NUMERO: 1945A - ANNO: 2016

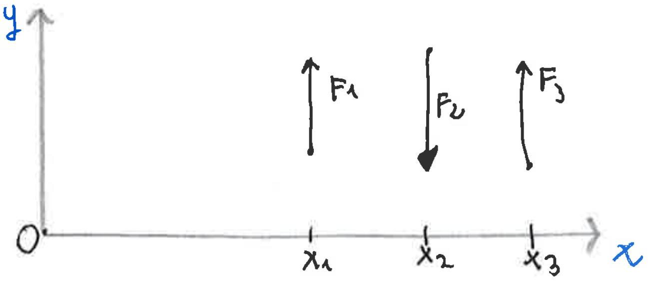
A P P U N T I

STUDENTE: Stoppelli Federico

MATERIA: Fondamenti di meccanica strutturale - (Esercitazioni) - prof. Goglio

FOMS – ESERCITAZIONE 1 –

1)



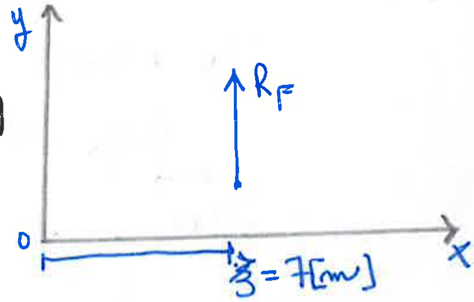
$F_1 = 500$ N
$F_2 = 800$ N
$F_3 = 1000$ N
$x_1 = 5$ m
$x_2 = 7$ m
$x_3 = 8$ m

$R_F, \xi = ?$

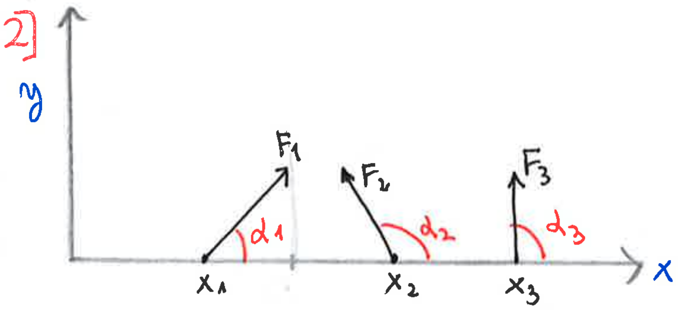
$$R_F = \sum_{i=1}^{m} F_i = F_1 - F_2 + F_3 = 500 - 800 + 1000 = 700 \, [N]$$

$$R_{MO} = \sum_{i=1}^{m} M_i = F_1 \cdot x_1 - F_2 x_2 + F_3 x_3 = 500 \cdot 5 - 800 \cdot 7 + 1000 \cdot 8 = 4900 \, [N \cdot m]$$

$$\xi = \frac{R_{MO}}{R_F} = \frac{4900 \, [N \cdot m]}{700 \, [N]} = 7 \, [m]$$



2)



$F_1 = 100$ N $\quad F_2 = 200$ N $\quad F_3 = 50$ N
$x_1 = 2$ m $\quad x_2 = 3$ m $\quad x_3 = 4$ m
$y_1 = y_2 = y_3 = 0$
$\alpha_1 = 60°$ $\quad \alpha_2 = 135°$ $\quad \alpha_3 = 90°$

- $F_{3x} = F_3 \cos \alpha_3 = 50 \cdot \cos(90°) = 0 \, [N]$

  $F_{3y} = F_3 \sin \alpha_3 = 50 \sin(90°) = 50 \, [N]$

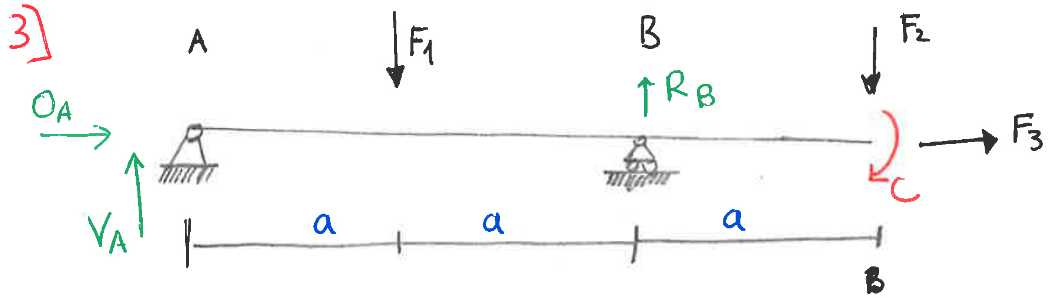
- $F_{1x} = F_1 \cdot \cos \alpha_1 = 100 \cdot \cos(60°) = 50 \, [N]$

  $F_{1y} = F_1 \sin \alpha_1 = 100 \cdot \sin(60°) = 86{,}6 \, [N]$

- $F_{2x} = F_2 \cos(180 - \alpha_2) = 200 \cdot \cos(45°) = 141{,}42 \, [N]$

  $F_{2y} = F_2 \sin(180 - \alpha_2) = 200 \sin(45°) = 141{,}42 \, [N]$

3]



$C = 60$ kN·mm
$F_1 = 2$ kN
$F_2 = 1$ kN
$F_3 = 0{,}5$ kN
$a = 80$ mm

- Reazioni vincolari

$\rightarrow$: $O_A + F_3 = 0$

$\uparrow$: $V_A - F_1 + R_B - F_2 = 0$

A ↻ : $F_1 \cdot a - R_B \cdot 2a + F_2 \cdot 3a + C = 0$

$O_A = -F_3 \qquad |O_A| = |F_3| = 0{,}5$ kN

$$R_B = \frac{F_1 \cdot a}{2a} + \frac{F_2 \cdot 3a}{2a} + \frac{C}{2a} = \frac{F_1}{2} + 3\frac{F_2}{2} + \frac{C}{2a}$$

$$V_A = F_1 + F_2 - R_B = F_1 + F_2 - \frac{F_1}{2} - \frac{3}{2}F_2 - \frac{C}{2a} =$$

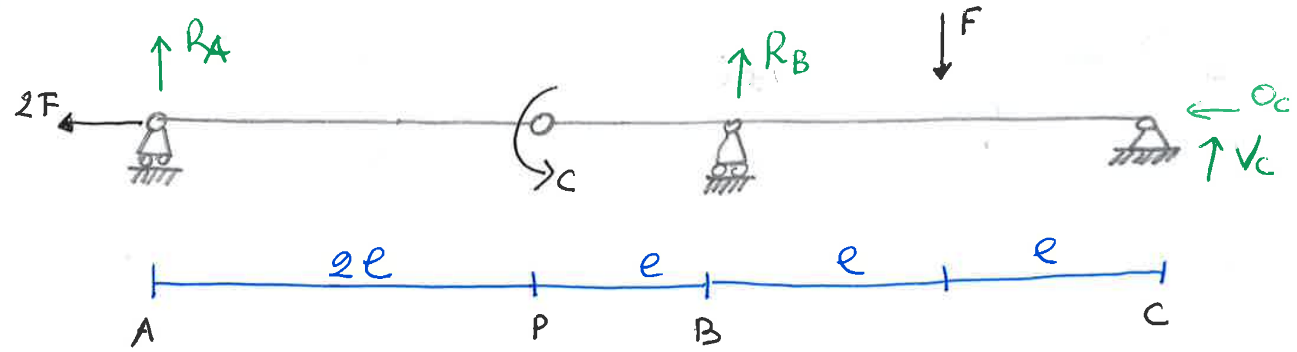
$$= \frac{1}{2}F_1 - \frac{1}{2}F_2 - \frac{C}{2a}$$

$$R_B = \frac{2}{2} + \frac{3 \cdot 1}{2} + \frac{60}{2 \cdot 80} = 2{,}875 \text{ [kN]}$$

$$V_A = \frac{2}{2} - \frac{1}{2} - \frac{60}{2 \cdot 80} = 0{,}125 \text{ [kN]}$$

FOMS - ESERCITAZIONE 2 -

1)



$F = 2000\ N$

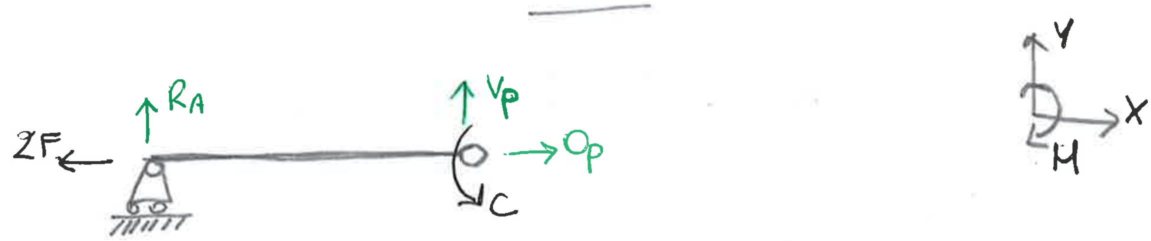
$C = 1000\ N \cdot m$

$\ell = 0{,}5\ m$

$\rightarrow: \quad -2F - O_C = 0 \qquad O_C = -2F \qquad |O_C| = 2F$

$\uparrow: \quad R_A + R_B - F + V_C = 0$

$C \circlearrowright: \quad -R_A \cdot 5\ell - C - R_B \cdot 2\ell + F \cdot \ell = 0$

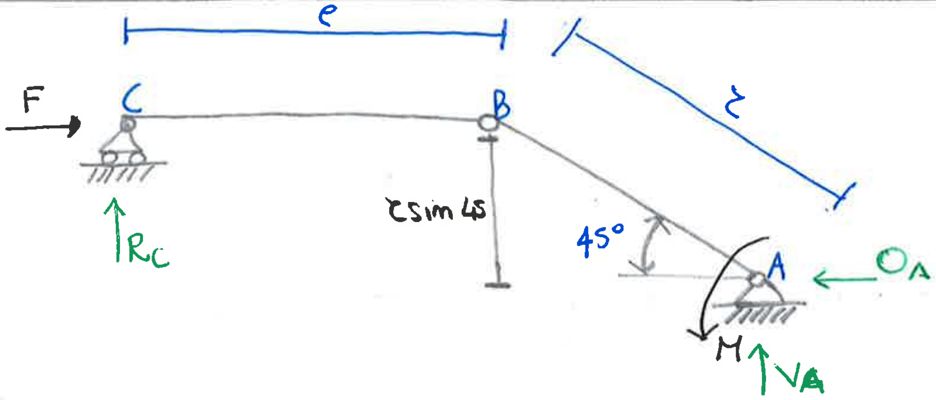


$\rightarrow: \quad O_P - 2F = 0 \qquad O_P = 2F$

$\uparrow: \quad R_A + V_P = 0 \qquad R_A = -V_P = \frac{C}{2\ell}$

$A \circlearrowright: \quad -V_P \cdot 2\ell - C = 0 \qquad V_P = -\frac{C}{2\ell} \qquad |V_P| = \frac{C}{2\ell}$

2]

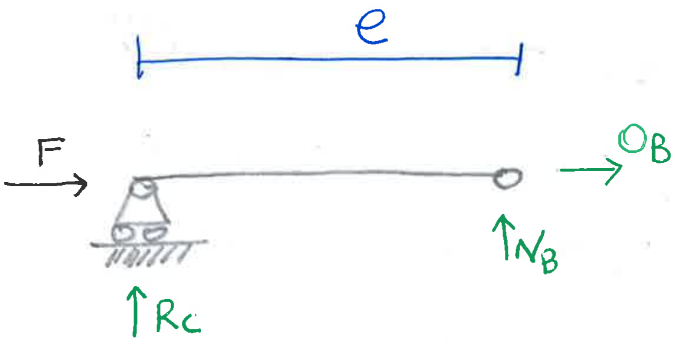


$\rightarrow$: $F - O_A = 0$ $\qquad O_A = F$

$\uparrow$: $R_C + V_A = 0$ $\qquad V_A = -R_C$

$\circlearrowleft A$: $M - F \cdot z \sin 45° = 0 \Rightarrow M = F \cdot z \sin 45° =$

