NUMERO: 1565A - ANNO: 2015

A P P U N T I

STUDENTE: Botta

MATERIA: Meccanica Applicata alle Macchine. Prof.Ferraresi

MECCANICA APPLICATA

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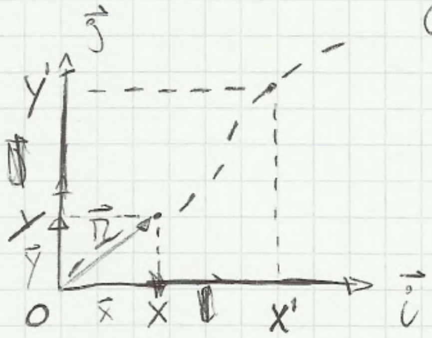
Sistemi di riferimento:

- Cartesiano
- Locale
- Polare

Coordinate cartesiane



$x = x(t)\ ;\ y = y(t)$

$$\vec{r} = \vec{x} + \vec{y} = x\vec{i} + y\vec{j}$$

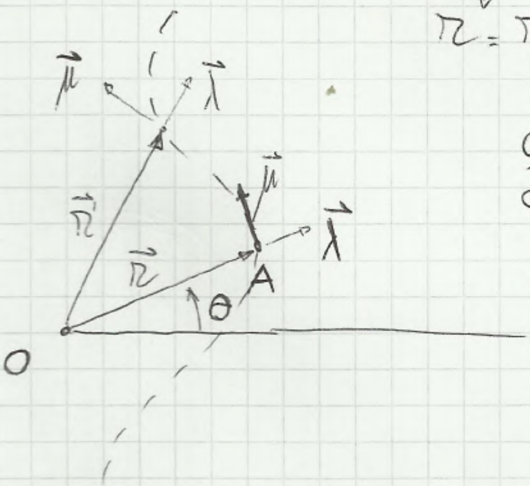
$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} = \vec{v}_x + \vec{v}_y$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{x}\vec{i} + \ddot{y}\vec{j} = \vec{a}_x + \vec{a}_y$$

Coordinate polari

raggio anomalia

$r = r(t)\ ;\ \theta = \theta(t)$

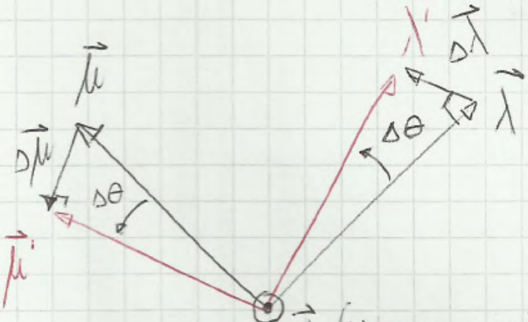


$$\frac{dr}{dt} \qquad \frac{d^2r}{dt^2}$$

$$\frac{d\theta}{dt} \qquad \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\vec{\lambda}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{r}\lambda + r\frac{d\lambda}{dt} \quad *$$



$\frac{\Delta\theta}{\Delta t} = \dot{\theta}$ velocità angolare

$$\dot{\vec{\theta}} = \dot{\theta}\vec{\nu}$$

$\vec{\nu}$ (versore uscente dal piano)

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Moto uniforme: $V =$ costante

$V = \frac{dr}{dt}$ equazione del moto

$r = r(t) \leftarrow$ legge del moto

$\int_{x_0}^{x} dx = v\int_{t_0}^{t} dt \qquad x = x_0 + v(t-t_0)$

Moto uniformemente accellerato: $a =$ costante

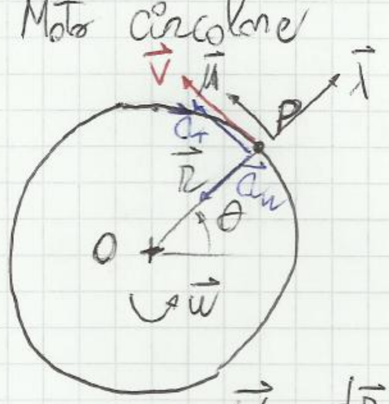
$a = \frac{dv}{dt}$ eq. del moto

$\int_{v_0}^{v} dv = a\int_{t_0}^{t} dt \qquad v = v_0 + a(t-t_0)$ legge del moto rispetto a v

$v = \frac{dx}{dt} \qquad \int_{x_0}^{x} dx = \int_{t_0}^{t} v\,dt = \int_{t_0}^{t} v_0\,dt + \int_{t_0}^{t} a t\,dt$

$x = x_0 + v_0(t-t_0) + \frac{1}{2}a(t-t_0)^2$ legge del moto rispetto ad x

Moto circolare



$\vec r = r\vec\lambda$

$r =$ costante

$\dot{\vec\theta} = \vec\omega$

$\ddot{\vec\theta} = \dot{\vec\omega}$

tangenziale

centripeta

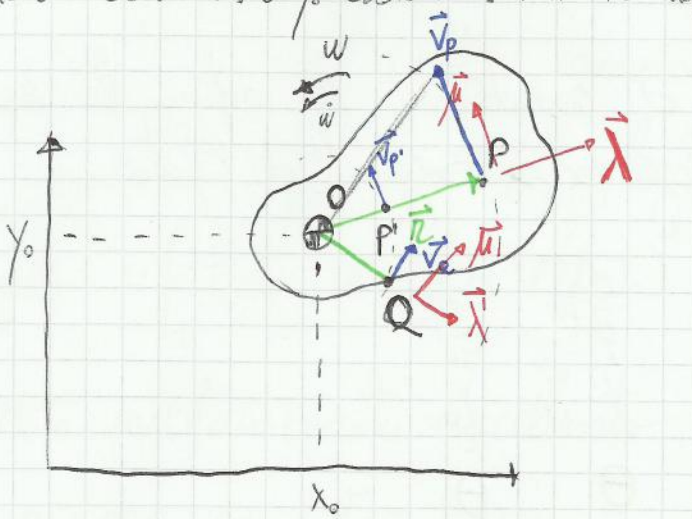
$\vec V = \frac{d\vec r}{dt} = r\frac{d\vec\lambda}{dt} = r\omega\vec\mu$

$\vec a = \frac{d\vec v}{dt} = r\dot{\omega}\vec\mu + r\omega\omega(-\vec\lambda) = r\dot\omega\vec\mu - r\omega^2\vec\lambda = \vec a_\mu + \vec a_\lambda = \vec a_T + \vec a_N$

$\vec a_T = r\dot\omega\vec\mu$ la direzione dipende dal segno di $\dot\omega$

$\vec a_N = -r\omega^2\vec\lambda$ sempre verso il centro di rotazione

Moto con X_0 e Y_0 costanti: moto rotatorio

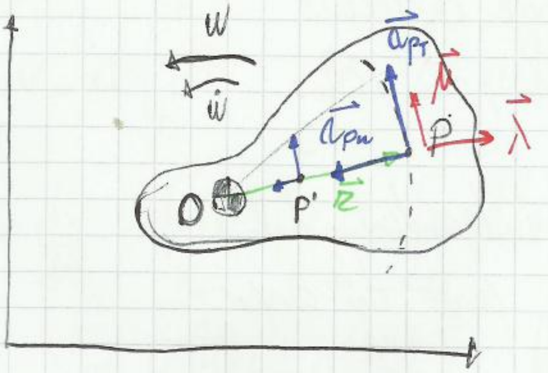


$\vec V_p = \omega r \vec\mu$

$\vec V_{p'} = \omega \overline{OP}\,\vec\mu$

$\overline{OQ} = \overline{OP}$

$\vec V_Q = \omega \overline{OQ}\,\vec\mu'$



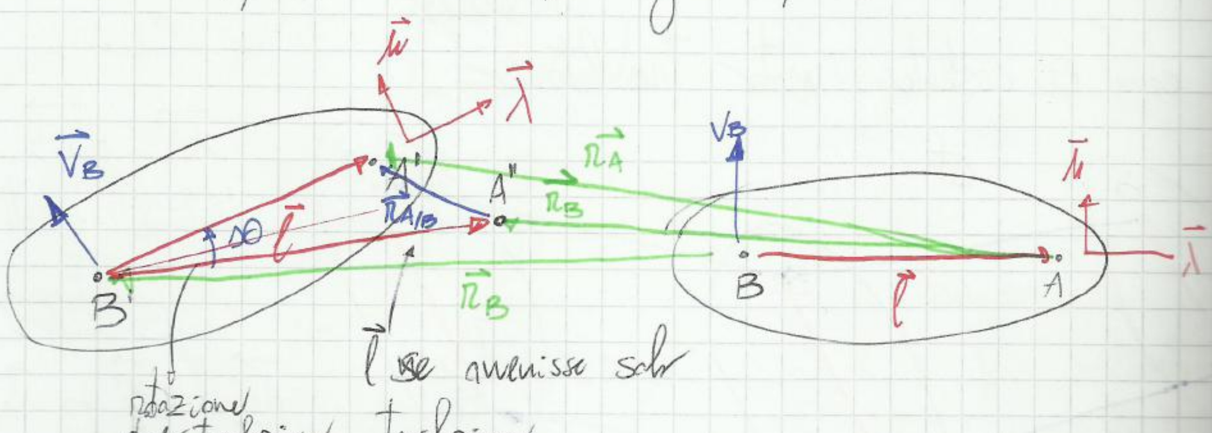
$\vec a_p = \vec a_{pn} + \vec a_{pt}$

$\vec a_{pn} = r\omega^2(-\vec\lambda)$

$\vec a_{pt} = r\dot\omega\vec\mu$

$\vec V$ e $\vec a$ hanno valori proporzionali alla distanza da O

Moto con $X(t), Y(t), \theta(t)$: moto piano generico/rototraslatorio



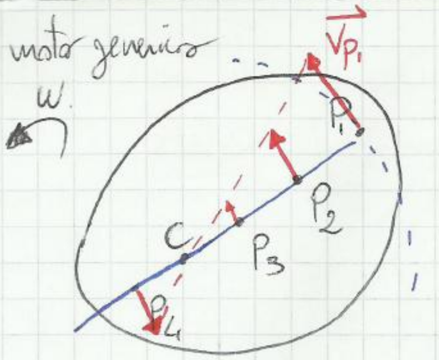
$\vec r_A = \vec r_B + \vec r_{A/B}$

$\vec r_{A/B}$ = A intorno a B (moto circolare)

$\lim_{\Delta t \to 0} \frac{\vec r_A}{\Delta t} = \vec V_A \quad \Rightarrow \quad \vec V_A = \vec V_B + \vec V_{A/B}$ formula fondamentale della cinematica

Moto d'istantanea rotazione

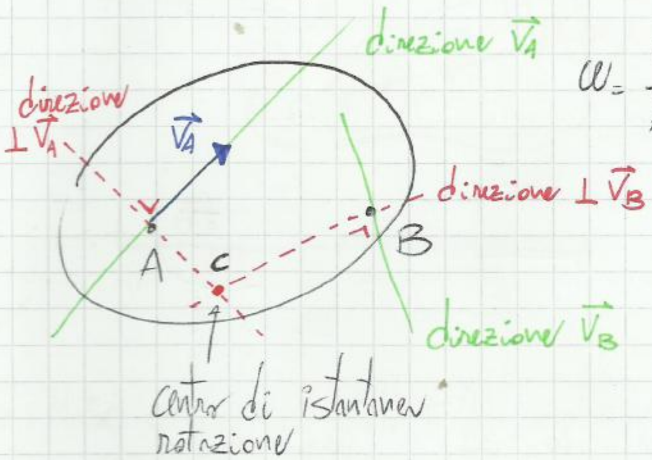
In un certo istante la velocità di un punto solidale al corpo la cui velocità sia nulla.
Il punto diventa il centro di rotazione degli altri punti nell'istante di tempo t.



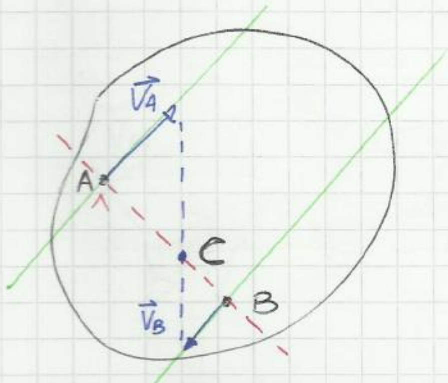
$\vec{V}_C = 0 \quad (t)$

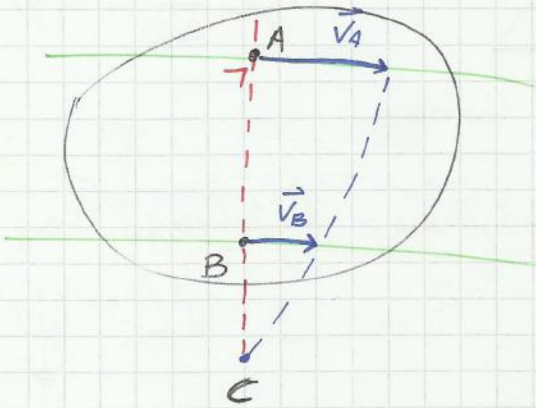
$|\vec{V}_{P_1}| = \omega \overline{CP_1}$
$V_{P_2} = \omega \overline{CP_2}$
$V_{P_3} = \omega \overline{CP_3}$
$V_{P_4} = \omega \overline{CP_4}$

tutte parallele tra loro

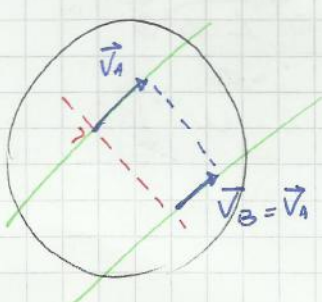


$\omega = \frac{V_A}{AC}$ ↻ $\qquad V_B = \omega \overline{BC}$



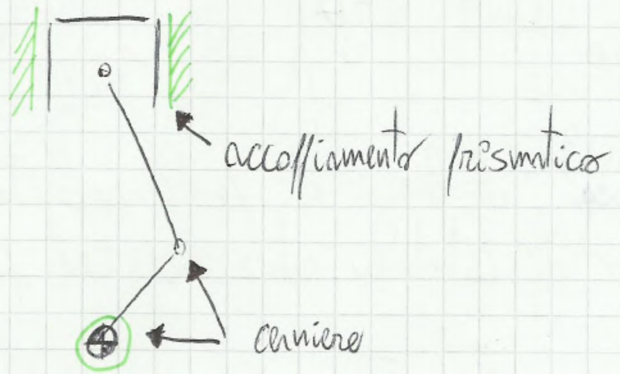


A e B devono essere sulla stessa retta ⊥ alle loro direzioni delle velocità
C si troverà anch'esso sulla retta



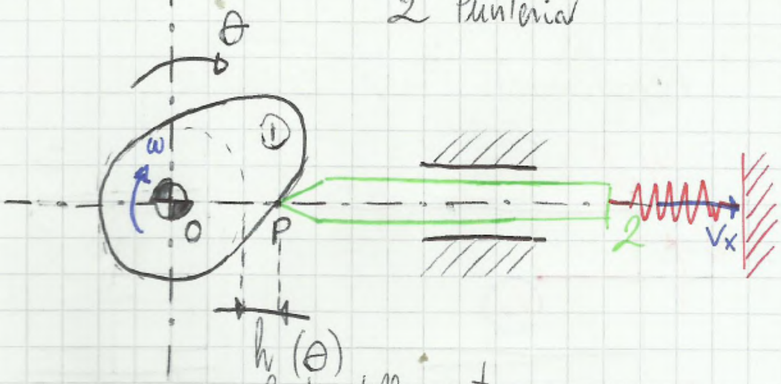
$V_A = V_B \quad AC = BC = \infty \quad \omega = \frac{V_A}{\infty} = 0$

Sistema biella manovella

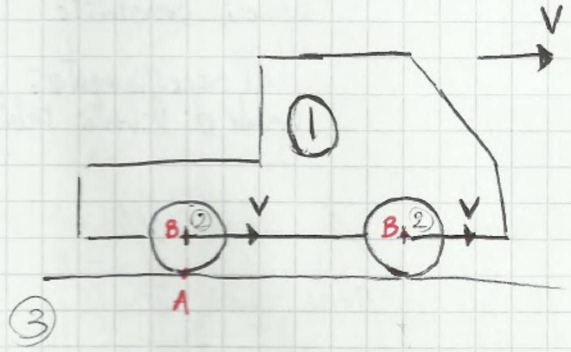


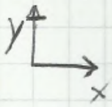
Accoppiamento di forza: le forze tra i corpi determinano i moti

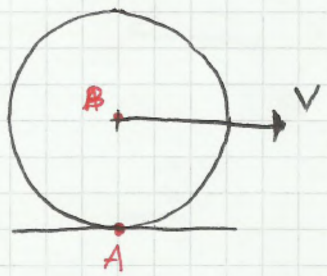
1 Camma a disco
2 Punteria



alzata della punteria





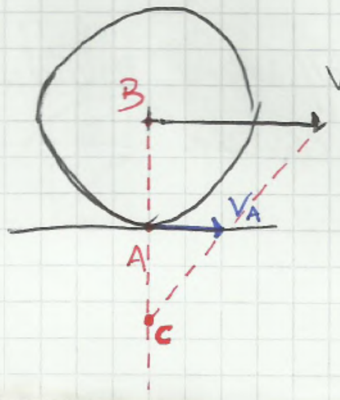


$V_A = 0 \rightarrow$ no slittamento

$\Downarrow$

A centro d'istantanea rotazione

$$\omega = \frac{V}{AB} = \frac{V}{r}$$



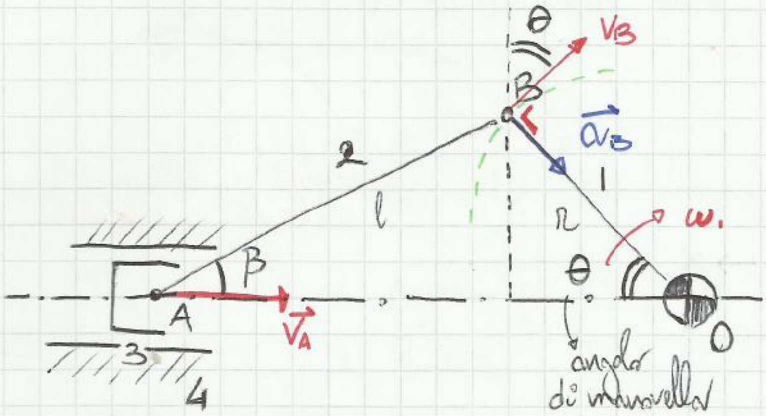
$V_A \neq 0 \rightarrow$ slittamento

C centro d'istantanea rotazione

$$\omega' = \frac{V}{BC} = \frac{V}{r+AC} < \frac{V}{r}$$

quando C è a contatto con il terreno $\theta = 0$:

$$\begin{cases} x_C = 0 \\ \dot{x}_C = 0 \\ \ddot{x}_C = 0 \end{cases} \qquad \begin{array}{l} y_C = 0 \\ \dot{y}_C = 0 \Rightarrow \vec{V}_C = 0 \text{ centro d'istantanea rotazione} \\ \ddot{y}_C = r\omega^2 \Rightarrow \vec{a}_C = r\omega^2 \vec{j} \end{array}$$



Biella-manovella

r, l, θ

$\omega_1 = 1500$ rpm

$\dot{\omega}_1 = 0$

$\vec{V}_A, \vec{a}_A = ?$

$\omega_2, \dot{\omega}_2 = ?$

$r\sin\theta = l\sin\beta \rightarrow \beta$

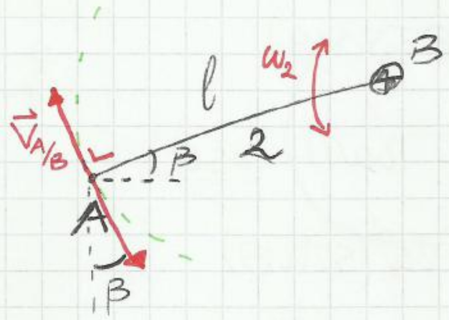
La formula f. cinematica si può applicare solo a punti dello stesso corpo.

