NUMERO: 1354 ANNO: 2014

A P P U N T I

STUDENTE: Preatto

MATERIA: Fisica I Eserc., Prof.Scarfone

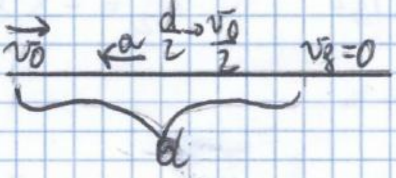
PROBLEMI DI MECCANICA - MAZZOLDI - CINEMATICA

1.1 $v_0 > 0$

$a < 0 \qquad v_f = 0$

d

A $\frac{d}{2} \rightarrow \frac{v_0}{2} = v_f$



① a costante $\quad v_f^2 - v_0^2 = 2ad \qquad 2a = -\frac{v_0^2}{d}$

$x = \frac{d}{2} \qquad v_f^2 - v_i^2 = 2a\frac{d}{2} \mapsto v^2 = \frac{v_0^2}{2} + 2\frac{d}{2}\cdot\frac{v_0^2}{d} = \frac{3}{2}v_0^2$

② $a = -kv \qquad v_f^2 - \frac{v_0^2}{2} = 2a\frac{d}{2} \mapsto v^2 = v_0^2 - kv\frac{d}{2} = \frac{v_0}{2}$

1.2. $T = 4{,}6\ s$

$v_s = 340\ m/s \qquad h_{pozzo}?$

$v_0 = 0$

g cost $= 9{,}8\ \frac{m}{s^2}$

TEMPO DI CADUTA $t_c = \sqrt{\frac{2h}{g}}$ TEMPO DI SALITA $t_s = \frac{h}{v_s}$

$T = t_c + t_s$

$T = \sqrt{\frac{2h}{g}} + \frac{h}{v_s} \qquad \sqrt{\frac{2h}{g}} = T + \frac{h}{v_s} \qquad T^2 + \frac{h^2}{v_s^2} - \frac{2hT}{v_s} = \frac{2h}{g}$

$\frac{h^2}{v_s^2} - \frac{2h(gT + v_s)}{g v_s} + T^2 = 0 \qquad hg^2 + 2hg(v_s + gT) - T^2 v_s^2 g = 0$

$$h_{1,2} = \frac{+2hv_s \pm \sqrt{v_s(2v_s + gT)^2 + 4(g^2)(v_s^2)(T^2)}}{2g} =$$

$$= \frac{+(2v_s + gT) \pm \sqrt{v_s^2 + 2gTv_s}}{2g} = 99{,}6\ m$$

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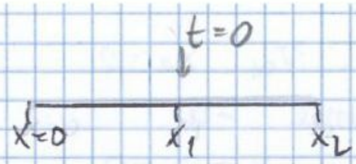
1.4. MUA



$v_1 = 1,9\ m/s$

$x_2 = x_1 + \Delta x \qquad v_2 = 8,2\ m/s$

$\Delta x = 10\ m = x_1 - x_2$

$x_2 = x(t) = x_1 + v_1 t + \frac{1}{2} a t^2 \qquad v_1(t) = \frac{dx}{dt} = v_1 + at$

t^* ISTANTE IN CUI PASSA PER x_2

$x(t^*) = x_2 + v_2 (t - t^*) + \frac{1}{2} a (t - t^*)^2$

$\begin{cases} x_2 - x_1 = v_1 t^* + \frac{1}{2} a t^{*2} \\ v_1 + a t^* = v_2 \end{cases} \leftarrow$ MRUA

$\begin{cases} t^* (v_1 + \frac{1}{2} a t^*) = \Delta x \\ a t^* = v_2 - v_1 \end{cases}$

$t^* \left(v_1 + \frac{1}{2}(v_2 - v_1)\right) = \Delta x$

$\begin{cases} t^* = \dfrac{2 \Delta x}{v_2 + v_1} \\ a = \dfrac{v_2 - v_1}{t^*} \end{cases}$

$t^* = \frac{2 \cdot 10\ m}{(1,9 + 8,2) \frac{m}{s}} = 1,98\ s$

$a = 1 : \frac{1,98\ s}{(8,2 - 1,9) \frac{m}{s}} = 3,18\ \frac{m}{s^2}$

1.7. Moto armonico

$T = 4,4\ s$

$t(0)$ $\quad x(0) = 0,28\ m$

$v(0) = -2,5\ \frac{m}{s}$ $\qquad v_{max}?\quad a_{max}?$

Eq del moto $y(t) = A\,sen\,(\theta + \cancel{\omega t})$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4,4\ s} = 1,43\ rad/s$$

$$\begin{cases} x(0) = A\,sen\,\theta \\ v(0) = +\omega A \cos\theta \end{cases} \qquad \begin{cases} x(0) = A\,sen\,\theta \\ A = \dfrac{v(0)}{\omega\cos\theta} \end{cases}$$

$$\begin{cases} x(0) = \dfrac{sen\,\theta}{\cos\theta}\,\dfrac{v(0)}{\omega} \\ A = \dfrac{v(0)}{\omega\cos\theta} \end{cases} \qquad tg\,\theta = x(0)\,\frac{\omega}{v(0)}$$

$$\begin{cases} \theta = arctg\,\dfrac{x(0)\,\omega}{v(0)} = arctg\,\dfrac{0,28\ \cancel{m}\cdot 1,43\ \cancel{s}}{-2,5\ \frac{\cancel{m}}{\cancel{s}}} = -9,9 \\ \theta = 180 - 9,09 = 170,9° \\ A = \dfrac{v(0)}{\omega\cos\theta} \end{cases}$$

$$A = \frac{-2,5\ \frac{m}{\cancel{s}}}{1,43\ \frac{1}{\cancel{s}}\cdot\cos 171} = 1,77\ m$$

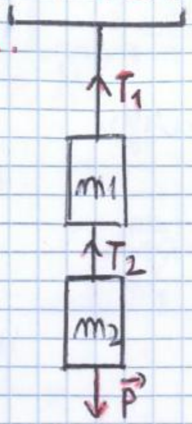
$v_{max} \rightarrow$ se $\theta = \frac{\pi}{2} \rightarrow sen\,\frac{\pi}{2} = 1$

$$v_{max} = A\omega = 1,77\ m \cdot 1,43\ \frac{1}{s} = 2,53\ m/s$$

$$a(t) = A\omega^2 = (1,43)^2 \cdot 1,77 = 3,62\ m/s^2$$

DINAMICA DEL PUNTO MATERIALE

2.1.



$T_1 ?$ $T_2 ?$ equilibrio

$-T_1 + m_1 g + m_2 g = 0$

$\begin{cases} T_1 = (m_1 + m_2) g \\ -T_2 + m_2 g = 0 \end{cases} \rightarrow T = m_2 g$

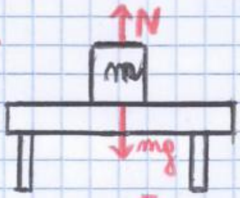
taglio filo $T_1 = 0$

$\begin{cases} m_2 g - T_2 = m_2 a \\ m_1 g + T_2 = m_1 a \end{cases}$

$(m_1 + m_2) a = m_1 g + m_2 g + \cancel{T_1} - \cancel{T_1}$

$T = 0$ filo NON è teso

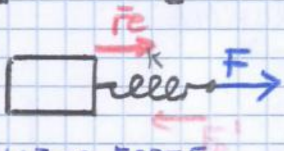
2.2.



$\vec{N}_1 = m_1 g$ blocchetto

$4 \vec{N}_2 = m_1 g + Mg$ blocchetto più tavolo su 4 gambe

2.3.



ANALIZZO FORZE SEPARATAMENTE

Δx molla

M massa molla

M massa blocchetto

applico forza F

forza su blocco $\vec{F'} = ma = kx$ (1)

forza su molla $F - F' = Ma$

$F = Ma + ma$

$a = \frac{F}{M+m}$ → sostituisco in (1) e determino k

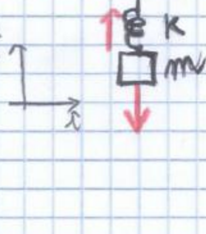
2.4.

$ma = T - mg$

$T = m(a+g)$

se $T > 10$ $a_{max}?$ $T = 10$ $a = \frac{T - mg}{m}$

2.5.



x_0 = lunghezza a riposo

$ma = -mg + \overset{F_{el}}{k\Delta x}$

$\Delta x = \frac{mg + ma}{k} = \frac{m(g-a)}{k}$

→ $a = g$ $\Delta x = 0$ caduta libera

→ $a < g$ si allunga $\Delta x > 0$

→ $a > g$ si accorcia $\Delta x < 0$

$\Delta x > x_0$

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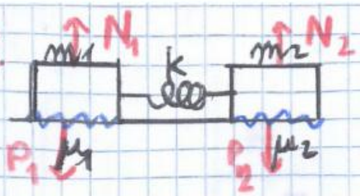
2.8.



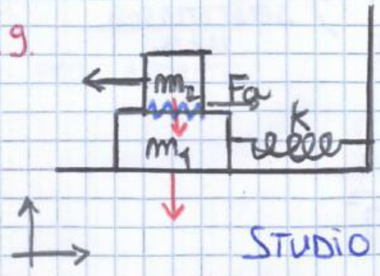
k a riposo

$kx_1 < \mu_1 m_1 g$

$kx_2 < \mu_2 m_2 g$

$x_2 < x_1 \to x_2$

2.9.



m_1, m_2

k, μ_s

max x? (perché m_2 non cade)

STUDIO FORZE

$$\begin{cases} m_1 a_1 = kx - \mu_s m_2 g & (1) \\ \cancel{m_2} a_2 = \mu_s \cancel{m_2} g \end{cases}$$

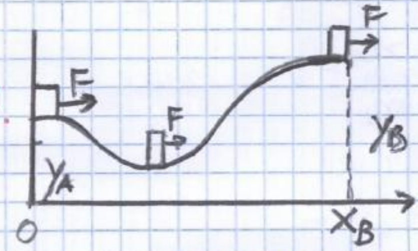
RICAVO $a_2 = a_1$ (perché NON ci DEVE ESSERE MOTO RELATIVO)

$a_2 = \mu_s g$ SOSTITUISCO IN (1)

$$m_1 \mu_s g = kx - \mu_s m_2 g$$

$$x = \frac{(m_1 + m_2)\mu_s g}{k}$$

2.10.



m x_A, x_B, y_A, y_B $\mathcal{L}$? v_B?

$$\Delta U = mg y_B - mg y_A = mg(y_B - y_A)$$

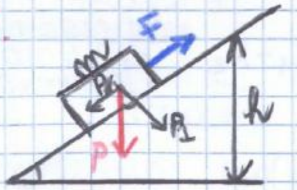
CONSERVAZIONE ENERGIA

$$\mathcal{L} = \underset{\text{ASSE } y}{-\Delta U} + \underset{\text{ASSE } x}{F\Delta x} = -mg(y_B - y_A) + F x_B$$

TEOREMA FORZE VIVE

$$\mathcal{L} = \frac{1}{2} m v_B^2$$

2.11.



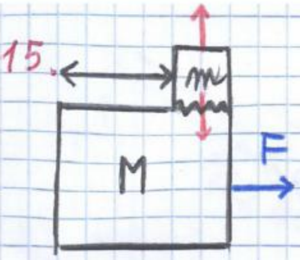
m $\vec{v_0}$ cost Δh Δt

$\mathcal{L}$? W?

$\mathcal{L} = -\Delta U = -mgh_i + mgh_F$ CONSERVAZIONE ENERGIA

$$W = \frac{\Delta \mathcal{L}}{\Delta t}$$

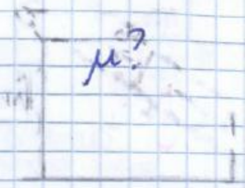
2.15.



NO ATTRITI

IST. INIZIALE → FORZA F A CUBO M

DOPO t CUBO CADE A DISTANZA d



$$x = x_0 + v_0 t + \frac{1}{2} a_S t^2$$

STUDIO FORZE

$$\begin{cases} M a_M = F - \mu m g \\ m a_m = \mu m g \end{cases}$$

→ PRINCIPIO AZIONE-REAZIONE

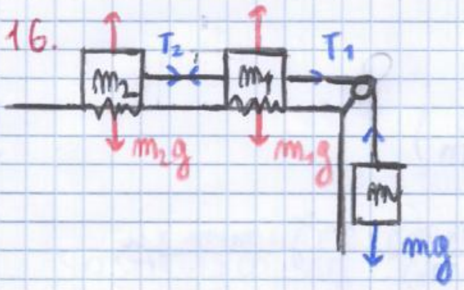
$$a_S = a_m - a_M$$

$$a_S = \frac{\mu m g}{m} - \frac{F}{M} + \frac{\mu m g}{M}$$

RICAVO μ

$$(m+M) g \mu = a_S M + F$$

2.16.



CONDIZIONE MOTO UNIFORME $a = 0$ T_1? T_2?

