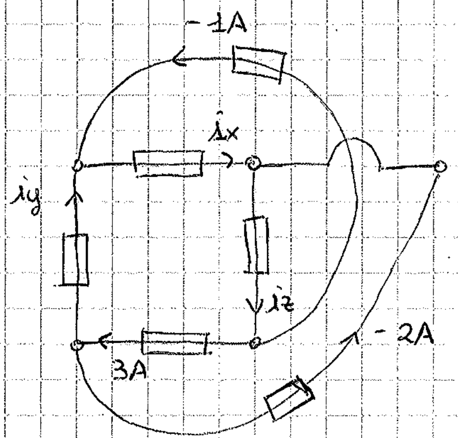
NUMERO: 940 DATA: 15/04/2014

A P P U N T I

STUDENTE: Contadin

MATERIA: Elettrotecnica + temi d'esame

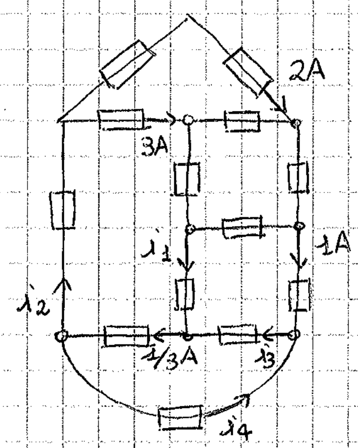
Prof. Lombardi



$i_y - 3A - 2A = 0 \rightarrow i_y = 5A$

$i_x - i_y + 1A = 0 \rightarrow i_x = 4A$

$-i_z + 3A - 1A = 0 \rightarrow i_z = 2A$

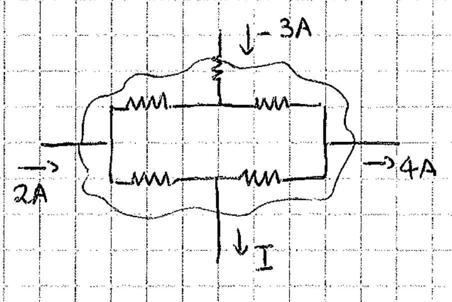


$-i_2 + 3A + 2A = 0 \rightarrow i_2 = 5A$

$-\frac{1}{3}A + 5A + i_4 = 0 \rightarrow i_4 = -\frac{14}{3}A$

$i_3 + \frac{14}{3}A - 1A = 0 \rightarrow i_3 = -\frac{11}{3}A$

$-i_1 + \frac{1}{3}A + \frac{11}{3}A = 0 \rightarrow i_1 = 4A$



$3A - 2A + 4A + I = 0 \rightarrow I = -5A$

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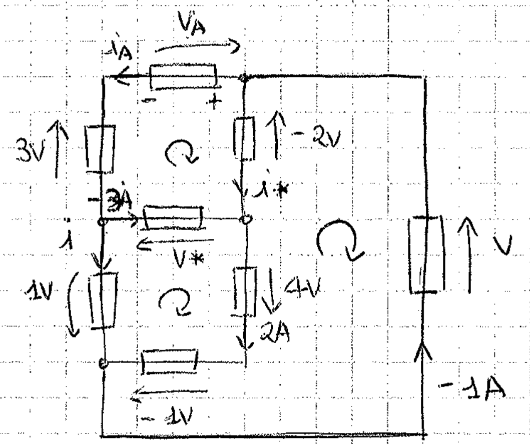


$-i^* + 2A + 3A = 0 \to i^* = 5A$

$i_A + 5A + 1A = 0 \to i_A = -6A$

$i - 3A + 6A = 0 \to i = -3A$

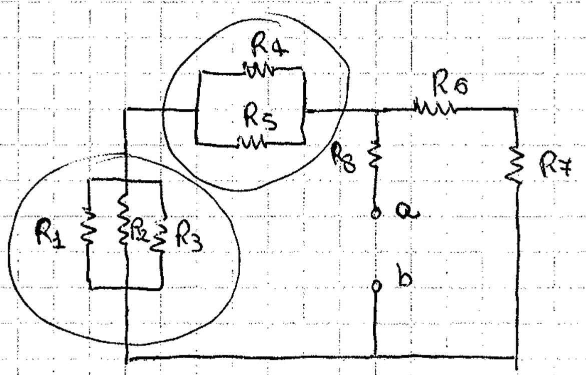
$+1V - 4V - 2V - V = 0 \to V = -\cancel{5}V \quad (-7V)$

$-V^* + 4V - 1V - 1V = 0 \to V^* = 2V$

$V_A + 2V + 2V + 3V = 0 \to V_A = -7V$

$P_A^{(a)} = V_A \cdot i_A = (-7V)(-6A) = 42\,W$

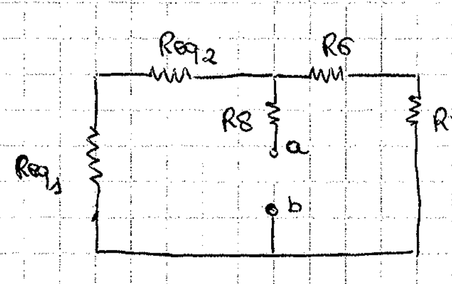
$$P_{k \neq A}^{(a)} = (-6A)\cdot 3V + (-3A)\cdot 2V + (-2V)\cdot 5A - 2A\cdot 4V - (-3A)\cdot 1V - (-1V)\cdot 2A + $$
$$- (-1A)(-5V) = -18 - 6 - 10 - 8 + 3 + 2 - 5 = -42\,W$$



$$Geq_1 = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_2R_3 + R_1R_3 + R_1R_2}{R_1R_2R_3}$$

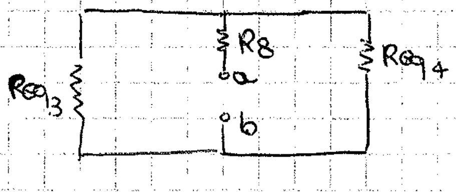
$$Req_2 = \frac{R_4R_5}{R_4+R_5}$$

$$Req_1 = \frac{1}{Geq_1} = \frac{R_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}$$



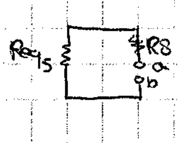
$Req_3 = Req_1 + Req_2$

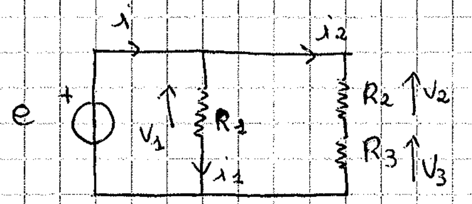
$Req_4 = R_6 + R_7$



$$Req_5 = \frac{Req_3\, Req_4}{Req_3 + Req_4}$$

$Req_{tot} = Req_5 + R_8$





$e = 4V$

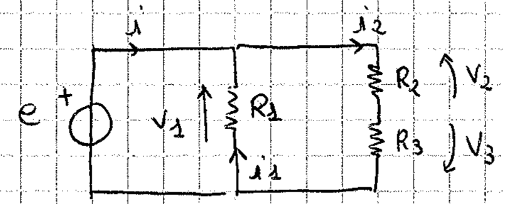
$R_1 = 1\Omega$
$R_2 = 1\Omega$
$R_3 = 3\Omega$

$R_{eq_1} = R_2 + R_3 = 4\Omega$

$R_{eq_2} = \frac{R_{eq_1} R_1}{R_{eq_1} + R_1} = \frac{4 \cdot 1}{4+1} = \frac{4}{5}\Omega$ $\qquad e = V_1 = 4V = V_{eq_2} = V_{eq_1}$

$i_1 = \frac{V_1}{R_1} = 4A \qquad i_2 = \frac{V_{eq_2}}{R_{eq_1}} = 1A$

$i = \frac{V_{eq_1}}{R_{eq_2}} = 5A \qquad V_2 = i_2 R_2 = 1V$
$V_3 = i_2 R_3 = 3V$



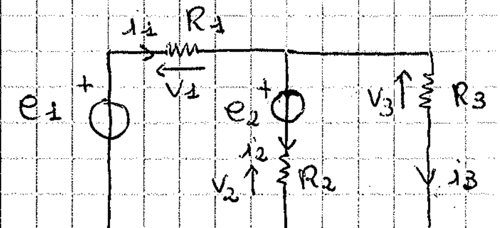
$V_2 = \frac{e R_2}{R_2 + R_3} = 1V$

$V_3 = \frac{e R_3}{R_2 + R_3} = 3V$

$i_2 = \frac{V_2 + V_3}{R_2 + R_3} = 1A$

$V_1 = e = 4V \qquad i_1 = -\frac{V_1}{R_1} = -4A$

$-i - i_1 + i_2 = 0 \Rightarrow i = 4A + 1A = 5A$



$R_1 = R_3 = 25\Omega$
$R_2 = 50\Omega$
$e_1 = 100V$
$e_2 = 200V$

$$\begin{cases} -i_1 + i_2 + i_3 = 0 \\ e_1 - V_1 - e_2 - V_2 = 0 \\ e_2 - V_3 + V_2 = 0 \end{cases}$$

$V_1 = i_1 R_1$
$V_2 = i_2 R_2$
$V_3 = i_3 R_3$

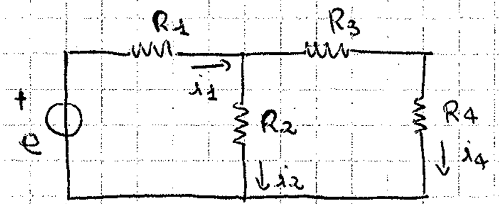
$p^{(a)} = V \cdot i_2 = 25\ W$

$\sum p_k^{(a)} = 6V \cdot 0{,}5 + 4 \cdot 0{,}5 - 10 \cdot 3 = -25\ W$

Il generatore di tensione si comporta come un utilizzatore

Esercitazione ①

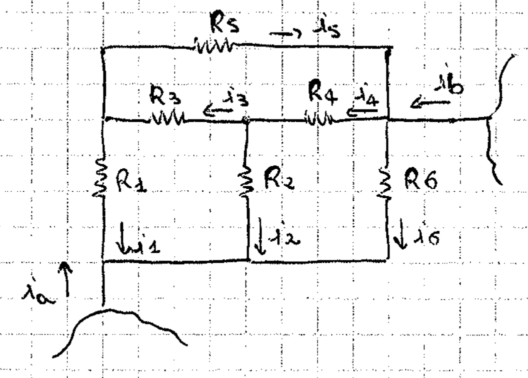
1.1



$i_1 = 2A$
$i_2 = 0{,}7\,A$
$i_4\,?$

$-i_1 + i_2 + i_4 = 0 \Rightarrow i_4 = i_1 - i_2 = 1{,}3\,A$

1.2



Note i_a, i_4, i_3, i_5

$i_6, i_1, i_2\,?$

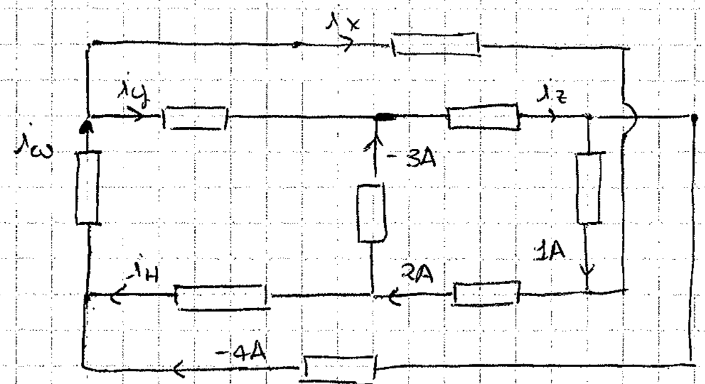
$i_1 - i_3 + i_5 = 0 \rightarrow i_1 = i_3 - i_5$

$i_2 + i_3 - i_4 = 0 \rightarrow i_2 = i_4 - i_3$

$i_5 + i_4 - i_b - i_5 = 0 \rightarrow i_6 = i_b - i_4 + i_5$

$i_b = -i_a \rightarrow i_6 = -i_a - i_4 + i_5$

1.3



$i_x, i_y, i_z, i_w\,?$

$-i_x - 1A + 2A = 0 \rightarrow i_x = 1A$

$-i_z - 4A + 1A = 0 \rightarrow i_z = -3A$

$-i_y + \cancel{3A} - \cancel{3A} = 0 \rightarrow i_y = 0$

$-i_w + \cancel{i_y} + i_x = 0 \rightarrow i_w = 1A$

4. $i_2 R_2 - i_4 R_4 - i_3 R_3 = 0$
2. $i_4 = i_3 - i_g$

$\Rightarrow i_2 R_2 - i_3 R_4 + i_g R_4 - i_3 R_3 = 0$
$20 i_2 - 90 i_3 + 900V - 10 i_3 = 0$
$i_2 = \dfrac{100 i_3 - 900V}{20} = 5 i_3 - 45 A$

3. $200V - i_1 R_1 - i_2 R_2 = 0$
$200V - 25 i_1 - 20 i_2 = 0$
1. $i_1 = i_2 + i_3$

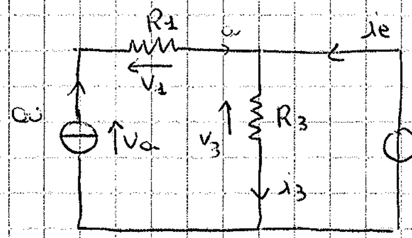
$200V - 25 i_2 - 25 i_3 - 20 i_2 = 0$
$200V - 45 i_2 - 25 i_3 = 0$

Sostituisco i_2: $200V - 45(5 i_3 - 45) - 25 i_3 = 0$
$200V - 225 i_3 + 2025V - 25 i_3 = 0$
$250 i_3 = 2225 V \rightarrow i_3 = 8{,}9 A$

$i_4 = -1{,}1 A$ $\qquad i_2 = -0{,}5 A$ $\qquad i_1 = 8{,}4 A$

$V_S = -i_4 R_4 = 99 V$

1.8



$R_1 = 2\Omega$
$R_3 = 1\Omega$
$a = 3A$
$e = 4V$

i_e? P?

1. $-a - i_e + i_3 = 0$
2. $V_a - V_1 - V_3 = 0$
3. $V_3 - e = 0 \Rightarrow V_3 = e = 4V$

$V_1 = i_1 R_1$ $\qquad i_1 = i_a$
$V_3 = i_3 R_3$

$i_3 = \dfrac{V_3}{R_3} = 4A$ $\quad$ Sostituisco in 1: $-3A - i_e + 4A = 0 \rightarrow i_e = 1A$

$P_3 = V_3 \cdot i_3 = 16 W$

$V_1 = i_1 R_1 = 3 \cdot 2 = 6V \rightarrow P_1 = V_1 \cdot i_1 = 18 W$

2. $V_a = V_1 + V_3 = 10 V \rightarrow P_a = -V_a \cdot a = 10 \cdot 3 = -30 W$
$\hookrightarrow$ gen.

