



Corso Luigi Einaudi, 55 - Torino

**Appunti universitari**

**Tesi di laurea**

**Cartoleria e cancelleria**

**Stampa file e fotocopie**

**Print on demand**

**Rilegature**

NUMERO: 1013

DATA: 14/07/2014

# **A P P U N T I**

STUDENTE: Scuttari

MATERIA: Fondamenti di Elettrotecnica Esercizi

Prof. Lombardi

Il presente lavoro nasce dall'impegno dell'autore ed è distribuito in accordo con il Centro Appunti.

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ATTENZIONE: QUESTI APPUNTI SONO FATTI DA STUDENTIE NON SONO STATI VISIONATI DAL DOCENTE.  
IL NOME DEL PROFESSORE, SERVE SOLO PER IDENTIFICARE IL CORSO.

ESERCIZI

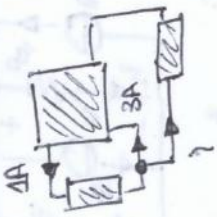
SVOLTI

ELETTROTECNICA

(DA PERFETTI ED ESERCITAZIONI [IN PARTE])

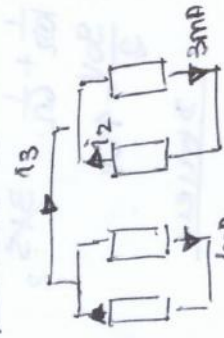
ESERCIZI ELETTRONICA

6



$i_1 = 1A - 3A = -2A$

7



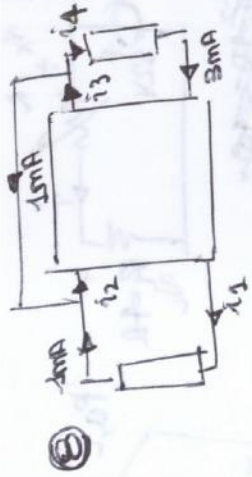
$i_1 = 1mA$   
 $i_2 = 3mA$   
 $i_3 = 0$

8  $i_2 = 0$

9  $i_3 = 3mA$

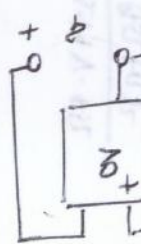
10  $i_1 = 1mA$

11  $i_2 = 3mA$



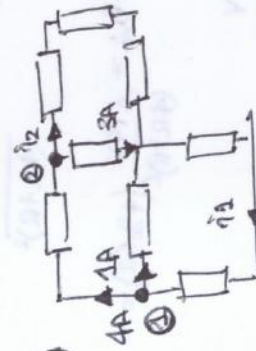
$i_1 = 1mA$   
 $i_2 = 2mA$   
 $i_3 = 4mA$   
 $i_4 = 3mA$

3



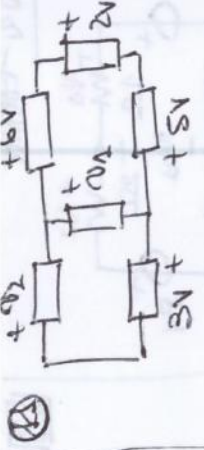
$v = 4V$

4



$i_1 = 5A$

$i_2 = 1A$



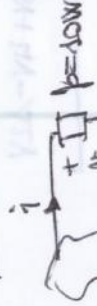
$v_1 = -5V + 2V + 6V = 3V$   
 $v_2 = -v_1 - 3V = -6V$

13



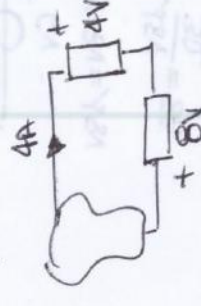
$v = 2V, i = 2A \rightarrow p = 4W$   
 $v = 2V, i = -3A \rightarrow p = -6W$   
 $v = -5V, i = 1A \rightarrow p = -5W$

14

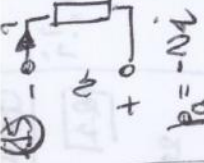


$i = 2A \rightarrow v = 5V$   
 $i = -4A \rightarrow v = -2.5V$

15

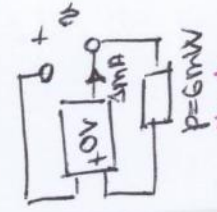


16 ammeter  $-16W$   
17 egera  $32W$



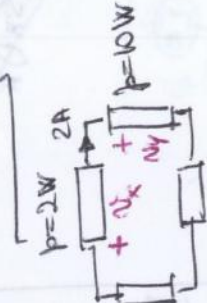
$p_a = -v \cdot i$

19



$p = 60mW$   
 $v_x = 6V$   
 $i_x = 3mA$

20

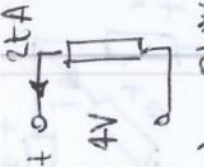


$v_x = 2W = 2V$   
 $i_x = 2A$   
 $v_y = 10W = 5V$   
 $i_y = 2A$

21

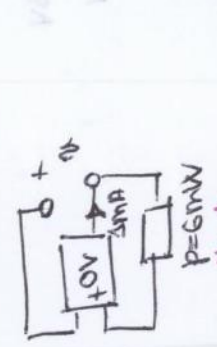


$\sum p_{a_j} = 0$   
 $p_{a5} = +9W$   
 $p_{a7} = 9W$   
 $p_{a3} = -5W$   
 $v_x = \frac{2V}{8} \cdot \frac{2}{8} = \frac{1}{8}V$   
 $i_y = 8A - 1A = 7A$   
 $p_{a4} = -9.1W$   
 $p_{a1} = 1.5W$



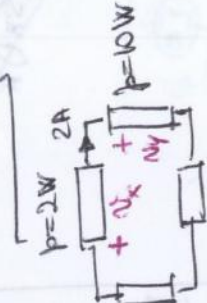
$p_a = -8W$

23



$p = 60mW$   
 $v_x = 6V$   
 $i_x = 3mA$

24



$v_x = 2W = 2V$   
 $i_x = 2A$   
 $v_y = 10W = 5V$   
 $i_y = 2A$

25

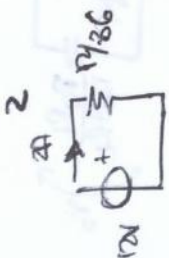
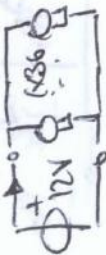


$\sum p_{a_j} = 0$   
 $p_{a5} = +9W$   
 $p_{a7} = 9W$   
 $p_{a3} = -5W$   
 $v_x = \frac{2V}{8} \cdot \frac{2}{8} = \frac{1}{8}V$   
 $i_y = 8A - 1A = 7A$   
 $p_{a4} = -9.1W$   
 $p_{a1} = 1.5W$



2.7 36 compare in parallelo

12V  
2A  
2A  
?R ?Pa

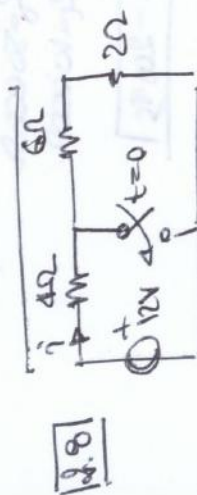


$12V = R_r \cdot 2$   
 $R_r = \frac{36}{2} = 18$

$R_r = 216 \Omega$

~~$P_r = 216 \cdot 2 = 432 W$~~

$\rightarrow P_r = \frac{(12V)^2}{216 \Omega} = \frac{9}{9} W = 1 W$



2.8

$i(t < 0), i(t > 0)$

$t < 0$   
 $i = \frac{12V}{4 \Omega} = 3A$

$t > 0$   
 $i = \frac{12V}{4 \Omega} = 3A$

2.9



$i = \frac{50V}{100 + \frac{2 \cdot 1000}{1002}} \approx 0.49 A$

2.10

$R = 1 \Omega$

$V = 1V, P = 1/3 W$



$R_{eq} = \frac{V}{i} = \frac{1V}{500mA} = 2 \Omega$

→ due R in serie

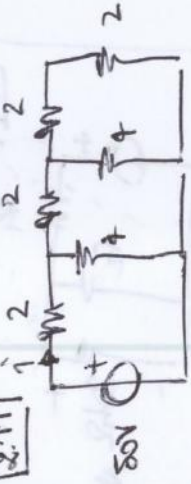


$R_{eq} = \frac{V}{i} = \frac{1V}{3 \cdot 500mA} = \frac{2}{3}$



$\frac{1}{3} = \frac{R}{2 + R}$

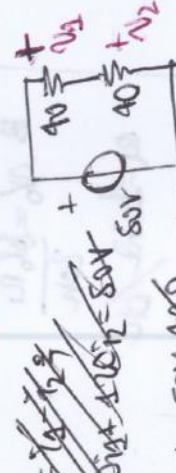
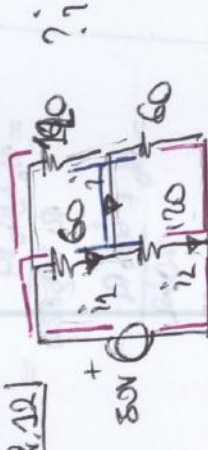
2.11



$i = \frac{50V}{R_{eq}} = \frac{50V}{4 \Omega} = 12.5 A$

$R_{eq} = (2+2) \parallel (4+2) \parallel (4+2) =$   
 $= [4 \parallel 4] \parallel 4 =$   
 $= 2 \parallel 4 = 1.33 \Omega$

2.12

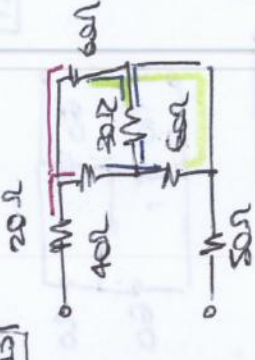


$V_1 = V_2 = \frac{50V \cdot 40 \Omega}{80 \Omega} = 25V$

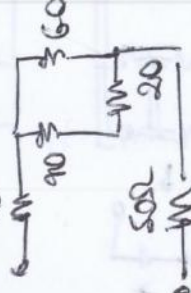
$i_x = \frac{25V}{4 \Omega} = 6.25 A$

$i_y = \frac{25V}{4 \Omega} = 6.25 A$

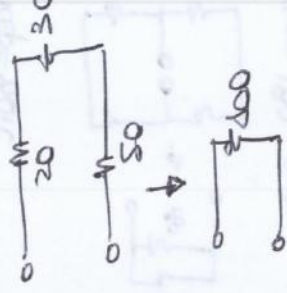
2.13



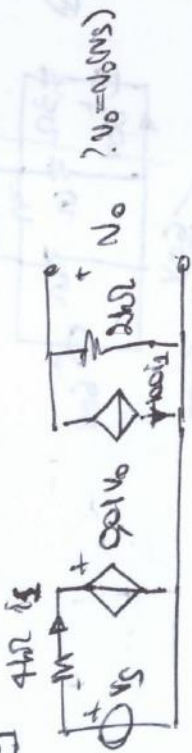
$60 \cdot \frac{36}{303} = 20$



$\rightarrow (40+20) \parallel 60 = 30$



2.18



i)  $v_o = -2k\Omega \cdot 100i_s \Rightarrow i_s = \frac{v_o}{-200k\Omega}$

ii)  $i_s = 90i_v + 4k\Omega i_s$

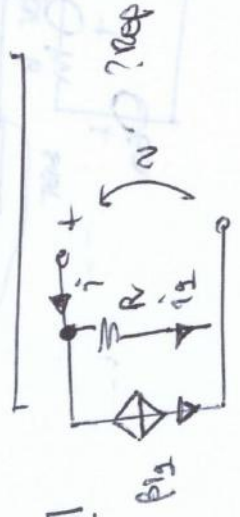
$\Rightarrow i_s = 90i_v + \frac{4k\Omega \cdot v_o}{200k\Omega}$

$i_s = 90i_v - \frac{v_o}{50}$

$50i_s = 90i_v - v_o = -95v_o$

$\Rightarrow v_o = \frac{50}{95} i_s = \frac{100}{19} i_s$

2.19

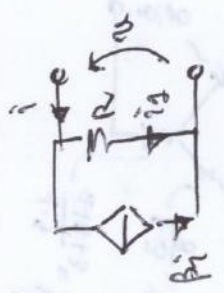


$i_1 = i_1(\beta + 1)$

$R = \frac{v_o}{i_1} = \frac{v_o(\beta + 1)}{i_1}$

$\Rightarrow \frac{v_o}{i_1} = \frac{R(\beta + 1)}{1} = R_{eq}$

2.20



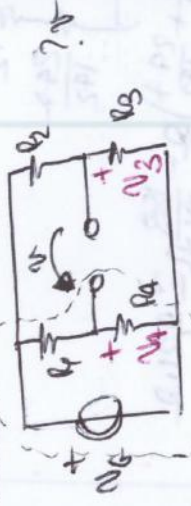
$i_1 = \beta i_1 + i_1$

$i_1 = i_1(\beta + 1)$

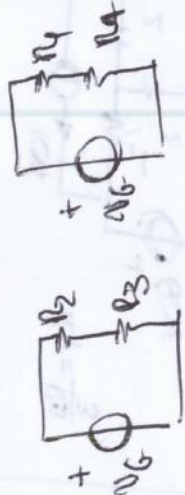
$v_o = R i_1 = R i_1(\beta + 1)$

$\frac{v_o}{i_1} = \frac{R(\beta + 1)}{1} = R_{eq}$

2.21



$v_o = v_1 - v_3$



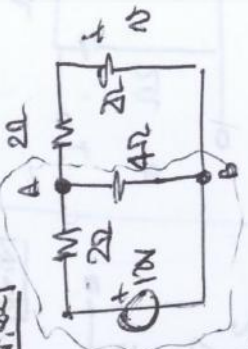
$v_1 = \frac{v_o R_3}{R_2 + R_3}$

$v_3 = \frac{v_o R_4}{R_4 + R_6}$

$\Rightarrow v_o \rightarrow v_o = v_1 - v_3 = \frac{R_3}{R_2 + R_3} v_o - \frac{R_4}{R_4 + R_6} v_o$

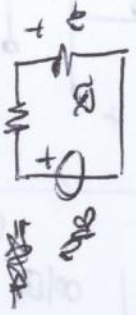
$v_o \cdot v_o = 0 \Rightarrow \frac{R_3}{R_2 + R_3} v_o - \frac{R_4}{R_4 + R_6} v_o = \frac{R_3(R_4 + R_6) - R_4(R_2 + R_3)}{(R_2 + R_3)(R_4 + R_6)} v_o = 0$

2.22

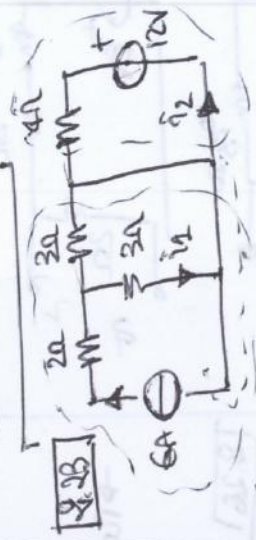


trovare:

$v_o = \frac{6A}{\frac{1}{2} + \frac{1}{4} + \frac{1}{4}} = \frac{6A \cdot 4\Omega}{2 + 1 + 1} = 6V$

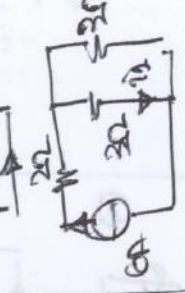


$v_o = \frac{v_o R_2}{2} = 3V$



trovare:

$i_1 = \frac{12V}{4\Omega} = 3A$

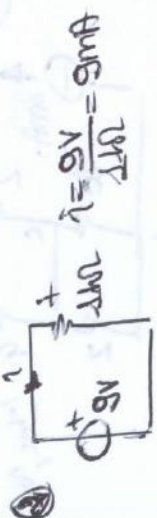


$v_o = 6A \cdot \frac{2\Omega}{2} = 6V$

$\rightarrow i_1 = 5A / 3\Omega = 3A$



ESERCIZI CAPITOLO 2: CIRCUITI LINEARI

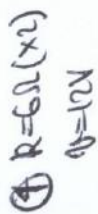


$i = \frac{9V}{1A} = 9mA$

$P_{max} = 9V \cdot 9mA = 81mW$

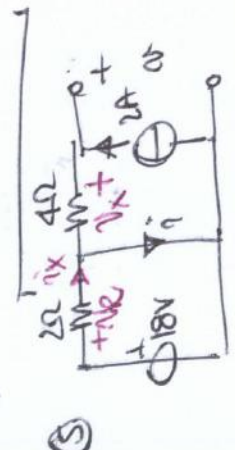


$V = 2mA \cdot 1A = 2V$   
 $P_{max} = 2mA \cdot 2V = 4mW$



$i = \frac{12V}{3\Omega} = 4A$

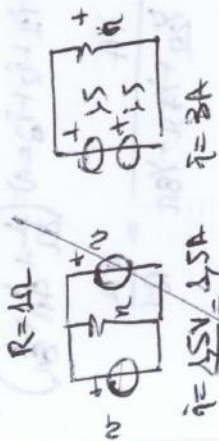
$P = 12V \cdot 4A = 48W$



$V_R = 18V \rightarrow i_x = 9A \rightarrow i = 1A$

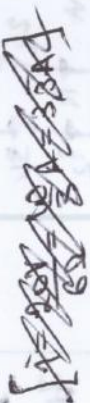
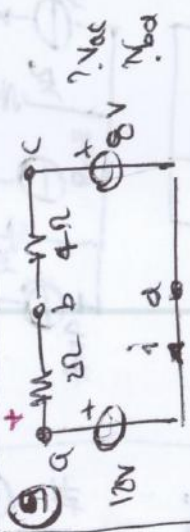
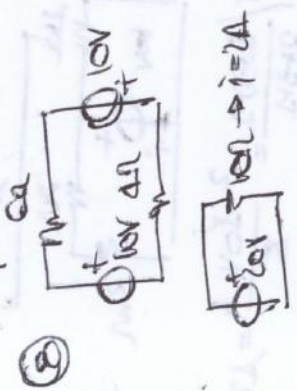
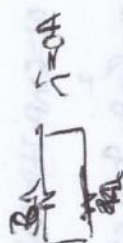
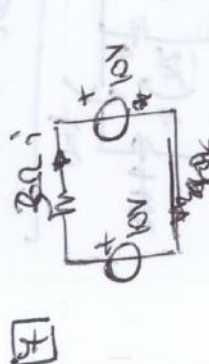
$V = 2A \cdot 4\Omega = 8V$   
 $V = 18V - 18V + 8V = 8V$

$V = 15V (x1)$



$i = \frac{15V}{4\Omega} = 3.75A$

$P_{max} = (4.5)^2 = 20.25W$



$V_{oc} = -8V + 10V = 2V$

$V_{sc} = 10V + 0V = 10V \rightarrow \frac{10V}{3A} = 3.33A$

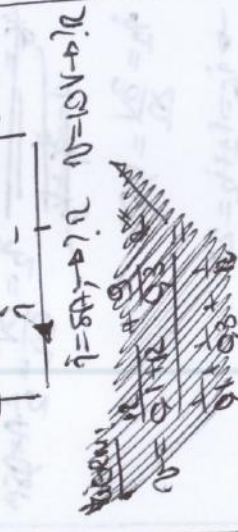
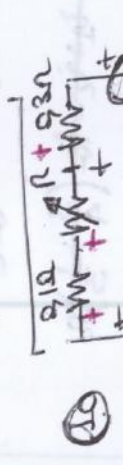
$V_{oc} = 10V + 20V = 30V$

$V_{sc} = 8V + 0V = 8V \rightarrow \frac{8V}{3A} = 2.67A$

$V_{oc} = 8V + 0V = 8V \rightarrow \frac{8V}{3A} = 2.67A$

$i = \frac{4V}{6\Omega} = \frac{2}{3}A$

$V_{oc} = 10V + 0V = 10V \rightarrow \frac{10V}{3A} = 3.33A$



1)  $V = 9V + 0.3A \cdot 5\Omega = 1.5V$   
 $V_{oc} = 9V$

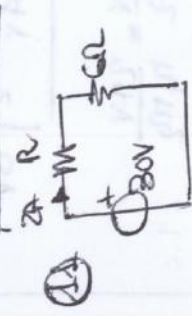
$V_{sc} = -1V + 1V = 0V$   
 $V_{oc} = -10V + 1V = -9V$   
 $R = \frac{V_{oc}}{I_{sc}} = \frac{9V}{3A} = 3\Omega$

$R = \frac{V_{oc}}{I_{sc}} = \frac{9V}{3A} = 3\Omega$

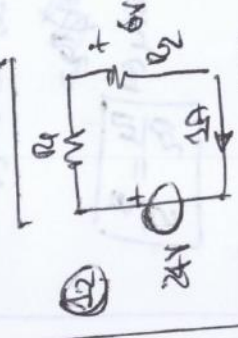
2)  $V_{oc} = 1V \rightarrow i = \frac{1V}{3\Omega} = \frac{1}{3}A$

$V_{oc} = -10V + 1V = -9V \rightarrow \frac{-9V}{3\Omega} = -3A$

$R = \frac{V_{oc}}{I_{sc}} = \frac{9V}{3A} = 3\Omega$



$P_{eq} = \frac{30V}{2A} = 15\Omega$   
 $\rightarrow V = 9V$



$V_{oc} = 18V \rightarrow i = 1.5A$   
 $V_{sc} = 6V \rightarrow i = 6A$



24V

$$N_b = \frac{24}{(1-\alpha)R} = \frac{1}{R} + \frac{1}{\alpha R} + \frac{1}{(1-\alpha)R}$$

$$= \frac{24}{(1-\alpha)R} \cdot \frac{(1-\alpha)R\alpha}{(1-\alpha)R\alpha + 1-\alpha + \alpha} = \frac{24\alpha}{\alpha + 1 - \alpha^2}$$

23

$$R_{eq} = R + \frac{R \cdot 2R}{R + 2R} = R + \frac{2R}{3}$$

24

$$R_{eq} = R + \frac{R \cdot R}{R + R} = R + \frac{R}{2}$$

25

$$R_{eq} = R + \frac{R \cdot R}{R + R} = R + \frac{R}{2}$$

26

$$R_{eq} = R + \frac{R \cdot R}{R + R} = R + \frac{R}{2}$$

27

$$R_{eq} = R + \frac{R \cdot R}{R + R} = R + \frac{R}{2}$$

28

$$R_{eq} = R + \frac{R \cdot R}{R + R} = R + \frac{R}{2}$$

29

$$R_{eq} = R + \frac{R \cdot R}{R + R} = R + \frac{R}{2}$$

30

$$R_{eq} = R + \frac{R \cdot R}{R + R} = R + \frac{R}{2}$$

31

$$R_{eq} = R + \frac{R \cdot R}{R + R} = R + \frac{R}{2}$$

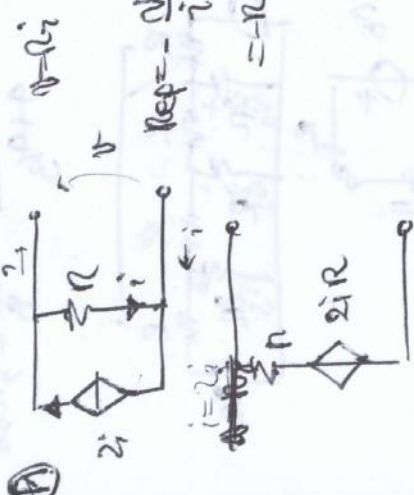
32

$$R_{eq} = R + \frac{R \cdot R}{R + R} = R + \frac{R}{2}$$





$i_x = 0 \rightarrow \text{hep} = 0$

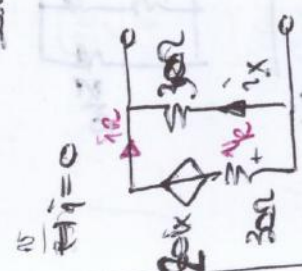


$v = R \cdot i$

$\text{hep} = -\frac{v}{i} = -R$



$3i_x = i + i_2 = \frac{3v}{20\Omega} = i + \frac{v}{10\Omega}$



$v = 30\Omega i_x$

$v_R = v - 20i_x$

$i_R = \frac{v - 20i_x}{30\Omega} = \frac{10\Omega i_x}{30\Omega} = \frac{1}{3}i_x$

$i_x = \frac{1}{3}i_x + \frac{v}{30\Omega} = \frac{2}{3}i_x + \frac{v}{30\Omega}$

$i = \frac{1}{3}i_x \Rightarrow i_x = \frac{3}{2}i$

~~$\text{hep} = \frac{v}{i} = \frac{30\Omega \cdot \frac{3}{2}i}{i} = 45\Omega$~~

$v = 30\Omega \cdot \frac{3}{2}i = \frac{90}{2}i = 45\Omega i$

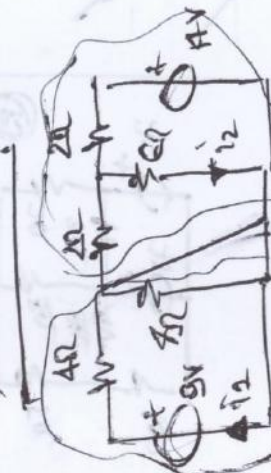


$i_1 = \frac{6A \cdot 4\Omega}{7\Omega} = \frac{24}{7}A \approx 3.43A$

$i_2 = \frac{6A \cdot 3\Omega}{7\Omega} = \frac{18}{7}A \approx 2.57A$



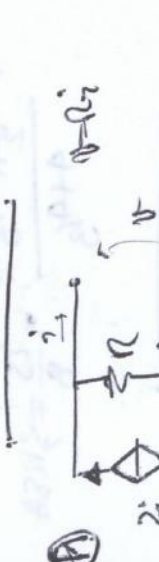
$v = \frac{36V \cdot 3\Omega}{15\Omega} = 7.2V$



$i_2 = \frac{9V}{4\Omega} = 2.25A$



$v = 10\Omega i$



$\text{hep} = \frac{v}{i} = \frac{10\Omega i}{i} = 10\Omega$

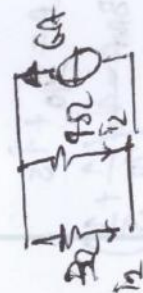


$v = 10\Omega (i - 3i_x) = 30\Omega i_x$

$i_x = \frac{1}{3}i - \frac{v}{30\Omega} = \frac{1}{3}i - \frac{10\Omega i}{30\Omega} = \frac{1}{3}i - \frac{1}{3}i = 0$

