NUMERO: 1542A - ANNO: 2015

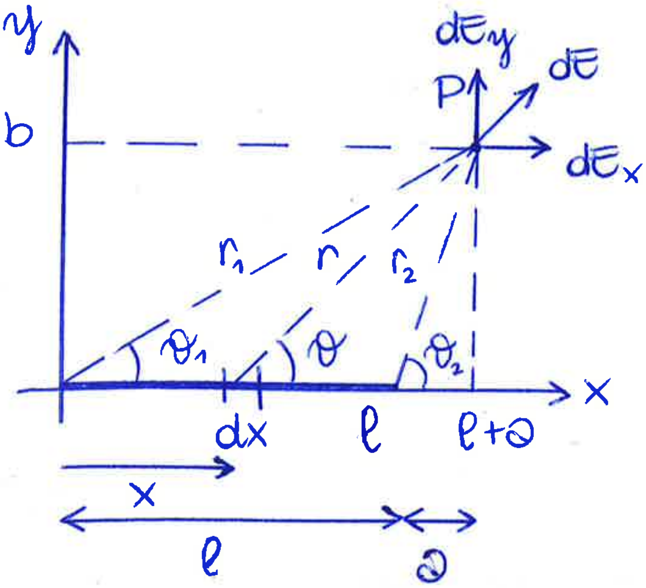
A P P U N T I

STUDENTE: Zago C.

MATERIA: Fisica II Eserc. Prof.Kaniadakis

ESERCITAZIONE 0

1



Determinare il campo elettrico in P creato da una sbarretta carica uniformemente di lunghezza $P(\ell+a, b)$ con densità di carica $\lambda = \frac{q}{\ell}$

$$dE = k\frac{dq}{r^2} = k\frac{\lambda dx}{r^2}$$

con $\lambda = \frac{q}{\ell}$

$$\left.\begin{aligned} r\cos\vartheta &= \ell+a-x \\ r\sin\vartheta &= b \end{aligned}\right\} \quad tg\,\vartheta = \frac{b}{\ell+a-x} \Rightarrow \ell+a-x = \frac{b\cos\vartheta}{\sin\vartheta}$$

$$x = \ell+a-\frac{b\cos\vartheta}{\sin\vartheta} \Rightarrow dx = \frac{b}{\sin^2\vartheta}\,d\vartheta$$

$$\frac{1}{r^2} = \frac{\sin^2\vartheta}{b^2}$$

$$dE = k\lambda \cancel{\frac{\sin^2\vartheta}{b^2}}\,\cancel{\frac{b}{\sin^2\vartheta}}\,d\vartheta = \frac{k\lambda}{b}\,d\vartheta$$

$$dE_x = \frac{k\lambda}{b}\cos\vartheta\, d\vartheta \Rightarrow E_x = \frac{k\lambda}{b}\int_{\vartheta_1}^{\vartheta_2}\cos\vartheta\, d\vartheta =$$

$$= \frac{k\lambda}{b}(\sin\vartheta_2 - \sin\vartheta_1)$$

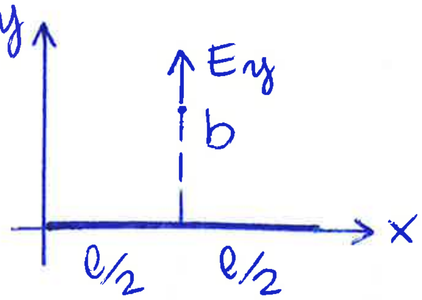
$$dE_y = \frac{k\lambda}{b}\sin\vartheta\, d\vartheta \Rightarrow E_y = \frac{k\lambda}{b}\int_{\vartheta_1}^{\vartheta_2}\sin\vartheta\, d\vartheta =$$

$$= \frac{k\lambda}{b}(\cos\vartheta_1 - \cos\vartheta_2)$$

$$\sin\vartheta_1 = \frac{b}{\sqrt{(\ell+a)^2+b^2}}$$

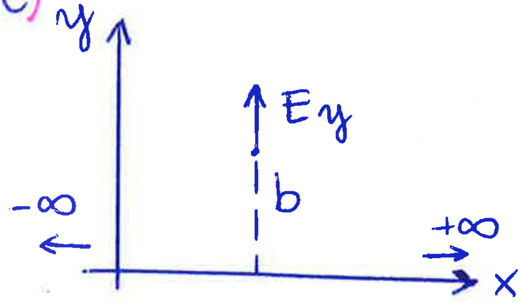
$$\begin{cases} E_x = k\lambda \left[\dfrac{1}{\sqrt{a^2+b^2}} - \dfrac{1}{\sqrt{(\ell-a)^2+b^2}} \right] \\ E_y = \dfrac{k\lambda}{b} \left[\dfrac{\ell-a}{\sqrt{(\ell-a)^2+b^2}} + \dfrac{a}{\sqrt{a^2+b^2}} \right] \end{cases}$$

d)



$$\begin{cases} E_x = 0 \\ E_y = \dfrac{2k\lambda}{b} \dfrac{\ell}{\sqrt{\ell^2+4b^2}} \end{cases}$$

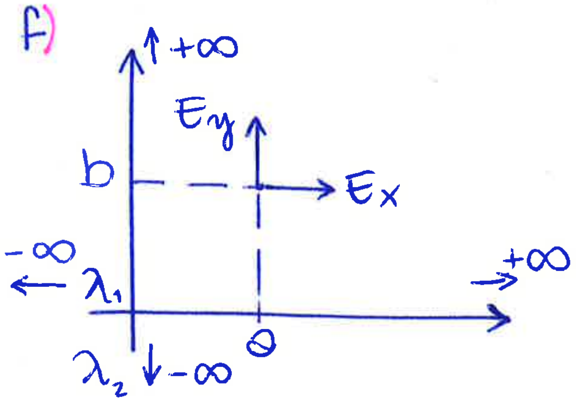
e)



$$\begin{cases} E_x = 0 \\ E_y = \dfrac{2k\lambda}{b} \end{cases}$$

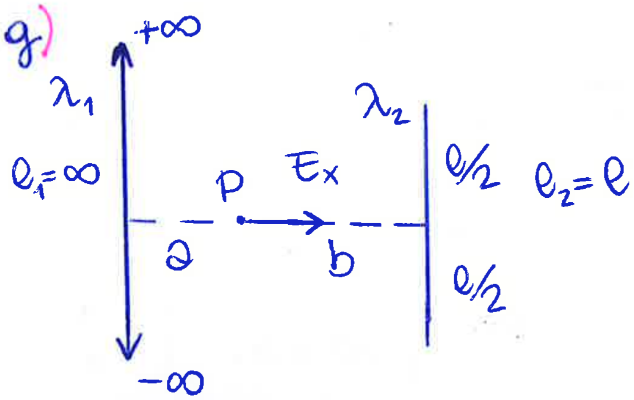
$\ell \to \infty$

f)



$$\begin{cases} E_x = \dfrac{2k\lambda_2}{a} \\ E_y = \dfrac{2k\lambda_1}{b} \end{cases}$$

g)

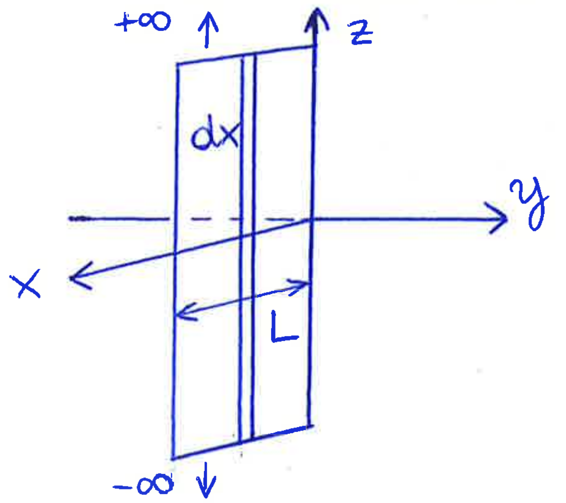


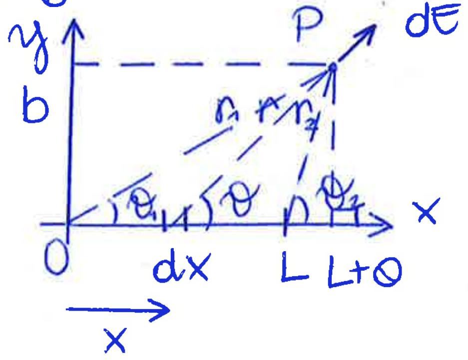
$$E_x = E_{x_1} - E_{x_2}$$

$$E_x = \frac{2k\lambda_1}{a} - \frac{2k\lambda_2}{b} \frac{\ell}{\sqrt{\ell^2+4b^2}}$$

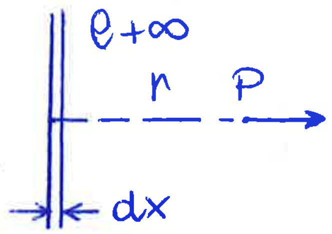
ESERCITAZIONE 1

(1) Determinare il campo $\vec{E}$ creato da un nastro indefinito di larghezza L portante una carica distribuita uniformemente con densità σ





$\sigma, L, \ell \rightarrow +\infty$



$$\boxed{dE = 2k\frac{dq}{r} = \frac{2k\sigma\,dx}{r}}$$

con $dq = \sigma\,dx$

$$\left.\begin{aligned} r\cos\theta &= L+a-x \\ r\sin\theta &= b \end{aligned}\right\} \quad tg\,\theta = \frac{b}{L+a-x}$$

$$L+a-x = \frac{b}{tg\,\theta} \Rightarrow x = L+a-\frac{b\cos\theta}{\sin\theta} = \frac{L\sin\theta + a\sin\theta - b\cos\theta}{\sin\theta}$$

$$\begin{cases} dx = \dfrac{\cancel{L\cos\theta\sin\theta} - \cancel{L\sin\theta\cos\theta} + \cancel{a\cos\theta\sin\theta} - \cancel{a\sin\theta\cos\theta} + b\sin^2\theta + b\cos^2\theta}{\sin^2\theta}\,d\theta \Rightarrow dx = \dfrac{b}{\sin^2\theta}\,d\theta \\ r = \dfrac{b}{\sin\theta} \end{cases}$$

$$dE = 2k\sigma\frac{\cancel{b}}{\sin^2\theta}\,d\theta\cdot\frac{\cancel{\sin\theta}}{\cancel{b}} = \frac{2k\sigma}{\sin\theta}\,d\theta$$

$$\begin{cases} dE_x = dE\cos\theta = \dfrac{2k\sigma}{\sin\theta}\cos\theta\,d\theta = 2k\sigma\,\mathrm{cotg}\,\theta\,d\theta \\ dE_y = dE\sin\theta = \dfrac{2k\sigma}{\cancel{\sin\theta}}\cancel{\sin\theta}\,d\theta = 2k\sigma\,d\theta \end{cases}$$

$$E_x = \int_{\theta_1}^{\theta_2} 2k\sigma\,\mathrm{cotg}\,\theta\,d\theta = 2k\sigma\ln\frac{\sin\theta_2}{\sin\theta_1}$$

$$E_x = 2k\sigma \int_{\vartheta_1}^{\pi/2} \cot g\,\vartheta \, d\vartheta = 2k\sigma \ln \frac{\sin \pi/2}{\sin\vartheta_1} = 2k\sigma \ln \frac{1}{\sin\vartheta_1}$$

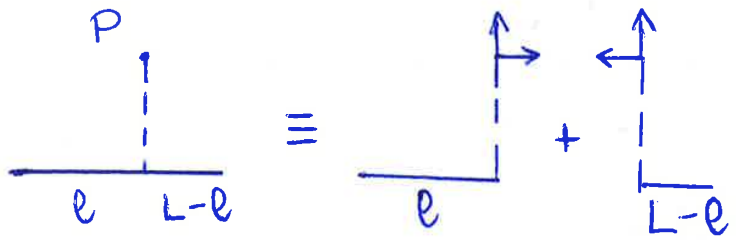
$$E_y = 2k\sigma \int_{\vartheta_1}^{\pi/2} d\vartheta = 2k\sigma\left(\frac{\pi}{2} - \vartheta_1\right)$$

con $\sqrt{L^2+b^2}\,\sin\vartheta_1 = b \Rightarrow \sin\vartheta_1 = \dfrac{b}{\sqrt{L^2+b^2}}$

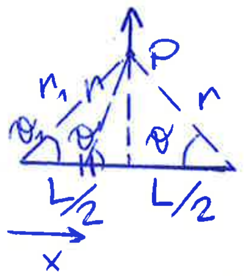
$$\vartheta_1 = \arcsin \frac{b}{\sqrt{L^2+b^2}}$$

$$\vec{E} = 2k\sigma \ln \frac{\sqrt{L^2+b^2}}{b}\,\vec{i} + 2k\sigma\left(\frac{\pi}{2} - \arcsin\frac{b}{\sqrt{L^2+b^2}}\right)\vec{j}$$

b)



in particolare



$E_x = 0$

$$dE = \frac{2k\sigma\,dx}{r}\,2$$

$$\left.\begin{aligned} r\cos\vartheta &= \frac{L}{2} - x \\ r\sin\vartheta &= b \end{aligned}\right\} \quad tg\,\vartheta = \frac{b}{\frac{L}{2} - x} \Rightarrow \frac{L}{2} - x = \frac{b}{tg\,\vartheta}$$

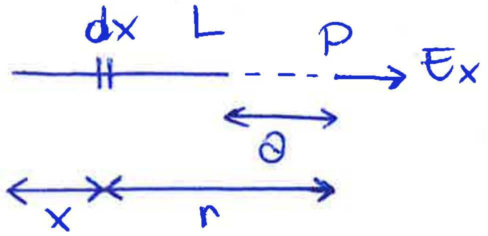
$$x = \frac{L}{2} - \frac{b\cos\vartheta}{\sin\vartheta} \Rightarrow dx = \frac{b\sin^2\vartheta + b\cos^2\vartheta}{\sin^2\vartheta}\,d\vartheta = \frac{b}{\sin^2\vartheta}\,d\vartheta$$

$$r = \frac{b}{\sin\vartheta}$$

$$dE = \frac{4k\sigma}{\cancel{b}}\,\cancel{\sin\vartheta}\,\frac{\cancel{b}\,d\vartheta}{\sin^{\cancel{2}}\vartheta} = \frac{4k\sigma}{\sin\vartheta}\,d\vartheta$$

$$dE_y = \frac{4k\sigma}{\cancel{\sin\vartheta}}\,\cancel{\sin\vartheta}\,d\vartheta = 4k\sigma\,d\vartheta$$

→ con calcolo diretto



$$dr = -dx$$

$$x = 0 \Rightarrow r = L + a$$

$$x = L \Rightarrow r = a$$

$$E = \int dE = \int \frac{2k\,d\lambda}{r} = \int_0^L \frac{2k\sigma\,dx}{L+a-x} = \int_a^{L+a} \frac{2k\sigma\,dr}{r} =$$

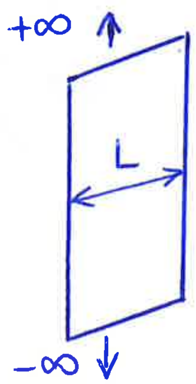
$$= 2k\sigma \int_a^{L+a} \frac{dr}{r} = 2k\sigma \ln\left(\frac{L+a}{a}\right)$$

$$\vec{E}_x = 2k\sigma \ln\left(\frac{L+a}{a}\right)\vec{i}$$

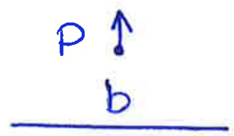
$L \to 0$ $\quad E_x \simeq \frac{2k\sigma L}{a} = \frac{2k\lambda}{a}$? $\quad$ OK (no punto)

$a \to \infty$ $\quad E_x \simeq \frac{2k\sigma L}{a} = \frac{2k\lambda}{a}$? $\quad$ OK

d) piano carico uniformemente



$$\vec{E} = 2k\sigma \ln \frac{\sin\theta_2}{\sin\theta_1}\,\vec{i} + 2k\sigma(\theta_2 - \theta_1)\,\vec{j}$$



$$\vec{E} = 4k\sigma\left(\frac{\pi}{2} - \arcsin\frac{b}{\sqrt{b^2 + \frac{L^2}{4}}}\right)\vec{j}$$

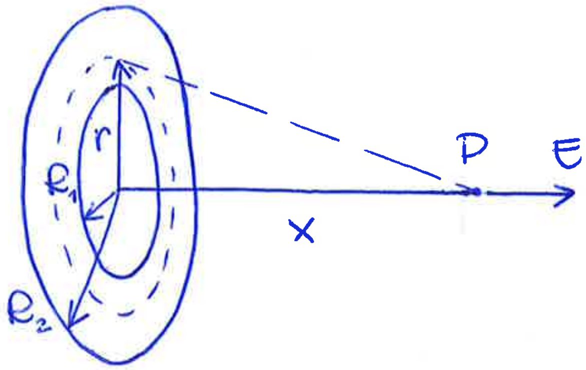
$$\lim_{L \to \infty} \vec{E} = 2k\sigma\pi = \frac{\sigma}{2\varepsilon_0}$$

$$\sin\vartheta_2 = \frac{b}{\sqrt{a^2+b^2}}$$

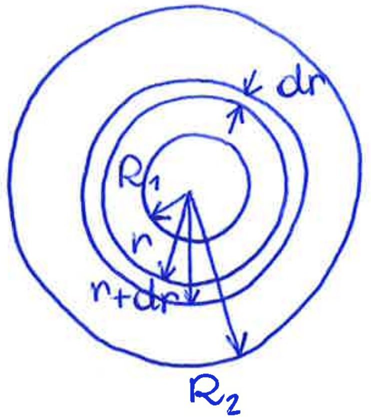
$$\cos\vartheta_1 = \frac{L+a}{\sqrt{(L+a)^2+b^2}}$$

$$\cos\vartheta_2 = \frac{a}{\sqrt{a^2+b^2}}$$

3



Calcolare il campo $\vec{E}$ creato sull'asse di una corona circolare di raggi R_1, R_2 carica uniformemente e portante una carica totale Q.



