NUMERO: 1496A - ANNO: 2015

A P P U N T I

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MATERIA: Numerical Methods, Prof.Barla

NUMERICAL METHODS

LECTURE 1

GEOTECHNICAL MODEL: it is a schematic representation of reality able to describe the fundamentals aspects of the behaviour of a soil or a rock mass. It is obtained combining together all the information provided by the geotechnical characterisation process.

DESIGN METHODS: ① Empirical ⇒ mainly based on previous experience or derived from construction of rock structures similar to the one to be designed (use classification methods RMR, Q, GSI); ② Limit Equilibrium ⇒ applied by performing a limit equilibrium analysis, based on the strength characteristics of the rock mass and of discontinuities; ③ Stress Analysis ⇒ it consists in applying the state of stress-strain in the engineering structure, with consideration given to the strength and deformability properties of the rock mass and of the discontinuities

STRESS ANALYSIS: is the most complete method as both stress and strain distributions are obtained following computation.

The soil/rock mass is represented as a CONTINUUM OR DISCONTINUUM.

Equivalent Continuum Model: the rock mass is treated as a continuum with equal in all directions input data for the strength and deformability properties, which define a given constitutive relation for the medium

Discontinuum model: The rock mass is represented as a discontinuum and most of the attention is devoted to the characterisation of the rock elements and the rock joints/discontinuities. The modelling approach consists in considering the ~~model~~ blocky nature of the system being analysed. Each block may interact with the neighbouring blocks troughe the joints.

DESIGN ANALYSIS: an initial design based on the most probable conditions together with predictions of behaviour is to be developed in order to identify, contingency plans and trigger values for the monitoring system.

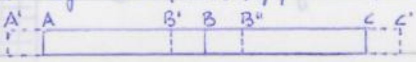
Equilibrium: if we consider that the element is define along 3 direction (x, y, z) we have 3 equation of equilibrium

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0$$

Physical Compatibility: compatible deformation in a continuum mechanics involves no overlapping (sovrapposizioni) of material and no generation of holes (buchi; fratture).



If we push we don't want overlapping

If we pull we don't want generation of holes

Mathematical Compatibility: we have 6 equations of compatibility

Constitutive Behaviour: combining the equilibrium and compatibility conditions, gives

- Unknowns (6 stress + 6 strains + 3 displacements = 15)
- Equations (3 equilibrium + 6 compatibility = 9)

To obtain a solution therefore requires 6 more equations; These come from the stress-strain equations.

3D Problems: The mechanical behaviour of soils and rocks in general is nonlinear and irreversible. It is therefore more realistic and appropriate to write the above constitutive law in incremental terms: $[\Delta\varepsilon] = [D_E][\Delta\sigma]$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix}$$

The above constitutive law can be re-written by introducing the shear modulus G and the bulk modulus K

K = legato a cambiamento volume $= \frac{E}{3(1-2\nu)}$

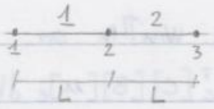
E = Young Module $= \frac{\sigma - \text{imposto}}{\varepsilon - \text{misurato}}$

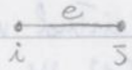
G = legato a cambiamento forma $= \frac{E}{2(1+\nu)}$

ν = poisson ratio = minus $\varepsilon_z, \varepsilon_x$

If we want to do a simplification we can study the problem in 2D where we have a plane strain or plane stress

For each node we have 1 DOF, in Total I have a system with 2 DOF





2-STEP: The displacement $u(x)$ is selected as The primary variable. Its components are assumed To have a simple polynomial form, where the order of The polynomial depends on The number of nodes in The element. $u(x) = \alpha_0 + \alpha_1 x$

$$u(x) = [1 \ \ x]\begin{bmatrix}\alpha_0\\ \alpha_1\end{bmatrix} \Rightarrow u(x) = [\phi]^T[\alpha]$$

For The nodes i and j we have

$$u(0) = u_i = [1 \ \ 0]\begin{bmatrix}\alpha_0\\ \alpha_1\end{bmatrix} \qquad u(L) = u_j = [1 \ \ L]\begin{bmatrix}\alpha_0\\ \alpha_1\end{bmatrix} \Rightarrow \begin{bmatrix}u_i\\ u_j\end{bmatrix} = \begin{bmatrix}1 & 0\\ 1 & L\end{bmatrix}\begin{bmatrix}\alpha_0\\ \alpha_1\end{bmatrix}$$

The vector of nodes is: $[u]_e = [A][\alpha]$

$$[\alpha] = [A]^{-1}[u]_e$$

Solving for generic point any x we have $u(x) = [\phi]^T[\alpha] = [\phi]^T[A]^{-1}[u]_e = [H]^T[u]_e$

$[H]$ SHAPE FUNCTION (funzione forma)

$$[H]^T = [1 \ \ x]\begin{bmatrix}1 & 0\\ -\frac{1}{L} & \frac{1}{L}\end{bmatrix}$$

$$u(x) = [(1 - x/L) \ \ x/L]\begin{bmatrix}u_i\\ u_j\end{bmatrix}$$

3-STEP: The axial strain is given by: $\varepsilon = \frac{du(x)}{dx} = \underbrace{\frac{d}{dx}[H]^T}[u]_e$

$[\varepsilon] \Rightarrow \varepsilon = [\phi]^T[A]^{-1}[u]_e = [B][u]_e$ $\qquad$ $[B]$ ELEMENT STRAIN MATRIX $= [-1/L \ \ 1/L]$

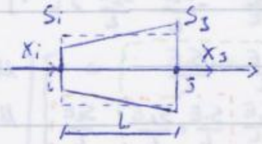
The axial stress is given on The basis of The constitutive law by:

$E \Rightarrow [C]$

$[\sigma] = [C][\varepsilon]$ $\qquad$ $\sigma = E\varepsilon = E[-1/L \ \ 1/L]\begin{bmatrix}u_i\\ u_j\end{bmatrix}$

If we want To define The constitutive law we will define The finite elastic constant $[C]$.

In order To determine The Stiffness Matrix we define a relationship between $[X]_e$ (The nodal forces at The nodes) and $[u]_e$ Taken from structural analysis.



$[X]_e = \begin{bmatrix}X_i\\ X_j\end{bmatrix}$ Nodal forces at nodes i, j

$[X]_e = \underbrace{[K]_e}_{\text{stiffness matrix}}[u]_e$ $\qquad$ $[K]_e = \begin{bmatrix}k_{ii} & k_{ji}\\ k_{ij} & k_{jj}\end{bmatrix}$ symmetric

Applying The principle of Virtual Work:

$$\underbrace{\int_{V_e}[\bar{\varepsilon}]_e^T[\sigma]_e\, dV_e}_{\text{Internal Virtual Work}} = \underbrace{X_i\bar{u}_i + X_j\bar{u}_j}_{\text{External Virtual Work}}$$

virtual strain $\qquad$ virtual nodal displacement

strain energy = Virtual work due To nodal Force

$$\int_{V_e}[\bar{\varepsilon}]_e^T[\sigma]_e\, dV_e = [\bar{u}_i \ \ \bar{u}_j]\begin{bmatrix}X_i\\ X_j\end{bmatrix} = [\bar{u}]_e^T[X]_e$$

5-STEP: Which are the boundary conditions? Define the displacement for each nodes.

