



Corso Luigi Einaudi, 55 - Torino

Appunti universitari

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NUMERO: 1836A -

ANNO: 2016

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STUDENTE: Gonella Anna

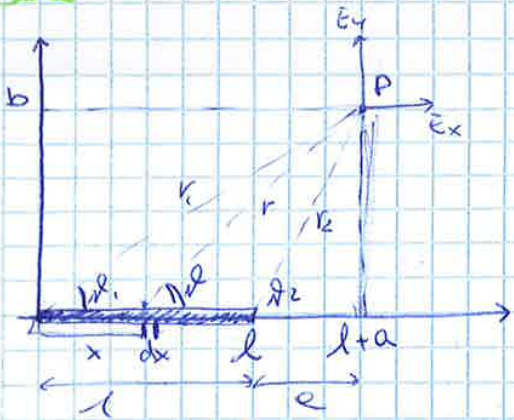
MATERIA: Fisica II - prof. Kaniadakis

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**ATTENZIONE: QUESTI APPUNTI SONO FATTI DA STUDENTIE NON SONO STATI VISIONATI DAL DOCENTE.
IL NOME DEL PROFESSORE, SERVE SOLO PER IDENTIFICARE IL CORSO.**

E+1



$$\lambda = \frac{dq}{dx} \quad dq = \lambda dx$$

$$d\vec{E} = \frac{k dq}{r^2} = k \lambda \frac{dx}{r^2}$$

- $\sin \phi = \frac{b}{r}$
- $\cos \phi = \frac{l+a-x}{r}$
- $dx = \frac{b}{\sin^2 \phi} d\phi$

$r = \frac{b}{\sin \phi}$
 $r = \frac{l+a-x}{\cos \phi}$

derivata (Pone)

$$d\vec{E} = k \lambda \frac{dx}{\sin^2 \phi} \frac{b^2}{\sin^2 \phi r^2} = \frac{k \lambda}{b} d\phi$$

$$\int \vec{E}_x = \frac{k \lambda}{b} [\sin \phi_2 - \sin \phi_1]$$

$$\int \vec{E}_y = \frac{k \lambda}{b} [\cos \phi_1 - \cos \phi_2]$$

→ scambiati perché vengono da $\int_{\phi_1}^{\phi_2} \sin \phi d\phi$

$$\sin \phi_1 = \frac{b}{r_1} = \frac{b}{\sqrt{(l+a)^2 + b^2}} \quad \sin \phi_2 = \frac{b}{r_2} = \frac{b}{\sqrt{a^2 + b^2}}$$

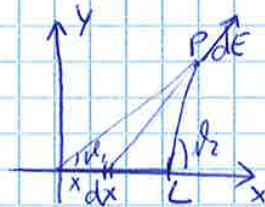
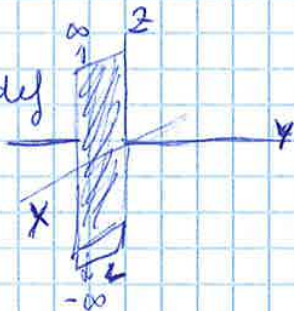
$$\cos \phi_1 = \frac{l+a}{r_1} = \frac{l+a}{\sqrt{(l+a)^2 + b^2}} \quad \cos \phi_2 = \frac{a}{r_2} = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\int \vec{E}_x = \frac{k \lambda}{b} \left[\frac{b}{\sqrt{a^2 + b^2}} - \frac{b}{\sqrt{(l+a)^2 + b^2}} \right] = k \lambda \left[\frac{1}{\sqrt{a^2 + b^2}} - \frac{1}{\sqrt{(l+a)^2 + b^2}} \right]$$

$$\int \vec{E}_y = \frac{k \lambda}{b} \left[\frac{l+a}{\sqrt{(l+a)^2 + b^2}} - \frac{a}{\sqrt{a^2 + b^2}} \right]$$

E+2

lastro indef



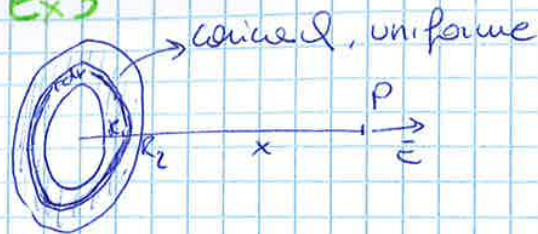
$$\sigma = \frac{q}{S_{\text{sup}}}$$

$$\int \vec{E}_y = 2k\sigma (r_2 - r_1) \quad \int \vec{E}_x = 2k\sigma l \ln \frac{\sin \phi_2}{\sin \phi_1}$$

$$d\vec{E} = 2k\sigma \sqrt{(r_2 - r_1)^2 + l^2} \frac{\sin \phi_2}{\sin \phi_1}$$



Ex 3



Se E di un anello raggio r , carica q $\Rightarrow E = \frac{kq x}{(r^2 + x^2)^{3/2}}$

\rightarrow Calcoliamo di raggio r e $r+dr$ approssimabile per $dr \rightarrow 0$ ad un anello

$\hookrightarrow dE = \frac{k dQ x}{(r^2 + x^2)^{3/2}}$

$\sigma = \frac{Q}{\pi R_2^2 - \pi R_1^2} \Rightarrow dQ = \sigma dA = \sigma 2\pi r dr$
 Sup. Corona $\rightarrow \frac{1}{\sqrt{r^2 + x^2}}$

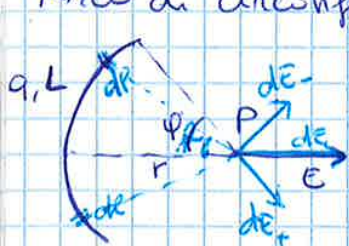
$\Rightarrow dE = \frac{k x \sigma 2\pi r dr}{(r^2 + x^2)^{3/2}} \Rightarrow E = \frac{k \sigma 2\pi x}{2\pi \epsilon_0} \int_{R_1}^{R_2} \frac{r dr}{(r^2 + x^2)^{3/2}} = \frac{\sigma x}{2\epsilon_0} \left(\frac{1}{\sqrt{x^2 + R_1^2}} - \frac{1}{\sqrt{x^2 + R_2^2}} \right)$

• disco $\rightarrow R_1 = 0, R_2 = R \Rightarrow E = \frac{\sigma x}{2\epsilon_0} \left(\frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}} \right)$

• piana \rightarrow disco con $R \rightarrow \infty \Rightarrow E = \frac{\sigma}{2\epsilon_0} \left(\frac{x}{x} - \frac{x}{\sqrt{x^2 - R^2}} \right)$

Ex 4

Arco di circonferenza



$\varphi, q, L, r \Rightarrow E = ?$
 P ed'centro

$dq = \lambda dl = \frac{q}{L} dl$

$dE_- = dE_+ = \frac{k dq}{r^2} = \frac{kq dl}{L r^2}$

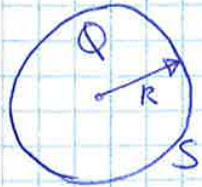
$dE = dE_x + dE_y = \frac{2kq}{L r^2} dl \cos \theta$

$dE = \frac{2kq}{L r^2} \cos \theta dl = \frac{2kq}{L r^2} \cos \theta r d\theta = \frac{2kq}{L r} \cos \theta d\theta$

$E = \int dE = \int \frac{2kq}{L r} \cos \theta d\theta = \frac{2kq}{L r} \int_0^\varphi \cos \theta d\theta = \frac{2kq}{L r} \sin \varphi$

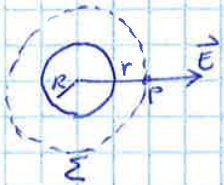
Ex 6

sfera carica Q distribuita uniformemente all'interno. $\vec{E} = ?$



$\vec{E} \parallel \vec{r} \quad |\vec{E}| = E \quad E = E(r)$

• Fuori dalla sfera $r > R$



$$\begin{cases} \Phi = 4\pi r^2 E \\ \Phi = \frac{Q}{\epsilon_0} \end{cases}$$

$$4\pi r^2 E = \frac{Q}{\epsilon_0} \rightarrow \boxed{E = \frac{Q}{4\pi \epsilon_0 r^2}} \quad \underline{r > R}$$

• Interno alla sfera $r < R$



$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$

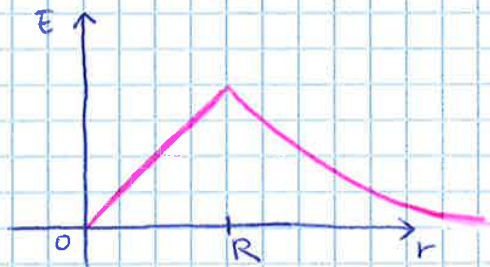
$q = \rho \frac{4}{3}\pi r^3 = Q \frac{r^3}{R^3}$

$$\begin{cases} \Phi = 4\pi r^2 E \\ \Phi = \frac{q}{\epsilon_0} \end{cases}$$

$$4\pi r^2 E = \frac{1}{\epsilon_0} \rho \frac{4}{3}\pi r^3 \rightarrow 4\pi r^2 E = \frac{1}{\epsilon_0} Q \frac{r^3}{R^3}$$

$$\Rightarrow \boxed{E = \frac{Qr}{4\pi \epsilon_0 R^3}} \quad \underline{r < R}$$

$$R(r) \begin{cases} r < R & \frac{Q}{4\pi \epsilon_0 R^3} r \\ r > R & \frac{Q}{4\pi \epsilon_0} \frac{1}{r^2} \end{cases}$$

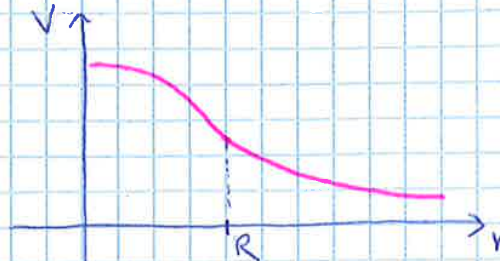


$E(r) = -\frac{dV(r)}{dr} \quad V(r) = -\int E(r) dr + C$

$$V(r) \begin{cases} r < R & -\frac{Q}{8\pi \epsilon_0 R^3} r^2 + C_1 \\ r > R & \frac{Q}{4\pi \epsilon_0} \frac{1}{r} + C_2 \end{cases}$$

$$\begin{cases} V(R^-) = V(R^+) \rightarrow -\frac{Q}{8\pi \epsilon_0 R^3} R^2 + C_1 = \frac{Q}{4\pi \epsilon_0 R} \rightarrow C_1 = \frac{3Q}{8\pi \epsilon_0 R} \\ V(\infty) = 0 \rightarrow C_2 = 0 \end{cases}$$

$$V(r) \begin{cases} r < R & \frac{3Q}{8\pi \epsilon_0 R} - \frac{Q}{8\pi \epsilon_0 R^3} r^2 \\ r > R & \frac{Q}{4\pi \epsilon_0} \frac{1}{r} \end{cases}$$



Ex 8



• $r > R$



$$\phi = \frac{q}{\epsilon_0}$$

$$2\pi r L E = \frac{1}{\epsilon_0} \rho \pi R^2 L$$

Superficie

$$E = \frac{\rho R^2}{2\epsilon_0 r}$$

$$\rho = \frac{dq}{dV} \rightarrow \pi R^2 L$$

• $r < R$

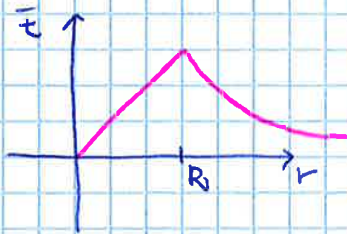


$$\phi = \frac{q}{\epsilon_0}$$

$$2\pi r L E = \frac{1}{\epsilon_0} \rho \pi r^2 L$$

$$E = \frac{\rho r}{2\epsilon_0}$$

$$E(r) \begin{cases} r < R & \frac{\rho}{2\epsilon_0} r \\ r > R & \frac{\rho R^2}{2\epsilon_0} \frac{1}{r} \end{cases}$$



Ex 9



• $r < R_1$ *Superficie* $4\pi r^2 E = 0 \rightarrow E = 0$ *Superfici sferiche*

• $R_1 < r < R_2$ $4\pi r^2 E = \frac{q_1}{\epsilon_0} \rightarrow E = \frac{q_1}{4\pi r^2 \epsilon_0}$

• $r > R_2$ $4\pi r^2 E = \frac{q_1 + q_2}{\epsilon_0} \rightarrow E = \frac{q_1 + q_2}{4\pi \epsilon_0 r^2}$

$$E(r) \begin{cases} r < R_1 & 0 \\ R_1 < r < R_2 & \frac{q_1}{4\pi r^2 \epsilon_0} \\ r > R_2 & \frac{q_1 + q_2}{4\pi \epsilon_0 r^2} \end{cases}$$

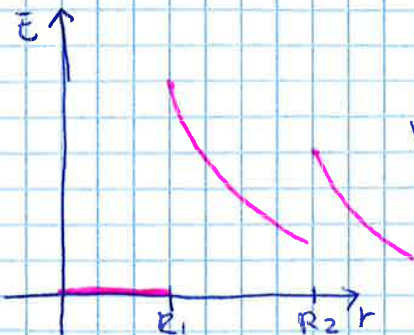
$$V(r) \begin{cases} r < R_1 & 0 + C_1 \\ R_1 < r < R_2 & \frac{q_1}{4\pi r \epsilon_0} + C_2 \\ r > R_2 & \frac{q_1 + q_2}{4\pi \epsilon_0 r} + C_3 \end{cases}$$

$$C_1 = \frac{q_1}{4\pi R_1 \epsilon_0} + \frac{q_2}{4\pi \epsilon_0 R_2}$$

$$C_2 = \frac{q_1 + q_2}{4\pi \epsilon_0 R_2} - \frac{q_1}{4\pi \epsilon_0 R_1}$$

$$C_3 = \frac{q_2}{4\pi \epsilon_0 R_2}$$

condizioni: $V(R_1^-) = V(R_1^+) \rightarrow C_1 = \frac{q_1}{4\pi \epsilon_0 R_1} + \frac{q_2}{4\pi \epsilon_0 R_2}$
 $V(R_2^-) = V(R_2^+) \rightarrow C_2 = \frac{q_2}{4\pi \epsilon_0 R_2}$
 $V(\infty) \rightarrow C_3 = 0$



$$V(r) \begin{cases} r < R_1 & \frac{q_1}{4\pi \epsilon_0 R_1} + \frac{q_2}{4\pi \epsilon_0 R_2} \\ R_1 < r < R_2 & \frac{q_1}{4\pi \epsilon_0 r} + \frac{q_2}{4\pi \epsilon_0 R_2} \\ r > R_2 & \frac{q_1 + q_2}{4\pi \epsilon_0 r} \end{cases}$$



Ex 11

Guscio sferico R_i, R_e con carica q distribuita uniformemente.

