it is smaller when the pier is reached.

DETAILS ON PRECAST SEGMENTS



The image represents the geometry of a classical hammer, in which we can see some little blocks, that contain the anchors for the temporary tendons.



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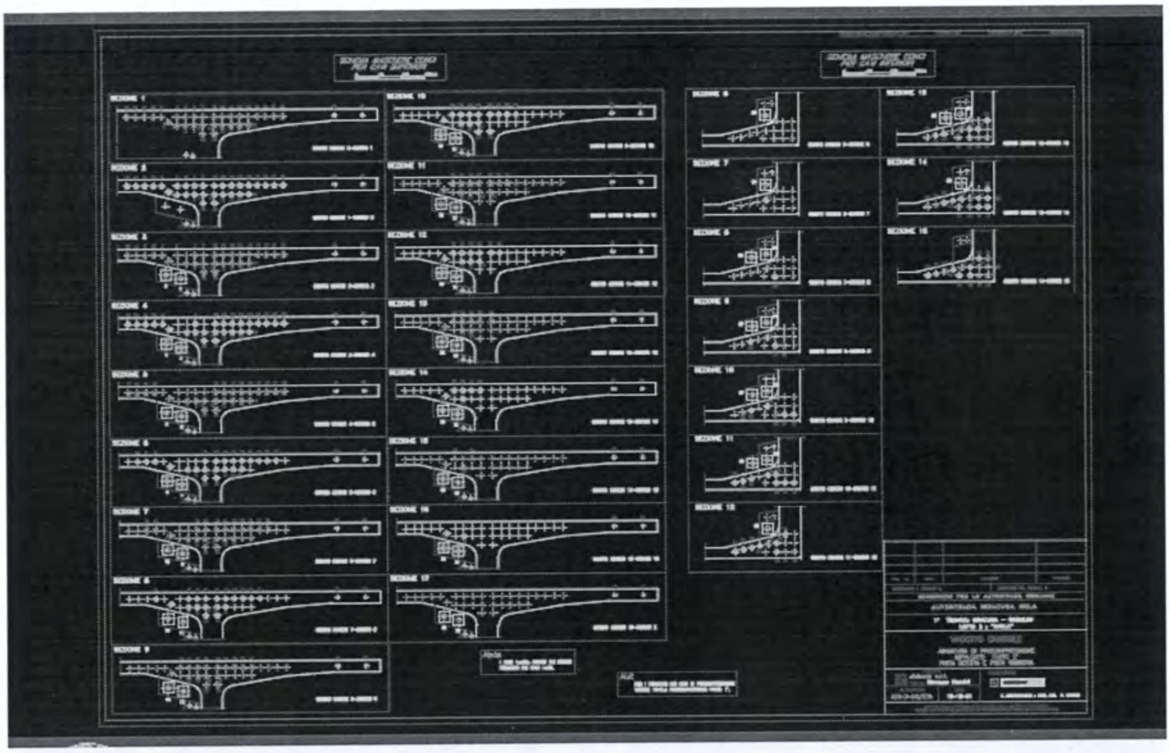
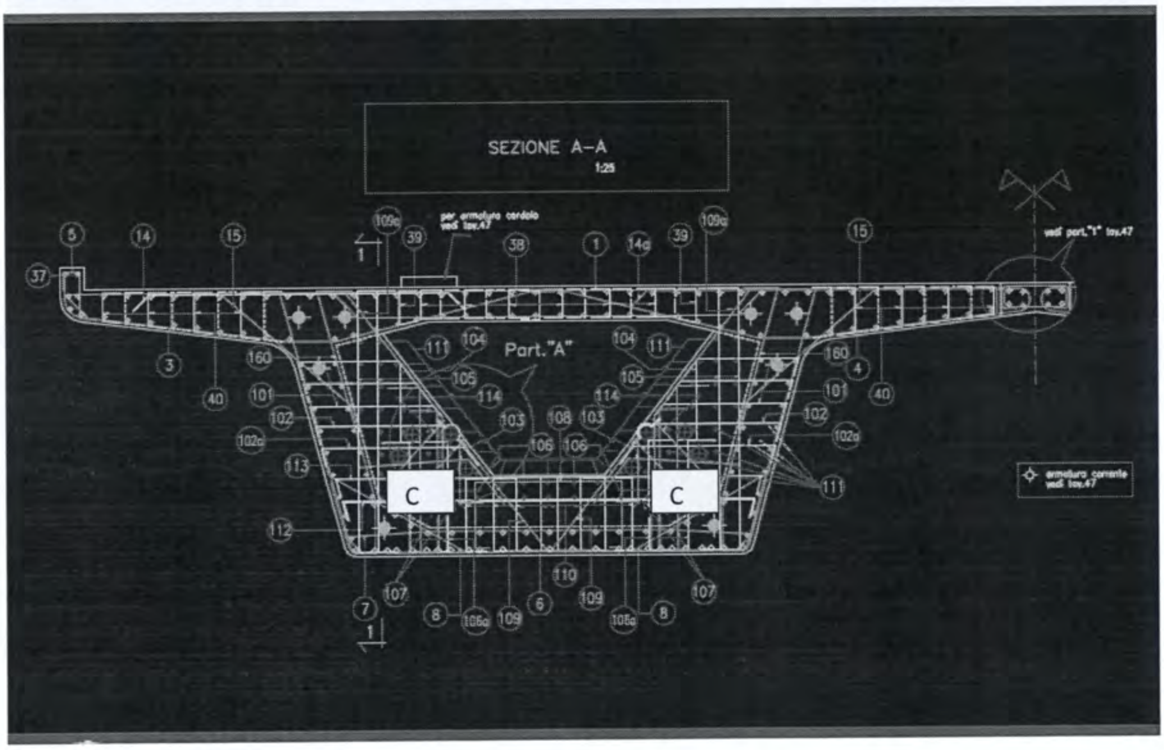
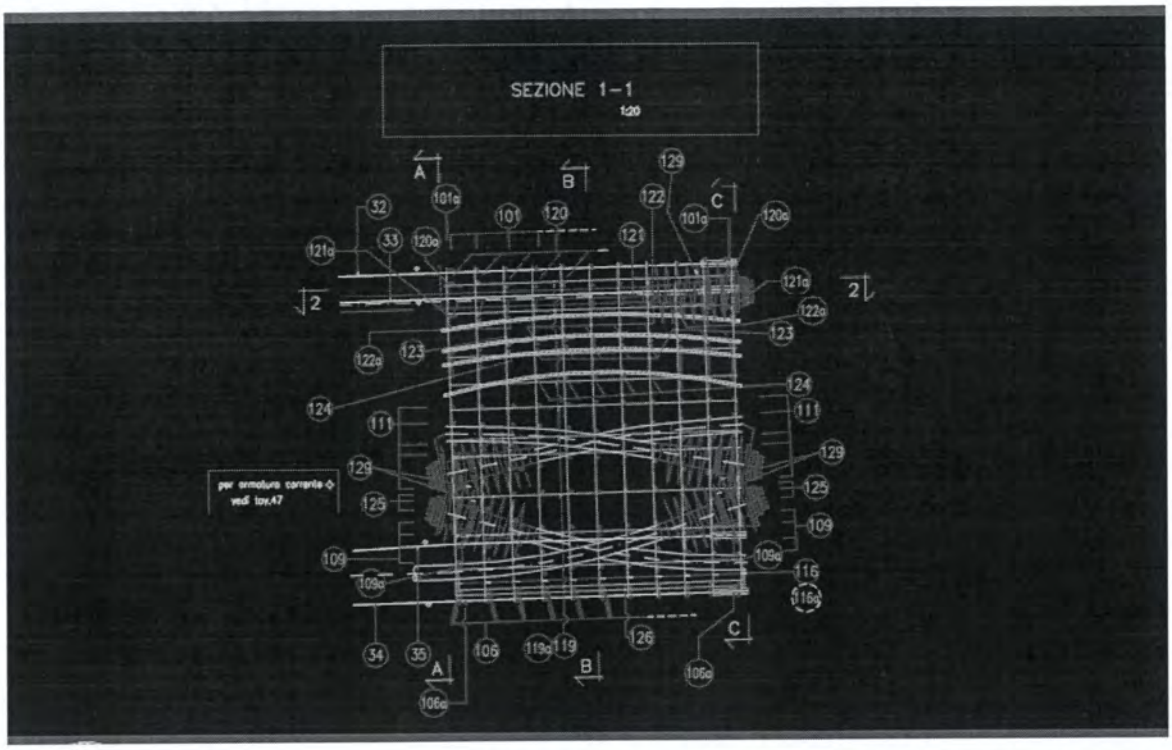
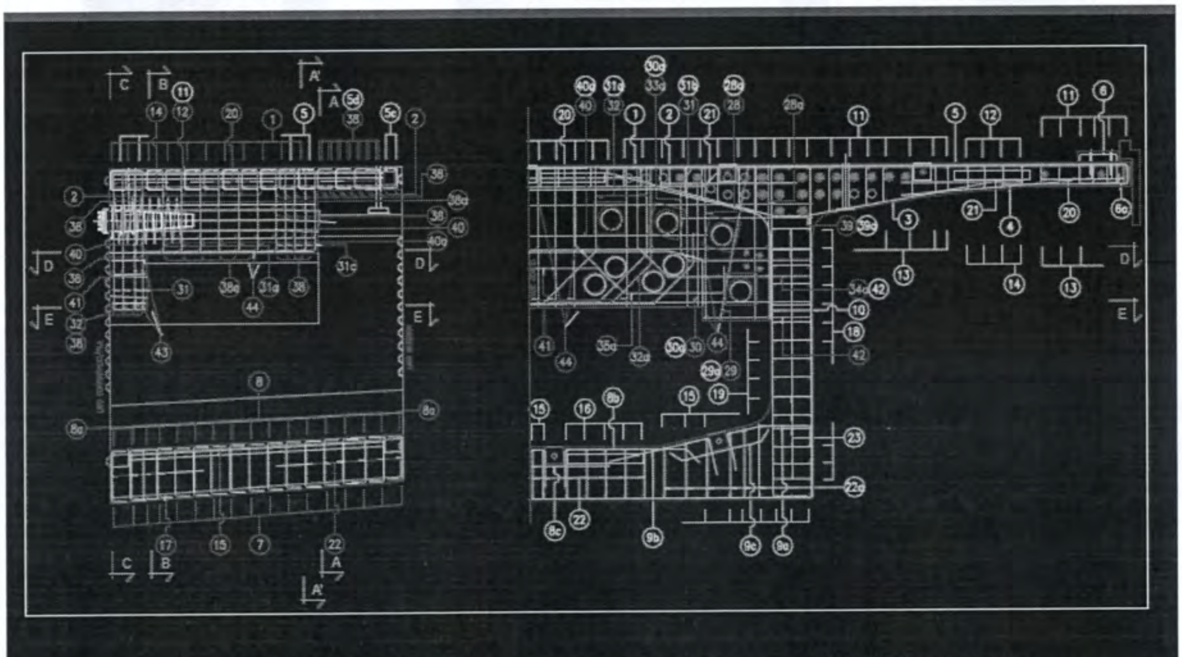


This drawing shows the holes in the section that will receive tendons, in order to know where to place the pipes in the formwork before the casting. The holes are represented with different colours, in order to distinguish the ones that will be filled with tendons and the ones that will not be used – they do not exist but they are represented in order to make the drawing easier to understand.



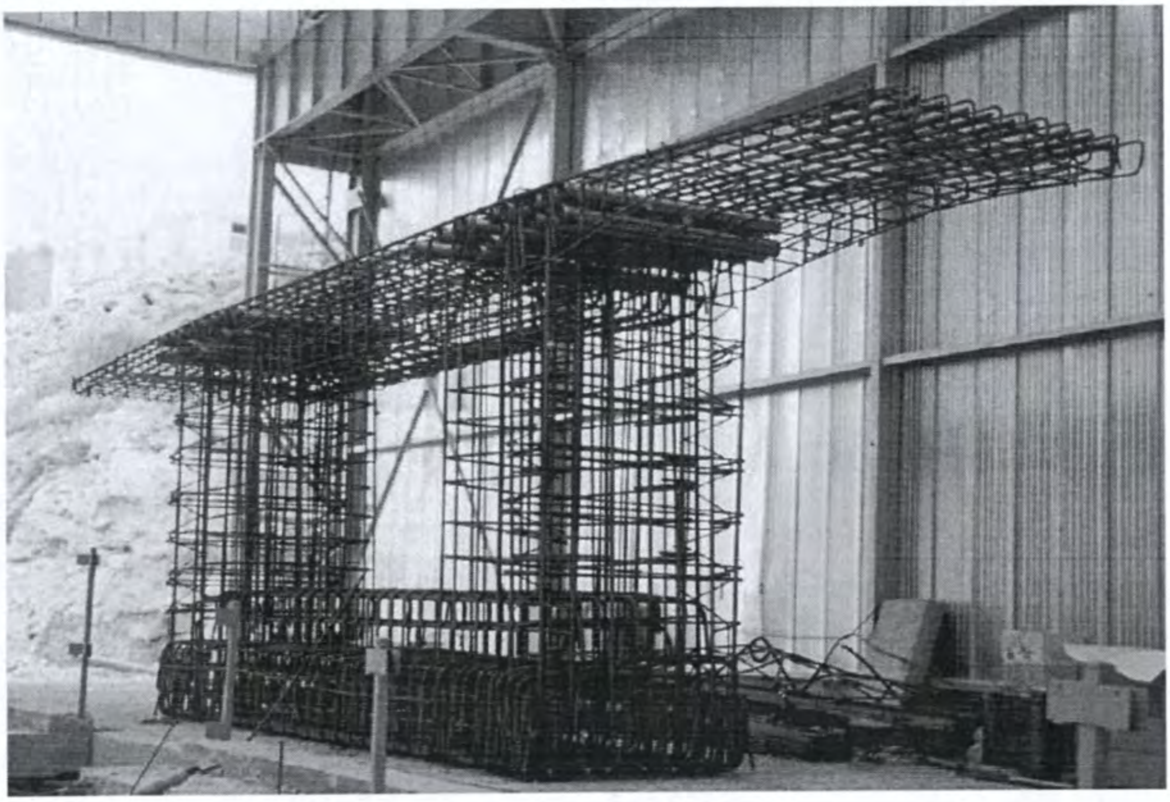


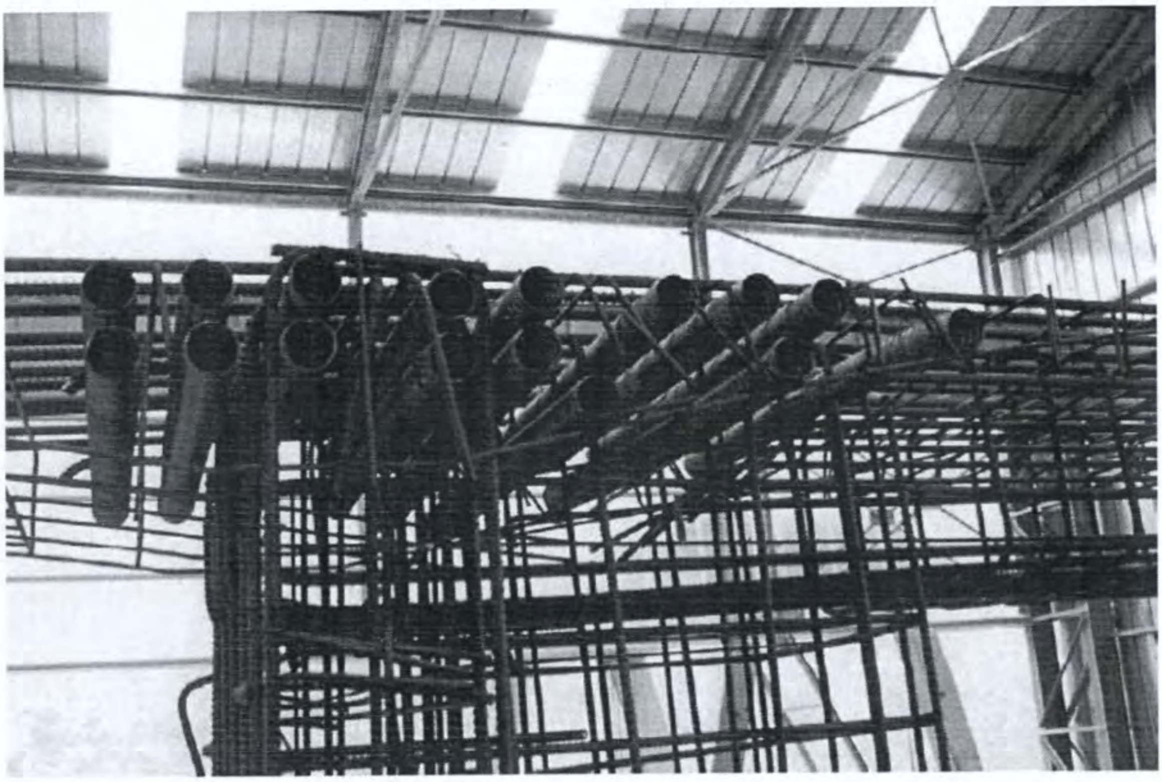
This detail shows couplers, that are special devices receiving anchors from the two sides and prestressing tendons are crossed, in order to have continuity of prestressing and connect the spans. In this zone, more reinforcement is required.



Modica Viaduct : anchorages and externs cables deviators

Here is a detail of external tendons anchored to the span.





As regards formwork, the pictures show details about the front, the reinforcement cage made of welded reinforcement and the tubes to insert tendons.

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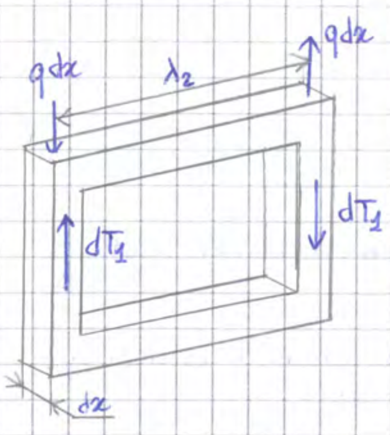


Then, if we consider a segment with infinitesimal length dx, we can put in evidence the differential value of internal actions which arise for an infinitesimal variation of torque moment dM_x:

the torque moment corresponds to a couple of forces qdx.

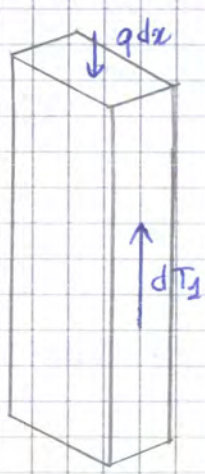
$$dM_x = q \lambda_2 dx$$

The effect of this torque moment can be evaluated using the formulations of Bredt theory introduced previously.

$$dT_1 = \frac{dM_x}{2\lambda_2} = \frac{q\cancel{\lambda_2}dx}{2\cancel{\lambda_2}} = \frac{q\,dx}{2}$$

$$dT_2 = \frac{dM_x}{2\lambda_1} = \frac{q\lambda_2 dx}{2\lambda_1} = \frac{\lambda_2}{\lambda_1}\frac{q\,dx}{2}$$

If the whole section is in equilibrium under these forces, also each part of the section should be in equilibrium, if we apply to this part the forces that it exchanges with the other ones.



Actually, if we analyze the equilibrium of a wall element, on this element the acting forces are

→ $q\,dx$

→ $dT_1 = \frac{q\,dx}{2}$

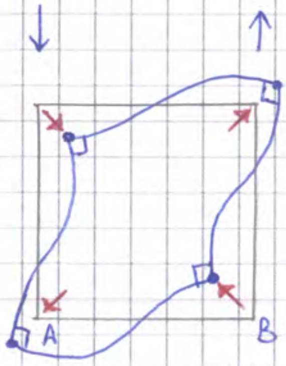
⇒ there is no equilibrium

It means that, when we separate the elements, to re-establish the equilibrium, we should consider other force.

This is the so-called Bredt anomaly, that appears because the Bredt formulation is referred to the whole elementary segment and not to its single parts.

In a box section, the effect of transverse eccentricity of loads is not only Bredt flow of tangential stresses inside the section, but also a variation of shape of the section.

What kind of distorsion?



From the original shape, point A is moved outwards and point B is moved inwards.

The distorsion should be taken into account in addition to Bredt because, due to distorsion, the axis line of each wall is not straight but it assumes a curved shape. By consequence, bending moment and shear arise inside the walls.
↓
curved with double curvature

Moreover, only some sections have eccentric loads - due to live load - and have distorsion. The other sections collaborate or distorsion is localized only in the section invested by the load?

2 Operative steps of the analysis in transversal direction

Once we have introduced Bredt anomaly, we have to find a way to evaluate internal actions.

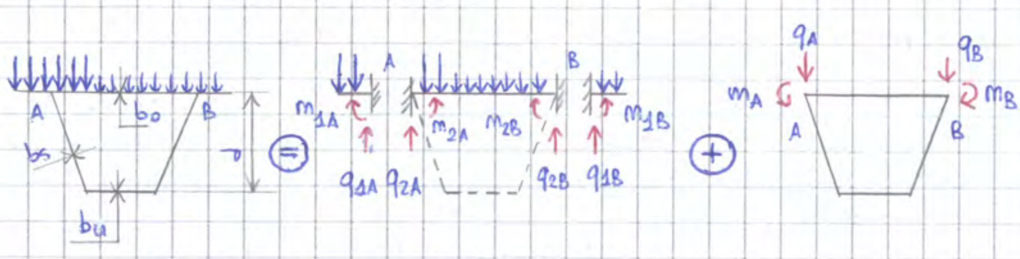
We consider a generic box section with one axis of symmetry, composed by a

→ top slab depth = b_o

→ bottom slab depth = b_u

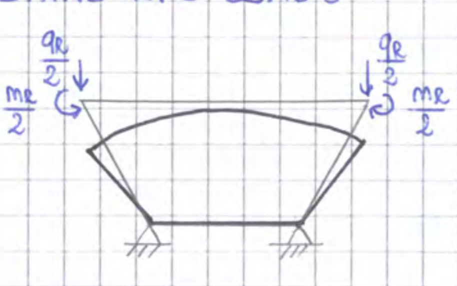
→ webs depth = b_s inclination = α

The box section is subjected to an eccentric load.



Thus, we have reduced a non-symmetrical distribution of forces on the frame to a system of symmetric forces and anti-metric forces.

SYMMETRIC LOADS



In the segment under symmetrical loading, we have a symmetrically loaded symmetric structure and deformation will be symmetric. By consequence, there is no variation of diagonal's length, i.e. there is no diagonalization of the section.

So, we can evaluate immediately the internal actions in the section due to the symmetric part of the load, if we superpose this stresses to symmetric part of the fully restraint actions.

↳ in the computation, we can consider the segment as supported by the webs (it is in equilibrium thanks to the difference of the shears in the webs)

ANTI-METRIC LOADS

As symmetric loads produce symmetric deformation, anti-metric loads will be responsible of diagonalization of section.

