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# **A P P U N T I**

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**MATERIA: Teoria e progetto dei ponti (I parte) - Prof. Mancini Bertagnoli**

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## HISTORICAL NOTES ON BRIDGE CONSTRUCTION

1 Commonly, history of bridges is divided into 3 eras

→ **I age**, that goes since the beginning of history to the beginning of XIX century and it is characterised by transportation by animals or on foot.

→ **II age**, that goes between the beginning of XIX century and the end of the world war I and it is characterised by rail transportation.

→ **III age**, that goes since the end of the World War I to today and it is characterised by transportation by rubber.

↳ classification on the base of MEANS OF TRANSPORTATION

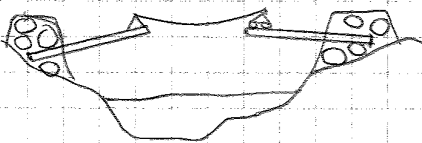
### 2 First age

Bridges were built using hands and using only natural materials, like trunks, natural fibers, etc.

Transportation was by animals and bridges were built across rivers or other natural obstacles.

In this age, cantilever bridges made with bulk or bamboo were built in China and Nepal.

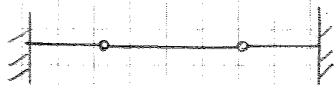
A cantilever stands for "mensola".



If there is an obstacle, firstly we realize abutments, made with stone.

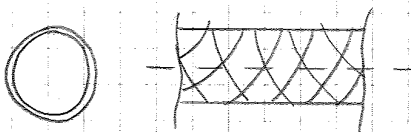
Then we fix some trees by using the weight of stones in the abutments as a counterweight. The trunks should be fixed into the abutments. For an adequate length, in order to provide full restraintment inside the abutments.

In this way, we have 2 cantilevers and on them we place an element, which is a beam hinged to the cantilevers.




This way of construction is good if it's not possible to work below the bridge, as there is a narrow valley or lots of current.

In this situation, we can also realize a framed pipe bridge.



For instance, in Nepal there is a bridge that consists of a circular section pipe and it is realized with flexible wooden elements.

Its size allows people to walk inside and it is stiff enough.

1st Age [Since the beginning to ~ 1800]  Transport by animals

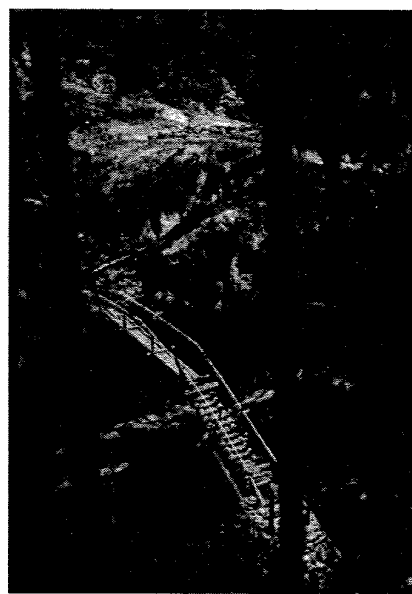
- Bridges → Crossing of rivers and/or natural obstacles
- Natural materials : Wood ( Trunks, Beams, Wooden decks... )  
Natural fibers ( Liane, hemsps... )  
Stone



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Primitive cantilever  
bridge built in Nepal

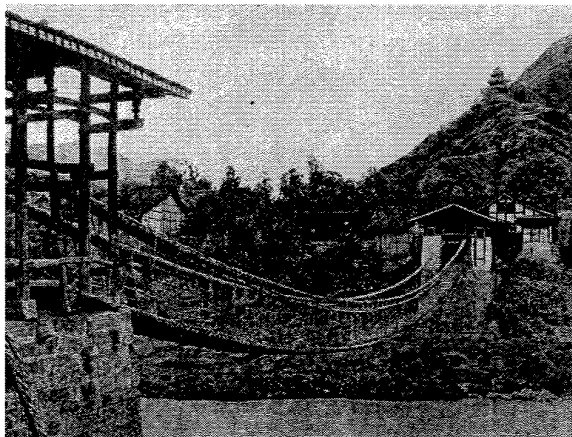


Cantilever bridge built  
in bamboo in China

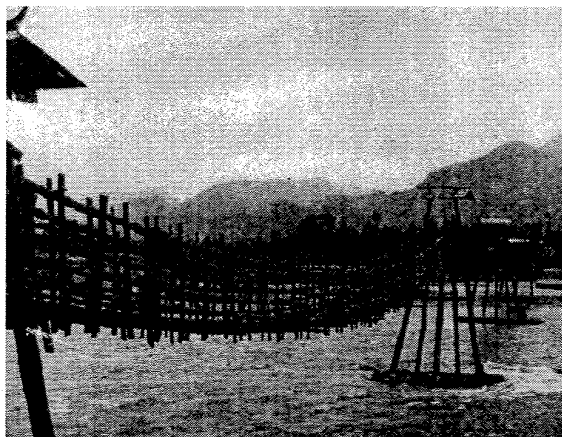


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**1** Historical notes on bridge construction **7/39**



Catenary bridge built in Szechuan (China) with hemp ropes



Catenary bridge built on Min river within the Szechwan Province (China) in 1930 (built with bamboo ropes; total span: 540m)

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**1** Historical notes on bridge construction **8/39**

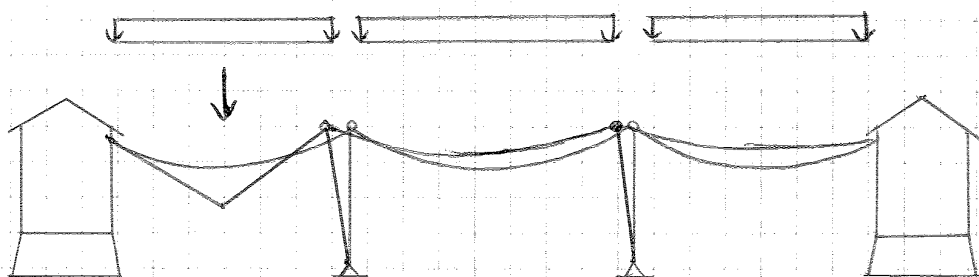
- Christian Age
  - Stone round arch bridges (span ~ 30m)
  - Stone original ogive arch bridges (Turkey)
  
- Middle Age
  - Depressed arches (span ~ 50m)
  - Scaligero Bridge (Verona, 1354)
  - Rodano Bridge (Avignon)
  - Karol Bridge (Prague)
  - Donau Bridge (Regensbur)
  
- XVI ÷ XVIII Centuries
  - Chain bridges (China, 1750)
  - Wooden bridges (Reno, 1758; span: 118 m)
  - Stone bridges (Goltzschtal, span: 578 m, depth: 78 m)

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this system is a kind of damping system because, if somebody walks on the first catenary, the other catenary help him not to vibrate too much. In other words, energy is transferred from one catenary to the other one and is dissipated.

Moreover, each pier is forced to bend and takes part of energy, so also piers help not to vibrate too much.



If we image a hinge - easy kind of restraint to be realized - , the static scheme works again but now piers don't bend, but only present a rigid rotation.

So, in this case, we lose the bending moment in the piers as a stabilizing action and only self-weight counterbalances payload.

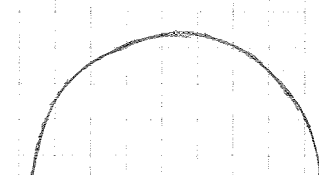
Hopefully, the bending moment is small compared to the force applied by catenary:

a load of 100 kg generates a 2 tons force in the ropes. If self-weight of 30 m long catenary is 2 tons, in the ropes we have a force of 40 tons. So, in the ropes we have a force of 40 tons applied by gravity and 2 tons applied by the payload and the movement is small compared to what we would have without the catenary.

Then, we focus on Europe and here we can divide the I age within 3 periods

→ Christian-Roman Age

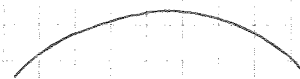
This period is characterised by stone arch bridges



Round arch

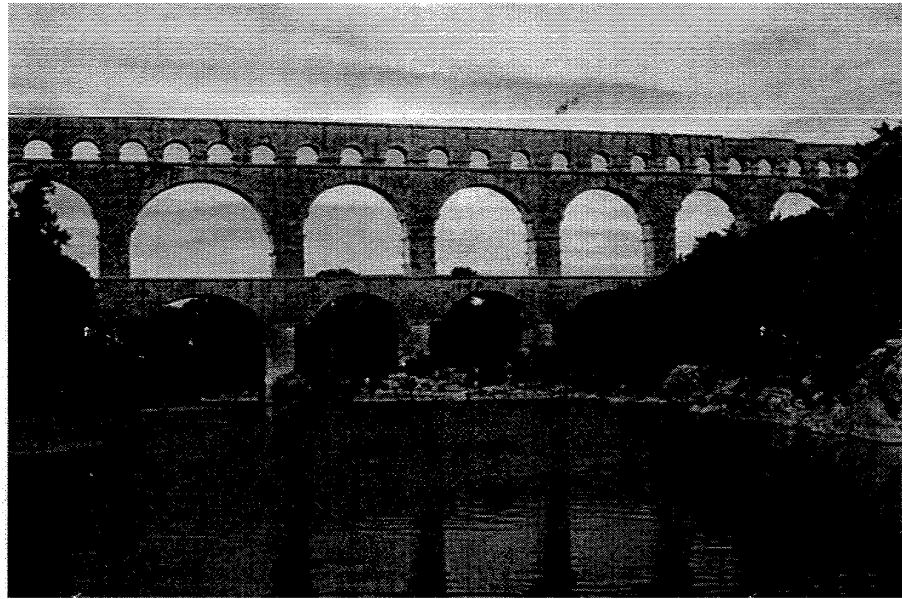


ogive arch



depressed arch

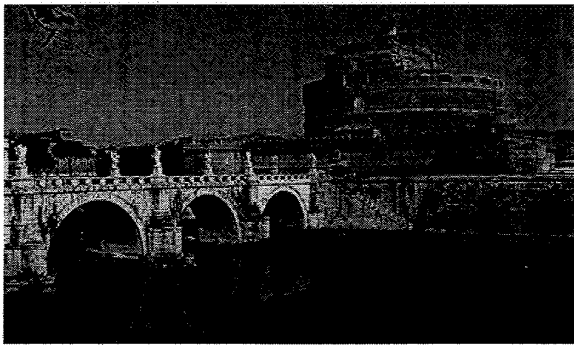
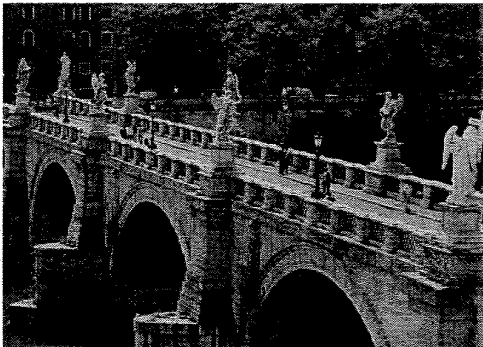
Pont du Gard  
span: 275 m  
depth: 48 m



( Roman water-supply-system built in Nimes by emperor Agrippa in 19 b.C.; flow: 35000 m<sup>3</sup> per day )



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S. Angelo Bridge (Rome): built by emperor Adriano in 134 a.C. (the three central arches are original, the others were rebuilt)



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## → Middle Age

This age is mainly characterised by **STONE ARCH BRIDGES**.

### → Regensburg bridge (Germany)

Built in the XII century, it presents 16 arches and total span is 350 m.

So, each span is about 20 m but, as the piers are very thick, free span between the piers is small.

⇒ there was no engineering development between Roman bridges and this bridge, because the same kind of structures were used in 1000 years.

### → Pont St. Bénézet (near Avignone, France)

It was built in 1184 - the same period of Regensburg bridge - but it was 850 m long with 22 arches and each arch had a span of 40 m.

So, the engineers that were building it were doubling the span that was considered safe. In this way, we had a smarter and a more elegant bridge but it was also too daring, as in that period they didn't have materials and engineering techniques developed in order to build this kind of structure.

↓ now S.F.  $\ll 20$

Indeed, this bridge was destroyed 2 times during floods.

### → Scaligero bridge (Verona, Italy)

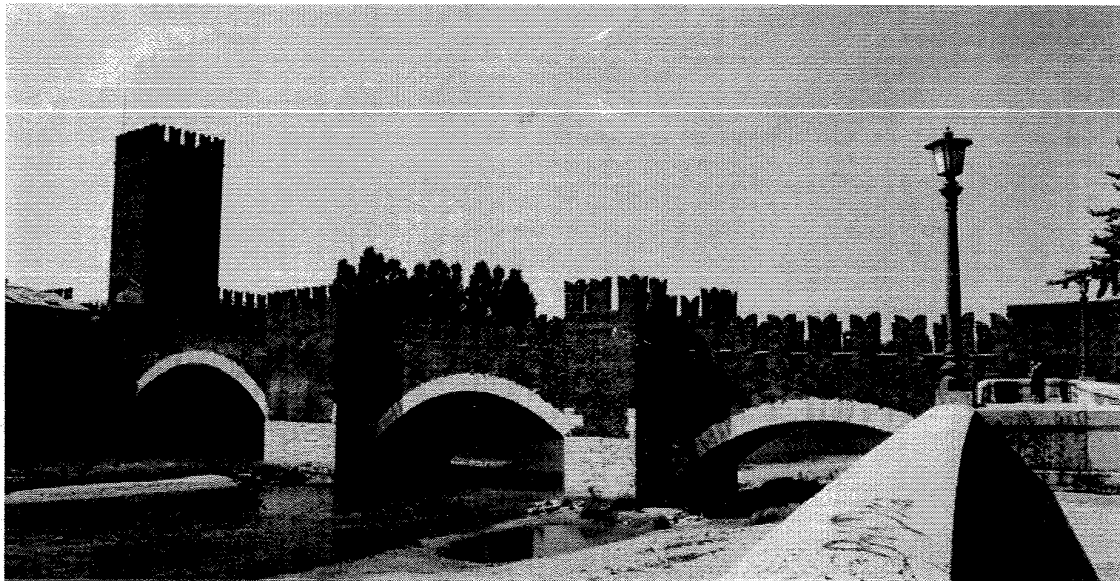
Built in the XIV century, it is characterised by short span, so this bridge is heavy and massive.

### → Karol bridge (Prague, Czech Republic)

It is a very solid arch bridge, built with round arches with 30 m of span, in a similar way of S. Angelo bridge - after 1000 years.



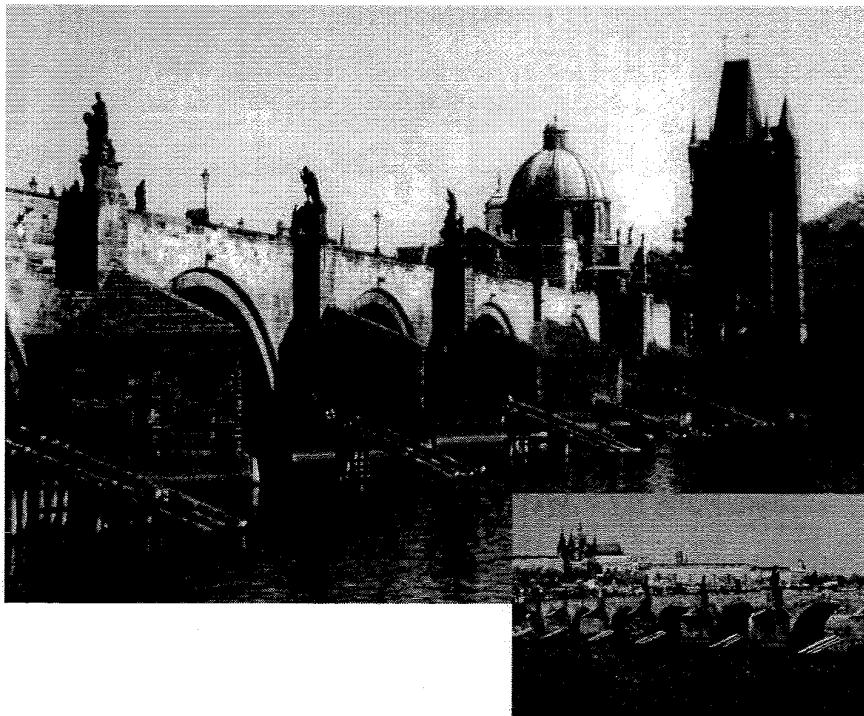
**1** Historical notes on bridge construction **15/39**



Scaligero bridge, built under Casagrande II (Verona 1354-1357)

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**1** Historical notes on bridge construction **16/39**



Karol bridge  
Prague, 1357  
total span:  
520m

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→ Lavertezzo Cliff bridge (Canton Ticino, Switzerland)

It is not a Roman bridge but was built in the XVII century on Roman ruins with the same static scheme and materials.

→ Kintai - kyo bridge (Japan)

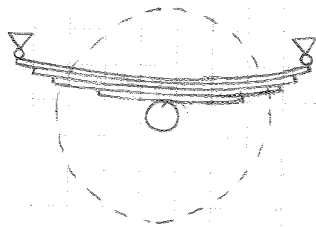
It is an arch bridge, made of 5 wooden arches and each one is 42 m span and 5 m wide.

It was built in 1673 - during the Japanese Renaissance - , destroyed by a fire and rebuilt in 1953.

↳ in Japan, every building ~~of~~ from Japanese Renaissance was made in wood and almost of them were destroyed by fire, especially after seismic events. That's why there are no historical centers in the cities nowadays.

From the static point of view, this bridge is interesting because it's a variable depth bridge.

The variable depth leads to a redistribution of stresses, concentrating them close to the piers and taking them away from the central way.



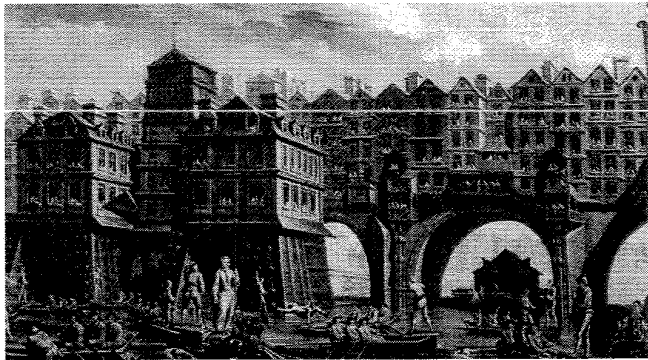
Variable depth is realized by means of several wooden elements coupled together with vertical elements. This mechanism is known as cross-bow suspension, made of variable stiffness elements which provide a big section where bending moment is big.

↳ "la sospensione dell'automobile"

→ Ponte degli Scalzi (Venice, Italy)

Built by Miozzi in 1931, it is a particular case because it <sup>is</sup> an arch bridge but technology and construction control allowed to realize a very big span.

**1** Historical notes on bridge construction **19/39**



Notre Dam Bridge  
(Paris – XVIII century)



Marie Bridge  
(Paris – XVIII century)

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**1** Historical notes on bridge construction **20/39**



Roman Bridge in Mostar  
span: 29m  
depth: 25m  
( Jugoslavia, ~ 1500 on old  
roman ruins, rebuilt in 2004)

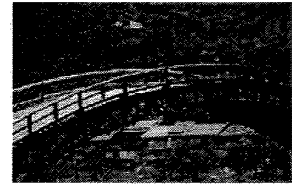
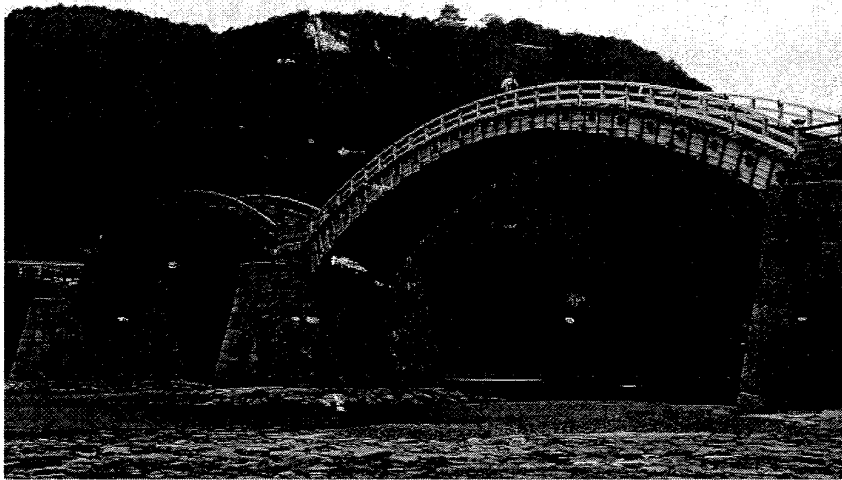


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Historical notes on bridge construction

23/39



Kintai – Kyo Bridge: 5 wooden arches spanning 42m (width 5m), built in Iwakuni, Japan, in 1673, destroyed in 1950 and rebuilt in 1953



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Historical notes on bridge construction

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Ponte degli Scalzi – Venice – 1931 ( eng. Miozzi )

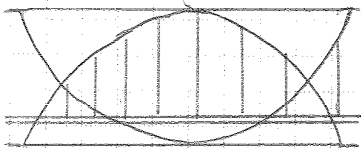


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The scheme of rectangular framed hollow core beam was also adopted in Marienburg bridge and Koln bridge, in Germany.

An evolution of the scheme happened at the end of the XIX century:

thinking about the box section and the arch, engineers proposed a combination of the 2 solutions, that is a hybrid system called lenticular truss bridge.



The traffic moves inside the structure and the deck is connected to 2 simple arches by means of vertical elements.

The arch over works like a stone arch and it is compressed, while the arch below works like to a catenary and it is tensed.

The arches can be simple - Smithfield Street Bridge, Pittsburgh (USA) - or reticular - Hamburg bridges (Germany).

Now we see some particular cases

→ Kinzua Creek viaduct (Pennsylvania, USA)

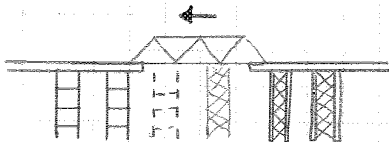
Built in 1882, the original viaduct was very tall and with very light deck because it was retained enough for the trains used in that period and in order to build it as quickest as possible.

20 years after, traffic increased and trains became heavier and the original viaduct began to show its limits.

So, they had to replace the old viaduct and, as they couldn't stop traffic, they did it by strengthening the structure and substituting element as quickest as possible.

↳ in this way, there was only a small closure time

They used a launching crane which was moving on the piers of the old bridge.



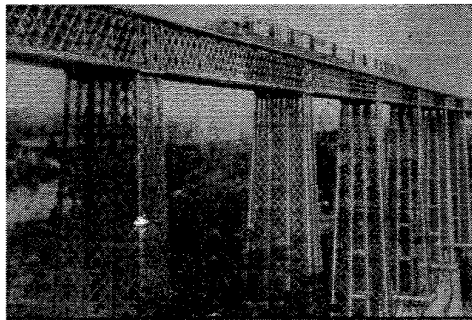
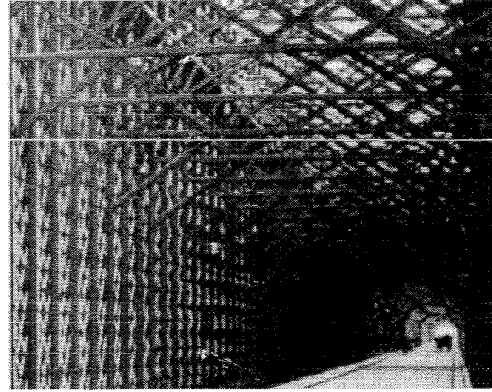
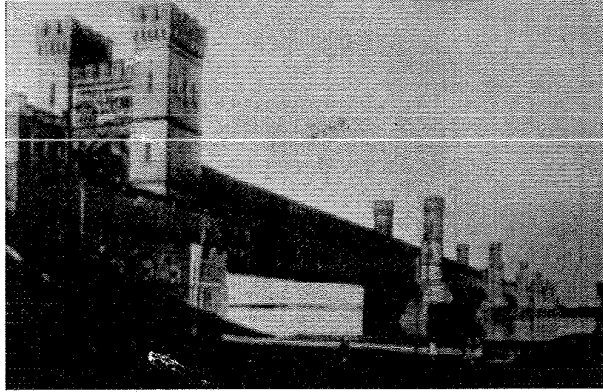
After placing the crane, they demolished the old piers below this and built the new ones. Then, they placed new beams.

So, they used the old bridge as a temporary structure to build the new one.

1

Historical notes on bridge construction

27/39



Dirschau Bridge, on Vistola River  
6 x 131m continuous spans  
8.3t/m of steel  
Destroyed in 1940 (Poland, 1857)

Grandfey Viaduct  
near Friburg - Swiss - 1862



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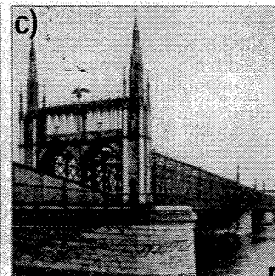
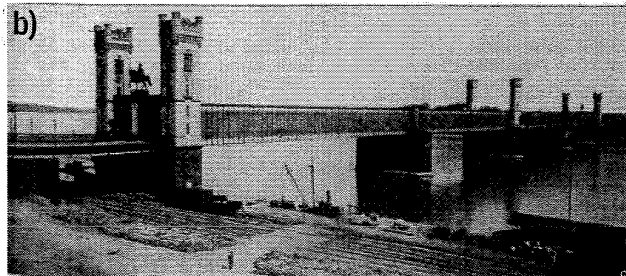
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Historical notes on bridge construction

28/39



a) Marienburg  
Bridge  
on Nogat River  
(Germany, 1857)

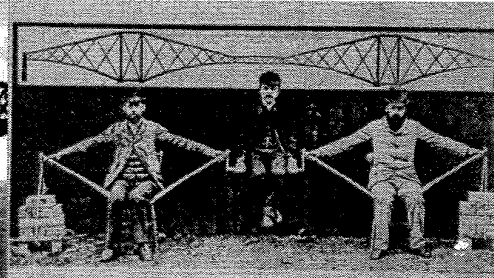
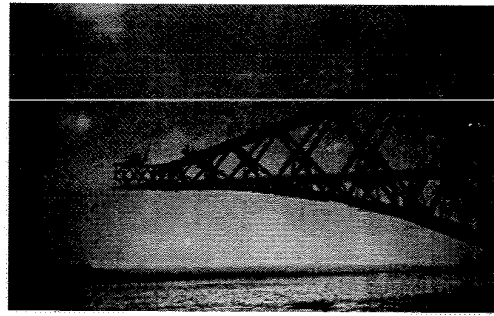
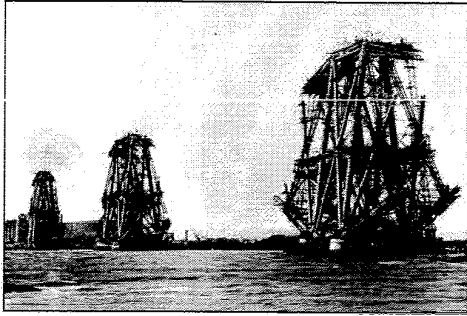


b) Koln Bridge on Reno River (Germany, 1857)  
c) Reno Bridge between Kehl and Strasburg (Germany, 1858)



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**1** Historical notes on bridge construction **31/39**



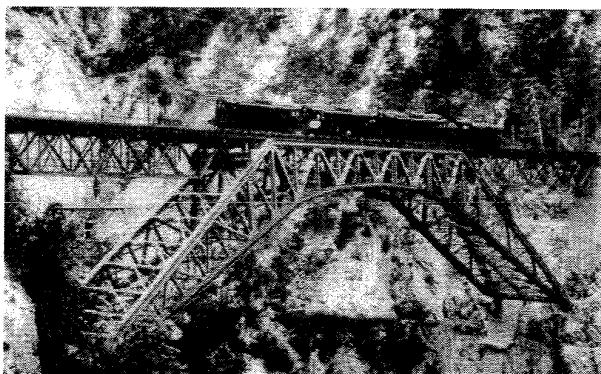
Firth of Forth bridge (1889 ÷ 1890) max span: 521m

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**1** Historical notes on bridge construction **32/39**



Assopos Bridge  
span: 80m  
(Greece, 1908)

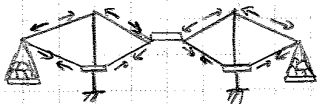
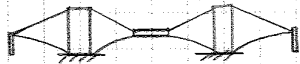


Bietschal Bridge on Brig river  
span: 94m  
(Swiss, 1913)

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## → Firth of Forth bridge (UK)

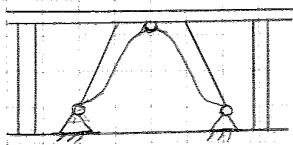
It was built between 1889 and 1890 and central span is 521 m.



They started to build the towers - they are the central part of the piers - and from them they built 2 big cantilevers on the 2 sides. Then, they placed the conjunction elements, which are simply supported beams based on the two cantilevers.

Static scheme is characterised by compressed elements and tensioned ones in the piers and on the two sides of the piers we have 2 counterweights, in order to avoid the collapse of the system by rotation.

## → Assopos bridge (Greece)

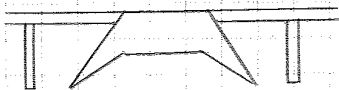


Built at the beginning of the XX century, it is an example of three-hinge arch bridge, that is an isostatic scheme.

The structure is made of 2 steel elements that are hinged together on the top.

Between abutments and the main span, the deck is made by simply supported beams on the piers.

## → Bietschal Bridge (Switzerland)

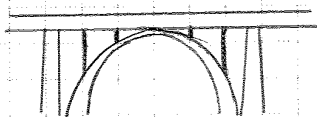


In this case, we have a two-hinge arch bridge, - that is a hyperstatic scheme - , as we have only one element in the arch.

Then, we have simply supported elements at the beginning and the end to connect the central element to the other piers.

## → Wupper Valley bridge (Germany)

It is the tallest railway bridge in Germany.



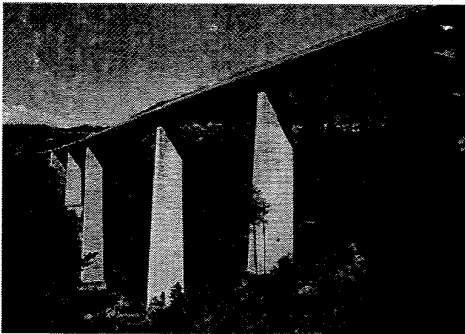
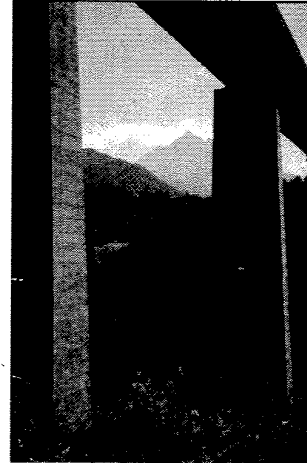
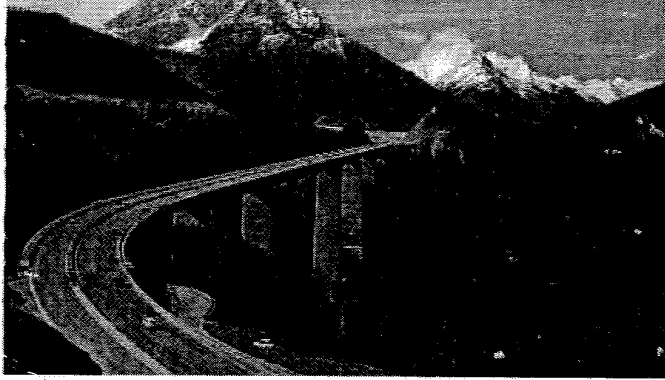
The static scheme is a reticular arch - TRUSS SYSTEM and the deck is supported by piers and vertical elements that are compressed and connect the arch to the deck.



