NUMERO: 1212 DATA: 27/10/2014

A P P U N T I

STUDENTE: Bettale

MATERIA: Elettrotecnica, Temi D'Esame + Eserc.

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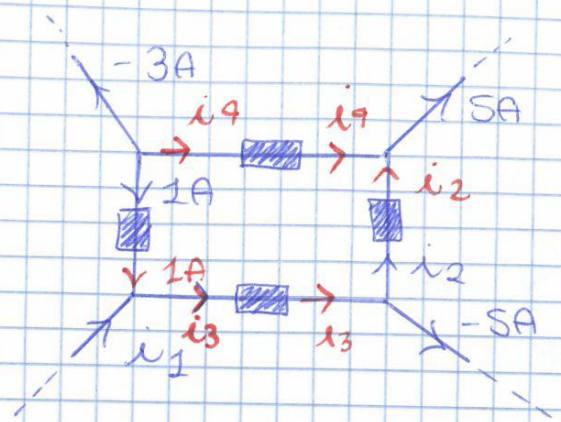
TELEFONO ING. BIOMEDICA

③ KCL (da sx)

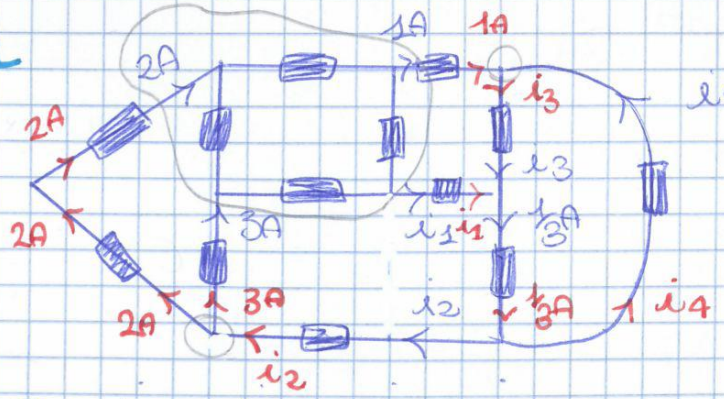


RISPOSTA
$i_1 = -3A$
$i_2 = 3A$

CALCOLO i_1 e i_2:

- $i_4 + 1 - 3 = 0$
  $i_4 = 2\,A$
- $i_2 + i_4 = 5$
  $i_2 + 2 = 5$
  $i_2 = 3A$
- $i_3 = i_2 - 5$
  $i_3 = 3 - 5 = -2$
- $i_1 + 1 = i_3$
  $i_1 = -3A$

④ KCL



$i_1 = 4A$

CALCOLO i_1, i_2, i_3, i_4:

- CERCO SUPERFICI CHIUSE X i_1
  $i_2 = 2 + 3 = 5A$
  $i_4 - 5 = \frac{1}{3}$ $\quad i_4 = -\frac{14}{3}A$
  $i_3 = 1 - \frac{14}{3} = -\frac{11}{3}A$

$\left.\begin{array}{l} i_1 - \frac{11}{3} = \frac{1}{3} \\ i_1 - \frac{12}{3} = 4A \end{array}\right\}$

CALCOLO V_t, V_y, V_x, V_w

$V_w + 2 = 1 + 1$

$V_w = 0\,V$ ✓

$V_t - 3 = 1$

$V_t = 4\,V$ ✓

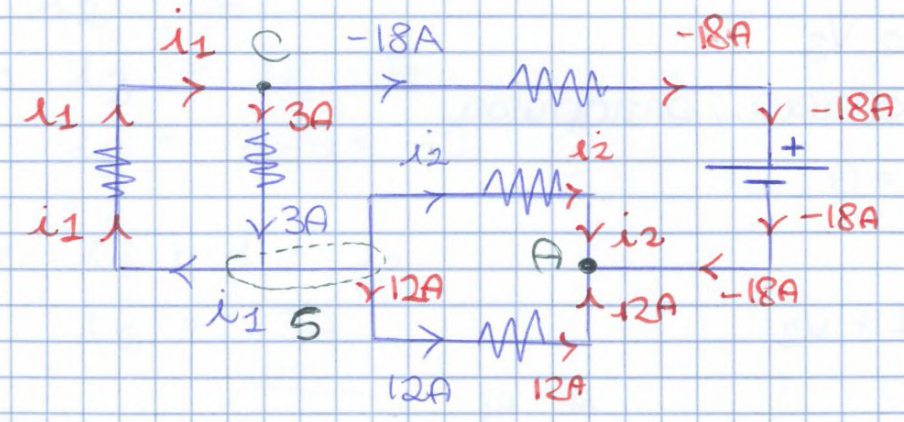
$V_y + V_t = 2$

$V_y = -2\,V$ ✓

$V_x + 1 = 2 + V_y$

$V_x = 1 - 2 = -1\,V$ ✓

⊕ KCL



CALCOLO i_1 e i_2:

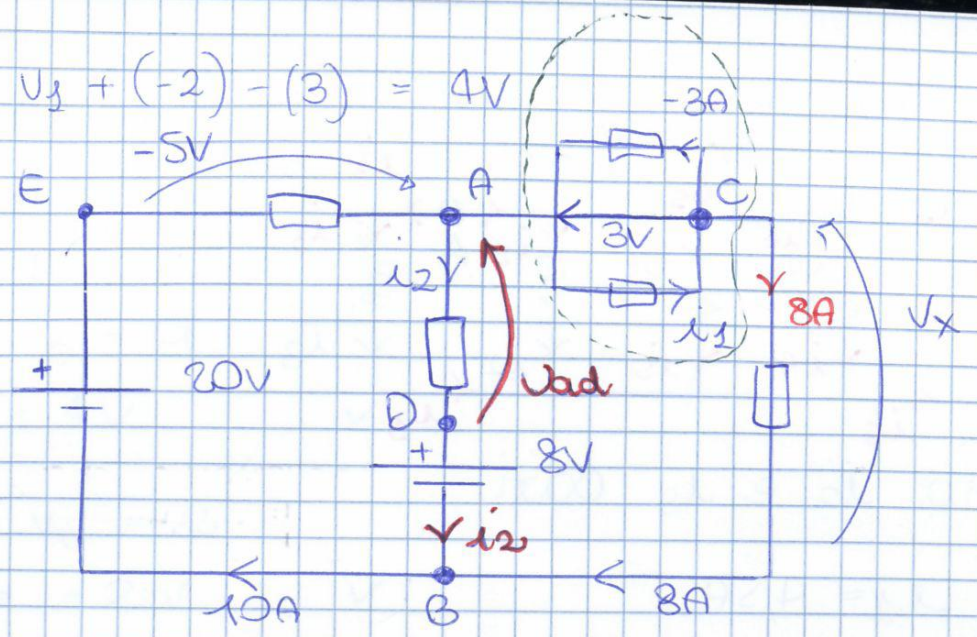
Ⓐ $i_2 + 12 - 18 = 0$ NODO A

$i_2 = 6A$

Ⓢ $i_1 = 3 - i_2 - 12$ SUPERFICIE S

$i_1 = -15A$ ATTORNO NODO C

$V_{bc} = V_1 + (-2) - (3) = 4V$



10

KCL +

KVL

CALCOLO i_1, i_2, V_{ad}, V_x

$i_2 + 8 = 10$

$i_2 = 2A$

$i_1 = 8 - 3 = 5A$

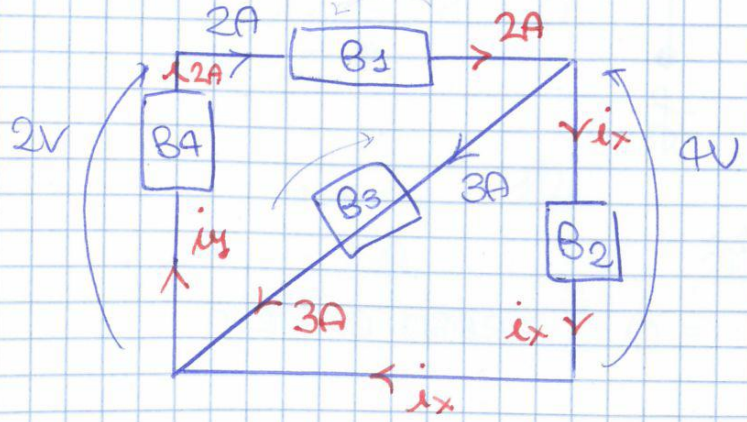
(B) $20 - 5 = V_{ad} + 8$

$V_{ad} = 7V$

(CB) $-5 + 20 = 3 + V_x$

$V_x = 12V$

1) CALCOLO (POTENZE)



RISPOSTE:

$p_1 = -4W$

$p_2 = -4W$

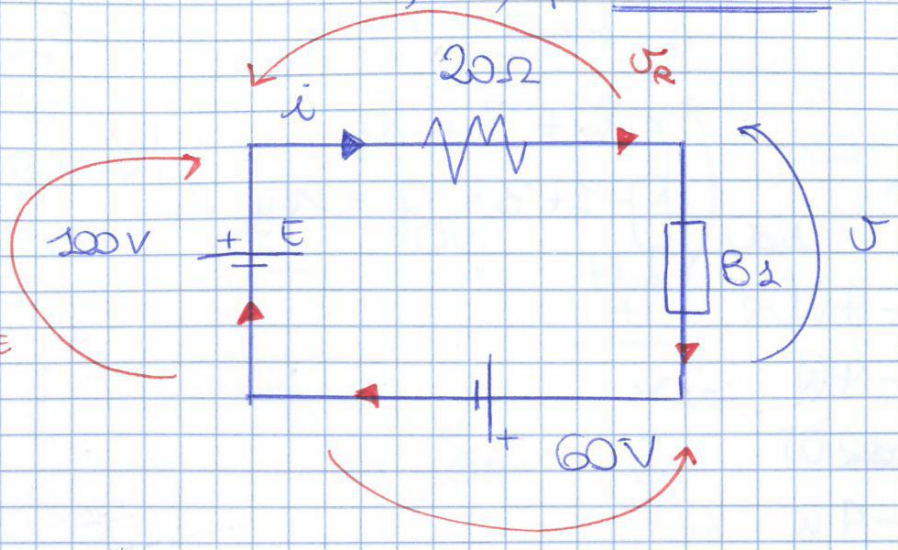
$p_3 = 12W$

$p_4 = -4W$

TOT = 0

CALCOLO LA POTENZA FORNITA A B1, B2, B3, B4

12) E EROGA $p_E = 100\,W$.
CALCOLO V, i, p ASSORBITA DA B_1



$P_E = V_E\, i$

$i = \frac{P_E}{V_E} = \frac{100}{100} = 1A$ ✓

$V_R = iR = 1 \cdot 20 = 20\,V$

$V + 60 + 20 - 100 = 0$

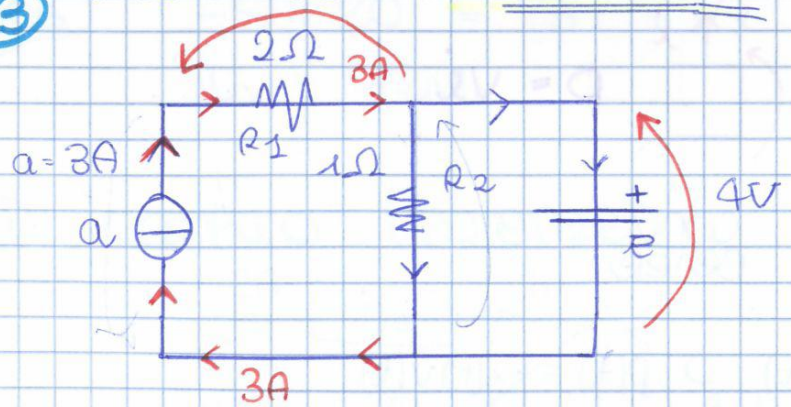
$V = 20\,V$ ✓

$P_{b_1} = +V i = 20\,W$ ✓

VERIFICA CON BILANCIO POTENZE

$P_R = 20\,W$	}	TOT = 0
$P_{B_1} = 20\,W$		
$P_E = -100\,W$		
$P_{E_2} = 60\,W$		

13) CALCOLO POTENZA FORNITA AD OGNI ELEMENTO



$P_{R_1} = i^2 R_1 = 9 \cdot 2 = 18\,W$ ✓

$V_{R_1} = i R_1 = 6\,V$

$P_{R_2} = 4$ $\quad i_{R_2} = \frac{V_{R_2}}{R_2} = 4A$

$P_{R_2} = i^2 R_1 = 16\,W$ ✓

$P_a = V \cdot i = 3(-4-6) = -30\,W$ ✓

$P_e = V \cdot i = 4(3-4) = -4\,W$ ✓

$18 + 16 = -(-30-4)$

$34 = 34$ ✓

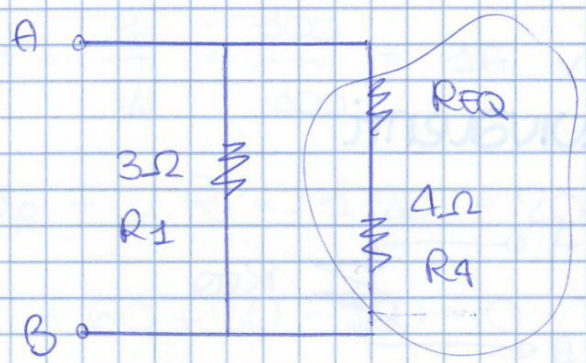
R_2 e R_4 non sono in serie xke non sono percorsi dalla stessa corrente

R_2 e R_4 non sono in // xke non hanno i 2 terminali in comune

R_2 e R_3 sono in //

$$\frac{1}{R_{EQ}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6} + \frac{1}{3} = \frac{1+2}{6} = \frac{1}{2}\ \Omega$$

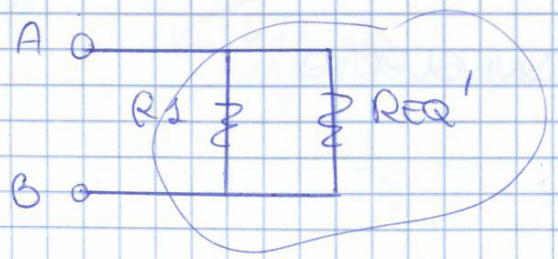
$R_{EQ} = 2\ \Omega$



serie

R_{EQ} e R_4 sono in serie

$R_{EQ}' = R_{EQ} + R_4 = 2 + 4 = 6\ \Omega$

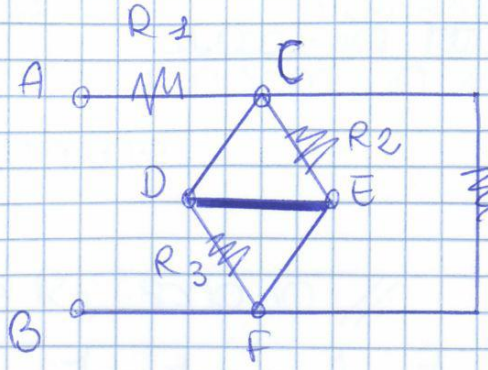


// R_{EQ}' e R_1 sono in //

$$\frac{1}{R_{EQ}''} = \frac{1}{R_{EQ}'} + \frac{1}{R_1} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}\ \Omega$$

$R_{EQ}'' = 2\ \Omega$

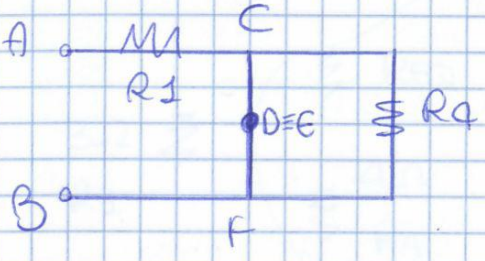
③ Resistenza tra A e B:



E ≡ D !!!

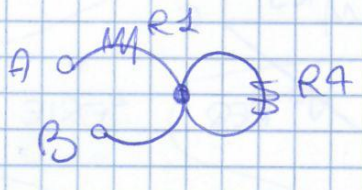
Se R ha un ∥ un corto circuito
→ la resistenza equivalente = 0

R_2 e R_3 si annullano! ∥ con corto circuiti



R_4 si annulla, xke ∥ con corto circuito

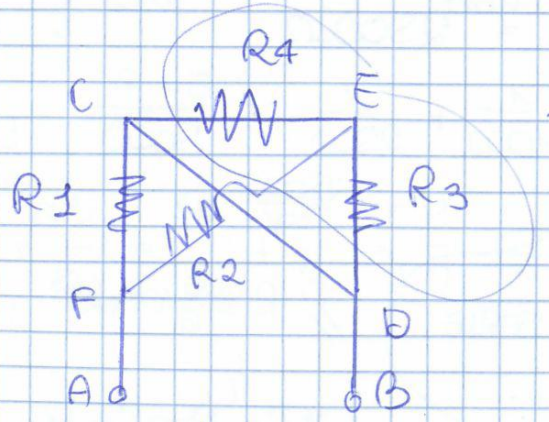
xk C ≡ D ≡ E ≡ F !



$R_{AB} = R_1$

④ Calcolo R_{AB} con

$R_1 = 10\Omega$, $R_2 = 20\Omega$, $R_3 = 40\Omega$, $R_4 = 40\Omega$.