$\rho = ?$



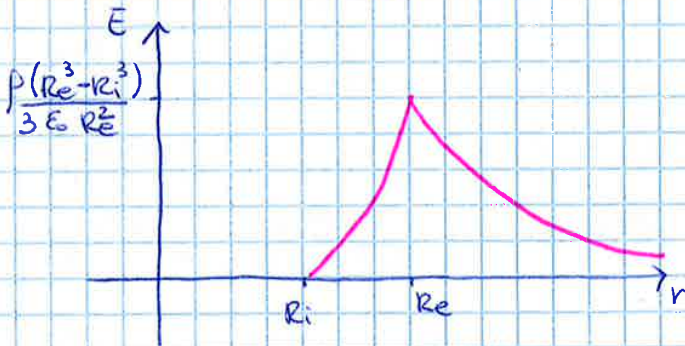
$\rho = \frac{q}{\frac{4}{3}\pi(R_e^3 - R_i^3)}$

$\Sigma_1 =$ sfera di raggio $r < R_i$ $\phi = \frac{Q}{\epsilon_0} \rightarrow 4\pi r^2 E = \frac{Q}{\epsilon_0} \rightarrow E = 0$

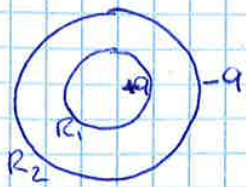
$\Sigma_2 =$ sfera di raggio $R_i < r < R_e$ $\phi = \frac{Q}{\epsilon_0} \rightarrow 4\pi r^2 E = \frac{\rho \frac{4}{3}\pi(r^3 - R_i^3)}{\epsilon_0} \rightarrow E = \frac{\rho(r^3 - R_i^3)}{3\epsilon_0 r^2}$

$\Sigma_3 =$ sfera di raggio $r > R_e$ $\phi = \frac{Q}{\epsilon_0} \rightarrow 4\pi r^2 E = \frac{Q}{\epsilon_0} = \frac{1}{\epsilon_0} \rho \frac{4}{3}\pi(R_e^3 - R_i^3)$
 $E = \frac{\rho(R_e^3 - R_i^3)}{3\epsilon_0 r^2}$ \rightarrow la carica è solo fra R_e e R_i

$$E = \begin{cases} r < R_i & 0 \\ R_i < r < R_e & \frac{\rho(r^3 - R_i^3)}{3\epsilon_0 r^2} \\ r > R_e & \frac{\rho(R_e^3 - R_i^3)}{3\epsilon_0 r^2} \end{cases}$$



Ex 14

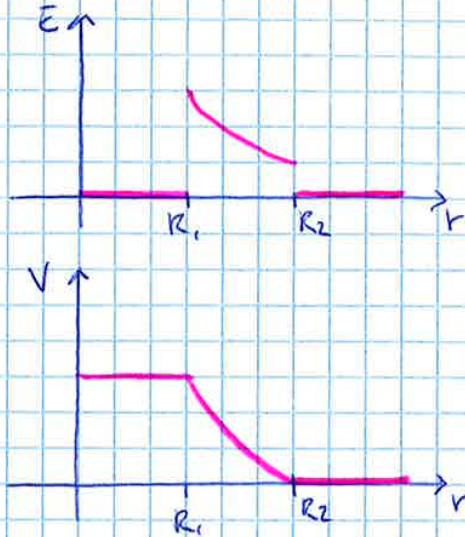


Calcolare la capacità di due superfici conduttrici sferiche concentriche, raggio R_1, R_2 , portanti cariche $+q$ e $-q$

$$\vec{E} = \begin{cases} r < R_1 & 0 \\ R_1 < r < R_2 & \frac{kq}{r^2} \\ r > R_2 & 0 \end{cases}$$

$$V = \begin{cases} r > R_2 & C_3 & V(\infty) = 0 \rightarrow C_3 = 0 \\ R_1 < r < R_2 & \frac{kq}{r} + C_2 & V(R_2^+) = V(R_2^-) \rightarrow C_2 = -\frac{kq}{R_2} \\ r > R_1 & C_1 & V(R_1^+) = V(R_1^-) \rightarrow C_1 = kq \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \end{cases}$$

$$V = \begin{cases} r < R_1 & kq \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\ R_1 < r < R_2 & \frac{kq}{r} - \frac{kq}{R_2} \\ r > R_2 & 0 \end{cases}$$



$$\Delta V = V_{int} - V_{ext} = kq \left(\frac{1}{R_1} - \frac{1}{R_2} \right) - 0 = kq \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$C = \frac{q}{\Delta V} = \frac{q}{\frac{1}{4\pi\epsilon_0} kq \left(\frac{1}{R_1} - \frac{1}{R_2} \right)} = \frac{4\pi\epsilon_0}{\frac{1}{R_1} - \frac{1}{R_2}}$$

Ex 16

Condensatore piano; si allontanano le due armature, da $d_1 \rightarrow d_2$

Calcolare la variazione di energia accumulata nei due casi.

caso A) variazione di d , rimane costante la V fra le armature.

caso B) variazione di d , rimane costante la Q delle armature.

* $W = U = \frac{1}{2} CV^2$ $C = \epsilon_0 \cdot \frac{\text{Superficie}}{\text{distanza}}$ (questa deriva dall'analisi di $C = \frac{Q}{\Delta V}$)
 (Vale solo per condensatori piani, in altri casi cambia!)

$C = \frac{Q}{\Delta V} \Rightarrow Q = CV \Rightarrow V = \frac{Q}{C}$

caso A) $\Delta U = U_2 - U_1 = \frac{1}{2} C_2 V^2 - \frac{1}{2} C_1 V^2 = \frac{1}{2} \epsilon_0 \frac{A}{d_2} V^2 - \frac{1}{2} \epsilon_0 \frac{A}{d_1} V^2 = \frac{1}{2} \epsilon_0 A V^2 \left(\frac{1}{d_2} - \frac{1}{d_1} \right)$

Variazione carica $\Delta Q = Q_2 - Q_1 = C_2 V - C_1 V = \epsilon_0 \frac{A}{d_2} V - \epsilon_0 \frac{A}{d_1} V = \epsilon_0 A V \left(\frac{1}{d_2} - \frac{1}{d_1} \right)$

$V = \text{cost per ipotesi} = \frac{2 \Delta U}{\Delta Q}$

$\Rightarrow \Delta U = \frac{1}{2} CV^2 \Rightarrow \frac{1}{2} \frac{Q}{V} V^2 \Rightarrow \Delta U = \frac{1}{2} QV \Rightarrow 2 \Delta U = \Delta Q V \Rightarrow \Delta Q = \frac{2 \Delta U}{V}$ Relazioni che lega ΔU e ΔQ con V costante

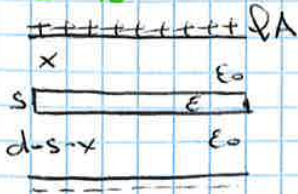
caso B) * $U = \frac{1}{2} CV^2 \Rightarrow \frac{1}{2} \frac{Q}{V} V^2 \Rightarrow \frac{1}{2} Q \cdot \frac{Q}{C} \Rightarrow \frac{1}{2} \frac{Q^2}{C}$ $C = \epsilon_0 \frac{A}{d}$

Variazione energia $\Delta U = U_2 - U_1 = \frac{1}{2} \frac{Q^2}{C_2} - \frac{1}{2} \frac{Q^2}{C_1} = \frac{1}{2} \frac{Q^2}{\epsilon_0 \frac{A}{d_2}} - \frac{1}{2} \frac{Q^2}{\epsilon_0 \frac{A}{d_1}} \Rightarrow \Delta U = \frac{1}{2} \frac{Q^2}{\epsilon_0 A} (d_2 - d_1)$

$Q = \text{cost per ipotesi} \Rightarrow Q = \frac{2 \Delta U}{\Delta V}$ relazione che lega ΔU e ΔV con Q costante

$\Delta V = V_2 - V_1 = \frac{Q}{C_2} - \frac{Q}{C_1} = Q \left(\frac{1}{\epsilon_0 \frac{A}{d_2}} - \frac{1}{\epsilon_0 \frac{A}{d_1}} \right) = \frac{Q}{\epsilon_0 A} (d_2 - d_1)$

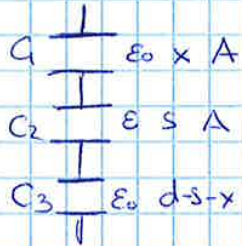
Ex 18



Inserimento di un dielettrico fra due armature di un condensatore

$Q, A, d, \epsilon, \epsilon_0, s, x \rightarrow \text{noti}$

ΔU dopo l'inserimento del dielettrico = ?



$$C_1 = \frac{\epsilon_0 A}{x}$$

Condensatori in parallelo!

$$C_2 = \frac{\epsilon A}{s}$$

$$C_3 = \frac{\epsilon_0 A}{d-s-x}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{x}{\epsilon_0 A} + \frac{s}{\epsilon A} + \frac{d-s-x}{\epsilon_0 A}$$

$$= \frac{d-s}{\epsilon_0 A} + \frac{s}{\epsilon A} \Rightarrow C = \frac{\epsilon_0 A}{d-s} + \frac{\epsilon A}{s}$$

prima $C(s) = \frac{A \epsilon_0}{d}$

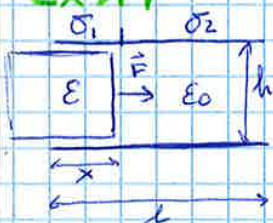
dopo $C = \frac{\epsilon_0 A}{d-s} + \frac{\epsilon A}{s}$

Q cost

$$\Delta U = U_{(s)} - U_{(C)} = \frac{Q^2}{2C(s)} - \frac{Q^2}{2C} = \frac{Q^2}{2} \left(\frac{d-s}{\epsilon_0 A} + \frac{s}{\epsilon A} - \frac{d}{\epsilon_0 A} \right) = \frac{Q^2}{2} \left(\frac{\epsilon d - \epsilon s + \epsilon_0 s}{\epsilon \epsilon_0 A} \right)$$

$$= \frac{Q^2 s (\epsilon_0 - \epsilon)}{2 A \epsilon_0 \epsilon} < 0 \quad U_{(s)} > U_{(C)}$$

Ex 19



$A = l \cdot L$

Inserimento di un dielettrico tra le armature di un condensatore

$h, L, l, x, \epsilon_0, \epsilon, Q_{in} \rightarrow \text{noti}$

\vec{F} di richiamo del dielettrico all'interno = ?



1) $V_1 = V_2 \Rightarrow E_1 = E_2 \Rightarrow \frac{\sigma_1}{\epsilon} = \frac{\sigma_2}{\epsilon_0}$

2) $Q = \sigma_1 L x + \sigma_2 L (l-x)$ ($\sigma = \frac{Q}{\text{Superficie}}$ *)

$A_1 = xL \quad A_2 = (l-x)L$

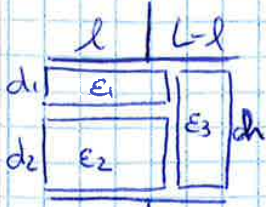
$C_1 = \frac{\epsilon L x}{h} \quad C_2 = \frac{\epsilon_0 (l-x)L}{h}$

3)
$$\begin{cases} \epsilon_0 \sigma_1 - \epsilon \sigma_2 = 0 \\ \sigma_1 x + \sigma_2 (l-x) = \frac{Q}{L} \end{cases} \dots \begin{cases} \sigma_1 = \frac{Q}{L} \frac{\epsilon}{\epsilon_0 l + (\epsilon - \epsilon_0) x} \\ \sigma_2 = \frac{Q}{L} \frac{\epsilon_0}{\epsilon_0 l + (\epsilon - \epsilon_0) x} \end{cases}$$

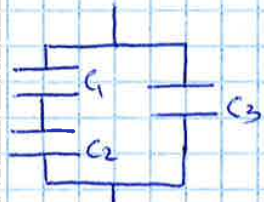
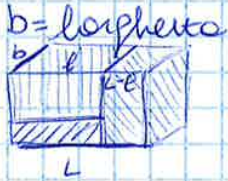
$C(x) = C_1 + C_2 = \frac{L(\epsilon x + \epsilon_0 l - \epsilon_0 x)}{h}$

* $V = E \cdot h$

Ex 20



Sono noti $\epsilon_1, \epsilon_2, \epsilon_3$ dei metalli dielettrici che occupano lo spazio fra le armature del condensatore. Calcolare C



C_1, C_2 in serie

$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow C_{tot} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_1 = \frac{\epsilon_1 b l}{d_1}$$

$$C_2 = \frac{\epsilon_2 b l}{d_2}$$

$$C = C_3 + C_{12} = \frac{C_1 C_2}{C_1 + C_2} + C_3$$

Sotto parallelo

$$C_3 = \frac{\epsilon_3 b (L-l)}{d_1 + d_2}$$

Ex 21



Sfera metallica raggio R_1 , ricoperta da un guscio spesso $R_2 - R_1$, dielettrico, con costante ϵ

Calcolare capacità

$$\oint_{\Sigma} \vec{D} \cdot d\vec{\sigma} = q$$

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon}$$

$$D = \begin{cases} r < R_1 & 0 \\ R_1 < r < R_2 & \frac{q}{4\pi r^2} \\ r > R_2 & \frac{q}{4\pi r^2} \end{cases}$$

