NUMERO: 1520A - ANNO: 2015

A P P U N T I

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MATERIA: Power Electronics + Eserc. Prof.Maddaleno

What is power electronics? We have power electronics every time that we are concerning with the EFFICIENCY.

In this field we have electrical energy conversion. So, we can have different types of conversion:

1. DC → DC (CONVERTER)
2. DC → AC (INVERTER)
3. AC → DC (RECTIFIER)
4. AC → AC (CYCLOCONVERTERS)

We are interested in (1) and (3).

EFFICIENCY DISTINGUISHES POWER ELECTRONICS FROM NOT POWER ELECTRONICS.

What is efficiency?

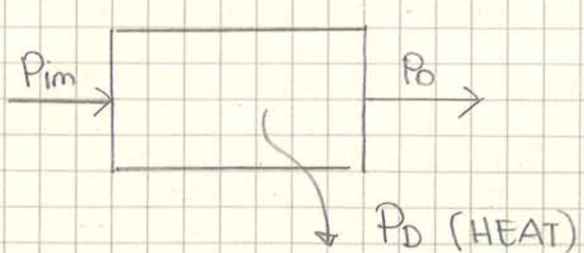
$$\eta = \frac{P_o}{P_{in}}$$

It's important for:

- money, cost

(If $\eta > 1$ we invent perpetual motion :))

$P_{in} - P_o$ → this difference is the power that stays inside; the dissipated power P_D.



NB: the relation $P_{in} - P_o$ is not instantaneous.

2) Let's consider AC adaptors. How many AC adapt. exist in Europe?

POWER SAVING

3) How much does electric energy for ~~ma~~ the main cost?

20 cents for KW/hours

But! The energy of batteries is more expensive! Three order of magnitude than the energy from the mains. (20 $ KW/h)

Our devices must have an high battery efficiency.

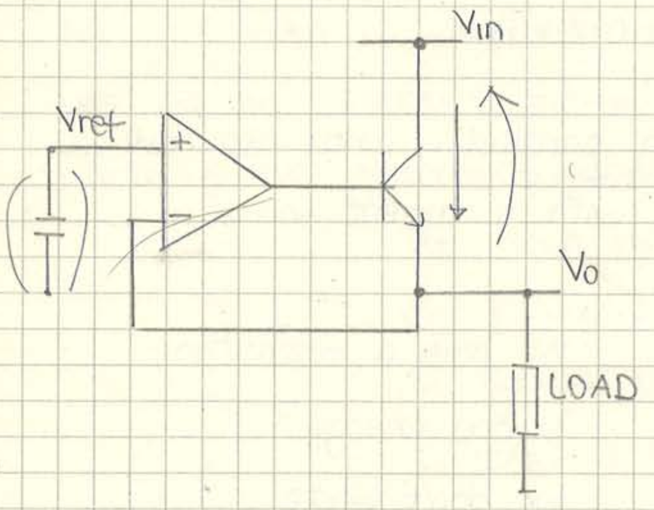
PORTABLE DEVICES

LONGER ENDURANCE

Let's start to analyze two type of conversion, their advantages and disavantages.

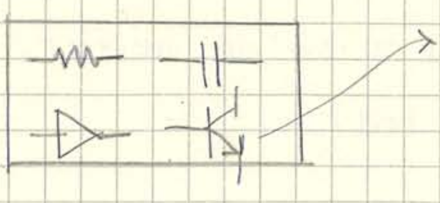
The standard way is to use a standard transistor working in linear region:

LINEAR "POWER SUPPLY"



⊖ efficiency
⊖ Vo RELATED Vin } DISADV.

⊕ EASY
⊕ FAST
⊕ E.M. NOISE LOW } ADV.



This transistor works in linearity; in RAD zone we have a power dissipation (V·I is power!)

What are the differences between switch mode conversion and linear power supply? (+) advantages of switch mode; (−) disadvantages:

(+) efficiency (we have no elements washing power)
(weight, cost, temperature, life ...)

(+) Vo not related in magnitude and sign with the input voltage (we can have Vo larger, smaller, with opposite sign respect to Vin)

(+) Multiple outputs

NEGATIVE POINTS:

(−) complicated

(−) They are a speedy feedback system. An important parameter is the crossover frequency (smaller than linear system)

(−) Slow system (slower than linear system)

(−) Electromagnetic noise (very high)

NB: we need to use inductors (in order to have high efficiency).
Can we use synthesize inductors? No.
First of all groundend inductor:

The main reason is that a generator needs of an OP-AMP. That's fine. An inductor carries a current that the order of AMPS (large current) so we need a POWER OP-AMP.
But this component has a very bad efficiency. ⚠

BASIC TOPOLOGIES - REFRESH -

HYPOTHESES :

- IDEAL SWITCHES

$V = 0$

- RC or $\frac{L}{R}$ TIME CONSTANTS (τ) $\tau \gg T_{SW}$

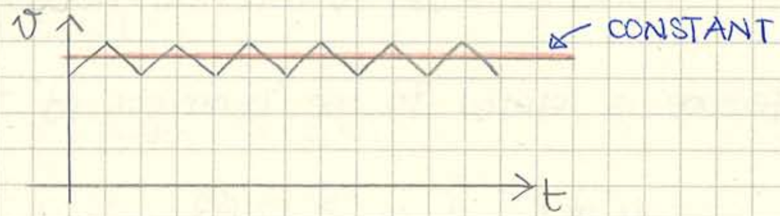
What does it mean? It means that waveform linear ; otherwise exponential. → EQUATIONS EASIER
(NB: R can be the load; the parass. of the L)

→ EFFICIENCY = 1 (IDEAL CASE)

So, for the beginning $\eta = 1$

- OUTPUT VOLTAGE (OF A POWER CONVERTER) IS CONSTANT.

NO RIPPLE



- There is a sort of "steady state" ; sort because there is a switch that open and close). → It's called CYCLOSTATIONARY CONDITION (each cycle is equal to the other one).
(It's equal to the DC bias point for linear circuit)

TWO BASIC TOPOLOGIES :

- BUCK $\frac{V_o}{V_i} < 1$ ($V_o < V_i$) → BUT DON'T WRITE IN THIS WAY! $|V_o| < |V_i|$

DUAL

- BOOST $\frac{V_o}{V_i} > 1$

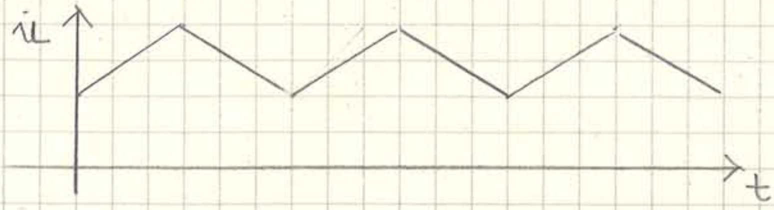
There are 2 more topologies :

- BUCK - BOOST (FLY BACK) $\frac{V_o}{V_{in}} < 0$ V_o OPPOSITE POLARITY OF V_{in}

We can do the average of voltage:

$$\bar{v}_L = \overline{L \frac{di_L}{dt}} \xleftrightarrow[\text{OPERATOR}]{\text{LINEAR}} = L \frac{d\bar{i}_L}{dt}$$

If we consider cyclostationary conditions ($\bar{i}_L$ is equal in each cycle):



THE AVARAGE IS ALWAYS THE SAME.

The average is the DC component of the current.

In cyclost. cond. the average is the same; so the derivative is zero:

$$\bar{v}_L = 0 \qquad \left(\frac{d\bar{i}_L}{dt} = 0 \right)$$

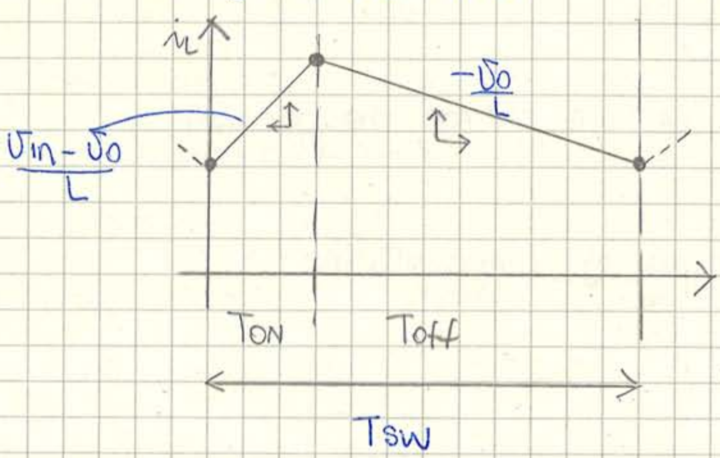
An inductor for DC component is a piece of wire; a real inductor has a resistance. (short circuit)

$$\bar{i}_C = C \frac{d\bar{v}_C}{dt} = 0 \qquad (\text{ OPEN CIRCUIT})$$

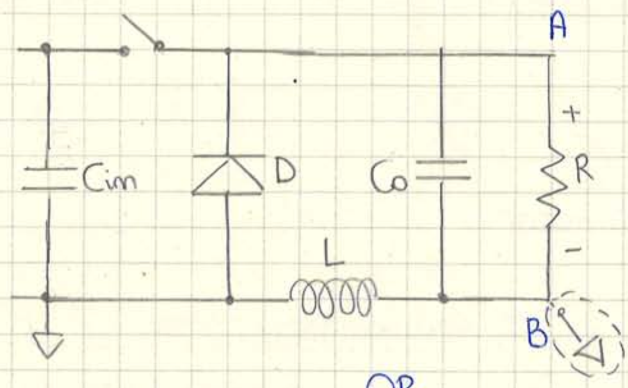
What happens if we apply a constant voltage to C and we have a constant current? → C is failed. It's time to change it.

$$\left.\begin{array}{l} \bar{v}_L = 0 \\ \bar{i}_C = 0 \end{array}\right\} \text{CYCLOSTATIONARY CONDICTION.}$$

We can find out $i_L(t)$



1



$V_o = V_{in} \cdot D$

SW ON

A → Vin B → Vin − Vo

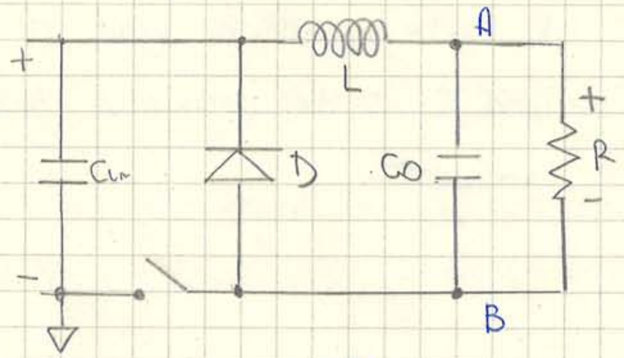
SW OFF A → 0

B → −Vo

NO

OR

2



SW ON

A → Vo B → Ø V

SW OFF

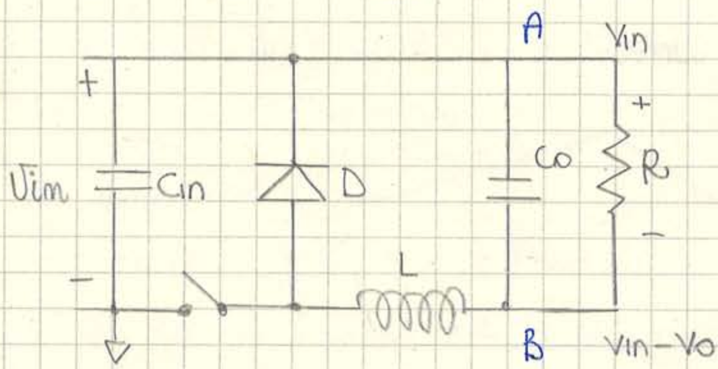
A → Vin + Vo

B → Vin

NO

3

OR



A → Vin

B → Vin − Vo

{ BOTH IN SW ON AND SW OFF

The potential is constant!

OK

1, 2 e 3 are all buck converter.

Voltage reference is usual connected to a large part of PCB; it could be connected to metal box. (GROUND). Let's see circuit 1 what happens? ~~If we look to the output voltage refers to ground~~ we see that there is a swing up and down (Vin) very fast.

↳ High speed changing voltage; so it's generate a lot of electromagnetic noise (the load goes up and down of quantity equal to Vin).

For solving this problem I can connect B to the ground, L became a short circuit!

We see that the equations are different.

$$\frac{V_{in} - V_{SW} - V_O}{\cancel{L}} \cdot \underbrace{\frac{T_{on}}{T_{SW}}}_{\to D} - \frac{V_D + V_O}{\cancel{L}} \underbrace{\frac{T_{off}}{T_{SW}}}_{\to 1-D} = 0$$

NB: If V_O doesn't depend on L, I can put $L = 0$? No! If we shorts the inductor; in the moment that we close the switch we apply to the ~~V_O~~ $V_{in} - V_O$, and it's not a good idea. And L can't be $\emptyset$; it's in the denominator!

So, solving the equation, we find out V_O:

- $V_{O,REAL} = (V_{in} - V_{SW}) D - V_D (1 - D)$

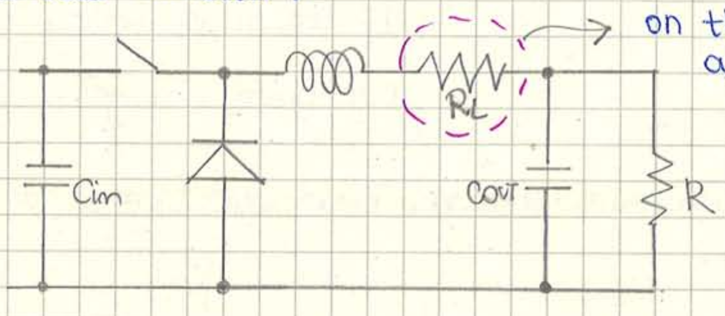
NB The presence of $-$, because V_O is smaller than the ideal

And the real duty cycle:

- $D_{REAL} = \dfrac{V_O + V_D}{V_{in} - V_{SW} + V_D}$ $\quad (D_{REAL} > D_{IDEAL})$

If $V_O >> V_D$ and $V_{in} > V_{SW} + V_D$ there is a sort of cancellation. But if V_{in} is 0.9 V this is not valid!

If we have losses our output voltage is smaller. By the way, If we have a real inductor:



on this resistance I have a voltage drop!

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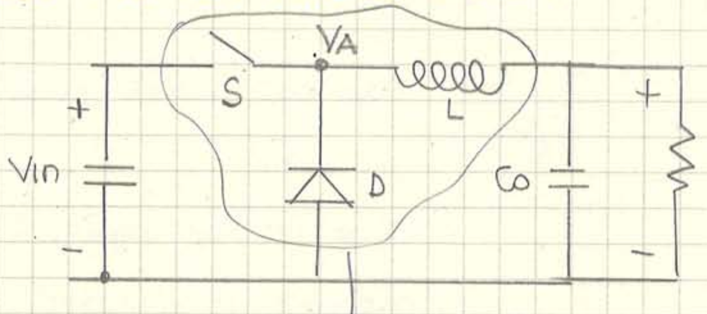
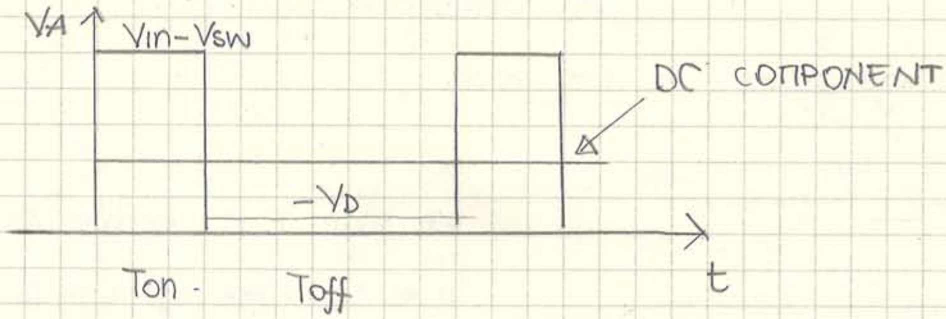
Let's see how our converter works:

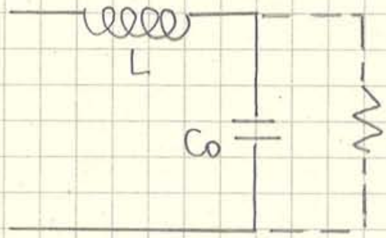


↳ The buck conv. is just this part!

NB: I can't change the polarity because of diode! ⚠



Let's see the 2nd part of the circuit:



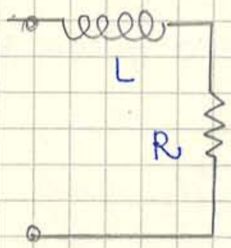
This is a resonance circuit.

Low pass filter (II order)

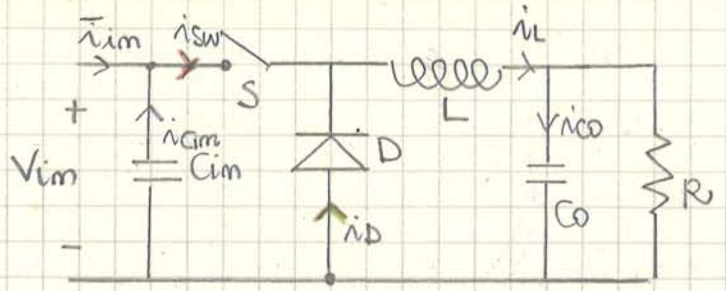
↳ AC component is stopped.

So, this low pass filter passes the DC component of VA. (the average!)

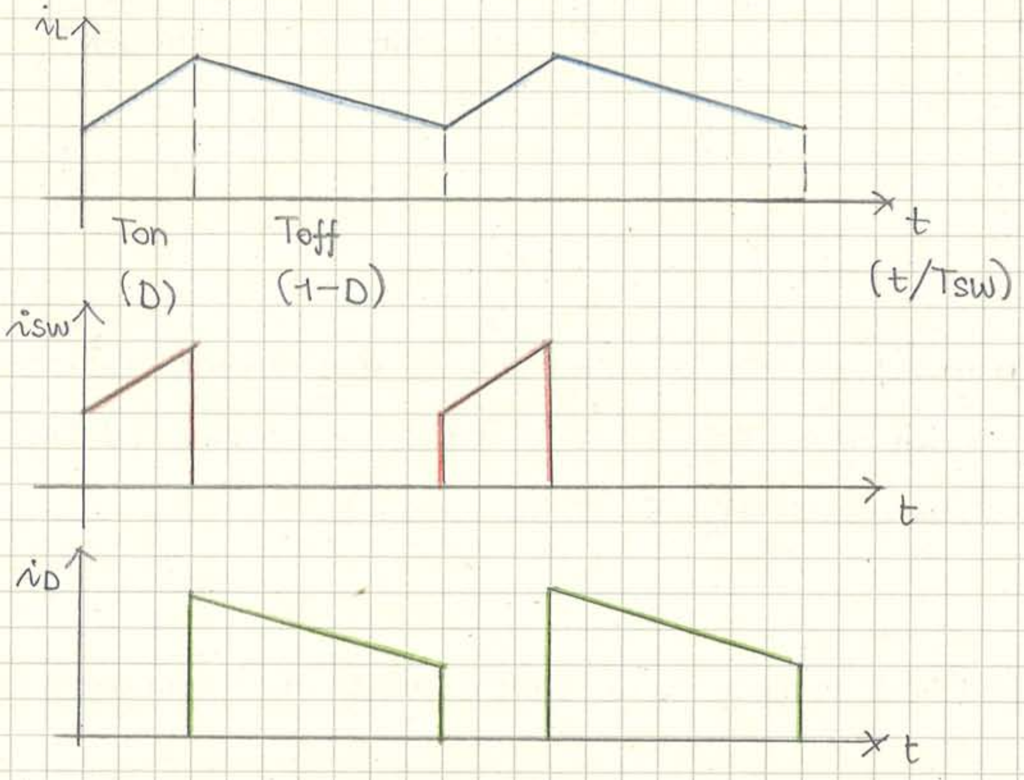
If I want to make a 1st order.



In this case if I want the same frequency, capability of the II order → L is large and large...

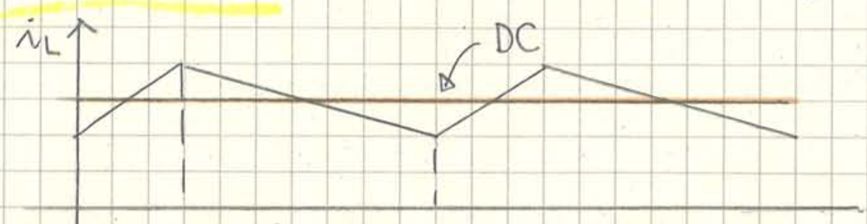


WRITE WAVEFORMS IN THE CIRCUIT.



EQ: I forget to put the diode? What happens to the circuit?
Or if I put the diode reversed? (cc Vin)

Let's go to see current in Co (i_{CO}). If the output voltage is constant $i_{CO} = \emptyset$. But Vo is not constant! There is ripple.

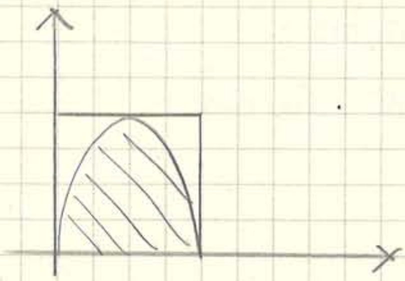


The DC component goes in R.

What about AC component? It goes in Co.

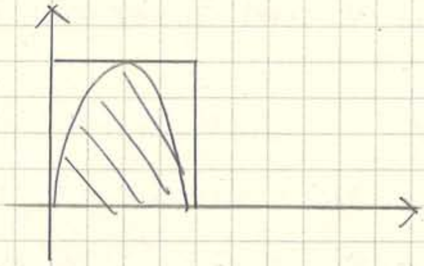
Can we use current divider rule? No.

Why?



PARABOLA $\qquad x(1-x)$

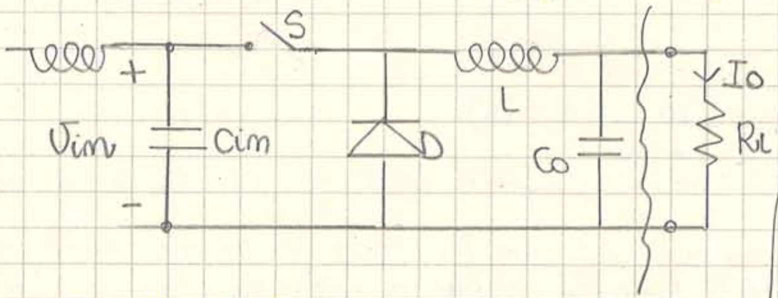
$$A_{PAR} = \frac{2}{3} \cdot b \cdot h$$



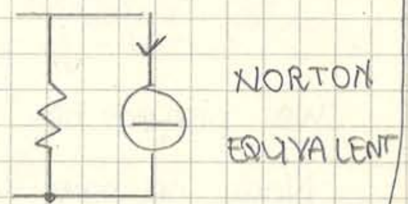
SINUSOIDAL

$$A_{SIN} = \frac{2}{\pi} b \cdot h$$

Let's take our standard configuration of buck converter:



In other case we can rapresent it as:

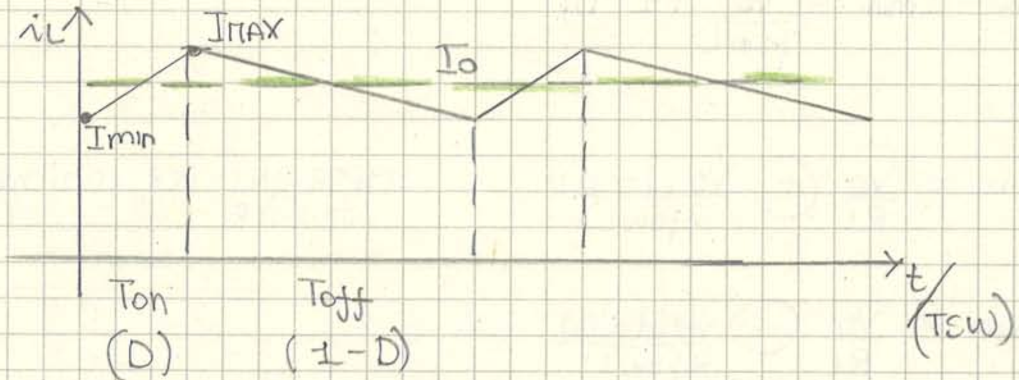


(or non-linear load)

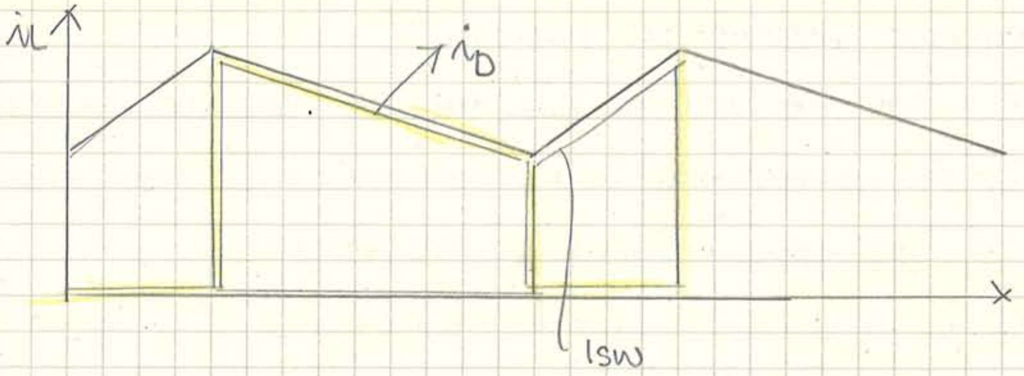
Remember that V_{in} is not ideal, otherwise C_{in} is useless!

The inductor at the input is one part of the E.M filter and it's designed by E.M. engineers.

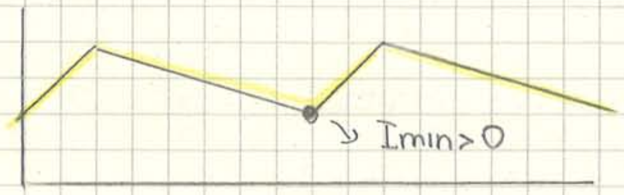
We know that the inductor current in CCM:



In order to find i_L, I need to find out I_{MAX} and I_{min}.
I need it because of the stress; for example I_{max} is the peak current flowing to the L, that is the peak flowing to the switch; and so on.



Let's try to derive the only design equation for buck converter.
In general, we want to work it in CCM (because it's easy to control it). CCM means that i_L must be larger than $\emptyset$.



So, in order to be sure that our buck converter works in CCM we must garantee that Imin is larger than $\emptyset$.
So, Imin>0 :

$$\frac{V_O}{R_L} - \frac{V_O(1-D)}{2 f_{SW} L} > 0$$

We have to find the critical condition ; the boundary between CCM and DCM ; and this condition is:

$$\frac{\cancel{V_O}}{R_L} - \frac{\cancel{V_O}(1-D)}{2 f_{SW} L} = 0$$

$$\boxed{\frac{1}{R_L} - \frac{1-D}{2 f_{SW} L} = 0}$$ → If it is satisfied we have the boundary between CCM and DCM.

R_L is not under our control! The customers decides how much current he wants. What about D?

$D = \frac{V_O}{V_{in}}$ → D is not our degree of freedom!

I want to find D and R that garantee ~~that~~ the value of L for staying still in CCM.

I want $I_{min} > 0$ in the worst case.

$$I_{min} = \frac{V_o}{R_L} - \frac{V_o(1-D)}{2 f_{sw} L} > 0$$

(I take the smallest possible)

↳ $R_{L MAX}$ and D_{min} CCM

So:

$$L_{CRIT} = \frac{(1-D_{min}) R_{MAX}}{2 f_{sw}}$$

→ IT'S THE ONLY DESIGN EQUATION OF BUCK CONVERTER

If I want to design a buck converter working in DCM, this situation is reversed considering R_{min} and D_{MAX}; and L should be smaller.

And if we remove load from the converter?

$I_{MAX} \to \infty$; and L should be ∞! Too much expensive!

(And then we pass from CCM to DCM) GUARANTEED!

On the contrary if R becames smaller and smaller we stay in CCM.

What about switch and diode? We have a range. (No stress)

What about Cin and Cout? They have a value, a stress!

We need to find:

- PEAK CURRENT
- AVERAGE CURRENT
- RMS CURRENT

Stressed depends on:

V ← electric field

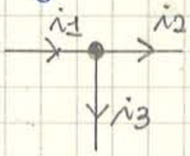
I_{PK} ← critical for bonding wires in transistor (magnetic saturation)

I_{AVE}, I_{RMS} → DISSIPATED POWER HEAT

What is important is:

$$p(t) = v(t) \cdot i(t)$$

Let's go into details:



. KCL $\Rightarrow$ $\sum_{NODE} i = 0$

i → INSTANTANEOUS $\sum i(t)$

→ AVERAGE CURRENTS $\overline{\sum i} = \bar{0} \Rightarrow \sum \bar{i} = 0$

(we have already done it!)

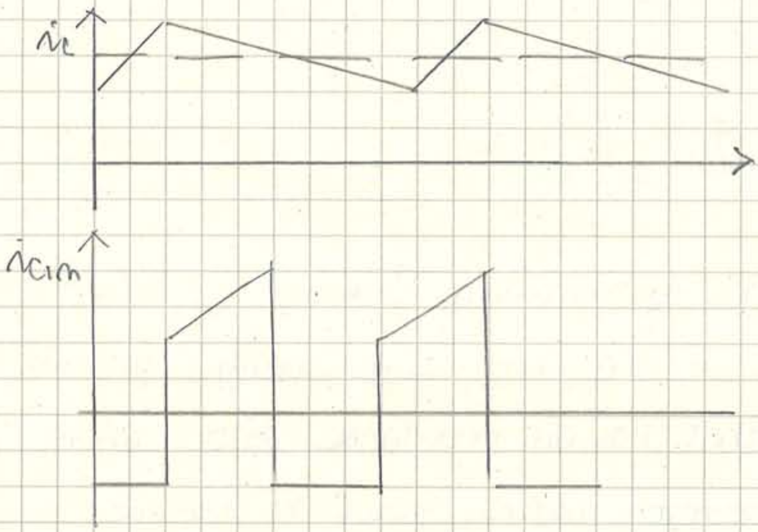
→ RMS $\sum i_{RMS} = 0$?

IS IT TRUE? NO. THE SQUARE IS NOT LINEAR.

$RMS(\sum i) = 0$

We can use a sort of KCL for RMS current.

Let's consider i_{Cin} and i_L:



$i_{L\,RMS}$

VERY IMPORTANT PARAMETERS

$i_{C\,RMS}$

It's hard to find RMS. We can do something better. Let's consider:

$i(t) = I_{DC} + i_{AC}(t)$

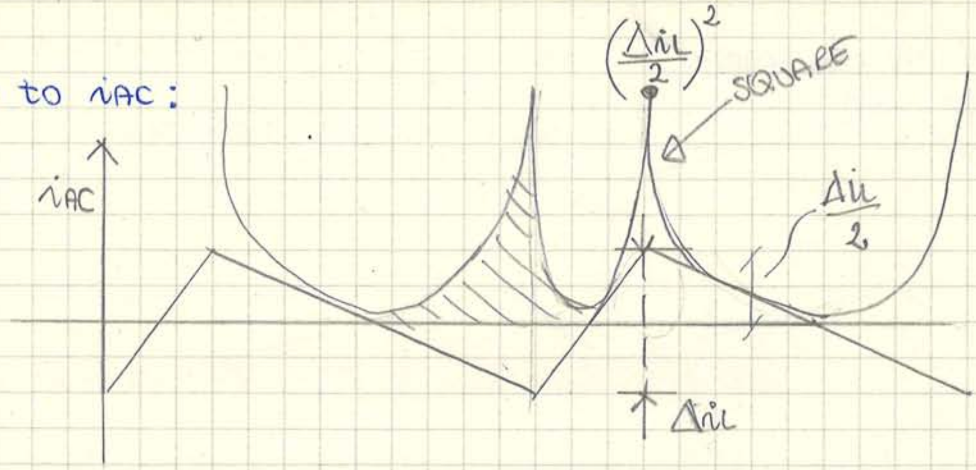
$\bar{i}_{AC}(t) = 0$

So, we try to find out RMS value from $i(t)$:

$$RMS[i(t)]^2 = \frac{1}{T_{SW}} \int_0^{T_{SW}} (I_{DC} + i_{AC}(t))^2 \, dt$$

$$= \frac{1}{T_{SW}} \int_0^{T_{SW}} I_{DC}^2 \, dt + \underbrace{\frac{1}{T_{SW}} \int_0^{T_{SW}} 2 I_{DC} \cdot i_{AC}(t) \, dt}_{\emptyset \ (\bar{i}_{AC} \text{ is } \emptyset!)} + \int_0^{T_{SW}} i_{AC}^2 \, dt$$

Zoom to i_{AC}:



In order to find RMS value:

- square waveform
- evaluate the peak
- Find the area and spread it over one cycle

AREA : $\frac{BASE}{3}\left(\frac{\Delta i_L}{2}\right)^2$

- AVERAGE : $\frac{BASE}{3}\left(\frac{\Delta i_L}{2}\right)^2 \cdot \frac{1}{BASE}$

So, without integrals we find out:

$$i_{AC_{RMS}}^2 = \frac{\Delta i_L^2}{12}$$

(Why equation ★ works? DC component and AC component are function in Hilbert space ; so I_{DC} and $I_{AC_{RMS}}$ are orthogonal.)

Substituting:

$$I_{L_{RMS}}^2 = I_o^2 + \underbrace{\frac{\Delta i_L^2}{12}}_{i_{AC_{RMS}}^2}$$

$$I_{L_{RMS}} = \sqrt{I_o^2 + \frac{\Delta i_L^2}{12}}$$

in case of CCM this term is negligible

$I_{L_{RMS}} \simeq I_o$ (!) CCM

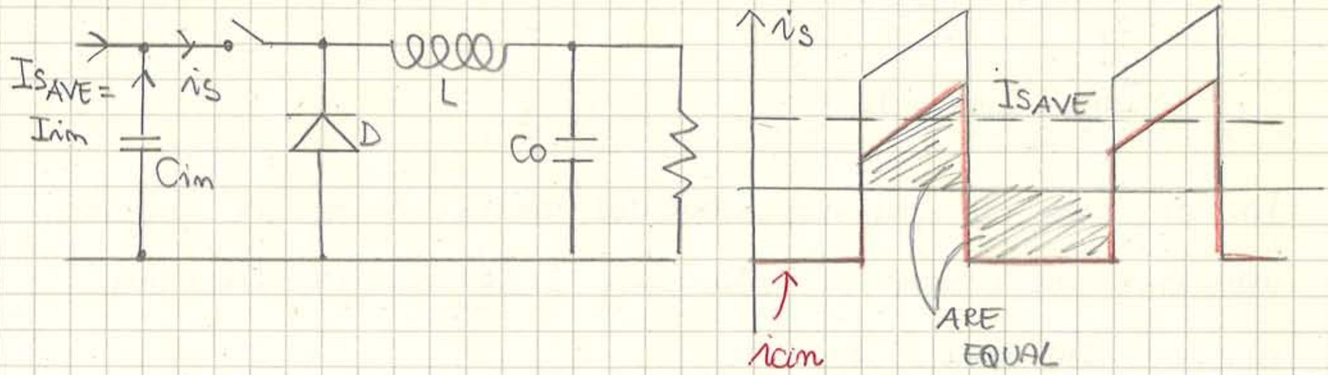
{NB ⚠ we did an assumption : all the AC component flows to C_o (Not totally true)}

True if there isn't ripple

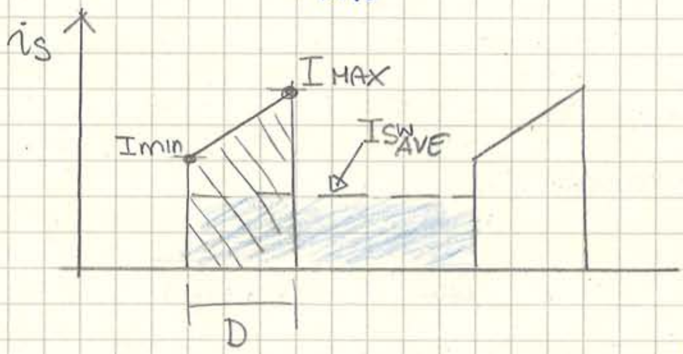
No. We can't do it. $I_{L_{RMS}}^2$ and I_o^2 are very close each other.
So, this calculation has no meaning.

$I_{L_{RMS}} \simeq I_{L_{AVE}}$ CCM

Let's analyze the input of the buck converter:



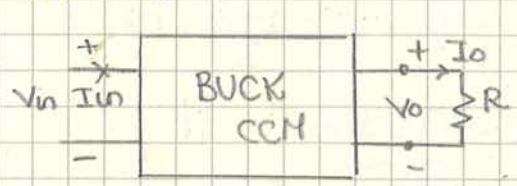
Let's start to evaluate $I_{SW_{AVE}}$:



(Remember: the average is the area)

$$I_{SW_{AVE}} = \frac{I_{min} + I_{MAX}}{2} \cdot D = I_o \cdot D = I_{in}$$ ⚠ IMPORTANT

IMPORTANT:



$V_o = V_{in} \cdot D$ ← IT LOOKS LIKE A TRANSFORMER! (DC)

$I_{in} = I_o \cdot D$

$I_o = I_{in} \cdot \frac{1}{D}$

$$\underbrace{V_o \cdot I_o}_{P_o} = \underbrace{V_{in} \cdot I_{in}}_{P_{in}}$$

POWER CONSERVATION
It's not true! We have losses!

We use these equations not to design our circuit but to check it.

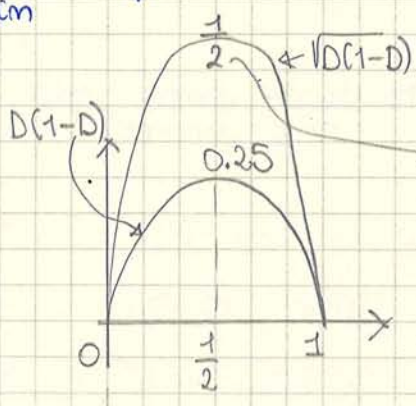
$I_{AC_{RMS\,Cin}} = ?$ Can we use $I_{AC_{RMS\,Cin}} = \sqrt{I_{TOT_{RMS}}^2 - I_{DC}^2}$?

In this case, we haven't numerical cancellation. ⚠

Cin IS UNDER HEAVY STRESS

$$I_{AC_{RMS\,Cin}} = \sqrt{(I_o \cdot \sqrt{D})^2 - I_o^2 D^2} = I_o \sqrt{D - D^2}$$

$$= I_o \sqrt{D(1-D)}$$

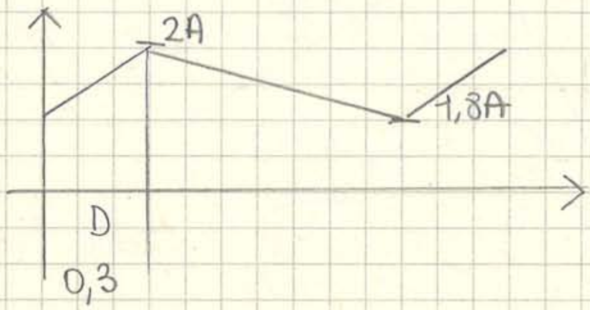


Bad news! Very strong stress for the Cin.

Suppose:

$I_o = 1{,}9\,A$

$\Delta i_L = 0.2$



- $I_{Co_{RMS}} = \frac{\Delta i_L}{\sqrt{12}} = 57\,mA$
- $I_{Cin_{RMS}} \simeq \frac{1}{2} I_o \simeq 0.95\,A$!! VERY HIGH CURRENT

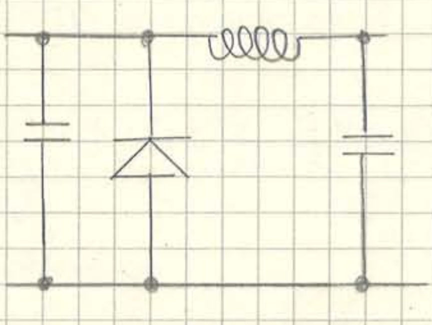
Different condition.

Cin VERY STRESSED ⚠

Without approximation:

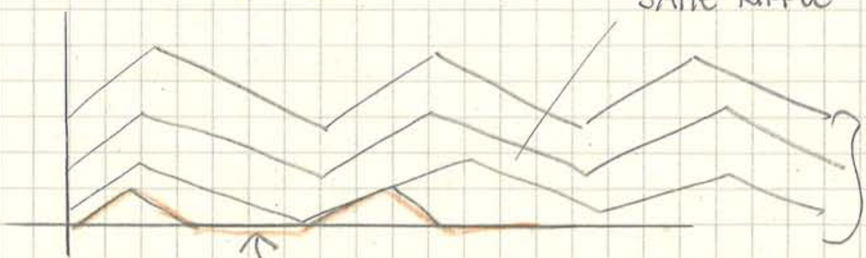
$$1.9\,A \sqrt{\underset{\substack{\uparrow \\ D}}{0.3} \times \underset{\substack{\uparrow \\ 1-D}}{0{,}7}} = 0{,}87\,A$$

Let's find now the stress of the diode:



DCM has a larger average value than CCM. (With the same duty cycle).

If the load is very light:

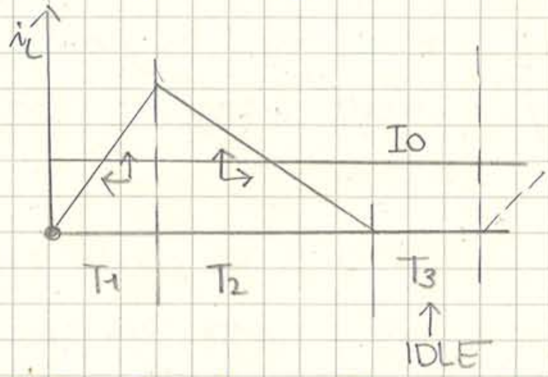


If I make the load still lighter → DCM

- CCM HEAVY MODE
- DCM LIGHT MODE

(Also called in this way)

If we want to analyze our circuit in DCM we have to consider:



$$\frac{V_{in}-V_o}{L}T_1 + -\frac{V_o}{L}T_2 = 0$$

$$\frac{V_o}{V_{in}} = \frac{T_1}{\underbrace{T_1+T_2}_{\neq T_{SW}}}$$

$T_1 + T_2 < T_{SW}$ because of T_3

So, we don't solve the problem and we need a II equation.

Idle is not useful because the circuit doesn't work in this time; we lose effectiveness.

1) $M = \underbrace{\frac{V_o}{V_{in}}}_{\text{ALWAYS}} = \frac{T_1}{T_1+T_2}$ We decide how long is T_1; but we don't know T_2.