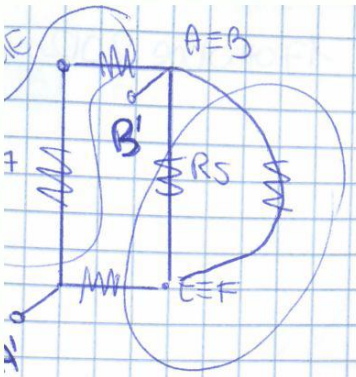


∥

C ≡ D

$$\frac{1}{R_{EQ}} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{40} + \frac{1}{40} = \frac{2}{40} = \frac{1}{20}\ \Omega$$

$R_{EQ} = 20\Omega$



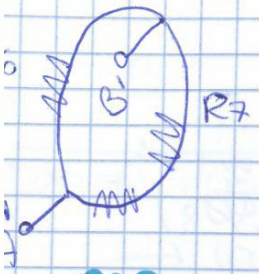
- $R_6 = R_4 + R = \frac{2}{3} + 1 = \frac{5}{3}\,\Omega$

$\frac{1}{R_7} = \frac{1}{R_5} + \frac{1}{R} = 1 + 1 = 2\,\Omega$

- $R_7 = \frac{1}{2}\,\Omega$
- $R_8 = R_7 + 1 = \frac{1}{2} + 1 = \frac{3}{2}\,\Omega$

$\frac{1}{R_{AB}} = \frac{1}{R_6} + \frac{1}{R_8} = \frac{3}{5} + \frac{2}{3} = \frac{9+10}{15}\,\Omega$

$R_{AB} = \frac{15}{19}\,\Omega$ ✓

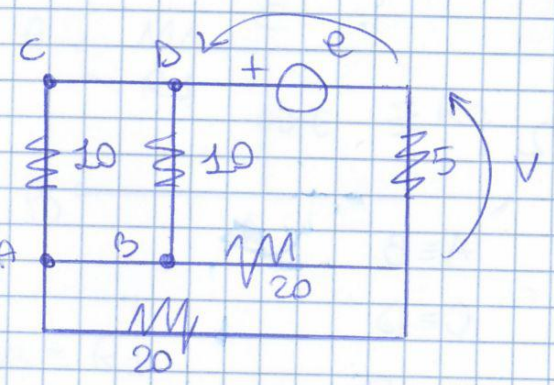


RICORDO $R_1 // R_2$

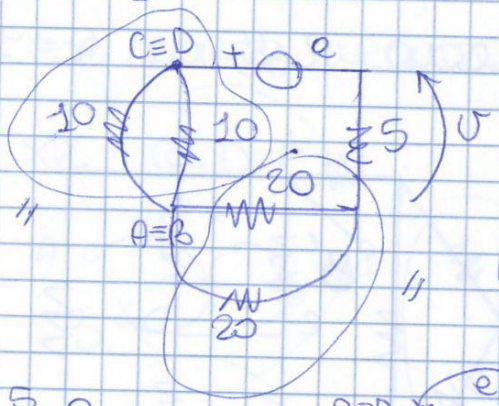
$R_{EQ//} = \frac{R_1 R_2}{R_1 + R_2}$

PARTITORE DI CORRENTE

5) CALCOLO $v(t)$ PRODOTTA DA $e(t) = E_0 \sin(\omega t + \theta)$



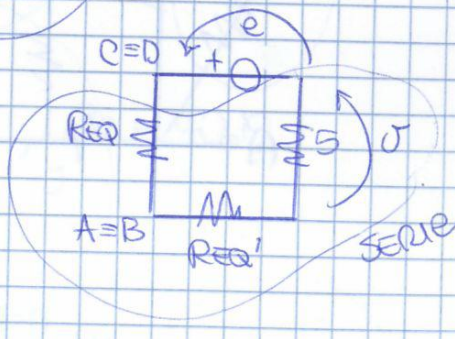
A=B C=D



TRASFORMO IL CIRCUITO MA NON REALMENTE QUELLO DEVO TROVARE LA MIA INCOGNITA!

$\frac{1}{R_{EQ}} = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}\,\Omega \qquad R_{EQ} = 5\,\Omega$

$R_{EQ}' = \frac{1}{20} + \frac{1}{20} = \frac{1}{10}\,\Omega \qquad R_{EQ}' = 10\,\Omega$

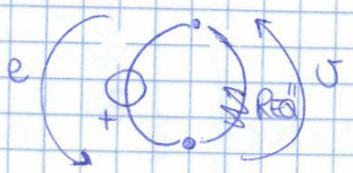


CIRCUITO DI GENERATORE DI TENSIONE E RESISTORI IN SERIE!!

$v = -e\,\frac{5}{5 + R_{EQ} + R_{EQ}'} = -e\,\frac{5}{20} = -\frac{e}{4}$

$v(t) = -\frac{E_0}{4}\sin(\omega t + \theta)$

IL MENO È XKÉ LA TENSIONE DEVE ESSERE NEL SENSO OPPOSTO AL GENERATORE!



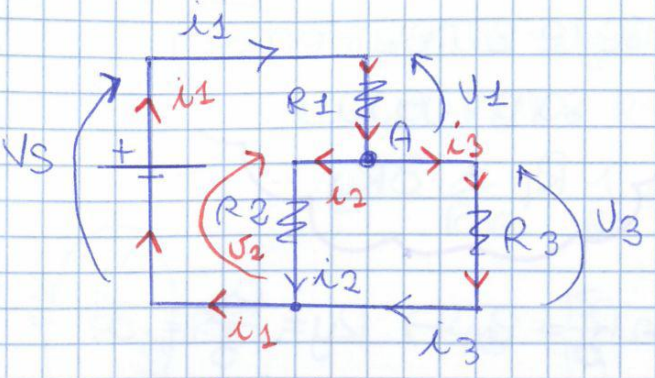
$R_{EQ}'' = 5 + R_{EQ} + R_{EQ}' = 20\,\Omega$

$$v_0 = \frac{R_i}{\Sigma R_k}\,e$$

③ PARTITORE DI TENSIONE

② USO IL PARTITORE DI TENSIONE (DOPO $R_2 // R_3$) X TROVARE V_1 E V_3

CALCOLO V e i INDICATE MEDIANTE PARTITORI



$R_1 = 19\,\Omega$ $\quad i_1 = ?$

$R_2 = 30\,\Omega$ $\quad i_3 = ?$ $\quad i_2 = ?$

$R_3 = 70\,\Omega$ $\quad V_1 = ?$

$V_S = 60\,V$ $\quad V_3 = ?$

$i_2 = V_3$ $\quad R_2 // R_3$

① FACCIO LA RESISTENZA EQUIVALENTE E CALCOLO i_1 COSÌ TROVO V_1 KVL X V_3, POI OHM X i_2

$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{30} + \frac{1}{70} = \frac{7+3}{210} = \frac{10}{210} = \frac{1}{21}$ $\quad R_{23} = 21\,\Omega$

R_1 SERIE R_{23} $\quad R_{123} = R_1 + R_{23} = 19 + 21 = 40\,\Omega$!

$i_1 = \frac{V_S}{40} = \frac{60}{40} = \frac{3}{2}A$ $\quad \rightarrow V_1 = R_1 \frac{V_S}{R_1 + R_{23}} = 19 \frac{60^3}{40_2} = \frac{57}{2}\,V$

$V_1 = R_1 i_1$

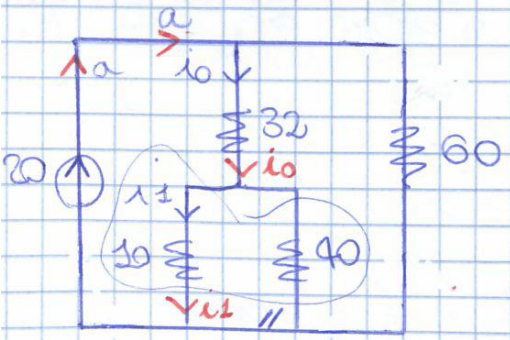
$V_{23} + V_1 - V_S = 0$ $\quad V_{23} = V_3 = 60V - \frac{57}{2}V = \frac{63}{2}V$

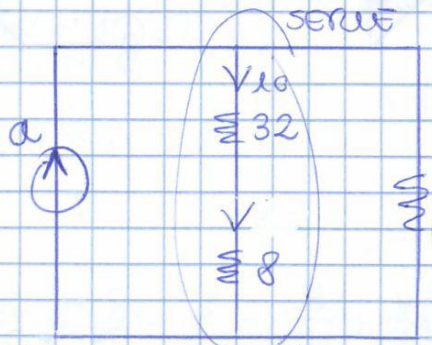
$i_3 = \frac{V_{23}}{R_3} = \frac{63/2}{70} = \frac{9}{20}\,A$ $\quad i_2 = \frac{V_{23}}{R_2} = \frac{63/2}{30} = \frac{21}{20}\,A$

NON NECESSARIO

PARTITORE DI CORRENTE

③ i_1 MEDIANTE PARTIZIONI DI CORRENTE



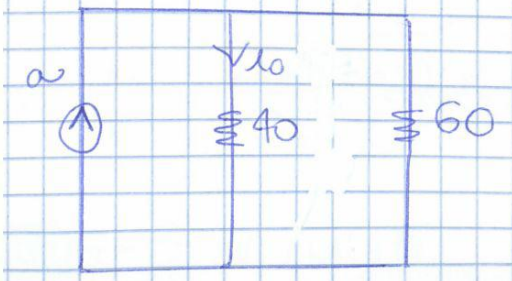


$\frac{1}{R_{EQ}} = \frac{1}{10} + \frac{1}{40} = \frac{4+1}{40} = \frac{1}{8}$

$R_{EQ} = 8\,\Omega$

$R_{EQ}' = 32 + 8 = 40\,\Omega$

$\frac{1}{R_{EQ//}} = \frac{1}{40} + \frac{1}{60} = \frac{3+2}{120} = \frac{1}{24}\,\Omega$

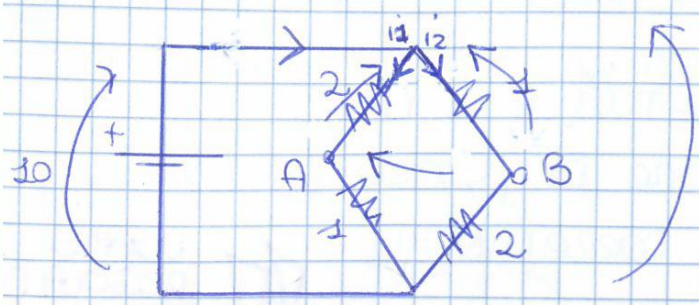


$i_0 = \frac{a}{40} \cdot 24^{12 \cdot 3} = \frac{20}{8} \cdot 3 = 12\,A$

$20 \cdot 2 = 5$

LA CORRENTE DEVE AVERE IL SEGNO OPPOSTO A @ STRUMENTI (−)

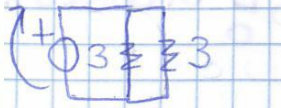
④ CALCOLARE V_{ab}



$V_{ab} = -10/3\ 0$

$V_{ab} = -3{,}33V$

CONTROLLO BENE FRECCE E VERSI ✓

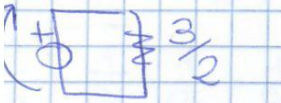


PARTITORE CORRENTE

$i_1 = i\frac{3}{6} = \frac{1}{2}\frac{20}{3} = \frac{10}{3}$

$(i = i_1 + i_2 = 20/3)$

$i_2 = \frac{10}{3}$



$V_1 = i_1 \cdot 1 = \frac{10}{3}$

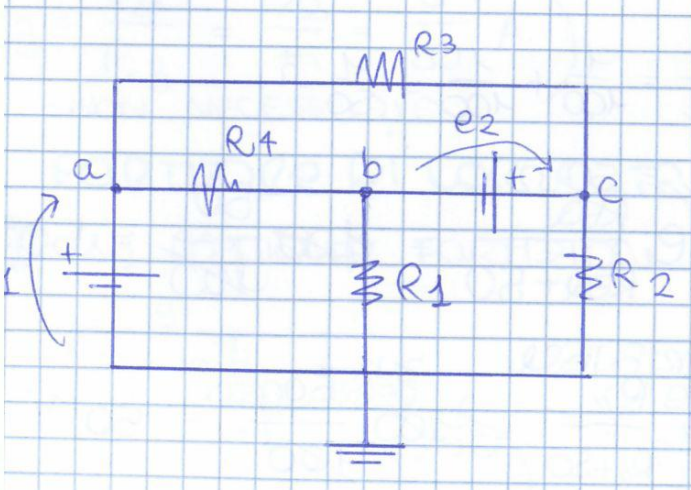
$V_2 = \frac{20}{3}$

$i = \frac{10 \cdot 2}{3} = \frac{20}{3}$

$V_{ab} = V_1 - V_2 = \frac{10}{3}$ ✓

SOVRAPPOSIZIONE EFFETTI

② CALCOLARE V_a, V_b, V_c MEDIANTE IL TEOREMA DI SOVRAPPOSIZIONE DEGLI EFFETTI



$E_1 = 12V$

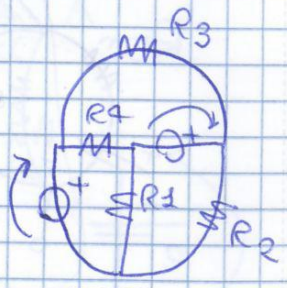
$E_2 = 40V$

$R_1 = 12\Omega$

$R_2 = 8\Omega$

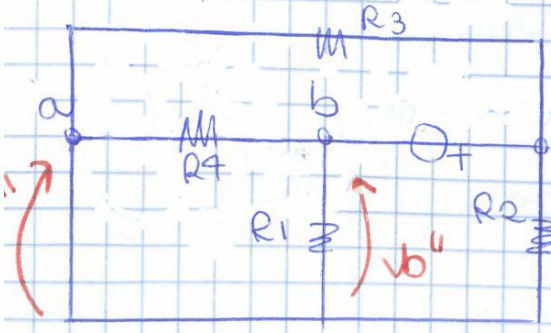
$R_3 = 8\Omega$

$R_4 = 2\Omega$



VISTA MIGLIORE

ANNULLO e_1 $(e_1 = 0)$ → CORTOCIRCUITO



$V_b'' = 0$

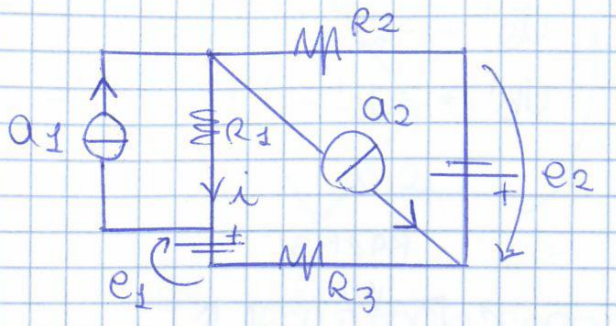
$R_4 /\!/ R_1 = \frac{24}{14} = \frac{12}{7}\Omega$

13 i con sovrapposizione effetti

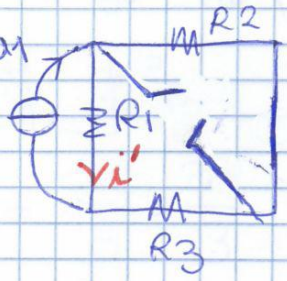
$a_1 = 2A$ $a_2 = 4A$ $e_1 = 120V$ $e_2 = 80V$

$R_1 = 10\Omega$ $R_2 = 20\Omega$ $R_3 = 40\Omega$

4 generatori → posso annullare 3 → posso annullare 2



annullo e_1, e_2, a_2

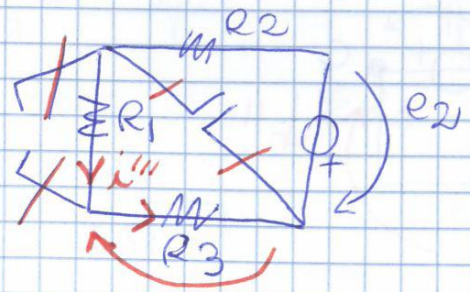


$R_2 + R_3 = 60\Omega$

partitore corrente

$$i' = a_1 \frac{(R_2 + R_3)}{R_1 + R_2 + R_3} = 2 \cdot \frac{60}{10} = \frac{12}{7} A \quad \checkmark$$

annullo a_1, a_2, e_1



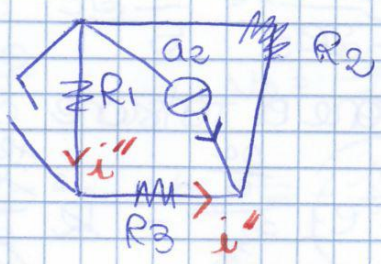
partitore di tensione

$$i'' = \frac{-V_3}{R_3} = \frac{1}{R_3} e_2 \frac{-R_3}{R_1 + R_2 + R_3}$$

$$i'' = \frac{-80}{10} = \frac{-8}{7} A \quad \checkmark$$

$$i = \frac{12}{7} - \frac{12}{7} - \frac{8}{7} - \frac{8}{7} = \left(-\frac{16}{7} A\right) \quad \checkmark$$

annullo e_1, e_2, a_1

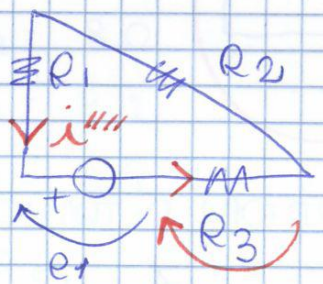


$R_1 + R_3 = 50\Omega$

partitore corrente

$$i'' = -4 \cdot \frac{20}{R_1 + R_3 + R_2} = \left(-\frac{8}{10}\right) \quad \checkmark$$

annullo a_1, a_2, e_2



partitore di tensione

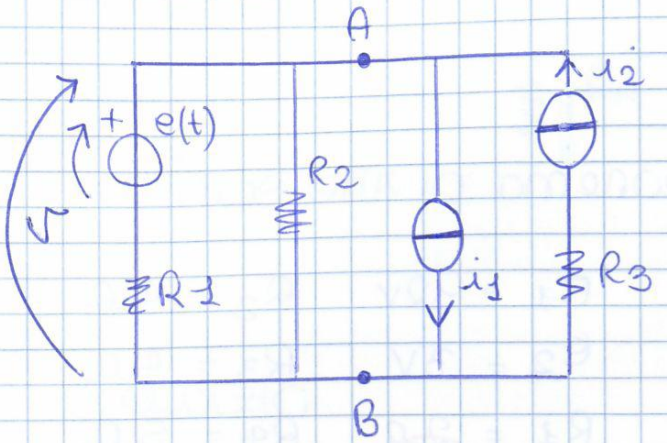
$$i'''' = \frac{-V_3}{R_3} = -\frac{e_1 R_3}{R_3 10} =$$

$$i'''' = -\frac{120}{10} = \left(-\frac{12}{7} A\right) \quad \checkmark$$

CAPITOLO 3

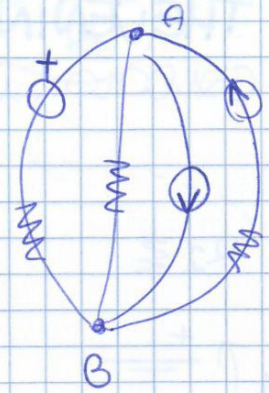
① V con MILLMANN

ELEMENTI IN PARALLELO SERIE x CALCOLARE AB
GEN. COR + RES = E' EQUIVALENTE AL SOLO GENERATORE DI CORR.
G.T + RES. RES. G.C (+ RES ✓