$u_1 = u_1^*$ KNOWN $\quad X_1 =$ UNKNOWN

$u_2 = ?$ UNKNOWN $\quad X_2 =$ KNOWN

$u_3 = u_3^*$ KNOWN $\quad X_3 =$ UNKNOWN

$$\begin{bmatrix} X_2 = 0 \\ X_1 = ? \\ X_3 = ? \end{bmatrix} = E/L \begin{bmatrix} S_1+S_2 & -S_1 & -S_2 \\ -S_1 & S_1 & 0 \\ -S_2 & 0 & S_2 \end{bmatrix} \begin{bmatrix} u_2 \\ u_1^* \\ u_3^* \end{bmatrix}$$

We don't have internal forces and then $X_2 = 0$

$$0 = -\frac{S_1 E}{L} u_1^* + u_2 \frac{S_1 E}{L} + u_2 \frac{S_2 E}{L} - \frac{S_2 E}{L} u_3^* \quad \rightarrow \quad u_2 = \frac{S_1 u_1^* + S_2 u_3^*}{S_1 + S_2}$$

$$X_1 = \frac{S_1 S_2}{S_1 + S_2} \frac{E}{L} (u_1^* - u_3^*) = -X_3$$

6-STEP: computation to obtain stress and strain thanks to constitutive law

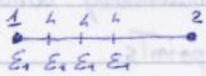
ELEMENT 1 - $\varepsilon_1 = [B][u]_1 = [-1/L \quad 1/L] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

$$\varepsilon_1 = \frac{S_2}{S_1+S_2} \frac{u_3^* - u_1^*}{L} \rightarrow [\sigma] = [C][\varepsilon] \rightarrow \sigma_1 = E\varepsilon_1$$

ELEMENT 2 - $\varepsilon_2 = [B][u]_2 = [-1/L \quad 1/L] \begin{bmatrix} u_2 \\ u_3 \end{bmatrix}$

$$\varepsilon_2 = \frac{S_1}{S_1+S_2} \frac{(u_3^* - u_1^*)}{L} \rightarrow \sigma_2 = E\varepsilon_2$$

If we look at element 1 what is the value of the strain in point 4?



N.B. the stress and strain are constant in the any small element and then this is an approximation, this is the motifs.

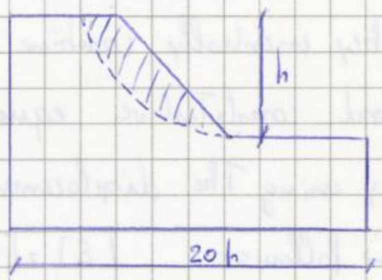
SUMMARY: The essential features of FEM have been described by taking the axial stiffness element as case example. The procedure show and the different equations derived apply in general. In the summary we have:

1. Element discretization
2. Displacement approximation $\quad u(x) = [H]^T [u]_e$
3. Computation of strains and stresses $[\varepsilon] = [B][u]_e$; Computation element stiffness $[K]_e = \int [B]^T [C][B] d$
4. Globale equilibrium equations $[X] = [K][u]$
5. Boundary conditions
6. Computation of strain and stress $[\varepsilon] = [B][u]_e \quad [\sigma] = [C][B][u]_e$

EXAMPLE

We consider a rock slope and we have to garantie the stability of the rocks

How we can do to solve this problem with FEM?



• Define where are boundaries (and the dimension of model, I can use equivalent diameter or elevation h.

• I select the shape of elements and the fenomeno, which happens in slope and I can utilise the continuum elastic frame and discontinuum frame

• This is a prediction but if the surface is not real I have to discretise with small element near the surface

2. DISPLACEMENT APPROXIMATION

In the displacement based finite element method the primary unknown quantity is the displacement field which varies over the problem domain. Strain and stresses are treated as secondary quantities which can be found from the displacement field once it has been determined. The variation of displacement component must satisfy the conditions of compatibility.

$$[u(x,y,z)] = [H(x,y,z)][u]_e$$

If we use this function for the elements in the mesh we obtain a FEM approximation of the equilibrium equations. To this end, we apply PLV. The essential feature of the element wise approximation is that the variation of the unknown displacements within an element is expressed as a simple function of the displacements at the nodes. These nodal displacements are refered to as the unknown degrees of freedom. For 2D plane strain problem there are two DOF at each node.

We have to increase the DOF each increase of dimension 1D, 2D, 3D.

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5. BOUNDARY CONDITIONS

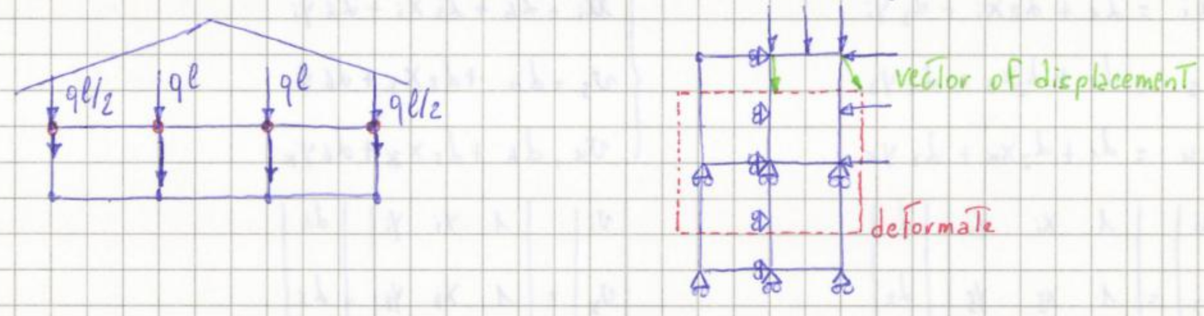
There are the load and displacement conditions which fully define the boundary value problem being analysed.

a. Loading conditions

b. Displacement boundary conditions

c. Combination of loading and displacements conditions.

In both to have the nodes in the intersection of structure and soil, I have to associate the load for each nodes something like this



6. SOLVE GLOBAL EQUATIONS

There are several different mathematical techniques for solving large system of equations: - direct methods

- iterative methods

Flow Diagram for FEM: Real Problem → FEM MESH → Approximate displacement [u] → Compute Stiffness matrix → Equilibrium equation → Boundary conditions → [u] [ε] [σ]

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We obtain : $u(x,y) = \frac{1}{2A}\left[(a_i + b_i x + c_i y)u_i + (a_j + b_j x + c_j y)u_j + (a_k + b_k x + c_k y)u_k\right]$

$v(x,y) = \frac{1}{2A}\left[(a_i + b_i x + c_i y)v_i + (a_j + b_j x + c_j y)v_j + (a_k + b_k x + c_k y)v_k\right]$

where $a_i = x_j y_k - x_k y_j$ $b_i = y_j - y_k$ $c_i = x_k - x_j$

with the remaining coefficients $a_j, b_j \dots c_k$ obtained by a cyclic permutation in order i, j, k

$2A = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix}$ = Area of Triangle (i, j, k)

3. ELEMENT EQUATIONS

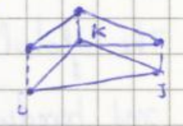
Strain and stiffness computation :

