NUMERO: 1618A - ANNO: 2015

A P P U N T I

STUDENTE: Botta

MATERIA: Dynamic Of Electrical Machine. Prof.Boglietti

DYNAMICS OF ELECTRICAL MACHINES

1. MAGNETIC FIELD PROPERTIES

The main quantities of the magnetic field are the flux density B[T] and the magnetic field H[A/m].
These quantities are related by the following relation:

$B = \mu H = \mu_r \mu_0 H$ with $\mu_0 = 4\pi \cdot 10^{-7} \frac{H}{m}$ vacuum permeability

μ_r relative permeability

The magnetic field can be studied starting from the Maxwell equations, in particular the 2nd and the 4th:

2nd: $\mathrm{div} B = 0$ B is solenoidal

4th: $\mathrm{rot} H = J$ J current density $[A/m^2]$

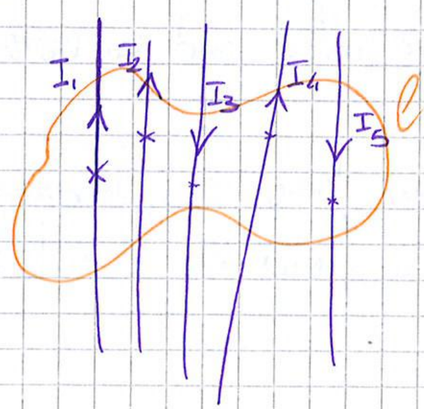
Appling Stokes' theorem to the 4th we get:

$\oint_\ell H \cdot dl = \int_S J \cdot ds$

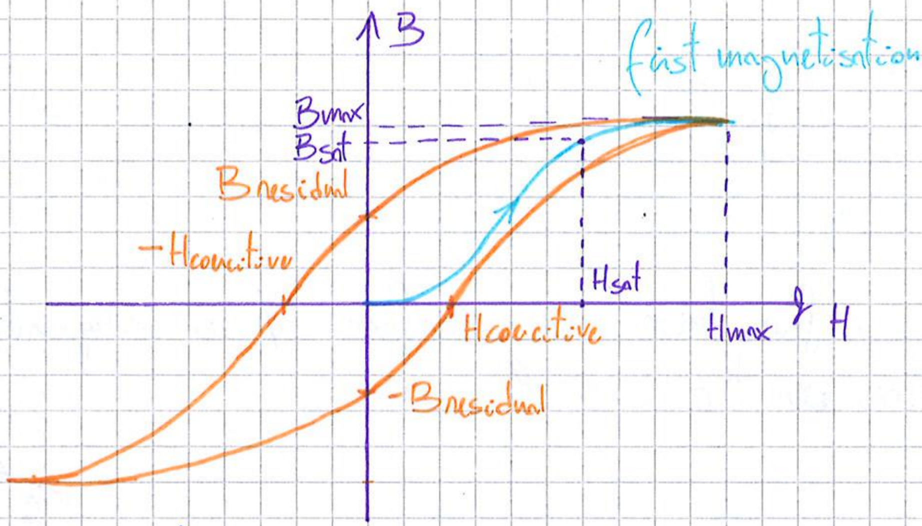
The integral $\int_S J \cdot ds$ is the current inside the surface S,
since currents are always inside a conductor we can simplify the integral to a finite sum:

$\int_S J \cdot ds = \sum_i^N I_i = NI$

usually equal currents



N: n° of linked wire



Material with a little cycle (little H_c and B_r) are used as electromagnet since is easier to magnetize and smagnetize. Bigger cycle instead are better for permanent magnet.

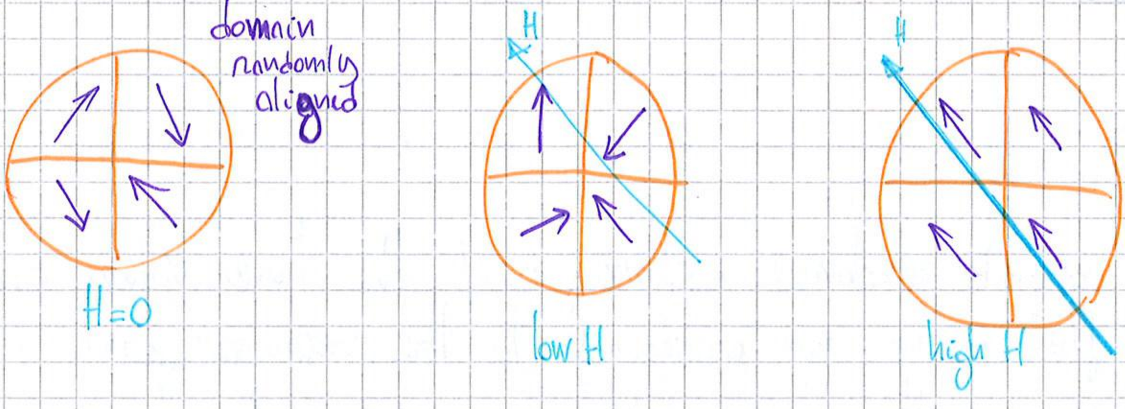
The area inside the loop represents the power losses due to hysteresis.

$$P_{hyst} = K_{hyst} \, f \, B_{max}^{x} \qquad x = 1.6 \div 2$$

frequency

material constant

Hysteresis losses can be seen as a magnetic friction in the rotation of the magnetic domain:



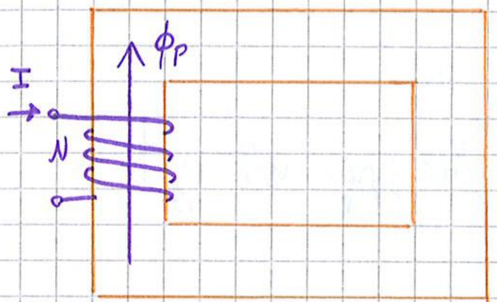
3. RELUCTANCE

Let's take the 4th equation in integral form:

$$\oint_l H \times dl = NI$$

4. LINKED FLUX, SELF AND MUTUAL INDUCTACE

Given the following circuit we can define the linked flux ϕ_L as:

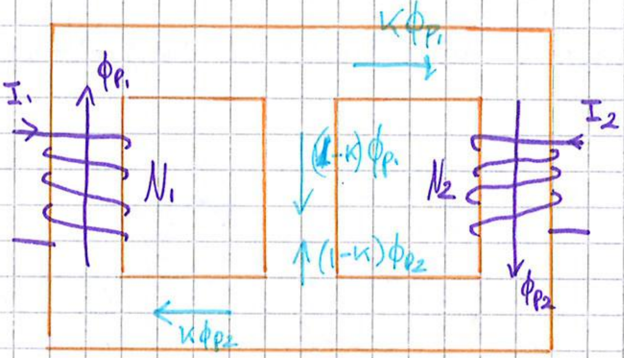


produced flux $\phi_P = \frac{N}{\Re} I$

linked flux with N turns: $\phi_L \equiv N\phi_P$

$\phi_L = \frac{N^2}{\Re} I = L I$ $\quad$ L (self) inductance [H]

If we have 2 windings on the circuit they mutually induce themself:



if $I_2 = 0$ and $I_1 \neq 0$

$\phi_{P1} = \frac{N_1 I_1}{\Re}$ $\quad$ not all ϕ_{P1} is linked $K = 0 \div 1$

$\phi_{L2} = K N_2 \phi_{P1} = K \frac{N_2 N_1}{\Re} I_1 = M_{21} I_1$

$\phi_{L2} = \phi_{21}$ $\quad$ linked to, produced by

if $I_1 = 0$ and $I_2 \neq 0$

$\phi_{12} = \phi_{L1} = N_1 K \phi_2 = K \frac{N_1 N_2}{\Re} I_2 = M_{12} I_2$

M_{ij} mutual inductance [H]

It can be demonstrated that $M_{12} = M_{21} = M$. The mutual inductance is a signed value, if the produced fluxes are concord M is positive, viceversa M is negative.

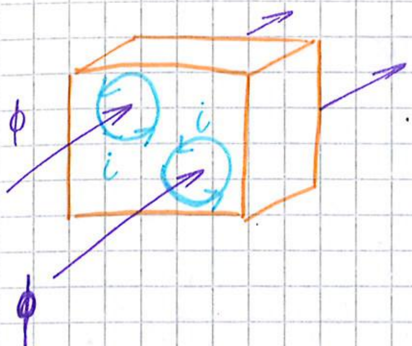
It's important to note L is always positive.

Superposing self and mutual inductance we get:

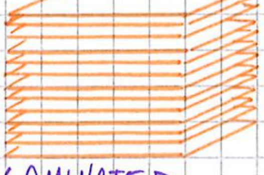
$$\begin{cases} \phi_1 = L_1 I_1 + M I_2 \\ \phi_2 = L_2 I_2 + M I_1 \end{cases} \qquad \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

linked flux $\qquad$ inductance matrix

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 7 di 78



Since most ferromagnetic materials are conductive a flux passing throught them produces an emf and a current. These currents are called eddy currents and they dissipate energy throught joule effect.



LAMINATED BLOCK

To reduce losses we have to increase the electrical resistence. We can add some Si (no more than 6%) to the iron but we increase its fragility. A better improvement is to use a laminated block so we have a long lenght and a small area (higher resistance).

$$P_{joule} = \frac{V^2}{R_{eq}} \qquad R_{eq} = \rho \frac{l_{eq}}{S_{eq}}$$

Eddy currents losses can be computed as:

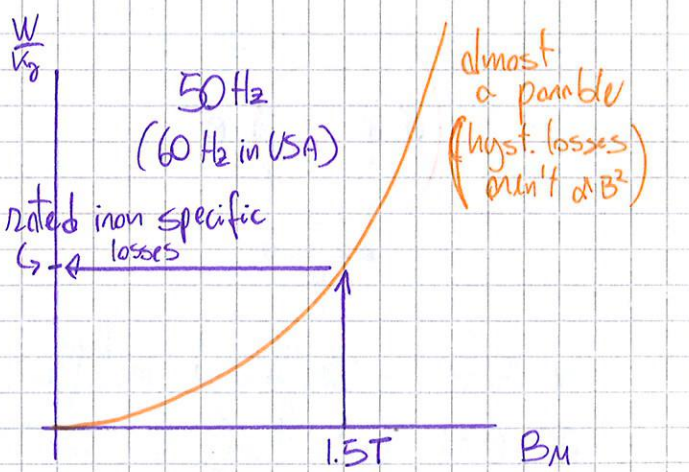
$$P_{eddy} = K_{eddy} f^2 B_M^2$$

The whole iron losses are:

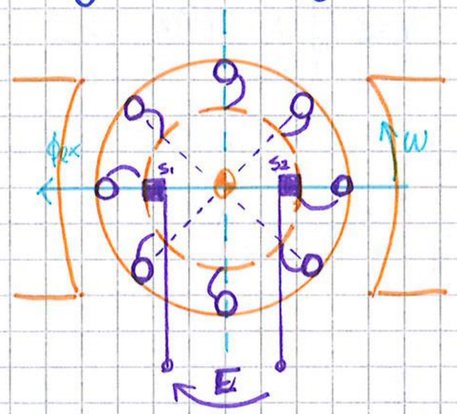
$$P_{Fe} = P_{hyst} + P_{eddy} = K_{hyst} f B_m^x + K_{eddy} f^2 B_m^2$$

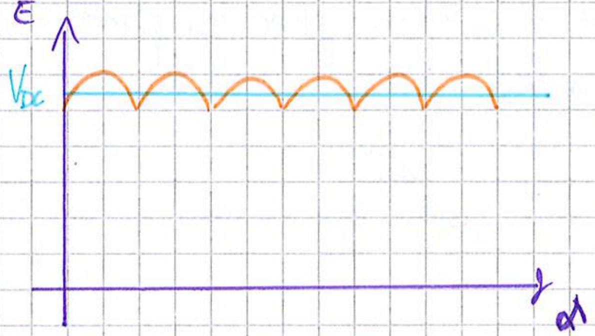
Lamination are classified using iron specific losses (cifra di perdita) $\left[\frac{W}{kg}\right]$.

- $8 \frac{W}{kg}$ ~ 0.65 mm low quality
- $2.4 \frac{W}{kg}$ ~ 0.50 mm common
- $1.3 \frac{W}{kg}$ ~ 0.35 mm high frequency application
- 0.18 ÷ 0.15 mm very high freq.



an emf always positive and therefore with a mean value different from zero, our DC voltage.
When $\alpha = \pm 90$ the machine is in commutation: the brushes touch both rings and make a short circuit, but since $emf(\alpha = \pm 90) = 0$ there isn't any current.
To use all the surface of the rotor and to reduce ripple of emf we add more coils, the 2 half rings become now a segmented ring.





