NUMERO: 1885A - ANNO: 2016

APPUNTI

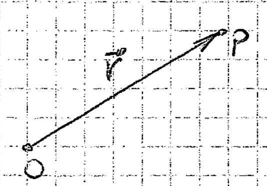
STUDENTE: Borghi Alessandro

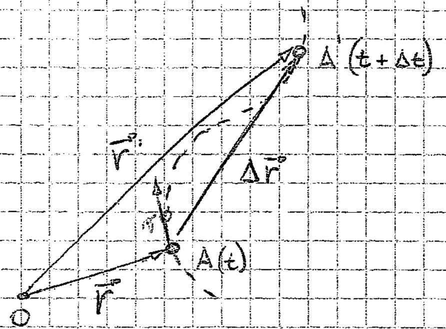
MATERIA: Meccanica applicata alle macchine - Prof. Ferraresi

vettore posizione $\vec{r} = \underset{\hookrightarrow \text{scalare}}{r}\,\underbrace{\vec{i}}_{\text{versore}}$

vettore velocità $\vec{v} = v\vec{j}$

vettore accelerazione $\vec{a} = a\vec{k}$





$\boxed{\Delta\vec{r} = \vec{r}' - \vec{r}}$ ⟹ equazione vettoriale

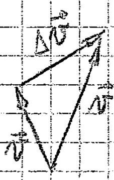
$|\Delta\vec{r}| \neq \Delta s$ (traiettoria)

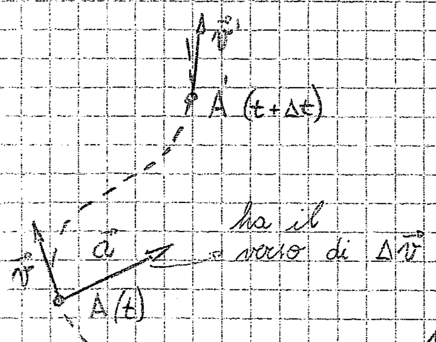
$\frac{\Delta\vec{r}}{\Delta t} = \vec{v}_m$ ⟹ velocità media

$\lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \vec{v}$ ⟹ velocità istantanea

$\vec{v}$ è sempre $\boxed{\text{tangente}}$ alla traiettoria

$\Delta\vec{v} = \vec{v}' - \vec{v}$





$\frac{\Delta\vec{v}}{\Delta t} = \vec{a}_m$ ⟹ accelerazione media

$\lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = \vec{a}$ ⟹ accel. istantanea

$\vec{a}$: può avere componente $\boxed{\text{tangenziale}}$ alla traiettoria

e componente $\boxed{\text{normale}}$ alla traiettoria

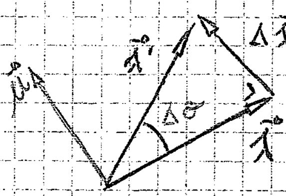
$$\boxed{\vec{r} = r\vec{\lambda}}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\vec{\lambda} + \underbrace{r\frac{d\vec{\lambda}}{dt}}$$

$$\boxed{\vec{v} = \dot{r}\vec{\lambda} + r\dot{\theta}\vec{\mu}} = \vec{v}_\lambda + \vec{v}_\mu$$

$$\vec{a} = \ddot{r}\vec{\lambda} + \dot{r}\dot{\theta}\vec{\mu} + \dot{r}\dot{\theta}\vec{\mu} + r\ddot{\theta}\vec{\mu} + r\dot{\theta}\dot{\theta}\underset{(-\lambda)}{}$$

$$\boxed{\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\vec{\lambda} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\vec{\mu}} = a_\lambda + a_\mu$$



$$\vec{\lambda}' = \vec{\lambda} + \Delta\vec{\lambda}$$

$$\Delta\vec{\lambda} = \Delta\theta \cdot |\vec{\lambda}| \cdot$$

$$\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \dot{\theta} = \omega \Longrightarrow \Delta\theta = \Delta t \cdot \dot{\theta}$$

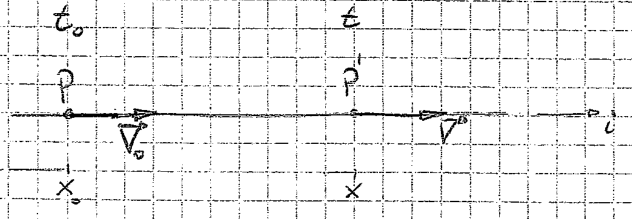
$$\Delta\vec{\lambda} = \Delta t \cdot \dot{\theta}\,\vec{\mu}$$

$$\boxed{\frac{d\vec{\lambda}}{dt} = \dot{\theta}\vec{\mu}}$$ ⇒ derivata del versore

$$\boxed{\frac{d\vec{\mu}}{dt} = \dot{\theta}(-\vec{\lambda})}$$ ⇒ derivata di θ

Poisson

$$\frac{d\vec{\lambda}}{dt} = \vec{\omega} \wedge \vec{\lambda} = \omega\underbrace{\vec{\nu} \wedge \vec{\lambda}}_{\to \vec{\mu}} = \omega\vec{\mu}$$



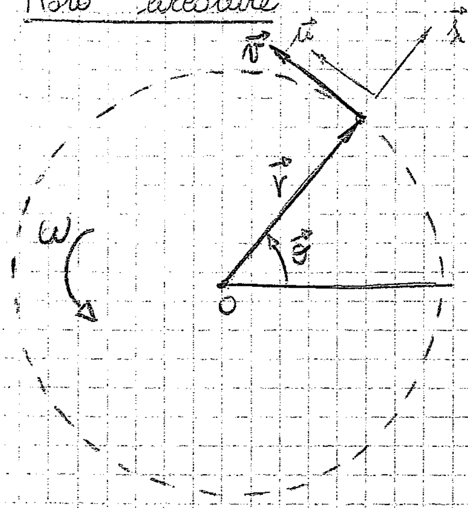
$$\vec{V} = \frac{d\vec{r}}{dt} = \dot{x}\vec{i}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \dot{V}\vec{i} = \ddot{x}\vec{i}$$

→ equazioni del moto

il versore $\vec{i}$ è comune
a tutte le quantità
quindi possiamo sottointenderlo

Moto circolare



$\vec{r} = r\vec{\lambda}$ $\quad r = \text{cost}$

$\vec{v} = \cancel{\dot{r}\vec{\lambda}}_{\to 0} + r\frac{d\vec{\lambda}}{dt} \Longrightarrow \boxed{\vec{v} = r\omega\vec{\mu}}$

$\vec{a} = \frac{d\vec{v}}{dt} = r\dot{\omega}\vec{\mu} + r\omega\frac{d\vec{\mu}}{dt} \Longrightarrow$

$\Longrightarrow \boxed{\vec{a} = r\dot{\omega}\vec{\mu} - r\omega^2\vec{\lambda}}$

$\vec{a} = \vec{a}_\mu + \vec{a}_\lambda = \vec{a}_t + \vec{a}_n$

- $\boxed{\vec{a}_n = r\omega^2(-\vec{\lambda})}$ sempre rivolta come $-$
quindi verso il centro
(accel. centripeta)

- $\boxed{\vec{a}_t = r\dot{\omega}\vec{\mu}}$
(accel. tangenziale)

$\theta(t)$
$\dot{\theta}(t) = \omega(t)$
$\ddot{\theta}(t) = \dot{\omega}(t)$
$\boxed{\vec{\omega} = \omega\vec{\nu}}$ $\odot^{\vec{\nu}}$

uniforme

$\omega = \text{cost}$

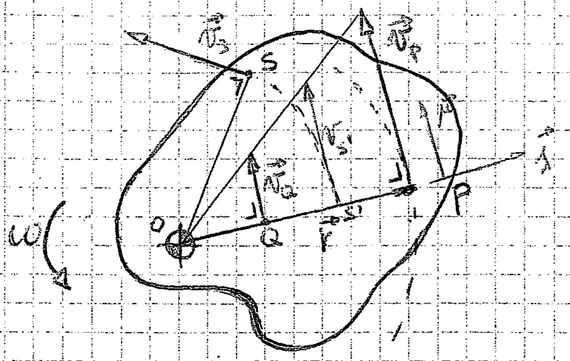
$\omega = \frac{d\theta}{dt}$ $\qquad \boxed{\theta = \theta_0 + \omega t}$

uniformemente accelerato

$\dot{\omega} = \frac{d\omega}{dt} = \text{cost}$

$\boxed{\omega = \omega_0 + \dot{\omega}t}$

$\boxed{\theta = \theta_0 + \omega_0 t + \frac{1}{2}\dot{\omega}t^2}$



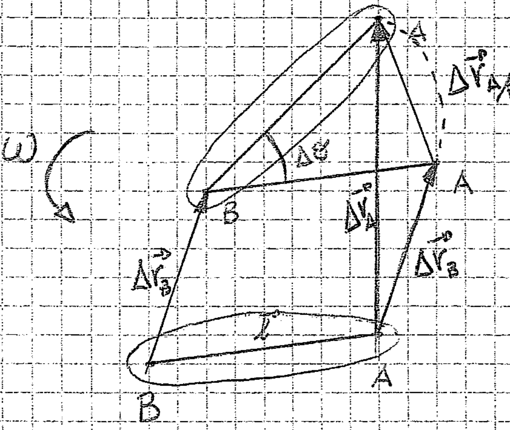
1 grado di libertà
$$\begin{cases} X_0 = \text{cost} & \dot{X}_0 = \ddot{X}_0 = 0 \\ y_0 = \text{cost} & \dot{y}_0 = \ddot{y}_0 = 0 \\ \theta(t) & \dot\theta = \omega \quad \ddot\theta = \dot\omega \end{cases}$$

P → moto <u>circolare</u>

corpo → moto <u>rotatorio</u>

$\vec r = r\vec\lambda$

$\vec v = r\omega\vec\mu$ la velocità varia in modo lineare



moto piano generico

<u>moto rototraslatorio</u>

rotazione di A intorno a B (A/B)

$\Delta\vec r_A = \Delta\vec r_B + \Delta\vec r_{A/B}$ nel tempo Δt

$\Delta\theta = \omega\cdot\Delta t$

$$\frac{\Delta\vec r_A}{\Delta t} = \frac{\Delta\vec r_B}{\Delta t} + \frac{\Delta\vec r_{A/B}}{\Delta t} \longrightarrow \lim_{\Delta t\to 0} \longrightarrow \boxed{\vec v_A = \vec v_B + \vec v_{A/B}}$$

<u>formula fondamentale della cinematica</u>

il moto di A/B è un moto circolare

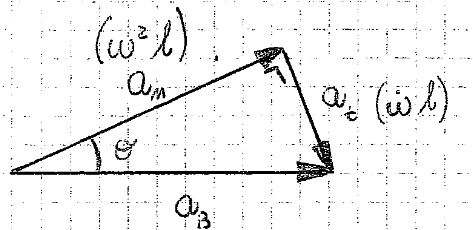
$\vec v_{A/B} = \omega\cdot l\vec\mu$

$\boxed{\vec a_A = \vec a_B + \vec a_{A/B}}$ <u>Teorema di Rivals</u>

$\vec a_{A/B} = \vec a_{A/B_t} + \vec a_{A/B_n}$

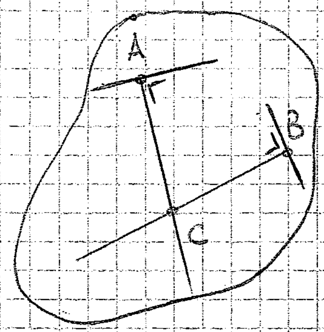
$\vec a_t = \dot\omega\, l\,\vec\mu$

$\vec a_n = \omega^2 l\,(-\vec\lambda)$

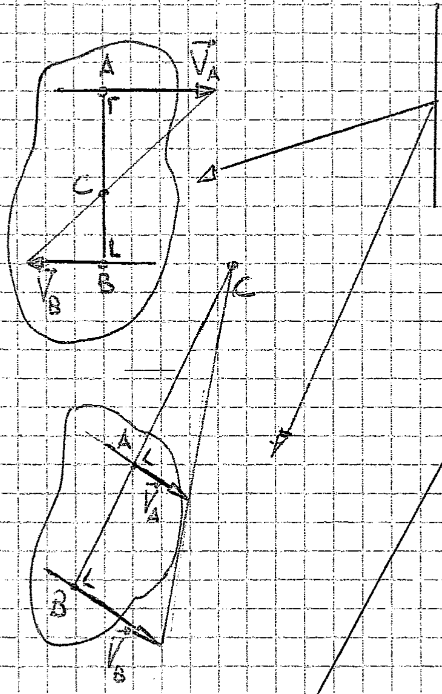


$$a_B = \frac{a_n}{\cos\theta} = \frac{\omega^2 l}{\cos\theta}$$

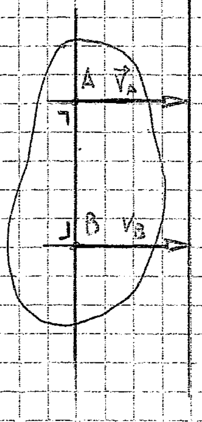
$$a_t = \sqrt{a_B^2 - a_n^2} = \dot\omega l \Longrightarrow \dot\omega = \frac{\sqrt{a_B^2 - a_n^2}}{l}$$



se conosco le direzioni delle velocità di due punti tracciando la loro perpendicolare trovo il centro di istantanea rotazione ($v_c = 0$)



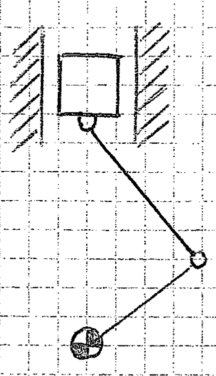
se le direzioni sono // ho bisogno di conoscere anche il verso e i moduli

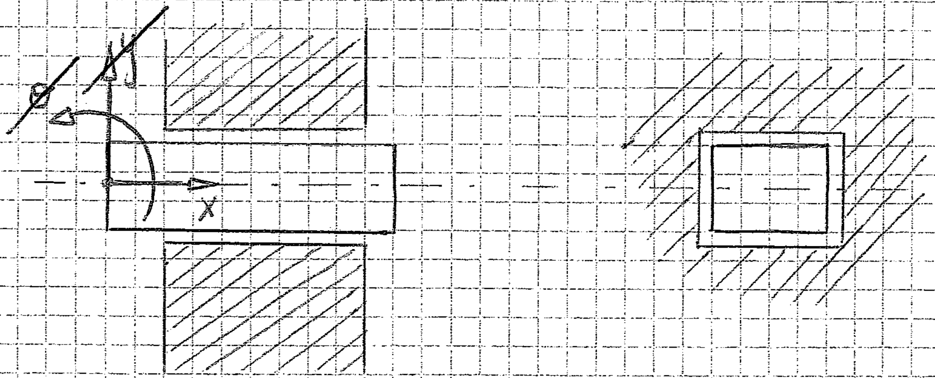


$$CA = CB = \infty$$

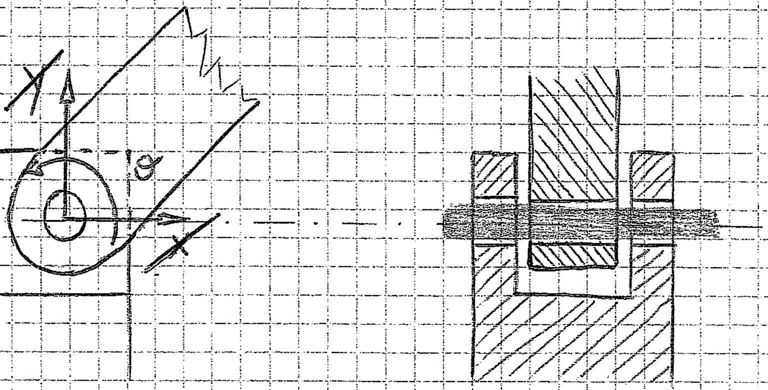
$$\omega = \frac{V_A}{AC} = \frac{V_B}{BC} = 0$$