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Historical notes on bridge construction

35/39



Europa Bridge (Brennero Highway)  
Max span: 218m  
Total span: 820m  
Max depth: 190m  
(Innsbruck, 1962 ÷ 1963)

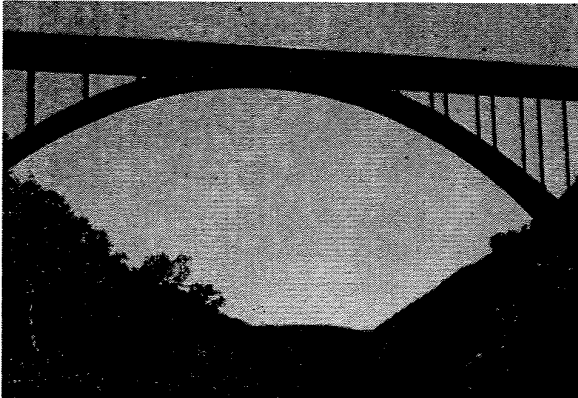


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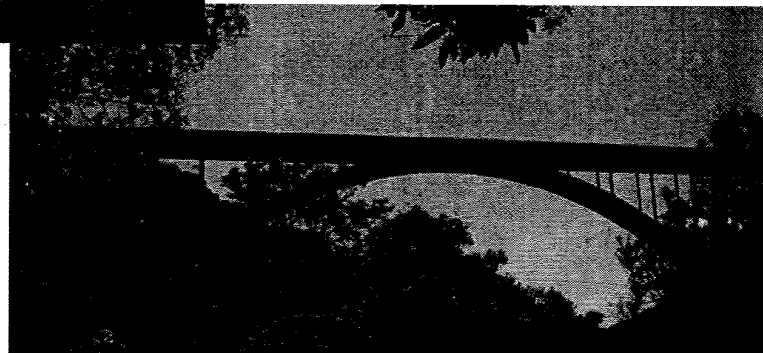
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Historical notes on bridge construction

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Van Stadens bridge  
span: 200m  
(1971 ÷ 1973 South Africa)



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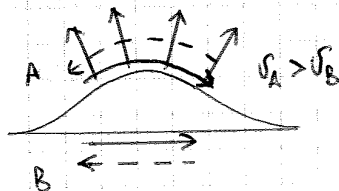
④ The deck arrives in segments on a boat and, using a crane that is moving on the ropes, they are lifted and connected to the other segments.  
 The advantage is that elements can be prefabricated completely and we have only to install them. In this way, due to the limited number of operations on the site, we have no problems about quality control, we don't need skilled people and procedures are cheaper.

to install, a low-level squad is enough

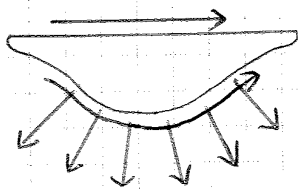
→ CROSS SECTION



The cross-section looks like to the one of the wing of an airplane.  
 A wing has a certain shape because, in this way, the path of the stream of air above the wing is longer than the path below. So, the speed of the air above the wing is higher than below and there is a depression which drags upwards the wing - it's sucked above.



Bridge's section has the opposite section of a wing - and it's symmetrical, because a wing is good for moving in one direction, while a bridge has to be good in 2 directions - because it should work like a wing <sup>with</sup> the opposite direction:



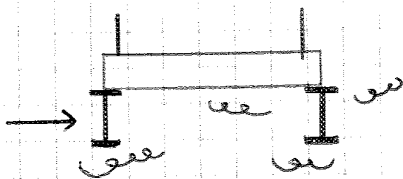
the speed of air below the deck is bigger and we have a depression dragging the deck downwards, like the spoilers of a car.

This is good because the bridge is suspended and linked to the ropes and, if the deck is dragged down, ropes are working against it. otherwise, if deck is lifted up, they don't work.

The reason of it is that ropes work fine and are stiff only if pulled.

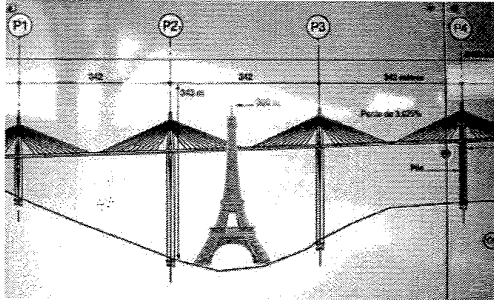
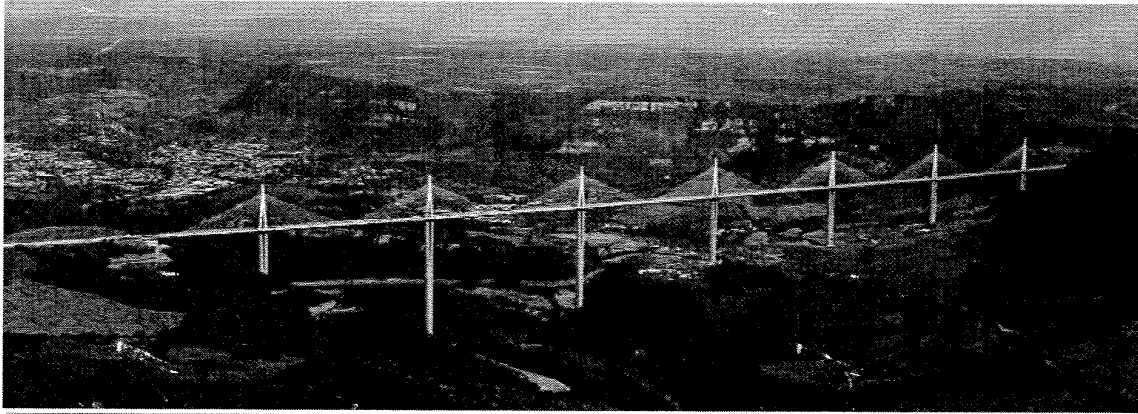
⇒ SUSPENDED STATIC SCHEME WORKS WELL UNTIL ALL ROPES ARE TENSED

So the shape is intentional in order to wind increase tension in ropes and stabilize the bridge.



At the beginning, shape was different and the deck was supported by 2 beams.

This section was abandoned because, when wind is arriving, there's turbulence and the deck starts moving and rolling. If wind's frequency is close to the natural frequency of the deck, it enters in resonance and collapses.



Millau viaduct  
(France, 2005)



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## BASIS OF DESIGN

1. We see some fundamental aspects, in order to choose the most sustainable solution and to fulfil bridge's request, that is cross something with certain characteristics.

To describe what are the main aspects in design, we can quote Marco Polo's "Il Milione":

Marco Polo was describing an arch bridge in stone to the Kublai Khan and he explained that it was built stone against stone. Then, Kublai Khan asked which of the stones was the most important one. No single stone was important, but all stones and the shape are important. So, Kublai Khan asked why he was talking about stones. Indeed, during realization the important element is not material but the shape. On the other side, Polo said the opposite because in the Middle Age there was no other material.

## 2. Input data needed in bridge design

→ planimetrical configuration

→ LAYOUT OF THE BRIDGE, with curvature radius and first attempts of total span and single span - put columns in some positions as hypothesis.

Nowadays, before the design of the bridge, there's the design of the infrastructure, on which a line that crosses the region is traced. From this, we take the first information and we'll design the bridge in order to respect infrastructural requirements.

→ OBSTACLE to be crossed:

the kind of obstacle rules maximum thickness in order to leave enough clearance, once we know the pavement's level

→ TRACK POSITION

→ altimetrical configuration

→ having observed the obstacle, we can define MAXIMUM AND MINIMUM DEPTH

→ SLOPE

→ CURVATURE RADIUS in the vertical plane, which is a not changeable input because imposed by codes (for visibility reasons).

There are also other aspects, like the presence of landslides and the stratigraphy, that are typical engineering matters.

→ local conditions for buildability (control of local situation)

→ ACCESS ROADS: if we want to use precast elements 35 m long, we have to be sure that the road is adapt

→ LOCAL AVAILABILITY OF MATERIALS

→ LOCAL AVAILABILITY OF COMMON AND SPECIALIZED WORKERS:

if a bridge will be built in a place where there are no specialized workers, we can't use sophisticated techniques because we should transfer many people for a long time and this is expensive.

→ environmental and meteorological conditions during the construction.

→ EXPECTED WATER LEVEL:

in this case, we refer to the common water level that we have every day - not the flow expected every 100 years. Indeed, every year we could have a flow of  $2500 \text{ m}^3/\text{s}$ . If width is 200 m and speed is 2,5 m/s, we have a wall of water which is 200 m x 5 m.

For this reason, during the construction of the bridge, it is better avoid having elements that stay in water - if possible - due to flow and the risk of being transported away or creating a damp.

⇒ we should build on the real abutments and not built any former wall inside the river during construction, except piers.

→ EXPECTED TIDE LEVEL, that is sea level and can vary of metres.

→ EXPECTED DROUGHT PERIOD (= "siccità") and EXPECTED TEMPERATURE DURING THE CONSTRUCTION:

in case of drought and/or high temperature, we have to take particular care in preparing concrete and cost increases.

## → LIMITATION AND CONTROL OF VIBRATION AMPLITUDE AND FREQUENCY

This is a problem of safety.

In railway bridges, vibration could cause the liquefaction of the ballast - it moves - and we have problems of safety and maintenance.

In pedestrian bridges, due to people, we can see movements due to the small torsional moment coupled to the vertical movement.

soil, otherwise we could find water inside the underpass

- flyover: it is used to overpass URBAN AREAS.  
In this case, design is influenced by
  - urban constraints, that imposes different spans
  - hindrances to the execution, connected with noise and occupation of roads, and that make work organisation more difficult and influence costs.

## II Service class

We can define many categories.

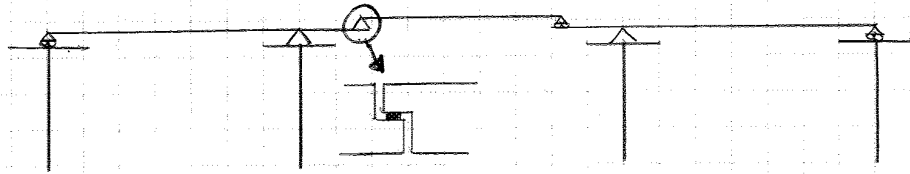
- I category road bridges, that can carry full traffic load
- II category road bridges, that have some limits for trucks.  
This category has been abolished in the last version of Italian Codes due to problems in repairs and accidents that happen because drivers don't take into account the limitation of capacity of the bridge.
- pedestrian bridges
- railway bridges, that can carry only passenger traffic or mixed traffic.  
Loads are different, as a TGV is 3,5 ton/m ~~we~~ heavy and heavy trains are 15 ton/m heavy. So, if we have only passenger traffic, we can use light structures; in mixed traffic, we use heavy structures.
- channel bridges or installation-bearing bridge, used to transfer water or tube.

## III Material to be used

We can classify bridges as

- REINFORCED CONCRETE BRIDGES
- PRE-STRESSED CONCRETE BRIDGES
- STEEL BRIDGES

→ GERBER SCHEME

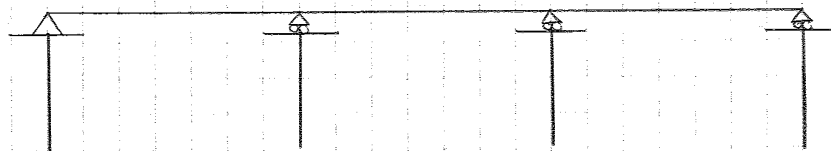


If there is a span bigger than the other ones to be crossed we can use first spans with cantilevers and put a simply supported beam in the central span. That is an isostatic scheme - forbidden for railways -, so we have.

- no internal actions due to settlements or thermal variations
- good internal actions distribution due to the presence of cantilevers.

When we realize Gerber's hinge, we have to reduce the height of the beam and we obtain a point which is difficult to inspect, as we have to uplift the beam to mount here machines for extra maintenance.

→ CONTINUOUS SCHEME



This scheme implies the best use of materials because in isostatic beams depth is decided from one section - the most solicited one -, whereas in continuous beam we can divide the isostatic bending moment between positive and negative moments and use reduced sections.

On the opposite, when there are settlements or thermal variations, the axis line of the bridge becomes curve and we have internal flexural problems - solicitations.

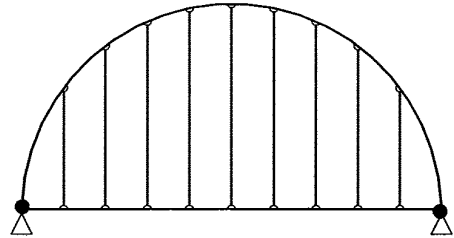


Before 1950s - 1960s, this scheme wasn't used because machinery was poor and there wasn't any advanced technique of realization of foundations, so isostatic scheme was preferred. Nowadays, capacity of machinery is higher, we use deep

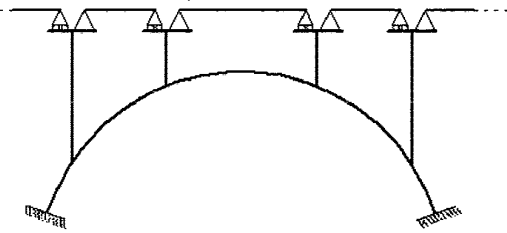


### Arch bridges

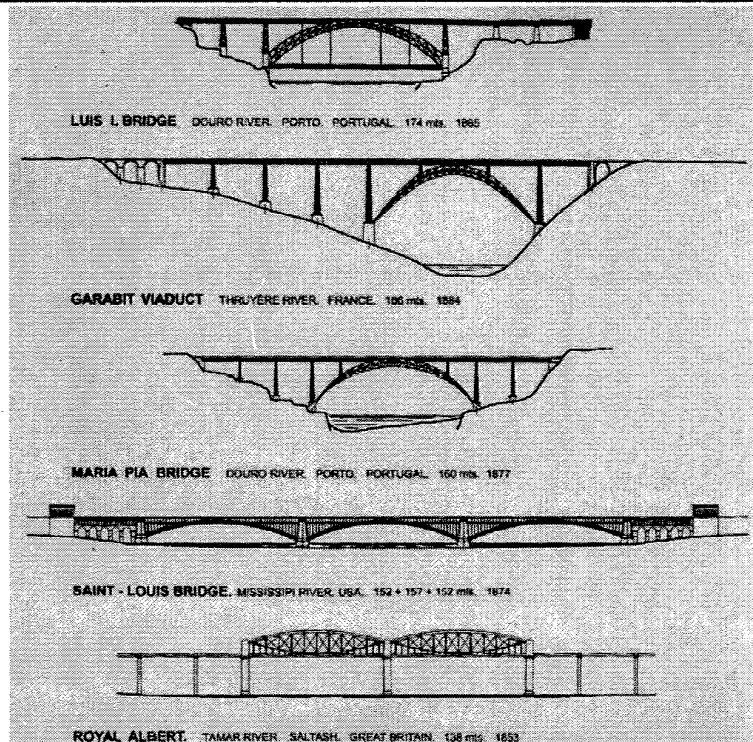
- Bow string  
(bottom deck)



- Fully restrained arch

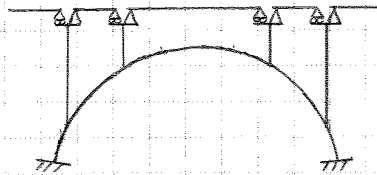


### The largest arch bridges (1853-1885)



Used for long spans, it is an elegant solution in which a horizontal thrust is placed, as arch transmits a high horizontal action.  
Sometimes, trust is realized with a girder of prestressed concrete.

### → FULLY RESTRAINED ARCHES



The scheme is similar to the first ones, but it presents a high level of hyperstaticity, so we have to be sure about soil conditions.  
This scheme usually is adopted in deep mountains where we can use rock, that grants a full restraint.

Arch scheme was abandoned in the 1950s - 1960s because building an arch presents 2 problems.

On the one side, we have to realize a structure of sustain, that is a huge scaffolding (= "cassera") put down before the work and onto the arch and the deck are casted, because they have no resistance when casted. Yet, this structure is expensive.  
On the other side, when scaffolding is realized, floods can destroy it when the arch is not completely built and can't sustain itself yet. Moreover, the scaffolding can become a damp.

If we analyze the largest arch bridges realized, we can define 3 periods.

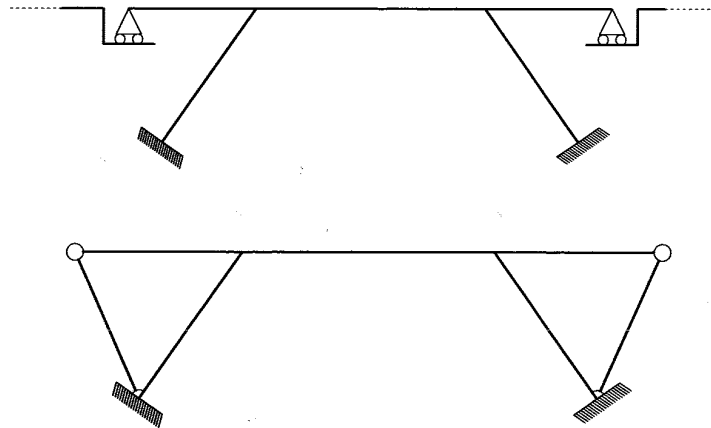
Between 1853 and 1885, steel was dominating and important bridges were built, mainly for railway transportation, like the Luis I Bridge in Portugal (length = 174 m).

Between 1886 and 1976, steel was still used with bigger results, like the New River Bridge in USA (length = 518 m - that is a record).

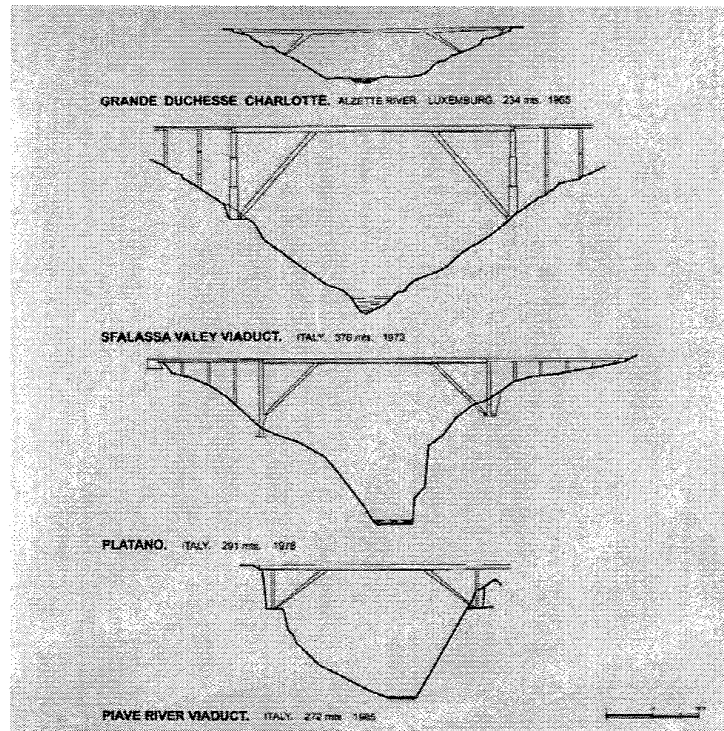
Between 1927 and 2000, we can see the introduction of concrete as construction material for arch bridges. An interesting case is the one of Krk Bridge (Croatia), which had lots of problems of durability due to the small cover and now it's on repair.

### Trestle bridges

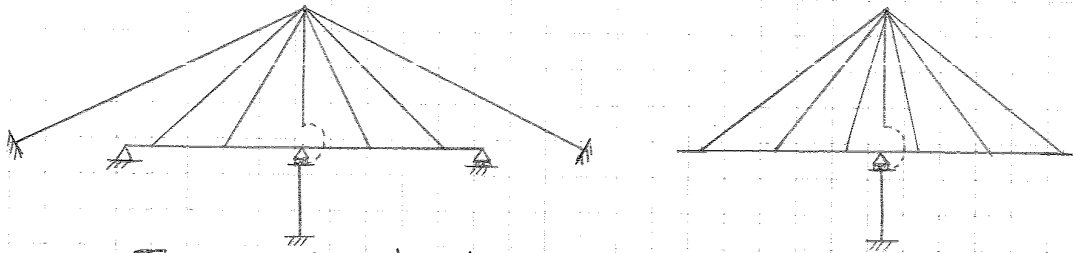
- Single trestle
- Tied trestle



### The largest trestle bridges



→ stay-cable bridges

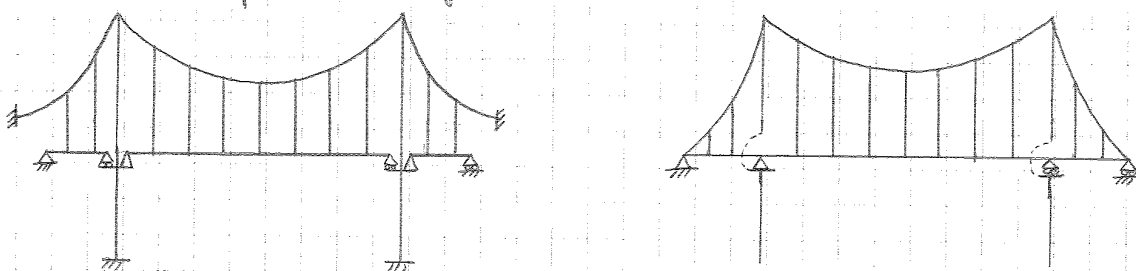


They were introduced after World War I but then abandoned because, at that time, stays were realized with chains, whereas stays have to be realized with high strength materials and loaded very highly in order to be straight. For this reason, many bridges collapsed and Navier suggested not to build this type of bridges because too much dangerous. Then, after World War II, high-stress steel was introduced and stay-cable bridges were built again.

Stays can be anchored to the soil or to the deck and they are elastic restraints that introduce an axial force - this bridges are subjected to global instability.

Examples of cable-stayed bridges are Duisburg - Neunkamp Bridge, where cables don't depart from the same point - semi-fan cables - ; Barrias de Luna, which has the largest concrete span (460 m); Bratan Bridge, that is the first bridge with central suspension - very elegant solution - ; Morandi Bridge, which presents one stay in prestressed concrete that conceptually should break itself.

→ Suspended bridges

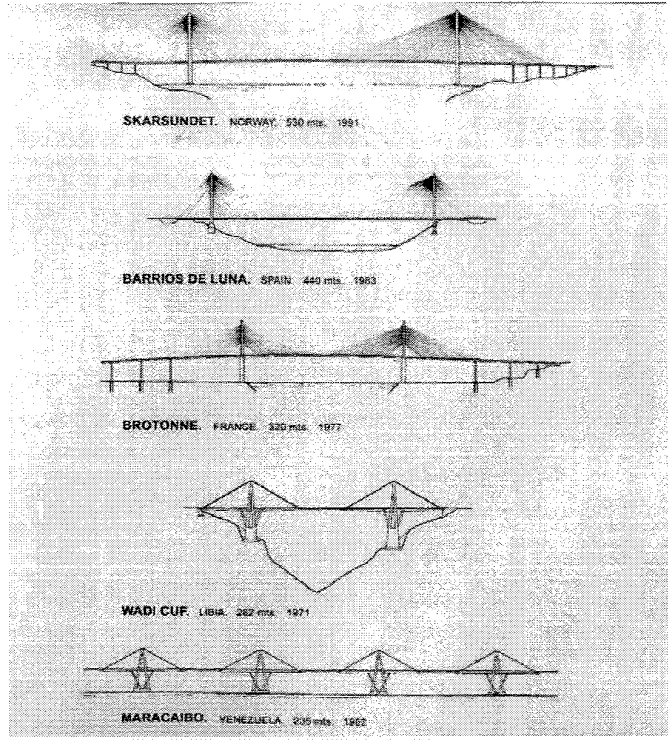


They present the deck, a suspended rope and a number of ties on which deck is suspended. The rope may be anchored to soil or to the deck.

Lots of suspended bridges were built in the XIX century, like the Brooklyn Bridge.

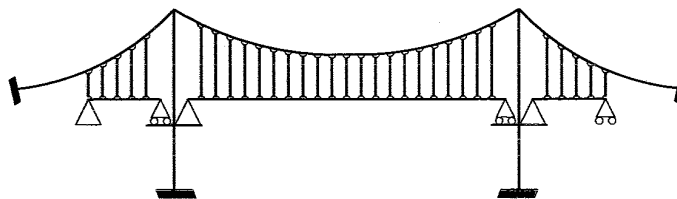
An interesting example is the Akashi-Kaikyo Bridge, which is the longest bridge in the world.

The largest stay-cable  
Bridges with concrete  
deck  
(1962 – 2000)

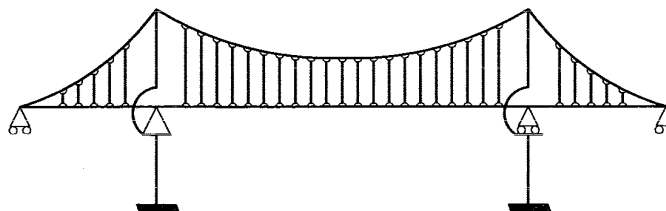


Suspended bridges

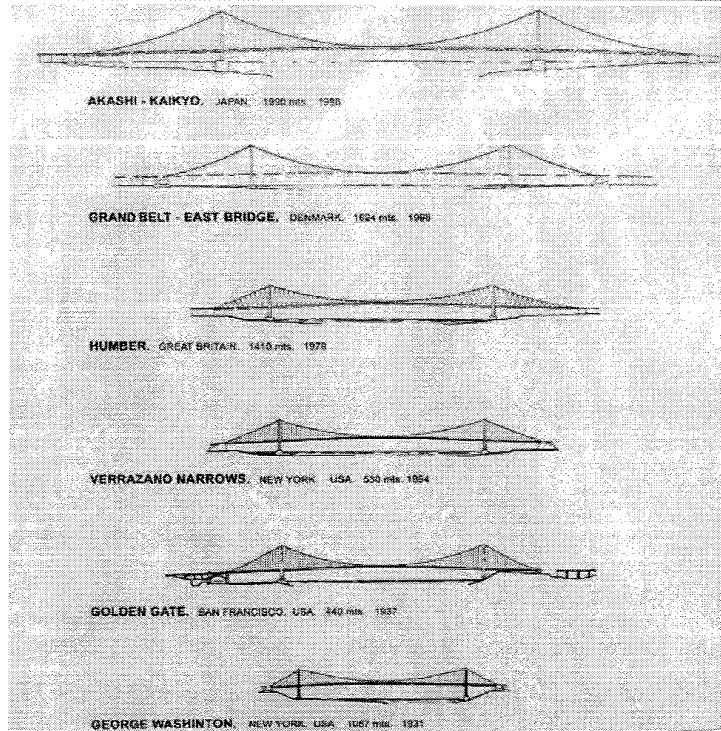
- Anchored to the ground



- Self-anchored (to the deck)



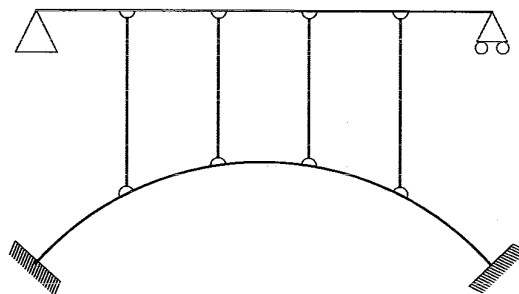
### The largest suspended bridges (1931 – 2000)



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### Arch-beam bridges

- Inverted suspended bridge



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## Construction systems

### 1 Moving scaffolding

Firstly, we have to introduce 2 definitions

→ Formwork (= cassetta):

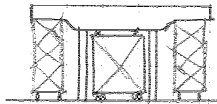
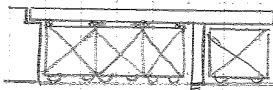
it is the structure that contains fresh concrete.

→ Falsework (or SCAFFOLDING):

it is the temporary structure that sustains the formwork and supports with limited deformations the weight of concrete and of the formwork.

This construction system may be realized in 2 ways

→ FORMWORK ON FIXED FALSEWORK, where scaffolding is solidar to soil.



This system usually is adopted in continuous beam bridges, where portions of different spans are casted. In particular, we cast the first span and 25% of the second span, so that then we cast 75% + 25% of the next spans - in this way, we have the same positive and negative moments.

Sometimes, the fixed falsework has wheels - it's "fixed" because it's directly supported by soil - , that allow to remove it and transfer to the other spans. Looking at the transverse section, we can see the disposition of scaffolding.

This system presents 3 problems

→ scaffolding safety, because it is a temporary structure and it needs controls

→ deformation during the casting, because we cast many  $m^3$  of concrete and it takes many time to harden.

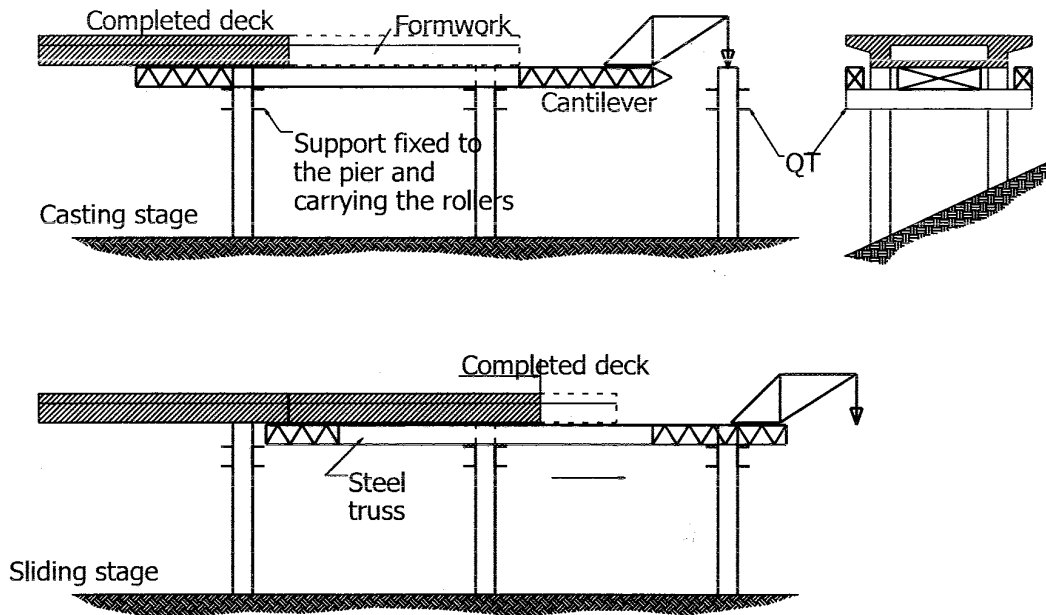
So, we have to control also the casting.

For instance, in a simply supported beam, we cast before the central part - in this way, we produce maximum rotation at the extremes - and then we cast the other parts.

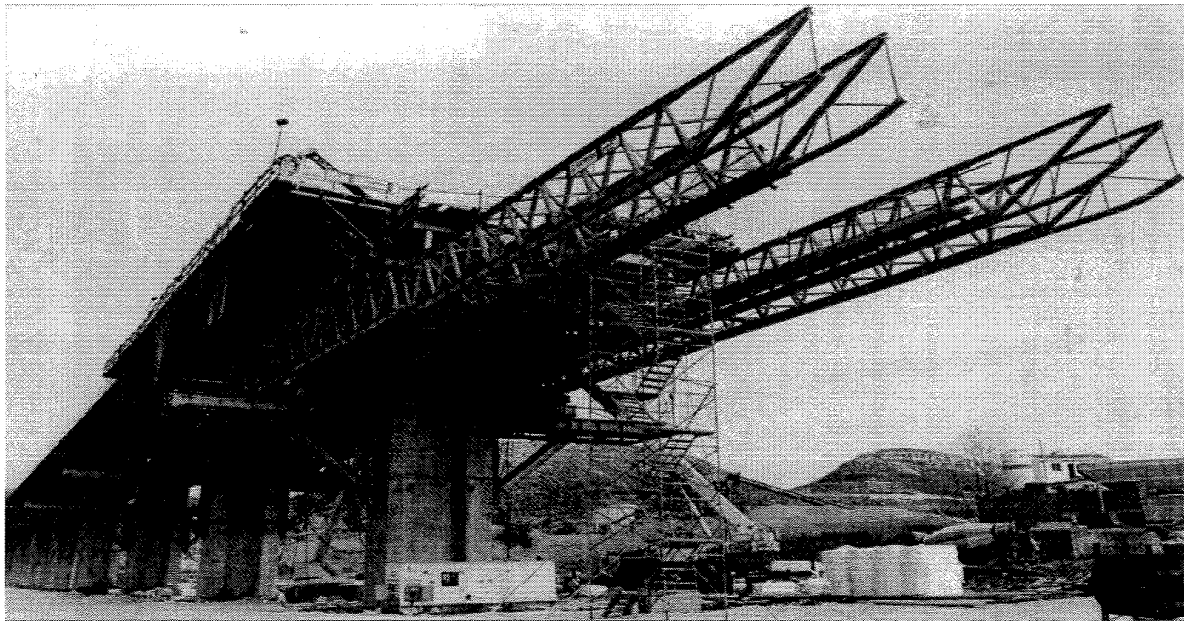
→ removal of scaffolding, that is a delicate operation because in it we apply self-weight.