$= F \cdot z \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{F \cdot z \cdot 2}{2\sqrt{2}} = \frac{F \cdot z}{\sqrt{2}}$



$\rightarrow$ $F + O_B = 0$ $\qquad O_B = -F \qquad |O_B| = F$

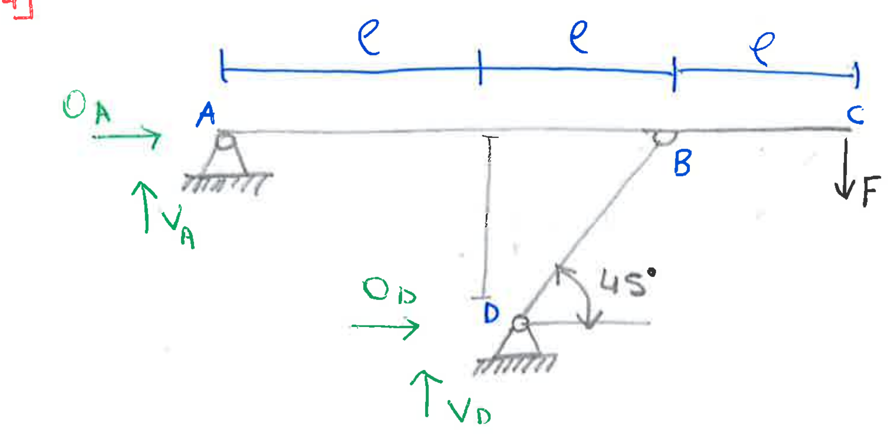
$\uparrow$ $R_C + V_B = 0$

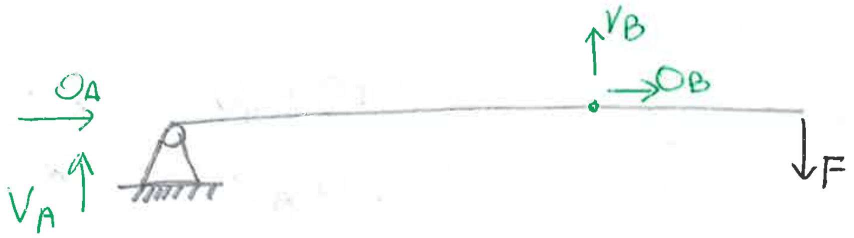
$\circlearrowright$ $-V_B \cdot e = 0$ $\qquad V_B = 0 \Rightarrow R_C = 0$

$V_A = -R_C = 0$

4

4





$\uparrow: \quad V_A + V_B - F = 0$

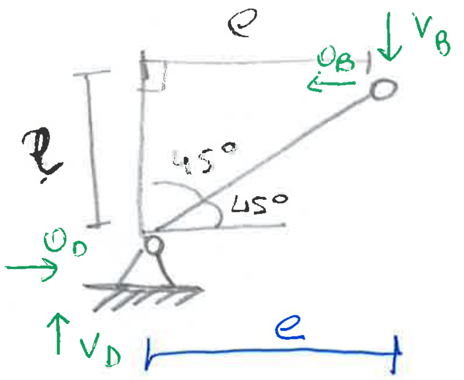
$\rightarrow: \quad O_A + O_B = 0$

$A): \quad F \cdot 3e - V_B \cdot 2e = 0$

$V_A = F - V_B = F - F\frac{3}{2} = F\left(1 - \frac{3}{2}\right) = -\frac{1}{2}F \qquad |V_A| = \frac{1}{2}F$

$V_B = \frac{F \cdot 3e}{2e} = \frac{3}{2}F$

$O_A = -O_B \qquad |O_A| = O_B$



$\uparrow: \quad V_D - V_B = 0 \qquad V_B = V_D = \frac{3}{2}F$

$\rightarrow: \quad O_D - O_B = 0 \qquad O_D = O_B$

$D): \quad V_B \cdot e - O_B \cdot e = 0 \qquad O_B = \frac{V_B e}{e} = V_B$

5

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \end{bmatrix} \cdot \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix} = \begin{bmatrix} 250 \\ 310 \\ -180 \end{bmatrix} \cdot \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 250 \\ 0 \\ 0 \end{Bmatrix}$$

$$\sigma = \sqrt{(\sigma_{xx} \cdot n_x)^2 + (\sigma_{yy} \cdot n_y)^2 + (\sigma_{zz} \cdot n_z)^2} =$$

$$= \sqrt{(250 \cdot 1)^2 + (310 \cdot 0)^2 + (-180 \cdot 0)^2} =$$

$$= 250 \text{ MPa}$$

$$\begin{bmatrix} \tau_{xy} & \tau_{xy} & \tau_{xz} \\ \tau_{xz} & \tau_{zx} & \tau_{yz} \end{bmatrix} \cdot \begin{Bmatrix} n_x \\ n_y \\ n_y \end{Bmatrix} = \begin{bmatrix} -90 & -90 & 110 \\ 110 & 0 & 0 \end{bmatrix} \cdot \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} = \begin{pmatrix} -90 \\ 110 \\ 0 \end{pmatrix}$$

$$= -90 \cdot 1 + (-90) \cdot 0 + 110 \cdot 0 + 110 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 =$$

$$= -90 + 110 + 0$$

$$\tau = \sqrt{\tau_x^2 + \tau_y^2 + \tau_z^2} = \sqrt{(-90)^2 + (110)^2 + 0^2} = 142{,}13 \text{ MPa}$$

$\tau_{xy} < 0$

$\sigma_{xx} < \sigma_{yy}$

$45° < \alpha < 90°$

$$|\tan 2\alpha| = \left| \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}} \right|$$

$$2\alpha = \operatorname{arctg}\left(\frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}} \right) =$$

$$= \operatorname{arctg}\left(\frac{2 \cdot (-150)}{120 - 240} \right) = 68{,}19°$$

$$\alpha = \frac{1}{2}\, 68{,}19° = 34{,}099°$$

$$\alpha^* = 90 - 34{,}099 \cong 56°$$

$$\tau_{max} = \frac{(\sigma_1 - \sigma_3)}{2} = \frac{408 - 18}{2} = 195 \text{ [MPa]}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{165 - (-15)}{2} = 90 \, [MPa]$$

$$\tan 2\alpha = \left| \frac{2 \tau_{xz}}{\sigma_{xx} - \sigma_{zz}} \right|$$

$$2\alpha = \operatorname{arctg}\left(\frac{2 \cdot 50}{0 - 150} \right) = -33{,}7^\circ \simeq -34^\circ$$

$$\alpha = -17^\circ$$ ← ok non si cambia

$$-45^\circ < \alpha < 0^\circ$$

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$$\tan 2\alpha = \left| \frac{2\,\tau_{yz}}{\sigma_{yy} - \sigma_{zz}} \right|$$

$$2\alpha = \tan^{-1} \left| \frac{2 \cdot (100)}{350 - 170} \right| = 48^\circ \qquad 0 < \alpha < 45^\circ$$

~~α = 24,8°~~ ← ~~considerare deve essere 0 < α < 45°~~

$\alpha^* = 24^\circ$ → va bene perché compreso

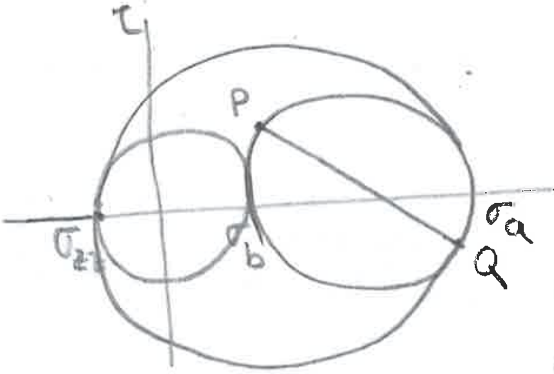
~~α* = 45 - 24,8°~~

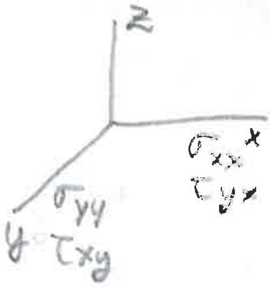
(b) trovare le tensioni e le deformazioni principali

$\tau_{xz} = 0 \quad \tau_{yz} = 0$ z è principale

$\tau_{xy} < 0$

$\sigma_{yy} > \sigma_{xx}$





$45° \leq \alpha \leq 90°$

$P(\sigma_{xx}, -\tau_{xy})$

$Q(\sigma_{yy}, \tau_{xy})$

$P(280, 160)$

$Q(320, -160)$

$$\sigma_{a,b} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} =$$

$$= \frac{280 + 320}{2} \pm \sqrt{\left(\frac{280 - 320}{2}\right)^2 + (120)^2} \Rightarrow$$

$$\tan 2\alpha = \left|\frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}\right|$$

$$2\alpha = \tan^{-1}\left|\frac{2 \cdot (-120)}{280 - 320}\right| = 80{,}5°$$

$\alpha = 40{,}27°$ non compreso

$\alpha^* = 90 - 40{,}27° = 49{,}7°$

$\sigma_a = 300 + 121{,}66 = 421{,}66$

$\sigma_b = 300 - 121{,}66 = 178{,}35$

$\sigma_1 = \sigma_a = 421{,}66 \quad [N/mm^2]$

$\sigma_2 = \sigma_b = 178{,}35$

$\sigma_3 = \sigma_{zz} = -160$

$$\varepsilon_1 = +\frac{1}{E}\sigma_1 - \frac{\nu}{E}\sigma_2 - \frac{\nu}{E}\sigma_3 + \cancel{\alpha \Delta T} = \frac{1}{6{,}8 \cdot 10^4}\left(421{,}66 - 0{,}31 \cdot 178{,}66 - 0{,}31 \cdot (-160)\right) =$$

$$= 6{,}16 \cdot 10^{-3}$$

$$\varepsilon_2 = -\frac{\nu}{E}\sigma_1 + \frac{1}{E}\sigma_2 - \frac{\nu}{E}\sigma_3 = \frac{1}{6{,}8 \cdot 10^4}\left(-0{,}31 \cdot 421{,}66 + 178{,}66 - 0{,}31 \cdot (-160)\right) =$$

$$= 1{,}43 \cdot 10^{-3}$$

$$\varepsilon_3 = -\frac{\nu}{E}\sigma_1 - \frac{\nu}{E}\sigma_2 + \frac{1}{E}\sigma_3 + \cancel{\alpha \Delta T} = \frac{1}{6{,}8 \cdot 10^4}\left(-0{,}31 \cdot 421{,}66 - 0{,}31 \cdot 178.35 - 160\right) =$$

$$= -5{,}089 \cdot 10^{-3}$$

~~$\varphi = \frac{1}{2}\tau_{xy}\gamma_{xy} = \frac{1}{2} \cdot (-120) \cdot (-4{,}623 \cdot 10^{-3}) =$~~

3) Prova di trazione su una provetta a sezione rettangolare 4×10 mm. ACCIAIO serie Sxxx con risultati della prova seguenti:

$F_{eH} = 12$ kN
$F_m = 17,6$ kN
$L_u = 45$ mm

Calcolare R_{eH}, R_m e A e individuare il materiale.

$$R_{eH} = \frac{F_{eH}}{S_0} = \frac{12 \text{ kN}}{40 \text{ mm}^2} = \frac{12 \cdot 10^3 \text{ N}}{40 \cdot 10^{-6} \text{ m}^2} = 300 \text{ [MPa]}$$

$$S_0 = 4 \times 10 \text{ mm}^2 = 40 \text{ mm}^2$$

$$R_m = \frac{F_m}{S_0} = \frac{17,6 \cdot 10^3 \text{ N}}{40 \cdot 10^{-6} \text{ m}^2} = 440 \text{ [MPa]}$$