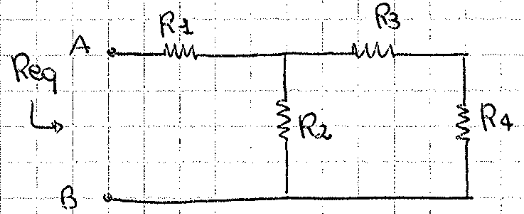
$P_e = e \cdot i_e = -4 \cdot 1 = -4 W$
$\hookrightarrow$ gen.

$$P_{R1} = \frac{V_1^2}{R_1} \rightarrow R_1 = \frac{V_1^2}{P_{R1}} = 3\Omega \qquad V_1 = V_2 = 18\,V$$

$$R_2 = \frac{V_2^2}{P_{R2}} = 6\Omega$$

$$V_{AB} = V_1 = 18\,V$$

1.11



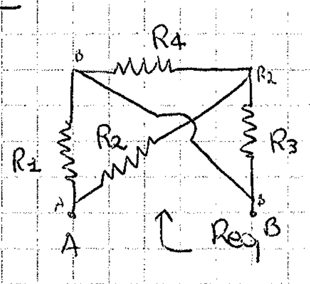
$R_1 = 5\Omega$
$R_2 = 3\Omega$ Req?
$R_3 = 4\Omega$
$R_4 = 2\Omega$

$$R_A = R_3 + R_4 = 6\Omega$$

$$R_B = \frac{R_2 R_A}{R_2 + R_A} = \frac{6 \cdot 3}{6+3} = 2\Omega$$

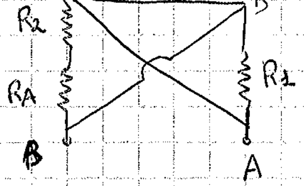
$$R_{eq} = R_1 + R_B = 2\Omega + 5\Omega = 7\Omega$$

1.12



$R_1 = 10\Omega$
$R_2 = 20\Omega$ Req?
$R_3 = 40\Omega$
$R_4 = 40\Omega$

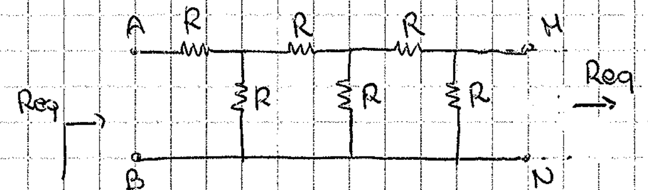
$$R_A = \frac{40 \cdot 40}{40+40} = 20\Omega$$



$$R_B = R_2 + R_A = 40\Omega$$

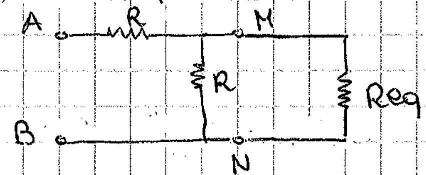
$$R_{eq} = \frac{R_1 R_B}{R_1 + R_B} = \frac{10 \cdot 40}{10+40} = 8\Omega$$

1.15



Data una cascata infinita Req vista da A-B deve essere uguale a quella vista a destra di M-N.

CIRCUITO EQUIVALENTE



$$R_{eq} = R + \frac{R \cdot R_{eq}}{R + R_{eq}}$$

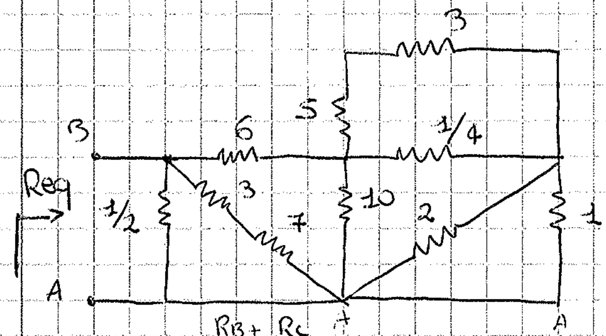
$$R R_{eq} + R^2_{eq} = R^2 + R \cdot R_{eq} + R \cdot R_{eq}$$

$$R^2_{eq} - R \cdot R_{eq} - R^2 = 0$$

$$R_{eq} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$R_{eq} = \frac{1 + \sqrt{5}}{2}$$

1.16



Req ?

$$R_A = 5 + 3 = 8\,\Omega$$

$$R_B = \frac{1/4 \cdot 8}{1/4 + 8} = \frac{8}{33}\,\Omega$$

$$R_C = \frac{2}{3}\,\Omega$$

$$R_D = \overset{R_B + R_C}{\frac{2}{3} + \frac{8}{33}} = \frac{10}{11}$$

$$R_E = \frac{R_D \cdot 10}{R_D + 10} = \frac{5}{6}\,\Omega$$

$$R_F = \frac{5}{6} + 6 = \frac{41}{6}$$

$$R_G = \frac{41/6 \cdot 10}{41/6 + 10} = \frac{410}{101}$$

$$R_{eq_1} = \frac{\frac{410}{101} \cdot \frac{1}{2}}{\frac{410}{101} + \frac{1}{2}} = \frac{410}{921\Omega} = 0{,}445\,\Omega$$

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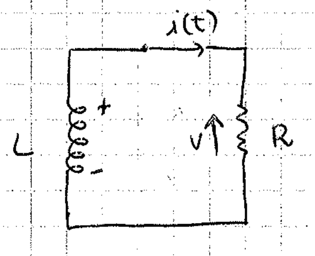
1.18



$i(t) = e^{-t}\,A$ R? L?

$W_M(0) = 1J$

$$w(t) = \frac{1}{2} L\,(i(t))^2$$

$$L = \frac{2\,w(t)}{i(t)^2} \quad \text{in } t=0$$

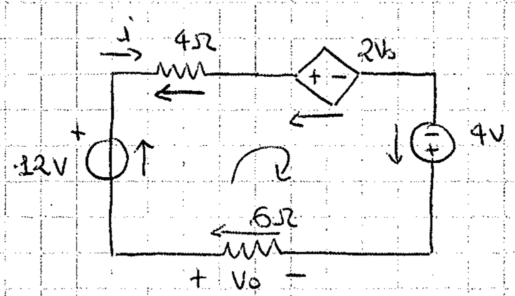
$$\rightarrow L = \frac{2 \cdot 1}{1} = 2H$$

$$v(t) = -L\,\frac{di(t)}{dt}$$

$$v(t) = -L(-e^{-t}) \rightarrow \text{in } t=0 \quad V=-2V$$

$$R = -2\Omega$$

1.19



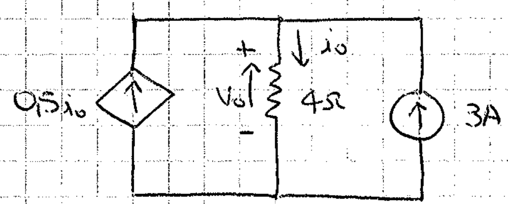
V_0? i?

$$\begin{cases} 12V - 4i - 2V_0 + 4V + V_0 = 0 \\ V_0 = -6i \end{cases}$$

$$12V - 4i + 12i + 4V - 6i = 0 \rightarrow i = -8A$$

$$V_0 = 48V$$

1.20



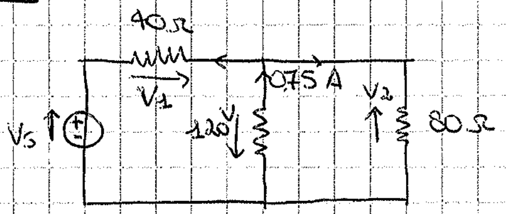
V_0? i_0?

$$-0{,}5i_0 + i_0 - 3A = 0$$

$$0{,}5i_0 = 3A \rightarrow i_0 = 6A$$

$$V_0 = 4i_0 = 24V$$

1.24

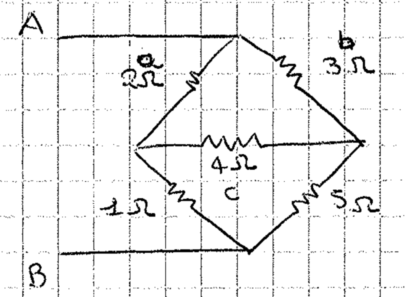


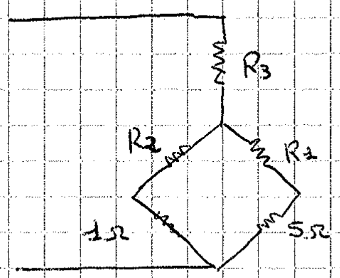
120 V 0,75 A → c.n. lampadina
V_s ?

$V_1 = 40 \cdot 0{,}75 = 30V$
$V_2 = 80 \cdot 0{,}75 = 60V$

$V_s = V_1 + V_2 + c.n. = 60 + 30 + 120 = 210V$

Calcolare Req





$R_2 = \dfrac{2 \cdot 4}{2+4+3} = \dfrac{8}{9}\,\Omega$

$R_1 = \dfrac{3 \cdot 4}{9} = \dfrac{4}{3}\,\Omega$

$R_3 = \dfrac{2 \cdot 3}{9} = \dfrac{2}{3}\,\Omega$

$R_2 + 1\Omega = \dfrac{8}{9} + 1 = \dfrac{8+9}{9} = \dfrac{17}{9}\,\Omega$

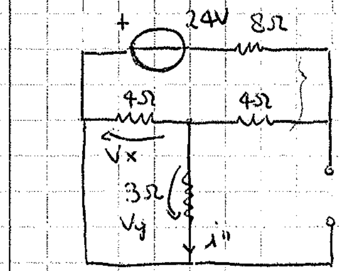
$R_1 + 5\Omega = \dfrac{4}{3} + 5 = \dfrac{4+15}{3} = \dfrac{19}{3}\,\Omega$

$R_{eq} = \dfrac{\frac{17}{9} \cdot \frac{19}{3}}{\frac{17}{9} + \frac{19}{3}} + \dfrac{2}{3} = \boxed{\dfrac{157}{74}\,\Omega}$

$\cancel{R_3 + R_2 + 1\Omega = \frac{2}{3} + \frac{8}{9} + 1 = \frac{6+8+9}{9} = \frac{23}{9}\Omega}$

$\cancel{R_1 + 5\Omega = \frac{4}{3} + 5 = \frac{4+15}{3} = \frac{19}{3}\Omega}$

$\cancel{R_{eq} = \frac{\frac{23}{9} \cdot \frac{19}{3}}{\frac{23}{9} + \frac{19}{3}} = }$

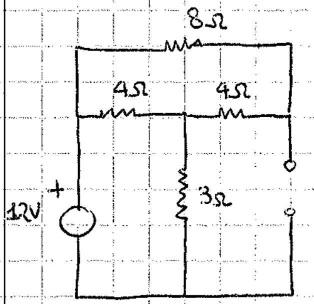


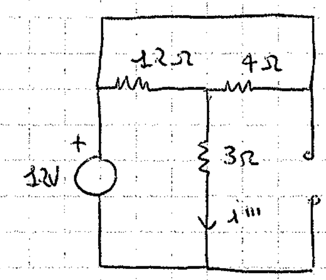
$R_1 = 8+4 = 12\,\Omega$

$R_2 = \frac{4\cdot 3}{4+3} = \frac{12}{7}\,\Omega$

$V_x = 24 \frac{12/7}{12+12/7} = 3V = V_y$

$i'' = -\frac{3}{3} = -1A$





$R_1 = 8+4 = 12\,\Omega$

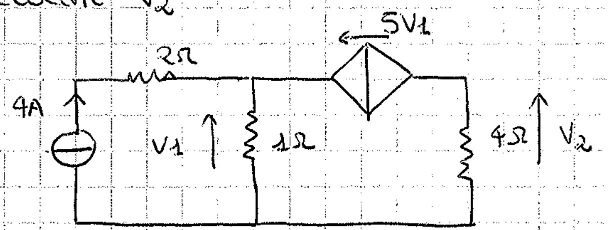
$R_2 = \frac{12\cdot 4}{12+4} = 3\,\Omega$

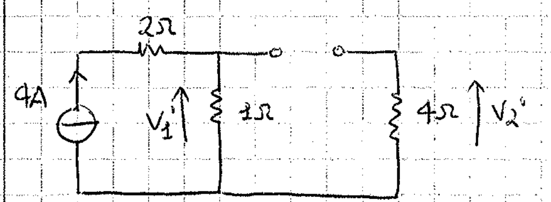
$R_3 = 3+3 = 6\,\Omega$

$i = \frac{V}{R_{eq}(3)} = \frac{12}{6} = 2A$

$i = 1-1+2 = 2A$

Calcolare V_2

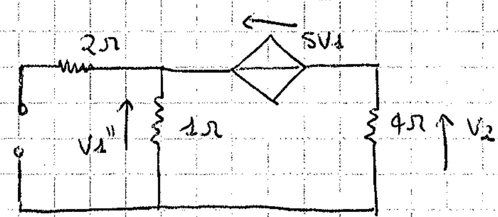




$V_2' = 0\ V$

$R_{eq} = 3\,\Omega$

$V_1' = 4\cdot 1 = 4\ V$



$V_1'' = 5V_1 \cdot 1 = 5V_1$

$V_2'' = -5V_1 \cdot 4 = -20V_1$

$V_1 = V_1' + V_1'' = 4 + 5V_1 \Rightarrow V_1 = -1V$

$V_2 = V_2' + V_2'' = 20V$

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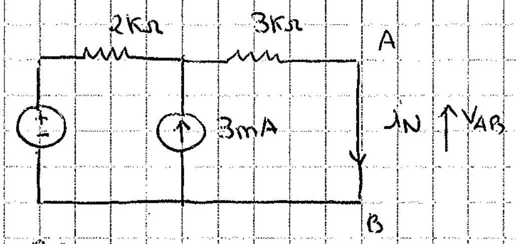
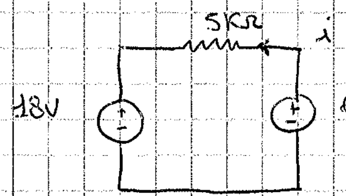
N



$$V_{AB} = \frac{\frac{12}{2k\Omega} + 3}{\frac{1}{2k\Omega} + \cancel{\frac{1}{3k\Omega}}} = \cancel{\frac{84}{5}}\,V \; 18V$$

$$i_N = \underset{\cancel{84/5}}{18V} \cdot \frac{1}{5k\Omega} = 3{,}6\ mA$$

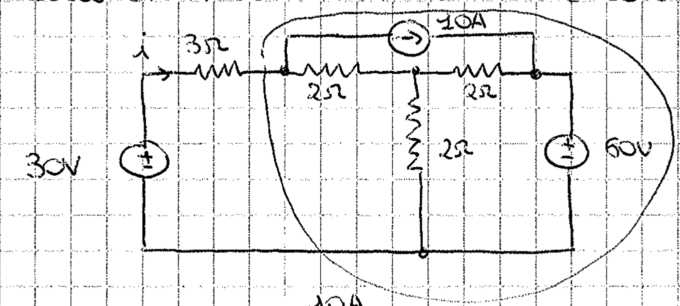
$R_N = R_T = 5k\Omega$

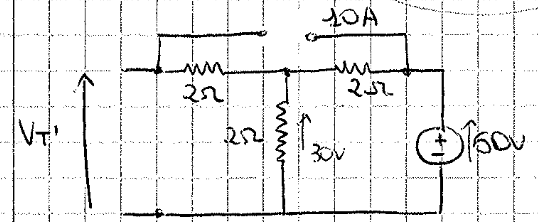


$$i = \frac{-18+6}{5 \cdot 10^3} = -\frac{12}{5}\ mA$$

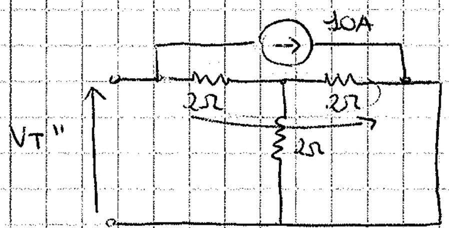
$$p^{(+)} = -\frac{12}{5} \cdot 6 = -\frac{72}{5}\ W$$

Calcolare modello Thevenin. Calcolare i.