$\rightarrow$ la coroncina infinitesimale di raggio interno r ed esterno $r+dr$ è approssimabile per $dr \rightarrow 0$ ad un anello

$$\sigma = \frac{Q}{A} \Rightarrow \sigma = \frac{Q}{\pi(R_2^2 - R_1^2)}$$

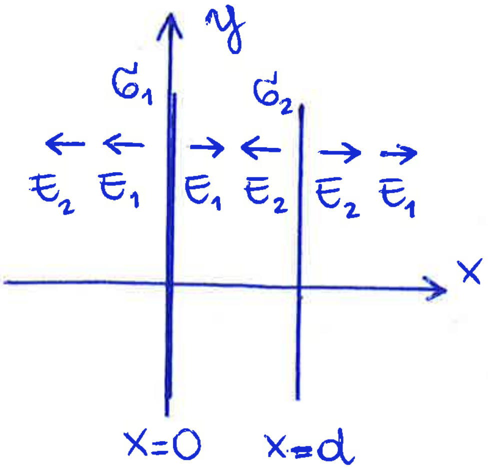
$$dq = \sigma\, dA = \sigma \cdot 2\pi r\, dr$$

$$\boxed{dE = k\, dq \frac{x}{(r^2+x^2)^{3/2}}} = \frac{k\sigma 2\pi r x\, dr}{(r^2+x^2)^{3/2}}$$

$$E = k\sigma 2\pi x \int_{R_1}^{R_2} \frac{r\,dr}{(r^2+x^2)^{3/2}} = k\sigma 2\pi x \left(\frac{1}{\sqrt{x^2+R_1^2}} - \frac{1}{\sqrt{x^2+R_2^2}}\right)$$

$$E = \frac{1}{{}_2\cancel{4\pi}\varepsilon_0}\sigma \cancel{2\pi} x \left(\frac{1}{\sqrt{x^2+R_1^2}} - \frac{1}{\sqrt{x^2+R_2^2}}\right) = \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{x^2+R_1^2}} - \frac{1}{\sqrt{x^2+R_2^2}}\right)$$

5



2 piani // ∞, σ_1, σ_2

$E(x) = ?$

$V(x) = ?$

$$\begin{cases} x<0 & E = -\dfrac{\sigma_1}{2\varepsilon_0} - \dfrac{\sigma_2}{2\varepsilon_0} = \dfrac{-\sigma_1-\sigma_2}{2\varepsilon_0} \\ 0<x<d & E = \dfrac{\sigma_1}{2\varepsilon_0} - \dfrac{\sigma_2}{2\varepsilon_0} = \dfrac{\sigma_1-\sigma_2}{2\varepsilon_0} \\ x>d & E = \dfrac{\sigma_1}{2\varepsilon_0} + \dfrac{\sigma_2}{2\varepsilon_0} = \dfrac{\sigma_1+\sigma_2}{2\varepsilon_0} \end{cases}$$

$$E = -\frac{dV}{dx} \Rightarrow V = -\int E\,dx + C \Rightarrow V = -Ex + C$$

$$\begin{cases} x<0 & V = \dfrac{\sigma_1+\sigma_2}{2\varepsilon_0}x + C_1 \\ 0<x<d & V = \dfrac{\sigma_2-\sigma_1}{2\varepsilon_0}x + C_2 \\ x>d & V = -\dfrac{\sigma_1+\sigma_2}{2\varepsilon_0}x + C_3 \end{cases}$$

$$V(0^-) = V(0^+) \Rightarrow C_1 = C_2$$

$$V(d^-) = V(d^+) \Rightarrow \frac{\sigma_2-\sigma_1}{2\varepsilon_0}d + C_2 = -\frac{\sigma_1+\sigma_2}{2\varepsilon_0}d + C_3$$

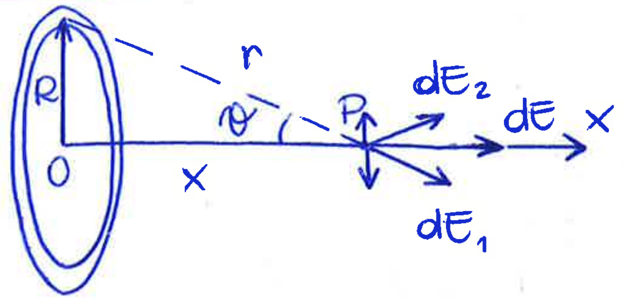
$$\frac{\sigma_2-\sigma_1}{2\varepsilon_0}d + \frac{\sigma_1+\sigma_2}{2\varepsilon_0}d = C_3 - C_2$$

$$C_3 - C_2 = \frac{\sigma_2 d - \cancel{\sigma_1 d} + \cancel{\sigma_1 d} + \sigma_2 d}{2\varepsilon_0} = \frac{\sigma_2 d}{\varepsilon_0}$$

se $C_2 = 0 \Rightarrow C_1 = C_2 = 0$

$$C_3 = \frac{\sigma_2 d}{\varepsilon_0}$$

ES AGGIUNTO



Una carica q è distribuita uniformemente su un sottile anello di raggio R. Calcolare il campo $\vec{E}$ sull'asse dell'anello.

$$dE_y = 0$$

$$dE_x = 2\,dE_1 \cos\vartheta$$

con $dE_1 = dE_2 = k \dfrac{dq}{r^2}$

$$dE_1 = k \frac{\lambda\, dl}{r^2}$$

con $dq = \lambda\, dl$ e $\lambda = \dfrac{q}{2\pi R}$

$$E_x = \frac{2k\lambda}{r^2} \cos\vartheta \int_0^l dl = \frac{2k\lambda \cos\vartheta}{r^2} \pi R$$

con $r^2 = x^2 + R^2$ e $\cos\vartheta = \dfrac{x}{\sqrt{x^2+R^2}}$

$$E_x = \frac{2k\lambda}{x^2+R^2} \frac{x}{\sqrt{x^2+R^2}} \pi R = \frac{\cancel{2}^{1}\lambda}{{}_{2}\cancel{4}\cancel{\pi}\varepsilon_0} \frac{x}{(x^2+R^2)^{3/2}} \cancel{\pi} R =$$

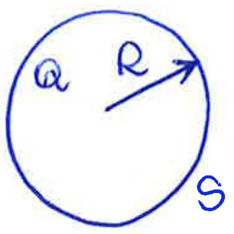
$$= \frac{\lambda R}{2\varepsilon_0} \frac{x}{(x^2+R^2)^{3/2}} = \frac{q\,\cancel{R}}{4\pi\varepsilon_0 \cancel{R}} \frac{x}{(x^2+R^2)^{3/2}} = kq \frac{x}{(x^2+R^2)^{3/2}}$$

$$\vec{E} = kq \frac{x}{(x^2+R^2)^{3/2}} \hat{u}_x$$

$$\vec{E}(x \gg R) = kq \frac{1}{x^2} \hat{u}_x$$

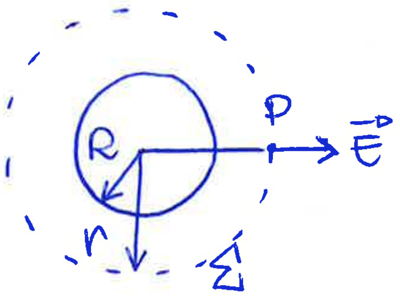
ESERCITAZIONE 2

1



Sfera con carica Q distribuita uniformemente al suo interno. Calcolare il campo $\vec{E}(r)$.

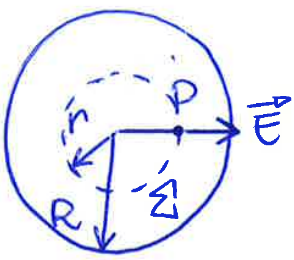
$r > R$



$$\begin{cases} \Phi(E) = \dfrac{Q}{\varepsilon_0} \\ \Phi(E) = \int_\Sigma \vec{E}\, d\vec{\sigma} = E\, 4\pi r^2 \end{cases}$$

$$E\, 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$r < R$



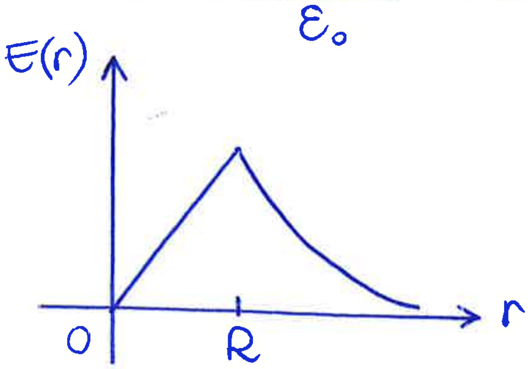
$$Q = \rho V = \rho \frac{4}{3}\pi R^3 \Rightarrow \rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$q = \rho V_\Sigma = \rho \frac{4}{3}\pi r^3 = \frac{Q}{\cancel{\frac{4}{3}}\pi R^3}\,\cancel{\frac{4}{3}}\,\pi r^3$$

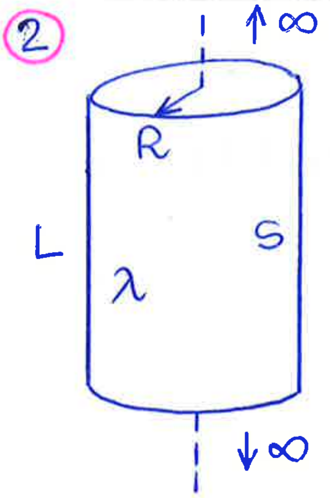
$$q = Q\frac{r^3}{R^3}$$

$$\begin{cases} \Phi(E) = \dfrac{q}{\varepsilon_0} \\ \Phi(E) = \int_\Sigma \vec{E}\, d\vec{\sigma} = E\, 4\pi r^2 \end{cases}$$

$$E\, 4\pi r^2 = \frac{Q\frac{r^3}{R^3}}{\varepsilon_0} \Rightarrow E = \frac{Q r^{\cancel{3}}}{4\pi \cancel{r^2} \varepsilon_0 R^3} = \frac{1}{4\pi\varepsilon_0} Q \frac{r}{R^3}$$



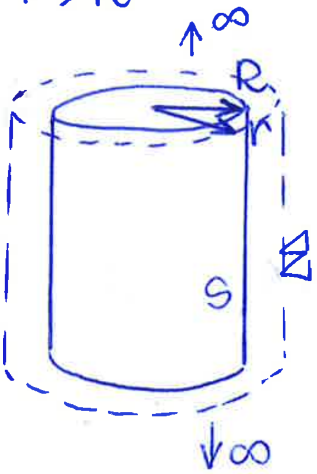
2



Determinare il campo $\vec{E}$ creato da una carica distribuita uniformemente su una superficie cilindrica indefinita di raggio R, densità di carica lineare λ e superficie S.

$$q = \lambda L$$

$r > R$

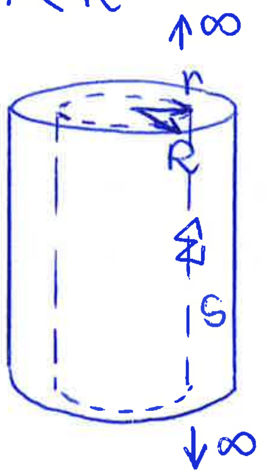


$$\begin{cases} \Phi(E) = \dfrac{q}{\varepsilon_0} \\ \Phi(E) = \displaystyle\int_{\Sigma} \vec{E}\, d\vec{\sigma} = E \cdot 2\pi r L \end{cases}$$

$$E \cdot 2\pi r L = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{1}{2\pi\varepsilon_0}$$

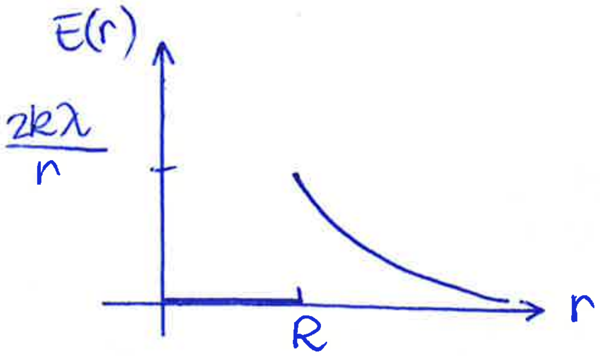
$$E = \frac{2k\lambda}{r}$$

$r < R$

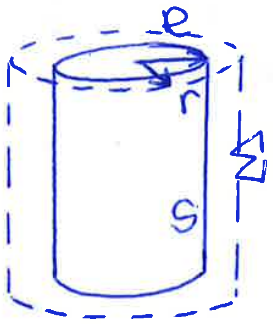


$$\begin{cases} \Phi(E) = \dfrac{q}{\varepsilon_0} = 0 \\ \Phi(E) = \displaystyle\int_{\Sigma} \vec{E}\, d\vec{\sigma} = E \cdot 2\pi r L \end{cases}$$

$$E\, 2\pi r L = 0 \Rightarrow E = 0$$



$r > R$

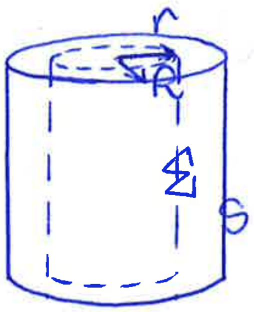


$$\begin{cases} \Phi(E) = \dfrac{q}{\varepsilon_0} \\ \Phi(E) = \int_\Sigma \vec{E}\, d\vec{\sigma} = E\, 2\pi r L \end{cases}$$

$$E\, 2\pi r L = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{2\pi\varepsilon_0 r L} = \frac{\rho \cancel{\pi} R^2 \cancel{L}}{2\cancel{\pi}\varepsilon_0 r \cancel{L}}$$

$$E = \frac{\rho R^2}{2\varepsilon_0 r}$$

$r < R$

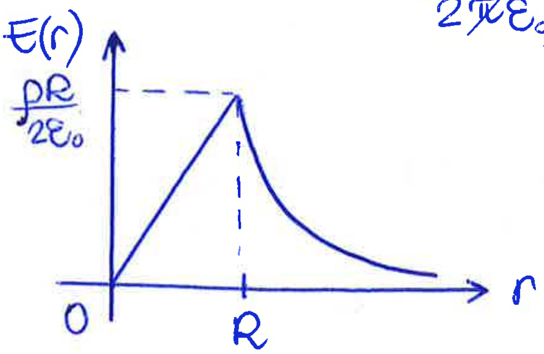


$$\begin{cases} \Phi(E) = \dfrac{Q}{\varepsilon_0} \\ \Phi(E) = \int_\Sigma \vec{E}\, d\vec{\sigma} = E\, 2\pi r L \end{cases}$$

con $Q = q\dfrac{r^2}{R^2}$

$$E\, 2\pi \cancel{r} L = \frac{q\, r^{\cancel{2}}}{\varepsilon_0 R^2} \Rightarrow E = \frac{q}{2\pi\varepsilon_0 L}\,\frac{r}{R^2}$$

$$E = \frac{\rho \cancel{\pi} \cancel{R^2} \cancel{L}}{2\cancel{\pi}\varepsilon_0 \cancel{L}}\,\frac{r}{\cancel{R^2}} = \frac{\rho r}{2\varepsilon_0}$$



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$V(r) = -\int E(r)\,dr + C$

$r < R_1 \qquad V(r) = C_1$

$R_1 < r < R_2 \qquad V(r) = -\int \frac{2k\lambda_1}{r}\,dr = -2k\lambda_1 \ln r + C_2$

$r > R_2 \qquad V(r) = -\int \frac{2k(\lambda_1+\lambda_2)}{r}\,dr = -2k(\lambda_1+\lambda_2)\ln r + C_3$

per $r = +\infty$ avrei un potenziale pari a $-\infty$ che è assurdo, quindi porto dall'interno a calcolare

$\Rightarrow$ pongo $=0$ il potenziale all'interno

$\Rightarrow C_1 = 0$

$V(R_1^-) = V(R_1^+) \Rightarrow 0 = -2k\lambda_1 \ln R_1 + C_2 \Rightarrow C_2 = +2k\lambda_1 \ln R_1$

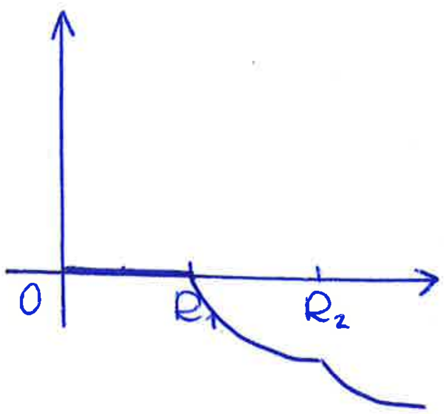
$V(R_2^-) = V(R_2^+) \Rightarrow 2k\lambda_1 \ln \frac{R_1}{R_2} = 2k(\lambda_1+\lambda_2)\ln R_2 + C_3$

$C_3 = 2k\lambda_1 \ln \frac{R_1}{R_2} + 2k(\lambda_1+\lambda_2)\ln R_2$

$$V(r) = \begin{cases} r < R_1 & 0 \\ R_1 < r < R_2 & -2k\lambda_1 \ln r + 2k\lambda_1 \ln R_1 = 2k\lambda_1 \ln \frac{R_1}{r} \\ r > R_2 & -2k(\lambda_1+\lambda_2)\ln r + 2k\lambda_1 \ln \frac{R_1}{R_2} + \end{cases}$$

$+ 2k(\lambda_1+\lambda_2)\ln R_2 =$

$= 2k(\lambda_1+\lambda_2)\ln \frac{R_2}{r} + 2k\lambda_1 \ln \frac{R_1}{R_2}$



$r < R_1 \qquad V(r) = C_1 \neq 0!$ *

$R_1 < r < R_2 \qquad V(r) = -\frac{q_1}{4\pi\varepsilon_0}\int_0^r \frac{1}{r^2}\,dr + C_2 = \frac{q_1}{4\pi\varepsilon_0}\frac{1}{r} + C_2$