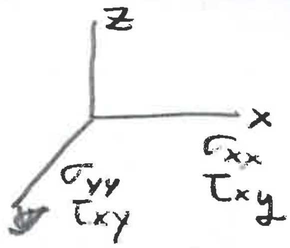
$$L_0 = 5,65\sqrt{S_0} = 5,65 \cdot \sqrt{40} = 35,73 \text{ mm}^2$$

$$A = \frac{L_u - L_0}{L_0} \cdot 100 = \frac{45 - 35,73}{35,73} \cdot 100 = 25,9\%$$

b

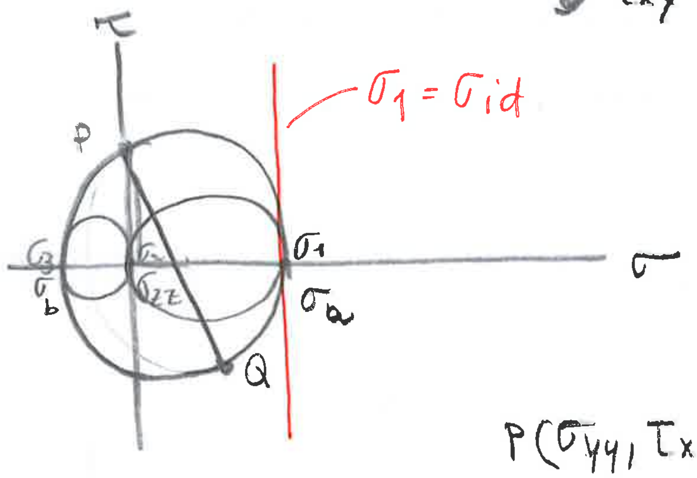
$\sigma_{xx} = 40$ MPa $\quad \sigma_{yy} = 0 \quad \sigma_{zz} = 0$

$\tau_{xy} = 150$ MPa $\quad$ Z è principale



$\tau_{xy} > 0$

$\sigma_{xx} > \sigma_{yy}$



$P(\sigma_{yy}, \tau_{xy})$

$Q(\sigma_{xx}, -\tau_{xy})$

$P(0, 150)$

$Q(40, 150)$

$$\sigma_{a,b} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + (\tau_{yx})^2} =$$

$$= \frac{40 + 0}{2} \pm \sqrt{\left(\frac{40}{2}\right)^2 + (150)^2}$$

$\sigma_a = 20 + 151{,}34 = 171{,}34$

$\sigma_b = 20 - 151{,}34 = -131{,}34$

$\sigma_{id} = 171{,}34$ [MPa]

Caso: Energia di distorsione

$$\sigma_{id} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}$$

$\sigma_1 = 180,26$ come prima

$\sigma_2 = 0$

$\sigma_3 = -210,26$ come prima

$$\sigma_{id} = \frac{1}{\sqrt{2}} \cdot \sqrt{\sigma_1^2 + (-\sigma_3)^2 + (\sigma_1 - \sigma_3)^2} =$$

$$= \frac{1}{\sqrt{2}} \sqrt{(180,26)^2 + (+210,26)^2 + (180,26 + 210,26)^2} =$$

$$= 338,53 \text{ MPa}$$

$$C_S = \frac{\sigma_{lim}}{\sigma_{id}}$$

$$\sigma_{lim} = C_S \cdot \sigma_{id} = 1,5 \cdot 338,53 = 507,8 \text{ [MPa]}$$

7] Un componente in acciaio da bonifica è sottoposto, nel punto più sollecitato, alle tensioni

$\sigma_{xx} = 0$ MPa $\tau_{xy} = 0$

$\sigma_{yy} = 0$ MPa $\tau_{xz} = 90$

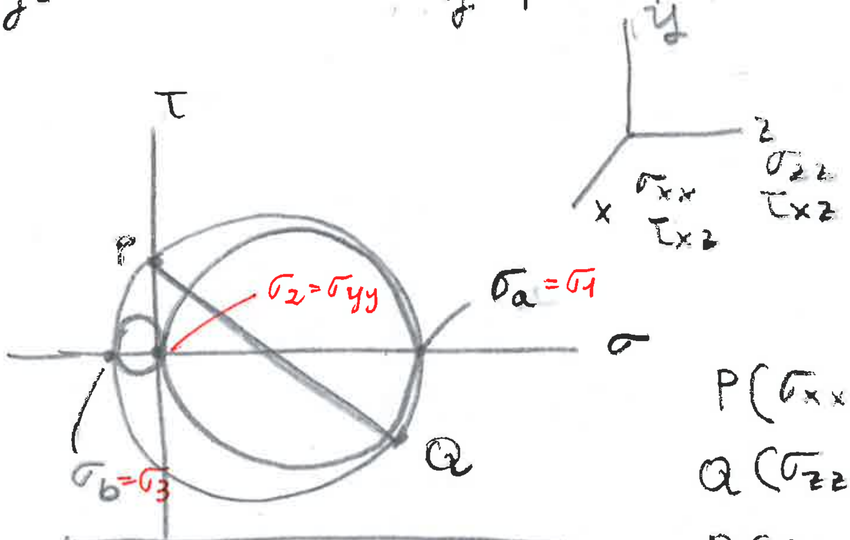
$\sigma_{zz} = 160$ MPa $\tau_{yz} = 0$

Scegliere un materiale adatto per realizzare un coeff. di sicurezza di almeno 1,5 rispetto allo snervamento.

$\tau_{xy} = 0$ $\tau_{yz} = 0$ y principale

$\tau_{xz} > 0$

$\sigma_{zz} > \sigma_{xx}$



$P(\sigma_{xx}, \tau_{xz})$

$Q(\sigma_{zz}, -\tau_{xz})$

$P(0, 90)$

$Q(160, -90)$

$$\sigma_{a,b} = \frac{\sigma_{xx} + \sigma_{zz}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{zz}}{2}\right)^2 + \tau_{xz}^2}$$

$$\sigma_{a,b} = \frac{0 + 160}{2} \pm \sqrt{\left(\frac{0 - 160}{2}\right)^2 + (90)^2}$$

$\sigma_a = 80 + 120,42 = 200,42 = \sigma_1$

$\sigma_b = 80 - 120,42 = -40,41 = \sigma_3$

$\sigma_{yy} = \sigma_2 = 0$

$\sigma_{id} = \sigma_1 - \sigma_3 = 200,42 - (-40,41) = 240,83$

$\sigma_{lim} = C_S \cdot \sigma_{id} = 1,5 \cdot 240,83 = 361,25$ [MPa]

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(b) Considerando i riferimenti con origine in A, determinare quello principale $\alpha(p_1^A p_2^A)$ e valutare i relativi momenti di inerzia.

riferimento non baricentrico $Axy \Rightarrow J_{xy} \neq 0$

$$J_{xx}^{(A)} = y_G^2 A + J_{\xi\xi}^{(G)} = 5^2(200) + 1666,66 = 6666,667$$

$$J_{yy}^{(A)} = x_G^2 A + J_{\eta\eta}^{(G)} = 10^2(200) + 6666,66 = 26666,67$$

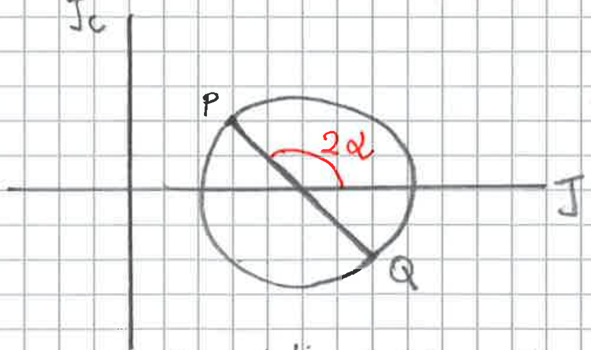
$$J_{xy}^{(A)} = x_G y_G A = 10 \cdot 5 \cdot 200 = 10000 \ mm^4$$

$J_{xy}^A > 0$ $\quad J_{xx}^A < J_{yy}^A$

$P(J_{xx}, J_{xy})$

$Q(J_{yy}, -J_{xy})$

$45° \leq \alpha < 90°$



$$\alpha = \frac{1}{2} \tan^{-1}\left|\frac{2J_{xy}}{J_{xx}-J_{yy}}\right| = \frac{1}{2}\tan^{-1}\left|\frac{2 \cdot 10000}{6666,66 - 26666,66}\right| = 22,5° \text{ non compreso}$$

$$\alpha^* = 90 - 22,5 = 67,5° \qquad \alpha^*(p_1^A p_2^A) = 67,5°$$

$$J_{1,2} = \frac{J_{xx}+J_{yy}}{2} \pm \sqrt{\left(\frac{J_{xx}-J_{yy}}{2}\right)^2 + J_{xy}^2}$$

$$J_{1,2} = \frac{6666,67 + 26666,67}{2} \pm \sqrt{\left(\frac{6666,67 - 26666,67}{2}\right)^2 + (10000)^2}$$

$$J_1^A = 16666,67 + 14142,13 = 30808 \ mm^4$$

$$J_2^A = 16666,67 - 14142,13 = 2524,5 \ mm^4$$

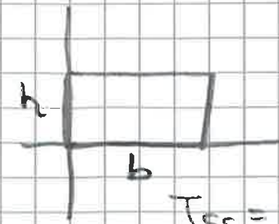
Momento d'inerzia nel riferimento baricentrico Gxy

nel riferimento baricentrico le coordinate sono diverse

$x_1 = X_1 - X_G = 0$

$x_2 = X_2 - X_G = 0$

$x_3 = X_3 - X_G = 0$



$J_{\xi\xi} = \frac{b h^3}{12}$

$J_{\eta\eta} = \frac{b^3 h}{12}$

$y_1 = Y_1 - Y_G = 240 - 108 = 132$

$y_2 = Y_2 - Y_G = 125 - 108 = 17$

$y_3 = Y_3 - Y_G = 10 - 108 = -98$

$$J_{xx}^{(G)} = \sum_i \left(y_i^2 A_i + J_{\xi_1\xi_1} \right) =$$

$$= y_1^2 A_1 + J_{\xi_1\xi_1} + y_2^2 A_2 + J_{\xi_2\xi_2} + y_3^2 A_3 + J_{\xi_3\xi_3} =$$

$$= 132^2 \cdot 2600 + \frac{130 \cdot (20)^3}{12} + 17^2 \cdot 6300 + \frac{30 \cdot 210^3}{12} +$$

$$+ (-98)^2 \cdot 4600 + \frac{230 \cdot 20^3}{12} = 11{,}47 \cdot 10^7 \, [mm^4]$$

$$J_{yy}^{(G)} = \sum_i \left(A_i x_i^2 + J_{\eta_i\eta_i} \right) =$$

$$= \underbrace{x_1^2 A_1}_{=0} + J_{\eta_1\eta_1} + \underbrace{x_2^2 A_2}_{=0} + J_{\eta_2\eta_2} + \underbrace{x_3^2 A_3}_{=0} + J_{\eta_3\eta_3} =$$

$$= \frac{130^3 \cdot 20}{12} + \frac{30^3 \cdot 210}{12} + \frac{230^3 \cdot 20}{12} = 2{,}44 \cdot 10^7 \, [mm^4]$$