Again, the distribution of load is substituted with a combination of two distributions.

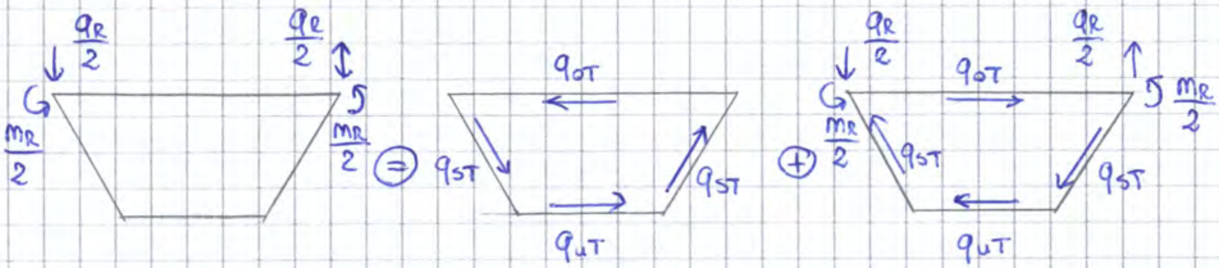
→ torsional part, corresponding to Bredt solution, with 4 membrane forces.

$$q_{oT} = \frac{dM_T}{2A_k} b_o \qquad q_{sT} = \frac{dM_T}{2A_k} b_s \qquad q_{uT} = \frac{dM_T}{2A_k} b_u$$

where

$$dM_T = \frac{q_R}{2} b_o + m_R \qquad A_k = \frac{b_o + b_u}{2} d$$

→ original distribution with Bredt forces with opposite sign.

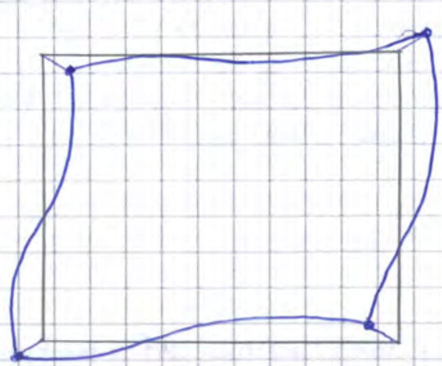


The first scheme has a known solution, because it corresponds to De Saint Venant torsion, which is carried by 4 shears in the webs.
This can be analysed in the LONGITUDINAL ANALYSIS:

when we perform the longitudinal analysis, the bridge is seen as a line in the space - there is no transversal effect - which is subjected to torque moment due to load's eccentricity and this produces transverse shears in the plates.

The second part is more complex and it is responsible of the variation of section, i.e. it produces the folded plate action - behaviour of plates which are folded to conform the section.
To evaluate the internal actions, we adopt two different limit approaches, that do not give the real solution.

(A) We evaluate internal actions by applying forces to the frame and assuming high deformability in the transverse direction, i.e. we evaluate internal actions with isolated section.



Due to eccentricities, the section changes its shape and there is diagonalization, whereas the remaining part of the bridge is undeformed.
It is impossible because we should have discontinuity of deformation, but we assume continuity.

⇒ the section is forced to change its shape due to the forces applied but it can't change it freely and the remaining part of bridge helps to reduce the overall deformation by changing itself the shape (partially)

Thus, there is collaboration of the remaining parts of the bridge so that the deformation in the bridge will remain continuous.

As regards this behaviour, we distinguish

→ CONCRETE BOX SECTION BRIDGES

slabs and webs have a certain thickness

$t_o, t_u \geqslant 30$ cm $\qquad t_s \geqslant 40$ cm

Thanks to this thickness, nodes are able to transmit bending moment - if there is reinforcement, of course -, and we have a frame that can take the transversal behaviour.

→ STEEL BOX SECTION BRIDGES

webs and slabs are plates coupled together and bending moment can not be transmitted in the nodes.
Due to this, we can not account for the transversal bending capacity of the section.
So, we introduce DIAGONALS at a short distance - about $1,5 \div 2$ times the depth -, so that the folded plate actions are taken by diagonals and they guarantee the absence of variation of shape of the section - otherwise, the section would fail.

In concrete bridges, we do not need diagonals because there is a capacity in transverse direction. We just need to close at the extremities the variation of section. To do this, diagonals are not convenient and we put a WALL, which restrains the shape of the final section.

↳ wall is also introduced due to principal torsion, but it is used also to maintain the section

Now, we define some geometrical and mechanical parameters that describe the section's shape

→ ratio between flexural rigidities in the longitudinal and transverse direction

$$\eta_o = \frac{\bar{I}_s b_o}{\bar{I}_o b_s}$$

$$\eta_u = \frac{\bar{I}_s b_u}{\bar{I}_u b_s}$$

→ geometrical parameters that describe the shape of the section

$$\alpha_o = \frac{t_o b^3}{t_s b_s b_o^2}$$

$$\alpha_u = \frac{t_u b_u}{t_s b_s}$$

→ effect of inclination of webs

$$\beta = \frac{b_u}{b_o}$$

→ diagonal length

$$g = \sqrt{d^2 + \left(\frac{b_o + b_u}{2}\right)^2}$$

Then, to get the analytical solution, we introduce some HYPOTHESES

→ as longitudinal behaviour is governed by flexural behaviour of the plates, they should behave like Bernoulli beams and so the length of each wall should be bigger than 4 times the width of the wall.

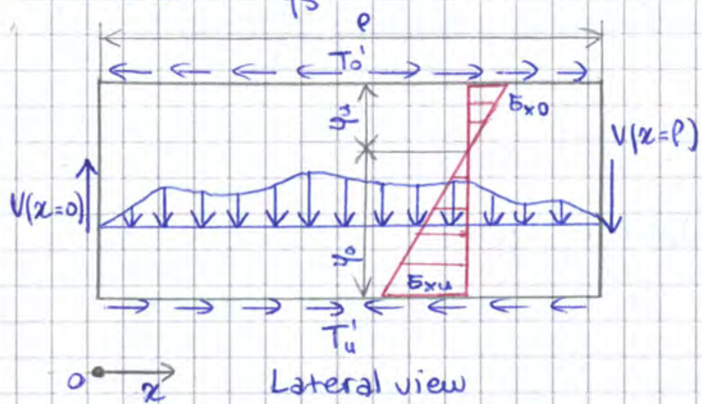
$$\ell > 4b$$

Moreover, to remain in Bernoulli beam model, thickness should be smaller than width, for each layer.

$$t < b$$

→ SYMMETRIC SECTION WITH RESPECT TO THE VERTICAL AXIS AND CONSTANT ALONG THE SPAN.

→ FORCE q_s^*



We extract the wall from the full section and we consider a lateral view of the web.

The web is subjected to the distribution of load $q_s^*(x)$, which is the amount of load $q_s(x)$ carried in longitudinal direction, and it is applied inside the web

The web is fully supported at the extremities, where shear is applied

$$V(x=0) \qquad V(x=\ell)$$

Moreover, due to the connection of the web to the other walls, shears T_o' and T_u' are transmitted between the connections and they are distributed along the corners of the layers.
In the analysis of longitudinal behaviour, we assume that section is made by 4 plates mutually connected by means of longitudinal cylindrical hinges, as bending moments are carried in the transversal direction.
These further forces along the connections contribute to equilibrium and compatibility of deformations between the web and the other plates.

In the imposition of the compatibility of deformations of the web with the other plates, all happens like we analysed a plate with not the real inertia, but an ideal moment of inertia which takes into account that the plate is not free in its deformation, but it is connected to the other walls.
Due to this restriction, the plate behaves in a more rigid way than it was free in the space and so we can analyse the behaviour with an ideal moment of inertia - not the real one -, involving geometrical and mechanical parameters.
This ideal moment of inertia is derived from the real one by multiplying it by a coefficient bigger than 1.

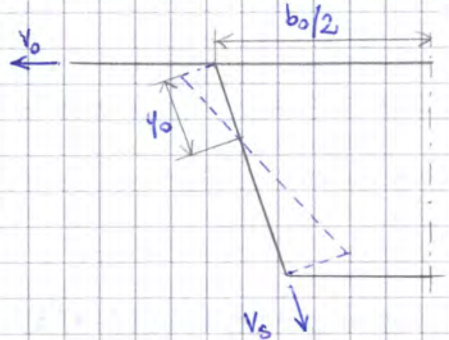
$$I_{s,i} = \frac{2\beta[(\alpha_o+2)(\alpha_u+2)-1]}{(1+\beta)(3+3\beta+\alpha_o+\alpha_u\beta)} I_s$$

Thus, the effect of load $q_s^*(x)$ on this beam with ideal moment of inertia is to produce longitudinal stresses σ_x.
They follow De Saint Venant, but the moment of inertia is different in order to account the behaviour of the other walls.

So, knowing the displacement V_s, we are able to get the load $q_s^*(x)$.

→ FORCE $\overline{q}_s$

As longitudinal stresses diagrams along the web are linear and the stresses value are the same in the hinges, also the curvature and the deformations are mutually proportional.

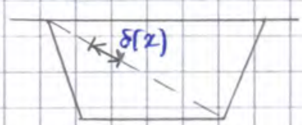


For instance, the ratio between the horizontal displacement of the top slab v_0 and the in-plane displacement of the web V_s is equal to the ratio of the corresponding distances from the neutral axis.

$$\frac{V_0}{V_s} = \frac{y_0}{\frac{b_0}{2}}$$

In other words, all is linear in the behaviour.
Thanks to this, by knowing only Vs, we can evaluate the displacements of the nodes and know the geometry of the section.

We can also know the diagonalization of the section, i.e. how much an ideal diagonal is lengthened or shortened.
The variation $\delta(x)$ of length of diagonals can be written as proportional to the web deformation $V_s(x)$.



$$\delta(x) = \frac{2 b_s}{\beta g} k_1 V_s(x)$$

where

$$k_1 = \frac{(1+\beta)(2+2\beta+2\beta^2+\alpha_0+\alpha_u\beta^2)}{3+3\beta+\alpha_0+\alpha_u\beta}$$

The resistant contribute $\overline{S}$ - amount of diagonal force carried in the transverse direction - to the diagonal deformation $\delta(x)$ can be evaluated by computing the δ effect of a force S applied on a unit length segment.
We obtain that $\overline{S}$ and then $\overline{q}_s$ is

$$\overline{q}_s = \frac{\overline{S} b_s}{g} = \frac{12 g E \overline{I}_s}{b_u b_0 d^2} k_2 \delta$$

where

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In this equation, we can define a characteristic length L, which defines the distance between the points with nil bending moment.

$$L = \frac{1}{\lambda} = \sqrt{\frac{4EI_{s,i}}{k}}$$

If we integrate the equation and apply the boundary conditions, we can derive the displacement function.

$$v_s(x)$$

By the II derivative, we can define the distribution of the bending moment M_s in the longitudinal direction.

$$M_s(x) = -EI_{s,i}\frac{d^2 v_s(x)}{dx^2}$$

Then, we derive the consequent longitudinal stresses σ_x.

$$\sigma_{x0} = -\frac{M_s(x)}{I_{s,i}}y_0 \qquad \sigma_{x,u} = \frac{M_s(x)}{I_{s,i}}y_u$$

At the same time, we evaluate the amount of diagonal force $\bar{S}$ carried transversely and the consequent transversal stresses.

$$\bar{S}(x) = k\frac{g}{b_s}v_s(x)$$

SCHEME

$$\begin{tikzcd}
 & & M_s(x) \arrow[r] & \sigma_x \\
EI_{s,i}\frac{d^4 v_s}{dx^4} + k v_s = q_s \arrow[r] & v_s(x) \arrow[ur] \arrow[dr] & & \\
 & & \bar{S}(x) \arrow[r] & \text{Transversal stresses}
\end{tikzcd}$$

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Thus, in the preliminary analysis, we can analyse the bridge only in the longitudinal direction and add the effect of the folded plate action.

Procedure for taking into account the transversal effects

I) IDENTIFICATION OF THE LOAD CASE and determi

II) DETERMINATION OF THE FULL RESTRAINED REACTIONS ACTING ON THE SLAB, which is supposed perfectly restrained.

III) APPLICATION OF THE FULL RESTRAINED REACTION AT THE BOX GIRDER, WITH INVERSE SIGN and THEIR SUBDIVISION IN SYMMETRIC AND ANTI-SYMMETRIC PARTS

IV) ANALYSIS OF THE FRAME UNDER SYMMETRIC ACTIONS and, taking into account the local actions, determination of the relative actions S_{symm}.

V) ANALYSIS OF THE FRAME SUPPOSING NOT DEFORMABLE UNDER ANTI-SYMMETRIC ACTIONS and, taking into account the local actions, determination of the corresponding actions S_{asymm} and diagonal actions.

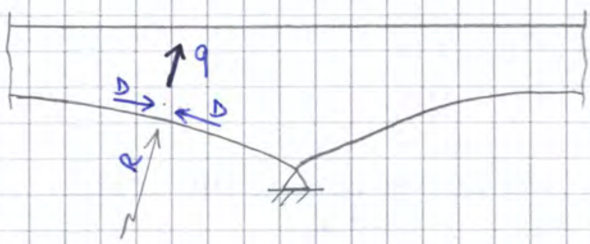
VI) ANALYSIS OF THE WEBS LIKE BEAMS ON ELASTIC SUPPORT SUBJECTED TO THE COMPONENT q_s of S and determination of longitudinal stresses σ_x and transversal actions S_{fp}.

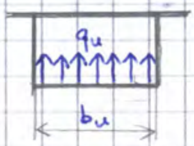
VII) SUM OF σ_x TO THOSE DERIVING FROM THE LONGITUDINAL ANALYSIS and of S_{fp} to S_{symm} and S_{asymm}

$$S = S_{fp} + S_{symm} + S_{asymm}$$

Transversal actions induced on the box girders at the variable depth

When span is very long, it is convenient to adopt a variable depth box girder - generally with a parabolic profile.
Yet, this solution shows some problems.





We can notice that compression changes continuously the direction as it follows the deck's shape.
So, if we consider a segment of the compressed chord, the compressive forces have a different direction and the resultant is a transverse force q; directed along the radial direction and which is function of the radial curvature.

$$q = \frac{D}{R}$$

At ULS, we assume that the bottom slab works at the maximum level of stress σ_u and we can write

$$q = \frac{\sigma_u t_u}{R}$$

$$q = \frac{\sigma_u t_u}{R}$$

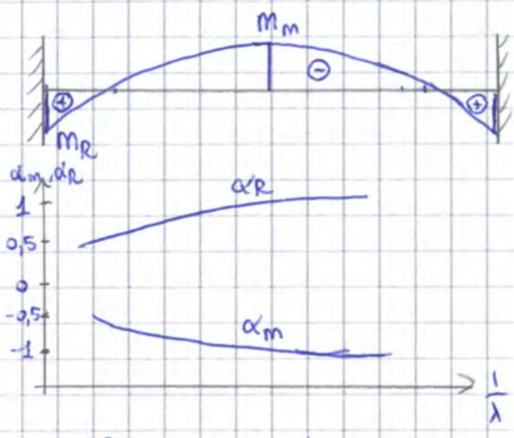
By consequence, in the analysis of the transverse section, we have a system of forces upwards, acting against the weight of the slab.

⇒ there is a bending moment in the bottom slab that counterbalance the one due to self-weight and it may change the sign of the overall bending moment.

The value of mean stress in the bottom slab is

$$\sigma_{u,m} = \frac{M_x z_{u,i}}{I_{u,i}}$$

where $z_{u,i}$ and $I_{u,i}$ are evaluated with respect to the ideal section obtained with $b_{u,i}$.



The bending moment in the bottom span by effect of the upward load q is evaluated assuming that the slab is fully restrainet in the webs, with values

Mid-span $\qquad m_m = \dfrac{q b_u^2}{24} \alpha_m$

Restraints $\qquad m_R = \dfrac{q b_u^2}{12} \alpha_R$

Coefficients α_m and α_R represent corrections that take into account the fact that q is not uniformly distributed in the bottom slab - due to the non-uniformity of compressive stresses - and it is bigger near the webs and smaller in the middle.

Indeed, in some bridges we can see cracks in the bottom slab near the webs because non uniformity was not considered in the design.

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INTRODUCTION TO PRESTRESSING

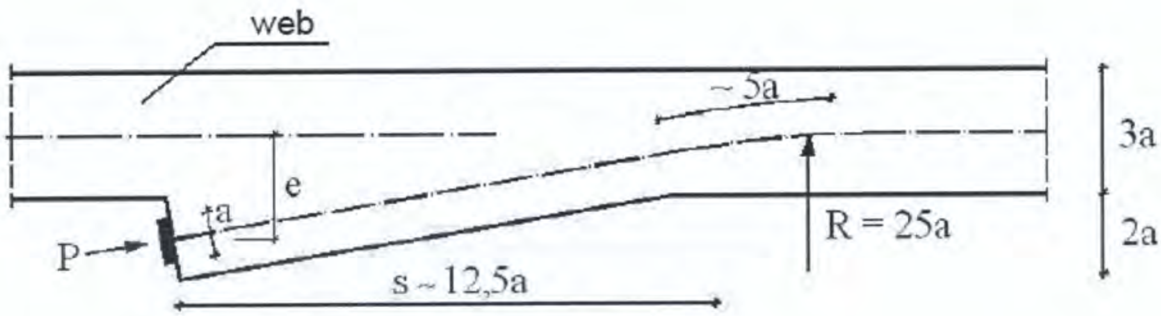
When we introduce prestressing, we have high forces localized in a concentrated area and the detailing of these forces is very important.

ANCHORAGE ZONE

We analyse an anchorage of the tendon on the web, internally to it – we see a horizontal section of the web.
In presence of concentrated forces, we are not in Bernoulli-Navier conditions and we have to design a particular region, called D-region.

Firstly, we see some suggestions for the geometry of the anchor. Because, to allow the flow of forces to enter inside the web – we apply the prestressing in a point – we need to respect some geometrical requirements. Generally, the minimum are 12 strands of 0,6 mm, each one initially tensed at 20 tons and we have 240 tons concentrated in a small region.
The steel plate to which we apply prestressing is a and the length of the increase of the web should be about 12,5a. It means that the angle to the longitudinal axis should be smaller than 10-20°.
Then, there is a length – depending on the anchoring type, in which tendon has to be straight because here there is the anchorage device, made by a tube which is straight. Then tendon deviate with a curvature radius which should be about 25a and be bigger than 5,5 m-6 m – otherwise, we would have failure of the wires due to tension because flexural tensions become important and should be sum to the longitudinal stress.
The transversal enlargement should be 2a wide and the web should be 3a wide.
This is the optimal geometry.

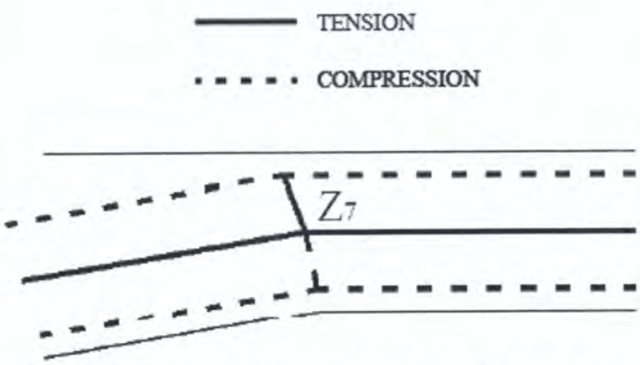
Horizontal section of the anchorage zone.



With this geometry, we can evaluate 3 types of regions.

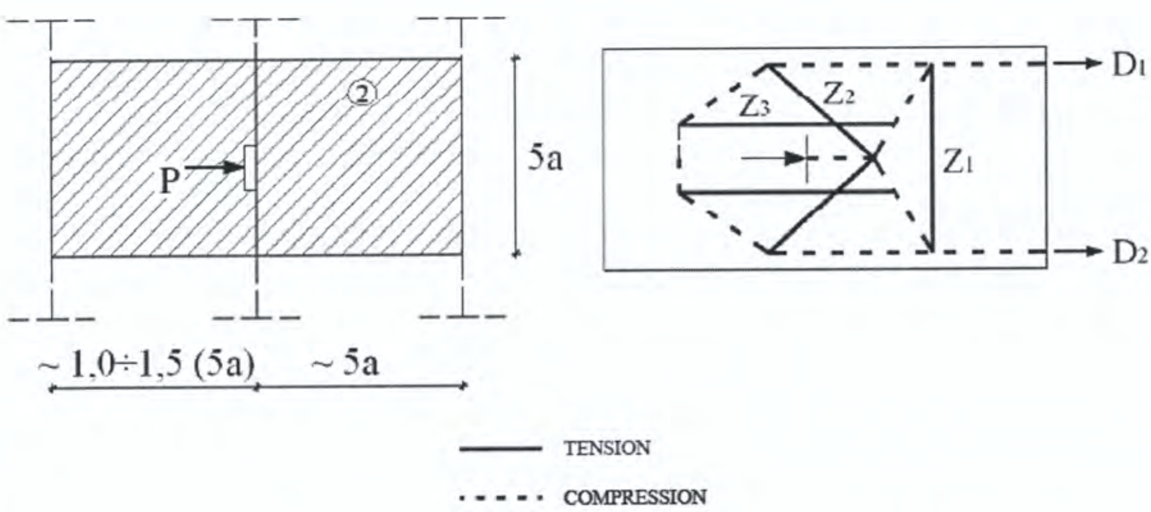
- Regions 1 and 3, that are regions in which we have discontinuity due to the applied force (1) or to the curved force applied by the tendons (3).
- In region 2, we have no applied force and no curved tendons and we can assume that as a Bernoulli region – B-region.

Region 1 is a D-region because, by tendons, we apply a concentrated force here.
Region 3 is a D-region because when the tendons are in tension and curve, locally we have transversal forces because tendons try to change their direction and then they exchange transversal forces with the concrete.



If we analyse the frontal view of the web, we can see the drag effect: the applied force is transferred backwards and there is an arch which is compressed and it is equilibrated by tensed ties.

