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A P P U N T I

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MATERIA: Physics I Eserc.(GB) Prof.Iazzi

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PHYSICS I

Exercises

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Asunto PHYSICS I Fecha

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$\frac{d\,(\text{distance})}{dt}$ = speed (velocity if you know that direction)

$f(x) = x$ $\quad \frac{d\,f(x)}{dx} = 2 \cdot x^{2-1} = 2x$

$f(x) = x^2$

Partial derivatives ↳ when you have more than two variables

ex

$f(x,z) = axz - \sin(2x)$

$\frac{df}{dx} = a \cdot 1 \cdot x^{1-1} z - 2\cos(2x)$

$x = 30$
$z = 10$
$a = 0{,}5$

$f(x,z) = (0{,}5)(30)(10) - \sin(60) = 14.9$

$\frac{df}{dx} = (0{,}5)(10) - 2\cos(60) = 4$

$\frac{df}{dz} = axz^{1-1} = ax = 15$

Asunto Fecha

Length $= L = (3{,}70 \pm 0{,}01)$ cm

Width $= W = (2{,}30 \pm 0{,}01)$ cm

Area = ?

ΔA = ?

$$A = L \cdot W = 8{,}51 \text{ cm}^2$$

$$\% \text{ error} = \% \frac{\Delta L}{L} = \frac{0{,}01}{3{,}70} = 0{,}2\ \%$$

$$\% \text{ error} = \% \frac{\Delta W}{W} = \frac{0{,}01}{2{,}30} = 0{,}43\ \%$$

$$\frac{\% \Delta A}{A} = \% \text{ error } A = \frac{\% \Delta W}{W} + \frac{\% \Delta L}{L} = 0{,}2\% + 0{,}43\% = 0{,}63\ \%$$

$$\Delta A = A \cdot 0{,}63\% = 0{,}05 \text{ cm}^2$$

$$\Longrightarrow \boxed{A = 8{,}51 \pm 0{,}05 \text{ cm}^2}$$

ex

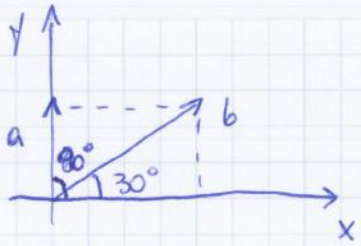
8000 m = ? feet

1 foot = 12 inches

1 inch = 2,54 cm = 0,00254 m

$8000 : x = 0{,}03048 : 1$ $\qquad x = 262467{,}19$ feet

Asunto | Fecha



$a_x = 0$

$a_y = a$

$b_x = \frac{\sqrt{3}}{2} b = \sqrt{3}\, a$

$b_y = \underbrace{a = \frac{1}{2} b}_{(b = 2a)}$

$b = \sqrt{a^2 + 3a^2} = 2a$

$\vec{a} + \vec{b} = a\sqrt{3}\,\vec{i} + 2a\,\vec{j}$

$\vec{a} - \vec{b} = -a\sqrt{3}\,\vec{i}$

$\vec{a} + 2\vec{b} = 2a\sqrt{3}\,\vec{i} + 3a\,\vec{j}$

$\vec{a} - 2\vec{b} = -2a\sqrt{3}\,\vec{i} - a\,\vec{j}$

PRODUCT

① Scalar product $\vec{a} \cdot \vec{b} = |b| \times (|a| \cos\theta)$: dot/inner product

② Vector product $\vec{a} \times \vec{b} = |b| \times (|a| \sin\theta)$: cross/outer product

ex $\vec{a} = 2\vec{i} + 3\vec{j}$

$\vec{b} = 5\vec{i} + \vec{j}$

$\vec{a} \cdot \vec{b} = ?$ $\qquad \vec{a} \times \vec{b} = ?$

$\vec{a} \cdot \vec{b} = (2\vec{i} + 3\vec{j})(5\vec{i} + \vec{j})$

$= (2\times5)\,\vec{i}\vec{i} + (2\times1)\,\vec{i}\vec{j} + (3\times5)\,\vec{j}\vec{i} + (3\times1)\,\vec{j}\vec{j}$

$= 10\,\vec{i}\vec{i}\cos 0^\circ + 2\,\vec{i}\vec{j}\underbrace{\cos 90^\circ}_{0} + 15\,\vec{j}\vec{i}\underbrace{\cos 90^\circ}_{0} + 3\,\vec{j}\vec{j}\cos 0^\circ$

$= 10 + 3 = 13$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 5 & 1 & 0 \end{vmatrix}$

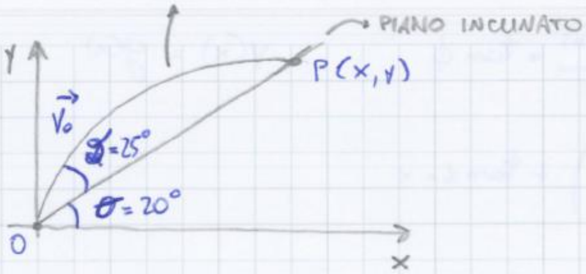
det

$= \hat{i}(3\times0 - 0\times1) - \hat{j}(2\times0 - 0\times5) + \hat{k}(2\times1 - 3\times5) = 0 - 0 + \hat{k}(-13) = -13\hat{k}$

Asunto | Fecha

IMP

(ex)



$\overline{OP} = ?$

$|\vec{v_0}| = 100 \frac{m}{s}$

Let $\alpha = \theta + \phi = 45°$

$\dot{x} = V_0 \cos\alpha$

$\dot{y} = V_0 \sin\alpha$

$\dot{z} = 0$

Position:

$x(t=0) = 0$

$y(t=0) = 0$

$z(t=0) = 0$

Acceleration $\ddot{x} = 0$ Integrating, $\int \ddot{x}\, dt = v_x$ $\boxed{\dot{x} = c_1}$

$\ddot{y} = -g \longrightarrow$ $\int \ddot{y}\, dt = \int -g\, dt$ $\boxed{\dot{y} = -gt + c_2}$

$\ddot{z} = 0$ $\boxed{\dot{z} = c_3}$

$c_1 = v_0 \cos\alpha$

$c_2 = v_0 \sin\alpha$

$c_3 = 0$

Integrating,
$x(t) = c_1 t + c_4$

$y(t) = -\frac{gt^2}{2} + c_2 t + c_5$

$z(t) = c_3 t + c_6$

Asunto Fecha

ex

CAR CAR

$A \leftrightarrow B \rightarrow$

$0,186\ \text{Km} = d = 186\ \text{m}$

$A \Rightarrow |\vec{v}_A| = 24,4\ \frac{m}{s}$

$B \Rightarrow |\vec{v}_B| = 18,6\ \frac{m}{s}$

1^{st} case) → same direction
2^{nd} case) → opposite direction } WHAT HAPPENS TO t ?

$$v_A = \dot{x}_A = \frac{dx_A}{dt}$$

integrate

$$x_A = v_A t + c_A$$

$@t=0 \quad x_A = c_A$

$c_A = 0$

$$-v_B = -\dot{x}_B = -\frac{dx_B}{dt}$$

integrate

$$x_B = -v_B t + c_B$$

$@t=0 \quad x_B = d$

$c_B = d$

To meet/catch $x_A = x_B$

$$v_A t = -v_B t + d$$

$$t = \frac{d}{v_A + v_B} = 32\ s$$

$= 4,3\ s$

Asunto   Fecha

ex

$\tau = 20\,s$

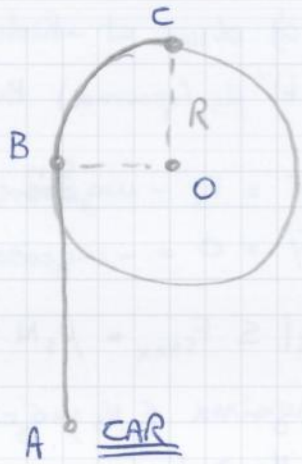
$R = 150\,m \quad a_0 = 4\,m/s^2$

At $t=0$ → full acceleration.

Then the pedal is released at a rate $a(t) = a_0 e^{-t/\tau}$.

A → B after $t = 50\,s$

Find: 1) $\overline{AB}$
2) $\vec{a}$ before B
3) $\vec{a}$ after B



1) $v(t) = \int a(t) = a_0 \cdot (-\tau) \cdot e^{-t/\tau} + c_1$ $c_1 = +\tau \cdot a_0$

$-\frac{t}{\tau} = Q$

$-\frac{1}{\tau} dt = dQ$

$dt = -\tau\, dQ$

$s(t) = \int v(t) = \tau^2 \cdot a_0 \cdot e^{-t/\tau} + \tau \cdot a_0 \cdot t + c_2$ $c_2 = -\tau^2 \cdot a_0$

$s(t) = 400 \cdot 4 \cdot e^{-50/20} + 20 \cdot 4 \cdot 50 - (400 \cdot 4)$

$= \boxed{2531\,m}$

2) $a(t) = a_0 e^{-t/\tau} = \boxed{0{,}33\,m/s^2}$

3) $\vec{v} = \dot{s}\,\vec{\tau}$

$\vec{a} = \ddot{s}\,\vec{\tau} + \frac{\dot{s}^2}{R}\vec{m}$

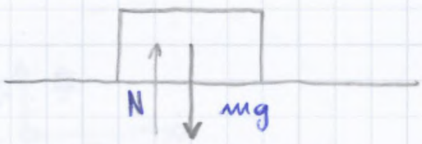
$= (0{,}33)\vec{\tau} + \left(\frac{\dot{s}^2}{150}\right)\vec{m}$ $\dot{s} = a_0(-\tau) \cdot e^{-t/\tau} + \tau a_0 = 73{,}43\,m/s$

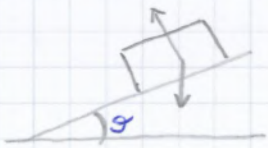
$= (0{,}33)\vec{\tau} + (35{,}95)\vec{m}$

$= \sqrt{0{,}33^2 + 35{,}95^2} = \boxed{35{,}95\,m/s^2}$

Asunto | Fecha

- STRONG FORCES
- WEAK FORCES
- FRICTIONAL FORCES (OPPOSITE)
- GRAVITATIONAL FORCE $(m \cdot g)$





$$\left.\begin{aligned} F &= -mg\vec{j} \\ ma_y &= -mg\vec{j} \\ a_y &= -g\vec{j} \end{aligned}\right\} \text{in some problem} \qquad \begin{aligned} \ddot{x} &= 0 \\ \ddot{y} &= -g\vec{j} \\ \ddot{z} &= 0 \end{aligned}$$

$$\vec{F}_T = \frac{d\vec{p}}{dt} \qquad \vec{p} = m\vec{v} \longrightarrow \text{MOMENTUM (mom. velocity)}$$

$$\vec{F}_T = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} \qquad \vec{F}_T = m\vec{a}$$

Asunto Fecha

$$Circ = 2\pi R = 6{,}28\ m$$

$$N^\circ \text{ of rounds} = \frac{20\ m}{6{,}28\ m} = 3{,}18$$

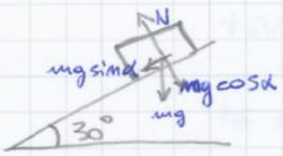
$$0{,}18 : 1 = x : 2\pi \qquad x = 1{,}13\ rad$$

$$\boxed{\vartheta\ (\text{degrees}) = 64{,}8^\circ}$$

$$1{,}13 : 6{,}28 = x : 360$$
$$x = 64{,}8^\circ$$

ex

A body of mass m is dropped from the top of a frictionless plane (30°), 2 m long. Find the t necess. for the body to reach the end of the plane



For x-axis $\quad mg\sin\alpha = a_x m$

y-axis $\quad -mg\cos\alpha + N = \cancel{a_y m}$

$$\Rightarrow \boxed{N = mg\cos\alpha}$$

$$\boxed{a_x = g\sin\alpha}$$

$$x(t) = x_0 + v_0 t + \frac{1}{2} a_x t^2$$

$$x(t) = \frac{1}{2}(g\sin\alpha)t^2$$

$$x(t = \ell) = \frac{1}{2}(g\sin\alpha)t^{*2} = \ell$$

$$t^* = \sqrt{\frac{2\ell}{g\sin\alpha}} = 0{,}9\ s$$

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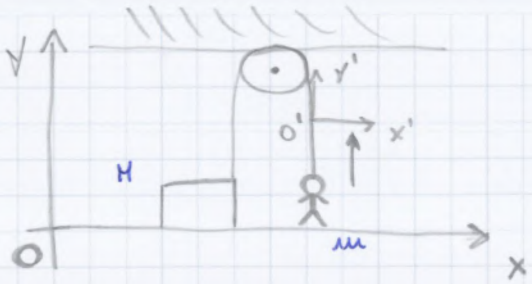
Asunto Fecha 02/04/14



$m < M$

The person starts to climb the rope

$m = 60\,kg$ $M = 100\,kg$

2 reference frame

↳ static (x, y), origin in O

↳ dynamic (x', y') origin in O'

Acceleration of man in (x', y') as $\ddot{y}' = a$

" " " in (x, y) as $\ddot{y}$

" of block in (x, y) = $\ddot{Y}$

$$\ddot{y} = \ddot{y}' - \ddot{Y} \quad (\text{no terms in } \omega \, (=0))$$

$$= a - \ddot{Y}$$

$$(\ddot{y}' + \ddot{y}_{O'})$$

$-mg + T = m\ddot{y}$ for man

$-Mg + T = M\ddot{Y}$ for block

$-mg + T = m(a - \ddot{Y})$

correaanayak

Asunto | Fecha

$$a = \frac{m_A g - T_1}{m_A}$$

$a = 1{,}42 \ m/s^2$

$T_1 = 85{,}7 \ N$

$T_2 = 64{,}3 \ N$

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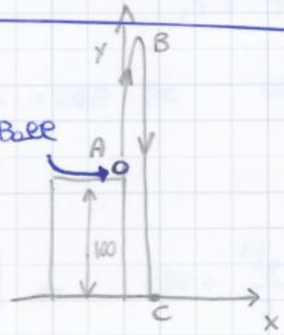


$x(0) = 100 \ m$

$v_0 = 98 \ m/s$

MAX HEIGHT = ?