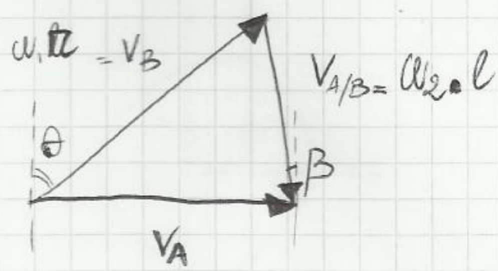
$V_B = \omega_1 r$

$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$

M $\quad ? \quad \omega_1 r \quad (\omega_2 l)?$

D/V —— 





$\omega_1 r\cos\theta = \omega_2 l\cos\beta \rightarrow \omega_2$ verso

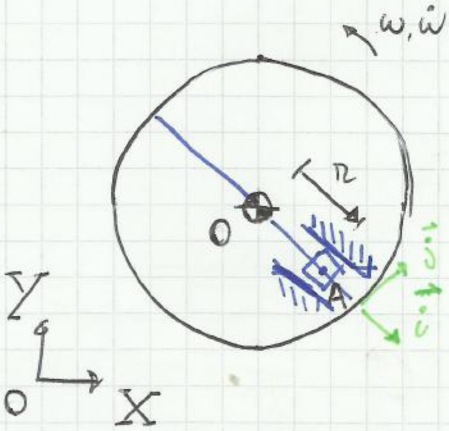
$V_A = \omega_1 r\sin\theta + \omega_2 l\sin\beta \rightarrow V_A$

$\vec{V}_{TP} = \vec{V}_A + \vec{V}_{T\,P/A}$ $\qquad$ $\vec{V}_{TP/A} = \omega_T r \vec{\mu}$

moto rotatorio attorno ad A

$\vec{a}_P = \vec{a}_{rP} + \vec{a}_{tP} + \vec{a}_{cP}$ $\qquad$ $\vec{a}_{TP} = \vec{a}_A + \vec{a}_{tP/A}$ $\qquad$ $\vec{a}_{t\,P/A} = \omega_t^2 r(-\vec{\lambda}) + \dot{\omega}_T r \vec{\mu}$

complementare o di Coriolis $= 2\vec{\omega}_t \times \vec{V}_{rP}$



$\omega = 4$ rad/s $\qquad$ $\dot{\omega} = -10$ rad/s^2

$r = \overline{OA} = 150$ mm

$\dot{r} = 125$ mm/s

$\ddot{r} = 2025$ mm/s^2

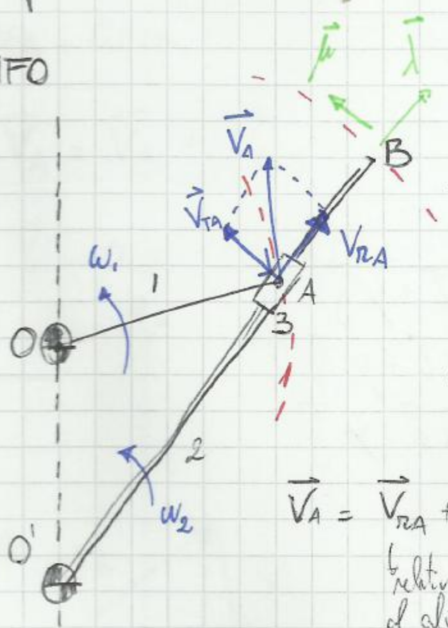
$\vec{V}_A,\ \vec{a}_A = ?$

$$\vec{V}_A = \vec{V}_{rA} + \vec{V}_{TA} = \dot{r}\vec{i} + r\omega\vec{j}$$

$$\vec{a}_A = \vec{a}_{rA} + \vec{a}_{tA} + \vec{a}_{cA} = \ddot{r}\vec{i} + \omega_t^2 r(-\vec{i}) + \dot{\omega}_t r\vec{j} + 2\omega_t\dot{r}\vec{j}$$

$$|\vec{V}_A| = \sqrt{\dot{r}^2 + r^2\omega^2} \qquad |a_A| = \sqrt{(\ddot{r} - r\omega_t^2)^2 + (r\dot{\omega}_t + 2\omega_t\dot{r})^2}$$

GLIFO



1- manovella

2- glifo

3- corsoio

A cerniera tra 1 e 3 e ruota attorno ad O

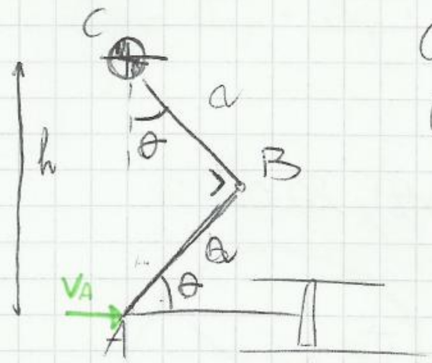
B ruota attorno a O'

$\vec{V}_A = \vec{V}_{rA} + \vec{V}_{TA}$ $\qquad$ $\omega_1 \overline{OA} = \vec{V}_A$

relativa al glifo $\qquad$ $\omega_2 \overline{O'A} = \vec{V}_{TA}$

$$\vec{a}_A = \omega_1^2 OB\cos\theta + \omega_2^2 OB\cos\beta - \dot{\omega}_2 AB\sin\beta = 318.5\ m/s^2$$

1.10



(A e C non allineati)

$V_A = 0.5$ m/s $\quad h = 175$ mm

$a = 125$ mm

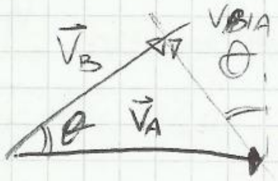
ω_{BC}, ω_{AB}

$$h = a\cos\theta + a\sin\theta \qquad h^2 = a^2(1 + \underbrace{2\cos\theta\sin\theta}_{\sin 2\theta})$$

$$\theta = \frac{1}{2}\arcsin\left(\frac{h^2}{a^2} - 1\right) = 36.9°$$

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$

	$\vec{V}_B$	$\vec{V}_A$	$\vec{V}_{B/A}$
M	?	V_A	?
D	$\perp BC$	—	$\perp AB$
V	?	$\rightarrow$	?



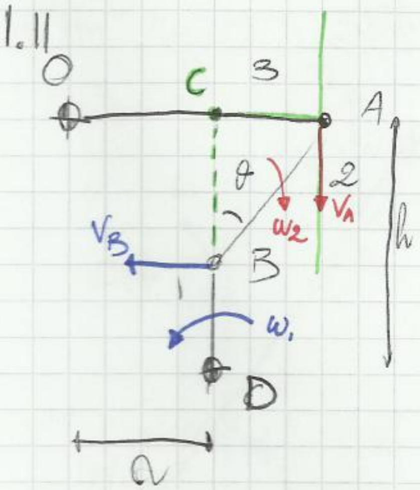
$$V_{B/A} = V_A\sin\theta$$

$$\omega_{AB} AB = V_A\sin\theta \qquad \omega_{AB} = 2.4\ \frac{rad}{s}$$

$$V_B = V_A\cos\theta$$

$$\omega_{BC} = \frac{V_A\cos\theta}{a} = 3.2\ \frac{rad}{s}$$

1.11



$\omega_1 = 2$ rad/s $\qquad \omega_2, \omega_3, a_A = ?$

$OA = 100$ mm

$DB = 75$ mm

$h = 250$ mm

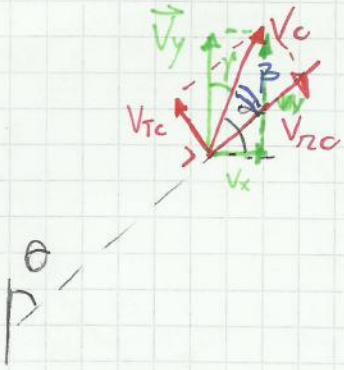
$a = 50$ mm

$$tg\theta = \frac{OA - 50}{250 - BD} \Rightarrow \theta = 15.95°$$

$$V_B = \omega_1 BD = 0.15\ m/s$$

C centro d'istantanea rotazione di AB

$$V_B = \omega_2 BC \qquad \omega_2 = \frac{V_B}{BC} = 0.87\ \frac{rad}{s}$$



$\beta = 15°$

$V_{rc} = V = 0{,}1\ m/s$

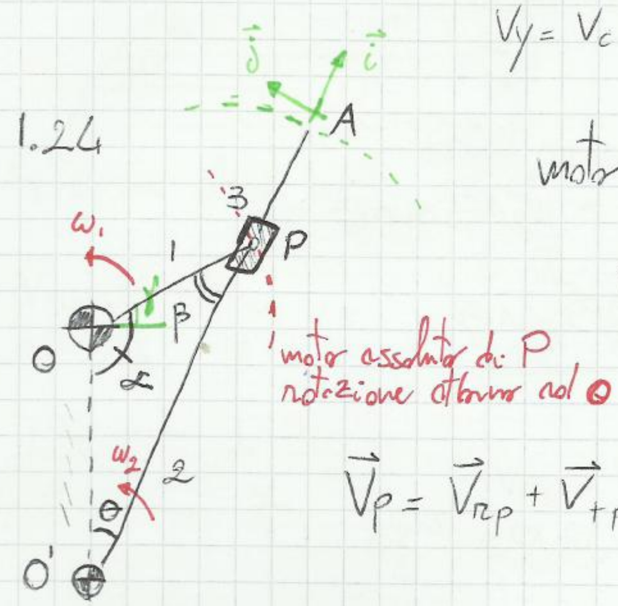
$V_c = \dfrac{V_{rc}}{\cos\beta} = \dfrac{0{,}1}{\cos 15°}$

$V_{Tc} = \omega OC = V_{rc}\, tg\beta$

$\gamma = 30°$

$V_y = V_c \cos\gamma = \dfrac{V\cos\gamma}{\cos\beta} = \dfrac{\cos 30°}{\cos 15°} = 0{,}09\ m/s$

1.24



moto relativo di P: scorrimento // a $\vec{i}$

moto di trascinamento di P: rotazione attorno ad O'

$\vec{V}_P = \vec{V}_{rP} + \vec{V}_{tP}$

$\overline{OP} = 0.3\ m$

$\overline{O'A} = 0.8\ m$

$\overline{OO'} = 0.4\ m$

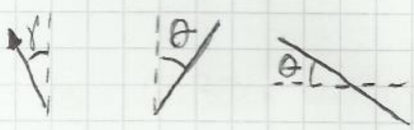
$\theta = 25°$

$\omega_1 = 100\ \dfrac{rad}{s}$ cost. $\Rightarrow \dot{\omega}_1 = 0$

$\vec{\omega}_2,\ \vec{V}_A,\ \dot{\vec{\omega}}_2,\ \vec{a}_A$?

$\vec{V}_P = \vec{V}_{rP} + \vec{V}_{tP}$

$\omega_1 OP$? $(\omega_2 O'P)$?



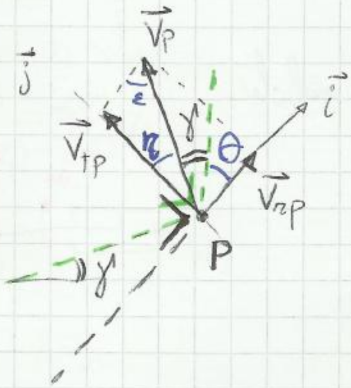
$\dfrac{OO'}{\sin\beta} = \dfrac{OP}{\sin\theta} = \dfrac{O'P}{\sin\alpha}$

$\sin\beta = \dfrac{OO'}{OP}\sin\theta \rightarrow \beta$

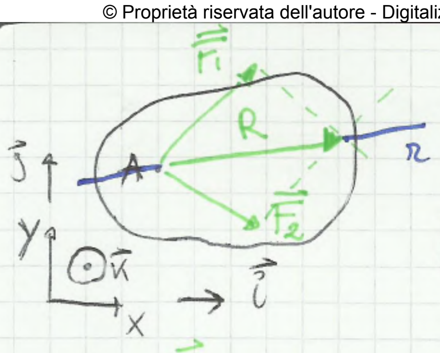
$\alpha = 180° - \theta - \beta$

$O'P = \dfrac{OP\sin\alpha}{\sin\theta}$

$\gamma = \alpha - 90°$

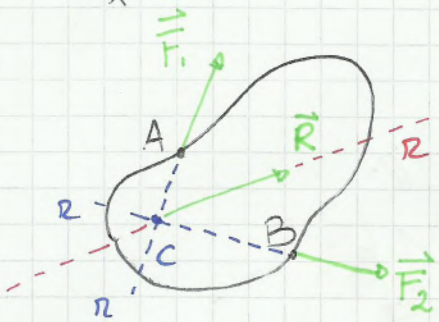


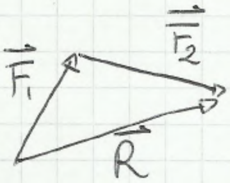
$\varepsilon = \gamma + \theta$

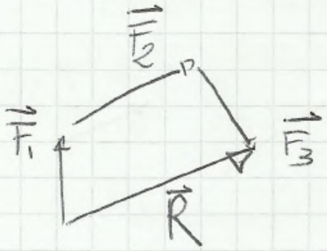


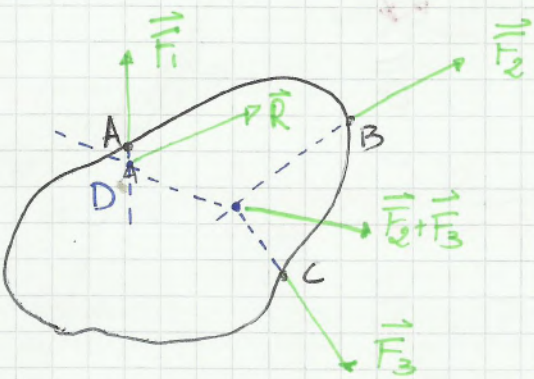
Se le forze appartengono allo stesso piano il moto si svolgerà nel medesimo piano

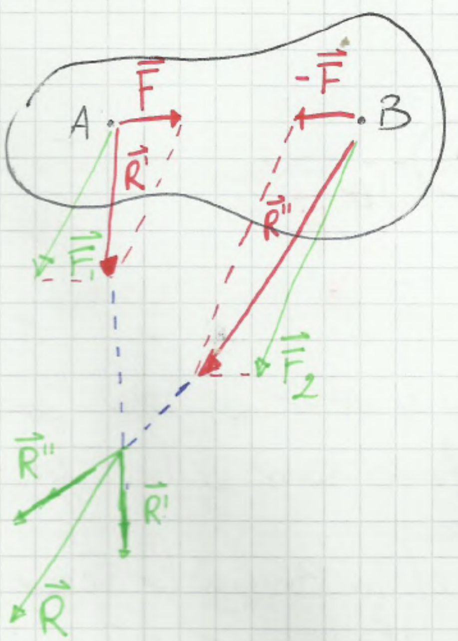
$$\vec{R} = \vec{F_1} + \vec{F_2}$$







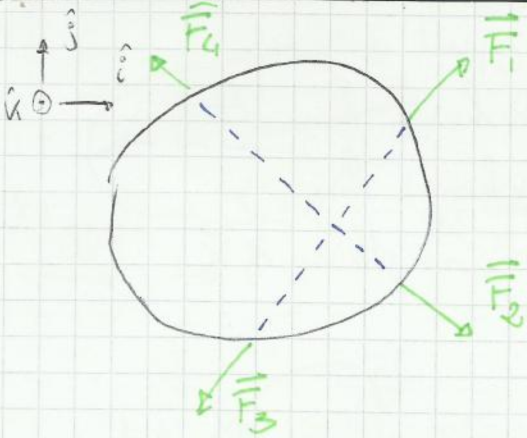




$\vec{F_1} // \vec{F_2}$

F forza arbitraria

$\vec{R} // \vec{F_1} // \vec{F_2}$

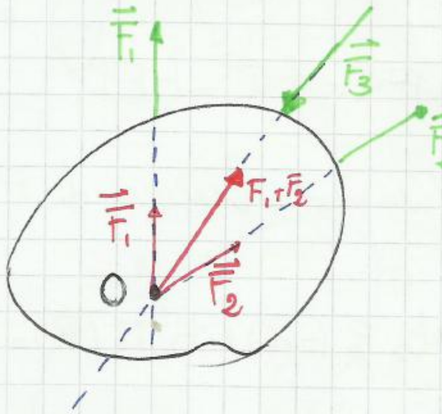


equilibrio $\sum \vec F = 0$

$\sum \vec M_p = 0$

punto $\forall$

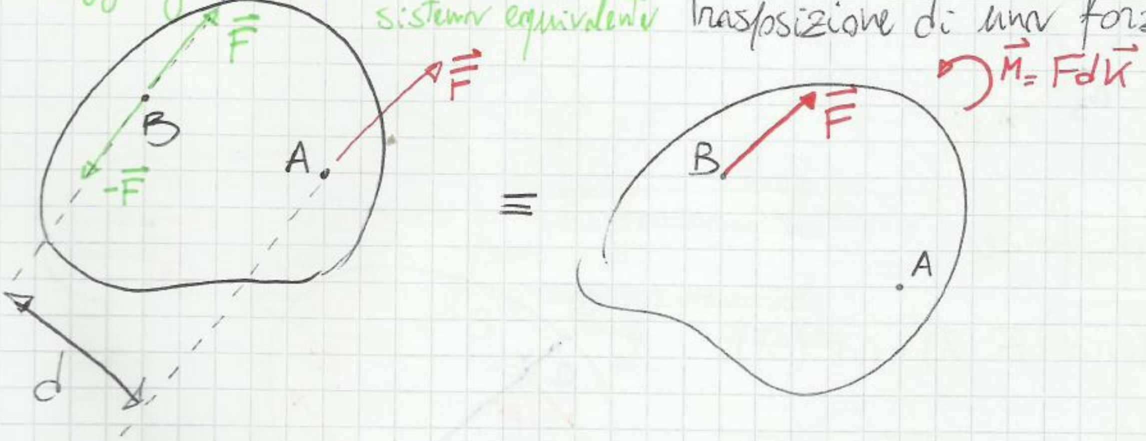
2 forze in equilibrio sono uguali in modulo, opposte in verso ed allineate alla stessa retta d'azione



3 forze sono in equilibrio se la loro risultante è nulla e se concorrono tutte allo stesso punto O

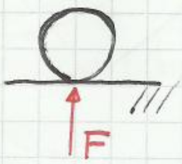
aggiungendo $\vec F$ e $-\vec F$ si ottiene un sistema equivalente

trasposizione di una forza



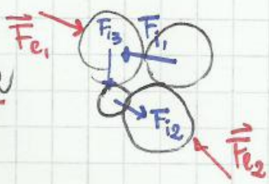
Tipi di Forze

Forze concentrate:



Forze distribuite

Forze interne ed esterne



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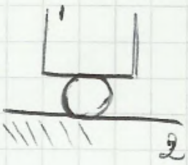
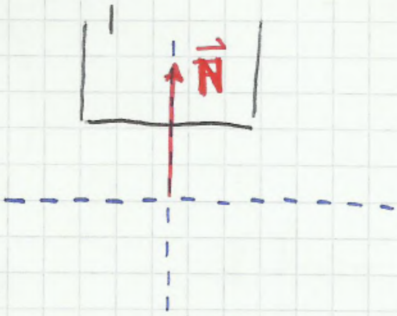
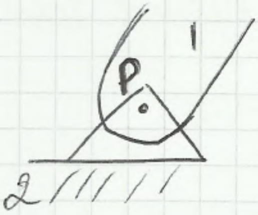
appoggio semplice

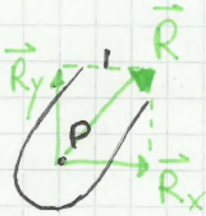




direzione forza: ⊥ al piano
verso: premente

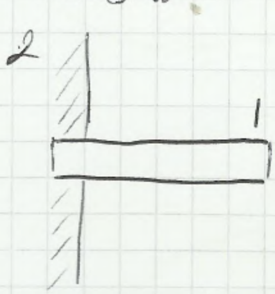
Cerniera

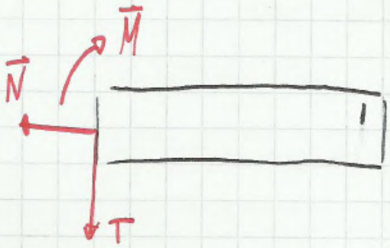




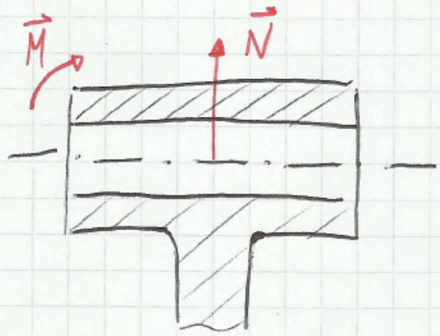
direzione forza: qualunque nel piano del moto relativo

Incastro

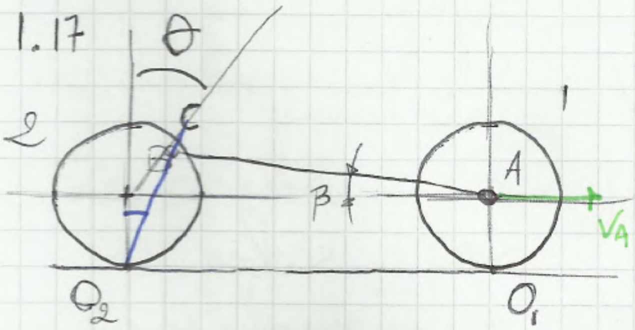




Accoppiamento prismatico



1.17



$AC = 800$ mm

$r = 250$ mm

$BC = 200$ mm

$V_A = 5$ m/s

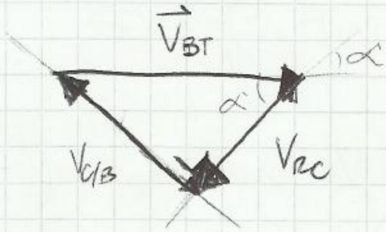
con $\theta = 0$

ω_{AC}, V_C, V_B

$V_A = \omega_1 AO_1 \qquad \omega_1 = \frac{V_A}{r} = 20 \frac{rad}{s}$

$\vec{V}_C = 0 = \vec{V}_{RC} + \vec{V}_{TC} = \vec{V}_{RC} + \vec{V}_{B_T} + \vec{V}_{C/B_T}$

	$\vec{V}_{RC}$	$\vec{V}_{B_T}$	$\vec{V}_{C/B_T}$
M	?	$\omega_1 OB$	$(\omega_2 CB)$
D	$\angle\alpha$	—	$\alpha \perp$
V	?	$\rightarrow$	?



