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A P P U N T I

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MATERIA: Fondamenti di Macchine Termiche e Idrauliche

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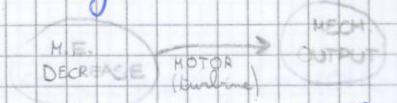
04/03

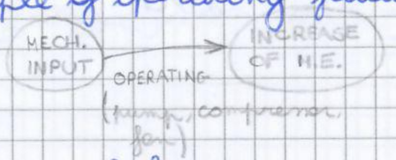
FLUID MACHINE

↳ system or a machine with a fluid working in it

- $\boxed{\text{MOTOR}}$: displays work on a shaft (turbine)



- $\boxed{\text{OPERATING}}$: it absorbs work from the outside to transfer to the fluid (a fan is an example of operating fluid machine and a compressor)



What are the energy sources?

- FOSSIL FUELS: energy is in chemical bonds;
- SOLAR ENERGY: available and clean ↑;
  not enough ↓;
- WATER: potential energy of water;
- WINDS: see solar energy;
- TIDES: not easy to be controlled;

The main goal is to increase the overall efficiency of the machine.

Fluid machines can be divided into:

- $\boxed{\text{THERMAL}}$: we consider the thermal issues connected with the fluid ($\rho \neq const.$)
- $\boxed{\text{HYDRAULIC}}$: deals with an uncompressible fluid ($\rho = const$) and thermal phenomena can be neglected
- $\boxed{\text{VOLUMETRIC}}$: machine where all the interactions within it are connected to a change of volume (internal combustion engine)
- $\boxed{\text{TURBO}}$: fluid flows continuously into the machine without significant change in volume (the flow has not to be steady but just continuous).

$$e_{mech} = \frac{p}{\rho} + \frac{V^2}{2} + gz$$

The choice of the system to study is always up to us but there is always a more or less convenient choice of the system.

Properties

- $\boxed{\text{INTERNAL STATE}}$: T and p define in a unique way our system (= HOMOGENUOUS SYSTEM);
- $\boxed{\text{EXTERNAL STATE}}$: $\underbrace{\text{position in space}}_{\text{BARICENTER}}$ of my system and velocity.

The HOMOGENEITY of the system is fundamental to allow us to write:

$V = m \cdot v$;

otherwise we should write:

$V = \int_{m_c} v\, dm$, remark that $v = \frac{1}{\rho}$

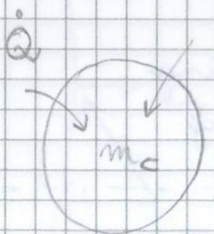
Also:

$\mathcal{L}, \mathcal{Q}$: extensive values

L, Q : intensive values

1st PRINCIPLE OF THERMODYNAMICS for CLOSED SYSTEMS

If we have a system that goes under an evolution from an initial state (i) at time t to the final (f) at time t+dt we can say:



$$\boxed{\delta Q + \delta \mathcal{L} = d\mathcal{E} = \mathcal{E}_f - \mathcal{E}_i}$$

$$\boxed{Q + \mathcal{L} = \Delta\mathcal{E} = \mathcal{E}_f - \mathcal{E}_i}$$

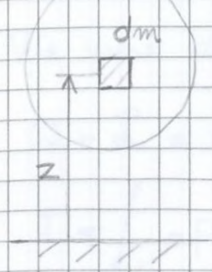
with

$\delta Q = \dot{Q}\, dt$, $\delta \mathcal{L} = \dot{\mathcal{L}}\, dt$

RMK the difference!

When the process executed by the closed system in a cycle:

$$\oint \delta Q + \oint \delta \mathcal{L} = 0$$



$\bar{c}$

$$\mathcal{E}_g = \int_{m_c} g z \, dm = g z m_c$$

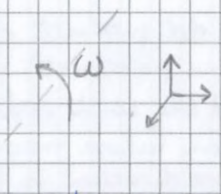
$$\mathcal{E}_k = \int_{m_c} \frac{c^2}{2} dm$$

The energy storage from centrifugal force is:

$$\mathcal{E}_\omega = \int_{m_c} - \frac{u^2}{2} dm = \int_{m_c} - \frac{\omega^2 r^2}{2} dm$$

tangential velocity

parte dell'energia viene spesa per tenere compatto il sistema



The 1st law of thermodynamics becomes:

$$Q + \mathcal{L} = \Delta \mathcal{E} =$$

$$= \Delta \mathcal{U} + \Delta \mathcal{E}_g + \Delta \mathcal{E}_k + \Delta \mathcal{E}_\omega$$

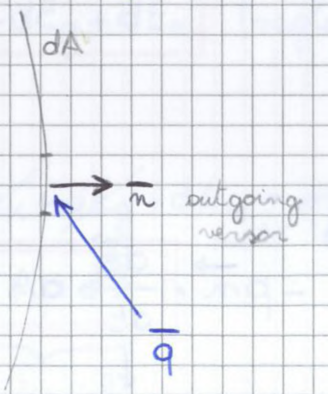
$$Q + L = \Delta U + \Delta E_g + \Delta E_k + \Delta E_\omega$$ INTEGRAL FORM

$$\delta Q + \delta L = dU + dE_g + dE_k + dE_\omega$$ DIFFERENTIAL FORM

WORK AND HEAT

Work and heat are exchanged at the boundary.

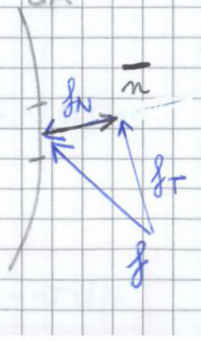
$$\delta Q = \dot{Q} \, dt$$



$\bar{q}$: heat per unit time per unit of surface

$$\dot{Q} = \int_A - \bar{q} \cdot \bar{n} \, dA$$

$$\delta \mathcal{L} = \dot{\mathcal{L}} \, dt$$

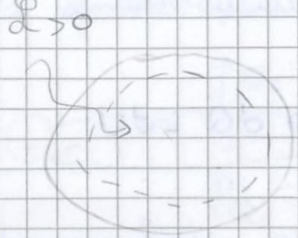


We were about 1^{st} law of thermodynamics:

$$Q + \mathcal{L} = \Delta \mathcal{E} = \Delta \mathcal{U}^* + \Delta \mathcal{E}_k + \Delta \mathcal{E}_g + \Delta \mathcal{E}_\omega$$

$$\mathcal{L} = \int_V -p\,dV \Rightarrow \delta \mathcal{L} = -p\,dV$$

what is - sign for?



- if $dV < 0$ (compression) $\Rightarrow \mathcal{L} > 0$
- if $dV > 0$ (expansion) $\Rightarrow \mathcal{L} < 0$

Actually the complete expression for work is:

2) $\boxed{dL = -p\,dv + dE_k + dE_g + dL_w}$

p è la pressione esterna al sistema chiuso

due to viscous losses ($dL_w \geq 0$)

δQ and δL will be converted into dQ and dL

1) $\boxed{dQ + dL = dU + dE_g + dE_k}$ (neglecting dE_ω)

So:

$$dQ - p\,dv + \cancel{dE_k} + \cancel{dE_g} + dL_w = dU + \cancel{dE_g} + \cancel{dE_k}$$

$$\boxed{\underbrace{dQ + dL_w}_{\text{interactions with the surrounding}} = \underbrace{dU + p\,dv}_{\text{properties of the system}}}$$

we define:

$$h = U + pv$$
$$dh = dU + p\,dv + v\,dp$$

So:

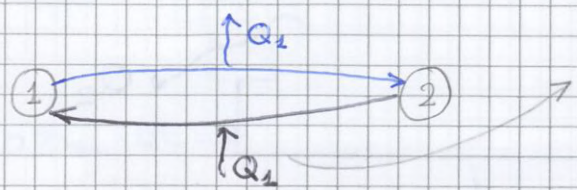
$$\boxed{dQ + dL_w = dh - v\,dp}$$

The 2nd principle can be also expressed in terms of Clausius inequality:

for a CYCLE (initial point = final point) we have:

$$\boxed{\oint \frac{dQ}{T} \leq 0}$$

- equality applies if the cycle is reversible:



if a opposite value of heat and/or work is introduced the system goes back to the initial state

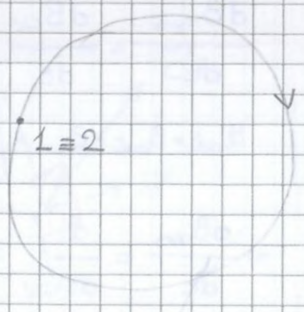
- S is a property ⇒ if we have to calculate the variation of entropy between two states we can calculate it through any process between the two same states (even a reversible process)

- for a cycle $\Delta S_{cycle} = 0$

$$\Rightarrow \oint \frac{dQ}{T} + \oint \frac{dL_w}{T} = 0$$

$$\boxed{\oint \frac{dQ}{T} = -\oint \frac{dL_w}{T}}$$ Clausius's inequality works

Let us consider a CYCLE:



$$dQ + dL = dU + dE_g + dE_k + dE_\omega$$

$$Q + L = \cancel{\Delta U} + \cancel{\Delta E_g} + \cancel{\Delta E_k} + \cancel{\Delta E_\omega}$$

$$= 0 \text{ (cycle)}$$

$$\Rightarrow \boxed{\oint dQ + \oint dL = 0}$$

$$m_c(t+dt) = m_{cv}(t+\Delta t) - \underset{(1-1')}{dm_{in}} + \underset{(2-2')}{dm_{out}}$$

$$m_{cv}(t) = m_{cv}(t+\Delta t) - dm_{in} + dm_{out}$$

$$m_{cv}(t+\Delta t) - m_{cv}(t) + dm_{out} - dm_{in} = 0$$

$$dm_{out} = \dot{m}_{out} \cdot dt$$
$$dm_{in} = \dot{m}_{in} \cdot dt$$

So:

$$dm_{cv} + \dot{m}_{out} \cdot dt - \dot{m}_{in} \cdot dt = 0$$

$$\frac{dm_{cv}}{dt} + \dot{m}_{out} - \dot{m}_{in} = 0$$

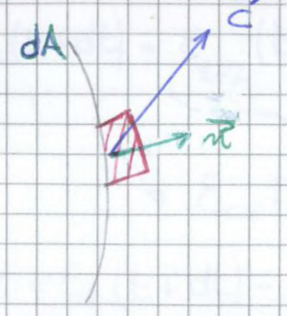
$$\frac{\partial m_{cv}}{\partial t} + \dot{m}_f = 0, \quad \dot{m}_f = \sum_i \dot{m}_{out,i} - \sum_j \dot{m}_{in,j}$$

↓
the use of a partial derivative is due to possible changes of mass within the control volume (it is not exactly the same mass: same amount but different mass)

so: if $\dot{m}_{in} = \dot{m}_{out} \Rightarrow \frac{\partial m_{cv}}{\partial t} = 0 \Rightarrow$ same amount of mass but different mass

So:

$$\boxed{\frac{\partial}{\partial t}\int_{CV} \rho \, dV + \dot{m}_f = 0}$$



$t \rightarrow t + dt$

$$d^2 m = \rho \, d^2 V, \quad d^2 V = dA \, \vec{n} \cdot d\vec{s}$$

$$\Rightarrow d^2 m = \rho \, dA \, \vec{n} \cdot d\vec{s}$$

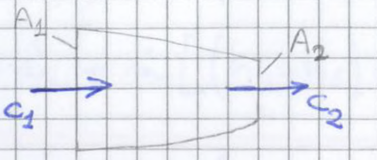
$$\frac{d^2 m}{dt} = \rho \, dA \, \vec{n} \cdot \vec{c}$$

$$d \frac{dm}{dt} = \rho \, dA \, \vec{n} \cdot \vec{c}$$

$$\boxed{\rho_2 c_2 \cos\vartheta_2 A_2 = \rho_1 c_1 \cos(\pi - \vartheta_1) A_1}$$

- If the fluid is <u>INCOMPRESSIBLE</u>

$$\boxed{c_{n_2} A_2 = c_{n_1} A_1} \quad \text{(CONTINUITY EQUATION)}$$



1^{st} PRINCIPLE for OPEN SYSTEM

$$\frac{\partial}{\partial t} m_{cv} + \dot{m}_g = 0$$

$$\frac{\partial}{\partial t}\int_{cv} \rho dV + \int_A \rho(\vec{c}\cdot\vec{n}) dA = 0$$

Let us consider a mass portion:

$m_c = m_{cv}(t+dt) + \ldots$

the 1^{st} law of thermodynamics holds:

$dQ + dL = dE$

$dQ + d\mathcal{L} = d\mathcal{E} = \mathcal{E}_c(t+dt) - \mathcal{E}_c(t)$,

$\mathcal{E}_c(t) = \mathcal{E}_{cv}(t)$

$$\mathcal{E}_c(t+dt) = \mathcal{E}_{(1'-2)}(t+dt) + \mathcal{E}_{2-2'}(t+dt)$$

$$\mathcal{E}_c(t+dt) = \underbrace{\mathcal{E}_{1-2}(t+dt)} + \mathcal{E}_{22'}(t+dt) - \mathcal{E}_{11'}(t+dt)$$

$$= \mathcal{E}_{cv}(t+dt) + d\mathcal{E}_{out} - d\mathcal{E}_{in}$$

$$d\mathcal{E} = \mathcal{E}_c(t+dt) - \mathcal{E}_c(t) =$$

$$= \mathcal{E}_{cv}(t+dt) + d\mathcal{E}_{out} - d\mathcal{E}_{in} - \mathcal{E}_{cv}(t)$$

So:

$$dQ + d\mathcal{L} = d\mathcal{E}_{cv} + \dot{\mathcal{E}}_{out}dt - \dot{\mathcal{E}}_{in}dt$$

$$\boxed{\dot{Q} + \dot{\mathcal{L}} = \frac{\partial \mathcal{E}_{cv}}{\partial t} + \dot{\mathcal{E}}_g}$$ flux term

- For 1D flow (= properties, at a given cross section, do not depend on the points of the cross section):

$$\dot{Q} + \dot{\mathcal{L}}_i = \underbrace{\rho_2 c_2 \cos\vartheta_2 A_2}_{\dot{m}_2}\left(h_2 + E_{K_2} + \dots\right) - \underbrace{\rho_1 c_1 \cos(\pi - \vartheta_1) A_1}_{\dot{m}_1}\left(h_1 + E_{K_1} + \dots\right)$$

but at steady state: $\dot{m}_2 = \dot{m}_1 = \dot{m}$

$$\dot{Q} + \dot{\mathcal{L}}_i = \dot{m}\left[(h_2 - h_1) + (E_{K_2} - E_{K_1}) + \dots\right]$$

$$\boxed{\dot{Q} + \dot{\mathcal{L}}_i = \dot{m}\left(\Delta h + \Delta E_K + \Delta E_g + \Delta E_\omega\right)}$$

operating motor

$$Q \pm L_i = \Delta h + \Delta E_K + \Delta E_g + \Delta E_\omega$$

If we make a comparison with the expression for closed system:

$$Q \pm L = \Delta U + \Delta E_K + \Delta E_g + \Delta E_\omega$$

we may notice:

- L refers to superficial work (moving boundary) while L_i is internal work (shaft work);
- ΔU for closed system is a difference between initial and final times

$$t_1 \longrightarrow t_2$$

$$\Delta U = U_{t_2} - U_{t_1}$$

- Δh for open system is a difference between sections 1 and 2 (= difference in space and not in time).

The 1^{st} principle of t states that:

$$dQ + dL_W = dh - v\,dp.$$

Let us consider the mass between sections 1 and 1' which evolves up to region 2 and 2'; since it is a closed system:

$$Q_{t_1 \to t_2} + L_W = h(t_2) - h(t_1) - \int_{t_1}^{t_2} v\,dp$$

since enthalpy is a property and we are considering a steady state flow the temperature of the mass does not change at the and if $L_i = 0$ (no shaft) we get:

$$\boxed{\frac{\Delta p}{\rho} + \Delta E_k + g\,\Delta z = 0}$$ BERNOULLI'S EQUATION

2^{nd} PRINCIPLE for OPEN SYSTEMS

$$dS = \frac{dQ}{T} + \frac{dL_w}{T}$$

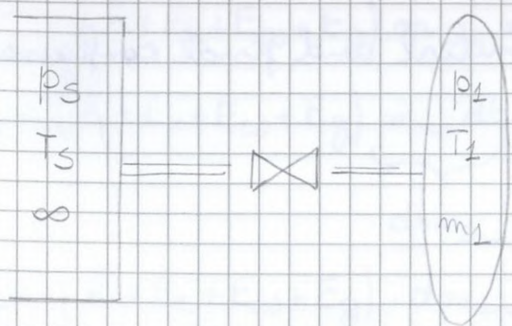
$$\int_{t_1}^{t_2} dS = \underbrace{S_2 - S_1}_{\text{for steady state flow}} = \int_{t_1}^{t_2} \frac{dQ}{T} + \int_{t_1}^{t_2} \frac{dL_w}{T} = \int_1^2 \frac{dQ}{T} + \int_1^2 \frac{dL_w}{T}$$

When we consider a COMPRESSIBLE FLUID (such a gas) we always neglect ΔE_g because to appreciate change of $1\,kcal/_{kg}$ we need a elevation change of 427 m.