So, we have to take care of the sequence of removal in order to apply deadload in a progressive way and avoid dynamic application (dynamic

b) Falsework sliding on rollers from pier to pier



a) Falsework sliding on rollers from pier to pier





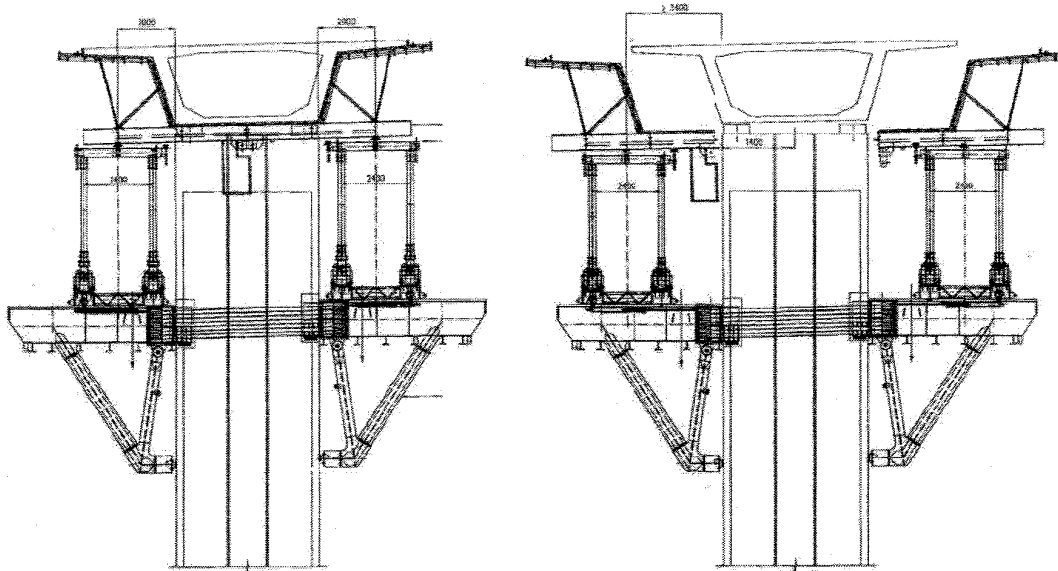
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Basis of design 41/91

Transverse view of scaffolding movement: transverse slide

Closed position: concreting

Open position: avancement



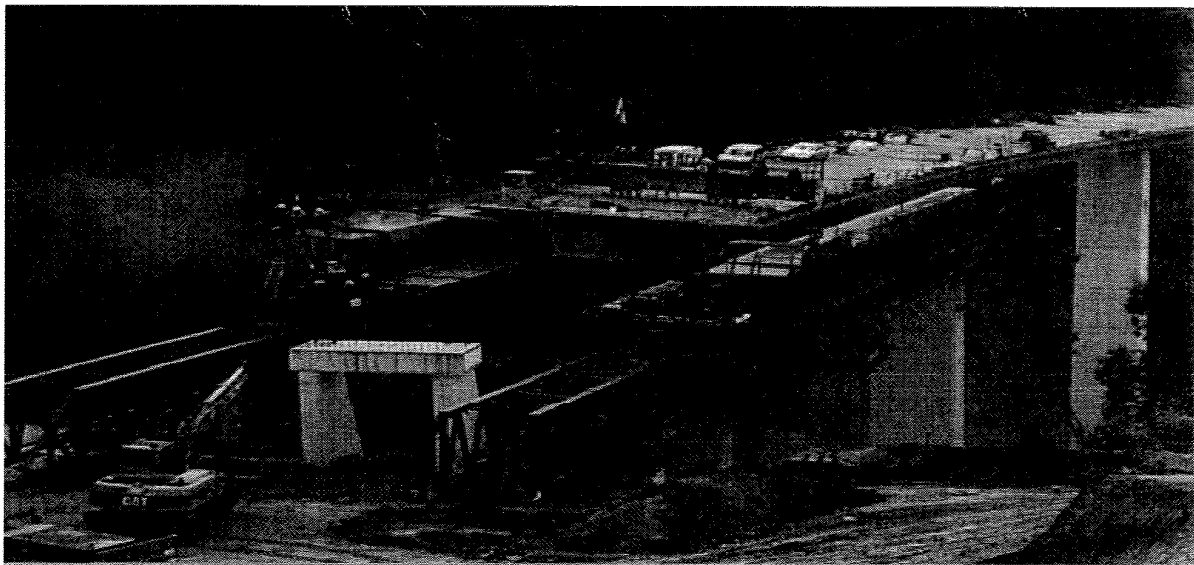
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Transverse view of scaffolding movement: transverse slide

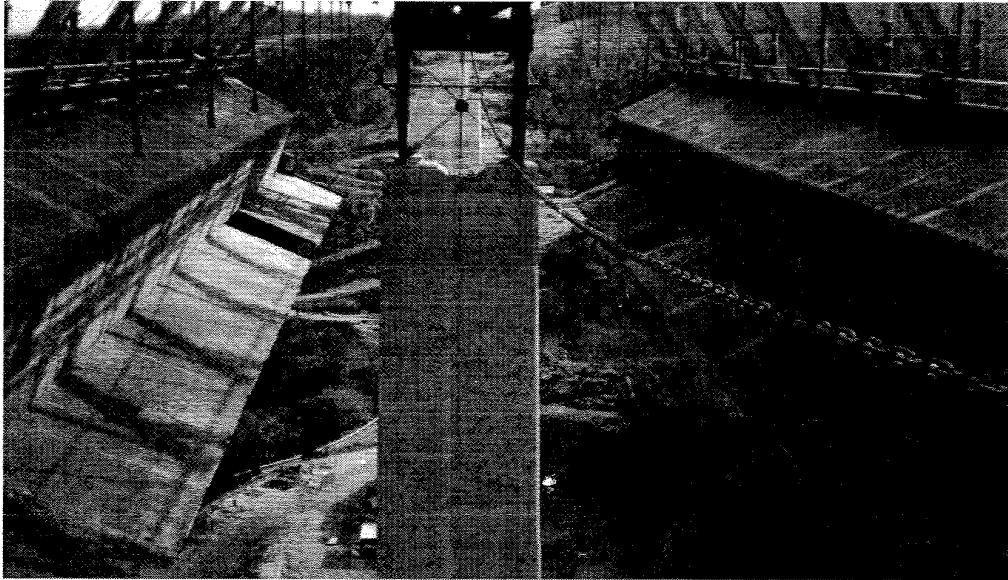
Open position: advancement



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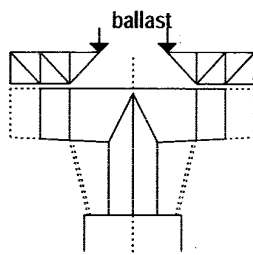
Transverse view of scaffolding movement: opening with rotation

Open position: advancement (view from the inside of the scaffolding)

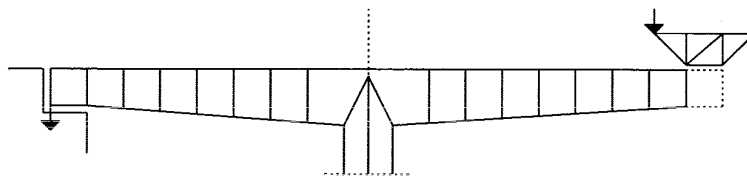


### 3.2) Free cantilevering [Finsterwalder / dywidag]

a) Classical cantilever with two ballasted falsework and formwork



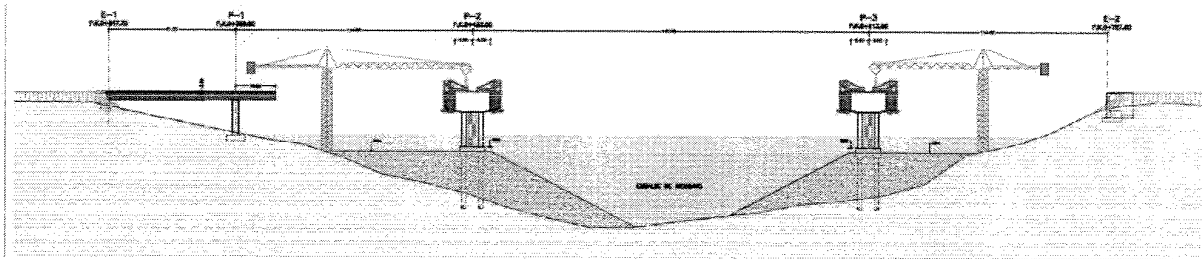
Equilibrium by means of full restraint to the pier or external ties



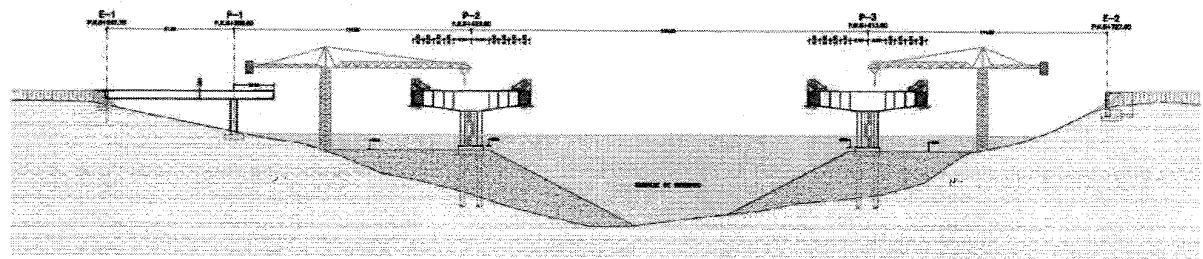
Maximum span: 240 m  
(Hamana/Giappone)

Complete cantilever; when the key is closed the cantilever is made continuous

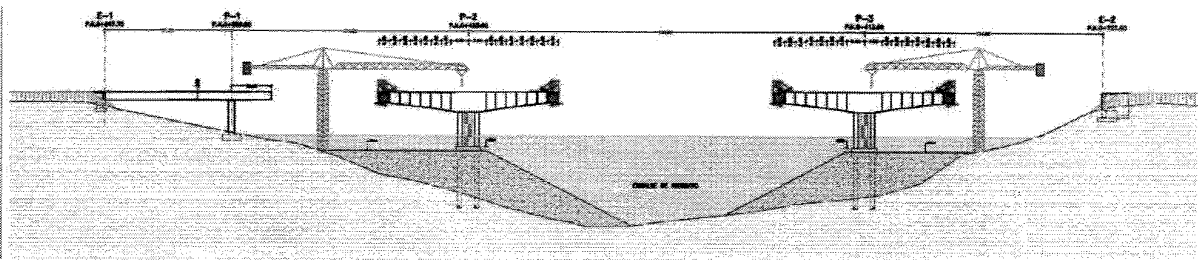
### 5) Concreting of segments 1



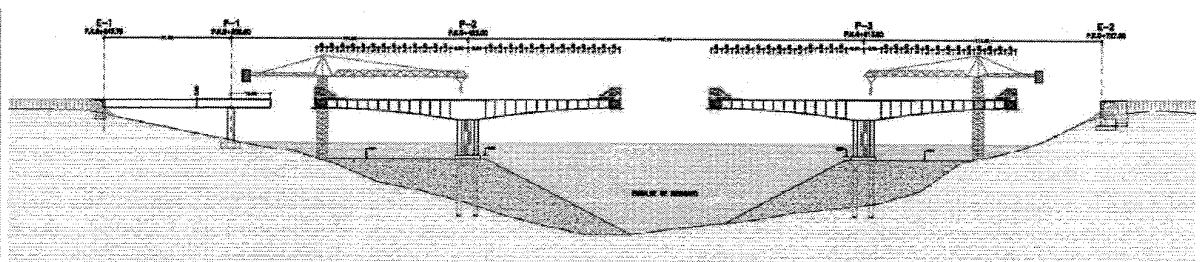
### 6) Concreting of following segments



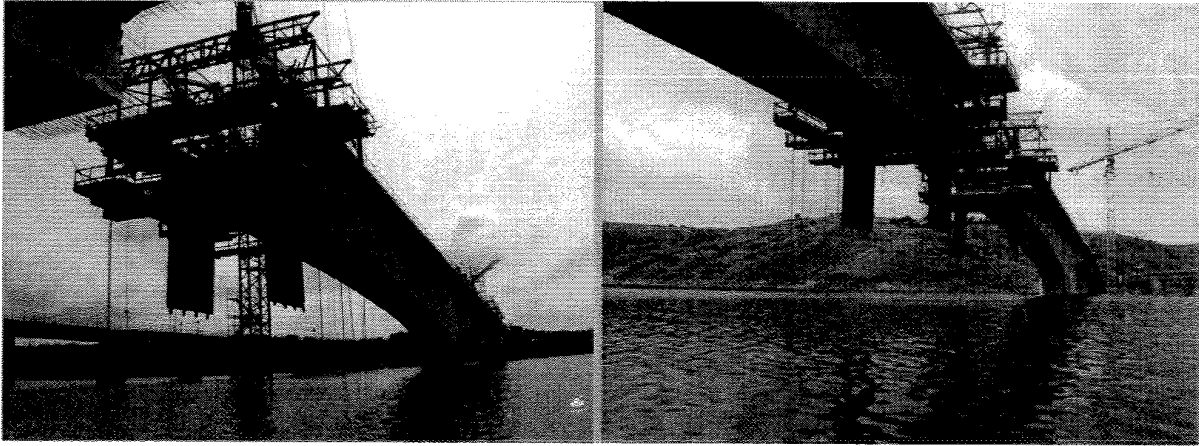
### 7)



### 8)

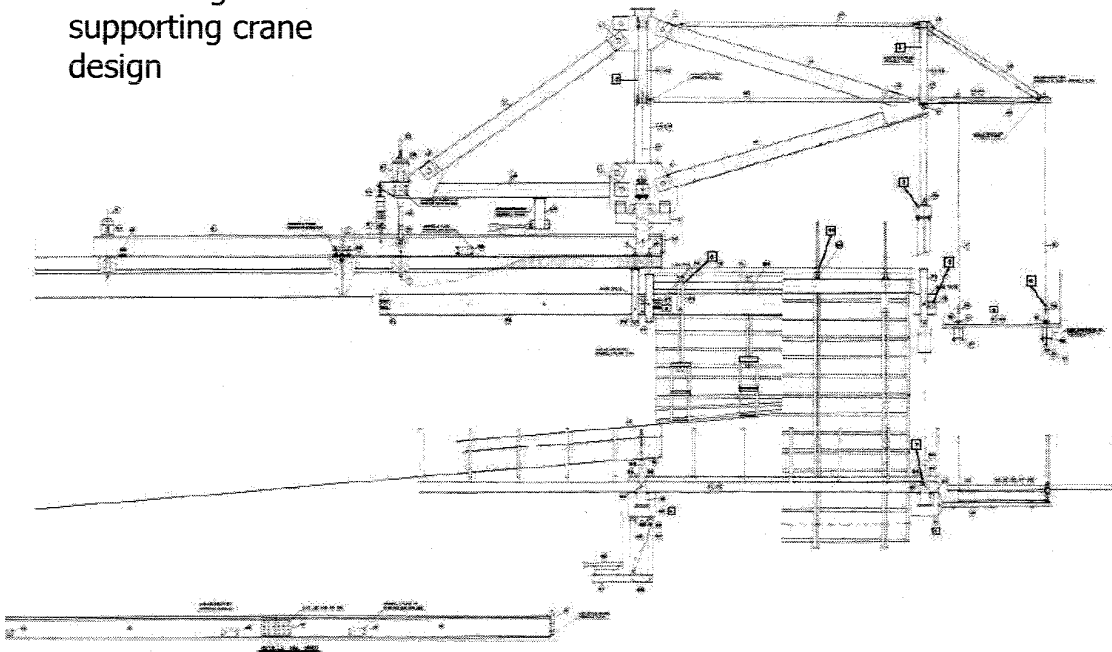


### Scaffolding movement and concreting of successive segments



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### Scaffolding supporting crane design

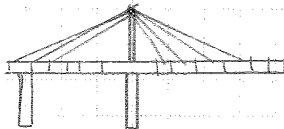


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In this system, we can define a certain construction process

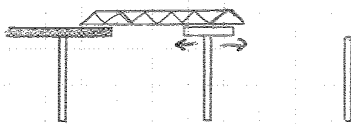
- 1) Building of foundations and temporary embankments to build the piers.
- 2) Construction of piers
- 3) Construction of pier segments, that are precast segments from which we start the construction of the deck.
- 4) Positioning of cranes used for scaffolding
- 5) Construction of the deck, segment by segment
- 6) Introduction of continuity tendons and final static scheme.

### 3 Cantilever with auxiliary tendons



If we go out of symmetric conditions, we can mount temporary tendons to put strength and suspend segments.

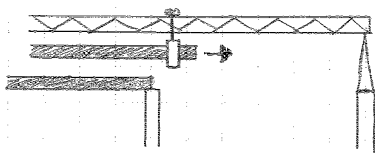
### 4 Cantilever with auxiliary truss beam



When we have already built part of the bridge, we realize a truss beam supported by the deck already built, to which a mobile formwork is suspended and casts segments with the cantilevering system.

Once casting is completed, the beam is moved longitudinally and formwork is transferred.

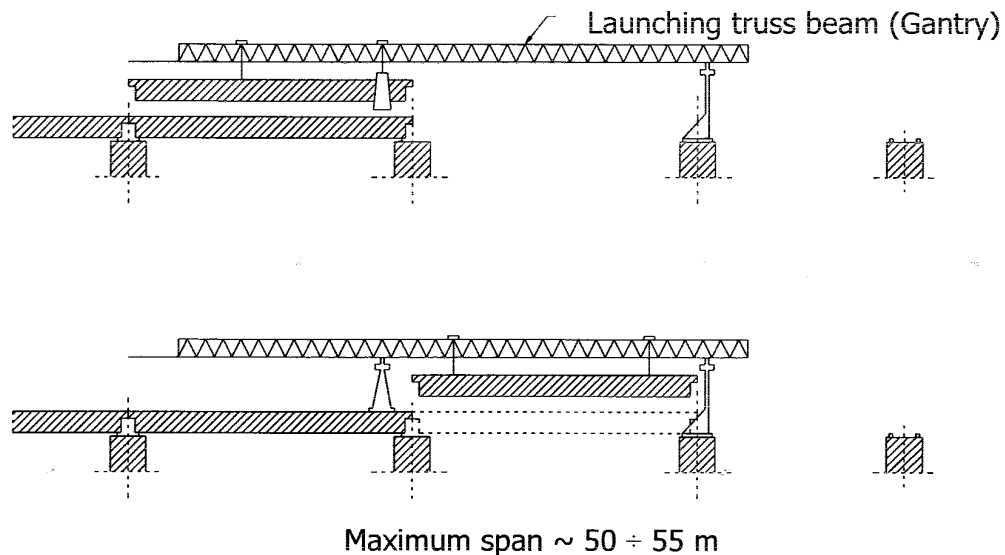
### 5 Full span prefabrication



In this case, each span is built on the one side of the bridge and it is transferred and put in position by a truss beam.

This beam has a maximum weight that it can support.

### 3.5) Full span prefabrication

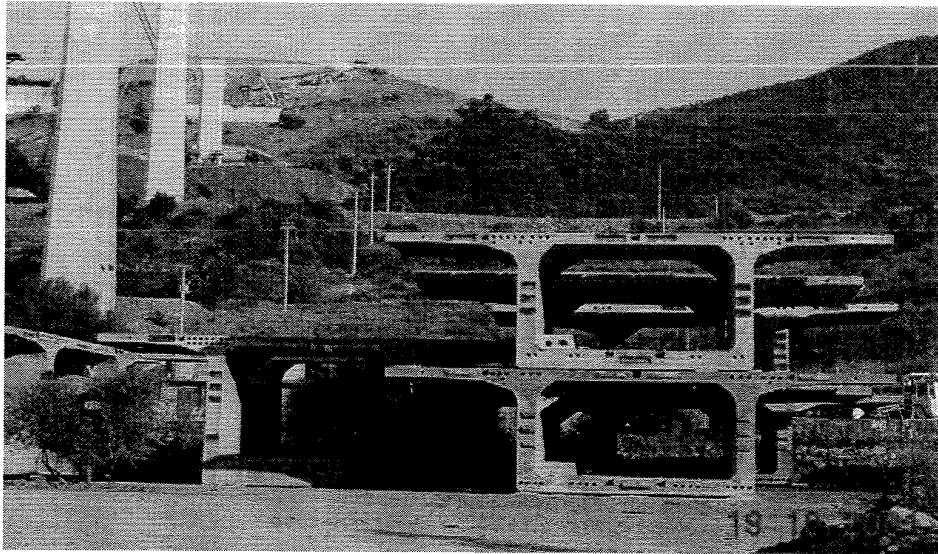


Both **overhead travelling cranes** and **gantry cranes** are types of Crane which lift objects by a **hoist** which is fitted in a trolley and can move horizontally on a rail or pair of rails fitted under a beam. An overhead travelling crane, also known as an overhead crane or as a suspended crane, has the ends of the supporting beam resting on wheels running on rails at high level, usually on the parallel side walls of a factory or similar large industrial building, so that the whole crane can move the length of the building while the hoist can be moved to and fro across the width of the building. A gantry crane has a similar mechanism supported by uprights, usually with wheels at the foot of the uprights allowing the whole crane to traverse.

A **hoist** is a device used for lifting or lowering a load by means of a drum or lift-wheel around which rope or chain wraps. It may be manually operated, electrically or pneumatically driven and may use chain, fiber or wire rope as its lifting medium.

1

Basis of design 61/91



Prefabrication yard



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Basis of design 62/91



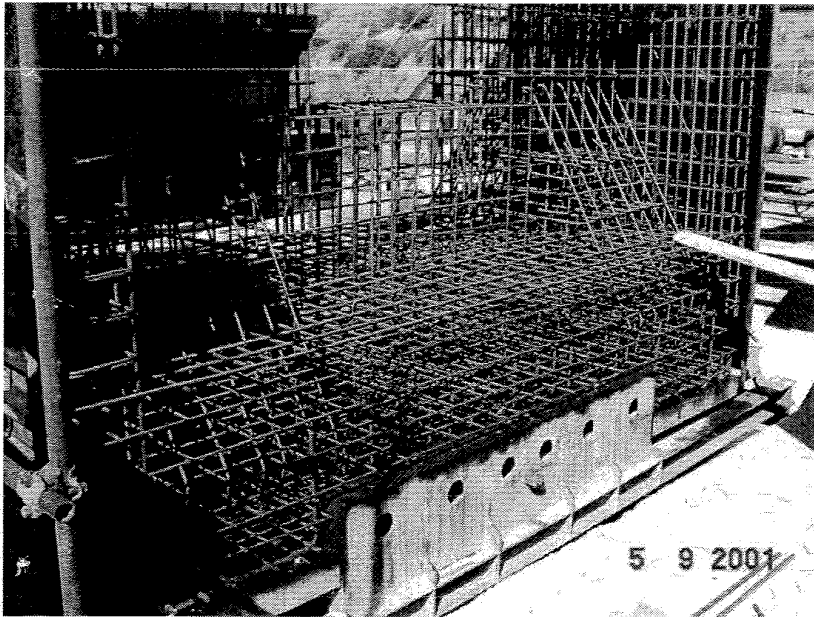
Prefabrication yard



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Basis of design 65/91



Pier-segment reinforcement



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Basis of design 66/91



Crane



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## 6 Precast segmental construction

This system of construction started in the middle '70s.

It consists of the PRECASTING OF SEGMENTS of the bridge THAT ARE ASSEMBLED by means of a grouting material, like resin, and temporary prestressing. Then, they are maintained in position with permanent prestressing.

ADVANTAGES → segments can be precast when foundation is being built, reducing the time of construction.

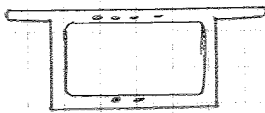
→ casting is made in a protected area, dedicated to the construction of segments - no problem with the external conditions.

APPLICATION: this system is generally applied to build framed bridges or continuous beam bridges, with a maximum span of

$$S_{max} = 135 \div 160 \text{ m}$$

Beyond this limit, the weight of the segments becomes very important (it is limited to 100 tons)

SEGMENTS: Segments are in a storage area and they present shear keys.



The reason of it is that segments are built one against the other, i.e. once the first segment is casted, this is used as a formwork for the second one and so on.

The presence of shear keys force us to respect the initial position of segments during the construction, but it helps to assembly them in a better way.

Segments present ~~at~~ same holes at the top and the bottom for the pre-stressing tendons.

A particular segment is "Segment 0", which is put on the top of the pier and, from it, we start to build symmetrically on the two sides.

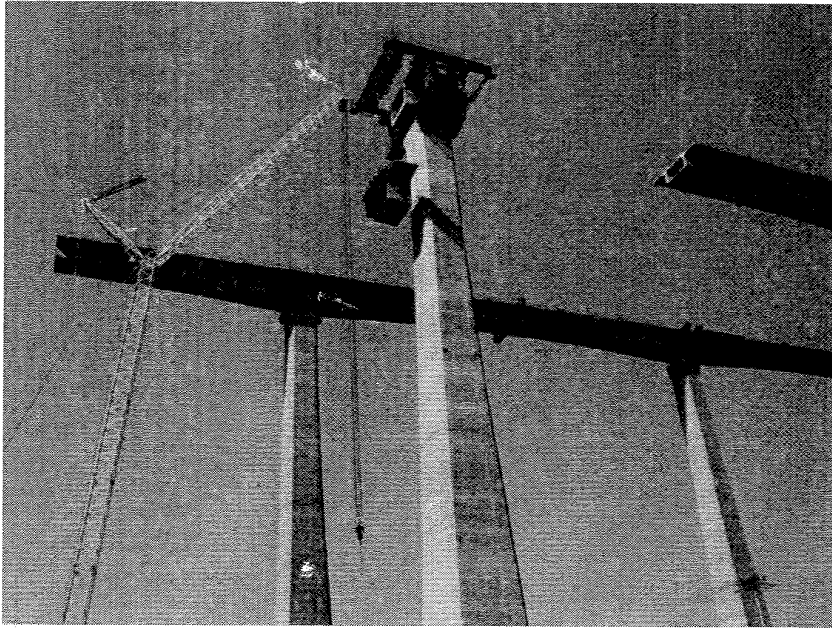
It presents a robust diaphragm because torsion that arrives from the current section of the bridge at this segment should be transformed in an equilibrium of 2 vertical reactions and, for this purpose, we need a rigid diaphragm.

As far as the reinforcement system is concerned, at the top of this segment, all the tendons cross the top of the pier and there is the maximum number of holes for tendons.

As regards ordinary reinforcement, it is a frame with an enlargement at its base, because we need to collect the reaction from beams and enlarge the contact area between the segment and the pier - not only a point for the bearing. Otherwise, the system will not work - we need a mass of concrete to introduce these forces inside the diaphragm.

1

Basis of design 69/91



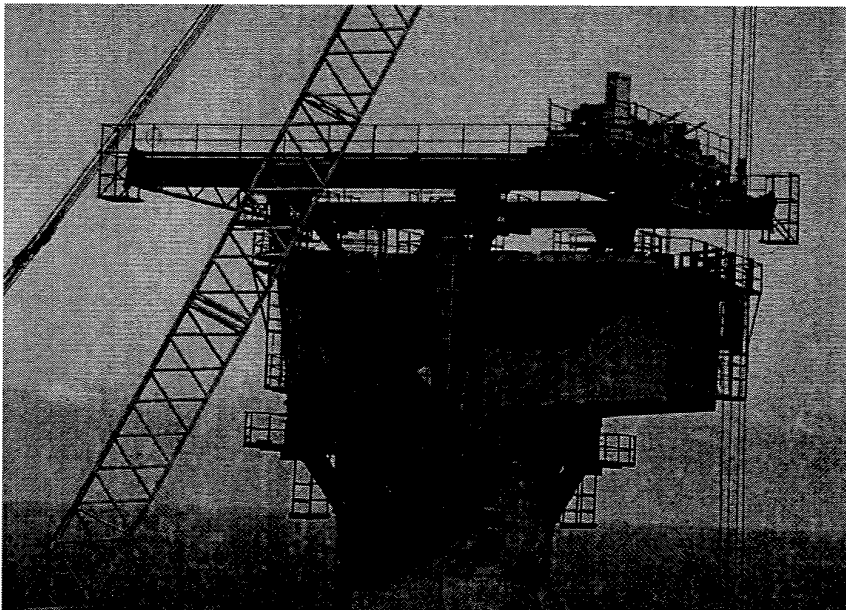
Pollina Viaduct:  
first segment  
outside the pier



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1

Basis of design 70/91



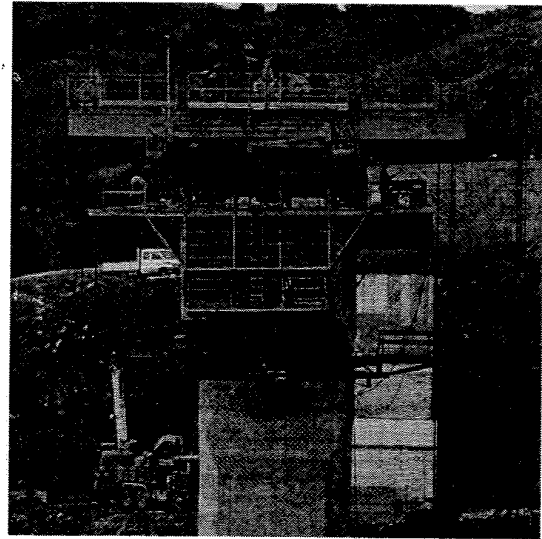
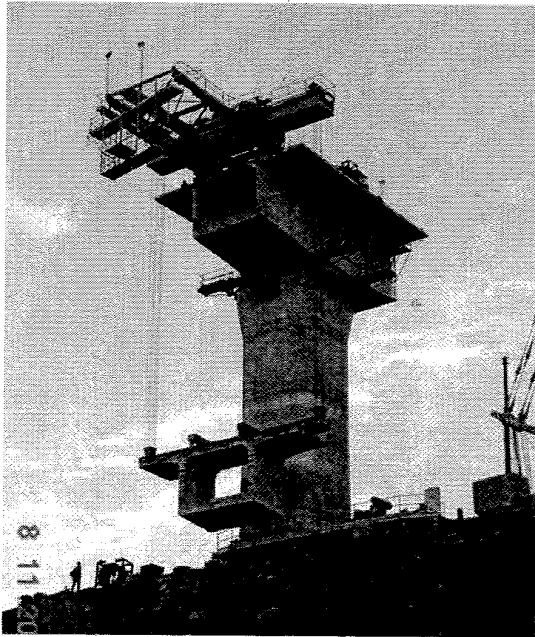
Positioning of  
segment two (1)



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1

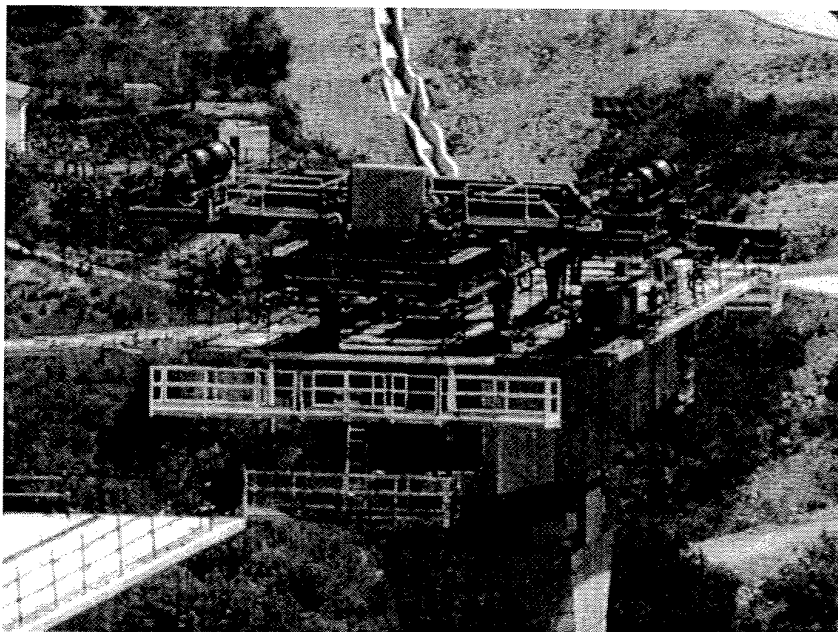
Basis of design 73/91



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"Bridge design"

1

Basis of design 74/91



Pier-cap with first  
segments

Tusa Viaduct



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Basis of design 77/91



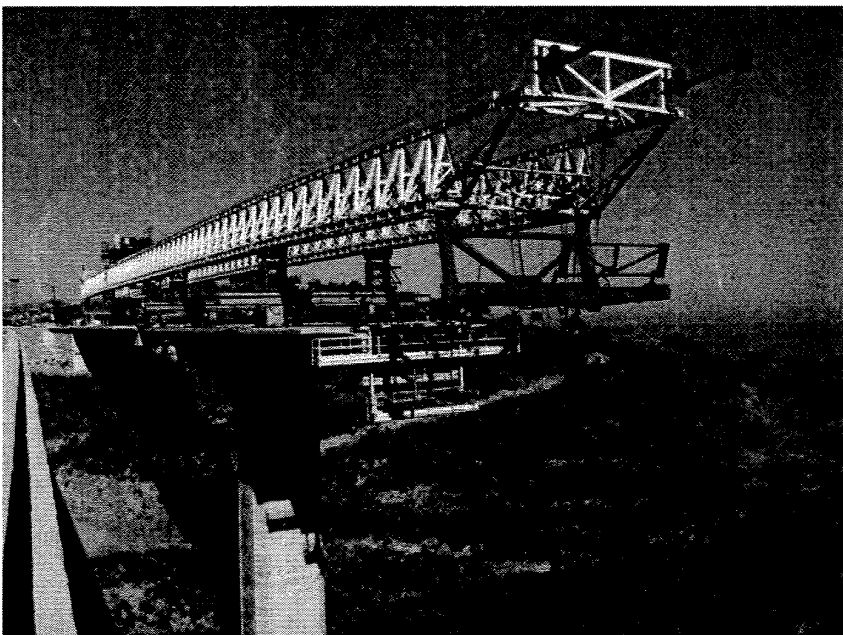
Vallone Marzo  
Viaduct:  
Launching girder



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Basis of design 78/91



Launching girder  
Ortora Viaduct



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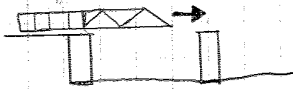
## → LAUNCHING GIRDER

This is a truss that carries along the top current the mobile crane which takes the segment, moves it along the truss and places it. The system is carried by the bridge and transmits to it a relevant weight.