$\text{hep} = \frac{v}{i} = \frac{10\Omega i}{i} = 10\Omega$

$\rightarrow v = 0$

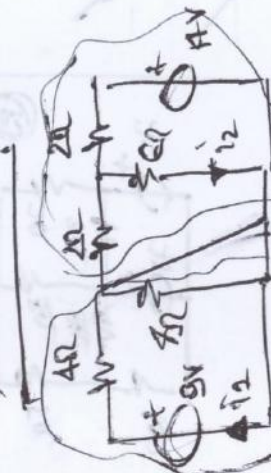


$i_1 = \frac{6A \cdot 4\Omega}{7\Omega} = \frac{24}{7}A \approx 3.43A$

$i_2 = \frac{6A \cdot 3\Omega}{7\Omega} = \frac{18}{7}A \approx 2.57A$

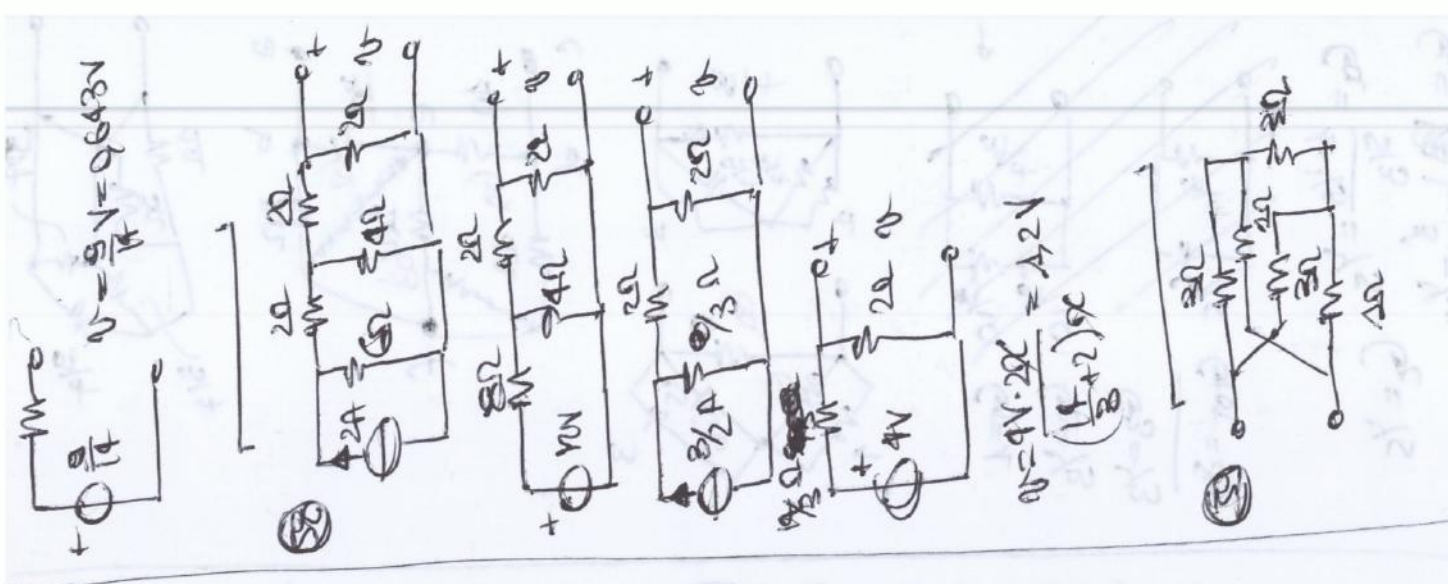
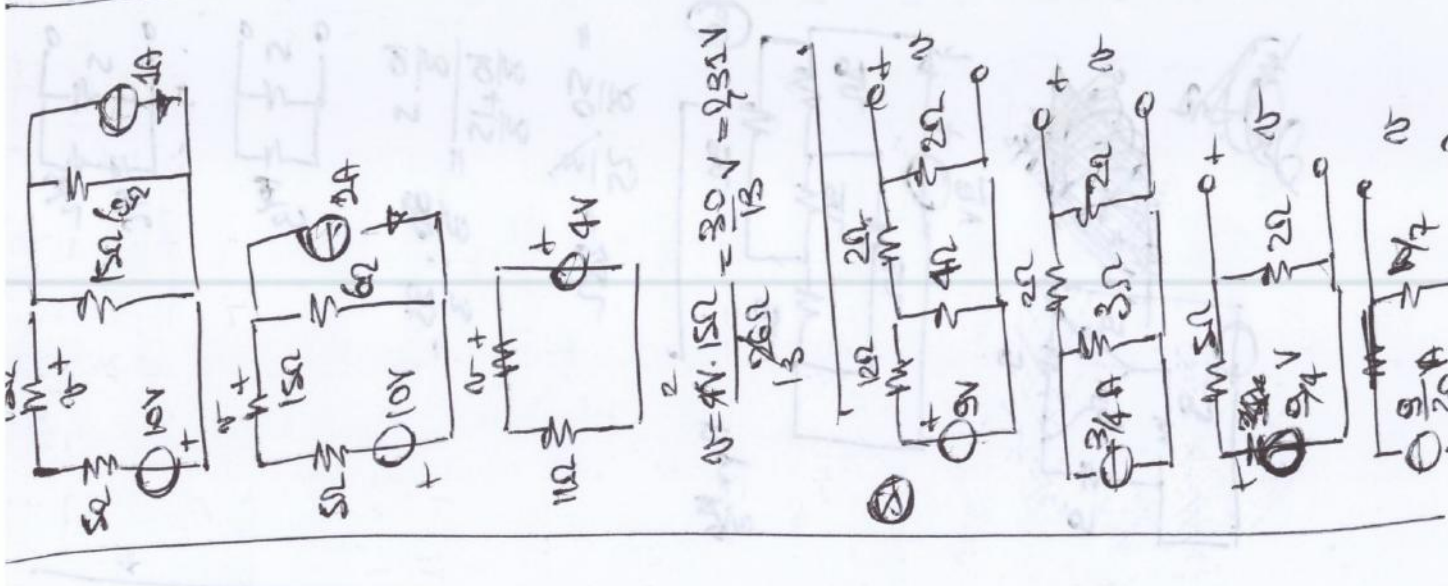
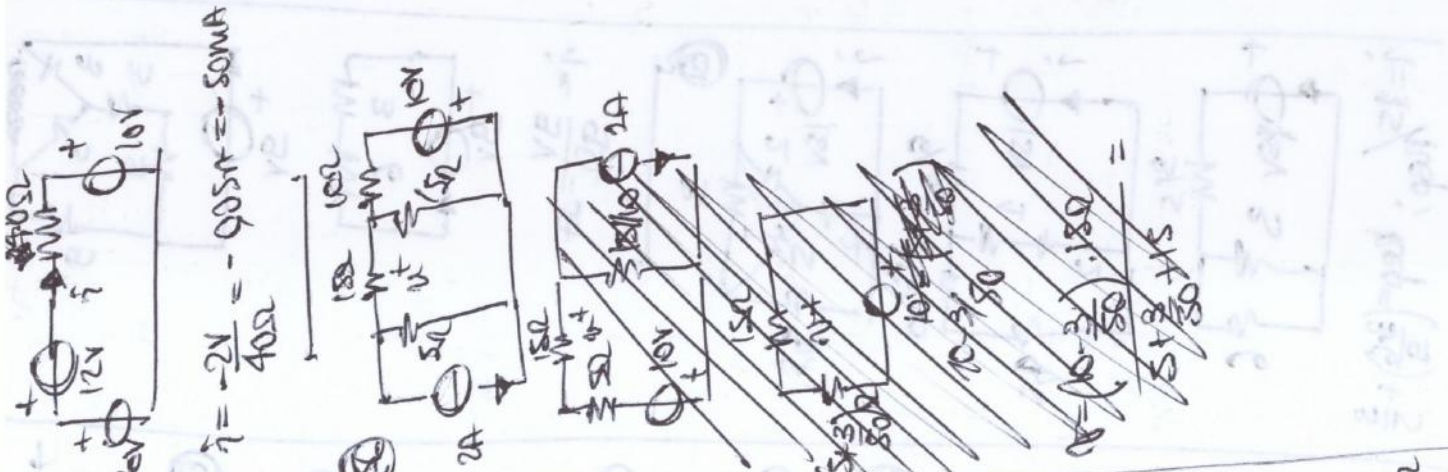
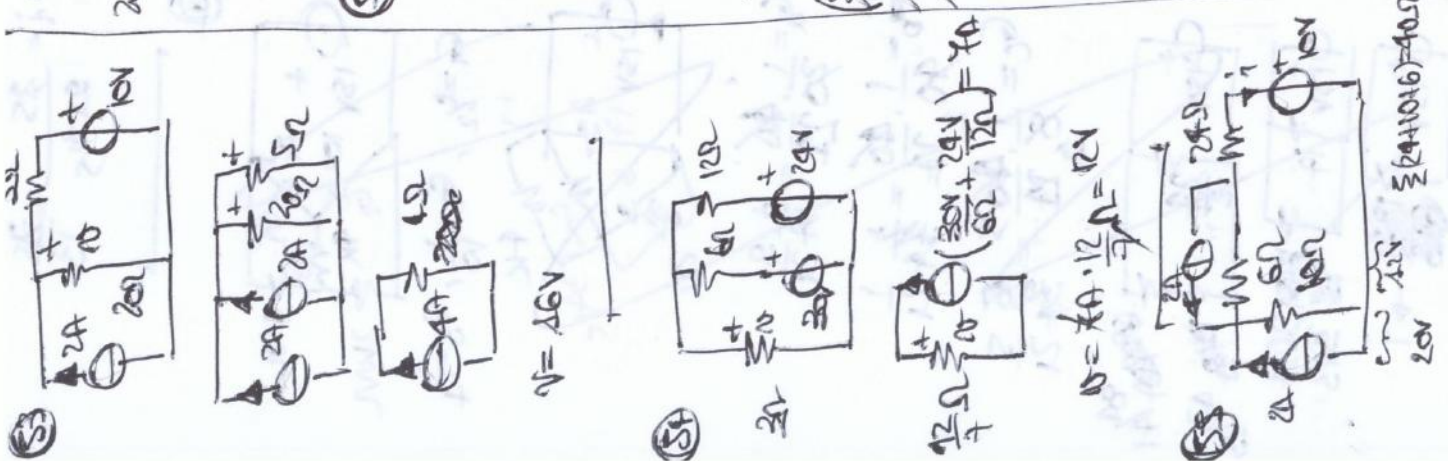


$v = \frac{36V \cdot 3\Omega}{15\Omega} = 7.2V$



$i_2 = \frac{9V}{4\Omega} = 2.25A$







CIRCUITO 3: ANPUBLI NODALE



$$G_1 = \frac{1}{2k} \quad G_2 = \frac{1}{4k} \quad G_3 = \frac{1}{1k}$$

$$G_0 = \frac{G_1 G_2}{G_1 + G_2 + G_3} = \left( \frac{1}{2} \cdot \frac{1}{10} \right) : \frac{17}{20} = \frac{1}{20} \cdot \frac{20}{17} = \frac{1}{17}$$

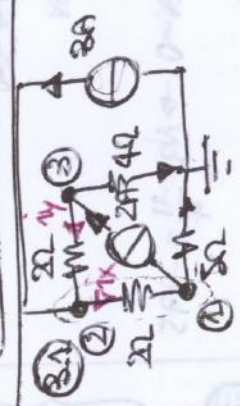
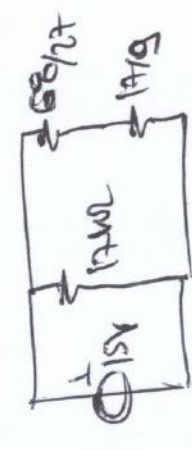
$$G_B = \frac{G_2 G_3}{G_1 + G_2 + G_3} = \left( \frac{1}{10} \cdot \frac{1}{4} \right) : \frac{17}{20} = \frac{1}{40} \cdot \frac{20}{17} = \frac{1}{34}$$

$$G_C = \frac{G_1 G_3}{G_1 + G_2 + G_3} = \left( \frac{1}{4} \cdot \frac{1}{2} \right) : \frac{17}{20} = \frac{1}{8} \cdot \frac{20}{17} = \frac{5}{17}$$

$R_A = 17k\Omega, R_B = 34k\Omega, R_C = 34 \frac{1}{5} k\Omega$



$$i = \frac{15 \cdot 2}{7} = \frac{30}{7} = 4,286A$$



$$\begin{pmatrix} I_1 \\ -I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} v_1 - \frac{1}{4} v_2 \\ \frac{1}{4} v_2 - \frac{1}{2} v_3 \\ 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$I_1 = \frac{1}{2} v_1 - \frac{1}{4} v_2 = \frac{1}{5}$$

$$I_2 = -3A = -\frac{1}{2} v_1 + \frac{1}{4} v_2 - \frac{1}{2} v_3$$

$$I_3 = -\frac{1}{2} v_2 + \frac{1}{4} v_3 = \frac{1}{3} / 4$$

~~$I_1 = \frac{1}{2} v_1 - \frac{1}{4} v_2 = \frac{1}{5}$~~   
 ~~$I_2 = -3A = -\frac{1}{2} v_1 + \frac{1}{4} v_2 - \frac{1}{2} v_3$~~   
 ~~$I_3 = -\frac{1}{2} v_2 + \frac{1}{4} v_3 = \frac{1}{3} / 4$~~

~~$\frac{1}{2} v_1 - \frac{1}{4} v_2 = \frac{1}{5}$~~   
 ~~$-3A = -\frac{1}{2} v_1 + \frac{1}{4} v_2 - \frac{1}{2} v_3$~~   
 ~~$-\frac{1}{2} v_2 + \frac{1}{4} v_3 = \frac{1}{3} / 4$~~

$$\frac{v_1}{2k} + 2A = \frac{v_2 - v_1}{4k}$$

$$\frac{3A + \frac{1}{2} v_2 - \frac{1}{2} v_1}{2k} = \frac{v_2 - v_1}{4k}$$

$$\frac{v_3 - v_2}{2k} + \frac{v_3}{4k} = 2A$$

$$\frac{2v_1}{k} + 20V = 5v_2 - 8v_1$$

$$6V + \frac{1}{2} v_3 - v_2 = \frac{1}{2} v_2 - v_1$$

$$2v_3 - 2v_2 + \frac{1}{2} v_3 = 8V$$

$$7v_1 - 8v_2 = -20V$$

$$v_1 - 2v_2 + \frac{1}{2} v_3 = -6V$$

$$-2v_2 + \frac{1}{2} v_3 = 8V$$

$$v_1 - 2v_2 + \frac{1}{2} v_3 = -6V$$

$$v_2 = \frac{2v_1 - 8V}{2}$$

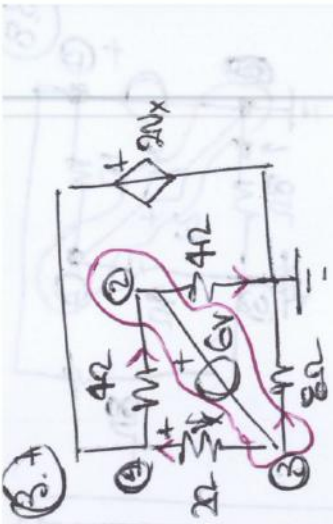
$$-98V + 10v_3 - \frac{5}{2}(2v_1 - 8V) = -20V$$

$$v_1 = -14V + 2v_3$$

$$v_2 = \frac{2v_1 - 8V}{2}$$

$$-186 + 20v_3 - 15v_1 + 40V = -40V$$





$$\begin{aligned}
 \text{① } 0 &= \sum I = I_1 - I_2 + I_3 - I_4 \\
 \text{② } 0 &= \sum I = I_2 - I_3 + I_4 - I_5 \\
 \text{③ } 0 &= \sum I = I_5 - I_6 + I_7 - I_8
 \end{aligned}$$

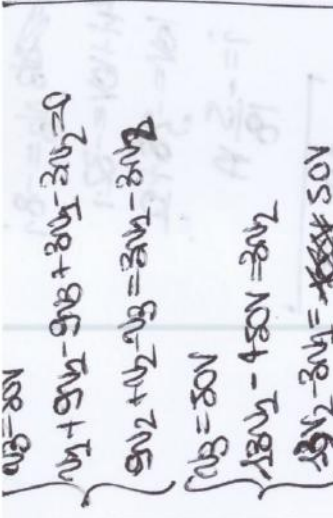
$$\begin{aligned}
 I_1 &= I_2 + I_3 \\
 I_2 &= I_3 + I_4 \\
 I_3 &= I_4 + I_5
 \end{aligned}$$

$$\begin{aligned}
 I_4 &= I_5 + I_6 \\
 I_5 &= I_6 + I_7 \\
 I_6 &= I_7 + I_8
 \end{aligned}$$

$$\begin{aligned}
 I_7 &= I_8 + I_9 \\
 I_8 &= I_9 + I_{10} \\
 I_9 &= I_{10} + I_{11}
 \end{aligned}$$

$$\begin{aligned}
 I_{10} &= I_{11} + I_{12} \\
 I_{11} &= I_{12} + I_{13} \\
 I_{12} &= I_{13} + I_{14}
 \end{aligned}$$

$$\begin{aligned}
 I_{13} &= I_{14} + I_{15} \\
 I_{14} &= I_{15} + I_{16} \\
 I_{15} &= I_{16} + I_{17}
 \end{aligned}$$



$$\begin{aligned}
 \text{① } 0 &= \sum I = I_1 - I_2 + I_3 - I_4 \\
 \text{② } 0 &= \sum I = I_2 - I_3 + I_4 - I_5 \\
 \text{③ } 0 &= \sum I = I_5 - I_6 + I_7 - I_8
 \end{aligned}$$

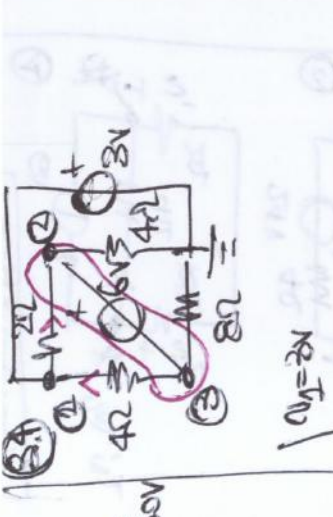
$$\begin{aligned}
 I_1 &= I_2 + I_3 \\
 I_2 &= I_3 + I_4 \\
 I_3 &= I_4 + I_5
 \end{aligned}$$

$$\begin{aligned}
 I_4 &= I_5 + I_6 \\
 I_5 &= I_6 + I_7 \\
 I_6 &= I_7 + I_8
 \end{aligned}$$

$$\begin{aligned}
 I_7 &= I_8 + I_9 \\
 I_8 &= I_9 + I_{10} \\
 I_9 &= I_{10} + I_{11}
 \end{aligned}$$

$$\begin{aligned}
 I_{10} &= I_{11} + I_{12} \\
 I_{11} &= I_{12} + I_{13} \\
 I_{12} &= I_{13} + I_{14}
 \end{aligned}$$

$$\begin{aligned}
 I_{13} &= I_{14} + I_{15} \\
 I_{14} &= I_{15} + I_{16} \\
 I_{15} &= I_{16} + I_{17}
 \end{aligned}$$



$$\begin{aligned}
 \text{① } 0 &= \sum I = I_1 - I_2 + I_3 - I_4 \\
 \text{② } 0 &= \sum I = I_2 - I_3 + I_4 - I_5 \\
 \text{③ } 0 &= \sum I = I_5 - I_6 + I_7 - I_8
 \end{aligned}$$

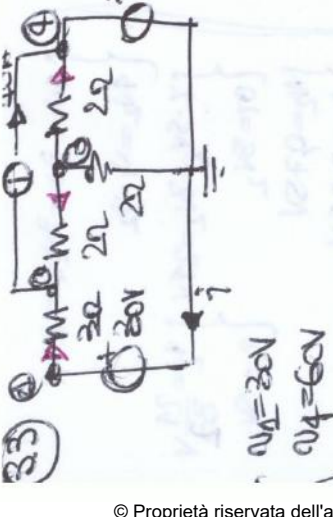
$$\begin{aligned}
 I_1 &= I_2 + I_3 \\
 I_2 &= I_3 + I_4 \\
 I_3 &= I_4 + I_5
 \end{aligned}$$

$$\begin{aligned}
 I_4 &= I_5 + I_6 \\
 I_5 &= I_6 + I_7 \\
 I_6 &= I_7 + I_8
 \end{aligned}$$

$$\begin{aligned}
 I_7 &= I_8 + I_9 \\
 I_8 &= I_9 + I_{10} \\
 I_9 &= I_{10} + I_{11}
 \end{aligned}$$

$$\begin{aligned}
 I_{10} &= I_{11} + I_{12} \\
 I_{11} &= I_{12} + I_{13} \\
 I_{12} &= I_{13} + I_{14}
 \end{aligned}$$

$$\begin{aligned}
 I_{13} &= I_{14} + I_{15} \\
 I_{14} &= I_{15} + I_{16} \\
 I_{15} &= I_{16} + I_{17}
 \end{aligned}$$



$$\begin{aligned}
 \text{① } 0 &= \sum I = I_1 - I_2 + I_3 - I_4 \\
 \text{② } 0 &= \sum I = I_2 - I_3 + I_4 - I_5 \\
 \text{③ } 0 &= \sum I = I_5 - I_6 + I_7 - I_8
 \end{aligned}$$

$$\begin{aligned}
 I_1 &= I_2 + I_3 \\
 I_2 &= I_3 + I_4 \\
 I_3 &= I_4 + I_5
 \end{aligned}$$

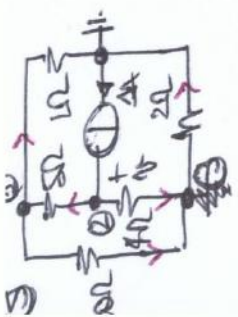
$$\begin{aligned}
 I_4 &= I_5 + I_6 \\
 I_5 &= I_6 + I_7 \\
 I_6 &= I_7 + I_8
 \end{aligned}$$

$$\begin{aligned}
 I_7 &= I_8 + I_9 \\
 I_8 &= I_9 + I_{10} \\
 I_9 &= I_{10} + I_{11}
 \end{aligned}$$

$$\begin{aligned}
 I_{10} &= I_{11} + I_{12} \\
 I_{11} &= I_{12} + I_{13} \\
 I_{12} &= I_{13} + I_{14}
 \end{aligned}$$

$$\begin{aligned}
 I_{13} &= I_{14} + I_{15} \\
 I_{14} &= I_{15} + I_{16} \\
 I_{15} &= I_{16} + I_{17}
 \end{aligned}$$





$$① \frac{V_1 + V_1 - V_3}{10\Omega} = \frac{V_2 - V_1}{5\Omega}$$

$$② \frac{V_2 - V_1 + V_2 - V_3}{5\Omega} = 1A$$

$$③ \frac{V_3}{2\Omega} = \frac{V_2 - V_3}{4\Omega} + \frac{V_1 - V_3}{10\Omega}$$

$$10V_1 + V_1 - V_3 = 2V_2 - V_1$$

$$4V_2 - 4V_1 + 5V_2 - 5V_3 = 20V$$

$$10V_3 = 5V_2 - 5V_3 + 2V_1 - 2V_3$$

$$12V_1 - 2V_2 - V_3 = 0$$

$$-4V_1 + 9V_2 - 8V_3 = 20V$$

$$2V_1 + 5V_2 - 14V_3 = 0$$

$$V_3 = 12V_1 - 2V_2$$

$$-4V_1 + 9V_2 - 60V_1 + 10V_2 = 20V$$

$$2V_1 + 5V_2 - 204V_1 + 34V_2 = 0$$

$$V_3 = 12V_1 - 2V_2$$

$$-64V_1 + 19V_2 = 20V$$

$$-202V_1 + 39V_2 = 0$$

$$V_2 = \frac{64 \cdot 39}{202} + 19 = 20V$$

$$V_1 = \frac{39}{202} V$$

$$V_3 = 9 \cdot \frac{39}{202} = 17.47V$$

$$V_2 = \frac{2020}{641}$$

$$V_1 = \frac{390}{641}$$

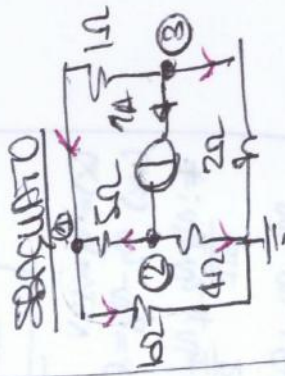
$$V = -V_3 + V_A =$$

$$= -V_3 + V_1 + V_2 =$$

$$= -17.47V + 0.608V + 3.15V =$$

$$V_3 = V_3 - V_1 = 9.87V$$

$$V_A = V_1$$



$$① \frac{V_1}{10\Omega} = \frac{V_2 - V_1 + V_3 - V_1}{5\Omega} + \frac{V_3 - V_1}{15\Omega}$$

$$② V_1 = 2V_2 - 2V_1 + 10V_3 - 10V_1$$

$$18V_1 = 2V_2 + 10V_3$$

$$③ \frac{V_2}{4\Omega} + \frac{V_2 - V_1}{5\Omega} = 1A$$

$$5V_2 + 4V_2 - 4V_1 = 20V$$

$$1 - 4V_1 + 9V_2 = 20V$$

$$④ 1A + \frac{V_3 - V_1}{10\Omega} + \frac{V_3}{5\Omega} = 0$$

$$2V_1 + 2V_3 - 2V_1 + 14V_3 = 0$$

$$-2V_1 + 8V_3 = -2V_1$$

$$2V_1 - 34V_3 = 2V_1$$

$$V_1 = 4V_1 + \frac{3}{2}V_3$$

$$-4(4V_1 + \frac{3}{2}V_3) + 9V_2 = 20V$$

$$13(4V_1 + \frac{3}{2}V_3) = 20V + 10V_3$$

$$V_1 = 4V_1 + \frac{3}{2}V_3$$

$$6V_3 + 9V_2 = 20V$$

$$13V_1 + \frac{39}{2}V_3 = 20V + 10V_3$$

$$V_3 = 4V_1 + \frac{3}{2}V_2$$

$$13V_1 + \frac{39}{2}(4V_1 + \frac{3}{2}V_2) = 20V + 10(4V_1 + \frac{3}{2}V_2)$$

$$13V_1 = 20V + 10V_2 - 14V_1 - 14V_3 = 0$$

$$405V_1 - 14225V_2$$

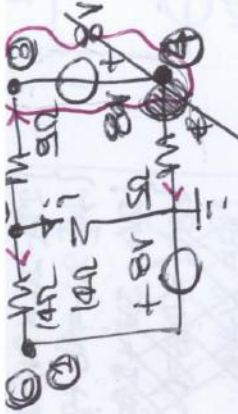
$$V_2 = \frac{405V_1}{14225}$$

$$13(2V_1 - \frac{19}{2}(4V_1 + \frac{3}{2}V_2)) = 20V$$

$$26V_1 - 19V_2 = 20V$$

$$28 = 19 \cdot \frac{405V_1}{14225} + 26V_1$$

$$V_2 = \frac{100}{202} = 2.04V$$



$$6A = 1A = 8V$$

$$9A = 1A = 8V = 8V \rightarrow V_A = 8V + V_B$$

$$\frac{V_2 - V_1 + V_2 - V_3 + V_2}{14\Omega} = \frac{V_2 - V_3}{5\Omega} + \frac{V_2}{14\Omega}$$

$$\frac{3V_2 - V_1 - V_3}{14\Omega} = \frac{V_2 - V_3}{5\Omega}$$

$$9V_2 - 9V_1 + 14V_2 - 14V_3 + 14V_3 = 0$$

$$8V_2 - 9V_1 = 9V_1$$

$$32V_2 - 9V_1 + 14V_3 = 0$$

$$V_2 = \frac{9}{5}V_1 + V_3$$

$$32V_2 - 8V_1 + 14V_3 = 0$$

$$V_2 = \frac{8}{5}V_1 + V_3$$

$$32V_2 - 8V_1 + 14V_3 = 0$$

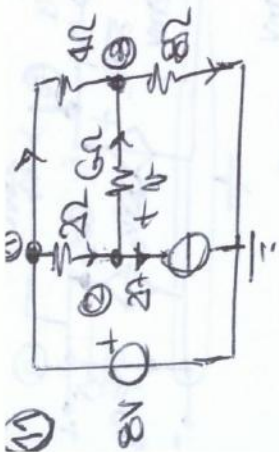
$$V_2 = \frac{9}{5} \cdot 8V_1 + 14V_3$$

$$B = \frac{1}{14}(81V_1 - 32V_2)$$

$$V_2 = \frac{9 \cdot 81}{5}V_1 + \frac{1}{5}(81V_1 - 32V_2)$$

$$\frac{32V_2}{5} = \frac{81V_1}{5} + \frac{81V_1}{5}$$





(\*) :  $v_1 = 8V$   
 (\*\*):  $v_2 - v_3 = 0$   
 (3) :  $\frac{v_3}{8\Omega} = \frac{v_1}{2\Omega} + \frac{v_2 - v_3}{4\Omega}$   
 (2) :  $\frac{v_1 - v_2}{2\Omega} = 2A + \frac{v_1}{6\Omega}$

$$\begin{cases} 3v_3 = 4v_1 + 6v_2 - 6v_3 \\ 3v_2 - 3v_3 = 12V + v_1 \\ 9v_3 = 4v_1 + 48V \end{cases}$$