$$V_T' = 60 \cdot \frac{2}{2+2} = 30\ V$$



$Req_1 = 1\Omega$

$Req_2 = 1+2 = 3\Omega$

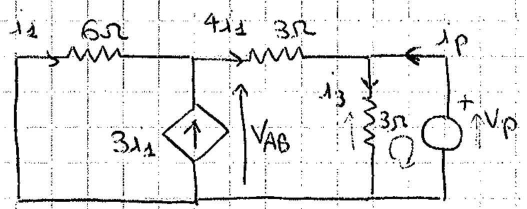
$V_{3\Omega} = 10 \cdot 3 = 30V$ } ragionamento

$VT'' + V_{3\Omega} = 0 \rightarrow V_T'' = -30V$ } con disegno con Req_2

$V_T = V_T' + V_T'' = 0V$

R_T

$R_T = 3\ \Omega$



$$i_p = i_3 - 4i_1$$

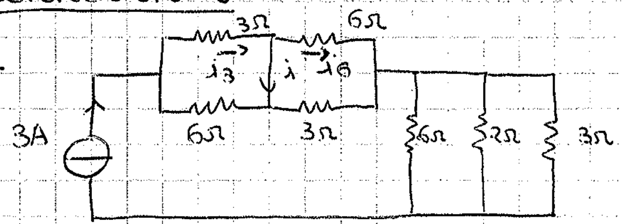
$$i_p = \frac{V_p}{3} - 4i_1 = \frac{V_p}{3} + \frac{\cancel{4}^{2}}{\cancel{18}_{9}} V_p = \frac{3+2}{9} V_p$$

$V_p = V_3$

$$i_p = \frac{5}{9} V_p \implies R_T = \frac{V_p}{i_p} = \frac{9}{5}\ \Omega$$

Esercitazione 2

2.1

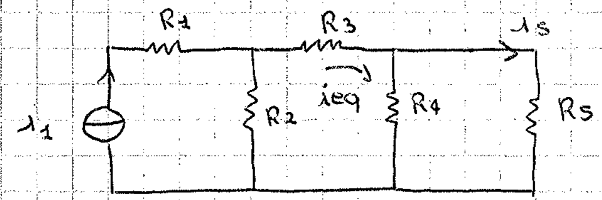


$$i_3 = \frac{3 \cdot 6}{3+6} = 2A$$

$$i_6 = \frac{3 \cdot 3}{3+6} = 1A$$

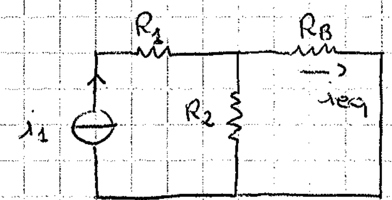
$$i = i_3 - i_6 = 1A$$

2.2



i_s ?

$$R_A = \frac{R_4 R_5}{R_4 + R_5} \qquad R_B = \frac{R_4 R_5}{R_4+R_5} + R_3 = \frac{R_4R_5 + R_3R_4 + R_3R_5}{R_4+R_5}$$



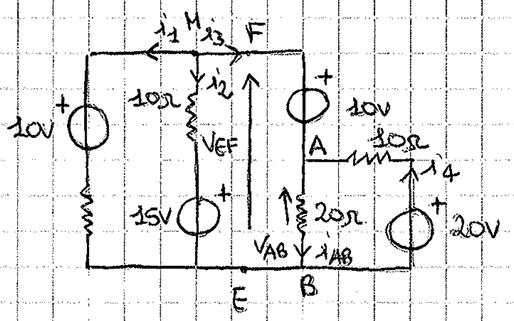
$$i_{eq} = \frac{i_1 R_2}{R_2 + R_B} = \frac{i_1 R_2}{R_2 + \frac{R_4R_5 + R_3R_4 + R_3R_5}{R_4+R_5}}$$

$$= i_1 R_2 \frac{R_4+R_5}{R_2R_4 + R_2R_5 + R_4R_5 + R_3R_4 + R_3R_5}$$

$$i_s = i_{eq} \frac{R_4}{R_4+R_5} = i_1 R_2 \frac{\cancel{R_4+R_5}}{R_2R_4 + R_2R_5 + R_4R_5 + R_3R_4 + R_3R_5} \cdot \frac{R_4}{\cancel{R_4+R_5}}$$

$$= \frac{i_1 R_2 R_4}{R_2R_4 + R_2R_5 + R_4R_5 + R_3R_4 + R_3R_5}$$

$$= \frac{i_1 R_2 R_4}{R_2(R_4+R_5) + R_3(R_4+R_5) + R_4R_5} = \frac{i_1 R_2 R_4}{(R_2+R_3)(R_4+R_5) + R_4R_5}$$



KCL a M : $i_3 = -i_1 - i_2 = -\frac{10}{9} - \frac{1}{18} = -\frac{7}{6}$ A

KVL : $V_{EF} - 10 - V_{AB} = 0$

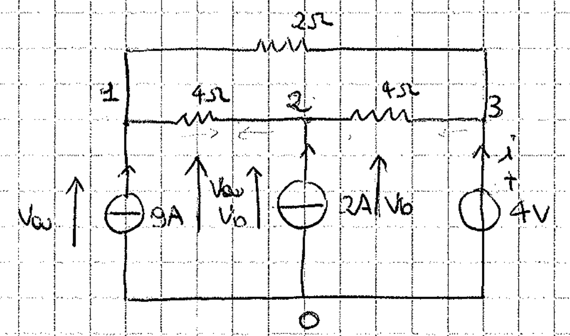
$V_{AB} = V_{EF} - 10$

$20 i_{AB} = V_{EF} - 10 \Rightarrow i_{AB} = \frac{V_{EF} - 10}{20} = \frac{5}{18}$ A

KCL ad A : $i_{AB} = i_3 + i_4$

$i_4 = i_{AB} - i_3 = \frac{5}{18} + \frac{7}{6} = \frac{13}{9}$ A

2.4 / 3.12



Va? Vb?

$V_a = \dfrac{1+9}{\frac{1}{4}+\frac{1}{4}} = 20V$

$V_b = V_a - 4V = 16V$

$$\begin{bmatrix} \frac{3}{4} & -\frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & -\frac{1}{4} & \frac{3}{4} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \\ i \end{bmatrix}$$

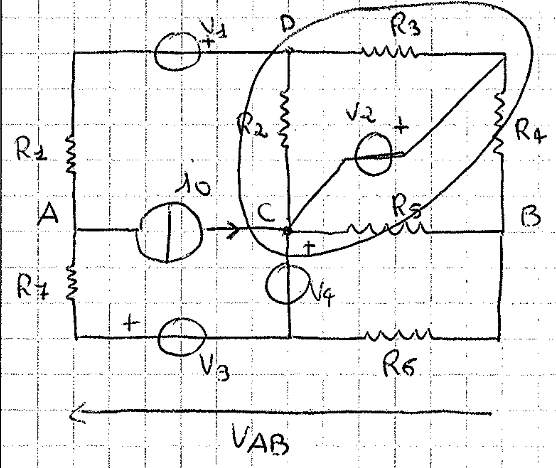
1) $\frac{3}{4}V_a - \frac{1}{4}V_b - 2 = 9 \Rightarrow V_b = \left(\frac{3}{4}V_a - 11\right) \cdot 4 = 3V_a - 44 = 16V$

2) $-\frac{1}{4}V_a + \frac{1}{2}V_b - 1 = 2$

$-\frac{1}{4}V_a + \frac{3}{2}V_a - 22 - 1 = 2$

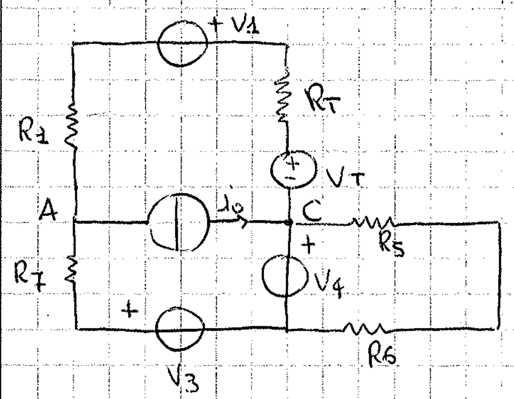
$\frac{5}{\cancel{4}}V_a = \cancel{25}^{5} \cdot \frac{4}{8} \Rightarrow V_a = 20\,V$

Calcolare V_{AB}



Thevenin

$$V_{CB} = -\frac{\frac{V_2}{R_4} - \frac{V_4}{R_6}}{\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}}$$

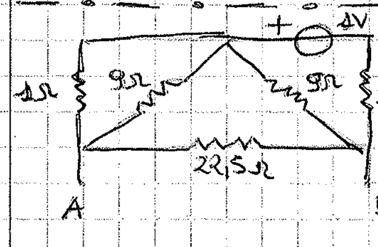


$$V_T = V_2 \frac{R_2}{R_2+R_3}$$

$$R_T = \frac{R_2 \cdot R_3}{R_2+R_3}$$

$$V_{AC} = \frac{\frac{V_3-V_4}{R_7} - i_0 + \frac{V_T - V_1}{R_1+R_T}}{\frac{1}{R_7} + \frac{1}{R_1+R_T}}$$

$$V_{AB} = V_{CB} + V_{AC}$$



$$R_{eq1} = \frac{9}{10}\,\Omega$$

$$V_{9\Omega} = 1 \cdot \frac{9}{9+3} = \frac{3}{4}\,V$$

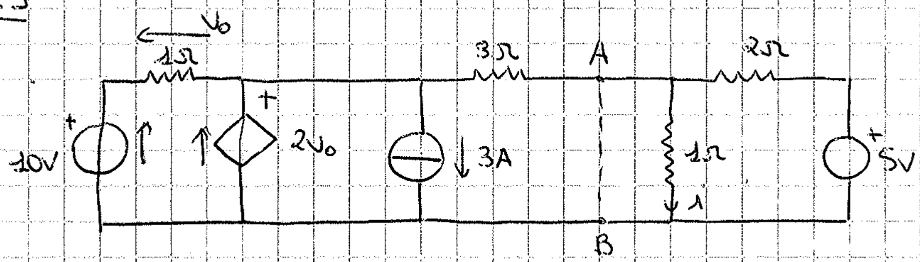
$$V_{22,5} = -V''_{AB} = V_{9\Omega} \cdot \frac{22,5}{22,5+\frac{9}{10}} = -\frac{75}{104}\,V$$

$$V_{AB} = V'_{AB} + V''_{AB} = \frac{18}{11} - \frac{75}{104} = 0,92\,V \rightarrow V_{Th}$$

$$I_N = \frac{V_{Th}}{R_{Th}} = 0,33\,A$$

$$i' = \frac{3V}{10\Omega} = 0,3\,A$$

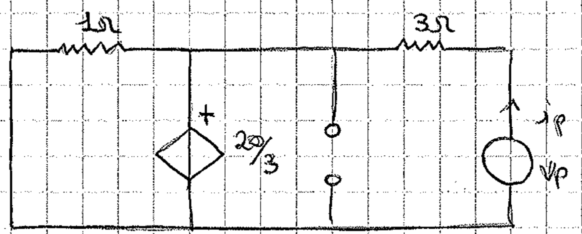
2.9



$$10V - V_0 - 2V_0 = 0 \rightarrow V_0 = \frac{10}{3} V$$

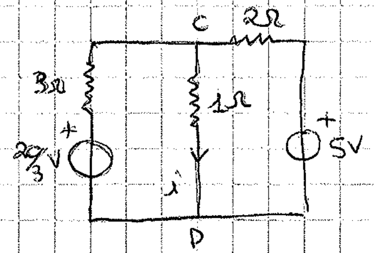
$$2V_0 = \frac{20}{3} V$$

$$V_T = \frac{20}{3} V$$



$$V_p = 3 i_p$$

$$\frac{V_p}{i_p} = R_T = 3\Omega$$



$$V_{CD} = \frac{\frac{20}{3} \cdot \frac{1}{3} + \frac{5}{2}}{1 + \frac{1}{3} + \frac{1}{2}} = \frac{85}{33} V$$

$$i = \frac{85}{33} A$$

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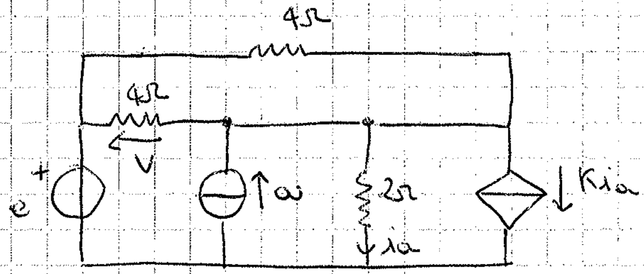
$i_a = \frac{e+2a}{2(K+2)}$ $V = \frac{e-2a+Ke}{K+2}$

2.12



i_a? V?

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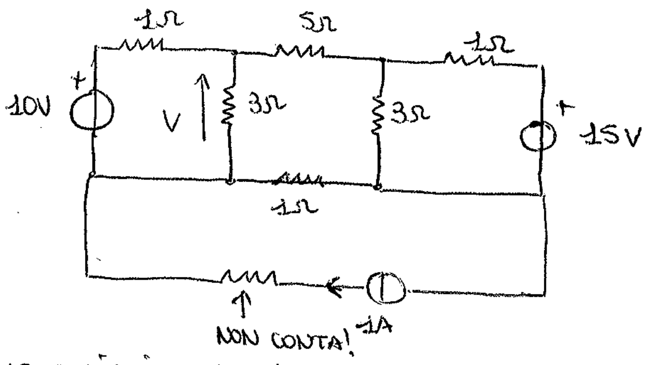
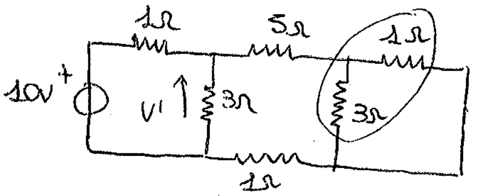
2.16



V ?