Momenti d'inerzia rispetto a Gxy

$x_1 = X_1 - X_G = 2 - 26{,}2 = -24{,}2$

$x_2 = X_2 - X_G = 24 - 26{,}2 = -2{,}2$

$x_3 = X_3 - X_G = 46 - 26{,}2 = 19{,}8$

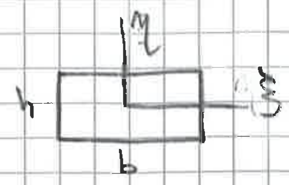
$y_1 = Y_1 - Y_G = 30 - 34 = -4$

$y_2 = Y_2 - Y_G = 30 - 34 = -4$

$y_3 = Y_3 - Y_G = 40 - 34 = 6$

$J_{\eta\eta} = \frac{b^3 h}{12}$

$J_{\xi\xi} = \frac{b h^3}{12}$



$$J_{xx}^{(G)} = \sum_i (y_i^2 A_i + J_{\xi_i\xi_i}) =$$

$$= (-4)^2 \cdot 240 + \frac{4(60)^3}{12} + (-4)^2 \cdot 240 + \frac{40 \cdot 6^3}{12} + (6)^2 \cdot 320 + \frac{4 \cdot (80)^3}{12} =$$

$$= 2{,}626 \cdot 10^5 \ [mm^4]$$

$$J_{yy}^{(G)} = \sum_i (x_i^2 A_1 + J_{\eta_i\eta_i}) =$$

$$= (-24{,}2)^2 \cdot 240 + \frac{60 \cdot 4^3}{12} + (-2{,}2)^2 \cdot 240 + \frac{6 \cdot 40^3}{12} + (19{,}8)^2 \cdot 320 +$$

$$+ \frac{80 \cdot 4^3}{12} = 2{,}999 \cdot 10^5 \ [mm^4]$$

$$J_{xy}^{(G)} = \sum_i (x_i y_i A_i + J_{\xi_i\eta_i}) =$$ in rif baricentrico $J_{\eta\xi} = 0$

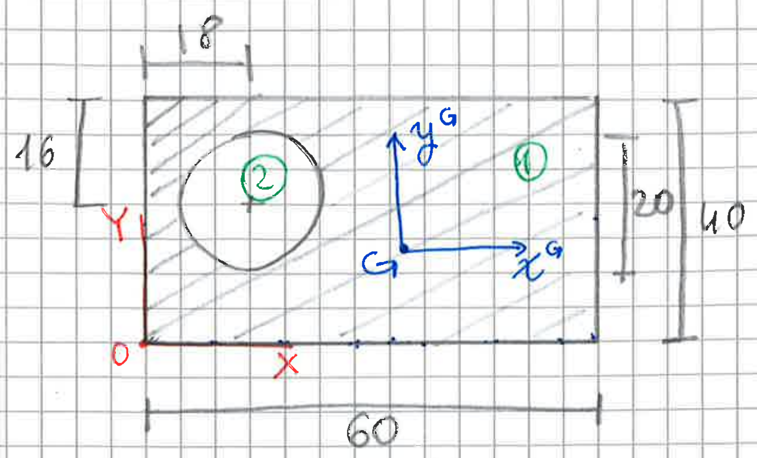
$$= x_1 y_1 A_1 + x_2 y_2 A_2 + x_3 y_2 A_3 =$$

$$= (-24{,}2) \cdot (-4) \cdot 240 + (-2{,}2)(-4)\, 240 + 19{,}8 \cdot 6 \cdot 320 =$$

$$= 63360 \ [mm^4] = 0{,}634 \cdot 10^5 \ [mm^4]$$

continua...

4]



- posizione del baricentro

$A_1 = 60 \cdot 40 = 2400 \ mm^2$

$A_2 = \frac{\pi}{4}(20)^2 = 314,159 \ mm^2$

Rispetto a OXY

$S_{Y1} = A_1 \cdot X_1 = 2400 \cdot 30 = 72000$

$S_{Y2} = A_2 \cdot X_2 = 314,159 \cdot 18 = 5654,87$

$S_{X1} = A_1 \cdot Y_1 = 2400 \cdot 20 = 48000$

$S_{X2} = A_2 \cdot Y_2 = 314,159 \cdot 24 = 7539,8$

$$X_G = \frac{\sum S_{Yi}}{\sum A_i} = \frac{S_{Y1} - S_{Y2}}{A_1 - A_2} = \frac{72000 - 5654,87}{2400 - 100\pi} = 31,8$$

$$Y_G = \frac{\sum_i S_{Xi}}{\sum A_i} = \frac{S_{X1} - S_{X2}}{A_1 - A_2} = \frac{48000 - 7539,8}{2400 - 100\pi} = 19,4$$

Momenti d'inerzia rispetto a Gxy

$x_1 = X_1 - X_G = 30 - 31,8 = -1,8$ $\quad y_1 = Y_1 - Y_G = 20 - 19,4 = 0,6$

$x_2 = X_2 - X_G = 18 - 31,8 = -13,8$ $\quad y_2 = Y_2 - Y_G = 24 - 19,4 = 4,6$

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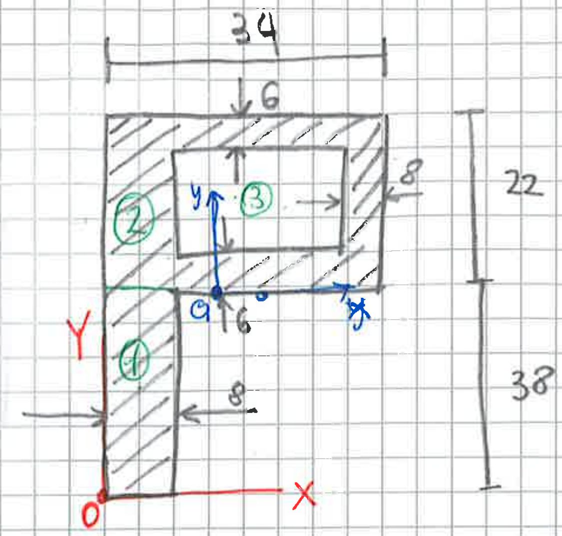
5]



- Posizione del Baricentro

$Y_1 = 19$ ✓ $X_1 = 4$ ✓ $A_1 = 8 \cdot 38 = 304$

$Y_2 = 49$ ✓ $X_2 = 17$ ✓ $A_2 = 34 \cdot 22 = 748$

$Y_3 = 49$ ✓ $X_3 = 17$ ✓ $A_3 = 26 \cdot 10 = 260$

$S_{Y1} = X_1 \cdot A_1 = 1216$ $S_{X1} = Y_1 \cdot A_1 = 5776$

$S_{Y2} = X_2 \cdot A_2 = 12716$ $S_{X2} = Y_2 \cdot A_2 = 36652$

$S_{Y3} = X_3 \cdot A_3 = 4420$ $S_{X3} = Y_3 \cdot A_3 = 12740$

$$X_G = \frac{\sum_i S_{Yi}}{\sum_i A_i} = \frac{S_{Y1} + S_{Y2} - S_{Y3}}{A_1 + A_2 - A_3} =$$

$$= \frac{1216 + 12716 - 4420}{304 + 748 - 260} = 12,01$$

$$Y_G = \frac{\sum_i S_{xi}}{\sum_i A_i} = \frac{5776 + 36652 - 12740}{304 + 748 - 260} = 37,48$$

In riferimento baricentrico: Gxy

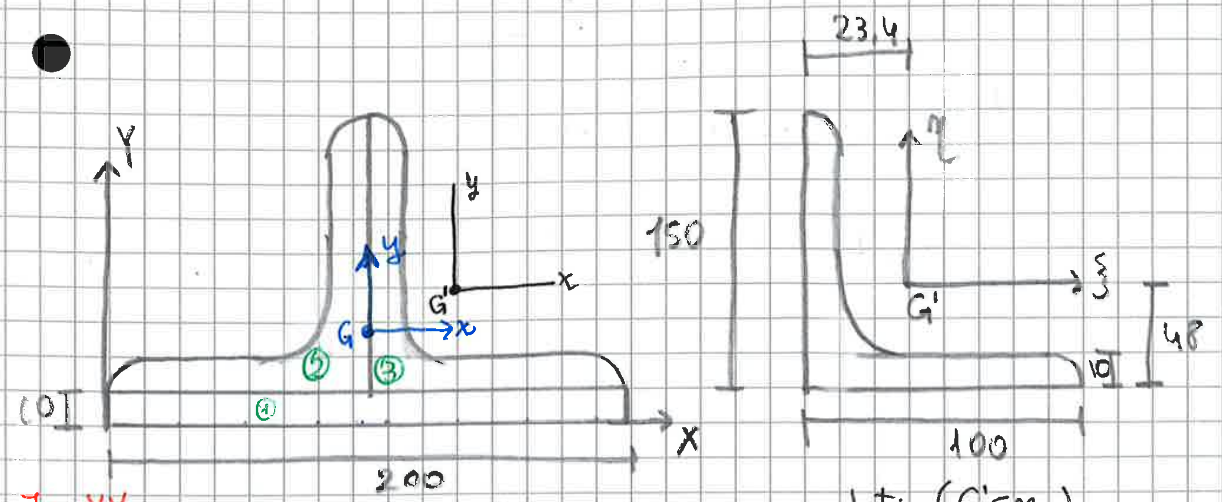
$x_1 = X_1 - X_G = 4 - 12,01 = -8,01$ $y_1 = Y_1 - Y_G = 19 - 37,48 = -18,48$

$x_2 = X_2 - X_G = 17 - 12,01 = 4,99$ $y_2 = Y_2 - Y_G = 49 - 37,48 = 11,52$

$x_3 = X_3 - X_G = 17 - 12,01 = 4,99$ $y_3 = Y_3 - Y_G = 49 - 37,48 = 11,52$

20

6] • posizione del baricentro
• momenti di inerzia nel sistema centrale principale.



In XY

$A_1 = 200 \cdot 10 = 2000$

$X_1 = 100$

$Y_1 = 5$

dati ($G'\xi\eta$)

$A = 2.42 \cdot 10^3$ mm²

$J_{\xi\xi} = 5{,}52 \cdot 10^6$ mm⁴

$J_{\eta\eta} = 1{,}98 \cdot 10^6$ "

$J_{\xi\eta} = -1{,}91 \cdot 10^6$ "

$A_2 = A_3 = 2{,}42 \cdot 10^3$

$X_2 = 100 - 23{,}4 = 76{,}6$ $X_3 = 100 + 23{,}4 = 123{,}4$

$Y_2 = 48 + 10 = 58$ $Y_3 = 58$

$S_{x1} = A_1 \cdot Y_1 = 10000$ $S_{y1} = A_1 \cdot X_1 = 200000$

$S_{x2} = A_2 \cdot Y_2 = 140360$ $S_{y2} = A_2 \cdot X_2 = 185372$

$S_{x3} = A_3 \cdot Y_3 =$ ~~[illegible]~~ $\downarrow =$ $S_{y3} = A_3 \cdot X_3 = 298628$

$$X_G = \frac{S_{y1} + S_{y2} + S_{y3}}{A_1 + A_2 + A_3} = \frac{200000 + 185372 + 298628}{2000 + 2420 + 2420} = 100 \quad ✓$$

$$Y_G = \frac{S_{x1} + S_{x2} + S_{x3}}{A_1 + A_2 + A_3} = \frac{10000 + 140360 + 140360}{2000 + 2420 + 2420} = 42{,}5 \quad ✓$$

$G(100; 42{,}5)$

$x_1 = X_1 - X_G = 100 - 100 = 0$ $y_1 = Y_1 - Y_G = 5 - 42{,}5 = -37{,}5$

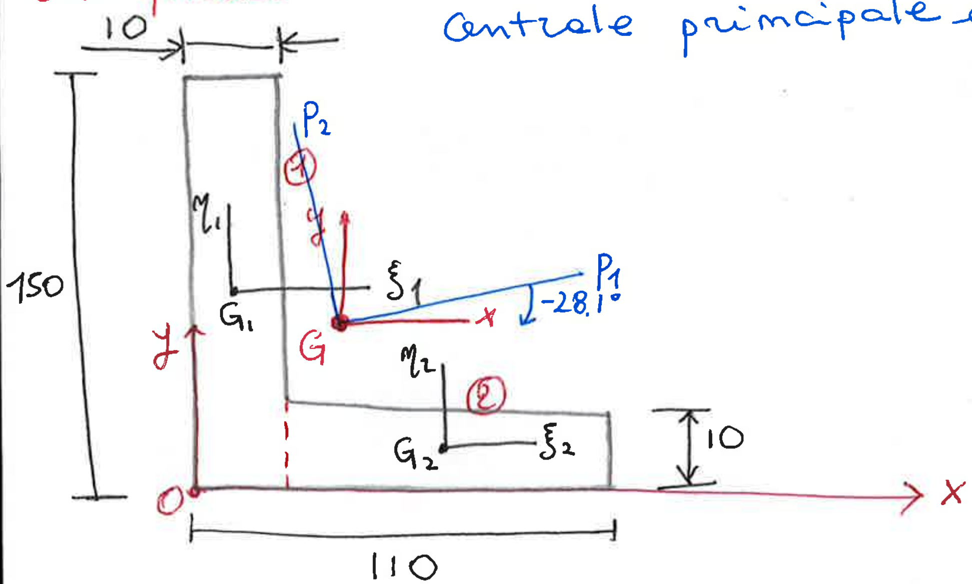
$x_2 = X_2 - X_G = 76{,}6 - 100 = -23{,}4$ $y_2 = Y_2 - Y_G = 58 - 42{,}5 = 15{,}5$

$x_3 = X_3 - X_G = 123{,}4 - 100 = 23{,}4$ $y_3 = Y_3 - Y_G = 58 - 42{,}5 = 15{,}5$

29/10/2015 (3)

Caso pratico

Determinare riferimento centrale principale e J_1, J_2.