$r > R_2 \qquad V(r) = -\frac{q_1+q_2}{4\pi\varepsilon_0}\int_0^r \frac{1}{r^2}\,dr + C_3 = \frac{q_1+q_2}{4\pi\varepsilon_0}\frac{1}{r} + C_3$

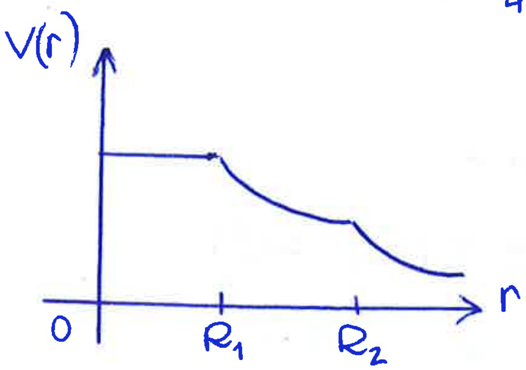
$$V(\infty) = 0 \implies C_3 = 0$$

$$V(R_2^-) = V(R_2^+) \implies \frac{q_1}{4\pi\varepsilon_0}\frac{1}{R_2} + C_2 = \frac{q_1+q_2}{4\pi\varepsilon_0}\frac{1}{R_2} \implies$$

$$C_2 = \frac{q_1+q_2}{4\pi\varepsilon_0}\frac{1}{R_2} - \frac{q_1}{4\pi\varepsilon_0}\frac{1}{R_2} = \frac{q_2}{4\pi\varepsilon_0 R_2}$$

$$V(R_1^-) = V(R_1^+) \implies C_1 = \frac{q_1}{4\pi\varepsilon_0}\frac{1}{R_1} + \frac{q_2}{4\pi\varepsilon_0}\frac{1}{R_2}$$

$$V(r) = \begin{cases} r < R_1 & \frac{q_1}{4\pi\varepsilon_0 R_1} + \frac{q_2}{4\pi\varepsilon_0 R_2} \\ R_1 < r < R_2 & \frac{q_1}{4\pi\varepsilon_0 r} + \frac{q_2}{4\pi\varepsilon_0 R_2} \\ r > R_2 & \frac{q_1+q_2}{4\pi\varepsilon_0 r} \end{cases}$$

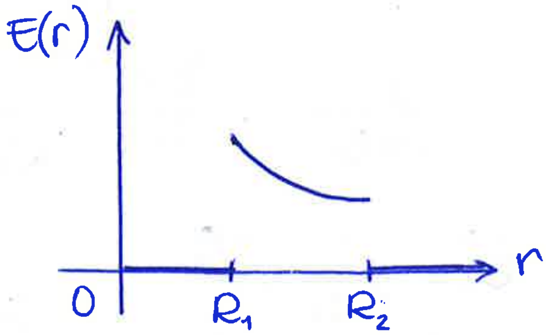


* nella sfera non ho il problema del potenziale pari a $-\infty$ per $r = +\infty$ quindi $C_1 \neq 0$ e $V(\infty) = 0$

CASO SPECIALE

$$q_2 = -q_1 = -q$$

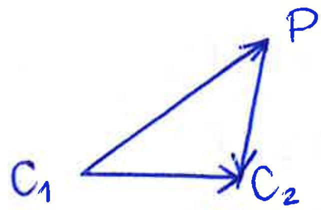
$$E(r) = \begin{cases} r < R_1 & 0 \\ R_1 < r < R_2 & \frac{q}{4\pi\varepsilon_0 r^2} \\ r > R_2 & 0 \end{cases}$$



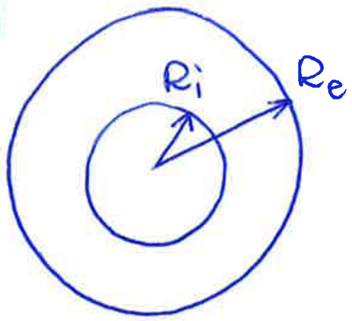
$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\overrightarrow{C_1P} - \frac{\rho}{3\varepsilon_0}\overrightarrow{C_2P} = \frac{\rho}{3\varepsilon_0}\left(\overrightarrow{C_1P} - \overrightarrow{C_2P}\right) =$$

$$= \frac{\rho}{3\varepsilon_0}\left(\overrightarrow{C_1P} + \overrightarrow{PC_2}\right) = \frac{\rho}{3\varepsilon_0}\overrightarrow{C_1C_2}$$

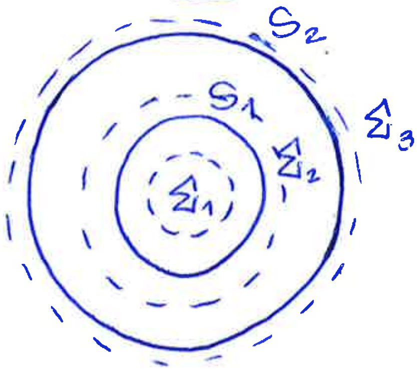


4



Goscio sferico R_i, R_e con carico q distribuito uniformemente.

$E = ?$



$$q = \rho \frac{4}{3}\pi\left(R_e^3 - R_i^3\right) \Rightarrow \rho = \frac{q}{\frac{4}{3}\pi\left(R_e^3 - R_i^3\right)}$$

$$Q = \rho V_\Sigma = \frac{q}{\frac{4}{3}\pi\left(R_e^3 - R_i^3\right)} \cdot \frac{4}{3}\pi\left(r^3 - R_i^3\right)$$

$r < R_i \rightarrow \Sigma_1$ $\qquad \Phi(E) = \frac{q}{\varepsilon_0} = 0 \Rightarrow E = 0$

$R_i < r < R_e \rightarrow \Sigma_2$

$$\begin{cases} \Phi(E) = \frac{Q}{\varepsilon} \\ \Phi(E) = \int \vec{E}\, d\vec{\sigma} = E\, 4\pi r^2 \end{cases}$$

$$E\, \cancel{4\pi} r^2 = \frac{\rho \cdot \frac{\cancel{4}}{3}\cancel{\pi}\left(r^3 - R_i^3\right)}{\varepsilon_0} \Rightarrow$$

$$E = \frac{\rho\left(r^3 - R_i^3\right)}{3\varepsilon_0 r^2}$$

$r > R_e \rightarrow \Sigma_3$

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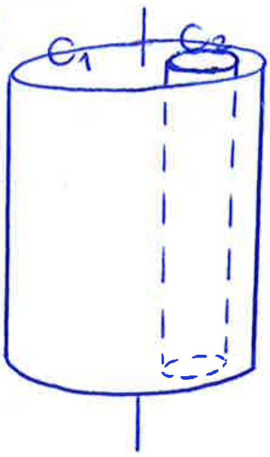
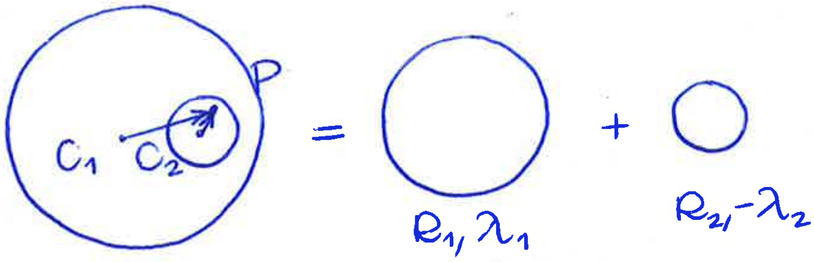
ESERCITAZIONE 4

1

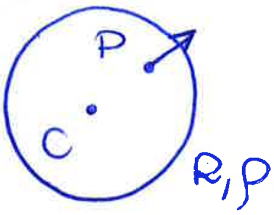


All'interno di un cilindro indefinito di raggio R_1 portante una carica distribuita uniformemente con densità di carica λ, è stata ricavata una cavità cilindrica eccentrica di raggio R_2. Determinare il campo $\vec{E}$ all'interno della cavità. $\vec{E}(P) = ?$



→ cilindro pieno di altezza L e raggio R

$$q = \lambda L = \rho \pi R^2 L \quad \Rightarrow \lambda = \rho \pi R^2$$



$$\vec{E}(P) = \frac{\rho}{2\varepsilon_0} \overrightarrow{CP} \;*$$

$$E(\vec{P}) = \vec{E}_+(P) + \vec{E}_-(P) = \frac{\rho}{2\varepsilon_0} \overrightarrow{C_1P} - \frac{\rho}{2\varepsilon_0} \overrightarrow{C_2P} =$$

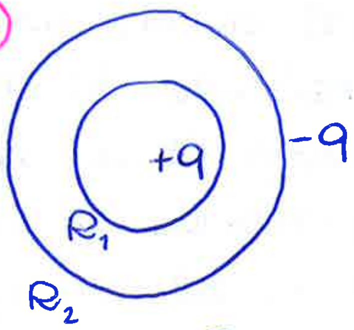
$$= \frac{\rho}{2\varepsilon_0} \left(\overrightarrow{C_1P} - \overrightarrow{C_2P} \right) = \frac{\rho}{2\varepsilon_0} \overrightarrow{C_1C_2}$$

* da

$$\begin{cases} \Phi(E) = \int_\Sigma \vec{E}\, d\vec{\sigma} = E\, 2\pi R L \\ \Phi(E) = \dfrac{q}{\varepsilon_0} = \dfrac{\rho \pi R^2 L}{\varepsilon_0} \end{cases}$$

$$E\, 2\cancel{\pi} \cancel{R} \cancel{L} = \frac{\rho \cancel{\pi} R^{\cancel{2}} \cancel{L}}{\varepsilon_0} \Rightarrow E = \frac{\rho}{2\varepsilon_0} R$$

3



Calcolare la capacità di un sistema di due superfici conduttrici sferiche concentriche di raggi R_1, R_2 portanti cariche $+q$ e $-q$ con $|+q| = |-q|$

$$E(r) = \begin{cases} r < R_1 & 0 \\ R_1 < r < R_2 & \dfrac{kq}{r^2} \\ r > R_2 & \dfrac{k(q-q)}{r^2} = 0 \end{cases}$$

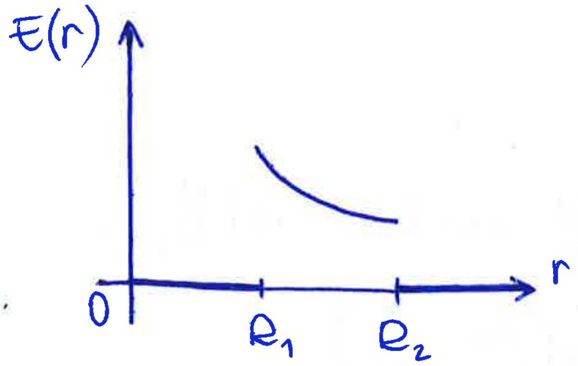
$$V(r) = \begin{cases} r < R_1 & C_1 & \Rightarrow \dfrac{kq}{R_1} - \dfrac{kq}{R_2} \\ R_1 < r < R_2 & + \dfrac{kq}{r} + C_2 & \Rightarrow \dfrac{kq}{r} - \dfrac{kq}{R_2} \\ r > R_2 & C_3 & \Rightarrow 0 \end{cases}$$

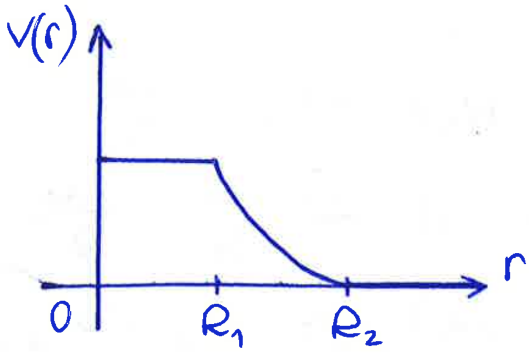
$$V(\infty) = 0 \quad \Rightarrow \quad C_3 = 0$$

$$V(R_2^+) = V(R_2^-) \Rightarrow \frac{kq}{R_2} + C_2 = 0 \Rightarrow C_2 = -\frac{kq}{R_2}$$

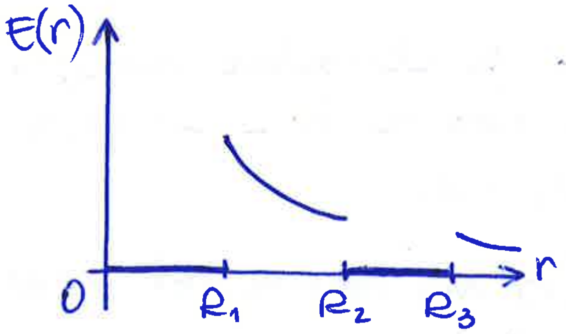
$$V(R_1^+) = V(R_1^-) \Rightarrow \frac{kq}{R_1} + C_2 = C_1 \Rightarrow \frac{kq}{R_1} - \frac{kq}{R_2} = C_1$$

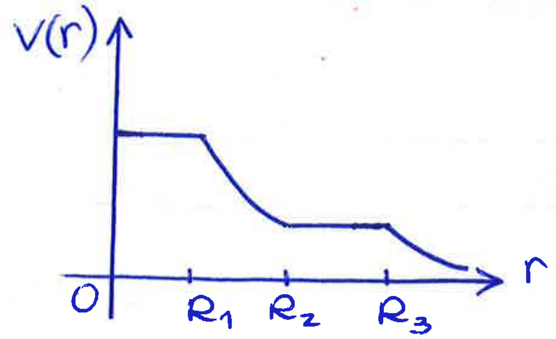
$$\Delta V = V_{in} - V_{est} = \frac{kq}{R_1} - \frac{kq}{R_2} - 0 = kq\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$





$$C = \frac{q}{\Delta V} = \frac{\cancel{q}}{k\cancel{q}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} = \frac{4\pi\varepsilon_0}{\frac{1}{R_1} - \frac{1}{R_2}}$$





$$\Delta V = V_{int} - V_{est} = V_{sfera} - V_{guscio}$$

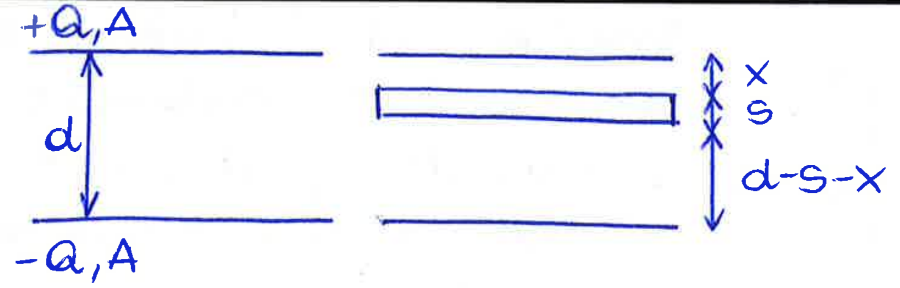
con $V_{sfera} = kq\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3}\right)$

$V_{guscio} = kq\left(\frac{1}{R_3}\right) \longrightarrow V(r)$ per $R_2 < r < R_3$

$$\Delta V = kq\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3}\right) - kq\left(\frac{1}{R_3}\right) = kq\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$C = \frac{q}{\Delta V} = \frac{\cancel{q}}{k\cancel{q}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} = \frac{4\pi\varepsilon_0}{\frac{1}{R_1} - \frac{1}{R_2}}$$

2)

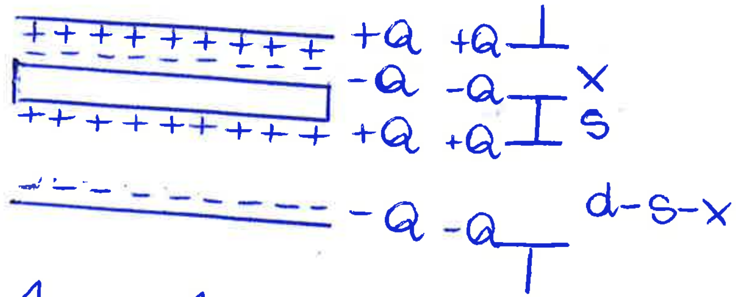


Si inserisce uno spessore metallico tra le armature di un condensatore piano. (Conosco A)

A) Calcolare ΔU nel caso in cui il generatore di tensione tra le armature è scollegato $\Rightarrow Q = cost$

B) Calcolare ΔU nel caso in cui il generatore di tensione tra le armature è collegato $\Rightarrow V = cost$

A) $Q = cost$, $V =$ varia



$$C_x = \frac{\varepsilon_0 A}{x}$$

$$C_{d-s-x} = \frac{\varepsilon_0 A}{d-s-x}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_x} + \frac{1}{C_{d-s-x}} = \frac{x}{\varepsilon_0 A} + \frac{d-s-x}{\varepsilon_0 A} = \frac{d-s}{\varepsilon_0 A}$$

$$C_2 = \frac{\varepsilon_0 A}{d-s}$$

$$C_1 = \frac{\varepsilon_0 A}{d}$$

$$\Delta V = V_2 - V_1 = \frac{Q}{C} - \frac{Q}{C_1} = Q\left(\frac{1}{C_2} - \frac{1}{C_1}\right)$$

$$= Q\left(\frac{d-s}{\varepsilon_0 A} - \frac{d}{\varepsilon_0 A}\right) = -\frac{Qs}{\varepsilon_0 A}$$

$$\Delta U = \frac{1}{2}\frac{Q^2}{C_2} - \frac{1}{2}\frac{Q^2}{C_1} = \frac{1}{2}Q^2\left(\frac{1}{C_2} - \frac{1}{C_1}\right) =$$

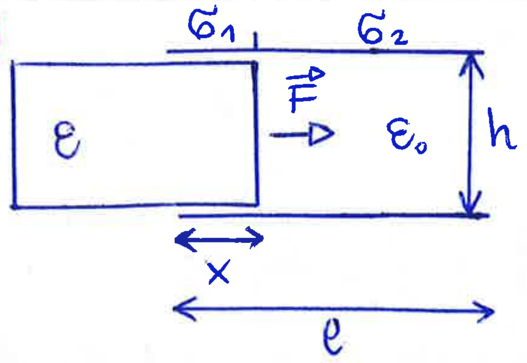
$$= \frac{1}{2}Q^2\left(\frac{d-s}{\varepsilon_0 A} - \frac{d}{\varepsilon_0 A}\right) = -\frac{1}{2}\frac{Q^2 s}{\varepsilon_0 A}$$

B) $Q =$ varia, $V = cost$

$$\Delta U = \frac{1}{2}C_2 V^2 - \frac{1}{2}C_1 V^2 = \frac{1}{2}V^2(C_2 - C_1) = \frac{1}{2}V^2\left(\frac{\varepsilon_0 A}{d-s} - \frac{\varepsilon_0 A}{d}\right)$$

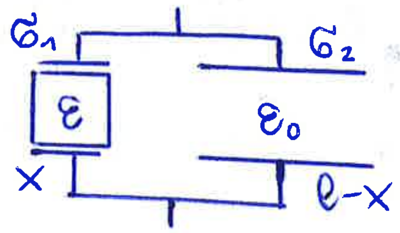
$$\Delta Q = Q_2 - Q_1 = C_2 V - C_1 V = V(C_2 - C_1) = \varepsilon_0 A V\left(\frac{1}{d-s} - \frac{1}{d}\right)$$

4



Inserimento di un dielettrico tra le armature di un condensatore. Si conoscono $h, L, \ell, x, \varepsilon_0, \varepsilon, Q_{iniziale}$. Calcolare la $\vec{F}$ di richiamo del dielettrico all'interno del condensatore.