The girder presents a pendulous leg. When a cantilever is built, machinery moves along the axis to reach next pier. When ~~at~~ the girder arrives at it, the leg is put in vertical position and same jacks are inserted on the piers and they enter in force.

Thus, girder has a new bearing on the following pier and a bearing on the already built part of the bridge and the girder can be transferred.

## 7 Incremental launching



In this system, we assembly on the rear of abutment a LAUNCHING NOSE.

It is a steel structure, lighter than the bridge. Then, segments are assembled on the rear of the launching nose and more segments are casted, more the bridge is pushed or transferred with tendons - the jack is placed against the front face of the wall of the abutment and there is a tendon on the rear - up to arriving over the first pier.



to transfer the load, we need to use stainless steel plate - with teflon inside - that theoretical friction coefficient will be  $0,02 \div 0,03$ .

Yet, the sliding surface is irregular and design friction coefficient is  $0,08 \div 0,1$ , so we need to apply a horizontal force that is  $8 \div 10\%$  of the weight.

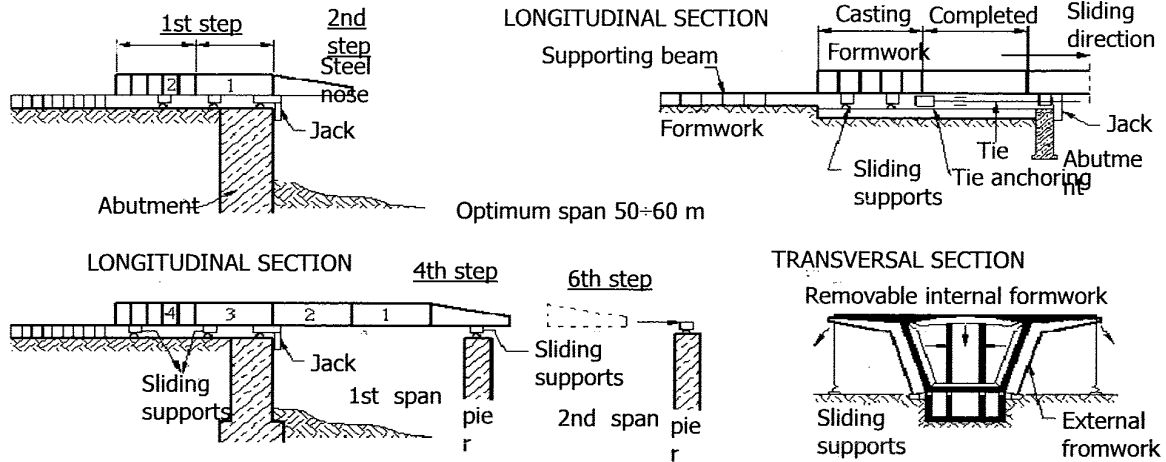
The launching nose is useful to reduce the weight of the cantilevering part of the bridge.

LIMITATIONS → in planimetry, curvature radius has to be infinite (straight bridge)

→ in altimetry, vertical curvature radius should be very large

→ the limit span is  $600 \div 700$  m, otherwise friction becomes too much high.

### 3.7) Incremental launching



The principle of incremental launching:  
 casting in segment on the rear of the  
 abutment and trusting from pier to pier.  
 Limit span: 600÷700 m

Limitations {  
 planimetric:  $R = \text{cost} = \infty$   
 altimetric:  $R$  very large

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### Criteria for span choice

- Controlling parameter  $\Rightarrow$  Performance  $\mu = \frac{\Delta\sigma_{var}}{\Delta\sigma_{var} + \Delta\sigma_{perm}} \Rightarrow \mu = 0$
- Economical limit of performance {  
 $STEEL : \mu_{lim} \cong 0.5$   
 $CONCRETE : \mu_{lim} = 0,10 - 0,15$   
Limit span
- Comparison of different performances for bridge type and materials

Type	Material	Economical limit span [m]	Maximum span realized [m]
Wall web and continuous beam	P.C.	~250	240 (Japan)
Wall web and continuous beam	STEEL	~350	345 (Jugoslavia)
Stay cable	P.C.	~500	400 (Thailand)
Stay cable	STEEL	~1800	404 (France)
Trussed gerber beam	STEEL	550÷600	549 (Canada)
Arch	STEEL	350÷400	366 (Canada)
Trussed arch	STEEL	~700	511 (U.s.a.)
Arch	R.C.	~400	~390 (Jugoslavia)
Suspended	STEEL	3500	~1900 (Japan)

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Another parameter is aesthetic:

in order to have equilibrium from the aesthetic point of view, we should take care that span is bigger than the depth of the piers.

$$p > h$$

## 2 Bridge transverse section shape (section of the deck)

The choice of the section of the deck is influenced by many parameters

→ SPAN, in relationship to statical scheme

→ SLENDERNESS RATIO REQUIRED  $p/h$ :

for instance, in a continuous beam with cantilever with concrete, the transversal profile is variable - if the spans are different - and maximum depth is about  $1/20$  times the span.  
There could be also some requirements from environmental authorities.

→ AVAILABLE TECHNOLOGY FOR EXECUTION:

it is a non-sense use a complex shape if there is no technology.

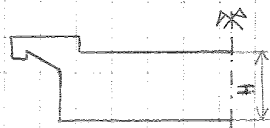
→ COST

→ RATIO BETWEEN LIVE LOAD AND DEAD LOAD:

it is important for dynamic behaviour, that has a role in slender bridges and should be avoided.

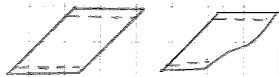
Considering these parameters, we have to choose among different kinds of section.

→ Slab bridges cast in situ



The section is simple but it can be used only for LITTLE SPANS

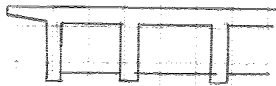
$$s \leq \begin{cases} 20 \text{ m} & \text{Isostatic} \\ 30 \text{ m} & \text{Continuous} \end{cases}$$



For this reason, it is a good solution for a stew crossing of an obstacle and in case of irregular geometries.

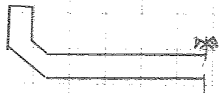
generate torsion and it should be equilibrated by 2 opposite reactions. Yet, if the beam is thin, the 2 forces can't be put in the section and there is no equilibrium for the torque moment and bridge overturns.

So, we need to introduce a diaphragm at the end, that is a deep beam supported at extremities and loaded at the span by the web of the bridge.



Another configuration is the multi-beam, where there are different webs with a slab connecting them and also transversal beams.

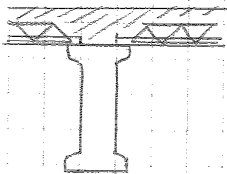
→ Inverted T-beams



Used for channel bridges, the appearance from external is that they're heavy - with a circular line, the impression is different.

From statical point of view, they don't work correctly in an isostatic scheme, because it would be necessary a lot of concrete on the top - high compression - and less in the bottom - there is tension. Indeed, this section is used only in case of particular requirements, for instance there mustn't be anything under the deck.

→ T or V precast beams



In this case, deck presents a beam and a partially precast slab. In particular, there is a small concrete slab reinforced with a truss and it is connected to the beam by casting.

→ box girder beams



It is the best solution for the deck because it has a high performance. Performance coefficient is the ratio between the distance between the extremes of inertial core and the depth of section.

$$\gamma \sim 0,5$$

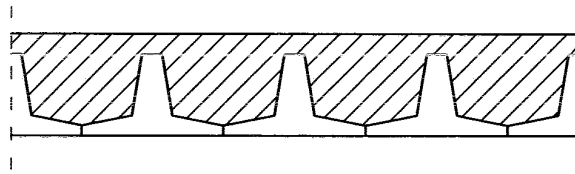
Slenderness ratio should be

$$\frac{p}{h} \leq 30 \text{ in continuous scheme}$$



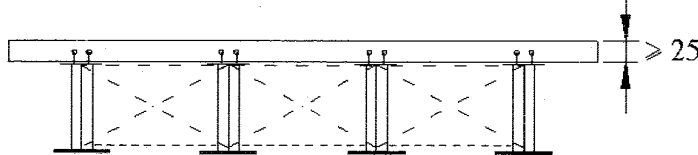
**1** **Basis of design 89/91**

- Slabs with infilled beams



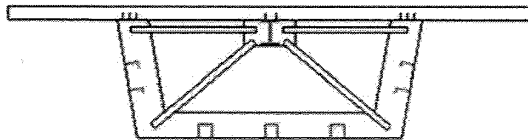
$$l = 10 \div 15 m$$

- Composite steel-concrete deck, with double T-beams



$$l \leq 50 \div 60 m$$

- Composite steel-concrete deck, with box girder beam



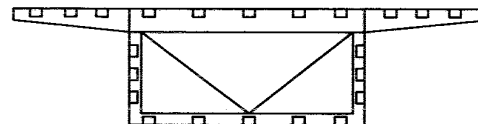
$$l \geq 50 m$$

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**1** **Basis of design 90/91**

- Steel bridges with horticropic deck

$$l \geq 120 \div 150 m$$



<b>Solution for continuous beam</b>	 $100 \leq l \leq 200 m$	 <b>Deck bridge</b> Box girder with horticropic deck
<b>Solution for stay-cable bridges</b>	 $200 \leq l \leq 300 m$	 <b>Stay-cable bridge with flexural rigidity deck</b> Deck suspended in the central region
	 $300 \leq l \leq 1500 m$	 <b>Stay-cable bridge with truss behaviour</b>

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for spans that are

$$l \leq 50 \div 60 \text{ m}$$

Above this limit, a full section of steel is needed.

→ orthotropic deck:

it presents 2 orthogonal preference directions of behaviour.

## SLAB BRIDGES

1 It is a common solution for SMALL SPANS, especially in presence of IRREGULAR GEOMETRY - due to irregularity, it can be difficult to determine internal actions.

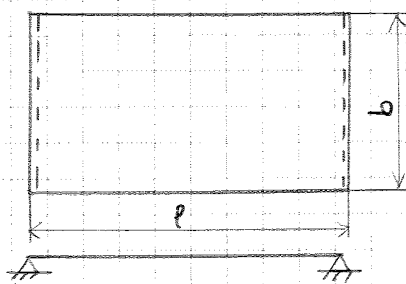
The main problem with slab bridges is not analysis - we have solutions especially for regular geometry - but there is a problem of design:

final element has to be designed with a combination of several internal actions.

Generally, slabs present 5 internal actions. If there's also pre-stressing reinforcement, other 3 components of internal action are added - 2 membrane internal forces and a shearing internal force.

So we have 8 internal actions and with them we have to check concrete and design.

## 2 Massive slabs with orthogonal edges



They represent a very simple case.

In this case, we can follow some considerations

→ internal actions may be derived with the SLAB THEORY, thanks to regularity.

→ we can also use, as an approximation, BEAM THEORY.

In this case, we analyze one strip that carries its live-load and so we design an equivalent beam.

Yet, with this system there are no tools to design the transverse reinforcement, as we design only in one direction.

So, we assume that bending moment in transverse direction is equal to 25% of principal bending moment and we design secondary reinforcement for this percentage.

$$M_t = 25\% M_o$$

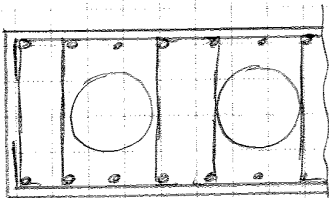
The approximation is valid if the ratio between span and width is

$$\frac{l}{b} \geq 2$$

### 3. Voided slabs with orthogonal edges

If span increases, thickness can become very big and so it could be convenient to use voided slabs.

How structural analysis performs in presence of holes?



Internal actions are calculated in the same way used with massive slab and anisotrop effect isn't taken into account - we consider an homogenous slab.

On the other side, we have to take in mind that, in transverse direction, the flow of stresses is disrupted by holes

⇒ we introduce transversally some regions of full concrete in which there are no voids (in 5 ÷ 6 positions along the span)

voids can't be continuous along the beam, <sup>in</sup> particular in the regions of bearings, where are important bending moments and we need full section.

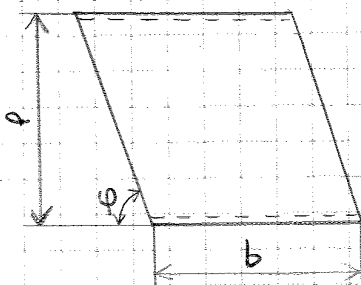
As regards shear, voids can't carry shear and there's an increase of tangential stresses, so STIRRUPS ARE NECESSARY.

In voided slabs, we can also introduce PRESTRESSING and tendons need to be concentrated where there are the interruptions of the holes, to cross concrete.

### 4. Skew slabs

They are very used because road designers ask us to respect geometry and obstacles usually are crossed in a skew direction.

↳ infrastructure is governing the structure



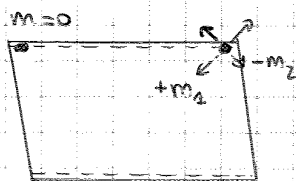
A skew slab is defined by the angle  $\varphi$  between the free edge and the edge of the bearings. This angle is generally included in the range

$$\varphi = 20^\circ \div 70^\circ$$

If  $\varphi > 70^\circ$ , the skew slab behaves as an orthogonal slab and so it can be calculated as an **ORTHOGONAL SLAB**.

Here, there is the maximum negative moment due to the presence of continuous support and transverse restraint exercised by the continuous support in the free deformation of the slab. At this point, a significative amount of reinforcement is required on the top level.

When there is skewness, we have also to take care of negative bending moment in the OBTUSE ANGLE and to the non-uniform distribution of reactions along the supported edge.



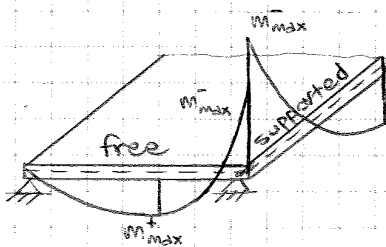
For instance, in a skew bridge with  $\varphi = 60^\circ$  under a uniformly distributed load, if we sketch the principal directions, we can notice that

→ on the top left, bending moment is zero.

→ on the top right corner, principal directions are oriented to  $45^\circ$ .

Now, we'll see the best way to put RESTRAINTS on a skew slab.

What if we sketch bending moment diagrams along the edges.



Along the free edge, we have a zero on one point on the bearings. Then we have a maximum in a non-symmetric point and finally a negative moment.

Along the supported edge, there is a big negative bending moment and then a positive one.

↓ in case of uniformly distributed load

If  $\varphi = 30^\circ$  - very skew bridge - along the free edge we have

$$m_{max}^+ = 0,12 q l^2$$

$$m_{max}^- = -0,07 q l^2 \rightarrow \text{half of positive moment along the free edge (high value)}$$

Along the supported edge we have

$$m_{max}^- = -0,16 q l^2$$

which is 50% more than positive bending moment along the free edge and this is very high.

So, we need to pay attention to reinforce section correctly to the bending moment - for cracks - and also to the uplift of the bearings, that determines a change of static scheme.

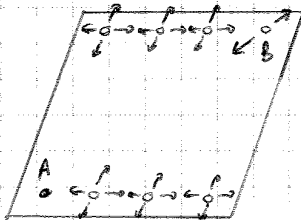
Then, we have to deal with REINFORCEMENT and we distinguish 2 cases.

→  $\varphi \geq 60^\circ$      $\frac{b_0}{p} \geq 0,5$

Longitudinal and transverse reinforcements are parallel to the borders. As there is also torque moment at the corner - because there are two negative bending moments -, we have to introduce stirrups along the free edges.

→  $\varphi < 60^\circ$

As the slab is skew, reactions in the bearings are not symmetrical and the higher vertical reaction is concentrated in the obtuse angles and the lower one is in the acute angles.



The reason of it is that load prefers to take the shortest way to cross the structure, because it is the most rigid way.

Indeed, all loads external to the line orthogonal to the bearing will be transferred by the first line which is orthogonal to the bearing and this makes the obtuse angle overloaded.

With concentrated bearings, we have also to decide which type of bearing to introduce.

As behaviour is complex, it should be better to leave the maximum freedom for the deformations and introduce JUST AND ONLY THE RESTRAINTS NECESSARY FOR THE EQUILIBRIUM:

the slab is a rigid solid in its plane and only 3 degrees of freedom need to be restrained.

If a hinge is put in A, the slab will be able only to rotate and not to translate. In order to stop rotation, we introduce a monodirectional bearing, able to let the translation along its axis, whereas transversal displacements are not allowed - in this way, rotation around A is not possible.

The other bearings are free.

In this way, we avoid internal actions - e.g. due to thermal variations - that engage the resistance of the slab.

## GIRDER BRIDGES

1 They represent the most used and abused solution in the past and today and for spans up to

$$l \sim 35 \text{ m}$$

it can be considered the most economical solution.

2 Girder bridge = "ponte a graticcio"

It presents

→ LONGITUDINAL BEAMS

→ TRANSVERSE BEAMS

→ a TOP SLAB. The top slab has different tasks

→ It receives directly the traffic load and TRANSFERS THE TRAFFIC LOAD TO THE BEAMS.

→ as it is connected to the beams, it is the COMPRESSED CHORD OF LONGITUDINAL BEAMS in one direction and OF TRANSVERSE BEAMS in the other direction, if the girder is isostatic.

→ it contributes to the DISTRIBUTION OF THE CONCENTRATED LOADS - e.g. traffic - to the beams  
Indeed, in a hyperstatic system, a beam doesn't carry the traffic pertinent to it but the girder works all together and a load applied on a beam will be engaged by the other beams and the connection is made by the slab and the transverse beams

↳ if a beam is required to have a displacement due to traffic loads, the other ones are involved in the displacement due to the slab's continuity and they will carry part of the load

So, it is statically convenient, but it is a big problem the evaluation of the load's distribution.

→ it works in its plane like a RIGIDE MEMBRANE and it is useful in case of horizontal forces, like earthquake or centrifugal forces from traffic in curved bridges.

→ combining a rectangular web with a slab, the INTERNAL LEVER ARM IS INCREASED.  
Indeed, the gravity center is moved towards the slab and the internal lever arm of the section is increased. So less reinforcement is needed with respect to a rectangular section.

⇒ the tendency is PUT TRANSVERSE BEAMS ONLY AT THE EXTREMITIES

To give an idea, having 2 transverse beams in  $l/3$  produces the same effect of a transverse beam in the mid-span.

Nevertheless, there are some authorities - like in railways - that prefer transverse beams for prudence, as the interruption of the line in case of damage of bridge has huge consequences.

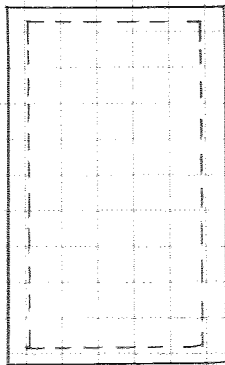
→ web thickness

If we increase the number of beams, we can reduce web thickness but torsional rigidity is reduced.

From a first analysis, it appears that is convenient to place the same amount of concrete in a large web, with respect to distribute it in small webs, because elements have a high torsional rigidity. Yet, with normal or reinforced concrete, due to shear and torque moment, generally there are cracks and torsional rigidity is very reduced - 5 ÷ 10 times

⇒ we have to take into account torsional rigidity, because it reduces in a very short time.

#### 4. Slab design



Generally, we have a long slab, which slab length is equal to the span of the bridge and width is equal to the one of transverse beams. The slab is continuously supported in the longitudinal direction, while they are usually supported only on the extreme transverse beams in the transverse direction. If there are also transverse beams along the span, they are separated from the slab by a gap. Thanks to this absence of connection with transverse beam, the slab has the same behaviour in every section.

When slab is designed - or also the transverse section of the beam - , structure can be optimised:

if the last beam is put at the end of the slab, the slab will work mainly with a positive bending moment in the first field.

So, it would be convenient having a cantilevering part of the slab because



Different methods are used to analyze the structure

→ Courbon method: based on the simplifying assumption that the shape of the deformed transverse section doesn't change - the slab remains straight after the deformation -, is valid for long and narrow bridges.

$$\frac{l}{b} > 10$$

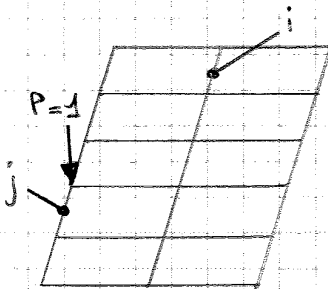
→ Massonet method: it is based on a more refined hypothesis and considers the stiffness of longitudinal beams both in torsion and bending, but it is obsolete.

→ Finite Element Method (F.E.M.):

it is the best method, able to solve all girders considering them as plane structures or 3D structures.

In the analysis, we have a space system that, in order to simplify computation, can be degenerated into a plane system. To do this, we can assume that the longitudinal axis of the slab and the one of transverse beams are the same and, in this way, we reduce to zero the III dimension.

The main problem in design the girder is evaluate the transversal distribution of a load applied to the system between the slab, the longitudinal beams and the transverse beams.



So, we consider a simple girder that has a load applied on beam  $i$ :

what is the percentage of load that is carried by the beam  $j$ ?

In this way, we can define the transversal distribution of actions and, in this way, we can design beams.

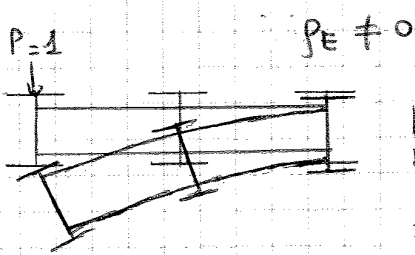
This percentage is expressed by the coefficient  $\rho_{ij}$ , which follows some properties

→  $\rho_{ij} \leq 1$

→ due to equilibrium in vertical direction, as we apply a unit force, we have

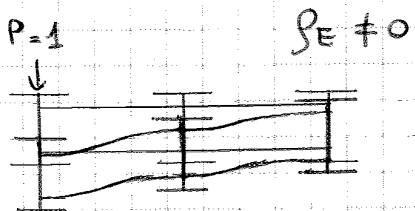
$$\sum_{i=1}^n \rho_{ij} = 1$$

III TRANSVERSE BEAMS WITH FINITE FLEXURAL RIGIDITY AND LONGITUDINAL BEAMS WITH NULL TORSIONAL RIGIDITY.



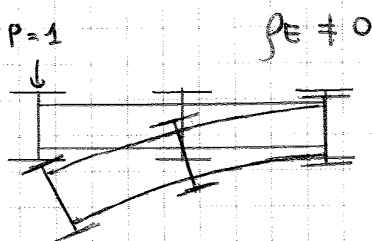
In this case, there is the bending of the transverse beam and a rotation of longitudinal beams, which remain orthogonal to the axis of transverse beams without opposing resistance.

IV TRANSVERSE BEAMS WITH FINITE FLEXURAL RIGIDITY AND LONGITUDINAL BEAMS WITH INFINITE TORSIONAL RIGIDITY (boxed sections)



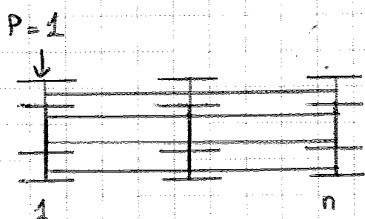
Due to the eccentricity of the load, we have the bending of the transverse beams but longitudinal beams can't rotate.

V TRANSVERSE BEAMS WITH FINITE FLEXURAL RIGIDITY AND LONGITUDINAL BEAMS WITH FINITE TORSIONAL RIGIDITY

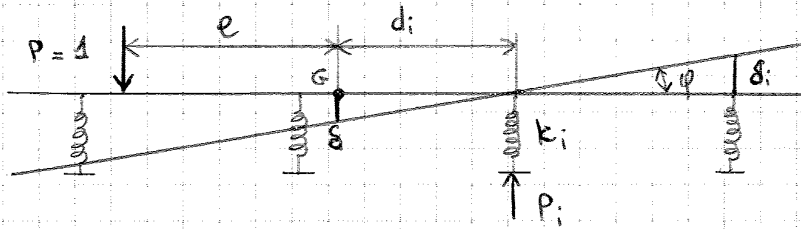


In this case, transverse beams should bend and longitudinal beams should accept a torsional rotation. This is the most realistic case.

VI TRANSVERSE BEAMS WITH INFINITE FLEXURAL RIGIDITY AND LONGITUDINAL BEAMS WITH INFINITE TORSIONAL RIGIDITY



It is a theoretical case in which transverse beams should remain straight and longitudinal beams can't rotate, so the deformation is a pure translation.



A unit load is applied with an eccentricity \$e\$ with respect to the gravity center \$G\$ of the spring rigidities.  
We call

\$\delta\$ = displacement of the transverse beam in correspondence of the rotation center \$G\$

\$\varphi\$ = rotation angle of the transverse beam

\$\delta\$ and \$\varphi\$ govern the displacement field, because the beam is rigid

\$d\_i\$ = distance of the longitudinal beam \$i\$ from the gravity center \$G\$ of the spring rigidities.

\$\delta\_i\$ = displacement of the generic spring.

Analysing the global displacement, the displacement of the generic spring is given by \$\delta\$ and a contribution of rotation.

$$\delta_i = \delta + \varphi d_i \quad \rightarrow \text{tangent is confused with the angle} \quad (1)$$

The force in the generic spring is

$$P_i = k_i \delta_i = k_i (\delta + \varphi d_i) \quad (2)$$

We write the equilibrium equation in vertical direction, remembering that a unit load is applied.

$$\sum_{i=1}^n P_i = 1 \quad (3)$$

$$\sum_{i=1}^n (k_i \delta + k_i \varphi d_i) = 1 \quad (3) + (2)$$

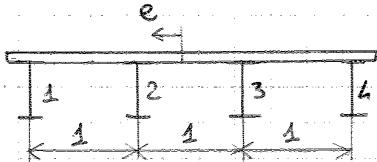
$$\delta \sum_{i=1}^n k_i + \varphi \sum_{i=1}^n k_i d_i = 1$$

static moment of the rigidities with respect to the gravity center of rigidities, that is 0 by definition

$$p_{ie} = \frac{1}{n} + \frac{ed_i}{\sum d_i^2} \quad \text{Courbon expression (8)}$$

In this way, for a given beam longitudinal beam - it means  $d_i = \text{cost}$  - we can draw the INFLUENCE LINE OF  $p_{ie}$  for variable  $e$  and it will represent the load on beam  $i$  for a variable position of a unit load.

### Ex (APPLICATION OF COURBON METHOD)



A girder bridge presents 4 longitudinal beams with an interaxis equal to 1 m. Evaluate  $p$  coefficients with Courbon method.

Due to symmetry, we have to evaluate two coefficients only

→ beams 1 and 4

The coefficient is

$$p_{1e} = \frac{1}{n} + \frac{ed_1}{\sum d_i^2} = \frac{1}{4} + \frac{1,5e}{2 \cdot (1,5^2 + 0,5^2)} = 0,25 + 0,3e$$

Term  $e$  is the eccentricity of the load and we try to apply it on the different beams.

Beam 1  $p_{11} = 0,25 + 0,3 \cdot 1,5 = 0,70$  → if load is put on beam 1, 70% of it is carried by beam 1

Beam 2  $p_{12} = 0,25 + 0,3 \cdot 0,5 = 0,40$

Beam 3  $p_{13} = 0,25 - 0,3 \cdot 0,5 = 0,10$

Beam 4  $p_{14} = 0,25 - 0,3 \cdot 1,5 = -0,2$  → if load is put on beam 4, beam 1 is uplifted

$$\sum p_{ie} = 1$$

Same thing happens on beam 4.

We can try to apply Betti - Maxwell theorem:

when a load is applied in positions A and B, the work done by load A in B is equal to the work done by load B in A.

Courbon formulation can be simplified by introducing eccentricity  $e$  and position  $d_i$  with the following expressions

$$e = e' b_0 \quad d_i = d'_i b_0$$

$b_0$  = distance between beams

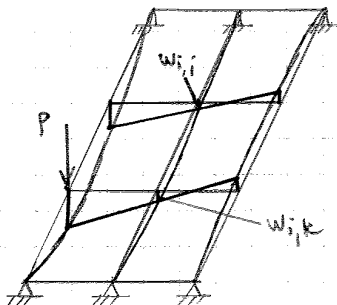
By substituting in equation (2), we get

$$\begin{aligned} \beta_{ie} &= \frac{1}{n} + \frac{e d_i}{\sum d_i^2} = \\ &= \frac{1}{n} + \frac{e' b_0 \cdot d'_i b_0}{\sum d_i^2 b_0^2} = \frac{1}{n} + \frac{e' d'_i}{\sum d_i'^2} \\ \beta_{ie} &= \frac{1}{n} + \frac{e' d'_i}{\sum d_i'^2} \quad (3) \end{aligned}$$

The expression is independent by  $b_0$  and it means that THE RIPARTITION OF LOADS IS INDEPENDENT BY THE INTERAXES OF THE BEAMS - if it is constant.

There is a table of transversal ripartition coefficients - valid for uniform distance between the beams and identic beams (general situation) -, in relationship with the number of beams.

PROPERTIES : a consequence of having assumed that transverse stiffness is infinite and longitudinal stiffness is nil is that THE TRANSVERSAL DISTRIBUTION OF LOAD IS NOT DEPENDENT BY THE SECTION ANALYSED. In other words, the coefficient of ripartition will be the same if load is applied in different sections, independently from what happens in the different spans.



In particular, if a load  $P$  is applied on the external beam in a certain position, the transverse ripartition of the load is given by the table (figure ) and the ripartition will be the same in the other transverse beams, as the deck should remain plane.

For this reason, the displacement in transverse beam  $k$  due to the load on beam  $i$  has the same ratio with respect to the displacement in transverse beam  $j$  due to the load on beam  $i$ .

$$\frac{w_{i,j}}{w_{i,k}} = \text{cost}$$

**3 Girder bridges 17/25**

For beams with the same mutual distance the coefficient  $\rho_{ij}$  is independent from that distance; then the influence lines of the coefficient of distribution may be drawn as a function of the beam number.

the value of coefficient  $\rho_{ij}$  for  $i = \text{cost}$  and  $2 \leq n \leq 10$  are presented in the following table.