$X_1 = 5$ mm $\quad Y_1 = 75$

$X_2 = 60 \quad Y_2 = 5$

$A_1 = 150 \cdot 10 = 1500$ mm^2

$A_2 = 100 \cdot 10 = 1000$ mm^2

$A_{TOT} = 2500$ mm^2

Momenti statici secondo OXY

$$S_x = A_1 \cdot Y_1 + A_2 \cdot Y_2 = 1500 \cdot 75 + 1000 \cdot 5 = 1.175 \cdot 10^5 \text{ mm}^3$$

$$S_Y = A_1 X_1 + A_2 X_2 = 1500 \cdot 5 + 1000 \cdot 60 = 6.75 \cdot 10^4 \text{ mm}^3$$

$$X_G = \frac{S_Y}{A_{TOT}} = \frac{6.75 \cdot 10^4}{2500} = 27{,}0 \text{ mm}$$

$$Y_G = \frac{S_x}{A_{TOT}} = \frac{1.175 \cdot 10^5}{2500} = 47.0 \text{ mm}$$

coordinate del Baricentro

Gxy riferimento Baricentrico

$x_1 = X_1 - X_G = 5 - 27 = -22 \qquad x_2 = X_2 - X_G = 60 - 27 = 33$

$y_1 = Y_1 - Y_G = 75 - 47 = 28 \qquad y_2 = Y_2 - Y_G = 5 - 47 = -42$

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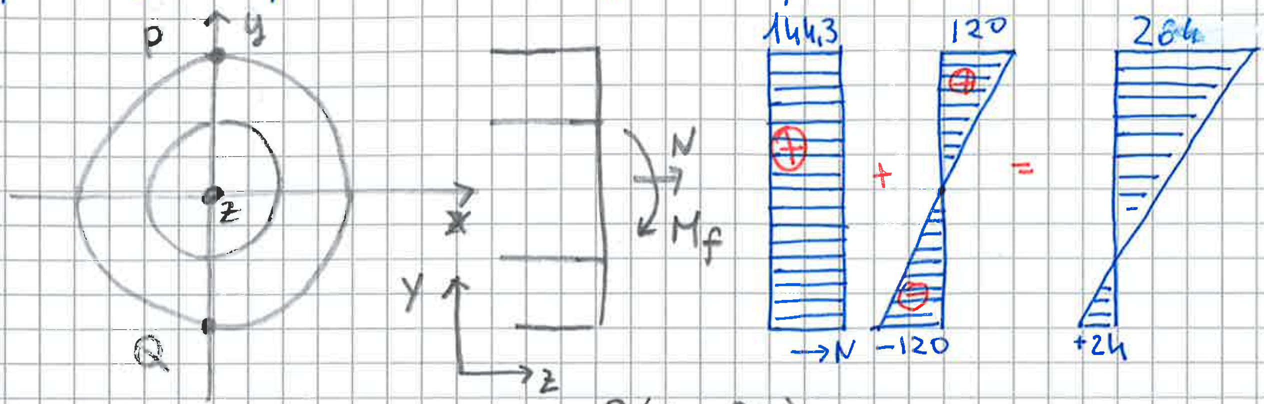
FOMS - ESERCITAZIONE 6 -

1) La sezione di un albero avente $d = 10$ e $D = 40$ è soggetta al momento flettente $M_f = 750$ kNmm e alla forza normale $N = 170$ kN.

- Tracciare il diagramma della tensione normale σ_{zz} agente sulla sezione e calcolare il coefficiente di sicurezza rispetto alla plasticizzazione per il punto più sollecitato.



$P(0, 20)$

$Q(0, -20)$

estensione

$$\sigma_{zz} = \frac{N}{A}$$

$$A = \frac{\pi}{4}D^2 - \frac{\pi}{4}d^2 = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(40^2 - 10^2) = 1178{,}09 \text{ mm}^2$$

$$\sigma_{zz} = \frac{N}{A} = \frac{170 \cdot 10^3}{1178{,}09 \cdot 10^{-6}} = 144{,}3 \text{ MPa}$$

torsione

$$C_s = \frac{\sigma_{lim}}{\sigma_{id}} = \frac{R_m}{\sigma_{zz_{N+e}}} = \frac{740}{264} = 2{,}8$$

$$= R_{p02} > 540$$

$$\sigma_{zz} = \frac{M_x}{J_{xx}} y$$

$$J_{xx} = \frac{\pi}{64}D^4 - \frac{\pi}{64}d^4 = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(40^4 - 10^4) = 12{,}51 \cdot 10^4 \text{ mm}^4$$

$$\sigma'_{zz} = \frac{750 \cdot 10^3}{12{,}51 \cdot 10^4} \cdot 20 = 119{,}9 \text{ MPa} \simeq 120 \text{ MPa}$$

$$\sigma''_{zz} = \frac{750 \cdot 10^3}{12{,}51 \cdot 10^4} \cdot (-20) = -119{,}9 \text{ MPa} \simeq -120 \text{ MPa}$$

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CASO B

$\xi_B = -28,2$

$\eta_B = -28,2$

$x_B = \xi_B \cos\alpha + \eta_B \sin\alpha = -28,2 \cos 45 - 28,2 \sin 45 = -39,9$

$y_B = -\xi_B \sin\alpha + \eta_B \cos\alpha = +28,2 \sin 45 - 28,2 \sin 45 = 0$

$\sigma_{zz}^B = \frac{M_x}{J_{xx}} y_B - \frac{M_y}{J_{yy}} x_B =$

$= \frac{1,768 \cdot 10^6}{2,8 \cdot 10^6} \cdot 0 - \frac{-1,768 \cdot 10^6}{0,73 \cdot 10^6} (-39,9) = -96,6 \ N/mm^2$

CASO C

$\xi_C = 71,8$

$\eta_C = -28,2$

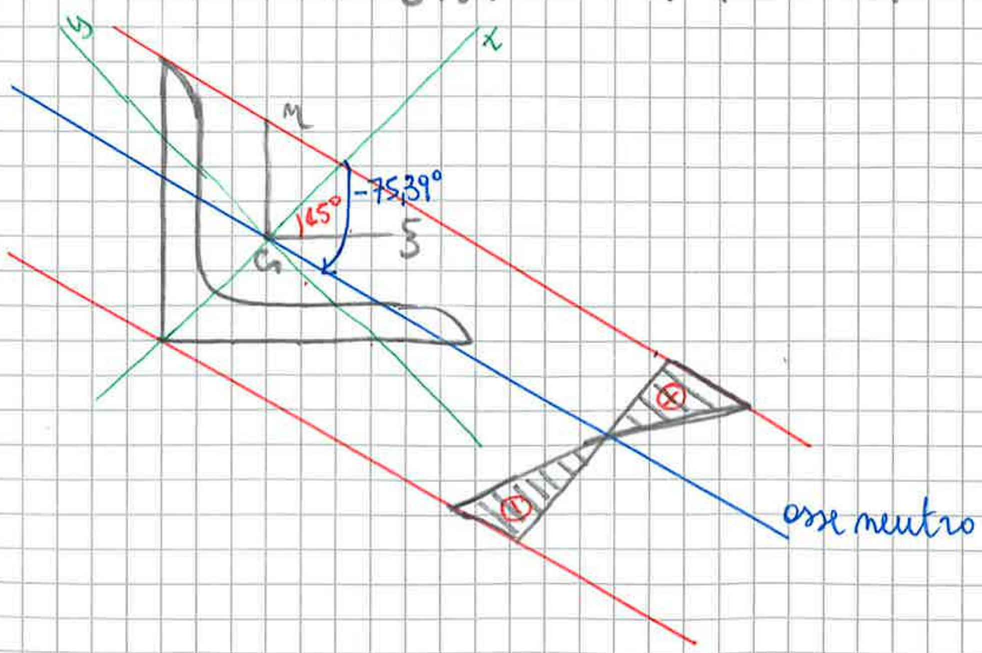
$x_C = \xi_C \cos\alpha + \eta_C \sin\alpha = 71,8 \cdot \cos 45 - 28,2 \sin 45 = 30,83$

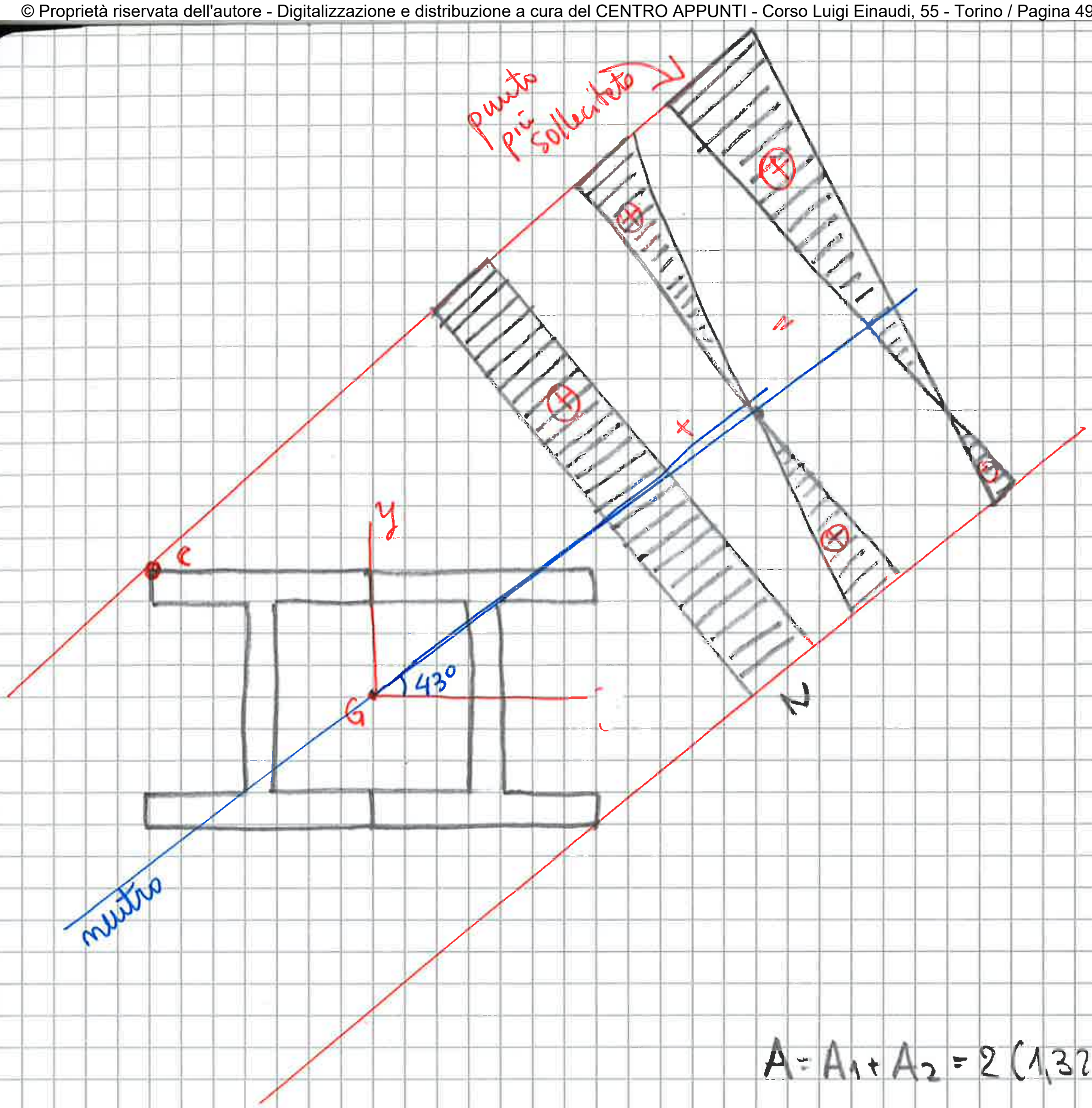
$y_C = -\xi_C \sin\alpha + \eta_C \cos\alpha = -71,8 \sin 45 - 28,2 \cos 45 = -70,71$

