



Corso Luigi Einaudi, 55 - Torino

Appunti universitari

Tesi di laurea

Cartoleria e cancelleria

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Rilegature

NUMERO: 1368A -

ANNO: 2015

A P P U N T I

STUDENTE: Cipiti

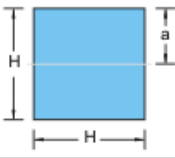
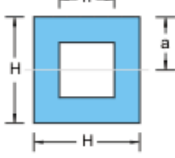
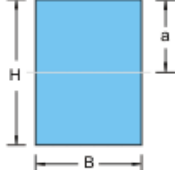
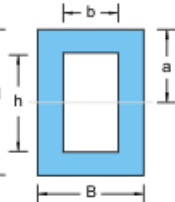

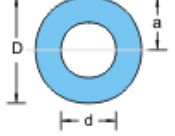
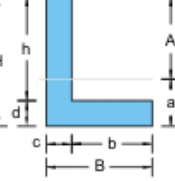
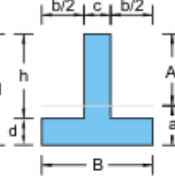
MATERIA: Fondamenti di Meccanica Strutturale + formulario,
Prof.Cura

Il presente lavoro nasce dall'impegno dell'autore ed è distribuito in accordo con il Centro Appunti.

Tutti i diritti sono riservati. È vietata qualsiasi riproduzione, copia totale o parziale, dei contenuti inseriti nel presente volume, ivi inclusa la memorizzazione, rielaborazione, diffusione o distribuzione dei contenuti stessi mediante qualunque supporto magnetico o cartaceo, piattaforma tecnologica o rete telematica, senza previa autorizzazione scritta dell'autore.

**ATTENZIONE: QUESTI APPUNTI SONO FATTI DA STUDENTIE NON SONO STATI VISIONATI DAL DOCENTE.
IL NOME DEL PROFESSORE, SERVE SOLO PER IDENTIFICARE IL CORSO.**

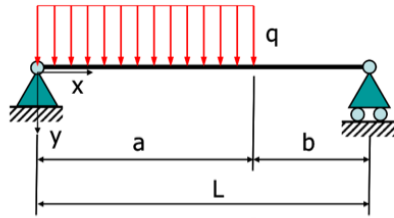
Fondamenti di Meccanica Strutturale

Sezione	Area della sezione A	Distanza dal baricentro a	Momento di inerzia J	Modulo di resistenza W
	cm ²	cm	cm ⁴	cm ³
	H^2	$\frac{H}{2}$	$\frac{H^4}{12}$	$\frac{H^3}{6}$
	$H^2 - h^2$	$\frac{H}{2}$	$\frac{H^4 - h^4}{12}$	$\frac{H^4 - h^4}{6H}$
	$B \cdot H$	$\frac{H}{2}$	$\frac{B \cdot H^3}{12}$	$\frac{B \cdot H^2}{6}$
	$BH - bh$	$\frac{H}{2}$	$\frac{1}{12} \cdot (BH^3 - bh^3)$	$\frac{1}{6H} \cdot (BH^3 - bh^3)$
	$\frac{\pi \cdot D^2}{4}$	$\frac{D}{2}$	$\frac{\pi \cdot D^4}{64}$	$\frac{\pi \cdot D^3}{32}$
	$\frac{\pi \cdot (D^2 - d^2)}{4}$	$\frac{D}{2}$	$\frac{\pi \cdot (D^4 - d^4)}{64}$	$\frac{\pi \cdot (D^4 - d^4)}{32 \cdot D}$
	$BH - bh$	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h - A)^3 + cA^3}{3}$	$W_A = \frac{I}{A}$ $W_a = \frac{I}{a}$
	$BH - bh$	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h - A)^3 + cA^3}{3}$	$W_A = \frac{J}{A}$ $W_a = \frac{J}{a}$

Materiali	Modulo di elasticità kN/mm ²
<i>Laterizi:</i>	
Mattoni	6 ÷ 10
Pignatte per solai	14 ÷ 28
<i>Legnami:</i>	
Abete bianco e rosso	9 ÷ 10
Betulla	12 ÷ 14
Faggio	15 ÷ 16
Noce	11 ÷ 12
Pino silvestre	10 ÷ 11
Pioppo	8 ÷ 9
Pitch pine	11 ÷ 12
Quercia	12 ÷ 12
<i>Metalli e leghe:</i>	
Acciai da costruzione	210
Alluminio	60 ÷ 65
Bronzo	90 ÷ 100
Ghisa	80 ÷ 120
Ottone	90 ÷ 110
Piombo	7 ÷ 15
Rame	120 ÷ 130
Zinco	80 ÷ 90
<i>Muratura e calcestruzzi:</i>	
Muratura ordinaria di mattoni	1 ÷ 5
Calcestruzzo di cemento normale	12 ÷ 25
Calcestruzzo di cemento ad alta resistenza	25 ÷ 35
<i>Pietre naturali:</i>	
Calcarei	60 ÷ 70
Graniti	15 ÷ 40
Marmi	20 ÷ 90

Materiali	E (N/m ²) a 20 °C
Acciaio	2,1×10 ¹¹
Alluminio	7×10 ¹⁰
Argento	7,5×10 ¹⁰
Ferro	2×10 ¹¹
Gomma	5÷80×10 ⁶
Ottone	9,1×10 ¹⁰
Piombo	1,4×10 ¹⁰
Platino	1,5×10 ¹¹
Polietilene	1÷14×10 ⁸
Rame	1,1×10 ¹¹
Tungsteno	3,5×10 ¹¹
Vetro per finestre	7×10 ¹⁰
Calcestruzzo	2,3×10 ¹⁰
Granito	2,5×10 ¹⁰

Freccia elastica per trave semplicemente appoggiata con diversi tipi di carichi

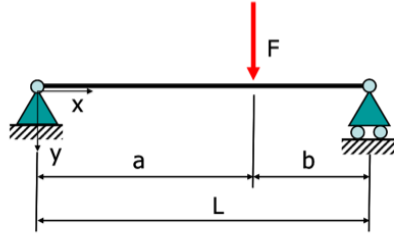


$$y = \frac{q}{EJ_z} \frac{2 \cdot (L^2 - b^2) \cdot (L^2 - x^2) + L \cdot x^3 - L^4 + b^4}{24 \cdot L} x$$

$$0 \leq x \leq a$$

$$y = \frac{q}{EJ_z} \frac{2x \cdot (L^2 - b^2) \cdot (L^2 - x^2) + L \cdot [x^4 - (x-a)^4] - x \cdot (L^4 + b^4)}{24 \cdot L}$$

$$a \leq x \leq b$$



$$y = \frac{F}{EJ_z} \frac{b \cdot x \cdot (L^2 - b^2 - x^2)}{6 \cdot L}$$

$$0 \leq x \leq a$$

$$y = \frac{F}{EJ_z} \frac{L \cdot (x-a)^3 + b \cdot x \cdot (L^2 - b^2 - x^2)}{6 \cdot L}$$

$$a \leq x \leq b$$

• Tensione dovuta a momento torcente: (sezione circolare)

$$\tau = \frac{M_t r}{I_p}$$

\leftarrow momento torc.
 \leftarrow raggio
 \leftarrow momento di inerzia polare

momento polare; $I_p = \frac{\pi D^4}{32}$
 sez. circolare

• Tensione dovuta a momento torcente: (sezione rettangolare)

$$\tau = \frac{M_t}{W}$$

\leftarrow momento torcente
 \leftarrow modulo di resistenza (tabulato)

• Tensione dovuta a sforzo normale:

$$\sigma = \frac{N}{A}$$

\leftarrow sforzo normale
 \leftarrow Area della sezione

• tensione dovuta a momento flettente:

$$\sigma = \frac{M_f \cdot y_{max}}{I_x}$$

Calcol di K_t

$$K_t = \frac{\sigma_p}{\sigma_n} \leftarrow \text{sigma di picco (sigma principale nel caso di trazione)}$$

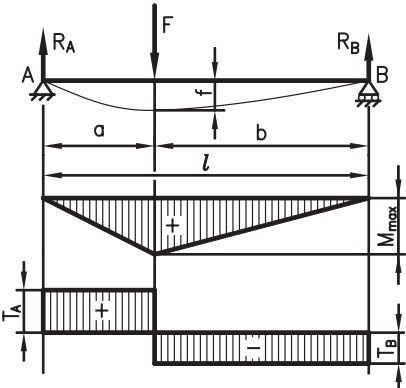
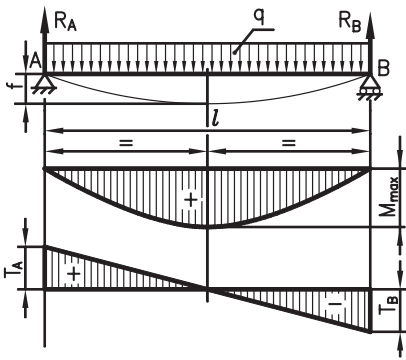
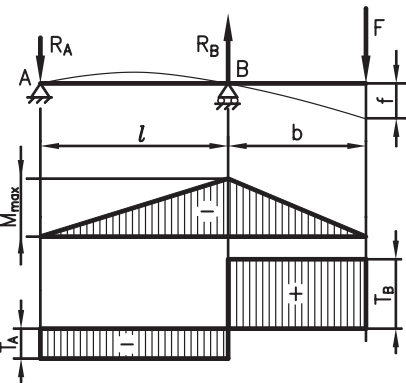
$$\sigma_n \leftarrow \sigma_{nominale} = \frac{4F}{A_n} \rightarrow \text{Area nominale}$$

Si usa soltanto in caso di fragilità del materiale

Carico di punta di Eulero:

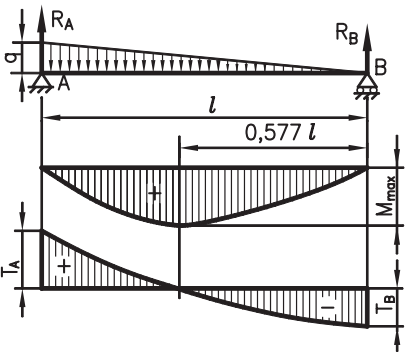
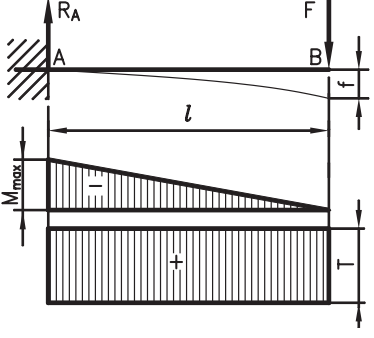
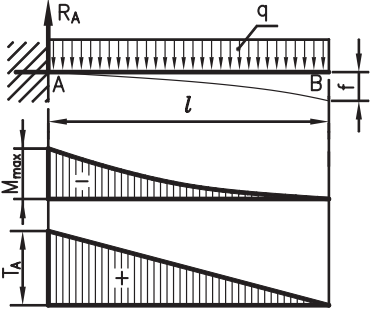
$$P_{ce} = \left(\frac{\pi}{2l} \right)^2 EI_{min} \rightarrow \text{trave a sbalzo}$$

Tabella H.1 Travi variamente vincolate e sottoposte ai tipi di carichi più comuni

Schema	Relazioni
<p>- Trave appoggiata agli estremi con carico concentrato</p> 	$R_A = \frac{b}{l} \cdot F; \quad R_B = \frac{a}{l} \cdot F$ $T_A = R_A; \quad T_B = -R_B$ $M_A = M_B = 0$ $M_{max} = R_A \cdot a = R_B \cdot b = \frac{a \cdot b}{l} \cdot F$ $f = \frac{1}{3} \cdot \frac{F \cdot a^2 \cdot b^2}{E \cdot I \cdot l}$
<p>- Trave appoggiata agli estremi con carico uniformemente distribuito</p> 	$R_A = R_B = \frac{q \cdot l}{2}$ $T_A = R_A; \quad T_B = -R_B$ $M_A = M_B = 0$ $M_{max} = \frac{1}{8} \cdot q \cdot l^2$ $f = \frac{1}{384} \cdot \frac{q \cdot l^4}{E \cdot I}$
<p>- Trave con un appoggio intermedio e con carico concentrato all'estremo del tratto a sbalzo</p> 	$R_A = \frac{b}{l} \cdot F; \quad R_B = \frac{l+a}{l} \cdot F$ $T_A = -\frac{b}{l} \cdot F; \quad T_B = F$ $M_A = 0$ $M_{max} = -F \cdot b$ $f = \frac{F}{E \cdot I} \cdot \frac{(l+b) \cdot b^2}{3}$

