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A P P U N T I

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INTRODUCTION

The things that are observed are the quantities: a quantity is observed through an instrument, which, operating on the quantity gives a real number as response. This operation is called MEASUREMENT.

As an example of quantity are length, time or mass (and so on). Also mathematical operations have been accepted as "objective tools" that can define new quantities. So a given set of quantities $x, y, z...$, every function $f(x, y, z..)$ of $x, y, z..$ can be defined as a new quantity.

We can identify two kind of measurements:

1. DIRECT : the value of the quantity is obtained as a result of an operation of the associated instrument.
2. INDIRECT: $x, y, z,$. are directly measured quantities, then each quantity defined as a mathematical function $f(x, y, z..)$ of x, y, z is called "indirect measurement" (through operations)

DIMENSIONS & UNITS

Each quantity has its own dimension: quantities measurable by the same instruments are called HOMOGENEOUS QUANTITIES; therefore they have the same DIMENSION

6. INTEGRAL $\int f(x) \cdot dx$

$$\left[\int f(x)\, dx\right] = \left[\sum f(x) \Delta x\right] = \left[f(x) \cdot \Delta x\right] = \left[f(x)\right]\left[\Delta x\right] = \left[f(x)\right]\left[x\right]$$

The dimension of every physical quantity can be expressed as a monomial of few fundamental dimensions:

1. LENGTH $[L]$ m
2. TIME $[T]$ s
3. MASS $[M]$ Kg
4. EL. CHARGE $[Q]$ C
5. TEMPERATURE $[\Theta]$ K or C°

ex.

$[v] = \frac{L}{T} = m/s$ $\quad [a] = \frac{L}{T^2} = m/s^2$ $\quad [F] = \frac{M \cdot L}{T^2} = \frac{m \cdot Kg}{s^2}$ $\quad [v] = \sqrt{V_0^2 + gh} = \sqrt{\frac{L^2}{T^2} + \frac{L^2}{T^2}} = \left[\frac{L}{T}\right] = \frac{L}{T}$

ERROR IN MEASUREMENTS

DIRECT ERROR

It is usual that, repeting N measurements of a physical quantity, the results obtained are not all the same. This situation is described assuming that:

each observation is always affected by an error.

There are two kind of it:

1. SYSTEMATIC ERROR:
   it depends on the precision of instruments; so better is the instrument lower will be the error (due to the resolution)

2. STATISTICAL ERROR:
   due to small changes of the quantity that have to be measured, to small differences in the way of using the instrument...

INDIRECT ERROR

In this case we deal with measured quantities $(x, y, z, ..)$ with mean values and errors $\bar{x}+\Delta x$, $\bar{y}+\Delta y$, $\bar{z}+\Delta z, \dots$, then the error is defined as a mathematical function of x, y, z:

$$\Delta f(x,y,z) = \left|\frac{\delta f}{\delta x}\right| \cdot |\Delta x| + \left|\frac{\delta f}{\delta y}\right| \cdot |\Delta y| + \left|\frac{\delta f}{\delta z}\right| \cdot |\Delta z| + \dots$$

Relative error

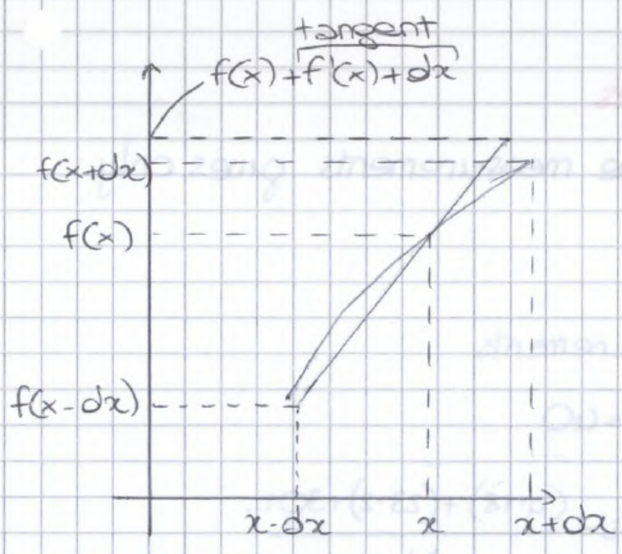
$$\left|\frac{\Delta f(x,y,z,\dots)}{f(x,y,z..)}\right|$$ * (dietro)

ex.

$$E = \frac{1}{2} m v^2 \qquad \Delta E = \left|\frac{\delta E}{\delta m}\right| \cdot \Delta m + \left|\frac{\delta E}{\delta v}\right| \cdot \Delta v$$

$$= \frac{1}{2}\bar{v}^2 \Delta m + \bar{m}\,|\bar{v}|\,\Delta v$$

DIFFERENTIAL OF A FUNCTION



$$\Delta f = f(x+dx) - f(x)$$

$$df = f'(x)\,dx$$

So, in this example, we consider different notes and we group them which are less than the nb of marks: (4)

so $M \leq N$

f_1 = number of marks (in this case) equal to x_1 — x_1 = mark, f_1 = frequency

f_k = " " " " " x_k

$$1 \leq k \leq M$$

So the average mark can be written as:

$$\bar{x} = \frac{1}{N}\sum_{k=1}^{n} x_k = \frac{1}{N}\sum_{k=1}^{M} f_k x_k \; ! \qquad f_k = \text{frequency}$$

$$= (f_1 x_1) + (f_2 x_2) + \dots$$

$\frac{f_k}{N}$ = normalized frequency

$$\bar{x} = \sum_{k=k_{min}}^{k_{MAX}} \frac{f_k}{N} \cdot x_k \qquad \sum_{k=1}^{M} f_k = N \qquad \sum_{k=1}^{M} \frac{f_k}{N} = 1$$

Measurements getting continuous values

Let's consider the case when the quantity assumes continuous values in the range x_{min} & x_{MAX}. If we have small distance (c'è poca diff. tra i valori) between lots of measurements (N is big) we can use the same concept of frequency.

ex. N = very big — x_i = age of a person

N ≃ 6 billion people

We want to divide people depending on their ages in minutes

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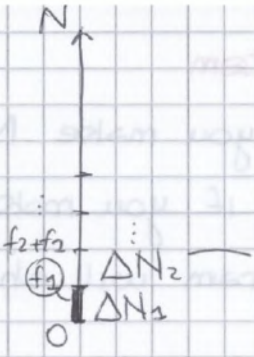
$$\bar{x} = \frac{1}{N}\sum_{k=1}^{M} \frac{f_k}{\Delta x_k}\,\Delta x_k \cdot x_k$$

$$= \sum_{k=1}^{M} \frac{f_k}{N\Delta x_k}\, x_k \cdot \Delta x_k$$

$$= \sum_{k=1}^{M} \left(\frac{\Delta N_k}{N\Delta x_k}\right) x_k \cdot \Delta x_k$$



intervals on the variable N

$\Delta x \to dx$

infinitesimal (d) instead of k, lower bound

$$\int_{x_{min}}^{x_{MAX}} \frac{dN(x)}{N\,dx}\cdot x \cdot dx$$

$\frac{dN}{N\,dx}$ = normalized frequency

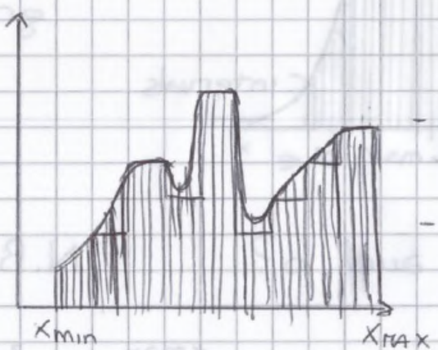
$\frac{dN}{dx}$ = density of frequency = distribution of the variable x

⇓ CONTINUOUS FUNCTIONS

For discrete values:

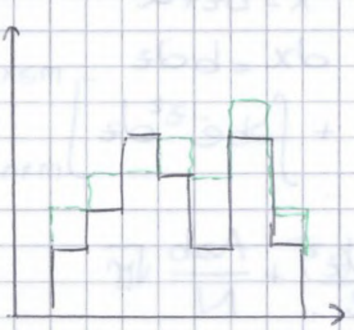
$$\bar{x} = \frac{1}{N}\sum_{k=1}^{M} \frac{f_k}{\Delta x}\cdot \Delta x \cdot x_k$$

$$= \frac{1}{N}\int_{x_{min}}^{x_{MAX}} \left(\frac{dN}{dx}\right)\cdot x \cdot dx$$



N meas.
- I shortened the intervals
- thinner are the intervals, more continuous is the function.

If we add N' to N



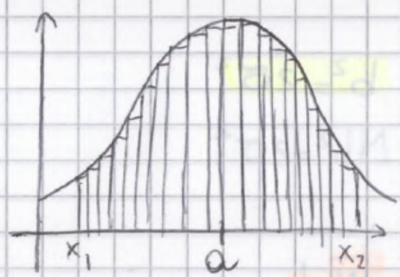
- change each column, the addiction changes the shape of the column

$$\sum_{k=1}^{M} f_k = N \longrightarrow \int_{-\infty}^{+\infty} \frac{dN}{dx} \cdot dx = N$$

$$N = \int_{-\infty}^{+\infty} A \cdot e^{-\frac{(x-a)^2}{b^2}} dx = \int_{-\infty} A e^{-z^2} (b\, dz) = Ab \int_{-\infty}^{+\infty} e^{-z^2} dz = Ab\sqrt{\pi}$$

$$A = \frac{N}{b\sqrt{\pi}}$$

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2 \qquad N(x_1, x_2) = \int_{x_1}^{x_2} \frac{dN}{dx} \cdot dx$$



SUMMARY

$$\bar{x} = \frac{A a b\sqrt{\pi}}{N} = \int_{-\infty}^{+\infty} \frac{dN}{dx} \cdot x \cdot dx$$

$$N = a b\sqrt{\pi} = \int_{-\infty}^{+\infty} \frac{dN}{dx} dx$$

$$\bar{x} = \frac{A a b\sqrt{\pi}}{N} = \frac{a N}{N} = a$$

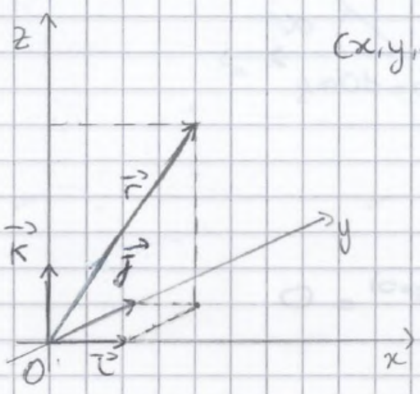
the density of frequency of the function $f(x_i)$ is equal to the one of the independent variable x_i.

$$\frac{dN}{dx} = A \cdot e^{-\frac{(x-\bar{x})^2}{b^2}} \qquad f(x) = (x-\bar{x})^2 \Longrightarrow \sigma^2 = \int_{-\infty}^{+\infty} \frac{dN}{dx} \cdot (x-\bar{x})^2 dx$$

↓

density of frequency of x_i and $f(x_i)$

KINEMATICS



$\vec{r} \equiv x\vec{i} + y\vec{j} + z\vec{k} \iff$ Points (P)

① $\vec{r}(t) \equiv x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k} \iff P(t)$

knowing the displacement as a function of the time means to know the past, present and future position of a physical point; V & a by derivative

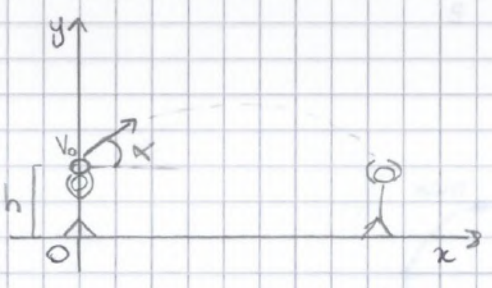
② $\vec{v}(t) \equiv \frac{d\vec{r}(t)}{dt} = \frac{dx(t)}{dt}\vec{i} + \frac{dy(t)}{dt}\vec{j} + \frac{dz(t)}{dt}\vec{k}$

$\equiv V_x\vec{i} + V_y\vec{j} + V_z\vec{k}$

③ $\vec{a}(t) \equiv \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{r}(t)}{dt^2} = \frac{d^2x(t)}{dt^2}\vec{i} + \frac{d^2y(t)}{dt^2}\vec{j} + \frac{d^2z(t)}{dt^2}\vec{k}$

$= \frac{d}{dt}V_x\vec{i} + \frac{d}{dt}V_y\vec{j} + \frac{d}{dt}V_z\vec{k}$

EX. A SOCCER PLAYER THROWING A BALL



$V_0 = 10\,m/s$

$\alpha = 45°$

$h = 2m$

$\vec{a} = -g\vec{j}$

$V_x = \int a_x\,dt + c_1 = V_0\cos\alpha$

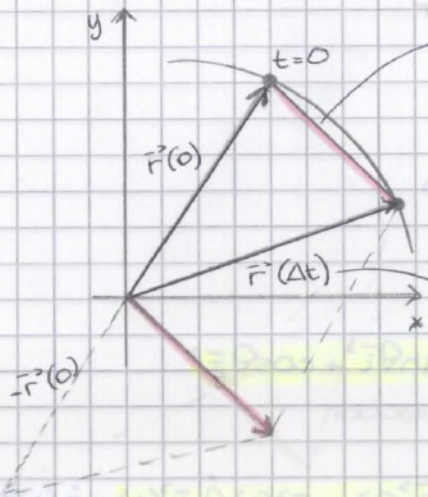
$V_y = \int a_y\,dt + c_2 = -gt + V_0\sin\alpha$

$V_z = \int a_z\,dt + c_3 = 0$

$$\begin{cases} V_x(0) = V_0\cos\alpha \\ V_y(0) = V_0\sin\alpha \\ V_z(0) = 0 \end{cases}$$

Properties of the vector velocity

① The velocity of a point is always tangent to the trajectory



$\vec{r}(\Delta t) - \vec{r}(0)$ it is parallel to the other one, tangent to the trajectory

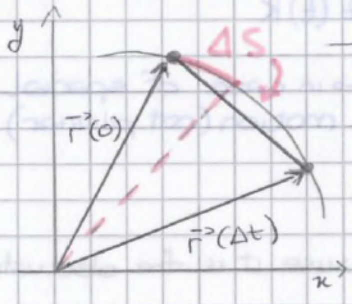
displacement

and if we consider an elapsed time Δt:

$$\lim_{\Delta t \to 0} \frac{\vec{r}(\Delta t) - \vec{r}(0)}{\Delta t} = \frac{d\vec{r}(t)}{dt} = \vec{v}(t)$$

this is tangent to the graph.