11/03

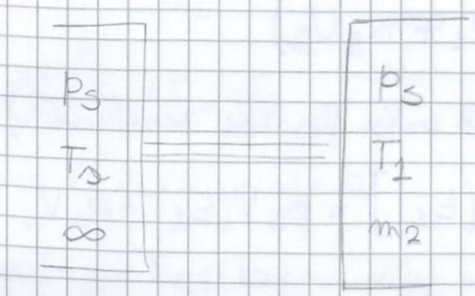
EXERCISE

Let us consider an infinite capacity tank (= property does not change) at temperature T_S and pressure p_S and a cylinder at pressure p_1 and temperature T_1. The valve is opened to allow fluid flow through the pipe.



t_1 $p_S >> p_1$

FLUID FLOW stops when the pressures get equal, as shown:



t_2

$p_2 = p_S$

$T_1 = ?$

m_2

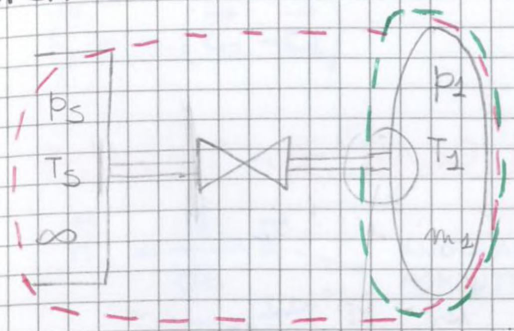
So:

$$p_S m_S v_S = m_2 U_2 - (m_1 U_1 + m_S U_S)$$

$$m_S (U_S + p_S v_S) = m_2 U_2 - m_1 U_1$$

$$m_S h_S = m_2 U_2 - m_1 U_1$$

OPEN-SYSTEM APPROACH

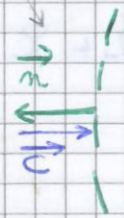


The open system chosen is not steady; so:

$$\cancel{Q} + \mathcal{L}_i = \frac{\partial}{\partial t}\int_{CV} \rho E \, dV + \int (h + E_K + E_\omega + E_g) \rho (\vec{c} \cdot \vec{n}) dA$$

$=0$

- $Q = 0$ because the process is adiabatic;
- $\mathcal{L}_i = 0$ (null) because there is no shaft.



let's suppose we have a 1D flow

$$\frac{\partial \mathcal{E}_{CV}}{\partial t} - (h + E_K + E_\omega + E_g)\dot{m}_{in} = 0$$

$$d\mathcal{E}_{CV} - (h + E_K + E_\omega + E_g)\underbrace{\dot{m}_{in} dt}_{dm_{in}} = 0$$

$$d\mathcal{E}_{CV} - (h + E_K + E_\omega + E_g) \cdot dm_{in} = 0$$

$$\int_{t_i}^{t_f} d\mathcal{E}_{CV} - \int_{t_1}^{t_f} (h + E_K + \underbrace{\cancel{E_\omega}}_{=0} + \underbrace{\cancel{E_g}}_{=0}) \, dm_{in} = 0$$

$$\mathcal{E}_{CV} \rightarrow \mathcal{U} + \mathcal{E}_K + E_\omega + E_g$$

since $\mathcal{E}$ is a property:

$$\mathcal{E}_{CV}(t_f) - \mathcal{E}_{CV}(t_i) -$$

• at steady state: $\frac{\partial}{\partial t}\int_{CV} \rho \vec{c}\, dV = 0$

$dm = 0 \Rightarrow \dot{m}_{in} = \dot{m}_{out}$

so we get:

$$\boxed{\vec{F} - \int_A p\vec{n}\, dA + \int_V \rho \vec{g}\, dV = \dot{m}(\vec{c}_2 - \vec{c}_1)}$$

$$\vec{R} = \dot{m}(\vec{c}_2 - \vec{c}_1)$$

MOMENT OF MASS MOMENTUM

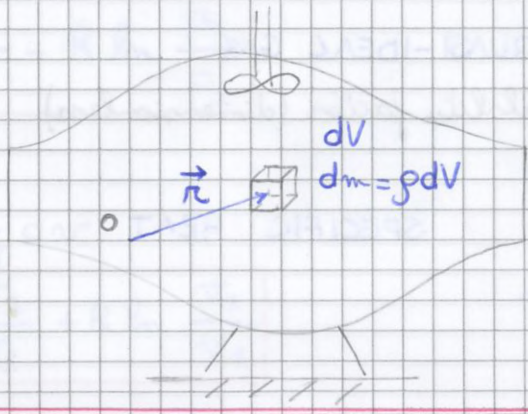
$$\vec{M}_o = \underbrace{\frac{d}{dt}}_{\text{SUBSTANTIAL DERIVATIVE}} \vec{K} \longrightarrow \text{ANGULAR MOMENTUM}$$

$$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d}{dt}(I\vec{\omega}) = \frac{d}{dt}(\vec{H})$$

$$\vec{M} = \vec{r} \wedge \vec{R} = \vec{r} \wedge m\frac{d\vec{V}}{dt} = m\frac{d}{dt}(\vec{r} \wedge \vec{V})$$

Using RTT we can say:

$$\frac{d\vec{K}}{dt} = \frac{\partial \vec{K}}{\partial t} + \dot{K}_f$$



$$\boxed{\vec{M}_o = \frac{\partial}{\partial t}\int_{CV} (\vec{r} \wedge \vec{c})\, \rho\, dV + \int_A (\vec{r} \wedge \vec{c})\, \rho(\vec{c} \cdot \vec{n})\, dA}$$

$\vec{r} \times \vec{R}$

At STEADY STATE:

$$\vec{M}_o = \int_A (\vec{r} \wedge \vec{c})\, \rho(\vec{c} \cdot \vec{n})\, dA$$

for 1D FLOW:

$$\vec{M}_o = \dot{m}(\vec{r}_{out} \wedge \vec{c}_{out} - \vec{r}_{in} \wedge \vec{c}_{in})$$

$$dh - du = p\,dv + v\,dp$$

$$c_p\,dT - c_v\,dT = d(vp)$$

$$c_p\,dT - c_v\,dT = d(RT)$$

$$c_p\,dT - c_v\,dT = R\,dT$$

$$\boxed{c_v = \frac{R}{K-1} \qquad c_p = \frac{K}{K-1}R}$$

$\boxed{c_p - c_v = R}$ Mayer's relation (for ideal gases only)

Then:

$$dS = \frac{dQ + dL_w}{T}$$

$$\boxed{T\,dS = dQ + dL_w = dh - v\,dp = dU + p\,dv}$$

1st and 2nd Gibbs equations

1) $$T\,dS = c_p\,dT - \frac{RT}{p}dp$$

$$dS = c_p\frac{dT}{T} - \frac{R}{p}dp$$

$$\Delta S_{1-2} = \int_1^2 c_p\frac{dT}{T} - \int_1^2 \frac{R}{p}dp$$

$$\boxed{\Delta S_{1-2} = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}}$$

2) $$T\,dS = dU + p\,dv$$

$$dS = c_v\frac{dT}{T} + \frac{RT}{vT}dv$$

$$\boxed{\Delta S_{1-2} = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}}$$

PROCESSES

1) $p = \text{const}$ ISOBARIC

$pv = RT \Rightarrow \dfrac{T}{v} = \text{const}$

2) $v = \text{const}$ ISOCHORIC

$pv = RT \Rightarrow \dfrac{T}{p} = \text{const.}$

3) $T = \text{const}$ ISOTHERMAL

$pv = \text{const}$

4) ADIABATIC ($Q=0$)
REVERSIBLE $\Rightarrow$ ISENTROPIC ($S = \text{const}$)

$$c\,dT = c_p\,dT - \frac{RT}{p}\,dp$$

$$(c_p - c)\frac{dT}{T} - (c_p - c_v)\frac{dp}{p} = 0 \qquad (1)$$

And recalling:

$$T\,dS = dU + p\,dv$$

$$\Rightarrow \quad (c_v - c)\frac{dT}{T} + (c_p - c_v)\frac{dv}{v} = 0 \qquad (2)$$

For isentropic processes:

$$c_v\frac{dT}{T} + (c_p - c_v)\frac{dv}{v} = 0$$

$$(I) \quad \frac{dT}{T} + (k-1)\frac{dv}{v} = 0$$

For polytropic processes:

$$(c_v - c)\frac{dT}{T} + (c_p - c_v)\frac{dv}{v} = 0$$

$$(P) \quad \frac{dT}{T} + \left(\frac{c_p - c_v}{c_v - c}\right)\frac{dv}{v} = 0$$

$$\boxed{m - 1 = \frac{c_p - c_v}{c_v - c}} \Rightarrow m = \frac{c_p - c}{c_v - c}$$

POLYTROPIC INDEX

$$\Rightarrow \quad (P) \quad \frac{dT}{T} + (m-1)\frac{dv}{v} = 0$$

So we can write the following equations for POLYTROPIC PROCESSES:

$p v^{m} = \text{const.}$
$T v^{m-1} = \text{const.}$
$\dfrac{T}{p^{\frac{m-1}{m}}} = \text{const.}$

se hai il coefficiente m di un processo puoi applicare direttamente queste relazioni perché esse valgono anche per $dQ \neq 0$ e $dL_w \neq 0$

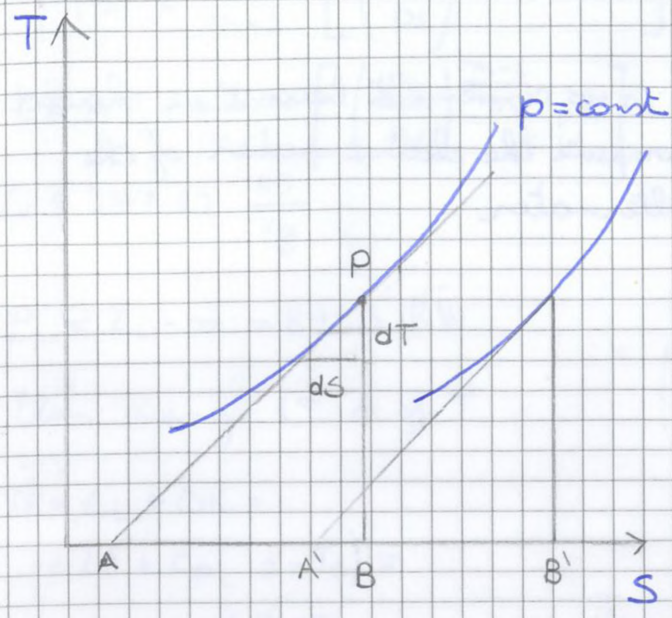
$$T_1\, p_1^{\frac{1-m}{m}} = T_2 \cdot p_2^{\frac{1-m}{m}}$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{1-m}{m}} = \frac{T_1}{T_2} \Rightarrow \frac{1-m}{m} = \log_{p_2/p_1}\frac{T_1}{T_2} \Rightarrow \frac{1}{m} = 1 + \log_{p_2/p_1}\frac{T_1}{T_2}$$

$$\Uparrow$$

$$m = \frac{1}{1 + \log_{p_2/p_1}\frac{T_1}{T_2}}$$

Moreover:



- nel diagramma T,S le isobare e le isocore sono curve congruenti, cioè che possono essere sovrapposte con un movimento rigido
- poiché

  $\Delta S = c_p \ln \Delta T$

  le isobare sono curve a pendenza crescente

  e le isocore lo sono ancora di più

$$T\,dS = c_p\,dT$$

$$\frac{dS}{dT} = \frac{c_p}{T} \qquad (1) \qquad\qquad \frac{dT}{T} = \frac{dS}{c_p} \Rightarrow \Delta S = c_p \ln\frac{T_2}{T_1}$$

from geometry:

$$\frac{dS}{\overline{AB}} = \frac{dT}{\overline{PB}} \Rightarrow \frac{dS}{\overline{AB}} = \frac{dT}{T} \qquad (2)$$

from (1) and (2) we get:

$$c_p = \overline{AB} = \overrightarrow{A'B'}$$

APPLIED LECTURE 1

2) Data:

$\dot{m} = 3\ \frac{kg}{s}$

$p_1 = 10$ bar

$T_1 = 500°C$

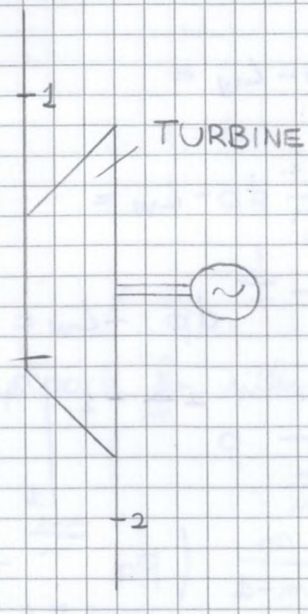
$p_2 = 1$ bar

$pv^{1.5} = \text{const}$

$\Delta E_k \cong 0$

$L_W = 62\ \frac{kJ}{kg}$

c_p, R are known



Unknowns:

P_i ?

Q ?

$$= -p_1 v_1 \cdot \frac{m}{m-1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}} - 1\right] - L_w =$$

$$= -RT_1 \cdot \frac{m}{m-1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}} - 1\right] - L_w$$

$$\int_1^2 v\,dp = RT_1 \cdot \frac{m}{m-1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}} - 1\right]$$

$$L_i = 297.12 \ \frac{KJ}{Kg}$$

$$P_i = L_i \cdot \dot{m} = 891.3 \ KW$$

Then using 1^{st} L. of T.

$$Q = L_i + \Delta h =$$

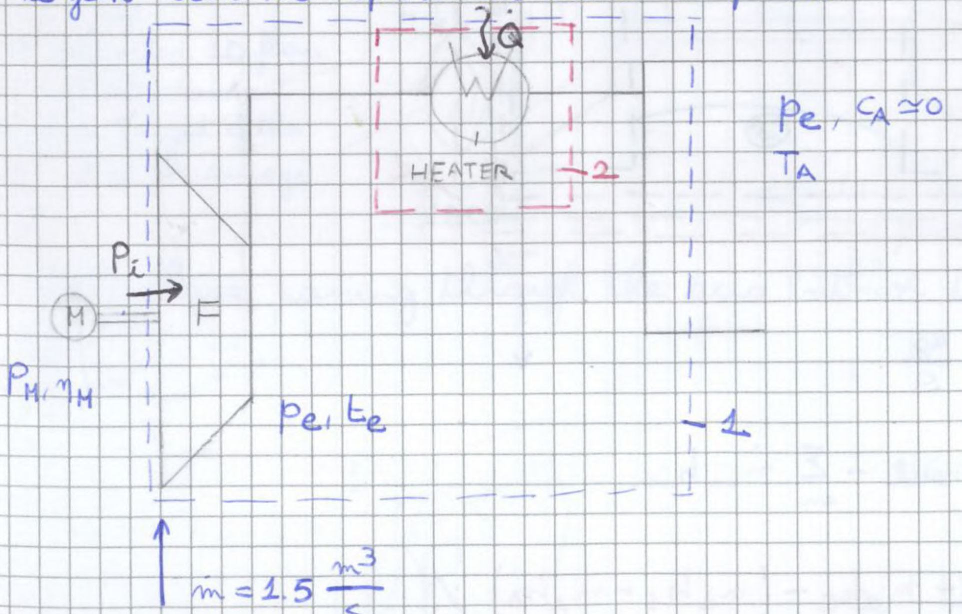
$$= L_i + c_p(T_2 - T_1) =$$

$$= L_i + c_p\left(\frac{p_2 v_2}{R} - T_1\right), \quad \frac{T}{p^{\frac{m-1}{m}}} = \text{const.}$$

$$= L_i + c_p\left[T_1\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}} - T_1\right] = -158,5 \ \frac{KJ}{Kg}$$

we are refrigerating our turbine from the outside

3) A fan can be represented as a compressor:



We choose the Eulerian approach for either 1 or 2 of the CV chosen:

$$Q + L_i = \Delta h + \Delta E_k + \ldots$$

$$\dot{Q} + P_i = \dot{m}(h_{out} - h_{in})$$

$$P_i = P_M \cdot \eta_M = 3.59 \ KW$$

1) is wrong choice becouse

$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4$

$\dot{m}_a = \dot{m}_b$

2) we don't know $\dot{m}_a = \dot{m}_b = ?$

4) right choice

Using CV # 4:

$P_i = \dot{m}\left(c_p T_2 + c_p T_4 - c_p T_1 - c_p T_3\right)$, c_p normally assumed as $1004.5 \ \frac{J}{kg \cdot K}$

$$P_M = \frac{P_i}{\eta_m} = 37 \ KW$$

13/03

TURBO-MACHINES

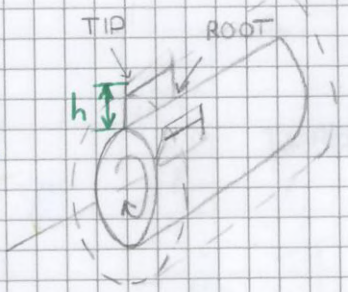
A turbo-machine is made of blades arranged around an axis.