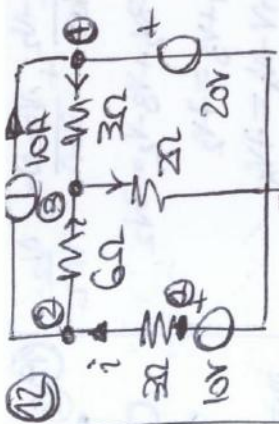
$$\begin{cases} v_2 = 0 + v_3 \\ v_3 = 4v_1 + 48V \\ 12V - 3v_1 - 3v_3 = 0 \end{cases}$$

$$\rightarrow 12V - 3(4v_1 + 48V) = 0$$

$$12V - 4v_1 - 144V = 0$$

$$16v_1 = -132V$$

$$v_1 = -8.25V$$



(\*)  $v_1 = 40V$   $i = \frac{v_1 - v_2}{3\Omega}$   
 (\*\*):  $v_4 = 20V$   
 (2) :  $\frac{v_1 - v_2}{3\Omega} = \frac{v_2 - v_3}{6\Omega} + 10A$   
 (3) :  $\frac{v_2 - v_3}{6\Omega} + \frac{v_4 - v_3}{3\Omega} = \frac{v_3}{2\Omega}$

$$\begin{cases} 2v_1 - 2v_2 = 3v_2 - 3v_3 + 60V \\ v_2 - v_3 + 2v_4 - 2v_3 = 3v_3 \end{cases}$$

$$\begin{cases} 2v_1 - 3v_2 + v_3 = 60V \\ v_2 - 6v_3 + 40V = 0 \end{cases}$$

$$\begin{cases} v_3 = 40V + 3v_2 \\ v_2 - 2(40V + 3v_2) - 18(40V + 3v_2) + 40V = 0 \end{cases}$$

$$v_2 = -\frac{200V}{19}$$

$$i = \frac{40V - \frac{200V}{19}}{3\Omega} = \frac{560V}{57\Omega}$$

(2) :  $\frac{v_1 - v_2}{3\Omega} = 10A + \frac{v_2 - v_3}{6\Omega}$   
 (3) :  $\frac{v_2 - v_3}{6\Omega} + \frac{v_4 - v_3}{3\Omega} = \frac{v_3}{2\Omega}$

$$\begin{cases} 2v_1 - 2v_2 = 60V + v_2 - v_3 \\ v_2 - v_3 + 2v_4 - 2v_3 = 3v_3 \end{cases}$$

$$\begin{cases} 2v_1 - 3v_2 + v_3 = 60V \\ v_2 - 6v_3 + 40V = 0 \end{cases}$$

$$\begin{cases} -3v_2 + v_3 = 40V \\ v_2 - 6v_3 = -40V \end{cases}$$

$$-2v_2 - 3v_3 = 0$$

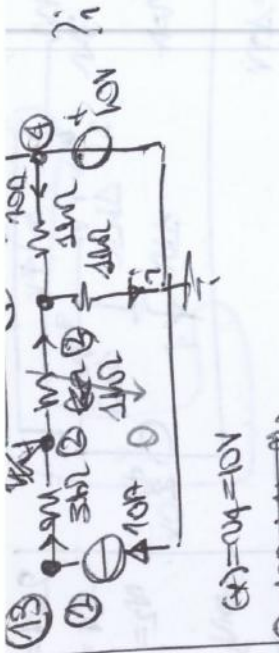
$$v_3 = -\frac{2}{5}v_2$$

$$\rightarrow v_2 + \frac{2}{5}v_2 \cdot 6 = -40V$$

$$17v_2 = -40V$$

$$v_2 = -\frac{200V}{17}$$

$$i = \frac{v_1 - v_2}{3\Omega} = \frac{80V - \frac{200V}{17}}{3\Omega} = \frac{720V}{51}$$



(\*) :  $v_4 = 10V$   
 (2) :  $10A + \frac{v_1 - v_2}{3\Omega} = \frac{v_2 - v_3}{4\Omega}$   
 (3) :  $v_2 = v_3$

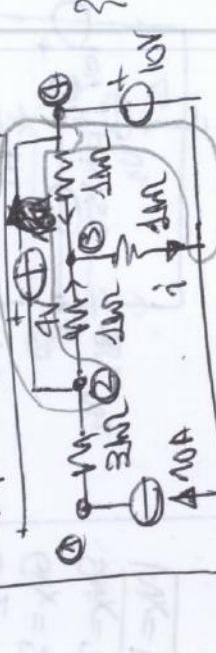
$$\frac{v_2 - v_3}{4\Omega} + \frac{v_4 - v_3}{4\Omega} = \frac{v_3}{10\Omega}$$

$$10A + \frac{v_1 - v_2}{3\Omega} = \frac{v_2}{10\Omega}$$

$$v_1 - v_2 = 2v_3$$

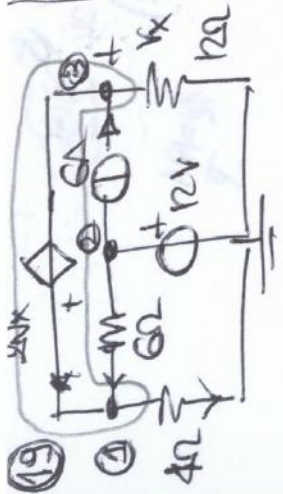
$$v_3 = 5V$$

$$i = 14V - 2v_3 = 14V - 10V = 4V$$

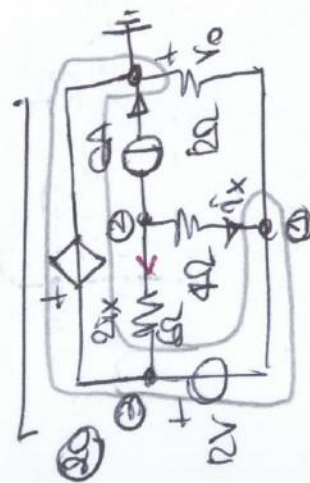


(\*) :  $v_4 = 10V$   
 (\*\*):  $v_2 - v_1 = 1V \rightarrow v_2 = 14V$   
 ~~$v_1 = 10V$~~   
 ~~$v_2 = 10V$~~   
 ~~$v_3 = 10V$~~   
 ~~$v_4 = 10V$~~   
 ~~$v_5 = 10V$~~   
 ~~$v_6 = 10V$~~   
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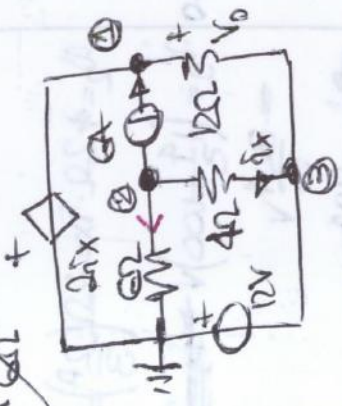


$v_2 = 12V$   
 $v_3 = v_6 = 2v_x$   
 $v_1 = 3v_x$   
 $v_4 + v_x = \frac{12V - v_4 + 6A}{4\Omega}$   
 $3v_1 + v_x = 24V - 2v_1 + 6V$   
 $5v_1 + v_x = 30V$   
 $15v_x + v_x = 96V$   
 $v_x = \frac{96V}{16} = 6V$

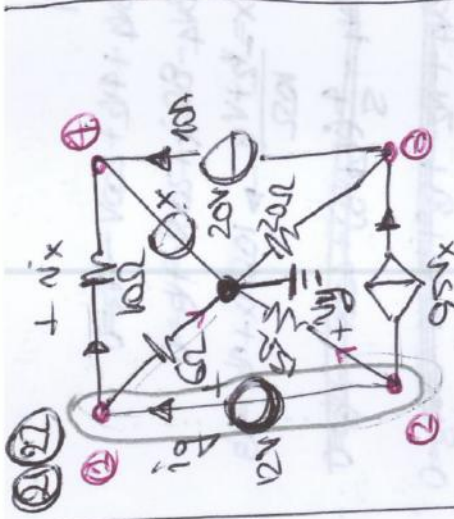


$v_5 = v_6$   
 $v_3 = 2v_x$   
 $\frac{v_2 - v_3}{6\Omega} + 6A + \frac{v_2 - v_4}{4\Omega} = 0$   
 $2v_2 - 2v_3 + 36V + 3v_2 - 3v_1 = 0$

$\frac{v_2 - v_1}{4\Omega} = i_x = \frac{v_2 + v_0}{4\Omega}$   
 $10A = v_2 + v_0$   
 $5v_2 - 4v_1 + 12V + 2v_1 + 3v_0 = 0$   
 $\frac{v_2 - v_3}{6\Omega} +$

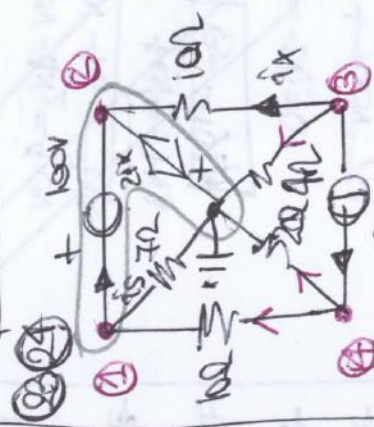


$v_3 = 12V$   
 $v_1 = -2v_x$   
 $6A + v_2 + \frac{v_2 - v_3}{4\Omega} = 0$   
 $7v_2 + 2v_1 + 3v_2 - 3v_3 = 0$   
 $4v_2 + 5v_1 - 3v_3 = 0$   
 $7v_2 + 5v_1 + 9v_1 = 0$   
 $108V = -5v_2$   
 $v_2 = -\frac{108V}{5}$   
 $i_x = \frac{v_2 - v_3}{4\Omega} = \frac{-108 + 12V}{4\Omega} = -24A$   
 $v_0 = 16.8V$



$6A + v_1 = 20V$   
 $6A + 2v_1 - v_2 = 12V$   
 $\frac{v_1 + v_1 - v_4}{10\Omega} + \frac{v_2}{15\Omega} + \frac{9.5v_x}{\Omega} = 0$   
 $5v_1 + 3v_1 - 3v_4 + 2v_2 + 15v_x = 0$   
 $8(v_2 + 12V) - 6v_1 + 2v_2 + 15v_x = 0$   
 $10v_2 + 36V + 15v_x = 0$   
 $v_x = v_2 - v_4 = v_2 + 12V - 2v_1$   
 $9v_x = v_2 - 8V$   
 $10v_2 + 36V + 15(v_2 - 12V) - 12v_1 = 0$   
 $25v_2 = 84V$   
 $v_2 = \frac{84V}{25} = 3.36V$   
 $v_x = -4.64V$   
 $i_y = \frac{v_4}{15\Omega} = 0.224A$

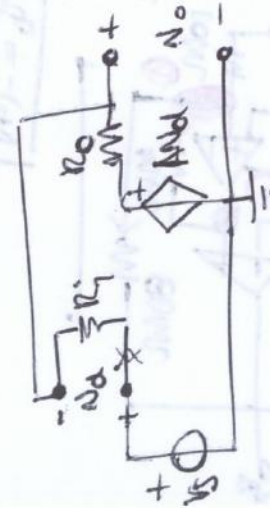
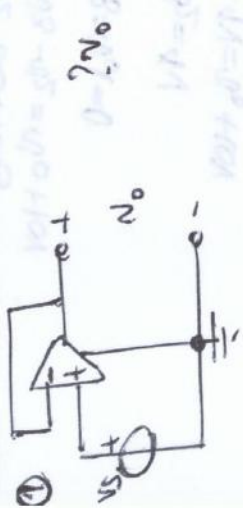
$i_g = i_y \neq 0, 5v_x =$   
 $= 0.224A + 2.32A =$   
 $= 2.546A$



$6A + v_2 = -2.5v_x$   
 $\frac{v_4 - v_1}{10\Omega} + \frac{v_4}{20\Omega} + \frac{v_3}{4\Omega} + \frac{v_3 - v_2}{10\Omega} = 0$   
 $2v_4 - 2v_1 + v_4 + 5v_3 + 2v_3 - 2v_2 = 0$   
 $3v_4 - 2v_1 + 7v_3 - 2v_2 = 0$   
 $(KVL): v_1 - v_2 = 100V$   
 $v_1 = 100V + v_2$   
 $3v_4 - 2(100V + v_2) - 4v_2 + 2v_3 = 0$   
 $3v_4 - 200V - 6v_2 + 2v_3 = 0$   
 $3v_4 - 2v_3 = 200V$   
 $3v_4 - 2v_3 = 40V$   
 $3v_4 - 2v_3 = 40V$   
 $3v_4 - 2v_3 = 40V$   
 $40V + 2v_2 - 2v_2 + 5v_3 = 0$   
 $40V + 5v_3 - 2v_2 = 0$



ESEMPIO CAP. 4: AMPLIFICATORI OPERAZIONALI

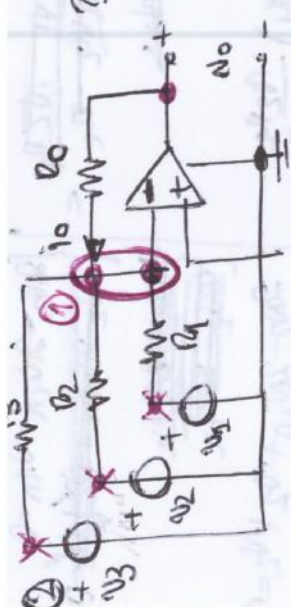


$$N_o = \frac{v_s + \frac{A v_i}{R_i}}{\frac{1}{R_i} + \frac{1}{R_o}} = \frac{v_s R_o + A v_i R_o}{R_o + R_i} \cdot \frac{R_o + R_i}{R_o + R_i}$$

$$\rightarrow N_o = \frac{v_s R_o + A v_i R_i}{R_o + R_i} = \frac{v_s R_o + A (v_s - N_o) R_i}{R_o + R_i}$$

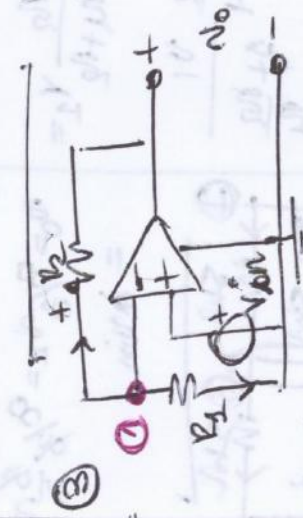
$$N_o (R_o + R_i + A R_i) = v_s R_o + A v_s R_i$$

$$N_o = \frac{v_s (R_o + A R_i)}{R_o + R_i + A R_i}$$



i)  $\frac{v_o - v_{o1}}{R_4} + \frac{v_2 - v_{o1}}{R_2} + \frac{v_3 - v_{o1}}{R_3} + \frac{v_o - v_{o1}}{R_o} = 0$   
 ii)  $v_{o1} = 0$

$$-\left(\frac{v_1}{R_4} + \frac{v_2}{R_2} + \frac{v_3}{R_3}\right) R_o = v_o$$



$$\frac{v_{o1} + v_{in} - v_o}{R_2} = 0$$

$$v_o = R_2 v_{in} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$v_o = v_{in} \left( 1 + \frac{R_2}{R_1} \right) > v_{in}$$

4. TERMOCOPIA

?  $N_o$   $v_T = \alpha (T - T_0)$ ,  $\alpha = 40 \mu V / ^\circ C$   
 $\rightarrow v_T = 100 mV$ ,  $\alpha T = T_0 + 100 ^\circ C$

~~term = 40  $\mu V / ^\circ C$  (T - T<sub>0</sub>)~~

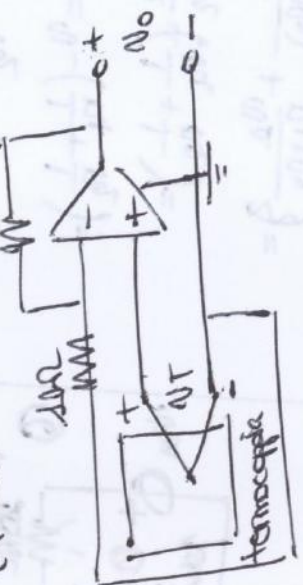
~~$v_T = 40 \mu V / ^\circ C$~~   
 ~~$v_T = 40 \mu V / ^\circ C$~~

$$v_T (100 ^\circ C) = 40 \mu V (100 + 100 - T_0) = 8 mV$$

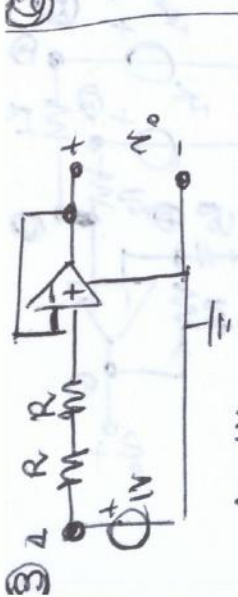
Il fattore di amplificazione è  $\frac{100}{4} = 25$   
 Se usi un amplificatore non invertente:

$$1 + \frac{R_2}{R_1} = 25 \Rightarrow \frac{R_2}{R_1} = 24 \Rightarrow R_2 = 24 R_1$$

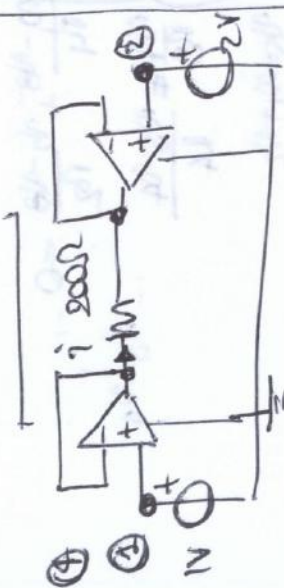
$$\left. \begin{matrix} R_2 = 24 k\Omega \\ R_1 = 1 k\Omega \end{matrix} \right\}$$



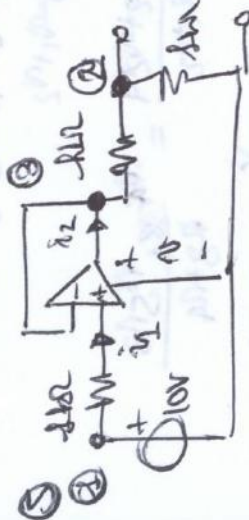




$v_o = 1V$



$v_1 = 1V$   
 $v_2 = 2V$   
 $i = \frac{v_1 - v_2}{200\Omega} = -5mA$



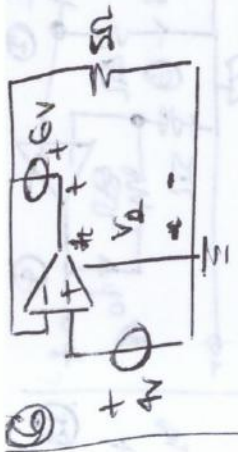
$i_1 = 0A, v_1 = 0V = v_3$

$i_2 = \frac{v_3 - v_2}{1M\Omega} = \frac{0V - v_2}{1M\Omega}$

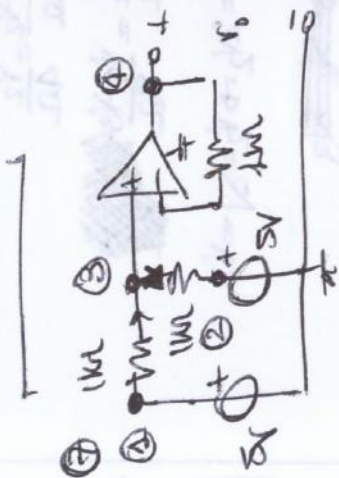
$v_3 = v_2 = \frac{v_2}{1M\Omega} = \frac{v_2}{1M\Omega}$

$v_3 = 2V \rightarrow v_2 = 2V$

$\rightarrow i_2 = \frac{v_3 - v_2}{1M\Omega} = \frac{0V - 2V}{1M\Omega} = -2\mu A$



$v_1 = 10V$



$v_1 = 5V$   
 $v_2 = 5V$

$v_1 = v_3 + v_2 = \frac{v_3}{1M\Omega} + \frac{v_2}{1M\Omega} = 0$   
 $v_1 + v_2 = 10V = 2v_3$   
 $v_3 = 5V$

$v_1 = v_3 + v_2 = \frac{v_3}{1M\Omega} + \frac{v_2}{1M\Omega} = 0$

$v_1 = v_3 + v_2 = \frac{v_3}{1M\Omega} + \frac{v_2}{1M\Omega} = 0$

$v_3 = 14 = v_6 = 5V$



$v_8 = v_6 = v_4$   
 $v_2 = v_3 = 5V$

$v_4 = v_2 + \frac{v_4 - v_3}{40\Omega} = \frac{v_2 - v_1}{60\Omega} + \frac{v_3 - v_1}{20\Omega}$   
 $8V - 2v_2 + 2v_4 - 2v_3 = 2v_2 - 2v_1 + 6v_3 - 6v_1$   
 $6v_4 - 5v_2 - 2v_3 + 8v_1 = 0$

$1: \frac{v_4 - v_2}{40\Omega} + \frac{v_4 - v_3}{40\Omega} = \frac{v_5 - v_4}{40\Omega}$

$10v_4 + 10v_4 - 10v_2 - 10v_3 = 15 - v_4$

$20v_4 - 10v_2 - 10v_3 = 15 - v_4$

$2: \frac{v_2 - v_1}{60\Omega} + \frac{v_3 - v_1}{20\Omega} = 0$

$v_2 - v_1 + 3v_3 - 3v_1 = 0$

$v_2 + 3v_3 = 4v_1$

$20v_4 + 10v_2 - 10v_3 = 0 \rightarrow v_1 = 0$

$6v_4 - 5v_2 - 2v_3 + 8v_1 = 0$

$v_5 = v_3 + 5V$

$6v_4 - 5v_2 - 2v_3 + 8v_1 = 0$

$6v_4 - 5v_2 - 2v_3 + 8v_1 = 0$

$6v_4 - 5v_2 - 2v_3 + 8v_1 = 0$

~~$50V - 2v_1 + 2v_2 = 0$~~   
 ~~$v_2 = v_3 + 5V$~~   
 ~~$v_2 + 2v_3 = 4v_1$~~   
 ~~$20v_4 - 10v_2 - 10v_3 = 0$~~   
 ~~$6v_4 - 5v_2 - 2v_3 + 8v_1 = 0$~~

$v_2 = v_3 + 5V$   
 $v_2 + 2v_3 = 4v_1$   
 $20v_4 - 10v_2 - 10v_3 = 0$   
 $6v_4 - 5v_2 - 2v_3 + 8v_1 = 0$

$4v_1 = 4v_1 - 9V \neq$

$v_3 = v_1 - 9V$   
 $20v_4 - 20(v_1 - 9V) - 20v_1 = 0$   
 $6v_4 - 14(v_1 - 9V) - 45V + 8V = 0$

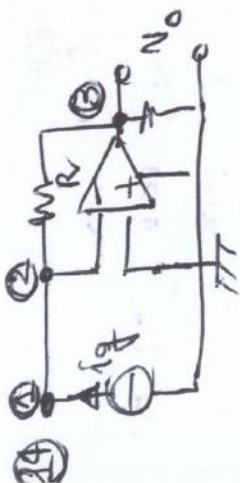
$716.8$   
 $\frac{1}{k_2}$

$20v_4 - 20v_1 - 45V = 0$   
 $6v_4 - 6v_1 + 63V - 45V = 0$

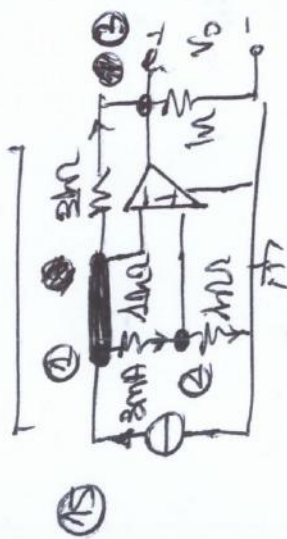
$v_1 = v_4 - \frac{45V}{2}$

$6v_4 - 6v_1 + 63V - 45V = 0$





$v_2 = v_3 = 1g$   
 $v_2 - v_3 = 1gR$   
 $v_0 = v_2 - 1gR$   
 $\rightarrow N_0 = -1gR$

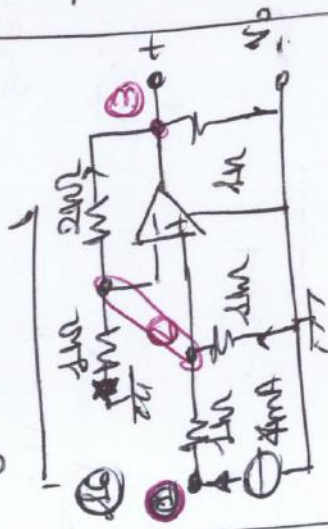


$v_2 = v_3 = v_1$   
 $\frac{v_2 - v_3}{1k\Omega} + \frac{v_2 - v_3}{3k\Omega} = 3mA$   
 $\frac{v_2 - v_3}{1k\Omega} = 3mA \rightarrow v_2 - v_3 = 3V$   
 $v_0 = v_2 - 3V = 3V$   
 $3mA = \frac{v_2}{1k\Omega} + \frac{v_2 - 3V}{3k\Omega}$

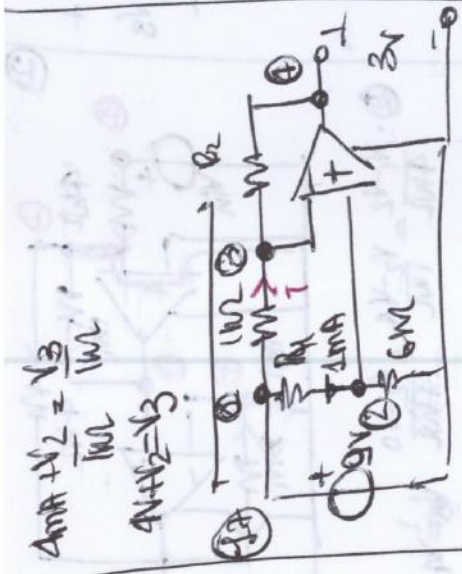
~~$v_1 = v_2 = 3V$~~   
 ~~$v_3 = 3V$~~   
 ~~$v_2 - v_3 = 0$~~   
 ~~$v_2 = 3V$~~   
 ~~$v_3 = 3V$~~   
 ~~$v_2 - v_3 = 0$~~

$\frac{v_1 - v_2}{1k\Omega} + \frac{v_1 - v_3}{3k\Omega} = 3mA$   
 $\frac{v_1 - v_2}{1k\Omega} = \frac{v_2}{1k\Omega} \rightarrow v_2 = 0$

$v_2 - v_3 = 3V$   
 $v_3 = -3V$



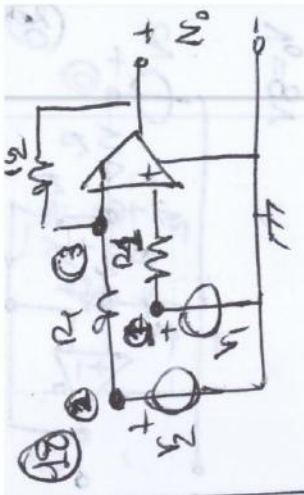
$4mA = \frac{v_1 - v_2}{1k\Omega}$   
 $2k\Omega \cdot \frac{v_1 - v_2}{1k\Omega} + \frac{v_2}{1k\Omega} = 0$   
 $2v_1 - v_2 + v_2 = 0 \rightarrow v_1 = 0$   
 $2v_2 + v_2 - v_3 = 0$   
 $3v_2 - v_3 = 0 \rightarrow v_3 = 3v_2$



$4mA + i_2 = \frac{v_3}{1k\Omega}$   
 $4mA + i_2 = v_3$

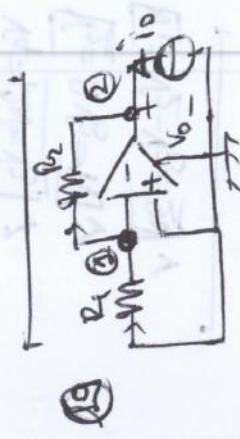
$v_1 = 5V, v_4 = 2V$

$i_2 = \frac{v_1 - v_2}{1k\Omega} = \frac{v_2}{1k\Omega} = 4mA$   
 $4mA + i_2 = v_3$   
 $8mA = v_3$   
 $\rightarrow v_4 = \frac{v_1 - v_2}{1k\Omega} = \frac{3V - 1V}{1k\Omega} = 2mA$   
 $v_0 = \frac{2V}{2k\Omega} = 1mA$