STUDIO FORZE

$$\begin{cases} m_2: & -\mu_2 m_2 g + T_2 = m_2 a = 0 \\ m_1: & -\mu_1 m_1 g - T_2 + T_1 = m_1 a = 0 \\ m: & -T_1 + mg = ma = 0 \end{cases}$$

$$-\mu_2 m_2 \cancel{g} + \cancel{T_2} - \mu_1 m_1 \cancel{g} - \cancel{T_2} + \cancel{T_1} - \cancel{T_1} + m\cancel{g} = 0$$

$$m = \mu_1 m_1 + \mu_2 m_2$$

VERIFICO CHE LA RELAZIONE SIA SODDISFATTA DA SISTEMA SOSTITUENDO I DATI FORNITI. OK

DETERMINO $T_1 = mg$

$T_2 = \mu_2 m_2 g$

m SI STACCA

CONTROLLO SE FILO É TESO STUDIANDO FORZE TRA m_1 E m_2

$$\begin{cases} -\mu_1 m_1 g - T_2 = m_1 a & (1) \\ T_2 - \mu_2 m_2 g = m_2 a \end{cases}$$

DETERMINO a

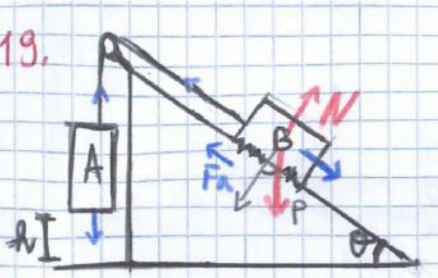
$$-\mu_1 m_1 g - \cancel{T_2} + \cancel{T_2} - \mu_2 m_2 g = (m_1 + m_2) a$$

$$a = \frac{(-\mu_1 m_1 - \mu_2 m_2) g}{m_1 + m_2}$$

SOSTITUISCO a IN (1)

OSSERVO CHE $T > 0 \Rightarrow$ IL FILO É TESO

2.19.



θ, h, $m_A = m_B = m$ μ_d

distanza d percorsa da B?

Conservazione energia → Teorema forze vive

$$\frac{1}{2} m v_B^2 \pm \underbrace{+mg\,\mathrm{sen}\theta\,\Delta x}_{\Delta U} + \mu_d\, mg \cos\theta\, \Delta x$$

$\mathcal{L}_{NC}$ → Forza attrito

Per trovare v $\quad v^2 = \underset{0}{v_0^2} + 2a(h - \underset{0}{h_0})$ $\quad (1)$

Studio le forze
$$\begin{cases} m_A: & -mg - T = ma \\ m_B: & +mg\,\mathrm{sen}\theta - mg\,\mu_d\cos\theta + T = ma \end{cases}$$

$$2ma = -mg - \cancel{T} + mg\,\mathrm{sen}\theta - \mu_d \cos\theta\, mg + \cancel{T}$$

$$a = \frac{\cancel{m}g\,(-1 + \mathrm{sen}\theta - \cos\theta\,\mu_d)}{2\cancel{m}}$$

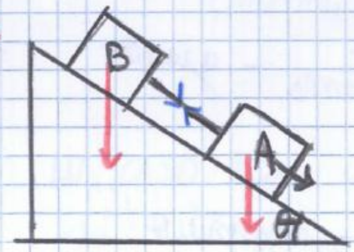
sostituisco a in (1) e determino $v^2 = 2ah$

SOSTITUISCO v_B IN TEOREMA FORZE VIVE

E DETERMINO $\Delta x = \frac{1}{2} \frac{\cancel{m} v_B^2}{\cancel{m} g\,(\mathrm{sen}\theta + \mu_d \cos\theta)}$

Distanza totale PERCORSA $\quad d = \Delta x + h =$

2.20.



T? μ_A? μ_B? $\qquad m_A = m_B$

t_1 filo si tende B inizia a scivolare

t_0: si muove solo A

$$mg\,\mathrm{sen}\theta - \mu_A\, mg \cos\theta = ma$$

LEGGE ORARIA $\quad x = \underset{0}{x_0} + \underset{0}{v_0 t} + \frac{1}{2} a t^2 \qquad a = \frac{2x}{t_1^2}$ $\qquad$ SOSTITUISCO a IN

E DETERMINO $\quad \mu_A = \frac{g\,\mathrm{sen}\theta - a}{g\cos\theta}$

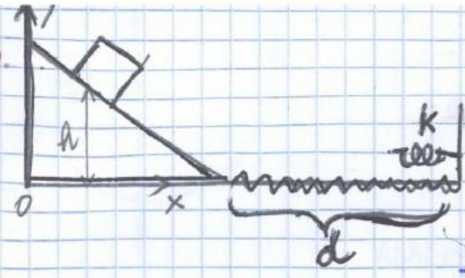
equilibrio:
$$\begin{cases} B: & mg\,\mathrm{sen}\theta - \mu_B\, mg\cos\theta + T = 0 \\ A: & mg\,\mathrm{sen}\theta - \mu_A\, mg\cos\theta - T = 0 \end{cases}$$

$$\cancel{m}\cancel{g}\,\mathrm{sen}\theta - \mu_B \cancel{mg}\cos\theta + \cancel{mg}\,\mathrm{sen}\theta - \mu_A \cancel{mg}\cos\theta = 0 \rightarrow$$

$$\mu_B = 2\,\mathrm{tg}\,\theta - \mu_A$$

$T = \mu_A\, mg\cos\theta - mg\,\mathrm{sen}\,\theta$

2.23



m PIANO INCLINATO LISCIO

μ_d, l_0, k, d

h AFFINCHÉ TOCCHI PARETE DEL VINCOLO → SI FERMA ISTANTANEAMENTE

TEOREMA FORZE VIVE

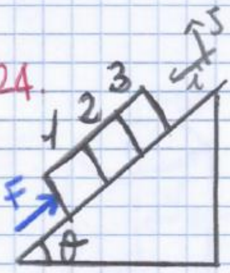
$$\Delta E = \mathcal{L}_{NC}$$

$$\underbrace{mgh}_{\text{FORZA PESO}} + \frac{1}{2} m \underbrace{v_0^2}_{0} + mg\underbrace{h_F}_{0} - \underbrace{\frac{1}{2} k x^2}_{\text{MOLLA}} = + \mu_d m g d$$

$$mgh = + \mu_d m g d + \frac{1}{2} k x^2$$

$$h = \mu_d d + \frac{1}{2} \frac{k x^2}{mg}$$

2.24



m_1, m_2, m_3

$\theta = 40°$ F?

F_2

determina $a \Rightarrow F_2 = m_2 a_2$ $\quad a_2 = \frac{m_2}{F_2}$

Studio forze

$$\begin{cases} (1)\ m_1 a = m_1 g \operatorname{sen}\theta - F \\ (2)\ m_2 a = m_1 g \operatorname{sen}\theta \\ (3)\ m_3 a = m_3 g \operatorname{sen}\theta \end{cases}$$

$$(m_1+m_2+m_3)a = (m_1+m_2+m_3) g \operatorname{sen}\theta - F$$

$$F = (m_1+m_2+m_3)(g \operatorname{sen}\theta - a)$$

IN CASO NON CI SIA F μ_3, μ_2 $\quad \mu_1$?

Studio Forze

$$\begin{cases} m_1 a = m_1 g \operatorname{sen}\theta - m_1 g \mu_{d_1} \cos\theta \\ m_2 a = m_2 g \operatorname{sen}\theta - m_2 g \mu_{d_2} \cos\theta \\ m_3 a = m_3 g \operatorname{sen}\theta - m_3 g \mu_{d_3} \cos\theta \end{cases}$$

$$m_1 g \mu_d \cos\theta = (m_1+m_2+m_3)(g \operatorname{sen}\theta - a) - g(\mu_{3d} m_1 \cos\theta + m_2 \mu_{2d} \ldots$$

$$\mu_1 = \frac{(m_1+m_2+m_3)(g \operatorname{sen}\theta - a)}{m_1 g \cos\theta} - \frac{(\mu_3 m_3 - \mu_2 m_2)}{m_1}$$

2.28.

parabola $y = 1{,}28x - 0{,}31x^2$

m

v_f

$E_{k\,x=0}$? F?

$$\frac{dy}{dx} = 1{,}28 - 2(0{,}31)x$$

calcolo $f'(0) = tg\,\theta \rightarrow \theta = arctg\,f'(0) \rightarrow$ angolo pendenza

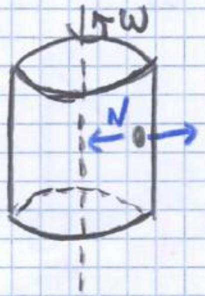
$v = v_0 \cos\theta \rightarrow$ determino v_0

$$E_k = \frac{1}{2} m v_0^2$$

PER TROVARE F DETERMINO a: da equazione parabola

$$a = \frac{0{,}31}{2v_0^2 \cos^2\theta} \Rightarrow F = ma$$

2.29.



cilindro cavo

μ_s min?

studio forze

$$F_a = \mu_s m \underbrace{a_{cp}}_{\text{accelerazione centripeta}} = \mu_s m \omega^2 r$$

perché corpo rimanga attaccato

$$\underbrace{\cancel{m}g}_{\text{Peso}} \le \mu_s \cancel{m} \omega^2 r \qquad \mu_s \ge \frac{g}{\omega^2 r}$$

2.30. m MCU $a_t = 0$ v cost

a_{cp}

α, R $\vec{a} = a_{cp} = \alpha R$

F conservativa?

$$F = m a_{cp} = m\alpha R$$

$$\mathcal{L} = \oint F \, \underbrace{ds}_{\text{moto circolare uniforme} \rightarrow \text{percorro una circ}} = 2\pi r F \neq 0 \Rightarrow \text{la forza NON è conservativa}$$

2.34. 

MCU

R, T $\theta = 0 \to$ F ATTRITO $\omega = \omega_0 - b\theta$

FINO A FERMARSI IN θ_2

$|\vec{a}|$? θ_1 $a = \sqrt{a_{cp}^2 + a_T^2}$

determino ω_0 $T = \frac{2\pi}{\omega_0}$ $\omega_0 = \frac{2\pi}{T}$

determino il coefficiente b $\underset{0}{\omega} = \omega_0 - b\theta_2$

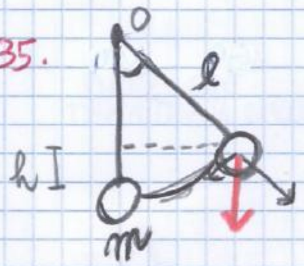
$b = \frac{\omega_0}{\theta_2} \to$ costante

così trovo $\omega(\theta_1) = \omega_0 - b\theta_1$

$a_{cp} = \omega R^2$

$a_T = \alpha R = \frac{d\omega}{dt} R = \frac{d\omega}{d\theta} \frac{d\theta}{dt} = -b\omega R$

$a = \sqrt{a_T^2 + a_{cp}^2}$

2.35. 

m, $\theta = \underbrace{0{,}085}_{\theta_0} \operatorname{sen} \underbrace{4{,}35}_{\omega} t$

DATO CHE È PENDOLO VALE $\omega^2 = \frac{g}{l}$

max ΔU?

$\Delta U = -mgh + mgl = mgl - mgl\cos\theta =$

$= mgl(1 - \cos\theta) = mg \frac{g}{\omega^2}(1 - \cos\theta) = \frac{mg^2}{\omega^2}(1 - \cos\theta)$

T_{max}? T_{min}?

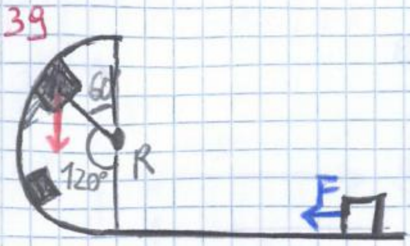
$T_{min} = mg\cos\theta$

$T_{max} = mg(3\cos\theta - 2\cos\theta_0) = mg(3 - 2\cos\theta_0)$

quando $\theta = 0°$

a_x? $t = \frac{\pi}{4\omega}$ $ma_x = T\operatorname{sen}\theta$ $a_x = \frac{T\operatorname{sen}\theta}{m}$

2.39



m, F Δt
piano orizzontale liscio
guida circolare liscia
R_N? dopo $\theta = 120°$

STUDIO FORZE

$F = ma$ $\quad a = \frac{F}{m} = \frac{\Delta v}{\Delta t} m \rightarrow \Delta v = \frac{F}{m} t$

APPLICO CONSERVAZIONE ENERGIA → PER AVERE v_0

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_f^2 + mgR(1-\cos\theta)$$

$$v_0^2 = v_f^2 + 2gR(1-\cos\theta) \quad 60$$

APPLICO 3ª LEGGE ORARIA

$$v_f^2 = v_0^2 + 2gR(1-\cos\theta)$$

$$v_f^2 = v_f^2 + 4gR(1-\cos\theta) \rightarrow v_f^2 = 2gR(1-\cos\theta)$$

DATO CHE TRAIETTORIA CIRCOLARE $R_N = m a_{cp} = m \frac{v^2}{R} - mg\cos\theta$ $60°$

Intervento forza peso

2.40.

m, R, v_0 $\quad F_f$ SI oppone
R_N? guida

APPLICO CONSERVAZIONE ENERGIA → DISSIPAZ PER ATTRITO

$$-\frac{1}{2} m v_0^2 + \frac{1}{2} mgR + \frac{1}{2} m v_f^2 = -\frac{2F_p}{m} \frac{\pi}{2} R$$

$\mathcal{L}_{NC}$

DETERMINO

$$v_f^2 = v_0^2 + 2gR - \pi R \frac{F_a}{m}$$ SOSTITUISCO IN

CONSERVAZ IN ULTIMO TRATTO

$$mgR + \frac{1}{2} m v_f^2 = mR(1-\cos\theta) + \frac{1}{2} v_{ff}^2 m$$

$30°$

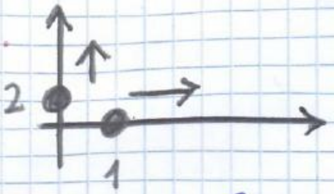
$$\frac{1}{2} v_{ff}^2 = gR + \frac{1}{2}\left(v_0^2 - \pi R \frac{F_a}{m_a} + 2gR\right) - gR(1-\cos\theta)$$

$$v_{ff}^2 = 2gR + v_0^2 - \frac{\pi R F}{m} + 2gR\cos\theta$$

$F_{cp} = F_p + R_N$ $\qquad R_N = \frac{v_f^2}{R} m - mg\cos\theta$

MOTI RELATIVI

3.1.



