NUMERO: 1941A - ANNO: 2016

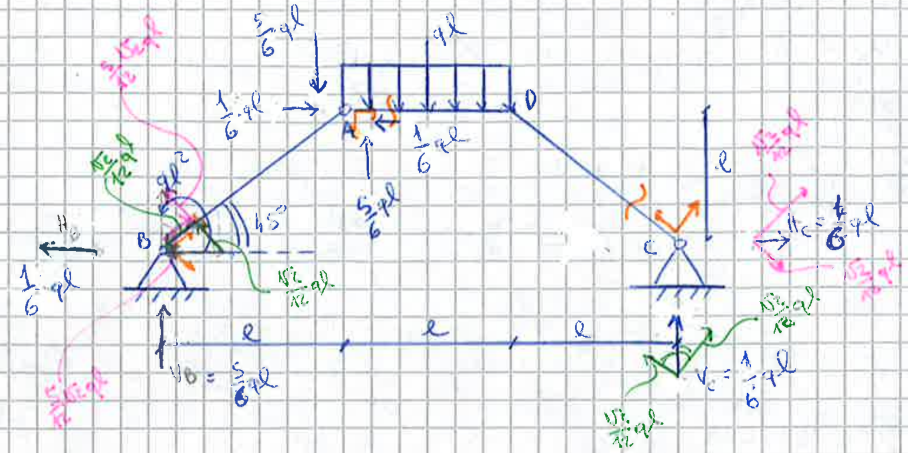
A P P U N T I

STUDENTE: Prette Arianna

MATERIA: Scienza delle costruzioni - prof. Surace

TEMA D'ESAME 6/9/05

ES. 1



1) REAZIONI VINCOLARI:

$\rightarrow \quad H_B + H_C = 0 \qquad H_C = \cancel{-\frac{3}{2}ql} + \frac{1}{6}ql$

$\uparrow \quad V_B + V_C - ql = 0 \quad \rightarrow \quad V_B = \cancel{\frac{5}{2}ql + \frac{3}{2}ql = \frac{5}{2}ql} = \frac{6}{6}ql - \frac{1}{6}ql = \frac{5}{6}ql$

$M(B) \rightarrow ql^2 - ql \cdot \frac{3}{2}l + V_C \cdot 3l = 0 \quad \rightarrow \quad V_C = \frac{\left(-\frac{2}{2}ql^2 + \frac{3}{2}ql^2\right)}{3l}$

$V_C = \frac{+\frac{1}{2}ql^2}{3l} = \cancel{-\frac{3}{2}ql} + \frac{1}{6}ql$

equazione ausiliaria corpo 1

$M(A) = -V_B \cdot l + H_B \cdot l + ql^2 = 0$

$-\frac{5}{6}ql^2 + H_B \cdot l + ql^2 = 0 \qquad H_B = \frac{-\left(\frac{6}{6}ql^2 + \frac{5}{6}ql^2\right)}{l}$

$H_B = \cancel{\frac{\frac{3}{2}ql^2}{l}} = \cancel{\frac{3}{2}ql} - \frac{1}{6}ql$

SOLLECITAZIONI

TRATTO $\overline{BA}$:

$N = \frac{1}{6}ql \cdot \frac{\sqrt{2}}{2} - \frac{5}{6}\frac{\sqrt{2}}{2}ql \quad \rightarrow \quad N = \frac{1\sqrt{2}}{12}ql - \frac{5\sqrt{2}}{12}ql = -\frac{4\sqrt{2}}{12}ql = -\frac{\sqrt{2}}{3}ql$

$T = \frac{\sqrt{2}}{2}\frac{1}{6}ql + \frac{\sqrt{2}}{2}\frac{5}{6}ql \quad \rightarrow \quad T = \frac{\sqrt{2}}{12}ql + \frac{5\sqrt{2}}{12}ql = \frac{\sqrt{2}}{2}ql$

$M = -ql^2 + \frac{\sqrt{2}}{12}ql \cdot z + \frac{5}{12}\sqrt{2}ql \cdot z = -ql^2 + \frac{\sqrt{2}}{2}qlz \qquad M(0) = M_B = -ql^2$

$\rightarrow M(l\sqrt{2}) = -ql^2 + \frac{\sqrt{2}}{2}ql\, l\sqrt{2}$

$M_A = M(l\sqrt{2}) = -ql^2 + ql^2 = 0$

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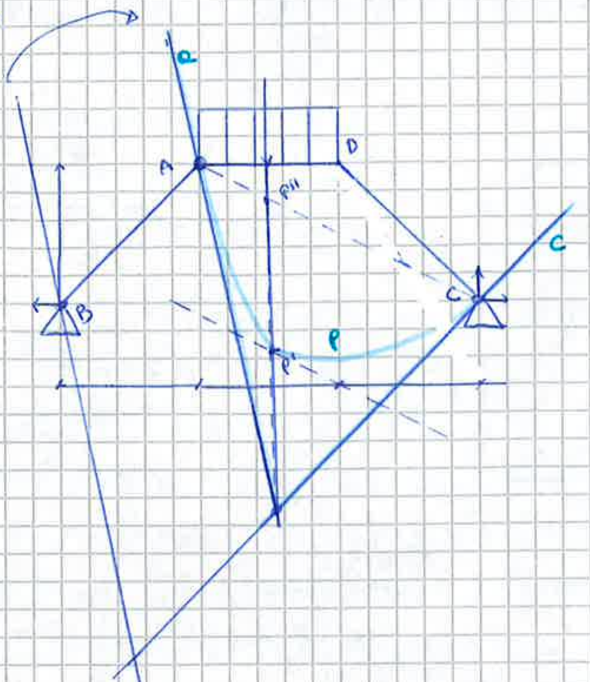
2)

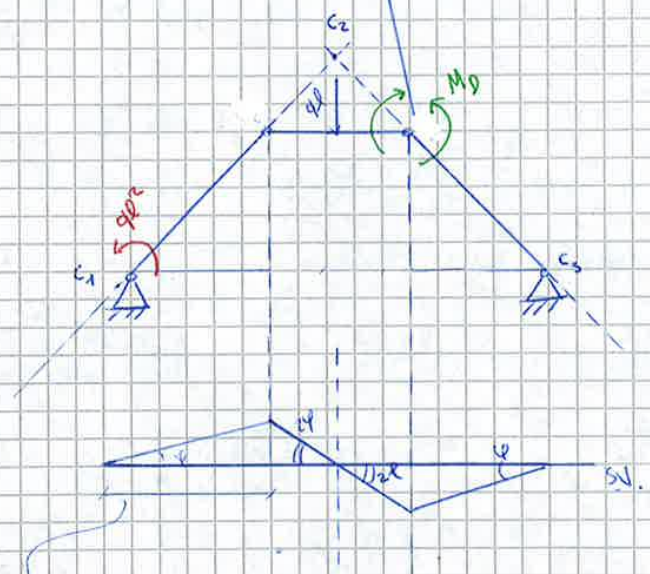
la traccia in A

perchè deve passare per la cerniera



tratto	cdp
AB	retta a
AD	parabola p
DC	~~parabola~~ retta c

3)



S.D.

NB.

→ centro di rotazione assoluta = spostamento nullo!

$\ell\varphi = \vartheta \cdot \frac{\ell}{2}$

$\vartheta = 2\varphi$

$$q\ell^2\cdot\varphi + q\ell\cdot 0 + M_D\cdot\varphi + M_D\cdot 2\varphi = 0$$

$$q\ell^2\varphi + M_D\, 3\varphi = 0 \rightarrow M_D = -\frac{q\ell^2}{3}$$

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$$x_1 = \frac{-2\ell^2 \lambda t}{\frac{8}{3EI}\ell^3} = -2\lambda t \cdot \frac{3EI}{8\ell} = -\frac{3EI}{4\ell}\lambda t$$

$$\int_s \frac{M_1^2}{EI}dz = \frac{1}{EI}\cdot\frac{8}{3}\ell^3 + \frac{1}{EI}\frac{\ell}{6}8\ell^2 = \frac{1}{EI}\left(\frac{16\ell^3 + 8\ell^3}{6}\right)$$

$$= \frac{1}{EI}\left(\frac{24}{6}\ell^3\right) = \frac{4\ell^3}{EI}$$

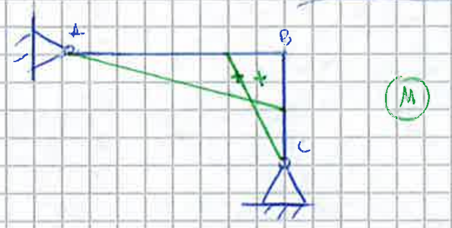
$$x_1 = \frac{+2\ell^2\lambda t}{\frac{4\ell^3}{EI}} = +\frac{EI\,\lambda t}{2\ell} = -\frac{EI\,\alpha\Delta T}{\ell h}$$

poichè:

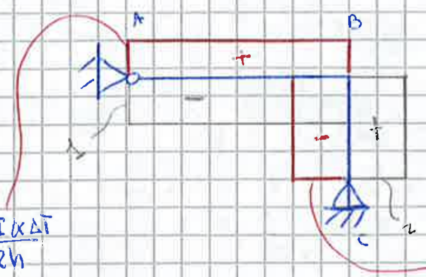
$$\lambda t = -\frac{2\alpha\Delta T}{h}$$

diagramma momento $M = M_1 x_1$

$$-2\ell \cdot \frac{-EI\alpha\Delta T}{h} = +\frac{2EI\alpha\Delta T}{h}$$



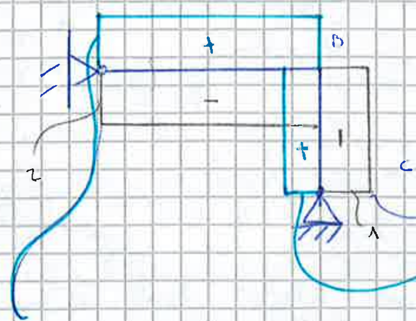
$T = T_1 x_1$



$$-1\cdot\frac{-EI\alpha\Delta T}{\ell h} = +\frac{EI\alpha\Delta T}{\ell h}$$

$$2\cdot\left(\frac{-EI\alpha\Delta T}{\ell h}\right) = -\frac{2EI\alpha\Delta T}{\ell h}$$

$N = N_1 x_1$



$$-1\cdot\left(\frac{-EI\alpha\Delta T}{\ell h}\right) = +\frac{EI\alpha\Delta T}{\ell h}$$

$$-2\cdot\left(-\frac{EI\alpha\Delta T}{\ell h}\right) = \frac{2EI\alpha\Delta T}{\ell h}$$

$$\sigma_z = \frac{N}{A} + \frac{M_x}{I_x} y$$

$$\sigma_{z\,max} = \frac{M_B}{I_x} \cdot y + \frac{N_B}{A}$$

$$A_{tot} = h \cdot \delta + \cancel{2}\,\delta \left(\frac{h}{\cancel{2}}\right) = 2h\delta$$

$$\sigma_{z\,max} = \frac{\frac{\cancel{2}EI\alpha\Delta T}{h}}{\frac{\delta h^3}{3}} \cdot \frac{\cancel{h}}{\cancel{2}} + \frac{\frac{\cancel{2}EI\alpha\Delta T}{\ell h}}{2h\delta}$$

$$\sigma_{z\,max} = 3\frac{EI\alpha\Delta T}{\delta h^3} + \frac{EI\alpha\Delta T}{\delta \ell h^2}$$

$$\sigma_{z\,max} = \frac{\cancel{3}E\cancel{I}\alpha\Delta T}{\cancel{\delta h^3}}\left(1 + \frac{h}{3\ell}\right) = E\alpha\Delta T\left(1 + \frac{h}{3\ell}\right)$$

ma $h << \ell$ $\quad \sigma_{z\,max} \cong E\alpha\Delta T$ $\quad$ contributo di N è molto inferiore a quello del momento flettente

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TRATTO EB

$\leftarrow \quad N = -2ql + \frac{7}{4}ql = \frac{-8ql + 7ql}{4} = -\frac{ql}{4}$

$\uparrow \quad T = -\frac{3}{2}ql$

$\circlearrowleft \quad M = -\frac{3}{2}ql \cdot z + \frac{7}{4}ql \cdot 2l - 2ql \cdot l \qquad M(0) = M_E = \frac{7}{2}ql^2 - 2ql^2$

$= \frac{7ql^2 - 4ql^2}{2} = \frac{3}{2}ql^2$

$M_B = M(l) = -\frac{3}{2}ql \cdot l + \frac{7}{2}ql^2 - 2ql^2$

$= -\frac{3}{2}ql^2 + \frac{7}{2}ql^2 - 2ql^2 = \frac{-3 + 7 - 4}{2}ql^2 = 0$

TRATTO DC

$\leftarrow \quad N = -\frac{5}{2}ql$

$\uparrow \quad T = \frac{1}{4}ql$

$\circlearrowleft \quad M = \frac{1}{4}ql \cdot z \qquad M(0) = 0 = M_C$

$M(2l) = M_D = \frac{1}{4}ql \cdot 2l = \frac{1}{2}ql^2$

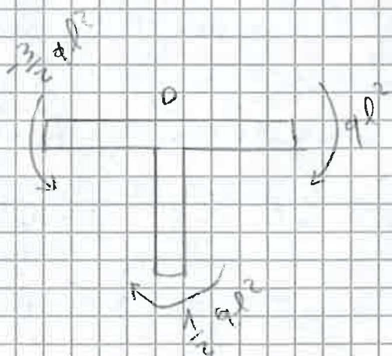
TRATTO DF

$\leftarrow \quad N = 0$

$\uparrow \quad T = ql$

$\circlearrowleft \quad M = ql \cdot z \qquad M(0) = M_F = 0$

$M(l) = M_D = ql^2$



TRATTO BD

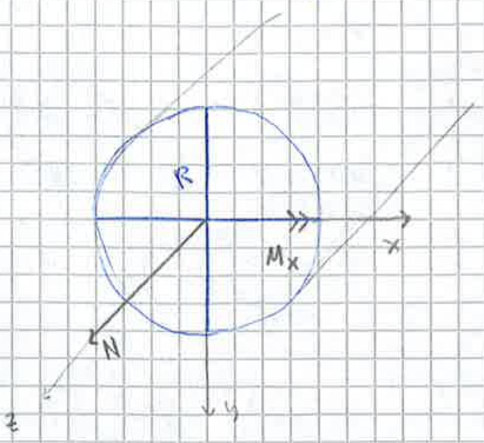
$\leftarrow \quad N = -\frac{ql}{4}$

$\uparrow \quad T = -\frac{3}{2}ql$

$\circlearrowleft \quad M = -\frac{3}{2}ql \cdot z \qquad M(0) = M_B = 0$

$M(l) = M_D = -\frac{3}{2}ql^2$

La sezione dove il momento flettente è max è la F $\left(M\left(\frac{7}{4}l\right)\right)$
che si trova a $\frac{7}{4}l$ da A



$$\sigma_z = \frac{N}{A} + \frac{M}{I_x}\,y$$

$$I_x = \frac{\pi R^4}{4}$$

trascuro

$$\sigma_z = \cancel{\frac{\frac{3}{2}ql}{\pi R^2}} + \frac{\frac{49}{32}ql^2}{\frac{\pi R^4}{4}}\cdot R$$

$$M\left(\frac{7}{4}l\right) = \frac{7}{4}ql\cdot\left(\frac{7}{4}l\right) - \frac{q}{2}\left(\frac{7}{4}l\right)\left(\frac{7}{4}l\right) = \frac{49}{16}ql^2 - \frac{q}{2}\left(\frac{49}{16}l^2\right) = \frac{49}{16}ql^2 - \frac{49ql^2}{32}$$

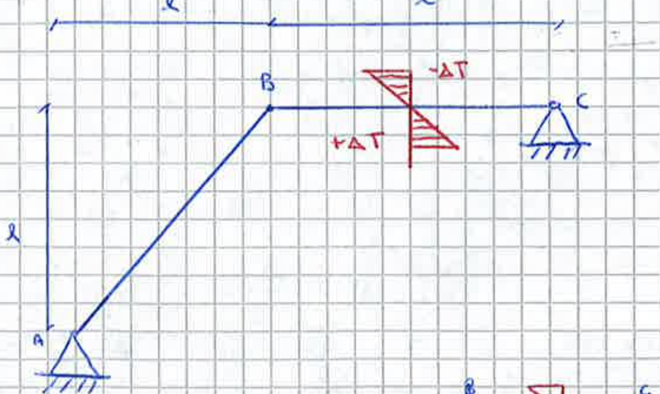
$$= \frac{98-49}{32}ql^2 = \frac{49}{32}ql^2$$

$$\sigma_z = \frac{49}{32}ql^2 R\cdot\frac{4}{\pi R^4} = \frac{49}{8\pi}\frac{ql^2}{R^3}$$

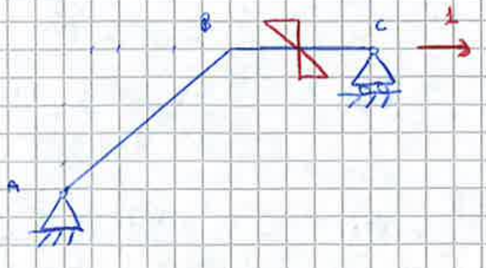
ES. 2

$g = 3$
$v = 4$

$g - v = 1$
1 volta iperstatica



$$\chi_t = \frac{+2\alpha\Delta T}{h}$$



$$2)\ \frac{1}{EI}\int_A^B M_1^2\,dz = \frac{l\sqrt2}{6EI}\cdot\left(0\cdot 0 + \left(\frac12 l\right)\left(\frac12 l\right) + \cancel{4}\left(\frac{l}{\cancel{4}}\right)\left(\frac{l}{4}\right)\right)$$

$$= \frac{l\sqrt2}{6EI}\cdot\left(\frac14 l^2 + \frac{l^2}{4}\right) = \cancel{\frac{l\sqrt2}{6EI}\left(\frac{9l^2+4l^2}{36}\right)}$$

$$= \cancel{\frac{l\sqrt2}{6EI}\cdot\frac{13}{36}l^2 = \frac{13\sqrt2}{216}\frac{l^3}{EI}} = \frac{l\sqrt2}{6EI}\cdot\frac{2}{4}l^2 = \frac{\sqrt2}{12}\frac{l^3}{EI}$$

$$\frac{1}{EI}\int_B^C M_1^2\,dz = \frac{l}{6EI}\left(\frac12 l\cdot\frac12 l + 4\left(-\frac{l}{4}\right)\left(-\frac{l}{4}\right) + 0\cdot 0\right)$$

$$= \frac{l}{6EI}\left(\frac14 l^2 + \frac{l^2}{4}\right) = \frac{l}{6EI}\cdot\frac12 l^2 = \frac{1}{12}\frac{l^3}{EI}$$