$[\varepsilon] = \begin{vmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{vmatrix} = \begin{vmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \partial u/\partial y + \partial v/\partial x \end{vmatrix}$ The matrix is independent of the position within the element, and hence strains are constant throught

$$[\varepsilon] = \underbrace{\frac{1}{2A}\begin{vmatrix} b_i & 0 & b_j & 0 & b_k & 0 \\ 0 & c_i & 0 & c_j & 0 & c_k \\ c_i & b_i & c_j & b_j & c_k & b_k \end{vmatrix}}_{[B]_e \text{ within the element}} \begin{vmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{vmatrix}$$

[B] is constant V, strain doesn't vary along the element because I know the displacement of the nodes too

NB For this reason I call "CONSTANT STRAIN TRIANGLE" (CST)



$[\sigma] = [C]_e[\varepsilon] = \underbrace{[C]_e[B]_e[u]_e}$, constitute behaviour

$[k]_e = \int_{V_e} [B]_e^T [C]_e [B]_e \, dV = [B]_e^T [C]_e [B]_e A t$ A = area t = thickness

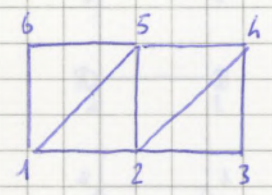
4. BOUNDARY CONDITIONS

5. GLOBAL EQUATIONS

Next step in the formulation of finite element equation is the assembly of the separate element equilibrium equations into a set of global

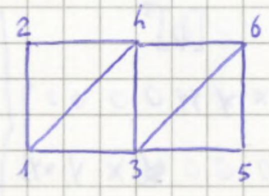
EXAMS EXERCICE

How we can reduce The BandwidTh ?



$$\begin{bmatrix} K_{11} & K_{12} & 0 & 0 & K_{15} & K_{16} \\ & K_{22} & K_{23} & K_{24} & K_{25} & 0 \\ & & K_{33} & K_{34} & 0 & 0 \\ & & & K_{44} & K_{45} & 0 \\ & & & & K_{55} & K_{56} \\ & & & & & K_{66} \end{bmatrix}$$

BandwidTh = 6



$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & 0 & 0 \\ & K_{22} & 0 & K_{24} & 0 & 0 \\ & & K_{33} & K_{34} & K_{35} & K_{36} \\ & & & K_{44} & 0 & K_{46} \\ & & & & K_{55} & K_{56} \\ & & & & & K_{66} \end{bmatrix}$$

BandwidTh = 4

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Strain in the element are a function of the nodal displacements

$\varepsilon_x = \frac{\partial u}{\partial x}$ $\varepsilon_y = \frac{\partial v}{\partial y}$ $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

Therefore, defining $[u(x,y)] = [N(x,y)]^T [u]_e$

You get the following $[\varepsilon] = [B][u]_e$

$[B] = [\phi']^T [A]^{-1}$ STRAIN MATRIX

The stiffness matrix is: $[K]_e = \int_A [B]^T [C][B]\, t\, dx\, dy$ t = Thickness of element equal to 1 plane strain

$$[C] = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{vmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{vmatrix}$$ plane strain conditions

Global Stiffness Matrix

$$[K]_{6\times6} = \begin{vmatrix} x & x & x & x & 0 & 0 \\ & x & x & x & 0 & 0 \\ & & x & x & x & x \\ & & & x & x & x \\ & & & & x & x \\ & & & & & x \end{vmatrix}$$

Boundary conditions, to be determined

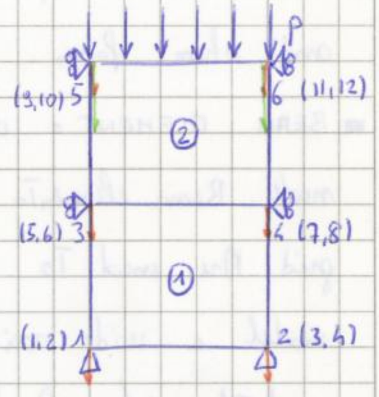
$[R]$: point nodal forces

$[R]_v$ Body nodal forces $[R]_v = \int_s [N]^T [f^v]\, dS$

$[R]_s$ Surface nodal forces $[R]_s = \int_s [N]^T [f^s]\, dS$

$[K]_{6\times6}$ Stiffness matrix

$[K]_{12\times12}$ Stiffness matrix



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- CABLE ELEMENT = one - dimensional axial elements That may be anchored at a specific point in The grid or grouted so That The cable elements develops forces along its length as The grid deforms. Cable element can yield in tension or compression, but They cannot sustain a bending moment. Cable element may be initially pre-Tensioned. Used To model rock bolts, cable bolts, and Tiebacks.
- PILE ELEMENT = Two - dimensional element That can Transfer normal and shear forces and bending moments To grid. Piles offer The combined features of beams and cables. Shear forces act parallel To The element and normal forces perpendicular To The element.
  The element doesn'T yield axially, but plastic hinges can develop.
- SUPPORT MEMBERS = are intended To model hydraulic props, wooden props or wooden packs. Is a spring connected between Two boundaries. The support member has no independent DoF: it simply imposes forces on The boundaries To which it is connected.

INTERFACES

In any ground - structure interaction situation, relative movement of The structure with respect To The soil can occur.
Continuum elements do not allow relative movement at The soil - structure interface. Joint elements or interfaces can be used To model soil-structure boundaries.
There are many method To model discontinuous behaviour at The soil structure interaction.
- use continuum elements
- use of springs To model interface
- interaction model
- use special element (zero Thickness)

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orthotropic matrix

$$\begin{bmatrix} 1/E_1 & -\nu/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\ & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0 \\ & & 1/E_3 & 0 & 0 & 0 \\ & & & 1/G_{12} & 0 & 0 \\ & & & & 1/G_{23} & 0 \\ & & & & & 1/G_{31} \end{bmatrix}$$

9 independent elastic constants.

isotrop geomedia

$$\frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ & 1 & -\nu & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ & & & 2(1+\nu) & 0 & 0 \\ & & & & 2(1+\nu) & 0 \\ & & & & & 2(1+\nu) \end{bmatrix}$$

2 independent elastic constant

$E_1 = E_2 = E_3 = E \qquad G_{12} = G_{13} = G_{23} = G$

$\nu_{21} = \nu_{23} = \nu_{31} = \nu \qquad G = \dfrac{E}{2(1+\nu)}$

NON LINEAR ELASTIC BEHAVIOUR

Soil and Rock mass exhibit a behaviour wich is much more complex than the linear elastic behaviour (LE) described so far. We have to use more complex models to solve the problem. Non Linear Elastic Model (NONLE) where the material parameters which define the compliance/stiffness matrix depend on the stress/strain level attained during the solution of a given problem. The material parameters to be used are either E, ν or K, G which can be retained as constant only for a given stress/strain increment.