CONDIZIONI V

$$V(\infty) = 0 \rightarrow C_3 = -\frac{q}{4\pi\epsilon_0\infty} \rightarrow C_3 = 0$$

$$V(R_2^-) = V(R_2^+) \rightarrow C_2 = \frac{q}{4\pi\epsilon R_2} - \frac{q}{4\pi\epsilon R_2}$$

$$V(R_1^-) = V(R_1^+) \rightarrow C_1 = \frac{q}{4\pi\epsilon R_1} + \frac{q}{4\pi} \left(\frac{1}{\epsilon R_2} - \frac{1}{\epsilon R_2} \right)$$

$$E = \begin{cases} r < R_1 & 0 \\ R_1 < r < R_2 & \frac{q}{4\pi\epsilon r^2} \\ r > R_2 & \frac{q}{4\pi\epsilon_0 r^2} \end{cases}$$

$$V = \begin{cases} r > R_2 & C_1 \rightarrow \frac{q}{4\pi} \left(\frac{1}{\epsilon R_1} + \frac{1}{\epsilon_0 R_2} - \frac{1}{\epsilon R_2} \right) \\ R_1 < r < R_2 & \frac{q}{4\pi\epsilon r} + C_2 \rightarrow \frac{q}{4\pi} \left(\frac{1}{\epsilon r} + \frac{1}{\epsilon_0 R_2} - \frac{1}{\epsilon R_2} \right) \\ r > R_2 & \frac{q}{4\pi\epsilon_0 r} + C_3 \rightarrow \frac{q}{4\pi\epsilon_0 r} \end{cases}$$

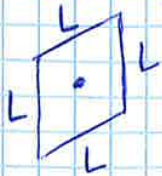
$$\Delta V = V(R_1) - V(\infty) = V(R_1) = \frac{q}{4\pi} \left(\frac{1}{\epsilon R_1} + \frac{1}{\epsilon_0 R_2} - \frac{1}{\epsilon R_2} \right)$$

$$C = \frac{q}{\Delta V} = \frac{q}{\frac{q}{4\pi} \left(\frac{1}{\epsilon R_1} + \frac{1}{\epsilon_0 R_2} - \frac{1}{\epsilon R_2} \right)} = \frac{4\pi}{\frac{1}{\epsilon R_1} + \frac{1}{\epsilon_0 R_2} - \frac{1}{\epsilon R_2}} = \frac{4\pi\epsilon_0 R_1}{1 + \frac{\epsilon - \epsilon_0}{\epsilon_0} \frac{R_1}{R_2}}$$

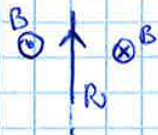
Se viene a mancare il dielettrico

$$R_2 \rightarrow R_1 \quad C = 4\pi\epsilon_0 R_1 \quad \epsilon = \epsilon_0$$

Ex 24



Campo creato al centro di una spira rettangolare di lato L percorso da i



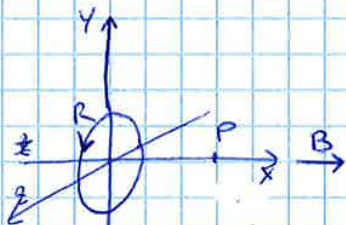
$$\vec{B}_1 = \hat{u}_\varphi \frac{\mu_0 i}{2\pi R} \frac{L}{\sqrt{L^2 + 4R^2}} \rightarrow \begin{matrix} \leftarrow \\ \otimes \\ \rightarrow \end{matrix}$$

con $R = \frac{L}{2}$ $\vec{B} = \hat{u}_\varphi \frac{\mu_0 i}{2\pi \frac{L}{2}} \frac{L}{\sqrt{L^2 + 4 \frac{L^2}{4}}} = \hat{u}_\varphi \frac{\mu_0 i}{\pi \sqrt{2} L}$

$$\vec{B} = 4 \cdot \vec{B}_1$$

$$\vec{B} = 4 \cdot \hat{u}_\varphi \frac{\mu_0 i}{\pi \sqrt{2} L} = \hat{u}_\varphi \frac{2\sqrt{2} \mu_0 i}{\pi L}$$

Ex 25



Campo creato sull'asse di una spira circolare

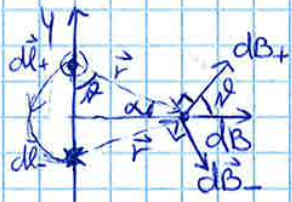
$$dB = \frac{\mu_0 i}{4\pi} \frac{|d\vec{l} \times \vec{r}|}{r^3} \cos\varphi = \frac{\mu_0 i}{2\pi} \frac{dl \cdot r \cdot \sin\frac{\pi}{2}}{r^3} \cos\varphi$$

$$= \frac{\mu_0 i}{2\pi} \frac{dl \cdot r \cdot \sin\frac{\pi}{2}}{r^3} \cos\varphi$$

$$= \frac{\mu_0 i}{2\pi r^2} \cos\varphi dl$$

$$B = \frac{\mu_0 i}{2\pi r^2} \cos\varphi \int_{\text{mezza circ.}} dl = \frac{\mu_0 i}{2\pi r^2} \cos\varphi \pi R$$

$$\begin{aligned} \alpha + \pi + \beta &= \pi \\ \alpha &= \pi - \beta \\ \beta &= \frac{\pi - \alpha}{2} \end{aligned}$$



$$\vec{B} = \frac{\mu_0 i}{2 r^2} \cdot B \cos\varphi$$

$$r^2 = x^2 + R^2$$

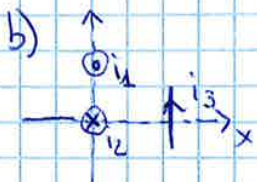
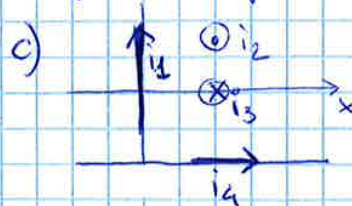
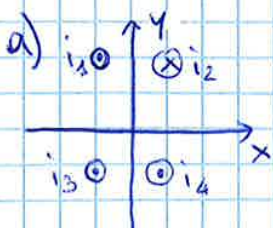
$$\cos\varphi = \frac{R}{\sqrt{x^2 + R^2}}$$

$$\vec{B} = \frac{\mu_0 i}{2 \cdot (x^2 + R^2)} \cdot R \cdot \frac{R}{\sqrt{x^2 + R^2}} \Rightarrow \frac{\mu_0 i R^2}{2(x^2 + R^2)^{3/2}} \vec{u}$$



Ex 26

Calcolare \vec{B} in vari phi del piano, creato dai conduttori

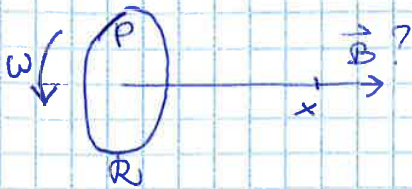


Si parte dal risultato

$$\frac{\mu_0 i dl}{4\pi r^2} \otimes \vec{B}$$

$$B = \frac{\mu_0 i}{2\pi d}$$

Ex 28



Calcolare il \vec{B} sull'asse del disco
raggio R, ruotante con ω e
portante una carica unif. σ



$$\int dq = \sigma dA = \sigma 2\pi r dr$$

$$T = \frac{2\pi}{\omega}$$

$$di = \frac{dq}{T} \quad di = \frac{\omega}{2\pi} \sigma 2\pi r dr = \omega \sigma r dr$$

$$i = \int di = \omega \sigma \int_0^R r dr = \frac{\omega \sigma R^2}{2}$$

Spira



di = ωσr dr dall'es. 25

$$d\vec{B} = \frac{\mu_0 di}{2} \frac{r^2}{(r^2+x^2)^{3/2}} \vec{n} = \frac{\mu_0 \omega \sigma}{2} \frac{r^3 dr}{(r^2+x^2)^{3/2}} \vec{n}$$

Disco

$$\vec{B} = \int d\vec{B} = \vec{n} \frac{\mu_0 \omega \sigma}{2} \int_0^R \frac{r^3 dr}{(r^2+x^2)^{3/2}} = \vec{n} \frac{\mu_0 \omega \sigma}{2} \times \int_0^{R/x} \frac{z^3 dz}{(1+z^2)^{3/2}}$$

$z = \frac{r}{x} \quad r=0 \rightarrow z=0$
 $r=R \rightarrow z = \frac{R}{x}$

$$\Rightarrow \vec{B} = \frac{\mu_0 \omega \sigma}{2} \left(\frac{2x^2+k^2}{-1+x^2+k^2} - 2x \right) \vec{n}$$

$$i = \frac{1}{2} \omega \sigma R^2$$

al centro $\vec{B} = \frac{\mu_0 \omega \sigma R}{2} \quad (x=0)$

5) Superficie indefinita $L \rightarrow \infty$

$$\vec{B} = \mu_0 w \sigma R \Rightarrow \vec{B} = \mu_0 i_n$$



Solenoid indefinito con n spire per unità di lunghezza

$$i_n = i_1 \cdot n \Rightarrow \vec{B} = \mu_0 n i_1 \quad n = \frac{N}{L}$$

Campo interno \rightarrow uniforme $B = \mu_0 n i_1 = w \sigma R$

Campo esterno \rightarrow nullo $B = 0$

Ex 30

Calcolare il campo \vec{B} creato da un cilindro indefinito di raggio R , carico uniformemente con densità ρ e che ruota con vel. ang. ω

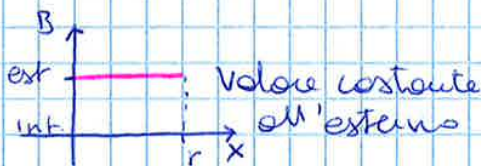


1) Superficie cilindrica raggio r indefinita con σ , ruotante con vel. ω



$$B = w \sigma r = \mu_0 i_n$$

$$B_{int} = 0$$



2) Cilindro raggio $R =$ unione di strati cilindrici di raggio $r, r+dr$, equivalenti a delle sup. cilindriche

per la lunghetta L : $dq = \rho L 2\pi r dr$
carica nello strato cilindrico

$$di = \frac{dq}{L} = \frac{\rho}{2\pi} \frac{dq}{L} = \frac{\rho}{2\pi} L 2\pi r dr = \rho w r dr$$

$$dB = \mu_0 di_n$$

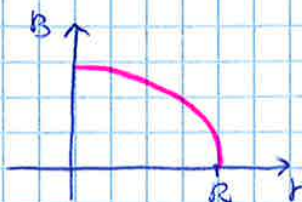
$$di_n = \frac{di}{L} = \frac{\rho w r dr}{L} = \rho w r dr$$

$$dB = \mu_0 \rho w r dr$$



$$B = \int dB = \mu_0 \rho w \int_r^R r dr$$

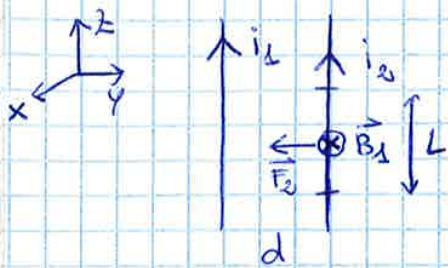
$$B = \frac{1}{2} \mu_0 \rho w (R^2 - r^2)$$



CILO SONO MARTINA COME SGLI'IO GENE

Ex 32

[Interazione magnetica fra conduttori]



Calcolare la forza di interazione per unità di lunghezza di due conduttori rettilinei, paralleli, indefiniti, percorsi dalle correnti i_1 e i_2 concordi

$$\vec{B}_1 = -\frac{\mu_0 i_1}{2\pi d} \hat{u}_x$$

$$d\vec{F}_2 = i_2 d\vec{l}_2 \times \vec{B}_1 = i_2 dz \hat{u}_z \times \left(-\frac{\mu_0 i_1}{2\pi d} \hat{u}_x\right) = \frac{\mu_0 i_1 i_2}{2\pi d} \hat{u}_x \times \hat{u}_z dz$$

$$= -\frac{\mu_0 i_1 i_2}{2\pi d} \hat{u}_y dz$$

$$\vec{F}_2 = \int d\vec{F}_2 = -\frac{\mu_0 i_1 i_2}{2\pi d} \hat{u}_y \int_0^L dz = -\frac{\mu_0 i_1 i_2}{2\pi d} L \hat{u}_y$$

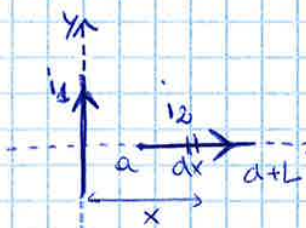
$$\frac{\vec{F}_2}{L} = \text{forza per unità di lunghezza} = -\frac{\mu_0 i_1 i_2}{2\pi d} \hat{u}_y$$

Ampère - Laplace $d\vec{B}_2 = \frac{\mu_0 i_2}{4\pi} \frac{d\vec{l}_2 \times \vec{r}}{r^3}$

$$d\vec{F}_2 = i_2 d\vec{l}_2 \times d\vec{B}_1 \rightarrow 2^\circ \text{ Laplace}$$

$$d\vec{F}_2 = i_2 d\vec{l}_2 \times \left(\frac{\mu_0 i_1}{4\pi} \frac{d\vec{l}_1 \times \vec{r}}{r^3}\right) = \frac{\mu_0}{4\pi} i_1 i_2 d\vec{l}_2 \times \left(\frac{d\vec{l}_1 \times \vec{r}}{r^3}\right)$$

Ex 33



Il conduttore percorso da i_1 è indefinito quello percorso da i_2 è lungo L , l'estremità sinistra dista a dal primo.