TIME TO GET TO MAX = ?

VELOCITY ON THE GROUND = ?

$t_{TOT} = ?$

$v_y = \int a_y \, dt$

$v_y = -gt + c_1$

$v_0 = 98 \ m/s$

@ $t = 0 \rightarrow v_0 = -gt + c_1 \rightarrow v_0 = c_1 = 98 \ m/s$

$\Rightarrow v_y = -gt + c$

$v = 0$ when ball @ max, so: $0 = -gt + v_0 \qquad t = \frac{v_0}{g} = \boxed{10 \ s}$

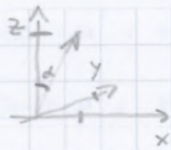
To find MAX HEIGHT, $\int v_1 = \int -gt + v_0 = -\frac{gt^2}{2} + v_0 t + c_2$

Apply init. cond's. : @ $t = 0$ $\quad r(0) = 100 \ m$, $v(0) = 0 \ m/s \quad \Rightarrow 100 = c_2$

$$r_x = -\frac{gt^2}{2} + v_0 t + c_2 = \frac{-10(10)^2}{2} + (98)(10) + 100 = \boxed{580 \ m}$$

Asunto Fecha

VECTORS



2.3) $v_x = 2\,m/s$
$v_y = 3\,m/s$
$v_z = 5\,m/s$

$|v| = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{38} = 6{,}16\,m/s$

α w/ THE Z-AXIS = ?

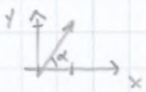
$v_z = v \cdot \cos\alpha \qquad \cos\alpha = \frac{v_z}{v} \qquad \alpha = \arccos\left(\frac{v_z}{v}\right) \qquad \alpha = 35{,}74°$

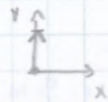
ϑ w/ x,y PLANE = ?

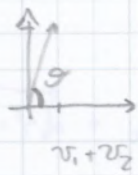
$\cancel{v_x = v \cdot \cos\vartheta} \qquad \cancel{\vartheta = \cos^{-1}\left(\frac{v_x}{v}\right)}$

$\vartheta = \frac{\pi}{2} - \alpha = 54{,}26°$

2.4) $v_1 = (2,3)$







$v_2 = (0,5)$

$v_1 = \sqrt{2^2+3^2} = 3{,}6 \qquad v_2 = 5$

$\cancel{\alpha = \cos^{-1}\left(\frac{2}{v_1}\right) = 56{,}25}$

$v_1 + v_2 = (2,8) = \sqrt{2^2+8^2} = 8{,}25$

$(v_1 + v_2)\cos\vartheta = 2 \qquad \vartheta = \cos^{-1}\left(\frac{2}{v_1+v_2}\right) = 75{,}97°$

~~VECTOR~~ DOT PRODUCT: $\vec{a} \cdot \vec{b} = (a_x \cdot b_x) + (a_y \cdot b_y)$ → = 0 IF PERPENDICULAR

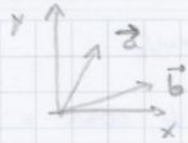
$= |\vec{a}| \cdot |\vec{b}| \cdot \cos(\theta)$ BOTH OK

VECTOR/CROSS PRODUCT: $\vec{a} \times \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \sin(\theta) \cdot \vec{m}$ ↳ BETWEEN THEM ↳ DIRECTION ⊥ TO a, b PLANE

$=$ MATRIX → = 0 IF PARALLEL

Asunto Fecha

2.9) $\vec{a} = 2\vec{i} + 3\vec{j}$ $\vec{b} = 5\vec{i} + \vec{j}$

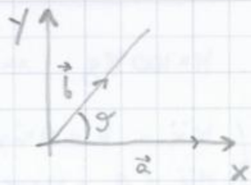


$\vec{a} \cdot \vec{b} = \cancel{(2 \cdot 5)\vec{i} + (3 \cdot 1)\vec{j} + 10\vec{i} + 3\vec{j}}$

$(2 \cdot 5) + (3 \cdot 1) = 10 + 3 = 13$

$$\vec{a} \times \vec{b} = \begin{array}{c|ccc|ccc} & \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} & \vec{k} \\ \vec{a} & 2 & 3 & 0 & 2 & 3 & 0 \\ \vec{b} & 5 & 1 & 0 & 5 & 1 & 0 \end{array} = (2-15)\vec{k} = -13\vec{k}$$

2.11) $|\vec{a}| = 12\,cm$ $|\vec{b}| = 5{,}8\,cm$ FORM ANGLE $\theta = 55°$



$\vec{a} \cdot \vec{b} = 12 \cdot 5{,}8 \cdot \cos(\theta) = 39{,}92$

OR

$= (12 \cdot 3{,}33) + (0 \cdot (5{,}8 \cdot \sin\theta)) = 39{,}96$

$\vec{a} \times \vec{b} = 12 \cdot 5{,}8 \cdot \sin\theta = 57\vec{k}$

$$\begin{array}{cccc} & \vec{i} & \vec{j} & \vec{k} \\ \vec{a} & 12 & 0 & 0 \\ \vec{b} & 3{,}33 & 4{,}75 & 0 \end{array} \Rightarrow 57\vec{k}$$

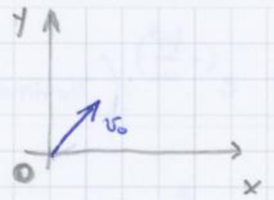
Asunto Fecha

ex

Gun in O

$v_0 = 100\ m/s$ $r = 4\ mm$ (of bullet)

$\alpha = 45°$ $m = 5g$



Viscosity of air (coeff.)

$\beta = \eta\gamma$ (η: ETA)

$\eta = 1.5 \cdot 10^{-5}\ kg/m \cdot s$

$\gamma = 6\pi r$

$$\left.\begin{aligned} y(0) &= 0 \\ x(0) &= 0 \\ \dot{y}(0) &= v_0 \sin\alpha \\ \dot{x}(0) &= v_0 \cos\alpha \end{aligned}\right\} \text{INIT. COND.}$$

POSITION OF BULLET AT MAX HEIGHT ?

$F_v = -\beta v$ $\hookrightarrow (\dot{x} + \dot{y})$

$$\begin{cases} m\ddot{y} = -\beta\dot{y} - mg \\ m\ddot{x} = -\beta\dot{x} \end{cases} \qquad \begin{cases} \ddot{y} + \frac{\beta}{m}\dot{y} = -g \\ \ddot{x} + \frac{\beta}{m}\dot{x} = 0 \end{cases}$$

$\beta = 1{,}13 \cdot 10^{-6}$

$z = \dot{y}$; $\dot{z} = \ddot{y}$

$$\dot{z} + \frac{\beta}{m} z = -g$$

$$\downarrow$$

(After solving)

$$\dot{y} = z = c \cdot e^{-\frac{\beta}{m}t} - \frac{mg}{\beta}$$

Apply init. cond. to find c

$$v_0 \sin\alpha = c \cdot e^{-\frac{\beta}{m}0} - \frac{mg}{\beta} \Rightarrow \boxed{c = v_0 \sin\alpha + \frac{mg}{\beta}}$$

$$\Rightarrow \dot{y} = v_0 \sin\alpha\, e^{\left(-\frac{\beta t}{m}\right)} + \frac{mg}{\beta}\left(e^{-\frac{\beta t}{m}} - 1\right)$$

Asunto Fecha

Now I work on x

$$p = \dot{x} \quad ; \quad \dot{p} = \ddot{x}$$

$$\Rightarrow \quad \dot{p} + \frac{B}{m} p = 0$$

$$p = \dot{x} = c\, e^{\left(-\frac{B}{m} t\right)}$$

Apply init. cond. to find c_2 $\quad \dot{x}(0) = v_0 \cos\alpha = c_2 e^{\left(-\frac{B}{m} 0\right)}$

$$\Rightarrow \quad c_2 = v_0 \cos\alpha$$

$$\Rightarrow \quad \dot{x} = v_0 \cos\alpha\, e^{\left(-\frac{B}{m} t\right)}$$

$$x = \int \dot{x}\, dt = v_0 \cos\alpha\, e^{\left(-\frac{B}{m} t\right)} \cdot \left(-\frac{m}{B}\right) + c_3$$

$$x(0) = 0 = v_0 \cos\alpha\, e^{\left(-\frac{B}{m} \cdot 0\right)} \cdot \left(-\frac{m}{B}\right) + c_3$$

$$c_3 = \frac{m}{B} v_0 \cos\alpha$$

$$\Rightarrow \quad x = v_0 \cos\alpha\, e^{\left(-\frac{B}{m} t\right)} \left(-\frac{m}{B}\right) + \frac{m}{B} v_0 \cos\alpha$$

$$= \frac{m v_0}{B} \cos\alpha \left[1 - e^{\left(-\frac{B}{m} t\right)}\right]$$

Plot $t^* = 7{,}1$ and get x_{MAX}

$$X = 500\, m$$

Asunto Fecha

EX. 3.11.1

ex ✓

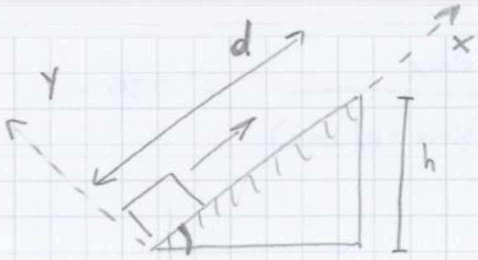
$m = 1\,kg$

$\mu_d = 0{,}4$

$\mu_s = 0{,}5$

$\alpha = 30°$

$|\vec{v}_0| = 3\ m/s$



MAX HEIGHT = ?

DOES IT COME BACK ?