Accoppiamento tra corpi

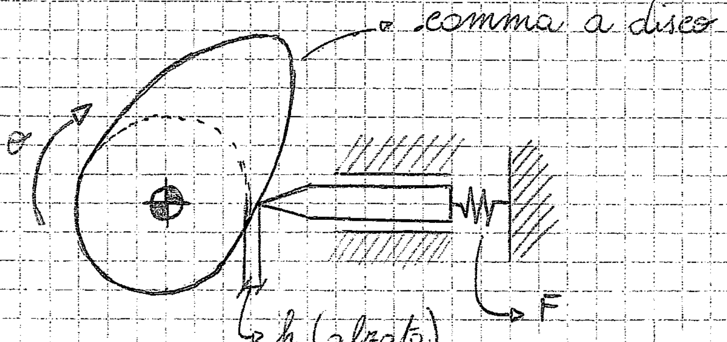




accoppiamento **PRISMATICO**

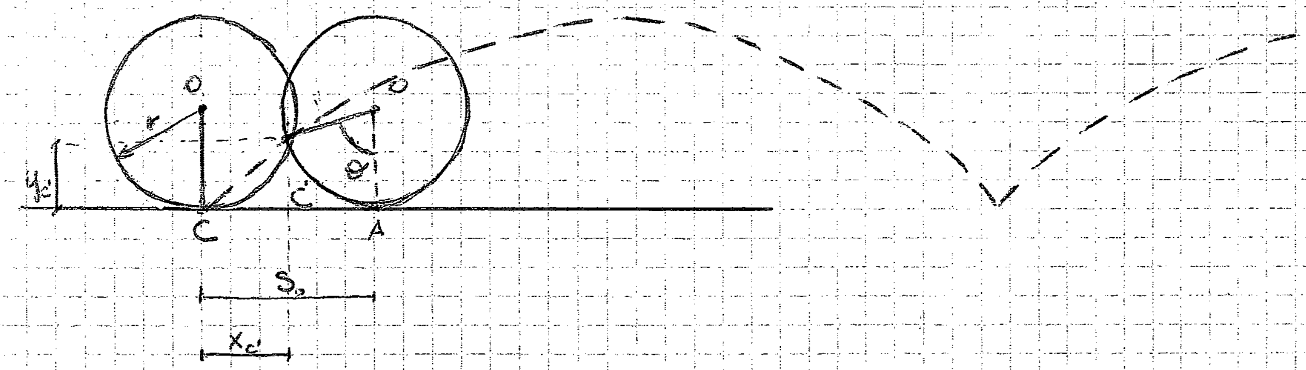


accoppiamento **ROTOIDALE**



accoppiamenti di **FORZE**

$h = h(\theta)$ dipende dall'angolo



cinematica di O e di C

$$S_o = \overline{AC} = \widehat{AC'} = r\vartheta$$

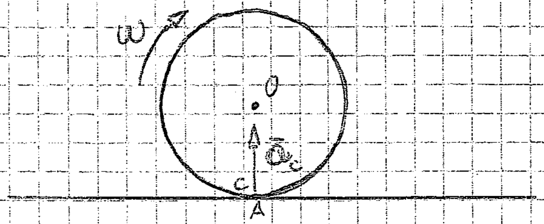
$$v_o = \frac{ds_o}{dt} = r\dot{\vartheta} = r\omega$$

$$a_o = r\ddot{\vartheta} = r\dot{\omega}$$

$$\begin{cases} x_{c'} = r\vartheta - r\,\mathrm{sen}\,\vartheta = r(\vartheta - \mathrm{sen}\,\vartheta) \\ \dot{x}_{c'} = r\omega(1 - \cos\vartheta) \\ \ddot{x}_{c'} = r\dot{\omega}(1 - \cos\vartheta) + r\omega^2\,\mathrm{sen}\,\vartheta \end{cases} \qquad \begin{cases} y_{c'} = r(1 - \cos\vartheta) \\ \dot{y}_{c'} = r\omega\,\mathrm{sen}\,\vartheta \\ \ddot{y}_{c'} = r\dot{\omega}\,\mathrm{sen}\,\vartheta + r\omega^2\cos\vartheta \end{cases}$$

$\vartheta = 0$

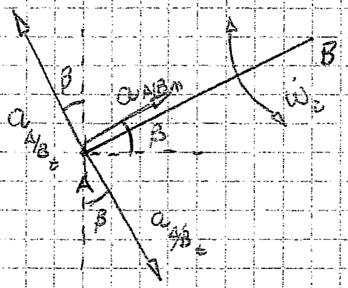
$$\begin{cases} x_{c'} = 0 \\ \dot{x}_{c'} = 0 \\ \ddot{x}_{c'} = 0 \end{cases} \qquad \begin{cases} y_{c'} = 0 \\ \dot{y}_{c'} = 0 \\ \ddot{y}_{c'} = r\omega^2 \end{cases}$$



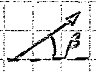
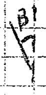
$$\vec{a}_c = r\omega^2\vec{j}$$

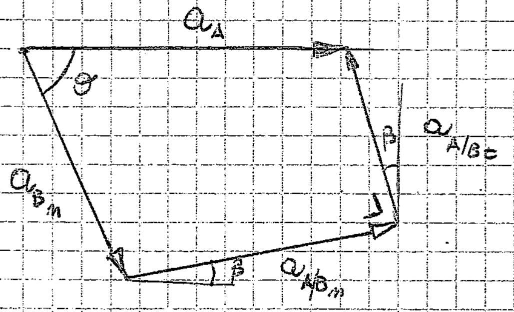
$$\vec{a}_A = \underset{n \quad \cancel{t}}{\vec{a}_B} + \underset{n \quad t}{\vec{a}_{A/B}}$$

↳ $\omega = \text{cost} \Rightarrow \dot{\omega} = 0$



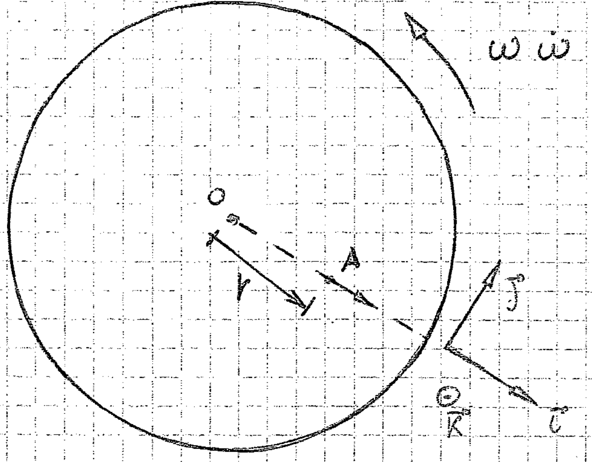
$$\vec{a}_A = \vec{a}_{B_n} + \vec{a}_{A/B_n} + \vec{a}_{A/B_t}$$

M	?	$r\omega^2$	$l\omega_2^2$	$(l\dot{\omega}_2)$?
D/V	——			



$$\begin{cases} a_A = a_{B_n} \cos\theta + a_{A/B_n} \cos\beta - a_{A/B_t} \sin\beta \\ a_{B_n} \sin\theta = a_{A/B_n} \sin\beta + a_{A/B_t} \cos\beta \end{cases}$$

$$\begin{cases} a_A = r\omega^2 \cos\theta + l\omega_2^2 \cos\beta - l\dot{\omega}_2 \sin\beta \\ r\omega^2 \sin\theta = l\omega_2^2 \sin\beta + l\dot{\omega}_2 \sin\beta \end{cases} \Longrightarrow \dot{\omega}_2 \Longrightarrow a_A$$



$\vec{V}_A = ?$ $\quad \vec{a}_A = ?$

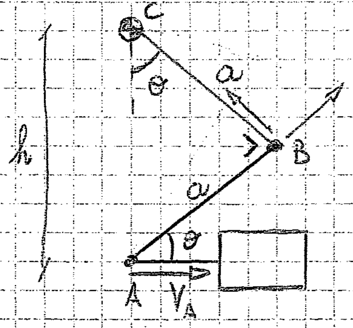
$$\vec{V}_A = \vec{V}_{r_A} + \vec{V}_{t_A} = \dot{r}\vec{i} + r\omega\vec{j}$$

$$|\vec{V}_A| = \sqrt{\dot{r}^2 + r^2\omega^2}$$

$$\vec{a}_A = \underbrace{\vec{a}_{r_A} + \vec{a}_{t_A} + \vec{a}_{c_A}} \longrightarrow \begin{cases} \vec{a}_{r_A} = \ddot{r}\vec{i} \\ \vec{a}_{t_A} = r\dot{\omega}\vec{j} - r\omega^2\vec{i} \\ \vec{a}_{c_A} = 2\vec{\omega}\wedge V_{r_A} = 2\omega\vec{k}\wedge\dot{r}\vec{i} = 2\dot{r}\omega\vec{j} \end{cases}$$

$$|\vec{a}_A| = \sqrt{(\ddot{r} - r\omega^2)^2 + (r\dot{\omega} + 2\dot{r}\omega)^2}$$

ES 1.10



$V_A = 0,5\ m/s$

$a = \overline{BC} = \overline{AB} = 125\ mm$

$h = 175\ mm$

$\omega_{AB} = ?$ $\omega_{BC} = ?$

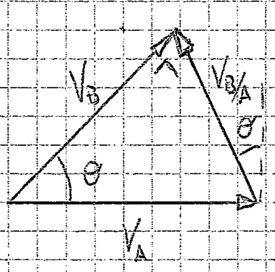
$$h = \overline{BC}\cos\theta + \overline{AB}\sin\theta = a(\cos\theta + \sin\theta)$$

$$h^2 = a^2(\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta) \Longrightarrow h^2 = a^2(1 + 2\cos\theta\sin\theta)$$

$$h^2 = a^2(1 + \sin 2\theta)$$

$$\frac{h^2}{a^2} - 1 = \sin 2\theta \Longrightarrow \theta = \frac{1}{2}\arcsin\left(\frac{h^2}{a^2} - 1\right) = 36,9^\circ$$

	$V_B =$	V_A +	$V_{B/A}$
M	?	V_A	?
D/V	$\angle\theta$	$\rightarrow$	$\perp\theta$



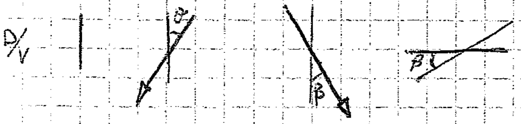
$$V_{B/A} = V_A \sin\theta = \omega_{AB}\, a \Longrightarrow \omega_{AB} = \frac{V_A}{a}\sin\theta$$

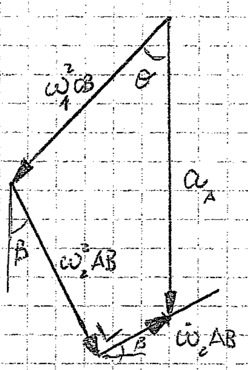
$$V_B = V_A \cos\theta = \omega_{BC}\, a \Longrightarrow \omega_{BC} = \frac{V_A}{a}\cos\theta$$

$$a_A = \underbrace{a_B}_{n \quad \cancel{t}} + \underbrace{a_{A/B}}_{n \quad t}$$

$$a_A = a_B + a_{A/B_n} + a_{A/B_t}$$

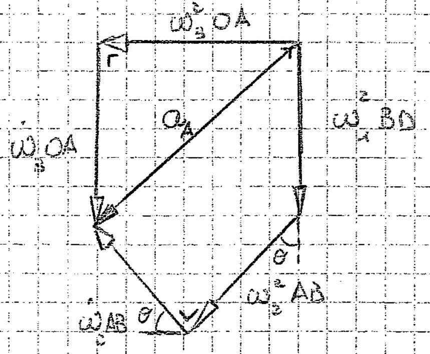
M ? $\omega_1^2 OB$ $\omega_3^2 AB$? $(\dot\omega_2 AB)$





$$\omega_1^2 OB \sin\theta = \omega_2^2 AB \sin\beta + \dot\omega_2 AB \cos\beta \Longrightarrow \dot\omega_2 = \frac{\omega_1^2 OB \sin\theta - \omega_2^2 AB \sin\beta}{AB \cos\beta}$$

$$a_A = \omega_1^2 OB \cos\theta + \omega_2^2 AB \cos\beta - \dot\omega_2 AB \sin\beta$$

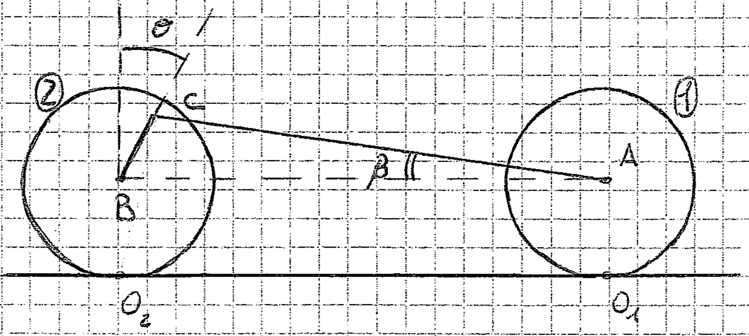


$$\begin{cases} \dot\omega_3 OA = \omega_1^2 BD + \omega_2^2 AB \cos\theta - \dot\omega_2 AB \sin\theta \\ \omega_3^2 OA = \omega_2^2 AB \sin\theta + \dot\omega_2 AB \cos\theta \end{cases} \Longrightarrow$$

$\Longrightarrow$ $\dot\omega_2$ ↺

$\dot\omega_3$ ↻

ES 1.17



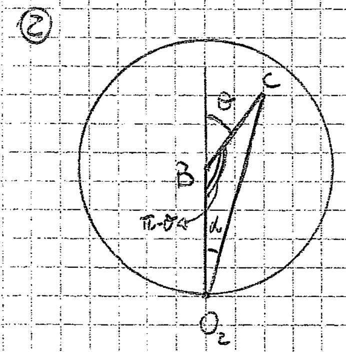
$V_A = 5$ m/s

$AC = 800$ mm

$r = 250$ mm

$BC = 200$ mm

se $\theta = 0 \Longrightarrow \begin{cases} \omega_{AC} = ? \\ V_C = ? \\ V_B = ? \end{cases}$



$$O_2C = \sqrt{r^2 + BC^2 - 2rBC\cos(\pi-\theta)}$$

$$O_2C = \sqrt{r^2 + BC^2 - 2rBC\cos\theta} \quad ①$$

$$\frac{BC}{\sin\alpha} = \frac{O_2C}{\sin(\pi-\theta)} = \frac{O_2C}{\sin\theta} \Longrightarrow \sin\alpha = \frac{BC}{O_2C}\sin\theta \quad ②$$

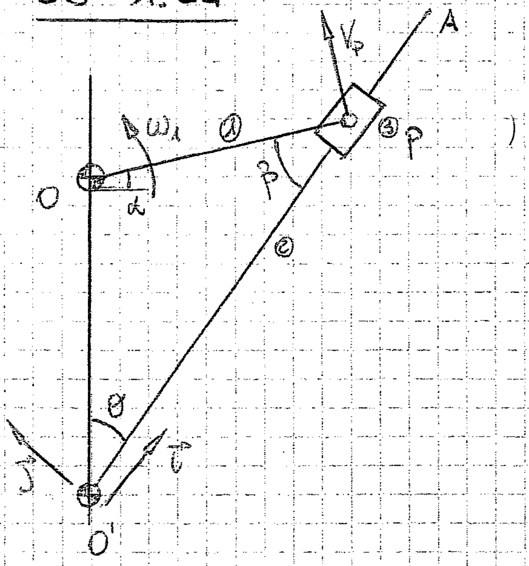
$$\frac{BC}{\sin\beta} = \frac{AC}{\sin\left(\frac{\pi}{2}-\theta\right)} \Longrightarrow \sin\beta = \frac{BC}{AC}\cos\theta \quad ③$$

