



Corso Luigi Einaudi, 55 - Torino

Appunti universitari

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Rilegature

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A P P U N T I

STUDENTE: Margaria

MATERIA: Dynamic of Electrical Machine (INGLESE)

Prof. Boglietti

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**ATTENZIONE: QUESTI APPUNTI SONO FATTI DA STUDENTIE NON SONO STATI VISIONATI DAL DOCENTE.
IL NOME DEL PROFESSORE, SERVE SOLO PER IDENTIFICARE IL CORSO.**

\vec{B} = induction [T]

\vec{H} = magnetic field

$$\rightarrow \vec{B} = \mu \vec{H}$$

μ = permeability (= attitudine del materiale a magnetizzarsi in presenza di un campo magnetico)

$$\mu_r = \frac{\mu}{\mu_0}$$

$$\rightarrow \vec{B} = \mu_r \cdot \mu_0 \cdot \vec{H}$$

$$\text{ROT } \vec{H} = \vec{J} \quad (1)$$

dove \vec{J} = current density [A]

$$\text{DIV } \vec{B} = 0 \quad (2)$$

$$\Phi = \int_S \vec{B} \cdot d\vec{s} \quad \rightsquigarrow \text{scalenoidal}$$

we can write the same equations in an integral form:

$$(1) \oint_l \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{s} \quad \rightarrow \text{la circolazione del vettore } \vec{H} \text{ lungo la linea chiusa } l \text{ e' = al flusso del vettore } \vec{J} \text{ attraverso la sup. } S \text{ delimitata dalla linea chiusa } l.$$

where l is the border line of the surface S



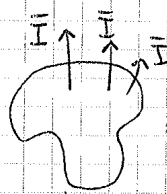
There are lots of surfaces which have l as border line.

The left side of the equation is simple *

The right side is easy because

$$\int_S \vec{J} \cdot d\vec{s} = \vec{I}_1 + \vec{I}_2 + \vec{I}_3 = NI$$

$$I_1 = I_2 = I_3 = I$$



wire

The wires must be inside the surface to be considered.

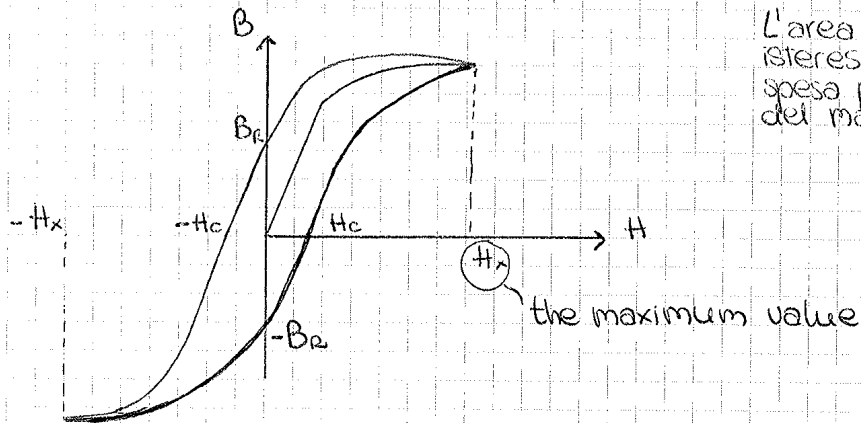
* In our conditions, we can simplify writing:

$$\oint_l \vec{H} \cdot d\vec{l} = \sum_{i=1}^K H_i \cdot l_i$$

$$\Rightarrow \sum_{i=1}^K H_i \cdot l_i = N \cdot I$$

$$(2) \Phi = \int_S \vec{B} \cdot d\vec{s} \quad \rightarrow \text{il flusso del vettore } \vec{B} \text{ che entra in una sezione di un tubo di flusso deve essere = a quello che esce da un'altra sezione dello stesso tubo di flusso.}$$

In order to have $B=0$ we have to apply a negative magnetic field which is called coercitive field ($-H_c$)



L'area racchiusa nel ciclo di isteresi rappresenta l'energia spesa per la magnetizzazione del materiale.

The closed loop is called hysteresis loop. All the magnetic material have one.

If the hysteresis loop is small means we have less losses inside the material.

$P_H = \alpha_H \cdot f \cdot B_x^x$ \rightarrow total losses are proportional to the frequency because we have losses every cycle
 power losses

α_H : coefficient depending on the material

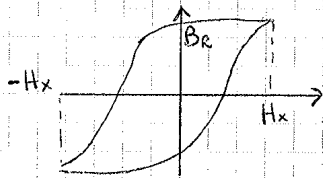
$1.6 < x < 2.2$: depending on the material

hanno un basso attrito magnetico: sono facilmente magnetizzabili/smagnetizzabili

alloys based on iron are not all ferromagnetic, but the majority are.

based on iron, only rarely on cobalt because it is too expensive

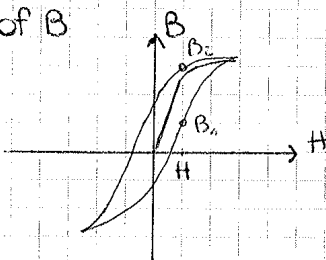
1. SOFT MAGNETIC MATERIALS: all the materials used in electrical machines they have a very small hysteresis loop \rightarrow great efficiency
2. HARD MAGNETIC MATERIALS: materials used in permanent magnets



una volta magnetizzati, mantengono una elevata induzione residua

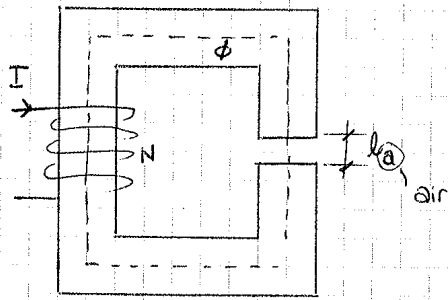
* In genere, i produttori classificano i materiali in base ad H_c e B_R , e forniscono al posto del ciclo di isteresi, la curva normale di magnetizzazione (= luogo dei vertici dei cicli di isteresi simmetrici)

But the relation between B and H is not only one: for each H , we have two values of B



$$\mu = \frac{B}{H}$$

B used to define μ is that on the curve obtained linking all the maximum value (H_x) of the infinite hysteresis loops. *



$$R = \frac{1}{\mu S} l$$

$$\Phi \cdot R = N \cdot I \rightarrow \Phi = \frac{N I}{R}$$

Now we have two sections: one with iron and one with air.

$$\bullet R_{Fe} = \frac{1}{\mu_{Fe} S_{Fe}} \cdot l_{Fe}$$

$$\Rightarrow R_t = R_{Fe} + R_a$$

$$\bullet R_a = \frac{1}{\mu_0 S_a} \cdot l_a$$

$$\Phi = \frac{N \cdot I}{R_t} = \frac{N \cdot I}{R_{Fe} + R_a}$$

But we know that $S_{Fe} = S_a$:

$$\bullet R_{Fe} = \frac{1}{\mu_{Fe} \textcircled{S}} \cdot l_{Fe}$$

$$\bullet R_a = \frac{1}{\mu_0 \textcircled{S}} \cdot l_a$$

$$R_{Fe} = R_a \quad \text{if} \quad \frac{l_{Fe}}{\mu_{Fe}} = \frac{l_a}{\mu_0}$$

We know that $\mu_{Fe} \gg \mu_0$ ($\mu_{Fe} = 1000 \mu_0$)

then $\boxed{l_{Fe} = 1000 l_a}$

we need to reduce R_t , that means to reduce l_a above all

To maintain $\Phi = \text{cost}$ reducing MMF, we need to reduce the length of air

Knowing that $\oint H \times dl = N I$ we can write:

infatti, usare materiali con μ alta significa avere R bassa e quindi spendere poca forza magnetomotrice per generare il Φ .

$$H_{Fe} \cdot l_{Fe} + H_a \cdot l_a = N I$$

If I want to reduce the current, I need to reduce l_a

$$\Phi \cdot R = N \cdot I$$

$$\downarrow \downarrow \quad \downarrow$$

$$I \cdot R = \checkmark$$

There is a relationship between magnetic and electrical laws.

So we can change the circuit in the following way:

$$\phi_p = \frac{NI}{R}$$

Flusso principale = quello che circola nel circuito, prodotto dalla FMM
 this is the actual flux in the magnetic circuit (produced by the MMF)

But we can define the flux in another way:

$$\phi_c = N \phi_p \rightarrow \text{this is the linked flux}$$

'concatenato'

(NB) Il flusso che circola nel circuito magnetico e' sempre quello principale

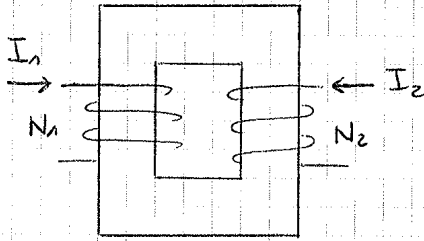
The two fluxes (ϕ_p) are the same:

$$\phi_c = \frac{N \cdot NI}{R} = \left(\frac{N^2}{R} \right) I$$

$L = \text{INDUCTANCE} = L = L(I)$ if we have magnetic materials it isn't linear.

$$\phi_c = L \cdot I$$

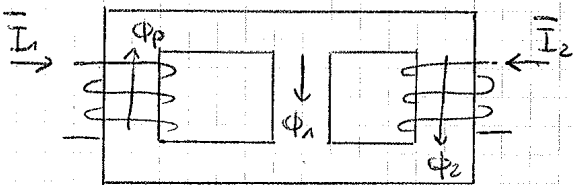
per gli stessi motivi per cui non lo e' la riluttanza.



If $I_2 = 0$ and $I_1 \neq 0$: $\phi_p = \frac{N_1 I_1}{R}$

$$\rightarrow \phi_{2,1} = N_2 \phi_p = \left(\frac{N_2 \cdot N_1}{R} \right) I_1$$

$M = \text{MUTUAL INDUCTANCE} = M_{ij} = \frac{N_i N_j}{R_{ij}}$



$$\phi_p = \phi_1 + \phi_2$$

$$M_{2,1} = \frac{N_1 N_2}{R}$$

$$M_{2,1} = \left(K \right) \frac{N_1 N_2}{R}$$

tiene conto del fatto che il flusso non si concatena perfettamente con tutte le spire.
 dipende dalla struttura del circuito magnetico e fornisce il rapporto tra il ϕ_p prodotto da N_1 e quello che interessa l'avvolgimento N_2 .

Ripetendo il calcolo considerando l'avvolgimento 2 come generatore del ϕ_p e calcolando il ϕ_c con l'avvolgimento 1 si ricava $M_{1,2}$, che e' numericamente = a $M_{2,1}$.

Questo significa che, a parita' di I circolante nei due avvolgimenti, i ϕ_c sono =.

LENZ LAW

→ Si consideri una spira immersa in un Φ magnetico; se detto Φ , rispetto alla spira, varia (in modulo, direzione o verso) nasce in essa una forza elettromotrice che si oppone alla variazione.

$$e = - \frac{d\Phi_e}{dt} \quad [V] \quad \rightarrow \quad \oint e$$

A variable flux in the time creates an electromotive force.

- means that the electromotive force generates a current which is opposite to the flux variation

⊖ when the generator works as a generator

⊕ " " " " " " motor

$$e = - \frac{d\Phi_e}{dt} = - \frac{d(Li)}{dt} = - L \frac{di}{dt} - i \frac{dL}{dt}$$

- $L \frac{di}{dt}$ = TRANSFORMER ELECTROMOTIVE FORCE

- $i \frac{dL}{dt}$ = MOTIONAL ELECTROMOTIVE FORCE (every time we have a rotation)

$$\left\{ \begin{aligned} L &= f(\theta) \rightarrow \frac{dL}{dt} = \frac{dL}{d\theta} \frac{d\theta}{dt} = \omega \frac{dL}{d\theta} \end{aligned} \right.$$

The current produced by the variation of the flux is called EDDY CURRENT (= parassita) : it introduces losses

⊕ it heats the material which can be used in some ways

→ In electrical machines we see Eddy current as losses:

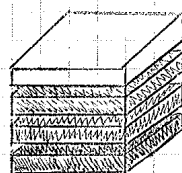
$$P_{Ec} = K_{Ec} B_{max}^2 f^2 \approx \frac{V^2}{R_{eq}} \approx \frac{e^2}{R_{eq}} \approx \frac{\left(\frac{d\Phi}{dt}\right)^2}{R_{eq}}$$

Is it possible to reduce P_{Ec} ? Yes, increasing R_{eq} (→ insulating material haven't got eddy current).

$$R_{eq} = \rho \frac{l}{S}$$

① → so we want to increase ρ : we have to add a percentage of Si to the ferromagnetic material (but it increases fragility) → 6-6.5% is the maximum percentage of Si

② → so we want to increase l and decrease S : we have lamination:



0.65 mm

0.5 mm

0.35 mm

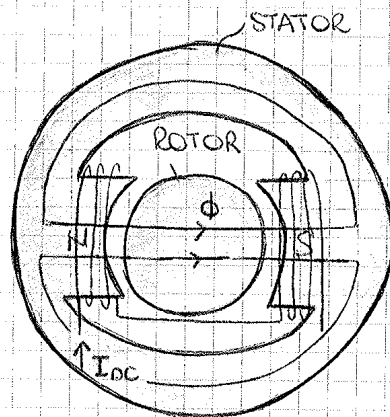
COEFF. DI STIPAMENTO = rapporto tra la sup. netta del solo ferro e quella lorda di tutto l'impiacchellamento.

→ si realizza il nucleo magnetico come pacco di lamierini, imponendo alle I un percorso obbligato.

DC MACHINE

Structure:

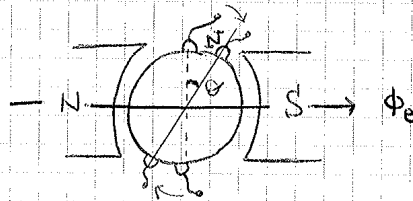
- rotor (in magnetic material, μ_r)
- stator with poles



2 poles

I_{dc} = excitation

We are interested only in the rotor, because in the stator the flux is constant.



$$\phi_e = N I_{dc} \phi_e$$

linked excitation flux

When we move the rotor, ϕ_e changes with θ : $\phi_e = N \phi_e \cos \theta$

If $\theta = 0 \Rightarrow \phi_e = (\phi_e)_{MAX}$

If $\theta = 90^\circ \Rightarrow \phi_e = 0$

Using Lenz law:

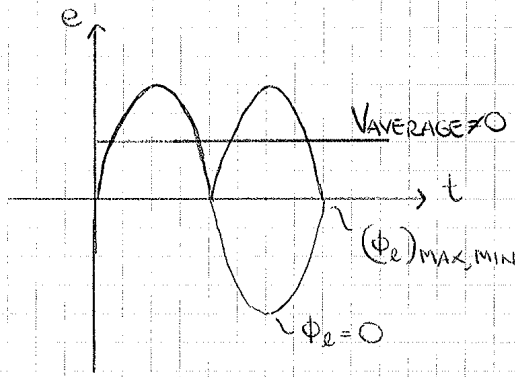
$$e = - \frac{d\phi_e}{dt} = - \frac{d(N\phi_e \cos \theta)}{dt}$$

The rotor moves at ω , so $\theta = \omega t$

$$\rightarrow e = - \frac{d}{dt} (N\phi_e \cos \omega t) = N\phi_e \omega \sin \omega t \rightarrow \text{forza elettromotrice indotta nella spira}$$

This means that the electromotive force has a sinusoidal behaviour

So, using the two half-rings, from the brushes point of view: -



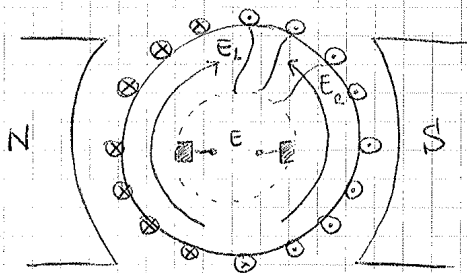
Now we can say we have a DC machine because the average voltage is different to 0.

If I want a constant current (or a constant voltage) I need a battery.
We can't produce it with an electrical machine.

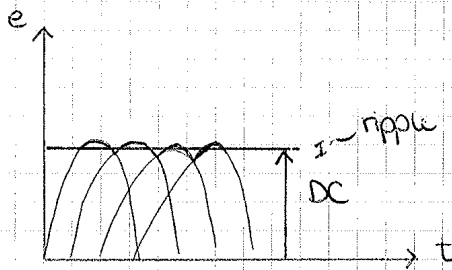
When the rotor rotates of 90° we have short circuit because the two half-rings are touching both brushes → dangerous!

In order to make a real machine we need lots of coils

→ Nella realtà l'avvolgimento del rotore viene realizzato utilizzando un numero elevato di spire opportunamente connesse tra loro ognuna delle quali risulta collegata a delle lamelle isolate che formano il sistema di commutazione meccanico chiamato collettore a lamelle.

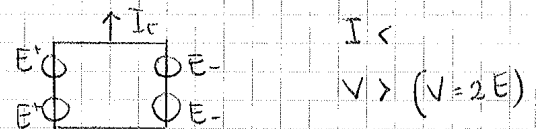
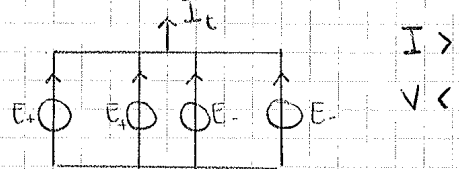
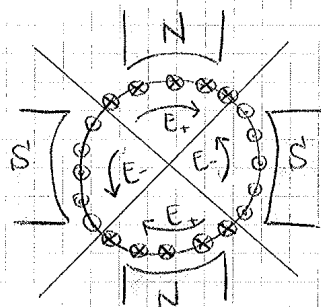


We can have two generators in // because they have the same value



Dipendentemente dal tipo di avvolgimento utilizzato i generatori equivalenti possono essere inseriti in // o in serie //.

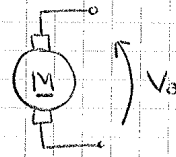
If, instead of 2 poles, we have 4 poles:



In short:

- ① $E = k \phi \omega$ electrical and mechanical
- ② $V_a = E + I R_a$ "
- ③ $T = k_a \phi I$ " " "

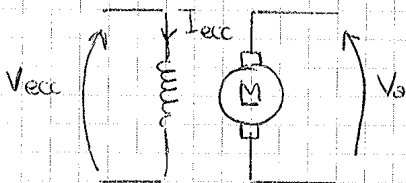
The symbol of a DC machine is



We can subdivide DC machines in

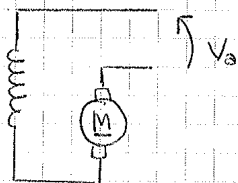
- separated excitation ①
- series excitation ②
- parallel excitation ③ ← (DC generator) we're not interested
- permanent magnets ④

① SEPARATED EXCITATION



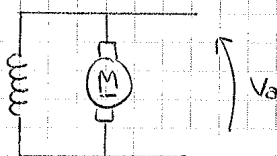
→ Il circuito di eccitazione risulta alimentato in modo separato dall'armatura della macchina.

② SERIES EXCITATION



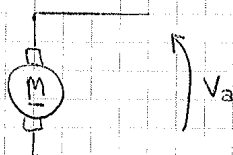
→ Il circuito di eccitazione è connesso in serie con la macchina, per cui la corrente di armatura risulta anche la corrente di eccitazione.

③ PARALLEL EXCITATION



→ Il circuito di eccitazione è connesso in parallelo alla macchina e risulta quindi alimentato dalla stessa tensione di armatura.

④ PERMANENT MAGNETS



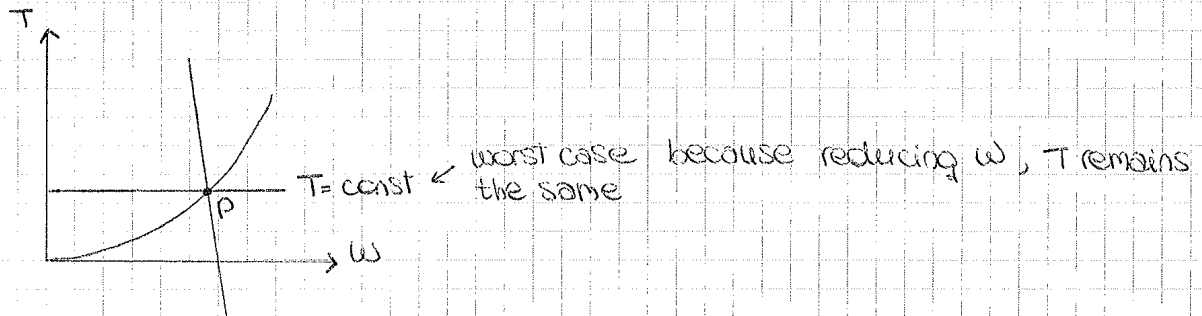
→ Non esiste un circuito di eccitazione: il flusso viene ottenuto mediante l'utilizzo di magneti permanenti.

In order to buy the right motor I have to know the load characteristic.

* Because $T = k_a \phi \frac{V_a}{R_a} - \frac{k_a^2 \phi^2 \omega}{R_a}$ and R_a is small. (to reduce losses)

If $R_a = 0$, ω would be constant at T variations.

We always consider the worst case for the motor:



Considering the starting torque we can define the STARTING CURRENT:

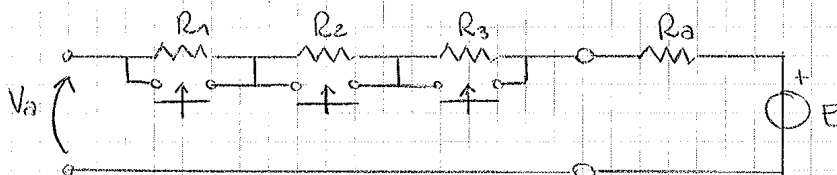
$$T_s = k_a \phi \frac{V_a}{R_a} = k_a \phi I_s$$

$\Rightarrow I_s = \frac{V_a}{R_a}$ When the machine starts to work it absorbs a high current. We want to limit it in order to avoid damages.

↳ nel caso in cui V_a sia pari alla tensione nominale V_{nom}

How can we do this?

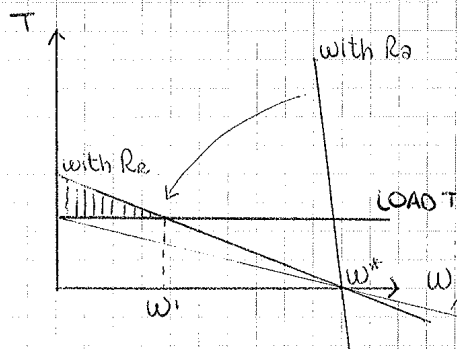
We can add several resistances in series with the machine:



Queste resistenze vengono cortocircuitate gradualmente mediante opportuni contattori.

$$I_s^* = \frac{V_a}{(R_1 + R_2 + R_3) + R_a}$$

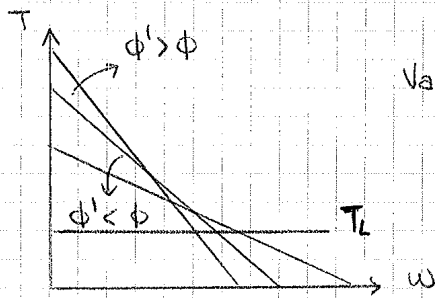
where $R_e = R_1 + R_2 + R_3$ is the reostatic equivalent resistance.



If the T_s is equal or smaller than the load T the motor doesn't start to work

$$T_m - T_L = J \frac{d\omega}{dt}$$

② increase/decrease ϕ with $V_a = \text{const}$: FLUX REGULATION



$V_a = \text{const}$

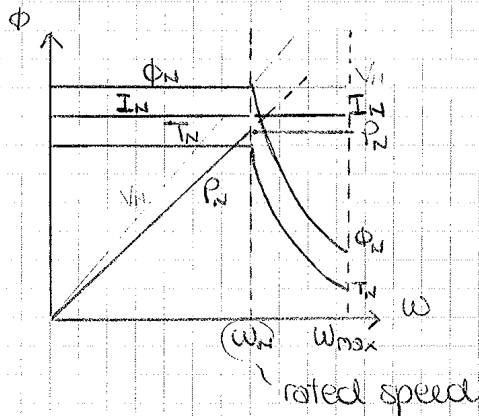
All variare di ϕ :

$$\bullet T_s = \left(K_a \phi \frac{V_a}{R_a} \right) \propto \phi$$

$$\bullet \omega^* = \left(\frac{V_a}{K_a \phi} \right) \propto \frac{1}{\phi_e}$$

⇒ Speed regulation is always made with voltage regulation.

* →



$$P = \omega \cdot T \rightarrow T = \frac{P}{\omega}$$

$$E = K \phi \omega \rightarrow \phi = \frac{E}{K \omega}$$

same behaviour

$0 < \omega < \omega_N$: CONSTANT TORQUE REGULATION

$\omega_N < \omega < \omega_{max}$: CONSTANT POWER REGULATION

In a good machine $\frac{\omega_{max}}{\omega_N} \approx 2$; in the best ones $\frac{\omega_{max}}{\omega_N} \approx 2,5$.

So we have some limitations from the mechanical point of view.

Is it possible to have a constant power load? Yes!

For example, the traction load (in order to reach a high speed, you have to change gear = marcia; every gear has a ratio $\frac{\omega_{max}}{\omega_N}$)

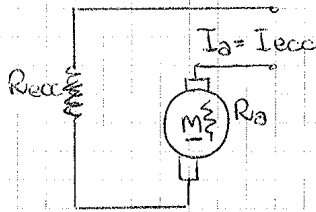
Another application is the lathe (= tornio): • sovrassatura → ↓ω, ↑T

• finitura: ↑ω, ↓T.

Another application is the winding of a sheet of paper.

In conclusion, we always use the first region, while we use the second one only if we have a constant power load

SERIES EXCITATION DC MACHINE



$$E = K_a \phi \omega$$

$$V_a = E + R_a I$$

$$T = K_a \phi I$$

$$\phi = K_{ec} I$$

$$E = (K_a K_{ec}) I \omega = (K_t) I \omega \quad (1)$$

$$T = K_a K_{ec} I I = K_a K_{ec} I^2 = K_t I^2 \quad (2)$$

$$V_a = E + R_t I \quad (*)$$

$$R_t = R_a + R_{recc}$$

RESISTENZA TOTALE IN SERIE ALLA MACCHINA

Substituting (1) in (*)

$$V_a = K_t I \omega + R_t I = I (K_t \omega + R_t) \quad (3)$$

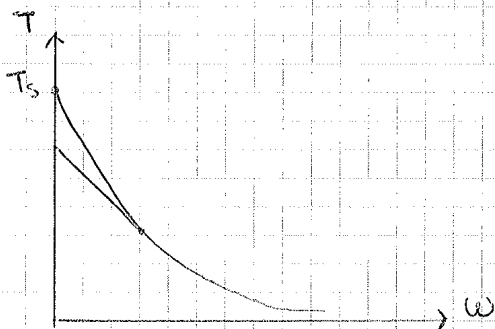
$$I = \frac{V_a}{K_t \omega + R_t} \rightarrow \text{corrente assorbita dalla macchina}$$

Then, (2) become:

$$T = K_t \frac{V_a^2}{(K_t \omega + R_t)^2} \quad \left\{ \begin{array}{l} T \propto V_a^2 \\ T \propto \frac{1}{\omega^2} \end{array} \right.$$

• If $\omega = 0 \Rightarrow T_s = K_t \frac{V_a^2}{R_t^2} = K_t I_s^2$ (higher than in the separated excitation)

• When $\omega \neq 0 \Rightarrow$ as we want a good efficiency R_t is as small as possible.
 $K_t \omega \gg R_t$



• If we decrease T_L motor will increase its speed.

So this is a motor that can't work in no-load conditions ($\omega \rightarrow \infty$)

• As $T = K_t (\cdot)^2$ it isn't possible to change the sign of the torque: in order to do this we have to change the configuration.

On the motor label we can find also the insulation class, which defines the maximum working temperature

When the motor is realized it has to be tested in order to verify if the temperature is lower than that corresponding to the insulation class.

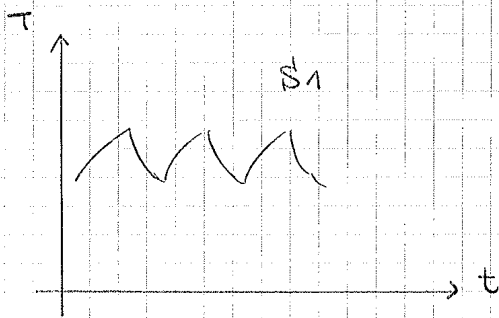
If it is higher I can:

- re-design the machine
- change P_N

P_N also depends on the cooling system of the machine.

The same machine can have different P_N according to the cooling system.

If I improve the cooling system P_N will increase



When the motor has time to reduce its temperature the maximum value is lower than that of the insulating class.

The service is usually written on the label

We suppose $\eta = 1$.

We can write $P = E \cdot I = k_a \Phi \overset{\leftarrow}{\omega I} = k_a (B \cdot S_{Fe}) \omega \cdot d \cdot S_{Cu}$

where B is the flux density and d is the current density, S_{Fe} is the section of the iron and S_{Cu} is the section of the copper.

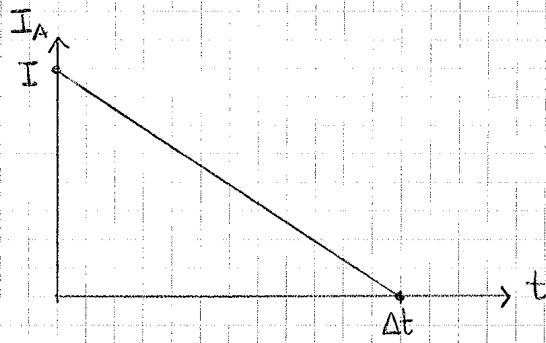
We can also write $P = T \cdot \omega$

$$\Rightarrow T \cdot \omega = k_a B (S_{Fe}) \omega d (S_{Cu})$$

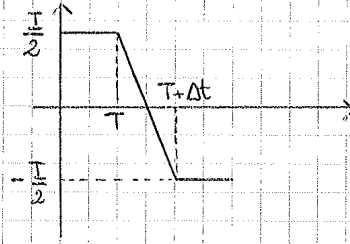
In conclusion, the size of the machine depends on the torque (and so on the n_N), and doesn't depend on the power.

$$P = k_a B d \omega \underset{l^2}{S_{Fe}} \underset{l^2}{S_{Cu}} = k_a B d \omega \cdot l^4$$

If I double the size of the machine, power will be $P' = k_a B d \omega (2l)^4 = 2^4 P$

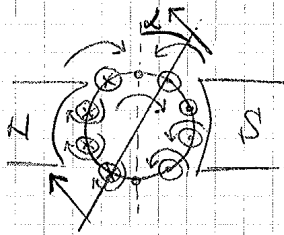


La corrente che percorre tale spira varia nel tempo, poiché deve passare da un valore $\frac{I}{2}$ a $-\frac{I}{2}$.



We assume that current is proportional to the surface of the touched segment of the commutator (by the brush)

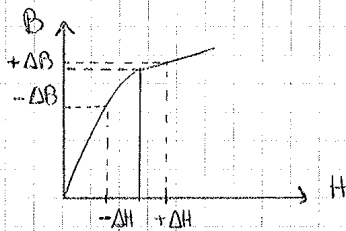
That's why we call this commutation "LINEAR COMMUTATION"



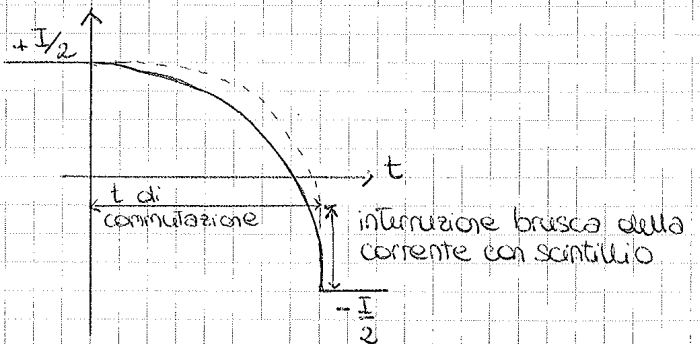
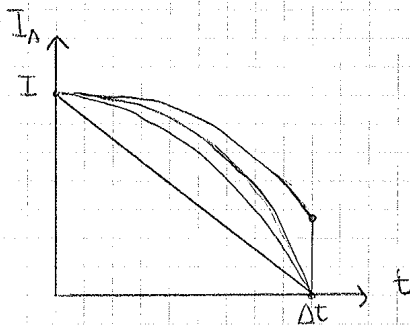
La commutazione è il problema principale delle macchine DC in quanto commutazioni non ottimali porterebbero a una rapida usura delle spazzole e del collettore a causa di un molto dannoso fenomeno di scintillio tra spazzole e collettore.

We have only one electromotive force which is that produced by the linked flux due to the excitation.

But in the actual machine we have two electromotive forces: we have also the one produced by the rotor



In the actual machine the current is in delay compared with linear commutation



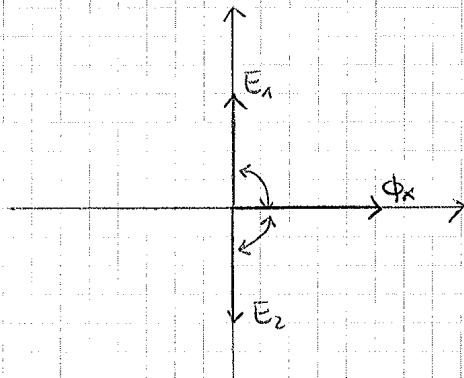
In conclusion:

$$e_1 = N_1 \frac{d}{dt} (\Phi_x \sin \omega t) = N_1 \omega \Phi_x \cos \omega t \quad \rightarrow$$

$$e_2 = -N_2 \frac{d}{dt} (\Phi_x \sin \omega t) = -N_2 \omega \Phi_x \cos \omega t \quad \rightarrow$$

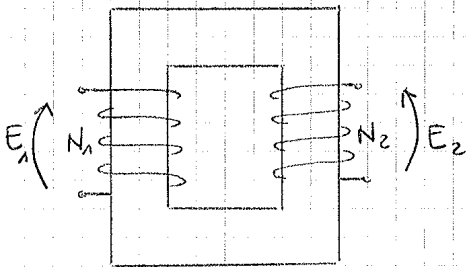
N : n° di spire dell'avvolgimento
 Φ_x : valore max del flusso magnetico
 ω : pulsazione elettrica

From now on we aren't in DC conditions!



$$E = N \omega \frac{\Phi_x}{\sqrt{2}} = N \frac{4.44j}{\sqrt{2}} f \Phi_x$$

- E_1 e' in anticipo di 90° rispetto a Φ_x
- E_2 e' in ritardo di 90° rispetto a Φ_x

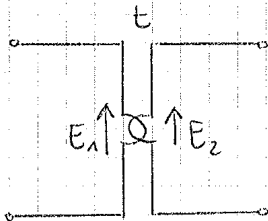


$$\vec{E}_1 = 4.44j f N_1 \Phi_x$$

$$\vec{E}_2 = 4.44j f N_2 \Phi_x$$

$$\frac{\vec{E}_1}{\vec{E}_2} = \frac{N_1}{N_2} = t$$

TRANSFORMATION RATIO



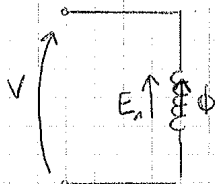
symbol of the ideal transformer

ideal transformer =

- $\mu_{Fe} \approx \infty$
- $\eta = 1$

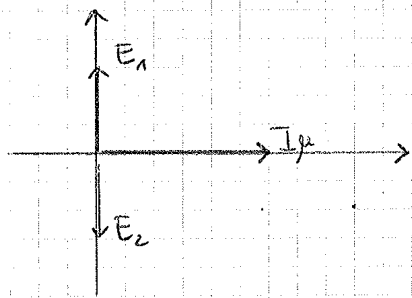
Now we want to represent the real transformer with an equivalent circuit.

The relation between absorbed current and linked flux is described by Hopkinson law: $NI = R\Phi$



The electromotive force is produced by the flux and is equal to the voltage.

If $V = \text{const} \Rightarrow \Phi = \text{const}$ (in term of amplitude)



$\eta = 1 \Leftrightarrow \text{losses} = 0$ (false!)

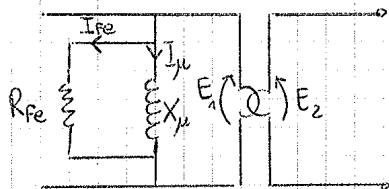
Iron losses:

- hysteresis : P_{hys}
- Eddy current : P_{ec}

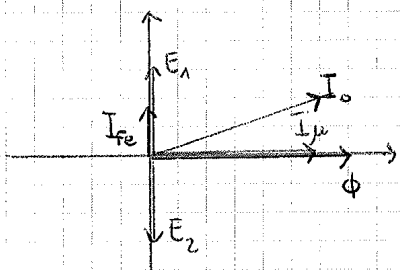
Both losses depend on B^2 , which means on Φ^2 and then E^2

We can say that $P_{\text{Fe}} = \frac{E^2}{R_{\text{Fe}}}$

R_{Fe} is an equivalent resistance which represents iron losses (it isn't an actual resistance)



In the resistance R_{Fe} the current is in phase with the voltage:

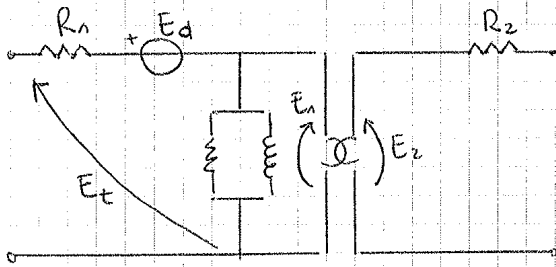


$$I_0 = I_{\mu} + I_{\text{Fe}}$$

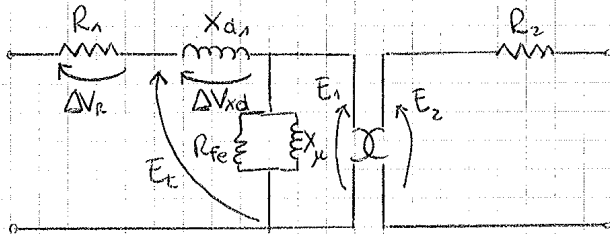
where I_0 is the NO-LOAD CURRENT, which is the current absorbed when the machine isn't linked to a load.

But $I_{\text{Fe}} \ll I_{\mu} \Rightarrow I_0 \approx I_{\mu}$

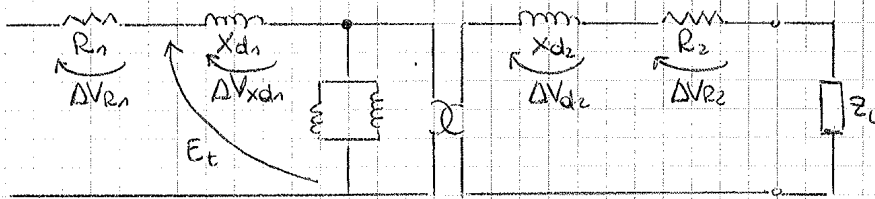
Anyway, we can't neglect P_{Fe} .



But we know that $\phi_e = L \cdot I$.



The same thing can be said for the secondary winding:



This is the best model to represent the real situation because V_L computed is equal to V_L measured. (in working conditions: $f = 50$ Hz)

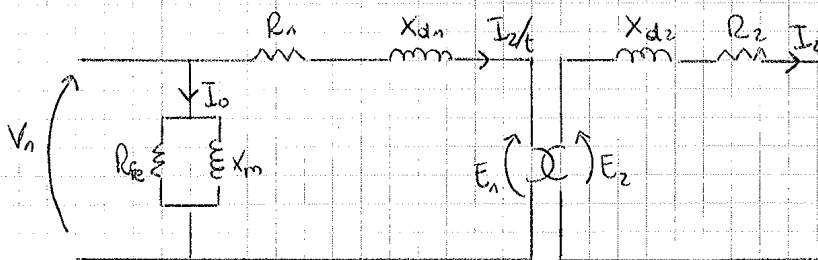
This is valid only in low-frequency conditions.

In higher frequency $X (= j\omega L)$ would be an open circuit. Then, components used in this model work well in low-frequency conditions.

We can't use it in higher frequency conditions!

Even if this is a very good model, from the solution-circuit point of view is not so easy. The problem is the node before no load components.

The solution is to change the position of these components:



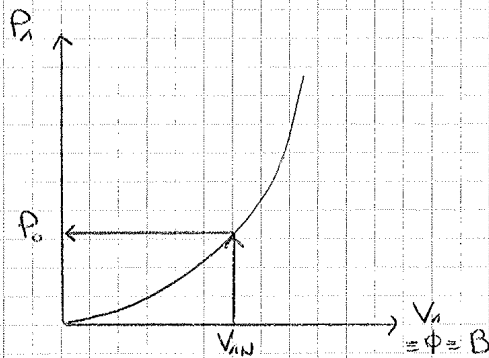
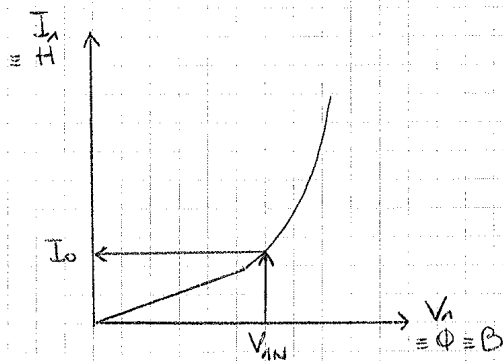
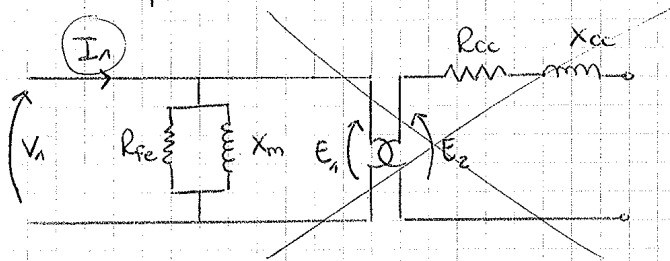
Doing this, we accept some errors.

The current which crosses R_1 and X_{d1} is now I_0 .

The machine is supplied by a voltage (an increasing voltage).

$$t = \frac{V_1}{V_2}$$

As we are in no-load conditions, the current I_1 is that absorbed by no-load components.



Since we know the rated voltage V_{1N} , which is defined by the transformer, after this test we also know I_0 and P_0 .

Once we know V_{1N} and P_0 , we can write: $P_0 = \frac{V_{1N}^2}{R_{fe}}$ so that we can compute R_{fe} :

$$R_{fe} = \frac{V_{1N}^2}{P_0}$$

Moreover, we know that: $P_0 = V_{1N} I_0 \cos \phi_0$ so that we can find $\cos \phi_0$:

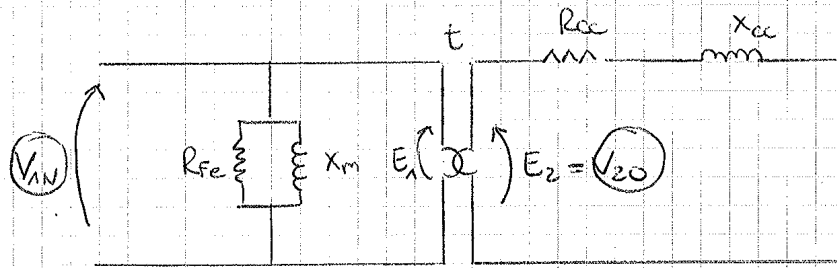
$$\cos \phi_0 = \frac{P_0}{V_{1N} I_0} \Rightarrow \tan \phi_0$$

Finally, knowing that: $Q_0 = P_0 \tan \phi_0$ and $Q_0 = \frac{V_{1N}^2}{X_m}$ we can find X_m :

$$X_m = \frac{V_{1N}^2}{Q_0}$$

RATED VOLTAGE IN THE PRIMARY WINDING = voltage applied to the primary winding

RATED VOLTAGE IN THE SECONDARY WINDING = voltage equal to no-load voltage at the secondary winding when we have the rated voltage in the primary winding

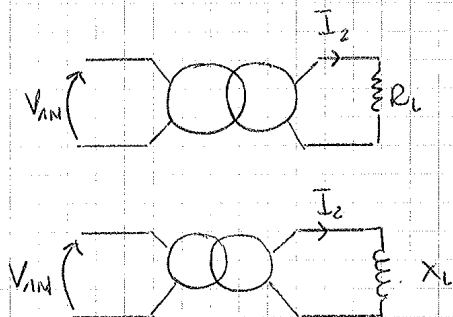


In the transformer we have 2 kinds of losses:

- iron losses: $P_{Fe} = B^2 = \phi^2 = V^2$
- Joule losses (copper losses): $P_{Cu} = I^2$

The rated power depends on thermal behaviour of the machine.

We can define the APPARENT POWER: $A_N = V_{1N} \cdot I_{1N} = V_{20} \cdot I_{2N}$ [VA]



$$\eta = \frac{P_{act}}{P_{act} + \Sigma \text{ losses}} = \frac{P_2}{P_2 + P_{Fe} + P_{Cu}} = \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + \frac{V_{1N}^2}{R_{Fe}} + R_{cc} I_2^2}$$

Both transformers are in rated conditions, indeed the apparent power is the same.

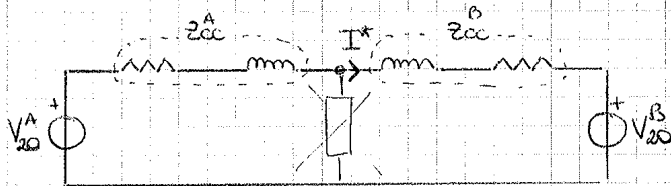
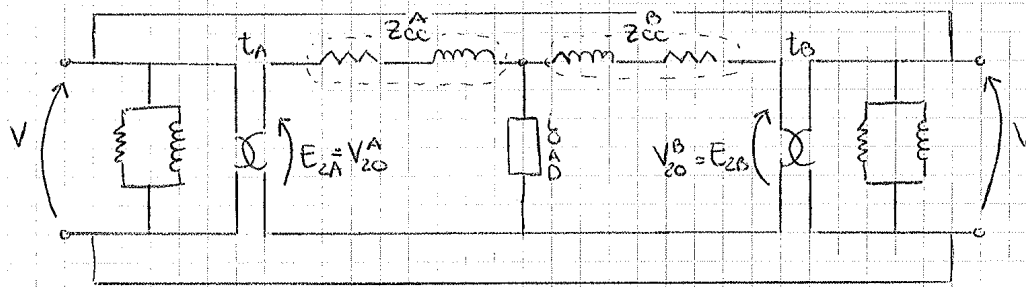
The great difference is that while $\eta_1 \neq 0$, I'm sure that $\eta_2 = 0$ because the active power is zero.

We can define all the rated values in percentage (as they are written in the transformer label):

- $I_0 \% = \frac{I_0}{I_N} 100$
- $P_0 \% = \frac{P_0}{A_N} 100$
- $V_{cc} \% = \frac{V_{cc}}{V_N} 100$
- $P_{cc} \% = \frac{P_{cc}}{A_N} 100$

We have 2 main reasons to connect two transformers in parallel:

- to increase the power
- to avoid stop production in the case a transformer doesn't work



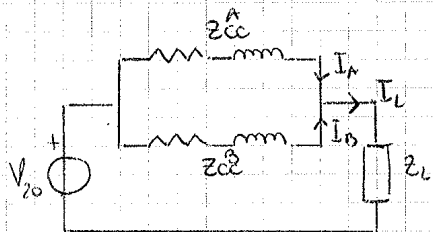
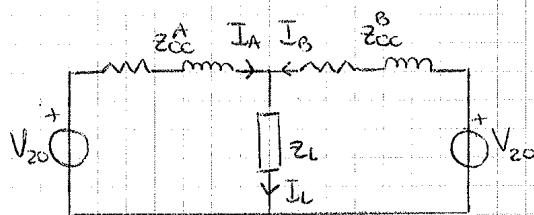
$$I^* = \frac{\frac{V_{20}^A}{z_{cc}^A} - \frac{V_{20}^B}{z_{cc}^B}}{\frac{z_{cc}^A}{t_A} + \frac{z_{cc}^B}{t_B}} = 0 \quad \text{NO-LOAD CURRENT}$$

⇒ if I want $I^* = 0 \rightarrow t_A = t_B$

So, we can connect in parallel two transformers only if they have the same transformation ratio. ①

A second condition is to connect the corresponding terminals* ②

If we connect a load:



↓ Mandatory conditions

- ⊗ collegamento dei morsetti della stessa polarità alle stesse barre di primario e secondario

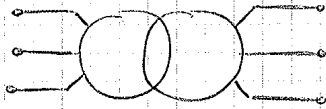
Partitore di corrente:

$$\bar{I}_A = \bar{I}_L \frac{\frac{z_{cc}^B}{z_{cc}^A + z_{cc}^B}}{\frac{z_{cc}^A}{z_{cc}^A + z_{cc}^B}} = \bar{I}_L \frac{|z_{cc}^B| e^{j\phi_{cc}^B}}{|z_{cc}^A| e^{j\phi_{cc}^A} + |z_{cc}^B| e^{j\phi_{cc}^B}}$$

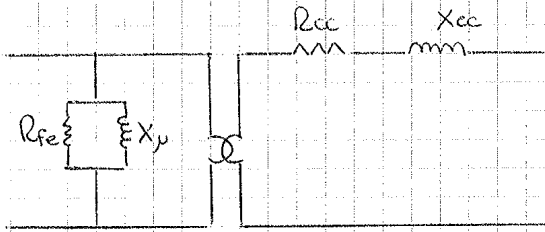
$$\bar{I}_B = \bar{I}_L \frac{z_{cc}^A}{z_{cc}^A + z_{cc}^B}$$

We want $\phi_{cc}^A = \phi_{cc}^B$ in order to produce the minimum current. ③

↳ minimizzare la corrente di circolazione tra i trasformatori in parallelo



Since $I_1 = I_2 = I_3$ we can study the equivalent single-phase transformer.



we can have two configurations of both windings:

- Y connection
- Δ connection

le combinazioni dei collegamenti del primario e del secondario e la presenza del neutro a terra oppure isolato condizionano il comportamento del trasformatore dal punto di vista delle armoniche e da quello delle dissimmetrie di carico.

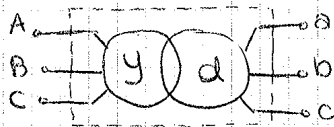
So we can have 4 types of transformer:

- Y/d
- Y/y
- Δ/y
- Δ/d

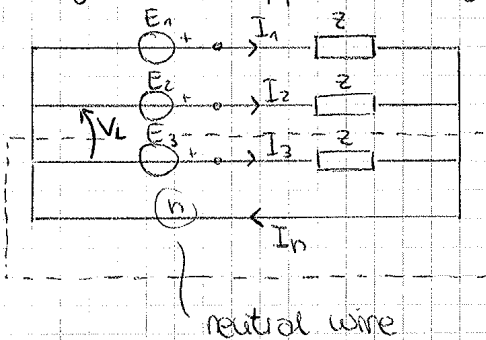
CIRCUITO EQUIVALENTE DEI TRASFORMATORI TRIFASE

Nell'ipotesi di considerare esclusivamente carichi equilibrati indipendentemente dal tipo di collegamento a stella del trasf. trifase è possibile rappresentare il trasf. trifase mediante un circuito equivalente monofase a stella. In questo modo l'analisi di una fase rappresenta, a meno degli sfasamenti di 120° tra le fasi, l'analisi di tutte e 3 le fasi. Il modello usato x il trasf. monofase risulta quindi valido anche x il trasf. trifase ricordando che esso rappresenta l'eq. monofase con collegamento a stella del trasf. monofase di partenza. ⊕ ↷

$$z_1 = z_2 = z_3$$



The theory of this approach says that I can consider only one phase. Indeed:

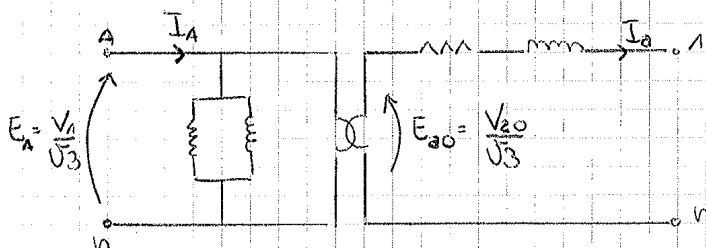


$$I_1 + I_2 + I_3 = 0 \Rightarrow I_n = 0$$

V_L = line voltage (o tensione concatenata)

E = phase voltage

$$V = \sqrt{3} E$$



⊗ I have to change voltages but not currents!

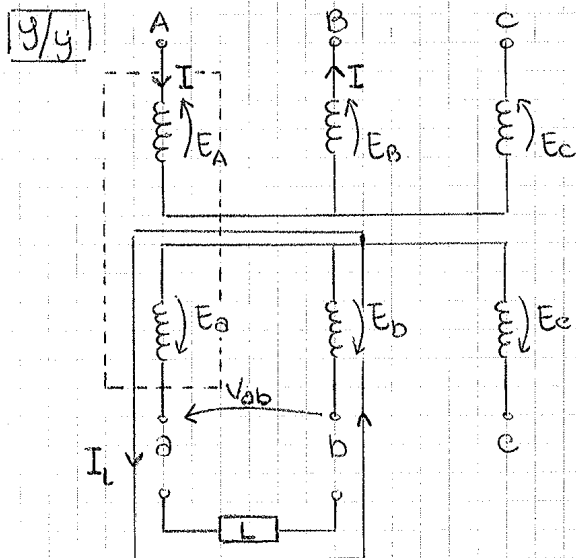
⊗ Con qst modello le correnti calcolate sono coincidenti con le correnti di linea del trasf. trifase, mentre le tensioni sono le tensioni di fase e non le tensioni concatenate ai morsetti del trasformatore

$$t = \frac{V_{1N}}{V_{2N}} = \frac{V_{1N}}{V_{20}}$$

three phase wire

three phase wire + earth wire

The best configuration is y/y_n , but without the balance of the currents it doesn't work.

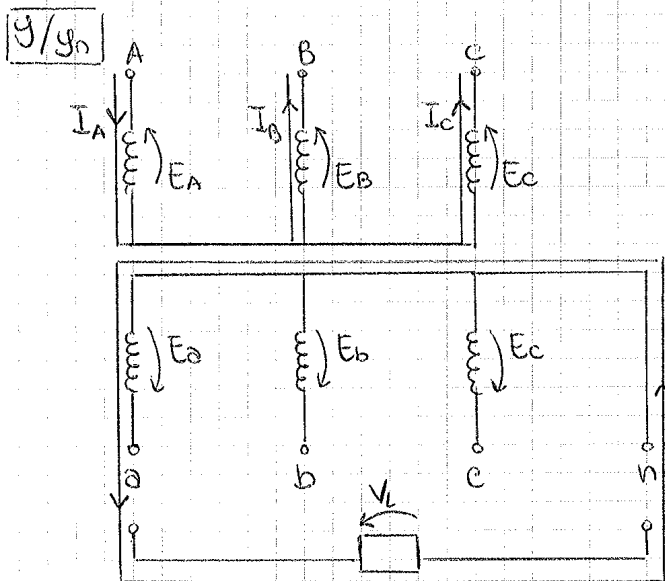


This is the configuration to have balanced electromotive forces. In the phase C there won't be current

A load is balanced when $\begin{cases} |z_A| = |z_B| = |z_C| = |z| \\ \phi_A = \phi_B = \phi_C = \phi \end{cases}$

The worst unbalanced load is a single phase (for example, an impedance between 'a' and 'b')

The configuration y/y works well even if there's not balance



The phases B and C haven't balanced electromotive forces.

$$V_L = V_{\text{phase}} = 400V$$

$$\begin{aligned} (N_1 I_1)_A &= (N_2 I_2)_a \\ (N_1 I_1)_B &= 0 \rightarrow I_{2b} = 0 \\ (N_1 I_1)_C &= 0 \rightarrow I_{2c} = 0 \end{aligned}$$

without a balanced load it doesn't work, so we have to change the configuration of the primary winding from y to D .

② SHORT-CIRCUIT TEST → Viene effettuato collegando in corto circuito i 3 morsetti secondari.

From measurement we know: V_{cc} , $I_{2N} = I_{1N} \cdot t$, P_{cc}

Since we know that: $P_{cc} = 3 R_{cc} I_{2N}^2$

V_{cc} : tensione di corto circuito
 I_{2N} : corrente nominale secondaria
 P_{cc} : potenza di corto circuito

we can compute: $R_{cc} = \frac{P_{cc}}{3 I_{2N}^2}$

Moreover, as: $P_{cc} = \sqrt{3} V_{cc} I_{1N} \cos(\varphi_{cc})$

$$\cos(\varphi_{cc}) = \frac{P_{cc}}{\sqrt{3} V_{cc} I_{1N}} \rightarrow \tan \varphi_{cc}$$

$$Q_{cc} = P_{cc} \tan \varphi_{cc}$$

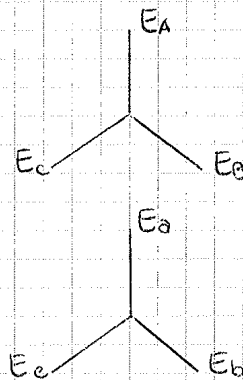
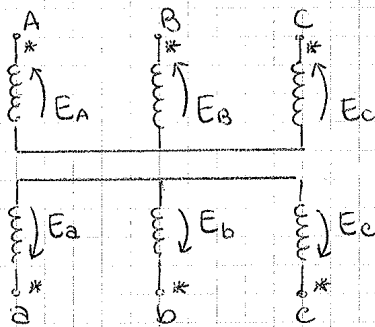
$$Q_{cc} = 3 X_{cc} I_{2N}^2$$

we can compute: $X_{cc} = \frac{Q_{cc}}{3 I_{2N}^2}$

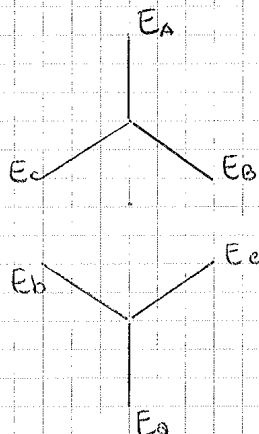
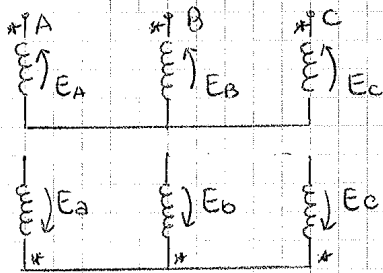
Nei trasformatori trifase puo' sussistere uno sfasamento tra la terna delle tensioni primarie e secondarie, sfasamento che nasce dalle possibili combinazioni delle connessioni primario-secondario.

y/y

The two three phase systems are in phase



In this other case, the transformer is the same (y/y) but the voltages are shifted of 180° .



EFFICIENCY OF A THREE PHASE TRANSFORMER

$$\eta = \frac{P_{out}}{P_{in}}$$

$$P_{in} - P_{out} = \Sigma \text{ losses} = P_{Fe} + P_{Cu}$$

$P_{Fe} \Rightarrow$ no-load test

$$P_{Cu} = 3 R_{cc} I_2^2$$

$$P_{out} = \sqrt{3} V_L I_2 \cos \phi_L$$

load

$$\Rightarrow \eta = \frac{\sqrt{3} V_L I_2 \cos \phi_L}{\sqrt{3} V_L I_2 \cos \phi_L + P_{Fe} + 3 R_{cc} I_2^2}$$

η depends on

- power factor: $\cos \phi_L$
- current absorbed by the load: I_2

We know that $V_L \neq V_{20}$
 \downarrow
 V_{2N}

But if we accept this error we can write:

this isn't the real efficiency

↗ but we want to know how it behaves

$$\eta = \frac{\sqrt{3} V_{2N} I_2 \cos \phi_L}{\sqrt{3} V_{2N} I_2 \cos \phi_L + P_{Fe} + 3 R_{cc} I_2^2}$$

↗ rapporto di carico: $\frac{\text{corrente erogata sul carico}}{\text{corrente nominale}}$

we can define $\alpha = \frac{I_2}{I_{2N}}$, which describes the current situation.

$$\rightarrow I_2 = \alpha I_{2N}$$

$$\eta = \frac{\sqrt{3} V_{2N} \alpha I_{2N} \cos \phi_L}{\sqrt{3} V_{2N} \alpha I_{2N} \cos \phi_L + P_{Fe} + 3 R_{cc} \alpha^2 I_{2N}^2}$$

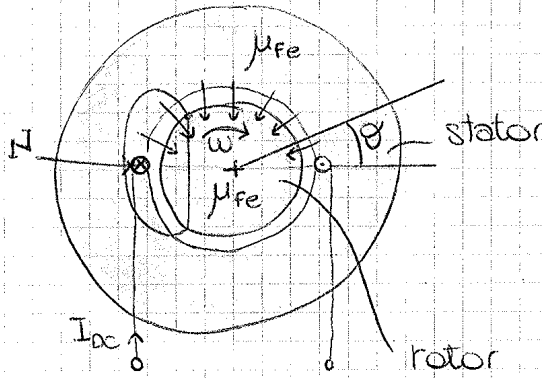
But we know that $A_N = \sqrt{3} V_{2N} I_{2N}$ and $P_{cc} = 3 R_{cc} I_{2N}^2$

$$\eta = \frac{A_N \alpha \cos \phi_L}{A_N \alpha \cos \phi_L + P_{Fe} + \alpha^2 P_{cc}}$$

In order to find η_{max} we have to solve $\frac{d\eta}{d\alpha} = 0$:

$$\frac{A_N \cos \phi_L (A_N \alpha \cos \phi_L + P_{Fe} + \alpha^2 P_{cc}) - A_N \alpha \cos \phi_L (A_N \cos \phi_L + 2\alpha P_{cc})}{(A_N \alpha \cos \phi_L + P_{Fe} + \alpha^2 P_{cc})^2} = 0$$

AC MACHINES



Following the red line:

$$N \cdot I_{oc} = H_{fe} \cdot l_{rfe} + H_a \cdot l_a + H_{sfe} \cdot l_{sfe} + H_a \cdot l_a$$

Si consideri una struttura magnetica realizzata mediante uno statore e un rotore in materiale ferromagnetico ed una bobina di N spire avvolte diametralmente nello statore. Si supponga la bobina percorsa da una corrente continua I_{oc} .

We can consider

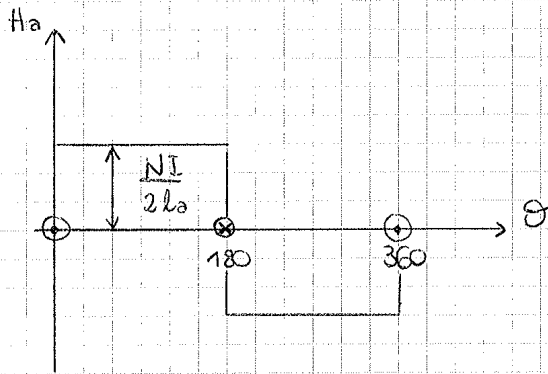
- $l_a = \text{const}$
- $\mu_{fe} = \infty$

so that we can neglect $H_{rfe} \cdot l_{rfe}$ and $H_{sfe} \cdot l_{sfe}$

$$N I_{oc} = 2 H_a \cdot l_a \quad \Rightarrow \quad H_a = \frac{N \cdot I_{oc}}{2 l_a} \quad \text{campo magnetico nel traferro}$$

If we change line to follow, the only difference is the length.

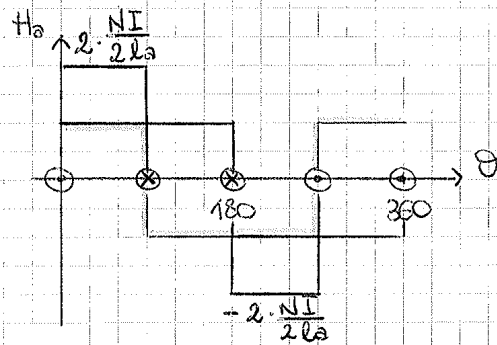
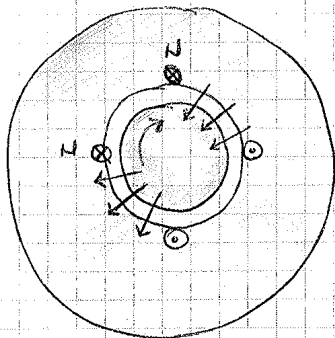
We know that H_a is constant in modulus.



Nella parte superiore della macchina il campo sarà uscente dallo statore ed entrante nel rotore. Nella parte inferiore viceversa.

La distribuzione del campo magnetico al traferro e' di tipo rettangolare.

Ora si aggiunge una seconda bobina sempre di N spire \perp nello spazio alla bobina precedente.



- the sum of the two magnetic fields

Since H_a - magnetic field, also B_a is.

$B_a = B_x \sin \theta$ flux density

And this means that also the flux Φ_a is sinusoidal.

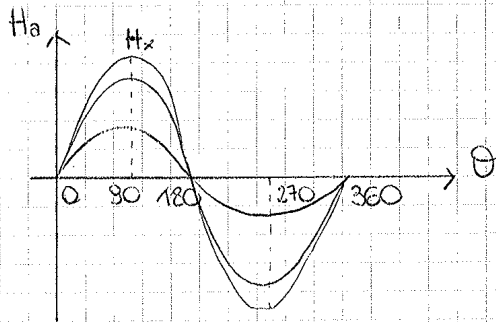
$\Phi_a = \Phi_x \sin \theta$

What happens if the current is AC instead of DC?

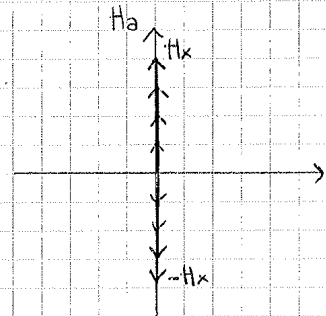
$I = I_x \sin \omega t$ (ω e' la pulsazione)

$H_a = \frac{N e q \cdot I_x \sin \omega t}{2 l a} \sin \theta$
 • time • space

→ 2 variables: il campo magnetico e' funzione sia del tempo che dello spazio.



t = const
 t = const
 t = const



The maximum and the minimum value is always when $\theta = 90^\circ \pm k\pi$

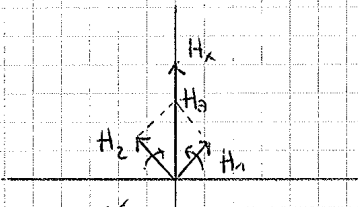
$H_a = \frac{N e q \cdot I_x}{2 l a} \sin \theta \sin \omega t$

↳ esprimibile come...

$H_a = \frac{1}{2} \frac{N e q \cdot I_x}{2 l a} [\cos(\theta - \omega t) - \cos(\theta + \omega t)]$

↳ Il campo H_a e' la somma di 2 espressioni funzioni dello spazio e del tempo, che rappresentano ognuna un'onda. Le due onde si muovono nello spazio con una velocita' angolare ω_m ricavabile dalle seguenti espressioni:

① $\theta - \omega t = 0 \rightarrow \theta = \omega t$ onda 1
 ↳ derivative
 $\frac{d\theta}{dt} = \frac{d}{dt} \omega t = \omega$
 ω_m mechanical ω_e electrical



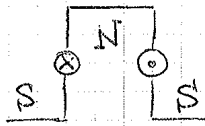
② $\theta + \omega t = 0 \rightarrow \theta = -\omega t$ onda 2
 $\frac{d\theta}{dt} = \frac{d}{dt} (-\omega t) = -\omega$
 ω_m - ω_e

↳ Il campo magnetico (che e' la somma delle due onde) non ruota.

→ Le 2 onde si muovono nello spazio con una velocita' angolare uguali e opposte: pari alla pulsazione della corrente di alimentazione:

- $\omega_m = \omega_e$
- $\omega_m = -\omega_e$

when we have 2 poles, the mechanical angle is equal to the magnetic angle:



$$\theta_{\text{mech}} = 360^\circ$$

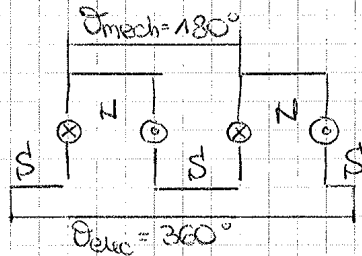
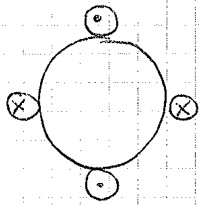
$$\theta_{\text{elec}} = 360^\circ$$



$$\omega_{\text{mech}} = \omega_{\text{elec}} : \frac{2\pi n}{60} = 2\pi f$$

$$n = 60 f$$

When we have more than 2 poles, the two angles aren't the same angle.



$$\omega_{\text{mech}} = \frac{\omega_{\text{elec}}}{2}$$

$$\omega_{\text{mech}} = \frac{\omega_{\text{elec}}}{P}$$

$$\rightarrow n = \frac{60 f}{P}$$

where P = number of pole pairs

$$P = 1 \rightarrow 2 \text{ poles}$$

$$n = 3000 \text{ rpm}$$

$$P = 2 \rightarrow 4 \text{ poles}$$

$$n = 1500 \text{ rpm}$$

$$P = 3 \rightarrow 6 \text{ poles}$$

$$n = 1000 \text{ rpm}$$

$$P = 4 \rightarrow 8 \text{ poles}$$

$$n = 750 \text{ rpm}$$

The speed rotation can't change continuously. So if we are working at a constant frequency we can't change the speed rotation in a continuous way. The only way is to change frequency.

→
 * Dal punto di vista strutturale il motore asincrono è costituito da uno statore nel quale è alloggiato un avvolgimento trifase ed un rotore in cui alloggiato un altro sistema di avvolgimento, che può anche essere non trifase. L'avvolgimento di rotore può infatti essere con un n° di fasi diverso da quello di statore purché venga rispettato il n° di poli dell'avvolgimento di statore.

$$S = \frac{\omega_s - \omega_m}{\omega_s}$$

ABSOLUTE SUP (= scorrimento)

- $\omega_m = 0$ (rotore fermo) : $S = 1$
- $\omega_m = \omega_s$: $S = 0$

$$S\% = \frac{\omega_s - \omega_m}{\omega_s} \cdot 100 \quad \text{RELATIVE SUP}$$

If we have these other hypothesis: in the same direction of the rotating field

- ① $I_r \neq 0$ → The rotor starts to rotate in order to reduce the flux variation $(\frac{d\phi}{dt})$ it tries to catch the field
- ② $\omega_m \neq 0$

It isn't possible to have $\omega_s = \omega_m$ because this would mean that $|\vec{E}_r| = 0$ and then $I_r = 0$.

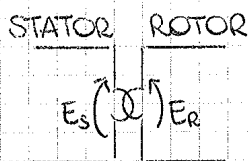
⇒ The slip is really small (in no-load conditions)

Now we want to represent this machine with an equivalent circuit.

$$\begin{array}{l} \text{If } I_r = 0 \\ \omega_m = 0 \end{array} \Rightarrow \begin{array}{l} |\vec{E}_s| = K_s \cdot \phi \cdot \omega_s \\ |\vec{E}_r| = K_r \cdot \phi \cdot \omega_s \end{array} \Rightarrow \frac{|\vec{E}_s|}{|\vec{E}_r|} = \frac{K_s}{K_r} = t$$

↳ The induction machine is working as a transformer

But in the transformer, the amplitude of the flux is varying, while the phase isn't. On the contrary, in the induction machine the amplitude is constant, but since the flux is rotating the machine sees different flux amplitude. Anyway, in both machines $\frac{d\phi}{dt} \neq 0$.



The big difference between the transformer and the induction machine is that in the transformer the primary and the secondary windings are working at the same frequency, while in the induction machine the stator and the rotor aren't.

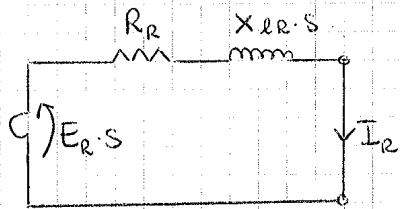
This happens only if:

• $\omega_m = 0$ → $S = 1$ ⇒ $\omega_s = \omega_r$, $f_s = f_r$

If $\omega_m = \omega_s$ → $S = 0$ ⇒ $f_r = 0$ because $\frac{d\phi}{dt} = 0$ (and $\vec{E} = 0$)

⇒ $f_r = s \cdot f_s$ perche' $f_r = \frac{(n_s - n_m)P}{60} = \frac{(n_s - n_m)P}{60} \cdot \frac{n_s}{n_s} = s \cdot \frac{n_s \cdot P}{60} = s \cdot f_s$

= pulsazione elettrica del rotore



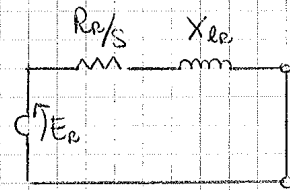
← Trascurando il perdite (s=1)

L'eq. alla maglia di rotore e':

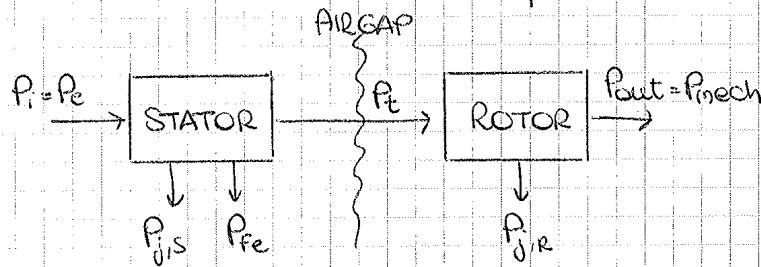
$$E_r \cdot s = (R_r + jX_{lr} \cdot s) \cdot I_r$$

$$E_r = \left(\frac{R_r}{s} + jX_{lr} \right) \cdot I_r$$

what is the physical meaning?



To answer we have to consider the power balance:



$$P_i = P_e = \sqrt{3} V_s I_s \cos \phi_s$$

Nello statore vengono dissipate:

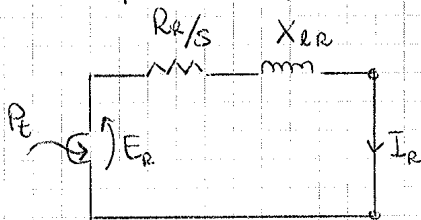
- $P_{j/s} = 3 R_s I_s^2$

- $P_{fe} = \frac{E_s^2}{R_{fe}}$

P_i = input power (la potenza assorbita dallo statore)

$$P_e = P_i - P_{j/s} - P_{fe}$$

P_t = transmitted power (la potenza trasmessa dallo statore al rotore)



la potenza trasmessa viene a ritrovarsi nella resistenza R_r/s :

$$P_t = 3 \cdot \frac{R_r}{s} \cdot I_r^2$$

3-phase

$$P_{j/r} = 3 R_r \cdot I_r^2$$

Il rotore presenta cmq una resistenza reale $R_r \times$ cui nel rotore saranno presenti le perdite Joule:

La differenza tra la potenza trasmessa e le perdite Joule di rotore e' pari alla potenza meccanica convertita P_{out} :

$$P_{out} = P_t - P_{j/r} = 3 \cdot \frac{R_r}{s} I_r^2 - 3 R_r \cdot I_r^2 = 3 \left[R_r \left(\frac{1-s}{s} \right) \right] \cdot I_r^2$$

e' la resistenza equivalente alla potenza meccanica convertita

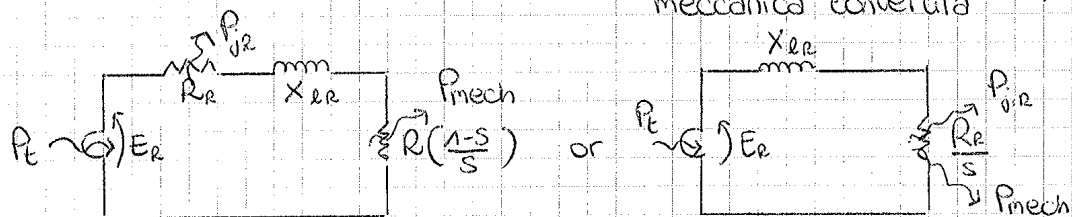
Thanks to this new factor we can understand the meaning of $\frac{R_r}{s}$:

$$\frac{R_r}{s} = R_r + R_r \left(\frac{1-s}{s} \right)$$

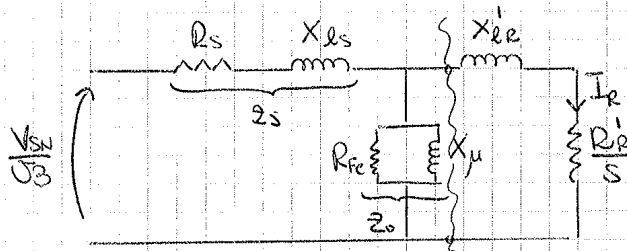
← somma di: <

- resistenza reale di rotore

- " equivalente alla potenza meccanica convertita



MECHANICAL CHARACTERISTIC



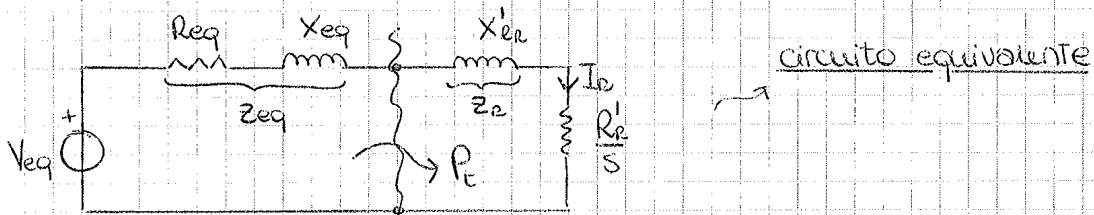
Thevenin equivalent circuit

$$\bar{V}_{eq} = \frac{\bar{V}_s}{\sqrt{3}} \cdot \frac{\bar{z}_0}{\bar{z}_s + \bar{z}_0}$$

TENSIONE EQUIVALENTE
← partitore di tensione

$$\bar{z}_{eq} = \frac{\bar{z}_s \cdot \bar{z}_0}{\bar{z}_s + \bar{z}_0} = R_{eq} + jX_{eq}$$

IMPIEDENZA EQUIVALENTE



$$|I_r| = \frac{|V_{eq}|}{\sqrt{(R_{eq} + \frac{R'_r}{s})^2 + (X_{eq} + X'_{le})^2}}$$

corrente di rotore

$$P_t = 3 \frac{R'_r}{s} \cdot I_r^2 = 3 \frac{R'_r}{s} \cdot \frac{V_{eq}^2}{(R_{eq} + \frac{R'_r}{s})^2 + (X_{eq} + X'_{le})^2}$$

potenza trasmessa da statore a rotore

this is the transmitted power, that is in the air gap where there is the rotating magnetic field

$$T_t = \frac{P_t}{\omega_s} = \frac{3 \frac{R'_r}{s}}{\omega_s} \cdot \frac{V_{eq}^2}{(R_{eq} + \frac{R'_r}{s})^2 + (X_{eq} + X'_{le})^2}$$

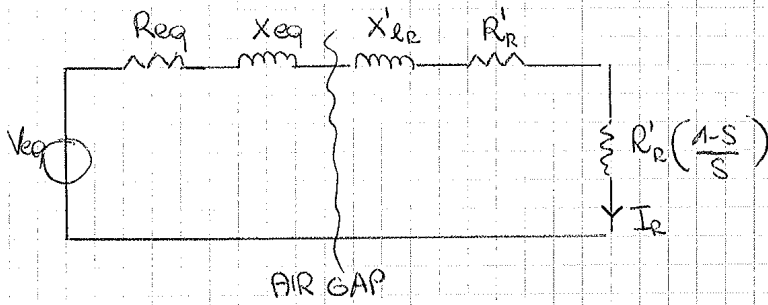
coppia elettromagnetica prodotta al traferro

the mechanical speed is hidden here

But we know that $\omega_s = \frac{\omega_e}{p}$

$$T_t = \frac{3 \frac{R'_r}{s} \cdot p}{\omega_e} \cdot \frac{V_{eq}^2}{(R_{eq} + \frac{R'_r}{s})^2 + (X_{eq} + X'_{le})^2}$$

So if we increase the number of poles we reduce the rotation speed and we increase the torque.



$$P_m = 3 R'_R \left(\frac{1-s}{s} \right) I_r^2$$

$$T_m = \frac{P_m}{\omega_m} = \frac{3 R'_R \left(\frac{1-s}{s} \right) I_r^2}{\omega_m} = \frac{3 R'_R \left(\frac{1-s}{s} \right) I_r^2}{\omega_s (1-s)} = T_t$$

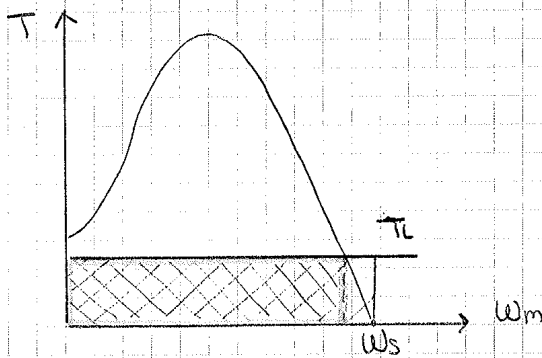
indeed $s = \frac{\omega_s - \omega_m}{\omega_s} \rightarrow \omega_m = \omega_s (1-s)$

È stata dimostrata l'uguaglianza tra coppia elettromagnetica al traferito T_t e coppia meccanica prodotta sul rotore T_m

coppia meccanica

Then the mechanical torque is exactly the transmitted torque.

Not to be confused, I have to consider parameters in the air gap: T_t and rotating magnetic field (ω_s).



$$P_t = T \cdot \omega_s$$

$$P_m = T \cdot \omega_m$$

But we know that:

$$P_t = P_m + P_{j,r}$$

so that:

$$P_t - P_m = P_{j,r}$$

While $T_t = T_m$, $P_t \neq P_m$.

Since $\omega_s = \text{const}$ (is imposed by the supply) if we want to increase P_t we have to increase T .

The slip is very important because it imposes how much power is mechanical and how much power is lost in Joule losses.

We have to understand if the current is a starting current or a short-circuit current. The difference is that:

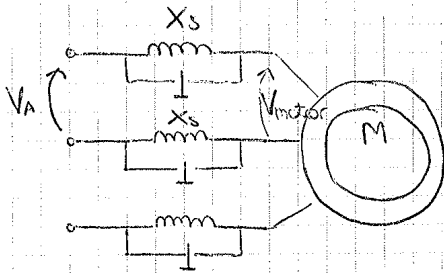
- starting current lasts only a few seconds
- short-circuit current is permanent ← we want to interrupt it because it is dangerous

Ponendo $s=1$ (cioè, $\omega_m = 0$) ottengo le cond. di avviamento:

$$I_s = \frac{V_{eq}}{\sqrt{\left(R_{eq} + \frac{R_r}{s}\right)^2 + \left(X_{eq} + X_{lr}\right)^2}} \quad \textcircled{1} \quad \text{corrente di avviamento}$$

$$T_s = \frac{P_{T,s}}{\omega_s} = \frac{3 \frac{R_r}{s-1} \cdot I_s^2}{\frac{2\pi n_s}{60}} \quad \textcircled{2} \quad \text{coppia di avviamento}$$

How can we reduce the starting current?

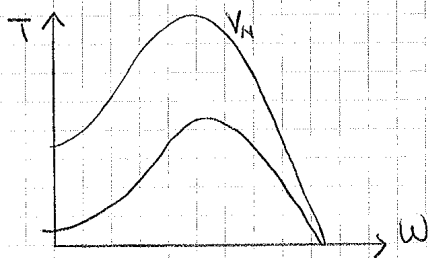


①

If we want to reduce the current, we have to reduce the voltage: $I_s \propto V_{eq}$

Come? Con l'inserimento di reattanze in serie al motore opportunamente cortocircuitate dopo l'avviamento del motore.

Since $T \propto V_{eq}^2$



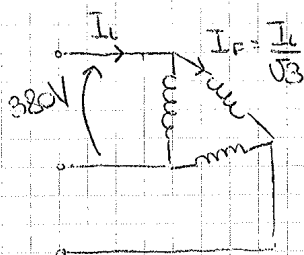
If we are reducing the voltage $V' = \frac{1}{2} V_n$

the new torque will be $T' = \frac{1}{4} T$

so the machine could not start.

There is another way to reduce the starting current.

② The motor must be linked in Δ connection. L'avviamento però avviene con un collegamento a stella.



In questo modo, la tensione applicata a ogni fase viene ridotta di un fattore pari a $\frac{1}{\sqrt{3}}$ con conseguente riduzione della corrente di linea di un fattore 3.

Sostituendo il valore di scorrimento di coppia massima nella relazione della coppia e' possibile ricavare il valore della coppia massima prodotta dal motore:

$$T_x = \frac{3P V_{eq}^2}{\omega_e} \frac{R_r / s_x}{(R_{eq} + \frac{R_r}{s_x})^2 + X_{lt}^2} = \frac{3P V_{eq}^2}{\omega_e} \frac{R_r \cdot \frac{X_{lt}}{R_r}}{(R_{eq} + R_r \cdot \frac{X_{lt}}{R_r})^2 + X_{lt}^2} = \frac{3P V_{eq}^2}{\omega_e} \frac{X_{lt}}{2X_{lt}^2} = \frac{3P V_{eq}^2}{\omega_e 2X_{lt}}$$

If we want to increase the current in the rotor we have to decrease X_{lt} but the problem is that in this way the starting current increases.

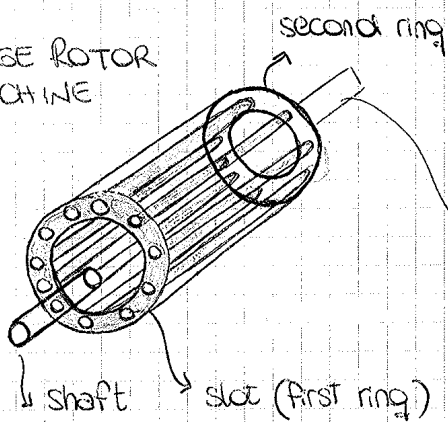
There are 2 kinds of rotor *

- ① wound rotor ← not used anymore (= rotore avvolto)
- ② cage rotor (= rotore a gabbia di scoiattolo)

⊖ high cost (copper)

⊖ it isn't possible to fill completely the slot
filling factor is defined as $k_f = \frac{\text{Area}}{\text{Area}}$

② CAGE ROTOR MACHINE



The material ($T_{F,Al} < T_{F,Cu}$)
The Al is in slots.

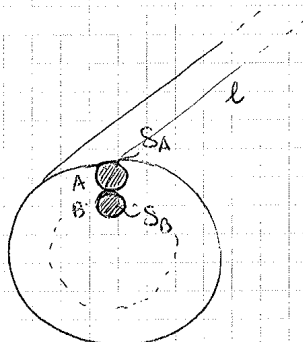
I rotor a gabbia di scoiattolo hanno l'avvolgimento di rotore costituito da un sistema di sbarre di rame o di alluminio collocate nelle cave di rotore, e collegate sulle due estremita' frontali a due anelli saldati dello stesso materiale.

In particolare, la costruzione degli avvolgimenti di rotore in Al avviene mediante un processo di pressofusione che realizza contemporaneamente sia le sbarre che gli anelli.

Since the two rings are linked they're in short-circuit.

⊕ low cost

⊕ high robustness (we haven't insulation materials so we haven't thermal problems → we will burn the stator, but never the rotor.)



- cage A
- cage B

⇒ hyp: $S_A = S_B$

A doppia gabbia

The external dimensions of the machines are all the same, so that you can change a motor with another without any problems.

In order to buy a machine you have only to check the shape of the rotor slots because they define $\frac{I_s}{I_N}$, $\frac{T_m}{T_N}$ and $\frac{I_s}{I_N}$.

solo a macchine con rotore avvolto

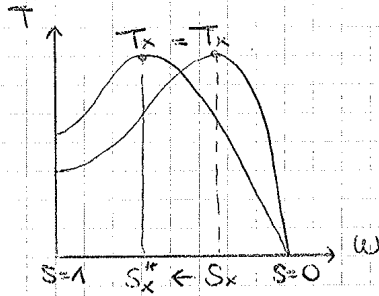
① WOUND ROTOR MACHINE

⊛

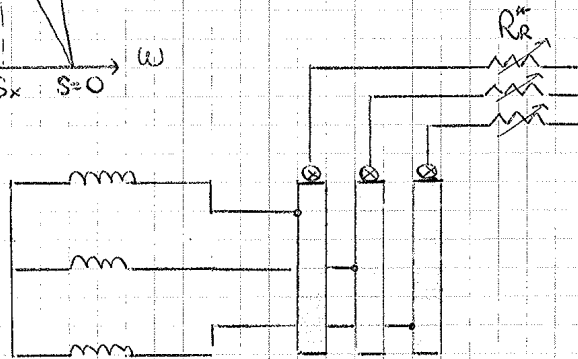
REGOLAZIONE DELLA VELOCITA'

Ⓐ REGOLAZIONE REOSTATICA SULLA RESISTENZA DI ROTORE

If we increase R_r , the peak of the torque goes on the left.



$$s_x \approx \frac{R_r}{X_{et}}$$

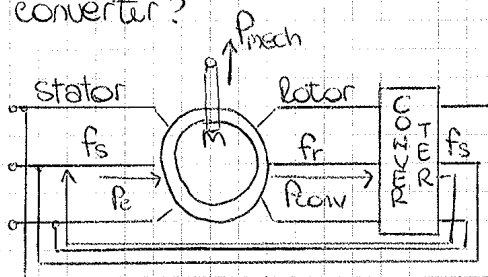


⊛ L'avvolgimento di rotore è realizzato mediante un avvolgimento del tutto simile a quello di statore: possono presentare n° di fasi diverso, ma usual n° di poli. Gli avvolgimenti di rotore fanno capo a 3 anelli rotanti che permettono l'accesso mediante spazzole striscianti verso eventuali circuiti elettrici esterni. In particolare, i 3 anelli consentono l'inserimento di opportune R in serie all'avvolgimento di rotore durante la fase di avviamento, riducendo I_s e aumentando T_s . Poi vengono corto circuitati.

xche' $P_m \downarrow$ e $P_{gr} \uparrow$

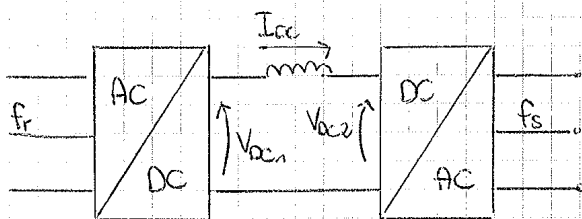
But in this way we reduce the efficiency and in addition it's not possible to regulate the speed from 0 to a value but only in a specific range.

Since we have so many power losses, why don't we recover them through a power converter?



$$P_e = P_{gr} + P_{mech} + P_{conv}$$

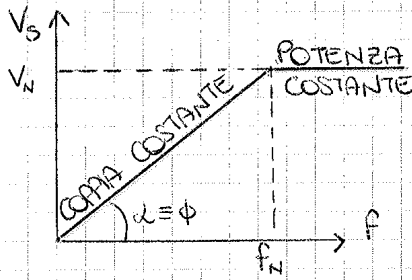
The converter is used to convert the rotor frequency into the stator one.



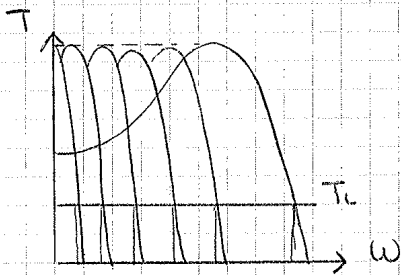
$$V_{dc1} > V_{dc2}$$

⊖ the speed regulation isn't so fast

Then, if we want to change f_e , we have to modify V_s too.



we have to follow this straight line if we want to be sure to leave the same flux.



- torque characteristic of the machine at $f = 50 \text{ Hz}$
- torque characteristic if we change f and V_s following the straight line

In this way we solve the problem of the starting conditions.

If at the beginning we have a very low f (\rightarrow very low ω) we have the starting torque equal to the pick torque. Then by changing the f we can move to the maximum speed with the pick torque.

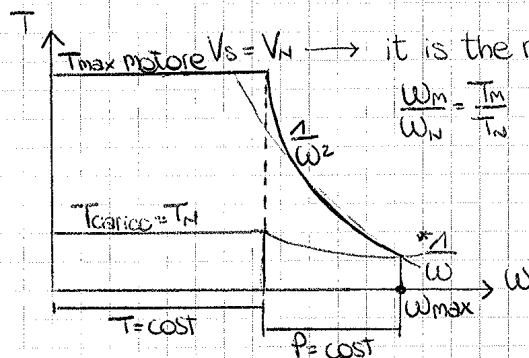
Since we know:

$$T_x = \frac{3p}{\omega_e} \frac{V_{eq}^2}{2X_{lt}}$$

we can put in evidence the f :

$$T_x = \frac{3p}{(2\pi f_e)^2} \frac{V_{eq}^2}{2 \cdot (2\pi f_e L_{lt})} = \frac{3p}{8\pi^2 L_{lt}} \left(\frac{V_{eq}^2}{f_e^2} \right)$$

A tensione di alimentazione costante: $T_x \propto \frac{1}{f_e^2}$



If we increase the speed (\rightarrow we can't also increase V_s) we have a constant power load (a decreasing torque)

we can have at maximum ω_{max} (above it, we need a decreasing power)

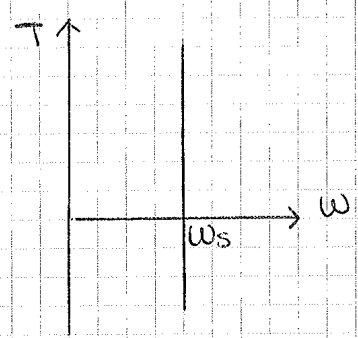
* Abbiamo ipotizzato $T_{carico} \propto \frac{1}{\omega}$ (potenza costante)

$\omega_{max} \rightarrow f_{max}$: f max di regolazione a potenza costante

SYNCHRONOUS MACHINE

The machine can produce a torque only if the rotor is rotating at the same speed of the rotating magnetic field. (opposite in comparison with the induction machine)

The stator is equal to that of the induction machine.



La velocità di sincronismo dipende da:

- frequenza di alimentazione
- n° di poli della macchina

$$\omega_s = \frac{2\pi f_e}{p}$$

Le macchine sincrone sono macchine perfettamente reversibili e sono in grado, quindi, di funzionare sia da generatore (in tal caso, e' detta ALTERNATORE) sia da motore. L'applicazione + comune delle macchine sincrone e' a produzione di energia elettrica nelle centrali.

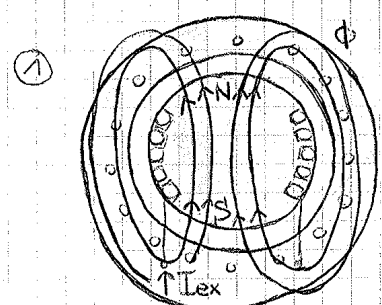
The rotors can be subdivided into 2 types:

- ① ISOTROPIC (the air gap is constant)
- ② ANISOTROPIC (" " isn't constant)

Dal punto di vista costruttivo:

- statore in corrente alternata *
- rotore alimentato in corrente continua

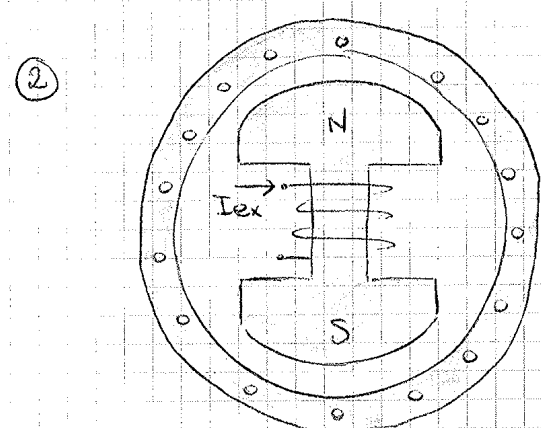
* e' sede di un sistema trifase di avvolgimenti



The rotor winding is only one.

$I_{ex} = I_{dc} \rightarrow$ constant flux
excitation

→ vengono realizzati con un n° ridotto di coppie polari (1-2 al max) perché accoppiate a turbine veloci.



$$n = \frac{60 f_e}{p} = \frac{3000}{p} \quad 50 \text{ Hz}$$

→ vengono realizzate con un elevato n° di coppie polari perché accoppiate a turbine più lente.

It is used in low-speed generators and of big dimensions (indeed, if p is high, n is low)

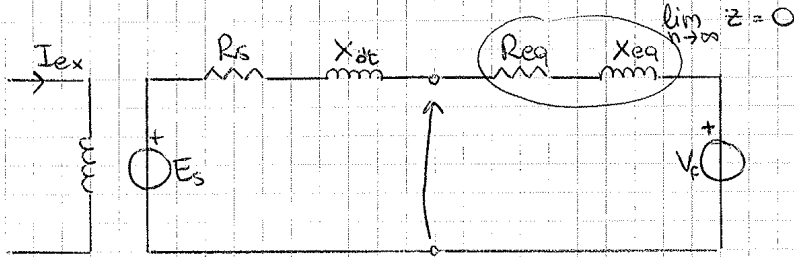
We will study only the isotropic machine, in linear conditions. (no saturazione)
The equations we'll find are valid for both.

The synchronous generators are inside the power stations.

All the power stations are linked together through the transmission line.

So from one generator you can see all the power stations around Europe.

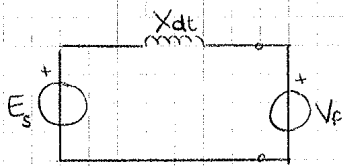
This means that all the generators are in parallel (\rightarrow all the impedances are in parallel) and so the equivalent impedance goes to zero: for the first time we can consider an ideal load.



with n = number of generators in parallel

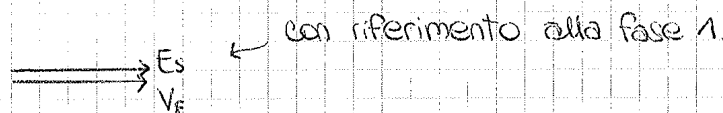
Even if our generator is huge, it is small if compared with the total system: this means we can't change the voltage because it is imposed by the total system.

In term of voltage drop we can neglect R_s (not in term of power) in comparison with the reactance.



\rightarrow Ipotesi di avere la macchina sincrona in // alla rete di alimentazione e che la terna delle tensioni di fase della macchina risulti uguale in modulo e fase alla terna delle tensioni di fase della rete di alimentazione.

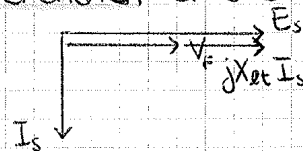
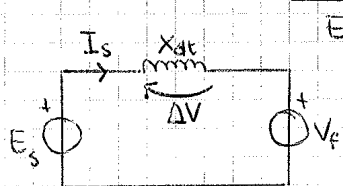
$$\vec{E}_s = \vec{V}_f$$



This means that the current is zero. (perche' $E_s - I_s j X_{dt} = V_f$)

We can control E_s by changing the excitation current ($\rightarrow \Phi_{ex}$)

If we increase the flux: $\Phi_{ex}^* > \Phi_{ex}$ \rightarrow over-excitation of the machine



In queste condizioni, la corrente prodotta dalla macchina e': $I_s = \frac{E_s - V_f}{j X_{dt}}$ con $E_s > V_f$

The current will be 90° in delay in comparison with V_f

We are producing only reactive power: $Q = V_f I_s$ di tipo induttivo

Since the current is in delay of 90° we are producing an inductive power.

Since Φ_R is due to the stator, it is in phase with I_s

We can find $\bar{\Phi}_{ex} = \bar{\Phi}_{rot} - \bar{\Phi}_R$

Once we know $\bar{\Phi}_{ex}$ we know the rotor position.

\bar{E}_r and $jX_{dt}I$ have the same direction: this means they're linked.

But for the reactance we use the load convention, while for the electromotive force the generator one. So we have to change the sign..

$$E_r = jX_r \cdot I_r$$

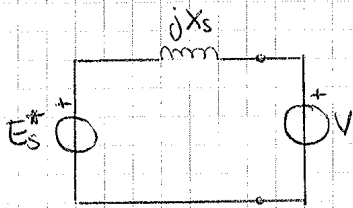
$$\Rightarrow X_s = X_{dt} + X_r \quad \text{synchronous reactance}$$

$$X_{dt} \ll X_r$$

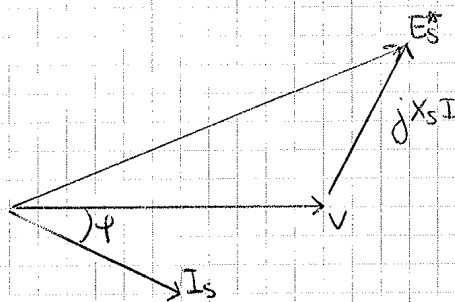
Then the previous equivalent circuit in which we considered only X_{dt} was wrong.

$$\frac{X_s \cdot I_N}{V_N} \cdot 100 > 100\%$$

The new equivalent circuit is the following one:

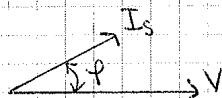


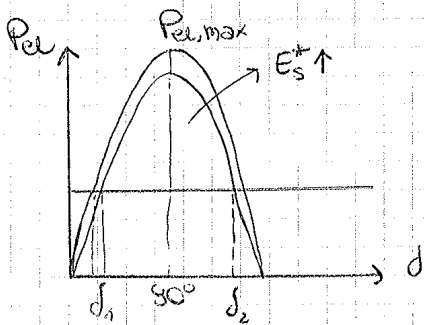
The new phasor diagram:



If we have a capacitive current

the excitation flux is increased instead of being reduced





$E_s^* = \text{constant} \rightarrow \text{requested power} \propto \delta$

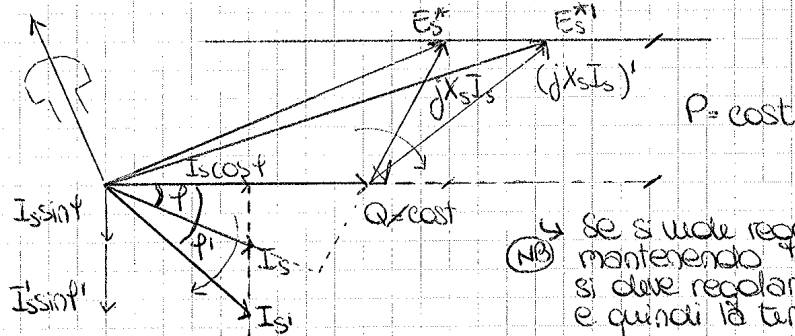
If we increase E_s^* , we decrease δ because we have a new working point.

But this doesn't mean that we change the produced electrical power 'cause it is constant.

La macchina viene fatta funzionare con $\delta \cong 45^\circ$

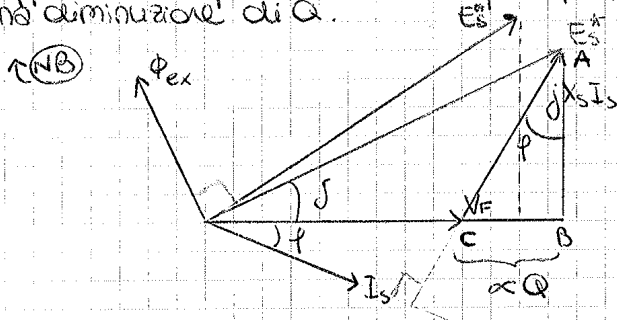
Since $P_{el} = 3V_f I_s \cos \rho$ and $3V_f$ is constant, P_{el} depends only on $I_s \cos \rho$, which is constant. (look at the sketch below)

AB can change only if we change the transmitted torque (but we are working with constant torque). This means that AB is constant and can move only on the right and on the left.



Se si vuole regolare la potenza reattiva Q mantenendo costante la potenza attiva P, si deve regolare la corrente di eccitazione, e quindi la tensione E_s^*

Perche' $I_{exc} = \text{cost}$, rimane costante anche l'ampiezza del vettore E_s^* . L'aumento della coppia e della potenza attiva si accompagna a una diminuzione di Q.



Potenza reattiva erogata:

$$Q_e = 3V_f I_s \sin \rho$$

$$|CB| = X_s I_s \sin \rho$$

$$I_s \sin \rho = \frac{|CB|}{X_s}$$

$$|CB| = E_s^* \cos \delta - V_f$$

$$Q_e = 3V_f \left(\frac{E_s^* \cos \delta - V_f}{X_s} \right)$$

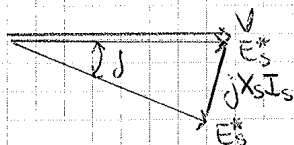
This means that by changing E_s^* we are only changing the reactive power. *

If we disconnect the machine from the turbine, the rotor will try to move in the opposite direction:

* Perche' cambiando E_s^* , cambia anche, in modo contrario, l'angolo di carico δ ($P_{el} = \text{const}$)

Quindi:

- regolazione di coppia \rightarrow regolazione della potenza attiva
- regolazione dell'eccitazione \rightarrow regolazione della potenza reattiva



- Abbiamo eccitazione separata in macchine grandi, quando vogliamo un ampio range di velocità (quando la macchina lavora in constant power mode).
- Quando la macchina lavora in constant torque mode si usano i magneti permanenti.

(permanent magnets)

Quando il flusso è costante, L_{ecc} non ha effetto,

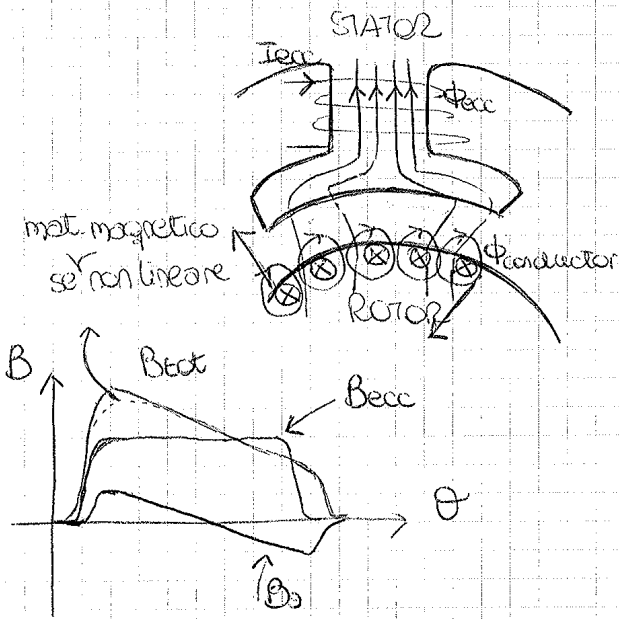
k : come sono collegati gli avvolgimenti dipende dalle caratteristiche geometriche dell'armatura

Solo nei permanent magnets abbiamo $\phi(t) = \text{cost}$ quindi possiamo chiamare $k\phi = k_v \Rightarrow k_{\omega} = k_T$
fem torque

k_v : si trova dal no-load test

k_T : " " " stand-still test ($V < V_N$ e $i_a = i_N$)

$\Rightarrow k_v$ sono diversi nelle misure ($k_T < k_v$)



Dove le espansioni polari finiscono, rimane solo il Φ dei conduttori

EQ. MECCANICA

se non accelera/decelera $\int \frac{dw}{dt} = 0$ inertia term

$\beta \omega$: friction term (\rightarrow calore)

$$\frac{1}{\tau_a} = \frac{\mathcal{J}R_a + \beta L_a}{\mathcal{J}L_a} = \frac{R_a}{L_a} + \frac{\beta}{\mathcal{J}} = \frac{R_a}{L_a} \left(1 + \frac{\beta L_a}{\mathcal{J} R_a}\right)$$

$$\tau_a = \frac{L_a}{R_a} \frac{1}{1 + \frac{\beta L_a}{\mathcal{J} R_a}}$$

$$\frac{1}{\tau_a \tau_m} = \frac{K_v K_T + \beta R_a}{\mathcal{J} L_a} = \frac{K_v K_T}{\mathcal{J} L_a} \left(1 + \frac{\beta R_a}{K_v K_T}\right)$$

$$\tau_m = \frac{1}{\tau_a} \frac{\mathcal{J} L_a}{K_v K_T} \frac{1}{1 + \frac{\beta R_a}{K_v K_T}} = \frac{R_a}{L_a} \frac{\mathcal{J} L_a}{K_v K_T} \frac{1 + \frac{\beta L_a}{\mathcal{J} R_a}}{1 + \frac{\beta R_a}{K_v K_T}} = \frac{R_a \mathcal{J}}{K_v K_T} \frac{\frac{\mathcal{J} R_a + \beta L_a}{\mathcal{J} R_a}}{\frac{K_v K_T + \beta R_a}{K_v K_T}} = \frac{\mathcal{J} R_a + \beta L_a}{K_v K_T + \beta R_a}$$

$$s_{1,2} = (-\zeta \pm \sqrt{\zeta^2 - 1}) \cdot \omega_0$$

A seconda di ζ (damping factor) i poli possono essere reali ($\zeta > 1$), uguali e pari a 0 ($\zeta = 1$) o complessi ($\zeta < 1$)

$$\zeta = \frac{1}{2} \sqrt{\frac{\tau_m}{\tau_a}} = \frac{1}{2} \sqrt{\frac{\mathcal{J} R_a}{K_v K_T} \cdot \frac{R_a}{L_a}} = \frac{1}{2} R_a \sqrt{\frac{\mathcal{J}}{K_v K_T L_a}}$$

Losses

le macchine + grandi oscillano di + ($\zeta < 1$), quelle + piccole + stabili ($\zeta > 1$)

Ora consideriamo:

- input: la coppia resistente
- output: la velocità di rotazione

→ Il denominatore è lo stesso di prima (→ stessi poli), ma cambia il numeratore (→ cambiano gli zeri)

trascurare $\beta \Leftrightarrow$ trascurare le perdite

Dal punto di vista:

- del rendimento: non vogliamo perdite
- del controllo: vogliamo perdite x che stabilizzano il sistema

GI = generator of impulses

AR = regulation amplifier

$$G_R(s) = K_R + K_{RI} \frac{1}{s} = \frac{K_R s + K_{RI}}{s} = K_R \frac{s \frac{K_{RI}}{K_R}}{1 + s \frac{K_{RI}}{K_R}} \rightarrow \tau_i = \frac{K_R}{K_{RI}}$$

DC MOTOR CONVERTERS

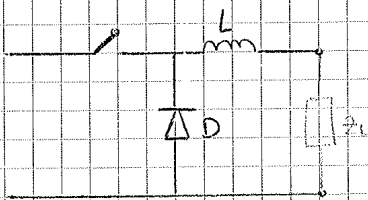
Chopper: DC/DC (1)

Bridge with thyristor: AC/DC (2)

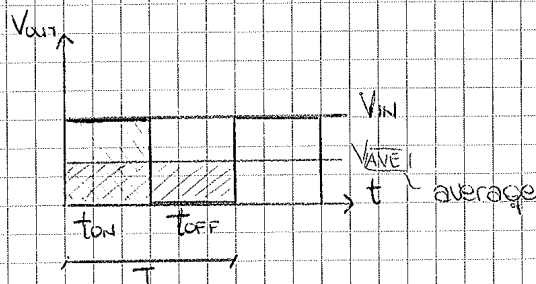
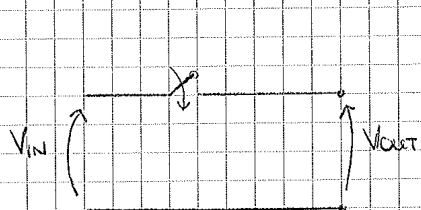
(1) BUCK CONVERTER

3 components:

- switch / transistor
- diode
- inductance



step-down converter (= convertitore riduttore)

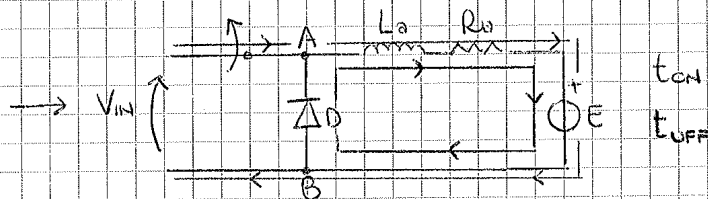
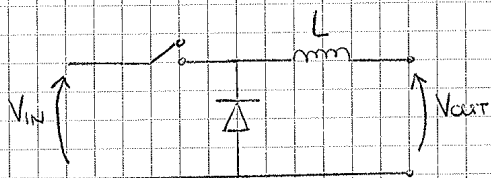


ON: closed
OFF: opened

DUTY CYCLE

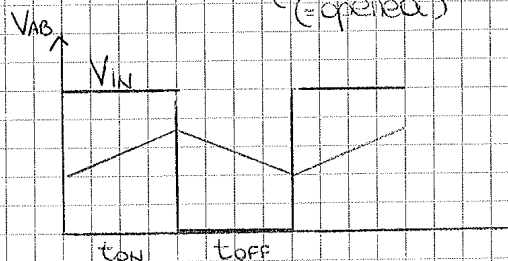
$$\alpha = \frac{t_{ON}}{T} \quad 0 \leq \alpha \leq 1$$

$$V_{AVE} \cdot T = V_{IN} \cdot t_{ON} \quad \rightarrow \quad V_{AVE} = V_{IN} \frac{t_{ON}}{T} = V_{IN} \cdot \alpha$$



It works only when $V_{IN} > V_{OUT}$

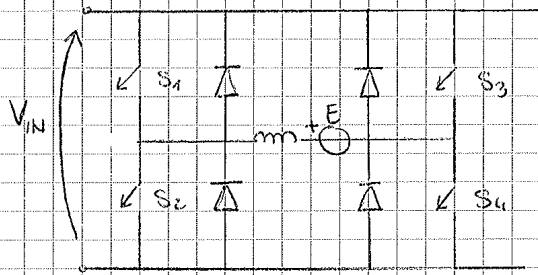
when the switch is off we are supplying the machine (= opened)



-- current

we can't change the sign of the voltage and of the current.

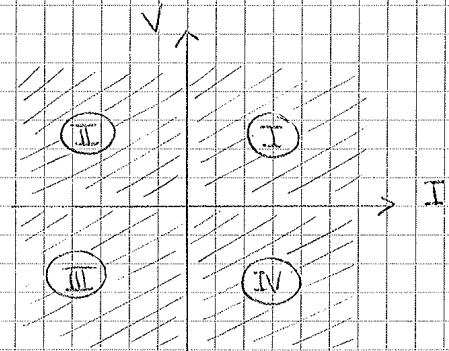
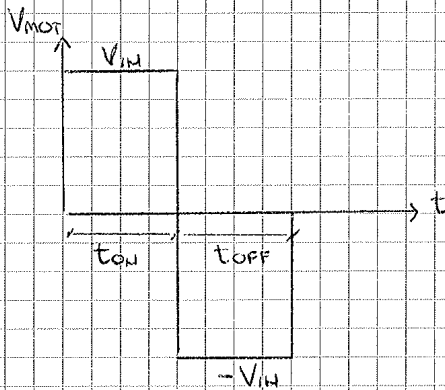
The following converter can work in 4 quadrants.
 CHOPPER A 4 QUADRANTI



$$\Delta V = V_{IN} - (-\bar{V}_{IN}) = 2 V_{IN}$$

$$t_{ON} > t_{OFF} : V_{MOT(AVER)} > 0$$

$$t_{ON} < t_{OFF} : V_{MOT(AVER)} < 0$$



Definizioni:

CONVERTITTORE = dispositivo elettronico che cambia la tensione di una sorgente di potenza elettrica.

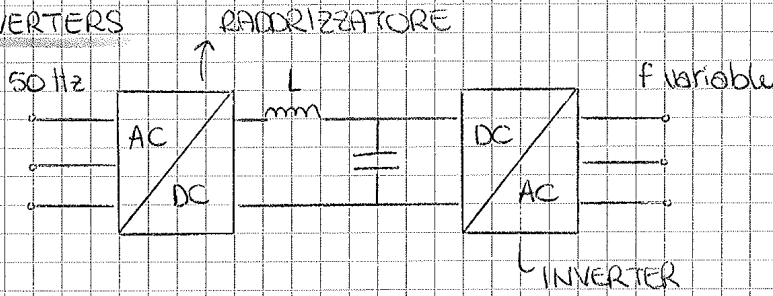
CONVERTITTORE DC-DC = circuito che converte una sorgente di corrente continua da una tensione ad un'altra.

CHOPPER / FRAZIONATORE ELETTRONICO = convertitore statico che permette di regolare il valore della tensione in un circuito elettrico di potenza in corrente continua.

Viene usato x trasformare una tensione continua in una tensione continua di diverso valore, svolgendo cioè una funzione simile a quella del trasformatore nei circuiti a corrente alternata.

La regolazione viene effettuata chiudendo e aprendo in sequenza un interruttore elettronico allo scopo di fornire energia al carico soltanto durante l'intervallo di chiusura dello stesso, detto TEMPO DI CONDUZIONE, e di lasciarlo disconnesso dal generatore durante l'intervallo di apertura, detto TEMPO DI INTERRUZIONE. La somma del tempo di conduzione e del tempo di interruzione è detta PERIODO DI COMMUTAZIONE e il suo inverso

INVERTERS



2 kinds: adatti x potenze che vanno dal kW alle centinaia di kW

- VSI (= Voltage Source Inverter): the most used
 - CSI (= Current Source Inverter): the less used
- adatti x potenze dell'ordine dei MW

VSI:

la forma d'onda della tensione alternata imposta al motore e' rettangolare di ampiezza regolabile.

1. SIX-STEP or SQUARE WAVE

... e' costituito da impulsi di ampiezza costante e frequenza modulati in durata

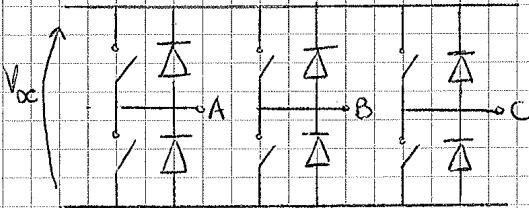
2. PWM (= Pulse Width Modulation): the most used

= Modulazione di larghezza di impulso

SIX-STEP

The inverter can be considered as an evolution of a two quadrant's DC/DC converter:

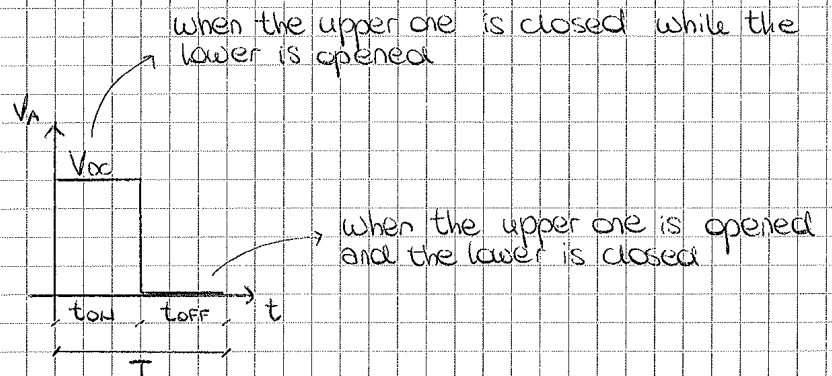
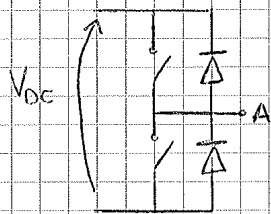
Questo e' l'inverter ibrido motore ed e' costituito da 3 gambe in ognuna delle quali sono disposti due tasti funzionanti in modo complementare, che fornisce una terna simmetrica di tensioni concatenate in onda quadra e ne regola la f. I 6 tasti, di cui 3 (uno x gamba) sono attivi ad ogni istante, sono costituiti da transistori, con in anti-parallelo dei diodi.



The input is the voltage coming from the bus.

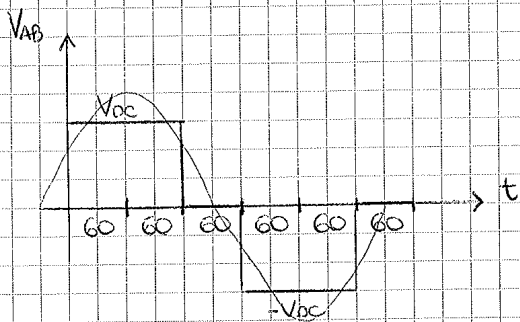
It isn't possible to create a sinusoidal wave, but an alternati one yes.

If we consider one leg:



We obtain a square wave.

If we close both switches we have a short circuit.



This is the line voltage between two terminals.

$$\rightarrow V_{AB} = V_A - V_B$$

= root mean square
= valore efficace

this is what we can measure *

$$V_{1^{st} RMS} = V_{RMS} \cdot 0,955$$

→ So the difference between $V_{1^{st} RMS}$ and V_{RMS} is small.

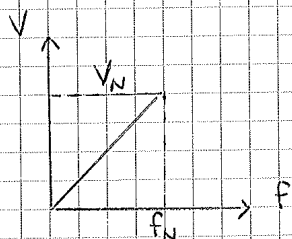
first harmonic

The harmonic content is the same both in the phase and in the line voltage.

* But we have to supply the machine with $V_{1^{st} RMS}$ so we need $V_{RMS} = \frac{V_{1^{st} RMS}}{0,955}$

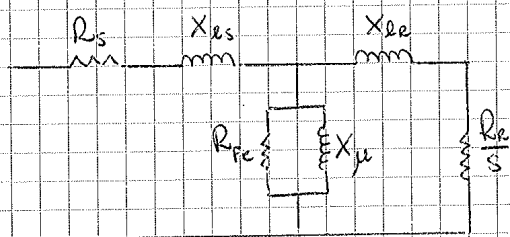
Anyway the error would be very small (but this is true only for this wave form which has a ratio ~ 1)

All these voltages depend on V_{oc} which is the voltage supplied by the ^{DC} bus.



When we change the frequency we have to change also the voltage of the bus, because they're directly proportional. and the output voltage (which depends on v. bus)
So it is very important to have the possibility of changing the bus voltage

The steady-state model of the induction machine is the following one:



$$X_{es} = 2\pi f \cdot L_{es}$$

$$X_{er} = 2\pi f \cdot L_{er}$$

$$X_{\mu} = 2\pi f \cdot L_{\mu}$$

↳ If we increase f , we also increase the reactance

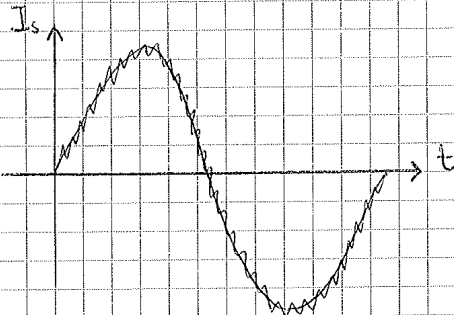
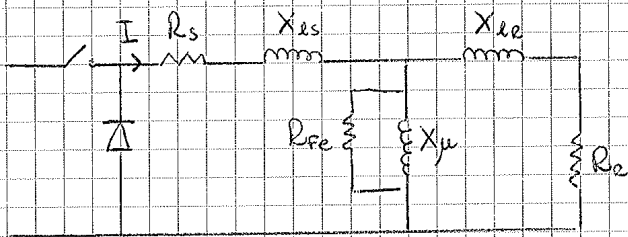
$$s = \frac{\omega_s - \omega_m}{\omega_s} \quad \rightsquigarrow \text{slip of the } 1^{st} \text{ harmonic}$$

We can have only the 5th, 7th, 11th, 13th... harmonics (non i multipli di 3)

$$s_{5^a} = \frac{5\omega_s - \omega_m}{5\omega_s}$$

$$s_{n^a} = \frac{n \cdot \omega_s - \omega_m}{n \cdot \omega_s}$$

$$s_{7^a} = \frac{7\omega_s - \omega_m}{7\omega_s}$$



- ripple due to the commutation

Core losses are more or less the same
 Core losses we have in a sinusoidal supply.

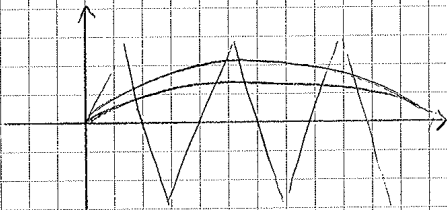
But we have an increase of the Eddy current losses.

Six-step : \uparrow core losses, the same iron losses

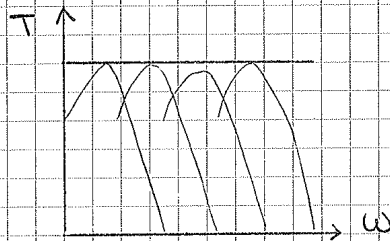
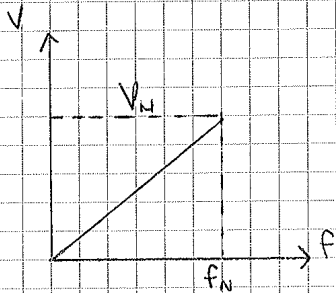
PWM : \uparrow iron losses, the same core losses

If we increase the switching frequency we improve the wave form of the current which is always more similar to the sinusoidal one.

But since $I_e = c \frac{d\psi}{dt}$ we have to reduce $\frac{d\psi}{dt}$ if we don't want to damage the machine, because our model isn't valid for high frequency



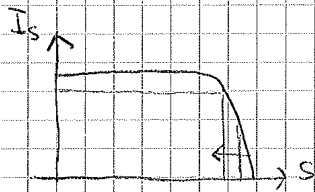
Changing the modulation : wave form
 we can change the frequency and the amplitude so we don't need a variable bus in the AC/DC converter (lower cost)



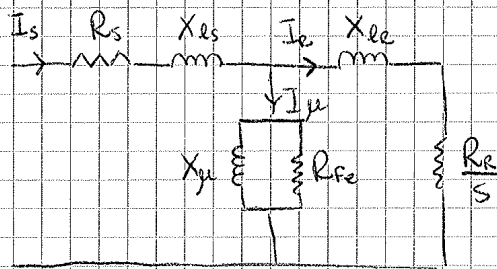
$$T = k_t \phi_{ex} I_a \quad (\text{DC machine})$$

If I increase the current the torque will increase in a proportional way.

But in this case (induction machine), if we want to increase the torque we have to accept a higher slip (\rightarrow higher current)



But the current which increases is not the torque current, but the stator one.



$$T = k \cdot I_r \quad \text{but we can't measure } I_r, \text{ but only } I_s$$

But this model is valid only in steady-state conditions. Now we have to use the dynamic model

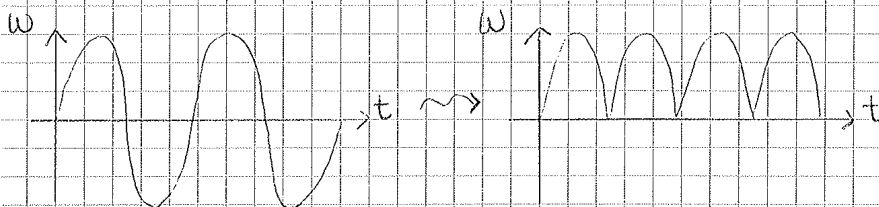
$$T = k_t \cdot \underset{\substack{\uparrow \\ \phi_{ex}}}{I_a^s} \cdot \underset{\substack{\uparrow \\ I_a}}{I_\phi^s}$$

The only quantities we can measure are: V_s, I_s, ω_m

Definizioni:

INVERTER = apparato elettronico di ingresso/uscita in grado di convertire una corrente continua in ingresso in una corrente alternata in uscita. E' funzionalmente il dispositivo antitetico rispetto a un rettificatore o raddrizzatore di corrente. $\otimes \rightarrow$

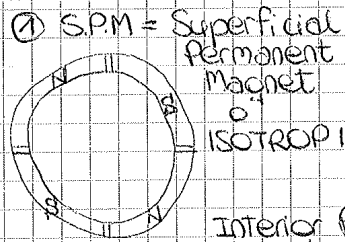
RETTIFICATORE / RADDRIZZATORE DI CORRENTE = dispositivo che serve a raddrizzare un segnale alternato, e quindi a trasformarlo in un segnale non negativo. Il raddrizzatore, collegato ad altri componenti, e' usato \times trasformare la corrente alternata in corrente continua.



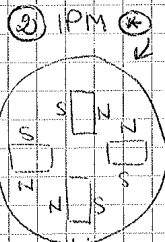
BRUSHLESS MACHINE

= e' una macchina sincrona a magneti permanenti in cui il Φ di eccitazione e' generato da una serie di magneti disposti sul rotore

(*) (*)



① S.P.M = Superficial Permanent Magnet o ISOTROPI



② IPM (*)

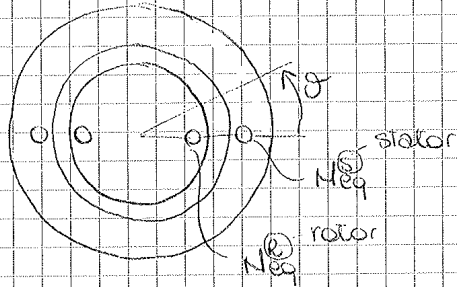
Interior Permanent Magnet (magneti inseriti o annegati) o ANISOTROPI

- ⊕ data la costante presenza del Φ al traferro, vi e' immediata disponibilita' di coppia motrice
- ⊕ a parita' di coppia disponibile all'albero, motori leggeri, compatti e con basso momento di inerzia rotorico

⊖ il problema principale e' la presenza di componenti armoniche di coppia, soprattutto a basse velocita'.

l'induzione al traferro e' sinusoidale

- stator: $B(\theta) = B_x \cos \theta$
- rotor: $I_r(\theta) = I_x \cos(\theta \pm \gamma)$



- un motore brushless e' costituito da uno statore e un rotore, entrambi in mat. ferro-magnetico laminato:
- statore: sede degli avvolgimenti trifase
- rotore: " dei magneti permanenti responsabili del Φ di eccitazione
- li sono quindi 2 sorgenti di F.M.M.:
- rotore (sempre) • statore (se alimentato)

$$F = B \cdot (L) \cdot i = B_x \cos \theta \cdot I_x \cos(\theta \pm \gamma) \cdot L = B_x \cdot I_x \cdot L \cdot \cos \theta \cos(\theta \pm \gamma)$$

length of the conductor

Lo statore viene alimentato da una terna di I sinusoidali sfasati fra loro di 120°

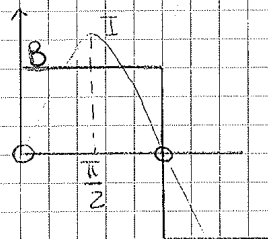
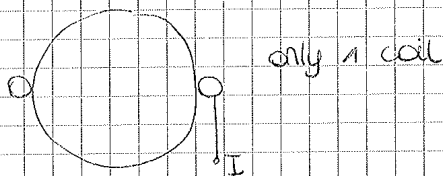
$$F_m = B_x \cdot I_x \cdot L \cdot \cos \gamma$$

medium

We have the maximum force when $\gamma = 0$

But when we have the maximum force we also have the maximum torque.

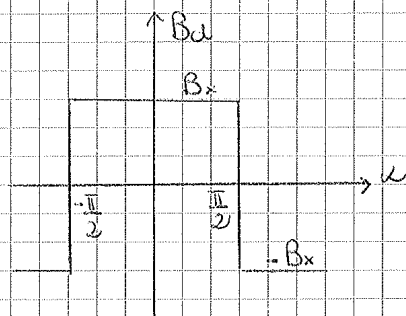
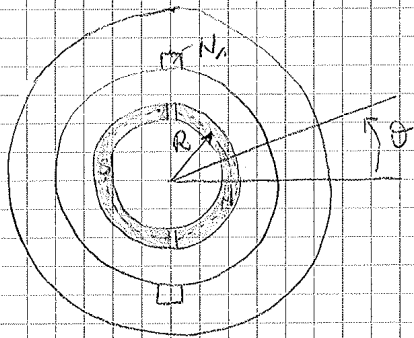
The angle between I and MMF is always 90°.



$$F = B_x \cdot FMM_x \cdot L \cdot \sin \gamma$$

The force is maximum when $\sin \gamma = 1$ ($\gamma = 90^\circ$)

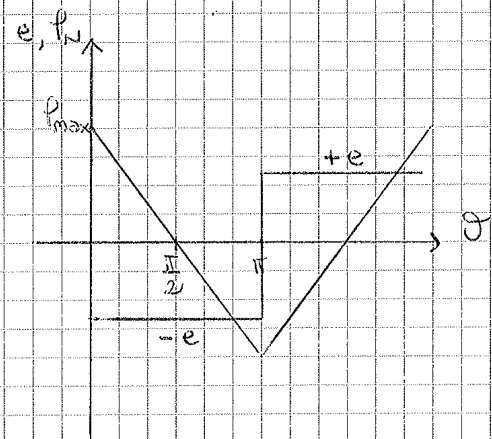
(*) (*) I motori sinusoidali si suddividono in:



$$f_{max} = N_1 \int_{-\pi/2}^{\pi/2} B(\alpha) \cdot R \, d\alpha = N_1 \int_{-\pi/2}^{\pi/2} B_x \, R \, d\alpha = B_x N_1 R \ell \pi$$

$$f_N = f_{max} \left[1 - \frac{\theta}{\frac{\pi}{2}} \right] \begin{cases} \text{if } \theta = 0 \Rightarrow f_N = f_{max} \\ \text{if } \theta = \frac{\pi}{2} \Rightarrow f_N = 0 \end{cases}$$

$$= f_{N_x}(\theta) = B_x N_1 R \ell \pi \left[1 - \frac{\theta}{\frac{\pi}{2}} \right]$$



$$e = + \frac{df}{dt} = \frac{df}{d\theta} \frac{d\theta}{dt} = \frac{df}{d\theta} \cdot \omega_m$$

$$e = - B_x N_1 R \ell \pi \cdot \frac{2}{\pi} \cdot \omega_m$$