In many cases we found a relationship between I_o and i_L. I_o is the DC component of i_L. So:

2) $I_o = \frac{T_1+T_2}{2T_{SW}} \cdot \underbrace{\left(\frac{V_{in}-V_o}{L}\right)T_1}_{i_{LMAX}} = \frac{V_o}{R}$ (AREA SPREADS IT OVER ONE CYCLE)

↓ AVERAGE OF i_L

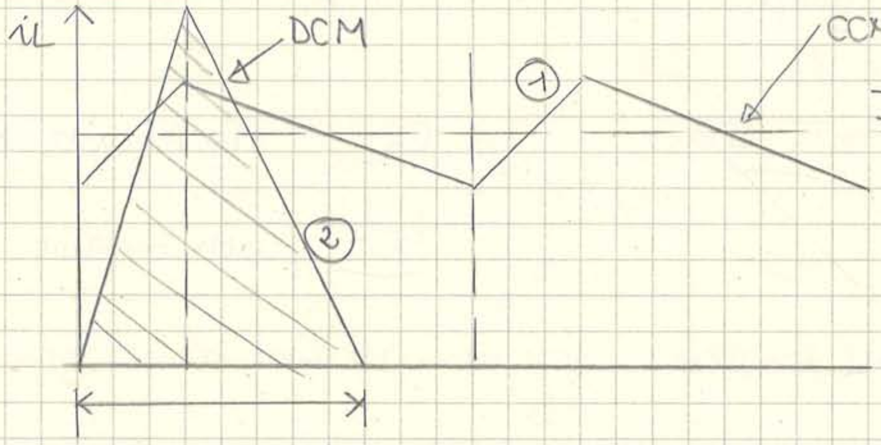
What does $D = 1$ mean? It means that the switch is closed all the time,

D is $1 \rightarrow V_o = V_{in}$. This means that we are not in DCM and the last equation found doesn't help me! (We are in CCM)

What happens if $R \rightarrow \infty$ (I remove the load)? $\frac{V_o}{V_{in}} = 1$.
We are still in DCM because through inductor there is no current.
So the equation can be used.

M IS DIMENSIONLESS!

Why we don't use buck converter in DCM?



② Curve of DCM design @ same parameters of CCM.
① CCM
The average value in DCM is still Io, but the peak is at least twice the peak of CCM (because we want to have the same area). This is a real strong stress for each component.
There is a possible advantage (but very very small):

$$L_{CRIT} = \frac{R(1-D)}{2f_{SW}} \quad \begin{cases} \text{CCM} & L > \frac{R_{max}(1-D_{min})}{2f_{SW}} \\ \text{DCM} & L < \frac{R_{min}(1-D_{max})}{2f_{SW}} \end{cases}$$

$L_{DCM} < L_{CCM}$ ← ADVANTAGE OF DCM

$$V_o = \frac{V_{in}\ 2}{1 + \sqrt{1 + \frac{8 L f_{sw}}{D^2 \underbrace{\left(\frac{V_o}{I_o}\right)}_{R}}}}$$

So, if we want to find the derivative we have to find out V_o and then find derivative

$V_o = f(f_{sw}, D, L, I_o)$ and then find out the output resistance

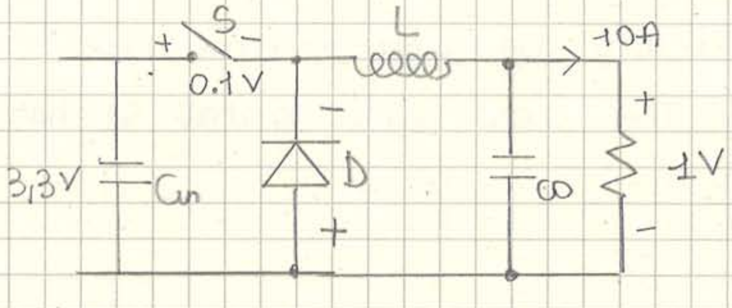
There is another way

1. Numerically:
2. Dini's Theorem

Most of the time when we get the derivative we obtain a negative value of resistence. $R < 0$. It means that:

$\left.\begin{array}{l} I_o \uparrow \\ V_o \downarrow \end{array}\right\}$ R negative. Why? Sign convention.

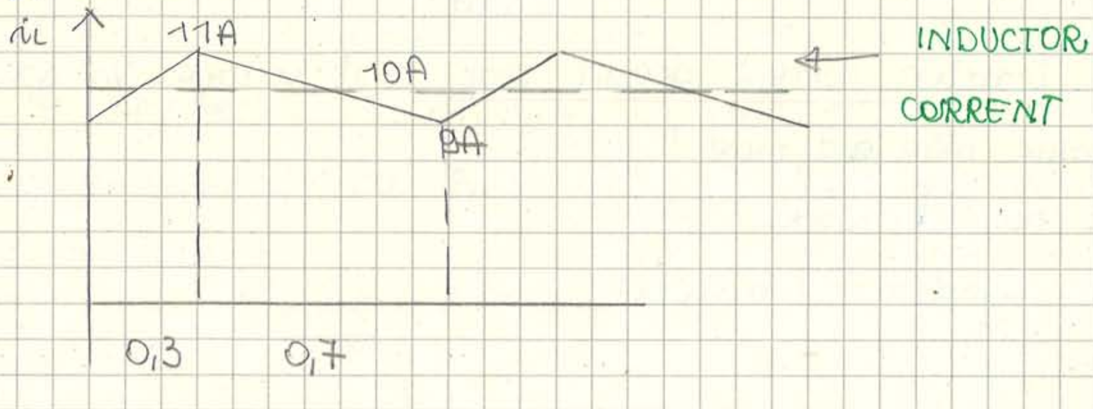
Let's consider a buck converter with some topological changes:

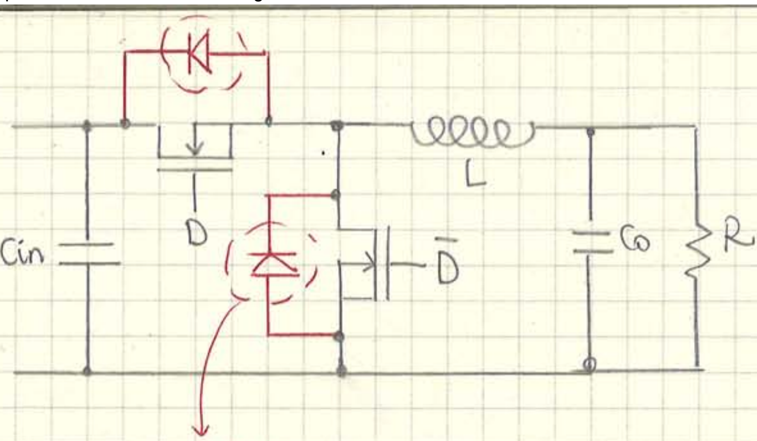


$D = \frac{1V}{3{,}3V} = 0{,}3$

Maybe not! Because if V_o is so ~~bad~~ low the voltage drop at diode starts to be important. ⚠

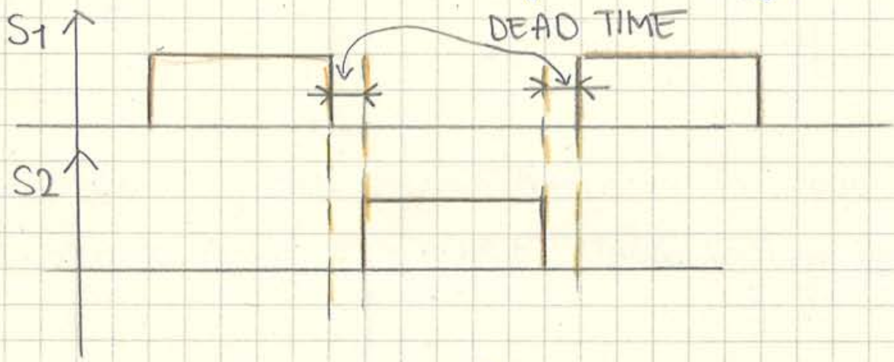
Let's suppose $I_o = 10A$





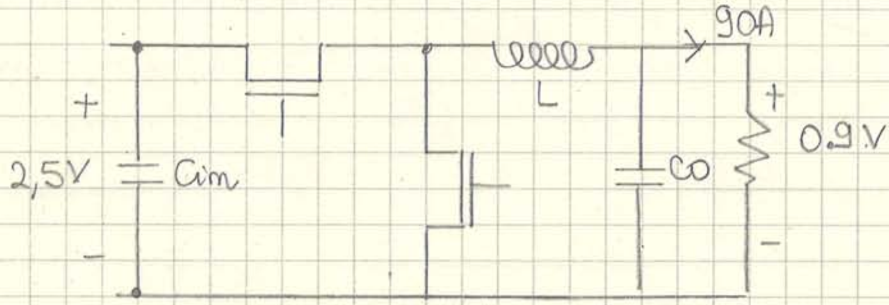
SYNCHRONOUS BUCK (RECTIFICATION)

BODY DIODE (derives from technology)



What happens in dead time? The current flows to the body diode and produces a voltage drop of 1 V but for a very short time!

On the other hands, let's try to design a buck like this:



$D \simeq 0.36$

The main problem is: how much current flows through Cin? more or less 45 A. It's a lot! There are capacitors able to support this value but are large, expensive! And also L must carry 90 A!
Instead of having 1 converter ables to support 90 A we can have 3 converterters (30 A each) in parallel.
Smaller components, but 3 of them! In this way we reduce the stress.

→

if the switches conduct together

So, the total current is something like this:

The RMS value of this current is quite low because the duty cycle should be the D of DC component and D of the small parts → OVERLAP.

If I have D of 36% and we have 3 circuits together this overlap will be 0.03%.

So the RMS current through C is very reduced.

This is the way that uses motherboard to supply processors.

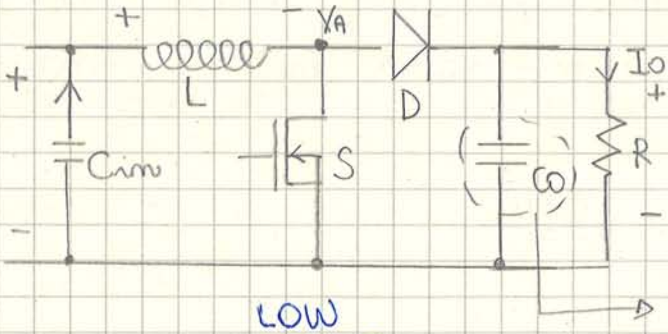
This way is called MULTIPHASE. (Low voltage very high current)

BOOST (STEP UP)

$\frac{V_O}{V_{in}} \geq 1$ The output voltage is larger than the input voltage (Yeah!)

The bad part is that it's very difficult to control it

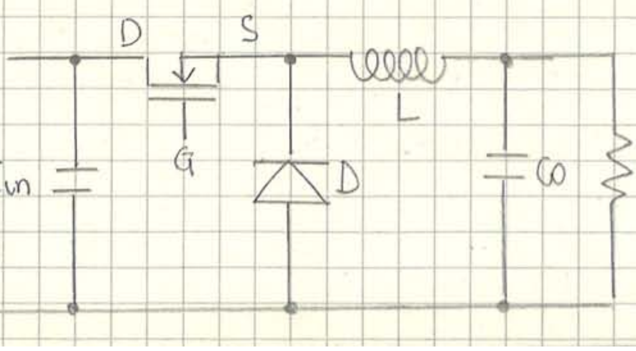
In many cases (high power) we use other type of converter; but boost converter for example has many applications like mp3.



LOW SIDE

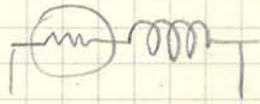
IN THIS CASE C_O IS A PART OF CIRCUIT.

Let's compare it with the buck converter:



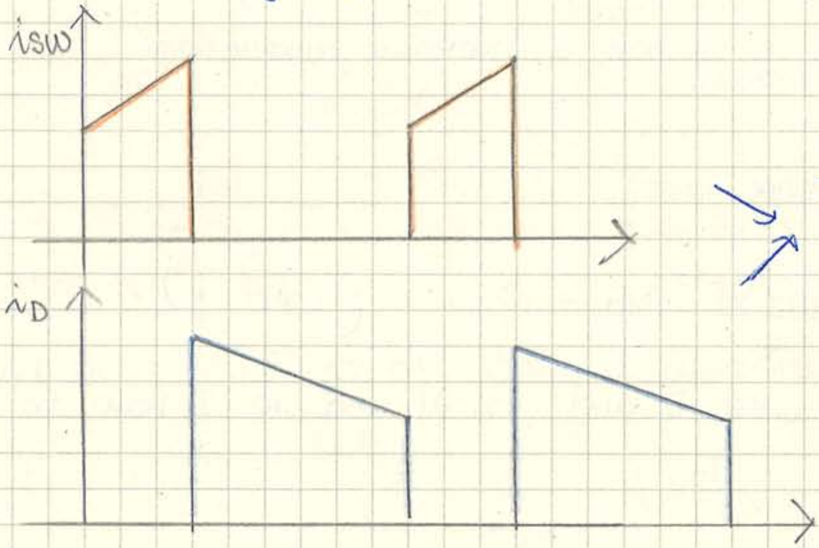
With nMOS

- Ton → switch is closed; so the voltage drop on L is V_{in}. If we want be more precise $V_L = V_{in} - V_{SW} - V_{RES\_PAR}$
  ↓ of the wire



- Toff → through L the voltage drop is $V_{in} - V_o - V_D$
  $V_{in} - V_o$ is negative!

Deriving the other waveforms:

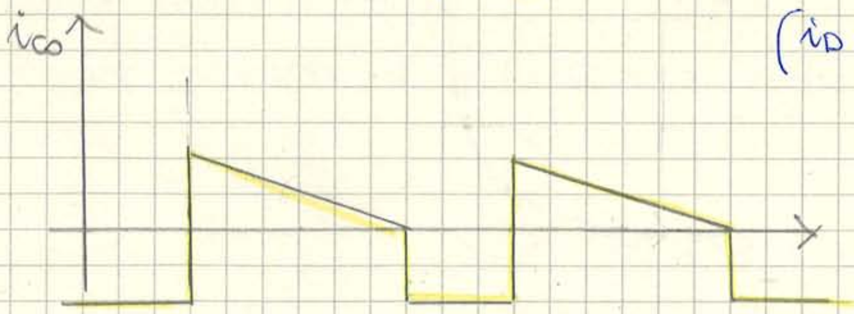


IDENTICAL TO THE BUCK CONVERTER CASE

icin and ico are different.
If we want to be very precise, if Vin is ideal, $i_{cin} = 0$; but:

i_L minus the DC comp.

And ico that is the AC comp. of the current flowing to the diode:
(i_D shift down of I_o)



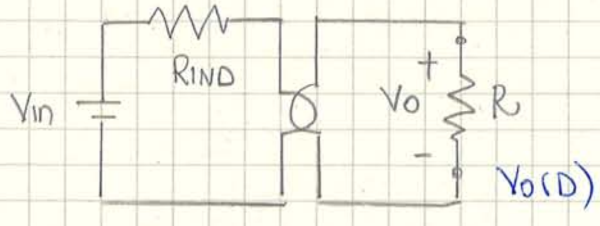
Let's consider an ideal boost:

$$\begin{cases} V_o = \dfrac{V_{in}}{1-D} \\ I_o = I_{in}(1-D) \end{cases}$$ POWER CONSERVATION

$1 : \frac{1}{1-D}$

↑ like a transformer!

But let's consider for example one parasitic element (R_{IND} of the wire), I have:

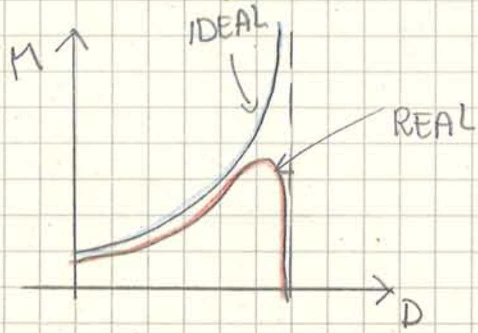


$1 : \frac{1}{1-D}$

transformer that transform the DC component.

If $D = 1$ I have an infinite output voltage. No!

If we find out $V_o(D)$ we get that it can't reach any value we want:



$\frac{1}{1-D}$

With one parasitic element!

In our boost converter we want to stay in this area: where losses are not important yet.

So now we can find the three derivatives:

- $$\frac{\partial V_o}{\partial D} = \frac{\partial}{\partial D}\left(V_{in} \cdot \frac{1}{1-D}\right) = \frac{V_{in}}{(1-D)^2} \cdot (+1) = \frac{V_{in}}{(1-D)^2}$$

If I want to express it if function of V_o $\left(\frac{V_o}{V_{in}} = \frac{1}{1-D}\right)$

$$= \frac{V_o}{1-D} = V_o \cdot \frac{V_o}{V_{in}} = \frac{V_o^2}{V_{in}}$$

$\rightarrow$ If V_o is constant I can see what happens if V_{in} changes.

Boundary condition:

$I_{min} = 0 \Rightarrow \frac{V_o}{R(1-D)} = \frac{D(1-D)V_o}{2 f_{sw} L}$

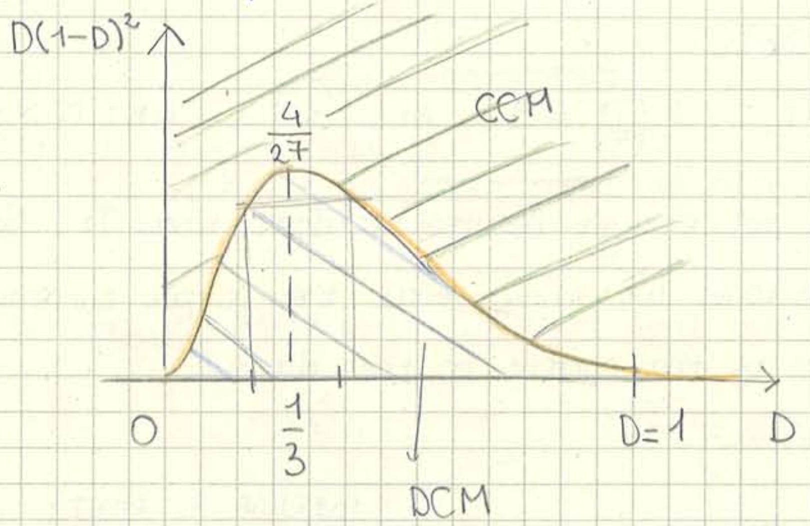
- f_{sw} degree of freedom
- D : $M = \frac{1}{1-D} \rightarrow D = \frac{M-1}{M}$ → IT'S A CONSTRAIN !
- R is a constrain of the customer.

So, fixed f_{sw} we can derive L : (or the contrary).

$L_{CRIT} = \frac{D(1-D)^2 R_{(MAX)}}{2 f_{sw}}$ → CCM

$L > L_{CRIT}$	CCM
$L < L_{CRIT}$	DCM

And what about D? D_{MAX} or D_{min}? What is the worst case?
(DCM is good for low power application)



So, we have $L_{CRIT\ BOOST} = \frac{D(1-D)^2 R}{2 f_{sw}}$

If we want $\frac{V_o}{V_i} > 1$ we have to stay in DCM. But in this mode the stress is very high with respect CCM and so we have to use it in low power voltage. If we need a large power we need to use other topology.

Now analyze the boost converter in DCM.

So we obtain :

$$\frac{M}{R} = \frac{1}{2}\frac{D}{L}\frac{T_1}{M-1} \qquad \left(\frac{D}{f_{SW}} = T_1\right)$$

$$M(M-1) = \frac{1}{2} R \frac{D^2}{L f_{SW}}$$

$$M^2 - M - \frac{RD^2}{2L f_{SW}} = 0$$

$$M = \frac{1 \pm \overbrace{\sqrt{1 + \frac{2RD^2}{L f_{SW}}}}^{\text{LARGER THAN } 1}}{2}$$

I can't take the negative sign because the second quantity is larger than 1.

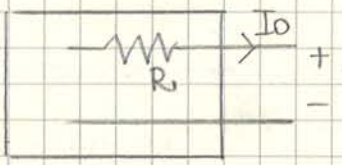
NB: In every DCM mode M depends on R ⚠ This is means that it's not an ideal voltage source (like CCM mode).

So we can evaluate the derivatives :

- $\frac{\partial V_o}{\partial V_{in}} = M$
- $\frac{\partial V_o}{\partial I_o} \neq 0 \quad < 0$ OUTPUT RESISTANCE

$$\left(\frac{V_o}{V_{in}} = \frac{1 + \sqrt{1 + 2\frac{V_o}{I_o}\frac{D^2}{L f_{SW}}}}{2} \text{ and we derive } \frac{\partial V_o}{\partial I_o} \ldots\right)$$

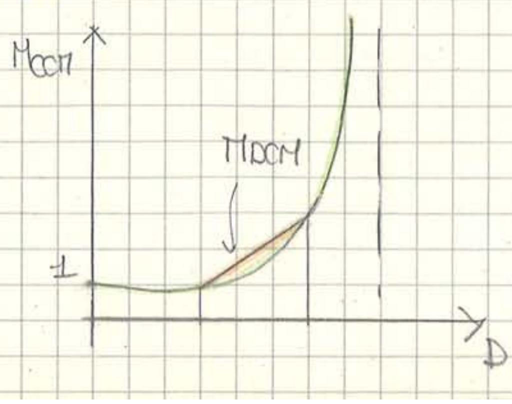
It's not a negative resistance! It's just found out with these signs :



← SIGN CONVENTION

- $\frac{\partial V_o}{\partial D} = \text{GAIN}(V_{in}, R, D, \ldots)$

Let's see what happens if I want to compare the M in CCM and the M in DCM

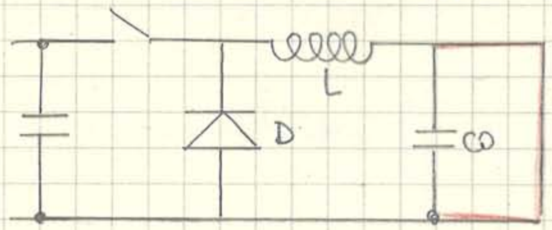


$$M_{CCM} = \frac{1}{1-D}$$

Unfortunately our circuit is not protected by S.C.

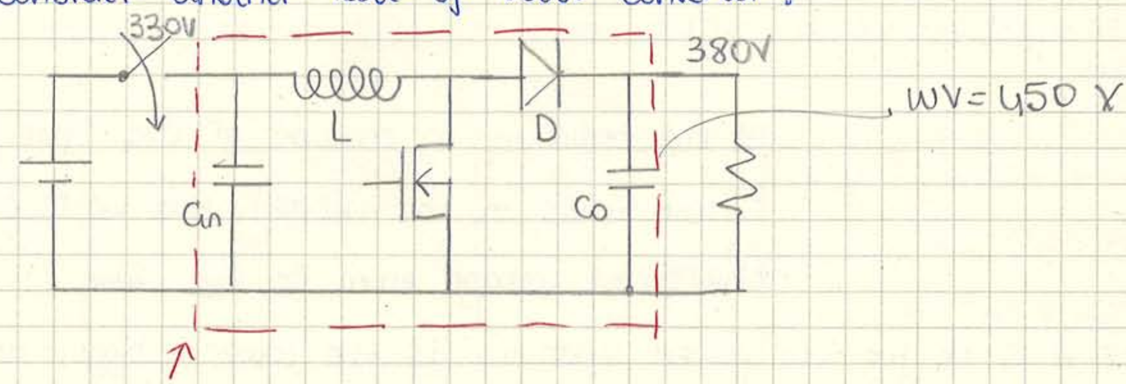
If the switch is opened there is a current that flows into the circuit. So we have no protection. Not good!

For the buck converter is totally different:

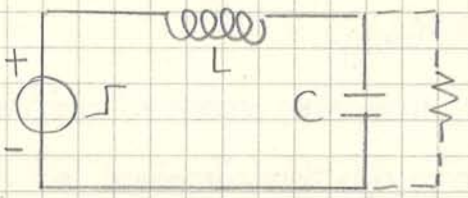


If we have a S.C at the output we can open the switch and we isolate the input from the short.

Let's consider another issue of boost converter:

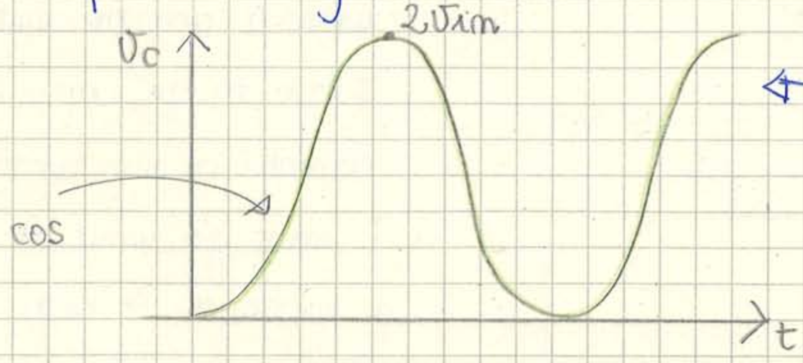


we apply a voltage step to this circuit:



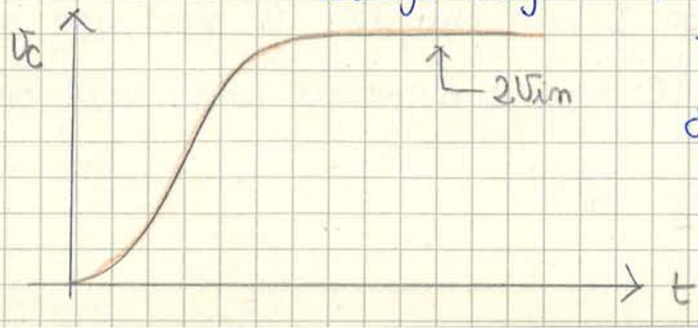
← RESONANT CIRCUIT

And at capacitor voltage:



← OSCILLATIONS

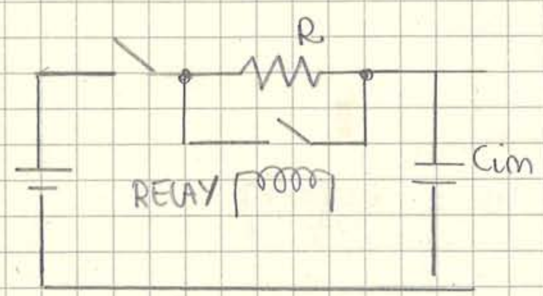
But we have diode and the voltage stays at $2V_{in}$:



The capacitor doesn't discharge.

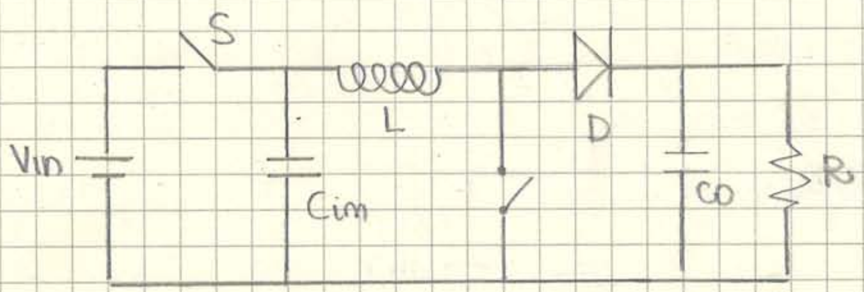
When it is HOT, the resistance of this component goes down and its power is negligible.

A second solution used for large power:



RELAY → I can close and open the switch. But this costs. If our converter works with many loads, we have to wait in order to relay works.

There is another way to avoid the over voltage at the output side.



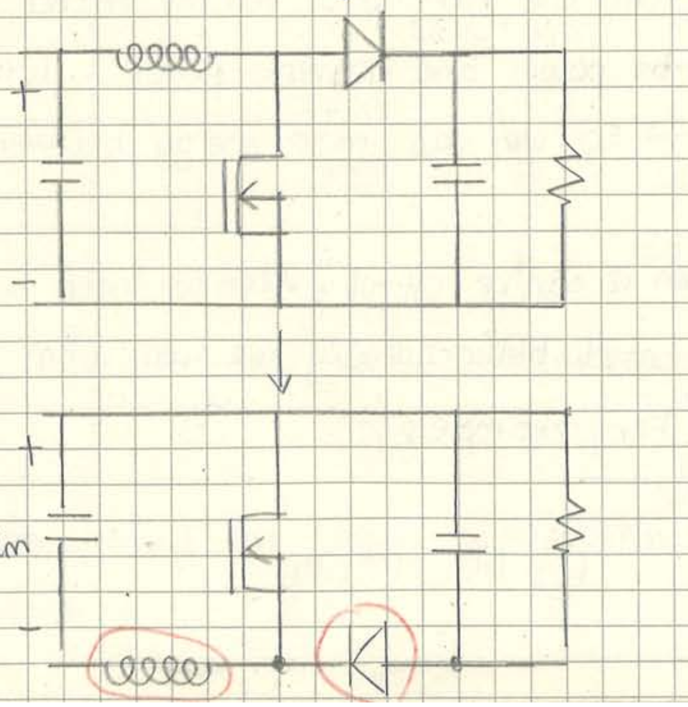
$\sqrt{LC} \simeq$ "τ"
TIME CONSTANT OF THE PULSE

$\tau \gg T_{SW}$

$T_{SW} = 200\,KHz$, $5\,\mu s$ → τ must be of the order of 50 ms

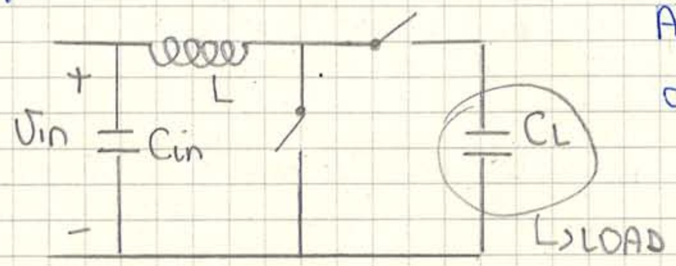
It means that the current goes up forever and when switch is opened we have a saturation of L and Co!

Let's make some changes:



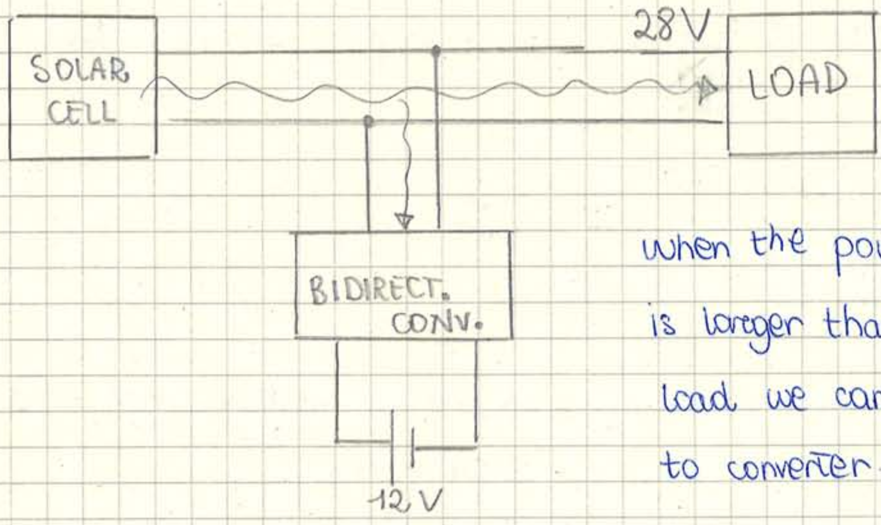
I have to move both!!
But it's not a good idea!
The output voltage is always the same.

Or:



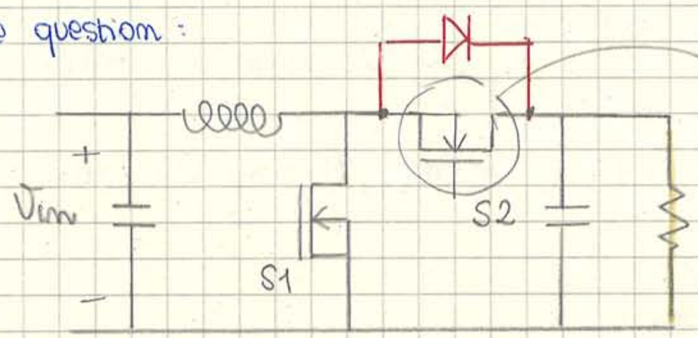
A boost converter only for charging or dischargig a capacitance.

Or:

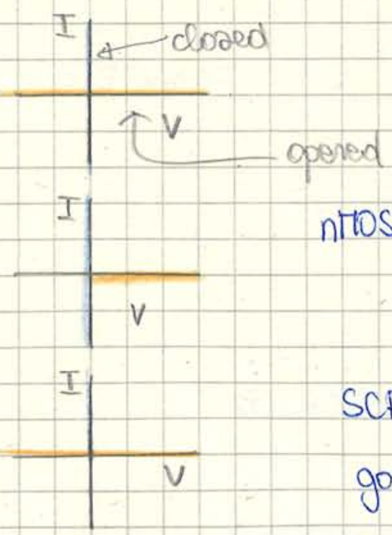


when the power of solar cell is larger than the power of the load we can have a flow power to converter.

One question:



If I open this switch can I handle a s.c. to the output? No! Because of body diode



IDEAL SWITCH

nMOS SWITCH → current both directions
voltage only one direction

SCR (we can't switch it off until current goes to zero)

For having an ideal switch behaviour we can have 2 nMOS in series:

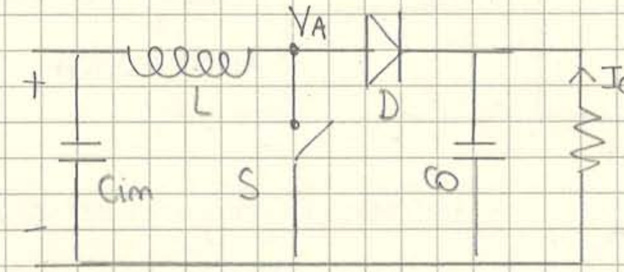
What is the disadvantage of the inverter?

During ⌐ we have a period of time which both of MOS conducts at the same time (CROSS-CONDUCTION or SHOOT-THROUGH). So we need to apply some kind of delay.

However, most of the time I prefer to use nMOS transistor because they have better electrical characteristics of pMOS transistor.

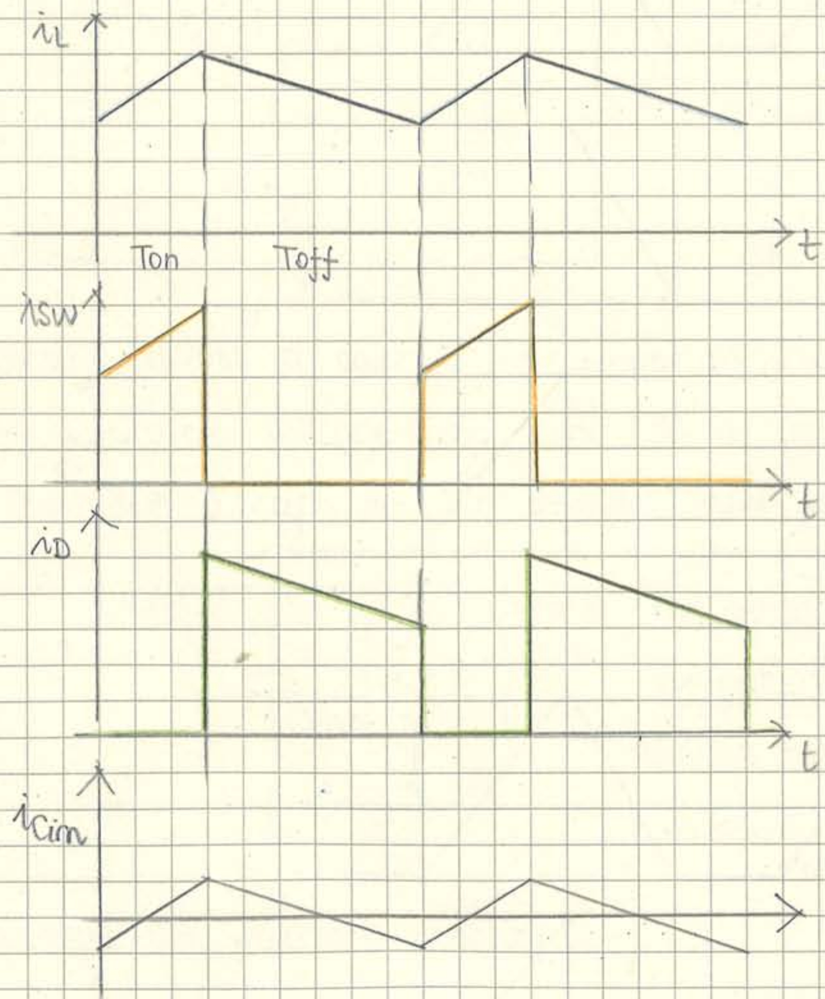
pMOS has $\frac{W}{L}$ larger than nMOS because of mobility.

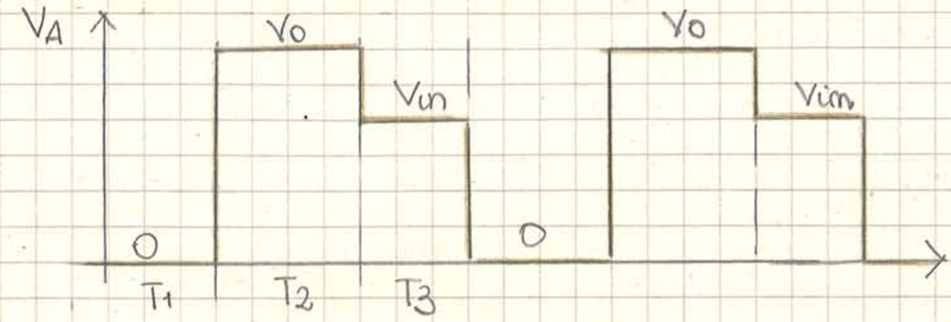
Let's come back to boost converter: *



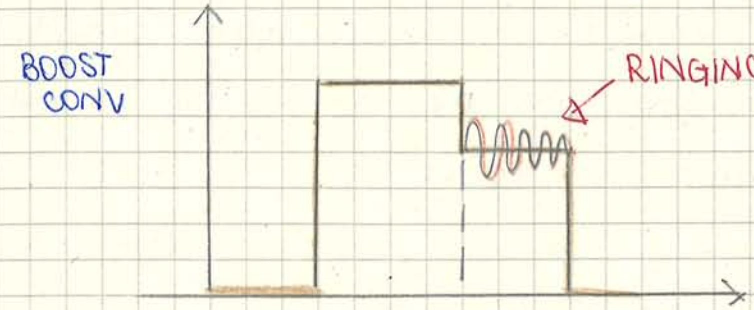
Let's find the waveforms.

• CCM





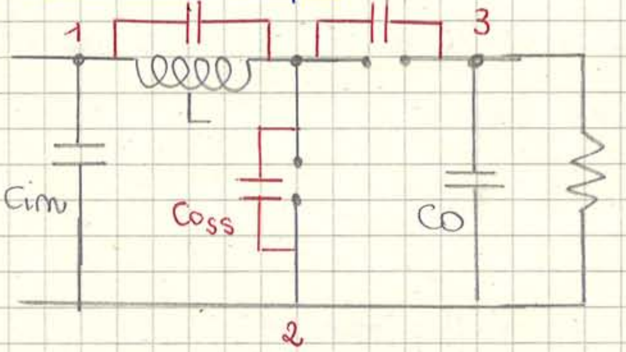
We saw this waveform: (like any converter)



RINGING that generates e.m. noise

How is this ringing generated?

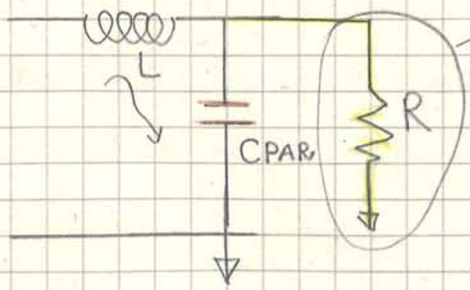
Switch and diode open:



PARASITIC CAPACITANCE

All these 3 paras. caps are in parallel.

The voltage across a cap is not important. what is important is the variation voltage. So node 3 is constant (like 2 and 1). So the 3 caps are in parallel and give me a resonator:



For avoid this oscillation we put a resistor (in parallel). But it takes a lot of DC component

So we have an important quantity:

$$f_{RES} = \frac{1}{2\pi\sqrt{LC}}$$

→ RESONANCE FREQUENCY

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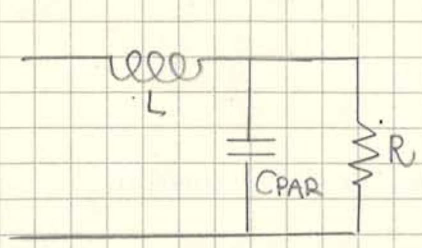
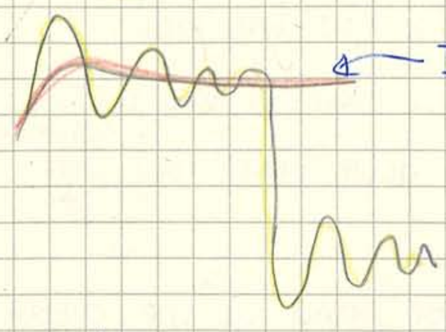


$Z_0 = \sqrt{\dfrac{L}{C_{PAR}}}$ And now we can find R:

$Q = \dfrac{Z_0}{R} \overset{OR}{=} \dfrac{R}{Z_0}$? Are both correct.

if in my circuit $R \to \infty$, $Q \to \infty$ and then: $Q = \dfrac{R}{Z_0}$



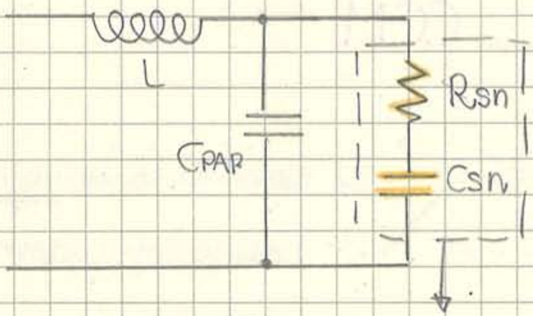
← I want a Q something like this

so $Q = 0{,}7 \to 1$

and I derive R.

↳ that is R_{sn}

But I have a capacitor in series:



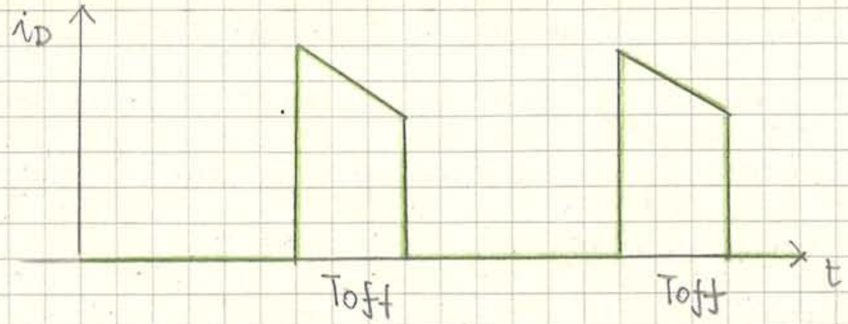
← 3rd order circuit

RC snubber

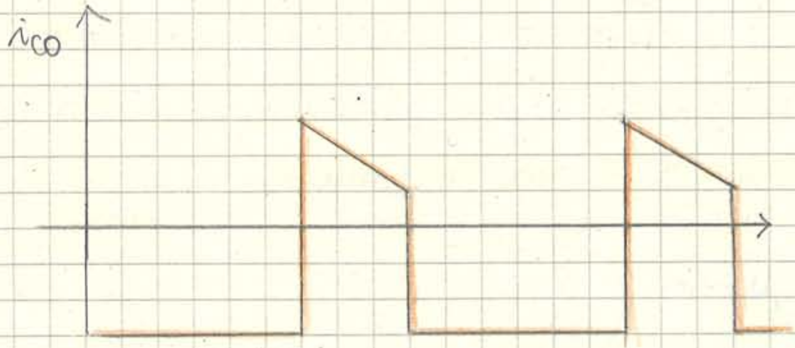
For minimize loss power.

- If $C_{sn} \to 0$, R_{sn} doesn't work;
- If $C_{sn} \to \infty$ (C_{sn} SC) and so I have DC power dissipation.

