NUMERO: 1167

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A P P U N T I

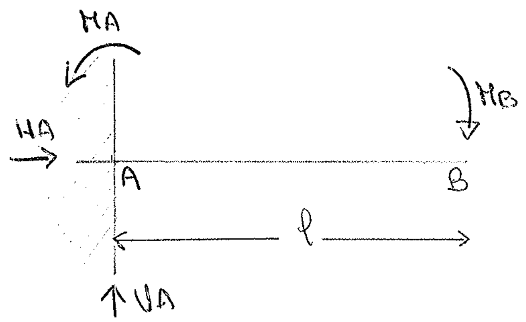
STUDENTE: Borgognone

MATERIA: Scienza delle Costruzioni + Temi + Eserc.

Prof. Valente

CORSO DI LAUREA INGEGNERIA EDILE - ESERCIZI DI SCIENZA DELLE COSTRUZIONI

ESERCIZIO 1 - MENSOLA (17/03/2014)



M_A, V_A e H_A sono le REAZIONI VINCOLARI

La TRAVE A MENSOLA si trova in equilibrio sotto queste condizioni:

$\rightarrow)$ $H_A = 0$

$\uparrow)$ $V_A = 0$

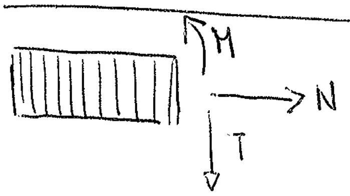
$\curvearrowleft A)$ $M_A = M_B$

N.B.) $\rightarrow)$ = equilibrio alla traslazione orizzontale

$\uparrow)$ = " " " verticale

$\curvearrowleft)$ = equilibrio alla rotazione attorno a un punto noto

CONVENZIONE PER LA RAPPRESENTAZIONE DEI DIAGRAMMI



- N: SFORZO NORMALE

  N > 0 se risulta essere uno sforzo di trazione

- T SFORZO DI TAGLIO

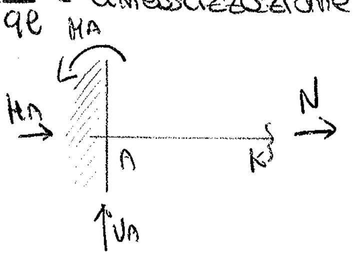
  T > 0 se tende a far ruotare in senso orario, il "concio" di trave sezionato.

- M MOMENTO FLETTENTE

  M > 0 dove tende le fibre. M si rappresenta in prossimità delle fibre tese

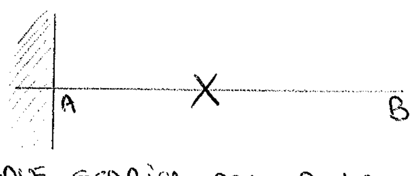
(N) DIAGRAMMA SFORZO NORMALE

$\frac{N}{ql}$ = linearizzazione del diagramma.



$\rightarrow)$ $N - H_A = 0$

$N = H_A = 0$



TRAVE SCARICA DAL PUNTO DI VISTA DELLO SFORZO NORMALE

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(N) SFORZO NORMALE

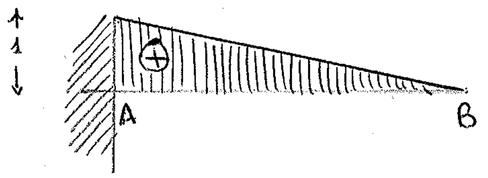
$\rightarrow)\ N + H_A = 0 \qquad N = -H_A = 0$

$\dfrac{N}{q\ell}$

(T) SFORZO DI TAGLIO

$\uparrow)\ V_A - T - qz = 0 \qquad -T = qz - V_A \;\therefore\; T = V_A - qz = q\ell - qz = q(\ell - z)$ $\begin{cases} z=0 & T = q\ell \\ z=\ell & T=0. \end{cases}$

Lo sforzo di taglio risulta lineare lungo la trave



$\dfrac{T}{q\ell}$ (linearizzazione diagrammi)

N.B. Per convenzione si è deciso di mettere lo sforzo tagliante positivo sopra l'asta della trave.

(M) MOMENTO FLETTENTE

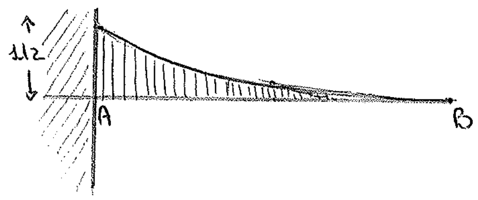
$\curvearrowleft H)\ -m + qz\left(\frac{z}{2}\right) + M_A - q\ell(z) = 0$

$-m + qz\left(\frac{z}{2}\right) + \frac{q\ell^2}{2} - q\ell(z) = 0 \qquad -m = -qz\left(\frac{z}{2}\right) - \frac{q\ell^2}{2} + q\ell(z)$

$m = qz\left(\frac{z}{2}\right) + \frac{q\ell^2}{2} - q\ell(z) \qquad m = \frac{q}{2}\left(z^2 - 2\ell z + \ell^2\right) \qquad m = \frac{q}{2}(\ell - z)^2$

$m(z=0) \Rightarrow \dfrac{q\ell^2}{2} \qquad m(z=\ell) \Rightarrow m=0 \qquad m\left(z=\frac{\ell}{2}\right) = \dfrac{q\ell^2}{8}$

$\dfrac{M}{q\ell^2}$



È utile ricordare che il taglio è la derivata prima del momento, quindi si ha che:

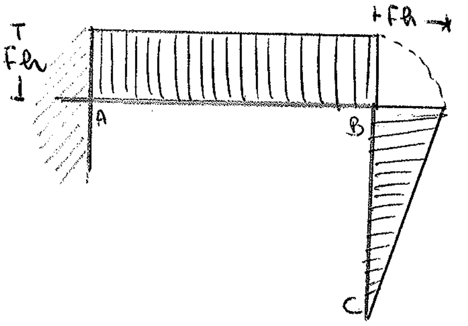
$\boxed{\begin{array}{l} M \text{ PARABOLICO} \Leftrightarrow T \text{ LINEARE} \\ M \text{ LINEARE} \Leftrightarrow T \text{ COSTANTE} \\ M \text{ COSTANTE} \Leftrightarrow T \text{ NULLO.} \end{array}}$

(M) MOMENTO FLETTENTE

$\curvearrowleft H)$ $M + M_A - V_A \cdot z = 0$ $\qquad M = M_A = Fh$ (1° tratto)

$\curvearrowleft K)$ $M + M_A - H_A \cdot z = 0 \Rightarrow M = H_A \cdot z - M_A \Rightarrow M = F(z) - Fh$

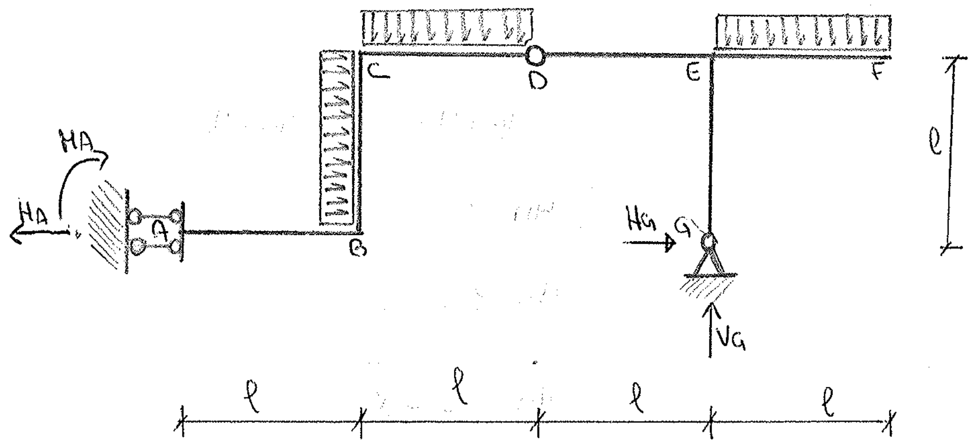
$M = F(z - h)$ $\quad \begin{cases} z = 0 & M = -Fh \text{ (cambio segno)} \\ z = h & M = 0 \end{cases}$



N.B. Im prossimità di un gomito il momento si ribalta di 90°.

ESERCIZI STRUTTURE ISOSTATICHE A MAGLIA APERTA

ESERCIZIO 1 (19/03/14)



GRADI DI LIBERTA

$2 \text{(aste)} \times 3 = 6$ Gdl

GRADI DI VINCOLO

$2 + 2 + 2 = 6$ GdV (cerniera interna, cerniera esterna, dop. pendolo)

$6 - 6 = 0$ STRUTTURA ISOSTATICA

REAZIONI DI EQUILIBRIO GLOBALE

$\rightarrow)$ $-H_A + H_G + ql = 0$ $\qquad \uparrow)$ $V_G - ql - ql = 0$ $\qquad \boxed{V_G = ql}$

VERIFICA ALLA ROTAZIONE

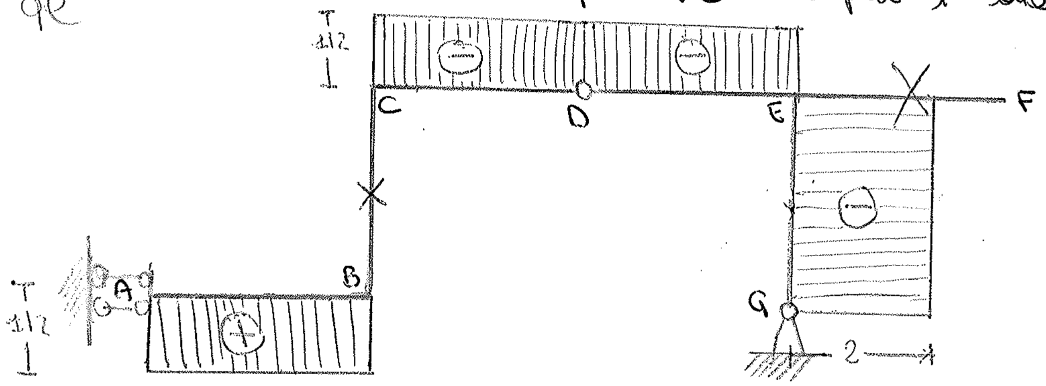
A) $-\frac{ql^2}{2} - ql\left(\frac{l}{2}\right) - ql\left(l+\frac{l}{2}\right) - ql\left(3l+\frac{l}{2}\right) + 2ql(3l) = 0$

$-\frac{ql^2}{2} - \frac{ql^2}{2} - \frac{3}{2}ql^2 - \frac{7}{2}ql^2 + 6ql^2 = 0$

$-ql^2 - 5ql^2 + 6ql^2 = 0 \qquad 0=0$ ok!

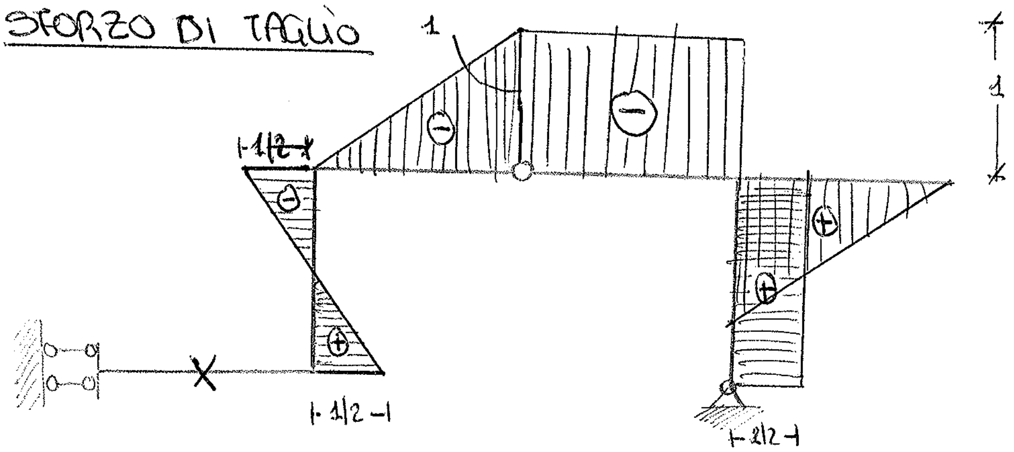
(N) SFORZO NORMALE (Il principio per la determinaz. degli sforzi è analogo ai casi precedenti. Si riporta d'ora in poi solo più i diagrammi)

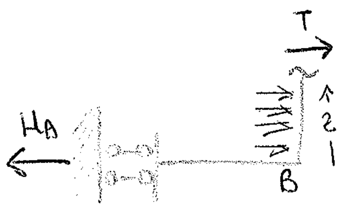
$\frac{N}{ql}$



(T) SFORZO DI TAGLIO

$\frac{T}{ql}$

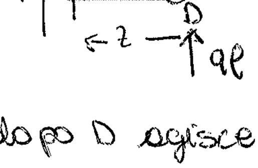




$T - H_A + qz = 0 \qquad T = -qz + \frac{ql}{2}$

$T(z=l)\ -ql/2$

$T = q\left(\frac{l}{2} - z\right) \begin{cases} z=0 & T = q\frac{l}{2} \\ z=\frac{l}{2} & T=0 \end{cases}$



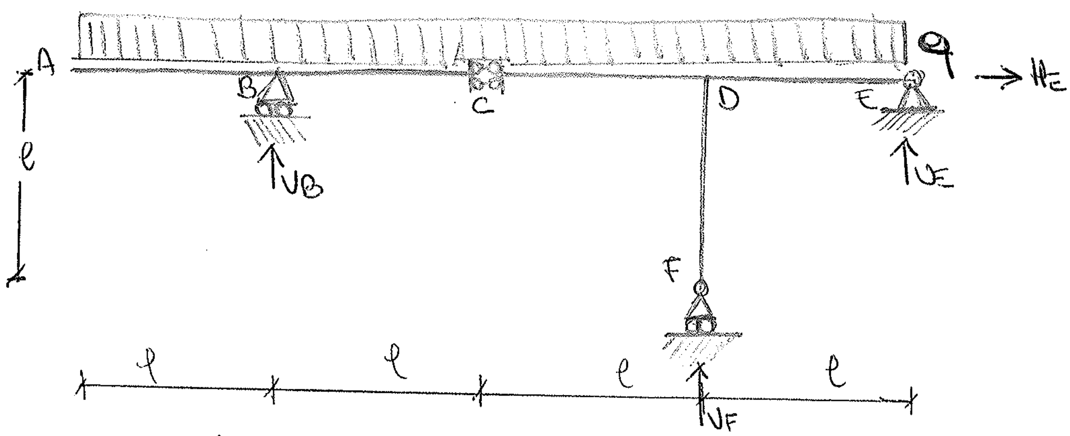
$T - qz + ql = 0$

$\begin{cases} z=0 & \cancel{T=ql}\ T=-ql \\ z=l & T=0 \end{cases}$

Appena dopo D agisce ql, poi inizia ad agire il carico distribuito

(7)

ESERCIZIO (2) 01/03/88 I



GRADI DI LIBERTÀ: 2 (aste) × 3 = 6 gdl

" " VINCOLO: 1+2+1+2 = $\underline{6 \text{ gdv}}$ ⇒ STRUTTURA ISOSTATICA.

EQUILIBRIO GLOBALE:

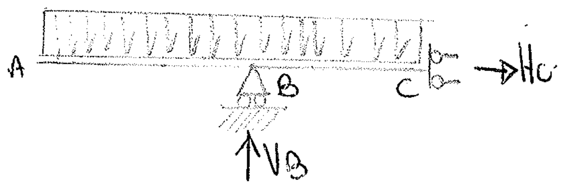
$\uparrow)\ V_B + V_F + V_E - 4ql = 0$

$\rightarrow)\ H_E = 0$

$\overset{\curvearrowleft}{E})\ -V_B \cdot 3l + 4ql \cdot 2l - V_F \cdot l = 0 \qquad -V_F \cdot l = +V_B \cdot 3l - 4ql \cdot 2l \qquad V_F \cdot l = V_B 3l + 4ql \cdot 2l$ $\boxed{V_F = 2ql}$

troppe incognite passo a esplodere la struttura e trovo le reazioni interne.

TRATTO ABC



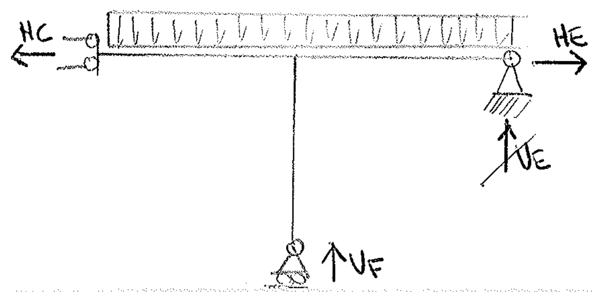
$\uparrow)\ V_B - ql - ql = 0 \qquad \boxed{V_B = 2ql}$

$\rightarrow)\ H_C = 0$

$\overset{\curvearrowleft}{C})\ -M_C + ql\left(l + \frac{l}{2}\right) - V_B \cdot l + ql\frac{l}{2} = 0$

$-M_C = -ql\left(\frac{3}{2}l\right) + V_B \cdot l - ql\frac{l}{2} \qquad \therefore M_C = \frac{3}{2}ql^2 - 2ql^2 + \frac{ql^2}{2} \qquad \boxed{M_C = 0}$

TRATTO CDEF



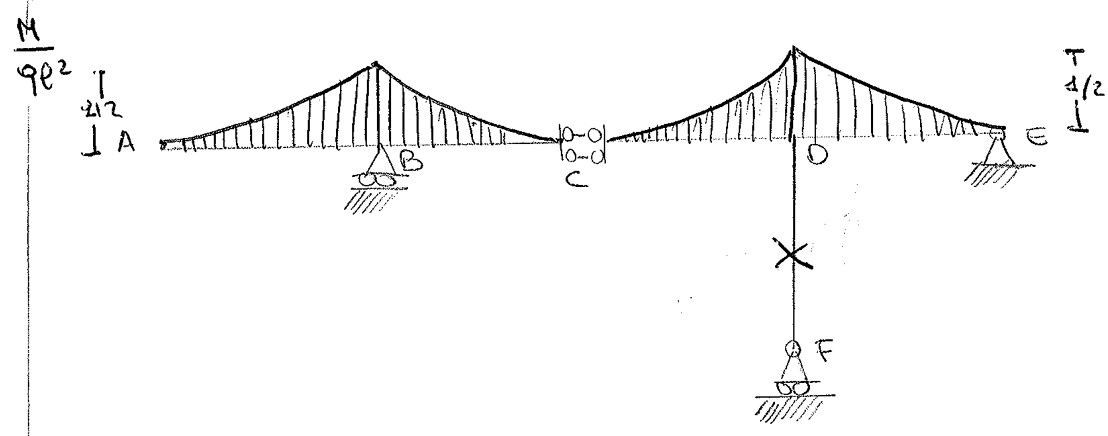
$\uparrow)\ V_F + V_E - 2ql = 0 \qquad V_E = 2ql - V_F$

$\rightarrow)\ H_E = H_C = 0 \qquad \boxed{H_E = 0}$

$\boxed{V_E = 0}$

9

(M) MOMENTO FLETTENTE



$\curvearrowleft K)$ $M - qz\frac{z}{2} = 0$ $\quad M = \frac{qz^2}{2}$ $\quad z=0 \; M=0$; $z=\ell \; M=\frac{q\ell^2}{2}$

$M(z=\ell/2) = \frac{q\ell^2}{8}$

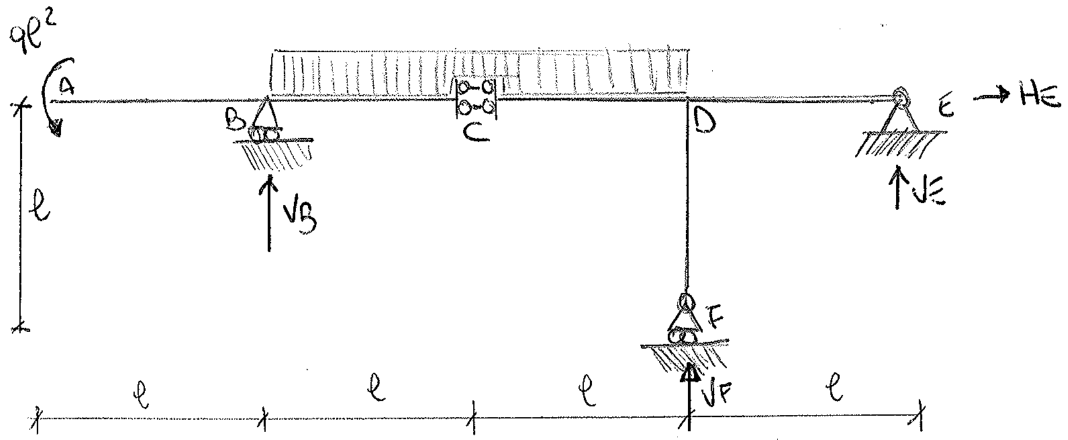
$\curvearrowleft H)$ $M - q\ell\left(\frac{\ell}{2}+z\right) + 2q\ell z - qz\left(\frac{z}{2}\right) = 0$

$\uparrow 2q\ell$

$M(z=0) \Rightarrow M = \frac{q\ell^2}{2}$ $\quad M(z=\ell) = -\frac{3}{2}q\ell^2 + 2q\ell^2 - \frac{q\ell^2}{2} = 0$

$M\left(z=\frac{\ell}{2}\right) = \cancel{-q\ell^2} \cancel{+q\ell^2} + \frac{q\ell^2}{8}$

ESERCIZIO (3) 01/03/88 - II



GRADI DI LIBERTÀ: 2(ASTE) × 3 = 6 GdL

// // VINCOLI: 1 + 2 + 1 + 2 = 6 GdV

0 ⇒ STRUTTURA ISOSTATICA

EQUILIBRIO GLOBALE

$\uparrow)$ $V_B - 2q\ell + V_F + V_E = 0$

$\rightarrow)$ $H_E = 0$

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↑) $V_F + V_E - q\ell = 0$

→) $H_E - H_C = 0$

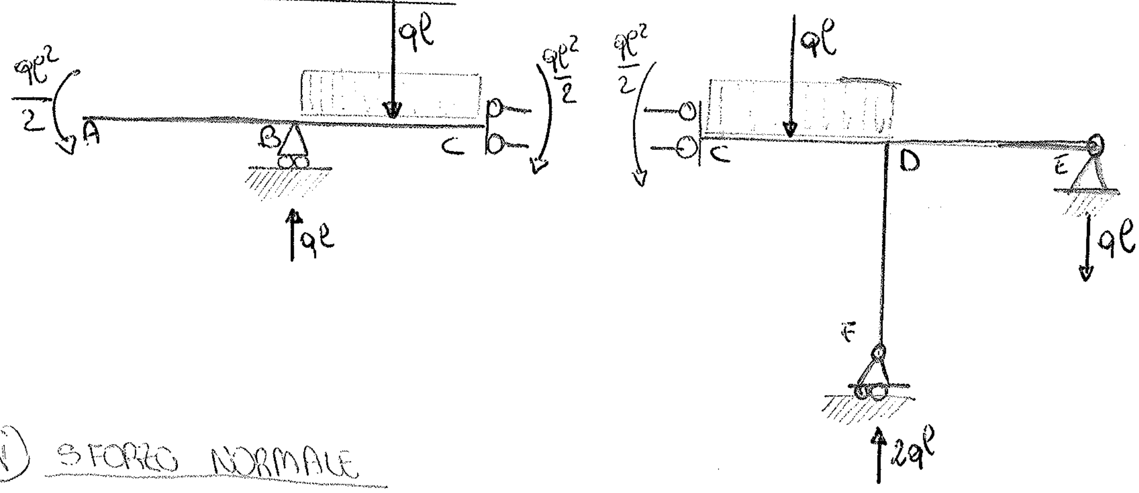
↺C) $M_C - q\ell\left(\frac{\ell}{2}\right) + V_F(\ell) + V_E(2\ell) = 0$

$V_E = q\ell - V_F$ $\boxed{V_E = -q\ell}$ (cambio verso nel disegno, ma non nelle equazioni!)