[Due to the Betti-Maxwell theorem the influence lines of  $\rho_{ij}$  ( $i = \text{cost}$ ) coincide with the transverse deformed configuration of deck ( $j = \text{cost}$ )]



**3 Girder bridges 18/25**

Pay attention! It is valid for identical beams with constant mutual distance

n° of beams	beam	Load in:									
		1	2	3	4	5	6	7	8	9	10
2	1	1	0								
	2	0,833	0,333	-0,166							
3	1	0,7	0,4	0,1	-0,2						
	2	0,4	0,3	0,2	0,1						
4	1	0,6	0,4	0,2	0	-0,2					
	2	0,4	0,3	0,2	0,1	0					
	3	0,2	0,2	0,2	0,2	0,2					
5	1	0,524	0,381	0,238	0,095	-0,048	-0,19				
	2	0,382	0,296	0,21	0,124	0,037	-0,049				
	3	0,24	0,21	0,161	0,152	0,123	0,094				
6	1	0,463	0,357	0,25	0,143	0,36	-0,71	-0,178			
	2	0,355	0,285	0,214	0,143	0,072	-0,001	-0,07			
	3	0,25	0,215	0,179	0,143	0,107	0,071	0,035			
7	1	0,416	0,333	0,25	0,167	0,083	0	-0,083	-0,166		
	2	0,331	0,273	0,213	0,133	0,095	0,037	-0,022	-0,082		
	3	0,251	0,215	0,179	0,143	0,107	0,071	0,035	-0,001		
8	1	0,187	0,155	0,143	0,131	0,119	0,107	0,095	0,083		
	2	0,38	0,312	0,245	0,178	0,111	0,044	-0,023	-0,09	-0,157	
	3	0,31	0,261	0,211	0,161	0,111	0,061	0,011	-0,039	-0,089	
9	1	0,243	0,21	0,177	0,144	0,111	0,078	0,045	0,012	-0,02	
	2	0,18	0,162	0,145	0,128	0,111	0,094	0,077	0,06	0,043	
	3	0,111	0,111	0,111	0,111	0,111	0,111	0,111	0,111	0,111	
10	1	0,343	0,289	0,235	0,181	0,129	0,073	0,019	-0,035	-0,089	-0,143
	2	0,289	0,249	0,205	0,163	0,121	0,079	0,037	-0,005	-0,047	-0,089
	3	0,235	0,205	0,175	0,145	0,115	0,085	0,055	0,025	-0,005	-0,035
10	4	0,181	0,163	0,145	0,127	0,109	0,091	0,073	0,055	0,037	0,019
	5	0,127	0,121	0,115	0,109	0,103	0,097	0,091	0,085	0,079	0,073

Due to the hypotheses that  $I_{\text{traverso}} = \infty$  and  $J_{\text{t trave}} = 0$  there is no mutual influence of transverse beams in the load distribution effect.



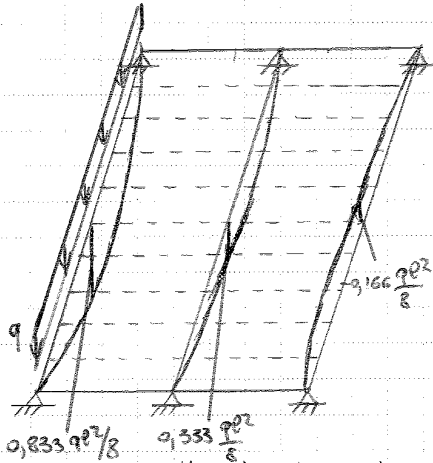
## 8 Evaluation of internal actions on longitudinal beams

Now, we analyze the internal actions in longitudinal beams and we can use 2 approaches

### ① Courbon / Albenga approach

The main hypothesis is that there is a INFINITE NUMBER - high number - OF RIGID TRANSVERSE BEAMS.

In other words, we remain within the Courbon approach



In case of rigid connection between the longitudinal beams - by rigid transverse beams - , a load is distributed along one of the longitudinal beams. What is the bending moment in longitudinal beams.

Due to the hypothesis, the transverse repartition of the load is the same in each section and it is divided among the longitudinal beams by Courbon approach.

As all the load is divided between the beams with the same repartition, it means that

→ maximum bending moment in the mid-span of beam 1 is

$$0,833 \frac{qL^2}{8}$$

→ maximum bending moment in the mid-span of beam 2 is

$$0,333 \frac{qL^2}{8}$$

→ maximum bending moment in the mid-span of beam 3

$$0,166 \frac{qL^2}{8}$$

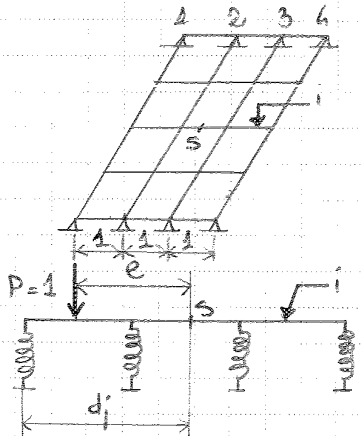
In other words, at each distribution of internal actions in each longitudinal beam, we apply the correspondent Courbon repartition coefficient.

The hypothesis is rough but the approach gives good results in case of

→ RIGID TRANSVERSE BEAMS (high flexural rigidity)

→ THICK SLAB

### 9 Evaluation of internal actions on transverse beams



Considering a deck with 5 transverse beams and 4 longitudinal beams, we can start to analyse the situation in which THE LOAD IS MOVED TRANSVERSELY ALONG THE TRANSVERSE BEAM.

The static scheme of the transverse beam is a continuous beam on elastic beddings. A unit load  $P=1$  is applied with eccentricity  $e$  with respect to the gravity center and the bearings are placed at a position  $d_i$ .

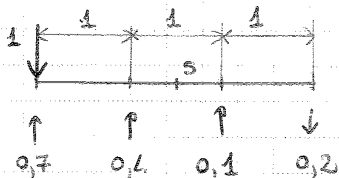
In the mid-span section, the bending moment is

$$M_s = \sum_j p_{ij} d_j - 1 \cdot e$$

The term  $p_{ij}$  is the Courbon coefficient of repartition of the load applied to the beams that precede the section  $S$ . This term is multiplied by the distance between the spring and the section.

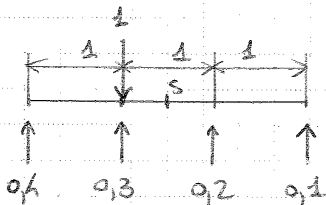
$$p_{ij} = \begin{cases} 0,7 & 0,4 & 0,1 & -0,2 & \text{Beams 1 and 4} \\ 0,4 & 0,3 & 0,2 & 0,1 & \text{Beams 2 and 3} \end{cases}$$

If load is applied on beam 1, the bending moment in  $S$  will be

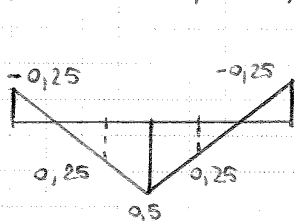


$$M_s^1 = \sum_j p_{ij} d_j - e = [0,7 \cdot 1,5 + 0,4 \cdot 0,5] - 1,5 = -0,25$$

If load is applied on beam 2, the bending moment in  $S$  will be



$$M_s^2 = \sum_j p_{ij} d_j - e = [0,4 \cdot 1,5 + 0,3 \cdot 0,5] - 0,5 = 0,25$$



In this way, we know bending moment  $M^v$  in some points and, from this, we can evaluate the relative influence line, as it is straight and symmetric. We notice that maximum negative bending moment is 0,25 - for a unit load - and the maximum positive one is 0,5.



In conclusion, we get a surface which is the INFLUENCE SURFACE of bending moment in the mid-span section of the transverse beam  $AA'$  for whichever position of the load in longitudinal beams and transverse beams.

↓ in other words, we have the distribution for ~~tran~~ repartition on transverse beams and, in order to consider every position of the load along the longitudinal beam, this diagram should be multiplied by the value of reaction in point A for every position of the load along the beam.

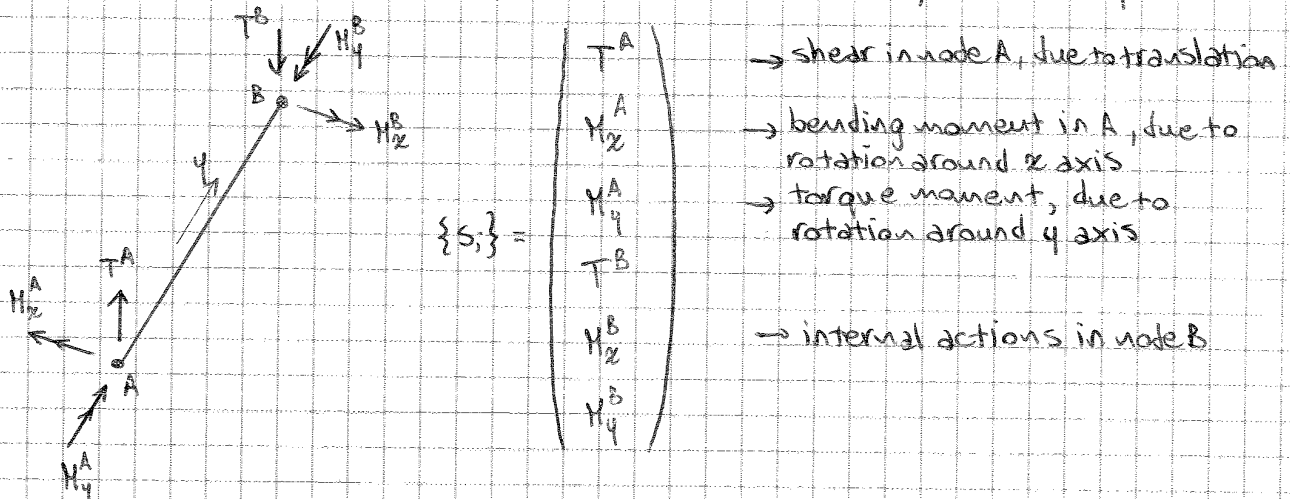
The ordinates of influence surface are null when the load is located over the other transverse beams, as they are not able to influence mutually themselves.

What happens on a single beam of the girder?

For every beam, we can define a relationship between knot displacements and knot internal actions.

$$\{S_i\} = [k_i] \{\delta_i\}$$

$\{S_i\}$  = vector of internal actions in the nodes, with 6 components.



$\{\delta_i\}$  = vector of nodal displacements, referred to nodes A and B, that are unknown

$$\{\delta_i\} = \begin{bmatrix} z^A \\ \varphi_z^A \\ \varphi_y^A \\ z^B \\ \varphi_z^B \\ \varphi_y^B \end{bmatrix}$$

$[k_i]$  = stiffness matrix (6x6) of the beam element.

It contains the flexural rigidity and the torsional rigidity of the single element.

As regards torsional rigidity, we have to pay attention to cracking, as this rigidity may change a lot.

Sometimes, also shear rigidities are included.

→ To represent the behaviour of the slabs, in the girder we have to introduce

→ 2 EDGE LONGITUDINAL BEAMS, due to the flow of stresses

→ INTERNAL LONGITUDINAL BEAMS, that should be at a mutual distance smaller than  $\frac{1}{6}$  of effective span and at least 5 longitudinal beams should be introduced (edge beams included)

$$i_{lb} \leq \frac{1}{6} l \quad n_{lb} \geq 5$$

→ TRANSVERSE BEAMS, that have the same prescriptions.

$$i_{tb} \leq \frac{1}{6} l \quad n_{tb} \geq 5$$

So, in order to have the full description of the behaviour of the slab, we should put at least 5 longitudinal beams - 2 on the edges, 3 equally distributed on the width - and 5 transverse beams.

→ skew decks ( $\theta \geq 20^\circ$ )

If the skew angle is up to  $20^\circ$ , they may be described as with orthogonal edges.

If skewness is high, it is necessary a SKEW MESH where transverse beams are aligned to the direction of reinforcement.

→ refinement in limited regions

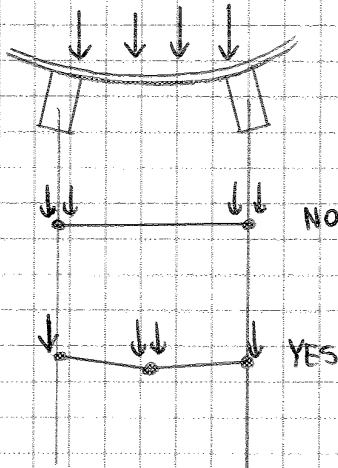
If we need a refinement in a limited region of the deck - For instance, bearings or particular load conditions - , in order to avoid a model with high refinement for all the elements, we can use

→ a SIMPLIFIED MODEL to solve all the girder, with a rough mesh

→ a REFINED MESH OF THE REGION, to which are applied as impressed deformations the ones derived from the simplified model, together with the directly applied loads.

The flexural rigidity shall be divided between the two beams, whereas torsional rigidity should be evaluated for the box section.

A node is also put in the mid-span of the slab.



If a load is applied directly to the slab and longitudinal beam elements are put in correspondence of the beams, the load is transferred to the beams and there are no internal actions in the slab.

So, this model is not correct.

To describe correctly the internal actions are reproduced the way in which the actions are transmitted by the slab to the beams, a further node is placed and we get a better distribution of the load:

$\frac{1}{4}$  of the load will go to one beam,  $\frac{1}{4}$  will go to the other one and one half of the load is on the slab.

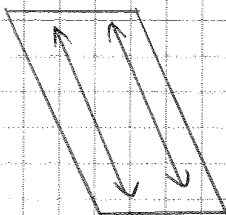
#### 4 Mesh for skew bridges

In this case, as seen for slab bridges, there is a high reaction in obtuse angle, whereas the uplift in acute angle is possible. As regards bending moments,

- in the central region, maximum positive moment is acting along the direction orthogonal to the supported edges
- in the free edges, the positive bending moment is acting along the skew edge.

There is also a high torque moment along the borders.

In this situation, to decide how to describe the slab, an important role is played by **PLANIMETRIC GEOMETRICAL CONDITIONS**.



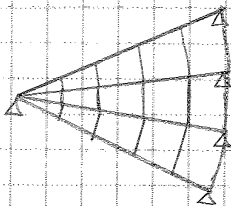
In case of **NARROW SLAB**, we may have longitudinal beam elements parallel to the free edges.

As seen for slabs, 2 elements are placed on free edges and maybe 3 are internal.

The mesh has also to be oriented along the expected direction of internal forces - struts.

## 5 Mesh for curved bridges

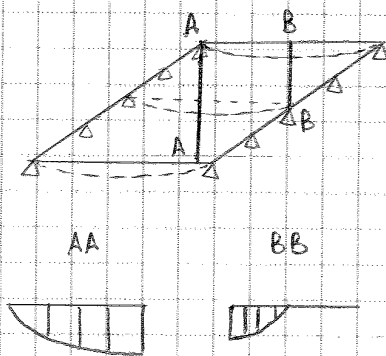
In curved bridge, there is a strict relation between bending moment and torque moment in each node and it is not only important the span, but also the TOTAL ANGLE covered by the bridge.



To evaluate the internal actions, the mesh should contain RADIAL ELEMENTS and CIRCUMFERENTIAL ELEMENTS.

The radial elements correspond to longitudinal beams and circumferential elements are straight segments.

### OBSERVATION



Moreover, if there is a high torque moment in the skew bridge and a section AA is performed, along this we can notice a certain deformed mesh that passes from null displacement to a big one.

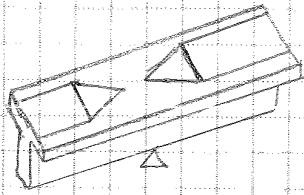
IF the same direction of section is performed in correspondence of the center of the bearings, the deformed shape see the maximum displacement at the border and null displacement on the mid-span.

So, there is a large difference of displacement in the slab along the same alignment and this difference is managed by the presence of a high torque moment.

In this situation, it is suggested not to introduce beams with high torsional resistance because they increase torsional stiffness and, as the difference of displacement is anyway high, also torque moment is increased and there will always be failure.

The unique solution is REDUCING TORSIONAL STIFFNESS, by using beams with null torsional stiffness, as they won't produce a torque moment.

This problems applies on VERY SKEW BRIDGES and, in this case, open section beams are used.



A similar situation occurs in a continuous beam:

the stresses arrive with a uniform distribution and then they concentrate with a compressive strut to the bearing.

Again, a transverse reinforcement is required in the slab.

This aspect is important because, at the bearings, compressive struts carry a high stress and, if we exceed with permanent load, also NON LINEAR CREEP should be considered.

### 3 Single beam deck

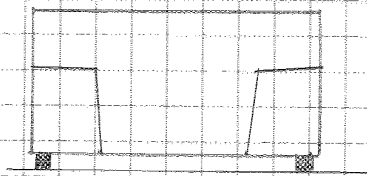
This kind of deck is used in case of SMALL WIDTH - e.g. exit of a highway - , where

$$W = 6 - 7 \text{ m}$$

The main problem consists of closing the torque moment at the end of the span.

Indeed, in this kind of section, 2 bearings may not be able to equilibrate the torque moment because it has to be equilibrated through 2 forces and if compression due to permanent loads is smaller than the tension due to torsion, the uplift of the deck occurs.

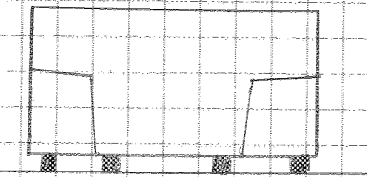
↓ problem of equilibrium (primary torsion)



If 2 bearings are not enough, we could put a diaphragm supported by 2 external bearings, enlarging the distance between them. In this way, with these bearings both torque moment and vertical reaction are closed.

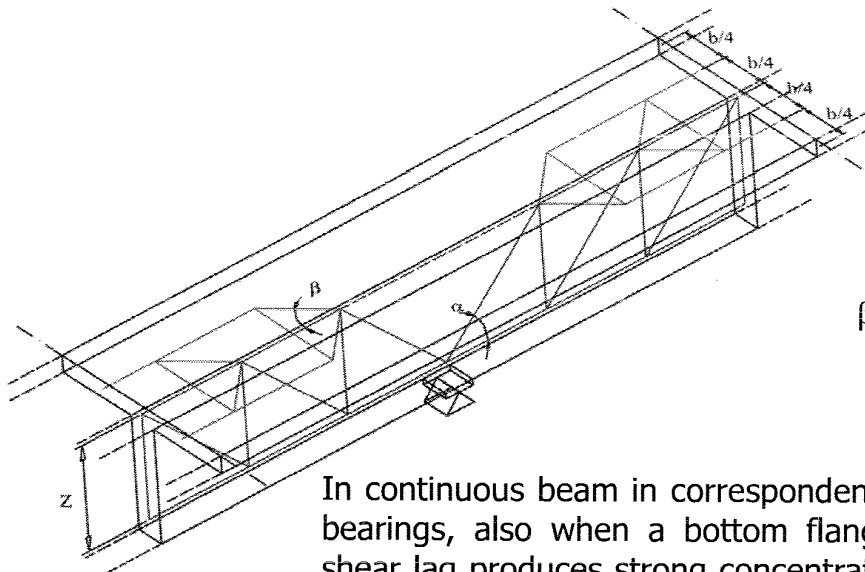
We could also introduce a diaphragm on 4 bearings.

This is a hyperstatic bearing system and, to solve the real reactions, vertical stiffness of the bearings should be introduced:



we insert springs instead of bearings and, in this way, we derive correctly the distribution of actions.

↓ usually, bearings have a settlement of 1 mm at full capacity of the bearing



$$\beta \cong 40^\circ \div 45^\circ$$

In continuous beam in correspondence of intermediate bearings, also when a bottom flange is present, the shear lag produces strong concentration of longitudinal stresses within the webs and increment of creep effects in serviceability conditions. In some cases also the bearing capacity may be reduced.

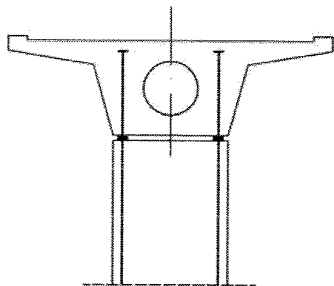

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Single beam deck → used for  $b \leq 6 \div 7$  m.

The web should be designed to carry the torsion coming from live loads with their eccentricity (primary torsion, governing the equilibrium!)



In the support regions the torsion should be equilibrated by means of two bearings with enough distance; if necessary, use a transverse beam to increase the lever arm of support reactions.



If it is impossible the equilibrium with dead and live loads (light beams), the deck shall be connected rigidly to the pier.

If necessary, use tensioned prestressing bars, crossing the bearings. Those bars should be extended within the pier so that they are anchored in a section on which the dead load of the upper part of the pier is enough.

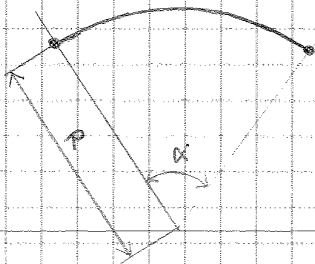

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Due to the small radius of curvature, high local stresses arise and TRANSVERSAL REINFORCEMENT is required along the tendon. Moreover, the counterflexure point - the point where there is the change of bent - has to be chosen in a proper way - otherwise, the tension at the bottom is too much big and cracks appear. These cracks will be closed when further load is applied but the durability is reduced.

So, if the deviation is not large, ordinary reinforcement is introduced to control the crack opening.

## 5 Curved beams

This solution can be adopted if not too much expensive in case of curved bridges.



The main parameter is the ANGLE TO THE CENTER  $\alpha$ , whereas radius  $R$  plays a less important role.

Curved decks can be adopted for small values of  $\alpha$

→  $\alpha \leq 20^\circ \Rightarrow$  SIMPLY SUPPORTED BEAM

→  $\alpha \leq 40^\circ \Rightarrow$  CONTINUOUS BEAM

For large values of  $\alpha$ , it is better to use small and straight beams.

In curved bridges, the end transverse beams should be very strong and transverse beams are also required along the span in order to limit the torque moment in longitudinal beams.

Generally, curved beams are not an efficient solution and

→ for small spans, it is better to use slab decks anyway

→ for medium or high spans, it is better to use box girder beams.



$s =$  slab thickness

In this formulation, a distribution of  $45^\circ$  in the direction of  $a$  in the pavement and the slab is assumed.

This imprint is evaluated at the gravity center of the slab.

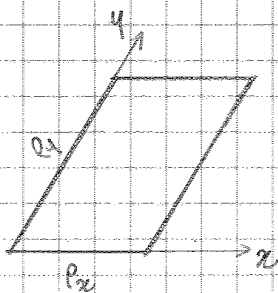
The same aspect is valid in the direction of  $b$ .

$$b = b_0 + 2s_1 + s$$

Experiments focus that the thickness  $s$  can be increased of  $2d$ , where  $d$  is the depth of the slab at the level of the reinforcement.

In other words, the distribution of  $45^\circ$  can be continued inside the thickness of the slab until its reinforcement.

In the model, we have to distinguish different types of slab, basing on geometrical condition and the position of restrains along the edge.



A slab is beared on transverse beams and longitudinal beams and we define

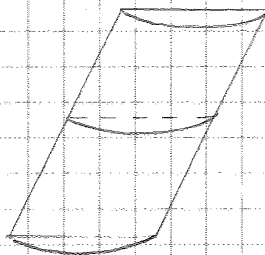
$l_y =$  length of the longitudinal beam

$l_x =$  distance between longitudinal beams (length of the transverse beam)

If the ratio of the borders is

$$\frac{l_y}{l_x} > 2$$

the slab can degenerate in a equivalent beam.



In particular, in case of uniformly distributed load applied along the slab, the deformed shape is a cylinder, that is the main behaviour is that one of a beam supported in one direction. In this condition, the slab can be designed - a UNIT WIDTH STRIP AS A SIMPLY SUPPORTED BEAM ON LONGITUDINAL BEAMS.

We should also add the transversal bending moment  $M_y$ , that derives by the restraining the transverse deformation.

Then, in case of POINT LOAD  $P$ , we can get the bending moment per unit length on the border in case of cantilevering slab.

$$M_x = \frac{Pa}{2a} = \frac{P}{2} \quad \rightarrow 2a = \text{length of the border}$$

This solution is valid for a constant thickness of the slab, whereas in case of variable thickness - cantilever with important slab - the distribution is not so favorable and the bending moment in the restrained edge increases.

$\rightarrow$  if  $\frac{s}{s'} = 2$

$s$  = thickness at the restrained border

$s'$  = thickness at the free border

the slab is more rigid and there is a less favorable distribution of load, i.e. a more concentrated load due to high rigidity and we get

$$M_x = 0,576P$$

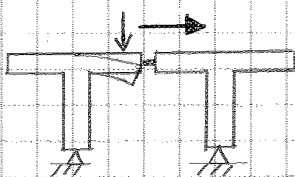
$\rightarrow$  if  $\frac{s}{s'} = 3$

we get

$$M_x = 0,631P$$

**OBSERVATION 1** : when we arrive to the border of the slab, we can't use the full width  $B$  because the bridge finishes. For this reason, in the last field double reinforcement is put, as here there is just one half of resisting structure.

**OBSERVATION 2** : cantilever in longitudinal direction



After the last transverse beam, in the cantilevering region very often there are fatigue problems:

given 2 cantilevers connected by a joint when the wheel is on the first cantilever, there is a displacement whereas the second deck is undeformed.

→ For a LINEAR LOAD along line  $s$

$$G(x_0; y_0) = \int_s p(s) \Delta(x_0; y_0; x; y) ds \quad (10)$$

If linear load  $p$  is uniform, we get

$$G(x_0; y_0) = p \int_s \Delta(x_0; y_0; x; y) ds = p \Omega$$

$$G(x_0; y_0) = p \Omega \quad (11)$$

$\Omega$  = area intersected by a plane crossing the influence surface and which imprint is the line  $s$ .

→ For a LOAD DISTRIBUTED OVER AN AREA  $A$ .

$$G(x_0; y_0) = \iint_A p(x; y) \Delta(x_0; y_0; x; y) dx dy \quad (12)$$

In case of uniformly distributed load, we get

$$G(x_0; y_0) = p \iint_A \Delta(x_0; y_0; x; y) dx dy = pV$$

$$G(x_0; y_0) = pV \quad (13)$$

$V$  = volume of the influence surface intercepted by a cylinder having the base coincident with the imprint of the load.

A fundamental property that simplifies the use of influence surface is the invariability of the influence surface for slabs with the same ratio of edges:

given the influence surface for a  $\bar{p}_x \times \bar{p}_y$  slab, in order to apply it to a  $p_x \times p_y$  slab, we define the size ratio.

$$\frac{\bar{p}_x}{p_x} = \frac{\bar{p}_y}{p_y} = k \quad \rightarrow \text{the ratio of edges is the same}$$

Then, the load is reduced by this factor

Linear load	$\frac{s}{k^2}$
Surface load	$\frac{A}{k^2}$

In this way, from the influence surface we evaluate  $\bar{G}$  and the real parameter  $G$  will be

→ in case of CONCENTRATED LOADS

$$G = \bar{G} \quad \rightarrow \text{there is no correction}$$

→ in case of LINEAR LOAD, this was reduced by  $k$ , so the parameter is increased.

$$G = k\bar{G}$$

→ in case of SURFACE LOAD

$$G = k^2\bar{G}$$

How can the influence surface be obtained?

We start from the differential equation of displacements typical of the slabs.

$$\frac{\partial^6 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = -\frac{p}{B}$$

$p$  = distributed load

$B$  = flexural rigidity of the slab

The parameters of influence surface are obtained by this equation, so we have to integrate this equation in order to get displacements and the derivatives will provide rotation, shear and bending moment.

The most known method used to drawing the influence surface is the Pucher method (or singularities method), that consists of dividing the deformed shape in 2 contributions.

$$w = w_1 + w_2 \quad (14)$$

$w_1$  = deformed shape in proximity to the load of a circular plate loaded in the center.

For this problem, the analytical solution is known.

$$w = \frac{r^2}{8\pi B} \ln\left(\frac{r}{a}\right) \quad (15)$$

$r$  = distance from the center of the plate

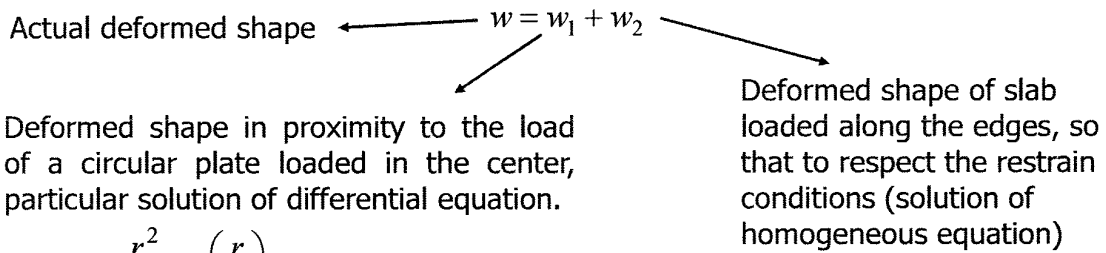
$a$  = diameter of the plate

➤ Drawing procedures of influence surfaces

All the influence surfaces of "G" parameters are obtained by the derivatives of displacement equation. It is necessary the integration of Lagrange equation:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = -\frac{p}{B}$$

The most known solution method is the Pucher one (Singularities method)

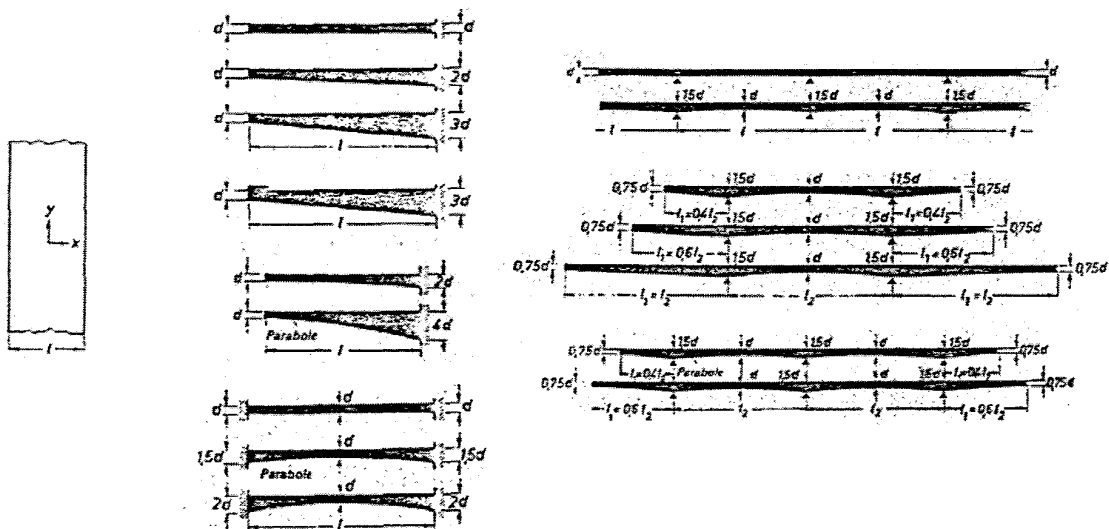


$$w = \frac{r^2}{8\pi B} \ln\left(\frac{r}{a}\right) \quad a = \text{diameter}$$

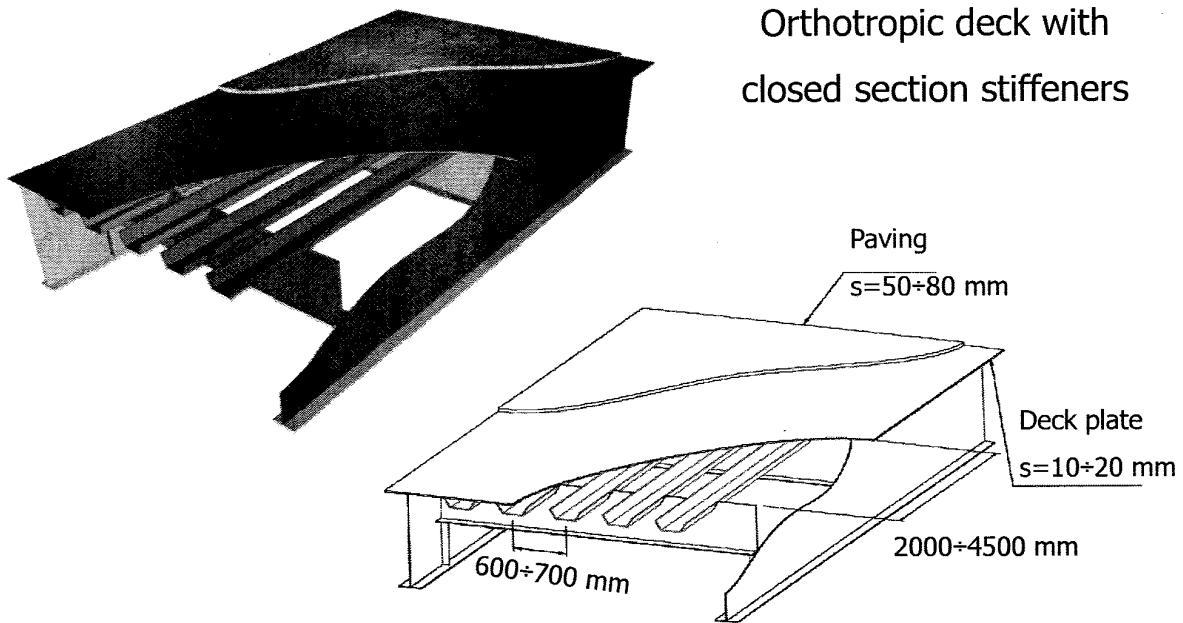


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Most important cases for which influence surfaces are available.




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Orthotropic deck with closed section stiffeners



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### 5.1 Plate behaviour (Klöppel)

Four steps are outlined following the load increase up to the failure:

- 1st step: serviceability loads → plate within the elastic field; it behaves like an isotropic slab on several bearings. Flexural stresses are prevailing on membrane ones.
- 2nd step: important membrane stresses take place for increasing load and the structure remains in the elastic field also for load levels considerably greater than the ones that can be predicted by means of thin slab theory.
- 3rd step: membrane stresses continue to increase, assuming the same order of magnitude of flexural ones. Elastic-plastic field is reached.
- 4th step: plastic hinges appear along the longitudinal stiffeners and the membrane behaviour is maintained up to the failure.



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## BEHAVIOUR OF ORTHOTROPIC DECK

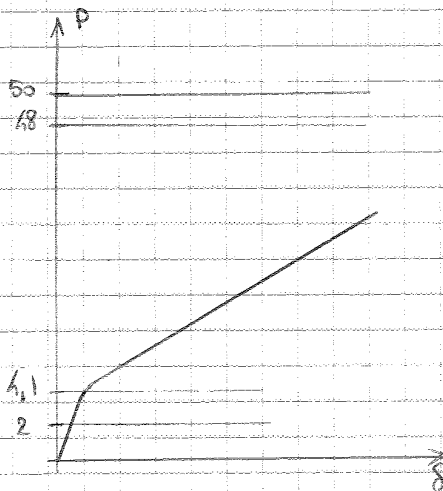
Its behaviour is described in the Klöppel experiment.

Klöppel realized a plate 1,4 m wide and 3,75 m long with 6 transverse beams and 10 longitudinal stiffeners.

The plate has been loaded with a concentrated load, applied on a small area in the center of the plate.

The system was designed with an elastic approach and by using the plate theory, i.e. only considering flexural behaviour. With this model, Klöppel designed the thickness of the plate and the size of stiffeners.

Then, Klöppel started to apply the load and he plotted the value of load  $P$  with respect to the displacement  $\delta$  under this load.



Klöppel appreciated a linear elastic behaviour under a load of 2 tons.

Then he tried to go beyond 4,10 tons, that was the theoretical limit of elastic behaviour as, at that time, the safety factor for collapse was 2 and was applied at the yielding stress that corresponds to a load of 4,10 tons - 2 ton is the permissible load.

After this load, as the slab was beyond the yielding, high displacements were expected. Actually, the behaviour continues to be linear, even if rigidity is different.

What happens at the corner point?

When yielding strength is reached in correspondence of the borders of the plate - at the connection with longitudinal beams -, plastic hinges has been formed there and a system of membrane forces intervenes inside the plate.

⇒ the mechanism changes from flexural to a MEMBRANAL MECHANISM and, even if rigidity is smaller, membrane forces increase the bearing capacity

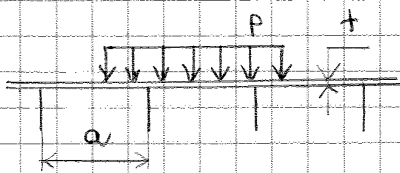
In this way, the diagram continues and the limit load where is the first failure of stiffeners is 48 tons - much bigger than 4 tons -, whereas the failure of the plate occurs at 50 tons.

So, the safety factor of this composite system was very large with respect to the expected one if only the plate theory is used because the restraint to large deformation is exerted by longitudinal stiffeners and transverse beams activate large membrane forces. It means that the local stresses between the stiffeners are small and the most important aspect is the OVERALL BEHAVIOUR OF THE DECK.

By consequence, we can reduce as much as possible the thickness of the top layer.

Yet, we also should take into account the problem of deformability of the layer between longitudinal stiffeners because, if thickness is too much small, problems of vibrations arise and it is impossible to maintain the pavement connected to the deck - problem of maintenance.

⇒ thickness should be designed for SERVICEABILITY LIMIT STATE and not for ULS.



Considering a top layer with thickness  $t$  with longitudinal stiffeners at a distance  $a$ , the tests on the pavement demonstrated that the maximum displacement allowed is

$$w = \frac{a}{300} \quad \text{Maximum displacement in the top layer}$$

Beyond this limit, the pavement can't be maintained in service.

The displacement is evaluated by using Boobnov Solution - experimentally checked by Klöppel.

$$w = \frac{1}{6} \cdot \frac{5}{384} \frac{Pa^4}{EZ} = \frac{a}{300}$$

This solution is valid if only the flexural effect is considered and the layer is fully supported by the stiffeners.

The slab is dealt as an equivalent strip supported on longitudinal supports in an isostatic scheme and the factor is  $1/6$  to take into account of this. In case of full restraint, the factor is  $1/5$  and the factor is smaller in isostatic scheme due to the presence of membrane force that opposes to deformation.

$I$  = moment of inertia per unit of length of the strip

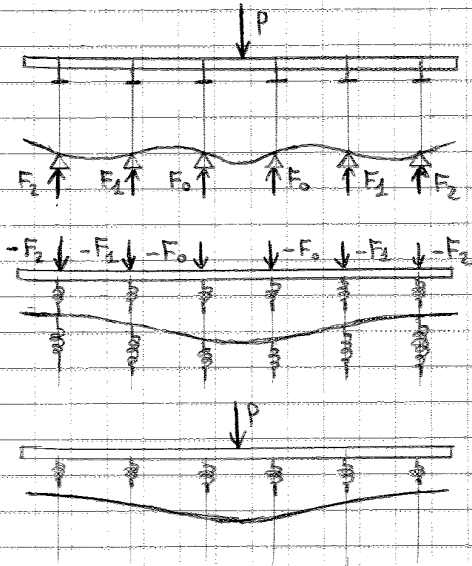
$$I = \frac{t^3}{12}$$

$E$  = Young modulus of steel

$$E = 2,1 \cdot 10^6 \text{ kgcm}^{-2}$$



Let's see the Pelikan - Esslinger method in detail.



Given the plate on longitudinal stiffeners, firstly, transverse beams are assumed to be infinitely rigid.

By consequence, the plate will assume a certain deformed shape and reactions can be evaluated - in this case, they're symmetric.

Then, these reactions are applied as actions to the plate, in which now the actual rigidities of transverse beams - dealt as springs - are introduced. These reactions are applied with opposite sign.

In this way, we get the actual distribution of reactions.

The final solution is given by the superposition of the 2 ones evaluated before.

### OVERALL BEHAVIOUR OF THE SECTION

In case of orthotropic deck, generally open sections or box sections are realized.

Considering the overall behaviour of the section, we can identify 3 systems of internal stresses.

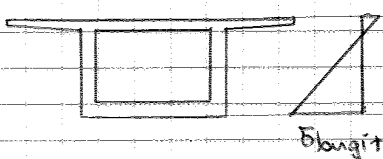
→ SYSTEM  $E_2$  :

it is a system that arises locally when the plate is supported by longitudinal stiffeners and transverse beams.

→ SYSTEM  $E_I$  :

in this case, the plate is a part of longitudinal ribs - the webs - and this generates mainly longitudinal stresses.

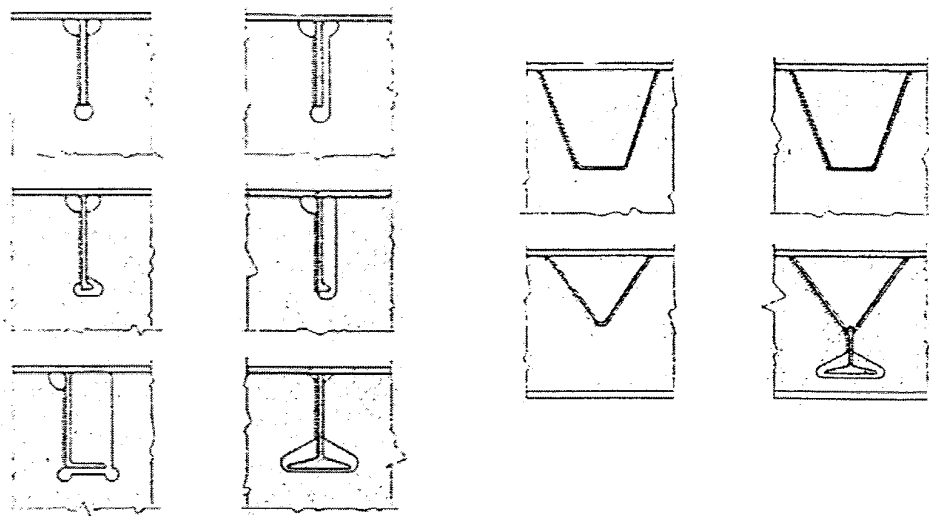
→ SYSTEM  $E_{III}$  :



This system corresponds to the overall behaviour of the section in which plate is bent in the longitudinal direction.

This system could lead to problems in local behaviour of the slab, e.g. local instability.

### Details for different stiffeners geometry.



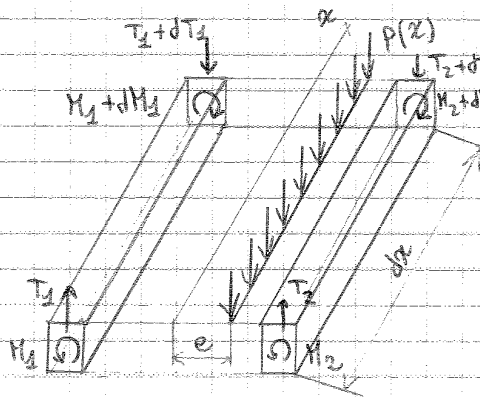
Same thing is valid for beam 2

$$EI \frac{\partial^4 w_2}{\partial x^4} = EI \left( \frac{\partial^4 w}{\partial x^4} + b \frac{\partial^4 B}{\partial x^4} \right) = -P_2 \quad (21)$$

By adding together the two differential equations (20) and (21), we obtain

$$2EI \frac{\partial^4 w}{\partial x^4} = -(P_1 + P_2) = -P \quad (22)$$

Then, considering a longitudinal beam <sup>segment</sup> with infinitesimal length  $dx$ , we analyze its equilibrium.



This element has the beams 1 and 2 and an extremity is carried with shears  $T_1$  and  $T_2$  and torque moments  $M_1$  and  $M_2$ . The opposite extremity is carried with shear  $T_1 + dT_1$  and  $T_2 + dT_2$  and torque moments  $M_1 + dM_1$  and  $M_2 + dM_2$ . The load is applied with the eccentricity  $e$ , which is positive towards the beam 2.

We impose the equilibrium to the rotation around longitudinal axis  $x$ .

$$T_1 b - T_2 b - (T_1 + dT_1) b + (T_2 + dT_2) b - M_1 - M_2 + M_1 + dM_1 + M_2 + dM_2 + m dx = 0$$

↳ the clockwise rotation is considered as positive

$$\Rightarrow -dT_1 b + dT_2 b + dM_1 + dM_2 + m dx = 0$$

where term  $m$  is the distributed moment given by load  $P$ .

$$m = p(x) \cdot e$$

Then, the equilibrium equation is divided by the length  $dx$ .

$$\left( -\frac{dT_1}{dx} + \frac{dT_2}{dx} \right) b + \frac{dM_1}{dx} + \frac{dM_2}{dx} + m = 0 \quad (23)$$

$$-2EIb^2 \frac{\partial^4 \beta}{\partial x^4} + 2GJ_T \frac{\partial^2 \beta}{\partial x^2} + m = 0$$

The term  $2J_T$  corresponds to the torsional inertia of total section  $\bar{J}_T$ .

$$\bar{J}_T = 2J_T$$

$$\Rightarrow -2EIb^2 \frac{\partial^4 \beta}{\partial x^4} - G\bar{J}_T \frac{\partial^2 \beta}{\partial x^2} = m(x)$$

$$2EIb^2 \frac{\partial^4 \beta}{\partial x^4} - G\bar{J}_T \frac{\partial^2 \beta}{\partial x^2} = m(x) \quad (27)$$

This is a **IV** order ODE in the rotation angle  $\beta$  of deck and, at the first member, there are 2 components and their analysis allows to give an interpretation of the behaviour of the deck.

→ first component is given by a flexural term multiplied by the **IV** derivative of rotation angle  $\beta$  with respect to direction  $x$ .  
The term

$$2EIb^2$$

can be interpreted as the moment of inertia of the flexural rigidities of the beams.

Indeed, if  $EI$  was an area,  $EI$  multiplied by  $b^2$  would be a moment of inertia, i.e. the moment of inertia of the rigidity with respect to the axis of the deck.

Since this value is multiplied by 2, we get the total moment of inertia of the flexural rigidities of the beams.

→ second component is given by torsional stiffness multiplied by the **II** derivative of rotation angle  $\beta$  with respect to direction  $x$ .  
This global torsional rigidity should be evaluated in agreement to De Saint Venant theory

So, the moment  $m_x$  along  $x$  axis is carried by 2 different behaviours

→ **FLEXURAL BEHAVIOUR**, with a moment of inertia of rigidities - it's a contribution due to the bending

→ **TORSIONAL RIGIDITY** of the full deck

In these equations, we can identify 2 limit cases.

→  $\bar{J}_t \sim 0$

It is the case of a deck composed by a slab over 2 double-T steel beams.

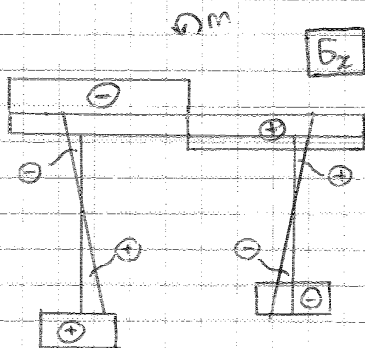
In this case, the torque moment applied to deck by the eccentricity of load is equilibrated by flexural behaviour.

This situation is characterised by a bimoment, i.e. bending moments with opposite signs, as the beams undergo opposite displacements and the eccentricity of load is transferred in a behaviour of alternative bending of the beams, without any torsional contribution.

→  $I_{yz} \sim 0$

It is the case of closed box section, in which torsional inertia is much bigger than the flexural one.

So, the torsional behaviour prevails and the solution is given by De Saint Venant theory.



In the case of

$\bar{J}_t \sim 0$

with a bending force applied downwards, the flange presents longitudinal stresses - the plot is referred to the stresses orthogonal to the sheet plane - , whereas there are tensions and compressions in the web and compressions at the bottom in the first beam.

On beam 2, the flange is compressed, webs are compressed and tensed and the bottom plate is tensed.

In this situation, the torsional behaviour of the deck is substituted by a flexural secondary behaviour, where the longitudinal bending moment in each beam is related to the derivative of the relative displacement between the deck and the beam, through the elastic line equation.

$$M^* = -EI \frac{\partial^2 (w_2 - w)}{\partial x^2} = -EI \frac{\partial^2 (w - w_1)}{\partial x^2} =$$

(beam 2)

(beam 1)

On the other side, if torsional rigidity is small, i.e.

$$J_t \rightarrow 0$$

the equation (31) will become

$$\frac{\partial^2 M^*}{\partial x^2} = - \frac{m(x)}{2b}$$

The equation is integrated one time and we get the first derivative of flexural moment, that is a shear.

$$\begin{aligned} T^* &= \frac{\partial M^*}{\partial x} = \\ &= - \int \frac{m(x)}{2b} dx = \\ &= - \frac{1}{2b} \int m(x) dx = - \frac{M}{2b} \end{aligned}$$

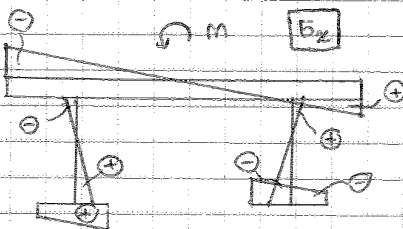
$M$  = global bending moment due to the distributed bending moment  $m(x) = p(x) \cdot e$

On the other side, the load applied on each beam - if they are identical - is given by the torque moment  $p(x) \cdot e$  divided by the total distance between them  $2b$ .

$$q = \frac{pe}{2b}$$

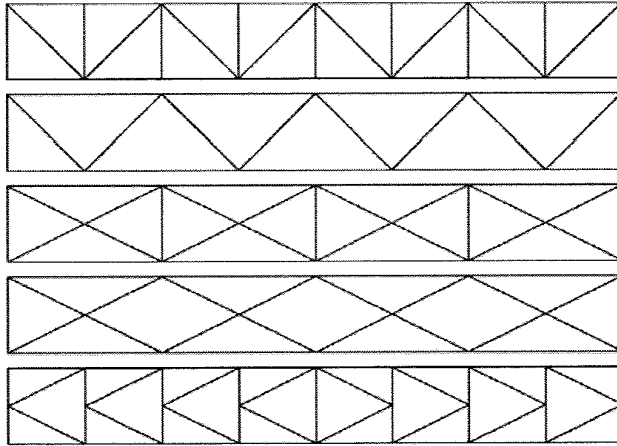
In practical terms, we fall into the situation of transversely rigid girder and COURBON APPROACH is valid. Indeed, its main hypothesis was null torsional rigidity.

$$J_t \rightarrow 0 \Rightarrow \text{Courbon approach}$$



Actually, the true solution doesn't present constant stresses along the flange but there is also a variation of stresses along it, which is small for practical applications.

- In case of a deck made of two beams if we close the bottom (i.e. by means of a truss beam), we may change its behaviour in a torsional one. In such a case the analysis should be performed like a box girder section having a fictitious thickness "s" of the bottom wall, which may be evaluated imposing the equality of internal actions and displacements between the actual and the fictitious wall.



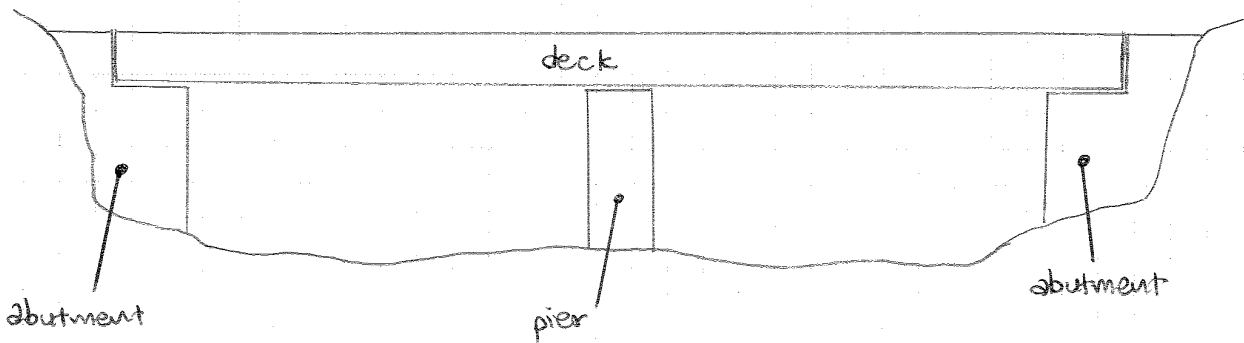
Most usual geometry of bottom stiffening stresses.



On the piers and on the abutments, supported by them, there is the deck = "impalcato".  
It is the part of the bridge that carries <sup>the</sup> traffic loading over piers and abutments and on which you can go.

If you want to inspect the deck, generally there is a special permanent access to the deck, called inspection gangway.  
That's a safety way on which you walk to inspect the bridge and it is located over the bridge, on the side of it or under it.  
This element is always present in rail bridges, whereas in road bridges this is not necessary because there is already a protected footpath on the side.

If you want to make more detailed inspection to the bridge, you need a special platform called by-bridge.  
Typically, it is a lorry with a train, which has a basket and moves under the bridge.



### 3 Road bridges

→ carriageway = "carreggiata"

It is the part of the road surface on which you can drive and is supported by a single structure (for instance, the deck).  
It includes all traffic lanes ("corsie"), hard shoulders, hard strips ("baucine") and marker strips.

→ central reservation:

it is an open space between the carriageways protected by guardrail

→ hard shoulder = "corsia di emergenza":

a part of the carriageway, usually with the same width of one traffic lane, located on the side and used only in emergency situations.



physical notional lanes, because it depends on the SPEED OF THE ROAD (in highways, vehicles need more space than in city roads).

↳ the bridge is loaded referring to theoretical notional lanes

→ remaining area:

what remains on the deck after having placed the theoretical notional lanes.

In fact, the width of the deck is designed to accept the real load and the physical notional lanes. As the two types of notional lanes are not equal, there is some extra space for the road which doesn't fall into the design notional lanes.

→ tandem system:

it is a theoretical model used to apply a load on a bridge in a design face (not real).

It is an assembly of 2 consecutive axles with 2 wheels on them (so, there are 4 wheels) considered to be simultaneously loaded and represents the back wheels of a really heavy lorry.

In design we use this model and don't apply a load which is similar to the traffic because it's simpler and calibrated to generate the same effects.

## 4 Rail bridges

→ Footpath:

a strip located between the tracks and the parapet, on which you can go.

→ track = "binario"

The track is made of 3 elements: we have 2 RAILS = "rotale", at a distance of about 1,5 m, that are connected by transverse elements in wood or concrete, called SLEEPERS.

Under the sleepers, we have a bed of selected stone, called BALLAST.

→ maximum line speed at the site:

maximum permitted speed at the site, generally limited by the characteristics of the infrastructure or railway operating safety requirements.

Usually, there's an automatic control of the speed, keeping it below the limit.

$d = 3,2 \text{ m}$  We need a permission from Military and Fire Brigade

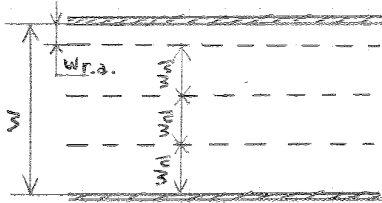
### → RIVER UNDER THE BRIDGE

In this case, we have to refer to the maximum level of the river, which is usually given by a 200÷500-years flood.

To this level, it is advised to add an extra space (0,5÷1 m) to let solid elements (vegetation, detrites, rubbish...) pass.

### 3 Traffic actions

Traffic actions are applied on theoretical notional lanes, in order to simplify the analysis, as the width of notional lanes is very variable.



A carriageway, which is limited by physical barriers, has width  $w$ .

The width is divided in a number of notional lanes, which is given by a table

- Ⓘ If width is small (less than 5,4 m), we consider only 1 notional lane which is 3 m wide and the remaining area is what remains

$$w < 5,4 \text{ m} \Rightarrow 1 \text{ t.n.l. } w_{n1} = 3 \text{ m}$$

- Ⓜ If width varies between 5,4 m and 6 m, we place 2 notional lanes that are  $w/2$  wide.

$$5,4 \text{ m} < w < 6 \text{ m} \Rightarrow 2 \text{ t.n.l. } w_{n1} = \frac{w}{2} = 2,7 \div 3 \text{ m}$$

- Ⓝ If width is bigger than 6 m, we place a number of notional lanes that's the integer of  $w/3$  (rounded down) with width of 3 m and the remaining area is the width of carriageway reduced by the width of the notional lanes ( $3n$ ).

$$w > 6 \text{ m} \Rightarrow n = \text{INT}\left(\frac{w}{3}\right) \quad w_{n1} = 3 \text{ m}$$

Generally, notional lanes' width is 3 m.

On theoretical lines we can place traffic loads, which can be

→ VERTICAL LOADS, due to the weight of the vehicles.

→ when vehicles move on the road, they also transmit HORIZONTAL FORCES, that come from traction, braking and centrifugal force (= transverse force that comes from moving on a curve).

Just to have in idea, the braking system of a car can give an acceleration of  $1g$  and transfer to road a horizontal force which is equal to the weight of the car. In a curve, we transmit a lateral acceleration of  $0,4g$ , which corresponds to a horizontal force of  $0,4$  times the weight

⇒ HORIZONTAL FORCES ARE IMPORTANT FORCES

## I Vertical forces

Vertical forces are function of vehicles and it is complicated to link forces to vehicles.

So, Eurocodes consider only 6 models of vehicles that represent the everyday traffic and some of them are used for global verifications, other ones for local verifications and other ones for both verifications.

→ LOAD MODEL 1 : it presents a tandem load and a distributed load in order to represent general traffic.  
It is used both in global and local verifications.

→ LOAD MODEL 2 : it presents a tandem load and represents the effect of a very heavy vehicle.  
It is used both in global and local verifications.

→ LOAD MODEL 3 : it presents a concentrated load on a  $0,4 \times 0,4$  m footprint and represents the effect of a very heavy vehicle.  
It is used ~~to~~ only in local verifications.

→ LOAD MODEL 4 : it presents a concentrated load on a  $0,1 \times 0,1$  m footprint, in order to represent the effects of small vehicles on a pedestrian bridge (maintenance vehicles).  
It is used only in local verifications.

→ LOAD MODEL 5 : it presents a distributed crowd load and it is used in both verifications.

→ LOAD MODEL 6 : it presents a distributed load and it is used for long span bridges.

The loads can be concentrated or distributed and both concentrated and distributed forces can be multiplied for a factor  $\alpha$ , which is function of the RELEVANCE OF THE BRIDGE.

In past, in Italy existed 2 categories

→ I category

$$\alpha_I = 1$$

→ II category

$$\alpha_{II} = 0,8 \quad \rightarrow 80\% \text{ of the I category bridges' load}$$

The distinction was made due to economic reasons and so the load ~~we~~ that we apply are

$$\alpha_Q Q_k \quad \alpha_q q_k$$

Nowadays, all new bridges belong to the I category but 80% of business is old structures (many of them are at the end of service life) and, in repairing an old bridge, maybe this belongs to II class and it is designed with different coefficients.

## Road bridges category

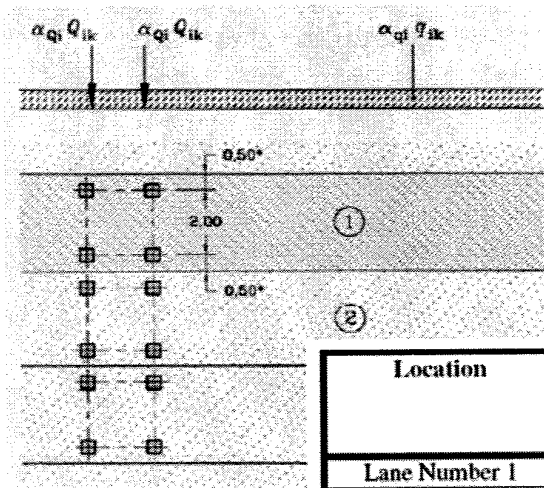
$$\alpha_Q Q_k$$

$$\alpha_q q_k$$

- 1° Category bridges  $\alpha_Q = \alpha_q = 1.0$
- 2° Category bridges  $\alpha_Q = \alpha_q = 0.8$

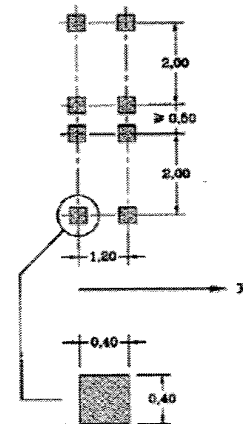

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## Load model 1 - LM1



$$\alpha_Q Q_k$$

$$\alpha_q q_k$$

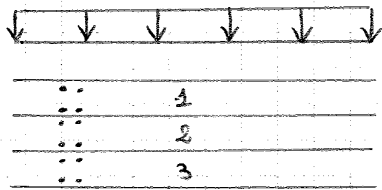


Location	Tandem system TS	UDL system
	Axle loads $Q_k$ (kN)	$q_k$ (or $q_{lk}$ ) (kN/m <sup>2</sup> )
Lane Number 1	300	9
Lane Number 2	200	2,5
Lane Number 3	100	2,5
Other lanes	0	2,5
Remaining area ( $q_{lk}$ )	0	2,5


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$Q_{ik}$  = axle load (the tandem system's load is the double)

Which is the lane's position?



It is free and I can swap them.  
It depends on the effects that I'm studying

⇒ I study different positions in order to maximize/minimize the effects

**OBSERVATION**: according to the Italian law, the heaviest lorry that can travel is 40 tons heavy.

The system in LM1 is 60 tons heavy (heavier than the heaviest lorry) and it is concentrated in a small area. So, the effect is equivalent to a 15 m vertical lorry multiplied for 1,5.

Also the distributed load doesn't simulate cars because they are 1 ton heavy and their area is  $4\text{ m} \times 2\text{ m} = 8\text{ m}^2$ . It means that the distributed load is  $1/8\text{ ton/m}^2$  and not  $10\text{ ton/m}^2$ .

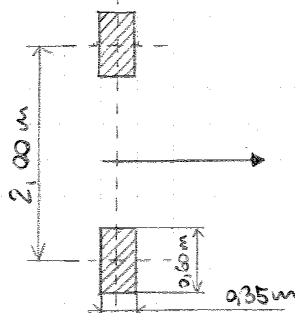
Even  $2,5\text{ ton/m}^2$  is bigger than a car and we also have to consider that the model of distributed load is correct only in traffic jams, where vehicles touch themselves.

The  $9\text{ ton/m}^2$  distributed load simulates a traffic jam of lorries full of iron.

⇒ LM1 stand for a traffic jam and a vertical lorry in a special point.

→ Load model 2 (LM2)

It is used for both verifications, even if it is very uncommon using it for global verification.

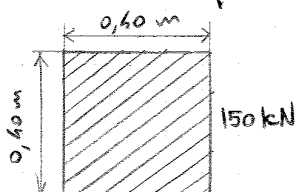


It presents a tandem that transfer a very heavy load (400 kN axle load) on a large foot print and it is equivalent to a lorry that enters to the bridge

⇒ huge local effect but not important in global behaviour, except in short bridges.

If more conservative, a single tyre (contact area) of 200 kN may be used.

→ Load model 3 (LM3)



It is usually used for local verifications and presents a concentrated force of 150 kN over a square surface.

## Load model 6 – LM6

May be used instead of LM1 for global verification of bridges with span > 300 m

$$q_{L,a} = 128,95 \left( \frac{1}{L} \right)^{0,25} \text{ [kN/m];}$$

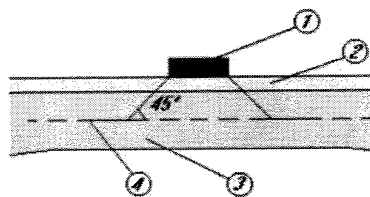
$$q_{L,b} = 88,71 \left( \frac{1}{L} \right)^{0,38} \text{ [kN/m];}$$

$$q_{L,c} = 77,12 \left( \frac{1}{L} \right)^{0,38} \text{ [kN/m].}$$

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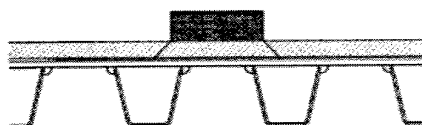
## Dispersal of concentrated loads


Dispersal of concentrated loads through pavement and a concrete slab



- Key**
- 1 Wheel contact pressure
  - 2 Pavement
  - 3 Concrete slab
  - 4 Middle surface of concrete slab

Dispersal of concentrated loads through pavement and orthotropic decks



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## II Horizontal forces

We can see different components.

→ braking and accelerating

They produce longitudinal forces, that are calculated as a percentage of vertical ones corresponding to LM1 likely to be applied on lane 1 (the heaviest lane).

$$Q_k = 0,6 \alpha_{Q1} (2Q_{1k}) + 0,10 \alpha_{Q1} q_{1k} w_{1k} L \leq 900 \text{ kN}$$

$w_{1k}$  length of the deck

This force is given by the 60% of concentrated load and 10% of distributed load and varies between a minimum of  $180\alpha_{Q1}$  kN and a maximum of 900 kN. Indeed, in normal vehicles, braking system is more efficient than engine and provides a bigger acceleration, for safety reasons. So, the force is calibrated on the brakes and, if we imagine some lorries going down from a mountain on a bridge and they are braking, they transmit to the bridge a force that is a percentage of lorry's weight and of the inclination, which can be assume a maximum value of 6%.

The force is applied horizontally at the level of the pavement.

So the force is not applied at the centre of the beam and, as slabs are big, the lever-arm is not neglectable.

Usually, this force is a concentrated one applied under the tandem, along the axis of any lane.

→ centrifugal forces  $Q_{tk}$

These forces are present only if bridge has a curvature and should be taken as a transverse force.

They are again calculated as a percentage of vertical forces, calculated with LM1, but now they are also function of the radius  $r$  of curvature of the carriageway centre line: the closest is the radius, the highest are the forces and stresses in the bridge.

$Q_{tk}$ [kN]	$r$ [m]
$0,2 Q_v$	$< 200$
$\frac{40}{r} Q_v$	$200 \div 1500$
$0$	$> 1500$

Centrifugal forces are horizontal forces applied to the level of the pavement (pushing outwards of the curve - bridge is keeping the cars inwards because they are going outwards) and they are orthogonal to the axis of the bridge.