Sovrapposizione degli effetti: $V = V' + V'' + V'''$

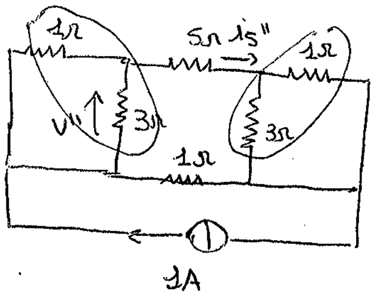


$R_A = \frac{3}{4}\Omega$

$R_B = \frac{3}{4} + 5 + 1 = \frac{27}{4}$

$R_{eq} = \frac{\frac{27}{4} \cdot 3}{\frac{27}{4} + 3} = \frac{27}{13}\Omega$

$V' = 10 \cdot \frac{27/13}{27/13 + 1} = \frac{27}{4} V$ → partitore di tensione

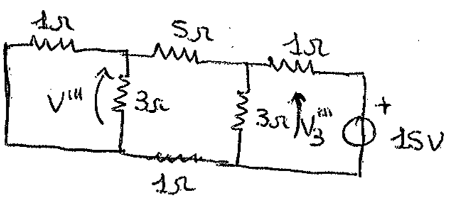


$R_A = R_B = \frac{3}{4}\Omega$

$R_{eq} = 2R_A + 5\Omega = \frac{13}{2}\Omega$ → dispongo le tensioni in parallelo per applicare il partitore di corrente

$i_S'' = 1A \cdot \frac{1}{1 + \frac{13}{2}} = \frac{2}{15} A$ → partitore di corrente

$V'' = -R_A \cdot i_S'' = -\frac{\cancel{3}}{\cancel{4}_2} \cdot \frac{\cancel{2}}{\cancel{15}_5} = -\frac{1}{10} V$

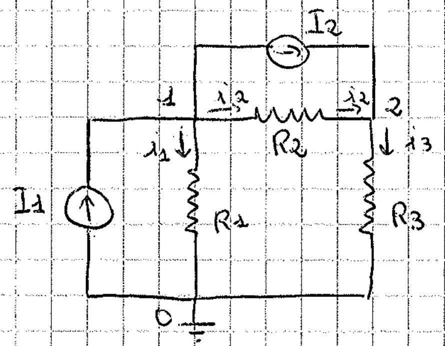


$R_A = \frac{3}{4}\Omega$ $R_B = \frac{27}{4}$ $R_{eq} = \frac{27}{13}$

$V_3''' = 15 \cdot \frac{27/13}{1 + \frac{27}{13}} = \frac{81}{8} V$ → partitore di tensione

$V''' = V_3''' \cdot \frac{R_A}{R_A + 1 + 5} = \frac{81}{8} \cdot \frac{3/4}{1 + 3/4 + 5} = \frac{9}{8} V$

$V = V' + V'' + V''' = \frac{27}{4} - \frac{1}{10} + \frac{9}{8} = \frac{311}{40} V$



$$\begin{bmatrix} G_1+G_2 & -G_2 \\ -G_2 & G_2+G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1-I_2 \\ I_2 \end{bmatrix}$$

① $\frac{V_1}{R_1}$ ② $\frac{V_1-V_2}{R_2}$ ③ $\frac{V_2}{R_3}$

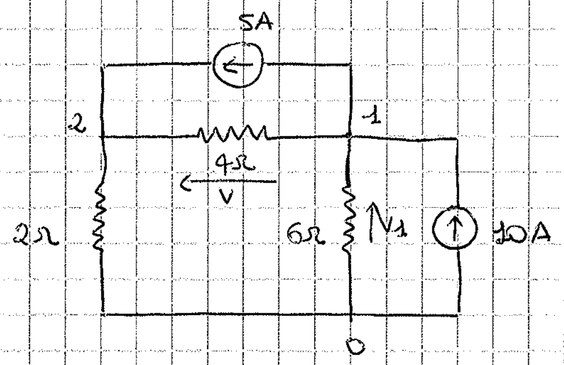
① $-I_1 + i_1 + i_2 + I_2 = 0$

② $i_3 - i_2 - I_2 = 0$

$$I_1 = \frac{V_1}{R_1} + \frac{V_1-V_2}{R_2} + I_2$$

$$I_2 = \frac{V_2}{R_3} - \frac{V_1-V_2}{R_2}$$

Calcolare V



$$\begin{bmatrix} \frac{1}{6}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{2}+\frac{1}{4} \end{bmatrix} \begin{bmatrix} V_1 \\ V \end{bmatrix} = \begin{bmatrix} \overbrace{10-5}^{5} \\ 5 \end{bmatrix}$$

$$\frac{5}{12}V_1 - \frac{1}{4}V = 5$$

$$-\frac{1}{4}V_1 + \frac{3}{4}V = 5 \quad \Rightarrow V_1 = \left(5 - \frac{3}{4}V\right)(-4) = -20 + 3V$$

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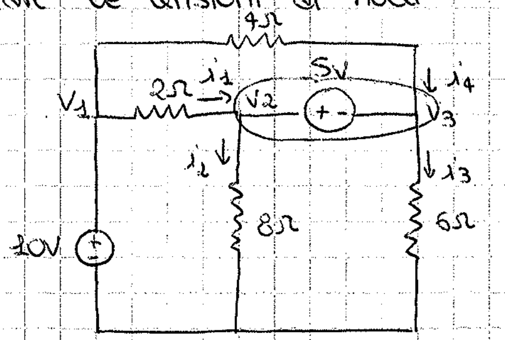
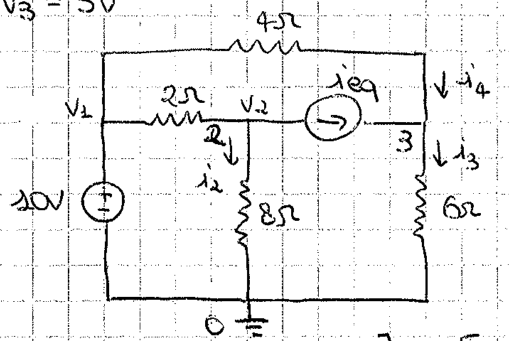
Calcolare le tensioni ai nodi



Sostituzione:

(4) $V_2 - V_3 = 5V$



$V_1 = 10V$
$V_2 = \frac{46}{5}V$
$V_3 = \frac{21}{5}V$

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2}+\frac{1}{8} & 0 \\ -\frac{1}{4} & 0 & \frac{1}{6}+\frac{1}{4} \end{bmatrix} \begin{bmatrix} 10 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -i_{eq} \\ i_{eq} \end{bmatrix}$$

(1) $\frac{3}{\cancel{4}2} \cdot \cancel{10}^{5} - \frac{1}{2}V_2 - \frac{1}{4}V_3 = 0$ → non si utilizza ma si usa la relazione costitutiva della sostituzione (a meno che manchi un'incognita)

(2) $-5 + \frac{5}{8}V_2 = -i_{eq}$

(3) $-\frac{5}{2} + \frac{5}{12}V_3 = i_{eq}$

$-5 + \frac{5}{8}V_2 = +\frac{5}{2} - \frac{5}{12}V_3$

$\Rightarrow V_2 = \left(\frac{15}{2} - \frac{5}{12}V_3\right)\cdot\frac{8}{5} = 12 - \frac{2}{3}V_3$

(4) $12 - \frac{2}{3}V_3 - V_3 = 5$

$-\frac{5}{3}V_3 = -7 \Rightarrow V_3 = \frac{21}{5}V$

(3) $V_2 = \frac{46}{5}V$

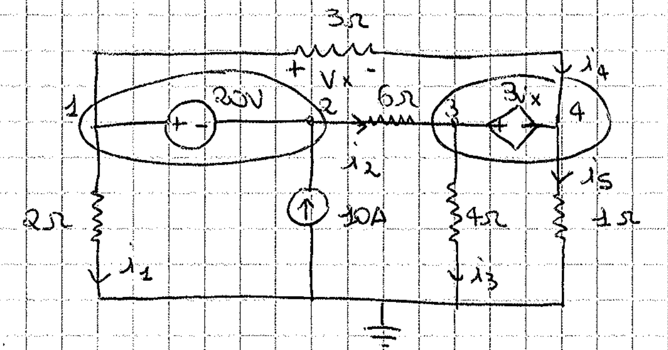
② $\frac{10}{7} - \frac{5}{14} V_2 + V_2 - \frac{4}{3} - \frac{1}{3} V_2 = 3$

$\frac{\cancel{13}}{\cancel{42}} V_2 = \frac{61}{\cancel{21}} \cdot \frac{\cancel{42}^{\,2}}{13} = \frac{122}{13} V = 9{,}384 V$

$V_1 = \frac{50}{13} V = 3{,}846 V$

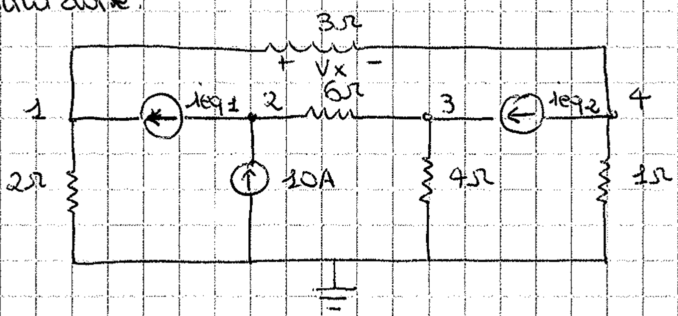
$V_3 = \frac{116}{13} V = 8{,}923 V$

Calcolare le tensioni di nodi



$V_1 - V_4 = V_x$
$V_1 - V_2 = 20V$
$V_3 - V_4 = 3V_x$

Sostituzione:



$$\begin{bmatrix} \frac{1}{2}+\frac{1}{3} & 0 & 0 & -\frac{1}{3} \\ 0 & \frac{1}{6} & -\frac{1}{6} & 0 \\ 0 & -\frac{1}{6} & \frac{1}{6}+\frac{1}{4} & 0 \\ -\frac{1}{3} & 0 & 0 & 1+\frac{1}{3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} i_{eq1} \\ 10 - i_{eq1} \\ i_{eq2} \\ -i_{eq2} \end{bmatrix}$$

① $\frac{5}{6} V_1 - \frac{1}{3} V_4 = i_{eq1}$

② $\frac{1}{6} V_2 - \frac{1}{6} V_3 = 10 - i_{eq1}$

$\frac{1}{6} V_2 - \frac{1}{6} V_3 = 10 - \frac{5}{6} V_1 + \frac{1}{3} V_4$

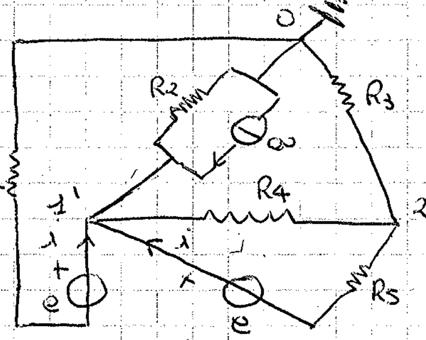
③ $-\frac{1}{6} V_2 + \frac{5}{12} V_3 = i_{eq2}$

④ $-\frac{1}{3} V_1 + \frac{4}{3} V_4 = -i_{eq2}$

$-\frac{1}{3} V_1 + \frac{4}{3} V_4 = \frac{1}{6} V_2 - \frac{5}{12} V_3$

$$\left.\begin{aligned} &① \quad \frac{4}{3}V_4 + \frac{5}{12}V_3 = \frac{V_1}{3} + \frac{V_2}{6} \\ &② \quad 10 = \frac{5}{6}V_1 + \frac{V_2}{6} - \frac{V_3}{6} - \frac{V_4}{3} \end{aligned}\right\} \text{OK!}$$

$V_2 = V_x + V_4 - 20V$
$V_1 = V_x + V_4$
$V_3 = 3V_x + V_4$



$V_3 = V_1' - e$
$V_2 - V_1' = e + R_5$

$$\begin{bmatrix} \frac{1}{R_2}+\frac{1}{R_1}+\frac{1}{R_4} & -\frac{1}{R_4} \\ -\frac{1}{R_4} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_{1'} \\ V_2 \end{bmatrix} = \begin{bmatrix} a + 2i \\ 0 \end{bmatrix}$$

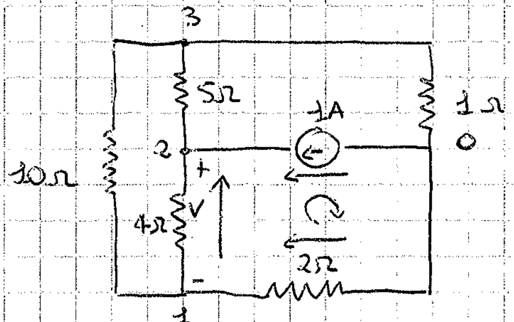
$e + R_1$ ↑ $V_{1'}$

$$\left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_4}\right)V_1' - \frac{1}{R_4}V_2 = a + 2i$$

$$-\frac{1}{R_4}V_1 + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)V_2 = 0$$

Esercitazione 3

3.1



V?

$V_1 + V - V_2 = 0 \Rightarrow V = V_2 - V_1$

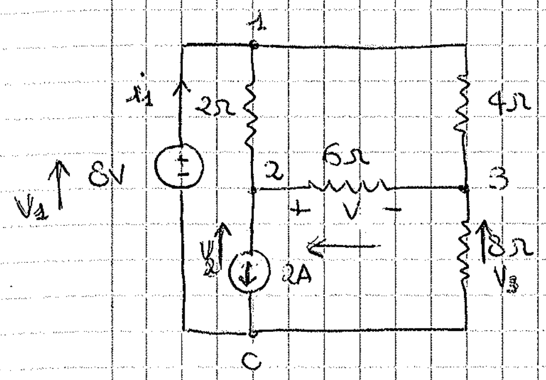
$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4}+\frac{1}{10} & -\frac{1}{4} & -\frac{1}{10} \\ -\frac{1}{4} & \frac{1}{4}+\frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{10} & -\frac{1}{5} & 1+\frac{1}{10}+\frac{1}{5} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

① $\frac{17}{20}V_1 - \frac{1}{4}V_2 - \frac{1}{10}V_3 = 0 \Rightarrow V_3 = \left(\frac{17}{20}V_1 - \frac{1}{4}V_2\right) \cdot 10 = \frac{17}{2}V_1 - \frac{5}{2}V_2$

② $-\frac{1}{4}V_1 + \frac{9}{20}V_2 - \frac{1}{5}V_3 = 1$

③ $-\frac{1}{10}V_1 - \frac{1}{5}V_2 + \frac{13}{10}V_3 = 0$

3.3



⇒ Quando ho un solo gen. di tensione dispongo i nodi in modo che risulti collegato ad un nodo di riferimento (oppure quando ne ho due collegati allo stesso nodo)

V_2?

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2}+\frac{1}{6} & -\frac{1}{6} \\ -\frac{1}{4} & -\frac{1}{6} & \frac{1}{6}+\frac{1}{4}+\frac{1}{8} \end{bmatrix} \begin{bmatrix} 8 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} i_1 \\ -2 \\ 0 \end{bmatrix}$$

(1) $6 - \frac{1}{2}V_2 - \frac{1}{4}V_3 = i_1$

(2) $-4 + \frac{2}{3}V_2 - \frac{1}{6}V_3 = -2 \Rightarrow V_3 = \left(2 - \frac{2}{3}V_2\right)(-6) = -12 + 4V_2 = 5V$

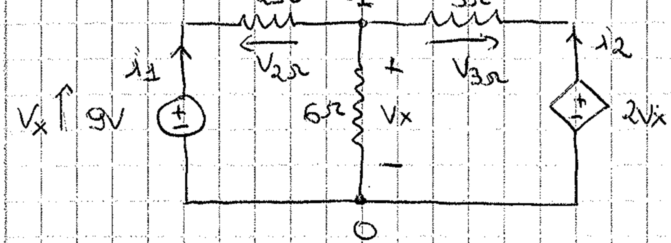
(3) $-2 - \frac{1}{6}V_2 + \frac{13}{24}V_3 = 0$

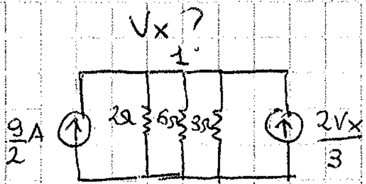
$-2 - \frac{1}{6}V_2 - \frac{13}{2} + \frac{13}{6}V_2 = 0$

$2V_2 = \frac{17}{2} \Rightarrow V_2 = \frac{17}{4}V$

KVL: $V = V_2 - V_3 = -\frac{3}{4}V = -0{,}75V$

3.6





$i_1 = \frac{9}{2}A$ $i_2 = \frac{2V_x}{3}$ → Trasformo in bipoli Norton

$V_x = \frac{9}{2} + \frac{2V_x}{3}$

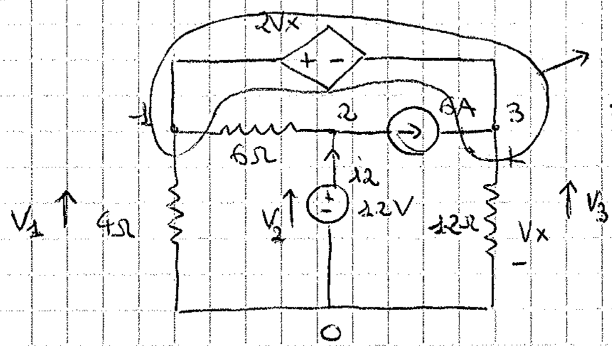
$V_x - \frac{2}{3}V_x = \frac{9}{2}$

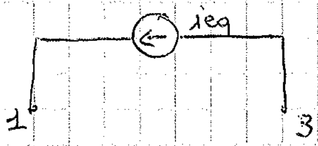
$\frac{1}{3}V_x = \frac{9}{2} \Rightarrow V_x = \frac{27}{2}V = 13{,}5\ V$

$V_x = \dfrac{\frac{9}{2} + \frac{2V_x}{3}}{1} \Rightarrow V_x = 13{,}5\ V$

↓
Millman

3.7





$V_1 - V_3 = 2V_x$
$V_3 = V_x$
$V_1 = 3V_x$

V_x ?

$$\begin{bmatrix} \frac{1}{4}+\frac{1}{6} & -\frac{1}{6} & 0 \\ -\frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{12} \end{bmatrix} \begin{bmatrix} 3V_x \\ 12 \\ V_x \end{bmatrix} = \begin{bmatrix} i_{eq} \\ i_2 - 6 \\ 6 - i_{eq} \end{bmatrix}$$

① $\frac{5}{4}V_x - 2 = i_{eq}$

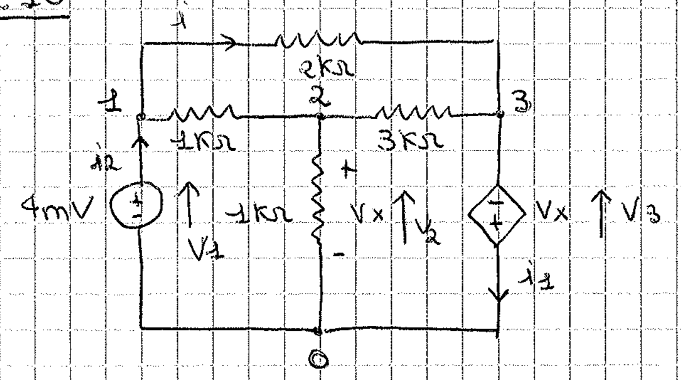
② $-\frac{1}{2}V_x + 2 = i_2 - 6A$

③ $\frac{1}{12}V_x = 6 - i_{eq}$

$\frac{1}{12}V_x = 6 - \frac{5}{4}V_x + 2$

$\frac{4}{3}V_x = 8 \Rightarrow V_x = 6V$

3.10



i? Vx?