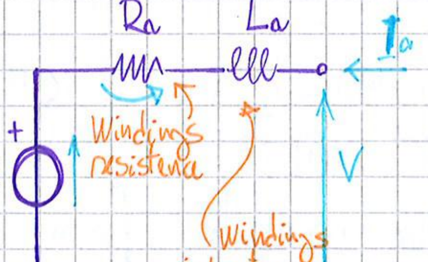
When we have more than 2 poles the coil winding can be connected in different ways

8. DC MACHINE MODEL

From the previous analisys we can say that the produced emf is equal to:

$$E = K_a \phi_{ex} \omega$$

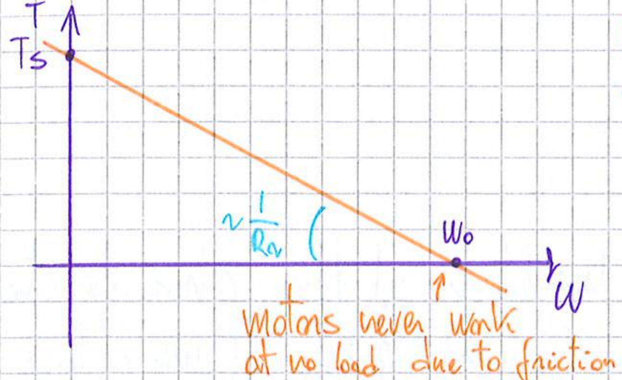
armature constant function of how the machine is built



motor configuration
(DC generator (dynamos) are rarely used nowadays)
can be neglected in steady state

Adding the armature resistence R_a and the supply we get the equivalent circuit. Its governing equations are:

the mechanical characteristic is a line with negative slope.
Knowing 2 points the line is known.



if $W = 0$

$T = T_S = \frac{K\phi_{ex} V}{R} = K\phi_{ex} I_S$

starting torque

starting current

if $T = 0$ ($I_a = 0 \rightarrow V = E$)

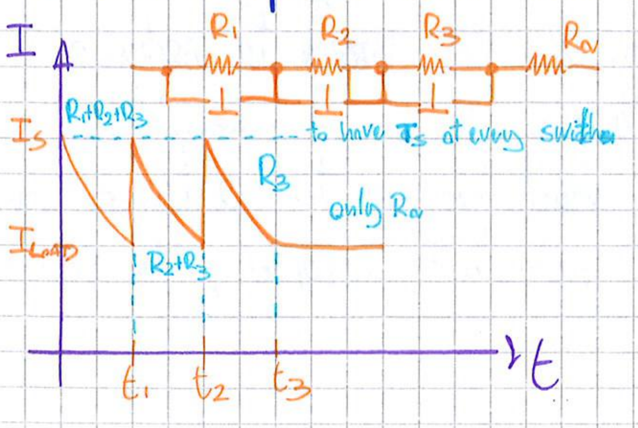
$W = W_0 = \frac{V}{K\phi_{ex}}$

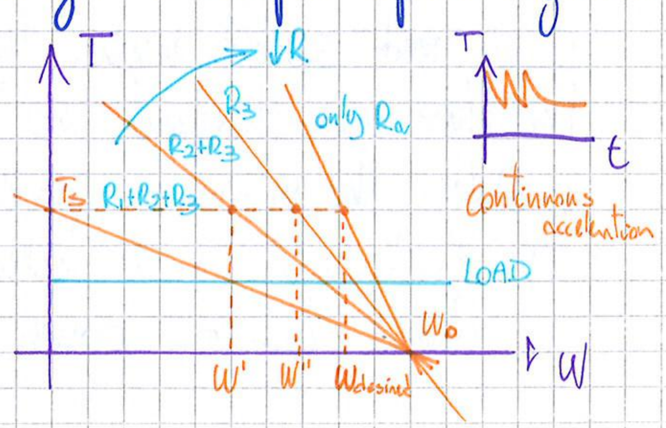
no load speed

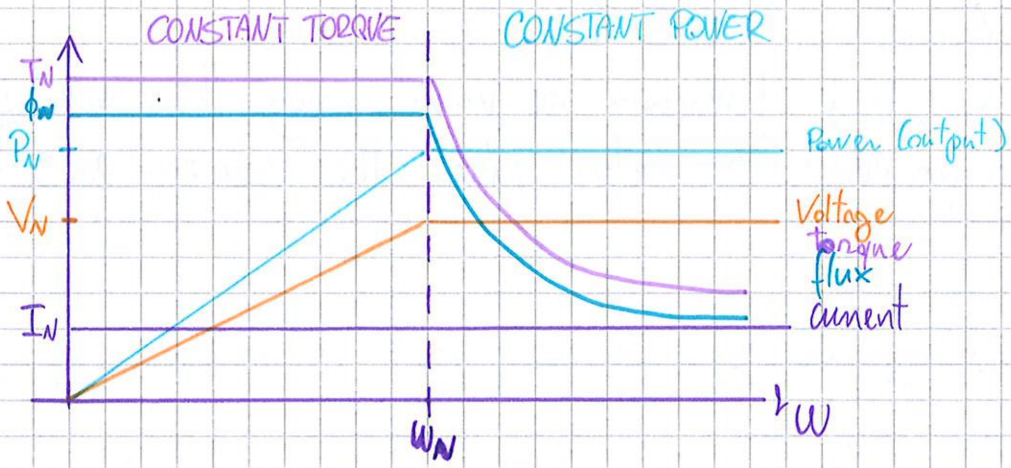
Since R_a is very little the line is very steep and therefore we have little change in speed changing the working point. The working point is given by crossing the motor and the load characteristics.

Again since R_a is little I_S is huge, as a short circuit current. This creates overheat issues and can produce sparks during the commutation.

To reduce the starting current if we don't use a variable voltage supply we can increase R_a adding more resistence in series. But torque is proportional to the current so the starting current can't be too little. So we add a series of resistence that can be short circuited, this reduce I_S to an acceptable value and we get a simple speed regulation



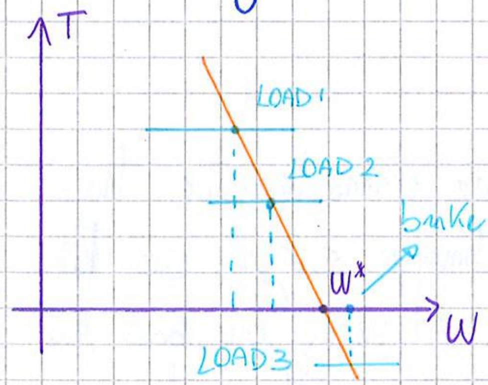


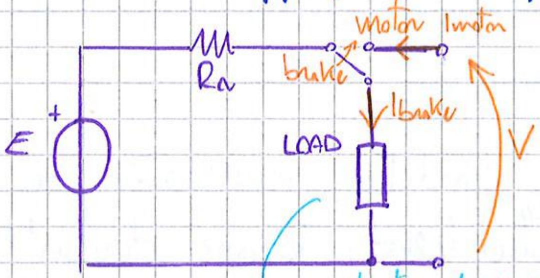


Permanent magnet machines have the same characteristic but they have constant flux so we can use only constant torque speed regulation.

10. WORKING QUADRANTS

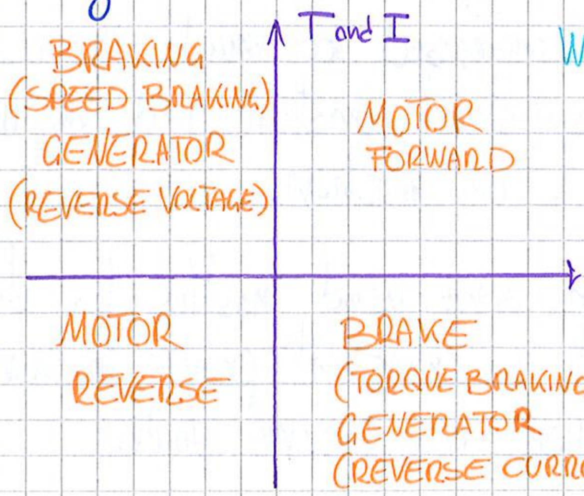
If the speed is increased over ω^* the motor works as a brake / generator. The transition is automatic: Torque and current become negative so we need an appropriate supply.





electrical energy produced by the braking must be used

We have 4 Working conditions



We need a proper supply to work in different conditions

This motor is also know as universal motor because it can work also in AC supply:

$T = K_{ex} I^2$

$I = I_x \sin \omega t \Rightarrow T = K I_x^2 \sin^2 \omega t$ ← always positive current, $T_{mean} \neq 0$

Unfortunelly AC machines have trouble with commutation

12. RATED VALUES OF A DC MACHINE

- V_N rated voltage: maximum voltage that can be applied without overheating issue.
- P_N rated power: output power in rated conditions. It's mechanical power for motors, electrical for generators.
- I_N rated current: maximum current that can be absorbed without overheating problems.
- $\eta = \frac{P_{OUT}}{P_{IN}} = \frac{P_N}{V_N I_N}$ rated efficency: for extend excitation we have the input power equal to $P_{in} = V_N I_N + R_{ex} I_{ex}^2$. Must be computed.
- n_N rated speed [rpm]: speed when the machine is in rated conditions.
- T_N rated torque $T_N = \frac{P_N}{2\pi n_N} 60$: torque in rated conditions. Must be computed.

13. LOSSES IN THE MACHINE AND SIZING

We have 3 kind of losses in a DC machine:

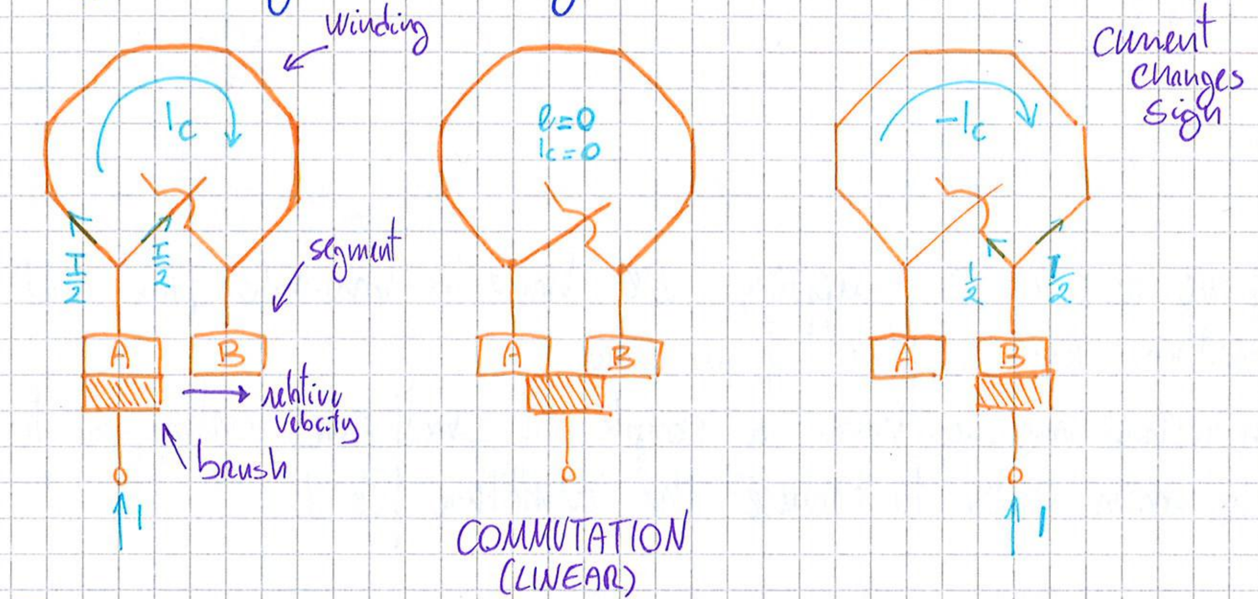
$$\begin{cases} P_{Je} = R_a I_a^2 & \text{armature joule losses} \\ P_{Fe} + P_{mech} = P_0 & \text{no load losses} \\ P_{ex} = R_{ex} I_{ex}^2 & \text{excitation joule losses} \end{cases}$$

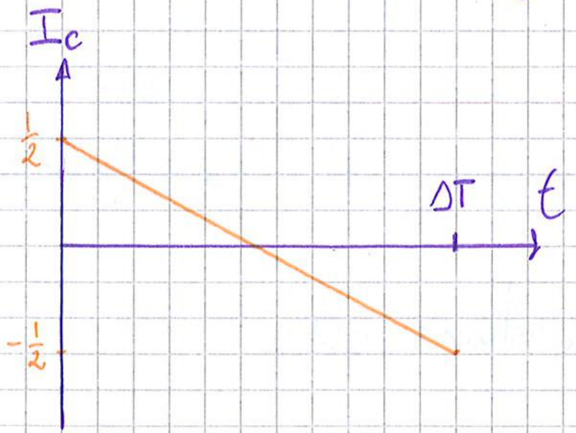
Losses produce heat on the machine. The insulating material

(l^2), so a longer machine can have overheating issues without the right coding system.

14. COMMUTATION

The machine is under commutation when a brush is touching 2 different segment closing the coil in short circuit.





If we suppose a linear commutation the emf is zero when we have the commutation and the current is proportional to the touching section.

In real cases emf is no zero and the commutation isn't linear. This is due to the coil self inductance:

$$e = -L_c \frac{dI_c}{dt} \neq 0$$

but also due to the rotation of the neutral axis. This rotation is produced by the current in the coils: it creates an armature reaction field and a corresponding flux.

1) $V_a(s) = R_a i_a(s) + sL_a i_a(s) + e(s)$
2) $e(s) = K_v W(s)$
3) $T_m = K_T i_a(s)$
4) $sJW(s) + \beta W(s) = T_m(s) - T_r(s)$

The current is:

$$I_a = \frac{V_a - E}{R_a + sL_a} \longrightarrow \frac{I_a}{V_a - E} = \frac{1}{R_a + sL_a}$$

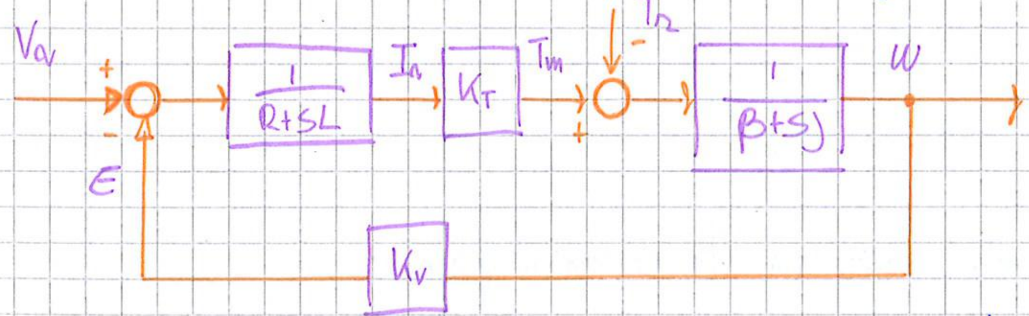
so the transfer function between torque and voltage difference is:

$$\frac{T_m}{V_a - E} = \frac{K_T}{R_a + sL_a}$$

the transfer function between speed and resulting torque is

$$\frac{W}{T_m - T_r} = \frac{1}{\beta + sJ}$$

Remembering that $\frac{E}{W} = K_v$, the block diagram is now completed:



At first we study the system with one input at time (V_a or T_r), then throught superposition we can obtain the total analisys.
If we consider the no load condition ($T_r = 0$), the armature voltage is the input, the speed is the output.