① se $\theta = 0 \Longrightarrow O_2C = r + BC$

② // $\Longrightarrow \alpha = 0$

③ // $\Longrightarrow \sin\beta = \frac{BC}{AC}$

ES 1.24



$OP = 0,3\ m$

$O'A = 0,8\ m$

$O'O = 0,4\ m$

$\theta = 25°$

$\omega_1 = 100\ rad/s$

$\omega_2 = ?\quad \dot{\omega}_2 = ?$

$V_A = ?$

moto assoluto di P ⟹ circolare intorno a O (ω_1)

moto glifo ⟹ rototorio intorno a O' (ω_2)

moto relativo di P ⟹ $/\!/\ \vec{i}$

moto traslatorio di P ⟹ circolare intorno a O' (ω_2)

$$\frac{OP}{\sin\theta} = \frac{O'P}{\sin(90°+\alpha)} = \frac{OO'}{\sin\beta}$$

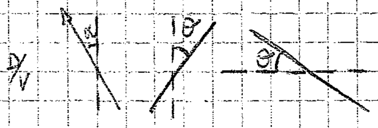
$$\beta = \arcsin\left(\frac{OO'}{OP}\sin\theta\right)$$

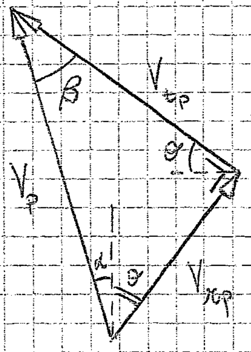
$$O'P = OP\,\frac{\sin(90°+\alpha)}{\sin\theta}$$

$$\alpha = 180° - 90° - \beta - \theta$$

$$\vec{V}_P = \vec{V}_{rP} + \vec{V}_{tP}$$

M $\quad \omega_1 OP \quad ? \quad ?(\omega_2 O'P)$

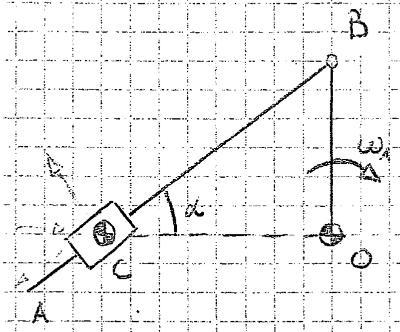




$$V_{tP} = V_P \cos\beta \Longrightarrow \omega_1 OP \cos\beta = \omega_2 O'P \Longrightarrow \omega_2 = \omega_1 \frac{OP}{O'P}\cos\beta$$

$$V_A = \omega_2\, O'A$$

ES 1.25



OB = 250 mm

OC = 600 mm

ω_1 = 5 rad/s

V_B = ?

V_{r_C} = ?

ω_2 = ?

$$CB = \sqrt{OB^2 + OC^2}$$

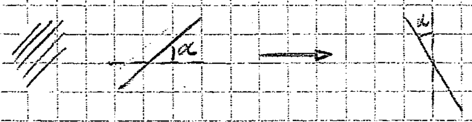
$$\frac{OB}{\sin\alpha} = \frac{CB}{1} \Longrightarrow \sin\alpha = \frac{OB}{CB} \Longrightarrow \alpha = \arcsin\left(\frac{OB}{CB}\right)$$

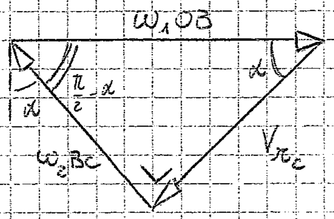
$$V_C = 0$$

$$V_C = V_{r_C} + \overbrace{V_{t_B} + V_{t_{C/B}}}^{V_{t_C}}$$

M $\quad 0 \quad ? \quad \omega_1 OB \quad (\omega_2 BC)?$

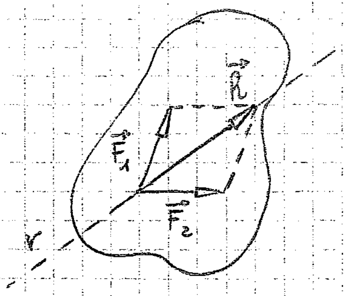
D/V



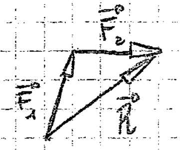


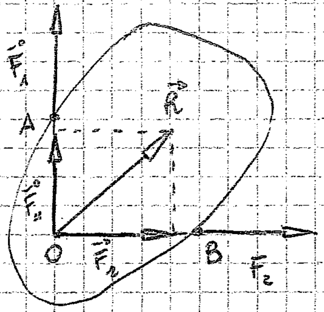
$$V_{r_C} = \omega_1 OB \cos\alpha$$

$$\omega_1 OB \cos\left(\frac{\pi}{2} - \alpha\right) = \omega_2 BC \Longrightarrow \omega_2 = \omega_1 \frac{OB}{BC} \sin\alpha$$

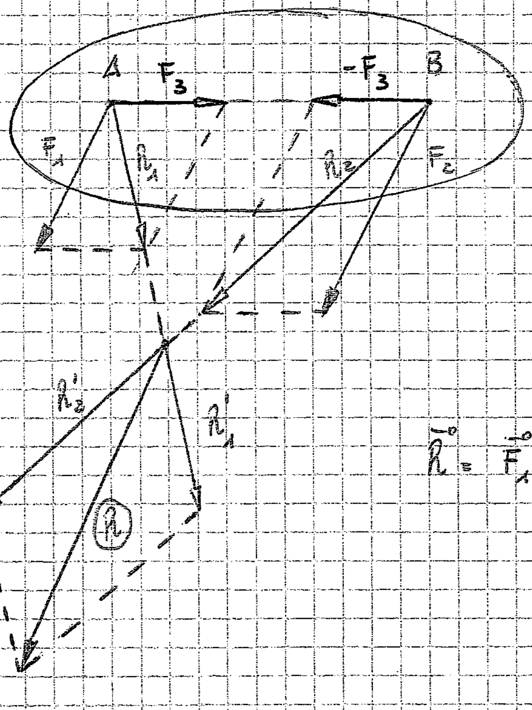


$\vec{R} = \vec{F}_1 + \vec{F}_2$





posso far scorrere le forze
lungo le loro rette d'azione



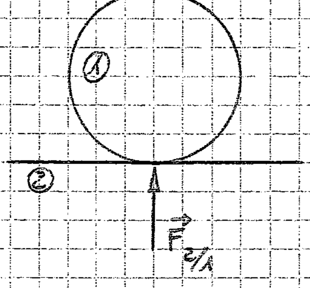
$\vec{R} = \vec{F}_1 + \vec{F}_2$

$\vec{R} /\!/ \vec{F}_1 /\!/ \vec{F}_2$

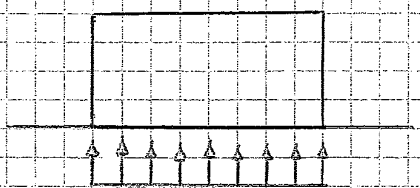
Equilibrio di un sistema di forze

$$\begin{cases} \vec{R} = \Sigma \vec{F} = 0 \Longrightarrow \text{il corpo non } \underline{\text{trasla}} \\ \vec{M} = 0 \Longrightarrow \text{il corpo non } \underline{\text{ruota}} \end{cases}$$

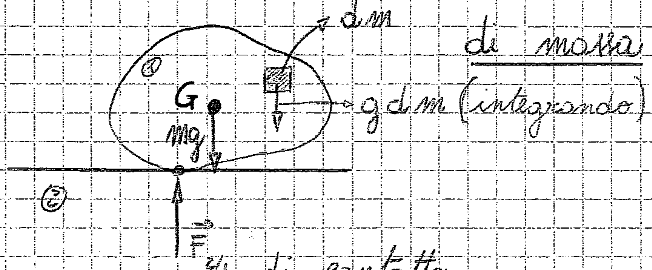
Tipiche forze nei sistemi meccanici



concentrata

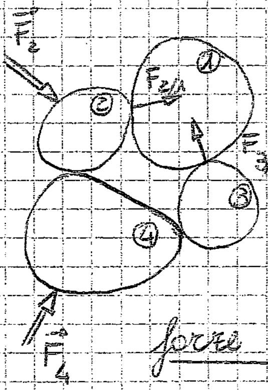


distribuita



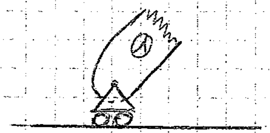
di massa

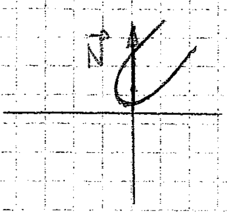
di contatto



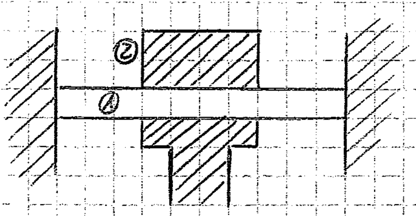
forze interne

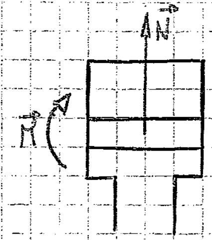
forze esterne



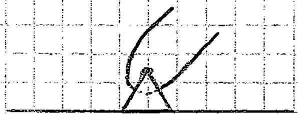


$\vec{N}$ ⇒ · ⊥ al piano di appogg
· premente

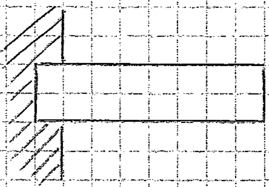


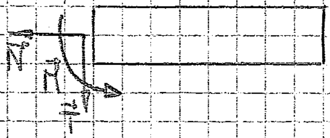


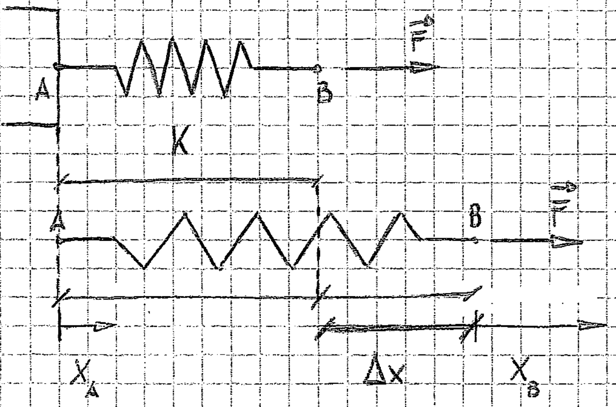
$\vec{N}$ ⇒ · ⊥ all'asse
· verso qualunque



direzione qualunque

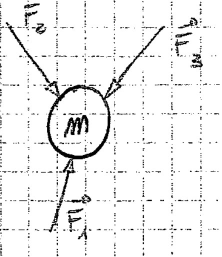






$$\vec{F} = K \underbrace{(x_B - x_A)}_{\Delta x}$$

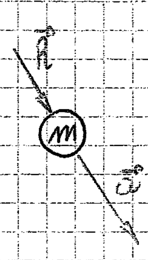
Equazioni cardinali o di Newton



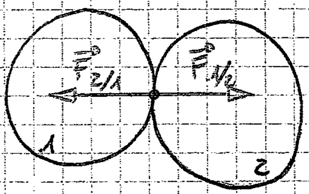
$$\vec{R} = \sum \vec{F}_e = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \begin{cases} = 0 & \text{①} \\ \neq 0 & \text{②} \end{cases}$$

① se $\vec{v} = 0 \Longrightarrow$ il corpo rimane fermo
se $\vec{v} \neq 0 \Longrightarrow$ moto rettilineo uniforme ($\vec{v}$ = cost)

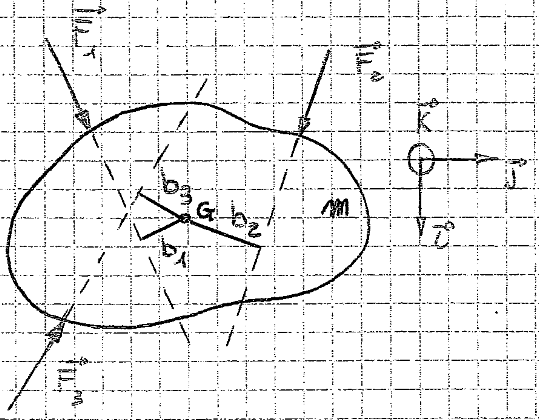
② $\sum \vec{F}_e = m\vec{a} = \vec{R}$



③



$$\vec{F}_{2/1} = -\vec{F}_{1/2}$$



$\sum \vec{F}_e = \vec{R} \neq 0 \Longrightarrow$

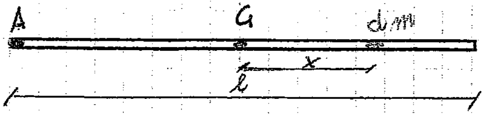
$\Longrightarrow \boxed{\sum \vec{F}_e = m\vec{a}_G}$ valida anche nello spazio

$\sum \vec{M}_{e_G} = b_1 F_1 \vec{K} - b_2 F_2 \vec{K} - b_3 F_3 \vec{K} \Longrightarrow$

$\Longrightarrow \boxed{\sum \vec{M}_{e_G} = I_G \dot{\vec{\omega}}}$ valida solo nel piano

I_G = momento d'inerzia baricentrico del corpo

$[I_G] = Kg \cdot m^2$



$$I_G = \int x^2\, dm$$

$m = \boxed{\mu}\, l$ → densita $dm = \mu\, dx$

$$I_G = \mu \int_{-l/2}^{l/2} x^2\, dx = \mu \frac{l^3}{12} = \frac{m l^2}{12}$$

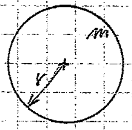
$$I_A = \mu \int_0^l x^2\, dx = \frac{\mu l^3}{3} = \frac{m l^2}{3}$$

Teorema

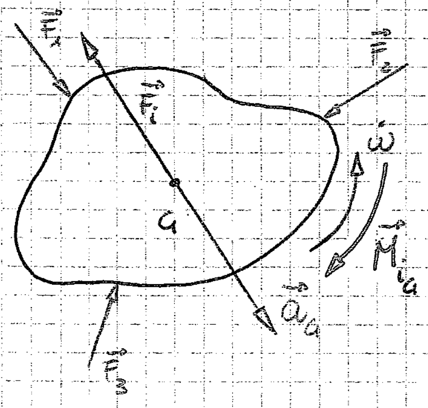
$$\boxed{I_A = I_G + m\,\overline{AG}^2} \Longrightarrow \frac{m l^2}{12} + m \frac{l^2}{4} = \frac{m l^2}{3}$$

raggio di inerzia

$$\boxed{I_G = m \rho_G^2}$$



$$I_G = \frac{m r^2}{2} \Longrightarrow \rho = \frac{r}{\sqrt{2}}$$



$$\begin{cases} \Sigma \vec{F}_e = m \vec{a}_G \\ \Sigma \vec{M}_e = I_G \dot{\vec{\omega}} \end{cases}$$

$$\begin{cases} \Sigma \vec{F}_e \boxed{- m\vec{a}_G} = 0 \\ \Sigma \vec{M}_e \boxed{- I_G \dot{\vec{\omega}}} = 0 \end{cases}$$

$$\begin{cases} \Sigma \vec{F}_e + \vec{F}_i = 0 \\ \Sigma \vec{M}_e + \vec{M}_i = 0 \end{cases}$$

$\vec{F}_i$ → risultante forze d'inerz

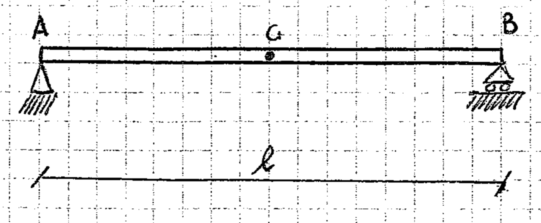
$\vec{M}_i$ → momento risultante dell forze d'inerzia