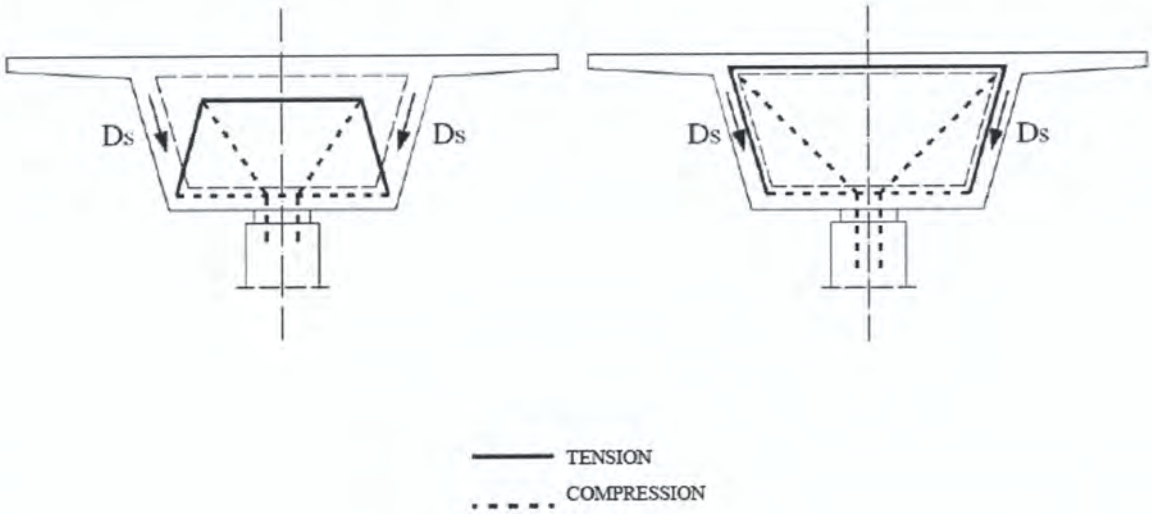
Frontal view of the anchorage zone.



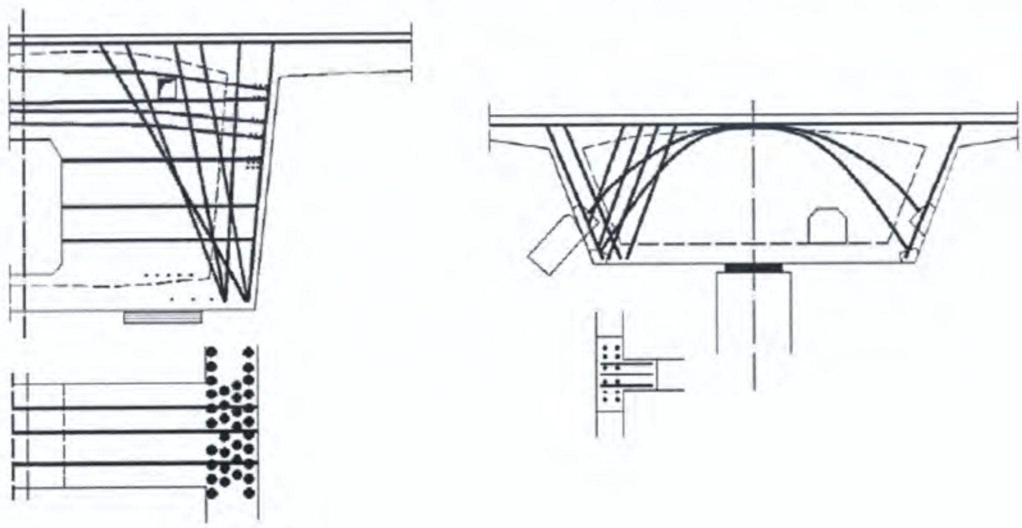
In the vertical section of the web, we see the force due to deviation in the horizontal plane of the tendon – when tendon is curved. The force is spread with an angle of 45° – in the elastic field – and tries to fail the concrete along this corner, either at the top or the bottom (there is a concentrated force inside the concrete). To give the equilibrium, we introduce two struts and the force should be applied to the other part of the section and it is transferred by means of ties Z_7. Otherwise, there would be failure of concrete, i.e. the concrete cone would be removed and the tendon would become linear and enter inside the section.

DIAPHRAGMS WITH UNIQUE CENTRAL SUPPORT

Sometimes, diaphragms are obliged, for geometrical reasons – no space in the cord which is small, whereas deck is big – and there is just one bearing.
In this case, torsion can not be beared with only one bearing and so, for the torsion, the pier does not exist. On the other side, the pier has to receive the vertical load – shear – and shear is transferred by the webs Ds and should go in the pier. Also here, we need a strut and tie system in order that the force Ds is transferred to the pier by the inclined strut. For that, we need a truss: the force has been suspended at the top and then it is transferred in compression. To close the equilibrium, there is a compression at the bottom and tension at the top. So we need reinforcement along the black lines.



Another system consists of transferring the load directly in the webs and the strut starts before the pier and a lot of stirrups are put inside the webs so that the load arrives at the bearing. So we do not enter in the diaphragm and there is a suspension of the load.
Without this truss, there is a failure with opening of the deck.
To realize this truss, ties may be prestressed bars or ordinary reinforcement – lot of transverse bars and vertical bars to suspend the load. In case of full wall, we can use tendons anchored on both sides. They are curved in order to follow the inclination of the struts.



2 TYPOLOGIES OF DIAPHRAGMS

In terms of geometry and span, we distinguish 3 types of diaphragms.

→ enlargement in the webs

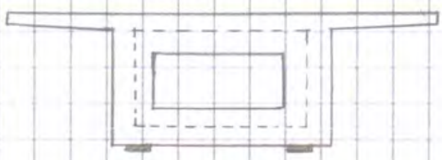
It is the minimal solution, in which just an enlargement of the width of webs is adopted.
Bearings remain within the enlargements, in order not to create punching in the slab.

Application — SMALL / MEDIUM SPAN

HIGH RADIUS OF CURVATURE - low torsional action

NARROW DECK

→ ponderous frame



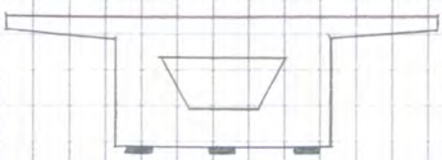
In this case, we increase both the width of slabs and webs, obtaining a frame with a port-hall inside.

Application — LARGE SPAN

IMPORTANT BENDING MOMENT

SMALL CURVATURE RADIUS - high primary torsion

→ pierced wall



This solution can be adopted in case of support on 3 bearings.
It consists of a wall with a port-hall inside. In order to facilitate the flow of forces from the webs to the central bearing, the wall width is reduced at the top and enlarged at the bottom.

This is not a very good solution but we are obliged to adopt it in case there is no enough space in the pier and we should use 3 bearings - with 2 bearings, we would risk the overturning of the deck.

Design of plates: sandwich model

1 An important aspect in boxed sections is the interaction between longitudinal shear and transverse bending moment in the webs:

webs have to transfer shearing actions due to primary torsion and the folded-plate action but they are also transversely bent.

In the past, this interaction was disregarded and engineers considered interaction by means of two different designs

→ design of stirrups for shear

→ with the same slab, design of the stirrups for the transversal bending moment.

Then, they summed the amounts of stirrups but, in the two designs, the concrete is the same and its resistance is accounted twice, whereas it should resist to vertical compression due to transversal bending and to the inclined compression fields due to shear.

By consequence, with this design approach, there is a lot of safety for reinforcement because, for instance, when ~~the~~ shear is applied, only the relative amount of stirrups is used, whereas concrete resistance is engaged more than the allowed.

So, we have to consider interaction and we adopt the SHELL ELEMENT THEORY, that analyses the behaviour of a shell loaded in its plane by vertical forces - due to shear - and bending forces - due to external bending moment.

Thus, the sandwich degenerates into two panels of thickness z_1 and z_2.

In each panel is acting a membrane force - so that the sum gives v^*_{xy} or v^*_{yx} - and an axial force which is the effect of bending moment

Membrane Forces $\quad n_{yx,1} \quad n_{yx,2} \quad n_{xy,1} \quad n_{xy,2}$

Axial Forces $\quad n_y$

By using equilibrium equations, we can derive the internal actions in each panel from the external actions v^*_{xy} and m^*_x

Translation along x axis $\quad n_{xy,1} + n_{xy,2} = v^*_{xy}$

Rotation of top face around x direction $\quad n_y\left(b_w - \frac{z_1+z_2}{2}\right) = m^*_x$

Rotation of front face around x direction $\quad n_{xy,1}\frac{b_w - z_1}{2} = n_{xy,2}\frac{b_w - z_2}{2}$

z_1, z_2 = lever-arm with respect to the gravity center of the panel

$$\Rightarrow \quad n_y = \frac{m^*_x}{b_w - \frac{z_1+z_2}{2}}$$

$$n_{xy,1} = v^*_{xy}\,\frac{b_w - z_2}{2b_w - z_1 - z_2}$$

$$n_{xy,2} = v^*_{xy}\,\frac{b_w - z_1}{2b_w - z_1 - z_2}$$

As each panel is subjected only to membrane forces, we can design separately each panel.

The inclined face is cutting the reinforcement and so here applies the resisting force of the reinforcement, equal to

$$\rho_y \sigma_{s,y} \qquad \qquad \rho_x \sigma_{s,x}$$

ρ = geometrical percentage of reinforcement - actual area of reinforcement divided by the area of the section in the vertical or horizontal direction

The force $\rho\sigma$ represents a distributed force along the inclined face.

At this point, we can write the equilibrium conditions of the element

→ translation in the horizontal direction

$$\underbrace{\tau_{yx} \cdot (1 \cdot \cos\vartheta)}_{\text{length of horizontal face}} - \underbrace{\rho_x \sigma_{s,x} \cdot (1 \cdot \sin\vartheta)}_{\text{length of vertical face}} = 0$$

$$\Rightarrow \tau_{xy} \cos\vartheta - \rho_x \sigma_{s,x} \sin\vartheta = 0$$

$$\tau_{xy} = \tau_{yx}$$

→ translation in the vertical direction

$$\tau_{xy} \cdot (1 \cdot \sin\vartheta) + \sigma_y (1 \cdot \cos\vartheta) - \rho_y \sigma_{s,y} \cdot (1 \cdot \cos\vartheta) = 0$$

$$\Rightarrow \tau_{xy} \sin\vartheta + \sigma_y \cos\vartheta - \rho_y \sigma_{s,y} \cos\vartheta$$

Globally, we have 4 equilibrium conditions but, in a plane problem, only 3 conditions are independent and one equation will be a linear combination of the other ones.

From the equation ( ), we can derive directly the tangential stress τ_{xy}

$$\tau_{xy} \cos\vartheta - \rho_x \sigma_{s,x} \sin\vartheta = 0$$

$$\Rightarrow \quad \tau_{xy} = \rho_x \sigma_{s,x} \tan\vartheta$$

The tangential stress τ_{xy} can be derived also from equations

$$\tau_{xy} \sin\vartheta + \sigma_y \cos\vartheta - \rho_y \sigma_{s,y} \cos\vartheta = 0$$

$$\Rightarrow \tau_{xy} = \left(\rho_y \sigma_{s,y} - \sigma_y\right) \operatorname{cotan}\vartheta$$

$$\tau_{xy} \cos\vartheta - \sigma_y \sin\vartheta + \rho_y \sigma_{sy} \sin\vartheta - \sigma_c \sin\vartheta = 0$$

$$\Rightarrow \tau_{xy} = \sigma_c \tan\vartheta - \left(\rho_y \sigma_{s,y} - \sigma_y\right) \tan\vartheta =$$

$$= \sigma_c \tan\vartheta - \underset{(\ )}{\frac{\tau_{xy}}{\operatorname{cotan}\vartheta}} \tan\vartheta = \sigma_c \tan\vartheta - \tau_{xy} \tan^2\vartheta$$

$$\Rightarrow \quad \tau_{xy} = \frac{\tan\vartheta}{1+\tan^2\vartheta}\sigma_c = \sigma_c \sin\vartheta \cos\vartheta$$

In this way, we have obtained 3 independent equations that describe the relationship

$$\tau_{xy} = \tau_{xy}\left(\sigma_y ; \sigma_c ; \sigma_{s,x} ; \sigma_{s,y} ; \vartheta\right)$$

Thanks to these equations, we are able to apply the limitations of the stress state

→ in reinforcement

$$\sigma_{s,x} \leq f_{yd} \qquad \sigma_{s,y} \leq f_{y,d}$$

→ in concrete

$$\sigma_c \leq f_{cd,2} \quad \rightsquigarrow f_{cd,2} = 0,6\, f_{cd,1}$$

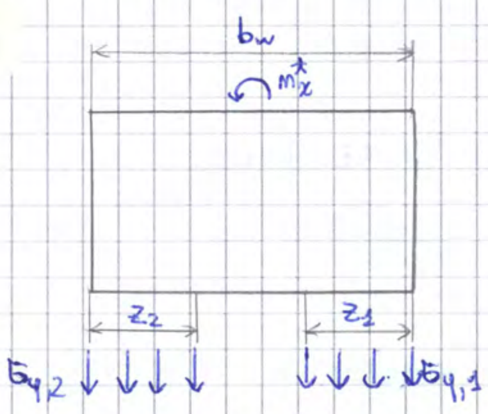
($f_{cd,1}$ is used for longitudinal analysis)

Focusing for instance ~~for~~ on A - similar results are valid for A_y -, through equilibrium we get the correction of the amounts of reinforcement in the two panels.

$$A_{x,1} = \rho_{x,1} z_1 \frac{b_w - \frac{z_1}{2} - c}{b_w - 2c} + \rho_{x,2} z_2 \frac{\frac{z_2}{2} - c}{b_w - 2c}$$

$$A_{x,2} = \rho_{x,1} z_1 \frac{\frac{z_1}{2} - c}{b_w - 2c} + \rho_{x,2} z_2 \frac{b_w - \frac{z_2}{2} - c}{b_w - 2c}$$

OBSERVATION: evaluation of σ_y stress in each panel



The σ_y stress is obtained through equilibrium of the element.

$$\sigma_{y,1} = \frac{m_x^*}{\left(b_w - \frac{z_1 + z_2}{2}\right) z_1}$$

$$\sigma_{y,2} = \frac{m_x^*}{\left(b_w - \frac{z_1 + z_2}{2}\right) z_2}$$

same equation of ( ) and ( )

$$\tau_{xy,1} = \frac{V_{xy}^{*}}{z_1} \frac{b_w - z_2}{2b_w - z_1 - z_2}$$

$$\sigma_{y,2} = \frac{m_x^{*}}{\left(b_w - \frac{z_1 + z_2}{2}\right) z_2}$$

$$\tau_{xy,2} = \frac{V_{xy}^{*}}{z_2} \frac{b_w - z_1}{2b_w - z_1 - z_2}$$

IV) We introduce these values in the equations ( ) and ( ) and we find the unknowns $\rho_{y,1}$, $\rho_{y,2}$, ϑ_1 and ϑ_2

$$\tau_{xy,1} \leq \left(\rho_{y,1} f_{yd} - \sigma_{y,1}\right) \cot\vartheta_1 \qquad \tau_{xy,2} \leq \left(\rho_{y,2} f_{yd} - \sigma_{y,2}\right) \cot\vartheta_2$$

$$\tau_{xy,1} \leq f_{cd,2} \sin\vartheta_1 \cos\vartheta_1 \qquad \tau_{xy,2} \leq f_{cd,2} \sin\vartheta_2 \cos\vartheta_2$$

It is possible to have

$$\vartheta_1 \neq \vartheta_2$$

but it is not a problem.

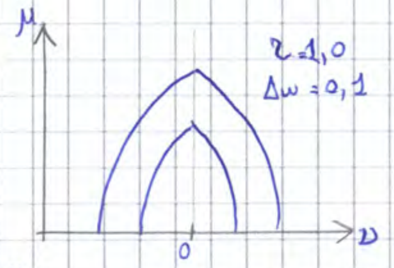
V) We apply the correction in the amount of reinforcement due to its eccentricity.

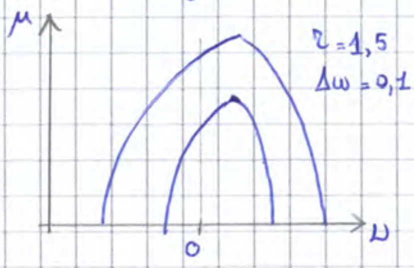
$$A_{y,1} = \rho_{y,1} z_1 \frac{b_w - \frac{z_1}{2} - c}{b_w - 2c} + \rho_{y,2} z_2 \frac{\frac{z_2}{2} - c}{b_w - 2c}$$

$$A_{y,2} = \rho_{y,1} z_1 \frac{\frac{z_1}{2} - c}{b_w - 2c} + \rho_{y,2} z_2 \frac{b_w - \frac{z_2}{2} - c}{b_w - 2c}$$

VI) We compute the ratio of reinforcement.

$$r_y = \frac{A_{y,1}}{A_{y,2}}$$





In this way, we get an influence surface in a 3D space.
Performing sections with planes having constant values of mechanical reinforcement ratio ω, we get curves representing the interaction domain. Curves are defined in a $\nu - \mu$ plane, for different values of τ

→ with $\tau = 1$, curves are symmetrical. The most internal area represents the zone where no reinforcement is required

→ with $\tau = 1,5$, curves are not symmetrical

With this approach, we design the vertical reinforcement A_y.

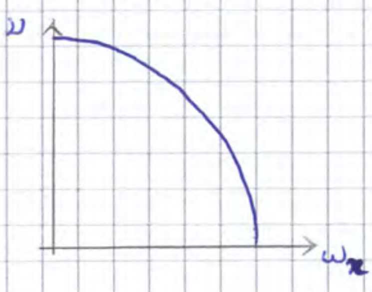
As regards the longitudinal reinforcement, we assume

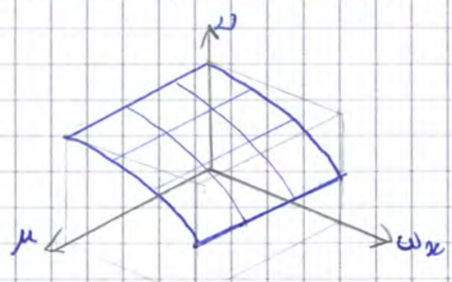
$$\tau_x = 1$$

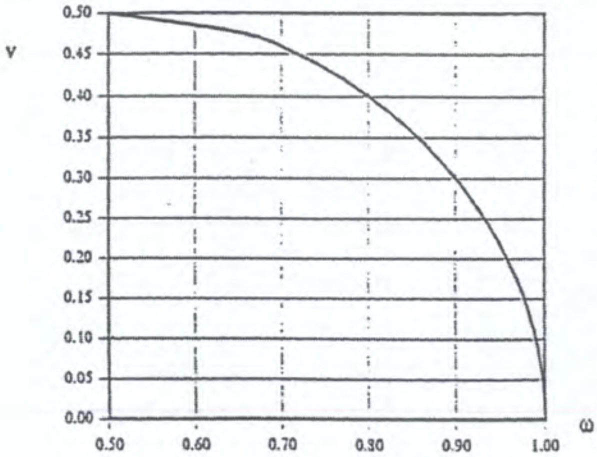
i.e. the longitudinal reinforcement is the same in the two faces, and we determine the minimum value of ω_x with the same procedure, remembering that

$$\omega_x = \omega_{x,1} + \omega_{x,2} = \frac{(A_{x,1} + A_{x,2})\, f_{yd}}{\Delta y \; b_w f_{cd,2}}$$

In this case, we obtain a cylindrical surface.
It means that LONGITUDINAL REINFORCEMENT DOES NOT DEPEND BY BENDING MOMENT because longitudinal reinforcement is required to equilibrate the tensile field due to shear. By consequence, the section - i.e. the interaction diagram - is the same with whichever value of bending moment.

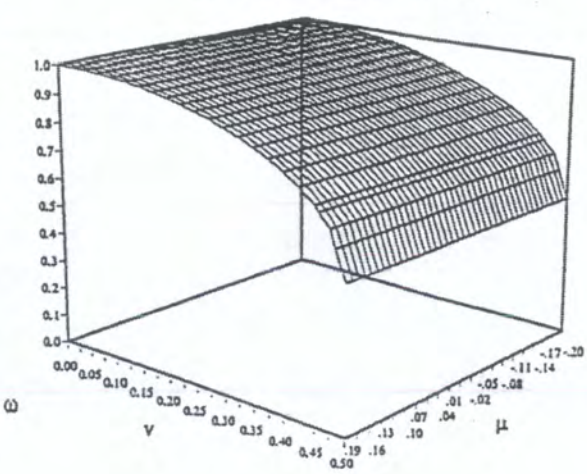






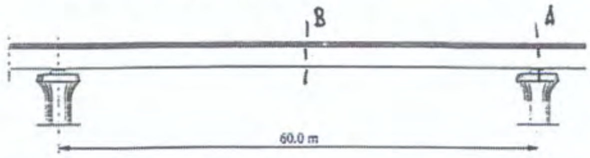
Project dyagram for longitudinal reinforcing

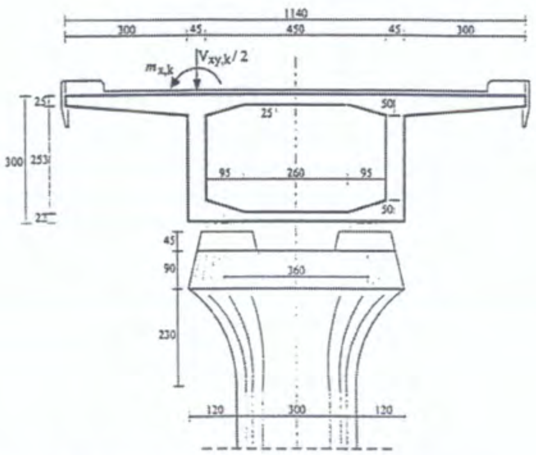
Longitudinal reinforcing ω_x



Interaction surface (μ_i, v_{xy}, ω_x)

3.3 NUMERICAL EXAMPLE





$z = 0{,}9d = 2{,}655$ m

$\Delta_y = 0{,}5$ m

concrete C35 $f_{cd2} = 12{,}04$ MPa

Politecnico di Torino
Department of structural and geotechnical engineering
"Bridge design"

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POINT	$V_{xy,Sd}$ $[kN]$	$m_{x,Sd}$ $[kNm]$
A	706,2	375
B	141,2	375

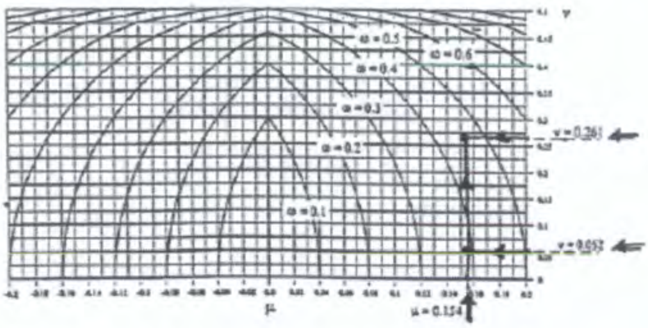
We evaluate them in non-dimensional terms.

POINT	$v_{xy,Sd}$	$m_{x,Sd}$
A	0,261	0,154
B	0,052	0,154

In order to get an idea of the effect of different design approaches, we assume two conditions.