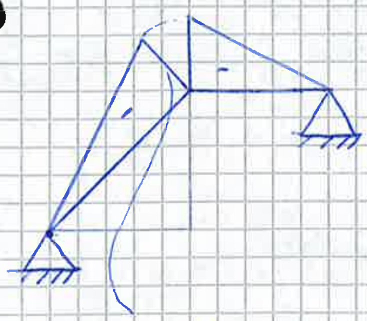
$$\int_S \frac{M_1^2}{EI}\,dz = \left(\frac{\sqrt2}{12}+\frac{1}{12}\right)\cdot\frac{l^3}{EI} = \left(\frac{\sqrt2+1}{12}\right)\frac{l^3}{EI}$$

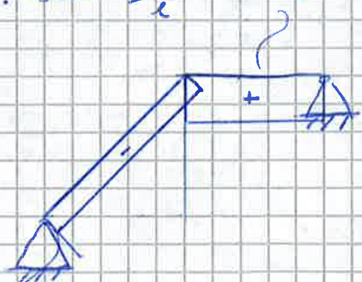
$$x_1 = \frac{-\frac{l^2}{4}\chi t}{\left(\frac{\sqrt2+1}{12}\right)\frac{l^3}{EI}} = -\frac{\cancel{l^2}}{\cancel{4}}\chi t\cdot\frac{\overset{3}{\cancel{12}}\,EI}{(\sqrt2+1)\,l^{\cancel{3}}} = -\frac{3\chi t EI}{(\sqrt2+1)\,l} = -3(\sqrt2-1)\frac{EI\chi t}{l}$$

Momento reale: $M = M_1 x_1$

Taglio reale: $T = x_1 T_1$

$$-\frac12\cdot -3(\sqrt2-1)\frac{EI\chi t}{l} = +\frac32(\sqrt2-1)\frac{EI\chi t}{l}$$





$$\frac12 l\cdot\left(-3(\sqrt2-1)\frac{EI\chi t}{l}\right) = -\frac32(\sqrt2-1)EI\chi t$$

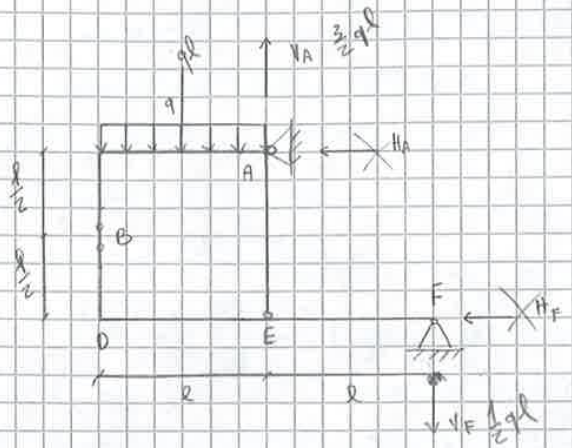
$$\frac{\sqrt2}{4}\cdot -3(\sqrt2-1)\frac{EI\chi t}{l} = -\frac{3(2-\sqrt2)}{4}\frac{EI\chi t}{l}$$

CORREZIONE

TEMA D'ESAME 18/7/2013

ES. 1

1.1)

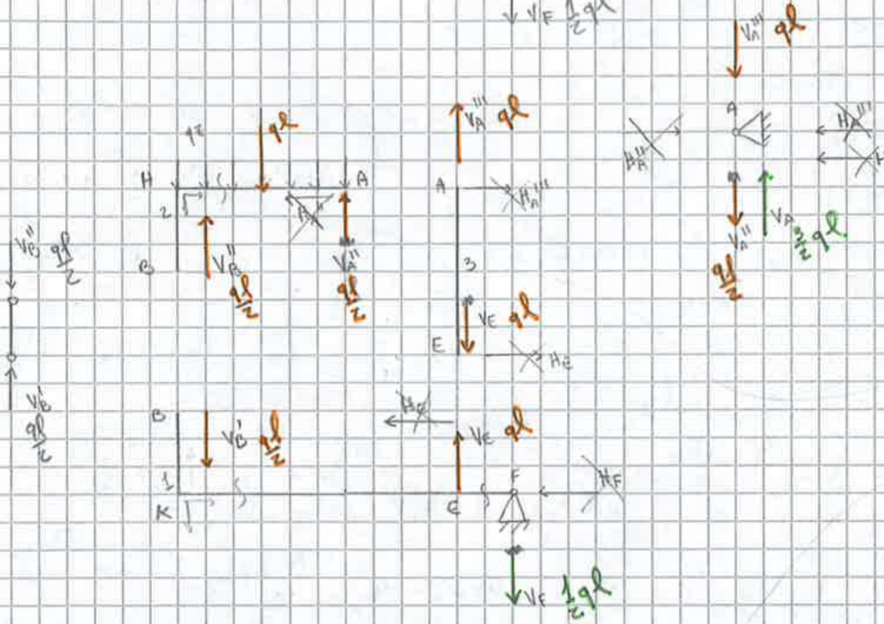


reazioni vincolari:

→ $-H_A - H_F = 0$

↑ $-ql + V_A + V_F = 0$

↺ $M(F) = ql \cdot \frac{3}{2}l - V_A \cdot l + H_A \cdot l$



3 come fosse biella: no reaz. H

corpo 2:

→ $-H_A'' = 0 \rightarrow H_A'' = 0$

↑ $V_B'' - V_A'' - ql = 0 \rightarrow V_A'' = -\frac{ql}{2}$

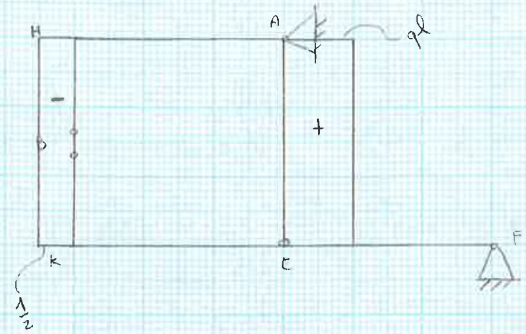
↺ $M(A) = -V_B'' \cdot l + ql \cdot \frac{l}{2} = 0$

$-V_B'' \cdot l = -\frac{ql^2}{2} \rightarrow V_B'' = \frac{ql}{2}$

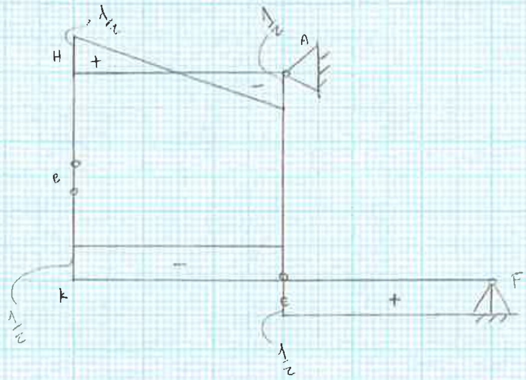
se $H_A = 0$ $\quad M(F) = \frac{3}{2}ql^2 - V_A \cdot l + \cancel{H_A l} = 0 \rightarrow V_A = \frac{3}{2}ql$

equilibrio nodo A:

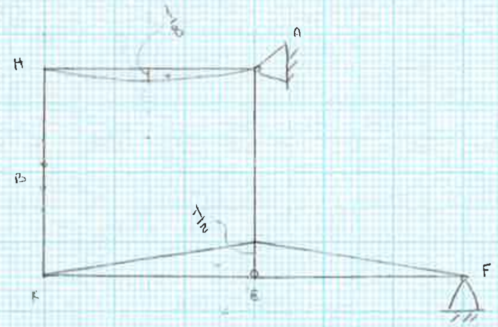
↑ $-\frac{ql}{2} + \frac{3}{2}ql - V_A''' = 0 \rightarrow V_A''' = ql$



$\frac{N}{ql}$

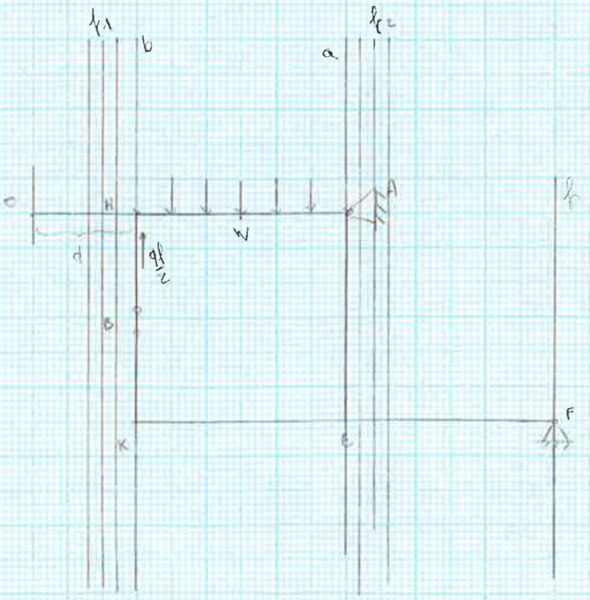


$\frac{T}{ql}$



$\frac{M}{ql^2}$

1.2)



$$\circlearrowleft M(C) = \frac{ql}{2} \cdot d - qz\left(\frac{z}{2} + d\right) = 0$$

$$\frac{ld}{2} - zd - \frac{z^2}{2} = 0$$

$$ld - 2zd - z^2 = 0$$

$$d(l - 2z) - z^2 = 0$$

$$d(z) = \frac{z^2}{(l - 2z)}$$

$z < \frac{l}{2}$ $\quad d(z) > 0 \rightarrow$ sx di H

$z > \frac{l}{2}$ $\quad d(z) < 0 \rightarrow$ dx di H

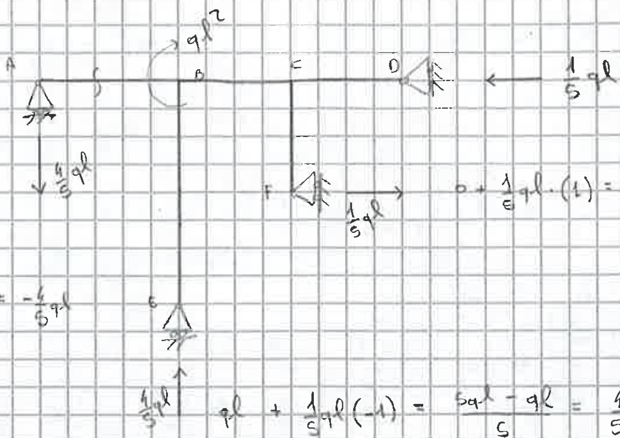
$z \rightarrow^- \frac{l}{2}$ $\quad d(z) = \frac{\left(\frac{l}{2}\right)^2}{0^+} = +\infty$

$z \rightarrow^+ \frac{l}{2}$ $\quad d(z) = \frac{\left(\frac{l}{2}\right)^2}{0^-} = -\infty$

tratto	cdp
FE	retta f
EK	retta a
BH	retta b
HW	fascio f1
WA	fascio f2
AC	retta a

2 fasci esterni ai vincoli per HW → fascio a sx di H
WA → fascio a dx di A

reazioni vincolari: $R = R_0 + R_1 X_1$ $\qquad X_1 = \frac{1}{5} q l$

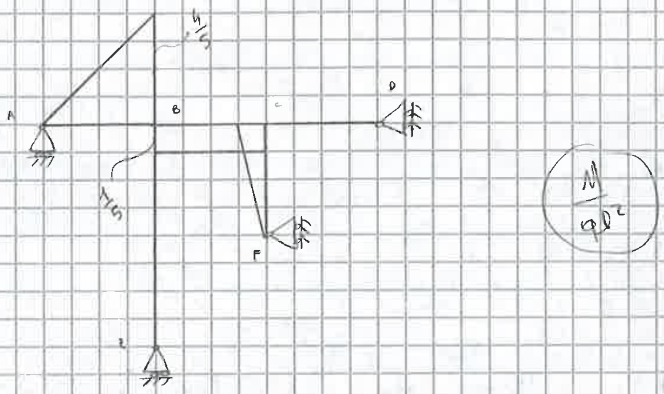


$-ql + \frac{1}{5}ql(+1) = \frac{-5ql + ql}{5} = -\frac{4}{5}ql$

$0 + \frac{1}{5}ql \cdot (1) = \frac{1}{5}ql$

$ql + \frac{1}{5}ql(-1) = \frac{5ql - ql}{5} = \frac{4}{5}ql$

$M = M_0 + M_1 X_1$

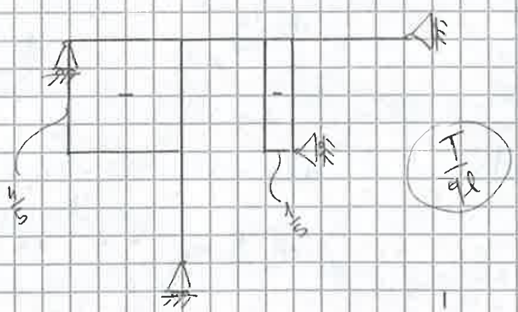


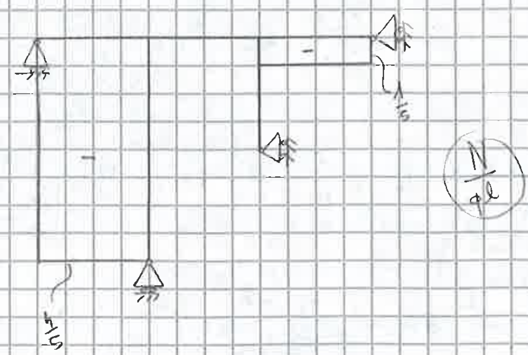
$\overline{AB}$ $\quad M = -\frac{4}{5} ql \cdot z$

$M(0) = M_A = 0$

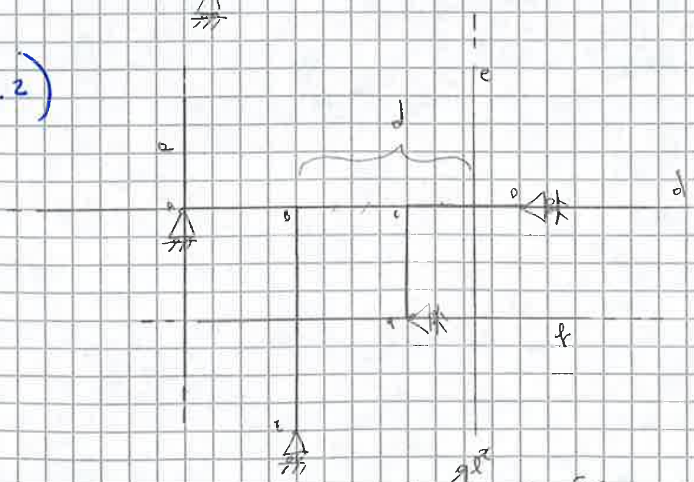
$M(l) = M_B = -\frac{4}{5} ql^2$

$\overline{BC}$ $\quad M = 0 + l \cdot \frac{1}{5} ql = \frac{ql^2}{5}$





2.2)



$d = \frac{ql^2}{\frac{1}{5}ql} = 5l$

tratto	cdp
AB	retta a
EB	retta e
FC	retta f
CD	retta d

$$\eta^{(2)} = H_E = \int_S \frac{M(f) \cdot M^{(2)}}{EI}\, dz$$

$$\frac{1}{EI}\int_A^B M_f \cdot M_2\, dz = \frac{\ell}{6}\left[0 \cdot 0 + 4 \cdot \left(\frac{\ell}{\cancel{2}}\right)\left(-\frac{\cancel{2}}{5}q\ell^2\right) + (\ell)\left(-\frac{4}{5}q\ell^2\right)\right]$$

$$= \frac{\ell}{6}\left[-\frac{4}{5}q\ell^3 - \frac{4}{5}q\ell^3\right] = \frac{\ell}{\cancel{6}_3} \cdot \left(-\frac{\cancel{8}^4}{5}q\ell^3\right) = -\frac{4}{15}q\ell^4 \cdot \frac{1}{EI}$$

$$\frac{1}{EI}\int_B^C M_f \cdot M_2\, dz = \frac{\ell}{6}\left[\ell \cdot \left(\frac{1}{5}q\ell^2\right) + 4(\ell)\left(\frac{1}{5}q\ell^2\right) + \ell\left(\frac{1}{5}q\ell^2\right)\right]$$

$$= \frac{\ell}{\cancel{6}}\left(\frac{\cancel{6}}{5}q\ell^3\right) = \frac{q\ell^4}{5} \cdot \frac{1}{EI}$$

$$\frac{1}{EI}\int_F^C M_f \cdot M_2\, dz = \frac{\ell}{6}\left[0 \cdot 0 + \cancel{4}^2\left(\frac{\ell}{\cancel{2}}\right) \cdot \left(-\frac{q\ell^2}{\cancel{10}_5}\right) + \ell \cdot \left(\frac{q\ell^2}{5}\right)\right]$$

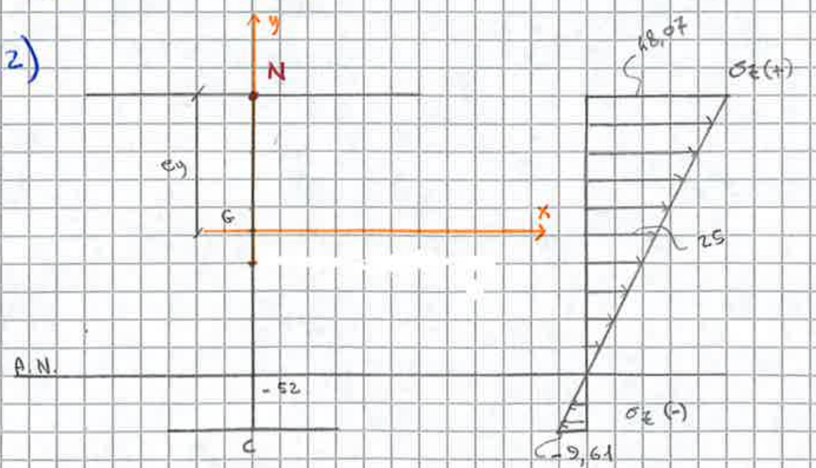
$$= \frac{\ell}{6}\left[-\frac{q\ell^3}{5} - \frac{q\ell^3}{5}\right] = \frac{\ell}{\cancel{6}_3} \cdot \left(-\frac{\cancel{2}q\ell^3}{5}\right) = -\frac{q\ell^4}{15} \cdot \frac{1}{EI}$$

$$H_E = \left(-\frac{4}{15} + \frac{1}{5} - \frac{1}{15}\right)\frac{q\ell^4}{EI} = \left(\frac{-4 + 3 - 1}{15}\right)\frac{q\ell^4}{EI} = -\frac{2}{15}\frac{q\ell^4}{EI}$$