## 4 Weather actions

→ snow : we use the same models of buildings, but snow can not be combined with traffic because, compared with it, it's only a little percentage.

The only cases where snow is important are footbridges in mountain and covered bridges, in which there is a combination of actions.

Snow is important also in BRIDGES DURING CONSTRUCTION, as they need long time construction and during this we can have snow, wind, etc. on the bridge

→ wind : we use the same models of buildings.

wind has a small effect in small concrete bridges, while it is a dominating action in long bridges due to their flexibility. This issue is valid especially on railway bridges, because they have higher performances.

To model this action, we assume that the surface offered by vehicles moving on the bridge as a rectangular continuous stripe 3 m high from the pavement.

## 5 Fatigue load models

5 different fatigue load models were developed.

Firstly, the codes divide roads in 4 traffic categories in relation to the traffic.

1 = roads and motorways with  $\geq 2$  lanes per direction with high flow rates of lorries

$$N = 2,0 \cdot 10^6$$

2 = road and motorways with medium flow rates of lorries

$$N = 0,5 \cdot 10^6$$

3 = main roads with low flow rates of lorries

$$N = 0,125 \cdot 10^6$$

4 = local roads with low flow rates of lorries

$$N = 0,05 \cdot 10^6$$

$N$  = number of lorries per year and per lane

With this table, we have the first parameter of fatigue, i.e. the number of passages.

Then we have to define loads and there are 5 models

→ Fatigue load model 1

It is generally the worst one and is taken from LM1, with the values of the axle loads equal to  $0,7Q_k$  and the values of the uniformly distributed loads equal to  $0,3q_k$ .

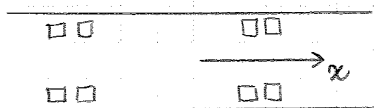
$$0,7Q_k + 0,3q_k$$

→ Fatigue load model 2

It is used for more detailed Fatigue loads and consist of a set of idealised lorries, called FREQUENT LORRIES.

Each frequent lorry is defined by the number of axles, their spacing and their frequent load and the wheel contact areas and the transverse distance between them ("wheel type")

→ Fatigue load model 3



This model consists of a theoretical lorry having two axles in front and two axles in back.

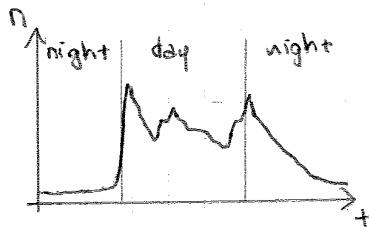
The weight of each axle is 120 kN (they are equal) and the contact area of each wheel is a square of side 0,40 m.

→ Fatigue load model 4

It is the more realistic one, as it consists of sets of standard lorries which together produce effects equivalent to those of typical traffic. Loads are realistic and they also include a percentage with respect to traffic in relation with distance.

→ Fatigue load model 5

It consists of direct application on the bridge of a recorded history on traffic.



So, we use a graph in function of the time which gives us the tons or the number of vehicles.

## Fatigue load models

### Fatigue load model 1 – (similar to LM1)

(1) Fatigue Load Model 1 has the configuration of the characteristic Load Model 1 defined in 4.3.2, with the values of the axle loads equal to  $0,7Q_k$  and the values of the uniformly distributed loads equal to  $0,3q_k$  and (unless otherwise specified)  $0,3q_{rk}$ .

### Fatigue load model 2

(1) Fatigue Load Model 2 consists of a set of idealised lorries, called "frequent" lorries, to be used as defined in (3) below.

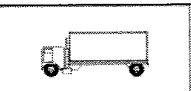
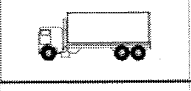
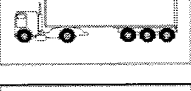
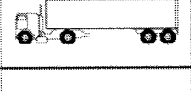
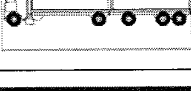
(2) Each "frequent lorry" is defined by :

- the number of axles and the axle spacing (Table 4.6, columns 1+2),
- the frequent load of each axle (Table 4.6, column 3),
- the wheel contact areas and the transverse distance between wheels (column 4 of Table 4.6 and Table 4.8).

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## Fatigue load models

### Fatigue load model 2

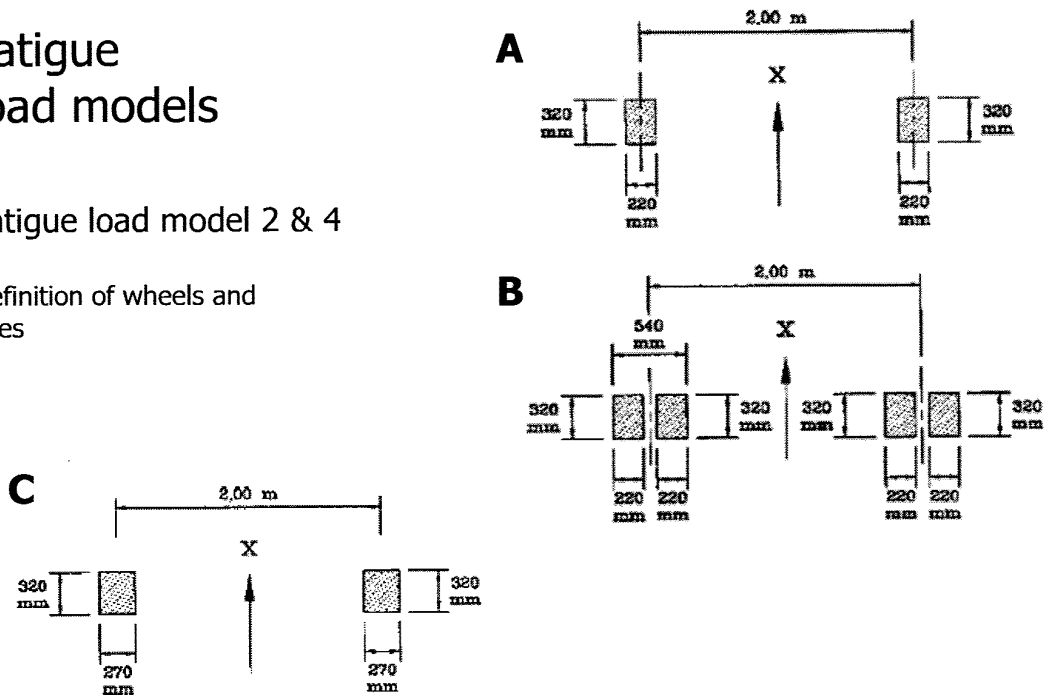
1	2	3	4
LORRY SILHOUETTE	Axle spacing (m)	Frequent axle loads (kN)	Wheel type (see Table 4.8)
	4,5	90 190	A B
	4,20 1,30	80 140 140	A B B
	3,20 5,20 1,30 1,30	90 180 120 120 120	A B C C C
	3,40 6,00 1,80	90 190 140 140	A B B B
	4,80 3,60 4,40 1,30	90 180 120 110 110	A B C C C

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# Fatigue load models

## Fatigue load model 2 & 4

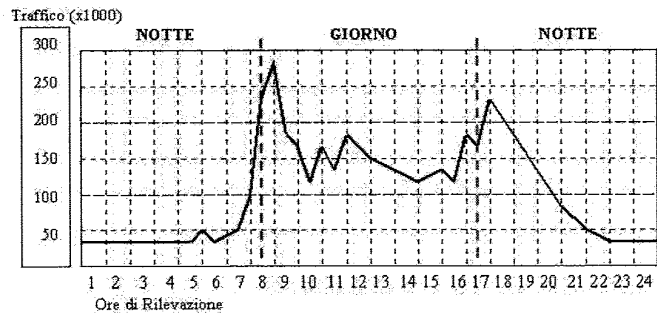
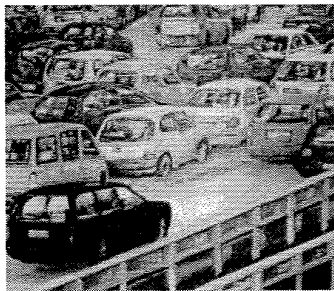
Definition of wheels and axes



# Fatigue load models

## Fatigue load model 5

(1) Fatigue Load Model 5 consists of the direct application of recorded traffic data, supplemented, if relevant, by appropriate statistical and projected extrapolations.



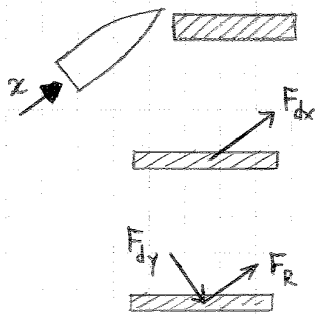
Its value depends on clearance  $h$  and collision force  $F_{dx}$ . It is generally smaller than the last one because, when a vehicle collides against the deck, generally it doesn't stop and so it doesn't apply all the kinetic energy.

$$F = r F_{dx} \quad r = \begin{cases} 1 & h \leq 5 \text{ m} \\ 1 - (h - 5) & 5 \text{ m} < h \leq 6 \text{ m} \\ 0 & h > 6 \text{ m} \end{cases}$$

The impact area is a square area which side is equal to 0,25 m.

→ impact of boats on piers

This accidental situation happens only on bridges that have piers alongside a river, a lake or the sea.



In this case, if  $z$  is the sailing direction (estimated with far approximation), the effect of the collision is simulated by 2 different systems of forces acting not simultaneously on the pier.

→ single force  $F_{dx}$  acting on  $z$  direction

→ combination of 2 forces,  $F_R = 0,4 F_{dy}$  acting on  $z$  direction and  $F_{dy} = 0,5 F_{dx}$  acting orthogonally to the navigation direction.

The value of the force  $F_{dx}$  is given by a table, depending on the kind of boat that could travel under the bridge. In order to simplify analysis, boats are divided into 4 classes according to mass and length.

Boat class	Length [m]	Tonnage [t]	$F_{dx}$ [kN]
Small	50	3'000	30'000
Medium	100	10'000	80'000
Large	200	40'000	240'000
Huge	300	100'000	660'000

## → impact of vehicles on a barrier on the bridge

Barriers are fixed to the deck and we have to design the deck in a way to prevent it to have damage when a vehicles collides on a barrier. In this way, if the base isn't damaged, we can remove the barrier and fix a new one in the same holes used for the old one.

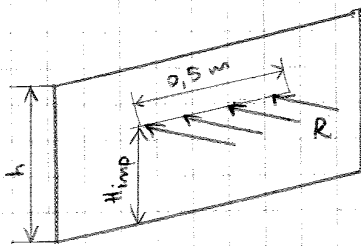
the idea is that the barrier can be replaced easily and so DAMAGE AND ENERGY DISSIPATION SHOULD BE CONCENTRATED INSIDE THE BARRIER, WHILE CONNECTION ELEMENTS between barrier and deck (bolts = "bulloni") SHOULD REMAIN IN ELASTIC STATE AND IN SERVICE AFTER THE IMPACT.

Thus, replacement is not expensive; otherwise, bridge will need special maintenance.

So, barriers are soft and bendable elements, while connection elements remain elastic.

This is a typical philosophy of CAPACITY DESIGN, in which we design connection not in function of the force applied by the lorry in the barrier, but in function of plastic capacity of the barrier and it has to be strong enough to sustain an internal action that corresponds to the full plasticity of the barrier.

Generally, barriers are classified with European standards in function of the maximum force. So, barriers aren't designed but chosen from a catalogue, which gives a maximum force that corresponds to full plasticity of the barrier itself (given by DH 2367 of 21/06/2004).

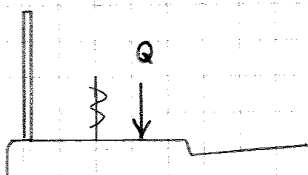


When certification quality isn't developed in the catalogue, we have to apply an horizontal force  $R$  of 100 kN from the inside to the outside of the bridge, at a distance  $h_{imp}$  from the pavement that is connected to the height of the barrier.

$$h_{imp} [m] = \min \{ h [m] - 0,1 ; 1,0 \}$$

## → vehicles on footways and cycle tracks

Another accidental situation is connected to a position of a vehicle on the bridge that's not correct



Generally, we have a pedestrian area between the parapet and the road restraint system, which is protected and vehicles can't generate loads on it. On the other side, there's a cycle track on the kerb and may happen that maintenance vehicles pass where traffic is not present.

**1 Practice & exercises 1: Traffic actions on bridges 45/119**

### SLU actions safety factors


		Coeff.	EQU <sup>(1)</sup>	A1 STR	A2 GEO
Dead load	FAV	$\gamma_{G1}$	0,90	1,00	1,00
	UNFAV		1,10	1,35	1,00
Permanent loads	FAV	$\gamma_{G2}$	0,00	0,00	0,00
	UNFAV		1,50	1,50	1,30
Traffic variable loads	FAV	$\gamma_Q$	0,00	0,00	0,00
	UNFAV		1,35	1,35	1,15
Other variable loads	FAV	$\gamma_{Qi}$	0,00	0,00	0,00
	UNFAV		1,50	1,50	1,30
Design imposed deformations	FAV	$\gamma_{e1}$	0,90	1,00	1,00
	UNFAV		1,00 <sup>(3)</sup>	1,00 <sup>(4)</sup>	1,00
Other imposed deformations (creep, temperature, settlements...)	FAV	$\gamma_{e2}, \gamma_{e3}, \gamma_{e4}$	0,00	0,00	0,00
	UNFAV		1,20	1,20	1,00



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**1 Practice & exercises 1: Traffic actions on bridges 46/119**

Actions	Description	$\Psi_0$ Charact.	$\Psi_1$ Freq.	$\Psi_2$ Q. Perm.
Traffic actions	LM1 Tandem	0,75	0,75	0,0
	LM1-5-6 Distributed	0,40	0,40	0,0
	LM3-4	0,40	0,40	0,0
	LM2	0,0	0,75	0,0
	2	0,0	0,0	0,0
	3	0,0	0,0	0,0
Wind	4	---	0,75	0,0
	5	0,0	0,0	0,0
	Wind – bridge unloaded SLU and SLE	0,6	0,2	0,0
Snow	In construction phases	0,8	---	0,0
	Wind – loaded bridge	0,6		
Temperature	SLU and SLE	0,0	0,0	0,0
	Execution	0,8	0,6	0,5
	$T_k$	0,6	0,6	0,5



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Yet, a bridge is a solid element - not a line - and we have to check stresses in all directions. So, we should pre-stress in longitudinal and transverse direction.

$$\sigma_1 < 0$$

#### → CRACK FORMATION

Decompression is a severe limit state and it is generally asked for railway bridges.

In road bridges, we can admit small tensile stresses, but these must not be close to material's resistance.

$$\sigma_1 < \frac{f_{ctm}}{1,2}$$

#### → CRACK OPENING

We admit crack formation, but they must not be bigger than one of these 3 quantities

$$w_1 = 0,2 \text{ mm} \quad w_2 = 0,3 \text{ mm} \quad w_3 = 0,4 \text{ mm}$$

Crack control is related to the aggressivity of the environment and limit state is given by a table, depending on environmental class (aggressivity) and sensibility of reinforcement - pre-stressing reinforcement is more sensible than ordinary reinforcement.



## SLE Cracking control

### Definition of cracking limit states

Decompression:  $\sigma_1 < 0$

Crack formation:  $\sigma_1 < \frac{f_{ctm}}{1.2}$

Crack opening:  
 $w_1 = 0,2 \text{ mm}$   
 $w_2 = 0,3 \text{ mm}$   
 $w_3 = 0,4 \text{ mm}$




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## SLE Cracking control

### Environmental class groups

Group	Environmental classes
Standard	X0, XC1, XC2, XC3
Aggressive	XC4, XD1, XS1
Very aggressive	XD2, XD3, XS2, XS3



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## Action on railway bridges

1 Italian codes reflect the <sup>actions</sup> ~~act~~ given by Model Code.

2 Permanent actions:

there are no many differences from road bridges and they include

→ dead load  $g_1$

→ carried permanent loads  $g_2$ :

they're related to tracks, barriers, parapets and **ballast** (= pebbles under the sleepers)

18  $\text{kNm}^3$  in case of straight track  
20  $\text{kNm}^3$  in case of curved track, because track is inclined towards to the interior of the curve to counterbalance centrifugal force and more pebble is needed

→ other permanent loads  $g_3$  (ground and hydraulic loads)

3 Imposed deformations

→ design imposed deformations  $\varepsilon_1$

→ creep and shrinkage  $\varepsilon_2$

→ thermal actions  $\varepsilon_3$

→ ground settlements  $\varepsilon_4$

4 Traffic actions

Models intend to represent theoretical trains - not real - in order to realize internal actions in the bridge bigger than real ones.

Five models of railway loading are given

① Load Model 71 (LM 71)

It is given to represent the static effect of vertical loading due to PASSENGER TRAINS.

### III Unloaded train

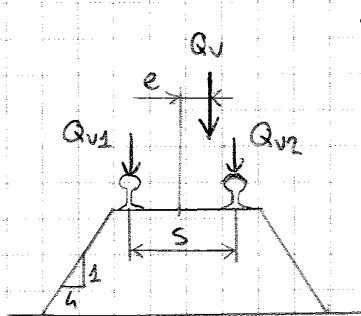
It represents the static effect of a GOOD EMPTY TRAIN and consists of a uniformly distributed load with a characteristic value of  $10 \text{ kNm}^{-1}$ , without concentrated loads.  
It is used for special verifications.

### IV Load Model HSLM

It represents the loading from passengers trains at speeds exceeding  $200 \text{ km/h}$ .

How traffic loads are distributed from the tracks to the structure?

Tracks are made of 2 rails, sleepers and ballast.  
The distance between the rails is called SCAM = "scartamento" and in Europe a standard value is adopt, in order to let trains move from one nation to the other.



$$S = 1435 \text{ mm}$$

In case of CH71 and CHSW/0, there is a linear load  $Q_v$  that is divided on the 2 rails.  
This load can also be not divided one half and one half, but there is a maximum ratio in the division

$$\frac{Q_{v2}}{Q_{v1}} \leq 1,25$$

In this way, eccentricity of the load, speed and centrifugal forces are taken into account.

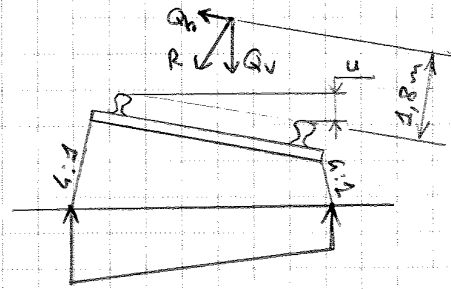
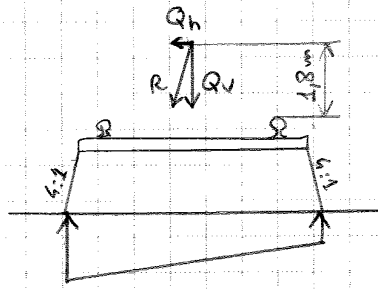
The maximum ratio corresponds to a maximum eccentricity of the vertical load that is

$$e = \frac{S}{18}$$

Then, we have to see the distribution of the loads

→ LONGITUDINAL DISTRIBUTION OF CONCENTRATED FORCES BY THE RAIL

If there is a concentrated force from the wheel to the rail, it is distributed on the sleepers by means of the rails' stiffness.



Another traffic action is the action from non public footpaths:

besides the bridge there is always a gangway - a footpath -, only used by authorized personnel to inspect and maintenance.

This action is represented by a uniform distributed load of  $10 \text{ kNm}^2$ .

this load is reached only if some goods like rails or sleepers are let here.

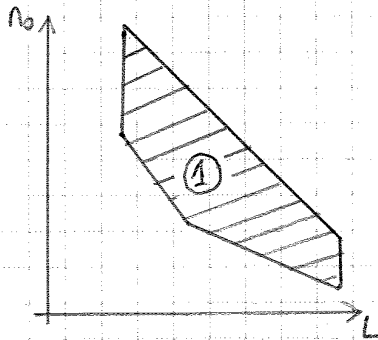
## 5 Dynamic effects

Dynamic effects are important in railway bridges because one train has a big mass, which is moving all together.

In road bridges, cars are little masses that move independently, so there is a big dispersion.

So, with respect to road bridges, there is a different kind of traffic and different dynamic actions and attention is needed.

Firstly, we have to evaluate if the limits of bridge natural frequency are respected.



We have to calculate the FIRST NATURAL FREQUENCY  $\nu_0$ , connected to the I eigen mode (= "I modo di vibrazione").

Then this value is inserted into a graph where

→  $x$  = LENGTH OF THE SPAN - the graph is intended only for simply supported bridges -

→  $y$  = FIRST NATURAL FREQUENCY  $\nu_0$

If the representative point falls inside the area ①, according to the railway the bridge is safe.

If this point falls outside this area, there's a risk of troubles and we have to follow special design procedures.

The coefficient  $\phi(\phi_2, \phi_3)$  can NOT be used for:

- The Unloaded train
- Real trains
- Trains for fatigue analysis

For steel deck without ballast (track directly connected to the deck) should be considered a coefficient  $\beta$  additional to  $\phi(\phi_2, \phi_3)$

$$\beta = 1.0 \quad \text{for } L_\phi < 8m \quad \text{and} \quad L_\phi > 90m$$

$$\beta = 1.1 \quad \text{for } 8m < L_\phi < 90m$$



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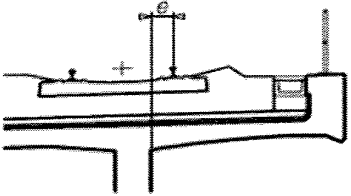
## Determinant length

Steel deck plate: closed deck with ballast bed (orthotropic deck plate) (for local and transverse stresses)		
	Deck with cross girders and continuous longitudinal ribs:	
1.1	Deck plate (for both directions)	3 times cross girder spacing
1.2	Continuous longitudinal ribs (including small cantilevers up to 0,50 m) <sup>a</sup>	3 times cross girder spacing
1.3	Cross girders	Twice the length of the cross girder
1.4	End cross girders	3,6m <sup>b</sup>



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## Determinant length

Concrete deck slab with ballast bed (for local and transverse stresses)		
4.1	Deck slab as part of box girder or upper flange of main beam <ul style="list-style-type: none"> <li>- spanning transversely to the main girders</li> <li>- spanning in the longitudinal direction</li> <li>- cross girders</li> <li>- transverse cantilevers supporting railway loading</li> </ul>	3 times span of deck plate  3 times span of deck plate  Twice the length of the cross girder   <ul style="list-style-type: none"> <li>- <math>e \leq 0,5</math> m: 3 times the distance between the webs</li> <li>- <math>e &gt; 0,5</math> m: <sup>a</sup></li> </ul>

## Determinant length

Concrete deck slab with ballast bed (for local and transverse stresses)		
4.2	Deck slab continuous (in main girder direction) over cross girders	Twice the cross girder spacing
4.3	Deck slab for half through and trough bridges: <ul style="list-style-type: none"> <li>- spanning perpendicular to the main girders</li> <li>- spanning in the longitudinal direction</li> </ul>	Twice span of deck slab + 3m  Twice span of deck slab
4.4	Deck slabs spanning transversely between longitudinal steel beams in filler beam decks	Twice the determinant length in the longitudinal direction

## Determinant length

Main girders		
5.3	Portal frames and closed frames or boxes:	Consider as three-span continuous beam (use 5.2, with vertical and horizontal lengths of members of the frame or box)
	- single-span	
	- multi-span	Consider as multi-span continuous beam (use 5.2, with lengths of end vertical members and horizontal members)
5.4	Single arch, archrib, stiffened girders of bowstrings	Half span
5.5	Series of arches with solid spandrels retaining fill	Twice the clear opening
5.6	Suspension bars (in conjunction with stiffening girders)	4 times the longitudinal spacing of the suspension bars



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## Determinant length

Structural supports		
6	Columns, trestles, bearings, uplift bearings, tension anchors and for the calculation of contact pressures under bearings.	Determinant length of the supported members



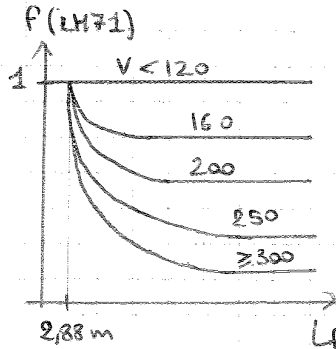
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$f$  = reduction factor, that should be taken as

$$f = 1,0 \quad \text{UNLOADED TRAIN and LM SW/2}$$

$$f = 1 - \frac{V-120}{1000} \left( \frac{814}{V} + 1,75 \right) \left( 1 - \sqrt{\frac{288}{L_f}} \right) \quad \text{LM 71}$$

$L_f$  = length of the loaded part of curved track on the bridge because we can load only a part of the length of the bridge in order to maximize actions



The relation of reduction factor with  $L_f$  is given by a graph which presents many curves in relation to the maximum speed  $V$

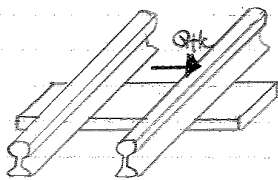
→ if  $V < 120$  km/h or  $L_f < 2,88$  m (very small length), there is no reduction

→ if  $V > 120$  km/h, a reduction is applied and the bigger is the loaded length, the bigger is the reduction.

The centrifugal force shall not be multiplied by the dynamic factor  $\Phi_2$  or  $\Phi_3$ .

→ nosing force (= "serpeggio")

It is the action that everyone can feel walking on the train, as it is moving inside the rails due to a gap between the wheels and the rail. For this reason, the train comes on one side and sometimes it jumps and comes closed to the other side. These displacement are small but the high speed makes us to feel a strong transversal acceleration.



Nosing force corresponds to a force applied by the train to the rail and it is a concentrated force of

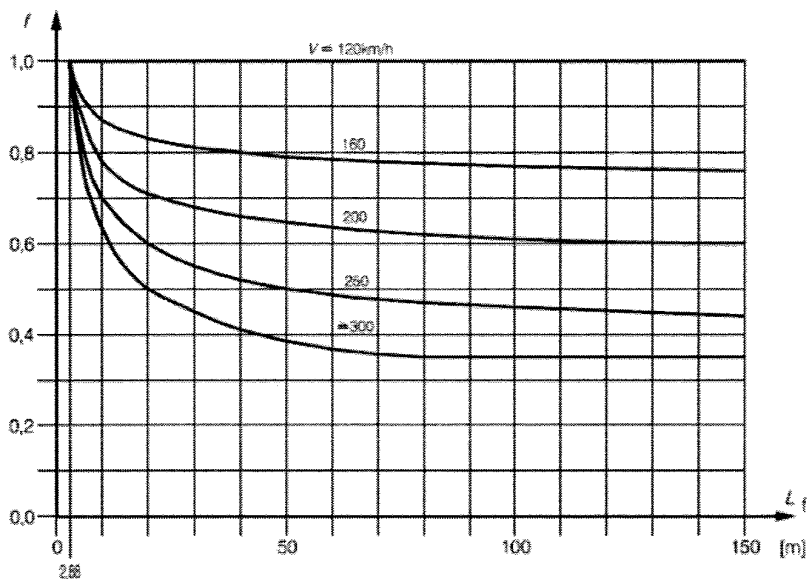
$$Q_{tk} = 100 \text{ kN}$$

applied transversally at the level of the track.


The nosing force shall always be combined with a traffic vertical load.



### Centrifugal forces



Factor  $f$  for Load Model 71 and SW/0

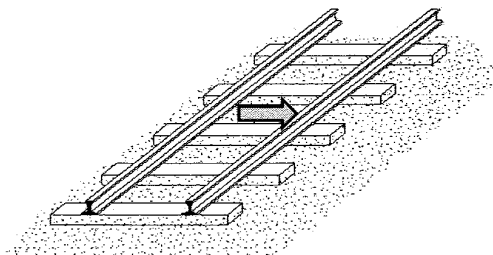

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### Horizontal actions

#### 2. Nosing force

(2)P The characteristic value of the nosing force shall be taken as  $Q_{sk} = 100$  kN. It shall not be multiplied by the factor  $\phi$  (see 6.4.5) or by the factor  $f$  in 6.5.1(4).

(4)P The nosing force shall always be combined with a vertical traffic load.




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→ traction and braking forces

As seen for road bridges, braking is much bigger than traction and this aspect is more evident in rail bridges

→ in a car, there is engine and traction is applied to 2 wheels, while braking is applied to 4 wheels.

→ in a train, traction is applied only by the locomotive, while all the coaches are supported by a braking system and so the power of the braking system is much bigger.

The characteristic values are

→ TRACTION

$$Q_{bk} = 33 \text{ [kNm}^{-1}\text{]} \cdot L_{a,b} \text{ [m]} \leq 1000 \text{ kN}$$

→ BRAKING

LM71, LM SW/0, HSLM  $Q_{bk} = 20 \text{ [kNm}^{-1}\text{]} \cdot L_{bk} \text{ [m]} \leq 6000 \text{ kN}$

LM SW/2  $Q_{bk} = 35 \text{ [kNm}^{-1}\text{]} \cdot L_{bk} \text{ [m]}$

Braking force has to be applied to all the length of the train.

The characteristic values of traction and braking forces shall not be multiplied by the dynamic factor.

In case of BRIDGES WITH MORE THAN 1 TRACK, we shall combine these forces

→ Bridges with 2 tracks: traction is applied to the I track; braking is applied to the II track.

So, we are combining 2 forces in the same direction, that corresponds to the unusual situation of having two trains at the same time on a small bridge ~~that~~ and one is accelerating and the other one is braking.

→ Bridges with more than 2 tracks (bridges near train stations):

I track : braking is applied

II track : traction is applied

III track : 50% of braking is applied

≥ IV track : nothing is applied

In design, thermal actions are divided in

→ uniform thermal variation:

in this case, there is no gradient inside the bridge and this action is connected with seasonal behaviour.

Concrete deck	$\pm 15^{\circ}\text{C}$
Composite steel-concrete deck	$\pm 15^{\circ}\text{C}$
Steel deck with ballast	$\pm 20^{\circ}\text{C}$
Steel deck without ballast	$\pm 25^{\circ}\text{C}$
Other concrete structures	$\pm 15^{\circ}\text{C}$

We have different values because materials have a different thermal inertia.

For instance, massive structures have a temperature similar to the average temperature of the day and sun can't produce a variation of temperature during the day because it has not enough power.

On the other side, steel is highly conductive thermal material and the variation of temperature is much bigger.

⇒ it is a matter of ratio of thermal capacity, thermal conductivity and thickness

- ▶ a thick element of a high capacity and low conductivity material presents a low variation
- ▶ a thin element of a low capacity and high conductivity material presents a big variation

For instance, the ballast is a layer of rock with a high thermal inertia and it reduces the thermal variation.

→ non uniform thermal variation

It is connected to daily behaviour, when the sun heats a side, whereas the other one is in shadow and cold.

This action has a period of 1 day.

Between intrados and extrados of the deck	$\pm 5^{\circ}\text{C}$
Between inside and outside of box section decks (inside it's warm in winter and cool in summer)	$\pm 5^{\circ}\text{C}$
Between concrete slab and steel beam	$\pm 5^{\circ}\text{C}$
Between inside and outside of box section piers	$\pm 10^{\circ}\text{C}$
Between the pier and its foundation (Foundation has the temperature of the ground)	$\pm 5^{\circ}\text{C}$

## 8 Interaction effects among track, ballast, piers and foundation

When the rails are continuous over discontinuities in the support - e.g. in the link between deck and abutment, where there is an expansion joint for the structure but not for the track - and a longitudinal action due to traction or braking, the rails transfer the action in part to the deck and in part to the embankment behind the abutment. To do this, STRUCTURE AND RAILS SHOULD JOINTLY RESIST.

Moreover, as rails aren't free to move, the deformations that the structure can have due to thermal variations, creep, etc. produce longitudinal forces that are distributed partly in the rails and partly in the fixed bridge bearings.

So the combined response of the structure and the track is difficult to evaluate because we should take into account the effect in terms of rigidity and resistance given by the track, the deck, the piers and the foundations.

The combined response is affected by many parameters.

→ configuration of the structure

→ statical scheme of the deck, e.g. simply supported beam, continuous beam or a series of beams.

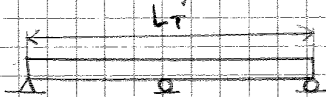
→ number of individual decks and their length.

→ number of spans and their length.

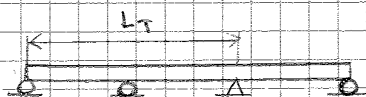
→ position of fixed bearings.

→ position of thermal bearing ("thermal fixed point")

→ EXPANSION LENGTH  $L_T$ , that is the length between the thermal fixed point and the end of the deck.



In the I case,  $L_T$  corresponds to the total span of the beam.



In the II case, the fixed point is placed on the III bearing and  $L_T$  is given by the maximum of the two lengths departing from this point.