① PUNTO SI MUOVE MRUA x $a_1 = 3 \ m/s^2$

② PUNTO SI MUOVE MRUA y $a_2 = 4 \ m/s^2$

$t=0 \quad v_0 = 0$

$t = 10s \quad r_{2,1}? \quad v_{2,1}? \quad a_{2,1}?$ di ② RISPETTO A ①

LEGGE ORARIA PER SINGOLI CORPI

① $x_1 = \frac{1}{2} a_{1x} t^2 \qquad v_{1x} = a_1 t \qquad a_{1x} = a_{1x}$

$y_1 = v_{1x} = a_{1y} = 0$ (SI MUOVE SU x)

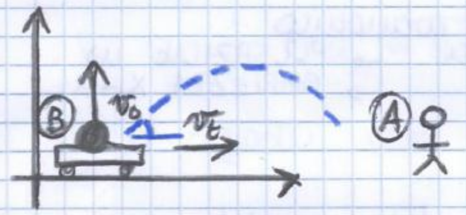
② $x_2 = v_{2x} = a_{2x} = 0$ (SI MUOVE SU y) $\quad y_2 = \frac{1}{2} a_2 t^2 \quad v_{2y} = a_{2y} t$

$\vec{r} = \vec{r}_2 - \vec{r}_1 = \sqrt{\left(\frac{1}{2} a_2 t^2\right)^2 + \left(\frac{1}{2} a_2 t^2\right)^2}$

$\vec{a}_{2,1} = \vec{a}_2 - \vec{a}_1 = \sqrt{a_{2y}^2 + a_{1y}^2} = \sqrt{(3^2+4^2)\frac{m^2}{s^4}} = 5 \ m/s$

$\vec{v}_{2,1} = \vec{v}_2 - \vec{v}_1 = \sqrt{v_{2y}^2 + v_{2x}^2} = \sqrt{(a_1 t)^2 + (a_2 t)^2}$

3.2.



v_0' PALLINA

v_t CARRELLO

v_p'? MOTO RELATIVO

Ⓐ OSSERVATORE FERMO → MOTO PARABOLICO

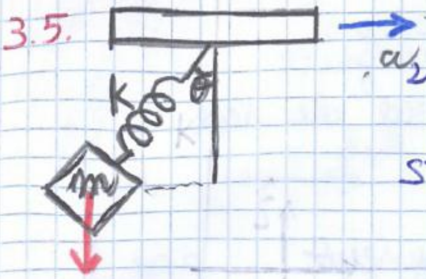
$\vec{v}_p' = \vec{v}_t + \vec{v}_0 = \sqrt{v_t^2 + v_0^2}$

$tg\,\theta = \frac{v_p'}{v_t} \qquad \theta = arctg \frac{v_p}{v_t}$

Ⓑ OSSERVATORE SU CARRELLO CHE SI MUOVE CON v_A

$\vec{v}_p' = (\vec{v}_t - \vec{v}_A) + \vec{v}_0 = \sqrt{(v_t - v_A)^2 + v_0^2}$

3.5.



m, k, a_2

Δl

studio FORZE SU ASSE

$F_{el} + F_p = 0$

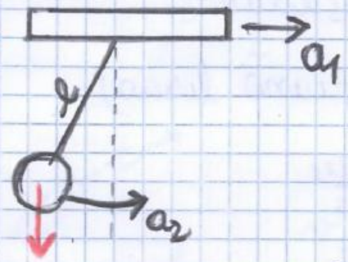
$k \Delta l \cos\theta - mg = 0$

$\Delta l = \frac{mg}{k \cos\theta}$

determino θ $\boxed{tg\,\theta = \frac{a_2}{g}}$ $\theta = \operatorname{arctg} \frac{a_2}{g}$

$\Delta l = mg \cdot \frac{1}{k \cos\theta}$

3.6.



PENDOLO l, a_1

θ? T? rispetto a posizione equilibrio

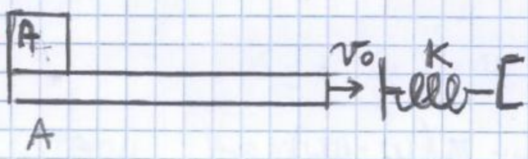
$tg\,\theta = \frac{a_1}{g}$ $\theta = \operatorname{arctg} \frac{a_1}{g}$

$a_2 = \sqrt{a_1^2 + g^2}$

Per PENDOLO vale $T = \frac{2\pi}{\omega}$ $\omega = \sqrt{\frac{a_2}{l}}$

$T = 2\pi \sqrt{\frac{l}{a_2}}$

3.7.



m_{piatt} MRU v_{0p}

No attrito $v_{0A} = 0$

$t=0$ moto frenato da molla a riposo

$\vec{r}_{AP}(t_1)$? t_1

$\vec{v}_{AP}(t_2)$ $t_2 = 2t_1$

moto armonico $\omega = \sqrt{\frac{k}{m}}$ per molla $A = 0,3$

$x = A \operatorname{sen}(\omega t + \phi)$ con $\phi = 0$

$x = A$

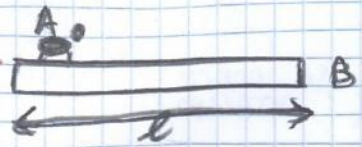
$v = A\omega \operatorname{sen} \omega t$

leggi orarie MRU $t = t_1$ $x = x_0 + v t$ $\Delta x = v_0 t_1 - x_1$

$t = t_2$ $v = v_0$ $\Delta v = 2 v_0$

DINAMICA DEI SISTEMI DI PUNTI

4.1.



m_A, l, m_B
v_A cost
t_1? per raggiungere estremo

Sistema isolato: conservazione quantità di moto

$$m_1 v_1 = m_2 v_2$$

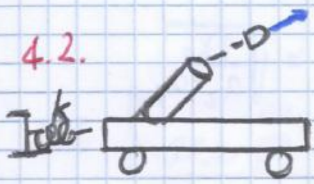
$$v_B = \frac{m_A v_A}{m_B}$$

$\vec{v} = v_A + v_B$ (somma algebrica perché sono su stesso asse) $v_1 // v_2$

MRU $X = X_0 + v t$, $X_0 = 0$ $\quad x = l$

$$t = \frac{l}{v}$$

4.2.



M_C, m_P, v_P
v_C? E_K? k? molla che arresta il cannone Δl

Conservazione q di moto: sist isolato $P_0 = P_f$

$$M_C v_C + m v_P = M v_C + m_P v_P \quad (v_C = 0,\ v_P = 0 \text{ iniziali})$$

$$v_C = -\frac{m_P}{M_C} v_P$$

$$K_C = \frac{1}{2} M_C v_C^2$$

Sistema frenato da molla → su sistema agisce forza conservativa F_{el}
Conservazione energia

$$K_i + U_i = K_f + U_f \quad (U_i = 0 \text{ molla a riposo},\ K_f = 0)$$

$$\frac{1}{2} M_C v_C^2 = \frac{1}{2} k \Delta l^2$$

$$k = \frac{M_C v_C^2}{\Delta l^2}$$

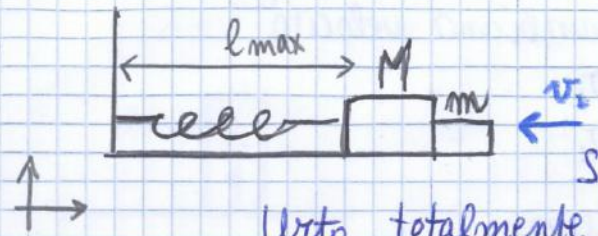
4.5.

M, K

A ampiezza oscillazioni

l_{max} colpita da m, v_2

urto anelastico, masse unite



v_{sist}? A'? sistema dopo urto

Sistema conservativo $U_i + K_i = U_f + K_f$

Urto totalmente anelastico → conservo q di moto

$$M \cdot \underset{0}{\underset{\parallel}{v_1}} + m v_2 = (M+m) v_{sist}$$

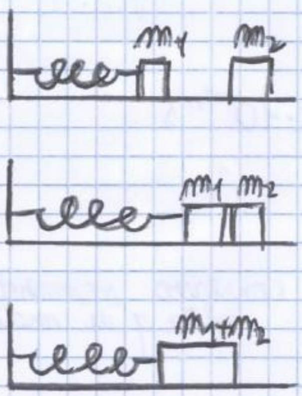
$$v_{sist} = \frac{m}{M+m} v_2$$

Dopo urto applico conservaz energia

$$\frac{1}{2} k \underset{\Delta l}{\underset{\parallel}{A}}^2 + \frac{1}{2}(M+m) v_{sist}^2 = \frac{1}{2} k A'^2$$

$$A' = \sqrt{\frac{kA^2 + (M+m) v_{sist}^2}{k}}$$

4.6.



① i) COMPRESSIONE MOLLA

② ISTANTE URTO

③ SUBITO DOPO URTO

m_1, m_2, x_0

m_2 fermo

urto anelastico

max x?

① sist. conservativo $K_i + \underset{0}{\underset{\parallel}{U_i}} = \underset{0}{\underset{\parallel}{K_f}} + U_f$

$$\frac{1}{2} m_1 v_-^2 = \frac{1}{2} k x^2 \qquad v_- \to \text{velocità subito prima di urto}$$

② urto tot anelastico → conservo $P_i = P_f$

$$m_1 v_- + m_2 \underset{0}{\underset{\parallel}{v_2}} = (m_1+m_2) v_+$$

v_+ velocità subito dopo urto

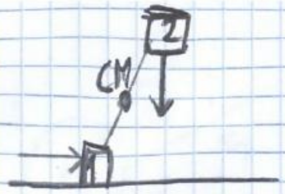
$$v_+ = \frac{m_1}{m_1+m_2} v_- = \frac{m}{m_1+m_2} \sqrt{\frac{k}{m}}\, x$$

③ dopo urto conservazione energia

$$\underset{0}{\underset{\parallel}{U_i}} + K_i = U_f + \underset{0}{\underset{\parallel}{K_f}} \qquad \frac{1}{2}(m_1+m_2) v_+^2 = \frac{1}{2} k x_{max}^2$$

$$x_{max} = \sqrt{\frac{(m_1+m_2)}{k} \cdot \frac{m_1^2}{(m_1+m_2)^2} \frac{k}{m} x^2} = \sqrt{\frac{m_1}{m_1+m_2}}\, x$$

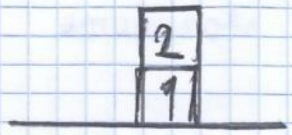
4.9.



moto CM? R_V?

prima dell'urto

CM = centro della retta che collega m_1 e m_2



$$v_{CM} = \frac{v_1}{2}\hat{u}_x + \frac{v_2}{2}\hat{u}_y$$

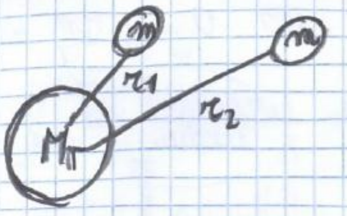
$$= \frac{v_1}{2}\cdot\cos 45 + \frac{v_1}{2}\operatorname{sen}45 = \frac{\sqrt{2}}{2}(v_1+v_2) = \sqrt{2}\,v_1$$

Dopo urto $\quad v_y = 0 \quad v_{CM} = \frac{v_1}{2}$

Reazione a guida $\quad N = \frac{dp}{dt}$

$$N = \frac{2m\,\frac{v}{2}}{t} = \frac{mv}{t}$$

5.3.