Two such models will be discussed in the following:

1- Bilinear Elastic Model, is the case where stress-strain curve is nonlinear, and in these cases we adopt the bilinear model:

- first interval, with initial elastic modulus E_i
- second interval, with E_y constant modulus

$[\Delta\sigma] = [C_i][\Delta\varepsilon] \rightarrow \sigma < \sigma_y$

$[\Delta\sigma] = [C_y][\Delta\varepsilon] \rightarrow \sigma > \sigma_y$

In this model poisson ratio ν is taken as a constant

With this model K and G are chosen to be constant up to the failure

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LECTURE 10-11-12

ELASTIC PERFECTLY PLASTIC MODELS

Non linear and irreversible behaviour, general structure of elastic perfectly plastic models

- Stress and strain invariants: (x, r, z) (x_1, x_2, x_3) (α, σ, θ)

Tensor have properties which are independent of the reference system used = invariant

1) Effective mean stress → axial symmetry $p' = \frac{\sigma_a' + 2\sigma_r'}{3}$

→ 3D $p' = \frac{\sigma_1' + \sigma_2' + \sigma_3'}{3} = \frac{I_{1\sigma}}{3}$

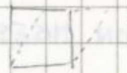
2) Deviatoric stress → axial symmetry $q = \sigma_a - \sigma_r$

→ 3D $q = \sqrt{3}\left[(\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2\right] = \sqrt{3 J_{2d}}$

3) Volumetric strain → axial symmetry $\varepsilon_p = \varepsilon_a + 2\varepsilon_r$

→ 3D $\varepsilon_p = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = I_{1\varepsilon}$

4) Deviatoric strain → axial symmetry $\varepsilon_q = \frac{2}{3}(\varepsilon_a - \varepsilon_r)$



→ 3D $\varepsilon_q = \frac{2}{\sqrt{3}}\left[\left(\varepsilon_1 - \frac{\varepsilon_v}{3}\right)^2 + \left(\varepsilon_2 - \frac{\varepsilon_v}{3}\right)^2 + \left(\varepsilon_3 - \frac{\varepsilon_v}{3}\right)^2\right]^{0.5}$

- Tests to characterize the behaviour are: TX, Oedometric Tests, shear Test, and Mx is not representative for soil.
- Evidence of non linear and irreversible behavior:

If we consider a result of drained triaxial compression $\sigma_a > \sigma_r$, we can look that the response of geomaterials are complex, the non linearity irreversibility, stress path dependence need to be taken into account by constitutive equations, but constitutive equations do not represent universal law of nature but they rather define "idealized materials". Constitutive models can be used to predict soil behavior under any possible circumstance; the quality of predictions depends on the ability of defining a suitable idealization for real materials, for the class of loading paths of practical interest

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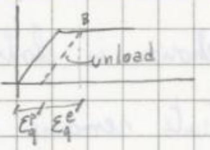
HOEK - BROWN MODEL - 2D $\sigma_1 = \sigma_3 + \sigma_c \left(m_b \frac{\sigma_3}{\sigma_c} + s \right)^{\alpha}$

▶ Yield function f is defined as a mathematical function separating a domain where the behavior is purely elastic from domains where the behavior is elastic - plastic -> $f(\sigma) < 0$, $f(\sigma) = 0$, $f(\sigma) > 0$ (material cannot sustain it)

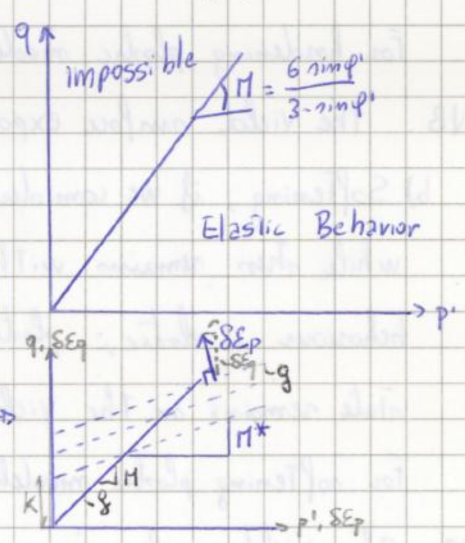
Lo at yielding $\delta\varepsilon = \delta\varepsilon^e + \delta\varepsilon^p$



ELASTIC PERFECTLY PLASTIC MOHR - COULOMB MODEL

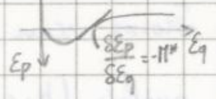
Parameters of yield function $f(\sigma) = f(p', q) = q - M p'$

$g(\sigma) = q - M^* p' - K$



The plastic potential of the ELPLA MC is usually a line characterized by a slope M^* = dilatancy

$M^* = \frac{6 \sin\psi}{3 - \sin\psi}$ ψ = angle of dilation

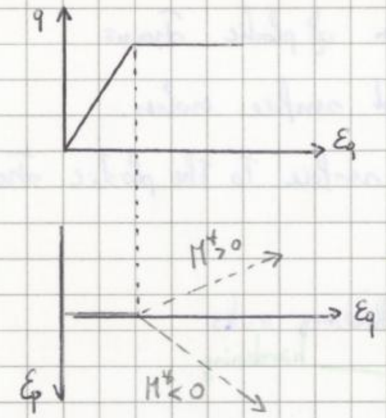


direction of the plastic vector $\frac{\delta\varepsilon_p^p}{\delta\varepsilon_q^p} = M^*$

$M^* > 0$ ($\psi > 0$) implies that when shearing at yield the volume increases

$M^* < 0$ ($\psi < 0$) implies " " " " " " " reduces

The special case is $\psi = \varphi$ and then $M = M^*$ is called Associated Flow



$M^* > 0$ ($\psi > 0$) implies that when shearing at yield the volume increases

$M^* < 0$ ($\psi < 0$) implies that when shearing at yield the volume reduces

$\psi = \varphi$ called Associated Flow

$M = M^*$ implies that no energy is dissipated during an increment of plastic deformation

unrealistic

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EXTENDED MOHR COULOMB MODEL

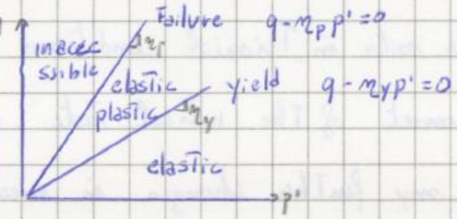
a) Elastic domain (NON LE) $G \propto p'^{1/2}$

b) Yield function $f = f(\sigma', \chi) = f(p', q, \eta_y) = 0$ $f = q - \eta_y p'$

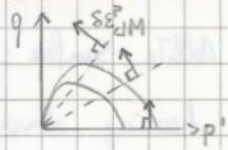
with η_y = hardening parameter it is not a constant but depends on the deviatoric plastic strain.

Failure and yield are different concepts.