We can have 3 cases:

1) $\tau_m > 4\tau_a$ $(\zeta > 1)$ Aperiodic and damped answer, real and separate poles

2) $\tau_m = 4\tau_a$ $(\zeta = 1)$ Aperiodic and damped answer, real and coincident poles

3) $\tau_m < 4\tau_a$ $(\zeta < 1)$ Oscillating and damped answer complex pole pair

ζ ς α β γ δ ε ξ η ϑ ι κ λ μ ν ξ ο π e
σ τ υ φ χ ψ ω

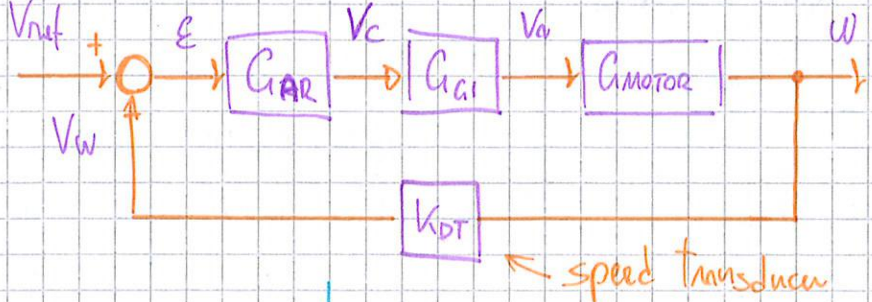
- 3.1) $2\tau_a < \tau_m < 4\tau_a$ $(0.707 < \zeta < 1)$ Oscillations with negligible amplitude
- 3.2) $0 < \tau_m < 2\tau_a$ $(0 < \zeta < 0.707)$ Underdamped oscillations
- 3.3) $\tau_m = 0$ $(\zeta = 0)$ not damped oscillations

Generally for small power motor we have $\tau_a < \tau_m$ $(\zeta > 1)$ and so a better step response. Instead for bigger power $\tau_a > \tau_m$ $(\zeta < 1)$ ~~and~~ so an oscillating step response.

Now we set $V_a = 0$ and analize the system with T_r as input. It's convinient to rearrange the block diagram to obtain a simpler one.

generator produces the impulses for the converter from the reference V_a^*.

The block diagram is the following:



$$G_{motor} = \frac{\frac{1}{K_v}}{1 + s\tau_m + s^2 \tau_a \tau_m}$$ as seen before

$$\simeq \frac{\frac{1}{K_v}}{(1+s\tau_a)(1+s\tau_m)}$$

$$G_{GI} = \underbrace{K_C}_{\text{converter gain}} \underbrace{\frac{1}{1+s\tau_y}}_{\text{delay}}$$

$G_{AR} = \frac{V_c}{\varepsilon}$ P | PI | PID controller if P $\Rightarrow$ $G_{AR} = K_R$

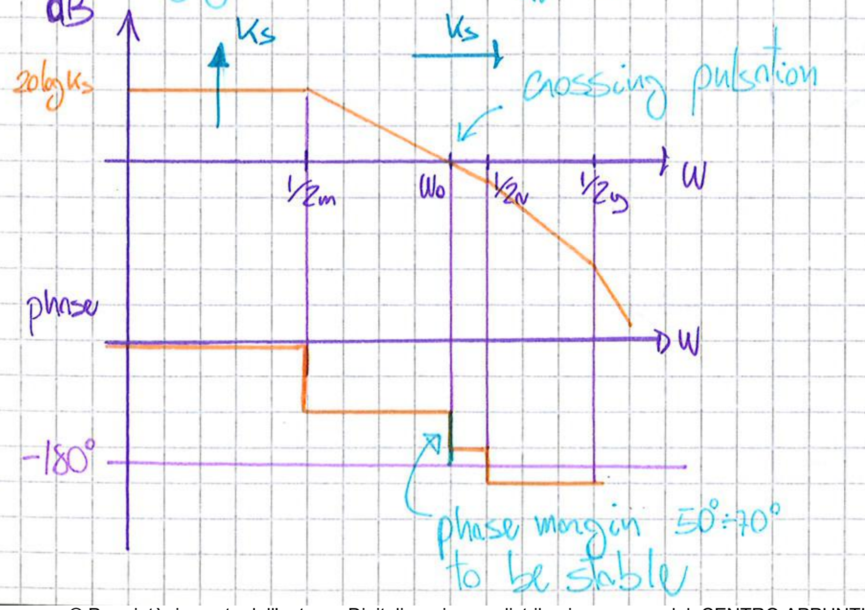
The direct transfer function is:

$$G_a(s) = K_R \frac{K_C}{1+s\tau_y} \frac{\frac{1}{K_v}}{(1+s\tau_a)(1+s\tau_m)} K_{DT}$$

0 type

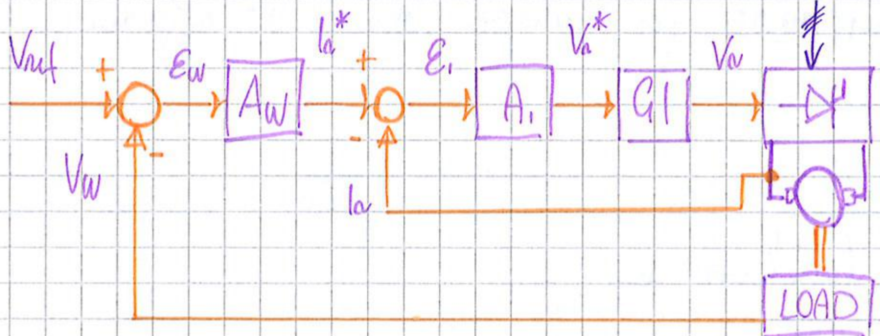
3rd order

steady gain $= K_s = K_R K_C \frac{1}{K_v} K_{DT}$

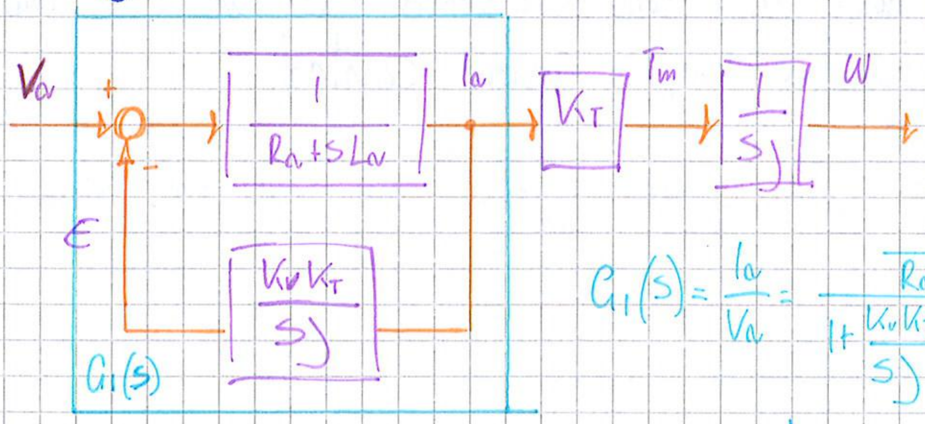


increasing K_s:
- ω_0 increases
- faster responses
- littler phase margin

The rise time is defined and the maximum overshoot too, these parameters are related to the step variation of the speed.
If the system is fast to respond, the absorbed current can be very high, it's advisable to limit the current to avoid overheating.
This limitation define the maximum motor torque and the acceleration.
A possible control system can be:



to build the block diagram ~~why~~ we have to do a little rearrangement to the motor block:

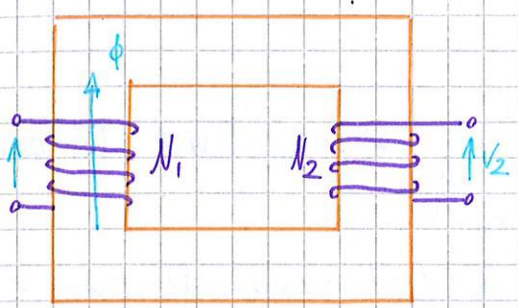


$$G_1(s)=\frac{I_a}{V_a}=\frac{\frac{1}{R_a+sL_a}}{1+\frac{K_vK_T}{sJ}\frac{1}{R_a+sL_a}}=$$

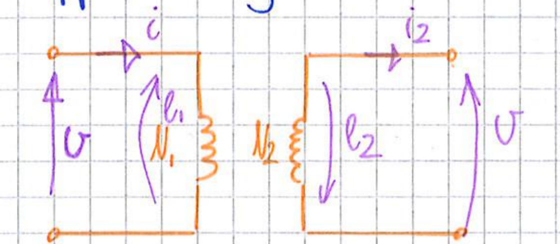
$$=\frac{\frac{1}{R_a\left(1+s\frac{L_a}{R_a}\right)}}{1+\frac{K_vK_T}{sJR_a}\frac{1}{1+s\frac{L_a}{R_a}}}=\frac{\frac{1}{R_a(1+s\tau_a)}}{1+\frac{1}{s\tau_m}\frac{1}{(1+s\tau_a)}}$$

$$G_1(s)=\frac{s\tau_m}{R_a(1+s\tau_m+s^2\tau_a\tau_m)}\cong\frac{s\tau_m}{R_a(1+s\tau_a)(1+s\tau_m)}$$

17. SINGLE PHASE TRANSFORMER



A transformer is a machine capable of converting electrical energy in electrical energy using the emf produced by a variable flux. To get a variable flux the main circuit (higher supply tension) is supplied by an AC source.



PRIMARY (LOAD CONVENTION)

SECONDARY (SOURCE CONVENTION)

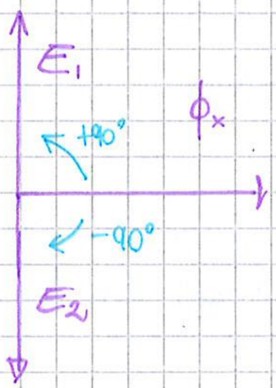
$\phi = \phi_x \sin\omega t$

load convention

$e_1 = +N_1 \frac{d}{dt}(\phi_x \sin\omega t) = +N_1 \phi_x \omega\cos\omega t$

$= N\phi_x \omega \sin(\omega t + 90°)$

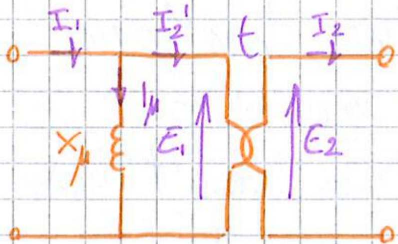
$e_2 = -N_2 \phi_x \omega t = N_2 \phi_x \omega \sin(\omega t - 90°)$



The transformer ratio is defined as:

$$t = \frac{|E_1|}{|E_2|} = \frac{N_1 \cancel{\frac{\phi_x}{\sqrt{2}}\omega}}{N_2 \cancel{\frac{\phi}{\sqrt{2}}\omega}} = \frac{N_1}{N_2}$$

To create the flux we need a mmf and therefore a current. This current is called magnetising current I_μ:



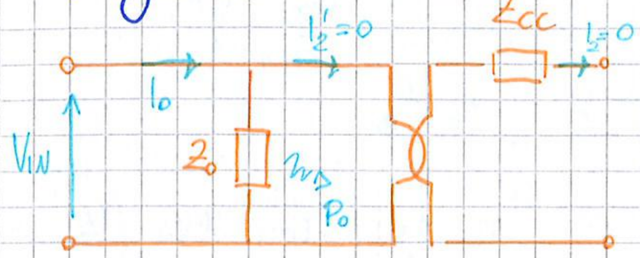
$\mathcal{R}\phi = N_1 I_\mu \rightarrow N_1^2 I_\mu = N\mathcal{R}\phi = \mathcal{R}\phi_L$

$\phi_L = \frac{N_1^2}{\mathcal{R}} I_\mu = L_\mu I_\mu$ model as an impedance

I_2' is I_2 moved to the primary and it's equal to:

$N_1 I_1 - N_2 I_2 = N_1 I_\mu \rightarrow I_1 - \frac{N_2}{N_1} I_2 = I_\mu \quad I_1 - \frac{1}{t} I_2 = I_\mu \quad I_1 - I_2' = I_\mu$

To obtain the equivalent circuit parameters some tests data are given:

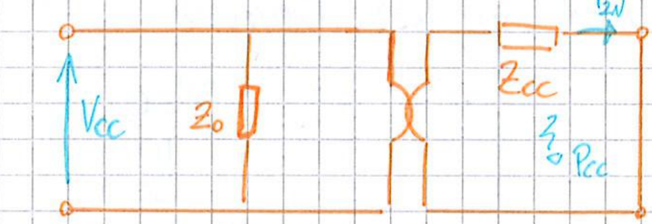


NO LOAD TEST

Measured value:

- V_{1N} rated primary voltage
- $P_0\%$ absorbed power $P_0\% = \frac{P_0}{A_N} \cdot 100$
- $I_0\%$ absorbed current $I_0\% = \frac{I_0}{I_N} \cdot 100$
- $\cos\varphi_0$ power factor

$$R_{Fe} = \frac{V_{1N}^2}{P_0} \qquad X_\mu = \frac{V_{1N}^2}{P_0 \tan\varphi_0}$$



SHORT CIRCUIT TEST

Measured value:

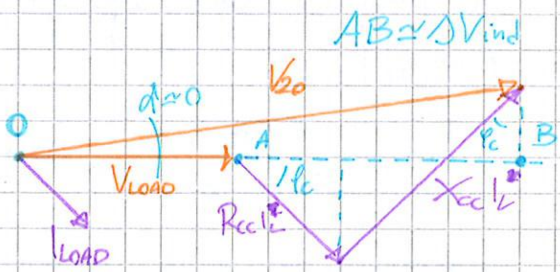
- V_{cc} short circuit voltage
- $P_{cc}\%$ absorbed power $P_{cc}\% = \frac{P_{cc}}{A_N} \cdot 100$
- I_{2N} rated secondary current
- $\cos\varphi_{cc}$ power factor

$$R_{cc} = \frac{P_{cc}}{I_{2N}^2} \qquad X_{cc} = \frac{P_{cc} \tan\varphi_{cc}}{I_{2N}^2}$$

Normatives state that the voltage drop on Z_{cc} can be computed in a simpler way:

$\Delta V_{real} = Z_{cc} I_{load}$

$\Delta V_{industrial} = I_{load} (R_{cc} \cos\varphi_{load} + X_{cc} \sin\varphi_{load})$



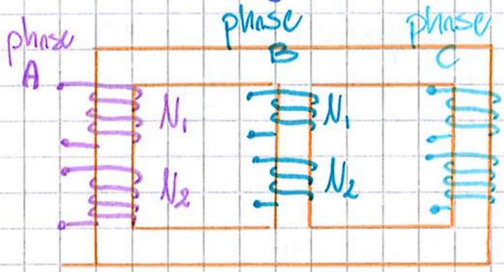
19. PARALLEL OF TRANSFORMERS

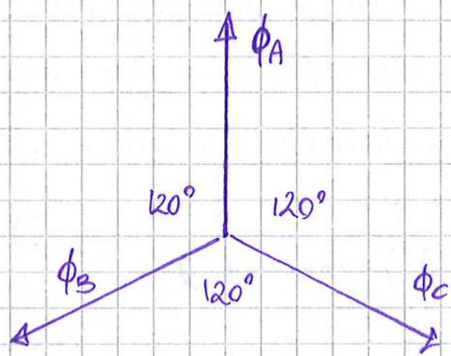
Sometimes is useful to use two transformer in parallel to get the requested power or to avoid problem for a faillure. For a good connection we have to minimize the circulating current:

20. 3 PHASES TRANSFORMER

We can obtain a 3 phases transformer connecting together 3 single phase transformers. This solution is mostly adopted in the US because the system still work if one transformer has a failure, but the whole machine costs a lot.