$A = \ell L$



$$V_1 = V_2 \Rightarrow E_1 = E_2 \Rightarrow \frac{\sigma_1}{\varepsilon} = \frac{\sigma_2}{\varepsilon_0}$$

$$A_1 = xL$$

$$A_2 = (\ell - x)L$$

$$C_1 = \frac{\varepsilon_0 A_1}{h} = \frac{\varepsilon\, xL}{h}$$

$$C_2 = \frac{\varepsilon_0 A_2}{h} = \frac{\varepsilon_0(\ell - x)L}{h}$$

$$C_{eq} = C_1 + C_2 = \frac{\varepsilon x L}{h} + \frac{\varepsilon_0(\ell - x)L}{h} = \frac{\varepsilon x L + \varepsilon_0(\ell - x)L}{h}$$

$$Q = q_1 + q_2 = \sigma_1 A_1 + \sigma_2 A_2 = \sigma_1 xL + \sigma_2(\ell - x)L$$

$$\begin{cases} \sigma_1\varepsilon_0 - \sigma_2\varepsilon = 0 & \Rightarrow \sigma_2 = \dfrac{\sigma_1\varepsilon_0}{\varepsilon} \\ \sigma_1 x + \sigma_2(\ell - x) = \dfrac{Q}{L} & \Rightarrow \sigma_1 x + \dfrac{\sigma_1\varepsilon_0}{\varepsilon}(\ell - x) = \dfrac{Q}{L} \end{cases}$$

$$\sigma_1\left(x + \frac{\varepsilon_0}{\varepsilon}(\ell - x)\right) = \frac{Q}{L} \Rightarrow \sigma_1\left(\frac{\varepsilon x + \varepsilon_0(\ell - x)}{\varepsilon}\right) = \frac{Q}{L}$$

$$\begin{cases} \sigma_1 = \dfrac{Q}{L}\,\dfrac{\varepsilon}{\varepsilon x + \varepsilon_0(\ell - x)} \\ \sigma_2 = \dfrac{\sigma_1\varepsilon_0}{\varepsilon} = \dfrac{Q}{L}\,\dfrac{\varepsilon_0}{\varepsilon x + \varepsilon_0(\ell - x)} \end{cases}$$

$$U(x) = \frac{1}{2}C(x)V^2 = \frac{1}{2}C(x)E^2h^2 = \frac{1}{2}C(x)E_1^2h^2 = \frac{1}{2}C(x)\frac{\sigma_1^2}{\varepsilon^2}h^2 =$$

$$= \frac{1}{2}\,\frac{L(\varepsilon x + \varepsilon_0(\ell - x))}{\cancel{h}}\,\frac{\sigma_1^2}{\varepsilon^2}h^{\cancel{2}} = \frac{h\cancel{L}}{2\cancel{\varepsilon^2}}\,\frac{\cancel{(\varepsilon x + \varepsilon_0(\ell - x))}}{(\varepsilon x + \varepsilon_0(\ell - x))^{\cancel{2}}}\,\frac{Q^2\cancel{\varepsilon^2}}{L^{\cancel{2}}} =$$

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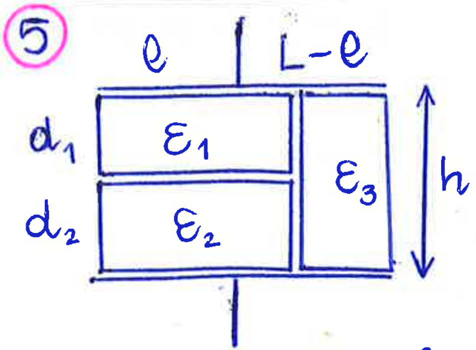
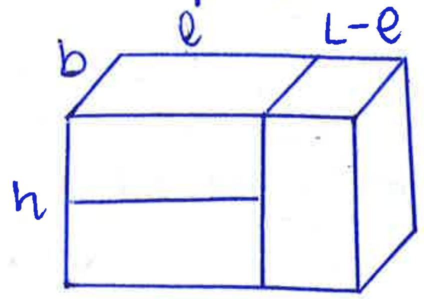
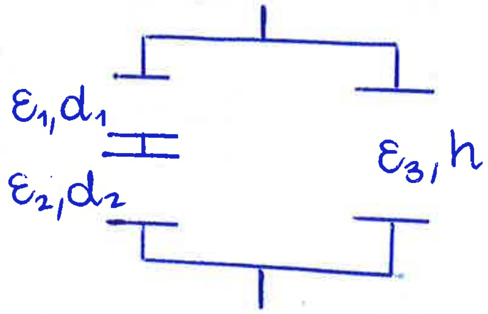
5



Sono note le $\varepsilon_1, \varepsilon_2, \varepsilon_3$ dei mezzi dielettrici che occupano lo spazio tra le 2 armature del condensatore. Calcolare la sua C





$$C_1 = \frac{\varepsilon A_1}{d_1} = \frac{\varepsilon_1 b\ell}{d_1}$$

$$C_2 = \frac{\varepsilon_2 A_2}{d_2} = \frac{\varepsilon_2 b\ell}{d_2}$$

in serie

$$C_3 = \frac{\varepsilon_3 A_3}{h} = \frac{\varepsilon_3 (L-\ell) b}{h}$$

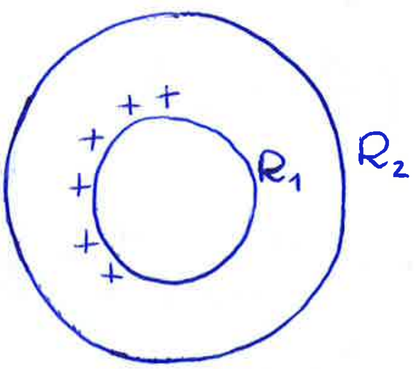
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{\varepsilon_1 b\ell} + \frac{d_2}{\varepsilon_2 b\ell} = \frac{1}{b\ell}\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right)$$

$$C_{eq} = \frac{b\ell}{\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right)}$$

in parallelo con C_3

$$C_{eq} + C_3 = \frac{b\ell}{\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right)} + \frac{\varepsilon_3 (L-\ell) b}{h} = C_{tot}$$

6



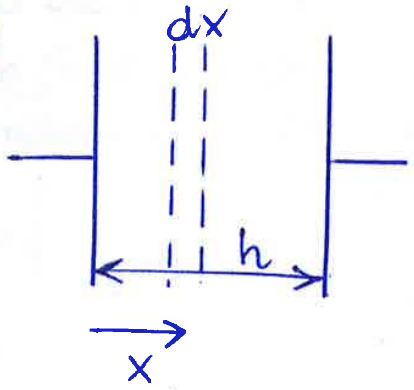
Una sfera metallica di raggio R_1 è ricoperta da un guscio di spessore $R_2 - R_1$ dielettrico di costante dielettrica ε. Calcolare la capacità del condensatore.

→ se non ho il dielettrico $R_2 \to R_1$

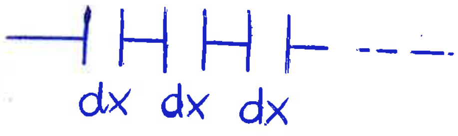
$\varepsilon = \varepsilon_0$

$\Rightarrow C = 4\pi\varepsilon_0 R_1$

7



Tra le armature di un condensatore esiste un mezzo dielettrico con $\varepsilon(x)$. Calcolare la capacità del condensatore.



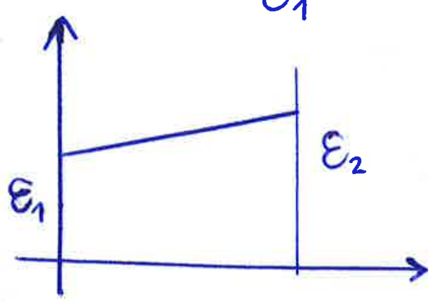
$$C(x) = \frac{A\varepsilon(x)}{dx}$$

$$\frac{1}{C} = \sum_i \frac{1}{C(x)}$$

$$\frac{1}{C} = \int \frac{1}{C(x)} = \frac{1}{A}\int_0^h \frac{dx}{\varepsilon(x)} \Rightarrow C = \frac{A}{\int_0^h \frac{dx}{\varepsilon(x)}}$$

→ ad es se $\varepsilon(x) = \varepsilon_1 + \frac{\varepsilon_2 - \varepsilon_1}{d} x$

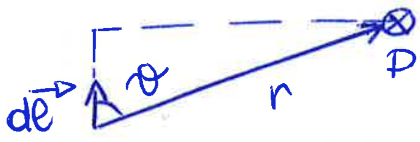
$$C = A \frac{\varepsilon_2 - \varepsilon_1}{h \ln \frac{\varepsilon_2}{\varepsilon_1}}$$



$$\cos\vartheta_1 = -\cos(\pi - \vartheta_1) = -\frac{L/2}{\sqrt{\left(\frac{L}{2}\right)^2 + R^2}}$$

$$\vec{B} = \hat{u}_r \frac{\mu_0 i}{4\pi R} \frac{L/2}{\sqrt{\frac{L^2}{4} + R^2}}$$

b) contributo del tratto inferiore



$$d\vec{B} = \frac{\mu_0 i}{4\pi} \frac{d\vec{l} \wedge \vec{r}}{r^3} = \frac{\mu_0 i}{4\pi} \frac{d\vec{l} \wedge \hat{u}_r}{r^2} =$$

$$= \frac{\mu_0 i}{4\pi} \frac{dl \sin\vartheta \, \hat{u}_r}{r^2}$$

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 i}{4\pi} \hat{u}_r \int_{\vartheta}^{\pi/2} \frac{dl}{r^2} \sin\vartheta$$

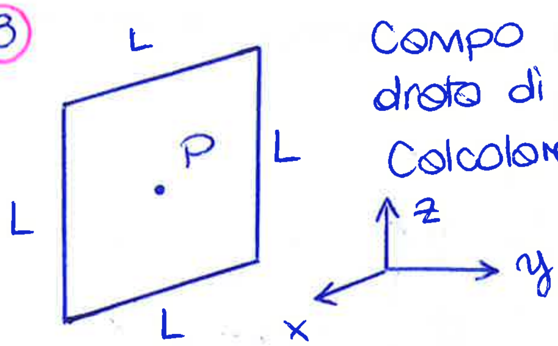
$$\vec{B} = \hat{u}_r \frac{\mu_0 i}{4\pi R} \frac{L/2}{\sqrt{\frac{L^2}{4} + R^2}}$$

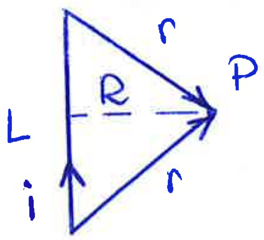
$$\vec{B}_{tot} = 2 \cdot \hat{u}_r \frac{\mu_0 i}{4\pi R} \frac{L/2}{\sqrt{\frac{L^2}{4} + R^2}} = \hat{u}_r \frac{\mu_0 i}{2\pi R} \frac{L/2}{\sqrt{\frac{L^2}{4} + R^2}}$$

2) Filo rettilineo indefinito percorso da i crea un campo $\vec{B}$ ad una distanza R.

$$\vec{B} = \frac{\mu_0 i}{2\pi R} \hat{u}_r \qquad \text{LEGGE di BIOT-SAVART}$$

3) Campo creato al centro di una spira quadrata di lato L percorsa da i.
Calcolare $\vec{B}(P)$.





$$\vec{B}_1 = \hat{u} \frac{\mu_0 i}{2\pi R} \frac{L}{\sqrt{L^2 + 4R^2}}$$

con $R = \frac{L}{2}$

$$\vec{B}_1 = \hat{u}_\varphi \frac{\mu_0 i}{2\pi \frac{L}{2}} \frac{L}{\sqrt{L^2 + L^2}} = \hat{u}_\varphi \frac{\mu_0 i}{\pi\sqrt{2} L}$$

$$\vec{B} = 4\vec{B}_1 = \hat{u}_\varphi \frac{2\sqrt{2}\mu_0 i}{\pi L}$$

$$\vec{B}_1 = \frac{\mu_0 i}{2\pi r}\left(\cos\vartheta\,\hat{u}_x + \sin\vartheta\,\hat{u}_y\right)$$

$$\vec{B}_2 = \frac{\mu_0 i}{2\pi r}\left(\cos\vartheta(-\hat{u}_x) + \sin\vartheta\,\hat{u}_y\right)$$

$$\vec{B}_3 = \frac{\mu_0 i}{2\pi r}\left(\cos\vartheta(-\hat{u}_x) + \sin\vartheta\,\hat{u}_y\right)$$

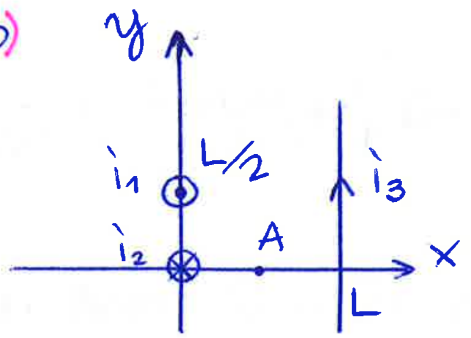
$$\vec{B}_4 = \frac{\mu_0 i}{2\pi r}\left(\cos\vartheta(-\hat{u}_x) + \sin\vartheta(-\hat{u}_y)\right)$$

con $r = L\sqrt{2}$ e $\vartheta = \frac{\pi}{4} = 45°$

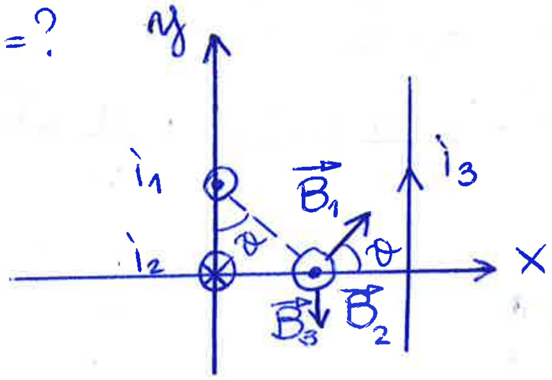
$$\vec{B}(0) = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = \frac{\mu_0 i \cdot \cancel{2}}{\cancel{2}\pi\sqrt{2}L}\left(\cos\vartheta(-\hat{u}_x) + \sin\vartheta\,\hat{u}_y\right) =$$

$$= \frac{\mu_0 i}{\pi L}\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\left(-\hat{u}_x + \hat{u}_y\right) = \frac{\mu_0 i}{2\pi L}\left(-\hat{u}_x + \hat{u}_y\right)$$

b)



$\vec{B}(A) = ?$



$$\vec{B}_1 = B_1\,\hat{u}_\vartheta = \frac{\mu_0 i_1}{2\pi r_1}\left(\cos\vartheta\,\hat{u}_x + \sin\vartheta\,\hat{u}_y\right)$$

$$\vec{B}_2 = B_2(-\hat{u}_y) = \frac{\mu_0 i_2}{2\pi r_2}(-\hat{u}_y)$$

$$\vec{B}_3 = B_3(\hat{u}_z) = \frac{\mu_0 i_3}{2\pi r_3}(\hat{u}_z)$$

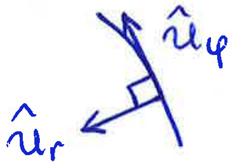
con $r_1 = \sqrt{(L/2)^2 + (L/2)^2} = \sqrt{\frac{L^2}{2}} = \frac{L}{\sqrt{2}}$