$R_1 = \frac{1}{2}\Omega$
$R_2 = \frac{1}{3}\Omega$
$R_3 = 1\Omega$
$i_1(t) = 7A$
$i_2(t) = 2A$
$e(t) = 10V$

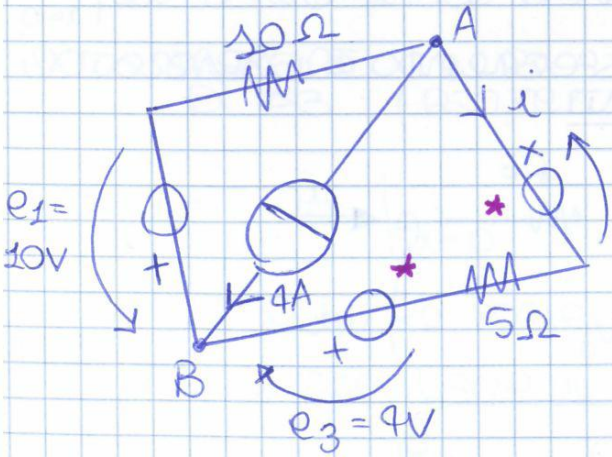
VISTO MEGLIO



CIRCUITO IN // CON RAMI IN SERIE:

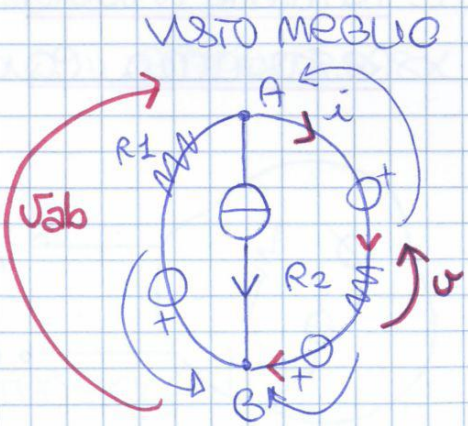
$$V = V_{ab} = \frac{\frac{e}{R_1} + i_2 - i_1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{20 + 2 - 7}{2 + 3} = \frac{15}{5} = 3V \checkmark$$

② i con MILLMANN



I 2 GENERATORI IN SERIE POSSO SOSTITUIRLI CON UNO COSÌ DIVIDERE ϕ ↑
2 − 4 = −2V

VISTO MEGLIO



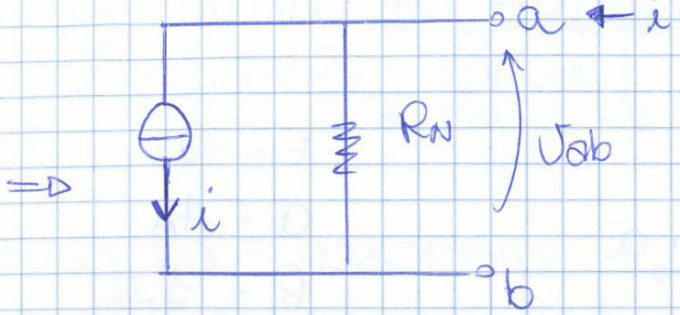
CIRCUITO IN // CON RAMI IN SERIE:

$$V_{ab} = \frac{-\frac{e_1}{R_1} - a_1 + \frac{e_2}{R_2} - \frac{e_3}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{-1 - 4 + \frac{2}{5} - \frac{4}{5}}{\frac{1}{10} + \frac{1}{5}} = \frac{\frac{-25 + 2 - 4}{5}}{\frac{1+2}{10}} =$$

$$V_{ab} = -\frac{27}{5} \cdot \frac{10}{3}^{2} = -18V \checkmark$$

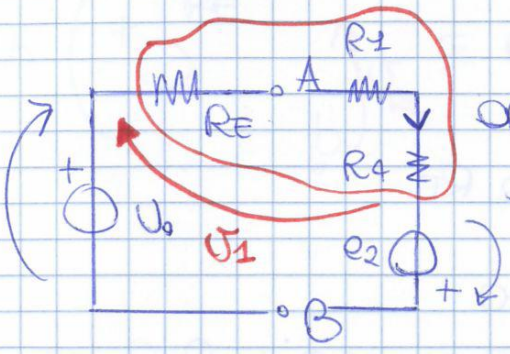
CORRENTE CORTOCIRCUITO: CORRENTE SU A CON $V_{ab}=0$ $i_N = i\big|_{V_{ab}=0}$

RESISTENZA EQ: CALCOLATA SPEGNENDO TUTTI I GENERATORI INDIPENDENTI $R_E = R_N$



$$i_n = \frac{V_0}{R_E}$$

ESERCIZIO



ORA CALCOLO V e RE

i SONO NELLA SERIE RE/R1/R4

$V_1 = V_0 - e_2$

$$i = \frac{V_1}{R_E+R_1+R_4} = \frac{V_0-e_2}{R_E+R_1+R_4} = \frac{V_0-3}{R_E+7} = \frac{6-3}{2+7} = \frac{3}{9}$$

× TROVARE V e RE, CONSIDERO SOLO IL CIRCUITO A SX DI AB (PENTE' VERDE 4-)

* $V_0 = V_{ab}\big|_{i=0}$

SE $i=0$, R_2 serie R_3 → 8Ω

POSSO APPLICARE IL PARTITORE DI TENSIONE!

• $V_{ab} = e_1 \frac{R_3}{8} = {}^{6}\!\not{12}\,\frac{4}{\not{8}_{2}} = 6V = V_0$

* R TRA AB CON I GENERATORI SPENTI:

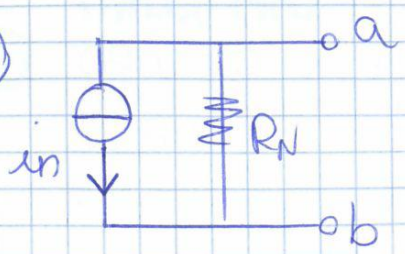
SE $e_1=0$ (CORTOCIRCUITO) $R_2 // R_3 \rightarrow \frac{4\cdot4}{8} = 2\Omega$

• $R_E = 2\Omega$

$$\Longrightarrow i = \frac{V-3}{R_E+7} = \frac{6-3}{9+7} = \frac{3}{9} = \frac{1}{3}\,A$$

SUL FOGLIO $i = 1A$

(b)

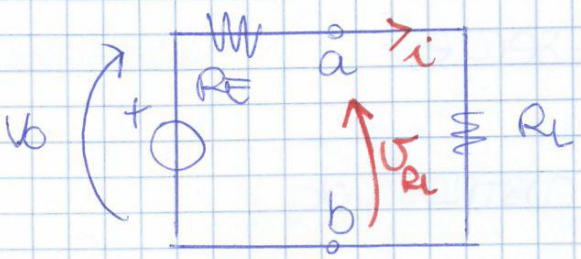


CALCOLO i_n e R_n:

★ $i_n = \frac{V_0}{R_E} = \frac{e - R a}{R} = \frac{8}{1} = 8\,A$ ✓

★ $R_n = R_e = R = 1\,\Omega$ ✓

(c) FORNITA DA = ASSORBITA (CON THEVENIN)



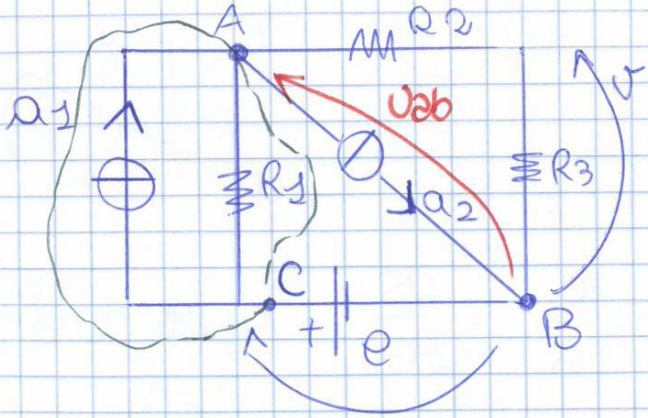
$P = i\,V_{RL} = R_L\,i^2$

con $i = \frac{V_0}{R_E + R_L}$

$P = R_L \left(\frac{V_0}{R_E + R_L}\right)^2 = R_L \frac{64}{(1+R_L)^2}$

se $R_L = 1\,\Omega \rightarrow P = 16\,W$

(6) RICAVARE LA CADUTA DI TENSIONE V.



$R_1 = 1\,\Omega$ $a_1 = 4\,A$

$R_2 = 2\,\Omega$ $a_2 = 5\,A$

$R_3 = 3\,\Omega$ $e = 6\,V$

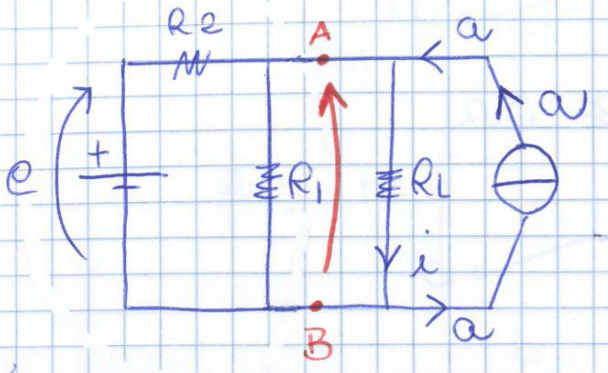
se conoscessi V_{ab} (applicata a R_2 serie R_3) potrei applicare il partitore di tensione

$V = \frac{R_3}{R_2 + R_3} V_{ab} = \frac{3}{5} V_{ab}$ quindi devo calcolare V_{ab}!

NON si può calcolare Millmann..

MILLMAN

⑥ CALCOLO i IN R_L IN FUNZIONE DEL VALORE DEL RESISTORE STESSO.

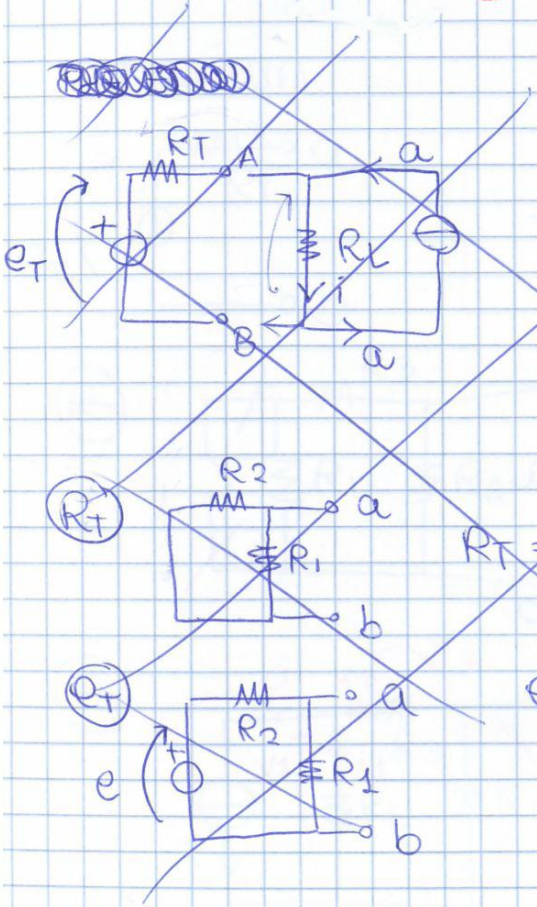


$E = 10\,V$

$a = 5A$

$R_1 = 4\,\Omega$

$R_2 = 6\,\Omega$



$i_1 = i - a$ $\qquad V_1 = (i-a)R_T$

$V_{R_L} = e_T - V_1 = e_T - iR_T + aR_T$

$iR_L = e_T - iR_T + aR_T$

$$i = \frac{e_T + aR_T}{R_L + R_T}$$

$$R_T = R_1 /\!/ R_2 = \frac{24}{10} = \frac{12}{5}\,\Omega$$

$e_T = V_{ab}$

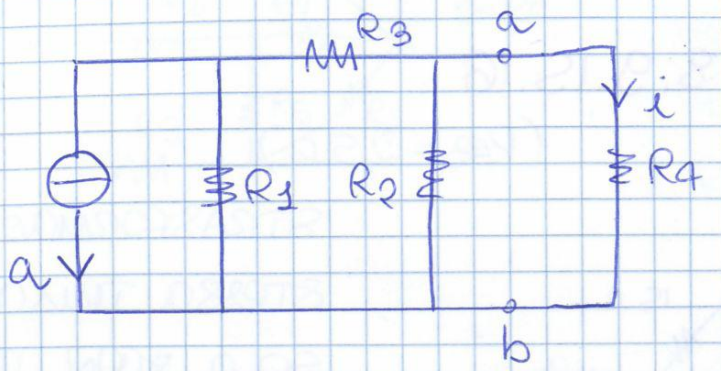
millman!

$$V_{ab} = \frac{\frac{E}{R_2} + a}{\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_L}} = \frac{\frac{10}{6}^{5} + 5}{\frac{1}{6} + \frac{1}{4} + \frac{1}{R_L}} = \frac{20}{3}\,\frac{12R_L}{(2R_L + 3R_L + 12)} = \frac{80R_L}{\cdots}$$

$$i = \frac{V_{ab}}{R_L} = \frac{80}{5R_L + 12}$$ ✓

⑧ THEVÉNIN

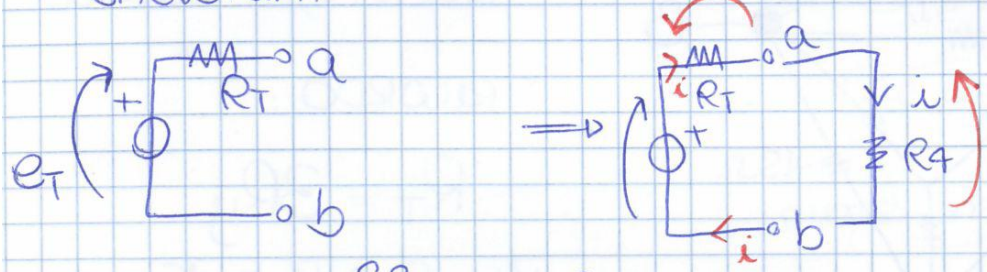
RICAVARE IL CIRCUITO EQUIVALENTE DI THEVÉNIN DEL BIPOLO A SX DI AB. TROVA i



$a = 1A$
$R_1 = 2\Omega$
$R_2 = 4\Omega$
$R_3 = 6\Omega$
$R_4 = 8\Omega$

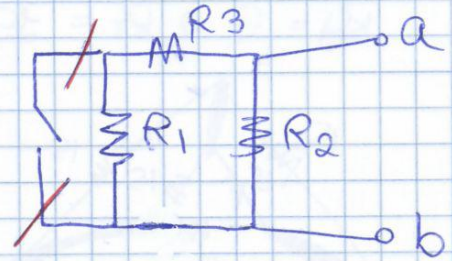
$e_T = -\frac{2}{3}V$
$R_T = 8/3\,\Omega$
$i = -\frac{1}{16}A$

Thevénin:



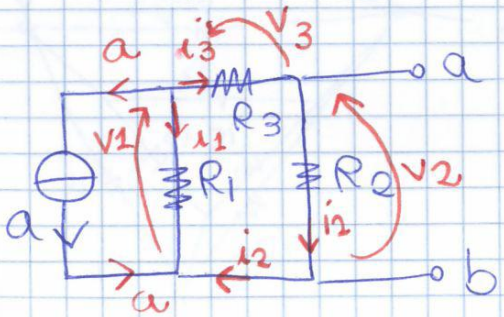
$e_T = iR_T + iR_4$
$i = \frac{e_T}{R_T + R_4}$
(DA PARTITORE TENSIONE)

(R_T)

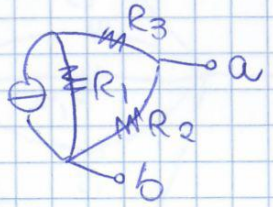


R_1 serie $R_3 = 8\Omega$
$8 // R_2 = \frac{32}{12} = \frac{8}{3}\Omega$
$R_{ab} = R_{eq} = \frac{8}{3}\Omega$ ✓

(e_T)