In EU it's used a three phase transformer like the following one:





Primary and secondary can be connected in delta or star connections. Depending on the connection combination, the transformation ratio can be different from the turns ratio:

Y/y $\quad t = t_{turns}$

Y/d $\quad t = \sqrt{3}\, t_{turns}$

D/d $\quad t = t_{turns}$

D/y $\quad t = \frac{1}{\sqrt{3}}\, t_{turns}$

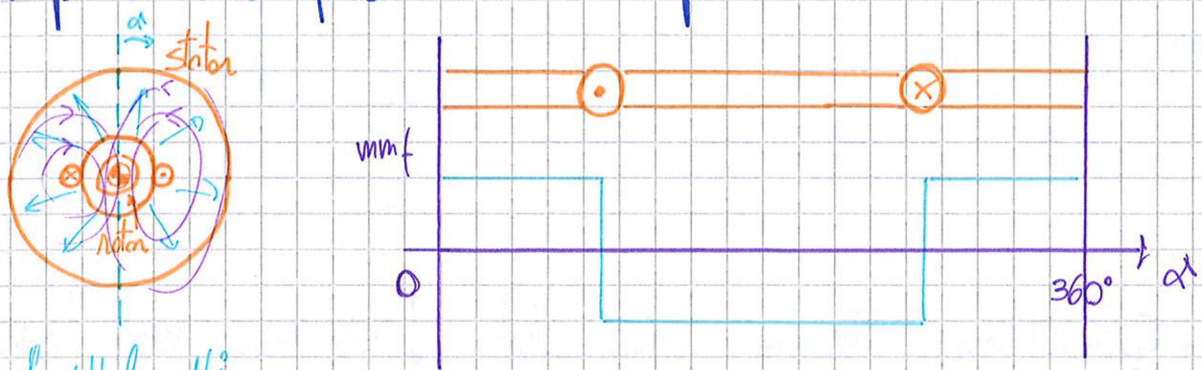
If the system is symmetric and balanced we can analyze it as an equivalent single phase where all the sources and load are in star connection.

The equivalent circuit is the same as the single phase, but the rated voltages and currents are line voltages and currents.

Also the tests are the same.

21. ROTATING MAGNETIC FIELD

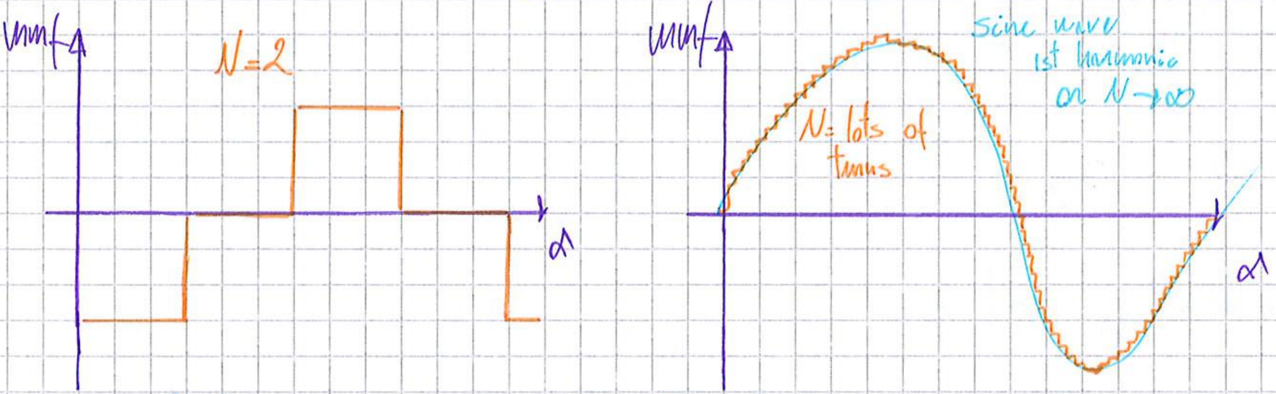
Let's consider a ~~rotating~~ coil with a current I: this system can produce a field variable in space.



$H_{Fe} l_{Fe} + H_a l_a = Ni$

$2 H_a l_a = Ni \Rightarrow H = \frac{Ni}{2 l_{air}}$

If we add more coils we get a different distribution in space:



To create a sinusoidal distribution we should need a sinusoidal distribution of turns. Using lots of turns we get an almost sinusoidal mmf distribution, where the first harmonic is sinusoidal.

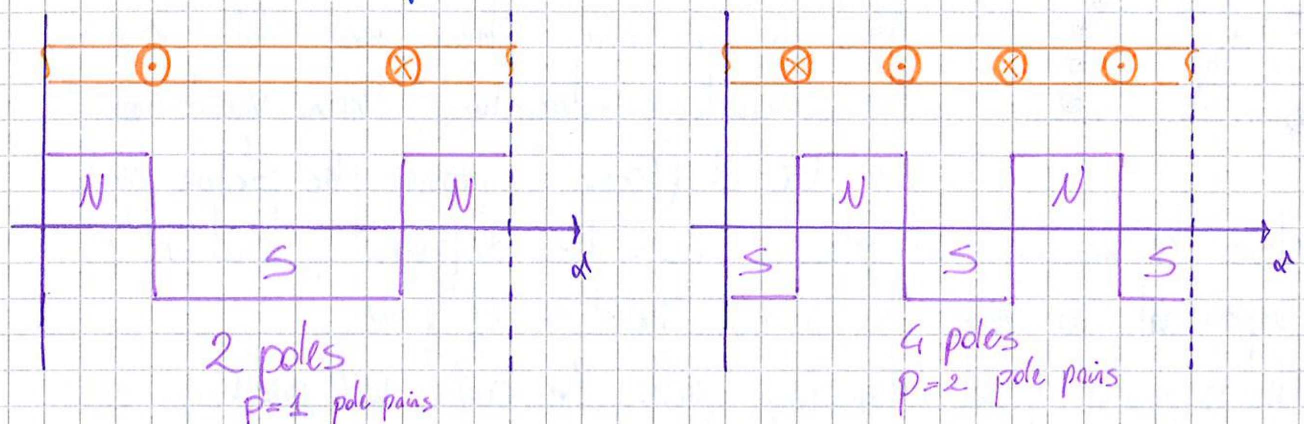
$n(\alpha) = N_x \sin\alpha \Rightarrow mmf = n(\alpha)\, i = N_x \sin\alpha\, i$

If the current is sinusoidal we get a pulsing mmf in time and space:

$i = I_x \sin\omega t \Rightarrow mmf = I_x N_x \sin\alpha \sin\omega t$

phases system currents must be shifted by $\frac{2\pi}{n}$ radians and also the windings distribution must be shifted by $\frac{2\pi}{n}$ radians.

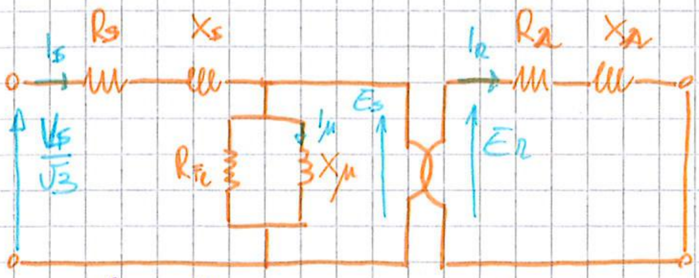
Equating the angular speed of the magnetic field and the electric pulsation of the current, it's possible to obtain the rpm of the field:



$$\omega_{mech} = \frac{\omega_{elect}}{p} \Rightarrow \frac{2\pi n}{60} = \frac{2\pi f}{p} \Rightarrow n = \frac{60 f}{p}$$

if the slip is one the rotor is still if it is zero the machine is in synchronous condition. The working point is really close to zero.

The system can be seen as a three phase transformer with some little difference.



$$f_r = \frac{\omega_r}{2\pi}\frac{\omega_s}{\omega_s} = \frac{\omega_s}{2\pi}\frac{\omega_r}{\omega_s} = f_s S$$

freq. $f_s = \frac{\omega_s}{2\pi}$ freq $f_r = \frac{\omega_r}{2\pi}$

First of all primary and secondary are at different frequency, to overcome this issue we do the following:

$$E_r = K_r \phi f_r = S K \phi f_s = S E_r(f_1)$$

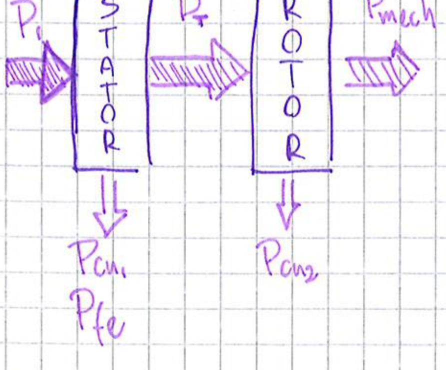
$$X_r = \omega_r L_r = 2\pi f_r L_r = S\, 2\pi f_r L_r = S X_r(f_1)$$

$$S E_{r f_1} = I_r R_r + I_r j X_{r f_1} S \Rightarrow E_{r f_1} = I_r \frac{R_r}{S} + I_r j X_{r f_1}$$

neglected notation for now on

It's also important to note that the airgap require an high magnetising current so we can't move anymore R_s and X_s to the secondary.

To model the mechanical power is convenient to follow this procedure:



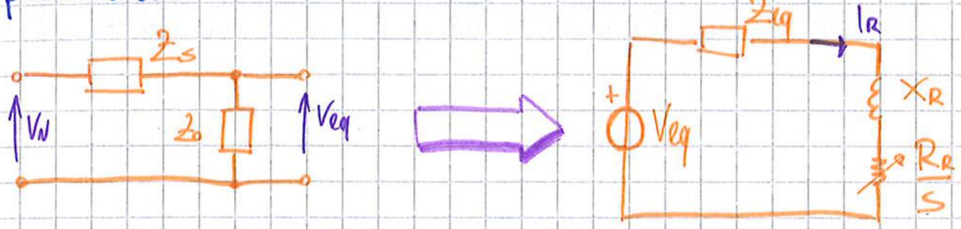
$$P_1 = \sqrt{3} V_1 I_1 \cos\varphi$$

$$P_T = 3 \frac{R_2}{S} I_2^2$$

$$P_{mech} = P_T - P_{cu_2} = 3\frac{R_2}{S} I_2^2 - 3 R_2 I_2^2 = 3 R_2 \left(\frac{1-S}{S}\right) I_2^2$$

22. MECHANICAL CHARACTERISTIC OF INDUCTION MOTOR

Let's start using the Thevenin equivalent before the rotor parameter:



$$V_{eq} = \frac{V_N}{Z_s + Z_0} Z_0 \qquad Z_{eq} = Z_s // Z_0 = \frac{Z_0 Z_s}{Z_0 + Z_s}$$

$$I_R = \frac{V_{eq}}{\sqrt{\left(\frac{R_R}{S} + R_{eq}\right)^2 + (X_{eq} + X_R)^2}}$$

The torque is found from the power:

$$P_T = 3 \frac{R_R}{S} I_R^2 = 3 \frac{R_R}{S} \frac{V_{eq}}{\sqrt{\left(\frac{R_R}{S} + R_{eq}\right)^2 + (X_{eq} + X_R)^2}} = T_T \omega_s$$

$$\omega_s = \frac{\omega_{elect}}{P}$$

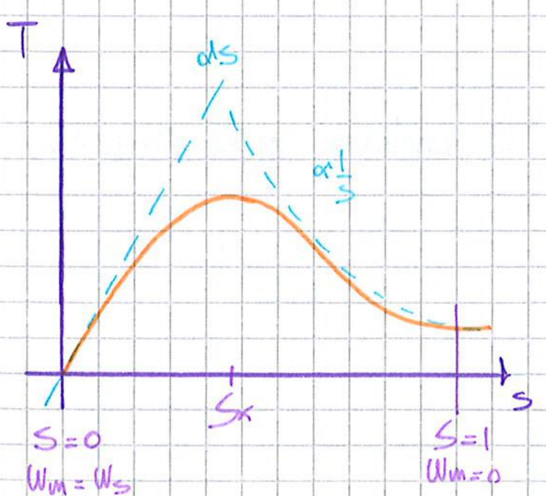
$$T = 3 \frac{R_R}{S} \frac{P}{\omega_{elect}} \frac{V_{eq}}{\sqrt{\left(\frac{R_R}{S} + R_{eq}\right)^2 + (X_{eq} + X_R)^2}}$$