$\boxed{C_{sn} = 6 \to 8\ C_{PAR}}$



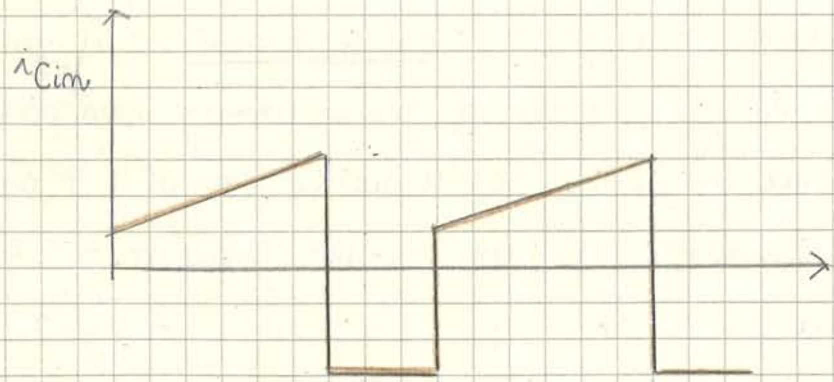
What is i_{Co} current? It's the current through the diode minus the DC component.



Is Co STRESSED?

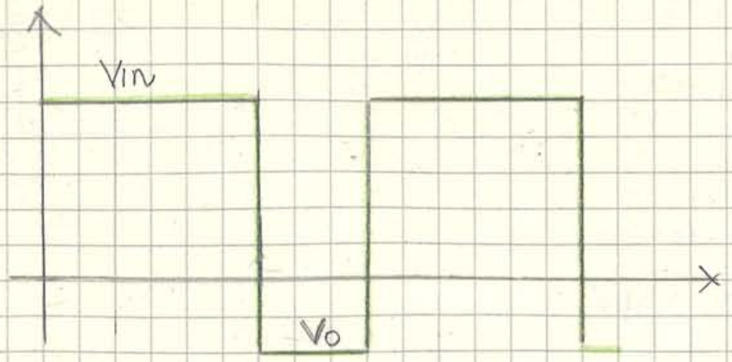
The current through Co has not a small ondulation! It's an impulsive current exactly like the one of the boost converter. So Co is under heavy stress.

What about Cin?



Also Cin is stressed.

And V_A?



Vo IS NEGATIVE RESPECT Vin

$$\begin{cases} I_{MAX} - I_{min} = \dfrac{V_{in}}{L} \cdot T_{on} \underset{OR}{=} -\dfrac{V_o}{L} \cdot T_{off} \\ \dfrac{V_o}{R} = -\bar{i}_D = -\dfrac{I_{MAX}+I_{min}}{2}\underbrace{(1-D)}_{\frac{T_{off}}{T_{SW}}} \end{cases}$$

Because we need $I_{MAX} - I_{min}$

(e' la media di i_D)

the direction is opposite!

$$\begin{cases} I_{MAX} - I_{min} = -\dfrac{V_o}{f_{SW}L}(1-D) \\ I_{MAX} + I_{min} = -\dfrac{2V_o}{R(1-D)} \end{cases}$$

- $I_{MAX} = -\dfrac{V_o}{R(1-D)} - \dfrac{V_o(1-D)}{2f_{SW}L}$
- $I_{min} = -\dfrac{V_o}{R(1-D)} + \dfrac{V_o(1-D)}{2f_{SW}L}$

And now we found out boundary conditions.

$$I_{min} = 0 = -\dfrac{\cancel{V_o}}{R(1-D)} + \dfrac{\cancel{V_o}(1-D)}{2f_{SW}L}$$

What are my degrees of freedom? f_{SW} and L. We usually choose f_{SW} and than find out L. (Remember that $|V_o|$ can be > or < than V_{in}!).

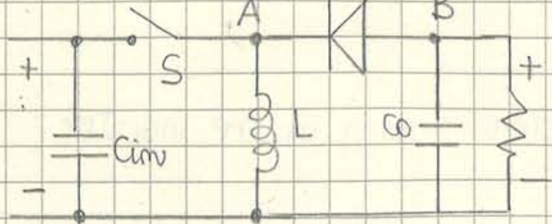
So:

$$L_{CRIT} = \dfrac{R(1-D)^2}{2f_{SW}}$$ → it's a monotonic expression; if $D\uparrow$, $L\downarrow$, always!

CCM → $L > L_{CRIT}$

DCM → $L < L_{CRIT}$ → $L < \dfrac{R_{min}(1-D_{MAX})^2}{2f_{SW}}$

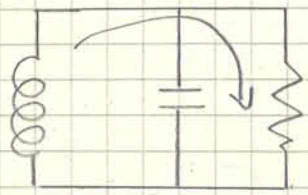
Now consider:



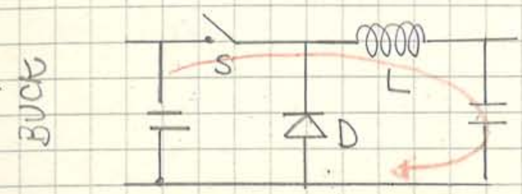
We can use quadratic KCL for both A and B. (Because we don't have smooth current).

Then :

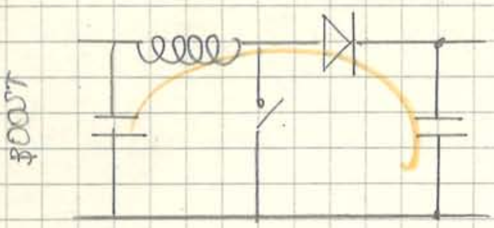
Toff → SWITCH OPEN, AND THE ENERGY STORED INSIDE INDUCTOR MOVES TO THE OUTPUT (and input is disconnected)



It's different for buck and boost converter :



During Ton we put energy to inductor but at the same time energy goes to the output !

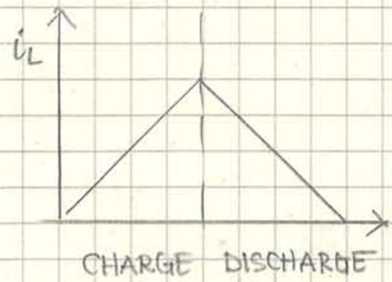


Same story for this.

Buck and boost converter are called DIRECT CONVERTER.

On the other hand , in a buck-boost converter we have 2 separate phases : charge and store energy and then deliver energy → INDIRECT CONVERTER.

① CHARGING $\quad \mathcal{E}_L = \frac{1}{2} L I^2$



NB: we have to use the maximum current because the energy is given by peak current.

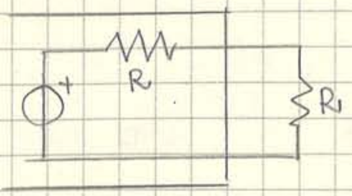
So: $\mathcal{E}_L = \frac{1}{2} L \left(\frac{V_{in}}{L} \cdot T_{on} \right)^2 = \frac{1}{2} \frac{V_{in}^2 D^2}{L f_{sw}} = \frac{V_{in}^2 D^2}{2 L f_{sw}^2}$

ENERGY STORED IN L IN ONE CYCLE

Instead of $\frac{V_{in}}{V_o}$:

$$\frac{\partial V_o}{\partial I_o} = -\frac{D^2 R^2}{2 f_{sw} L} \cdot \underbrace{\frac{2 f_{sw} L}{D^2 R}}_{\frac{1}{\Pi^2}} = -R$$

The internal resistance of our converter is equal to the load resistance:

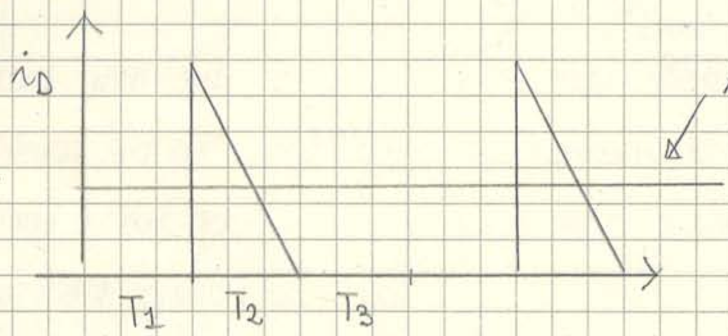


Is this means that the efficiency is 50%?

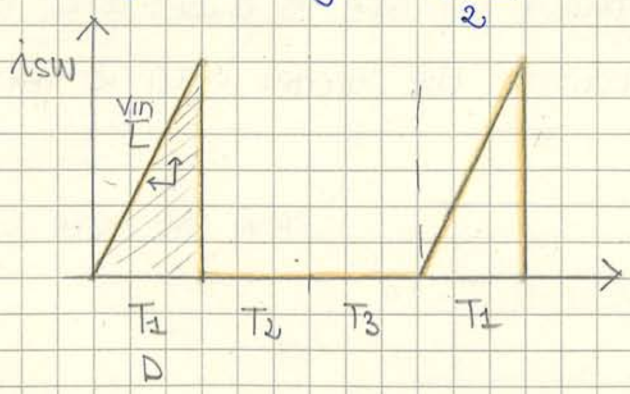
In circuity theory yes; but on the other hand if we reduce Rint we can increase efficiency. But how can is possible? They are equal, we derive it!

Rint = R comes out from a derivative; it's a DIFFERENTIAL RESISTANCE, not linear! It has nothing to do with power dissipation.

Let's go back to DCM mode:



How can we find average? $\frac{T_2 \cdot I}{2}$



DC current coming from input

$$\bar{i}_{SW} = \underbrace{\underbrace{T_1 \cdot \frac{V_{in}}{L}}_{\text{height}} \cdot \frac{T_1}{2}}_{\text{AREA}} \cdot \frac{1}{T_{SW}} =$$

$$= \frac{V_{in} D^2}{2 f_{sw} L} \quad \text{(A) EQ}$$

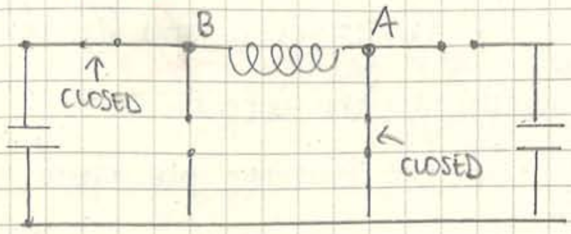
Why it's called buck-boost converter?

It's a merge of buck and boost.

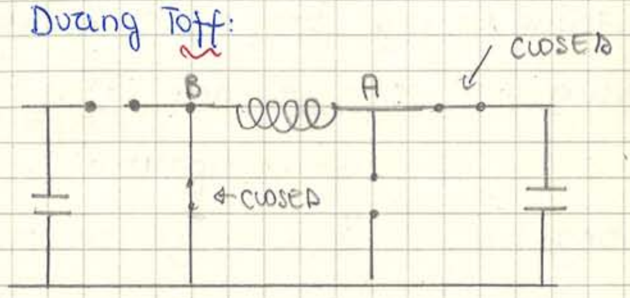
Let's consider a buck followed by boost.

How can we go from this topology to the standard one?

During T_{on}:



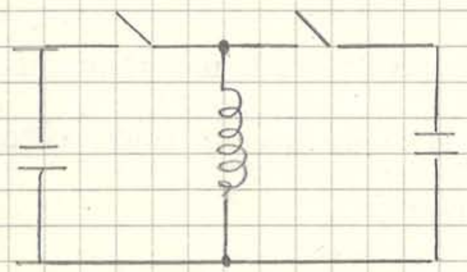
During T_{off}:



Energy stored from input to L — Energy moves from L to the output

During T_{on} A is connected to the ground; during T_{off} B is connected to ground.

So we can re-draw the circuit like this:

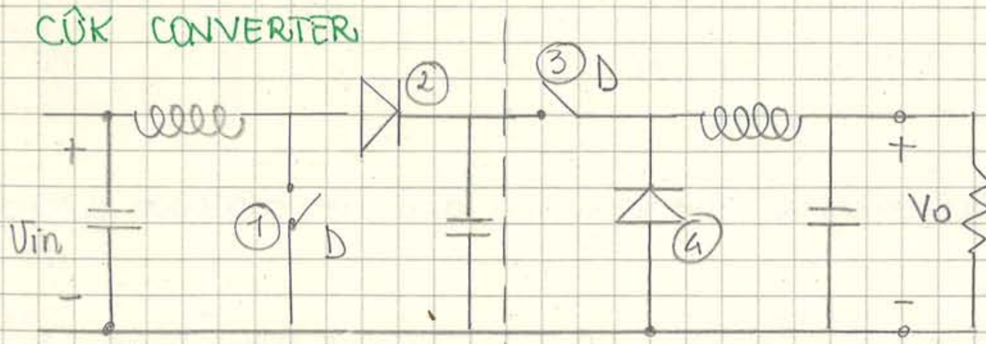


STANDARD BB CONVERTER

The difference between the previous circuit is that L, now, doesn't flip up and down. And this means that V_o has negative sign respect V_{in}.

And what about the boost-buck?

CÛK CONVERTER



$M = \frac{1}{1-D}$ $\qquad$ $M = D$

$$M_{TOT} = \frac{D}{1-D}$$

V_o has the same sign of V_{in}.

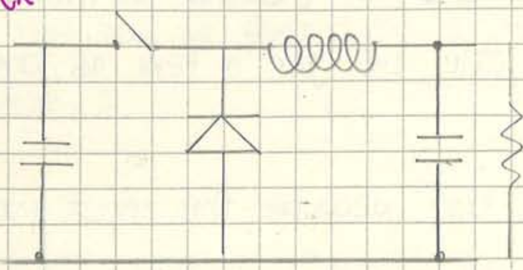
Both of input and output currents are smooth current. However, it's a son of boost, so it's difficult to drive.

We have 2 L, 4 switches and the capacitor ~~doesn't can be~~

Good news: the two inductors can be done on the same magnetic core.
Bad news: it's complicated to do.

OVERVIEW OF DIFFERENT TOPOLOGIES

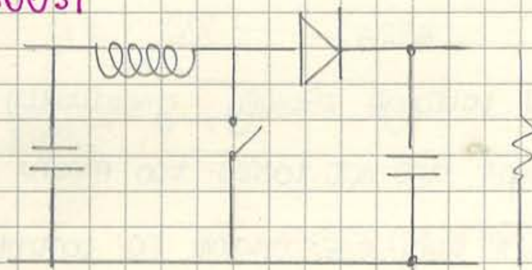
BUCK



- C_{in} STRESSED, C_o NOT STRESSED
- OVERVOLTAGE: NO SURVIVE
- OUTPUT SC: OK
- CL: OK (There is the switch)
- SOFT START: OK

D

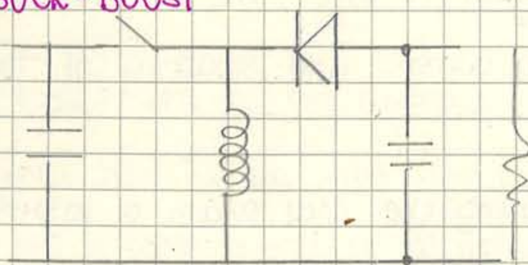
BOOST



- C_{in} UNSTRESSED, C_o STRESSED
- OVERVOLTAGE: survive
- OUTPUT SC: NO
- CL: NO
- SOFT START: NO

$\frac{1}{1-D}$

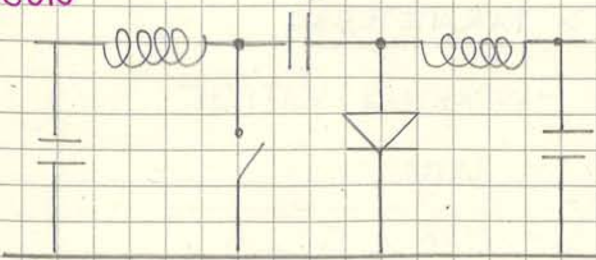
BUCK-BOOST



- C_{in} STRESSED, C_{out} STRESSED
- OVERVOLTAGE: NO SURVIVE
- OUTPUT SC: OK
- CL: OK
- SOFT START: OK

$\frac{D}{D-1}$

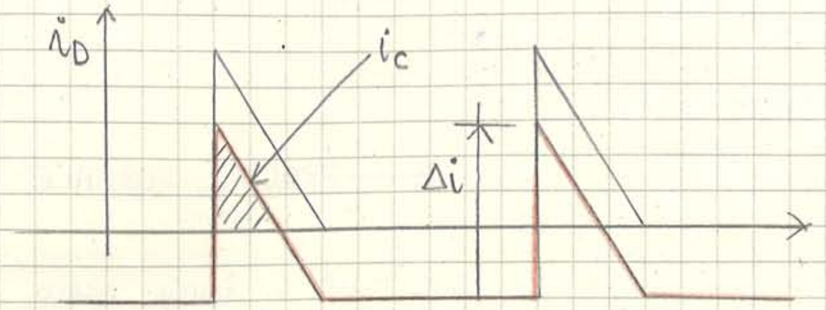
ĈUK



- C_{in} UNSTRESSED, C_o UNSTRESSED
- OVERVOLTAGE: SURVIVE
- OUTPUT SC: C is like an open circuit, so OK
- CL: MMH... OK
- SOFT START: OK

$\frac{D}{D-1}$

In DCM :



If the cap is electrolytic I have to specify the ESR to control the output ripple :

$$\boxed{V_{OUT\,RIPPLE\,PP} = \Delta i \cdot ESR}$$

What about if I use a film or ceramic caps? ESR is not a issue.

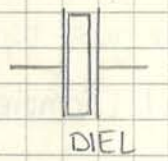
So I need to specify C :

$\Delta V = \frac{\Delta Q}{C}$ — AREA

RIPPLE

(IN CCM BUCK $C = \frac{\Delta i_L}{\Delta V_C \cdot 8 f_{SW}}$)

ELECTROLYTIC CAP



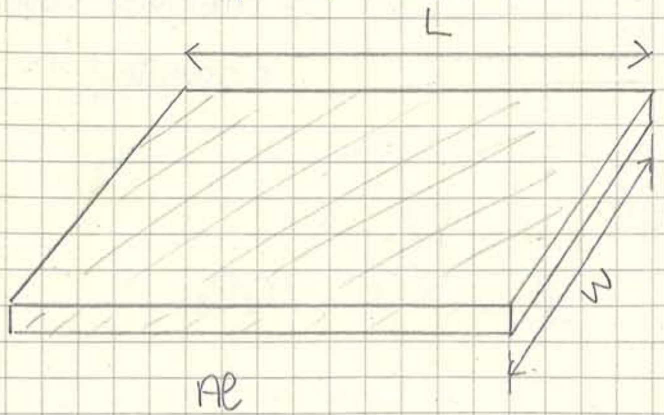
$C = \varepsilon \cdot \frac{A}{d}$

$\varepsilon_{r\,AIR} \simeq 1$

$\varepsilon_{r\,PLAST} \simeq 2 \div 2{,}5$ (depends on plastic)

(water has a big dielectric constant.) There are some ceramic materials that have a big value of ε_r.

Let's take a foil of aluminium :



If we use this foil for doing capacitor, we compute the area as : W·L.

we discover that it is low

How can we have more area? We can skretch it.

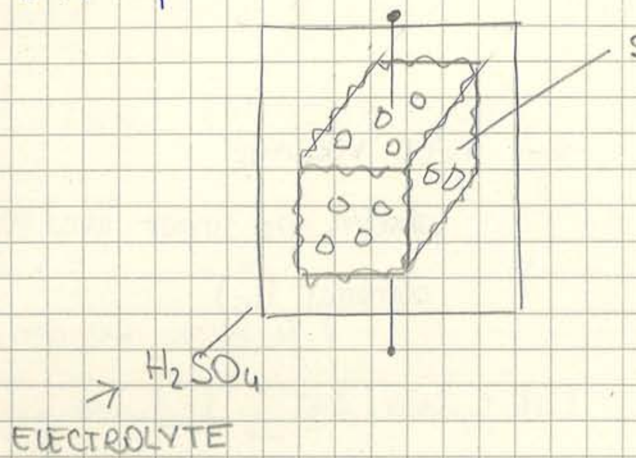
Why electr. cap has a positive and negative sign?

$Al_2^+ O_3^-$ → The down side must be positive and the other side negative

↳ In order to have this we want that the oxygen must be on the Aluminum oxide.

If we reverse the polarity the cap is destroyed.

Tantalum caps are similare:



The area is ~ $1 m^2$ (large area in a small surface)

The oxide is Ta_2O_5

But with this cap we can't reach ~~ove~~ voltage over 30 ÷ 35 V.

$v(t) = V_{PK} \cos(\omega t)$
$\hookrightarrow 2\pi \cdot 50Hz$

i_c is a periodic waveform and we can use Fourier series:

$$i_t = \sum_{n=1}^{\infty} I_{n_{PK}} \cos(n\omega t + \varphi_n)$$ ← INPUT CURRENT

$$I^2_{RMS} = \frac{1}{T}\int_0^T i(t)^2\, dt$$
$\hookrightarrow$ we measure with the scope

OR:

$$I^2_{RMS} = \frac{1}{T}\int_0^T \left(\sum_1^{\infty} I_{n_{PK}} \cos(n\omega t + \varphi_m)\right)^2 dt$$

We have $(\omega t\ 2\omega t\ 3\omega t \dots) \times (\omega t\ 2\omega t\ 3\omega t \dots)$

$$\left((\cos(2\omega t))^2 = \frac{1+\cos 4\omega t}{2} \right)$$

So we can write it:

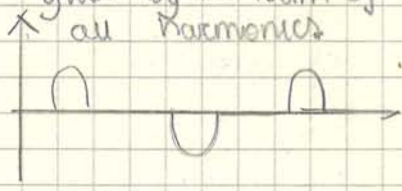
$$= \frac{1}{T}\int_0^T \sum \cos(n\omega t + \varphi_n) + \frac{1}{2}(\dots)$$

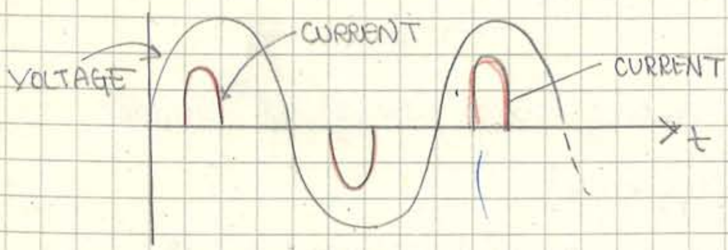
So now swapping:

$$\sum \frac{1}{T}\int_0^T \left(\underbrace{I_j I_k \cancel{\cos(n\omega t)}}_{=0} + \frac{1}{2} I^2_{n_{PK}} \right) dt$$

$$I^2_{RMS} = \sum \frac{1}{T}\int_0^T \underbrace{\frac{1}{2} I^2_{n_{PK}}}_{\text{CONSTANT}} dt = \sum_{n=1}^{\infty} \frac{1}{2} I^2_{n_{PK}}$$

the RMS current is given by the sum of all harmonics





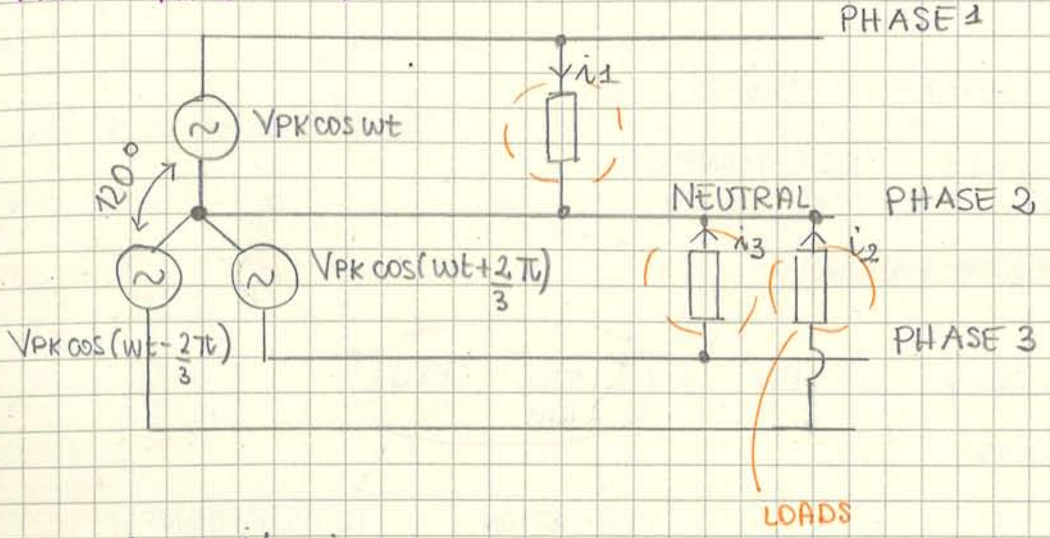
$$\begin{cases} V_{PK}\cos\omega t = v \\ \sum I_{n_{PK}} \cos(n\omega t + \varphi_n) = i \end{cases}$$
$\hookrightarrow$ we can find the average power

AVERAGE POWER

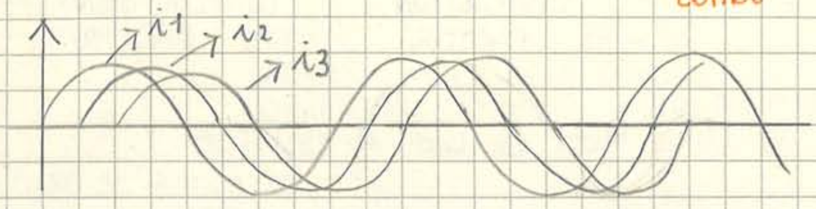
$$\bar{P} = \frac{1}{T}\int_0^T v \cdot i\, dt = \frac{1}{T}\int_0^T V_{PK}\cos\omega t \cdot \sum I_{n_{PK}} \cos(n\omega t + \varphi_n)\, dt$$

$$= \frac{1}{T} V_{PK} \int_0^T \cos\omega t \sum_1^{\infty} I_{n_{PK}} \cos(n\omega t + \varphi_n)\, dt =$$

THREE-PHASE SYSTEM

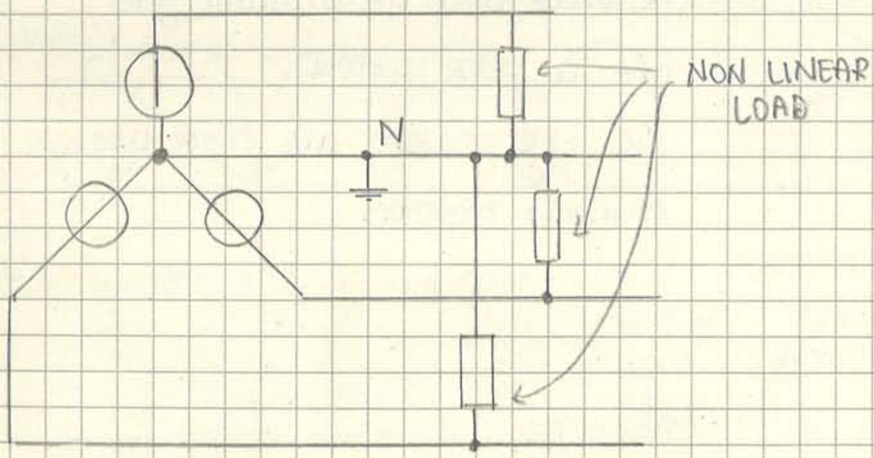


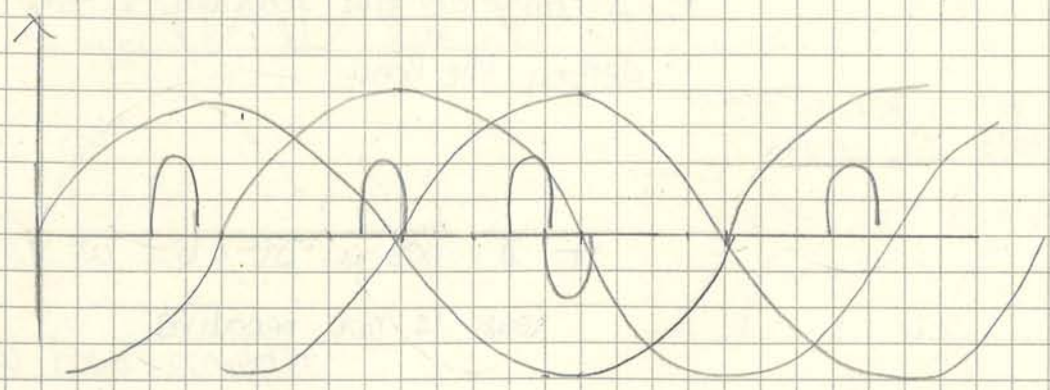
we see only 1 phase



If we sum these currents they cancelled out (in the node of neutral)

What happens if we have a non-linear load?



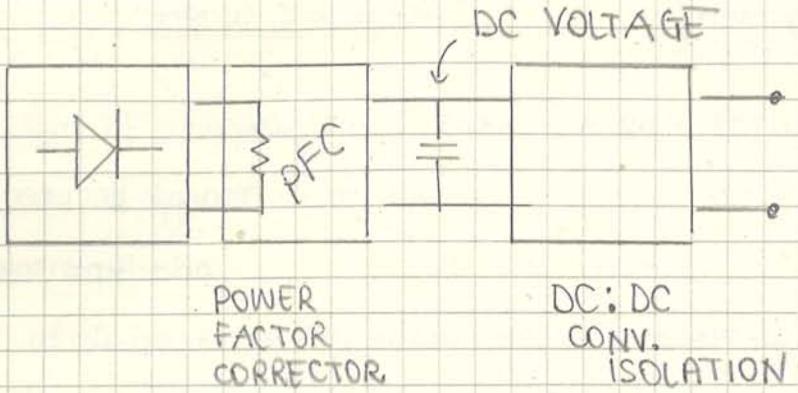


Currents are out of phase and they don't cancel out.

1) The voltage is not a DC voltage
2) It's not isolated.

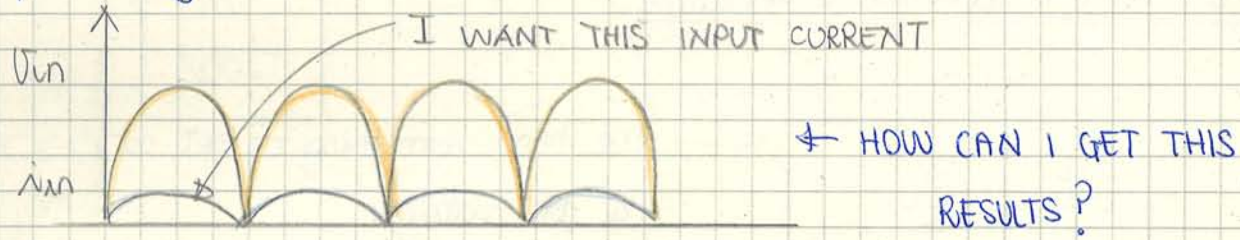
But if I put the load, it's not generates harmonics.

We need to have a DC voltage and we need to introduce an isolation.

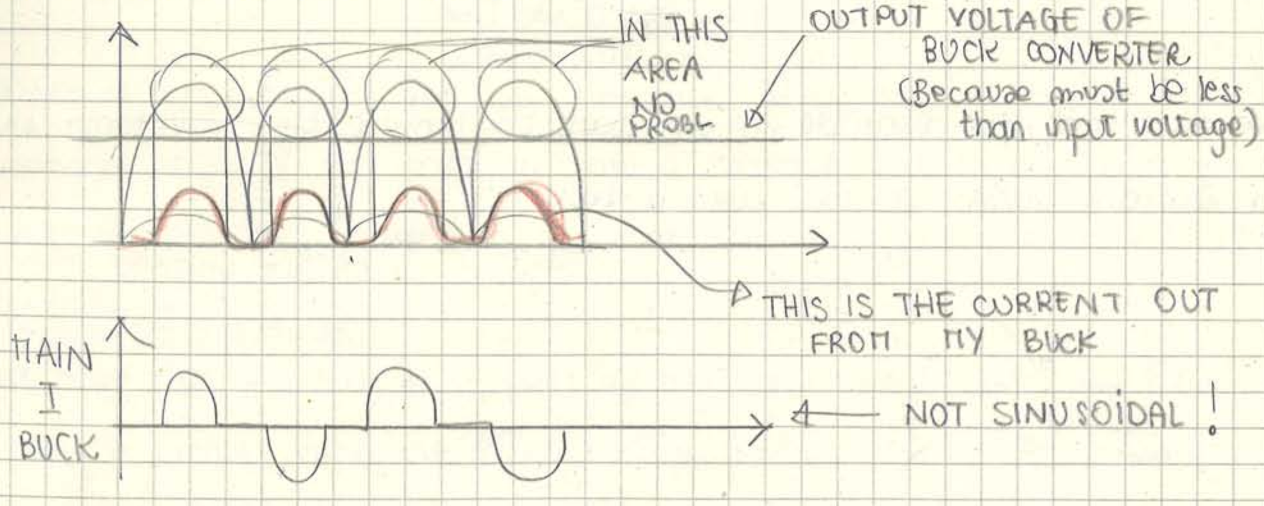


Now we're looking to PFC.

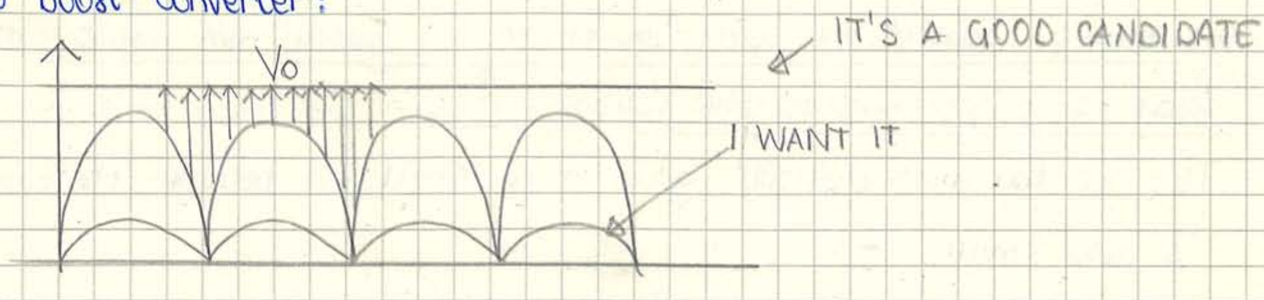
Input voltage to PFC is:



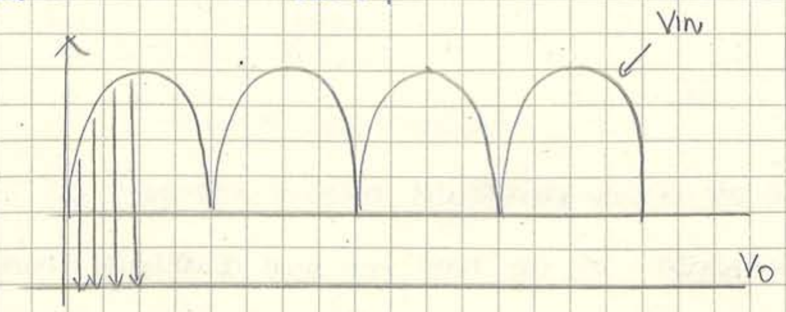
Let's consider a buck converter:



Now boost converter:

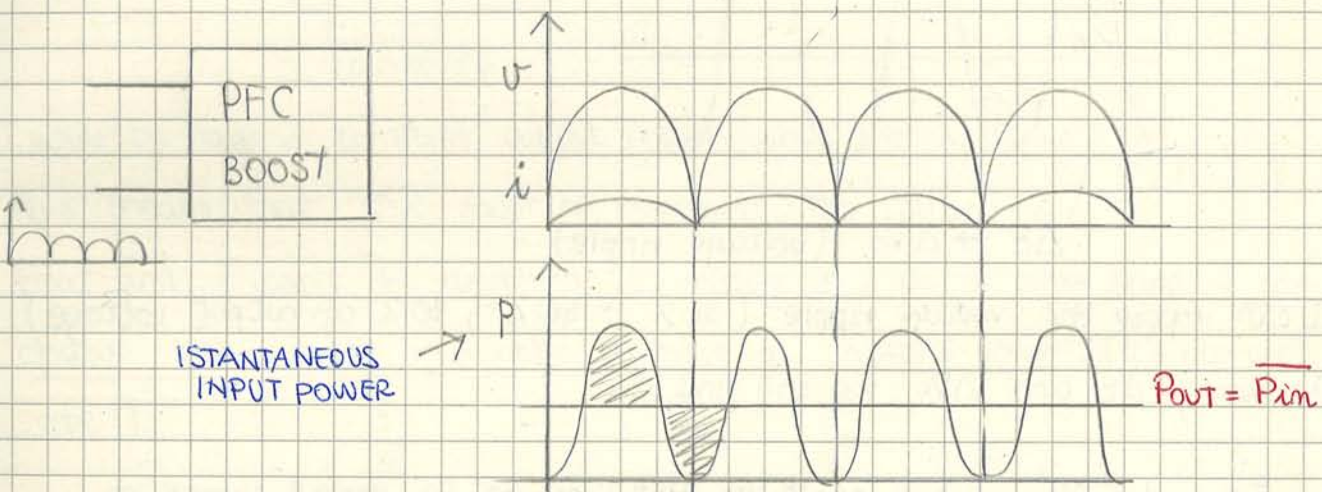


And what about buck-boost?



It is also a good candidate for PFC.

The problem of boost is that Vo is very high. BB hasn't this problem; than it is ok. (we change only the reference). What is the problem of BB as PFC?
BB has a pulsating input current that generates noise. So in general we prefer boost, and if the voltage is too much high we work in CCM because we have less stress.



power is not constant! Too bad. Because after PCF we have a DC-DC converter, and if we want to have a constant voltage the input power must be constant! So we can say that the output power is equal to the average of input power.

We have that for the first part of the cycle there is some ~~som~~ power that is greater than the average and in the second part is smaller. But, the 2 areas are equals.

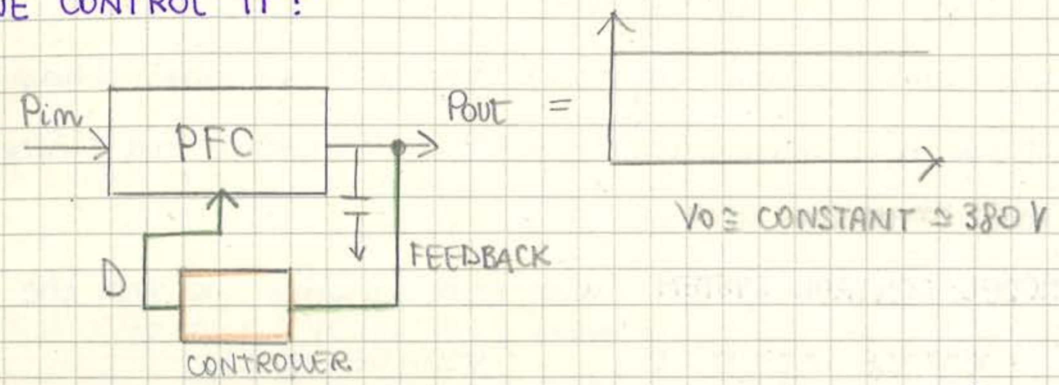
How can we store energy? CAPACITOR (Co) that is used to store energy for the first part and then releases in the second part of the cycle.

$$= \frac{1}{4f_{LINE}} \cdot P_{OUT} \cdot \underbrace{\frac{2}{\pi}}_{\text{SINUSOIDAL FUNCTION}} = \frac{P_{OUT}}{2\pi f_{LINE}}$$ → And then we find Co

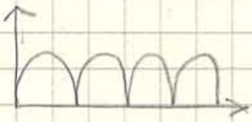
Other way to solve it is to say that cap has a reactance and :

$$V_{RIPPLE} = \frac{I_{RIPPLE}}{j\underbrace{\omega}_{2\pi \cdot f_{SW}} C}$$

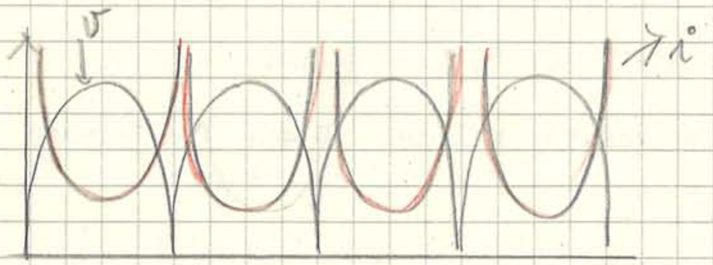
HOW WE CONTROL IT ?



I want to have a constant output power when the input is



This means that $v \cdot i$ must be constant. The voltage is that and I can't do anything to modify it. On the other hand, I can change the current in order to have a constant power. (It's the opposite!)



So, I need that the output voltage has some ripple because this ripple makes the difference between a PFC and a standard DC-DC converter. How can I control the output voltage but not the ripple? Filter. (L.P. FILTER)
The bandwidth of the loop gain must be narrow otherwise I correct the output ripple. So the crossover frequency is very small (~10 Hz)
(Boost has a RHP and so we have to cross before).

If I need to design this compensator (the transfer function of circuit) what do I need? I need to know the dynamic relationship between V_o and D. it's not $V_o = V_{in} \cdot D$ this is for steady state!

I need to know the transfer function! In order to get a stable system.

I need Laplace tranform $\frac{V_o}{D}$ in this domain!

Our converter has switches, so it's time variant circuit (the schematic changes). And the relationship is not linear. ⚠

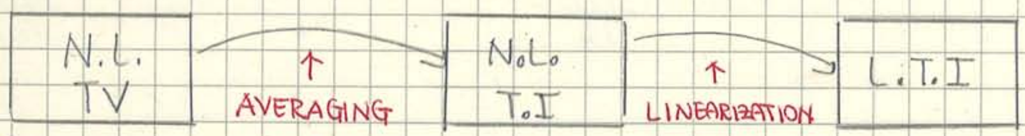
How can we find transfer function?

- STATE SPACE AVERAGING (MIDDLEBROOK) - CCM - eq. differenziali

- SWITCH AVERAGING (VORPERIAN) easier to apply for many topologies.

- CIRCUIT AVERAGING it's not a method, even if is very easy to apply

Why averaging?



we start from this point

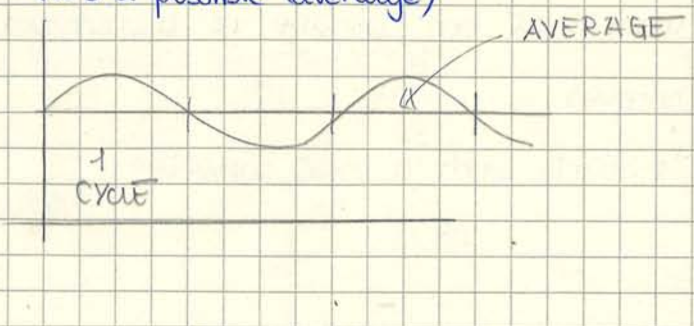
Averaging changes from method to method (applying an approximation)

For example we can have:

$$\overline{x(t)} = \frac{1}{T_{SW}} \int_{t}^{t+T_{SW}} x(\tau)\, d\tau$$ → MOVING AVERAGE

time continous average

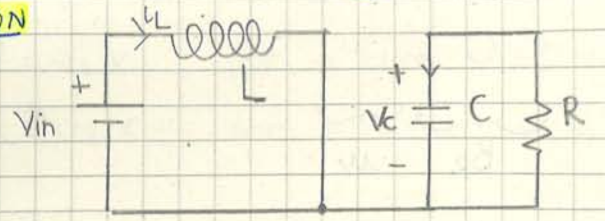
(it's a possible average)



In CCM we have: MIDDLEBROOK

$\boxed{\text{BOOST}}$

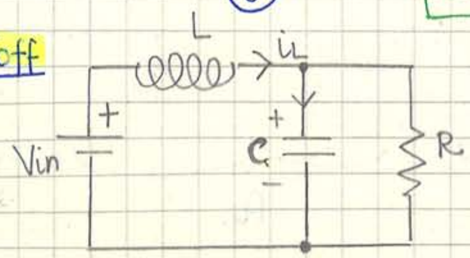
1 Ton



SW ON
DIODE OFF

$T_{on} = D \cdot T_{sw}$

2 Toff



SW OFF
DIODE ON

$T_{off} = (1-D)T_{sw}$

Both of them are linear and time invariant.