$\sigma_{zz}^C = \frac{M_x}{J_{xx}} y_C - \frac{M_y}{J_{yy}} x_C = \frac{1,768 \cdot 10^6}{2,8 \cdot 10^6} \cdot (-70,71) - \frac{-1,768 \cdot 10^6}{0,73 \cdot 10^6} \cdot 30,83 =$

$= 30 \ N/mm^2$

$\psi = \tan^{-1}\left(\frac{M_y}{M_x} \cdot \frac{J_{xx}}{J_{yy}}\right) = \tan^{-1}\left(\frac{-1,768}{1,768} \cdot \frac{2,8}{0,73}\right) = -75,39°$





$A = A_1 + A_2 = 2\,(1{,}32 \cdot 10^3) =$
$= 2{,}64 \cdot 10^3$

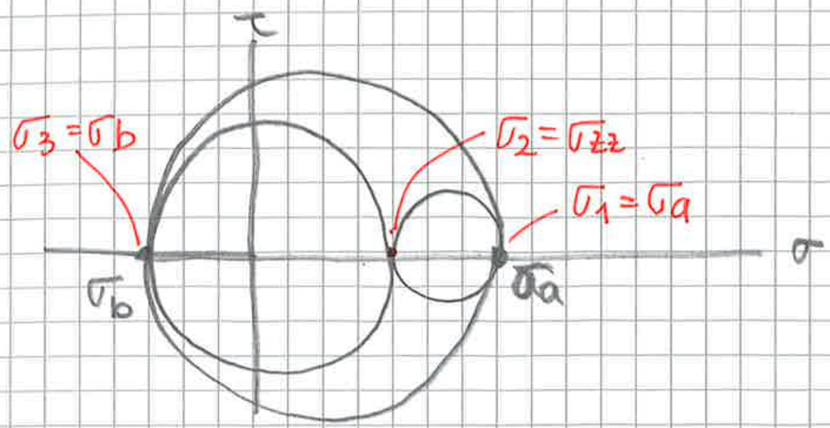
c) $x_C = -64$

$y_C = 60$

$$\sigma^{C}_{zz,max} = \frac{N}{A} + \frac{M_x}{J_{xx}}\,y - \frac{M_y}{J_{yy}}\,x =$$

$$= \frac{8 \cdot 10^4\ (N)}{2{,}64 \cdot 10^3} + \frac{5{,}3 \cdot 10^6}{6{,}36 \cdot 10^6} \cdot 60 - \frac{2{,}5 \cdot 10^6}{3{,}26 \cdot 10^6} \cdot (-64) =$$

$$= 129\ [MPa]$$



$$C_S = \frac{\sigma_{lim}}{\sigma_{id}} = \frac{R_m}{\sigma_{eq}}$$

σ_{eq} dipende dal materiale (GHISA)
⇒ GALILEO, RANKINE

$$\sigma_{id} = \sigma_1$$

$$C_S = \frac{300}{80} = 3,75 \ ✓$$

$$\tau_{1,picco} = \frac{M_t}{J_{t,tot}} \cdot s_1 = \frac{1{,}8 \cdot 10^4}{2284{,}27} \cdot 4 = 31{,}52 \text{ MPa}$$

$$\tau_{2,picco} = \frac{M_t}{J_{t,tot}} \cdot s_2 = \frac{1{,}8 \cdot 10^4}{2284{,}27} \cdot 5 = 39{,}39 \text{ MPa}$$

$$\tau_{3,picco} = \frac{M_t}{J_{tot}} \cdot s_3 = \frac{1{,}8 \cdot 10^4}{2284{,}27} \cdot 4 = 31{,}52 \text{ [MPa]}$$

$$\Delta\theta = \frac{M_t}{G J_t} \cdot L = \frac{1{,}8 \cdot 10^4}{2{,}556 \cdot 10^4 \cdot 2284{,}28} \cdot 600 = 0{,}1849$$

$$G = \frac{E}{2(1+\nu)} = \frac{6{,}8 \cdot 10^4}{2(1+0{,}33)} = 2{,}556 \cdot 10^4$$

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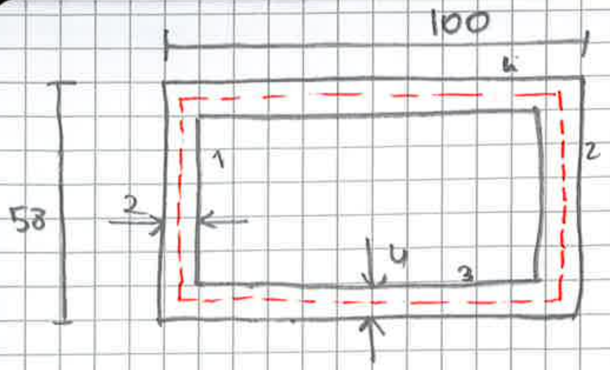

$l_1 = 58 - \frac{4}{2} - \frac{4}{2} = 54$

$J_t = \frac{1}{3} l s^3 = \frac{1}{3} 54 \cdot 2^3 = 144$

$l_2 = l_1 = 54$ $\quad J_t = 144$

$l_3 = 100 - \frac{2}{2} - \frac{2}{2} = 98$ $\quad J_t = \frac{1}{3} 98 \cdot 4^3 = 2090$

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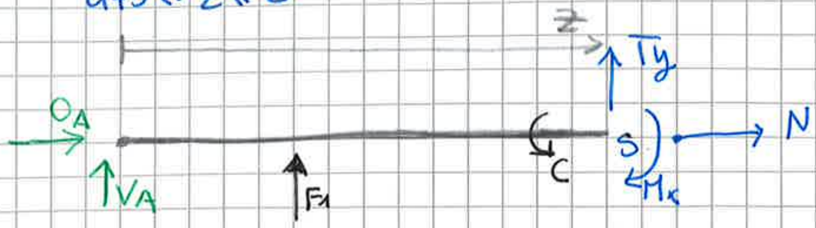
$a+b \leq z \leq \ell$



$\rightarrow$: $O_A + N = 0$ $\qquad O_A = -N = F_2$

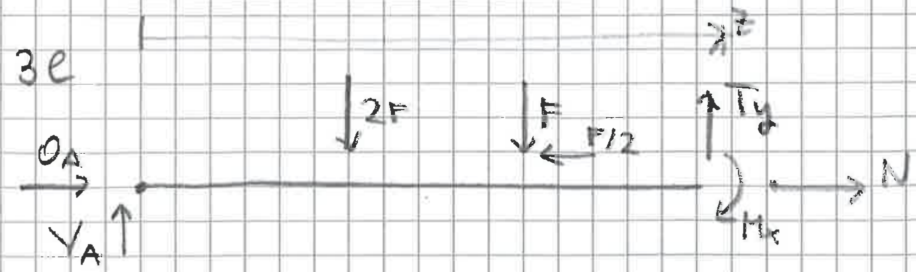
$\uparrow$: $V_A + F_1 + T_y = 0$ $\qquad T_y = -V_A - F_1 = -F_1 \frac{a}{\ell} - F_1 = -F_1\left(\frac{a}{\ell}+1\right)$

s): $M_x - C + V_A \cdot z + F_1 \cdot (z-a) = 0$

$$M_x = C - F_1(z-a) - V_A z.$$

poi si disegnano le strutture

$0 \leq z \leq 3\ell$



$\uparrow: \quad V_A - 2F - F + T_y = 0 \qquad T_y = 3F - \frac{5}{3}F = \frac{4}{3}F$

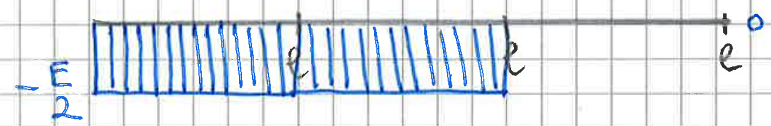
$\rightarrow: \quad O_A - \frac{F}{2} + N = 0 \qquad N = -O_A + \frac{F}{2} = -\frac{F}{2} + \frac{F}{2} = 0$

$S): \quad M_x + V_A \cdot z - 2F(z - 2\ell) - F(z - \ell) = 0$

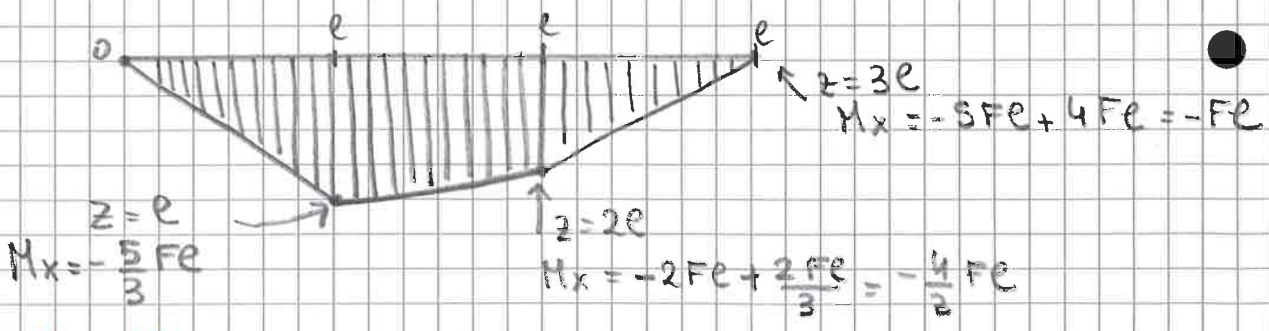
$M_x = Fz - F\ell + 2Fz - 4F\ell - \frac{5}{3}Fz =$