Calcolare \vec{F}_2 subito dal 2° conduttore

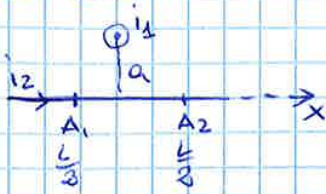
1) $\vec{B}_1(x) = -\frac{\mu_0 i_1}{2\pi x} \hat{u}_z$ (Biot-Savart)

2) Forza sull'elemento dx

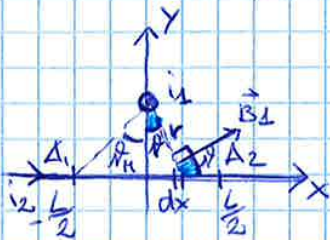
$$d\vec{F}_2 = i_2 d\vec{l}_2 \times \vec{B}_1 = i_2 (dx \hat{u}_x) \times \left(-\frac{\mu_0 i_1}{2\pi x} \hat{u}_z\right) = \frac{\mu_0 i_1 i_2}{2\pi} \frac{dx}{x} \hat{u}_z \times \hat{u}_x$$

$$\vec{F}_2 = \int d\vec{F}_2 = \hat{u}_y \int_a^{L+a} \frac{\mu_0 i_1 i_2}{2\pi} \frac{dx}{x} \Rightarrow \vec{F}_2 = \hat{u}_y \frac{\mu_0 i_1 i_2}{2\pi} \ln\left(1 + \frac{L}{a}\right)$$

Ex 35



Determinare il momento delle forze agenti sul tratto A1 A2 del conduttore percorso da i_2 orientato lungo x in seguito all'azione del conduttore percorso da i_1 orientato lungo z



$d\vec{M}$ rispetto all'origine O (dM = momento meccanico)

$$d\vec{M} = (x\hat{u}_x) \times d\vec{F} = \frac{\mu_0 i_1 i_2 \sin^2 \varphi}{2\pi r} dx \hat{u}_x \times \hat{u}_z$$

$$\vec{B}_1 = B_1 \hat{u} =$$

$$= B_1 \cos \varphi \hat{u}_x + B_1 \sin \varphi \hat{u}_y$$

$$dl_2 = dx \hat{u}_x \quad B_1 = \frac{\mu_0 i_1}{2\pi r}$$

$$d\vec{F} = i_2 dl_2 \times \vec{B}_1 =$$

$$= i_2 dx B_1 \hat{u}_x \times (\cos \varphi \hat{u}_x + \sin \varphi \hat{u}_y) =$$

$$= i_2 B_1 \sin \varphi dx \hat{u}_x \times \hat{u}_y =$$

$$= i_2 B_1 \sin \varphi dx \hat{u}_z =$$

$$= \frac{\mu_0 i_1 i_2 \sin^2 \varphi}{2\pi r} dx \hat{u}_z^*$$

$$= -\frac{\mu_0 i_1 i_2 \sin^2 \varphi}{2\pi r} dx \hat{u}_y$$



⇒ Rotazione attorno a y , senso orario

$$\begin{cases} x = a \operatorname{tg} \varphi \\ dx = \frac{a}{\cos^2 \varphi} d\varphi \\ x = r \sin \varphi \rightarrow r = \frac{x}{\sin \varphi} \\ \varphi = \operatorname{arctg} \frac{x}{2a} \end{cases}$$

$$dx = \frac{a}{\cos^2 \varphi} d\varphi$$

$$x = r \sin \varphi \rightarrow r = \frac{x}{\sin \varphi}$$

$$\varphi = \operatorname{arctg} \frac{x}{2a}$$

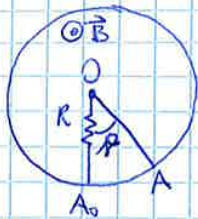
$$\int \operatorname{tg}^2 \varphi d\varphi = \operatorname{tg} \varphi - \varphi$$

$$d\vec{M} = -\frac{\mu_0 i_1 i_2 a}{2\pi} \operatorname{tg}^2 \varphi d\varphi \hat{u}_y$$

$$\vec{M} = -\hat{u}_y \frac{\mu_0 i_1 i_2 a}{2\pi} \int_{-\varphi_1}^{+\varphi_1} \operatorname{tg}^2 \varphi d\varphi$$

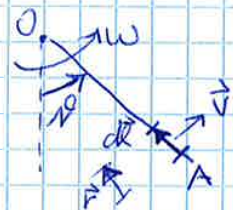
$$\vec{M} = -\hat{u}_y \frac{\mu_0 i_1 i_2 a}{2\pi} \left[\frac{L}{2a} - \operatorname{arctg} \frac{L}{2a} \right]$$

Ex 38



$\vec{B} = B \hat{u}_z$

$\vec{\omega} = \omega \hat{u}_z$



[Faraday] 4° Maxwell $\oint_C \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{\sigma}$ $\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Asta OA è vincolata a ruotare attorno allo suo estremo O con velocità angolare $\vec{\omega}$ l'estremità A si trova in contatto elettrico con l'anello di raggio $OA=l$ e centro O

Determinare la corrente i che circola nel circuito di resistenza R , $A_0 A_+ \equiv A_0 A_-$

ambivalenza $\oint \vec{v} \cdot d\vec{l} = 0$

$d\vec{l} = dr \hat{u}_r$ $\vec{v} = v \hat{u}_\theta$ $v = \omega(l-r)$

1) Forza di Lorentz

$\vec{F} = q \vec{v} \wedge \vec{B} = q v \hat{u}_\theta \wedge B \hat{u}_z = q v B (\hat{u}_\theta \wedge \hat{u}_z) = -q v B \hat{u}_r$

elettroni: $q = -e$ ($e =$ modulo della carica)

$\vec{F} = e v B \hat{u}_r \rightarrow$ gli e^- si spostano da A verso O

$\vec{E} = \frac{\vec{F}}{q} = -\frac{q v B \hat{u}_r}{q} = -v B \hat{u}_r$

$V_{A_0} = \underline{\underline{\mathcal{E}}} = \int_{A_0} \vec{E} \cdot d\vec{l} = \int_{A_0} -v B \hat{u}_r \cdot d\vec{l} = -B \hat{u}_r \int_{A_0} v d\vec{l} = -B \hat{u}_r \int_{A_0} v dr \hat{u}_r =$

$= -B \int_{A_0} v dr = -B \int_{A_0} \omega(l-r) dr = B \int_{A_0} \omega(r-l) dr = B \omega \int_{A_0} (r-l) d(r-l) =$

$= B \omega \int_{-l}^0 \omega dw = B \omega \left[\frac{1}{2} \omega^2 \right]_{-l}^0 = B \omega \left[0 - \frac{1}{2} l^2 \right] = \underline{\underline{-\frac{1}{2} B \omega l^2}}$

* $\int_{l \rightarrow A} \omega = r-l$
 $\int_{l \rightarrow O} \omega = 0$

2) Faraday

$\phi_{\text{sup}} \mathcal{E} = -\frac{\partial \phi_{\text{sup}}}{\partial t} = -\frac{\partial}{\partial t} \left(B \frac{1}{2} l^2 \right) = -\frac{1}{2} B l^2 \frac{\partial \omega}{\partial t} = -\frac{1}{2} B \omega l^2$

$\phi_{\text{inf}} \mathcal{E} = -\frac{\partial \phi_{\text{inf}}}{\partial t} = -\frac{\partial}{\partial t} \left(B \left(\pi l^2 - \frac{1}{2} l^2 \omega \right) \right) = \frac{1}{2} B l^2 \frac{\partial \omega}{\partial t} = \frac{1}{2} B \omega l^2$

$i = \frac{|\mathcal{E}|}{R} = \frac{1}{2} \frac{\omega B l^2}{R}$

Ex 4.1



Calcolare lo \mathcal{E} indotto

$$\vec{B} = -\frac{\mu_0 I}{2\pi x} \hat{u}_z$$

$$\vec{v} = v \hat{u}_x$$

$$\vec{F} = q \vec{v} \wedge \vec{B}$$

1) Lorentz

$$\hat{u}_z \wedge \hat{u}_x = \hat{u}_y$$

A3A4 $\vec{F} = q \vec{v} \wedge \vec{B} = q v \hat{u}_x \wedge \frac{\mu_0 I}{2\pi x} (-\hat{u}_z) = \frac{q v \mu_0 I}{2\pi x} \hat{u}_y$ non è attivo

A1A2 analogo a A3A4, non è attivo

A2A3 $\vec{F} = q \vec{v} \wedge \vec{B} = q v \hat{u}_x \wedge \frac{\mu_0 I}{2\pi x} (-\hat{u}_z) = \frac{q v \mu_0 I}{2\pi x} \hat{u}_y$ $x = L + a(t)$

$$\vec{E} = \frac{\vec{F}}{q} = \frac{v \mu_0 I}{2\pi (L+a(t))} \hat{u}_y = \frac{v \mu_0 I}{2\pi (L+a(t))} \hat{u}_y \quad d\vec{l} = dy (-\hat{u}_y)$$

$$\hat{u}_y \cdot \hat{u}_y = 1$$

$$\mathcal{E} = \int_{A2A3} \vec{E} \cdot d\vec{l} = -\frac{\mu_0 I v}{2\pi (L+a(t))} \int_{A2A3} \hat{u}_y dy \hat{u}_y = -\frac{\mu_0 I v h}{2\pi (L+a(t))} \rightarrow \underline{\text{attivo}}$$

A4A1 $\vec{F} = q \vec{v} \wedge \vec{B} = \frac{q v \mu_0 I}{2\pi x} \hat{u}_y$ $x = a(t)$

$$\vec{E} = \frac{\vec{F}}{q} = \frac{v \mu_0 I}{2\pi a(t)} \hat{u}_y \quad d\vec{l} = dy \hat{u}_y$$

$$\mathcal{E} = \int_{A4A1} \vec{E} \cdot d\vec{l} = \frac{\mu_0 I v h}{2\pi a(t)} \quad \underline{\text{attivo}}$$

$$\mathcal{E}_{\text{tot}} = \mathcal{E}_{A2A3} + \mathcal{E}_{A4A1} = \frac{\mu_0 I v h}{2\pi} \left(\frac{1}{a(t)} - \frac{1}{L+a(t)} \right)$$

2) Faraday

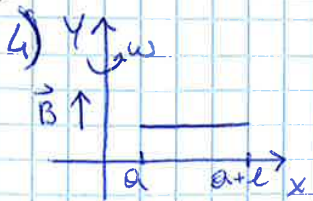
$$\Phi(B)(t) = \int B \cdot d\vec{\sigma} \quad * \quad \mathcal{E} = -\frac{\partial \Phi(t)}{\partial t}$$

$\Phi(t) = \frac{\mu_0 I h}{2\pi} \ln \left(1 + \frac{L}{a(t)} \right) = \frac{\mu_0 I h}{2\pi} [\ln(L+a(t)) - \ln(a(t))]$

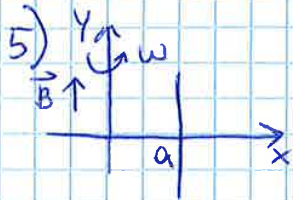
$\frac{\partial \Phi(t)}{\partial t} = \frac{\mu_0 I h}{2\pi} \left[\frac{da(t)}{dt} \frac{1}{L+a(t)} - \frac{da(t)}{dt} \frac{1}{a(t)} \right] = \frac{\mu_0 I h v}{2\pi} \left[\frac{1}{L+a(t)} - \frac{1}{a(t)} \right]$

variazione area = dσ → base · h cost

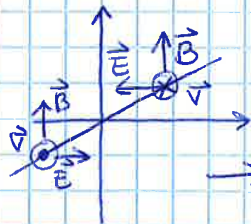
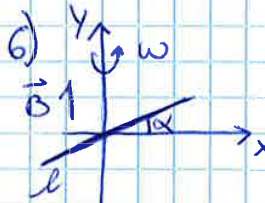
$$\mathcal{E} = \frac{\mu_0 I h v}{2\pi} \left[\frac{1}{a(t)} - \frac{1}{L+a(t)} \right] \quad \text{!!! Invertiti perché } \mathcal{E} = -\frac{\partial \Phi(t)}{\partial t}$$



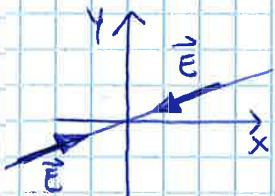
$\alpha = 0 \rightarrow \cos \alpha = 1$
 $E = \frac{1}{2} \omega B (l^2 + 2al)$



$E = 0$

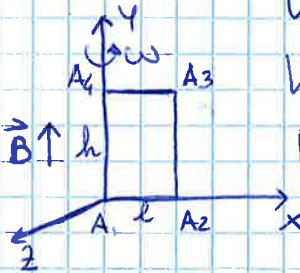


$\vec{E} = \vec{v} \wedge \vec{B} = v \hat{u}_z \wedge B \hat{u}_y = vB \hat{u}_x$



$E = 0$

Ex 43



Il circuito $A_1A_2A_3A_4$ si trova sul piano xy
 Il piano xy ruota attorno all'asse y con $\vec{\omega}$
 Il campo $\vec{B} = B \hat{u}_y$

Determinare E sul circuito (Lorentz) \rightarrow

A_1A_2 : $E = \frac{1}{2} B \omega l^2$
 $\alpha=0 \alpha=0$

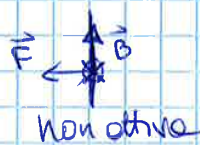


$\vec{F}_{A_1A_2} = q(-v \hat{u}_z) \wedge B \hat{u}_y = qvB \hat{u}_x$

$\vec{E}_{A_1A_2} = vB \hat{u}_x = \vec{E}_{A_3A_4}$

A_3A_4 : $E = \frac{1}{2} B \omega l^2$
 $\alpha=0 \alpha=0$

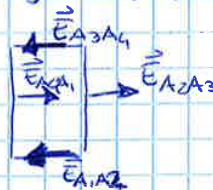
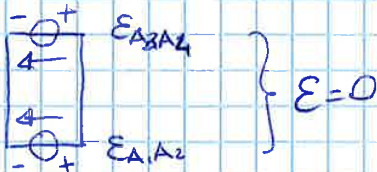
A_2A_3 : $E = 0$
 $\alpha=l \alpha=\frac{\pi}{2}$



$\vec{F}_{A_2A_3} = q(-v \hat{u}_z) \wedge B \hat{u}_y = qvB \hat{u}_x$

$\vec{E}_{A_2A_3} = vB \hat{u}_x = \vec{E}_{A_4A_1}$

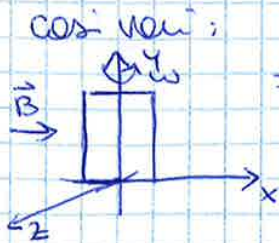
A_1A_4 : $E = 0$
 $\alpha=0 \alpha=\frac{\pi}{2}$



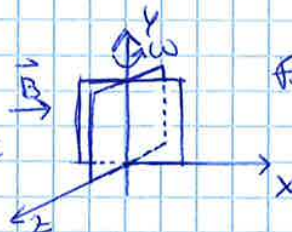
Faraday:

$E = \frac{\partial \Phi}{\partial t} = - \frac{\partial}{\partial t} 0 = 0$

\vec{B} non investe mai il circuito durante la rotazione



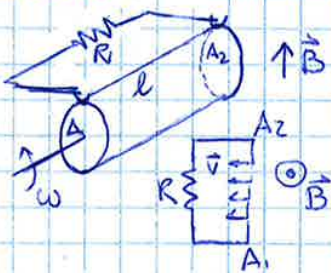
Faraday: $E = -BA\omega \cos \omega t$



Faraday: $E = -BA\omega \cos \omega t$
 $E = -BA\omega \sin \omega t$
 $E = -BA\omega \cos \omega t$
 $E = -BA\omega \cos(\omega t + \frac{\pi}{2})$

con possibili - + *

Ex 47



Un cilindro di raggio r e lunghezza l ruota con ω in presenza di $\vec{B} \perp$ al suo asse. Due piastre A_1, A_2 forate parte di una generatrice del cilindro si trovano in contatto ohmico tramite una resistenza R . Determinare come varia ω sapendo

che a $t=0 \rightarrow \omega(0) = \omega_0$ e come varia $i(t)$

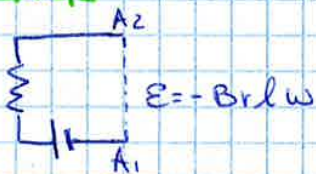
$\vec{v} = \vec{\omega} \times \vec{r} = v \hat{\phi} = v \sin \frac{\pi}{2} \hat{\phi}$
 $\vec{E} = \vec{v} \times \vec{B} \quad \vec{E} = vB = \omega r B \rightarrow E = \int \omega B r dl \rightarrow i = -\frac{Brl}{R} \omega \quad (i = \frac{E}{R})$
 $E = \frac{d\Phi}{dt}$

Il tratto $A_1 A_2$ sente la $F = i l B \Rightarrow$ momento meccanico $\tau = r F$

$\tau = r F = r i l B = -\frac{B^2 l^2 r^2}{R} \omega \Rightarrow \tau = -\frac{B^2 l^2 r^2}{R} \omega$ è frenante, per il collettore ω

eq. moto: $I \frac{d\omega}{dt} = -\frac{B^2 l^2 r^2}{R} \omega \rightarrow \omega(t) = \omega_0 e^{-\frac{B^2 l^2 r^2}{IR} t} \Rightarrow i(t) = \frac{Brl \omega_0}{R} e^{-\frac{B^2 l^2 r^2}{IR} t}$

Ex 48



Determinare $\omega(t)$ e $i(t)$ nel circuito precedente quando viene inserito un generatore di V_0 esterno ($V = \cos t = V_0$)

$i = \frac{V_0 - Brl\omega}{R}$

$F = i l B \quad \tau = r F \rightarrow \tau = \frac{V_0 - Brl\omega}{R} r l B$

eq. moto $I \frac{d\omega}{dt} = \frac{V_0 - Brl\omega}{R} r l B \quad \frac{d\omega}{dt} = \frac{V_0 r l B}{RI} - \frac{B^2 l^2 r^2}{RI} \omega$

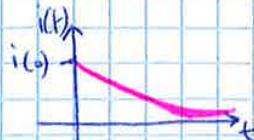
$\frac{d\omega}{dt} = C_1 - C_2 \omega \Rightarrow \omega(t) = \frac{C_1}{C_2} - \left(\frac{C_1}{C_2} - \omega_0\right) e^{-C_2 t}$

$i(t) = \frac{V_0 - Brl\omega(t)}{R}$

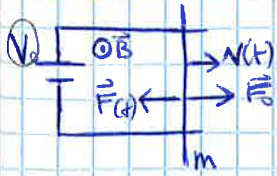
$\omega(\infty) = \frac{C_1}{C_2} = \frac{V_0}{Brl}$

$i(\infty) = 0$

$i(0) = \frac{V_0 - Brl\omega_0}{R}$



Ex 52



Studiare il moto dell'asta sapendo che $v_0 = V_0$

V_0 potenziale
 $I_0 = \frac{V_0}{R}$
 $F_0 = BlI_0 = \frac{BlV_0}{R}$

$$\begin{cases} \mathcal{E}(t) = -\frac{\partial \phi}{\partial t} = -Blv(t) \\ I(t) = \frac{\mathcal{E}(t)}{R} = -\frac{Blv(t)}{R} \\ F(t) = BlI(t) = -\frac{B^2 l^2 v(t)}{R} \end{cases}$$

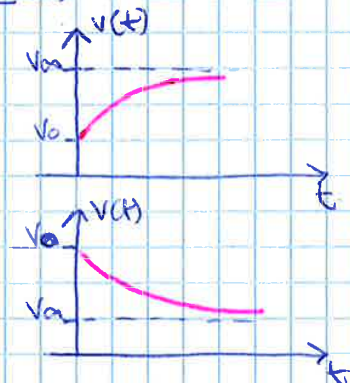
Potenza totale agente su m:

$$\vec{F}_T(t) = \vec{F}_0 + \vec{F}(t) = \frac{BlV_0}{R} - \frac{B^2 l^2 v(t)}{R}$$

$$I_T(t) = \frac{I_0 + I(t)}{R} = \frac{V_0}{R} - \frac{Blv(t)}{R}$$

eq. moto:

$$m \frac{dv(t)}{dt} = F_T(t) \quad \frac{dv(t)}{dt} - \frac{F_T(t)}{m} = \frac{BlV_0}{Rm} - \frac{B^2 l^2}{Rm} v(t)$$

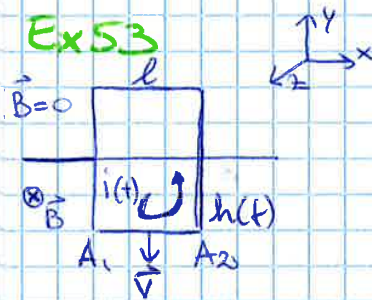


$$v(\infty) \Rightarrow \frac{dv(t)}{dt} = 0 \Rightarrow \frac{BlV_0}{Rm} - \frac{B^2 l^2}{Rm} v_{\infty} = 0 \Rightarrow v_{\infty} = \frac{V_0}{Bl}$$

$$I(\infty) \Rightarrow \frac{V_0}{R} - \frac{Bl}{R} v(\infty) \Rightarrow I(\infty) = \frac{V_0}{R} - \frac{Bl}{R} \frac{V_0}{Bl} \Rightarrow I(\infty) = 0$$

a seconda del valore di V_0

Ex 53



Sotto l'azione del suo peso,

la spira di massa m e resistenza R entra in una regione del piano xy dove è presente il campo \vec{B} . Determinare le v di discesa.

$$1) \mathcal{E}(t) = -\frac{\partial \phi(t)}{\partial t} = -\frac{\partial}{\partial t} Bl h(t) = -Bl v(t)$$

$$i(t) = \frac{\mathcal{E}(t)}{R} = -\frac{Bl}{R} v(t)$$

2) Forza su $A_1 A_2$ $\vec{F}_1 = q \vec{w} \wedge \vec{B} = q w \hat{u}_x \wedge B(-\hat{u}_z) = -q w B \hat{u}_x \wedge \hat{u}_z = q w B \hat{u}_y$
 $\vec{F}_{A_1 A_2} = N \vec{F}_1 = N S l (q w B \hat{u}_y) = q N S l w B \hat{u}_y = \rho S l w B \hat{u}_y = j S l B \hat{u}_y = i B l \hat{u}_y$



Eq di moto

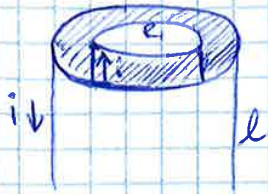
$$\downarrow \frac{dv(t)}{dt} = mg - F_{A_1 A_2} \quad \frac{dv}{dt} = mg - \frac{B^2 l^2}{R} v$$

In condizioni stazionarie $v(\infty)$ si ha

$$\frac{dv(t)}{dt} = 0 \Rightarrow mg - F_{A_1 A_2} = 0 \Rightarrow v(\infty) = \frac{mg R}{B^2 l^2}$$

Ex 57

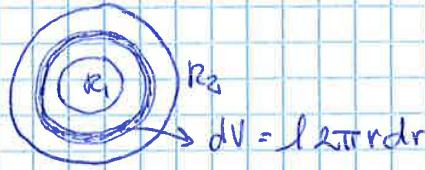
Calcolare l'UB del sistema



$$B = \begin{cases} r < R_1 & 0 \\ R_1 < r < R_2 & \frac{\mu_0 i}{2\pi r} \\ r > R_2 & 0 \end{cases}$$



$$u_B = \begin{cases} r < R_1 & 0 \\ R_1 < r < R_2 & \frac{\mu_0 i^2}{8\pi^2 r^2} \\ r > R_2 & 0 \end{cases}$$



pg 99
MUSIMINO

$$U_B = \int_V u_B dV = \frac{1}{2\mu_0} \int_V B^2 dV = \frac{\mu_0 i^2}{8\pi^2} \int_V \frac{dV}{r^2} = \frac{\mu_0 i^2}{48\pi^2} \int_{R_1}^{R_2} \frac{l 2\pi r dr}{r^2} =$$

$$= \frac{\mu_0 i^2 l}{4\pi} \ln\left(\frac{R_2}{R_1}\right) = \frac{1}{2} \left(\frac{\mu_0 l}{2\pi} \ln\left(\frac{R_2}{R_1}\right) \right) i^2 = \frac{1}{2} L i^2$$

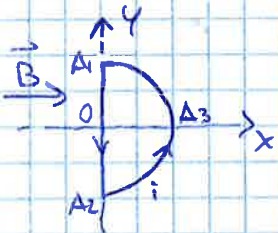
Induttanza

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{R_2}{R_1}\right) = \frac{\Phi}{i}$$

$$\Phi = \int_S B ds = \frac{\mu_0 i}{2\pi} \int_{R_1}^{R_2} \frac{ds}{r} = \frac{\mu_0 i}{2\pi} \ln\left(\frac{R_2}{R_1}\right)$$

$$L = \frac{\Phi}{i} = \frac{\mu_0 l}{2\pi} \ln\left(\frac{R_2}{R_1}\right)$$

Ex 58



Calcolare la F magnetica che agisce sui due rami del circuito percorsi da i e immerso in \vec{B}

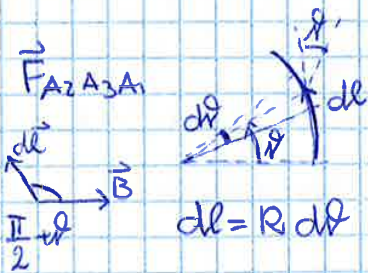
$$\vec{B} = B \hat{u}_x$$

$$\vec{F}_{A_1 A_2} = i \int_C d\vec{l} \wedge \vec{B}$$

2° Laplace $\hat{u}_x \wedge \hat{u}_y = \hat{u}_z$

$$d\vec{F} = i (dy \hat{u}_y) \wedge (B \hat{u}_x) = i B dy \hat{u}_z$$

$$\vec{F}_{A_1 A_2} = i B 2R \hat{u}_z$$



$$d\vec{F} = i d\vec{l} \wedge B \hat{u}_x = -i dl B \sin\left(\frac{\pi}{2} + \theta\right) \hat{u}_z =$$

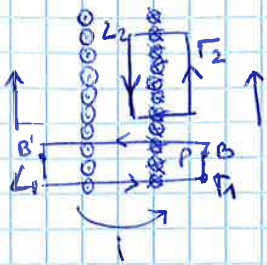
$$= -i B \cos \theta dl \hat{u}_z = -i B \cos \theta R d\theta \hat{u}_z$$

$$\vec{F}_{A_2 A_3} = -\hat{u}_z i B R \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos \theta d\theta = -2 i B R \hat{u}_z$$

$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos = \sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) = 2$



Ex 61



Calcolare il campo \vec{B} creato da un solenoide indefinito contenente n spire per unità di lunghezza e percorso della corrente i :

1) Γ_1 $L_1 B - L_1 B' = \mu_0 (nL_1 i - nL_1 i)$ Pall'infinito $\rightarrow B=0$
 $B=B'$ dunque $B=B'=0$

2) \uparrow Circuitazione lungo Γ_2 , contribuisce solo il tratto interno
 verticale $\vec{B} \parallel d\vec{\ell}$
 $\rightarrow B L_2 = \mu_0 (L_2 n i) \Rightarrow B = \mu_0 n i$

Ex 62

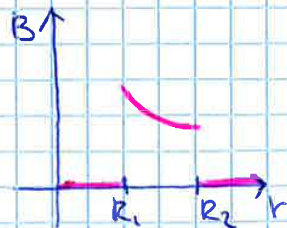
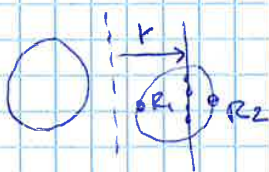


Calcolare il campo B creato da un solenoide toroidale di raggi R_1 e R_2 creata N spire e percorso della corrente i :

1) Calcolo di B per $r < R_1$: Γ_1 $2\pi r B = \mu_0 \cdot 0 \Rightarrow B = 0$

2) Calcolo di B per $r > R_2$: Γ_3 $2\pi r B = \mu_0 (N i - N i) \Rightarrow B = 0$

3) Calcolo di B per $R_1 < r < R_2$: Γ_2 $2\pi r B = \mu_0 N i \Rightarrow B = \frac{\mu_0 N i}{2\pi r}$



$$F = k \frac{Q_1 Q_2}{r^2} \quad E = \frac{F}{q} = k \frac{Q}{r^2} \quad U = k \frac{Q Q}{r} \quad V = \frac{U}{q} = k \frac{Q}{r}$$

$$L = F \cdot ds = -q \Delta V \rightarrow \Delta U \quad E = -\nabla V$$

Th. Gauss - 1° Maxwell

$$\Phi_{(\vec{E})} = \int_{\Sigma} \vec{E} \cdot d\vec{\sigma} = \vec{E} \cdot \vec{S} = E S \cos \theta = \sum \frac{Q_{int}}{\epsilon_0} = \int_V \nabla \cdot \vec{E} dV$$

Dim integrale: se le cariche sono interne: $\frac{Q}{\epsilon_0}$
 se le cariche sono esterne: i contributi si annullano

Dim differenziale: $\nabla \cdot \vec{E}$

$$d\phi = \frac{dq}{\epsilon_0} \quad \underline{d\phi} = \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right) dx dy dz \quad \rho(x, y, z) = \frac{dV}{dq}$$

$$\hookrightarrow \nabla \cdot \vec{E} \cdot V = \frac{Q}{\epsilon_0} \rightarrow \rho V \rightarrow \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \rightarrow \epsilon_0 \nabla \cdot \left(\frac{\partial \vec{E}}{\partial t} \right) = \frac{\partial \rho}{\partial t}$$

