NUMERO: 1493A - ANNO: 2015

# A P P U N T I

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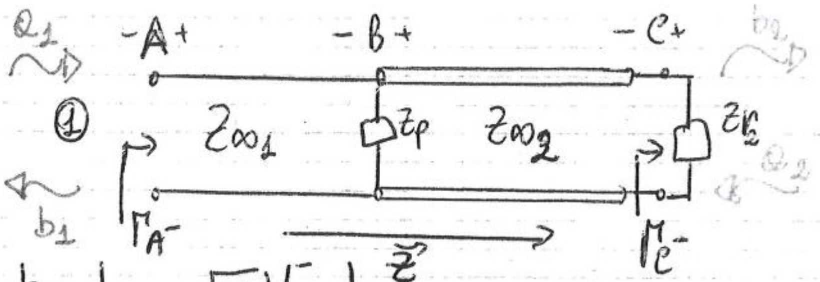
MATERIA: Guiding Electromagnetic System + temi + Eserc. Prof.Pirinoli

Assignment N° 1 BECCARIA MICHELE (Enroll. N° 209013)

1st sheet

Problem N° 1

To compute $S_{11}$ parameter I have to use this circuit:




$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \left.\frac{\sqrt{Z_{r1}}\, V_{A^-}^-}{\sqrt{Z_{r1}}\, V_{A^-}^+}\right|_{V_{C^+}^-=0} = \Gamma_{A^-}$$

To find $\Gamma_{A^-}$ I started from C:

$$\Gamma_{C^-} = \frac{Z_{r_2} - Z_{\infty 2}}{Z_{r_2} + Z_{\infty 2}} = 0 \quad \text{matched condition}$$

So I have that:

$$\Gamma_{B^+} = \Gamma_{C^-}\, e^{-j2Kl_{BC}} = 0 \Longrightarrow Z_{B^+} = 60\,\Omega$$

$$Z_{B^-} = Z_{B^+} // Z_p = \frac{(60\cdot 60)\,\Omega^{\cancel{2}}}{(60+60)\,\cancel{\Omega}} = 30\,\Omega$$

Now I compute $\Gamma_{B^-}$

$$\Gamma_{B^-} = \frac{Z_{B^-} - Z_{\infty 1}}{Z_{B^-} + Z_{\infty 1}} = 0$$

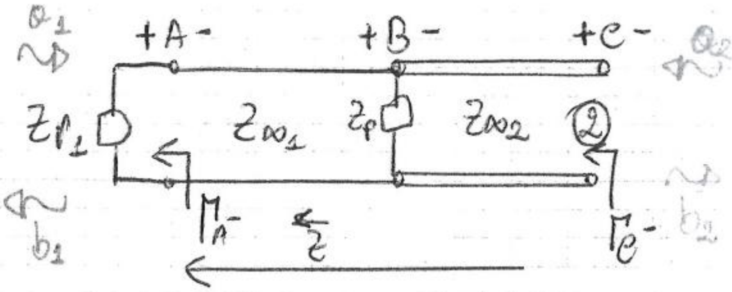
Again I have that:

$\Gamma_{A^+} = \Gamma_{B^-} = 0$ This implies $\Longrightarrow Z_{A^+} = 30\,\Omega$

Finally i compute $\Gamma_{A^-}$

$$\Gamma_{A^-} = \frac{Z_{A^+} - Z_{r_1}}{Z_{A^+} + Z_{r_1}} = 0$$

So i found that $S_{11} = 0$




Starting to compute $S_{22}$:

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} = \left.\frac{V_{C^-}^-/\cancel{\sqrt{Z_{r_2}}}}{V_{C^-}^+/\cancel{\sqrt{Z_{r_2}}}}\right|_{V_{A^+}^-=0} = \Gamma_{C^-}$$

As for the previous calculus I will have:

$$\Gamma_{A^-} = \frac{Z_{r_1} - Z_{\infty_1}}{Z_{r_1} + Z_{\infty_1}} = 0$$

$$\Gamma_{B^+} = \Gamma_{A^-} e^{-j2K l_{AB}} = 0 \Longrightarrow Z_{B^+} = 30\,\Omega$$

$$Z_{B^-} = Z_{B^+} /\!/ Z_p = \frac{(30 \cdot 60)\,\Omega^{\cancel{2}}}{(30+60)\,\cancel{\Omega}} = 20\,\Omega$$

$$\Gamma_{B^-} = \frac{Z_{B^-} - Z_{\infty_2}}{Z_{B^-} + Z_{\infty_2}} = -0{,}5$$

$$\Gamma_{C^+} = \Gamma_{B^-} e^{-j2K l_{BC}} = -0{,}5\, e^{-j\cancel{2}\frac{2\pi}{\cancel{\lambda}} \cdot \frac{3\cancel{\lambda}}{8_{\cancel{2}}}} = -0{,}5j$$

$$Z_{C^+} = Z_{\infty_2} \cdot \frac{1 + \Gamma_{C^+}}{1 - \Gamma_{C^+}} = (36 - j48)\,\Omega$$

$$\Gamma_{C^-} = \frac{Z_{C^+} - Z_{r_2}}{Z_{C^+} + Z_{r_2}} = -j0{,}5$$

$$\underline{S_{22} = -j0{,}5} = \underline{-0{,}5\, e^{-j270^\circ}}$$

Because this circuit is made only with passive components the circuit is reciprocal, so $S_{21} = S_{12}$ but to demonstrate it I will compute however $S_{12}$

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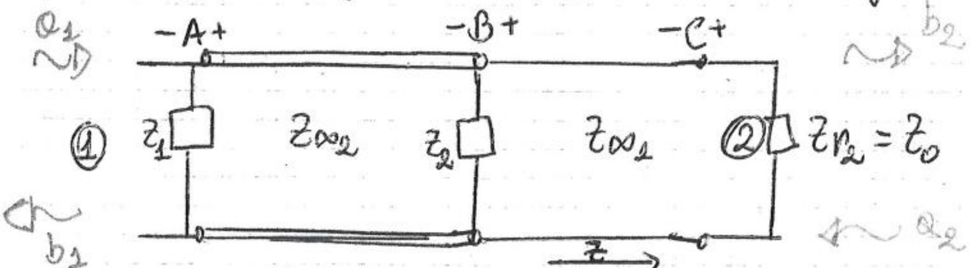
## Problem N°2

I transformed the given circuit in the following one:




(For the notation the reader has to see the complete circuit only to understand how I computed $Z_1$ and $Z_2$)

• $Z_1$

Because we have an open circuit $\Gamma_D = 1$

$$\Gamma_{AD} = \Gamma_D \, e^{-j2K\, l_{AD}} = 1 \cdot e^{-j \frac{\cancel{4}\pi}{\cancel{\lambda}} \cdot \frac{5}{8_2}\lambda} = -j$$

$$Z_{AD} = Z_1 = Z_{\infty_3} \cdot \frac{1+\Gamma_{AD}}{1-\Gamma_{AD}} = -j \frac{Z_0}{2}$$

• $Z_2$

$$\Gamma_E = \frac{Z_E - Z_{\infty_1}}{Z_E + Z_{\infty_1}} = \frac{j\cancel{Z_{\infty_1}} - \cancel{Z_{\infty_1}}}{j\cancel{Z_{\infty_1}} + \cancel{Z_{\infty_1}}} = \frac{j-1}{j+1} = j$$

$$\Gamma_{BE} = \Gamma_E \, e^{-jK\, l_{BE}} = 1$$

$$Z_{BE} = Z_{\infty_1} \cdot \frac{1+\Gamma_{BE}}{1-\Gamma_{BE}} = \infty \Rightarrow Y_{BE} = 0 \Rightarrow Z_2 = \infty \text{ and } Y_2 = 0$$