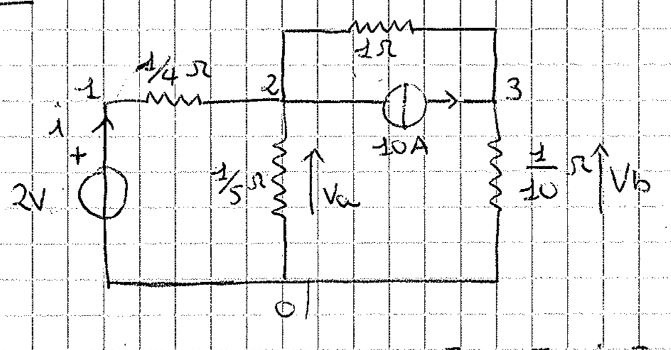
$$10^3 \cdot \begin{bmatrix} \frac{3}{2} & -1 & -\frac{1}{2} \\ -1 & \frac{7}{3} & -\frac{1}{3} \\ -\frac{1}{2} & -\frac{1}{3} & \frac{5}{6} \end{bmatrix} \begin{bmatrix} \frac{4}{10^3} \\ V_x \\ -V_x \end{bmatrix} = \begin{bmatrix} i_2' \\ 0 \\ -i_1 \end{bmatrix}$$

(2) $-4 + \frac{7}{3} V_x + \frac{1}{3} V_x = 0$

$-4 + \frac{8}{3} V_x = 0 \Rightarrow \frac{8}{\cancel{3}} V_x = 4 \cdot \frac{3}{8_2} \text{ mV} = 1{,}5 \text{ mV}$

$i' = \frac{V_1 - V_3}{2 \cdot 10^3} = \frac{(4+1{,}5) \cdot 10^{-3}}{2 \cdot 10^3} = 2{,}75\ \mu A$

3.11



Va? Vb?

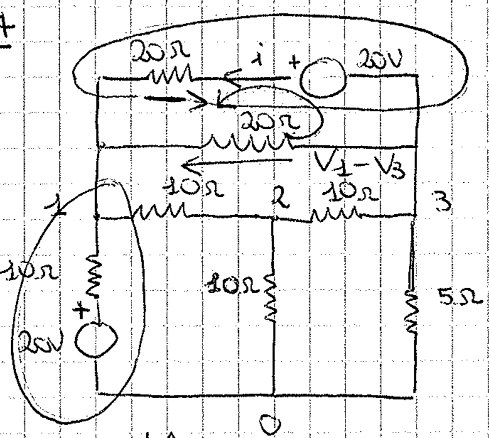
$$\begin{bmatrix} 0 & -4 & 0 \\ -4 & 10 & -1 \\ 0 & -1 & 11 \end{bmatrix} \begin{bmatrix} 2 \\ V_a \\ V_b \end{bmatrix} = \begin{bmatrix} i \\ -10 \\ 10 \end{bmatrix}$$

(1) $-4V_a = i$

(2) $-8 + 10V_a - V_b = -10$

(3) $-V_a + 11V_b = 10 \Rightarrow V_a = 11V_b - 10$

3.14



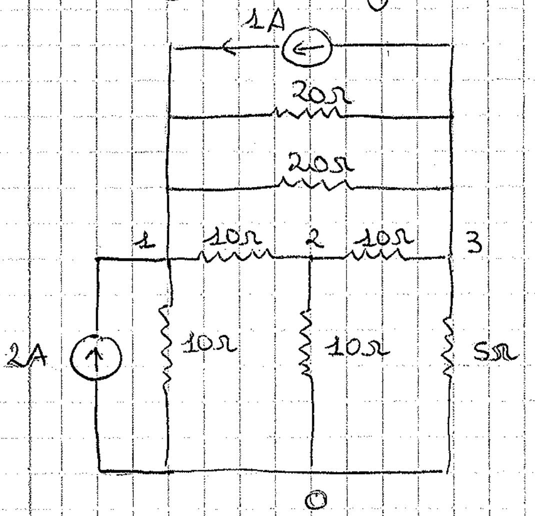
i? V_1? V_2? V_3?

$V_1 = \frac{145}{12}$ V

$V_2 = \frac{55}{12}$ V

$V_3 = \frac{5}{3}$ V

$i = \frac{23}{48}$ A



$$\begin{bmatrix} \frac{3}{10} & -\frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{3}{10} & -\frac{1}{10} \\ -\frac{1}{10} & -\frac{1}{10} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$$

① $\frac{3}{10} V_1 - \frac{1}{10} V_2 - \frac{1}{10} V_3 = 3$

② $-\frac{1}{10} V_1 + \frac{3}{10} V_2 - \frac{1}{10} V_3 = 0 \Rightarrow V_2 = \left(\frac{1}{10} V_1 + \frac{1}{10} V_3\right) \cdot \frac{10}{3} = \frac{1}{3} V_1 + \frac{1}{3} V_3 = \frac{55}{12}$ V

③ $-\frac{1}{10} V_1 - \frac{1}{10} V_2 + \frac{2}{5} V_3 = -1$

① $\frac{3}{10} V_1 - \frac{1}{30} V_1 - \frac{1}{30} V_3 - \frac{1}{10} V_3 = 3$

$\frac{4}{15} V_1 = \left(\frac{2}{15} V_3 + 3\right) \cdot \frac{15}{4} = \frac{1}{2} V_3 + \frac{45}{4} = \frac{145}{12}$ V

③ $-\frac{1}{10} V_1 - \frac{1}{30} V_1 - \frac{1}{30} V_3 + \frac{2}{5} V_3 = -1$

$-\frac{2}{15}\left(\frac{1}{2} V_3 + \frac{45}{4}\right) + \frac{11}{30} V_3 = -1$

$-\frac{1}{15} V_3 - \frac{3}{2} + \frac{11}{30} V_3 = -1 \Rightarrow \frac{3}{\cancel{10}} V_3 = \frac{1}{\cancel{2}} \cdot \frac{\cancel{10}^{5}}{3} = \frac{5}{3}$ V

$V_3 - V_1 + 20 - 20i = 0 \Rightarrow i = \frac{V_3 - V_1 + 20}{20} = \frac{23}{48}$ A

PONTE DI WHEATSTONE

$R_1 = 1K\Omega$, $R_3 = 0 - 1100\Omega$ step 1Ω, $R_2 = 10^n K\Omega$ con $n = 1,2,3$

a) Quale è il valore di R_x quando $R_3 = 732\Omega$, $R_2 = 10K\Omega$?

$$R_x = \frac{R_2}{R_1} \cdot R_3 = \frac{10K\Omega}{1K\Omega} \cdot 732\Omega = 7320\Omega$$

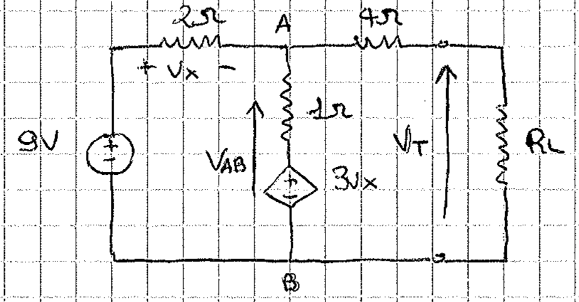
b) Quale è il massimo valore misurabile di R_x?

$$R_{x\,max} = \frac{R_{2\,max}}{R_1} \cdot R_{3\,max} = 1.1 \cdot 10^6 \Omega = 1.1 M\Omega$$

c) Quale incremento deve essere dato a R_x per assicurare un bilanciamento perfetto del ponte quando $R_2 = 1M\Omega$?

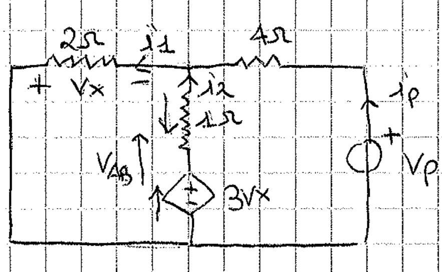
$$R_{x\,inc} = \frac{R_2}{R_1} \cdot R_{3\,inc} = 1K\Omega$$

Calcolare R_L per avere max trasf. di potenza. Calcolare potenza max trasferibile al carico



$$V_{AB} = \frac{\frac{9}{2} + \frac{3V_x}{1}}{\frac{1}{2} + 1} = \left(\frac{9}{2} + \frac{3V_x}{1}\right) \cdot \frac{2}{3} = 3 + 2V_x = 7V = V_T$$

$9V - V_x - V_{AB} = 0 \Rightarrow V_x = 9 - 3 - 2V_x \Rightarrow V_x = 2V$



$$V_{AB} = \frac{\frac{V_p}{4} + 3V_x}{\frac{7}{4}} = \frac{V_p}{7} + \frac{12}{7}V_x$$

$V_x = -V_{AB} \Rightarrow V_x = -\frac{V_p}{7} - \frac{12}{7}V_x \Rightarrow V_x = -\frac{V_p}{19}$

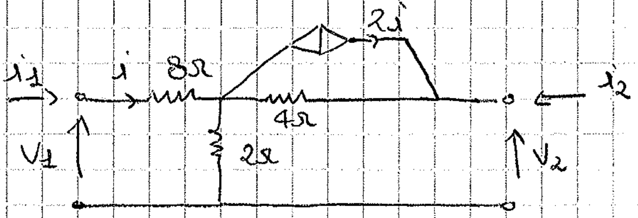
$i_p = i_1 - i_2 = -\frac{V_x}{2} - 3V_x + V_{AB}$

$i_1 = -\frac{V_x}{2}$

$i_p + i_2 = i_1$

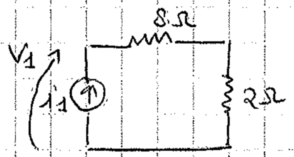
$i_p = i_1 - i_2$

$V_{AB} + i_2 - 3V_x = 0 \rightarrow i_2 = 3V_x - V_{AB}$



$$V_1 = R_{11} i_1 + R_{12} i_2$$
$$V_2 = R_{21} i_1 + R_{22} i_2$$

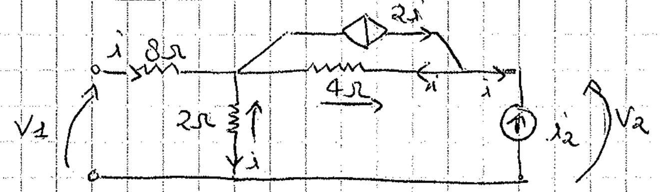
$$R_{11} = \left.\frac{V_1}{i_1}\right|_{i_2=0}$$



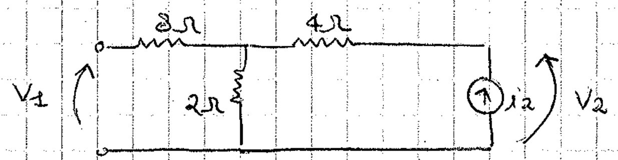
$V_1 = 10 i_1$ $\qquad i = i_1$

$R_{11} = 10\Omega$

$$R_{12} = \left.\frac{V_1}{i_2}\right|_{i_1=0}$$



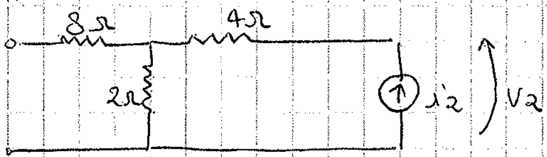
$i = 0$



$V_1 = 2 i_2$ $\qquad R_{12} = \frac{V_1}{i_2} = 2\Omega$

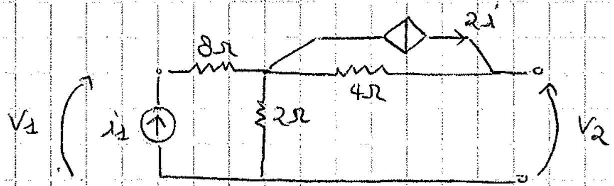
$$R_{22} = \left.\frac{V_2}{i_2}\right|_{i_1=0}$$