For describing these circuits we can use differential equation. The basic equations are:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

STATE, INPUT, MATRIX, OUTPUT

The state variables are i_L and V_C (that is the output voltage across R)

1

$$\begin{cases} L\dfrac{di_L}{dt} = V_{in} \\ C\dfrac{dV_C}{dt} = -\dfrac{V_C}{R} \end{cases}$$

2

$$\begin{cases} L\dfrac{di_L}{dt} = V_{in} - V_C \\ C\dfrac{dV_C}{dt} = i_L - \dfrac{V_C}{R} \end{cases}$$

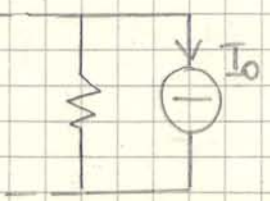
$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{V_{in}}{L} \\ \dfrac{dV_C}{dt} = -\dfrac{V_C}{RC} \end{cases}$$

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{V_{in} - V_C}{L} \\ \dfrac{dV_C}{dt} = \dfrac{i_L}{C} - \dfrac{V_C}{RC} \end{cases}$$

→ From them we have to write down matrixes

1
$$\underbrace{\begin{bmatrix} \dot{i}_L \\ \dot{V}_C \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -1/RC \end{bmatrix}}_{A_1} \underbrace{\begin{bmatrix} i_L \\ V_C \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 1/L \\ 0 \end{bmatrix}}_{B_1} \underbrace{V_{in}}_{u}$$

NB: If I have on the output:



I_o is an input!

Let's consider (for a moment) if our result is resonable or not. For example ~~the~~ let's consider a simple system:

$\dot{x} = a_1 x$ for a time $D \cdot T_{SW}$

$\dot{x} = a_2 x$ for a time $(1-D) T_{SW}$

SOLUTION: $\begin{cases} x(t) = x(0) e^{at} \\ \dot{x} = ax \end{cases}$

Solving:

1) $x(DT_{SW}) = x(0) e^{a_1 D T_{SW}}$

2) $x(T_{SW}) = \underbrace{x(0) e^{a_1 D T_{SW}}}_{\text{INIT. COND.}} \cdot e^{a_2 (1-D) T_{SW}} = x(0) e^{\overbrace{[a_1 D + a_2(1-D)]}^{a} T_{SW}}$

$(1-D)$

a is the reciprocal of time constant; it's an avenage like that we found first! So, is it true? Does it work? No, it's wrong. Why? Solution must be something like:

$$e^{A_1 D T_{SW}} \cdot e^{A_2 (1-D) T_{SW}} \neq \cancel{e^{(A_1 D + A_2 (1-D) T_{SW}}}$$

IT'S VALID ONLY FOR SCALAR, NOT FOR MATRIXES!

But in this case it works. Let's consider:

$$e^{A_1 D T_{SW}} = I + A_1 D T_{SW} + \frac{1}{2}(A_1^2 D^2 T_{SW}^2) + \ldots$$

$$e^{A_2(1-D) T_{SW}} = I + A_2 (1-D) T_{SW} + \frac{1}{2} A_2^2 (1-D)^2 T_{SW}^2 + \ldots$$

If we multiply these two:

$$e^{(A_1 D + A_2(1-D)) T_{SW}} = I + A_1 D T_{SW} + A_2 (1-D) T_{SW} + \cancel{A_1 A_2 D (1-D) T_{SW}^2}$$

Remember our assumption $\tau \gg T_{SW}$; in our expression we have time constants, so we can neglect the other terms (we have T_{SW}^2 etc.. etc..). So:

$$\boxed{e^{(A_1 D + A_2 (1-D)) T_{SW}} \simeq I + A_1 D T_{SW} + A_2 (1-D) T_{SW}} \leftarrow \text{IT'S COMMUTATIVE}$$

↑ IT'S A GOOD APPROXIMATION.

$$\begin{cases} \dfrac{d\bar{i}_L(t)}{dt} = \dfrac{d-1}{L}\bar{v}_C(t) + \dfrac{\bar{V}_{in}}{L} \\ \dfrac{d\hat{v}_C(t)}{dt} = \dfrac{1-d}{C}\bar{i}_L(t) - \dfrac{\bar{V}_C}{RC} \end{cases} \rightarrow \text{NON LINEAR}$$

What is advantage? Matlab can solve it very faster because it's time continous differential equation. We can't put it in spice. We can't apply LaPlace transf. (not in easy way); so we have to linearize it considering small variations around all the variables. How can we do it?

SMALL SIGNAL MODEL:

- Taylor series
- Different approach; for each part:

$$\bar{i}_L = I_L + \hat{i}_L \qquad d = D + \hat{d} \qquad \bar{v}_C = V_C + \hat{v}_C$$

$$\bar{V}_{in} = V_{in} + \hat{v}_{in}$$

TOTAL, DC VALUE, VAR.

$$\begin{cases} \dfrac{d(I_L+\hat{i}_L)}{dt} = \dfrac{D+\hat{d}-1}{L}(V_C+\hat{v}_C) + \dfrac{V_{in}+\hat{v}_{in}}{L} \\ \dfrac{d(V_C+\hat{v}_C)}{dt} = \dfrac{1-D-\hat{d}}{C}(I_L+\hat{i}_L) - \dfrac{(V_C+\hat{v}_C)}{RC} \end{cases} \quad \text{NON LINEAR}$$

$i_L = I_L + \hat{i}_L$ if $\hat{i}_L << I_L$ SMALL SIGNAL CONDITIONS

When I multiply 2 terms of small signal, I can neglect them.
For example $\hat{i}_L \cdot \hat{d}$ (2^{nd} order terms)

Let's write DC terms:

$$\begin{cases} \dfrac{dI_L}{dt} = \dfrac{(D-1)}{L}V_C + \dfrac{V_{in}}{L} = 0 \rightarrow V_C = \dfrac{V_{in}}{1-D} \rightarrow V_{in} = V_C(1-D) \\ \dfrac{dV_C}{dt} = \dfrac{(1-D)}{C}I_L - \dfrac{V_C}{RC} = 0 \rightarrow \dfrac{V_C}{R} = I_L(1-D) \end{cases}$$

$$\frac{V_C}{R(1-D)} = I_L$$

$$I_L = \frac{V_{in}}{R(1-D)^2}$$

STEADY STATE SOLUTION (ONLY FOR CHECKING)

I_L and V_C are constants

How can we solve it? LaPlace transform ($\mathcal{L}$):

$$\begin{cases} s\hat{i}_L(s) = \hat{v}_C(s)\dfrac{D-1}{L} + \hat{d}\dfrac{V_C}{L} \\ s\hat{v}_C(s) = \hat{i}_L(s)\cdot\dfrac{1-D}{C} - \dfrac{\hat{v}_C(s)}{RC} - \hat{d}(s)\dfrac{I_L}{C} \end{cases}$$

$$\frac{\hat{v}_C(s)}{\hat{d}(s)} = ?$$

NB: we don't need initial condition because if we have to find transfer function we don't need it.

For exercise let's check dimension.

Substituting 1^{st} in the 2^{nd} equation:

$$s\hat{v}_C = \frac{1-D}{C}\left[\hat{v}_C(s)\frac{D-1}{sL} + \frac{\hat{d}V_C}{sL}\right] - \frac{\hat{v}_C}{RC}(s) - \hat{d}(s)\frac{I_L}{C}$$

with: $V_C = \dfrac{V_{in}}{1-D}$ and $I_L = \dfrac{V_{in}}{R(1-D)^2}$

We obtain:

$$s\hat{v}_C = \hat{v}_C\left(-\frac{(1-D)^2}{sLC} - \frac{1}{RC}\right) + \hat{d}\left(\frac{V_{in}\,\hat{d}}{\cancel{(1-D)}sLC} - \frac{V_{in}}{RC(1-D)^2}\right)$$

Dividing and multip. for SLC:

$$\hat{s}LC\hat{v}_C = \hat{v}_C\left(-(1-D)^2 - \frac{sL}{R}\right) + \hat{d}\left(V_{in} - \frac{sL}{R}\frac{V_{in}}{(1-D)^2}\right)$$

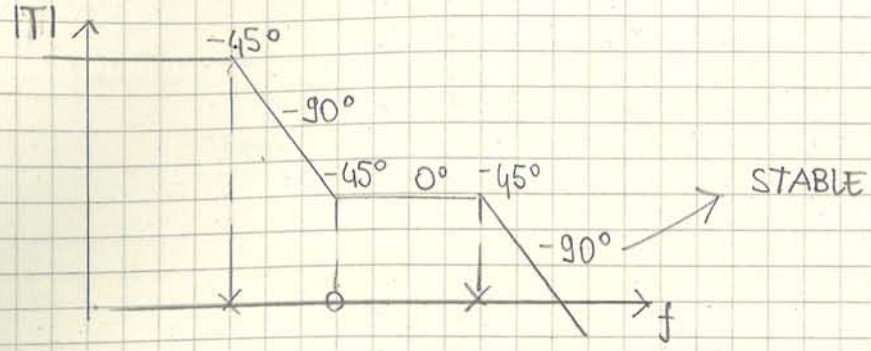
$$\hat{v}_C\left(s^2LC + (1-D)^2 + \frac{sL}{R}\cdot\frac{1}{(1-D)^2}\right)$$

$$\frac{\hat{v}_C}{\hat{d}} = \frac{V_{in}}{(1-D)^2}\cdot\frac{1 - \frac{sL}{R}\cdot\frac{1}{(1-D)^2}}{\frac{s^2LC}{(1-D)^2} + s\frac{L}{R(1-D)^2} + 1}$$

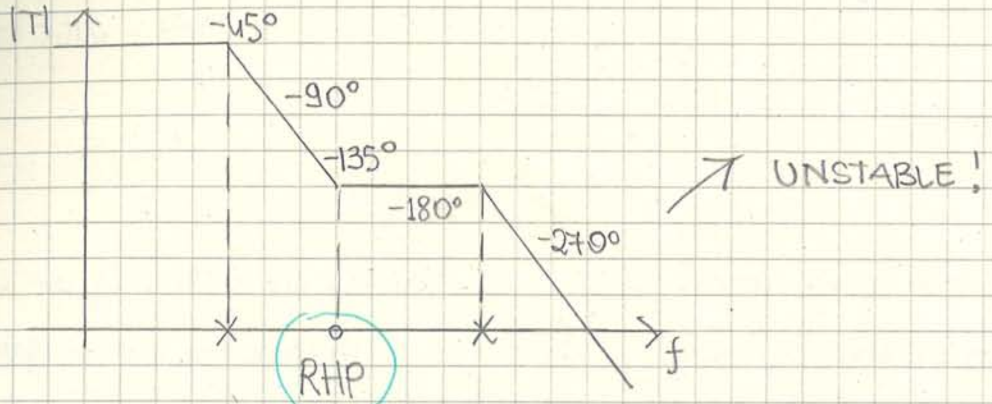
⚠ SON OF A BOOST!

2 poles $f_p = \dfrac{1-D}{2\pi\sqrt{LC}}$

Loop gain |T| is:



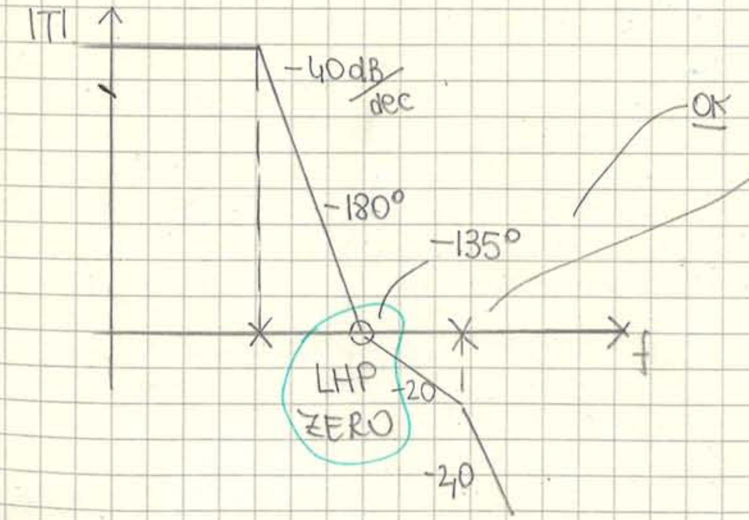
Now let's suppose that is an extra zero: (that is RHP zero)



Bad idea: putting a LHP in order to cancel RHP. It doesn't work! It works only for high precision system.

The only way to get it stable it's crossover before RHP. So we have to look for the minimum of this frequency f_z and crossover before. It happens for boost, buck-boost converter... Boost is used as PFC; in the PFC my bandwidth has to be less than the line frequency. For example I have to crossover at 10Hz maximum.

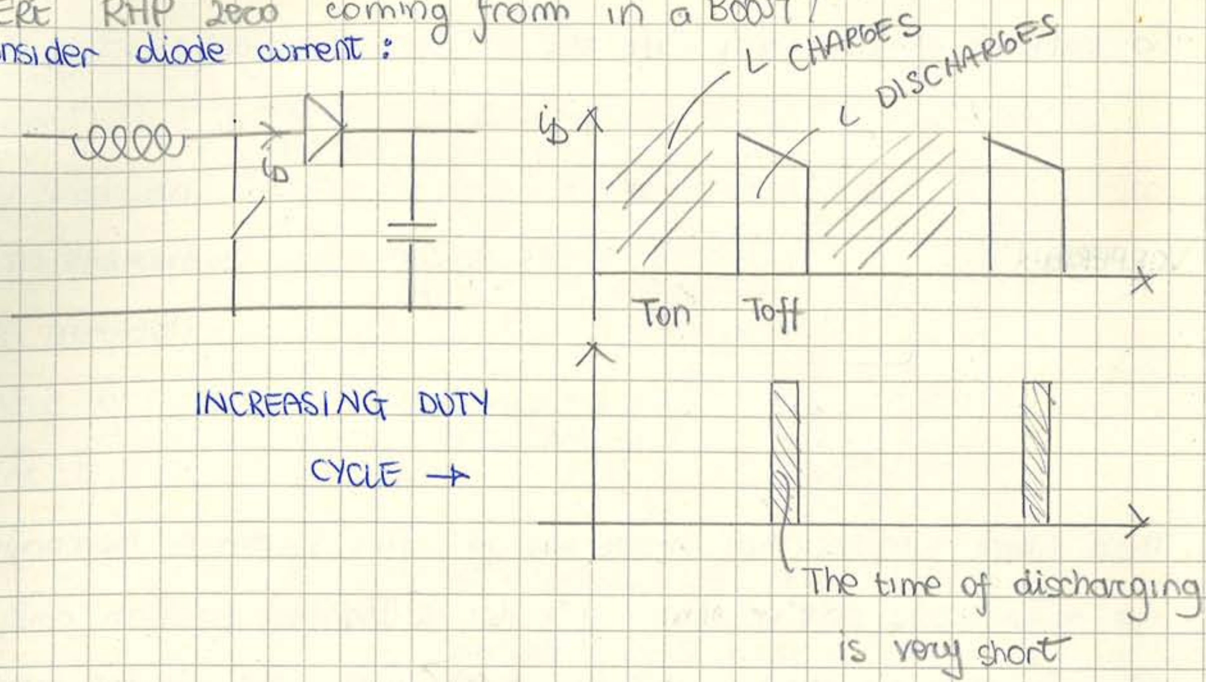
Let's consider another transfer function:



If this pole is not too far from crossover frequency. This pole decrease my phase margin → UNSTABLE

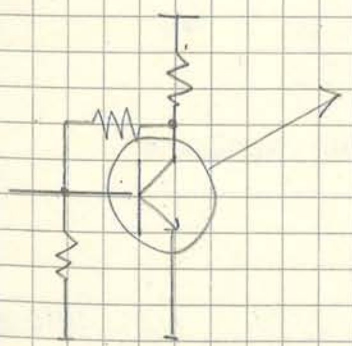
WHERE RHP ZERO coming from in a BOOST?

Let's consider diode current:



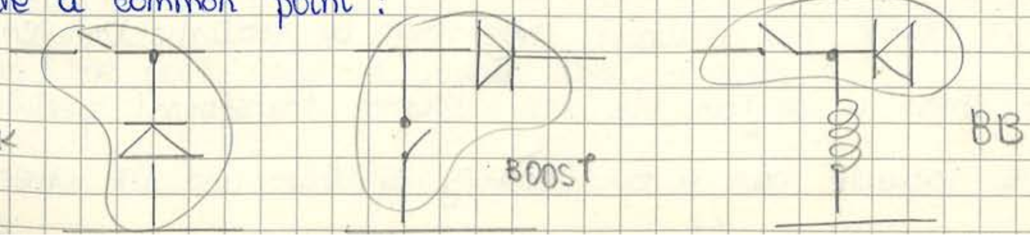
Let's change method to analyze our circuit in order to find transfer function.
With the previous method we have to write down the differential equations for all part of the circuit. and then we have to average it.
we have to write down diff. equat. for switches that are non-linear and we to stabilize them.

For example if we have this circuit:



We do the same now with switches.

In all topologies we find the same structure of switch and diode that have a common point:



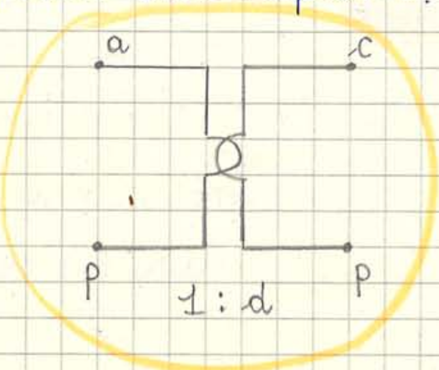
frequency spectrum they are stat. independent. In our case ic doesn't change too much (is the current through the inductor). So we can do this approximation. During 1 cycle q changes and v_{ap} and i_c not too much! → Reasonable approximation for our circuit.

So:

$$\begin{cases} \overline{v_{cp}} = \overline{v_{ap}} \cdot \overline{q} \\ \overline{i_a} = \overline{i_c} \cdot \overline{q} \end{cases} \qquad \begin{cases} \overline{v_{cp}} = \overline{v_{ap}} \cdot d(t) \\ \overline{i_a} = \overline{i_c} \cdot d(t) \end{cases} \qquad (\overline{q} = d)$$

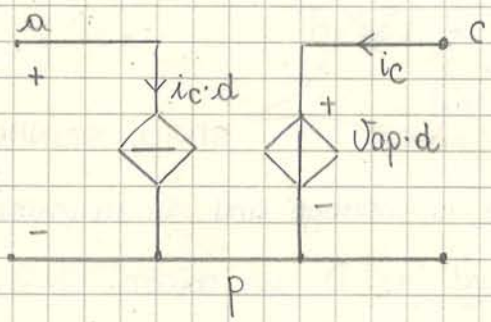
↳ NON LINEAR EQUATIONS, $\overline{i_c}$ AND $\overline{v_{ap}}$ CHANGE! (AND ALSO d)

what does represent these equations? → TRANSFORMER



It's a strange transformer because the turn ratio d changes! (And it also transform the DC)

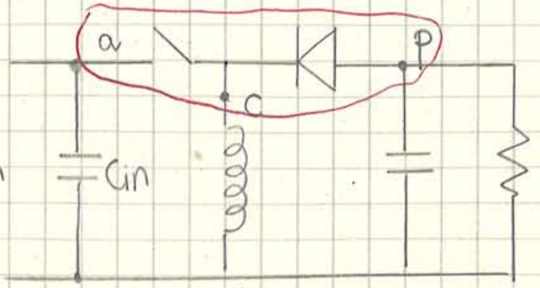
So we can model our tripole with a transformer; so it's easier doing computation with it. Better:



MODEL OF AN IDEAL TRANSF. that we can put in spice and simulate it faster than having switch and diode.

This is still NON LINEAR; but this isn't a problem for spice.
But it has no changes because there aren't switches. → TIME INVARIANT.
Now we have to linearize it. What are advantages and disadvanges?
We can use LaPlace transform. But we lost generality because we study only one point and a small area around it.

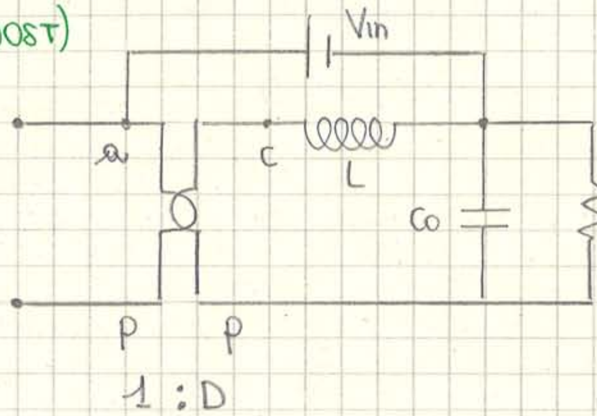
Let's see buck-boost :



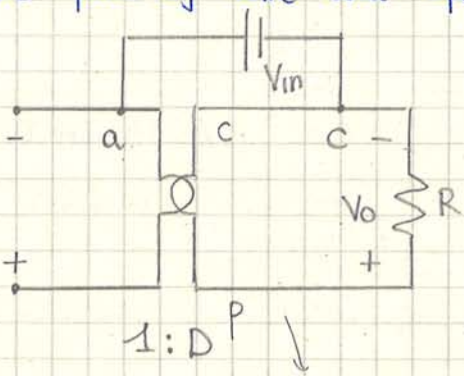
$M = \frac{V_o}{V_i}$ CCM ?

Substituting this tripole with transformer :

- DC (BUCK BOOST)



But L and C are piece of wire and open circuit. (in DC) :



$\frac{V_o}{D}$

using transf. equation

NB: the output is on p

$\frac{V_o}{D} + V_{in} = V_o$

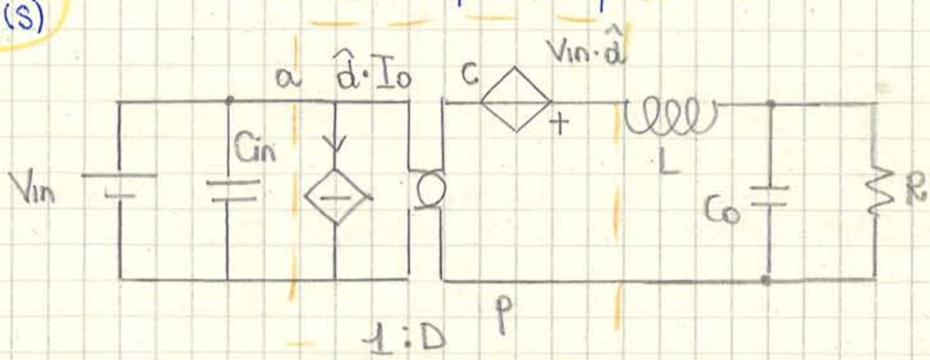
$V_{in} = V_o - \frac{V_o}{D} =$

$V_{in} = V_o \left(1 - \frac{1}{D}\right)$

$\frac{V_o}{V_{in}} = \frac{D}{D-1}$ → OK !

- AC (BUCK)

$\frac{\hat{v}_o(s)}{\hat{d}(s)}$ ← this is the important point



So, removing bias voltage sources :

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We haven't RHP zero ! This is the reason why we design it in CCM

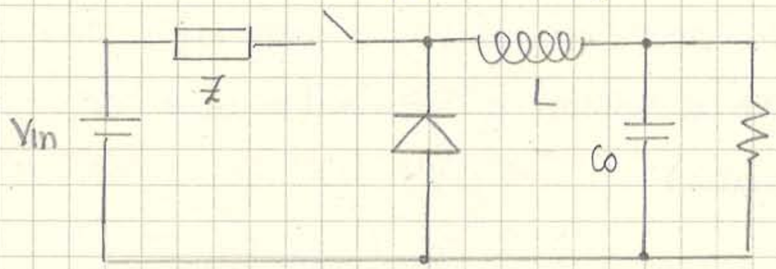
If we put Co ESR the mainly effect is :

$$\frac{\hat{V}_o(s)}{\hat{d}(s)} = V_{in} \frac{1 + s \cdot C \cdot ESR}{\underbrace{s^2 LC + s\frac{L}{R} + 1}}$$

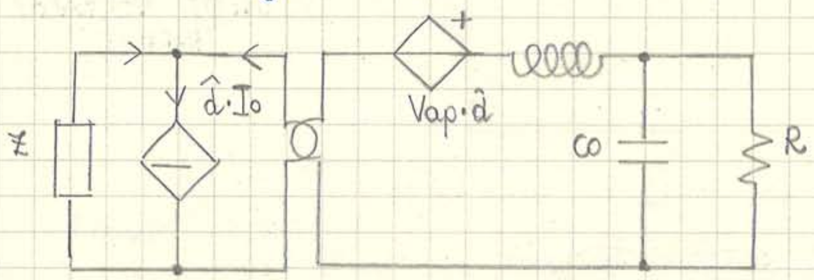
↳ Actually we have ESR is these term, but it's negligible ; so we MUST neglect them!

Bad news.

I assume an ideal voltage. Actually :



If I look small signal model :



$d\uparrow \rightarrow \hat{V}_o\uparrow$

Using superposition and removing $V_{ap} \cdot \hat{d}$, when $\hat{d}$, the current sinked is much more. The current comes from source and the primary ; so on the secondary I have current flowing to the left and V_o will decrease.

So :
- $V_{ap} \cdot \hat{d} \rightarrow d\uparrow, V_o\uparrow$
- $\hat{d} \cdot I_o \rightarrow d\uparrow, V_o\downarrow$

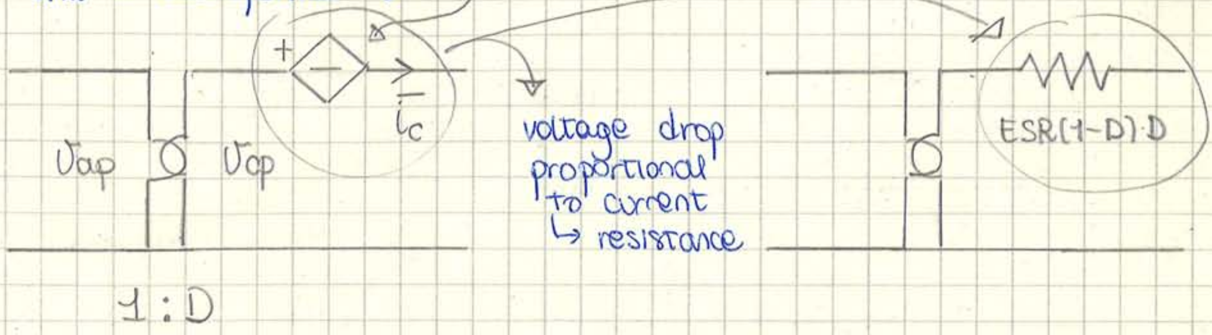
} SUPER POSITION

~~At high frequency It could happen that both of them are working~~

It could happen that $\hat{d} \cdot I_o$ works at high frequency and $V_{ap} \cdot \hat{d}$ at low frequency ; so I have positive gain and negative gain. It means that there is a RHP zero. It could happen.

$\bar{V}_{cp} = \bar{V}_{ap} \cdot D \underbrace{- (1-D)\, ESR \cdot \bar{i}_c \cdot D}_{\text{EXTRA TERM}}$ ← what is an electrical representation of this equation?

This is still a transformer :

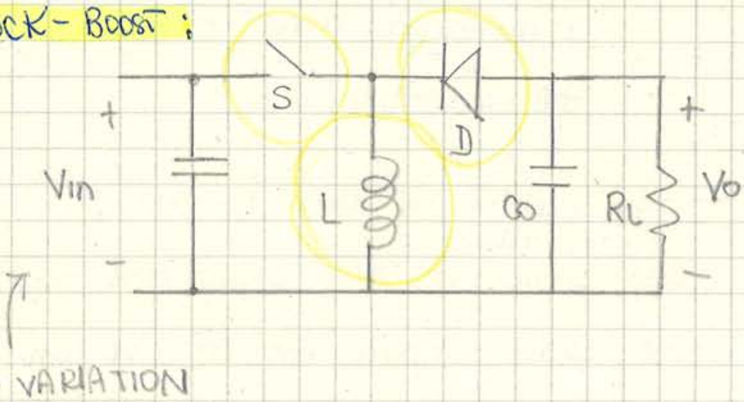


Resistance decrease Q factor of L-C circuit. who cares? Electrolytic caps have a toll of ±20% and ESR isn't a very precise number and it goes up when cap becomes old. Also the inductor has ±20%; and its value decreases as DC current decreases (we are close to saturation). So this change of Q is negligible.

Averaging DCM converter : CIRCUIT AVERAGE

As first study we use the most important topology used in DCM:

BUCK-BOOST:



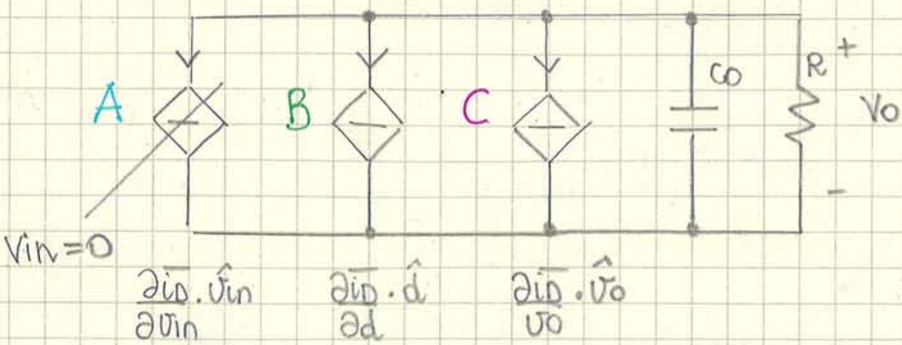
What happens for each component when it is averaged?

What is average of C → a capacitor! Linear; time invariant.

// // // L → an inductor!

// // // RL → is still output load!

And the diode and switch? This method is not systematic like previous method that we have seen; we need to substitute this diode (that is a switch on and off) with something that has a voltage across it and current through it. So it can be either a control current source or it could a control voltage source.



← LINEAR; so I can use superposition (Assuming v_{in} constant because I have to find out $\frac{\hat{v}_o(s)}{\hat{d}(s)}$)

A) $\frac{\partial \bar{i_D}}{\partial v_{in}} = A = -\frac{\cancel{2} v_{in} \cdot d^2}{\cancel{2} f_{sw} L v_o} = -\frac{v_{in}\, d^2}{f_{sw} L v_o}$ (A.S.)

B) $\frac{\partial \bar{i_D}}{\partial d} \doteq B = -\frac{2 v_{in}^2 \cdot d}{2 f_{sw} L v_o} = -\frac{v_{in}^2 d}{f_{sw} L v_o}$ (GAIN) B

C) $\frac{\partial \bar{i_D}}{\partial v_o} = C = \frac{v_{in}^2 \cdot d^2}{2 f_{sw} L v_o^2}$

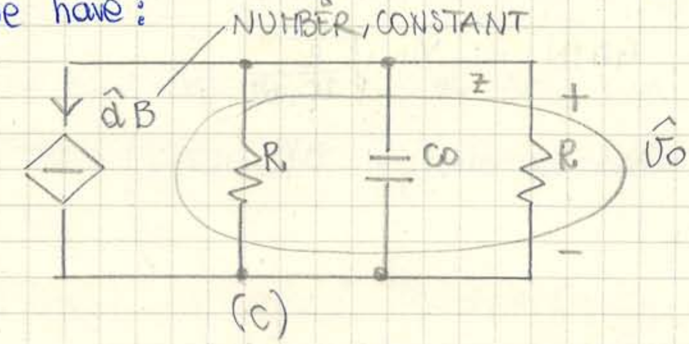
Remember that: $\frac{V_o}{V_{in}} = -D\sqrt{\frac{R}{2 f_{sw} L}}$ (FOUND WITH ENERGY)

$$\left(\frac{V_{in}}{V_o}\right)^2 = \frac{2 f_{sw} L}{D^2 R}$$

And so I can write:

$\frac{\partial \bar{i_D}}{\partial v_o} = \frac{\cancel{2 f_{sw} L\, d^2}}{\cancel{2 f_{sw} L d^2} R} = \frac{1}{R}$ (Actually is negative but the current in the load is coming down and it's ok)

So we have:



This is linear. I'm looking for $\hat{v}_o$. We can use Laplace transf.

$\hat{v}_o(s) = -\hat{d}(s) \cdot B \cdot Z$

$$\boxed{\frac{\hat{v}_o(s)}{\hat{d}(s)} = -B \cdot Z}$$

↳ let's see Bode function.

The pole frequency changes ! $\dfrac{1}{2\pi C\left(ESR + \frac{R}{2}\right)}$ → NEGLIGIBLE

So the pole frequency doesn't change too much.

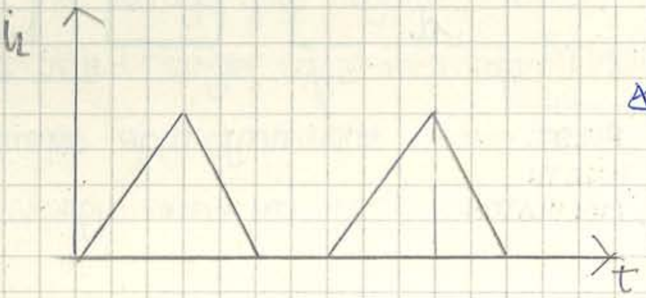
On the other hand, if R becomes lighter, the pole moves to the left. AND GAIN ↑

What happens to gain? It changes with Vin and R.

The zero is fixed but we don't know where it is. (And it moves because C becomes old.)

Maybe there is something wrong. We have L and C but we have found only one pole (that related to capacitor). We "lost" the other one. When we have a pole it means that is an element storing energy; it has memory; and I describe it with differential equation..

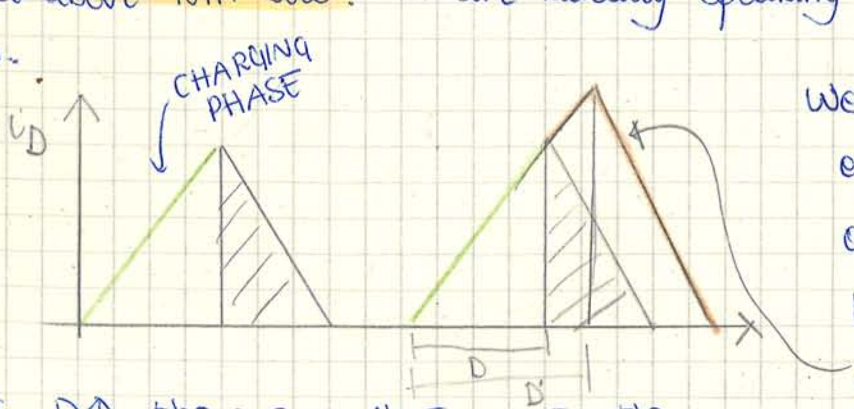
Let's consider inductor current in time domain:



It has memory, but it's limited to stay in 1 cycle. This memory is cancel before the end of the ~~con~~ cycle.

This means that we have a pole; but it's a fast pole and it is above $\frac{1}{2}$ of fsw and we can't describe it. And we can neglect it because we work far below $\frac{f_{sw}}{2}$. For our issue all convert. working in DCM have only one pole.

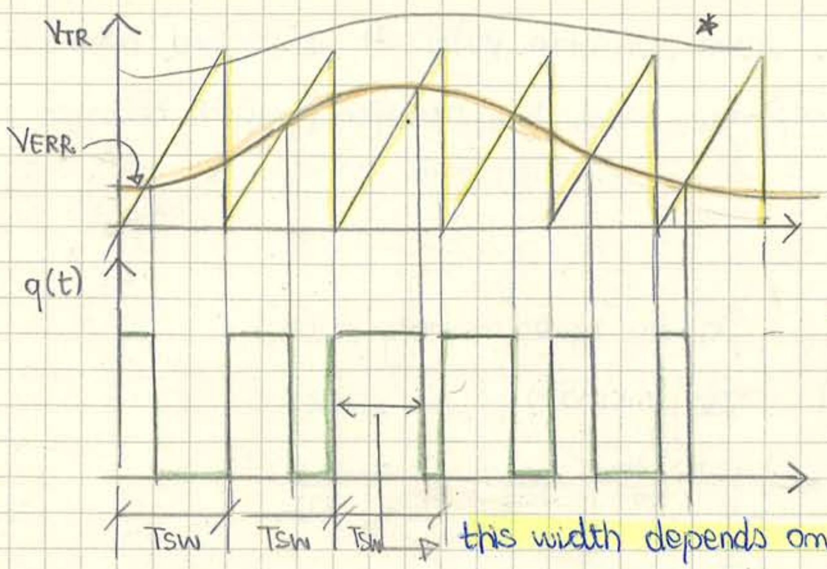
What about RHP zero? mathematically speaking this topology has RHP zero.



We are int. in i_D because energy goes to the output.

If we increase D, input current goes higher

So, if D↑, the energy that goes to the load increases because peak current goes up.



SINGLE EDGE MODULATION

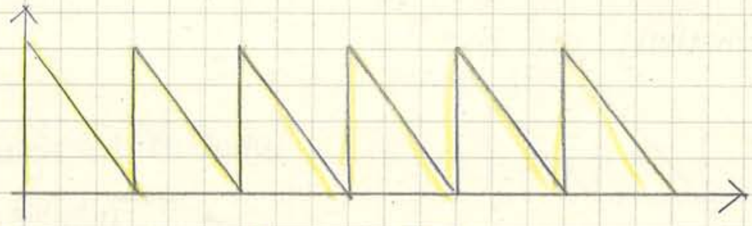
this width depends on how much is higher the voltage error ($V_{ERR}\uparrow$, WIDTH $\uparrow$)

PERIOD IS ALWAYS TSW (CONSTANT TIME)

(In each cycle the switch is on and off)

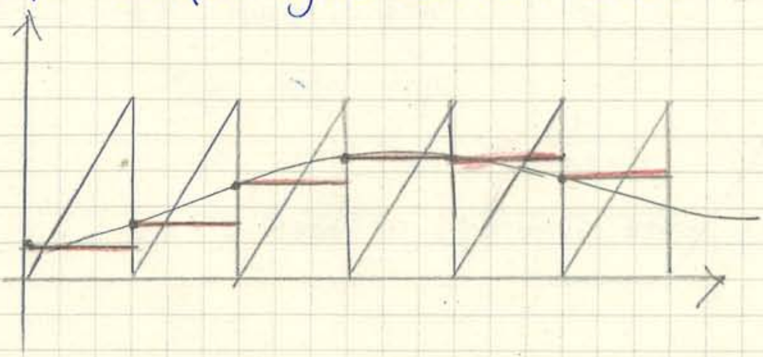
What happens if Vin goes above the peak? * D goes to 1.
And the voltage goes negative? Below the zero? D becomes 0.
} SATURATION

It is called single edge modulation because the time is periodic and the end of this pulses is modulated by the amplitude of the error signal.
There is another way to use sawtooth; for example in this way:



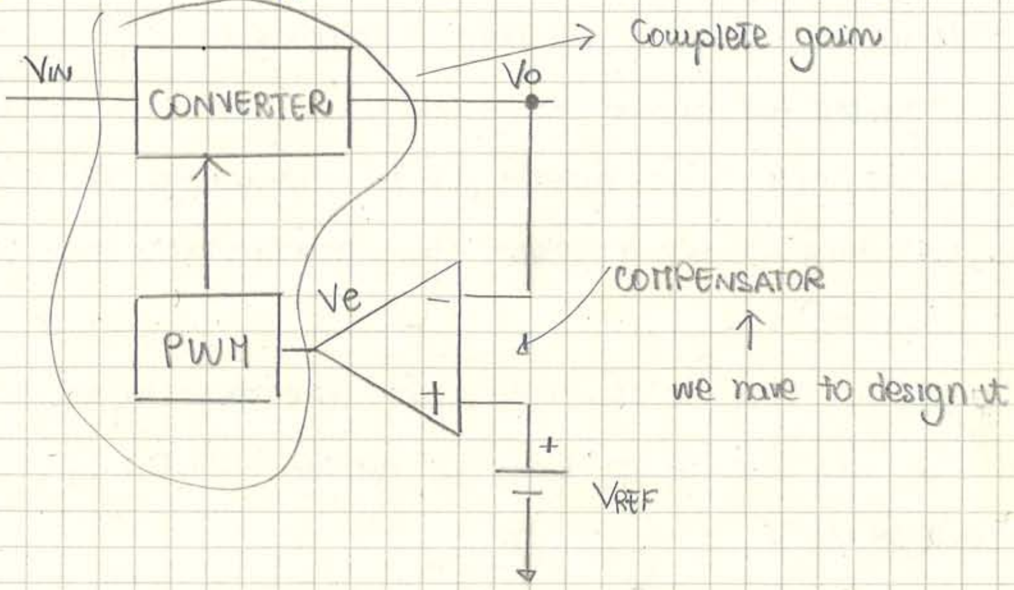
It's still a single edge modulation.
(In the AC-DC converter they use symm. waves).

In some cases, instead of having modulator, in order to avoid noise we can put on the error signal a S/H circuit. It keeps V_{ERR} constant for all full cycle. So we obtain:



It generates a lot of problems.

So, my system is something like this:



I want to close a loop because I want output voltage it's a fixed voltage, constant voltage. The important quantity that we have to study is: LOOP GAIN $|T|$. We have to define what kind of $|T|$ we want.

And for the loop gain the important parameters are:

STABILITY
- phase margin
- steady state error → High gain DC means low steady state error → PRECISION (ACCURACY)
- Bandwidth (speed of Vo when par. change; if the load changes quickly I need to recover fast).
- (High bandwidth)

So we want:

- High DC gain for precision;
- Phase margin for stability;
- Bandwidth for fast recovery;

Limitations: 1) There is $\frac{f_{SW}}{2}$

Our model stops to work here

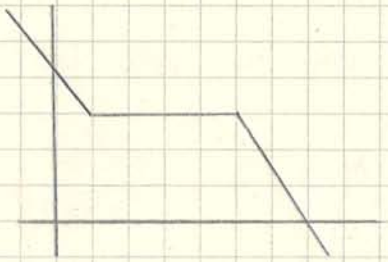
We have to stay quite lower than $\frac{f_{SW}}{2}$.