$v_2 - v_3 = \frac{v_3 - v_0}{1k\Omega}$   
 $v_2 - v_1 = \frac{v_3 - v_0}{1k\Omega}$

$v_2 - v_1 - v_2 + v_3 = -v_0$   
 $v_0 = v_1 - v_2 + v_3 - v_0 = -v_0$   
 $v_0 = v_1(1 + \frac{1}{k_1}) - v_2 \frac{1}{k_1} = -v_2 \frac{1}{k_1}$



$v_1 = \frac{v_1 - v_2}{1k\Omega}$

$v_1 = 0 \rightarrow v_2 = v_0 = 0V$







PARTE 5: LINEARITÀ E SOVRAPPONIBILITÀ

esempio



$v_o = v_s$

①:  $\frac{v_1}{90} + \frac{v_1 - v_3}{30} = \frac{v_2 - v_1}{10}$

$v_1 + 3v_1 - 3v_3 = 9v_2 - 9v_1$

$13v_1 - 3v_3 = 9v_2$

②:  $\frac{v_3}{10} = \frac{v_1 - v_3}{30} + \frac{v_2 - v_3}{90}$

$9v_3 = 3v_1 - 3v_3 + v_2 - v_3$

$13v_3 = 3v_1 + v_2$

$13v_1 - 3v_3 = 9v_2$

$13v_3 = 3v_1 + v_2$

$v_3 = \frac{13v_1 - 9v_2}{3}$

$13 \left( \frac{13v_1 - 9v_2}{3} \right) = 3v_1 + v_2$

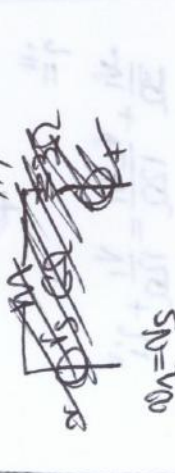
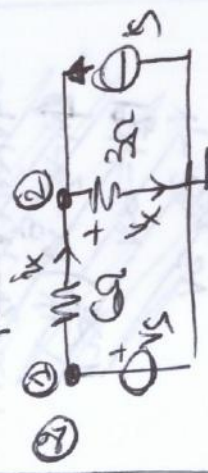
$v_3 = \frac{13v_1 - 9v_2}{3}$

$16v_1 - 11v_2 = 9v_1 + 3v_2$

$v_3 = \frac{13 \cdot 0.75 \text{ V} - 9 \text{ V}}{3} = \frac{1 \text{ V} - 9 \text{ V}}{3} = -0.75 \text{ V}$

$16v_1 = 9v_2 + 3v_3 \rightarrow 0 = \frac{3}{4}v_2 - 0.75 \text{ V}$

$i = \frac{v_1 - v_3}{30} = \frac{0.5 \text{ V} - (-0.75 \text{ V})}{30} = \frac{1.25 \text{ V}}{30}$



$v_1 = v_s$

①:  $\frac{v_1 - v_2}{90} + i_s = \frac{v_2}{30}$

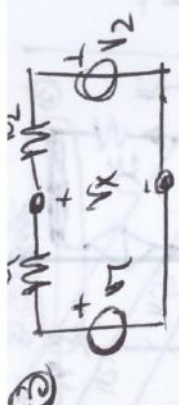
$v_1 - v_2 + 60i_s = 2v_2$

$60i_s + 60i_s = 3v_2$

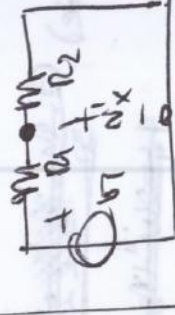
$60i_s = v_2$

$i_x = \frac{v_1 - v_2}{90} = \frac{v_s - v_2 - 2v_2}{90} = \frac{v_s - 3v_2}{90}$

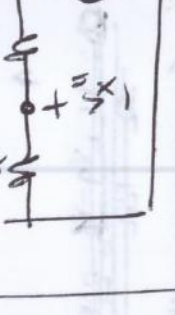
$= \frac{1}{9}v_s - \frac{1}{30}v_2$



SOVRAPPONIBILITÀ:

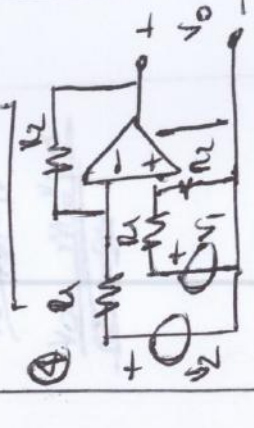


$v_x'' = \frac{v_2 v_1}{R_1 + R_2}$

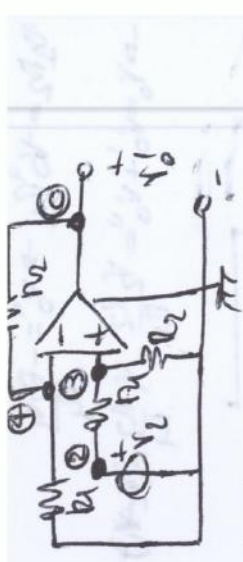


$v_x'' = \frac{v_2 v_1}{R_1 + R_2}$

$v_x = v_x' + v_x'' = \frac{v_1 v_2 + v_2 v_1}{R_1 + R_2}$



?  $v_o$ , approssimazione  
SOVRAPPONIBILITÀ:



$v_2 = v_1$

③:  $\frac{v_2 - v_3}{10} = \frac{v_3}{90}$

$v_2 v_2 - v_2 v_3 = v_3 v_2$

$v_2 v_2 = v_3 (v_2 + v_2)$

$v_3 = \frac{v_2}{2} = \frac{v_1}{2}$

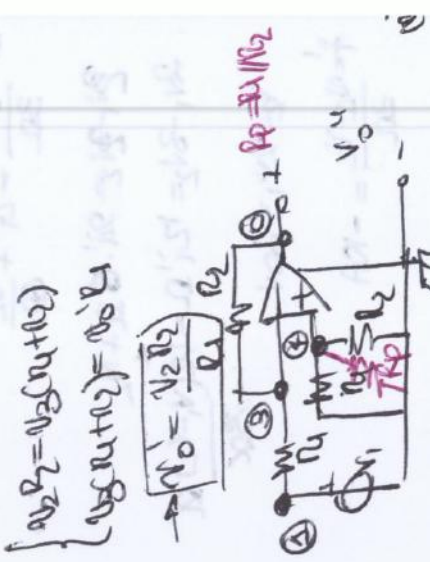
$v_4 v_2 = v_1 v_2 - v_4 v_1$

$v_4 (v_2 + v_2) = v_1 v_1$

$v_2 v_2 = v_2 (v_1 + v_2)$

$(v_2 v_2 + v_2) = v_1 v_1$

$\rightarrow v_1' = \frac{v_2 v_2}{v_1}$



⑤:  $v_1 - v_3 = \frac{v_3 v_1}{10} + v_1 v_2 = v_3 (v_1 + v_2)$

④:  $v_1 - v_3 = \frac{v_3 v_1}{10} + v_1 v_2 = v_3 (v_1 + v_2)$









$v_1 = v_1 i_1$

$v_1 i_1 = \frac{v_2 - v_3}{R_1} \quad v_2 = 0 \quad v_3 = 0$

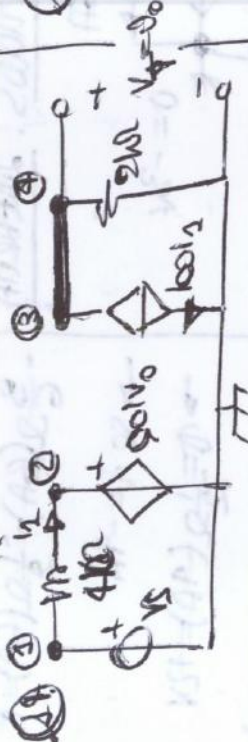
$v_1 i_1 R_2 = v_2 (R_1 + R_2) - v_3 R_1$

$v_1 i_1 R_2 = v_2 \left( \frac{R_1 + R_2}{R_1} \right) - v_3 \left( \frac{R_2}{R_1} \right)$



$v_2 = v_2 i_2$      $v_1 = 0$

$v_2 = v_1 \frac{R_2}{R_1} \rightarrow v_2 = 0$



~~$v_1 = v_1 i_1$~~

~~$v_2 = v_2 i_2$~~

~~$v_3 = v_3 i_3$~~

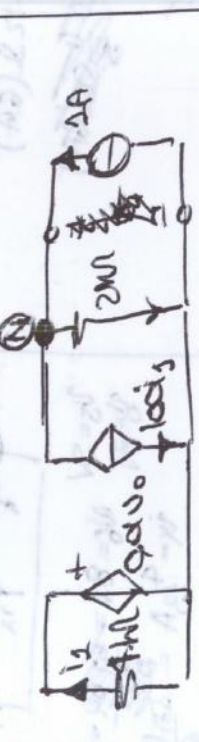
$i_2 = \frac{v_3 - 0.01 v_0}{4k\Omega}$

$\rightarrow v_1 = -2k\Omega \cdot 100(0.01 v_0) = -200 v_0$

$= -50(0.01 v_0) = -0.5 v_0 + 0.5 v_1$

$950 v_1 = -80 v_0$

$v_1 = -1000 v_0$



$i_0 = -\frac{0.01 v_0}{4k\Omega}$

$100 i_0 = 1A \cdot \frac{v_0}{2k\Omega} = -\frac{v_0}{4k\Omega} \quad v_0 = 4k\Omega$

$1k\Omega - 2k\Omega = -1k\Omega \quad v_0 = 4k\Omega \rightarrow v_0 = 4k\Omega$



$v_1 = 12V$

$\frac{12V - v_2}{6\Omega} + 3 \left( \frac{12V - v_2}{6\Omega} \right) = \frac{v_2 - v_1}{3\Omega}$

$\frac{24}{3\Omega} (12V - v_2) = \frac{v_2 - v_1}{3\Omega}$

$24V - 2v_2 = v_2 - v_1$

$24V - 3v_2 = -v_1$

$v_1 = 3v_2 - 24V$

$\frac{v_2 - v_1}{3\Omega} = \frac{v_2 - (3v_2 - 24V)}{3\Omega}$

$\rightarrow v_1 = 6v_2 - 24V$

$-5v_1 = -24V$

$v_1 = \frac{24}{5} = 4.8V$



$-\frac{v_1}{6\Omega} + 3 \left( \frac{v_1}{6\Omega} \right) = \frac{v_1 - v_2}{3\Omega}$

$\frac{24}{3\Omega} v_1 = \frac{v_1 - v_2}{3\Omega}$

$-9v_1 = v_1 - v_2 \quad v_1 = 3v_2$

$\frac{v_1 v_0}{3\Omega} + 1A = \frac{v_0}{3\Omega}$

$v_1 v_0 + 3A = v_0$

$\frac{v_0}{3} - 2v_0 = -3A$

$-5v_0 = -9A$

$v_0 = \frac{9}{5}V$

$\rightarrow R_0 = \frac{9}{5}\Omega$



**④**

$V_0 = 800 \text{ k} \Omega \cdot i_1$   
 $V_1 = 15 - 0,01 V_0 \rightarrow V_0 \cdot 80 \Omega = 800 \Omega (15 - 0,01 V_0)$   
 $2 \text{ k} \Omega \cdot V_0 = 800 \Omega \cdot 15$   
 $\frac{V_0}{80} = \frac{800 \cdot 15}{80} = -150$

**⑤**  $i = 2A$   $\begin{cases} i_1 \text{ ON} \\ i_2 \text{ OFF} \end{cases}; i = 2A \begin{cases} i_1 \text{ ON} \\ i_2 \text{ OFF} \end{cases}$   
 $i = ? \begin{cases} i_1 \text{ OFF} \\ i_2 \text{ ON} \end{cases}$   
 $i = i_1 = 2A$   
 $i = i = 2A = i_1 + i_1$   
 $i^u = i - i_1 = 2A - 2A = 0A$

**⑥**

$i = 2A; i^u = 2A;$   
 $i^i + i^u = 1A.$

**⑦**

**PSE 1:**

①:  $4mA = \frac{V_1}{2\Omega} + \frac{V_1 - V_2}{4\Omega}$   
 $48N = 4V_1 + 2i_1 - 2i_2$

②:  $\frac{V_1 - V_2}{4\Omega} = \frac{V_2}{1\Omega}$   
 $V_1 - V_2 = 4V_2 \rightarrow V_1 = 5V_2$   
 $48N = 85V_2 - 2i_2$   
 $V_2 = \frac{48N}{32} = \frac{3}{2} V$   
 $V_1 = \frac{15N}{2}$

**PSE 2:**

$V_2^u = \frac{8V \cdot 1\Omega}{8\Omega} = 1V$   
 $V_2^m = \frac{-8V \cdot 2\Omega}{8\Omega} = -2V$

**PSE 3:**

①:  $2mA + \frac{V_1}{1\Omega} + \frac{V_1 - V_2}{4\Omega} = 0$   
 $8V_1 + 4V_2 + V_1 - V_2 = 0$

②:  $\frac{V_1 - V_2}{4\Omega} = \frac{V_2}{2\Omega}$   
 $3V_1 - 3V_2 = 4V_2$

$3V_1 = 7V_2$   
 $8V_1 + 8V_2 = 4V_2$   
 $8V_1 = -4V_2$   
 $V_1 = -0,5V_2$

$V_1^m = -56V / 32 = -1,75V$   
 $V_2^m = 8V - 35 \cdot 1,75V = -3,75V$

$V_1 = V_1^u + V_1^m + V_1^m = \frac{15}{2} - 3 - \frac{3}{2} = 3,75V$   
 $V_2 = V_2^u + V_2^m + V_2^m = \frac{8}{2} + 1 - 7,5 = 0,95V$



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$i_2 = \frac{V_1}{2k} + \frac{V_2}{1k} = \frac{V_0 - V_0}{3k}$   
 $3k i_1 + 6V_1 - 6V_2 = 2k \cdot 2V_0$   
 $1V_1 - 6V_2 = 2V_0$   
 $i_2 = \frac{V_1 - V_2}{1k} = i_1$   
 $V_0 = 1k i_1 + V_2$   
 $1k i_1 + 1V_2 - 6V_2 = 2V_0$   
 $1k i_1 + 5V_2 = 2V_0$   
 $1k \left( \frac{V_1 - V_2}{1k} \right) + 5V_2 = 2V_0$

$3V = 4(V_1 + 3V) - 3V_2 - V_1$   
 $2V = 3V_1 - 3V_2 + V_1 - V_1$   
 $3V = 4V_1 - 3V_2 - V_1$   
 $2V_1 - 3V_2 = 2V$   
 $2V_1 - 2V_2 = 2V + 2V$   
 $V_1 - V_2 = 2V$   
 $V_1 + 3V = 0$   
 $V_1 = -3V$   
 $V_2 = -3V - 2V = -5V$   
 $2V_1 + 5V_2 = 2(-3V) + 5(-5V) = -6V - 25V = -31V$   
 $-3V = V_1$

$i_1 = 1mA = \frac{V_1 - V_2}{1k} = \frac{-3V - (-5V)}{1k} = \frac{2V}{1k} = 2mA$   
 $V_2 = 2V$   
 $V_1 = -3V$   
 $V_3 = 3V$   
 $V_T = V_2 + V_3 + V_1 = 2V + 3V - 3V = 2V$

20

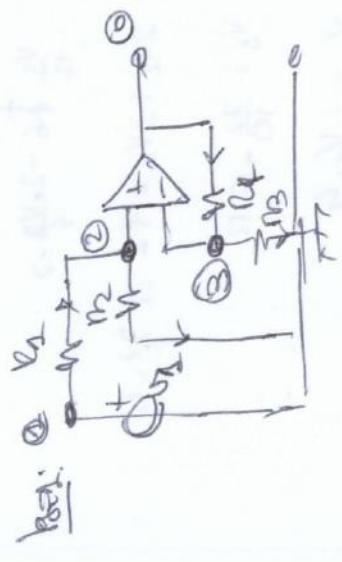
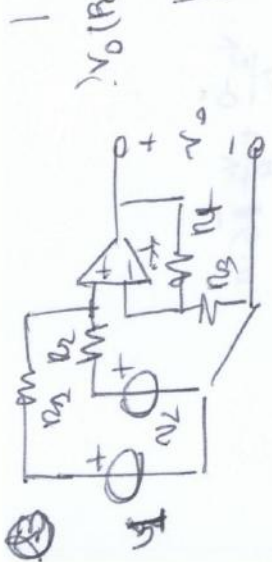
$R_T = \frac{2 \cdot 8}{2 + 8} = 1.6k\Omega$   
 $R_T = 6k\Omega$   
 $V_1 - 3V_2 = 3V - V_1 - V_1$   
 $V_2 + 2V_1 = 2V_1 + 2V_2$   
 $V_1 + 3V_2 = 0$   
 $3V_1 - 3V_2 = 3V - V_1 - V_1$   
 $4V_1 - 3V_2 = 3V - 2V_1$   
 $3V_2 = 2V_1 - 6V + 3V$   
 $3V_2 = 2V_1 - 3V$   
 $V_1 = V_1 - 3V$   
 $V_T = 4V - 10V + 8V = 2V$   
 $V_2 = 2V - 9V = -7V$

21

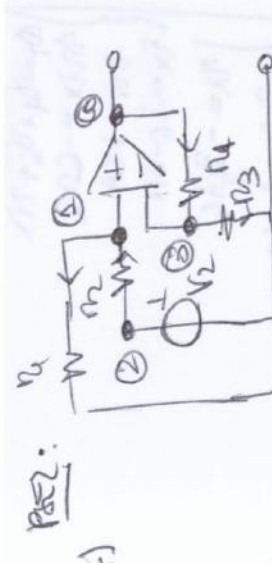
$10k + 1.25A = 1.25A \cdot 2k$   
 $0.8k\Omega$   
 $9V = 9.8k\Omega$   
 $V_T = 9V$   
 $4V_T = V_1 - V_T$   
 $5V_T = V_1 - 4V_T = 3V$

22

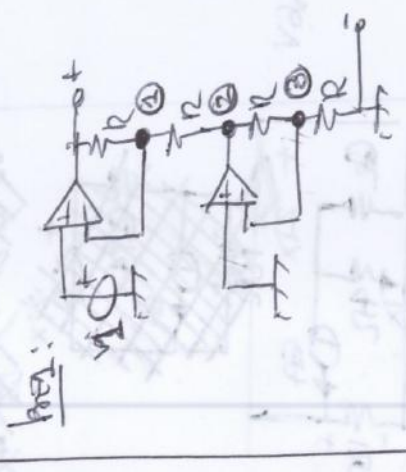
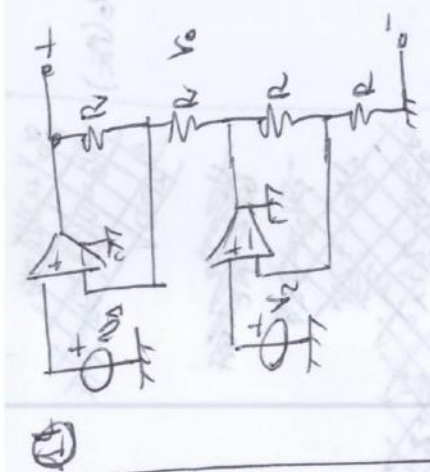
$V_T = 9V$   
 $4V_T = V_1 - V_T$   
 $5V_T = V_1 - 4V_T = 3V$



$V_1 = V_2 = V_1$   
 $V_1 = V_2 = \frac{V_2}{R_1}$   
 $V_1 = V_2 = V_2 (R_1 + R_2)$   
 $V_1 = V_2 = \frac{V_2 R_2}{R_1}$   
 $V_1 = V_2 = \frac{V_2 R_2}{R_1 + R_2}$

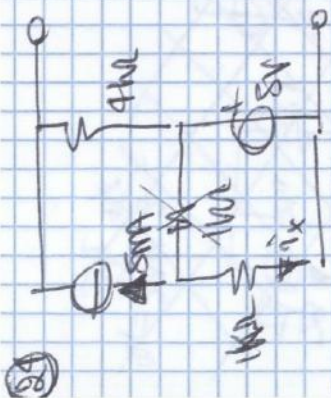


$V_1 = V_2 = V_2$   
 $V_1 = V_2 = \frac{V_2 R_1}{R_1 + R_2}$   
 $V_1 = V_2 = \frac{V_2 R_1}{R_1 + R_2}$   
 $V_1 = V_2 = V_2$   
 $V_1 = V_2 = \frac{V_2 R_1}{R_1 + R_2}$   
 $V_1 = V_2 = V_2 =$   
 $= \frac{V_2 R_1 (R_3 + R_4) + V_2 R_1 (R_3 + R_4)}{R_1 R_2 + R_1 R_3}$   
 $= \frac{V_2 R_1 (R_3 + R_4)}{R_1 R_2 + R_1 R_3}$



$V_1 = 2V_2 \dots$

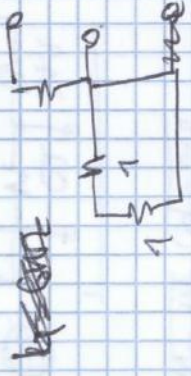




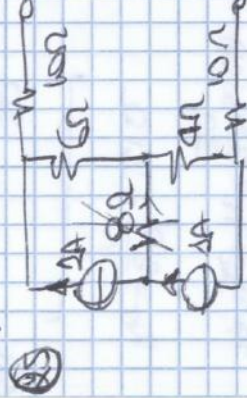
$i_x = 5mA$

$u_4 = 20V$

$u_T = 5V + u_4 = 25V$

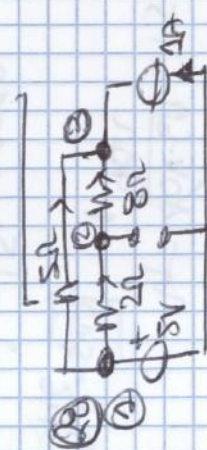
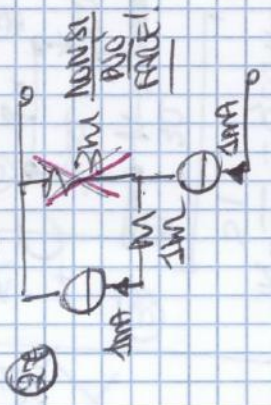
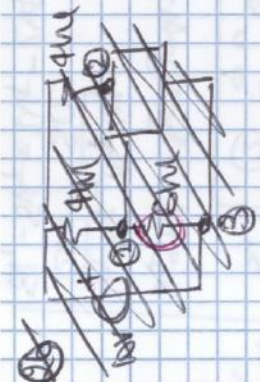


$i = \frac{4V}{4k + 100k} = 4\mu A$



$u_T = 4A \cdot 4k + 6k \cdot 4A = 40V$

$i_T = 30A$



$i_T = (5V/2) / 2 = 1.25A$

$u_1 = 5V$

$\frac{u_1 - u_2}{2\Omega} = \frac{u_2 - u_3}{8\Omega}$

$4u_1 - u_2 = u_2 - u_3$

$\frac{5u_3}{8\Omega} + \frac{u_1 - u_3}{5\Omega} + 1A = 0$

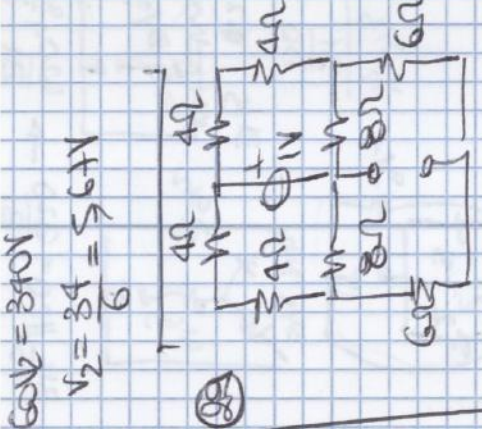
$5u_2 - 5u_3 + 8u_1 - 8u_3 + 40V = 0$

$5u_2 + 80V - 13u_3 = 0$

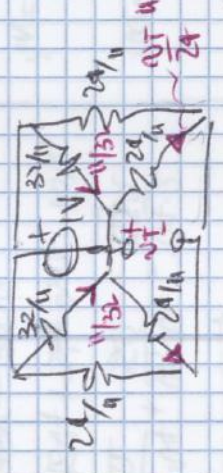
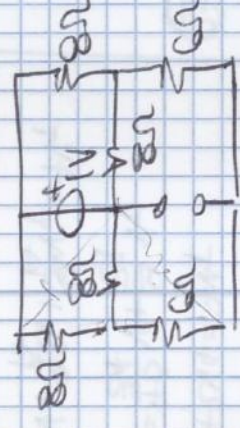
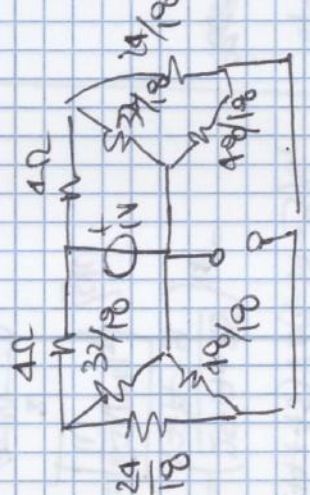
$-20V + 5u_2 = u_3$

$5u_2 + 80V - 13(-20V + 5u_2) = 0$

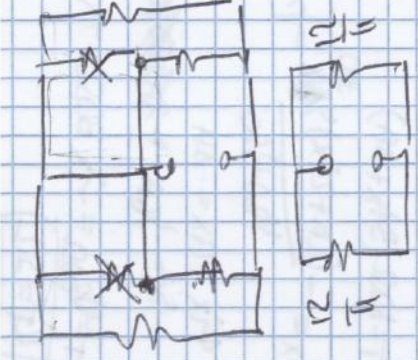
$5u_2 + 80V + 260V - 65u_2 = 0$



$60V = 340V$   
 $i_2 = \frac{34}{6} = 5.67A$



$\frac{11}{32} + \frac{11}{24} u_T = 0 \Rightarrow u_T = -\frac{243}{4} V$



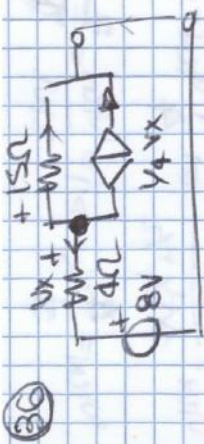
$\frac{12 \cdot 12}{12 + 12} = 6$   
 $\frac{12 \cdot 6}{12 + 6} = 4$   
 $\frac{12 \cdot 4}{12 + 4} = 3$

$80 + 20V$

$u_T = -\frac{80}{20/3} = -12V$

$\frac{3}{80} + \frac{1}{20} u_T = 0$

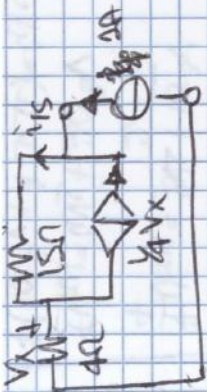




~~$i_N = \frac{8V}{4\Omega}$~~   
 ~~$i_N = \frac{8V}{4\Omega} - \frac{150i_N}{150\Omega}$~~   
 ~~$i_N = \frac{8V}{4\Omega} + \frac{150i_N}{150\Omega}$~~   
 ~~$i_N = \frac{8V}{4\Omega} + \frac{8V \cdot 150}{150 \cdot 4\Omega}$~~

~~$600i_N = 150i_N + 8000i_N$~~   
 ~~$40i_N = 150i_N + 8000i_N$~~   
 ~~$i_N = \frac{8000i_N}{40 - 150}$~~   
 ~~$i_N = \frac{8000i_N}{-110}$~~   
 ~~$i_N = -\frac{8000i_N}{110}$~~   
 ~~$i_N = -\frac{8000i_N}{110}$~~

~~$U_T = 8V + U_x - U_{150\Omega} = 8V + U_x - i_N \cdot 150\Omega = 8V + U_x - 150\Omega \left( \frac{8V}{4\Omega} - U_x \right) = 8V + U_x - 150\Omega \left( \frac{2}{1} - U_x \right) = 8V + U_x - 300V + 150\Omega U_x \rightarrow U_T = 8V$~~