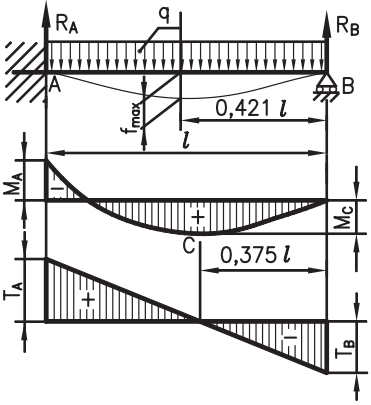
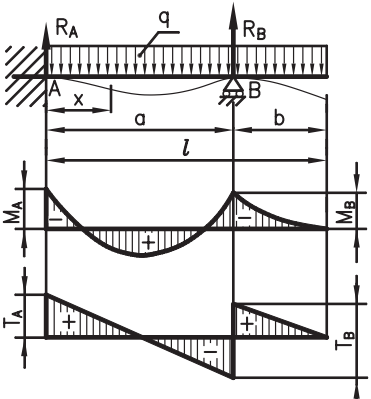
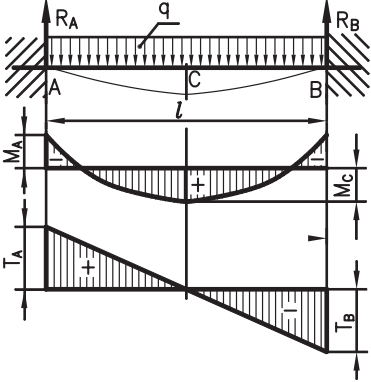
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Tabella H.1 Travi variamente vincolate e sottoposte ai tipi di carichi più comuni

Schema	Relazioni
<p>- Trave appoggiata agli estremi con carico distribuito variabile con legge lineare (triangolare)</p> 	$R_A = \frac{q \cdot l}{3} \quad ; \quad R_B = \frac{q \cdot l}{6}$ $T_A = R_A; \quad T_B = -R_B$ $M_A = M_B = 0$ $M_{max} = \frac{16}{250} \cdot q \cdot l^2$ $f_{max} = 0,013 \cdot \frac{q \cdot l^4}{2E \cdot I}$
<p>- Trave incastrata a un estremo con carico concentrato all'estremo libero</p> 	$R_A = F$ $T = T_A = T_B = R_A$ $M_{max} = M_A = -F \cdot l$ $f = \frac{1}{3} \cdot \frac{F \cdot l^3}{E \cdot I}$
<p>- Trave incastrata a un estremo con carico uniformemente distribuito</p> 	$R_A = q \cdot l$ $T_A = R_A$ $M_{max} = M_A = -\frac{q \cdot l^2}{2}$ $f = \frac{1}{8} \cdot \frac{q \cdot l^4}{E \cdot I}$

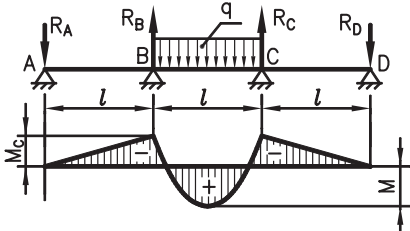
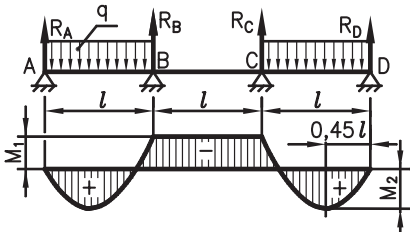
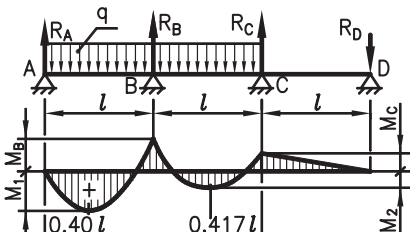
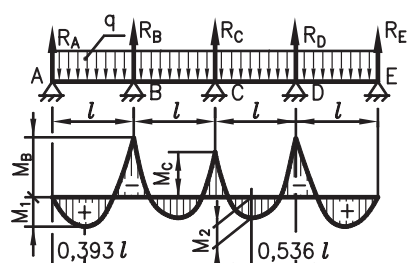
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Tabella H.1 Travi variamente vincolate e sottoposte ai tipi di carichi più comuni

Schema	Relazioni
<p>- Trave incastrata a un estremo, appoggiata all'altro e con carico uniformemente distribuito</p> 	$R_A = \frac{5}{8} \cdot q \cdot l; \quad R_B = \frac{3}{8} \cdot q \cdot l$ $T_A = R_A; \quad T_B = -R_B$ $M_A = \frac{1}{8} \cdot q \cdot l^2$ $M_C = \frac{9}{158} \cdot q \cdot l^2$ $f = \frac{1}{185} \cdot \frac{q \cdot l^4}{E \cdot I}$
<p>- Trave incastrata a un estremo, con un appoggio intermedio e con carico uniformemente distribuito</p> 	$R_A = \frac{q \cdot l}{2} \cdot \left(3 - \frac{3l}{2a} - \frac{a}{4l} \right)$ $R_B = \frac{q \cdot l}{2} \cdot \left(\frac{3l}{2a} + \frac{a}{4l} - 1 \right)$ $T_A = R_A; \quad T_B = R_B $ $M_A = q \cdot l \cdot \frac{2b^2 - a^2}{8l}$ $M_B = -\frac{q \cdot b^2}{2}$ $M_A = M_{max} \quad \text{se } a > \sqrt{6} \cdot b$ $M_B = M_{max} \quad \text{se } a < \sqrt{6} \cdot b$
<p>- Trave incastrata ai due estremi e con carico uniformemente distribuito</p> 	$R_A = R_B = \frac{q \cdot l}{2}$ $T_A = R_A; \quad T_B = -R_B$ $M_A = M_B = -\frac{1}{12} \cdot q \cdot l^2$ $M_C = \frac{1}{24} \cdot q \cdot l^2$ $f = \frac{1}{384} \cdot \frac{q \cdot l^4}{E \cdot I}$

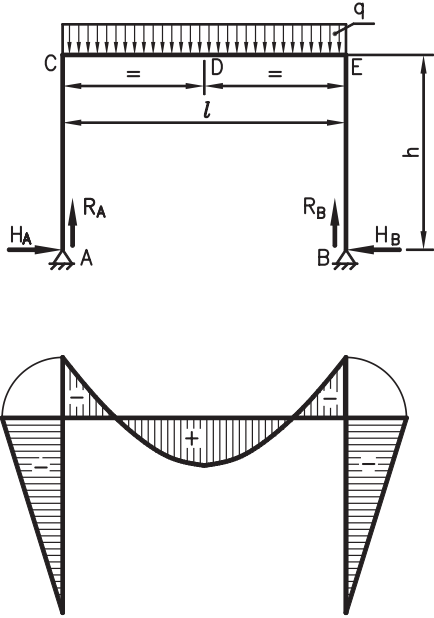
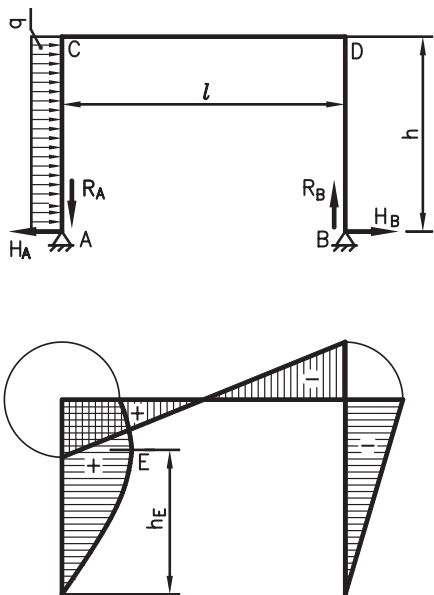
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Tabella H.1 Travi variamente vincolate e sottoposte ai tipi di carichi più comuni

Schema	Relazioni
<p>- Trave su quattro appoggi simmetrici e con carico uniformemente distribuito sulla campata centrale</p> 	$R_A = R_D = 0,005 \cdot q \cdot l$ $R_B = R_C = 0,550 \cdot q \cdot l$ $M = \frac{5}{67} \cdot q \cdot l^2$ $M_A = M_B = M_C = -\frac{1}{20} \cdot q \cdot l^2$
<p>- Trave su quattro appoggi simmetrici e con carico uniformemente distribuito sulle campate laterali</p> 	$R_A = R_D = 0,450 \cdot q \cdot l$ $R_B = R_C = 0,550 \cdot q \cdot l$ $M_1 = -\frac{1}{20} \cdot q \cdot l^2$ $M_2 = \frac{10}{99} \cdot q \cdot l^2$
<p>- Trave su quattro appoggi simmetrici e con carico uniformemente distribuito sulle campate laterali</p> 	$R_A = 0,383 \cdot q \cdot l; R_B = 1,2 \cdot q \cdot l$ $R_C = 0,450 \cdot q \cdot l; R_D = 0,033 \cdot q \cdot l$ $M_1 = \frac{1}{12,7} \cdot q \cdot l^2; M_B = -\frac{1}{8,55} \cdot p \cdot l^2$ $M_C = -\frac{1}{30,3} \cdot q \cdot l^2; M_2 = \frac{1}{18,3} \cdot q \cdot l^2$
<p>- Trave su cinque appoggi simmetrici e con carico uniformemente distribuito</p> 	$R_A = 0,393 \cdot q \cdot l; R_B = 1,143 \cdot q \cdot l$ $R_C = 0,929 \cdot q \cdot l; R_D = 1,143 \cdot q \cdot l$ $R_E = 0,393 \cdot q \cdot l$ $M_1 = \frac{1}{13} \cdot q \cdot l^2; M_B = -\frac{1}{8,55} \cdot p \cdot l^2$ $M_C = -\frac{1}{14,1} \cdot q \cdot l^2; M_2 = \frac{1}{27,75} \cdot q \cdot l^2$

Continua

Tabella H.1 Travi variamente vincolate e sottoposte ai tipi di carichi più comuni

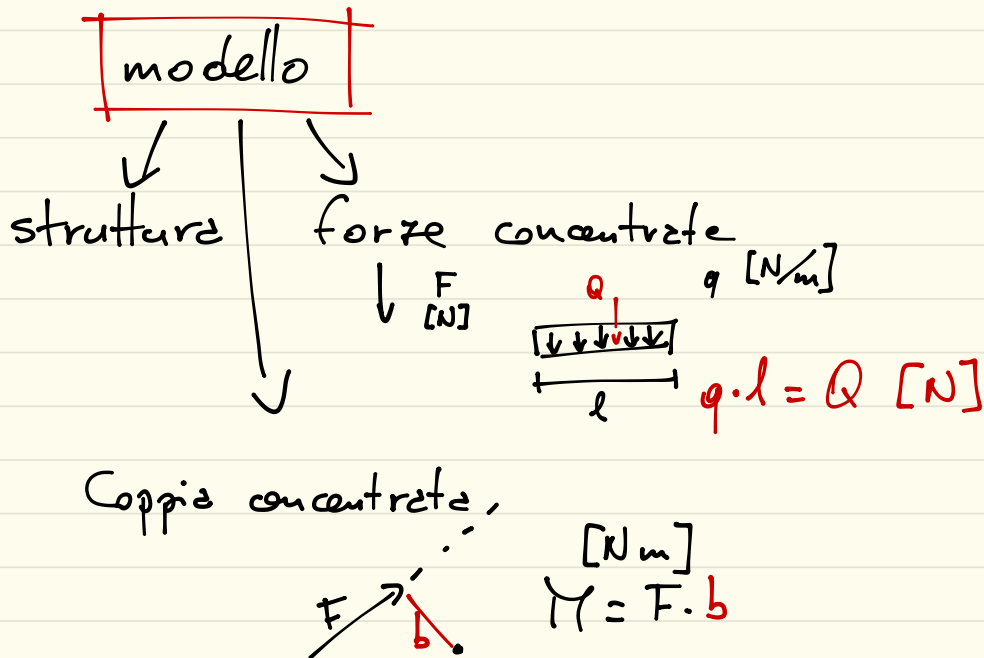
Schema	Relazioni
<p data-bbox="236 427 791 483">- Telaio incernierato agli estremi e con carico uniformemente distribuito sulla parte orizzontale</p> 	$R_A = R_B = \frac{q \cdot l}{2}$ $H_A = H_B = \frac{q \cdot l^2}{4h \cdot \left(2 \frac{h}{l} + 3\right)}$ $M_D = \frac{q \cdot l^2}{8} \cdot \frac{2h + l}{2h + 3l}$ $M_C = M_E = \frac{q \cdot l^2}{4 \cdot \left(2 \frac{h}{l} + 3\right)}$
<p data-bbox="236 1196 791 1252">- Telaio incernierato agli estremi e con carico uniformemente distribuito su una parte verticale</p> 	$R_A = R_B = \frac{q \cdot h^2}{2l}$ $H_A = \frac{q \cdot h}{8} \cdot \frac{11h + 18l}{2h + 3l}$ $H_B = q \cdot h - H_A$ $M_E = \frac{q \cdot h^2}{128} \cdot \left(\frac{11k + 18}{2k + 3}\right)^2; \quad k = \frac{h}{l}$ $h_E = \frac{h}{8} \cdot \left(\frac{11k + 18}{2k + 3}\right)$ $M_C = \frac{3}{8} \cdot q \cdot l^2 \cdot \frac{h + 2l}{2h + 3l}$ $M_D = -\frac{q \cdot h^2}{8} \cdot \frac{5k + 6}{2k + 3}$