What are the problems? Pole is moving → gain is moving. I have to look for the plant transf. function that gives me the maximum crossover frequency fc. It depends on Vin, R → so I have to look for the worst case what happens to the curve (3) if Vin decreases? Curve shift down. And what about crossover frequency? It moves to the left. So the worst condition is:

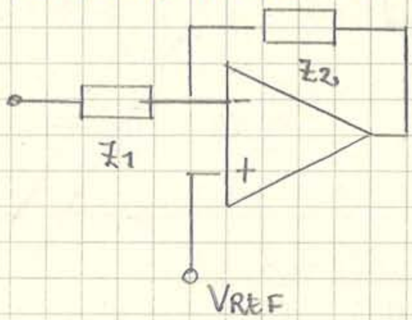
MOST CRITICAL → $\boxed{\begin{array}{c} V_{in\,MAX} \\ R_{min} \end{array}}$ → Because we consider $\frac{1}{R}$ (POLE) and not $\sqrt{R}$ (GAIN)

$\left(\frac{1}{R} \text{ wins}\right)$

So, now looking for a circuit that give me a transfer function like this:

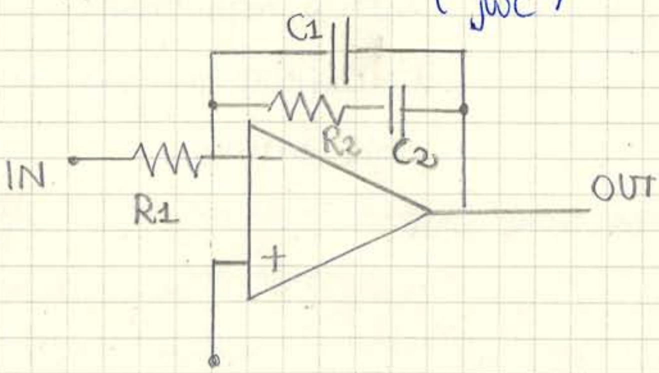


COMPENSATOR TYPE 2



$\frac{Z_2}{Z_1} = ?$
↓
2 poles; so I use 2 caps

I want to come down with 2 capacitors; they must be in Z_2 because an impedance decreases $\left(\frac{1}{j\omega C}\right)$. I want an integrator:



There isn't DC feedback. We have to find 4 parameters; but we have 3 requirements:
- Zero frequency;
- pole frequency
- Band gain

The last equation comes from values of the components.

1) In low freqency band; C_2 has an impedance that is larger than R_2 and so we can neglect R_2

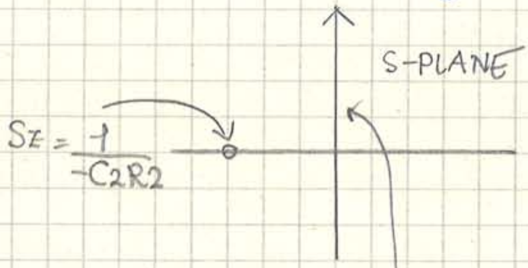
2) when frequency goes up, C_2 impedance decreases and the branch is dominated by R_2

3) The parallel is dominated by C_1.

- $f_z = \frac{1}{2\pi C_2 R_2}$ → IS IT RIGHT? OR IS IT AN APPROXIMATION?

Zero means that function goes to zero, that there is no output (for a given frequency). What is the condition in the circuit that gives me zero output? when R_2 and C_2 become short circuit I have zero output. And when they become s.c? when C_2 has an impedance that is opposite of R_2; and when we find this frequency we obtain (when $R_2 = -\frac{1}{sC_2}$)

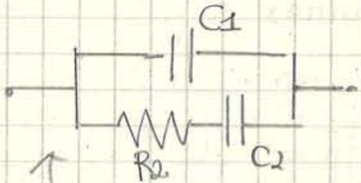
$$s_z = -\frac{1}{C_2 R_2}$$ → There is something strange!



There is negative sign; so my zero is in left plane; but how is it possible that our zero doesn't give us an output that is no zero if we look transf. function?

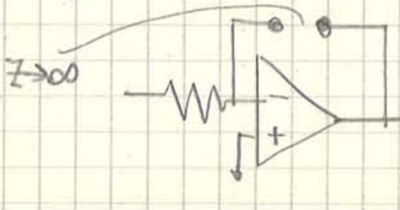
Because we're visiting s-plane only along the $j\omega$ axis; we don't hit the zero; we don't pass through the zero. Zero is on real axis; and the effect of zero is that we have a slope of 20 dB/dec.

- $f_p = \frac{1}{2\pi C_1 R_2}$ → IS IT RIGHT?



Yes. Because actually is $\frac{1}{2\pi \underbrace{\left(\frac{C_1 C_2}{C_1 + C_2}\right)}_{\simeq C_1} R_2}$

C1 AND C2 ARE IN SERIES



f_z : we have an idea of ESR is C is electrolytic. (At the exam we find ESR from output ripple and we estimate it.)
f_z can be $\simeq$ 3 kHz (about 2 decades from f_p).

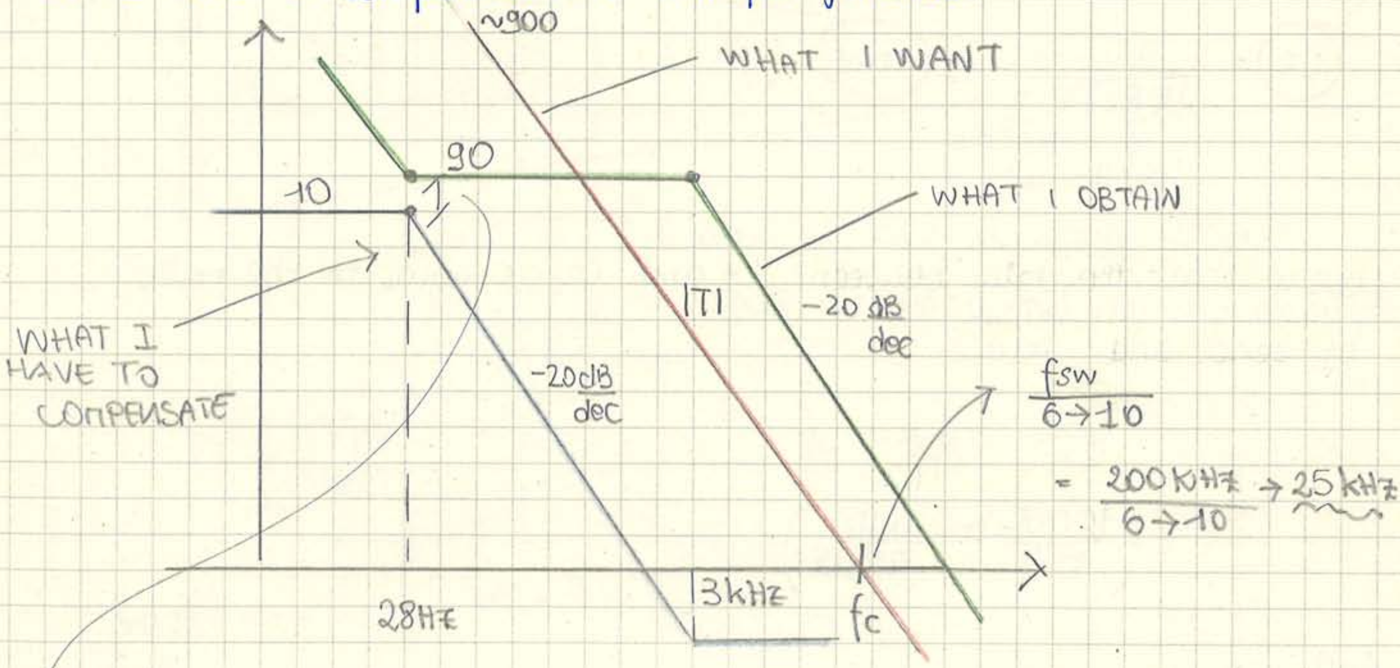
Is f_z fixed? more or less. Is f_z accurate? No; because it depends on the parasitic elements of capacitors.

GAIN : $V_{in_{MAX}} \cdot \sqrt{\dfrac{R_{Lmin}}{2 f_{sw} L}}$

$$= 18V \cdot \sqrt{\frac{24\Omega}{2 \cdot 200\,kHz \cdot 22\mu H}} = 29{,}73 \simeq 30V$$

$\dfrac{30V}{3V} \simeq 10$ times $\rightarrow$ 20 dB

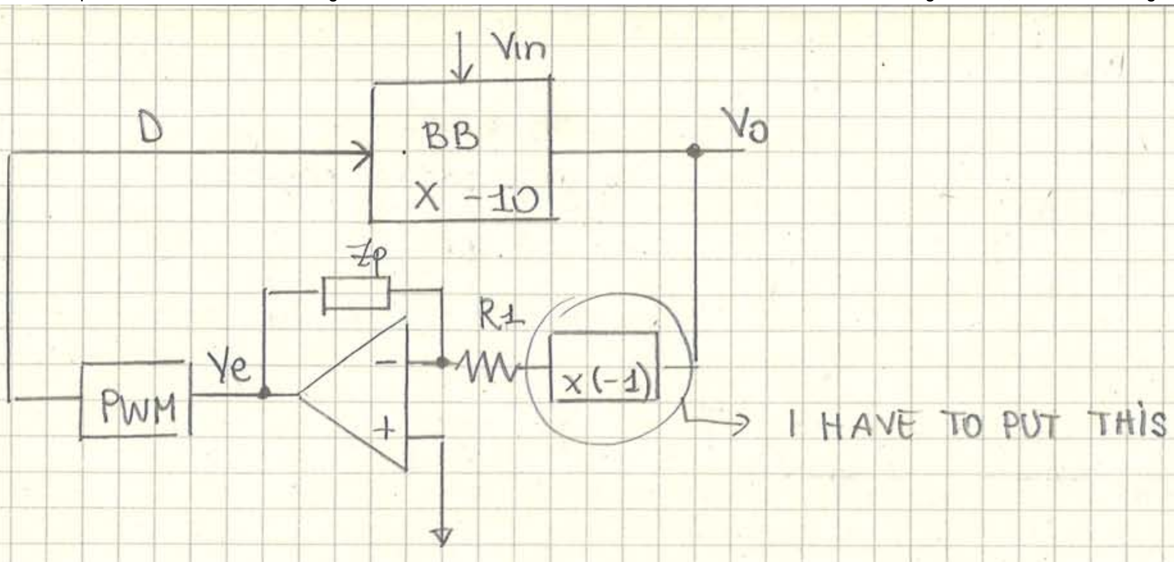
So I have to compensate the transfer function.



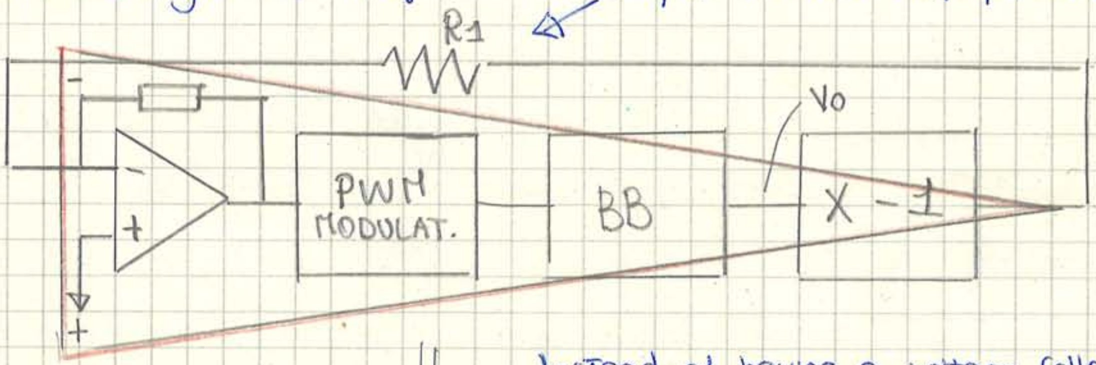
$\left(\dfrac{25\,kHz}{28\,Hz}\right) = \dfrac{x}{1} \Rightarrow x = 900$ (gain of T @ 28 Hz)

$\dfrac{900}{10} = 90 \rightarrow$ gain of my curve (DIFFERENCE; BUT I HAVE TO DIVIDE!)

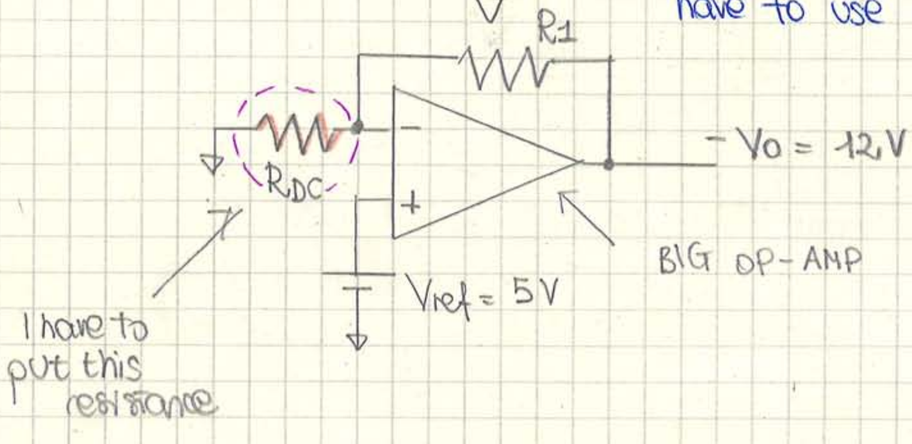
And so, my compensator gives me a transfer function that is:



Now, how can I guarantee 12V at the output? There is nothing in this circuit that give me this! I put R1 in a diff. position



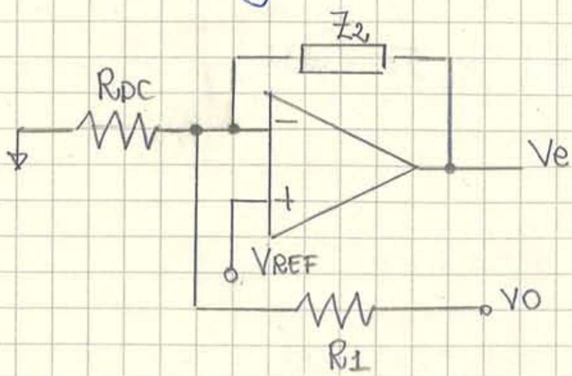
Instead of having a voltage follower I have to use a NI op-amp amplifier.



RDC sets only DC gain.

Does RDC change my stability; phase margin, crossover frequency? No.

Look at the original OP-AMP:



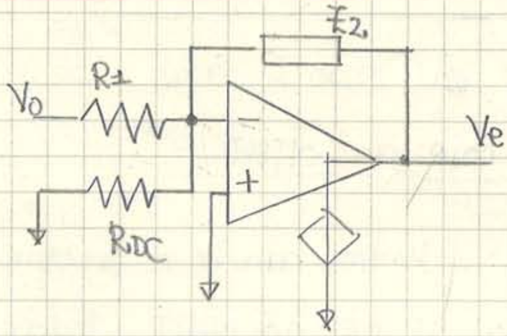
In order to analyze loop gain I need to analyze a feedback path a starting from one point and arriving at the same point.

$$\frac{V_e}{V_O} = -\frac{Z_2}{R_1}$$ → there isn't RDC!

Since RDC there isn't in the calculation the gain doesn't change with RDC and so all our deals are valid.

However, adding RDC has a defect on this circuity:

let's consider in this system (simplified explanation) just the first op-amp; that give us a transfer function – what I get –, there is a feedback system. So I want to study how well this op-amp provides me the transfer function that I want:



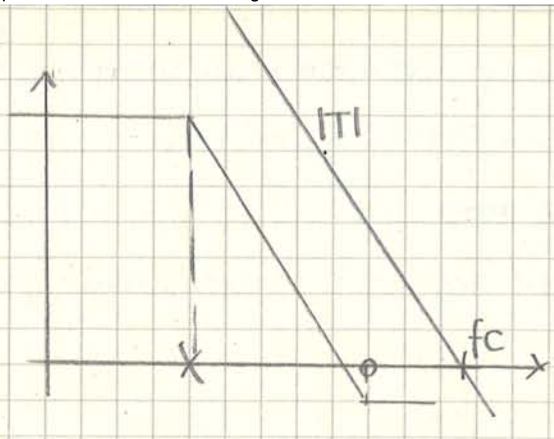
What is the loop gain of this compensator:

$$A_d \cdot \frac{R_1 // R_{DC}}{R_1 // R_{DC} + Z_2}$$!

What happens if $R_{DC} \to 0$? Very high DC gain.

↳ Loop gain → 0! So we don't have this transfer function anymore.

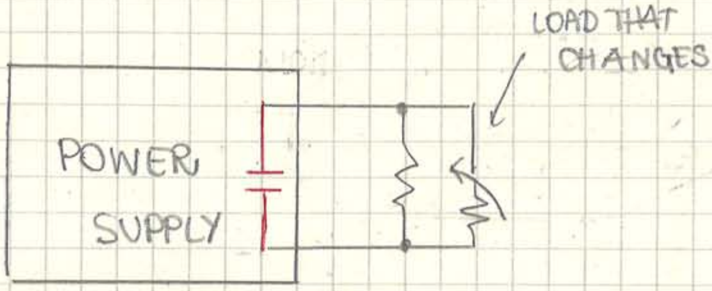
Let's suppose an output voltage = 400 V and $V_{REF} = 2.5$ V → we need a gain factor that is 160! and $R_{DC} < R_1$ and loop gain goes down; it means that the bandwidth of this compensator becomes smaller and smaller!



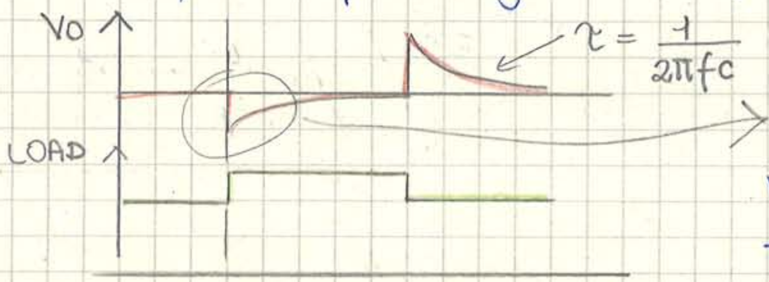
why we want an high fc? If I have only DC amplifier?

I want a system that recover from fast variation of output voltage.

So:

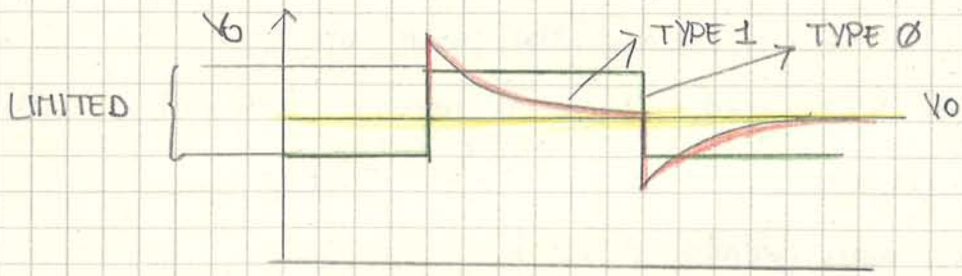


If I change the load, the output voltage instead of constant is:



Vo GOES DOWN WHEN I INCREASE THE LOAD, AND THE LOOP CAN'T CONTROL IT!

There is a case when I don't like this kind of behaviour:

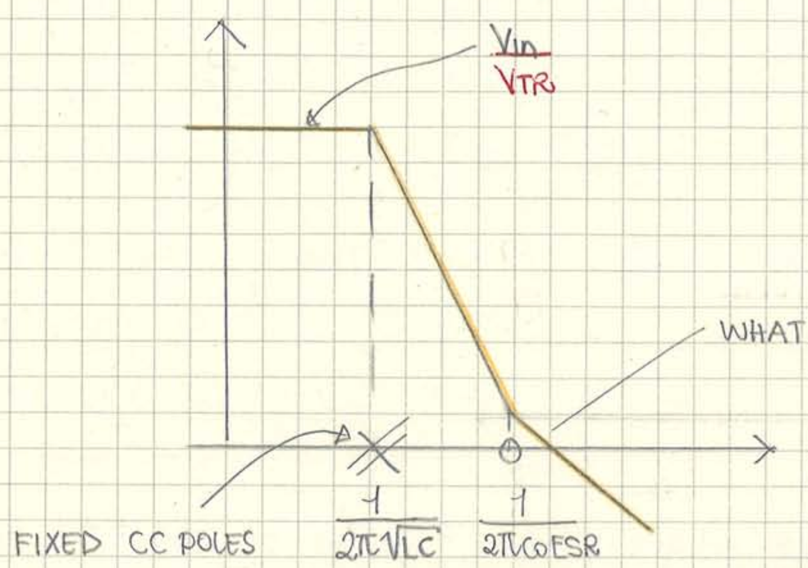


Which we prefer? It depends of what kind of load we have. If we look the integral of the area of Type 1 curve we have a short period of time when we have error; on the other hand the type Ø curve has a large period of time when we have a constant error.

But I have the load at the end and it could be ~~be~~ unhappy if it receives a peak of error!

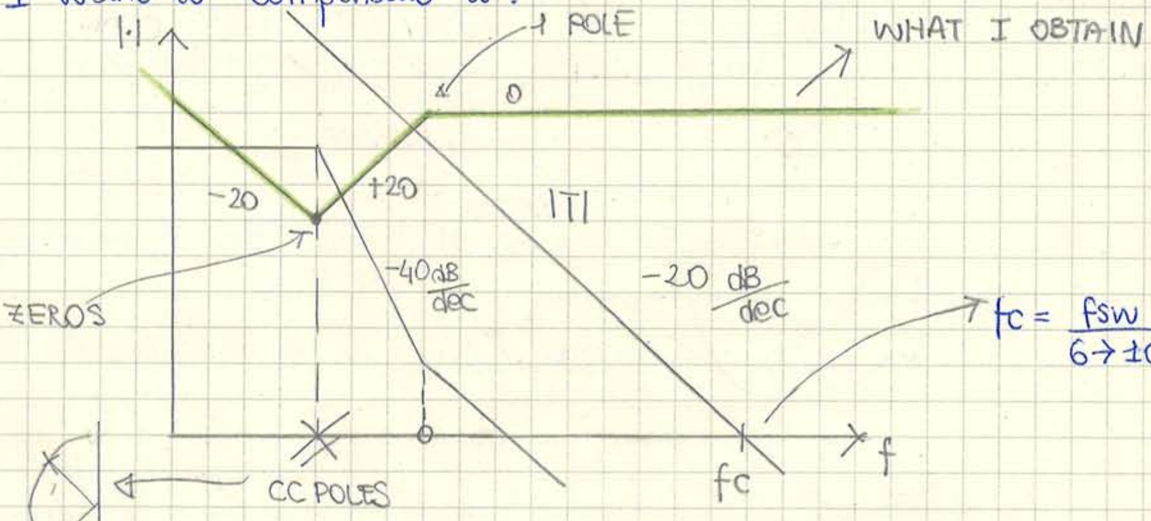
What's next step? We want to compensate buck converter voltage mode:

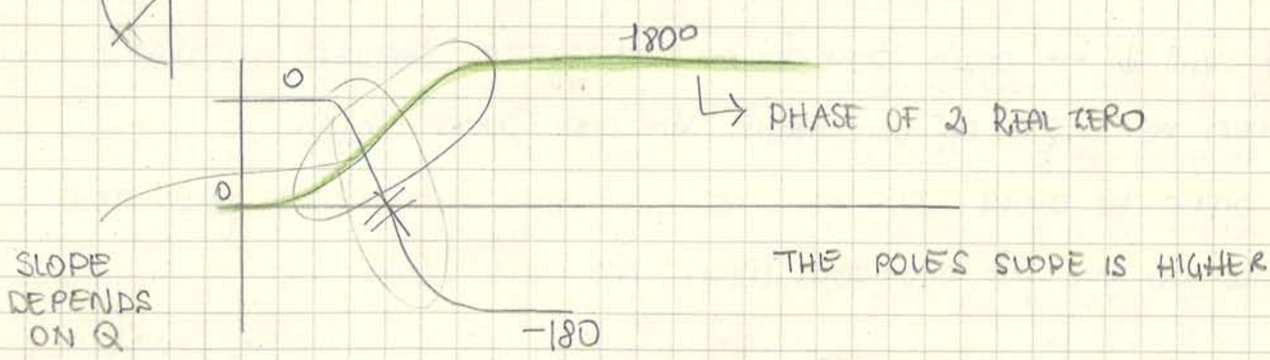
BUCK VM CCM



The only thing that changes is DC gain; so the curve moves up and down. (only shift).

So I want to compensate it:





So we have a region when the phase is −180 and the gain is greater than 1: $\longrightarrow$

So, to compensate a B.C. I need 3 poles → TYPE 3 compensator:

- 1 POLE ORIGIN (in order to reduce steady state error)
- 1 POLE over the Co ZERO
- 1 CLOSURE POLE $> 3 f_c$, $< \frac{f_{SW}}{2}$ (NB: $3 \cdot 2 = 6$ from $\frac{f_{SW}}{6 \to 10}$)

And 2 zeros:

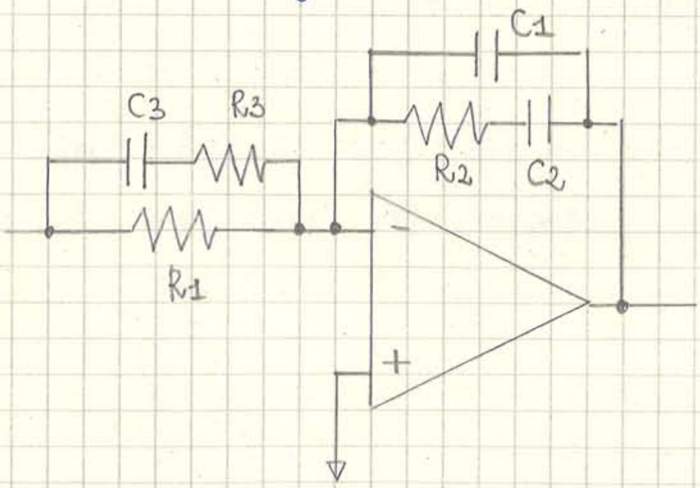
- 1 ZERO @ POLES OF B.C
- 1 ZERO @ $\frac{1}{2}$ C.C. POLES OF B.C

When f_c moves to the left? When transfer function moves down. So the most critical case for stability is when the buck gain is its maximum. When I have the maximum curve; so:

$V_{in\,MAX}$

And the load? who cares! It doesn't change the gain.

So the schematic to get a TYPE 3 compensator:



GAIN: $-\frac{Z_2}{Z_1}$

For increasing gain I can Increase Z_2 or decrease Z_1.

- So Z_2 can be (inductor) NO!
- or Z_1 can be (capacitor) OK!

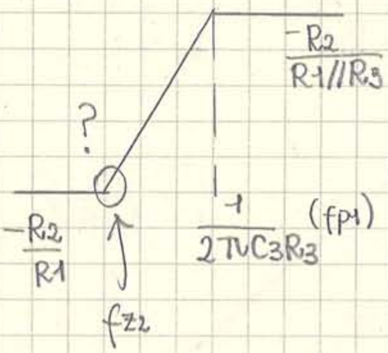
$$\begin{cases} \text{5 equations (not 6 because } A_2/A_1 = f_H/f_L \text{ so I have only 5 indip.)} \\ \text{6 elements} \end{cases}$$

↳ 1 degree of freedom (we choose C_1 [$10\mu F \to 1nF$]

What does zero mean? It means that there is nothing to the output. There is zero voltage at the output. In order to have it $Z_2 \to 0$ (or at the input an infinite impedance).

$$Z_1 = R_1 // \left(R_3 + \frac{1}{sC} \right) = \infty$$ → So we find the s value and the zero.

Or I can say that I know the transfer function that I get:



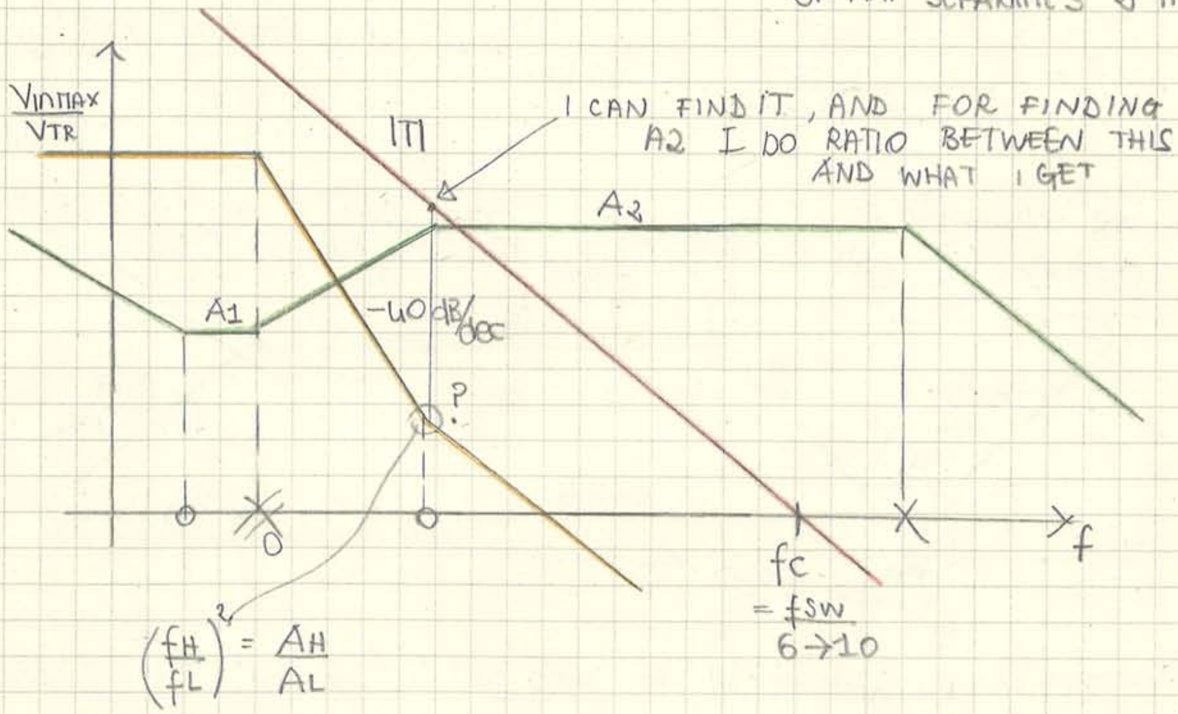
The only unknown is f_{Z2}:

$$\frac{\frac{-R_2}{R_1//R_3}}{\frac{-R_2}{R_1}} = \frac{\frac{1}{2\pi C_3 R_3}}{f_{Z2}}$$

↓

$$f_{Z2} = \frac{1}{2\pi C_3 (R_1 + R_3)}$$

NB: THIS IS EXACTLY (BECAUSE OP-AMP SEPARATES 2 IMP.)



Now let's find components values.

- Let's assume $C_1 = 100\ pF$ (Don't use it at the exam)!

- $f_{p2} = \frac{1}{2\pi C_1 R_2} = 19\ kHz = \frac{1}{2\pi \cdot 100\ pF \cdot R_2} \Rightarrow R_2 = 83{,}7\ k\Omega$

  ↳ $R_2 = 82\ k\Omega$

  ↳ %5 is OK

- $C_2 = \frac{1}{2\pi f_{z1} \cdot R_2} = \frac{1}{2\pi \cdot 170 Hz \cdot 82 k\Omega} = 11\ nF$

  ↳ $C_2 = 10\ nF$ or $12\ nF$

  ↳ zero moves little but left

associated to first zero

- $R_1 = \frac{R_2}{A_1} = \frac{82\ k\Omega}{1{,}84} \simeq 47\ k\Omega$ %1 MUST BE PRECISE! Because it determines the precision of output voltage!

- $f_{p1} = \frac{1}{2\pi C_3 R_3}$ $f_{z2} = \frac{1}{2\pi C_3 (R_1 + R_3)}$ } 2 EQUATIONS; I HAVE TO FIND R3 AND C3

Dividing these equations:

$$\frac{3\ kHz}{340\ Hz} = \frac{R_1 + R_3}{R_3}$$

$$= 7{,}82 = \frac{R_1}{R_3} = \frac{47 k\Omega}{R_3} \Rightarrow R_3 = \frac{47 k\Omega}{7{,}82} = 6\ k\Omega$$

↳ $R_3 = 6.2\ k\Omega$

- $f_{z2} = \frac{1}{2\pi C_3 (R_1 + R_3)}$

  $C_3 = \frac{1}{2\pi \cdot 340 Hz \cdot (47 k\Omega + 6.2 k\Omega)} = 8.8\ nF$

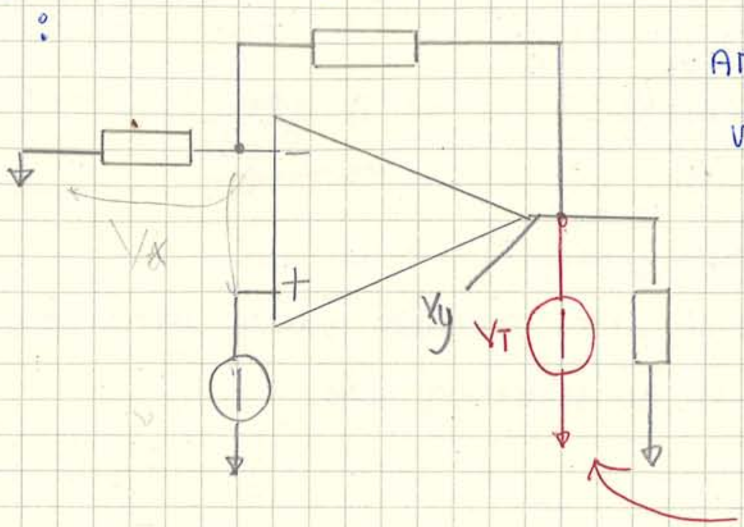
  ↳ $C_3 = 10\ mF$

It's better 10 nF because this cap gives me fz frequency (small period).

Now let's check the gain: $\frac{R_2}{R_1 // R_3} = \frac{82\ k\Omega}{47 k\Omega // 6.2 k\Omega} = 14.9$. (A_2)

What is missing? R_{DC}.

So, we have to measure loop gain $|T|$ (L for control guys) of our feedback system. (for checking if it's ok). ~~Les~~ Let's consider any feedback system :

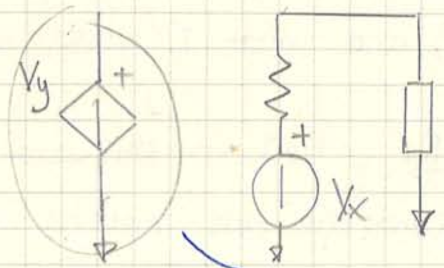


AMPLIFIER WITH FEEDBACK.

We want to measure loop gain of this system. (it could be a compensator + pwm ...). So, for do it we put a test signal V_T and we compare V_y :

$$\boxed{-\frac{V_y}{V_x} = T}$$ CUTTING

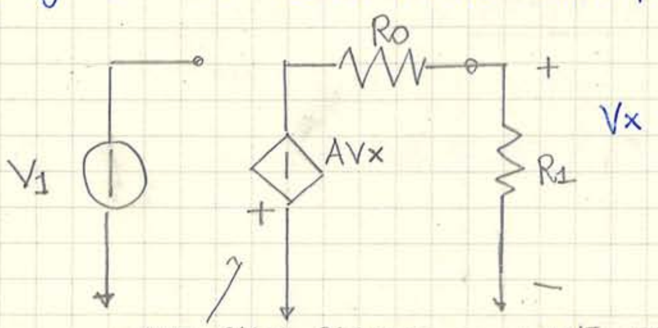
On the other hand, if ~~we cut~~ our system saturate; ~~and~~ I don't measure anything. There is another way to evaluate T → Roserstark method.
Inside we have a current or voltage controlled source and I substitute it with an independent one and then the ratio between V_y and V_x is T



$T = -\frac{V_y}{V_x}$

↳ this is right

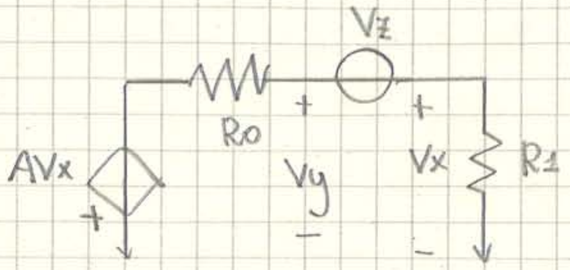
Unfortunately this system can't be used because this is only a model; a symbol ! And so it is not good for doing measure in the lab.
Let's go to the details to see what feedback system is :



Vx control the voltage source

THIS SIGN BECAUSE I WANT THIS NEGATIVE FEEDBACK

I put a voltage source $V_Z = V_y - V_x$ that has exactly the same voltage between the 2 nodes and so nothing changes!
The idea of Middlebrook is: if I remove V_x probably system is in the same position as before and $T = -\frac{V_y}{V_x}$. Is it true?



$V_y = -A V_x$? Not true, I have a current in Ro!

$$V_y = -AV_x - R_o \cdot \frac{V_x}{R_1}$$

↳ the current is the same!

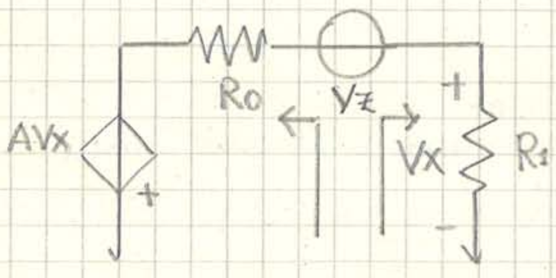
$$V_y = -V_x\left(A + \frac{R_o}{R_1}\right)$$

And so: $-\frac{V_y}{V_x} = T = A + \frac{R_o}{R_1}$ (3)

$$\boxed{\text{If } \frac{R_o}{R_1} << A \rightarrow T_3 \simeq T_2 \simeq T_1}$$

GOOD APPROXIMATION

This method works in the lab; and we found a method that is a good approximation. We can use this method if $R_o << R_1$ (for passing from T_2 to T_1) and $\frac{R_o}{R_1} << A$ (for passing from T_3 to T_2).



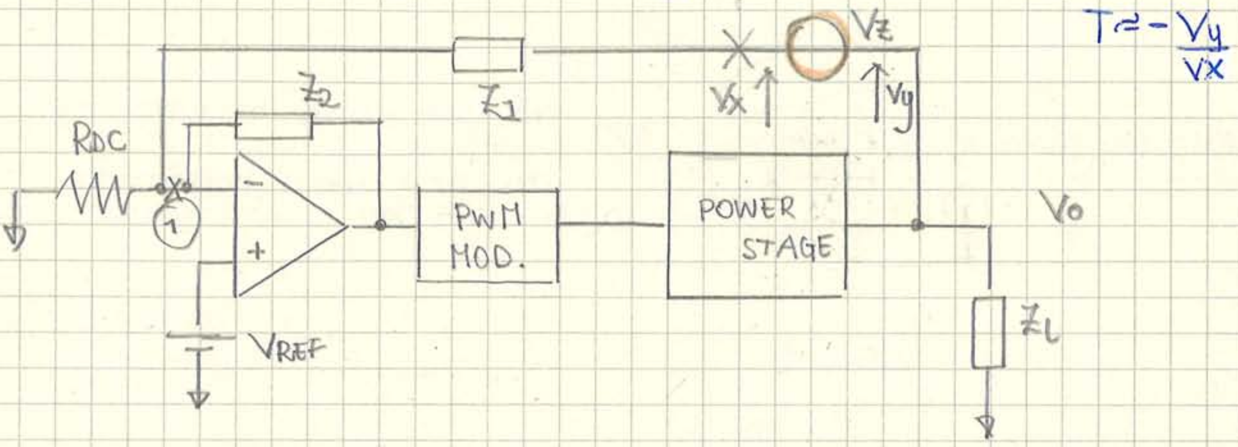
← It is still in linearity because I don't cut it.

- $R_o << R_1$ (imp. looking backword far far smaller than imp. looking forward). If it is true we can do the T measure in this way. In power electronics this condition is always satisfied.

Is it better (1) or (2)? From imp. mismatch both are ok. The offset is not a big deal because we do our measure in AC. The problem is the noise! We want to maximize $\frac{S}{N}$! In (1) we have almost ϕ; so it's not good!

And so our measurement is something like this:

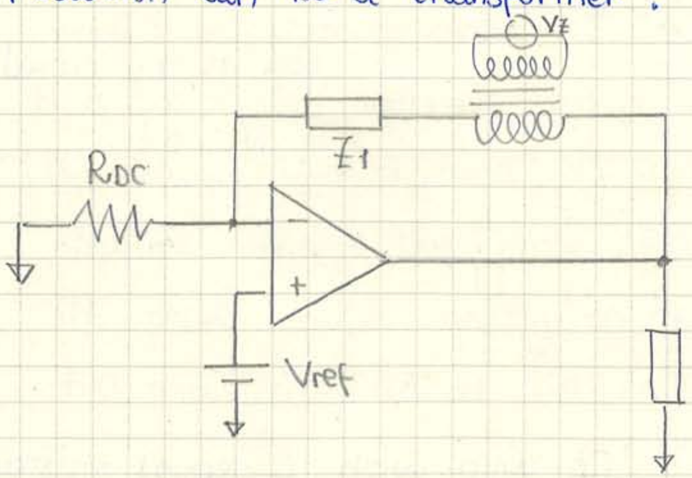
$T \simeq -\frac{V_y}{V_x}$



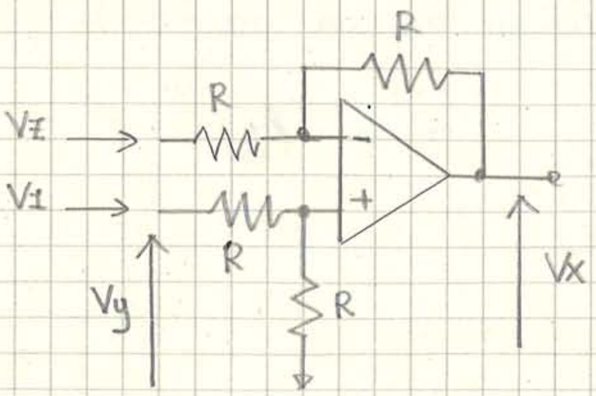
There are cases where it's not too easy to measure. For example if we have high voltage we don't have voltage source that works with this voltage and in this case we have to use (1). So we need:

1) HOW TO INJECT V_Z
2) HOW TO MEASURE $-\frac{V_y}{V_x}$

1) We need a signal source and it must be floating.
A solution can be a transformer:



But we have limitations.



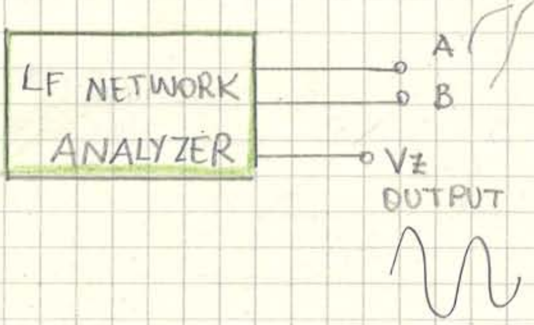
$V_{SUM} = V_1 \mp V_z$ ($V_1 \pm V_z$; NOTHING CHANGES !)

It works only with small voltages!

Let's try to make a transformer using op-amps and it must works exactly as a transformer. But it's more difficult.

2) $T = -\frac{V_y}{V_x}$?

A)



2 PROBES
Vy
Vx

(using a transformer or an adder for inject it)

20 000 $!!! Very expensive.