$= -5F\ell + \frac{4}{3}Fz$

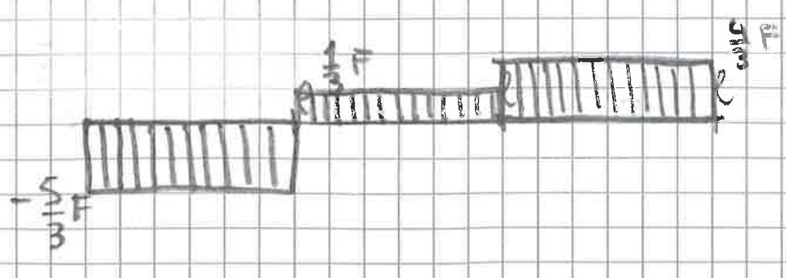
NORMALE

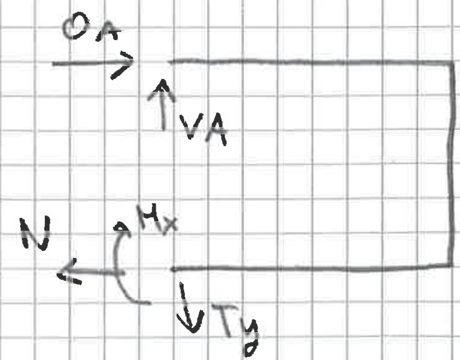


Momento flettente M_x



Taglio T_y

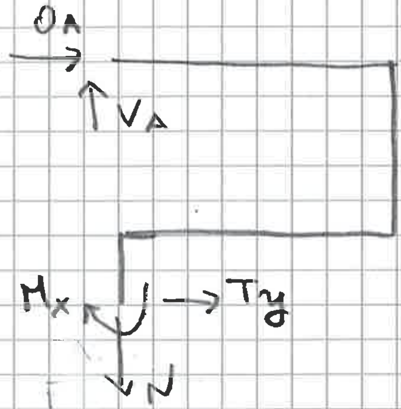




$\uparrow \quad V_A - T_y = 0 \qquad T_y = V_A = -F$

$\rightarrow \quad O_A - N = 0 \qquad N = 0$

$\circlearrowright \quad M_x + V_A z = 0 \qquad M_x = Fz$

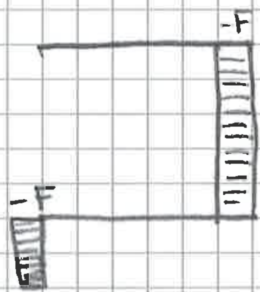


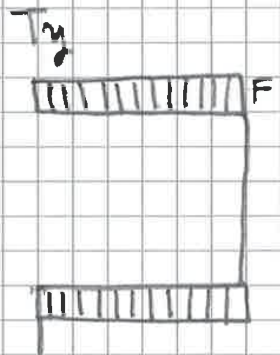
$O_A + T_y = 0$

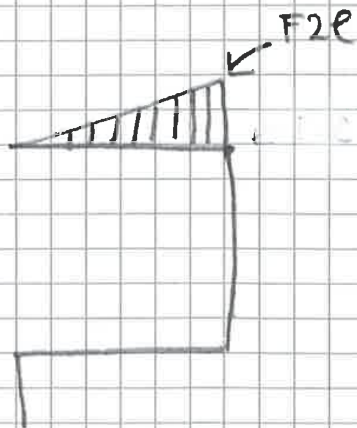
$V_A - N = 0 \qquad N = V_A = -F$

$M_x + O_A \left(z - \frac{3}{2}\ell\right) = 0$

N :







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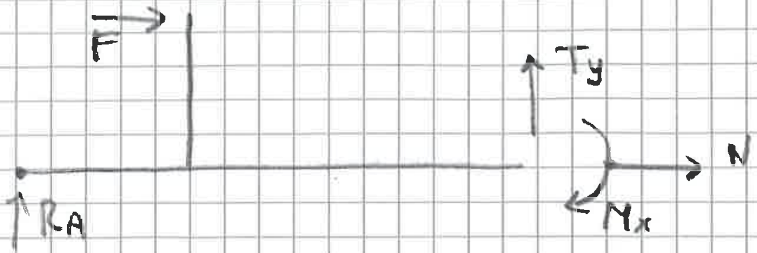
$0 \le z \le \ell$



$\uparrow: \quad R_A + T_y = 0 \qquad T_y = +R_A = +\frac{1}{3}F$

$\rightarrow: \quad F + N = 0 \qquad N = -F$

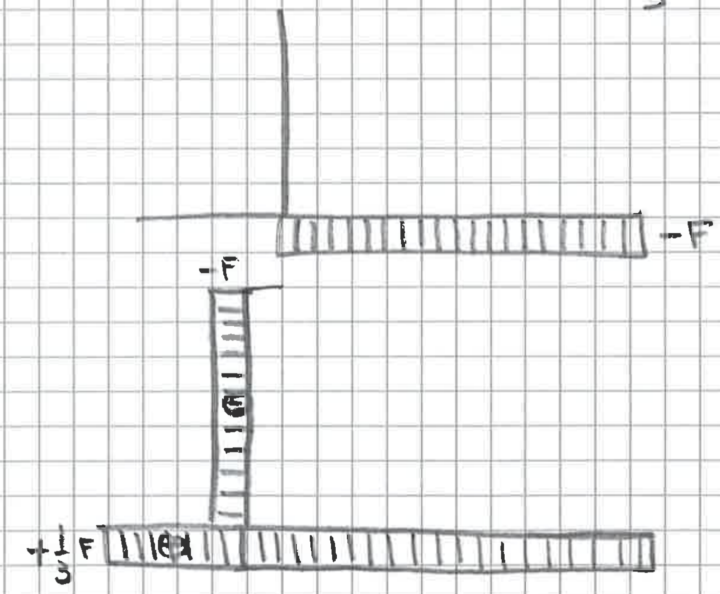
$S\circlearrowright: \quad M_x + R_A \cdot z + F\left(z - \frac{2}{3}\ell\right) = 0$

$M_x = -R_A z - Fz + F\frac{2}{3}\ell = +\frac{1}{3}Fz - Fz + \frac{2}{3}F\ell =$

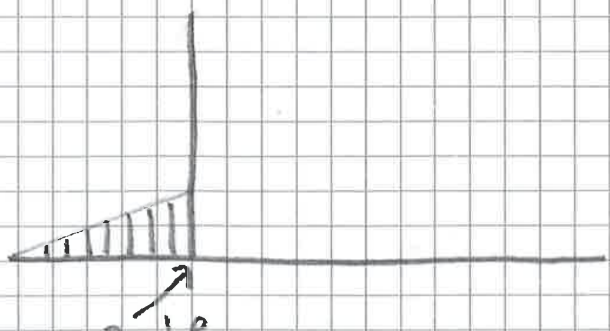
$= \left(\frac{+1-3}{3}\right)Fz + \frac{2}{3}F\ell =$

N:

$= -\frac{2}{3}Fz + \frac{2}{3}F\ell$



M_x



$z = \frac{1}{3}\ell$

$M_x = \frac{1}{3}F\frac{\ell}{3} = \frac{F\ell}{9}$

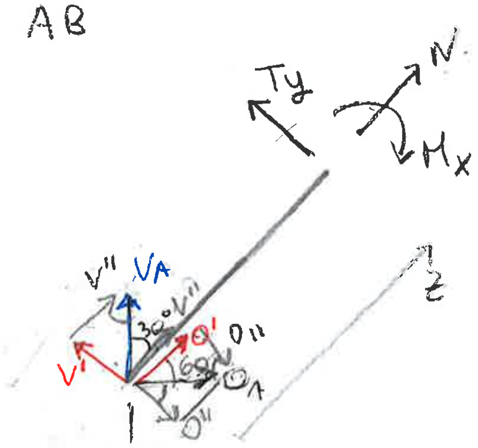
$z = \frac{1}{3}\ell$

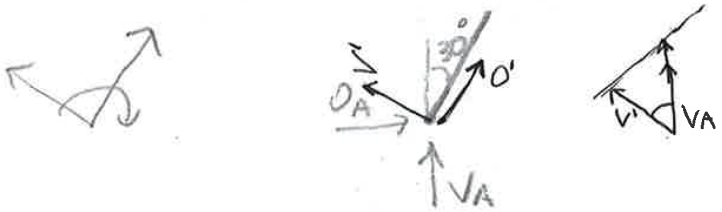
$M_x = -\frac{2}{3} \cdot F \cdot \frac{1}{3}\ell + \frac{2}{3}F\ell$

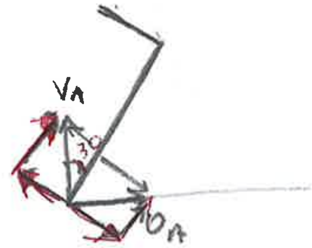
$= -\frac{2}{9}F\ell + \frac{2}{3}F\ell$

$\frac{-2+4}{9}$

AB







$V' = V_A \sin 30°$

$O' = O_A \sin 30°$

$V'' = V_A \cos 30°$

$O'' = O_A \cos 30°$

$N + V'' + O' = 0$

$N = -V_A \cos 30° - O_A \sin 30°$

$N = F\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} F \cdot \frac{1}{2} = \frac{2F\sqrt{3} + \sqrt{3}F}{4} = \frac{3\sqrt{3}F}{4}$

$T_y + V' - O'' = 0$

$T_y = O'' - V' = O_A \cos 30 - V_A \sin 30° =$

$= -\frac{\sqrt{3}}{2} F \frac{\sqrt{3}}{2} + F\frac{1}{2} = -\frac{3}{4}F + \frac{1}{2}F = \frac{-3F + 2F}{4} = -\frac{1}{4}F$

$M_x + V'z - O'' \cdot z = 0$

$M_x + (V' - O'')z = 0$

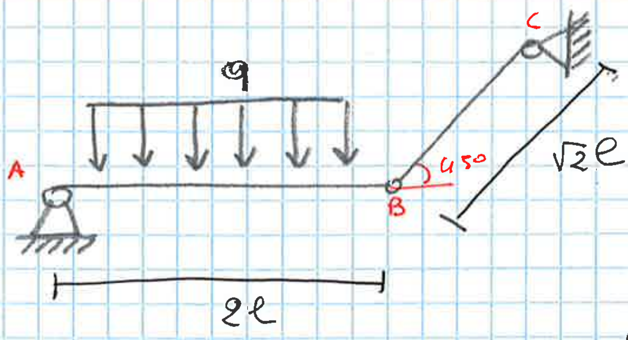
$M_x = -(V' - O'')z =$

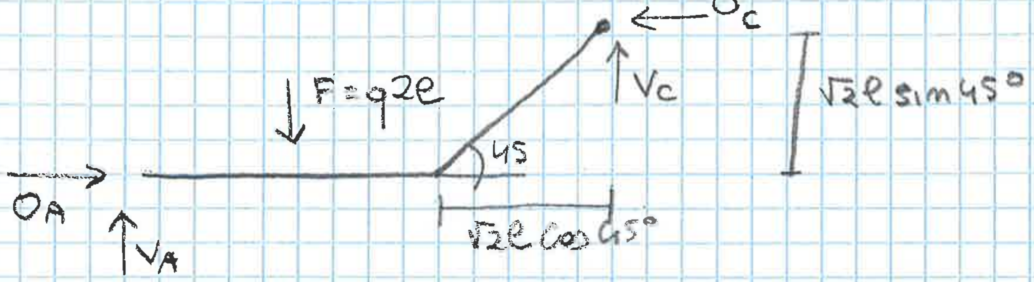
$= -(V_A \sin 30° - O_A \cos 30°)z =$

$= \left(+F \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} F \frac{\sqrt{3}}{2} = \frac{1}{2}F - \frac{3}{4}F\right)z =$

$= \left(\frac{2F - 3F}{4}\right)z = -\frac{F}{4}z$

ES2

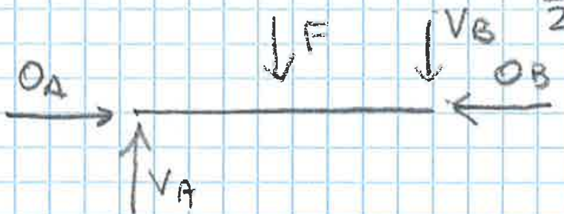




$\rightarrow \quad O_A + O_C = 0 \qquad O_C = -O_A = -O_B$

$\uparrow \quad V_A + V_C - F = 0 \qquad V_C = F - V_A = F - \frac{F}{2} = \frac{F}{2} = q\ell$

$A) \quad -F \cdot \ell + V_C \sqrt{2}\ell \underbrace{\cos 45}_{\frac{\sqrt{2}}{2}} + O_C \sqrt{2}\ell \underbrace{\sin 45°}_{\frac{\sqrt{2}}{2}} = 0$



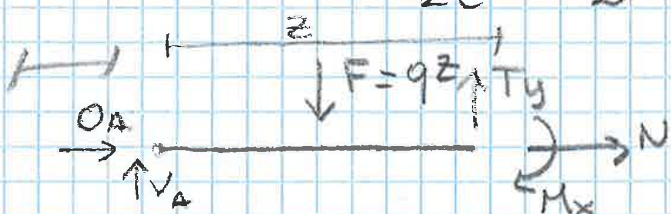
$$O_C = \frac{F\ell - \frac{F}{2} \cdot \sqrt{2}\ell \cdot \frac{\sqrt{2}}{2}}{\sqrt{2}\ell \cdot \frac{\sqrt{2}}{2}} = \frac{F}{2} = q\ell$$

$O_A - O_B = 0 \qquad O_A = O_B$

$V_A - V_B - F = 0 \qquad V_A = F + V_B = F - \frac{F}{2} = \frac{F}{2} = q\ell$

$A) \quad -F\ell - V_B \cdot 2\ell = 0 \qquad V_B = -\frac{F\ell}{2\ell} = -\frac{F}{2} = -\frac{q2\ell}{2} = -q\ell$

Sollecitazioni **AB**



$N + O_A = 0 \qquad N = -O_A = +q\ell$

$T_y + V_A - F = 0 \qquad T_y = F - V_A = qz - q\ell = q(z - \ell)$

$M_x + V_A z - F \cdot \frac{z}{2} = 0 \qquad M_x = qz\frac{z}{2} - q\ell z = q\left(\frac{z^2}{2} - \ell z\right)$

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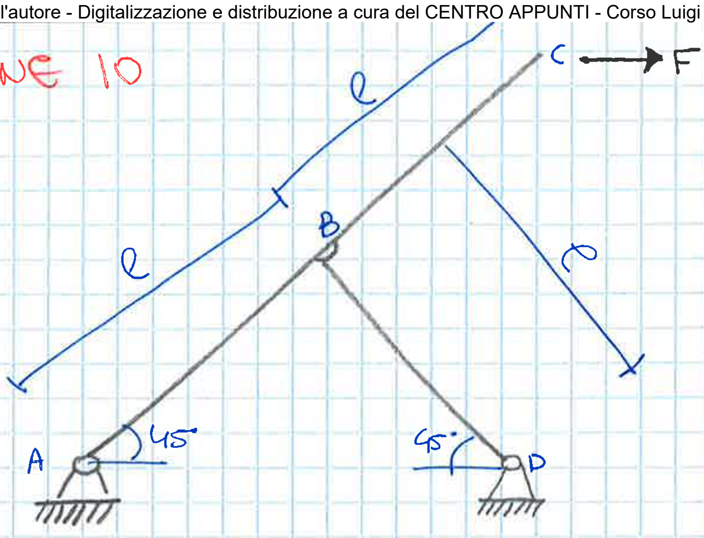
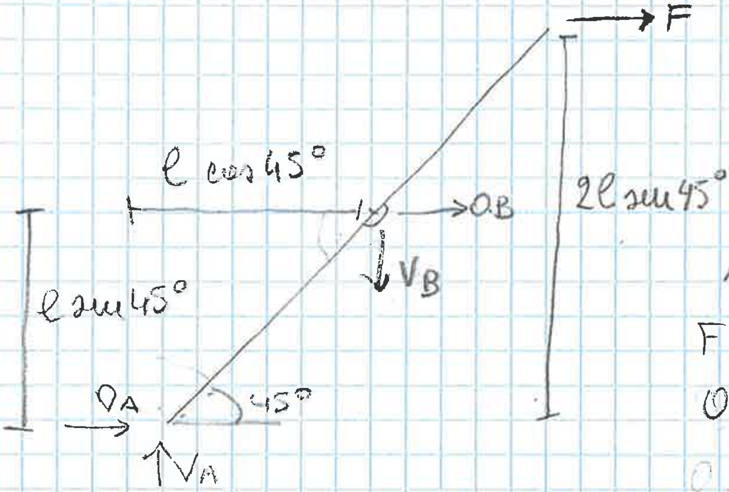
ESERCITAZIONE 10





$O_A + O_B + F = 0$

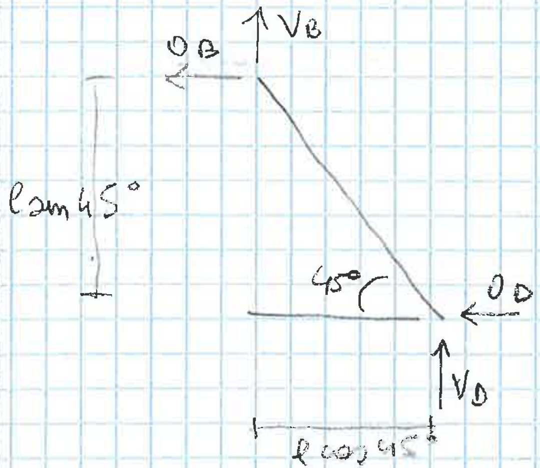
$V_A - V_B = 0$

A) $F \cdot 2l \sin 45° + O_B\, l \sin 45° + V_B\, l \cos 45° = 0$ ($V_B\, l \cos 45° = O_B\, l \sin 45°$)

$F \cdot 2l \sin 45° + 2 O_B\, l \sin 45° = 0$

$O_B = -\frac{F\, 2l \sin 45°}{2l \sin 45°} = -F$

$O_A = -F - O_B = -F + F = 0$



$-O_D - O_B = 0 \qquad O_D = -O_B$

$V_D + V_B = 0 \qquad V_D = -V_B = V_A$

D): $O_B \cdot l \sin 45° - V_B\, l \cos 45° = 0$

$$O_B = \frac{V_B \frac{\sqrt{2}}{2} l}{l \frac{\sqrt{2}}{2}} = V_B$$

$O_B = -F$

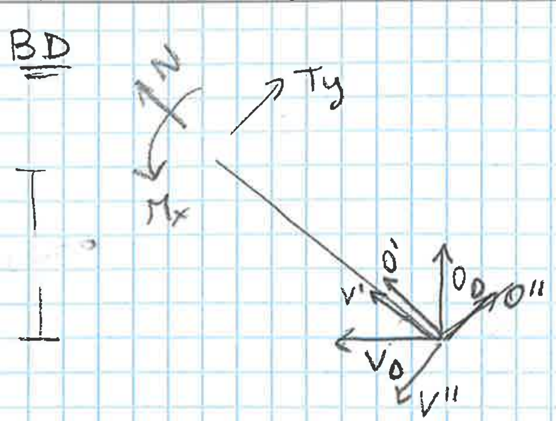
$V_A = V_B = O_B = -F$

$O_A = 0$

$O_D = -O_B = F$

$V_D = -V_B = -O_B = F$

BD



$V' = O' = V_D \cdot \cos 45°$

$O'' = O_D \text{ sen } 45°$

$V'' = V_D \cdot \text{sen } 45°$

$-N - O' - V' = 0$

$T_y + O'' - V'' = 0$

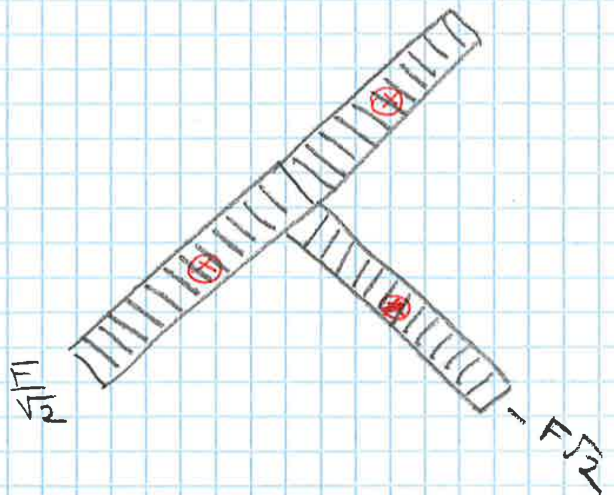
$M_x - V''z + O''z = 0$

$N = -O' - V' = -V_D \cos 45 - V_D \cos 45° = -2F \frac{\sqrt{2}}{2} = -F\sqrt{2}$

$T_y = V'' - O'' = V_D \text{ sen } 45° - O_D \text{ sen } 45° =$
$= F\frac{\sqrt{2}}{2} - F\frac{\sqrt{2}}{2} = 0$

$M_x = (V'' - O'')z = (V_D \text{ sen } 45 - O_D \text{ sen } 45)z =$
$= \left(F\frac{\sqrt{2}}{2} - F\frac{\sqrt{2}}{2}\right)z = 0$

N



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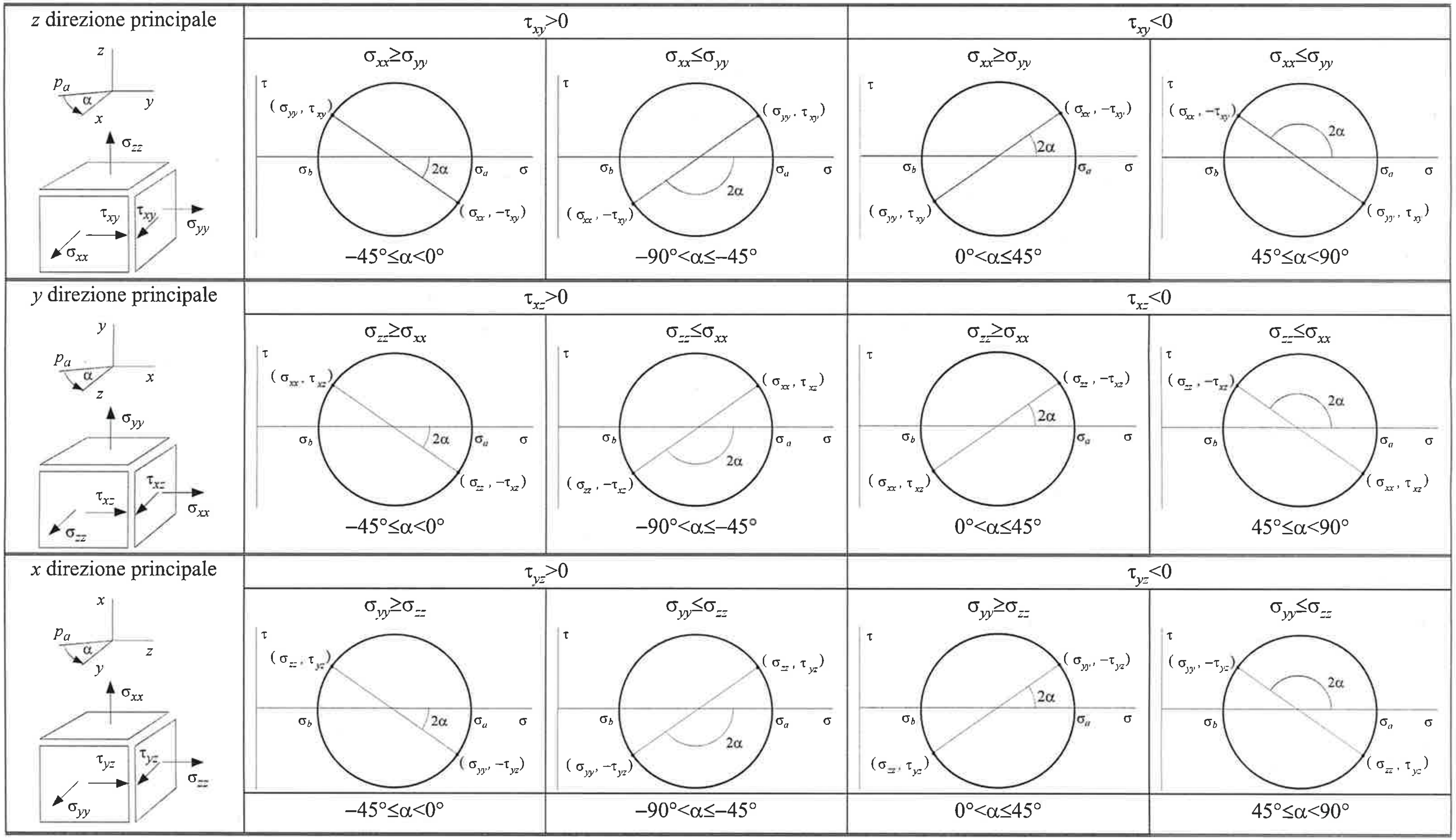
CERCHI DI MOHR

Determinazione dell'angolo formato tra il riferimento principale e quello xyz (α misurato dall'asse principale p_a all'asse x, z, y a seconda del caso).



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