$V_{RC} = V_B \cos\alpha = 1{,}15 \; m/s$

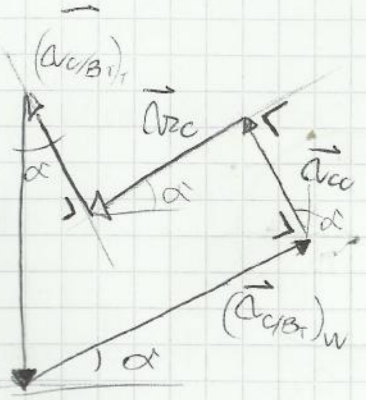
$V_{C/B} = V_B \sin\alpha = 0.68 \; m/s$

$\omega_2 = \frac{V_{C/B}}{BC} = 0.74 \; \frac{rad}{s}$ ↻

$\vec{a}_C = 0 = \vec{a}_{RC} + \vec{a}_{TC} + \vec{a}_{CC} \qquad \vec{a}_{c} = 2\vec{\omega}_T \times \vec{V}_R = 2\vec{\omega}_2 \times \vec{V}_R$

$0 = \vec{a}_{RC} + \vec{a}_{B_T} + (\vec{a}_{C/B_T})_N + (\vec{a}_{C/B_T})_T + \vec{a}_{CC}$

	$\vec{a}_{RC}$	$\vec{a}_{B_T}$	$(\vec{a}_{C/B_T})_N$	$(\vec{a}_{C/B_T})_T$	$\vec{a}_{CC}$
M	?	$\omega_1^2 OB$	$\omega_2^2 BC$	$(\dot\omega_2 BC)$	$2\omega_2 V_{RC}$
D	$\angle\alpha$	$\mid$	$\angle\alpha$	$\alpha\perp$	$\perp\mu$
V	?	$\downarrow$	$\nearrow$	?	$\alpha\perp$

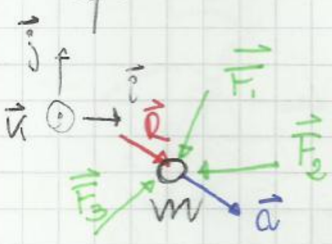


$(a_{C/B_T}) + a_{CO} = a_{B_T} \cos\alpha$

$\dot\omega_2 BC + 2\omega_2 V_{RC} = \omega_1^2 OB \cos\alpha$

$\dot\omega_2 = 6{,}25 \; \frac{rad}{s}$ ↻

Equazioni cardinali della dinamica

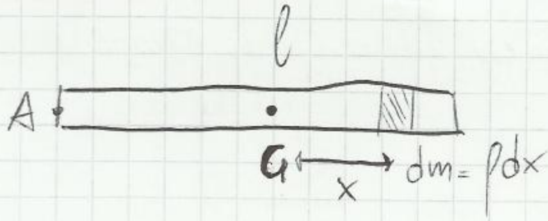


$\vec{R} = \sum \vec{F}_e = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

se $\vec{R} = 0 \rightarrow \vec{V} = cost \begin{cases} 0 \\ M.R.U \end{cases}$ 1ª legge della dinamica

se $\vec{R} \neq 0 \quad \sum \vec{F}_e = m\vec{a}$ 2ª legge della dinamica (o di Newton)

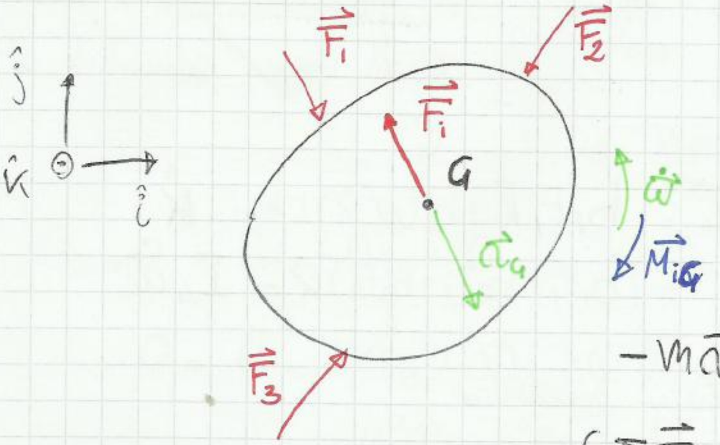
$\vec{a}$ è // ed ha lo stesso verso di $\vec{R}$



$$I_G = \int x^2 dm = \rho \int_{-\frac{l}{2}}^{\frac{l}{2}} x^2 dx = \rho \frac{l^3}{12} = \frac{ml^2}{12}$$

$$I_A = \int x^2 dm = \rho \int_0^l x^2 dx = \rho \frac{l^3}{3} = \frac{ml^2}{3}$$

$$I_A = I_G + m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{ml^2}{3}$$



$$\begin{cases} \sum F_e = m\vec{a}_G \\ \sum M_{eG} = I_G \dot{\vec{\omega}} \end{cases}$$

equazioni cardinali della dinamica (Newton)

$$-m\vec{a} = \vec{F}_{inerzia}$$

$$\begin{cases} \sum \vec{F}_e + \vec{F}_i = 0 \\ \sum M_{eG} + M_{iG} = 0 \end{cases}$$

equilibrio dinamico

$$-I_G \dot{\vec{\omega}} = \vec{M}_{inerzia}$$

momento risultante delle forze d'inerzia

equazioni cardinali della dinamica (d'Alembert)

$$\begin{cases} \sum \vec{F}_e + \vec{F}_i = 0 \\ \sum M_{eG} + M_{iG} = 0 \end{cases}$$

$$\begin{cases} \bullet \ \sum F_{ex} + F_{ix} = 0 \\ \bullet \ \sum F_{ey} + F_{iy} = 0 \\ \sum F_{ez} + F_{iz} = 0 \\ \sum M_{eGx} + M_{iGx} = 0 \\ \sum M_{eGy} + M_{iGy} = 0 \\ \bullet \ \sum M_{eGz} + M_{iGz} = 0 \end{cases}$$

• nel piano ij

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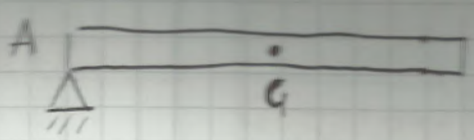
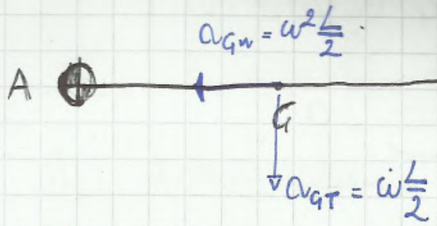


$m = \rho L$

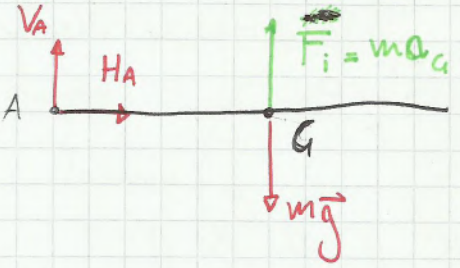
moto incipiente

$\omega = 0$ e $\dot\omega \neq 0$ a $t=0$

$a_{G\omega} = 0$ a $t = 0$



$\theta, \omega, \dot\omega$



$M_{iG} = I_G \dot\omega$

$$\begin{cases} \sum F_{ex} + F_{ix} = 0 \\ \sum F_{ey} + F_{iy} = 0 \\ \sum M_e + M_i = 0 \end{cases}$$

$$\begin{cases} \rightarrow H_A = 0 \\ \uparrow V_A + m a_G - mg = 0 \\ A \circlearrowleft \; m a_G \frac{L}{2} - mg\frac{L}{2} + I_G \dot\omega = 0 \end{cases}$$

$$a_G = \frac{L}{2}\dot\omega \qquad I_G = \frac{mL^2}{12}$$

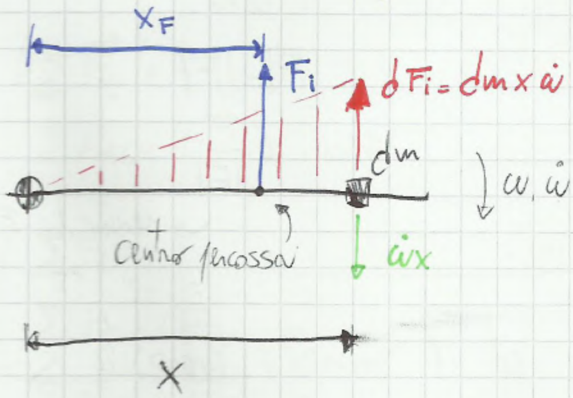
$$\cancel{m}\frac{L}{2}\frac{L}{2}\dot\omega - \cancel{m}g\frac{\cancel{L}}{2} + \frac{\cancel{m}L^{\cancel{2}}}{12}\dot\omega = 0$$

$$\dot\omega\left(\frac{L}{12} + \frac{L}{4}\right) = \frac{g}{2} \qquad \dot\omega = \frac{g}{\cancel{2}}\frac{3}{\cancel{2}L} = \frac{3}{2}\frac{g}{L}$$

$$a_G = \frac{L}{2}\frac{3}{2}\frac{g}{L} = \frac{3}{4}g$$

$$V_A = mg - m a_G = m\left(g - \frac{3}{4}g\right) =$$

$$V_A = \frac{mg}{4}$$



$\omega, \dot\omega$

$$|F_i| = \int dF_i = \int \dot\omega x\, dm = \rho\dot\omega \int_0^L x\, dx$$

$$|F_i| = \rho\dot\omega \frac{L^2}{2} = m\dot\omega\frac{L}{2} = m a_G$$

$$|M_{iA}| = \int x\, dF_i = \rho\dot\omega \int_0^L x^2 dx$$

$$|M_{iA}| = \rho\dot\omega\frac{L^3}{3} = m\dot\omega\frac{L^2}{3}$$

$$|M_{iA}| = x_F F_i = x_F m \frac{L}{2}\dot\omega$$

$$x_F m \frac{L}{2}\dot\omega = m\dot\omega\frac{L^2}{3} \qquad x_F = \frac{2}{3}L$$

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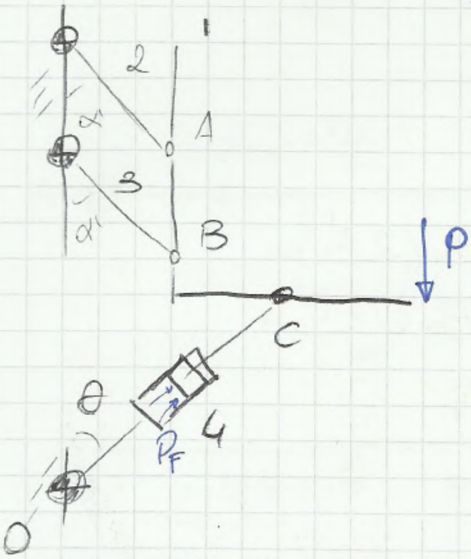
2.



$\theta = 45°$

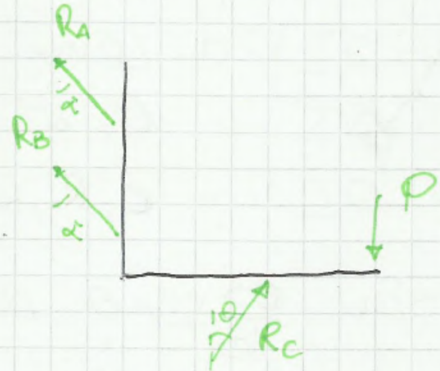
$\alpha = 60°$

$\phi = 5\ cm$

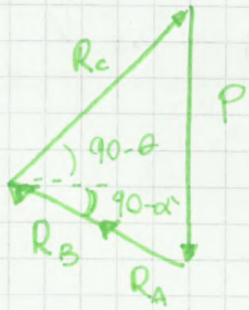
$P = 500\ Kg$

P_F ?

PALA



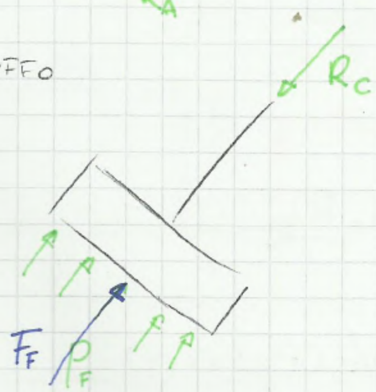
$$\vec{R_A} + \vec{R_B} + \vec{R_C} + \vec{P} = 0$$



$$\frac{P}{\sin(180-\alpha-\theta)} = \frac{P}{\sin(\alpha+\theta)} = \frac{R_C}{\sin\alpha}$$

$$R_C = \frac{P}{\sin(\alpha+\theta)} \sin\alpha = 4398\ N$$

STANTUFFO

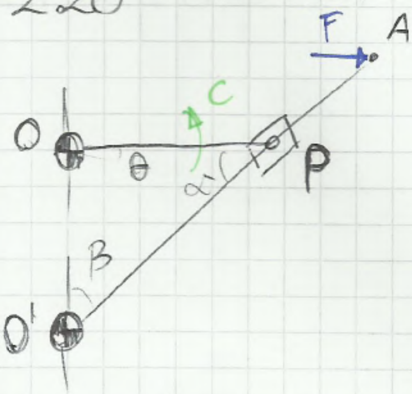


$$F_F = pA = p\frac{\pi d^2}{4}$$

$$F_F = R_C$$

$$p = R_C \frac{4}{\pi \phi^2} = 2.24\ MPa$$

2.20



$OP = 0.3m$

$O'A = 0.8m$

$O'O = 0.4m$

$\theta = 25°$

$F = 100N$

$C, R_O = ?$

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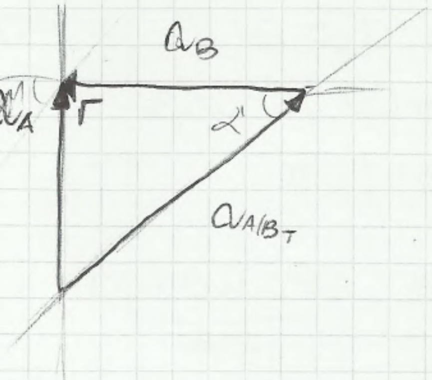
$$\vec{a}_A = \vec{a}_B + (\vec{a}_{A/B})_n + (\vec{a}_{A/B})_T$$

?	$\omega_1^2 OB$	0	$(\dot{\omega}_2 AB)$
|	—		α
?	←		?



$a_B = \omega_1^2 OB = 4{,}1195 \ m/s^2$

$a_A = a_B \, tg\alpha = 1805 \ m/s^2$

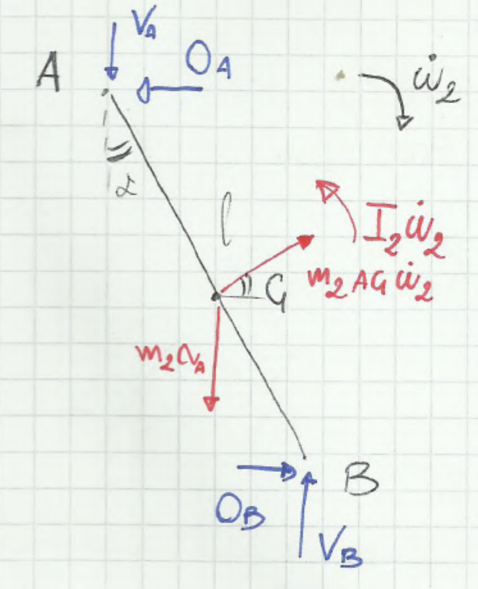
$a_{A/B} \cos\alpha = a_B \qquad a_{A/B} = 44850 \ m/s^2$

$\dot{\omega}_2 = \dfrac{a_{A/B}}{AB}$ ~~[illegible]~~ ?

$$\vec{a}_G = \vec{a}_A + (\vec{a}_{G/A})_n + (\vec{a}_{G/A})_t$$

?	a_A	0	$\dot{\omega}_2 AG$
?	|		α
?	↑		↙

BIELLA



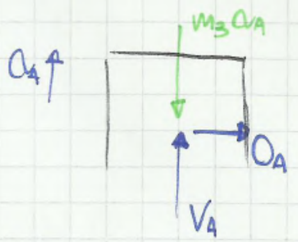
$I_2 = m_2 \rho_2$

$V_A l \sin\alpha + O_A l \cos\alpha + I_2 \dot{\omega}_2 + m_2 a_A BG \sin\alpha +$
$- m_2 AG \dot{\omega}_2 GB = 0$

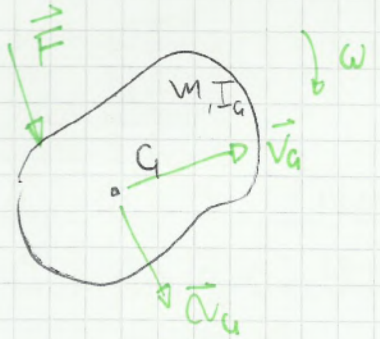
$O_A + m_2 AG \dot{\omega}_2 \cos\alpha - O_B = 0$

PISTONE

$V_A = m_3 a_A$

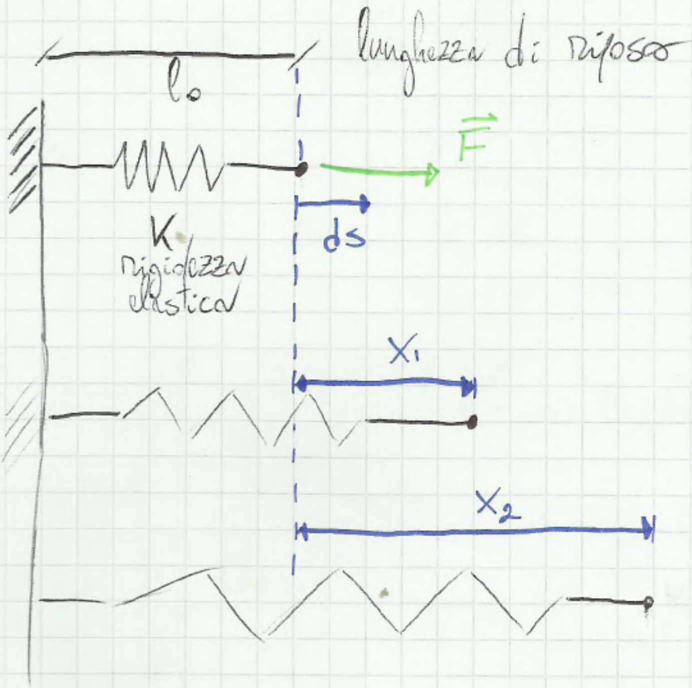


$$L_{1-2} = \int_{v_1}^{v_2} m v \, dv = \frac{1}{2} m (v_2^2 - v_1^2) = \Delta E_c \quad \frac{1}{2} m v^2 = E_c$$



$$E_c = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

$$L_{1-2} = \Delta E_c$$



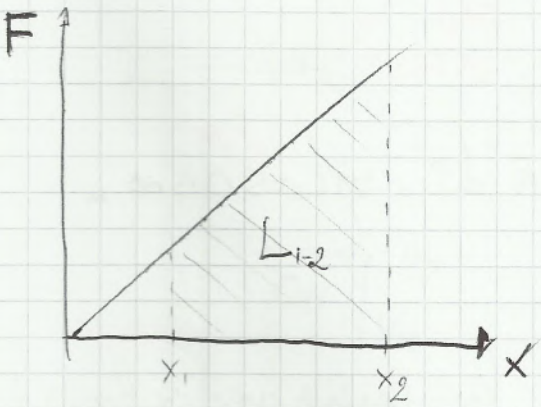
$$F = Kx$$

$$dL = F ds = Kx dx$$

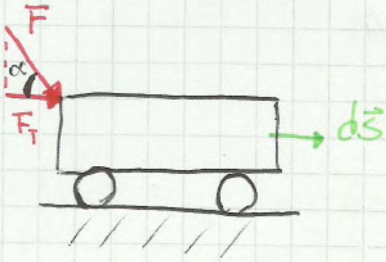
$$L_{1-2} = \int_{x_1}^{x_2} Kx \, dx = \frac{1}{2} K (x_2^2 - x_1^2)$$