Symmetrical transversal reinforcement design ($r_y = 1,0$)

Assuming symmetrical reinforcement, we start from the actions and we get the mechanical amount of reinforcement ω_y by linear interpolation in the interaction diagram.

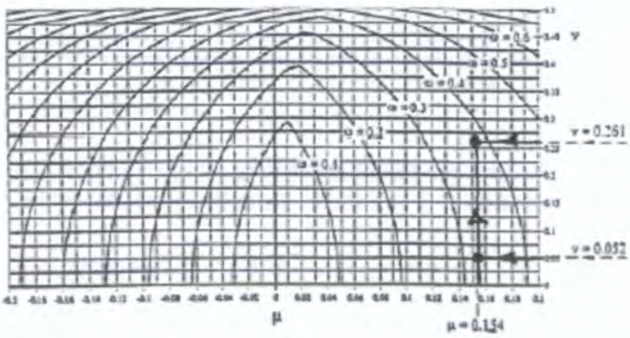


By consequence, we obtain the area $A_{xy,1/2}$ of each leg of the stirrup – stirrups has two legs in the web –, in correspondence of the pier axis and of the mid-span section.

POINT	ω_y $[-]$	$A_{xy,1/2}$ $[cm^2/m]$
A	0,458	32,43
B	0,387	27,40

Symmetrical transversal reinforcement design ($r_y = 1,5$)

Assuming then a reinforcement ratio between the two legs equal to 1,5, the interaction diagram changes and, if the points are the same – as they are defined by the internal actions –, curves change and move in the right direction.



DISCONTINUITY REGIONS

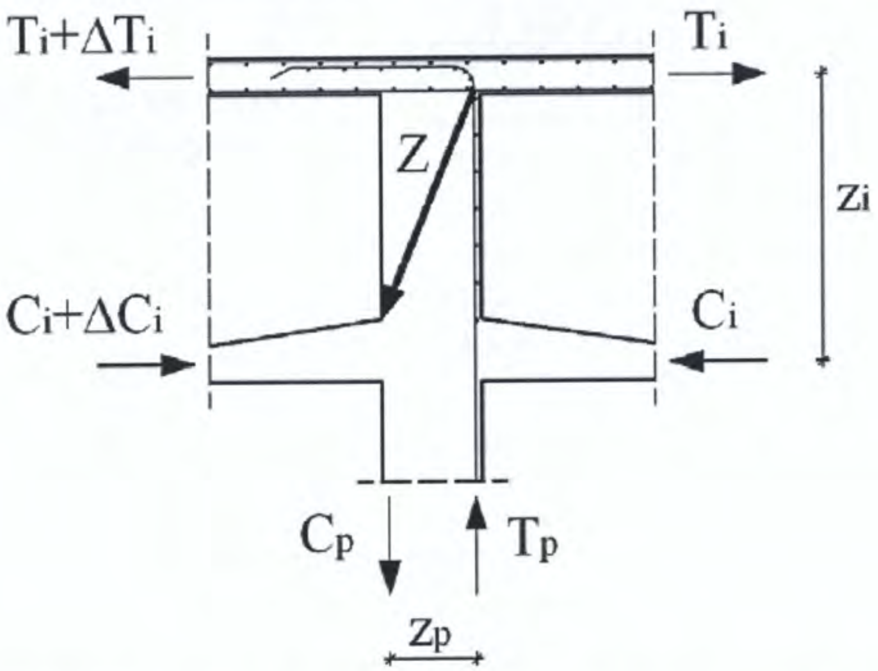
Transfer of the actions from deck to pier

We can distinguish different situations.

Box girder with diaphragm pier

A box section deck is integral in the pier, i.e. there are no bearings but there is full connection between the pier – square solid pier – and the deck.

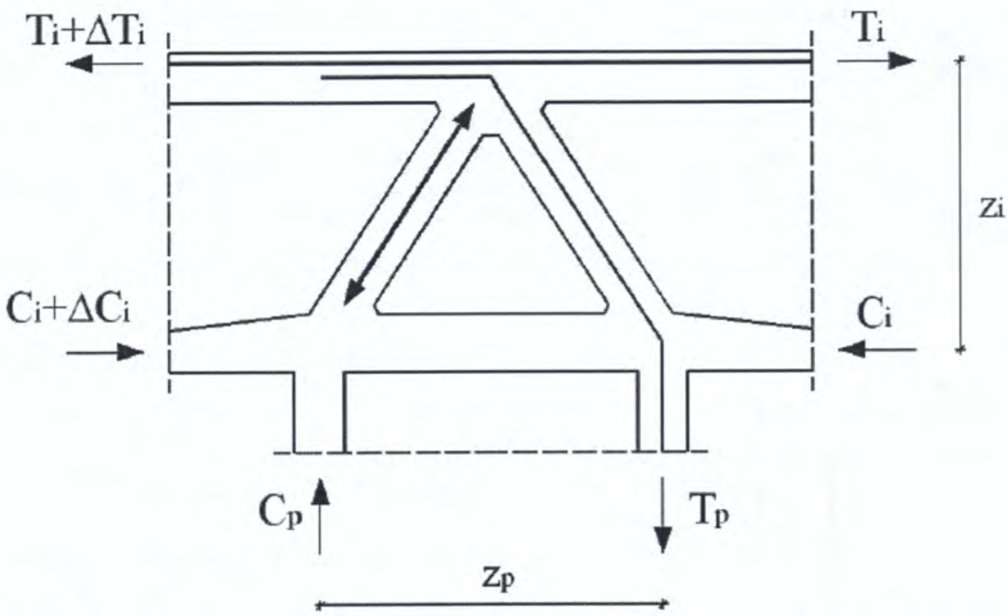
The longitudinal section shows the diaphragm, the top slab and the bottom slab. The bottom slab has an increase in its thickness due to negative bending moment.



On the right part of the deck, a bending moment is acting and this is represented by a couple of forces, with a tensile force T_i and a compressive force C_i.
On the other side, a different bending moment is acting and it is represented by the forces $T_i + \Delta T_i$ and $C_i + \Delta C_i$.
The difference between the two bending moments should be found in the pier, as the node is globally equilibrated. So, the difference of bending moment, represented by ΔT_i and ΔC_i, goes into the pier and generates a tensile force T_p and a compressive force C_p, having an internal lever-arm equal to z_p.

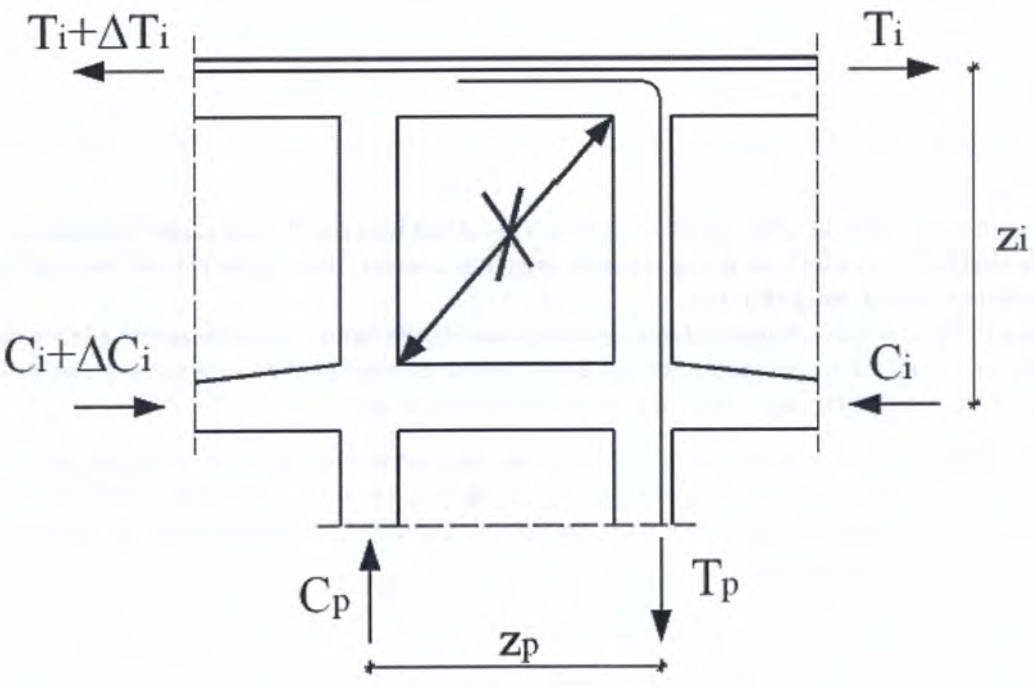
This is the effect of the transfer of bending moment from the deck to the pier due to the continuity between them.

How could this bending moment be transferred?
We need a compressive field, i.e. compression should cross the diaphragm to reach the compressive filed inside the pier and, due to equilibrium of the node, tension arises. By consequence, we need reinforcement on the right face. This reinforcement is prolonged beyond the end point of the diaphragm because, in the strut and tie approach, we assume that the force in the ties is constant. To ensure this, reinforcement has to be fully mobilised and so we must provide anchoring beyond that point.



Again, there is a variation of the bending moment between the two opposite faces of the deck and the difference should be transferred to the pier. We define as z_i the lever-arm of the couple of forces inside the deck and as z_p the lever-arm of the couple of forces inside the pier.
The best way to transfer this action is to introduce two inclined diaphragms and compression will enter on a diaphragm and will deviate at the angles to generate compression inside the pier and tension is taken by the other diaphragm. The reinforcement should be anchored beyond the intersection point, so that there is the maximum force in the strut and tie model at that point.

This is an elegant and clever solution but it is complicated for the working side and we are obliged to adopt vertical diaphragms.



Diffusion of prestressing in the extreme zones of the box girder beams

A problem is that, when there is a longitudinal action on the longitudinal beam due to shear, at the bearings we have to equilibrate a force which, assuming isostatic scheme, is equal to

$$F_{Sd} = \frac{V_{Sd}}{2} \cot \vartheta$$

In the relationship, V_{Sd} is the shear action in the web and ϑ is the inclination of compressive fields.

This force may be equilibrated by tendons, but how much prestressing is active in the region of the bearings? This aspect is important because it gives a contribution in equilibrating this tensile force that means less ordinary reinforcement.

To answer to this question, we introduce two different phenomena, that are prestressing distribution and prestressing transmission in prestressed tendons.

- Prestressing diffusion – described with anchoring length – intervenes when we apply an over-stress to a tendon with respect to the stress that it has, as in ordinary reinforcement.
- Prestressing transmission – described with transmission length – intervenes when we release a tendon and prestressing diffuses in the beam.

Transmission is a phenomenon involving *pushing in* because the strand is released and pushes against the concrete to apply the prestressing, whereas diffusion involves *pull out* because, in case of crack opened, the stress in the tendon is increased and tendon is pulled out.
Of course, anchoring length and transmission length are different.

Prestressing dispersion and transmission length

The bond stress is function of the tensile strength of concrete at release time $f_{ctd}(t)$.

$$f_{bpt} = \eta_{\rho 1} \eta_1 f_{ctd}(t)$$

$\eta_{\rho 1}$ is 3,2 for 3 or 7 wires strands and η_1 is assumed 0,7 in bad bond conditions – prestressing applied on the top of the section with respect to casting direction – and 1 for good bond conditions – prestressing applied on the bottom of the section.

From this, we define the basic value of transmission length.

$$l_{pt} = \alpha_1 \alpha_2 \phi \frac{\sigma_{pm,0}}{f_{bpt}}$$

$\sigma_{pm,0}$ is the stress inside the tendon at the moment of release and ϕ is the diameter of the strands.
α_1 is equal to 1 if gradual release of prestressing is applied or 1,25 if the release is sudden with cut of the tendons (adopted in case of small precast elements). α_2 is equal to 0,19 for 3 or 7 wires strands.

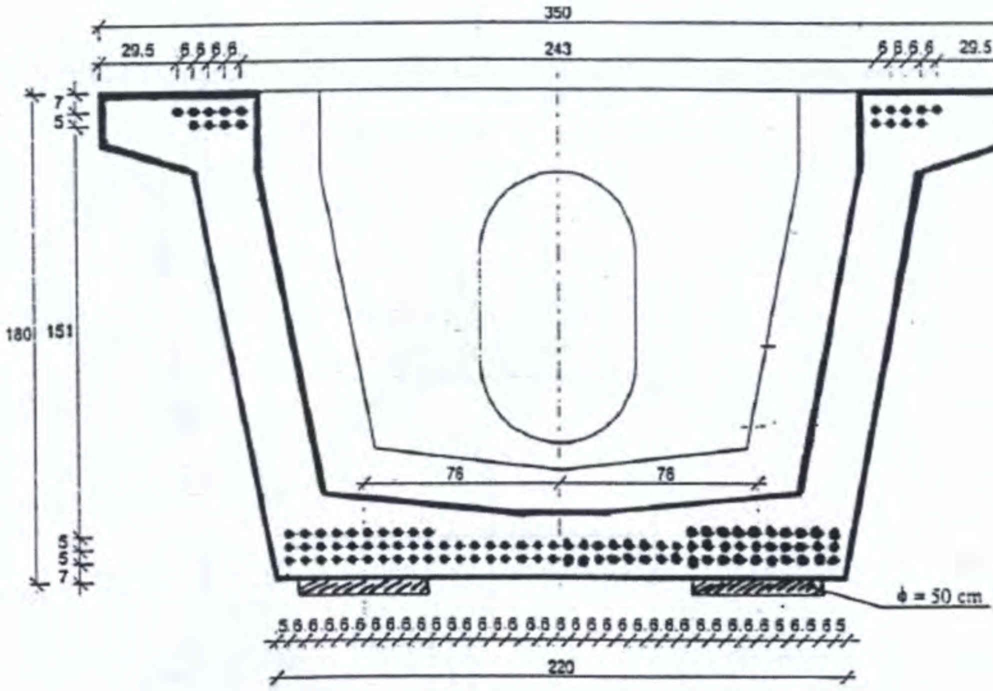
Then, we define the design values for transmission length from the mean value.

- Local value, for verifications of local stresses at release – transverse action is bigger.

$$l_{pt,1} = 0{,}8 l_{pt}$$

- Value at ULS, where slip is assumed and full prestressing will enter at a bigger distance.

$$l_{pt,2} = 1{,}2 l_{pt}$$



From the structural analysis, the value of acting shear is

$$V_{Sd} = 4125\ kN$$

The deck is working on an isostatic scheme and the longitudinal force in the tensed chord at the bearing due to shear will be the following – the angle ϑ is assumed as 30°.

$$F_{Sd} = \frac{V_{Sd}}{2}\cot\vartheta = \frac{4125\ [kN]}{2}\cot 30° = 3568\ kN$$

The properties of concrete and prestressing tendons are known.

Tensile strength of concrete f_{ctd}	1,6 MPa
Strands diameter Ø	15 mm
Tensioning stress $\sigma_{pm,0}$	1400 MPa
Strand cross section area A_s	139 mm^2
Force in each strand at release $F = \sigma_{pm,0}\ A_s$	194,6 kN
Force in each strand after long term prestressing losses $F_\infty \cong 0{,}85F$	165,4 kN

We evaluate the anchoring length, by following the procedure defined previously.

$$Tensile\ strength\ of\ concrete \quad f_{bpt} = \eta_{\rho 1}\eta_1 f_{ctd}(t)$$

$$7\ wires\ strands + good\ bond\ conditions \implies f_{bpt} = \eta_{\rho 1}\eta_1 f_{ctd}(t) = 3{,}2 \times 1 \times 1{,}6\ [MPa] = 5{,}15\ MPa$$

$$Basic\ value\ of\ transmission\ length \qquad l_{pt} = \alpha_1\alpha_2\phi\frac{\sigma_{pm,0}}{f_{bpt}}$$

$$7\ wires\ strands + gradual\ release \implies l_{pt} = \alpha_1\alpha_2\phi\frac{\sigma_{pm,0}}{f_{bpt}} = 1 \times 0{,}19 \times 15\ [mm] \times \frac{1400}{5{,}15} = 775\ mm$$

$$Transmission\ length\ (global\ effects) \quad l_{pt,2} = 1{,}2 l_{pt} = 1{,}2 \times 775\ [mm] = 930\ mm$$

So, the black line defines the forces that can enter inside the bearing, strand by strand, i.e. the distance in proportion of 80 cm defines the amount of force applied inside the bearing – the remaining part is transmitted far away.
The mean available length will be 400 mm in region a – one half –, 800 mm in region b and 710 mm in region c.

By consequence, the horizontal force available in the gravity centre of the bearing will be given by the available length over the total transmission length multiplied by the number of strands in each region.

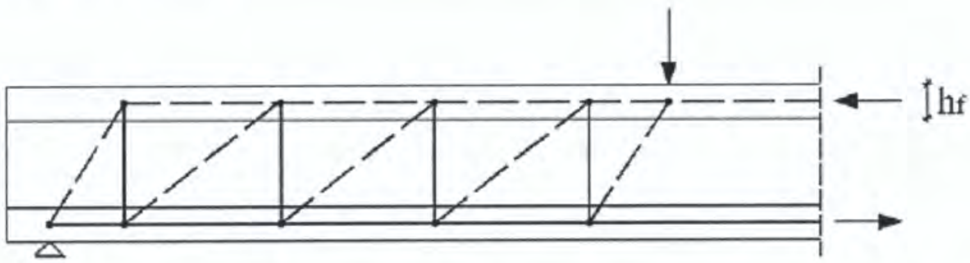
$$F_{Rd} \cong \left(\frac{400}{930} \times 16 + \frac{800}{930} \times 27 + \frac{710}{930} \times 3\right) \times 165{,}44 = 32{,}4 \times 16{,}4 = 5359\ kN > F_{Sd}$$

As the resisting force due to prestressing – the already entered force – is bigger than the required one, we do not need any longitudinal ordinary reinforcement because here is 80 cm of distance bearing-end of the beam and, in this distance, a large amount of prestressing is already entered.

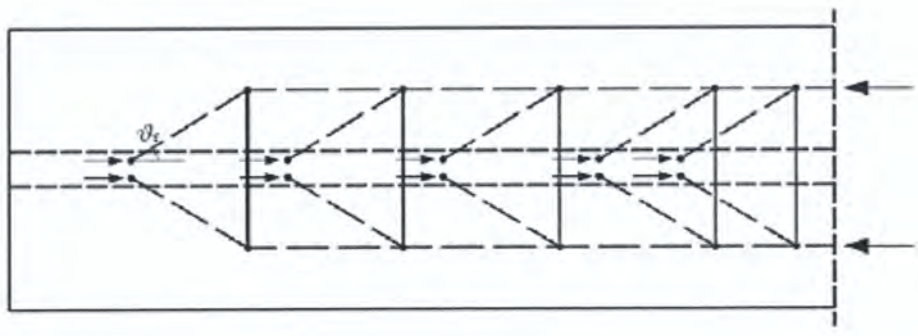
Flange-web connection

In presence of double-T and box sections, there is the necessity to connect the slab with the web.

The resisting mechanism consists of a truss in the slab – it is presented in the case of positive bending, i.e. top slab compressed and bottom slab tensed.
Focusing on the web, in presence of a concentrated force applied on a point, this is transferred by a strut to the neighbouring stirrup. Then, it is suspended and transferred to an other stirrup with the angle of truss for shear and so on until the bearing.



In the top slab, there are two compressive longitudinal forces applied on the gravity centre of each half slab that arrive to a certain point. Here, two inclined compressed struts start and there are tensile forces that need to be equilibrated. In the other point, due to the strut which is arriving here, there are two compressed struts and we need again transversal reinforcement. Thus, at each point, there is dispersion of load along the slab.



OBSERVATION: Value of η

An approach may be to assume that, given two strips of slab, which is b wide, and the web with width b_w, the percentage defined by the width ratio will enter directly in the web.

$$\eta = \frac{b_w}{b}$$

Actually, other values can be assumed.

If the beam is subjected to high concentrated loads – it means high shear – in regions having high bending moment, at the same time there will be strong longitudinal action in the slab and the necessity of transmitting also high shear on the slab. It means that in the region of the slab over the web, where there is a maximum resistance in the longitudinal direction equal to $b_w h_f \times 0{,}85 f_{cd}$, we may have to transmit a bigger compression due to the diffusion of the load. So, maybe it is not possible to transmit shear over the web in the slab and so we assume

$$\eta = 0$$

It means that all the compression due to shear is transmitted to the slab.
It is a limit case, corresponding to assume that nothing remains on the region of the slab which is over the web.

Generally, there are more distributed loads – high bending moments with not high shear - and total width is in excess with respect to the bending required one. In this case, in the half flanges on each side of the web, we can transmit a force equal to the shear minus the maximum longitudinal force which can be carried by the flange.

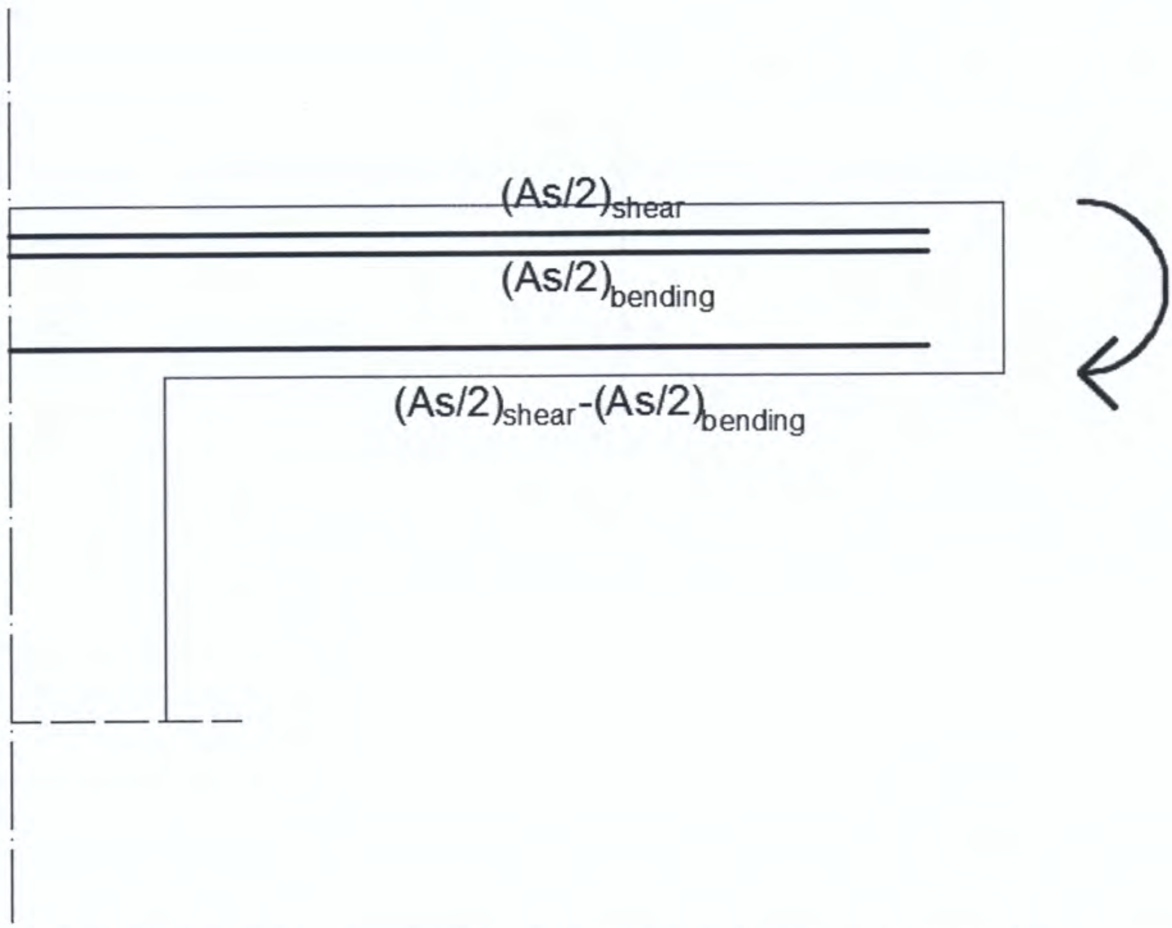
$$\frac{1}{2}\frac{M_{Sd}}{z} - b_w h_f \times 0{,}85 f_{cd}$$

In this situation, part of the longitudinal force is going inside the web and the remaining is divided between the two adjacent parts. The advantage is that there is less compression in the slab and, by consequence, less need of transversal reinforcement.