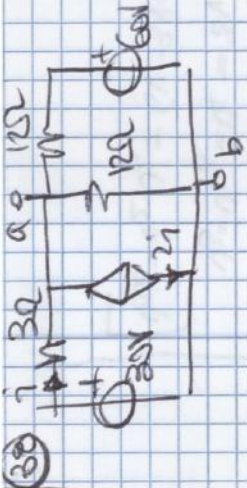


$U_x = 4V$   
 $i_{15} = \frac{1}{4} U_x + 1A = 2A$   
 $U_{15} = i_{15} \cdot 15 = 30V$   
 $U_{4A} = 34V \rightarrow R_T = 34\Omega$

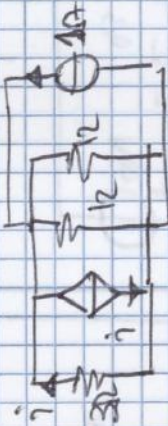


$i_N = \frac{20A + 6A - 3A}{Y_3 + Y_6} = \frac{10A + 3(N/3\Omega)}{Y_3 + Y_6}$   
 $\frac{1}{2} i_N = 10A + N_T$   
 $-10A = \frac{1}{2} N_T \Rightarrow N_T = -20V$

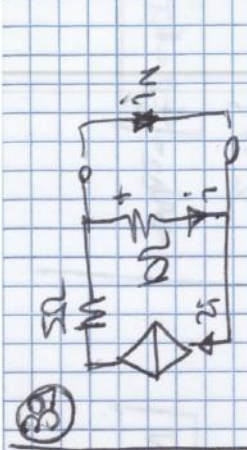
$U_{4A} = \frac{20A + 6A}{Y_3 + Y_6} + 1A = \frac{1}{2} U_T$   
 $U_{4A} = -2V \rightarrow R_T = -2\Omega$



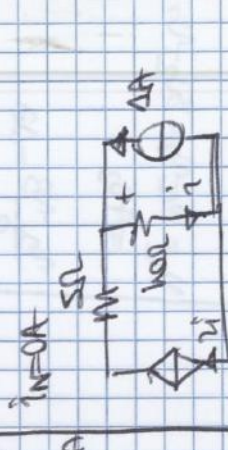
$U_{bb} = N_T = \frac{60V + 30V - 2i}{Y_3 + Y_2 + Y_1}$   
 $\frac{1}{2} N_T = \frac{60V}{12\Omega} + \frac{30V}{3\Omega} - 2 \left( \frac{N_T}{3\Omega} \right)$   
 $\left( \frac{1}{2} + \frac{2}{3} \right) N_T = 8A + 10A + 20A = 38A$   
 $N_T = 30V$



$U_{1A} = \frac{15A + U_{30\Omega}}{Y_3 + Y_6}$   
 $\frac{1}{2} U_{1A} = 15A + \frac{1}{2} U_{30\Omega}$   
 $-\frac{1}{60} U_{1A} = 15A \rightarrow U_{1A} = -90V$



$i_N = i - 2i = -i = -\frac{U_{10V}}{10\Omega}$   
 $U_{10\Omega} = 0V$   
 $10\Omega i = 0V$



$2i + 4A = i$   
 $i = -4A$   
 $U_{10\Omega} = 0V = -4A \cdot 10\Omega = -40V$   
 $R_T = -10\Omega$



$v_2 = \frac{19V_3 - 60V}{4}$   
 $14 \left( \frac{19V_3 - 60V}{4} \right) = 60V + 11V_3$   
 $266V_3 - 840V = 240V + 44V_3$   
 $280V_3 = 1080V$   
 $V_3 = 4.32V$   
 $V_2 = 5.52V$   
 $\boxed{i = \frac{1.2V}{5\Omega} = 0.24A}$

$3mA = \frac{V_2 + V_3}{2k} + \frac{V_3}{4k}$   
 $\frac{V_2 - V_3}{4k} + \frac{19V_3 - 60V}{1k} = \frac{V_3}{2k}$   
 $12V = 2V_2 + V_2 - V_3$   
 $V_2 - V_3 + 19V_3 - 60V = \frac{V_3}{2}$   
 $12V = 3V_2 - V_3$   
 $V_2 + 48V = 4V_3$   
 $V_3 = 3V_2 - 12V$   
 $V_2 + 48V = 2V_2 - 84V$   
 $120V = 3V_2 - 4V_3 = 6.6$

$(75 + 2000 \cdot 10^3) v_2 = 19250v_1$   
 $v_2 = \frac{2000 v_1 + 10000v_1}{3000}$   
 $v_T = \frac{(75 + 2000 \cdot 10^3) [2000 v_1 + 10000v_1]}{1925 \cdot 3000}$   
 $v_T [1 + 1000(75 + 2000 \cdot 10^3)] = 2000 v_1$   
 $v_T = \frac{2000}{1 + 1000(75 + 2000 \cdot 10^3)} v_1 = 9057 v_1$

$v_T = 12V \quad i = \frac{v_2 - v_3}{5\Omega}$   
 $\textcircled{1}: v_2 + \frac{v_2 - v_3}{4\Omega} = \frac{v_1 - v_2}{4\Omega}$   
 $8v_2 + v_2 - v_3 = 5v_1 - 5v_2$   
 $14v_2 - v_3 = 5v_1$   
 $\textcircled{2}: \frac{v_2 - v_3}{4\Omega} + \frac{v_2 - v_3}{5\Omega} = \frac{v_3}{2\Omega}$   
 $5v_1 - 5v_2 + v_2 - v_3 + 4v_2 - 4v_3 = 10v_3$   
 $5v_1 - 4v_2 - 4v_3 = 10v_3$   
 $5v_1 - 4v_2 = 14v_3$

$v_2 = v_1 - v_2 = 0 \Rightarrow v_2 = v_1$   
 $v_4 = 10^5 v_2$   
 $\textcircled{2}: v_1 - v_2 = \frac{v_2}{1M} + \frac{v_2 - v_T}{2M}$   
 $2000 v_1 - 2000 v_2 = 2v_2 + 1000v_2 - 1000v_T$   
 $2000 v_1 = 2002v_2 - 1000v_T$   
 $\frac{v_2 - v_4}{2M} = \frac{v_2 - v_4}{1M}$

$v_T = v_2 = v_3 = v_T$   
 $v_{VA} = 0V \quad v_T = \frac{\Delta V}{200A} = 0$   
 $i_5 = v_1 - v_2 = \frac{v_2}{R_1} + \frac{v_2}{R_2}$   
 $i_3 = \frac{v_2 - v_T}{R_3} = \frac{v_2 - v_T}{R_3}$   
 $i_4 = \frac{v_T - v_2}{R_4} = \frac{v_T - v_2}{R_4}$   
 $v_2 = v_T = -15v_1$



80:  $U_T = 30V$



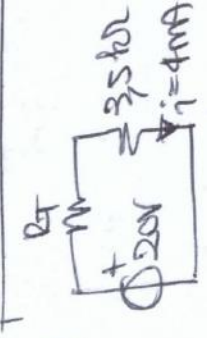
$I = \frac{10V \cdot 4\Omega}{10\Omega + 4\Omega} = 10A$

$4\Omega + 4\Omega = 10\Omega$

$R_T = \frac{6}{4} \Omega$

$U(3\Omega) = \frac{10V \cdot 3\Omega}{(3+6/4)\Omega} = 6.67V$

81



$N(3.5\Omega) = 14V$

$10V = \frac{20V \cdot 3.5\Omega}{R_T + 3.5\Omega}$

$10\Omega + 4\Omega = 14\Omega$

$R_T = \frac{21}{14} = 1.5\Omega$

$N(1.5\Omega) = \frac{20V \cdot 1.5\Omega}{2.5\Omega} = 8mA$

82:  $R_1 = 2\Omega, i_1 = 4mA$

$U_1 = 8V$   
 $U_2 = 5\Omega, i_2 = 2mA$   
 $U_3 = 10V$

?  $U_4 = 3\Omega, i_4$

$8V = U_T \frac{2\Omega}{0.5 + 2\Omega}$

$10V = U_T \frac{5\Omega}{0.5 + 5\Omega}$

$8\Omega + 16 = 2U_T$

$10\Omega + 50 = 5U_T$

$10U_T = 40\Omega + 8$

$10\Omega + 50 = 20\Omega + 40$

$10 = 10\Omega \Rightarrow U_T = 10V$

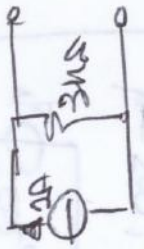
$U_T = 10V$

$N(3\Omega) = \frac{10V \cdot 3\Omega}{4\Omega} = 7.5W$

$i_4 = \frac{9V}{3\Omega} = 3mA$

83

$P_{max} = \frac{U_T^2}{4R_T}$



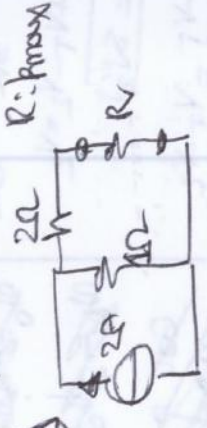
$U_T = 8kV$

$R_T = 3\Omega$

$P_{max} = \frac{8kV^2}{4 \cdot 10^3} = 0.75 \cdot 10^6 = 750W$

$= 750W$

84

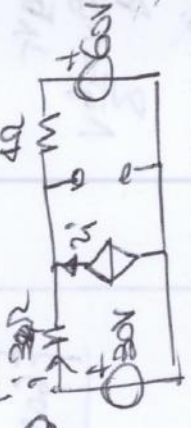


$U_T = 2V$

$R_T = 3\Omega$

$P_{max} = \frac{4W}{4 \cdot 3} = \frac{1}{3}W$

85



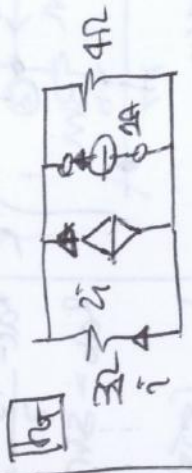
$i_{4\Omega} = 3i \rightarrow N_{4\Omega} = 12i\Omega$

$U_{60V} = -12i\Omega - 60V + 30V = -12i\Omega - 30V$

$i = -10A \Rightarrow i = -2A$

$U_{4\Omega} = -2A \cdot 4\Omega$

$N_T = 60V \cdot 2A = 36V \cdot kA$



$U_{4\Omega} = 0.9A \cdot 4\Omega = 3.6V$

$= 452 (2A + 2 \cdot \frac{10V}{3\Omega}) = 4A + 11.4\Omega$

$N_{4\Omega} = \frac{3.6V \cdot 0.9A}{2} = 1.62W$

$N_{2A} = \frac{1A + 2 \cdot \frac{10V}{3\Omega}}{1.5 + 4\Omega}$

$\frac{7 \cdot 10V}{12} = 1A + \frac{2 \cdot 10V}{3}$

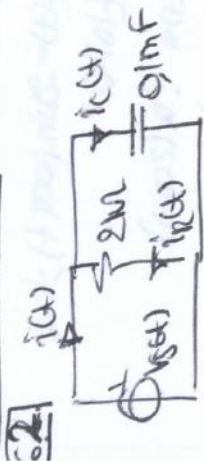
$\frac{5 \cdot 10V}{4} = 1A + N_{2A} = 1A + 11.4\Omega = 12.4\Omega$

$R_T = 9.8\Omega$

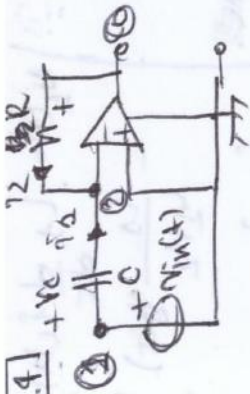
$P_{max} = \frac{36^2}{4 \cdot 9.8} W = 40.5W$



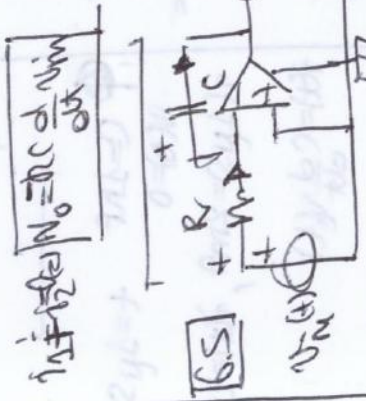
ESEMPIO CIRCUITO 6:



$v_s(t) = \cos(\omega t)$   
 $i(t) = i_R(t) + i_C(t)$   
 $i_R(t) = \frac{\cos(\omega t)}{2000} = 0,5 \cos(\omega t) \text{ mA}$   
 $i_C(t) = C \frac{dv_s(t)}{dt} = 10^{-5} \text{ F} (-\sin(\omega t) \cdot \omega) = 10^{-5} (-\sin(\omega t)) \text{ A}$   
 $i(t) = [0,5 \cos(\omega t) - \sin(\omega t)] \text{ mA}$   
 $C = 1 \text{ mF} = 10^{-3} \text{ F}$   
 $v(t) = (2 + \Delta) \text{ V}$   
 $p(t) = i(t) \cdot v(t) = (0,5 \cos(\omega t) - \sin(\omega t)) \cdot C \frac{dv(t)}{dt} = (2 + \Delta) \cdot 10^{-3} \text{ W}$   
 $= 2(2 + \Delta) \text{ mW}$   
 $W(t) = \frac{1}{2} C v^2 = 0,5 (2 + \Delta)^2 \text{ mJ}$



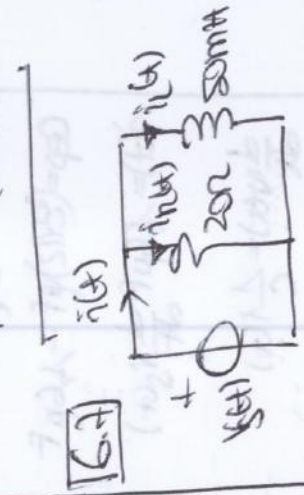
$i_R = i_C = i$   
 $v_C = N_C \dot{q} = N_C i$   
 $v_R = N_R \dot{q} = N_R i$   
 $v = v_C + v_R = (N_C + N_R) \dot{q}$   
 $i = \frac{v}{R + \frac{1}{C \omega}}$



$i_R = i_C = i$   
 $v_C = N_C \dot{q} = N_C i$   
 $v_R = N_R \dot{q} = N_R i$   
 $v = v_C + v_R = (N_C + N_R) \dot{q}$   
 $i = \frac{v}{R + \frac{1}{C \omega}}$

6.6

$L = 6 \text{ mH}$   
 $i(t) = \cos(100\pi t) \text{ A}$   
 $v(t) = 6 \cdot 100\pi \sin(100\pi t) \text{ mV} = -600\pi \sin(100\pi t) \text{ mV} = -0,6\pi \sin(100\pi t) \text{ V}$



$N_S(t) = \cos(\omega t) \text{ V}$   
 $i_L(t) = \Delta \text{ A}$   
 $i(t) = i_R(t) + i_L(t)$   
 $i_R(t) = \frac{v_s(t)}{20\Omega}$   
 $N_S(t) = 50 \text{ mH} \frac{di(t)}{dt}$   
 $\int_0^t v_s(t) dt = 50 \text{ mH} (i_L(t) - \Delta)$   
 $= \int_0^t \cos(\omega t) dt = \frac{\sin(\omega t)}{\omega}$   
 $i_L(t) = \Delta + \frac{\sin(\omega t)}{95} \text{ A}$

$i(t) = \frac{\cos(\omega t)}{20} + \Delta + \frac{\sin(\omega t)}{95}$   
 $= \Delta + 2 \sin(\omega t) + 0,5 \cos(\omega t)$

$L = 6 \text{ mH}$   
 $i(t) = \cos(100\pi t)$   
 $\Delta t = 20 \text{ mS} \rightarrow W(t)$   
 $W(t) = i(t)v(t) = i(t) L \frac{di(t)}{dt}$   
 $= 6 \text{ mH} (\cos(100\pi t) \cdot (-\sin(100\pi t)) \cdot 100) = -0,6 (\cos(100\pi t) \sin(100\pi t)) = -0,3 \sin(200\pi t)$   
 $p(t) = \frac{dW(t)}{dt} = 20\pi \cos(200\pi t)$   
 $W(t) = -0,3 \int_0^{20 \text{ mS}} \sin(200\pi t) dt = \frac{0,3}{200} \cos(200\pi t) \Big|_0^{20 \text{ mS}} = \frac{0,3}{200} [\cos(200\pi \cdot 0,02) - 1] = 0 (35,9 \mu\text{J})$



③  $C = 5 \cdot 10^{-6} F$

$i_C(t) = 1 - e^{-t}$   
 $i_C(t=0) = 1 - 0 = 1 \text{ mA}$

$$v_C(t) = \frac{1}{C} \int_0^t (1 - e^{-\tau}) d\tau$$

$$v_C(50 \mu s) = \frac{10^6}{5} \int_0^{50 \mu s} (1 - e^{-\tau}) d\tau =$$

$$= \frac{10^6}{5} \left[ \int_0^{50 \mu s} 1 d\tau + \int_0^{50 \mu s} (-e^{-\tau}) d\tau \right] =$$

$$= \frac{10^6}{5} \left[ 50 \mu s + \left[ e^{-\tau} \right]_0^{50 \mu s} \right] =$$

$$= \frac{10^6}{5} \left[ 50 \mu s + e^{-50 \mu s} - 1 \right] =$$

$$= \frac{10^6}{5} \left[ 5 \cdot 10^{-5} - 1 \right]$$

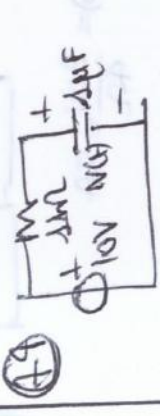
**NO**

④ TEMPI CARATTERISTICI:



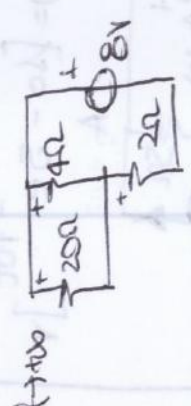
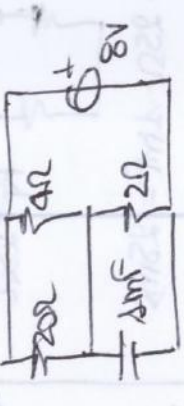
$v_C(0) = 4V$   
 $v_C(\infty) = 1V$   
 $v_C(t) = v_C(\infty) e^{-t/\tau} + v_C(0)$   
 $1V = 4V e^{-t/1000 \cdot 10^{-7}}$   
 $\frac{1}{4} = \exp\left(-\frac{t}{10^{-4}}\right)$   
 $\ln\left(\frac{1}{4}\right) = -\frac{t}{10^{-4}}$   
 $\tau = -10^{-4} \ln\left(\frac{1}{4}\right) = 0,108 \text{ ms}$

⑤



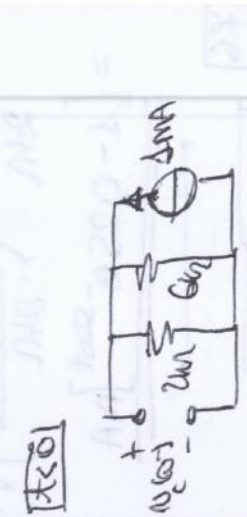
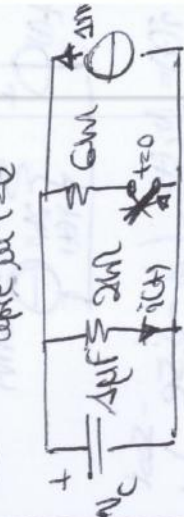
$v_C(t) = 1V e^{-t/\tau}$   
 $v_C(10^{-8}) = (10 - 1V) e^{-\frac{t}{\tau}} + 1V =$   
 $= -9V e^{-\frac{t}{10^{-8}}} + 10V =$   
 $= \dots - 9V + 10V = (10 - 9V) e^{-t/\tau} =$   
 $= 6,6899V$

⑥ Ricavare gli risposte numeriche:



~~$i_{20\Omega} = \frac{10V}{20\Omega} = 0,5 \text{ mA}$~~   
 ~~$i_{4\Omega} = \frac{10V}{4\Omega} = 2,5 \text{ mA}$~~   
 $i_{20\Omega} = \frac{8V \cdot 2\Omega}{\left(\frac{10}{3} + 2\right)\Omega} = 8V \cdot \frac{10/3}{\left(\frac{10}{3} + 2\right)} = 8V \cdot \frac{10/3}{10/3 + 6/3} = 8V \cdot \frac{10/3}{16/3} = 8V \cdot \frac{10}{16} = 5V$   
 $i_{4\Omega} = \frac{3}{4} A = 1,5 \text{ mA}$   
 $i_{20\Omega} = 0,25 \text{ A}$

⑦



$i_{10V}(t) = \frac{10V}{2\Omega} = 5 \text{ mA}$   
 $v_C(t) = v_C(\infty) = v_C(0) = 10V$   
 $v_C(t) = 10V e^{-t/\tau}$   
 $v_C(10^{-8}) = 10V e^{-\frac{t}{\tau}} = 9V$   
 $10V e^{-\frac{t}{\tau}} = 9V$   
 $e^{-\frac{t}{\tau}} = \frac{9}{10}$   
 $-\frac{t}{\tau} = \ln\left(\frac{9}{10}\right)$   
 $t = -\tau \ln\left(\frac{9}{10}\right) = -10^{-8} \ln\left(\frac{9}{10}\right) = 1,05 \cdot 10^{-8} \text{ s} = 10,5 \text{ ns}$

**NO**

$R_{Th} = 2\Omega + 2\Omega = 4\Omega$   
 $\tau = R_{Th} C = 4\Omega \cdot 10^{-8} \text{ F} = 4 \cdot 10^{-8} \text{ s} = 40 \text{ ns}$   
 $v_C(t) = (0,5 e^{-\frac{t}{40 \text{ ns}}} + 2) V$



$i_L(t) = 4A - e^{-12t} A$

$v_2 = 0V$

$30V - v_2 + \frac{30V - v_2}{10\Omega} = \frac{v_2}{30\Omega} + i_L(t)$

$30V - v_2 + 30V - 2v_2 = v_2 + 30i_L(t)$

$120V - 30i_L(t) = 5v_2$

$v_2 = 24V - 6i_L(t)$

KCL:  $i_L(t) = i_1(t) + \frac{v_2}{30\Omega} - \frac{30V - v_2}{30\Omega} =$

$= \frac{30\Omega i_1(t) + v_2 - 30V + 2v_2}{30\Omega} =$

$= \frac{-30\Omega i_1(t) - 30V + 48A + 12i_L(t)}{30\Omega}$

$= \frac{18A + 12i_L(t)}{30\Omega}$

$= \frac{1}{2} (4A + 4A e^{-12t}) =$

$= 4A e^{-12t}$

$3A - 0,6 e^{-10t} A$

17.10

Il circuito è instabile.

2A

20Ω

30V

20Ω

17.10

2A

20Ω

30V

20Ω

Il circuito è instabile.

2A

20Ω

30V

20Ω

$v_C(0^+) = 10V$

$\tau = RC = 2ms$

Dopo quanto tempo  $v_C$  è il 50% di  $v_C(0^+)$ ?

$0,5 v_C(0^+) = v_C(t) e^{-t/\tau}$

$\ln(0,5) = -t/\tau$

$t = -\tau \ln(0,5) = -2ms \ln(0,5) = 1,386ms$

Dopo quanto tempo  $v_C$  è il 50% di  $v_C(0^+)$ ?

$t = -\tau \ln(0,5) = 5,991ms$

$C = 10\mu F$  Dopo quanto tempo mettiamo R e manovri?

$R = 200k\Omega$

$v_C(0^+) = 12V$

$t = -\tau \ln(0,5) = 20,713s$

$= -3m(900) = 20,713s$

17.10

2A

20Ω

30V

20Ω

Il circuito è instabile.

2A

20Ω

30V

20Ω

$v_C(0^+) = 12V$

$\tau = RC = 2ms$

Dopo quanto tempo  $v_C$  è il 50% di  $v_C(0^+)$ ?