$\frac{1}{2} K x^2 = E_E$ energia potenziale elastica

$$L_{1-2} = \Delta E_E = E_{E2} - E_{E1}$$

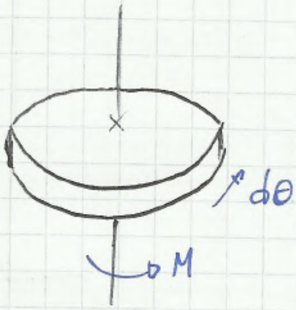


$\frac{dE}{dt} = P$ potenza istantanea



$dL = \vec{F} \cdot d\vec{s} = F_T\, ds$

$P = \frac{dL}{dt} = \frac{F_T\, ds}{dt} = F_T V$

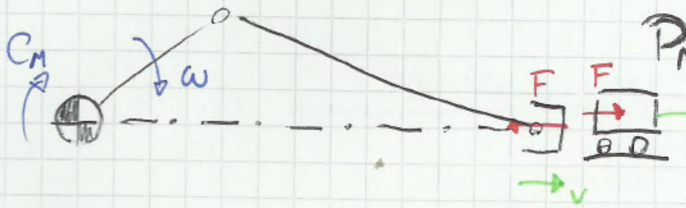


$dL = \vec{M}\, d\vec{\theta} = M d\theta$

$P = \frac{dL}{dt} = \frac{M d\theta}{dt} = M\omega$

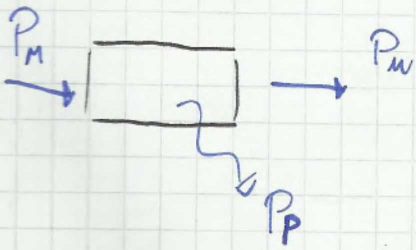
$[L] = N \cdot m = J$

$[P] = \frac{N \cdot m}{s} = \frac{J}{s} = W$



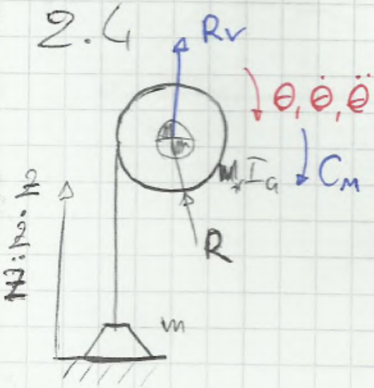
$P_M = C_M \omega$

$P_u = FV$



$\eta = \frac{P_u}{P_M} < 1$

2.4



$z = R\theta$

$m_T = 100\ kg$

$R = 0.15\ m$

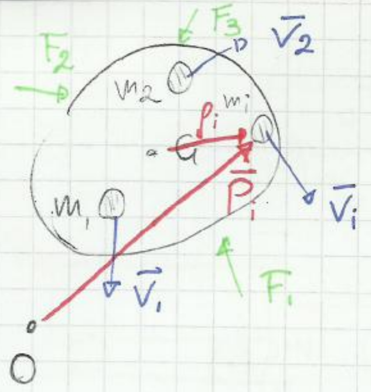
$I_G = \frac{m_T R^2}{2}$

$m = 200\ kg$

$h = 2\ m$

$V = 1\ m/s$

$C_M = ?$



$\sum \vec{M}_{ea}$, $\sum \vec{M}_{eo}$

$Q_i = m_i \vec{V}_i$

$\vec{K}_o = \sum \vec{p}_i \times m_i \vec{V}_i$

$\vec{K}_a = \sum \vec{\rho}_i \times m_i \vec{V}_i$

$$\sum \bar{M}_{ea} = \frac{d\vec{K}_a}{dt}$$

Se O è fisso vale la relazione:

$$\sum \bar{M}_{eo} = \frac{d\vec{K}_o}{dt}$$

$$\begin{cases} \sum \vec{F}_e + \vec{F}_i = 0 \\ \sum \vec{M}_{ea} + \vec{M}_{ia} = 0 \end{cases}$$

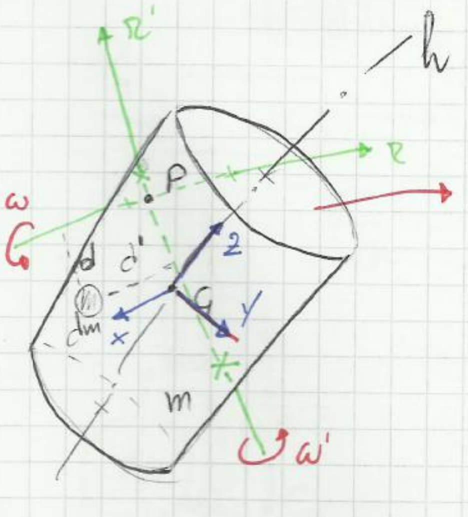
$$\sum \vec{F}_e = \frac{d\bar{Q}}{dt} = \frac{d(m\vec{v}_a)}{dt} = m\frac{d\bar{v}_a}{dt} = m\bar{a}_a$$

$$\vec{F}_i = -\frac{d\bar{Q}}{dt}$$

nel piano $\vec{M}_{ia} = -I_a \dot{\bar{\omega}}$ le forze e le velocità appartengono allo stesso piano

nello spazio $\sum \vec{M}_{ea} = \frac{d\vec{K}_a}{dt} \rightarrow \vec{M}_{ia} = -\frac{d\vec{K}_a}{dt}$

$\vec{M}_{io} = -\frac{d\vec{K}_o}{dt}$ se O è fisso



$\bar{\omega}$

$dI_r = d^2 \cdot dm$

$I_r = \rho_r^2 \cdot m$ ρ_r raggio d'inerzia (si ipotizza tutta la massa concentrata a distanza ρ_r dall'asse)

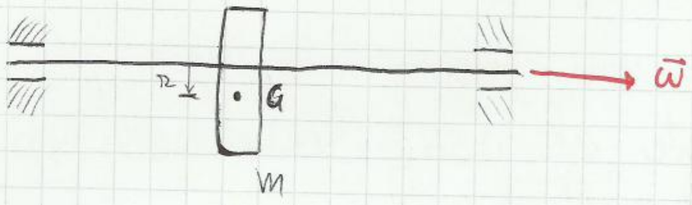
$dI'_r = d'^2 \cdot dm$

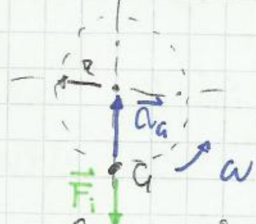
$I'_r = \rho_r'^2 \cdot m$

×P

Esistono infiniti assi di rotazione r, quindi infiniti momenti d'inerzia. Tra questi esiste un massimo $I_{p\,max}$ ed un minimo $I_{p\,min}$, tra loro perpendicolari ~~tra loro~~. Trovando un terzo asse perpendicolare ai due precedenti (con

Rotore con squilibrio statico (staticamente esiste una condizione d'equilibrio

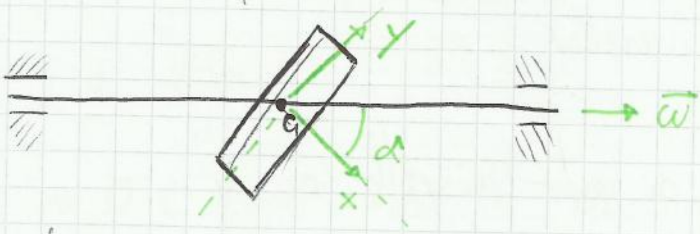




$a_G = r\omega^2$

$F_i = -m a_G$

Rotore con squilibrio dinamico

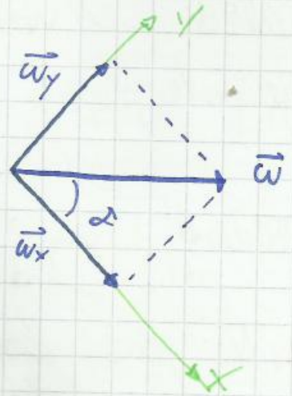


$a_G = 0$

staticamente qualunque posizione è d'equilibrio, se messo in moto genera ~~forze~~ momenti d'inerzia che creano vibrazioni

$\vec{M}_{iG} = -\dfrac{d\vec{K}_G}{dt}$ $\qquad \vec{K}_G = I_x \omega_x \hat{i} + I_y \omega_y \hat{j} + I_z \omega_z \hat{k}$

I_x, I_y, I_z noti ; $\vec{\omega}$ cost.



$$\begin{cases} \omega_x = \omega\cos\alpha \\ \omega_y = \omega\sin\alpha \\ \omega_z = 0 \end{cases}$$

$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = \dot{\omega} = 0$

$\dfrac{d\hat{i}}{dt} = \vec{\omega} \times \hat{i} = \omega\sin\alpha(-\hat{k})$

$\dfrac{d\hat{j}}{dt} = \vec{\omega} \times \hat{j} = \omega\cos\alpha(\hat{k})$

$\dfrac{d\vec{K}_G}{dt} = I_x\, \omega\cos\alpha\, \omega\sin\alpha(-\hat{k}) + I_y\, \omega\sin\alpha\cos\alpha(\hat{k}) =$

$= \omega^2\cos\alpha\sin\alpha(I_y - I_x)\hat{k} = -\vec{M}_{iG}$

$M_{iG} = (I_x - I_y)\,\omega^2\cos\alpha\sin\alpha\,\hat{k}$

se è un disco sottile $I_x \simeq 2I_y \Rightarrow M_{iG}\hat{k} = \vec{M}_{iG}$

$$\frac{d\vec{\lambda}}{dt} = \vec{\omega}_T \times \vec{\lambda} = \omega_1 \vec{k} \times \vec{\lambda} = \omega_1 \underset{\uparrow \; \sin\left(\frac{\pi}{2}-\alpha\right)}{\cos\alpha} (-\vec{\nu})$$

$$\frac{d\vec{\mu}}{dt} = \vec{\omega}_T \times \vec{\mu} = \omega_1 \sin\alpha \, \vec{\nu}$$

$$\vec{K}_a = I_\lambda (\omega_1 \sin\alpha - \omega_2)\vec{\lambda} + I_\mu \, \omega_1 \cos\alpha \, \vec{\mu}$$

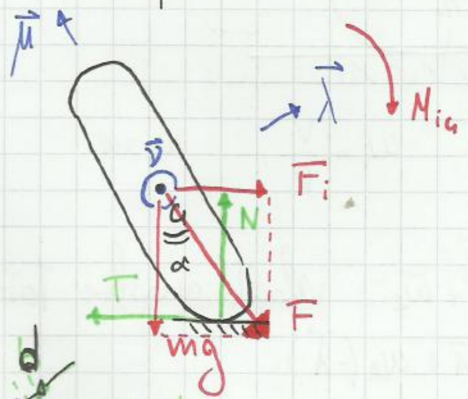
$$\vec{M}_{ia} = -\frac{d\vec{K}_a}{dt} = +I_\lambda(\omega_1 \sin\alpha - \omega_2)\,\omega_1 \cos\alpha \, \vec{\nu} - I_\mu \, \omega_1 \cos\alpha \, \omega_1 \sin\alpha \, \vec{\nu}$$

Se si approssima la ruota ad un disco sottile $I_\lambda \simeq 2 I_\mu$

$$\vec{M}_{ia} = \left(I_\lambda \omega_1^2 \sin\alpha\cos\alpha - I_\lambda \omega_1 \omega_2 \cos\alpha - \frac{I_\lambda}{2} \omega_1^2 \sin\alpha\cos\alpha\right)\vec{\nu}$$

$$\vec{M}_{ia} = \left(\frac{I_\lambda}{2} \omega_1^2 \sin\alpha\cos\alpha - I_\lambda \omega_1 \omega_2 \cos\alpha\right)\vec{\nu} = I_\lambda \omega_1 \cos\alpha \left(\frac{\omega_1 \sin\alpha}{2} - \omega_2\right)\vec{\nu}$$

$\omega_2 > \omega_1$ quindi $\vec{M}_{ia} = |\vec{M}_{ia}|(-\vec{\nu})$

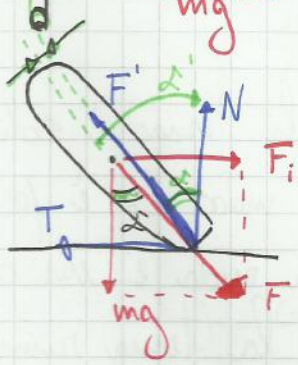


$N = mg$

$T = F_i = m\frac{v^2}{R}$

se non si considera M_{ia}

$$tg\,\alpha = \frac{F_i}{mg} = \frac{mv^2}{Rmg} = \frac{v^2}{gR}$$



F e F' formano una coppia

$Fd = F'd = M_{ia}$

$\alpha' > \alpha$

$L_i + L_e = \Delta E_c + \Delta E_E + \Delta E_g = \Delta E_M$ — in L_i, L_e non si considerano il peso e le forze d'inerzia

se $L_i + L_e = 0 \Rightarrow E_M = E_c + E_E + E_g = cost.$

$\sum \vec{F}_e = \frac{d\vec{Q}}{dt}$ se $\sum \vec{F}_e = 0$ $\vec{Q} = cost.$

$\sum \vec{M}_{ea(o)} = \frac{d\vec{K}_{a(o)}}{dt}$ se $\sum \vec{M}_{ea(o)} = 0$ $\vec{K}_{a(o)} = cost.$

se $f_a = 0$ non c'è perdita di energia (tutta la massa in G - pendolo semplice)

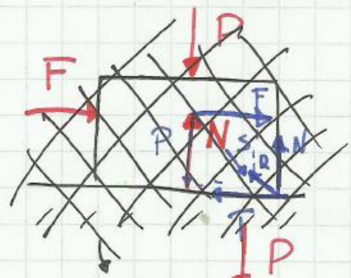
Attrito

- secco → attrito di strisciamento
- fluido → fluido tra le superfici di contatto
- interno → deformazione dei materiali

ATTRITO SECCO

$N = P = mg$

$T = F = N \operatorname{tg} \beta$



Per iniziare il moto $F \geq F_{lim}$

$F_{lim} = N \operatorname{tg} \beta_{lim} \geq T$

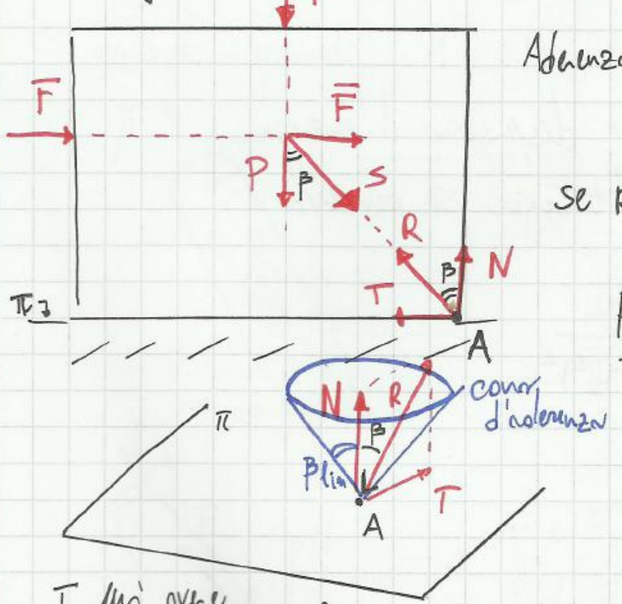
Aderenza: $V_R = 0$ se $F \leq F_{lim} \Rightarrow \beta \leq \beta_{lim}$

↳ velocità relativa

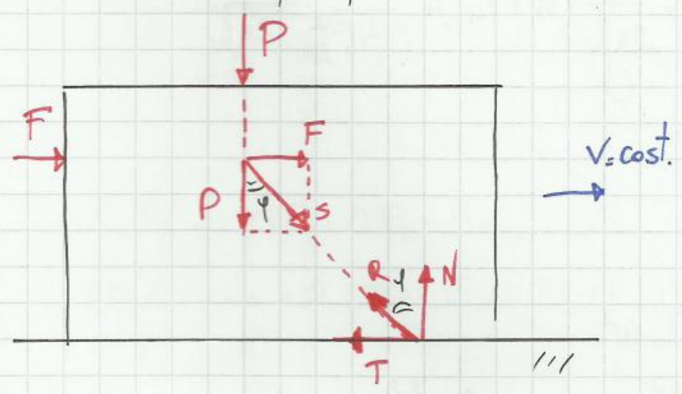
se R è fuori dal cono non c'è più aderenza

$\beta_{lim} = \varphi_A$ angolo di aderenza (o di attrito statico)

$\operatorname{tg} \varphi_A = f_A$ coefficiente di aderenza (o di attrito statico)



T può avere una direzione qualunque su π



Durante lo strisciamento $\varphi = cost.$

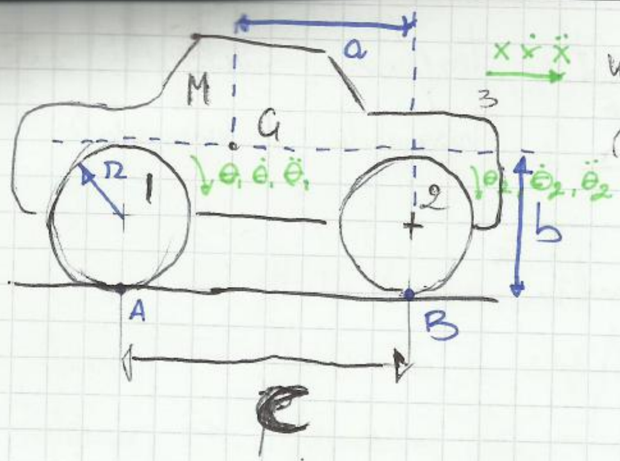
$N = P = mg$ cost.

$T = N \operatorname{tg} \varphi = mg \operatorname{tg} \varphi$ cost.

$F = T + m\ddot{x}$ ← si genera un'accelerazione

$\varphi =$ angolo di attrito (dinamico)

$\operatorname{tg} \varphi = f$ coefficiente di attrito (dinamico)

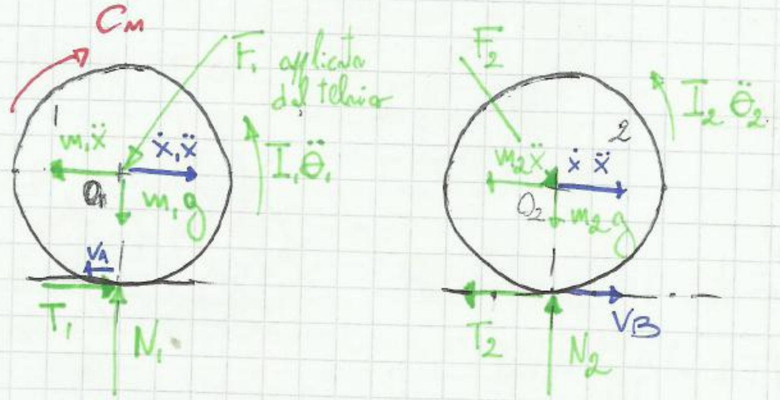


m_1, m_2, M ← massa Totale

a, b, c, r $\quad f, f_A$

C_M (posteriore)

$\ddot{x} = ?$



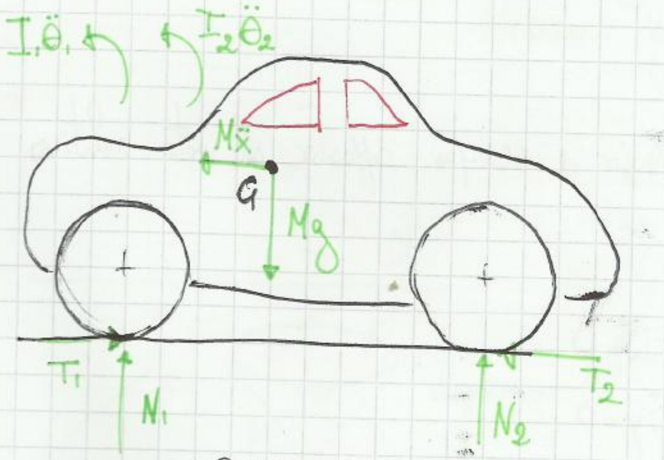
$\rightarrow\; T_1 - T_2 - M\ddot{x} = 0$

$\uparrow\; N_1 + N_2 - Mg = 0$

$B:\; I_1\ddot{\theta}_1 + I_2\ddot{\theta}_2 + M\ddot{x}b + Mga - N_1 c = 0$

$O_1:\; C_M - I_1\ddot{\theta}_1 - T_1 r = 0$

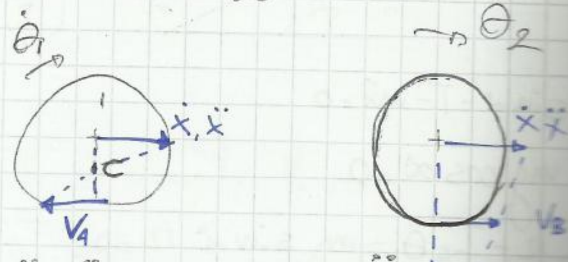
$O_2:\; I_2\ddot{\theta}_2 - T_2 r = 0$



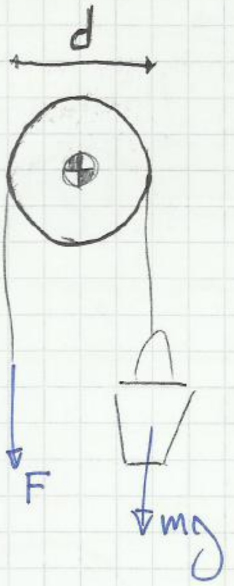
4 situazioni:

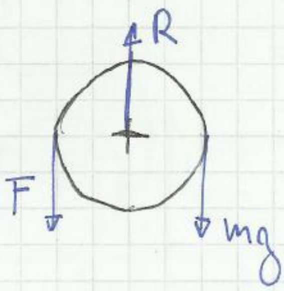
a': aderenza 1
a'': strisciamento 1
b': aderenza 2
b'': strisciamento 2

	IPOTESI	VERIFICA
a'	$\ddot{x} = r\ddot{\theta}_1$	$T_1 \le f_A N_1$
a''	$T_1 = f N_1$	$\ddot{x} < r\ddot{\theta}_1$
b'	$\ddot{x} = r\ddot{\theta}_2$	$T_2 \le f_A N_2$
b''	$T_2 = f N_2$	$\ddot{x} > r\ddot{\theta}_2$



$\ddot{\theta}_1 > \frac{\ddot{x}}{r}$ $\qquad$ $\ddot{\theta}_2 < \frac{\ddot{x}}{r}$

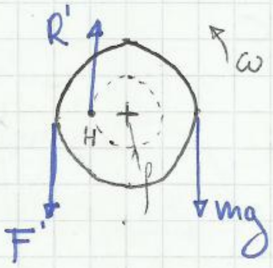




o) $F\frac{d}{2} - mg\frac{d}{2} = 0$

$F = mg$ × fare spostare il carico a velocità costante

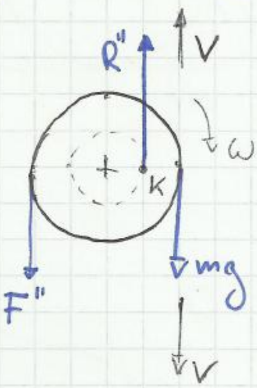
CON ATTRITO



$R' = mg + F'$

$H'^{\circlearrowleft}\ F\left(\frac{d}{2} - \rho\right) - mg\left(\frac{d}{2} + \rho\right)$

$F' = mg\,\dfrac{\frac{d}{2} + \rho}{\frac{d}{2} - \rho}$ $F' > mg$

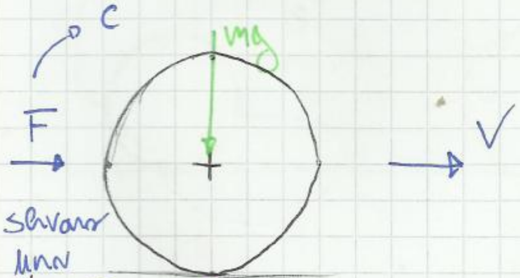


$R'' = mg + F''$

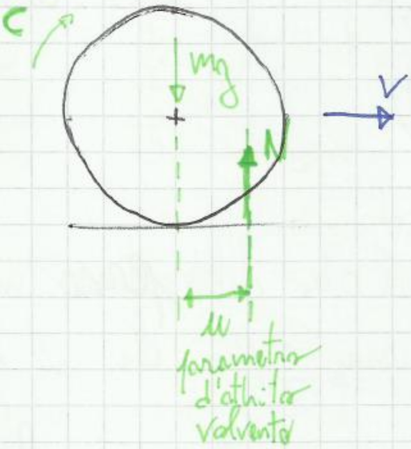
$K^{\circlearrowleft}\ F''\left(\frac{d}{2} + \rho\right) - mg\left(\frac{d}{2} - \rho\right)$

$F'' = mg\,\dfrac{\frac{d}{2} - \rho}{\frac{d}{2} + \rho}$ $F'' < mg$

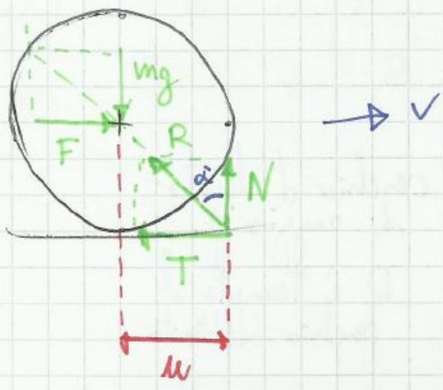
ATTRITO VOLVENTE

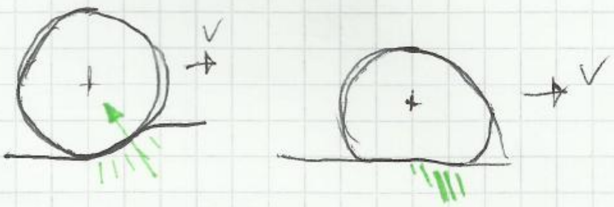


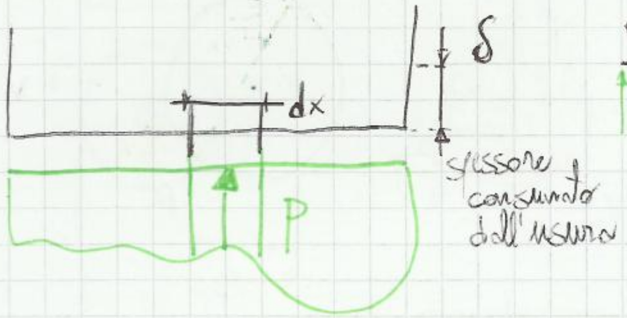
servono una forza F o una coppia C per evitare l'arresto

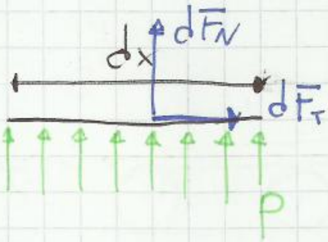


L'attrito volvente dipende dalla deformabilità dovuta al contatto, aumentando la superficie di contatto









in dx $p = cost.$

$$dF_N = p\,dA$$

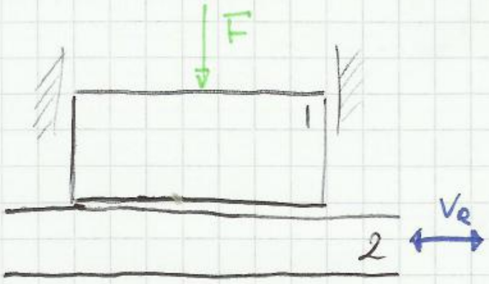
$$dF_T = f\,p\,dA$$

$$\frac{dL}{dt} = dF_T \frac{dx}{dt} = f\,p\,dA\,V_R$$

$Vol = \delta\,dA$

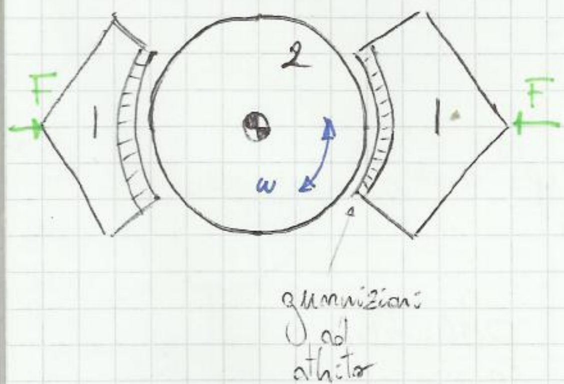
$\delta\,dA \propto f\,p\,dA\,V_R$ ipotesi dell'usura

FRENO A PATTINO

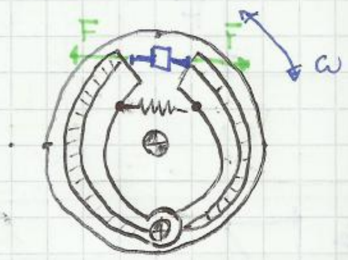


1: pattino

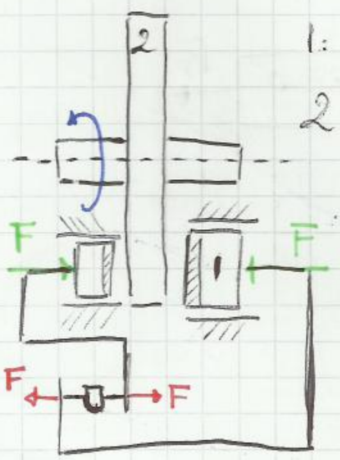
FRENO A TAMBURO (A CEPPO)



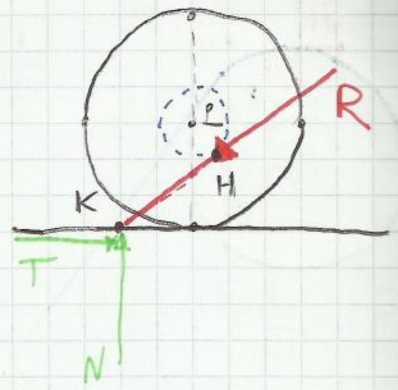
1: ceppo (ganascia)
2: tamburo



FRENO A DISCO



1: pastiglia (pattino)
2: disco



(1) $N = Q - F\cos\beta$

(2) $T = F\sin\beta$

(3K) $F\cos\beta(l+u) + F\sin\beta\, h - Q(a+u) = 0$

(4) $N(u+\rho) = Tr$

(4) $T = N\dfrac{u+\rho}{r} = (Q - F\cos\beta)\dfrac{u+\rho}{r}$

$\rho = r_p \sin\varphi$

$\to$(2) $F = \dfrac{T}{\sin\beta} = \dfrac{(Q - F\cos\beta)}{\sin\beta}\dfrac{u+\rho}{r}$

$r \cdot F\sin\beta = (u+\rho)(Q - F\cos\beta)$

$$F = \frac{(u+\rho)Q}{r\sin\beta + (u+\rho)\cos\beta}$$

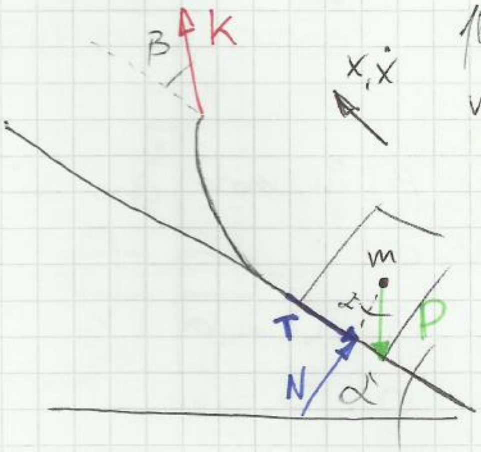
$\to$(3) $\dfrac{Q(u+\rho)\cos\beta(l+u)}{r\sin\beta + (u+\rho)\cos\beta} + \dfrac{hQ(u+\rho)\sin\beta}{r\sin\beta + (u+\rho)\cos\beta} - Q(a+u) = 0$

$\ldots \operatorname{tg}\beta = \dfrac{(u+\rho)[(a+u) - (u+e)]}{h(u+\rho) - (a+u)r} \Rightarrow \beta = 2.86°$

$\Downarrow$

$F = 444\ N$

3.17



pendenza 30%. β / K sia minimo

$m = 500\ kg$

$f = 0.2$

$\tan\alpha = 0.3 \Rightarrow \alpha$

$T = K\cos\beta - P\cos\alpha$

$N = P\cos\alpha - K\sin\beta$

$T = fN$

$T = mg\sin\alpha - m\ddot{\theta}$ $\begin{cases} \alpha = 10° & T = 8284.9\,N \\ \alpha = 45° & T = 26967.2\,N \end{cases}$

per $\alpha = 10°$ la condizione d'aderenza è verificata
per $\alpha = 45°$ si ha strisciamento

(4bis) $T = fN$

$$\ddot{x} = \frac{mg\sin\alpha - fmg\cos\alpha}{m} = 5.89\ m/s^2$$

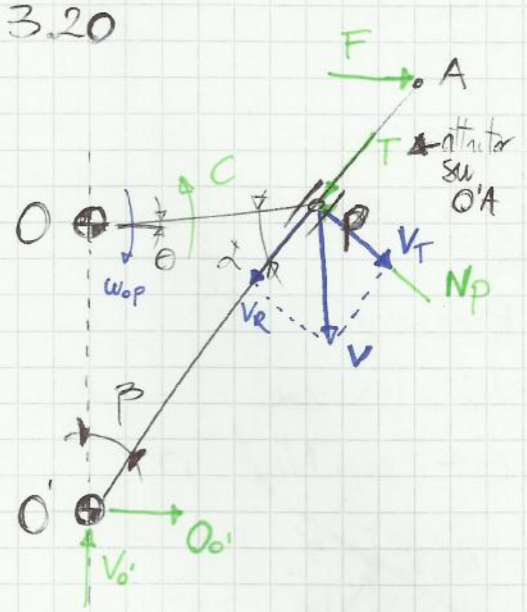
$$\ddot{\theta} = \frac{(fmgr - mgu)\cos\alpha}{\frac{mr^2}{2}} = 3.05\ \frac{rad}{s^2}$$

$x(t) = \underbrace{x_0}_{=0} + \underbrace{\dot{x}_0}_{=0} t + \frac{1}{2}\ddot{x}t^2 \qquad x(t) = \frac{1}{2}\ddot{x}t^2$

$x^* = 200\ m \quad t^* = ? \quad t^* = \sqrt{\frac{2x^*}{\ddot{x}}} = \begin{cases} \alpha = 10° & t^* = 21.3\ s \\ \alpha = 45° & t^* = 8.2\ s \end{cases}$

$\theta^*(t) = \frac{1}{2}\ddot{\theta}t^{*2} = \begin{cases} \alpha = 10° & \theta(t^*) = 63.6\ \text{giri} \\ \alpha = 45° & \theta(t^*) = 16.48\ \text{giri} \end{cases}$

3.20



$OP = 0.3\ m$
$OO' = 0.4\ m$
$\theta = 25°$
$F = 100N$
$f = 0.5$
$\beta = 27.3°$
$\alpha = 37.7°$
$O'P = 0.59\ m$

$C = ? \begin{cases} \omega_{op}\ ↻ \\ \omega_{op}\ ↺ \end{cases}$

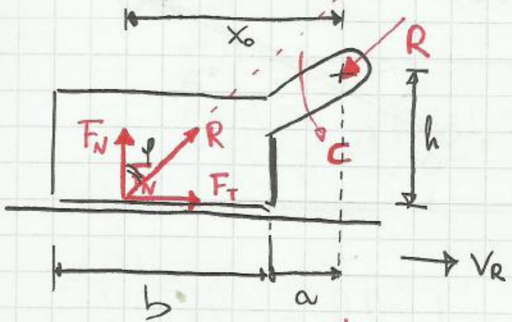
$O' ↑ \quad O'A\cos\beta = N_P O'P$
$N_P = 119.92\ N$
$T = fN_P = 59.86\ N$

$$F_N = \int_A p\,dA = \int_a^{a+b} kx\,dx = \frac{k}{2}\left[(a+b)^2 - a^2\right]$$

$$F_N x_0 = \int_A dF_N\, x \qquad F_N x_0 = \int x\,p\,dA = \int_a^{a+b} kx^2\,dx = \frac{k}{3}\left[(a+b)^3 - a^3\right]$$

$$x_0 = \frac{k}{3}\left[(a+b)^3 - a^3\right] \frac{2}{k} \frac{1}{\left[(a+b)^2 - a^2\right]}$$

$$x_0 = \frac{2}{3}\frac{(a+b)^3 - a^3}{(a+b)^2 - a^2}$$

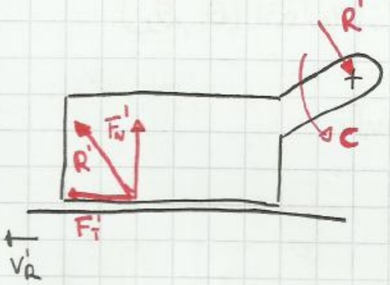


$R = R$

$O \circlearrowleft \quad C + F_T h - F_N x_0 = 0$

$F_N = \frac{F_T}{f}$

$C = F_T\left(\frac{x_0}{f} - h\right) \qquad F_T = \frac{C}{\frac{x_0}{f} - h}$



$R' = R'$

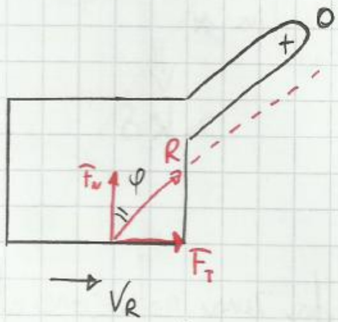
$O \circlearrowleft \quad C - F_T' h - F_N' x_0$

$F_T' = f F_N'$

$C = F_T'\left(\frac{x_0}{f} + h\right)$

$$F_T' = \frac{C}{\frac{x_0}{f} + h}$$

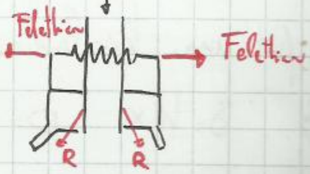
$F_T > F_T'$

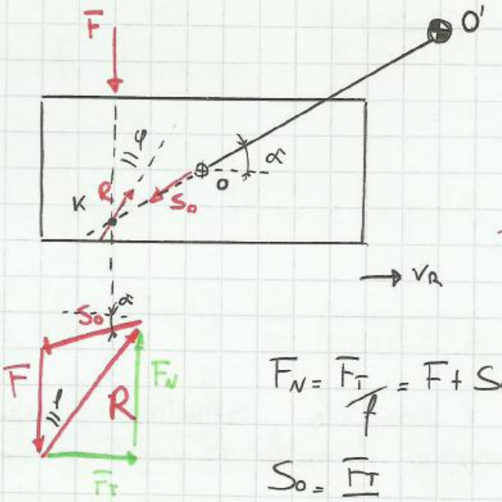


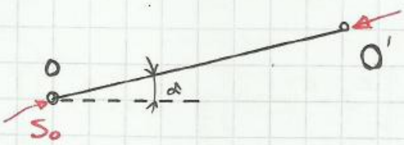
fenomeno dell'impuntamento: il freno si impunta anche se non azionato.

La direzione di R è sotto la cerniera O, quindi crea una coppia che favorisce l'impuntamento

Sono usati come freni d'emergenza per gli ascensori





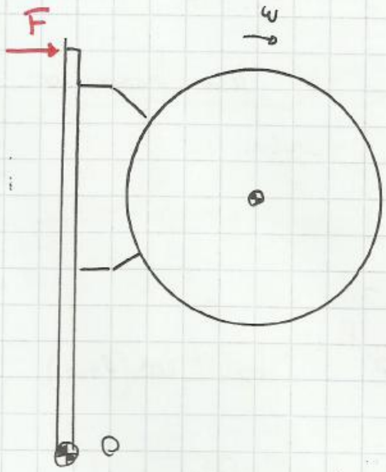


$F_N = \frac{F_T}{f} = F + S_0 \sin\alpha$

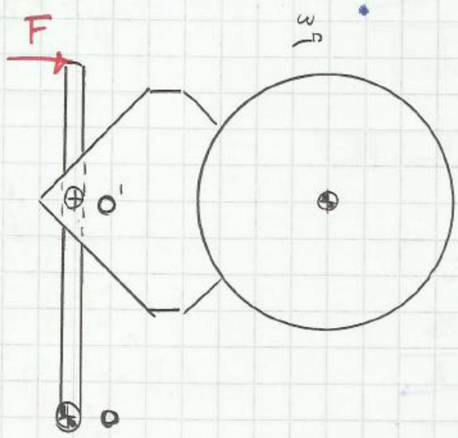
$S_0 = \frac{F_T}{\cos\alpha}$ $\qquad \frac{F_T}{f} = F + F_T \tan\alpha$

$F = F_T\left(\frac{1}{f} - \tan\alpha\right) \qquad F_T = \frac{F}{\frac{1}{f} - \tan\alpha}$

Freno a tamburo

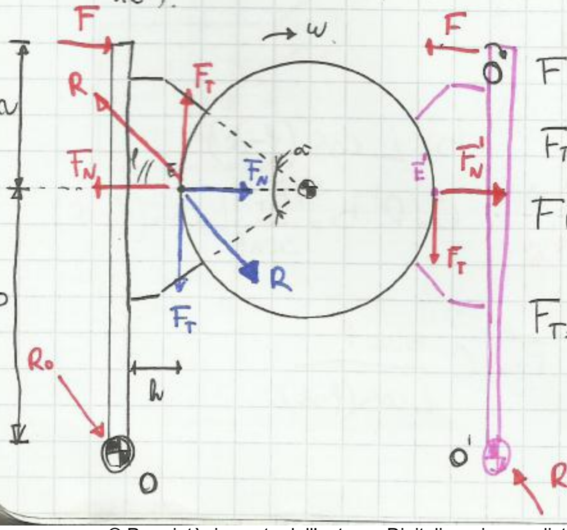


1 GDL: accostamento rigido



2 GDL: accostamento libero

per il freno ad accostamento rigido si potrebbe usare l'ipotesi dell'usura. Si usa una semplificazione: R è applicata in E (+ piccolo è α migliore è l'approssimazione).