$V_{ab} = e_T = V_2$



$V_2 = V_1 - V_3 = i_1R_1 - i_3R_3 = i_1R_1 + aR_3 + i_1R_3$

$\begin{cases} i_2 - i_3 = -a - i_1 \\ i_3 = i_2 - -a - i_1 \end{cases}$

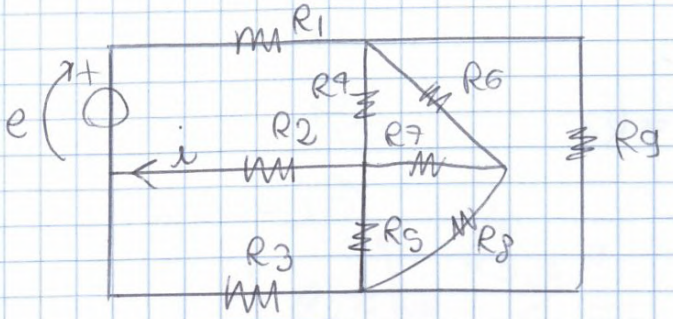
$\rightarrow V_2 = i_2R_2 = -aR_2 - i_1R_2$

$\Rightarrow -aR_2 - i_1R_2 = i_1R_1 + aR_3 + i_1R_3$

$i_1 = \frac{-a(R_2+R_3)}{R_1+R_2+R_3}$ → FORMULA PARTITORE DI TENSIONE!! $-\frac{10}{12} = -\frac{5}{6}A$

$i_2 = -1 + \frac{5}{6} = -\frac{1}{6}A \rightarrow V_2 = V_{ab} = -\frac{1}{6}\cdot 4 = -\frac{2}{3}V$

(29) calcolo i $(i = 20A)$



$R_1 = 5\,\Omega$ $R_5 = 30\,\Omega$

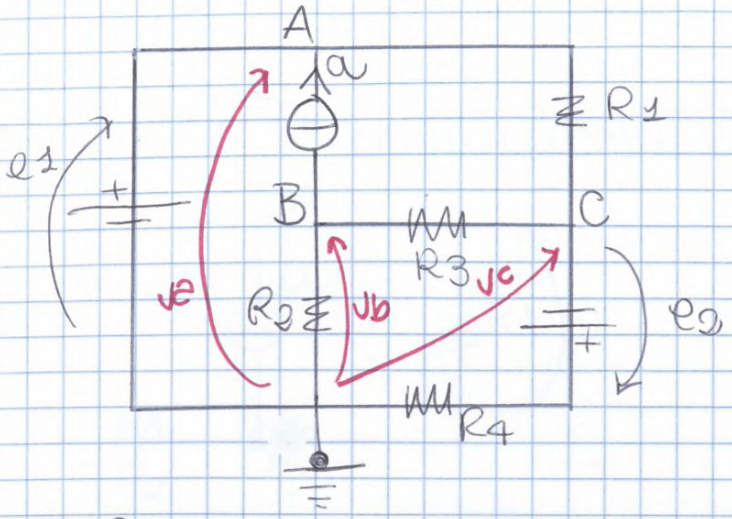
$R_2 = 5\,\Omega$ $R_6 = 10\,\Omega$

$R_3 = 5\,\Omega$ $R_7 = 10\,\Omega$

$R_4 = 30\,\Omega$ $R_8 = 10\,\Omega$

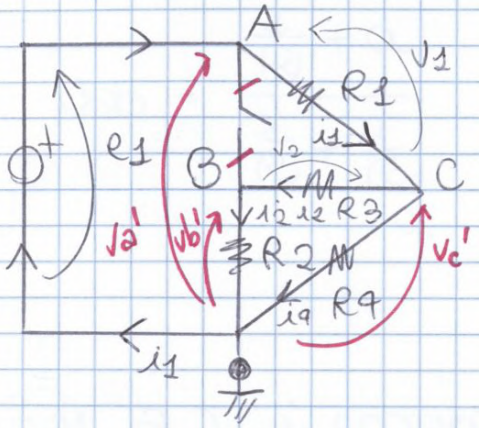
$R_g = 30\,\Omega$ $e = 300V$

(26) calcolo V_a, V_b, V_c con sovrapposizione effetti



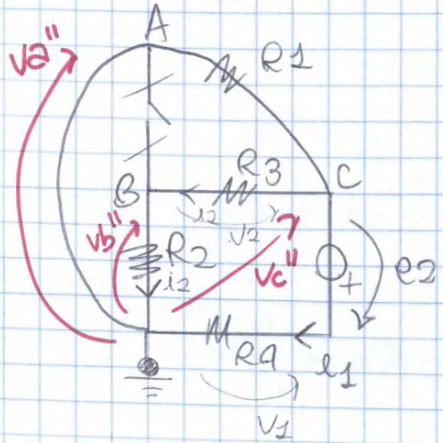
$(V_b = 6V)$
$(V_c = 16V)$
$a = 7A \quad R_1 = 1\Omega$
$e_1 = 60V \quad R_2 = 2\Omega$
$e_2 = 30V \quad R_3 = 30\Omega$
$R_4 = 10\Omega$

tengo e_1:



$V_b' = e_1 = 60V$
$V_b' = e_1 = 60 - V_1 + V_c'$
$60 = i_1 R_1 + i_4 R_4$
$i_1 = i_2 + i_4$

tengo e_2:

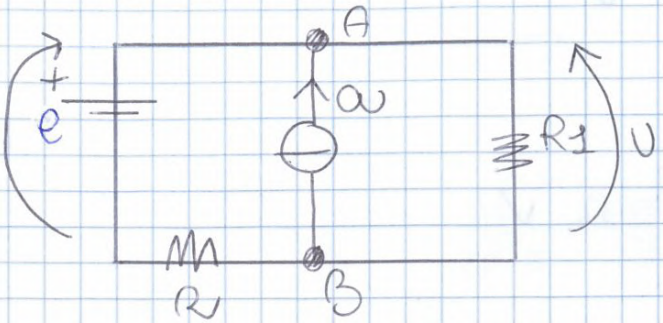


$V_b'' = 0$
$V_c'' + e_2 = V_1 = i_1 R_4$
$V_c'' = V_b'' + i_2 R_3 = i_2 R_2 + i_2 R_3$

NUOVI CAPITOLO ③. 3.4.7.8

(33) nella rete $R \to 0$

calcolo V con MILLMANN GIUSTIFICANDO IL RISULTATO



formula millmann

$$V_{ab} = \frac{\frac{e}{R_1} + a}{\frac{1}{R} + \frac{1}{R_1}}$$

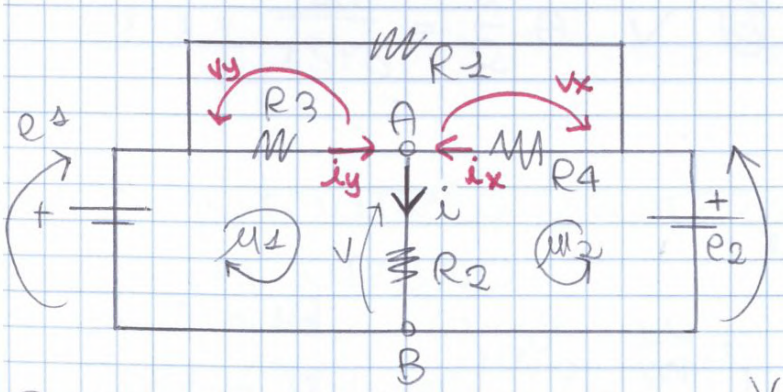
ma se $R \to 0$ $V_{ab} = e$ ✓

xk $R=0$, $V = iR = 0 \Rightarrow$ CORTOCIRCUITO

(34) a) i con KCL in A e KVL in M_1 e M_2

b) i se $R_1 = 7\,\Omega$

c) TOLGO R2, RICAVO L'EQ. THEVENIN DA AB RICALCOLO i CON L'EQUIVALENTE



$e_1 = 4V$ $R_1 = 2\Omega$

$e_2 = 8V$ $R_2 = 2\Omega$

$R_3 = 2\Omega$ $R_4 = 4\Omega$

② KCL: $i = i_x + i_y \to \frac{V}{R_2} = \frac{V_x}{R_4} + \frac{V_y}{R_3}$

KVL: $e_1 = V_y + V \to \frac{e_1}{R_2} - \frac{V_y}{R_2} = \frac{V_x}{R_4} + \frac{V_y}{R_3}$

$e_2 = V_x + V$

$V = e_1 - V_y = e_2 - V_x$

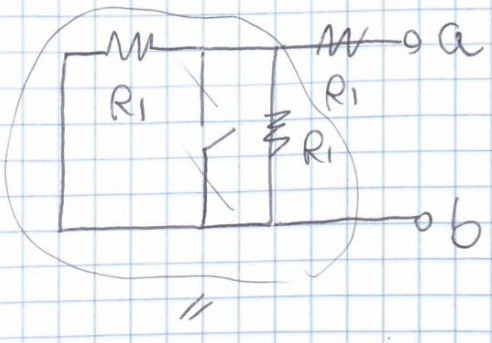
$V_y = V_x - e_2 + e_1$

$\frac{e_1}{R_2} - \frac{V_x}{R_2} + \frac{e_2}{R_2} - \frac{e_1}{R_2} = \frac{V_x}{R_4} + \frac{V_x}{R_3} - \frac{e_2}{R_3} + \frac{e}{R}$

$V_x\left(\frac{5}{4}\right) = 2 + 4 - 2 + 4 - 2 = 6\left(\frac{4}{5}\right) = \frac{24}{5}$

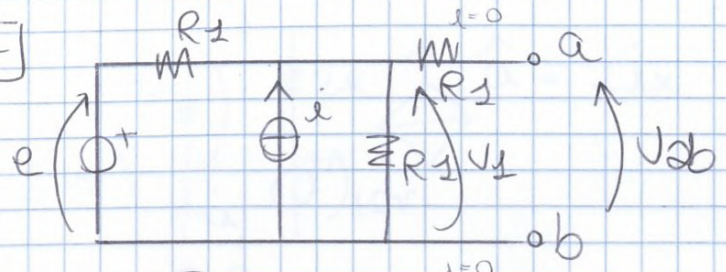
EQUIVALENTE di THEVENIN

R_T



$$R_T = R_1 /\!/ R_1 + R_1 = \frac{\frac{5}{100}}{20} + 10 = 15\,\Omega$$ ✓

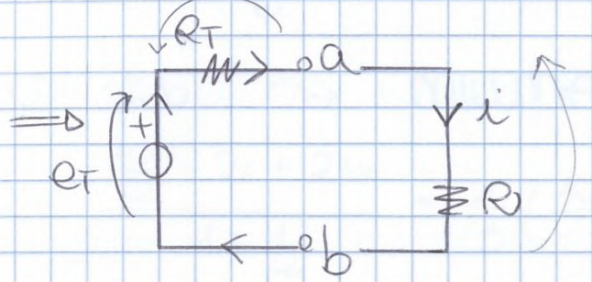
e_T



$V_{2b} = V_1$

METODO NN

$$V_1 = \frac{\frac{e}{R_1} + i}{\frac{1}{R_1} + \frac{1}{R_1}} = \frac{2}{\frac{1}{5}} = 10\,V$$ ✓

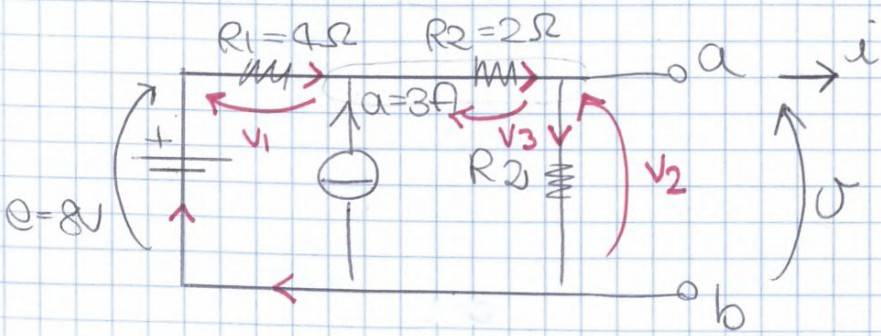


$$e_T = iR_T + iR \longrightarrow i = \frac{e_T}{R_T + R}$$ PARTITORE TENSIONE

(a) $i = \frac{10}{15+15} = \frac{1}{3}\,A$ ✓ (b) $i = \frac{10}{15+10} = \frac{2}{5}\,A$ ✓

(c) $i = \frac{10}{15+5} = \frac{1}{2}\,A$ ✓ (d) $i = \frac{10}{15+0} = \frac{2}{3}\,A$ ✓

(38) TRACCIARE LA CARATTERISTICA TENSIONE/CORRENTE DEL CIRCUITO



THEVENIN:

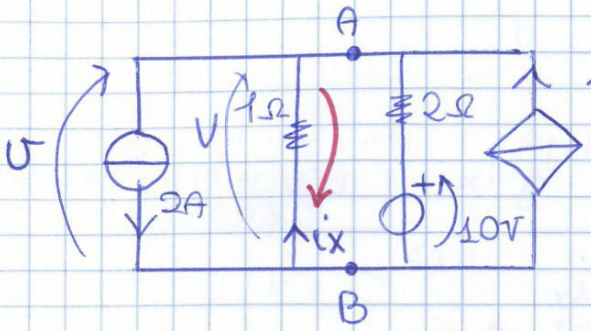
R_T $(R_1+R_2) /\!/ R_2 =$

$$R_T = \frac{12}{8} = \frac{3}{2}\,\Omega$$

CAPITOLO ④ RETI con GENERATORI PILOTATI

MILLMANN

① calcolare v.



GENERATORE PILOTATO DI CORRENTE che DIPENDE DA CORRENTE i_x

ATTENZIONE AI SEGNI MENO

$v = v_{2b}$ ⇝ MILLMANN!

$$v = \frac{\frac{10}{2} - 2 + 2i_x}{1 + \frac{1}{2}} = \frac{2}{3}\left(\frac{10 - 4 + 4i_x}{2}\right) = 2 + \frac{4}{3}i_x$$

$v = v_{2b} = -1 \cdot i_x$ ⇝ $i_x = -v$

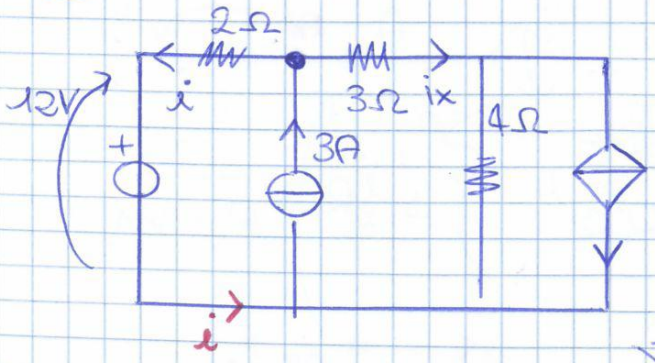
EGUAGLIO LE RELAZIONI

$2 + \frac{4}{3}i_x = -i_x$

$\frac{7}{3}i_x = -2$ $\quad i_x = -\frac{6}{7}$ A ✓ ⇝ $v = v_{2b} = -i_x = \frac{6}{7}$ V ✓

PRINCIPIO DI SOVRAPPOSIZIONE

② calcolare i



$i = 3 - i_x$

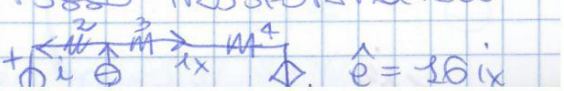
SERVE X IL CONTROLLO FINALE

+ MILLMANN

GENERATORE PILOTATO DI CORRENTE che DIPENDE DA CORRENTE i_x

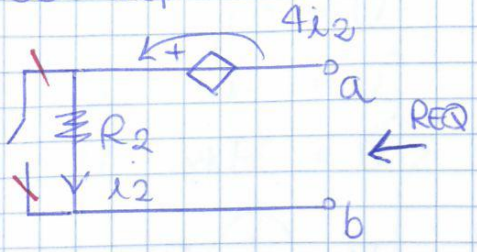
NEL PRINCIPIO DI SOVRAPPOSIZIONE
- NON SPENGO MAI IL GENERATORE PILOTATO
- TRATTO IL PILOTATO COME INDIPENDENTE E LO SPENGO ECC. (SUPPONGO i_x NOTO)

POSSO TRASFORMARLO



$V_{ab} = e_T = i_2 R_2 - 4i_2 = 10 \cdot 2 - 4 \cdot 10 = -20\ V$ ✓

* CERCO R_T

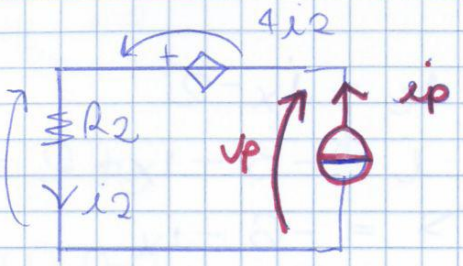


METTO IN ab UN GENERATORE DI CORRENTE O DI TENSIONE DI PROVA V_p o i_p (A SECONDA DELLA SITUAZIONE)

$$R_{EQ} = \frac{V_p}{i_p} \qquad V_p = R_{EQ}\, i_p$$

LA RESISTENZA RO VENUTA NEGATIVA

E SPESSO CAPITA GENER. IND!!