$i = 0$



$V_2 = 6 i_2 \quad \Rightarrow \quad R_{22} = 6\Omega$

$$R_{21} = \left.\frac{V_2}{i_1}\right|_{i_2=0}$$

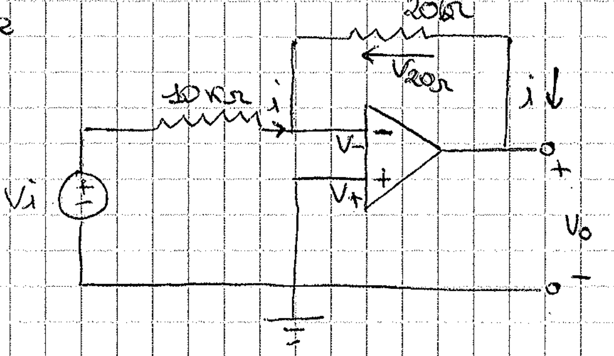


$V_2 = 2 i_1$ $\qquad R_{21} = 2\Omega$

$$V_1 = 10 i_1 + 2 i_2$$
$$V_2 = 2 i_2 + 6 i_2$$

Calcolare il guadagno ad anello chiuso A_c e i per $v_i = 2V$

1) ideale



$i_+ = i_- = 0$

$V_+ - V_- = 0 \Rightarrow V_+ = V_- = 0$

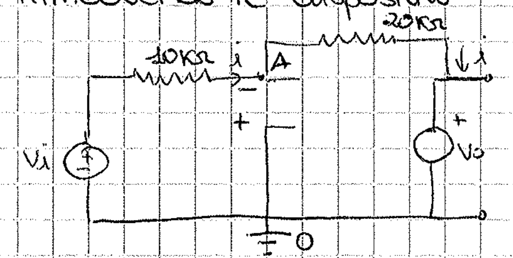
$A_c = \frac{V_o}{V_i}$

$$i = \frac{2V}{10k\Omega} = 0{,}2 \text{ mA}$$

$$V_{20\Omega} = 0{,}2 \cdot \cancel{10^{-3}} \cdot 20 \cdot \cancel{10^3} = 4V$$

$$V_o = -V_{20\Omega} = -4V \qquad \Rightarrow A_c = \frac{V_o}{V_i} = -2$$

Rimuovendo il dispositivo

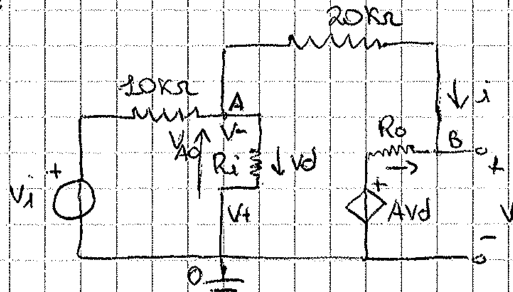


$$V^-_{(A0)} = -V_d = \frac{\frac{V_i}{10k\Omega} + \frac{V_o}{20k\Omega}}{\frac{1}{10k\Omega} + \frac{1}{20k\Omega}} = 0$$

$V_o = -2V_i$

$A_c = -2$

2) Reale



$A = 200.000$

$R_i = 2M\Omega$

$R_o = 50\Omega$

$$V_{A0} = \frac{\frac{V_i}{10^4} + \frac{2 \cdot 10^5 V_d}{50 + 20 \cdot 10^3}}{\frac{1}{10^4} + \frac{1}{20 \cdot 10^3 + 50} + \frac{1}{2 \cdot 10^6}} = -V_d$$

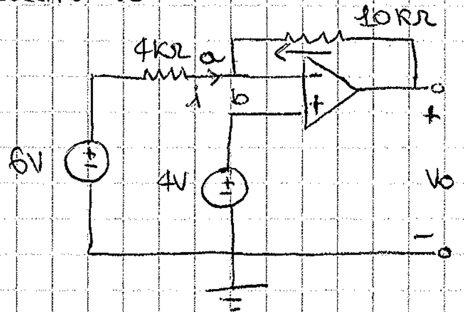
$$\frac{V_i}{10^4} + \frac{2 \cdot 10^5 V_d}{50 + 20 \cdot 10^3} = -V_d \left(\frac{1}{10^4} + \frac{1}{20 \cdot 10^3 + 50} + \frac{1}{2 \cdot 10^6} \right)$$

$$\frac{4000}{401} V_d + (1{,}50 \cdot 10^{-4}) V_d = -2 \cdot 10^{-4} \Rightarrow 9{,}98 V_d = -2 \cdot 10^{-4} \Rightarrow V_d = -20{,}06 \cdot 10^{-4} V$$

$$i = \frac{V_{A0} - AV_d}{20 \cdot 10^3 + 50} = 0{,}02 A \qquad V_o = AV_d + R_o i = -400{,}2V$$

$$A_c = -\frac{400{,}2}{2} = -200$$

Calcolare V_o



$V_a = V_b = 4V$

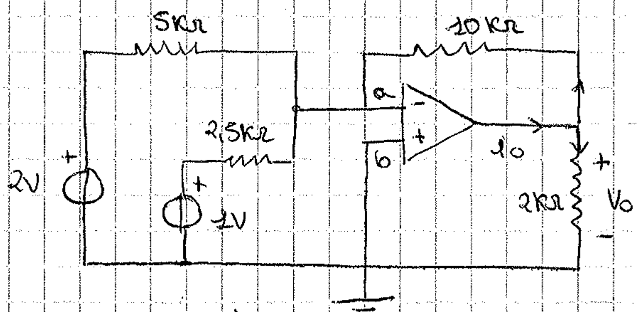
$i = \frac{6V - 4V}{4 \cdot 10^3} = 5 \cdot 10^{-4}\ A$

$V_{10} = i \cdot 10^4 = 5\ V$

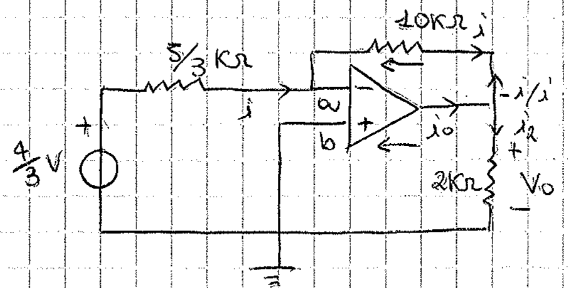
$V_o + V_{10} - 4V = 0 \Rightarrow V_o = -1V$

Oppure $V_o = \underbrace{\left(\frac{R_f}{R_1} + 1\right) V_i}_{\text{non invertente}} - \underbrace{\frac{R_f}{R_1} V_i}_{\text{invertente}} = \left(\frac{10}{4} + 1\right) 4 - \frac{10}{\cancel{4}_2} \cdot \cancel{6}^3 = 10 + 4 - \frac{30}{2} = \frac{28-30}{2} = -1V$

Calcolare i_o e V_o



$V_{TH} = \dfrac{\frac{2}{5 \cdot \cancel{10^3}} + \frac{1}{2{,}5 \cdot \cancel{10^3}}}{\frac{1}{5 \cdot \cancel{10^3}} + \frac{1}{2{,}5 \cdot \cancel{10^3}}} = \frac{4}{3} V \qquad R_{TH} = \dfrac{5 \cdot 5/2}{5 + 5/2} = \frac{5}{3}\ k\Omega$



$i = \frac{4}{\cancel{3}} \cdot \frac{\cancel{3}}{5 \cdot 10^3} = 0{,}8\ mA$

$V_{10} = 8V$

$V_o = -8V$

$i_2 = \frac{V_o}{2 \cdot 10^3} = \frac{-8}{2 \cdot 10^3} = -4\ mA$

$i_o = -i + i_2 = -0{,}8 - 4 = -4{,}8\ mA$

$V_o = -\frac{R_f}{R_1} V_i = -\overset{2}{\cancel{10}} \cdot \frac{\cancel{3}}{\cancel{5}} \cdot \frac{4}{\cancel{3}} = -8V$

$V_{10} = -V_o = 8V \qquad i = -\frac{V_{10}}{10k\Omega} = -0{,}8\ mA$

$i_2 = \frac{V_o}{2k\Omega} = -4\ mA \qquad \Rightarrow i_o = i + i_2 = -4{,}8\ mA$

$$V_A = \frac{\frac{0,2}{10^3} + \frac{V_{o1}}{10^4} + \frac{V_o}{50 \cdot 10^3}}{\frac{1}{10^3} + \frac{1}{10^4} + \frac{1}{50 \cdot 10^3}} = 0$$

$$V_B = V_o \cdot \frac{1}{1+4} = \frac{V_o}{5} \Rightarrow V_B = V_{o1}$$

$$\frac{0,2}{10^3} + \frac{V_o}{5} \cdot \frac{1}{10^4} + \frac{V_o}{50 \cdot 10^3} = 0$$

$$\cancel{4 \cdot 10^{-5}} V_o = \frac{\cancel{-2}}{\cancel{4}_{2}} \cdot \frac{\cancel{10^{-4}}}{10^{-5}} = -5V$$

Metodo ai nodi:

$V_A = 0$
$V_B = V_{o1}$
$V_C = V_o$

$$\begin{bmatrix} \frac{1}{10^3} + \frac{1}{10^4} + \frac{1}{50 \cdot 10^3} & -\frac{1}{10^4} & -\frac{1}{50 \cdot 10^3} \\ -\frac{1}{10^4} & \frac{1}{10^3} + \frac{1}{4 \cdot 10^3} & -\frac{1}{4 \cdot 10^3} \\ -\frac{1}{50 \cdot 10^3} & -\frac{1}{4 \cdot 10^3} & \frac{1}{50 \cdot 10^3} + \frac{1}{1,35 \cdot 10^3} \end{bmatrix} \begin{bmatrix} 0 \\ V_{o1} \\ V_o \end{bmatrix} = \begin{bmatrix} \frac{0,2}{10^3} \\ 0 \\ 0 \end{bmatrix}$$

① $-10^{-4} V_{o1} - 2 \cdot 10^{-5} V_o = 2 \cdot 10^{-4}$

$$(2 \cdot 10^{-5}) V_o = -2 \cdot 10^{-4} - 10^{-4} V_{o1}$$

② $1,35 \cdot 10^{-3} V_{o1} - (4 \cdot 10^{-3}) V_o = 0$

$$V_{o1} = \frac{4 \cdot 10^{-3}}{1,35 \cdot 10^{-3}} V_o$$

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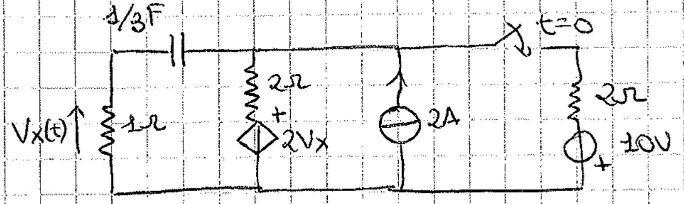
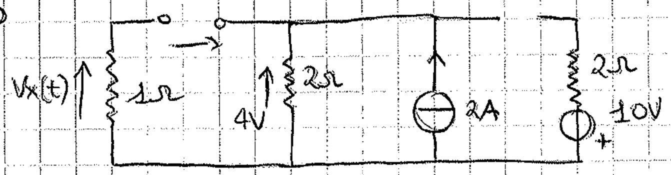
Calcolare e disegnare $V_x(t)\ \forall t$

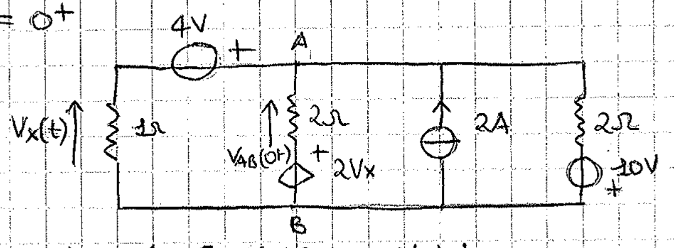


- $t<0$



$V_x(0^-)=0 \qquad V_C(0^-)=-4V$

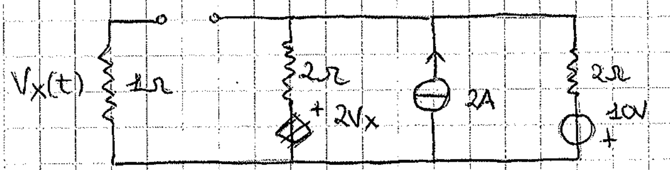
- $t=0^+$



$$V_{AB}(0^+)=\frac{4-5+V_x+2}{2}=\frac{V_x+1}{2}$$

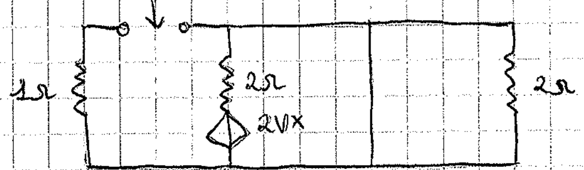
$$V_x+4V-\frac{V_x}{2}-\frac{1}{2}=0 \Rightarrow \frac{V_x}{\cancel{2}}=-\frac{7}{\cancel{2}} \Rightarrow V_x(0^+)=-7V$$

- $t\to+\infty$



$V_{x\infty}(t)=0V$

R_T



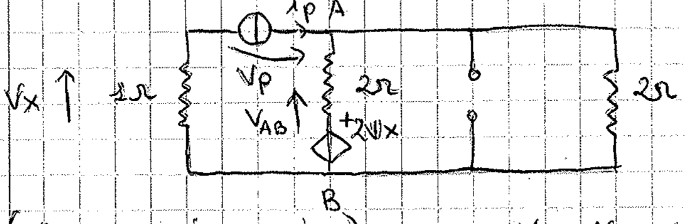
Oppure con V_p:

$$V_{AB}=\frac{V_p+\frac{2V_x}{2}}{2}=\frac{V_p}{2}+\frac{V_x}{2}$$

$V_x=-i_p$

$V_x+V_p-\frac{V_p}{2}-\frac{V_x}{2} \Rightarrow V_x=-V_p$

$\Rightarrow R_T=1\Omega$



$$V_{AB}=\frac{i_p+\frac{2V_x}{2}}{1}=i_p+V_x$$

$(V_x=-1i_p=-i_p)$

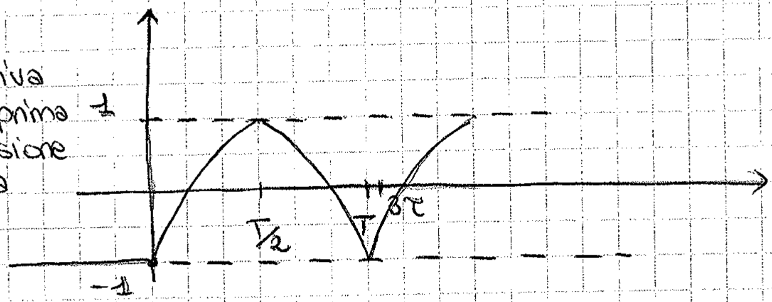
$V_p=V_{AB}-V_x=i_p+\cancel{V_x}-\cancel{V_x}$

$R_{TH}=1\Omega$

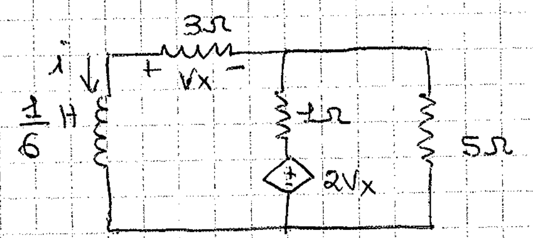
Con $f = 100\,KHz$

$$T = \frac{1}{f} = \frac{1}{100 \cdot 10^3} = 1 \cdot 10^{-5}$$

Non si arriva a regime prima dell'inversione dell'onda



Dato $i(0) = 5A$ calcolare $i(t)$

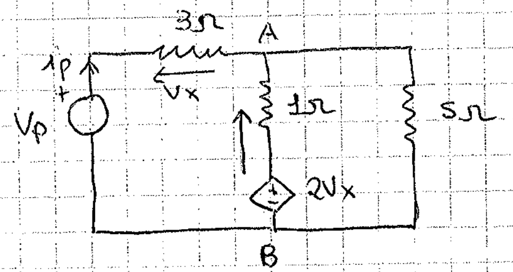


$i(0) = 5A$

$i_\infty = 0A \rightarrow$ non ci sono termini forzanti

Se non ci sono generatori indipendenti e il circuito rimane chiuso $i_\infty = 0A$ $\Rightarrow$ come fosse RL autonomo

R_T



$$V_{AB} = \frac{\frac{V_p}{3} + 2V_x}{\frac{1}{3} + 1 + \frac{1}{5}} = \left(\frac{V_p}{3} + 2V_x\right)\frac{15}{23}$$