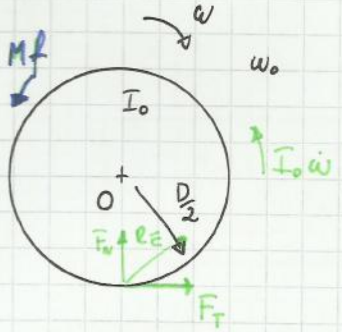
$F(a+b) - F_T h - F_N b = 0$ $\qquad$ O') $F(a+b) - F_N' b + F_T' h = 0$

$F_T = f F_N$ $\qquad F_T' = f F_N'$

$F(a+b) = F_T\left(\frac{b}{f} + h\right)$ $\qquad F(a+b) = F_T'\left(\frac{b}{f} - h\right)$

$F_T = F\frac{a+b}{\frac{b}{f} + h}$ $\qquad F_T' = F\frac{a+b}{\frac{b}{f} - h}$

$F_T' > F_T$ il freno di DX frena di più



$$M_f + I_0\dot{\omega} = 0$$

$$\dot{\omega} = -\frac{M_f}{I_0}$$

oppure

$$F_T\frac{D}{2} + I_0\dot{\omega} = 0$$

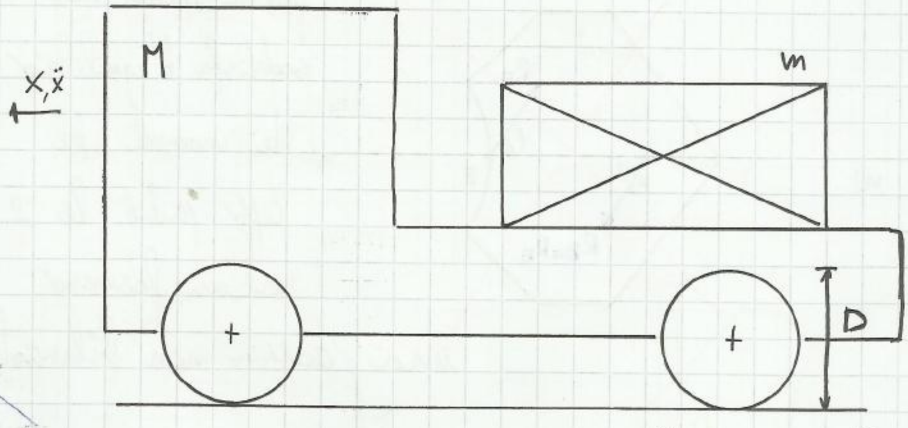
$$\dot{\omega} = -\frac{F_T\frac{D}{2}}{I_0}$$

$$\omega = \omega_0 + \dot{\omega}t$$

$$0 = \omega_0 + \dot{\omega}t_F$$

$$t_F = -\frac{\omega_0}{\dot{\omega}}$$

4.12



frena con decelerazione cost. 3 m/s²

⇓

$\ddot{X} = -3\ m/s^2$

$\dot{X}_0 = 50\ km/h = \frac{50}{3.6}\ m/s$

$M = 3600\ kg$

$m = 400\ kg$

$f = 0.25$ nel freno

$D = 0.8\ m$

$d = 0.6\ m$

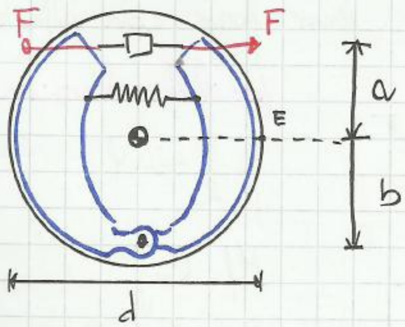
$a = 0.2\ m$

$t_{frenata}, s_{frenata} = ?$

$f_A(min)$ causa aderenza $= ?$

$F = ?$ freni

freni: solo sulle ruote posteriori

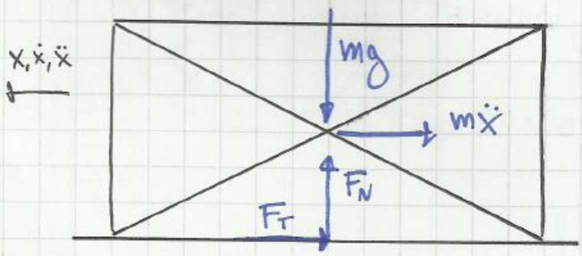


$$\dot{x} = \dot{x}_0 + \ddot{x}t$$

$$0 = \dot{x}_0 + \ddot{x}t_f \qquad t_f = -\frac{\dot{x}_0}{\ddot{x}} = -\frac{50}{3.6(-3)} = 4.63\ s$$

$$x = \dot{x}_0 t + \frac{1}{2}\ddot{x}t^2$$

$$s_f = \dot{x}_0 t_f + \frac{1}{2}\ddot{x}t_f^2 = 32\ m$$



$$F_N - mg = 0$$

$$F_T + m\ddot{x} = 0$$

$$\frac{F_T}{F_N} \le f_A \qquad f_A(min) = \frac{F_T}{F_N} = \frac{-\cancel{m}\ddot{x}}{\cancel{m}g} = -\frac{(-3)}{9.81} = 0.306$$

$F = K(R_e - R_i)\alpha$ forza azionante

$dF_T = f\,dF_N \qquad R\,dF_T = dM$

$$\int_A dM = \int R f K \alpha R\, dR\, d\theta = fK \int_{R_i}^{R_e} \int_{\theta_1}^{\theta_2} R\, dR\, d\theta = fK\alpha \frac{R_e^2 - R_i^2}{2} = M$$ momento frenante totale

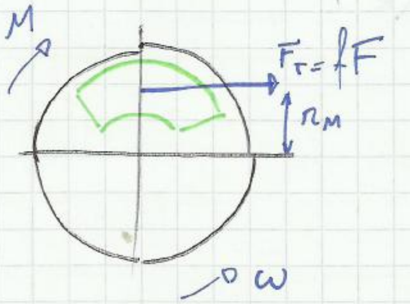
$$\frac{M}{F} = \frac{f\cancel{K\alpha}}{2} \frac{\cancel{(R_e - R_i)}(R_e + R_i)}{\cancel{K(R_e - R_i)\alpha}}$$

$$M = fF\frac{R_e + R_i}{2}$$

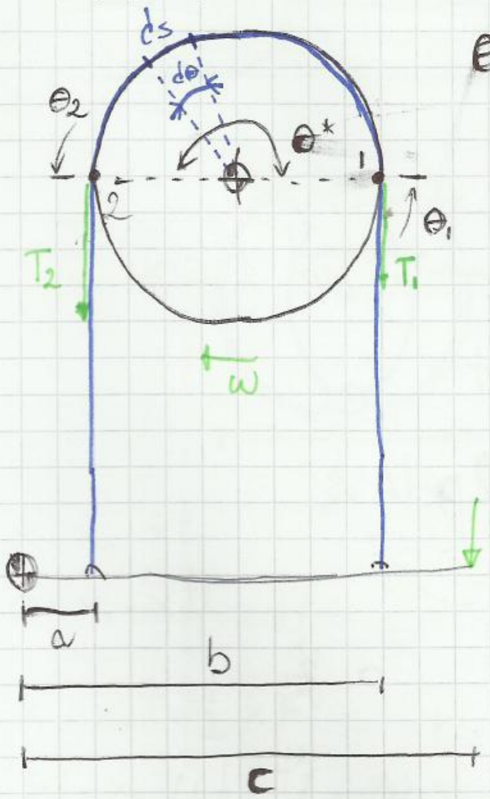
$fF = F_T$ forza tang. complessiva

$$R_M = \frac{R_e + R_i}{2}$$

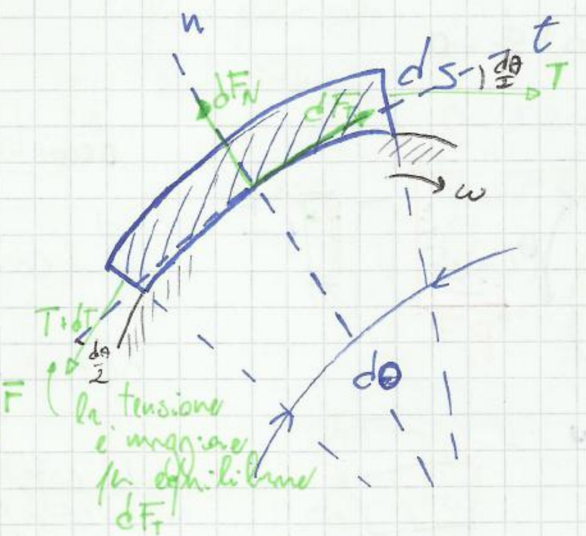
$$M = F_T R_M$$



Freno a nastro



θ* arco d'avvolgimento



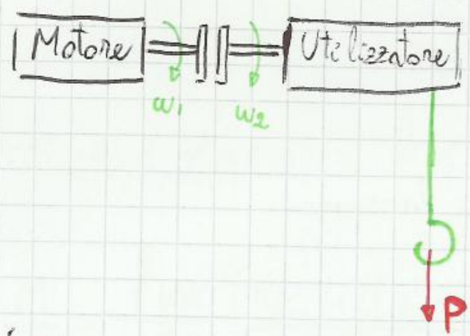
$dF_T = f\,dF_N$

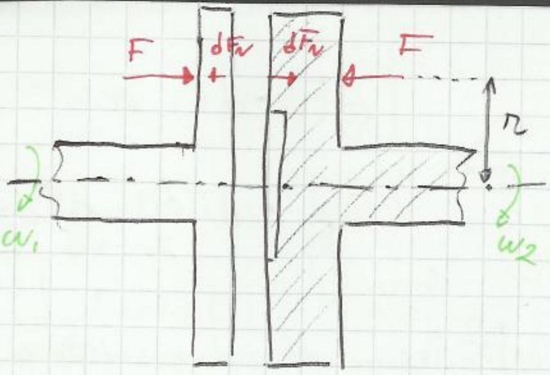
$T_2 > T_1$

$$dF_T + T\cos\frac{d\theta}{2} - (T + dT)\cos\frac{d\theta}{2} = 0$$

$$\frac{dF_T}{f} - T\sin\frac{d\theta}{2} - T + dT\sin\frac{d\theta}{2} = 0$$

Frizioni



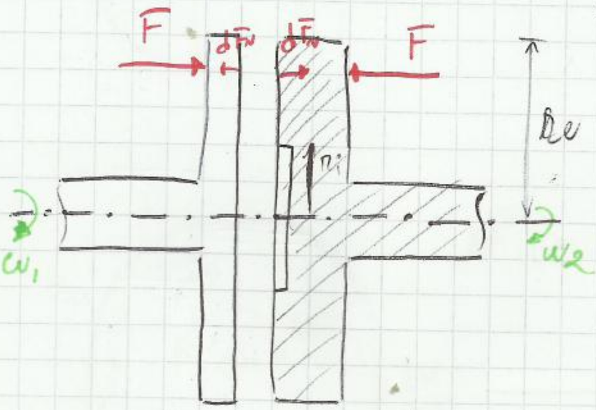


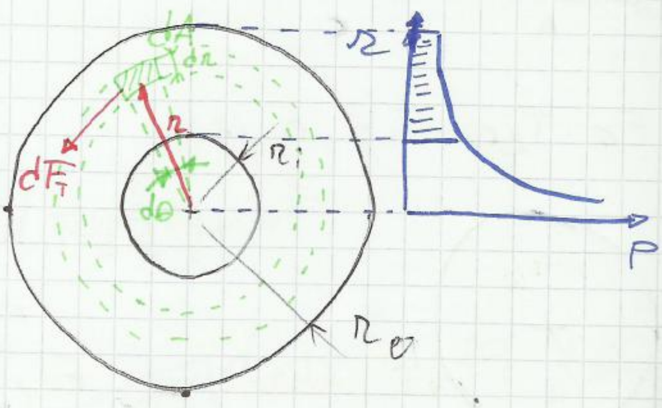
$dF_N = p\,dA$

$dF_T = f\,dF_N$ durante lo strisciamento $V_R \neq 0 \Rightarrow \omega_1 \neq \omega_2$

$dF_T\, r = dM$ momento trasmesso dalla frizione

in condizioni di aderenza $\frac{dF_T}{dF_N} \le f_A$ $\quad V_R = 0 \Rightarrow \omega_1 = \omega_2$





$\omega_1 \neq \omega_2$

$dF_N = p\,dA$

$\underbrace{\delta}_{cost.}\, \cancel{dA} \propto f\,p\,\cancel{dA}\, \underbrace{V_R}_{\omega_R r,\ \omega_R\ cost.} \Rightarrow \quad p r = k \quad p = \frac{k}{r} \quad dF_N = \frac{k}{r} 2\pi r\,dr$

$\omega_R = \omega_1 - \omega_2$

$dA = r\,dr\,d\theta$

$dA = \Sigma\, dA_{(r)} = 2\pi r\,dr$

$\int_A dF_N = F = 2\pi k (r_e - r_i)$

$dF_N f = dF_T$

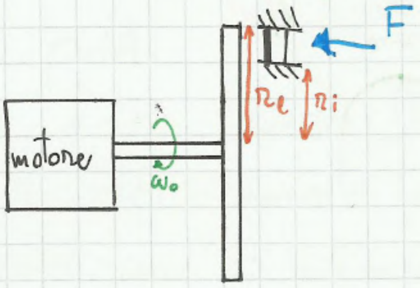
$dF_T\, r = dM$

$M = \int_A dM = \int r\,dF_T = \int r f p\,dA = \int r f \frac{k}{r} 2\pi r\,dr = 2\pi f k \frac{r_e^2 - r_i^2}{2}$

$$\frac{M}{F} = \frac{2\pi f k (r_e - r_i)(r_e + r_i)}{\sin\alpha \; 2\pi k (r_e - r_i) 2} = \frac{f}{\sin\alpha}\,\frac{r_e + r_i}{2}$$

$$M = \frac{f}{\sin\alpha} F \frac{r_e + r_i}{2} \qquad f' = \frac{f}{\sin\alpha} \qquad f' \geq f$$

4.9



$M = 100$ Kg

$\rho = 0.3$ m

$\omega_0 = 1500$ r/m

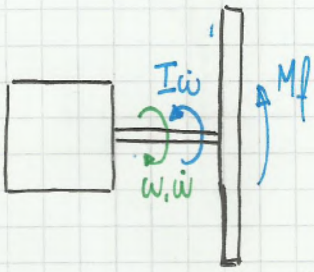
$r_e = 20$ cm

$r_i = 15$ cm

$f = 0.3$

F / t_{stop} 10s

$I = M\rho^2 = 9$ Kg m^2



$M_f + I\dot{\omega} = 0$

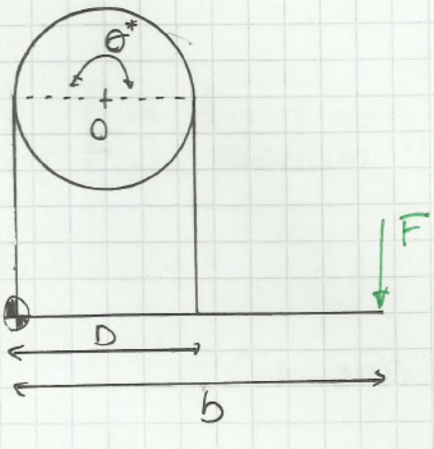
$\omega = \omega_0 + \dot{\omega} t$

a $t = t_s$ $\omega(t_s) = 0$ $\qquad 0 = \omega_0 + \dot{\omega} t_f \qquad \dot{\omega} = -\frac{\omega_0}{t_f} = -15.707$

$M_f = -I\dot{\omega} = 141.37$ Nm

$M_{f\,disco} = f F \frac{R_e + R_i}{2} \qquad F = 2693$ N

4.11



$D = 0.4$ m
$b = 1$ m

$\theta^* = \pi$ rad

$f = 0.25$

$F = 200$ N

M_f, R_0 per ω ↻ e ω ↺

$\frac{T_2}{T_1} = e^{f\theta^*} \qquad T_2 > T_1$

$M_f = (T_2 - T_1) r$

(leva)

$R_{Ay} + R\cos\varepsilon = F$

$R_{Ax} = R\sin\varepsilon$

(Tamburo)

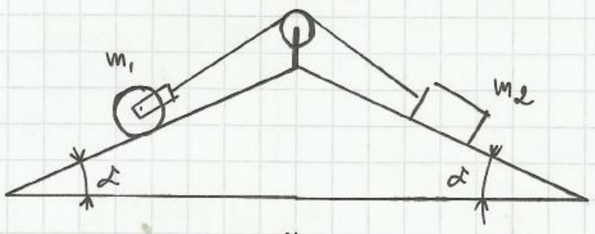
$R\rho = C$

A) $R\cos\varepsilon\, a - R\sin\varepsilon\, h - F(a+b) = 0$

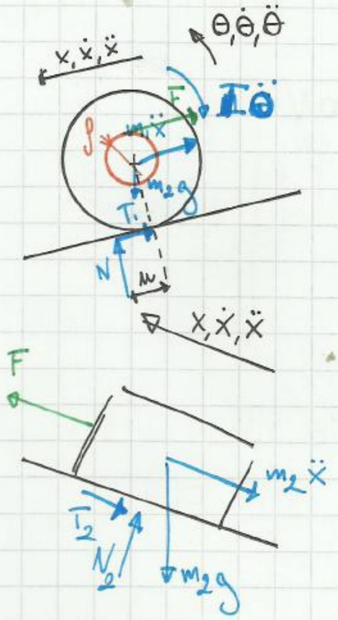
$R = 310.14\ N \quad R_{Ax} = 86.716\ N \quad R_{Ay} = -228.9\ N \quad C = 13.877\ Nm$

3.16

attrito rullo-piano, blocco-piano, perno. $\alpha, r, w, f, m_1, I, r_{perno}, f_{perno}$

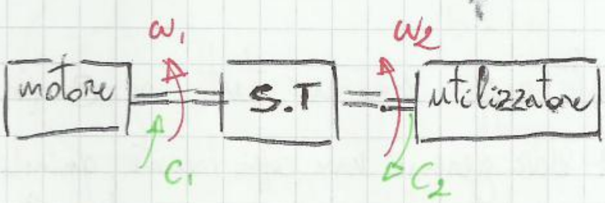


- trovare m_2 in modo che m_1 si muova a velocità costante.
- Assegnato m_2 calcolare l'accellerazione del blocco e trovare f in modo che il moto sia di rotolamento puro.



per il primo punto i contributi inerziali dell'equilibrio sono nulli.