B) We have to measure the magnitude and the phase of 2 sinusoidal voltages. We can use the scope. It takes a long time! We have to repeat the procedure for many frequencies!
It is nice to use when we have to determine the crossover freq.
[it means $|T| = 1$ and Vy and Vx has same amplitude; then we measure the phase diff @ fc and we find the phase margin directly on the scope (because we have the minus that cancel out 180°)]

There are other techiniques for measure indirectly the loop gain.

$$A_F = A_\infty \cdot \underbrace{\frac{T}{1+T}}_{D}$$ → we find T from this equation (after measured A_F and A_∞)

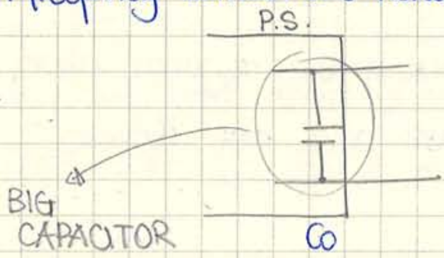
If $|T| \gg 1$ it's difficult to determine

THIS INPUT DOESN'T EXIST

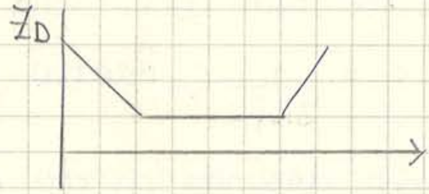
What we could do instead of measuring the gain, we can try to measure the output impedance:

$$Z_o(s) = Z_D \cdot \frac{(1+T_{sc})}{(1+T_{oc})}$$ BLACKMAN

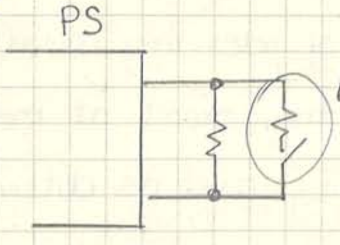
In frequency domain it's hard



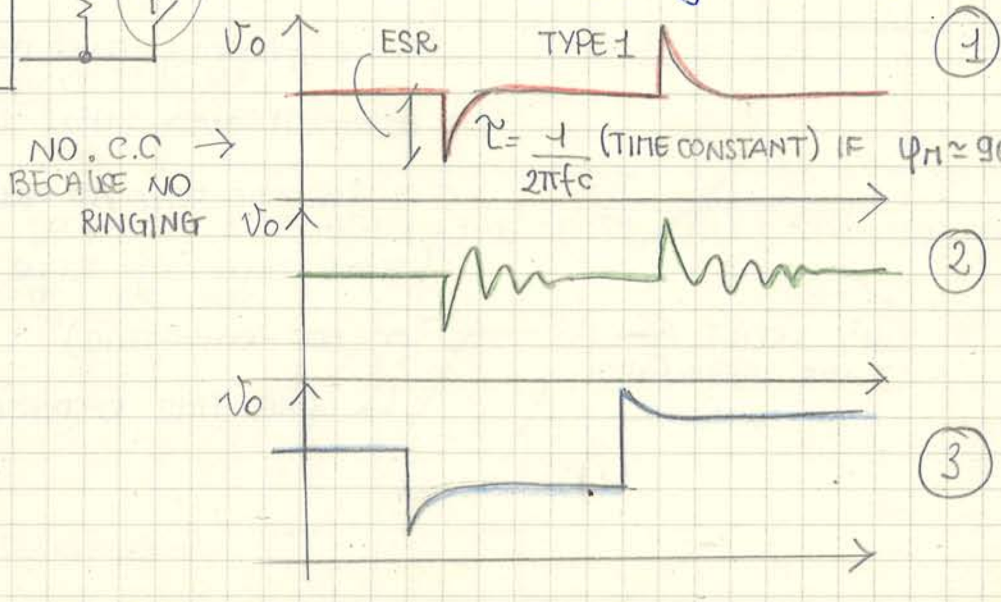
This big capacitor has low impedance and it is modify by feedback



Another way is in time domain looking at the output voltage when load changes!



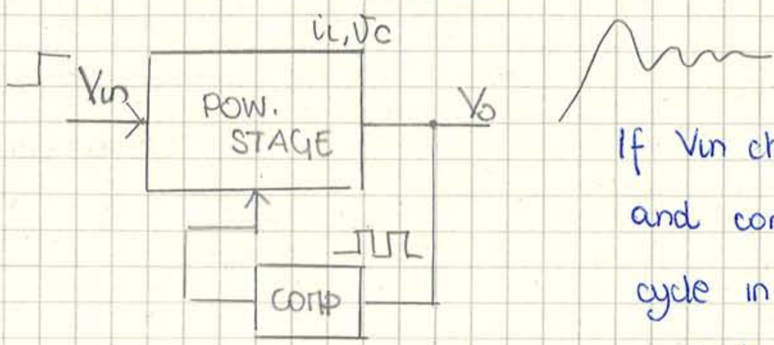
we put a second load in order to change load.
We can see something like these:



In ② we have open loop output impedance. Since we have a ringing it means that φ_M is low

In ③ we have type ∅ system; it doesn't return to the nominal value

CONTROL TECHNIQUES



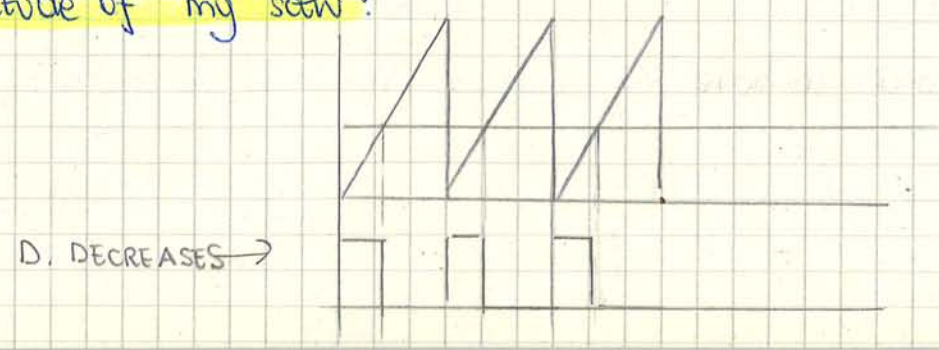
If Vin changes Vo changes and comp. decreases duty cycle in order to have Vo constant.

But for this method it takes sometimes before that control system recognized that there is something wrong; But there is a techinique that "feel" the changes in real time and we don't lose too much time and if:

$V_{in} \uparrow \rightarrow D \downarrow$

In same cases could be helpful. If we have PWM MODULATOR:

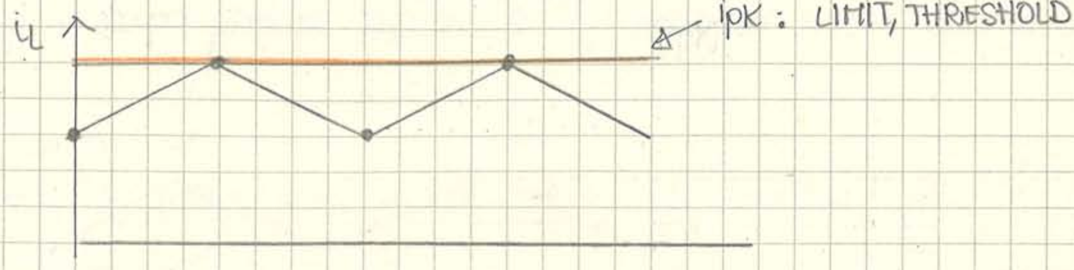
HOW CAN I DECREASE IMMEDIATLY THE DUTY CYCLE? We can increase the amplitude of my saw:



The next step is to measure the inductor current and the output voltage and we can design a better control system. Now see CURRENT MODE

The idea is:

if I consider inductor current in a buck converter:



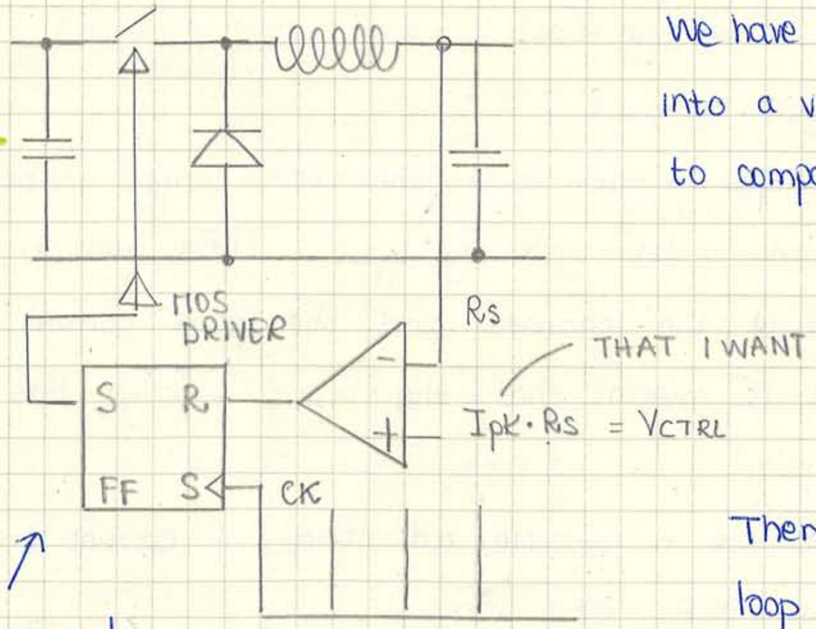
When i_L reaches i_{pk} limit, I open the switch. Basic working steps are:

- AT THE BEGINNING OF THE CYCLE CLOSE THE SWITCH
- WAIT UNTIL i_L REACHES i_p
- OPEN THE SWITCH
- WAIT UNTIL THE BEGINNING OF THE NEXT CYCLE

STEPS FOR C.M.

NB: we can assume that $\bar{i_L}$ and i_{pk} are almost the same if ripple is very small. The circuit to implement this method (this behaviour) is: (it's valid for any topology). For example for BUCK:

CM CONTROLLER



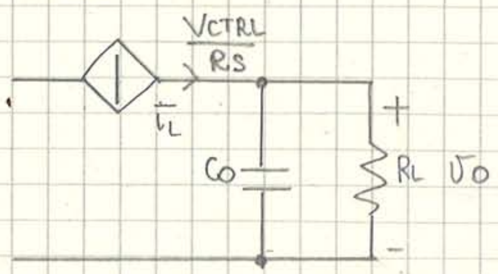
We have to convert i_L into a voltage in order to compare it.

switch on and "wait"

There is a feedback loop because we measure a current and control the switch.

It means that:

$\frac{V_{CTRL}}{R_S} \simeq \bar{i}_L$ the inductor becomes a controlled current source:



This is because I close a loop and it keeps my current constant.

So this is my system and inside the symbol there are all the other components.

$$V_O = \bar{i}_L \left(R_L // \frac{1}{sC} \right) \cong \frac{V_{CTRL}}{R_S} \left(R_L // \frac{1}{sC} \right)$$

$$= \frac{V_{CTRL}}{R_S} \cdot \frac{R_L}{1 + sCR_L} \Rightarrow \frac{V_O}{V_{CTRL}} = \frac{R_L}{R_S} \cdot \frac{1}{1 + sCR_L}$$

Actually, I have to consider ESR and my transf. function becomes:

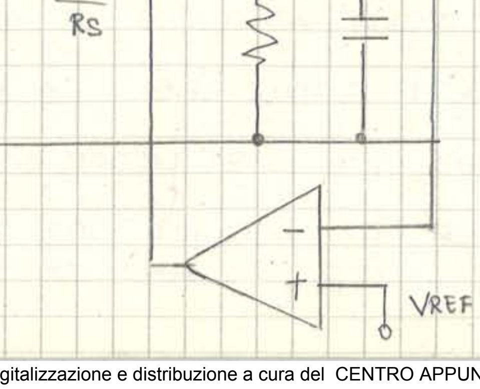
$$= \frac{1 + sCESR}{1 + sC(R + \underbrace{ESR}_{\ll R})}$$

I need to have a L.T.I system!

We have done the ~~sp~~ steps for linearizing; one is this: $\frac{V_{CTRL}}{R_S} \simeq \underbrace{\bar{i}_L}_{\text{AVERAGE}}$

$\bar{i}_L$ is linear (already).

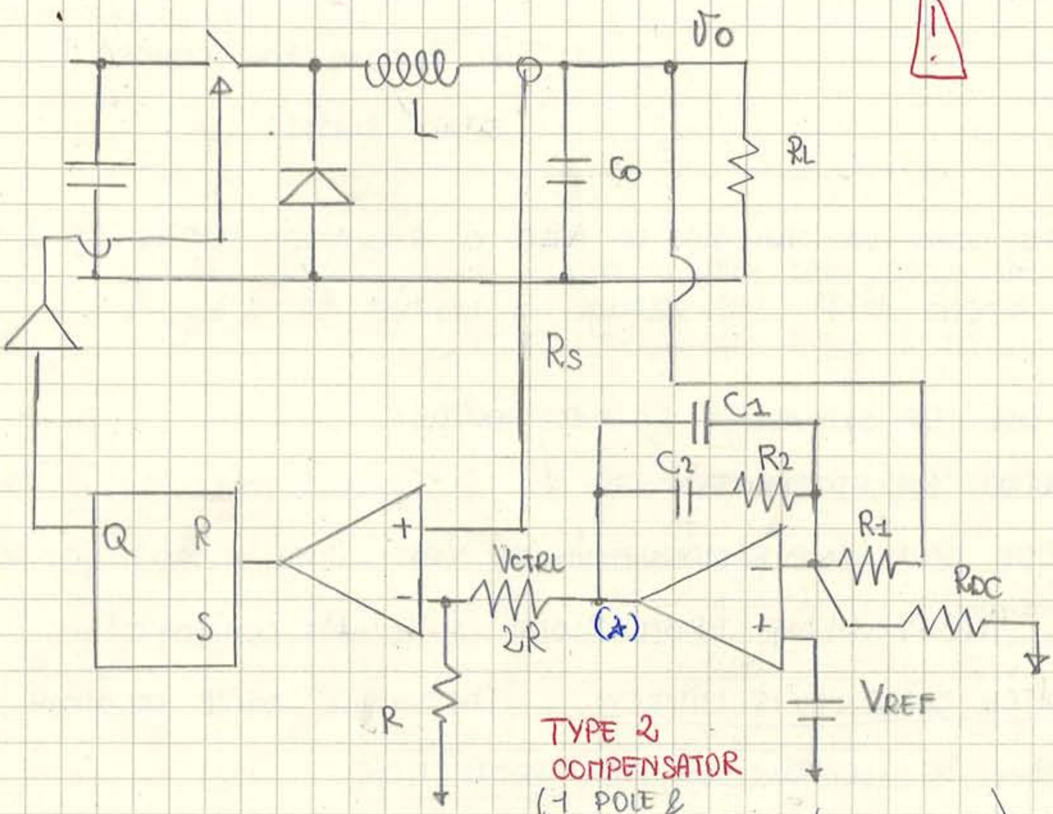
So: $\frac{V_O}{V_{CTRL}} = \frac{R}{R_S} \cdot \frac{1 + sCESR}{1 + sCR_L}$ → TRANSF. FUNCTION BETWEEN OUTPUT VOLTAGE AND PEAK CURRENT



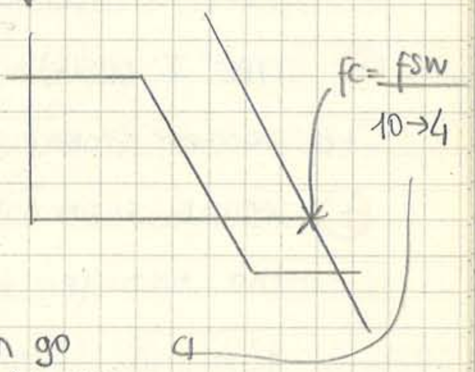
This has one pole and one zero.

The HF behaviour is independent from the load. (Yeah!) And from Vin!

So, let's go back to the original circuit:



TYPE 2 COMPENSATOR (1 POLE & 1 ZERO)



HOW DOES IT WORK?

The clock put my switch on periodically.

VCTRL comes from the compensator and it adjusts the controlled current source go up or go down.

we can go higher because fc is constant (more or less)

We have 2 loops: one fast (internal loop) and the other one is slow.

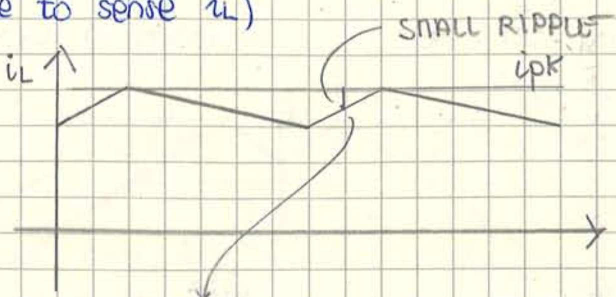
NON LINEAR

I want a large voltage in (A) because I have to compare it. In this way I haven't too much problem with noise. On the other hand I need a big voltage on Rs! It means much losses. (large power dissipation on Rs). The standard is $i_L \cdot R_{S\,MAX} = 1\,V$. So we put 2 resistors (a voltage divider); and the compensator can go up to 3V. In some cases we put a diode before 2R:

And disadvantages?

(−) COST (because we have to sense i_L)

(−) NOISE SENSITIVITY

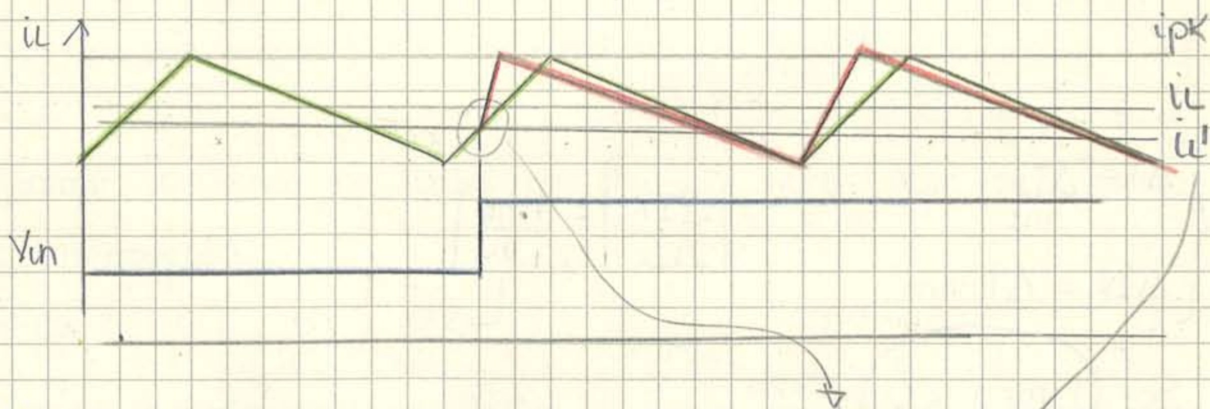


If I have a spike here, since the ripple is very small, if there is some noise immediately the switch is on!

It's not a big deal because if we decrease duty cycle the voltage starts to decrease and then the compensator adjust it.

(−) $\bar{i}_L \neq i_{pk}$

For steady state condition is not a big deal because if the current that we provide is too small the voltage will be too small and the int. voltage loop increase the requested current i_{pk} value in order to get exactly the output voltage. So this error can be compensated by voltage loop; but let's consider for example:



If Vin increases, I have that D decreases immediately and the average current goes down and so we have an error of average current.

$m_1 = \frac{\Delta I_{pp}}{T_{on}}$ $m_2 = \frac{\Delta I_{pp}}{T_{off}}$

So :

$\frac{\Delta I_{pp}}{T_{off}} \cdot \frac{T_{on}}{\Delta I_{pp}} < 1$ $\boxed{\frac{T_{on}}{T_{off}} < 1}$ CONDITION FOR STABILITY

It means that duty cycle must be < 50% ⚠

If D > 0.5 it happens that the final error is larger that the initial error. (and final error will be larger and larger than initial error ...) Incredibly Vo stays the same.

For example, instead of having:

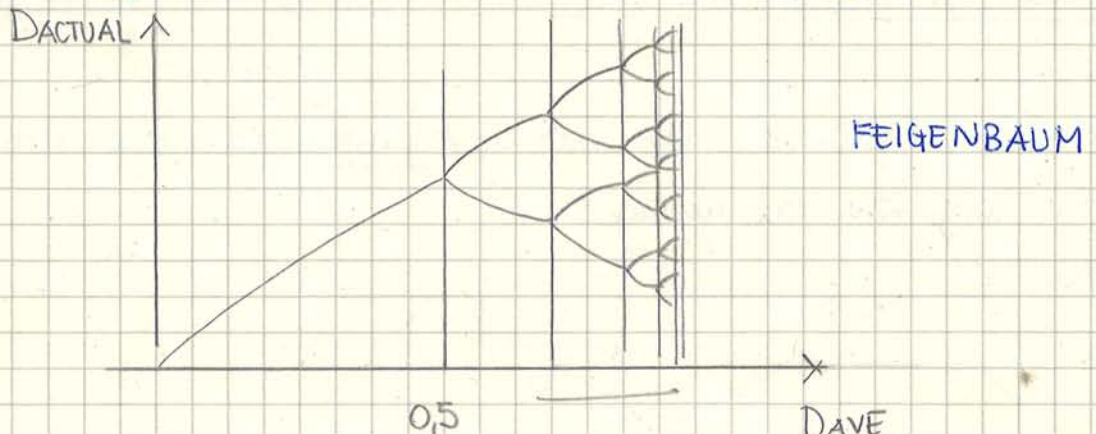
D1 D2 D1 D2

we have: (with instability)

D1 D2 D3 D4

70% 60% 85% 55% (Now the instability is $\frac{1}{4}$ of fsw)

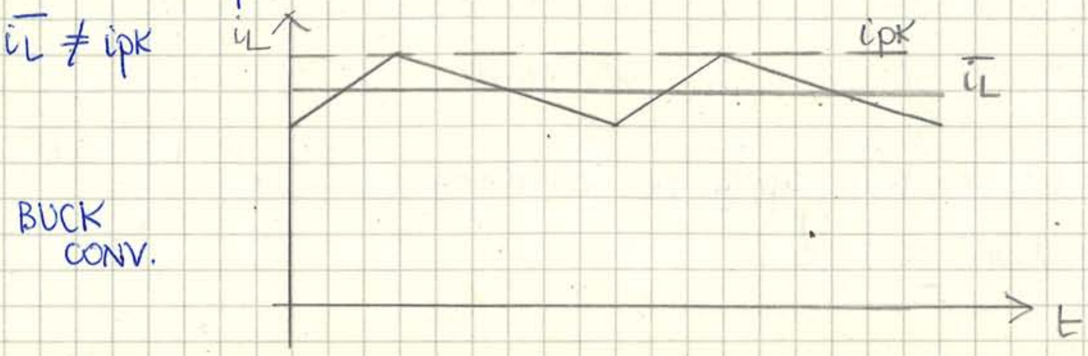
So if we go more and more we have many cycle!

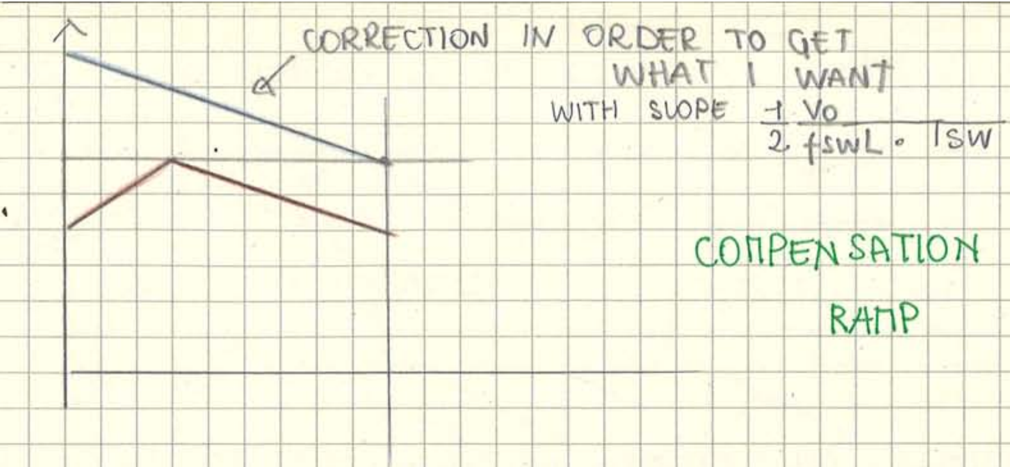


↳ The interval Became smaller)

The same correction for subharmonic instability works also for $\bar{i_L} \neq i_{pK}$. (Same techiniques).

- $\bar{i_L} \neq i_{pK}$

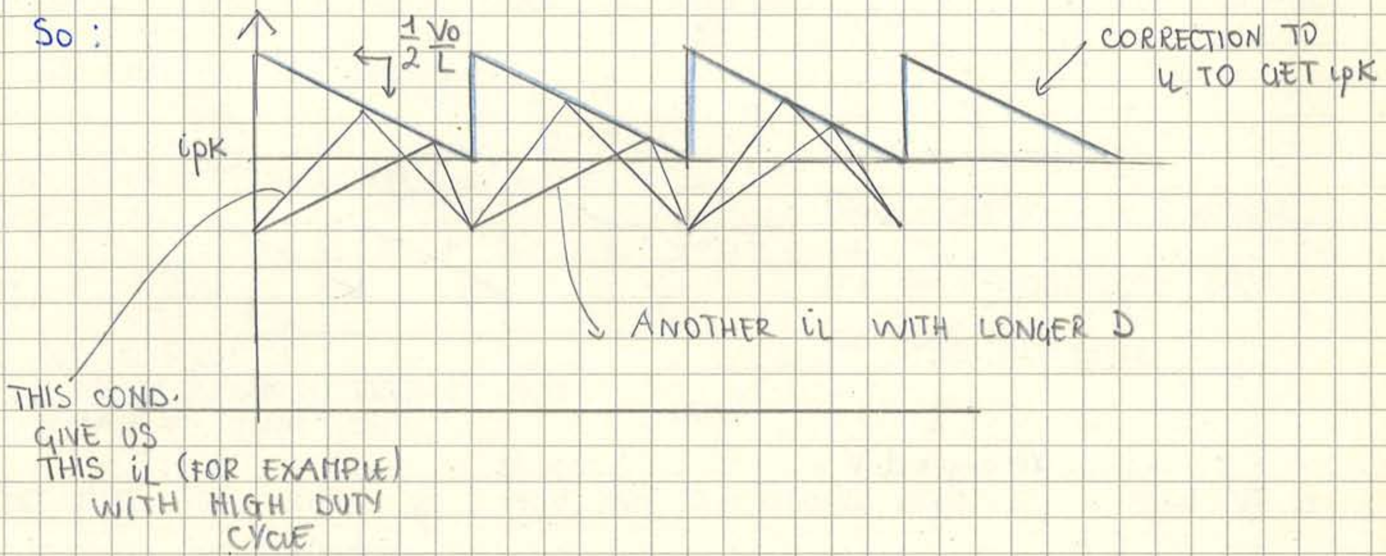




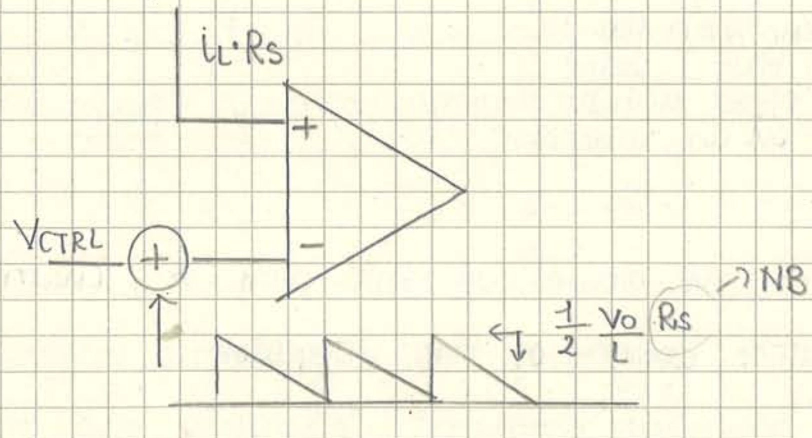
COMPENSATION RAMP

SLOPE : $\frac{1}{2}\frac{V_0}{L f_{sw} T_{sw}} = \frac{1}{2}\frac{V_0}{L}$ (our slope doesn't depend on D and Vin!)

So :

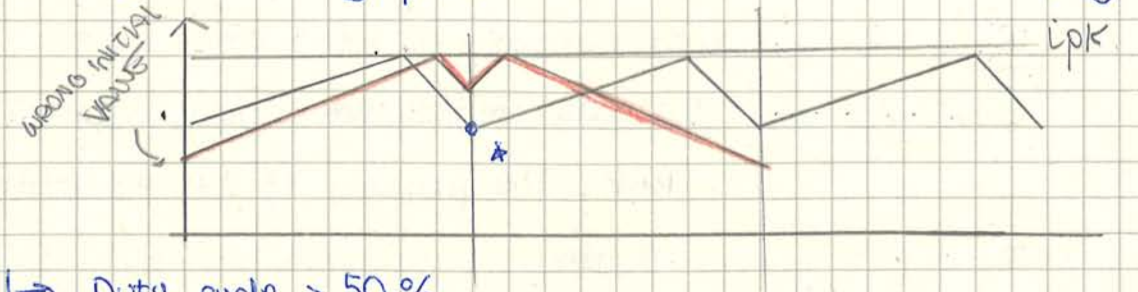


A circuit that implements this is :



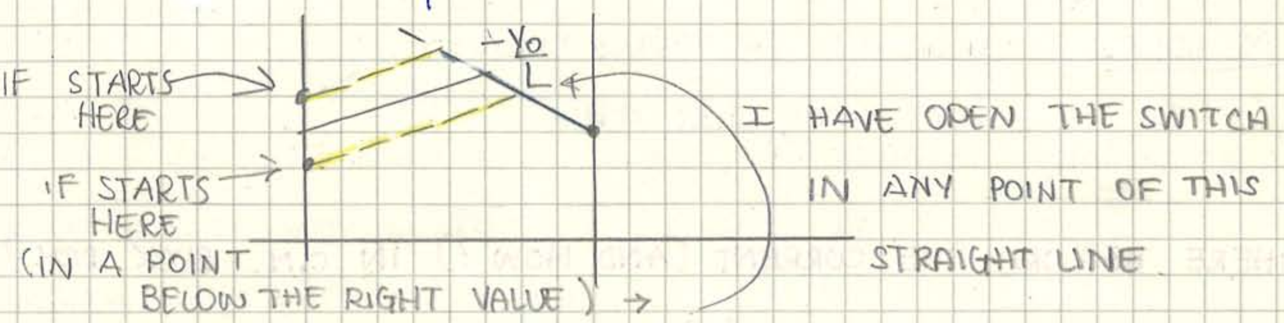
it is never done because this waveform is not easy to generate.
It's better put a compensation ramp on the other side of the comparator. But in this way I have to flip down the ramp :

We saw that a big problem is the subharmonic instability:



↳ Duty cycle > 50 %

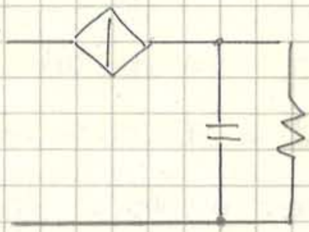
A way to correct this error is the compensation ramp. Compensation ramp for correcting sub-harmonic instability is different than we saw last time. I want that at the end of the cycle the current is here A. For example:



So with the slope $-\frac{V_o}{L}$ I kill the subharmonic instability. Since V_O is almost constant the slope is fixed.

POLES in CM (CCM). How many poles do we have?

Buck is (with a good approximation)



1 POLE @ $\frac{1}{2\pi RC}$

In the original schem. if I have a feedback it only moves the pole! (it doesn't add anything). Somebody discovered that this pole moves to high frequency; graphically speaking:

- 1 POLE
- 2 POLES : $\frac{1}{2\pi C_o R}$; HIGH FREQUENCY $\left(\frac{f_{SW}}{\pi}\right)$.

↙ it can decrease phase margin but no too much; because at $f_c = \frac{f_{SW}}{5}$

peak current during Ton ! (like 11)

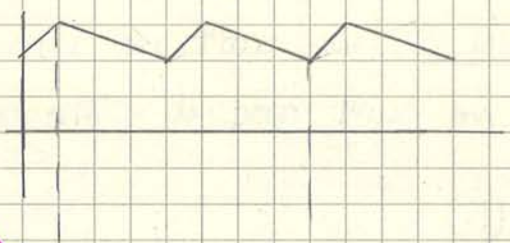
- 8, NO ! Because it's just the average value!
- 7, NO !: It's only AC current; I need the total, the actual current.
- 9, I have the inductor current here; it could be a good point but remember; If I connect it to the ground it's a problem!
- 10 ; same story
- 13, NO we have only AC value.

SO THE GOOD POINTS ARE 2, 3, 4, 5 .

THEY SHARE SOME PROPRIERTIES

- 3,4 : In 2 we have Vin that is a DC voltage. The voltage in 3 and 4 is a square wave. So they have a VNOISE (up and down 0 to Vin etc...)
- 2,5 : VQUIET (DC voltage)

- 4,5 : they have in common the current. But it is:



- 2,3 :

common voltage is Vin

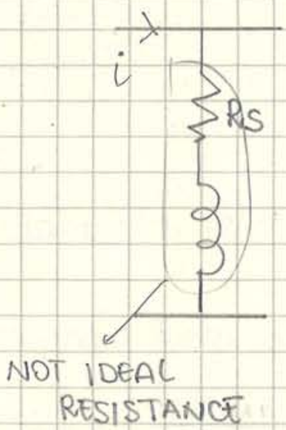
lower DC average value, high frequency noise

The number 4 is not used. So which one do we choose? It depends on the technique that we use to sense the current.

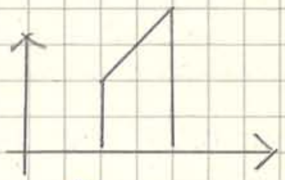
In 2 power dissipation is smaller than 5, because in 5 the current flows all the time! (In 2 only in D).
So 2 is better from pow. dissipation point of you.
But in 2 we have a smaller DC value ($V_{in} < V_o$). So measure when the voltage is higher is better. Inside an IC the resistor is put in 5 and the measure is easier. Quantity that we have to define:

- $R_S = \frac{V_{SENSE\,MAX}\ (1V)}{I_{L\,PK}}$ → $R_{S\,ACTUAL}$ must be smaller, otherwise $I_{L\,PK}$ decrease.
- $P_D = \underset{\text{ACTUAL}}{R_S} \cdot I^2_{RMS}$

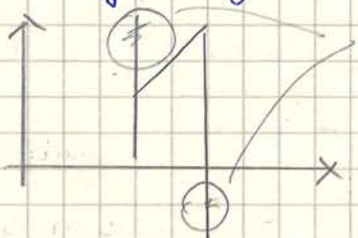
R_S must be a non-inductive resistor.
What happens if:



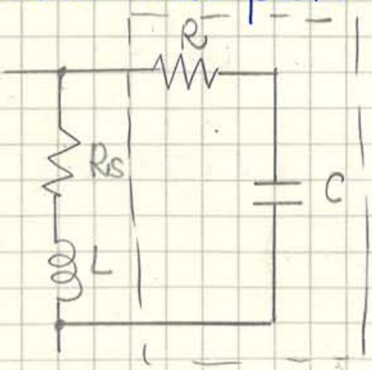
If we send an i that is:



What kind of voltage we have?



We can control these peaks with a low pass filter:



→ this low pass filter remove the peaks. In order to cancel it:

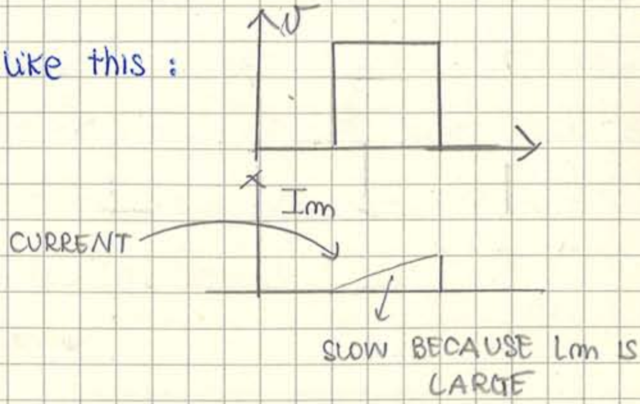
$$\frac{L}{R_S} = RC$$

But in many cases:

$R = 1\,k\Omega$
$C = 470\,pF$
} FOR CANCEL INDUCTIVE EFFECT OF R_S

AT THE OUTPUT.

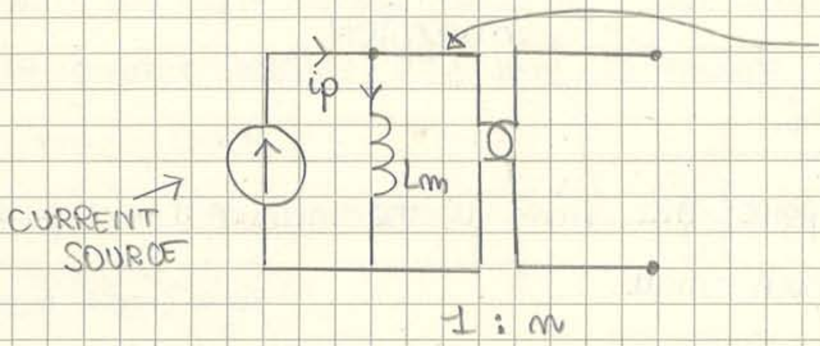
But ... if we apply a voltage like this:



And so at the output we obtain more or less the input pulse.

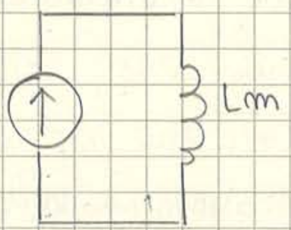
NB: After a certain number of pulses it saturates; but the idea is ok.

But in our case we want to measure a current; so:



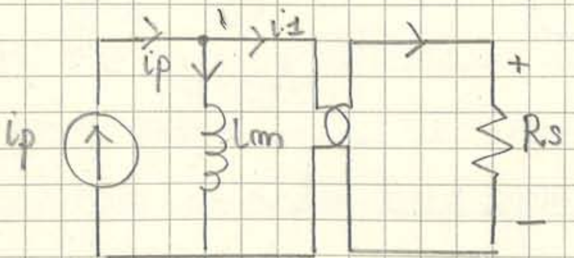
IN THIS POINT we have $i_1 = 0$, because $i_1 = i_2 \cdot m$ (but $i_2 = 0$)

And so we have:



In this way we have a big spike of voltage.

For this reason we need to put a load resistance at the secondary:



in order to make a good measurement I want that:

$i_m <<< i_1$

↳ in this way i_m is not important

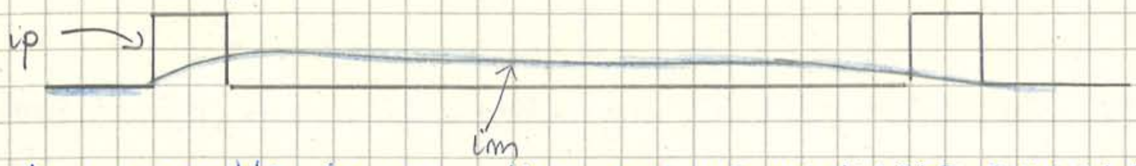
And so at the output we have:

$$V_0 = i_p \cdot \frac{1}{N} \cdot R_s$$

discharge Lm.

Bad news : im is a small current, and when it goes to the output it generates a very small voltage drop and when it comes back is not enough to discharge Lm. It means that if we want to discharge Lm with this voltage we need looooong time !

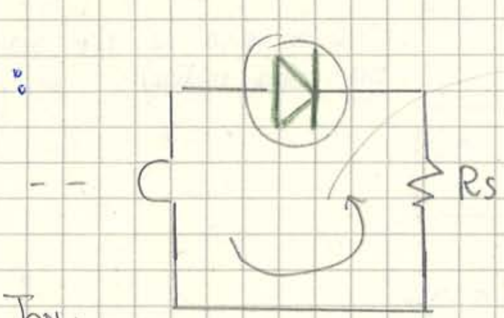
So the next pulse must be:



So it is impossible to use. HOW CAN WE DISCHARGE QUICKLY AN INDUCT? with an high voltage across it. We should increase Rs. But it can't be greater because it's here to measure the ip current not the im! OUCH !

So we can put a diode:

DURING TON I WANT THAT THE CURRENT FLOWS IN THE RIGHT WAY.



IL DIODO NON CONSENTE ALLA CORRENTE DI FLUIRE IN QUESTO MODO

So during the phase $\overset{T_{ON}}{\checkmark} I_p$: $V_o = R_s \cdot \frac{I_p}{N}$ (DURING TON);

then switch open and the current goes to Ø and current flows to the secondary side but there is the diode; so the inductance is open ; and there is a voltage spike (we don't destroy any component)

Is Vo still the same if I add a diode? it's the same! it's a current! We have some extra im... but who cares!

So we can have an higher duty cycle.

What are advantages of curr. transformer?

(+) ISOLATION

(+) Rs (SECONDARY) HAS A PD LOWER BY N

(−) COST

So:

$$P_{R'_S} = \frac{I^2_{SW_{RMS}} \cdot R_S}{m} = \frac{P_{R_S}}{m}$$

So the power dissipated by R's is < than Rs.

EX:

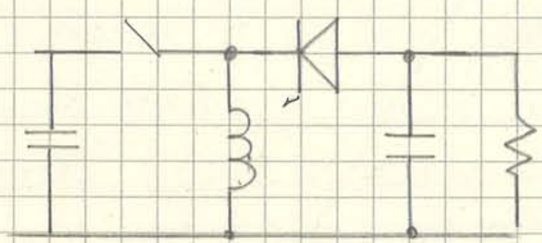
$$P_{R'_S} = \frac{25W}{50} = \frac{1}{2} W \quad \text{ok!}$$

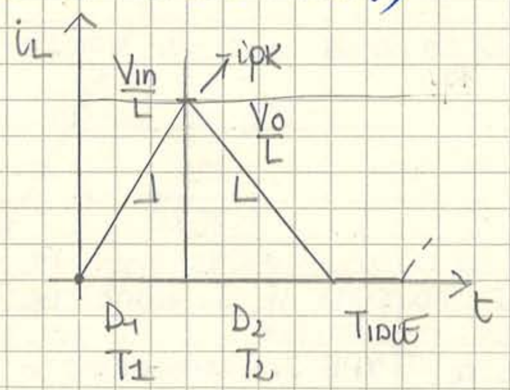
So we have to use current transformer when the power dissipated by Rs is too much. (Otherwise we loose efficiency!)

We saw that C.M removes one pole from CCM transfer function (see BUCK conv.). What about boost or buck-boost in CCM? We have 2 poles and if we close a loop around inductor, probably inductor poles moves to high frequency as well. But what are problems of these topology used in CCM? RHP zero. And it can't be removed by C.M? Unfortunately no. (Also in V.M.)

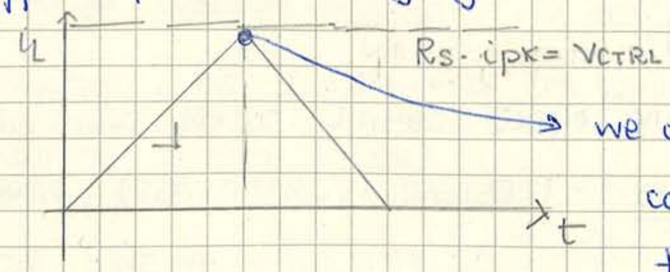
What about DCM?

BB DCM (can we use current mode?)





What happens if instead duty cycle we control ipk?



we control this value, so I control the energy inside the inductor ($\mathcal{E}_L$)

$$\mathcal{E}_L = \frac{1}{2} L \, I_{pk}^2$$

, And so:

$$\mathcal{E}_L = \frac{1}{2} \cdot L \cdot \left(\frac{V_{CTRL}}{R_S}\right)^2$$

DERIVED TOPOLOGIES
(BASIC TOP. + TRANSFORMER)

- BUCK + TRANSF. → : there are many topologies
  - FORWARD
  - PUSH PULL
  - HALF BRIDGE
  - FULL BRIDGE
- BOOST + TRANSF. → Not used very much. (Boost is only used as PFC)
  ↳ CURRENT FED CONVERTERS
- BUCK-BOOST + TRANSF → FLY BACK conv. (is the most used topology in the world). But it can only work in DCM.