An other problem in the connection slab-web is that generally there is a transversal load on the slab and, by consequence, transversal bending, together with longitudinal shear coming from the overall behaviour of the structure.

Considering the web axis, there are an additional bending moment m – transverse bending moment per unit length – due to the local effects.
The horizontal force on the tensed side of a slab (generally evaluated at the beginning of the cantilever of the section) to be equilibrated will be given by the contribution of dispersion effect – due to longitudinal shear per unit length ν – and the contribution of transverse bending moment m.

$$\frac{\nu \tan\vartheta}{2} + \frac{m}{z_f}$$



In the support regions, the longitudinal bars in the web or close to it must bear the following force.

$$N_{Sd} = \frac{V_{Sd}}{2} \cot an\, \vartheta$$

Finally, when we consider the tensed region, the shift of longitudinal forces with respect to the compressed slab causes a translation of the region where shear is evaluated. By consequence, when we evaluate the required amount of reinforcement in one section due to bending, we have to anchor beyond an other section due to two struts of dispersion of longitudinal force inside the bottom slab. Thus, with respect to the verification performed in section A, reinforcement has to be prolonged up to position B.

EXTERNAL PRESTRESSING

1 External prestressing has been introduced in codes very recently - about 1990s - but the first application of this technique was adopted by Esslinger to build a bridge over a railway station in 1930s.
At that time, behaviour of prestressing steel in presence of corrosion was not completely known and no sophisticated protection of tendons has been adopted - just a paper with bitumen put around the tendon.
This protection system was not efficient and, due to the smoke coming from locomotives, tendons corroded. So
So, the bridge has been demolished and a new one has been built, with internal prestressing.

Today, we have an higher level of protection and a good knowledge about durability.

2 OLD AND NEW CODES

The indications about external prestressing in codes followed an evolution in time related to the evolution of knowledge about the structural behaviour in presence of external prestressing.

→ DM96:

external prestressing should be considered when there is no contact between concrete and prestressing steel and there is a relative slip between them.

→ EC2 : 1

in case of external prestressing, we should take into account the overall behaviour of the structure and the deformations of the whole structural element in the ULS for bending.
For instance, if the tendon is placed at the gravity center of the beam, the force in the tendon does not change and will not increase at ULS.

→ EC2 : 1.5

There are more specific indications about the use of prestressing tendons

→ in external prestressing, as steel and concrete have not the same deformation, ~~we~~ new models should be introduced - different from the ones used for internal prestressing.

→ in external tendons, ductility is ignored and no stress redistribution is allowed (as their effect is seen as external forces) in structures built with joints without crossing reinforcement.

→ EC2 : 2

Some rules change because

→ monolitic model is not ~~more~~ valid for segments with not reinforced joints

→ the limit of the compressed zone in joints is not valid

This is in contrast with the previous indications

→ CEB - FIB · MC90 - similar to EC2 : 1.5

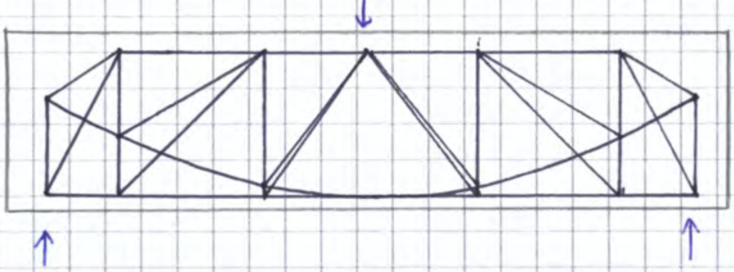
In conclusion, there is a good level of knowledge for ULS for bending, whereas there are lots of uncertainties for shear and torque moment.

We can also define two kind of structure, in relation with which elements there are in addition to external tendons

→ structures with ordinary reinforcement

→ structures with only external tendons

we will refer to MC90 model.



So, there are two models

→ arch model in case of thick web, that passes to truss system in case of cracking

→ division of prestressing force in case of thin web, that passes to truss system.

In any case, we define

λ = amount of load transferred with the initial mechanism

$1-\lambda$ = amount of load in the truss system

At this point, we can analyze what happens in the truss system

→ TENSED CHORD

The force to be equilibrated longitudinally is given by the expression of the truss system in case of only ordinary reinforcement reduced by the prestressing force in the tensed chord - not the whole prestressing chord because it migrates.
Assuming - for sake of simplicity - a beam with internal tendons, the resisting force is given by a contribution of longitudinal reinforcement and the residual strength available in tendons - the variation of stress between $\sigma_{t,\infty}$ and f_{ptd}.

$$(1-\lambda)\frac{M_{sd}}{z} + (1-\lambda)\frac{V_{sd}}{2}(\cot\vartheta - \cot\alpha) - F_{pb} = A_s f_{ytd} + A_p f_{pyd,net}$$

In case of external prestressing, acting forces are the same, whereas the increase of prestressing force in tendons is very small and this resisting contribution should be neglected.

$$A_p f_{pyd,net} = 0$$

$$(1-\lambda)\frac{M_{sd}}{z} + (1-\lambda)\frac{V_{sd}}{2}(\cot\vartheta - \cot\alpha) - F_{pb} = A_s f_{ytd}$$

In conclusion, the only difference between unbonded tendons and bonded tendons is the longitudinal force in the evaluation of shear, in order to compensate the lack of increase of prestressing force in case of external tendons, as they are unable to increase their stress.

This model has been confirmed by experimental tests realized on beam with the same geometry and same prestressing force applied on internal or external tendons.
The results of these tests show two different families of shear failure

→
- shear-compression, in which occur yielding of stirrups and then failure of the compressed chord.
- diagonal tension, in which occur yielding of stirrups and then failure in tension of the tensed chord

This family is characterised by the yielding of stirrups

→ web crushing, in which occurs failure due to compression before yielding of stirrups.

In case of web crushing, experimental results show no differences between beam with internal tendons and beam with external tendons.

In case of failure due to shear-compression or diagonal tension, the resistance of the beams with external prestressing is about 10% smaller than the resistance with internal prestressing, due to the bigger deformability of tensed chord, which does not take the contribution of the stress increase in the cables.

Thus, the model is valid in all cases and, in the second case, we should increase the resistance in the tensed chord:

to get the same value of resistance, we cover the gap by adding more ordinary reinforcement.

$$A_s' f_{ytd} = A_s f_{ytd} + A_p f_{ptd,net}$$

In other words, we can say that the only difference between external prestressing and internal prestressing is that, with external prestressing, we need to increase longitudinal reinforcement in beams to take into account the need to equilibrate the tensed chord at ULS.

As torsion has been transformed in shear, the mechanism in a wall is the same seen before with

→ EQUILIBRIUM IN THE LONGITUDINAL DIRECTION

The axial force and the longitudinal contribution of shear should be equilibrated by longitudinal reinforcement and the contribution due to the increase of prestressing force.

$$N_{sd,i} + V_{sd,i} \cot\theta = A_{s,i} f_{yd} + A_{p,i} f_{pyd,net}$$

In case of external prestressing, the contribution is null and we should increase the area of longitudinal reinforcement $A_{s,i}$ to equilibrate the loss.

$$N_{sd,i} + V_{sd,i} \cot\theta = A_{s,i} f_{yd}$$

→ COMPRESSION FIELD in the web

The equation does not change in case of external prestressing.

$$\frac{V_{sd,i}}{\sin\theta_i} = f_{cd,2}\, t_i\, z_i \cos\theta_i$$

t_i = wall thickness

→ TENSION FIELD

The equation does not change in case of external prestressing.

$$V_{sd,i} = A_{sw,i} f_{yd} \cot\theta_i \frac{z_i}{s}$$

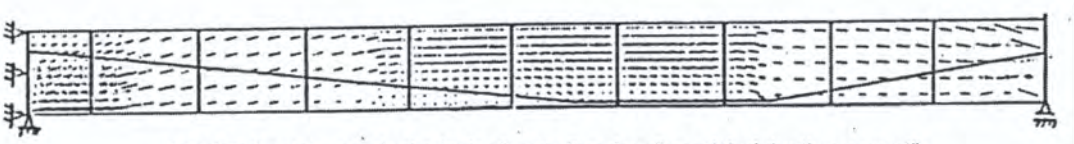
$A_{sw,i}$ = stirrups area s = stirrups spacing

In conclusion, in case of unbonded tendons, in design for shear and torsion in monolitical element with ordinary reinforcement, the only difference is the design in the tensed chord because, in case of bonded tendons, we would have an additional resisting contribution, which is the increase of stress in tendons.
In external tendons, this increase is small and it is assumed null for safety.

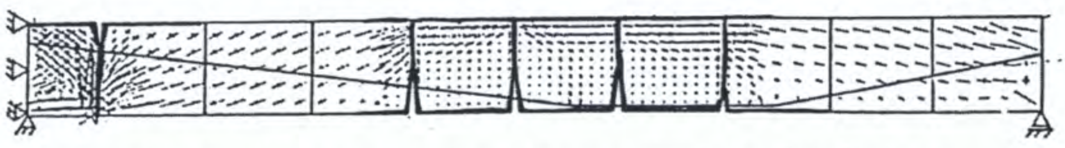
COMPORTAMENTO A TAGLIO DI ELEMENTI PREFABBRICATI A GIUNTI NON ARMATI

Tali strutture alla S.L.U. tendono a comportarsi come un insieme di blocchi rigidi connessi da zone di minor resistenza e deformabilità nelle quali si concentra la non-linearità della risposta di insieme.

Questi tipo di comportamento è stato individuato sia per via sperimentale (Fouré et al.), che per via numerica (Eibl et al.).



(a) Vista frontale: tensioni principali sotto il carico di servizio (giunti non aperti)



(b) Vista frontale: tensioni principali sotto 1,75 volte il carico di servizio (giunti aperti)

Occorre quindi individuare i meccanismi resistenti per mezzo dei quali il taglio può trasmettersi attraverso i giunti compressi su una limitata estensione dell'altezza where

$$c = 0{,}5\, f_{ctd}$$

$$\tan\psi = 0{,}5\, f_{ctd}^{1/3}$$

$$\sigma = \frac{N}{A}$$ Stress orthogonal to the key – if exists –, given by the compressive force N in the shear key

If we want to write the expression of the full shear strength of the shear keys we should consider that some keys fall in the tensed region and there is not the stress σ, whereas some keys fall in the compressed region.
Thus, the resistance due to shear keys is

$$V_{Rd} = N \tan\varphi + \sum A_{k,i} \left(c + \sigma_i \tan\psi \right) + \sum A_{k,i}\, c$$

- $N \tan\varphi$: shear transmitted by friction
- $\sum A_{k,i} (c + \sigma_i \tan\psi)$: shear transmitted by keys in the compressed region of the joint
- $\sum A_{k,i}\, c$: shear transmitted by keys in the tensed region of the joints

To evaluate numerically V_{Rd}, we need to know how many shear keys are in the tensed region and how many shear keys are in the compressed region and the axial force closing the joints.

⇒ we need to solve the problem of bending and define, from local analysis,

→ neutral axis, to identify the keys in the tensed region and the keys in the compressed region.

→ axial force, given by the bending moment over the internal lever-arm.

There is a compensation between these two mechanisms because, if the opening takes a big value - opening is bigger than the required to equilibrium - and M_{sd}/z decreases, the rising of the compressive field will be smaller than the required by the shear increase.

At this point, we can see the verification of resistance of the material:

the resistance of the compressive field is given by the same formulation of monolitic structures.

$$\sigma_c = \frac{V_{sd}}{b_w h_{red} \sin\vartheta \cos\vartheta} \leq f_{cd,2} = 0{,}6\left(1 - \frac{f_{ck}}{250}\right)$$

h_{red}: it is placed instead of z because section is partly opened and its depth is reduced.

According to this formulation, the maximum opening of joint allowed is reached when

$$\vartheta = 45° \quad \text{and} \quad \sigma_c = f_{cd,2}$$

because this produces the maximum value of $\sin\vartheta\cos\vartheta$ and, to keep σ_c equal to $f_{cd,2}$, h_{red} should assume its minimum value, i.e. the opening is maximized.

By putting these conditions, we derive the value h_{red}, which is the minimum extent of the compressed region in the joint which is necessary to transmit shear.
From it, as the residual depth h_{red} depends on bending, we could derive the bending moment.

In conclusion, there are two levels of verification

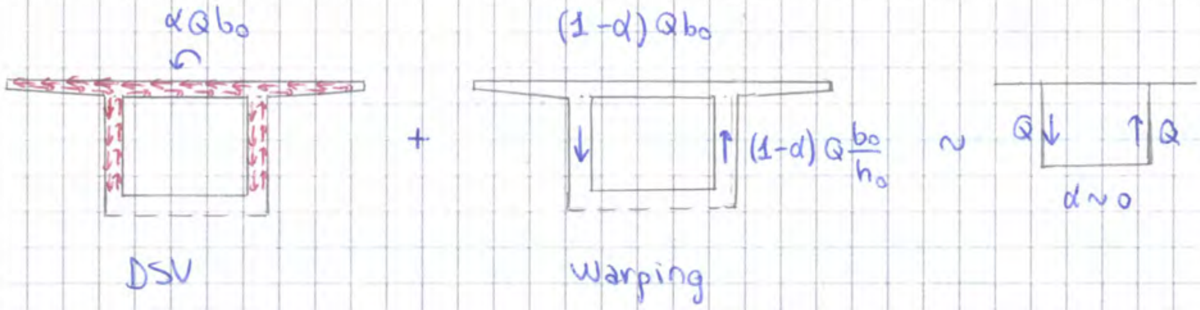
→ verification related to rigid blocks, from which we derive

$$h_{red,1}$$

→ verification related to concrete resistance

$$h_{red,2}$$

In design, the bigger value dominates - generally, material verification governs design - and from it we evaluate the resisting bending moment M_{Rd}.



The amount carried by each mechanism is defined by the parameter α

$$0 \leq \alpha \leq 1$$

With the usual dimensions for the webs and the slabs, we have

$$\alpha \sim 0$$

and the bigger amount of torsion is carried by the warping torsion mechanism.
At the limit case - $\alpha = 0$ -, the forces Q and Q are directly applied in the webs.
So, the difference between these two situations is that

→ when joint is closed, load $Q/2$ is applied in the webs

→ when joint is open, at limit, load Q is applied in the webs.

So, when the joint is openend, along the webs there is a discontinuity of shear because the torsional contribution is doubled passing from the segment to the joint.

In conclusion, design for torsion is performed by introducing internal actions discontinuity due to change of torsional mechanism at the opened joints.

A solution consists of adopting shear keys in the slabs, in order to transmit shear also in the bottom slab when the joint is opened.
In this way, as horizontal shear can be transmitted between segments, we remain in the first situation, i.e. Bredt mechanism.

$$A_{s,min} = \frac{M_{up}}{z_s f_{yk}}$$

M_{up} = cracking moment evaluated with f_{ctm} and no prestressing. In case of segmental construction, f_{ctm} is null and there is no reinforcement in the joints.

Minimum reinforcement should be placed in the regions in which, at the characteristic combination, concrete is tensed and tensile stress reaches f_{ctm}.

As regards prestressing, its contribution is neglected in case of isostatic scheme whereas hyperstatic bending is accounted because it derives from overall behaviour and it is not influence by the local events in the section.

In case of PRETENSED BEAMS, as corrosion starts from the external layers, we can consider as active the strands with a cover c which is

$$c > 2c_{min}$$

c_{min} = minimum cover prescribed for durability reasons

and they work with $f_{p,01,k}$ with a limit of $\Delta\sigma_p \leq 500$ MPa, if $\sigma_p \leq 0{,}6 f_{ptk}$.

In case of continuous beam, reinforcement should be extended to bearings, in order to avoid brittle failure there.

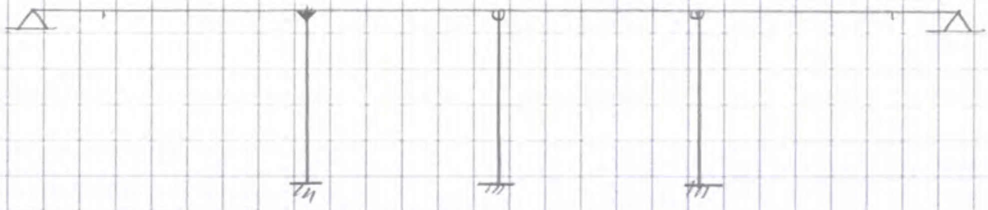
In case of BOX GIRDER BRIDGES, this extension is not necessary if at ULS failure occurs only in steel, that corresponds to impose

$$\underbrace{A_s f_{yk} + A_p f_{p,01,k}}_{\text{Reinforcement contribution in tensed chord}} < \underbrace{t_{inf}\, b_0\, \alpha_{cc} f_{ck}}_{\text{Resistance of compressed chord}}$$

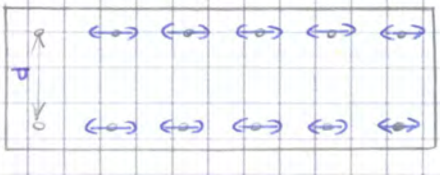
t_{inf} = lower slab's thickness

In this way, we get a ductile behaviour because there are cracks.

→ we ensure the possibility to control and substitute tendons in case of corrosion problems.
It means to adopt external tendons.



GEOMETRICAL CONFIGURATION OF BEARINGS: TRANSVERSAL ARRANGEMENT



If the transverse distance of bearings is not big, i.e.

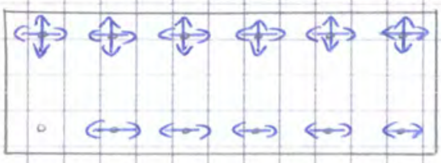
$$d < 5 \text{ m}$$

2

we can introduce fixed bearings on the abutment and all the other bearings will be one-directional. These allow the movement only in the longitudinal direction, whereas they restrain the transverse movement.

In case of large deck, i.e.

$$d > 8 \text{ m}$$

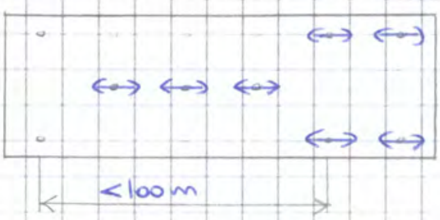


one alignement is made free. So, there will be only one fixed point, an alignment in which longitudinal movement is allowed and the other alignment is completely free.

Sometimes, we need to introduce small piers along the span, during construction stages.

IF the two alignments of piers are at a smaller distance than 100 m, we can have just one pier with just the longitudinal movement free.

In intermediate bearings, there is no control of torque moment, which is closed between the abutment and the first alignment where there are two bearings - intermediate bearings do not exist for torque moment.



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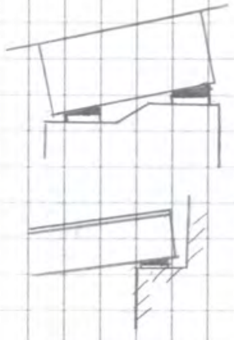


In case of deck inclination in the longitudinal or in the transverse direction, bearing should work in the horizontal plane in order to avoid the rise of transversal or longitudinal actions due to permanent loads.
So, we need a casting for the compensation of inclination, made with high strength concrete and epoxide resine.

Sliding surface

The design of the sliding surface is important because deck moves with respect to bearings and this surface gives the possibility to movement. If the surface was not enough, bridge would fall down from the bearings.

The design of the sliding surface is based on the entity of displacements of the bridge, with different contributions:

→ CREEP

$$\varepsilon_S = 0{,}2 \div 0{,}3 ‰$$

→ ELASTIC STRAIN DUE TO PRESTRESSING, at the level of the bottom chord.

→ SHRINKAGE due to prestressing, in presence of permanent loads.

→ ELASTIC STRAIN due to external actions

→ THERMAL EFFECTS, depending on situ conditions and assumed as

$$\Delta T = \pm 30°$$

between winter and summer.

We see now the analysis of movement in a bridge over 5 bearings that has inversion of fixed point from the construction to the end of construction.
We want to design the sliding surface of the medium bearing, so we analyse the movements here.

→ during construction, we have movements due to prestressing, partial creep and partial shrinkage, occurring towards the fixed point. There is also the movement due to thermal effects, that can be in both directions depending on the season.
We may also consider the wind contribution, only if the bridge is sensible to it.

For very large movements, we use a machinery having tilts that move along guide wheels.

Elimination of water from the deck

The system of water elimination should be designed accounting the expected maximum amount of precipitation for m^2 on the deck, according to a return period of 200 ÷ 500 years.
Design should be "generous" and we should not assign only the neat section, because it is available only when cleaned, but it would need frequent maintenance.

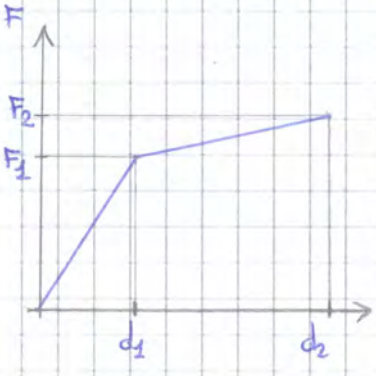
Sometimes, for aesthetic reasons, water may be collected and carried inside the box section but it must be avoided because no liquid should enter inside it.