FIND t @ WHICH IT STOPS

$\cancel{m\ddot{x} = -mg\sin\alpha - \mu_d N}$

$\cancel{m\ddot{y} = -mg\cos\alpha = -N}$

$F_{g,x} = -mg\sin\alpha$

$F_d = -\mu_d N = -\mu_d\, mg\cos\alpha$

$F_{g,y} = -mg\cos\alpha = -N$

$$W_{0,d} = \int_0^d \vec{F} \cdot d\vec{r}$$

$$= \int_0^d (F_{g,x} + F_d)\, dx\, \vec{i}$$

$$= \int_0^d -mg(\sin\alpha + \mu_d\cos\alpha)\, dx$$

$$= -mg(\sin\alpha + \mu_d\cos\alpha)\int_0^d dx$$

$$= \text{- - - - - - -}\ [x]_0^d$$

$$= \text{- - - - - - -}\ [d - 0]$$

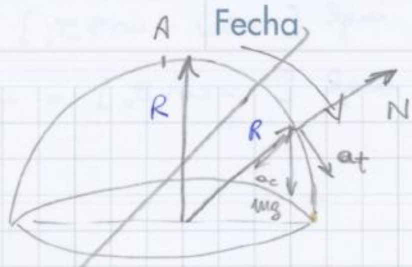
$$W_{0,d} = -mg(\sin\alpha + \mu_d\cos\alpha)\, d \qquad (1)$$

$$W_{0,d} = E_{k,d} - E_{k,0}$$

$$= 0 - \frac{1}{2} m v_0^2 \qquad (2)$$

Equate (1) and (2)

Asunto Fecha



along tan | $mg\sin\alpha - N = m a_c$

along n | $mg\cos\alpha = m a_t$ → $g\cos\alpha$

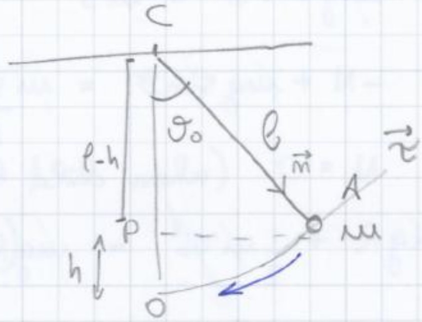
$mg\sin\alpha^* = m a_c$

$mg\sin\alpha^* = \dfrac{m v^{*2}}{R}$

$R = \dfrac{v^{*2}}{g\sin\alpha^*}$

$E_A = E_P$

ex EX. 3.11.6



✓ Work done by gravity from A to O ? $W_g(A,O) = ?$

$m\vec{g} = -mg\cos\vartheta_0\,\vec{n} - mg\sin\vartheta_0\,\vec{\tau}$

$dW_g = -mg(\cos\vartheta\,\vec{n} + \sin\vartheta_0\,\vec{\tau})\,d\vec{r}$

$= -mg(\cos\vartheta\,\vec{n} + \sin\vartheta\,\vec{\tau})\,\vec{v}\,dt$

$= -mg(\cos\vartheta\,\vec{n} + \sin\vartheta\,\vec{\tau})\,v\vec{\tau}\,dt$ → $\vec{\tau}\,ds$

$= -mg\sin\vartheta\,\vec{\tau}\vec{\tau}\,ds$

$= -mg\sin\vartheta\,ds$

$= -mg\sin\vartheta \cdot \ell \cdot d\vartheta$

$\vartheta = \dfrac{\text{arch}}{\text{radius}}$

$d\vartheta = \dfrac{ds}{\ell}$

pag. 129 ex. 7.5

$m = 0{,}80 \text{ kg}$

$\alpha = 30°$

$\mu_d = 0{,}30$

$F = ?$ to move uphill: uniform motion (1); $a = 0{,}10 \text{ m} \cdot s^{-2}$ (2)

to move downhill: uniform motion (3); $a = 0{,}10 \text{ m} \cdot s^{-2}$ (4)

x, y (1)
$$\begin{cases} m\ddot{x} = 0 = F - mg\sin\alpha - \mu_d N \\ m\ddot{y} = -mg\cos\alpha + N = 0 \end{cases}$$

$$\begin{cases} N = mg\cos\alpha \\ F = mg\sin\alpha + \mu_d(mg\cos\alpha) \end{cases} \rightarrow F = 5{,}96 \text{ N}$$

x, y (2) $F = m\ddot{x} + mg\sin\alpha + \mu_d(mg\cos\alpha) = 6{,}04 \text{ N}$

x', y' (3)
$$\begin{cases} m\ddot{x} = 0 = F + mg\sin\alpha - \mu_d N \\ m\ddot{y} = 0 = N - mg\cos\alpha \end{cases}$$

$$\begin{cases} N = mg\cos\alpha \\ F = \mu_d(mg\cos\alpha) - mg\sin\alpha = -1{,}88 \text{ N} \end{cases}$$

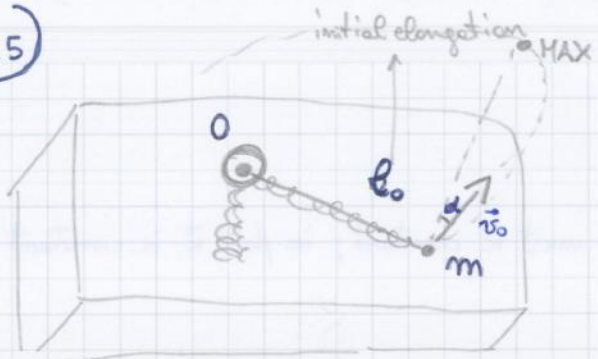
x', y' (4) $F = m\ddot{x} - 1{,}88 \text{ N} = -1{,}80 \text{ N}$

Asunto

Fecha

(EX. 3.II.5)

ex



TOP VIEW

spring is bent, elongated on the plane and kicked w/ $\vec{v}_0$

$\alpha = 10°$

$m = 0{,}5$ kg

$k = 100$ N/m

$v_0 = 10$ m/s

$\ell_0 = 2$ m

$\ell_{MAX} = ?$

v_{fin} @ $\ell_{MAX} = ?$

NO FRICTION
NO VISCOSITY

CYLINDRICAL COORDINATES

only force = Elastic force

$$\vec{F}_e = -k r \vec{\lambda}$$

$$(\text{torque})\ \vec{\tau}_e = (\vec{r} - \underbrace{\vec{r}_0}_{0}) \times \vec{F}_e = r\vec{\lambda} \times (-kr)\vec{\lambda} = 0$$

$$\overset{\|}{0} = \frac{d\vec{L}_0}{dt} \Rightarrow \vec{L}_0 = \text{const}$$

CONSERVATION OF ENERGY

$$E_{init} = \frac{1}{2} m v_0^2 + \frac{1}{2} k r_0^2$$

$$E_{fin} = \frac{1}{2} m v_{fin}^2 + \frac{1}{2} k r_{MAX}^2$$

$$\Longrightarrow \cancel{\frac{1}{2}} m v_0^2 + \cancel{\frac{1}{2}} k \ell_0^2 = \cancel{\frac{1}{2}} m v_{fin}^2 + \cancel{\frac{1}{2}} k \ell_{MAX}^2 \quad *$$

Asunto **Ex. 3.11** Fecha

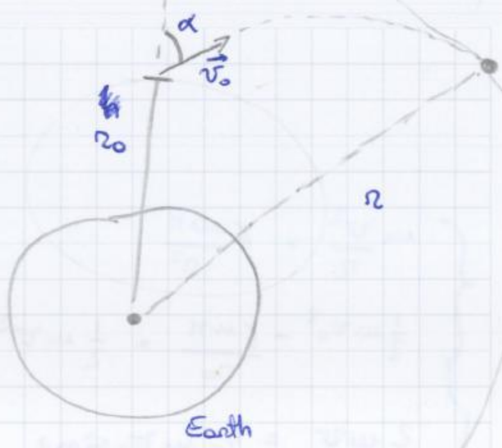
ex

$\gamma = 6,67 \cdot 10^{-11}$ m

$M_E \cong 6 \cdot 10^{24}$ Kg

$m_{SATELLITE} =$ [illegible]

$r = 30'000$ km $= 3 \cdot 10^7$ m



How do you locate the satellite to make it do a circular orbit?

A rocket will take it to height r_0

$r_{0\,MIN}$ (to save fuel) = ?

Don't care about time, just position.

Only force = GRAVITY → conservative
→ central (angular momentum is conserved)

INTRINSIC COORD'S

$$\cancel{m} \frac{v^2}{r} = \gamma \frac{\cancel{m} M}{r^2}$$

FINAL VELOCITY

ACCELERATION IN INTRINSIC C.

$$E_{init} = \frac{1}{2} m v_0^2 - \frac{\gamma m M}{r_0}$$

$$E_{fin} = \frac{1}{2} m v_f^2 - \frac{\gamma m M}{r}$$

$$\Rightarrow \frac{1}{\cancel{2}} m v_0^2 - 2\frac{\gamma m M}{r_0} = \frac{1}{\cancel{2}} m v_f^2 - 2\frac{\gamma m M}{r}$$

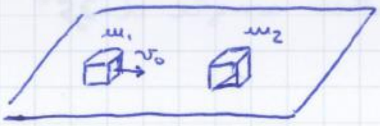
Asunto ESERCITAZIONE Fecha

ex

$m_1 = 1\,kg \qquad |\vec{v}_0| = 5\ m/s$

$m_2 = 2\,kg \qquad |\vec{v}| = ? \qquad |\vec{V}| = ?$

	m_1	m_2
before	v_0	0
after	v	V



Kinetic energy and linear momentum are conserved:

$$\begin{cases} m_1 v_0 + 0 = m_1 v + m_2 V & \text{(linear momentum)} \\ \frac{1}{2} m_1 v_0^2 + 0 = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 V^2 & \text{(kinetic energy)} \end{cases}$$

$$\begin{cases} 1 \cdot 5 + 0 = 1 \cdot v + 2 \cdot V \\ \frac{1}{2} \cdot 1 \cdot 25 + 0 = \frac{1}{2} \cdot 1 \cdot v^2 + \frac{1}{2} \cdot 2 \cdot V^2 \end{cases}$$

$$\begin{cases} v + 2V = 5 \\ \frac{25}{2} = \frac{v^2}{2} + V^2 \end{cases} \qquad \begin{cases} v = 5 - 2V \\ \frac{25}{2} = \frac{25 + 4V^2 - 20V}{2} + V^2 \end{cases}$$

$$V^2 + 2V^2 - 10V = 0 \rightarrow 3V^2 - 10V = 0 \rightarrow V(3V - 10) = 0$$

CASE 1) $V = 0\ m/s \quad v = 5\ m/s$

CASE 2) $V = \frac{10}{3}\ m/s \quad v = -\frac{5}{3}\ m/s$

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Asunto Fecha

Conservation of E_p / E_k during the displacement of the cube

$$\left(\cancel{E_{p\,in}} + E_{k\,in} = E_{p\,fin} + \cancel{E_{k\,fin}} \right) \qquad \cancel{E_{p\,in}} = 0, \quad \cancel{E_{k\,fin}} = 0$$

$$\frac{1}{2}(m+M)\,V^2 = (m+M)gh$$

$$\frac{1}{2}(m+M)\left(\frac{mv_0}{m+M}\right)^2 = (m+M)gh$$

$$v_0 = \sqrt{2gh}\cdot\left(\frac{m+M}{m}\right)$$

$$\boxed{v_0 = 189{,}8\ m/s \qquad V = \frac{mv_0}{m+M} = 1{,}88\ m/s}$$

$$L\cdot\cos\alpha = (L-h)$$

$$\cancel{\cos\alpha = [illegible]}$$

$$h = -L\cdot\cos\alpha + L$$

$$h = 0{,}18\ m$$

NON-MASSLESS PULLEY

Asunto **ESERCITAZIONE** Fecha

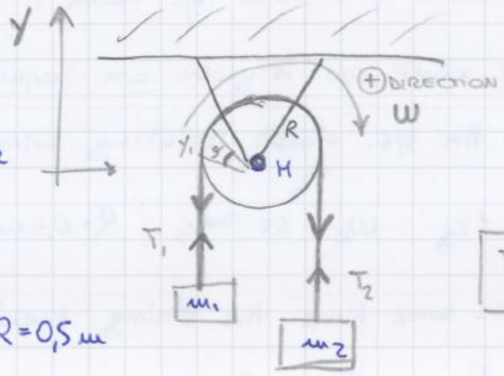
ex

Beginning: system at rest

Later, @ $t=0$ m_1, m_2 are left free

$\vec{a}_{m_1} = ?$ $\vec{a}_{m_2} = ?$ $\dot{\omega}_M = ?$

$M = 5\,kg$ $m_1 = 2\,kg$ $m_2 = 4\,kg$ $R = 0{,}5\,m$



$\boxed{T_1 \neq T_2}$

$$\begin{cases} m_1\ddot{y}_1 = -m_1 g + T_1 \\ m_2\ddot{y}_2 = -m_2 g + T_2 \\ \ddot{y}_1 = -\ddot{y}_2 \\ I\dot{\omega} = R(T_2 - T_1) \\ \ddot{y}_1 = \dot{\omega} R \end{cases}$$

$\tau = I\dot{\omega}$

$= -RT_1 + RT_2$

$\vartheta = \frac{arc}{radius}$

$\vartheta = \frac{y_1}{R}$

$\omega = \dot{\vartheta} = \frac{\dot{y}_1}{R}$

$\dot{\omega} = \ddot{\vartheta} = \frac{\ddot{y}_1}{R}$

$\Rightarrow$ $\ddot{y}_1 = \dot{\omega} R$

Recall: $I = \frac{1}{2} M R^2$

$$\dot{\omega} = \frac{(m_2 - m_1)\, g}{R\left(m_1 + m_2 + \frac{M}{2}\right)}$$

$$\boxed{\dot{\omega} = 4{,}6\ rad/s^2 \qquad \ddot{y}_1 = 2.3\ m/s^2 \qquad \ddot{y}_2 = -2{,}3\ m/s^2}$$

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Asunto EX. 4.14.1 Fecha

ex

A homog. cylinder (r, h, m_1) can rotate on a horizontal plane around a vertical axis fixed on the plane.