$H_E = H_C$ $\boxed{H_E = 0}$

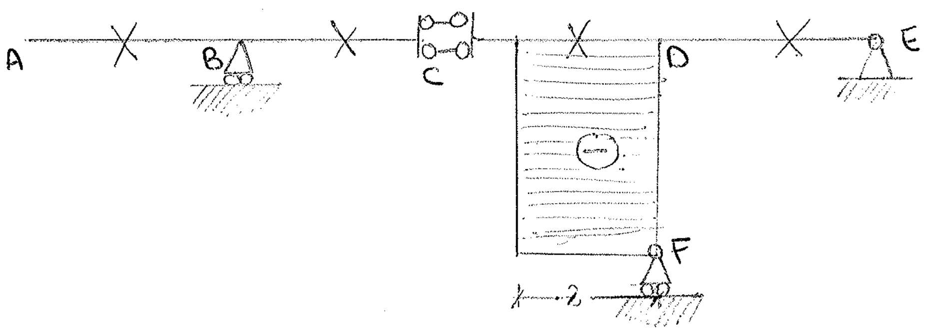
$M_C = \frac{q\ell^2}{2} - \cancel{2q\ell^2} + 2q\ell^2$ $\boxed{M_C = \frac{q\ell^2}{2}}$ OK! VERIFICATA

• DIAGRAMMA "CORPO LIBERO"

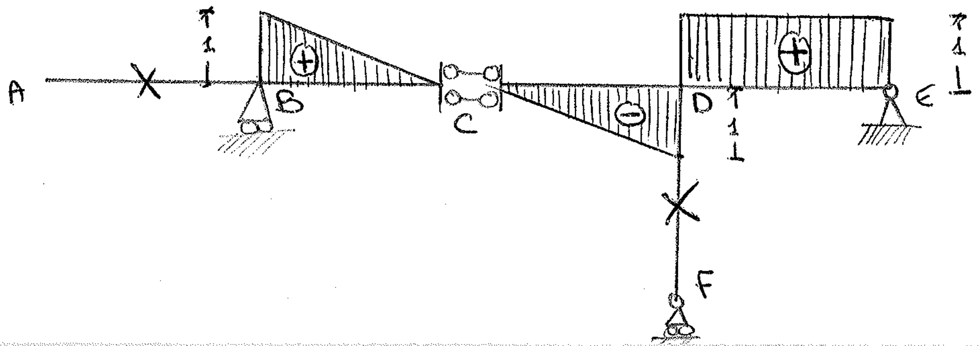


(N) SFORZO NORMALE

$\frac{N}{q\ell}$



(T) SFORZO DI TAGLIO

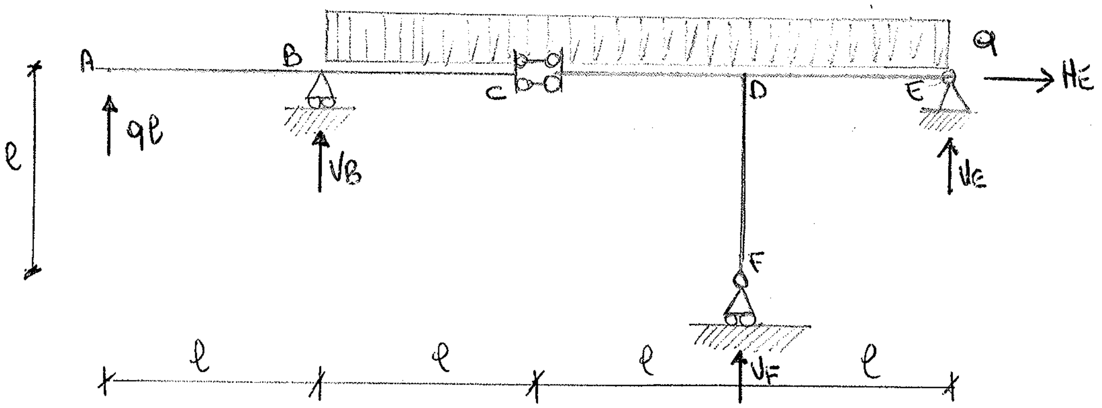


$M(z=0) \Rightarrow -\frac{q\ell^2}{2} - \frac{q\ell^2}{2} = -q\ell^2$

$M(z=\frac{\ell}{2}) = q\ell^2 - \frac{q\ell^2}{2} - q\ell\left(\frac{\ell}{2}+\frac{\ell}{2}\right) = \cancel{q\ell^2} - \frac{q\ell^2}{2} - \cancel{q\ell^2} = -\frac{q\ell^2}{2}$

$M(z=\ell)\quad 2q\ell^2 - \frac{q\ell^2}{2} - q\ell\left(\frac{\ell}{2}+\ell\right) = 2q\ell^2 - \frac{q\ell^2}{2} - \frac{3}{2}q\ell^2 = 0$

ESERCIZIO ④ 01/03/88 - III



- GRADI DI LIBERTÀ: 2(aste) × 3 = 6 GdL
- GRADI DI VINCOLO: 1 + 2 + 1 + 2 = 6 GdV (carrello - doppio pendolo / // - cerniera)

0 ⇒ STRUTTURA ISOSTATICA

REAZIONI DI EQUILIBRIO GLOBALE

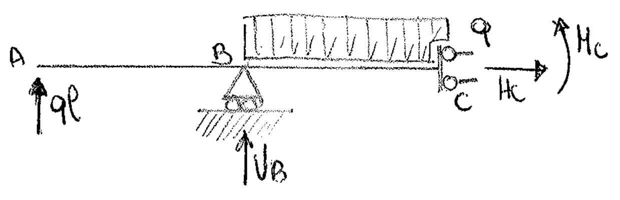
$\rightarrow)\ H_E = 0$

$\uparrow)\ q\ell + V_B + V_F - 3q\ell + V_E = 0$

$\overset{\curvearrowleft}{E})\ -q\ell(4\ell) - V_B(3\ell) + 3q\ell\cdot\left(\frac{3}{2}\ell\right) - V_F\cdot\ell = 0$

Troppe ~~equaz~~ incognite, rispetto alle equazioni, conviene spezzare la struttura e trovare le reazioni interne

TRATTO ABC



$\uparrow)\ V_B + q\ell - q\ell = 0 \quad \boxed{V_B = 0}$

$\rightarrow)\ \boxed{H_C = 0}$

$\overset{\curvearrowleft}{C})\ -q\ell(2\ell) + q\ell\left(\frac{\ell}{2}\right) + M_C = 0 \quad M_C = 2q\ell^2 - \frac{q\ell^2}{2} \quad \boxed{M_C = \frac{3}{2}q\ell^2}$

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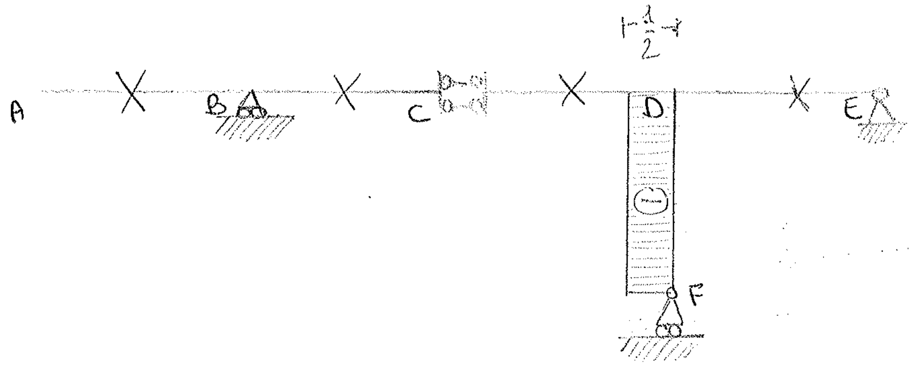
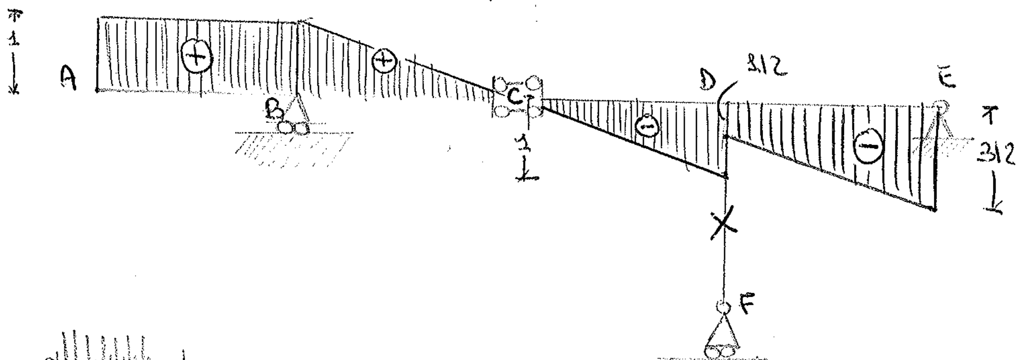
(N) SFORZO NORMALE

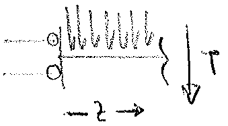
$\frac{N}{q\ell}$



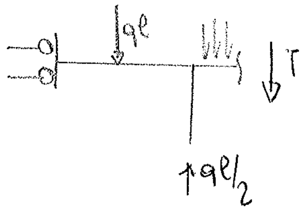
(T) SFORZO DI TAGLIO

$\frac{T}{q\ell}$





$-T-qz=0 \qquad -T=qz \qquad T=-qz$

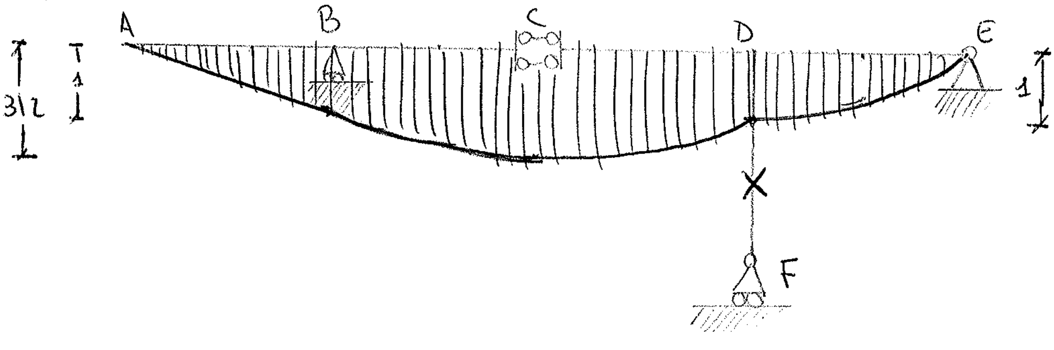


$-T-q\ell+\frac{q\ell}{2}-qz=0$

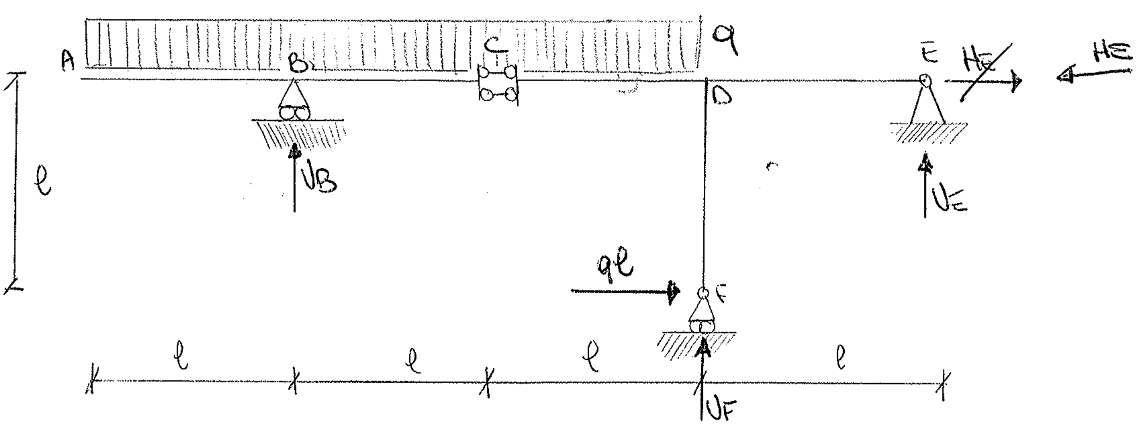
$-T=qz+q\ell-q\frac{\ell}{2} \qquad T=q\frac{\ell}{2}-qz-q\ell$

$T=q\left(-\frac{\ell}{2}-z\right) \quad \begin{cases} z=0 & T=-q\frac{\ell}{2} \\ z=\ell & T=-\frac{3}{2}q\ell \end{cases}$

(M) MOMENTO FLETTENTE



ESERCIZIO ⑤ 01/03/08 - IV



- GRADI DI LIBERTÀ: 2 (aste) × 3 = 6 GdL
- " " VINCOLO: 1 + 2 + 1 + 2 = 6 GdV

$\qquad\qquad\qquad 0 \Rightarrow$ struttura isostatica.

- <u>REAZIONI DI EQUILIBRIO GLOBALE</u>

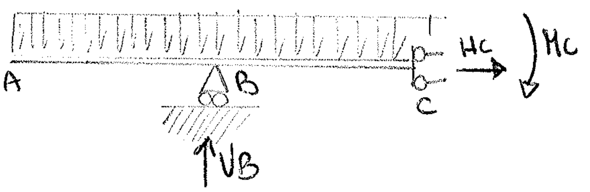
$\rightarrow)\ H_E + q\ell = 0 \quad \boxed{H_E - q\ell}$ (cambio verso)

$\uparrow)\ V_B + V_F + V_E - 3q\ell = 0$

$\overset{\curvearrowleft}{E})\ -V_B(3\ell) + 3q\ell\left(2\ell + \frac{\ell}{2}\right) - V_F(\ell) + q\ell^2 = 0$

Essendoci più incognite di equazioni, conviene esplodere la struttura e cercare le reaz. interne.

TRATTO ABC



$\uparrow)\ V_B - 2q\ell = 0 \quad \boxed{V_B = 2q\ell}$

$\rightarrow)\ H_C = 0$

$\overset{\curvearrowleft}{C})\ 2q\ell^2 - V_B \cdot \ell - M_C = 0$

$-M_C = -2q\ell^2 + V_B \cdot \ell \qquad M_C = -2q\ell^2 + 2q\ell^2 \quad \boxed{M_C = 0}$

Una volta determinata la reazione vincolare V_B, posso calcolarmi la reaz. V_F, dall'equaz. d'equilibrio alla rotazione attorno al punto E.

$\overset{\curvearrowleft}{E})\ -V_B(3\ell) + 3q\ell\left(\frac{5}{2}\ell\right) - V_F(\ell) + q\ell^2 = 0 \quad \Big|\ -6q\ell^2 + \frac{15}{2}q\ell^2 - V_F(\ell) + q\ell^2 = 0$

$-V_F(\ell) = -\frac{15}{2}q\ell^2 + 6q\ell^2 - q\ell^2 \quad \Big|\ V_F(\cancel{\ell}) = \frac{15}{2}q\ell^{\cancel{2}} - 6q\ell^{\cancel{2}} + q\ell^{\cancel{2}}$

$V_F = \frac{15-12+2}{2}q\ell \qquad \boxed{V_F = \frac{5}{2}q\ell}$

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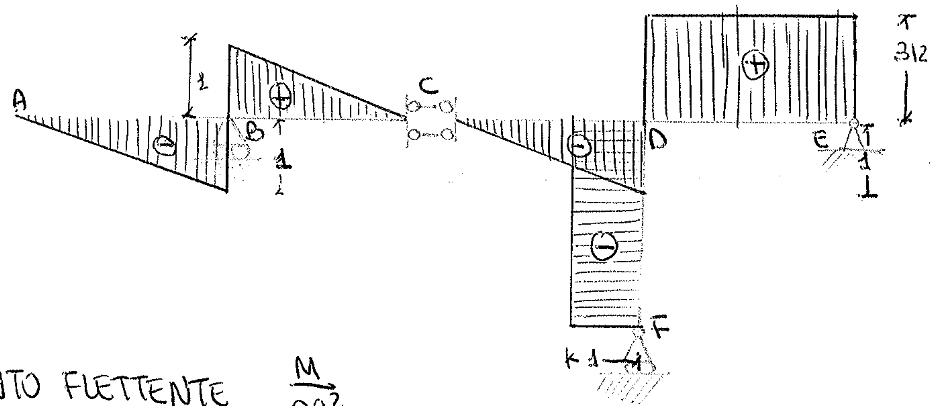
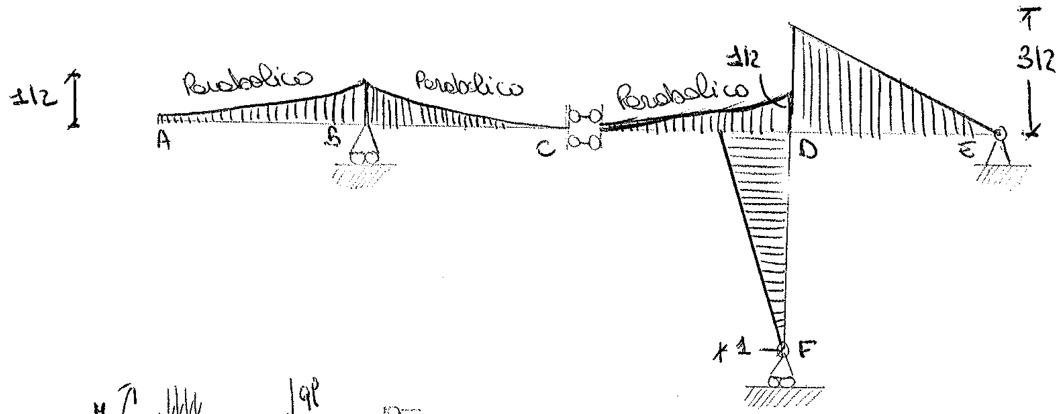
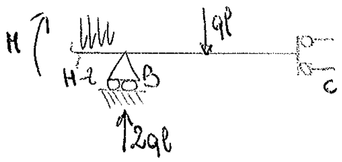
(T) SFORZO DI TAGLIO

$\frac{T}{q\ell}$



(M) MOMENTO FLETTENTE $\frac{M}{q\ell^2}$





$$-M - \frac{qz^2}{2} + 2q\ell(z) - q\ell\left(\frac{\ell}{2}+z\right) = 0$$

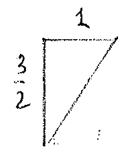
$$-M = \frac{qz^2}{2} - 2q\ell(z) + q\ell\left(\frac{\ell}{2}+z\right) \qquad M = 2q\ell(z) - \frac{qz^2}{2} - q\ell\left(\frac{\ell}{2}+z\right)$$

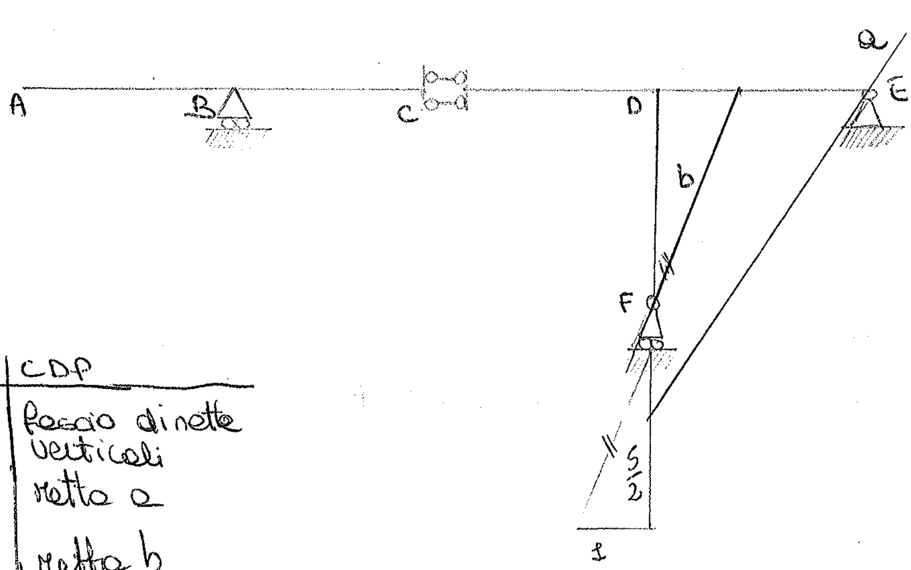
$z=0 \quad M = -\frac{q\ell^2}{2}$

$z=\frac{\ell}{2} \quad \Rightarrow M = -\frac{q\ell^2}{8}$

$z=\ell \quad M=0$

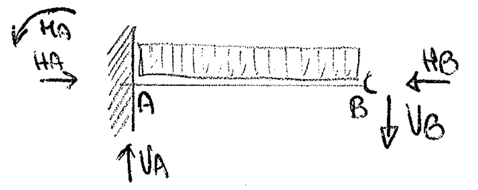
CURVA DELLE PRESSIONI





TRATTO	CDP
AB - BC - CD	fascio di rette verticali
DE	retta a
DF	retta b

TRATTO AB



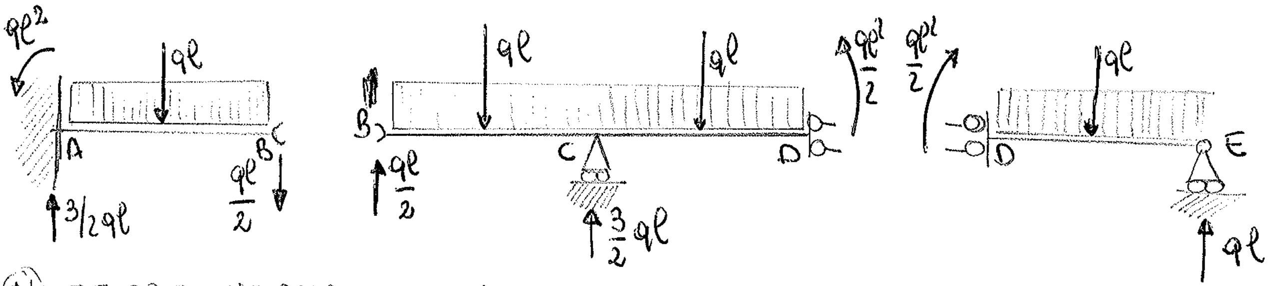
$\rightarrow)\ H_A = H_B \qquad \boxed{H_A = 0}$

$\uparrow)\ V_A - ql - V_B = 0 \qquad V_A = ql + V_B \qquad \boxed{V_A = \frac{3}{2}ql}$

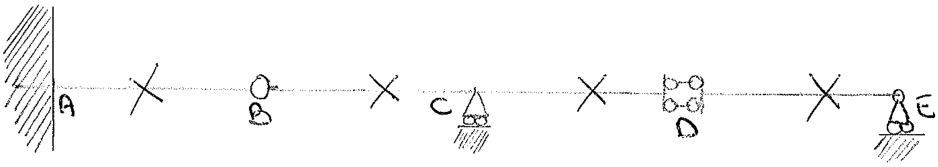
verifico il valore di MA con un'equaz. alla rotazione attorno ad A

$\curvearrowleft A)\ M_A - \frac{ql^2}{2} - V_B(l) = 0 \qquad M_A = \frac{ql^2}{2} + V_B(l) \qquad M_A = \frac{ql^2}{2} + \frac{ql^2}{2} \qquad M_A = ql^2$ ok!

DIAGRAMMA CORPO LIBERO.

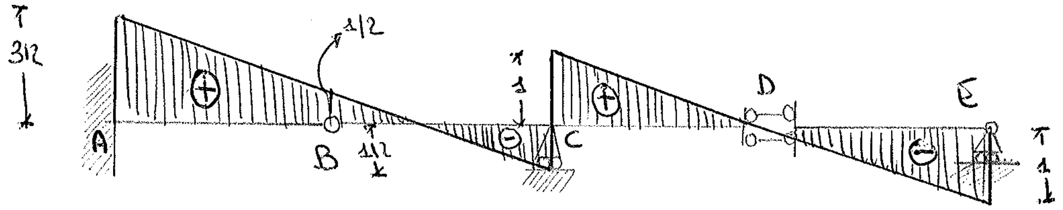


(N) SFORZO NORMALE N/ql

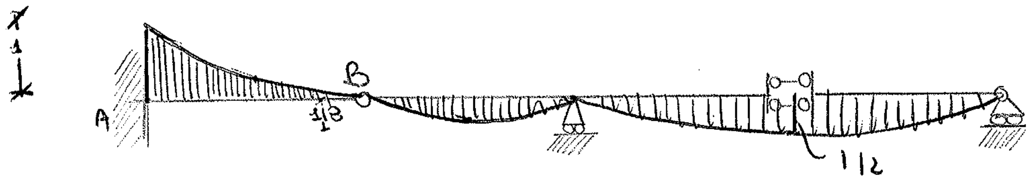


TRAVE SCARICA ALLO SFORZO NORMALE

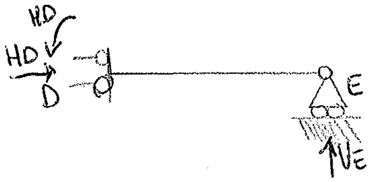
(T) SFORZO DI TAGLIO



(M) MOMENTO FLETTENTE



- TRATTO DE

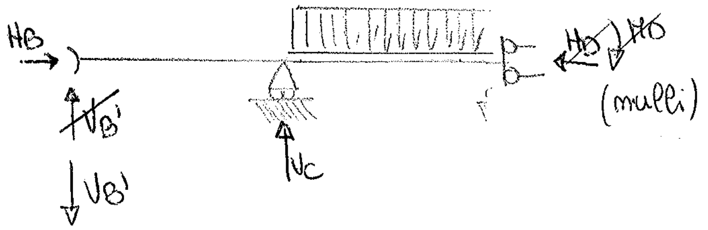


$\rightarrow)\ \boxed{H_D = 0}$

$\uparrow)\ \boxed{V_E = 0}$

$\overset{\frown}{D})\ M_D + V_E \cdot l = 0 \qquad \boxed{M_D = 0}$

- TRATTO BDC



N.B.| Il "NODO" B o "CERNIERA B" risulta caricato dalla forza ql ecco perché chiamo le reazioni V_B' e V_B'' perché occorrerà anche equilibrare la cerniera B.