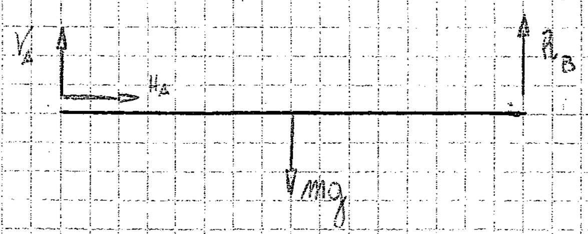
sistema d'Alambert

i momenti devono riferirsi allo stesso polo

equilibrio statico e dinamico del sistema

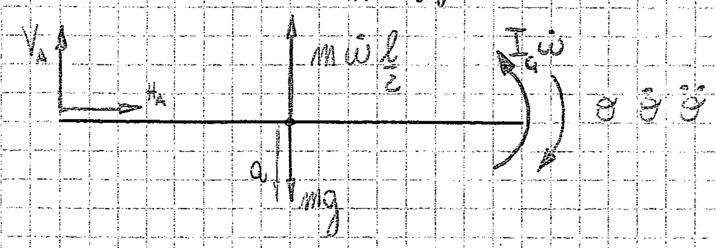
ES 2.21





$$\begin{cases} H_A = 0 & \rightarrow \\ V_A + R_B - mg = 0 & \uparrow \\ R_B\, l - mg\,\frac{l}{2} = 0 & \circlearrowleft_A \end{cases} \qquad \begin{cases} H_A = 0 \\ V_A = R_B = \frac{mg}{2} \end{cases}$$

eliminiamo l'appoggio B



$$t=0 \begin{cases} \theta = 0 \\ \dot{\theta} = 0 \\ \ddot{\theta} \neq 0 \end{cases}$$

moto incipiente
(che sta per inizi

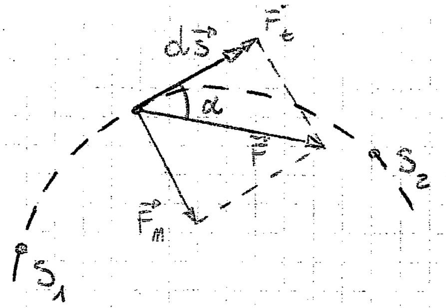
$$a_G = a_{G_n} + a_{G_t} \qquad I_G = \frac{m\,l^2}{12}$$

$$\cancel{\omega^2 \frac{l}{2}} + \dot{\omega}\,\frac{l}{2} \downarrow$$

$$\begin{cases} H_A = 0 & \rightarrow \\ V_A + m\dot{\omega}\frac{l}{2} - mg = 0 & \uparrow \\ m\dot{\omega}\frac{l^2}{4} - mg\frac{l}{2} + I_G\dot{\omega} = 0 & \circlearrowleft_A \end{cases} \Longrightarrow \dot{\omega}\frac{l}{3} = \frac{g}{2} \Rightarrow \dot{\omega} = \frac{3}{2}\frac{g}{l}$$

$$a_G = \dot{\omega}\,\frac{l}{2} \Rightarrow \dot{\omega} = \frac{2}{l}a_G \qquad \Longrightarrow a_G = \frac{3}{4}g$$

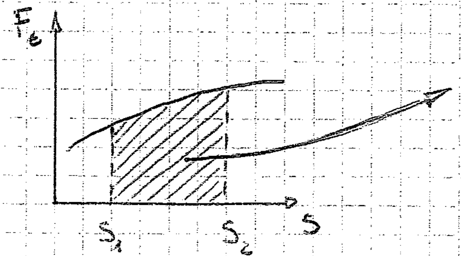
$$V_A + m\,\frac{3}{4}g - mg = 0 \Rightarrow V_A = \frac{mg}{4}$$



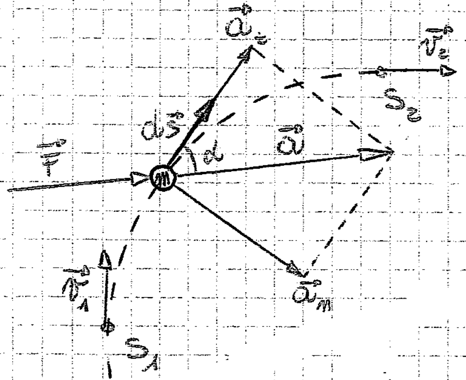
$$dL = \vec{F} \cdot d\vec{s}$$

$$dL = F \cdot ds \cdot \cos\alpha = F_t \, ds$$

[svolge lavoro solo la componente lungo lo spostamento



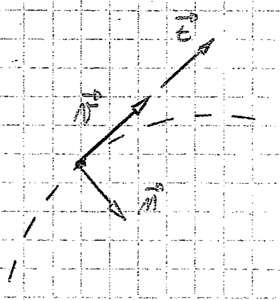
$$L_{1-2} = \int_{S_1}^{S_2} F_t \, ds$$



$$\vec{F} = m\vec{a}$$

$$dL = \vec{F} d\vec{s} = m\vec{a} \cdot d\vec{s} = m a \cos\alpha \, ds \Longrightarrow$$

$$\Longrightarrow dL = m a_t \, ds$$



velocità sempre tangenziale

$$\vec{v} = v \, \vec{t}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \underbrace{\frac{dv}{dt}\vec{t}}_{\vec{a}_t} + \underbrace{v\frac{d\vec{t}}{dt}}_{\vec{a}_n}$$

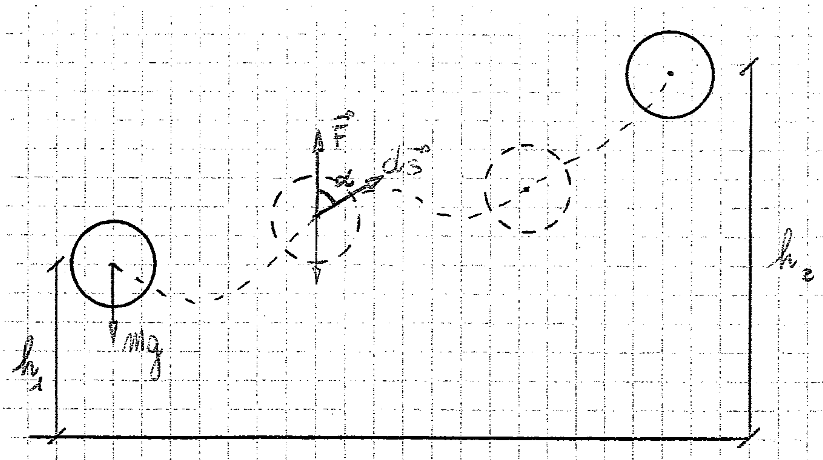
$$dL = m \frac{dv}{dt} ds = m v \, dv$$

$$L_{1-2} = m \int_{v_1}^{v_2} v \, dv = \boxed{\frac{1}{2} m \left(v_2^2 - v_1^2\right)}$$

$$\Downarrow$$

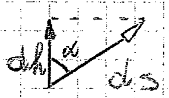
<u>energia cinetica</u> $\Longleftarrow \boxed{E_c = \frac{1}{2} m v^2}$

$$\boxed{L_{1-2} = \Delta E_c} \Longrightarrow \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$



$F = mg$

$dL = \vec{F}\, d\vec{s} = F \cdot ds \cdot \cos\alpha$



$dL = mg\, dh$

$$L_{1-2} = mg \int_{h_1}^{h_2} dh = \boxed{mg(h_2 - h_1)}$$

$\Downarrow$

energia potenziale gravitaz. $\Longleftarrow$ $\boxed{E_g = mgh}$

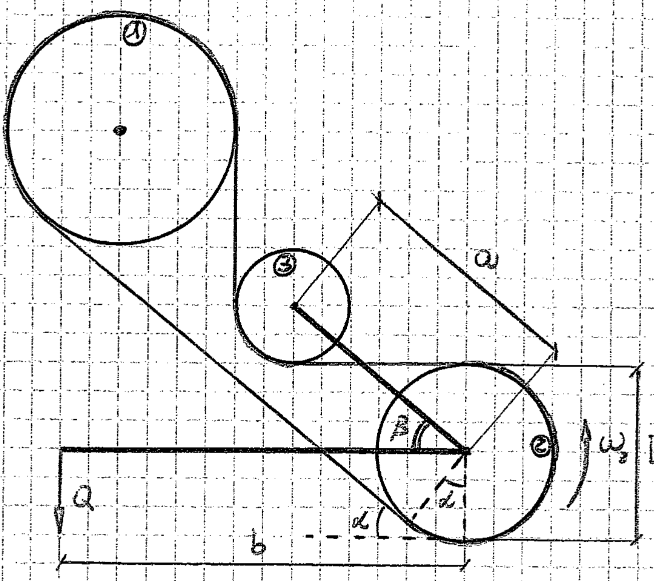
$\boxed{L_{1-2} = \Delta E_g} \Longrightarrow mgh_2 - mgh_1$

$\boxed{E_M = E_c + E_e + E_g}$ $\Longrightarrow$ energia meccanica

sistema conservativo $\Longrightarrow$ E_M = cost.

sistema con dissipazione $\Longrightarrow$ E_M si riduce (una parte viene dissipa
es. calore

$$\boxed{L_i + L_e = \underbrace{\Delta E_c}_{\frac{1}{2} m (v_f^2 - v_i^2) + \frac{1}{2} I_a (\omega_f^2 - \omega_i^2)} + \underbrace{\Delta E_e}_{\frac{1}{2} K (x_f^2 - x_i^2)} + \underbrace{\Delta E_g}_{mg(h_f - h_i)} = \Delta E_M}$$ $\Longrightarrow$ equazione dell'energia



$\omega_2 = 360$ rpm

$Q = 180$ N

$D_2 = 300$ mm

$a = 300$ mm

$b = 405$ mm

$\alpha = 30°$

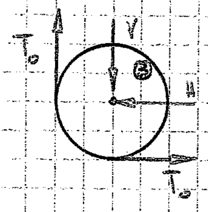
$\beta = 60°$

$f_a = 0{,}3$

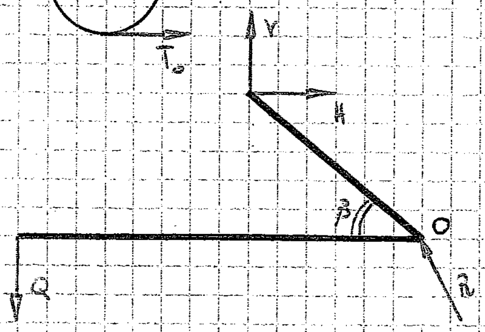
$T_1 = ?$

$T_0 = ?$

$W = ?$ potenza trasmessa



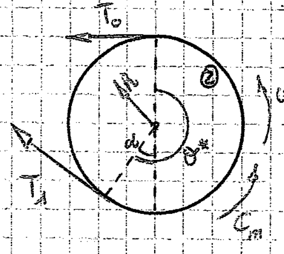
$$\begin{cases} V = T_0 \\ H = T_0 \end{cases}$$



$\circlearrowleft_O \quad Qb = V a \cos\beta + H a \operatorname{sen}\beta$

$Qb = T_0 a \cos\beta + T_0 a \operatorname{sen}\beta$

$$T_0 = \frac{Qb}{a(\cos\beta + \operatorname{sen}\beta)} = 178 \text{ N}$$

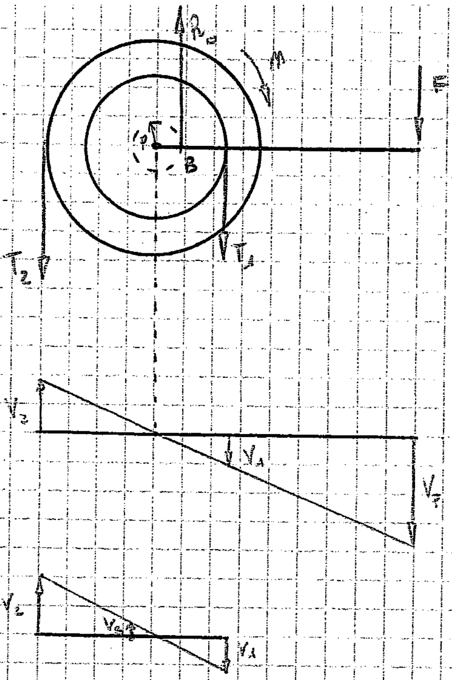


$\vartheta^* = \pi + \alpha$

$$\frac{T_1}{T_0} = e^{f\vartheta^*}$$

$T_1 = 534$ N

$W = C_m \cdot \omega_2 = (T_1 - T_0)\frac{D}{2}\omega_2 = 2015$ W



$$Q_B \quad F(a-\rho) + T_1\left(\frac{d}{2}-\rho\right) - T_2\left(\frac{D}{2}+\rho\right) = 0$$

$F = 312 \text{ N}$

$\omega = \frac{2\pi}{60} n$

$V_F = \omega a = 1{,}57 \text{ m/s}$

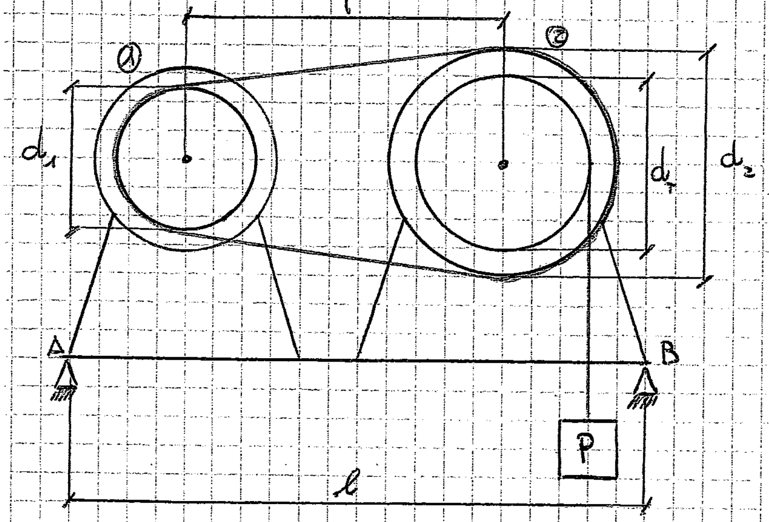
$V_1 = \omega \frac{d}{2} = 0{,}628 \text{ m/s}$

$V_2 = \omega \frac{D}{2} = 0{,}785 \text{ m/s}$

$$V_S = \frac{V_2 - V_1}{2}$$

$$\eta = \frac{P\, V_S}{F\, V_F} = 0{,}8$$

ES 5,19



$C_m = 600 \text{ Nm}$

$P = 3000 \text{ N}$

$\ell = 1{,}2 \text{ m}$

$i = 0{,}6 \text{ m}$

$d_1 = 250 \text{ mm}$

$d_2 = 500 \text{ mm}$

$d_T = 400 \text{ mm}$

$m_1 = 50 \text{ Kg}$

$m_2 = 50 \text{ Kg}$

$I_1 = 0{,}05 \text{ Kg m}^2$

$I_2 = 0{,}25 \text{ Kg m}^2$

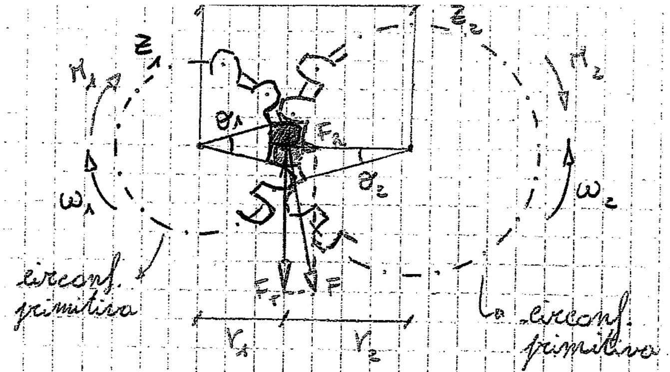
$\ddot{x} = ?$

$R_A = ?$

$R_B = ?$

velocità periferiche uguali

$$\omega_1 \frac{d_1}{2} = \omega_2 \frac{d_2}{2}$$



$M_1 = F_T r_1$

$M_2 = F_T r_2$

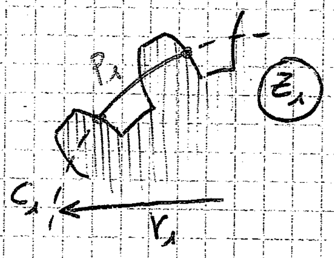
$r_1 = ?$

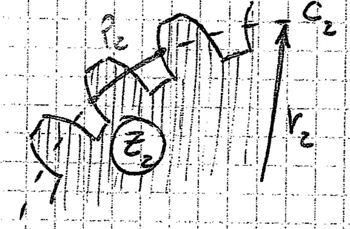
$r_2 = ?$

$i = \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = ?$

$$\begin{cases} \theta_1 = \omega_1 \Delta t = \frac{2\pi}{z_1} \\ \theta_2 = \omega_2 \Delta t = \frac{2\pi}{z_2} \end{cases} \Longrightarrow \frac{\omega_1}{\omega_2} = \frac{z_2}{z_1} = i \qquad \left.\begin{matrix}\theta_1 \\ \theta_2\end{matrix}\right\} \text{passi angolari}$$