$V_x = 3\,i_p$

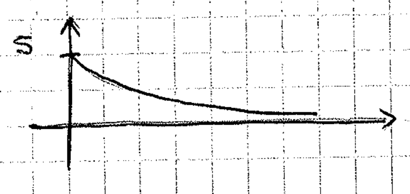
$V_p - V_x - V_{AB} = 0$

$V_p - V_x - \frac{15}{69}V_p - \frac{30}{23}V_x = 0$

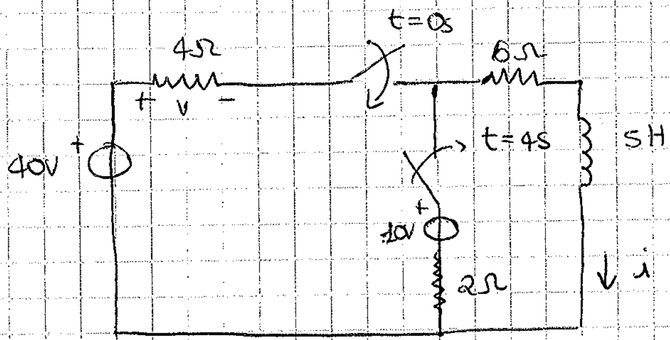
$\frac{18}{\cancel{23}}V_p = \frac{53}{\cancel{23}}V_x \Rightarrow V_p = \frac{53}{\cancel{18}\,6}(\cancel{3}\,i_p) \Rightarrow R_T = \frac{53}{6}\,\Omega$

$$\tau = \frac{L}{R} = \frac{1}{6} \cdot \frac{6}{53} = \frac{1}{53}\ \text{sec}$$

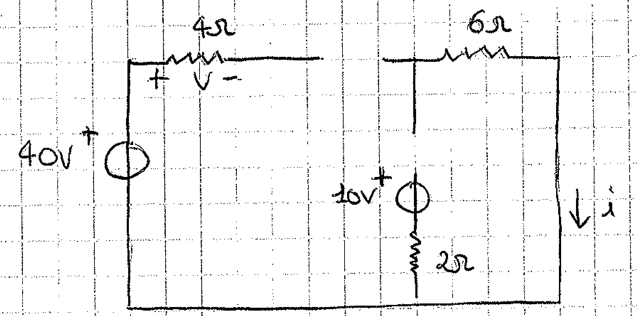
$i(t) = 5e^{-53t}\ A$



Calcolare e disegnare $v(t)$ e $i(t)$ $\forall t$

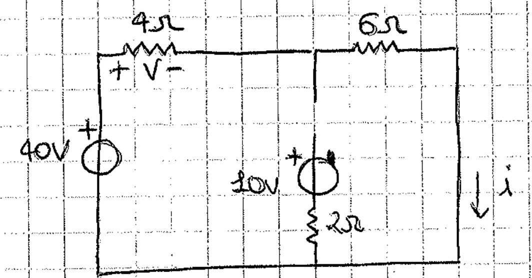


- $t<0$



$i(0^-) = 0\,A$
$V(0^-) = 0\,V$

- $0<t<4$



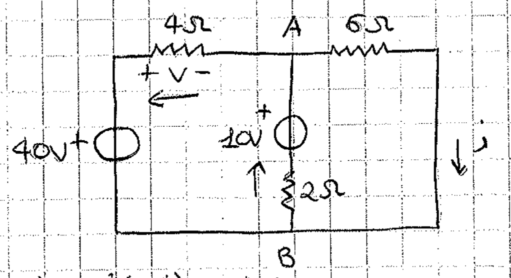
$i(0^-) = i(0^+) = 0A$

$V(0^+) = 40 \cdot \frac{4}{10} = 16V$

$i(4^-) = \frac{40}{10} = 4A$

$V(0^+) = V(4^-)$

- $t>4$ $(t \to +\infty)$



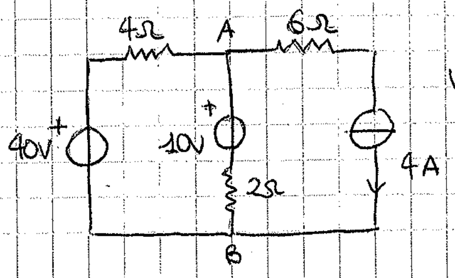
$V_{AB(\infty)} = \dfrac{10+5}{\frac{1}{4}+\frac{1}{2}+\frac{1}{6}} = \frac{180}{11}\,V$

$i_\infty = \frac{V_{AB}}{6} = \frac{30}{11}\,A = 2{,}72\,A$

$i(4^-) = i(4^+) = 4A$

$40 - V - V_{AB} = 0 \Rightarrow V_\infty = 40 - \frac{180}{11} = \frac{260}{11}V$ $23{,}63\,V$

$t=4^+$



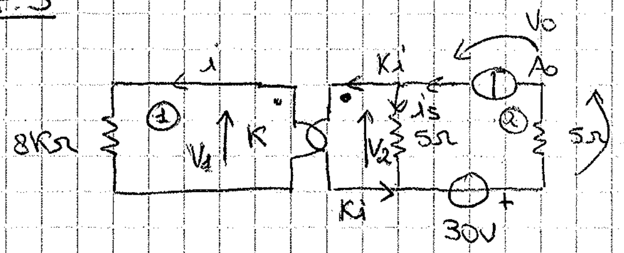
$V_{AB}(4^+) = \dfrac{10+5-4}{\frac{1}{4}+\frac{1}{2}} = \frac{44}{3}\,V$

$V_{4^+} = 40 - V_{AB} = 40 - \frac{44}{3} = \frac{76}{3}\,V$ $(25{,}33\,V)$

$R_A = 18\ \Omega$

$$V_{AB} = \frac{\frac{100}{10} - \frac{60}{6} + \frac{120}{18}}{\frac{1}{10} + \frac{1}{6} + \frac{1}{18}} = \frac{600}{29}\ V$$

4.3



$K = 20$
$i = 10\,mA$ $\quad A_0$?
Calcolare potenza erogata da A_0

$V_1 = 8K\Omega \cdot 10\,mA = 80\,V$

$V_2 = \frac{V_1}{K} = 4\,V$

$i_S = -Ki + A_0$

$V_2 = 5 \cdot i_S = 5(-Ki + A_0) = -5Ki + 5A_0$

$4 = -5Ki + 5A_0 \Rightarrow 4 = -1 + 5A_0 \Rightarrow A_0 = 1A$

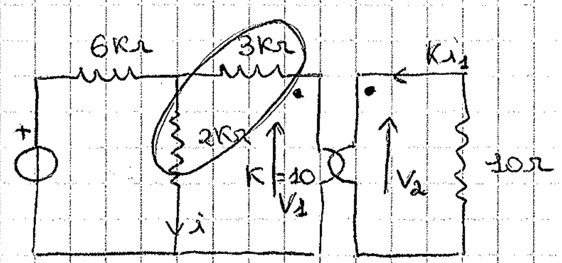
KVL: $V_2 - V_0 - V_3 - 30 = 0$ $\qquad V_3 = -5A_0 = -5V$

$V_0 = V_2 - V_3 - 30 \Rightarrow -21\,V$

$P = Vi = -21\,W$

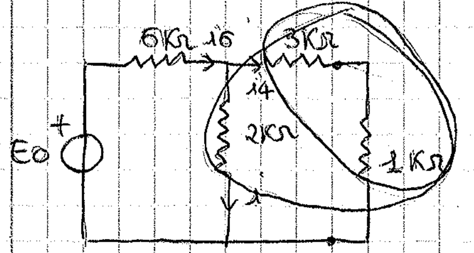
$P = V_2 \cdot Ki = \frac{V_1}{\cancel{K}} \cdot \cancel{Ki}$ $\qquad i = 22{,}72\,mA$

4.4



$i = 10\,mA$
E_0 ? $\quad P_{10\Omega}$?

$R_{eq} = K^2 \cdot 10\Omega = 1\,K\Omega$

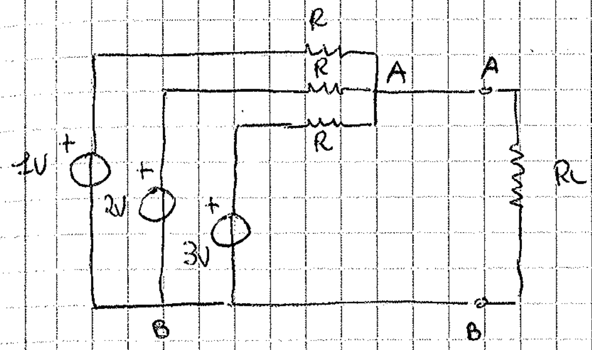


$V_{2K\Omega} = 20\,V$
$R_A = 4\,K\Omega$
$R_B = \frac{4}{3}\,K\Omega$

$i_{4K\Omega} = \frac{20}{4 \cdot 10^3} = \frac{1}{200}\,A$

$i_{6K\Omega} = i_{4K\Omega} + i = \frac{3}{200}\,A$

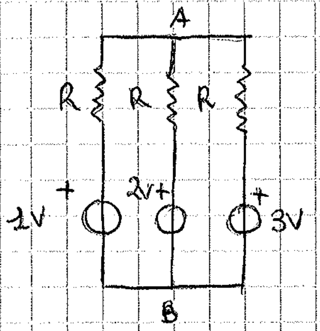
4.6



R tale da avere $p = 3mW$ su R_L

$R_L = R_T$

$$p = \frac{V_{th}^2}{4R_{th}} \Rightarrow R_{th} = \frac{V_{th}^2}{4p}$$



$$V_{AB} = \frac{\frac{1}{R} + \frac{2}{R} + \frac{3}{R}}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R}} = \frac{\frac{\cancel{6}^2}{\cancel{R}}}{\frac{\cancel{3}}{\cancel{R}}} = 2V$$

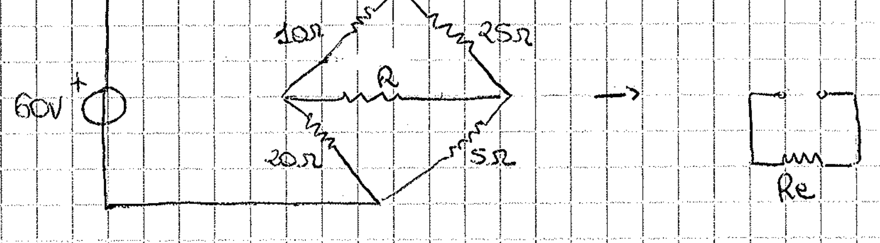
$$R_{th} = \frac{R \cdot R}{R+R} \parallel R = \frac{R^{\cancel{2}}}{2\cancel{R}} \parallel R = \frac{\frac{R}{2} \cdot R}{\frac{R}{2} + R} =$$

$$= \frac{R^{\cancel{2}}}{\cancel{2}} \cdot \frac{\cancel{2}}{3\cancel{R}} = \frac{R}{3}$$

$$R_{th} = \frac{\cancel{4}}{\cancel{4} \cdot 3 \cdot 10^{-3}} = \frac{1000}{3}$$

$$\frac{R}{\cancel{3}} = \frac{1000}{\cancel{3}} \Rightarrow R = 1k\Omega$$

4.7 → Max potenza al carico R



$$V_{20} = 60 \cdot \frac{20}{30} = 40V$$

$$V_5 = 60 \cdot \frac{5}{30} = 10V$$

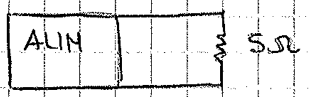
$\rightarrow V_R = V_{20} - V_5 = 30V$

$$R_A = \frac{10 \cdot 20}{30} = \frac{20}{3}\Omega \qquad R_B = \frac{25 \cdot 5}{30} = \frac{25}{6}\Omega$$

$$R_{eq} = R_A + R_B = \frac{65}{6}\Omega$$

? **4.9**

$p = 20W$
potenza dissipata su 15Ω



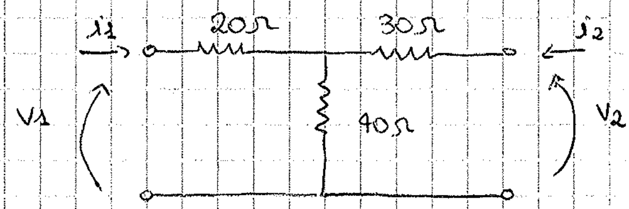
$V_{ALIM} = V_{TH} \Rightarrow V_{TH} = \sqrt{p \cdot R_{TH} \cdot 4} = 20\ V$

$V_{15\Omega} = 20 \cdot \frac{15}{15+5} = 15\ V$

$P_{15\Omega} = \frac{V^2}{R} = \frac{(15)^{\cancel{2}}}{\cancel{15}} = 15\ W$

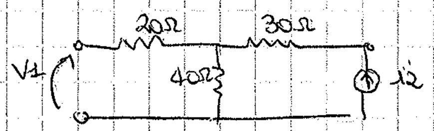
4.10

Matrice R



$R_{11} = \left.\frac{V_1}{i_1}\right|_{i_2=0} = 60\Omega$

$R_{12} = \left.\frac{V_1}{i_2}\right|_{i_1=0} = 40\Omega$



$V_1 = \cancel{70}\, i_2 \cdot \frac{40}{40+30} = 40 i_2$

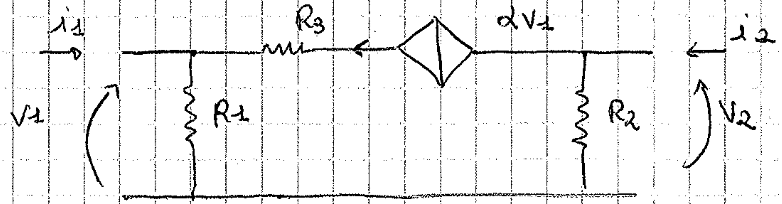
$R_{21} = \left.\frac{V_2}{i_1}\right|_{i_2=0} = 40\Omega$

$R_{22} = 70\Omega$

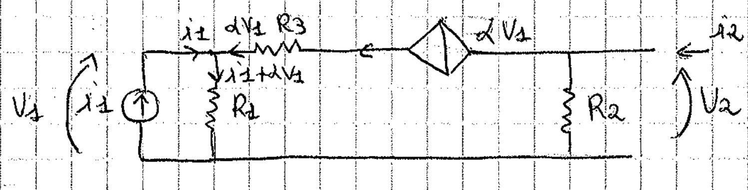
$$\begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix}\Omega$$

4.11

Matrice R?



$R_{11} = \left.\frac{V_1}{i_1}\right|_{i_2=0}$



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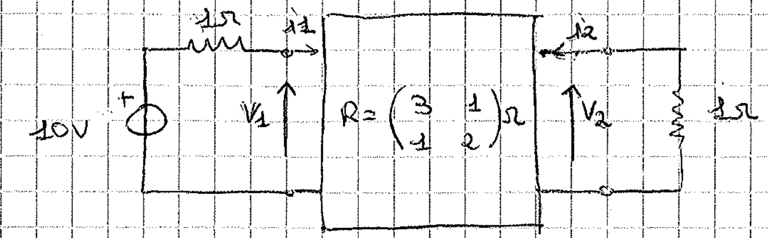
4.12



Pass?

$$\left.\begin{aligned} V_1 &= 3i_1 + i_2 = 10 - i_1 \\ V_2 &= i_1 + 2i_2 = -i_2 \end{aligned}\right\} \rightarrow -4i_1 + 10 = i_2 = -\frac{10}{11}\,A$$

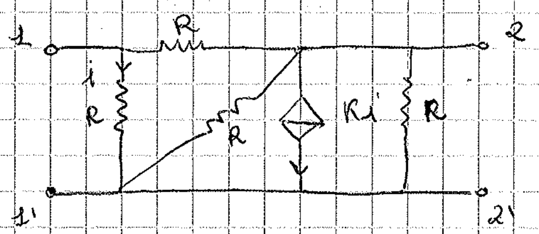
$$i_1 - 8i_1 + 20 - 4i_1 + 10 = 0$$

$$+11 i_1 = +30 \rightarrow i_1 = \frac{30}{11}\,A$$

$$V_1 = 10 - i_1 = \frac{80}{11}\,V \qquad V_2 = \frac{10}{11}\,V$$

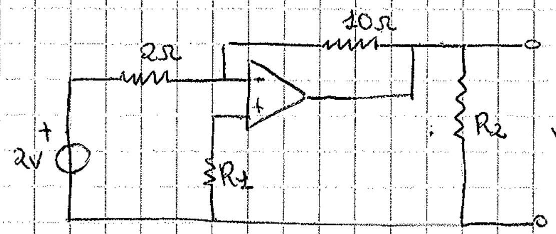
$$p = V_1 i_1 + V_2 i_2 = \frac{80}{11}\cdot\frac{30}{11} + \frac{10}{11}\left(-\frac{10}{11}\right) = \frac{2300}{121}\,W$$

4.13



Matrice G

4.17

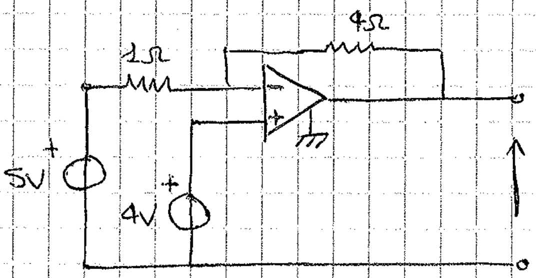


Vu?