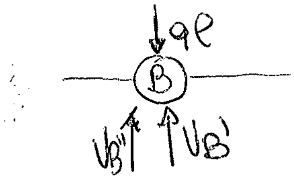
Proprio per questo motivo, non è detto che V_B'' sia opposto in segno e uguale in modulo a V_B'; ogniqualvolta vi è un nodo carico, la "regola" di "forze uguali e opposte" all'interno della cerniera non è più valida.

$\rightarrow)\ \boxed{H_B = 0}$

$\uparrow)\ V_B' + V_C - ql = 0 \qquad V_B' = ql - V_C \qquad \boxed{V_B' = -\frac{ql}{2}}$

$\overset{\frown}{B})\ V_C(l) - ql\left(l + \frac{l}{2}\right) = 0 \qquad V_C(l) = ql\left(l + \frac{l}{2}\right) \qquad \boxed{V_C = \frac{3}{2}ql}$

- EQUILIBRIO NODO B

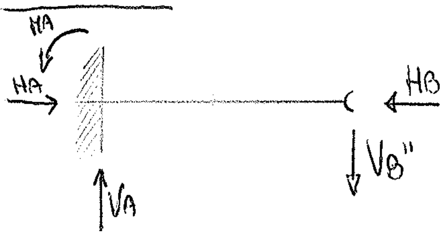


$V_B' - ql + V_B'' = 0 \qquad V_B'' = +ql - V_B'$

$V_B'' = \frac{ql}{2}$

N.B.| V_B' opposta a quella trovata nel tratto ~~BB~~ BDC, perché qui si considera tutta la cerniera, e non la si seziona a metà come nel tratto BDC

- TRATTO AB



$\rightarrow)\ H_A = 0$

$\uparrow)\ V_A = V_B'' \qquad \boxed{V_A = \frac{ql}{2}}$

$\overset{\frown}{A})\ M_A - V_B''\, l = 0$

$\boxed{M_A = \frac{ql^2}{2}}$

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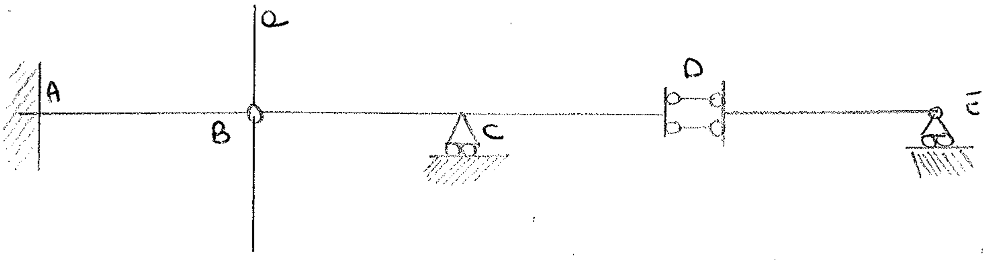
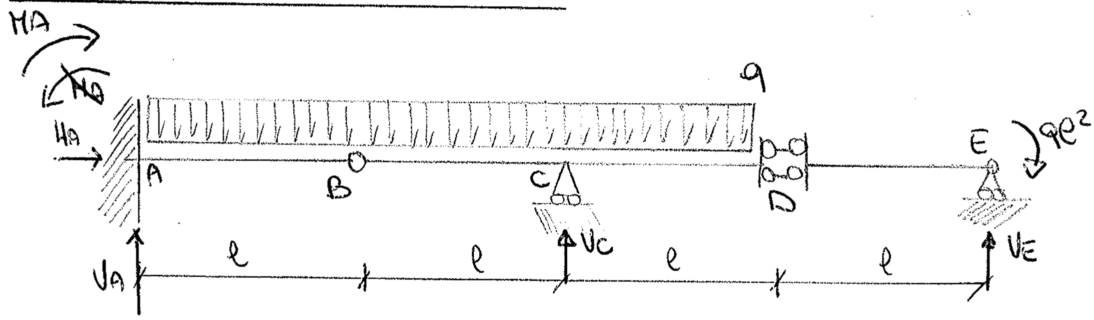
- CURVA DELLE PRESSIONI



TRATTO	C.D.P.
AB - BC	retta a
CD	fascio rette vert.
DE	scarico

ESERCIZIO ⑧ 24/01/89 - III



- GRADI DI LIBERTÀ: 3(aste) × 3 = 9 GdL +
- " " VINCOLO: 3+2+1+2+1 = 9 GdV =

0 => struttura isostatica

- REAZIONI DI EQUILIBRIO GLOBALE DELLA STRUTTURA

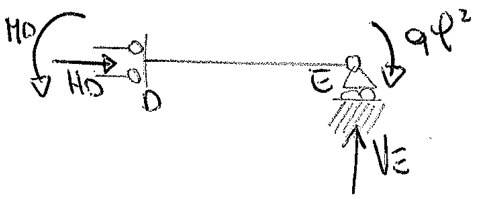
$\uparrow)\ V_A - 3q\ell + V_C + V_E = 0$

$\rightarrow)\ \boxed{H_A = 0}$

$\overset{\curvearrowleft}{A})\ M_A - 3q\ell\left(\frac{3}{2}\ell\right) + V_C(2\ell) + V_E(4\ell) - q\ell^2 = 0$

Passo a esplodere la struttura

- TRATTO DE

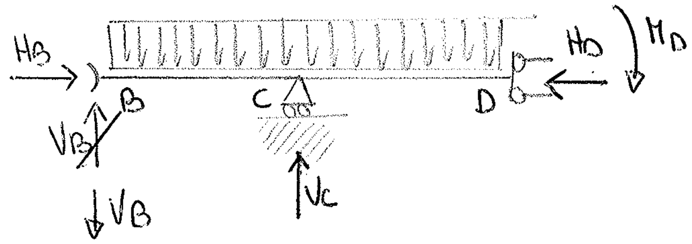


$\uparrow)\ \boxed{V_E = 0}$

$\rightarrow)\ \boxed{H_D = 0}$

$\overset{\curvearrowleft}{D})\ M_D - q\ell^2 = 0 \quad \boxed{M_D = q\ell^2}$

- TRATTO BDC



$\rightarrow)\ H_B = H_D \quad \boxed{H_B = 0}$

$\uparrow)\ V_B + V_C - 2q\ell = 0$

$\overset{\curvearrowleft}{B})\ -2q\ell^2 + V_C(\cancel{2}\ell) - M_D = 0$

$\cancel{-2q\ell^2}\ V_C(\ell) = M_D + 2q\ell^2 \quad \boxed{V_C = 3q\ell}$

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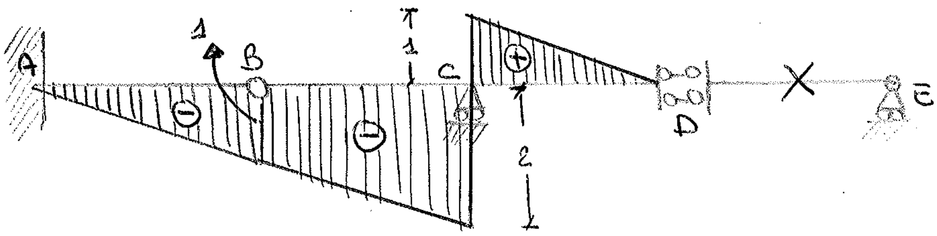
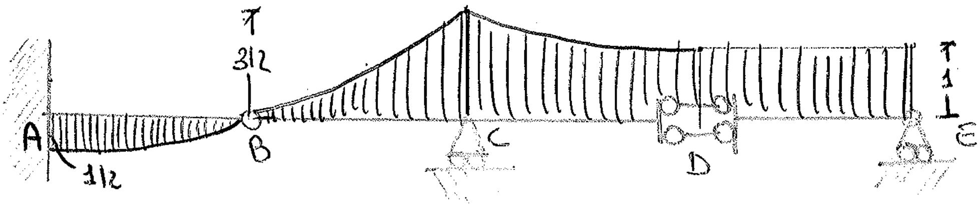
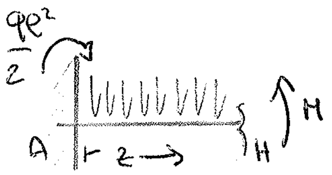
Ⓣ SFORZO DI TAGLIO $\quad T/q\ell$



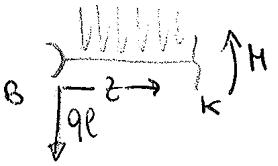
Ⓜ MOMENTO FLETTENTE $\quad M/q\ell^2$





$\overset{\frown}{H})\ M + qz\left(\frac{z}{2}\right) - \frac{q\ell^2}{2} = 0 \qquad M = \frac{q\ell^2}{2} - \frac{qz^2}{2} \quad \begin{cases} z=\ell & M=0 \\ z=0 & M=\frac{q\ell^2}{2} \end{cases}$

$M(z=\ell/2) \qquad M = q\ell^2/2 - q\ell^2/8 \qquad \cancel{M = 3/8\, q\ell^2} \qquad M = \left(3/8\right) q\ell^2$



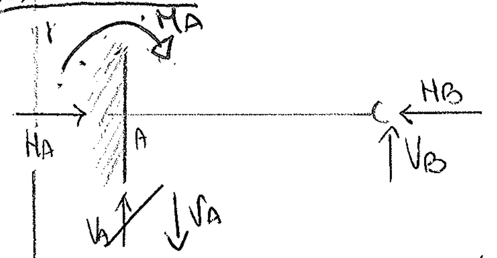
$\overset{\frown}{K})\ M + q\ell(z) + qz\left(\frac{z}{2}\right) = 0$

$M = qz\left(\frac{z}{2}\right) + q\ell(z) \quad \begin{cases} z=0 & M=0 \\ z=\ell/2 & M=\frac{q\ell^2}{8}+\frac{q\ell^2}{2} \quad M=+\frac{5}{8}q\ell^2 \\ z=\ell & M=\frac{3}{2}q\ell^2 \end{cases}$

- CURVA DELLE PRESSIONI

TRATTO	C.D.P.
DE	non definita
$\overline{AB}$ - $\overline{BC}$ - $\overline{CD}$	fascio di rette verticali

TRATTO AB



$\uparrow)\ V_A + V_B = 0 \qquad V_A = -V_B \qquad \boxed{V_A = -\frac{q\ell}{2}}$

$\rightarrow)\ H_A = H_B = 0 \qquad \boxed{H_A = 0}$

$\overset{\curvearrowleft}{A})\ -M_A + V_B(\ell) = 0$

Calcolo MA dall'equilibrio globale esterno della struttura

$M_A - q\ell^2 + V_C(2\ell) - q\ell\left(3\ell + \frac{\ell}{2}\right) + V_E(4\ell) = 0$

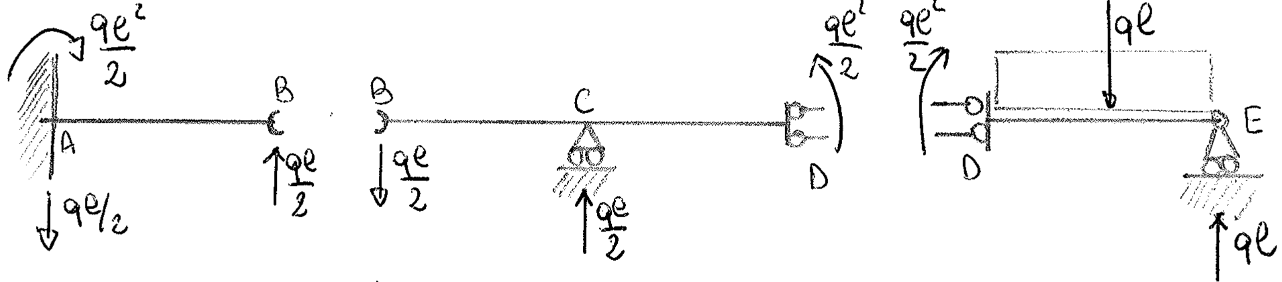
$M_A = q\ell^2 - V_C(2\ell) + q\ell\left(3\ell + \frac{\ell}{2}\right) - V_E(4\ell) = 0$

$M_A = \cancel{q\ell^2} - \cancel{q\ell^2} + \frac{7}{2}q\ell^2 - 4q\ell^2 \qquad \boxed{M_A = -\frac{q\ell^2}{2}}$

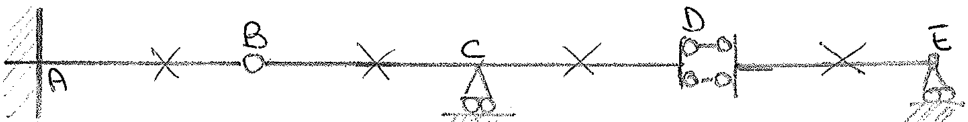
Verifico MA dalla rotazione attorno ad A al tratto AB

$-M_A = -V_B(\ell) \qquad M_A = V_B \cdot \ell \qquad \boxed{M_A = \frac{q\ell^2}{2}}$ OK!

- DIAGRAMMA CORPO LIBERO

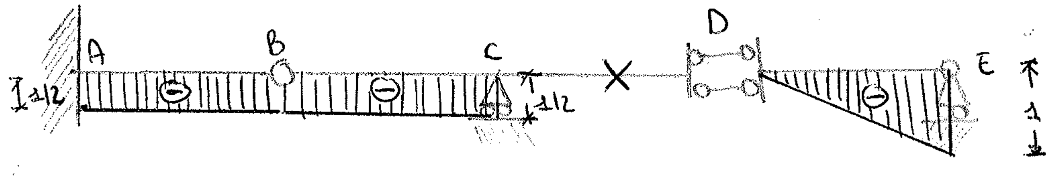


(N) SFORZO NORMALE $\frac{N}{q\ell}$

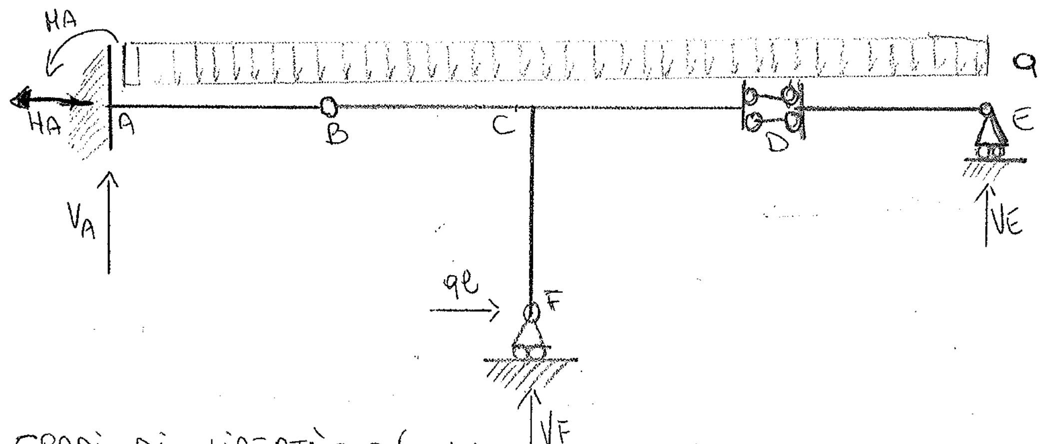


scarico allo sforzo normale

(T) SFORZO DI TAGLIO



ESERCIZIO 10 28/02/89 - I



GRADI DI LIBERTÀ: 3(aste) x 3 = 9 GdL

GRADI DI VINCOLO: 3 + 2 + 1 + 2 + 1 = 9 GdV = 0 ⇒ struttura isostatica

REAZIONI DI EQUILIBRIO GLOBALE

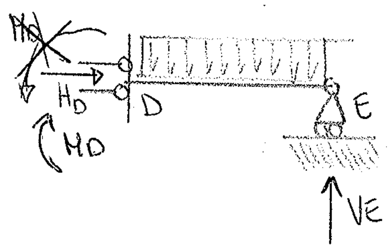
$\rightarrow)\ -H_A + ql = 0 \qquad \boxed{H_A = ql}$

$\uparrow)\ V_A + V_F + V_E - 4ql = 0$

$\curvearrowleft A)\ M_A - 4ql(2l) + V_F(2l) + ql^2 + V_E(4l) = 0$

Passo a determinare le reazioni interne, esplodendo la struttura

TRATTO DE

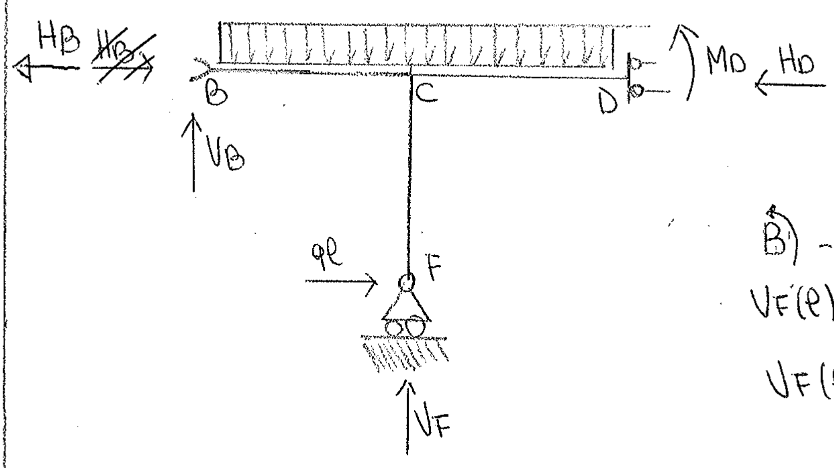


$\rightarrow)\ H_D = 0$

$\uparrow)\ V_E - ql = 0 \qquad \boxed{V_E = ql}$

$\curvearrowleft D)\ M_D - \frac{ql^2}{2} + ql^2 = 0 \qquad \boxed{M_D = -\frac{ql^2}{2}}$ cambio verso

TRATTO BCD



$\rightarrow)\ H_B + ql - H_D = 0 \qquad \boxed{H_B = -ql}$

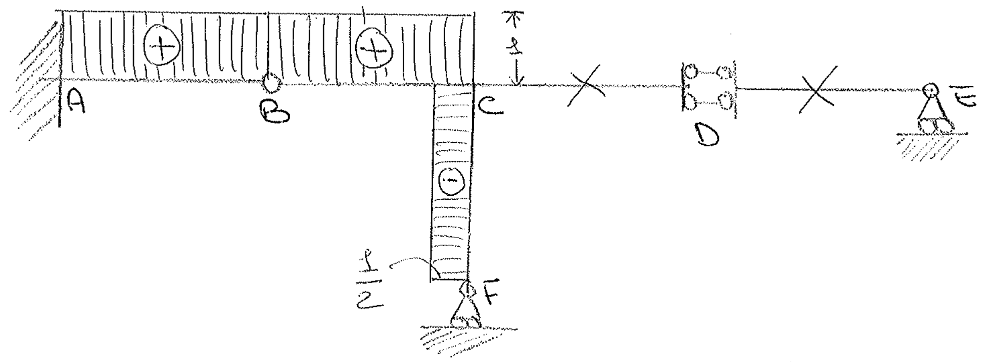
$\uparrow)\ V_B + V_F - 2ql = 0$

$\curvearrowleft B)\ -2ql^2 + V_F(l) + M_D + ql^2 = 0$

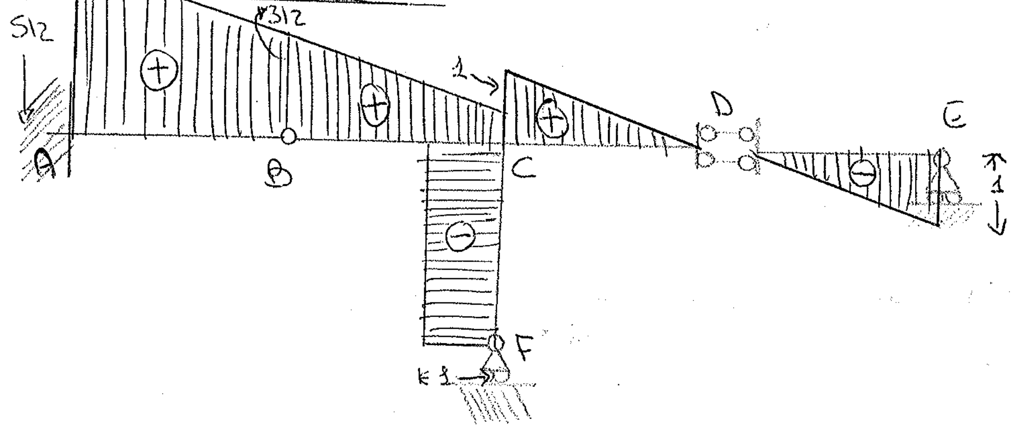
$V_F(l) = ql^2 - M_D$

$V_F(l) = \frac{ql^2}{2} \qquad \boxed{V_F = \frac{ql}{2}}$

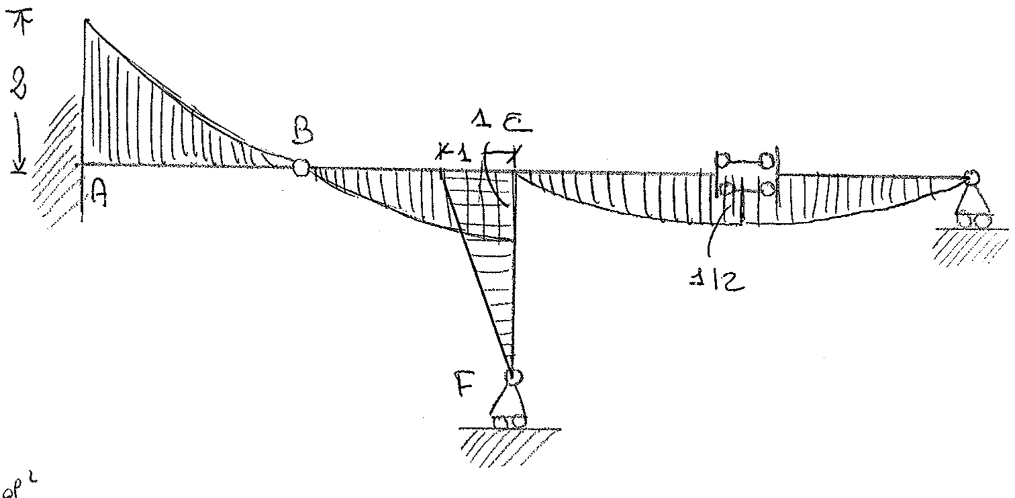
(N) SFORZO NORMALE

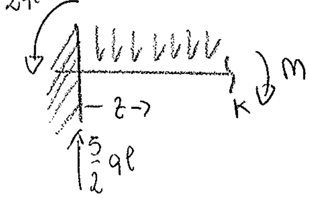


(T) SFORZO DI TAGLIO



(M) MOMENTO FLETTENTE





$-M + 2ql^2 - \frac{5}{2}qlz + \frac{qz^2}{2} = 0 \qquad -M = -2ql^2 + \frac{5}{2}qlz - \frac{qz^2}{2}$

$M = 2ql^2 + \frac{qz^2}{2} - \frac{5}{2}qlz \qquad \begin{cases} z = 0 & M = 2ql^2 \\ z = l & M = 0 \end{cases}$

$M\left(z = \frac{l}{2}\right) = 2ql^2 + \frac{ql^2}{8} - \frac{5}{4}ql^2 \qquad \frac{16+1-10}{8}ql^2 = \frac{7}{8}ql^2$

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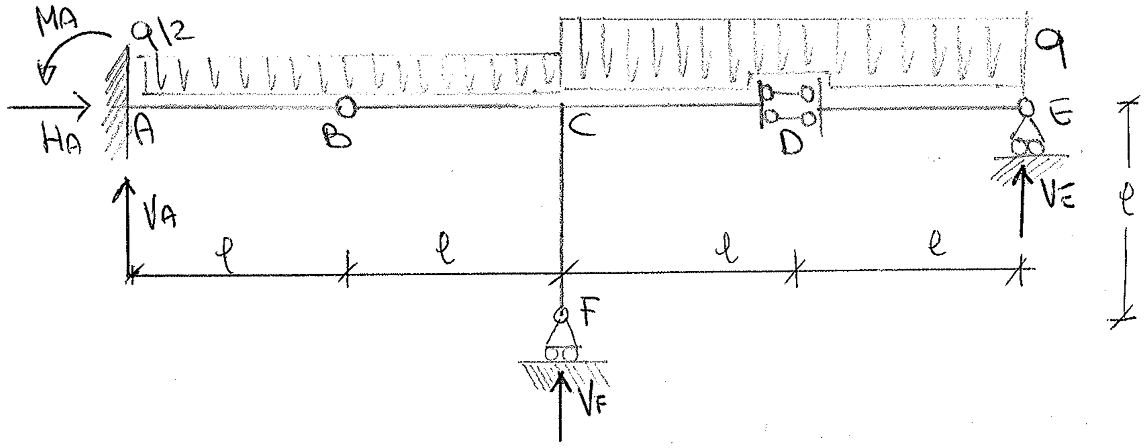
~~ESER~~ ESERCIZIO 11 28/02/89 - II



- GRADI DI LIBERTÀ: 3(aste) × 3 = 9 GdL −
- GRADI DI VINCOLO: 3+2+1+2+1 = 9 GdV =

0 struttura isostatica

- EQUILIBRIO GLOBALE DELLA STRUTTURA

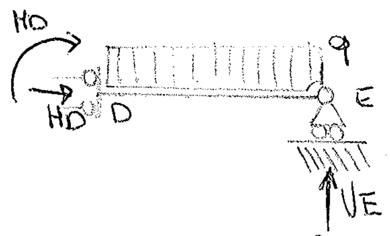
$\uparrow)\ V_A - q\ell - 2q\ell + V_F + V_E = 0$

$\rightarrow)\ \boxed{H_A = 0}$

$\circlearrowleft A)\ M_A - q\ell^2 + V_F(2\ell) - 2q(3\ell) + V_E(4\ell) = 0$

Passo a determinare le reaz. interne, esplodendo la struttura.