Beginning: ω_0 , no friction cyl/plane/axis

Later: another cyl (r, h, m_2) is put down slowly on top of the 1st

Between 2 cyl's there is friction (μ_s, μ_d)

Find ω of upper cyl and loss of energy of the system after a very long time.

CONSERVATION OF $\vec{L}_0$

$$L_i = I_d \omega_0$$

$$L_f = (I_d + I_u)\omega$$

$$\Rightarrow I_d \omega_0 = (I_d + I_u)\omega$$

$$\Rightarrow \boxed{\omega = \frac{I_d \omega_0}{I_d + I_u}}$$

$$E_i = \frac{1}{2} I_d \omega_0^2$$

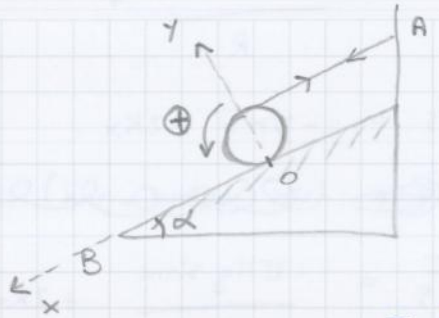
$$E_f = \frac{1}{2}(I_d + I_u)\omega^2$$

$$\boxed{W = \Delta E^k = \frac{1}{2} I_d \omega_0^2 - \frac{1}{2}(I_d + I_u)\left(\frac{I_d \omega_0}{I_d + I_u}\right)^2}$$

Asunto **ESERCITAZIONE** Fecha

ex

EX. 4.14.5



Elastic string

sphere → $m = 2kg$, $r = 20$ cm

$\alpha = 45°$ $k = 200$ N/m

$\mu_s = 0{,}5$ $\mu_d = 0{,}4$

$I = \frac{2}{5} MR^2$

$\boxed{\nu = \frac{2}{5}}$

Beginning : system at rest

sphere → is at the elastic rest length

Max displacement of sphere, rolling, not sliding = ?

$F_{el} = -k(x + x)$ → every x of displacement of the sphere, there is another x caused by the rolling EASY

(1) $\sum \vec{F}^{(E)} = Mg\sin\alpha + (F_{el} + F_f)$

(2) $\sum \tau^{(E)} = R \times (\vec{F}_{el} + \vec{F}_f)$

$= -R\,(F_{el} + |F_f|)$

(2) $\tau = I\alpha = I\dot{\omega} = -R\,(F_{el} + |F_f|)$

At the beginning, $F_f = |F_s|$

$$-\frac{I\dot{\omega}}{R} = \underbrace{(F_{el} + F_s)}$$

(1) $M\ddot{x} = Mg\sin\alpha - \frac{I\dot{\omega}}{R}$ $\ddot{x} = g\sin\alpha - \frac{I\dot{\omega}}{RM}$

$\ddot{x} = g\sin\alpha - \frac{2}{5}R\dot{\omega} = g\sin\alpha - \nu R\dot{\omega}$

$\ddot{x} = g\sin\alpha - \nu\ddot{x}$ $\ddot{x} = \frac{g\sin\alpha}{1+\nu} = \dot{\omega}R$

$\vartheta = \frac{arc}{R} = \frac{x}{R}$

$x = \vartheta R$

$\dot{x} = \dot{\vartheta}R = \omega$

$\ddot{x} = \ddot{\vartheta}R = \dot{\omega}$

Asunto Fecha

(1) $Ma_c = +W - 2T$

$\sum \vec{\tau} = I\vec{\alpha}$

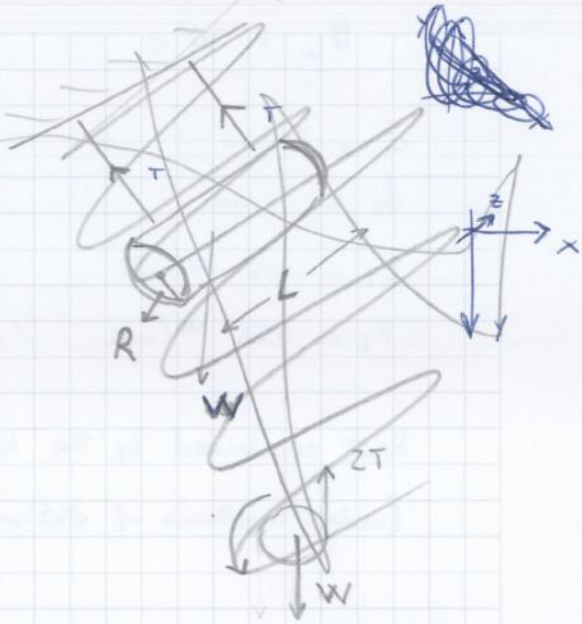
$\vec{\alpha} = -\alpha\vec{k}$

$\sum \vec{\tau} = -I\alpha = (\vec{R} \times 2\vec{T})$

$= -R\vec{i} \times 2T\vec{j}$

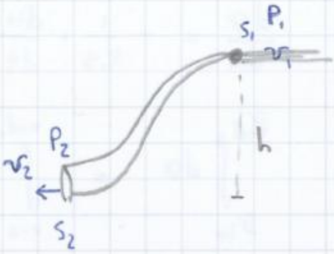
$-I\alpha = -2RT\vec{k}$

$\boxed{I\alpha = 2RT}$



(ex) Water flows inside a tube

$v_1 = ?$ $v_2 = ?$



Continuity equation:

$v_1 S_1 = v_2 S_2$

$v_1 = \frac{v_2 S_2}{S_1}$

$h = (3.11 \pm 0.19)\ m$

$P_1 = (0.05 \pm 0.005) 10^5\ Pa$

$\frac{S_1}{S_2} = \frac{1}{2}$ $\rho = 10^3\ kg/m^3$

$P_2 = 1atm = 1.013 \cdot 10^5\ Pa$

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h = P_2 + \frac{1}{2}\rho v_2^2 + 0$

$v_1^2 = v_2^2 + \frac{2}{\rho}(P_2 - P_1) - 2gh$

$v_1^2 = \frac{v_1^2 S_1^2}{S_2^2} + \frac{2}{\rho}(P_2 - P_1) - 2gh$

$v_1 = \cancel{\sqrt{\frac{\left[\frac{2}{\rho}(P_2-P_1) - 2gh\right]}{\left(1 - \left(\frac{S_1}{S_2}\right)^2\right)}}}$ $\cancel{\sqrt{\frac{v_1^2 S_1^2}{S_2^2} + \frac{2}{\rho}(P_2-P_1) - 2gh}}$ $= 8{,}86\ m/s$

$4{,}43\ m/s$

$v_2 = \frac{v_1 S_1}{S_2} =$

THERMODYNAMIC TRANSFORMATIONS

~~CALORIMETRY~~

Asunto $\boxed{\text{EX. ----}}$ Fecha

V_A contains n moles of a diatomic ideal gas @ T_A

~~CH_4~~

$T_A = 300$ K

$n = 0,3$ mol

Burning a mass M of methane, you get a power P.

$P = 13271$ Kcal/kg

Find: 1) $P_A = ?$ $P_B = ?$ 2) W produced by the gas = ?

1) $T_A \rightarrow (2T_A = T_B)$ Temperature changes

Also, $\frac{T^2}{V} = \text{const}$

$P_A = ?$

$P_B = ?$

$PV = nRT$

$$\begin{cases} P_A = \frac{nRT_A}{V_A} \\ P_B = \frac{nRT_B}{V_B} \end{cases}$$

$$\frac{T_A^2}{V_A} = \frac{T_B^2}{V_B} \rightarrow \frac{T_A^2}{V_A} = \frac{(2T_A)^2}{V_B}$$

$$\rightarrow V_B = 4V_A$$

$$P_A = \frac{nRT_A}{V_A}$$

$$P_B = \frac{nR(2T_A)}{4V_A} = \frac{nRT_A}{2V_A} = \frac{P_A}{2}$$

$$\boxed{\begin{aligned} P_A &= 0.82 \text{ atm} \\ P_B &= 0,41 \text{ atm} \end{aligned}}$$

CALORIMETRY

Asunto **EX 6.2.3** Fecha

In an isolated container there is:

H_2O $m_{H_2O} = 2kg$ $\vartheta_{H_2O} = 20°c$

Then we add Cu $\rightarrow m_{Cu} = 0,1 \text{ kg}$, $\vartheta_{Cu} = 70°c$

and ICE $\rightarrow m_{ICE} = 1 \text{kg}$, $\vartheta_{ice} = -10°c$

$c_{H_2O} = 0.998 \text{ kcal/kg°c}$ | $c_{Cu} = 0.093 \text{ kcal/kg°c}$ | $c_{ICE} = 0,5 \text{ kcal/kg}$

Final Temperature = ? How much ice remains ?

t° of change of state

$$Q_{ICE} = m_{ICE} \cdot c_{ICE} \cdot (\vartheta_i - \vartheta_c)$$

$$= 1 \cdot 0,5 \cdot (-10° - 0°) = -5 \text{ kcal}$$

$$Q_{H_2O} = m_{H_2O} \cdot c_{H_2O} \cdot (\vartheta_i - \vartheta_c) = 39,92 \text{ kcal}$$

$$Q_{Cu} = 0,651 \text{ kcal}$$

$$Q_{TOT} = Q_{H_2O} + Q_{ICE} + Q_{Cu} = 35,571 \text{ kcal}$$

$$Q_{ICE} = m_{ICE} \cdot \lambda$$

$$\lambda = 79,7 \text{ kcal/kg}$$

$$Q_{ICE} = 79,7 \text{ kcal}$$

$$M_{ICE, MELTED} = \frac{Q_{TOT}}{\lambda} = 0,44 \text{ kg}$$

$$M_{ICE, LEFT} = 1 \text{kg} - M_{ICE, MELTED} = 0,56 \text{ kg}$$

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Ex. 2.6.5

Car accelerates for 6s, $a = 2\ m/s^2$ then $v = const$

At same time, a truck passes w/ const speed $= 10\ m/s$

$t^* = ?$ $x^* = ?$ will the car and the truck meet again?

Init. cond.: $x_{A_0} = 0$ $v_{A_0} = 0$

$x_{B_0} = 0$ $v_{B_0} = 10\ m/s$

$\vec{v}_A = \int \vec{a}_A = a_A t + c_1$ | $v_B = a_B t + c_3$

$x_A = \int v_A = \frac{1}{2} a_A t^2 + c_1 t + c_2$ | $x_B = a_B \frac{t^2}{2} + c_3 t + c_4$

$x_A(0) = c_2 = 0$ | $v_B(0) = c_3 = 10\ m/s$

$v_A(0) = c_1 = 0$ | $x_B(0) = c_4 = 0$

After 6s, $x_A = \frac{1}{2}(2\ m/s^2)(6s)^2$ $x_B = c_3 t$

$\boxed{x_A = 36\ m \quad v_A = 12\ m/s \quad x_B = 60\ m \quad v_B = 10\ m/s}$ NEW COND.'S

$x_A = v_A t + c_5$ $x_B = v_B t + c_6$

@ $t = 6s$ $36 = 72 + c_5 \rightarrow = -36$ $60 = 60 + c_6 \rightarrow 0$

When they meet, $x_A = x_B$

$\Rightarrow v_A(6+t) - 36 = v_B(6+t)$

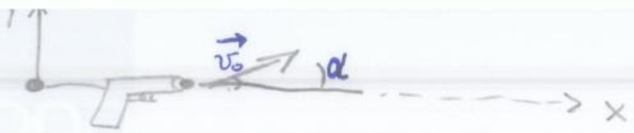
$6v_A + v_A t = 36 + 6v_B + v_B t$

$t = \frac{6v_B + 36 - 6v_A}{v_A - v_B} = 12s$

$\boxed{t^* = 18s}$

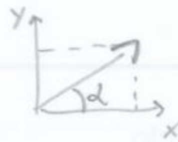
$\boxed{x_A^* = x_B^*} = v_A(6+12) - 36 = \boxed{180\ m}$

Ex. 2.6.7



$v_0 = 200\ m/s$ $\quad (x,y)_{BULLET}\ (t=20s) = ?$

$\alpha = 40°$ $\quad v_{BULLET}\ (t=20s) = ?$



$\ddot{y} = -g\vec{j}$ $\quad \dot{y} = -gt + c_1$ $\quad y = -\frac{1}{2}gt^2 + c_1 t + \underbrace{c_2}_{0\ (I.C.)}$