η_y can change up to η_p



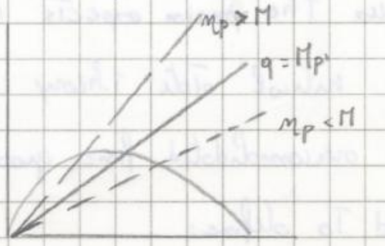
c) Flow rule, associate flow conditions would be $\frac{\delta\varepsilon_p^p}{\delta\varepsilon_q^p} = -\eta_y$, implies that the dilatancy increases with stress obliquity, not realistic, non physical.



direct shear test on sand would suggest $\frac{\delta\varepsilon_p^p}{\delta\varepsilon_q^p} = M - \frac{q}{p'} = M - \eta_y$

Volume decrease at low stress ratios, volume increase at high stress ratios.

d) Hardening rule, current size of the yield locus is expressed as a function of deviatoric strains. $\frac{\eta_y}{\eta_p} = \frac{\varepsilon_q^p}{a + \varepsilon_q^p}$ it is an hyperbolic law



$\eta_p < M$ compression, reduction of volume

$\eta_p = M$ critical state (constant volume)

$\eta_p > M$ expansion

$$\eta_p = \frac{q}{p'} = M = \frac{6 \sin\varphi_p}{3 - \sin\varphi_p}$$

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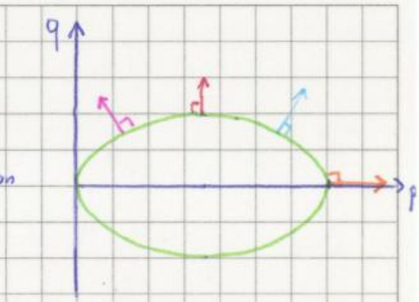
$\eta = 0 \rightarrow \frac{\delta\varepsilon_p^p}{\delta\varepsilon_q^p} = \infty$ $\quad \delta\varepsilon_q^p = 0$ only volume deformation

$\eta = M \rightarrow \frac{\delta\varepsilon_p^p}{\delta\varepsilon_q^p} = 0$ $\quad \delta\varepsilon_p^p = 0$ only distorsional deformation

$\eta < M \rightarrow \frac{\delta\varepsilon_p^p}{\delta\varepsilon_q^p} > 0$ $\quad$ compression + distorsion

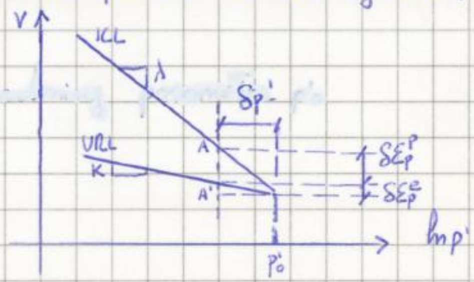
$\eta < M \rightarrow \frac{\delta\varepsilon_p^p}{\delta\varepsilon_q^p} > 0$ $\quad$ expansion + distorsion



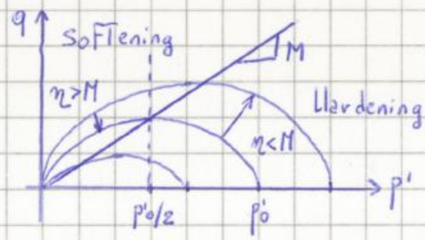
c) Hardening, is related to the plastic component of the volumetric deformations

$\frac{\partial p'_0}{\partial\varepsilon_p^p} = \frac{v p'_0}{\lambda - k}$ This relationship provides the expression for the hardening law,

distorsionic plastic strains have no effect on the hardening parameter p'_0



c) Hardening law:



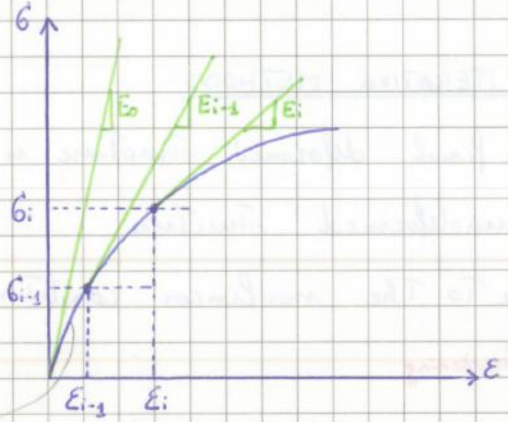
PARAMETERS OF THE MODEL

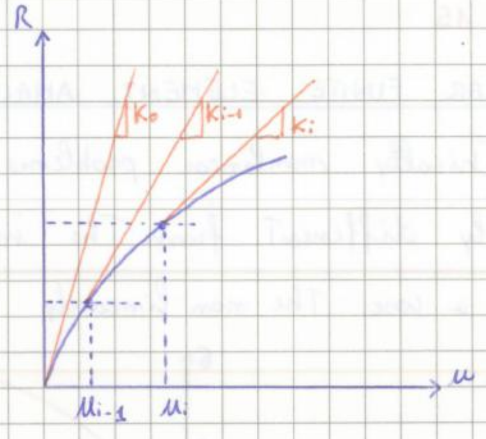
Elastic domain: $k \rightarrow K = \frac{vp'}{k}$; G (either constant or variable)

2 plastic parameters $M \rightarrow$ slope of Critical State Line in $(p';q)$ plane

$\lambda \rightarrow$ slope of Virgin Compression Line in $(v; \ln p')$ plane

1 further condition $N \rightarrow$ reference specific volume (at $p' = 1 kPa$)

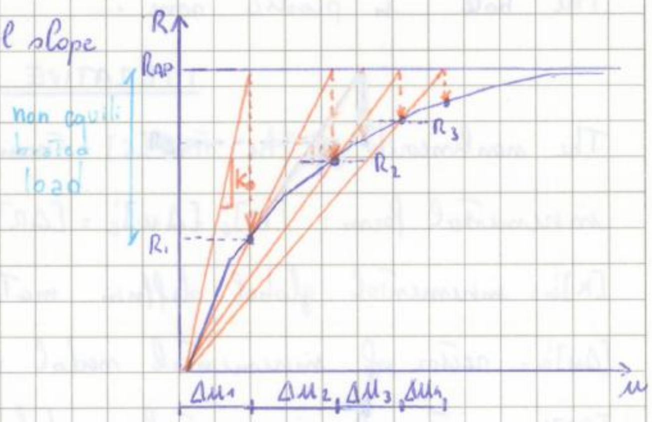




Constitutive behaviour is non linear for this material

E_0 is my initial Young modulus and when I apply the load my modulus will change, also k will change

If we apply a R_{AP} = load applied at the Top of a element, we have to define the initial stiffness K_0, is given by initial slope (Tangent) and I can have u_1



1° ITERATION $\quad [u_1]_1 = [K_0]^{-1}[R_{AP}]$

$\Delta R_1 = R_{AP} - R_1$

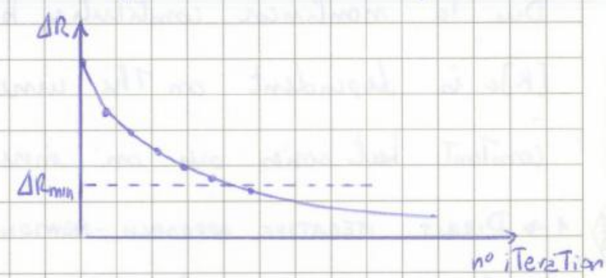
2° ITERATION $\quad [u]_2 = [K]_1^{-1}[R]_{AP}$

$\Delta R_2 = R_{AP} - R_2$

I have to do more than one iteration to compute all displacements

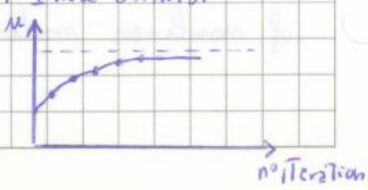
We reduce unbalanced force for each iteration if I consider this process I have this graph:

First reduction is much bigger than second and other



NB I have:

- reduction of difference between real behaviour and computed behaviour
- reduction of displacement between the first, second, third... I have asimptot
- I have to define the limit ΔR_{min} or Δu



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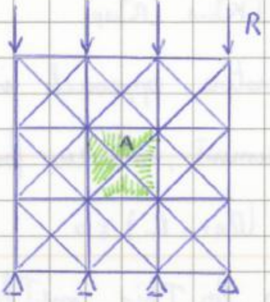
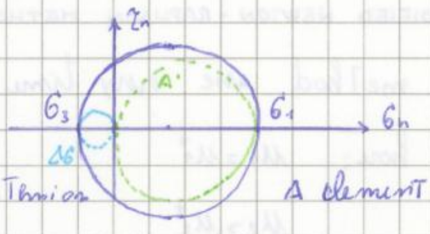
LECTURE 16

ITERATIVE METHODS AND INCREMENTAL METHODS



Application to a no tension material.
The specimen is loaded axially. The element shown attain a no-tension behaviour; The other elements follow a linearly elastic law (ILE).
We looks like is similar to brazilian test but is not similar we expect traction tension.
It is known from a FEM-ILE solution in plane strain conditions that element A will undergo a state of stress

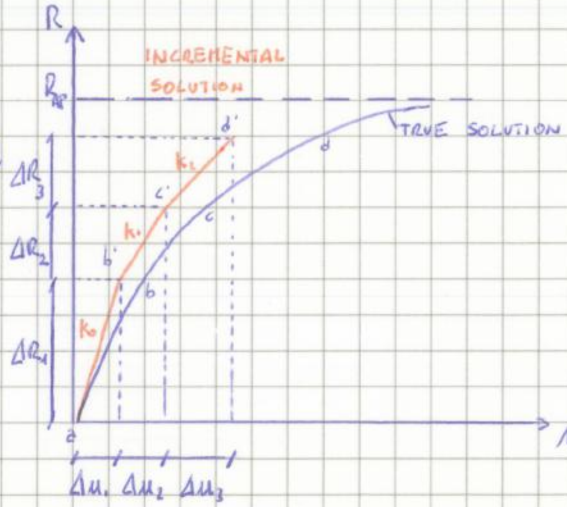


with $\sigma_3 < 0$ (tensile stress) and $\sigma_1 > 0$ (compressive)
State of stress in A is not in line with the no-tension stress strain law. The unbalanced state of stress in A is to be transferred to the surrounding elements as illustrated in figure, so that the new state of stress is as shown in A'. $\Delta\sigma$ = unbalanced state which is to be transferred from the element A to the surrounding elements.

INCREMENTAL METHODS

- TANGENT STIFFNESS METHOD

In this method we are computed increment of displacement Δu
2° Technique to deal with no-linearity.
We have non linear behaviour, at first we divided the load in a lot of increments and we didn't apply these in one step.
We consider 3 increments of load R.



R

$$[\Delta u]_1 = [K]_0^{-1} [\Delta R]_1$$

$$[\Delta u]_2 = [K]_1^{-1} [\Delta R]_2$$

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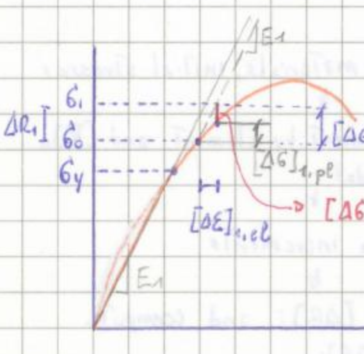
LECTURE 17

METHOD FOR ELASTO-PLASTIC MATERIAL, MIXED METHOD

- The constitutive law for an elasto-plastic material is $[\Delta\sigma] = [C^{ep}][\Delta\varepsilon]$
- We apply that equation when the state of stress is such that $F([\sigma],[k]) = 0$ yield Funct
- The incremental total strains $[\Delta\varepsilon]$ can be split into $[\Delta\varepsilon] = [\Delta\varepsilon^{el}] + [\Delta\varepsilon^{pl}]$

Initial Stress approach:

We assume an increment of load ΔR_1, starting with the state of stress σ_0, assuming that greater than the yield stress σ_y. σ_y assumed after defined the resistance criterion

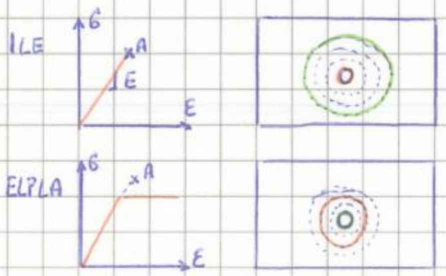


$[\Delta\sigma]_1 = [\Delta\sigma]_{1,el} - [\Delta\sigma]_{1,pl}$ unbalanced stress

$[\Delta\sigma]_{1,pl} = [C^{ep}][\Delta\varepsilon]_{1,el}$ from which $[R]_1 = \int_{V_e} [H]^T [\Delta\sigma]_1 \, dV$

$[\Delta u]_1 = [K]^{-1}[R]_1$, $[\Delta\varepsilon]_2 = [B][\Delta u]_1$, $[\Delta\sigma]_2 = [\Delta\sigma]_{2el} - [\Delta\sigma]_{2pl} = [C][\Delta\varepsilon]_2 - [\Delta\sigma]_{2,ep}$

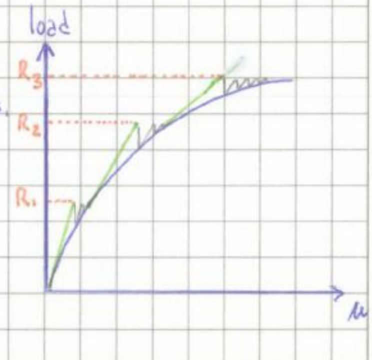
EXAMPLE: We assume to analyse a problem two constitutive behaviours



For away I only have initial state of stress. Near the excavation is probably to have plastic zone R_{pl} big because I have the redistribution of unbalanced load with iterative method.

MIXED METHOD

We reduce the error by increase (1) iteration and increase increments. Very efficient solution approach for solving nonlinear problem in the increment-iterative approach.



1) N-R METHOD and N-R MODIFIED 2) TANGENT STIFF. METHOD

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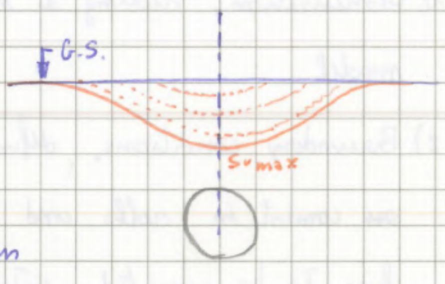
EXAMPLE SHALLOW TUNNEL

Peck's = determine displacement profile for the surface in function of the excavation of tunnel $S_v = S_{v,max}\, e^{(-y^2/2i^2)}$



We can compute the displacement and then we have to compare the results.

I have 2 stages: 1- gravity turn on and no excavation

2- excavation tunnel

We have to check the displacement, principal stress.

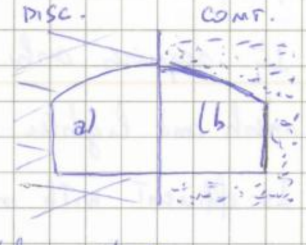
We compare Peck's results and finite element results.