Restraint elements

There are restraint elements able to dissipate energy and they are used in presence of earthquakes.
For instance, there is a hydraulic system realized with two rooms in communication through a pipestone having very small holes and the system is filled with oil.
When there is an external action, oil should move from one room to the other passing through these holes and there is a viscous resistance which dissipates energy.
Actually, after few cycles, temperature increases as mechanical energy becomes heat and viscosity gets smaller. So, efficiency is reduced.
Moreover, to evaluate its efficiency, this device needs continuous maintenance.

Another device presents a rubber element having hyper-elastic behaviour.

There are also special bearings made of rubber elements with inside lead (= "piombo").
The rubber element allows movement and lead reduces deformability.
This device, at the end of oscillation, put the structure in the initial position - recentering.



Efficiency of devices is measured on the ENERGY DISSIPATION, that corresponds to the area under the curve describing the behaviour in the force - displacement diagram.
In case of elasto-plastic behaviour, the efficiency is

$$\Omega = \frac{\text{Cycle area}}{4 F_2 d_2}$$

→ with 4, we consider all the signs for force and displacements

Dettagli costruttivi per l'eliminazione delle acque meteoriche dal piano stradale

La soluzione con spargimento diretto dell'acqua deve essere valutata accuratamente per evitare che le acque di scolo investano le strutture.

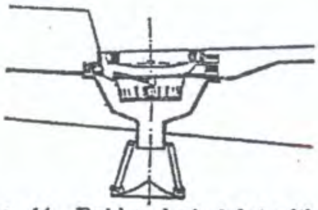


Fig. 44 Bridge drain inlet with vertical drain and disperser

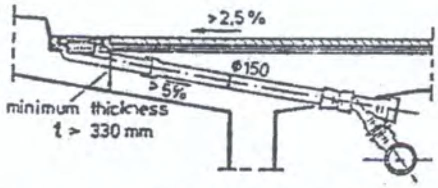
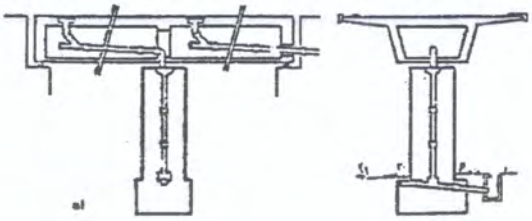


Fig. 45 Bridge drain inlet with side drain



Dispositivi per collegamenti rigidi → realizzano, senza assorbimento di energia, la trasmissione delle forze tra due elementi strutturali con spostamenti relativi trascurabili. Nel dimensionamento un fattore di iperesistenza rispetto agli elementi da collegare $\gamma_{oh} \approx 1,5$

Dispositivi reagenti a forze di tipo impulsivo → forniscono una risposta "R" dipendente dalla velocità con cui viene imposto uno spostamento relativo x agli elementi accoppiati In pratica $R \approx 0$ per valori piccoli di dx/dt (effetti di ritiro, fluage e variazioni di temperatura); $R \neq 0$ e $x_{rel} = 0$ per azioni impulsive (sisma, forze di frenatura,..)

Dispositivi a comportamento prevalentemente elastico → forniscono una risposta funzione lineare dello spostamento relativo presente (assenza di dissipazione, indipendenza dalla storia di carico e dalla dx/dt, capacità di recuperare la configurazione iniziale). Possono anche avere una certa dissipazione. Vengono usati per spostare il periodo proprio della struttura isolata in un intervallo di valori in cui il sisma ha meno effetto ($1 \leq T \leq 3$ sec). Possono essere realizzati anche con apparecchi in gomma armata funzionanti a taglio o a compressione, per sforzo normale.

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PIERS

1 MATERIALS → steel (rare - used for urban highways)

→ reinforced concrete or prestressed concrete (slender piers or segmental piers assembled with prestressing)

2 Geometry of the piers

As regards transverse section, we distinguish

→ monolithic section - full concrete section - for small depth, up to 20 m.

→ box section with one or two cells - if one or two decks are carried - and variable profile, with section reduction, when depth is bigger than 20 m

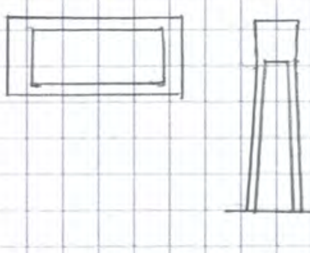
Sometimes, twin piers are adopted for small structures because of aesthetic reason and rational position under the webs. Indeed, with this solution we avoid the use of big piers, that look like walls. We may also adopt single piers remembering that, in presence of a box section deck, connection needs attention due to torsion.

The typical section shapes are circle, ellipse - useful in the current of river, as it opposes less resistance to water flow (if current changes position frequently, it would be better a circular section) -, square and squad piers.

As regard box sections, the minimum thickness of each wall is

$$t_{min} = 30\ cm$$

and section changes along the depth.



At the top, there is a small increase in the section and a segment in full concrete because here bearings are placed and there is concentration of actions.
Generally, at the top we use 2 bearings.

1. Description

Materials

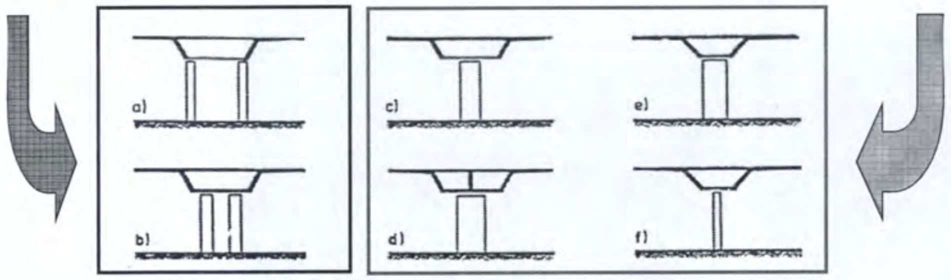
- Steel
  - Generally not used except for urban highways
- R.C. / P.C.
  - Reinforced concrete is more used. Prestressed concrete is limited to segmental piers assembled with prestressing

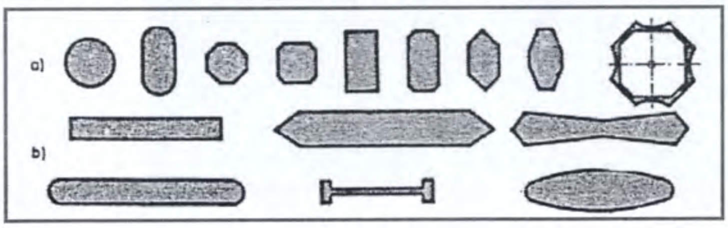
Geometry

- Monolithic sections for small hights10 ÷ 20 m)
- Box shaped sections (one or more cells) for bigger hights. Variable profile and thickness may be adopted (self rising formworks can be used)
- Coupled bearings (on wide piers) to improve the bending restraint of the spans of long span bridges.

Twin piers are to be preferred for small structures because of aesthetic and rational position under the webs.

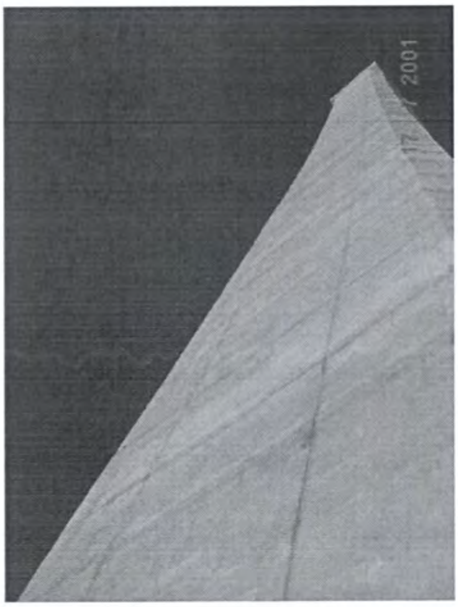
Single pier may result charming but gives rise to transverse problems related to diaphragms and torsion.



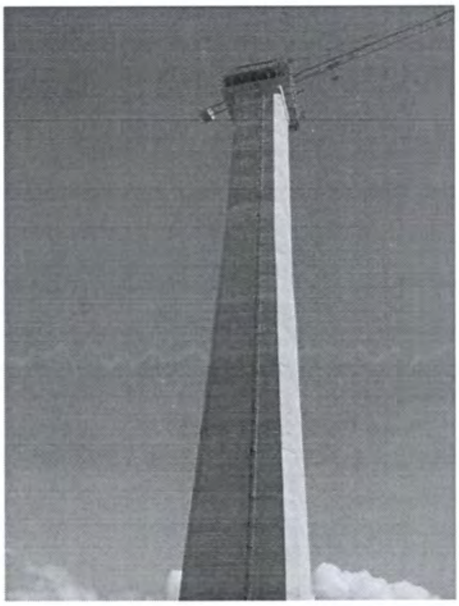


Circular, elliptic and polygonal sections are to be preferred to rectangular ones for aesthetics.

Bottom prospective with crane



Bottom prospective



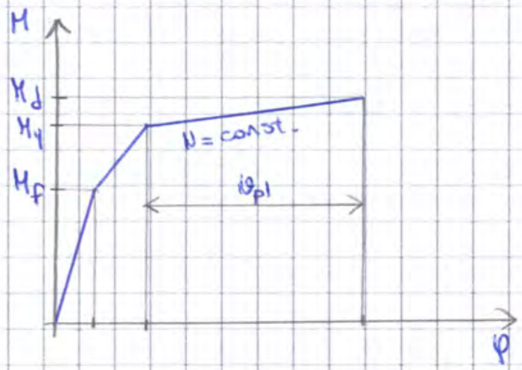
Pier and crane

- ice pressure exerted at the moment of it's formation

5 Evaluation of internal actions

Once external actions have been defined, we evaluate the internal actions at ULS.
In this evaluation, we have to take into account of two types of non linearities

→ mechanical non linearity



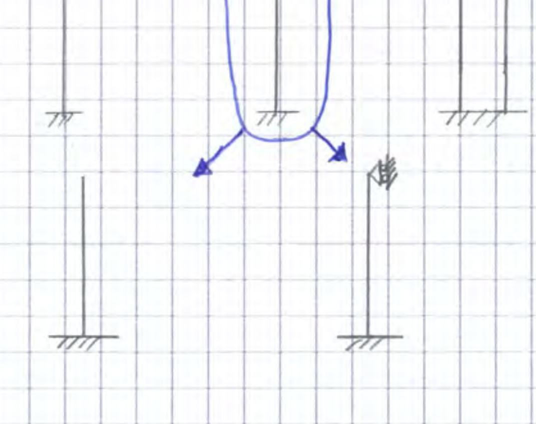
Increasing the load of bending moment, axial force and shear, we have cracking and plastic rotation, that are non linear phenomena.
In presence of linear elements – 1D elements –, non linearity is described in the bending moment – rotation diagram.
The diagram presents a linear behaviour up to cracking bending moment, than the slope assumes a new value up to yielding bending moment. Finally, there is the plastic stage of deformation, with plastic rotation ϑ_{pl} and a new slope, up to failure.

→ geometrical non linearity

Geometrical non linearity intervenes due to

→ SLENDERNESS of the pier, for which the position of loads at ULS is different from the beginning. So, we have to take into account structure deformation to evaluate II order effects.

→ VARIATION OF STATIC SCHEME during the increase of loads.

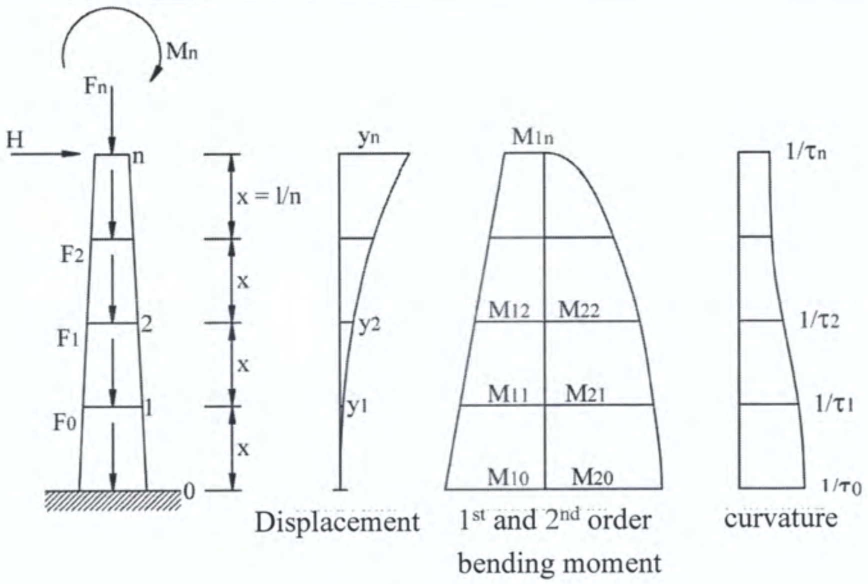


Considering a bridge with isostatic beams, each pier is a cantilever up to a certain level of horizontal force.
Increasing this force, the horizontal displacement gets bigger ~~and~~ but the abutment has a limited strock.
By consequence, at a certain point, it becomes a horizontal bearing and the pier does not move anymore.
Thus, the pier becomes a cantilever with a horizontal bearing at the top.

Case	Idealized column and buckling mode	Restraints: Location	Position	Rotation	Effective height, l_0
1	L	Top Bottom	Full Full	Full * Full *	0,70 L
2	L	Top Bottom	Full Full	None Full *	0,85 L
3	L	Top Bottom	Full Full	None None	1,0 L
4	L Elastomeric bearing	Top Bottom	None * Full	None * Full *	1,3 L
5	L	Top Bottom	None Full	None Full *	1,4 L
6	L	Top Bottom	None Full	Full * Full *	1,5 L
7	L or L	Top Bottom	None Full	None Full *	2,3 L

Table 1

* assumed value



Taylor series approximation of the displacement of the pier:

$$y_n = y_{n-1} + xy'_{n-1} + \frac{x^2}{2} y''_{n-1}$$

The pier is divided into a number of segments, having height equal to the bigger depth of the section.
Each segment is subjected to the horizontal force H, bending moment M_n and axial force F_n, given by the value at the top combined with self-weight of the pier.

Pier displacement y_i is described by a certain diagram.
Bending moment has a I order contribution, which is trapezoidal – "it starts from M_0 and it increases due to the horizontal force –, and a II order contribution, given by axial force F_i working with an eccentricity y_i.
Considering the deformative contributions, the result is a curvature diagram.

The approach starts from the approximation of the displacement y_n of the pier, by applying Taylor series.

$$y_n = y_{n-1} + x\, y'_{n-1} + \frac{x^2}{2} y''_{n-1}$$

The derivatives are obtained as finite differences.

$$y'_{n-1} = \frac{1}{2x}\left(y_n - y_{n-2}\right)$$

$$y''_{n-1} = \frac{1}{r_{n-1}} \rightarrow \text{curvature of the section}$$

We substitute the derivatives as finite differences in Taylor approximation.

$$y_n = y_{n-1} + x\, y'_{n-1} + \frac{x^2}{2} y''_{n-1} =$$

$$= y_{n-1} + \cancel{x} \cdot \frac{1}{2\cancel{x}}\left(y_n - y_{n-2}\right) + \frac{x^2}{2}\frac{1}{r_{n-1}} =$$

$$= y_{n-1} + \frac{y_n}{2} - \frac{y_{n-2}}{2} + \frac{x^2}{2}\frac{1}{r_{n-1}}$$

$$\Rightarrow \frac{1}{2} y_n = y_{n-1} - \frac{y_{n-2}}{2} + \frac{x^2}{2}\frac{1}{r_{n-1}}$$

$$\Rightarrow y_n = 2y_{n-1} - y_{n-2} + \frac{x^2}{r_{n-1}}$$

With the same expression, we can write y_{n-1}

$$y_{n-1} = 2y_{n-2} - y_{n-3} + \frac{x^2}{r_{n-2}}$$

As a first approximation, we assume that all sections have the same curvature

$$\frac{1}{r_i} = \frac{1}{r_0} \qquad r_0 \text{ is assumed}$$

In this way, we obtain the deformed shape

$$y_1 = x^2\left(\frac{1}{2r_0}\right) = \frac{1}{2r_0}x^2$$

$$y_2 = x^2\left(\frac{2}{2r_0} + \frac{1}{r_0}\right) = x^2\left(\frac{4}{2r_0}\right) = \frac{1}{2r_0}\cdot(2x)^2$$

$$y_3 = x^2\left(\frac{3}{2r_0} + \frac{2}{r_0} + \frac{1}{r_0}\right) = x^2\left(\frac{9}{2r_0}\right) = \frac{1}{2r_0}(3x)^2$$

$$(\ldots)$$

$$y_n = \frac{1}{2r_0}(nx)^2$$

At this point, knowing the I order bending moment and the deformed shape, we can evaluate the total bending moment M_i.

$$M_0 = M_{1,0} + F_n y_n + F_{n-1} y_{n-1} + \ldots + F_1 y_1$$

$$M_1 = \underbrace{M_{1,1}}_{\text{I order}} + \underbrace{F_n(y_n - y_1) + F_{n-1}(y_{n-1} - y_1) + \ldots + F_2(y_2 - y_1)}_{\text{II order}}$$

From the values of total bending moment and axial force, we can derive a more correct value of curvature r_0 for all the section.
By introducing this new value, we get a new deformed shape and a new total bending moment and so on.

The procedure is repeated and the result may be

→ convergence, which is high and little influenced by the starting value r_0.

→ divergence, which means that there is no equilibrium and failure.

In case of BIAXIAL ECCENTRICITY, we can perform a separate verifications in which we start from the actions in each direction and we evaluate the maximum bending moment in each direction for the same value of axial force.

As a first approximation can be chosen: $\frac{1}{\tau_i} = \frac{1}{\tau_0}$ Obtaining:

$$\begin{cases} y_1 = 0.5x^2 \frac{1}{\tau_0} \\ y_2 = \frac{x^2}{2}\left(\frac{2}{\tau_0} + \frac{2}{\tau_0}\right) = 0.5\left(2x^2\right)\frac{1}{\tau_0} \\ y_3 = \frac{x^2}{2}\left(\frac{3}{\tau_0} + \frac{4}{\tau_0} + \frac{2}{\tau_0}\right) = 0.5\left(3x^2\right)\frac{1}{\tau_0} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_n = 0.5\left(nx^2\right)\frac{1}{\tau_0} \end{cases}$$

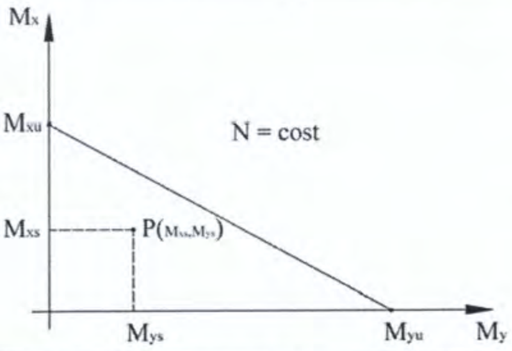
So, as the first order bending moments M_{1i} are known, the total bending moments M_i are:

$$\begin{cases} M_0 = M_{10} + F_n y_n + F_{n-1} y_{n-1} + \ldots\ldots + F_1 y_1 \\ M_1 = M_{11} + F_n\left(y_n - y_1\right) + F_{n-1}\left(y_{n-1} - y_1\right) + \ldots\ldots + F_2\left(y_2 - y_1\right) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{cases}$$

The curvatures $\frac{1}{\tau_i} = f\left(M_i, N_i\right)$ can be calculated and the procedure can be iterated as far as the results don't change from one step to the other.

The convergence of the procedure is high and little influenced by the starting value $\frac{1}{\tau_0}$

In the very common case of biaxial eccentricity, a linear interaction diagram can be used once the resisting bending moment in the two principal directions X and Y are known:



The point that represent the internal actions P(Mxs , Mys) should be inside the linear domain.

This procedure is *generally* conservative

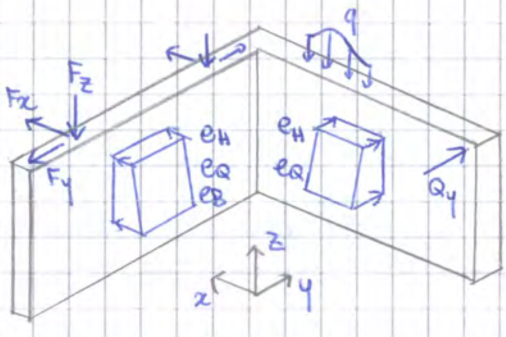
reinforcement at the bottom.
Indeed, the force arrives from the bearing in the middle of the pier and it becomes a uniform distribution at a distance d_2 from the top.
The forces line are bottle-like and tension is on the bottom.
The force Z is

$$Z = 0{,}2P$$

Then, the working stresses of the members of the strut & tie system need to be reduced of 20 ÷ 25 % as these models respect the equilibrium but not the congruence of the deformations.

2 Actions on the abutment

In design of abutment, we should consider the following actions



→ SELF WEIGHT

→ ACTIONS COMING FROM THE DECK F_x, F_y and F_z

→ ACTIVE EARTH PRESSURE coming from the embankment against the walls e_H

→ increase of earth pressure due to TRAFFIC LOADS on the embankment e_Q

→ increase of earth pressure due to BRAKING FORCES e_B over the embankment

→ service actions on the side wall q, e.g. footway supported by the side wall

→ lateral CRASH ACTIONS Q_y transferred by the safety barriers.

3 Verifications over an abutment

We have to satisfy different verifications

→ GLOBAL EQUILIBRIUM of the abutment as rigid body - overturning and translation along the longitudinal and transversal direction

→ BEARING CAPACITY of the ground to foundation or piles

→ SETTLEMENTS CALCULATION in serviceability condition, at short and long terms, due to permanent actions and, if important, variable actions.
In this way, we predict the settlement and place the wall in the correct position.

→ STRUCTURAL VERIFICATION OF THE ABUTMENT, designed with a static scheme depending on geometry of the abutment - plate or cantilever.

→ CRACK OPENING VERIFICATION, to grant durability

DESIGN OF REINFORCED CONCRETE SHELL ELEMENTS

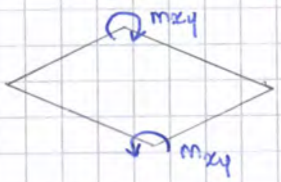
1 We focus on SLAB BRIDGES, in which deck is not made of beams but it presents a solid element where thickness is smaller than the other dimensions.
we will see a design procedure that can be used at ULS - not serviceability conditions -, since plasticity theory is adopted.
In design at SLS, approximate models are used.