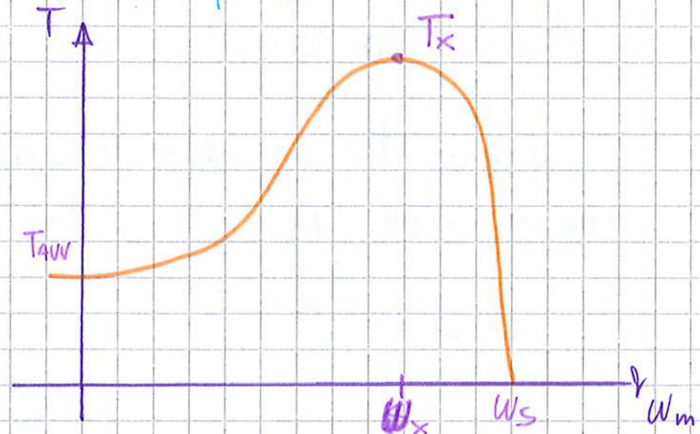
if $S \to 0$

$$T = 3 \frac{P}{\omega_{elect}} \frac{V_{eq}^2}{R_R} S$$

if $S \to \infty$

$$T = 3 \frac{P}{\omega_{elect}} \frac{V_{eq}^2}{R_{eq}^2 + X_R} \frac{R_R}{S}$$





So the maximum torque is:

$$T_x = \frac{3p}{\omega_e} V_{eq}^2 \frac{R_R/s_x}{\left(R_{eq}+\frac{R_R}{s_x}\right)^2 + X_{tL}^2} \simeq \frac{3p}{\omega_e} V_{eq} \frac{\frac{R_R}{R_R/X_{tL}}}{\left(R_{eq}+\frac{R_R}{R_R/X_t}\right)^2 + X_{tL}^2} =$$

$$= \frac{3p}{\omega_e} V_{eq}^2 \frac{X_{tL}}{X_{tL}^2 + X_{tL}^2} = \frac{3p}{\omega_e} \frac{V_{eq}^2}{2X_{tL}^2}$$

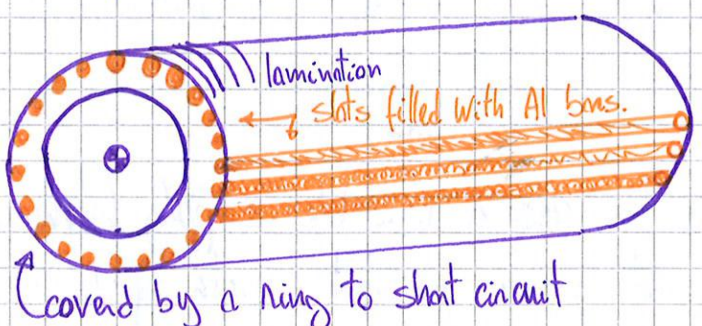
To increase the torque X_{tL} must be lowered, but an higher X_{tL} reduces the starting current.

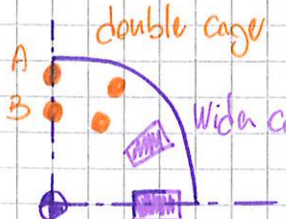
23. ROTOR STRUCTURES

The rotor of induction machine can be built in 2 different ways:

- WOUND ROTOR: classical winding, similar to the stator one (same number of poles). It's rarely used in industrial application, except for wind generator.

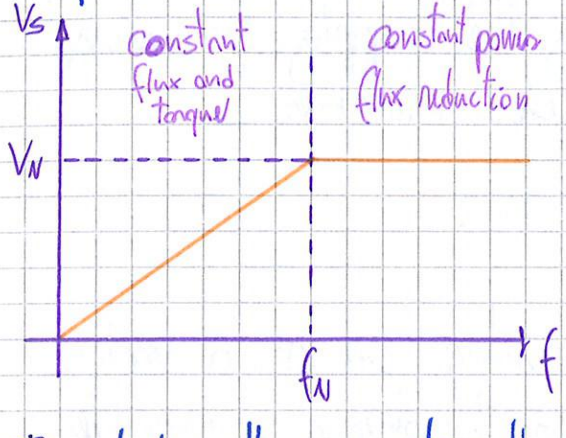
- SQUIRREL CAGE or CAGE ROTOR: this is the preffered solution, since it's more robust, there aren't any insulation problem and the iron losses can be neglected thanks to the low frequency.

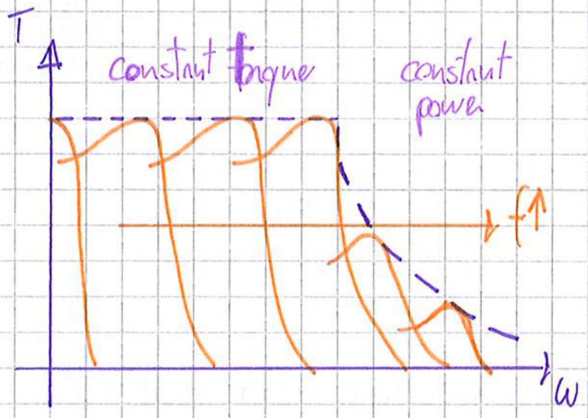




$\phi_A < \phi_B$ because A is closer to the air gap
⇓
$X_{lA} < X_{lB}$

It's possible to obtain a variable rotor resistence in starting conditions using a double cage (or a wider cage for smaller motor): at start ($f_r = f_s = 50\,Hz$) the current try to move from B to A, not all if the motor is supplied with a frequency higher than the nominal one the voltage can't be higher than the nominal one so we get a flux reduction.

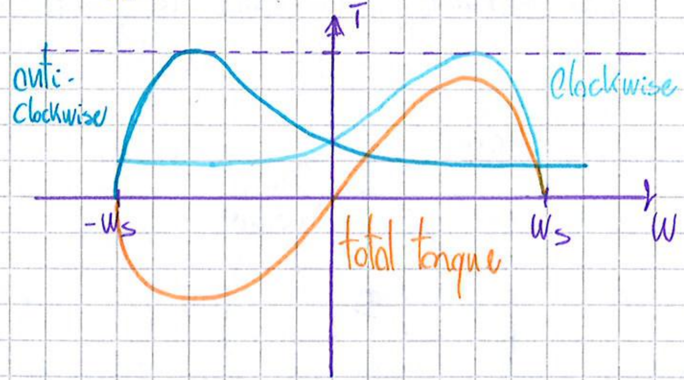


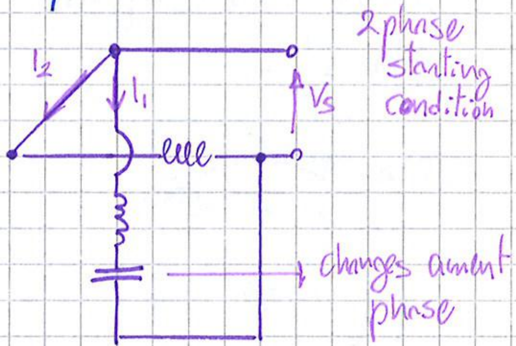


Supplied with an inverter the induction machine mechanically behave like a DC motor.

25. SINGLE PHASE INDUCTION MACHINE

To have a rotating magnetic field we need at least 2 phases, with only one phase ~~when~~ we have an unidirectional pulsating field. It can be seen as a sum of 2 rotating field (one clockwise, the other anticlockwise), each of them produce a component of torque: the difference between them is the total torque:





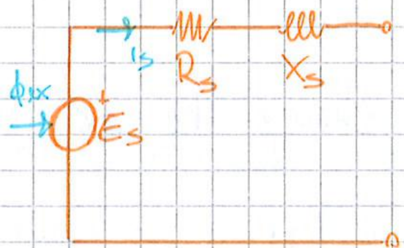
A single phase machine cannot start itself ($T_s = 0$), to solve this issue it's possible to start the machine as a 2 phase machine and then switch back to single phase.

27. SYNCHRONOUS GENERATOR'S WORKING PRINCIPLE

Since the speed is function of the desired frequency, the produced fem only depends on the excitation flux and therefore the excitation current.

$$E_s = K_s \phi_{ex} \omega_s = K_s K_{ex} I_{ex} \omega_s$$

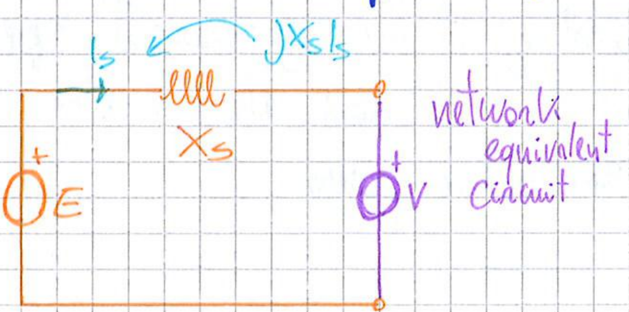
The equivalent circuit is made as follow:



$$|\Delta V_R| << |\Delta V_x|$$

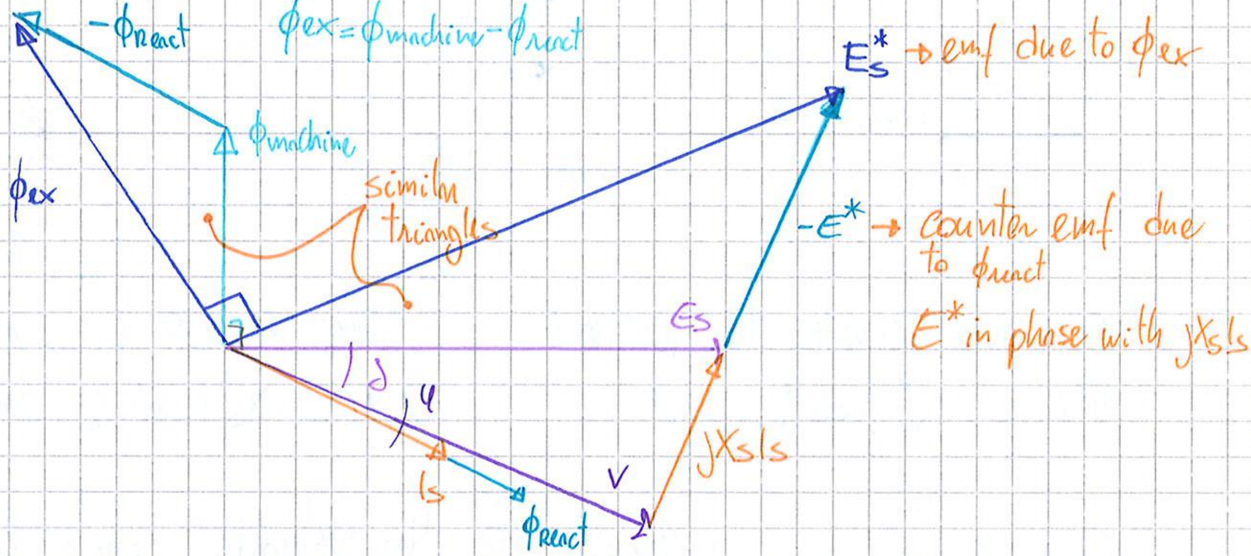
in synchronous condition the voltage drop on X_s is higher than the drop on R_s: the resistence can be neglected from the voltage computation, but not its Joule losses.

The whole Europe's electric network is connected together, and all power sources are connected in parallel. From a generator point of view the network imposes a voltage that cannot be changed and an almost null impedence (lots of impedences in parallel).



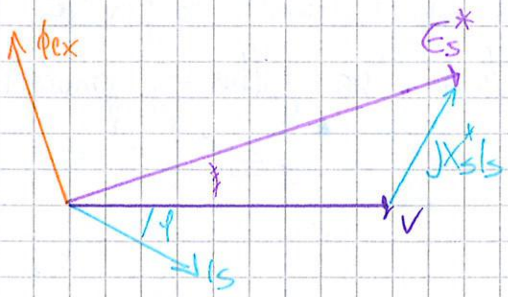
28. MAGNETIC FLUXES IN THE MACHINE

Adding a load to the generator produces a current in the stator. The rotor current (excitation) produces the excitation flux, the stator current produces the armature reaction flux. The total flux is named machine flux and produce E_s:

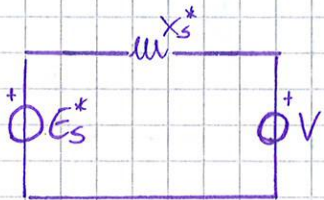


Since $-E^*$ and jX_sI_s are in phase it's possible to see their sum as a voltage drop on an equivalent reactance X_s^* called synchronous reactance:

$$\Delta V_{X_s^*} = jX_s^* I_s = jX_s I_s - E^* = jI_s(X_s + X^*)$$

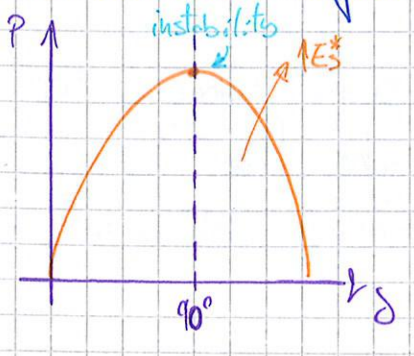


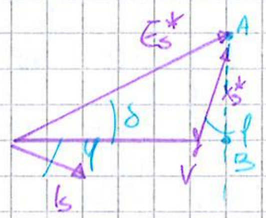
$$X^* > 9 \div 10\, X_s \Rightarrow X_s^* \simeq X^*$$



29. POWER REGULATION AND NETWORK CONNECTION

As mentioned before, active power depends on the load angle δ





$P = 3VI_s\cos\varphi \qquad \overline{AB} = X_s^* I_s \cos\varphi$

$\overline{AB} = E_s^* \sin\delta$

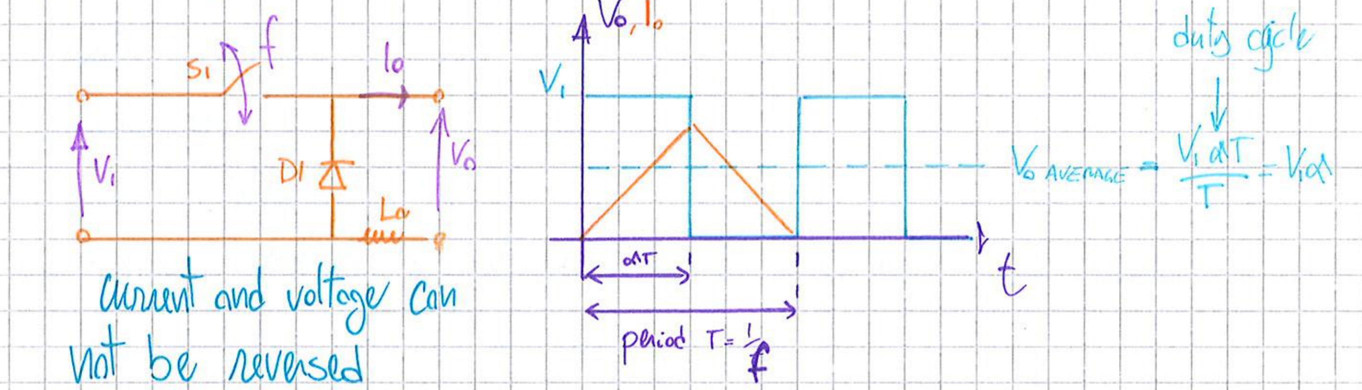
$$P = \frac{3V}{X_s^*} E_s^* \sin\delta$$

With angular speed proportional to $f_{net} - f_s$. When the measured ΔE is zero the systems are in phase and the switches can be closed. Now the generator is synchronized and we can start to regulate active and reactive powers.