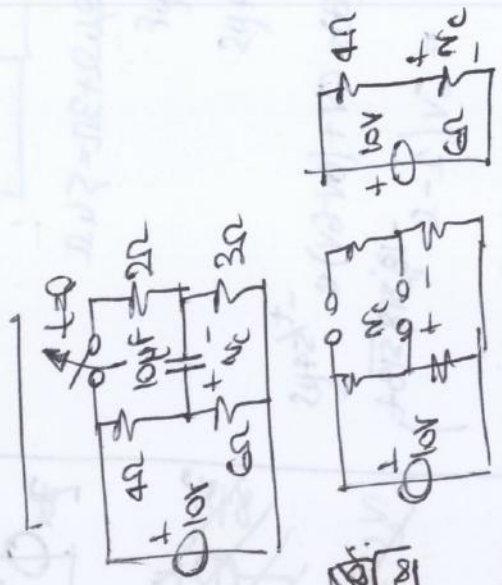
$0,5 v_C(0^+) = v_C(t) e^{-t/\tau}$

$\ln(0,5) = -t/\tau$

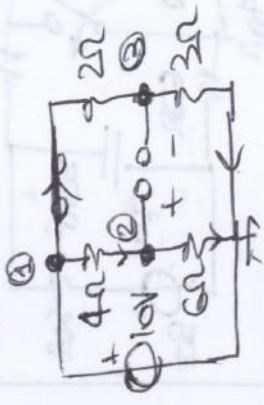
$t = -\tau \ln(0,5) = -2ms \ln(0,5) = 1,386ms$



$T = 45 \mu s \cdot \Delta \omega F = 1,5 \mu s$   
 $N(t) = 1,5 e^{-t/1,5 \mu s}$

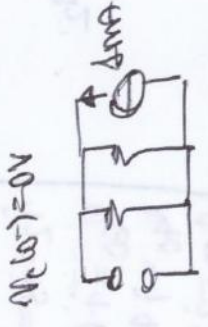
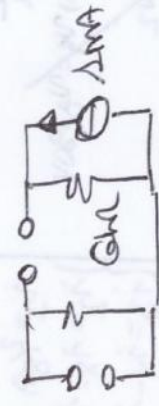
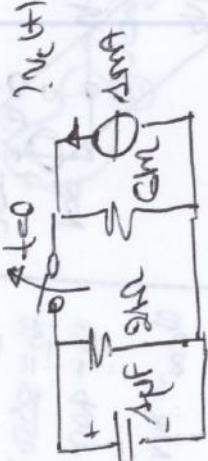
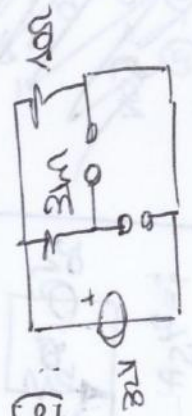
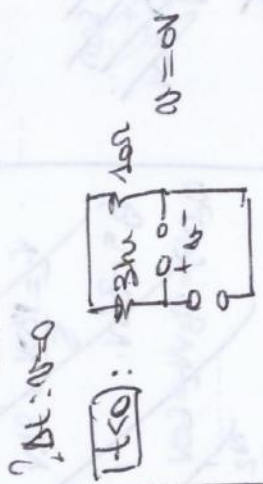
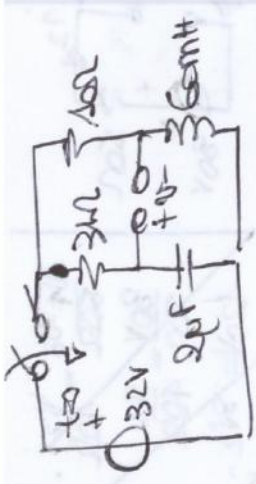


$N_1 = 10V \cdot 10\mu F = 10^{-4} V \cdot s$



$N_1 = 10V$   
 $N_2 = 10V \cdot 2\Omega = 20V \cdot \Omega$   
 $N_3 = 10V \cdot 3\Omega = 30V \cdot \Omega$   
 $N_4 = 10V \cdot 6V = 60V^2$

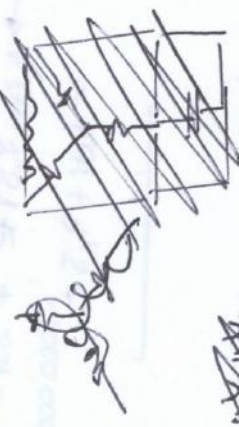
$N_0 = 8N_1 + 2N_2 = 100V \cdot \Omega$   
 $N_9 = 10V \cdot 5 = 50V \cdot \Omega$   
 $N_{10} = 8N_1 + 2N_2 = 100V \cdot \Omega$



$N_1 = 10V \cdot 2\mu F = 20 \mu V \cdot s$   
 $N_2 = 10V \cdot 2\mu F = 20 \mu V \cdot s$   
 $N_3 = 10V \cdot 2\mu F = 20 \mu V \cdot s$



$\frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$   
 $\frac{2}{9} \cdot \frac{1}{2} = \frac{1}{9}$   
 $\frac{1}{9} \cdot 1 = \frac{1}{9}$



$\frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$   
 $\frac{2}{9} \cdot \frac{1}{2} = \frac{1}{9}$   
 $\frac{1}{9} \cdot 1 = \frac{1}{9}$

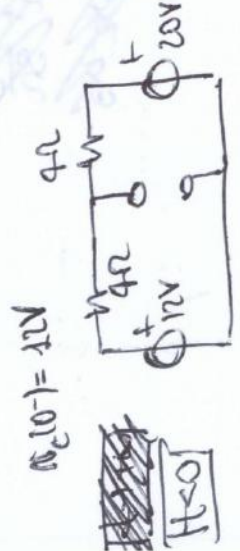
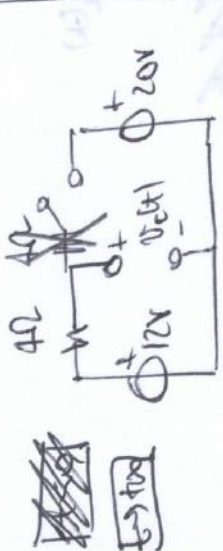
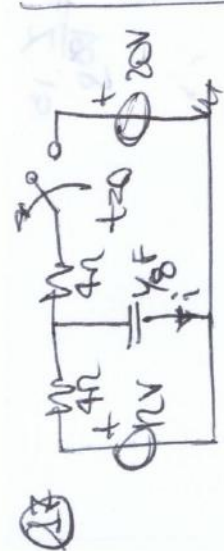


$\frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$   
 $\frac{2}{9} \cdot \frac{1}{2} = \frac{1}{9}$   
 $\frac{1}{9} \cdot 1 = \frac{1}{9}$

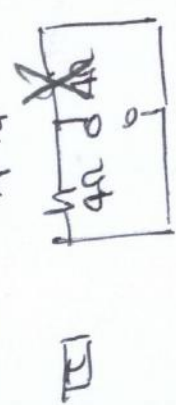
$\frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$   
 $\frac{2}{9} \cdot \frac{1}{2} = \frac{1}{9}$   
 $\frac{1}{9} \cdot 1 = \frac{1}{9}$

$5 \cdot 10 = 50$   
 $10 \cdot 1 = 10$   
 $10 \cdot 1 = 10$

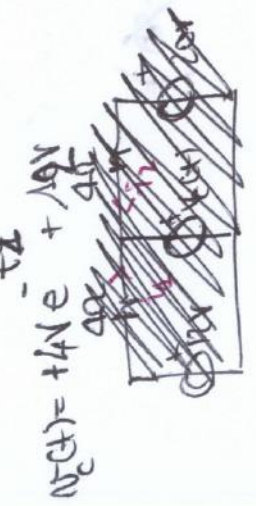




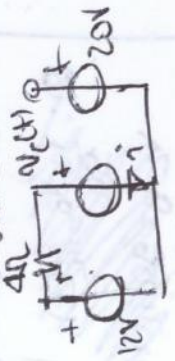
$N_C(\infty) = \frac{3A + 8A}{4 + 4} = 1.6V$



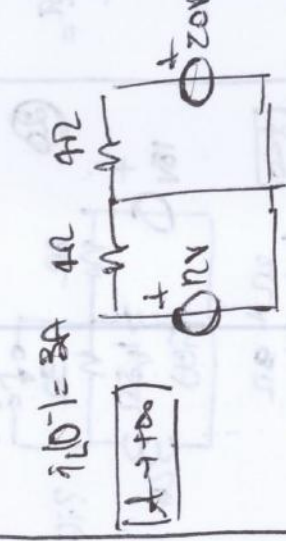
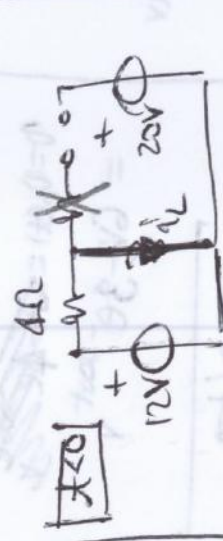
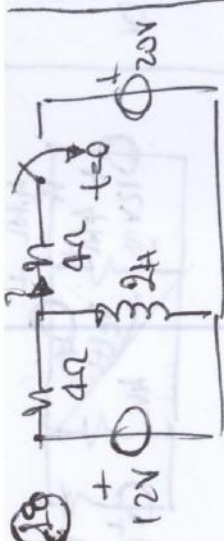
$\tau = 4\Omega \cdot \frac{1}{8} = \frac{1}{2}s = 0.5s$



~~$i_1 = 12V \cdot \frac{1}{4\Omega} = 3A$~~   
 ~~$i_2 = 20V \cdot \frac{1}{4\Omega} = 5A$~~   
 ~~$i = 3A + 5A = 8A$~~   
 ~~$N_C(t) = 8A \cdot 4\Omega = 32V$~~   
 ~~$N_C(t) = 12V - 16V e^{-t/0.5}$~~   
 ~~$N_C(t) = 12V - 16V e^{-2t}$~~   
 ~~$N_C(t) = 12V - 16V e^{-2t}$~~

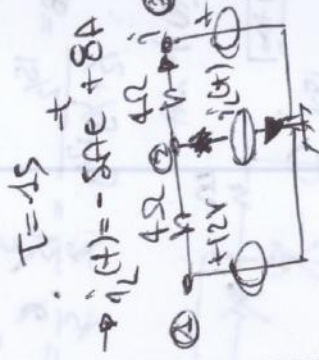


$i = \frac{12V - N_C(t)}{4\Omega} = \frac{12V - 4V e^{-2t} - 16V}{4\Omega} = -e^{-2t} A$



$i_L(\infty) = 3A + 5A = 8A$

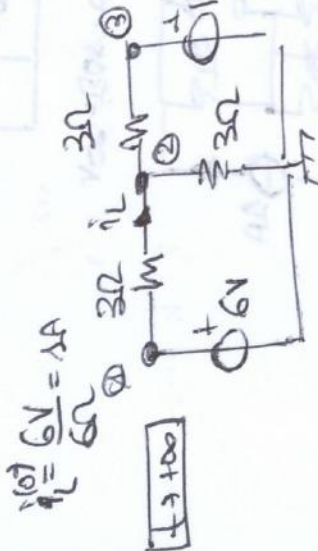
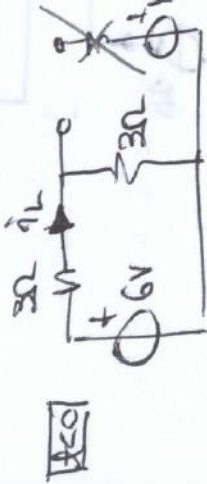
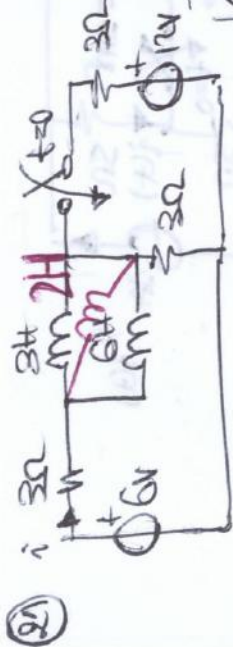
$\tau = 1s$



$N_C(t) = \frac{12V - 16V}{4\Omega} = \frac{12V - 16V}{4\Omega} = -1V$   
 $N_C(t) = 12V - 16V e^{-t/1} = 12V - 16V e^{-t}$

$2V \cdot 4A = 8V$   
 $16V \cdot 2A = 32V$   
 $N_C(t) = \frac{8V - 32V}{4\Omega} = -6V$   
 $N_C(t) = \frac{16V - 32V}{4\Omega} = -4V$   
 $N_C(t) = \frac{16V - 32V}{4\Omega} = -4V$   
 $N_C(t) = \frac{16V - 32V}{4\Omega} = -4V$   
 $N_C(t) = \frac{16V - 32V}{4\Omega} = -4V$   
 $N_C(t) = \frac{16V - 32V}{4\Omega} = -4V$



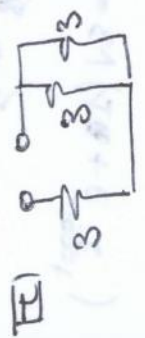


$i_L(0) = \frac{6V}{3\Omega} = 2A$

$12V = 2i_2$

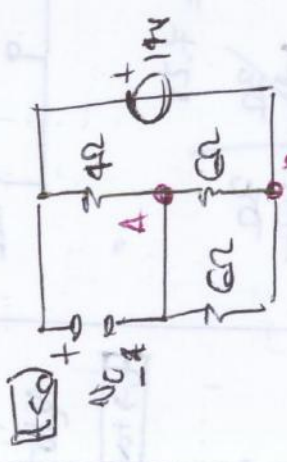
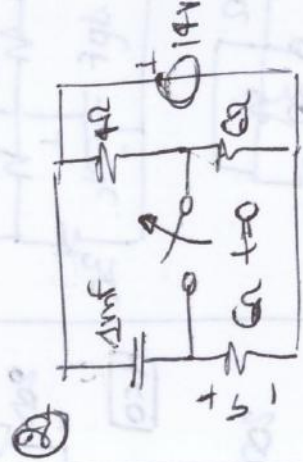
$i_2 = 6V$

$i_L(\infty) = \frac{6V - 6V}{3\Omega} = 0A$



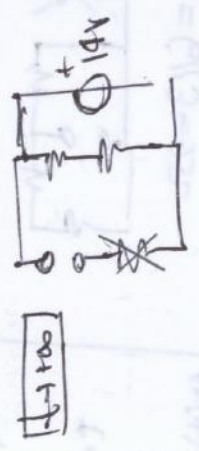
$\tau = \frac{L}{R} = \frac{2H}{3\Omega} = \frac{2}{3}s$

$i_L(t) = e^{-\frac{t}{\tau}} \cdot 2A = 2e^{-\frac{3}{2}t}$



$u_{AB}(0) = \frac{4V}{6+4} \cdot 6 = 2.4V$

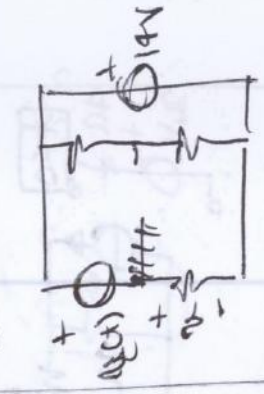
$u_{AB}(\infty) = 14V - 6V = 8V$



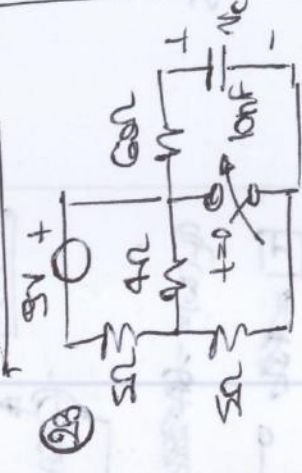
$u_{AB}(\infty) = 14V$



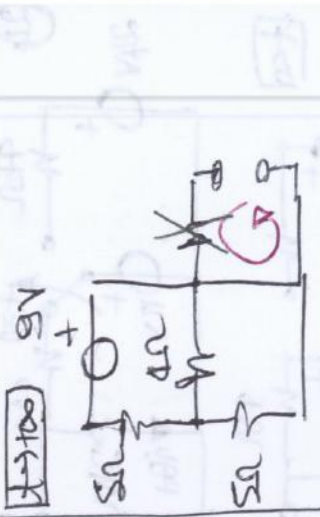
$R_T = 6\Omega$   
 $\tau = 6\Omega \cdot 10\mu F = 60\mu s$   
 $u_C(t) = -6Ve^{-\frac{t}{60\mu s}} + 14V$



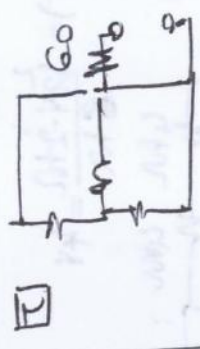
$u_C(t) = 14V - u_C(t) = 6Ve^{-\frac{t}{60\mu s}}$



$u_C(\infty) = \frac{9V \cdot 4\Omega}{4\Omega + 5\Omega} = 4.18V$



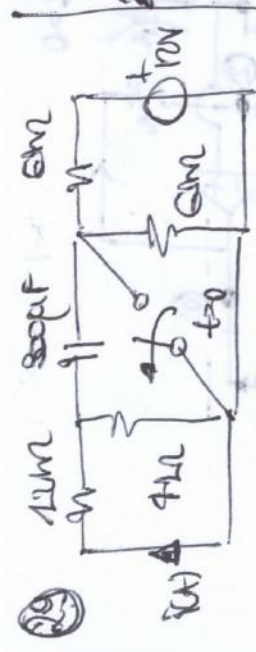
$u_C(\infty) = 0V$

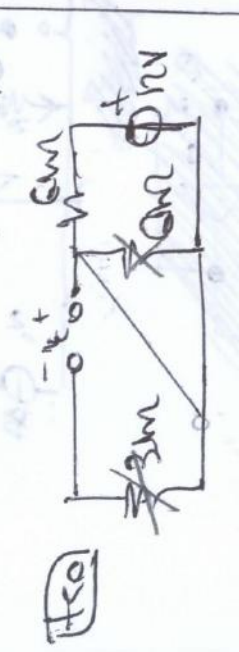


$[5(14) + 5] / (10 + 60) = 60\Omega = 1\mu s$   
 $\tau = 60\Omega \cdot 10\mu F = 6 \cdot 10^{-7}s = 0.6\mu s$

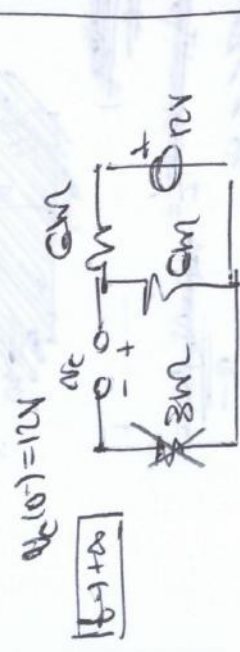
$u_C(t) = 4e^{-\frac{t}{0.6\mu s}}$



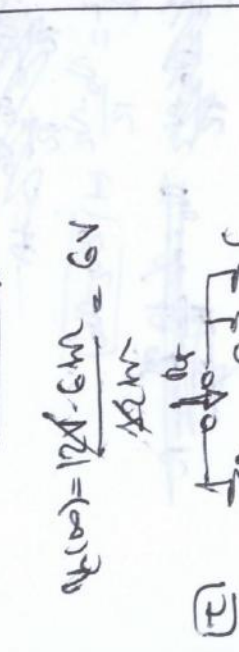
(29) 

(30) 

$v_c(t) = 12V$

(31) 

$v_c(t) = 12V \cdot 6\Omega = 6V$

(32) 

$R_T = (6||6) + 3\Omega = [3+3]\Omega = 6\Omega$

$I = 6V : 300\mu F = 4,8S$

$v_c(t) = 6V [e^{-t/180S} + 1] = 6V [e^{-\frac{st}{18}} + 1]$

$v = v_{AB} - v_c(t)$

$v_{AB} = \frac{v_c(t) + 12V}{3\Omega} + \frac{12V}{6\Omega} = \frac{3}{2} \left( \frac{v_c(t)}{3\Omega} + \frac{12V}{6\Omega} \right) = \frac{1}{2} v_c(t) + 6V$

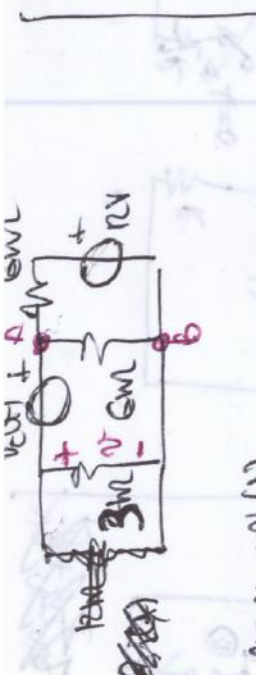
$= \frac{v_c(t)}{2} + 6V$

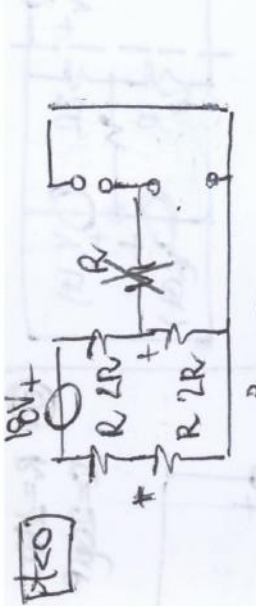
$v_c(t) = v(t) = \frac{3}{2} (6V - v_c(t))$

$v_c(t) = \frac{1}{2} (6V - v_c(t))$

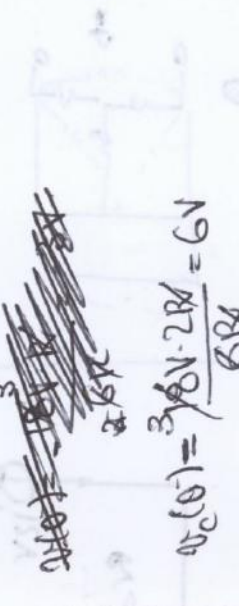
$= -\frac{1}{2} (6V e^{-st/9}) = 0,5 \text{mA} e^{-st/9}$

$= 0,25 \text{mA} e^{-st/9}$


(33) 

(34) 


$v_c(t) = \frac{10V \cdot 2R}{4R} = 6V$

(35) 

$v_c(t) = 0V$

(36) 

$v_c(t) = 0V$

(37) 

$2R||2R = R$

$v_c(t) = \frac{15V}{2R} = 7,5V$

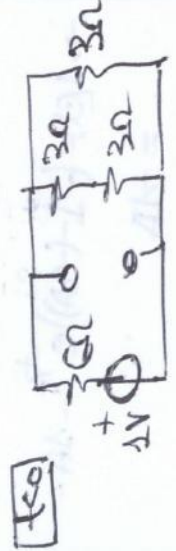
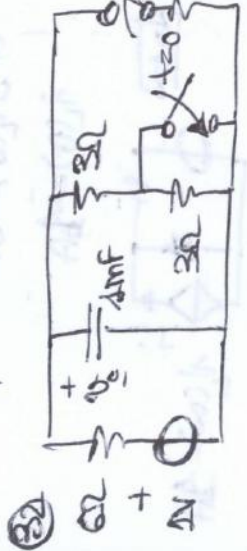
$\rightarrow v_c(t) = 6e^{-t/4CR} V$



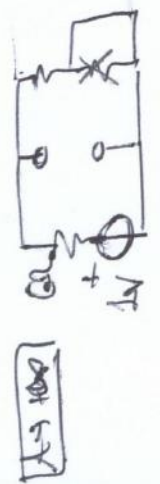


$$v_o(t) = v_o(0^+) + \frac{v_o - v_o(0^+)}{R} = 0$$

$$v_o(t) = v_o(0^+) = 8V e^{-sot}$$



$$v_o(\infty) = 1V \cdot \frac{30}{30+30} = 0.5V$$



$$v_o(\infty) = 1V \cdot \frac{30}{60} = 0.5V$$

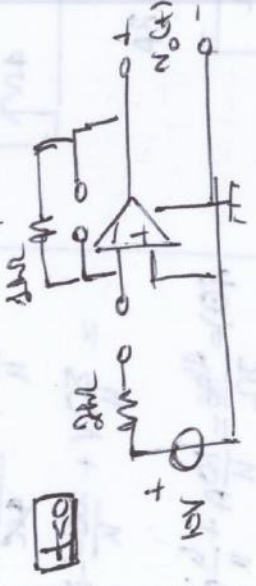
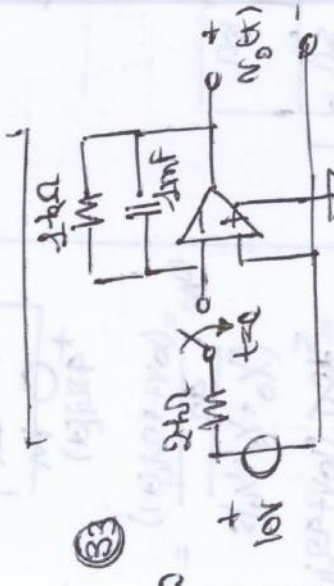


$$R_T = 2\Omega$$

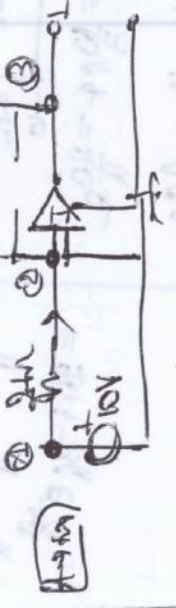
$$\tau = 2ms$$

$$v_o(t) = \left[ \frac{1}{2} e^{-\frac{t}{2ms}} + \frac{1}{3} \right] V$$

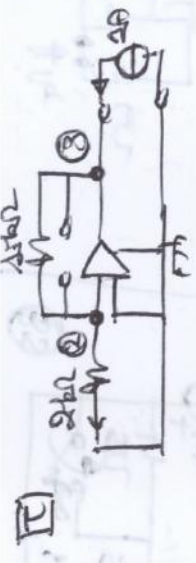
$$= \left[ \frac{1}{3} - \frac{1}{2} e^{-sot} \right] V$$



$$v_o(\infty) = 0V$$



$$v_o(\infty) = 10V \cdot \frac{0}{2+0} = 0V$$



$$\frac{0.2 - v_o(0^+)}{2k\Omega} = \frac{v_o(0^+)}{2k\Omega}$$

$$0.2 = 2v_o(0^+) - 2v_o(0^+) = 0$$

$$v_o(0^+) = 0.2V$$

$$v_o(0^+) = 2V$$

$$v_o(0^+) = 2V$$

$$v_o(0^+) = 2V$$

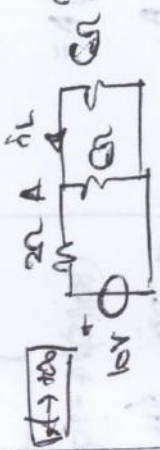
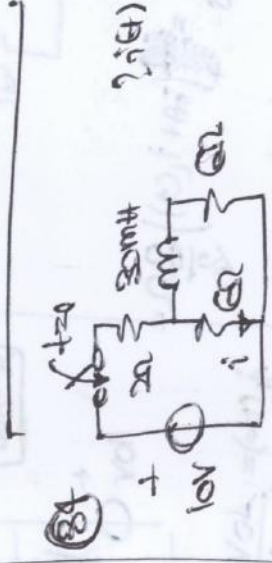
$$v_o(0^+) = 2V$$

$$v_o(0^+) = 2V$$

$$\tau = 2k\Omega \cdot 1\mu F = 2ms$$

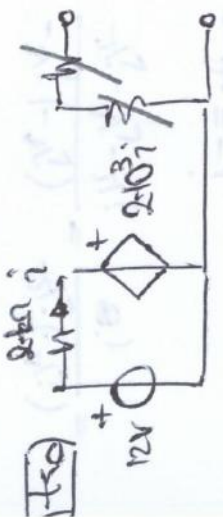
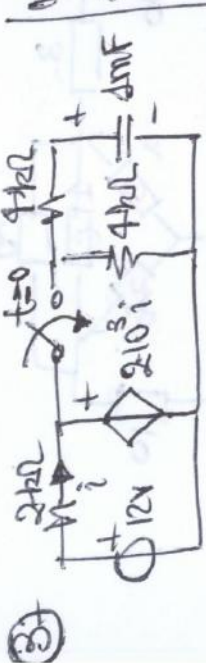
$$v_o(t) = [0.2 + v_o(0^+) - 0.2] e^{-\frac{t}{\tau}}$$

$$= 2V (e^{-\frac{t}{2ms}} - 1)$$

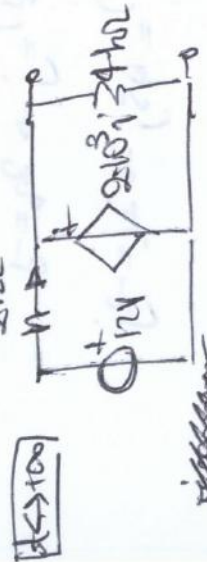


$$v_o(\infty) = -6V / 1.5 = -4A$$

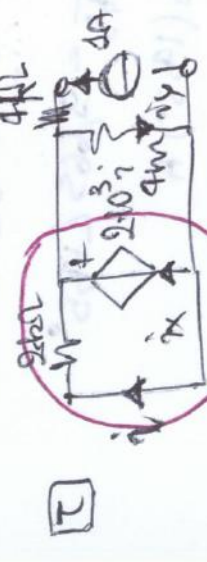




$v_c(t) = 8V$

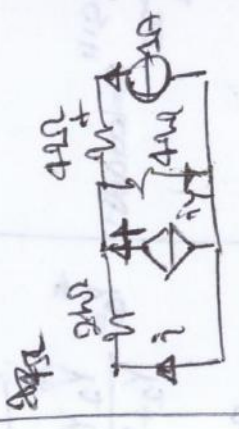


$v_c(\infty) = \frac{12V \cdot 4k\Omega}{6k\Omega} = 8V$



~~$v_c(t) = 8V + (12V - 8V)e^{-t/\tau}$~~   
 ~~$\tau = \frac{4k\Omega \cdot 4mF}{2}$~~   
 ~~$\tau = 8ms$~~   
 ~~$v_c(0) = 12V$~~   
 ~~$v_c(\infty) = 8V$~~

$v_{AB} = 210^3 i$   
 $i = \frac{210^3 i}{4k\Omega} = \frac{1}{2} i$

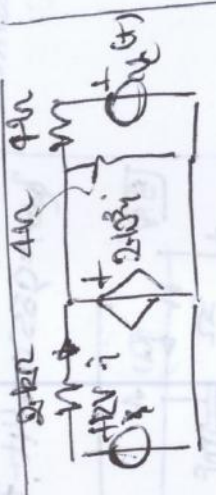


$v_{4k} = 4kN$   
 $v_T = 210^3 i + 4kN =$   
 ~~$= \frac{210^3 \cdot 4kN}{4k\Omega} + 4kN =$~~   
 ~~$= \frac{210^3 \cdot 4kN}{4k\Omega} + 4kN$~~



$v_T = 210^3 (4kN - 4kN) + 4kN =$   
 $= \frac{210^3 \cdot 4kN}{4k\Omega} + 4kN$   
 $= \frac{1}{2} v_T - 2kN + 4kN$   
 $\frac{1}{2} v_T = 2kN$   
 $v_T = 4kN$   
 $\tau = 4s$

$v_{CE}(t) = 8V (1 - e^{-t/4})$

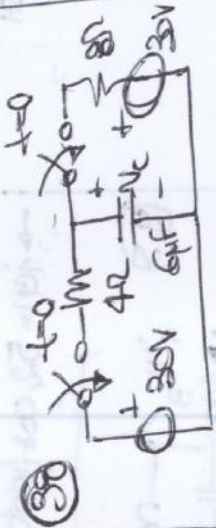


$v_{AB} = \frac{12V}{4k\Omega + 210\Omega} = 210^3 i$   
 $1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{210}$