TRATTO DE



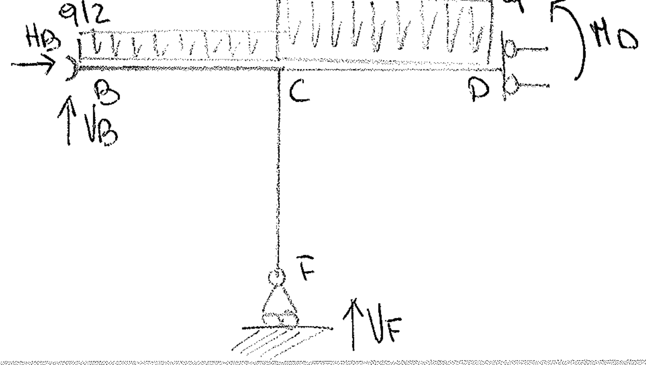
$\uparrow)\ V_E - q\ell = 0 \qquad \boxed{V_E = q\ell}$

$\rightarrow)\ \boxed{H_D = 0}$

$\circlearrowleft D)\ -M_D - \frac{q\ell^2}{2} + V_E(\ell) = 0$

$-M_D = -V_E(\ell) + \frac{q\ell^2}{2} \qquad M_D = V_E(\ell) - \frac{q\ell^2}{2} \qquad \boxed{M_D = \frac{q\ell^2}{2}}$

TRATTO BCDF

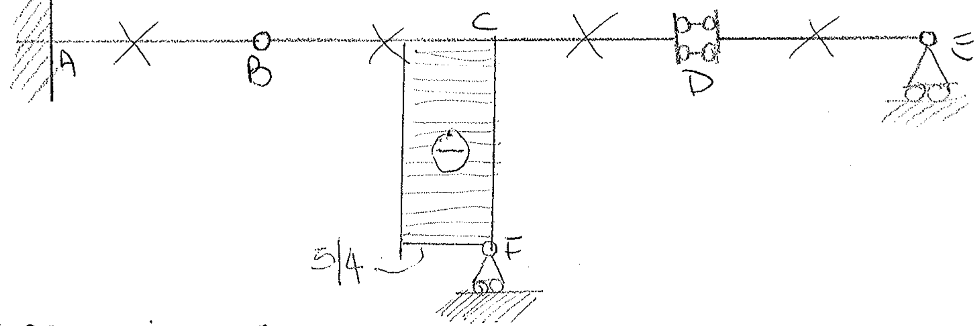


$\uparrow)\ V_F + V_B - q\ell - \frac{q\ell}{2} = 0$

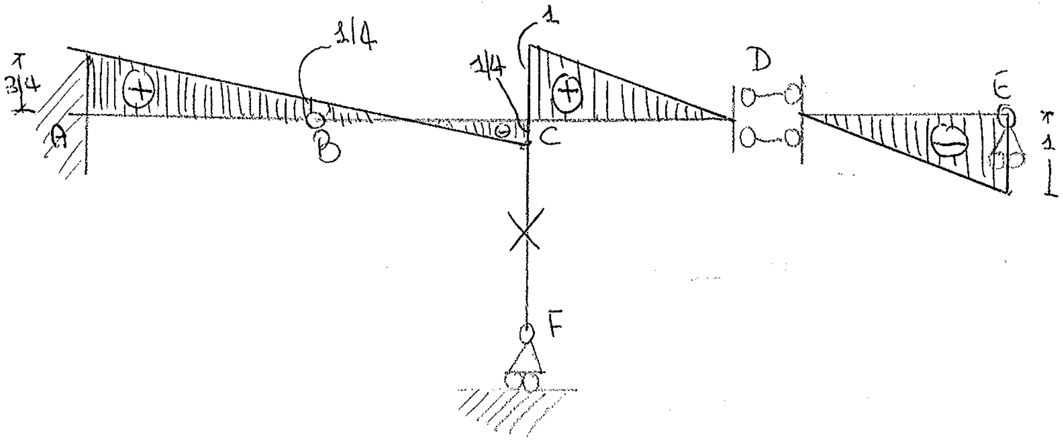
$\rightarrow)\ \boxed{H_B = 0}$

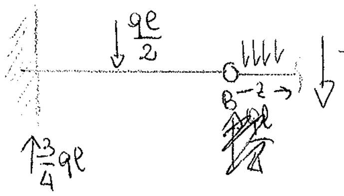
$\circlearrowleft B)\ -\frac{q\ell^2}{4} + V_F(\ell) - q\ell\left(\ell + \frac{\ell}{2}\right) + M_D = 0$

(N) SFORZO NORMALE



(T) SFORZO DI TAGLIO



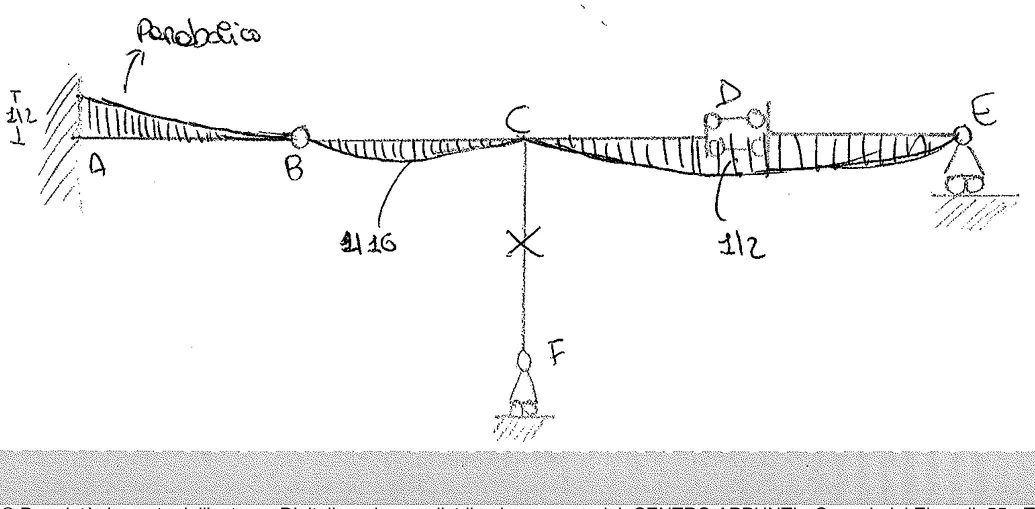


$-T + \frac{3}{4}q\ell + \cancel{\frac{q\ell}{4}} - \frac{qz}{2} - \frac{q\ell}{2} = 0$ $\qquad -T = -\frac{3}{4}q\ell - \cancel{\frac{q\ell}{4}} + \frac{qz}{2} + \frac{q\ell}{2}$

$T = \frac{3}{4}q\ell + \cancel{\frac{q\ell}{4}} - \frac{qz}{2} - \frac{q\ell}{2}$ $\qquad T = \frac{3 \cancel{+4} - 2}{4}$

$T = \frac{q\ell}{4} - \frac{qz}{2}$ $\qquad T(z=0)$ $\quad T = \frac{q\ell}{4}$ $\qquad T(z=\ell) \Rightarrow T = -\frac{q\ell}{4}$

(M) MOMENTO FLETTENTE



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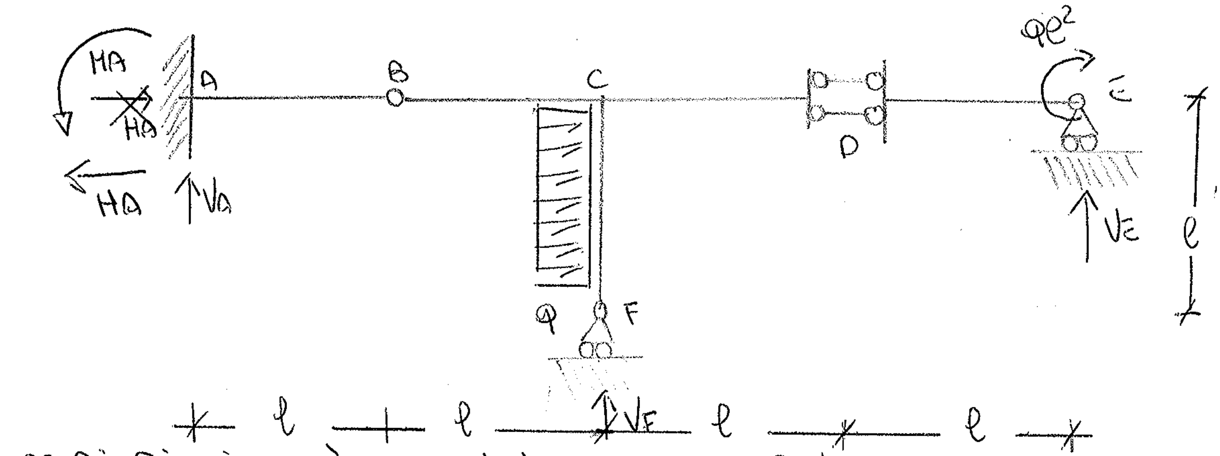
ESERCIZIO (12) 23/02/99 - III



- GRADI DI LIBERTÀ: 3(aste) × 3 = 9 Gdl -
- GRADI DI VINCOLO: 3 + 2 + 1 + 2 + 1 = 9 Gdv =

0 ⇒ struttura isostatica

- EQUILIBRIO GLOBALE DELLA STRUTTURA

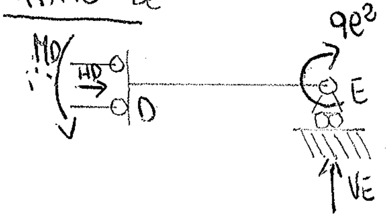
$\rightarrow)\ H_A + q\ell = 0 \qquad \boxed{H_A = -q\ell}$

$\uparrow)\ V_A + V_F + V_E = 0$

$\circlearrowleft A)\ M_A + q\ell\left(\frac{\ell}{2}\right) + V_F(2\ell) + V_E(4\ell) - q\ell^2 = 0$

Passo a determinare le reazioni interne, esplodendo la struttura

TRATTO DE

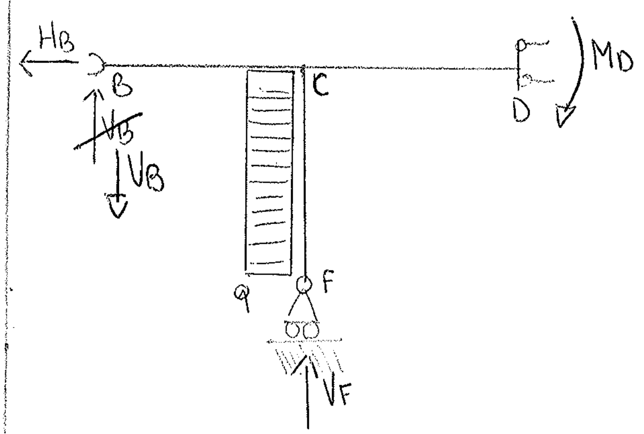


$\uparrow)\ \boxed{V_E = 0}$

$\rightarrow)\ H_D = 0$

$\circlearrowleft D)\ M_D - q\ell^2 = 0 \qquad \boxed{M_D = q\ell^2}$

TRATTO BCDF



$\uparrow)\ V_B + V_F = 0$

$\circlearrowleft B)\ +\frac{q\ell^2}{2} + V_F(\ell) - M_D = 0$

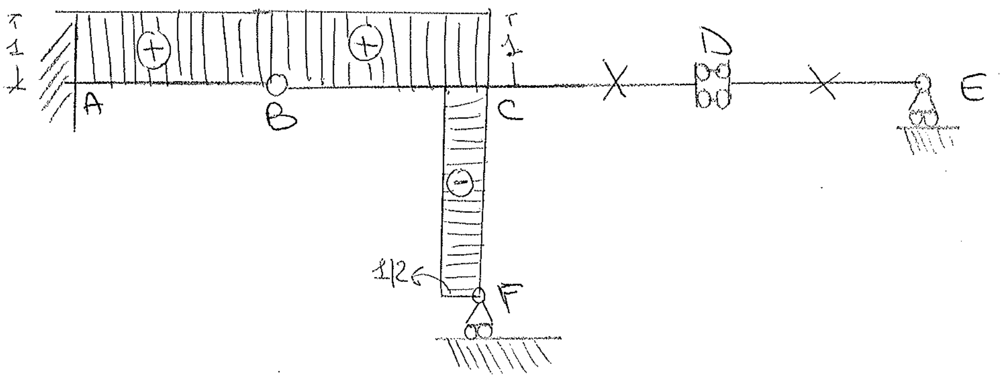
$\rightarrow)\ -H_B + q\ell = 0 \qquad \boxed{H_B = q\ell}$

$V_F(\ell) = M_D - \frac{q\ell^2}{2} \qquad V_F(\ell) = \frac{q\ell^2}{2}$

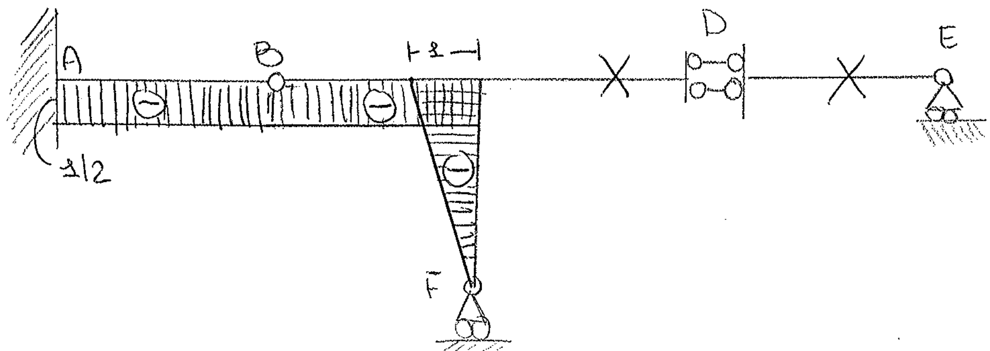
$\boxed{V_F = \frac{q\ell}{2}}$

(N) SFORZO NORMALE

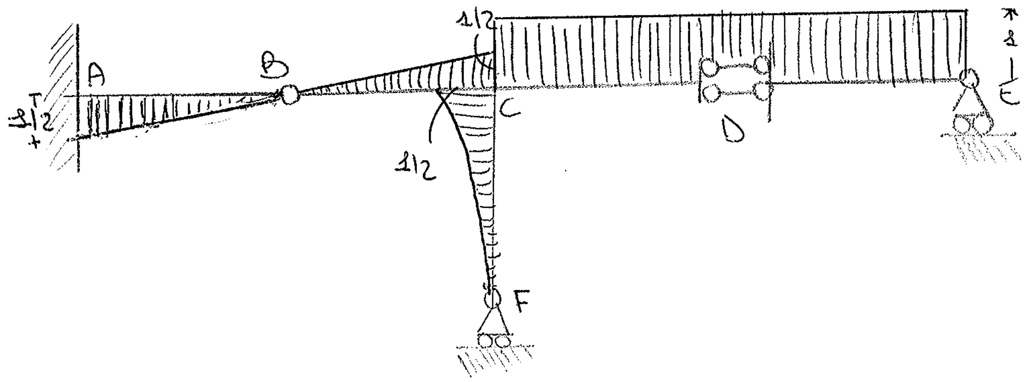
$\frac{N}{q\ell}$

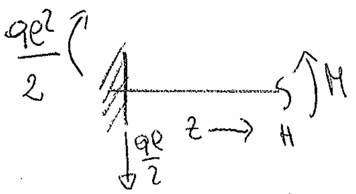


(T) SFORZO DI TAGLIO $T/q\ell$



(M) MOMENTO FLETTENTE $M/q\ell^2$





$\circlearrowleft A$: $M - \frac{q\ell^2}{2} + \frac{q\ell}{2}(z) = 0$ $\qquad M = \frac{q\ell^2}{2} - \frac{q\ell}{2}(z)$

$M(z=0)\ \frac{q\ell^2}{2}$ $\qquad M(z=\ell)\ 0$ $\qquad M(z=\frac{\ell}{2})\ \frac{q\ell^2}{2} - \frac{q\ell^2}{8} = \frac{3}{8}q\ell^2$

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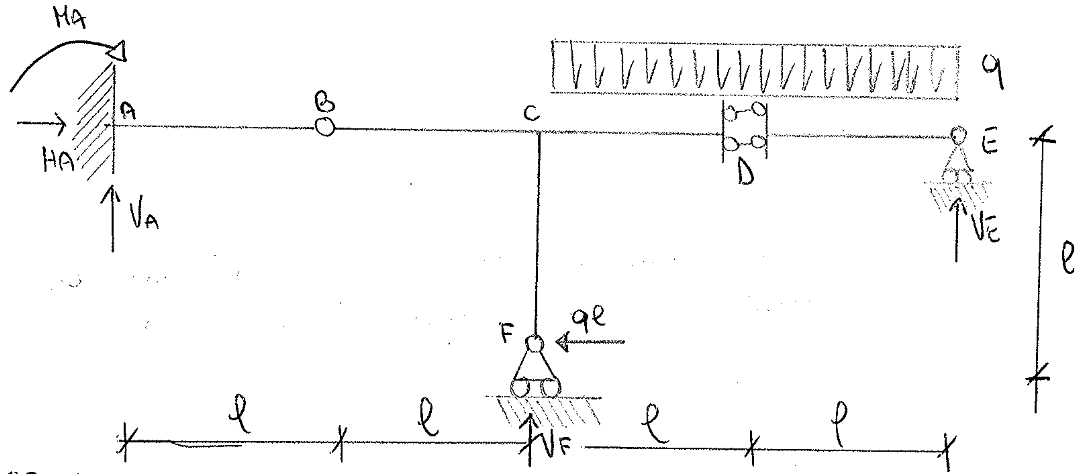
ESERCIZIO 13 28/02/09 - IV



GRADI DI LIBERTÀ: 3(aste) × 3 = 9 Gdl -
GRADI DI VINCOLO: 3+2+1+2+1 = 9 Gdv =

= 0 => STRUTTURA ISOSTATICA

- EQUILIBRIO GLOBALE DELLA STRUTTURA

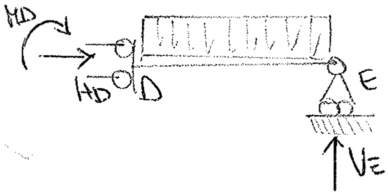
$\rightarrow)\ H_A - q\ell = 0 \qquad \boxed{H_A = q\ell}$

$\uparrow)\ V_A + V_F + V_E - 2q\ell = 0$

$\circlearrowleft A)\ -M_A + V_F(2\ell) - q\ell^2 - 2q\ell(2\ell+\ell) + V_E(4\ell) = 0$

Passo a determinare le equazioni interne.

TRATTO ED



$\rightarrow)\ H_D = 0$

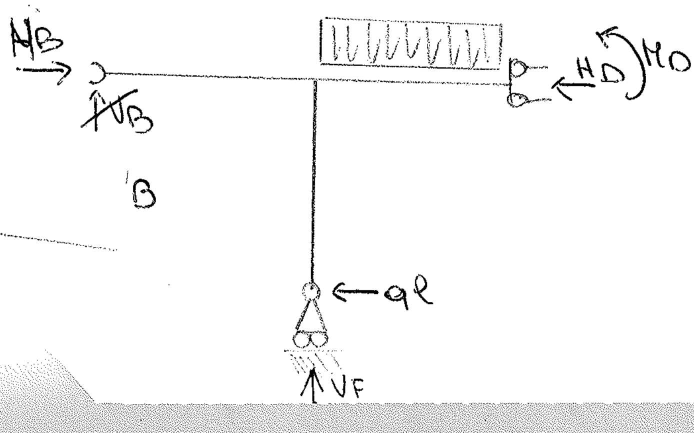
$\uparrow)\ V_E - q\ell = 0 \qquad \boxed{V_E = q\ell}$

$\circlearrowleft D)\ V_E(\ell) - \frac{q\ell^2}{2} - M_D = 0$

$-M_D = -V_E(\ell) + \frac{q\ell^2}{2}$

$M_D = q\ell^2 - \frac{q\ell^2}{2} \qquad \boxed{M_D = \frac{q\ell^2}{2}}$

TRATTO DCBF



$\rightarrow)\ H_B - H_D - q\ell = 0$

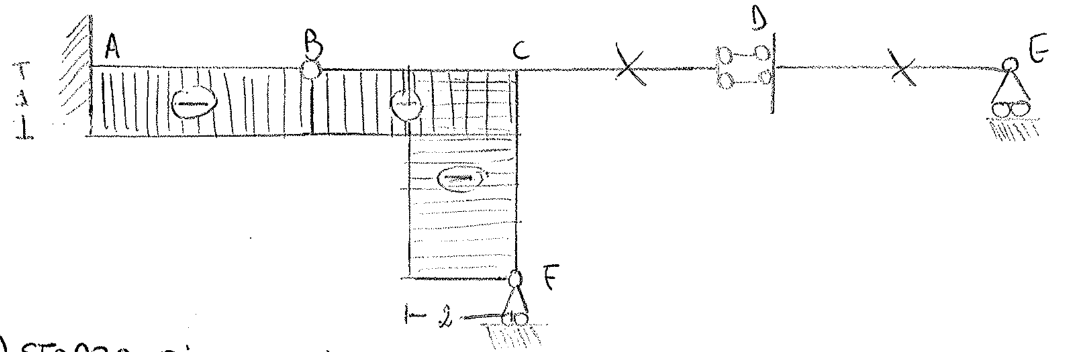
$\boxed{H_B = q\ell}$

$\uparrow)\ V_B + V_F - q\ell = 0$

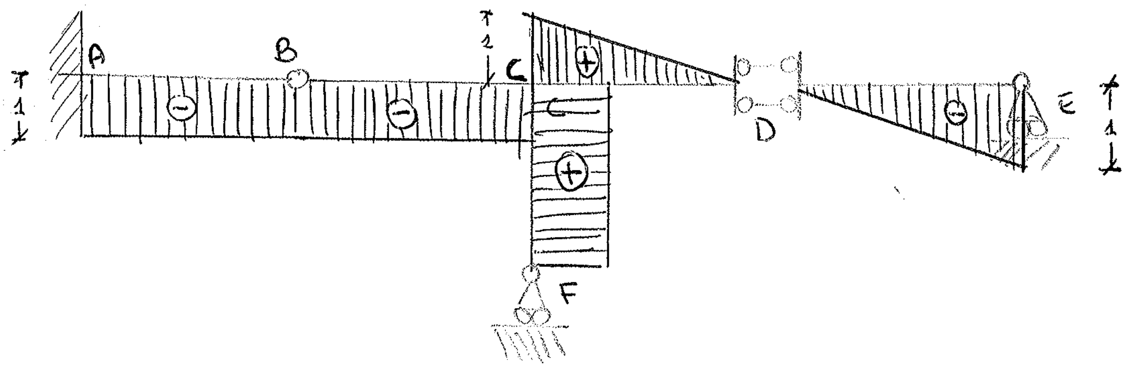
$\circlearrowleft B)\ -q\ell^2 + V_F(\ell) - q\ell^2\left(\ell + \frac{\ell}{2}\right) + M_D = 0$