Sistemi di trasmissione



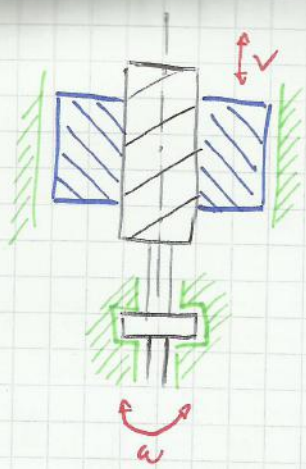
rapporto di trasmissione

$i = \frac{\omega_1}{\omega_2}$ ← motore, ← utilizzatore

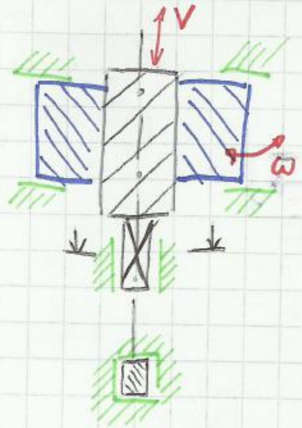
$P_M = C_1\omega_1$

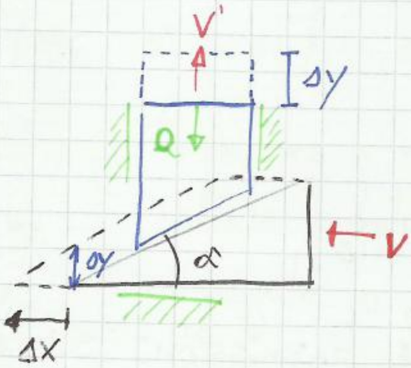
$P_u = C_2\omega_2$

$\eta = \frac{P_u}{P_M} = \frac{C_2\omega_2}{C_1\omega_1}$

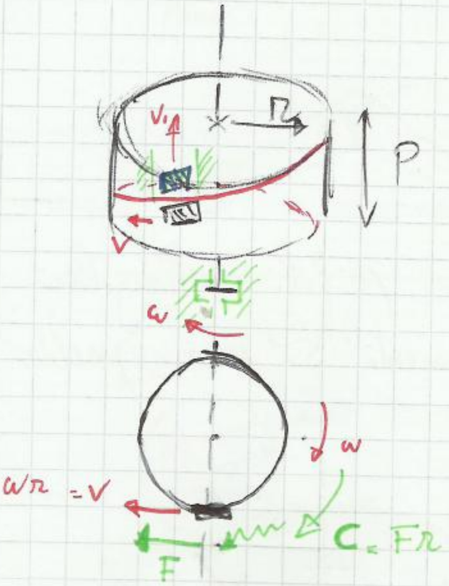


oppure



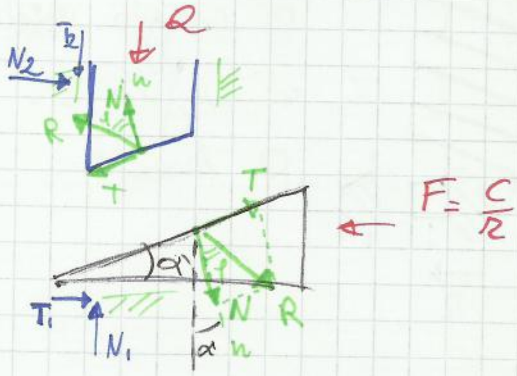


modello dei cunei equivalenti:



$\Delta y = \Delta x \tan\alpha$

$\frac{\Delta y}{\Delta t} = \frac{\Delta x}{\Delta t}\tan\alpha \qquad V' = V\tan\alpha = \omega r \tan\alpha$



$P_M = C\omega$

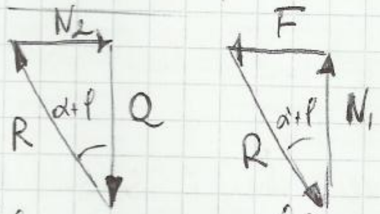
$P_u = QV'$ $\qquad \eta = \frac{QV'}{C\omega}$

$P_M = FV$

per semplificare:

$T_1 = f_1 N_1 \quad f_1 = 0$

$T_2 = f_2 N_2 \quad f_2 = 0$



blocco superiore

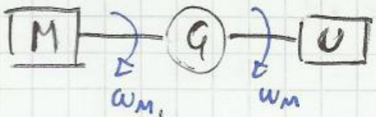
blocco inferiore

$R = \frac{Q}{\cos(\alpha+\varphi)} = \frac{F}{\sin(\alpha+\varphi)}$

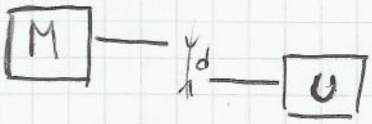
$\frac{F}{Q} = \tan(\alpha+\varphi) = \frac{C}{rQ}$

$C = Qr\tan(\alpha+\varphi)$ coppia necessaria a sollevare Q

GIUNTI DI TRASMISSIONE

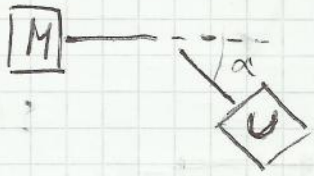


disassamento

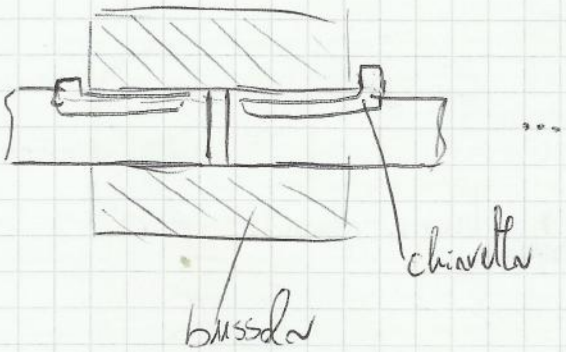


giunti fissi

disallineamento

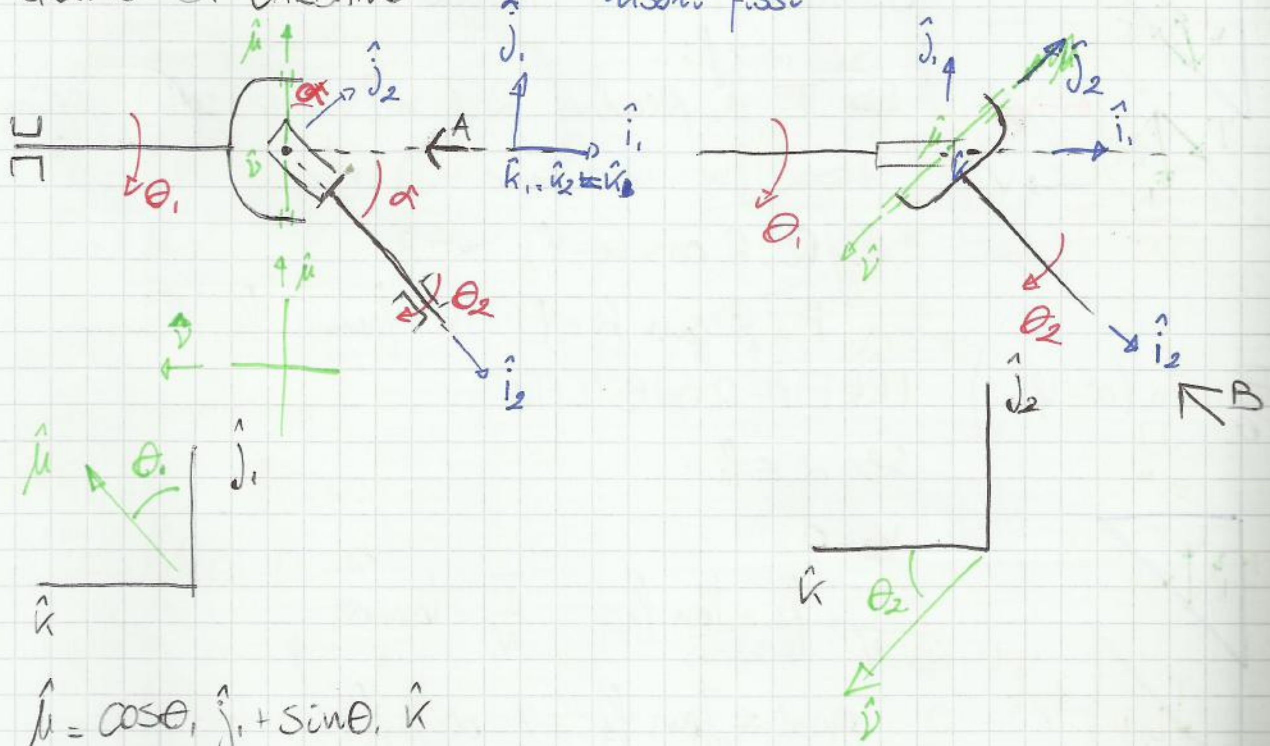


ES: FISSO: bussola inchiavettata



GIUNTO DI CARDANO

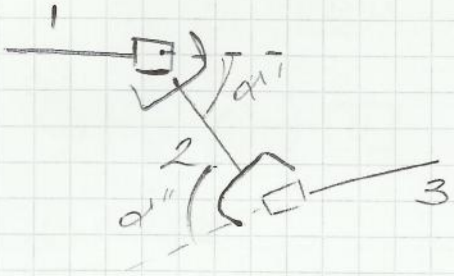
versori fissi



$$\hat{\mu} = \cos\theta_1 \, \hat{j}_1 + \sin\theta_1 \, \hat{k}$$

$$\hat{\nu} = -\sin\theta_2 \, \hat{j}_2 + \cos\theta_2 \, \hat{k}$$

$$\hat{\mu} \cdot \hat{\nu} = 0 \quad (\hat{\mu} \perp \hat{\nu})$$



$$\frac{\tan\theta_1}{\tan\theta_2} = \cos\alpha'$$

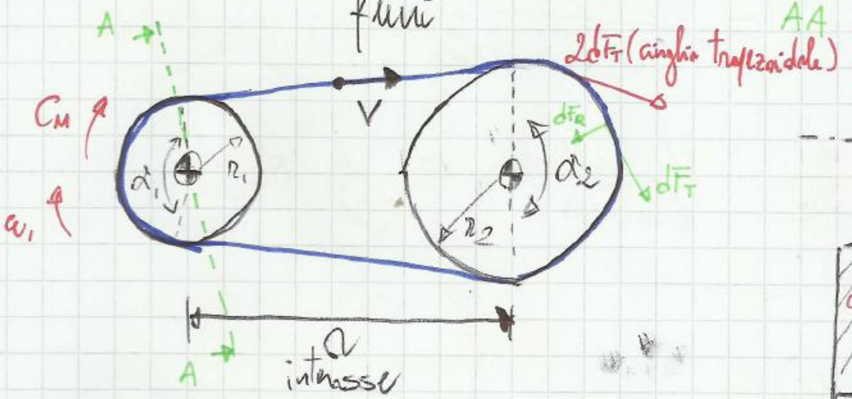
$$\frac{\tan\theta_3}{\tan\theta_2} = \cos\alpha''$$

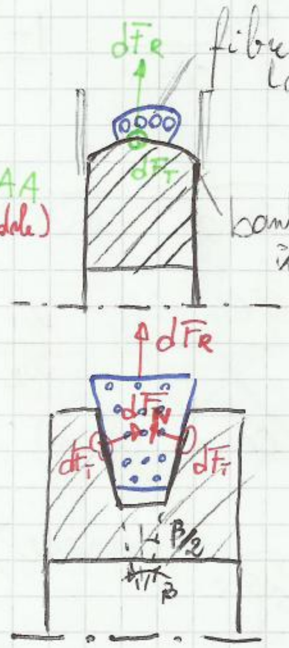
$$\frac{\tan\theta_1}{\tan\theta_3} = \frac{\cos\alpha'}{\cos\alpha''} = 1 \qquad \dot\theta_1 = \dot\theta_3 \Rightarrow \alpha' = \alpha''$$ giunto omocinetico

i bracci delle crociere dell'albero intermedio devono essere //

Trasmissioni con flessibili

Flessibile: cinghie, catene, funi





fibre x aumentare la rigidità

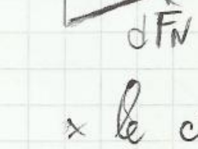
bandella x stabilizzare la cinghia al centro
→ piana

il contatto deve avvenire sui fianchi, non sul fondo.

$dF_T = f\,dF_R$ x cinghia piana

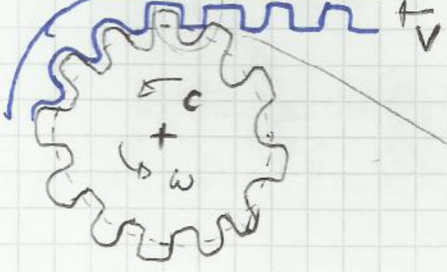
$$2dF_T = 2dF\cdot f = f\frac{dF_R}{\sin\frac{\beta}{2}} = \frac{f}{\sin\frac{\beta}{2}}dF_R = f'dF_R$$

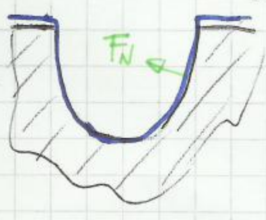
$f' \gg f$



$dF_R = 2dF_N \sin\frac{\beta}{2}$

x le cinghie dentate è la F_N che determina la trasmissione



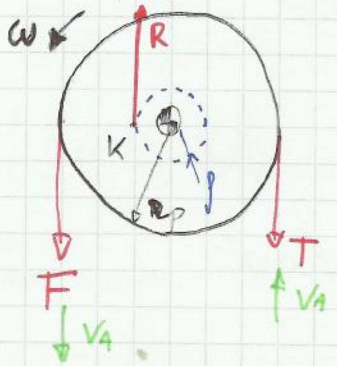


dall'equilibrio dell'equipaggio mobile

$$Q = T_1 + T_2 + T_3 + T_4 = 4F$$

$$\eta = \frac{P_u}{P_M} = \frac{Q\, v_p}{F\, v_A} = \frac{4}{4} = 1$$

se il perno della puleggia avesse attrito:

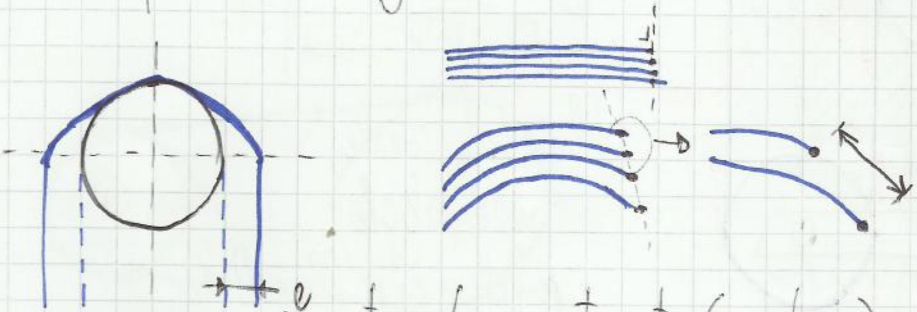


$$\rho = r_p \sin\varphi$$

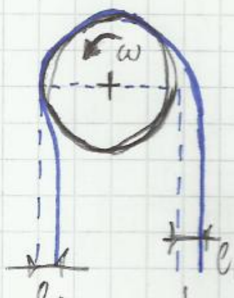
$$F(r-\rho) = T_1(r+\rho) \qquad T_1 < F$$

$$\eta_p = \frac{P_u}{P_M} = \frac{V_A T_1}{V_A F} = \frac{r-\rho}{r+\rho}$$

se la fune è rigida:

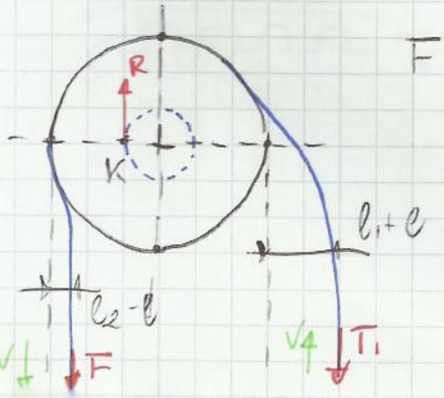


parametro di scostamento (rigidezza) elastico



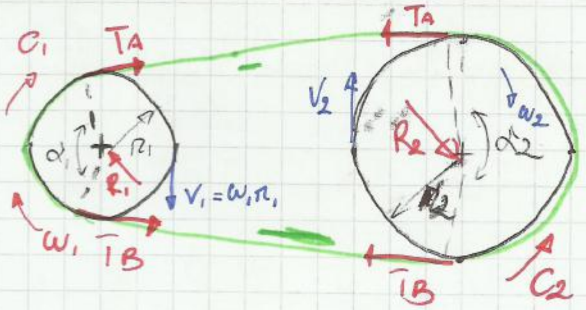
e_1 parametro di scostamento anelastico in ingresso

e_2 parametro di scostamento anelastico in uscita



$$F(r - \rho - e_2 + e) = T_1(r + \rho + e + e_1)$$

$$\eta = \frac{r + e - \rho - e_2}{r + e + \rho + e_1}$$



flessibile ideale:

$V_1 = \omega_1 r_1$

1) aderenza cinghia/α_1

$V_R = 0$

$V = V_1$

2) aderenza cinghia/α_2

$V_R = 0$

$V_2 = V = V_1$

$V_2 = \omega_2 r_2 = \omega_1 r_1$

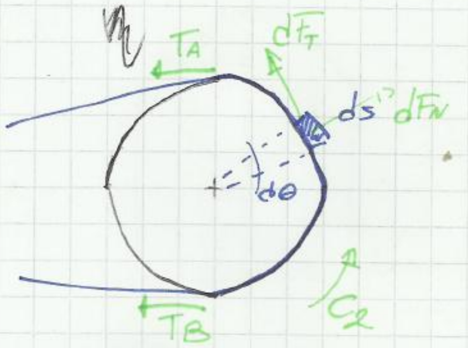
$$\eta = \frac{C_2 \omega_2}{C_1 \omega_1}$$

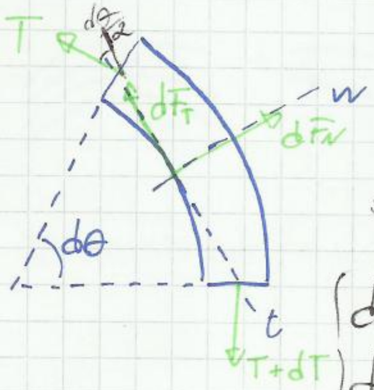
$$i = \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1}$$

$C_1 = (T_B - T_A) r_1$

$C_2 = (T_B - T_A) r_2$

$$\eta = \frac{\cancel{(T_B - T_A)} r_2 \omega_2}{\cancel{(T_B - T_A)} r_1 \omega_1} = \frac{V_2}{V_1} = 1$$





$$\begin{cases} dF_N - T\sin\frac{d\theta}{2} - (T+dT)\sin\frac{d\theta}{2} = 0 \\ dF_T + T\cos\frac{d\theta}{2} - (T+dT)\cos\frac{\theta}{2} = 0 \end{cases}$$

$\sin\frac{d\theta}{2} = \frac{d\theta}{2}$ $\cos\frac{d\theta}{2} = 1$

$$\begin{cases} dF_N = T d\theta \\ dF_T = dT \end{cases}$$

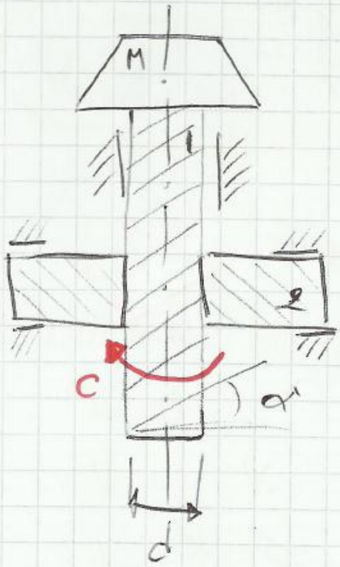
$\dfrac{dF_T}{dF_N} = \dfrac{dT}{T d\theta} \le f_A$ per soddisfare l'ipotesi d'aderenza

$\dfrac{dT}{T} \le f_A d\theta \Rightarrow \ln\dfrac{T_B}{T_A} \le f_A \alpha$ $\quad \dfrac{T_B}{T_A} \le e^{f_A \alpha}$ $\quad (T_B > T_A)$

$\dfrac{T_B}{T_A} \le e^{f_A \alpha_1} < e^{f_A \alpha_2}$ la verifica dell'aderenza va verificata solo sulla puleggia più piccola.