2 Shell (= "guscio"):

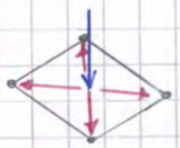
a shell is a 2-dimensional element - thickness is smaller than the other 2 dimensions - which is subjected to the following internal actions

→ 3 membrane components → axial force in x direction n_x
→ axial force in y direction n_y
→ in-plane shear $n_{xy} = n_{yx}$, that tends to deform and change the angle in the plane

→ 5 plate components → 2 bending moments in 2 orthogonal directions m_x and m_y



→ a torque moment $m_{xy} = m_{yx}$
→ 2 out of the plane shears in 2 orthogonal directions t_x and t_y



when a shell is loaded and it is supported on 4 bearings, shear will go in the bearings but it can be decomposed in t_x and t_y

In other words, SHELL ARE A SUPERPOSITION OF 2 STRESS STATES - PLATE STATE and MEMBRANE STATE - and they are made of internal actions working out of the plane and internal actions working in the plane.

3 Internal actions

The analysis of 2D elements and computation of internal actions is done only by using FEM today.
In the past, closed-form solution were used, but they were available only for simple conditions.
There are also tabular methods and the most known is Massonet method. Massonet used finite difference method and he solved by hands a huge number of plates under different load and bearing conditions.

From internal actions, we calculate reinforcement in the elements. Some codes allow to calculate it in an automatic way but internal actions are got perfectly right - according to the model used - because it is a simple calculation, whereas computation of reinforcement needs the introduction of many hypotheses and simpler model would lead to wrong results.

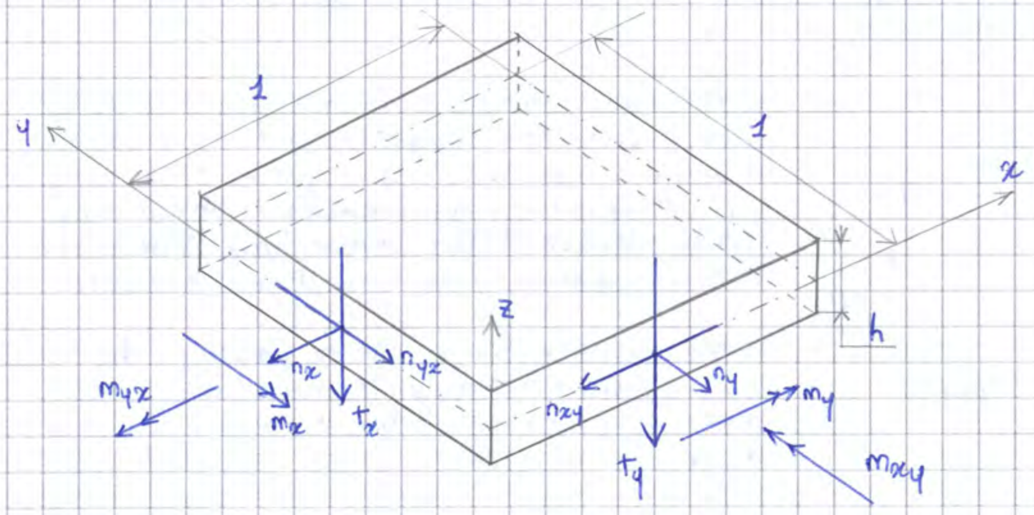
We have seen that they are 8 internal actions and they are given for unit length - for a length of 1 m

→ 3 membrane components $n_x, n_y, n_{xy} = n_{yx}$ kN/m

→ 2 bending moments m_x, m_y kNm/m

→ 1 torque moment $m_{xy} = m_{yx}$ kNm/m

→ 2 out of plane shears t_x, t_y kN/m



→ BENDING MOMENTS m_x, m_y

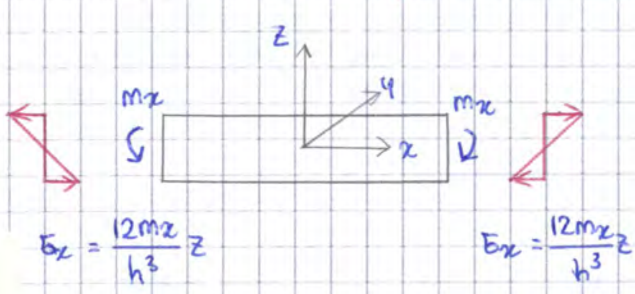
In beams, we follow the rule according with which bending moment is given the name of the axis around which we have rotation. For instance, M_x is the bending moment that generates rotation around x axis and it gives rise to σ_z or σ_y stresses.
In 2D elements, m_x is the bending moment that gives rise to σ_x stresses and it causes a rotation around y axis.
This new convention is adopted as it relates bending moment and stresses coming from it and it is more intuitive.

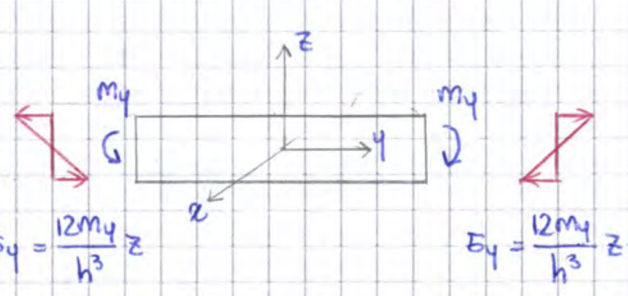
Bending moment m_x is positive if it gives rise to tensile stresses σ_x on the side with positive z axis.
This definition works on the local axis of the element, i.e. z is not a vertical axis but it is orthogonal to the plane of the element and it identifies the positive face - top face.

$$m_x > 0 \quad \text{if} \quad \sigma_x(z) > 0,\ z > 0$$

Bending moment m_y is positive if it gives rise to tensile stresses σ_y on the side with positive z axis.

$$m_y > 0 \quad \text{if} \quad \sigma_y(z) > 0,\ z > 0$$





→ TORQUE MOMENT m_{xy}

Torque moment m_{xy} generates τ_{xy} tangential stresses applying on the surfaces and it is positive if

→ on the $x > 0$ face, τ_{xy} stress is in the positive y direction on the top surface and in the negative y direction on the bottom surface

→ on the $y < 0$ face, τ_{xy} stress is in the negative x direction on the top surface and in the positive x direction on the bottom surface.

Sandwich model

Sandwich model is the resisting model used and here the shell is decomposed into a sandwich, on the basis of the plasticity theory.

In using this model, the starting point is given by the internal actions

$$m_x \quad m_y \quad m_{xy} \quad n_x \quad n_y \quad n_{xy} \quad t_x \quad t_y$$

provided by the finite element code.

Then, we have to follow many steps.

I) Check of cracking

The thickness of the shell is divided into several layers - e.g. 20 layers - and stresses in these layers are evaluated in uncracked conditions.
In these conditions, stresses can be derived from the internal actions as follows.

$$\sigma_x = \frac{n_x}{h} + \frac{12 m_x}{h^3} z$$

$$\sigma_y = \frac{n_y}{h} + \frac{12 m_y}{h^3} z$$

$$\tau_{xy} = \tau_{yx} = -\frac{n_{xy}}{h} + \frac{12 m_{xy}}{h^3} z$$

$$\tau_{yz,max} = \frac{3}{2} \frac{t_y}{h}$$

$$\tau_{xz,max} = \frac{3}{2} \frac{t_x}{h}$$

Thickness is divided into layers as, due to bending, stresses vary in the thickness.

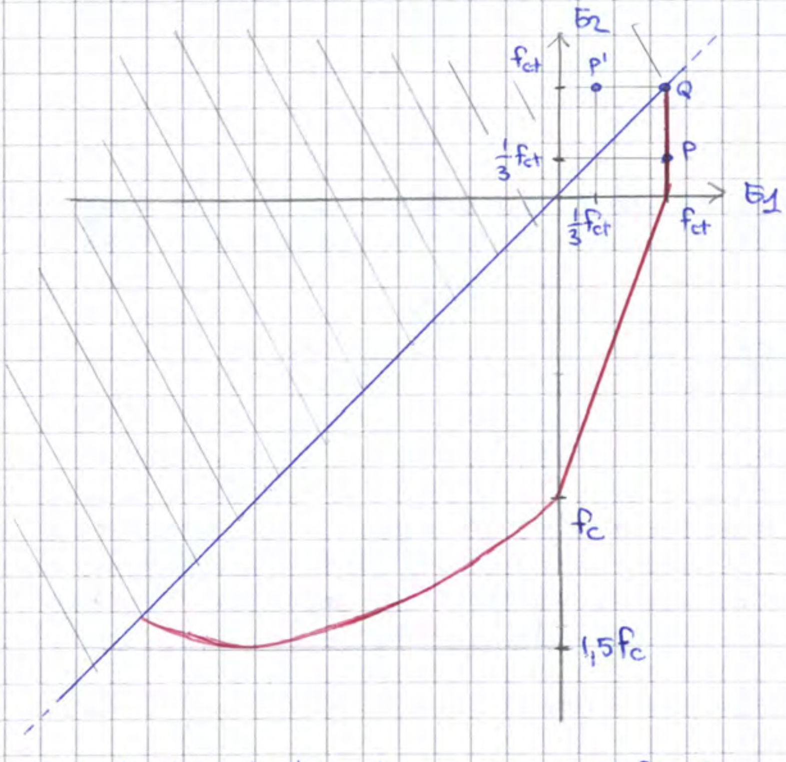
Once stresses have been evaluated, we have to verify if the element is cracked.

How to check cracking in multiaxial stress state?

In the second case, in order to establish if there is cracking or not, we have to refer to a failure criterion.

Just to have in idea, we introduce the kupfer & Gerstle failure curve, that represents the failure envelope on a 2D space.
On a shell element, stress state is tri-axial, but we introduce this criterion to keep in touch with the problem.

The curve is represented in the principal plane $\sigma_1 \sigma_2$ - in this case, σ_1 is the horizontal axis and σ_2 is the vertical axis, but they can be switched.
On these axes, we introduce the tensile strength f_{ct}.



Not all points in this plane can be defined.

→ point P, having

$$\sigma_1 = f_{ct} \qquad \sigma_2 = \frac{1}{3} f_{ct}$$

can exist.

→ point P', having

$$\sigma_1 = \frac{1}{3} f_{ct} \qquad \sigma_2 = f_{ct}$$

can't exist because σ_1 should be bigger than σ_2.

→ if failure occurs in an intermediate situation, we define a line that separates uncracked state and cracked state according
According to this line, concrete can bear same tension and compression together without crack and, more it is compressed in one direction, less we can tense it in the other direction as specimen is already tensed due to Poisson's effect.
So, there will be cracking for small tension stresses when they are together with high compression stresses and for small compression stresses when they are together with high tensile stresses - it doesn't support any compression because concrete is already detached.

Kupfer and Gerstle also defined a curve in the region of

COMPRESSION + COMPRESSION

If specimen is subjected to a biaxial compression, it becomes tough and doesn't tend to crash.
The ideal condition is the one of a sphere subjected to a hydrostatic pressure - triaxial compression - and here the specimen will never crash.
On a biaxial compression, as

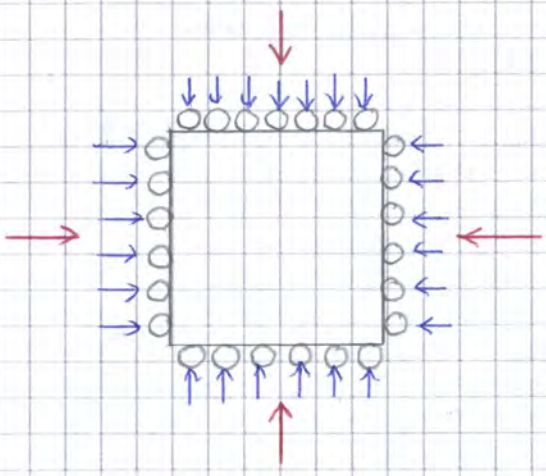
$$\sigma_3 = 0$$

the specimen swells outside the plane and crashes due to the swelling.
The behaviour is described now by a curve, which maximum point is at

$$1,5 f_c$$

This means that, on a biaxial compression, the compressive strength of the material is 50% bigger than the uniaxial one.

In building this curve, Kupfer and Gerstle used a BIAXIAL TEST.



Here, a cubic specimen is subjected to a stress state by means of an equipment with 2 hydraulic jacks (= "pistoni"), one working on the horizontal plane and one working on the vertical plane.
The force is transferred to concrete by means of thin needles set in order no transfer friction in the loading:

applying a compressive force, the specimen will shorten and the needles will move with it.

↓

with a steel plate, part of compression goes in friction, whereas with needles, all forces go in the cube

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Once we have introduced these models, we have to see how cracking is verified in practical terms:

according to Model Code 90, concrete is cracked if the following condition is realized.

$$\Phi = \alpha \frac{J_2}{f_{cm}^2} + \lambda \frac{\sqrt{J_2}}{f_{cm}} + \beta \frac{I_1}{f_{cm}} - 1 > 0$$ Cracking condition

I_1 = first invariant of stress state - sum of the principal stresses

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

J_2 = second invariant of the deviatoric stress state

$$J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2\right]$$

Indeed, when a body is deformed, it changes its volume and shape. Changing only in volume keeping shape as constant is called hydrostatic strain and it is related to hydrostatic stress. Changing only in shape keeping volume as constant is called deviatoric strain and it is related to pure deviatoric stress. Generally, in a deformation process we identify 2 contributions

→ hydrostatic contribution, related to mean stress σ_m

$$\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

that changes only the volume.

→ stress state in excess with respect to mean stress, related to the variation of shape.

The equation of J_2 is similar to the one of the Von Mises resistance criterion:

Von Mises was studying metals and he invented a failure criterion based on pure deviatoric stresses, as metals don't feels hydrostatic stresses and go to failure due to shape variation.
So, he associated failure to deviatoric stresses alone, especially to the II invariant J_2.

Given a generic 3D carthesian space of principal stresses, here a point P is identified by means of 3 coordinates

$$P(\sigma_1 ; \sigma_2 ; \sigma_3)$$

In the same space, the point can be identified by a different reference system that should include 3 information.
~~One~~
One is the POLAR REFERENCE SYSTEM, with coordinates

$$\xi \quad r \quad \vartheta$$

How can these coordinates be defined?

Firstly, we introduce a line identified by $\{e\}$ vector, along which all stresses are equal.
This is the hydrostatic line and it corresponds to a line inclined of 45° with respect to the 3 axes, lying in the middle of the I optant.

The planes orthogonal to this line are called deviatoric planes.

To define the position of point P, we have to choose the deviatoric plane passing by this point.
Then, if we move to the view from this plane in the orthogonal direction - hydrostatic line becomes a point, we can identify point P on deviatoric plane, which has a certain distance from the hydrostatic line N (intersection hydrostatic line and deviatoric plane)

From this construction, we can define the coordinates.

ξ = distance from the origin O to point N of intersection between hydrostatic line and the deviatoric plane containing point P.

r = distance between points P and N on the deviatoric plane.

ϑ = starting from point N, if we walk at distance r, we will run a circumference.
So, we need an information about the direction along which we walk, called lode angle.

In terms of equations, ~~Tresca~~ Von Mises criterion is expressed as

$$f(\sigma_{cu}) = \frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2\right] - \sigma_{cu} = 0$$

We can write that with the deviatoric stress invariant J_2.

$$f(\sigma_{cu}) = \sqrt{6J_2} - \sigma_{cu} = 0$$

According to Tresca criterion, failure is achieved when the difference between principal stresses is equal to a given value σ_{cu}.

$$\sigma_3 - \sigma_2 = \pm\sigma_{cu} \qquad \sigma_2 - \sigma_1 = \pm\sigma_{cu} \qquad \sigma_1 - \sigma_3 = \pm\sigma_{cu}$$

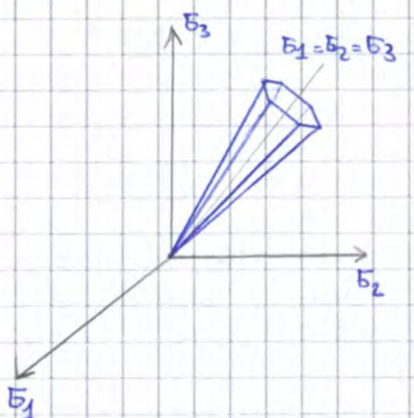
This condition corresponds to a regular hexagon in the Von Mises circle in the deviatoric plane.
It means that, walking from the ~~devi~~ hydrostatic line to an edge, we will walk a certain distance; walking from the hydrostatic line to a vertex, we will walk a bigger distance.
So, according to Tresca, fail is reached sooner or later in function of the direction we go.

⇒ LODE ANGLE ϑ IS IMPORTANT IN TRESCA CRITERION and, as lode angle corresponds to a ratio of stresses, it means that strength of the material is a function of the stress ratio.

Actually, the two criteria are not so different and the difference can almost be neglected, as the maximum difference is small compared to stress values.
On the other side, the phylosophy of these criteria is different

→ in Von Mises, lode angle ϑ is not important.

→ in Tresca, lode angle ϑ is important.



Passing to Mohr-Coulomb criterion, it is represented by an hexagon with dimensions varying in fun of the hydrostatic stress:

if hydrostatic stress increases - i.e. compression increases and stabilizes the material -, the strength increases.

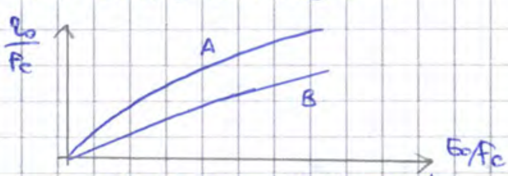
This section presents also a symmetry because there are slices of 60° in which the distances of the point from the center are the same and the equations of the curves are the same.
Thanks this polar-simmetry, we only have to define a portion of curve of 60°.

On the failure surface, we can identify

→ parallels, that are curves with the same hidrostatic stress.

→ meridians, that are lines going from the origin up to the section of interest and, along these, hydrostatic stress changes.
Meridians OA and OB define a slice of the failure surface and they correspond to

$A \rightarrow \frac{\tau_{oc}}{f_c}$ $\qquad B \rightarrow \frac{\tau_{oe}}{f_c}$



Once we have defined this slice, we can obtain the whole failure surface thanks to simmetry.

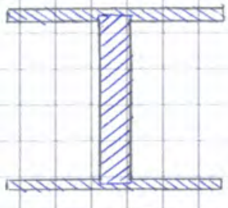
The equation of the curve is

$$r = \sqrt{3}\,\tau_{ou} =$$

$$= \frac{2\tau_{ou}(\tau_{oc}^2 - \tau_{oe}^2)\cos\vartheta + \tau_{oc}(2\tau_{oe} - \tau_{oc})\sqrt{4(\tau_{oc}^2 - \tau_{oe}^2)\cos^2\vartheta + 5\tau_{oe}^2 - 4\tau_{oc}\tau_{oe}}}{4(\tau_{oc}^2 - \tau_{oe}^2)\cos^2\vartheta + (\tau_{oc} - 2\tau_{oe})^2}$$

II If element is not cracked, we have only to check the maximum compressive stress.
If element is cracked at least in one layer, we should refer to a model called Sandwich model.

Similar models are used in steel, with nil interaction and same formulations.



$M_{Rd} = f_y A_f z$ $\qquad A_f$ = chord's area

$V_{Rd} = b_w z \cdot \frac{f_y}{\sqrt{3}}$ $\qquad b_w z$ = area of the web

Moreover, in both cases the idea is that we use plasticity static theorem:

if, inside the structure, we find an arbitrary distribution of stresses that respects only equilibrium, the solution is smaller than the real one.

GEOMETRY AND INTERNAL ACTIONS

In the sandwich model, we identify 3 layers

→ top layer, with thickness t_s (s = "superiore")

→ middle layer, with thickness t_c

→ bottom layer, with thickness t_i (i = "inferiore")

Then, we define some distances

→ y_{xi} = distance between the centroid of the she and the centroid of the bottom layer in x direction, i.e. the reinforcement that bears m_x and n_x in this layer

→ y_{xs} = distance between the centroid of the shell and the centroid of the top layer in x direction.

→ y_{yi} = distance between the centroid of the shell and the centroid of the bottom layer in y direction, i.e. the reinforcement that bears m_y and n_y in this layer.

→ y_{ys} = distance between the centroid of the shell and the centroid of the top layer.

From these distances, we identify some internal lever arms

→ internal lever arm in x direction, given by the distance between the centroid of the top layer and the one of the bottom layer in x direction.

$$z_x = y_{xs} + y_{xi}$$

Generally, we use the same thicknesses and distances both for membranal internal actions and plate internal actions, for sake of simplicity.

↳ actually, they could be different as bending moment in x direction may require a neutral axis which is different from the one necessary for bending moment in y direction (different approaches in different directions)

Once we have identified the sandwich, we evaluate the INTERNAL ACTIONS IN EACH LAYER according to some formulations.

→ membrane actions

Axial forces are split in 2 components, one on the upper layer and one on the bottom layer

→ axial force in x direction in the top layer n_{xs}:

it is given by the axial force in the x direction multiplied by the "influence area" of the top layer.
To this, we add the contribution of bending moment. Bending moment is divided by the lever arm z_x in order to decompose it into a couple of forces. Assuming positive bending moment, it gives arise tensile stresses in the positive face and compressive stresses in the other one.

$$n_{xs} = n_x \frac{z_x - y_{xs}}{z_x} + \frac{m_x}{z_x}$$

→ axial force in x direction in the bottom layer n_{xi}

$$n_{xi} = n_x \frac{z_x - y_{xi}}{z_x} - \frac{m_x}{z_x}$$

→ axial force in y direction in the top layer n_{ys}

$$n_{ys} = n_y \frac{z_y - y_{ys}}{z_y} + \frac{m_y}{z_y}$$

→ axial force in y direction in the bottom layer n_{yi}

$$n_{yi} = n_y \frac{z_y - y_{yi}}{z_y} - \frac{m_y}{z_y}$$

Component n_{xs} and n_{xi}, n_{ys} and n_{yi} are equal if the thickness of the layers is the same, as axial forces are equally divided between the layers.
If thicknesses are different, the bigger amount will go on the thicker layer.

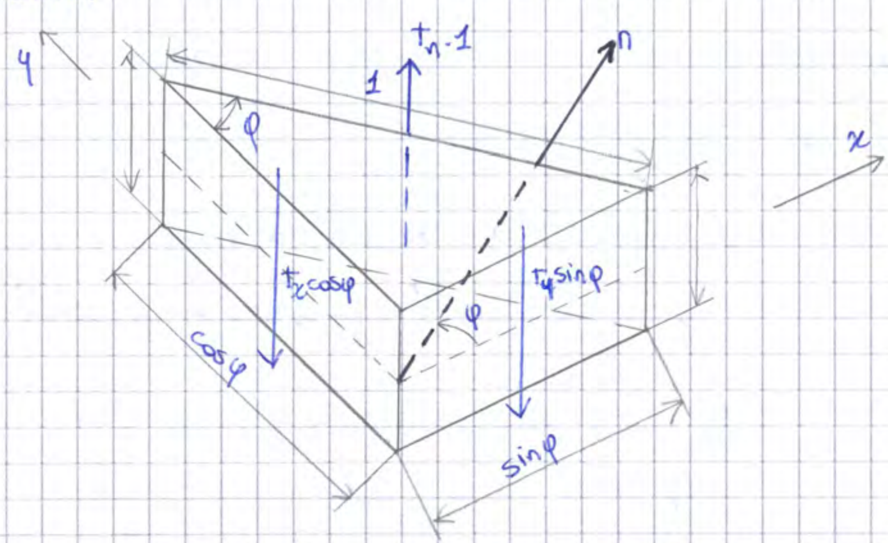
If these thicknesses are the same, z_c will go from the centroid of top layer and the centroid of the bottom layer and it will mean that it is equal to

$$z_c = t_c + \frac{1}{2} t_s + \frac{1}{2} t_i$$

INTERNAL LAYER DESIGN

In presence of a shear t_x in x direction and a shear t_y in y direction, we are able to compute the value of shear in a generic direction n - i.e. shear on a face orthogonal to direction n, belonging to xy plane.