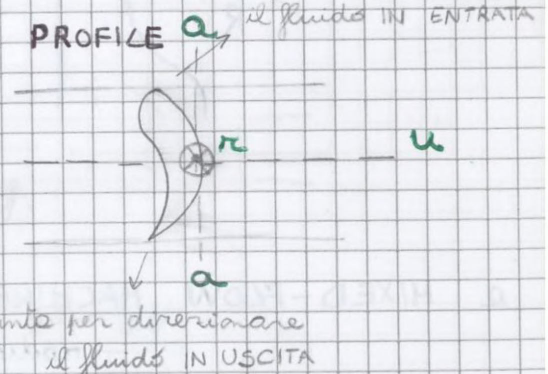
1^{st}) cylindrical surface with bigger diameter

$\frac{h}{d} << 1$

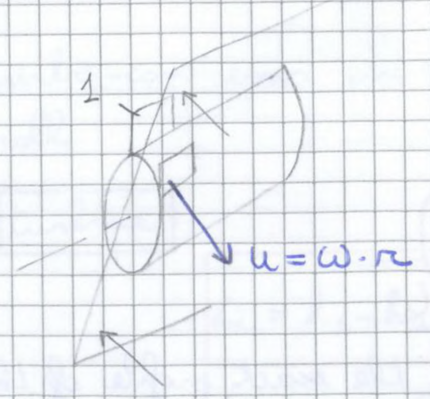
⇓

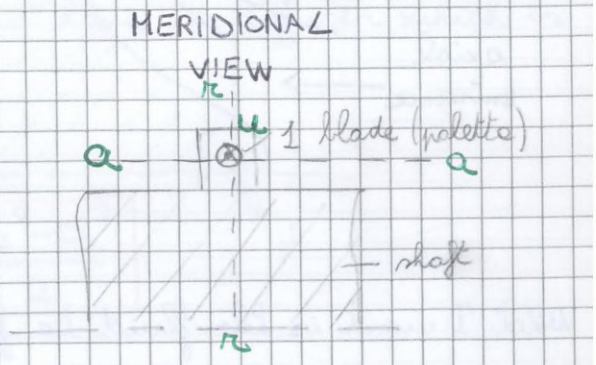
we'll consider 1D flow with velocity constant along the height of the blade set at the average height





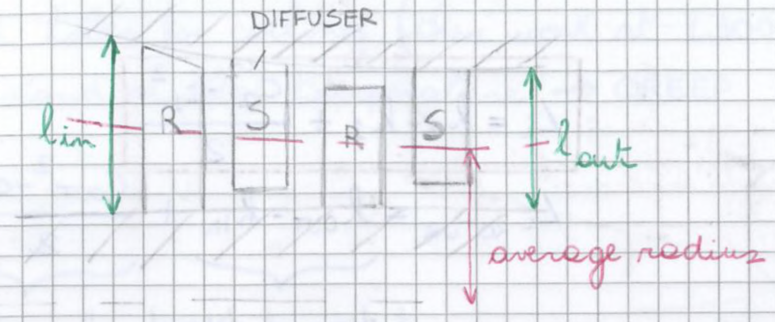
2^{nd}) plane passing through the axis but not touching the blades



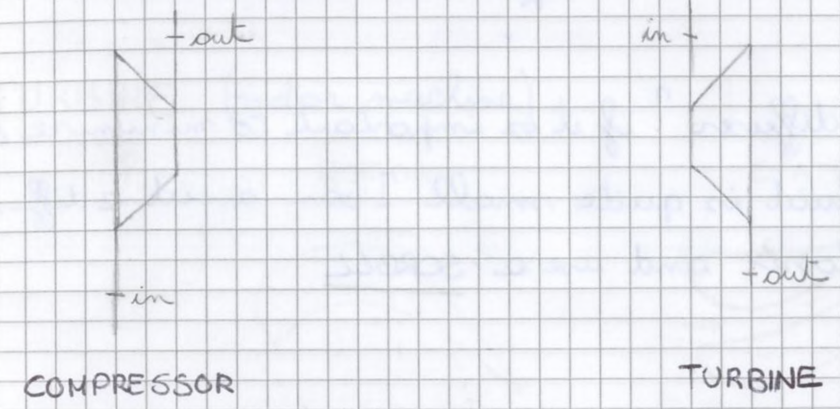


A type of MERIDIONAL VIEW is the following:

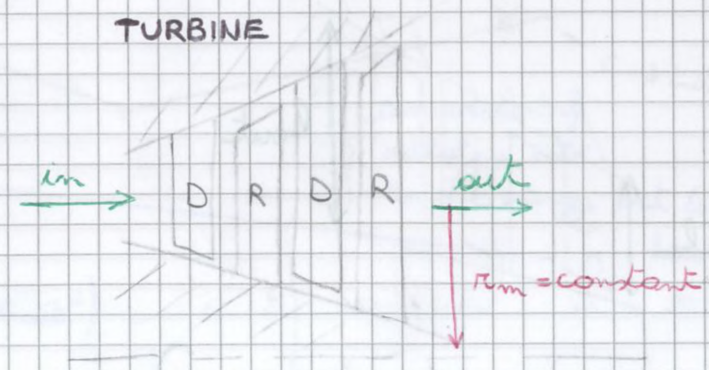
AXIAL TURBO-COMPRESSOR



That is the reason why:

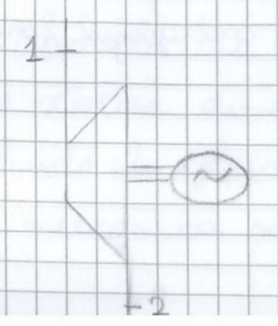


TURBINE



Quite often in a turbo-compressor you do not have the diffuser which is replaced by a scroll.

Let us consider a turbine



$$\boxed{L_i = h_1 - h_2 + \frac{c_1^2 - c_2^2}{2}}$$

$L_{i_{STATOR}} = 0$

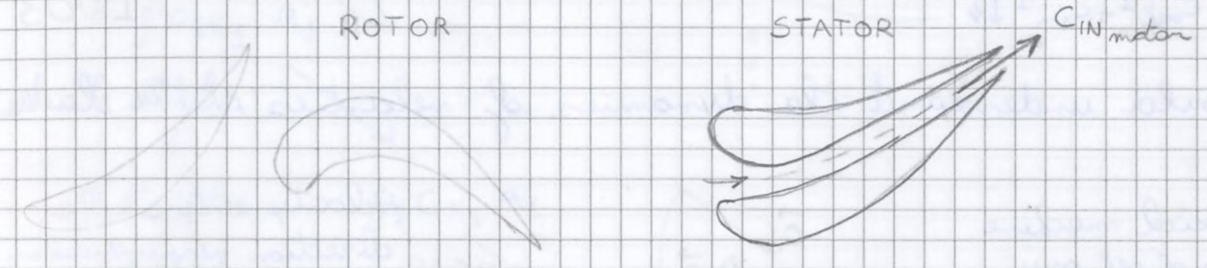
$$L_{i_{ROTOR}} = h_{IN} - h_{OUT} + \frac{c_{IN}^2 - c_{OUT}^2}{2}$$

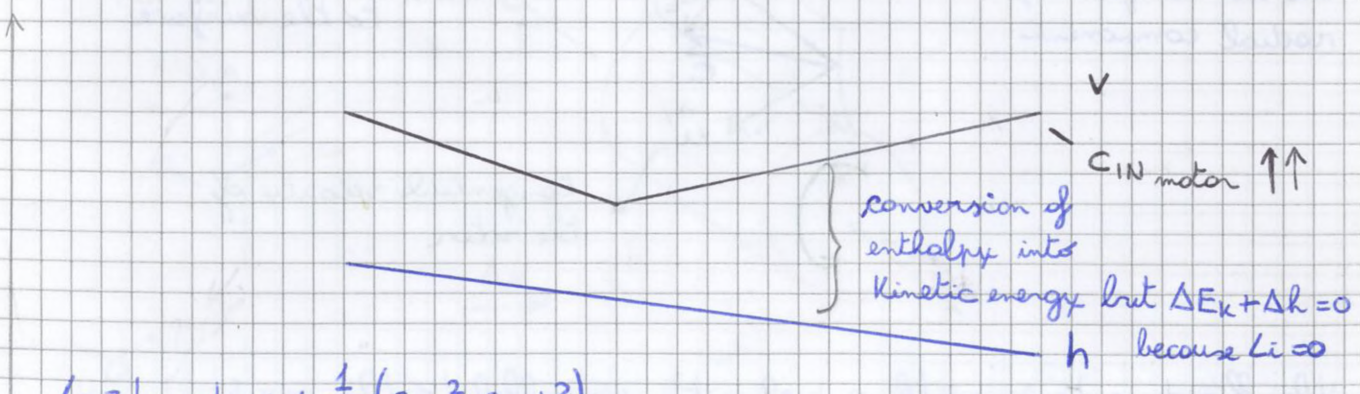
to maximise L: $c_{IN} \uparrow\uparrow$, $c_{OUT} = 0$

In this last case we use a TWISTED PROFILE ; it is also possible to have both twisted blades and not-twisted blades for compressor and turbine.

Turbines have holes for cooling (they work at 1300°C) because they continuously stand at that temperature → CREEP

TURBINE (motor machine)





$L_i = h_{in} - h_{out} + \frac{1}{2}(c_{in}^2 - c_{out}^2)$

$L_i \uparrow\uparrow \Rightarrow c_{in}^2 - c_{out}^2 \uparrow\uparrow$

La prima palettatura fissa dello statore (distributore) serve ad aumentare l'energia cinetica del fluido; poi il rotore fa diminuire velocità e entalpia del fluido trasformando la variazione di energia meccanica in lavoro per l'albero; se vi fosse un nuovo rotore erro utilizzerebbe una velocità in entrata troppo bassa e quindi le palette dello statore aumenta la velocità in entrata del rotore.
Il fluido che colpisce le pale del rotore lo fa muovere e genera energia meccanica sotto forma di rotazione dell'albero.

The mass flow rate equation is:

$$\underbrace{\cancel{\frac{\partial m_{cv}}{\partial t}}}_{\text{transient term}} + \underbrace{\dot{m}_f}_{\text{flux term}} = 0$$

talking with TURBOMACHINES we'll deal with STEADY STATE flow; so:

$$\dot{m}_f = 0 \Rightarrow \dot{m} = \text{const.}$$

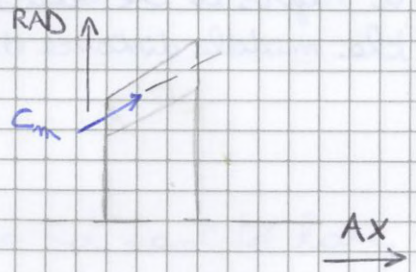
$$\dot{m} = \int_A \rho \left(\vec{C} \cdot \vec{n} \right) dA$$

we assume the flow to be 1D flow; so:

$$\dot{m} = \rho_1 \vec{C}_1 \cdot \vec{n} \, A_1 =$$
$$= \rho \, c_{m_1} \cdot A_1$$

$\rho_1 \, c_{a_1} A_1$ AXIAL

$\rho_1 \, c_{x_1} A_1$ RADIAL



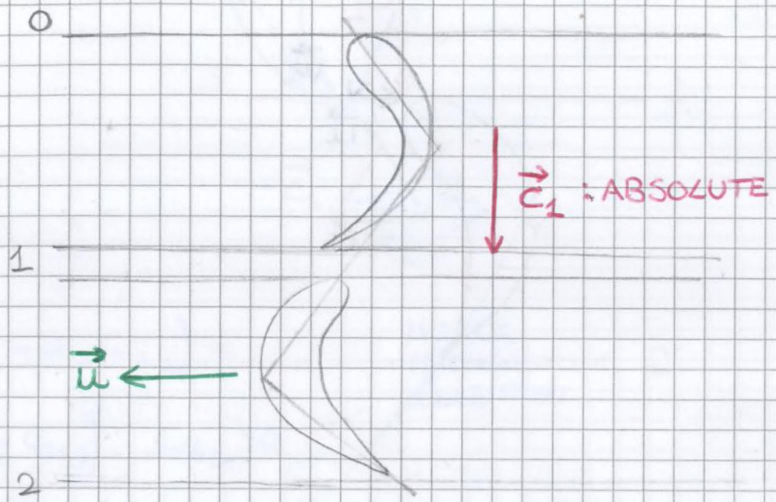
What about the area A to be considered for the mass flow rate?

since $l << d$ we can compute it as:

$$\frac{\rho_1}{\rho_2} = \frac{c_{a_2}}{c_{a_1}}$$

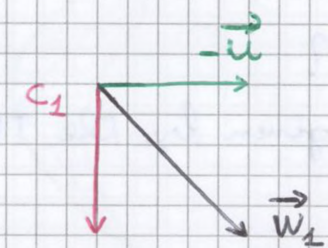
· if $\rho = \text{const} \Rightarrow c_a = \text{const}$.

VELOCITY TRIANGLE



$\vec{C_1} = \vec{u} + \vec{w}$

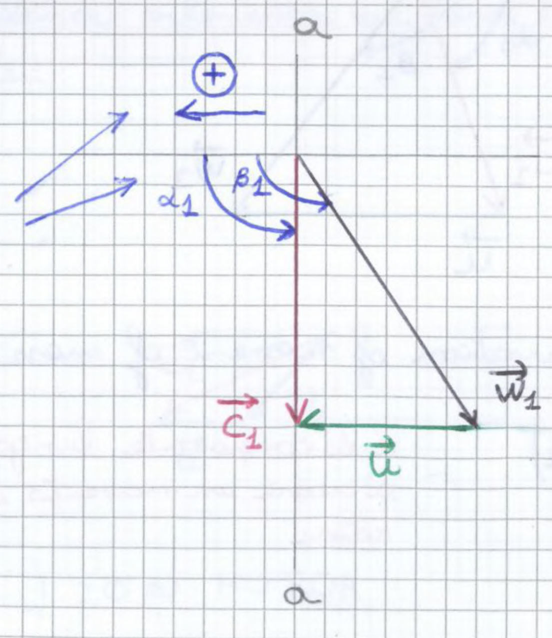
$\vec{w_1} = \vec{C_1} - \vec{u}$



velocity of the fluid relative to the blade

we do not want to have $-\vec{u}$, so:

α_1 is not necessarily equal to $\frac{\pi}{2}$

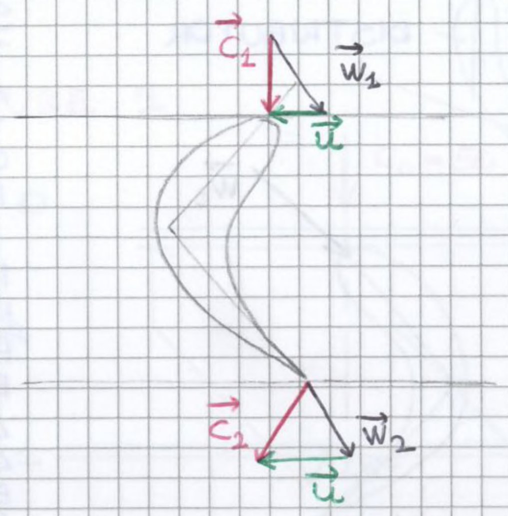


conventions assumed if u is directed leftwards

$L_i = u(c_{u_2} - c_{u_1})$

$\boxed{L_i = u_2 c_{u_2} - u_1 c_{u_1}}$ EULER

CIAO

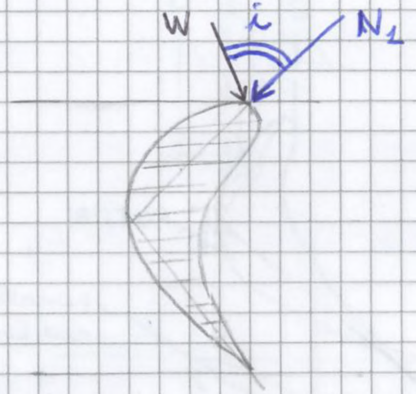


$L_i = (u_1 c_{u_1} - u_2 c_{u_2}) =$

MOTOR MACHINE CONVENTION

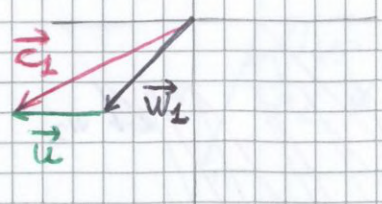
$= u(\cancel{c_{u_1}} - c_{u_2}), \quad c_{u_2} > 0$