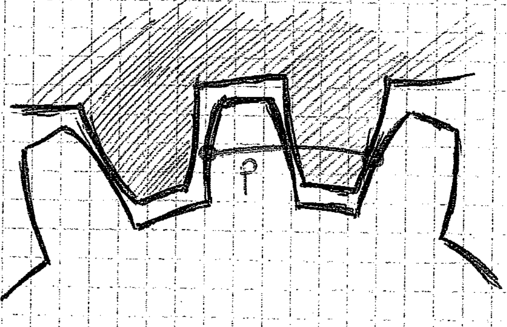
$$\begin{cases} i = \frac{z_2}{z_1} = \frac{r_2}{r_1} \\ a = r_2 + r_1 \end{cases} \qquad \begin{cases} r_1 = \frac{a}{1+i} \\ r_2 = a \frac{i}{1+i} \end{cases} \qquad \text{raggi primitivi}$$



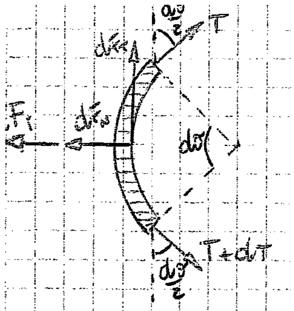


$\frac{p_1}{p_2} = \frac{r_1}{z_1} \cdot \frac{z_2}{r_2} = \frac{i}{i} = 1 \qquad p = p_1 = p_2$

$p \Longrightarrow$ passo dell'ingranaggio



lunghezza uguale
altezza uguale



$$dF_i = dm\,r\omega^2 = q\,dl\,\frac{V^2}{r} = q\,r\,d\vartheta\,\frac{V^2}{\cancel{r}} = qV^2 d\vartheta$$

$$\begin{cases} dF_T = dT\cos\frac{d\vartheta}{2} \\ dF_N = 2T\,\text{sen}\frac{d\vartheta}{2} + dT\,\cancel{\text{sen}\frac{d\vartheta}{2}} - qV^2 d\vartheta \\ \frac{dF_T}{dF_N} = f \quad \text{strisciamento} \end{cases}$$

$$\begin{cases} dF_T = dT \\ \frac{dF_T}{f} = T\,d\vartheta - qV^2 d\vartheta \end{cases} \Longrightarrow \frac{dT}{f} = (T - qV^2)\,d\vartheta$$

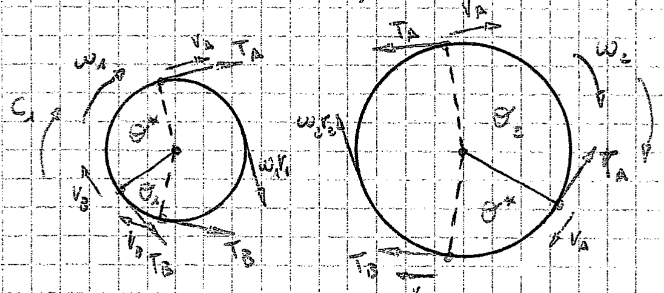
$$\int_{T_A}^{T_B} \frac{dT}{T - qV^2} = f\int_0^{\vartheta^*} d\vartheta \Longrightarrow \lg\frac{T_B - qV^2}{T_A - qV^2} = f\vartheta^* \Longrightarrow \boxed{\frac{T_B - qV^2}{T_A - qV^2} = e^{f\vartheta^*}}$$

se $q = 0 \Longrightarrow \frac{T_B}{T_A} = e^{f\vartheta^*}$ (freno a nastro)

$C_1 = (T_B - T_A)\,r_1 = T_A(e^{f\vartheta^*} - 1)\,r_1$ se $C_1 \uparrow$ anche $\vartheta^* \uparrow$

il max è quando $\vartheta^* = \alpha_1$

$C_{1_{MAX}} = T_A(e^{f\alpha_1} - 1)$



$i = \frac{\omega_1}{\omega_2}$ $\qquad V_B = \omega_1 r_1 \qquad V_A = \omega_2 r_2$

$$i = \frac{\omega_1}{\omega_2} = \frac{V_B}{r_1}\cdot\frac{r_2}{V_A} = \frac{r_2}{r_1}\cdot\frac{V_B}{V_A} = \frac{r_2}{r_1}\,\frac{1 + \frac{T_B}{ES}}{1 + \frac{T_A}{ES}} \quad \left(> \frac{r_2}{r_1}\right)$$

$$\eta = \frac{C_2\,\omega_2}{C_1\,\omega_1} = \frac{\cancel{(T_B - T_A)}\,r_2\omega_2}{\cancel{(T_B - T_A)}\,r_1\omega_1} = \frac{V_A}{V_B} = \frac{1 + \frac{T_A}{ES}}{1 + \frac{T_B}{ES}} \quad (< 1)$$

$C_1 = (T_B - T_A)\, r_1$ $\frac{T_B}{T_A} \le e^{f_a \alpha_1}$

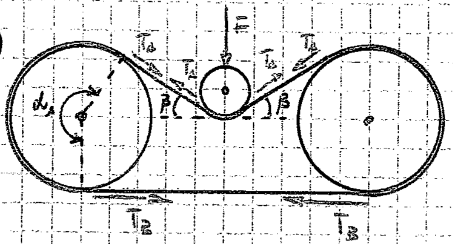
aderenza limite $T_B = T_A\, e^{f_a \alpha_1}$

$C_{1MAX} = T_A \left(e^{f_a \alpha_1} - 1 \right) r_1$

C_1 è in funzione di T_A

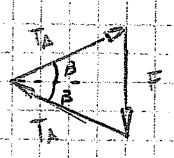
si fa in modo che la cinghia sia sempre in tensione (T_A)

①



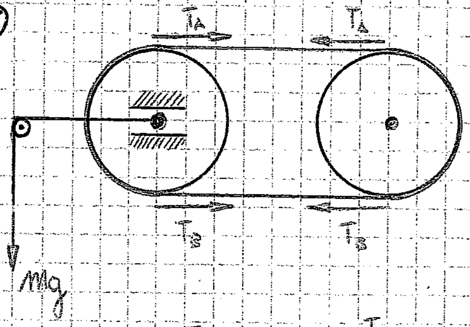
Galoppino

aumento della tensione T_A e aumento di α_1 } → aumento C_{1MAX}



$F = 2 T_A \sin\beta$

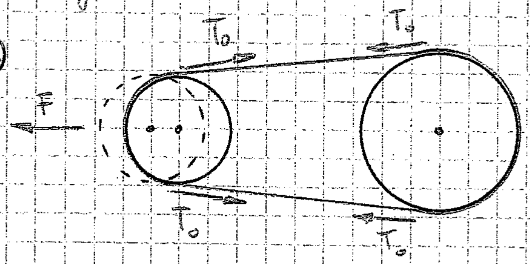
②



Tenditore

$Mg = T_A + T_B$

③



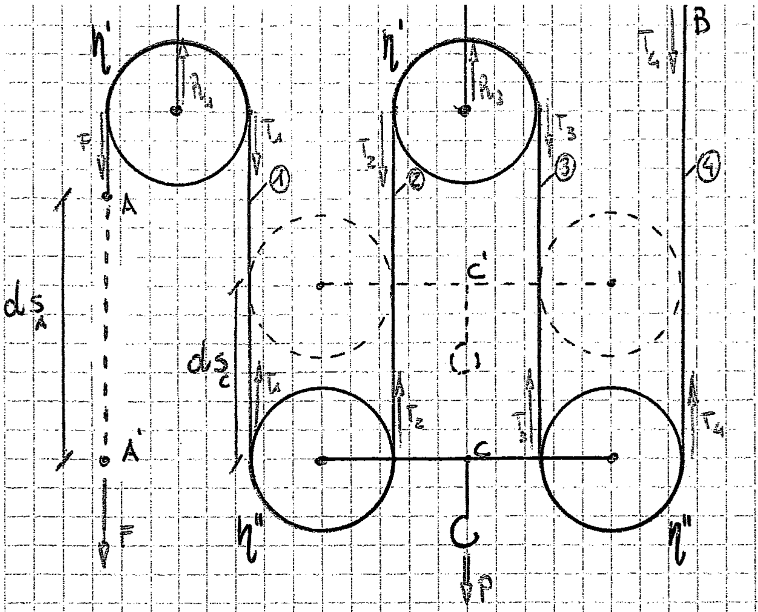
Tensionamento iniziale

(Forzamento)

→ T_0 si ridistribuirà in T_A e T_B

$T_0 \cong \frac{T_A + T_B}{2}$

① ② ③ tre metodi per tendere un flessibile ad attrito e permettergli di sfruttare la coppia motrice



Paranco

(fune inestensibile) AB = cost

4 tratti di funi portanti (n)

$$\frac{\Delta S_A}{\Delta t} = 4\frac{\Delta S_c}{\Delta t} \Longrightarrow V_A = 4V_c$$

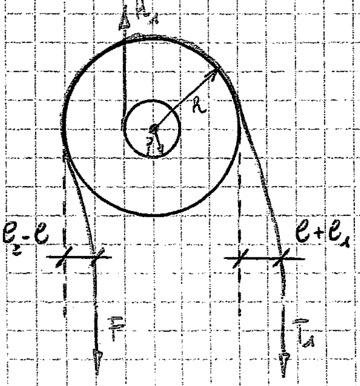
ipotesi: $\eta' = \eta'' = 1 \Longrightarrow$

$\Longrightarrow F = T_1 = T_2 = T_3 = T_4$

si considero solo le pulegge mobili

$P = T_1 + T_2 + T_3 + T_4$ con l'ipotesi del rendimento unitario

$P = 4F$ $P = nF$



in realtà

$$\eta' = \frac{R + e - e_2 - \rho}{R + e + e_1 + \rho} < 1$$

$T_1 = \eta' F$

$T_2 = \eta'' T_1 = \eta' \eta'' F$

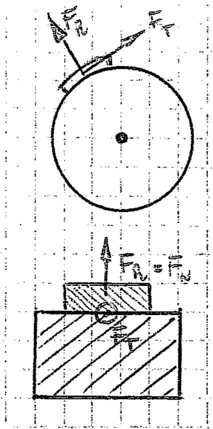
$T_3 = \eta' T_2 = \eta'^2 \eta'' F$

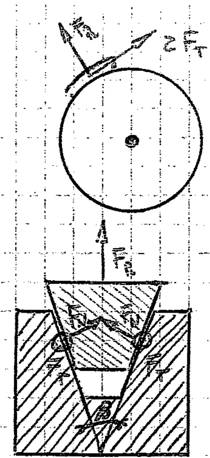
$T_4 = \eta'' T_3 = \eta'^2 \eta''^2 F$

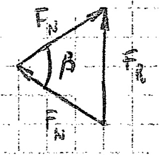
$P = \sum T_i = F\left(\eta' + \eta'\eta'' + \eta'^2\eta'' + \eta'^2\eta''^2\right)$ tutti componenti < 1

$$\eta_{TOT} = \frac{P_U}{P_M} = \frac{P v_c}{F V_A} = \frac{\left(\eta' + \eta'\eta'' + \eta'^2\eta'' + \eta'^2\eta''^2\right)}{4} < 1$$

→ sommati sono < 4







$$F_T = F_N f$$

$$\frac{F_T}{F_R} = f$$

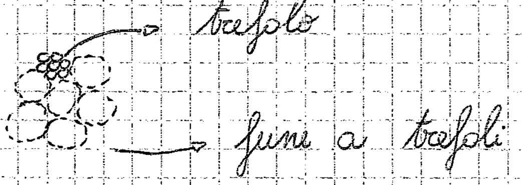
$$C = F_T r$$

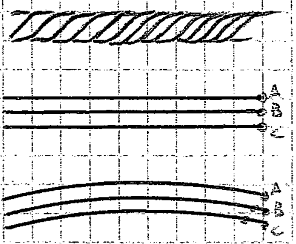
$$F_R = 2 F_N \sin\frac{\beta}{2} = 2 \frac{F_T}{f} \sin\frac{\beta}{2}$$

$$\frac{2F_T}{F_R} = \frac{f}{\sin\frac{\beta}{2}} = f' \; (> f)$$

$$C = 2 F_T r$$

Funi

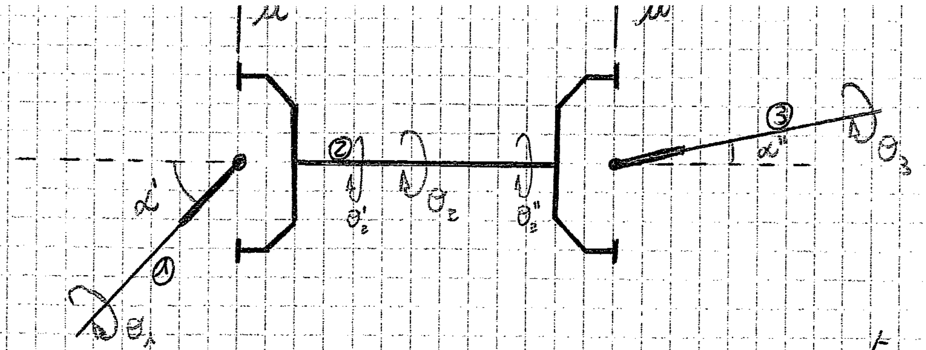




Se i fili non fossero intrecciati ogni volta che passano da una curva a un pezzo ret
sfregano ⟹ attrito

(fenomeno anaelastico
resistenza)

in più i fili sono metallici
e hanno una loro resistenza
elastica



trasmissione
omocinetica

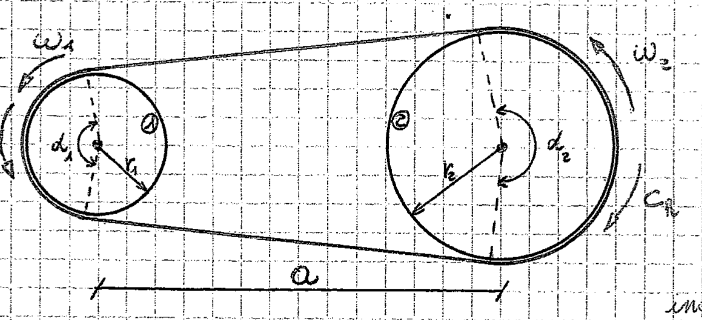
$$\boxed{\vec{u}' = \vec{u}''} \longrightarrow \theta_2 = \theta_2' = \theta''$$

$$\begin{cases} tg\,\theta_1 = tg\,\theta_2 \cos\alpha' \\ tg\,\theta_3 = tg\,\theta_2 \cos\alpha'' \end{cases} \longrightarrow \frac{tg\,\theta_1}{tg\,\theta_3} = \frac{\cos\alpha'}{\cos\alpha''}$$

se $\boxed{\alpha' = \alpha''} \longrightarrow \theta_1 = \theta_3$

Trasmissione con flessibili

cinghie
funi
catene



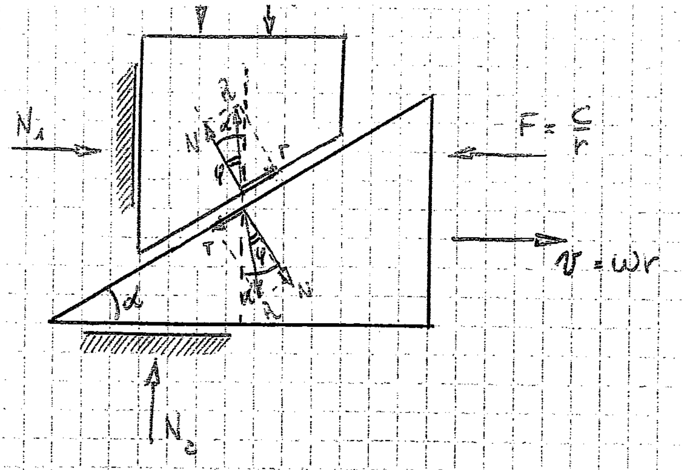
r_1 r_2 raggi pulegge
a interasse
α_1 α_2 angoli di avvolgimento