$m_1, \quad r_1, \quad r_2$

$\mathcal{L}? \quad \Delta T?$

$M_T = 5{,}9 \cdot 10^{24}$ kg

$F = m a_{cp} = -m \omega^2 r$

$F = -\gamma \frac{Mm}{r^2}$

$-\gamma \frac{Mm}{r^2} = -m \omega^2 r \qquad \omega = \frac{v}{r}$

$\gamma \frac{M\cancel{m}}{r^{\cancel{2}}} = \cancel{m} \frac{v^2}{\cancel{r}}$

$v^2 = \frac{\gamma M}{r}$

per orbita circolare $v = r\omega \qquad \omega = \frac{2\pi}{T}$

(elevo a quadrato per evitare radice) $\quad \frac{v^2}{r^2} = \frac{4\pi^2}{T^2} \rightarrow T = \sqrt{\frac{4\pi^2 r^2}{v^2}} = \sqrt{\frac{4\pi r^2}{\frac{\gamma M}{r}}}$

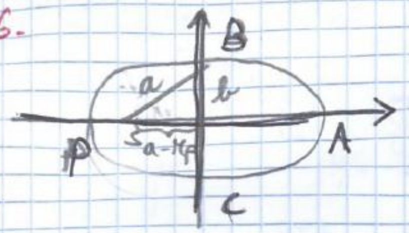
$T = \sqrt{\frac{4\pi r^3}{\gamma M}}$

$T_1 = \sqrt{\frac{4\pi r_1^3}{\gamma M_T}} \qquad T_2 = \sqrt{\frac{4\pi r_2^3}{\gamma M_T}}$

$\Delta T = T_2 - T_1$

$\mathcal{L} = \Delta E_m = +\frac{1}{2}\gamma \frac{mM}{r_1} - \frac{1}{2}\gamma \frac{mM}{r_2} = \frac{1}{2}\gamma mM \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

5.6.



v_a velocità afelio
v_p velocità perielio

$\overline{PA} = 2a$

$\overline{BC} = 2b$

$\overline{PS} = r_1$ $\quad \overline{SA} = r_2$

$r_1 + r_2 = 2a \qquad r_2 = 2a - r_1$

$\overline{SB} = a^2 \qquad a^2 = b^2 + (a - r_1)^2$

$(a - r_1)^2 = a^2 - b^2$

$r_1 = a - \sqrt{a^2 - b^2}$

Conservazione momento angolare della q di moto

$\cancel{m} r_1 v_P = \cancel{m} r_2 v_a$

$r_1 v_p = (2a - r_1) v_a \qquad v_p = \frac{(2a - r_1)}{r_1} v_a$

6.3.

CILINDRO CAVO

R_1^{int}, R_2^{ex} $\qquad m_1, m_2$ (ex, int)

ℓ

per cilindro pieno $\qquad$ cilindro RIMOSSO INTERNO

$$I = \frac{1}{2} m R^2 \rightarrow I = \frac{1}{2} m_1 R_2^2 - \frac{1}{2} m_2 R_1^2$$

sfrutto $\rho = \frac{m}{V}$

$$\begin{cases} \rho = \frac{m_2}{\pi (R_2^2 - R_1^2)\ell} & (1) \\ \rho = \frac{m_1}{\pi R_1^2 \ell} & m_1 = \rho \pi R_1^2 \ell \quad (2) \end{cases}$$

sostituisco (1) in (2)

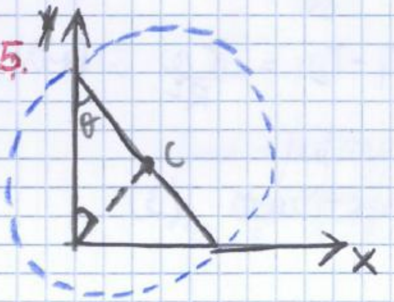
$$m_1 = \frac{m_2 R_1^2}{\pi (R_2^2 - R_1^2)} \frac{\pi \ell}{\ell} = \frac{m_2 R_1^2}{(R_2^2 - R_1^2)}$$

$$I = \frac{1}{2} m_2 R_2^2 + \frac{1}{2} R_2^2 R_1^2 \frac{m_2}{R_2^2 - R_1^2} - \frac{1}{2} R_1^4 \frac{m_2}{R_2^2 - R_1^2} =$$

$$= \frac{1}{2} m_2 \left(R_2^2 + \frac{R_1^2 R_2^2}{R_2^2 - R_1^2} - \frac{R_1^4}{R_2^2 - R_1^2} \right) = \frac{1}{2} m_2 \left(\frac{R_2^4 \cancel{- R_2^2 R_1^2} \cancel{+ R_2^2 R_1^2} - R_1^4}{R_2^2 - R_1^2} \right)$$

$$= \frac{1}{2} m_2 \frac{\cancel{(R_2^2 - R_1^2)}(R_2^2 + R_1^2)}{\cancel{(R_2^2 - R_1^2)}} = \frac{1}{2} m (R_2^2 + R_1^2) \quad \text{cilindro cavo}$$

6.5.



Dimostra che il centro dell'asta descrive una circonferenza di raggio $\frac{\ell}{2}$

$$y_C = \frac{\ell}{2} \sin\theta \qquad x_C = \frac{\ell}{2} \cos\theta$$

trovo raggio circonferenza

$$x_C^2 + y_C^2 = \frac{\ell^2}{4} \cos^2\theta + \frac{\ell^2}{4} \sin^2\theta =$$

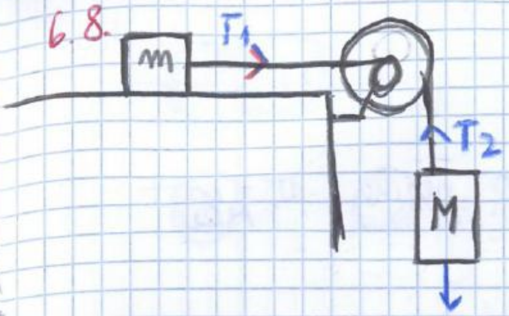
$$= \frac{\ell^2}{4} (\sin^2\theta + \cos^2\theta) = \frac{\ell^2}{4}$$

$$r^2 = \frac{\ell^2}{4} \rightarrow r = \frac{\ell}{2}$$

$$v_{CM} = \omega r = \omega \frac{\ell}{2} \qquad \omega = \frac{d\theta}{dt} \qquad v_{CM} = \frac{d\theta}{dt} \frac{\ell}{2}$$

$$a_{CM} = \omega r^2 = \omega \frac{\ell^2}{4} \qquad a_{CM} = \frac{d\theta}{dt} \frac{\ell^2}{4}$$

6.8



m, M
r, R $\qquad I$

v? di m dopo che è scesa di $h = 1\ m$

T_1? T_2?

III LEGGE ORARIA $\quad v^2 = 2ah \qquad v = \sqrt{2ah}$

Studio forze

$$\begin{cases} ma = T_m & (1) \\ Mg - T_M = Ma & (2) \rightarrow M(a-g) = T_M \\ I\alpha = Tr \end{cases}$$

$$Tr = T_M R - T_m r$$

sostituisco (1) e (2)

$$I\alpha = -M(\alpha - g)R^2 + m\alpha r^2 - MgR$$

$$(I + MR^2 + mr^2)\alpha = MgR$$

$$\alpha = \frac{MgR}{I + MR^2 + mr^2}$$

$$v = \sqrt{2ha} = \sqrt{2h\alpha R}$$

$$T_M = M(g - \alpha R)$$

$$T_m = m\alpha r$$

attrito di m come cambia v? μ_d

$$T_m - \mu_d mg = m_1 a \qquad a = \frac{T_m - \mu_d m_1 g}{m_1}$$

$$v = \sqrt{2h\alpha R}$$

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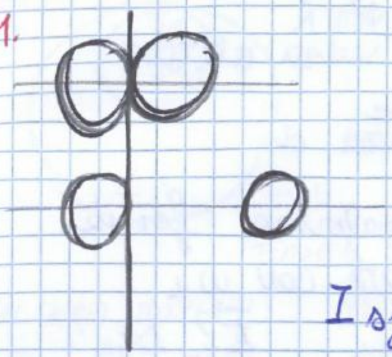
6.11.



sfera piena R_1, m_1

guscio $R_2 = R_1$, $m_2 = \frac{m_1}{4}$

$\mathcal{L}$?

$\omega_0 = 0 \rightarrow \omega_1$

$$I_{sfera} = I_{CM} + md^2 = \frac{2}{5} m_1 R^2 + m_1 R^2 = \frac{7}{5} m_1 R^2$$

$$I_{guscio} = I_{CM} + md^2 = \frac{2}{3} m_2 R^2 + m_2 R^2 = \frac{5}{3} m_2 R^2$$

$$I_{TOT} = I_{sfera} + I_{guscio} = \frac{7}{5} m_1 R^2 + \frac{5}{3} m_2 R^2$$

TEO FORZE VIVE

$$\mathcal{L}_1 = \frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{7}{5} m_1 R^2 + \frac{5}{2} m_2 R^2 \right) (\omega_1)^2$$

DOPO $\omega_1 \rightarrow$ sist ruota guscio si stacca $d = 2R$

$\mathcal{L}$?

calcolo $\underset{\substack{\uparrow \\ sist}}{I'_{TOT}} = \frac{7}{5} m_1 R^2 + \frac{2}{3} m_2 R^2 + (m_2 R^2 + 2 m_2 R_2^2)$

Conservazione momento angolare

$$I_1 \omega_1 = I_2 \omega_2$$

$$\omega_2 = \frac{I_1 \omega_1}{I_2}$$

$$\mathcal{L} = \frac{1}{2} I_1 \omega_1 - \frac{1}{2} I_2 \omega^2 = \mathcal{L}_1 - \frac{1}{2} I'_{TOT} \omega_2^2$$

6.13. 

m_d R DISCO RIGIDO
RUOTA ATTORNO A ASSE
VERTICALE A DISTANZA d

CONTATTO CON DISCO → ANELLO $r = R+d$

F_{att} RADENTE

FERMI → $t=0$ $\vec{M}$ MOMENTO COSTANTE DA MOTORE

1) CALCOLO MOMENTI: ω? $t_1 = 10$ s

$I_d = \frac{1}{2} mR^2 + md^2$

$I_A = mr^2 = m(R+d)^2$

$\vec{M}^{ex} = rF_{att}$

CONSERVAZIONE MOMENTO DI FORZE $\frac{dL}{dt} = \vec{M}$

$\int \frac{dL}{dt} = \int M$ $\int dL = \int M dt$

$I\frac{\omega}{t} = M - rF$

$\omega_1 = \left(\frac{M - rF}{I}\right) t_1$

2) t_1 ANELLO INIZIA A RUOTARE

t_2 ω_{anello}?

$M = I_A \alpha$

$rF = I_A \frac{d\omega}{dt}$

$\omega_2 = \left(\frac{rF}{I_A}\right) dt = \frac{rF}{I_A}(t_2 - t_1)$

3) t_2 MOTORE STACCATO $\omega_D = \omega_A$

E_k? → t_3 DEVO TROVARE ω_3

CONSERVAZIONE MOMENTO ANGOLARE $I_D\omega_3 + I_A\omega_2 = (I_d + I_A)\omega_{sist}$

$\omega_3 = \alpha_1 t = \frac{\omega_1}{t_1} t_2$ ↗ SOSTITUISCO E RICAVO

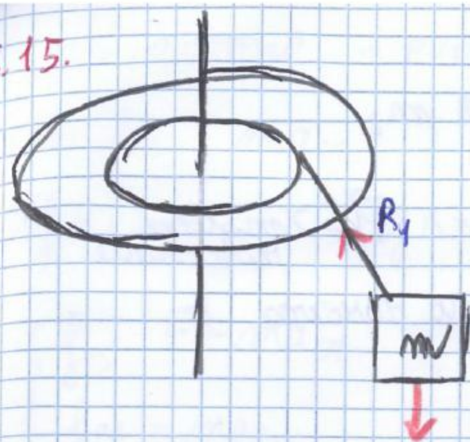
$\omega_{sist} = \frac{I_D \omega_3 + I_A\omega_2}{I_d + I_A}$

4) $\mathcal{L}_{int}$?

$\mathcal{L}_f = \Delta E_K$ $\mathcal{L}^{ex} = \underset{\text{MOTORE}}{M}\theta = M \boxed{\frac{1}{2}\alpha t_2^2}$

$\mathcal{L}_{int} = \mathcal{L}_f - \mathcal{L}^{ex}$ <0 perché il sistema è messo in moto da forze esterne

6.15.