In reality I have monitoring data to compare with computation.

DEM 1

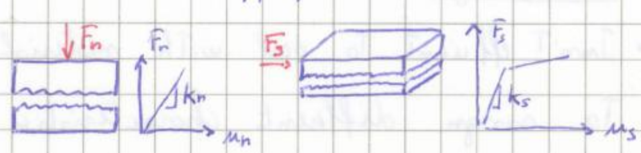
Method that permit us to move from continuum to discontinuum (because rock masses are discontinuum, and we need a method to allow the interaction between the blocks).

The scale of the problem is really important.



▷ Excavation response: a) each block moves like a discont. behaviour
b) gradual increase of displacements continuum

▷ We have to define a constitutive law to the joints to define stiffness properties (shear stiffness and normal stiffness) I have to apply a load and look deformation but not on intact rock but on the joints



Advantages:

- Allow to reproduce finite displacements and rotations of elements
- Rockmass represented as an assembly of discrete blocks.
- Joints represented as an interface between blocks

If we combine FEM and DEM we can consider not rigid but also deformable rock mass

Distinct element method is based on the concept that the timestep Δt is sufficiently small that, during a single step, disturbances cannot propagate between one discrete element and its immediate neighbours as this would need a larger time to occur.

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FDEM

Combination of FEM and DEM, there is finite element mesh associated with each discrete element

Combined FDEM:

- Deformability of material (FEM)
- Fracture model FEM to DEM
- Contact detection } DEM
- Contact interaction } DEM
  - penetration
  - Frictional interaction

Advantages:

- Combines advantages of both continuum and discontinuum methods
- Can simulate intact fracture propagation and fragmentation of jointed media
- Can be used to study the evolution of a landslide

Disadvantages:

- Computer demanding
- Is a powerful method but slightly more than an advanced research tool and needs to be validated for application to engineering practice.
- Yet to be coupled with groundwater

Steps: discontinuum blocks and FEM mesh / boundary condition / properties of block, joints / state of stress / compute / interpret

* SSRM = apply to FEM (or FDEM) -> reduce ϕ and c' until failure, estimation of F_s

The passage from continuum to discontinuum is done through fracture and fragmentation process: mode I -> opening mode — tensile strength normal to the fracture

mode II -> sliding mode — shear strength parallel to the fracture

Validation of model is given by lab tests, and mesh sensitivity

* The failure surface is not defined a priori but is a result of the computation

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The main disadvantages being the difficulty in obtaining undisturbed samples.
The essential requirement for element testing is to simulate field conditions as closely as possible.
SOIL TEST (Oedometer, Direct shear, Ring shear, TX TxT, Hollow cylinder)
ROCK TEST (Sonic velocity, Point load, Brasilian, MX, TX)
To study creep behaviour we use shear tests

BACK ANALYSES

May be effectively used to verify a number of aspects of numerical models. they allow to get feedback and improved confidence on the set of parameters selected before using them to run predictive analysis.
Back analysis may also be used to determine geomechanical parameters in some specific situations.

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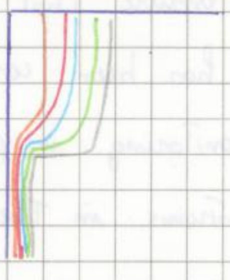
To monitor landslides, Tunnels, diaphragm, retaining walls
Steps: drill a hole, useful we use borehole, place The inclinometer Tube, with a specific shape circle with some Tracks To permit To insert The instrument. When we are going down we read The horizontal displacements, we also could have The automatic instruments.
With inclinometer we can see The failure surface, we have different colors because represent The different time measure



- Crackmeter: intended To measure movement across surface rocks and joints.
It is installed by grouting, bolting, or bonding Two Threaded anchors on opposite side of The rock and Then attaching The ends of The gage To The anchors. To monitor joints, rocks
- Piezometer: designed To measure fluid pressure such as ground water elevations and pore pressures. A porous element allows water To filter inside The instrument where pressure is measured by a pressure Transducer. To monitor landslide, Tunnel, dams.
- Load cell and pressure cell: They measure load or pressure inside a structural element. They are insert in The concrete lining. To monitor anchors, linings and concrete structures.

We use Ground Based Radar Interferometry To model or measure distance but vegetation is not good To measure because reflectation is worse
Monitoring plan: must Take into account Scope, Configuration, Monitored sections Precision Time interval, Time range, Staff qualification, Cont...

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CASE STUDIES

LECTURE 25

Construction of The Tunnel should not result in unacceptable damage To building and prior To construction, ground movements should be predicted and Their potential effects assessed.

TUNNELLING METHODS

1) CONVENTIONAL, NATM (New Austrian Tunnelling MeThod), by dividing non section in more little section and we install The preliminary support and later we place The final support. The face stability is given Thanks To fibreglass, we install 3 fibreglass in one hole like This 

2) MECHANISED, we can use Two Type of closed shield:

- Slurry shield when we have To define The pressure To apply
- Epb shield (EarTh Pressure Balance), we have a screw conveyor, if rotate fastest we remove more material, we define The pressure in function of velocity of This screw.
- Air pressure, oTher Technique.

STABILITY CONDITIONS

Tunnels in urban environment are "near surface" Tunnels and in most cases They are excavated in "soft ground". Two Topics must be addressed: stability and induced ground movements.

1) UNDRAINED STABILITY, we have To define The stability ratio N, based on That coefficient Cassin and Mair gave a set of curves To compute The displacement at The surface, crown and face.

2) DRAINED STABILITY, we evaluate The stability at The face in Term of σt by using c' and φ

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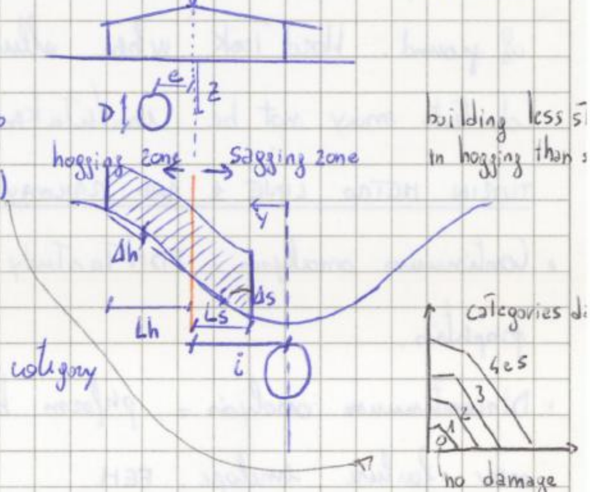
LECTURE 26

DEFORMATION OF A BUILDINGS ABOVE A TUNNEL

We don't have a gaussian distribution because the building has own stiffness and the problem is not symmetric

Relationship of damage category to deflection ratio and horizontal tensile strain for hogging ($L/H = 1$)



Deflection ratio $= \frac{\Delta h}{L_h}$ or $\frac{\Delta s}{L_s}$

Deflection ratio permit to have different damage category

TURIN METRO

- Subsoil and geological cross section
- Construction sequence (with and cover) TBM-EPB depth 16m and $\phi = 8m$
- Section of interest :

1 - Corso Francia , settlement monitoring (not greenfield situation, large dispersion, Ed ↑)

2 - Porta Susa , pietro micca's Tunnel interaction with metro Tunnel , solution is to increa consolidated slab

3 - Piazza statuto

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LECTURE 28

TUNNELLING IN RANDOMLY CEMENTED GROUND - MICROTUNNELLING

MicroTunnelling is a Trenchless Technology, it allows The remote construction of small Tunnels without The need of open cuts and with minimum disturbance To ground surface activities.
Field monitoring appears To be essential for gaining accurate experimental data and improve prediction methods.