$i_p = i_2$

$V_p = i_2 R_2 - 4i_2$

$V_p = i_p (R_2 - 4)$

$V_p = i_p (-2)$

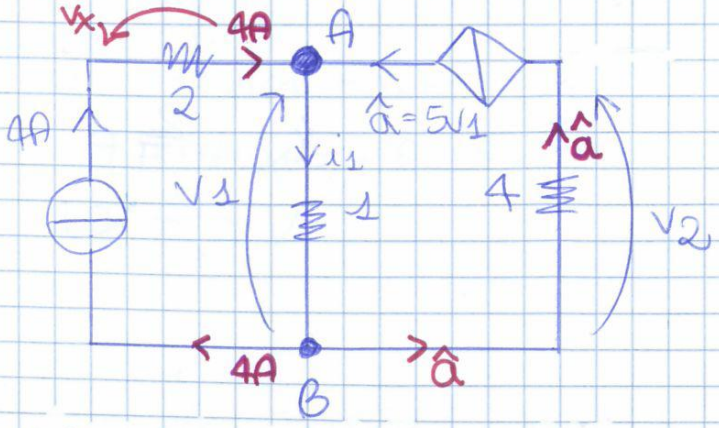
$R_T = -2\Omega$ ✓

ATTENZIONE: CONVENZIONE DEI GENERATORI SUI GEN. DI PROVA X QUESTO CONV. UTILIZZ. SU CIRCUITI

DATO L'EQUIVALENTE DI THEVENIN:

$$i_R = \frac{e_T}{R_1 + R_T} = \frac{-20}{R_1 - 2}$$ ✓

④ CALCOLARE V_2.



GENERATORE DI CORRENTE DIPENDENTE DA TENSIONE V_1

$V_2 = \hat{a}\,4 = -20 V_1$ OHM

$i_1 = 4 + 5V_1$ KCL A

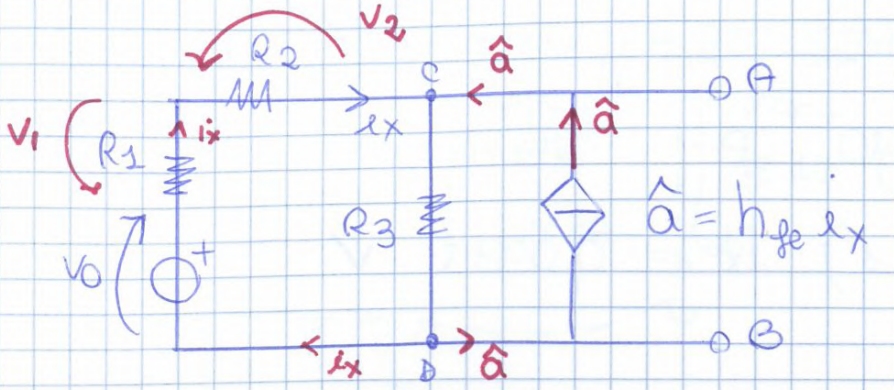
OPPURE POTEVO USARE MMAN SU AB

$\frac{V_1}{1} = 4 + 5V_1$

$V_1 = -1\ V$

$V_2 = +20\ V$ ✓

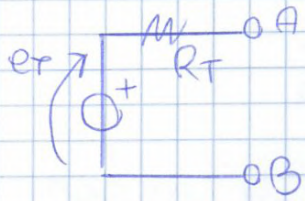
⑥ THEVENIN su AB



$\hat{a} = h_{fe}\, i_x$

$R_1 = 1\,K\Omega$
$R_2 = 15\,K\Omega$
$R_3 = 5\,K\Omega$
$V_o = 0{,}1\,V$
$h_{fe} = 50$

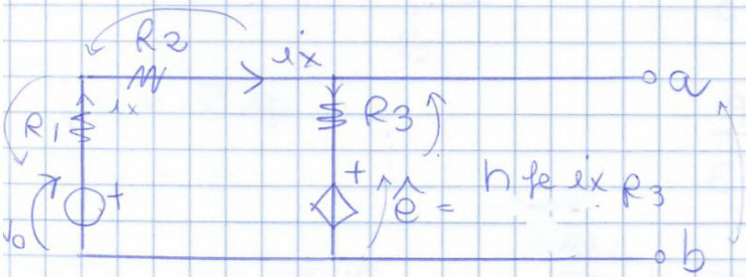
THEVENIN



CERCO e_T:

- MAGLIA
- TRASFORMO $\hat{a}$ IN $\hat{e}$

$\hat{e} = \hat{a} R$



$e_T = V_{ab} = i_x R_3 + h_{fe}\, i_x R_3$

$V_{ab} = -i_x R_1 - i_x R_2 + V_o$

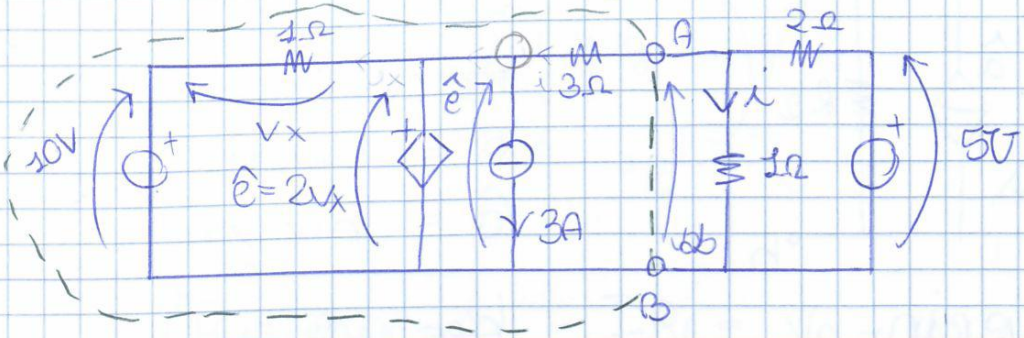
$V_o = (h_{fe} R_3 + R_3 + R_1 + R_2)\, i_x$

$i_x = \dfrac{0{,}1}{50(5000) + 7500}$

$i_x = 3{,}8835 \cdot 10^{-7}\,A$

⑦ THEVENIN

calcolo i con THEVENIN



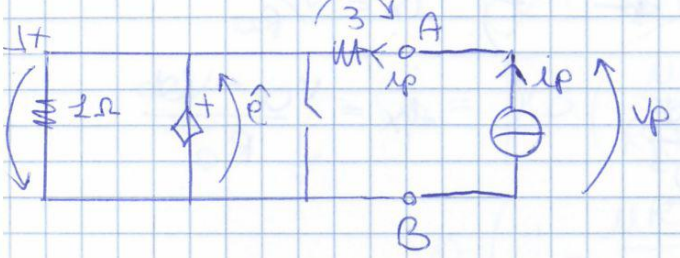
$(i = 2{,}576\,A)$

THEVENIN:

cerco e_T: $\quad \hat{e} + V_x = 10 \;,\; 2V_x + V_x = 10 \;,\; V_x = \frac{10}{3}\,V$

$\hat{e} = \frac{20}{3}\,V$

cerco R_T



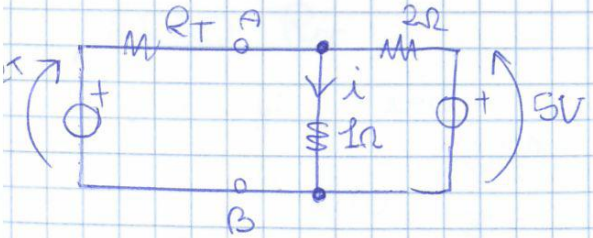
$\hat{e} = -V_x \rightarrow 2V_x = -V_x \rightarrow V_x = 0$

$\hat{e} = 0$

$V_p = 3\,i_p$

$R_T = 3$

EQ THEVENIN:

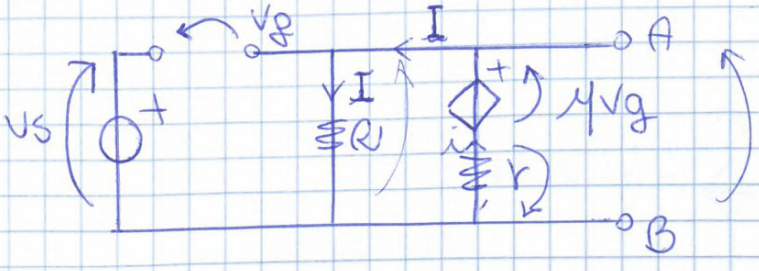


× MILLMANN:

$$i = V_{2b} = \frac{\frac{e_T}{R_T} + \frac{5}{2}}{\frac{1}{R_1} + \frac{1}{2} + 1}$$

$$i = \frac{\frac{95 + 40}{18}}{\frac{2+3+6}{6}} = \frac{95}{18_3}\cdot\frac{6}{11} = 2{,}88 \;(NO)$$

9 TROVO THEVENIN AD AB



CERCO e_T

$e_T = V_{ab} = \mu V_g - \frac{i}{r} = IR = V_S - V_g$ $\qquad i = I = ?$

$$V_p = i_p \left(\frac{\mu R + (R+r+\mu R)}{R+r-\mu R} \right) \left(\frac{Rr}{R+r} \right)$$

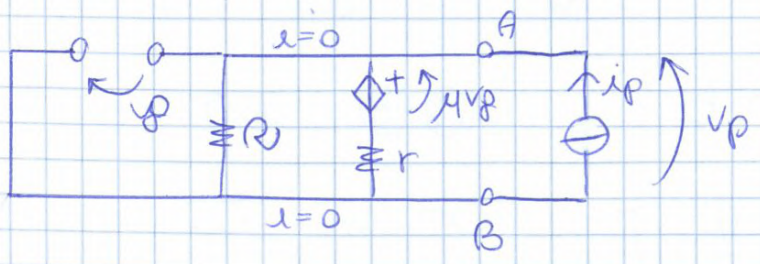
$$V_p = \left(\frac{\mu}{\beta} i_p \frac{Rr}{R+r-\mu R} + i_p \right) \left(\frac{Rr}{R+r} \right)$$

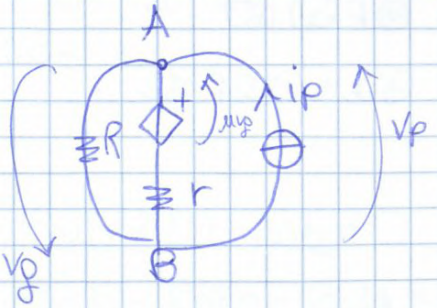
$$V_p = \left[i_p \left(\frac{\mu R}{R+r-\mu R} + 1 \right) \right] \left(\frac{Rr}{R+r} \right)$$

$$V_g \left(1 - \frac{\mu R}{R+r} \right) = i_p \left(\frac{Rr}{R+r} \right) \left(\frac{R+r}{R+r-\mu R} \right)$$

$$V_g = i_p \frac{Rr}{R+r-\mu R}$$

CERCO R_T





$V_g = V_p$

$\mu V_g = \mu V_p$

$$\text{CHIAMO } V_p = \frac{\frac{\mu V_g}{r} + i_p}{\frac{1}{R} + \frac{1}{r}}$$

$$\text{MENO } V_g = \left(\frac{\mu V_g}{r} + i_p \right) \left(\frac{Rr}{r+R} \right)$$

$$\text{TROVO } V_g = i_p \frac{Rr}{R+r-\mu R}$$

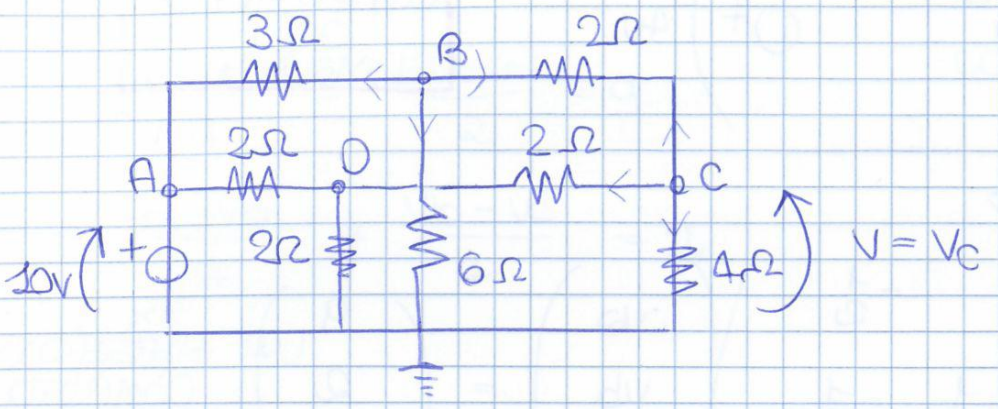
SOSTITUISCO $= V_p = i_p \left(\frac{Rr}{R+r-\mu R} \right)$

CAPITOLO 5: METODI AUTOMATICI DI ANALISI

13 novembre '12

METODO DEI NODI

① calcolare V.



TRATTO OGNI NODO COME UN SUPERNODO DA CUI FARE KCL CON CORRENTI TUTTE USCENTI

Ⓐ $V_a = 10V$

Ⓑ $\frac{V_b - V_a}{3} + \frac{V_b}{6} + \frac{V_b - V_c}{2} = 0$ SUPERNODO

$V_b\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{2}\right) - V_a\left(\frac{1}{3}\right) - V_c\left(\frac{1}{2}\right) = 0$

Ⓒ $\frac{V_c - V_D}{2} + \frac{V_c}{4} + \frac{V_c - V_B}{2} = 0 \iff V_c\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{2}\right) - \frac{V_b}{2} - \frac{V_d}{2} =$

Ⓓ $\frac{V_D - V_a}{2} + \frac{V_D}{2} + \frac{V_D - V_c}{2} = 0 \iff V_d\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right) - \frac{10}{2} - \frac{V_c}{2} =$

sistema ⒷⒸⒹ

$$\begin{cases} 6V_b - 20 - 3V_c = 0 \\ 5V_c - 2V_b - 2V = 0 \\ 3V_D - 10 - V_c = 0 \quad * \end{cases}$$

① / 3·②

$$\begin{cases} 6V_b - 20 - 3V_c = 0 \\ 15V_c - 6V_b - 6V_d = 0 \end{cases}$$

$* \quad 12V_c - 6V_d - 20 = 0$

$-20 - 2V_c + 12V_c - 20 = 0$

$V_c = 4V$ ✓

ALTRA POSSIBILE MATRICE X QST ESERCIZIO

$$\begin{pmatrix} \frac{1}{4}+\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4}+\frac{1}{4} \end{pmatrix}\begin{pmatrix} V_a \\ V_b \end{pmatrix} = \begin{pmatrix} 9+\frac{V_c}{2} \\ 2+\frac{V_c}{4} \end{pmatrix}$$

× ISPEZIONE RIDOTTA

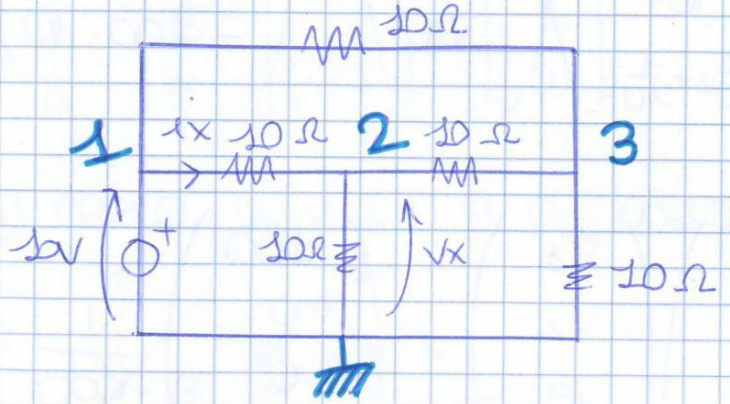
OPPURE:

$$\begin{cases} \frac{V_a - V_b}{4} + \frac{V_a - V_c}{2} = 9 \\ \frac{V_b - V_a}{4} + \frac{V_b - V_c}{4} = 2 \\ V_c = 4V \end{cases}$$

come KVL

METODO DEI NODI

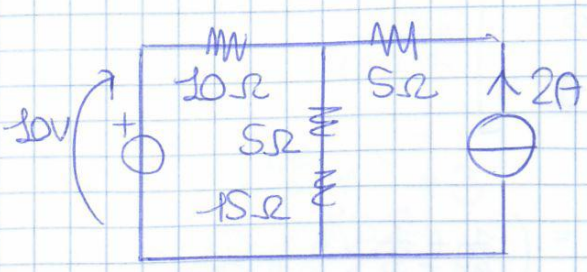
③ IMPOSTARE IL SISTEMA DI EQ AI NODI

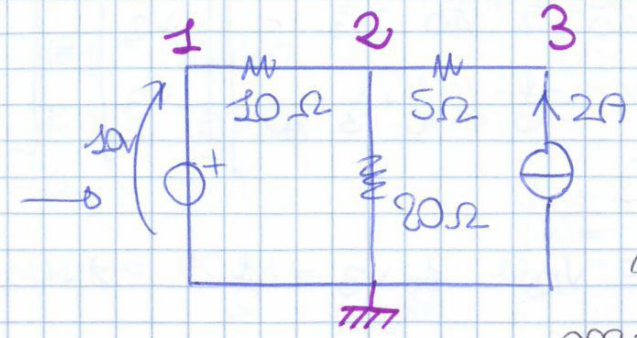


$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{10} & \frac{1}{10}+\frac{1}{10}+\frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & -\frac{1}{10} & \frac{1}{10}+\frac{1}{10}+\frac{1}{10} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{3}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{3}{10} \end{bmatrix}\begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{10}\cdot 10 \\ 1 \end{bmatrix}$$