$v_{0x} = v_0 \cos\alpha = 153{,}2\ m/s$

$v_{0y} = v_0 \sin\alpha = 128{,}6\ m/s$

$\Rightarrow \quad v_{0y} = \dot{y}(t=0) \quad 128{,}6 = -gt + c_1 \quad c_1 = 128{,}6$

$\dot{y}(t=20s) = -g(20) + 128{,}6\ m/s$

$= -67{,}6\ m/s$

$y(t=20s) = -\frac{1}{2}gt^2 + c_1 t$

$= 610\ m$

$x = (v_0 \cos\alpha)(t) + \underbrace{c_3}_{0}$

$$\begin{cases} v_x = \int a_x = \cancel{a_x}^{0} t + c_1 \\ v_y = \int a_y = -gt + c_2 \end{cases} \qquad \begin{cases} x = c_1 t + c_3 \\ y = -\frac{1}{2}gt^2 + c_2 t + c_4 \end{cases}$$

I.C.

$$\begin{cases} v_{0x} = v_0 \cos\alpha \\ v_{0y} = v_0 \sin\alpha \end{cases} \qquad \begin{cases} x(0) = 0 \\ y(0) = 0 \end{cases}$$

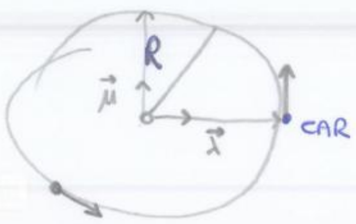
$c_1 = v_0 \cos\alpha$ $\qquad c_3 = 0$

$c_2 = v_0 \sin\alpha$ $\qquad c_4 = 0$

$$\Rightarrow \boxed{\begin{aligned} x(20s) &= v_0 \cos\alpha\,(20s) = 3064\ m \\ y(t=20s) &= -\frac{1}{2}g(20s)^2 + (v_0 \sin\alpha)(20s) = 609\ m \end{aligned}}$$

$$\boxed{\begin{aligned} v_x(t=20s) &= 153\ m/s \\ v_y(t=20s) &= -68\ m/s \\ v(t=20s) &= 167\ m/s \end{aligned}}$$

EX. 2.6.9.a



$R = 2Km$

@ $t=0$ CAR at rest

Pressing the accelerator, → change of velocity α →

→ $\tau = a \cdot \sin(\omega \cdot t)$ $\quad a = 6$

$\omega = \pi/20$

CYLINDRICAL COORDINATES

1) DIMENSIONS OF a, ω, τ = ?

2) MODULUS OF VELOCITY @ $t = 20s$

3) ACCELERATION (MODULUS + DIRECTION) @ $t = 10 s$

1) $t \Rightarrow s$ $\quad a \Rightarrow \frac{m}{s}$

$\omega \Rightarrow \frac{1}{s}$ $\quad \tau \Rightarrow \frac{m}{s}$

$\vec{v} = \dot{r}\vec{\lambda} + r\dot{\vartheta}\vec{\mu}$

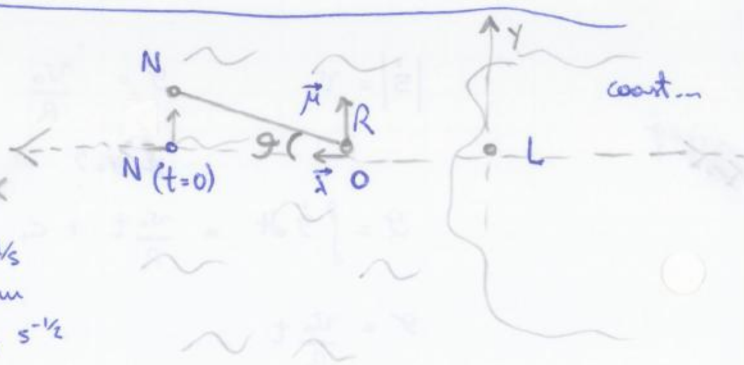
$\vec{a} = (\ddot{r} - r\dot{\vartheta}^2)\vec{\lambda} + (2\dot{r}\dot{\vartheta} + r\ddot{\vartheta})\vec{\mu}$

2) $r = R \quad \dot{r} = 0$

$\vec{v} = R\dot{\vartheta}\vec{\mu}$ $\quad R\dot{\vartheta}\vec{\mu} = a \cdot \sin(\omega t)$

???

EX. 2.6.10 $\quad$ N = boat, R = radar



√

$r = \overline{NR}$

$r(t)$; $\vartheta(t)$

$\vartheta(t) = c \cdot \sqrt{t}$ $\quad r(t) = a \cdot t + b$ $\quad a = 10\ m/s$, $b = 0{,}5\ m$, $c = 1/60\ s^{-1/2}$

$a \to \left[\frac{L}{T}\right]$ $\quad b \to [L]$

$c \to \left[\frac{1}{T}\right]^{1/2}$

CYL. COORDINATES

1) Position of N after $t_0 = 4h$ $\quad$ 2) velocity of N after $t_0 = 4h$ $\quad$ 3) $\vec{a}_N$ @ $t_0 = 4h$

$\vec{v} = \dot{r}\vec{\lambda} + r\dot{\vartheta}\vec{\mu}$ $\quad \vec{a} = (\ddot{r} - r\dot{\vartheta}^2)\vec{\lambda} + (2\dot{r}\dot{\vartheta} + r\ddot{\vartheta})\vec{\mu}$

$\dot{r}(t) = a$ $\quad \ddot{r}(t) = 0$ $\quad \dot{\vartheta}(t) = \frac{1}{2}c\,t^{-1/2}$ $\quad \ddot{\vartheta}(t) = -\frac{1}{4}c\,t^{-3/2}$

$\vec{r}(t=0) = b\vec{\lambda} = 0{,}5\,m$ $\quad \vec{r}(t_0) = 144'000\ m$ $\quad \vartheta(t_0) = 2\ rad$

$\vec{v}(t_0) = a\vec{\lambda} + (at_0 + b)\left(\frac{1}{2}c(t_0)^{-1/2}\right)\vec{\mu} = \left(10\frac{m}{s}\right)\vec{\lambda} + \left(10\frac{m}{s}\right)\vec{\mu}$

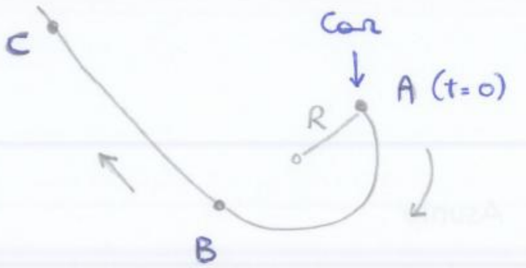
$\vec{a}(t_0) = \left[(0 - (at_0 + b))\left(\frac{c^2}{4t_0}\right)\right]\vec{\lambda} + \left(2a \cdot \frac{1}{2}ct_0^{-1/2}\right) + \left[(at_0 + b)\left(-\frac{1}{4}ct^{-3/2}\right)\right]\vec{\mu} = \ldots$

Ex. 2.6.12

$R = 20\,m \qquad \overline{BC} = 1\,km$

$v(t) = b t^{\frac{3}{2}}$

$b = 1 \frac{m}{s \cdot s^{3/2}}$



In which position the modulus of $\vec{a}$ is MAX ? INTRINSIC COORDINATES

$\vec{v} = \dot{s}\,\vec{\tau} \qquad \vec{a} = \ddot{s}\,\vec{\tau} + \frac{\dot{s}^2}{R}\vec{n}$

$v(t) = b t^{3/2} \rightarrow s(t) = \frac{2}{5} b t^{5/2} \qquad \ddot{s}(t) = \frac{3}{2} b t^{1/2}$

POINT B : $\pi R = \frac{2}{5} b t^{5/2}$

$\sqrt{t^5} = \frac{\pi R}{b} \cdot \frac{5}{2} \qquad t = \sqrt[5]{\left(\frac{\pi R 5}{2b}\right)^2} = 7{,}56\ s$

POINT C : $\pi R + \overline{BC} = \frac{2}{5} b t_c^{5/2} \qquad \sqrt{t^5} = \frac{\pi R + \overline{BC}}{b} \cdot \frac{5}{2} \qquad t = \sqrt[5]{\left(\frac{5\pi R + 5\overline{BC}}{2b}\right)^2} = 23{,}43\ s$

$\vec{a}(t_B) = \left(\frac{3}{2} b t_B^{1/2}\right)\vec{\tau} + \left[\frac{\left(b t_B^{3/2}\right)^2}{R}\right]\vec{n} = (4{,}12)\vec{\tau} + (21{,}60)\vec{n}$

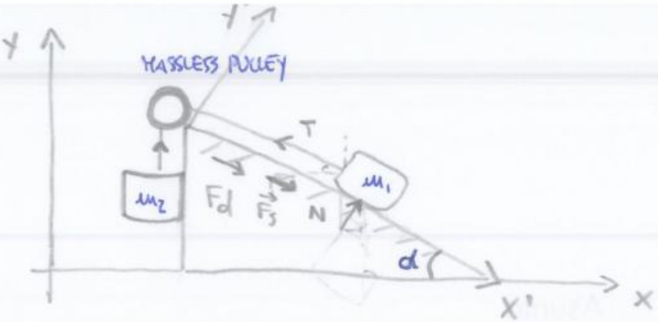
$\downarrow =$

$\sqrt{(4{,}12)^2 + (21{,}60)^2} = \boxed{21{,}99 \frac{m}{s^2}}$

$\vec{a}(t_c) = \left(\frac{3}{2} b t_c^{1/2}\right)\vec{\tau} = (7{,}26)\vec{\tau} = 7{,}26 \frac{m}{s^2}$

MAX ACCEL. @ B

EX. 3.3.10



$\alpha = 30°$

√ $m_1 = 1kg$

$m_2 = 0{,}75\ kg$

$\mu_s = 0{,}25 \quad \mu_d = 0{,}2$

$|\vec{a}_{m_1}| = |\vec{a}_{m_2}| = ?$

$$\begin{cases} m_1\ddot{x} = -T\cos\alpha + F_s\cos\alpha + N\sin\alpha \\ m_1\ddot{y} = -m_1 g + N\cos\alpha + T\sin\alpha + F_s\sin\alpha \\ 0 = m_2\ddot{x} = 0 \\ m_2\ddot{y} = -m_2 g + T \end{cases}$$

$$\boxed{F_{s\,max} = \mu_s \cdot N}$$

IF NO MOVEMENT $\Rightarrow \ddot{x} = \ddot{y} = 0$

$\Rightarrow T = m_2 g$

$$\Rightarrow \begin{cases} m_2 g\cos\alpha = F_s\cos\alpha + N\sin\alpha \\ m_1 g = N\cos\alpha + m_2 g\sin\alpha + F_s\sin\alpha \end{cases}$$

$$F_s = \frac{m_2 g\cos\alpha - N\sin\alpha}{\cos\alpha}$$

$N = (m_1 g - m_2 g\sin\alpha - (m_2 g\cos\alpha - N\sin\alpha$

$$\begin{cases} m_1\ddot{x}' = -T + F_s + m_1 g\sin\alpha \\ 0 = m_1\ddot{y}' = N - m_1 g\cos\alpha \\ 0 = m_2\ddot{x} = 0 \\ m_2\ddot{y} = T - m_2 g \end{cases}$$

IF NO MOTION $\ddot{x}' = \ddot{y} = 0$

$$\begin{cases} N = m_1 g\cos\alpha \\ T = F_s + m_1 g\sin\alpha \\ T = m_2 g \end{cases} \qquad \begin{aligned} N &= 8{,}49\ N \\ F_s &= 2{,}45\ N \\ T &= 7{,}35\ N \end{aligned}$$

$|F_s| \le F_{s\,max}$

$2{,}45 \le \underset{\mu_s}{(0{,}25)}\underset{N}{(8{,}49)}$

NOT VALID

→ MASSES MOVE

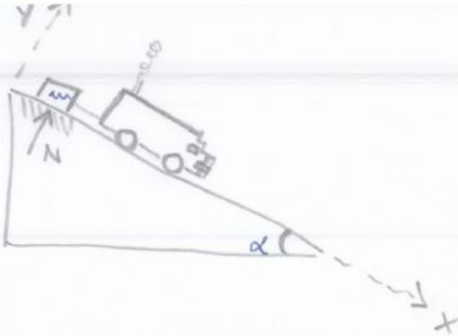
$$\begin{cases} m_1\ddot{x}' = -T + F_d + m_1 g\sin\alpha \\ \boxed{N = m_1 g\cos\alpha} \\ m_2\ddot{y} = T - m_2 g \\ \ddot{x}' = \ddot{y} \end{cases}$$