$r_2 = \frac{L}{2}$

$r_3 = \frac{L}{2}$

$\vartheta = \frac{\pi}{4} = 45°$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \frac{\mu_0}{2\pi}\left(\frac{i_1}{r_1}\frac{\sqrt{2}}{2}(\hat{u}_x + \hat{u}_y) + \frac{i_2}{r_2}(-\hat{u}_y) + \frac{i_3}{r_3}\hat{u}_z\right)$$



$$\hat{u}_\varphi \wedge \hat{u}_r = \hat{u}_z$$

$$d\ell = R\,d\vartheta$$

$$\vec{B}_2 = \frac{\mu_0 i \varphi}{4\pi R_2}\hat{u}_z$$

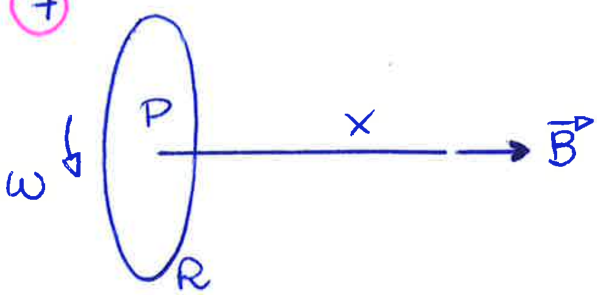
$$d\vec{B}_3 = \frac{\mu_0 i}{4\pi}\frac{d\ell\,\hat{u}_r \wedge \hat{u}_r}{r^2} = 0$$

$$d\vec{B}_4 = \frac{\mu_0 i}{4\pi}\frac{d\ell\,\hat{u}_\varphi \wedge \hat{u}_r}{R_1^2} = \frac{\mu_0 i}{4\pi R_1^2}\,d\ell\,\hat{u}_z$$

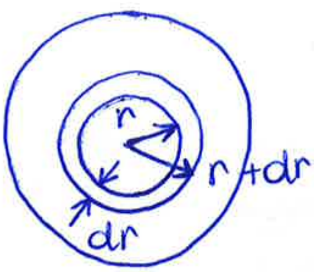
$$\vec{B}_4 = \int d\vec{B}_4 = \frac{\mu_0 i}{4\pi R_1^{\cancel{2}}}\cancel{R_1}\,\hat{u}_z\int_\varphi^0 d\vartheta = -\frac{\mu_0 i \varphi}{4\pi R_1}\hat{u}_z$$

$$\vec{B} = \vec{B}_2 + \vec{B}_4 = \frac{\mu_0 i \varphi}{4\pi}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)\hat{u}_z$$

7



Calcolare il campo $\vec{B}$ sull'asse del disco R rotante con ω e portante una carica uniformemente distribuita σ.



$$dq = \sigma\,dA = \sigma\,2\pi r\,dr$$

$$T = \frac{2\pi}{\omega}$$

$$di = \frac{dq}{T} = \frac{dq\,\omega}{2\pi} = \frac{\sigma \cancel{2\pi} r\,dr\,\omega}{\cancel{2\pi}}$$

$$di = \sigma\omega r\,dr$$

$$i = \int di = \sigma\omega\int_0^R r\,dr = \frac{\sigma\omega R^2}{2}$$

$\rightarrow$ campo magnetico per una spira sull'asse $\sim$ campo $\vec{E}$

$$q = \frac{\sigma}{A} \Rightarrow \sigma = \frac{q}{A} = \frac{q}{2\pi R L}$$

$$dq = \sigma \cdot 2\pi R\, dt$$

$$T = \frac{2\pi}{\omega}$$

$$di = \frac{dq}{T} = \frac{\sigma \cdot \cancel{2\pi} R\, dt\, \omega}{\cancel{2\pi}} = \omega \sigma R\, dt$$

$$B_{spira} = \frac{\mu_0 i}{2} \frac{R^2}{(R^2 + (z-t)^2)^{3/2}}$$

$$dB = \frac{\mu_0 di}{2} \frac{R^2}{(R^2 + (z-t)^2)^{3/2}} =$$

$$B = \int dB = \frac{\mu_0 \omega \sigma R^3}{2} \int_0^L \frac{dt}{(R^2 + (z-t)^2)^{3/2}}$$

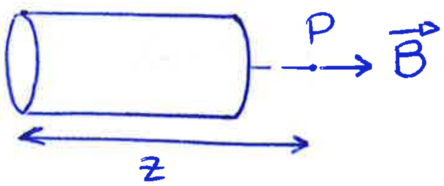
$\rightarrow$ sostituisco $\quad W = \dfrac{z-t}{R} \Rightarrow t = z - WR \Rightarrow dt = -R\,dW$

$t = 0 \quad \Rightarrow W = \dfrac{z}{R}$

$t = L \quad \Rightarrow W = \dfrac{z-L}{R}$

$$B = \frac{\mu_0 \omega \sigma R}{2} \int_{z-L/R}^{z/R} \frac{dW}{(1+W^2)^{3/2}} = \frac{\mu_0 \omega \sigma R}{2} \left[\frac{W}{\sqrt{1+W^2}}\right]_{z-L/R}^{z/R}$$

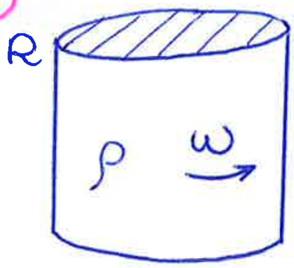
$$B = \frac{\mu_0}{2} \omega \sigma R \left(\frac{z}{\sqrt{R^2+z^2}} - \frac{z-L}{\sqrt{R^2+(z-L)^2}} \right)$$



$d = z - \dfrac{L}{2}$ $\qquad$ d = distanza di P dal centro del sist.

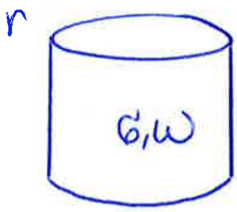
$$B = \frac{\mu_0}{2} \omega \sigma R \left(\frac{L/2 + d}{\sqrt{R^2 + (d + L/2)^2}} + \frac{L/2 - d}{\sqrt{R^2 + (d - L/2)^2}} \right)$$

9



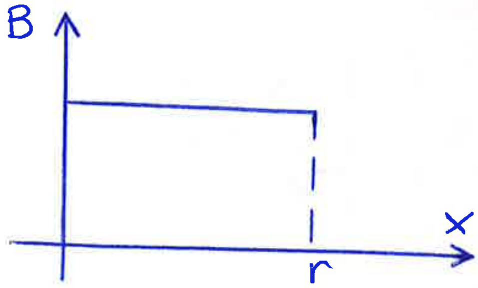
Calcolare il campo B creato da un cilindro indefinito di raggio R che è carico uniformemente con densità di carica ρ e che ruota con velocità angolare ω.

→ superficie cilindrica di raggio r indefinita con σ ruotante con velocità ω ~ solenoide

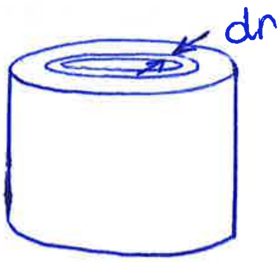


$B_{int} = \mu_0 i_u = \mu_0 \omega \sigma r$

$B_{est} = 0$



→ il cilindro di raggio R si vede come unione di strati cilindrici di raggio r, r+dr equivalenti a delle superfici cilindriche



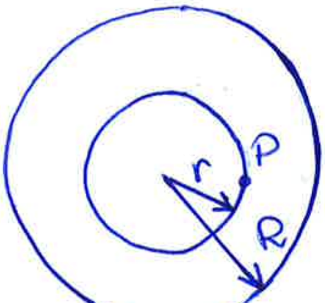
→ per una lunghezza L carica nello strato cilindrico

$dq = \rho dV = \rho L 2\pi r dr$

$di = \frac{dq}{T} = \frac{\omega dq}{2\pi} = \frac{\omega \rho L \cancel{2\pi} r dr}{\cancel{2\pi}} = \omega \rho L r dr$

con $T = \frac{2\pi}{\omega}$

→ voglio calcolare $\vec{B}$ in P



$di_u = \frac{di}{L} = \frac{\omega \rho \cancel{L} r dr}{\cancel{L}} = \omega \rho r dr$

$d\vec{B} = \mu_0 di_u = \mu_0 \omega \rho r dr$

$\vec{B} = \int d\vec{B} = \mu_0 \omega \rho \int_r^R r dr = \frac{1}{2} \mu_0 \omega \rho (R^2 - r^2)$

contribuiscono solo gli anelli esterni

$$dB = \frac{\mu_0 di}{2} \frac{x^2}{(x^2+(z-x\cot g\alpha)^2)^{3/2}} =$$

$$= \frac{\mu_0 \omega q}{4\pi L} \frac{x^2}{(x^2+(z-x\cot g\alpha)^2)^{3/2}} dl =$$

$$= \frac{\mu_0 \omega q}{4\pi L} \frac{x^2}{(x^2+(z-x\cot g\alpha)^2)^{3/2}} \frac{dx}{\sin\alpha}$$

$$B = \frac{\mu_0 \omega q}{4\pi L \sin\alpha} \int_0^{L\sin\alpha} \frac{x^2}{(x^2+(z-x\cot g\alpha)^2)^{3/2}} dx$$

$l = 0 \quad \Rightarrow x = 0$

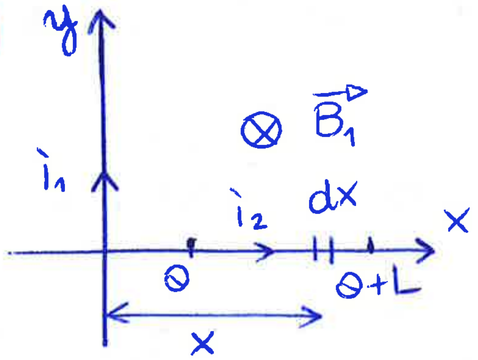
$l = L \quad \Rightarrow x = L\sin\alpha$

→ al centro $z = 0$

→ superficie conica ruotante

$$\frac{\vec F_{12}}{L} = -\frac{\mu_0 i_1 i_2}{2\pi d}\,\hat u_y$$

3)



Il conduttore percorso da i_1 è indefinito, quello percorso da i_2 ha lunghezza L e la sua estremità sx dista una distanza a dal primo. Calcolare la $\vec F_2$ che sente il secondo conduttore.

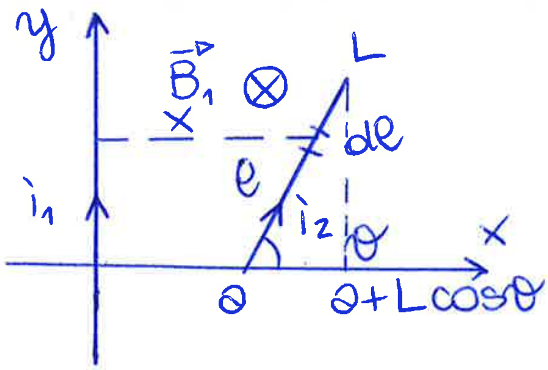
$$\vec B_1 = B(-\hat u_z) = \frac{\mu_0 i_1}{2\pi x}(-\hat u_z)$$

$$d\vec F_{12} = i_2\, d\vec\ell \wedge \vec B = i_2\, dx\, \hat u_x \wedge \vec B(-\hat u_z) = i_2 B\, dx\, \hat u_y$$

$$\vec F_{12} = \frac{\mu_0 i_1 i_2}{2\pi}\,\hat u_y \int_a^{a+L} \frac{dx}{x} = \frac{\mu_0 i_1 i_2}{2\pi}\,\hat u_y \ln\frac{a+L}{a}$$

$$\vec F_{12} = \frac{\mu_0 i_1 i_2}{2\pi} \ln\left(1+\frac{L}{a}\right)\hat u_y$$

4)



i_1 conduttore indefinito. Calcolare $\vec F_2$ agente sul conduttore di lunghezza L percorso da i_2

$$\vec B_1 = B_1(-\hat u_z) = \frac{\mu_0 i_1}{2\pi x}(-\hat u_z) = -\frac{\mu_0 i_1}{2\pi(a+\ell\cos\theta)}\,\hat u_z$$

$$d\vec\ell = d\ell\cos\theta\,\hat u_x + d\ell\sin\theta\,\hat u_y$$

$$d\vec F_2 = i_2\, d\vec\ell \wedge \vec B_1 = i_2(d\ell\cos\theta\,\hat u_x + d\ell\sin\theta\,\hat u_y)\wedge B_1(-\hat u_z) =$$

$$= i_2 B_1\, d\ell(\cos\theta\,\hat u_y - \sin\theta\,\hat u_x)$$

$$\vec F_2 = \frac{\mu_0 i_1 i_2}{2\pi}\int_0^L \frac{d\ell}{a+\ell\cos\theta}(\cos\theta\,\hat u_y - \sin\theta\,\hat u_x)$$

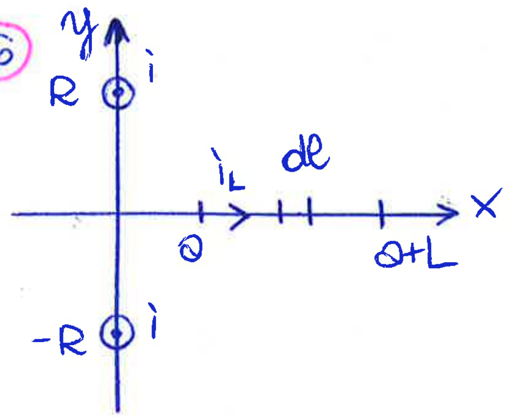
$$d\vec{M} = -\frac{\mu_0 i_1 i_2 \sin^2\vartheta\, \cancel{\ell}}{2\pi \cancel{\ell}} \frac{a}{\cos^2\vartheta}\, d\vartheta\, \hat{u}_y = -\frac{\mu_0 i_1 i_2 a}{2\pi}\, tg^2\vartheta\, d\vartheta\, \hat{u}_y$$

$$\vec{M} = \int dM = -\frac{\mu_0 i_1 i_2 a}{2\pi}\, \hat{u}_y \int_{-\vartheta_M}^{\vartheta_M} tg^2\vartheta\, d\vartheta =$$

$$= -\frac{\mu_0 i_1 i_2 a}{2\pi}\, \hat{u}_y \left[tg\vartheta - \vartheta \right]_{-\vartheta_M}^{\vartheta_M} =$$

$$= -\frac{\mu_0 i_1 i_2 a}{2\pi}\, \hat{u}_y \left(\frac{L}{2a} - arctg\, \frac{L}{2a} \right)$$

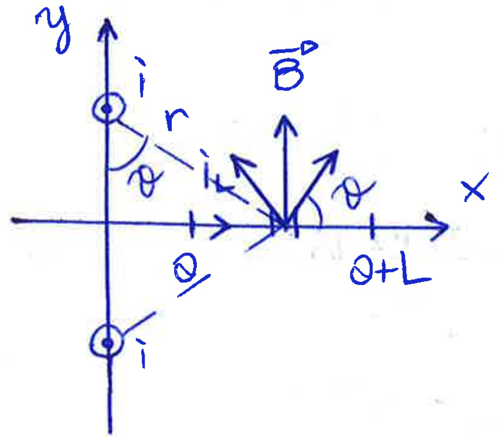
6



Calcolare la $\vec{F}$ sul conduttore L, i_L immerso nel campo $\vec{B}$ creato da due correnti // ∞, i, i.

$$\vec{B} = B\hat{u}_y = 2\, \frac{\mu_0 i}{2\pi r} \sin\vartheta\, \hat{u}_y$$

$$\vec{B} = \frac{\mu_0 i}{\pi r} \sin\vartheta\, \hat{u}_y$$



$$d\vec{F}_L = i_L\, d\vec{\ell} \wedge \vec{B} = i_L\, d\ell\, \hat{u}_x \wedge B\, \hat{u}_y =$$

$$= i_L B\, d\ell\, \hat{u}_z$$

con $r = \sqrt{\ell^2 + R^2}$

$$\sin\vartheta = \frac{\ell}{r} = \frac{\ell}{\sqrt{\ell^2+R^2}}$$

$$d\vec{F}_L = \frac{\mu_0 i i_L\, \ell\, d\ell}{\pi \sqrt{\ell^2+R^2}\sqrt{\ell^2+R^2}}\, \hat{u}_z = \frac{\mu_0 i i_L\, \ell\, d\ell}{\pi(\ell^2+R^2)}\, \hat{u}_z$$

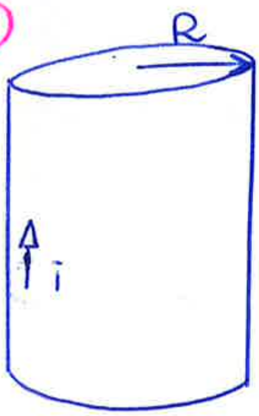
$$\vec{F}_L = \int dF_L = \frac{\mu_0 i i_L}{\pi}\, \hat{u}_z \int_a^{a+L} \frac{\ell}{(\ell^2+R^2)}\, d\ell =$$

$$= \frac{\mu_0 i i_L}{2\pi}\, \hat{u}_z \ln\left(\frac{(a+L)^2 + R^2}{a^2+R^2} \right)$$

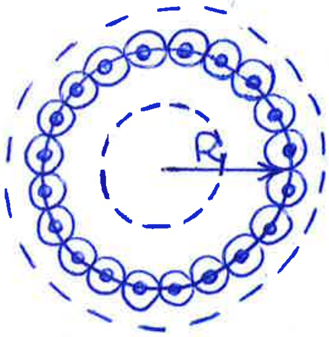
se $a = -\frac{L}{2} \Rightarrow \vec{F} = 0$

ESERCITAZIONE 8

1



Calcolare il campo magnetico $\vec{B}$ creato da una superficie cilindrica indefinita di raggio R percorsa da una corrente i distribuita uniformemente lungo la sua lunghezza.