$P_M = C_M \cdot \omega_1$
$P_U = C_R \cdot \omega_2$

$u = \frac{\omega_1}{\omega_2}$ ingresso uscita $\Longrightarrow$ rapporto di trasmissione

$\eta = \frac{P_M}{P_U} = \frac{C_M \omega_1}{C_R \omega_2}$ (<1)

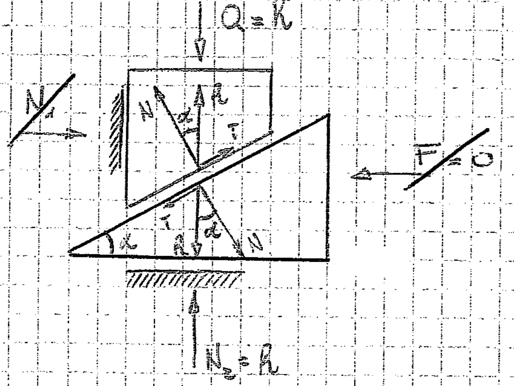
se $u > 1 \Longrightarrow \omega_1 > \omega_2$ (riduttore di velocità)



$$\begin{cases} Q = R \cdot \cos(\alpha - \varphi) \\ F = R \cdot \sin(\alpha - \varphi) \end{cases}$$

$$\boxed{\frac{F}{Q} = tg(\alpha - \varphi)} \qquad (\alpha > \varphi)$$

se vogliamo che la forza F sia nulla (cric per auto)
$(\alpha \leq \varphi) \Longrightarrow$ irreversibilità

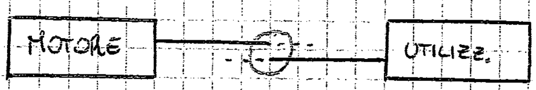


$$\begin{cases} \frac{T}{N} = tg\,\alpha \\ \frac{T}{N} \leq f_a \end{cases} \longrightarrow tg\,\alpha \leq f_a$$

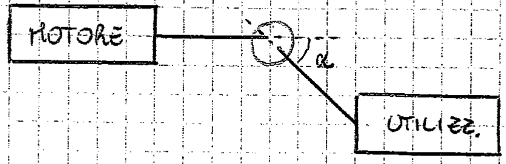
Giunti di trasmissione

sono dei dispositivi che permettono di trasmettere il moto senza trasformarlo (rotatorio → rotatorio) e senza cambiarne il rapporto ($\omega_1 \to \omega_1$)

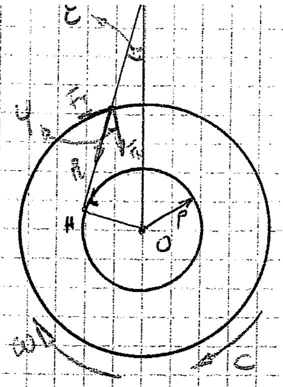
permettono di collegare il motore con l'utilizzatore anche quando



gli alberi sono disassati



disallineati

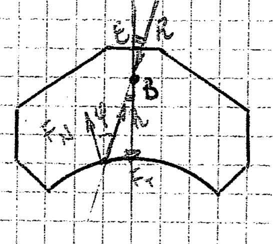


$\operatorname{tg}\varphi = f \Longrightarrow \varphi = 22°$

$\rho = r \sin\varphi = 4\,cm$

$\frac{\sin\varepsilon}{\rho} = \frac{1}{h+\frac{d}{2}} \Longrightarrow \sin\varepsilon = \frac{\rho}{h+\frac{d}{2}} \Longrightarrow \varepsilon = 14°$

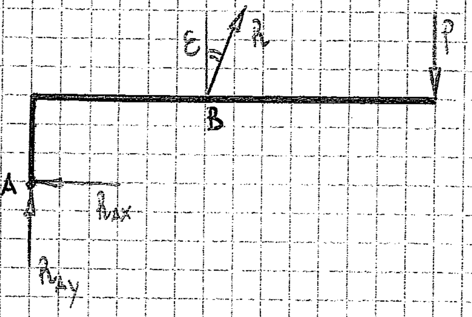
$C = R \cdot \rho = F_T\, r$



$\circlearrowright_A \; R \cos\varepsilon\, a - P(a+b) = 0$

$R = \frac{P(a+b)}{a \cdot \cos\varepsilon} = 340\,N$

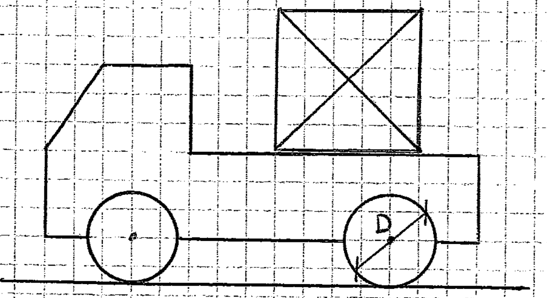
$C = R \cdot \rho = \frac{P(a+b)}{\cos\varepsilon} \cdot \frac{\rho}{a} = 13\,Nm$



$$\left(\begin{array}{l} F_T = R \sin\varphi \\ C = R \sin\varphi \cdot \frac{d}{2} \end{array} \right)$$

$$\begin{cases} R_{Ay} + R\cos\varepsilon - P = 0 \\ R_{Ax} = R \sin\varepsilon \end{cases} \Longrightarrow R_{Ax},\ R_{Ay}$$

ES 4.12



$a = -3\ m/s^2$

$v_0 = 50\ Km/h \longrightarrow 13{,}8\ m/s$

$M = 3600\ Kg$

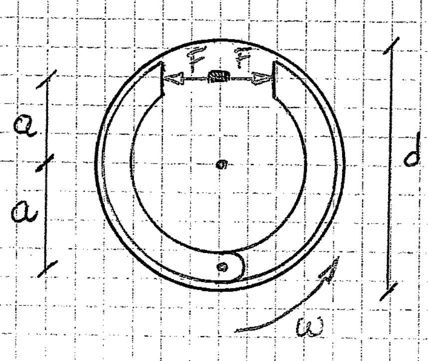
$m = 400\ Kg$

$f = 0{,}25$ (cep.-tamb.)

$D = 0{,}8\ m$

$d = 60\ cm$

$a = 20\ cm$



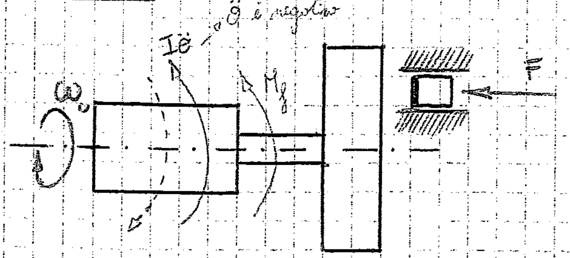
$t_{STOP} = ?$

$x_{STOP} = ?$

f(cassa-pianale) = ? ⟹ perché rimanga ferma

$F = ?$

ES 4.9



$M = 100\,Kg$

$\rho = 0,3\ m$

$\omega_0 = 1500\ rpm \Rightarrow 157\ rad/s$

$r_e = 20\ cm$

$r_i = 15\ cm$

$f = 0,3$

$F = ? \Rightarrow t_{stop} = 10\,s$

$I = M\rho^2 = 9\ Kg\,m^2$

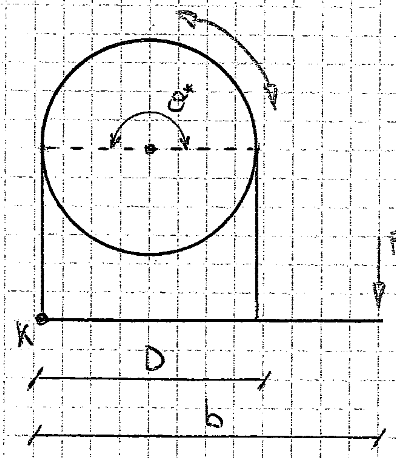
$$\ddot{\theta} = -\frac{\omega_0}{t_{stop}} = -15,7\ \frac{rad}{s^2}$$

$$M_f + I\ddot{\theta} = 0 \Rightarrow M_f = -I\ddot{\theta} \longrightarrow M_f = 141,3\ Nm$$

$$M_f = F f \frac{r_e + r_i}{2}$$

$$F = \frac{2M_f}{f(r_e + r_i)} = 2691\ N$$

ES 4.11



$D = 0,4\ m$

$b = 1\ m$

$\theta^* = \pi\ rad$

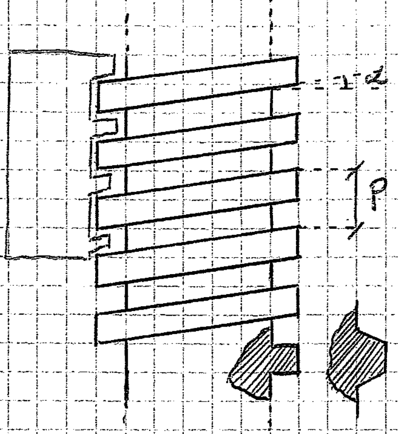
$f = 0,25$

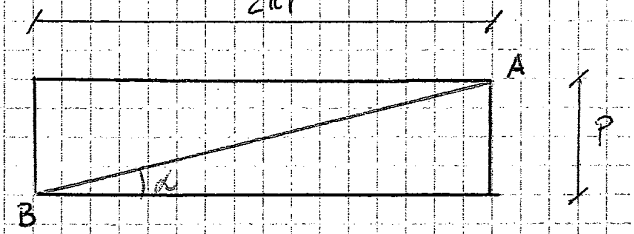
$F = 200\,N$

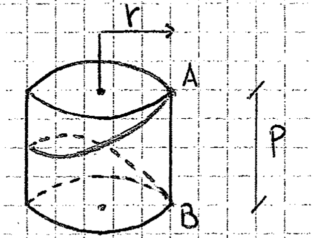
$M_f = ?$ $\begin{matrix}\omega\circlearrowright \\ \omega\circlearrowleft\end{matrix}$

Sistemi di trasmissione

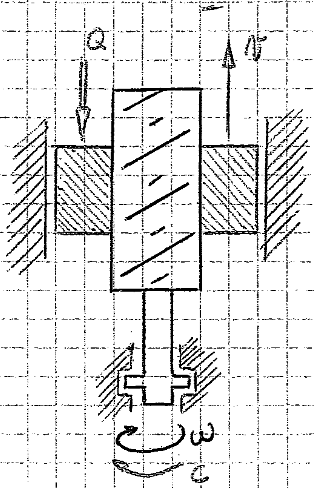
Vite - Madrevite





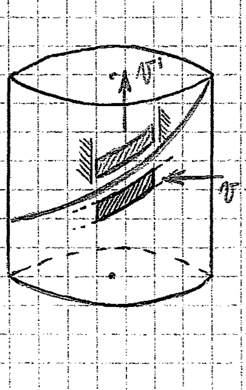


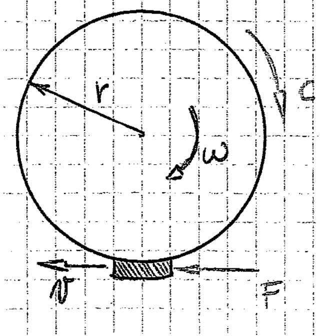
$$\boxed{p = 2\pi r \, tg\, \alpha} \longrightarrow \text{passo}$$



- la vite è vincolata e può solamente ruotare.
- la madrevite può solamente traslare in verticale.

applicando una coppia alla vite possiamo trasmettere il moto rotazionale in traslazione verticale e permettere alla madre vite di vincere un carico



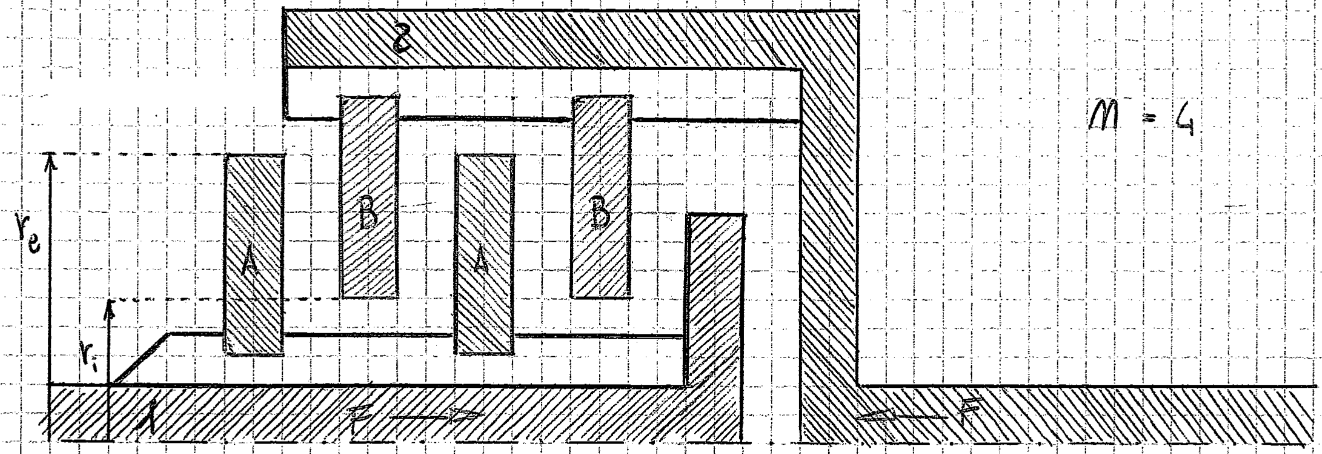


$$F = \frac{C}{r}$$

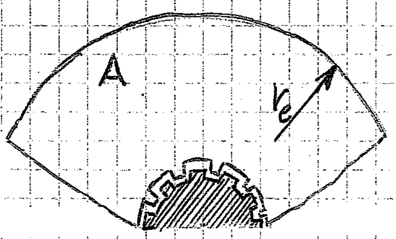
$$v = \omega \cdot r$$

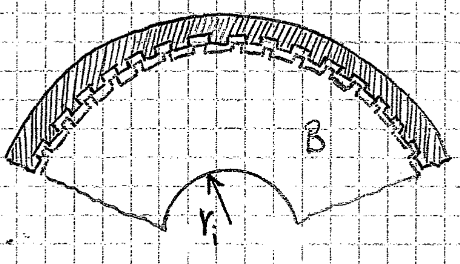
nella fase di innesto $\omega_1 = \omega_2$ ci troviamo in condizioni di aderenza

$$\begin{cases} dF_T \leqslant f_a \, dF_N \\ M_{f(MAX)} = f_a F \frac{r_e + r_i}{2} \end{cases}$$