④ SCRIVERE IL SISTEMA DI EQ DI NODI





C'È UN GEN. TENSIONE

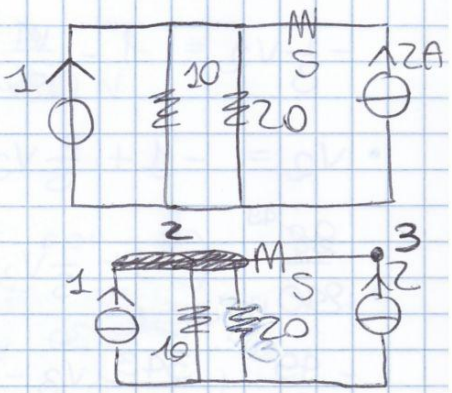
- LO SPOSTO E SCALGO IL NODO 1
- SEMPLIFICO LA MATRICE

$V_1 = 10\,V$

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{10} & \frac{1}{10}+\frac{1}{5}+\frac{1}{20} & -\frac{1}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 2 \end{bmatrix}$$

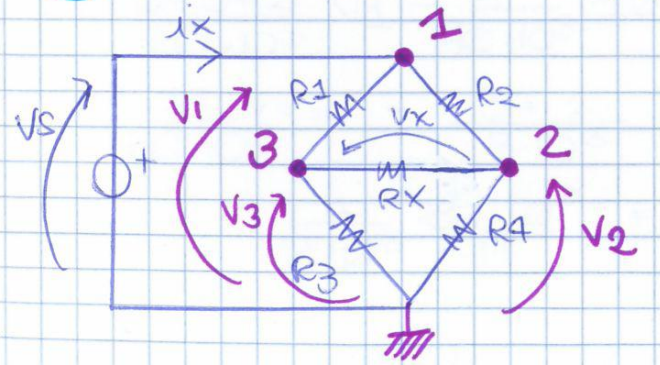
$$\begin{bmatrix} \frac{7}{20} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

↓ APPLICO LA TRASFORMA



$$\begin{pmatrix} \frac{7}{20} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{5} \end{pmatrix} \begin{pmatrix} V_2 \\ V_3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

⑥ CERCO V_X e I_X e P_{R2} ASSORBITA



$R_n = n \cdot 10\ k\Omega$

$R_X = 3k\Omega$

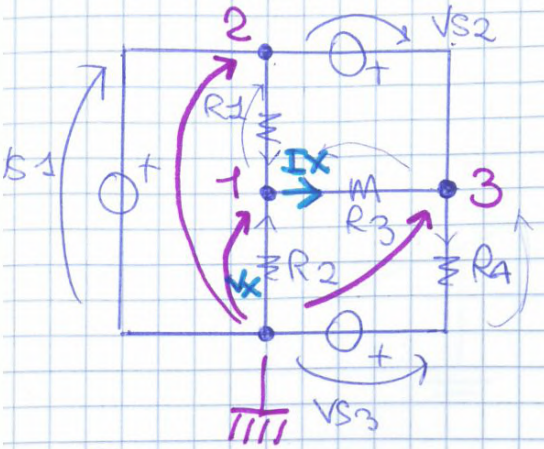
$V_S = 10V$

$(V_X = -1)$

$V_1 = V_S = 10\,V$

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_X}+\frac{1}{R_4} & -\frac{1}{R_X} \\ -\frac{1}{R_1} & -\frac{1}{R_X} & \frac{1}{R_1}+\frac{1}{R_X}+\frac{1}{R_3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

⑦ CALCOLO V_x e I_x e P FORNITA A R_1



$R_1 = 20\,k\Omega$

$R_2 = 20\,k\Omega$

$R_3 = 30\,k\Omega$

$R_4 = 40\,k\Omega$

$V_{S1} = V_{S2} = V_{S3} = 10\,V$

$V_x = 7{,}27\,V$

$I_x = -0{,}92\,mA$

$P_{R1} = 9{,}5\,mW$

$V_2 = V_{S1} = 10\,V$

$V_3 - V_2 = V_{S2}$

$V_3 = V_2 + V_{S2} = 20\,V$

$V_{R4} = V_3 - V_{S3} = 10\,V$

$V_1 = V_{R2}$

$V_{R3} = V_1 - V_3$

$V_{R1} = V_0 - V_1$

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_1} & -\frac{1}{R_3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}$$

$$\frac{6+3+2}{60\cdot10^3} V_1 - \frac{1}{10\cdot10^3}\,10 - \frac{1}{30\cdot10^3}\,20 = 0$$

$$\frac{11}{60\cdot10^3} V_1 = \frac{3+1}{3\cdot10^3} = \frac{4}{3\cdot10^3}$$

$$V_1 = \frac{4}{3\cdot10^3}\cdot\frac{60\cdot10^3}{11} = \frac{80}{11} = 7{,}27\,V = V_x \quad \checkmark$$

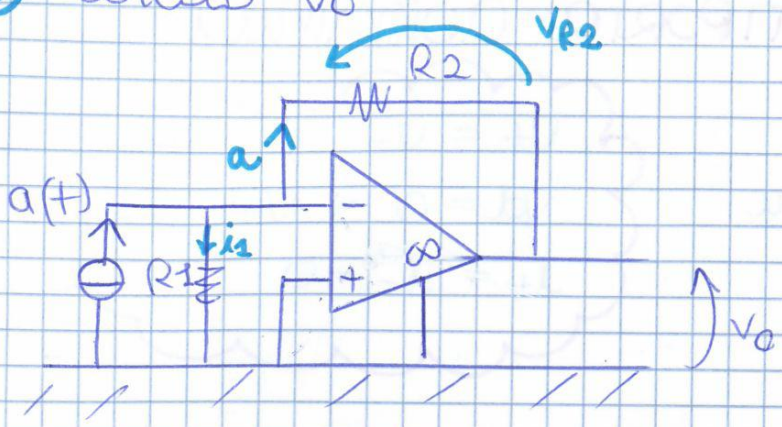
KCL ① $I_1 + I_2 = I_X = \frac{V_2 - V_1}{R_1} + \frac{V_3}{R_2} = \frac{2{,}73}{10\cdot10^3} + \frac{7{,}27}{20\cdot10^3} =$

$6{,}365$

$I_X = \frac{12{,}73}{20\cdot10^3} = 0{,}63\,mA$ (NO)

SBAGLIATI I NODI E RIFARLI!!

⑤ CALCOLO V_0



$R_1 = 100\ k\Omega$

$R_2 = 10\ k\Omega$

$a(t) = 0{,}5 \sin(500t)\ mA$

$V_+ = 0 = V_d$

$V_- + V_d = V_+$

$V_- = 0$

$V_{R1} = 0 \quad = \quad i_1 = 0$

$a(t)$ passa su R_2

$V_{R2} = a(t)\, R_2 = 0{,}5 \sin(500t)\, R_2$

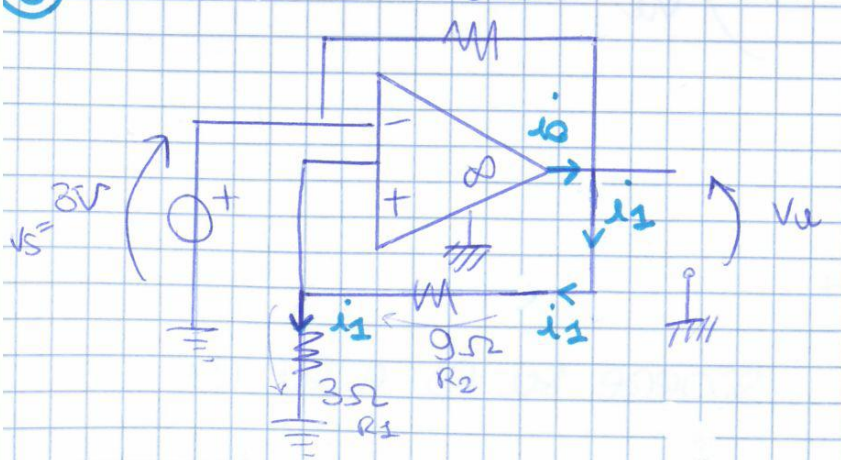
$V_{R2} = 0{,}5 \sin(500t) \cdot 10^{-3} \cdot 10 \cdot 10^{3}$

$V_{R2} = 5 \sin(500t)\ V$

$V_0 = -V_{R2} = -5 \sin(500t)\ V$!!

AMPLIFICATORE INVERTENTE.

⑥ CALCOLO V_u:



$V_u = i(R_1 + R_2)$

$V_u = \frac{V_S}{R_1}(R_1 + R_2)$

$V_- = 3V \qquad V_d = 0 \qquad V_+ = 3V = V_1$

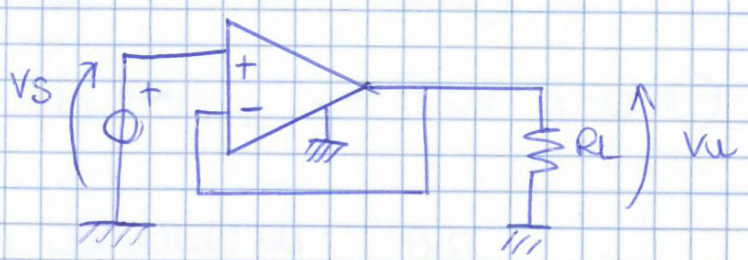
$i_1 = \frac{V_1}{3} = 1A$, $i_+ = 0$, QUINDI i_1 PASSA SU 9 Ω

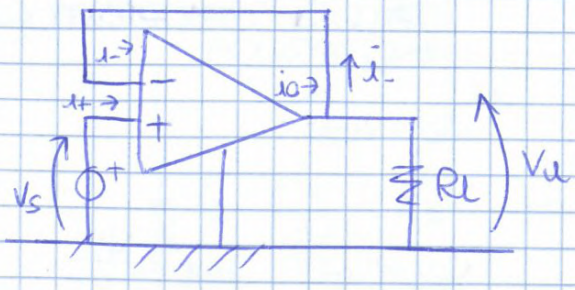
$V_u = +V_1 + V_2 = +3 + 9 = +12V$

9 VALUTARE V_u/V_S E LA POTENZA ASSORBITA DA R_L E QUELLA EROGATA DA V_S.

$V_S = 3V$

$R_L = 3K\Omega$





$V_+ = V_- = V_S = 3V$

$V_S = V_u = 3V$

$V_S/V_u = 1$ ✓

i_o passa su R_L

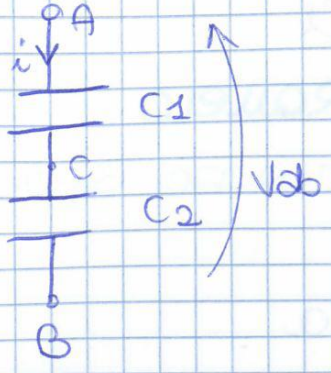
$$i_o = \frac{V_u}{R_L} = \frac{3}{3\cdot 10^3} = 1mA \rightarrow P_{RL} = R_L i^2 = 3\cdot 10^3 \cdot 1\cdot 10^{-6} = 3mW$$ ✓

$$P_{VS} = V_{Si}\cdot i = 0$$

CAPITOLO 7
CIRCUITI CON ELEMENTI NON LINEARI

② CONDENSATORI IN SERIE E IN PARALLELO

ⓐ



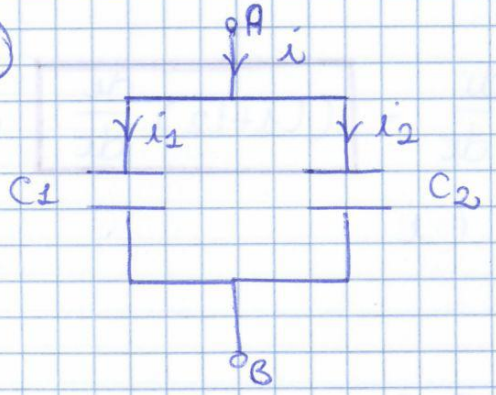
È EQUIVALENTE A

$$C' = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

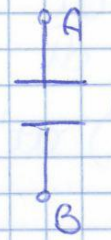
$$V_{ab} = V_{ac} + V_{cb} = \frac{1}{C_1}\int_{t_0}^{t} i(t')\,dt' + \frac{1}{C_2}\int_{t_0}^{t} i(t')\,dt' =$$

$$V_{ab} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_{t_0}^{t} i(t')\,dt' = \frac{1}{C'}\int_{t_0}^{t} i(t')\,dt'$$

ⓑ



È EQUIVALENTE A

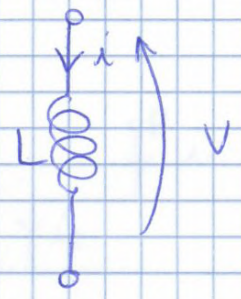


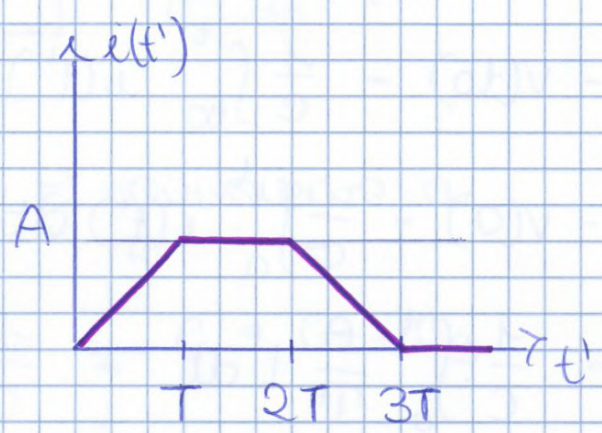
$$C' = C_1 + C_2$$

$$i = i_1 + i_2 = C_1\frac{dV}{dt} + C_2\frac{dV}{dt} = (C_1 + C_2)\frac{dV}{dt} = C'\frac{dV}{dt}$$

$$V = V_0 + \frac{1}{C'}\int_{t_0}^{t} i(t')\,dt'$$

(b)





$$v = L\frac{di}{dt} \qquad i = i_0 + \frac{1}{L}\int_{t_0}^{t} v(t')\,dt$$

DEVO DERIVARE IL GRAFICO X AVERE $v(t)$:

- $0 < t < T$

  $i(t') = \frac{A}{T}t' \rightarrow v = L\frac{A}{T}$

- $T < t < 2T$

  $i(t') = A \rightarrow v = 0$

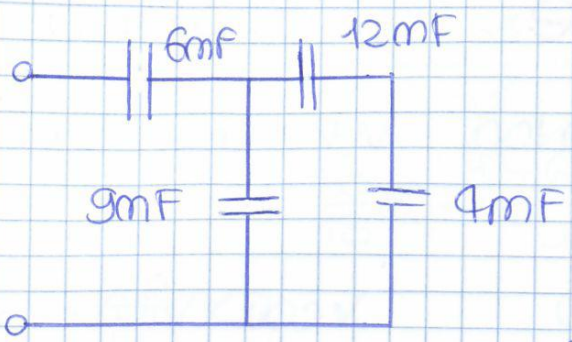
- $2T < t < 3T$

  $i(t') = -\frac{A}{T}t' \rightarrow v = -L\frac{A}{T}$

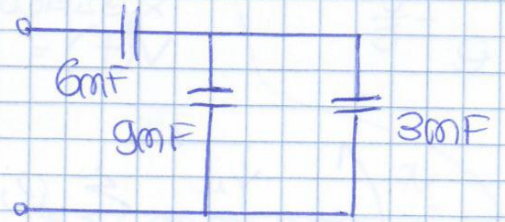
- $t > 3T$

  $i(t') = 0 \rightarrow v = 0$

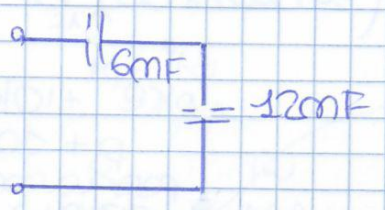
⑤ CAPACITÀ EQUIVALENTE



12mF serie 4mF $= \frac{12^{3} \cdot 4^{1}}{16_{4_1}} = 3mF$



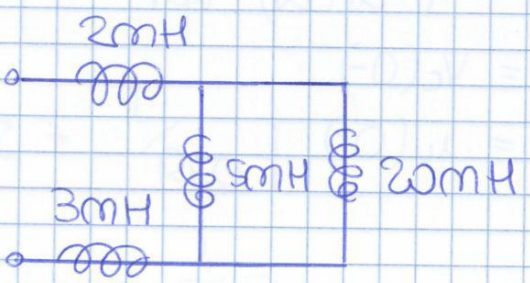
3mF // 9mF = 3+9 = 12mF

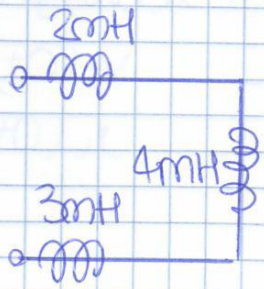


6mF serie 12mF $= \frac{6^{1} \cdot 12^{4}}{18_{3_1}} = 4mF$

$C_{EQ} = 4mF$ ✓

⑥ INDUTTANZA EQUIVALENTE





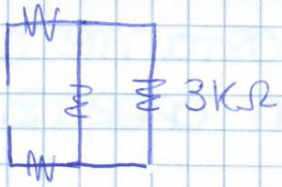
5mH // 20mH $= \frac{^{4}20 \cdot 5^{1}}{25_{5_1}} = 4mH$

2mH serie 4mH serie 3mH = 2+4+3 = 9mH

$L_{EQ} = 9mH$ ✓

$1K\Omega + 1K\Omega + 1K\Omega = 3K\Omega$

$10K\Omega \,//\, 3K\Omega = \frac{30 \cdot 10^6}{13 \cdot 10^3} = \frac{30}{13} K\Omega$