TRANSFORMER (HF)

Studiare bene per l'esame (teoria) ⚠⚠⚠

(+) ISOLATION (very important, don't forget it!)

(+) MULTIPLE OUTPUTS

(+) NO LIMITATIONS ON V_o VS V_{in} (very important because for example, if we want boost a voltage we can use a buck derived topology!) and V_o can be larger than V_{in}.

(+) EXTRA DEGREE OF FREEDOM $\frac{N_p}{N_s}$

→ CHOOSE A "CONVENIENT" DC. A good choice is take a DC larger as possible. (for some topology 80%, because the switch is cheaper)

→ MOVE STRESSES AROUND : we can decrease the stress of some component; but I increase it on other part. (we just move them to another place)

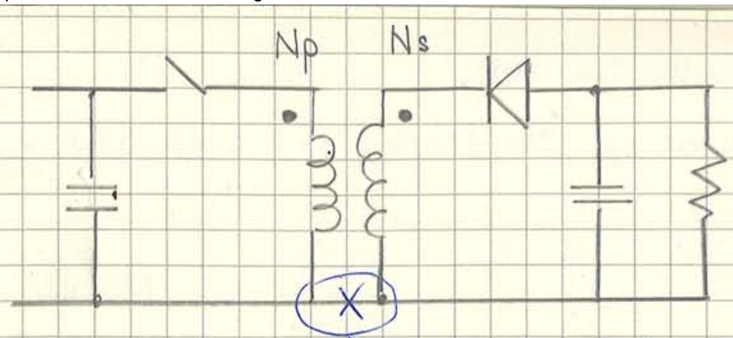
(+) LIGHT AND SMALL COMPARED TO 50 Hz TRANSFORMER

(−) COST, WEIGHT, VOLUME, $

(−) DIFFICULT TO DESIGN

(−) PARASITIC ELEMENTS (LOSSES)

(PEAK LOST), (LEAKAGE INDUCTANCES THAT LIMIT f_{sw})

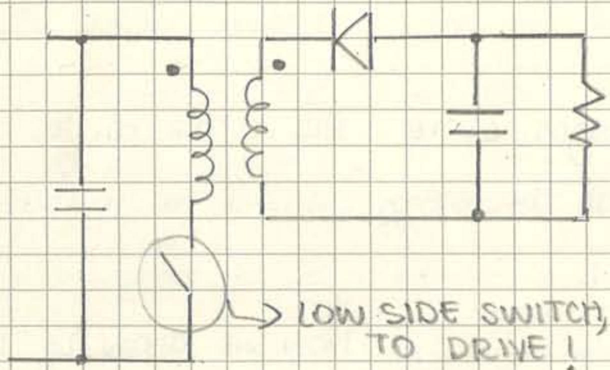


FLY BACK

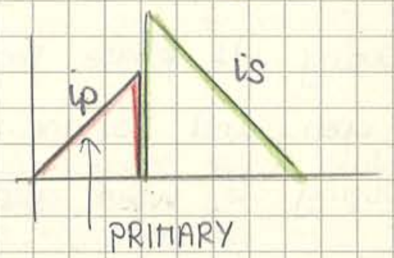
↳ If I remove this ground they are isolated.

The output voltage has no relationship with the input!

We can do some changes: switch and primary L are connected in series and then I can swap them:



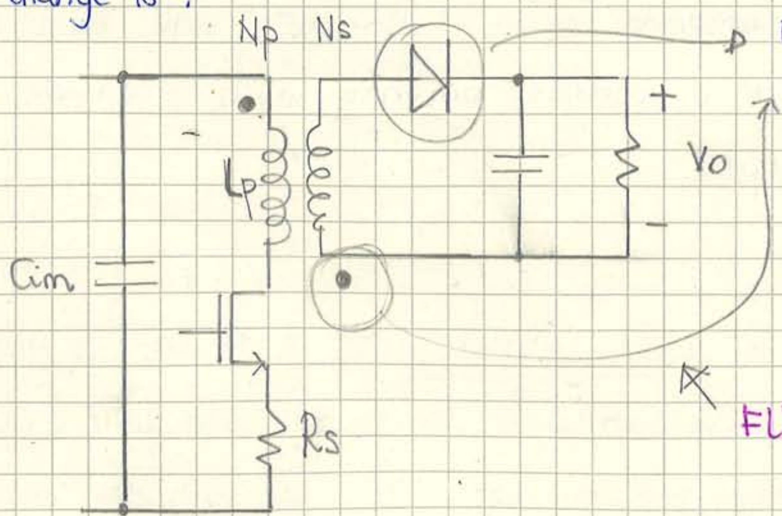
It is still a flyback conv.



We can control it in voltage mode and current mode. In the last mode we can sense the current with R_s in series with the switch: with 1 V on R_s, I don't lose any drive capability.

Can I use $R_{DS\,on}$ in order to measure the current? We can't use it in discrete case. R_{DS} changes with temperature!

Last change is:

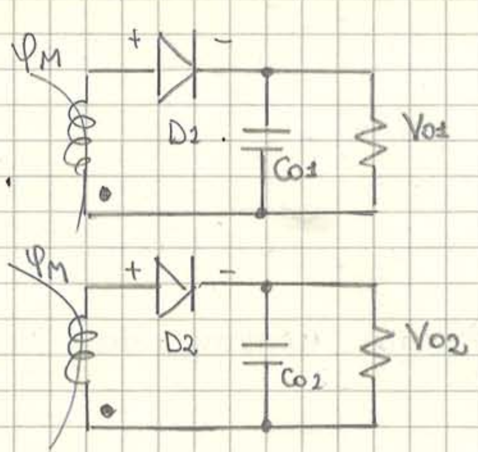


in order to change the polarity of Vo

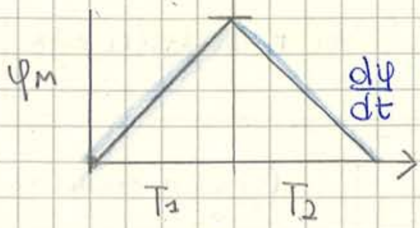
$$L_S = L_P\left(\frac{N_S}{N_P}\right)^2$$

FLYBACK

- $\frac{N_{S1}}{N_{S2}}$:



Both of them feel the same flux φ_M. It means that when I open the switch the flux starts :



$\frac{d\varphi}{dt} \to$ VOLTAGE PER TURN

$\frac{d\varphi}{dt}$ is the same for each turn of the winding. If I want that V_{o1} has double voltage respect V_{o2} we can image that the first one has double turns respect the second one. We have to consider the voltage drop across the diode ; so :

$$\boxed{\frac{N_{S1}}{N_{S2}} = \frac{V_{o1} + V_{D1}}{V_{o2} + V_{D2}}}$$

We put two different voltage diode because we can have two diff. output voltage ; for example if we have $V_{o1} = 5V$ we can use a Shottky diode ($\sim 0.5\,V$).

N_{S1} and N_{S2} are integer numbers ⚠

- $L_p \Rightarrow$

We can choose our duty cycle with the turn ratio.

To find out L_p we have to do an initial choice : DCM. It means that

$D_1 + D_2 < 1$. So my initial choice is that D_1 must be $\simeq 40\%$ and $D_2 \simeq 40\%$.

f_{sw} ($\simeq 300$ kHz).

We have to find the TOTAL OUTPUT POWER and then the TOTAL INPUT POWER. They are the same? In a real world no! We have losses! We can estimate it. And then we can find the ENERGY PER CYCLE and then L_p.

But $\mathcal{E}_p$ is equal to $\frac{P_S}{f_{SW}\,\eta_{MAGN}}$

So we have:

$$\frac{P_S}{f_{SW}\,\eta} = \frac{1}{2} L_p \left[\frac{(V_{in\,min} - V_{SW} - V_{RS})\cdot D_1}{L_p^2 \cdot f_{SW}^2}\right]^2$$

And finding out L_p:

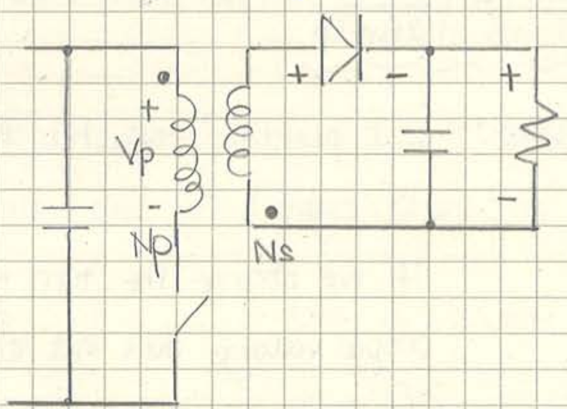
ONLY IN C.M. !!!

$$\boxed{L_p = \frac{[(V_{in\,min} - V_{SW} - V_{RS})\cdot D_1]^2\,\eta_{MAGN}}{2 f_{SW}\cdot P_S}}$$

We need the turn ratio $\frac{N_{S1}}{N_p}$.

In steady state across any winding: $\overline{V} = 0$ If we have it $\neq 0$, the current starts to increase!

In some cases we have to guarantee it. Let's consider:



V_p during D_1:

- $V_p = V_{in} - V_{SW} - V_{RS}$

they are not constant. They depend on current! But we take the maximum for worst case.

V_p during D_2: (switch open)

is given by the secondary voltage reflected to the primary side.

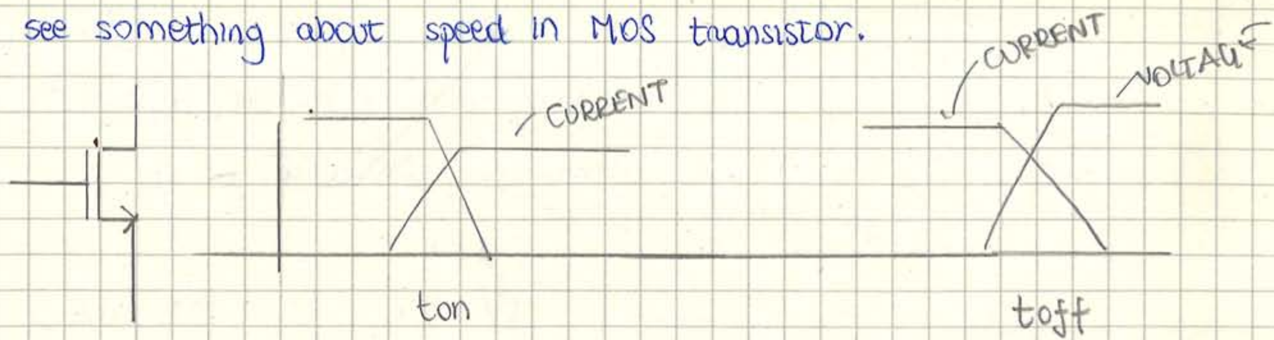
- $V_p = -(V_O + V_D)\cdot\frac{N_p}{N_S}$

For designing any magnetic component I need $\overline{V_p} = 0$:

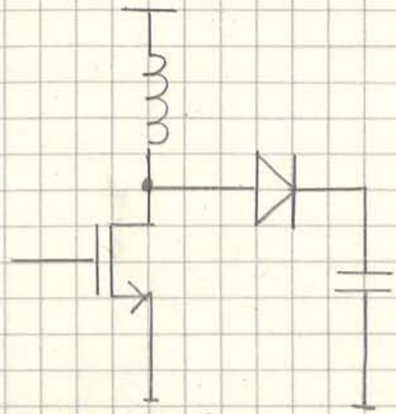
$$= D_{1MAX}(V_{in\,Min} - V_{SW} - V_{RS}) - D_{2MAX}(V_O + V_D)\frac{N_p}{N_S}$$

So finding: $$\boxed{\frac{N_S}{N_p} = \frac{D_{2MAX}\,(V_O + V_D)}{D_{1MAX}\,(V_{in\,MAX} - V_{RS} - V_{SW})}}$$

Let's see something about speed in MOS transistor.

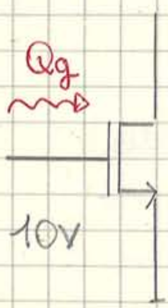


How can we drive the MOS transistor in order to have a short ton and toff?



⚠ MOS transistor are device controlled by charge (also bipolar transistor). It means that we have to inject charge quickly! Fast! and extract is quickly as well. (otherwise toff and ton are too long)

A normal MOS transistor need 10 V for being in full conduction (ohmic region); so we need to inject charge. It depends on the size of silicon

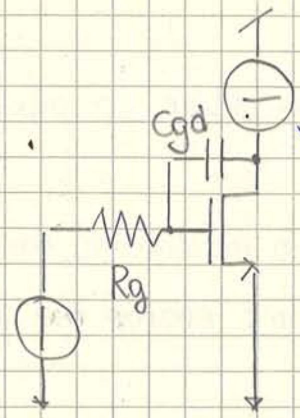


Qg is on the order (10 mC → 100 mC)
I want to inject (for example) 1 mC im 30 ms:

$\frac{1 \text{ mC}}{30 \text{ ms}}$ → CURRENT = 30 mA

Let's suppose : 30 nC in 30 ns → 1 A ! OUCH!

NB: The DC current in the gate is zero! So we need a MOS DRIVER able to deliver more than 1 A very fast (For few ms it's ok otherwise it burns!). Since the charge comes from power supply we need to ~~provide~~ provide the storage of charge and send to the MOS transistor. → CAPACITOR (across MOS driver). It is a fast capacitor (ceramic capacitor).

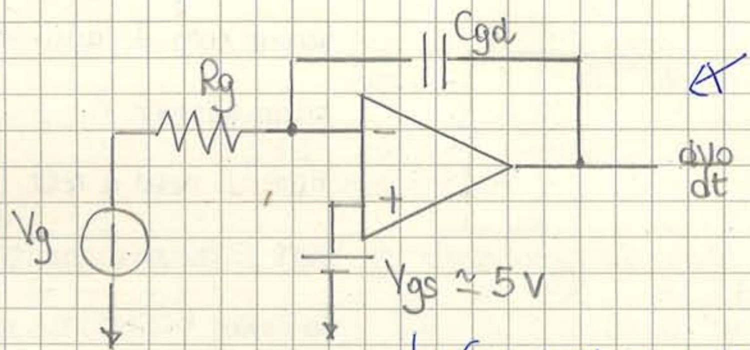


Looking for $\frac{dV_o}{dt}$?

An inductance for short time looks like a current source.

Cgd give us a very strong Miller Effect.

But let's go to applied electronics. This is a common source; so we can model our circuit like:



THIS MODEL IS OK DURING THE SWITCHING TIME (because MOS is linearity)

↳ (we need to apply a large V_{gs} in order to have the current that we want.)

Stay in time domain because it's easier.

$I_G = \frac{V_G - V_{GS}}{R_g}$ this current goes through capacitor $\frac{dV_c}{dt} = \frac{I_G}{C}$

And so: $\boxed{\frac{dV_c}{dt} = \frac{V_G - V_{GS}}{R_g \cdot C_{gd}}}$ $\rightarrow I_G$

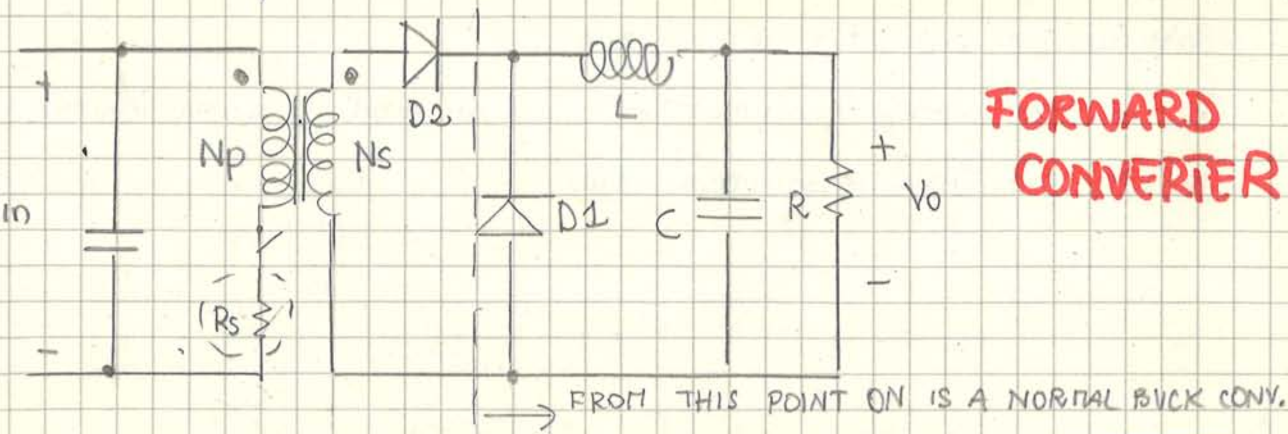
The OP-AMP has a dc-gain $10^4 \div 10^6$, but we have only a C.S. Can we use it? It's an approximation.

$V_G \simeq 10\ V \rightarrow 15\ V$ $\qquad V_{GS} = 5\ V$

$C_{gd} \simeq 200\ pF$ $\qquad 2\Omega < R_g < 15\Omega$

Let's see what happens:

$$\frac{dV_C}{dt} = \frac{0.5\cancel{A}V}{10\Omega \cdot 200\,pF} = \frac{0.5A}{200\,pF} = 2{,}5\ \frac{GV}{s} = 2{,}5\ \frac{V}{ns}$$



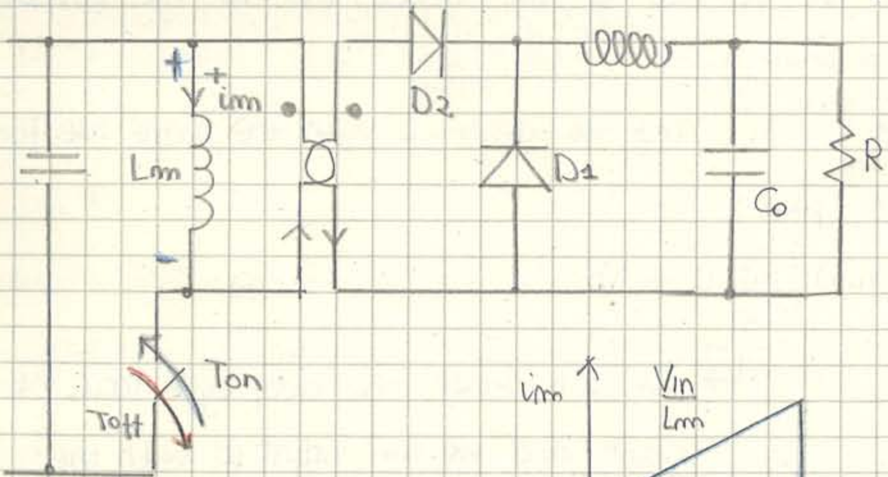
FORWARD CONVERTER

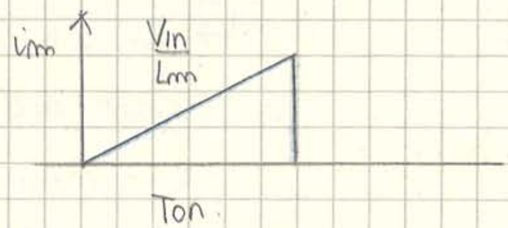
We put D2 because we need that during Toff the voltage is reversed and there is no connection.

$$V_O = \underbrace{(V_{in} - V_{SW} - V_{RS}) \cdot \frac{N_S}{N_P}}_{\text{INPUT VOLTAGE OF NEW BUCK CONV.}} \cdot D - V_D \qquad \rightarrow \text{GAIN OF FORWARD CONVERTER}$$

V_D " V_{D2} OR V_{D1}

The switch generates a lot of electromagnetic noise. We have a transformer (in this case) and $L_m \to \infty$. If we consider it we get:





When the switch is closed (Ton) we have a current im (not so large) because Lm is large. During Toff we have a current going down through Lm; it can't go on switch and it enter in a "no dot". And it means that in the secondary it wants to enter in a "dot"; and so it wants to go like this (look at schem). Is it possible? No! There is the diode D2! And we can't remove it. Looking for a way to discharge it.

Can I put $V_{in\,MAX}$ and D_{min}? Yes. But it's better on the contrary, because V_{SW} and V_{RS} becomes important when V_{in} is minimum.

We have to increase $\frac{N_S}{N_P}$ by 5% (because the transformer is real)

We also need $\frac{N_T}{N_P}$. For finding it we must guarantee that the average voltage across N_P must be zero:

$$\overline{V_P} = \underbrace{V_{in} \cdot D}_{\text{SWITCH CLOSE}} - \underbrace{V_{in} \frac{N_P}{N_T}(1-D)}_{\text{SWITCH OPEN}} = 0 \quad \Rightarrow \quad \frac{N_P}{N_T}(1-D) = D$$

FOR DOT CONVENTION

$$\boxed{\frac{N_P}{N_T} = \frac{D}{1-D}}$$

if $D_{MAX} = 0.5$

$$\left.\frac{N_P}{N_T}\right|_{D_{MAX}} = 1$$

The primary winding has a large current; in the N_T just the magnetizing current so, the tertiary is very small:

→ we can do these windings together: the par. inductances are very small ⚠

The adv. are isolation, V_O not related to V_{in}. But it's expensive! (Respect to flyback; now we have 3 windings). But we can go up in power.

VOLTAGE STRESSES

- D_1 $\quad V_{D1} = V_{in\,MAX} \frac{N_S}{N_P}$ (DURING T_{on})
- D_2 $\quad V_{D2} = V_{in\,MAX} \frac{N_S}{N_T} = V_{in\,MAX} \frac{N_S}{N_P} \cdot \frac{N_P}{N_T}$ (DURING T_{off})

During T_{off} D_3 is ON, on the tertiary winding is V_{in} and so we determine the voltage across D_2.

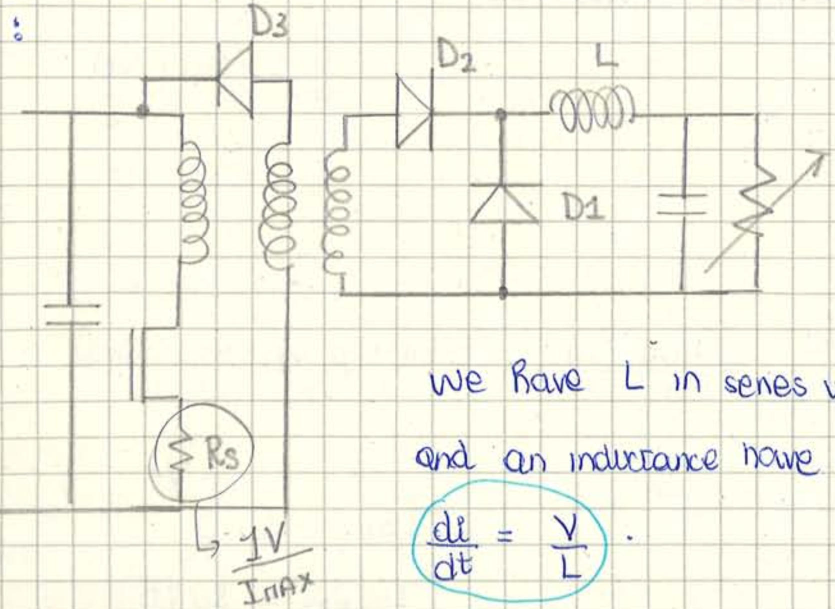
- D_3 $\quad V_{D3} = V_{in\,MAX} - (-V_{in}) \frac{N_T}{N_P}$ (DURING T_{on})

V_{in} is also applied to the tertiary winding (also on the secondary one) but in opposite way.

Let's set $D_{MAX} = 0.7$ for our forward converter. What happens?

$\frac{N_p}{N_t} = \frac{0.7}{0,3} = 2,3$ and so the V_{sw} becomes larger (3 V_{in} !!!)

We have to stay quite low than 0.5 because this is a buck derived converter:



We have L in series with the output; and an inductance have:

$$\frac{di}{dt} = \frac{V}{L}.$$

Now let's suppose that our load changes from minimum to maximum, the output current goes from a low value to a large value:

where this current coming from? From Co; but in steady state this current should came from the inductance.

Can an inductance change its current suddenly? No way. For doing this we need high voltage and time. If the voltage across L is not too much high, the current starts to increase in a very very long time.

So where does 10A come from? From capacitor. What does capacitor delivering in current? CHARGES! And the output voltage goes down.
But we have a controller that increases duty cycle in order to keep output voltage constant. So, for this reason we have to stay quite low than the maximum duty cycle. (Because of transient).

comes from the input.

$V_{in} \cdot I_{in}$ is the input power. Since we have to deliver a tot power and the current doesn't change! (MUSTN'T CHANGE).

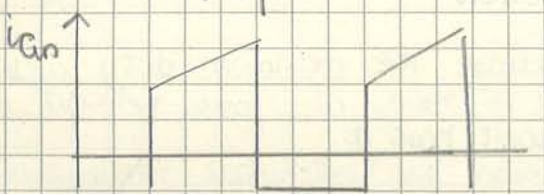
CAPACITORS

C_o NO STRESSED
C_{in} HEAVY STRESSED } like a buck converter

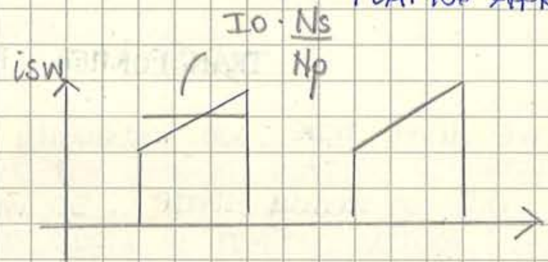
$$I_{Co_{RMS}} = \frac{\Delta I_{Lpp}}{\sqrt{12}}$$

$$I_{Cin_{RMS}} = I_o \cdot \frac{N_s}{N_p} \sqrt{D(1-D)}$$

$\rightarrow I_p$



FLAT-TOP APPROX



How much is the stress?

$\left(\text{If D is close to } 0.5 = \frac{I_o}{2} \frac{N_s}{N_p} \right)$

In CM control, with R_s we have advantages. We protect the switch and, if the transformer for any reason saturates, it opens the switch even if we have done an error in our design.

we have done with stresses, but not with losses.

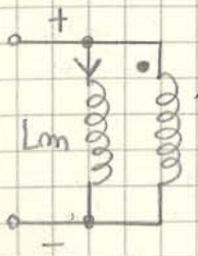
LOSSES

- $P_{D1} = V_{D1} \cdot \underbrace{I_o (1 - D_{min})}_{\text{AV. CURRENT}}$
- $P_{D2} = V_{D2} \cdot I_o \cdot D_{MAX}$

} TOTAL POWER of the two diode IS $V_D \cdot I_o$

- $P_{MOS} = P_{conduction} + P_{SWITCH} = \underbrace{r_{DS,on\,HOT} \cdot I^2_{SW\,RMS}}_{P_{COND}} + \underbrace{\frac{f_{SW}}{2} \left(V_1 \cdot I_1 t_{on} + V_2 I_2 t_{off} \right)}_{P_{SWITCH}}$

We saw that in a forward converter ; a buck converter can handle a power up to 200 W. (especially offline) ; the transformer is not used very well , but only during 40 % of the time (Ton) because voltage stresses are too high. We need the resitant time in order to reset the magnetization. But there is another way to reset the magnetization inside the core. (without no-wasted time of transformer) ; let's consider this :

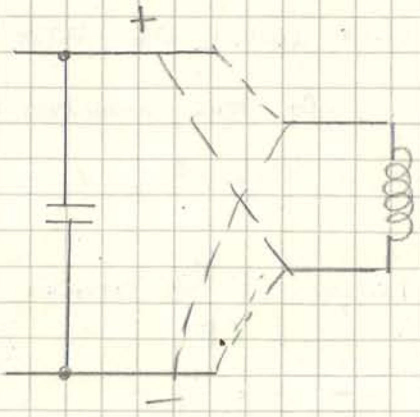


During Ton we apply positive on the dot and then we need a long Toff to reset the magnetize current by the tertiary winding. (old way).

Let's see another way. In order to reset the magn. current (otherwise the transformer saturates) we have to apply a reverse voltage (with opposite polarity). So instead of using a tertiary winding let's suppose to be able to apply to the same winding a polarity that is reversed. (for the same time, for example for 40% of the time).
This reset the core. For example we apply positive voltage for a 40% of the time and then on the secondary we rectify it and then we apply a reverse voltage for 40% of the time and rectify it. In this way we can work with duty cycle that is 80%.
What we need on secondary is a FULL WAVE RECTIFIER.
(In this way we can have an output that is twice the old frequency.)

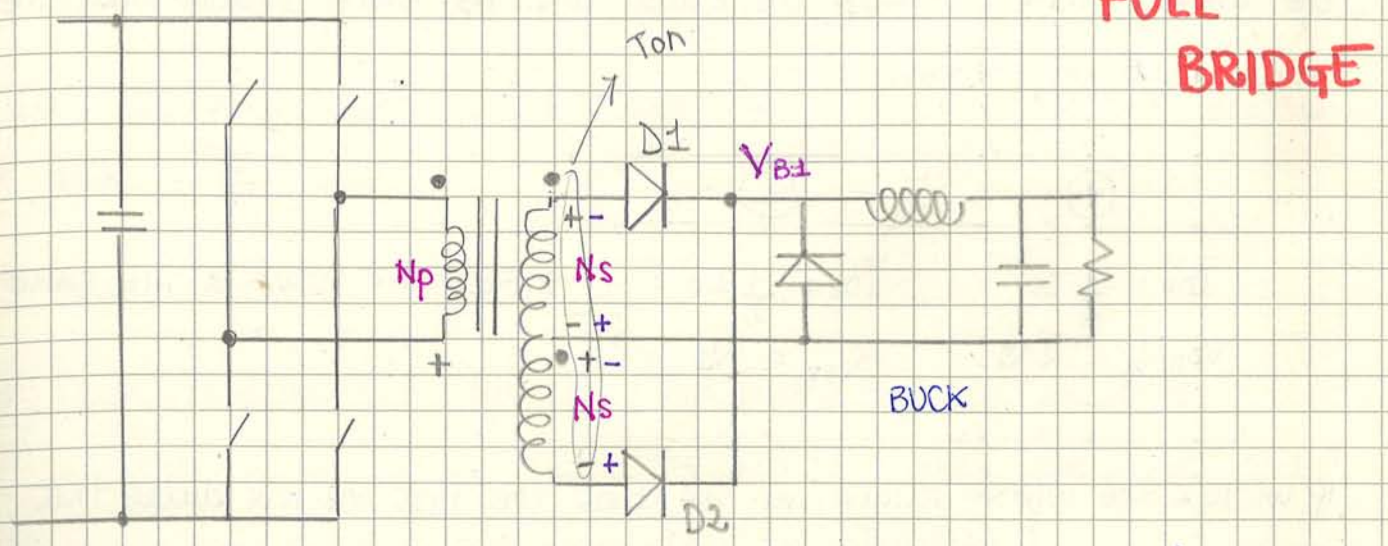
How can we swap + and − ?

4 switches



FULL BRIDGE

1

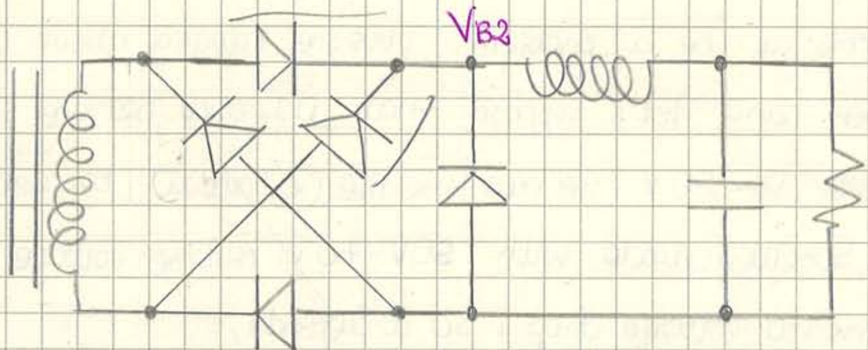


On the secondary side, current flowing (comes out from the dot) and D2 is OFF because we have negative polarity of the anode. This happens during the first half wave. During the other half wave we have on the secondary the positive on not-dot and D2 conducts, and the current flows in the opposite way. So, we have a full wave rectification.

NB: the free-wheel diode is not really needed. We see better after.

Ouch! We have two secondary winding but 2 diodes. If we want only one secondary winding:

2



GRÄTZ BRIDGE

So we have 2 possibilities. Which one is more convenient? It depends on the output voltage. How much voltage do we lose with diodes?

$$V_{B1} = V_S - V_D$$

$$V_{B2} = V_S - 2V_D$$

For low output voltage ($<30\,V$) is better the first solution.

But it's not the only consideration. We have also to consider

- $D_{SW} = \frac{T_{on\,sw}}{T_{sw}} < 0{,}5$ (otherwise {BOOM}!)

Let's see what happens at the part that is the original buck converter:

- $T_{BUCK} = \frac{T_{sw}}{2}$
- $f_{BUCK} = 2 f_{sw}$
- $D_{BUCK} = 2 D_{sw}$ Yeah!!! This means that I can obtain (for the buck!) a duty cycle close to 1! And my efficiency ↑.

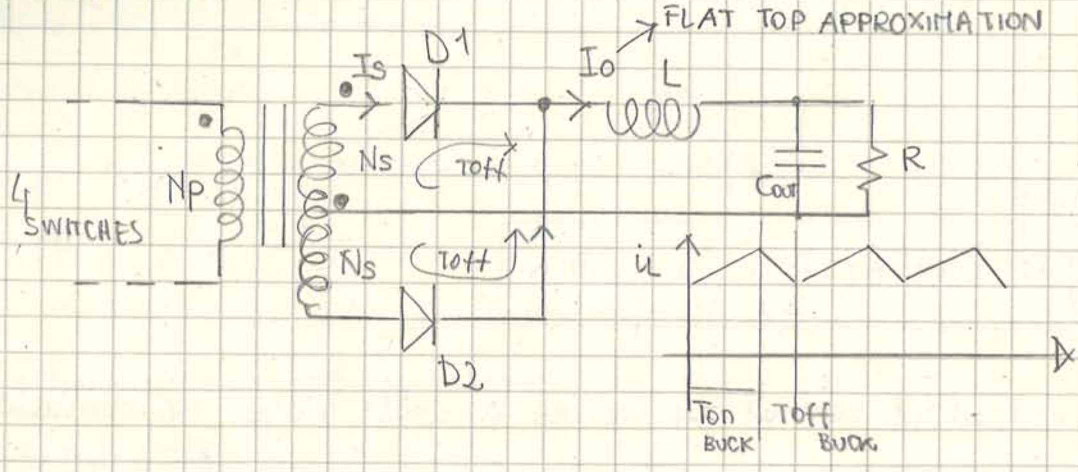
If our load decreases suddenly we need to apply a large V_B in order to ramp up quickey the current to the inductor.

We design a buck in CCM. To stay in CCM:

$$L > \frac{R_{max}(1 - D_{min})}{2 f_{sw_{BUCK}}}$$

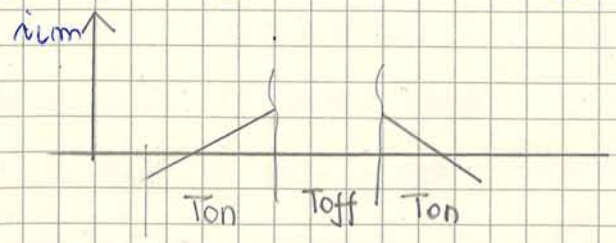
$f_{sw_{BUCK}}$ → the buck switching frequency

Do we really need the freewheel diode? We discover that our circuit works without this diode:



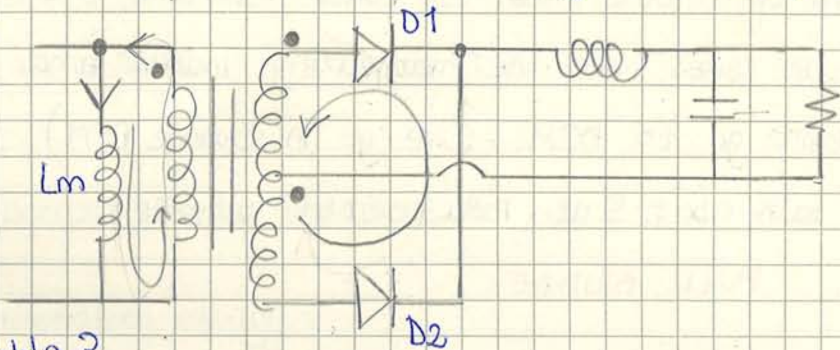
During Toff, all switches are open, and there is no current flowing in the primary and also there is no current flowing in the secondary winding. But there is a current flowing through L; if I don't put the freewheel diode, where this current coming from? One part from D1 and one part from D2. (This current is Io basically).

i_{Lm} goes up during one Ton and goes down during ~~th~~ other Ton.



What happens during Toff?
The 4 switches are open.
Let's suppose that i_{Lm} goes down.

During Toff i_{Lm} can't go to the input (switches are open), the current enter in a "no-dot" and on the secondary comes out from the dot:



Is it possible?

No, there is D2. This means that i_{Lm} must go up and on the secondary the current goes on the contrary.

Is it possible?

No, there is D1.

There is something wrong.

If I can't give a path to the magn. current, Lm has a very high strong voltage spike.

But! How is it possible? we can't use superposition! It's NONLINEAR

We saw a moment ago ~~the magn. curren~~ the output current that is divided between the two diodes.

So I have to consider the 2 currents together. During Toff:

I have Io through L, and in D1 and in D2 I also have the $\frac{I_o}{2}$ and the magn. current. In D1: $\frac{I_o}{2} + I_m$ and in D2: $\frac{I_o}{2} - I_m$ and both diode are conducting (They are ON by the output current).

We can put Rs in (1) and (2).

(1) It has some disadvantages. The common voltage goes from zero to Vin; there is a lot of switch and it's difficult to measure a voltage that swings up and down. Another dis. is that the voltage across it is bipolar (because the current flows in one sense when 2 pair of switches are closed and in the other sense when are closed the other pair).

(2) When S1 and S4 are closed (and S3 and S2 are closed) the current always flows to the left. So I put resistor Rs on (2) point.

So if current mode we have to put Rs.

So the secondary voltage:

$$V_S = (V_{in} - 2V_{SW} - V_{RS}) \cdot \frac{N_S}{N_P}$$

And from this point on we have a buck converter and so for the output voltage:

$$\boxed{V_O = V_S \cdot D_{BUCK} - \#V_D}$$

$\#V_D \rightarrow$ in C.T. is 1

So for the last equation we have to find the turn ratio $\frac{N_S}{N_P}$ with:

- V_{SW} = 1 % or 2 % of V_{in}
- D_{BUCK} : we choose it.

$$\boxed{\frac{N_S}{N_P} = \frac{V_O + \#V_D}{(V_{in} - \#V_{SW} - V_{RS}) \cdot D_{BUCK}} + 5\,\% \left(\text{of } \frac{N_S}{N_P}\right)}$$

In this formula we put $D_{BUCK\,MAX}$ and $V_{in\,min}$.

If $\frac{N_S}{N_P} \uparrow$, D_{BUCK} must be a little bit smaller.

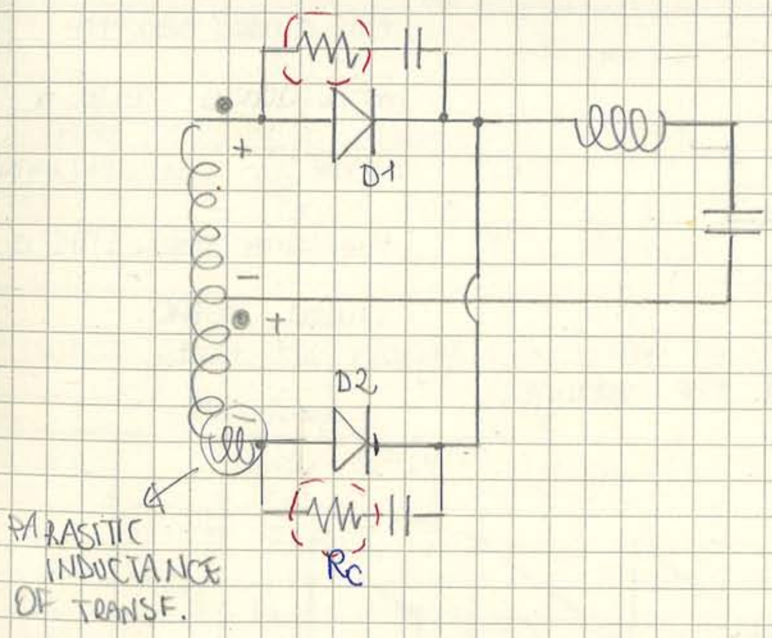
During Toff there is no driving voltage on the primary side and the current flows like (1). The current is divided between the 2 pairs of diode and this division is not guarantee to 50 %.
There is also Im that comes down. But remember! Im can't go alone down (between the 2 pairs of diode); infact we can't use superposition!

$$V_S = V_{in_{MAX}} \cdot \frac{N_S}{N_P}$$

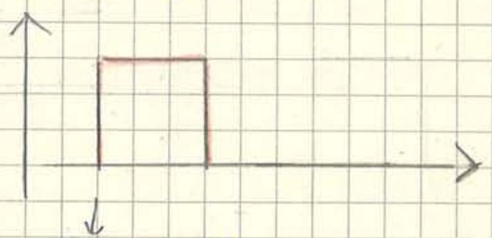
who cares about Rs and VSWITCH!

- CT $\quad V_{D_{MAX\,REV}} = 2 \cdot V_{in_{MAX}} \cdot \frac{N_S}{N_P} + V_{SPIKES\,RINGING}$
- GB. $\quad V_{D_{MAX\,REV}} = V_{in_{MAX}} \cdot \frac{N_S}{N_P}$

We have to add $V_{RINGING}$ on the central tap because:



If Vs is:



we have a big voltage on the secondary, and D2 is not conducting and it sees VSEC + VSEC. So we have the parasitic inductance of the secondary; an open diode (that has a capacitor) and so they starts to resonate, ringing. In many cases, to avoid this problem put a resistance Rc across each diode (snubber). And sometimes it can be put across the secondary winding. It's not very easy to design because we have to estimate the parasitic inductance.

if we close the first pair of switches, suppose that the voltage on the primary winding is positive (I_m ramps up, because we have positive voltage across the inductance). Then (during T_{off}) the 4 switches are open, and the current stays constant. Then applying a negative voltage on the primary side, I_m ramps down.

So:

$$I_{SW_{RMS}} = I_o \cdot \frac{N_s}{N_p} \cdot \sqrt{D_{SW}}$$

$$I_{SW_{AVE}} = I_o \cdot \frac{N_s}{N_p} \cdot D_{SW}$$

Not important for the MOS, but I find it for checking the input power.

$\downarrow$

$$I_{SW_{AVE}} = I_{in}$$

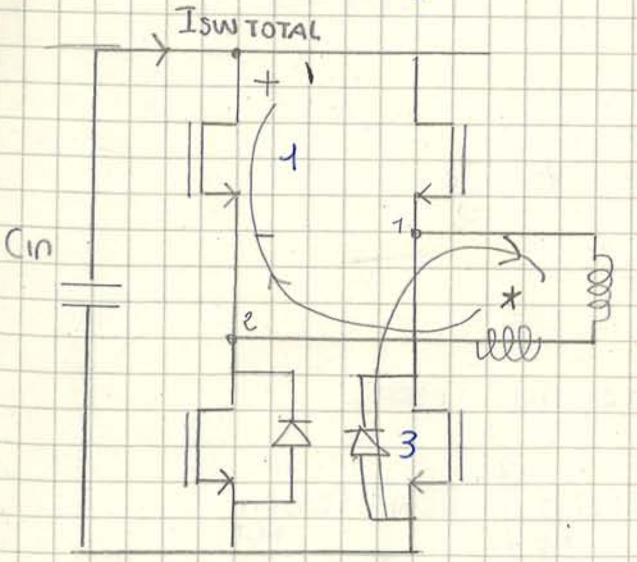
So,:

$$I_{in} = I_o \cdot \frac{N_s}{N_p} \cdot \underbrace{2 D_{SW_{MAX}}}_{D_{BUCK\ MAX}}$$

And the input power:

$$\boxed{P_{in} = V_{in_{min}} \cdot I_o \cdot \frac{N_s}{N_p} \cdot D_{BUCK_{MAX}}}$$ → ONLY FOR CHECKING!