Now I can compute $S_{11}$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \left.\frac{V_A^- / \cancel{\sqrt{Z_{R_1}}}}{V_A^+ / \cancel{\sqrt{Z_{R_1}}}}\right|_{V_C^+=0} = \Gamma_{A^-}$$

$$= V_{A^+}^{+} \frac{(1+\Gamma_{B^-})}{(1+\Gamma_{B^+})} (1+\Gamma_{e^-}) e^{-jK(l_{AB}+l_{Be})} =$$

$$= \frac{V_{A^+}}{1+\Gamma_{A^+}} \frac{(1+\Gamma_{B^-})}{(1+\Gamma_{B^+})} (1+\Gamma_{e^-}) e^{-jK(l_{AB}+l_{Be})} =$$

$$= V_{A^-}^{+} \frac{(1+\Gamma_{A^-})}{(1+\Gamma_{A^+})} \frac{(1+\Gamma_{B^-})}{(1+\Gamma_{B^+})} (1+\Gamma_{e^-}) e^{-jK(l_{AB}+l_{Be})}$$

So we have:

$$S_{21} = \sqrt{\frac{\varepsilon_{r_2}}{\varepsilon_{r_1}}} \frac{(1+\Gamma_{A^-})}{(1+\Gamma_{A^+})} \frac{(1+\Gamma_{B^-})}{(1+\Gamma_{B^+})} (1+\Gamma_{e^-}) e^{-j\frac{2\pi}{\lambda}\left(\frac{5}{4}\lambda\right)}$$

$$\downarrow$$

$$= 1 \cdot \frac{(1+0{,}9e^{-j127^\circ})}{\cancel{1+\frac{11}{13}}} \cdot \frac{\cancel{1+\frac{11}{13}}}{1+\frac{1}{3}} \left(1-\frac{1}{3}\right) \cdot e^{-j\frac{5}{2}\pi}$$

$$= -0{,}36 - j\,0{,}22 = 0{,}42\, e^{-j148^\circ}$$

$$S_{11} = 0{,}9\, e^{-j127^\circ}$$

$$S_{21} = 0{,}42\, e^{-j148^\circ}$$

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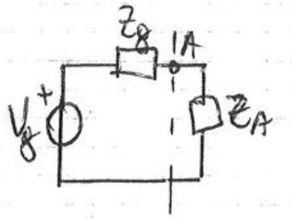
3rd sheet

... continue Problem 3

To find $P_A$ I consider this circuit




Where $Z_A$ is:

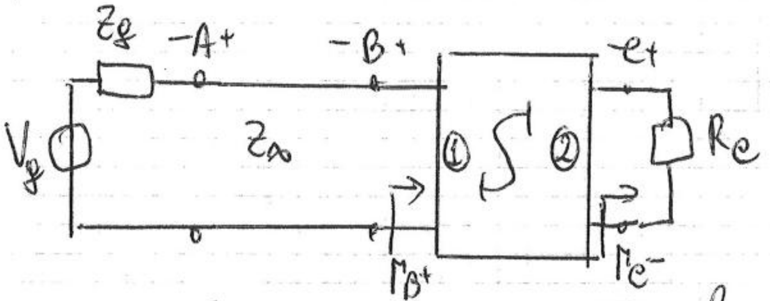
$$Z_A = Z_{P_1} \cdot \frac{1+\Gamma_{A^+}}{1-\Gamma_{A^-}} = Z_0 \cdot \frac{1+0{,}25}{1-0{,}25} = 1{,}67\,Z_0$$

$$P_A = \frac{1}{2}\,\mathrm{Re}\{Z_A\}\,|I_A|^2 = \frac{1}{2}\,\mathrm{Re}\{Z_A\}\cdot\left|\frac{V_g^2}{(Z_A+R_g)^2}\right| = \frac{1}{2}\cdot 1{,}67\cancel{Z_0}\cdot\frac{100\,V^2}{4{,}70\,Z_0^{\cancel{2}}} =$$

$$= \frac{17{,}73\,V^2}{Z_0}$$

$$P_C = P_A \cdot 0{,}68 = \frac{12{,}09\,V^2}{Z_0}$$

Problem 4




To compute the maximum power in the load $R_C$ taking into account the maximum reflected power that the generator can withstand I'll do this:
Considering that

$$\frac{P_C}{P_B = P_A} = \frac{|S_{21}|^2}{|1-S_{22}\Gamma_{C^-}|^2}\cdot\frac{(1-|\Gamma_{C^-}|^2)}{(1-|\Gamma_{B^+}|^2)}$$

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ASSIGNMENT N°2

## Problem n°1

To design a Low Pass filter with maximally flat response we use Butterworth filter. We start to find the prototype filter:

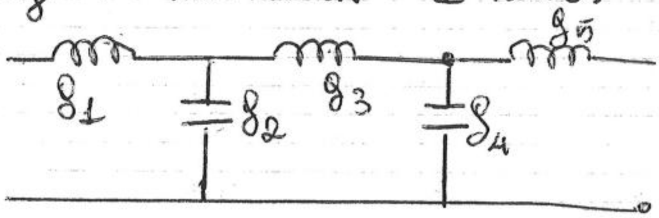
$$\Omega = \frac{\omega}{\omega_c} = \frac{\cancel{2\pi} \cdot 3,75\,\cancel{GHz}}{\cancel{2\pi} \cdot 2,5\,\cancel{GHz}} = 1,5$$

$$|\Omega| - 1 = 0,5$$

We use the graph shown us in theory lesson that plots the frequency behaviour of $P_{LR}$ out of pass-band and we find the value of the point $(|\Omega|-1, IL)$, in order to find the order N of the filter

N = 5 (actually the point would be N=4 but to respect the specification of $IL \geq 15dB$, we choose N=5)

So, using T-network I have:



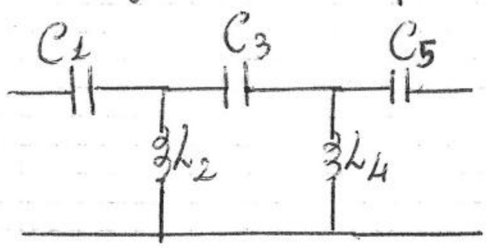

In theory lessons we saw that to find values of $g_N$ we have to see the table in wich are located these values corresponding every order of the filter N.
For N=5

$g_1 = g_5 = 0,618$ $\quad g_3 = 2$

$g_2 = g_4 = 1,618$

Now we can find the circuit wit lumped elements:

So Now I Transform The prototype with ~~The~~ real high pass filter:




Where:

$$C_1 = C_5 = \frac{1}{\omega_c R_0 g_1} = 0{,}36\,pF$$

$$L_2 = L_4 = \frac{R_0}{\omega_c \cdot g_2} = 4{,}17\,nH$$

$$C_3 = \frac{1}{\omega_c R_0 g_3} = 0{,}28\,pF$$

Problem n°3

I use a Butterworth filter because of "maximally flat response". I shall compute $\Omega_1$ and $\Omega_2$

$$\omega_1 = 2\pi \cdot f_1 = 15{,}08 \cdot 10^9 \frac{rad}{s} \qquad \omega_0 = \sqrt{\omega_1 \cdot \omega_2} = 15{,}39 \cdot 10^9 \frac{rad}{s}$$

$$\omega_2 = 2\pi \cdot f_2 = 15{,}71 \cdot 10^9 \frac{rad}{s}$$

$$\omega_{R_1} = 2\pi \cdot f_{R_1} = 12{,}06 \cdot 10^9 \frac{rad}{s}$$

$$\omega_{R_2} = 2\pi \cdot f_{R_2} = 18{,}85 \cdot 10^9 \frac{rad}{s}$$

$$\Omega_1 = \frac{\omega_0}{\omega_2 - \omega_1} \cdot \left( \frac{\omega_{R_1}}{\omega_0} - \frac{\omega_0}{\omega_{R_1}} \right) = -12{,}06$$

$$\Omega_2 = \frac{\omega_0}{\omega_2 - \omega_1} \cdot \left( \frac{\omega_{R_2}}{\omega_0} - \frac{\omega_0}{\omega_{R_2}} \right) = 10$$

$$|\Omega_1| - 1 = 11{,}06$$

$$|\Omega_2| - 1 = 9$$

So even if I have Two specifications and 1 grade of degree I however have $N=2$

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ASSIGNMENT 3 MICHELE BECCARIA 209013 SHEET 1