$\Rightarrow (m_1 + m_2)\ddot{x} = \cancel{-T} + \cancel{T} + F_d + m_1 g\sin\alpha - m_2 g$

$$\ddot{x} = \frac{m_1 g\sin\alpha - m_2 g + \mu_d N}{m_1 + m_2}$$

$$\boxed{\ddot{x}' = -0{,}43\ m/s^2} \qquad \boxed{\ddot{y} = -0{,}43\ m/s^2}$$

EX. 3.3.13



$m = 2000\ kg$

$\alpha = 10°$

$\mu_s = 0{,}8 \qquad \mu_d = 0{,}4$

Truck is moving with $v_0 = const$, then it decelerates with constant $\vec{a}$ with modulus a.
$\vec{a}_{MAX}$ s.t. the mass doesn't hit the truck = ?

$$\begin{cases} 0 = m\ddot{y} = N - mg\cos\alpha \\ m\ddot{x} = -ma = F_s + mg\sin\alpha \end{cases}$$

$$\begin{cases} N = mg\cos\alpha = 19302\ N \\ F_s = -ma - mg\sin\alpha \end{cases}$$

$$|F_s| \le F_{s_{MAX}} = \mu_s N = 15442\ N$$

$$|-ma - mg\sin\alpha| \le F_{s_{MAX}}$$

$$(2000)a + (2000)(g)(\sin\alpha) \le 15442$$

$$\boxed{|a| \le 6{,}02\ m/s^2} \implies \boxed{\vec{a} \le -6{,}02\ m/s^2}$$

$S(t) = 0$

$$\Rightarrow \frac{\mu_d N}{\beta} t + 10 \frac{m}{\beta} e^{-\frac{\beta}{m}t} + \frac{\mu_d N m}{\beta^2} e^{-\frac{\beta}{m}t} = 10 \frac{m}{\beta} + \frac{\mu_d m N}{\beta^2}$$

$$A t + 10^6 e^{-\frac{\beta}{m}t} + B e^{-\frac{\beta}{m}t} = 10^6 + B$$

$$A = \frac{\mu_d m N}{\beta} = 392'400$$

$$B = \frac{\mu_d m N}{\beta^2} = 39'240'000'\ldots$$

$\dot{S}(t) = 0$

$$\frac{\mu_d N}{\beta} = \left(10 + \frac{\mu_d N}{\beta}\right) e^{-\frac{\beta}{m}t}$$

$$-\frac{\beta}{m} t = \ln\left(\frac{\mu_d N}{10 \cdot \beta} + 1\right)$$

$$\frac{\beta}{m} t = -10{,}58 \quad \Rightarrow \quad t_0 = -1057748 \text{ s}$$

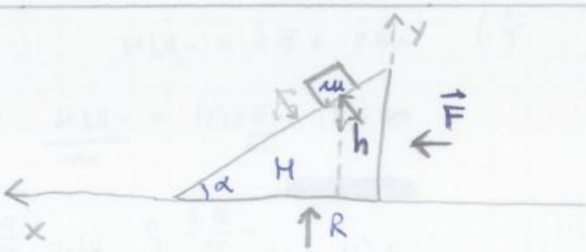
$$\Rightarrow t_0 = 1057'748 \text{ s}$$

I can plot t_0 in the equation $S = \ldots (t)$ and find s at which the mass stops.

Ex. 3.3.16

$m = 1$ kg $M = 5$ kg

$F = 500$ N $\alpha = 30°$ $h = 4$ m

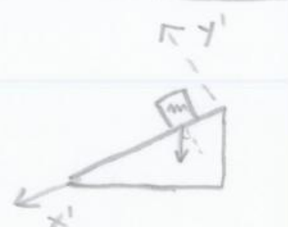


NO FRICTION

acceleration of M, m = ?

$$\begin{cases} m\ddot{x} = N\sin\alpha \\ m\ddot{y} = -mg + N\cos\alpha \\ M\ddot{X} = F - N\sin\alpha \\ 0 = M\ddot{Y} = -N\cos\alpha + R - Mg \end{cases} \Rightarrow \begin{cases} m\ddot{x} = N\sin\alpha \\ m\ddot{y} = N\cos\alpha - mg \\ M\ddot{X} = F - (\sin\alpha)(N) \\ R = Mg + N\cos\alpha \end{cases}$$

AND $\ddot{x} = \ddot{x}' + \ddot{X}$



$$\begin{cases} m\ddot{x}' = mg\sin\alpha \\ 0 = m\ddot{y}' = N - mg\cos\ldots \end{cases}$$

$\ddot{x}' = g\sin\alpha$

$\ddot{x}' = g/2$

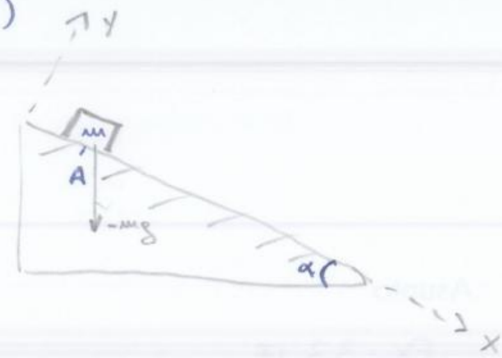
$$\begin{cases} m\ddot{x} = N\sin\alpha \\ m\ddot{y} = N\cos\alpha - mg \\ M\ddot{X} = F - N\sin\alpha \\ R = Mg + N\cos\alpha \\ \ddot{x} = \ddot{X} + g/2 \end{cases}$$

M) $\ddot{X} = 82{,}52 \text{ m/s}^2$

m) $\ddot{x} = 87{,}42 \text{ m/s}^2$

$\ddot{y} = -141{,}61 \text{ m/s}^2$

EX. 3.3.20 (Exam April 30^{th}, 2007)



$\alpha = 30°$ $\quad \beta = 0{,}1\ kg/s$

$\mu_s = 0{,}4$ $\quad m = 4\,kg$

$v_0 = 0\ m/s$ $\quad \mu_d = 0{,}3$

1) Does the cube slide down?

$$\begin{cases} m\ddot{x} = -F_s + mg\sin\alpha - \beta v \\ 0 = m\ddot{y} = -mg\cos\alpha + N \end{cases} \qquad \begin{cases} F_s = -m\ddot{x} + mg\sin\alpha - \beta v \\ N = mg\cos\alpha \end{cases}$$

$$|F_s| \le F_{s\,max} = \mu_s N = \mu_s (mg\cos\alpha)$$

$$|\cancel{m}g\sin\alpha| \le \mu_s(\cancel{m}g\cos\alpha) \rightarrow \frac{1}{2} \le 0{,}4\left(\frac{\sqrt{3}}{2}\right) \quad \text{NO}$$

So m moves!

2) v_{MAX} (in case of infinite plane)?

$$\left\{ \underbrace{m\ddot{x}_\infty}_{=0} = F_d + mg\sin\alpha - \beta\dot{x}_\infty \right.$$

$$\hookrightarrow \dot{x}_\infty = \frac{-\mu_d N + mg\sin\alpha}{\beta} = \frac{mg(\sin\alpha - \mu_d\cos\alpha)}{\beta} = 94\ m/s$$

3) $\dot{x}(t=10\,s) = ?$

$$\left\{ m\ddot{x} = F_d + mg\sin\alpha - \beta\dot{x} \right. \qquad \ddot{x} = \dot{u} \quad \dot{x} = u$$

$$\dot{u} + \frac{\beta}{m}u = \frac{F_d + mg\sin\alpha}{m}$$

$F_d = -\mu_d N = -\mu_d mg\cos\alpha$

$$u = e^{-\frac{\beta}{m}t} \int \left(\frac{F_d}{m} + g\sin\alpha\right)\left(e^{\frac{\beta}{m}t}\right) dt$$

$$u = \left(e^{-\frac{\beta}{m}t}\right)\left(g[-\mu_d\cos\alpha + \sin\alpha]\right)\left(\frac{m}{\beta}e^{\frac{\beta}{m}t}\right) + c_1$$

$$u(0) = 0 = 1\left[g(-\mu_d\cos\alpha + \sin\alpha)\right]\cdot 1 + c_1$$

$$\Rightarrow c_1 = g(\mu_d\cos\alpha - \sin\alpha)$$

$$\dot{x}(t^* = 10\,s) = e^{-\frac{\beta}{m}t^*}\left[g(-\mu_d\cos\alpha + \sin\alpha)\right]\cdot\left(\frac{m}{\beta}e^{\frac{\beta}{m}t^*}\right) + g(\mu_d\cos\alpha - \sin\alpha)$$

INTRINSIC COORD'S :

$$\vec{v} = \dot{s}\,\vec{\tau}$$

$$\vec{a} = \ddot{s}\,\vec{\tau} + \frac{\dot{s}^2}{R}\,\vec{m}$$

HORIZONTAL FRICTION

$$M(\vec{a}) = F_T$$

$$M\ddot{s} = -F_s$$

$$M \cdot \alpha \cdot R = -F_s = 0$$

$$\frac{\ddot{s}}{R} = \alpha$$

$$\ddot{s} = \alpha \cdot R$$

$$\cancel{-F_s = 89{,}6\ N}$$

$$\cancel{|F_s| \le F_{s\,MAX} \quad 89{,}6 \le \mu_s N}$$

$$\cancel{N = M \cdot R \cdot \omega^2}$$

$$\cancel{89{,}6 \le \mu_s \cdot M \cdot R \cdot \omega^2}$$

$$\rightarrow \cancel{89{,}6 \le 1024\,\omega^2}$$

$$\cancel{\omega \ge 0{,}30\ rad/s}$$ ~~for the man not to fall~~

$$\cancel{0{,}07\ rad : 1\ s^2 = 0{,}30\ rad : x\ s^2}$$

$\vec{k}$)

VERTICAL FRICTION

$$-Mg = F_s$$

$$|F_s| \le F_{s\,MAX} = \mu_s N \qquad N = M \cdot R\,\omega^2$$

$$Mg \le \mu_s M R \omega^2$$

$$\omega^2 \ge \frac{g}{\mu_s R}$$

$$\omega \ge \sqrt{\frac{g}{\mu_s R}} \qquad \text{but} \quad \omega = \alpha \cdot t$$

$$\alpha t \ge \sqrt{\frac{g}{\mu_s R}}$$

$$t \ge \frac{1}{\alpha}\sqrt{\frac{g}{\mu_s R}}$$

$$t \ge 12{,}51\ s$$

$$\hookrightarrow \boxed{t \le 12{,}51\ s}$$ for the man not to fall

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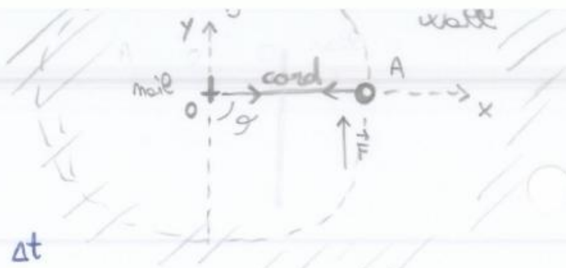
EX. 3.11.8



$m = 0{,}2\,kg \quad \ell = 1\,m \quad \Delta t = 10^{-4}\,s$

A force $\vec{F}$ is applied on m for a time interval Δt and m remains practically in the point A.

Force needed to make a complete round = ?

INTRINSIC COORD'S : $\vec{v} = \dot{s}\vec{\tau}$

$$\vec{a} = \ddot{s}\vec{\tau} + \frac{\dot{s}^2}{R}\vec{n}$$

$\vec{\tau}$) ~~Gravity : $-mg\sin\vartheta$~~

$\vec{n}$) ~~Gravity : $-mg\cos\vartheta$~~

Tension : T

(impulse) $\vec{I} = \vec{F}\cdot\Delta t \quad \rightarrow \quad m\vec{v}(A) - m\underbrace{v_0}_{=0} = \vec{F}\cdot\Delta t$

=0 (system at rest @t=0)

CONSERVATION OF TOT. MECH. ENERGY : (NO non-cons. forces)

$$mg(0) + \frac{1}{2}m v_{in}^2(A) = mg\ell + \frac{1}{2}m v_{fin}^2(B)$$

point B) $\quad m\cdot\frac{\dot{s}^2}{R}\vec{n} = mg\vec{n} + T\vec{n}$

$$m\frac{\dot{s}^2}{R} = mg + T$$

$$\begin{cases} \frac{1}{2}m v^2(A) = mg\ell + \frac{1}{2}m v^2(B) \\ \frac{\cancel{m}(v(B))^2}{\ell} = \cancel{m}g + \underbrace{\cancel{T}}_{=0} \end{cases}$$

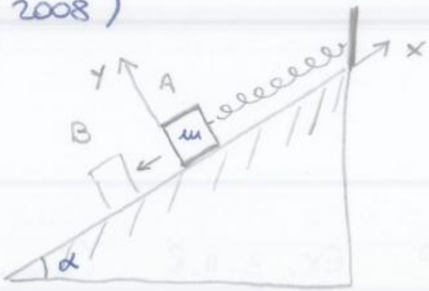
$$v(B) = \sqrt{g\cdot\ell} = 3{,}13\ m/s$$

$$v(A) = \sqrt{\frac{2mg\ell + \frac{1}{2}\cancel{m}v^2(B)}{\frac{1}{2}\cancel{m}}} = 5{,}42\ m/s$$

$$m\vec{v}(A) = \vec{F}\cdot\Delta t$$

$$\hookrightarrow \vec{F} = \frac{m\cdot\vec{v}(A)}{\Delta t} = \boxed{10850\ N}$$

EX. 3.11.10 (Exam June 5^{th} 2008)



√ 0 RESULTS

$t=0 \rightarrow m$ at rest

$\vec{I}$ is given toward down,
for a time Δt (short)

↓

After Δt the spring length is still the rest length.