② the path of a physical point divided by the elapsed time is the modulus of the velocity (not the velocity itself).



$$\longrightarrow \lim_{\Delta t \to 0} \frac{|\vec{r}(\Delta t) - \vec{r}(0)|}{|\Delta t|} = \lim_{\Delta t \to 0} \frac{|\Delta S|}{|\Delta t|}$$

$$\left| \lim_{\Delta t \to 0} \frac{\vec{r}(\Delta t) - \vec{r}(0)}{\Delta t} \right|$$

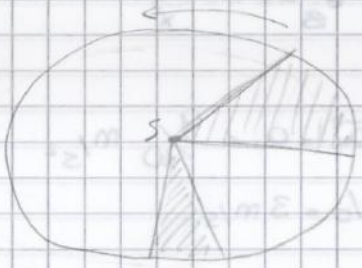
$$|\vec{v}(t)| = \lim_{\Delta t \to 0} \left| \frac{\Delta S}{\Delta t} \right|$$

First Law of Kepler

The orbit of a planet is an ellipse with the Sun at one of the two foci.

Second Kepler's law

If a physical point moves with acceleration having no transverse component then its displacement wipes equal areas in equal time intervals

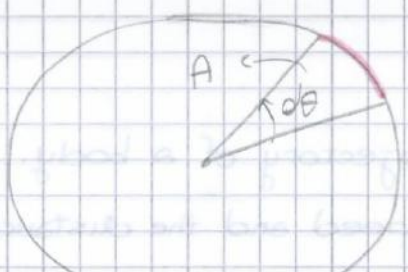


- the areal $\vec{v}$ of a planet is constant because, seen that the orbital is elliptical, in the parts in which the radius is longer, the planets decelerates, in the parts in which the radius is smaller, they accelerate.

$$\vec{a} = (\ddot{r}(t) - \dot{r}\dot{\theta}^2)\vec{\lambda} + (2\dot{r}(t)\dot{\theta} + r(t)\ddot{\theta})\vec{\mu}$$

if we suppose no components in $\vec{\mu}$, the direction will be parallel to $\vec{\lambda}$

$$y: \vec{a} \parallel \vec{\lambda} \Rightarrow (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{\mu} = 0$$



$d\theta$ = infinitesimal area

$$\underbrace{\frac{dA}{dt} = \frac{1}{2} r r \frac{d\theta}{dt}}_{\text{AREAL SPEED}} \quad \text{(it's a triangle)}$$

$$\frac{d\dot{A}}{dt} = \frac{1}{2} 2 r \dot{r} \dot{\theta} + \frac{1}{2} r^2 \ddot{\theta} = \frac{1}{2} r (2\dot{r}\dot{\theta} + r\ddot{\theta})$$

$$= \frac{1}{2} r \cdot a\mu$$

$\Downarrow$

if it is equal to zero, $\dot{A}$ is equal to a constant and it depends on the time.

3) $\vec{n}(t)$ is the unit vector perpendicular to $\vec{\tau}$, positive when directed to the concavity.

4) $\rho(t)$ is the radius of a special circumference ("osculant circle") and it has the same tangent of the trajectory

$$\vec{v} = \dot{s}(t)\,\vec{\tau}(t) \qquad \lim_{\Delta t \to 0} \left| \frac{s(t+\Delta t) - s(t)}{\Delta t} \right| = |\dot{s}(t)|$$

this component because it is tangent to the trajectory

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{s}(t)\vec{\tau}(t) + \dot{s}(t)\dot{\vec{\tau}}(t)$$
$$= \ddot{s}(t)\vec{\tau}(t) + \dot{s}(t)\,a\,\vec{n}$$
$$= \ddot{s}(t)\vec{\tau}(t) + \frac{\dot{s}^2(t)}{\rho(t)}\vec{n}$$

I have to find a formula to represent $\dot{\vec{\tau}}$ as a function of $\vec{\tau}$ & $\vec{n}$.

$1 = \vec{\tau}^2$ (because it is an unit vector)

$$0 = \frac{d}{dt}(\vec{\tau}^2) = \frac{d}{dt}(\vec{\tau}\cdot\vec{\tau}) = \cancel{\frac{d\vec{\tau}}{dt}\cdot\vec{\tau}} + 2\vec{\tau}\cdot\frac{d\vec{\tau}}{dt} \Rightarrow \vec{\tau} \perp \frac{d\vec{\tau}}{dt}$$

vector times its derivative $\perp$

$0 = \vec{\tau}\cdot\dot{\vec{\tau}}$ how can be zero this equation?

$= |\vec{\tau}||\dot{\vec{\tau}}|\cos\alpha$ if $\cos\alpha = 0$

$\vec{\tau} \perp \dot{\vec{\tau}} \Rightarrow \dot{\vec{\tau}} = a\cdot\vec{n}$ $\quad \dot{\vec{\tau}}$ parallel to $\vec{n}$

$|\dot{\vec{\tau}}| = |a|$ number positive or negative

$\vec{\tau}(t+\Delta t) - \vec{\tau}(t)$

$$\frac{|\Delta\vec{\tau}|}{|\vec{\tau}|} = \frac{|\Delta s|}{\rho}$$

modulus, length of the arc

radius

$$\frac{|\vec{\tau}(t+\Delta t) - \vec{\tau}(t)|}{1\,|\Delta t|} = \frac{|s(t+\Delta t) - s(t)|}{|\Delta t|\,\rho(t)}$$

mod of

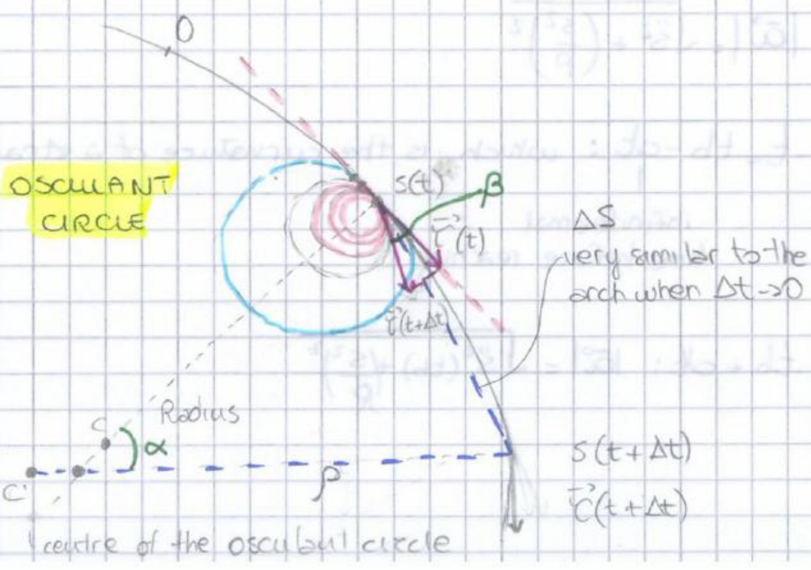
$$|\dot{\vec{\tau}}| = \frac{1}{\rho(t)}|\dot{s}|$$

$$|a| = \frac{1}{\rho(t)}|\dot{s}|$$

radius of the osculant circle

$$\vec{a} = \ddot{s}\vec{\tau} + \dot{s}^2(t)/\rho \cdot \vec{n}$$

OSCULANT CIRCLE



Circular trajectory

R = radius

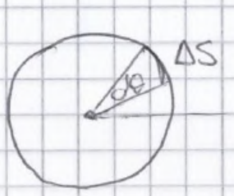
These coordinates are used in the case of a circular trajectory.

Cyl. $\vec{r} = R\vec{\lambda}$ $\quad \vec{v} = R\dot{\theta}\vec{\mu}$ $\quad \vec{a} = -R\dot{\theta}^2\vec{\lambda} + R\ddot{\theta}\vec{\mu}$

parallel to the radius ($\perp \vec{\mu}$)

tangent to the circumference

Intr. $\vec{v} = \dot{s}\vec{\tau}$ $\quad \vec{a} = \ddot{s}\vec{\tau} + \frac{\dot{s}^2}{R}\vec{n}$



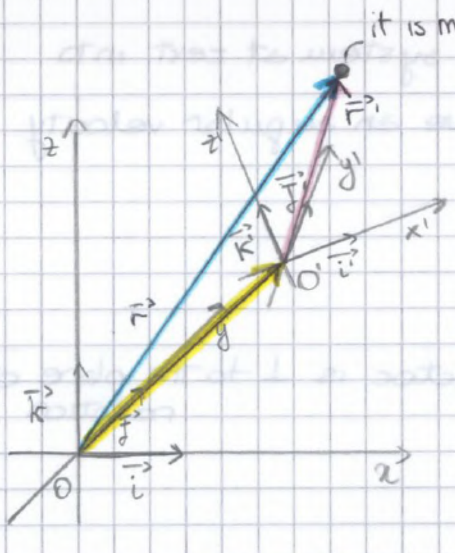
$\Delta S = R \cdot \Delta\theta \qquad R = \frac{\Delta S}{\Delta\theta}$

$\frac{\Delta S}{\Delta t} = R\frac{\Delta\theta}{\Delta t}$ $\qquad \frac{ds}{dt} = R\dot{\theta} = \dot{s} \Rightarrow$ so $\vec{\tau}$ & $\vec{\mu}$ correspond

are becoming smaller

Perfect interaction between cylindrical & intrinsical coordinates.

Relative motion



$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$\vec{r}' = x'\vec{i}' + y'\vec{j}' + z'\vec{k}'$

$\vec{r}_{O'} = x_{O'}\vec{i} + y_{O'}\vec{j} + z_{O'}\vec{k}$

↳ vector displacement of the reference system in motion w.r.t the one which is at rest.

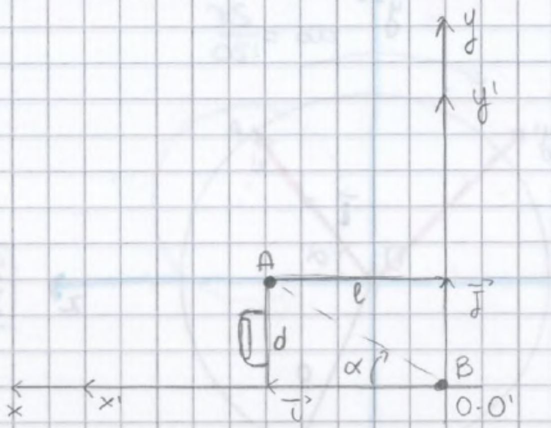
$\vec{r} = \vec{r}_{O'} + \vec{r}'$

$x\vec{i} = x_{O'}\vec{i} + x'\vec{i}'$

$y\vec{j} = y_{O'}\vec{j} + y'\vec{j}'$

$z\vec{k} = z_{O'}\vec{k} + z'\vec{k}'$

EXERCISE



- The man is walking from A to B with a pace of 1 m/s
- The track initially is at rest; then it starts moving with

$\vec{a}_T = bt^2 \quad (b = 0{,}1)$

$v' = 1\,m/s$ velocity of the man seen by the track

$\alpha = \arctan\frac{d}{\ell} \quad d = 5 \quad \ell = 15$

To get the velocity of the track:

$$\int a_T = V_T \qquad V_T = \Big(b\frac{t^3}{3} + \underbrace{C_1}_{=0,\text{ initially at rest}}\Big)\vec{i} \qquad a_T = bt^2$$

$$\vec{a} = \vec{a}_T + \underbrace{\emptyset}_{\text{acc. of the man}} + \underbrace{\emptyset}_{\text{no rotation}} + \underbrace{\emptyset}_{\vec{\omega}} = bt^2\,\vec{i} \longrightarrow \vec{a} = \vec{a}_{O'} + \vec{a}' + 2\vec{\omega}\times\vec{v}' + \dot{\vec{\omega}}\times\vec{r}' + \vec{\omega}\times(\vec{\omega}\times\vec{r}')$$

→ acceleration of the man

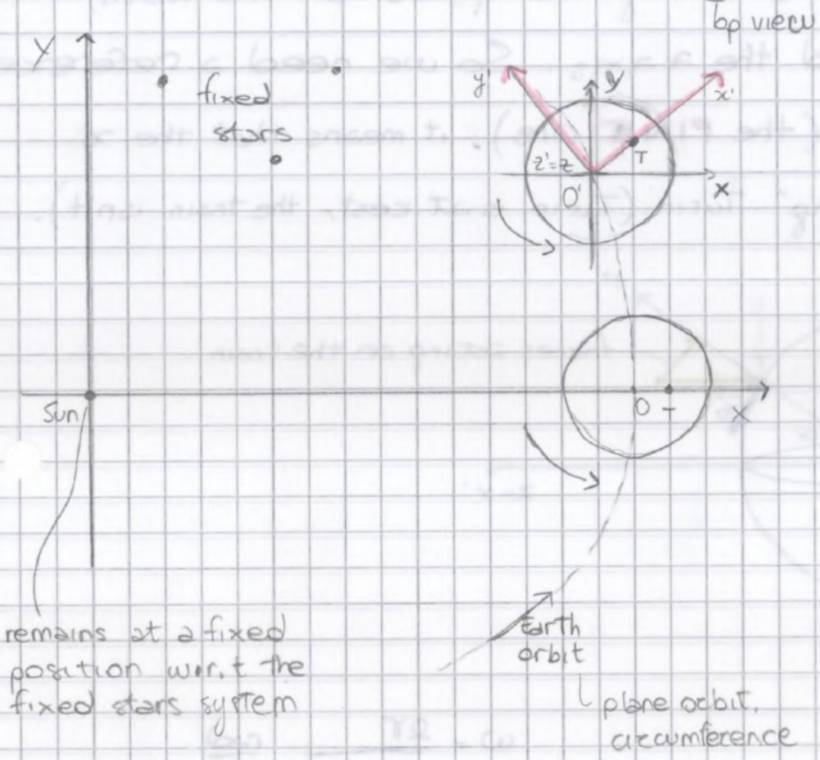
$$\vec{v} = V_T\,\vec{i} + \underbrace{\emptyset}_{\vec{\omega}\times\vec{r}'} - v'\big(\cos\alpha\,\vec{i}' + \sin\alpha\,\vec{j}'\big)$$

we cancel them 'cause they're parallel

we can get the trajectory by integration

Motion on the earth surface

Recalling the last exercise, we add a fixed system which is the "fixed stars" one.