Problem 1 - Low Pass filter

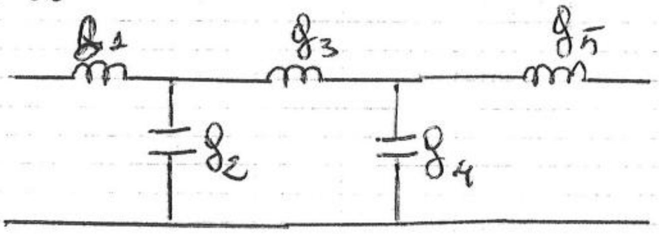
With microstrip Technology we work in This way:

$\Omega = \frac{\omega}{\omega_c} = 1{,}5$

$|\Omega| - 1 = 0{,}5$

So, from The Table of Butterworth filter we use $N = 5$ (because of maximally response)

The prototype will be:




$g_1 = g_5 = 0{,}6180$

$g_2 = g_4 = 1{,}6180$

$g_3 = 2$

So now we compute The parameters of The microstrip to realize This filter with stepped impedance:

$d = 1{,}8'' = 3{,}175$ mm

$\left(\frac{W}{d}\right)_{min} = 0{,}1575 \qquad \left(\frac{W}{d}\right)_{max} = 4{,}7244$

So now we compute $\varepsilon_{eff}$ for every values of $\frac{W}{d}$:

$$\varepsilon_{eff_2} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \cdot \frac{1}{\sqrt{1 + 12 \left(\frac{d}{W}\right)_{min}}} = 1{,}77 + 0{,}77 \cdot 8{,}76 \cdot 10^{-2} = 1{,}857$$

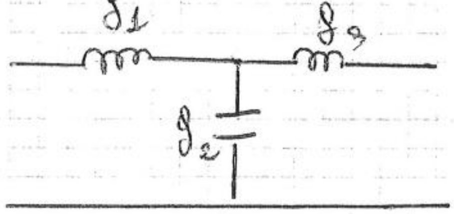
$$\varepsilon_{eff_1} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \cdot \frac{1}{\sqrt{1 + 12 \left(\frac{d}{W}\right)_{max}}} = 1{,}77 + 0{,}77 \cdot 0{,}409 = 2{,}080$$

$$l_2 = l_4 = \frac{\theta_2}{\beta_1} = 1,45\ cm$$

$$l_3 = \frac{\theta_3}{\beta_2} = 8,10\ mm$$

## Problem 2

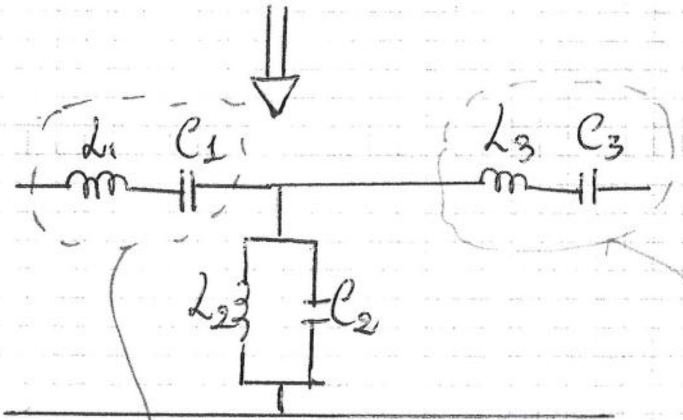
To design a band pass filter using three resonant $\lambda/4$ shunt stub, we know that $N=3$ in fact from the prototype



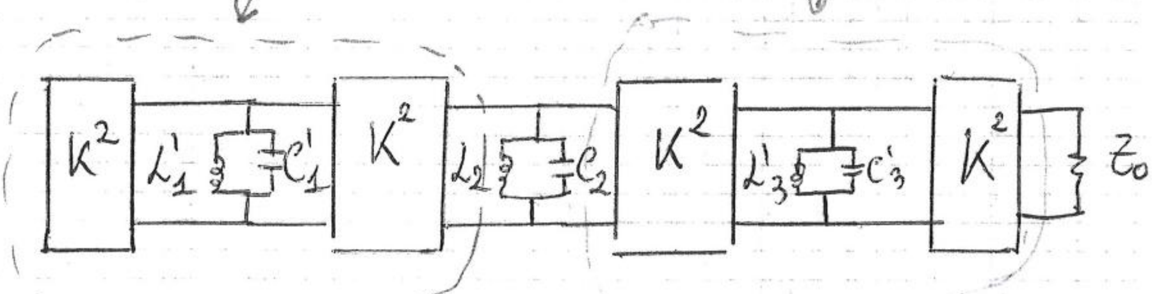

We have Chebychev filter with:

$g_1 = g_3 = 1,5963$

$g_2 = 1,0967$

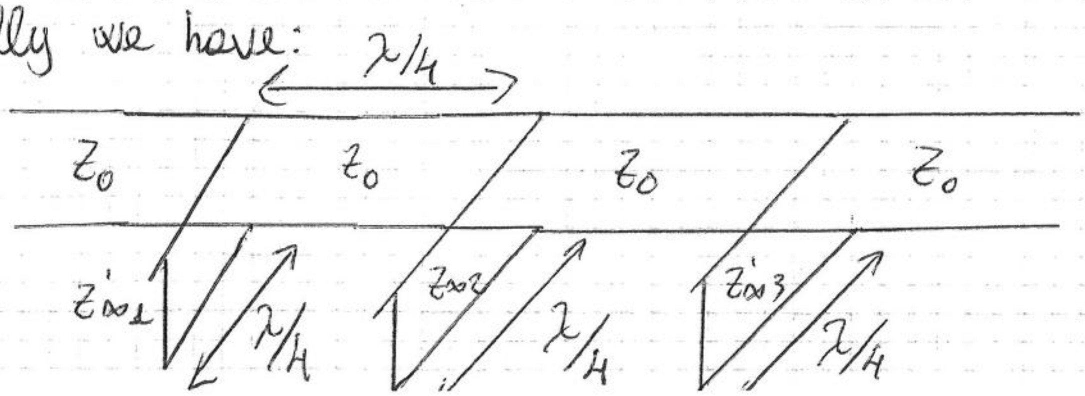



And we have:



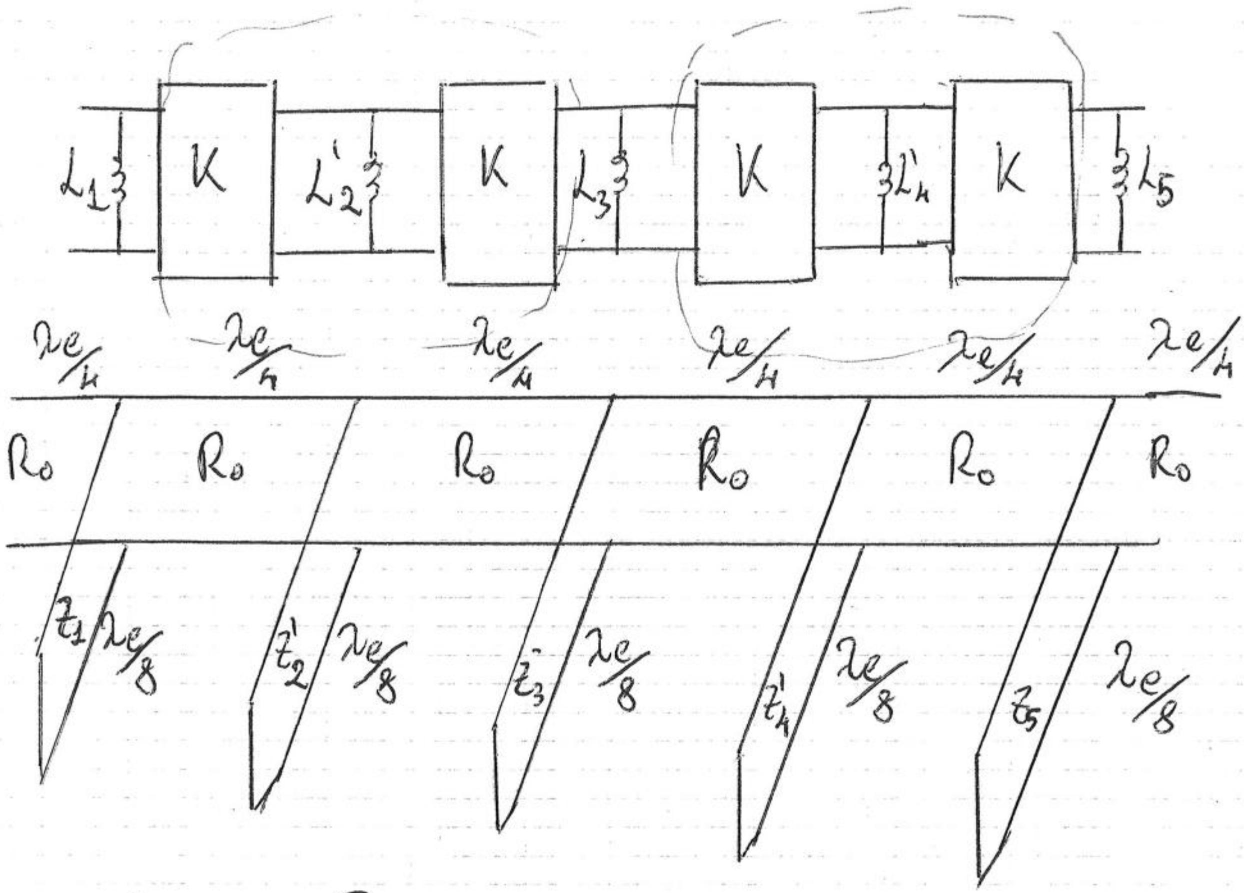

Where $K = Z_0$ and is a quarter wave impedance inverter

Finally we have:



... continuing Problem 3

We transform the previous circuit as:




$Z_1 = Z_5 = \omega_e L_1 = 81{,}7\ \Omega$

~~$Z_2 = Z_4' = \frac{1}{\omega_e C_2} = 81\ \Omega$~~

$Z_3 = \omega_e L_3 = 24{,}81\ \Omega$

$\overline{Z_2'} = C_2 \cdot Z_0^2 = \frac{1}{g_2 \cdot \cancel{Z_0} \omega_e} \cdot Z_0^{\cancel{2}} \Rightarrow \omega_e \cdot \overline{Z_2'} =$

$= \frac{Z_0}{g_2 \cancel{\omega_e}} \cdot \cancel{\omega_e} =$

$Z_2' = 30{,}7\ \Omega$



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$$Z_{\infty_2} = \frac{4 Z_0}{g_2 \pi \Delta} = 387\ \Omega$$

The computation of $Z_e$ and $Z_h$ for microstrip technology were the same of the previous assignment and we found:

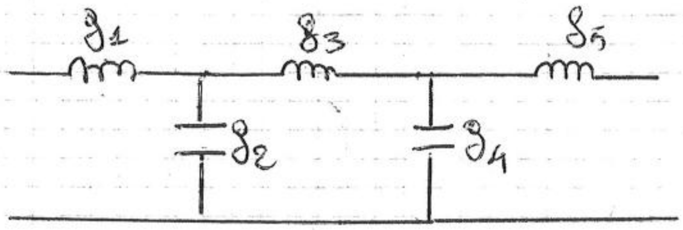
$Z_\ell = 34{,}82\ \Omega$

$Z_h = 172{,}97\ \Omega$

for this reason we can't use this technology because the impedances are different.

Problem 2 - Low Pass filter

Starting from the prototype: ($N=5$, $0{,}5$ dB ripple response)



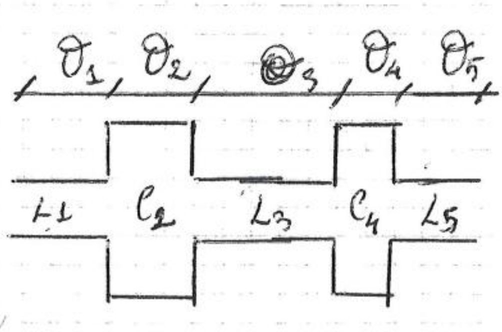

$g_1 = 1{,}7058 = g_5$

$g_2 = g_4 = 1{,}2296$

$g_3 = 2{,}5408$

$\ell = \lambda/8$

We have



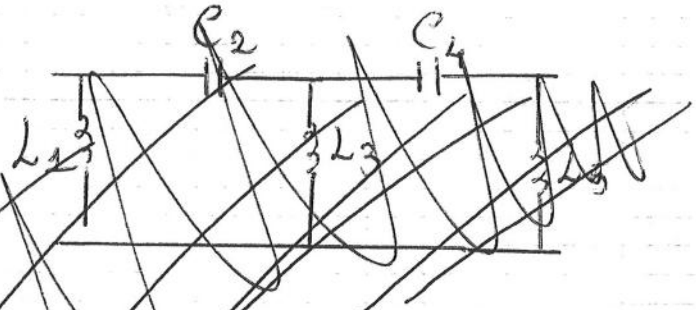

$$\theta_1 = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} = \frac{\pi}{4} = \theta_2 = \theta_3 = \theta_4 = \theta_5$$

$$Z_{h_1} = \frac{g_1 \cdot R_0}{\theta_1} = 108{,}6\ \Omega = Z_{h_5}$$

$$Z_{\ell_2} = Z_{\ell_4} = \theta_2 \cdot \frac{R_0}{g_2} = 32\ \Omega$$

## Problem 4

As for the previous assignment, ex N° 2 I have (lumped elements):