2 dischi

M_1, R_1
$M_2, R_2 = 2R_1$ } STESSO ASSE ROTAZ $M_2 = \frac{3}{2} M_1$

$m_1 \to$ IN $t=0$ lasciato cadere

$h =$ t? per percorrere h

(1) $t = \sqrt{\frac{2a}{h}}$ LEGGE ORARIA $h = \frac{1}{2} a t^2$

DEVO RICAVARE a

CONSIDERO MOMENTI ANGOLARI

$I_{D1} = \frac{1}{2} M_1 R_1^2$ $I_{D2} = \frac{1}{2} M_2 R_2^2 = \frac{1}{2} M_2 4 R_1^2$

$\vec{M} = T R_1$

$\vec{M} = I_{TOT}\, \alpha = \left(\frac{1}{2} M_1 R_1^2 + \frac{1}{2} \frac{3}{2} M_1 4 R_1^2\right) \alpha = \frac{7}{2} M_1 R_1^2 \alpha$

$I_{TOT}\, \alpha = T R_1$

$T = \frac{I_{TOT}\, \alpha}{R_1} = \frac{I_{TOT}\, a R_1}{R_1}$

STUDIO FORZE SU m

$mg - T = ma$

$mg = \frac{7}{2} M_1 R_1 a + ma$

$(7 M_1 R_1 + 2m) a = 2mg$ $a_1 = \frac{2mg}{7 M_1 R_1 + 2m}$ SOSTITUISCO a IN (1) E DETERMINO t

$t_0 = \sqrt{\frac{2a}{h}} = \sqrt{\frac{2}{h} \frac{2mg}{7 M_1 R_1 + 2m}}$

R_2 magnetino m_0 F magnetica
in t_0 è ancora attaccato?
DETERMINO $v_{caduta} = a_1 t_0$

IN t_0 $\vec{F} = m a_{cp} = m \frac{v_{cad}^2}{R_1}$ > F magn data SI STACCA PRIMA

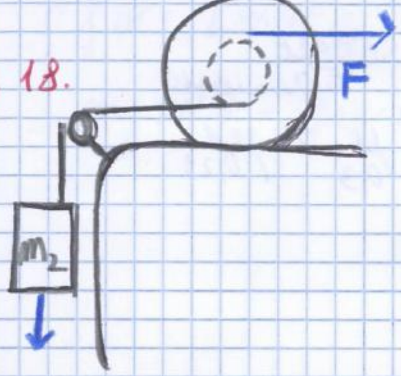
TAGLIO FILO → PURO ROTOLAMENTO

$$\begin{cases} F - f_{as} = m a_{cm} \\ f R = I \alpha \end{cases} \qquad f R = I \frac{a_{cm}}{R} \rightarrow f \cancel{R} = \frac{1}{2} m R^{\cancel{2}} \frac{a_{cm}}{\cancel{R}}$$

$$\begin{cases} f_{as} = \vec{F} - m a_{cm} \\ f_{as} = m a_{cm} \end{cases} \qquad \begin{cases} \vec{F} = 2 m a_{cm} \\ f = m a_{cm} \end{cases} \rightarrow f = \frac{F}{2}$$

$$\boxed{\underbrace{\mu_d m g \geq f_{as}}_{\text{CONDIZIONE PURO ROTOLAMENTO}}} \Rightarrow \mu_d \geq \frac{F}{2mg}$$

6.18.



SFERA R_1, m_1
r_1 → raggio scanalatura
μ_s coeff attr statico

APPLICO F EQ. STATICO
$m_{2\,max}$ PER EQUILIBRIO STATICO? F?

$$\begin{cases} T = m_2 g \\ F + f = T \\ F r + m_2 g r = f R \end{cases} \qquad \begin{cases} F + f = m_2 g \\ F r + m_2 g r = f R \end{cases}$$

$$\begin{cases} F = m_2 g - f \\ (m_2 g - f) r - m_2 g r = f R \end{cases}$$

$$2 m_2 g r - f r = f R$$

$$2 m_2 g r = f R + f R$$

$$m_2 = \frac{f R}{2 r g} + \frac{f}{2 g} = \frac{f}{2 g}\left(\frac{R}{r} + 1\right) =$$

$$= \frac{1}{2} \frac{\mu_d m g}{g} \left(1 + \frac{R}{r}\right)$$

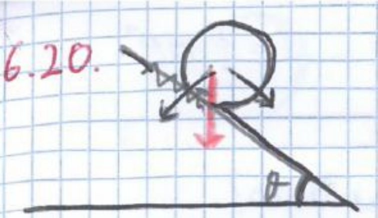
$$F = -\mu_d m_1 + m_2 g$$

TAGLIO FILO → m_1 avanza PURO ROTOLAMENTO?

$$\begin{cases} m_1 a_{cm} = -f_{as} + F \\ F r - f_{as} R = I \alpha \end{cases} \qquad F r - f_{as} R = \frac{2}{5} m_1 R^2 \frac{a_{cm}}{R} \qquad f_{as} = \frac{F r}{R} + \frac{2}{5} m_1 a_{cm}$$

RICAVO $a_{cm} = \frac{5}{7 m_1} F \left(1 + \frac{r}{R}\right) \xrightarrow[\text{SOST IN}]{} f_{as} < \mu m g \rightarrow$ puro rotol

6.20.



r, m anello $\quad I = mr^2$

μ

θ tale che $\boxed{\text{NO}}$ puro rotolamento?

$f_{as} > \mu_d mg\cos\theta$

Studio forze

$$\begin{cases} mg\,\text{sen}\theta - f_{as} = ma_{CM} \\ f r = I\alpha \end{cases}$$

$$\begin{cases} f\cancel{r} = \cancel{I} m\cancel{r^2}\dfrac{a}{\cancel{r}} \\ mg\,\text{sen}\theta - \cancel{f_{as}} = \cancel{f_{as}} \end{cases} \qquad \begin{cases} a_{CM} = \dfrac{f}{m} \\ f_{as} = \dfrac{1}{2} mg\,\text{sen}\theta \end{cases}$$

$$\frac{1}{2}\cancel{mg}\,\text{sen}\theta > \mu_d \cancel{mg}\cos\theta$$

$$\text{tg}\,\theta > 2\mu_d$$

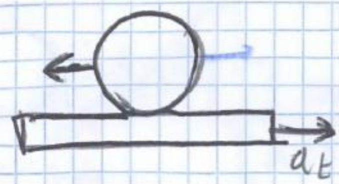
PARTE DA FERMO DA h DAL CM $\rightarrow$ PURO ROT ω?

CONSERVAZIONE ENERGIA

$$mg(h-r) = \frac{1}{2} I\omega^2$$

$$\omega = \sqrt{\frac{4\cancel{m}g(h-r)}{\cancel{m}r^2}} = \sqrt{\frac{4(h-r)g}{r^2}}$$

6.22



piattaforma a_t

cilindro $= \frac{1}{2} mR^2$

PURO ROTOLAMENTO

$a_{cilindro}$?

$a_r \rightarrow$ rispetto a piattaforma

$a_r = -\alpha R$ verso opposto

$$a = a_t + a_r$$

$$\begin{cases} ma = f_a \\ fr = I\alpha \end{cases}$$

$$fr = \frac{I(a_t - a)}{r}$$

$$f\cancel{r} = \frac{1}{2} m\cancel{r^2} \frac{(a_t - a)}{\cancel{r}}$$

$$ma = \frac{1}{2} m a_t - \frac{1}{2} ma$$

$$\frac{3}{2}\cancel{m}a = \frac{1}{2}\cancel{m}a_t \qquad a = \frac{a_t}{3}$$

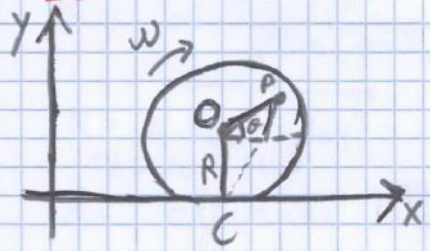
$$a_r = a_t - a$$

minimo μ_s? per puro rot

$$f_a = ma \qquad f \le \mu_d mg \qquad \cancel{m}a \le \mu_d \cancel{m}g$$

$$\mu_d \ge \frac{a}{g}$$

6.23



$P \in$ ruota

$x_p = r\cos\theta + x_{CM} \qquad y_p = r\sin\theta + y_{CM}$

v_p?

MOTO PURO ROTOLAMENTO

$$v_{CM} = \omega R$$

$$v_{xp}' = \omega \cancel{R} \cos\theta = \frac{v_{CM} r}{R}\cos\theta \qquad v_{yp}' = \omega \cancel{R} \sin\theta = \frac{v}{R} r \sin\theta$$

$$v_{cp_x} = v_{CM_x} + \frac{v_{CM} r\cos\theta}{R} = v_{CM}\left(1 + \frac{r}{R}\cos\theta\right)$$

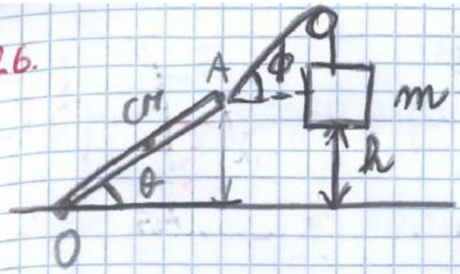
$$v_{cpy} = v_{CMy} + \frac{v_{CM} r \sin\theta}{R} = v_{CM}\left(1 + \frac{r}{R}\sin\theta\right)$$

$$v_p = \sqrt{v_x^2 + v_y^2} \qquad v_x^2 + v_y^2 = v_{CM}^2\left(\left(1+\frac{r}{R}\sin\theta\right)^2 + \left(1+\frac{r}{R}\cos\theta\right)^2\right) =$$

$$= v_{CM}^2\left(1 + \frac{r^2}{R^2}\cos^2\theta + 2\frac{r}{R}\cos\theta + 1 + \frac{r^2}{R^2}\sin^2\theta + \frac{r}{R}\cos\theta\right)$$

$$= v_{CM}^2 \left(R^2 + r^2 + 2rR\sin\theta\right) \frac{v_{CM}^2}{R^2} \quad R'^2 = \omega^2 R'^2$$

6.26.



$OA = \ell$

M incernierato ad O

$\theta = 35°$ $\phi = 75°$

m? R_N?

STUDIO FORZE + MOMENTI

$$\begin{cases} T = mg & (1) \\ T\cos\phi = R_{Nx} & (2) \\ -T\,sen\,\phi + Mg = R_{Ny} & (3) \\ \vec{M}_P : Mg\frac{\cancel{\ell}}{2}\cos\theta = T\cancel{\ell}\,sen(\phi-\theta) \end{cases}$$

momento forza peso

$\vec{M}_T$ momento della tensione

↓

ricavo T

$$T = \frac{Mg\cos\theta}{2\,sen(\phi+\theta)}$$

SOSTITUISCO IN (1) (2) E (3)

$$m = \frac{M\cancel{g}\cos\theta}{\cancel{g}\,2\,sen(\phi+\theta)}$$

$$R_{Nx} = \frac{Mg\cos\theta}{2\,sen(\phi-\theta)}\cos\phi$$

$$R_{Ny} = -\frac{Mg\cos\theta}{2\,sen(\phi-\theta)}\,sen\,\phi + Mg$$

$$R = \sqrt{R_{Nx}^2 + R_{Ny}^2}$$

TAGLIO FILO

h affinché $v_A = v_m$?

$v_m \to$ III LEGGE ORARIA $\quad v_m^2 = 2gh$

v_A PER CONSERVAZ DI ENERGIA

$$\frac{1}{2}I\omega^2 = \cancel{M}g\,h_{CM} \qquad h_{CM} = \frac{\ell}{2}\,sen\,\theta$$

$$\frac{1}{2}\cdot\frac{1}{12}\cancel{M}\frac{\ell^2}{4}\omega^2 = \cancel{M}g\,h_{CM} \qquad \omega^2 = 12g\frac{8\frac{\ell}{8}\,sen\,\theta}{\frac{\ell^2}{4}} = \frac{3g\,sen\,\theta}{\ell}$$

$$v_A^2 = \omega^2\ell^2 \Rightarrow v_A^2 = 3g\,sen\,\theta\,\ell$$

$$v_A^2 = v_m^2$$

$$3\cancel{g}\,sen\,\theta\,\ell = 2\cancel{g}h \qquad h = \frac{3}{2}\ell\,sen\,\theta$$

6.29

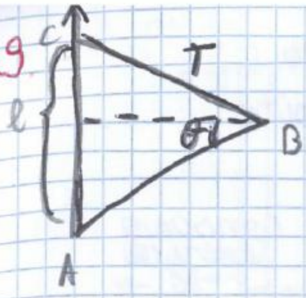


TAVOLA MASSA M, INCERNIERATA l

T_{fune}? R?

θ

EQUILIBRIO

$$A: \begin{cases} R_x - T\cos\theta = 0 \\ -R_y - Mg + T\sin\theta = 0 \end{cases}$$

CONSERVAZIONE MOMENTO ANGOLARE

$$Mg\,\frac{l}{2}\cos\theta = T\,\cancel{l\cos\theta}$$

$T = \frac{1}{2} Mg$ sostituisco

$$\begin{cases} R_x = \frac{1}{2} Mg\cos\theta \\ R_y = \frac{1}{2} Mg\cos\theta - Mg\sin\theta \end{cases}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

6.30

anello $I = mR^2$

R, v

ruota rispetto a centro con ω

urta contro P con raggio r

anello in quiete $v_f = 0$

URTO VINCOLATO

CONSERVO MOMENTO DELLA QUANTITÀ DI MOT ω?