$Y /\!/ y \quad X /\!/ x$

- We relate to the fixed stars system to calculate the acceleration of the moving system.
- Small x, y, z system has an acceleration w.r.t the big one because the trajectory is not a straight line.

The trajectory of the earth is a circumference so we are able to calculate the angular velocity of the centre of the earth:

$$\Omega = \frac{2\pi}{365 \cdot 24 \cdot 3600} = \frac{2}{10.043.312} \simeq \frac{1}{5.021.656} \ \mathrm{rad/s}$$

$$\simeq \frac{1}{1{,}14 \cdot 10^5} \ \mathrm{°/s}$$

In small x, y, z system, we don't have a rotation of the axis and the modulus of the velocity remains constant. If we measure the acceleration of the train on the surface of the earth w.r.t the small x, y, z system, ~~thus~~ we can notice that is the same acceleration that we can calculate in the big X, Y, Z system.

exercise

the train oscillates, so one part will be more consumed than the other because we have a higher force on that side (rails are the ones consumed)

in this case is the right side the one more consumed, because it undergoes a higher pressure

This example means that we can't use the cartesian system without taking into consideration these particularities.

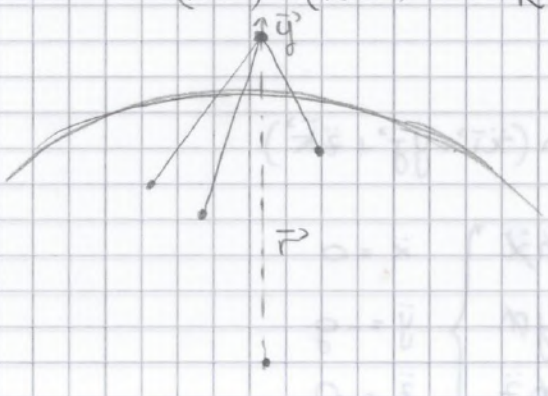
This force appears on an electrically charged physical point 1 when another charged physical point 2 with charge q_2 is at distance r

(3) GRAVITY ON EARTH SURFACE $\vec{F}_g$

This is the gravitational force evaluated on the earth surface, ie in a volume of area few Km 2 and height less than 1 Km. Direction and intensity of the force can be calculated from the F_G, assuming that the forces generated by all points of the earth which attract a point of mass m in the volume is equal to the force generated by a single point in the centre of the Earth containing all the Earth mass M.

$$\vec{F}_G = -\gamma \frac{mM}{(R+h)^2} \cdot \frac{(\overrightarrow{R+h})}{(R+h)} = -\frac{\gamma M}{R^2} \cdot m \cdot \frac{(\overrightarrow{R+h})}{(R+h)} \simeq \pm \frac{\gamma M}{R^2} m \vec{j} \equiv -g m \vec{j}$$

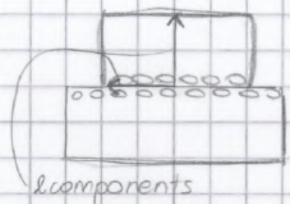
this value is negligible w.r.t R



it is a force acting on every m on the Earth, always toward the centre, always vertical

Forces again

1 NORMAL FORCE



- we have a body over another body which interact through the electrons in contact between the surfaces.

- this force has two components, the one perpendicular to the surface is the normal force ($\vec{N}$), the one along the surface is called the friction force. ($\vec{F_d}$)

2 FRICTION FORCE

- STATIC FRICTION

it acts along the contact plane, when both surfaces are at rest with respect to each other, and is given by:

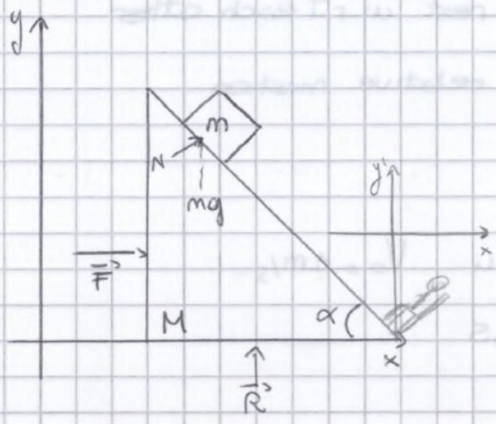
$$\vec{F_s} = \mu_s \cdot N$$

- DYNAMIC FRICTION

it acts along the contact plane, when both surfaces are in relative motion with respect to each other and is given by:

$$\vec{F_d} = -\mu_d N \frac{\vec{v}}{|\vec{v}|}$$

exercise



$M = 2\,Kg \quad m = 1\,Kg \quad \alpha = 30° \quad F = 10N$

$$\begin{cases} m\ddot{x} = N\sin\alpha \\ m\ddot{y} = -mg + N\cos\alpha \end{cases} \qquad \begin{cases} M\ddot{X} = \vec{F} - N\sin\alpha \\ M\ddot{Y} = -Mg - N\cos\alpha + R \end{cases}$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow M\ddot{Y} = 0$$

$$\begin{cases} \ddot{x} = \ddot{x}_{0'} + a'_x \\ \ddot{y} = \underbrace{\cancel{\ddot{y}_{0'}}}_{=0} + a'_y = -\underline{tg}\alpha\, a_{y'} \end{cases} \qquad R = Mg + N\cos\alpha$$

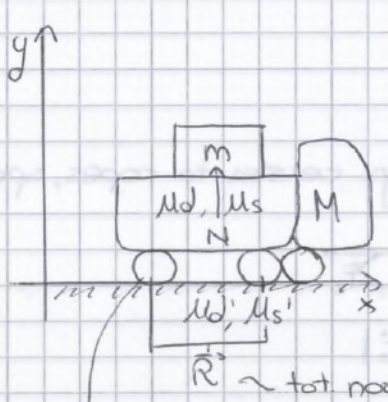
$\vec{v} = v_{x'}\vec{i}' + v_{y'}\vec{j}' \qquad \dfrac{v_{y'}}{v_{x'}} = -tg\alpha$ if we are in the moving reference system and the angle remains constant

$v_{y'} = -tg\alpha\, v_{x'} \implies \dot{v}_{y'} = a_{y'} = -tg\alpha\, a_{x'}$

seen we're considering a moving reference system we have

$$\vec{a} = \vec{a}_0 + \vec{a}'$$

exercise



$m = 100\,kg$ $\quad M = 10t \quad \vec{v}(0) = 0$ at the beginning

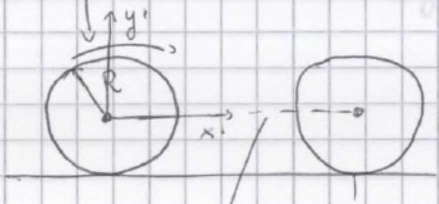
$\mu_d = 0{,}4 \quad \mu_d' = 0{,}5$

$\mu_s = 0{,}5 \quad \mu_s' = 0{,}8$

The wheels rotate and slide

there is more than one point on the wheel is contact with the surface

$\omega R > v \qquad v = \omega R$ NO



$V_{TOT} = -\omega R + V = 0$

velocity of the contact point

$v = \frac{2\pi R}{T} \quad \omega = \frac{2\pi}{T}$ (total rotation)

T: time in which the rotation occurs

$v = \omega R$

$$\begin{cases} M\ddot{X} = \mu_d' R + F_s \\ 0 = M\ddot{Y} = R - Mg - N \\ m\ddot{x} = -F_s \\ 0 = m\ddot{y} = N - mg \end{cases}$$

$\longrightarrow R - mg - Mg = 0$

$\longrightarrow N = mg$

$\ddot{X} = \ddot{x}$

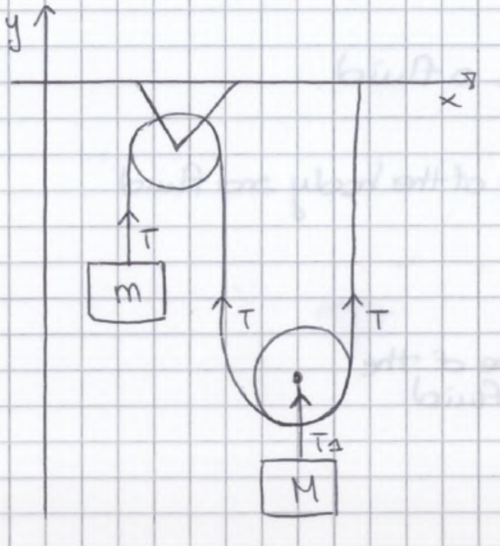
$$\begin{cases} M\ddot{x} = \mu_d' R + F_s \\ R - Mg - mg = 0 \\ m\ddot{x} = -F_s \\ N = mg \end{cases}$$

At the end you can calculate all the unknowns, also F_s. This one will be checked also trough

$|F_s| \leq F_{s\,MAX} = \mu_s N$

to know if it moves or not (m & M)

exercise



$$\begin{cases} m\ddot{y} = T - mg \\ M\ddot{Y} = T_1 - Mg \\ T_1 = 2T \end{cases}$$

$\downarrow$

$\mu\ddot{Y} = 2T - T_1 - \mu g \qquad \lim_{\mu \to 0} \quad 2T - T_1 \Rightarrow T_1 = 2T$

mass of the pulley (which is massless but it proves the equality of T)

To know all the ~~k~~ unknowns we must add another equation:

$$\begin{cases} m\ddot{y} = T - mg \\ M\ddot{Y} = T_1 - Mg \\ T_1 = 2T \\ \ddot{y} = 2\ddot{Y} \end{cases}$$

$\Rightarrow$ for example M increases 1m, m will decrease 2m because of the two T.

$$\left.\begin{cases} 2m\ddot{y} = 2T - 2mg \\ -\frac{M}{2} = 2T - Mg \end{cases}\right\} \text{subtr.} \Rightarrow \left(2m + \frac{M}{2}\right)\ddot{y} = (M - 2m)g \Rightarrow \ddot{y} = \frac{(M-2m)g}{\left(2m + \frac{M}{2}\right)}$$

$$T_1 = 2T \qquad \Rightarrow 2T = Mg - \frac{M}{2}$$

$$\ddot{y} = 2\ddot{Y}$$

if it is negative, it means that the lighter mass is decreasing

ex. $M = 2kg \quad m = 1{,}5 kg$

Another force

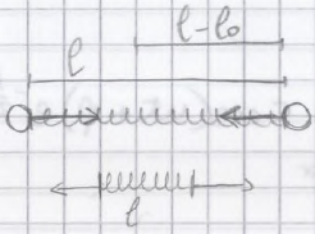
ELASTIC FORCE

It appears on both ends of an "extendible object" when it is elongated or shortened by Δx w.r.t. the "at rest" length

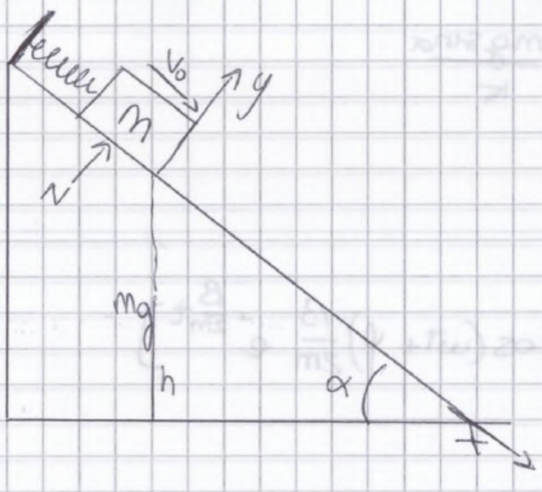
$F_e = -K \cdot (\ell - \ell_0)$ —> proportional to the elongation of the object

ℓ: after the elon. ℓ_0: at rest

K: elastic constant



exercise



$m = 1\,Kg$ $\quad V_0 = 10\,m/s$

$\alpha = 30°$ $\quad h = 2m$

$K = 200\,N/m$ $\quad \beta = 10^{-5}$

I.C.