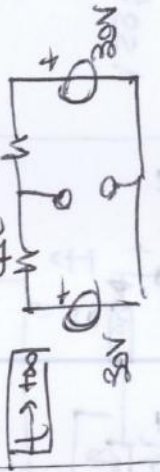
$v_{CE}(t) + 2kN = 8 \cdot 10^6 i(t)$

$i(t) = \frac{v_{CE}(t) + 2kN}{8 \cdot 10^6 \Omega} = [- \dots]$

~~$v_{CE}(t) = 8V$~~   
 NO RUCCHIENO



$v_{CE}(t) = 1kV$



$v_{CE}(t) = \frac{30V + 30V}{4 + 4} = 30V$



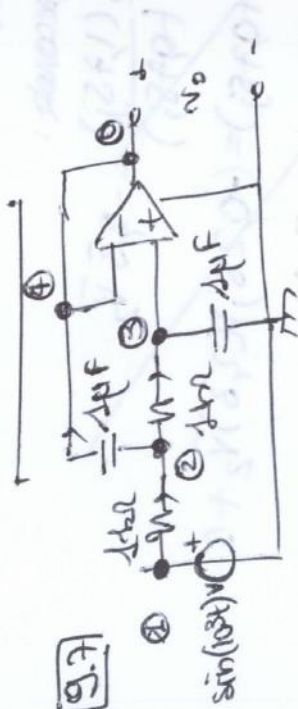
9.6)



$$\hat{V} = \frac{30}{3-j} \cdot \frac{-2+10}{\frac{1}{3j} + 1} = 10,918 \angle 44,703^\circ$$

$$|V_1(t)| = 10,918 \cos(44,703^\circ)$$

9.7)



$$\hat{V}_2 = \hat{V}, \hat{R} = 1k\Omega, \hat{C} = 4\mu F; \hat{V}_3 = \hat{V}_4 = \hat{V}_b$$

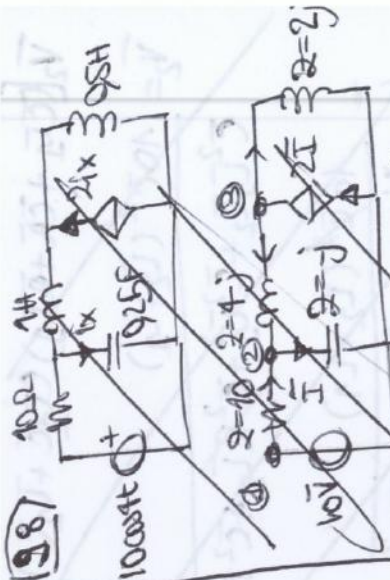
$$\hat{V}_1 \hat{C} - \hat{V}_2 = \hat{V}_2 \hat{R} - \hat{V}_4 \hat{R} + \hat{V}_2 \hat{C} - \hat{V}_3 \hat{C}$$

$$\hat{V}_1 \hat{C} - \hat{V}_2 \hat{C} = \hat{V}_2 \hat{R} - \hat{V}_4 \hat{R} + \hat{V}_2 \hat{C} - \hat{V}_3 \hat{C}$$

$$\hat{V}_1 \hat{C} = \hat{V}_2 (2\hat{C} + \hat{R}) - \hat{V}_3 (\hat{R} + \hat{C})$$

$$\hat{C} = \frac{-\hat{V}_1}{10^3} = -10^3 j = -1000j$$

9.8)



$$\hat{R} = 10, \hat{C} = j$$

$$\frac{10 - \hat{V}_2}{\hat{R}} + \frac{\hat{V}_3 - \hat{V}_2}{\hat{C}} = \frac{\hat{V}_2}{2\hat{C}}$$

$$\hat{V}_3 - \hat{V}_2 = 2\hat{I} = 2\frac{\hat{V}_2}{\hat{C}}$$

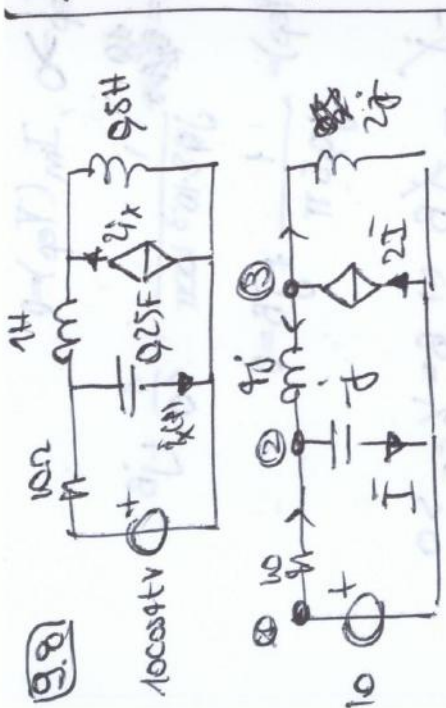
$$100\hat{C}\hat{V}_1 - \hat{V}_2\hat{C}\hat{R} + \hat{V}_3\hat{R}\hat{C} - \hat{V}_2\hat{R}\hat{C} = \hat{V}_2$$

$$\hat{V}_3\hat{R}\hat{C} - \hat{V}_2\hat{C}\hat{R} + \hat{V}_3\hat{R}\hat{C} = 2\hat{V}_2\hat{R}\hat{C}$$

$$\hat{V}_3(\hat{R}\hat{C} + \hat{R}\hat{C}) = \hat{V}_2(2\hat{R}\hat{C} + \hat{R}\hat{C})$$

$$\hat{V}_3(2\hat{R}\hat{C} + \hat{R}\hat{C}) = \hat{V}_2(2\hat{R}\hat{C} + \hat{R}\hat{C})$$





9.8

$$\textcircled{1}: \frac{10 - \bar{V}_2 + \bar{V}_3 - \bar{V}_2}{10} = \frac{\bar{V}_2}{2}$$

$$1 - 0.25j\bar{V}_3 + 0.25j\bar{V}_2 = \bar{V}_2j$$

$$1 - \frac{\bar{V}_2}{10} + \frac{\bar{V}_3}{4} + \frac{\bar{V}_2j}{4} = \bar{V}_2j$$

$$1 - \frac{\bar{V}_3j}{4} = \bar{V}_2(j - j/4 + j/10) = \bar{V}_2(0.1 + 0.75j) = u_{800}$$

$$(1 - 0.25j\bar{V}_3j = \bar{V}_2(0.1 + 0.75j)) = u_{800}$$

$$\textcircled{2}: \frac{\bar{V}_3 - \bar{V}_2}{4j} + \frac{\bar{V}_3}{2j} = 2\frac{\bar{V}_2}{2}$$

$$-\frac{\bar{V}_3j}{4} + \frac{\bar{V}_2j}{4} - \frac{\bar{V}_3j}{2} = 2\bar{V}_2j$$

$$\bar{V}_3(-j/4 - j/2) = \bar{V}_2(2j - j/4)$$

$$\bar{V}_3 = \bar{V}_2 \frac{-(7/4)}{3} = u_{800}$$

$$1 - 0.583j\bar{V}_2 = \bar{V}_2(0.1 + 0.75j)$$

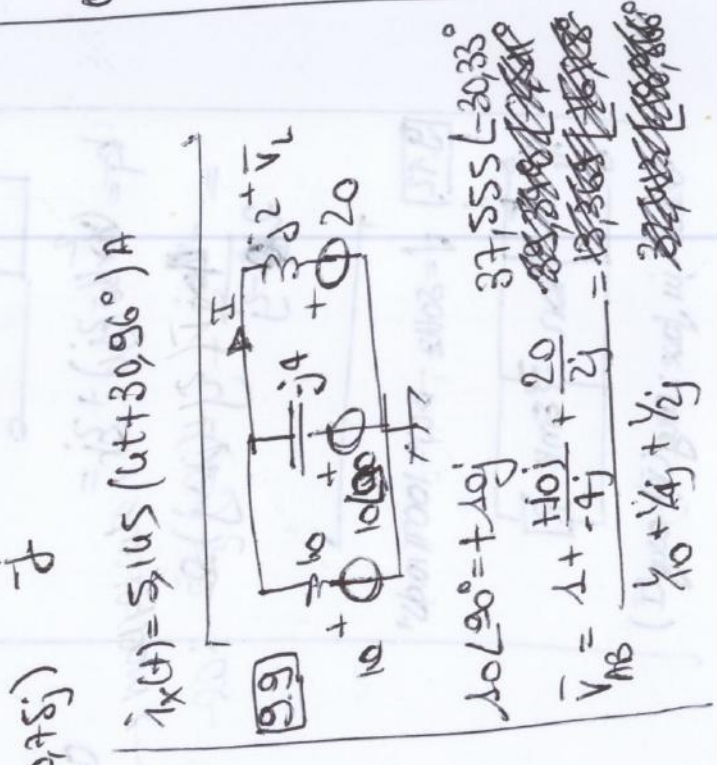
$$\bar{V}_2 = \frac{1}{0.417 - 0.583j}$$

$$1 - 0.25j(-\bar{V}_3)\bar{V}_2 = \bar{V}_2(0.1 + 0.75j)$$

$$1 + \frac{7}{12}j\bar{V}_2 = \bar{V}_2(0.1 + 0.75j)$$

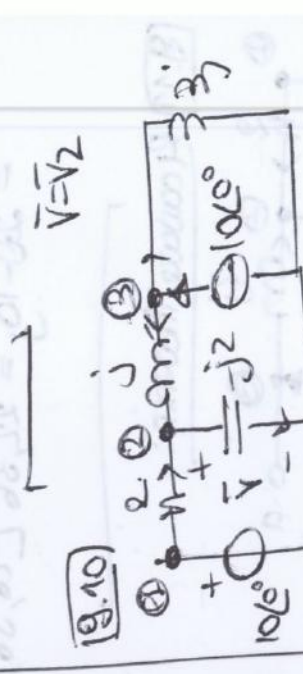
$$\bar{V}_2 = \frac{1}{0.417 - 0.583j - 0.71j} = 5.165 \angle -59.036^\circ$$

$$\bar{I} = \bar{V}_2 = 5.165 \angle 30.963^\circ$$



$$\bar{V}_L = \bar{V}_{AB} - 20 = 22.667 \angle -56.733^\circ$$

$$\bar{I} = \frac{\bar{V}_L}{2j} = 11.333 \angle -146.73^\circ$$



$$\textcircled{1}: \frac{10 - \bar{V}_2 + \bar{V}_3 - \bar{V}_2}{2} = \frac{\bar{V}_2}{-j}$$

$$8 - 0.5\bar{V}_2 + j\bar{V}_3 + j\bar{V}_2 = 0.5j\bar{V}_2$$

$$8 + \bar{V}_2(-0.5 + 0.5j) = j\bar{V}_3$$

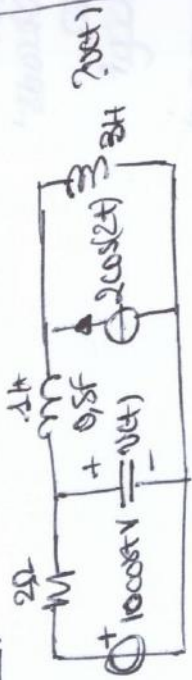
$$\textcircled{2}: \frac{\bar{V}_3 - \bar{V}_2 + \bar{V}_3}{j} = 10$$

$$-\bar{V}_3j + \bar{V}_2j - \bar{V}_3j = 10$$

$$\bar{V}_3 = \frac{(10 - \bar{V}_2j)j}{-j} = 7.5j + 9.75\bar{V}_2$$

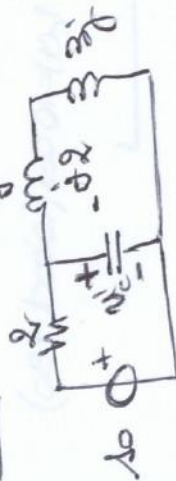


9.18)



CAPAC

PER 1:  $\omega = 2 \text{ rad/s}$



$$\vec{V}'_{AB} = \frac{10}{2} = 5 - 4j = 8,944 \angle 26,5^\circ$$

PER 2:  $\omega = 2 \text{ rad/s}$



$$\vec{V}''_{AB} + \vec{V}''_C = \frac{\sqrt{2} \vec{V}_C}{2}$$

$$0,5 \vec{V}''_C + j \vec{V}''_C = j 0,5 \vec{V}''_C - 0,5 j \vec{V}''_C$$

$$\vec{V}''_C = \frac{(0,5j - j - 0,5 \vec{V}''_C) (-1 + j) \vec{V}''_C}{0,5j}$$

$$\textcircled{2}: \frac{\vec{V} - \vec{V}''_C}{R_1} + \frac{\vec{V}''_C}{R_2} = 2$$

$$-0,5 j \vec{V}''_C + 0,5 j \vec{V}''_C - j (46) \vec{V}''_C = 2$$

$$-10,5 j + 46 j \vec{V}''_C - 2 = -0,5 j \vec{V}''_C$$

~~$$(10,5j + 46j) \vec{V}''_C - 2 = -0,5j \vec{V}''_C$$~~

~~$$(56,5j) \vec{V}''_C - 2 = -0,5j \vec{V}''_C$$~~

~~$$\vec{V}''_C = \frac{2}{57j} = -0,035 \angle -90^\circ$$~~

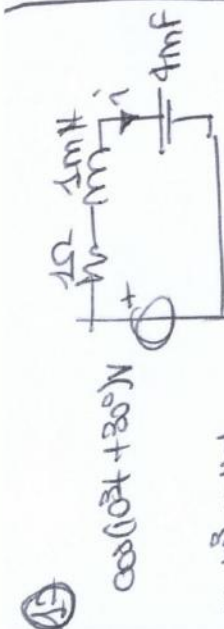
$$-10,5j + 46j (-1 + j) \vec{V}''_C - 2 = -0,5j \vec{V}''_C$$

$$(23 + 43j) \vec{V}''_C + 0,5j \vec{V}''_C = 2$$

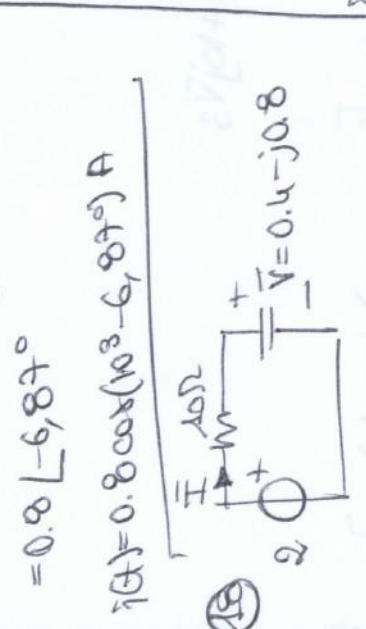
$$\vec{V}''_C = 1,49 \angle -60,3^\circ$$

$$\rightarrow \vec{V}''_C = 8,944 \cos(t - 26,5^\circ) + 1,49 \cos(t - 60,3^\circ)$$

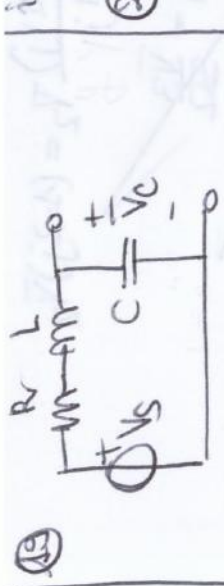




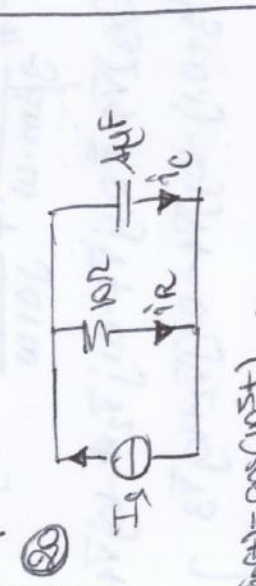
$\omega = 10^3 \text{ rad/s}$   
 $\bar{V} = 1 \angle 30^\circ \text{ V}$   
 $\bar{R} = 1 \Omega$   
 $\bar{L} = 10^3 \cdot 10^{-3} j \Omega = j \Omega$   
 $\bar{C} = (4 \cdot 10^{-6} \cdot 10^3) j \Omega = 4 j \Omega - 0.25 j \Omega$   
 $\bar{I} = \frac{\bar{V}}{\bar{R} + j\omega L + \frac{1}{j\omega C}} = \frac{1 \angle 30^\circ}{1 + j + 0.75 j} = 0.8 \angle -6.87^\circ$



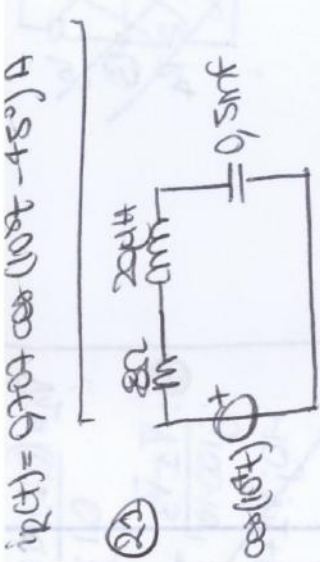
$\bar{I}_s = 0.8 \cos(10^3 t - 6.87^\circ) \text{ A}$   
 $\bar{I} = \frac{2 - \bar{V}}{10 \Omega} = \frac{2 - 0.4 + j0.8}{10} = 0.16 + j0.08 \text{ A}$



$\omega = 10^5 \text{ rad/s}$   
 $L = 0.1 \text{ mH} = 10^{-4} \text{ H}$   
 $C = 1 \mu\text{F} = 10^{-6} \text{ F}$   
 $R = 10 \Omega$   
 $\bar{L} = j10j$   
 $\bar{C} = \frac{1}{j10^5} = -j10j$   
 $\bar{V}_C = \frac{\bar{V}_S (10j)}{10} = -j\bar{V}_S$



$\bar{I}_S = 1 \text{ A}$   
 $\omega = 10^5 \text{ rad/s}$   
 $\bar{C} = \frac{1}{j10^5 \cdot 10^{-6}} = -j10j \Omega$   
 $\bar{I}_C = \frac{1}{10 \Omega + (-j10j \Omega)} = 0.707 \angle 45^\circ$   
 $\bar{I}_C(t) = 0.707 \cos(10^5 t + 45^\circ) \text{ A}$   
 $\bar{I}_R = \frac{\bar{V}_C}{10 \Omega} = 0.707 \angle 45^\circ$



$\omega = 10^4 \text{ rad/s}$   
 $\bar{R} = 8 \Omega$   
 $\bar{V} = 1 \text{ V}$   
 $\bar{L} = j10^4 \cdot 20 \cdot 10^{-6} = +j2j \Omega$   
 $\bar{C} = \frac{1}{j0.5 \cdot 10^3 \cdot 10^{-6}} = -j2j \Omega$   
 $\bar{V}_R = \frac{\bar{V} \bar{R}}{\bar{R} + j\omega L + \frac{1}{j\omega C}} = \frac{1 \text{ V} \cdot 8 \Omega}{8 \Omega} = 1 \text{ V}$   
 $\bar{V}_L = \frac{\bar{V} \bar{L}}{\bar{R} + j\omega L + \frac{1}{j\omega C}} = \frac{1 \text{ V} \cdot j2j \Omega}{8 \Omega} = 0.025 j \text{ V}$   
 $\bar{V}_C = \frac{\bar{V} \bar{C}}{\bar{R} + j\omega L + \frac{1}{j\omega C}} = \frac{1 \text{ V} \cdot (-j2j \Omega)}{8 \Omega} = -0.025 j \text{ V}$



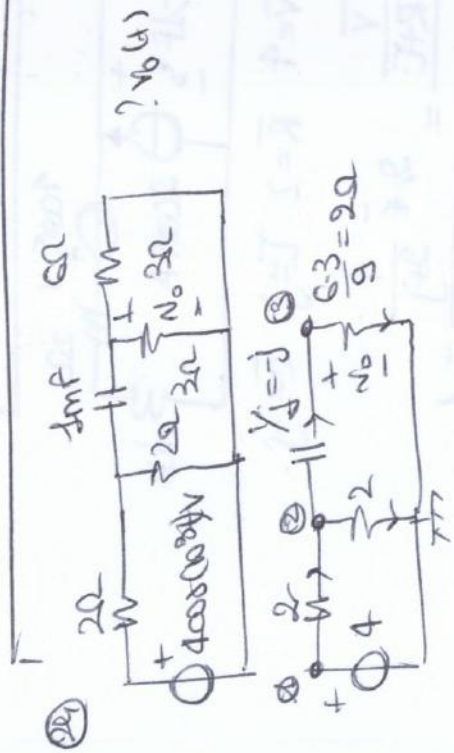
$$Y_{3N} = [(Y_{3a} - g_3) + j0.2] + j0.1 + \frac{(0.2 + j0.1)(0.1 - j0.1)}{(0.2 + j0.1)(-j0.1)}$$

$$Y_{3N} = 0.569689 - j0.0871539 = 0.576 \angle -8.6979^\circ$$

$$V_2 = \frac{(0.1j)(0.2 + j0.1)}{(0.2 + j0.1)(-j0.1)} F_3 = 0.2578 \angle 55.883^\circ$$

$$V = V_2 V_3 = 0.5206 \angle 114.737^\circ$$

→ Staggerato, fuculo.



$$V_1 = 4V$$

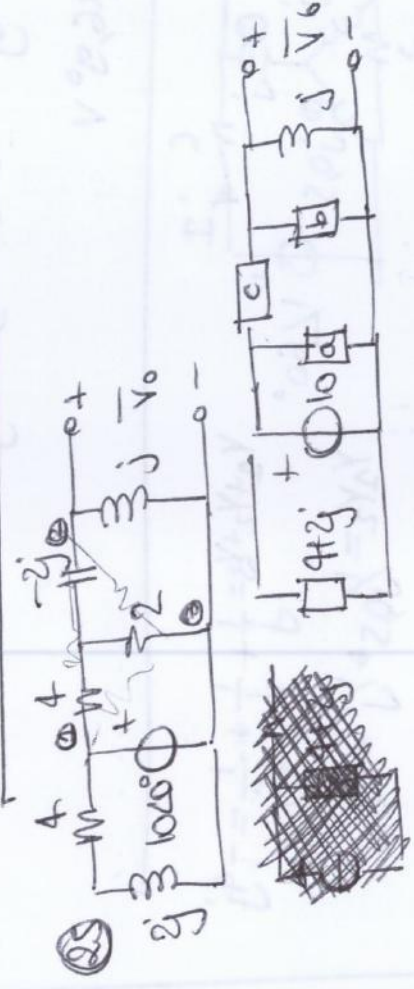
$$\textcircled{1} \frac{4V \cdot V_2}{2\Omega} = \frac{V_2}{2\Omega} + \frac{V_2 \cdot V_3}{2\Omega} \Leftrightarrow \frac{4}{2} - 0.5V_2 = 0.5V_2 + jV_2 - jV_3$$

$$\textcircled{2} \frac{V_2 \cdot V_3}{j} = \frac{V_3}{2\Omega} \Leftrightarrow jV_2 jV_3 = 0.5V_3$$

$$\begin{cases} \bar{I}_4 = (1+j)\bar{V}_2 - j\bar{V}_3 \\ j\bar{V}_2 = 0.5j\bar{V}_3 \\ \bar{I}_4 = (1.5 + 0.5j)\bar{V}_2 - j\bar{V}_3 \\ \bar{V}_2 = (1 - 0.5j)\bar{V}_3 \end{cases}$$

$$\bar{V}_3 = \frac{42}{(1.5 - 0.5j)} = 42 + j0.8 = \frac{42.65}{1.265} \angle 18.4^\circ V$$

$$N_0(t) = 42.65 \cos(10^3 t + 18.4^\circ) V$$



Staggerato:

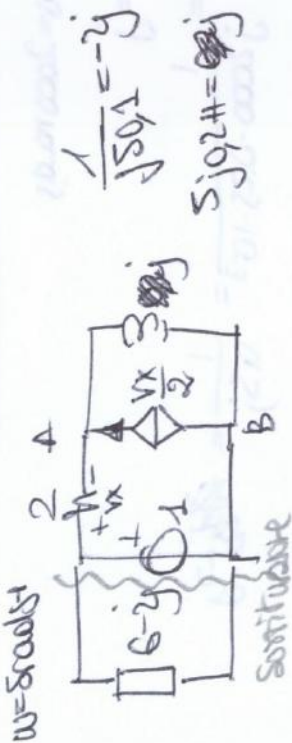
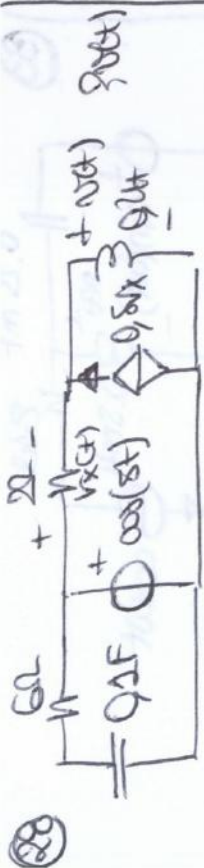
$$Y_1 + Y_2 + Y_3 = \frac{1}{4} + \frac{1}{2} - \frac{1}{2} = \frac{3}{4} + jY_2$$

$$Y_1 Y_3 = \frac{1}{4} \cdot Y_2 = Y_8 \Rightarrow Y_8 = \frac{Y_2}{3/4 + jY_2}$$

$$Y_1 Y_2 = \frac{1}{4} \cdot jY_2 = Y_0 j \Rightarrow Y_0 = \frac{jY_2}{3/4 + jY_2}$$

$$Y_2 Y_3 = Y_2 \cdot jY_2 = Y_4 j \Rightarrow Y_4 = \frac{jY_2}{3/4 + jY_2}$$





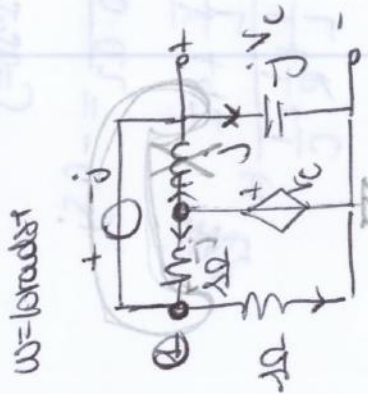
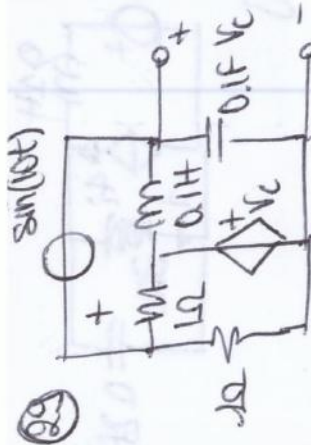
$$\bar{V} = \bar{V}_{AB} = \frac{1 - \frac{V_{AB}}{2}}{\frac{1}{2} + \frac{1}{j}} = \frac{1 - \frac{V_{AB}}{2}}{\frac{1}{2} + \frac{1}{j}}$$

$$\Rightarrow \left(\frac{1}{2} + \frac{1}{j}\right) \bar{V}_{AB} = 1 - \frac{V_{AB}}{2}$$

$$\bar{V}_{AB} \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{j}\right) = 1$$

$$\bar{V}_{AB} = \frac{1}{1 + \frac{1}{j}} = 0.5 \angle 45^\circ = 0.707 \angle 45^\circ$$

$$v_{AB}(t) = 0.707 \cos(5t + 45^\circ) \text{ V}$$



$$\bar{V}_c - j = \bar{V}_1$$

$$\frac{\bar{V}_1 + \bar{V}_c}{1\Omega} = \frac{\bar{V}_c - \bar{V}_1}{1\Omega}$$

$$\bar{V}_1 + \bar{V}_c = \bar{V}_c - \bar{V}_1$$

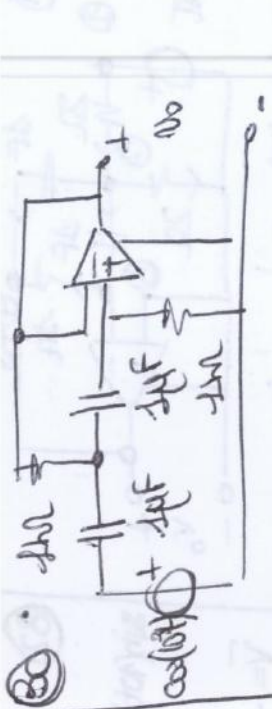
$$\sqrt{2} \bar{V}_c = \sqrt{2} (\angle 45^\circ)$$

$$2(\bar{V}_c j) = \bar{V}_c (\angle 45^\circ)$$

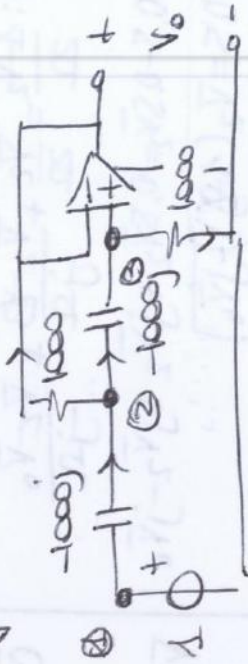
$$-2j = \bar{V}_c (\angle 45^\circ - 2)$$

$$\bar{V}_c = \frac{-2}{-j-1} = +1 + j = \sqrt{2} \cos(45^\circ)$$

$$\Rightarrow v_{AB}(t) = \sqrt{2} \cos(10t + 45^\circ) \text{ V}$$



$$\frac{1}{j10^3 \cdot 10^{-6}} = -1000j$$



$$\bar{V}_0 = \bar{V}_0$$

$$\frac{1 - \bar{V}_0}{-1000j} = \frac{\bar{V}_0 - \bar{V}_3}{1000}$$

$$j - j\bar{V}_0 = j\bar{V}_0 - j\bar{V}_3 + \bar{V}_0 - \bar{V}_3$$

$$j = (1 + 2j)\bar{V}_0 - \bar{V}_3 (1 + j)$$

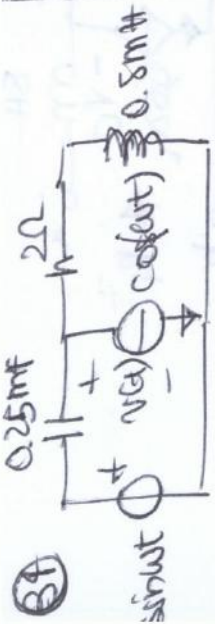
$$\frac{\bar{V}_0 - \bar{V}_3}{-1000j} = \frac{\bar{V}_3}{1000} \Rightarrow j\bar{V}_0 - j\bar{V}_3 = \bar{V}_3$$

$$\Rightarrow j\bar{V}_0 = \bar{V}_3 (1 + j) \Rightarrow \bar{V}_0 = (1 + j)\bar{V}_3$$

$$j = [(1 + j)(1 + j) - (1 + j)] \bar{V}_3$$

$$\Rightarrow \bar{V}_3 = 0.5j + 0.5(1) = \frac{1}{2} \cos(10^3 t + 90^\circ)$$





$\omega = 2000 \text{ rads}^{-1}$

$\bar{V} = j$   $\bar{R} = 2$

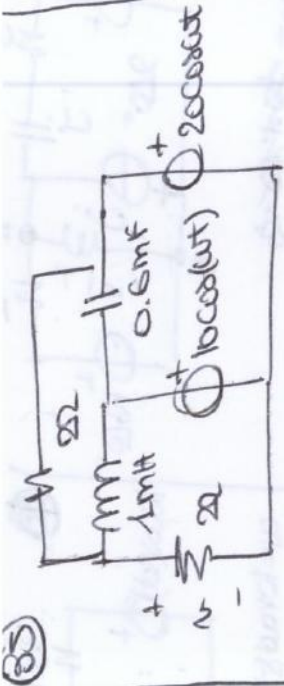
$\bar{I} = 1$   $\bar{L} = 0.5 \cdot 10^{-3} \cdot 2000j = j$