$r < R$

$$\begin{cases} \oint_{\Gamma_{int}} \vec{B}\, d\vec{\ell} = \mu_0 0 \\ \oint \vec{B}\, d\vec{\ell} = 2\pi r B \end{cases}$$

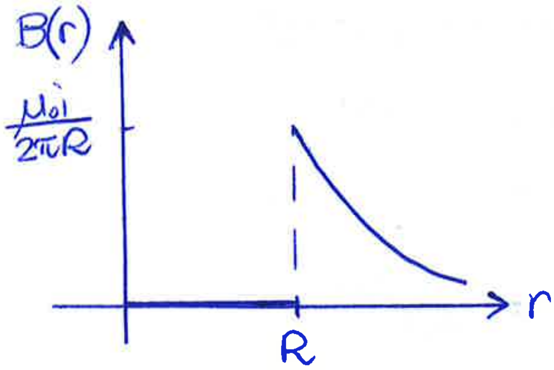
all'interno non ho nessuna corrente concatenata

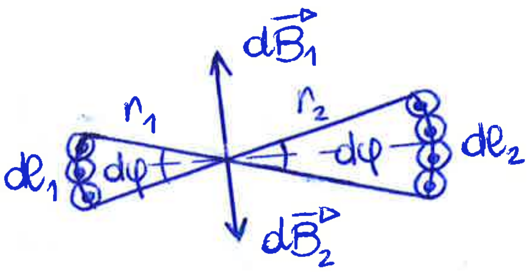
$$B\, 2\pi r = \mu_0 0 \Rightarrow \vec{B} = 0$$

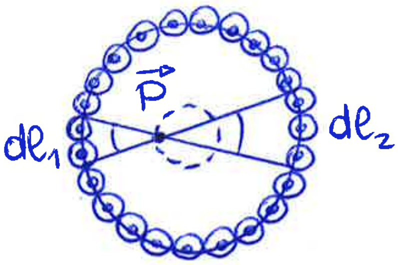
$r > R$

$$\begin{cases} \oint_{\Gamma_{est}} \vec{B}\, d\vec{\ell} = \mu_0 i \\ \oint_{\Gamma_{est}} \vec{B}\, d\vec{\ell} = 2\pi r B \end{cases}$$

$$B\, 2\pi r = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r}$$







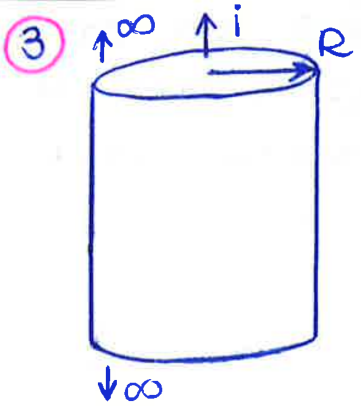
$$d\ell_1 = r_1 d\varphi \Rightarrow dB_1 = \frac{\mu_0 di_1}{2\pi r_1} = \frac{\mu_0 i \cancel{r_1}}{4\pi R \cancel{r_1}} d\varphi = \frac{\mu_0 i}{4\pi^2 R} d\varphi$$

$$d\ell_2 = r_2 d\varphi \Rightarrow dB_2 = \frac{\mu_0 di_2}{2\pi r_2} = \frac{\mu_0 i}{4\pi^2 R} d\varphi$$

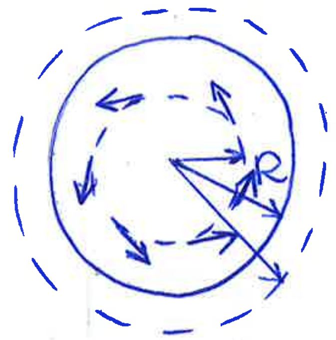
con $di_1 = i_L d\ell_1 = \frac{i}{2\pi R} d\ell_1 = \frac{i r_1}{2\pi R} d\varphi$

$|d\vec{B}_1| = |d\vec{B}_2| \Rightarrow$ in P $\vec{B} = 0$

3



Calcolare il campo $\vec{B}$ creato da un conduttore cilindrico indefinito di raggio R e percorso da una corrente i distribuito uniformemente nella sua sezione.



$r<R$ Ampere-Maxwell

$$2\pi r B = \mu_0 i(r)$$

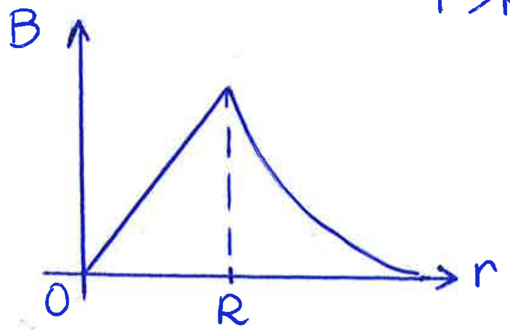
con $i(r) = j\pi r^2 = \frac{i}{\cancel{\pi} R^2}\cancel{\pi} r^2 = i\frac{r^2}{R^2}$

$$B = \frac{\mu_0 i(r)}{2\pi r} = \frac{\mu_0 i r^{\cancel{2}}}{2\pi \cancel{r} R^2} = \frac{\mu_0 i r}{2\pi R^2}$$

$r>R$

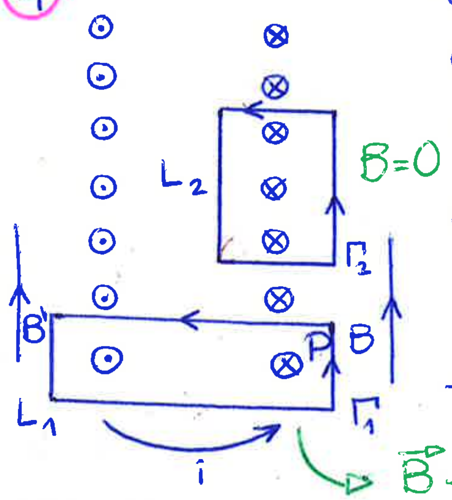
$$2\pi r B = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r}$$



4

Calcolare il campo $\vec{B}$ creato da un solenoide indefinito contenente n spire per unità di lunghezza e percorso dalla corrente i



1) Γ_1 : $L_1 B - L_1 B' = \mu_0 (n L_1 i - n L_1 i) = 0$

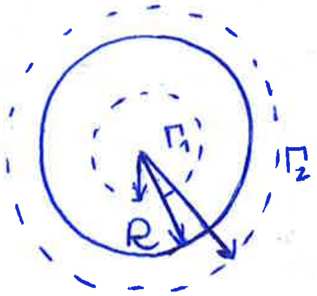
$\Rightarrow B = B'$

$\rightarrow$ se prendo P all'∞ $\Rightarrow B = 0$, $B' = 0$

$\vec{B} \perp$ al percorso, contributo nullo

2) $-BL_2 = \mu_0(-L_2 n i) \Rightarrow B = \mu_0 n i$

$\rightarrow$ circuitazione lungo Γ_2: contribuisce solo il tratto interno verticale $\vec{B} /\!/ d\vec{\ell}$



$$V = -\int E\,dr + C \Rightarrow E = -\frac{dV}{dr} = -\frac{V}{d}$$

$$E = -\frac{V_0 \sin\omega t}{d}$$

$$\oint_\Gamma \vec{B}\,d\vec{\ell} = \mu_0 i + \mu_0 \varepsilon_0 \frac{\partial \Phi(E)}{\partial t}$$

$$\oint_\Gamma \vec{B}\,d\vec{\ell} = 2\pi r B$$

$r < R_1$

$$\begin{cases} \oint \vec{B}\,d\vec{\ell} = \mu_0 0 + \mu_0 \varepsilon_0 \dfrac{\partial \Phi(E)}{\partial t} \\ \oint \vec{B}\,d\vec{\ell} = 2\pi r B \end{cases}$$

$$B\,2\pi r = \mu_0 \varepsilon_0 \frac{\partial \Phi(E)}{\partial t}$$

$$\Phi(E) = \int_\Sigma \vec{E}\,d\vec{\sigma} = E\pi r^2 = -\frac{V_0 \sin\omega t}{d}\pi r^2$$

$$B\,2\pi r = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(-\frac{V_0 \sin\omega t}{d}\pi r^2\right)$$

$$B\,2\cancel{\pi}\cancel{r} = \mu_0 \varepsilon_0 \left(-\frac{V_0 \cos\omega t \cdot \omega \cdot \pi r^{\cancel{2}}}{d}\right)$$

$$B = -\frac{\mu_0 \varepsilon_0 V_0 \cos\omega t \cdot \omega \cdot r}{2d}$$

$r > R_1$

$$B\,2\pi r = \mu_0 \varepsilon_0 \frac{\partial \Phi(E)}{\partial t}$$

→ al di fuori del condensa- tore piano, $\vec{E} = 0$

$$\Phi(E) = \int_\Sigma \vec{E}\,d\vec{\sigma} = E\pi R^2 = -\pi R^2 \frac{V_0 \sin\omega t}{d}$$

$$B\,2\pi r = -\mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(\frac{V_0 \sin\omega t}{d}\pi R^2\right) -$$

$$B\,2\cancel{\pi} r = -\mu_0 \varepsilon_0 \frac{V_0 \cos\omega t \cdot \omega\,\cancel{\pi} R^2}{d}$$

$$B = -\frac{\mu_0 \varepsilon_0 V_0 \cos\omega t \cdot \omega R^2}{2dr}$$

con $W = r - \ell$

in A : $W = -\ell$

in O : $W = 0$

→ oppure più semplicemente

$$\mathcal{E} = -\omega B \int_{AO} (\ell - r)\,dr = -\omega B \left[\ell r - \frac{r^2}{2} \right]_0^\ell = -\omega B \left[\ell^2 - \frac{\ell^2}{2} \right] =$$

$$= -\omega B \frac{\ell^2}{2}$$

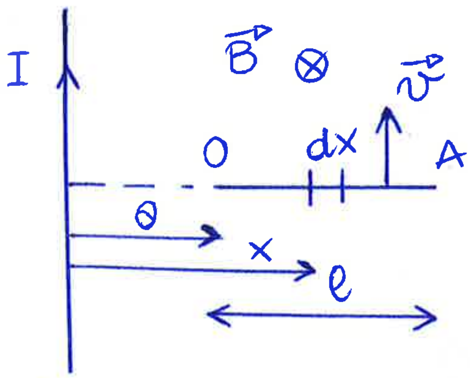
2) Faraday

Φ ↻ $\quad \mathcal{E} = -\frac{\partial \Phi}{\partial t} = -\frac{\partial}{\partial t}\left(B \frac{1}{2} \ell^2 \vartheta\right) = -\frac{1}{2} B \ell^2 \frac{\partial \vartheta}{\partial t} = -\frac{1}{2} B \omega \ell^2$

Φ ↺ $\quad \mathcal{E} = -\frac{\partial \Phi}{\partial t} = -\frac{\partial}{\partial t} B\left(\pi \ell^2 - \frac{1}{2} \ell^2 \vartheta\right) = \frac{1}{2} B \ell^2 \frac{\partial \vartheta}{\partial t} = \frac{1}{2} B \omega \ell^2$

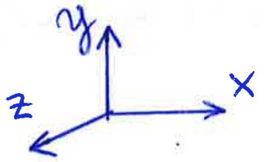
$$i = \frac{|\mathcal{E}|}{R} = \frac{\omega B \ell^2}{2R}$$

2



Un'asta di lunghezza ℓ è dotata di velocità $\vec{v} \parallel \vec{J}$ dove $\vec{J}$ è la densità di corrente di un conduttore rettilineo indefinito inoltre $\vec{v}$ è $\perp$ all'asta. Calcolare la forza elettromotrice indotta nell'asta OA.

$$\begin{cases} d\vec{\ell} = dx\,\hat{u}_x \\ \vec{v} = v\,\hat{u}_y \\ \vec{B} = -B\,\hat{u}_z \end{cases}$$



$$\vec{F} = q\vec{v} \wedge \vec{B} = q v \hat{u}_y \wedge B(-\hat{u}_z) = -q v B \hat{u}_x$$

$$B = \frac{\mu_0 I}{2\pi x}$$

$$E = \frac{\vec{F}}{q} = -vB\hat{u}_x = -\frac{\mu_0 I v}{2\pi x}\hat{u}_x$$

$\vec{B} = B(-\hat{u}_z)$

$\vec{v} = v\hat{u}_x$

→ Forza di Lorentz

$\vec{F} = q\vec{v} \wedge \vec{B} = qvB\,\hat{u}_x \wedge (-\hat{u}_z) = qvB\hat{u}_y = qv\frac{\mu_0 I}{2\pi x}\hat{u}_y$

$\vec{B} = \frac{\mu_0 I}{2\pi x}(-\hat{u}_z)$

$\vec{E} = \frac{\vec{F}}{q} = v\frac{\mu_0 I}{2\pi x}\hat{u}_y$

A_3A_4: $\mathcal{E} = \int_{A_3A_4} \vec{E}\,d\vec{\ell} = \int_{A_3A_4} v\frac{\mu_0 I}{2\pi x}\hat{u}_y\,dx(-\hat{u}_x) = \emptyset$

A_4A_1: $\mathcal{E} = \int_{A_4A_1} \vec{E}\,d\vec{\ell} = \int_{A_4A_1} v\frac{\mu_0 I}{2\pi x}\hat{u}_y\,dy\,\hat{u}_y = \int_{A_4A_1} v\frac{\mu_0 I}{2\pi a(t)}\,dy =$

$= v\frac{\mu_0 I h}{2\pi a(t)}$

A_1A_2: $\mathcal{E} = \int_{A_1A_2} \vec{E}\,d\vec{\ell} = \int_{A_1A_2} v\frac{\mu_0 I}{2\pi x}\hat{u}_y\,dx\,\hat{u}_x = \emptyset$

A_2A_3: $\mathcal{E} = \int_{A_2A_3} \vec{E}\,d\vec{\ell} = \int_{A_2A_3} v\frac{\mu_0 I}{2\pi(a(t)+L)}\hat{u}_y\,dy(-\hat{u}_y) =$

$= \int_{A_2A_3} -\frac{v\mu_0 I}{2\pi(a(t)+L)}\,dy = -\frac{v\mu_0 I h}{2\pi(a(t)+L)}$

$\mathcal{E} = \mathcal{E}_{41} + \mathcal{E}_{23} = \frac{\mu_0 I v h}{2\pi}\left(\frac{1}{a(t)} - \frac{1}{a(t)+L}\right)$

→ Faraday-Henry

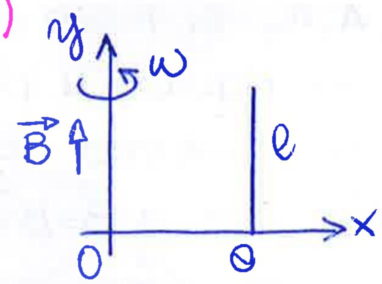
$\mathcal{E} = -\frac{\partial\Phi(t)}{\partial t}$

$\Phi(t) = \frac{\mu_0 I h}{2\pi}\ln\left(1 + \frac{L}{a(t)}\right) = \frac{\mu_0 I h}{2\pi}\left[\ln(L + a(t)) - \ln(a(t))\right]$

$\frac{\partial\Phi(t)}{\partial t} = \frac{\mu_0 I h}{2\pi}\left[\frac{1}{L+a(t)} - \frac{1}{a(t)}\right]\frac{da(t)}{dt} = \frac{\mu_0 I h v}{2\pi}\left[\frac{1}{L+a(t)} - \frac{1}{a(t)}\right]$

$\mathcal{E} = \frac{\mu_0 I h v}{2\pi}\left(\frac{1}{a(t)} - \frac{1}{L+a(t)}\right)$

c)



$\alpha = \frac{\pi}{2}$

$\mathcal{E} = 0$

$\vec{B} = B\hat{u}_y$

$\vec{v} = v_x\hat{u}_x + v_z\hat{u}_z$

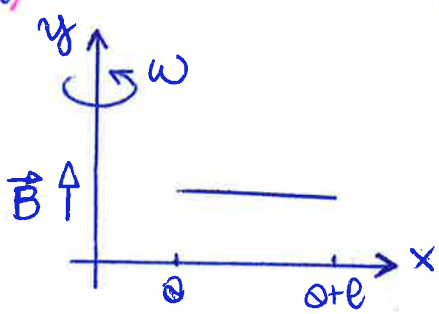
$d\vec{\ell} = dx\,\hat{u}_x$

$\vec{F} = q\vec{v} \wedge \vec{B} = q(v_x\hat{u}_x + v_z\hat{u}_z) \wedge B\hat{u}_y = qB(v_x\hat{u}_z - v_z\hat{u}_x)$

↳ non è attiva

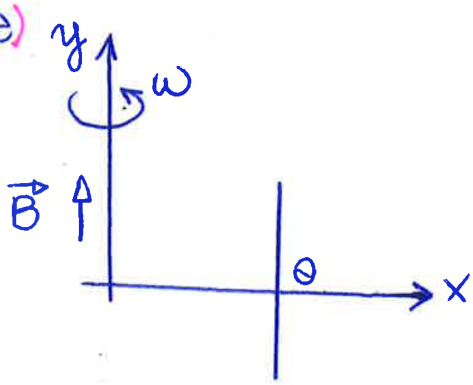
↳ poiché agisce ⊥ alla sbarretta

d)



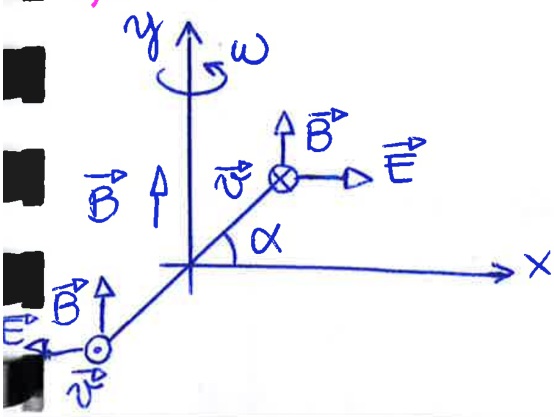
$\mathcal{E} = \frac{1}{2}B\omega\ell(\ell + 2a)$

e)



$\mathcal{E} = 0$

f)