$C_1 = (T_B - T_A) r$ in aderenza limite $\dfrac{T_B}{T_A} = e^{f_A \alpha_1}$

ES. 5.24



$M = 100\ kg$ $\quad$ $C / V = cost$

$d = 30\ mm$ $\quad$ se $C' = 5\ Nm$ $\ddot{x} = ?$

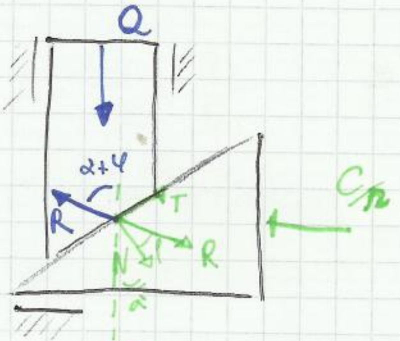
$\alpha = 3°$

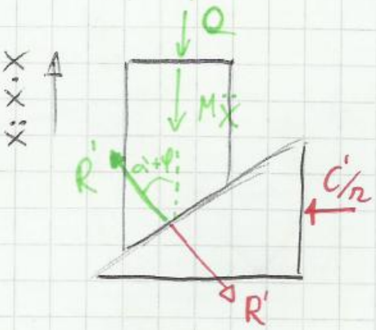
$f = 0{,}1 \rightarrow \varphi = \arctan f = 5.71°$

$$\begin{cases} \rightarrow \ \dfrac{C}{R} = R \sin(\varphi + \alpha) \\ \uparrow \ Q = R \cos(\alpha + \varphi) \end{cases}$$

$$\frac{C}{R} = Q \tan(\alpha + \varphi)$$

$$C = 225\ Nm$$



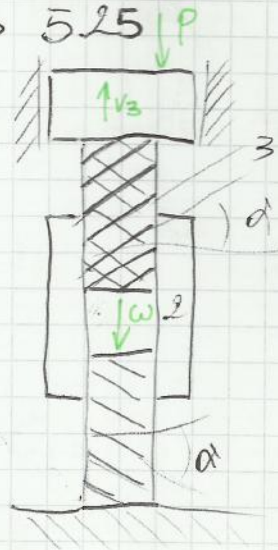


$$\begin{cases} \uparrow \ R' \cos(\alpha + \varphi) = Q + M\ddot{x} \rightarrow R' = \dfrac{Q + M\ddot{x}}{\cos(\alpha + \varphi)} \\ \rightarrow \ \dfrac{C'}{r} = R' \sin(\alpha + \varphi) \end{cases}$$

$$\frac{C'}{r} = (Q + M\ddot{x}) \tan(\alpha + \varphi)$$

$$\ddot{x} = \frac{C'}{r M \tan(\alpha + \varphi)} - \frac{Q}{M} = 11.95\ m/s^2$$

ES 5.25



$p = 6\ mm$

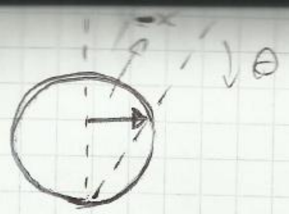
$d = 25\ m$

$f = 0.15$

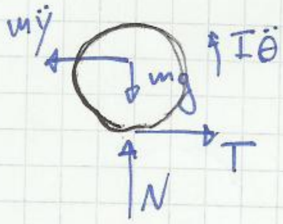
$f_A = 0.25$

$P = 10000\ N$

$C / V_3 = 0.5\ m/s$



$$\theta = \frac{y-x}{r}$$



$$A)\ r m\ddot{y} + I\ddot{\theta} = 0$$

$$m\ddot{y}r + I\frac{\ddot{y}-\ddot{x}}{r} = 0 \qquad \pm m\ddot{x}r$$

$$m(\ddot{y}-\ddot{x})r + m\ddot{x}r + \frac{I}{r}(\ddot{y}-\ddot{x}) = 0$$

$$-(\ddot{y}-\ddot{x})\underbrace{\left(mr + \frac{I}{r}\right)}_{A} = m\ddot{x}r$$

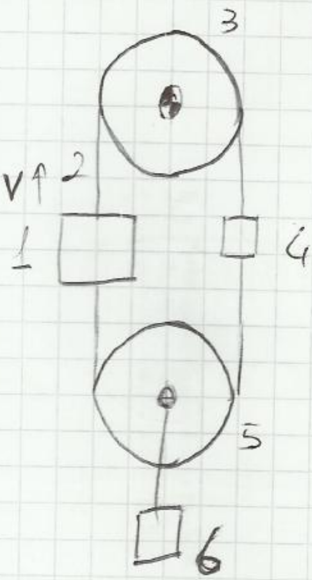
$$\ddot{x}-\ddot{y} = \frac{\ddot{x}}{A}$$

$$s = \frac{1}{2}\ddot{x}t^2 \qquad \frac{t^2}{2} = \frac{s}{\ddot{x}}$$

$$d = \frac{1}{2}(\ddot{x}-\ddot{y})t^2 = \frac{1}{2}\frac{\ddot{x}}{A}t^2 = \frac{t^2}{2}\frac{\ddot{x}}{A}$$

$$d = \frac{s}{\ddot{x}}\frac{\ddot{x}}{A} \rightarrow s = dA = d\left(\cancel{m}\,1 + \frac{I}{mr^2}\right)$$

Es. 5.16



$V = 1\,m/s$

$m_1 = 600\,Kg$

$m_4 = 300\,Kg$

$m_6 = 1500\,Kg$

$R_3 = R_5 = 300\,mm$

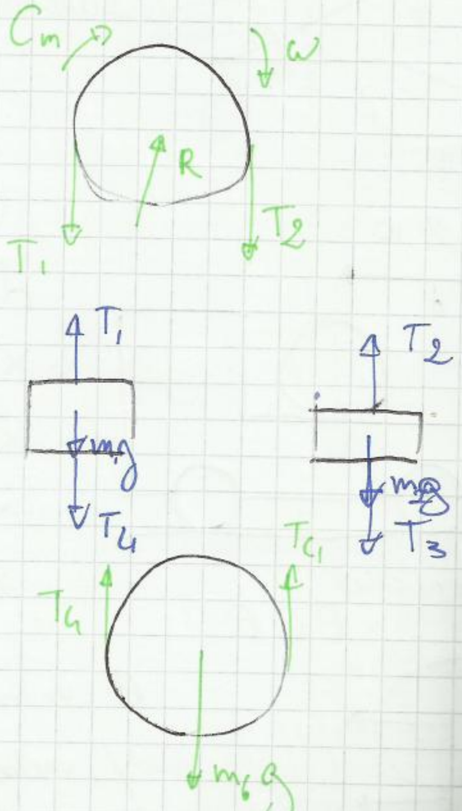
· $\cancel{W}$ / $V = 1\,m/s$

· f_a

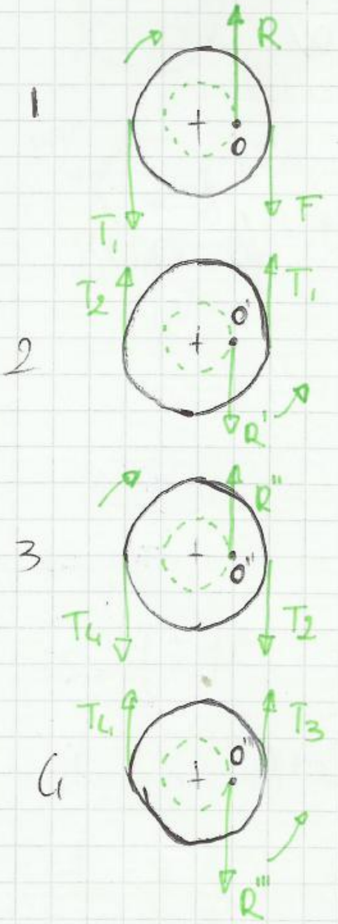
$l = 5\,mm$

$l_1 = l_2 = 8\,mm$

W ?



$$F = \frac{Q}{4} = 1962\ N \qquad \rho = r_p \sin \varphi_f$$



$$1\cdot\ T_1 (R_2 + \rho) = F'(R_2 - \rho)$$

$$2\cdot\ T_2 (R_1 + \rho) = T_1 (R_1 - \rho)$$

$$T_2 = F' \frac{R_2 - \rho}{R_2 + \rho} \frac{R_1 - \rho}{R_1 + \rho}$$

$$3\cdot\ T_3 (R_2 + \rho) = T_2 (R_2 - \rho)$$

$$4\cdot\ T_4 (R_1 + \rho) = T_3 (R_1 - \rho)$$

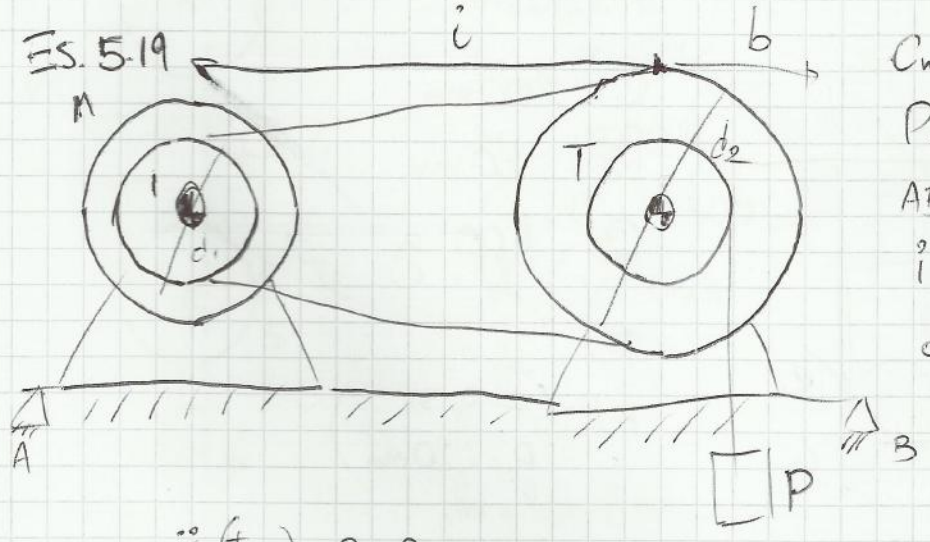
$$T_4 = F' \frac{(R_2 - \rho)^2}{(R_2 + \rho)^2} \frac{(R_1 - \rho)^2}{(R_1 + \rho)^2}$$

$$\underbrace{T_1 + T_2 + T_3 + T_4}_{f(F)} = Q$$

$$F = 2362\ N$$

$$\eta = \frac{F'}{F} = 0.838$$

Es. 5.19



$C_m = 600\ Nm$

$P = 3000\ N$

$AB = L = 1.2$

$i = 0.6\ m$

$d_1 = 250\ m$ $\qquad d_1 = 400\ mm$

$d_2 = 500\ mm$

$I_1 = 0.05\ kg\,m^2$

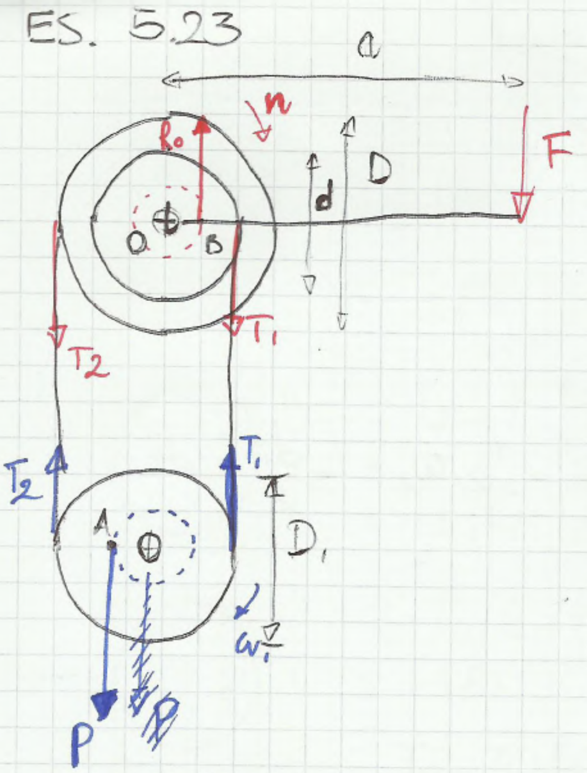
$I_2 = 0.025\ kg\,m^2$

$m = 50\ kg$

$m_2 = 50\ kg$

$\ddot{x}(t=0)$, R_A, R_B

ES. 5.23



$d = 600\,mm$

$D = 500\,mm$

$D_1 = 450\,mm$

$a = 500\,mm$

$r_p = 30\,mm$

$f = 0.1$

$P = 5000\,N$

$n = 30\,rpm$

$\rho = r_p \sin\varphi = 3\,mm$

$F = ?$

$V_s = ?$

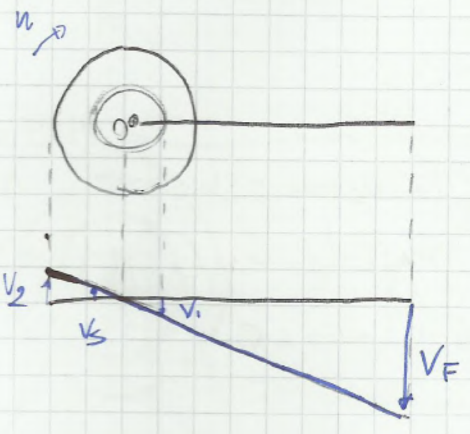
$$A \curvearrowleft \begin{cases} T_1\left(\frac{D_1}{2}+\rho\right) = T_2\left(\frac{D_1}{2}-\rho\right) \\ T_1 + T_2 = P \end{cases}$$

$T_1 = 2466\,N$

$T_2 = 2533\,N$

$$B \curvearrowleft \; F(a-\rho) + T_1\left(\frac{d}{2}-\rho\right) - T_2\left(\frac{D}{2}+\rho\right) = 0$$

$F = 312\,N$



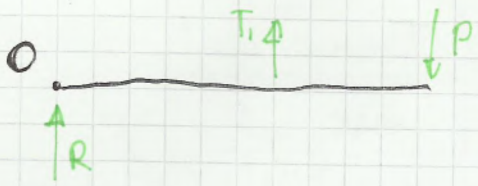
$V_1 = n\frac{d}{2} = 0.62\ m/s$

$V_2 = n\frac{D}{2} = 0.78\ m/s$

$V_F = na =$

$V_s = \frac{V_2 - V_1}{2} = 0.07\ m/s$

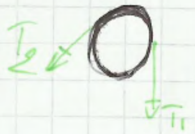
$\eta = \frac{P V_s}{F V_F} = 0.8$



O: $T_1 a = P(a+b)$

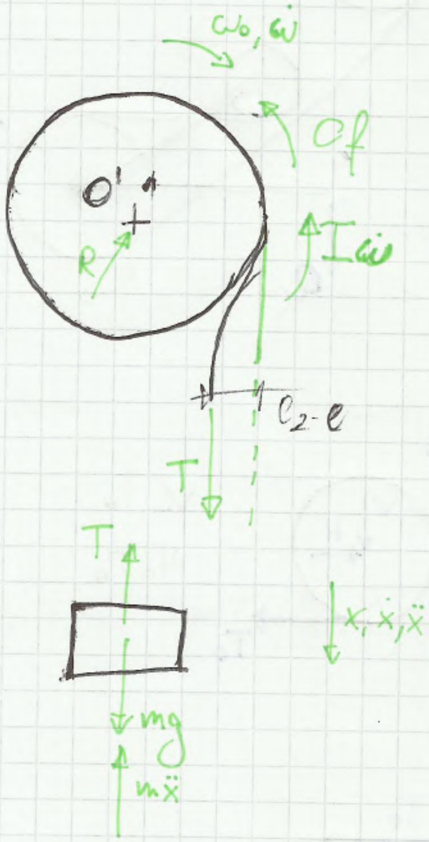
$T_1 = 2533\ N$

$\theta^* = \frac{\pi}{2} + \alpha = 150°$



$\frac{T_2}{T_1} = e^{f\theta^*} \rightarrow T_2$

$C_f = (T_2 - T_1)\frac{D_2}{2} = 789\ Nm$



$$\begin{cases} O': \ C_f + I\dot{\omega} - T\left(\frac{D_1}{2} - l_2 + l_1\right) = 0 \\ \uparrow \ T = mg - m\ddot{x} \\ \ddot{x} = \frac{D_1}{2}\dot{\omega} \end{cases}$$

$\dot{\omega} = -3.06\ \frac{rad}{s^2}$

$\omega(t) = \omega_0 + \dot{\omega} t \qquad \omega(t = t_{stop}) = 0$

$0 = \omega_0 + \dot{\omega} t_{stop} \qquad t_{stop} = -\frac{\omega_0}{\dot{\omega}} = 1.1\ s$

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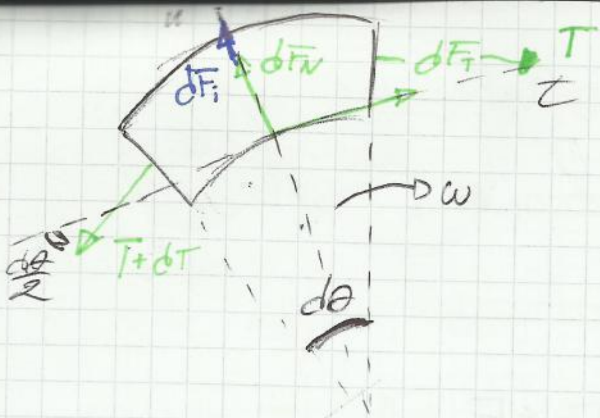


$m = l q$ → massa x unità di lunghezza

$dm = q\,ds$

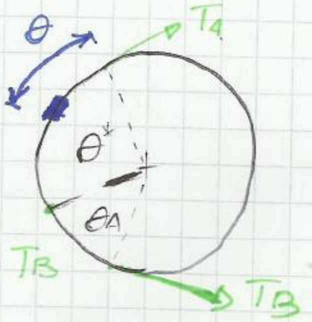
$dF_i = dm\, r\omega^2 = q\,ds\, r \frac{V^2}{r^2} = q\, r\, d\theta \frac{V^2}{r}$

$dF_i = q V^2 d\theta$

$$\begin{cases} dF_T + T\cos\frac{d\theta}{2} - (T+dT)\cos\frac{d\theta}{2} = 0 \\ dF_N + qV^2 d\theta - 2T\sin\frac{d\theta}{2} - \cancel{dT\sin\frac{d\theta}{2}} = 0 \end{cases}$$

$$dF_N = \frac{dF_T}{f}$$

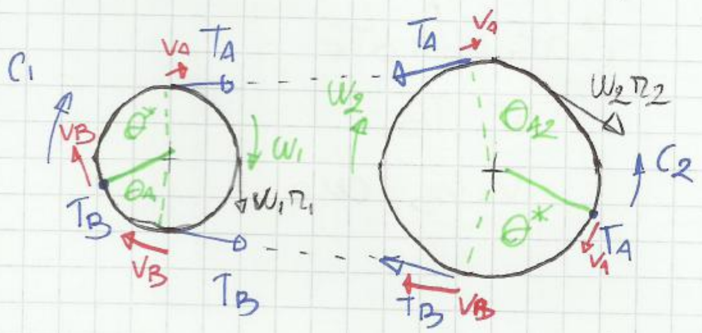
$$\begin{cases} dF_T = T \\ \frac{dT}{f} + qV^2 d\theta = T\,d\theta \end{cases} \qquad \frac{dT}{f} = (T - qV^2)\,d\theta$$



$$\int_{T_A}^{T_B} \frac{dT}{T - qV^2} = f\int_0^{\theta^*} d\theta$$

$$\ln\left(\frac{T_B - qV^2}{T_A - qV^2}\right) = f\theta^*$$

$$\frac{T_B - qV^2}{T_A - qV^2} = e^{f\theta^*}$$ se si trascura la massa $$\frac{T_B}{T_A} = e^{f\theta^*}$$



$\alpha_1 = \theta^* + \theta_{A1}$

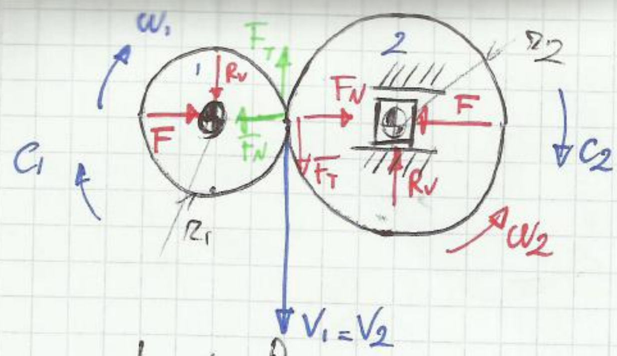
$\alpha_2 = \theta^* + \theta_{A2}$

rapporto ideale

$$i = \frac{\omega_1}{\omega_2} = \frac{V_B}{r_1}\frac{r_2}{V_A} = \frac{r_2}{r_1}\,\frac{1 + \frac{T_B}{ES}}{1 + \frac{T_A}{ES}}$$

$\omega_1 r_1 = V_B$

$\omega_2 r_2 = V_A$



$V_1 = \omega_1 r_1$

ipotesi aderenza 1-2

$V_1 = V_2 = \omega_2 r_2 = \omega_1 r_1$

$i = \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1}$

ruote di frizione (ad attrito)

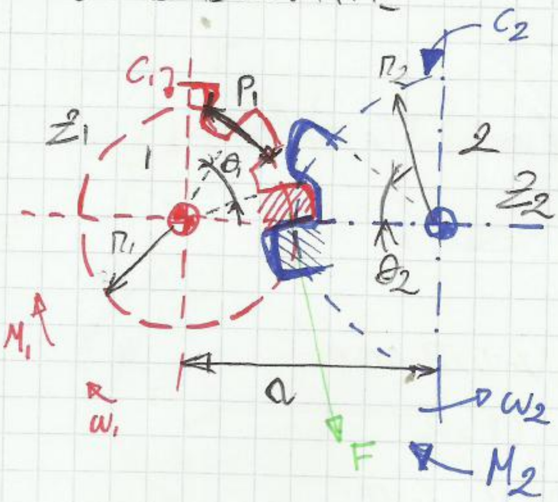
$C_1 = F_T r_1$

$C_2 = F_T \cdot r_2$

$\eta = \frac{C_2 \omega_2}{C_1 \omega_2} = \frac{F_T r_2 \omega_2}{F_T r_1 \omega_1} = 1$

$F_T \leq f_A F_N \Rightarrow F_N \geq \frac{F_T}{f_A}$

RUOTE DENTATE



$i = \frac{\omega_1}{\omega_2} = \left(\frac{r_2}{r_1}\right) = \frac{z_2}{z_1}$

non è nota il raggio dei cerchi polari

$\theta_1 = \frac{2\pi}{z_1} = \omega_1 \Delta t$

$\theta_2 = \frac{2\pi}{z_2} = \omega_2 \Delta t$

$$\begin{cases} i = \frac{z_2}{z_1} = \frac{r_2}{r_2} \\ r_1 + r_2 = a \end{cases} \qquad a = r_1 (1+i)$$

$r_1 = \frac{a}{1+i}$

$r_2 = a \frac{i}{1+i}$

raggi primitivi

C_1, C_2 circonferenze primitive

$P_1 = \frac{2\pi r_1}{z_1}$ $\qquad$ $P_2 = \frac{2\pi r_1}{z_1}$ $\qquad$ $\frac{P_1}{P_2} = \frac{r_1}{z_1} \frac{z_2}{r_2} = 1 \Rightarrow P_1 = P_2 = P$ passo dell'ingranaggio

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