ES. 3

3.1) ok (vedi compito)

3.2)



$$N = 22{,}5 \text{ kN} = 22{,}5 \cdot 10^3 \text{ N}$$

$$\sigma_z = \frac{N}{A} + \frac{M_x}{I_x} \cdot y - \cancel{\frac{M_y}{I_y} \cdot x}$$

$$M_x = N \cdot e_y = N \cdot (120 - 72) = N \cdot 48$$

$$M_y = -N \cdot e_x = 0$$

$$\text{A.N.} \quad 1 + \frac{e_y}{\rho_x^2} \cdot y + \underbrace{\cancel{\frac{e_x}{\rho_y^2} \cdot x}}_{=0} = 0 \qquad \rho_x^2 = \frac{I_x}{A} = \frac{2\,246\,400}{900} = 2\,496$$

$$1 + \frac{48}{2\,496} \cdot y = 0 \rightarrow 1 + 0{,}0192\, y = 0 \rightarrow y = -\frac{1}{0{,}0192} = -52{,}1$$

$S_x(72) = -216(72) + \frac{3}{2}(72)^2 - 12960 = -15552 + 7776 - 12960$
$= -20736$

$S_x(0) = -12960$

$$\tau_{zx}(\xi_1) = \frac{T_y \cdot (144 \cdot \xi_1)}{3 \cdot (2246400)} = \frac{15 \cdot 10^3 \cdot (144) \cdot \xi_1}{3\,(2246400)}$$

1) $\tau_{zx}(0) = 0$

2) $\tau_{zx}(60) = \frac{15 \cdot 10^3 \cdot 8640}{3 \cdot (2246400)} = 19{,}23$

$$\tau_{zy}(\xi_2) = \frac{15 \cdot 10^3 \cdot \left(144\,\xi_2 - \frac{3}{2}\xi_2^2 + 17280\right)}{3 \cdot (2246400)}$$

1) $\tau_{zy}(0) = \frac{15 \cdot 10^3 \cdot 17280}{3 \cdot (2246400)} = 38{,}46$

2) $\tau_{zy}(48) = \frac{15 \cdot 10^3 \cdot 20736}{3 \cdot (2246400)} = 46{,}15$

$$\tau_{zy}(\xi_1) = \frac{15 \cdot 10^3 \cdot (-216\,\xi_1)}{3 \cdot (2246400)}$$

1) $\tau_{zy}(0) = 0$

2) $\tau_{zy}(30) = \frac{15 \cdot 10^3 \cdot (-6480)}{3 \cdot (2246400)} = 14{,}42$

$$\tau_{zy}(72) = \frac{15 \cdot 10^3 \cdot (-20736)}{3 \cdot 2246400} = 46{,}15$$

$$\tau_{zy}(0) = \frac{15 \cdot 10^3 \cdot (-12960)}{3 \cdot 2246400} = 28{,}84$$

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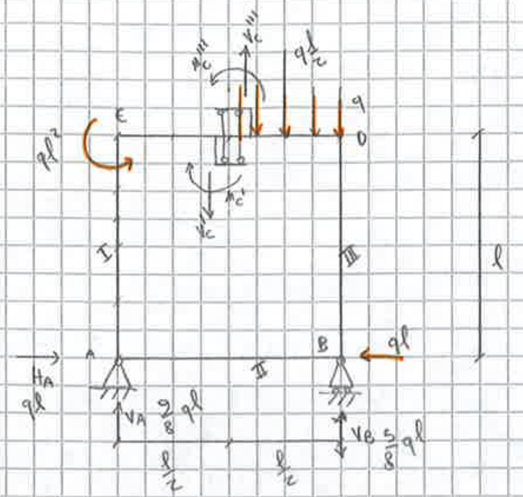
TEMA D'ESAME 28/6/2011 — COMPITO I

ES. 1)



$g = 3 \cdot 3 = 9$

$v = 3 + 2 + 2 + 2 = 9$

Isostatica

reazioni vincolari:

$\rightarrow \quad H_A - ql = 0 \quad \rightarrow \quad H_A = ql$

$\uparrow \quad V_A + V_B - \frac{ql}{2} = 0$

$V_B = \frac{4ql}{8} - \frac{9}{8} ql = -\frac{5}{8} ql$

ANT

$M(B) = -V_A \cdot l + ql^2 + \frac{ql}{2} \cdot \frac{l}{4} = 0$

$-V_A \cdot l + \frac{8}{8} ql^2 + \frac{ql^2}{8} = 0$

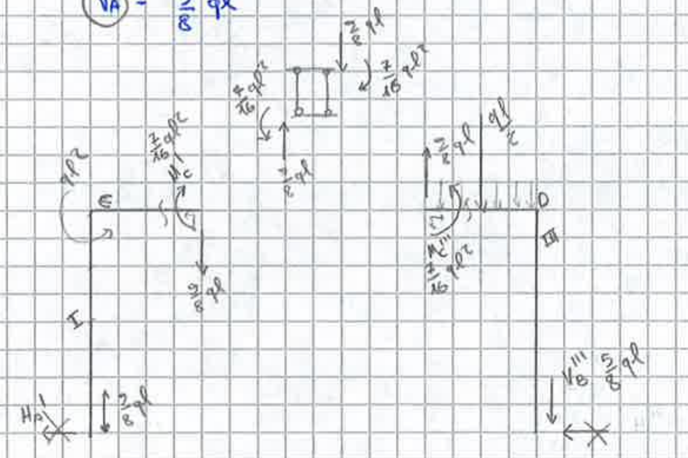
$V_A = \frac{9}{8} ql$

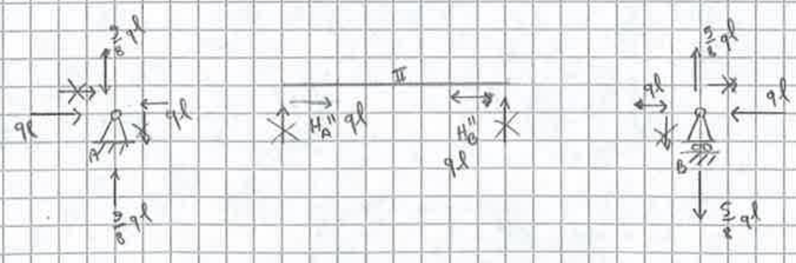
corpo I:

$M(C) = -\frac{7}{16} ql^2 + ql^2 - \frac{9}{8} ql \cdot \frac{l}{2} + H_A' \cdot l$

$= -\frac{7}{16} ql^2 + ql^2 - \frac{9}{16} ql^2 + H_A' \cdot l = 0$

$H_A' = \frac{(-7 + 16 - 9)}{16} ql = 0$





corpo III:

$\uparrow \quad \frac{9}{8} ql - \frac{ql}{2} - V_B''' = 0$

$V_B''' = \frac{9-4}{8} ql$

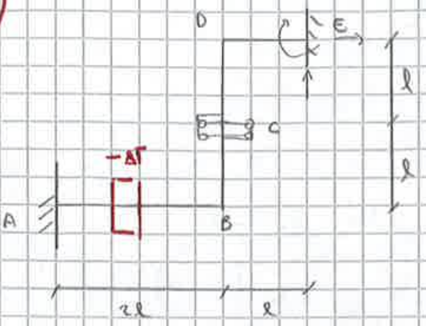
$= \frac{5}{8} ql$

corpo I:

$M(A) = ql^2 - M_c' - \frac{9}{8} ql \cdot \frac{l}{2} = 0$

$M_c' = ql^2 - \frac{9}{16} ql^2 = \frac{7}{16} ql^2$

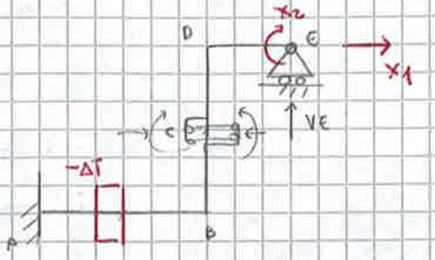
ES. 2)



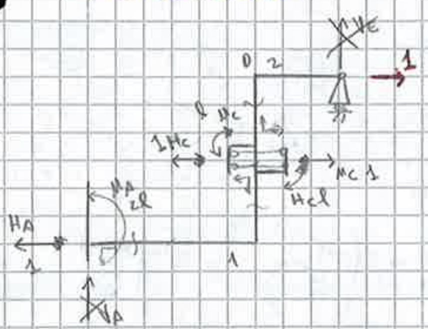
2 - volte iper

$g = 2 \cdot 3 = 6$

$V = 3 + 3 + 2 = 8$



sistema (e_1):



reazioni vincolari:

$\rightarrow \quad H_A + 1 = 0 \quad \rightarrow \quad H_A = -1$

$\uparrow \quad V_A + V_E = 0$

$\circlearrowright M(E) = -V_A \cdot 3l - 1 \cdot 2l + M_A = 0$

corpo 2: $\circlearrowright M(E) = -1 \cdot l - M_C = 0$

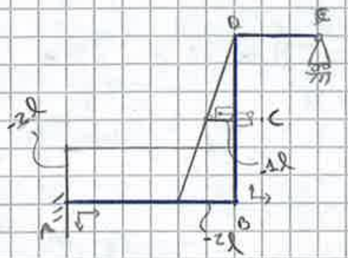
$M_C = -l$

corpo I: $A \circlearrowright M(A) = M_A - l - 1 \cdot l = 0$

$M_A = 2l$

$-V_A \cdot 3l - \cancel{2l} + \cancel{2l} = 0$

$V_A = 0$



M_1

$\overline{CB}$ $\quad M = l + 1 \cdot z$

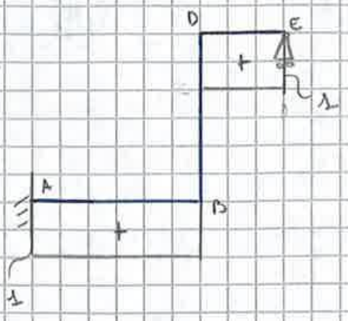
$M(0) = M_C = l$

$M(l) = M_B = 2l$

$\overline{CD}$ $\quad M = -l + 1 \cdot z$

$M(0) = M_C = -l$

$M(l) = M_D = 0$



N_1

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$$\int_A^B M_1 M_2 \, dz = \frac{2l}{6}\left(1\cdot(-1)+4(1)(-1)+(1)(-1)\right) = \frac{2l}{6}\left(-1-4-1\right) = -2l$$

$$\int_A^B M_1 M_2 \, dz = \frac{2l}{6}\left((-2)(-1)+4(-2)(-1)+(-2)(-1)\right) = \frac{2l}{6}\left(+2+8+2\right) = 4l^2$$

$$\int_B^C M_1 M_2 \, dz = \frac{2l}{6}\left((-2)(-1)+4(-1)(-1)\right) = \frac{2l}{6}\left(+2+4\right) = 2l^3$$

$$\int_S \frac{M_1 M_2}{EI} dz = \frac{6l^2}{EI}$$

$$\int_A^B M_2^2 \, dz = \frac{2l}{6}\left((-1)^2+4(-1)^2+(-1)^2\right) = 2l$$

$$\int_B^C M_2^2 \, dz = \frac{2l}{6}\left(\quad // \quad\right) = 2l$$

$$\int_C^D M_2^2 \, dz = \frac{l}{6}\left((-1)^2+4(-1)^2+(-1)^2\right) = l$$

$$\int_S \frac{M_2^2}{EI} dz = \frac{5l}{EI}$$

$$\begin{cases} 2l\,\varepsilon_t + \frac{32}{3}\frac{l^3}{EI}X_1 + 6\frac{l^2}{EI}X_2 = 0 \\ 6\frac{l^2}{EI}X_1 + 5\frac{l}{EI}X_2 = 0 \end{cases}$$

$$X_1 = -5\frac{\cancel{l}}{\cancel{EI}}X_2 \cdot \frac{\cancel{EI}}{l^2 6} = -\frac{5}{6l}X_2$$

$$2l\,\varepsilon_t + \frac{32}{3}\frac{l^3}{EI}\left(-\frac{5}{6}\frac{X_2}{l}\right) + 6\frac{l^2}{EI}X_2 = 0$$

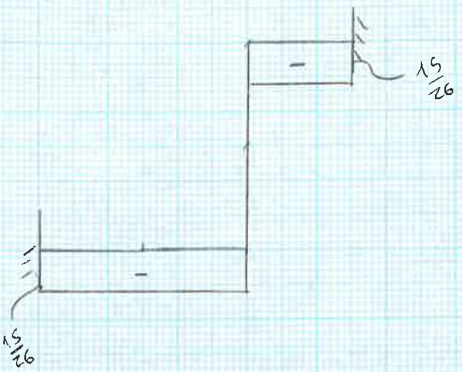
$$2l\,\varepsilon_t - \frac{160}{18}\frac{l^2}{EI}X_2 + 6\frac{l^2}{EI}X_2 = 0$$

$$\frac{-160\,l^2X_2 + 108\,l^2X_2}{18\,EI} = -2l\,\varepsilon_t$$

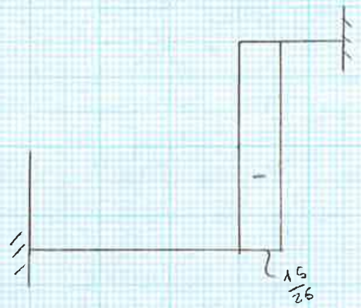
$$-\frac{52}{18}\frac{l^2X_2}{EI} = -2l^2\varepsilon_t \rightarrow X_2 = \cancel{2}\cdot\frac{18}{\frac{52}{26}}\cdot\cancel{l}\varepsilon_t\cdot\frac{EI}{\cancel{l}}$$

$$X_1 = -\frac{5}{{}_2 6}\cdot\frac{9^3}{13}\frac{\varepsilon_t EI}{l} = -\frac{15}{26}\frac{\varepsilon_t EI}{l}$$

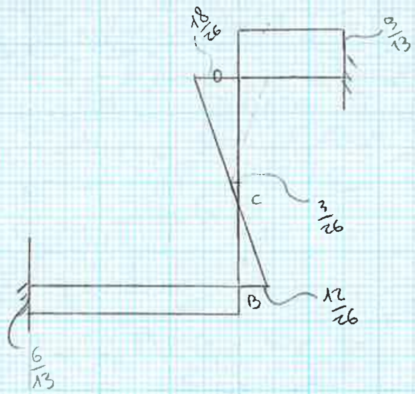
$$X_2 = \frac{9}{13}\varepsilon_t EI$$



$$\frac{N}{\frac{6+EI}{\ell}}$$



$$\frac{T}{\frac{6+EI}{\ell}}$$



$\overline{BC}$ $M = \frac{15}{26}\frac{6+EI}{\ell}\cdot z - \frac{3}{26}\frac{6+EI}{\ell}$

$\overline{CD}$ $M = -\frac{15}{26}\frac{6+EI}{\ell}\cdot z - \frac{3}{26}\frac{6+EI}{\ell}$

grafici ok
ma ribaltati

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$$\theta_0 = \frac{1}{2} \operatorname{arctg} \frac{2 \cdot \overset{I_{xy}}{86,49}}{(\underset{I_y}{333,94} - \underset{I_x}{480,30})} = -24,78°$$

$$\begin{matrix} I_\xi \\ I_\eta \end{matrix} = \frac{480,30 + 333,94}{2} \pm \frac{1}{2}\sqrt{(480,30 - 333,94)^2 + 4(86,49)^2}$$

$$= 406,62 \pm \frac{1}{2}\cancel{\sqrt{}}\ 113,62$$

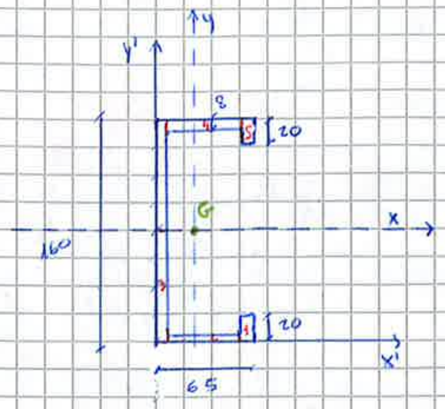
$I_\xi = 520,23$

$I_\eta = 293$

$I_x > I_y$

$I_\xi > I_\eta$

ES. 2



l'asse x è di simmetria perciò il baricentro G starà su di esso:

y_G noto $= \frac{160}{2}$ ⇒ distanza da asse x'

$x_G = \frac{S_{y'}}{A}$ ⇒ distanza da asse y'

asse di simmetria
(simmetria assiale retta)

Scomposta la figura in 5 rettangoli si ha:

$$S_{y'_1} = A \cdot x_{G1} = 20 \cdot 8 \cdot \left(65 - \frac{8}{2}\right) = 9'760$$

$$S_{y'_2} = A \cdot x_{G2} = (65 - 8 \cdot 2) \cdot 8 \cdot \left(\frac{65}{2}\right) = 12'740$$

$$S_{y'_3} = A \cdot x_{G3} = (160 \cdot 8) \cdot \frac{8}{2} = 5'120$$

$$S_{y'_{tot}} = 5'120 + 2 \cdot [9'760 + 12'740] = 50'120 \ mm^3$$

$$A_{tot} = 2 \cdot (20 \cdot 8 + (65 - 16) \cdot 8) + 160 \cdot 8 = 1'280 + 1'104 = 2'384 \ mm^2$$

$$x_G = \frac{S_{y'}}{A} = \frac{50'120}{2'384} = 21 \ mm$$

G(21 ; 80) → coordinate G rispetto ad x' e y'

Momenti di inerzia rispetto ad x'y' $\underbrace{\text{(assi baricentrici e principali)}}_{\text{assi centrali}}$:

I_x)

$$I_{x_G}^{(1)} = I_{xG_1} + A \cdot (y_G - y_{G1})^2 = \frac{20^3 \cdot 8}{12} + (20 \cdot 8) \cdot (10 - 80)^2 = 789'333,3$$

$$I_{x_G}^{(2)} = I_{xG_2} + A(y_G - y_{G2})^2 = \frac{8^3 \cdot 49}{12} + (49 \cdot 8) \cdot (80 - 4)^2 = 2'266'282,6$$

$$I_{x_G}^{(3)} = I_{xG_3} + A(y_G - y_{G3})^2 = \frac{160^3 \cdot 8}{12} + \cancel{(160 \cdot 8) \cdot (80 - 80)} = 2'730'666,6$$

$$I_{x_G \, tot} = 2 \cdot (I_{x_G}^{(1)} + I_{x_G}^{(2)}) + I_{x_G}^{(3)} = 8'841'898,4 \ mm^4$$