Continua

Unità di Misura

Coppie: Nm

Modelli:



- struttura reale **Corpo rigido**
- ↓
- modello (struttura, forze, vincoli)
- ↓
- calcolo delle forze "esterne" (reazioni vincolari)
- ↓
- calcolo delle forze "interne" (sforzi, diagrammi di sforzo)

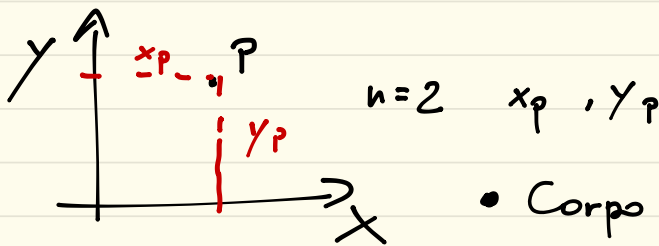
(prima parte del corso)
soo scritto!



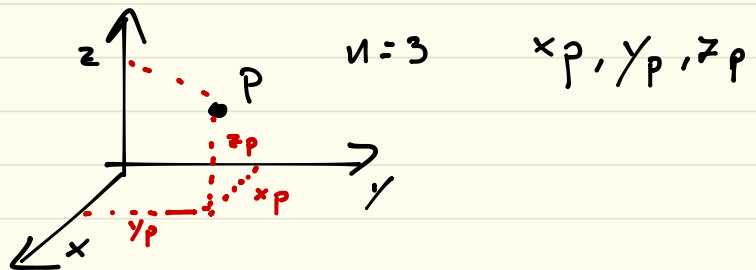
gradi di libertà \rightarrow Degrees of freedom

Coordinate n

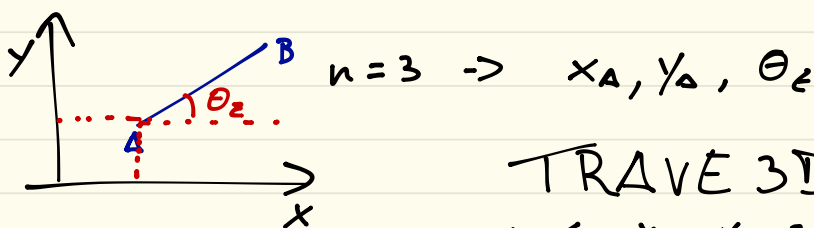
• Corpo puntiforme 2D



• Corpo puntiforme 3D



n coordinate
TRAVE 2D



TRAVE 3D

$n=6 \quad x_A, y_A, z_A, \theta_x, \theta_y, \theta_z$

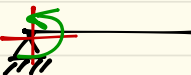
Coordinate n


libere Vincolate

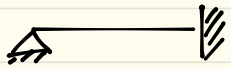
$\hookrightarrow l \rightarrow D.O.F.$ $\hookrightarrow m$ $n \geq \frac{m}{2} + l$ $n = m + l$
sistema generale

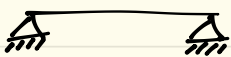
Esempi di calcolo di DOF_s (degrees of freedom)

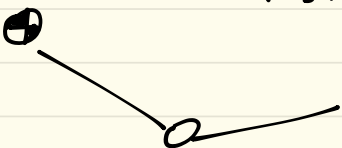
1.  $n=3$ $m=3$ $n=m+l \rightarrow l=0 \rightarrow$ *isostatico*


2.  $n=3$ $m=2$ $n=m+l \rightarrow l=1 \rightarrow$ *ipostatico*


3.  $n=3$ $m=3$ $n=m+l \rightarrow l=0 \rightarrow$ *isostatico*

4.  $n=3$ $m=5$ $n=m+l \rightarrow l=-2 \rightarrow$ *iperstatico*

5.  $n=3$ $m=4$ $n=m+l \rightarrow l=-1 \rightarrow$ *iperstatico*

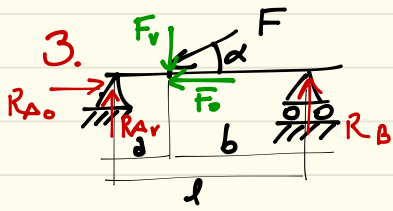
6.  $n=3+3=6$ $m=2+2=4$ $n=m+l \rightarrow l=2$
ipostatico

7.  *cerchelli lo schematizziamo* \rightarrow 

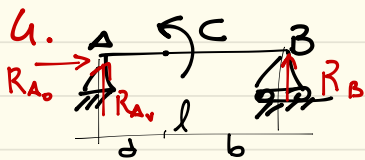
8.  $n=3$ $m=3$ $l=0$
ingenuerole del punto di vista teorico

del punto di vista cinematico \rightarrow labile

del punto di vista teorico \rightarrow iperstatico.

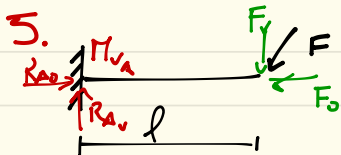


$$\begin{aligned} \xrightarrow{+} R_{A0} - F_0 &= 0 & R_{A0} &= F_0 \\ \uparrow^+ R_{Av} - F_v + R_B &= 0 & F_{v2} &= R_B l \\ \curvearrowright^+ -F_v b + R_B l &= 0 \end{aligned}$$



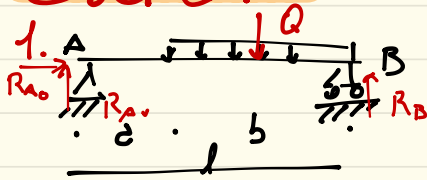
$$[C] = N_m$$

$$\begin{aligned} \xrightarrow{+} R_{A0} &= 0 & R_{Av} &= -R_B = -\left(-\frac{C}{l}\right) \\ \uparrow^+ R_{Av} + R_B &= 0 & R_{Av} &= +\frac{C}{l} \\ \curvearrowright^+ C + R_B l &= 0 & R_B &= \frac{C}{l} \end{aligned}$$



$$\begin{aligned} \xrightarrow{+} R_{A0} - F_0 &= 0 & R_{A0} &= F_0 \\ \uparrow^+ R_{Av} - F_v &= 0 & R_{Av} &= F_v \\ \curvearrowright^+ M_v - F_v l &= 0 & M_v &= F_v l \end{aligned}$$

Esercizi



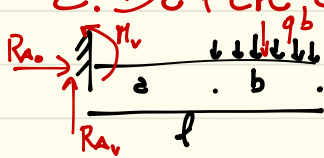
$$Q = qb$$

$$\rightarrow R_{A0} = 0$$

$$\uparrow R_A + R_B - Q = 0 \rightarrow R_A = \frac{qb^2}{2l}$$

$$\text{A)} \quad -qb\left(a + \frac{b}{2}\right) + R_B l = 0 \rightarrow R_B = \frac{qb}{l} \left(a + \frac{b}{2}\right)$$

2. Da fare a casa

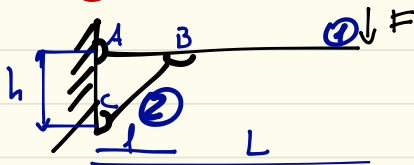


$$R_{A0} = 0$$

$$R_A = qb$$

$$M_V = qb \left(a + \frac{b}{2}\right)$$

Sistema articolato



$$h = 3 + 3 = 6$$

$$m = 2 + 2 + 2 = 6$$

\gg isostatico!

A e C sono cerniere esterne, B cerniera interna
 quando una trave è caricata solo alle estremità in corrispondenza delle due cerniere questa è un'asta!

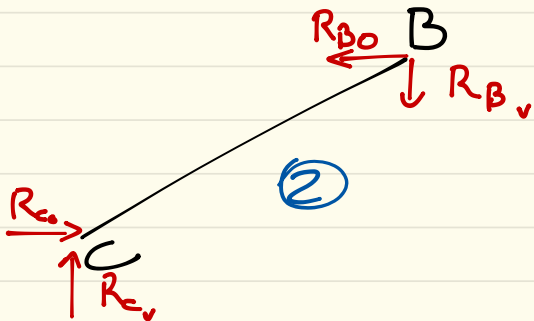
① trave

② asta

un'asta è caricata solo dalle reazioni vincolari peraltro uguali agli estremi

$\leftarrow \text{-----} \rightarrow$ asta tirante

$\rightarrow \text{-----} \leftarrow$ asta puntone

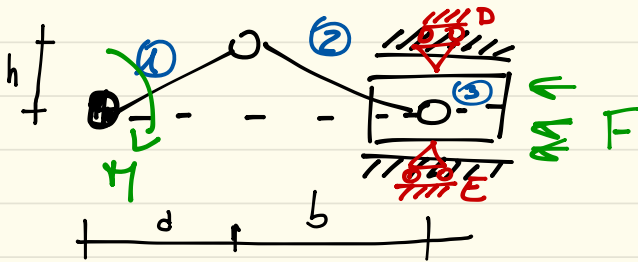


$$\begin{aligned} \rightarrow & R_{Co} - R_{Bo} = 0 \\ \uparrow & R_{Cv} - R_{Bv} = 0 \\ \curvearrowright & -R_{Bv} \cdot l + R_{Bo} \cdot h = 0 \end{aligned}$$

$$R_{Bv} = R_{Bo} \cdot \frac{h}{l} = \frac{FL}{l} \cdot \frac{h}{l}$$

$$R_C = \frac{FL}{l}, \quad R_{Av} = -\frac{F}{l} (L-l)$$

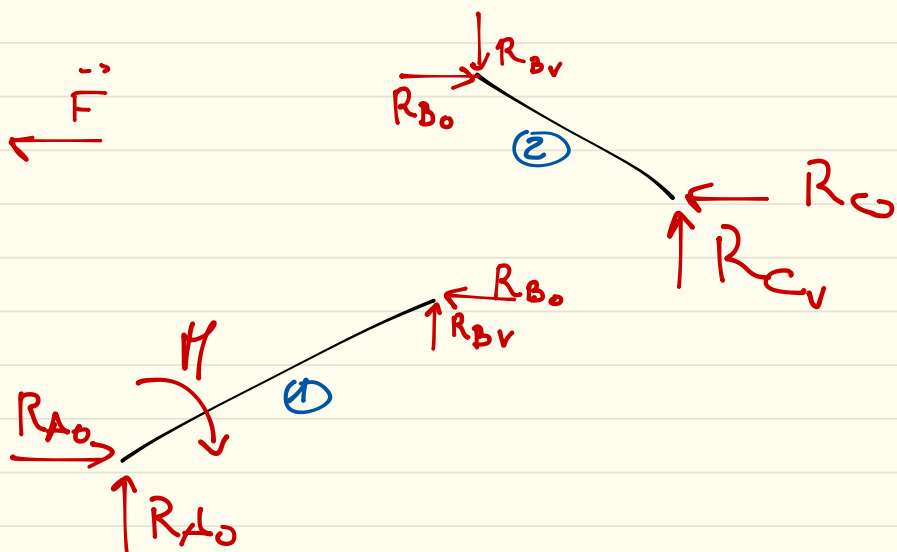
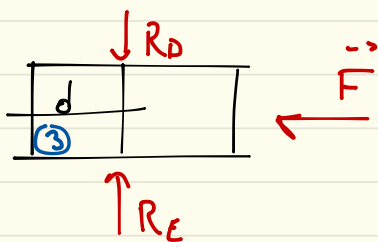
Sistema biella - menovelle (labile)