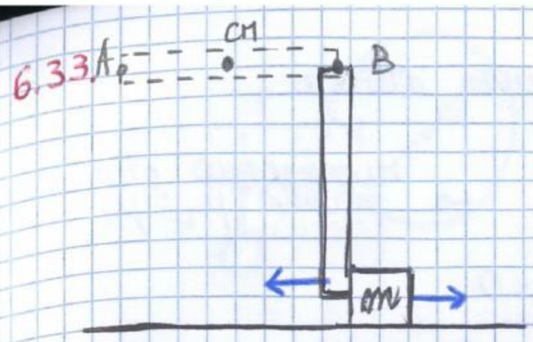
$$L_i = I\omega = mR^2\omega$$

$$L_f = r \wedge P = r \wedge mv = rmv$$

$$L_i = L_f \qquad \cancel{m}R^2\omega = \cancel{m}rv$$

$$\omega = \frac{rv}{R^2}$$

6.33



l, M incernierata a B

$t=0$ colpisce m

urto vincolato

ω? v_0? ΔE_k? I_m urto? (impulso

URTO VINCOLATO → CONSERVO MOMENTO $\vec{L}$

$$\vec{L}_{prima} = \vec{L}_{dopo}$$

$$I\omega = I\omega + lmv \quad (1)$$

SCELGO VINCOLO COME POLO E RICAVO → $I = \frac{1}{12} ml^2 + m\left(\frac{l}{2}\right)^2 = \frac{1}{12} Ml^2 + \frac{1}{4} Ml^2 = \frac{1}{3} Ml^2$ E SOSTITUISCO IN (1)

$$\frac{1}{3} Ml^{\cancel{2}}\omega = \cancel{l}mv$$

$$v = \frac{1}{3}\frac{M}{m} l\underbrace{\omega}$$

RICAVABILE DA PERIODO T

$$\omega = \sqrt{\frac{mlg}{I}} = \sqrt{\frac{Mlg}{\frac{1}{3}Ml^2}} = \sqrt{\frac{3g}{l}}$$

CONSERVO ENERGIA → URTO ELASTICO

$$E_i = E_f$$

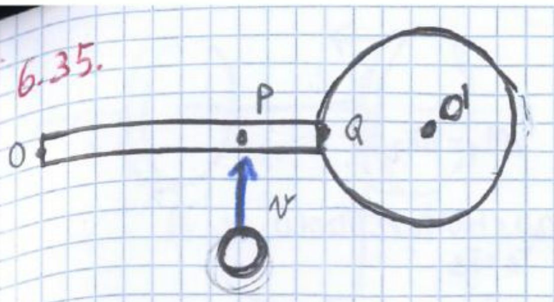
$$\Delta E_K \Rightarrow \quad \frac{1}{2} I\omega^2 = \frac{1}{2} mv^2$$

$$\Delta E_K = E_f - E_i = \frac{1}{2} mv^2 - \frac{1}{2}\frac{1}{3} Ml^2\omega^2 =$$

$$J = \Delta p = mv - Mv = mv - M\omega\frac{l}{2}$$

variazione q. di moto

6.35.



Asta m_1
disco $m_1 = m_2$
$R = \frac{d}{4}$ in quiete
M punto si muove con v_p
URTA P $\quad OQ = d \quad O'Q = \frac{1}{4}d$

raggio $r = OP = \frac{7}{8}d$

1) moto?

IPOTIZZO $x_{CM} = \dfrac{m_1 \frac{d}{2} + m_2\left(d + \frac{d}{4}\right)}{2m_1} = \dfrac{\cancel{m_1}\left(\frac{d}{2} + d + \frac{d}{4}\right)}{2\cancel{m_1}} = \frac{7}{8}d$

P è centro di massa $\Rightarrow$ moto traslatorio

2) v_{CM}? No conservaz energia
urto totalmente anelastico $\rightarrow$ conservo nell'urto q di moto

$$Mv_p = (M + 2m)v_{CM}$$

$$v_{CM} = \frac{Mv_p}{M+2m}$$

3) Corpo vincolato ad O attorno a cui può ruotare

$$I = \frac{1}{12}md^2 + m\frac{d^2}{4} + \frac{1}{2}m\frac{d^2}{16} + m\left(\frac{5}{4}d\right)^2 + m\left(\frac{7}{8}d\right)^2 = 3{,}2\,d^2 \text{ kg}$$

CONSERVO Q DI MOTO $\quad \frac{7}{8}d\,Mv_p = I\omega$

$$\frac{7}{8}\cancel{d}\,Mv_p = 3{,}2\,d^{\cancel{2}}\omega$$

$$v = \frac{7}{8}\omega d$$

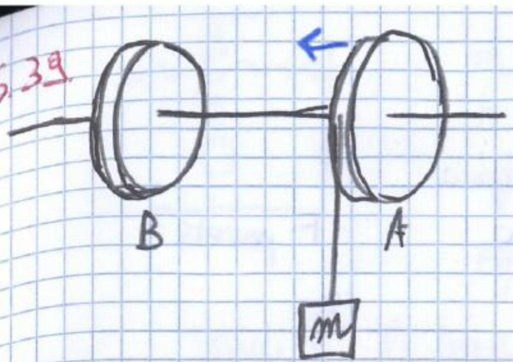
$$\frac{7}{8}Mv_p = 3{,}2\,\frac{8}{7}v_{CM}$$

$$v_{CM} = \frac{7}{8}\cdot\frac{7}{8}\cdot\frac{Mv_p}{3{,}2} \text{ kg}$$

4) J? Impulso

$$J = \Delta P = (M + 2m)v - Mv_p$$

6.39



DISCO M, R $v_{0B}=0$

ω_B ω_F?

Conservo q di moto per urto tot anelastico

$$I\omega_A + mR^2\omega_A = \underbrace{\frac{1}{2}MR^2}_{I_{disco}}\omega + \underbrace{\frac{1}{2}MR^2}_{I_d}\omega_A + \underbrace{mR^2}_{I_m}\omega_A$$

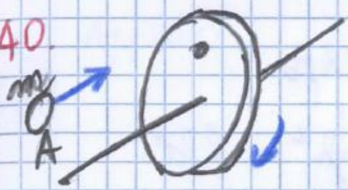
$$\left(\frac{1}{2}Mr^2 + m\right)\omega_A = mR^2 + MR^2\omega_F$$

$$\omega_F = \frac{\left(\frac{1}{2}Mr^2+m\right)\omega_A}{(m+M)R^2}$$

IMPULSO TRASMESSO A ASSE URTO

$$J = \Delta P = (2M+m)\,\omega_F R - (M+m)\,\omega_A R$$

6.40



DISCO M, R

MOMENTO D'ATTRITO → $\vec{M}_a \neq \mu$

$t=0$ ω_0, m, v_A

t_1? $\omega_F=0$ rotazione urto tot anelastico → conservo q di moto

Conservo momento angolare

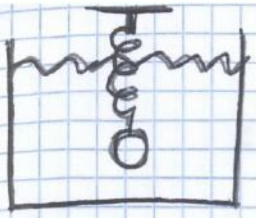
$$\begin{cases} I\omega_0 = (I + mr^2)\,\omega_{sist} \\ M_a = I_F\,\alpha = (I+mr^2)\alpha \end{cases} \qquad \alpha = \frac{M_a}{I+mr^2}$$

mr^2: PALLINA CHE SI È ATTACCATA

SOSTITUISCO ω_0 E α IN 3° LEGGE ORARIA

$$\omega = \omega_0 - \alpha\,(t_1 - t_0)$$

$$\left(\frac{M_a}{I+mr^2}\right)t_1 = \frac{(\cancel{I+mr^2})}{M_a}\frac{I\omega_0}{(\cancel{I+mr^2})}$$

8.2. 

m, R
k
sfera immersa in liquido
Eq statico x' ρ_L?

Studio le forze

$$kx - mg = 0$$

Determino $x = \frac{mg}{k}$ posizione molla SENZA fluido

$\Delta x = x - x'$ → posizione della molla sotto influenza del fluido

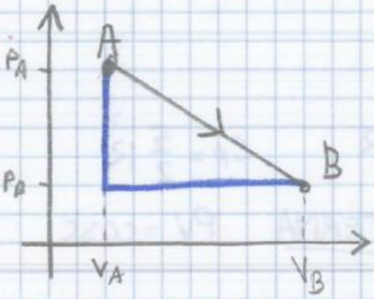
Applico $F_P = F_V$

$$kx' - mg = \rho_L \underbrace{V}_{\text{SPINTA DI ARCHIMEDE}} g$$

$$kx' - mg = \rho_L \frac{4}{3}\pi R^3 g$$

$$\rho_L = \frac{kx' - mg}{\frac{4}{3}\pi R^3 g}$$

11.2.



$P_1V_1 = nRT_1$

$P_2V_2 = nRT_2$

$T_1 = \frac{P_1V_1}{nR}$ $\quad T_2 = \frac{P_2V_2}{nR}$

$P = aV + b \rightarrow P_1 = aV_1 + b$

$P_2 = aV_2 + b$ $\quad P_2 < P_1$

$aV_2 - b < aV_1 + b$

$P_2 < P_1$

$aV_2 - aV_1 = P_2 - P_1 + \cancel{b} - \cancel{b}$

$a = \frac{P_2 - P_1}{V_2 - V_1} < 0$ $\quad a(V_2 - V_1) = (P_2 - P_1)$

$PV = nRT$

$T = \frac{PV}{nR} = \frac{aV^2 + bV}{nR}$

$T_1 = \frac{2aV + b}{nR} > 0$ $\quad 2aV > -b$

$V > -\frac{b}{2a}$

11.3 Adiabatico: $V = 10^{-2}\ m^3 = 10\ L$

oggetto metallico: $m = 0,8\ kg$

$c = 130\ J/kg\,K$ $\quad C_V = \frac{5}{2}R$

V trascurabile

$n = 2,5\ mol \rightarrow$ gas biatomico

$\xrightarrow{\text{riscaldo}}$ $T_{eq1} = 290\ K \rightarrow T_{e2} = 470\ K$

espansione $\rightarrow P_{ex} \rightarrow P_{atm} = 1\ atm$

Eq termico in contenitore

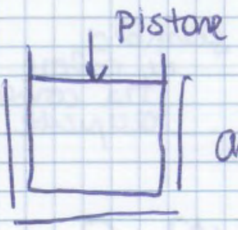
Eq meccanico a esterno $\quad T_1?\ T_f?\ V_f?$

① equilibrio T_{eq} $\xrightarrow{\text{Riscaldato}}$ ② $T_1 \rightarrow$ ③ Equilibrio $T_e = 470\ K$

$\underbrace{\text{② } T_1 \rightarrow \text{③}}_{\text{NON reversibile}}$

④ espansione

11.4



adiabatico $\quad n = 2$ mol $\quad$ gas ideale monoatomico

$c_v = \frac{3}{2}R$

$c_p = \frac{5}{2}R$

$V_0 \quad T_0$

compressione $V_1 = \frac{V_0}{10}$ $\longrightarrow$ espansione V_0

$\mathcal{L} = 27,7 \cdot 10^3 \, J \quad T_0 ? \; T_f ?$

$$\gamma = \frac{c_p}{c_v} = \frac{5}{3}\frac{\cancel{R}}{\cancel{R}}$$

$$TV^{\gamma-1} = TV^{\frac{2}{3}}$$

$$\boxed{T_0 V_0^{\gamma-1} = T_1 V_1^{\gamma-1}}$$

$$T_0 = \frac{T_1}{10^{\frac{2}{3}}} \frac{\cancel{V_0}}{\cancel{V_0}}$$

$$dU = n c_v \, dT \qquad dU = -\delta \mathcal{L}$$

$$n c_v \, dT = P dV$$

$$\Rightarrow dT = \frac{P dV}{n c_v} = \frac{\mathcal{L}}{n c_v} = \frac{27,7 \cdot 10^3}{\cancel{2} \cdot \frac{3}{\cancel{2}} R} = 1,1 \cdot 10^3$$

$$\Delta T = T_1 - T_0$$

$$\Delta T = T_1 - \frac{T_1}{10^{\frac{2}{3}}}$$

$$1,1 \cdot 10^3 = T_1 \Big/ \frac{10^{\frac{2}{3}} - 1}{10^{\frac{2}{3}}} \qquad T_1 = 1415 \, K$$

$$T_0 = \frac{T_1}{10^{\frac{2}{3}}} = 304 \, K$$

11.6. contenitore adiabatico

$V_1 = 2 \cdot 10^{-3}\ m^3$

$T_0 = 280\ K$ $\quad p_0$

sist assorbe calore $\quad T_1 = 448\ K$ $\quad$ IN A → V cost

espansione

$V_g \rightarrow 0{,}09$ mol $\quad V_2$, affinchè p_0? $\quad p_g = p_0$

n mol?

$V_1 p_1 = V_2 p_2$

$-\Delta L = \Delta U$

$\Delta U = n c_V \Delta T$

$\Delta L = P \Delta V$

Inizialmente V cost ⇒ isocora

$\frac{p_0}{T_0} = \frac{p_1}{T_1}$ finale $\quad \frac{p_1}{p_0} = \frac{T_1}{T_0} = \frac{448}{280} = 1{,}6$

espansione libera ⇒ isoterma $\quad pV_1 = p_g(V_2 + V_1)$

$p_g = p_0 \longrightarrow p_0 V_2 = pV_1 - p_0 V_1$

$V_2 = \left(\frac{p}{p_0} V_1 - V_1\right) = \left(\frac{p}{p_0} - 1\right) V_1$

$= (1{,}6 - 1) V = 1{,}2\ L$

$p_0 V = nRT$

ricavo n per determinare P

$\frac{V_1}{n_1} = \frac{V_2}{n_2} \qquad \frac{n_2}{n_1} = \frac{V_2}{V_1}$

$\frac{V_1}{V_2} = \frac{n_1}{n_2} \qquad n_2 = \frac{V_2}{V_1} \cdot 0{,}09 = 0{,}054$ mol

$p_0 = \frac{n_2 RT}{V_2} = \frac{0{,}054 \cdot 0{,}0821 \cdot 448}{1{,}2} = 1{,}65$ atm

11.8. $n = 5$ mol gas ideale pistone adiabatico

$S = 500\ cm^2 = 500 \cdot 10^{-4}\ m^2$

$m = 500$ kg scorre senza attrito

Eq. termodinamico T cost $h_0 = 1{,}3$ m

Poi rimuovo isolante termico $\rightarrow m_g = 106{,}5$ g

$T_0 = -18\ °C$
$\downarrow$
$T_2 = 0\ °C$ a eq termico T cost

$c_g = 2052$ J/kgK

gas monoatomico o poliatomico?