With a 2-span bridge with 2 fixed points,  $L_T$  is the distance between the fixed points because all the 2 spans can move, thanks to the joint in the mid-span.

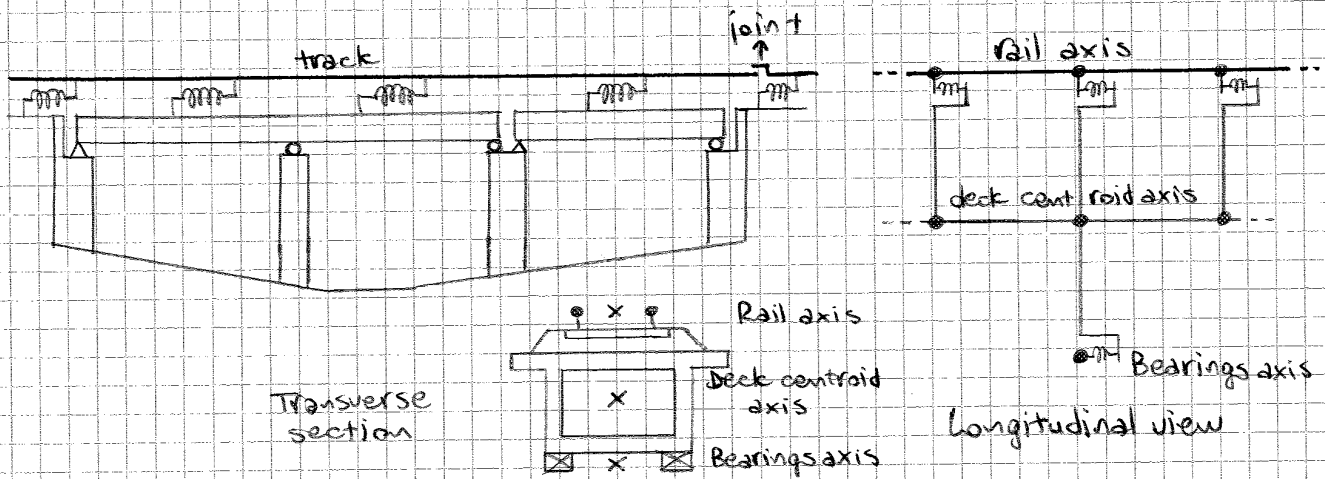
→ configuration of the track

→ ballasted track or non-ballasted track

→ vertical distance between the upper surface of the deck and the central axis of the rails:

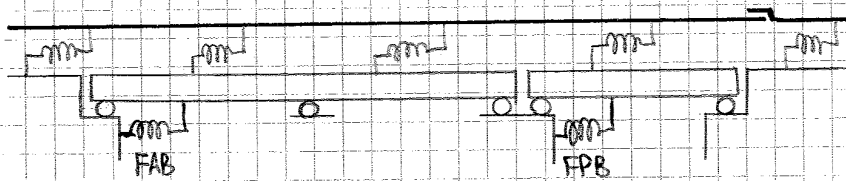
longitudinal actions generate in the deck a bending moment that depends on this distance, which is the lever-arm.

→ location of rail expansion joints.



From this, we pass to a simplified structural model where the support system is represented with 2 system of springs

- spring with stiffness equal to the one of foundation, abutments and bearings (FAB).
- spring with stiffness equal to the one of foundation, pier and bearings (FPB).

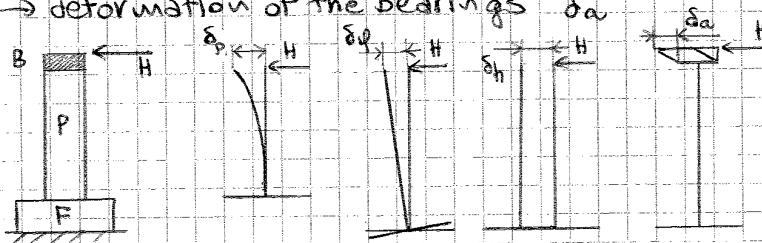


How can we evaluate the stiffness of these springs?

Starting from the structure, with reference to the FPB system, we apply a force  $H$  on the top of the pier:

the system will have a displacement, that is given by many contributions.

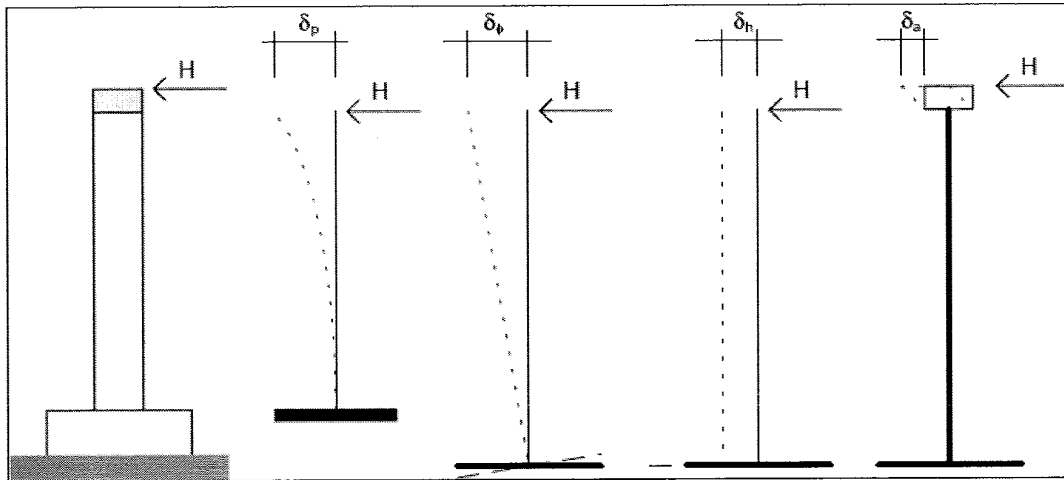
- deflection of the pier  $\delta_p$ , that depends on its stiffness
- rigid rotation of the foundation  $\delta_\varphi$
- rigid translation of the foundation  $\delta_h$
- deformation of the bearings  $\delta_a$



Total displacement will be the sum of these contributions.

$$\delta = \sum \delta_i = \delta_p + \delta_\varphi + \delta_h + \delta_a$$

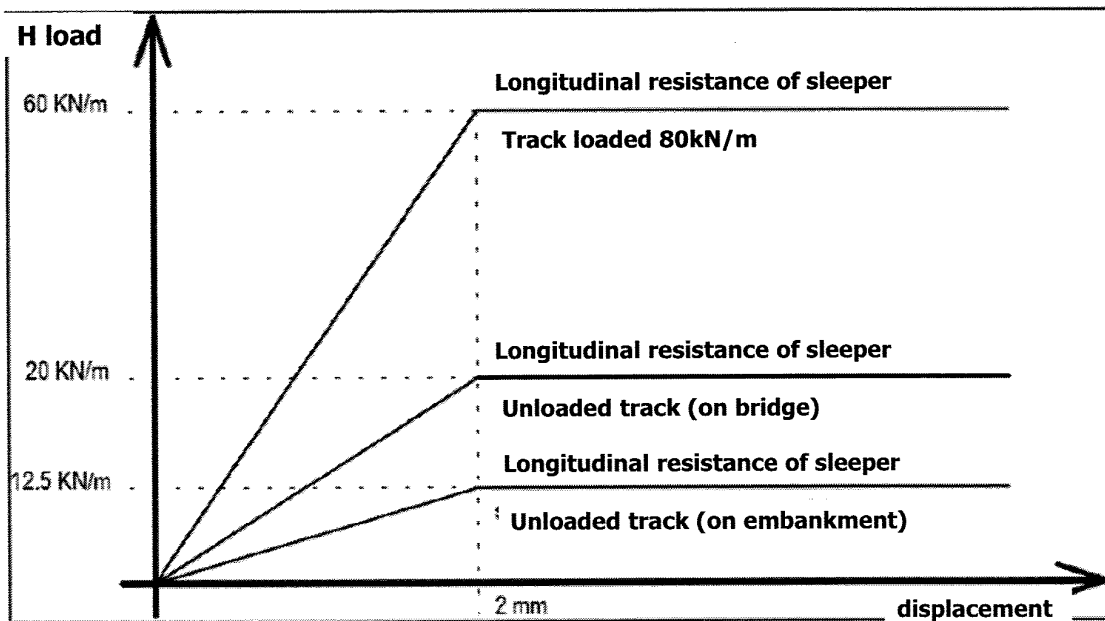
### Example: longitudinal total stiffness of FPB




$$K = H / \sum \delta_i \quad \sum \delta_i = \delta_p + \delta_\phi + \delta_h + \delta_a$$

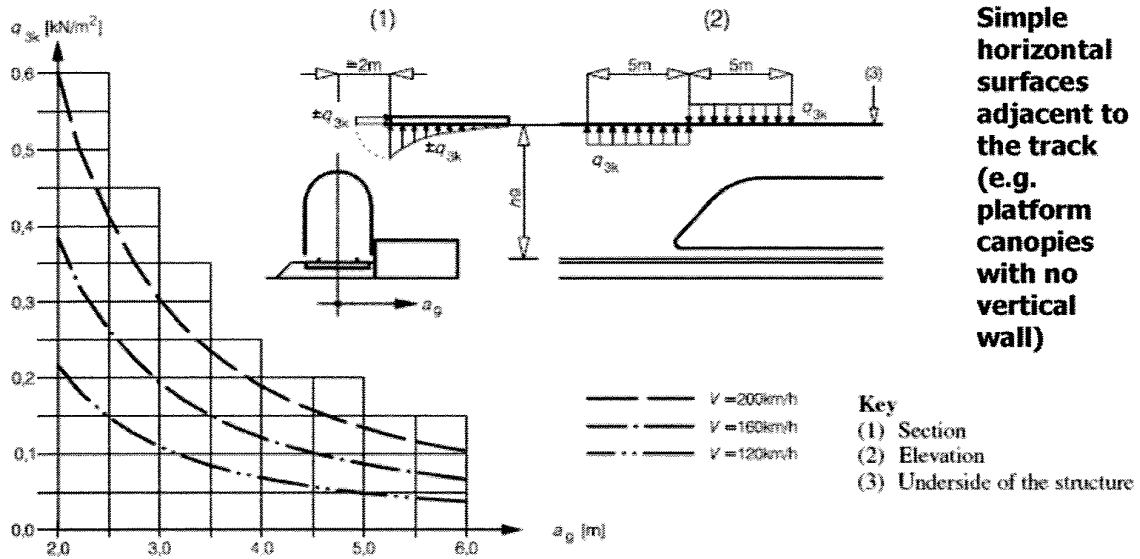

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### Ballast: variation of longitudinal shear force with vertical load for 1 track



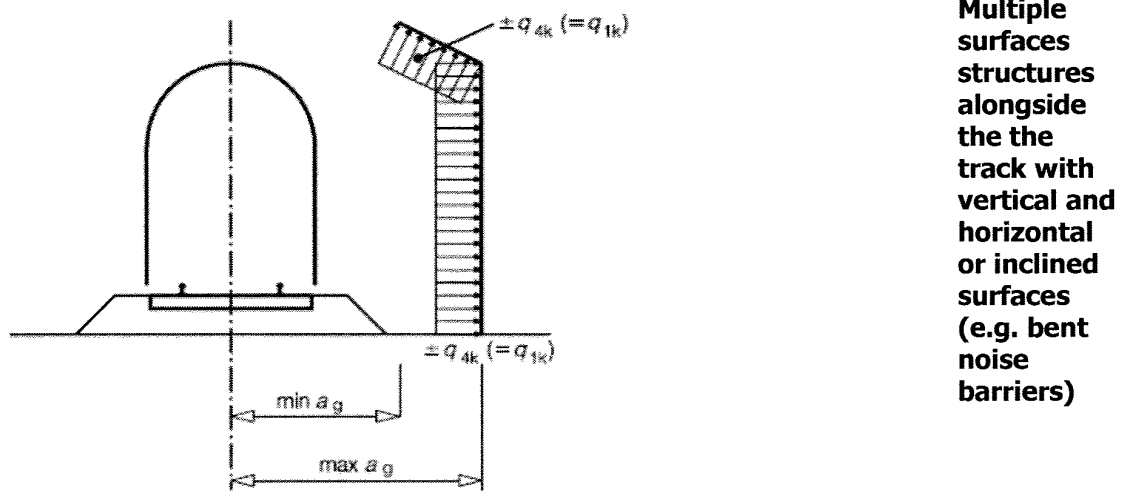

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Use values from figure in slide 101 using as distance from the track the lesser between:

$$a'_g = 0,6 \min a_g + 0,4 \max a_g \quad \text{or} \quad 6 \text{ m}$$

If  $\max a_g > 6 \text{ m}$  the value  $\max a_g = 6 \text{ m}$  should be used.


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**6.6.6 Surfaces enclosing the structure gauge of the tracks over a limited length (up to 20 m) (horizontal surface above the tracks and at least one vertical wall, e.g. scaffolding, temporary constructions)**

(1) All actions should be applied irrespective of the aerodynamic shape of the train:  
 – to the full height of the vertical surfaces:

$$\pm k_4 q_{1k} \tag{6.35}$$

where:

$q_{1k}$  is determined according to **slide 101**  
 $k_4 = 2$

– to the horizontal surfaces:

$$\pm k_5 q_{2k} \tag{6.36}$$

where:

$q_{2k}$  is determined according to **slide 102**  
 $k_5 = 2,5$  if one track is enclosed,  
 $k_5 = 3,5$  if two tracks are enclosed.

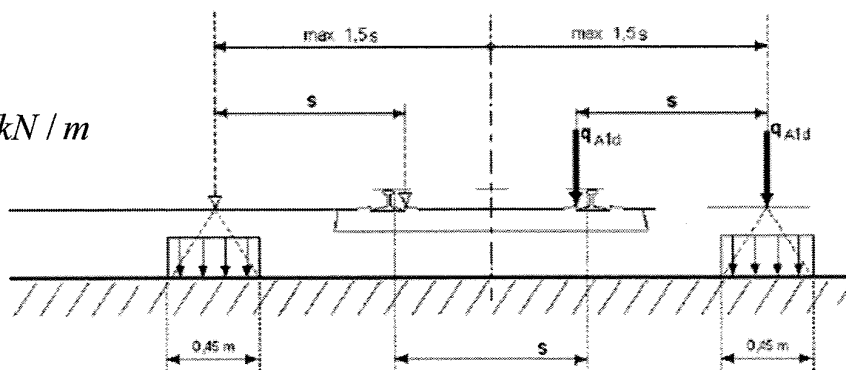
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**1 Practice & exercises 1: Traffic actions on bridges 106/119**

**Derailment over the bridge**


**Case 1**

$$q_{A1d} = 60 \text{ kN/m}$$



**The load includes dynamic effect and may be placed transversally in every position within the field  $\pm 1.5$  s**

**Only small entity damage can be accepted in order to re-open the line after light maintenance**

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## 10 Derailment over the bridge

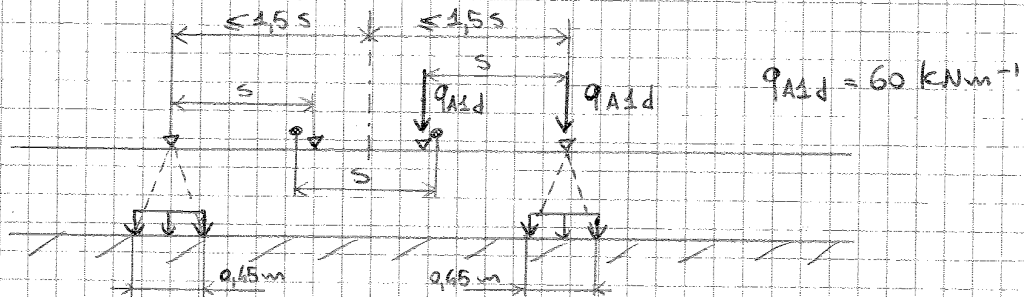
In this case, we want avoid an excessive damage in order to re-open the bridge immediately, after having done ~~the~~ small maintenance interventions.

We can define 2 cases

### → SMALL ENTITY DERAILMENT

In this case, a distributed load of  $60 \text{ kN/m}$  over a width of  $0,65 \text{ m}$  is applied in every position within the field of  $\pm 1,5s$  and with an interaxis of  $1,5 \text{ m}$  - that is the scam.

The load is applied in that field because, if the train derails ~~to~~ leftwards, it will impact leftwards and so it will move transversally of one scam and a half.



In this case, only small damage is accepted in order to re-open the line after light maintenance.

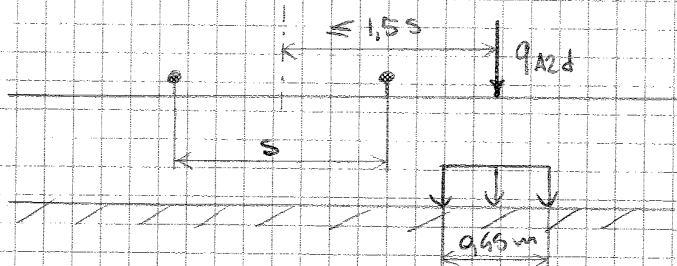
### → BIG ENTITY DERAILMENT

In this case, a distributed load of

$$q_{A2d} = 80 \cdot 1,6 \text{ kNm}^{-1}$$

→ the constant  $1,6$  was already considered in  $q_{A1d}$

is placed for a maximum longitudinal extension of  $20 \text{ m}$  and in every position within the field  $\pm 1,5s$ .

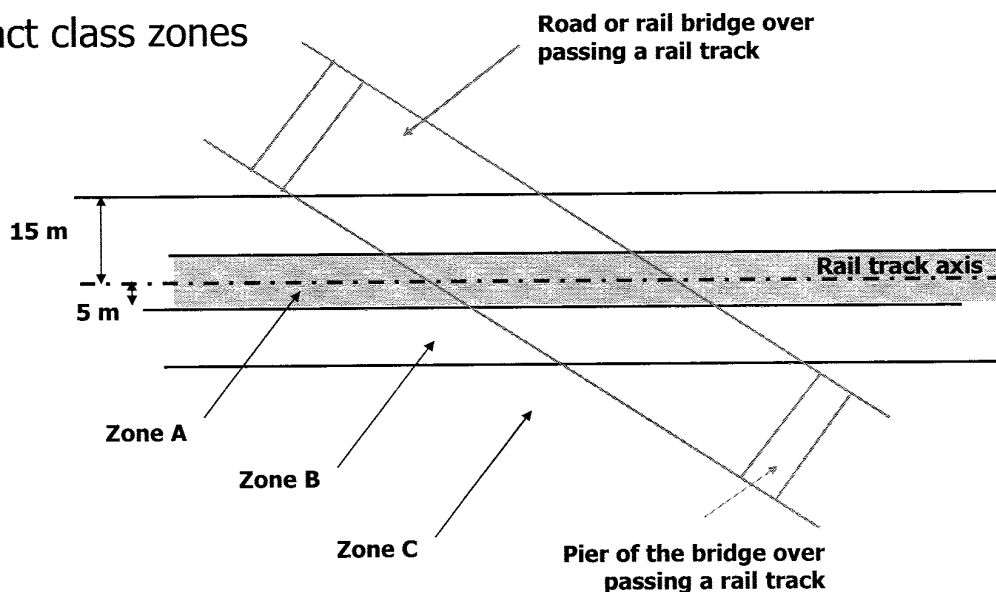



This approach is used only for GLOBAL VERIFICATIONS - e.g. equilibrium of the deck, collapse of the main structure - and severe damage in a portion is accepted, but the collapse must interest only this portion.

**1 Practice & exercises 1: Traffic actions on bridges 109/119**

## Derailment under a road or a rail bridge

Impact class zones



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**1 Practice & exercises 1: Traffic actions on bridges 110/119**

## Derailment under a road or a rail bridge

Impact forces

- Zone A**
  - 4000 kN parallel to the track
  - 1500 kN orthogonal to the track
- Zone B**
  - 2000 kN parallel to the track
  - 750 kN orthogonal to the track
- Zone C**
  - Nothing

**These forces should be considered at 1.80 m from the rail level and not acting simultaneously**

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The two components are not acting simultaneously and they are applied at 1,80 m from the rail level.

## 12 Values of the multi component actions

How can we proceed in case of more than 1 track and different kinds of loads?

Let's start from the NUMBER OF TRACKS TO BE LOADED:

a table is given and it presents several case basing on the number of tracks, the loaded track and the kind of load in relation with the traffic

→ 1 TRACK : in case of normal traffic, we choose the most disadvantageous between LM71 and LM SW/0 and to this we'll apply the coefficient 1,0.  
In case of heavy traffic, we use LM SW/2.

→ 2 TRACKS : in a first case, all tracks are loaded with LM71 or LM SW/0 - we have 2 cases and we choose the worst one.  
In case of heavy traffic, first track is loaded with SW/2 and the second track is loaded with LM71 or SW/0.  
So, we have 4 cases given by permutation of these loads

→ 3 TRACKS : in case a), two tracks are loaded and the other ones are unloaded.  
In case b), we have to apply permutations and a coefficient 0,75 is applied.  
As regards heavy traffic, there is no difference for the case of 2 tracks.

Then, we focus on LOAD GROUPS, that indicate us how to combine horizontal and vertical loads.

- GROUP 1 ⇒ maximum vertical and horizontal actions
- GROUP 2 ⇒ used for lateral stability
- GROUP 3 ⇒ maximum longitudinal actions
- GROUP 4 ⇒ used for crack control

For each group, a table gives the coefficients to be applied in the load combination.

The coefficients in the brackets are the values to be assumed if the action is advantageous, except in group 4 where these terms are applied in case of 2 or  $\geq 3$  tracks.

**1 Practice & exercises 1: Traffic actions on bridges 113/119**

## Values of the multi component actions

Values inside brackets should be considered when the action is favourable.

Group 4 should be considered only for crack control. Value 0.6 should be used for 2 tracks loaded; while value 0.4 should be considered for more than 2 tracks loaded.




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**1 Practice & exercises 1: Traffic actions on bridges 114/119**

## SLU actions safety factors

		Coeff.	EQU <sup>(1)</sup>	A1 STR	A2 GEO	Accidental situation	Seismic situation
Permanent actions	Fav.	$\gamma_{G1}$	0,90	1,00	1,00	1,00	1,00
	Unfav.		1,10	1,35	1,00	1,00	1,00
Non struct. <sup>(2)</sup> permanent actions	Fav.	$\gamma_{G2}$	0,00	0,00	0,00	1,00	1,00
	Unfav.		1,50	1,50	1,30	1,00	1,00
Ballast <sup>(3)</sup>	Fav.	$\gamma_B$	0,90	1,00	1,00	1,00	1,00
	Unfav.		1,50	1,50	1,30	1,00	1,00
Variable traffic loads <sup>(4)</sup>	Fav.	$\gamma_Q$	0,00	0,00	0,00	0,00	0,00
	Unfav.		1,45	1,45	1,25	0,20 <sup>(5)</sup>	0,20 <sup>(5)</sup>
Other variable actions	Fav.	$\gamma_{Qi}$	0,00	0,00	0,00	0,00	0,00
	Unfav.		1,50	1,50	1,30	1,00	0,00
Prestressing	Fav.	$\gamma_P$	0,90	1,00	1,00	1,00	1,00
	Unfav.		1,00 <sup>(6)</sup>	1,00 <sup>(7)</sup>	1,00	1,00	1,00




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**1 Practice & exercises 1: Traffic actions on bridges 115/119**

## SLU actions safety factors

- (1) Equilibrium which is independent from the resistance and deformability characteristics of the ground; otherwise GEO values apply.
- (2) If non structural loads are defined in detail, the same values as for permanent loads may be applied.
- (3) If sensible variations of the ballast load are expected, they should be explicitly taken into account in the design.
- (4) Traffic actions should be treated using the multi component actions shown in slide 112.
- (5) Ratio of traffic load to be taken into account.
- (6) 1.30 for instability in external prestressed structures.
- (7) 1.20 for local effects.




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**1 Practice & exercises 1: Traffic actions on bridges 116/119**

## SLS actions combination factors

Azioni		$\Psi_0$	$\Psi_1$	$\Psi_2$
Single traffic actions	Load on the embankment	0,80	0,50	0,0
	Aerodynamic actions	0,80	0,50	0,0
Multi Comp. actions	gr1	0,80 <sup>(2)</sup>	0,80 <sup>(1)</sup>	0,0
	gr2	0,80 <sup>(2)</sup>	0,80 <sup>(1)</sup>	-
	gr3	0,80 <sup>(2)</sup>	0,80 <sup>(1)</sup>	0,0
	gr4	1,00	1,00 <sup>(1)</sup>	0,0
Wind	F <sub>wk</sub>	0,60	0,50	0,0
Snow	In construction	0,80	0,0	0,0
	SLU and SLE	0,0	0,0	0,0
Temper.	T <sub>k</sub>	0,60	0,60	0,50



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## SLE Verifications for rail-traffic safety

- Horizontal deflection of the deck

(to be calculated with LM71 incremented with dynamic effect,  
Wind, nosing force, centrifugal force, temperature variation)

The curvature radius for single span deck is given by

$$R = \frac{L^2}{8 \delta_h}$$

Where  $\delta_h$  is the horizontal deflection

Speed [km/h]	Maximum angular variation	Minimum curvature radius	
		Single span	>1 span
$V \leq 120$	0,0035 rd	1700 m	3500 m
$120 < V \leq 200$	0,0020 rd	6000 m	9500 m
$200 < V$	0,0015 rd	14000 m	17500 m



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We start from the LOAD ANALYSIS:

the structure will be solved only for the multi-component action group 1 - actually, we should take into account the other groups

$$1 \left\{ \begin{array}{l} \text{Loads in the carriageway} = \text{characteristic value of U1-2-3-4-6} \\ \text{Loads on footway} = \text{uniform load of } 2,5 \text{ kNm}^2 \end{array} \right.$$

### (I) DEAD LOAD $g_1$ .

It is the self-weight of longitudinal beams, transverse beams and the slab.

The codes give us a value of specific weight of reinforced concrete

$$\gamma = 25 \text{ kNm}^{-3}$$

In this way, we can compute self-weight.

→ longitudinal beam

$$\begin{aligned} g_{1,lb} &= \gamma V_{lb} = \\ &= 25 [\text{kNm}^{-3}] \cdot 0,5 [\text{m}] \cdot 1,20 [\text{m}] \cdot 15 [\text{m}] = 225 \text{ kN} \end{aligned}$$

→ transverse beam

$$\begin{aligned} g_{1,tb} &= \gamma V_{tb} = \\ &= 25 [\text{kNm}^{-3}] \cdot 0,3 [\text{m}] \cdot 1,00 [\text{m}] \cdot \underbrace{(3 - 2,50 [\text{m}])}_{= 0,50 [\text{m}]} = \\ &= 56 \text{ kN} \end{aligned}$$

we don't consider 12 m, because a part of it is already counted in longitudinal beams.

→ slab

$$\begin{aligned} g_{1,sl} &= \gamma V_{sl} = \\ &= 25 [\text{kNm}^{-3}] \cdot 12 [\text{m}] \cdot 15 [\text{m}] \cdot 0,25 [\text{m}] = 1125 \text{ kN} \end{aligned}$$

Total weight of the girder is the sum

$$g_{1,tot} = 4 \cdot g_{1,lb} + 4 \cdot g_{1,tb} + g_{1,sl} =$$



$$g_{2, VRS} = \gamma P_{VRS} =$$

$$= 2 \text{ [kNm}^{-1}\text{]} \cdot 15 \text{ [m]} = 30 \text{ kN}$$

→ Pedestrian parapet, represented with a linear load of  $1,0 \text{ kNm}^{-1}$ .

$$g_{2, PP} = \gamma P_{PP} =$$

$$= 1,0 \text{ [kNm}^{-1}\text{]} \cdot 15 \text{ [m]} = 15 \text{ kN}$$

The permanent load on the outermost beam is

$$g_{2, ab} = \frac{1}{l_{ab}} \left( g_{2, k} + g_{2, p} + g_{2, VRS} + g_{2, PP} \right) =$$

$$= \frac{1}{15 \text{ [m]}} \left( 129 \text{ [kN]} + 67,5 \text{ [kN]} + 30 \text{ [kN]} + 15 \text{ [kN]} \right) = 16 \text{ kNm}^{-1}$$

In this calculation, we have assumed that the load of barriers and kerbs is fully given to the outermost beam. Actually, all the longitudinal beams bend according to the same shape and the weight of barriers and kerbs go also in the internal beams. This contribution is neglected, in order to simplify the design. Moreover, in this way, we assume that external beams are more loaded and it means that we'll design bigger beams than necessary - we are on the safe side.

We also notice that permanent load is about 46% of dead load. This is a limit value because

→ in very small concrete bridges, permanent load is less than 50% of dead load

→ in bigger bridges, this ratio goes down because  $g_1$  increases and  $g_2$  doesn't change very much

⇒ we never find a ratio bigger than 0,5

Otherwise, we have probably done a mistake.



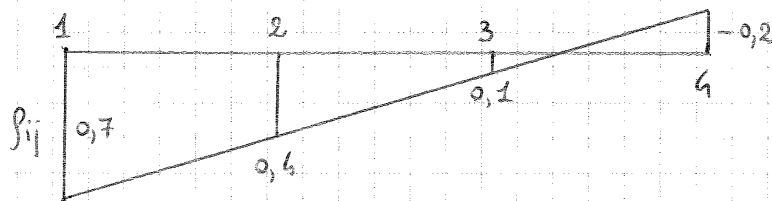
to Betti theorem, we get

$$M \cdot 1 = F \cdot v$$

$v$  = vertical displacement of the point of application

$$\Rightarrow M = Fv$$

On the other side, in this case we have a girder which presents also a transverse repartition of loads. We consider the transverse load repartition according to Courbon theory (case of  $h$  transverse beams)



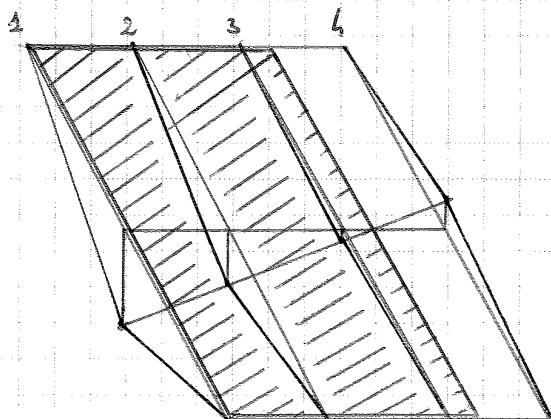
If a load is applied on beam 1, 70% of it will go on beam 1, 60% will go on beam 2, 10% will go on beam 3 and -20% will go on beam 4. This repartition happens because the structure is a girder and, if a load is applied on a beam, all the other beams will help the loaded one.

The coefficients of transverse distribution are called  $\beta_{ij}$  and they have two different meanings, as an effect of Betti theorem

$\rightarrow \beta_{ij}$  = amount of unit load applied on beam  $i$  that goes on beam  $j$  ( $j = 1 \div h$ )

$\rightarrow \beta_{ij}$  = amount of unit load applied on beam  $j$  ( $j = 1 \div h$ ) that goes on beam  $i$ .

By combining the influence line in the longitudinal direction with the transverse repartition, we get the influence surface on the whole deck, that has a certain shape in transverse direction and longitudinal direction.



This surface isn't smooth and has edges and, in order to get a positive bending moment on beam 1, the load should be placed in the area where the influence surface is positive - the dashed area.

Biggest bending moment is possible where there are the edges, because here the influence surface is maximum.

In this case, bending moment is equal to the work associated to the load and the deformed mesh, which corresponds to the load multiplied by the area of the triangle.  
 It is the same as coupling the resultants of the distributed load on each segment with relative displacements.

$$\begin{aligned}
 M_{s,q} &= 2 R_q \delta = \\
 &= 2 \cdot q \frac{p}{2} \cdot \frac{p}{8} = \\
 &= 2 \cdot q \cdot \frac{15}{2} \cdot \frac{15}{8} = 28q
 \end{aligned}$$

In order to define the values of P and q, we have to deal with TRANSVERSE DISTRIBUTION of loads :

as the carriageway is 9 m wide - in this case - according to LM1, the carriageway will be divided into 3 notional lanes and each one will be 3 m wide.  
 Each lane will be loaded according the following table.

Location	Tandem system - axle load	Distributed load
Lane n.º 1	300 kN	9 kNm <sup>-2</sup>
Lane n.º 2	200 kN	2,5 kNm <sup>-2</sup>
Lane n.º 3	100 kN	2,5 kNm <sup>-2</sup>
Other lanes	0 kN	2,5 kNm <sup>-2</sup>

To apply this table to the bridge, we consider the cross section of the bridge with Carbon ~~repartition~~ coefficients diagram.