$\vec{B} \cong 0$ ↘ $E^k_{in} \cong 0$

$m = 0{,}2$ kg $K = 200$ N/m

$\mu_d = 0{,}1$ $\mu_s = 0{,}3$ $\alpha = 30°$

MAX ELONGATION (ℓ) OF SPRING = ?

Distance d (from initial position) where m will stop at the end = ?

Forces:
- dynamic, static > friction → NC
- normal → NO WORK
- gravity → cons ($U = mgy$)
- elastic force → cons ($\frac{1}{2}kd^2 = U$)

$$\begin{cases} 0 = m\ddot{y} = N - mg\cos\alpha \\ 0 = m\ddot{x} = F_s - mg\sin\alpha + k(\ell) \end{cases}$$

$$F_s = mg\sin\alpha - k(\ell)$$

$$|F_s| \le F_{s\,MAX} = \mu_s N = \mu_s mg\cos\alpha$$

$$|mg\sin\alpha - k\ell| \le \mu_s mg\cos\alpha$$

$$|0{,}98 - 200\underbrace{\ell}_{=0 \text{ @ beginning}}| \le 0{,}51 \rightarrow m \text{ will move}$$

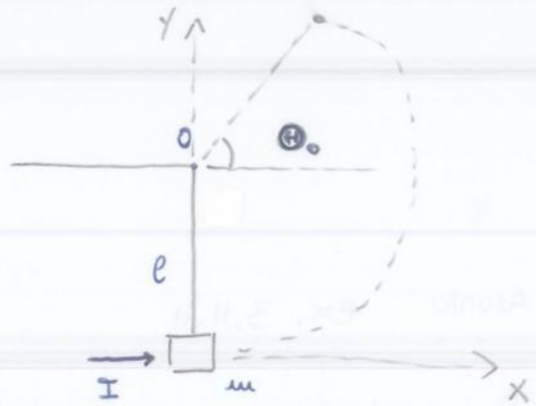
even @ ℓ_{MAX} $|0{,}98 - 200\ell| \le 0{,}51$ will not be satisfied → m will go back up

$$W_{A,B} = W^{NC}_{A,B} + mg\sin\alpha(\ell - d) + \frac{1}{2}k(\ell^2 - d^2)$$

$$W_{A,B} = -\mu_d mg\cos\alpha(d - \ell) + mg\sin\alpha(\ell - d) + \frac{1}{2}k(\ell^2 - d^2)$$

EX. 3.11.12

√



$\beta = 0$

Impulse for short Δt

$$\vec{I} = \vec{F} \cdot \Delta t$$

After Δt, m practically still vertical.

$$\begin{pmatrix} \ell = 1\,m \\ v_0 = 6\,m/s \\ m = 0{,}2\,kg \end{pmatrix}$$

?) Find modulus + direction of velocity @ Θ_0 and Θ_0 at which the mass leaves the circular trajectory.

$$\begin{cases} \dfrac{m v_f^2}{\ell} = T + mg \sin\Theta_0 & \rightarrow \text{components along } \vec{n} \\ \dfrac{1}{2} m v_0^2 + mg(0) = \dfrac{1}{2} m v_f^2 + mg\ell(1 + \sin\Theta_0) \\ T = 0 \end{cases}$$

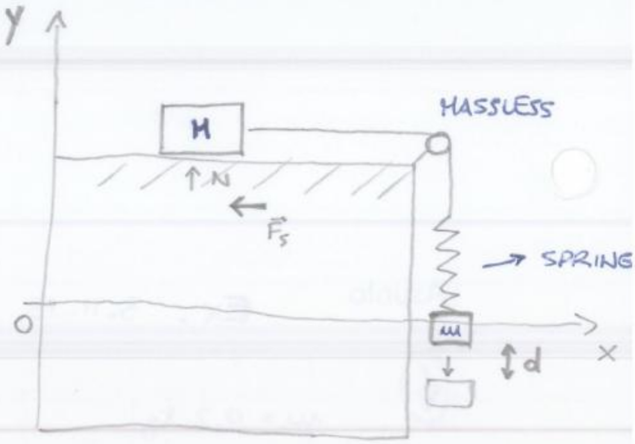
$$\hookrightarrow \begin{cases} v_f^2 = (\cancel{m} g \sin\Theta_0) \dfrac{\ell}{\cancel{m}} \\ \dfrac{1}{\cancel{2}} \cancel{m} v_0^2 = \dfrac{1}{2} \cancel{m} (g\ell \sin\Theta_0) + 2\cancel{m} g\ell(1 + \sin\Theta_0) \end{cases}$$

$$\hookrightarrow \sin\Theta_0 = \frac{v_0^2 - 2g\ell}{3g\ell} \rightarrow \boxed{\Theta_0 = 33{,}8^\circ}$$

$$\hookrightarrow \boxed{v_f = 2{,}34\ m/s}$$

EX. 3.11.15

√



$M = 0,1\ Kg$

$K = 200\ N/m$

$\mu_d = 0,1 \qquad \mu_s = 0,3$

REST LENGTH $= \ell_0$

System kept at rest by hand.
Then m is released, left free.

Find m_{MAX} that allows M to remain at rest.

$$\begin{cases} m\ddot{y} = -mg + k(\ell - \ell_0) & \rightarrow \text{not really useful} \\ 0 = M\ddot{x} = -F_s + T \\ 0 = M\ddot{y} = N - Mg & \rightarrow N = Mg \end{cases}$$

Forces acting on m? - Elastic force, - Gravity } CONSERVATIVE

TOTAL MECH. ENERGY (on m) IS CONSERVED

$$\frac{1}{2}m\underbrace{v_y^2}_{\to 0} + (mg\underbrace{y}_{\to 0} + \frac{1}{2}k\ell_0^2) = \frac{1}{2}m\underbrace{v_{y_{MIN}}^2}_{\to 0} + (mg\,y_{MIN} + \frac{1}{2}k\ell^2)$$

$$mg\,y_{MIN} = \frac{1}{2}k(\ell_0^2 - \ell^2) \quad \rightarrow \text{Consider } \ell_0 \to 0, \ \ell \to \ell_0 + d, \ \ell_0 \to 0, \ d \to -y_{MIN}$$

$$\Rightarrow mg\,y_{MIN} = \frac{1}{2}k(-y_{MIN}^2)$$

$$y_{MIN}\left(mg + \frac{1}{2}k\,y_{MIN}\right) = 0$$

1) ~~$y_{MIN} = 0$~~

2) $y_{MIN} = \dfrac{-mg}{\frac{1}{2}k} = \dfrac{-2mg}{k}$

$$\Rightarrow F_{el} = k(\ell - \ell_0) = k(y_{MIN}) = -2mg$$

The elastic force propagates through the spring to the cord, so:

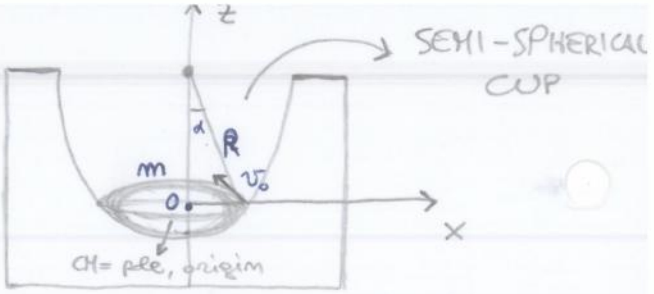
$T = -2mg \quad \Rightarrow \quad F_s = -2mg$

$|F_s| \le F_{sMAX} = \mu_s N = \mu_s Mg \quad \Rrightarrow \quad 2mg \le \mu_s Mg$

$$m \le \frac{\mu_s M}{2}$$

$$\boxed{m_{MAX} = 0,015\ Kg}$$

EX. 3.11.17



???

$m = 0{,}2\ kg \quad R = 15\ cm \quad \alpha = 30°$

BALL

No friction, no viscosity

Find $|v_0|$ to allow the ball to reach the upper edge

Forces: Normal } → NO WORK
Gravity } CONSERVATIVE

CONSERVATION OF MECH. ENERGY
~~ANGULAR MOMENTUM~~

$\vec{r} = r\cdot\vec{\lambda} + z\vec{k}$

$\vec{v} = \dot{r}\vec{\lambda} + r\dot{\vartheta}\vec{\mu} + \dot{z}\vec{k}$

$\vec{L}_o = (\vec{r} - \vec{r}_o) \times m\vec{v} = (r\vec{\lambda} + z\vec{k}) \times m(\dot{r}\vec{\lambda} + r\dot{\vartheta}\vec{\mu} + \dot{z}\vec{k}) =$

$= mr^2\dot{\vartheta}\vec{k} + mr\dot{z}(-\vec{\mu}) + mz\dot{r}\vec{\mu} + mzr\dot{\vartheta}(-\vec{\lambda}) =$

$= (-mrz\dot{\vartheta})\vec{\lambda} + (mz\dot{r} - mr\dot{z})\vec{\mu} + (mr^2\dot{\vartheta})\vec{k} =$

$= (-mrz\dot{\vartheta})\vec{\lambda} + m(z\dot{r} - r\dot{z})\vec{\mu} + (mr^2\dot{\vartheta})\vec{k}$

$E^k = \frac{1}{2}m(\dot{r}^2 + (r\dot{\vartheta})^2 + \dot{z}^2)$

$\tau_o^G = \vec{r} \times mg\vec{k} = (r\vec{\lambda} + z\vec{k}) \times mg\vec{k} = -rmg\vec{\mu}$

$\tau^G_{o(z)} = 0 \Rightarrow L_{o(z)} = const \Rightarrow mr^2\dot{\vartheta} = const$

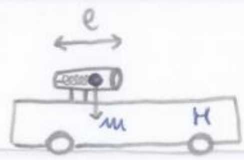
$r_i = R\cdot\sin\alpha \quad r_i\dot{\vartheta}_i = v_o \quad L^i_{o(z)} = mR\sin\alpha\cdot v_o \quad E_i = \frac{1}{2}mv_o^2 - mgR\cos\alpha$

$r_f = R \quad r_f\dot{\vartheta}_f = v_f \quad L^f_{o(z)} = mRv_f \quad E_f = \frac{1}{2}mv_f^2$

EX. 4.6.5

$\ell = 0{,}3\ m$

$m = 0{,}05\ kg$ $\quad M = 0{,}8\ kg$ (carriage + cannon)



NO FRICTION HERE / NO VISCOSITY

inside of cannon → frictionless

If wheels → fixed

v_0 of the ball = $10\ m/s$

Find v_{ball} when wheels are free to move

$V_{carriage}$

CONSERVATION OF TOTAL MECHANICAL ENERGY (wheels fixed)

$$E^T_{fin} = E^T_{in}$$

$$\cancel{\frac{1}{2}} m v_0^2 = \cancel{\frac{1}{2}} k (\ell)^2$$

$$\hookrightarrow k = \frac{m v_0^2}{\ell^2} = 55{,}56\ N/m$$

CONSERVATION OF LINEAR MOMENTUM ($\vec{p}$)

$$0 = MV + mv$$

CONSERV. OF TOT. MECH. ENERGY (wheels moving)

$$\cancel{\frac{1}{2}} MV^2 + \cancel{\frac{1}{2}} m v^2 = \cancel{\frac{1}{2}} k (\Delta x)^2$$

$$\begin{cases} V = -\frac{m}{M} v \\ \frac{m^2}{M^{\cancel{2}}} v^2 \cdot \cancel{M} + m v^2 = k\ell^2 \end{cases} \rightarrow v^2 = \frac{\overset{\nearrow 5}{k\ell^2}}{\underset{\searrow 0{,}0531}{m + \frac{m^2}{M}}} \rightarrow \boxed{\vec{v} = 9{,}7\ m/s}$$

$$\rightarrow \boxed{\vec{V} = -0{,}61\ m/s}$$

EX. 4.8.1

COLLISIONS

$v_0 = 5\ m/s \quad m_1 = 1\ kg \quad m_2 = 2\ kg$

$v_{m_1} = ? \quad V_{m_2} = ?$

m_2 moves parallel to m_1 after collision

~~Angular~~ CONSERV. OF LINEAR MOMENTUM

$$m_1 v_0 = m_1 v + m_2 V$$

1-DIMENSIONAL SCATTERING : $\quad m_1 v_0^2 - m_1 v^2 = m_2 V^2$

$$\begin{cases} 5 = v + 2V \\ 25 - v^2 = 2V^2 \end{cases} \qquad \begin{cases} v = 5 - 2V \\ 25 - 25 - 4V^2 + 20V - 2V^2 = 0 \end{cases}$$

$$-6V^2 + 20V = 0 \rightarrow V(-3V + 10) = 0 \rightarrow \cancel{V=0} \quad \boxed{V = \frac{10}{3}\ m/s}$$

$$v = -2V + 5 = \boxed{-\frac{5}{3}\ m/s = v}$$

EX. 4.8.3

NO FRICTION

$\vec{v}_0 = 5\ m/s \quad \vec{v} = 2\ m/s \quad m_1 = 1\ kg \quad m_2 = 2\ kg$

Bullet penetrates m_2 and leaves w/ $v = 2\ m/s$

Calculate $V = ?$ and work done by internal forces.