$V_i = S h_0 = (500 \cdot 10^{-4} \cdot 1{,}3)\ m^3 = 0{,}065\ m^3$

$P_{int} = \frac{\overset{Peso}{mg}}{S} + P_{atm} = \frac{500\ kg \cdot 9{,}8\ m/s^2}{500 \cdot 10^{-4}\ m^2} + 101325\ Pa =$

$1{,}993 \cdot 10^5$ Pa

$P_i V_i = nRT_i$

$T_i = \frac{P_i V_i}{nR} = \frac{1{,}993 \cdot 10^5 \cdot 6{,}5 \cdot 10^{-2}}{8{,}31 \cdot 5} = 311$ K

rimuovo isolante gas cede calore

$Q = n c_p\, dT$

① trasf isobara

ghiaccio assorbe calore $Q_g = m_g c_g\, dT$

equilibrio termico $Q_1 = -Q_g$

$n c_p \Delta T = -m_g c_g \Delta T$

$n c_p (T_2 - T_i) = -m_g c_g (T_2 - T_0)$

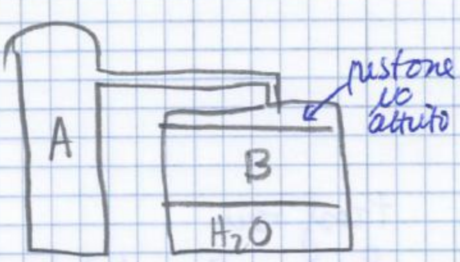
$c_p = -\frac{m_g c_g (T_2 - T_0)}{n (T_2 - T_i)} = \frac{0{,}106\ kg \cdot 2052\ J/kgK\ (285{,}15)\ K}{5\,(-311)\ K}$

$= 35{,}6$ J/molK

$\frac{5}{2} R = 20{,}8$ J/molK

gas monoatomico

11.9. Bombola pareti adiabatiche gas A
collegato a ~~valvola~~ recipiente adiabatico



$m_{H_2O} = 5g$ di ghiaccio $T_0 = 0°C$

$n = 0,5$ mol gas in B

$B \rightarrow V_0 = 10^{-2}\ m^3$

apro valvola → fluisce gas → ghiaccio si scioglie

δQ_B? V_{fB}? T cost isoterma

dU_A?

$$Q_B = m_{H_2O}\lambda = 5 \cdot 10^{-3}\ kg \cdot 3,3 \cdot 10^5\ J/kg = 16,5 \cdot 10^2\ J$$

Trasf isoterma $\Delta U = 0$

gas biatomico

$$\delta L = \delta Q = nRT_0 \log \frac{V_f}{V_0}$$

$$1,65 \cdot 10^{-3} = 0,5 \cdot 8,31 \log \frac{V_f}{V_0}$$

$$\log \frac{V_f}{V_0} = \frac{1,65 \cdot 10^{-3}}{0,5 \cdot 8,31\, T_0 = 0} \rightarrow V_f = 2,3 \cdot 10^{-3}\ m^3$$

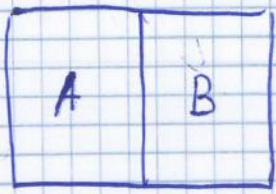
I PRINCIPIO $\Delta U = 0$

$\Delta Q = \Delta L$

$Q_A = 0$

$L = -L_B = -Q_B = -1,65\ kJ$

11.11



recipiente con pareti adiabatiche
AB diaterma
1 mol di gas ideale monoatomico
in ogni parte

$V_{Ai} = V_{Bi} = 10^{-2}\ m^3$

$T_i = 300\ K$

per esterna cost → $p_0 = 4\ bar = 4 \cdot 10^5\ Pa$

↓ B compressione

equilibrio T_f? V_{Bf}? $\Delta H? = Q_p$

INIZ. P cost

$$nRT_i = p_i V_i$$

$$p_i = \frac{nRT_i}{V_i} = \frac{1 \cdot 8{,}31 \cdot 300}{10^{-2}} = 2{,}49\ bar$$

I PRINCIPIO

$$dU_A = \delta Q_A - \delta L_A$$

$$\delta Q_A = n c_v\, dT$$

$$\delta L = -p_0 (V_{Bf} - V_i)$$

$$dU = n c_v\, dT$$

$$-n c_v\, dT = n c_v\, dT - \delta L$$

$$p_0 (V_{Bf} - V_i) = 2 c_v\, dT$$

$$\begin{cases} p_0 (V_{Bf} - V_i) = 2 c_v (T_f - T_i) \\ p_f V_f = \underset{1}{n} R T_f \end{cases}$$

$$\begin{cases} T_f = \frac{p_f V_f}{R} \\ p_0 V_f - p_0 V_i = 2 c_v \frac{p_f V_f}{R} - 2 c_v T_i \end{cases}$$

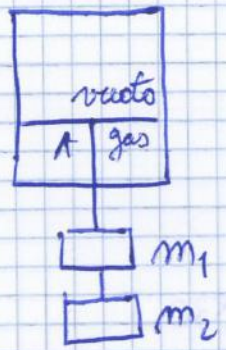
$$\left(p_0 - \frac{2 c_v p_f}{R}\right) V_f = p_0 V_i - 2 c_v T_i$$

$$V_f = \left(\frac{R}{p_0 R - 2 c_v p_f}\right) + 2 c_v T_i = 0{,}72 \cdot 10^{-2}\ m^3$$

$$T_f = 345\ K$$

$$\Delta H = \Delta U + p \Delta V = n c_v \Delta T + p \Delta V = 1130\ J + 780\ J = 1910\ J$$

11.13.



$n = 0{,}25$ mol

gas ideale monoatomico $c_P = \frac{5}{2}R$ $c_V = \frac{3}{2}R$

$T_0 = 283$ K

cilindro adiabatico

$m_1 = 36$ kg m_2

S.st eq termodinamico $h = 0{,}625$ m da fondo cilindro

m_2?

$$P = \frac{F_P}{S}$$

$$P_0 V_0 = nRT_0$$

$$\frac{F_P}{S} = \frac{nRT_0}{V_0} \qquad \frac{(m_1+m_2)g}{\cancel{S}} = \frac{nRT_0}{\cancel{S}h}$$

$$m_2 = -m_1 + \frac{nRT_0}{hg} = \frac{0{,}25 \cdot 8{,}31 \cdot 283}{0{,}625 \cdot 9{,}81} - 36 = 60 \text{ kg}$$

Taglio filo $\rightarrow$ $\mathcal{L} = (m_1 \quad)gh = (\quad 36)\,9{,}8 \cdot 0{,}625 = 220{,}5$ J

$V_f = 2V_0$

riattacco m_2

$$\mathcal{L} = (m_1+m_2)g\,\Delta x$$

$$\mathcal{L} = nc_V dT$$

$$(m_1+m_2)g\,\Delta x = nc_V dT$$

$$\begin{cases} (m_1+m_2)g\,\overset{2h-x}{\Delta x} = nc_V(T_2-T_1) \\ T_f = \frac{PV}{nR} \end{cases}$$

$x = 0{,}47$ m

$T_f = 353{,}5$ K

11.15. $n = 1$ mol
ciclo reversibile
espansione adiabatica $T_A = 600$ K $\rightarrow T_B = 300$ K
compressione isoterma $\rightarrow V_A$ $\quad T_B$ cost
trasformazione isocora $\rightarrow T_A$

η?

$$\eta = 1 - \frac{T_A}{T_B} \quad \text{IN CARNOT}$$

$$\eta = \frac{\mathcal{L}}{Q_{ass}} = 1 - \frac{Q_{ced}}{Q_{ass}}$$

① ADIABATICA $\quad T_A V_A^{1-\gamma} = T_B V_B^{1-\gamma} \rightarrow \frac{V_B}{V_A} = \frac{T_A}{T_B}^{\frac{1}{\gamma-1}}$

② COMPRESSIONE ISOTERMA
CEDO CALORE $\quad Q_C = -\underset{1}{n} R T_B \ln \frac{V_B}{V_A} = -nRT \ln \frac{T_A}{T_B}^{\frac{1}{\gamma-1}}$

③ TRASFORMAZIONE ISOCORA $\quad Q_a = \underset{1}{n} c_v \Delta T$

$$\eta = 1 - \frac{Q_C}{Q_a} = 1 - \frac{R T_B \ln (T_A/T_B)^{(\frac{1}{\gamma-1})}}{c_v \Delta T} =$$

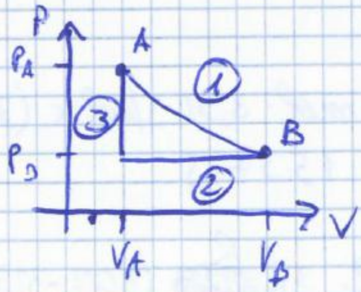
$$= 1 - \frac{\frac{RT}{\gamma-1} \ln \frac{T_A}{T_B}}{c_v (T_A - T_B)} = 1 - \frac{RT \ln \frac{T_A}{T_B}}{(\gamma-1) c_v (T_A - T_B)}$$

SOSTITUISCO $\gamma = \frac{c_p}{c_v} \qquad R = c_p - c_v$

$$\eta = 1 - \frac{c_p - c_v}{(\frac{c_p}{c_v} - 1) c_v} \frac{T_B \ln}{(T_A - T_B)} \frac{T_A}{T_B} =$$

$$= 1 - \frac{\cancel{c_p - c_v}}{\cancel{c_p - c_v}} \frac{T_B}{(T_A - T_B)} \ln \frac{T_A}{T_B} = 1 - \frac{300}{600 - 300} \ln \frac{600}{300} =$$

$$= 0{,}307$$



11.17. frigorifero reversibile $T_2 = 25\,°C \rightarrow T_1 = -4\,°C$

assorbe $\mathcal{L} = -400\ J$ in un ciclo

PONGO $m = 1\ kg$ di H_2O $T = 20\,°C$

numero di cicli per trasformare H_2O in ghiaccio?

$\lambda = 3{,}3 \cdot 10^5\ J/kg$

$Q_{a_{id}} = m c_s \Delta T + m\lambda = 1\ kg \cdot 4186\ J/kg \cdot K (20+4) + 1\,kg \cdot 3{,}3 \cdot 10^5$

$= 413 \cdot 10^3\ J$

$\eta = \frac{\mathcal{L}}{Q_a}$ $\eta = 1 - \frac{T_a}{T_B} = 1 - \frac{-4}{25} = 0{,}84$

$Q_a = \frac{\mathcal{L}}{\eta} = \frac{400}{0{,}84} = 476\ J$

$\frac{Q_{aid}}{Q_a} = \frac{413 \cdot 10^3}{476} = 120$ cicli

11.18. Gas ideale biatomico $n = 1{,}6\ mol$ $C_p = \frac{7}{2}R$

miscela H_2O e ghiaccio a T_A fusione $C_V = \frac{5}{2}R$

$p_A = 10^5\ Pa$

ESPANSIONE ISOTERMA $p_B = 0{,}64 \cdot 10^5\ Pa$

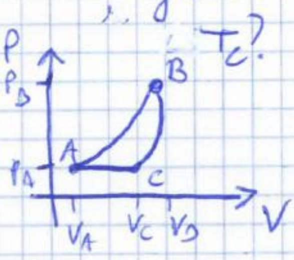
in stato C $(P_C = P_A)$

↓ ISOBARA

(A) a contatto termico con la miscela

fusi $8 \cdot 10^{-3}\ kg$ ghiaccio $\Delta Q = -246\ J$

T_C? $\mathcal{L}_{BC}$?



$Q = m c_s \Delta T + m\lambda$

$-m\lambda = Q_{AB} + Q_{CA}$

$Q_{CA} = -nRT \ln \frac{V_f}{V_i} - m\lambda = -4262\ J$

$Q_{AB} + Q_{BC} + Q_{CA} = \Delta Q$ $Q_{CA} = n c_p \Delta T = 1{,}6 \cdot \frac{7}{2} R \Delta T$

$Q_{BC} = 2394$ $(T_C - T_A) = \frac{-4262}{1{,}6 \cdot \frac{7}{2}R} + T_A$ $T_C = 364{,}5\ K$

$\mathcal{L}_{BC} = -\Delta U + Q = \Delta Q - n c_V \Delta T = 2394 - 1{,}6 \cdot \frac{5}{2} 8{,}31 \cdot T_C - T_B =$

$-61{,}8\ J$

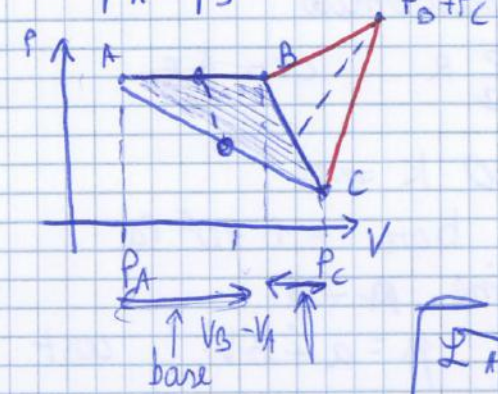
11.20. $n = 1$ mol gas ideale monoatomico

$p_A = p_B = 1$ bar $\quad p_C = 0{,}2$ bar

$V_A = 10^{-2}\ m^3$

$V_B = 3 \cdot 10^{-2}\ m^3$

$V_C = 4 \cdot 10^{-2}\ m^3$



$\mathcal{L}$? $\quad \eta$?