$-q\ell^2 + V_F(\ell) - \frac{3}{2}q\ell^2 + \frac{q\ell^2}{2} = 0$

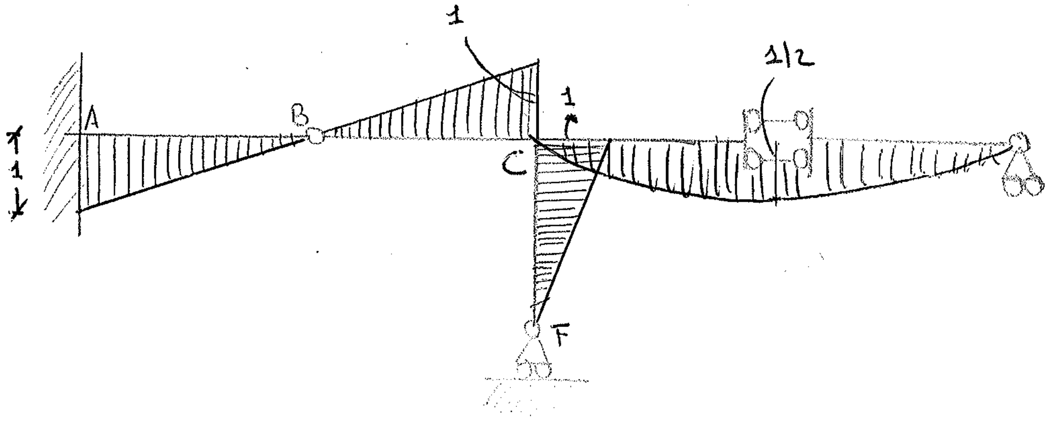
(N) SFORZO NORMALE $\quad N|q\ell$

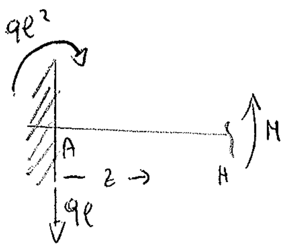


(T) SFORZO DI TAGLIO $\quad T|q\ell$



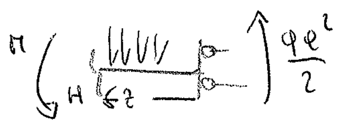
(M) MOMENTO FLETTENTE





(H) $M + q\ell z - q\ell^2 = 0$

$M = q\ell^2 - q\ell z$ (lineare



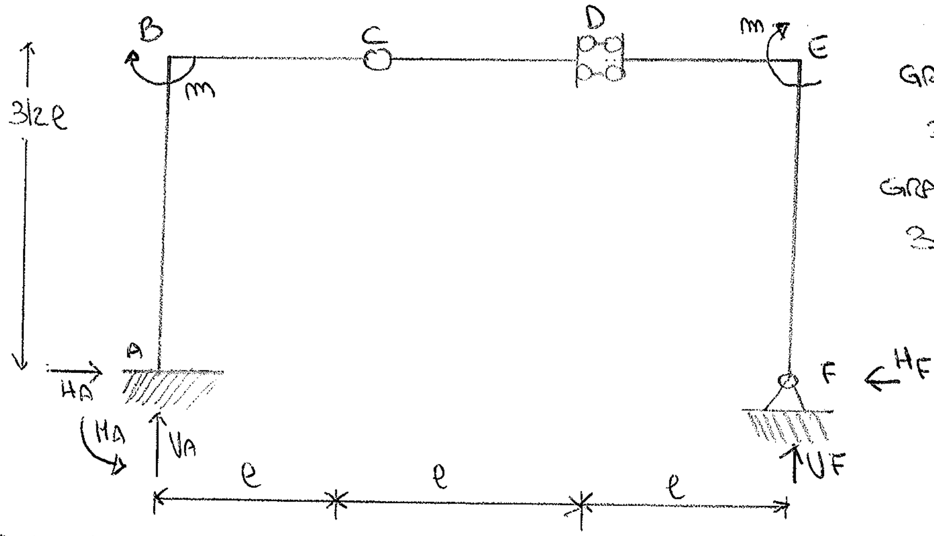
(H) $M - \frac{qz^2}{2} + \frac{q\ell^2}{2} = 0 \qquad M = \frac{qz^2}{2} - \frac{q\ell^2}{2}$

$z = \ell \quad M = 0$

$z = \frac{\ell}{2} \rightarrow \frac{q\ell^2}{8} - \frac{q\ell^2}{2}$

$\frac{1-4}{8} = -\frac{3}{8}$

ESERCIZIO 14 11/07/83 I



GRADI DI LIBERTÀ

$3 \times 3 = 9$ Gdl

GRADI DI VINCOLO

$3+2+2+2=6$ Gdv

struttura isostatica

- EQUILIBRIO GLOBALE DELLA STRUTTURA

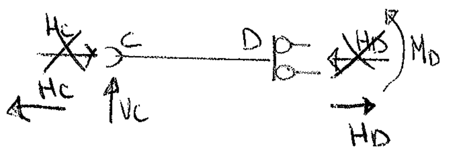
$\uparrow)\ V_A + V_F = 0$

$\rightarrow)\ H_A - H_F = 0$

$\circlearrowleft A)\ M_A + 2m + \cancel{H_F \frac{3}{2} l} - V_F\, 3l = 0$

Passo a cercare le reazioni interne, esplodendo la struttura

TRATTO CD

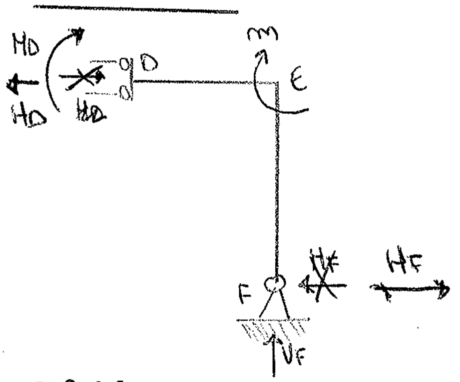


$\rightarrow)\ H_C - H_D = 0 \qquad H_C = H_D \qquad \boxed{H_C = -\frac{2}{3}\frac{m}{l}}$ cambio verso

$\uparrow)\ \boxed{V_C = 0}$

$\circlearrowleft D)\ +M_D \cancel{+} V_C \cdot l = 0 \qquad \boxed{M_D = 0}$

TRATTO DEF



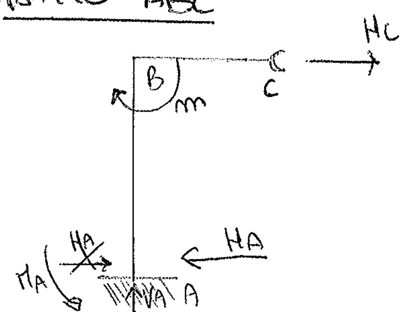
$\uparrow)\ \boxed{V_F = 0}$

$\rightarrow)\ H_F = H_D \qquad \boxed{H_F = -\frac{2}{3}\frac{m}{l}}$ cambio verso

$\circlearrowleft F)\ -M_D - m \cancel{+} H_D \frac{3}{2} l = 0 \qquad -H_D \frac{3}{2} l = m$

$H_D \frac{3}{2} l = -m \qquad \boxed{H_D = -\frac{2}{3}\frac{m}{l}}$ cambio verso

TRATTO ABC

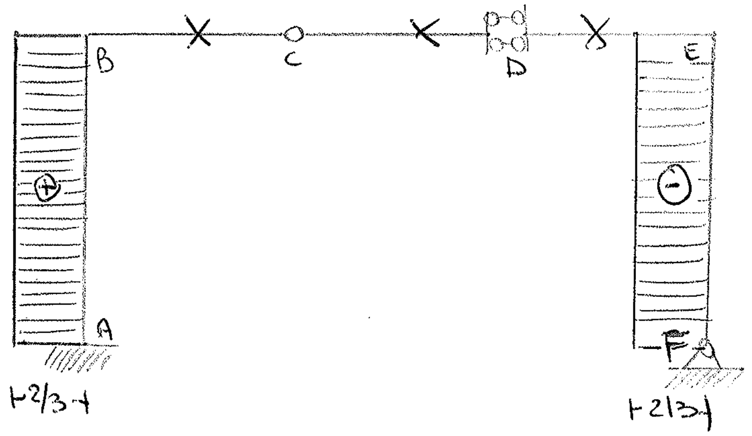


ricavo M_A dall'equilibrio globale

$\cancel{M_A = -2m - H_F \frac{3}{2} l}$

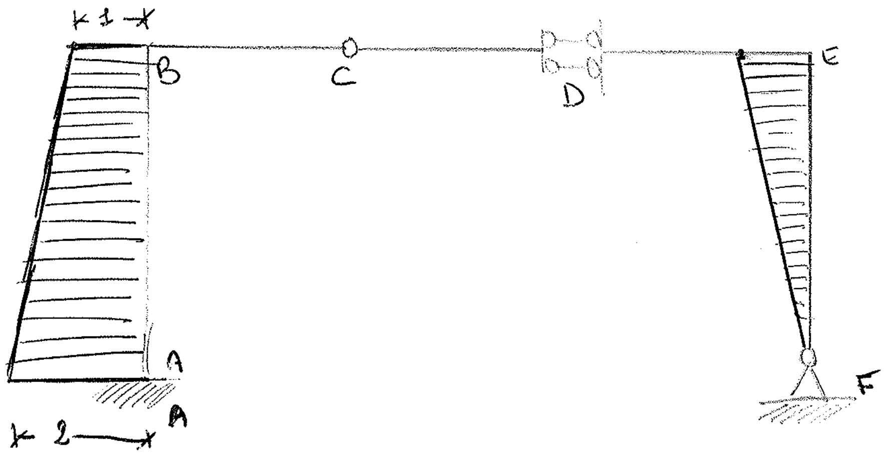
$\boxed{M_A = +2m}$

(T) SFORZO DI TAGLIO



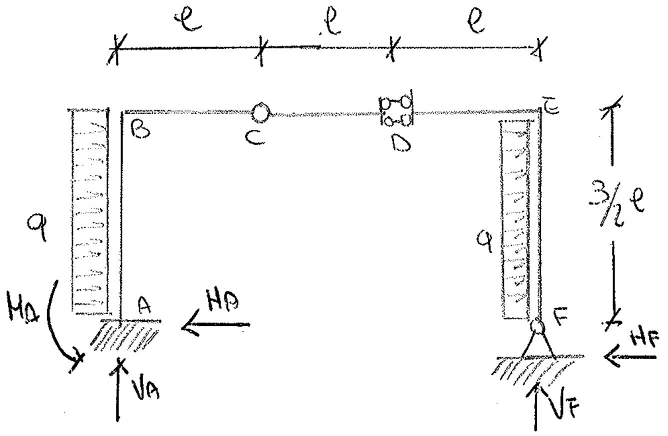
$\frac{Te}{m}$

(M) MOMENTO FLETTENTE



$\frac{N}{m}$

ESERCIZIO (15) 11/07/89 - II



GRADI DI VINCOLO:

3(aste) × 3 = 9 Gdv

GRADI DI LIBERTÀ:

3+2+2+2 = 9 Gdℓ

→ STRUTTURA ISOSTATICA

- REAZIONI DI EQUILIBRIO GLOBALE

↑) $V_A + V_F = 0$

→) $2q\ell - H_A - H_F = 0$

(A) $M_A - \frac{3}{2}q\ell\left(\frac{3}{4}\ell\right) - \frac{3}{2}q\ell\left(\frac{3}{4}\ell\right) + V_F(3\ell) = 0$

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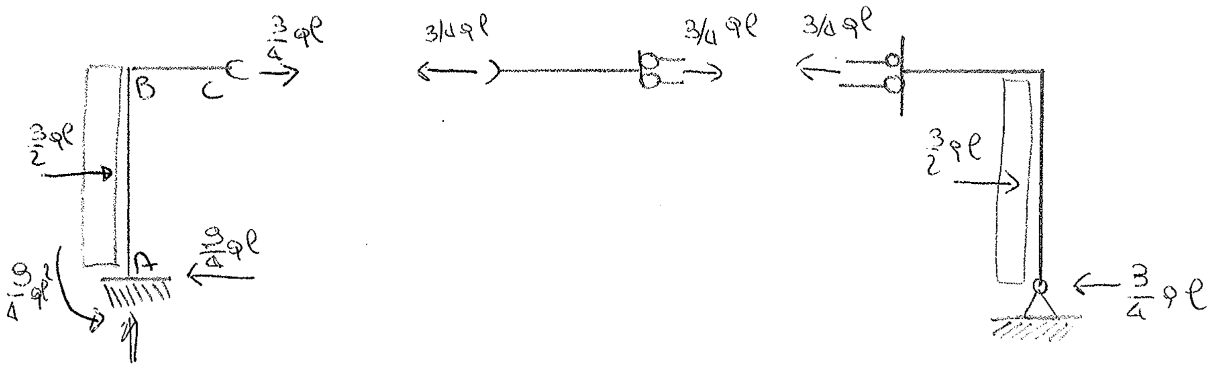
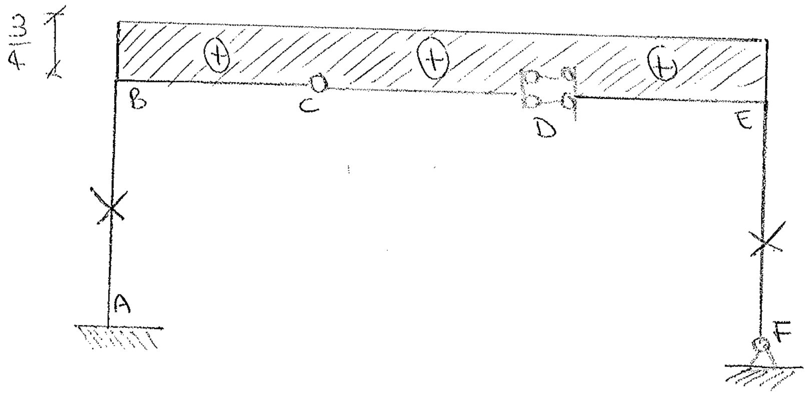
- DIAGRAMMA CORPO LIBERO

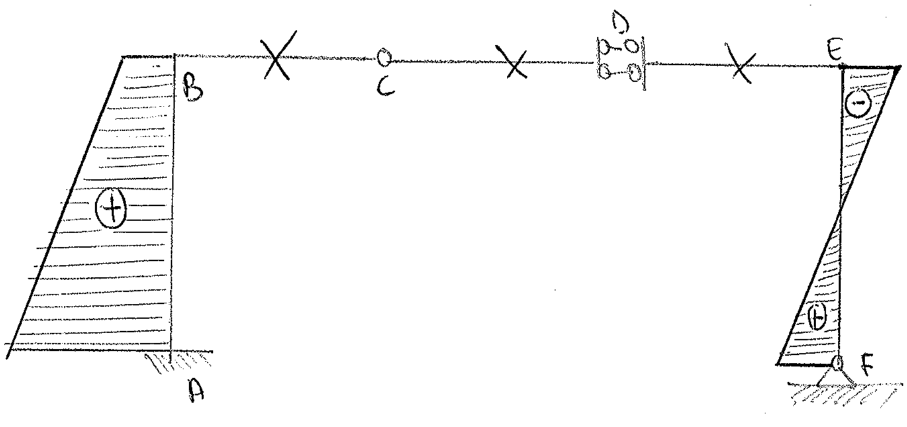


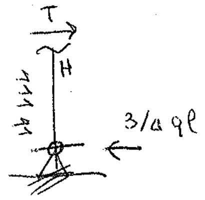
N SFORZO NORMALE



$\frac{N}{ql}$

T SFORZO DI TAGLIO

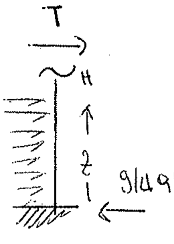




$T + qz - \frac{3}{4}ql = 0$

$T = -qz + \frac{3}{4}ql$

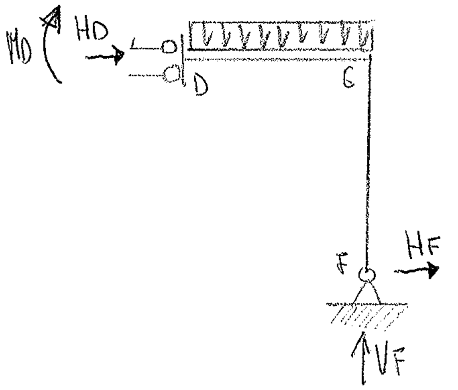
$T = q\left(-z + \frac{3}{4}l\right)$



$\rightarrow)\ T + qz - \frac{9}{4}ql = 0 \qquad T = \frac{9}{4}ql - qz$

$T(z = 3/2\,l) \quad T = \frac{3}{4}ql$

TRATTO DEF



$\uparrow)\ V_F - q\ell = 0 \qquad \boxed{V_F = q\ell}$

$\rightarrow)\ H_D + H_F = 0$

$(F)\ -M_D - H_D\left(\frac{3}{2}\ell\right) + \frac{q\ell^2}{2} = 0$

$-H_D\left(\frac{3}{2}\ell\right) = M_D - \frac{q\ell^2}{2}$

$H_D\left(\frac{3}{2}\ell\right) = \frac{q\ell^2}{2} - M_D \qquad H_D\left(\frac{3}{2}\ell\right) = 0 \qquad \boxed{H_D = 0}$

$\rightarrow)\ H_D + H_F = 0 \qquad \boxed{H_F = 0}$

dall'equazione alla traslazione orizzontale ricavo H_C:

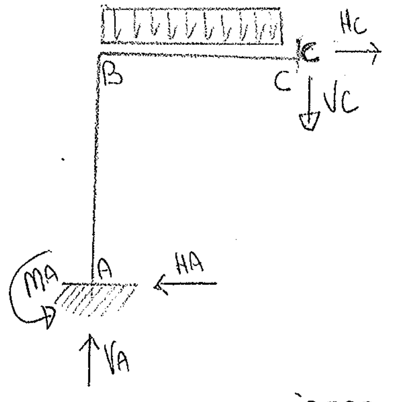
$\rightarrow)\ H_C = H_D \qquad \boxed{H_C = 0}$

Dall'equazione alla rotazione attorno ad A dell'equilibrio globale ricavo M_A:

$(A)\ M_A - 3q\ell\left(\ell + \frac{\ell}{2}\right) + V_F(3\ell) = 0 \qquad M_A = 3q\ell\left(\ell + \frac{\ell}{2}\right) - 3q\ell^2 \qquad M_A = \frac{9}{2}q\ell^2 - 3q\ell^2$

$\boxed{M_A = \frac{3}{2}q\ell^2}$

TRATTO ABC



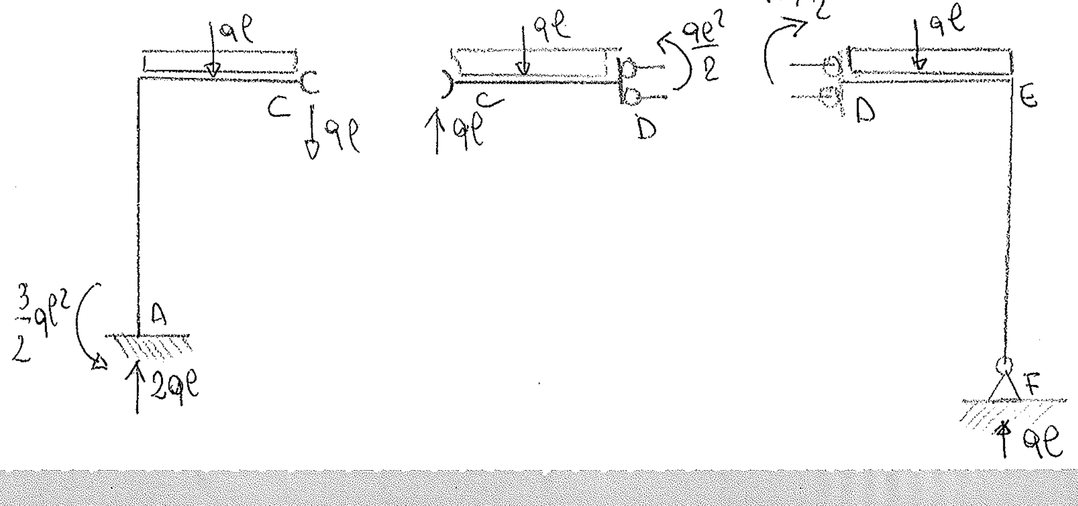
$\uparrow)\ V_A - q\ell - V_C = 0 \qquad \boxed{V_A = 2q\ell}$

$\rightarrow)\ H_A = H_C \qquad \boxed{H_A = 0}$

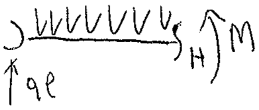
$(A)\ M_A - \frac{q\ell^2}{2} - V_C(\ell) = 0$

$M_A = \frac{q\ell^2}{2} + q\ell^2 \qquad M_A = \frac{3}{2}q\ell^2$ ok!

- DIAGRAMMA CORPO LIBERO



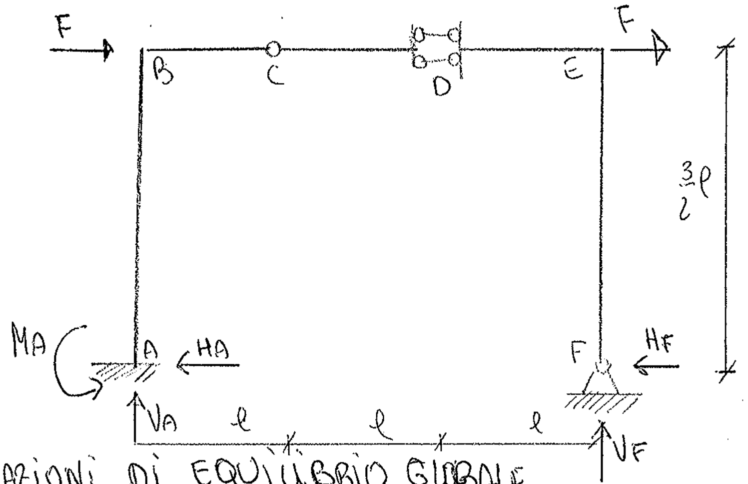
55



$\overset{\curvearrowleft}{H})\ M - q\ell z + \frac{qz^2}{2} = 0 \qquad M = q\ell z - \frac{qz^2}{2}$

$M(z=0)\quad M=0 \qquad M(z=\ell) = \frac{q\ell^2}{2} \qquad M(z=\frac{\ell}{2}) = \frac{q\ell^2}{2} - \frac{q\ell^2}{8} = \frac{3}{8}q\ell^2$

ESERCIZIO 17 11/07/89 - IV



- GRADI DI VINCOLO

$3(\text{aste}) \times 3 = 9\ \text{Gdv}$

- GRADI DI LIBERTÀ

$3+2+2+2 = 9\ \text{Gdl}$

→ struttura isostatica

- REAZIONI DI EQUILIBRIO GLOBALE

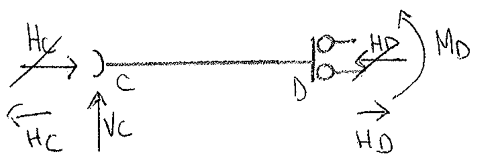
$\uparrow)\ V_A + V_F = 0$

$\rightarrow)\ F - H_A + F - H_F = 0$

$\overset{\curvearrowleft}{A})\ M_A - F(\frac{3}{2}\ell) - F(\frac{3}{2}\ell) + V_F(3\ell) = 0$

Passo a determinare le reazioni interne, esplodendo la struttura

TRATTO DC

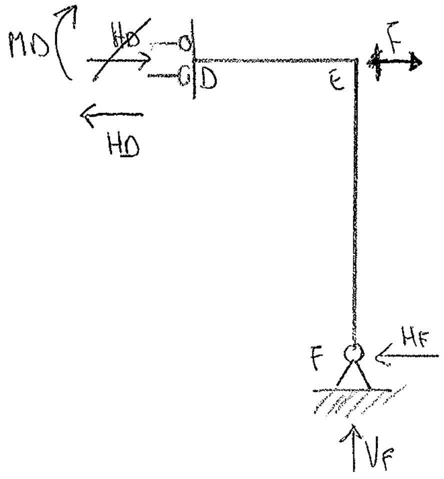


$\uparrow)\ \boxed{V_C = 0}$

$\rightarrow)\ H_C - H_D = 0$

$\overset{\curvearrowleft}{D})\ M_D - V_C(\ell) = 0 \qquad \boxed{M_D = 0}$

TRATTO DEF



$\uparrow)\ \boxed{V_F = 0}$

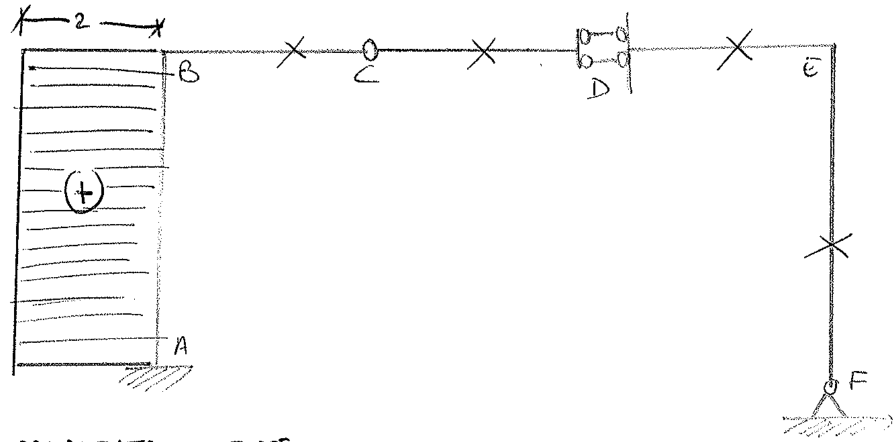
$\rightarrow)\ H_D + F - H_F = 0 \qquad \boxed{H_F = 0}$

$\overset{\curvearrowleft}{F})\ -F(\frac{3}{2}\ell) - H_D(\frac{3}{2}\ell) = 0$

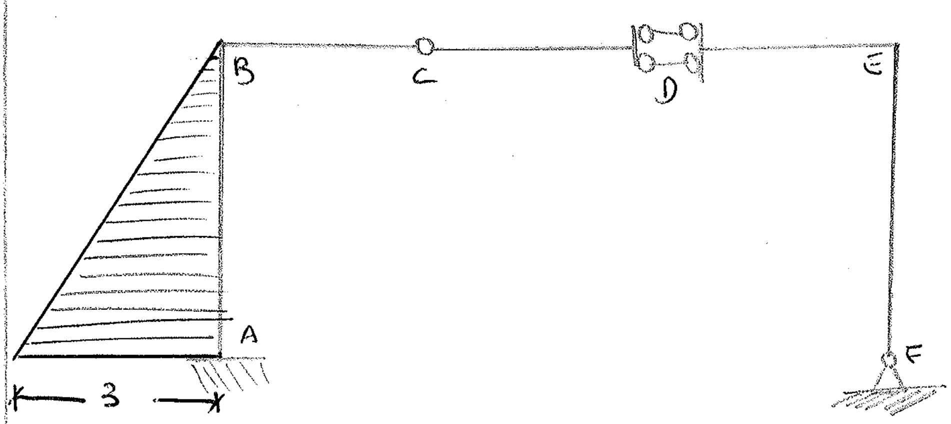
$-H_D(\frac{3}{2}\ell) = F(\frac{3}{2}\ell) \qquad H_D = -F\cancel{(\frac{3}{2}\ell)}\cancel{(\frac{2}{3}\ell)}$

$\boxed{H_D = -F}$

(T) SFORZO DI TAGLIO T/F

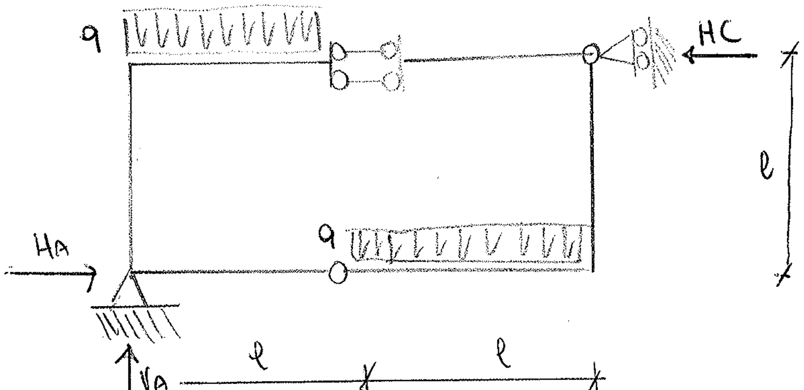


(M) MOMENTO FLETTENTE



STRUTTURE ISOSTATICHE A MAGLIA CHIUSA

ESERCIZIO (1) LEZIONE DEL 26/03/2014



- GRADI DI LIBERTA': 3(aste) × 3 = 9 GdL -
- GRADI DI VINCOLO: 2 + 2 + 2 + 2 + 1 = 9 GdV =

0 ⇒ STRUTTURA ISOSTATICA

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$\uparrow)$ $V_A + V_D - ql = 0$

$\rightarrow)$ $H_A - H_D - H_B = 0$

B) $M_B + \frac{ql^2}{2} + H_A(l) - V_A(l) - H_D(l) = 0$

$\uparrow)$ $V_B = ql - V_A$ $\quad \boxed{V_D = -ql}$ cambio verso.