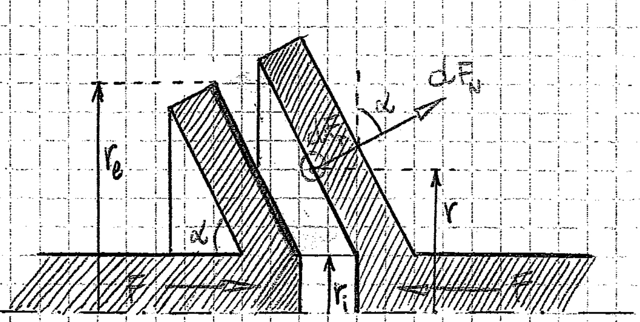
$n = 4$





$$\boxed{M_f = f F \frac{r_e + r_i}{2} \, n}$$

frizione a dischi multipli

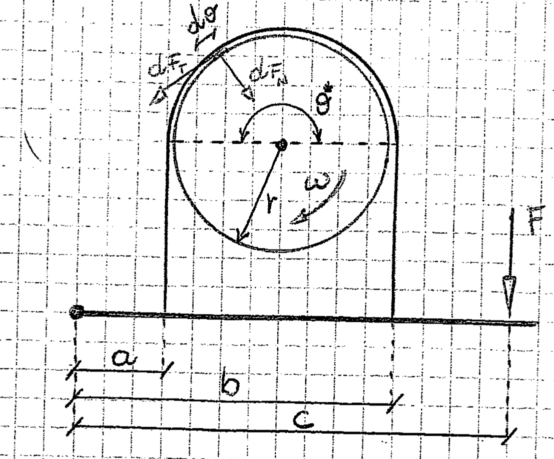


$$M_f = \int_A dF_T \cdot r$$

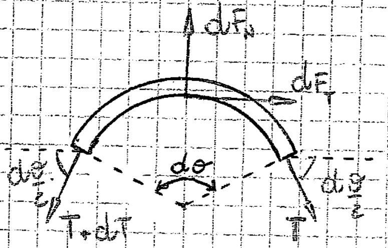
$$dF_T = f \, dF_N$$

$$F = \int_A dF_N \sin\alpha$$

Freni a nastro



$$\int_A dF_T \cdot r = M_f$$



$$\begin{cases} dF_N = 2T \text{ sen } \frac{d\theta}{2} + \cancel{dT \text{ sen } \frac{d\theta}{2}} \\ dF_T = dT \cos \frac{d\theta}{2} \\ dF_T = f\, dF_N \end{cases}$$

infinitesimo del 2° ordine

angoli piccoli

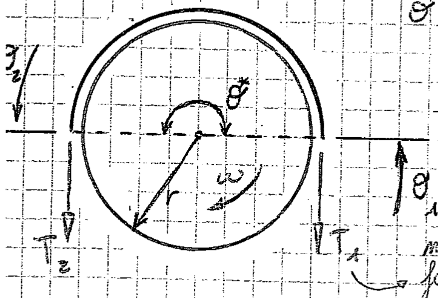
$$\boxed{\begin{aligned} \text{sen } \frac{d\theta}{2} &= \frac{d\theta}{2} \\ \cos \frac{d\theta}{2} &= 1 \end{aligned}}$$

$$\begin{cases} \frac{dF_T}{f} = T\, d\theta \\ dF_T = dT \end{cases}$$

$$\boxed{\frac{dT}{f} = T\, d\theta}$$

variazione della tensione lungo l'angolo di avvolgimento sul tamburo

$$\theta^* = \theta_2 - \theta_1$$



$$\boxed{M_f = (T_2 - T_1)\, r} \text{ ①}$$

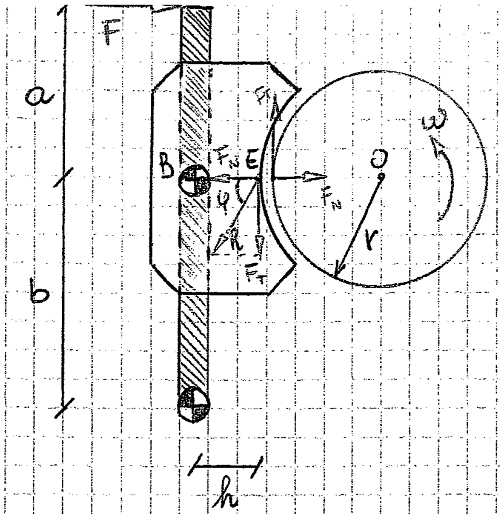
non frena anzi fa momento motrice

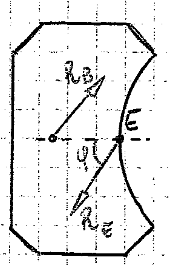
$T_2 > T_1$

$$\int_{T_1}^{T_2} \frac{dT}{T} = f \int_{\theta_1}^{\theta_2} d\theta \Longrightarrow \lg \frac{T_2}{T_1} = f(\theta_2 - \theta_1) = f\theta^*$$

$$e^{f\theta^*} > 1 \Longrightarrow T_1 > T_2$$

$$\boxed{\frac{T_2}{T_1} = e^{f\theta^*}} \text{ ②}$$

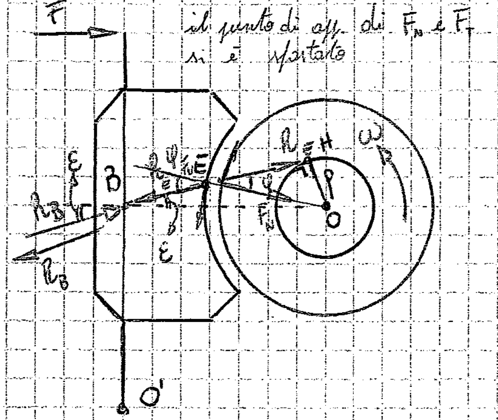




$\Sigma F = 0 \Longrightarrow$ OK

$\Sigma M \neq 0 \Longrightarrow$ NO

devo cambiare modello



in blu forze agenti sull'asta

possiamo immaginare che sia un modello di attrito secco (il perno e il tamburo)

$\Sigma F = 0$

$\Sigma M = 0$

se considero il triangolo $\overset{\Delta}{BHO}$ posso ricavare ε

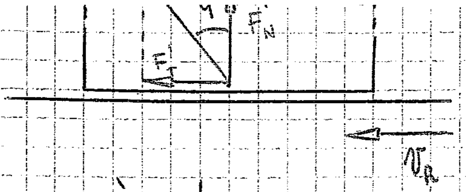
$$\operatorname{sen} \varepsilon = \frac{\rho}{h+r}$$

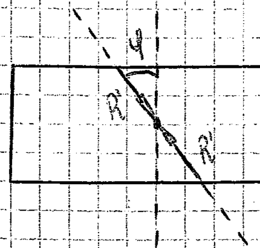
$$\circlearrowleft_{O'} \quad F(a+b) - R_B \cos\varepsilon \cdot b = 0$$

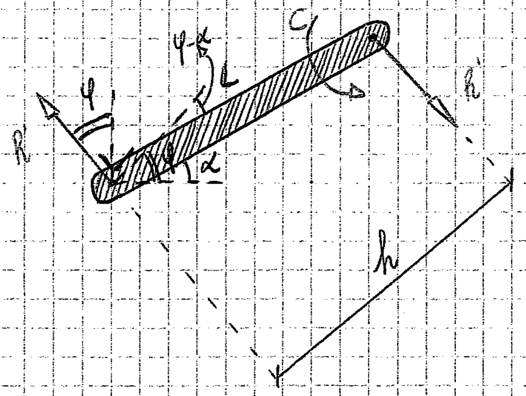
$R_E = R_B$

$F_T = R_E \operatorname{sen}\varphi$

$$\boxed{F_T = F \, \frac{a+b}{b} \, \frac{\operatorname{sen}\varphi}{\cos\varepsilon}}$$





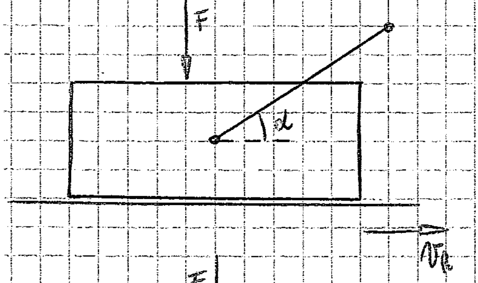


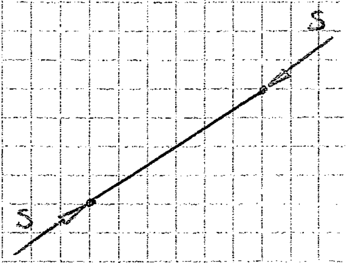
$$h = L\cos(\varphi-\alpha)$$

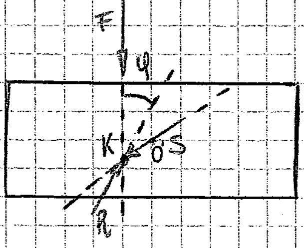
$$C = R' h = \frac{F_T'}{\operatorname{sen}\varphi} L\cos(\varphi-\alpha)$$

$$\boxed{F_T' = \frac{C}{L} \frac{\operatorname{sen}\varphi}{\cos(\varphi-\alpha)}}$$

$$\boxed{F_T' < F_T}$$



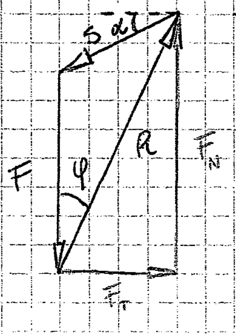




3 forze

① $\Sigma \vec{F} = 0$

② concorrenti nello stesso punto ⟶ K



$$F_N = \frac{F_T}{f} = F + S \operatorname{sen}\alpha$$

$$S = \frac{F_T}{\cos\alpha}$$

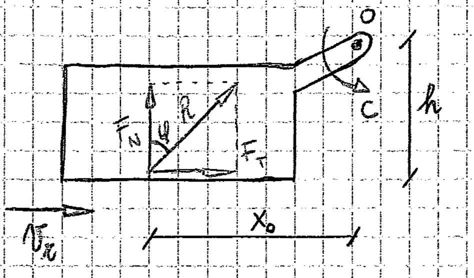
$$\frac{F_T}{f} = F + F_T \operatorname{tg}\alpha$$

$$\boxed{F_T = \frac{F}{\frac{1}{f} - \operatorname{tg}\alpha}}$$

la risultante F_N dovrà avere un punto di applicazione X_0 affinché venga equilibrata la coppia C

$$F_n \cdot X_0 = C = \int_A dF_N \cdot x = \int_a^{a+b} Kx^2\, dx = \frac{K}{3}\left[(a+b)^3 - a^3\right]$$

$$\boxed{X_0 = \frac{2}{3}\,\frac{(a+b)^3 - a^3}{(a+b)^2 - a^2}} \Longrightarrow$$ baricentro dell'area consumata

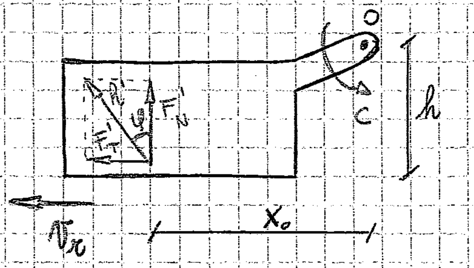


$$\begin{cases} C + F_T h - F_N X_0 = 0 \\ F_T = f F_N \Longrightarrow F_N = \frac{F_T}{f} \end{cases}$$

$$C + F_T h - F_T \frac{X_0}{f} = 0$$

$$C + F_T\left(h - \frac{X_0}{f}\right) = 0$$

$$\boxed{F_T = \frac{C}{\frac{X_0}{f} - h}}$$



$$\begin{cases} C - F'_T h - F_N X_0 = 0 \\ F'_N = \frac{F'_T}{f} \end{cases}$$

$$C - F'_T h - F'_T \frac{X_0}{f} = 0$$

$$\boxed{F'_T = \frac{C}{\frac{X_0}{f} + h}}$$

$$\boxed{F'_T < F_T}$$

$$T \left< \quad \alpha = 45° \Longrightarrow T = 24967\ N \right.$$

verifichiamo la nostra ipotesi

$$\frac{T}{N} < f_a \qquad \alpha = 10° \quad \text{OK}$$

$$\alpha = 45° \quad \text{NO} \Longrightarrow \text{ipotesi errata}$$

ci troviamo in un caso di strisciamento per $\alpha = 45°$

④ $T = fN$

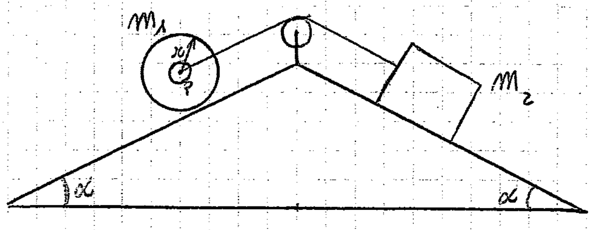
$$\alpha = 45° \Longrightarrow \ddot{X}_{45}$$

il moto è un moto uniformemente acc.

$$X(t) = \cancel{X_0} + \cancel{\dot{X}t} + \frac{1}{2}\ddot{X}t^2$$

$$X(t) = \frac{1}{2}\ddot{X}t^2$$

$$t = \sqrt{\frac{2 \cdot 200}{\ddot{X}}} \left< \begin{array}{l} \ddot{X}_{10} \Longrightarrow t \\ \ddot{X}_{45} \Longrightarrow t \end{array} \right.$$



$\alpha = 20°$

$r = 20$ cm

$u = 2$ cm

$f = 0{,}3$

$m_1 = 60$ Kg

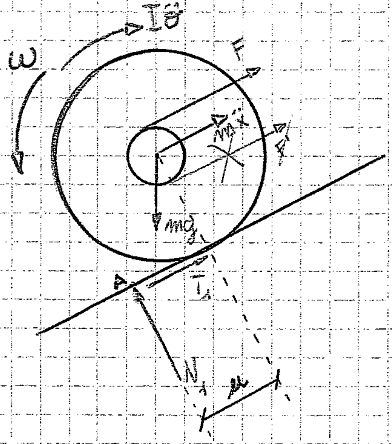
$I = 1{,}2$ Kg mm^2

$r_p = 4$ cm

$f_p = 0{,}4$

① $m_2 = ? \Longrightarrow \dot{x} = cost$

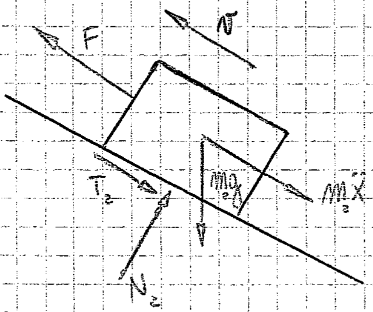
② se $m_2 = 20$ Kg $\quad \ddot{x} = ?$



$p = r \sin\varphi$

$tg\,\varphi = f_p$

$$C_A \begin{cases} F(r+p) + I\ddot{\vartheta} + m_1 g \cos\alpha \cdot u - m_1 g \sin\alpha \cdot r + \\ + m\ddot{x}\, r = 0 \\ \nearrow \quad F + m_1\ddot{x} + T_1 - m_1 g \sin\alpha = 0 \end{cases}$$ ① ②



$$\nearrow \begin{cases} N_2 - m_2 g \cos\alpha = 0 \\ T_2 = N_2 f \end{cases} \qquad \begin{matrix} N_2 = m_2 g \cos\alpha \\ T_2 = f m_2 g \cos\alpha \end{matrix}$$

① $\ddot{x} = 0$

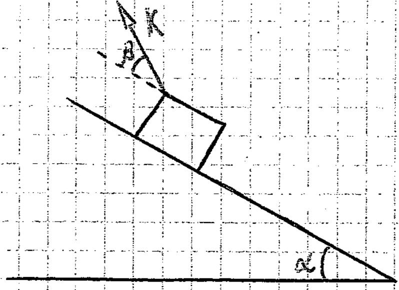
① $F(r+p) + \cancel{I\ddot{\vartheta}} + m_1 g \cos\alpha \cdot u - mg \sin\alpha \cdot r + \cancel{m\ddot{x}}\, r = 0$

$$F = \frac{m_1 g \sin\alpha \cdot r - m_1 g \cos\alpha \cdot u}{r+p}$$

② $F + \cancel{m_1\ddot{x}} + T_1 - m_1 g \sin\alpha = 0$

$T_1 = m_1 g \sin\alpha - F$

$T_1 = T_2 \Longrightarrow m_1 g \sin\alpha - F = f m_2 g \cos\alpha \Longrightarrow m_2$