$\vec{E}_+ = \vec{v} \wedge \vec{B} = v(-\hat{u}_z) \wedge B\hat{u}_y =$

$= vB\hat{u}_x$

$\vec{E}_- = \vec{v} \wedge \vec{B} = v\hat{u}_z \wedge B\hat{u}_y =$

$= vB(-\hat{u}_x)$

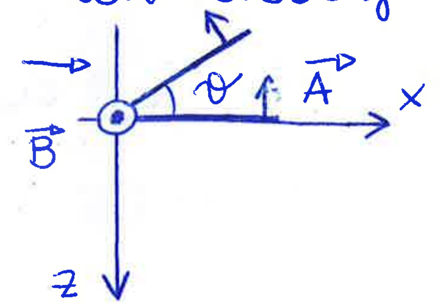
$\vec{E} = \vec{E}_+ + \vec{E}_- = 0$

$\mathcal{E} = 0$

$\mathcal{E}_{41} = 0$

$\mathcal{E}_{tot} = \mathcal{E}_{23} = -Bhl\omega\cos\omega t$

$\rightarrow$ con Faraday



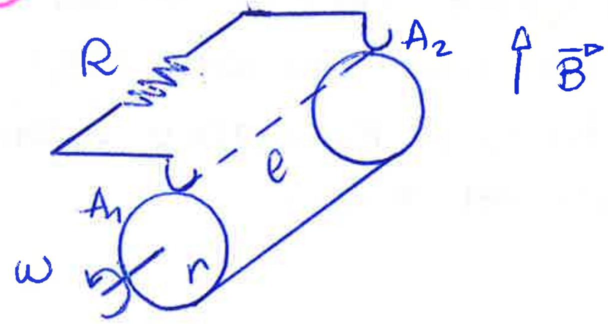
$\Phi = \vec{B}\vec{A} = B\cos\vartheta A = BA\sin\vartheta = BA\sin\omega t$

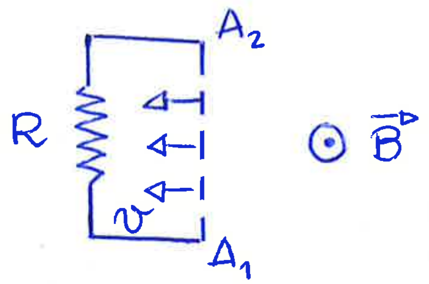
$\Phi = BA(t) \qquad$ con $\qquad A(t) = A\sin\omega t$

$$\mathcal{E} = -\frac{\partial\Phi}{\partial t} = -BA\frac{\partial\sin\omega t}{\partial t} = -Bhl\omega\cos\omega t$$

10

Un cilindro di raggio r e lunghezza l ruota con velocità angolare ω in presenza di un campo $\vec{B} \perp$ al suo asse. Due punti $A_1 A_2$ facenti parte di una generatrice del cilindro si trovano in contatto ohmico tramite una resistenza R. Determinare come varia la ω nel tempo sapendo che al tempo $t=0$ si ha $\omega(0) = \omega_0$ e anche la $i(t)$





$\vec{v} = v(-\hat{u}_x) \qquad$ con $\quad v = \omega r$

$\vec{B} = B\hat{u}_y$

$\vec{E} = \vec{v} \wedge \vec{B} = v(-\hat{u}_x) \wedge B\hat{u}_y) = -vB\hat{u}_z$

$\mathcal{E} = \int \vec{E}\,d\vec{l} = -\int_0^l vB\hat{u}_z\, dl\,\hat{u}_z = -vBl$

$i = \frac{\mathcal{E}}{R} = -\frac{\omega r B l}{R}$

$$\frac{d\omega}{dt} = C_1 - C_2\omega \Rightarrow \omega(t) = \frac{C_1}{C_2} - \left(\frac{C_1}{C_2} - \omega_0\right)e^{-C_2 t}$$

con $C_1 = \frac{V_0 r \ell B}{RI}$

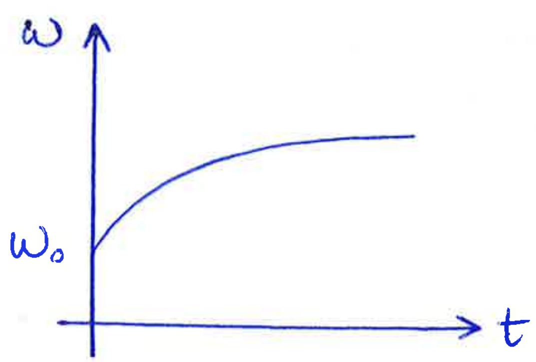
$$C_2 = \frac{B^2 r^2 \ell^2 \omega}{RI}$$

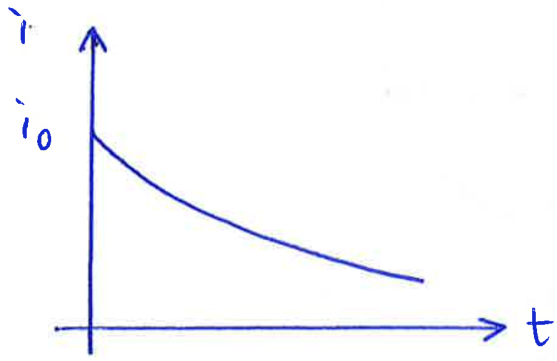
$$i(t) = \frac{V_0}{R} - \frac{Br\ell}{R}\omega(t)$$

$$\omega(\infty) = \frac{C_1}{C_2} = \frac{V_0}{Br\ell}$$

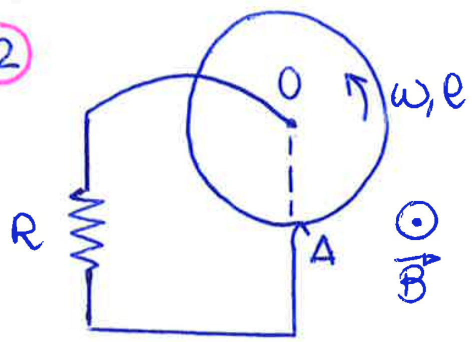
$$i(\infty) = 0$$

$$i(0) = \frac{V_0}{R} - \frac{Br\ell\omega_0}{R}$$

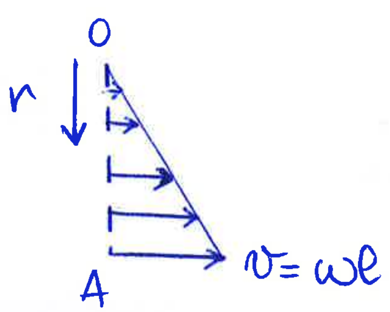




12



In presenza di un campo $\vec{B}$, un disco metallico di raggio ℓ ruota con velocità angolare ω. Il suo centro è in contatto ohmico tramite una resistenza R con la sua periferia. Determinare la i che attraversa la resistenza R.



→ OA si comporta come un'asta che ruota con velocità angolare ω

$$i(t) = \frac{B r_0^2 \omega_0}{2R} \exp\left\{-\frac{B^2 r_0^4}{4RI} t\right\}$$

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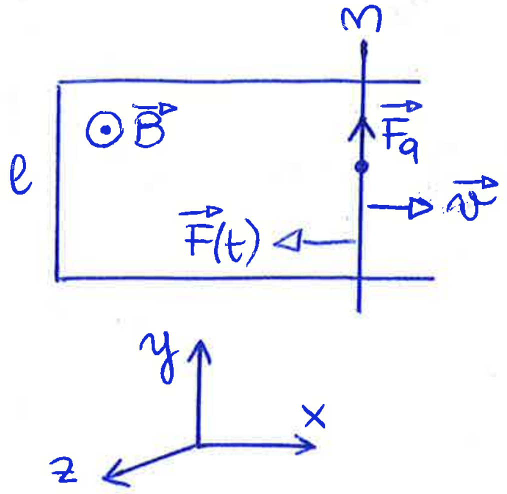


L'asta di massa m parte con v_0. Calcolare la $v(t)$ lungo x se il circuito è immerso in un $\vec{B}$

$\vec{B} = B\hat{u}_z$

$\vec{v} = v\hat{u}_x$

$\vec{E} = \vec{v} \wedge \vec{B} = v\hat{u}_x \wedge B\hat{u}_z =$
$= -vB\hat{u}_y$

$$\mathcal{E} = \int \vec{E}\, d\vec{\ell} = -\int_0^\ell vB\hat{u}_y\, d\ell\, \hat{u}_y = -B\ell v(t)$$

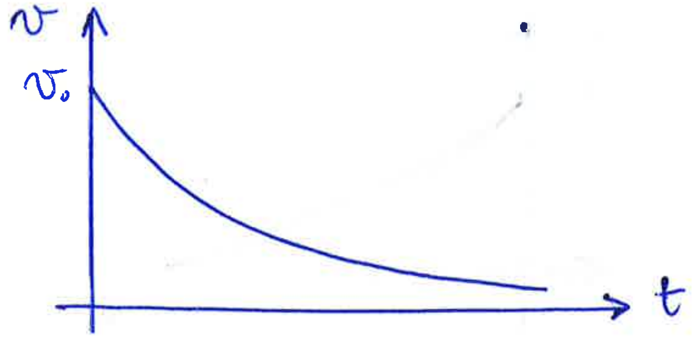
$$i(t) = \frac{\mathcal{E}(t)}{R} = -\frac{B\ell v(t)}{R}$$

$$F(t) = B\ell i(t) = B\ell \cdot \left(-\frac{B\ell v(t)}{R}\right) = -\frac{B^2\ell^2 v(t)}{R}$$

$$F = m \cdot a \quad \Rightarrow \quad F(t) = m \cdot \frac{dv(t)}{dt}$$

$$\frac{dv(t)}{dt} = -\frac{B^2\ell^2 v(t)}{Rm}$$

$$v(t) = v_0 \exp\left\{-\frac{B^2\ell^2}{Rm} t\right\}$$



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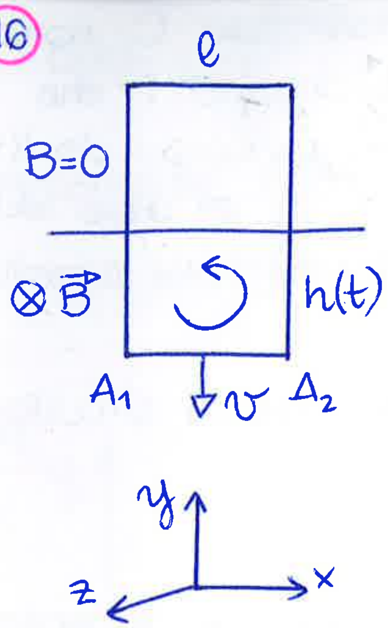


Sotto l'azione del suo peso la spira di massa m e resistenza R entra in una regione del piano xy dove è presente un campo $\vec{B}$. Determinare la velocità di discesa della spira.

$$\vec{v} = v(-\hat{u}_y)$$

$$\vec{B} = B(-\hat{u}_z)$$

$$\Phi(t) = -B \cdot A(t) = -B \cdot \ell h(t)$$

$$\vec{E} = \vec{v} \wedge \vec{B} = vB(-\hat{u}_y \wedge -\hat{u}_z) = vB\hat{u}_x$$

$$\mathcal{E} = -\frac{\partial \Phi(t)}{\partial t} = +\frac{\partial h(t)}{\partial t} B\ell = +vB\ell$$

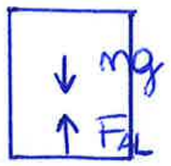
$$\mathcal{E} = \int \vec{E}\, d\vec{\ell} = \int vB\hat{u}_x\, dx\, u_x$$

$$i(t) = \frac{\mathcal{E}(t)}{R} = +\frac{vB\ell}{R}$$

$$F_L = q\vec{w} \wedge \vec{B} = qw\hat{u}_x \wedge B(-\hat{u}_z) = qwB\hat{u}_y$$

$$F_{AL} = N V_{A_1A_2} \vec{F}_1 = NS\ell\,(qwB\hat{u}_y) = qNS\ell\, wB\hat{u}_y =$$

$$= \rho S\ell\, wB\hat{u}_y = jS\ell B\hat{u}_y = i\ell B\hat{u}_y$$



$$m\frac{dv(t)}{dt} = mg - F_{AL} \Rightarrow \frac{dv(t)}{dt} = mg - i\ell B$$

$$m\frac{dv(t)}{dt} = mg - \frac{B^2\ell^2 v}{R}$$

$\rightarrow$ in condizioni stazionarie $v(\infty)$

$$\frac{dv(t)}{dt} = 0 \Rightarrow mg - F_{AL} = 0 \Rightarrow mg - \frac{B^2\ell^2 v(\infty)}{R} = 0$$

$$v(\infty) = \frac{mgR}{B^2\ell^2}$$

$$\mathcal{E}(t) = -\frac{\partial \Phi(t)}{\partial t} = -\frac{\partial i(t)}{\partial t} \mu_0 n \pi a^2$$

→ corrente $I(t)$ che circola su S

$$I(t) = \frac{\mathcal{E}(t)}{R} = -\frac{\mu_0 n \pi a^2}{R} \frac{\partial i(t)}{\partial t}$$

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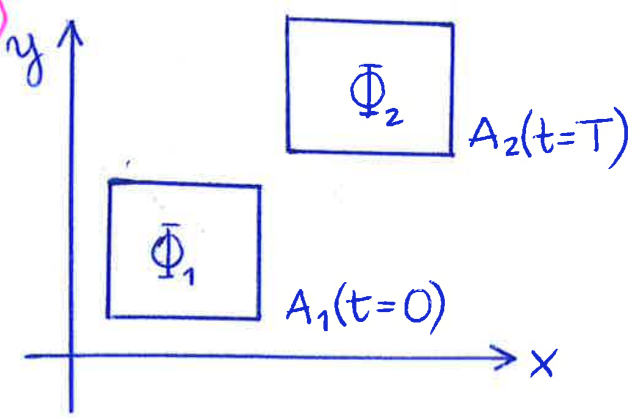

Calcolare la Q spostata nel circuito durante il suo spostamento dalla configurazione iniziale A_1 a quella finale A_2 in presenza di un campo $\vec{B}$

→ durante lo spostamento da A_1 ad A_2 cambia Φ e quindi circola corrente

$$\begin{cases} i(t) = -\dfrac{1}{R} \dfrac{d\Phi(t)}{dt} \\ i(t) = \dfrac{dq(t)}{dt} \end{cases}$$

$$\frac{dq(t)}{dt} = -\frac{1}{R}\frac{d\Phi(t)}{dt} \Rightarrow \int_0^Q dq(t) = -\frac{1}{R}\int_{\Phi_1}^{\Phi_2} d\Phi(t)$$

$$Q = -\frac{1}{R}(\Phi_2 - \Phi_1)$$

$$Q = \frac{\Phi_1 - \Phi_2}{R}$$

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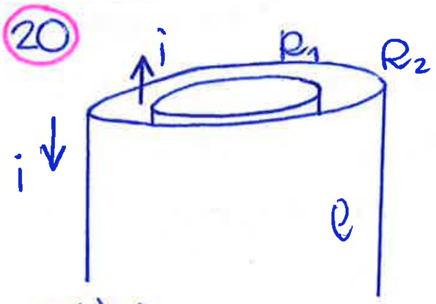
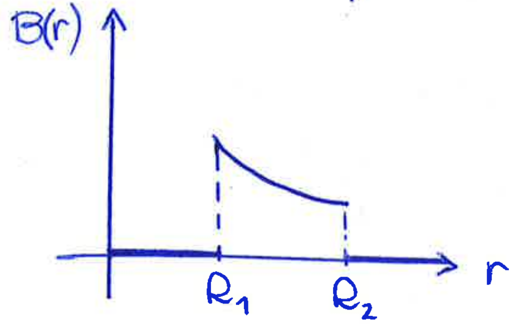
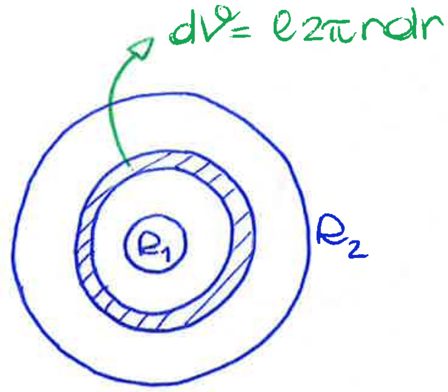


Calcolare l'U_B del sistema

$$B = \begin{cases} 0 & r < R_1 \\ \dfrac{\mu_0 i}{2\pi r} & R_1 < r < R_2 \\ 0 & r > R_2 \end{cases}$$



$$u_B = \begin{cases} 0 & r < R_1 \\ \dfrac{\mu_0 i^2}{8\pi^2 r^2} & R_1 < r < R_2 \\ 0 & r > R_2 \end{cases}$$



$$U_B = \int_V u_B \, dV = \frac{1}{2\mu_0} \int_V B^2 dV = \frac{\mu_0 i^2}{8\pi^2} \int_V \frac{dV}{r^2} =$$

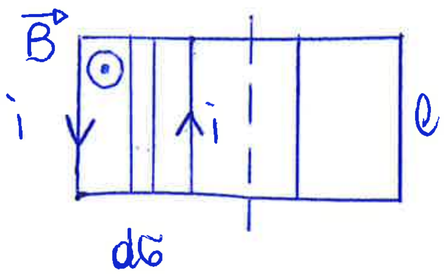
$$= \frac{\mu_0 i^2}{8\pi^2} \int_{R_1}^{R_2} \frac{\ell 2\pi r dr}{r^2} = \frac{\mu_0 i^2 \ell}{4\pi} \ln \frac{R_2}{R_1} = \frac{1}{2} \left(\frac{\mu_0 \ell}{2\pi} \ln \frac{R_2}{R_1} \right) i^2 =$$

$$= \frac{1}{2} L i^2$$

$$L = \ell \frac{\mu_0}{2\pi} \ln \frac{R_2}{R_1} \qquad L = \frac{\Phi}{i}$$

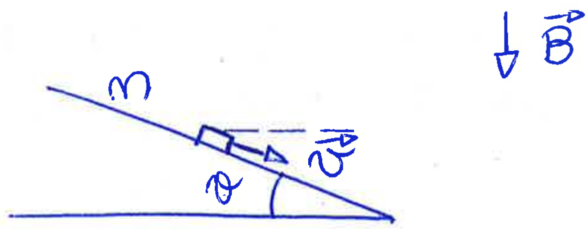
$$\Phi = \int_\Sigma B \, d\sigma = \frac{\mu_0 i}{2\pi} \int \frac{d\sigma}{r} = \frac{\mu_0 i}{2\pi} \int_{R_1}^{R_2} \frac{\ell dr}{r} = i \frac{\mu_0 \ell}{2\pi} \ln \frac{R_2}{R_1}$$