Trovato V_D posso passare a determinare V_C'' e H_C'' grazie alle equazioni del tratto CD

$\uparrow)$ $V_C'' = ql - V_D$ $\quad \boxed{V_C'' = 0}$

D) $-\frac{ql^2}{2} + H_C''(l) = 0$ $\quad H_C''(l) = \frac{ql^2}{2}$ $\quad \boxed{H_C'' = \frac{ql}{2}}$

$\boxed{H_D = H_C'' = \frac{ql}{2}}$

Determino V_A e H_B grazie all'equazioni del tratto BD

$\uparrow)$ $V_A = ql - V_D$ $\quad ql - (-ql) = 2ql$ OK! VERIFICATO

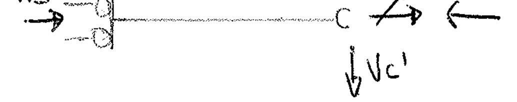
B) $M_B + \frac{ql^2}{2} + H_A(l) - V_A(l) - H_D(l) = 0$

$M_B = -\frac{ql^2}{2} - H_A(l) + V_A(l) + H_D(l)$ $\quad M_B = \cancel{-\frac{ql^2}{2}} \cancel{-2ql^2} \cancel{+2ql^2} \cancel{+\frac{ql^2}{2}}$

$\boxed{M_B = 0}$

$\rightarrow)$ $-H_B = -H_A + H_D$ $\quad H_B = H_A - H_D = 2ql - \frac{ql}{2}$ $\quad \boxed{H_B = \frac{3ql}{2}}$

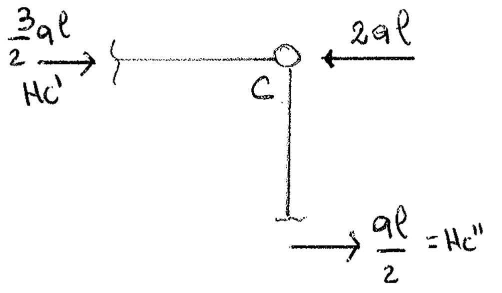
TRATTO BC



$\uparrow)$ $V_C' = 0$

$\rightarrow)$ $+H_C' + H_B = 0$ $\quad H_C' = -H_B$ $\quad H_C' = -\frac{3}{2}ql$

EQUILIBRIO AL NODO C

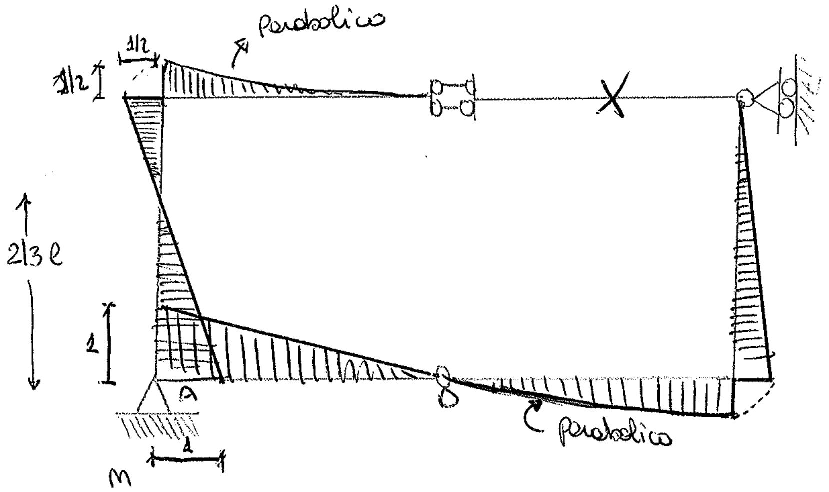


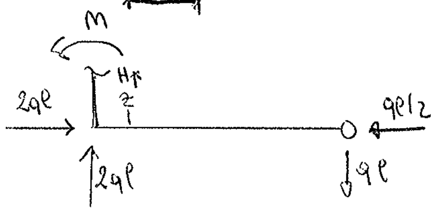
$-2ql + \frac{3}{2}ql + \frac{ql}{2} = 0$

$0 = 0$

NODO in equilibrio.

(M) MOMENTO FLETTENTE $M/q\ell^2$





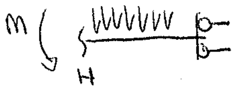
$(H)\ M + 2q\ell(z) - q\ell^2 - \frac{q\ell}{2} z = 0$

$M = q\ell^2 - 2q\ell z + \frac{q\ell}{2} z$

$z=0 \quad M = q\ell^2$

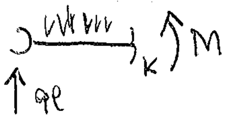
$(z=\ell) \quad M = q\ell^2 - 2q\ell^2 + \frac{q\ell^2}{2} \qquad M = -\frac{q\ell^2}{2} \qquad \cancel{z=\ell/2} \quad M=0 \qquad 2q\ell(z) - q\ell^2 - \frac{q\ell}{2}(z) = 0$

$z\left(2q\ell - \frac{q\ell}{2}\right) = q\ell^2 \qquad z\left(\frac{3}{2}q\ell\right) = q\ell^2 \qquad z = \frac{2}{3}\ell$



$(H)\ M - q\frac{z^2}{2} = 0 \qquad M = \frac{qz^2}{2} \qquad M(z=0) \quad M=0$

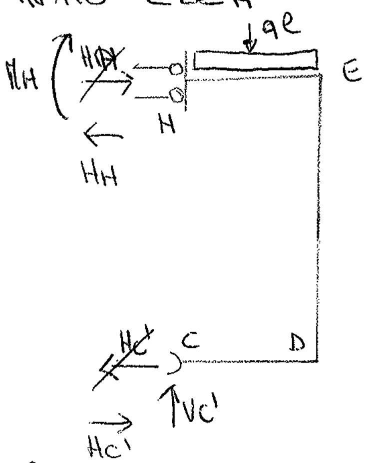
$M(z=\ell) \Rightarrow M = \frac{q\ell^2}{2} \qquad M\left(z=\frac{\ell}{2}\right) \quad M = \frac{q\ell^2}{8}$



$(K)\ M - q\ell(z) + \frac{qz^2}{2} = 0 \qquad M = q\ell z - \frac{qz^2}{2}$

$M(z=0) \quad M=0 \qquad M(z=\ell) \rightarrow M = \frac{q\ell^2}{2} \qquad M\left(z=\frac{\ell}{2}\right) \rightarrow M = \frac{q\ell^2}{2} - \frac{q\ell^2}{8} = \frac{3}{8}q\ell^2$

TRATTO CDEH



$\uparrow)\ \boxed{V_C' = q\ell}$

$\rightarrow)\ H_H - H_C' = 0$

$\circlearrowleft C)\ -\frac{q\ell^2}{2} - M_H - H_H(2\ell) = 0$

Ricavato V_C' sfrutto l'equazione di equilibrio ai nodi per calcolare V_C'':

$\uparrow)\ V_C - V_C' + V_C'' = 0 \qquad V_C'' = V_C' - V_C \qquad V_C'' = q\ell - q\ell \qquad \boxed{V_C'' = 0}$

Da V_C'' posso ricavarmi V_A'' grazie al tratto ABC

$\boxed{V_A'' = V_C'' = 0}$

Noto V_A'' grazie all'equazione alla rotazione attorno al punto C nel tratto ABC, posso ricavarmi H_A''

$H_A''(\ell) = V_A''(\ell) \qquad \boxed{H_A'' = 0}$

Dall'equazione dell'equilibrio al nodo A mi ricavo H_A'

$\rightarrow)\ H_A'' + H_A' - H_A = 0 \qquad H_A' = H_A - H_A'' \qquad \boxed{H_A' = \frac{q\ell}{2}}$

Dall'equaz. alla traslazione orizzontale nel tratto ABC, ricavo H_C''

$H_C'' = H_A'' \qquad \boxed{H_C'' = 0}$

Sfrutto l'equaz. dell'equilibrio ai nodi.

$\rightarrow)\ H_C' = -\frac{q\ell}{2} + 0 \qquad \boxed{H_C' = -\frac{q\ell}{2}}$ cambio verso

trovo H_H e M_H:

$H_H = H_C' = \boxed{H_H = \frac{-q\ell}{2}}$

$\circlearrowleft C)\ -\frac{q\ell^2}{2} - M_H - H_H(2\ell) = 0 \qquad -M_H = H_H(2\ell) + \frac{q\ell^2}{2} \qquad M_H = -H_H(2\ell) - \frac{q\ell^2}{2}$

$M_H = -\left(-\frac{q\ell}{2}\right)(2\ell) - \frac{q\ell^2}{2} \qquad \boxed{M_H = \frac{q\ell^2}{2}}$

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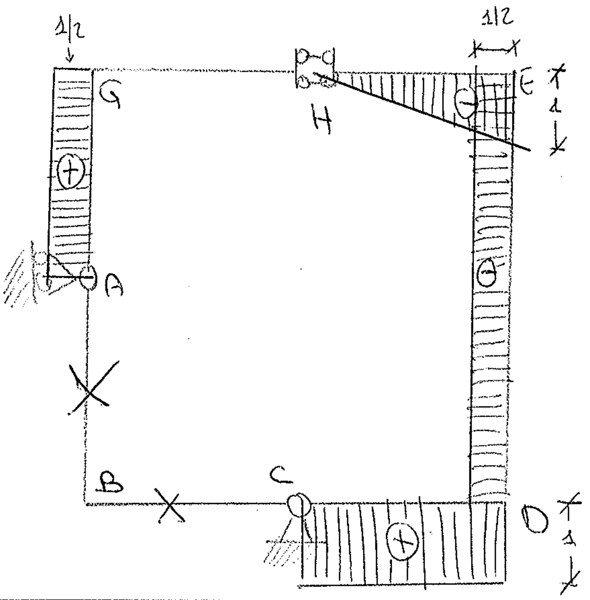
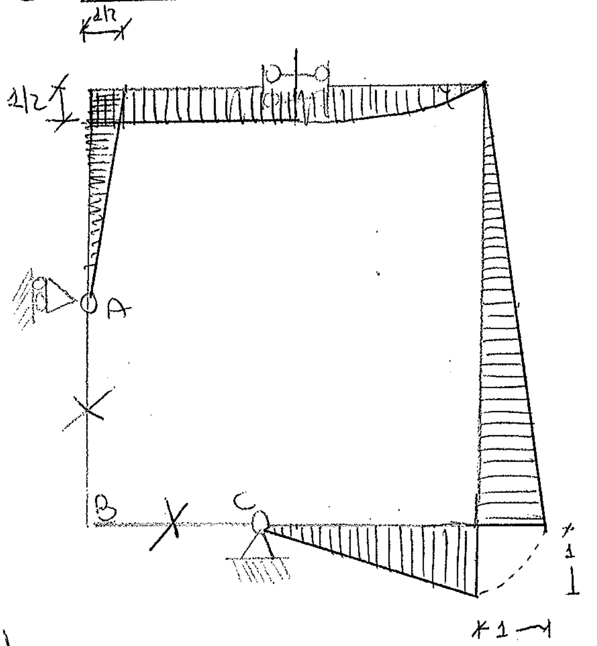
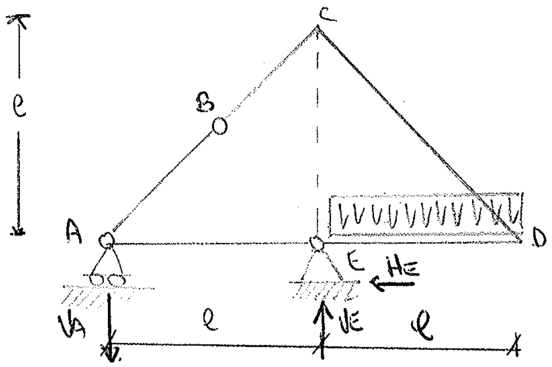
(T) SFORZO DI TAGLIO $T/q\ell$



(M) MOMENTO FLETTENTE $M/q\ell^2$



ESERCIZIO (3) LEZIONE DEL 31/03/14



GRADI DI LIBERTÀ:

3 (aste) × 3 = 9 GdL

GRADI DI VINCOLO

2 + 1 + 2 + 2 + 2 = 9 GdV.
esterni

→ struttura isostatica

EQUAZIONI DI EQUILIBRIO GLOBALE DELLA STRUTTURA

↑) $-V_A + V_E - q\ell = 0$

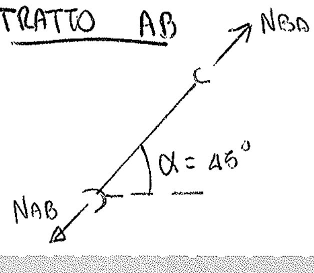
→) $-H_E = 0$

A) $+V_E \cdot \ell - q\ell^2 (\ell + \ell/2) = 0$ $\quad V_E \cdot \cancel{\ell} = q\ell^{\cancel{2}}\left(\ell + \frac{\ell}{2}\right)$ $\quad \boxed{V_E = \frac{3}{2} q\ell}$

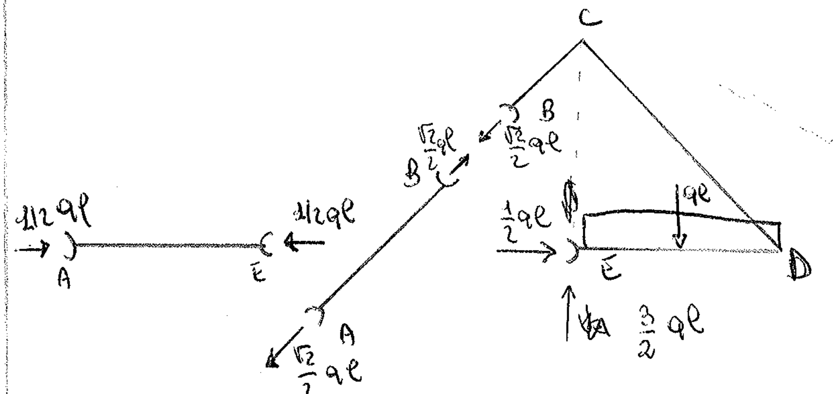
↑) $-V_A + V_E - q\ell = 0$ $\quad -V_A + \frac{3}{2} q\ell - q\ell = 0$ $\quad -V_A = -\frac{1}{2} q\ell$ $\quad \boxed{N_A = \frac{1}{2} q\ell}$

Una volta determinate le reazioni esterne di equilibrio globale occorre esplodere la struttura e andare a determinare l'equilibrio ai ~~nodi~~ nodi.

TRATTO AB



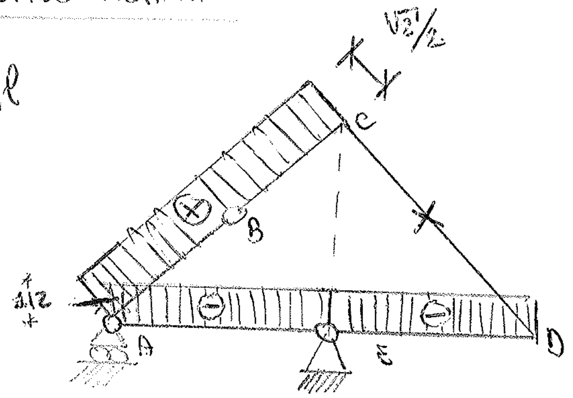
- DIAGRAMMA CORPO LIBERO



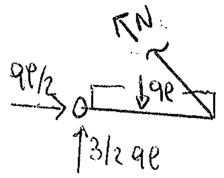
Tramite l'equilibrio ai nodi e il diagramma del corpo libero, si può osservare che l'asta AE è un'asta compressa, mentre quella AB è tesa.

(N) SFORZO NORMALE

N/ql



N.B. ASTA CD

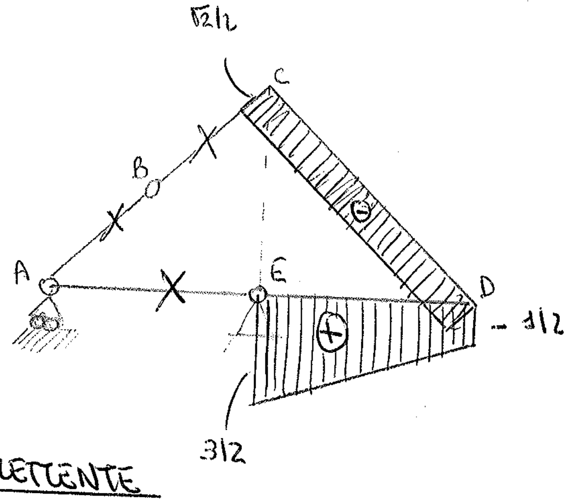


$\uparrow)\ 3/2\,ql - ql = 1/2\,ql$

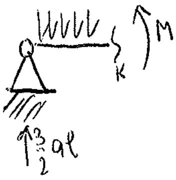
le due reaz. di $ql/2$ cui generano una reaz. ortogonale a CD e quindi ci da taglio e non sforzo normale.

(T) SFORZO DI TAGLIO

T/ql



(M) MOMENTO FLETTENTE



$M - \frac{3}{2}qlz + \frac{qz^2}{2} = 0$

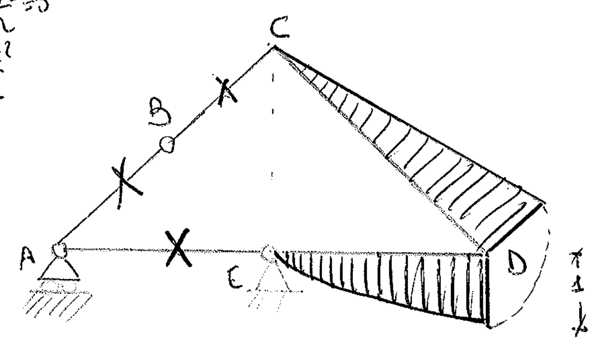
$M = \frac{3}{2}qlz - \frac{qz^2}{2}$

$M(z=0) : M=0$

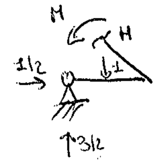
$M(z=l) \Rightarrow M = ql^2$

$M(z=l/2) \Rightarrow M = \frac{3}{4}ql^2 - \frac{ql^2}{8}$

$\frac{6-1}{8} = \frac{5}{8}ql^2$



M/ql^2



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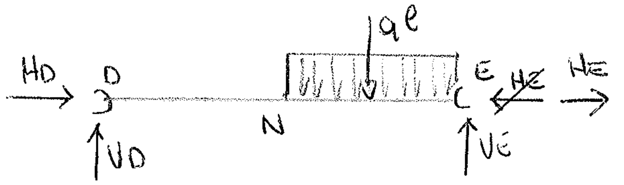
TRATTO DE



$\uparrow)\ V_D + V_E - q\ell = 0$

$\rightarrow)\ H_D - H_E = 0$

$\circlearrowleft E)\ \frac{q\ell^2}{2} - V_D 2\ell = 0$

$-V_D 2\ell = -\frac{q\ell^2}{2}$ $\quad V_D 2\ell = \frac{q\ell^2}{2}$ $\quad \boxed{V_D = \frac{q\ell}{4}}$

$\uparrow)\ V_E = q\ell - V_D = q\ell - \frac{1}{4}q\ell \Rightarrow \boxed{V_E = \frac{3}{4}q\ell}$

Una volta determinata V_D sfruttando le equazioni del tratto ADC mi ricavo le altre reaz. vincolari:

$\uparrow)\ V_A - V_D - V_C = 0$ $\quad -V_C = V_D - V_A$ $\quad V_C = V_A - V_D = \frac{3}{8}q\ell - \frac{q\ell}{4} = \frac{3-2}{8}q\ell$

$\boxed{V_C = \frac{1}{8}q\ell}$

$\boxed{H_E = H_D = -\frac{1}{4}q\ell}$ cambio verso, non uso H_E in modulo perchè fa riferimento alle equaz. prec.

~~$\circlearrowleft D)\ -H_A\ell - V_A\ell + V_C(\ell) + H_C(2\ell) = 0$~~

$\circlearrowleft D)\ \cancel{-H_A\cdot\ell} - V_A\cdot\ell - V_C\cdot\ell - H_C(2\ell) = 0$ $\quad -H_C(2\ell) = V_A\cdot\ell + V_C\cdot\ell$

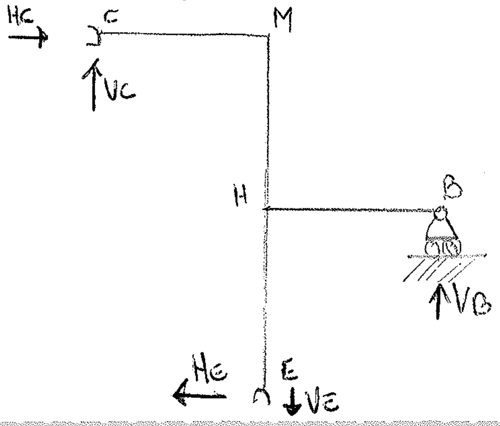
$H_C(2\ell) = -V_A\cdot\ell - V_C\cdot\ell$ $\quad 2H_C = -V_A - V_C$

$H_C = -\frac{V_A}{2} - \frac{V_C}{2} = -\frac{3}{16}q\ell - \frac{1}{16}q\ell$ $\quad \boxed{H_C = -\frac{1}{4}q\ell}$ cambio verso della reazione

$\rightarrow)\ \cancel{H_A} + H_C - H_D = 0$ $\quad H_D = H_C$ $\quad \boxed{H_D = -\frac{1}{4}q\ell}$ cambio verso della reazione

N.B.| In questo caso nella reazione H_C non e' stata messa in modulo, ma con il suo verso, perchè si faceva riferimento alle equazioni trattate all'inizio.

TRATTO CBE



Imposto già i versi corretti delle reazioni in tale tratto e faccio le verifiche

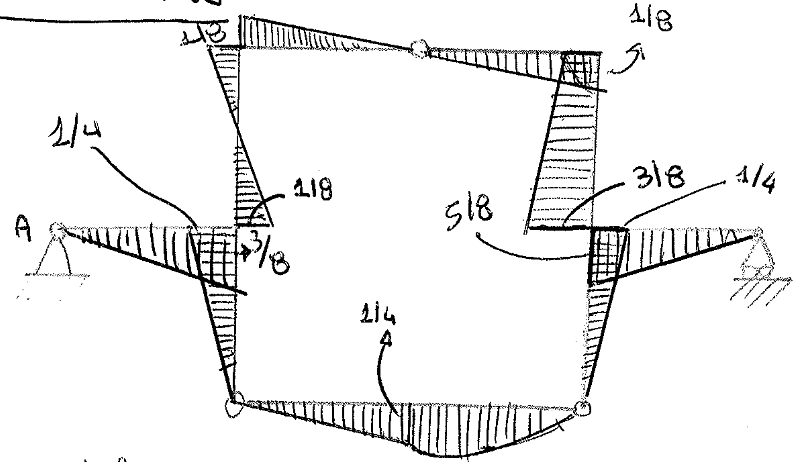
$\uparrow)\ V_C + V_B - V_E = 0$

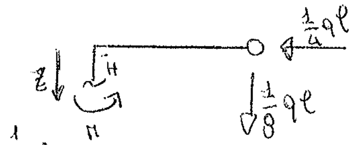
$\rightarrow)\ H_C - H_E = 0$

$\uparrow)\ \frac{1}{8}q\ell + \frac{5}{8}q\ell - \frac{3}{4}q\ell = 0$ $\quad \frac{3}{4}q\ell - \frac{3}{4}q\ell = 0$

$0 = 0$ ok!

(M) MOMENTO FLETTENTE

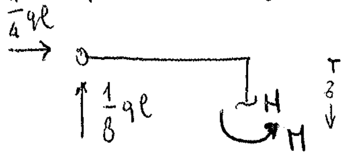




$M + \frac{1}{4} q\ell(z) - \frac{1}{8} q\ell^2 = 0 \qquad M = \frac{q\ell^2}{8} - \frac{q\ell}{4}(z)$

$M(z=\frac{\ell}{2}) \quad M=0$

$M(z=\ell) = M = -\frac{q\ell^2}{8}$

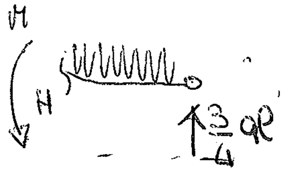


$M - \frac{1}{8} q\ell^2 - \frac{1}{4} q\ell(z) = 0$

$M = \frac{1}{8} q\ell^2 + \frac{1}{4} q\ell(z)$

$M(z=\ell) = \frac{3}{8} q\ell^2$

$M(z=0) = \frac{1}{8} q\ell^2$



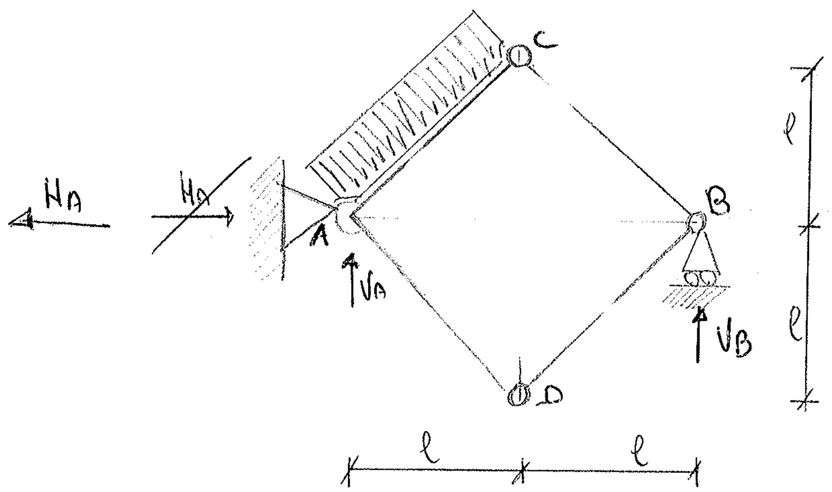
$M - \frac{qz^2}{2} + \frac{3}{4} q\ell(z) = 0 \qquad M = \frac{qz^2}{2} - \frac{3}{4} q\ell(z)$

~~M(z)~~ $M(z=0) \cdot M=0$

$M(z=\frac{\ell}{2}) = \frac{q\ell^2}{8} - \frac{3}{8} q\ell^2 = \frac{1-3}{8} q\ell^2 = -\frac{1}{4} q\ell^2$

$M(z=\ell) = \frac{q\ell^2}{2} - \frac{3}{4} q\ell^2 = \frac{2q\ell^2 - 3q\ell^2}{4} = -\frac{1}{4} q\ell^2$

ESERCIZIO (5) LEZIONE DELLO 07/04/14



- GRADI DI LIBERTÀ

3(aste) × 3 = 9 GdL

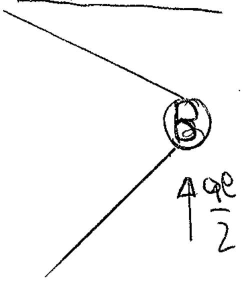
- GRADI DI VINCOLO

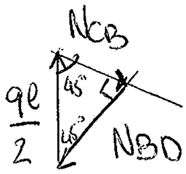
2 + 1 + 2 + 2 + 2 = 9
esterni interni

Gdv

STRUTTURA
ISOSTATICA

NODO B

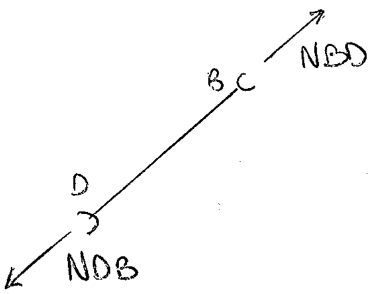




$$N_{CB} = \frac{ql}{2} \cdot \cos 45° = \frac{ql}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} ql$$

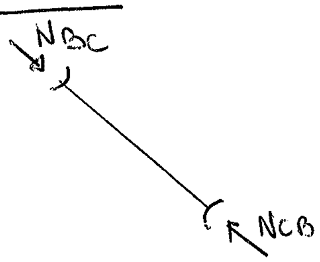
$$N_{BD} = \frac{ql}{2} \cdot \operatorname{sen} 45° = \frac{ql}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} ql$$

ASTA BD



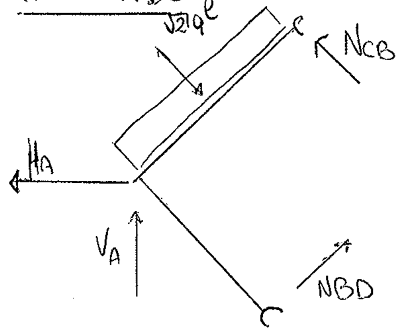
$$N_{DB} = N_{BD} = \frac{\sqrt{2}}{4} ql$$

ASTA CB



$$N_{BC} = N_{CB} = \frac{\sqrt{2}}{4} ql$$

ASTA ADC



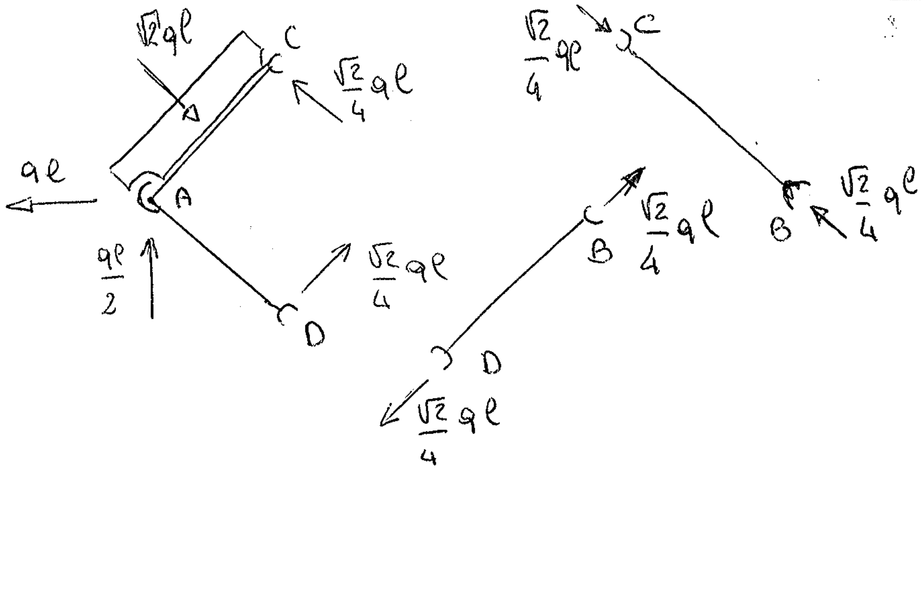
$N_{CB} = \frac{\sqrt{2}}{4} ql$

$N_{BD} = \frac{\sqrt{2}}{4} ql$

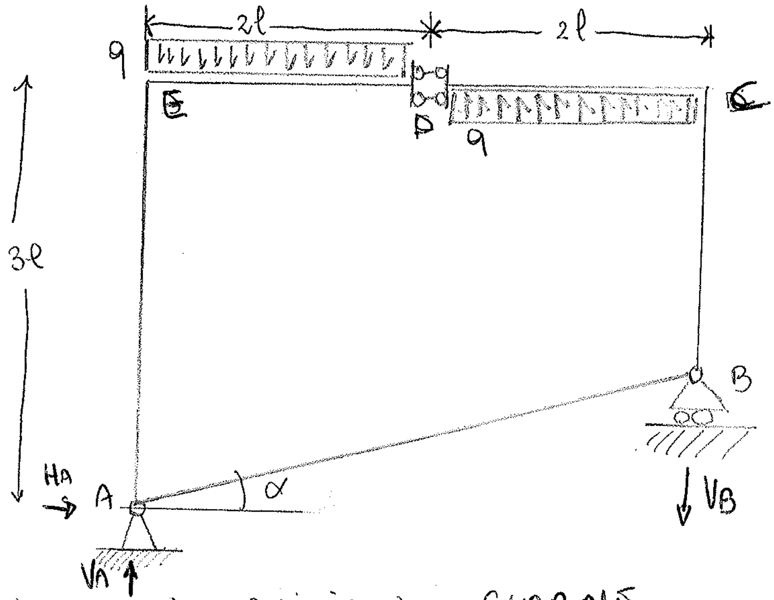
$V_A = \frac{ql}{2}$

$H_A = ql$

DIAGRAMMA CORPO LIBERO APPLICATO ALLA STRUTTURA



ESERCIZIO 6 LEZIONE 09/04/14



- GRADI DI LIBERTÀ

3(aste) × 3 = 9 GdL

- GRADI DI VINCOLO

$\underbrace{2+1}_{\text{esterni}} + \underbrace{2+2+2}_{\text{interni}} = 9$ GdV

STRUTTURA ISOSTATICA

- REAZIONI DI EQUILIBRIO GLOBALE

↑) $V_A \cancel{-2q\ell} \cancel{+2q\ell} - V_B = 0$

→) $H_A = 0$

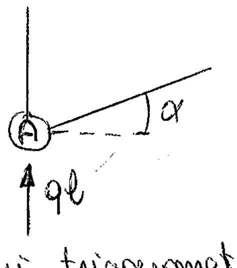
↶A) $-2q\ell^2 + 2q\ell(2\ell+\ell) - V_B \cdot 4\ell = 0$ $\qquad -2q\ell^2 + 6q\ell^2 - V_B \cdot 4\ell = 0$

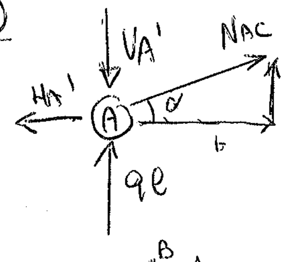
$-V_B\, 4\ell = -4q\ell^2 \qquad V_B\, 4\ell = 4q\ell^2 \qquad \boxed{V_B = q\ell}$

↑) $V_A = V_B \qquad \boxed{V_A = q\ell}$

Esplodo la struttura e passo a determinare le reazioni interne, osservando che le cerniere interne A e B sono caricate.

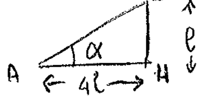
- EQUILIBRIO AL NODO A





↑) $q\ell - V_A' + N_{AC}\,\text{sen}\,\alpha = 0$

→) $-H_A' + N_{AC}\cos\alpha = 0$

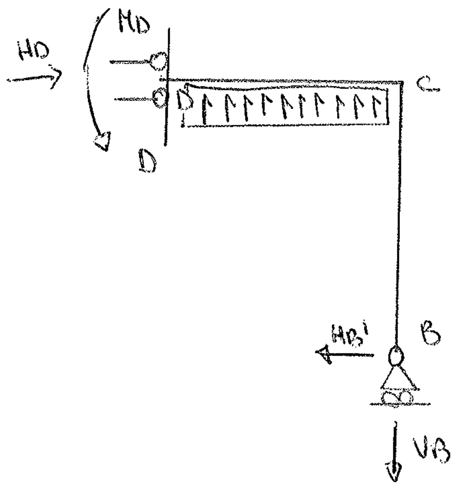


$\overline{AB} = \sqrt{(4\ell)^2 + \ell^2} = \sqrt{17\ell^2} = \sqrt{17}\,\ell$

Relazioni trigonometriche:

$4\ell = AB \cdot \cos\alpha \qquad \cos\alpha = \dfrac{4\ell}{\sqrt{17}\,\ell} = \dfrac{4}{\sqrt{17}} \qquad \text{sen}\,\alpha = \dfrac{\ell}{\sqrt{17}\,\ell} = \dfrac{1}{\sqrt{17}}$

trovato cos α e sen α, mi accorgo però che le incognite sono tre mentre le equazioni due, quindi il sistema non è risolvibile. occorre passare per un'altra via.



$\uparrow)\; +2q\ell - V_B = 0 \qquad 0=0$ ok!

$\rightarrow)\; H_D - H_B' = 0 \qquad 0=0$ ok!

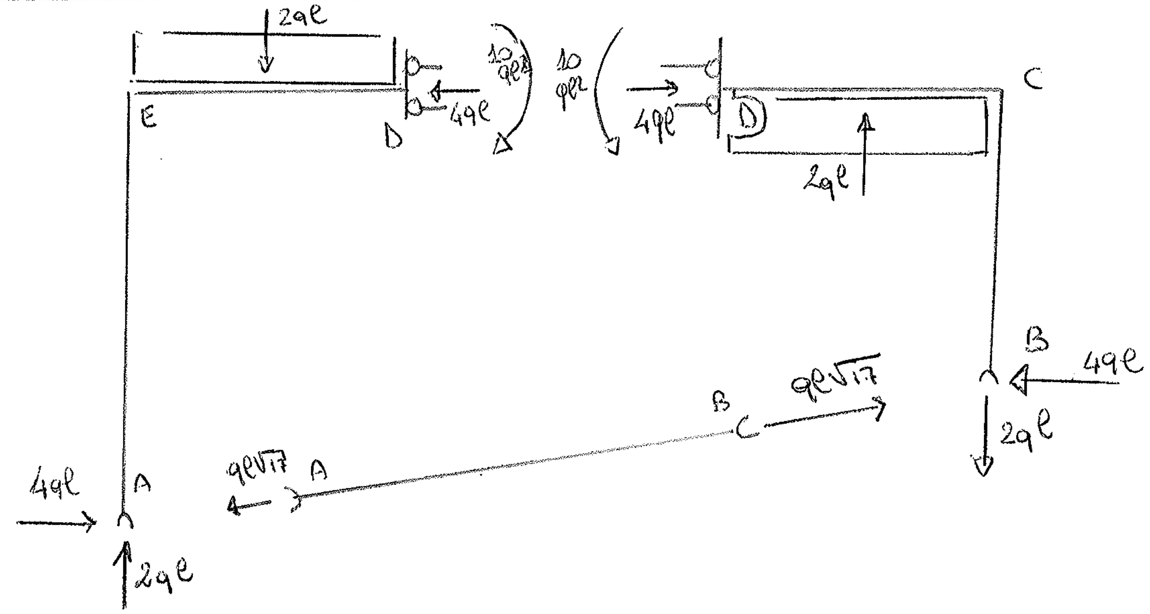
$\circlearrowright D)\; M_D + 2q\ell^2 - H_B'(2\ell) - V_B(2\ell) = 0$

$10q\ell^2 + 2q\ell^2 - 8q\ell^2 - 4q\ell^2 = 0$

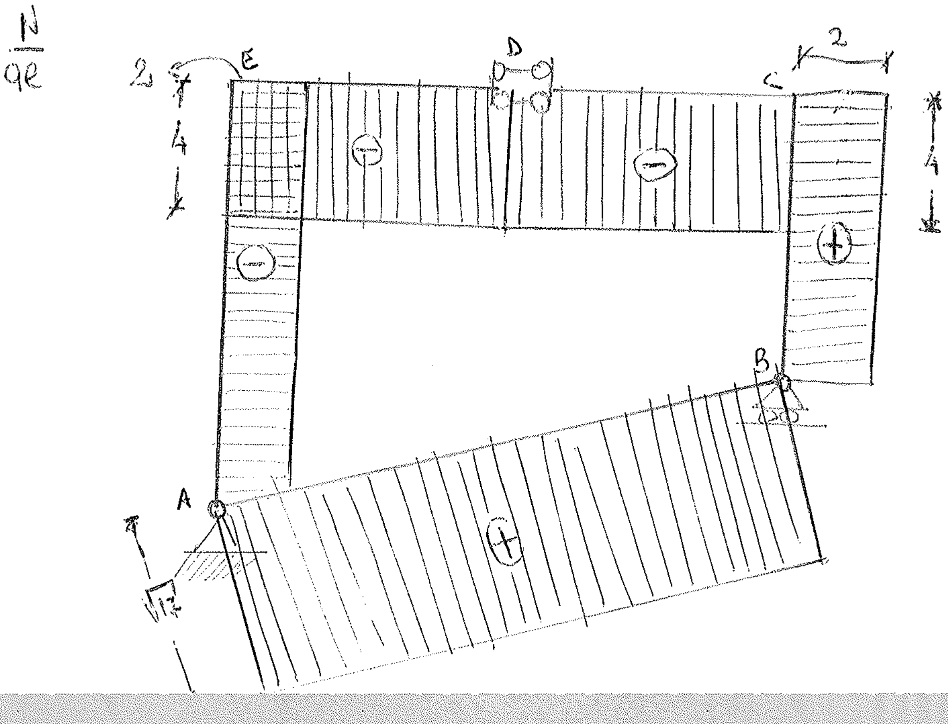
$0=0$ ok!

verificato!

DIAGRAMMA CORPO LIBERO APPLICATO ALLA STRUTTURA

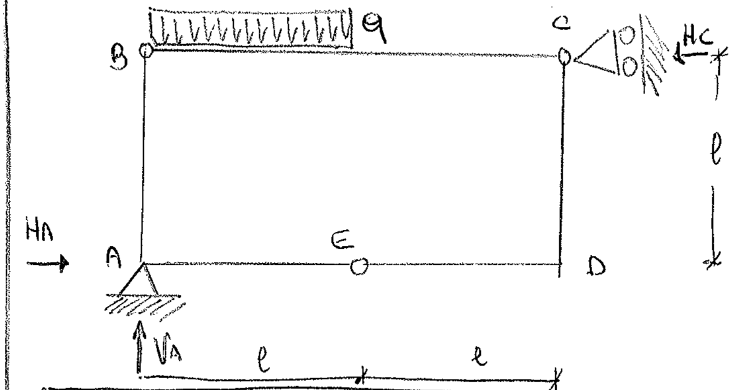


(N) SFORZO NORMALE



ESERCIZIO 7

• STRUTTURA



- GRADI DI LIBERTÀ:
  3(aste) × 3 = 9 gdl
- GRADI DI VINCOLO
  $2+1+2+2+2 = 9$ Gdv

⇒ STRUTTURA ISOSTATICA.

• REAZIONI DI EQUILIBRIO GLOBALE

$\uparrow)\ V_A - q\ell = 0 \qquad \boxed{V_A = q\ell}$

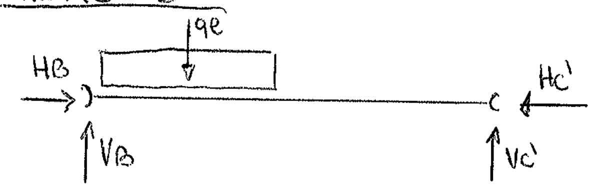
$\rightarrow)\ H_A - H_C = 0 \qquad H_A = H_C \qquad \boxed{H_A = \frac{q\ell}{2}}$

$\overset{\curvearrowleft}{A})\ -\frac{q\ell^2}{2} + H_C \cdot \ell = 0 \qquad H_C \cdot \ell = \frac{q\ell^2}{2} \qquad \boxed{H_C = \frac{q\ell}{2}}$

Passo a esplodere la struttura, e oltre a determinare le reazioni interne occorre anche verificare l'equilibrio al nodo C

• EQUILIBRIO NODO C

- TRATTO BC



$\uparrow)\ V_B + V_C' - q\ell = 0$

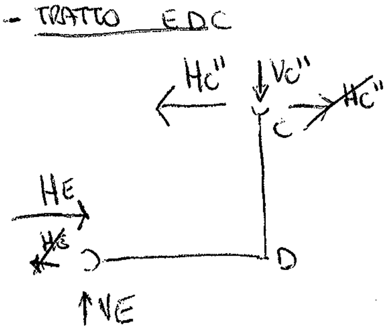
$\rightarrow)\ H_B - H_C'$

$\overset{\curvearrowleft}{C})\ -V_B(2\ell) + q\ell\left(\frac{3}{2}\ell\right) = 0$

$V_B(2\ell) = \frac{3}{2}q\ell^2 \qquad V_B 2 = \frac{3}{2}q\ell \qquad \boxed{V_B = \frac{3}{4}q\ell}$

$V_C' = q\ell - V_B \qquad \boxed{V_C' = \frac{1}{4}q\ell}$

- TRATTO EDC



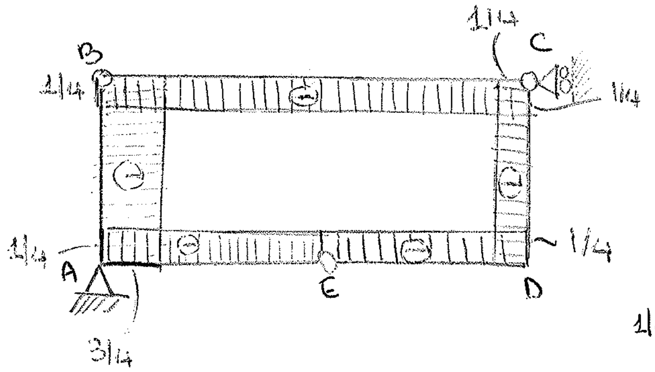
$\uparrow)\ V_E - V_C'' = 0$

$\rightarrow)\ H_C'' - H_E = 0$

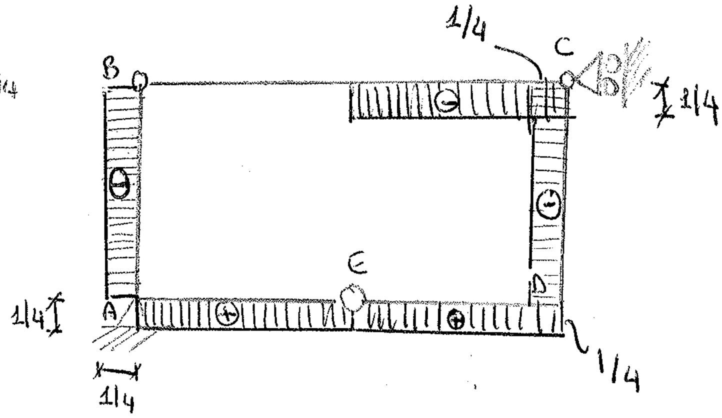
$\overset{\curvearrowleft}{E})\ H_E \cdot \ell + V_E \cdot \ell = 0$

(N) SFORZO NORMALE

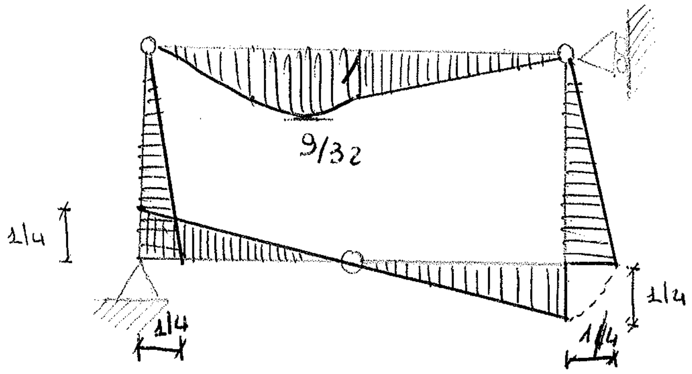
$\frac{N}{q\ell}$

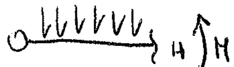


(T) SFORZO DI TAGLIO $T/q\ell$



(M) MOMENTO FLETTENTE $M/q\ell^2$





$\frac{3}{4} q\ell$

$$M - \frac{3}{4} q\ell z + \frac{qz^2}{2} = 0$$

$$M = \frac{3}{4} q\ell z - \frac{qz^2}{2}$$

$M(z=0)$ $M=0$

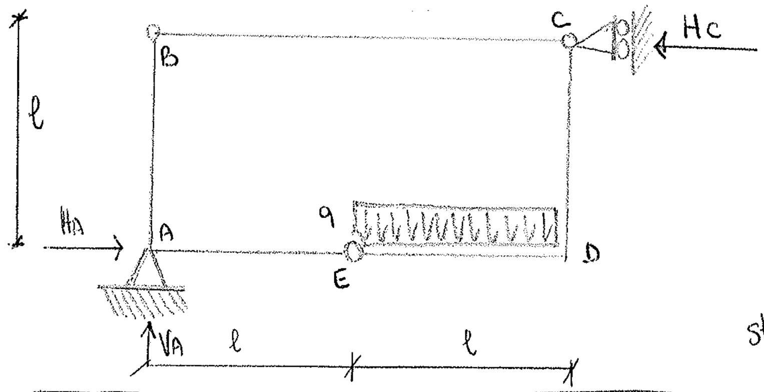
$M(z=\ell) = \frac{1}{4} q\ell^2$

$M\left(z=\frac{\ell}{2}\right) = \frac{3}{8} q\ell^2 - \frac{q\ell^2}{8} = \frac{3-1}{8} = \frac{1}{4} q\ell^2$

$M\left(z=\frac{3}{4}\ell\right) = \frac{3}{4} q\ell\left(\frac{3}{4}\ell\right) - \frac{q}{2}\left(\frac{3\ell}{4}\right)^2 = \frac{9}{16} q\ell^2 - \frac{9}{32} q\ell^2 = \frac{9}{32} q\ell^2$

ESERCIZIO (8)

- STRUTTURA



GRADI DI VINCOLO

$2 + 1 + 2 + 2 + 2 = 9$ GdV
est. int.