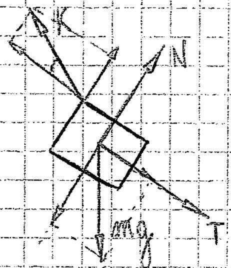
30%

$m = 500\ \text{Kg}$

$f = 0{,}2$

$\beta = ? \implies K_{minima} = ?$

$\text{tg}\,\alpha = 0{,}3$



$$\begin{matrix} \nearrow \\ \nwarrow \\ \\ \end{matrix}\begin{cases} N + K \operatorname{sen}\beta - mg \cos\alpha = 0 \\ K \cos\beta - T - mg \operatorname{sen}\alpha = 0 \\ T = f N \end{cases}$$

$$\begin{cases} N = p \cos\alpha - K \operatorname{sen}\beta \\ T = -p \operatorname{sen}\alpha + K \cos\beta \end{cases}$$

$$K \cos\beta - p \operatorname{sen}\alpha = f p \cos\alpha - f K \operatorname{sen}\beta$$

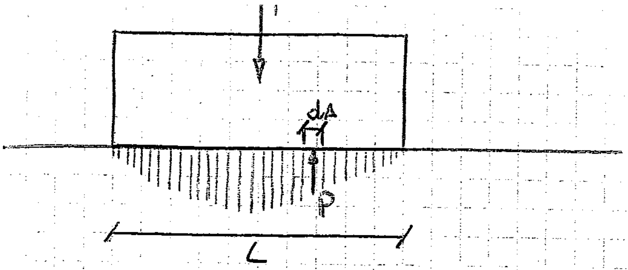
$$K = \frac{p \operatorname{sen}\alpha + f p \cos\alpha}{\underbrace{\cos\beta - f \operatorname{sen}\beta}_{\text{faccio la derivata}}} \longrightarrow \text{numero}$$

perchè K sia minimo il denominat
deve essere massimo

$$\hookrightarrow -\operatorname{sen}\beta + f \cos\beta = 0 \implies \text{tg}\,\beta = f \implies \beta = 11{,}3°$$

faccio la derivata
seconda per contro
re che sia un ma

$K = 2303\ \text{N}$

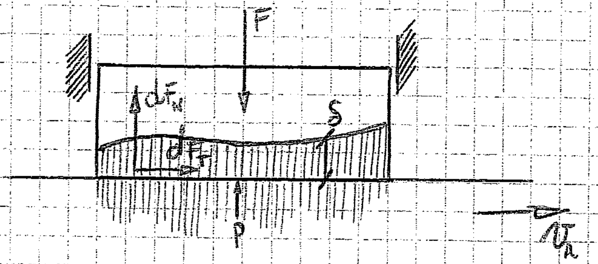


$$F = \int_A p\,dA = \int_0^L p(x)\,dx$$

dipende dall'andamento di p(x)

p(x)
- uniforme
- ipotesi dell'usura

il volume di materiale consumato in un intervallo di tempo è proporzionale al lavoro delle forze d'attrito nello stesso tempo



$$dF_N = p\,dA$$

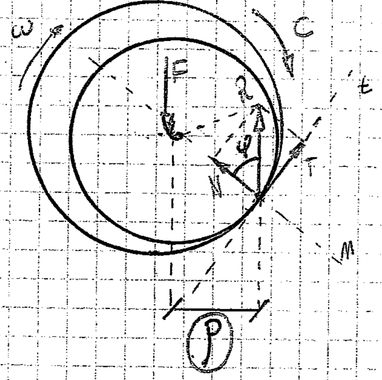
$$dF_T = f\,dF_N = f\,p\,dA$$

le forze che compiono lavoro sono solo quelle tangenziali

$$dL = dF_T \cdot dx$$

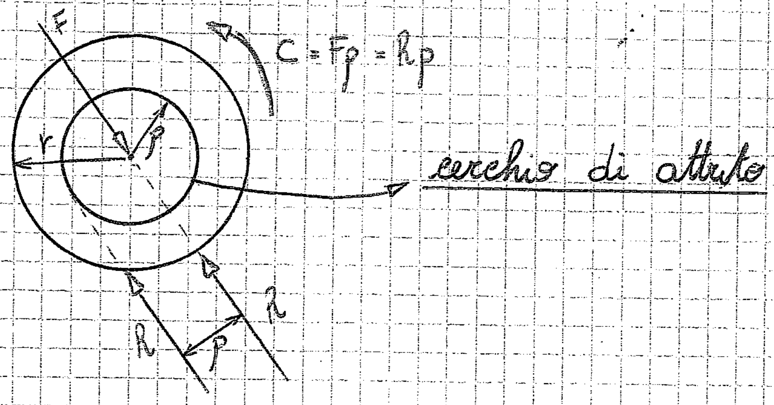
$$\underbrace{\delta\,dA}_{\text{volume consumato}} \propto f\,p\,dA\,\frac{dx}{dt} = \underbrace{f\,p\,dA\,v_R}_{\text{lavoro forze d'attrito}}$$

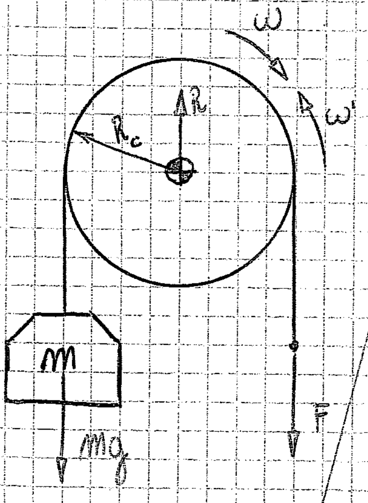
dobbiamo equilibrare la coppia C senza aggiungere forze orizzontali

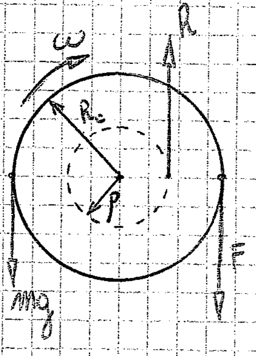


$$C = F\rho = R\rho$$

$$\boxed{\rho = r \sin\varphi}$$





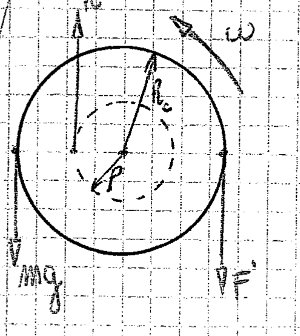


alzare il peso

$$F(R_c - \rho) = mg(R_c + \rho)$$

$$F = m\,\frac{R_c + \rho}{R_c - \rho} \Longrightarrow (F > mg)$$

la forza deve vincere anche l'attrito



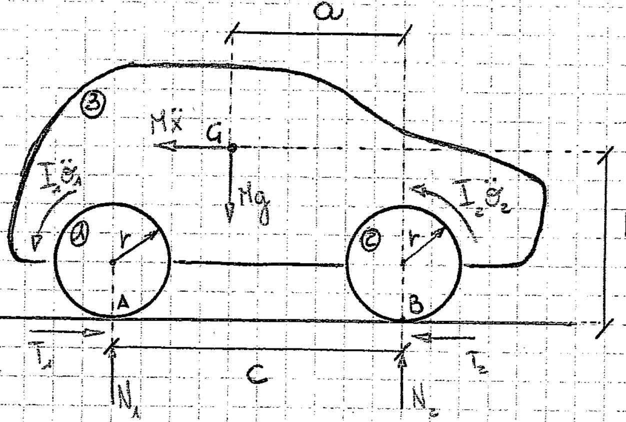
far scendere il peso (ω cost)

$$F'(R_c + \rho) = mg(R_c - \rho)$$

$$F' = mg\,\frac{R_c - \rho}{R_c + \rho} \Longrightarrow F' < mg$$

		/N
aderenza	0	$< f_a$
aderenza limite	0	$= f_a$
strisciamento	$\neq 0$	$= f$

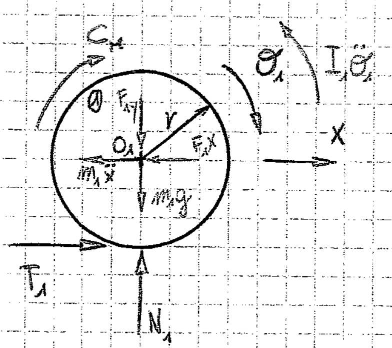
la C_M non si vede perchè annullata da quella del telaio sulla ruota



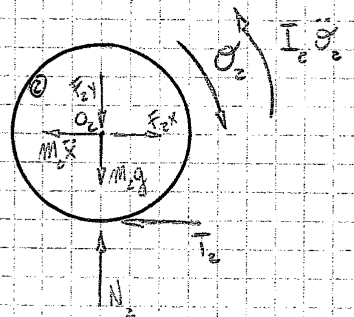
1 ruota motrice
2 ruota condotta

C_M coppia motrice

a, b, c, r
M massa totale
m_1, m_2 massa ruote
I_1, I_2 ruote
f, f_a



ruota motrice



ruota condotta

7 incognite
5 equazioni

③
$$\begin{cases} \rightarrow \quad \boxed{T_1} - \boxed{T_2} - M\boxed{\ddot{x}} = 0 \\ \uparrow \quad \boxed{N_1} + \boxed{N_2} - Mg = 0 \\ \circlearrowright_B \quad I_1\boxed{\ddot{\theta}_1} + I_2\boxed{\ddot{\theta}_2} + Mga + M\ddot{x}b - N_1 c = 0 \end{cases}$$

① $\circlearrowleft_{O_1}$ $C_M - I_1\ddot{\theta} - T_1 r = 0$

② $\circlearrowright_{O_2}$ $I_2\ddot{\theta}_2 - T_2 r = 0$

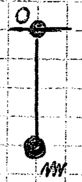
$E_p = E_f - E_i$

$E_i = \frac{1}{2} m v^2$

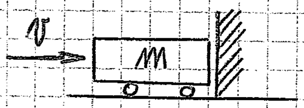
$E_f = \frac{1}{2} I_o \omega^2 = mgh(1-\cos\theta)$

$$E_p = \frac{1}{2} m v^2 - \frac{1}{2} m \frac{h^2 v^2}{\rho^2 + h^2} = \frac{1}{2} m v^2 \left(1 - \frac{h^2}{\rho^2+h^2}\right) = \frac{1}{2} m v^2 \frac{\rho^2}{\rho^2+h^2}$$

se $\boxed{\rho = 0}$ ⟹ $\boxed{0\%}$ energia persa (pendolo semplice) ideale



se $\boxed{h = 0}$ ⟹ $\boxed{100\%}$ energia persa (urto)



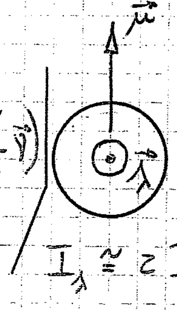
Attrito

- **secco** (radente, di strisciamento) ⟹ quando 2 superfici vengono fatte strisciare l'una sull'altra
- **fluido** ⟹ quando tra 2 superfici è presente uno strato di fluido (liquido o gas)
- **interno** ⟹ tra le molecole di un materiale dovuto alla deformazione

$$\frac{d\vec{\lambda}}{dt} = \omega_1 \vec{K} \wedge \vec{\lambda} = \omega_1 \cos\alpha \left(-\vec{\nu}\right)$$

$$\frac{d\vec{\mu}}{dt} = \omega_1 \vec{K} \wedge \vec{\mu} = \omega_1 \,\text{sen}\,\alpha\, \vec{\nu}$$

$$\frac{dK_G}{dt} = I_\lambda\left(\omega_1 \,\text{sen}\,\alpha - \omega_2\right) \omega_1 \cos\alpha \left(-\vec{\nu}\right) + I_\mu\, \omega_1^2 \cos\alpha \,\text{sen}\,\alpha\, \vec{\nu} = -\vec{M}_{i_G}$$

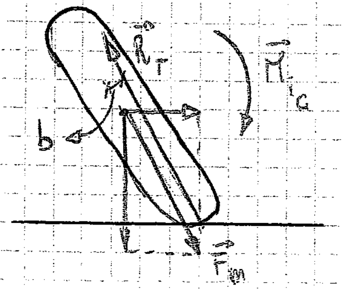


$$-M_{i_G} = I_\lambda\left(\omega_1^2 \cos\alpha \,\text{sen}\,\alpha - \omega_1\omega_2 \cos\alpha - \frac{\omega_1^2 \,\text{sen}\,\alpha \cos\alpha}{2}\right)\left(-\vec{\nu}\right)$$

$I_\lambda \cong 2 I_\mu$ ipotizzando che sia un disco sottile

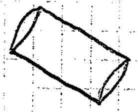
$$M_{i_G} = I_\lambda\left(\omega_1\omega_2 \cos\alpha - \frac{\omega_1^2 \,\text{sen}\,\alpha \cos\alpha}{2}\right)\left(-\vec{\nu}\right)$$

$$M_{i_G} = I_\lambda\, \omega_1 \cos\alpha \underbrace{\left(\omega_2 - \frac{\omega_1 \,\text{sen}\,\alpha}{2}\right)}_{>0 \ (\omega_2 > \omega_1)}\left(-\vec{\nu}\right)$$

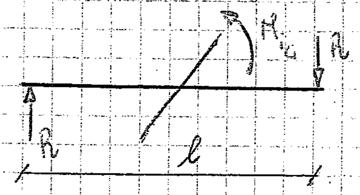


$$M_{i_G} = R_T \cdot b = F_m \cdot b$$

se $I_x > I_y$  $\Longrightarrow \vec{K}$ ↺ tende a raddrizzare

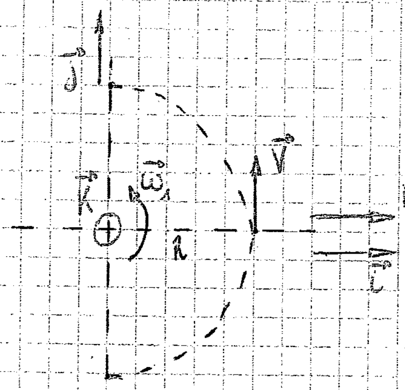
se $I_x < I_y$  $\Longrightarrow -\vec{K}$ ↻ tende a storgerlo

inoltre i cuscinetti avranno una reazione vincolare che tende a contrastare M_{ic}

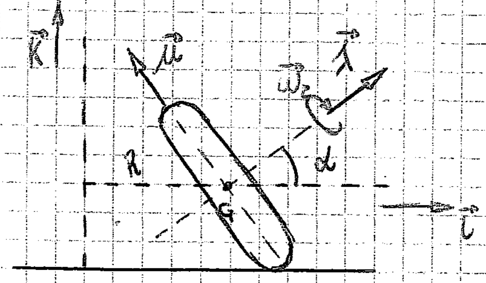


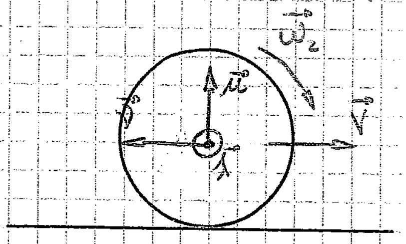
$$R = \frac{M_{ic}}{l}$$

moto in curva



$\vec{F}_i = m R \omega_1^2 = m \frac{V^2}{R}$





$\vec{\omega}_2 = \omega_2 (-\vec{\lambda})$

$\vec{\omega}_1 = \omega_1 \vec{K}$

$V = \omega_1 R = \omega_2 r \Longrightarrow (R > r) \Longrightarrow \omega_2 > \omega_1$