$x(0) = 0 \quad \dot{x}(0) = V_0$

$$\begin{cases} m\ddot{x} = mg\sin\alpha - Kx - \beta\dot{x} \\ m\ddot{y} = 0 = N - mg\cos\alpha \end{cases}$$

$m\ddot{z} = 0$

$m\ddot{x} + Kx + \beta\dot{x} = mg\sin\alpha$ (diff. equation)

$N = mg\cos\alpha$

$a = m \quad b = \beta \quad c = K$ —> $a\ddot{x} + b\dot{x} + cx = 0$ hom. & then particular (if you obtain complex results, they can't be applied)

HOMOG. SOLUTION (after that, you add the particular one)

In case of >0 $\quad \frac{b^2 - 4ac}{4a^2} = \Delta^2$ $\quad y(t)\, e^{-\frac{b}{2a}t}\left(Ae^{-\Delta t} + Be^{\Delta t}\right)$

<0 $\quad -\frac{(b^2 - 4ac)}{4a^2} = \omega^2$ $\quad y(t) = e^{-\frac{b}{2a}t}\left(A\sin(\omega t) + B\cos(\omega t)\right)$

$=0$ $\quad y(t) = e^{-\frac{b}{2a}t}(A + Bt)$ nearly impossible this case

$= e^{-\frac{b}{2a}t}\left(c \cdot \cos(\omega t + \varphi)\right)$

amplitude decreasing in time

Last force

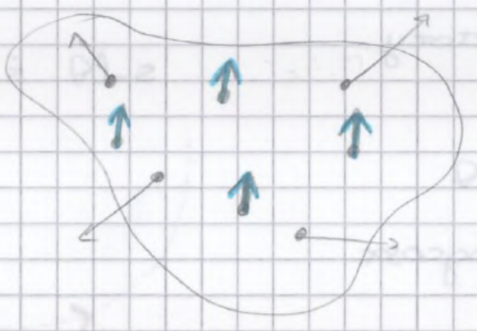
LORENTZ FORCE

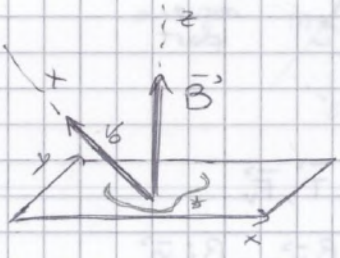
It appears when a charged point of mass m and charge q is moving inside a magnetic field with a velocity $\vec{v}$. $\vec{B}$ is present in every geometrical point of a region of the space.

$\vec{B}$ = magnetic field (in vacuum) → produced by charges in motion
↳ always equal to the same vector

$$\vec{F_L} = q \cdot \vec{v} \times \vec{B}$$

$\vec{B} = B \cdot \vec{k}$





MOTION OF THE PARTICLE:

$$\vec{v_0} = V_{0x}\vec{i} + V_{0y}\vec{j} + V_{0z}\vec{k}$$

$$\vec{r_0} = x_0\vec{i} + y_0\vec{j} + z_0\vec{k}$$

$$\vec{F_L} = q(\dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}) \times B\vec{k} = qB(-\dot{x}\vec{j} + \dot{y}\vec{i})$$

$$\begin{matrix} \dot{x}\cdot \\ \dot{y}\cdot \end{matrix}\begin{cases} m\ddot{x} = qB\dot{y} \\ m\ddot{y} = -qB\dot{x} \end{cases} \quad + \left(\begin{cases} \ddot{x}\dot{x} = \frac{qB}{m}\dot{y}\dot{x} \\ \ddot{y}\dot{y} = -\frac{qB}{m}\dot{x}\dot{y} \end{cases}\right. \qquad \ddot{x}\dot{x} + \ddot{y}\dot{y} = 0$$

RECALLING

$$a = \ddot{x}\vec{i} + \ddot{y}\vec{j} \qquad \vec{v} = \dot{x}\vec{i} + \dot{y}\vec{j}$$

$$v^2 = \dot{x}^2 + \dot{y}^2 \xrightarrow{\text{derivative}} \frac{d(v^2)}{dt} = 2\dot{x}\ddot{x} + 2\dot{y}\ddot{y} = 0$$

modulus change dir not intensity

$$\vec{a} \cdot \vec{v} = \ddot{x}\dot{x} + \ddot{y}\dot{y} = 0 \qquad \vec{a} \perp \vec{v}$$

$$|\vec{F_L}|^2 = q^2B^2\overbrace{(\dot{y}^2 + (-\dot{x})^2)}^{v^2} = q^2B^2v^2$$ proportional to v^2, so it is constant

*In intrinsic coordinates:

$$\vec{v} = \dot{s}\vec{\tau} \rightarrow \dot{s} = v \; ; \quad \vec{a} = \cancel{\ddot{s}\vec{\tau}} + \frac{\dot{s}^2}{\rho} \cdot \vec{n}$$

parallel to the velocity so = 0 because $\vec{v} \perp \vec{a}$

(supposing a circular motion)

ELLIPSE ← ($\vec{B}$ produces the motion of particles with constant velocity)

$$\begin{cases} \theta(t) = \underbrace{A}_{\text{maximum of the path}} \cdot e^{-\frac{\beta}{2m}t} \sin(\omega t + \varphi) \\ \quad \downarrow \\ \beta \text{ gets small values} \\ \quad \downarrow \\ \theta(t) = A \sin(\omega t + \varphi) \\ \dot{\theta}(t) = A\omega \cos(\omega t + \varphi) \end{cases} \Rightarrow \text{I.C.} \begin{cases} \theta(0) = \theta_0 \leq 6° \\ \dot{\theta}(0) = 0 \end{cases}$$

KINETIC ENERGY

It is a scalar quantity defined as:

$$E_K = \frac{1}{2} m (\vec{v})^2 = \frac{1}{2} m v^2 = \frac{1}{2} m (v_x^2 \cdot v_y^2 \cdot v_z^2)$$

If the particle is at rest is equal to zero and it is always positive.

WORK

It is a NOT CONSERVATIVE force

$$dW = \vec{F} \cdot d\vec{r} \equiv \underbrace{(F_x \vec{i} + F_y \vec{j} + F_z \vec{k})}_{\text{components of } \vec{F}} \cdot \underbrace{(dx \vec{i} + dy \vec{j} + dz \vec{k})}_{\text{components of } d\vec{r}}$$

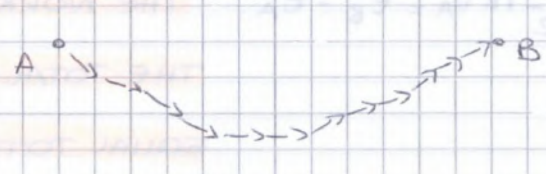
dW: infinitesimal work made by $\vec{F}$

$d\vec{r}$: differential vector displacement

$$\equiv F_x dx + F_y dy + F_z dz$$

$$W_{AB,\gamma} = \int_A^B dW = \int_{A,\gamma}^B \vec{F} d\vec{r} = \int_{A,\gamma}^B (F_x dx + F_y dy + F_z dz)$$

$W_{AB,\gamma}$: from A to B of a particle



γ = trajectory

THEOREM

$$\int_A^B \ddot{x}\, dx = \int_{A,\gamma}^B \frac{d}{dt}(\dot{x})\, \frac{dx}{dt}\, dt = \int_{A,\gamma}^B \frac{d(\dot{x})}{dt} \cdot \dot{x}\, dt = \int_{A,\gamma}^B \dot{x}\, \frac{d(\dot{x})}{dt}\, dt$$

$$= \int_{A,\gamma}^B \dot{x}\, d(\dot{x}) = \left[\frac{\dot{x}^2}{2}\right]_{A,\gamma}^B$$

POTENTIAL ENERGY

$U(x,y,z)$, it is CONSERVATIVE

$$\begin{cases} \dfrac{\partial U}{\delta x} = -F_x \\[2ex] \dfrac{\delta U}{\delta y} = -F_y \\[2ex] \dfrac{\delta U}{\delta z} = -F_z \end{cases}$$

=> Forces have their potential energy.

Let's calculate the pot. energy made by F from A to B.

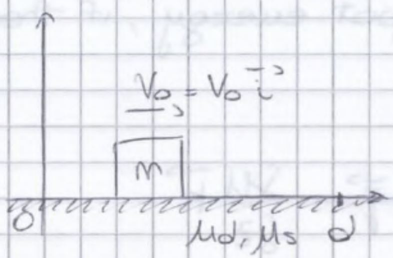
$$\int_{A,\gamma}^B (F_x\, dx + F_y\, dy + F_z\, dz) = -\int_{A,\gamma}^B \frac{\delta U}{\delta x} dx + \frac{\delta U}{\delta y} dy + \frac{\delta U}{\delta z} dz = -\int_{A,\gamma}^B dU$$

$$= U(A) - U(B)$$

example

$\mu_d = 0{,}45$

$v_0 = 3{,}7\ m/s$



(1st) WAY

$$\begin{cases} m\ddot{x} = F_d = -\mu_d N \\ m\ddot{y} = 0 = -mg + N \end{cases} \qquad \begin{cases} m\ddot{x} = -\mu_d mg \\ N = mg \end{cases}$$

$N = mg$

$\ddot{x} = -\mu_d g \qquad \dot{x} = \int \ddot{x} = -\mu_d g t \qquad x = \int \dot{x} = -\frac{\mu_d g t^2}{2}$ and so on..

(2nd) WAY (if you don't want to know everything)

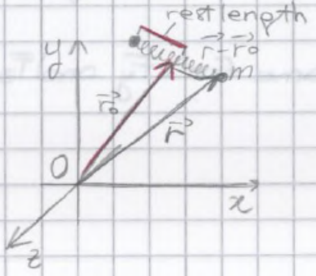
$$W_{0,d} = \int_0^d dW = \int_0^d \vec{F}_d \cdot d\vec{r} = \int_0^d \vec{F}_d \cdot dx = \int_0^d -\mu_d mg\, dx = -\mu_d mg d$$

$$W_{0,d}^{F_d} = E(d) - E(0) = 0 - \frac{1}{2} m v_0^2$$

$$\mu_d mg d = \frac{1}{2} m v_0^2 \qquad d = \frac{v_0^2}{2\mu_d g} = \frac{v_0^2}{2\mu_d g}$$

$$\begin{cases} F_x = 0 & -\dfrac{\delta U_F}{\delta x} = 0 \\ F_y = F & -\dfrac{\delta U_F}{\delta y} = +F \\ F_z = 0 & -\dfrac{\delta U_F}{\delta z} = 0 \end{cases}$$

(3) $\vec{F}_{el} = -K \cdot \text{elongation} \cdot \vec{n}$



$$\vec{F}_{el} = -K \cdot |\vec{r} - \vec{r_0}| \cdot \frac{\vec{r}-\vec{r_0}}{|\vec{r}-\vec{r_0}|}$$

$$= -K \cdot (\vec{r} - \vec{r_0})$$

$$= -K \cdot \left[(x-x_0)\vec{i} + (y-y_0)\vec{j} + (z-z_0)\vec{k}\right]$$

$$U_{el}(x,y,z) = \frac{1}{2} K \cdot \left[\underbrace{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}_{\text{modulus squared of } \vec{r}-\vec{r_0}}\right] = \frac{1}{2} K \cdot d^2$$ IT IS CONSERVATIVE

$$-\frac{\delta U_{el}}{\delta x} = -\frac{1}{2} K \cdot \left[2(x-x_0) + \cancel{0} + \cancel{0}\right] = -K(x-x_0)$$

and the same for the two other components.

If we talk about work:

$$W_\gamma(A,B) = E_B - E_A = W_\gamma(A,B) = W_C^\gamma + W_{NC}^\gamma = U(A) - U(B) + W_{NC}^\gamma$$

$$(E_B + U(B)) - (E_A + U(A)) = W_{NC}^\gamma(A,B)$$ TOTAL MECHANICAL ENERGY

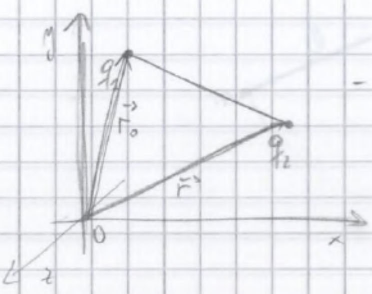
the variation of tot mec. energy = Work of noncons. forces

In case of absence of NON CONSERVATIVE forces, the total mechanical energy is constant

$$E_B + U(B) = E_A + U(A)$$

Pot. energy for culomb force

$$F_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{(\vec{r}-\vec{r}_0)^2} \frac{\vec{r}-\vec{r}_0}{[(\vec{r}-\vec{r}_0)^2]^{1/2}}$$

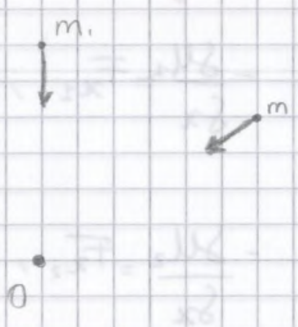


- with q_1 & q_2 with different sign we should keep ⊕ because we get the minus by the product between q_1 & q_2

$$U_C(x,y,z) = \frac{q_1 q_2 (x-x_0)}{4\pi\varepsilon_0 [(x-x_0)^2+(y-y_0)^2+(z-z_0)^2]^{3/2}} = F_x$$

Central forces

$|\vec{F}| = F(r)$



An example of central force is the gravitational one, (the one of the Earth!!) which points always toward the sun.

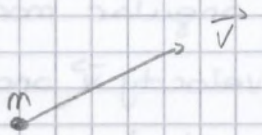
Another one is ~~the~~ culomb force or the elastic one; because they both have a fixed point toward which the force is pointed.

The gravitational force F_g, the one on the earth, is not central because it doesn't point toward a single centre, but always parallel.

F_G = YES F_C = YES F_{el} = YES F_g = NO

LINEAR MOMENTUM (quantità di moto)

$\vec{p} = m \cdot \vec{v}$



It is a vector quantity of a physical point of mass m moving with a velocity $\vec{v}$.