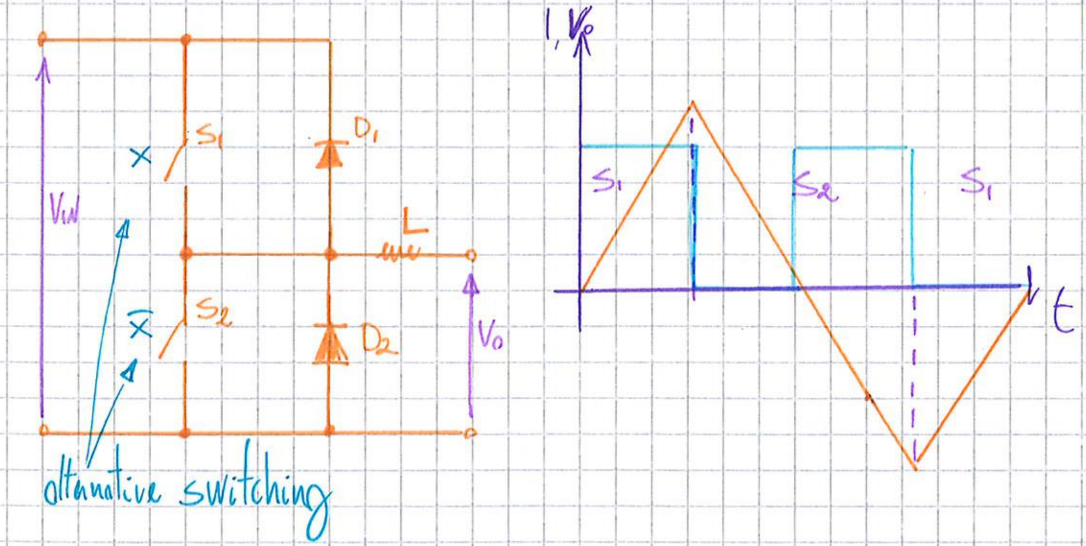
30. DC-DC CONVERTER

A DC-DC converter is defined by in how many quadrants (V-I plane) can work.

FIRST QUADRANT



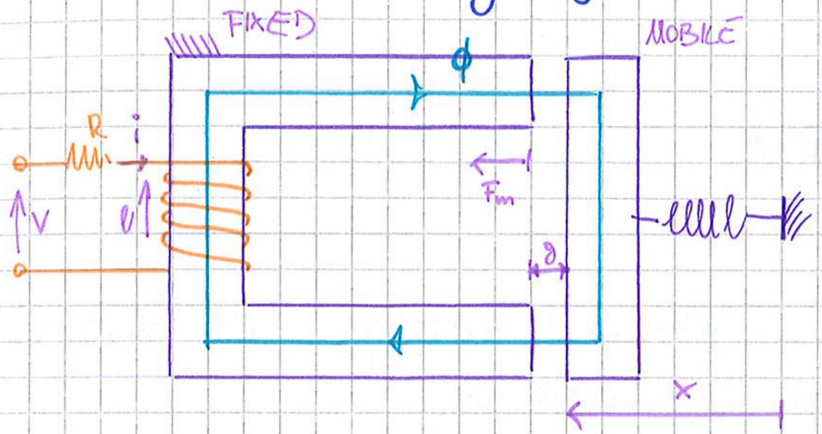
TWO QUADRANT



if S_1 is closed for more time the average current is $I > 0$.
the opposite for S_2

31. ELECTROMECHANICAL ENERGY CONVERSION

Let consider the following system. The total energy is given by:



$E_{elect} = E_{mech} + E_{magn} + E_{Losses}$

if we neglect the losses and we suppose the mobile part fixed we get:

$dW_{elect} = dW_{mech} + dW_{magn}$

$dW_{mech} = 0$

$dW_{elect} = dW_{magn}$

Considering the link between the emf e and the flux:

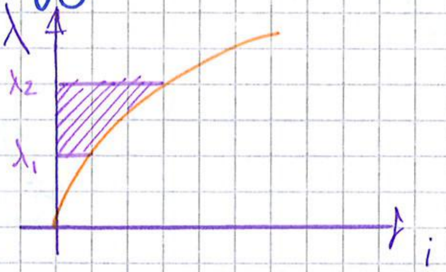
$dW_{elect} = e\,i\,dt = \frac{d\lambda}{dt}\,i\,dt = i\,d\lambda$

$\frac{d\,\text{linked flux}}{dt}$

$dW_{elect} = i\,d\lambda = dW_{magn}$

As consequence the electric energy is stored in magnetic energy:

$W_{elect} = W_{magn} = \int_{\lambda_1}^{\lambda_2} i\,d\lambda$



Remembering the mmf relations we can compute the current:

$$\left.\begin{aligned} Ni &= H_a l_a + H_{Fe} l_{Fe} \\ \lambda &= N\phi = NSB \end{aligned}\right\} \quad W_{magn} \int_{B_1}^{B_2} \frac{H_a l_a + H_{Fe} l_{Fe}}{\cancel{N}} \cancel{N} S\,dB = \int \frac{B l_a}{\mu_0} S\,dB + \int H_{Fe} l_{Fe} S\,dB$$

section

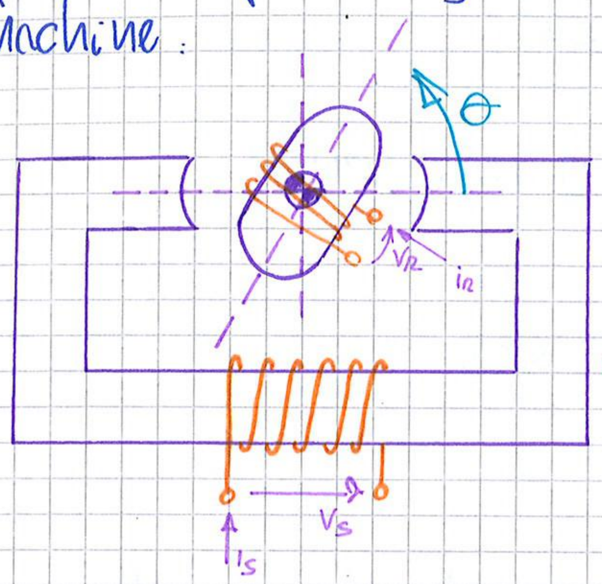
volume

volume

$$W_{magn} = \frac{1}{2}\frac{B^2}{\mu_0} l_a S + H_{Fe} B S l_{Fe} = W_{airgap} + W_{Fe}$$

As said previously due to the low value of μ_0 the energy is mostly stored inside the airgap.

If instead of linearly mobile part we have a rotating machine:



$dW_{magn} = e_s i_s + e_r i_r = i_s d\lambda_s + i_r d\lambda_r$

$\lambda_s = L_s i_s + M i_r$

$\lambda_r = L_r i_r + M i_s$

$dW_{magn} = i_s L_s di_s + i_s M di_r + i_r L_r di_r + i_r M di_s$

$W_{magn} = \frac{1}{2} L_s i_s^2 + \frac{1}{2} L_r i_r^2 + M i_s i_r$

$$T = \left| \frac{\partial W^*_{magn}(i,\theta)}{\partial \theta} \right|_{i=const}$$

$$T = \frac{1}{2} i_s^2 \frac{dL_s}{d\theta} + \frac{1}{2} i_r^2 \frac{dL_r}{d\theta} + i_s i_r \frac{dM}{d\theta}$$

We get 3 different ways to create torque.

For example if we supply only the stator where the variable inductance is defined by:

$L(\theta) = L_0 + L_1 \cos(2\theta) \rightarrow$ 2 cycle every revolution

L_0 → mean value; L_1 → amplitud

The torque is given by:

$$T = \frac{1}{2} i^2 \frac{dL}{d\theta} = \frac{1}{2} i^2 \frac{d}{d\theta}\left(L_0 + L_1 \cos(2\theta)\right) = -i^2 L_1 \sin 2\theta$$

if the system is supplied by a DC source the rotor tends to reach a stable position ($\theta = 0°$ or $\theta = 180°$, minimal energy condition), under perturbation the rotor creates torque to stay in stable condition. The average produced torque is zero, the rotors cannot rotate itself.

if instead we supply the system with an AC supply:

induction machine supply

if $\begin{cases} i_s = I_s \sin \omega_s t \\ i_r = I_R \sin \omega_R t \end{cases} \Rightarrow T = -I_s I_R M_x \sin\omega_s t \sin\omega_r t \sin\theta$

$\theta = \omega_m t + \delta \quad , \quad \omega_s = \omega_m - \omega_r$

$T_{AV} = \frac{1}{4} I_s I_R M_x \sin\delta$

32. TRANSFORMATIONS ~~FOR~~ 3 PHASE SYSTEMS

To describe 3 phase system usually ~~we~~ used 3×1 vectors and 3×3 matrices. To simplify the system is usefull to apply some transformation.

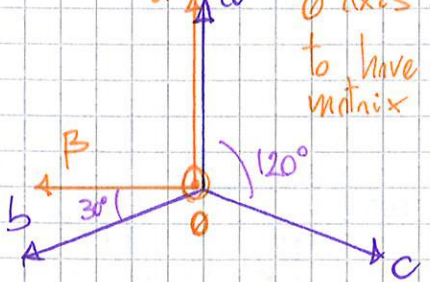
To apply a transformation we use a transformation matrix M, in general it is a 3×3 matrix and to mantain the formalism for electrical power computation it has to be orthonormal:

$M_{3\times3} \qquad M^{-1} = M^T \Leftrightarrow$ orthonormal

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = M \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \qquad \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = M \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$P_{123} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}^T \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \qquad P_{abc} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \left[M \vec{V}_{123} \right]^T M \vec{I}_{123} = \vec{V}_{123} \underbrace{M^T M}_{I} I_{123}$$

To reduce the number of equation is convenient to apply a "3 phase to 2 phase" transformation:



0 axis is needed to have a 3×3 matrix

$$V_\alpha = V_a - \frac{1}{2} V_b - \frac{1}{2} V_c$$

sin30°

cos30°

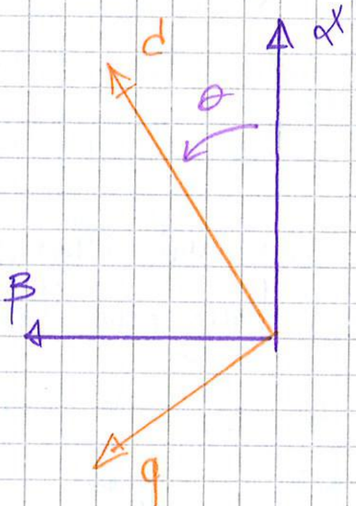
$$V_\beta = \frac{\sqrt{3}}{2} V_b - \frac{\sqrt{3}}{2} V_c$$

$$V_0 = V_a + V_b + V_c$$

3→2 phase transformation:

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

↳ rotation about θ axis



if $\theta = \omega t$

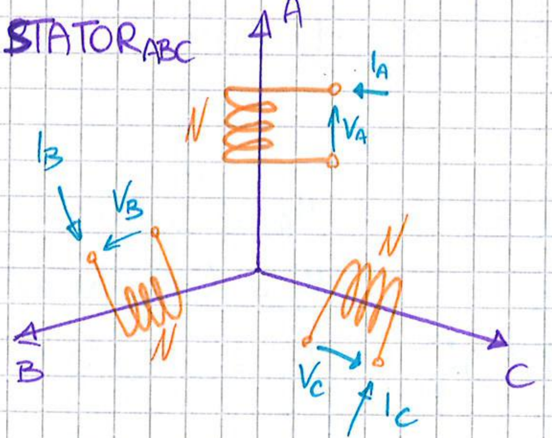
$$V_{\alpha\beta o} = \begin{bmatrix} \sqrt{\frac{3}{2}}\, V\cos\omega t \\ \sqrt{\frac{3}{2}}\, V\sin\omega t \\ 0 \end{bmatrix}$$

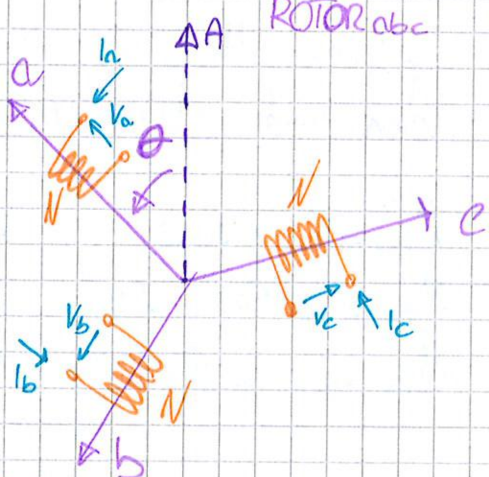
⤳ DC voltage → easier computation

$$V_{dqo} = R\, V_{\alpha\beta o} = \begin{bmatrix} \sqrt{\frac{3}{2}}\, V \\ 0 \\ 0 \end{bmatrix}$$

33. INDUCTION MOTOR DYNAMIC MODEL

An induction motor can be seen as a 3 phase stator winding magnetically coupled with the rotor ones.