Circulazione di \vec{E} stazionario

$$\oint_C \vec{E}_{staz} = 0 \quad \nabla \wedge \vec{E} = 0$$

$C = \frac{Q}{\Delta V}$ conduttore piano $C = \epsilon_0 \frac{Area}{d}$ conduttore corona mediana $C = \frac{4\pi \epsilon_0}{\frac{1}{r_1} - \frac{1}{r_2}}$

$$W = U = \frac{1}{2} CV^2$$

$E(r) = -\frac{dV(r)}{dr}$ $V(r) = -\int E(r) \cdot dr \rightarrow V = -E(r) \cdot r + C$ Bisogna discutere

$$i = \frac{Q}{R} \rightarrow E \text{ (fem)}$$

Th. Gauss \vec{B} 2° Maxwell

$$\Phi(\vec{B}) = \oint_{\Sigma} \vec{B} d\vec{\sigma} = 0 \quad \nabla \cdot \vec{B} = 0$$

Lorentz $\vec{F} = q \cdot \vec{v} \times \vec{B} = q v B \sin \theta$

\hookrightarrow 2° legge $\vec{F} = i d\vec{l} \wedge \vec{B}$

Dim $\vec{F}_2 = q \vec{v} \wedge \vec{B}$ corrente $\rightarrow e^-$ che circolano $q = -e \vec{F}_2 = -e \vec{v} \wedge \vec{B}$

in $dS \Sigma \rightarrow$ ho $n e^- \Rightarrow n dS \Sigma$ e su cui agisce \vec{F}_2

$$dF = n dS \Sigma \vec{F}_2 \rightarrow dF = \underbrace{-ne}_{\downarrow} dS \Sigma \underbrace{\vec{v} \wedge \vec{B}}_{\downarrow} \rightarrow dF = dS \Sigma \underbrace{j}_{\downarrow} \wedge \vec{B} \rightarrow dF = i d\vec{S} \wedge \vec{B}$$

Filo conduttore con i , lungo l , in P $\vec{B}_P(x) = \frac{\mu_0}{4\pi} \frac{i l_0}{\sqrt{x^2 + l_0^2}} \hat{u}_P$

il lim $\Delta l_0 \rightarrow \infty$ $\frac{\mu_0}{4\pi} \frac{i l_0}{\sqrt{x^2 + l_0^2}} \hat{u}_P \rightarrow \left[\vec{B}(x) = \frac{\mu_0 i}{2\pi x} \hat{u}_P \right]$ Biot-Savart

* $\vec{B} = \mu_0 i$ \rightarrow campo di un solenoide

Teorema Ampère

$dS = r dr \theta$

Biot-Savart \rightarrow circolazione $\rightarrow \oint_r \vec{B} d\vec{l} = \oint B dl \cos \theta \rightarrow dS = dl \cos \theta$

$\rightarrow \oint \vec{B} d\vec{l} = \oint B dS \rightarrow \oint \vec{B} d\vec{l} = \frac{\mu_0}{4\pi} i \oint d\theta$

linea chiusa concatenata al conduttore $\rightarrow \oint \vec{B} d\vec{l} = \mu_0 i$ 4° Maxwell integrale stazionario

linea chiusa NON concatenata \rightarrow contributo nullo

Formulazione differenziale 4° Maxwell

$\vec{B} = \sum \vec{B}_i \rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 \sum i \quad i = \iint_{\Sigma} \vec{j} \cdot \vec{n} d\Sigma \rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 \iint_{\Sigma} \vec{j} \cdot \vec{n} d\Sigma$

$\vec{j} = \sum \vec{j}_i \rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 \iint_{\Sigma} \vec{j} \cdot \vec{n} d\Sigma = \mu_0 \iint_{\Sigma} \vec{j} \cdot \vec{n} d\Sigma$ differenziale stazionario

Teorema Stokes $\rightarrow \oint \vec{B} \cdot d\vec{l} = \iint_{\Sigma} \nabla \wedge \vec{B} \cdot \vec{n} d\Sigma \Rightarrow \iint_{\Sigma} \vec{j} \cdot \vec{n} d\Sigma \Rightarrow \left[\nabla \wedge \vec{B} = \mu_0 \vec{j} \right]$

4° Maxwell integrale + differenziale NON stazionario

Ampère: $\oint \vec{B} \cdot d\vec{l} = \mu_0 i$ $\nabla \wedge \vec{B} = \mu_0 \vec{j}_0 \rightarrow$ condiz stat $\rho_0 = \text{cost}$ $i_0 = 0$

eq continuità \rightarrow conservazione carica $\rightarrow \nabla \cdot \vec{j} + \frac{\partial \rho_0}{\partial t} = 0 \Rightarrow \nabla \cdot \vec{j}_0 = 0$ stat

$\Rightarrow \nabla \cdot (\nabla \wedge \vec{B}) = 0$

Dim \rightarrow fra le armature di un condensatore c'è corrente data da

una variazione di flusso del \vec{E} - 1° Maxwell NON stat

$\Rightarrow \nabla \cdot \vec{E} = \frac{\rho_0}{\epsilon_0} \quad \rho_0 = \nabla \cdot (\epsilon_0 \vec{E}) \rightarrow \frac{\partial \rho_0}{\partial t} = \frac{\partial}{\partial t} (\nabla \cdot (\epsilon_0 \vec{E})) = \nabla \cdot \left[\epsilon_0 \frac{\partial \vec{E}}{\partial t} \right]$

$\Rightarrow \nabla \cdot \vec{j}_0 + \frac{\partial \rho_0}{\partial t} = \nabla \cdot \vec{j}_0 + \nabla \cdot \left[\epsilon_0 \frac{\partial \vec{E}}{\partial t} \right] = \nabla \cdot \left[\vec{j}_0 + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right] = 0$

Densità di corrente di spostamento $\vec{j}_s = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Corrente NON stazionaria $\Rightarrow \vec{j} = \vec{j}_0 + \vec{j}_s \Rightarrow \nabla \cdot \vec{j} = 0$ $\vec{j} \rightarrow$ solenoidale

$\nabla \wedge \vec{B} = \mu_0 \vec{j} = \mu_0 (\vec{j}_0 + \vec{j}_s) = \mu_0 \vec{j}_0 + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ $i_s = \epsilon_0 \frac{\partial \phi(\epsilon)}{\partial t}$

$\oint_r \vec{B} d\vec{l} = \mu_0 i = \mu_0 (i_0 + i_s) = \mu_0 i_0 + \mu_0 \epsilon_0 \frac{\partial \phi(\epsilon)}{\partial t}$

Effetto Hall $\rightarrow \vec{j} = \frac{i}{\text{sup}(a \cdot b) \text{ superficie}} \quad \vec{j} = Ne\vec{v} \Rightarrow \vec{v}$

$\mathcal{E} = - \int_0^b \vec{E}_B dz = - \frac{iB}{eNe}$ $\mathcal{E} = \frac{\mathcal{B}}{e} \rightarrow$ forza di Lorentz conv e $q = e$

Indotto \downarrow direzione b

TEOREMA DI GAUSS IN FORMA DIFFERENZIALE

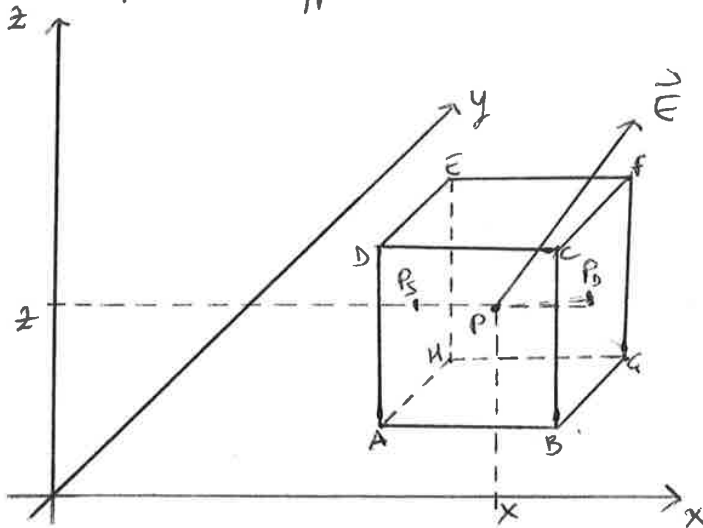


$$\phi = \frac{q}{\epsilon_0}$$

$$q = \int_V \rho dV$$

$$\phi = \oint_{\Sigma} \vec{E} \cdot d\vec{\Sigma}$$

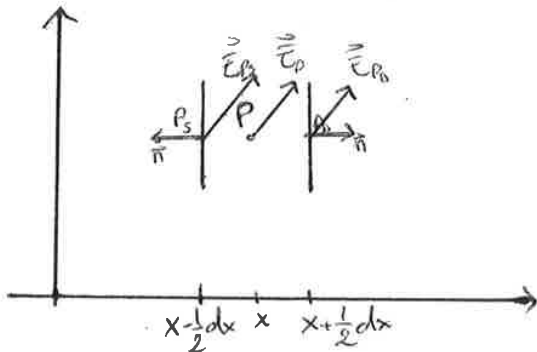
Gauss in forma differenziale



$$P: \vec{E} = \vec{E}(x, y, z) = E_x(x, y, z)\vec{i} + E_y(x, y, z)\vec{j} + E_z(x, y, z)\vec{k}$$

$$P_2: \vec{E}_{P_2} = \vec{E}(x + \frac{1}{2}dx, y, z)$$

$$P_1: \vec{E}_{P_1} = \vec{E}(x - \frac{1}{2}dx, y, z)$$



$$d\phi_x = E_x(x + \frac{1}{2}dx, y, z) dy dz - E_x(x - \frac{1}{2}dx, y, z) dy dz$$

$$d\phi_y = E_y(x, y + \frac{1}{2}dy, z) dx dz - E_y(x, y - \frac{1}{2}dy, z) dx dz$$

$$d\phi_z = E_z(x, y, z + \frac{1}{2}dz) dx dy - E_z(x, y, z - \frac{1}{2}dz) dx dy$$

$$d\phi = d\phi_x + d\phi_y + d\phi_z$$

$$d\phi = \frac{dq}{\epsilon_0}$$

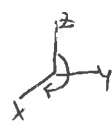


DEFINIZIONI

gradiente $\vec{\nabla} V = \frac{\partial V}{\partial x} \vec{i} + \frac{\partial V}{\partial y} \vec{j} + \frac{\partial V}{\partial z} \vec{k}$

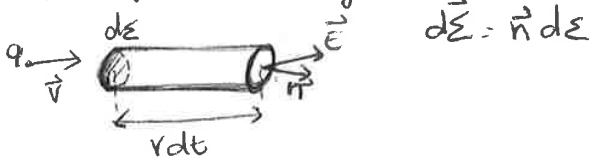
divergenza $\vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot (E_x \vec{i} + E_y \vec{j} + E_z \vec{k})$

rotore $\vec{\nabla} \wedge \vec{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) \vec{i} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \vec{j} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \vec{k}$

$$\vec{\nabla} \wedge \vec{E} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$


CORRENTE ELETTRICA

1) definizione \vec{j}, i



$$\left. \begin{aligned} dV &= \vec{v} dt d\vec{\xi} \\ dQ &= Ne dv \\ \vec{j} &= Ne \vec{v} = \rho \vec{v} \\ \rho &= Ne \end{aligned} \right\} dQ = \vec{j} d\vec{\xi} dt$$

$$\frac{dQ}{dt} = \vec{j} d\vec{\xi}$$

N = densità di particelle

ρ = densità di carica

Ne = densità di carica

\vec{j} = densità di corrente

dQ = carica nel volume dv

$$\vec{j} = \rho \vec{v}$$



$i = \oint_S (\vec{j} \cdot \vec{n})$

$i = \int_S \vec{j} \cdot d\vec{\xi}$

$$di = \vec{j} \cdot d\vec{\xi}$$

* se \vec{j} unif.

$\vec{j} \parallel \vec{n}$

$i = jS$

$j = \frac{i}{S}$

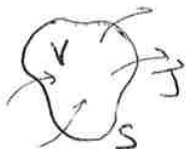


$dQ = \vec{j} \cdot d\vec{\xi} dt$

$dQ = di dt$

carica che attraversa $d\vec{\xi}$ in dt

i = intensità di corrente



\vec{j} positiva uscente

\vec{j} negativa dimmuisse

$$\Rightarrow i = - \frac{\partial q}{\partial t}$$

∂q = carica che attraversa la superficie finita S delimitante il volume V finito nell'intervallo ∂t = variazione di carica presente in V nel ∂t

4) Legge di Joule

Su ogni carica q si scende $\vec{F} = q\vec{E}$

durante lo spostamento di una carica $dW = \vec{F} d\vec{l}$

Scorre energia per fili circolari la corrente

→ Potenza spesa per fili circolari $1 e^-$

$$P_e = \frac{dW}{dt} = \frac{\vec{F} d\vec{l}}{dt} = \vec{F} \vec{v} = e\vec{E} \vec{v} \quad P_e = eE v \quad j = \sigma E$$

$$\sigma = \frac{1}{\rho} \cdot E = \rho j$$

→ Potenza spesa per fili circolari pl' e^- presenti nell'unità di volume (N el.)

$$P_N = N P_e = N e E v = N e E \frac{j}{N e} = E j = E \sigma E = \sigma E^2 = \rho j^2$$

→ Potenza per fili circolari pl' e^- nel volume del conduttore $dV = S dh$

$$dP = P_N dV = \rho j^2 S dh = \rho \frac{dh}{S} i^2 \quad \left(\frac{i}{S}\right)^2$$