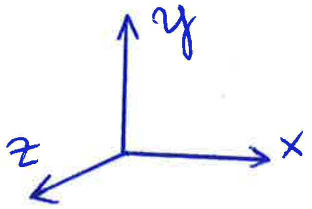
$$L = \frac{\Phi}{i} = \frac{\mu_0 \ell}{2\pi} \ln \frac{R_2}{R_1}$$

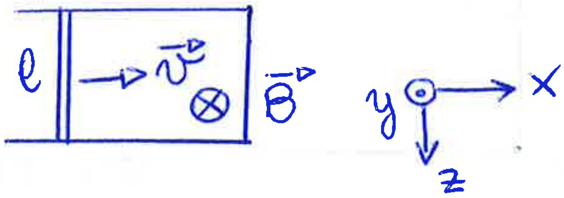


ES AGGIUNTO

Un conduttore di lunghezza l scende lungo due guide che formano l'angolo θ con l'orizzontale. La sbarretta ha massa m. Calcolare la velocità di discesa.







$$\vec{B} = B(-\hat{u}_y)$$

$$\vec{v} = v(\cos\theta\,\hat{u}_x - \sin\theta\,\hat{u}_y)$$

$$\vec{E} = \vec{v} \wedge \vec{B} = v(\cos\theta\,\hat{u}_x - \sin\theta\,\hat{u}_y) \wedge B(-\hat{u}_y) =$$

$$= vB(-\cos\theta\,\hat{u}_z) = -vB\cos\theta\,\hat{u}_z$$

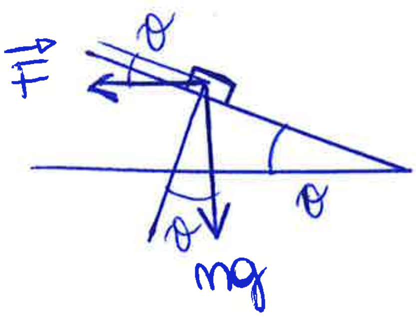
$$\mathcal{E} = \int \vec{E}\,d\vec{l} = \int E(-\hat{u}_z)\,dl\,\hat{u}_z = -\int_0^l vB\cos\theta\,dl =$$

$$= -vB\cos\theta\, l$$

$$i = \frac{\mathcal{E}}{R} = \left| -\frac{vB\cos\theta\, l}{R} \right| = \frac{vB\cos\theta\, l}{R}$$

$$\vec{F} = i\int_C d\vec{l} \wedge \vec{B} = i\int dl\,\hat{u}_z \wedge B(-\hat{u}_y) = iB\int_0^l dl\,\hat{u}_x =$$

$$= -\frac{vB^2\cos\theta\, l^2}{R}\,\hat{u}_x$$

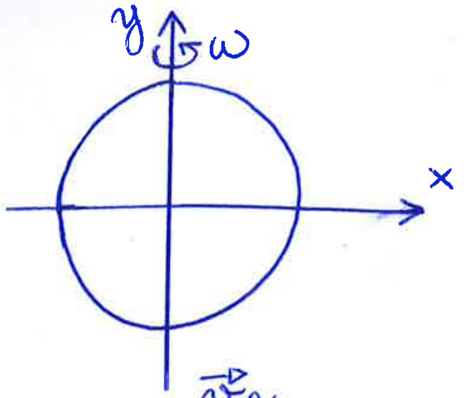


$$F\cos\theta = mg\sin\theta$$

$$\frac{vB^2\cos^2\theta\, l^2}{R} = mg\sin\theta$$

$$v = \frac{mg\sin\theta\, R}{B^2\cos^2\theta\, l^2}$$

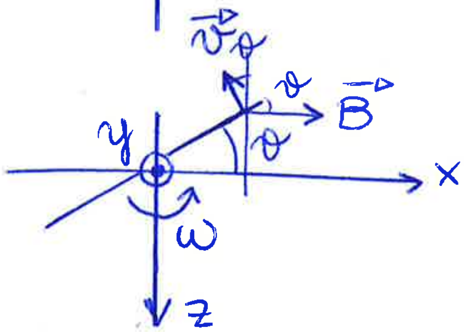
ES AGGIUNTO



$R,\ \omega = cost$

$\vec{B} = \alpha_1 t\,\hat{u}_x + \alpha_2 t^2\,\hat{u}_y$

$\mathcal{E} = ?$



$v = \omega r$

$\vec{B}_y = 0$ non influenza il flusso

$\Phi(t) = \vec{B}(t)\vec{A} = BA\sin\omega t$

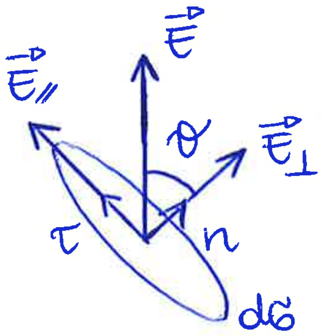
$\vartheta = \int_0^t \omega\, dt = \omega t$

$$\mathcal{E} = -\frac{\partial}{\partial t}\Phi(t) = -\frac{\partial}{\partial t}\left(\alpha_1 t\,\pi r^2 \sin\omega t\right) =$$

$$= -\left(\alpha_1 \pi r^2 \sin\omega t + \alpha_1 t\,\pi r^2 \cos\omega t \cdot \omega\right) =$$

$$= -\alpha_1 \pi r^2 \left(\sin\omega t + \omega t \cos\omega t\right)$$

TH. GAUSS - forma integrale



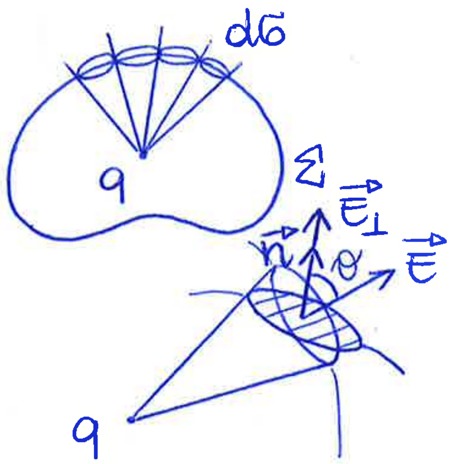
$$\vec{E} = k\frac{q}{r^2}\vec{u}_r$$

$$d\vec{\sigma} = d\sigma\,\vec{n}$$

$$d\Phi = \vec{E}\,d\vec{\sigma} = E\cos\vartheta\,d\sigma = (E_\perp\vec{n} + E_{/\!/}\vec{\tau})\,d\sigma\,\vec{n}$$

$$\vartheta = \begin{cases} \frac{\pi}{2} \Rightarrow d\Phi = 0 \\ 0 \Rightarrow d\Phi = \max \end{cases}$$

q interno



$$d\Phi = \vec{E}\,d\vec{\sigma} = k\frac{q}{r^2}\vec{u}_r\,d\sigma\,\vec{n} =$$

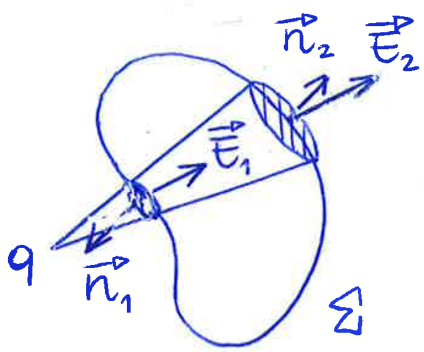
$$= k\frac{q}{r^2}\cos\vartheta\,d\sigma = kq\frac{d\sigma_n}{r^2}$$

$$\Phi = \int_\Sigma d\Phi = kq\int_\Sigma d\Omega = kq\,4\pi =$$

$$= \frac{1}{\cancel{4\pi}\varepsilon_0}\,q\,\cancel{4\pi} = \frac{q}{\varepsilon_0}$$

con $\Omega = \int d\Omega = \int \frac{d\sigma_n}{r^2} = 4\pi$

q esterno



$$d\Phi = d\Phi_1 + d\Phi_2 = \vec{E}_1 d\vec{\sigma}_1 + \vec{E}_2 d\vec{\sigma}_2 =$$

$$= \frac{kq}{r_1^2}(-d\sigma_{1n}) + \frac{kq}{r_2^2}(d\sigma_{2n}) =$$

$$= -\frac{kq}{r_1^2}d\sigma_{1n} + \frac{kq}{r_2^2}d\sigma_{2n} =$$

$$= -kq\,d\Omega_1 + kq\,d\Omega_2 =$$

$$= 0$$

con $d\Omega_1 = d\Omega_2 = 4\pi$

$\Rightarrow$ flusso attraverso Σ dipende solo dalle cariche interne

EQUAZIONE di POISSON

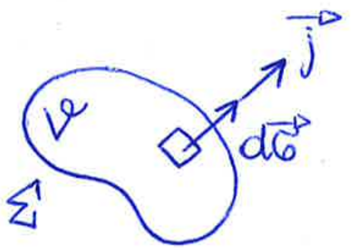
$$\begin{cases} \vec{\nabla}\vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \vec{E} = -\nabla V \end{cases}$$

$$\nabla(-\nabla V) = \frac{\rho}{\varepsilon_0}$$

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\varepsilon_0}$$

EQUAZIONE di CONTINUITÀ



$j = \rho v$ con $\rho = Ne$

$dq = \rho\, dv$ $\quad q = \int_v \rho\, dv$

$$\begin{cases} i = \oint_\Sigma \vec{j}\, d\vec{\sigma} \\ i = -\dfrac{\partial q}{\partial t} = -\dfrac{\partial}{\partial t}\displaystyle\int_v \rho\, dv \end{cases}$$

$$\oint_\Sigma \vec{j}\, d\vec{\sigma} = -\frac{\partial}{\partial t}\int_v \rho\, dv$$

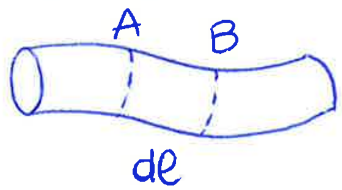
con $\oint_\Sigma \vec{j}\, d\vec{\sigma} = \int_v \vec{\nabla}\vec{j}\, dv$

$$\int_v \vec{\nabla}\vec{j}\, dv = -\frac{\partial}{\partial t}\int_v \rho\, dv$$

$$\int_v \vec{\nabla}\vec{j}\, dv + \int_v \frac{\partial \rho}{\partial t}\, dv = 0$$

$$\int_v \left(\vec{\nabla}\vec{j} + \frac{\partial \rho}{\partial t}\right) dv = 0$$

FORZA MAGNETICA - II LEGGE ELEMENTARE di LAPLACE



$$dV = \Sigma \, dl$$

$$d\vec{l} = dl \, \frac{\vec{v}}{v}$$

$$j = j \, \frac{\vec{v}}{v}$$

$$\begin{cases} \vec{j} = -Ne\vec{v} \\ \vec{j} = \rho \vec{v} \end{cases} \quad \text{con } \rho = -Ne$$

$\vec{F}_L = -e\vec{v} \wedge \vec{B}$ forza che agisce su una singola particella

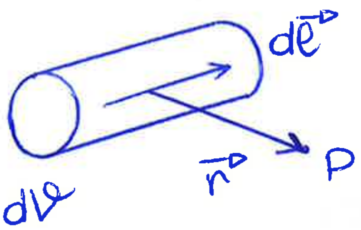
$d\vec{F} = \vec{F}_L \, N dV$ forza che risentono tutti gli e⁻ nel volume dV

$$d\vec{F} = \vec{F}_L N dV = -e\vec{v} \wedge \vec{B} N dV = -\underbrace{Ne\vec{v}} \wedge \vec{B} dV =$$

$$= \vec{j} \wedge \vec{B} \, \Sigma \, d\vec{l} = \Sigma \, \vec{j} \, d\vec{l} \wedge \vec{B} = i \, d\vec{l} \wedge \vec{B}$$

$$dF = i \, dl \wedge B \Rightarrow F = i \int_A^B dl \wedge B$$

I LEGGE ELEMENTARE di LAPLACE



$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \wedge \vec{r}}{r^3}$$

$$d\vec{B} = N dV \vec{B} = N dl \, \Sigma \, \frac{\mu_0}{4\pi} q \frac{\vec{v} \wedge \vec{r}}{r^3} =$$

$$= dl \, \Sigma \, \frac{\mu_0}{4\pi} \frac{Nq\vec{v} \wedge \vec{r}}{r^3} =$$

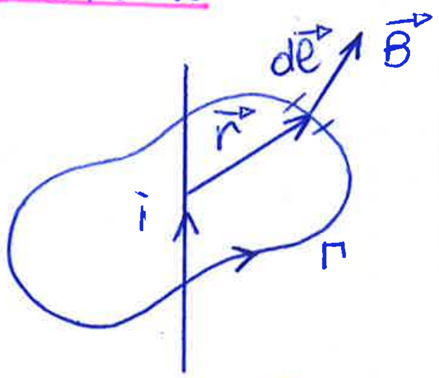
$$= dl \, \Sigma \, \frac{\mu_0}{4\pi} \frac{\vec{j} \wedge \vec{r}}{r^3} = \frac{\mu_0}{4\pi} \Sigma \frac{j \, d\vec{l} \wedge \vec{r}}{r^3} =$$

$$= \frac{\mu_0 i}{4\pi} \frac{d\vec{l} \wedge \vec{r}}{r^3}$$

$$B = \frac{\mu_0 i}{4\pi} \int \frac{d\vec{l} \wedge \vec{r}}{r^3}$$

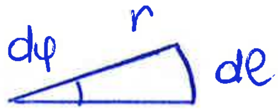
LEGGE di AMPERE

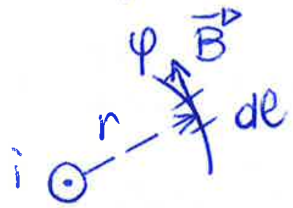
i interno



$$\oint_\Gamma \vec{B}\,d\vec{l} = \oint_\Gamma B\,dl\cos\varphi = \oint_\Gamma \frac{\mu_0 i}{2\pi r}\,dl = $$
$$= \frac{\mu_0 i}{2\pi}\oint_\Gamma \frac{dl}{r}$$

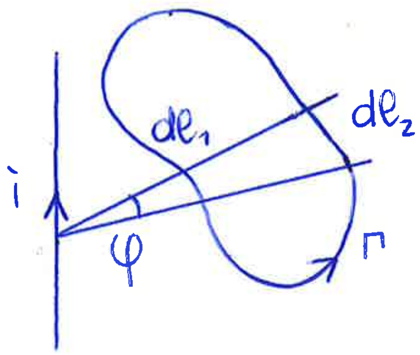
con $dl = r\,d\varphi$





$$\oint_\Gamma \vec{B}\,d\vec{l} = \frac{\mu_0 i}{2\pi}\oint_\Gamma d\varphi = \frac{\mu_0 i}{2\pi}\,2\pi = \mu_0 i$$

i esterno



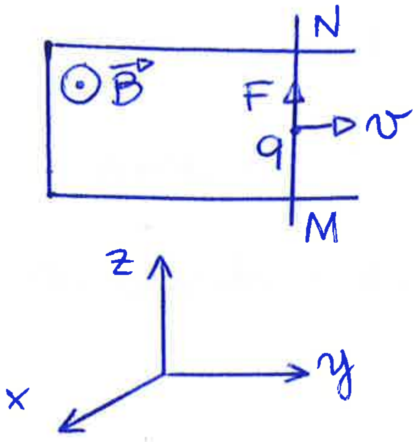
$$\vec{B}_1 d\vec{l}_1 = B_1\,dl_1\cos\pi = -B_1\,dl_1$$

$$\vec{B}_2 d\vec{l}_2 = B_2\,dl_2\cos 0 = B_2\,dl_2$$

$$\vec{B}\,d\vec{l} = -\frac{\mu_0 i}{2\pi r_1}\,dl_1 + \frac{\mu_0 i}{2\pi r_2}\,dl_2 =$$
$$= -\frac{\mu_0 i}{2\pi r_1}\,r_1\,d\varphi + \frac{\mu_0 i}{2\pi r_2}\,r_2\,d\varphi = 0$$

$$\oint_\Gamma \vec{B}\,d\vec{l} = 0$$

LEGGE di FARADAY-HENRY



$$\vec{v} = v\hat{u}_y$$

$$\vec{B} = B\hat{u}_x$$

$$\vec{F} = -e\vec{v} \wedge \vec{B} = -e\vec{v}\,\hat{u}_y \wedge B\hat{u}_x = $$
$$= evB\hat{u}_z$$

$$\vec{E} = \frac{\vec{F}}{-e} = -vB\hat{u}_z$$

$$\oint_\Gamma \vec{E}\,d\vec{l} = \oint_\Gamma \vec{v} \wedge \vec{B}\,d\vec{l} = -\int_{MN} vB\hat{u}_z\,dl\,\hat{u}_z = -\int_{MN} vB\,dl =$$
$$= -\frac{dy}{dt}Bl = -\frac{B\,dA}{dt} = -\frac{dBA}{dt} = -\frac{\partial}{\partial t}\Phi(B) =$$
$$= -\frac{\partial}{\partial t}\int_\Sigma \vec{B}\,d\vec{\sigma}$$

$$\oint_\Gamma \vec{E}\,d\vec{l} = -\frac{\partial}{\partial t}\int_\Sigma \vec{B}\,d\vec{\sigma}$$

$$\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$