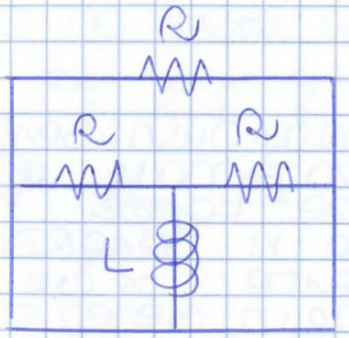


$1K\Omega + 5K\Omega + \frac{30}{13}K\Omega = \frac{13+65+30}{13} K\Omega$

$R_{EQ} = \frac{108}{13} K\Omega$ ✓

$\tau = 0,1 \cdot 10^{-6} \frac{108}{13} \cdot 10^3 = 0,83 \cdot 10^{-3} s = 0,83 ms$ ✓

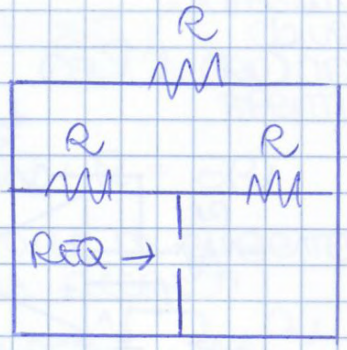
③ τ costante di tempo x induttore



$R = 1\Omega$

$L = \frac{1}{8} H$

⇒



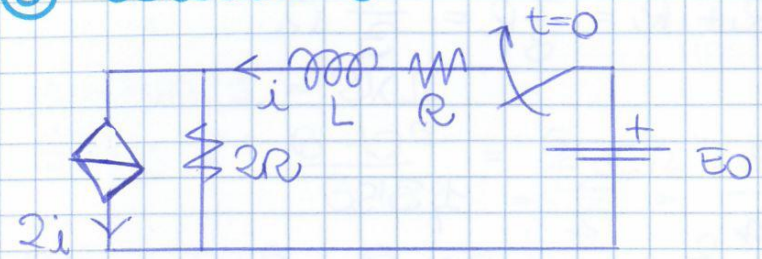
$\tau = L / R_{EQ}$

$R // R = \frac{1}{2}\Omega$

$R_{EQ} = \frac{1}{2}\Omega$

$\tau = \frac{1}{3} H \cdot 2\Omega = 0,667 s$

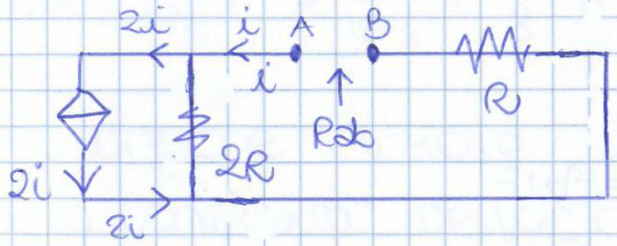
⑥ Calcolo τ



CIRCUITO RL

$\tau = \frac{L}{R_{EQ}}$

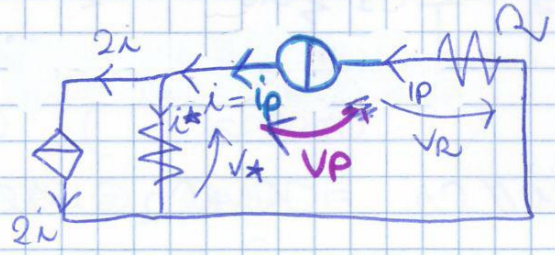
Come se stessi cercando $i_L(t)$, $t \geq 0$ (interruttore chiuso! Gen. ind. spenti



$R_{ab} = 2R + R = 3R$

Il generatore dipendente rimane acceso!

Avendo un generatore dipendente, metto un generatore di prova



$\frac{V_P}{i_P} = R_{ab}$

i = pilota = i_P → che fluisce su R

Ora calcolo V_P

$V_R = R i_P$

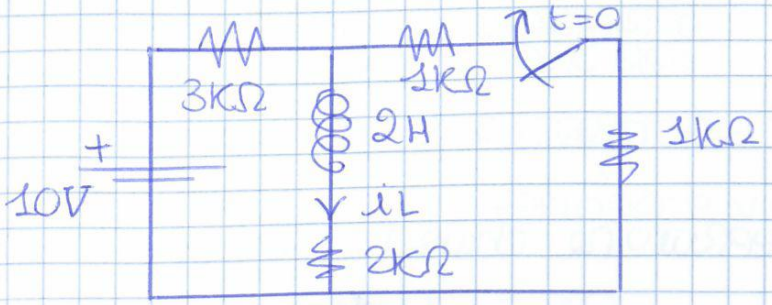
Su 2R circola $i^* = i - 2i = -i = -i_P$

e quindi $V_{2R} = V^* = -2R i_P$

$V_P = V^* + V_R = R i_P - 2R i_P = -R i_P$

$R_{EQ} = -R$ ✓ $\left(\tau = -\frac{L}{R}\right)$ ✓ quindi $e^{-\frac{t}{\tau}}$ tende a ∞ è instabile

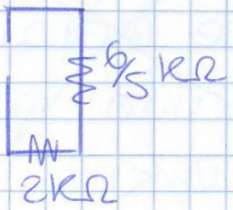
⑧ calcolo τ e la condizione iniziale $i_L(0)$



- $\tau = \frac{L}{REQ}$ per $t \geq 0$

serie 1 = 2KΩ

$2 // 3 = \frac{6 \cdot 10^6}{5 \cdot 10^3} = \frac{6}{5}$ KΩ



$REQ = 2 + \frac{6}{5} = \frac{16}{5}$ KΩ

$\tau = \frac{2}{16/5} = 2 \cdot \frac{5}{16_8} = \frac{5}{8}$ s

- CONDIZIONE INIZIALE $i_L(0+)$

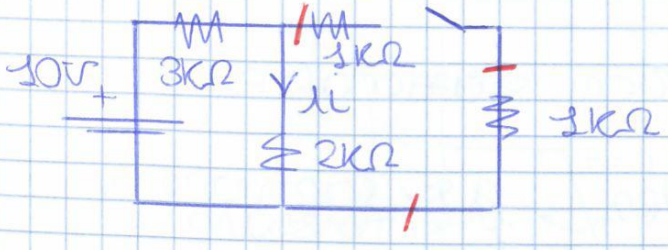
X LA CONTINUITÀ DELLA CORRENTE DELL'INDUTTORE

$i_L(0-) = i_L(0+)$

CERCO $i_L(0-)$, CIOÉ $t<0$ (INTERRUTTORE APERTO)

SIAMO IN REGIME STAZIONARIO, QUINDI

$i_L(t) = cost$, $V_L(t) = 0$ → CORTOCIRCUITO



a SX: PARTITORE DI TENSIONE

$i_L = \frac{V_2}{2 \cdot 10^3} = \frac{1}{2 \cdot 10^3} = \frac{2 \cdot 10^3}{5 \cdot 10^3}$

$i_L = 2 \cdot 10^{-3} = 2$ mA

$i_L(0-) = i_L(0+) = 2$mA

$$V_c(t) = -V_0 \frac{R2}{R1+R2} e^{-\frac{t(R1+R2)}{CR1R2}} + V_0 \frac{R2}{R1+R2} \quad \checkmark, \ t \geq 0$$

RICALCOLO DELL'INTERRUTTORE FUNZIONA AL CONTRARIO (CHIUSO X UN LUNGO PERIODO E VIENE APERTO IN $t=0$)

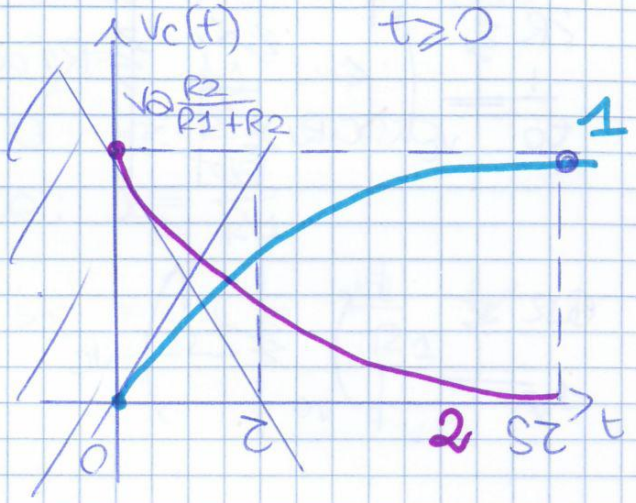
ⓒ $\tau = C R_{EQ}$, $t \geq 0$

$R_{EQ} = R2$

$\tau = CR2$ ✓

ⓐ $V_c(0^+)$ COND. INIZ.

$V_c(0^+) = V_0 \frac{R2}{R1+R2}$ ✓



ⓑ $V_c(\infty)$ COND. FIN. ↔ SI SONO SCAMBIATI

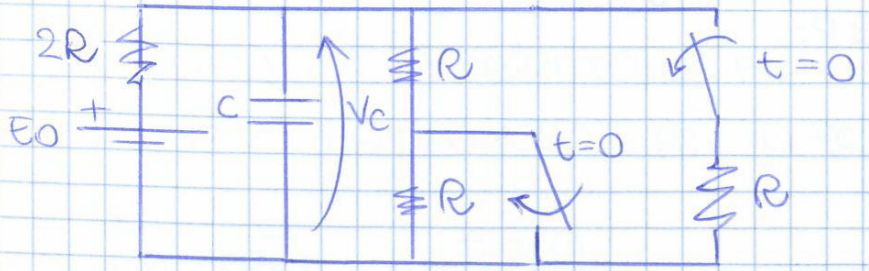
$V_c(\infty) = 0$ ✓

ⓓ $V_c(t)$

$$V_c(t) = [V_c(0) - V_c(\infty)] e^{-\frac{t}{R}} + V_c(\infty)$$

$$V_c(t) = V_0 \frac{R2}{R1+R2} e^{-\frac{t}{CR2}} \quad \checkmark, \ t \geq 0$$

⑨ CALCOLO τ E CONDIZIONE INIZIALE

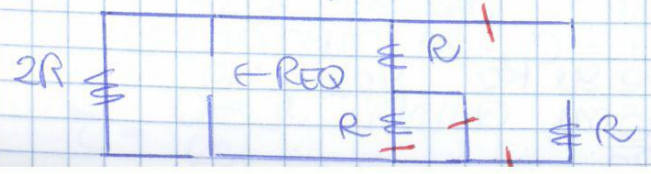


$R = 3\Omega$

$E_0 = 1V$

$C = 1mF$

- $\tau = C R_{EQ}$, $t \geq 0$



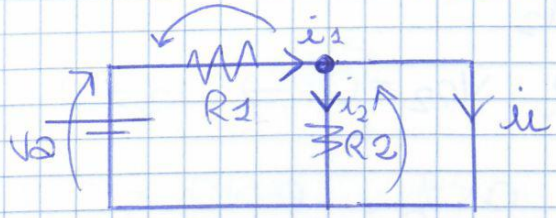
R // CORTO = CORTO CIRCUITO

$R_{EQ} = R // 2R = \frac{2}{3}R = \frac{2}{3}3 = 2\Omega$

$i_L(0^-) = i_L(0^+)$

⑥ CONDIZIONE FINALE $i_L(\infty)$

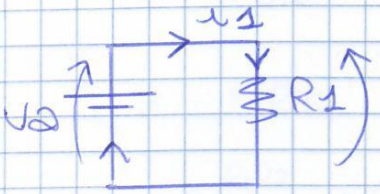
per $t \to \infty$ < CIRCUITO CHIUSO
CONDIZIONI STAZIONARIE



R2 // CORTO → CORTOCIRCUITO

$i_L(\infty) = i_1 - i_2$

$i_2 = 0$, xk R // CORTO



PARTITORE: $i_1 = \frac{V_g}{R_1}$

$i_L(\infty) = i_1 = \frac{V_g}{R_1}$ ✓

$i_L(\infty) = V_g \left(\frac{R_2 - R_1 + 1}{R_1 R_2} \right)$

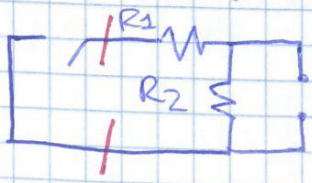
⑦ $i_L(t)$

$$i_L(t) = [i_L(0) - i_L(\infty)]\, e^{-\frac{t}{\tau}} + i_L(\infty)$$

$$i_L(t) = -\frac{V_g}{R_1}\, e^{-\frac{t R_1 R_2}{L(R_1+R_2)}} + \frac{V_g}{R_1}$$

RICALCOLO SE L'INTERRUTTORE FUNZIONA ALL'INCONTRARIO
(CHIUSO X "LUNGO PERIODO" POI APERTO IN $t=0$)

⑧ $\tau = \frac{L}{R_{EQ}}$, $t \geq 0$

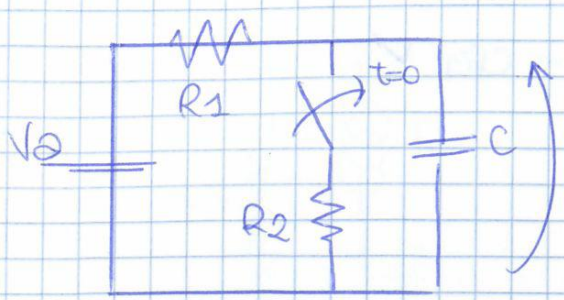


$R_{eq} = R_2$

$\tau = \frac{L}{R_2}$

(12) TUTTO CON CONDENSATORE

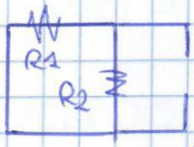
APERTO x "LUNGO PERIODO" e CHIUSO in $t=0$



SOLUZIONE:

$$v_c(t) = (v_c(0+) - v_c(\infty))\, e^{-\frac{t}{\tau}} + v_c(\infty)$$

* $\tau = C R_{EQ}$, $t \geq 0$ (REQ come di THONORIN)



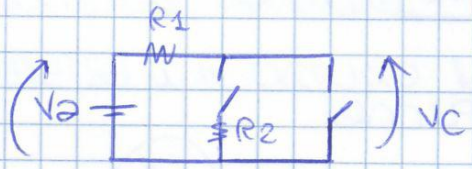
$$R_{EQ} = R_1 /\!/ R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$\tau = C \frac{R_1 R_2}{R_1 + R_2} \checkmark$$

* COND. INIZIALE

$v_c(0+) = v_c(0-)$

$t < 0$ ← INT. APERTO COND. STAZ. (C = C. APERTO)



SU R_1 e SU R_2 non CIRCOLA CORRENTE $V_{R1} = V_{R2} = 0$

$v_c(0+) = v_c(0-) = V_g$ ✓

* COND. FINALE

$t \to \infty$ ← INT. CHIUSO COND. STAZ. (C = C. APERTO)

$V_C = V_{R2}$ = PART. TENSIONE $= V_g \frac{R_2}{R_1 + R_2}$ ✓

* SOLUZIONE

$$v_c(t) = (v_c(0+) - v_c(\infty))\, e^{-\frac{t}{\tau}} + v_c(\infty), \quad t > 0$$

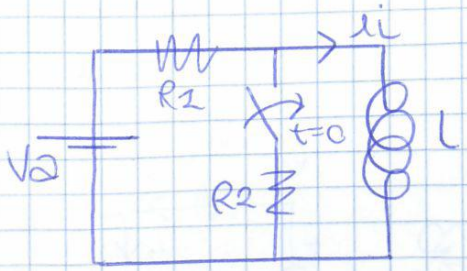
$$v_c(t) = \left(V_g - V_g \frac{R_2}{R_1+R_2}\right) e^{-\frac{t(R_1+R_2)}{C R_1 R_2}} + V_g \frac{R_2}{R_1+R_2}, \quad t \geq 0$$

$$v_c(t) = V_g \left[\frac{R_2}{R_1+R_2} + \left(1 - \frac{R_2}{R_1+R_2}\right) e^{-\frac{t(R_1+R_2)}{R_1 R_2}}\right], \quad t \geq 0$$

⇒ $v_c(t) = V_g$ per $t \leq 0$

(13) TUTTO CON L'INDUTTORE

APERTO x LUNGO PERIODO e CHIUSO in $t=0$

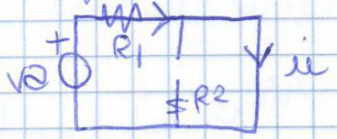


★ $\tau = \frac{L}{R_{EQ}}$, $t \geq 0$

$R_{EQ} = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2}$ $\qquad \tau = \frac{L(R_1+R_2)}{R_1 R_2}$ ✓

★ CONDIZ. INIZIALI

$i_L(0^+) = i_L(0^-)$ → PER $t<0$ ← INT. APERTO COND. STAZ (L=C.C.)