$= -u c_{u_2} < 0 \Rightarrow$ OPERATING



we have turbulence whenever we have incidence $\Rightarrow$ the optimum solution is when:

$\boxed{i = 0}$

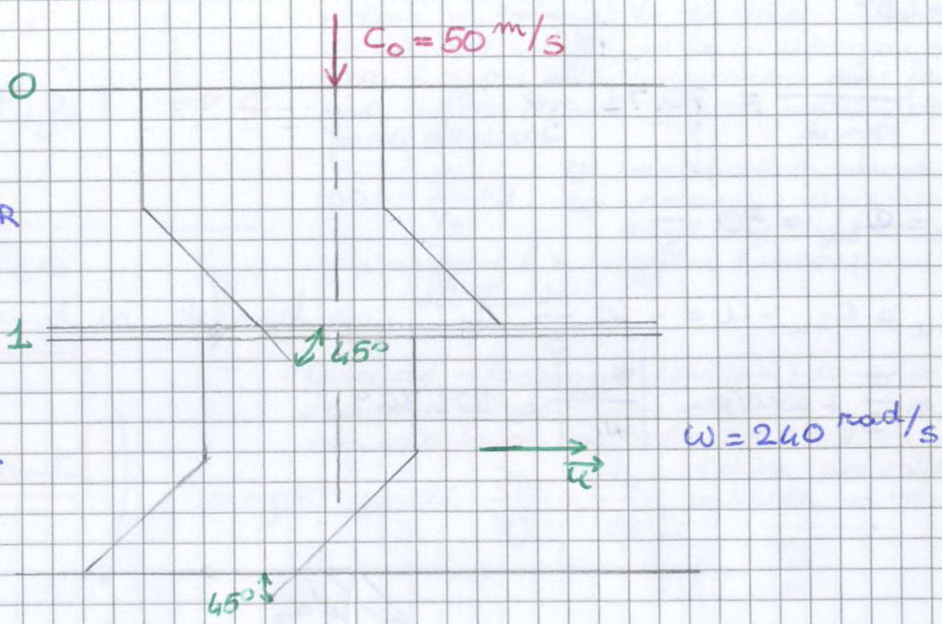


$L_i = u(c_{u_1} - c_{u_2}) > 0 \Rightarrow$ MOTOR

That is the reason why we have a smooth leading edge is that:

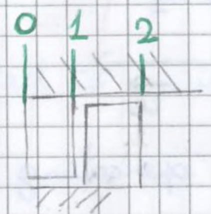
APPLIED LECTURE 2

1)

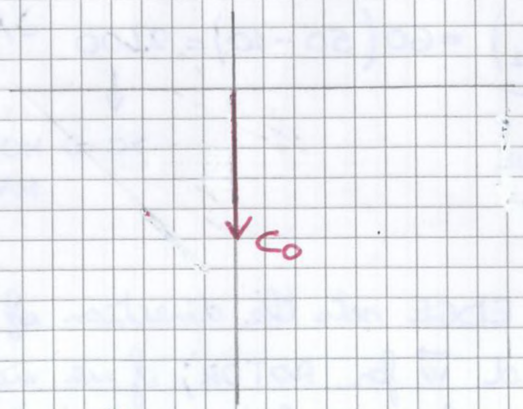


d, l const

WATER

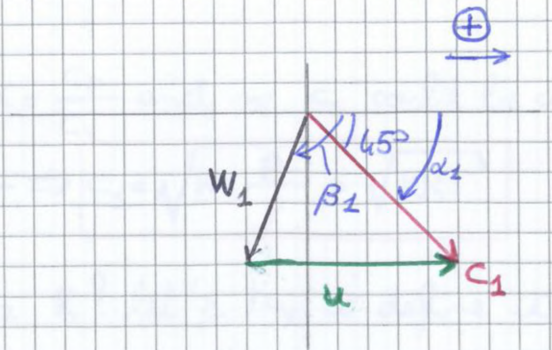


(0)



$$\vec{u} = \omega \cdot r = \omega \cdot \frac{d}{2} = 60\ \frac{m}{s}$$

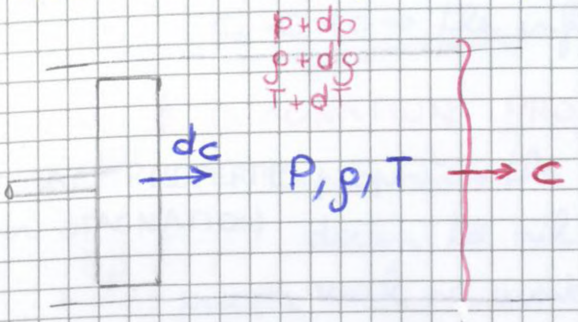
(1)



we are not working under design, so we cannot assume W_2 axially

We deal with a fixed channel flow.

Let us consider fluid properties in a channel:



- Se comprimiamo lentamente mediante un pistone un volume V_0 di aria contenuta in un cilindro indeformabile ci accorgiamo che essa si oppone alla compressione come farebbe una molla. L'aria reagisce mediante una forza elastica

$$\Delta p = -\frac{K}{V_0}\Delta V,$$ ΔV variazione di volume, Δp variazione di pressione all'interno del cilindro

$\Rightarrow$ maggiore è K maggiore l'incomprimibilità del fluido

The velocity of sound is defined as:

$$\boxed{c_s = \sqrt{\left.\frac{\partial P}{\partial \rho}\right|_{s=const}}}$$

- Quando rilasciamo il pistone l'aria torna ad occupare il suo volume originale in maniera non istantanea, ma rallentato dall'inerzia dell'aria, cioè dalla massa contenuta nel volume V_0

$p = \rho RT$, perché $\frac{\partial p}{\partial \rho} = \frac{dp}{d\rho}$

$$\rho = \frac{m}{V_0}$$

$$c_s = \sqrt{\left.\frac{dP}{d\rho}\right|_{s=const.}}$$

$$\Rightarrow c = \sqrt{\frac{K}{\rho}}$$

for a isentropic process applied to an ideal gas:

$$\frac{P}{\rho^K} = const$$

$$dP = d(const \cdot \rho^K)$$

$$dP = const \cdot d(\rho^K)$$

$$dP = const \cdot K \cdot \rho^{K-1} d\rho$$

$$dP = \frac{P}{\rho^K} \cdot K \cdot \rho^{K-1} d\rho$$

$$\boxed{\frac{dP}{d\rho} = K \cdot \frac{P}{\rho}}$$

- for an ideal gas $K = \frac{c_p}{c_v}$ and so c_s could be calculated as square root of $K \cdot \frac{P}{\rho} \Rightarrow \boxed{c_s = \sqrt{K \cdot \frac{P}{\rho}} = \sqrt{KRT}}$
- for vapour what is K? let's try to define it

$Ma < 1 \Rightarrow c < c_s$

$c_s - c > 0 \Rightarrow$ the information arrives

$Ma > 1 \Rightarrow c > c_s$

$c_s - c < 0 \Rightarrow$ the information does not arrive

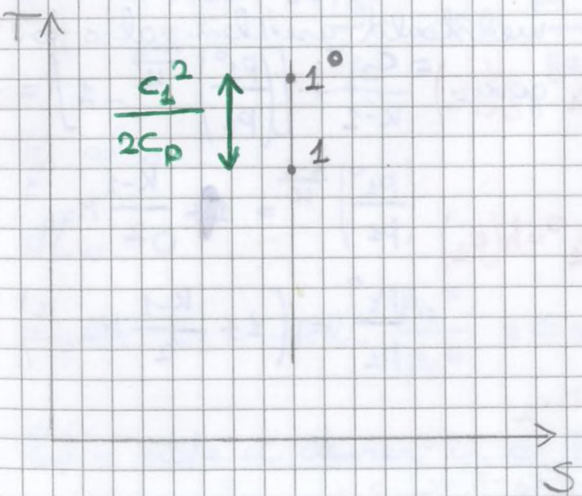
TOTAL & STAGNATION PROPERTIES

TOTAL PROPERTIES: (or STAGNATION) properties the fluid would get if decelerating down to nil velocity through an adiabatic reversible process with no work exchanged with environment.

$$\boxed{p_1, c_1, T_1 \xrightarrow[L_i = 0]{dQ = 0,\ dL_w = 0} c_1^\circ = 0,\ p_1^\circ,\ T_1^\circ,\ \rho_1^\circ}$$

Applying 1st l. of. t. to o.s.:

$$\cancel{dQ} + \cancel{dL_i} = dh + dE_k + \underbrace{\cancel{dE_g}}_{\simeq 0} + \cancel{dE_w}$$



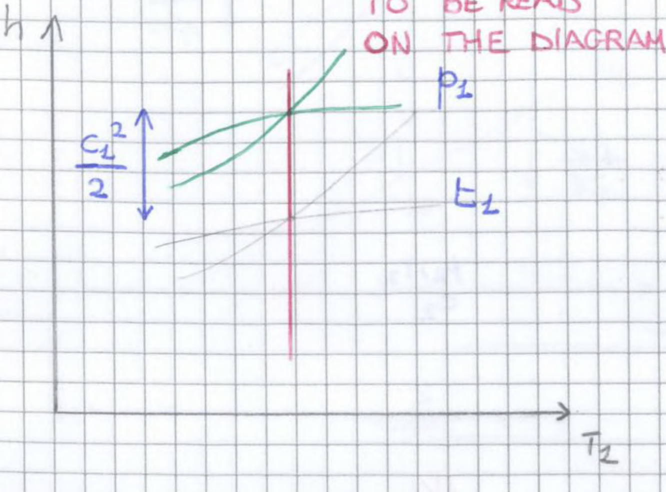
$$h_1^\circ - h_1 + \frac{\cancel{c_1^{\circ 2}} - c_1^2}{2} = 0$$

$$h_1^\circ = h_1 + \frac{c_1^2}{2}$$

$$c_p (T_1^\circ - T_1) = \frac{c_1^2}{2}$$

$$\boxed{T_1^\circ = T_1 + \frac{c_1^2}{2c_p}} \qquad T_1^\circ = f(T_1)$$

What about Mollier's diagram for vapour:



$t\,[°C]$

$T\,[K]$

$$dS = c_p \frac{dT}{T} - R\frac{dp}{p}$$

$$S_2 - S_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

$$S_2 - S_1 = S_2^\circ - S_1^\circ = c_p \ln\frac{T_2^\circ}{T_1^\circ} - R\ln\frac{p_2^\circ}{p_1^\circ}$$

Let us suppose the process to be $\boxed{\text{adiabatic}}$: $dQ = 0$ and let's apply 1st l. of t. for process $1 \to 2$:

$$dQ + dL_i = dh + dE_K$$

$$h_1 + \frac{c_1^2}{2} = h_2 + \frac{c_2^2}{2}$$

$$h_1^\circ = h_2^\circ$$

$$\boxed{h^\circ = \text{const.}}$$

$$\Rightarrow \boxed{T^\circ = \text{const.}}$$

- adiabatic
- no internal work

$h^\circ = \text{const}$

$\Rightarrow T^\circ = \text{const}$

$S_2 - S_1 = -R\ln\frac{p_2^\circ}{p_1^\circ}$

So:

$$S_2 - S_1 = c_p \ln\frac{T_2^\circ}{T_1^\circ} - R\ln\frac{p_2^\circ}{p_1^\circ} = -R\ln\frac{p_2^\circ}{p_1^\circ}$$

If also:

$$dL_W = 0$$

$$\Rightarrow S_2 - S_1 = -R\ln\frac{p_2^\circ}{p_1^\circ} = 0 \Rightarrow p^\circ = \text{const}$$

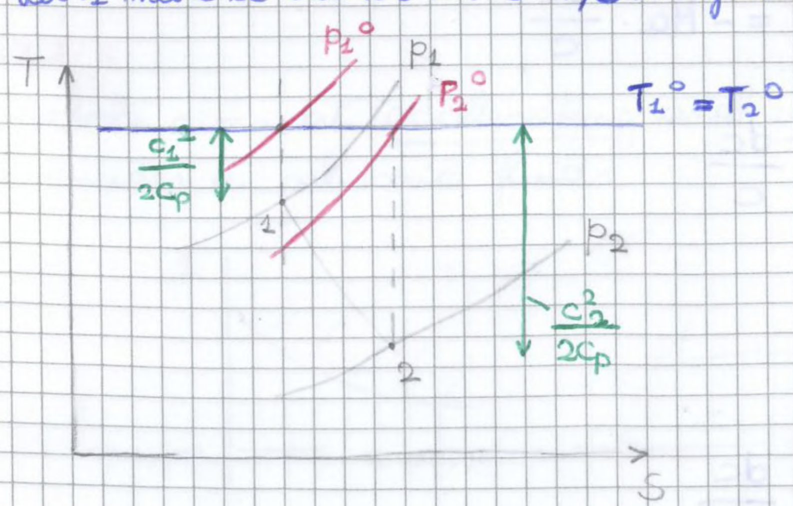
- adiabatic reversible
- no internal work

$h^\circ = \text{const.}$

$\Rightarrow T^\circ = \text{const}$

$p^\circ = \text{const.}$

Let's make it clearer in a T, S diagram



$$\frac{dp}{Kp} = -\frac{c^2}{c_s^2} \cdot \frac{dc}{c} \Rightarrow \frac{dc}{c} = -\frac{1}{KMa^2} \cdot \frac{dp}{p}$$

$$\boxed{\frac{dA}{A} = \frac{1-Ma^2}{KMa^2} \cdot \frac{dp}{p}}$$

un UGELLO o DIFFUSORE possono:
- accelerare e espandere
- decelerare e comprimere
non ci sono più casi (accelerare e comprimere o espandere e decelerare) perché

$$\frac{dc}{c} = -\frac{dp}{p} \cdot \underbrace{\frac{1}{KMa^2}}_{>0}$$

1) hp

$\frac{dA}{A} > 0$, $Ma < 1$ $\Rightarrow$ $\frac{dc}{c} < 0$, $\frac{dp}{p} > 0$ $\Rightarrow$ DIFFUSER = compressing + decelerating

2) hp

$\frac{dA}{A} > 0$, $Ma > 1$ $\Rightarrow$ $\frac{dc}{c} > 0$, $\frac{dp}{p} < 0$ $\Rightarrow$ NOZZLE: accelerating + expanding

3) hp

$\frac{dA}{A} < 0$, $Ma < 1$ $\Rightarrow$ $\frac{dc}{c} > 0$, $\frac{dp}{p} < 0$ $\Rightarrow$ NOZZLE: accelerating + expanding

4) hp

$\frac{dA}{A} < 0$, $Ma > 1$ $\Rightarrow$ $\frac{dc}{c} < 0$, $\frac{dp}{p} > 0$ $\Rightarrow$ DIFFUSER: compressing + decelerating

NOZZLE

A nozzle aims at accelerating and expanding the gas.
We consider a subsonic fluid:

CONVERGENT
subsonic flow

DIVERGENT
supersonic flow

$$1)\quad \boxed{c\left(\frac{p}{p_1^\circ}\right) = \sqrt{\frac{2k}{k-1}\,\frac{p_1^\circ}{\rho_1^\circ}\left[1-\left(\frac{p}{p_1^\circ}\right)^{\frac{k-1}{k}}\right]}}$$