SOIL PIPE INTERACTION

I use DEM because we have been based on equivalent continuum modelling.
Experience in modelling The Torino subsoil show The difficulties are encountered with continuum method.
Model reproduce cross section ⊥ To microTunnelling axis
I have To adding in situ state of stress. k_0, σ_v, σ_h
I have To adding cementation because we have randomly distributed areas where we have cemented material
We have To evaluate The disturbance of cementation of excavation

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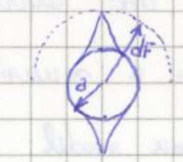
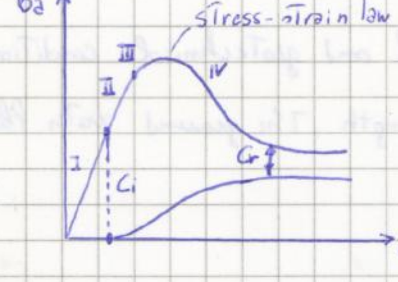
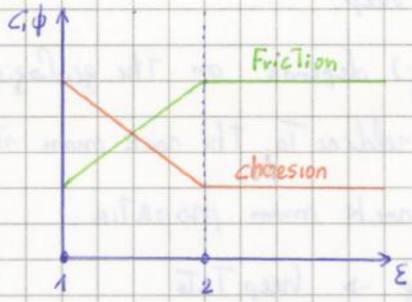
and / or crack propagation mechanisms
One common approach to estimate the yield potential and the depth of disturbance for a tunnel is the Hoek & Brown criterion for rock mass.
It is found that this approach is of limited reliability to predict brittle failure for rock masses of good quality, typically GSI > 70-75.
Failure zone around the tunnel may be determined with reference to the method developed for brittle rocks by Martin et al.

▷ MARTIN et AL. MODEL
In the Hoek e Brown criterion with $m = 0$ and $s = 0{,}11$ d_F = depth of failure zone due to spalling estimated by empirical relation: $\frac{d_F}{a} = 0{,}49\,(\pm 0{,}1) + 1{,}25\,\frac{\sigma_{max}}{\sigma_c}$



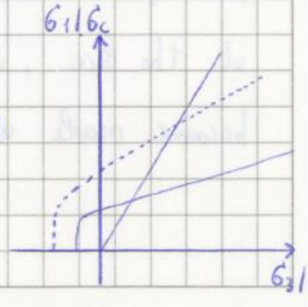
▷ CWFS MODEL
Cohesion Weaking, friction strengthening
The cohesive strenght is gradually destroyed by tensile cracking and crack coalescence. The frictional strength can be mobilized only when the cohesive strength is significantly reduced.





da 2 in avanti è deformazione che ho quando sono arrivato a campo residuale.

▷ DISL APPROACH
Damage initiation spalling loaded
Composite in situ strenght envelope for hard rock (solid unit), composed of segments corresponding to upper bound strenght (high confinements), lower bound strenght or damage initiation (low confinement) and transition zone.



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Installation of Invert Struts close to the face to control floor heave and to "close the ring", we use circular lining because is the best shape to support the stress without formation of the moment forces.
Use yielding elements, like steel sets fully imbedded in shotcrete except for 1m wide "window" along line of sliding joints to allow movement in these joints.

NB In phase 2 riproduco lining con elementi finiti e riproduco effetto giunti imponendo in certi punti una rigidezza bassa ma non nulla

ANALYSIS METHODS

Time dependent behaviour of rock mass need be taken into account in predictive modelling of the tunnel response. Simplified methods of design analysis consider the onset of yielding within the rock mass, as determined by the shear strenght parameters relative to the induced stress.

Monitoring

Steps can be taken in designing tunnels in squeezing ground?

- Ensure that you have the best possible geological model
- Establish some sort of criterion to identify the most likely failure mode
- Check the geology and in situ stress for zones of squeezing
- In zone of potential squeezing persuade your client to permit adequate over-excavation
- Design a robust temporary lining to ensure safe working
- Monitor convergence
- Add support capacity
- Design final lining taking into account long term loading

Saint Martin La Porte - Access Tunnel

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LECTURE 32

MICROTUNNELLING IN A FRACTURED ROCK MASS

Microtunnelling is a Trenchless Technology. It allows the remote construction of small tunnels without the need of open cuts and with minimum disturbance to ground surface activities.
Field monitoring appears to be essential for gaining accurate experimental data and improve prediction methods.

MARTINA FRANCA (TA)

- Jacking forces increased due to localized failure of rock blocks
- This could be minimized by appreciating the fracture pattern of the rock mass
- a continuum approach didn't allow to predict the jacking forces properly
- Only with discontinuum approach is possible to back analyse the tunnel behaviour

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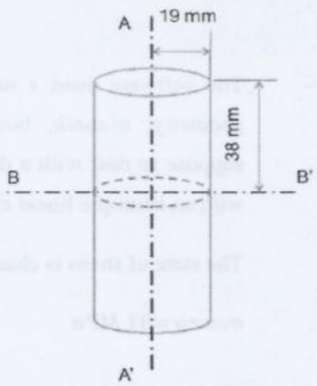
LABORATORY TEST



DIMENSION: We have to model a cylindrical specimen so we can benefit from the axial (AA') and horizontal (BB') symmetry. We will then simulate only the upper 1/8 of the specimen and run an Axisymmetric analysis.

The dimension of our specimen are illustrated in figure…

BOUNDARIES:

- In the <u>oedometric test</u> the specimen is prevented from moving in radial direction, it can only move in the axial direction. To represent this we put rollers at all the side, excluding the one where is applied the load. We set a 4-noded quadrilaterals mesh and 12 vertical distributed load stages:
  sigma'v = 50 ,100 ,200 ,300 ,400 ,100 ,400 ,800 ,1600 ,400 ,100 ,25 kPa.

- In the <u>triaxial test</u> the specimen is prevented from moving in radial and vertical direction because we have the confining pressure applied.

The boundaries conditions used to simulate the triaxial test have been rollers to block horizontal and vertical displacements and an hinge in the corner, to simulate the vertical and radial confinement I used the uniform distribuited load like in figure (…Mpa or kPa). [Fare figura boundary e load]

MESH: meshing is a simple two-step process, first we must discretize the boundaries and then the mesh can be generated. Mesh and discretization settings are:

Mesh type →
Elements Type →
Gradiation factor →
Number of nodes →

PROPERTIES: (Material) To define the material of specimen/rock mass/soil we have chosen as the elastic properties (Isotropic material, Young modulus E = …. , Poisson ratio's v = ….) and like strength parameters (Failure criterion = …. , type of behavior of material =…, the parameters which characterize the failure criterion are ……..)

(Excavation) The elements within the excavation boundary will disappear, because are typically used to simulate tunnel excavation, earthworks at different stages.