IMPULSE

$\vec{F} \cdot dt = d\vec{I}$

An infinitesimal impulse is defined as the force acting on a physical point in a certain time dt.

$\vec{F} = \vec{F_1} + \vec{F_2}$

$d\vec{I} = \vec{F} \cdot dt = (\vec{F_1} + \vec{F_2})dt = \vec{F_1}dt + \vec{F_2}dt = d\vec{I_1} + d\vec{I_2}$

So if we consider an impulse in a certain time we have:

$$I(t_0, t_1) \equiv \int_{t_0}^{t_1} \vec{F}(t)\, dt$$

$$I_T(t_0, t_1) = \int_{t_0}^{t_1} \vec{F_T}(t)\, dt = \int_{t_0}^{t_1} m \cdot \vec{a}\, dt = m \int_{t_0}^{t_1} \vec{a}\, dt = m \left[\vec{v}\right]_{t_0}^{t_1} = mv(t_1) - mv(t_0) = \vec{p_1} - \vec{p_0}$$

example 1

$m = 1\,kg$

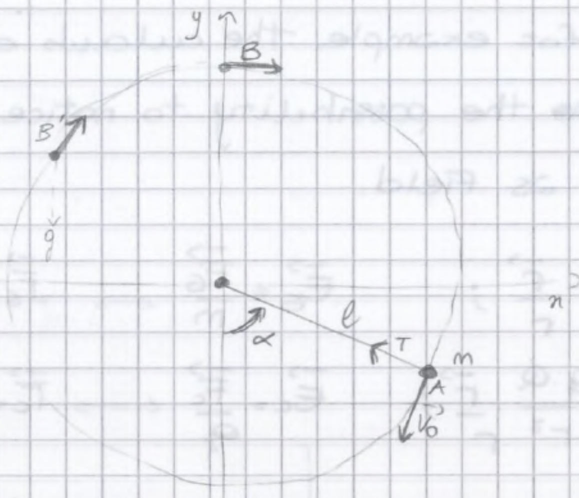
$\ell = 1\,m$

$\alpha = 60°$

$V_0 = 10\,m/s$

$\vec{V_0} = ?$ for which I have a complete round

$\vec{F_v} = 0$



$W_{NC} = E_{KB} + U_B - (E_A + U_A)$

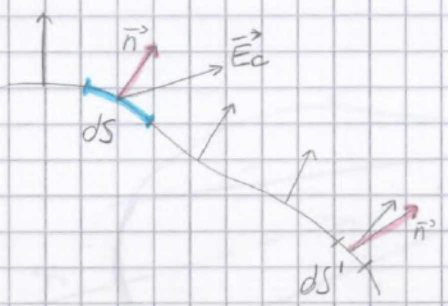
$U + E_{KB} = U_A + E_{AK}$

It can reach point B' if it doesn't have enough velocity to reach B.

$$\left.\begin{array}{ll} E_A^k = \frac{1}{2} m v_0^2 & U_A = -mg\ell\cos\alpha \\ E_B^k = \frac{1}{2} m v^2 & U_B = mg\ell \end{array}\right\} mg\ell + \frac{1}{2} m v^2 = -mg\ell\cos\alpha + \frac{1}{2} m v_0^2$$

$$\begin{cases} \frac{mv^2}{\ell} = mg + T & T = \frac{mv^2}{\ell} - mg \qquad T \geq 0 \\ mg\ell + \frac{1}{2} m v^2 = -mg\ell\cos\alpha + \frac{1}{2} m v_0^2 \end{cases}$$

FLUX OF A FIELD ON A SURFACE



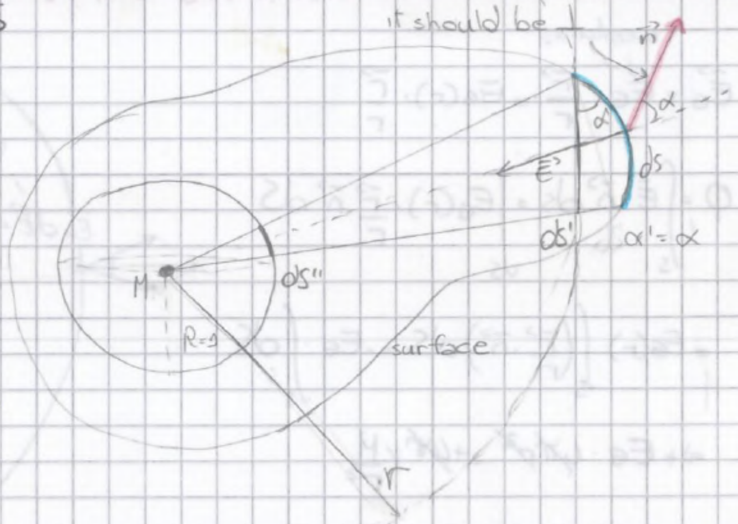
$$d\,\phi_S(\vec{E}) = \vec{E}(x,y,z)\cdot\vec{n}(x,y,z)\cdot dS \quad \text{INFINITESIMAL}$$

$$\phi_S(\vec{E}) = \int_S d\,\phi_S(\vec{E}) \quad \text{finite flux}$$

GAUSS THEOREM (M inside)

$$d\phi = \vec{E}\cdot\vec{n}\cdot dS = E\cdot|\vec{n}|\cos\alpha\, dS$$

$$dS' = dS\cos\alpha$$



$$\begin{cases} \vec{E_G} = -\gamma\dfrac{M}{r^2}\dfrac{\vec{r}}{r} \\ \vec{E_c} = \dfrac{Q}{4\pi\varepsilon_0}\dfrac{1}{r^2}\dfrac{\vec{r}}{r} \end{cases}$$

$$\frac{dS''}{R^2} = \frac{dS'}{r^2}$$

$$\phi_S(\vec{E_G}) = \int_S \vec{E}\cdot\vec{n}\cdot dS = \int_S +\frac{\gamma M}{r^2}\cdot\frac{\vec{r}}{r}\cdot\vec{n}\cdot dS = +\gamma M\int_S \frac{1}{r^2}\underbrace{(-\cos\alpha)}_{\frac{\vec{r}}{r}\cdot\vec{n}}\,dS$$

$$= -\gamma M\int_S \frac{dS'}{r^2} = -\gamma M\int_S \underbrace{\frac{\cancel{r^2}}{\cancel{r^2}}\cdot\frac{dS''}{R^2_{=1}}}_{\text{sum of all spherical pieces}} = -\gamma M\cdot 4\pi \underbrace{R^2}_{=1} = -\gamma M 4\pi$$

① The flux depends on the M, not on the surface

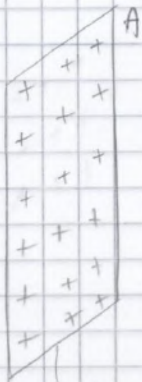
② It doesn't depend on the position of M if the surface is closed.

$$\phi_S(\vec{E_c}) = \frac{Q}{\varepsilon_0}$$

example

dA = infinitesimal area

dq = " charge



planar distribution of el. charges
↓
no thickness, no volume of the mass so no density

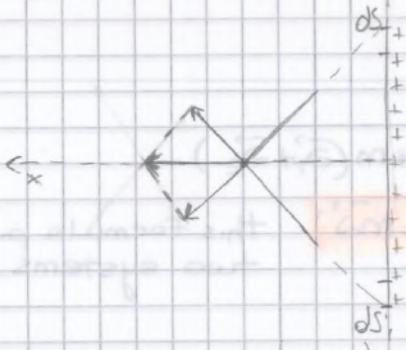
$$\sigma = \frac{dq}{dA}$$

$$Q = \int dq = \int \frac{dq}{dA} \cdot dA = \int \sigma \, dA$$

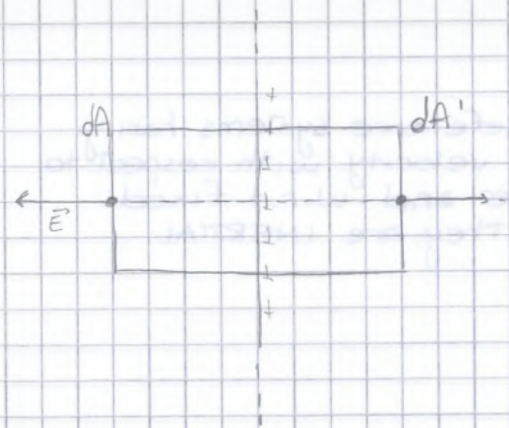
σ = uniform and constant
charge always the same

example

- $\vec{E_c}(x,y,z) = E_c(x,y,z) \cdot \vec{i}$
- It is an infinite plane so the field is always in the same direction, (in) every point we take.



- σ = const
- $\vec{E_c} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\text{pointlike charge}}{d^2} \cdot \vec{n}$



$$\Phi(\vec{E_c}) = \int_A \vec{E} \cdot \vec{n} \, dA = \int_A E_c(x) \cdot \vec{i} \, dA + \int_A \vec{E_c'} \cdot (-\vec{i}) \, dA'$$

$$+ \int_{LS} E_c(x) \cdot \underbrace{\vec{n}}_{\perp} \, dA =$$

lateral surface

$$= E_c(x) \cdot \int \underbrace{\vec{i} \cdot \vec{i}}_{1} \, dA + E_c' \int (-\vec{i})(-\vec{i}) \, dA' + \emptyset =$$

$$= E_c(x) \cdot A + E_c' A' = A \cdot 2 \cdot \vec{E_c}(x)$$

$E_c(x)$ & E_c' are equal in modulus

$$\Phi(\vec{E_c}) = 2\cancel{A} E_c(x) = \frac{1}{\varepsilon_0} \sigma \cdot \cancel{A}$$

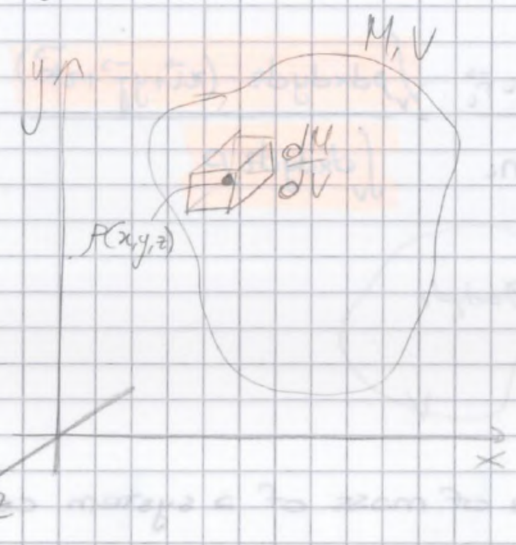
$$E_c(x) = \frac{\sigma}{2\varepsilon_0}$$ doesn't depend on the distance

SYSTEM OF POINTS GENERATING FORCES

In this case we have different points which can generate different forces in the same system. So we have:

$$\rho = \frac{M}{V}$$

ρ: whole system; M: whole mass; V: whole volume

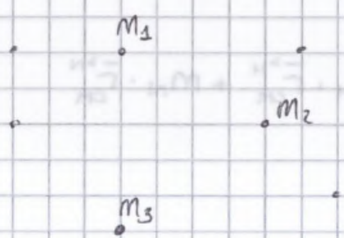


$$\rho(x,y,z) = \frac{dM}{dV}$$

$$dm = \rho(x,y,z) \cdot dV$$

$$dm + dm' + dm'' + \ldots = \int_V dm = \int_V \rho(x,y,z)\, dV = \int_V \rho(x,y,z) \cdot dx \cdot dy \cdot dz$$

SYSTEM OF POINTS



$m_i \; ; \; i = 1, \ldots, N$

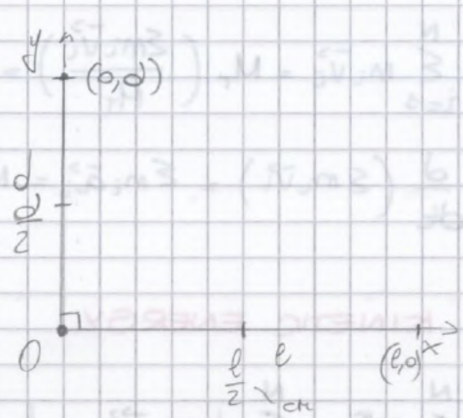
CM = is a point which doesn't belong to the system (usually) and it doesn't have mass (sometimes)

$$\vec{r}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i}$$

$$\begin{cases} x_{CM} = \dfrac{\sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i} \\ y_{CM} = \dfrac{\text{"} \; y_i}{\text{"}} \\ z_{CM} = \dfrac{\text{"} \; z_i}{\text{"}} \end{cases}$$

example

$\underset{\text{density}}{\rho} = \text{const} = 0{,}5 \ kg/m$



$$x_{CM} = \frac{\int_0^\ell x \cdot \rho dx + \int_0^d x dx \rho}{\int \rho dx + \int \rho dy} = \frac{\rho \frac{\ell^2}{2} + \emptyset}{\rho \cdot (\ell + d)} = \frac{1}{2} \frac{\ell^2}{(\ell+d)}$$

$$y_{CM} = \frac{\int_0^\ell y \rho dy + \int_0^d y \rho dy}{\rho(\ell+d)} = \frac{\emptyset + \rho \frac{d^2}{2}}{\rho(\ell+d)} = \frac{1}{2} \frac{d^2}{(\ell+d)}$$

we find the position of the CM

$$\left.\begin{array}{l} m_\ell = \rho \cdot \ell \\ m_d = \rho \cdot d \end{array}\right\} m = m_\ell + m_d$$

$x^\ell_{CM} = \frac{\ell}{2}$

$y^d_{CM} = \frac{d}{2}$

$$x_{CM} = \frac{m_\ell \cdot x^\ell_{CM} + \cancel{m_d} \overset{=\emptyset}{x^d_{CM}}}{m} = \frac{m_\ell}{m} \cdot \frac{\ell}{2} = \frac{\cancel{\rho}\ell}{\cancel{\rho}(\ell+d)} \cdot \frac{\ell}{2} = \frac{1}{2} \frac{\ell^2}{(\ell+d)}$$

$$y_{CM} = \frac{\cancel{m_\ell} \overset{=\emptyset}{y^\ell_{CM}} + m_d \, y^d_{CM}}{m} = \frac{1 d^2}{2(\ell+d)}$$

... CM ...