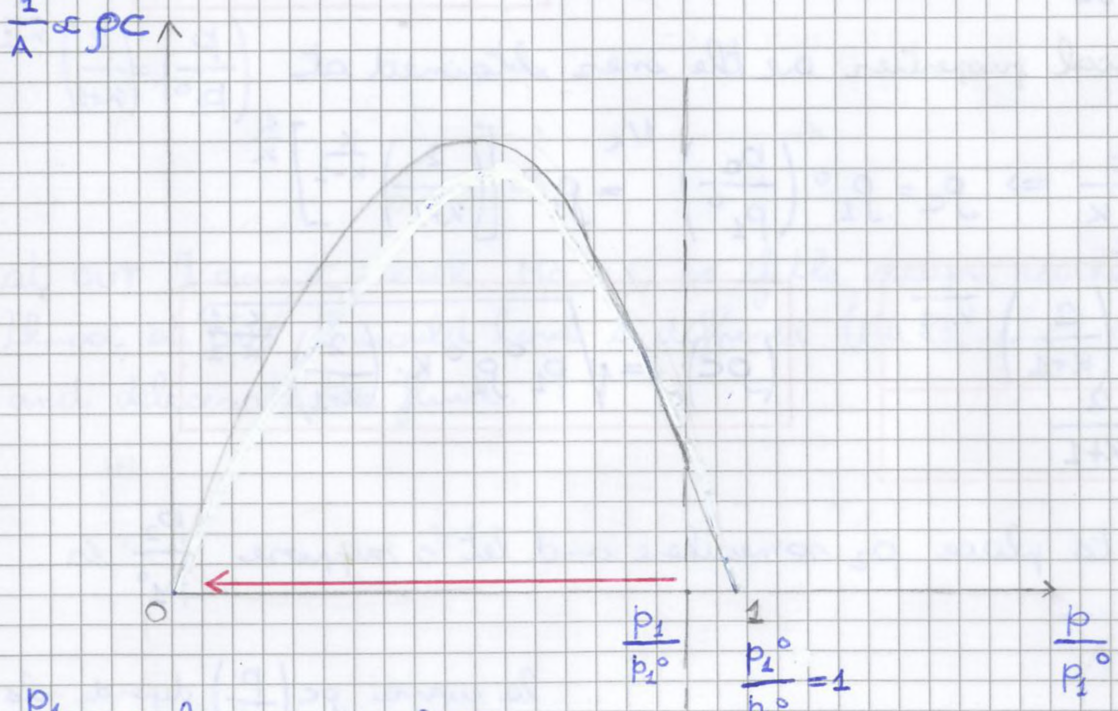
$$\frac{p_1^\circ}{\rho_1^{\circ k}} = \frac{p}{\rho^k} \Rightarrow \boxed{\rho\left(\frac{p}{p_1^\circ}\right) = \left(\frac{p}{p_1^\circ}\right)^{1/k} \cdot \rho_1^\circ} \quad 2)$$

chi mi garantisce che $\frac{p_1^\circ}{\rho_1^{\circ k}}$ = const ? perché per def le proprietà totali per definizione sono definite su un'isoentropica

$$3)\quad \boxed{\rho c\left(\frac{p}{p_1^\circ}\right) = \sqrt{\frac{2k}{k-1}\,p_1^\circ \rho_1^\circ\left[\left(\frac{p}{p_1^\circ}\right)^{\frac{2}{k}} - \left(\frac{p}{p_1^\circ}\right)^{\frac{k+1}{k}}\right]}}$$

20/03

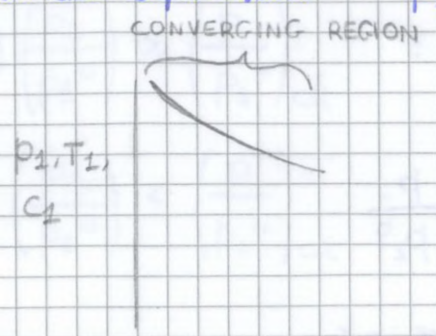
Let us represent relation 3) in a diagram:

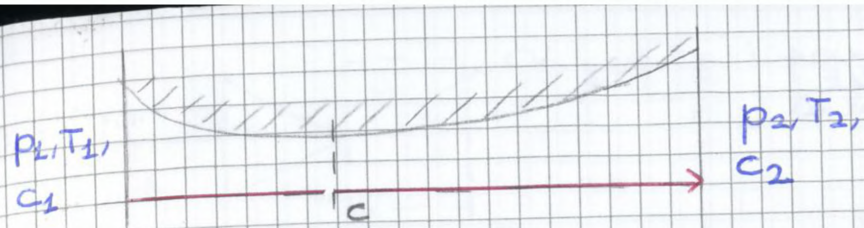


$\frac{p_1}{p_1^\circ} < 1$ because $p_1^\circ > p_1$

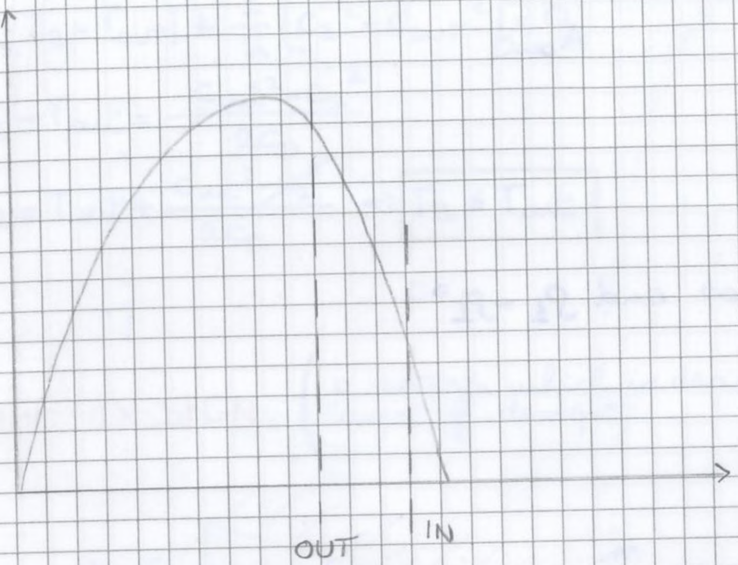
if I want to expand the fluid I would need to have ρc increase and therefore A has to decrease

in the expansion: $p_2 < p_1 \Rightarrow \rho c \uparrow \Rightarrow A \downarrow$

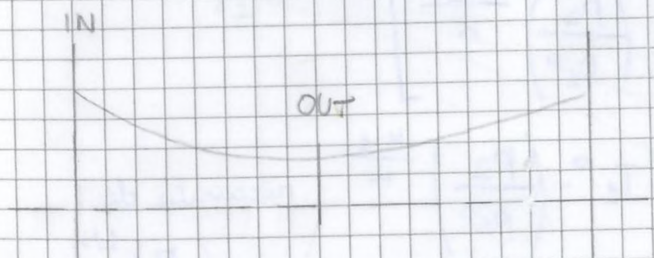




If I want to expand from IN to OUT



at OUT I do not reach $Ma = 1$; so if the section reaches its throat at OUT I would have a diffuser ($Ma < 1$) which compress and delecerate the fluid.



DESIGN OF NOZZLES

$\dot{m}$

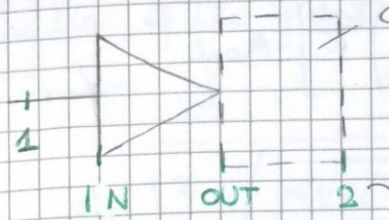
$p_1, T_1, c_1, \rho_1 \rightarrow p_1^\circ, T_1^\circ, \rho_1^\circ$

$p_2 < p_1$ because it is expanded

1) $\left(\frac{p_2}{p_1^\circ}\right) \geq \left(\frac{p}{p_1^\circ}\right)_{CR}$: PURE CONVERGENT

2) $\left(\frac{p_2}{p_1^\circ}\right) < \left(\frac{p}{p_1^\circ}\right)_{CR}$: CONVERGENT - DIVERGENT (DE LAVAL)

point 1 can be considered to be coincident with point IN



$2 \equiv OUT^{\circ}$ perché da OUT a 2 processo adiabatico, internamente reversibile e senza lavoro interno?

$$\underbrace{\cancel{Q}}_{=0} + \underbrace{\cancel{L_i}}_{=0} = \Delta h + \Delta E_K$$

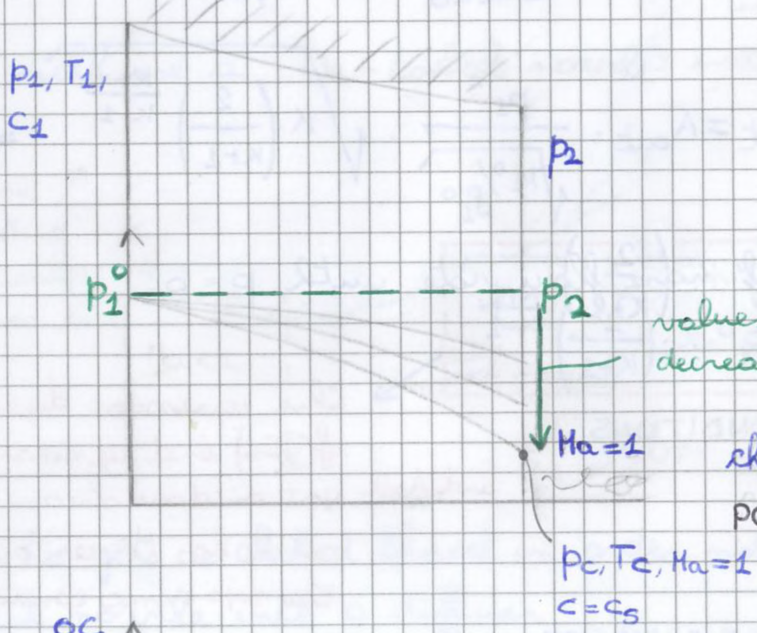
$$c_p(T_2 - T_{OUT}) + \frac{1}{2}(c_2^2 - c_{OUT}^2) = 0$$

$$T_2 - T_{out} = \frac{c_{out}^2 - c_2^2}{2c_p}$$

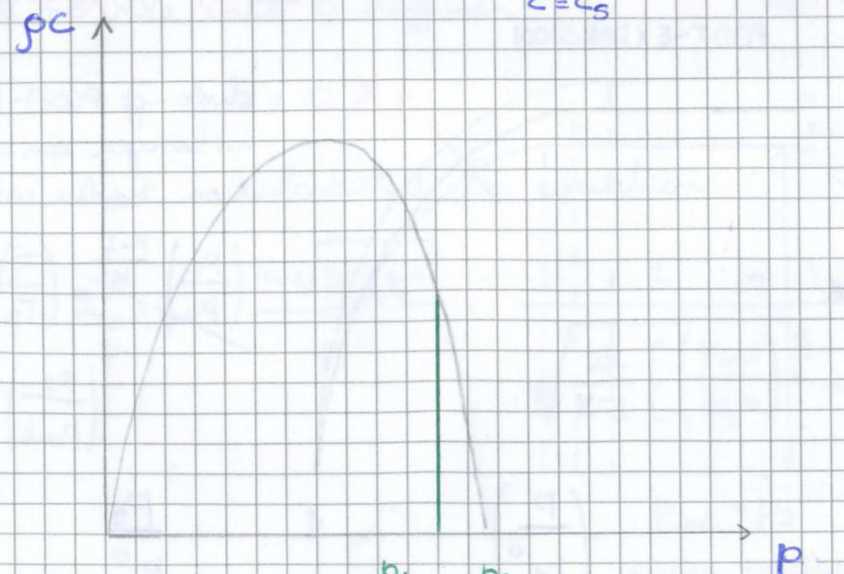
$$T_2 = T_{out} + \frac{c_{out}^2 - \cancel{c_2^2}}{2c_p} \Rightarrow \boxed{T_2 \neq T_{out}}$$

OFF-DESIGN

(a nozzle which is designed isentropically is used off-design)



chaotic expansion called POST EXPANSION



$$\frac{p_1}{p_1^{\circ}} = \frac{p_2}{p_2^{\circ}}$$
$$\Rightarrow p_1 = p_2$$

The portion of the graph characterized by the ellipse can be written as:

$$\boxed{\left(\frac{\frac{p_2}{p_1^\circ}-\frac{p_c}{p_1^\circ}}{1-\frac{p_c}{p_1^\circ}}\right)^2+\left(\frac{\frac{\dot{m}\sqrt{p_1^\circ/\rho_1^\circ}}{p_1^\circ\cdot A_{out}}}{\underbrace{\sqrt{k\cdot\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}}_{T}}\right)^2=1}$$

mass flow rate for subsonic flow conditions can also be evaluated according to the following equation: approssimazione buona ma non esatta

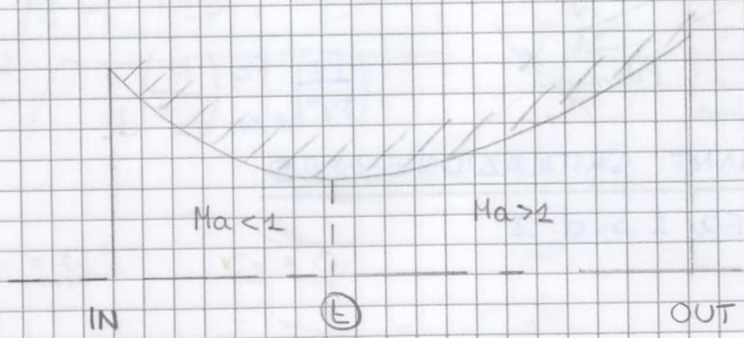
25/03

DE-LAVAL NOZZLE

Suppose we want to have:

$$\left(\frac{p_2}{p_1^\circ}\right)<\left(\frac{p}{p_1^\circ}\right)_{CR} \Rightarrow$$ we cannot longer use a SIMPLY CONVERGENT nozzle

but I will use a De-Laval nozzle:



I want to get a choked throat in order not to transform the divergent nozzle into a diffuser:

at t : $Ma = 1 \Rightarrow c = c_s$

OUT: A_{out} is calculated by equation:

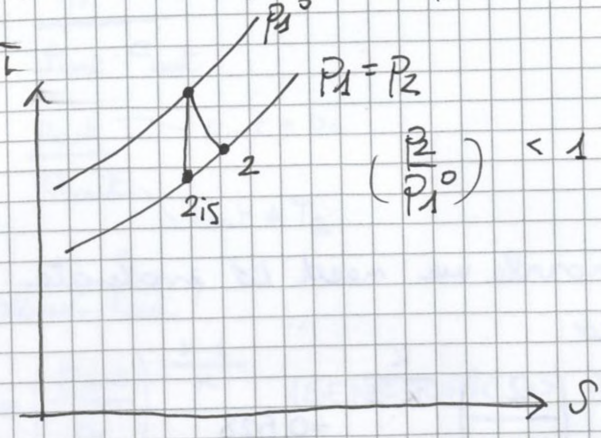
$$A_{out}=\frac{\dot{m}\sqrt{p_1^\circ/\rho_1^\circ}}{p_1^\circ}\cdot\frac{1}{\sqrt{\frac{2k}{k-1}\left[\left(\frac{p_{out}}{p_1^\circ}\right)^{\frac{2}{k}}-\left(\frac{p_{out}}{p_1^\circ}\right)^{\frac{k+1}{k}}\right]}}$$

$p_{out}=p_2$ under DESIGN

IN: $A_1=\frac{\dot{m}}{\rho\, c}$

$c_2 = 0$

$p_2 = p_1 \qquad c_1 \neq 0 \qquad p_1^0 > p_1$



$\left(\frac{p_2}{p_1^0}\right) < 1 \implies \dot{m} \neq 0$

$$\frac{dp}{\rho} = -c\,dc$$

$$\int_{1^0}^{2^0} \frac{dp}{\rho} = \int_{1}^{2} -c\,dc = \frac{c_2^2 - c_1^2}{2}$$

$$\frac{k}{k-1} \frac{p_1^0}{\rho_1^0}\left[\left(\frac{p_2}{p_1^0}\right)^{\frac{k-1}{k}} - 1\right]$$

$$c_2 = \sqrt{\qquad} \quad \cancel{+ c_1}$$

$p_2 = p_2^0 \qquad c_2 = 0 \qquad \infty$

$p_{out} = p_2 \qquad \cancel{p_{out}^0 = p_2^0}$

$p_{out} = p_2$ anche per $\left(\frac{p_2}{p_1^0}\right)_{DES} \leq \frac{p_2}{p_1^0} \leq \left(\frac{p_2}{p_1^0}\right)_d$

while

$p_{out} \neq p_2$ for $\left(\frac{p_2}{p_1^0}\right) \leq \left(\frac{p_2}{p_1^0}\right)_{design}$

What about A_{out}?

$$A_{out} \begin{cases} \text{eq. 1} \\ \dfrac{\dot{m}}{\rho_{out} \cdot c_{out}} \end{cases}$$

$$\rho_{out} = \frac{p_{out}}{RT_{out}} \qquad p_{out} = p_2 \qquad T_{out} \neq T_2$$

what about T_{out}?

$$\frac{T_{out}}{T_1} = \left(\frac{p_{out}}{p_1}\right)^{\frac{k-1}{k}} \quad \text{ISENTROPIC} \Rightarrow \quad T_{out} = T_1 \cdot \left(\frac{p_{out}}{p_1}\right)^{\frac{k-1}{k}} = 437.2\ K$$

and c_{out}?

$$\cancel{Q} + \cancel{L_i} = \Delta E_k + \Delta h$$

$$c_p(T_{out} - T_1) + \frac{1}{2}\left(c_{out}^2 - c_1^2\right)$$

$$c_{out} = \sqrt{c_1^2 + 2c_p(T_1 - T_{out})} = 369.0\ \frac{m}{s}$$

so:

$$A_{out} = \frac{\dot{m}}{\frac{p_{out}}{RT_{out}} \cdot c_{out}} = 102.01\ cm^2$$

Now check:

- $A_{out} < A_{in}$
- check $Ma_{out} < 1$

$$Ma_{out} = \frac{c_{out}}{c_{s_{out}}} = \frac{369.0}{\sqrt{kRT_{out}}} = 0.880 < 1$$

OFF-DESIGN (we use the nozzle designed with other conditions)
We need to evaluate the expansion ratio

$$\left(\frac{p_2}{p_1^0}\right) \quad \left(\frac{p}{p_1^0}\right)_{CR} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = 0.528$$

$$p_1^0 = p_1\left(1 + \frac{k-1}{2} \cdot \cancel{\frac{c_1^2}{kRT_1}}\right)^{\frac{k}{k-1}} = p_1 = 0.5\ MPa$$

NOTE

Quando progetto un ugello tale da avere un determinato rapporto di pressioni $\left(\frac{p_2}{p_1^0}\right)$ decido in base alla relazione:

$$\frac{p_2}{p_1^0} \lessgtr \left(\frac{p}{p_1^0}\right)_{CR}$$

se progettare un ugello di de Laval o un ugello puramente convergente.