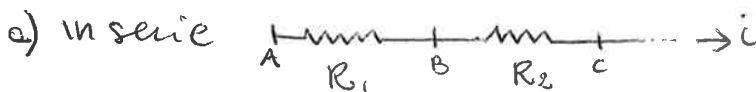
→ Potenza per fili circolari pl' e^- nel conduttore lungo h

$$P = \int dP = \rho \frac{i^2}{S} \int dh = \rho \frac{h}{S} i^2 \quad P = \rho \frac{h}{S} i^2 \Rightarrow \boxed{P = R i^2} \text{ legge Joule}$$

$$W = \int dW = \int \frac{dW}{dt} dt = \int_0^T P dt = R \int_0^T i^2 dt$$

$W =$ energia spesa nell'intervallo T per fili circolari corrente i

5) Collegamento fra resistenze

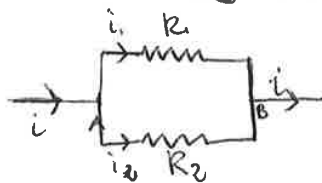


$$V_A - V_B = R_1 i$$

$$V_B - V_C = R_2 i$$

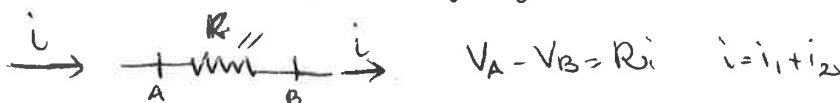
$$V_A - V_C = (R_1 + R_2) i \Rightarrow R_{AB} = R_1 + R_2$$

b) in parallelo



$$V_A - V_B = R_1 i_1 \Rightarrow i_1 = \frac{V_A - V_B}{R_1}$$

$$V_A - V_B = R_2 i_2 \Rightarrow i_2 = \frac{V_A - V_B}{R_2}$$



$$V_A - V_B = R i \quad i = i_1 + i_2$$

$$V_A - V_B = R i$$

$$V_A - V_B = R (i_1 + i_2)$$

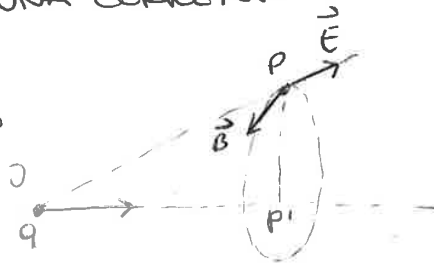
$$V_A - V_B = R \left(\frac{V_A - V_B}{R_1} + \frac{V_A - V_B}{R_2} \right)$$

$$1 = R \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

CAMPO MAGNETICO CREATO DA UNA CORRENTE

1) B creato da una carica in moto

$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \wedge \vec{r}}{r^3} \quad \vec{E} = \frac{1}{4\pi\epsilon_0} q \frac{\vec{r}}{r^3}$$



a) $\vec{B} = \mu_0 \epsilon_0 \vec{v} \wedge \left(\frac{1}{4\pi\epsilon_0} \frac{\vec{r}}{r^3} \right) = \mu_0 \epsilon_0 \vec{v} \wedge \vec{E}$

$$\mu_0 \epsilon_0 = \frac{1}{c^2}$$

$$\vec{B} = \frac{1}{c^2} \vec{v} \wedge \vec{E} \Rightarrow \vec{B} \perp \vec{E}$$

b) sull'asse di moto di q , il campo $\vec{B} = 0$

2) Legge Ampere - Laplace

$$\vec{B}_i = \frac{\mu_0}{4\pi} i \frac{d\vec{l} \wedge \vec{r}}{r^3}$$

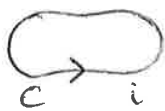
$$d\vec{B} = N dV B_i = N dl S \frac{\mu_0}{4\pi} i \frac{d\vec{l} \wedge \vec{r}}{r^3} = dl S \frac{\mu_0}{4\pi} \frac{Nq \vec{v} \wedge \vec{r}}{r^3} = dl S \frac{\mu_0}{4\pi} \frac{\vec{j} \wedge \vec{r}}{r^3} = \frac{\mu_0}{4\pi} S \frac{d\vec{j} \wedge \vec{r}}{r^3} =$$

$$= \frac{\mu_0}{4\pi} S \frac{j d\vec{l} \wedge \vec{r}}{r^3} = \frac{\mu_0}{4\pi} i \frac{d\vec{l} \wedge \vec{r}}{r^3}$$

Conduttore finito $\vec{B} = \frac{\mu_0}{4\pi} i \int_C \frac{d\vec{l} \wedge \vec{r}}{r^3}$

ENERGIA MAGNETICA

1) Autoinduzione



la corrente i genera un campo $\vec{B} = \frac{\mu_0}{4\pi} i \oint_C \frac{d\vec{l} \wedge \vec{r}}{r^3}$

che investe il circuito stesso $\phi = \int_S \vec{B} \cdot d\vec{\sigma}$

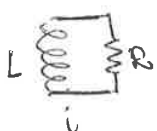
con conseguente generazione di una fem indotta nel circuito

$$\mathcal{E} = -\frac{\partial \phi}{\partial t} \quad \mathcal{E} = -\int_S d\vec{\sigma} \frac{\mu_0}{4\pi} \oint_C \frac{d\vec{l} \wedge \vec{r}}{r^3} \frac{di}{dt}$$

$$L = \frac{\mu_0}{4\pi} \int_S d\vec{\sigma} \oint_C \frac{d\vec{l} \wedge \vec{r}}{r^3} \quad \text{coefficiente di autoinduzione o induttanza}$$

$$L = \frac{\phi}{i} \quad \mathcal{E} = -L \frac{di}{dt}$$

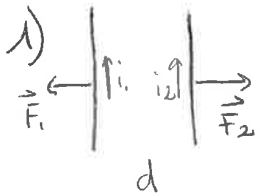
2) esempio del solenoide: calcolo di L $B = \mu_0 n i$



$$\phi = SB = NS \mu_0 n i = n^2 \mu_0 l S i = \mu_0 n^2 V i = L i \Rightarrow \phi = L i$$

$$\mathcal{E} = -\frac{\partial \phi}{\partial t} = -\mu_0 n^2 l S \frac{di}{dt} \quad \left. \begin{array}{l} L = \mu_0 n^2 l S = \mu_0 n^2 V \\ \mathcal{E} = -L \frac{di}{dt} \end{array} \right\}$$

FORZA MAGNETICA



1) i_1 crea \vec{B}_1
 i_2 immersa in \vec{B}_1 sente la forza \vec{F}_2
 e viceversa

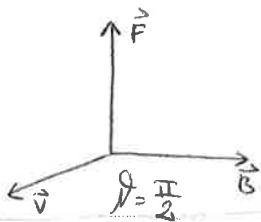
2) Forza magnetica su una carica in moto quando è in presenza di un campo \vec{B}

$|\vec{F} = q \vec{v} \wedge \vec{B}|$

a) $F = qvB \sin \vartheta$ $\vec{F} \perp \vec{v}, \vec{B}$

b) F massima se $\vartheta = \frac{\pi}{2}$

c) F nulla se $\vartheta = 0, \pi$ $\vec{v} \parallel \vec{B}$ $\vec{v} \parallel \vec{B}$

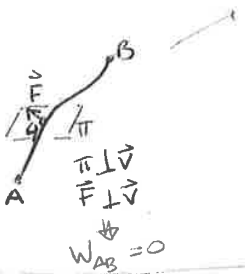


d) F non compie lavoro

$d\vec{l} : dW = \vec{F} d\vec{l}$ $W = \int_{AB} \vec{F} d\vec{l}$

Forza magnetica

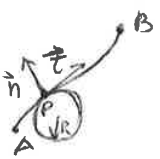
$W = \int_{AB} \vec{F} d\vec{l} = \int_{AB} q(\vec{v} \wedge \vec{B}) d\vec{l} = q \int_{AB} (\vec{v} \wedge \vec{B}) \frac{d\vec{l}}{dt} dt = q \int_{AB} (\vec{v} \wedge \vec{B}) \vec{v} dt = 0$



e) $W_{AB} = 0$

teorema energia cinetica $K_B - K_A = W_{AB}$

$K_B - K_A = 0 \Rightarrow K_B = K_A \Rightarrow v_B = v_A$



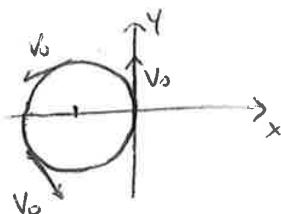
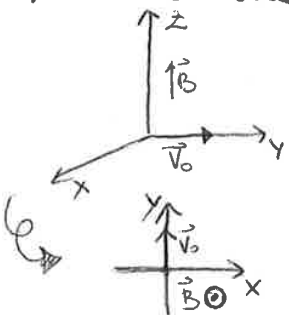
3) moto di una carica in un campo magnetico

$\vec{B} = B\vec{k}$

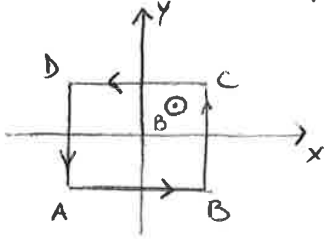
all'istante $t=0$ $\vec{v}_0 = v_0 \vec{j}$

$\vec{F} = m\vec{a}$ in altre coord $\begin{cases} \vec{v} = v\vec{e} \\ \vec{a} = a_t\vec{e} + a_n\vec{n} \\ a_t = \frac{dv}{dt} \\ a_n = \frac{v^2}{R} \end{cases}$

$\begin{cases} F_n = ma_n \\ F_t = ma_t \end{cases} \Rightarrow \begin{cases} qvB = m \frac{v^2}{R} \\ 0 = m \frac{dv}{dt} \end{cases} \Rightarrow \begin{cases} v = \frac{qBR}{m} \\ v = \text{cost} \end{cases} \Rightarrow \begin{cases} v = v_0 \\ R = \frac{mv_0}{qB} \end{cases}$



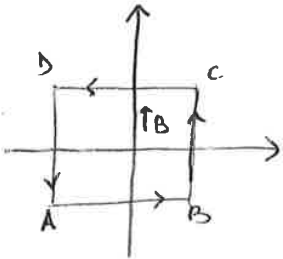
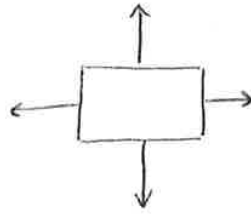
g) Spina rettangolare immersa in un campo B uniforme



$$\vec{B} = B \hat{u}_z$$

$$\vec{F}_{AB} = i \vec{l}_{AB} \wedge \vec{B} = i l_{AB} \hat{u}_x \wedge B \hat{u}_z = i l_{AB} B \hat{u}_x \wedge \hat{u}_z$$

$$\begin{cases} \vec{F}_{AB} = -i l_{AB} B \hat{u}_y \\ \vec{F}_{CD} = i l_{AB} B \hat{u}_y \\ \vec{F}_{BC} = i l_{BC} B \hat{u}_x \\ \vec{F}_{DA} = -i l_{BC} B \hat{u}_x \end{cases}$$

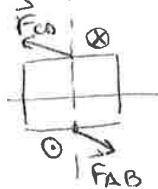


$$\vec{F}_{AB} = i \vec{l}_{AB} \wedge \vec{B} = i l_{AB} \hat{u}_x \wedge B \hat{u}_y = i l_{AB} B \hat{u}_z$$

$$\vec{F}_{CD} = -i l_{AB} B \hat{u}_z$$

$$\vec{F}_{BC} = 0$$

$$\vec{F}_{DA} = 0$$



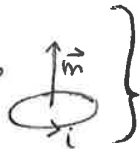
Momento meccanico \Rightarrow rotazione attorno x

$$\vec{M} = \vec{r}_{BC} \wedge \vec{F}_{CD} = l_{BC} \hat{u}_y \wedge (-i l_{AB} B \hat{u}_z) = i l_{AB} l_{BC} B \hat{u}_z \wedge \hat{u}_y =$$

$$= i (l_{AB} l_{BC} \hat{u}_z) \wedge B \hat{u}_y = i \vec{S} \wedge \vec{B}$$

$\vec{S} = S \hat{n}$ area vettore

$\vec{m} = i S \hat{n}$ momento magnetico delle spine



$$\vec{m} = i \vec{S}$$

$$\vec{M} = \vec{m} \wedge \vec{B}$$

$\vec{M} = 0 \Rightarrow \vec{m} \parallel \vec{B}$ $\varphi = 0$ stabile
 $\varphi = \pi$ instabile

se $B \parallel x \Rightarrow$ rotazione attorno y $\vec{M} = \vec{m} \wedge \vec{B}$

Spina in un campo elettrico \Rightarrow spostamento

Energia potenziale U $M = -\frac{dU}{d\varphi}$ $F = -\frac{dU}{dx}$

$$\vec{M} = \vec{m} \wedge \vec{B} \quad M = m B \sin \varphi$$

$$U = -\vec{m} \cdot \vec{B} \quad U = -m B \cos \varphi$$

Dipolo Elettrico		$\vec{M} = \vec{p} \wedge \vec{E}$	$U = -\vec{p} \cdot \vec{E}$
Dipolo magnetico		$\vec{M} = \vec{m} \wedge \vec{B}$	$U = -\vec{m} \cdot \vec{B}$

Esempio Solenoide

1) $B(t) = \mu_0 n i(t) = \mu_0 \frac{N}{l} i(t)$

2) $\Phi(B) = N S B(t) = N \pi r^2 B(t) = N \pi r^2 \mu_0 \frac{N}{l} i(t) = \mu_0 \frac{N^2 \pi r^2}{l} i(t)$

$E(t) = - \frac{\partial \Phi(B)}{\partial t} = - \mu_0 \frac{N^2 \pi r^2}{l} \frac{di(t)}{dt}$

3) $i_2(t) = \frac{E(t)}{R} = - \frac{\mu_0}{R} \frac{N^2 \pi r^2}{l} \frac{di(t)}{dt}$

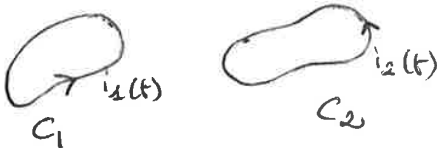
$L = \frac{\mu_0}{R} \frac{\pi r^2 N^2}{l}$

La $i_2(t)$ si oppone alla variazione di $i_1(t)$

$i_2(t) = -L \frac{di_1(t)}{dt}$

Induttanza del circuito dipende da $\frac{\mu_0}{R}$ come fattore moltiplicativo, un secondo fattore è tipico del circuito

Mutua induzione



1) $i_1(t) \Rightarrow \vec{B}_1(t) = i_1(t) \vec{F}_1(t, \vec{r})$

2) $\vec{B}_1(t)$ investe $C_2 \Rightarrow \Phi_2(\vec{B}_1(t)) = i_1(t) \int_{1,2}$

3) $E_2(t) = - \frac{\partial \Phi_2}{\partial t} = - \int_{1,2} \frac{\partial i_1(t)}{\partial t}$

4) $i_{2M} = \frac{E_2(t)}{R_2} = - \frac{\int_{1,2}}{R_2} \frac{\partial i_1(t)}{\partial t} = - M_{1,2} \frac{\partial i_1(t)}{\partial t}$

$i_{2M} = - M_{1,2} \frac{\partial i_1(t)}{\partial t}$

$M_{1,2}$ = mutua induttanza

5) $i_{1M} = - M_{2,1} \frac{\partial i_2(t)}{\partial t}$

6) $C_1 : i_1(t) \quad i_{1M}(t) \quad i_{1L}(t)$

$C_2 : i_2(t) \quad i_{2M}(t) \quad i_{2L}(t)$

TEOREMA STOKES



$$\oint_{\Gamma} \vec{A} d\vec{\ell} = \oint_{\Sigma} \vec{\nabla} \wedge \vec{A} d\vec{\sigma}$$

$$i = \oint_{\Sigma} \vec{J} d\vec{\sigma} \text{ definizione}$$

Legge Ampere - Maxwell $\oint_{\Gamma} \vec{B} d\vec{\ell} = \mu_0 i$ (A-M integrale)

$$\oint_{\Gamma} \vec{B} d\vec{\ell} = \mu_0 \int_{\Sigma} \vec{J} d\vec{\sigma}$$



Stokes

$$\int_{\Sigma} \vec{\nabla} \wedge \vec{B} d\vec{\sigma} = \mu_0 \int_{\Sigma} \vec{J} d\vec{\sigma} \quad \int_{\Sigma} (\vec{\nabla} \wedge \vec{B} - \mu_0 \vec{J}) = 0 \Rightarrow \vec{\nabla} \wedge \vec{B} - \mu_0 \vec{J} = 0$$

$$\hookrightarrow \vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} \text{ (A-M differenziale)}$$

Condizioni dipendenti dal tempo - Legge di A-M?