To do this, we analyze the equilibrium to the vertical direction of a prismatic triangular element obtained by sectioning the internal layer with a plane parallel to the z axis and orthogonal to the n direction, which is inclined of an angle φ with respect to the x axis.



As t_x and t_y are values referred for a unit length, on the faces along x and y direction - with lenght $\sin\varphi$ and $\cos\varphi$, respectively - the shear is

$$t_y \sin\varphi \qquad t_x \cos\varphi$$

From the equilibrium along the vertical direction, we get

$$t_n = t_x \cos\varphi + t_y \sin\varphi$$

The angle φ_0 defines the principal direction of shear and it can be calculated from the relationship

$$t_m = 0$$

$$t_m = -t_x \sin\varphi_0 + t_y \cos\varphi_0 = 0$$

$$\Rightarrow \tan\varphi_0 = \frac{t_y}{t_x}$$

$$\tan\varphi_0 = \frac{t_y}{t_x}$$

Along this direction, shear is maximum and it is coupled with an orthogonal shear which is null:

in a plate subjected to bending moments M_x and M_y with different slopes, shears V_x and V_y correspond to the slopes of bending moments in x and y directions, as shear is the first derivative of bending moment.
If we focus on a diagonal direction, we evaluate the slope of bending moment in this direction.
The angle φ_0 defines the direction of the maximum slope of bending moment, i.e. the maximum shear direction.
The orthogonal direction presents no shear and no variation of bending moment, so it is an equi-moment direction.

$\Rightarrow$ along the principal direction of shear, the shell element behaves like a beam because there is only one shear in this direction.

By consequence, we'll design the shell element in this direction because we are able to design structures with 1 dimension, by using beam theory in a direction.

Focusing on the case

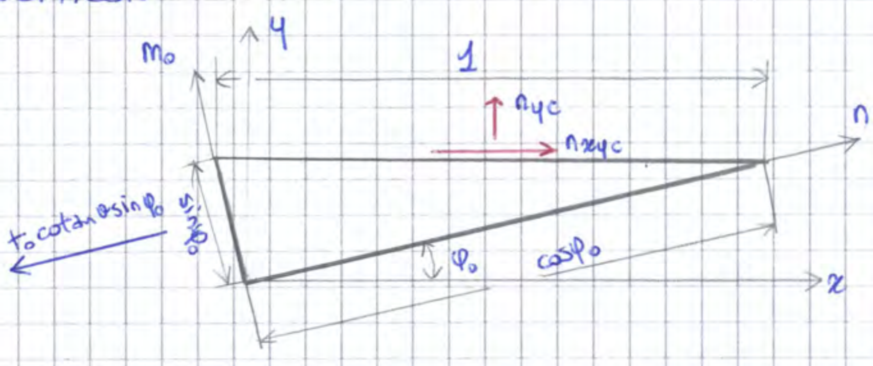
$$t_0 > V_{Rd,c}$$

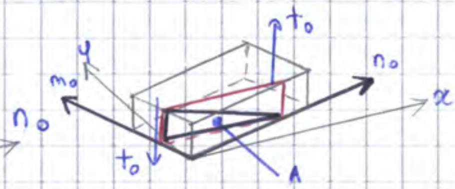
$V_{Rd,c}$ = concrete shear strength without stirrups

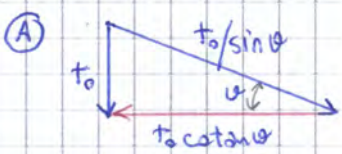
vertical reinforcement should be provided.
With stirrups, at ULS we have a Morsch truss system along the principal shear direction and we have

→ VERIFICATION OF COMPRESSED STRUTS, according to the formulation in beams.

Firstly, we cut the prism with a plane parallel to x axis, which is vertical.







If we see from above the new prism, cut has been done in order to have a cutting edge which is 1 long.
If cutting edge is long 1, the other two sides will be long

$$\sin\varphi_o \qquad \cos\varphi_o$$

In the left face of the initial prism, that has an unit length, is acting the shear t_o.
In the same face of the new prism, since the length is smaller and equal to $\sin\varphi_o$, shear will be

$$t_o \sin\varphi_o$$

Then, looking at the triangle of forces that represents the equilibrium in the vertical plane with n_o axis and the inclined strut in the thickness of the ideal beam, we have

→ vertical force t_o

→ inclined force $t_o/\sin\vartheta$ in the compressed strut

To close the triangle, we have to introduce the horizontal force $t_o \cot\vartheta$, that is the additional tension in the bottom chords - increase of effect of bending moment.

The horizontal force $t_o \cot\vartheta$ is applied in the n_o direction and it is related to the length of 1.
In the new prism, the amount of horizontal force will be

$$t_o \cot\vartheta \sin\varphi_o$$

On the other edge of the prism, nothing is applied because this edge is in m_o direction and no shear is applied there. So, no force is coming out from this edge.

$$= t_0 \cot\!an\,\vartheta \frac{\dfrac{t_y^2}{\cancel{t_x^2}}}{\dfrac{t_x^2+t_y^2}{\cancel{t_x^2}}} = t_0 \cot\!an\,\vartheta \frac{t_y^2}{t_x^2+t_y^2}$$

$$= t_0 \cot\!an\,\vartheta \frac{t_y^2}{t_x^2+t_y^2} =$$

$$= t_0 \cot\!an\,\vartheta \frac{t_y^2}{t_0^2} = \frac{t_y^2}{t_0} \cot\!an\,\vartheta \qquad \rightsquigarrow \quad t_x^2+t_y^2=t_0^2$$

$$n_{xyc} = t_0 \cot\!an\,\vartheta \sin\varphi_0 \cos\varphi_0 =$$

$$= t_0 \cot\!an\,\vartheta \frac{\tan\varphi_0}{1+\tan^2\varphi_0} =$$

$$= t_0 \cot\!an\,\vartheta \frac{\dfrac{t_y}{t_x}}{1+\dfrac{t_y^2}{t_x^2}} = \qquad \rightsquigarrow \quad \tan\varphi_0 = \frac{t_y}{t_x}$$

$$= t_0 \cot\!an\,\vartheta \frac{\dfrac{t_y}{\cancel{t_x}}}{\dfrac{t_x^2+t_y^2}{t_x^2}} =$$

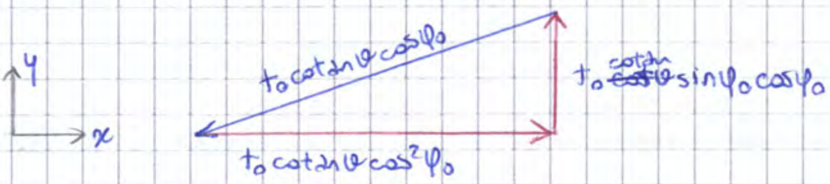
$$= t_0 \cot\!an\,\vartheta \frac{t_x t_y}{t_x^2+t_y^2} =$$

$$= \cancel{t_0} \cot\!an\,\vartheta \frac{t_x t_y}{t_0^2} = \frac{t_x t_y}{t_0} \cot\!an\,\vartheta \qquad \rightsquigarrow \quad t_x^2+t_y^2=t_0^2$$

$$n_{yc} = \frac{t_y^2}{t_0} \cot\!an\,\vartheta$$

$$n_{xyc} = \frac{t_x t_y}{t_0} \cot\!an\,\vartheta$$

To obtain their values, we decompose the horizontal force in x and y directions, by using the triangle of forces.



These forces can be written in a more elegant way.

$$n_{xc} = t_o \cot\theta \cos^2\varphi_o = \frac{t_x^2}{t_o}\cot\theta$$

$$n_{yxc} = t_o \cot\theta \sin\varphi_o \cos\varphi_o = \frac{t_x t_y}{t_o}\cot\theta = n_{xyc}$$

$$n_{xc} = \frac{t_x^2}{t_o}\cot\theta$$

$$n_{xyc} = n_{yxc} = \frac{t_x t_y}{t_o}\cot\theta$$

The forces parallel to the cuts are the same because the two cuts are orthogonal to each other and two shear forces in two orthogonal faces are equal to each other due to Cauchy.
Thanks to this, we have only 3 unknowns - n_{xc}, n_{yc}, $n_{xyc} = n_{yxc}$ - and the problem could be solved only with equilibrium equations.

In this way, we have decomposed the forces ΔF_{st} and ΔF_{sc} in n_{xc}, n_{yc} and n_{xyc}.
These forces should be added to the external actions n_x, n_y and n_{xy} and then divided between the two external layers.

↳ in the beam, the variation of tension in each chord is

$$\Delta T = \frac{V_{sd}}{2}(\cot\theta - \cot\alpha) = \frac{V_{sd}}{2}\cot\theta$$

(in case of vertical stirrups - $\alpha = 90°$)

Factor 2 is used because the force is distributed in the section and one half is assigned to the tensed chord, the half to the compressed chord

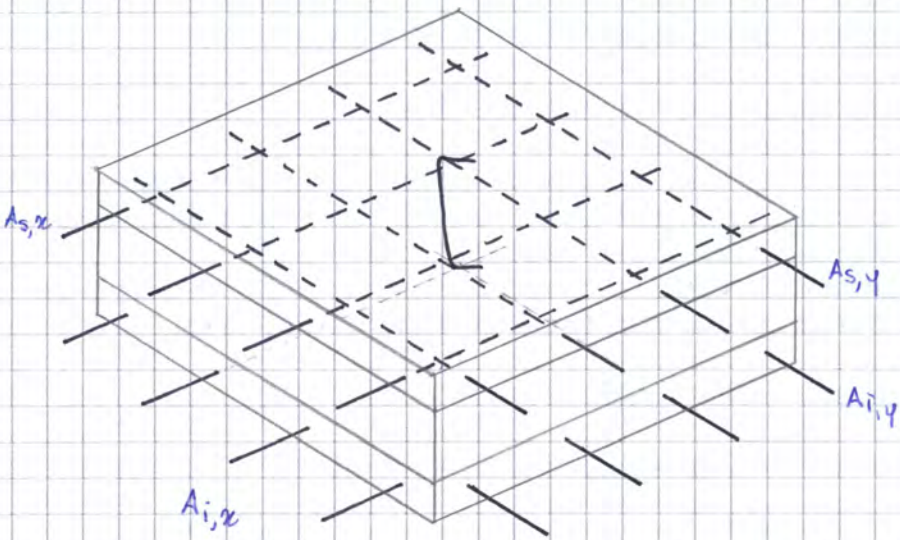
These forces lead to an increase of ~~as~~ longitudinal reinforcement.

$$V_{Sd,s} = n_{xy}\frac{z - y_s}{z} - \frac{m_{xy}}{z} + \frac{1}{2}\frac{t_x t_y}{t_0}\cot\vartheta$$

$$V_{Sd,i} = n_{xy}\frac{z - y_i}{z} + \frac{m_{xy}}{z} + \frac{1}{2}\frac{t_x t_y}{t_0}\cot\vartheta$$

From these internal actions, we'll design the reinforcement in the external layers.

What is the typical arrangement of the reinforcement in shells?



The typical arrangement present a mesh (= "rete") of reinforcement in the top layer and in the bottom layer.
This arrangement is useful in order to place vertical shear reinforcement, i.e. C-shaped hooks (= "spilli") placed inside one free space of reinforcement and turned in order that a part is above the reinforcement and a part is below - like in beams. Otherwise, hooks would not be efficient as they would not grab the reinforcement and it would be like not having shear reinforcement.

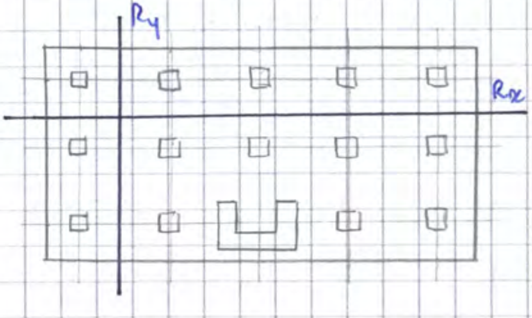
Generally, meshes are made of orthogonal reinforcement because it is easier to place than skew reinforcement - for instance, we may use a welded mesh (= "rete elettrosaldata"), made of orthogonal reinforcements.

The amount of the reinforcement can be different in different directions, as bending moment is not the same and, generally, there is a leading direction and a secondary direction.
Moreover, the amount of the reinforcement in the top slab can be different from the one in the bottom layer because, if bending moment is positive or negative, the top reinforcement is different from the bottom one - as in beams.

$\Rightarrow$ reinforcement in each direction in each layer can be different and, generally, we define 4 quantities

We can see some practical cases

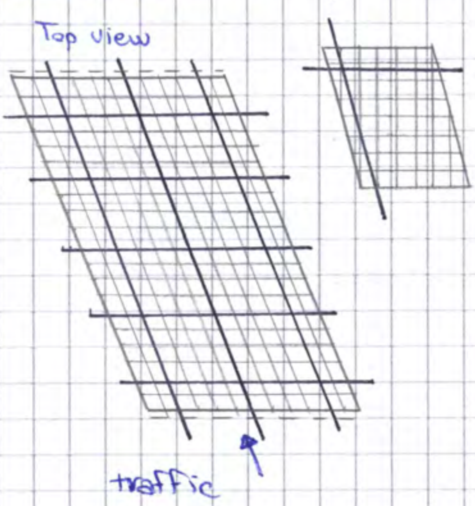
→ RECTANGULAR RAFT, i.e. foundations of a building



We consider a raft with the columns of the building, which sides are parallel to x and y axes.

In the analysis, the finite element model presents shell elements that are rectangular shaped, as we don't need to use skew elements.
Also the reinforcement will be placed along x and y direction – orthogonal reinforcement – both at the top and the bottom.

→ SKEW BRIDGE



The shape is quite easy – it's a parallelogram – but, if we want to mesh the structure with shell elements, a typical mesh presents parallelogram.
We may also adapt a different finite element mesh, which doesn't follow the actual geometry of the element.

Reinforcement will be placed along the edges, with a reinforcement in the transversal direction which is parallel to the supported edges

distribution of the internal actions and we get very far from linear elastic behaviour.
A similar situation occurs in beam, where internal actions are computed with linear elasticity and, with these, we design the beam at ULS assuming parabola-rectangle law and yielding of steel. The hypotheses are not congruent because stiffness changes with cracks and internal actions are different from the ones predicted in elastic field.
On the other side, we don't have other tools, so we are forced to use this approach.

At ULS, the internal actions are

$$n_{sd,x} \qquad n_{sd,y} \qquad V_{sd}$$

Angle ϑ_e defines the direction of crack, which is orthogonal to the maximum tensile stress before cracking.
So, ϑ_e is the inclination of the principal direction of compression with respect to x axis and the reference system 1-2 will be the principal stresses system, which axis 1 is rotated of ϑ_{el} from x axis.
From the internal actions, we can pass to the principal system and evaluate

$$n_1 \qquad n_2 \qquad \vartheta_{el}$$

generally, we compute the angle α between x axis and the direction of tension and ϑ_e will be

$$\vartheta_e = \alpha - 90°$$

As these values are referred to an elastic prediction - before cracking -, we have

$$n_1 > f_{ct} = n_{cr} \qquad n_{cr} = \text{cracking axial force}$$

So, we can scale down this value to cracking state and then scale down the other quantities of the ratio n_{cr}/n_1, getting the values of the internal actions before the I crack arises.

$$n_1 \quad n_2 \quad \vartheta_{el} \xrightarrow[\frac{n_{cr}}{n_1}]{} n_{cr} \quad n_{2,cr} \quad \vartheta_{el}$$

For instance, if n_1 is 10 MPa but concrete can't support it and cracking strength is 3 MPa, we scale down everything of a ratio $10/3$ and find the internal actions present just before the first crack arises.

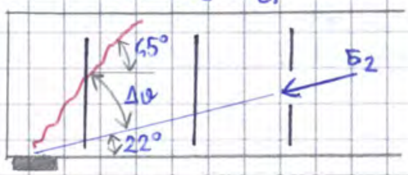
So, at ULS we define the angle ϑ_u, which is the angle between x axis and the principal compressive stress direction at failure.

$\vartheta_u \neq \vartheta_{el}$ → the difference is due to anis of the behaviour

What is the value of ϑ_u?

We can try to start from a similar situation, that is design of beams at shear with the Morsch truss system, where there is a variable ϑ approach.

$\Delta\vartheta = \vartheta_u - \vartheta_{el}$



Given a supported beam reinforced with stirrups, focusing on the bearing, the first crack will be inclined of 45° as it occurs in pure shear - nil bending moment.

At ULS, we can chose an inclination ϑ_u which can be different from ϑ_{el} and, in dimensioning the beam with the variable ϑ approach, ϑ_u can vary between

$$22° < \vartheta_u < 45°$$

→ $\vartheta_u < 45°$ because if we have pure shear cracking direction is inclined of 45° and we can't go beyond this

→ $\vartheta_u > 22°$ This is a limit introduced to control crack opening and involves extremely high tangential stresses.
So, this ϑ_u corresponds to the maximum variation of ϑ that we can ask to the beam as, in this case, we are placing too much small reinforcement and concrete strength to compression is smaller.

The main idea is that more ϑ_u is different from ϑ_{el}, smaller is the compressive strength of concrete and the variation of ϑ is something that measures the damage that occurs in concrete due to the rotation of compressive fields, starting from ϑ_{el} to a different value when load is increased.
This rotation is useful because it allows to use less reinforcement, but it can be used only if there is strength in the concrete side - if it is at maximum level, ~~we~~ they can't rotate anymore.

→ in y direction

$$n_{Rd,y} \cot\vartheta_u = n_{Sd,y} \cot\vartheta_u + V_{Sd}$$

$n_{Rd,y} \cot\vartheta_u$ = internal force in the reinforcement in y direction - force per unit length - multiplied by the length $\cot\vartheta_u$ of the segment

We express the force in the reinforcement as function of its area

$$n_{Sd,x} + V_{Sd} \cot\vartheta_u = A_{s,x}\,\sigma_{s,x}$$

$$n_{Rd,y} = n_{Sd,y} + \frac{V_{Sd}}{\cot\vartheta_u} = A_{s,y}\,\sigma_{s,y}$$

We have 2 equations for 3 unknowns

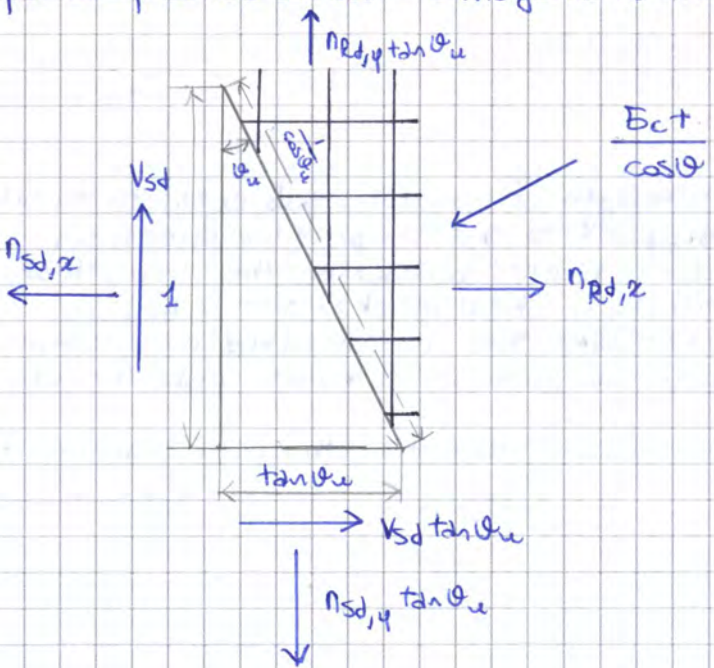
→ amount of reinforcement to be placed in x and y directions

$$A_{s,x} \qquad A_{s,y}$$

→ inclination ϑ_u, which is our choice

Stresses $\sigma_{s,x}$ and $\sigma_{s,y}$ are known because, at ULS, we want the reinforcement to be yielded and so we consider stresses to be equal to the yielding value.

So, we write the equation of a new cut element from the layer, obtained by cutting it with a plane which is orthogonal to the principal compressive stress - orthogonal to the previous cut.



Actually, this reduction is not enough if at least one reinforcement is yielded.
On the base of laboratory experiments, the correct value is

$$f_{cd,2} = f_{cd,2}^{MC90}\left(1 - 0{,}032\,|\Delta\varphi|\right) \quad \geq 1 \text{ reinforcement yielded}$$

It means that, in a uniaxial compressed concrete, strength is f_{cd}.
As soon as cracks arise due to tensile stresses orthogonal to the compressive ones, strength falls by a factor

$$\left(1 - \frac{f_{ck}}{250}\right) \cdot 0{,}6$$

In a C25/30 concrete, the factor is

$$\left(1 - \frac{25}{250}\right) \cdot 0{,}6 = 0{,}9 \cdot 0{,}6 \sim 0{,}55$$

So, compressive strength ~~falls~~ drops to 50% of its value as soon as first crack appears.
If reinforcement begins to get yielded and compressive fields rotate, strength of the material drops down again and, if $\Delta\varphi$ is equal to 10°, the factor is

one bar yielded ⇒ rotation of strut (demonstrated in laboratory)

$$\left(1 - 0{,}032 \cdot 10\right) \cdot 0{,}55 \sim 0{,}35$$

So, with a rotation of 10°, strength drops from f_{cd} to $0{,}35 f_{cd}$ and 65% of strength is lost - huge amount.

Laboratory tests also show that we have to run a different verification of concrete in case of no reinforcement yielded.

$$\sigma_c \leq f_{cd,2}^{MC90}\left(0{,}85\,\frac{f_{cd}}{f_{cd,2}^{MC90}} - \frac{\sigma_s}{f_{yd}}\left(0{,}85\,\frac{f_{cd}}{f_{cd,2}^{MC90}} - 1\right)\right) \quad \text{No reinforcement yielded}$$

Generally, at ULS we want the yielding of the reinforcement in order to have a ductile behaviour and place less steel.