$V_2 = V_{R1}$ PARTITORE TENSIONE

$i_L = \frac{V_2}{R_1}$ SCORRE SU TUTTO CIRCUITO ✓

★ CONDIZ. FINALI

$t \to \infty$ ← INT. CHIUSO COND. STAZ (L=C.C)

$V_{R2} = 0$, $V_{R1} = V_2$

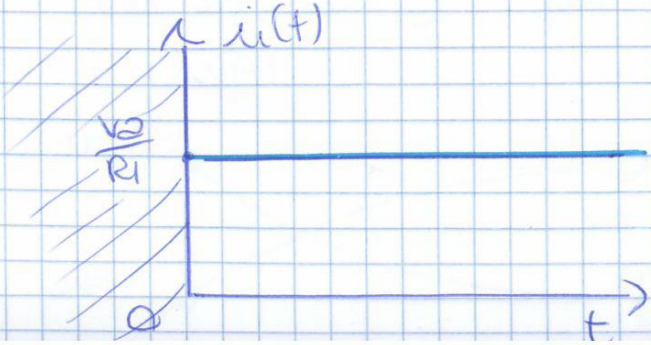
$i_2 = 0$, $i_{R1} = i_L$

$i_L = \frac{V_2}{R_1}$ ✓

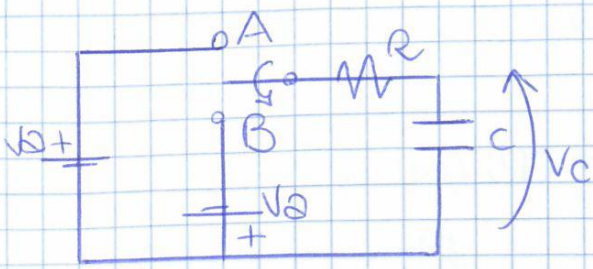
★ SOLUZ. FINALE

$i_L(t) = (i_L(0^+) - i_L(\infty))\, e^{-\frac{t}{\tau}} + i_L(\infty)$, $t \geq 0$

$i_L(t) = \left(\frac{V_2}{R_1} - \frac{V_2}{R_1}\right) e^{-\frac{t R_1 R_2}{L(R_1+R_2)}} + \frac{V_2}{R_1} = \frac{V_2}{R_1}$, $t \geq 0$ ✓

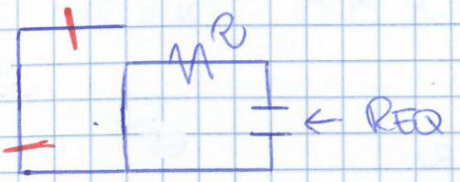


(14) INTERRUTTORE COMMUTATORE



IN A "LUNGO PERIODO", SI SPOSTA IN B IN $t=0$
CALCOLO $V_C(t)$
E INVERSO

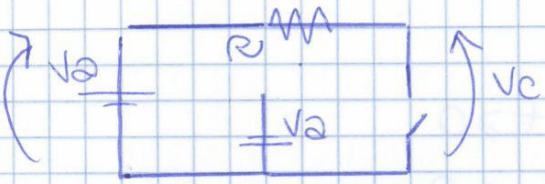
- $\tau = C R_{EQ}$, $t \geq 0$



$R_{EQ} = R$
$\tau = CR$ ✓

- CONDIZIONE INIZIALE $V_C(0^+)$

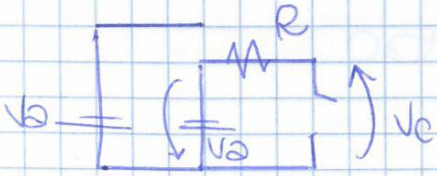
CERCO $V_C(0^-)$ < INTERRUTTORE IN A
COND. STAZIONARIE (C.A.)



SU R NON CIRCOLA CORRENTE
$V_C(0^-) = V_0$ ✓

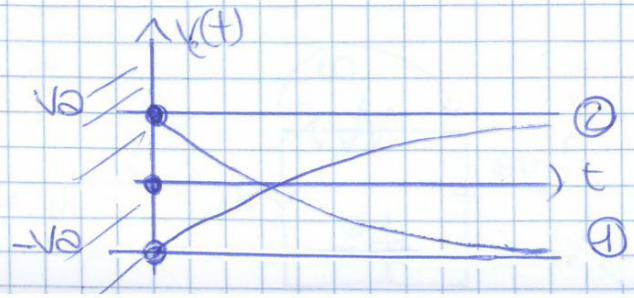
- CONDIZIONE FINALE $V_C(\infty)$

$t \to \infty$ < EXP → 0, COND. STAZ (C.A.)
INTERRUTTORE IN B



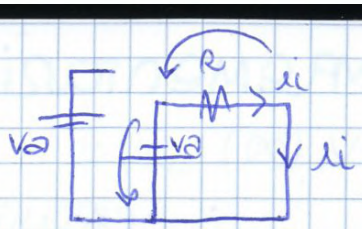
SU R NON CIRCOLA CORRENTE
$V_C(\infty) = -V_2$ ✓

- $V_c(t) = [V_0 - V_\infty] e^{-t/\tau} + V_\infty$ per $t \geq 0$
$V_c(t) = (V_0 + V_2) e^{-\frac{t}{CR}} - V_2$ per $t \geq 0$
$V_c(t) = V_2(-1 + 2e^{-\frac{t}{CR}})$ per $t \geq 0$ ✓



* CONDIZ. FINALE

$t \to \infty$ < INT. IN B / COND. STAZ



$$i_f = -\frac{Va}{R} = i_L(\infty)$$

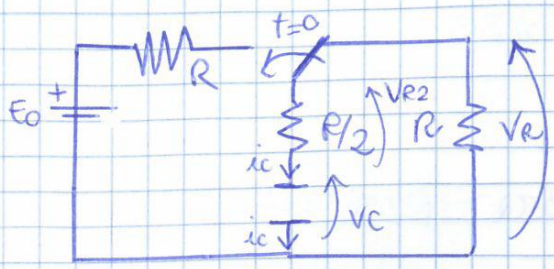
* SOLUZIONE

$$i_L(t) = 2\frac{Va}{R}e^{-\frac{Rt}{L}} - \frac{Va}{R} = \frac{Va}{R}\left(-1+2e^{-\frac{Rt}{L}}\right), \quad t \geq 0$$

INCONTRARIO: IN B X LUNGO TEMPO, IN A PER $t=0$

(17) altre variabili

Calcolo $v_R(0^+)$ e $v_R(\infty)$ e v_{R_2}



$R = 1\,k\Omega$

$E_0 = 1V$

$C = 1\,\mu F$

* $\tau = C \cdot R_{EQ}$

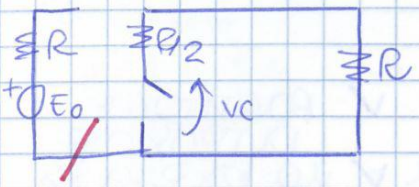
$R_{EQ} = R + \frac{R}{2} = \frac{3}{2}\,k\Omega$

$\tau = \frac{3}{2}\,ms = 1{,}5\,ms$ ✓

* cond. viz $v_R(0^+)$

Cerco $v_C(0^-) = v_C(0^+)$

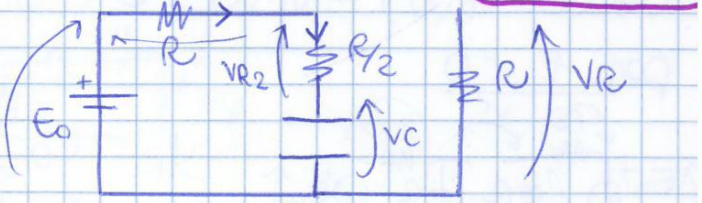
per $t<0$



circuito aperto!

$v_C(0^+) = 0$

per $t = 0^+$ → sostituisco C con un corto circuito



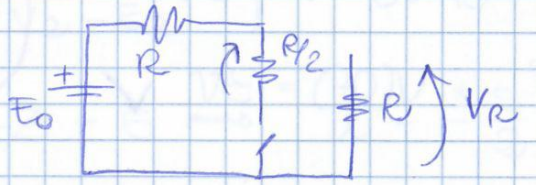
$v_{R_2} = E_0 \frac{R/2}{3/2R} = E_0 \frac{1}{2}\frac{2}{3} = \frac{E_0}{3}$

$v_{R_2}(0^+) = \frac{E_0}{3} = \frac{1}{3}V$ ✓

$v_R(0^+) = 0$ ✓ xk $i_R = 0$!

* cond. finale $v_R(\infty)$

per $t \to \infty$



$v_R(\infty) = 0$ ✓ $\quad v_{R_2}(\infty) = 0$ ✓

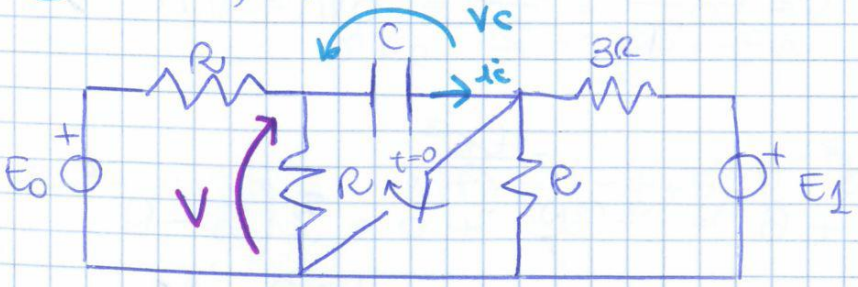
xk non c'è apporto di carica

* soluzione finale per $t>0$

$v_R(t) = 0$

$v_{R_2}(t) = \frac{1}{3} e^{-\frac{t}{1{,}5}}$ [ms]

(18) $v(t)$, per $t>0$.



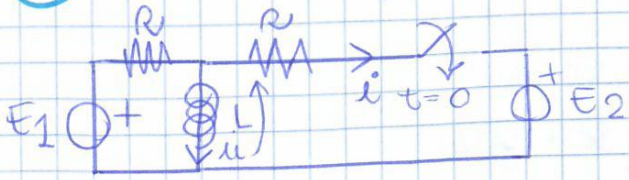
$E_0 = 12V$

$R = 4k\Omega$

$C = 1\mu F$

$$v(t) = [v(0^+) - v(\infty)]\, e^{-\frac{t}{\tau}} + v(\infty), \quad t>0$$

19 Cerco $i(t)$, $t>0$



$E_1 = 12V$
$E_2 = 20V$
$R = 4\Omega$
$L = 2H$

★ $\tau = L/R_{EQ}$, $t>0$

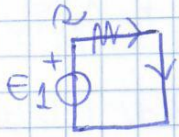
$R_{EQ} = R // R = \frac{R}{2} = 2\Omega$

$\tau = \frac{L}{R_{EQ}} = 1s$

★ Cond. iniziale $i(0^+)$

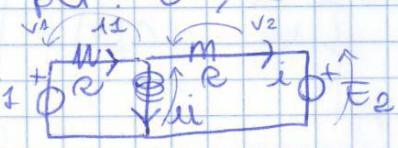
Cerco $i_L(0^-) = i_L(0^+)$

per $t<0$



$i_L = \frac{E_1}{R}$

per $t>0$



$i = i_1 - i_L = i_1 - 3A = \frac{V_1}{R} - 3A$

$V_1 + V_L = E_1 \rightarrow V_L = E_1 - V_1$

$V_L = V_2 + E_2$

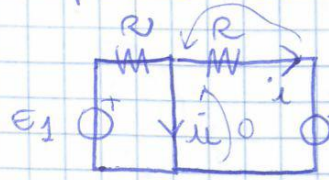
$E_1 - V_1 = V_2 + E_2$
$V_1 = E_1 - V_2 - E_2$

$i = \frac{E_1}{R} - \frac{iR}{R} - \frac{E_2}{R} - 3A$

$2i = \cancel{3} - \cancel{3} - 5 \Longrightarrow i = -\frac{5}{2}A$

★ Cond. finale

per $t \to \infty$

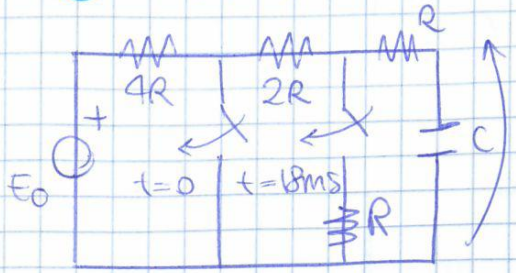


$E_2 = V_R$

$i = -\frac{E_2}{R} = -5A$

★ soluzione $i(t) = \left(-\frac{5}{2}+5\right)e^{-\frac{t}{1s}} - 5 = -5\left(1 - \frac{3}{2}e^{-\frac{t}{1s}}\right)$, $t>0$

91 $V_c(t)$, $t>0$ — 2 interruttori sfasati



$E_0 = 9V$
$R = 6\Omega$
$C = 1mF$

$$v_c(t) = [v_c(0^+) - v_c(\infty)]e^{-\frac{t}{\tau}} + v_c(\infty)$$

Primo pezzo $0<t<18ms$

* τ in $t>0$ ma $<18ms$

4R // corto
$\leftarrow R_{eb} = 3R = 18\Omega$
$\tau = CR_{eb} = 18ms$

* Condiz iniziali

$v_c(0^+) = v_c(0^-)$, $t<0$ (stazionario)

$v_c(0^+) = E_0 = 9V$

* Condizioni finali

$t\to\infty$ (stazionario)

Vc su un corto
$v_c(\infty) = 0$

$$v_c(t) = E_0 e^{-\frac{t}{18\cdot10^{-3}}} \quad \text{per } 0<t<T=18ms$$

$$v_c(t) = 9e^{-\frac{500t}{9}} \quad \text{per } 0<t<18\cdot10^{-3}s$$

Secondo pezzo $t>18ms$

$t>T$ Faccio cambiamento variabile

$t' = t - T$, $T = 18ms$

* τ in $t'>0$ $(t>T)$

4R // corto
2R // R
$R_{eb} = 4R = 10\Omega$

$\tau = CR_{eb} = 10\cdot10^{-3} = 10^{-2}s$

* Condizioni iniziali

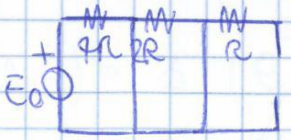
$t'<0$

$$v_c(T) = E_0 e^{-\frac{T10^3}{18}} = \frac{9}{e}$$

Siccome $v_c(t'=0^-) = v_c(t=0^+)$

* Condizione finale

$t'\to\infty$ (stazionario)



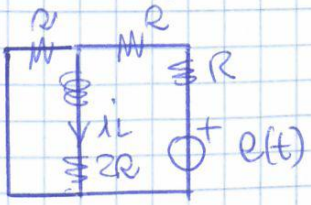
Vc su un corto
$v_c(\infty) = 0$

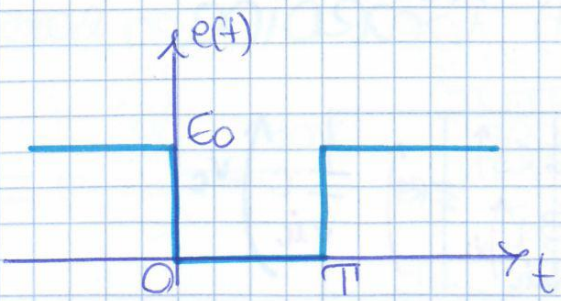
$$v_c(t) = \frac{9}{e} e^{-\frac{t'}{10^{-2}}} \quad \text{per } t>18ms$$

$$v_c(t) = \begin{cases} 9e^{-\frac{500t}{9}} & 0<t<18\cdot10^{-3}s \\ \frac{9}{e}e^{-100(t-18\cdot10^{-3})} & t>18\cdot10^{-3}s \end{cases}$$

$$\frac{9}{e}e^{(-100t+1,8)} = 9(e^{-1})e^{(-100t+1,8)}$$

R3 $i_L(t)$, $t>0$.

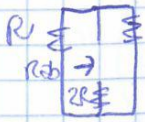




$E_0 = 24V$
$R = 1K\Omega$
$L = 1mH$
$T = 1ms$

PRIMO TRATTO $0<t<T$

★ $\tau = L/R_{EQ}$, $t>0$

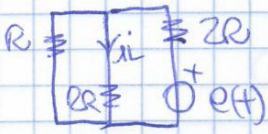


$R_{eb} = (R//2R)+2R$

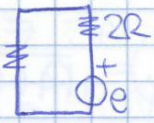
$R_{eb} = \frac{2}{3}R + \frac{6}{3}R = \frac{8}{3}R = \frac{8}{3}K\Omega$

$\tau = \frac{3}{8}10^{-6}s = \frac{3}{8}\mu s$

★ COND. INIZIALE $t<0$



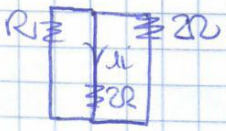
$R//2R = \frac{2}{3}R = \frac{2}{3}K\Omega$



$V_{2R} = e\frac{\frac{2}{3}R}{\frac{8}{3}R} = 24\cdot\frac{2}{8} = 6V$

$i_L = \frac{6}{2} = 3A$ $\quad i_L(0^-) = i_L(0^+) = 3A$

★ COND. FINALE



CIRCUITO INERTE
$e = 0$
$i_L(\infty) = 0$

★ SOLUZ.

$i_L(t) = 3e^{-\frac{8\cdot10^6 t}{3}}$ per $0<t<T$

SEGUE DISEGNO

TRATTO $t>T$

★ CAMBIAMENTO VARIABILE

$t' = t - T$

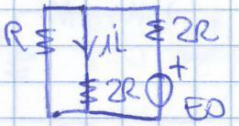
★ $\tau = L/R_{EQ}$, $t'>0$

$\tau = \frac{3}{8}\mu s$

★ COND. INIZIALE $t'<0$

$i_L(T) = 3e^{-\frac{8\cdot10^6\cdot10^{-3}}{3}} = 3e^{-\frac{8000}{3}}$

★ COND. FINALE $t'\to\infty$



$i_L = 3A$

★ SOLUZ.

$i_L(t) = \left[3e^{-\frac{8000}{3}} - 1\right]e^{-\frac{8\cdot10^6 t'}{3}} + 3$

$i_L(t) = 3\left(e^{-\frac{8000}{3}} - 1\right)e^{-\frac{8\cdot10^6 t'}{3}} + 3$

$i_L(t) = 3\left[\left(e^{-\frac{8000}{3}} - 1\right)e^{-\frac{8\cdot10^6 t'}{3}} + 1\right]$

$i_L(t) = 3\left[\left(e^{-\frac{8000}{3}} - 1\right)e^{-\frac{8\cdot10^6 (t-T)}{3}} + 1\right]$

$i_L(t) = 3\left[\left(e^{-\frac{8000}{3}} - 1\right)e^{-\frac{8\cdot10^6 t}{3} + \frac{8000}{3}} + 1\right]$

per $t>T$