CONSERV. OF LINEAR MOMENTUM $\quad m_1 v_0 = m_1 v + m_2 V \rightarrow \boxed{V = 1{,}5\ m/s}$

$$E^K_{in} = \frac{1}{2} m_1 v_0^2 = 12{,}5\ J$$

$$E^K_{fin} = \frac{1}{2}\left(m_1 v^2 + m_2 V^2\right) = 4{,}25\ J$$

$$W = E^K_{fin} - E^K_{in} = \boxed{-8{,}25\ J}$$

$m_A \ddot{x} = -\mu_d m_A g$

$\dot{x} = -\mu_d g t + c_1 \quad \rightarrow c_1 = 9{,}06 \ m/s \quad \rightarrow 0 = -\mu_d g t + c_1$

$x = -\mu_d g \frac{t^2}{2} + c_1 t + c_2 \quad \rightarrow c_2 = 0 \qquad t = 1{,}85 \ s$

$x = 8{,}39 \ m$

He should have stopped after 8,39 m after the collision, but he stopped (m_A) after 16 m.

$\Rightarrow$ HE WAS GOING FASTER THAN 50 km/h

~~WORK LOST DURING THE IMPACT:~~

Actual velocities:

$x = 16 \ m$

$$\begin{cases} x = 16 = t\left(-\mu_d g \frac{t}{2} + c_1\right) \\ \dot{x} = 0 = -\mu_d g t + c_1 \end{cases} \Rightarrow \begin{cases} 16 = t\left(-\mu_d g \frac{t}{2} + \mu_d g t\right) \\ c_1 = \mu_d g t \end{cases}$$

$\Rightarrow \quad 16 = \frac{1}{2}(\mu_d g t) \Rightarrow t = 6{,}52 \ s$

$\Rightarrow \boxed{c_1 = 32 \ m/s = v_A}$

$$\boxed{v_B = \frac{v_A \sin\vartheta \, m_A}{\sin\varphi \, m_B} = 16{,}42 \ m/s}$$

$$v_0 = \frac{v_A \cos\vartheta \, m_A + v_B \cos\varphi \, m_B}{m_A} = 49 \ m/s = 176{,}5 \ km/h$$

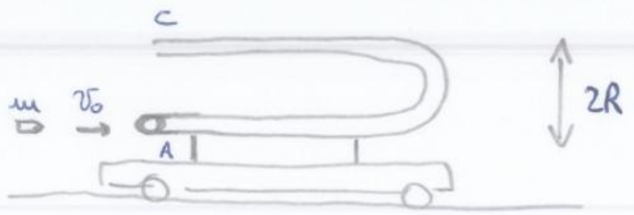
27063

WORK DONE DURING THE IMPACT:

$E^k_{in} = \frac{1}{2} m_A v_0^2 = 1'080'450 \qquad E^k_{fin} = \frac{1}{2}\left(m_A v_A^2 + m_B v_B^2\right) = 622570 \ J$

$$\boxed{W = E^k_{fin} - E^k_{in} = -457'880 \ J}$$

$\boxed{\text{EX. 4.89.}}$



√

$R = 0{,}010\ m$

$M = 2\ kg$ (trolley + pipe)

$m = 0{,}01\ kg$

$v_0 = 50\ m/s$

Find: v_m, V_M when the bullet exits from point C ?

$$\begin{cases} m v_0 = m v + M V \\ \frac{1}{2} m v_0^2 = \frac{1}{2} M V^2 + \frac{1}{2} m v^2 + m g (2R) \end{cases}$$

$$\begin{cases} V = \dfrac{m (v_0 - v)}{M} \\ \cancel{\frac{1}{2}} m v_0^2 = \cancel{\frac{1}{2}} M \dfrac{m^2 v_0^2 - m^2 v^2}{M^2} + \cancel{\frac{1}{2}} m v^2 + 2 m g (2R) \end{cases}$$

$$v_0^2 = \frac{m v_0^2 - m v^2}{M} + v^2 + 4gR$$

$$v = \sqrt{\frac{\left(v_0^2 - \frac{m v_0^2}{M} - 4gR\right)}{\left(1 - \frac{m}{M}\right)}} = \sqrt{\frac{2500 - 12{,}5 - 0{,}39}{0{,}995}} \cong \boxed{49{,}99\ m/s}$$

$$\boxed{V \cong 0\ m/s}$$

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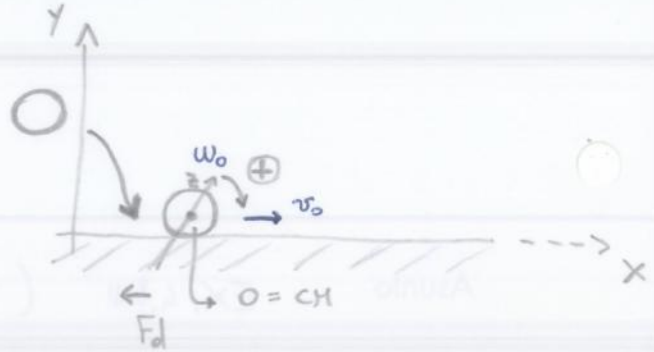
RIGID BODY

EX. 4.14.2



$I = \frac{2}{5} MR^2$

$\vec{v}_0 = v_0 \vec{i}$

$\omega_0 = -5$ rad/s

$R = 0.10$ m

$M = 2$ kg $\quad \mu_s = 0{,}8 \quad \mu_d = 0{,}4 \quad v_0 = 1$ m/s

Beginning : FRICTION IS DYNAMIC ; ⊖ ROTATION

$$\begin{cases} M \cdot \ddot{x} = F_x^E \\ M \cdot \ddot{y} = F_y^E = 0 \\ \tau_z^E = I_z \dot{\omega} \end{cases} \qquad \begin{cases} M\ddot{x} = -\mu_d N \\ 0 = N - Mg \\ I_z \dot{\omega} = +\mu_d MgR \end{cases} \qquad \begin{cases} \dot{x} = -\mu_d g t + c_1 \\ N = Mg \\ I_z \dot{\omega} = +R\mu_d Mg \end{cases}$$

$$\begin{cases} \dot{x} = -\mu_d g t + v_0 \rightarrow x = -\frac{1}{2}\mu_d g t^2 + v_0 t \qquad x = 0.08 \\ \omega = \frac{R\mu_d Mg}{I_z} t + \omega_0 \rightarrow \omega = \frac{5}{2} \mu_d g t + \omega_0 \end{cases}$$

$$\frac{5}{2}\mu_d g t R + \omega_0 R = -\mu_d g t + v_0$$

$$t\left(\frac{5}{2}\mu_d g R + \mu_d g\right) = v_0 - \omega$$

$t = 0.10$ s

$$\begin{cases} \frac{1}{2} M v^2 = \frac{1}{2} M V^2 + \frac{1}{2} I \omega^2 \qquad I = \frac{2}{5}MR^2 \\ -\mu_d Mg(x) = \frac{1}{2} M v^2 - \left(\frac{1}{2} M v_0^2 + \frac{1}{2} I \omega_0^2\right) \end{cases} \qquad \omega R = V$$

$$\begin{cases} v^2 = (\omega R)^2 + \frac{2}{5}(R\omega)^2 \\ -2\mu_d Mg x = \frac{1}{2} M\left(\frac{7}{5}\omega^2 R^2\right) - \frac{1}{2}\left(M v_0^2 + I \omega_0^2\right) \qquad I = \frac{2}{5}MR^2 \end{cases}$$

$$-0.6308 = (0.014)\,\omega^2 - 1.1$$

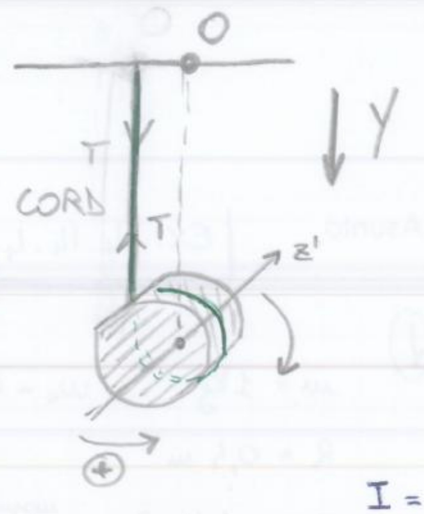
$\omega = 5.8$ rad/s

$\boxed{\text{EX. } 4.14.6}$

$M = 0{,}1\ kg$

$R = 0{,}07\ m$

$d = 0{,}3\ m$



Cylinder @ rest @ $t=0$
Then it is released.
Find t_0 when the cyl. will have reached a distance $\underline{d}$ from the ceiling.

$I = \frac{1}{2}MR^2$

$\cancel{\tau_O^z = I\dot{\omega}}$

$\cancel{\vec{\tau}_{O,z}^{G} = -Mg\, x_{CM}}$ 0,31

$\cancel{I\dot{\omega} = -Mg\, x_{CM}}$

$c' = O = CM$

$\cancel{\frac{1}{2}MR^2\dot{\omega} = -Mg\, x_{CM}}$

$\cancel{\frac{R^2\dot{\omega}}{2} = -g\, x_{CM}}$

$\dot{\omega} =$

$$\begin{cases} M\cdot\vec{a}_{CM\,(y)} = Mg - T & \longrightarrow \text{I CARD. EQ. } (y) \\ I_{z'}\cdot\dot{\omega}' = \cancel{Mg\,x_{CM}}\ RT & \longrightarrow \text{TORQUE ALONG } z \\ \ddot{y} = R\dot{\omega}' \end{cases}$$

$$\begin{cases} M R\dot{\omega}' = Mg - T \\ \frac{1}{2}MR^2\cdot\dot{\omega}' = RT \end{cases} \quad \begin{cases} T = Mg - MR\dot{\omega}' \\ \frac{1}{2}MR^2\dot{\omega}' = \cancel{RMg - RMR\dot{\omega}'} \end{cases}$$

$\dot{\omega}' =$

$$\begin{cases} M\ddot{y} = Mg - T \\ I_z\frac{\ddot{y}}{R} = RT \end{cases} \quad \begin{cases} \ddot{y} = g - \frac{T}{M} \\ \ddot{y} = RT\cdot\left(\frac{R}{I_z}\right) \end{cases} \quad g - \frac{T}{M} = R^2\frac{T}{I}$$

$T = g\Big/\left(\frac{R^2}{I}+\frac{1}{M}\right)$

$\dot{y} = gt + c_1$
$\Rightarrow c_1 = 0$

$\ddot{y} \approx g \Leftarrow T = 0{,}002\bar{3}\ N$

$y = \frac{1}{2}gt^2 + c_2 \quad y = \frac{1}{2}gt^2$
$\Rightarrow c_2 = 0$

$0{,}3 = \frac{1}{2}gt^2 \rightarrow \boxed{t = 0{,}247\ s}$