$$\begin{cases} V_A = \dfrac{d}{dt}\lambda_A + R_S I_A \\ V_B = \dfrac{d}{dt}\lambda_B + R_S I_B \\ V_C = \dfrac{d}{dt}\lambda_C + R_S I_C \end{cases}$$

$$\begin{cases} V_a = \dfrac{d}{dt}\lambda_a + R_R I_a \\ V_b = \dfrac{d}{dt}\lambda_b + R_R I_b \\ V_c = \dfrac{d}{dt}\lambda_c + R_R I_c \end{cases}$$

$$\begin{cases} V_{ABC} = R_S\, i_{ABC} + \frac{d}{dt} M_{SS}\, i_{ABC} + \frac{d}{dt} M_{SR}\, i_{abc} \\ V_{abc} = R_R\, i_{abc} + \frac{d}{dt} M_{RR}\, i_{abc} + \frac{d}{dt} M_{RS}\, i_{ABC} \end{cases}$$

Since R_i is a diagonal matrix the transformation has any effects on it.

$$R^i_{\alpha\beta o} = T R_i T^{-1} = R_i \quad \text{with } i = S, R$$

$$M^{SS}_{\alpha\beta o} = T M_{SS} T^{-1} = \begin{bmatrix} \frac{3}{2}L_o + L_{ds} & 0 & 0 \\ 0 & \frac{3}{2}L_o + L_{ds} & 0 \\ 0 & 0 & L_{ds} \end{bmatrix}$$

$$M^{RR}_{\alpha\beta o} = T M_{RR} T^{-1} = \begin{bmatrix} \frac{3}{2}L_o + L_{dr} & 0 & 0 \\ 0 & \frac{3}{2}L_o + L_{dr} & 0 \\ 0 & 0 & L_{dr} \end{bmatrix}$$

$$M^{SR}_{\alpha\beta o} = T M_{SR} T^{-1} = \begin{bmatrix} \frac{3}{2}\cos\theta & -\frac{3}{2}\sin\theta & 0 \\ \frac{3}{2}\sin\theta & \frac{3}{2}\cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} L_o$$

$$M^{RS}_{\alpha\beta o} = \left[M^{SR}_{\alpha\beta o}\right]^T$$

So we get (neglectin the 0 axis):

$$\begin{bmatrix} V^S_\alpha \\ V^S_\beta \end{bmatrix} = R_S \begin{bmatrix} i^S_\alpha \\ i^S_\beta \end{bmatrix} + M^{SS}_{\alpha\beta}\, p \begin{bmatrix} i^S_\alpha \\ i^S_\beta \end{bmatrix} + p\left[M^{SR}_{\alpha\beta o} \begin{bmatrix} i^R_\alpha \\ i^R_\beta \end{bmatrix}\right]$$

$$p = \frac{d}{dt}$$

$$\begin{bmatrix} V^R_\alpha \\ V^R_\beta \end{bmatrix} = R_R \begin{bmatrix} i^R_\alpha \\ i^R_\beta \end{bmatrix} + M^{RR}_{\alpha\beta}\, p \begin{bmatrix} i^R_\alpha \\ i^R_\beta \end{bmatrix} + p\left[M^{RS}_{\alpha\beta o} \begin{bmatrix} i^S_\alpha \\ i^S_\beta \end{bmatrix}\right]$$

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 61 di 78

need to substitute p with $j\omega_e$.
The motor parameter obtained during the test are still valid and we can use them in the dynamic model

The produced torque can be obtained ~~dir~~ from a power balance:

$$V_d^s i_d^s + V_q^s i_q^s + V_d^R i_d^R + V_q^R i_q^R = R_s\left({i_d^s}^2 + {i_q^s}^2\right) + L_s i_d^s p i_d^s + L_s i_q^s p i_q^s +$$
$$+ L_m i_q^s p i_q^R + R_R\left({i_d^R}^2 + {i_q^R}^2\right) + L_R i_d^R p i_d^R + L_m i_d^R p i_d^s + L_R i_q^R p i_q^R +$$
$$+ L_m i_q^R p i_q^s + \omega \lambda_q^R i_d^R - \omega \lambda_d^R i_q^R$$

$$P_{converted} = \omega \lambda_q^R i_d^R - \omega \lambda_d^R i_q^R$$

$$P_{mech} = P_{converted} = T\omega \Rightarrow T = \lambda_q^R i_d^R - \lambda_d^R i_q^R$$

if instead of a stator fixed reference frame we want a rotor fixed reference frame a rotation of $\theta_0 = \omega_0 t$ must be applied to the stator and a rotation of $\theta_0 - \theta$ to the rotor.

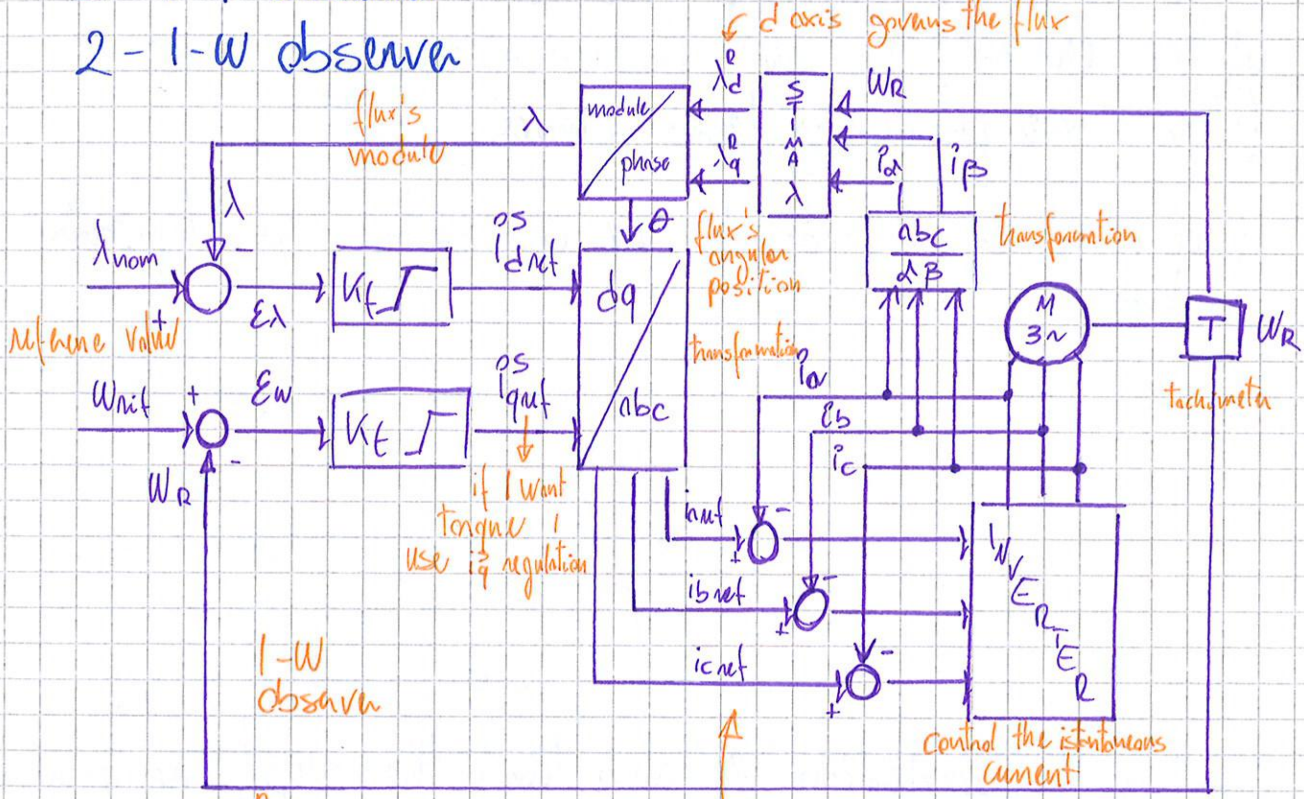
To build the dynamic model we have to add the mechanical equation:

$$T = \left(\lambda_q^R i_d^R - \lambda_d^R i_q^R\right) = Jp\omega + T_r$$

© Proprietà riservata dell'autore - Digitalizzazione e distribuzione a cura del CENTRO APPUNTI - Corso Luigi Einaudi, 55 - Torino / Pagina 63 di 78

the addition of the rotor angular position plus an angle due to the slip speed. A ~~flux~~ position transducer is requested.

- DIRECT METHOD: The flux is measured directly using the measures of some machine quantities. The flux observer can be divided in 2 categories:
  1 - V-I observer
  2 - I-ω observer



$$\lambda_d^R = \int -\frac{\lambda_d^R}{\tau_R} - \omega\lambda_q^R + \frac{L_m}{\tau_R} i_d^{os}\, dt$$

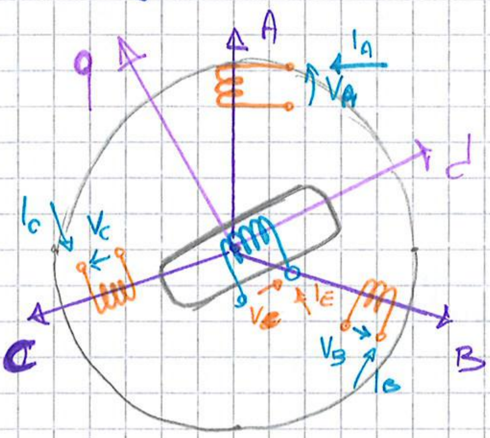
actual control must be done with a 3 phase system

$$\lambda_q^R = \int -\frac{\lambda_q^R}{\tau_R} - \omega\lambda_d^R + \frac{L_m}{\tau_R} i_q^{os}\, dt$$

A cheap and simple control is given by an open loop control. It consists of controlling the stator supply and its frequency to get the desired characteristic. To avoid overheating currents the

35. SYNCHRONOUS MACHINE DYNAMIC MODEL

The synchronous machine is defined by a 3 phase stator and a single phase rotor supplied a DC current for the production of the excitation flux.



$$V_A = p\lambda_A + R_S i_A$$
$$V_B = p\lambda_B + R_S i_B$$
$$V_C = p\lambda_C + R_S i_C$$
$$V_e = p\lambda_e + R_e i_e$$

The linked fluxes are:

$$\lambda_A = L_A i_A + M_{AB} i_B + M_{AC} i_C + M_{Ae} i_e$$
$$\lambda_B = L_B i_B + M_{BA} i_A + M_{BC} i_C + M_{Be} i_e$$
$$\lambda_C = M_{CA} i_A + M_{CB} i_B + L_C i_C + M_{Ce} i_e$$
$$\lambda_e = M_{eA} i_A + M_{eB} i_B + M_{eC} i_C + L_e i_e$$

All the inductances (self and mutual) depend on the mechanical angle θ

To simplify the system we impose some hypothesis:

- Stator windings with sinusoidal distribution:

  $n(\alpha) = \frac{N}{2}\sin(\alpha)$ $\quad N$ n° of coils

- Stator sinusoidal emf:

  $emf = \int_0^{2\pi} \omega_R R l \frac{N}{2} B(\alpha)\sin(\alpha)\, d\alpha$

- linear magnetic circuit

The produced mmf in a **dq** frame can be, under these

For the other phases we simply add the shifting angle:

$L_B = L_i + L_a \cos(2\theta - 240°)$

$L_C = L_i + L_a \cos(2\theta - 120°)$

In the same way we compute the stator mutual inductance and we get:

$M_{AB} = M_{BA} = -\frac{1}{2} L_i + L_a \cos(2\theta - 120°)$

$M_{BC} = M_{CB} = -\frac{1}{2} L_i + L_a \cos(2\theta)$

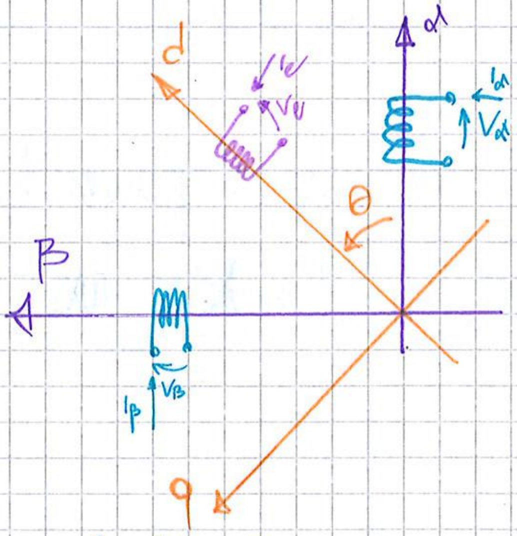
$M_{AC} = M_{CA} = -\frac{1}{2} L_i + L_a \cos(2\theta - 240°)$

$M_{Ae} = M_{eA} = L_d \cos\theta$

$M_{Be} = M_{eB} = L_d \cos(\theta - 120°)$

$M_{Ce} = M_{ec} = L_d \cos(\theta - 240°)$

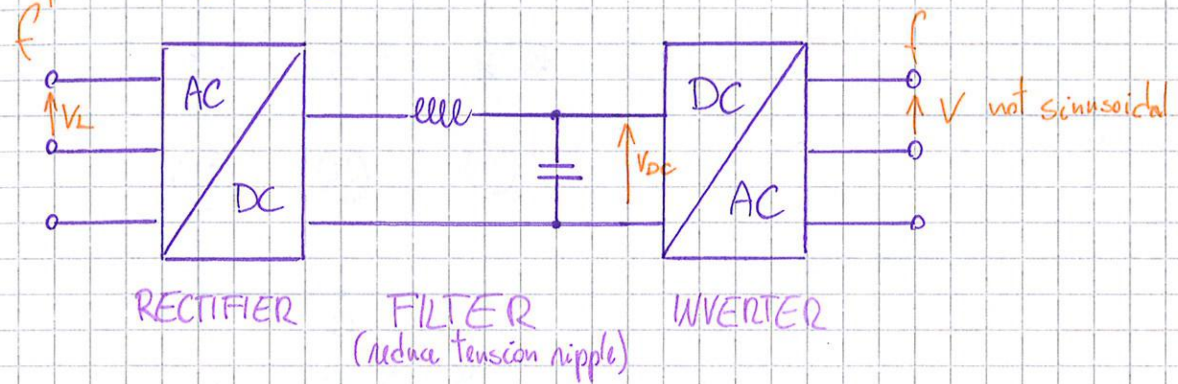
Like for the induction machine we need to apply some transformations to simplify the system.
First we do a 3→2 phase transformation:



With reference to the figure is possible to see the θ angle influence on the magnetic equation. With θ = 0° the system has the maximum inductance in the α axis and the minimum in the β axis. In this case α axis has the maximum