Potevo anche calcolarlo come differenza tra il momento di inerzia del rettangolo circoscritto e la somma tra il momento del rettangolo interno e del rettangolino di dimensioni 8 x 120:

$$I_x = \frac{65 \cdot 160^3}{12} - \left(\frac{49 \cdot 144^3}{12} + \frac{8 \cdot 120^3}{12}\right) = 8'841'898 \ mm^4$$

$I_{x_G\,tot} = 691\,934{,}08$

$I_{y_G}^{(1)} = I_{yG1} + A(x_G - x_{G1})^2 = \frac{45^3 \cdot 20}{12} + (45 \cdot 20) \cdot (16{,}33 - 22{,}5)^2 = 186\,137{,}01$

$I_{y_G}^{(2)} = I_{yG2} + A(x_G - x_{G2})^2 = \frac{15^3 \cdot 50}{12} + (50 \cdot 15) \cdot (16{,}33 - 7{,}5)^2 = 72\,539{,}17$

$I_{y_G}^{(3)} = I_{yG3} + A(x_G - x_{G3})^2 = \frac{20^3 \cdot 20}{\cancel{36}} + \frac{(20 \cdot 20)}{2} \cdot (16{,}33 - 21{,}67)^2 = 10\,147{,}56$

$I_{y_{G\,tot}} = 268\,823{,}74 \quad mm^4$

$I_{x_G y_G}^{(1)} = \cancel{I_{x_{G1} y_{G1}}}_{=0} + A \cdot (y_G - y_{G1})(x_G - x_{G1}) = +(20 \cdot 45)(25{,}99 - 10)(16{,}33 - 22{,}5)$
$= -88\,792{,}47$

$I_{x_G y_G}^{(2)} = \cancel{I_{x_{G2} y_{G2}}}_{=0} + A \cdot (y_G - y_{G2})(x_G - x_{G2}) = +(50 \cdot 15) \cdot (25{,}99 - 45) \cdot (16{,}33 - 7{,}5)$
$= -125\,893{,}72$

$I_{x_G y_G}^{(3)} = I_{x_{G3} y_{G3}} + A(y_G - y_{G3})(x_G - x_{G3}) = +\frac{20^2 \cdot 20^2}{72} + \frac{(20 \cdot 20)}{2}(25{,}99 - 26{,}66)(16{,}33 - 21{,}66)$
$= -1\,508{,}0$

$I_{x_G y_G} = -216\,194{,}2 \quad mm^4$

$\theta_0 = \frac{1}{2} \arctan\left(\frac{2\, I_{x_G y_G}}{I_{yy} - I_{xx}}\right) = \frac{1}{2} \arctan\left(\frac{2 \cdot (-216\,194{,}2)}{268\,823{,}74 - 691\,934{,}08}\right) =$
$= \frac{1}{2} \arctan(1{,}0219) = 22{,}81°$

$I_\xi = \frac{(I_x + I_y)}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4 I_{xy}^2} = 782\,852 \quad mm^4$

$I_\eta = \frac{(I_x + I_y)}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4 I_{xy}^2} = 177\,896 \quad mm^4$

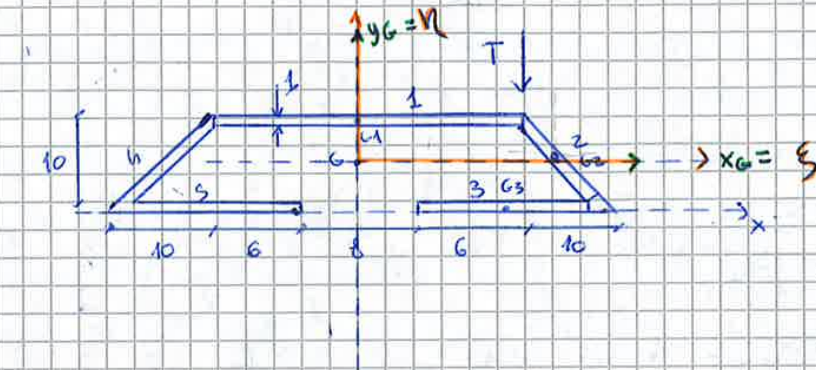
↳ assi baricentrali ≠ da assi principali

$$I_\xi = \frac{(194'667 + 72'000)}{2} + \frac{1}{2}\sqrt{(194'667 - 72000)^2 + 4\cdot(36'000)^2} = 204'000$$

$$I_\eta = 62'000$$

$$\theta_0 = \frac{1}{2}\arctan\left(\frac{2\cdot 36'000}{72'000 - 194'667}\right) = -15,2°$$

ES. 5 esame 26 luglio 2011



$$S_x^{(1)} = y_{G1}\cdot A_1 = 10\cdot(20) = 200$$
$$S_x^{(2)} = y_{G2}\cdot A_2 = 5\cdot(10\cdot\sqrt{2}) = 70,71$$
$$S_x^{(3)} = y_{G3}\cdot A_3 = 0\cdot(16\cdot 1) = 0$$

$$S_{xTOT} = 270,71 \quad 341,42$$
$$A_{TOT} = 20 + 10\sqrt{2} + 16 = 50,14$$
80,28

$$y_G = \frac{S_{xTOT}}{A_{TOT}} = \cancel{\frac{270,71}{50,14}} = \cancel{5,39}\ 4,25 \qquad G = (0\ ;\ \cancel{5,39}\ 4,25)$$

$$I_{x_G}^{(1)} = I_{xG1} + A(y_G - y_{G1})^2 = 0 + (20)\cdot(\cancel{5,39}\,4,25 - 10)^2 = \cancel{425,042}\ 661,25$$

$$I_{x_G}^{(2)} = I_{xG2} + A(y_G - y_{G2})^2 = \frac{1\cdot(10\sqrt{2})^3}{12}\sin^2 45° + (10\sqrt{2})\cdot(\cancel{5,39}\,4,25 - 5)^2 = \cancel{123,36}\ \cancel{167,124}\ 125,80$$

$$I_{x_G}^{(3)} = I_{xG3} + A(y_G - y_{G3})^2 = 0 + (16\cdot 1)\cdot(\cancel{5,39}\,4,25 - 0)^2 = \cancel{464,83}\ 289$$

$$I_{xTOT} = I_{x_G}^{(1)} + 2\left(I_{x_G}^{(2)} + I_{x_G}^{(3)}\right) = 1'490,85 = I_\xi$$

$$I_{y_G}^{(1)} = I_{yG1} + A(x_G - x_{G1})^2 = \frac{1\cdot 20^3}{12} + (20)\cdot(0) = 666,66$$

$$I_{y_G}^{(2)} = I_{yG2} + A(x_G - x_{G2})^2 = \frac{(10\sqrt{2})^3\cdot 1}{12}\cos^2 45 + (10\sqrt{2})\cdot 1\,(0 - 15)^2 = 3'299,83$$

$$I_{y_G}^{(3)} = I_{yG3} + A(x_G - x_{G3})^2 = \frac{16^3\cdot 1}{12} + (16)(0 - 12)^2 = 2'645,33$$

$$I_{yTOT} = 12'556,92 = I_\eta \qquad I_{x_G y_G} = 0 \quad \times \text{ simmetria}$$

(gli assi baricentrici sono anche assi principali ⇒ assi centrali)

$$I_{y_G}^{(1)} = I_{y_{G1}} + A(\underbrace{x_G - x_{G1}}_{=0})^2 = \frac{100^3 \cdot 5}{12} = 416'666,66$$

$$I_{y_G}^{(2)} = I_{y_{G2}} + A(\underbrace{x_G - x_{G2}}_{=0})^2 = 0$$

$$I_{y_G}^{(3)} = I_{y_{G3}} + A(\underbrace{x_G - x_{G3}}_{=0})^2 = \frac{50^3 \cdot 5}{12} = 52'083,33$$

$$I_{y_G}^{(4)} = I_{y_{G4}} + A(\underbrace{x_G - x_{G4}}_{=0})^2 = \cancel{\frac{47,5^3 \cdot 5}{12}} \; 0$$

$$I_{y_{G\,tot}} = 468'749,99$$

$$I_{x_G y_G}^{(1)} = I_{x_G y_G 1} + A(x_G - x_{G1})(y_G - y_{G1}) = 0 + 0 = 0$$

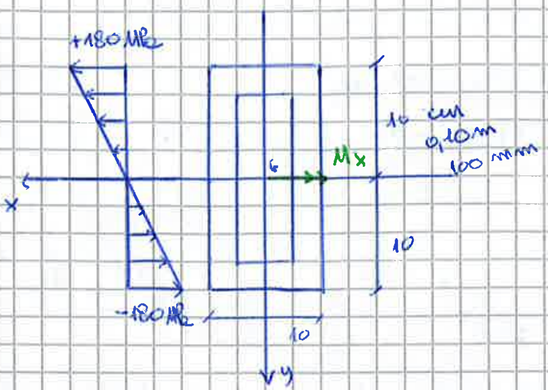
$$I_{x_G y_G}^{(2)} = I_{x_G y_G 2} + A(x_G - x_{G2})(y_G - y_{G2}) = 0 + 0 = 0$$

$$I_{x_G y_G}^{(3)} = 0$$

$$I_{x_G y_G}^{(4)} = 0$$

$$I_{x_G y_G} = 0$$

gli assi baricentrici sono anche assi principali (assi centrali)



I_x : 3'333,33 cm^4
3'333,33 $\cdot 10^4$ mm^4

$$\sigma_z = \frac{M_x}{I_x} \cdot y$$

M_x : -60 kN·m
$-60 \cdot 10^3$ kN·mm

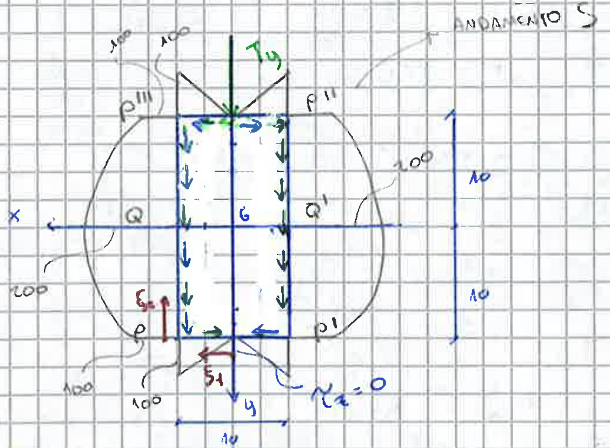
$$\sigma_z = \frac{-60 \cdot 10^3}{3'333,33 \cdot 10^4} \cdot y = -0,0018 \cdot y \ \frac{kN}{mm^3}$$
$$= -1,8 \cdot y \ \frac{N}{mm^3}$$

tensioni max in corrispondenza del lembo inferiore e superiore della trave

A.N. = asse x ⇒ y = 0 $\sigma_z = 0$

$$\sigma_{max} = \sigma_z(-100) = -1,8 \cdot (-100) = 180 \ \frac{N}{mm^2} \ (MPa)$$

$$\sigma_{max} = \sigma_z(100) = -1,8 \cdot (100) = -180 \ \frac{N}{mm^2} \ (MPa)$$



$$\tau_z = \frac{T_y \cdot S_x^{(A')}}{2 I_x \cdot b}$$

→ si può usare Jourawsky solo perché la sezione è simmetrica rispetto ad asse y

coordinate ξ_2 ξ_1
↳ si misura da punto in cui $\tau = 0$

PP':
$$S_x^{A'}(\xi_1) = 2 \cdot (1 \cdot \xi_1 \cdot 10) = 20\,\xi_1 \ cm^3 \qquad S_x^{A'}(5) = 2 \cdot 5 \cdot 10 = 100 \ cm^3$$

PP''':
$$S_x^{A'}(\xi_2) = 2 \cdot \left(\left(1 \cdot \xi_2 \cdot (10 - \xi_2/2)\right) + 5 \cdot 10\right)$$
$$= 20\,\xi_2 - \xi_2^2 + 100$$ ⤳ parabolico

momento statico max: $\frac{d S_x}{d \xi_2} = 0$ $20 - 2\xi_2 = 0$ → $\xi_2 = 10$ → in corrisp. delle 2 corde orizzontali

$$S_x^{A'}(10) = 20 \cdot 10 - 10^2 + 100 = 200 \rightarrow S_{x\,max}^{A'}$$

$$S_x^{A'}(0) = 100$$

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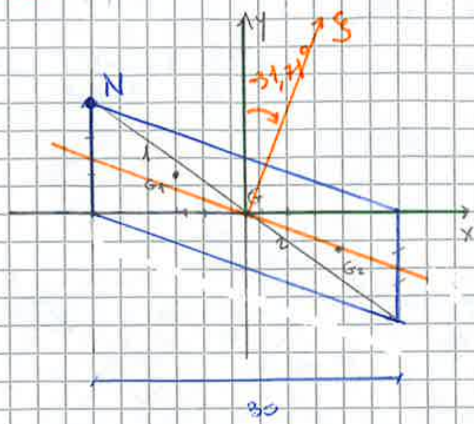
ES. 8 esame 26 GIUGNO 2012 es. 3



$A_{1,2} = \frac{30 \cdot 15}{2} = 225$

$A_{TOT} = 225 \cdot 2 = 450$

$S_x^{(1)} = y_{G1} \cdot A_1 = 5 \cdot \frac{30 \cdot 15}{2} = 1125$

$S_x^{(2)} = y_{G2} \cdot A_2 = (-5) \cdot \frac{30 \cdot 15}{2} = -1125$

→ sezione a simmetria polare (baricentro = centro geometrico)

1)

$I_{xG}^{(1)} = I_{xG1} + A \cdot y_0^2 = I_{xG1} + A(y_G - y_{G1})^2 = \frac{15^3 \cdot 30}{36} + \frac{30 \cdot 15}{2}(0-5)^2 = 8437,5$

$I_{xG}^{(2)} = I_{xG2} + A \cdot y_0^2 = I_{xG2} + A(y_G - y_{G2})^2 = \frac{15^3 \cdot 30}{36} + \frac{30 \cdot 15}{2}(0+5)^2 = 8437,5$

$I_{xG\,TOT} = 16875 \text{ mm}^4$

$I_{yG}^{(1)} = I_{yG1} + A \cdot (x_G - x_{G1})^2 = \frac{15 \cdot 30^3}{36} + \left(\frac{30 \cdot 15}{2}\right)(0+5)^2 = 16875$

$I_{yG}^{(2)} = I_{yG2} + A \cdot (x_G - x_{G2})^2 = \frac{15 \cdot 30^3}{36} + \left(\frac{30 \cdot 15}{2}\right)(0-5)^2 = 16875$

$I_{yG\,TOT} = 33750$

$I_{xGyG}^{(1)} = I_{xG1yG1} + A \cdot (y_G - y_{G1})(x_G - x_{G1}) = -\frac{30^2 \cdot 15^2}{72} + 225 \cdot (0-5) \cdot (0+5)$
$= -2812,5 - 5625 = -8437,5$

$I_{xGyG}^{(2)} = I_{xG2yG2} + A \cdot (y_G - y_{G2})(x_G - x_{G2}) = -\frac{30^2 \cdot 15^2}{72} + 225(0+5)(0-5) = -8437,5$

$I_{xGyG\,TOT} = -16875$

$I_\xi = \frac{(I_x + I_y)}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4 \cdot I_{xy}^2} = \frac{(16875 + 33750)}{2} - \frac{1}{2}\sqrt{(16875 - 33750)^2 + 4 \cdot (16875)^2}$

$= 25312,5 - 18866,82 = 6445,68$

$I_\eta = \frac{(I_x + I_y)}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4 \cdot I_{xy}^2} = 25312,5 + 18866,82 = 44179,32$

tensioni max nei punti + distanti da A.N. : N e K

$$\sigma_z = \frac{N}{A} + \frac{M_x}{I_\xi}\eta - \frac{M_y}{I_\eta}\xi$$

$M_x = N \cdot e_y$

$M_y = -N \cdot e_x$

$N \begin{cases} \eta = 20,64 \\ \xi = 4,88 \end{cases}$

$\begin{cases} e_y = 20,64 \\ e_x = 4,88 \end{cases}$

$K \begin{cases} \eta = -20,64 \\ \xi = -4,88 \end{cases}$

$I_\eta = 44\,179,32$

$I_\xi = 6\,445,68$

$A = 450$

K)

$$\sigma_z = \frac{N}{450} + \frac{N \cdot 20,64}{6\,445,68} \cdot (-20,64) - \frac{N \cdot 4,88}{44\,179,32} \cdot (-4,88)$$

$$\sigma_z = N \left(\frac{1}{450} - \frac{426,01}{6\,445,68} + \frac{23,81}{44\,179,32} \right)$$

$$\sigma_z = N(-0,00396) \quad \rightarrow \quad N = \frac{\sigma_z}{-0,00396}$$

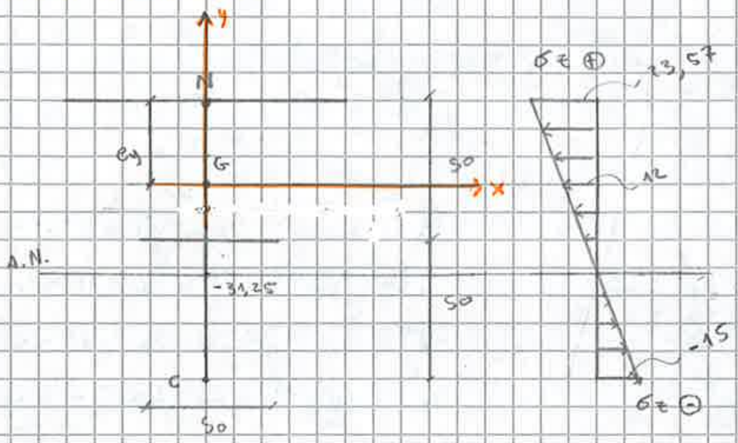
$\sigma_{amm} = 255\ \text{MPa}$ $\frac{N}{mm^2}$

$$N = \frac{255}{-0,00396} = 64\,393,93$$

$$I^{(1)}_{x_G y_G} = I_{x_{G1} y_{G1}} + A(y - y_{G1})\cancel{(x - x_{G1})}^{=0} = 0$$
$$I^{(2)}_{x_G y_G} = I_{x_{G2} y_{G2}} + A(y - y_{G2})\cancel{(x - x_{G2})}^{=0} = 0$$
$$I^{(3)}_{x_G y_G} = I_{x_{G3} y_{G3}} + A(y - y_{G3})\cancel{(x - x_{G3})}^{=0} = 0$$
$$I^{(4)}_{x_G y_G} = \underbrace{I_{x_{G4} y_{G4}}}_{\text{tutti} = 0} + A(y - y_{G4})\cancel{(x - x_{G4})}^{=0} = 0$$

assi baricentrici sono anche assi principali (assi centrali)

SFORZO NORMALE N:



$N = 15\ kN = 15 \cdot 10^3\ N$

$$\sigma_z = \frac{N}{A} + \frac{M_x}{I_x} y - \cancel{\frac{M_y}{I_y}} \cdot x$$

$M_x = N \cdot e_y = N \cdot (100 - 70) = 30 \cdot N$

$M_y = -N e_x = 0$

$$\underline{\underline{A.N.}} \quad 1 + \frac{e_y}{\rho_x^2} \cdot y + \cancel{\frac{e_x}{\rho_y^2}}^{=0} x = 0$$

$$\rho_x^2 = \frac{I_x}{A} \qquad \rho_y^2 = \frac{I_y}{A}$$

$$\rho_x^2 = \frac{1\,166\,666{,}66}{1\,250} = 933{,}33 \qquad e_y = 30$$

$$\rho_y^2 = \frac{468\,749{,}99}{1\,250} = 374{,}99$$

$$1 + \frac{30}{933{,}33} y = 0 \qquad 1 + 0{,}032\, y = 0$$

$$y = -\frac{1}{0{,}032} = -31{,}25$$

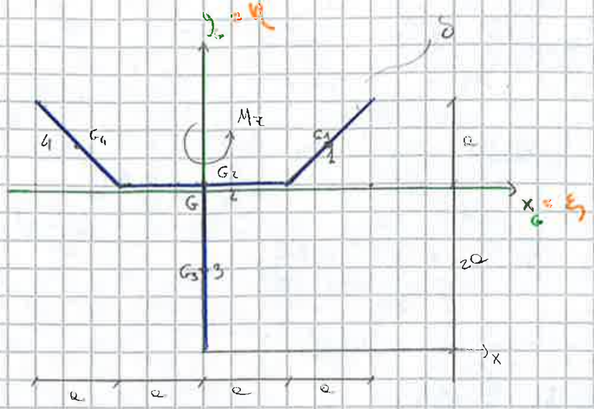
$$\sigma_z(y) = \sigma_z(N) = \sigma_z(30) = \frac{N}{A} + \frac{N \cdot e_y}{I_x} \cdot 30 = \frac{15 \cdot 10^3}{1\,250} + \frac{15 \cdot 10^3 \cdot 30}{1\,166\,666{,}66} \cdot 30 =$$
$$= 12 + 11{,}57 = 23{,}57 \frac{N}{mm^2} = MPa$$

$$\sigma_z(y) = \sigma_z(C) = \sigma_z(-70) = \frac{N}{A} + \frac{N \cdot e_y}{I_x} (-70) = 12 + \frac{15 \cdot 10^3 \cdot 30}{1\,166\,666{,}66} (-70) =$$
$$= 12 - 27 = -15 \frac{N}{mm^2} = MPa$$

$$\sigma_z(y) = \sigma_z(G) = \sigma_z(0) = 12$$

TEMA D'ESAME 29/9/1993

ES. 3)



$A_1 = a\sqrt{2}\,\delta$

$A_2 = 2a\,\delta$

$A_3 = 2a \cdot \delta$

$A_4 = a\sqrt{2} \cdot \delta$

$$A_{TOT} = \sum_{i=1}^{4} A_i = 2(a\sqrt{2}\delta) + 2 \cdot (2a\delta) = 2\sqrt{2}\,a\delta + 4a\delta = 2a\delta(\sqrt{2}+2)$$

$S_x^{(1)} = y_{G1} \cdot A^{(1)} = \left(2a + \frac{a}{2}\right) \cdot (a\sqrt{2}\,\delta)$

$S_x^{(2)} = y_{G2} \cdot A^{(2)} = 2a \cdot (2a\,\delta)$

$S_x^{(3)} = y_{G3} \cdot A^{(3)} = a \cdot (2a\,\delta)$

$S_x^{(4)} = y_{G4} \cdot A^{(4)} = \left(2a + \frac{a}{2}\right) \cdot (a\sqrt{2}\,\delta)$

$$S_{TOT} = (4a \cdot a\sqrt{2}\delta + a \cdot a\sqrt{2}\delta) + 4a^2\delta + 2a^2\delta = 4\sqrt{2}a^2\delta + \sqrt{2}a^2\delta + 6a^2\delta = 5\sqrt{2}a^2\delta + 6a^2\delta$$

$$y_G = \frac{5\sqrt{2}a^2\delta + 6a^2\delta}{2\sqrt{2}a\delta + 4a\delta} = \frac{a^2\delta(5\sqrt{2}+6)}{a\delta(2\sqrt{2}+4)} = 1{,}914\,a \qquad G(0; 1{,}914a)$$

$$I_{xG}^{(1)} = I_{xG1} + A^{(1)}(y_G - y_{G1})^2 = \frac{1}{12}\delta l^3 \left(\frac{\sqrt{2}}{2}\right)^2 + (a\sqrt{2}\delta)\left(1{,}914a - \left(2a + \frac{a}{2}\right)\right)^2$$

$$= \frac{1}{12}\delta l^3 \frac{1}{2} + (a\sqrt{2}\delta) \cdot (1{,}914a - 2a - 0{,}5a)^2$$

$$= \frac{1}{24}\delta l^3 + (a\sqrt{2}\delta) \cdot (-0{,}586a)^2 = \frac{1}{24}\delta(a\sqrt{2})^3 + 0{,}\cancel{448}\,486\,a^3\delta$$

$$= 0{,}117\,a^3\delta + 0{,}\cancel{448}\,486\,a^3\delta = \cancel{0{,}4485 a^3\delta}\ 0{,}602\,a^3\delta$$

$$I_{xG}^{(2)} = I_{xG2} + A^{(2)}(y_G - y_{G2})^2 = 0 + (2a\delta)(1{,}914a - 2a)^2 = 0{,}0147\,a^3\delta$$

$$I_{xG}^{(3)} = I_{xG3} + A^{(3)}(y_G - y_{G3})^2 = \frac{\delta \cdot (2a)^3}{12} + (2a\delta)(1{,}914a - a)^2 =$$

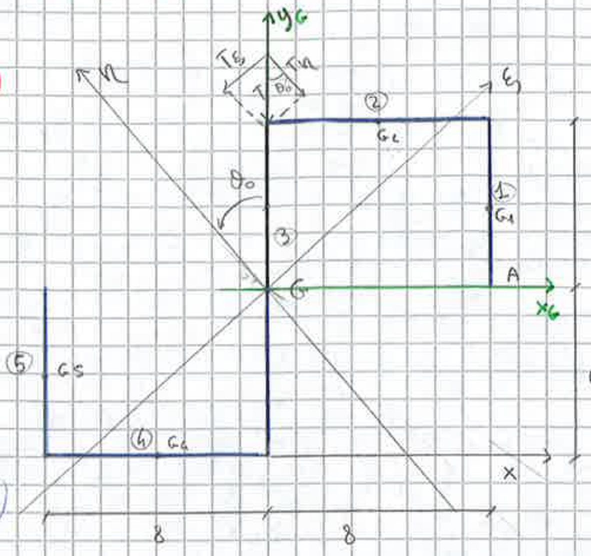
$$= 0{,}67\,a^3\delta + 1{,}67\,a^3\delta = 2{,}34\,a^3\delta$$

$$I_{x_{TOT}} = 2 \cdot (0{,}602\,a^3\delta) + 0{,}0147\,a^3\delta + 2{,}34\,a^3\delta = 3{,}56\,a^3\delta$$

TEMA D'ESAME 22/9/1994

ES. 3)

$\delta = 1$



$A^{(1)} = 6 \cdot 1 = 6$

$A^{(2)} = 8 \cdot 1 = 8$

$A^{(3)} = 12$

$A^{(4)} = 8$

$A^{(5)} = 6$

$A_{tot} = 40$

→ sezione a simmetria polare : baricentro = centro geometrico

$$I_{x_G}^{(1)} = I_{x_{G1}} + A^{(1)} (y_G - y_{G1})^2 = \frac{1 \cdot 6^3}{12} + 6 \cdot (0-3)^2 = 72$$

$$I_{x_G}^{(2)} = I_{x_{G2}} + A^{(2)} (y_G - y_{G2})^2 = 0 + 8 \cdot (0-6)^2 = 288$$

$$I_{x_G}^{(3)} = I_{x_{G3}} + A^{(3)} (y_G - y_{G3})^2 = \frac{1 \cdot 12^3}{12} + 12(0-0)^2 = 144$$

$$\to I_{x_G\,tot} = 2 \cdot 72 + 2 \cdot 288 + 144 = 864$$

$$I_{y_G}^{(1)} = I_{y_{G1}} + A^{(1)} (x_G - x_{G1})^2 = 0 + 6 \cdot (0-8)^2 = 384$$

$$I_{y_G}^{(2)} = I_{y_{G2}} + A^{(2)} (x_G - x_{G2})^2 = \frac{1 \cdot 8^3}{12} + 8 \cdot (0-4)^2 = 170{,}67$$

$$I_{y_G}^{(3)} = I_{y_{G3}} + A^{(3)} (x_G - x_{G3})^2 = 0 + 12(0-0)^2 = 0$$

$$\to I_{y_G\,tot} = 2 \cdot 384 + 2 \cdot 170{,}67 = 1109{,}34$$

$$I_{x_G y_G}^{(1)} = I_{x_{G1} y_{G1}} + A^{(1)} (y_G - y_{G1})(x_G - x_{G1}) = 0 + 6(0-3)(0-8) = 144$$

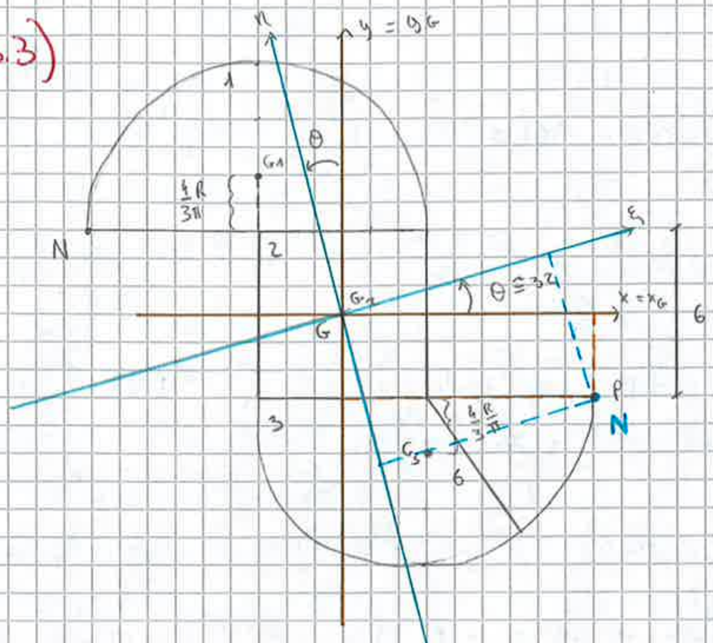
$$I_{x_G y_G}^{(2)} = I_{x_{G2} y_{G2}} + A^{(2)} (y_G - y_{G2})(x_G - x_{G2}) = 0 + 8(0-6)(0-4) = 192$$

$$I_{x_G y_G}^{(3)} = I_{x_{G3} y_{G3}} + A^{(3)} (y_G - y_{G3})(x_G - x_{G3}) = 0 + 12(0-0)(0-0) = 0$$

$$\to I_{x_G y_G\,tot} = 2(144 + 192) = 336 \cdot 2 = 672$$

TEMA D'ESAME 19/9/2011 COMPITO I

es. 3)



$$G_1 = \left(-3\ ;\ 3 + \frac{4}{3\pi}\cdot 6\right) = \left(-3\ ;\ 3 + \frac{8}{\pi}\right)$$

$$G_2 = (0\ ;0)$$

$$G_3 = \left(3\ ;-\left(3 + \frac{4}{3\pi}6\right)\right) = \left(3\ ;\ -3 - \frac{8}{\pi}\right)$$

$$N = 10\ kN$$

$$A_1 = A_3 = \frac{\pi}{2}R^2 = \frac{\pi}{2}\cdot 6^2 = \frac{36\pi}{2} = 18\pi$$

$$A_2 = 6\cdot 6 = 36$$

$$A_{TOT} = 2\cdot 18\pi + 36 = 36\pi + 36 = 36(\pi + 1)$$

$$S_x^{(1)} = y_{G1}\cdot A^{(1)} = \left(3 + \frac{8}{\pi}\right)\cdot 18\pi = 313{,}65$$

$$S_x^{(2)} = y_{G2}\cdot A^{(2)} = 0$$

$$S_x^{(3)} = y_{G3}\cdot A^{(3)} = \left(-3 - \frac{8}{\pi}\right)\cdot 18\pi = -313{,}65$$

$$S_{x_{tot}} = 0 \qquad y_G = 0$$

G baricentro $(0\ ;0)$ → coincide con G2

$$S_{y_{tot}} = 0 \qquad x_G = 0$$

$$I_{xG}^{(1)} = I_{xG1} + A^{1}(y_G - y_{G1})^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)6^4 + (18\pi)\cdot \left(3 + \frac{8}{\pi}\right)^2$$

$$= 142{,}24 + 1739{,}63 = 1881{,}87$$

$$I_{xG}^{(3)} = I_{xG}^{(1)} = 1881{,}87$$

$$I_{xG}^{(2)} = I_{xG_2} + A(y_G - y_{G2})^2 = \frac{1}{12}\cdot 6\cdot 6^3 + 0 = 108$$

$$\rightarrow I_{xG_{tot}} = 3871{,}74$$

$$\sigma_z = \frac{N}{A} + \frac{M_x}{I_x} \cdot y - \frac{M_y}{I_y} \cdot x \quad \rightarrow \quad \sigma_z = \frac{N}{A} + \frac{M_\xi}{I_\xi}\eta - \frac{M_\eta}{I_\eta} \cdot \xi$$

$$M_\xi = N \cdot e_\eta \qquad M_\eta = -N \cdot e_\xi$$

$$P_{xy} = (\overset{x}{9} \; ; \overset{y}{-3}) \qquad P_{\xi\eta} = ?$$

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 9 \\ -3 \end{bmatrix}$$

$$\xi = -3\cos\theta + 9\sin\theta = 9\cos 32{,}66 + 3\sin 32{,}66 = 5{,}96$$
$$\eta = +3\sin\theta + 9\cos\theta = -9\sin 32{,}66 + 3\cos 32{,}66 = -7{,}38$$

$$P_{\xi\eta} = (5{,}96 ; -7{,}38) \qquad \begin{matrix} \xi = e_\xi \\ \eta = e_\eta \end{matrix}$$

$$M_\xi = 10^4 \cdot -7{,}38 = -73{,}8 \cdot 10^3$$
$$M_\eta = -10^4 \cdot 5{,}96 = -59{,}6 \cdot 10^3$$

misure in cm → mm

sforzo in kN → N

$$\sigma_z = \frac{10 \cdot 10^3}{36(\pi+1) \cdot 10^2} + \frac{-73{,}8 \cdot 10^3}{5078{,}47} \cdot \eta - \frac{(-59{,}6) \cdot 10^3}{937{,}01} \cdot \xi$$

$$= \cancel{0{,}0067{,}07 - 0{,}01453\eta + 0{,}0063608\xi} \qquad \rightarrow \text{ in } P(5{,}96 ; -7{,}38)$$

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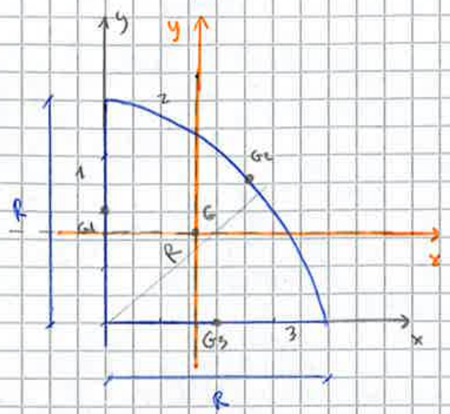
TEMA D'ESAME 10/5/1994

ES.3)



$G_1 = \left(0 ; \frac{R}{2}\right)$

$G_3 = \left(\frac{R}{2} ; 0\right)$

$G_2 = \left(\frac{2}{\pi}R ; \frac{2}{\pi}R\right)$

N.B. qualsiasi sistema di riferimento che passa per G è baricentrico ($S_x, S_y = 0$)

$$\left.\begin{aligned} A_1 &= R\delta \\ A_3 &= R\cdot\delta \\ A_2 &= \frac{\pi}{2}\delta R \end{aligned}\right\} \quad A_{tot} = R\delta\left(2+\frac{\pi}{2}\right) \to \frac{4+\pi}{2}$$

$G = (0{,}42R ; 0{,}42R)$

$$\left.\begin{aligned} S_x^{(1)} &= y_{G1}\cdot A^{(1)} = \frac{R}{2}\cdot R\delta = \frac{R^2\delta}{2} \\ S_x^{(2)} &= y_{G2}\cdot A^{(2)} = \frac{2}{\pi}R\cdot\left(\frac{\pi}{2}\delta R\right) = \delta R^2 \\ S_x^{(3)} &= y_{G3}\cdot A^{(3)} = 0 \end{aligned}\right\} \quad S_{x\,tot} = \frac{R^2\delta}{2} + \frac{2\delta R^2}{2} = \frac{3}{2}R^2\delta$$

$$y_G = \frac{\frac{3}{2}R^2\delta}{R\delta\left(2+\frac{\pi}{2}\right)} = \frac{3}{2}\cdot\frac{2}{4+\pi}R = \frac{3}{4+\pi}R = 0{,}42R$$

$$\left.\begin{aligned} S_y^{(1)} &= x_{G1}\cdot A^{(1)} = 0 \\ S_y^{(2)} &= x_{G2}\cdot A^{(2)} = \frac{2}{\pi}R\cdot\left(\frac{\pi}{2}\delta R\right) = \delta R^2 \\ S_y^{(3)} &= x_{G3}\cdot A^{(3)} = \frac{R}{2}\cdot R\delta = \frac{R^2\delta}{2} \end{aligned}\right\} \quad S_{y\,tot} = \frac{3}{2}R^2\delta$$

$$x_G = \frac{3}{4+\pi}R = 0{,}42R$$

$$I_{xG}^{(1)} = I_{xG1} + A^{(1)}(y_G - y_{G1})^2 = \frac{\delta\cdot R^3}{12} + R\delta\left(0{,}42R - \frac{R}{2}\right)^2 = \frac{1}{12}\delta R^3 + 6{,}4\cdot 10^{-3}R^3\delta = 0{,}089\,\delta R^3$$

• TRATTO BC :

$\overset{+}{\leftarrow}$ $N = \frac{ql}{4}$

$+\uparrow$ $T = \frac{ql}{4} - qz = q\left(\frac{l}{4} - z\right)$ $T(0) = T_B = \frac{ql}{4}$

$T(l) = T_C = q\left(\frac{l}{4} - l\right) = q\left(\frac{l}{4} - \frac{4l}{4}\right) = q\left(-\frac{3}{4}l\right) = -\frac{3}{4}ql$

$\circlearrowright^+$ $M = \frac{ql}{4} z + \frac{ql}{4} \cdot l - qz \cdot \frac{z}{2} = q\left(\frac{lz}{4} + \frac{l^2}{4} - \frac{z^2}{2}\right)$

$M(0) = M_B = \frac{ql^2}{4}$

$M(l) = M_C = q\left(\frac{l^2}{4} + \frac{l^2}{4} - \frac{l^2}{2}\right) = q\left(\cancel{\frac{l^2 + l^2 - 2l^2}{4}}\right) = 0$

altro punto parabola :

$\frac{dM}{dz} = 0$ $T = 0$ $q\left(\frac{l}{4} - z\right) = 0$ $\rightarrow z = \frac{l}{4}$ $\rightarrow M\left(\frac{l}{4}\right) = \frac{9}{32} ql^2$

• TRATTO DC

$\overset{+}{\leftarrow}$ $N = 0$

$+\uparrow$ $T = -ql$

$\circlearrowright^+$ $M = ql^2 - qlz = ql(l - z)$ $M(0) = M_D = ql^2$

$M(l) = M_C = \cancel{ql^2} - \cancel{ql^2} = 0$

• TRATTO CE

$\overset{+}{\leftarrow}$ $N = \frac{ql}{4} - ql = \frac{ql - 4ql}{4} = -\frac{3}{4} ql$

$+\uparrow$ $T = \frac{ql}{4} - ql = -\frac{3}{4} ql$ $T = \frac{dM}{dz}$ $M = \int T\, dz$

$\circlearrowright^+$ ~~$M = \frac{ql}{4} \cdot z + \frac{ql}{4} \cdot l - ql \cdot l - ql \cdot \left(z - \frac{l}{2}\right) - ql^2$~~

~~$= \frac{ql}{4} z + \frac{ql^2}{4} - ql^2 - qlz + \frac{ql^2}{2} - ql^2$~~

~~$= \frac{qlz + ql^2 - 4ql^2 - 4qlz + 2ql^2 - 4ql^2}{4} = \frac{5ql^2 - 3qlz}{4}$~~

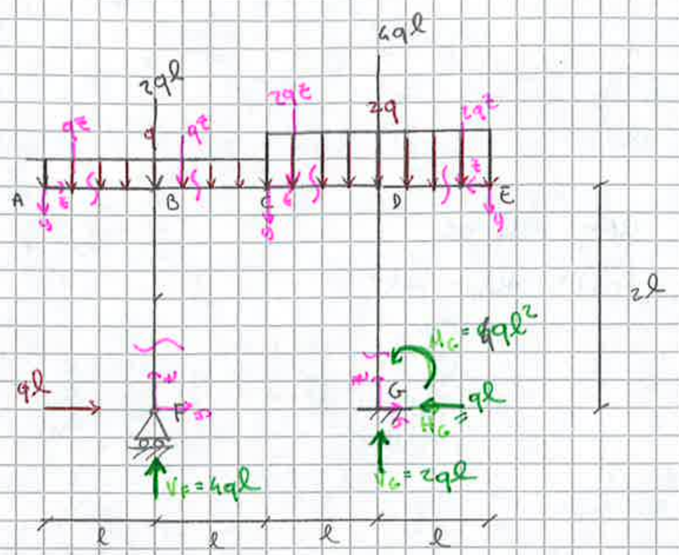
~~$M(l) = M_C = \frac{5ql^2 - 3ql^2}{4} = \frac{2ql^2}{4} = \frac{ql^2}{2}$~~

~~$M(2l) = M_E = \frac{5ql^2 - 6ql^2}{4} = -\frac{ql^2}{4} = -\frac{ql^2}{4}$~~

$M = -\frac{3}{4} qlz$ $M(0) = M_C = 0$

$M(l) = M_E = -\frac{3}{4}$

TEMA D'ESAME 8 settembre 1994 ESERCIZIO 1



4 incognite
⇓
4 equazioni

REAZIONI VINCOLARI

$\xrightarrow{+}$ $\quad ql + H_G = 0 \quad \rightarrow \quad H_G = -ql$

$+\uparrow$ $\quad V_F - 2ql - 4ql + V_G = 0 \quad \rightarrow \quad V_F + V_G - 6ql = 0$

NO

$M(F) = -4ql \cdot 2l + M_G + \cancel{H_G \cdot 2l} + V_G \cdot 2l$

$= -8ql^2 + M_G - \cancel{2ql^2} + V_G \cdot 2l$

$= -8ql^2 + M_G + V_G \cdot 2l$

eq. ausiliaria

$M(C)$ da sx $= -V_F \cdot l + 2ql \cdot l + ql \cdot 2l = 0$

$= -V_F \, l + 2ql^2 + 2ql^2 = 0$

$= -V_F \cdot l + 4ql^2 = 0 \quad \rightarrow \quad V_F = \frac{4ql^2}{l} = 4ql$

$4ql + V_G - 6ql = 0 \quad \rightarrow \quad V_G = 2ql$

$-8ql^2 + M_G + 2ql \cdot 2l = 0 \Rightarrow -8ql^2 + M_G + 4ql^2 = 0$

$M_G = 4ql^2$

TRATTO AB

$\overset{+}{\leftarrow}$ $N = 0$

$+\uparrow$ $T = qz$ $\quad T(0) = T_A = 0$

$T(\ell) = T_B = q\ell$

$\circlearrowleft +$ $M = -qz \cdot \frac{z}{2} = -q\frac{z^2}{2}$ $\quad M(0) = M_A = 0$

$M(\ell) = M_B = -\frac{q\ell^2}{2}$

TRATTO BF

$\overset{+}{\leftarrow}$ $N = -4q\ell$

$+\uparrow$ $T = -q\ell$

$\circlearrowleft +$ $M = -q\ell \cdot z$ $\quad M(0) = M_F = 0$

$M(2\ell) = M_B = -q\ell \cdot 2\ell = -2q\ell^2$

TRATTO BC

$\overset{+}{\leftarrow}$ $N = -q\ell$

$+\uparrow$ $T = -qz + 4q\ell$ $\quad T(\ell) = T_B = -q\ell + 4q\ell = 3q\ell$

$T(2\ell) = T_C = -q2\ell + 4q\ell = 2q\ell$

$\circlearrowleft +$ $M = 4q\ell \cdot (z - \ell) - q\ell \cdot 2\ell - qz \cdot \frac{z}{2}$

$= 4q\ell z - 4q\ell^2 - 2q\ell^2 - q\frac{z^2}{2}$

$= 4q\ell z - 6q\ell^2 - \frac{qz^2}{2}$ $\quad M(\ell)^B = 4q\ell^2 - 6q\ell^2 - \frac{q\ell^2}{2} = \frac{8q\ell^2 - 12q\ell^2 - \cancel{2\ell^2}}{2}$

$= \cancel{\left(-\frac{5}{2}q\ell^2\right) = \left(\frac{3}{2}q\ell^2\right)} = -\frac{5}{2}q\ell^2$

$M(2\ell)^C = 8q\ell^2 - 6q\ell^2 - \frac{q4\ell^2}{2} = (8 - 6 - 2)q\ell^2$

$= 0$

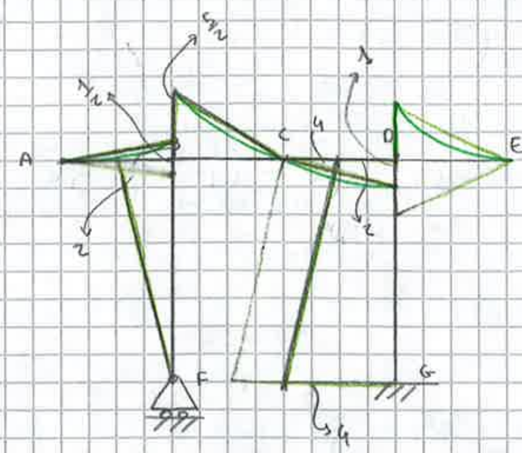
TRATTO CD

$\overset{+}{\leftarrow}$ $N = -q\ell$

$+\uparrow$ $T = 4q\ell - 2q\ell - 2qz = 2q\ell - 2qz$ $\quad T(0) = T_C = 2q\ell$

$T(\ell) = T_D = 0$

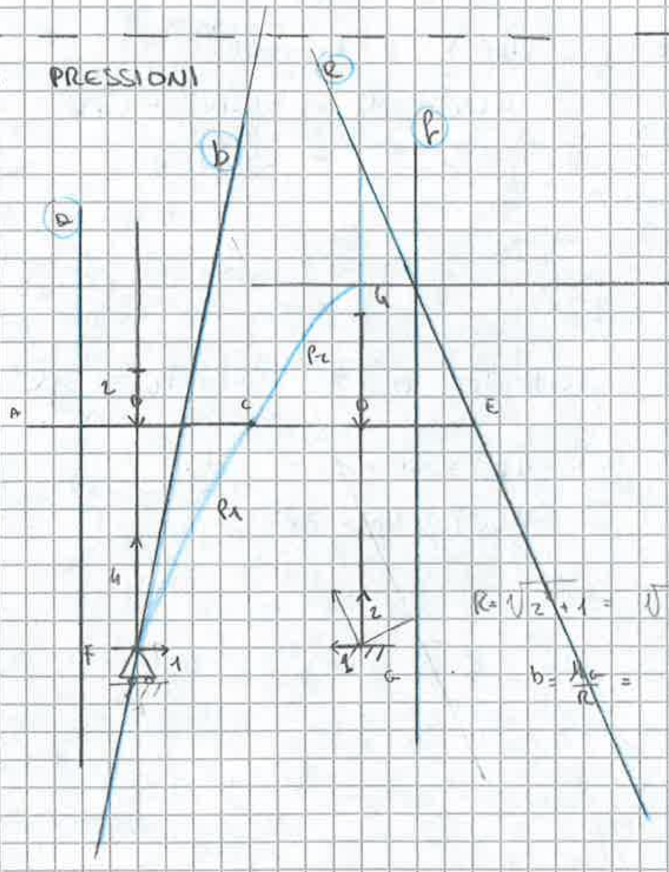
$\circlearrowleft +$ $M = 4q\ell \cdot (\ell + z) - 2q\ell \cdot (\ell + z) - q\ell \cdot 2\ell - 2qz \cdot \frac{z}{2}$



$$\frac{M}{ql^2}$$

— — — — — — — — — — — —

CURVA DELLE PRESSIONI



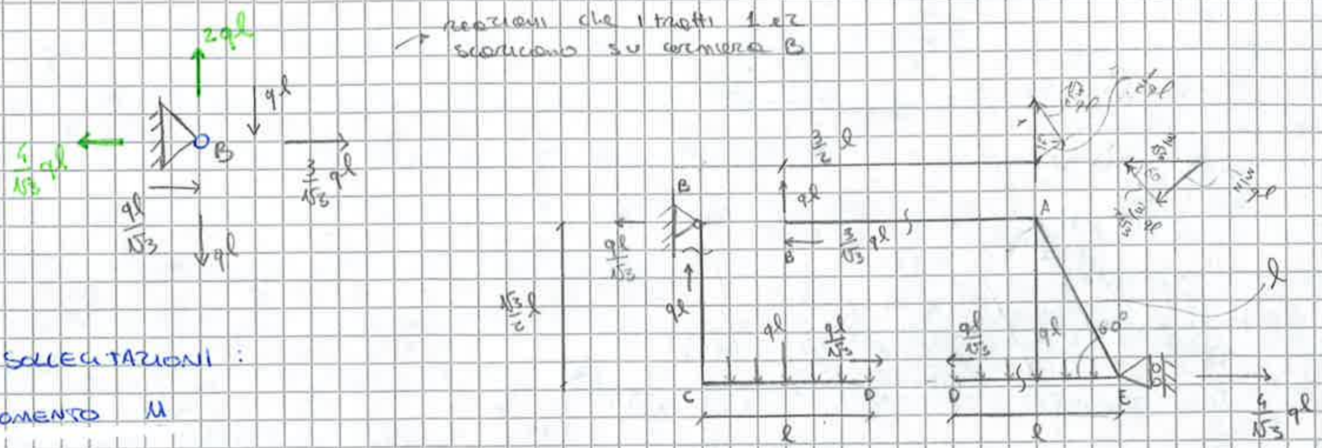
$$R = \sqrt{2^2+1} = \sqrt{5}\, ql$$

$$b = \frac{M_G}{R} = \frac{4ql^2}{\sqrt{5}\, ql} = \frac{4}{\sqrt{5}}\, l$$

$$\frac{1}{2}ql^2 - H_D \frac{\sqrt{3}}{2} l = 0 \rightarrow H_D = \frac{\frac{1}{2}ql^2}{\frac{\sqrt{3}}{2}l} \rightarrow H_D = \frac{1}{2}ql^2 \cdot \frac{2}{\sqrt{3}l} = \frac{ql}{\sqrt{3}}$$

Verifica nodo B:

reazioni che i tratti 1 e 2 scaricano su cerniera B



SOLLECITAZIONI:

- MOMENTO M

TRATTO BA

$\circlearrowright +$ $M = ql \cdot z$

$M(0) = M_B = 0$

$M\left(\frac{3}{2}l\right) = M_A = ql \cdot \left(\frac{3}{2}l\right) = \frac{3}{2}ql^2$

TRATTO AE

~~$\circlearrowright +$ $M = \frac{ql}{2} \cdot z - \frac{3}{2}ql \cdot z$~~

~~$M(0) = M_A = 0$~~

~~$M(l) = M_E = \frac{ql^2}{2} - \frac{3}{2}ql^2 = -\frac{2}{2}ql^2 = -ql^2$~~

TRATTO DE

$\circlearrowright +$ $M = qz \cdot \frac{z}{2}$

$M(0) = M_D = 0$

$M(l) = M_E = q\frac{l^2}{2}$

TRATTO BC

$\circlearrowright +$ $M = -\frac{ql}{\sqrt{3}} \cdot z$

$M(0) = M_B = 0$

$M\left(\frac{\sqrt{3}}{2}l\right) = M_C = -\frac{ql}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2}l = -\frac{ql^2}{2}$

- TAGLIO T

TRATTO BA

$+\uparrow$ $T = ql$

TRATTO DE

$+\uparrow$ $T = -qz$

$T(0) = 0 = T_D$

$T(l) = T_E = -ql$

TRATTO BC

$+\uparrow$ $T = -\frac{ql}{\sqrt{3}}$

TRATTO AE

$+\uparrow$ $T = \frac{1}{2}ql - \frac{3}{2}ql = -ql$

- SFORZO N

TRATTO BA

$\overset{+}{\leftarrow}$ $N = \frac{3}{\sqrt{3}}ql$

TRATTO DE

$\overset{+}{\leftarrow}$ $N = \frac{ql}{\sqrt{3}}$

TRATTO BC

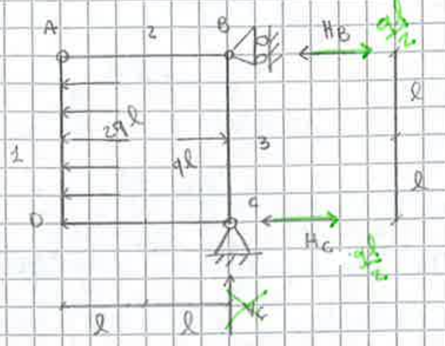
$\overset{+}{\leftarrow}$ $N = ql$

TRATTO AE

$\overset{+}{\leftarrow}$ $N = \frac{\sqrt{3}}{2}ql + \frac{3}{2\sqrt{3}}ql = \frac{3}{\sqrt{3}}ql$

TEMA D'ESAME 12/6/2007

ES. 1) ISOSTATICA



3 corpi:

$3 \cdot 3 = 9$ g

$\underbrace{(2 \times 3)}_{\text{interni}} + \underbrace{(1+2)}_{\text{esterni}} = 9$ v

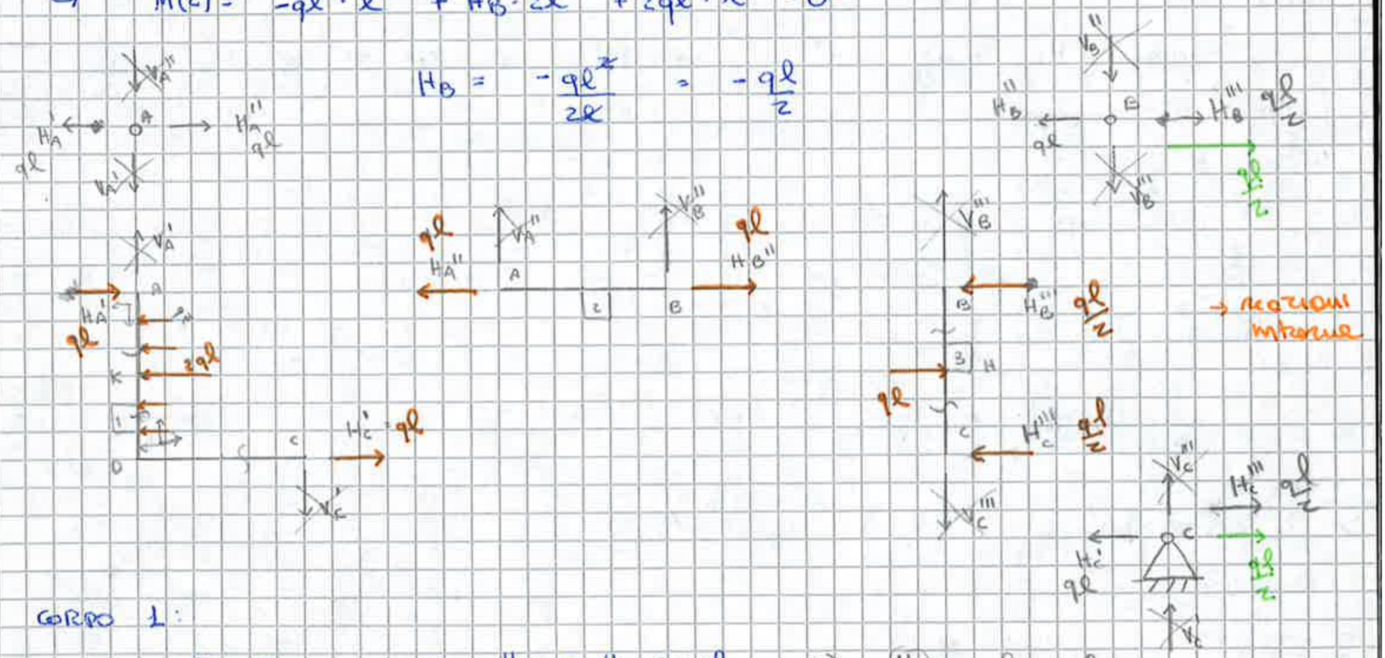
> REAZIONI VINCOLARI: → reazioni esterne

$\rightarrow^{+}$ $-H_C - H_B + ql - 2ql = 0 \rightarrow -H_C + \frac{ql}{2} + ql - 2ql = 0 \rightarrow H_C = -\frac{ql}{2}$

$+\uparrow$ $V_C = 0$

$\circlearrowleft^{+}$ $M(C) = -ql \cdot l + H_B \cdot 2l + 2ql \cdot l = 0$

$H_B = -\frac{ql^2}{2l} = -\frac{ql}{2}$



CORPO 1:

$\rightarrow$ $-H_A - 2ql + H_C = 0 \rightarrow H_C = H_A + 2ql \rightarrow H_C = -ql + 2ql = ql$

$\uparrow$ $V_A - V_C = 0 \rightarrow V_A = V_C \rightarrow V_C = 0$

$\circlearrowleft$ $M(A) = -2ql \cdot l - V_C \cdot 2l + H_C \cdot 2l = 0$

$\circlearrowleft$ $M(C) = 2ql \cdot l + H_A \cdot 2l - V_A \cdot 2l = 0 \rightarrow H_A = \frac{V_A 2l - 2ql^2}{2l} = \frac{2l(V_A - ql)}{2l}$

$H_A = -\frac{2ql^2}{2l} = -ql$

$-2ql^2 - V_A 2l + H_A 2l + 4ql^2 = 0$

$-2ql^2 - V_A 2l + (V_A - ql) 2l + 4ql^2 = 0$

$-2ql^2 - V_A 2l + V_A 2l - 2ql^2 + 4ql^2 = 0$

$V_A = 0$

(da equazione corpo 2)

• TAGLIO T

TRATTO AB

$\circlearrowleft$ $T = 0$

TRATTO BH

$\circlearrowleft$ $T = -\frac{ql}{2}$

TRATTO HC

$\circlearrowleft$ $T = -\frac{ql}{2} + ql = \frac{-ql + 2ql}{2} = \frac{ql}{2}$

TRATTO CD

$\circlearrowleft$ $T = 0$

TRATTO AD

$\circlearrowleft$ $T = ql - qz$

$T(0) = T_A = ql$

$T(l) = T_K = 0$

$T(2l) = T_D = -ql$

• SFORZO N

TRATTO AB

$\overset{+}{\leftarrow}$ $N = ql$

TRATTO BC

$\overset{+}{\leftarrow}$ $N = 0$

TRATTO CD

$\overset{+}{\leftarrow}$ $N = ql$

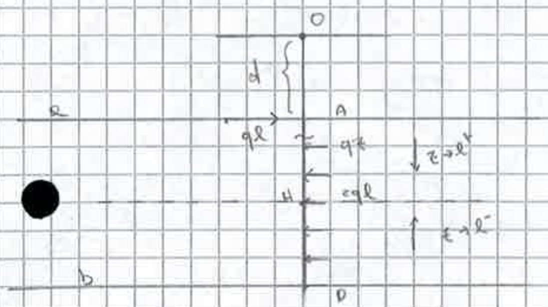
TRATTO AD

$\overset{+}{\leftarrow}$ $N = 0$

> CURVA DELLE PRESSIONI (uso reazioni interne):

TRATTO AD

Ci sono solo forze orizzontali, no forze verticali → non abbiamo + una parabola, ma la cdp degenera



$\circlearrowleft$ $M(O) = ql \cdot d - qz\left(d + \frac{z}{2}\right) = 0$

$qld - qzd - \frac{qz^2}{2} = 0$

$d(l - z) - \frac{z^2}{2} = 0$

$d = \frac{z^2}{2(l - z)}$ → distanza a cui è applicata la risultante delle forze

per $z < l$ $\quad d(z) > 0$ → retta a sx di A (sopra A)
$N > 0$
$D > 0$

per $z > l$ $\quad d(z) < 0$ → retta a dx di A (sotto A)
$N > 0$
$D < 0$

$z \to l^+$ $\quad d = +\infty$ fascio di rette da A verso $+\infty$

$z \to l^-$ $\quad d = -\infty$ fascio di rette da D verso $-\infty$

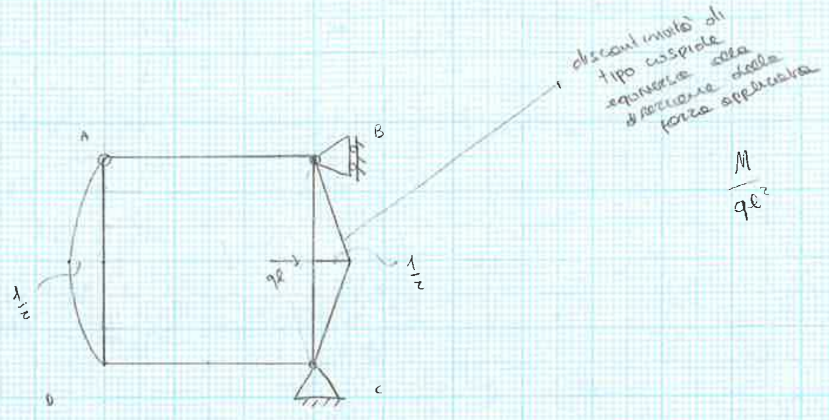
$d = \frac{(l)^{+2}}{2(l - (l^+))}$ → un po' + piccolo di l

$d = \frac{(l^+)^2}{0^+} = +\infty$

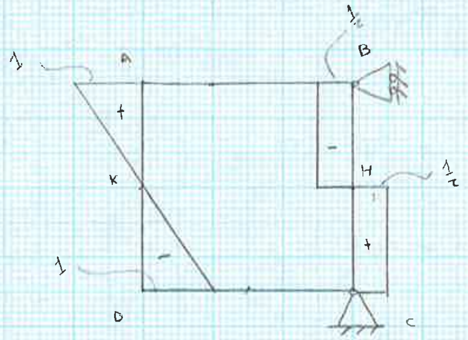
$d = \frac{(l)^{-2}}{2(l - (l^-))}$ → un po' + grande di l

$d = \frac{(l^-)^2}{0^-} = -\infty$

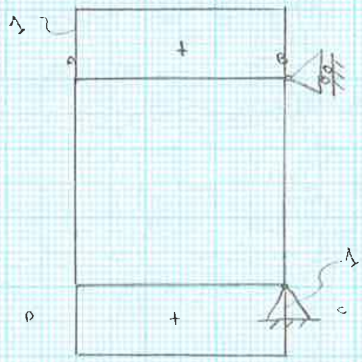
ES. 1 ISOSTATICA



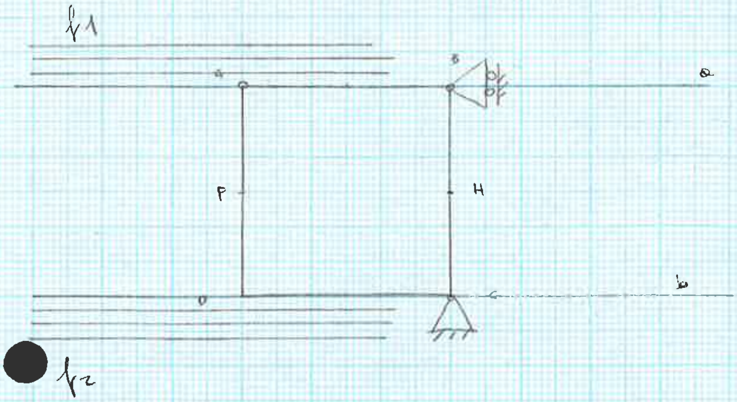
$\frac{M}{q\ell^2}$



$\frac{T}{q\ell}$



$\frac{N}{q\ell}$



TRATTO	cdp
AB	retta a
BH	retta a
HC	retta b
AF	f1
FD	f2

FLYGT

TRATTO BE

$M = \frac{5}{8} q\ell \cdot z$

TRATTO EC

$M = -\frac{5}{4} q\ell \cdot \frac{z\sqrt{2}}{2} + \frac{5}{8} q\ell \cdot \left(\ell + \frac{z\sqrt{2}}{2}\right)$

$M(0) = M_E = \frac{5}{8} q\ell^2$

$M(\ell\sqrt{2}) = M_C = 0$

• TAGLIO T ↑

TRATTO AD

$T = \frac{3}{8} q\ell - qz$

$T(0) = T_A = \frac{3}{8} q\ell$

$T(\ell) = T_D = \frac{3}{8} q\ell - q\ell = \frac{(3-8)}{8} q\ell$

$= -\frac{5}{8} q\ell$

TRATTO DC

$T = \frac{3}{16}\sqrt{2}\, q\ell + \frac{3\sqrt{2}}{8} q\ell \;\; \frac{\sqrt{2}}{2} q\ell = \frac{\sqrt{2}}{16}$

TRATTO BE

$T = \frac{5}{8} q\ell$

TRATTO EC

$T = -\frac{5}{4}\frac{\sqrt{2}}{2} q\ell + \frac{5}{8}\frac{\sqrt{2}}{2} q\ell = -\frac{5\sqrt{2}}{16}$

• SFORZO N ←

TRATTO AD

$N = -\frac{3}{4} q\ell$

TRATTO DC

$N = \frac{3\sqrt{2}}{16} q\ell - \frac{\sqrt{2}}{2} q\ell - \frac{3}{8}\sqrt{2} = -\frac{11}{16}\sqrt{2}$

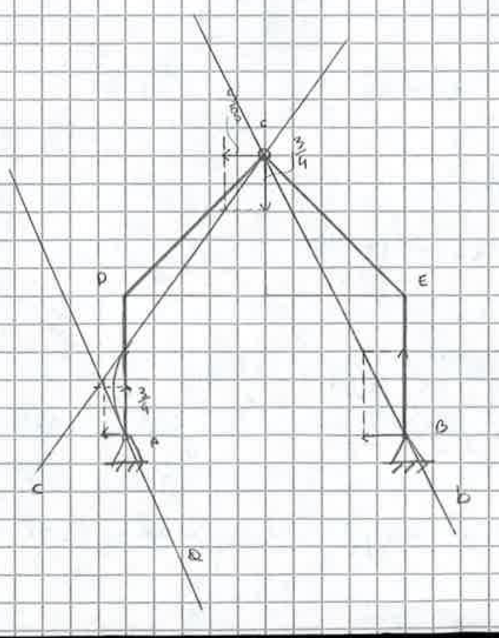
TRATTO BE

$N = -\frac{5}{4} q\ell \; \cancel{-\frac{5}{4}\frac{\sqrt{2}}{2} q\ell}$

TRATTO EC

$N = -\frac{5}{8}\frac{\sqrt{2}}{2} q\ell - \frac{5}{4}\frac{\sqrt{2}}{2} q\ell = -\frac{15\sqrt{2}}{16} q\ell$

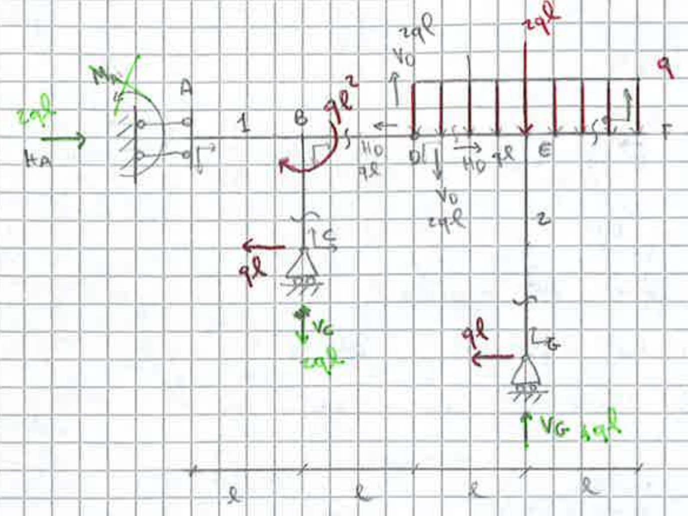
tratto	cdp
AD	parabola p
DC	retta c
CEB	retta b

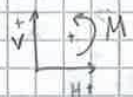


TEMA D'ESAME 30/6/2008

ES. 1)

ISOSTATICA





PUNTO 1) M, N, T e curva delle pressioni

REAZIONI VINCOLARI:

$\xrightarrow{+}$ $H_A - q\ell - q\ell = 0 \rightarrow H_A = 2q\ell$

$+\uparrow$ $V_C + V_G - 2q\ell = 0 \rightarrow V_G = 2q\ell + 2q\ell = 4q\ell$

$M(G) = -q\ell^2 + q\ell \cdot \ell - V_c \cdot 2\ell - H_A \cdot 2\ell + M_A = 0$

corpo 1:

$M(D) = M_A - q\ell^2 - q\ell \cdot \ell - V_c \cdot \ell = 0$

$M_A = V_c\ell + q\ell^2 + q\ell^2 = V_c\ell + 2q\ell^2 \rightarrow M_A = -2q\ell^2 + 2q\ell^2 = 0$

$\cancel{-q\ell^2} + \cancel{q\ell^2} - V_c \cdot 2\ell - (2q\ell)\cdot(2\ell) + (V_c\ell + 2q\ell^2) = 0$

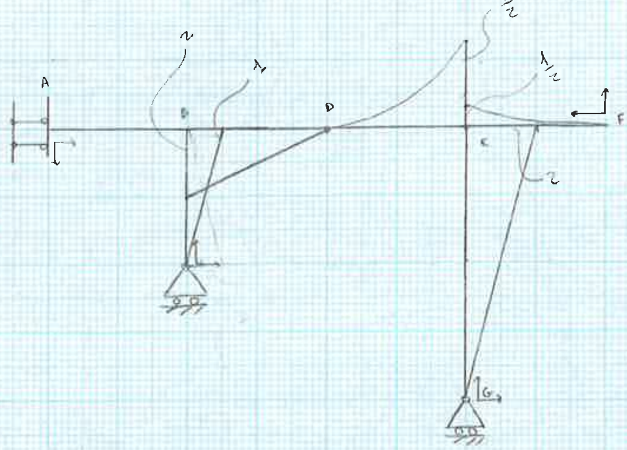
$- V_c \cdot 2\ell - 4q\ell^2 + V_c\ell + 2q\ell^2 = 0$

$-2q\ell^2 - V_c\ell = 0 \rightarrow V_c = -\frac{2q\ell^2}{\ell} = -2q\ell$

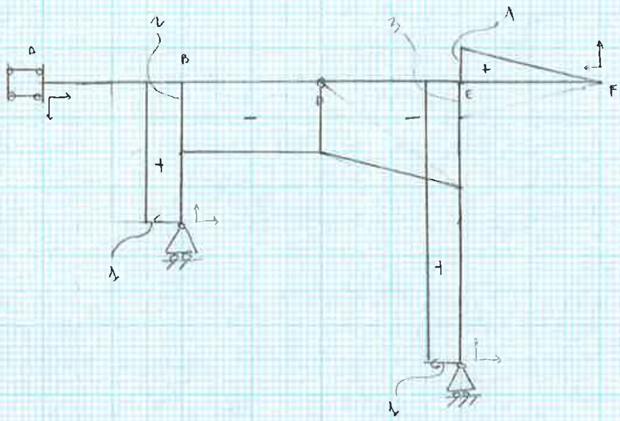
sist. di riferimento

fibre di sotto (M+)

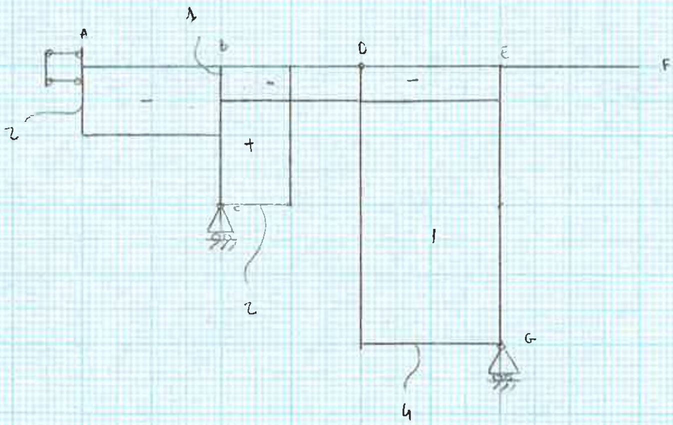
1



$\frac{M}{ql^2}$

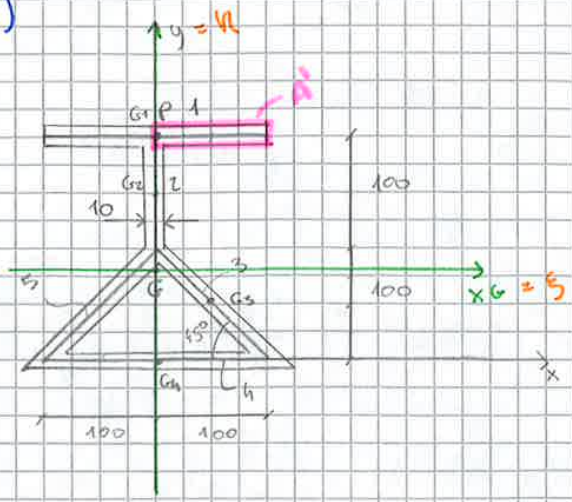


$\frac{T}{ql}$



$\frac{N}{ql}$

PUNTO 2)



$A^{(1)} = 200 \cdot 10 = 2'000$

$A^{(2)} = 100 \cdot 10 = 1'000$

$A^{(3)} = 100\sqrt{2} \cdot 10 = 1'414,21$

$A^{(4)} = 200 \cdot 10 = 2'000$

$A_{TOT} = A^{(1)} + A^{(2)} + 2A^{(3)} + A^{(4)}$

$= 7'828,42$

$S_x^{(1)} = y_{G1} \cdot A^{(1)} = 200 \cdot 2'000 = 400'000$

$S_x^{(2)} = y_{G2} \cdot A^{(2)} = 150 \cdot 1'000 = 150'000$

$S_x^{(3)} = y_{G3} \cdot A^{(3)} = 50 \cdot 1'414,21 = 70'710,5 \cdot 2$

$S_x^{(4)} = y_{G4} \cdot A^{(4)} = 0$

$S_{TOT} = \cancel{620'710,5}\ 691'421$

$y_G = \frac{S_{TOT}}{A_{TOT}} = \frac{\cancel{620'710,5}}{7'828,42}$

$= \cancel{79,29}\ 88,32$

$G = (0; \cancel{79,29}\ 88,33)$

$I_{x_G}^{(1)} = I_{x_{G1}} + A^{(1)} \cdot (y_G - y_{G1})^2 = 0 + (2000)(\cancel{79,29}\ 88,33 - 200)^2 = \cancel{29'141'808,2}\ 24'940'377,8$

$I_{x_G}^{(2)} = I_{xG2} + A^{(2)} (y_G - y_{G2})^2 = \frac{10 \cdot 100^3}{12} + (1'000)(\cancel{79,29}\ 88,33 - 150)^2 = \cancel{5'833'723,48}\ 4'636'522,23$

$3'803'188,9$

$I_{x_G}^{(3)} = I_{xG3} + A^{(3)} (y_G - y_{G3})^2 = \frac{10 \cdot (100\sqrt{2})^3}{12} \cdot \sin^2 45° + 1'414,21 \cdot (\cancel{79,29}\ 88,33 - 50)^2$

$1'178'511,302 \qquad 2'077'741,634$

$= \cancel{1'220'523,204}\ \cancel{1'221'880,180}\ 3'256'252,9$

$I_{x_G}^{(4)} = I_{xG4} + A^{(4)} (y_G - y_{G4})^2 = 0 + 2000(\cancel{79,29}\ 88,33 - 0)^2 = \cancel{1'882'380}$

$15'604'377,8$

$I_{x_G\,TOT} = I_{x_G}^{(1)} + I_{x_G}^{(2)} + 2I_{x_G}^{(3)} + I_{x_G}^{(4)} = \cancel{37'580'808,4}\ 51'693'783,63 \ mm^4$

$I_{y_G}^{(1)} = I_{yG1} + A^{(1)} (x_G - x_{G1})^2 = \frac{10 \cdot 200^3}{12} + 2000 \cdot (0-0) = 6'666'666,6$

$I_{y_G}^{(2)} = I_{yG2} + A^{(2)} (x_G - x_{G2})^2 = 0$

$I_{y_G}^{(3)} = I_{yG3} + A^{(3)} (x_G - x_{G3})^2 = \frac{10 \cdot (100\sqrt{2})^3}{12} \cos^2 45° + 1'414,21 \cdot (0-50)^2$

$= 4'714'036,3$

$I_{y_G}^{(4)} = I_{yG4} + A^{(4)} (x_G - x_{G4})^2 = \frac{10 \cdot 200^3}{12} = 6'666'666,6$

Von Mises: $\sigma_{eq} = \sqrt{\sigma_t^2 + 3\tau_t^2} = \sqrt{293{,}44^2 + 3 \cdot (13{,}14)^2} = 294{,}32 > \sigma_{amm}$

sezione
non verificata

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$\overline{DE}$ $M = -qz \cdot \frac{z}{2}$

$M(0) = M_D = 0$

$M(l) = M_E = -\frac{ql^2}{2}$

$\overline{FE}$ $M = \frac{ql}{2} \cdot z$

$M(0) = M_F = 0$

$M(l) = M_E = \frac{ql^2}{2}$

(T) $\overline{AC}$ $T = -\frac{ql}{2}$

$\overline{CD}$ $T = ql - qz$

$T(0) = T_C = ql$

$T(l) = T_D = 0$

$\overline{FE}$ $T = \frac{ql}{2}$

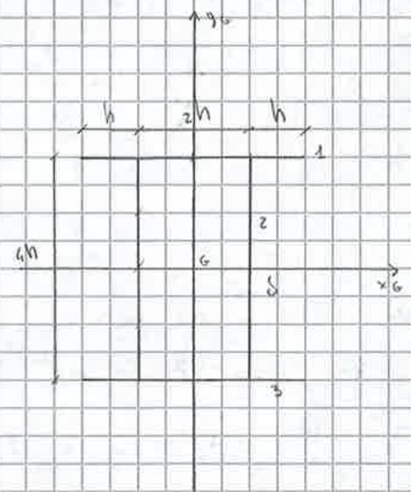
$\overline{DE}$ $T = -qz$

$T(0) = T_D = 0$

$T(l) = -ql$

(N) $\overline{AC}$ $N = -ql$ $\quad$ $\overline{DE}$ $N = -\frac{ql}{2}$ $\quad$ $\overline{AF}$ $N = \frac{ql}{2}$

$\overline{CD}$ $N = -\frac{ql}{2}$ $\quad$ $\overline{EB}$ $N = ql$



2) $T_{max} = T_C = T_E = ql$

sezione chiusa simmetrica $\Rightarrow \tau_z = \frac{T_y \cdot S_x^{(A')}}{2\, b\, I_x}$

$A_1 = 4h \cdot \delta$

$A_2 = 4h \cdot \delta$

$A_3 = 4h \cdot \delta$

$$\begin{cases} I_{xG}^{(1)} = I_{xG1} + A^{(1)}(y_G - y_{G1})^2 = 0 + 4h\delta(0 - 2h)^2 = 4h\delta \cdot (4h^2) = 16 h^3 \delta \\ I_{xG}^{(2)} = I_{xG2} + A^{(2)}(y_G - y_{G2})^2 = \frac{\delta \cdot (4h)^3}{12} + 4h\delta \cdot 0 = \frac{64\,\delta h^3}{12} \\ I_{xG}^{(3)} = I_{xG3} + A^{(3)}(y_G - y_{G3})^2 = 0 + 4h\delta(2h)^2 = 16 h^3 \delta \end{cases}$$

$$I_{xG\,TOT} = I_{xG}^{(1)} + 2 I_{xG}^{(2)} + I_{xG3} = 16 h^3\delta + 2 \cdot \frac{64\,\delta h^3}{12} + 16 h^3 \delta$$

$$= 32 h^3 \delta + \frac{128\,\delta h^3}{12} = \frac{384 + 128}{12} h^3 \delta = \frac{\cancel{512}}{\cancel{12}} h^3 \delta$$

$$= \frac{128}{3} h^3 \delta$$

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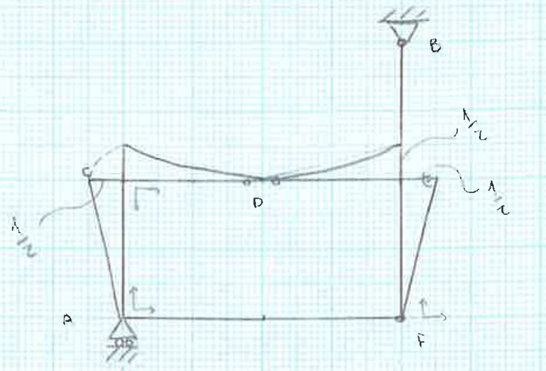
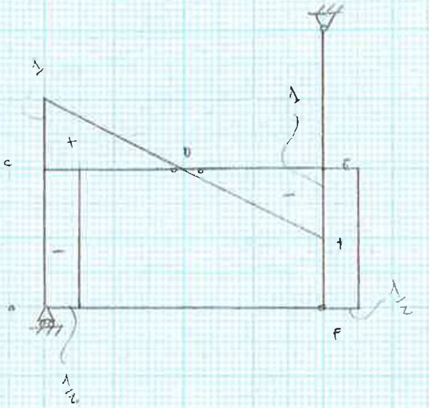
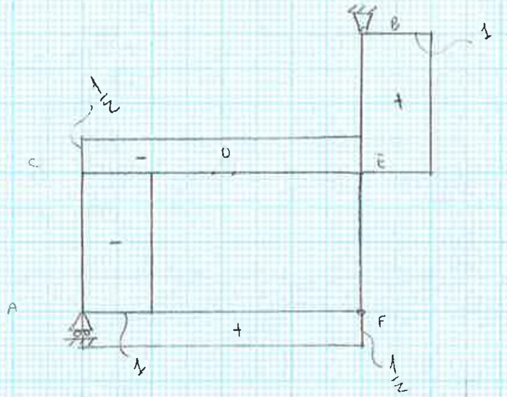


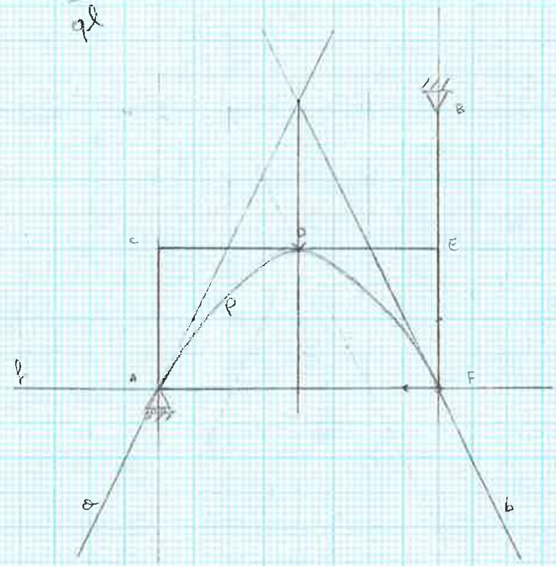
$\frac{M}{ql^2}$

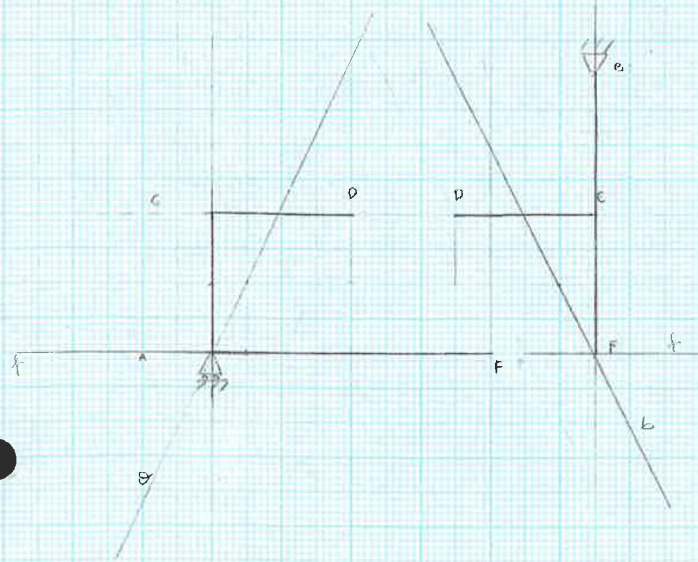


$\frac{T}{ql}$



$\frac{N}{ql}$

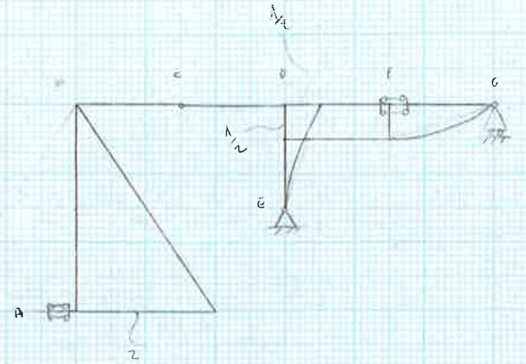




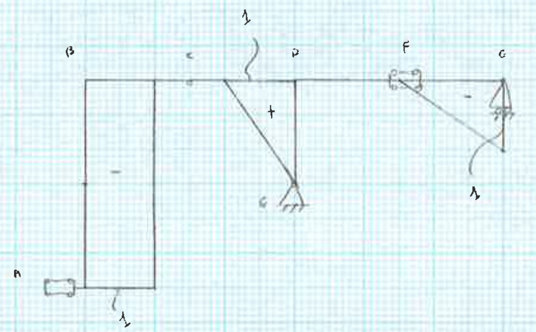
tratto	cdp
FA	retta f
AC	retta a
FE	retta f
BE	retta b
CDE	parabola p

risultante forze da F

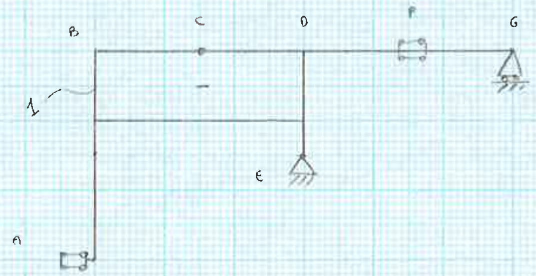
risultante forze da F



$\frac{M}{ql^2}$

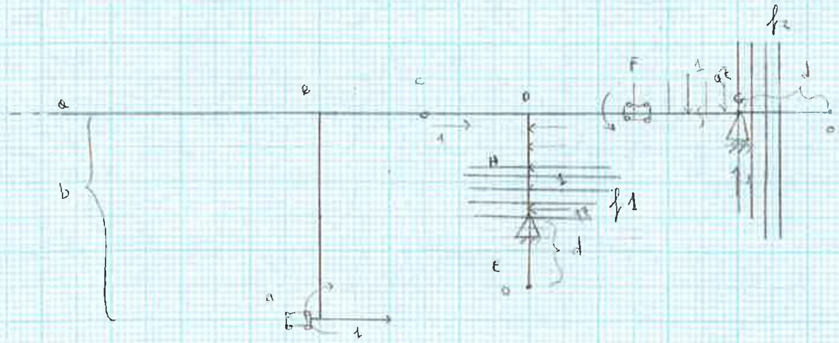


$\frac{T}{ql}$



$\frac{N}{ql}$

1.2



N.B. tratto FG :

$$M(O) = -ql \cdot d + qz \cdot \left(\frac{z}{2} + d\right)$$

$$-ld + \frac{z^2}{2} + zd = 0$$

$$d(-l + z) + \frac{z^2}{2} = 0$$

$$d = \frac{-\frac{z^2}{2}}{(-l+z)} = \frac{\frac{z^2}{2}}{(l-z)}$$

$0 < z < l$ → $z = 0$ $d = 0$; $z = l$ $d = +\infty$

f_2 tra G e l'∞ a dx di G

$$b = \frac{M}{R} = \frac{2ql^2}{ql} = 2l$$

N.B. tratto DE :

$$M(O) = qz\left(\frac{z}{2} + d\right)$$

$$\frac{qz^2}{2} + qzd = 0$$

$$\frac{z^2}{2} + zd = 0$$

$$d = \frac{-\frac{z^2}{2}}{z} = -\frac{z}{2}$$

$0 < z < l$ → $z = 0$ $d = 0$; $z = l$ $d = -\frac{l}{2}$

f_1 tra E e H

tratto	cdp
AD	retta a
DF	retta impropria all'∞
DE	fascio f1
FG	fascio f2

(T)

$\overline{AB}$ $T = 2ql$

$\overline{CD}$ $T = 2ql - 2q\frac{z}{l} = 0$

$T(0) = T_C = 2ql$
$T(l) = T_D = 0$

$\overline{GH}$ $T = -ql - \frac{3}{2}ql$
$= -\frac{5}{2}ql$

$\overline{DE}$ $T = -qz$

$T(0) = T_D = 0$
$T(l) = T_E = -ql$

$\overline{FG}$ $T = -ql$

(N)

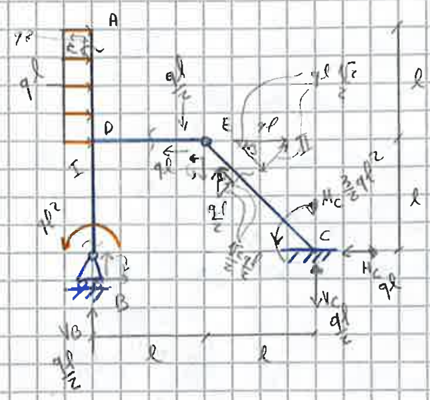
$\overline{CB}$ $N = +2ql$

$\overline{EF}$ $N = ql$

$\overline{FG}$, $\overline{GH}$ $N = -ql$

TEMA D'ESAME 3/7/2007 COMPITO II

ES. 1)



reazioni vincolari

$\rightarrow \quad ql + H_c = 0 \rightarrow H_c = -ql$

$\uparrow \quad V_B + V_c = 0$

$\circlearrowleft \; M(c) = -V_B \cdot 2l + ql^2 - ql\left(l + \frac{l}{2}\right) - M_c = 0$

corpo II

$\circlearrowleft \; M(E) = +V_c \cdot l + H_c \cdot l - M_c = 0$

$V_c \cdot l - ql^2 - M_c = 0 \rightarrow M_c = V_c \cdot l - ql^2 \qquad M_c = -\frac{ql^2}{2} - \frac{ql^2}{2} = -\frac{3}{2}ql^2$

$$\begin{cases} V_B + V_c = 0 \rightarrow V_B = -V_c \\ -V_B \cdot 2l + ql^2 - \frac{3}{2}ql^2 - V_c \cdot l + ql^2 = 0 \end{cases} \qquad V_B = \frac{ql}{2}$$

$V_c \cdot 2l - V_c \cdot l + 2ql^2 - \frac{3}{2}ql^2 = 0$

$V_c\, l + \frac{ql^2}{2} = 0 \rightarrow V_c = \frac{-\frac{ql^2}{2}}{l} = -\frac{ql}{2}$

interne : corpo I

$\rightarrow \quad ql - H_E = 0 \rightarrow H_E = ql$

$\uparrow \quad \frac{ql}{2} - V_E = 0 \rightarrow V_E = \frac{ql}{2}$

SOLLECITAZIONI

M

$\overline{BD} \quad M = -ql^2$

$\overline{AD} \quad M = qz \cdot \frac{z}{2}$

$M(0) = M_A = 0$

$M(l) = M_D = \frac{ql^2}{2}$

$\overline{DE} \quad M = -\frac{ql}{2} \cdot z$

$M_E = M(0) = -\frac{ql}{2} \cdot 0 = 0$

$M_D = M(l) = -\frac{ql^2}{2}$

$\overline{EC} \quad M = ql\frac{\sqrt{2}}{2} \cdot z + \frac{ql}{2}\frac{\sqrt{2}}{2} \cdot z$

$M(0) = M_E = 0$

$M(l\sqrt{2}) = M_c = ql\frac{\sqrt{2}}{2} \cdot (l\sqrt{2}) + \frac{ql}{4}\sqrt{2} \cdot (l\sqrt{2})$

$= ql^2 + \frac{ql^2}{2} = \frac{3}{2}ql^2$