$\bar{C} = \frac{1}{0.25 \cdot 2000j \cdot 10^{-3}} = \frac{1}{0.5j} = -2j$

$\bar{V}_{AB} = \frac{\bar{V}_C - \bar{I}}{\frac{1}{\bar{C}} + \frac{1}{\bar{I}} + \bar{R}} = \frac{j - 1}{-2j + 1 + 2} = \frac{j - 1}{1 - 2j}$

$= \frac{-j_2}{-1 + j} = 1 \angle 143^\circ$

$v(t) = \cos(2000t + 143^\circ)$

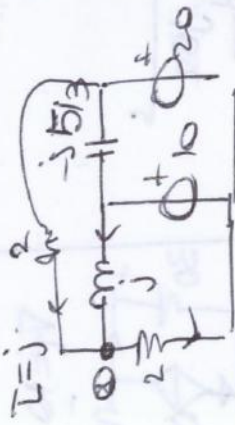


$\omega = 1000 \text{ rads}^{-1}$

$\bar{V}_1 = 10$

$\bar{V}_2 = 20$

$\bar{C} = \frac{1}{j0.5 \cdot 10^{-3} \cdot 10^3} = -j5/3$



$\bar{V} = \frac{10 - \bar{V}}{j\Omega} + \frac{20 - \bar{V}}{2\Omega}$

$0.5\bar{V} = -10j + j\bar{V} + 10 - 0.5\bar{V}$

$\bar{V}(1-j) = 10(1-j)$

$\bar{V} = 10$

$\rightarrow v(t) = 10 \cos(10^3 t) \text{ V}$



$2 \cos(10t) \rightarrow 2$

$-10 \sin(10t) \rightarrow j$

$Y_{S1} \rightarrow Y_{S1} \cdot 10 = 2j$

$Y_{10} \rightarrow \frac{10}{j10} = -j$

$2\bar{V}_0 + \frac{j - \bar{V}_0}{j} = 0$

$-j + 0.5\bar{V}_0 + 1 - j\bar{V}_0 = 0$

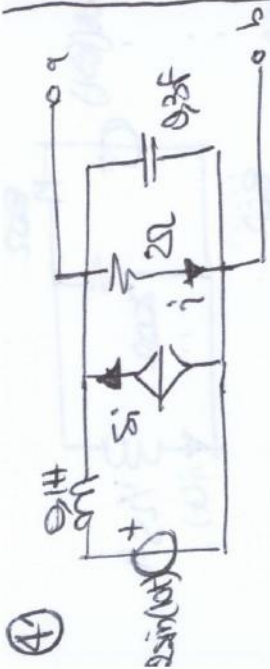
$-1j = j - 0.5j\bar{V}_0$

$\bar{V}_0 = \frac{-1j}{j - 0.5j} = \frac{-1j}{0.5j} = -2$

$= 2 \angle 180^\circ$

$\rightarrow v_0(t) = 2 \cos(10t + 180^\circ) \text{ V}$





$\omega = 10^3 \text{ rad/s}$

$\bar{V} = 2 - 2j$

$\bar{C} = 91 \cdot 10^{-6} j = j$

$\bar{C} = \frac{1}{93 \cdot 10^3} = -j/3$

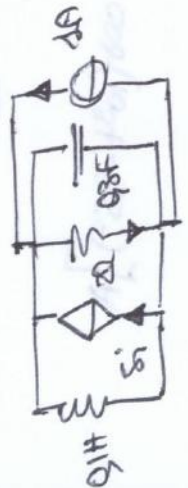
$\bar{V}_{AB} = \bar{V}_A = \frac{-2j + 5\bar{I}}{1 + \frac{1}{2} - \frac{3}{j}}$

$(\frac{1}{2} - \frac{2}{j}) \bar{V}_{AB} = -\frac{2j}{j} + \frac{5\bar{V}_{AB}}{20}$

$(\frac{1}{2} - \frac{2}{j}) \bar{V}_{AB} = \frac{-2j}{j} + \frac{5\bar{V}_{AB}}{20}$

$\bar{V}_{AB} = -0,5 + 0,5j = 0,707 \angle 45^\circ$

$\rightarrow \bar{V}_T = 0,909 \cos(10^3t + 45^\circ)$



$\bar{V}_P = \frac{1 + 5\bar{I}}{\frac{1}{2} + \frac{1}{j} - \frac{3}{j}}$

$(\frac{1}{2} - \frac{2}{j}) \bar{V}_P = 1 + 5\bar{I}$

$(-2 - \frac{2}{j}) \bar{V}_P = 1$

$\bar{V}_P = \frac{1}{-2 - 2/j} = -0,25 - 0,25j$

$-R_T = 0,25 - 0,25j$

$-R_T = 0,25 - 0,25j$



$X(\omega) \rightarrow$

$Z_{ep} = (2 || 20) + 20 =$

$\frac{1}{\frac{1}{j\omega C} + 20} + 20 =$

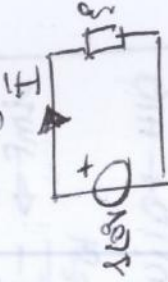
$= \frac{20 \cdot j\omega C}{j\omega C + 20} + 20 =$

$= \frac{20(1 + j\omega C)}{1 + 20j\omega C} + 20$

$\frac{20 - 400j\omega C + 20 + 8000\omega^2 C^2}{1 + 4000\omega^2 C^2}$

$\text{Im}(Z_{ep}) = \frac{-400j\omega C}{1 + 4000\omega^2 C^2}$

$\omega = 10^3 \text{ rad/s}$   
 $\bar{I} = 5j \text{ A}$



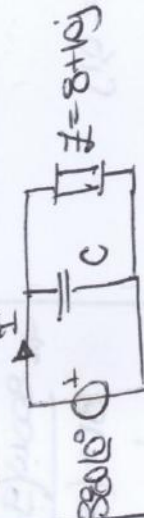
$\bar{Z} = \frac{1}{3j} = -j/3$

$X < 0 \rightarrow$  condensatore:

$\omega C = 3 = C \cdot 10^3$

$C = 3 \cdot 10^{-3} = 3 \text{ mF}$

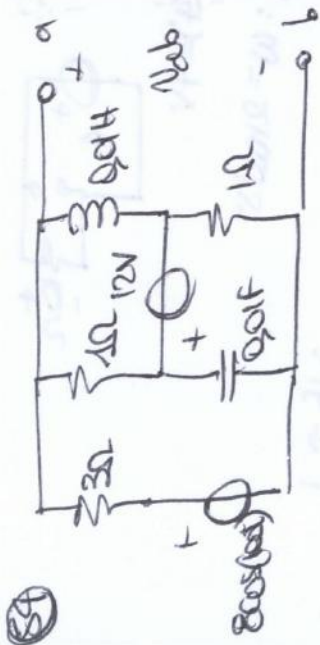
$f = 50 \text{ Hz} \rightarrow \omega = 100\pi \text{ rad/s}$



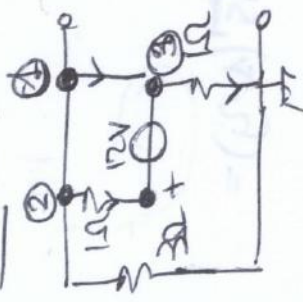
$\bar{C} = \frac{-j}{100\pi C} \rightarrow Z_{ep} = \frac{(8 + j\omega)(-j/100\pi C)}{8 + j\omega - j/100\pi C} =$

$= - \dots [ \dots ]$





Prof 1:  $\omega = 0$



$$\frac{10 \cdot 12}{1+1} = 12V - \bar{V}_1 -$$

$$\bar{V}_1 - 12V = 12V - \bar{V}_1$$

$$2\bar{V}_1 = 24V$$

$$\bar{V}_1 = 12V$$

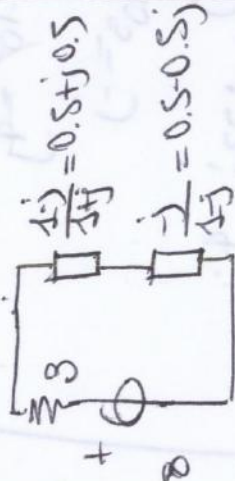
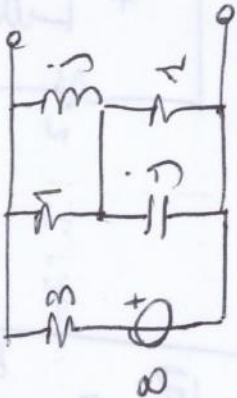
$$V_{ab} = 12V + 12V - 12V = 12V$$

~~$V_2 = 12V$~~

$$V_2 = \bar{V}_3 \rightarrow \bar{V}_1 = 0!$$

$$\rightarrow \bar{V}'(t) = 0V$$

Prof 2:  $\omega = 100 \text{ rad/s}$

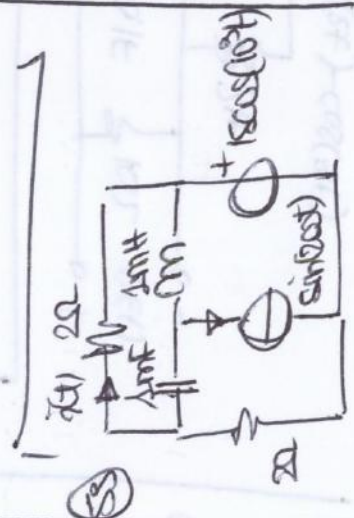


$$V_{ab} = \frac{8 \cdot 1}{1+3} = 2$$

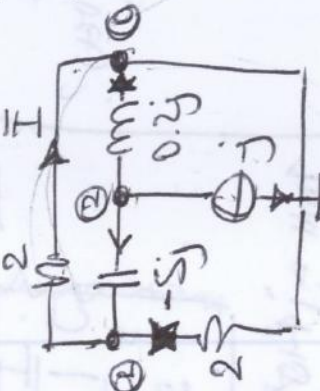
$$\rightarrow \bar{V}'(t) = 2 \cos(100t) V$$

$$\rightarrow \bar{V}(t) = \bar{V}'(t) + \bar{V}''(t) =$$

$$= 2 \cos(100t) V$$



Prof 1:  $\omega = 200 \text{ rad/s}$



$$\text{①: } \frac{\bar{V}_1}{2\Omega} + \frac{\bar{V}_1}{2\Omega} = \frac{\bar{V}_2 - \bar{V}_1}{-Sj}$$

$$\bar{V}_1 = 0.4 \bar{V}_2 - 0.2 j \bar{V}_1$$

$$\bar{V}_1 (1 + 0.2j) = 0.2 j \bar{V}_2$$

$$\text{②: } \frac{\bar{V}_2 - \bar{V}_1}{-Sj} - j + \frac{\bar{V}_2}{9j} = 0$$

$$0.2 j \bar{V}_2 - 0.2 j \bar{V}_1 - j - S j \bar{V}_2 = 0$$

$$(\bar{V}_2 (4.8j) - 0.4 j \bar{V}_1 - j) = 0$$

$$\bar{V}_2 = \bar{V}_1 (1 - Sj)$$

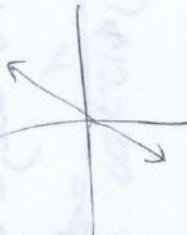
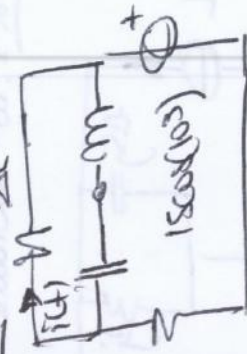
$$[(1 - Sj)(-4.8j) - 0.2j] \bar{V}_1 = j$$

$$\rightarrow \bar{V}_1 = 0.06 \angle -101.7^\circ$$

$$1 \text{ mH} \rightarrow j 10^{-3} \cdot 200 = 2j \cdot 10^{-1} = 0.2j$$

$$1 \text{ mF} \rightarrow \frac{1}{j 200 \cdot 10^{-3}} = -Sj$$

Prof 2:





ESEMP. CAPITOLO 10: POTENZA



$$\bar{V}_R = \frac{10V \cdot 8\Omega}{(4+j)j\Omega} = \frac{40V}{4+j} = 9,701 \angle -14,036^\circ$$

$$P_R = \frac{1}{2} \frac{V_m^2}{R} = \frac{(9,994)^2}{8} = 10W$$

$$P_{P,R} = 2 \cdot P_R = 20W$$

$$10.1 \quad \bar{I} = 5 \angle 30^\circ A$$

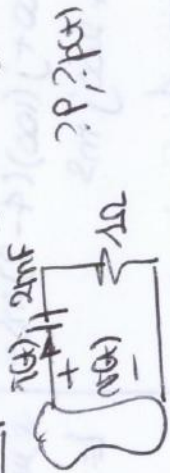


$$\bar{I}_P = 10 - 5 \angle 30^\circ = 6,197 \angle -23,794^\circ A$$

$$\bar{V} = 20\Omega \cdot \bar{I} = 100 \angle 30^\circ$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} \cdot 100 \cdot 6,197 \cos(30 + 23,794^\circ) = 193W$$

$$10.3 \quad i(t) = 2 \cos(1000t + 36^\circ) A$$



$$2mF \rightarrow \frac{1}{j1000 \cdot 0,002} = -0,5j$$

$$\bar{I} = 2 \angle 36^\circ A$$

$$\bar{Z}_{eq} = 1 - 0,5j = 1,118 \angle -26,57^\circ \Omega$$

$$\bar{V} = \bar{Z}_{eq} \cdot \bar{I} = 2,236 \angle 9,13^\circ V$$

$$P = \frac{1}{2} (2,236)(2) \cos(9,13^\circ - 36^\circ) = 1,118 \cos(-26,87^\circ) = 1,0W$$

$$p(t) = v(t) \cdot i(t) = P + \frac{1}{2} V_m I_m \cos(2000t + 45^\circ + 43^\circ) = 2,236 \cos(2000t + 45^\circ + 43^\circ) W$$

$$10.4 \quad P = 40W, Y_{eff} = 220\Omega$$

? R, ? I\_eff, ? I\_P

$$P = \frac{V_{eff}^2}{R} \Leftrightarrow R = \frac{V_{eff}^2}{P} = \frac{80^2}{40} = 160\Omega$$

$$P = I_{eff}^2 R \Leftrightarrow I_{eff} = \sqrt{\frac{P}{R}} = \sqrt{\frac{40}{160}} = 0,5A$$

$$I_{mF} = \sqrt{2} \cdot I_{eff} = 0,25\sqrt{2} A$$

$$10.5 \quad P_{out} = 200W = P_{in} = P$$

$$V_{out} = S V = \frac{P}{I_{eff}}$$

$$I = \frac{P}{S V} = 40A$$

$$I_{eff} = \frac{P}{V_{eff}} = \frac{200}{200} = 0,9A$$

$$10.6 \quad P = 12W, \theta_v - \theta_i$$

$$Q = S \sin \theta$$

? P\_max, ? P\_min

$$S = P + jQ \rightarrow S = \sqrt{P^2 + Q^2} = 13,4VA$$

$$\theta_v - \theta_i = \cos^{-1} \left( \frac{P}{S} \right) = 22,62^\circ$$

$$P_{max} = P + \frac{1}{2} V_m I_m = P + S = 25W$$

$$P_{min} = P - \frac{1}{2} V_m I_m = P - S = -11W$$



• generatore  $\cos(10t)V$ :



$$V = 1V$$

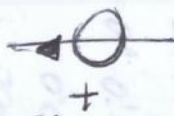
$$I_1 = 0.158 \angle 18.43^\circ A$$

$$S = \frac{1}{2} \bar{V} I_1^* = 0.079 \angle 18.43^\circ$$

$$P = S \cos \phi = 0.079 \cos(18.43^\circ) = 0.075 W$$

$$Q = S \sin \phi = 0.079 \sin(18.43^\circ) = 0.025 VAR$$

• generatore  $\sin(10t)V$ :



$$\bar{V} = -jV = 1 \angle -90^\circ$$

$$I_2 = 0.149 \angle -135^\circ A$$

$$I_2^* = 0.149 \angle 135^\circ A$$

$$S = \frac{1}{2} \bar{V} I_2^* = 0.074 \angle 45^\circ$$

$$P = S \cos \phi = 0.074 \cos(45^\circ) = 0.05 W$$

$$Q = S \sin \phi = 0.074 \sin(45^\circ) = 0.05 VAR$$

$$P = S \cos \phi = 0.074 \cos(45^\circ) = 0.05 W$$

$$Q = S \sin \phi = 0.074 \sin(45^\circ) = 0.05 VAR$$

• induttore:

$$I_1 = 0.158 \angle 18.43^\circ A$$

P=0

$$Q = \frac{1}{2} \omega L I_m^2 = 5 \cdot 1 \cdot 0.158^2 = 0.125 VAR$$

• condensatore:

$$I_2 = 0.149 \angle -135^\circ$$

P=0

$$Q = -\frac{1}{2} \frac{I_m^2}{\omega C} = -\frac{0.158^2}{90} = -0.062 \text{ (circled)} = -0.05 VAR$$

• resistore:

$$V_R = V_3 = 1.58 \angle -71.57^\circ$$

$$I_R = I_3 = 0.158 \angle 18.43^\circ$$

$$P = \frac{1}{2} R I_m^2 = 0.125 W$$

Q=0 VAR

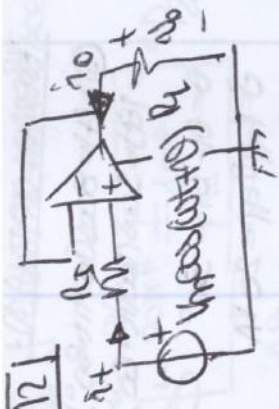
condensatore di P:

$$+0.075 - 0.05 = 0.125$$

condensatore di Q:

$$0.05 + 0.025 = 0.125 - 0.05$$

10.12



• generatore:

$$\bar{V} = V_m \angle 0$$

$$\bar{I}_1 = 0$$

$$P=0=Q=S=0$$

• R<sub>s</sub>:

$$\bar{I}_1 = 0 \rightarrow P=0$$

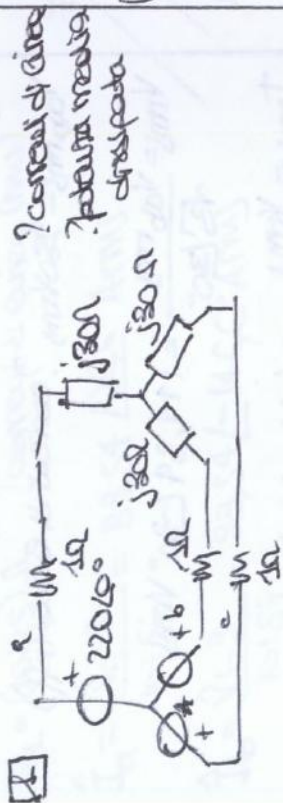
• R<sub>L</sub>:

$$V_0 = \bar{V} = V_m \angle 0$$

$$P = \frac{1}{2} R I_m^2 = \frac{V_m^2}{2R}$$



ESERCIZI CIRCUITO M: CIRCUITI TRIFASICI



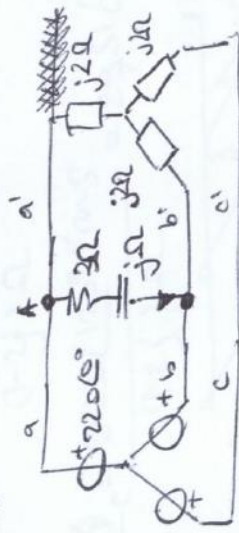
$$\hat{I}_a = \frac{220}{50 + j30} = 7,33 \angle -80,1^\circ \text{ A rms}$$

$$\hat{I}_b = 7,33 \angle -200,1^\circ \text{ A rms}$$

$$\hat{I}_c = 7,33 \angle 84,9^\circ \text{ A rms}$$

$$P = 3 \cdot (R I^2) = 164,15 \text{ W}$$

Es2) ricavarla le correnti di linea:



$$\hat{I}_a = \frac{220}{j20} = 11 \angle -90^\circ \text{ A rms}$$

$$\hat{I}_b = 11 \angle 150^\circ \text{ A rms}$$

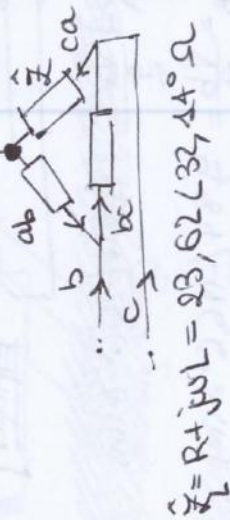
$$\hat{I}_c = 11 \angle 30^\circ \text{ A} = \hat{I}_c$$

$$\hat{V}_{ab} = \sqrt{3} V_m \angle 30^\circ = \sqrt{3} \cdot 220 \angle 30^\circ = 381,05 \angle 30^\circ$$

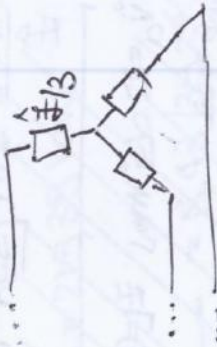
$$\hat{I}_{ab} = \frac{\hat{V}_{ab}}{Z} = 120,5 \angle 184^\circ \text{ rms}$$

A)  $\hat{I}_a = \hat{I}_{ab} \angle \hat{I}_a' = 82,4 \angle 13,9^\circ \text{ A rms}$   
 B)  $\hat{I}_b = \hat{I}_b' - \hat{I}_{ab} = 178,7 \angle -168,9^\circ \text{ A rms}$

E3)  $\hat{V}_a = 220 \angle 0^\circ \text{ V rms}$   $R = 20 \Omega$   $L = 40 \text{ mH}$   
 $f = 50 \text{ Hz}$   
 ? correnti nelle cariche  
 ? correnti di linea



equivalente pu:



$$\frac{\hat{Z}}{3} = 7,87 \angle 32,14^\circ \Omega$$

$$\hat{I}_a = 27,94 \angle -32,14^\circ \text{ A rms}$$

$$\hat{I}_b = 27,94 \angle -152,14^\circ \text{ A rms}$$

$$\hat{I}_c = 27,94 \angle 87,86^\circ \text{ A rms}$$

~~$\hat{I}_a + \hat{I}_b + \hat{I}_c = \hat{I}_{ab}$   
 $\hat{I}_b + \hat{I}_c = \hat{I}_{bc}$   
 $\hat{I}_c + \hat{I}_a = \hat{I}_{ca}$~~

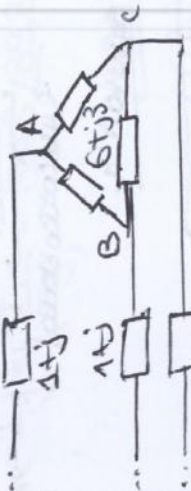
Per ricavare le correnti nei carichi.

$$\hat{I}_{ab} = \frac{\hat{V}_{ab}}{Z} = \sqrt{3} V_m \angle 30^\circ = 16,13 \angle 2,1^\circ \text{ A rms}$$

$$\hat{I}_{bc} = \frac{\sqrt{3} V_m \angle 90^\circ}{Z} = 16,13 \angle 123,1^\circ$$

$$\hat{I}_{ca} = \frac{\sqrt{3} V_m \angle 150^\circ}{Z} = 16,13 \angle 179^\circ \text{ A rms}$$

E4)  $\hat{V}_m = 220 \text{ V}$



Correnti nei carichi (non indovinate)

$$\hat{I}_{AB} = \frac{\sqrt{3} \cdot 220 \text{ V rms} \angle 30^\circ}{6 + j3} = 59,8 \angle -56,6^\circ \text{ A rms}$$

Correnti di linea:

$$\hat{I}_a = \frac{\hat{Z}_A}{3} = \frac{274 \angle 56^\circ}{3} \Omega$$

$$\hat{I}_{TOT} = \hat{I}_a + \hat{I}_b + \hat{I}_c = 56,1 \angle 33,7^\circ \Omega$$

$$\hat{I}_a = \frac{220 \text{ V rms}}{\hat{I}_{TOT}} = 61,02 \angle 33,7^\circ \text{ A rms}$$

$$\hat{I}_b = 61,02 \angle 153,7^\circ \text{ A rms}$$

$$\hat{I}_c = 61,02 \angle 89,3^\circ \text{ A rms}$$