Nella progettazione OFF-DESIGN nel caso di un ugello puramente convergente non ci son problemi fino al rapporto:

$$\left(\frac{p_2}{p_1^0}\right) \geq \left(\frac{p}{p_1^0}\right)_{CR}$$

e se $\left(\frac{p_2}{p_1^0}\right) = \left(\frac{p}{p_1^0}\right)_{CR}$ ottengo: $\underbrace{\begin{cases} c = c_C \\ \rho = \rho_C \\ \dot{m} = A_{out} \cdot \dfrac{p_1^0}{\sqrt{p_1^0/\rho_1^0}} \sqrt{K \cdot \left(\dfrac{2}{K+1}\right)^{\frac{K+1}{K-1}}} \end{cases}}$

questa espressione non dipende dal punto 2 (p_2, ρ_2: DOWNSTREAM CONDITIONS)

Quindi se diminuisco ulteriormente

$\frac{p_2}{p_1^0} < \left(\frac{p}{p_1^0}\right)_{CR}$, $\dot{m}$ rimane costante.

Cosa succede invece a c? Se utilizzo l'ugello con la condizione

$\frac{p_2}{p_1^0} = \left(\frac{p}{p_1^0}\right)_{CR}$ la $c_{out} = c_S$, cioè la velocità in uscita è sempre la velocità del suono; se lavoro a $\frac{p_2}{p_1^0} < \left(\frac{p}{p_1^0}\right)_{CR}$ ho la fase di POST-EXPANSION in cui:

- c rimane costante perché $c_C = \sqrt{\frac{2K}{K+1} \cdot \frac{p_1^0}{\rho_1^0}}$ non dipende dalle condizioni a valle e rimane pari a c_S
- $\dot{m}$ rimane costante

Quindi la post-expansion avviene fuori dall'ugello ? ed è una espansione non isentropica da p_{out} a p_2? ✓

Ricorda che la curva di $\dot{m}$ non è simmetrica.

Is it the ratio belonging to the choked non isentropic?

The two equations:

$$\dot m = A_{out} \sqrt{\frac{2k}{k-1} p_1^\circ \rho_1^\circ \left[\left(\frac{p_2}{p_1^\circ}\right)^{\frac{2}{k}} - \left(\frac{p_2}{p_1^\circ}\right)^{\frac{k+1}{k}} \right]} \qquad (1)$$

$$\dot m = A_t \sqrt{p_1^\circ \rho_1^\circ k \cdot \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} \qquad (2)$$

hold for both points (1) and (2).

So:

$$A_t \sqrt{p_1^\circ \rho_1^\circ k \cdot \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} = A_{out} \sqrt{\frac{2k}{k-1} p_1^\circ \rho_1^\circ \left[(x)^{\frac{2}{k}} - (x)^{\frac{k+1}{k}} \right]}$$

solving the equation we'd get $x = \left(\frac{p_2}{p_1^\circ}\right)_d$ but we miss values of A_t.

Another solution is:

① $p_2/p_1^\circ = 0.11 \Rightarrow \dot m_{choked} = 0.13$ Kg/s

② $p_2/p_1^\circ = 0.64 \Rightarrow \dot m = 0.232$ kg/s

if we compare them we can understand that the point under study is in the CHOKED NON-ISOENTROPIC; otherwise if $\dot m (p_2/p_1^\circ) < \dot m (0.11)$ we'd have been in the NON-CHOKED ISOENTROPIC.

What about c_{out}?

we cannot use $c = \sqrt{\frac{2k}{k-1} \, p_1^\circ/\rho_1^\circ \left[1 - \left(\frac{p_2}{p_1^\circ}\right)^{\frac{k-1}{k}}\right]}$

but

1) $\dot m = const = \rho_{out} \cdot A_{out} \cdot c_{out} = \frac{p_{out}}{RT_{out}} A_{out} \cdot c_{out}$ (we miss T_{out})

2) $dQ + dL_i = dh + dE_k$

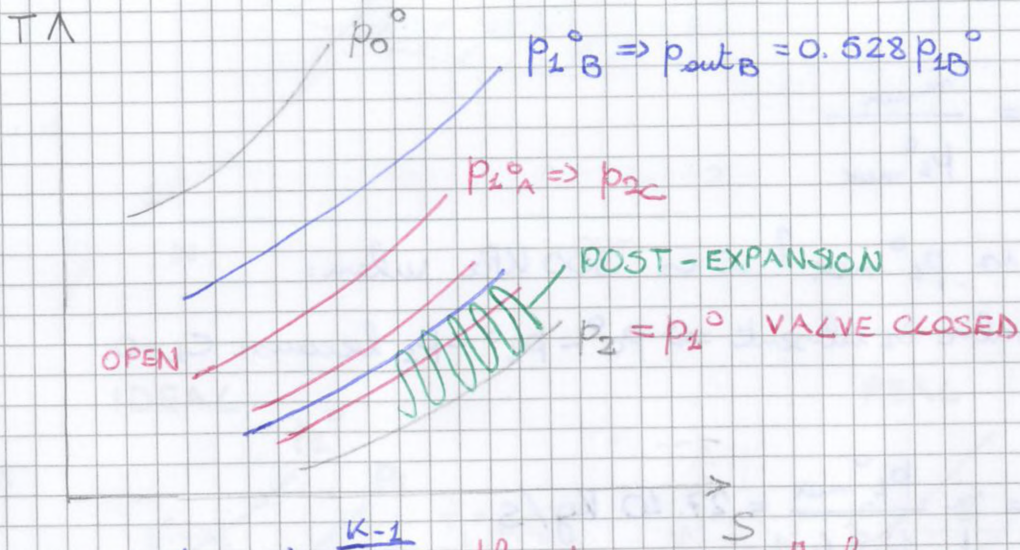
$dL_i = v\,dp + dE_k + dL_w$ (we do not use because dL_w is not known)

we use equation 2_1:

The max. possible velocity is:

$c_{S_{OUT}} = \sqrt{KRT_{OUT}}$ got at the throat.

Can I change the upstream conditions to affect the speed of sound at the outlet?



$$T_{OUT} = T_1^\circ \cdot \left(\frac{p_{OUT}}{p_1^\circ}\right)^{\frac{K-1}{K}}$$

$\| \quad T_0^\circ \qquad \downarrow$ const.

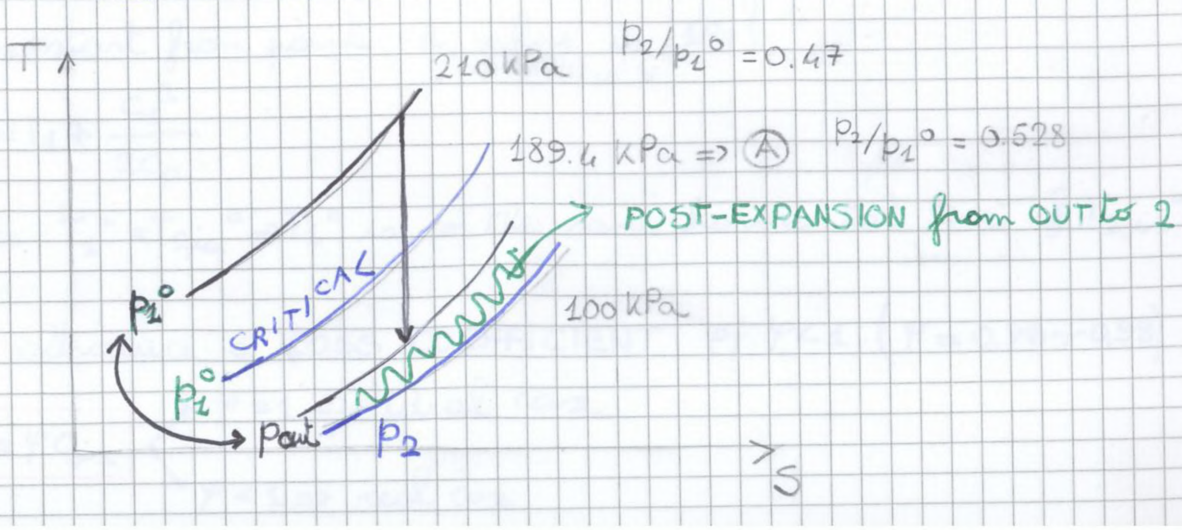
the idea is: can I have a greater velocity c_2 by varying p_1°? I know Ma = 1 at the throat ⇒ if I want to have a greater c_{OUT} I need to have a greater $c_{S_{OUT}}$ but $c_{S_{OUT}}$ depends on T_{OUT} which depends on $\frac{p_{OUT}}{p_1^\circ}$ whose maximum value is 0.528

So Tout does depend only on T_1° and not on the ratio $\left(\frac{p_{OUT}}{p_1^\circ}\right)^{\frac{K-1}{K}} \Rightarrow$

$T_{OUT} = 416.67$ K

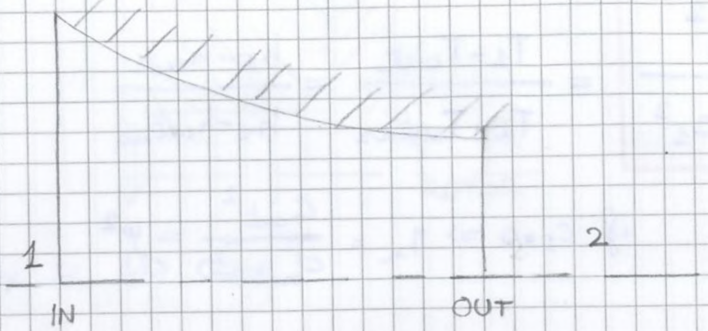
$c_{OUT} = \sqrt{KRT_{OUT}} = 409.17 \ \frac{m}{s}$

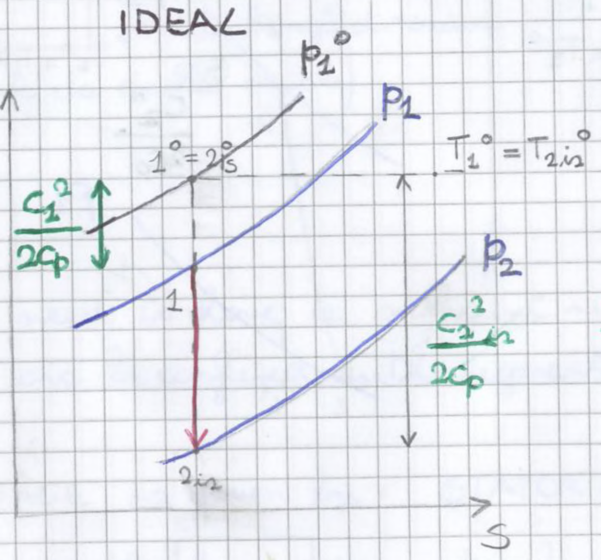
$p_1^\circ = p_2/0.528 = 189.4$ kPa

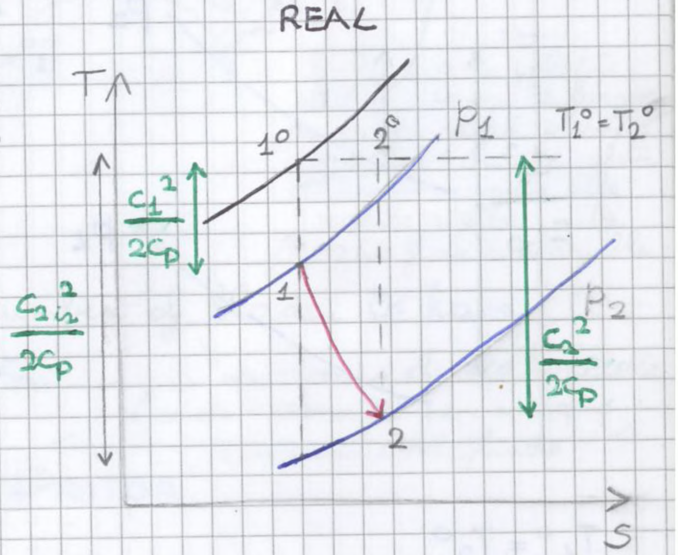


NOZZLE EFFICIENCY

The evolutions we've considered in a nozzle have always been considered as IDEAL, but this is not the case:







$dQ = 0$, $dL_w = 0 \Rightarrow dS = 0 \Rightarrow S = \text{const.}$

$\Rightarrow \begin{cases} T^\circ = \text{const.} \\ p^\circ = \text{const.} \end{cases}$

$dQ = 0$, $dL_w \neq 0 \Rightarrow dS \neq 0$

$\Rightarrow \begin{cases} T^\circ = \text{const.} \\ p^\circ \neq \text{const.} \end{cases}$

ID.) Let's start from point 1; where is 1°?

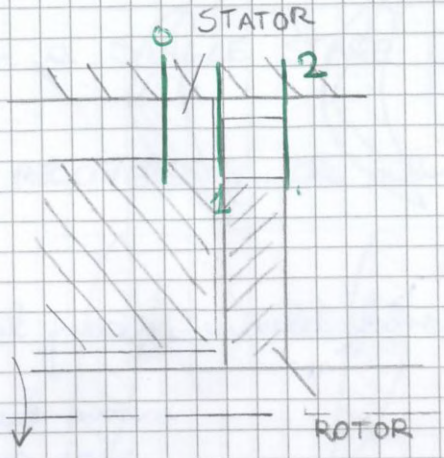
$$T_1^\circ = T_1 + \frac{c_1^2}{2c_p}$$

since $T_1^\circ = T_{2is}^\circ \Rightarrow T_1^\circ$ is on the same horizontal line of $T_{2,is}^\circ$

R.) We introduce a LOSS COEFFICIENT φ; $\varphi < 1$ ($\varphi = 0.96 \div 0.98$)

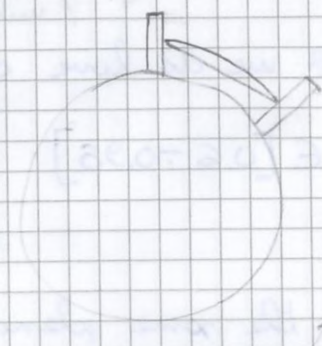
$\boxed{c_2 = \varphi c_{2is}}$ $\begin{cases} \varphi = 1 \Rightarrow \text{ideal case} \\ \varphi < 1 \Rightarrow \text{real case} \end{cases}$

TURBINE STAGE



Is it really a 1D case?
It is if

1) $\boxed{\frac{l}{d} << 1}$ (in modo da non avere perdite lungo direzioni diverse da quella assiale)



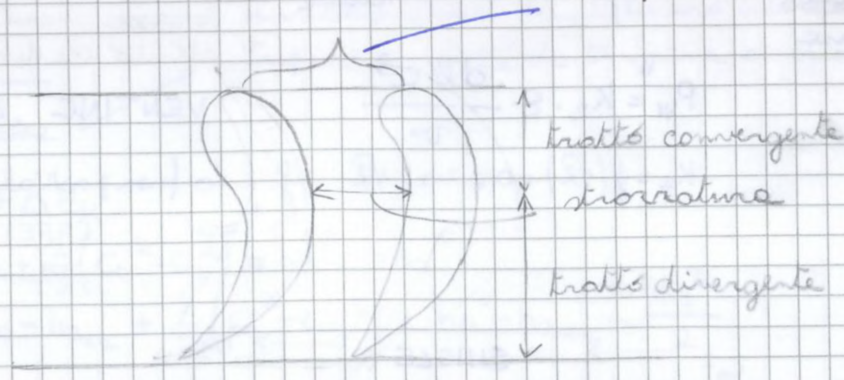
il numero di palette alto fa ritenere la lunghezza periferica del condotto delimitato da due palette consecutive uguale alla radice e alla punta
$\Rightarrow$ 1D flow is allowable

2) we need to have a sufficient number of blades to have the arc be confused with segment.

un numero elevato di palette è anche necessario per evitare l'inversione del fluido

A STAGE is given by: STATOR + ROTOR.
What are the losses in a stage?

shaped as a nozzle



- $\boxed{c_1 = \varphi \cdot c_{1,is}}$, $\varphi = f(Re, \text{shape of blades, roughness} \ldots)$
  $\varphi \in [0.9 \div 0.98]$

Let us suppose we have the same final kinetic energy

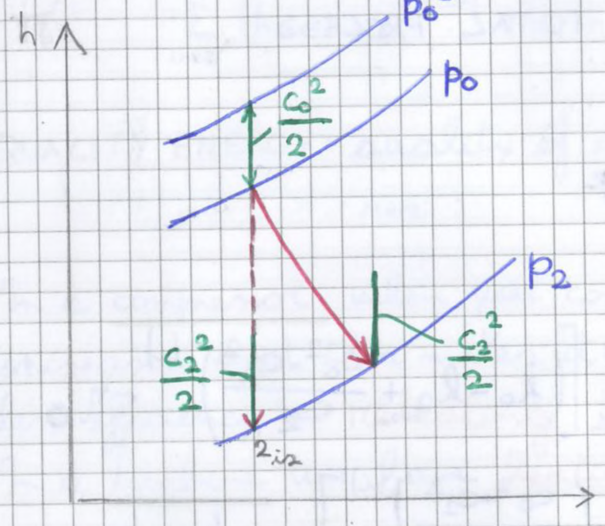
$\left(\frac{c_2^2}{2}\right)$: LOSS for a SINGLE STAGE

$\left(\frac{c_2^2}{2}\right)$: TO BE RECOVERED in the subsequent stage for a MULTI-STAGE

therefore an ideal process would have: $c_{2,is} = 0$

an ideal process has NO LOSS

We define the TOTAL TO STATIC EFFICIENCY η_{ts} and the TOTAL TO TOTAL EFFICIENCY η_{Θ}



TOTAL TO TOTAL

it is also called L_{ie} (lavoro interno limite), isentropico a parità di termine cinetico allo scarico

MOTOR CONVENTION $\eta_{\Theta} = \frac{L_i}{L_{i_{IDEAL}}}$

$\cancel{Q} - L_i = \Delta h + \Delta E_K$

$(0-2)$ $L_i = h_0 - h_2 + \frac{1}{2}(c_0^2 - c_2^2) = h_0^0 - h_2^0$

$(0-2_{is})$ $L_{i\,id} = h_0 - h_{2is} + \frac{1}{2}(c_0^2 - c_2^2) = h_0^0 - h_{2is} - \frac{c_2^2}{2}$

$$\eta_{\Theta} = \frac{h_0^0 - h_2^0}{h_0^0 - h_{2,is} - \frac{c_2^2}{2}}$$

since the outlet velocity of one stage is the same as the inlet velocity of the subsequent one, we have:

$$\sum_{i=1}^{N} \frac{c_0^2 - c_2^2}{2} = \frac{c_A^2 - c_B^2}{2} \approx 0$$

So we get:

$$\eta_{\ominus} = \eta_{\ominus}^{(i)} \cdot \underbrace{\left[\frac{\sum_{i=1}^{N} (h_0 - h_{2,is})^{(i)}}{h_A - h_{B,is}} \right]}_{\substack{\text{it is always greater} \\ \text{than 1 because}}} = \eta_{\ominus}^{(i)} \cdot \underset{\substack{\shortparallel \\ 1.1.}}{\Upsilon}$$

$$\sum_{i=1}^{N} (h_0 - h_{2,is})^{(i)} > h_A - h_{B,is}$$

because le isobare nel diagramma h, S divergono

QUALITY ENERGY: quality of energy is greater at high temperatu= res.

In a <u>compressor</u> when you compress a gas its temperature increases and this makes it expand and so the compression has less efficiency ⇒ reheating

In a <u>turbine</u> we have heating

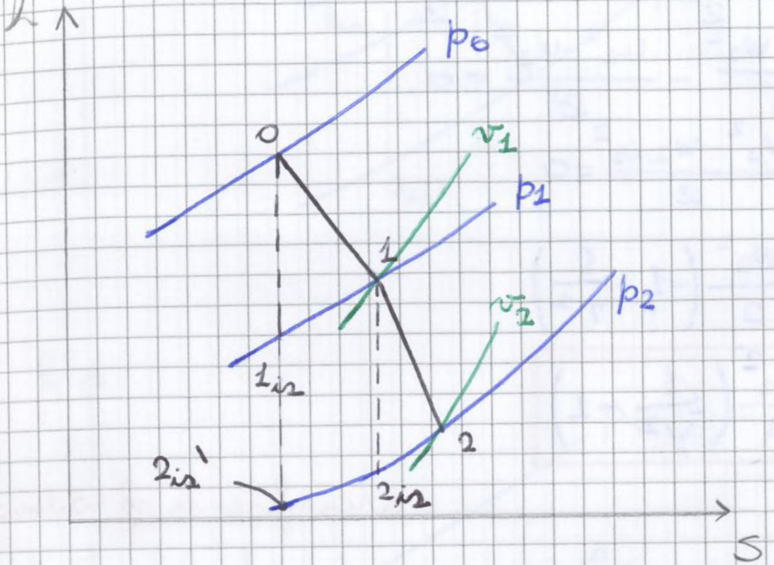
ACTION AND REACTION TURBINES

ACTION: no expansion in the impeller = the expansion drop occurs only in the distributor

REACTION: expansion is divided between the impeller and distributor

$$h_1 = h_{1,is} + \frac{\left(\frac{c_1}{\varphi}\right)^2 - c_1^2}{2}$$

$$\boxed{h_1 = h_{1,is} + \frac{c_1^2}{2}\left(\frac{1}{\varphi^2} - 1\right)}$$



remark that point $2_{is}'$ is the same point which was called as 2_{is} before

$(1-2_{is})$ $\quad \cancel{Q} - L_i = \Delta h + \Delta E_k + \cancel{\Delta E_g} + \Delta E_\omega$

adiabatic ⇓ $=0$ $\quad$ compressible flow ⇓ $=0$

- FIXED OBSERVER (you do not have any centrifugal force term)

$-L_i = \Delta h + \Delta E_k$

it is quite difficult to apply such an equation to the ideal case; so we prefer using a relative refrence of frame

- MOVING FRAME OR REFERENCE

$L_i = 0, \ \Delta E_\omega \neq 0$

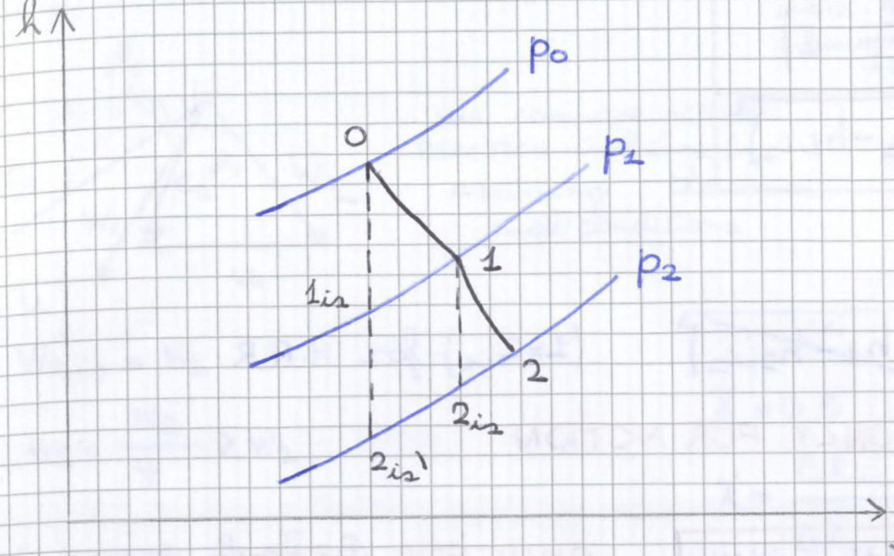
$\Rightarrow \Delta E_k + \Delta E_\omega + \Delta h = 0$

$$h_1 - h_{2,is} + \frac{w_1^2 - w_{2,is}^2}{2} - \underbrace{\left(\frac{u_1^2 - u_2^2}{2}\right)}_{\Delta E_\omega} = 0$$

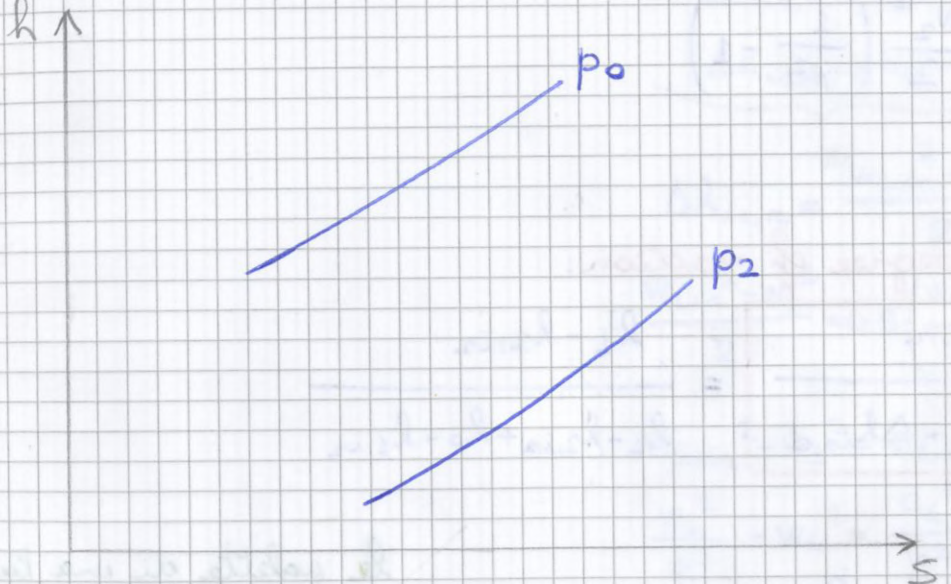
for a constant d blade $u_1 = u_2 \Rightarrow$

$$h_1 - h_{2,is} + \frac{w_1^2 - w_{2,is}^2}{2} = 0 \qquad \text{IMPELLER}$$

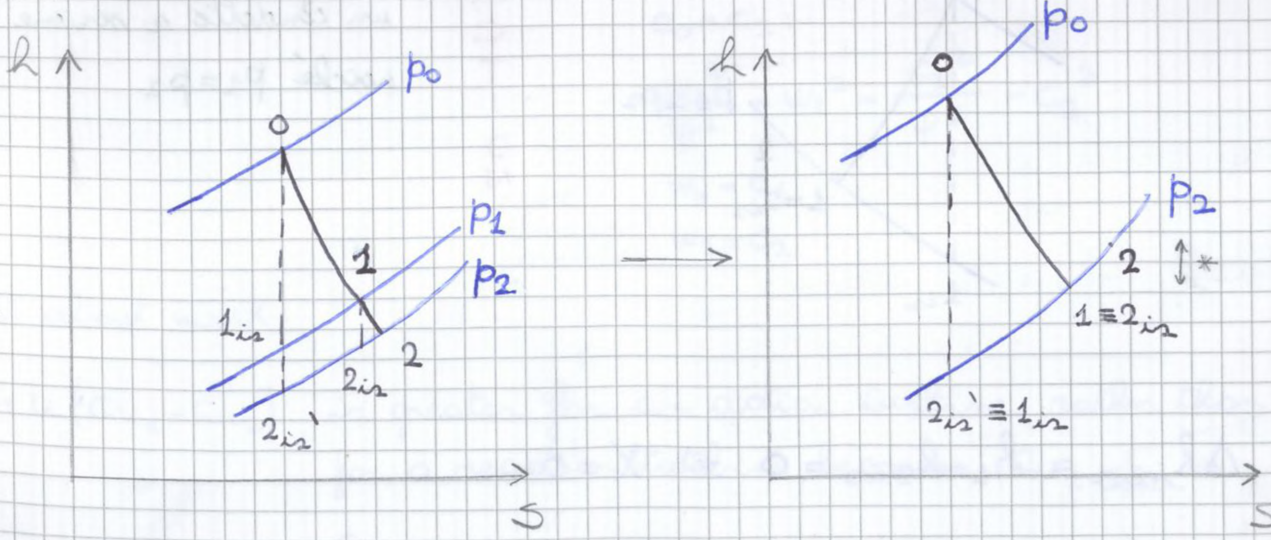
ACTION TURBINE



reaction turbine



action turbine

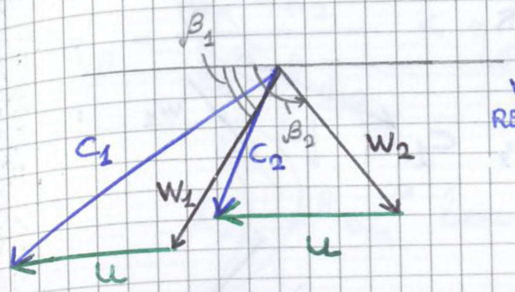


You have an EXPANSION in the distributor from 0 to 1 and then an ISOBARIC PROCESS from 1 to 2

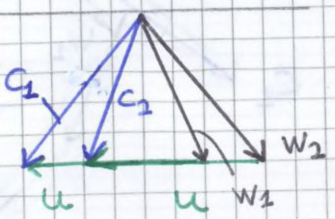
ACTION

REACTION

in questo caso ovviamente dobbiamo avere $l_1 \neq l_2$ perché se fosse $l_1 = l_2$ avremmo $\rho_1 = \rho_2$ ↑ che non è possibile



we compare ACTION-REACTION turbines by assuming the same final conditions



$$W_{2,is} = W_1$$

$$W_1 = \frac{W_2}{\psi} > W_2$$

we assume $\beta_2 = \pi - \beta_1$

$$\chi = 0.5 \text{ (assumption)}$$

$$\chi = \frac{\Delta h_{is,r}}{\Delta h_{is,r} + \Delta h_{is,d}}$$

$$\Delta h_{is,r} = h_1 - h_{2,is} = \frac{W_{2,is}^2 - W_1^2}{2}$$

$$\Delta h_{is,d} = h_0 - h_{1,is} = \frac{C_{1,is}^2 - C_0^2}{2}$$

$$\Delta h_{is,r} = \frac{W_{2,is}^2 - W_1^2}{2} = 0.5\left(\Delta h_{is,r} + \Delta h_{is,d}\right)$$

$$\frac{W_{2,is}^2 - W_1^2}{2} = 0.5\left(\frac{W_{2,is}^2 - W_1^2}{2} + \frac{C_{1,is}^2 - C_0^2}{2}\right)$$

$$W_{2,is}^2 - W_1^2 = C_{1,is}^2 - C_0^2$$

$$\frac{W_2^2}{\psi^2} - W_1^2 = \frac{C_1^2}{\varphi^2} - C_0^2$$

we assume $\psi = \varphi$ and since $C_0 = C_2$:

$$\frac{W_2^2}{\psi^2} - W_1^2 = \frac{C_1^2}{\psi^2} - C_2^2$$

unica soluzione possibile $\begin{cases} W_2 = C_1 \\ W_1 = C_2 \end{cases}$

What about work:

$L_i = u(C_{u_1} - C_{u_2})$ is greater for an action turbine rather than for a reaction turbine because $C_{u_1\,\text{action}} > C_{u_1\,\text{reaction}}$

So the TOTAL TO STATIC EFFICIENCY:

$$\eta_\theta = \frac{h_0^\circ - h_2^\circ}{h_0^\circ - h_{2is}} < \eta_\Theta$$

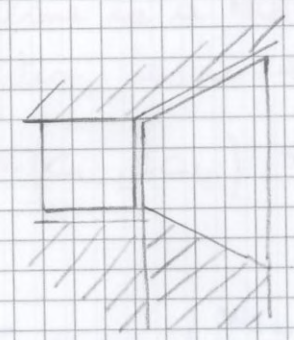
$\downarrow$

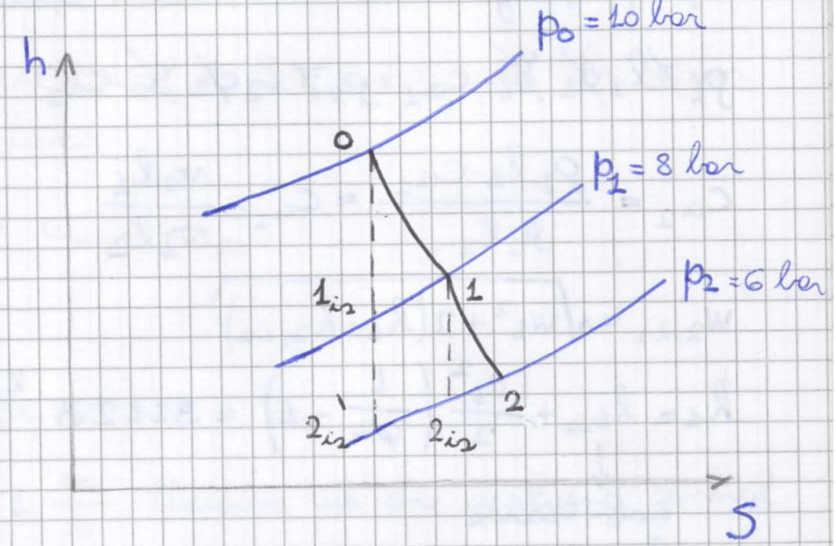
$h_{2is} = h_{2is}^\circ$ because $c_{2is} = 0$