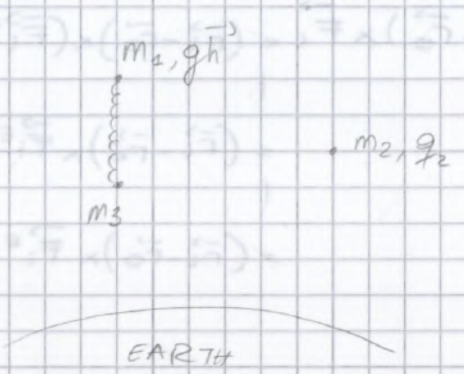
$$\vec{r}_{CM}(t) = \frac{\sum m_i \vec{r}_i(t)}{\underset{\text{total mass}}{M_T}}$$

$$\vec{v}_{CM}(t) = \dot{\vec{r}}_{CM} = \frac{1}{M_T} \cdot \sum m_i \dot{\vec{r}}_i(t) = \frac{1}{M_T} \cdot \sum m_i \vec{v}_i$$

$$\vec{a}_{CM}(t) = \dot{\vec{v}}_{CM} = \frac{1}{M_T} \sum m_i \dot{\vec{v}}_i(t) = \frac{1}{M_T} \sum m_i \vec{a}_i$$

FORCES IN A SYSTEM

Forces can be divided into two groups:



1. **EXTERNAL FORCES**: a force which is outside the system (ex. viscosity, gravity..)

2. **INTERNAL FORCES**: a force which belongs to the system.

$$\vec{F}_i = \sum \vec{F}_i^E + \sum_{j=1}^{N} F_{ij} = \vec{F}_i^E + \sum_{j=1}^{N} F_{ij}$$

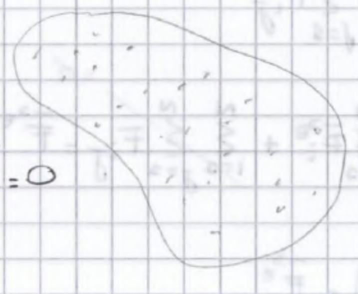
TOTAL FORCES

2 index, internal force from j to i

Considering a system:

$$\vec{F}^T = \sum_{i=1}^{N} \left(\vec{F}_i^E + \sum_{j=1}^{N} \vec{F}_{ij} \right) = \sum_{i=1}^{N} \vec{F}_i^E + \sum_{i=1}^{N} \sum_{j=1}^{N} \vec{F}_{ij} = \vec{F}_T^E + \sum_{pairs} \cancel{(\vec{F}_{ij} \, \vec{F}_{ji})}$$

$=0$ equal and opposite forces



$\vec{F}_{ii} = 0$

$$\vec{L}_O^T = \sum_{i=1}^{N} (\dot{\vec{r}}_i - \dot{\vec{r}}_O) \times m_i \vec{v}_i = \sum_{i=1}^{N} \left[(\dot{\vec{r}}_i - \dot{\vec{r}}_O) \times m_i \vec{v}_i + (\vec{r}_i - \vec{r}_O) \times m_i \vec{a}_i \right] =$$

$$= \sum_{i=1}^{N} \left[(\vec{v}_i - \vec{v}_O) \times m_i \vec{v}_i + (\vec{r}_i - \vec{r}_O) \times \vec{F}_i \right] =$$

$$= -\sum_{i=1}^{N} m_i v_i \times \vec{v}_O + \sum_{i=1}^{N} (\vec{r}_i - \vec{r}_O) \times \vec{F}_i$$

$$= -\sum_{i=1}^{N} v_O m_i \vec{v}_i + \vec{\tau}_O^E = \vec{\tau}_O^E - \vec{v}_O \times \sum m_i \vec{v}_i = \vec{\tau}_O^E - \vec{v}_O \times M_T \left(\frac{\sum m_i \vec{v}_i}{M_T} \right)$$

$$= \vec{\tau}_O^E - \vec{v}_O \times M_T \vec{v}_{CM} = \vec{\tau}_O^E - \vec{v}_O \times \vec{P}_T$$

SECOND CARDINAL EQUATION

TOTAL WORK

Made by the forces in the system.

$$W_T = \sum_{i=1}^{N} W_i = \sum_{i=1}^{N} \Delta E_i^K = \sum_{i=1}^{N} (E_i^{Kf} - E_i^{Ki}) = \sum_{i=1}^{N} E_i^{Kf} - \sum_{i=1}^{N} E_i^{Ki}$$

each single point (we don't distinguish ext. & int. forces)

$$= E_f^{TK} - E_i^{TK}$$

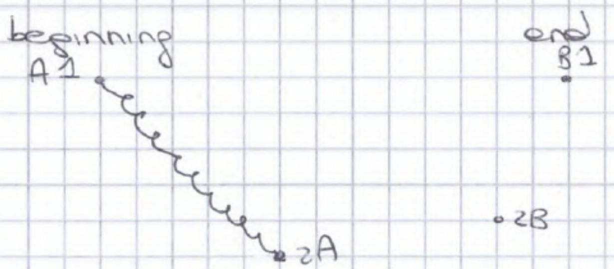
all forces are included

$$W_T^{NC} = (E_f^{TK} + U_f) - (E_i^{TK} + U_i)$$

variation of total mechanical energy

ex.

Let's suppose to have 2 points which are influenced by forces (conservative ones).

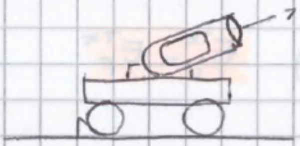


$U = \frac{1}{2} k (\Delta l)^2$ $\qquad$ $U = \frac{1}{2} k (\Delta l')^2$

another lenght

$$U(A_1; A_2) = U(\overline{A_1; A_2}) =$$
$$U(A_2; A_1) = U(\overline{A_2; A_1}) =$$

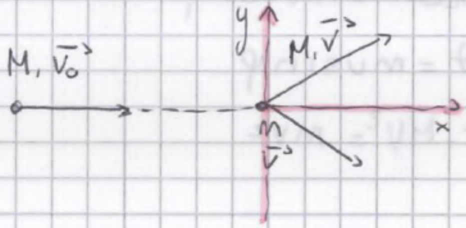
For example considering a canoon:



- the force that push out the bullet is an internal force

COLLISIONS OF TWO POINTS

Happens in the case in which two points interact with each other.



$$\begin{cases} \vec{V_0} = V_0 \vec{i} \\ \vec{V} = V_x \vec{i} + V_y \vec{j} \\ \vec{v} = v_x \vec{i} + v_y \vec{j} + v_z \vec{k} \end{cases}$$

$\vec{p_i} = M\vec{V_0} + m \cdot \emptyset = M V_0 \vec{i}$

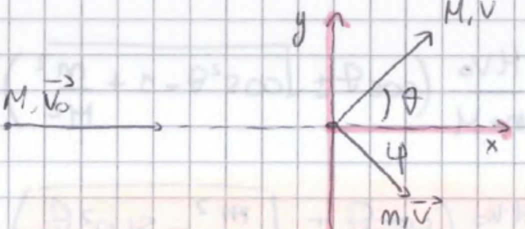
$\vec{p}f = M\vec{V_0} + m\vec{v} = \underline{MV_x \vec{i}} + \underline{MV_y \vec{j}} + \underline{mv_x \vec{i}} + \underline{mv_y \vec{j}} + mv_z \vec{k}$

x) $MV_0 = MV_x + mv_x$

y) $0 = MV_y + mv_y$

z) $0 = mv_z \vec{k}$ $\qquad v_z = 0$

example



$\vec{p_i} = MV_0 \vec{i}$

$\vec{p}_{f_M} = MV\cos\theta \vec{i} + MV\sin\theta \vec{j}$

$\vec{p}_{f_m} = mv\cos\varphi \vec{i} - mv\sin\varphi \vec{j}$

x) $MV_0 = MV\cos\theta + mv\cos\varphi$

y) $0 = MV\sin\theta - mv\sin\varphi$

$W_T = E_f^{TK} - E_i^{TK} = -\frac{1}{2}MV_0^2 + \frac{1}{2}MV^2 + \frac{1}{2}mv^2$

$= \frac{1}{2}MV^2 + \frac{1}{2}mv^2 - \frac{1}{2}MV_0^2$

Considering now 1 dimensional collision:

$$V = \frac{MV_0}{m+M}\left(1 \pm \frac{m}{M}\right)$$

$$= \frac{\cancel{M} V_0}{m+M} \frac{M \pm m}{\cancel{M}} = V_0 \frac{M \pm m}{m+M} \begin{cases} = V_0 \\ = V_0 \dfrac{M-m}{M+m} \end{cases}$$

$MV_0^2 - MV^2 = mv^2$ (to obtain small v)

If, at the end, we consider both masses with the same velocity:

$$MV_0 \vec{i} = (mV_x + MV_x)\vec{i} + (mV_y + MV_y)\vec{j}$$

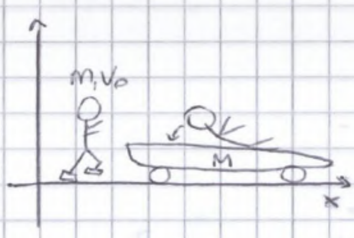
x) $MV_0 = mV_x + MV_x$

y) $0 = (m+M)V_y = 0$

$$E_i^K = \frac{1}{2} MV_0^2$$

$$E_f^K = \frac{1}{2} mV^2 + \frac{1}{2} MV^2 = \frac{1}{2}(m+M)v^2 = \frac{1}{2}(\cancel{m+M})\frac{M^2}{(m+M)^{\cancel{2}}} V_0^2 = \frac{1}{2}\frac{M^2}{m+M} V_0^2 = \frac{1}{2}\left(\frac{M}{m+M}\right) MV_0^2$$

ex. 1



$V_0 = 5 m/s$

$m = 80 Kg$

$M = 20 Kg$

- On x axis we don't have ext. forces so the linear momentum is constant.

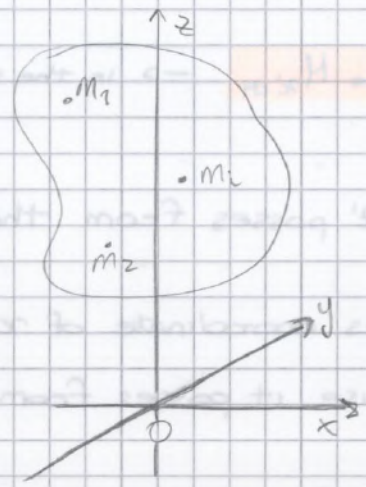
$$\left.\begin{aligned} \vec{P}_m &= m \cdot \vec{V_0}\,\vec{i} \\ \vec{P}_M &= 0 \end{aligned}\right\} \vec{P}^T = mV_0\vec{i}$$

$$\vec{P}^T_{m+M} = [m \cdot (V_0 + V) + MV]\vec{i}$$

$$\cancel{mV_0} = \cancel{mV_0} + mV + MV \quad V = 0$$

RIGID BODY

The rigid body is a body containing a number of points, which number can be both finite or infinite. The distance between two points in the rigid body is constant, not the position. So, in this case, the shape is always the same. What is releated to the shape is the so called:

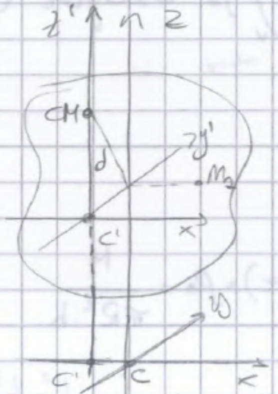


MOMENT OF INERTIA (w.r.t an axis): z

$$I_z = \sum_{i=1}^{N} m_i d_i^2 = \int_V dm \cdot d^2(x,y,z) = \int_V \rho(x,y,z) \cdot dx dy dz \cdot d^2(x,y,z)$$

distance from the axis — cont. distr. — density

ex



$$I_z = \sum_{i=1}^{N} m_i d_i^2$$

THEOREM OF HUYGHENS - STEINER

$$= \sum_{i=1}^{N} m_i (x_i^2 + y_i^2) = \sum_{i=1}^{N} m_i \left[(x'_i + x_{c'})\vec{i}\right]^2 +$$

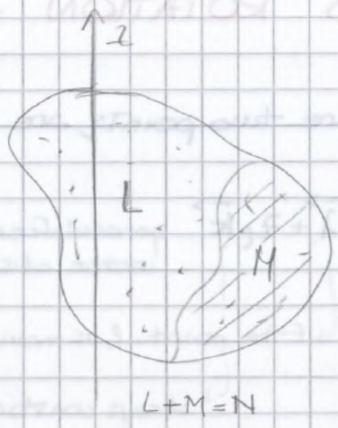
$$+ \left[\left(y'_i + y_{c'}\right)\vec{j}\right]^2 \qquad \vec{r} = \vec{r}' + \vec{r}_{c'} = x_c^2 \vec{i}$$

$$= \sum_{i=1}^{N} m_i \left(x_i'^2 + 2x_{c'} x'_i + x_{c'}^2 + y_i'^2 + 2y_{c'} y'_i + y_{c'}^2\right)$$

$$= \sum_{i=1}^{N} m_i x_i'^2 + 2x_{c'} \sum_{i=2}^{N} m_i x'_i + \left(\sum_{i=1}^{N} m_i\right) x_{c'}^2 + \dots$$

$$= \sum_{i=1}^{N} m_i x_i'^2 + \sum_{i=1}^{N} m_i y_i'^2 = \sum_{i=1}^{N} m_i (x_i'^2 + y_i'^2) = I_{z'}$$