What about the voltage across the switches?



$V_{DS_{MAX}} = V_{in}$ Why?

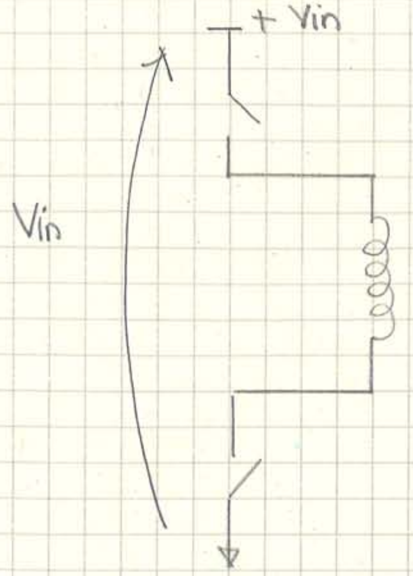
The node (2) can reach maximum (when goes down) is almost Ø (because of body diode).

The spikes that we can have are clamped because we have a leakage inductance *.

When we open the switches, it has some energy and it want to go somewhere. It can't go to the secondary side (because it is no concatened) and so

And we have a problem. Why?

Current it's easy to find. The problem is that we have 2 switches in series:



the total voltage Vin is shared between this 2 switches. We can't find it! It depends on the parassitic element, symmetry...

But we can say that we know the voltage (TOTAL) across the 2 switches is Vin.

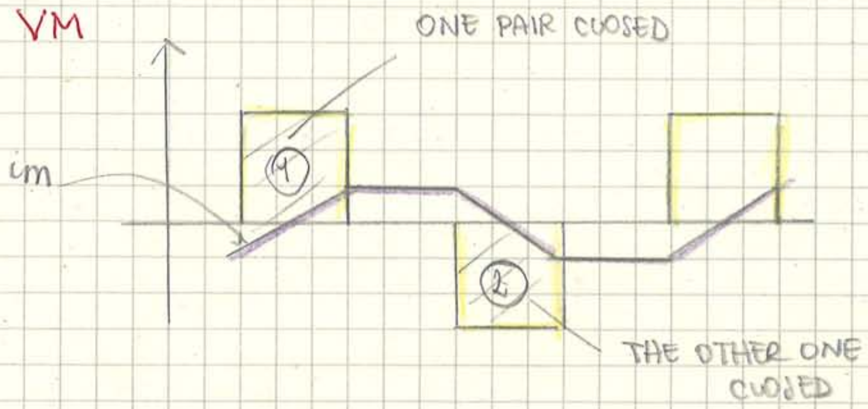
What we can say is that the power that we estimate is for one pair:

$$P_{SW\,PAIR} = f_{SW} \cdot \frac{1}{2}\left(I_1 V_{in} t_{on} + I_2 V_{in} t_{off}\right)$$

power dissipated by one pair of transistor

CONTROL (VM OR CM)

- VM

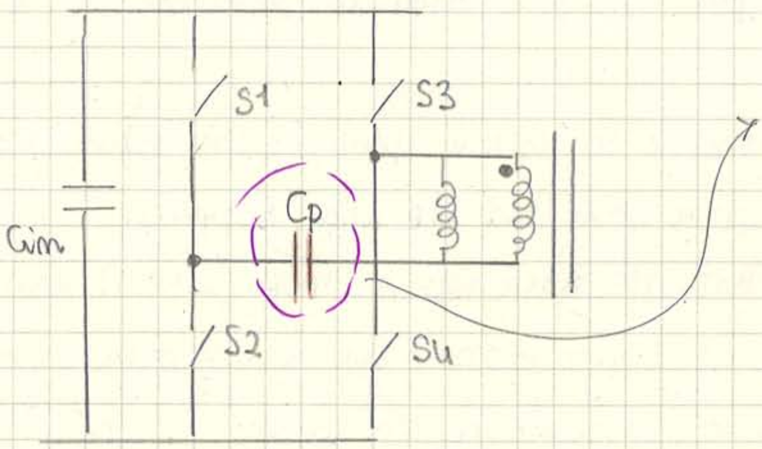


Let's suppose that some accident happens. For example the first pulse is higher than the other one (because for example the 2 switches have a lower a RDS on than the other pair).

Or we can have that the transistor are not symmetrycal and the pulse is a little bit longer.

We have many many pulses. It means that the area of pulse 1 is not exactly equal to the area 2 and Im ramps up.

→

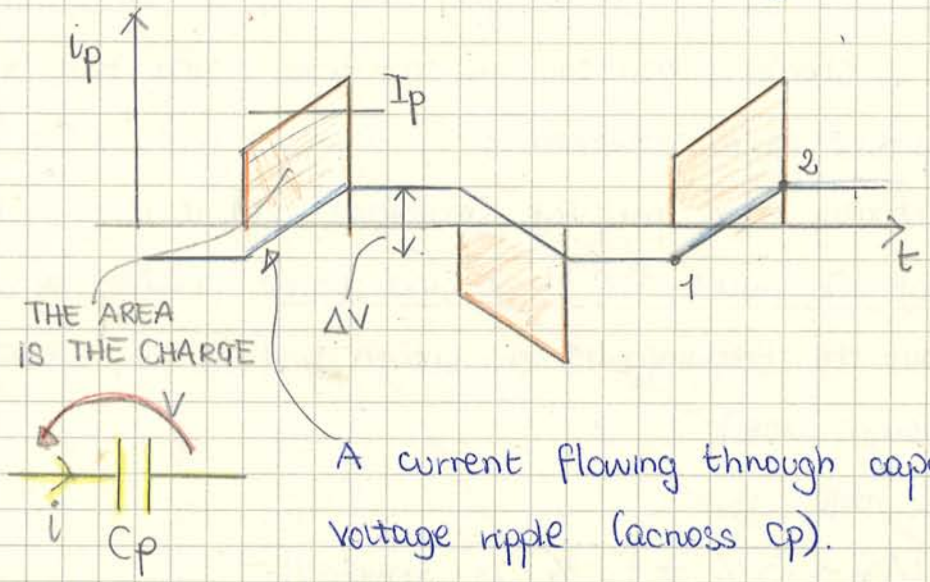


- What about WV? The voltage across Cp (the average) is a few volts; but when we switch on, at beginning of the cycle we can have here a large voltage, and for taking a safe decision let's take Vin.

$$\boxed{WV = V_{in}}$$

(Because we don't know what happens at start-up)

Cp is not an electr. capacitor, it is a film capacitor and so we have to evaluate its value. It is evaluated in this way:



A current flowing through capacitor generates voltage ripple (across Cp).

$\Delta Q = C_p \cdot \Delta V$ $\qquad \Delta V = \dfrac{\Delta Q}{C_p}$

ΔQ is the area: $\Delta Q = I_p \cdot T_{on}$ (we know everything)

How much is a suitable ΔV value over there?

A first idea is to say "I want a very small ripple voltage, so just getting Cp as larger as possible".

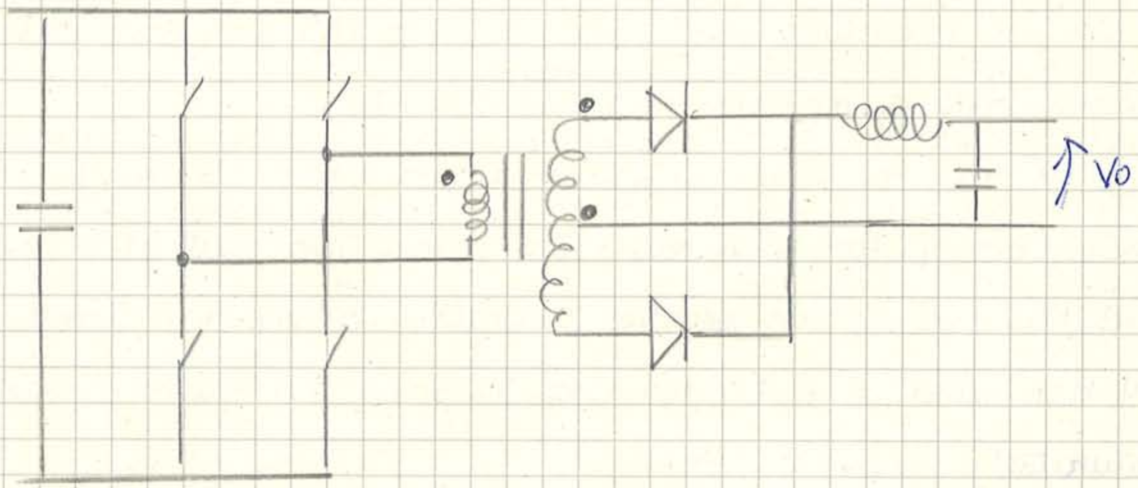
Deriving a practical equation (even if is unusefule):

$$\frac{\Delta Q}{\Delta V} = \frac{I_p \cdot T_{on}}{\frac{V_{in}}{K}} = \frac{I_o \cdot \frac{N_S}{N_p} \cdot \frac{D_{MAX}}{f_{SW}}}{\frac{V_{in}}{K}} = \frac{\overbrace{I_o \cdot \frac{N_S}{N_p}\frac{D_{BUCK}}{2 f_{SW}} \cdot V_{in}}^{\overline{I_{in}}}}{\frac{V_{in}}{K} \cdot V_{in}}$$

$$C_p = \frac{\Delta Q}{\Delta V} = \frac{P_{in}}{2 f_{SW} \frac{V_{in}^2}{K}}$$

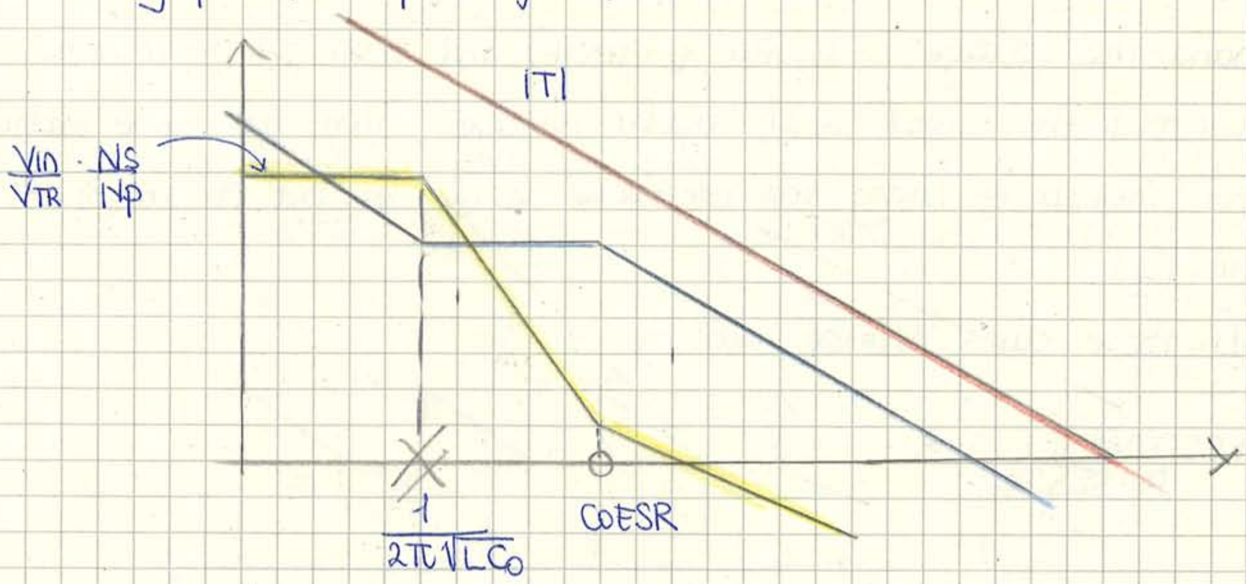
The order of magnitude of C_p is μF.

What about the transfer function between V_o and D?



D

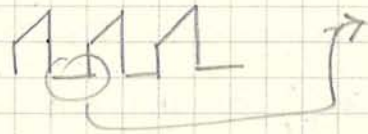
How many poles? 2 pole (fixed) and one zero



And we know how to compensate it.

Let's go in current mode.

Can we use a transformer over here? No, there is a DC! No! Yes we can because:



we have this period of time when we discharge a transformer, and so it is ok.

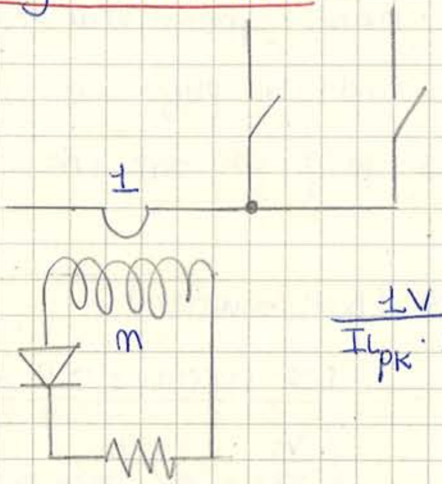
What is our goal? It's to measure over Rs 1 V.

How much is the Rs value?

$$V_{RS} = 1V \cdot I_{P_{MAX}} = R_S \cdot I_O \cdot \frac{N_S}{N_P}$$

+ RIPPLE (because we want consider very max value)

And what if we put a current transformer? It could happen, when we have a large current value.

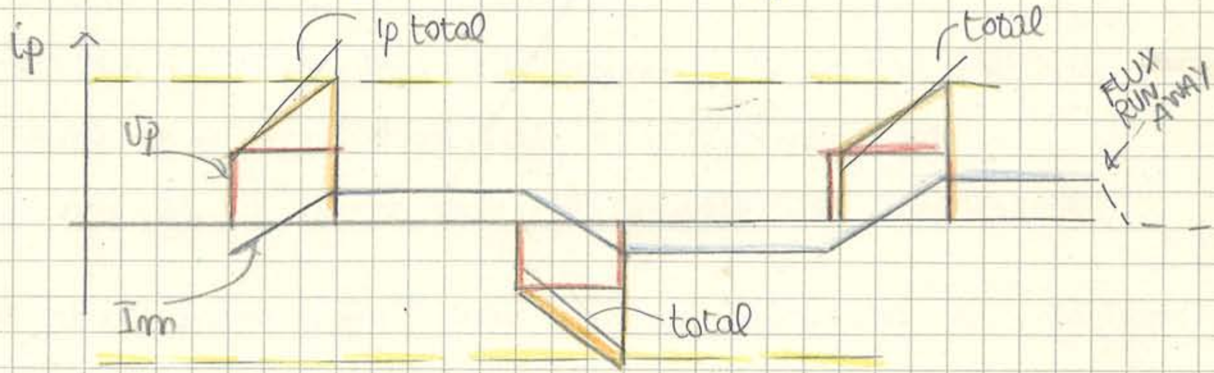


$$\frac{1V}{I_{L_{PK}} \cdot \frac{N_S}{N_P}} \cdot m = R_S$$

WITH CURRENT TRANSFORMER

So, it is a good idea to use full bridge in current mode? Yes! Because with Rs we protect the circuit from saturation, and we save Cp.

WHY DON'T WE NEED Cp IN CM? It's easy.

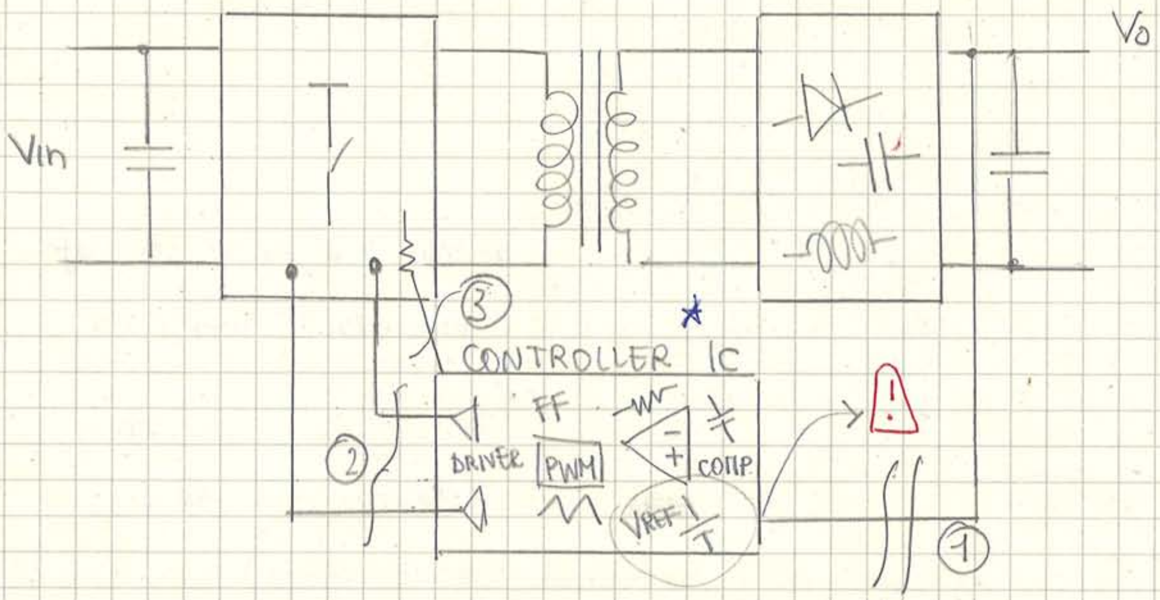


If Vp is not regular we can have flux runaway. For example if Vp is larger, smaller, higher.

Some remarks: this is an high voltage configuration. In addiction the switches "share" the stresses because the input current divides into the 2 branches. → Typical configuration for high power configuration.

HOUSE KEEPING (OF POWER SUPPLY)

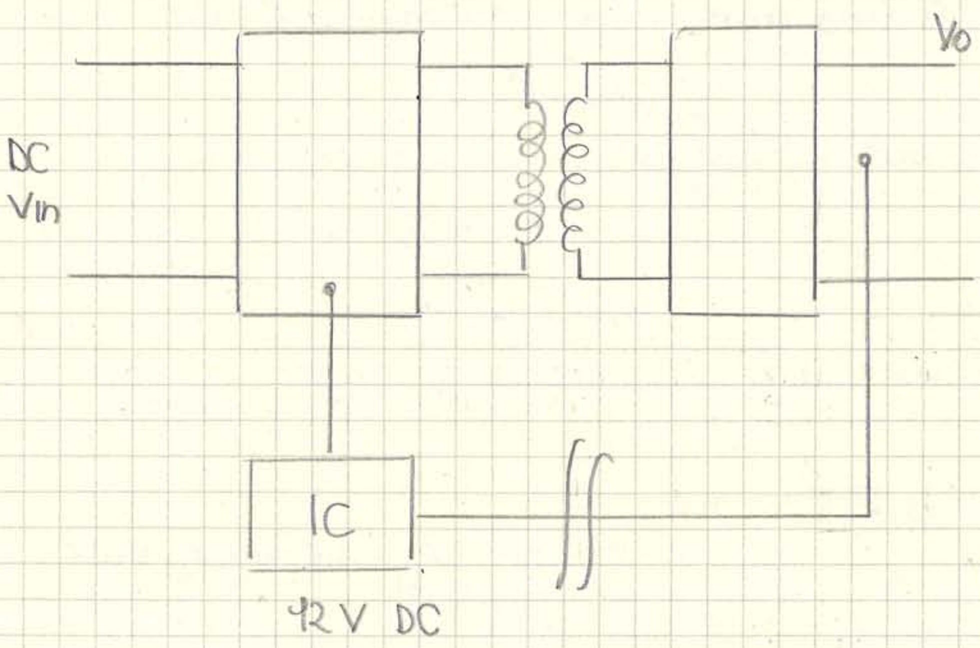
It's valid for any topology (isolated).



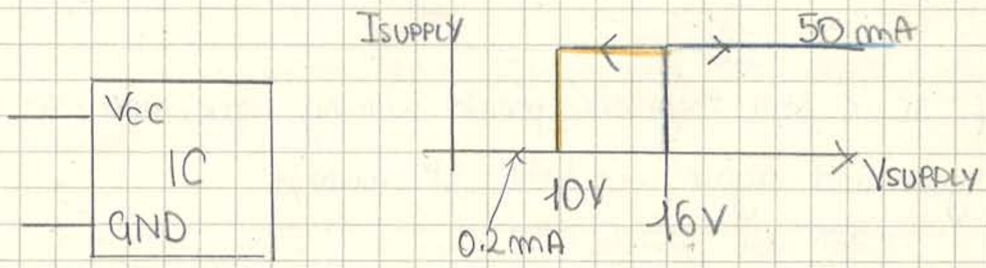
What about isolation? If we put VREF linked to the output the system is no longer isolated, so we need to place an isolation somewhere.
∫∫ → point where we can put isolation.

We can put isolation in (2); we can use driver transformer and in (3) we can have current transformer that give us isolation. These two solution changes the name of controller.
If we use (1) we say that the controller is on the PRIMARY SIDE. Because it is connected directly to the primary side.
If in (2) and (3) it is on the SECONDARY SIDE CONTROLLER.

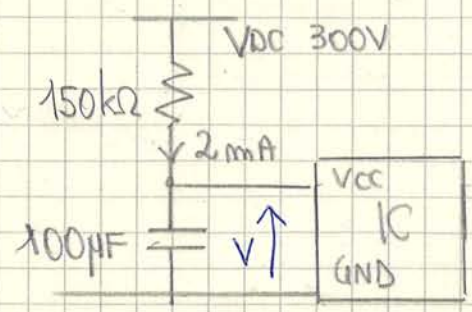
And what about power supply? How can we supply the IC controller? We put another power supply with another controller that needs power supply ☺. We can put another transformer here *, but



If we have Vin = 15 V DC we connect directly Vin to our IC and it's ok.
If Vin = 24V, 48V we can still use a linear regulator because IC doesn't take too much power. (~ 100 mW). And if Vin = 300 V ? We can't use a resistor to going down in voltage because, for example if I = 50 mA , P = 300 V · 50 mA = 15 W ! No, we don't want to ~~waste~~ this power just to supply the IC!
ICs have a specific feature that is called LOW START UP CURRENT. :



When the input voltage reaches a given threshold (for example 16V), ok, it's time to work. If the voltage comes back it stays up to another threshold (10 V) and then it stops. With these ICs we can do:



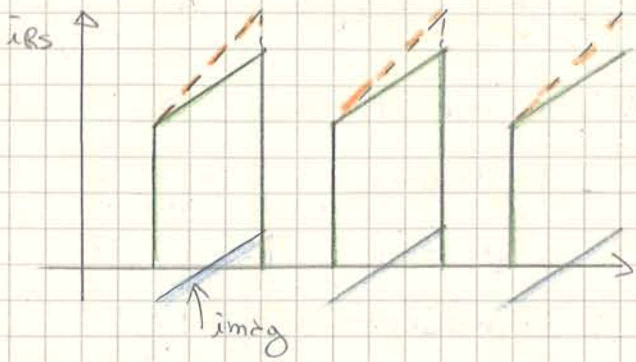
$2\,mA \cdot 300\,V = 0.6\,W$ (Better!)
C starts to charge slowly and it would charge to 300 V, but when V = 16 V, IC wakes up and it

Do we need a compensation ramp? We need it if $D > 50\%$. Yes! Because we have D_{BUCK}.

So we have to move the compensation ramp (that is in L) on the primary side. $\frac{V_o}{2L} \left(\frac{A}{\mu s}\right)$

Let's see the adv. and disadv. of using current transformer or only Rs.

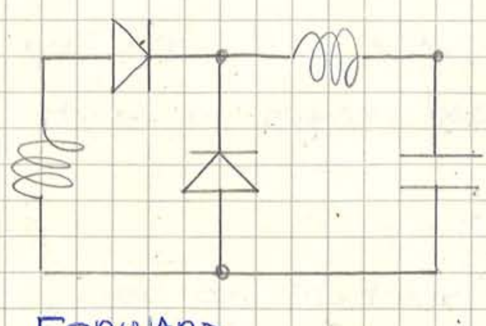
The current through Rs is:

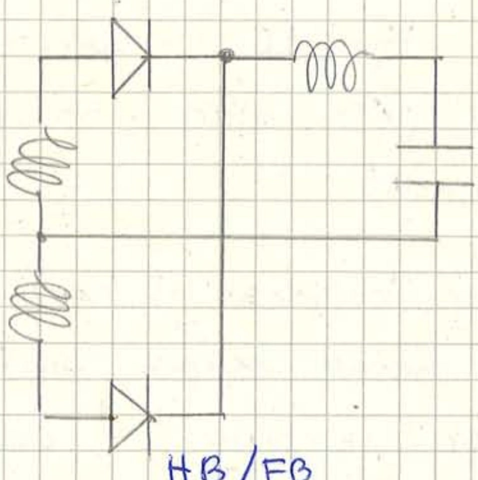


i_{RS} its the standard input waveform for a Buck converter. We have a very high D because we are measuring the current over there. We have to remember that what we have on the primary side is not only the secondary current moved on the primary side, but there is also the magnetizing current in top of it. This current has the effect of having a steeper i_{RS}.

We have to put a compensation ramp, but the magnetizing current is already helpful!

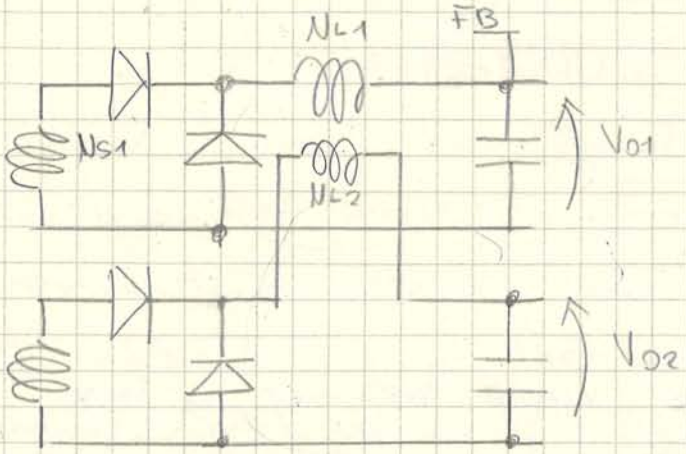
Let's consider the multiple outputs solution for Buck derived converters.





What if the first output passes from CCM to DCM?
The controller keep the output voltage constant reducing the D so the voltage n the second output goes down!

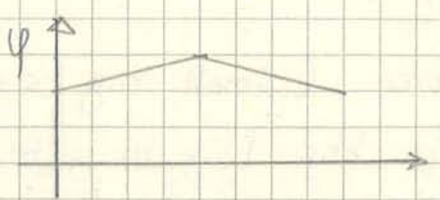
So the solution is this one:



$$\frac{N_{L1}}{N_{L2}} = \frac{V_{01}}{V_{02}} = \frac{N_{S1}}{N_{S2}}$$

A very huge advantage of this solution is that we have only one magnetic core

We still take the feedback from the first output but this time if the load in ② changes the change goes immediately in ① and so the controller can do something! So the dynamic behaviour is better.
We still have the problem CCM → DCM because for example if ② goes in DCM current tries to go backward, but there are the two diodes.
In this case is not so bad as before because the flux in the coupled inductors is unique and this flux $\varphi = L \cdot I$ is translated into a current ripple.



It is the so called Ripple-Steering

Flux goes up and down → current ripple
but we can steer the ripple on the output we want so ① never goes to DCM. ② has problem to go in DCM because the ripple is small (unless the current is very small). $\varphi = \varnothing$ → System DCM

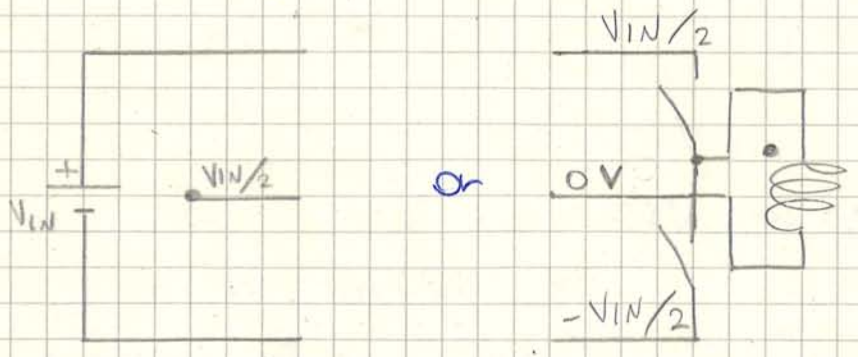
In many cases in multiple outputs Buck-derived converters, we can find one big transformer with all the outputs windings and small ring cores in series with the secodaries in order to make the ripple steering.

HALF BRIDGE

Why we moved from forward converter to full bridge? We want to reset the magnetic core without wasting time. We can force the input voltage to be reversed so at the same time we reset the magnetic core and we use the transformer to transfer power! We have to remember that if the reset isn't very precise we have the flux runaway. (So we have to put a capacitor in series).

The full bridge has a very big disadvantage: we have 4 switches. This is the reason why for low power the full-bridge is a waste of money (we have 2 switches in series with V_{IN}).

There are other ideas to solve this...
For example:



ADV: 2 SWITCHES
DIS: • ASYMM. SUPPLY
• $V_P = \frac{V_{IN}}{2}$

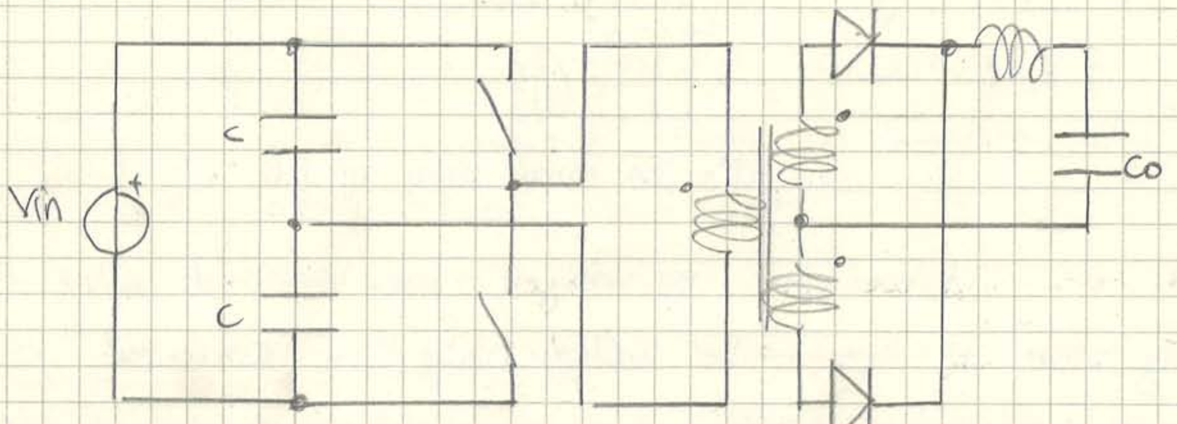
If we have $V_{IN}/2$, 0 and $-V_{IN}/2$ and we want to supply the primary with an AC we can use two switches!

* If V_p it's a little bit longer we have an extra current an so the voltage increases. (ACROSS C)
So $D_{S1} > D_{S2}$ V_C won't be @ $V_{IN}/2$ but
little bit
a little bit higher because the two capacitors absorb the unbalance

Half bridge has the advantage of having only two switches but the primary voltage is only $V_{IN}/2$ and we need the two capacitors.

Long time ago maybe two caps costed less than two switches

Obviously having a $V_p = V_{IN}/2$ requires a change in the turn ratio to get the same output voltage. It will also modify the current ratio.



I'm looking for $\frac{V_o}{V_{IN}}$ to design the transformer.
We have a Buck so:

$$\cancel{\frac{V_o}{\ldots}} = D \cdot V_{sec} - V_D = D \cdot V_p \cdot \frac{N_s}{N_p} - V_D = D \cdot \left(\frac{V_{IN}}{2} - V_{SW}\right)\frac{N_s}{N_p} - V_D$$

center tap

always a drop on a switch

So the $\boxed{I_{IN}\big|_{HB} = 2\, I_{IN}\big|_{FB}}$

because we have to deliver the same power with $V_{IN}/2$

so less voltage $\Rightarrow$ double current.

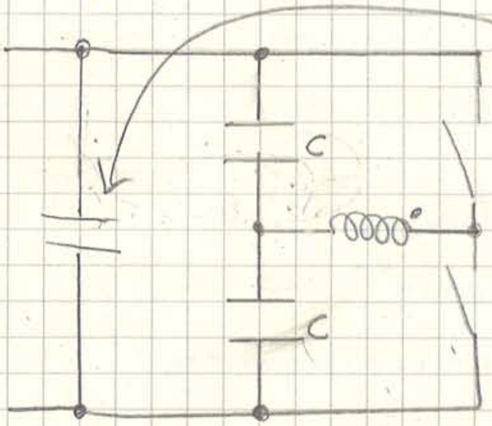
Let's remove one possible error from this design:

Do we control it voltage mode or current mode?

Current mode it is absolutely forbidden because we have a capacitor in series with VIN so we cannot put Rs.

It is voltage mode only.

Actually it can be done but it's very complicated since requires an extra winding. Let's see now input capacitor:



Have we to put it?

The capacitance seen by the input winding is $C_P = 2C$ because C & C are in parallel for the high frequency coming out from the winding.

How we design these two caps?

Exactly as we did it for the full bridge.

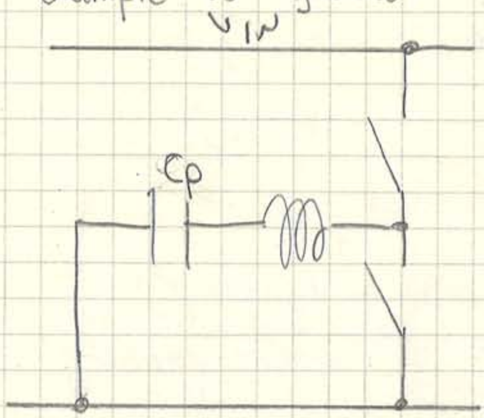
The two caps make $V_{IN}/2$ but are also used to absorb the unbalances without saturates the primary winding.

$$C_P = \frac{\Delta Q}{\Delta V} = \frac{t_{ON}\, I_P}{10\%\; V_{IN}/2} \qquad \left(V_P = \frac{V_{IN}}{2}\right)$$

$(\Delta V_{IN} \simeq 5\% \div 15\%\ V_{primary})$

COMPARING: $C_P\big|_{HB} = 4\, C_P\big|_{FB}$ because $\begin{cases} I_P\big|_{HB} = 2\, I_P\big|_{FB} \\ \Delta V\big|_{HB} = \frac{1}{2}\Delta V\big|_{FB} \end{cases}$

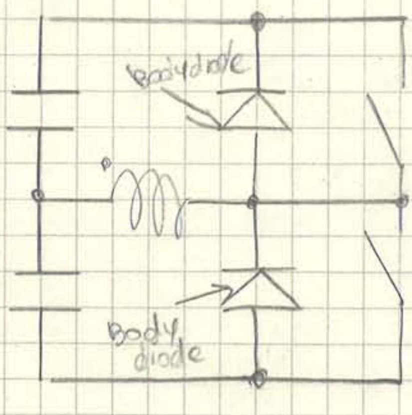
We have a DC in series with a cap
What's the effect?
When we have a cap connected to a fixed voltage point, we can move that point to any other fixed point in the circuit without changing nothing, for example to ground:



It seems that I can make this circuit and save one cap, but if I make it and I start the circuit... KABOOM!

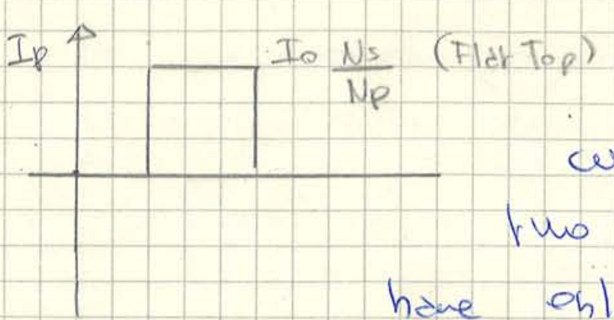
Wait a moment. I derived this circuit using two exact rules. Why it doesn't work?
There is something that seems to be right! But it isn't! → EQ

Let's go back to our circuit and in particular on the switches stresses



$\boxed{V_{DS\,MAX} = V_{IN}}$

like in a full bridge
It tells me that this topology is for high voltage! But for not so high current because of the current stresses. (ALL CURRENT GOES ALL IN ONE BRANCH)

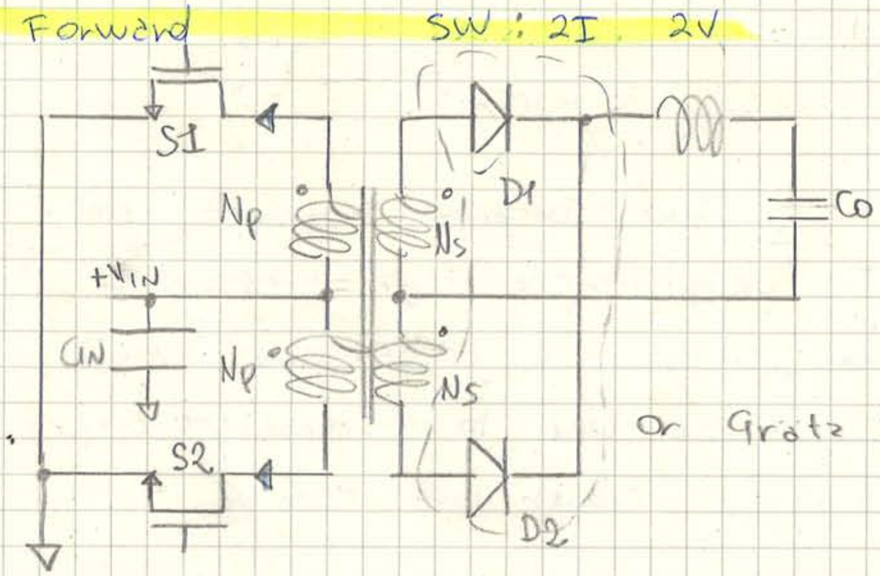


In a full bridge the DC current is shared between the two legs. In an HB we have only one leg! And all DC current comes over here.

Push - Pull topology

Before looking to the last topology let's make a comparison

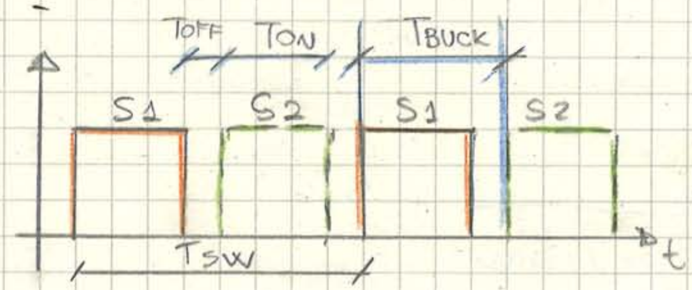
		MAXIMUM VOLTAGE STRESS
_ Full Bridge	SW : I	V
_ Half Bridge	SW : 2I	V
_ Push Pull	SW : I	2V ← medium power low V_{IN}
_ Forward	SW : 2I	2V



PUSH PULL

Or Gratz bridge

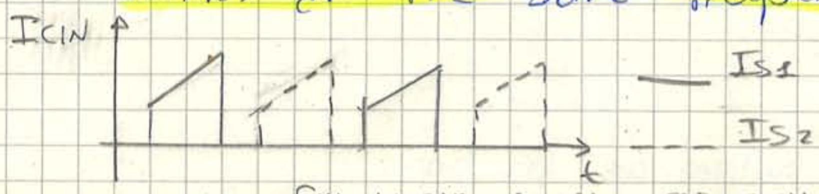
As usual we drive the switches in a non overlapping way



When we close S_1 we apply V_{IN} on non-dot.
So we have the positive on non-dot on the secondary and vice versa. (With S2 ON, magnetic flux starts to decrease because S2 ON, D2 OFF)

What about the input capacitor? What it seen as current?

It experiences both the current in S1 and S2. It works at the same frequency as Buck.



TWICE FREQUENCY OF S1 AND S2.

- CIN WORKS AT SAME FREQUENCY OF BUCK CONVERTER -

As usual the magnetizing current increase the slope but this is useless for the stresses

$\overline{I_{SW}} = I_o \frac{N_S}{N_P} \cdot D_{SW}$ ← BJTs and estimate P_{IN}

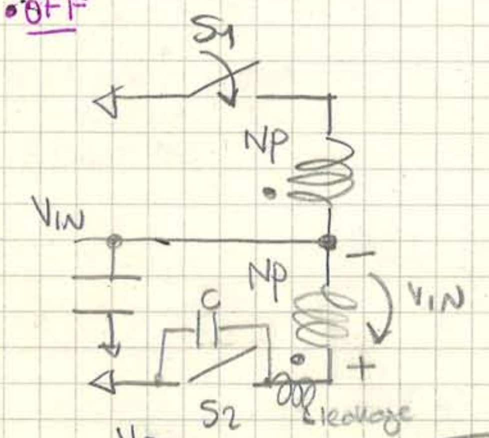
$I_{SW_{RMS}} \simeq I_o \frac{N_S}{N_P} \sqrt{D_{SW}}$ ← we can choose the wire gauge of our transformer

We can check our calculations:

$P_{IN} = V_{IN} \cdot \overline{I_{SW}}$

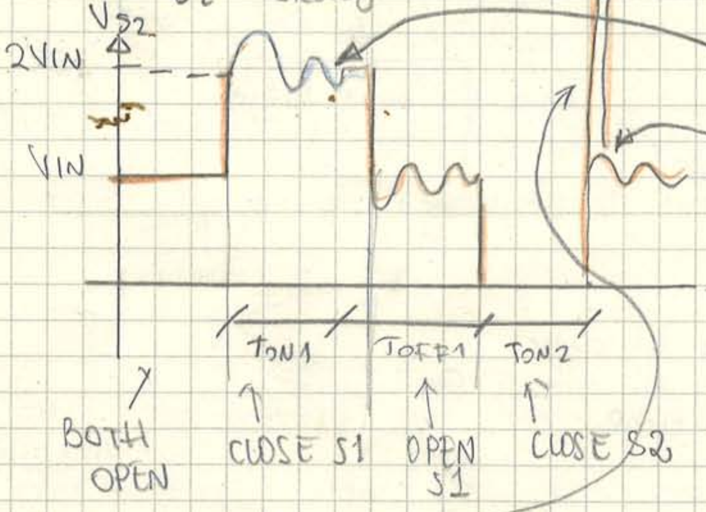
V_{SW} → ON few % of V_{IN} ; → OFF } we have 2 voltages for V_{SW}

• OFF



When we close the switch we have V_{IN} in series with V_{IN} so the open switch experiences $2V_{IN}$. (dot down per numeri positivi)

But when we close the switch: S1



We have an LC circuit. (AN OPEN SWITCH HAS A CAPACIT. AND LEAK)

When we open the switch S2 the voltage should go to V_{IN}, but there is an LC so we have a ringing. BUT! THERE IS A PROBLEM

Low voltage + Medium power → lot of current that flows in the leakage ind

Since there is a lot of current in the leakage inductor when we stop the flow the inductor get mad! There isn't a recycle path. so we have to put FOR BLOCK THE SPIKE a tranzorb or a zener. The spike is $\simeq 6V_{IN}$