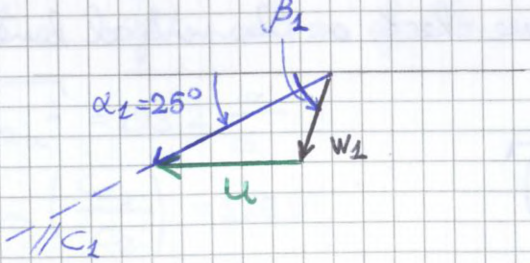
Exercises

5.1)

p_0	p_1	p_2
t_0	φ	ψ
c_0	α_1	l_2
n	l_1	d = const
	u	







From a Mollier diagram we can read h_0 and h_{1s}:

$h_0 = 3268$ KJ/Kg

$h_{is} = 3204$ KJ/Kg

This is a REACTION TURBINE; so:

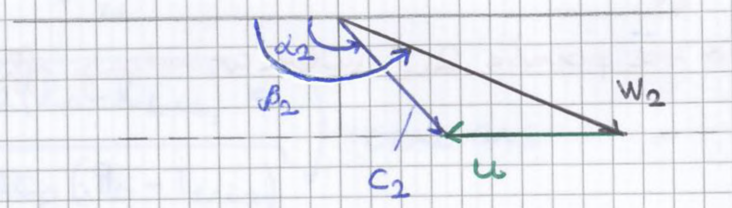
I cannot use OBVIOUSLY the ideal gas equation but we read v_1 and v_2 on the Mollier's diagram:

$v_1 = 0.35 \ m^3 \cdot kg^{-1}$

$v_2 = 0.45 \ m^3 \cdot kg^{-1}$ (after calculating $h_2 = h_{2,is} + \frac{w_2^2}{2}\left(\frac{1}{\psi^2} - 1\right) = 3146 \frac{kJ}{kg}$)

$\Rightarrow c_{a_2} = c_{a_1} \cdot \frac{v_2 \, l_1}{v_1 \, l_2} = 160.6 \ m \cdot s^{-1}$

We have 3 pieces of information; hence:



A turbine is a MOTOR MACHINE; so the work with the motor convention has to be positive:

$L_i = u(c_{u_1} - c_{u_2}) > 0 \Rightarrow c_{u_1} > c_{u_2}$

So:

$\beta_2 = \pi - \arcsin\left(\frac{w_{2a}}{w_2}\right) = 156.22°$

$c_{2u} = w_2 \cos\beta_2 + u = -116.1 \ \frac{m}{s}$ (because we are applying general equations regardless whether we know the position of c_2 in the left or right portion of the vertical line)

$c_2 = \sqrt{c_{2a}^2 + c_{2u}^2} = 198.2 \ \frac{m}{s}$

$\alpha_2 = \pi - \arcsin\left(\frac{c_{a_2}}{c_2}\right) = 125.9°$

The power is given by:

$P_i = \dot{m} L_i$,

$L_i = u(c_{u_1} - c_{u_2}) = 108.73 \ \frac{kJ}{kg}$

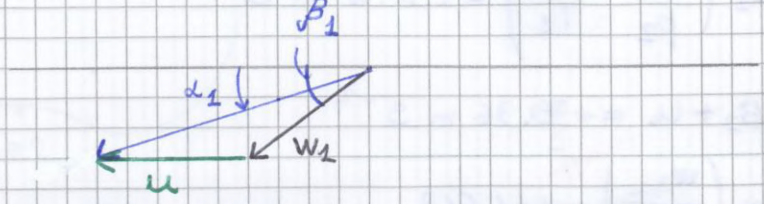
$\dot{m} = \xi_1 \pi d_1 l_1 \cdot c_{a_1} \cdot \frac{1}{v_1}$, $\xi = 0.98$

$u = \pi d n$

The internal power is:

$$P_i = \dot{m} L_i =$$

$$= \xi_1 d_1 l_1 \cdot \pi c_{a_1} \cdot \underbrace{\frac{1}{v_1}}_{\text{ideal gas}} \cdot u \underbrace{\left(c_{u_1} - c_{u_2} \right)}_{\text{velocity components}}$$



$$\left.\begin{aligned} c_{1,is} &= \sqrt{c_0^2 + 2\left(h_0 - h_{1,is}\right)} = \\ &= \sqrt{c_0^2 + 2c_p\left(T_0 - T_{1,is}\right)} \end{aligned}\right\} \text{IDEAL GAS}$$

$$\left\{\begin{matrix} R \\ K \end{matrix}\right. \Rightarrow c_p = c_v + R = \frac{c_p}{K} + R \Rightarrow c_p\left(1 - \frac{1}{K}\right) = R \Rightarrow c_p = \frac{R}{1 - \frac{1}{K}} = 1008 \ \frac{J}{kg \cdot K}$$

$$T_{1,is} = T_0 \cdot \left(\frac{p_1}{p_0}\right)^{\frac{K-1}{K}} = 555.68 \ K \longrightarrow$$

remark $pv^K = const$ is for ISOENTROPIC, not just for adiabatic

$$c_{1,is} = \sqrt{c_0^2 + 2c_p\left(T_0 - T_{1,is}\right)} = 420.7 \ m \cdot s^{-1}$$

$$c_1 = \varphi \cdot c_{1,is} = 403.9 \ m \cdot s^{-1}$$

$$h_1 = h_{1,is} + \frac{c_1^2}{2}\left(\frac{1}{\varphi^2} - 1\right)$$

$$T_1 = T_{1,is} + \frac{c_1^2}{2c_p}\left(\frac{1}{\varphi^2} - 1\right) = 562.7 \ K$$

$$v_1 = \frac{RT_1}{p} \Rightarrow \dot{m} = \xi_1 \cdot \pi l_1 d_1 c_1 \sin\alpha_1 \cdot \frac{p_1}{RT_1} = 38.6 \ kg \cdot s^{-1}$$

Then:

$$u = \pi \cdot \frac{n}{60} d = 157.1 \ m \cdot s^{-1}$$

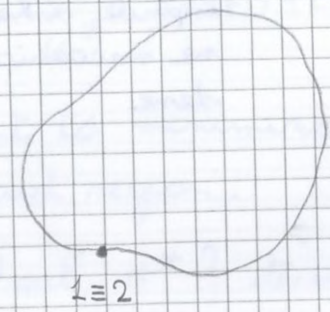
$$w_{1a} = c_1 \sin\alpha_1 = 138.1 \ m \cdot s^{-1}$$

$$w_{1u} = c_1 \cdot \cos\alpha_1 - u = 222.4 \ m \cdot s^{-1}$$

$$w_1 = \sqrt{w_{1a}^2 + w_{1u}^2} = 261.8 \ m \cdot s^{-1}$$

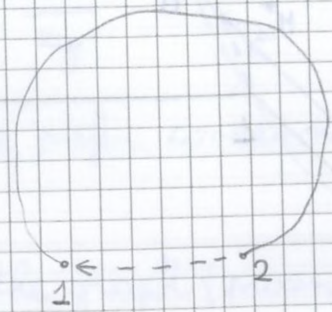
STEAM PLANTS

A CYCLE is a transformation which brings the system from an initial to a final condition which are equal.



same fluid performing the transformation

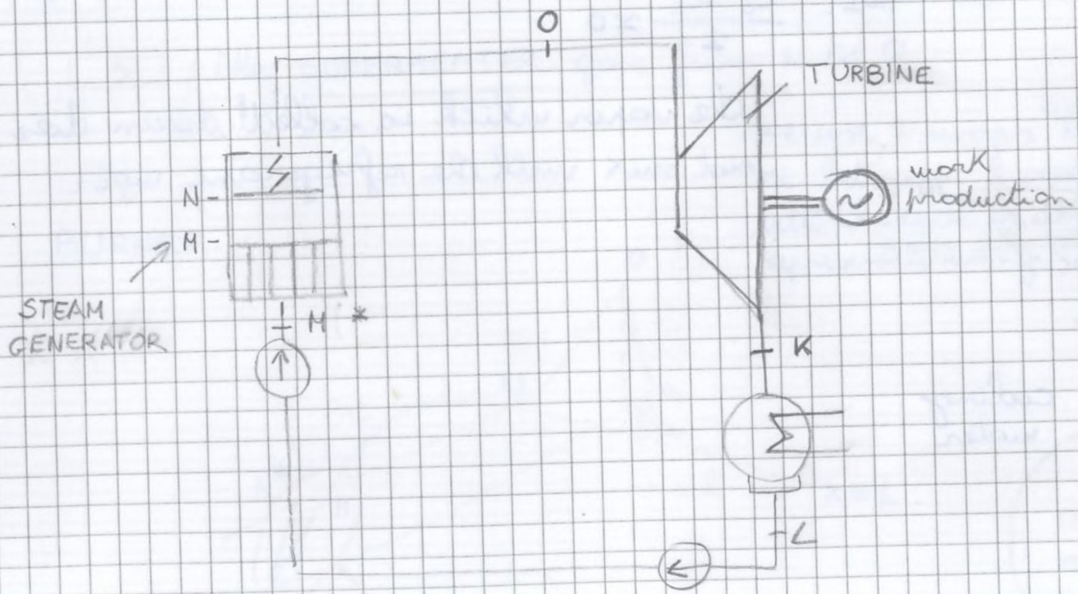
CYCLE

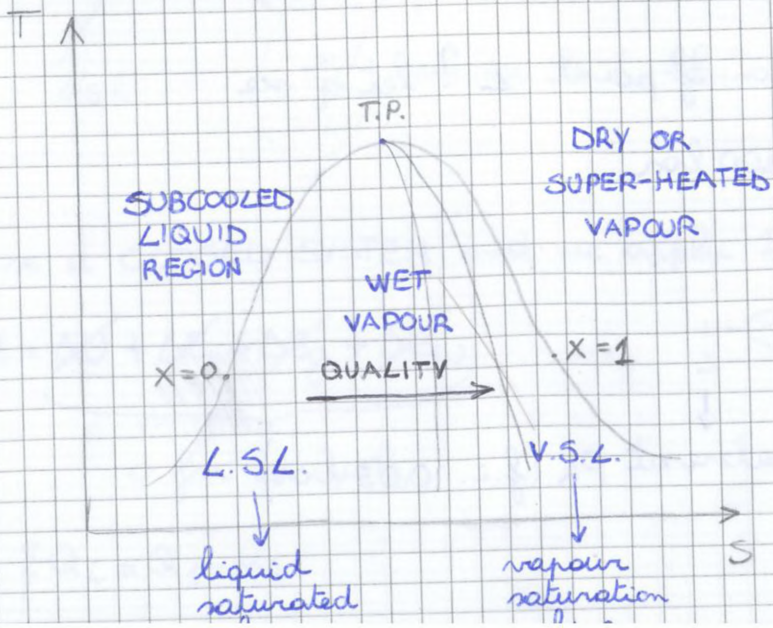


REFERENCE CYCLE

it is not the same fluid performing the transformation

RANKINE-HIRN CYCLE





$$L_i = \underbrace{\int v\,dp}_{} + \cancel{\Delta E_k} + L_w \qquad (\Delta E_g = \Delta E_\omega = 0)$$

perform the integration and solving we get a much higher value

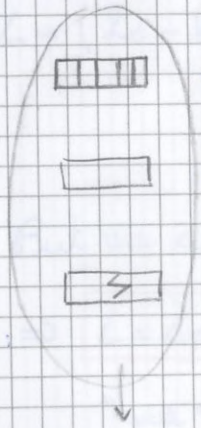
$$L_i = \frac{\Delta p}{\rho} + L_w \,, \quad \text{supposing} \quad L_w = 10\%\, L_i$$

$$L_i = \frac{(100-0.05)\cdot 10^5}{10^3} + 0.1 L_i \Rightarrow L_i \cong 11 \ \frac{kJ}{kg}$$

We want to minimize L_i => so we prefer condensing up to the liquid region.

What about M^*? it is shown in the graph (process has $L_w \neq 0$)

Then



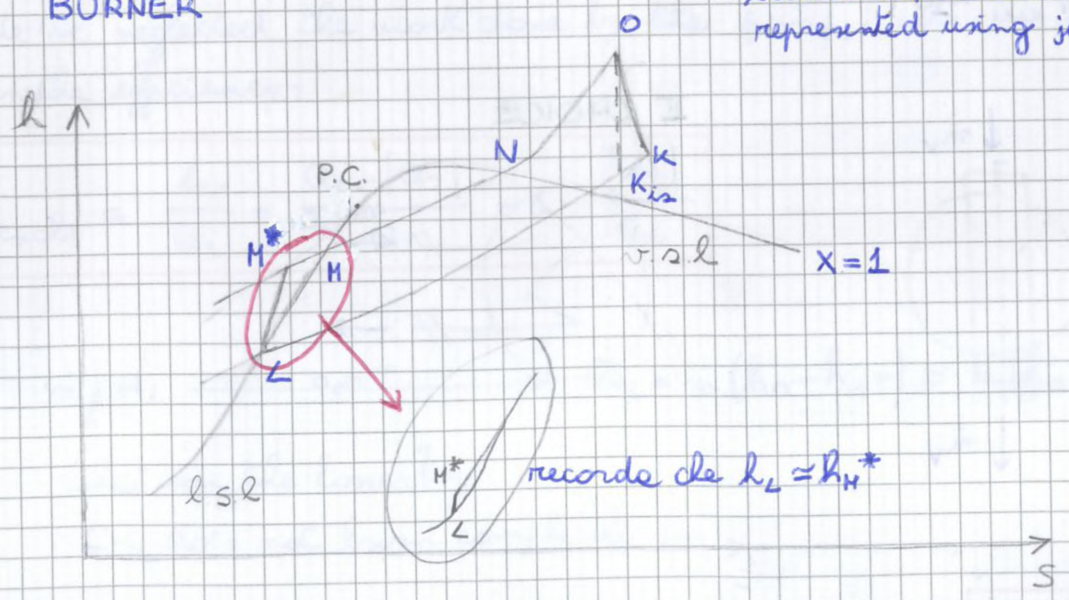
the HEATER heats the vapour up

the VAPORIZER goes from M to N

the SUPERHEATER goes from N to O

BURNER

we use 2 pumps because of distance between burner and condenser but the cycle is conventionally represented using just 1 pump

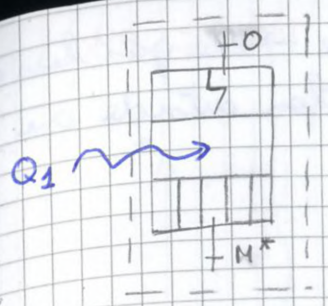


recorda che $h_L \simeq h_{M^*}$

We use a CLOSED SYSTEM and we apply 1st. l. of t.

$$Q - L = \underbrace{\cancel{\Delta U} + \cancel{\Delta E_k} + \cancel{\Delta E_g} + \cancel{\Delta E_\omega}}_{0 \text{ for a cycle}}$$

$$L = \sum_i Q_i = Q_1 - Q_2$$



$$\boxed{Q_1 = \Delta h = h_O - h_{M^*}}$$

Then

$$L = Q_1 - Q_2 =$$

$$= (h_O - h_{M^*}) - (h_K - h_L) = (h_O - h_K) + (h_L - h_{M^*})$$

$$L = (h_O - h_K) - \underbrace{(h_{M^*} - h_L)}_{L_p}$$

il bwr (back work ratio) per un ciclo Rankine:

$$bwr = \frac{L_{pump}}{L_{turbine}} = 1 \div 3\%$$

quindi L_{pump} é generalmente trascurabile e quindi:

$$L_{pump} = h_{M^*} - h_L \simeq 0 \Rightarrow h_{M^*} \simeq h_L$$

But we assume: $\boxed{h_{M^*} \simeq h_L}$

$$\Rightarrow \boxed{L = L_T = h_O - h_K}$$

$$\eta_{cycle} = \frac{h_O - h_K}{h_O - h_{M^*}}$$

08/04

We've neglected the work done by the pump; now we take them in the efficiency:

$$\boxed{\eta_{cycle} = \frac{L_T}{Q_1} = \frac{Q_1 - |Q_2|}{Q_1} = 1 - \frac{|Q_2|}{Q_1}}$$

$$\dot{m}_f H_i \longrightarrow \dot{Q}_1 = \dot{m}(h_O - h_{M^*}) \simeq \dot{m}(h_O - h_L)$$

what are the losses?

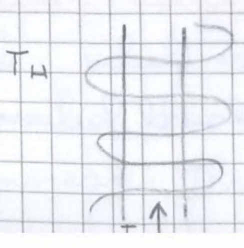
- CH_4 does not burn completely in:

$$CH_4 + O_2 \rightleftharpoons CO_2 + H_2O \Rightarrow \text{NO COMPLETE COMBUSTION}$$

- losses due to heat exchange through a finite ΔT

$\downarrow$

entropy increases because for the system heat is absorbed through a finite difference in temperature



$$T_H - T_L \neq dT$$