PROPERTIES



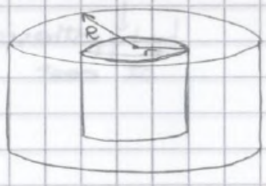
$$I_z = \sum_{i=1}^{N} m_i d_i^2$$

$$= \sum_{i=1}^{M} m_i d_i^2 + \sum_{i=M+1}^{M+L} m_i d_i^2$$

$$= I_z^M + I_z^L$$

ADDITIVE PROPERTY

ex. 1

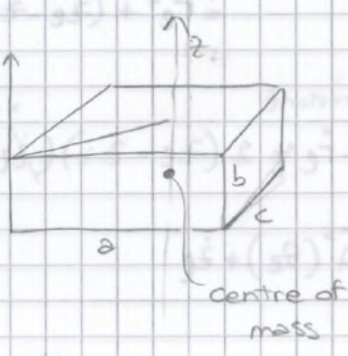


$$I_z + I_z^r = I_z^R$$

$$I_z = \frac{M^R}{2} \cdot R^2 - \frac{M^r}{2} \cdot r^2$$

$M^R = \rho_0 \cdot \underbrace{h \cdot \pi R^2}_{\text{vol. cyl.}}$; $M^r = \rho_0 \cdot 4 \cdot \pi r^2$

ex. 2



$$I_z = \frac{1}{12} M (a^2 + b^2) + M \left[\frac{1}{2} \sqrt{a^2 + c^2} \right]^2$$

ANGULAR MOMENTUM

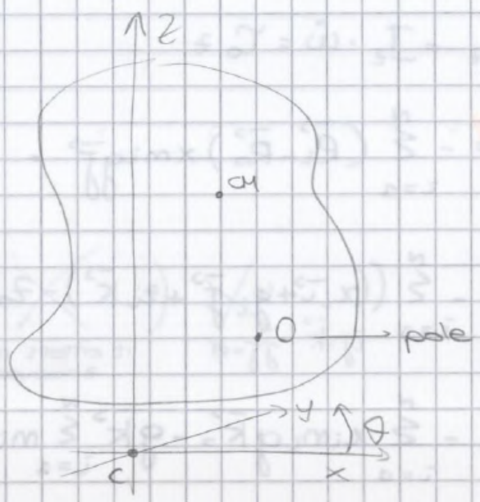
$$\vec{L}_0 = \sum_{i=1}^{N} (\vec{R}_i - \vec{R}_0) \times m_i \vec{v}_i =$$

$$= \sum_{i=1}^{N} \left(r_i \vec{\lambda}(\theta_i) + z_i \vec{k} - r_0 \vec{\lambda}(\theta_0) - z_0 \vec{k} \right) \times$$

$$\times m_i \omega r_i \vec{\mu}(\theta_i) =$$

$$= \left(\sum_{i=1}^{N} m_i r_i^2 \right) \omega \vec{k} - \sum_{i=1}^{N} (z_i - z_0) m_i r_i \omega \vec{\lambda}(\theta_0) +$$

$$+ \omega r_0 \vec{\lambda}(\theta_0) \times \sum_{i=1}^{N} m_i r_i \vec{\mu}(\theta_i)$$



pole on z $\Rightarrow r_0 = 0$

$$\vec{L}_0 = I_z \cdot \omega \cdot \vec{k} - \omega \sum_{i=1}^{N} m_i r_i (z_i - z_0) \vec{\lambda}(\theta_0)$$

on the plane x-y variable in time

$$\vec{\tau}_{0_r} = \frac{d\vec{L}_0}{dt} = \dot{L}_{0r} = I_z \cdot \dot{\omega}$$

$+ \vec{v}_0 \times \vec{P}_T = \emptyset$ because $\vec{V}_0$ is inside the axis

PROPERTY with kinetic energy

$$E^k = \sum_{i=1}^{N} \frac{1}{2} m_i v_i^2 = \sum_{i=1}^{N} \frac{1}{2} m_i \left(r_i \omega \vec{\mu}(\theta_i) \right)^2 = \frac{1}{2} \omega^2 \sum_{i=1}^{N} m_i r_i^2 = \frac{1}{2} I \omega^2$$

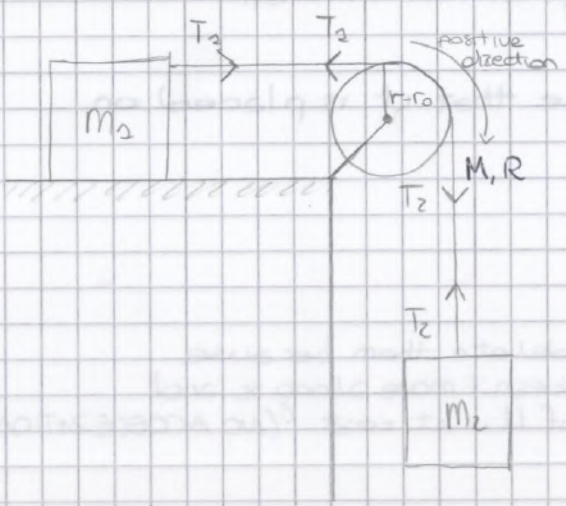
$$\tau_{0z} = I \dot{\omega}$$

$$(\vec{\tau}_0)_z = \left(\sum_{i=1}^{N} (\vec{R}_i - \vec{R}_0) \times \vec{F}_i^{(E)} \right)_z = \left(\sum_{i=1}^{N} (\vec{R}_i - \vec{R}_0) \times \left(\vec{F}_{ix}^{(E)} + \vec{F}_{iy}^{(E)} + F_{iz}^{(E)} \vec{k} \right) \right)$$

perpendicular to $\vec{k}$, so it lays on x-y plane

- I have to take in consideration only those forces which are not parallel to z axis

Exercises about the rotation around a fixed axis



$I = \frac{1}{2} M R^2$

the pulley is homogeneous so the CM corresponds to the rotating axis

$m_1 = 1\,kg$

$m_2 = 1\,kg$

$M = 0{,}2\,kg$

$\mu_s = 0{,}5$

$\mu_d = 0{,}4$

$$\begin{cases} m_1 \ddot{x}_1 = F_s + T_1 \\ m_2 \ddot{y}_2 = m_2 g - T_2 \\ 0 = m_1 \ddot{y}_1 = m_1 g + N \end{cases} \qquad \begin{cases} m_1 \ddot{x}_1 = F_s + T_1 \\ m_2 \ddot{y}_2 = m_2 g - T_2 \\ N = m_1 g \end{cases}$$

$$|\ddot{x}| = |-\ddot{y}| = \ddot{s}$$

$$\begin{cases} I\dot{\omega} = -RT_1 + RT_2 = R(T_2 - T_1) \\ R\dot{\omega} = \ddot{s} \\ N = m_1 g \\ -m_2 \ddot{s} = m_2 g - T_2 \\ m_1 \ddot{s} = m_1 g \mu_s + T_1 \end{cases}$$

you have to check if $|F_s| \leq \mu_s m_1 g$

$\Rightarrow$ if it isn't then m_2 will go down

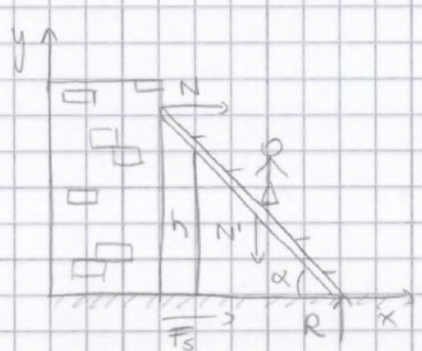
- If we consider τ and angular momentum:

$$\vec{\tau}^{(E)}_{0z} = \dot{\vec{L}}_{0z} = I\dot{\omega} \qquad \vec{\tau}_{1z} = |(\vec{r} - \vec{r}_0) \times \vec{T}_1|_z$$

ext. forces are: - gravity
- both tensions
- normal (but if the axel is very thin, it will be zero)
- friction

- After checking, we will see the behaviour:

exercises to explain the equilibrium



$M = 20\,kg$

$m = 80\,kg$

$l = 6\,m$

$\alpha = 45°$

$h = 4\,m \quad \frac{h}{l'} = \sin\alpha \quad l' = \frac{h}{\sin\alpha}$

$I_{cm} = 60\,kg\,m^2$

$\mu_s = ?$ minimum in order to avoid the sliding down of the stair

$\vec{F}^{(E)} = \vec{N} + \vec{R} + \vec{F_s} + \vec{G} + \vec{N'} = 0$ they must be equal to zero in order to have equilibrium

$\vec{N'} = N\hat{j} = mg$

$$\begin{cases} 0 = \vec{F}_x^{(E)} = N + F_s \\ 0 = \vec{F}_y^{(E)} = R + \underset{Mg}{\underset{\shortparallel}{G}} - mg = R - Mg - mg = R - (M+m)g \end{cases}$$

$0 = \vec{\tau}_t^{(E)} = \cancel{\tau_{F_s}} + \cancel{\tau_R} + \tau_N + \tau_G + \tau_{N'}$

$$\left.\begin{aligned} \tau_N &= lN\sin\alpha \\ \tau_G &= -\frac{l}{2}Mg\cos\alpha \\ \tau_{N'} &= -l'N'\cos\alpha \end{aligned}\right\} \quad 0 = \vec{\tau}_t^{(E)} = lN\sin\alpha - \frac{l}{2}Mg\cos\alpha + l'N'\cos\alpha$$

$N = -F_s$

$R = (M+m)g$

$lN\sin\alpha = \frac{l'\cos\alpha\, mg}{l\sin\alpha} + \frac{l\,Mg\cos\alpha}{2l\sin\alpha}$

$F_s = -N \Rightarrow |-N| \leq \mu_s (M+m)g$

$\frac{|-N|}{(M+m)g} = \mu_{s\,min}$

$$\frac{\frac{l'\cos\alpha\, mg}{l\sin\alpha} + \frac{l\,Mg\cos\alpha}{2l\sin\alpha}}{(M+m)g} = \mu_{s\,min}$$

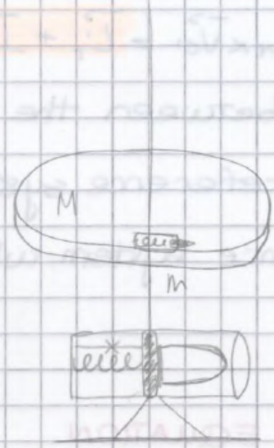
$\mu_{s\,min} = 0{,}17$

exercise

$$\begin{cases} I\omega = mRv \\ \frac{1}{2}Kx^2 = \frac{1}{2}mV_0^2 \\ \quad = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \end{cases}$$

TOTAL ENERGY BEFORE THE SHOOTING

total mech energy



$\frac{1}{2}mx^2 = \frac{1}{2}mV_0^2$

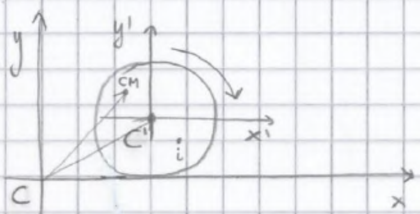
$\omega = \frac{mRV}{I}$ then you substitute in the second equality

$Kx^2 = mv^2 + \cancel{x}\, mRV$

$\sqrt{\frac{Kx^2 - mRV}{m}} = V$ and then, getting the value, you can calculate ω

ROTATION AROUND A NOT FIXED AXIS

The rigid body rotate around an axis, which is moving in the space maintaining the same direction, i.e. remaining parallel to its self.



$\vec{r}_{CM} = \vec{r}'_{CM} + \vec{r}_{C'}$ $\quad \vec{V}_{CM} = \vec{V}'_{CM} + \vec{V}_{C'}$

$\vec{r}_i = \vec{r}'_i + \vec{r}_{C'}$ $\quad \vec{V}_i = \vec{V}'_i + \vec{V}_{C'}$

$$\vec{L}_0 = \sum_{i=1}^{N} (\vec{r}_i - \vec{r}_0) \times \vec{v}_i m_i = \sum(\vec{r}'_i + \cancel{\vec{r}_{C'}} - \vec{r}'_0 - \cancel{\vec{r}_{C'}}) \times m_i(\vec{v}'_i + \vec{V}_{C'})$$
$$= \sum(\vec{r}'_i - \vec{r}'_0) \times (\vec{V}'_i + \vec{V}_{C'}) =$$
$$= \sum(\vec{r}'_i - \vec{r}'_0) \times \vec{v}'_i m_i + \sum(\vec{r}'_i - \vec{r}'_0) \times \vec{V}_{C'} m_i =$$
$$= \vec{L}'_0 + (\sum \vec{r}'_i m_i) \times \vec{V}_{C'} - \sum \vec{r}'_0 \times \vec{V}_{C'} m_i =$$
$$= \vec{L}'_0 + M\vec{r}'_{CM} \times \vec{V}_{C'} - M\vec{r}'_0 \times \vec{V}_{C'}$$

This the general relationship between the angular momentum with respect to a pole O in an inertial frame and a moving (non rotating) frame with parallel axes

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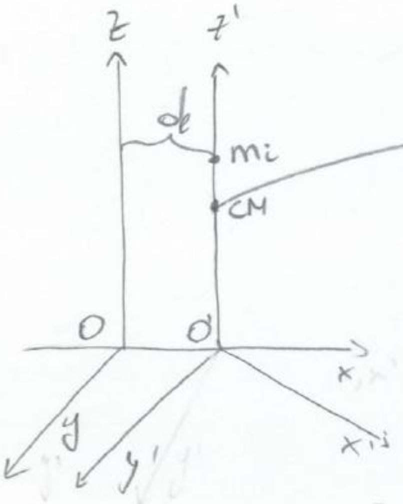
$O = c' \Rightarrow \vec{r}'_0 = \vec{r}'_{C'} = 0$



$x_{c'} = x'_{CM} = \emptyset \rightarrow M(x_{CM} + y_{CM}) = \emptyset$

$I_z = I_{z'}^{CM} + Md^2$

distance i-th point from the axis z

$$I_z = \sum_{i=1}^{N} m_i d_i^2 \quad (1) \text{ DISCRETE DIST. OF P.}$$

if cont. distr. of the masses

$$= \int_V dm \cdot d^2(x,y,z)$$

$$= \int_V \rho(x,y,z) \cdot dx\,dy\,dz \cdot d^2(x,y,z) \quad (2) \text{ CONT. DISTR. OF P.}$$

$\vec{i}' \parallel \vec{i}$

H.S.