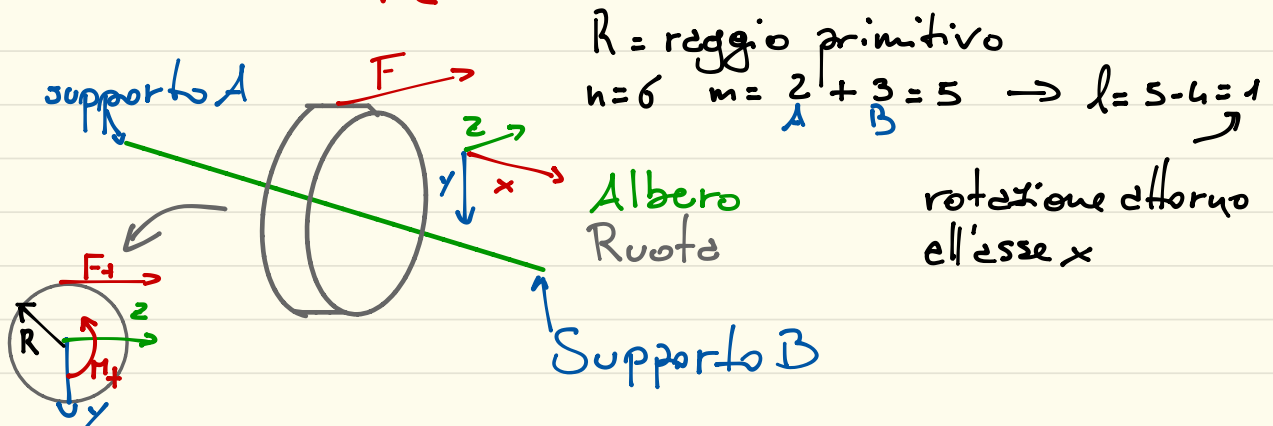
Provare a casa!

$$\begin{aligned} h &= 3 + 3 + 3 = 9 \\ m &= 2 + 2 + 1 + 1 + 2 = 8 \end{aligned}$$

$$n - m = l = 9 - 8 = 1$$



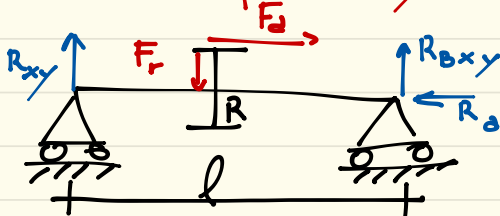
Reazioni vincolari di un sistema spaziale: albero con ruote dentate



Su una ruota e su un albero le forze è sempre risolte verso l'interno

equazione del moto $\rightarrow M_t - F_t R = 0 \rightarrow M_t = F_t R$

Forze nel piano xy



$M_c = F_d R$

$\rightarrow F_d - R_d = 0$

$\uparrow R_{Axy} + R_{Bxy} - F_r = 0$

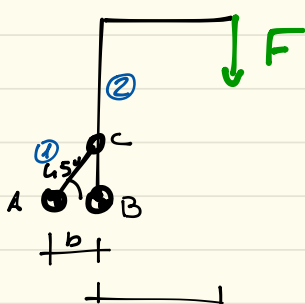
$\curvearrowright -M_A - F_r b + R_{Bxy} l = 0$

$R_A = F_d$

$R_{Bxy} = \frac{F_r b + M_c}{l}$

$R_{Axy} = \frac{F_r b - M_c}{l}$

Esercizi



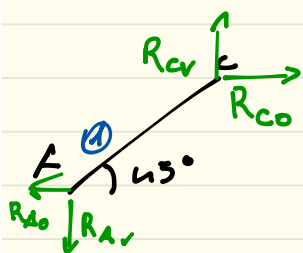
$$n = 3 + 3$$

$$m = 2 + 2 + 2$$

A B C

$$l = 0$$

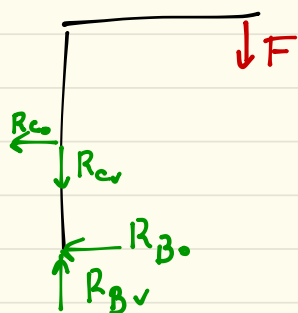
$$\begin{cases} R_{A0} & R_{Av} \\ R_{B0} & R_{Bc} \\ R_{C0} & R_{Cv} \end{cases}$$



$$R_{C0} - R_{A0} = 0$$

$$R_{Cv} - R_{Av} = 0$$

$$R_{Cvb} - R_{Cob} = 0$$

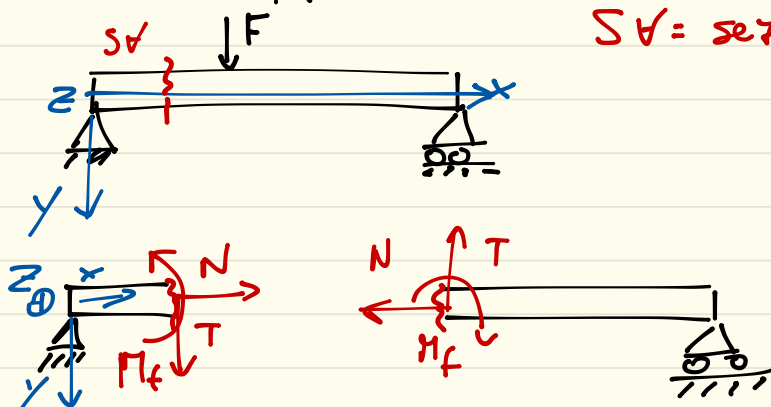


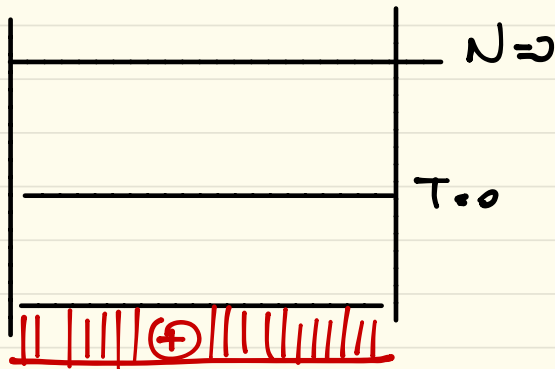
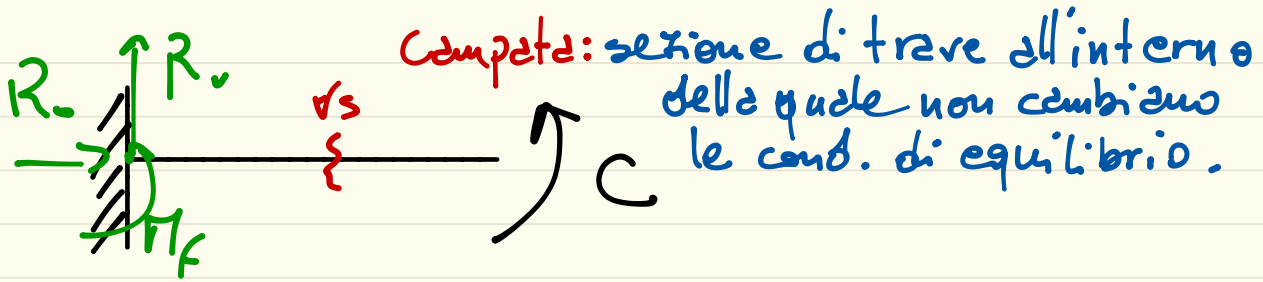
$$R_{B0} - R_{C0} = 0$$

$$R_{Bv} - R_{Cv} - F = 0$$

2D $N (N_x) \rightarrow$ Sforzo normale
 $T (T_y) \rightarrow$ Sforzo al taglio
 $M_f (M_z) \rightarrow$ Momento flettente

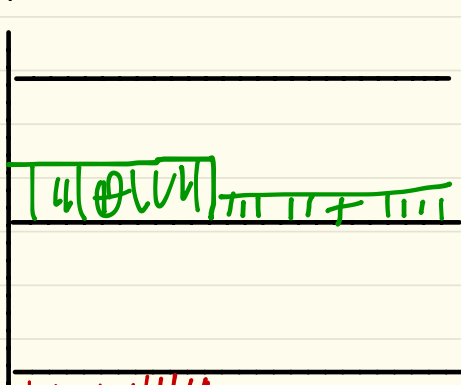
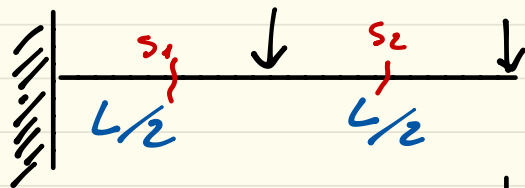
SA = sezione qualunque





$N=0 \quad T=0$
 $\cup M_f - C = 0 \Rightarrow M_f = C$

Altro esercizio



$N=0 \quad R_v = 2F$
 $M_v - F \cdot L/2 - FL \Rightarrow$
 $M_v = \frac{3}{2} FL$

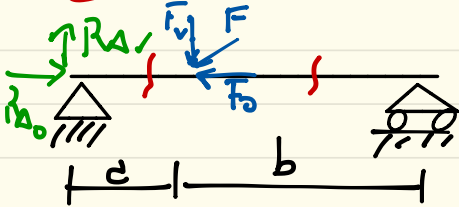
$T - R_v = 0 \rightarrow T = R_v = 2F$

$M_f + M_v - R_v x = 0$ $\begin{cases} x=0 \\ x=L/2 \end{cases}$

$T - F = 0 \Rightarrow T = F$

$M_f = -\frac{3}{2} FL$
 $M_{f_x} = -\frac{FL}{2}$

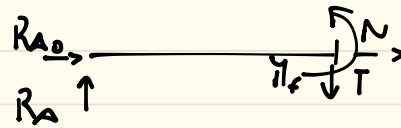
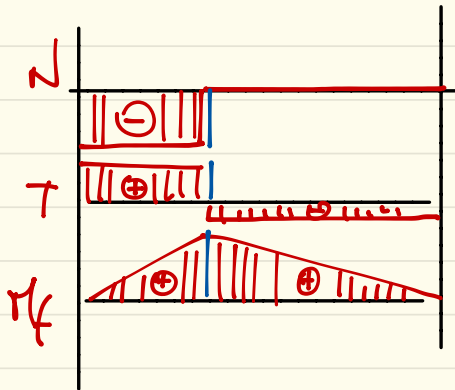
Esercizio



$$R_{A0} = F_o$$

$$R_{Av} = \frac{F_v b}{L}$$

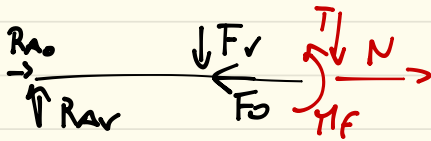
$$R_B = \frac{F_v e}{L}$$



$$T - R_{Av} = 0 \quad N + R_{B0} = 0$$

$$T = R_{Av} \quad N = -R_{A0} = -F_o$$

$$M_f - R_{Av} x = 0 \quad M_f = \frac{F_v b}{L} x$$



$$N - F_o + R_{A0} = 0$$

$$T + F_v - R_{Av} = 0$$

$$T = \frac{F_v b}{L} - F_v = -F$$

quindi $x = \frac{\sum m_i x_i}{\sum m_i} \rightarrow xM = \sum m_i x_i$

$\cdot S_x = \int_A y dA = y_G \int_A dA = y_G A$

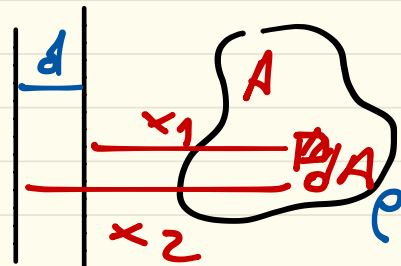
Analogamente per le altre direzioni

Teorema di trasposizione dei momenti statici

$S_{y_1} = \int_A x_1 dA$

$S_{y_2} = \int_A x_2 dA = \int_A (x_1 + d) dA =$

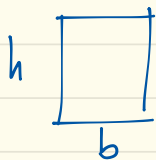
$= S_{y_1} + d \int_A dA = S_{y_1} + d \cdot A$
 ↳ Area della sezione



È comodo usare uno dei due assi come "baricentrico" in modo che uno dei momenti statici valda 0

N.B: se l'area possiede uno o più assi di simmetria il baricentro si troverà o lungo l'asse (nel caso di 1) o nella loro intersezione (più di uno)

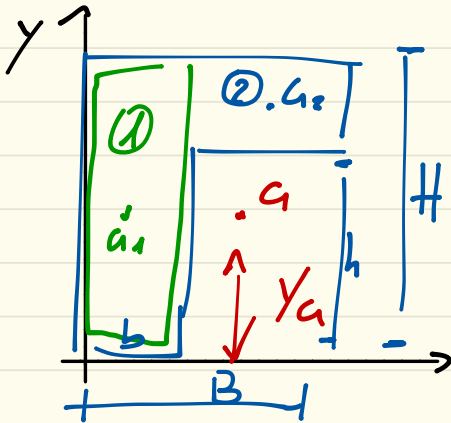
Caso pratico:



$y_G = ? \quad x_G = ?$

$y_G = \frac{\int_A y dA}{\int_A dA} = \frac{S_x}{A} \quad \rightarrow \quad S_x = \int_0^h y b dy = b \frac{y^2}{2} \Big|_0^h = \frac{bh^2}{2}$
 $\rightarrow y_G = \frac{S_x}{A} = \frac{bh^2}{2} \cdot \frac{1}{bh} = \frac{h}{2}$

Momento statico di una figura composta



$y_G = ?$
 possiamo procedere scomponendo la figura in "parti" più semplici e noi noti.

$$A_1 = bH \quad A_2 = (B-b)(H-h)$$

$$\sum x_{tot} \Rightarrow y_G = \frac{\sum x_{tot}}{A_{tot}}$$

$$\sum x_{tot} = \sum x_1 + \sum x_2 = A_1 \cdot y_{a1} + A_2 \cdot y_{a2} = A_1 \frac{H}{2} + A_2 \cdot \frac{(H+h)}{2}$$

$$y_G = \frac{\sum x_{tot}}{A_{tot}}$$

Proviamo con un altro riferimento: $A_{pieno} = B \cdot H$
 $A_{vuoto} = h(B-b)$

$$\sum x_{tot} = \sum x_{pieno} - \sum x_{vuoto} ?$$

$$\sum x_{pieno} = A_{pieno} \cdot y_{Gp} = B \cdot H \cdot \frac{H}{2} = \frac{B \cdot H^2}{2}$$

$$\sum x_{vuoto} = A_{vuoto} \cdot y_{Gv} = h(B-b) \cdot \frac{h}{2}$$

$$y_G = \frac{\sum x_{tot}}{A_{tot}} = \frac{\sum x_{pieno} - \sum x_{vuoto}}{A_{tot}}$$

• Momento centrifugo

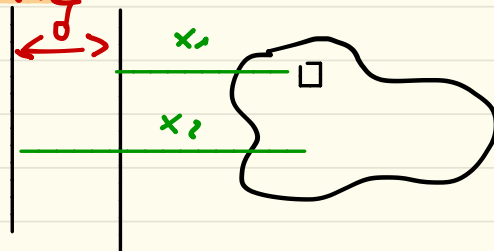
$$I_{xy} = \int_A x \cdot y \, dA \rightarrow \text{poco usato}$$

• Momento polare

$$I_P = \int_A r^2 \, dA \rightarrow \text{Molto usato}$$



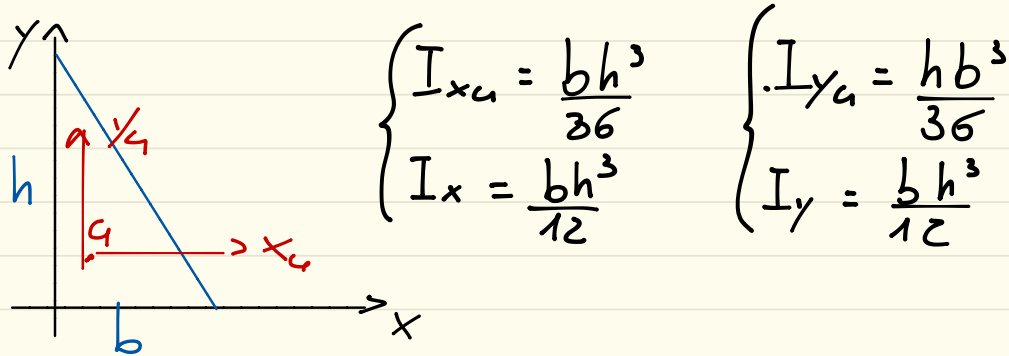
Teorema di trasposizione dei momenti d'inerzia



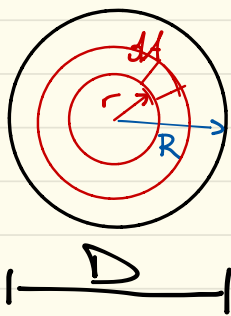
$$I_{y_2} = \int_A x^2 \, dA = \int_A (x_1 + d)^2 \, dA = \int_A x_1^2 \, dA + \int_A d^2 \, dA + 2 \int_A d x_1 \, dA =$$

$$I_{y_1} + d^2 A + 2d S_{y_1}$$

Se y_1 passa per G $S_{y_1} \rightarrow 0$ quindi scompare dalla formula.

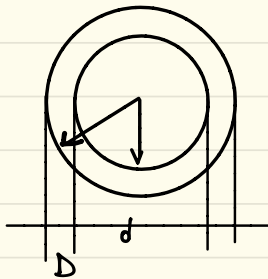


Ancora un'applicazione:



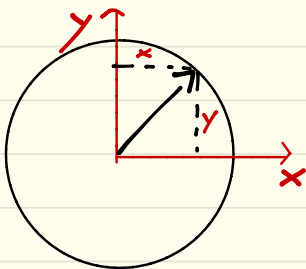
$$I_p = \int_A r^2 dA = \int_0^R r^2 2\pi r dr = 2\pi \frac{r^4}{4} \Big|_0^R = \pi \frac{r^4}{2} = \frac{\pi D^4}{32}$$

Sezione cava



$$I_p = I_{p_{pieno}} - I_{p_{vuoto}} = \frac{\pi D^4}{32} - \frac{\pi d^4}{32}$$

Un'altro caso

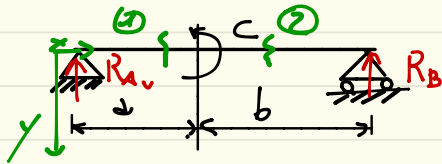


$$I_x = I_y = \frac{I_p}{2} = \frac{\pi D^4}{64}$$

$$I_r = \int_A r^2 dA = \int_A (x^2 + y^2) dA = \int_A 2x^2 dA = 2I_x$$

Esercizio 1

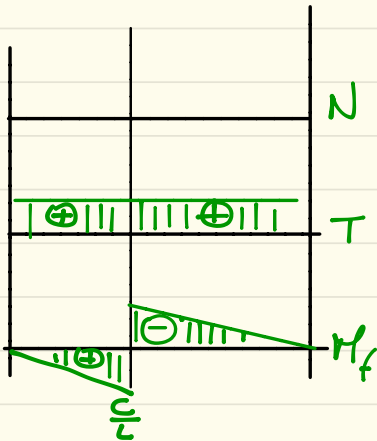
(Temid' esame ottime preparazione)



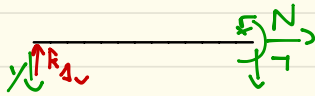
$$R_{A_0} = 0$$

$$R_{A_v} = \frac{C}{L}$$

$$R_B = -\frac{C}{L}$$



$$\textcircled{1} \begin{cases} T - R_{A_v} = 0 & T = R_{A_v} \\ M_f - R_{A_v} x = 0 & M_f = \frac{C}{L} x \end{cases} \begin{matrix} x=0 \\ x=d \end{matrix}$$



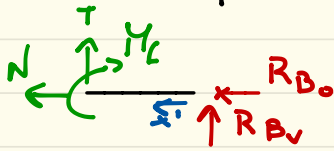
$$T - R_{A_v} = 0$$

$$M_f + C - R_{A_v} x = 0 \rightarrow M_f = R_{A_v} x - C \begin{matrix} x=d \\ x=d+b=L \end{matrix}$$

$$M_{f_{x=d}} = \frac{C}{L} d - C = \frac{C}{L} (d - L) = -\frac{C}{L} b$$

$$M_{f_{x=L}} = C - C = 0$$

3° campo



$$N + R_{B0} = 0 \rightarrow N = -R_{B0}$$

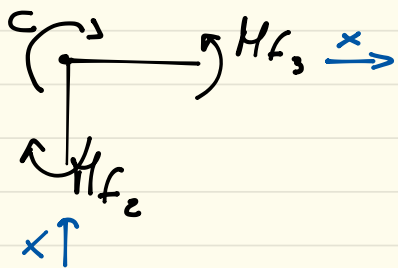
$$T + R_{Bv} - qx' = 0 \rightarrow T = qx' - R_{Bv}$$

$$M_f - R_{Bv}x' + qx' \cdot \frac{x'}{2} \rightarrow M_f = R_{Bv}x' - q \frac{x'^2}{2}$$

$$T = qx' - R_{Bv} = 0 \quad x' = \frac{R_{Bv}}{q} \quad x' = 3,33 \text{ m}$$

$$M_{f_{max}} = R_{Bv}x' - q \frac{x'^2}{2}$$

• Equilibrio al nodo → utilizziamo per verifica



$$+ \curvearrowright M_{f3} - C - M_{f2} = 0$$

$$-3400 - 600 + 10000 = 0 \text{ verificato}$$

Equilibrio al nodo

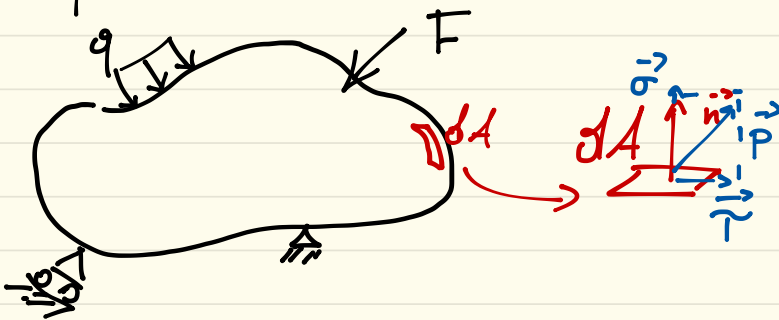
$$\begin{array}{l} M_{f_2} \ 1000 \text{ Nm} \\ | \\ M_{f_3} \ -3000 \text{ Nm} \\ | \\ M_{f_1} \ -2000 \end{array} \quad \sum_{i=1}^3 M_{f_i} = 0$$

$$\downarrow M_{f_2} - M_{f_1} + M_{f_3} = 0 \rightarrow 1000 + 2000 - 3000 = 0$$

verificato!

Stato di tensione di un punto

quello elementare



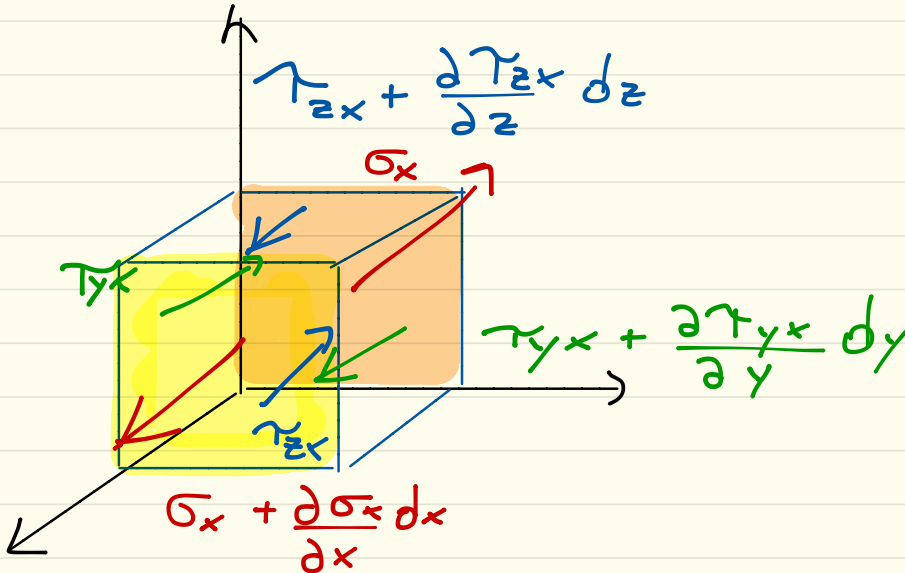
Def. stress: $\lim_{dA \rightarrow 0} \frac{dF}{dA} = \vec{p} \rightarrow \left[\frac{N}{mm^2} \right]$

$\vec{\sigma} \rightarrow$ tensione normale (normal stress)
 $\vec{\tau} \rightarrow$ tensione tangenziale (shear stress)

$$\sigma_{ij} \quad \tau_{ij}$$

il primo indice indica la "faccia" su cui è applicata,
 il secondo indice la "direzione"

Equazioni di equilibrio $\left\{ \begin{array}{l} 3 \text{ eq. traslaz.} \\ 3 \text{ eq. rotazione} \end{array} \right.$



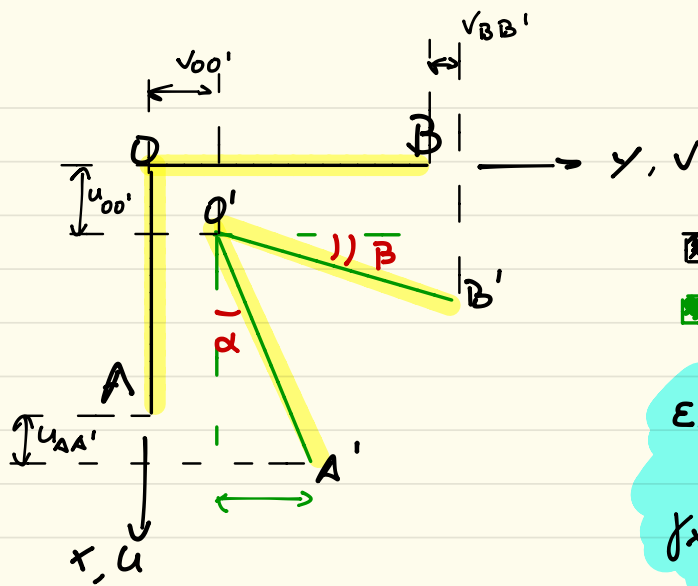
$$\left(\sigma_x + \frac{\partial \sigma_x}{\partial x} dx \right) dy dz - \sigma_x dy dz + \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy \right) dx dz -$$

$$\tau_{yx} dx dz + \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} dz \right) dx dy - \tau_{zx} dy dx = 0$$

$$\left\{ \begin{array}{l} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \end{array} \right.$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \frac{\partial \tau_{xy}}{\partial x} = 0$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = 0$$



■ prima della deformazione
■ dopo la deformazione

$$\epsilon_x = \frac{\partial u}{\partial x} \quad \epsilon_y = \frac{\partial v}{\partial y}$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$u_{AA'} - u_{00'} = \frac{\partial u}{\partial x} dx \rightarrow \frac{u_{AA'} - u_{00'}}{dx} = \frac{\partial u}{\partial x} = \epsilon_x$$

indice quanto si è deformato il mio elemento rispetto alle dimensioni iniziali.

$$v_{BB'} - v_{00'} = \frac{\partial v}{\partial y} dy \rightarrow \frac{\partial v}{\partial y} = \epsilon_y$$

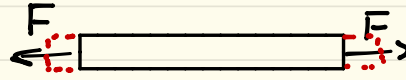
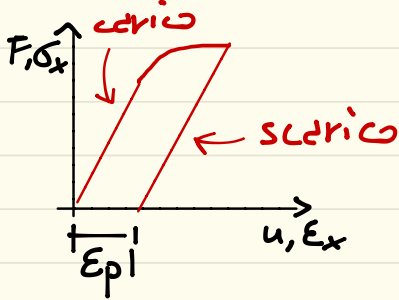
$$\alpha + \beta \approx \text{tg} \alpha + \text{tg} \beta$$

$$\text{tg} \alpha = \frac{v_{AA'} - v_{00'}}{dx} = \frac{\partial v}{\partial x}$$

$$\text{tg} \beta = \frac{u_{BB'} - u_{00'}}{dy} = \frac{\partial u}{\partial y}$$

$$\left\{ \begin{array}{l} \epsilon_x = \frac{\partial u}{\partial x} \\ \epsilon_y = \frac{\partial v}{\partial y} \\ \epsilon_z = \frac{\partial w}{\partial z} \end{array} \right. \quad \left\{ \begin{array}{l} \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \end{array} \right.$$

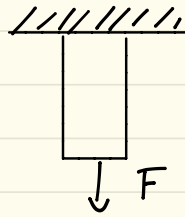
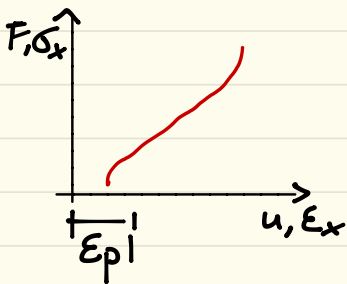
Legame Plastico



Nel regime plastico

- variazione di t influente
- Non recupero allo scarico
- Variazione dei legami al livello atomico

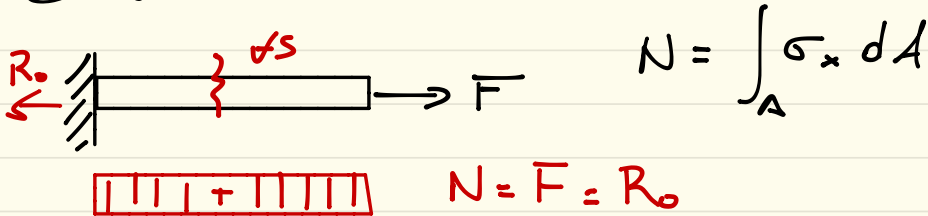
Legame Viscoso (Creep)



Nel regime viscoso:

- variazione di t importantissima
- non totale recupero allo scarico
- rottura legami atomici

1) Normale

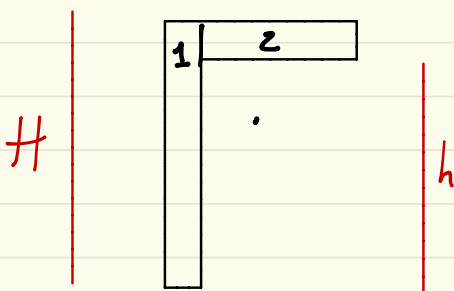


$$\sigma_x = \text{cost}$$

$$\sigma_x = \frac{N}{A}$$

Esercitazione geometria delle aree

B

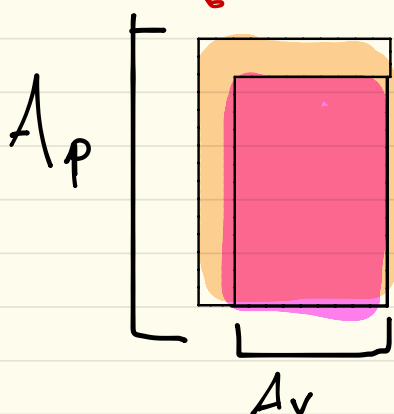


$$y_a = \frac{S_{x \text{ tot}}}{A_{\text{tot}}}$$

$$x_a = \frac{S_{y \text{ tot}}}{A_{\text{tot}}}$$

$$y_a = \frac{S_{\text{tot}x}}{A_{\text{tot}}} = \frac{A_1 y_{a1} + A_2 y_{a2}}{A_1 + A_2} =$$

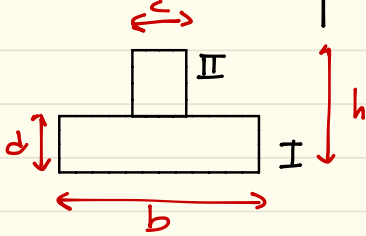
$$= \frac{b H \frac{H}{2} + (B-b)(h-b)}{A_1 + A_2}$$



$$y_a = \frac{A_p y_p - A_v y_v}{A_p - A_v} = \frac{B H \frac{H}{2} - (B-b) h \frac{h}{2}}{B H - (B-b) h}$$

$$x_a = B H \frac{H}{2} \frac{B}{2} - (B-b) h [b + (B-b)]$$

Altro esempio



$$x_u = \frac{b}{2}$$

$$y_u = \frac{S_{x \text{ tot}}}{A_{\text{tot}}} = \frac{A_1 y_{u1} + A_2 y_{u2}}{A_1 + A_2} =$$

$$= \frac{[b \cdot d \cdot \frac{d}{2}] + [c(h-d)(h - \frac{h-d}{2})]}{bd + c(h-d)}$$

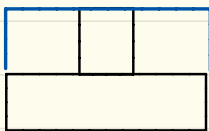
$$y_u = \frac{S_{x \text{ tot}}}{A_{\text{tot}}} = \frac{A_p \cdot y_p - A_v \cdot y_v}{A_p - A_v} = \frac{b \cdot h \cdot \frac{h}{2} - (h-d)(b-c)(h - \frac{h-d}{2})}{(b \cdot h) - (h-d)(b-c)}$$

$$I_{\text{tot } x} = I_{Ix} + I_{Iix}$$

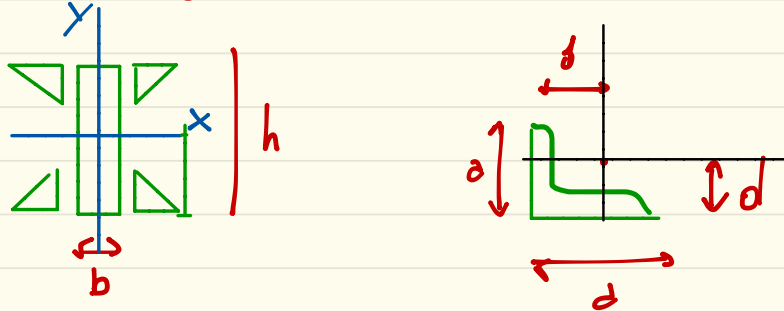
$$\hookrightarrow I_{Iix} = I_{Iixc2} + A_2 (y_{u2} - y_u)^2 =$$

$$= \frac{c(h-d)^3}{12} + c(h-d) \cdot d + \frac{(h-d)}{2} \cdot y_u^2$$

$$I_{Ix} = I_{Ixc1} + A_1 (y_{u1} - y_u)^2 = \frac{bd^3}{12} + bd \left(\frac{d}{2} - y_u \right)^2$$



Il cantonale



$$I_{x \text{ tot}} = I_{xp} + 4 I_{xc}$$

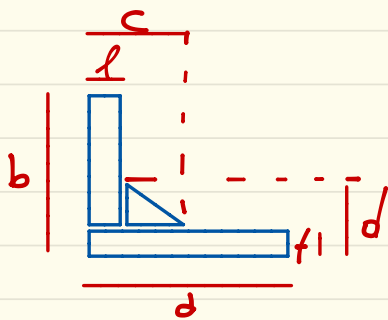
$$\downarrow$$

$$\frac{bh^3}{12}$$

$$\rightarrow [I_{xc} + A_c \cdot \left(\frac{h}{2} - d\right)^2]$$

↙ cantonale

Altro esempio



$$A_1 = lf$$

$$A_2 = l(b-f)$$

$$A_3 = \frac{(c-l)(d-f)}{2}$$

$$x_{c1} = d/2$$

$$y_{c2} = f/2$$

$$x_{c2} = l/2$$

$$y_{c3} = f + (b-f)/2$$

$$x_{c3} = l + (c-l)/3$$

$$y_{c3} = f + (d-f)/3$$

$$x_{cT} = \frac{S_{xT}}{A_T} = \frac{A_1 y_{c1} + A_2 y_{c2} + A_3 y_{c3}}{A_1 + A_2 + A_3}$$

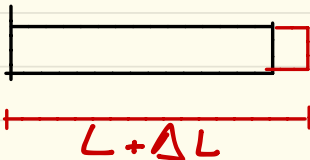
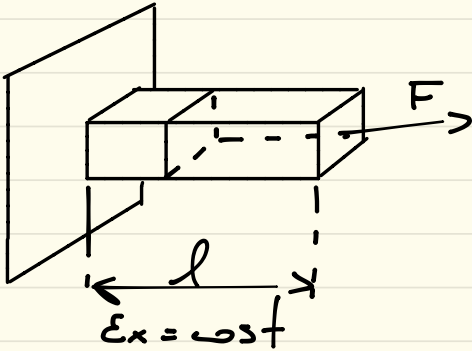
$$y_{cT} = \frac{S_{yT}}{A_T} = \frac{A_1 x_{c1} + A_2 x_{c2} + A_3 x_{c3}}{A_1 + A_2 + A_3}$$

Deformazioni

ricordiamo che σ_x costante

N risult. distr. σ_x

$N = \text{cost}$ lungo la
trave



$$\epsilon_x = \frac{\Delta L}{L}$$

$$\Delta L = \int_0^L \epsilon_x dx = \epsilon_x L$$

$$\Delta L = \epsilon_x L = \frac{\sigma_x}{E} L = \frac{N}{A E} L$$

← sforzo normale

↳ superficie o Area

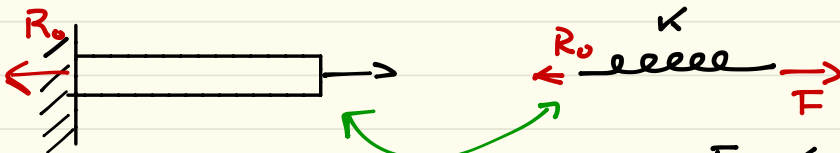
costante elastica (o di young)

$$E = 210'000 \text{ N/mm}^2$$

$$N \quad \sigma_x = \frac{N}{A}$$

$$\epsilon_x = \frac{\sigma_x}{E} = \frac{N}{EA}$$

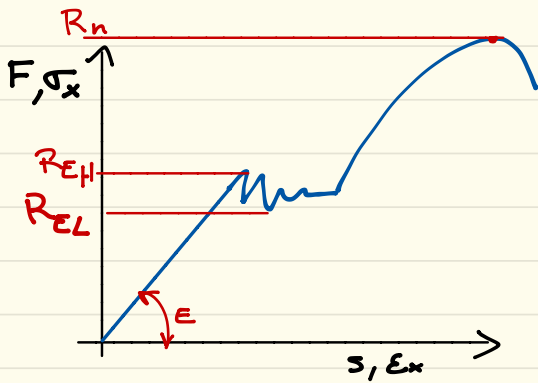
$$\Delta L = \frac{N}{EA} L$$



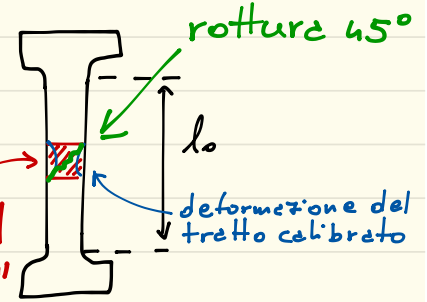
$$F = K \times \Delta L \Leftrightarrow N = \frac{EA}{L} \Delta L$$

$$[K] = \frac{N}{mm} \quad \left[\frac{EA}{L} \right] = \frac{N}{mm} \cdot \frac{mm^2}{mm}$$

Prove di trazione



acciaio duttile



Se si rompe nel tratto "calibrato" allora va bene

Provino per trazioni

$$\sigma_x = \frac{F}{A}$$

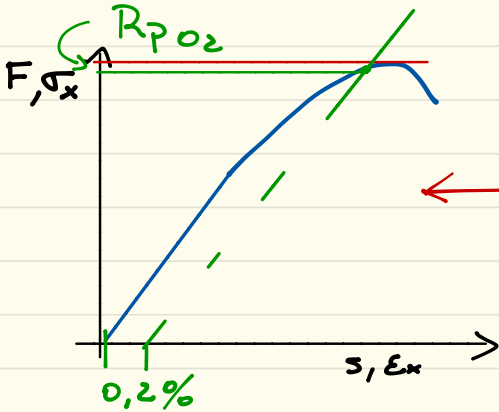
N.B. = Le forze si misura le σ_x si calcola

R_{EH} = limite elastico superiore

R_m = carico massimo

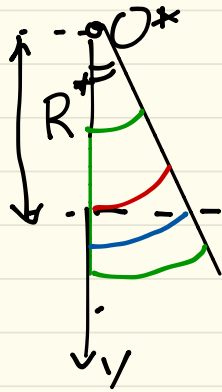
- R_{EH} R_m
- R_{EL} R_n
- R_s S_n
- S_y
- S_e

$A = \frac{l_u - l_0}{l_0} \% \quad A > 5\% \leftarrow \text{comp. duttile}$
 $A < 5\% \leftarrow \text{comp. fragile}$
 "Indice di fragilità o duttilità"
 (nome inventato)

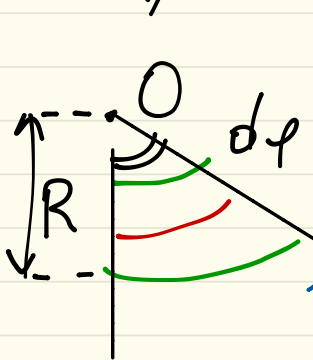


Nel caso in cui il materiale non si snerva

$$dl^* = (R^* + y) d\varphi^*$$



Applichiamo adesso un momento flettente



$$dl = (R + y) d\varphi$$

$$dl_n = R^* d\varphi^* = R d\varphi$$

deformazione nulla per il piano neutro

le fibre neutre non "sentono" il momento flettente

N.B: Deformazione adimensionale

$$\begin{aligned} \epsilon_x &= \frac{dl - dl^*}{dl^*} \rightarrow \epsilon_x = \frac{(R + y) d\varphi - (R^* + y) d\varphi^*}{(R^* + y) d\varphi^*} = \\ &= \frac{R d\varphi + y d\varphi - R^* d\varphi^* - y d\varphi^*}{(R^* + y) d\varphi^*} \end{aligned}$$

deformazione fibre neutre

$$\epsilon_x = \frac{y}{R^* + y} \cdot \frac{d\varphi - d\varphi^*}{d\varphi^*} = \frac{y}{R^* + y} \left(\frac{d\varphi}{d\varphi^*} - 1 \right) = \frac{y}{R^* + y} \left(\frac{R^*}{R} - 1 \right)$$

Dimostriamo che piano neutro \equiv piano baricentrico
 $\int_A \sigma_x dA = 0$

esprimiamo con lo schema:

integrando:

$$\int_A \frac{E}{R} y dA$$

$\rightarrow S_z$ (momento statico baricentrico sempre uguale a 0)



Ne consegue che ogni piano baricentrico è un piano neutro

Calcoliamo adesso il momento flettente:

$$M_f = M_z = \int_A \sigma_x y dA = \int_A \frac{E}{R} y \cdot y dA = \frac{E}{R} \int_A y^2 dA$$

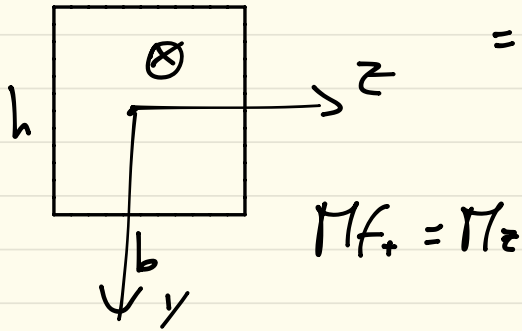
momento d'inerzia rispetto a z

quindi:

$$\left. \begin{aligned} M_f &= \frac{E}{R} I_z \\ \sigma_x &= \frac{E}{R} y \end{aligned} \right\}$$

$$\sigma_x = \frac{M_f}{I_z} y$$

Esempio
1) Rettangolare



$$\sigma_{x \max} = \frac{M_z}{I_z} y_{\max} =$$

$$= \frac{M_z}{\frac{bh^3}{12}} \cdot \frac{h}{2} = *$$

↑
distanza max
dal piano neutro

$$* = \frac{M_z \sigma}{bh^2} = \frac{M_z}{W_f}$$

$\frac{bh^2}{6} = W_f \rightarrow$ modulo di resistenza a flessione

$$M_z - \sigma_{x \max} = \frac{(-M_z)(-y_{\max})}{I_z} = \frac{M_z h/2}{\frac{bh^3}{12}} = \frac{6M_z}{bh^2}$$

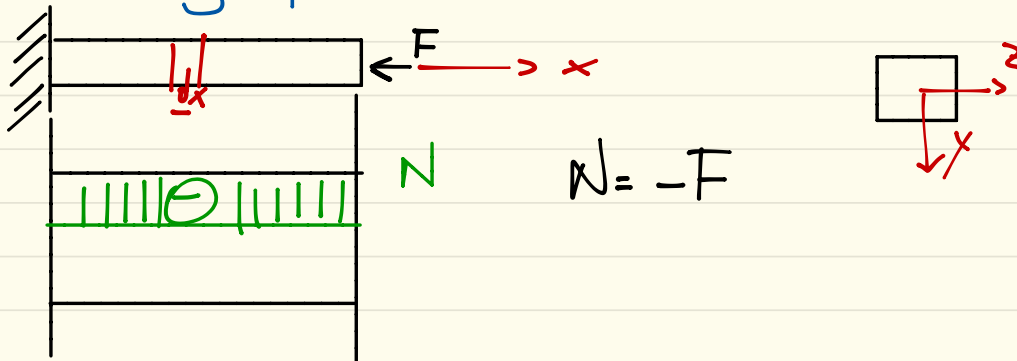
↑ Momento flettente negativo

Definiamo il modulo di resistenza alla torsione di una sezione circolare piena:

$$W_f = \frac{\pi D^3}{32}$$

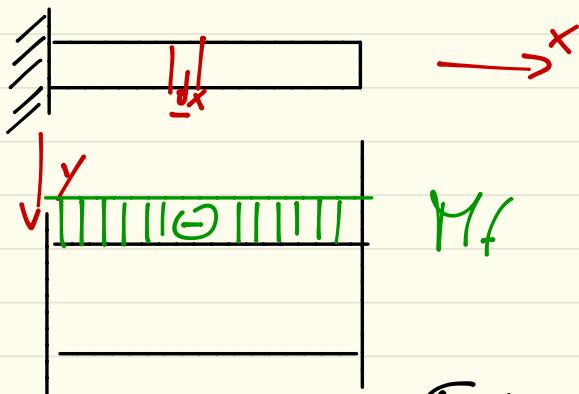
Esercizio 1 esercitazione 4

N.B. ogni punto ha il suo tensore delle tensioni



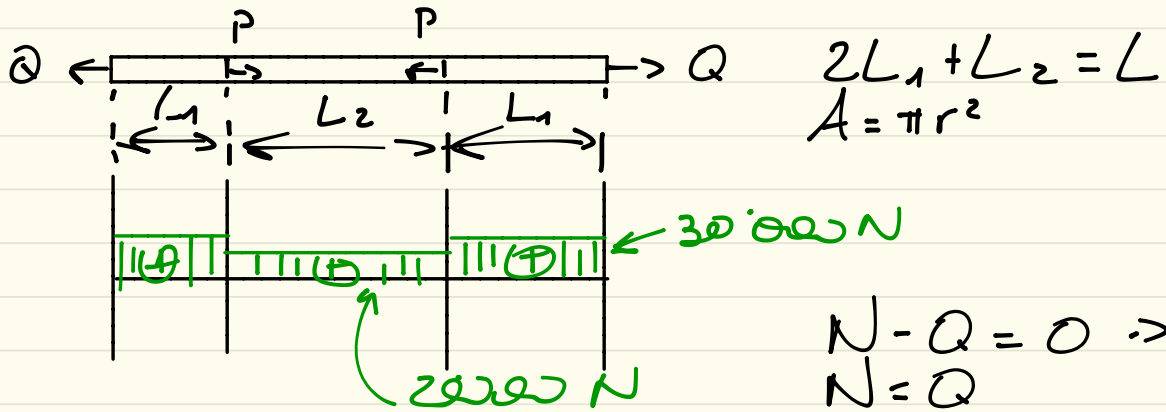
$N = -F$
 $\sigma_x = -\frac{E}{bh} N$

Esercizio 2



$M_2 \ominus$
 M_f
 $\sigma_{x \max} = \frac{-M_2}{bh^3/12} (-y_{\max}) =$
 $= -\frac{M_2}{bh^3/12} \left(-\frac{h}{2}\right) = \frac{6M_2}{bh^2}$
 $\sigma_{x \min} = \frac{-M_2}{bh^3/12} \left(\frac{h}{2}\right) = -\frac{M_2 \cdot 6}{bh^2}$

Esercizio 5



$$N = Q - P = 30000 - 10000$$

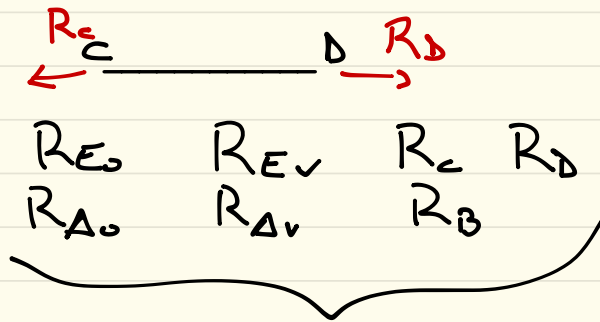
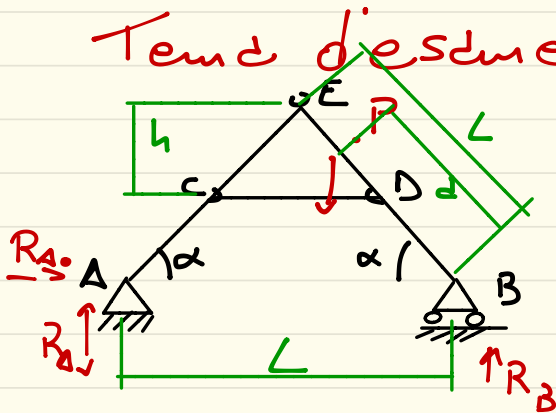
$$\sigma_{max} = \frac{Q}{\pi r^2}$$

$$\Delta L = \frac{N L}{EA}$$

$$\Delta L_{tot} = \frac{2QL_1}{EA} + \frac{(Q-P)L_2}{EA}$$

Tema d'esame

07-02-2013



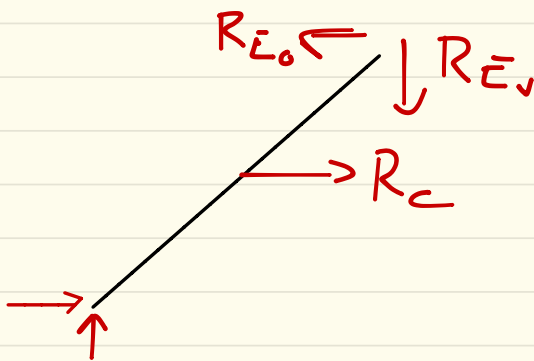
incognite

$$\rightarrow R_{A0} = 0$$

$$\uparrow R_{Av} + R_B - P = 0$$

$$\curvearrowright R_B L - P(L - d \cos \alpha) = 0 \rightarrow R_B = \frac{P(L - d \cos \alpha)}{L}$$

$$R_{Av} = \frac{P d \cos \alpha}{L}$$



$$R_C - R_{E0} = 0 \quad R_C = R_{E0}$$

$$R_{Av} - R_{Ev} = 0 \quad R_{Ev} = \frac{P d \cos \alpha}{L}$$

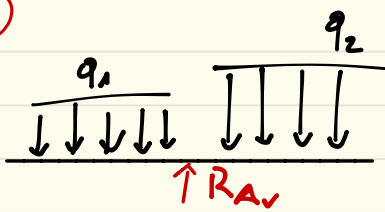
$$\curvearrowright R_C h - R_{Av} \frac{L}{2} = 0$$

$$R_C = R_{Av} \frac{L}{2h} = \frac{P d \cos \alpha}{L} \frac{L}{2h}$$

$$N = R_C$$

$$\sigma_x = \frac{N}{A} = \frac{N}{b^2}$$

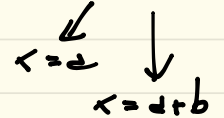
(2)



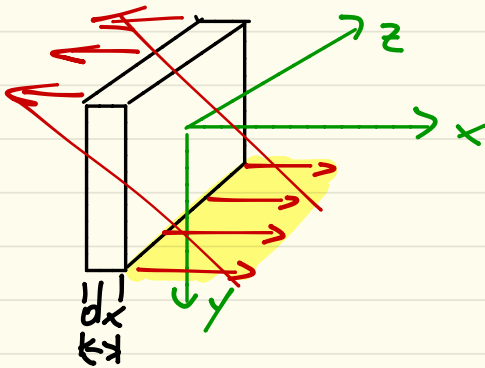
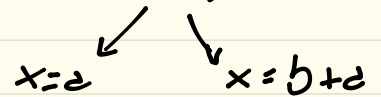
$$N=0$$

$$T + q_1 d + q_2 (x-d) - R_{Av} = 0$$

$$T = q_1 d + q_2 \frac{b}{2} - q_1 d - q_2 (x-d)$$



$$V_f + q_1 d \left(x - \frac{d}{2}\right) + q_2 \left(\frac{x-d}{2}\right)^2 - R_{Av} (x-d) = 0$$



$$\sigma_{x \max} = \frac{M_{f \max}}{W_f}$$

(\rightarrow modulo di resistenza alla flessione
 $W_f = \frac{I}{z}$)

quindi:

$$\varphi_x = \varphi_t = \frac{d\theta}{dl} G I_p$$

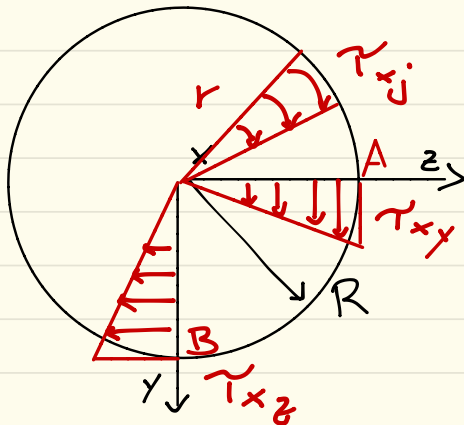
Inoltre:

$$\tau = \frac{\varphi_t}{I_p} \cdot r$$

e ancora:

$$\Delta\theta = \frac{M_t L}{G I_p}$$

ricordiamo che $I_p = \pi D^4 / 32$



A

0	τ_{yx}	0
τ_{xy}	0	0
0		0

B

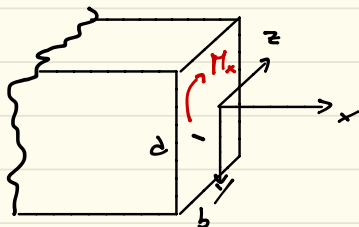
0	0	τ_{zx}
0	0	0
τ_{xz}	0	0

ricordiamo che $\tau_{ij} = \tau_{ji}$

$$\tau_{max} = \frac{M_t \cdot r_{max}}{I_p}$$

$$\tau_{max} = \frac{M_t D/2}{\pi D^4 / 32} = \frac{16 M_t}{\pi D^3}$$

Solidi non Assial simmetrici

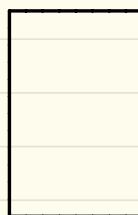


I_T

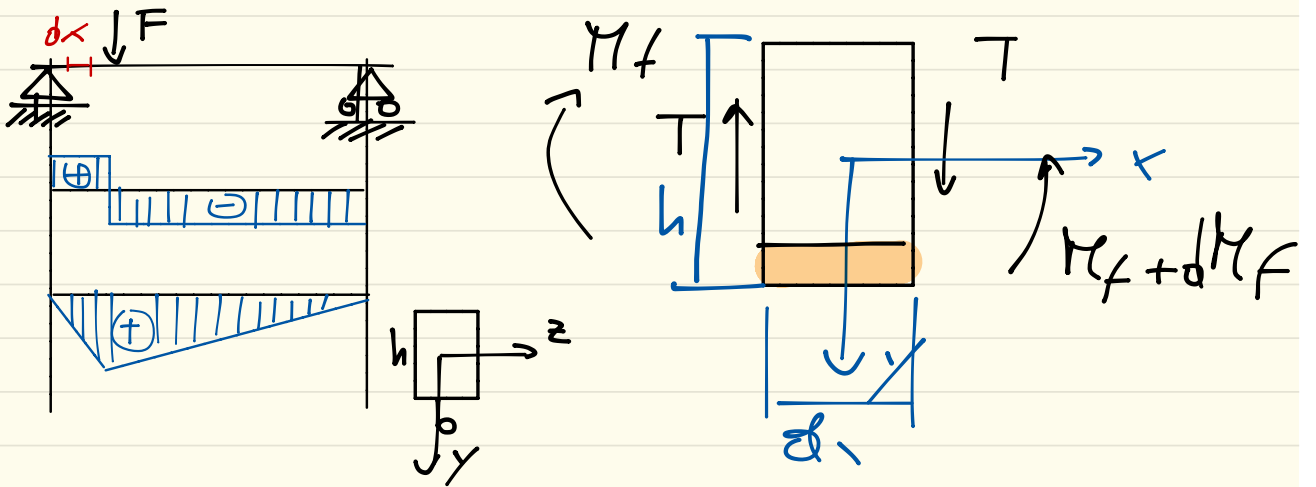
$$\Delta\theta = \beta \frac{M_T}{6cb^3}$$

$$\tau_{max} = \alpha \frac{M_T}{db^2}$$

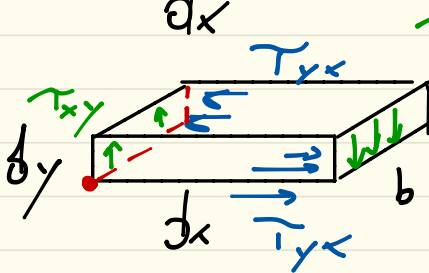
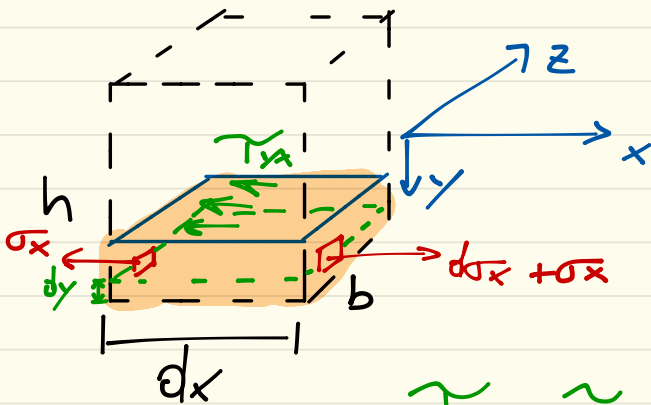
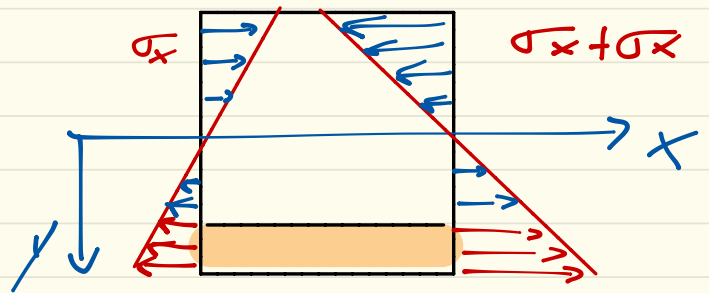
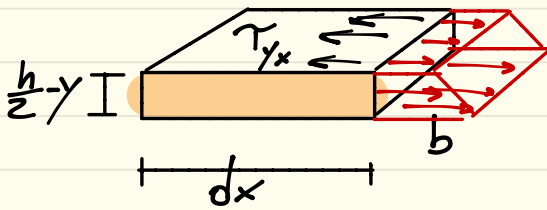
$$\omega_T = \frac{db^2}{\alpha}$$



$n=d/b$	1	1.1	-----	∞
d	1.804			
b				



↑ segue la convenzione delle forze di taglio



$$\tau_{yx} = \tau_{xy}$$

$$\tau_{xy} = \tau_{yx} = \text{costante}$$