36. INVERTER

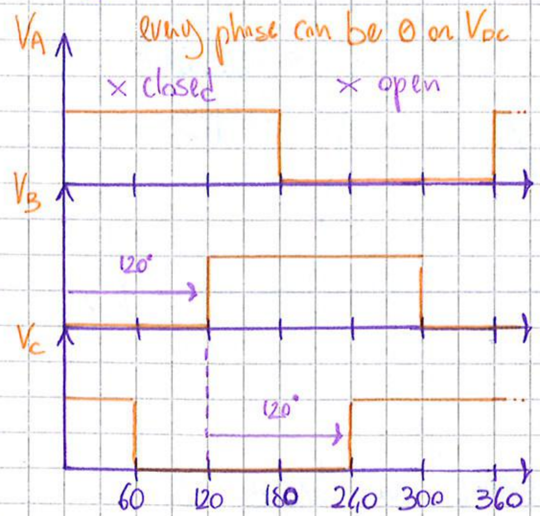
An inverter is a device that can produce a supply with variable frequency

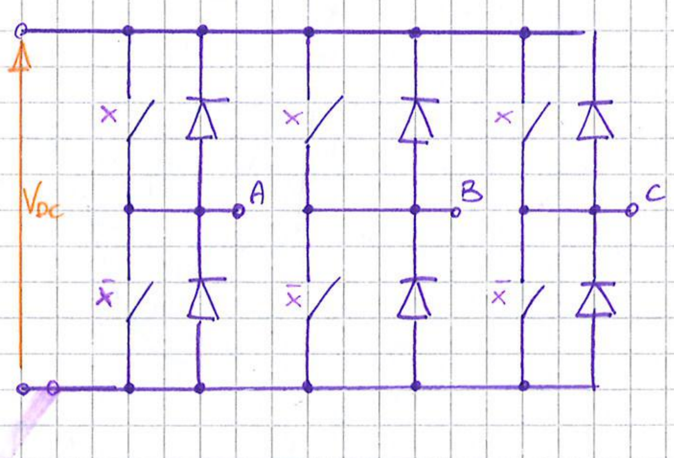


Depending on how we regulate the ~~current~~ frequency we can have different inverter:

- CSI - Current source inverter, used for high power machines
- VSI - Voltage source inverter
  - square wave or 6 steps inverter
  - PWM (pulse with modulation) inverter

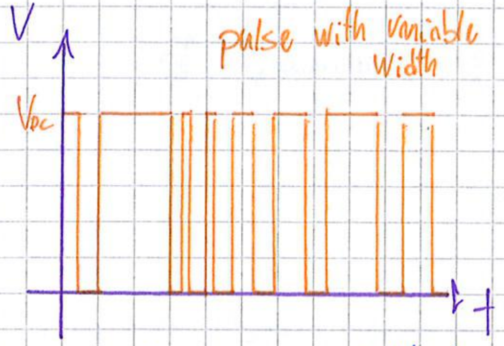
A square wave inverter produces 3 single phases shifted by 120°

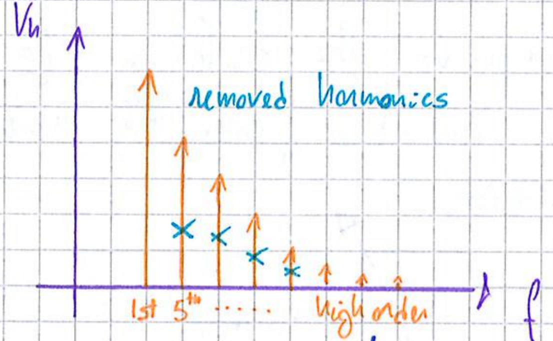




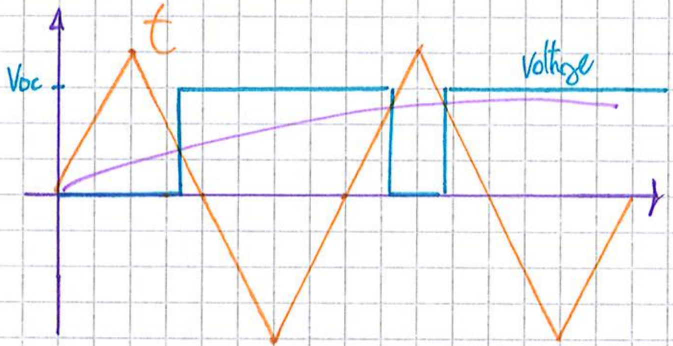
sinusoidal supply, but the joule losses are way more influent due to the high harmonics effect.

A good idea is to reduce the harmonic content produced by the inverter. PWM inverters do this increasing the number of swithes commutation in time:





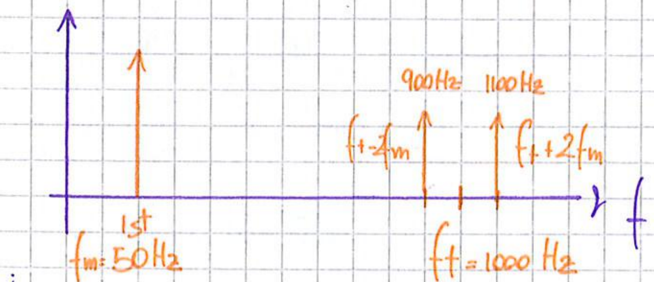
To produce a modulated voltage we compare a triangular wave with a modulation wave with the desired 1^{st} harmonic frequency



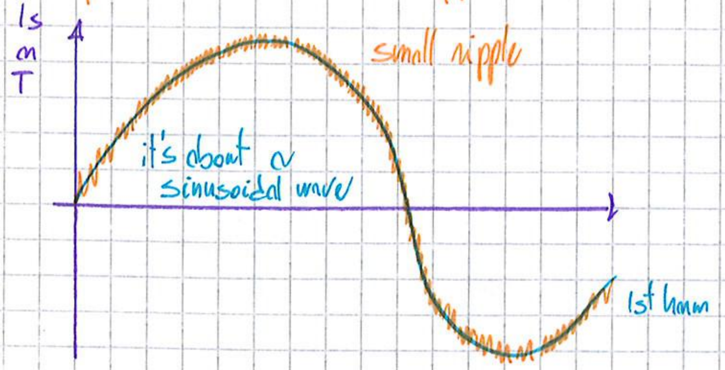
when $t > m$ $V = 0$

$t < m$ $V = V_{DC}$

$f_{triang} > 10\, f_{modulation}$

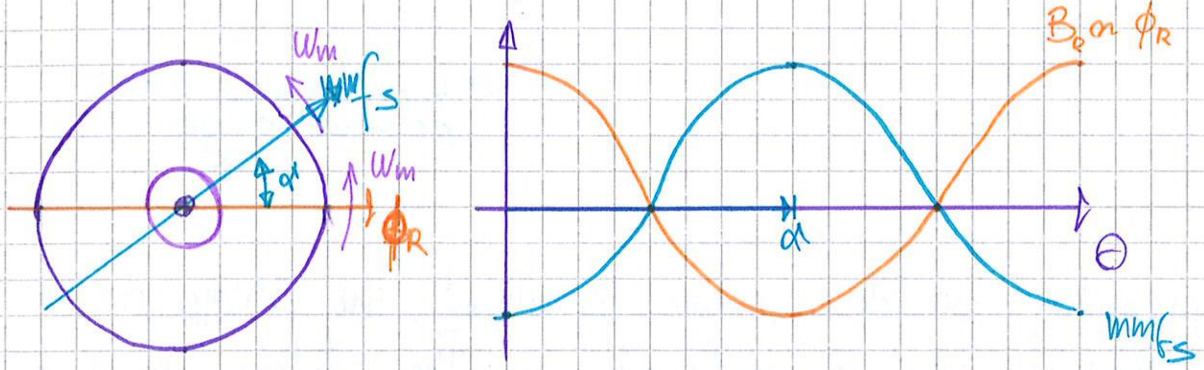


Increasing the triangular wave frequency the harmonics influence get reduced and therefore the current/torque ripple is highly reduced in amplitude. This produce an almost sinusoidal wave, so we get almost the same joule losses of a sinusoidal supply.



37. SINUSOIDAL BRUSHLESS MACHINE

A brushless motor is a synchronous motor with permanent magnet rotor and a 3phase stator winding driven by an inverter. The inverter is needed to obtain a



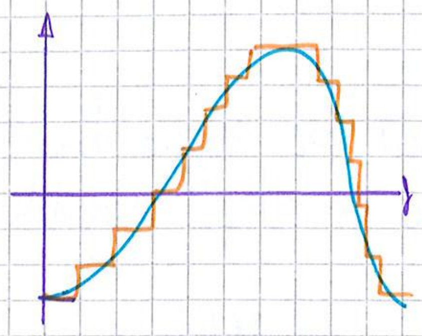
variable synchronous rotation speed.
In sinusoidal brushless motors the stator winding is made very accurately to obtain a sinusoidal distribution of coils that produce a sinusoidal distribution of current in space, this give us an almost sinusoidal back emf ~~and so a sinusoidal~~.
A brushless motor is really similar to a DC motor but now the commutation is made elethronically by the inverter instead of the brushes.
Like a DC machine the produced torque is given by:

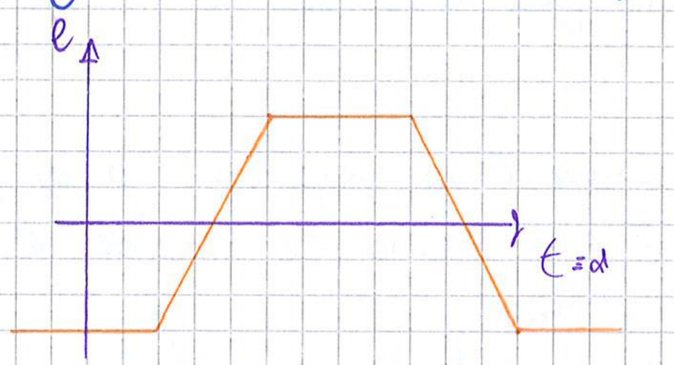
$$T = K_t \phi_R \, mmf_s \sin\alpha = K_t \phi_R N_{eqs} I_s \sin\alpha$$

T_{max} when $\alpha = 90°$

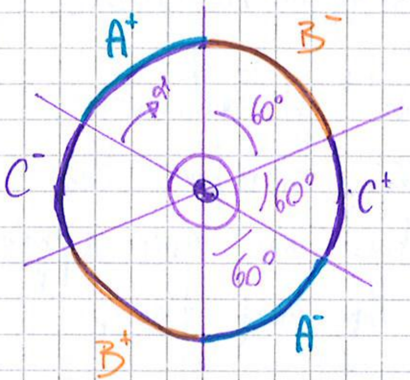
to mantain $\alpha = 90°$ it's needed to know the rotor field direction to use the inverter to produce an always orthogonal mmf. Our motor control need a resolver, a position sensor for the rotor

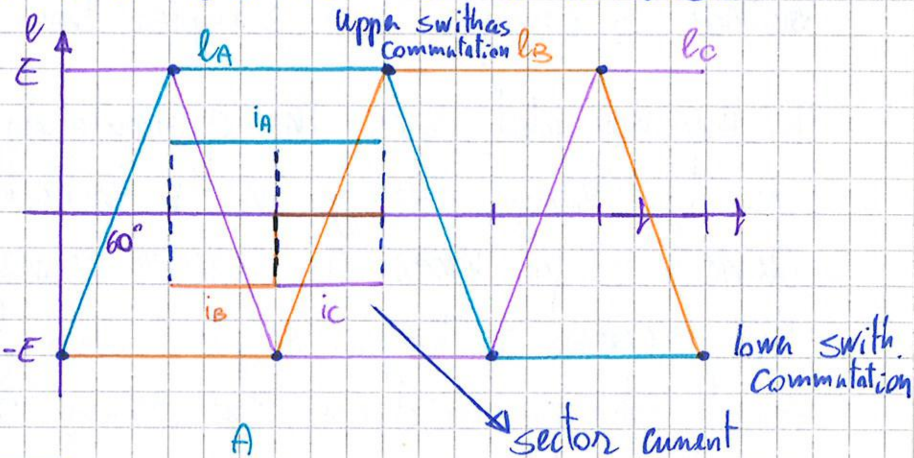
If we have a even distribution of coil with the same turns (not sinusoidal) we finally get the trapezoidal shape.

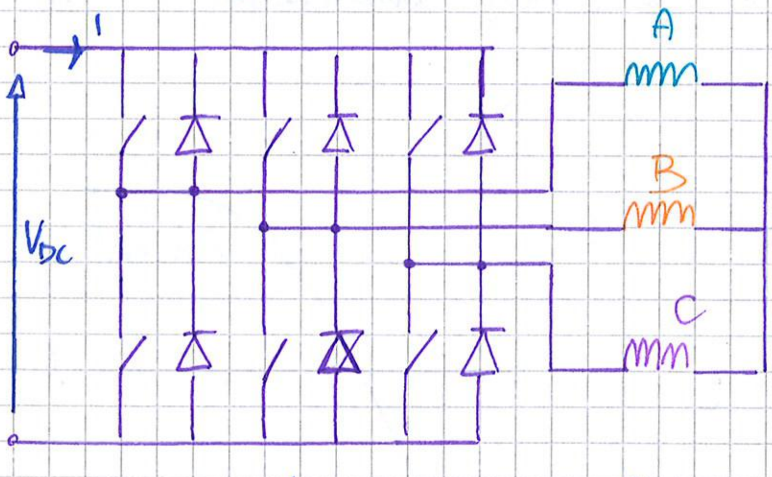


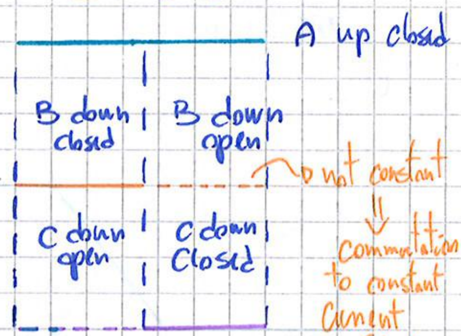


To create a 3 phase system we divide the rotor in 3 sectors, each one identical to the others:









We need a continuous commutation of swithches to mantain constant current and therefore emf. Only 2 phases are working at time.

If we compute the power in a 60° sector we get: