NUMERO: 1152

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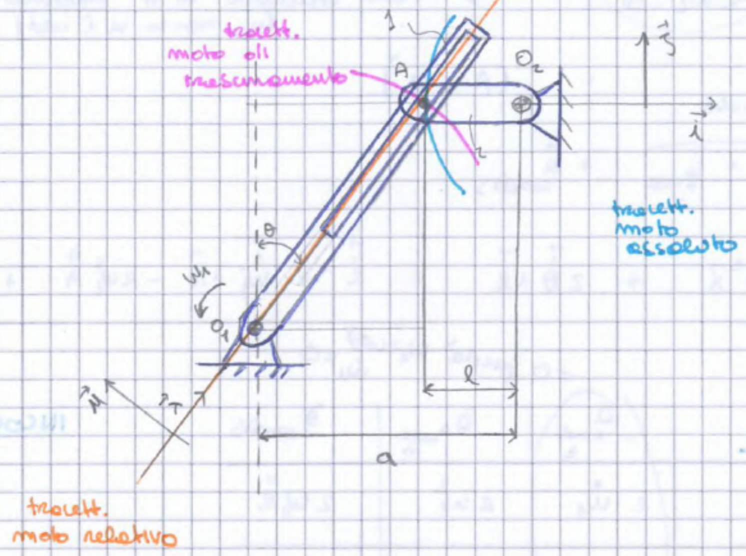
A P P U N T I

STUDENTE: Prette

MATERIA: Meccanica Applicata e Macchine + Eserc.

Prof. Giorcelli

ES. GUFO PAG. 28 (es. 2 esercizi svolti di Meccanica)



$a = 500\ mm = 0{,}5\ m$

$l = 250\ mm = 0{,}25\ m$

$\theta = 30° = \pi/6$

$\omega_1 = 5\ rad/s$

1) $\vec{V}_{A2}$ 3) $\vec{a}_{2A}$

2) $\vec{\omega}_2$ 4) $\vec{\dot\omega}_2$

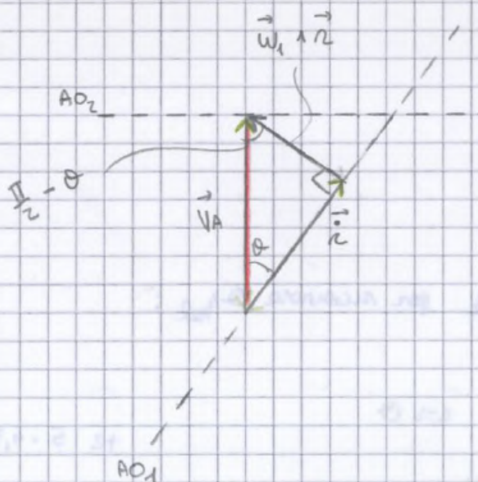
$$AO_1 \cdot \sin\theta = (a - l) \rightarrow AO_1 = \frac{(a-l)}{\sin\theta} = \frac{(0{,}5 - 0{,}25)}{\sin 30°} = 0{,}5\ m = r$$

$$V_A = V_{A_{rel}} + V_{A_{trasc}} = \dot{\vec{r}} + \underbrace{\cancel{\vec{V}_{O_1}}}_{=0} + V_{A/O_1} = \dot{\vec{r}} + \vec{\omega}_1 \wedge \vec{r} \quad (A \in 1)$$

$$V_A = \vec{\omega}_2 \wedge \vec{l} \quad (A \in 2)$$

$$\vec{\omega}_2 \wedge \vec{l} = \dot{\vec{r}} + \vec{\omega}_1 \wedge \vec{r}$$

V_A	V_{Arel}	V_{A/O_1}
$\omega_2 l$	$\dot r$	$\omega_1 r$ → da O_1 ad A
$\perp AO_2$	$// AO_1$	$\perp AO_1$
$\pm \vec{j}$	$\pm \vec{\lambda}$	$-\vec{\mu}$



$$\frac{V_A}{\sin\frac{\pi}{2}} = \frac{\omega_1 r}{\sin\theta} = \frac{\dot r}{\sin\left(\frac{\pi}{2} - \theta\right)}$$

$V_A = \omega_2 l$

$$\frac{3\pi - \pi}{6} = \frac{2}{6}\pi = \frac{1}{3}\pi$$

$$\vec{V}_{A2} = \dot r = \omega_1 \cdot r \cdot \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\sin\theta} = \frac{5 \cdot 0{,}5 \cdot \sin\left(\frac{\pi}{2} - \frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}\right)}$$

$$= \frac{5 \cdot 0{,}5 \cdot \sin\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{6}\right)} = 4{,}33\ \frac{m}{s}$$

$$\omega_2 = \frac{\omega_1 \cdot r \cdot \sin\frac{\pi}{2}}{l \sin\theta} = \frac{5 \cdot 0{,}5 \cdot \sin\frac{\pi}{2}}{0{,}25 \cdot \sin\frac{\pi}{6}} = 20\ rad/s$$

orario

$\circlearrowright \otimes$ dedotto da verso $\vec{V}_A$

Proietto il poligono su una retta verticale $\perp AO_2$ per ricavare $\dot{\omega}_2$:

$$a_{A_{rel}} \cdot \cos\theta + a_{coriolis} \cdot \sin\theta = a_{A/O_2 t} + a_{A_{trasc. n}} \cdot \sin\left(\frac{\pi}{2} - \theta\right)$$

$$287{,}5 \cdot \cos\left(\frac{\pi}{6}\right) + 2\omega_1 \dot{r} \sin\left(\frac{\pi}{6}\right) = l\dot{\omega}_2 + r\omega_1^2 \cdot \sin\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$$

$$\dot{\omega}_2 = \frac{287{,}5 \cdot \cos\left(\frac{\pi}{6}\right) + 2 \cdot 5 \cdot 4{,}33 \sin\frac{\pi}{6} - 0{,}5 \cdot 5^2 \cdot \sin\left(\frac{\pi}{3}\right)}{0{,}25} = 1039{,}2 \ rad/s^2$$

$\circlearrowright$ $\otimes$ orario

→ dedotto da verso $\vec{a}_{A/O_2 t}$

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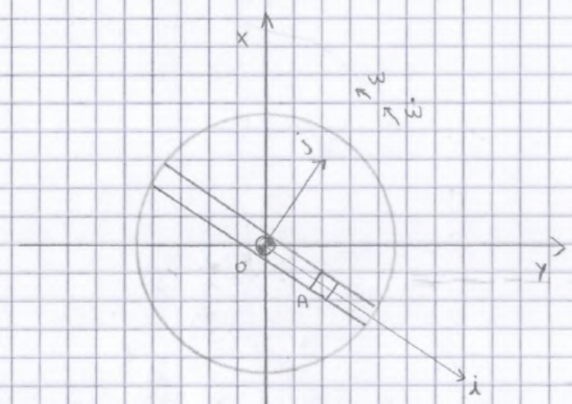


$\omega = 4$ rad/s

$\dot{\omega} = -10$ rad/s²

$OA = r = 150$ mm $= 0{,}15$ m

$\dot{r} = 125$ mm/s

$\ddot{r} = 2025$ mm/s²

V_A a_A ?

$$\vec{V}_A = \vec{V}_{Ar} + \vec{V}_{Atr} = \vec{V}_{Ar} + \underbrace{\cancel{\vec{V}_O}}_{=0} + \vec{V}_{A/O} = \dot{r}\vec{i} + r\omega\vec{j}$$

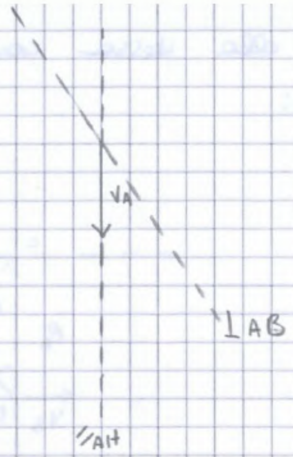
$$V_{Ar} = \dot{r}\vec{i} = 0{,}125\,\vec{i} \ m/s$$

$$V_{Atr} = r\omega\vec{j} = 0{,}15 \cdot 4\,\vec{j} = 0{,}6\,\vec{j} \ m/s$$

$$\vec{V}_A = 0{,}125\,\vec{i} + 0{,}6\,\vec{j} \qquad V_A = \sqrt{(0{,}125)^2 + (0{,}6)^2} = 0{,}6128 \ m/s$$

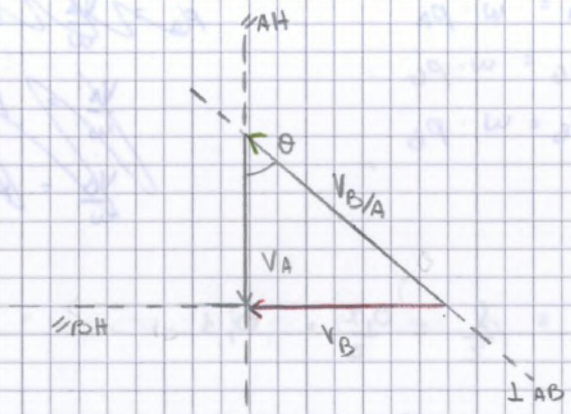
$$\vec{a}_A = \vec{a}_{Ar} + \vec{a}_{Atr} + \vec{a}_c = \ddot{r}\vec{i} + \underbrace{\cancel{\vec{a}_O}}_{=0} - r\dot{\theta}^2\vec{i} + r\ddot{\theta}\vec{j} + 2\dot{r}\dot{\theta}\vec{j}$$

V_G	V_A	$V_{G/A}$
?	2	$\omega \frac{l}{2}$ → incognita
?	// AH	⊥ AB
?	↓	$\pm \vec{u}$



$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$
$$= \vec{V}_A + \omega \cdot l$$

V_B	V_A	$V_{B/A}$
?	2	$\omega \cdot l$ → incognita
// BH	// AH	⊥ AB
←	↓	$\pm \vec{u}$



$$V_{B/A} \cdot \cos\theta = V_A \quad \rightarrow \quad V_{B/A} = \frac{V_A}{\cos\theta} = \frac{2}{\cos 30^\circ} = 2{,}309 \ \frac{m}{s}$$

$$V_{B/A} = \omega \cdot l \quad \Rightarrow \quad \omega = \frac{V_{B/A}}{l} = \frac{2{,}309}{0{,}2} = 11{,}54 \ \frac{rad}{s}$$

↻ ⊗ orario
(da verso $V_{B/A}$)

$$V_{G/A} = \omega \cdot \frac{l}{2} = 11{,}54 \cdot \frac{0{,}2}{2} = 1{,}154 \ \frac{m}{s}$$

↖ verso da $\omega \cdot \frac{l}{2}$
(adesso conosco ω)

$$\cancel{V_G = \sqrt{2^2 + 1{,}154^2} = \sqrt{5{,}33} = 2{,}309 \ \frac{m}{s}}$$

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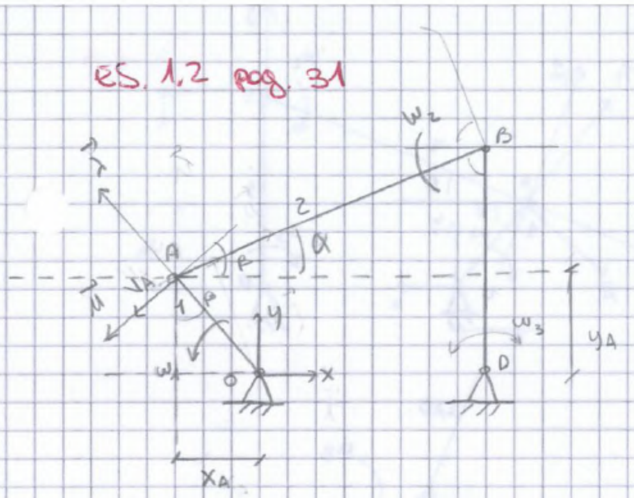


$\omega_1 = 10\ rad/s$ ⊙ $\dot{\omega}_1 = \emptyset$

$x_A = -60\ mm = -0{,}06\ m$

$y_A = 80\ mm = 0{,}08\ m$

$\omega_2 ?\quad \omega_3 ?\quad \dot{\omega}_2 ?\quad \dot{\omega}_3 ?$

$OA = 100\ mm = 0{,}1\ m$
$AB = 260\ mm = 0{,}26\ m$
$BD = 180\ mm = 0{,}18\ m$
$OD = 180\ mm = 0{,}18\ m$

$V_A = V_B + V_{A/B}$

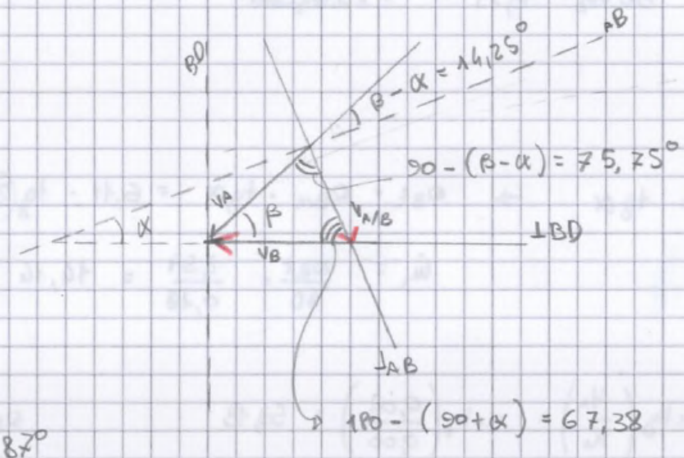
$\omega_1 \cdot AO = \omega_3 \cdot BD + \dot{\alpha} \cdot AB$

geometria: $AB \cdot \cos\alpha = (OD + |x_A|)$

$\cos\alpha = \dfrac{(OD + |x_A|)}{AB} = \dfrac{0{,}24}{0{,}26} = 0{,}923$

$\alpha = \arccos\alpha = 22{,}62°$

V_A =	V_B +	$V_{A/B}$
$\omega_1 \cdot AO$	$\omega_3 \cdot BD$	$\omega_2 AB$
$\perp AO$	$\perp BD$	$\perp AB$
$-\vec{\mu}$	?	?



$\beta = tg^{-1}\left(\dfrac{x_A}{y_A}\right) = tg^{-1}\left(\dfrac{0{,}06}{0{,}08}\right) = 36{,}87°$

$V_A = \omega_1 \cdot AO = 10 \cdot 0{,}1 = 1\ \frac{m}{s}$

$$\frac{V_A}{\sin(67{,}38)} = \frac{V_B}{\sin(75{,}75)} = \frac{V_{A/B}}{\sin(36{,}87)}$$

$V_B = V_A \cdot \dfrac{\sin(75{,}75)}{\sin(67{,}38)} = 1{,}05\ \frac{m}{s}$

$V_{A/B} = V_A \cdot \dfrac{\sin(36{,}87)}{\sin(67{,}38)} = 0{,}65\ \frac{m}{s}$

$V_B = \omega_3 \cdot BD \rightarrow \omega_3 = \dfrac{V_B}{BD} = \dfrac{1{,}05}{0{,}18} = 5{,}83\ \frac{rad}{s}$ ⊙

$V_{A/B} = \omega_2 \cdot AB \rightarrow \omega_2 = \dfrac{V_{A/B}}{AB} = \dfrac{0{,}65}{0{,}26} = 2{,}5\ \frac{rad}{s}$ ⊙

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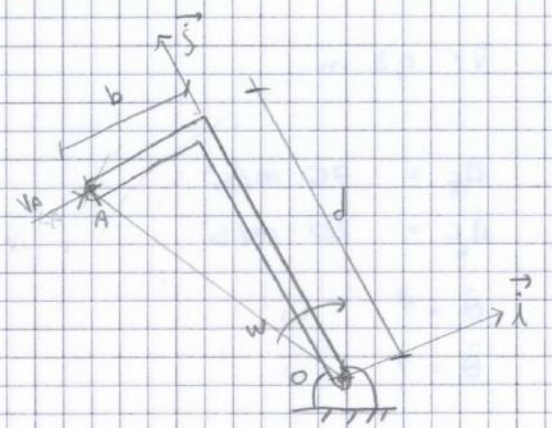

$\dot{\omega} = -2 \ rad/s^2$

$\vec{V}_A$? $\vec{a}_A$?

quando:

$\omega = 3 \ rad/s$

$b = 0,3 \ m$

$d = 0,4 \ m$

NO

$\overline{AO} = \sqrt{d^2 + b^2} = \sqrt{0,4^2 + 0,3^2} = 0,49$

$\vec{V}_A = \vec{\omega} \wedge \vec{r} = r\,\omega \ \vec{\ } = \overline{AO} \cdot 3 \ rad/s \cdot \vec{i} = 0,49 \cdot 3 \cdot \vec{i}$

$= 1,47 \ \vec{i} \ m/s$

$\vec{a}_A = -r\dot{\theta}^2 \vec{\lambda} + r\ddot{\theta}\vec{\mu} = -r\,\omega^2 \vec{j} + r \cdot \dot{\omega}\,\vec{i}$

$= -0,49 \cdot 3^2 \vec{j} + 0,49 \cdot -2 \ \vec{i}$

$= -4,41 \ \vec{j} - 0,98 \ \vec{i}$

vettore $\vec{OA} = -b\,\vec{i} + d\,\vec{j}$ $\qquad \vec{\omega} = \omega \cdot \vec{k}$, $\omega = -3 \ rad/s$

$$\vec{V}_A = \vec{\omega} \wedge \vec{r} = \vec{\omega} \wedge \vec{OA} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -\omega \\ -b & +d & 0 \end{vmatrix} = +\omega d\,\vec{i} + \vec{j}\,b\omega$$

$= +3 \cdot 0,4 \ \vec{i} + 0,3 \cdot 3 \ \vec{j}$

$= +1,2 \ \vec{i} + 0,9 \ \vec{j} \ m/s$

$\vec{a}_A = \vec{\dot{\omega}} \wedge \vec{OA} - \omega^2 \vec{OA}$

$= \dot{\omega}\vec{k} \wedge (-b\vec{i} + d\vec{j}) - \omega^2(-b\vec{i} + d\vec{j})$

+2

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & +2 \\ -b & +d & 0 \end{vmatrix} + (+\omega^2 b\,\vec{i} - \omega^2 d\,\vec{j}) = -2d\,\vec{i} - 2b\,\vec{j} + \omega^2 b\,\vec{i} - \omega^2 d\,\vec{j}$$

$= \underbrace{-2 \cdot 0,4 \cdot \vec{i}}_{} - \underbrace{2 \cdot 0,3\,\vec{j}}_{} + \underbrace{3^2 \cdot 0,3\,\vec{i}}_{} - \underbrace{3^2 \cdot 0,4\,\vec{j}}_{}$

$= +1,9 \ \vec{i} - 4,2 \ \vec{j} \ m/s^2$

$$\vec{a}_c = -\underbrace{r}_{0,36}\cdot\omega^2\,\vec{\lambda} + \underbrace{r}_{0,36}\,\dot{\omega}\,\vec{\mu}$$

V_A = V sulla puleggia $\qquad \vec{V}_A = \frac{d}{2}\cdot\dot{\theta}\,\vec{\mu} = \frac{d}{2}\,\omega\,\vec{\mu}$

$$\omega = \frac{V_A\cdot 2}{d} = \frac{2\cdot 2}{0,15} = 26,66\ \frac{rad}{s} \qquad \omega = -26,66\,\vec{K}$$

$a_B = a_t$ sulla puleggia $\qquad \vec{a}_B = \frac{D}{2}\,\ddot{\theta}\,\vec{\mu} = \frac{D}{2}\,\dot{\omega}\,\vec{\mu}$

$$\dot{\omega} = \frac{a_B}{D/2} = \frac{35}{\frac{0,80}{2}} = 87,50\ \frac{rad}{s^2}$$

$$\vec{a}_c = -0,36\cdot 26,66^2\,\vec{\lambda} + 0,36\cdot 87,50\,\vec{\mu} = -255,87\,\vec{\lambda} + 31,5\,\vec{\mu}$$

$$a_c = \sqrt{(255,87)^2 + (31,5)^2} = 257,8\ \frac{m}{s^2}$$

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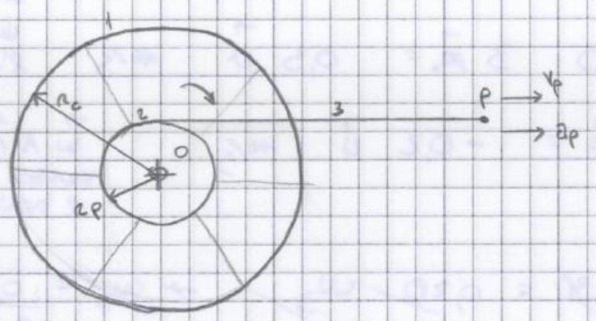

R_c = 0,12 m
R_p = 0,10 m
V_p = 2,2 m/s
a_p = 1,3 m/s²

ω ? $\dot{\omega}$? del castello

a_c ? a ? di 1 punto del castello

V_p = V su corpo 2

$$\vec{V}_p = R_p\cdot\omega\cdot\vec{\mu} \qquad \omega = \frac{V_p}{R_p} = \frac{2,2}{0,10} = 22\ \frac{rad}{s} \quad \otimes$$

$a_p = a_t$ su corpo 2

$$\vec{a}_t = r_p\,\ddot{\theta}\,\vec{\mu} = r_p\cdot\dot{\omega}\,\vec{\mu} \qquad \dot{\omega} = \frac{a_t}{r_p} = \frac{1,3}{0,10} = 13\ \frac{rad}{s^2}$$

$\vec{a}_o = -\overline{AO} \cdot \omega_3^2 \vec{\lambda} + \overline{AO} \cdot \dot{\omega}_3 \vec{\mu}$

$\vec{a}_B = \vec{a}_{B_n} + \vec{a}_{B_t} = -\overline{AB} \cdot \omega_3^2 \vec{\lambda} + (-0{,}2 \vec{\mu})$

$\vec{a}_{B_t} = \overline{AB} \cdot \dot{\omega}_3 \vec{\mu}$ $\qquad -0{,}2 \vec{\mu} = (0{,}20+0{,}10) \cdot \dot{\omega}_3 \vec{\mu}$

$\dot{\omega}_3 = \dfrac{-0{,}2}{(0{,}30)} = 0{,}6667 \ rad/s^2$

verso orario affinché a_{Bt} verso il basso

$\dot{\vec{\omega}}_3 \wedge \overrightarrow{AB}$

$\vec{a}_o = -0{,}10 \cdot 1^2 \vec{\lambda} + 0{,}10 \cdot 0{,}6667 \vec{\mu}$

$= -0{,}10 \vec{\lambda} + 0{,}0666 7 \vec{\mu}$ $\rightarrow$ $\vec{a}_{ot} = 0{,}0666 7 \ m/s^2$

$a_o = \sqrt{(0{,}10)^2 + (0{,}06667)^2} = 0{,}12 \ m/s^2$

accelerazione del carico = accelerazione tangenziale di O

CASO b)

$\vec{V}_o = \vec{V}_A + \vec{V}_{o/A}$

$\vec{V}_A = \vec{\omega}_1 \wedge \vec{r}_1 = r_1 \omega_1 \vec{\mu} = 0{,}10 \cdot 1 \ \vec{\mu} = 0{,}10 \vec{\mu} \ m/s$

verso l'alto ~~verso il basso~~

$\vec{V}_{o/A} = 0{,}1 \ m/s$

$\vec{V}_o = +0{,}10 \cancel{\vec{\mu}} + 0{,}10 = 0{,}20 \ \vec{\mu}$

$\vec{a}_o = \vec{a}_A + \vec{a}_{o/A} = \vec{a}_A + \vec{a}_{o/A_t} + \vec{a}_{o/A_n}$

$= \vec{a}_A + (-\overline{AO}\,\omega_3^2 \vec{\lambda} + \overline{AO}\,\dot{\omega}_3 \vec{\mu})$

$V_o = \overline{AO}\,\omega_3 \rightarrow \omega_3 = \dfrac{V_o}{AO} = \dfrac{0{,}20}{0{,}10} = 2 \ rad/s$

$\vec{a}_{At} = r_1 \cdot \dot{\omega}_1 \vec{\mu} = 0{,}10 \cdot 4 = 0{,}4 \vec{\mu}$

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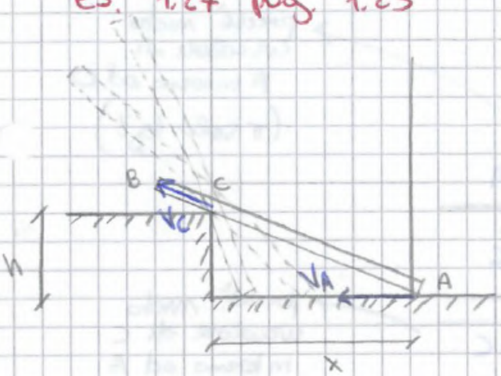
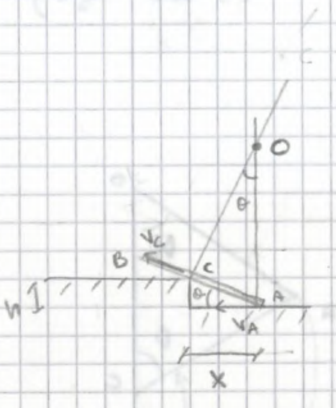


$V_A = 3\ m/s$

$x = 1\ m$

$h = 0,5\ m$

$\omega(x) = ?$



$\vec{V}_A = \vec{\omega} \wedge \vec{OA}$

$\vec{V}_C = \vec{\omega} \wedge \vec{CO}$

$AO \cdot \sin\theta = AC$

$$AO = \frac{AC}{\sin\theta} = \frac{\sqrt{h^2+x^2}}{h / AC} = \frac{\sqrt{h^2+x^2} \cdot \sqrt{h^2+x^2}}{h} = \frac{h^2+x^2}{h}$$

$AC \cdot \sin\theta = h$

$$\omega = \frac{V_A}{OA} = \frac{V_A}{\frac{h^2+x^2}{h}} = \frac{V_A \cdot h}{h^2+x^2}$$

$$\omega\left(V_A = 3\ m/s,\ x = 1\ m,\ h = 0,5\ m\right) = \frac{3 \cdot 0,5}{0,5^2 + 1^2} = 1,2\ \frac{rad}{s}$$

↻ orario

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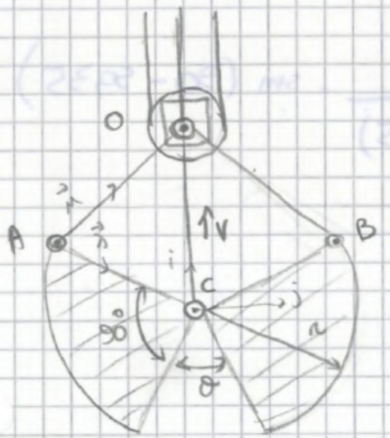

$V = 0,3\ m/s$

ω delle ganasce ?

$\theta = 45°$

$R = 0,5\ m$

$\overline{AO} = \overline{BO} = 0,6\ m$

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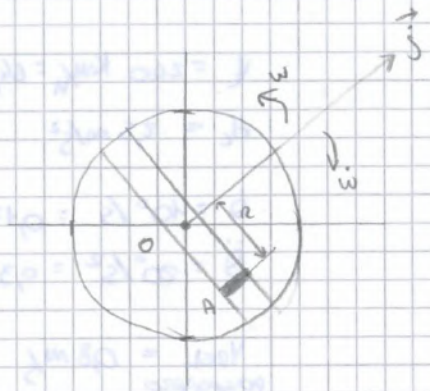


$\omega = 5\ rad/s$

$\dot{\omega} = -12\ rad/s^2$

$R = 0{,}18\ m$

$\dot{r} = 0{,}115\ m/s$

$\ddot{r} = 1{,}025\ m/s^2$

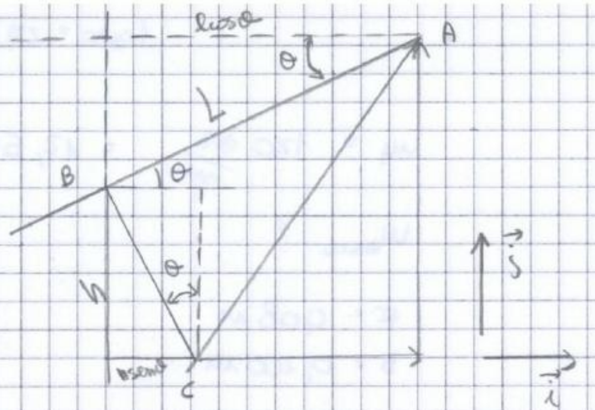
$V_A \quad a_A$

$$\vec{V}_A = \vec{V}_{Ar} + \vec{V}_{Atr} = \vec{V}_{Ar} + \cancel{\vec{V}_o} + \vec{V}_{A/o}$$
$$= \dot{r}\,\vec{\lambda} + r\cdot\omega\,\vec{\mu}$$
$$= 0{,}115\,\vec{\lambda} + 0{,}18\cdot 5\,\vec{\mu} = 0{,}115\,\vec{\lambda} + 0{,}9\,\vec{\mu}$$

$$V_A = \sqrt{(0{,}115)^2 + (0{,}9)^2} = 0{,}9073\ \frac{m}{s}$$

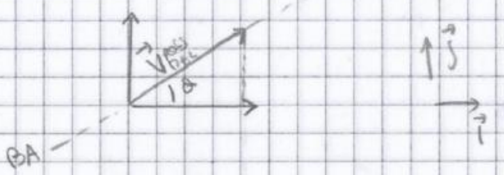
$$\vec{a}_A = \vec{a}_{Ar} + \vec{a}_{Atr} + \vec{a}_{coriolis}$$
$$= \ddot{r}\,\vec{\lambda} + \cancel{\vec{a}_o} - r\cdot\omega^2\,\vec{\lambda} + r\,\dot{\omega}\,\vec{\mu} + 2\cdot\dot{r}\cdot\omega\,\vec{\mu}$$
$$= 1{,}025\,\vec{\lambda} - 0{,}18\cdot 5^2\,\vec{\lambda} - 0{,}18\cdot 12\,\vec{\mu} + 2\cdot 0{,}115\cdot 5\,\vec{\mu}$$
$$= 1{,}025\,\vec{\lambda} - 4{,}5\,\vec{\lambda} - 2{,}16\,\vec{\mu} + 1{,}15\,\vec{\mu}$$
$$= -3{,}475\,\vec{\lambda} - 1{,}01\,\vec{\mu}$$

$$a_A = \sqrt{(3{,}475)^2 + (1{,}01)^2} = 3{,}619\ \frac{m}{s^2}$$



$$\vec{V}_{poss} = \vec{V}_{poss_{REL}} + \vec{V}_{poss_{t}}$$
$$= \vec{V}_{poss_{REL}} + \vec{V}_A + \cancel{\vec{V}_{poss/A}}$$
$$= \vec{V}_{poss_{REL}} + \vec{V}_C + \vec{V}_{A/C}$$
$$= \vec{V}_{poss_{REL}} + \vec{V}_C + \vec{\omega} \wedge \vec{CA}$$

$$\vec{CA} = (L\cos\theta - h\,\text{sen}\theta)\vec{i} + (L\,\text{sen}\theta + h\cos\theta)\vec{j} = 16{,}61\,\vec{i} + 7{,}557\,\vec{j}$$



$$V_{poss_{REL}} \cdot \cos\theta = 0{,}8 \cdot \cos 15^\circ = 0{,}773$$
$$V_{poss_{REL}} \cdot \sin\theta = 0{,}8 \cdot \sin 15^\circ = 0{,}207$$
$$\vec{V}_{poss_{REL}} = 0{,}773\,\vec{i} + 0{,}207\,\vec{j}$$

$$\vec{\omega} \wedge \vec{CA} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 0{,}174 \\ 16{,}61 & 7{,}557 & 0 \end{vmatrix} = -1{,}315\,\vec{i} + 2{,}890\,\vec{j}$$

$$\vec{V}_{poss} = 0{,}773\,\vec{i} + 0{,}207\,\vec{j} + 64{,}8\,\vec{i} - 1{,}315\,\vec{i} + 2{,}890\,\vec{j}$$
$$= 64{,}25\,\vec{i} + 3{,}1\,\vec{j}$$

$$\vec{a}_{poss} = \cancel{\vec{a}_{pr}} + \vec{a}_{ptr} + \vec{a}_{cor}$$
$$= \vec{a}_c - \dot{\theta}^2 \vec{AC} + \ddot{\theta} \wedge \vec{AC} + \cancel{2\vec{\omega} \wedge \vec{i}}$$
$$\cancel{= \vec{a}_c + \vec{a}_{A/c} - \dot{\theta}^2 \vec{AC} + \ddot{\theta} \wedge \vec{AC}}$$
$$\cancel{= \vec{a}_c + \vec{a}_{A/c\,t} + \vec{a}_{A/c\,n} - \dot{\theta}^2 \vec{AC} + \ddot{\theta} \wedge \vec{AC}}$$
$$= 2{,}5\,\vec{i} - 0{,}174^2 \cdot (16{,}61\,\vec{i} + 7{,}557\,\vec{j}) + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 0{,}349 \\ 16{,}61 & 7{,}557 & 0 \end{vmatrix}$$
$$= 2{,}5\,\vec{i} - 0{,}503\,\vec{i} - 0{,}229\,\vec{j} - 2{,}637\,\vec{i} + 5{,}797\,\vec{j}$$
$$= -0{,}64\,\vec{i} + 5{,}56\,\vec{j}$$

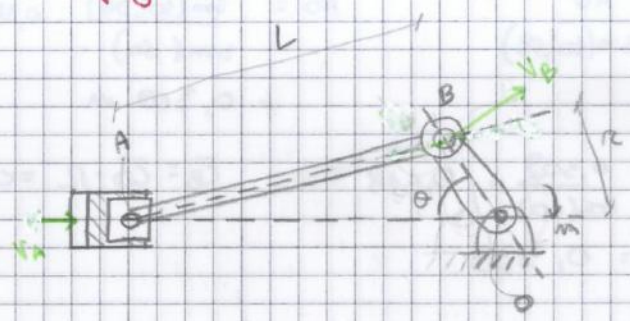
$V_B = \omega_A \cdot a = 17{,}57 \cdot 0{,}08 = 1{,}005 \ m/s = \omega_o \cdot \overline{OB}$

$\omega_o = \dfrac{1{,}005}{0{,}23856} = 4{,}213 \ \dfrac{rad}{s}$

$V_C = 4{,}213 \cdot \overline{OC} = 1{,}168 \ m/s$

$V_C = d\omega \rightarrow \omega = \dfrac{V_C}{d} = \dfrac{1{,}168}{0{,}36} = 3{,}244 \ rad/s$

ES. 1.37 pag 1.33



$\overline{AB} = L = 0{,}35 \ m$

$\overline{OB} = r = 0{,}125 \ m$

$n = 1500 \ \dfrac{giri}{min} = 157{,}08 \ \dfrac{rad}{s}$

$\theta = 60°$

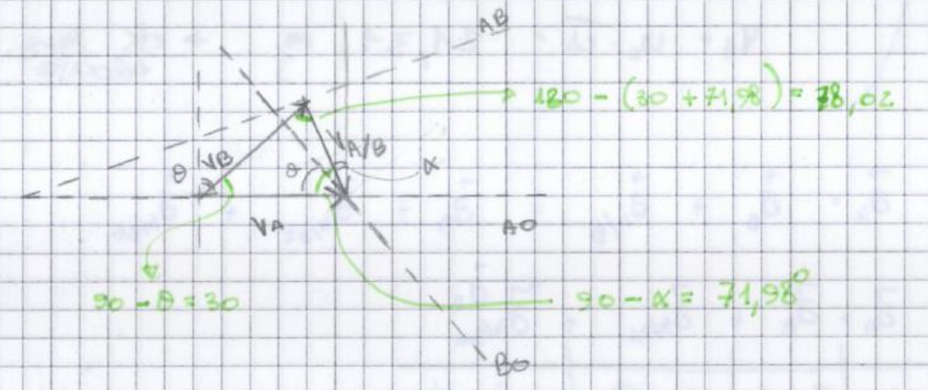
$v_A = ? \qquad a_A = ?$

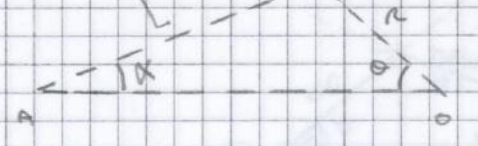
$\omega_{biella} ?$

$\dot{\omega}_{biella} ?$

$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$

?	$n \overline{OB}$	$\omega_{biella} \cdot \overline{AB}$
// AO	⊥ OB	⊥ AB
→	↗	? ↘





$\dfrac{L}{\sin\theta} = \dfrac{r}{\sin\alpha} \rightarrow \sin\alpha = \dfrac{r}{L} \cdot \sin\theta$

$\alpha = \arcsin\left(\dfrac{r}{L} \sin\theta\right) = 18{,}02°$

$\dfrac{V_B}{\sin(71{,}98)} = \dfrac{V_{A/B}}{\sin(30)} \rightarrow V_{A/B} = \dfrac{\sin(30)}{\sin(71{,}98)} V_B = 10{,}32 \ \dfrac{m}{s}$

$V_{A/B} = \omega_{biella} \cdot \overline{AB} \rightarrow \omega_{biella} = \dfrac{V_{A/B}}{\overline{AB}} = \dfrac{10{,}32}{0{,}35} = 29{,}48 \ \dfrac{rad}{s}$

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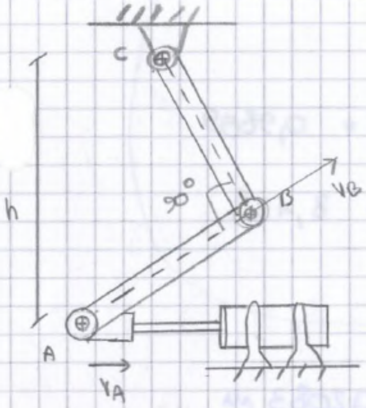
$\dot{\omega}_{biella} = 7742,559 \ \frac{rad}{s^2}$

$a_A = a_B \cdot \cos\theta + \left(a_{A/B_n} \cdot \cos\alpha - a_{A/B_t} \cdot \cos 71,98\right)$

$a_A = 1542,1329 + 289,2544 - 838,30339$

$= 993,08 \ \frac{m}{s^2}$

ES. 1.38 pag. 134



$V_A = 0,5 \ m/s$

$AB = BC = 0,125 \ m$

$h = 0,175 \ m$

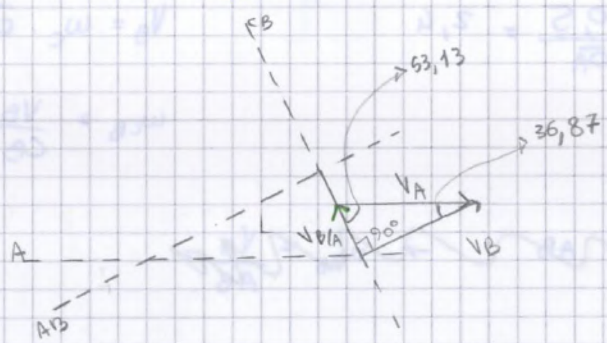
ω_{AB} ?

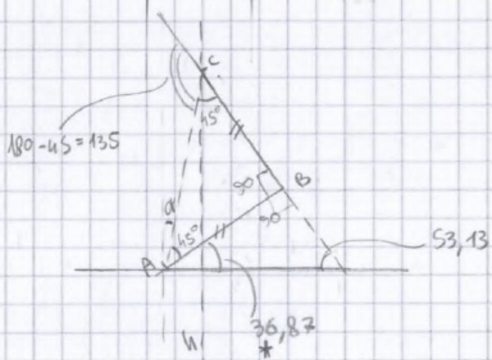
ω_{CB} ?

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

? $\omega_{CB}\,\overline{CB}$	0,5	? $\omega_{AB}\,\overline{AB}$
$\perp CB$	// A	$\perp AB$
↗	→	↖

$\vec{V}_B = \vec{\omega}_{CB} \wedge \vec{CB}$





$V_B = V_A \cdot \sin 53,13 = 0,3999$

$V_B = \omega_{CB} \cdot CB \rightarrow \omega_{CB} = \frac{V_B}{CB} = \frac{0,3999}{0,125}$

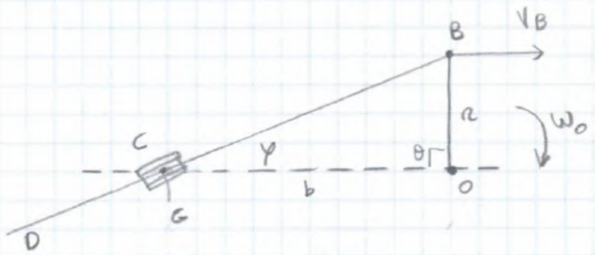
$= 3,199 \ \frac{rad}{s}$

$V_A \cdot \cos 53,13 = V_{B/A} = 0,30$

$V_{B/A} = \omega_{AB} \cdot AB$

$\omega_{AB} = \frac{V_{B/A}}{AB} = \frac{0,30}{0,125} = 2,4 \ \frac{rad}{s}$

ES. 1.41 pag. 1.38



$\omega_o = 5 \, rad/s$

$\theta = 90°$

$\dot{\varphi} = ?$

$\ddot{\varphi} = ?$

$r = 0,250 \, m$

$b = 0,600 \, m$

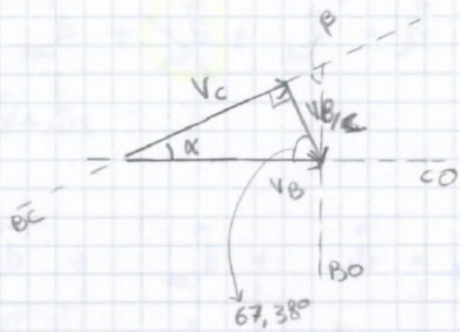
$\vec{V}_B = \vec{\omega}_o \wedge \vec{r}$

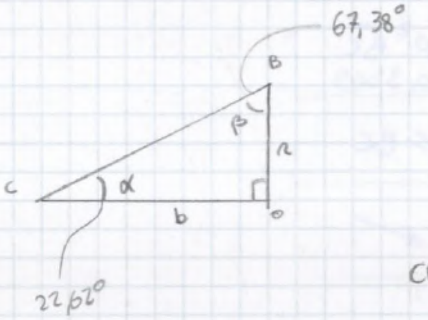
$\vec{V}_B = \vec{V}_C + \vec{V}_{B/C}$

$\rightarrow \vec{V}_{Cr} + \cancel{\vec{V}_{Cm}} = \vec{V}_{C_r} + \cancel{\vec{V}_G} + \cancel{\vec{V}_{C/G}}$

$\rightarrow$ = 0 perché la guida ruota attorno a G = C

$\omega_o r$	?	? $\dot{\varphi} \, \overline{CB}$
$\perp BO$	$// CB$	$\perp CB$
$\rightarrow$	$\nearrow$	$\searrow$





$\sqrt{b^2 + r^2} = CB = 0,65$

$CB \cdot \cos\alpha = b \rightarrow \cos\alpha = \frac{b}{CB}$

$\alpha = \arccos\left(\frac{b}{CB}\right) = 22,62°$

$\beta = 180 - (90 + 22,62) = 67,38°$

$V_C = V_B \cdot \cos\alpha = 5 \cdot 0,250 \cdot \cos 22,62 = 1,25 \cdot \cos\alpha = 1,154$

$V_{B/C} = V_B \cdot \sin\alpha = 1,25 \cdot \sin 22,62 = 0,4807$

$V_{B/C} = \dot{\varphi} \, \overline{CB} \rightarrow \dot{\varphi} = \frac{V_{C/B}}{\overline{CB}} = \frac{0,4807}{0,65} = 0,739 \, \frac{rad}{s}$ ⊗

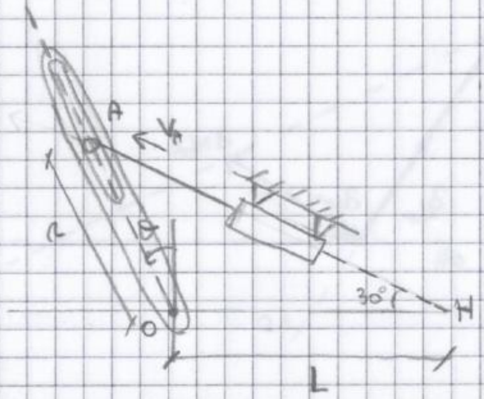
$$\begin{cases} a_B + a_{CR} \cdot \sin 22{,}62 = (a_{cor} + a_{B/Ct}) \cdot \cos 22{,}62 + a_{B/C_n} \cdot \sin 22{,}62 \\ a_{CR} \cdot \cos 22{,}62 + (a_{cor} + a_{B/Ct}) \cdot \sin 22{,}62 = a_{B/C_n} \cdot \cos 22{,}62 \end{cases}$$

$$\begin{cases} \underline{6{,}25} + a_{CR} \cdot 0{,}3846 = \underline{1{,}574} + \ddot{\varphi}\, 0{,}5999 + \underline{0{,}136} \\ a_{CR} \cdot 0{,}923 + \underline{0{,}6561} + \ddot{\varphi} \cdot 0{,}25 = \underline{0{,}3276} \end{cases}$$

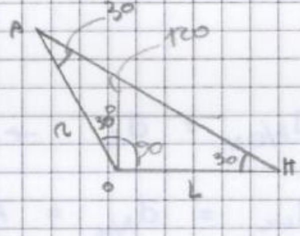
in realtà era

$$\begin{cases} 4{,}54 + a_{CR} \cdot 0{,}3846 - \ddot{\varphi}\, 0{,}5999 = 0 \\ 0{,}328 + a_{CR} \cdot 0{,}923 + \ddot{\varphi} \cdot 0{,}25 = 0 \end{cases} \rightarrow \begin{array}{l} a_{CR} = -2{,}04 \\ \ddot{\varphi} = +6{,}25 \end{array}$$

ES. 1.44 pag. 1.43



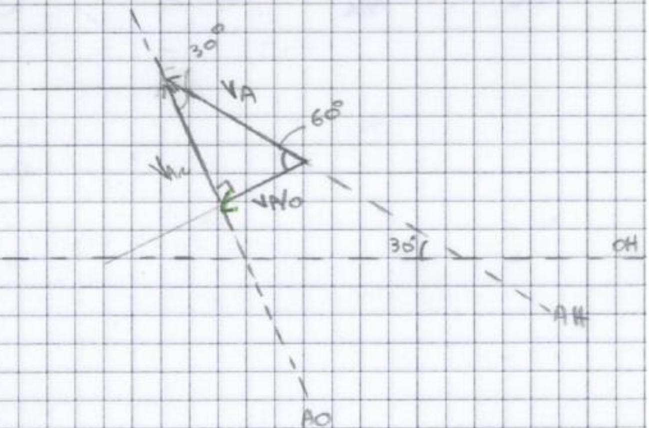
$V_A = 4$ m/s
$\dot{r} = ?$
$\ddot{r} = ?$
$\ddot{\theta} = ?$
$\theta = 30°$
$L = 0{,}300$ m



$$\vec{V}_A = \vec{V}_{AR} + \vec{V}_{A_{tr}} = \vec{V}_{AR} + \cancel{\vec{V}_O} + \vec{V}_{A/O}$$

$$\vec{V}_A = \vec{V}_{AR} + \vec{V}_{A/O}$$

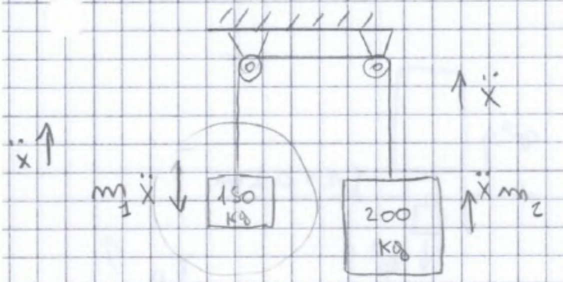
4	$\dot{r}$!	$\overline{AO} \cdot \dot{\theta}$!
// AH	// AO	⊥ AO
↖	↖	? ↙

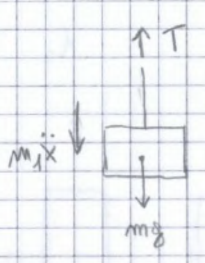


$V_A \cdot \sin 60° = V_{AR} = \dot{r} = 3{,}464$ rad/s

ES. 2.5 pag. 87

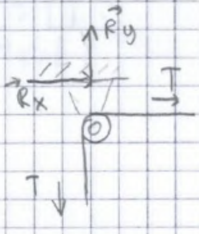
CASO A)





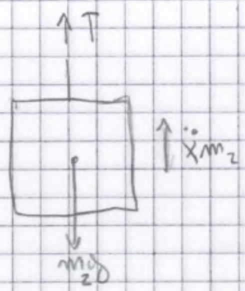
$+\uparrow \quad T + \ddot{x} m_1 - m_1 g = 0$

$T = m_1 g + \ddot{x} m_1$



$+\uparrow \quad \vec{R}_y - T = 0$

$\xrightarrow{+} \quad \vec{R}_x + T = 0$



$+\uparrow \quad T - m_2 g + \ddot{x} m_2 = 0$

$T = m_2 g + \ddot{x} m_2$

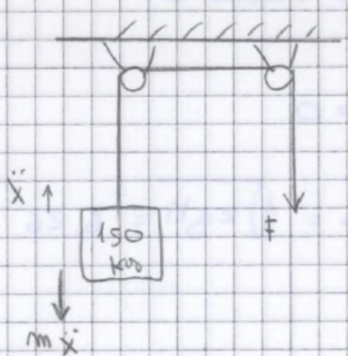
$m_1 g - \ddot{x} = m_2 g + \ddot{x} \rightarrow 2\ddot{x} = m_1 g - m_2 g$

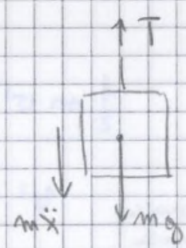
$\ddot{x} = \dfrac{(m_1 - m_2) g}{2} = -245{,}25$

$m_1 g + \ddot{x} m_1 = m_2 g + \ddot{x} m_2$

$m_2 g - m_1 g = (m_1 + m_2)\ddot{x} \qquad \ddot{x} = \dfrac{(m_2 g - m_1 g)}{m_1 + m_2} = 1{,}4 \ \dfrac{m}{s^2}$

CASO B)





$+\uparrow \quad T - mg - m\ddot{x} = 0$

$T = mg + m\ddot{x}$

$T = F = 200 \cdot 9{,}81 \text{ N} \qquad \ddot{x} = \dfrac{F - mg}{m} = 3{,}27 \ \dfrac{m}{s^2}$

$t_2 - t_3$

eq. cons. quant. di moto
$$m v' + \cancel{m_p v_p} = m v'' + m_p \cdot v_p''$$

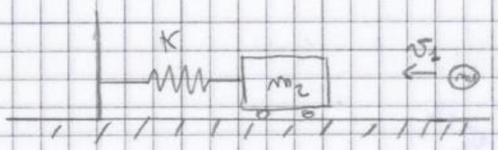
$$v_p'' = \frac{m v' - m v''}{m_p} = \frac{m(v' - v'')}{m_p} = 2{,}554 \ m/s$$

en. cinetica prima urto
$$E_c^- = \frac{1}{2} m v'^2$$

en. cinetica dopo urto
$$E_c^+ = \frac{1}{2} m v''^2 + \frac{1}{2} m_p v_p''^2$$

$$E_p\% = \frac{E_c^- - E_c^+}{E_c^-} = 0{,}45 \rightarrow 45{,}09\% \rightarrow$$ en. persa durante l'urto

ES 1 esercizi quad.

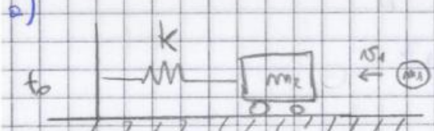


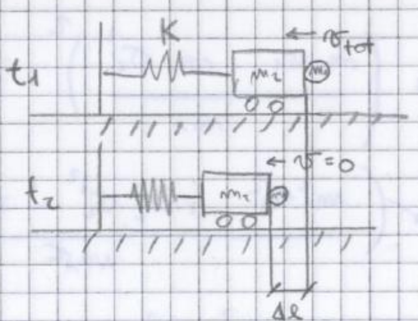
$m_2 = 50$ kg $v_1 = 500$ m/s
$m_1 = 5$ kg $k = 1000$ N/m
$v_2 = \emptyset$

a) urto anelastico
b) urto elastico

v_1' v_2' v_{tot} ? Δx max ?

a)





$t_0 - t_1$
$$m_1 v_1 = (m_1 + m_2) v_{fin} \rightarrow v_{fin} = \frac{m_1 v_1}{m_1 + m_2} = 45{,}45 \ \frac{m}{s}$$

$t_1 - t_2$
$$\Delta E_c + \cancel{\Delta E_g} + \Delta E_e = \cancel{\Delta L} + \cancel{\Delta L_e}$$

$$\underbrace{\underbrace{m_1}_{5} v_1' \underbrace{- m_1 v_1}_{2500} + \underbrace{\underbrace{m_2}_{50} \underbrace{v_2'}_{9,09}}_{454,5}}_{2045,5} = 0 \quad \rightarrow \quad v_1' = 409,1 \ m/s$$

$t_1 - t_2$

$$\Delta E_c + \Delta E_e = 0$$

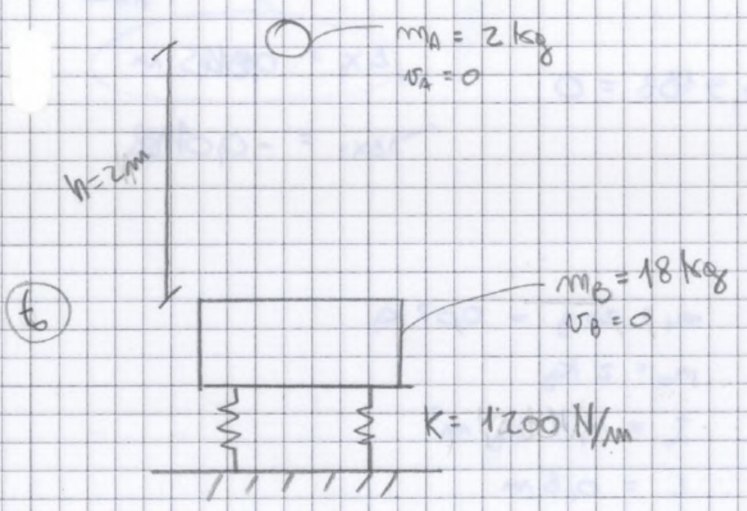
$$\Delta E_c = \left(\frac{1}{2} m_2 (v_2 = 0)^2 + \cancel{\frac{1}{2} m_1 v_1'}\right) - \left(\cancel{\frac{1}{2} m_1 v_1'} + \frac{1}{2} m_2 v_2'^2\right)$$

$$= -\frac{1}{2} m_2 v_2'^2$$

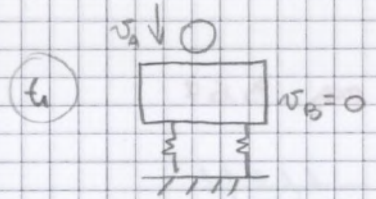
$$\Delta E_e = \frac{1}{2} k (\Delta x)^2 - \cancel{\frac{1}{2} k \cdot 0^2}$$

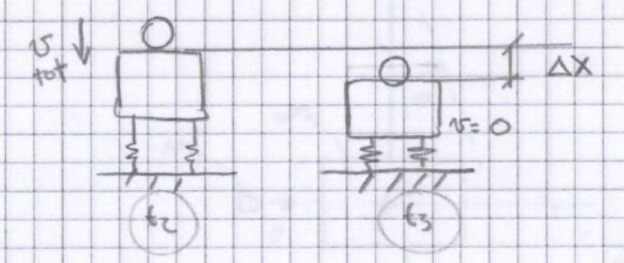
$$-\frac{1}{2} m_2 v_2'^2 + \frac{1}{2} k (\Delta x)^2 = 0 \qquad \Delta x = \sqrt{\frac{\cancel{\frac{1}{2}} m_2 v_2'^2 \cdot \cancel{2}}{k}} = \cancel{0,64} \ 2,032 \ m$$

ES. 3.68 pag. 3.64



ΔX molle!

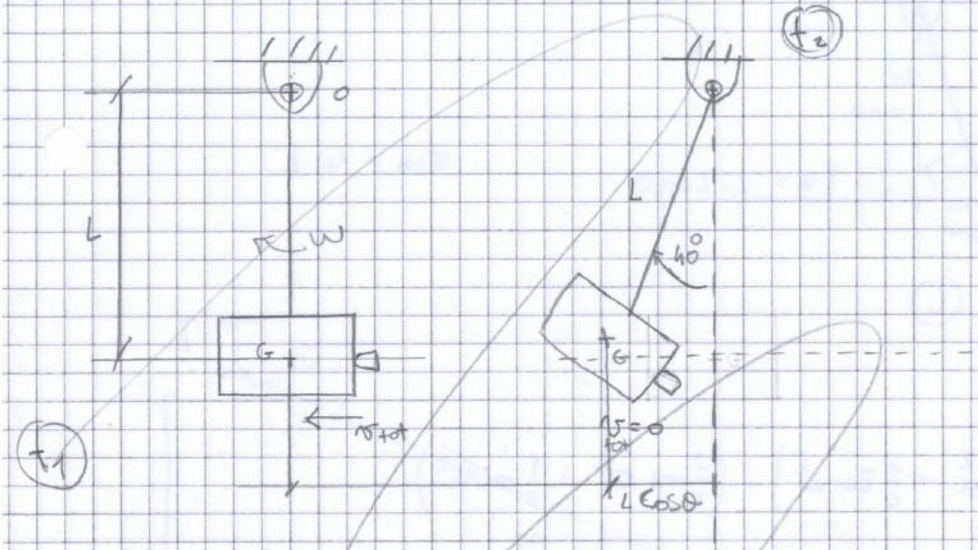




poco prima dell'urto ($t_0 - t_1$)

$$\Delta E_g = mg \cancel{h_1} - mg h_0 \qquad (h_1 = 0)$$

$$\Delta E_c = \frac{1}{2} m v_{A1}^2 - \cancel{\frac{1}{2} m v_{A0}^2}$$



$t_0 - t_1$

$$m_1 v_1 + \cancel{m_0 v_0} = m_{tot} \cdot v_{tot}$$

$$\frac{1}{2} m_1 v_1^2 + \cancel{\frac{1}{2} m_0 v_0^2} = \frac{1}{2} m_{tot} v_{tot}^2 + \frac{1}{2} I \omega^2$$

$t_1 - t_2$

$$\Delta E_c = \frac{1}{2} m_{tot} v_{tot}^2 + \frac{1}{2} I \omega^2 - \underbrace{\cancel{\frac{1}{2} m_{tot} v_{tot}^2}}_{=0} + \underbrace{\cancel{\frac{1}{2} I \omega^2}}_{=0}$$

$\Delta E_c = 0$ $\quad \frac{1}{2} m_{tot} v_{tot}^2 + \frac{1}{2} I \omega^2 = 0$

$$\frac{1}{2} m_{tot} (L \sin\theta\, \omega)^2 + \frac{1}{2} I \omega^2 = 0$$

$x = -L \cos\theta$

$v_{tot} = \dot{x} = + L \sin\theta\, \omega$ $\quad \frac{1}{2} m_{tot} L^2 \sin^2\theta\, \omega^2 + \frac{1}{2} I \omega^2 = 0$

$$\begin{cases} m_1 v_1 = m_{tot} \cdot (L \sin\theta\, \omega) \\ \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_{tot} (L \sin\theta\, \omega)^2 + \frac{1}{2} I \omega^2 \end{cases}$$

$$\begin{cases} m_1 v_1 - m_{tot} L \sin\theta\, \omega = 0 \\ \frac{1}{2} m_1 v_1^2 = \left(\frac{1}{2} m_{tot} L^2 \sin^2\theta + \frac{1}{2} I\right) \omega^2 \end{cases} \qquad \begin{matrix} 0{,}02 v_1 - 1{,}0387 \omega = 0 \\ 0{,}01 v_1^2 - 1{,}017 \omega^2 = 0 \end{matrix}$$

$v_1 = 51{,}935\, \omega$

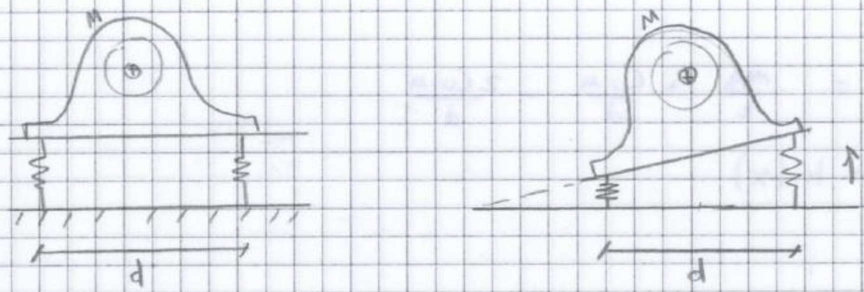
$0{,}01 (51{,}935 \omega)^2 - 1{,}017 \omega^2 = 0$ $\qquad 26{,}97\, \omega^2 - 1{,}017 \omega^2 = 0$

$$7{,}15 \cdot 10^{-7} \, v_1^2 - 838{,}95 \cdot 10^{-7} \, v_1^2 + 3{,}708 = 0$$

$$-831{,}8 \cdot 10^{-7} \, v_1^2 + 3{,}708 = 0$$

$$v_1 = \cancel{831{,}81 \cdot 10^{-7}} \sqrt{\frac{3{,}708}{831{,}8 \cdot 10^{-7}}} = 211{,}13 \ m/s$$

ES. 3.50 pag. 3.49 - es. 4 esercizi sul quad.



rotaz. antiorario piano

$P = 5 \ kW = 5000 \ W$

$\omega = 1500 \ giri/min = 157 \ rad/s$

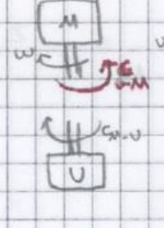
$K = 30 \ N/mm = 30000 \ N/m$

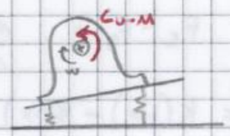
verso rotaz. motore ↑ $\delta = ?$

$d = 0{,}25 \ m$

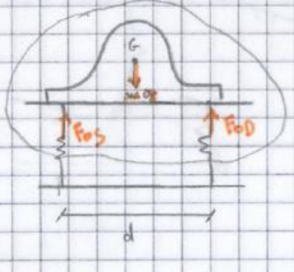
$$P = C_{M-U} \cdot \omega \rightarrow C_{M-U} = \frac{P}{\omega} = 31{,}847 \ N \cdot m$$

Le molle si schiacciano verso sinistra
quindi la coppia di reazione (utilizzatore - motore)
è in verso antiorario; questo significa
che la rotazione del motore è in verso orario (motore - utilizzatore)
per il principio di azione e reazione





MOTORE SPENTO



$$\uparrow \quad F_{0S} + F_{0D} - mg = 0$$

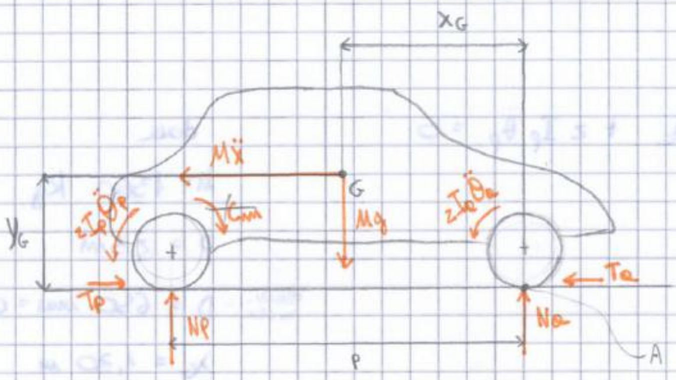
$$C_G \quad F_{0S} \cdot \frac{d}{2} - F_{0D} \cdot \frac{d}{2} = 0 \rightarrow F_{0S} = F_{0D}$$

$$F_{0S} + F_{0S} - mg = 0 \rightarrow F_{0S} = \frac{mg}{2}$$

$$F_{0D} + F_{0D} - mg = 0 \rightarrow F_{0D} = \frac{mg}{2}$$

$$F_{0S} = F_{0D} = \frac{mg}{2}$$

ES. 3.12 pag. 111



$M = 1360\ kg$

$p = 2,3\ m$

diametro ruote $D = 650\ mm = 0,65\ m$

$x_G = 1,30\ m$

$y_G = 0,72\ m$

massa di 1 ruota $m = 10\ kg$

raggio di inerzia $\rho = 0,2\ m$

$f_a = 1$

C_m ? $\ddot{x}$? $N_A = ?$ $T_A = ?$ $N_p = ?$ $T_p = ?$

incognite: $C_m, \ddot{x}, N_a, T_a, N_p, T_p, \ddot{\theta}_a, \ddot{\theta}_p$

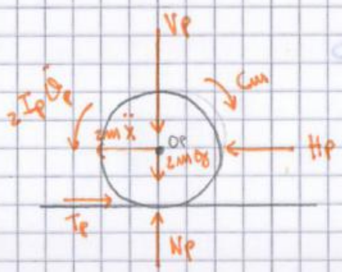
Sistema intero:

$\rightarrow \quad T_p - T_a - M\ddot{x} = 0 \quad (1)$

$\uparrow \quad N_p + N_a - Mg = 0 \quad (2)$

$A) \quad Mg \cdot x_G + M\ddot{x} \cdot y_G - N_p \cdot p + 2 I_a \ddot{\theta}_a + 2 I_p \ddot{\theta}_p = 0 \quad (3)$

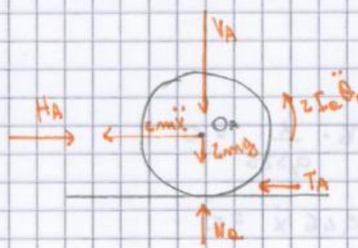
ruota posteriore:



$\rightarrow \quad T_p - H_p - 2m\ddot{x} = 0$

$\uparrow \quad N_p - V_p - 2mg = 0$

$O_p) \quad 2 I_p \ddot{\theta}_p + T_p \cdot \frac{D}{2} - C_m = 0 \quad (4)$



$\rightarrow \quad H_A - T_A - 2m\ddot{x} = 0$

$\uparrow \quad N_a - V_A - 2mg = 0$

$O_A) \quad 2 I_a \ddot{\theta}_a - T_a \cdot \frac{D}{2} = 0 \quad (5)$

Condizioni di aderenza limite ruote posteriori: $T_p = f_a \cdot N_p \quad (6)$ $\quad \ddot{x} = \frac{D}{2} \ddot{\theta}_p \quad (7)$

HP condizioni di aderenza ruote anteriori: $\ddot{x} = \frac{D}{2} \ddot{\theta}_a \quad (8)$ → verifica $T_a \leq f_a \cdot N_a$

$$\ddot{\theta}_e = \frac{\ddot{x}}{0,325} = \frac{8,02}{0,325} = 24,677$$

$$\ddot{\theta}_p = \frac{\ddot{x}}{0,325} = 24,677$$

$$T_e = \frac{0,8}{0,325} \cdot \ddot{\theta}_e = 60,74 \quad N$$

$$N_p = T_p = T_e + 1360 \cdot \ddot{x} = 10\,967,94 \ N$$

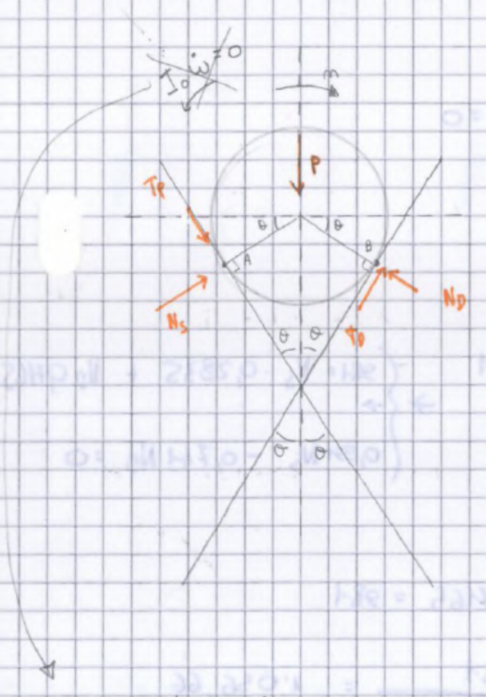
$$C_m = 0,8\ \ddot{\theta}_p + 0,325\ T_p = 3\,584,32 \quad N \cdot m$$

$$N_e = 13\,341,6 - N_p = 2\,373,66 \quad N$$

verifica: $\frac{T_e}{N_e} \le f_e$ verificata

$\frac{T_e}{N_e} \to 0,02$, $f_e \to 1$

ES. 3.19



$r = 15\ mm = 0,015\ m$

$\theta = 30°$

$m = 100$ giri/min

$P = 981\ N$

$\rho = 0,2\ m$

$f = 0,25$

C ? per vincere attriti ed $\omega = cost$

Q ? dissipata

T ? (tempo) affinché eliminando coppie esterne si fermi

se $\omega = cost \Rightarrow \dot{\omega} = 0$

$m = 100$ giri/min $\Rightarrow 0,166 \frac{giri}{s} \Rightarrow 1,66 \cdot 2\pi = 1,047 \frac{rad}{s}$

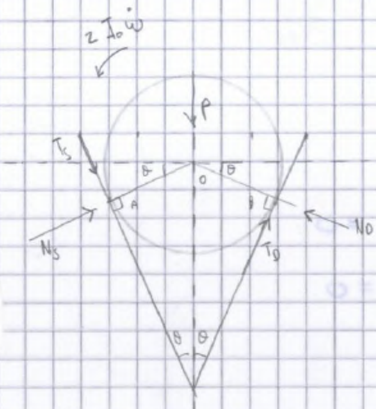
$\omega = 1,047 \frac{rad}{s}$

$P = C \cdot \omega = 6{,}92 \cdot 1{,}047 = 7{,}24 \ W \rightarrow \frac{J}{s}$

$1 \ cal = 4{,}186 \ J \qquad 7{,}24 \frac{J}{s} \rightarrow 1{,}729 \frac{cal}{s} \rightarrow \frac{1{,}729 \cdot 10^{-3} \ Kcal}{s}$

$0{,}238 \ cal = 1 \ J$

$1{,}729 \cdot 10^{-3} \ Kcal / 1s \rightarrow 1{,}729 \cdot 10^{-3} \cdot 3600 = 6{,}232 \frac{Kcal}{h}$



$I_0 = 2 M \rho^2 = 2 \ 100 \cdot 0{,}2^2 = 8 \ kg \cdot m^2$

è un albero

$P = M \cdot g \rightarrow M = \frac{P}{g} = 100 \ kg$

$L_i = \Delta E_c$

$E_{c_i} = \frac{1}{2} I_0 \omega^2 = 438{,}48 \qquad L_i = \underbrace{E_{c_f}}_{=0} - E_{c_i}$

$E_{c_f} = 0$

$P_m = \frac{L_i}{\Delta t} \qquad \Delta t = \frac{L_i}{P} = \frac{438{,}48}{72{,}4} = 6{,}05 \ s$

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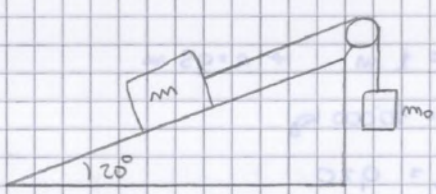


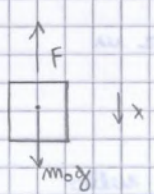
$m = 100 \ kg$

$f_a = 0{,}3$

A) m_0 oltre cui si ha risalita di m

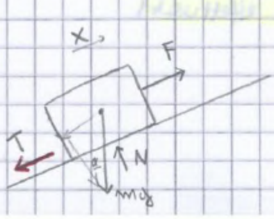
B) m_0 sotto cui si ha discesa di m

A)



$\uparrow \quad F - m_0 g = 0$

incognite F, m_0, T, N

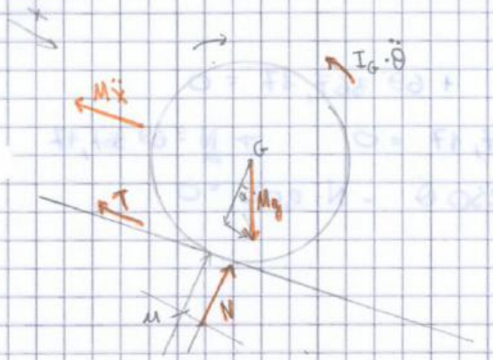


$\nearrow^{+} \quad F - T - mg \sin\alpha = 0$

$\nwarrow^{+} \quad N - mg \cos\alpha = 0$

$T = f_a \cdot N$

$\alpha = 10°$



$$-T - M\ddot{x} + Mg \cdot \sin\alpha = 0$$
$$N - Mg\cos\alpha = 0$$
$$\circlearrowleft G \quad T \cdot r - I_G \cdot \ddot{\theta} - N \cdot \mu = 0$$

incognite $T, N, \ddot{\theta}, \ddot{x}$

faccio HP di aderenza: $\ddot{x} = r\ddot{\theta}$ → verifica $T \le f_a \cdot N$

$I_G = m \cdot \frac{r^2}{2} = 1250$

$$\begin{cases} -T - 10000\,\ddot{x} + 17034{,}88 = 0 \\ N - 96609{,}64 = 0 \rightarrow \underline{\underline{N}} = 96609{,}64 \\ 0{,}5\,T - 1250\,\ddot{\theta} - N \cdot 0{,}02 = 0 \\ \ddot{x} = 0{,}5\,\ddot{\theta} \end{cases}$$

$$T = \frac{1250\,\ddot{\theta} + 1932{,}19}{0{,}5} = 2500\,\ddot{\theta} + 3864{,}38 \qquad \underline{\underline{T}} = 8254{,}38$$

$$-2500\,\ddot{\theta} - 3864{,}38 - 5000\,\ddot{\theta} + 17034{,}88 = 0$$

$$\underline{\underline{\ddot{\theta}}} = \frac{17034{,}88 - 3864{,}38}{(5000 + 2500)} = 1{,}756 \qquad \underline{\underline{\ddot{x}}} = 0{,}878$$

$\frac{T}{N} \le f_a$? $\qquad \frac{T}{N} = 0{,}085 < 0{,}20$ verificata

moto rettilineo unif. accelerato:

$$x = \frac{1}{2}\, a\, t^2 \qquad a \to \ddot{x}$$

$$t = \sqrt{\frac{2L}{\ddot{x}}} = \sqrt{\frac{2 \cdot 200}{0{,}878}} = 21{,}34 \text{ s}$$

$$x = r\,\theta \longrightarrow \theta = \frac{x}{r} = \frac{200}{0{,}5} = 400 \text{ rad} \qquad \frac{400}{2\pi} = 63{,}66 \text{ giri}$$

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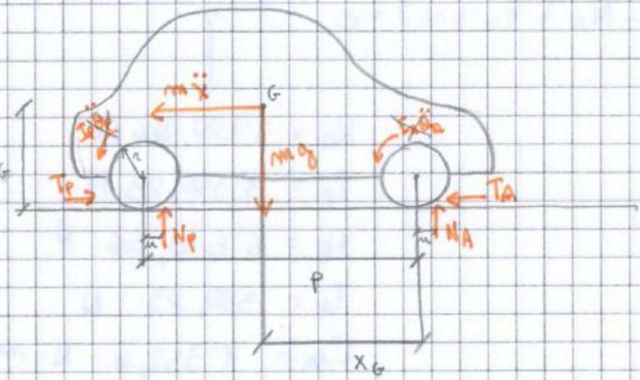


$p = 2{,}5\ m$

$x_G = 1{,}4\ m$

$y_G = 0{,}8\ m$

$r = 0{,}32\ m$

$u = 4\ cm = 0{,}04\ m$

$m = 1000\ kg$

$a = 3\ m/s^2 = \ddot{x}$

N_p ? N_A ? C_m ?

$f_{a\,min}$?

trascurare masse ruote

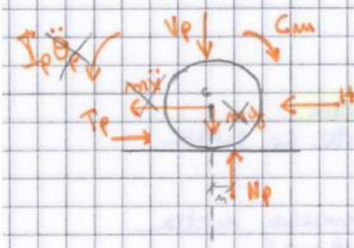
$I_p = \cancel{m_p \rho_p^2}$ $\quad I_a = \cancel{m_a \rho_a^2}$

$\xrightarrow{+}$ $\quad T_p - T_A - m\ddot{x} = 0 \quad$ ①

$\uparrow +$ $\quad N_p + N_A - mg = 0 \quad$ ②

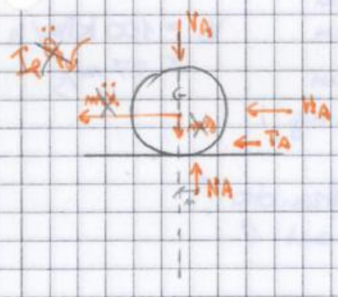
$\circlearrowleft G \quad -T_p \cdot y_G + T_A \cdot y_G + N_p(p - x_G - u) - N_A(x_G + u) = 0 \quad$ ③

ruota posteriore:



$\circlearrowleft G \quad -T_p \cdot r - N_p \cdot u + C_m = 0 \quad$ ④

ruota anteriore:



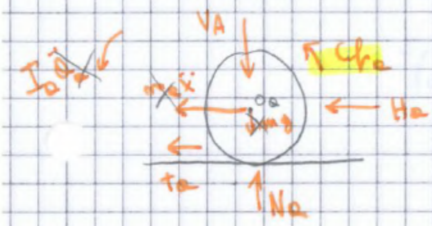
$\circlearrowleft G \quad T_A \cdot r - N_A \cdot u = 0 \quad$ ⑤

Condizioni di aderenza limite

$T_p = f_a \cdot N_p \quad$ ⑥ $\quad \ddot{x} = r\,\ddot{\theta}_p$

$T_a = f_a \cdot N_a \quad$ ⑦ $\quad \ddot{x} = r\,\ddot{\theta}_a$

ruota anteriore:



$$O_A) \quad C_{fa} - T_a \cdot r = 0 \quad (5)$$

aderenza limite

$$T_p = f_a \cdot N_p \quad (6) \qquad \ddot{x}_p = r\ddot{\theta}_p = \ddot{x} \quad (7)$$

$$T_a = f_a \cdot N_a \quad (8) \qquad \ddot{x}_a = r\ddot{\theta}_a = \ddot{x} \quad (9)$$

incognite $T_p \; T_a \; \ddot{x} \; N_p \; N_a \; C_{fp} \; C_{fa} \; \ddot{\theta}_p \; \ddot{\theta}_a$

$$\begin{cases} -T_p - T_a - 1000\ddot{x} = 0 \\ -9810 + N_p + N_a = 0 \qquad \rightarrow \quad N_p = 9810 - N_a \\ -T_p \cdot 0{,}8 - T_a \cdot 0{,}8 - N_p(1{,}1) + N_a \cdot 1{,}4 = 0 \\ C_{fp} = T_p \cdot 0{,}32 \\ C_{fa} = T_a \cdot 0{,}32 \\ T_p = 0{,}75\, N_p \qquad T_p = 0{,}75\,(9810 - N_a) \\ \ddot{x} = 0{,}32\, \ddot{\theta}_p \\ T_a = 0{,}75\, N_a \\ \ddot{x} = 0{,}32\, \ddot{\theta}_a \end{cases}$$

$$-0{,}75 \cdot (9810 - N_a) \cdot 0{,}8 - 0{,}75 N_a \cdot 0{,}8 - (9810 - N_a) \cdot 1{,}1 + N_a \cdot 1{,}4 = 0$$

$$-5886 + \cancel{0{,}6 N_a} - \cancel{0{,}6 N_a} - 10791 + 1{,}1 N_a + N_a\, 1{,}4 = 0$$

$$N_a = 6670{,}8 \; N \qquad T_A = 5003{,}1$$

$$C_{fa} = 1600{,}99 \; N \cdot m \qquad N_p = 3139{,}2 \qquad T_p = 2354{,}4$$

$$C_{fp} = 753{,}408 \; N \cdot m$$

$$\ddot{x} = \frac{-T_p - T_A}{1000} = -7{,}357 \; \frac{m}{s^2}$$

$N_S = N_D \cdot 0{,}396$

$0{,}095 \cdot N_D + N_D \cdot 1{,}016 = 490{,}5$

$N_D = 441{,}494$

$N_S = 174{,}8316$

$M = C = \frac{D}{2}\left(f N_S + f N_D \right) = 184{,}89 \cdot \frac{D}{2} = 83{,}2 \ N \cdot m$

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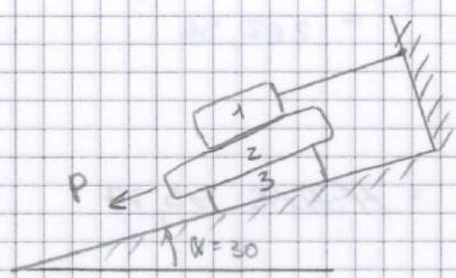


$m_1 = 30\,kg$

$m_2 = 50\,kg$

$m_3 = 40\,kg$

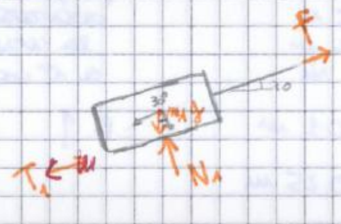
$f_{a12} = 0{,}3$

$f_{a23} = 0{,}4$

$f_{a3} = 0{,}45$

$\vec{P}_{max}$ senza movimento dei corpi

blocco 1



$-T_1 + F - m_1 g \sin 30° = 0$

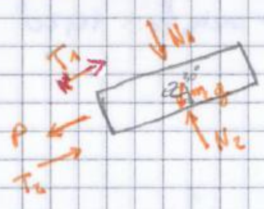
$N_1 - m_1 g \cos 30° = 0$

$T_1 = f_{a12} \cdot N_1$

$N_1 = 254{,}87$ $\quad F = \cancel{70{,}69}\ 223{,}61$

$T_1 = 76{,}46$

blocco 2



$+T_1 - P + T_2 - m_2 g \sin 30° = 0$

$N_2 - m_2 g \cdot \cos 30° - N_1 = 0$

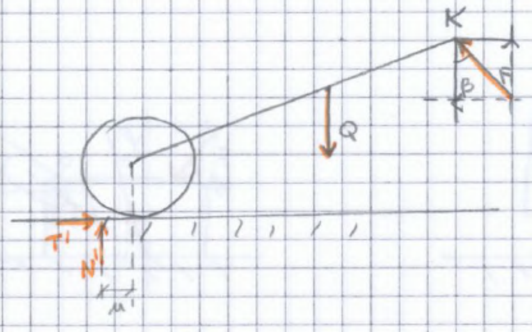
$T_2 = f_{a23} \cdot N_2$

blocco 3

$T_3 - T_2 - m_3 g \cdot \sin 30° = 0$

$N_3 - N_2 - m_3 g \cdot \cos 30° = 0$

$T_3 = f_{a3} \cdot N_3$



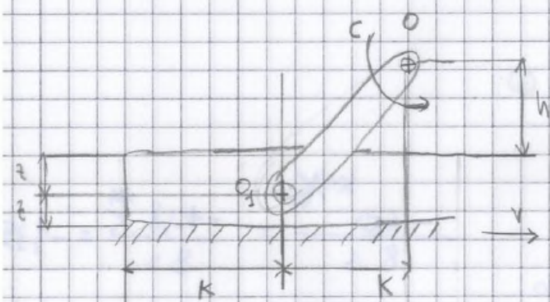
$\xrightarrow{+}$ $T' - F \sin\beta = 0$ 2

$\uparrow +$ $N' - Q + F\cos\beta = 0$ 3

$\circlearrowleft$ $T' \cdot h - N'(a+b+u) + Q \cdot b = 0$ 4

→ da qst 4 equazioni si ricavano N' T' F β

es. 4.1 pag. 132



$T = ?$

P_{diss} ?

$h = 0{,}10\ m$

$k = 0{,}07\ m$

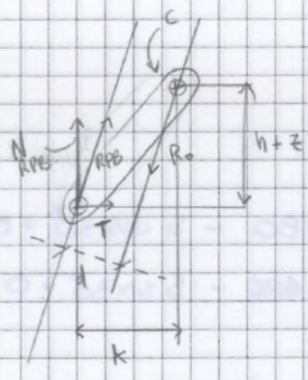
$z = 0{,}025\ m$

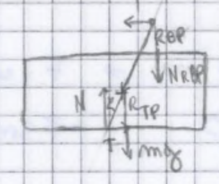
$f = 0{,}15$

$m = 7\ kg$

$v = 5\ m/s$

$C = 40\ N\cdot m$





$+\uparrow$ $N - mg - N_{RBP} = 0$

$N_{RBP} = N - mg$

$= N_{RPB}$

$T = f \cdot N$

$N = T/f$

$N - mg = \frac{T}{f} - mg$

C_O $-C + N_{RPB} K - T(h+z) = 0$

$T = \frac{C + mgK}{\left(\frac{K}{f} - (h+z)\right)} = \frac{40 + 4{,}8069}{\left(\frac{0{,}07}{0{,}15} - (0{,}10 + 0{,}025)\right)} = \cancel{117{,}07}\ 131{,}14\ N$

$P_{diss} = T \cdot v = \cancel{\ldots}\ 655{,}71$

~~E = 8.697,41 N~~

$$S = \frac{18.224}{(\cos 30^\circ + \sin 30^\circ)} = 13.340,89$$

$$F = 12.407,6 \ N$$

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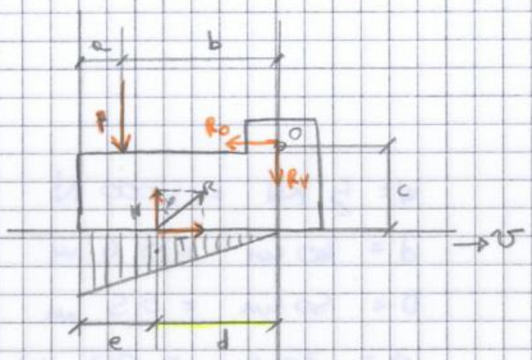


$T = ?$

R_o ? R_v ?

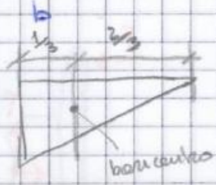
$a = 0,05$ m
$b = 0,175$ m
$c = 0,075$ m
$f = 0,1$
$F = 500$ N

$\xrightarrow{+}$ $T - R_o = 0$ ①

$+\uparrow$ $-F + N - R_v = 0$ ②

$\circlearrowleft$ $N \cdot d - T \cdot c - F \cdot b = 0$ ③ $\rightarrow$ $F = \dfrac{N \cdot d - T \cdot c}{b} = \dfrac{N \cdot (d - f \cdot c)}{b}$

$T = f \cdot N$ ④



oppure

$$-\frac{N \cdot (d - f \cdot c)}{b} + N - R_v = 0$$

$$N \cdot d = \int_0^{a+b} dN \cdot x = \int p\,dx \cdot x = k \int x^2 dx$$

$$= k \left[\frac{x^3}{3} \right]_0^{a+b} = k \cdot \frac{(a+b)^3 - a^3}{3}$$

~~$N = \dfrac{F \cdot b}{(d - f \cdot c)}$~~

$$N = \int_0^{a+b} dN = \int p\,dx = k \int x\,dx = k \left[\frac{x^2}{2} \right]_0^{a+b}$$

$$= k \, \frac{(a+b)^2}{2}$$

$$d = \frac{2}{3} \cdot \frac{(a+b)^3}{(a+b)^2} = 0,15 \quad ⑤$$

$\hookrightarrow$ 5 equaz. in 5 incognite

$$a = \ddot{x} = \frac{dV}{dt} = \frac{\underbrace{\cancel{V_f}}_{=0} - V_i}{t_f - \underbrace{\cancel{t_i}}_{=0}} = -\frac{V_0}{t_f}$$

$$x - \underbrace{\cancel{x_0}}_{=0} = V_0 (t_f - \underbrace{\cancel{t_0}}_{=0}) + \frac{1}{2}\ddot{x}(t_f - \underbrace{\cancel{t_0}}_{=0})^2$$

$$\begin{cases} x = V_0 t_f + \frac{1}{2}\ddot{x}\, t_f^2 \\ \ddot{x} = -\frac{V_0}{t_f} \end{cases}$$

2 eq.
2 inc.

$$x = V_0 t_f + \frac{1}{2}\left(-\frac{V_0}{\cancel{t_f}}\right) \cdot t_f^{\cancel{2}}$$

$$x = V_0 t_f - \frac{1}{2} V_0 t_f \rightarrow t_f = \frac{x}{\left(\frac{2}{2}V_0 - \frac{1}{2}V_0\right)} = \frac{1}{\frac{1}{2}V_0} = \frac{2}{0{,}2} = 10$$

V_0 → 0,2

$$\ddot{x} = -\frac{V_0}{t_f} = 0{,}02 \ \frac{m}{s^2}$$

$$\begin{cases} F \cdot 2a - N \cdot a = 0 \\ -f \cdot N \cdot \frac{D}{2} + W \cdot \frac{d}{2} - m\ddot{x} \cdot \frac{d}{2} = 0 \end{cases}$$

F e N uniche incognite

$$N = \frac{\left(-m\ddot{x}\frac{d}{2} + W \cdot \frac{d}{2}\right)}{\left(f \cdot \frac{D}{2}\right)} = 7983{,}69 \ N$$

$$F = \frac{N \cdot \cancel{a}}{2\cancel{a}} = 3991{,}84 \ N$$

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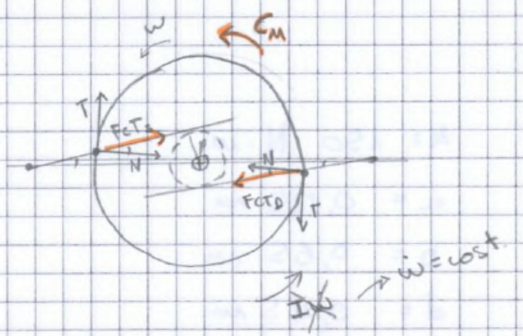

$$\circlearrowleft O \quad F_{CTS} \cdot \rho + F_{CTD} \cdot \rho - C_M = 0$$

$$C_M = (F_{CTS} + F_{CTD}) \cdot \rho = 21{,}498 \ \text{N·m}$$

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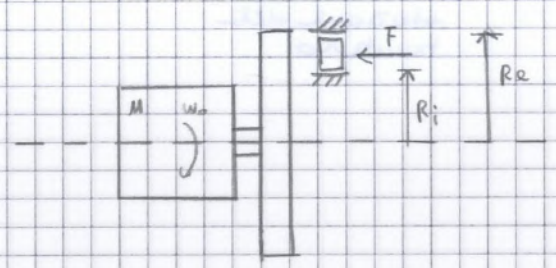


$M = 100$ kg

$\rho = 0{,}3$ m raggio medio

$\omega_0 = 1500$ g/min $= 157{,}08 \ \frac{rad}{s}$

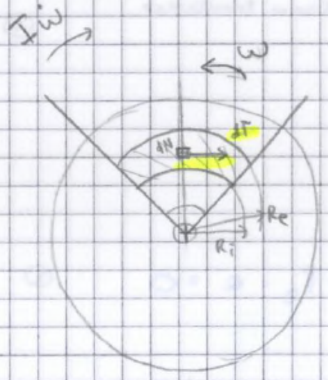
$R_e = 0{,}20$ m

$R_i = 0{,}15$ m

$f = 0{,}3$

$t = 10$ s ?

F x arrestare il sistema m

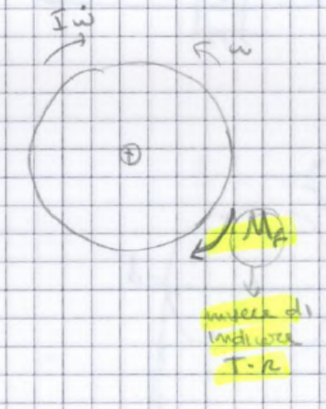


$F = N$

$$\dot{\omega} = \frac{d\omega}{dt} = \frac{\omega_f - \omega_0}{t_f - t_0} = -\frac{157{,}08}{10} = -15{,}708 \ \frac{rad}{s^2}$$

($\omega_f = 0$, $t_0 = 0$)

$$I = M \cdot \rho^2 = 9$$



$dM_f = dT \cdot r$
momento frenante

$$M_f = f \cdot F \frac{(R_e + R_i)}{2}$$

$$\circlearrowleft \quad I\dot{\omega} + M_f = 0 \rightarrow M_f = -I\dot{\omega} = 141{,}372$$

$$F = \frac{2 M_f}{f (R_e + R_i)} = 2\,692{,}8 \ \text{N}$$

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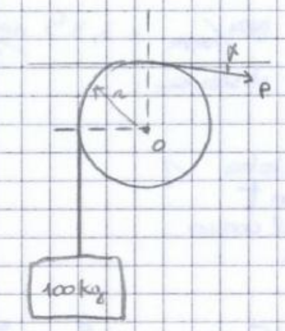
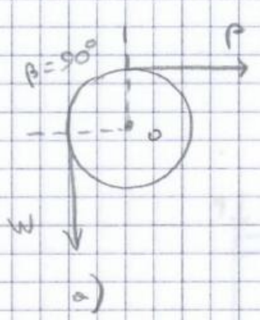
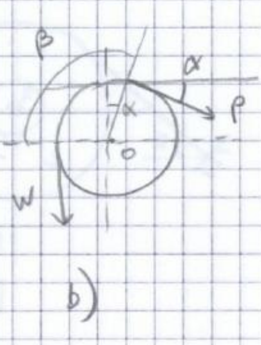






P garantisca equilibrio

$f_a = 0,3$

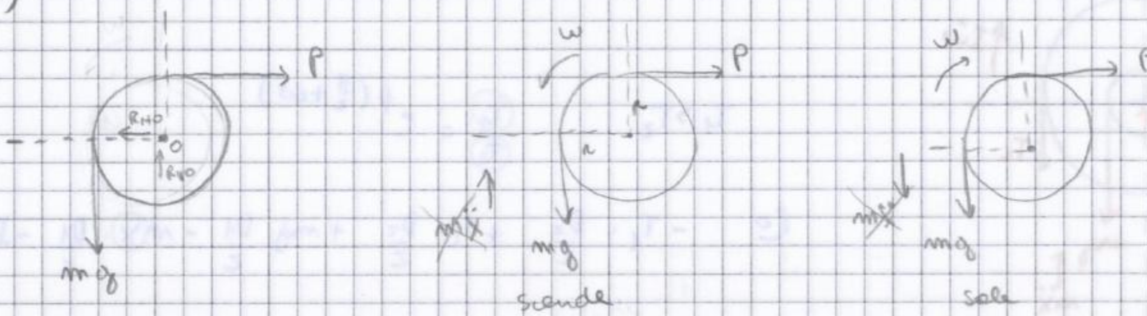
$\alpha = 0$

P_{max} e P_{min} affinché il carico non scende e non sale

$P = 500\ N$

α_{min} prima che il carico scenda

a)

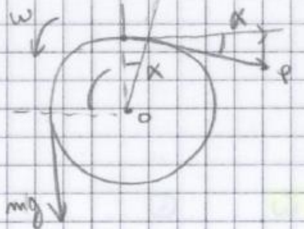


~~scende~~ sale:

$P > mg \qquad \frac{P}{mg} = e^{f\Delta\theta} \qquad P = mg \cdot e^{f\Delta\theta} \quad (\Delta\theta = \frac{\pi}{2}) \quad = 1571,54$

~~sale~~ scende: $mg > P \qquad \frac{mg}{P} = e^{f\Delta\theta} \qquad P = \frac{mg}{e^{f\Delta\theta}} = 612,36$

b)



$P \cdot \cos\alpha > mg$

$\Delta\theta = \left(\frac{\pi}{2} + \alpha\right)$

~~$\frac{P \cdot \cos\alpha}{mg} = e^{f\Delta\theta}$~~

~~$\alpha = \arccos\left(\frac{e^{f\Delta\theta} \cdot mg}{P}\right)$~~

scende

$\frac{mg}{P} = e^{f\left(\frac{\pi}{2}+\alpha\right)}$

$\ln\left(\frac{P}{mg}\right) = f\left(\frac{\pi}{2}+\alpha\right) \rightarrow \ln\left(\frac{P}{mg}\right) = f\frac{\pi}{2} + f \cdot \alpha$

$\alpha = \left[\ln\left(\frac{mg}{P}\right) - f\frac{\pi}{2}\right] \cdot \frac{1}{f} = 0,675\ rad = 38,7°$

$$T_2 = \frac{P \cdot (a+b)}{a} = 2'533,33$$

$$T_1 = T_2 \cdot e^{f\left(\frac{\pi}{2}+60\right)} = 4'506,38 \qquad \frac{\pi}{3}$$

$$\dot{\omega}\left(-I - m\frac{D_1^2}{4}\right)$$

$$-T_1 \cdot \frac{D_2}{2} + T_2 \cdot \frac{D_2}{2} + mg\frac{D_1}{2} - m\left(\frac{D_1}{2}\dot{\omega}\right)\cdot\frac{D_1}{2} - I\dot{\omega} = 0$$

$$\dot{\omega} = \frac{\left(-T_1 \cdot \frac{D_2}{2} + T_2 \cdot \frac{D_2}{2} + mg\frac{D_1}{2}\right)}{\left(I + m\frac{D_1^2}{4}\right)} = -1,05 \ \frac{rad}{s^2}$$

$$\ddot{x} = \frac{D_1}{2}\cdot\dot{\omega} = -0,183 \ \frac{m}{s^2}$$

$$C_f = (T_1 - T_2)\cdot\frac{D}{2} = 789,22 \quad N \cdot m$$

$$\dot{\omega} = \frac{d\omega}{dt} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{0 - 3,35}{\Delta t}$$

$$\Delta t = \frac{-3,35}{-1,05} = 3,19 \ s$$

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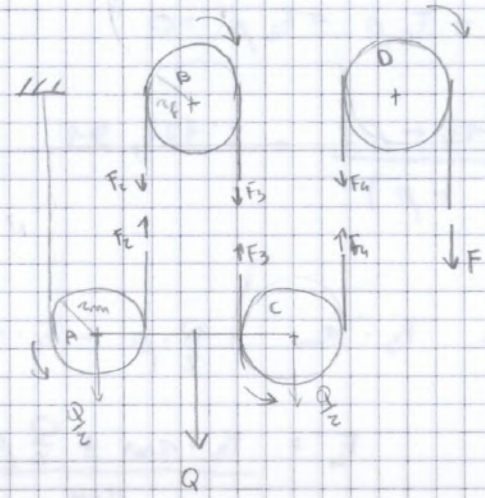
$r_m = 0{,}20$ m

$r_f = 0{,}15$ m

$m_q = 800$ kg

F necessaria x il sollevamento del carico senza attrito

F' con attrito perni
$f = 0{,}3 \quad r_p = 0{,}02$



senza attrito:

$L_e = F \cdot \ell$

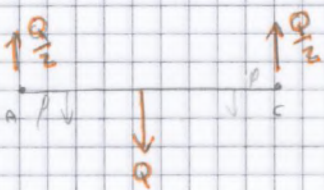
$L_U = Q \cdot s$

HP $\eta = 1 \Rightarrow F \cdot \ell = Q \cdot s$

$F \cdot 2m\cancel{s} = Q \cdot \cancel{s}$ → $F = \dfrac{Q}{2m} = 1'962$ N

n° pulegge mobili

7'848



$\rho = r_p \operatorname{sen} \varphi$

$f = \operatorname{tg} \varphi$

$\varphi = \operatorname{arc\,tg} f = 16{,}69°$ — 0,2914 rad

$\rho = 0{,}00574$ m

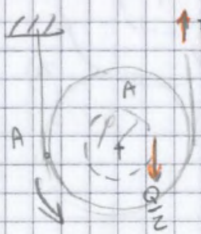
A: $\circlearrowleft^A \quad \frac{Q}{2} \cdot r_m - F_2 \cdot (2 r_m) = 0 \rightarrow F_2 = \dfrac{\frac{Q}{2} \cancel{r_m}}{2\cancel{r_m}} = \dfrac{Q}{4}$

$\circlearrowleft O_B \quad -F_2 \cdot r_f + F_3 \cdot r_f = 0 \rightarrow F_3 = \dfrac{F_2 \cdot \cancel{r_f}}{\cancel{r_f}} = \dfrac{Q}{4}$

$\circlearrowleft^C \quad F_4 = F_3 = \dfrac{Q}{4}$

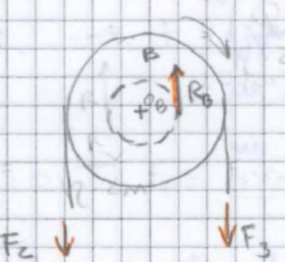
$\circlearrowleft O_D \quad F = F_4 = \dfrac{Q}{4} = 1'962$ N

NB le reaz. del perno si oppone al moto di rotazione



$\eta = \dfrac{Q \cdot v_q}{F \cdot v_F} = \dfrac{Q \cdot \cancel{v}}{F \cdot 2\cancel{v}}$

$F = \dfrac{Q}{\eta \cdot 2M}$



$+\uparrow R_B - F_2 - F_3 = 0$

$\circlearrowleft A \quad \frac{Q}{2} \cdot (r_m + \rho) - F_2 \cdot (2 r_m) = 0$

$F_2 = 2'018{,}2$

$\circlearrowleft O_B \quad -F_2 \cdot r_f + F_3 \cdot r_f - R \cdot \rho = 0$

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1) $+\uparrow \quad T - mg - m\ddot{x} = 0 \quad \rightarrow \quad T = mg + m\ddot{x} = 32.967\ N$

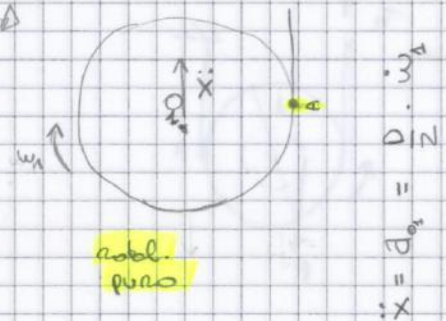
2) $\circlearrowleft O_A \quad F_2 \cdot \frac{D}{2} - F_1 \cdot \frac{D}{2} - I\dot{\omega}_1 - T \cdot \rho = 0$ ①

$+\uparrow \quad F_2 - T + F_1 = 0 \quad \rightarrow \quad F_1 = T - F_2$ ②

3) $\circlearrowleft O_B \quad -F \cdot \frac{D}{2} + R \cdot \rho + I\dot{\omega}_2 + F_2 \cdot \frac{D}{2} = 0$ ③

$+\uparrow \quad -F + R - F_2 = 0 \quad \rightarrow \quad R = F_2 + F$

relaz. cinematica $\quad \ddot{x} = \dot{\omega}_1 \cdot \frac{D}{2}$ ④



$\ddot{x} = \dot{\omega} \cdot \frac{D}{2}$

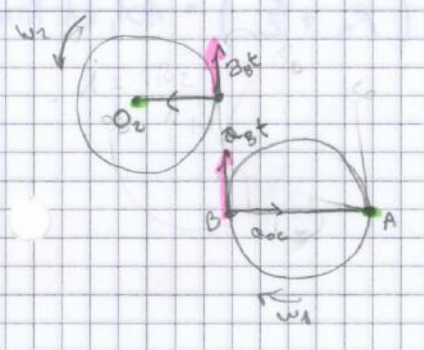
NB $\quad \rho = \frac{D}{2} \operatorname{sen} \varphi = 0{,}001594\ m \qquad \varphi = \operatorname{arctg} f$

$\dot{\omega}_1 = \frac{2\ddot{x}}{D} = \ 8 \ \frac{rad}{s^2}$

$F_2 \cdot \frac{D}{2} - (T - F_2)\frac{D}{2} - I \cdot \dot{\omega} - T \cdot \rho = 0$

$F_2 \cdot \frac{D}{2} - T\frac{D}{2} + F_2\frac{D}{2} - I\dot{\omega} - T\rho = 0$

$F_2 = \dfrac{T\rho + I\dot{\omega}_1 + T\frac{D}{2}}{\left(\frac{D}{2} + \frac{D}{2}\right)} = \dfrac{T\left(\rho + \frac{D}{2}\right) + I\dot{\omega}_1}{D} = 19.499\ N$



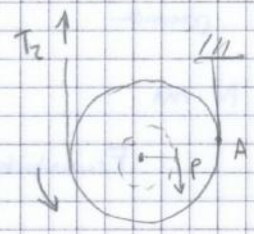
$a_{Bt} = D \cdot \dot{\omega}_1 = 4{,}8$ (rot. intorno ad A)

$a_{Bt} = \frac{D}{2} \cdot \dot{\omega}_2 \Rightarrow \quad \dot{\omega}_2 = \frac{a_{Bt} \cdot 2}{D} = 16\ \frac{rad}{s^2}$ ⑤

(rot. intorno a O_2)

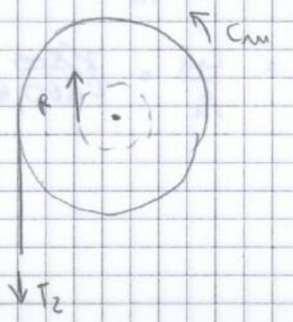
→ 5 eq 5 inc.

discesa:



$\circlearrowleft A \quad T_2 \cdot d + p\left(\frac{d}{2} - \rho_1\right) = 0$

$$T_2 = \frac{-p\left(\frac{d}{2} - \rho_1\right)}{d} = 496{,}683 \text{ N}$$



$\circlearrowleft O_3 \quad -C_m + p\rho_3 \underbrace{R}_{=T_2} - T_2 \cdot \frac{D}{2} = 0$

$$C_m = T_2\left(\rho_3 - \frac{D}{2}\right) = -98{,}6 \text{ N}$$

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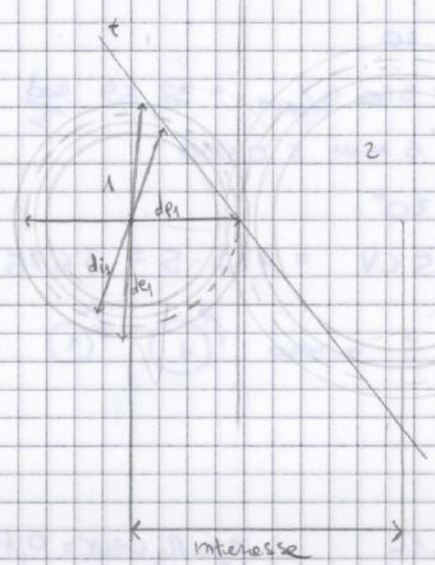

$i = 3$

$z_1 = 30$

$d_{e1} = 0{,}128$ m

$p = 0{,}01257$

m_1? = m_2?

d_{p1} d_{p2}? d_{e2}?

d_{i1} d_{i2}? z_2?

interasse?

$i = \frac{\omega_1}{\omega_2} = \frac{r_{p2}}{r_{p1}} = \frac{z_2}{z_1}$ $\qquad z_2 = i \cdot z_1 = 90$

$d_{p1} = 2 r_{p1} = 0{,}120$ m

$m_2 = m_1 = \frac{p}{\pi} = 0{,}0040011$ $\qquad p \cdot z_1 = 2\pi \cdot r_{p1} \rightarrow r_{p1} = \frac{p \cdot z_1}{2\pi} = 0{,}06$ m

$r_{p2} = \frac{p \cdot z_2}{2\pi} = 0{,}18$ $\qquad$ interasse $= r_{p2} + r_{p1} = 0{,}24$

$d_{p2} = 0{,}36$

$r_{i1} = r_{p1} - (1{,}25\,m) = 0{,}05499$ $\qquad d_{i1} = 0{,}109$

$r_{e2} = r_{p2} + m = 0{,}184$

$r_{i2} = r_{p2} - (1{,}25\,m) = 0{,}174$ $\qquad d_{i2} = 0{,}349$

$d_{e2} = 0{,}368$

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$F_{13m} = F_{13} \cdot \sin\alpha = \cancel{812,51}\ 295,73$

$F_{23t} = F_{23} \cdot \cos\alpha = 812,51$

$+\uparrow \quad -F_{13m} - F_{23t} + R_{3V} = 0 \quad \rightarrow \quad R_{3V} = F_{13m} + F_{23t} = 1108,24$

$F_{13t} = F_{13} \cdot \cos\alpha = 812,51$

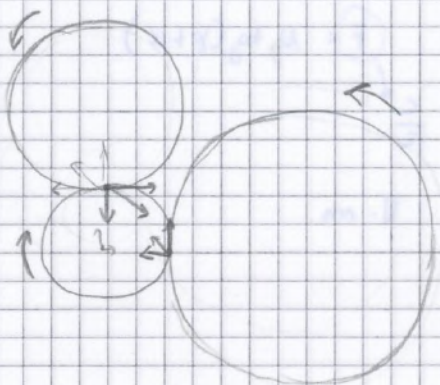
$F_{23m} = F_{23} \cdot \sin\alpha = 295,73$

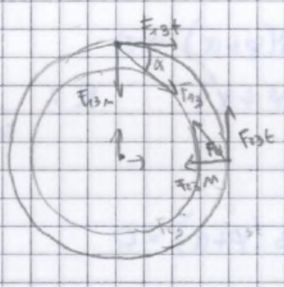
$\overset{+}{\rightarrow} \quad -F_{13t} - F_{23m} + R_{3H} = 0 \quad \rightarrow \quad R_{3H} = 1108,24$

$R = \sqrt{R_{H3}^2 + R_{V3}^2} = 1567,28$

2)

ROT. ANTIORARIA





$F_{13} = F_{31}$

$C_3 = F_{13} \cdot \rho_3 \quad \rightarrow \quad F_{13} = 864,66\ N$

$C_O \quad F_{13} \cdot \rho_3 + F_{23} \cdot \rho_3 = 0 \quad \rightarrow \quad F_{23} = -F_{13} = -864,66$

$F_{13m} = F_{13} \cdot \sin\alpha = 295,73$

$F_{23t} = F_{23} \cdot \cos\alpha = 812,51$

$+\uparrow \quad -F_{13m} + F_{23t} + R_{3V} = 0 \quad \rightarrow \quad R_{3V} = \cancel{1108,24}\ 516,78$

$F_{13t} = 812,51$

$F_{23m} = 295,73$

$\overset{+}{\rightarrow} \quad F_{13t} - F_{23m} + R_{3H} = 0 \quad \rightarrow \quad R_{3H} = -516,78$

$R = \sqrt{R_{H3}^2 + R_{V3}^2} = 730,83$

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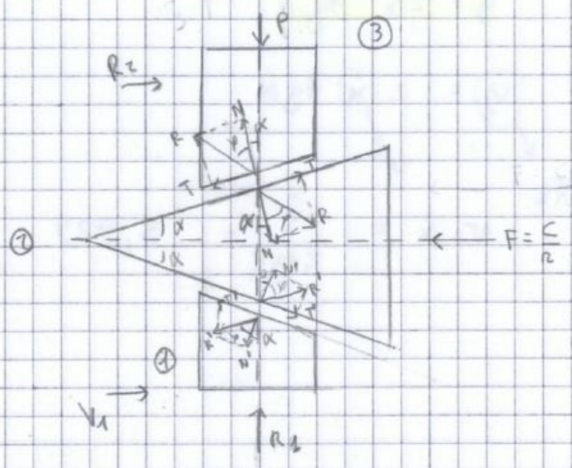


$p = 0,005$

$d = 0,025$

$f = 0,15$ → strisciam.

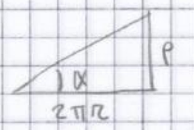
$f_a = 0,25$ → aderenza

$P = 10'000$ N

C ? per avanzamento a v costante

P per avanzamento a velocità $V_3 = 0,5$ mm/s !

Verifica irreversib. moto



$2\pi r \cdot tg\alpha = p$ $\qquad tg\alpha = \dfrac{p}{2\pi \cdot \frac{d}{2}} = 0,06366$

$\alpha = \cancel{1,82}$ $3,64°$

$\varphi = arctg\, f = 8,53°$

3) $+\uparrow \quad -P + R\cos(\varphi+\alpha) = 0$ ①

2) $\xrightarrow{+} \quad -F + R\sin(\alpha+\varphi) + R'\sin(\alpha+\varphi)$ ②

3) $+\uparrow \quad +R_1 - R'\cos(\alpha+\varphi) = 0$ ③

4 eq. 4 inc.

2) $+\uparrow \quad -R\cancel{\cos(\alpha+\varphi)} + R'\cancel{\cos(\alpha+\varphi)} = 0 \quad \rightarrow \quad R' = R$ ④

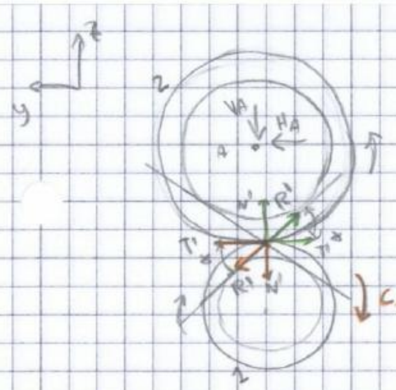
$$R = \frac{P}{\cos(\varphi+\alpha)} = 10'229,90$$

$$F = R\sin(\alpha+\varphi) + R\sin(\alpha+\varphi) = 4'313,19$$

$$F = \frac{C}{r} \rightarrow C = F\cdot r = 53,91 \text{ N}\cdot\text{m}$$

$$P_u = P \cdot v_3 = 5'000 \text{ W} \qquad \varphi_a = arctg\, f_a = 14°$$

$\underset{14°}{\varphi_a} > \underset{3,64°}{\alpha} \quad \Rightarrow$ sist. irreversibile

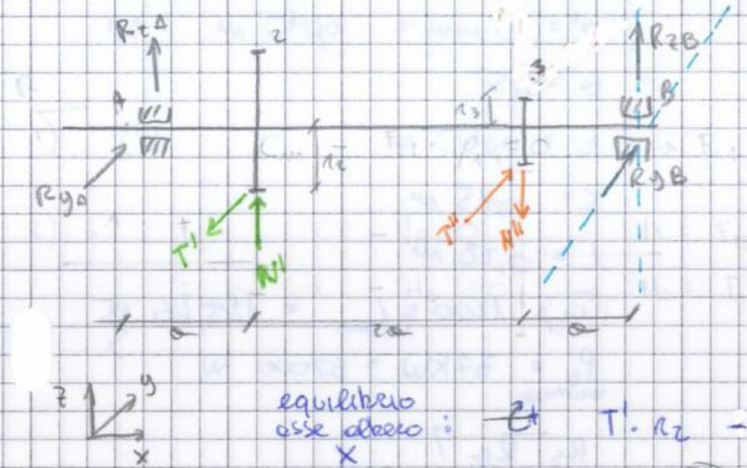


C_{O_1} r_1 $C_m - R' p_1 = 0$

$C_m - R' r_1 \cos\alpha = 0$

$\underbrace{R' \cos\alpha}_{T'} = \dfrac{C_m}{r_1} = 470{,}59 \text{ N}$

$N' = T' \, tg\,\alpha = 171{,}28$



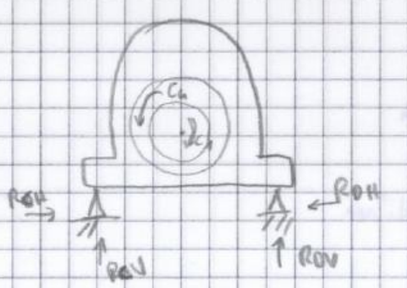
equilibrio asse albero X : $T' \cdot r_2 - T'' \cdot r_3 = 0$

$T'' = \dfrac{T' r_2}{r_3} = 1439{,}42$

$N'' = T'' \cdot tg\,\alpha = 523{,}90 \text{ N}$

Reaz. vincolari :

- asse y in B : $R_{zA} \cdot 4a + N' \cdot 3a - N'' \cdot a = 0$ $\quad R_{zA} = \dfrac{N'' a - N' 3a}{4a} = 2{,}5$ $\quad R_A = 7{,}34$
- asse z in B : $R_{yA} \cdot 4a - T' \cdot 3a + T'' \cdot a = 0$ $\quad R_{yA} = \dfrac{T' 3 - T''}{4} = -6{,}9$
- eq. y : $R_{yA} - T' + T'' + R_{yB} = 0$ $\quad R_{yB} = -961{,}03$
- eq. z : $R_{zA} + N' - N'' + R_{zB} = 0$ $\quad R_{zB} = 350{,}10 \rightarrow R_B = 1023{,}65$



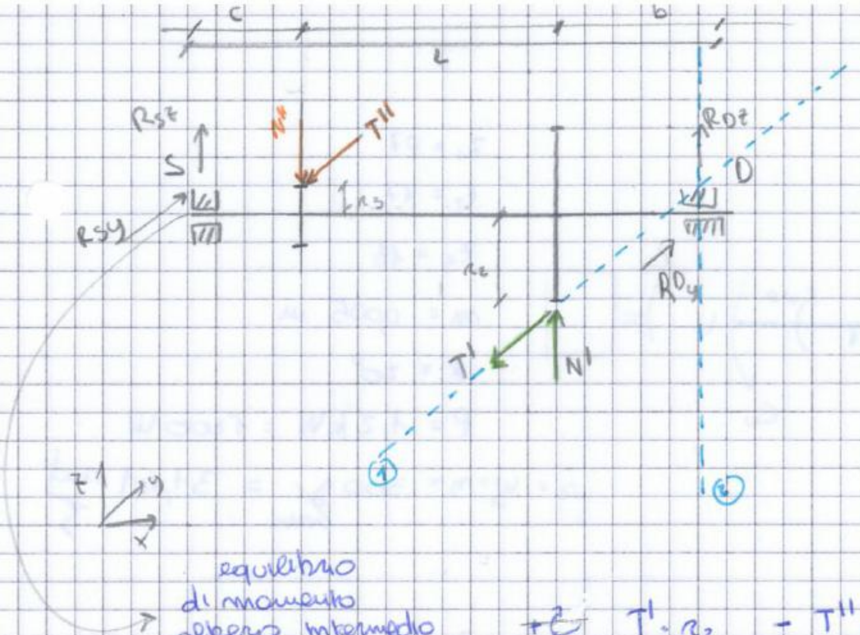
C_B $\quad C_1 - C_4 + R_{CV} \cdot b = 0$

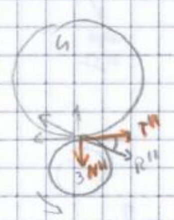
$R_{CV} = \dfrac{C_4 - C_1}{b} = 464{,}22 \text{ N}$?

$\uparrow$ $R_{CV} + R_{DV} = 0 \rightarrow R_{DV} = -R_{CV} = -464{,}22$

$\rightarrow$ $R_{CH} - R_{DH} = 0 \rightarrow R_{CH} = R_{DH}$

C_D $\quad C_1 - C_4$





equilibrio di momento albero intermedio interno al suo asse

$$+\circlearrowleft \quad T' \cdot R_2 - T'' \cdot R_3 = 0$$

$$T'' = \frac{T' R_2}{R_3} = 14\,721{,}9 \text{ N}$$

$$R'' \cdot \sin\alpha = N'' \qquad \frac{T''}{\cos\alpha} \cdot \sin\alpha = N'' \qquad T'' \cdot \operatorname{tg}\alpha = N'' = 6\,864{,}97 \text{ N}$$

Reazioni vincolari

- equazione momento interno ad asse passante per D e // y (1)

$$+\circlearrowleft \quad R_{Sz} \cdot L - N'' \cdot (L - c) + N' \cdot b = 0 \quad \rightarrow \quad R_{Sz} = \frac{N''(L-c) - N'b}{L} = 4515{,}35$$

- " passante per D e // z (2)

$$+\circlearrowleft \quad R_{Sy} \cdot L - T''(L-c) - T' \cdot b = 0 \quad \rightarrow \quad R_{Sy} = \frac{T'b + T''(L-c)}{L} = 11\,873{,}9$$

- equilibrio forze y

$$\xrightarrow{+} \quad R_{Sy} - T'' - T' + R_{Dy} = 0 \quad \rightarrow \quad R_{Dy} = 8\,982{,}16$$

- equilibrio forze z

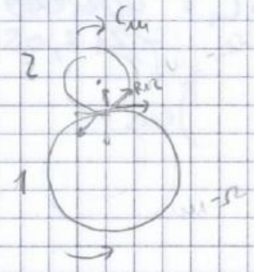
$$+\uparrow \quad R_{Sz} - N'' + N' + R_{Dz} = 0 \quad \rightarrow \quad R_{Dz} = -510{,}7 \text{ N}$$

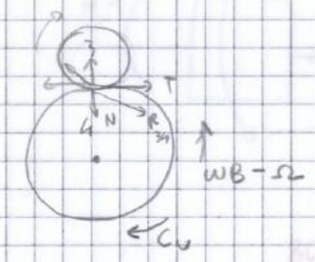
$$R_S = \sqrt{R_{Sy}^2 + R_{Sz}^2} = 12\,703{,}45 \text{ N}$$

$$R_D = \sqrt{R_{Dy}^2 + R_{Dz}^2} = 8\,996{,}66 \text{ N}$$

$r_1 = \frac{m \cdot z_1}{2} = 0{,}2425$ $\qquad r_3 = \frac{m \cdot z_3}{2} = 0{,}045$

$r_2 = \frac{m \cdot z_2}{2} = 0{,}0425$ $\qquad r_4 = \frac{m \cdot z_4}{2} = 0{,}24$





$C \quad C_m - R_{12} \cdot \rho_2 = 0$

$R_{12} = \frac{C_m}{r_2 \cos\alpha} = 956{,}50$

$C \quad C_u - R_{34} \cdot \rho_4 = 0$

$C_u - R_{34} \cdot r_4 \cos\alpha = 0$

$R_{34} = \frac{C_u}{r_4 \cos\alpha} = 2\,425{,}79 \text{ N}$

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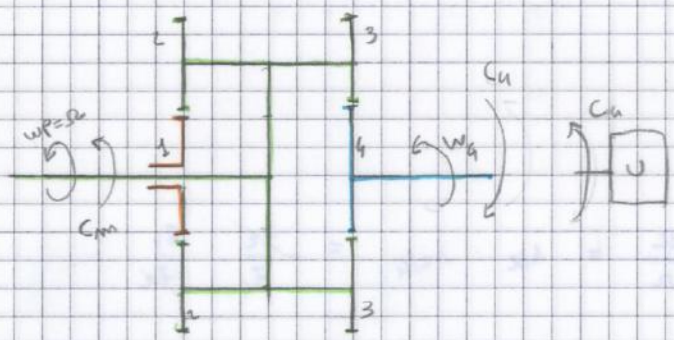

$\Omega = \omega_p = 800 \text{ g/min} = 83{,}77 \frac{rad}{s}$

$W = 8 \text{ kW} = 8\,000 \text{ W}$

$z_1 = 20$

$z_2 = 24$

$z_4 = 24$

$\omega_4 = ?$

C_1 su telaio fisso!

$P_S = C_m \cdot \omega_p \rightarrow C_m = \frac{P_S}{\omega_p} = 95{,}499 \text{ N} \cdot m$

$z_1 + z_2 = z_3 + z_4 \rightarrow z_3 = z_1 + z_2 - z_4 = 20$

$i_{14} = \frac{\omega_1 - \Omega}{\omega_4 - \Omega} = i_{12} \cdot i_{34} = -\frac{z_2}{z_1} \cdot -\frac{z_4}{z_3}$

da qui si capisce segno di ω_4 dato ω_p! (qui concordi)

HP $\omega_1 = 0$ $\qquad \frac{-\Omega}{\omega_4 - \Omega} = \frac{z_2}{z_1} \cdot \frac{z_4}{z_3}$

$-\Omega = \omega_4 \left(\frac{z_2}{z_1} \cdot \frac{z_4}{z_3} \right) - \Omega \left(\frac{z_2}{z_1} \cdot \frac{z_4}{z_3} \right)$

$i_{1,4}$

$\omega_4 = \frac{\Omega \left(\frac{z_2 z_4}{z_1 z_3} - 1 \right)}{\left(\frac{z_2}{z_1} \cdot \frac{z_3}{z_4} \right)} = 25\ldots \frac{rad}{s^2}$

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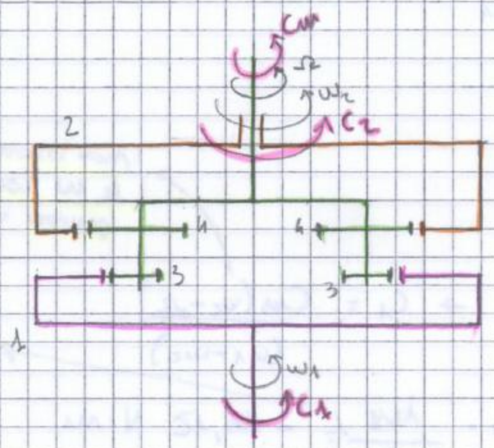

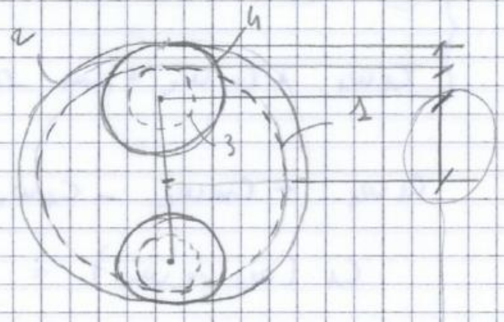

costanza interasse

$z_1 = 40 \quad z_3 = 14 \quad z_4 = 18$

$\Omega = 4000$ g/min $= 418,88 \ \frac{rad}{s}$

$W = 4\ kW = 4000\ W$

1) z_2 ? 2) ω_2 quando $\omega_1 = 0$ 3) ω_1 quando $\omega_2 = 0$

4) C_2 e C_1 !

1) ~~$z_4 + z_2 = z_3 + z_1$~~

$z_1 - z_3 = z_2 - z_4$

~~$z_4 = z_2 - z_1 + z_3$~~

$R_2 - R_4 = R_1 - R_3$

$\frac{m z_2}{2} - \frac{m z_4}{2} = \frac{m z_1}{2} - \frac{m z_3}{2}$

$z_2 = z_1 - z_3 + z_4 = 44$

2) $i_{2/1} = \frac{\omega_2 - \Omega}{\omega_1 - \Omega} = i_{2/4} \cdot i_{3/1} = \frac{z_4}{z_2} \cdot \frac{z_1}{z_3} = 1,1688$

$\omega_1 = 0$

$\frac{\omega_2 - \Omega}{-\Omega} = \frac{z_4}{z_2} \cdot \frac{z_1}{z_3} \rightarrow \omega_2 = \left[\left(\frac{z_4}{z_2} \cdot \frac{z_1}{z_3}\right) \cdot (-\Omega)\right] + \Omega$

$\omega_2 = \Omega - \Omega\left(\frac{z_4}{z_2} \cdot \frac{z_1}{z_3}\right)$

$\omega_2 = \Omega\left(1 - \frac{z_4}{z_2} \cdot \frac{z_1}{z_3}\right) = -70,7 \ \frac{rad}{s}$

3) $i_{2/1} = \frac{\omega_2 - \Omega}{\omega_1 - \Omega} = i_{2/4} \cdot i_{3/1} = \frac{z_4}{z_2} \cdot \frac{z_1}{z_3}$

$\omega_2 = 0$

$\frac{0 - \Omega}{\omega_1 - \Omega} = \frac{z_4}{z_2} \cdot \frac{z_1}{z_3} \rightarrow \omega_1 = \frac{-\Omega + \Omega\left(\frac{z_4}{z_2} \cdot \frac{z_1}{z_3}\right)}{\left(\frac{z_4}{z_2} \cdot \frac{z_1}{z_3}\right)} = 60,50 \ \frac{rad}{s}$

4) $P_s = C_m \cdot \Omega \rightarrow C_m = \frac{P_s}{\Omega} = 9,55\ N \cdot m$

~~$P_u = C_1 \cdot \omega_1 \rightarrow C_1 = \frac{P_u}{\omega_1} = 66,11\ N \cdot m$~~ ~~$C_m + C_1 + C_2 = 0$~~

~~$C_2 = -75,66$~~

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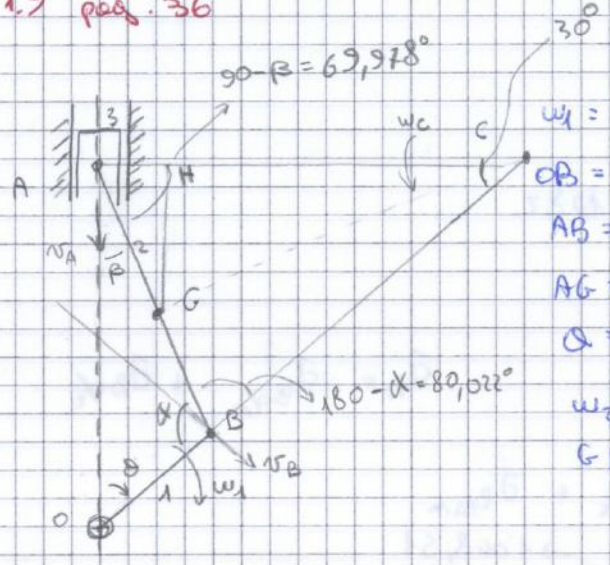


157,07 rad/s

$w_1 = 1500$ giri/min orario e cost.

$OB = 42,5$ mm $= 0,0425$ m

$AB = 0,1075$ m

$AG = 0,075$ m

$\vartheta = 60°$

$w_2 = ?$ $\quad \dot{w}_2 ?$ $\quad a_A ?$

G baricentro $\quad v_G ?$

$$\frac{AB}{\sin\vartheta} = \frac{BO}{\sin\beta} \rightarrow \cancel{\beta = \arcsin\left(\frac{AB}{\sin\vartheta}\right)} \qquad \sin\beta = \frac{BO}{AB}\cdot\sin\vartheta$$

$$\beta = \arcsin\left(\frac{BO}{AB}\cdot\sin\vartheta\right) = 20,022°$$

$$\alpha = 180 - \beta - \vartheta = 99,978°$$

$$\frac{AC}{\sin 80,022} = \frac{BA}{\sin 30°} \rightarrow AC = \frac{BA}{\sin 30°}\cdot\sin 80,022 = 0,211 \text{ m}$$

$$\frac{BC}{\sin 69,978} = \frac{BA}{\sin 30°} \rightarrow BC = \frac{BA}{\sin 30}\cdot\sin 69,978 = 0,202 \text{ m}$$

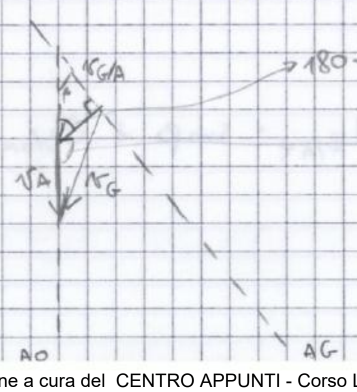
$v_A = w_C \cdot AC$ $\qquad v_A = 6,97$ m/s

$v_B = w_C \cdot BC$ $\qquad w_C \cdot BC = w_1 \cdot OB$

$= w_1 \cdot OB$ $\rightarrow$ $w_2 = w_C = \dfrac{w_1\, OB}{BC} = 33,048 \dfrac{rad}{s} = w_2$ antiorario

$v_G = w_C \cdot GC$ $\qquad v_G = v_A + v_{G/A} = v_A + w_2 \cdot AG$

v_G	v_A	$v_{G/A}$
	6,97	$w_2 \cdot AG$
	// AO	⊥ AG
↙	↓	



$180 - (90 + \beta) = 69,978$

$180 - 69,978 = 110,022$

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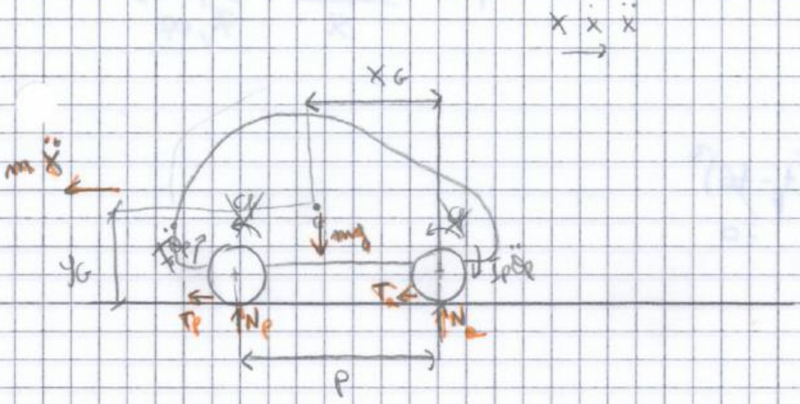
$x \; \dot{x} \; \ddot{x}$



frenando

4 ruote bloccate e strisciano

modello bidim.

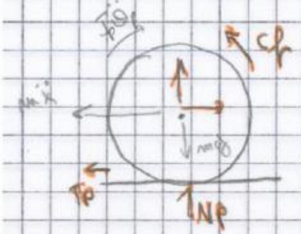
spazio frenata ?

Cf su ruote?

$$+\leftarrow \quad m\ddot{x} + T_p + T_a = 0 \qquad 1$$

$$+\uparrow \quad N_p + N_a - mg = 0 \qquad 2$$

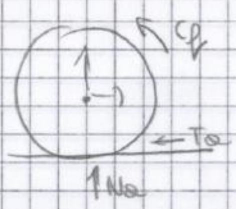
$$C_P \quad -N_a \cdot p + mg(p - x_G) - m\ddot{x}\, y_G = 0 \qquad 3$$



$$C_{O_p} \quad -C_{f_p} + T_p \cdot r = 0 \qquad 4$$

$C_{f_p} = 542{,}45$

$C_{f_a} = 713{,}22$



$$C_{O_a} \quad -C_{f_a} + T_a \cdot r = 0 \qquad 5$$

→ 7 eq
7 inc.

$T_p = f \cdot N_p$ 6 $= 1695{,}18$

$T_a = f N_a$ 7 $= 2228{,}83$

$\cancel{N_p = mg - N_a = mg - \frac{T_a}{f}}$

$\cancel{m\ddot{x} + f\cdot\left(mg - \frac{T_a}{f}\right) + T_a = 0}$

$$\ddot{x} = \frac{-T_p - T_a}{m} = \frac{-fN_p - fN_a}{m}$$

$N_p = mg - N_a = 4237{,}92$

$$\ddot{x} = \frac{-f(mg - N_a) - fN_a}{m} = \frac{-fmg - \cancel{2fN_a}}{m} = -3{,}924$$

$$-N_a\, p + mg(p - x_G) - m\cdot\left(\frac{-fmg - 2fN_a}{m}\right) y_G = 0$$

$$N_a = \frac{\cancel{-}m\ddot{x}\, y_G + mg(p - x_G)}{p} = 5572{,}08\ \text{N}$$

GRANDEZZE FISICHE E UNITA' DI MISURA

GRANDEZZA	ESPRESS.	S.T.	S.I.	NOTE
TEMPO	t	$\boxed{s}$	$\boxed{s}$	
LUNGHEZZA	l	$\boxed{m}$	$\boxed{m}$	angolo φ (rad)
VELOCITA'	$v=\frac{dl}{dt}$	$\frac{m}{s}$	$\frac{m}{s}$	vel. angolare $\omega=\frac{d\varphi}{dt}$ (rad/s)
ACCELERAZIONE	$a=\frac{dv}{dt}$	$\frac{m}{s^2}$	$\frac{m}{s^2}$	acc. angolare $\dot{\omega}=\frac{d\omega}{dt}$ (rad/s²)
MASSA	$M=\frac{F}{a}$	$\frac{kg_f \cdot s^2}{m}$ ×9,81	$\boxed{kg}$ ×9,81	$1\ \frac{kg_f \cdot s^2}{m}=9{,}81\ kg$
FORZA	$F=M \cdot a$	:9,81 $\boxed{kg_f}$	:9,81 $\frac{kg \cdot m}{s^2}$ (N)	$1\ kg_f=9{,}81\ N$
COPPIA	$C=F \cdot b$	$kg_f \cdot m$	$N \cdot m$	
LAVORO ENERGIA	$L=\bar{F} \cdot \bar{l}$ $L=C \cdot \varphi$	$kg_f \cdot m$	$N \cdot m$ (J)	$1\ kg_f \cdot m=9{,}81\ J$ $1\ cal=4{,}186\ J$
POTENZA	$\Pi=\frac{dL}{dt}$ $\Pi=F \cdot v$ $\Pi=C \cdot \omega$	$\frac{kg_f \cdot m}{s}$	$\frac{J}{s}$ (W)	$1\ \frac{kg_f \cdot m}{s}=9{,}81\ W$ $1\ CV=735\ W=75\ \frac{kg_f \cdot m}{s}$
MOMENTO D'INERZIA	$I=M \cdot \rho^2$	$kg_f \cdot s^2 \cdot m$	$kg \cdot m^2$	$1 kg_f \cdot s^2 \cdot m=9{,}81\ kg \cdot m^2$
QUANTITA' DI MOTO	$M \cdot v$	$kg_f \cdot s$	$\frac{kg \cdot m}{s}$	$1 kg_f \cdot s=9{,}81\ \frac{kg \cdot m}{s}$
ENERGIA CINETICA	$\frac{1}{2} M \cdot v^2$ $\frac{1}{2} I \cdot \omega^2$	$kg_f \cdot m$	$N \cdot m$ (J)	$1\ kg_f \cdot m=9{,}81\ J$
PRESSIONE	$p=\frac{F}{A}$	$\frac{kg_f}{m^2}$	$\frac{N}{m^2}$ (Pa)	$1\ \frac{kg_f}{cm^2}=10^4\ \frac{kg_f}{m^2}=98100\ Pa$ $1\ bar=10^5\ Pa=1{,}02\ \frac{kg_f}{cm^2}$
DENSITA'	$\rho=\frac{M}{V}$	$\frac{kg_f \cdot s^2}{m^4}$	$\frac{kg}{m^3}$	$1\ \frac{kg_f \cdot s^2}{m^4}=9{,}81\ \frac{kg}{m^3}$
PESO SPECIFICO	$\gamma=\frac{P}{V}$	$\frac{kg_f}{m^3}$	$\frac{N}{m^3}$	$1\ \frac{kg_f}{m^3}=9{,}81\ \frac{N}{m^3}$
VISCOSITA' DINAMICA	μ	$\frac{kg_f \cdot s}{m^2}$	$\frac{kg}{m \cdot s}$	$1\ \frac{kg_f \cdot s}{m^2}=9{,}81\ \frac{kg}{m \cdot s}$ $1\ Poise=1\ \frac{g}{cm \cdot s}=0{,}1\ \frac{kg}{m \cdot s}$
VISCOSITA' CINEMATICA	$\nu=\frac{\mu}{\rho}$	$\frac{m^2}{s}$	$\frac{m^2}{s}$	$1\ Stokes=1\ \frac{cm^2}{s}=10^{-4}\ \frac{m^2}{s}$

CF90

SCHEMI MECCANICA

N.B.

PRODOTTO SCALARE

$S = \vec{a} \cdot \vec{b}$

$S = |a| \cdot |b| \cos\theta$

dove $\vec{a} = (a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k})$
$\vec{b} = (b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k})$

$a \perp b \Rightarrow S = |a| \cdot |b| \cdot \underbrace{\cos 90°}_{\to 0}$
$S = 0$

nello spazio: $\vec{a} \cdot \vec{b} = (a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}) \cdot (b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k})$
$= a_1 b_1 + a_2 b_2 + a_3 b_3$

PRODOTTO VETTORIALE

$\vec{c} = \vec{a} \wedge \vec{b}$

$|c| = |a| \cdot |b| \cdot \operatorname{sen}\theta \rightsquigarrow \vec{c} \perp$ ad $\vec{a}$ e $\vec{b}$
verso regola mano dx

dove $\vec{a} = //$
$\vec{b} = //$

$a // b \Rightarrow |c| = |a| \cdot |b| \underbrace{\operatorname{sen}\theta}_{\to 0}$
$\vec{c} = 0$

nello spazio: $\vec{a} \wedge \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2 b_3 - a_3 b_2)\vec{i} - (a_1 b_3 - a_3 b_1)\vec{j} + (a_1 b_2 - a_2 b_1)\vec{k}$

$\boxed{\text{CINEMATICA}}$ → moto dei corpi, no cause

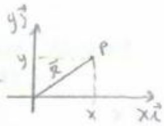
PUNTO

VELOCITÀ MEDIA $\vec{V}_m = \dfrac{\Delta \vec{r}}{\Delta t}$

VELOCITÀ ISTANTANEA $\vec{V}_p = \lim_{\Delta t \to 0} \dfrac{\Delta \vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt}$

sempre tg a traiettoria

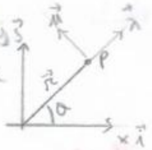
coordinate cartesiane



$\vec{V}_p = \dfrac{d(x\vec{i} + y\vec{j})}{dt} = \dot{x}\vec{i} + \underbrace{x \cdot \dfrac{d\vec{i}}{dt}}_{=0} + \dot{y}\vec{j} + \dots$

$\vec{V}_p = \dot{x}\vec{i} + \dot{y}\vec{j} = V_x \vec{i} + V_y \dots$

coordinate polari



$\vec{V}_p = \dfrac{d(r\vec{\lambda})}{dt} = \dot{r}\vec{\lambda} + r \cdot \dfrac{d\vec{\lambda}}{dt}$

$\vec{V}_p = \dot{r}\vec{\lambda} + r\dot{\theta}\vec{\mu}$

N.B. derivate versori $\vec{\mu}$ e $\vec{\lambda}$

$\dfrac{d\vec{\lambda}}{dt} = \dot{\theta}\vec{\mu} \qquad \dfrac{d\vec{\mu}}{dt} = -\dot{\theta}\vec{\lambda}$

ACCELERAZIONE MEDIA $\vec{a}_M = \dfrac{\Delta \vec{V}}{\Delta t}$

ACCELERAZIONE ISTANTANEA $\vec{a}_p = \lim_{\Delta t \to 0} \dfrac{\Delta \vec{V}}{\Delta t} = \dfrac{d\vec{V}_p}{dt} = \dfrac{d^2\vec{r}}{dt^2}$

coordinate cartesiane $\vec{a}_p = \dfrac{d(V_x \vec{i} + V_y \vec{j})}{dt} = a_x \vec{i} + a_y \vec{j}$

coordinate polari $\vec{a}_p = \dfrac{d(\vec{V}_p)}{dt} = \ddot{r}\vec{\lambda} + \dot{r}\dfrac{d\vec{\lambda}}{dt} + \dot{r}\dot{\theta}\vec{\mu} + r\ddot{\theta}\vec{\mu} + r\dot{\theta}\dfrac{d\vec{\mu}}{dt}$

$= \ddot{r}\vec{\lambda} + \dot{r} \cdot \dot{\theta}\vec{\mu} + \dot{r}\dot{\theta}\vec{\mu} + r\ddot{\theta}\vec{\mu} - r\dot{\theta}^2\vec{\lambda}$

$\vec{a}_p = \underbrace{(\ddot{r} - r\dot{\theta}^2)\vec{\lambda}}_{a_m} + \underbrace{(r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{\mu}}_{a_t}$

MOTO CIRCOLARE

N.B. $\dot{\theta} = \omega$ — $\odot$ uscente, antiorario
$\ddot{\theta} = \dot{\omega}$ — $\otimes$ uscente, orario

• GENERICO $r = \text{cost}$

$\vec{V} = \underbrace{\dot{r}\vec{\lambda}}_{=0} + r\dot{\theta}\vec{\mu} \longrightarrow \vec{V} = r\dot{\theta}\vec{\mu} = r\omega\vec{\mu}$

$\vec{V} = \vec{\omega} \wedge \vec{r}$

velocità solo tangenziale

CENTRIPETA — TANGENZIALE (solo se $\omega \neq \text{cost}$)

$\vec{a} = (\cancel{\ddot{r}} - r\dot{\theta}^2)\vec{\lambda} + (r\ddot{\theta} + \cancel{2\dot{r}\dot{\theta}})\vec{\mu} \to \vec{a} = -r\dot{\theta}^2\vec{\lambda} + r\ddot{\theta}\vec{\mu}$

$= -r\omega^2\vec{\lambda} + r\dot{\omega}\vec{\mu}$

sempre verso di rotazione

$\vec{a} = -\omega^2\vec{r} + \dot{\vec{\omega}} \wedge \vec{r}$

$|\vec{a}_p| = \sqrt{a_m^2 + a_t^2}$

• UNIFORME $r = \text{cost}$ $\omega = \text{cost}$

$\vec{V} = r\dot{\theta}\vec{\mu}$

$\vec{V} = \vec{\omega} \wedge \vec{r}$

CENTRIPETA

$\vec{a} = (\cancel{\ddot{r}} - r\dot{\theta}^2)\vec{\lambda} + (\cancel{r\ddot{\theta}} + \cancel{2\dot{r}\dot{\theta}})\vec{\mu} \to \vec{a} = -r\dot{\theta}^2\vec{\lambda}$

$= -r\omega^2\vec{\lambda}$

DINAMICA

$\sum \vec{F_e} + \vec{F_i} = 0$ $\qquad$ $\sum \vec{M}_{e_G} + \vec{M}_{iG} = 0$

$\sum \vec{F_e} \rightarrow m \cdot \vec{a}_G$ $\qquad$ $\vec{F_i} \rightarrow -m\,\vec{a}_G$

$\sum \vec{M}_{e_G} \rightarrow \dot{\omega}\, I_G$ $\qquad$ $\vec{M}_{iG} \rightarrow -I_G \vec{\dot{\omega}}$ $\qquad$ $I_G \rightarrow M\rho^2$

$$\begin{cases} \Sigma F_{ex} + F_{ix} = 0 \\ \Sigma F_{ey} + F_{iy} = 0 \\ \Sigma M_e + M_i = 0 \end{cases}$$

$F_i = 0$ in STATICA $\qquad$ $M_i = 0$ in STATICA

Procedimento: 1) costruz. del modello → 2) diagr. corpo libero → 3) equazioni di equilibrio → ↑ ↻

Oppure uso conservazione energia: $L_e + L_i = \Delta E_c + \Delta E_g + \Delta E_e = \Delta E_{TOT}$

ΔE_c ← f. inerzia $\qquad$ ΔE_g ← f. peso $\qquad$ ΔE_{TOT} ↓ se $i = 0$ sistema conservativo

$$\begin{cases} E_c = \underbrace{\frac{1}{2} m v_G^2}_{\text{traslazione}} + \underbrace{\frac{1}{2} I_G \cdot \omega^2}_{\text{rotazione}} \\ E_g = mgh \\ E_e = \frac{1}{2} k x^2 \end{cases}$$

Δ → differenza tra stato finale e iniziale del moto

ΔL_e = lavoro forze esterne → No peso, → No inerzia

ΔL_i = lavoro forze interne → attrito

quantità di moto: $\vec{Q} = m \cdot \vec{v}$

$\frac{d\vec{Q}}{dt} = \vec{R}$ ← risultante forze esterne $\qquad$ se $\vec{R} = 0$ $\quad$ $\vec{Q}$ = cost. nel tempo

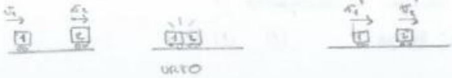
URTO

→ URTO ELASTICO

conservazione Q_x ($\Delta Q_x = 0$): $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$

equazione energia: $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$

$\Delta E_c + \cancel{\Delta E_g} + \cancel{\Delta E_e} = \cancel{\Delta L_i} + \cancel{\Delta L_e}$

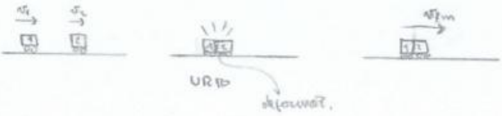


→ URTO ANELASTICO

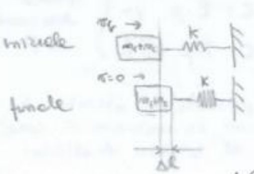
conservazione Q_x ($\Delta Q_x = 0$): $m_1 v_1 + m_2 v_2 = (m_1 + m_2) \cdot v_{fin}$

equazione energia: $\Delta E_c + \cancel{\Delta E_g} + \cancel{\Delta E_e} = \Delta L_i + \cancel{\Delta L_e}$

$\Delta E_c = \Delta L_i = \frac{1}{2}(m_1 + m_2) v_{fin}^2 - \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2\right)$



se ci fosse molla



$\Delta E_c + \cancel{\Delta E_g} + \Delta E_e = \cancel{\Delta L_i} + \cancel{\Delta L_e}$

$\Delta E_c + \Delta E_e = 0$

$\Delta E_c = \underbrace{\frac{1}{2}(m_1+m_2)\cdot(v=0)}_{E_c fin} - \underbrace{\frac{1}{2}(m_1+m_2) v_f^2}_{E_c iniz}$ $\qquad$ $\Delta E_e = \underbrace{\frac{1}{2} k (\Delta l)^2}_{E_e fin} - \underbrace{\frac{1}{2} K \cdot 0^2}_{E_e iniz}$

MOMENTO quantità di moto:

$\vec{K}_O = \vec{r} \wedge \vec{Q} = \vec{r} \wedge (m\vec{v})$

$\vec{K}_O = I_O \vec{\omega}$ (x corpo che ruota) $\qquad$ $I_O \rightarrow r^2 \cdot m$

$\frac{d\vec{K}_O}{dt} = \dot{\vec{r}} \wedge m\vec{v} + \vec{r} \cdot m\dot{\vec{v}} = \cancel{\vec{v} \wedge m\vec{v}} + \vec{r} \wedge m\vec{a} = \vec{r} \wedge \Sigma \vec{F_e} = M_O$

potenza → se agisce una FORZA $\quad P = F_t \cdot V$

→ se agisce una COPPIA $\quad P = M \cdot w$ (M → C)

conservaz. se $\vec{F_e} \parallel \vec{r}$ oppure $\vec{F_e} = 0$ $\quad \frac{d\vec{K}_O}{dt} = 0$ (sen0//=0) $\rightarrow \vec{K}_O$ = cost

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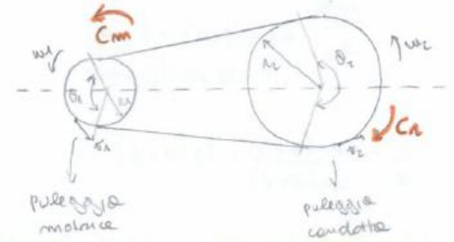
TRASMISSIONE DEL MOTO

CINGHIE



$v_1 = \omega_1 r_1 \quad v_2 = \omega_2 r_2 \rightarrow v_1 = v_2 = v_{flessibile} \Rightarrow \omega_1 r_1 = \omega_2 r_2$

$i = \dfrac{\omega_1}{\omega_2} = \dfrac{r_2}{r_1}$

equilibrio puleggia 1:

$\circlearrowleft O_1 \quad T_1 r_1 - T_2 r_1 - C_m = 0$

$C_m = (T_1 - T_2) r_1$

equilibrio puleggia 2:

$\circlearrowleft O_2 \quad -T_1 r_2 + T_2 r_2 + C_r = 0$

$C_r = (T_1 - T_2) r_2$

$\dfrac{C_r}{C_m} = \dfrac{(\cancel{T_1 - T_2}) r_2}{(\cancel{T_1 - T_2}) r_1} \Rightarrow \dfrac{C_r}{C_m} = \dfrac{r_2}{r_1} = \dfrac{\omega_1}{\omega_2} = i$

potenze:

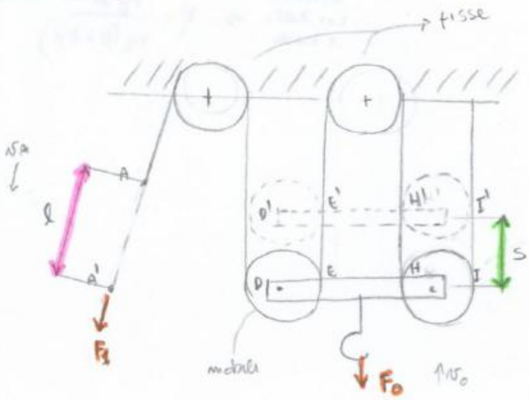
$P_u = C_r \omega_2$

$P_e = C_m \omega_1$

$\eta = \dfrac{P_u}{P_e} = \dfrac{C_r \omega_2}{C_m \omega_1} = 1$

HP aderenza $\quad \dfrac{T_1}{T_2} < e^{f\theta}$ → controllo su puleggia + piccola ($\theta <$)

PARANCHI



$AA' = D'D + EE' + H'H + II'$

$l = (2 \cdot S) \cdot m \Rightarrow l = 2 \cdot S \cdot 2 \rightarrow l = 4S$

spostamento filo x ogni puleggia — n° pulegge mobili

$v_A = 2 \cdot m \cdot v_0 \rightarrow v_A = 4 v_0$

$L_e = F_1 \cdot l$

$L_u = F_0 \cdot S$

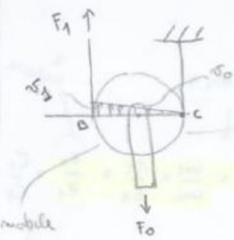
se $\eta = 1 \rightarrow L_e = L_u$

$F_1 l = F_0 S$

$F_1 \cdot 2 \cancel{S} m = F_0 \cancel{S} \Rightarrow F_1 = \dfrac{F_0}{2m}$

se $\eta < 1 \rightarrow F_1 = \dfrac{F_0}{\eta \cdot 2m}$

N.B. esempio sollevamento di 1 carico:



come circonferenza rotolamento puro

$v_0 = \omega \cdot r$

$v_1 = \omega \cdot 2r$

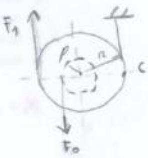
$v_0 = \frac{1}{2} v_1$ — vel. carico, vel. fune

no attrito su PERNO

$\circlearrowleft C \quad F_1 \cdot 2r - F_0 \cdot r = 0$

$F_1 = \dfrac{F_0}{2}$

attrito su PERNO



$\rho = r \sin\varphi$

$\circlearrowleft C \quad -F_0 (r + \rho) + F_1 \cdot 2r = 0$

$F_1 = F_0 \cdot \dfrac{(r+\rho)}{2r}$

$\eta = \dfrac{P_u}{P_e} = \dfrac{F_0 v_0}{F_1 v_1}$

no attrito su PERNO $\quad \eta = \dfrac{F_0 \cancel{v_0}}{(F_0/2)(2 \cancel{v_0})} = 1 \quad$ + pulegge: $\eta = \dfrac{F_0 v_0}{(F_0/2m) \cdot (v_0 2m)} = 1$

attrito su PERNO $\quad \eta = \dfrac{F_0 \cdot \cancel{v_0}}{\left(F_0 \cdot \dfrac{(r+\rho)}{2r}\right) \cdot (2 \cancel{v_0})} = \dfrac{r}{(r+\rho)} < 1 \quad$ + pulegge: $\eta = \dfrac{F_0 \cancel{v_0}}{F_1 \cancel{v_0} \cdot 2m} = \dfrac{F_0}{F_1 \cdot 2m}$