GRADI DI LIBERTÀ

3(aste) x 3 = 9 Gdl

struttura isostatica.

- REAZIONI DI EQUILIBRIO GLOBALE ESTERNO

$\uparrow)\ V_A - q\ell = 0$ $\boxed{V_A = q\ell}$

$\rightarrow)\ H_A - H_C = 0$ $H_A = H_C$ $\boxed{H_A = \frac{3}{2} q\ell}$

$\curvearrowleft A)\ -\frac{3}{2} q\ell^2 + H_C(\ell) = 0$ $H_C(\ell) = \frac{3}{2} q\ell^2$ $\boxed{H_C = \frac{3}{2} q\ell}$

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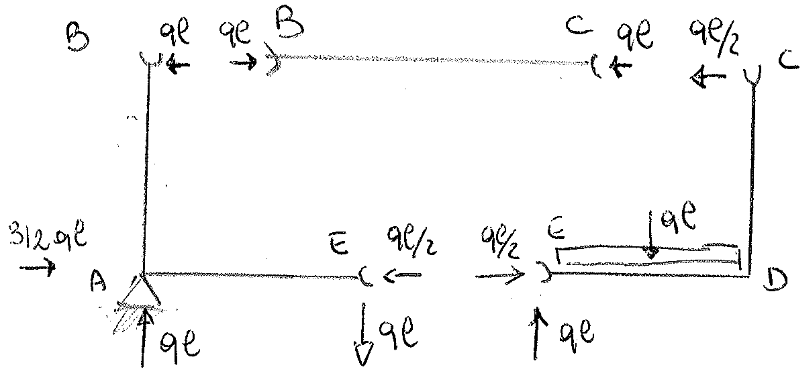
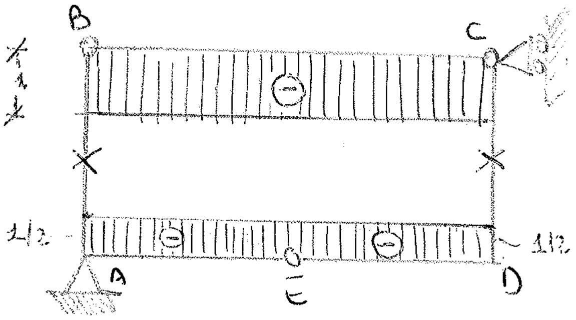
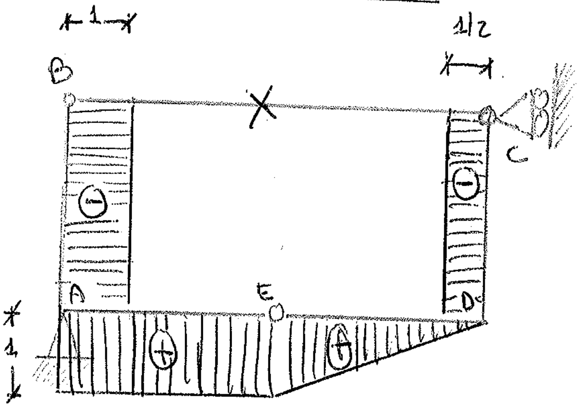
DIAGRAMMA CORPO LIBERO APPLICATO ALLA STRUTTURA



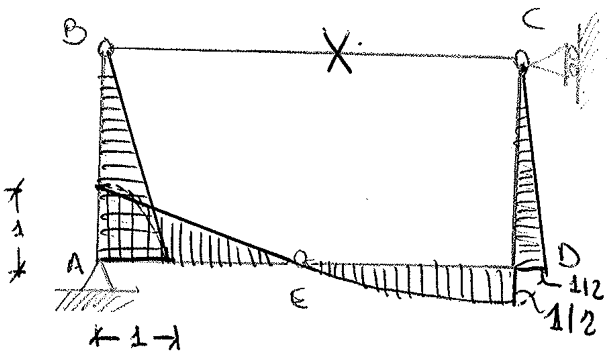
Ⓝ SFORZO NORMALE N/ql

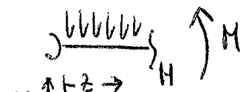


Ⓣ SFORZO DI TAGLIO T/ql



Ⓜ MOMENTO FLETTENTE





$$M - qlz + \frac{qz^2}{2} = 0$$

$$M = qlz - \frac{qz^2}{2}$$

$M(z=0)$ $M = 0$

$M(z=\frac{l}{2})$ $M = \frac{ql^2}{2} - \frac{ql^2}{8}$ $M = \frac{3}{8} ql^2$

$M(z=l) \Rightarrow M = \frac{ql^2}{2}$

Una volta determinate V_B e $V_{C'}$ posso passare a ricavarmi le altre reazioni del tratto BAE.

$\uparrow)\ V_A - V_B + V_E = 0 \qquad V_E = V_B - V_A \qquad V_E = \frac{q\ell}{4} - q\ell \qquad \boxed{V_E = -\frac{3}{4}q\ell}$ cambio verso.

Per il calcolo di H_B uso la rotazione attorno ad A, ricordandomi di introdurre il valore totale con il segno di V_E e non solo il modulo siccome le equaz. sono state scritte con i versi ipotizzati di partenza:

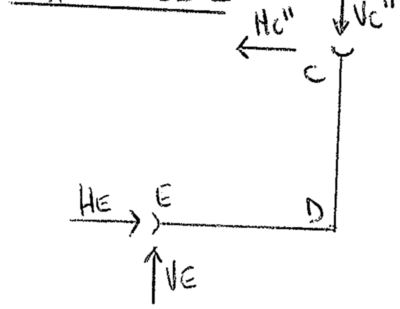
$\circlearrowleft A)\ V_E \cdot \ell + H_B \cdot \ell = 0 \qquad \cancel{H_B \ell}\ H_B \cdot \ell - \frac{3}{4}q\ell^2 = 0 \qquad \boxed{H_B = \frac{3}{4}q\ell}$

Ricavo H_E e $H_{C'}$ dal tratto BAE e BC:

$\rightarrow)\ -H_E = -H_A + H_B \qquad H_E = H_A - H_B = \frac{3}{2}q\ell - \frac{3}{4}q\ell = \boxed{\frac{3}{4}q\ell = H_E}$

$\rightarrow)\ H_B - H_{C'} = 0 \qquad H_{C'} = H_B \qquad \boxed{H_{C'} = \frac{3}{4}q\ell}$

TRATTO EDC

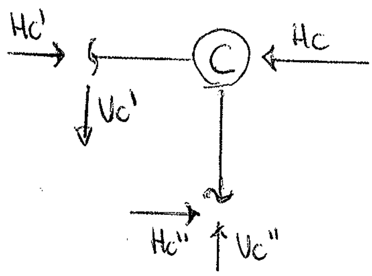


$\uparrow)\ V_E - V_{C''} = 0$

$\rightarrow)\ H_E - H_{C''} = 0 \qquad H_{C''} = H_E \qquad \boxed{H_{C''} = \frac{3}{4}q\ell}$

$V_{C''} = V_E \qquad \boxed{V_{C''} = \frac{3}{4}q\ell}$

EQUILIBRIO NODO C

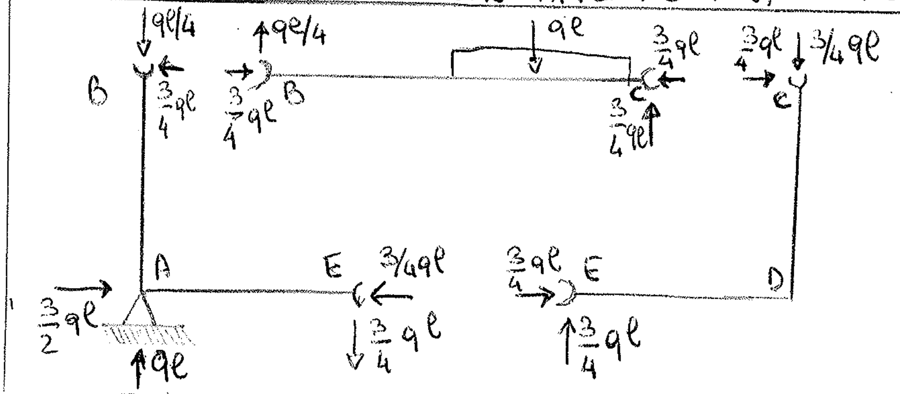


$\uparrow)\ -V_{C'} + V_{C''} = 0 \qquad -\frac{3}{4}q\ell + \frac{3}{4}q\ell = 0 \qquad 0=0$ OK!

$\rightarrow)\ H_{C'} + H_{C''} - H_C = 0 \qquad \frac{3}{4}q\ell + \frac{3}{4}q\ell - \frac{3}{2}q\ell = 0$

$\frac{6}{4}q\ell - \frac{3}{2}q\ell = 0 \qquad 0=0$ OK! NODO IN EQUILIBRIO.

DIAGRAMMA CORPO LIBERO APPLICATO ALLA STRUTTURA.



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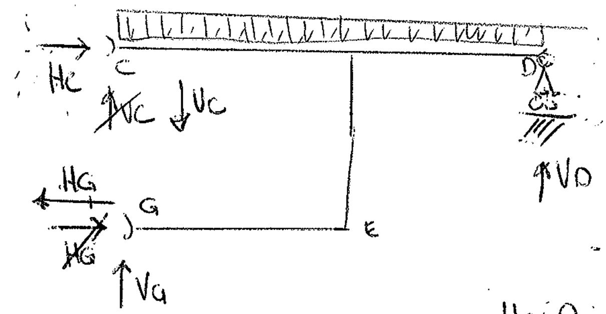
$\uparrow)\ V_A = 3ql - V_D = 3ql - \frac{15}{8}ql \qquad \frac{24-15}{8} \qquad \boxed{V_A = \frac{9}{8}ql}$

Passo a determinare le reaz. interne alla struttura.

- TRATTO CDEG



$\uparrow)\ V_D + V_C + V_G - 2ql = 0$

$\rightarrow)\ H_C + H_G = 0$

$G)\ -H_C \cdot l - 2ql^2 + V_D(2l) = 0$

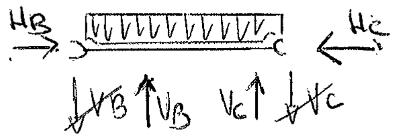
$-H_C \cdot l = 2ql^2 - V_D(2l)$

$H_C \cdot l = V_D(2l) - 2ql^2$

$H_C\, l = \frac{30}{8}ql^2 - 2ql^2 \qquad H_C \cdot l = \frac{14}{8}ql^2 \qquad \boxed{H_C = \frac{7}{4}ql}$

$H_G = -H_C \qquad \boxed{H_G = -\frac{7}{4}ql}$

- TRATTO BC



$\uparrow)\ -ql - V_B - V_C = 0$

$\rightarrow)\ H_B = H_C \qquad \boxed{H_B = \frac{7}{4}ql}$

$B)\ -\frac{ql^2}{2} - V_C(l) = 0$

$-V_C \cdot l = \frac{ql^2}{2} \qquad \boxed{V_C = -\frac{ql}{2}}$ cambio verso

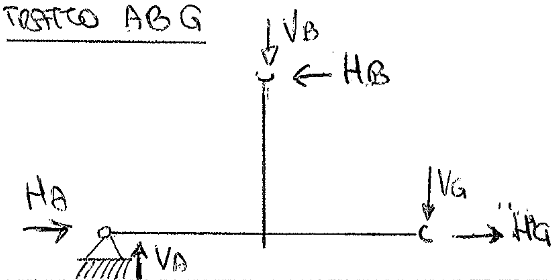
$\uparrow)\ -ql - V_B - V_C = 0 \qquad -V_B = V_C + ql \qquad V_B = -V_C - ql \qquad V_B = \frac{ql}{2} - ql \qquad \boxed{V_B = -\frac{ql}{2}}$ cambio verso.

Determino ora V_G, ricordandomi di usare non solo il modulo di V_C, ma anche il verso, perchè le equazioni fanno ancora riferimento alle reazioni ipotizzate.

$\uparrow)\ V_D + V_C + V_G - 2ql = 0 \qquad V_G = 2ql - V_D - V_C \qquad V_G = 2ql - \frac{15}{8}ql + \frac{ql}{2}$

$V_G = \frac{16 - 15 + 4}{8} \qquad \boxed{V_G = \frac{5}{8}ql}$

TRATTO ABG

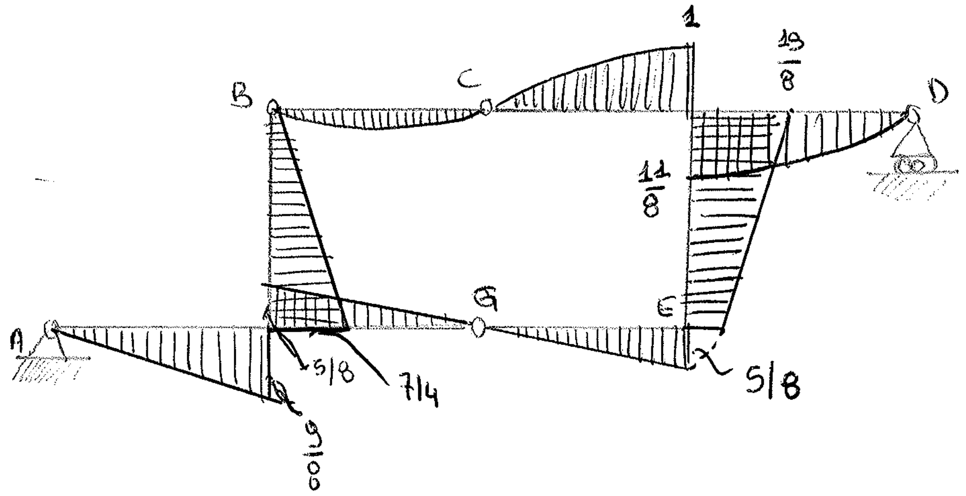


$\uparrow)\ V_A - V_G - V_B = 0 \qquad \frac{9}{8}ql - \frac{5}{8}ql - \frac{ql}{2} = 0$

$\frac{9-5-4}{8}ql = 0 \qquad 0 = 0$ OK!

verificato.

(M) MOMENTO FLETTENTE $\quad M/q\ell^2$



$$M - \frac{5}{8}q\ell^2 - \frac{7}{4}q\ell z = 0 \qquad M = \frac{5}{8}q\ell^2 + \frac{7}{4}q\ell z$$

$$M(z=0) = \frac{5}{8}q\ell^2 \qquad M(z=\ell) = \frac{5}{8}q\ell^2 + \frac{7}{4}q\ell^2 = \frac{19}{8}q\ell^2$$

$$M + \frac{7}{4}q\ell z = 0 \qquad M = -\frac{7}{4}q\ell z$$

$$M(z=0) = 0$$

$$\text{(H)}\ M - \frac{q\ell}{2}\cdot z + \frac{qz^2}{2} = 0 \qquad M = \frac{q\ell}{2}\cdot z - \frac{qz^2}{2}$$

$$M(z=\ell/2) = \frac{q\ell^2}{4} - \frac{q\ell^2}{8} = \frac{q\ell^2}{8}$$

$$M(z=\ell) = 0$$

$$\text{(K)}\ M + \frac{q\ell}{2}\cdot z + \frac{qz^2}{2} = 0 \qquad M = -\frac{q\ell}{2}(z) - \frac{qz^2}{2}$$

$$M(z=0) = 0$$

$$M(z=\tfrac{\ell}{2}) = -\frac{q\ell^2}{4} - \frac{q\ell^2}{8} = -\frac{3}{8}q\ell^2$$

$$M(z=\ell) = -q\ell^2$$

$$M - \frac{qz^2}{2} + \frac{15}{8}q\ell z = 0 \qquad M = -\frac{15}{8}q\ell z + \frac{qz^2}{2}$$

$$M(z=0) = 0 \qquad M(z=\tfrac{\ell}{2}) = -\frac{15}{16}q\ell^2 + \frac{q\ell^2}{8} = \frac{-15+2}{16} = -\frac{13}{16}q\ell^2$$

$$M(z=\ell) = -\frac{15}{8}q\ell^2 + \frac{q\ell^2}{2} = \frac{-15+4}{8} = -\frac{11}{8}q\ell^2$$

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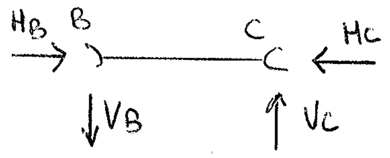
$\rightarrow)\ H_E - H_C = 0 \qquad H_E = H_C \qquad \boxed{H_E = -\frac{7}{4}q\ell}$ cambio verso

- TRATTO BC



$\uparrow)\ V_C - V_B = 0$

$\rightarrow)\ H_B - H_C = 0$

$\overset{\curvearrowleft}{B})\ V_C(\ell) = 0 \qquad \boxed{V_C = 0}$

$V_B = V_C \qquad \boxed{V_B = 0} \qquad H_B = H_C \qquad \boxed{H_B = \frac{7}{4}q\ell}$

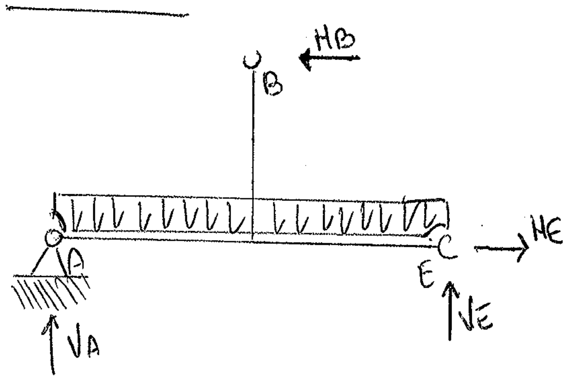
L'asta BC risulta essere un' asta compressa.

Una volta determinato V_C posso determinare V_E grazie all' equazione alla traslazione verticale del tratto CDE

$\uparrow)\ V_E = V_C + q\ell - V_D \qquad V_E = q\ell - \frac{9}{8}q\ell \qquad \boxed{V_E = -\frac{1}{8}q\ell}$

cambio segno sul tratto CDE

- TRATTO ABE



$\rightarrow)\ H_E - H_B = 0 \qquad \boxed{H_E = \frac{7}{4}q\ell}$

$\uparrow)\ V_A + V_E - 2q\ell = 0$

$\frac{15}{8}q\ell + \frac{1}{8}q\ell - 2q\ell = 0$

$2q\ell - 2q\ell = 0 \qquad 0 = 0$

verifico il valore di V_A grazie a un equaz. della rotazione.

$\overset{\curvearrowleft}{E})\ -V_A(2\ell) + 2q\ell^2 + H_B \cdot \ell = 0$

$-V_A(2\ell) = -2q\ell^2 - H_B \cdot \ell \qquad V_A(2\ell) = 2q\ell^2 + H_B \cdot \ell$

$V_A(2\ell) = 2q\ell^2 + \frac{7}{4}q\ell^2 \qquad V_A(2\ell) = \frac{15}{4}q\ell^2 \qquad V_A = \frac{15}{8}q\ell$ ok!

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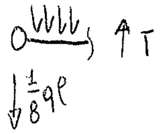
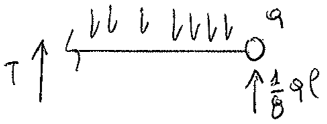


$T - qz - \frac{1}{8}ql = 0$

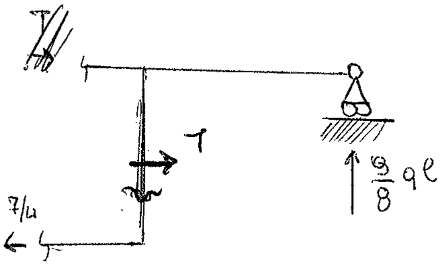
$T = qz + \frac{1}{8}ql$ $\qquad T = q(z + \frac{1}{8}l)$ $\quad z = l \quad T = \frac{9}{8}ql$ $\quad \cancel{T=q}\ z=$



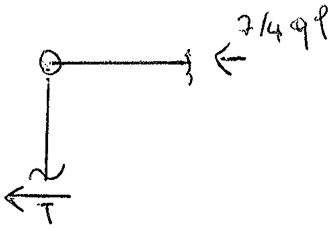
$T + \frac{1}{8}ql - qz = 0$

$T = qz - \frac{1}{8}ql$

$T = q(z - \frac{1}{8}l)$ $\quad z = l \quad T = \frac{7}{8}ql$ $\quad z = 2l \quad T = \frac{15}{8}ql$

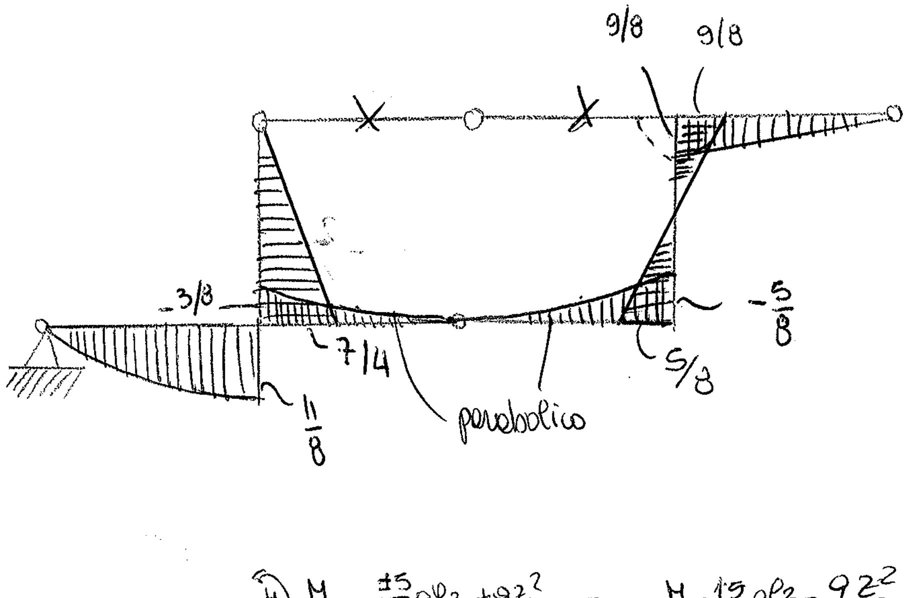


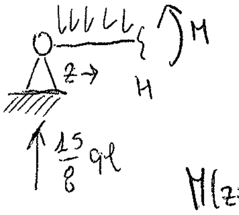
$T - \frac{7}{4} = 0 \qquad T = \frac{7}{4}ql$



$-T - \frac{7}{4}ql = 0 \qquad T = -\frac{7}{4}ql$

M - MOMENTO FLETTENTE $\frac{M}{ql^2}$





H) $M - \frac{15}{8}qlz + \frac{qz^2}{2} = 0 \qquad M = \frac{15}{8}qlz - \frac{qz^2}{2}$ $\quad M(z=0) = 0$ $\quad M(z=\frac{l}{2}) =$

$= \frac{15}{16}ql^2 - \frac{ql^2}{8} = \frac{15-2}{16}ql^2 = \frac{13}{16}ql^2$

$M(z=l) = \frac{15}{8}ql^2 - \frac{ql^2}{2} \quad \frac{15-4}{8} = \frac{11}{8}ql^2$