$\vec{r} = \vec{r}' + \vec{r}_C = x_c \vec{i}$

$= x'_i \vec{i}' + y'_i \vec{j}' + z'_i \vec{k}' + x_{c'} \vec{i} + y_{c'} \vec{j} + z_{c'} \vec{k}$

$$I_z = \sum_i^N m_i d_i^2$$

$(x-x_0)^2 + (y-y_0)^2$ (PROJECTION), $x_0 = \emptyset$, $y_0 = \emptyset$

$$= \sum_i^N m_i \left(\sqrt{x_i^2 + y_i^2}\right)^2 = \sum_i^N m_i (x_i^2 + y_i^2) = \sum_i^N m_i \left\{\left[(x'_i + x_{c'})\vec{i}\right]^2 + \left[(y'_i + y_{c'})\vec{j}\right]^2\right\}$$

we change the ref. system (at rest w.r.t each other)

$$= \sum_{i=1}^{N} m_i \left(x_i'^2 + x_{c'}^2 + 2x'_i x_{c'} + y'^2 + y_{c'}^2 + 2y'_i y_{c'}\right)$$

not dep on index i

$$= \sum_{i=1}^{N} m_i x_i'^2 + \left(\sum_{i=1}^{N} m_i\right) x_{c'}^2 + 2x_{c'} \sum_{i=1}^{N} m_i x'_i + \sum_{i=1}^{N} m_i y'^2 + \left(\sum_{i=1}^{N} m_i\right) y_{c'}^2 + 2y_{c'} \sum_{i=1}^{N} m_i y'$$

$$= \underbrace{\sum_{i=1}^{N} m_i x_i'^2 + \sum_{i=1}^{N} m_i y_i'^2}_{\sum_i^N m_i (x'_i + y'_i)^2 = I_{z'}} + \underbrace{\left(\sum_{i=1}^{N} m_i\right)(x_{c'}^2 + y_{c'}^2)}_{Md^2}$$

$2(x_{c'} + y_{c'}) \cdot M_T (x' + y')$

$\sum m_i x_i = M x_{CM}$ w.r.t new ref syst

$\sum m_i y'_i = M y_{CM}$

d: distance between old/new axis

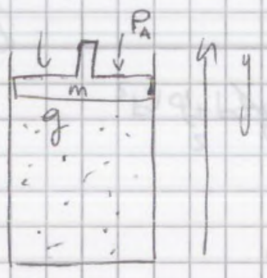
$$= \cancel{l_0 \pi r_0^2} + 2 l_0 r_0 \Delta r + l_0 \Delta r^2 + \Delta l \pi r_0^2 + \Delta l \pi 2 r_0 \Delta r + \Delta l \pi \Delta r^2 - \cancel{l_0 \pi r_0^2} =$$

This means that measuring the difference of length of the rope you get the temperature

GAY-LUSSAC LAW

$\Delta V = \text{constant} \cdot \Delta \Theta$ $\qquad p = \text{const.}$

It's a gaseous thermometer



$$V(\Theta) - V(0) = \text{const}(\Theta - 0)$$

$$V(\Theta) = V(0) + \text{const} \cdot \Theta$$

$$V(\Theta) = V(0)\left(1 + \frac{\text{const}\ \Theta}{V(0)}\right)$$

$$= V(0)(1 + \alpha \cdot \Theta)$$

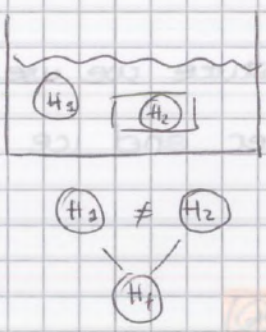
$$\alpha = \frac{1}{273.15}$$

$$P_G \cdot S - mg - P_A S = 0 \text{ if in equilibrium}$$

↳ lower limit which is 273,15 otherwise we get a negative value for the volume which is impossible

0^{th} LAW

$$C_1 \cdot (\Theta_1 - \Theta_f) + C_2(\Theta_2 - \Theta_f) = 0$$



C_1, C_2: thermal capacities

$$\frac{C_1 \Theta_1 + C_2 \Theta_2}{C_1 + C_2} = \cancel{(C_1 + C_2)}\, \Theta_f$$

Capacities depend on the volume and the material; there is also a relation between the mass and the capacities;

$$\underbrace{C = c}_{\text{specific heat}} \cdot m$$

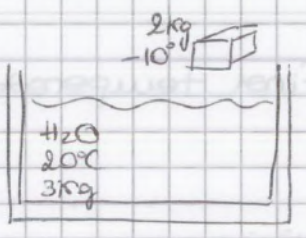
The 0^{th} law ~~can~~ can be written also as:

$$\underbrace{C_2(\Theta_2 - \Theta_f)}_{\text{higher T}} = \underbrace{C_1(\Theta_f - \Theta_1)}_{\text{lower T}}$$

ex.

H_2O at 20°
↳ 3 kg

2 kg ice at -10°



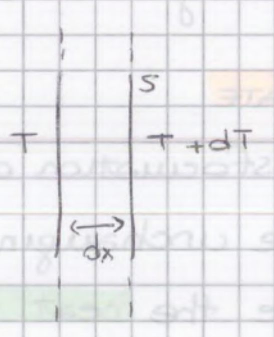
Heat transfer

Heat flows from the points at higher T to the adjacent points at lower T in the space, in order to maintain equilibrium. There are three kinds of heat transfer: internal, external transfer & radiation.

① INTERNAL TRANSFER

dx = thickness

How much heat flows from one side to the other?



$$dH \propto dt$$

$$\frac{dH}{dt} = -\cancel{dt} \cdot S \cdot \frac{dT}{dx} \cdot K \quad \text{FOURIER LAW}$$

$$(x,t)\frac{dH}{dt} = -k\,S\,\frac{dT}{dx}(x,t)$$

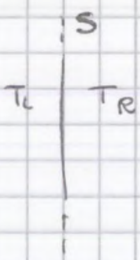
If we consider STD conditions (heat is the same in all positions)

$\frac{dH}{dt}$ doesn't depend on x

② EXTERNAL TRANSFER

$$\frac{dH}{dt} = h \cdot \cancel{dt} \cdot S \cdot (T_L - T_R)$$

h: newton coefficient



$T_L > T_R$ the heat flows toward right

$$\Rightarrow H_1 = H_E + B e^{-\frac{KA}{SC}t} = H_E + \left[H_0 - H_E\right] e^{-\frac{KA}{SC}t} = 25°$$

$$H_I(t=0) = H_0 \qquad H_0 = H_E + B \rightarrow B = H_0 - H_E$$

Thermodynamics

Before starting this topic we must introduce a new quantity:

CALORIC POWER = $\frac{\Delta H}{m}$ amount of heat produced by burning 1 kg of a material

it depends on the material

$\Delta H = m \cdot c \cdot \Delta H$

specific heat

$V = \text{const} \rightarrow c = c_v$

$P = \text{const} \rightarrow c = c_p$

they're different in case of gasses

Now we can begin the "title" topic. In thermodynamics we refer to three variables: P, V, T. When these 3 variables are defined, we can say that the system is in EQUILIBRIUM.

These variables can depend on time $\Rightarrow P(t), V(t), T(t)$, and they can related to each other through these equations:

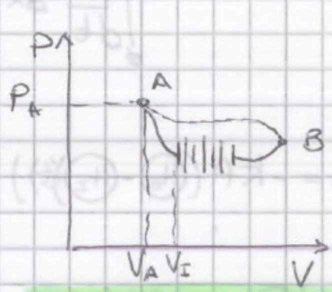
$f(V, P, T) = 0$ ex. $\ln P = \frac{V}{T^2} \rightarrow \ln P - \frac{V}{T^2} = 0$

$PV = nRT$

As a reference system we have:

$f(V_A, P_A, T) = 0 \rightarrow T_A$

We can get a transformation from one eq. point to another:



CLAPEYRON - PLANE

$f(P_A + dP_A, V_A + dV_A, T_A + dT_A) = 0$ it changes of an infinitesimal amount

1. REVERSIBLE TRANSF.

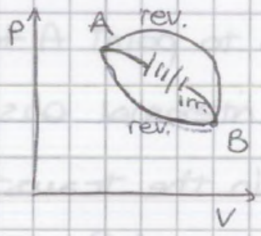
when the transformation has infinitesimal changes it can come back

(it occurs slowly)

Recalling:

(p, V, T) $\quad f(p, V, T) = 0$

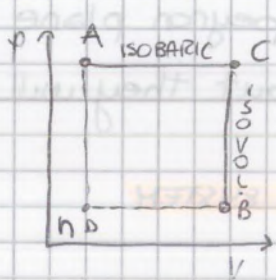
ΔH = exchanged heat
ΔW = work made by the system
↳ if the system expands, the work that is internal will be positive, the external one will be negative.



ex.

ISOBARIC → same pressure

ISOVOLUMIC → same volume



with this example we demonstrate that the exchange heat depends on the path.

ACB)

$$\Delta W_{ACB} = \int_A^C p\,dV + \int_C^B \cancel{p\,dV}^{=\emptyset} = p\int_a^c dV = p\,(V_c - V_a) = p_A(V_B - V_a)$$

ADB)

$$\Delta W_{ADB} = \int_A^D \cancel{p\,dV} + \int_D^B p\,dV = p\int_D^B dV = p(V_B - V_D) = p_B(V_B - V_A)$$

ACB) variation of heat

specific heat w.r.t. the moles

$$\Delta H_{AC} = n \cdot C_p^n \cdot (T_C - T_A)$$

$$\Delta H_{CB} = n \cdot C_V^n \cdot (T_B - T_C)$$

⇓

$$\Delta H_{AC} = \frac{n \cdot C_p^n}{nR}(P_C V_C - P_A V_A)$$

$$\Delta H_{CB} = \frac{n C_V^n}{nR}(P_B V_B - P_C V_C)$$

ADB)

$$\Delta H_{AD} = n C_V^n (T_D - T_A)$$

$$\Delta H_{DB} = n C_p^n (T_B - T_D)$$

⇓

$$\Delta H_{AD} = \frac{n C_V^n}{nR}(P_D V_D - P_A V_A)$$

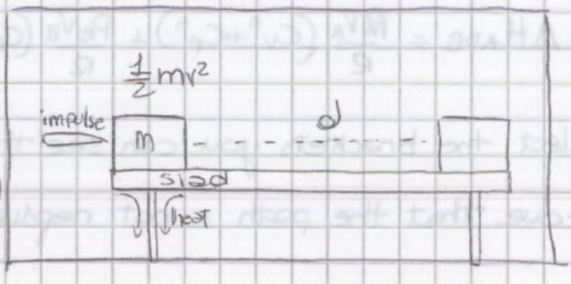
$$\Delta H_{DB} = \frac{n C_p^n}{nR}(P_B V_B - P_D V_D)$$

$$\frac{C_p^n}{R} P_A V_B - \frac{C_p^n}{R} P_A V_A + \frac{C_V^n}{R}(P_B V_B) - \frac{C_V^n}{R} P_A V_B = \Delta H_{ACB}$$

$$\frac{C_V^n}{R} P_B V_A - \frac{C_V^n}{R} P_A V_A + \frac{C_p^n}{R} P_B V_B - \frac{C_p^n}{R} P_B V_A = \Delta H_{ADB}$$

⇓

ex.



- It's an adiabatic system, so we don't have exchange of heat (no variation of entropy)
- In this case heat is produced through friction between m and the slab
- All the walls are mirror, in this way all the radiation produced will turn back.
- $W = \frac{1}{2}mv^2$ and, in this way, the friction work turns into heat.

ex. with ideal gases

$n = 1\,mol$ $\quad PV = RT$

$V = const$

$\delta H = p\cancel{dV} + dU(T)$

$C_v = \frac{\delta H}{dT} = \frac{dU(T)}{dT}$

in monoatomic gasses: $C_v = \frac{3}{2}R$ $\quad C_p = \frac{5}{2}R$

in biatomic gases $C_v = \frac{5}{2}R$ $\quad C_p = \frac{7}{2}R$

in triatomic gases $C_v = \frac{7}{2}R$ $\quad C_p = \frac{9}{2}R$

in n^{th}-atomic gases $C_v = \frac{1+2n}{2}R$

$P = const$ $\quad P = \frac{RT}{V}$

$\delta H = pdV + dU(T) = pdV + C_v dT$ $\qquad U = \int C_v dT$ (equal in all transf.)

$C_p = \frac{\delta H}{dT} = p\frac{dV}{dT} + C_v = \frac{\cancel{P}R}{\cancel{P}}\frac{\cancel{dT}}{\cancel{dT}} + C_v = R + C_v$ MAYER THEOREM