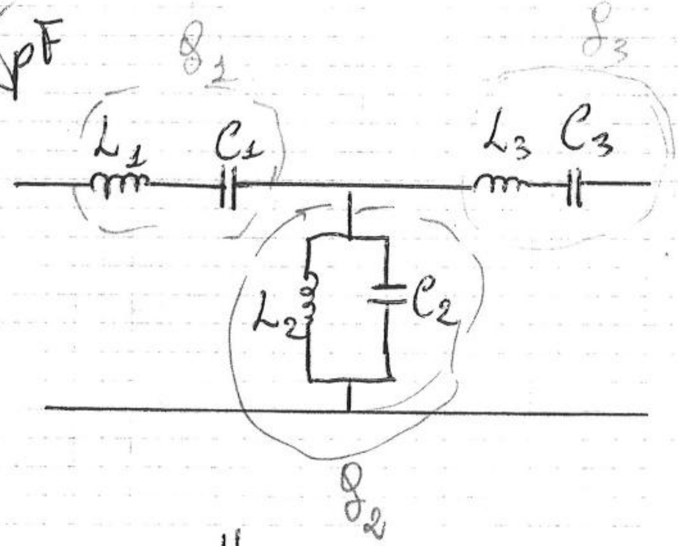



~~Where:~~

~~$L_1 = L_5 = 2,6$ nH~~

~~$L_3 = 0,79$ nH~~

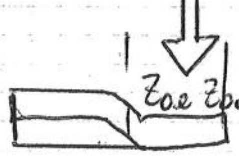
~~$C_2 = C_4 = 0,33$ pF~~

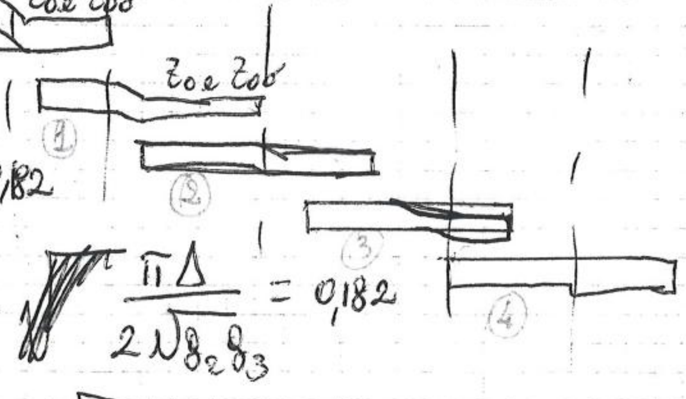



$\Delta = 0,154$

$g_1 = g_3 = 1,5963$

$g_2 = 1,0967$




$\delta Z_{01} = \sqrt{\frac{\pi \Delta}{2 g_1}} = 0,389$

$\delta Z_{02} = \frac{\pi \Delta}{2\sqrt{g_1 g_2}} = 0,182$

$\delta Z_{03} = \frac{\pi \Delta}{2\sqrt{g_2 g_3}} = 0,182$

$\delta Z_{04} = \sqrt{\frac{\pi \Delta}{2 g_3 g_4}} = 0,389$

$$\begin{cases} Z_{0e_1} = Z_0 \cdot [1 + Z_0 \delta + \delta Z_0^2] = 77\,\Omega = Z_{0e_4} \\ Z_{0o_1} = Z_0 \cdot [1 - Z_0 \delta + \delta Z_{01}] = 38,11\,\Omega = Z_{0o_4} \end{cases} \quad \begin{cases} Z_{0e_2} = 60,83\,\Omega = Z_{0e_3} \\ Z_{0o_2} = 42,518\,\Omega = Z_{0o_3} \end{cases}$$

Considering $\Gamma_2$:

$$\Gamma_2 = \frac{b_2}{a_2}$$

$$b_2 = S_{23} a_3 = S_{23} \Gamma_3 b_3 = S_{23} \Gamma_3 S_{31} a_1 = S_{23} \Gamma_3 S_{31} \Gamma_1 b_1 =$$
$$= S_{23} \Gamma_3 S_{31} \Gamma_1 S_{12} a_2$$

$$\Gamma_2 = S_{23} \Gamma_3 S_{31} \Gamma_1 S_{12}$$

$$\frac{P_3}{P_2} = \frac{|S_{31}|^2 \cdot |\Gamma_1|^2 |S_{12}|^2 (1 - |\Gamma_3|^2)}{(1 - |S_{23}|^2 |\Gamma_3|^2 |S_{31}|^2 |\Gamma_1|^2 |S_{12}|^2)}$$

$$\Gamma_1' = \frac{Z_e - Z_{P1}}{Z_e + Z_{P1}} \xrightarrow{(Z_e = 50\,\Omega)} 0$$

$$\Gamma_1'' \xrightarrow{(Z_e = 70 + j30)} 0{,}21 + j\,0{,}19$$

$$|\Gamma_1''|^2 = 0{,}084$$

$$\left(\frac{P_3}{P_2}\right)_{TX} (Z_e = 50\,\Omega) = 0$$

$$\left(\frac{P_3}{P_2}\right)_{TX} (Z_e = (70 + j30)\,\Omega) = 7{,}95 \cdot 10^{-2}$$

continues →

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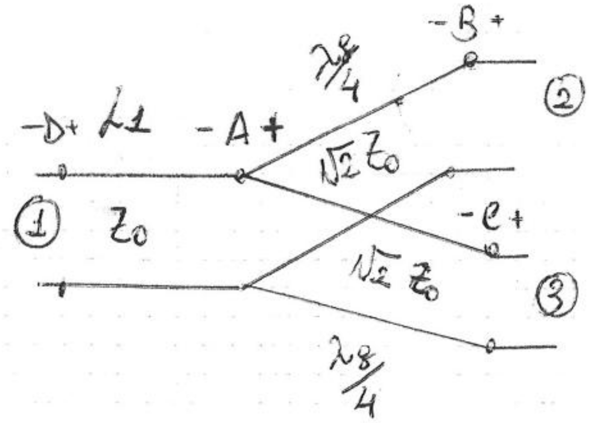
## PROBLEM 2

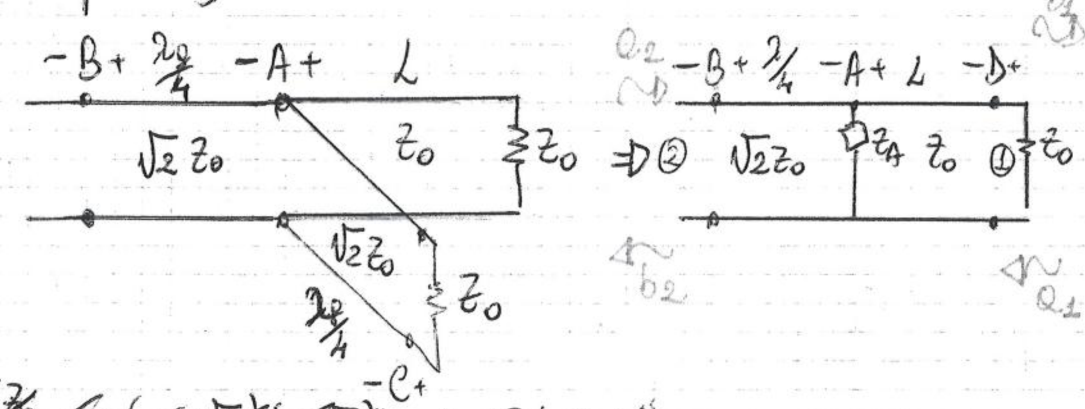



a) Scattering matrix

$$[S] = \begin{pmatrix} S_{11} & S_{12} & S_{12} \\ S_{12} & S_{22} & S_{23} \\ S_{12} & S_{23} & S_{22} \end{pmatrix}$$

$$S_{12} = \frac{b_1}{a_2}\bigg|_{a_1, a_3 = 0} \qquad S_{22} = \frac{b_2}{a_2}\bigg|_{a_1, a_3 = 0} \qquad S_{23} = S_{32} = \frac{b_2}{a_3}\bigg|_{a_2, a_1 = 0}$$

~~the~~ for semplicity I draw the circuit as:




$$\cancel{\Gamma_C = \frac{Z_0 - \sqrt{2}Z_0}{Z_0 + \sqrt{2}Z_0} = \frac{(1-\sqrt{2})(1-\sqrt{2})}{1-2} = -(1-\sqrt{2})^2}$$

$$Z_A = \frac{2Z_0^{\cancel{2}}}{\cancel{Z_0}} = 2Z_0 \quad \Rightarrow \text{because the line is } \frac{\lambda_g}{4}$$

$$S_{12} = \frac{b_1}{a_2}\bigg|_{a_1, a_3 = 0} = \frac{V_{D^+}^+/\cancel{\sqrt{Z_0}}}{V_{B^-}^+/\cancel{\sqrt{Z_0}}} =$$

$$V_{D^+}^+ = \frac{V_{D^+}}{1+\cancel{\Gamma_{D^+}}} = V_{D^-} = V_{B^-}^+(1+\Gamma_{D^-}) = V_{A^+}^+(1+\Gamma_{D^-})e^{-jkL} =$$

$$\frac{V_{A^+}}{1+\Gamma_{A^+}}(1+\Gamma_D)e^{-jkL} = V_{A^-}^+\frac{(1+\Gamma_{A^-})}{(1+\Gamma_{A^+})}(1+\Gamma_{D^-})e^{-jkL} \quad \text{continue}$$

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$$\frac{P_2}{P_1} = \frac{|S_{21}|^2}{1 - \underbrace{|\Gamma_{2+}|^2}_{\to 0}} = \frac{1}{2}$$

$$\frac{P_3}{P_1} = \frac{|S_{31}|^2}{1 - \underbrace{|\Gamma_{3+}|^2}_{\to 0}} = \frac{1}{2}$$

d) Find $Z_{03}$ and $Z_{02}$ with $P_2/P_3 = -2\,dB$

$$\begin{cases} \dfrac{1}{Z_0} = \dfrac{Z_0}{Z_{02}^2} + \dfrac{Z_0}{Z_{03}^2} \\[2ex] \dfrac{Z_{03}^2}{Z_{02}^2} = 0{,}631 \end{cases}$$

$Z_{0_2} = x \qquad Z_{03} = y$

$$\begin{cases} \dfrac{1}{Z_0} = \dfrac{Z_0}{x^2} + \dfrac{Z_0}{y^2} \\[2ex] \dfrac{y^2}{x^2} = 0{,}631 \end{cases} \qquad \begin{cases} \dfrac{Z_0}{x^2} + \dfrac{Z_0}{0{,}631x^2} - \dfrac{1}{Z_0} = 0 \\[2ex] y^2 = 0{,}631\, x^2 \end{cases}$$

$$\frac{Z_0^2 \cdot 0{,}631x^2 + Z_0^2 x^2 - 0{,}631 x^4}{\cancel{x^2 \cdot 0{,}631x^2 \cdot Z_0}} = 0$$

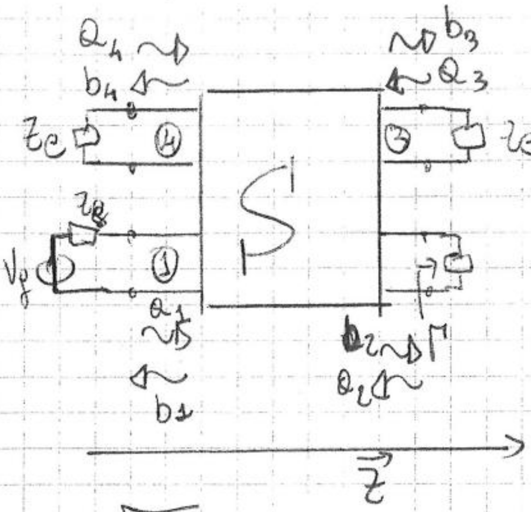
$$0{,}631\, x^{\cancel{4}\,2} = 1{,}631\, Z_0^2 \cancel{x^2}$$

$$x = 1{,}60\, Z_0 \Longrightarrow 1{,}60 \cdot 50\Omega = 80{,}4\Omega$$

$$y = 2{,}05\, Z_0 \Longrightarrow 1{,}27 \cdot 50\Omega = 63{,}54\Omega$$

# ASSIGNMENT 6 Beccaria Michele 209013

## Problem 1




$$\left|\frac{V_4}{V_3}\right| = 0{,}6 \qquad \angle V_4/V_3 = \frac{\pi}{8}$$

$$[S] = \begin{pmatrix} 0 & \sqrt{1-K^2} & jK & 0 \\ \sqrt{1-K^2} & 0 & 0 & jK \\ jK & 0 & 0 & K \\ 0 & jK & K & 0 \end{pmatrix} \text{ where } K = 10^{-\frac{3}{20}} = \frac{\sqrt{2}}{2}$$

Knowing that:

$$\frac{V_4}{V_3} = \frac{b_4}{b_3} \text{ because } \begin{cases} V_4 = (1+\underbrace{\Gamma_4}_{\to 0})V_4^+ = V_4^+ = b_4 \\ V_3 = (1+\underbrace{\Gamma_3}_{\to 0})V_3^+ = V_3^+ = b_3 \end{cases}$$

$$\begin{cases} b_4 = \cancel{S_{43}}\underbrace{a_3}_{\to 0} + S_{42}a_2 = S_{42}a_2 = S_{42}\Gamma_2 b_2 \\ b_3 = \cancel{S_{34}}\underbrace{a_4}_{\to 0} + S_{31}a_1 = S_{31}a_1 \\ b_2 = S_{21}a_1 + \cancel{S_{24}}\underbrace{a_4}_{\to 0} = S_{21}a_1 \end{cases}$$

$$\frac{b_4}{b_3} = \frac{S_{42}\Gamma_2 b_2}{S_{31}a_1} \Longrightarrow \Gamma_2 = \frac{b_4}{b_3}\cdot\frac{S_{31}}{S_{42}S_{21}} = 0{,}6\,e^{-j\frac{\pi}{8}}\cdot\frac{j\frac{\sqrt{2}}{2}}{j\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}} =$$

$$= 0{,}848\cdot e^{-j\frac{\pi}{8}}$$

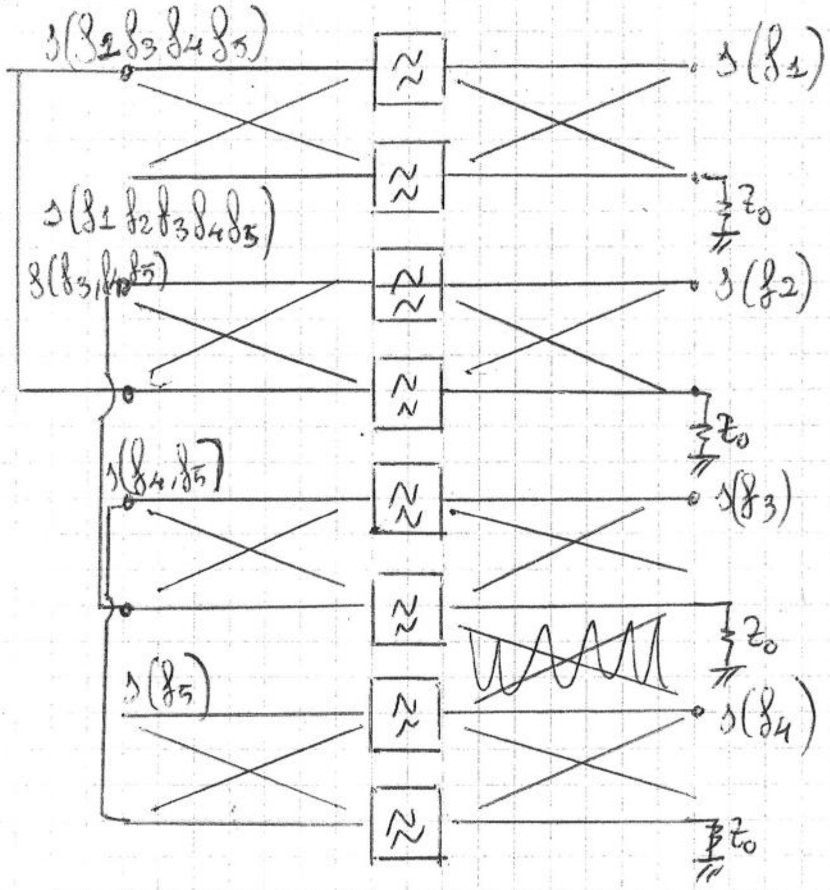
looking for the worst case we compute:

$|S_{34}^2| = -6{,}247\,dB$ ⇒ This is the worst case

$|S_{31}S_{34}| = -6{,}223\,dB$

$|S_{31}^2| = -6{,}199\,dB$

Problem 3




$|S_{11}|^2 = 0{,}95$

$|S_{12}|^2 = 0{,}98$

$IL_1 = |S_{12}|^2 = 0{,}98$

$IL_2 = |S_{11}|^2 |S_{12}|^2 = 0{,}931$

$IL_3 = |S_{11}|^4 |S_{12}|^2 = 0{,}884$

$IL_4 = |S_{11}|^6 |S_{12}|^2 = 0{,}840$

$IL_5 = |S_{11}|^8 = 0{,}814$

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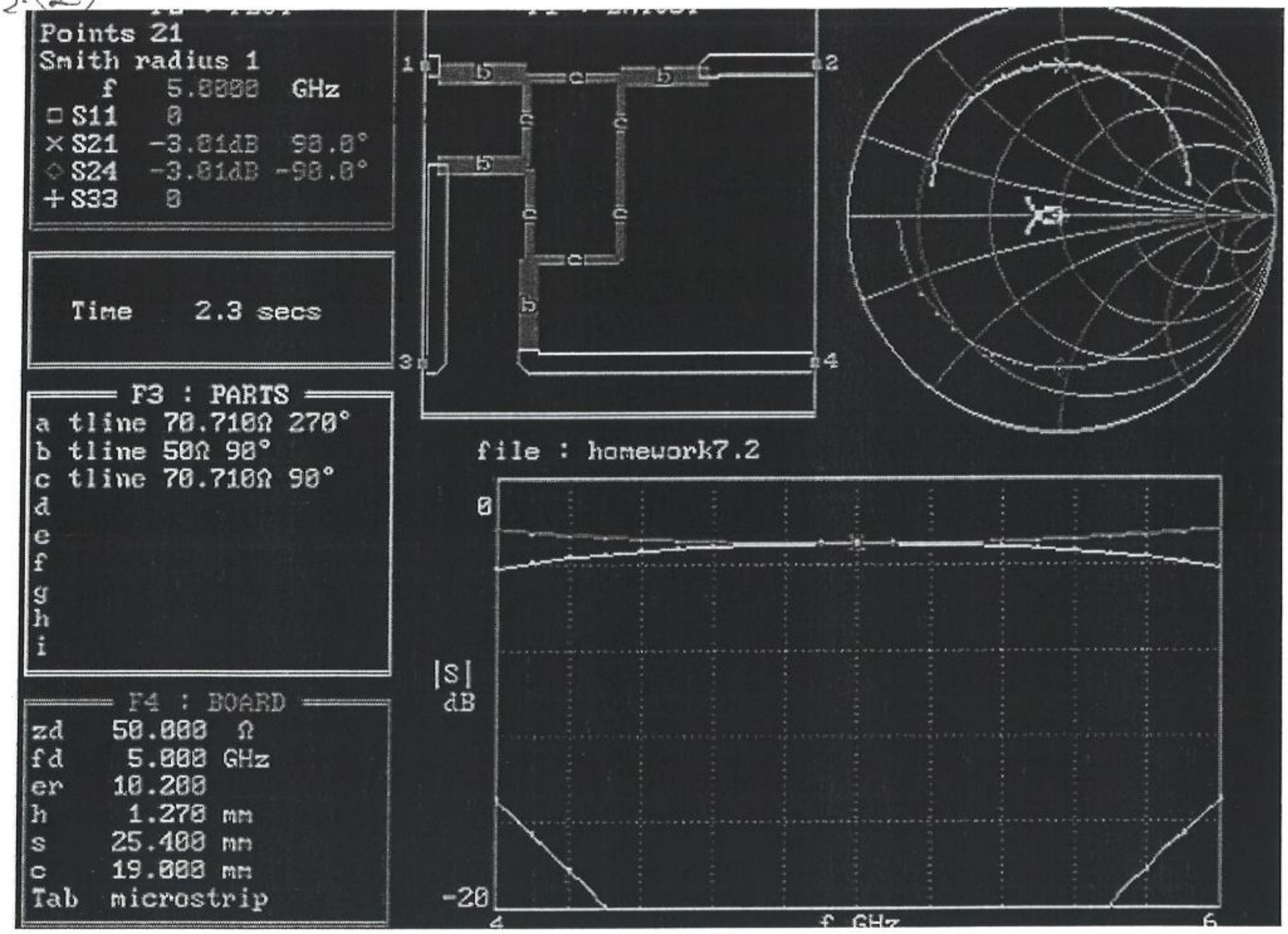
1.(2)



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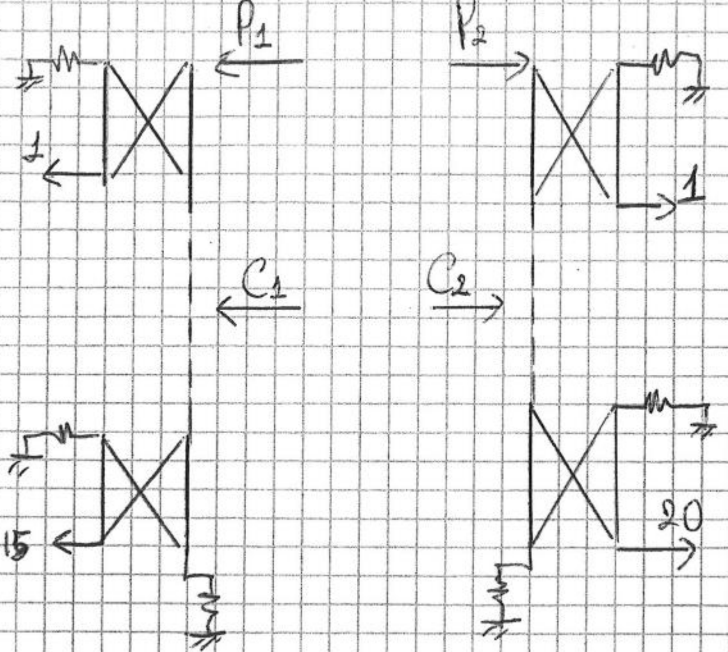
Problem 2




For both lines we have:

$$A\ell = 10^{-\frac{3}{10}} = 0{,}501$$

$$IL = 10^{-\frac{12}{10}} = 0{,}063$$

The receiver sensitivity has $-35$ dBm that are equal to $-65$d

So we have

$$P_N = -65\,dB + 5\,dB = -60\,dB = 10^{-6}\,W$$

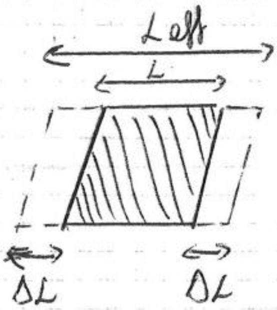
$$C_1 = \frac{1 - IL}{N_1} = 6{,}24 \cdot 10^{-2}$$

$$C_2 = \frac{1 - IL}{N_2} = 0{,}0468$$

$$P_{iN_1} = \frac{P_N}{C_1 A\ell (1 - C_1 - IL)^{14}} = 209\,\mu W = -36{,}8\,dB = -6{,}8\,dBm$$

$$P_{iN_2} = \frac{P_N}{C_2 A\ell (1 - C_2 - IL)^{19}} = 389\,\mu W = -34{,}1\,dB = -4{,}10\,dBm$$

## Problem 2




$\varepsilon_r = 2{,}33$

$h = 1/15'' = 0{,}17\,cm$

$f_0 = 2045\,MHz$

$$W = \frac{c}{2 f_0}\sqrt{\frac{2}{\varepsilon_r + 1}} = 5{,}68\,cm$$

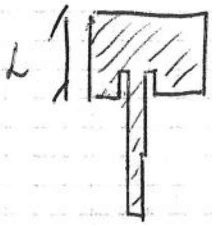
$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \cdot \frac{1}{\sqrt{\left(1 + 12\frac{h}{W}\right)}} = 2{,}664$$

$$L_{eff} = \frac{\lambda_g}{2} = \frac{1}{2}\,\frac{c/\sqrt{\varepsilon_{eff}}}{f_0} = 4{,}49\,cm$$

$$\frac{\Delta L}{h} = 0{,}514$$

$$\Delta L = 0{,}514 \cdot h = 0{,}87\,mm$$

$$L = L_{eff} - 2\Delta L = 4{,}31\,cm$$




I used this formula because $\frac{W}{d} \geq 1$

$$Z_\infty = \frac{120\pi}{\sqrt{\varepsilon_{eff}}\left[W/d + 1{,}393 + 0{,}667 \ln\left(\frac{W}{h} + 1{,}444\right)\right]} = 6{,}21\,\Omega$$

The equivalent transmission line model is:

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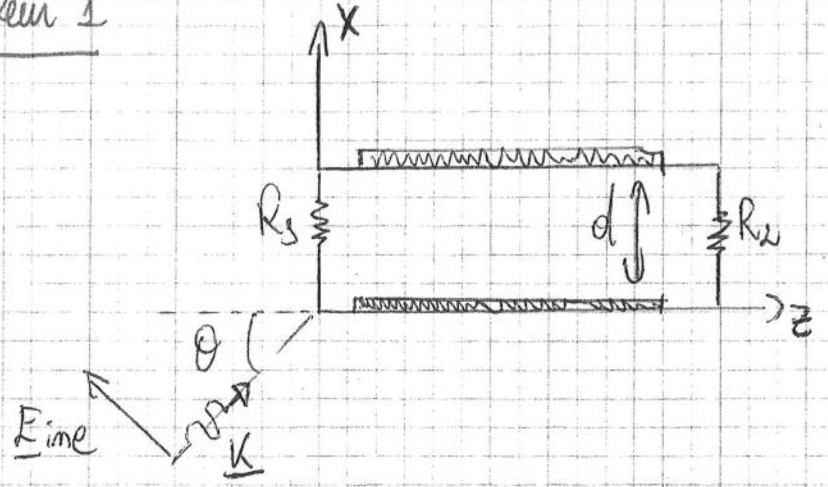
## Problem 1




Starting to compute K and the other parameters:

$$\underline{K} = K_0\left(\cos\theta^{inc}\hat{z} + \operatorname{sen}\theta^{inc}\hat{x}\right) = K_0 \cdot \left(\frac{\hat{z}}{2} + \frac{\sqrt{3}}{2}\hat{x}\right) = \frac{K_0}{2}\cdot\left(\hat{z} + \sqrt{3}\hat{x}\right)$$

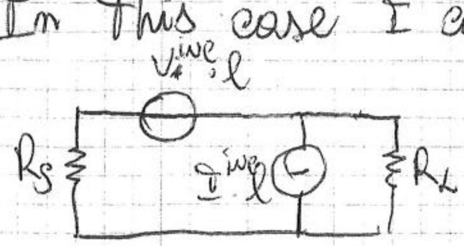
$$\underline{E} = \quad = E_0\left[\hat{x}\operatorname{sen}(90^\circ - \theta^{inc}) - \hat{z}\cos(90^\circ - \theta^{inc})\right] =$$

$$= E_0\left[\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{z}\right] = \frac{E_0}{2}\left[\hat{x} - \sqrt{3}\hat{z}\right]$$

$$\underline{H} = \frac{1}{Z_0}\hat{K}\wedge\underline{E} = \frac{1}{Z_0}\cdot\frac{E_0}{4}\cdot\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \sqrt{3} & 0 & 1\\ 1 & 0 & -\sqrt{3}\end{vmatrix} = \frac{1}{Z_0}\cdot\frac{E_0}{4}\cdot\left(\hat{y}+3\hat{y}\right) = \frac{E_0}{Z_0}\hat{y}$$

$$\lambda = \frac{c}{f} = \frac{3\cdot 10^8\ m/s}{100\ MHz} = 3\,m \gg 15\,cm$$

In this case I can use the short line approximation:



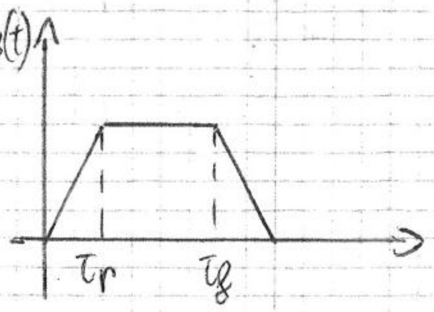

$$V^{inc}\cdot l = -j\omega_0\mu_0\, l\cdot d\cdot \vec{H}^{inc}\cdot\hat{n}$$
$$I^{inc}\cdot l = -j\omega_0\, C\, l\cdot d\cdot\vec{E}^{inc}\cdot\hat{\tau}$$

$\hat{n} = \hat{y}$

$\hat{\tau} = \hat{x}$

$$V^{inc}\cdot l = -j2\pi\cdot 1\cdot 10^8\cdot 4\pi\cdot 10^{-7}\cdot 0{,}15\cdot 5\cdot 10^{-3}\cdot\frac{10}{120\pi} = -j15{,}708\ mV$$
$$I^{inc}\cdot l = -j2\pi\cdot 1\cdot 10^8\cdot 40\cdot 10^{-12}\cdot 15\cdot 10^{-2}\cdot 5\cdot 10^{-3}\cdot\frac{10}{\ } = -j94{,}248\ \mu A$$

## Problem 3




We know that:

$V_{FE} = -j\omega \{ M_{FE}^{IND} + M_{CAP} \} V_S$ in ~~the~~ frequency domain,

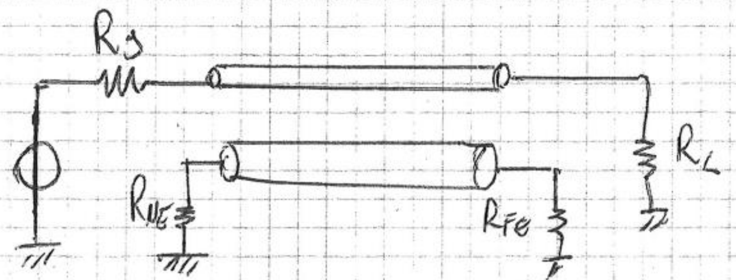
in time domain we have:

$$v_{FE}(t) = -\frac{d}{dt} v_S(t) \cdot \{ M_{FE}^{IND} + M^{CAP} \}$$

$$v_{NE}(t) = -\frac{d}{dt} v_S(t) \cdot \{ M_{FE}^{IND} + M_{CAP} \}$$

$$V_{FE_{MAX}} = \frac{5}{50 \cdot 10^{-9}} \{ 5 \cdot 10^{-9} + 6{,}25 \cdot 10^{-9} \} = 1{,}125 \text{ V}$$

## Problem 4




$\lambda = \frac{c}{f} = 300 \text{ m} \gg 2 \text{ m} \Rightarrow$ I can use short line approximati

$$\tau_S = \frac{L_{SH}}{R_{SH}} = 16\,\mu \Rightarrow \omega = 2\pi f = 6{,}283 \cdot 10^6 \frac{rad}{s}$$

$$\frac{1}{\tau_S} = 62{,}5 \cdot 10^3 \frac{rad}{s} \ll \omega$$

Thus:

$$\frac{1}{1 + j\omega\tau_S} \simeq \frac{1}{j\omega\tau_S}$$