A) $\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J}$

$\vec{\nabla} \cdot (\vec{\nabla} \wedge \vec{B}) = \mu_0 \vec{\nabla} \cdot \vec{J}$ divergenza

matematica: $\vec{\nabla} \cdot (\vec{\nabla} \wedge \vec{A}) = 0$

$0 = \mu_0 \vec{\nabla} \cdot \vec{J} \rightarrow \vec{\nabla} \cdot \vec{J} = 0$ Eq continuità $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$ condiz. dinamiche

$\vec{\nabla} \cdot \vec{J} = 0$ condiz. stazionarie

Conclusioni: legge A-M $\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J}$ vale in condiz. stazionarie

B) 1) Eq di continuità in condiz. dipendenti dal tempo

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

2) legge di Gauss

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho \Rightarrow \epsilon_0 \vec{\nabla} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{\partial \rho}{\partial t}$$

3) Combinando 1 e 2

$$\vec{\nabla} \cdot \vec{J} + \epsilon_0 \vec{\nabla} \cdot \frac{\partial \vec{E}}{\partial t} = 0 \rightarrow \vec{\nabla} \cdot \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) = 0 \rightarrow \vec{\nabla} \cdot \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) = \vec{\nabla} \cdot (\vec{\nabla} \wedge \vec{A})$$

$$\rightarrow \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \vec{\nabla} \wedge \vec{A} \rightarrow \vec{\nabla} \wedge \mu_0 \vec{A} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow \vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

ONDE ELETTROMAGNETICHE

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = 0 & (1) \text{ Gauss} \\ \vec{\nabla} \cdot \vec{B} = 0 & (2) \\ \vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t} & (3) \text{ Faraday} \\ \vec{\nabla} \wedge \vec{B} = \epsilon\mu \frac{\partial \vec{E}}{\partial t} & (4) \text{ Ampere} \end{cases}$$

non a caso sospesi

1) Soluzioni piane $\vec{E}(x,t)$ $\vec{B}(x,t)$

$$\frac{\partial E_x}{\partial x} = 0 \quad (1)$$

$$\frac{\partial B_x}{\partial x} = 0 \quad (2)$$

$$\frac{\partial B_x}{\partial t} = 0$$

$$\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x}$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}$$

(3)

$$\left. \begin{matrix} (1) \\ (2) \end{matrix} \right\} \Rightarrow E_x = \cos t \Rightarrow \text{Campo creato da una distribuzione di cariche piane}$$

$\Rightarrow E_x = \cos t \Rightarrow$ Campo creato da una distribuzione di cariche piane

$$\Downarrow \\ E_x = 0$$

$$\frac{\partial E_x}{\partial t} = 0$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\epsilon\mu} \frac{\partial B_z}{\partial x}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\epsilon\mu} \frac{\partial B_y}{\partial x}$$

(4)

$$\left. \begin{matrix} (3) \\ (4) \end{matrix} \right\} \Rightarrow B_x = 0$$

2) ? E_y, E_z, B_y, B_z

$$\frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t} \quad (a) \text{ F}$$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad (b) \text{ F}$$

$$\frac{\partial E_z}{\partial t} = +\frac{1}{\epsilon\mu} \frac{\partial B_y}{\partial x} \quad (c) \text{ A}$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\epsilon\mu} \frac{\partial B_z}{\partial x} \quad (d) \text{ A}$$

$$3) \frac{\partial}{\partial t} (a) \Rightarrow \frac{\partial^2 E_z}{\partial t \partial x} = \frac{\partial^2 B_y}{\partial t^2} \Rightarrow \frac{\partial^2 B_y}{\partial x^2} = \epsilon\mu \frac{\partial^2 B_y}{\partial t^2}$$

$$\frac{\partial}{\partial x} (c) \Rightarrow \frac{\partial^2 E_z}{\partial x \partial t} = \frac{1}{\epsilon\mu} \frac{\partial^2 B_y}{\partial x^2} \Rightarrow \frac{\partial^2 B_y}{\partial x^2} = \epsilon\mu \frac{\partial^2 B_y}{\partial t^2}$$

$$\left. \begin{matrix} \frac{\partial}{\partial t} (b) \\ \frac{\partial}{\partial x} (d) \end{matrix} \right\} \Rightarrow \frac{\partial^2 B_z}{\partial x^2} = \epsilon\mu \frac{\partial^2 B_z}{\partial t^2}$$

$$\left. \begin{matrix} \frac{\partial}{\partial x} (a) \\ \frac{\partial}{\partial t} (c) \end{matrix} \right\} \Rightarrow \frac{\partial^2 E_z}{\partial x^2} = \epsilon\mu \frac{\partial^2 E_z}{\partial t^2}$$

$$\left. \begin{matrix} \frac{\partial}{\partial x} (b) \\ \frac{\partial}{\partial t} (d) \end{matrix} \right\} \Rightarrow \frac{\partial^2 E_y}{\partial x^2} = \epsilon\mu \frac{\partial^2 E_y}{\partial t^2}$$

(17)

7) $\vec{E} = E_y(x-vt)\hat{u}_y + E_z(x-vt)\hat{u}_z$
 $\vec{B} = B_y(x-vt)\hat{u}_y + B_z(x-vt)\hat{u}_z$

misura del campo all'istante $t=0$
 sconosciuta ad ogni istante $t>0$

8) $u = x - vt \quad \frac{\partial u}{\partial x} = 1 \quad \frac{\partial u}{\partial t} = -v \quad dt = -\frac{1}{v} du$

$\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x} \quad (a) \quad = \frac{\partial u}{\partial x} \frac{\partial E_z}{\partial u} = \frac{\partial E_z}{\partial u}$

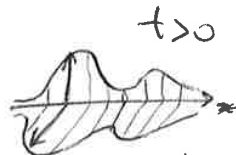
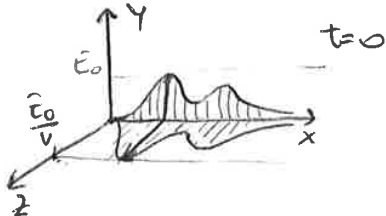
$B_y = \int \frac{\partial B_y}{\partial t} dt = \int \frac{\partial E_z}{\partial u} dt = -\frac{1}{v} \int \frac{\partial E_z}{\partial u} du = -\frac{1}{v} \int du E_z = -\frac{E_z}{v} + const = 0$

$B_y = -\frac{E_z}{v} \quad (A) \quad \text{prendendo da (a)} \rightarrow B_z = +\frac{E_y}{v} \quad (B)$

9) $\vec{E} = E_y(x-vt)\hat{u}_y + E_z(x-vt)\hat{u}_z$
 $\vec{B} = -\frac{1}{v}E_z(x-vt)\hat{u}_y + \frac{1}{v}E_y(x-vt)\hat{u}_z$

① $B^2 = \frac{E^2}{v^2} \rightarrow E = vB \quad \text{② } \vec{E} \cdot \vec{B} = 0 \rightarrow \vec{E} \perp \vec{B} \quad \text{③ } \vec{E} \wedge \vec{B} = \frac{E^2}{v} \hat{u}_x = vB^2 \hat{u}_x = EB \hat{u}_x$

10) $t=0 \quad \vec{E} = E(x)\hat{u}_y \quad \vec{B} \perp \vec{E} \quad B = \frac{1}{v}E \quad \vec{B} = \frac{1}{v}E(x)\hat{u}_z$
 $\begin{cases} \vec{E} = E(x-vt)\hat{u}_y \\ \vec{B} = \frac{1}{v}E(x-vt)\hat{u}_z \end{cases}$



onda che si propaga verso destra con v

11) caso in cui \vec{E} ha due componenti (y, z)

12) Onde armoniche

$t=0 \quad \vec{E} = E_{0y} \sin(kx)\hat{u}_y + E_{0z} \sin(kx + \delta)\hat{u}_z$
 $t>0 \quad \vec{E} = E_{0y} \sin(kx - \omega t)\hat{u}_y + E_{0z} \sin(kx - \omega t + \delta)\hat{u}_z$



$\omega = kv \rightarrow$ pulsazione
 $k =$ num. d'onda, vett. d'onda
 $\lambda = \frac{2\pi}{k}$ lunghezza d'onda
 $T = \frac{2\pi}{\omega}$ periodo
 $\lambda = vT \quad v = \frac{1}{T}$ frequenza
 $\lambda v = v \rightarrow v = \frac{\lambda}{T}$

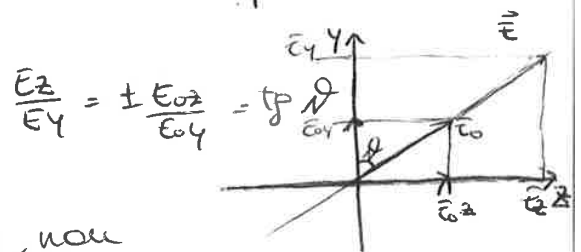
13) Polarizzazione onde armoniche

A) $\delta = 0 \quad \delta = \pi$ (polarizzazione rettilinea)

$\vec{E} = E_{0y} \sin(kx - \omega t)\hat{u}_y \pm E_{0z} \sin(kx - \omega t)\hat{u}_z$

rapporto \Rightarrow costante nel tempo

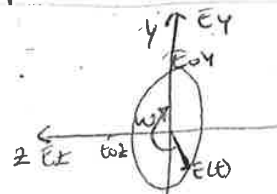
Il campo \vec{E} oscilla rimanendo sulla retta, non cambia direzione, verso si. Il campo \vec{B} è $\perp \vec{E}$



B) $\delta = \frac{\pi}{2} \quad \delta = \frac{3\pi}{2}$ (polarizzazione ellittica)

$\vec{E} = E_{0y} \sin(kx - \omega t)\hat{u}_y \pm E_{0z} \cos(kx - \omega t)\hat{u}_z$

eq. pol. ellisse $\begin{cases} E_y = E_{0y} \sin(kx - \omega t) \\ E_z = \pm E_{0z} \cos(kx - \omega t) \end{cases}$ eq. circ. ellisse $\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_z}{E_{0z}}\right)^2 = 1$



\vec{E} descrive nel tempo un'ellisse di semiassi E_{0y}, E_{0z} . Ruota con velocità angolare ω

$E_{0y} = E_{0z} \rightarrow$ ellisse \rightarrow circonferenza \Rightarrow polarizzazione circolare

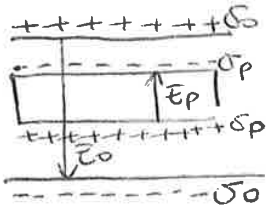
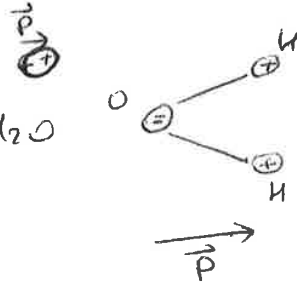
c) Onde non polarizzate per $\delta \neq 0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}$

Materiali dielettrici

In un campo elettrico si polarizzano

a) polarizzazione elettrica dell'atomo

b) polarizzazione per orientamento es. H₂O



1) cost dielettrica relativa

$$\epsilon_r = k = \frac{V_0}{V_k} > 1$$

$$\left. \begin{aligned} V_0 &= E_0 h \\ V_k &= E_k h \end{aligned} \right\} \Rightarrow k = \frac{E_0}{E_k} = \frac{\epsilon_0}{\epsilon_0 - \epsilon_p} > 1$$

2) cost dielettrica

$$\epsilon = k \epsilon_0 \quad \epsilon > \epsilon_0$$

$$E_k = E_0 - E_p$$

3) Suscettività elettrica

$$\chi = k - 1$$

$$V_k = \frac{1}{k} V_0$$

$$E_k = \frac{1}{k} E_0$$

$$C_k = k C_0$$

$$4) C_k = \frac{Q_0}{V_k} = \frac{Q_0}{E_k h} = \frac{Q_0}{\frac{E_0}{k} h} = k \frac{Q_0}{E_0 h} = k \frac{Q_0}{V_0} \Rightarrow$$

5) Induzione dielettrica

$$\vec{D} = \epsilon \vec{E}$$

6) Th Gauss

$$\int \phi(\vec{r}) = q$$

$$\int \vec{\nabla} \cdot \vec{D} = \rho$$

7) Energia elettrostatica

$$u = \frac{1}{2} \epsilon^2 \epsilon \quad U = \int_V u \, dV$$

8) metallo $E_k = 0 \rightarrow k = \infty \rightarrow \epsilon = \infty$