$\mathcal{L} = P\Delta V$

~~$\mathcal{L}_{AB} = 1 \cdot 10^5 \cdot (3 \cdot 10^{-2} - 10^{-2}) = 2 \cdot 10^3$ J~~

~~$\mathcal{L}_{BC} = (0{,}2 \cdot 10^5 - 1 \cdot 10^5)((4-3)\,10^{-2}) = 0{,}8 \cdot 10^3$ J~~

~~$\mathcal{L}_{CA} = (1 - 0{,}2)\,10^5((1-4) \cdot 10^{-2}) = 2{,}4 \cdot 10^3$ J~~

$\Delta\mathcal{L} =$ Area rettangolo

$$\Delta\mathcal{L} = \frac{1}{2}(V_B - V_A)(p_A - p_C) = 800\ J$$

$$Q_{AB} = n c_p (T_B - T_A) = \frac{5}{2} nR (T_B - T_A) = \frac{5}{2} p_A (V_B - V_A)$$

$$= \frac{5}{2}\, 1 \cdot 10^5 (3-1)\, 10^{-2} = 5 \cdot 10^3\ J$$

$$Q_{BC} = \Delta U + \mathcal{L}_{BC} = n c_V \Delta T + \Delta P \Delta V =$$

$$= \frac{3}{2} n R \Delta T + \Delta P \Delta V = \frac{3}{2}(P_C V_C - P_B V_B) + \frac{1}{2}(P_B + P_C)(V_C - V_B) = -2{,}7\ kJ$$

$$Q_{CA} = \mathcal{L} - Q_{AB} - Q_{BC} = -1{,}5\ J$$

$$\eta = 1 - \frac{Q_c}{Q_a} = 1 - \frac{2{,}7 + 1{,}5}{5} = 0{,}16$$

12.1. $m = 10$ kg caduta libera
$h = 10$ m
e si ferma

$T = 293$ K

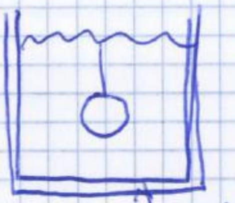
ΔS_u? Energia inutilizzabile?

$$\Delta S_u = \frac{\Delta Q}{T} = \frac{mgh}{T_i} = \frac{10 \cdot 10 \cdot 9{,}8}{293} = 3{,}34 \text{ J/K}$$

Energia inutilizzabile ΔQ della caduta

$$\Delta Q = mgh = 10 \cdot 10 \cdot 9{,}8 = 980 \text{ J}$$

12.2. Pendolo di torsione
immerso in liquido



pareti adiabatiche

$T_{eq} = 293$ K
$\theta = 20\pi \to$ libero
dopo un certo numero di giri
pendolo si ferma
dopo attriti

$K = 0{,}74$ N·m/rad

pendolo $m_1 = 0{,}85$ kg $\quad c_1 = 234$ J/kg·K
liquido $m_2 = 0{,}57$ kg $\quad c_2 = 1780$ J/kg·K

T_f? ΔS_u?

bilancio delle energie
en. elastica $\frac{1}{2} K \theta^2$ pendolo
calore pendolo $Q = m_1 c_1 \Delta T$
calore acqua $Q = m_2 c_2 \Delta T$

$$\frac{1}{2} K \theta^2 = m_1 c_1 (T_f - T_i) + m_2 c_2 (T_f - T_i)$$

$$\frac{1}{2} K \theta^2 = (m_1 c_1 + m_2 c_2)(T_f - T_i)$$

$$T_f - T_i = \frac{1}{2} \frac{K\theta^2}{(m_1 c_1 + m_2 c_2)} \to T_f = \frac{1}{2} \frac{K\theta^2}{(m_1 c_1 + m_2 c_2)} + T_i = 294{,}6 \text{ K}$$

$$\Delta S_u = \frac{\Delta Q}{T_i} \Rightarrow \frac{\Delta Q}{T_i} = mc \ln \frac{T_f}{T_i} = 4{,}2 \text{ J/K}$$

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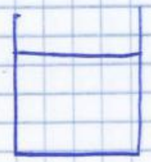
12.4. Gas ideale
in un cilindro con pistone



$V_1 = 10^{-3}\ m^3$
$p_1 = 2 \cdot 10^5\ Pa$
$T_1 = 300\ K$ inizi pistone bloccato

$p^{ex} = 1\ bar = 1 \cdot 10^5\ Pa$
T_1 Temperat ambiente

Lascio libero pistone $\rightarrow T$ cost

ΔQ? ΔS_{sist}? ΔS_a?

$V_1 p_1 = n R_1 T_1$

$$n = \frac{V_1 p_1}{R_1 T_1} = \frac{10^{-3} \cdot 2 \cdot 10^5}{8{,}31 \cdot 300} = 0{,}08\ mol$$

isoterma irreversibile

$$\left[\Delta Q = nRT_1 \ln \frac{p_A}{p_B}\right]$$

$\Delta U = 0$
$\Delta Q = \Delta L = p\,(V_2 - V_1)$

isoterma $V_1 P_1 = V_2 P_2$ $\quad V_2 = \frac{V_1 P_1}{P_2} = \frac{10^{-3} \cdot 2 \cdot 10^5}{1 \cdot 10^5}$

$P_2 = \cancel{p_1} - p_a^{ex} = (2-1)\,10^5$

$V_2 = 2 \cdot 10^{-3}\ m^3$

$$\Delta Q = p_2 (V_2 - V_1) = 1 \cdot 10^5 (2 \cdot 10^{-3} - 10^{-3}) = 100\ J$$

$$\Delta S_{sist} = nR \ln \frac{V_B}{V_A} = nR \ln 2 = 0{,}08 \cdot 8{,}31 \ln 2 = 0{,}46\ J/K$$

$$\Delta S_a = -\frac{Q}{T} = -\frac{100}{300} = -0{,}33\ J/K$$

12.7. $n = 5$ mol gas biatomico $\gamma = \frac{7}{5}$

$T_1 = 250\,°C = 523{,}15\ V_1$

espansione adiabatica

$V_1{}' = 4V_1$

↓ poi apro rubinetto

$V_2 = V_1{}'$

espansione libera

ΔU? ΔS?

$T_1 V_1{}^{\gamma} = T_2 V_1'{}^{\gamma}$

$T_1 V_1{}^{\gamma-1} = T_2 (4V_1)^{\gamma-1}$

$T_2^{\gamma-1} = \frac{(T_1 \cancel{V_1})^{\cancel{\gamma-1}}}{(4\cancel{V_1})^{\cancel{\gamma-1}}}$

$T_2 = 300\ K$

$\Delta S = 0$ rev

$\Delta U = n\, c_V\, \Delta T = 5 \cdot \frac{5}{2} R \cdot (300 - 523{,}15) = -23\ kJ$

poi espansione libera ΔT cost

$\Delta U = 0$

$\Delta S = nR \ln \frac{2V_1'}{V_1'} = 28{,}8\ J/K$

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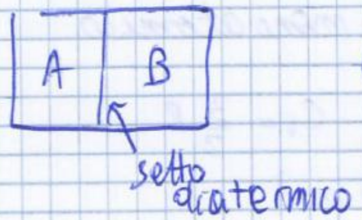
12.3 Contenitore pareti adiabatiche



(A) $2V_0$, $n = 0{,}57$ mol
gas ideale monoatomico
$c_p = \frac{5}{2}R$ $c_v = \frac{3}{2}R$
T_0

(2) $V_0 = 3{,}24 \cdot 10^{-3}\ m^3$
$n = 0{,}57$ mol
T_0

cedo calore porto a temperatura T
↓ equilibrio

aumento $\Delta p(A) = 1{,}033 \cdot 10^5$ Pa
$\Delta p(B) = -1{,}054 \cdot 10^5$ Pa
T_0? T? Q?

(1) $p_A = \frac{nRT_0}{2V_0}$

(2) $p_B = \frac{nRT_0}{V_0}$

apro foro $p = \frac{2nRT}{2V_0 + V_0}$

$$\begin{cases} \Delta P_A = \frac{2nRT}{3V_0} - \frac{nRT_0}{2V_0} = \frac{nR}{V_0}\left(\frac{2}{3}T - \frac{1}{2}T_0\right) \\ \Delta P_B = \frac{2nRT}{3V_0} - \frac{nRT_0}{V_0} = \frac{nR}{V_0}\left(\frac{2}{3}T - 1T_0\right) \end{cases}$$

$T = 320$ K
$T_0 = 285{,}4$ K

$Q = \Delta U = 2nc_v\Delta T = 2(0{,}57)\frac{3}{2}8{,}31 \cdot (320 - 285{,}4) = 492$ J

isocora irrev $\Delta S = nc_v \ln\frac{T_0}{T_0} + \frac{Q}{T_0} = 9 \cdot 10^{-3}$ J/K

$\Delta S_{sist} = 0 \rightarrow$ PER ESPANSIONE LIBERA REVERSIBILE

$$\underbrace{n C_V R \ln \frac{T_f}{T_i} + n R \ln \frac{V_f}{V_i}}_{A} + \underbrace{n c_v \ln \frac{T_f}{T_i}}_{B} = 0$$

ISOTERMA espansione libera

$$\cancel{2} \cdot \frac{3}{\cancel{2}} \cancel{R} \ln \frac{T_f}{T_i} + 2 R \ln \frac{V_f}{V_i} + 1 \cdot \frac{5}{2} R \ln \frac{T_f}{T_i} = 0$$

799   567

$$\left(3 \ln \frac{T_f}{T_i} + \frac{5}{2} \ln\left(\frac{T_f}{T_i}\right)\right) = -2 \ln \frac{V_f}{V_i}$$

799   567

$$3 \ln \frac{T_f}{799} + \frac{5}{2} \ln \frac{T_f}{567} = -2 \ln \frac{20}{5}$$

$$3 \frac{T_f}{799} + \frac{5}{2} \frac{T_f}{567} = e^{\underbrace{-2 \ln \frac{20}{5}}_{0,065}}$$

$$T_f = 0,065 \cdot \left(\frac{799}{3} + \frac{567 \cdot 2}{5}\right) = 308,2 \text{ K}$$

$$\Delta S = n_A c_v \ln \frac{T_f}{T_0} + n_B c_v \ln \frac{T_f}{T_0} = 2 \frac{3}{2} R \ln \frac{308}{280} + 1 \cdot \frac{5}{2} R \ln \frac{308}{280} =$$

$$= 5,7 \text{ J/K}$$

ISOCORA

12.12. Cilindro pareti adiabatiche
$V_0 = 72 \cdot 10^{-3}\ m^3$

gas ideale in due metà
$c_v = 16{,}6\ J/mol\,K$
$p_0 = 1 \cdot 10^5$ ~~atm~~ Pa
$T_0 = 0\,°C = 273{,}15\ K$

Compressione ~~ISOCORA~~ reversibile
$p_B = 2 \cdot 10^5\ Pa$
ΔS_A? ΔV? $\mathcal{L}$?

REVERSIBILE
$\Delta S = 0$
$\Delta S_A + \Delta S_B = 0$

$n = \frac{p_0 V_0}{R T_0} = 3{,}17$ in tutto il sist

$\Delta S_A = -\Delta S_B$

$n_A = n_B = \frac{3{,}17}{2} = 1{,}58$ mol

$n c_v \ln \frac{T_f}{T_0} + nRT \ln \frac{V_A}{V_0} = -nR \ln \frac{T_B}{T_0}$

isocora reversibile $\frac{T_f}{T_0} = \frac{p_B}{p_0}$ $T_B = \frac{p_B}{p_0} \cdot T_0 = 2 \cdot 273{,}15 = 546{,}3\ K$

$\Delta S_A = -\Delta S_B$

$\Delta S_B = nR\ln \frac{T_B}{T_0} = 1{,}58 \cdot 16{,}6 \cdot \ln \frac{546}{273} = 18{,}2$

$\Delta S_A = -18{,}1$

$n c_v \ln \frac{T_f}{T_0} - nR \ln \frac{V_A}{V_0} = -18{,}1$

$-nR \ln \frac{V_A}{V_0} = -18{,}1 - n c_v \ln \frac{T_f}{T_0} =$

$nR \ln \frac{V_A}{V_0} = 18{,}1 + 1{,}58 \cdot 16{,}6 \ln \left(\frac{546}{273}\right) = 36{,}27$

$\ln \frac{V_A}{V_0} = -2{,}76$ $\frac{V_A}{V_0} = 0{,}063$

$V_A = 0{,}063 \cdot 36 \cdot 10^{-3} = 2{,}27 \cdot 10^{-3}\ m^3$

$(V_A - V_0) = (2{,}27 \cdot 10^{-3}) - (36 \cdot 10^{-3}) = -33{,}7 \cdot 10^{-3}\ m^3$

$\mathcal{L} = p\Delta V = -\Delta U = -2 n c_v (T_f - T_0) = -2 \cdot 1{,}58 \cdot 16{,}6 \cdot (546 - 273)$
adiabatico $= -15\ kJ$