→ invertente

$$V_U = -\frac{R_f}{R_1} \cdot V_i = -\frac{10}{\cancel{2}} \cdot \cancel{2} = -10V$$

4.18

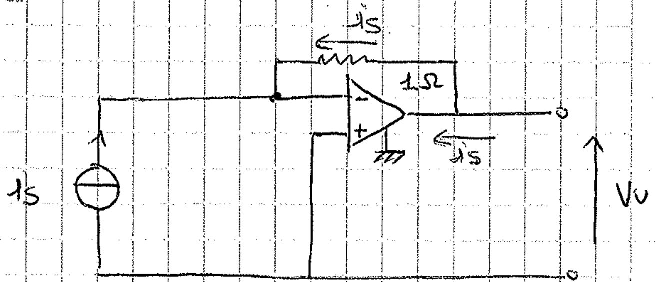


Vu?

→ invertente + non invertente

$$V_U = (1+4)4 - 4 \cdot 5 = 0V$$

4.19



$$\frac{V_U}{i_S} = ?$$

$$V_U = -i_S \Rightarrow \frac{V_U}{i_S} = -1$$

---

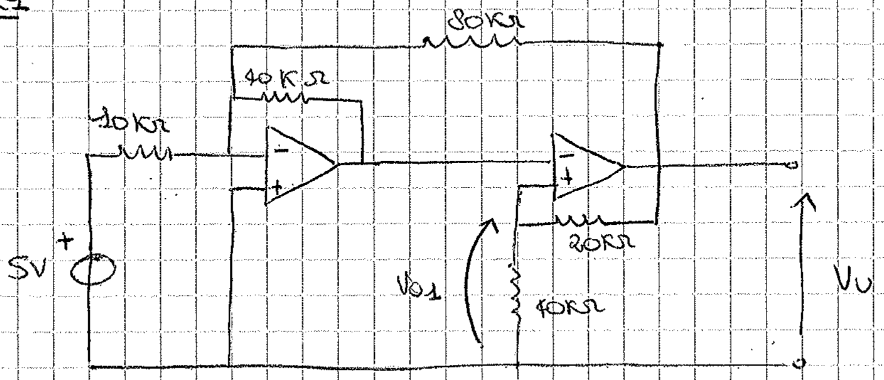
* $V_A = \frac{R_1}{R_2+R_1} V_{O1}$ portitore di tensione $V_A = V_1 \Rightarrow V_{O1} = V_1 \frac{R_1+R_2}{R_1}$

$$V_B = V_2 = \frac{\frac{V_{O1}}{R_3} + \frac{V_{OUT}}{R_4}}{\frac{1}{R_3}+\frac{1}{R_4}} \rightarrow \text{Millman} \Rightarrow V_2 = \frac{\frac{R_1+R_2}{R_1}\frac{V_1}{R_3} + \frac{V_{OUT}}{R_4}}{\frac{1}{R_3}+\frac{1}{R_4}}$$

$$\left(\frac{1}{R_3}+\frac{1}{R_4}\right)V_2 = \left(1+\frac{R_2}{R_1}\right)\frac{V_1}{R_3} + \frac{V_{OUT}}{R_4}$$

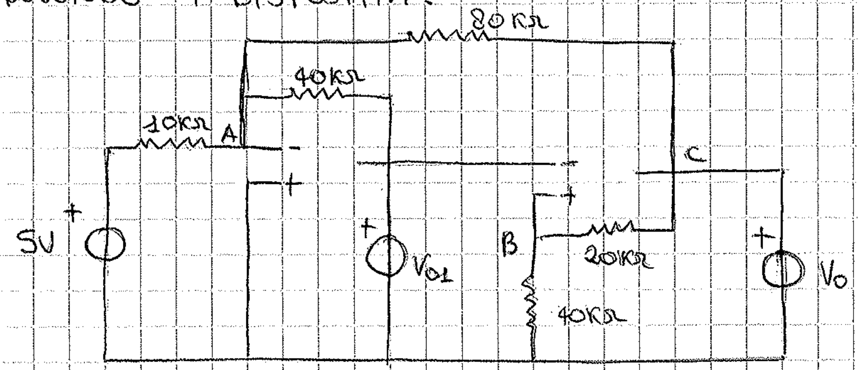
$$V_{OUT} = R_4\left[\left(\frac{1}{R_3}+\frac{1}{R_4}\right)V_2 - \frac{1}{R_3}\left(1+\frac{R_2}{R_1}\right)V_1\right] \Rightarrow \left[\left(1+\frac{R_2}{R_1}\right)V_1 - V_2\right]\left(-\frac{R_4}{R_3}\right) + V_2$$

4.21



V_{o1} ?

RIMUOVENDO I DISPOSITIVI:



$$V_A = \frac{\frac{5}{10\cdot 10^3} + \frac{V_{o1}}{40\cdot 10^3} + \frac{V_o}{80\cdot 10^3}}{\frac{1}{10\cdot 10^3} + \frac{1}{40\cdot 10^3} + \frac{1}{80\cdot 10^3}} = V_- = 0$$

$$\frac{5}{10\cdot 10^3} + \frac{V_{o1}}{40\cdot 10^3} + \frac{V_o}{80\cdot 10^3} = 0$$

$$V_B = V_{o1} = V_o \cdot \frac{24\cancel{0}\cdot \cancel{10^3}}{36\cancel{0}\cdot \cancel{10^3}} = \frac{2}{3} V_o$$

$$\frac{5}{10\cdot \cancel{10^3}} + \frac{2}{3} V_o \frac{1}{40\cdot \cancel{10^3}} + \frac{V_o}{80\cdot \cancel{10^3}} = 0$$

$$\frac{7}{240} V_o = -\frac{5}{10} \quad \Rightarrow V_o = -\frac{120}{7} V$$

$$V_{o1} = \frac{2}{3} V_o = -\frac{80}{7} V = -11,43 V$$

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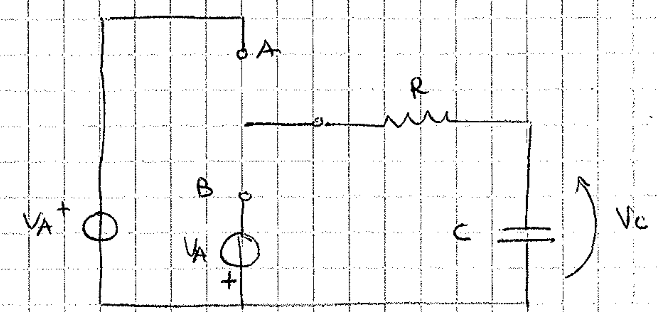
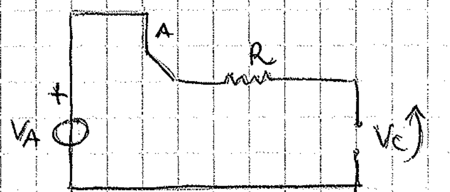
4.23



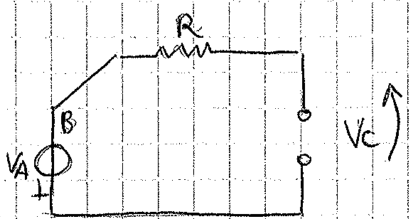
① Transizione da A a B

- $t<0$



$V_C(0^-) = V_C(0^+) = V_A$

- $t>0$

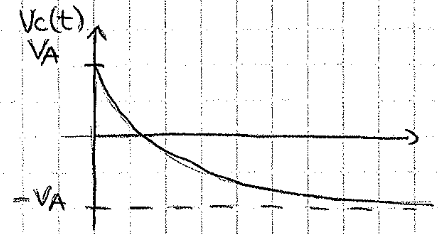


$V_{C\infty} = -V_A$

$\tau = RC$

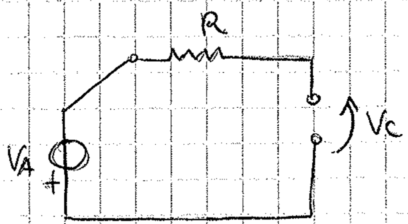
$$V_C(t) = \begin{cases} V_A & t \leq 0 \\ (V_A + V_A)e^{-t/\tau} - V_A & t \geq 0 \end{cases}$$

$2V_A e^{-t/\tau} - V_A$



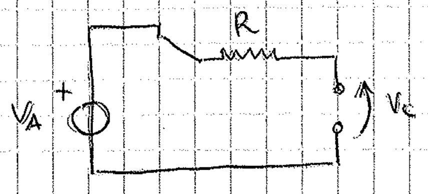
② Transizione da B ad A

- $t<0$



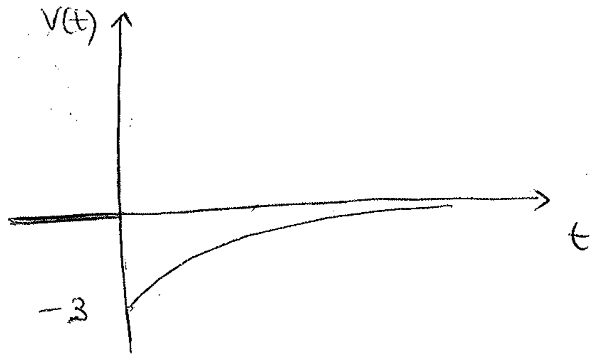
$V_C(0^-) = V_C(0^+) = -V_A$

- $t>0$



$V_{C\infty} = V_A$

$$V(t) = \begin{cases} 0 & t < 0 \\ -3e^{-\frac{1}{5}t} & t > 0 \end{cases}$$



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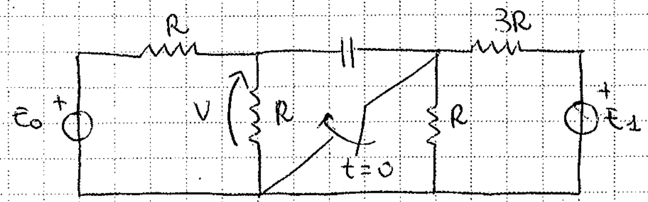
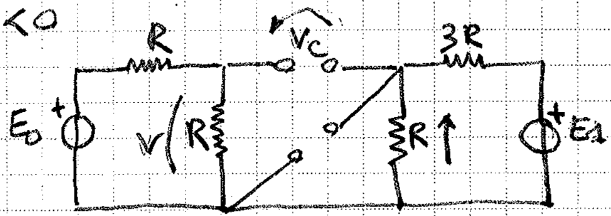
4.24

Calcolare e disegnare $v(t)\ \forall t$



$E_0 = E_1 = 12V$
$R = 4\,k\Omega$
$C = 1\,\mu F$

$t<0$



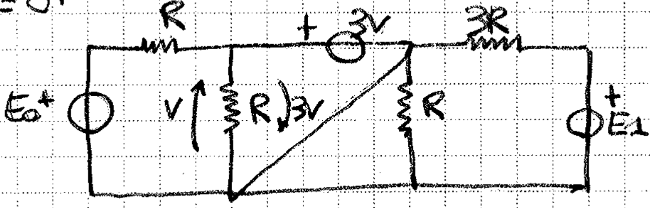
$$V(0^-) = 12 \cdot \frac{4k\Omega}{8k\Omega} = 6V$$

$$V_C(0^-) = V_C(0^+) = V - V_R = 3V$$

$$V - V_C - V_R = 0 \Rightarrow V_C = V - V_R$$

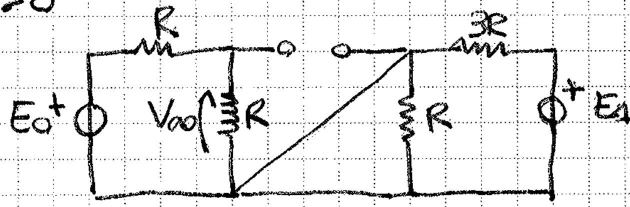
$$V_R = 12 \cdot \frac{4k\Omega}{16k\Omega} = 3V$$

$t=0^+$



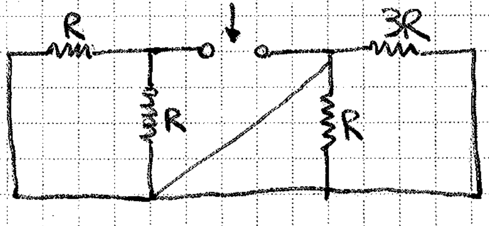
$$V(0^+) = 12 \cdot \frac{4k\Omega}{8k\Omega} - 3V = 3V$$

$t>0$



$$V(\infty) = 12 \cdot \frac{4k\Omega}{8k\Omega} = 6V$$

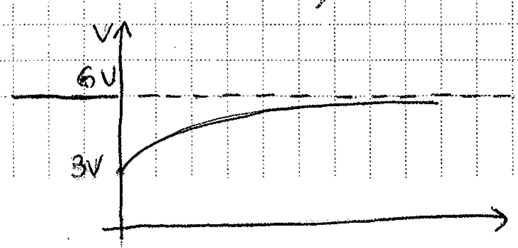
R_{TH}



$$R_{TH} = \frac{R \cdot R}{2R} = \frac{R}{2} = 2k\Omega$$

$$\tau = RC = 2k\Omega \cdot 1\mu F = 2ms$$

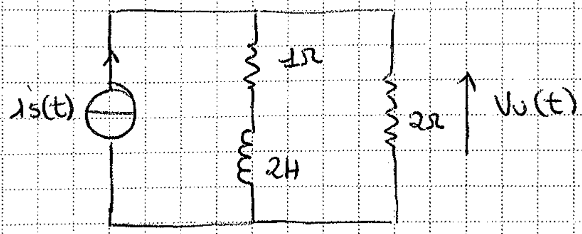
$$V(t) = \begin{cases} 6V & t<0 \\ (3-6)e^{-\frac{t}{2\cdot10^{-3}}} + 6 \Rightarrow -3e^{-\frac{t}{2\cdot10^{-3}}} + 6\ V \end{cases}$$



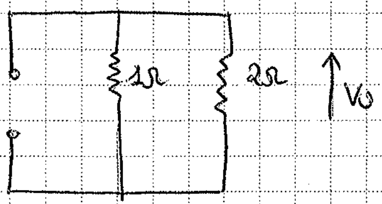
[redacted]

4.28

Calcolare e disegnare $V_u(t)$
data $i_s(t) = u(t)$ gradino unitario



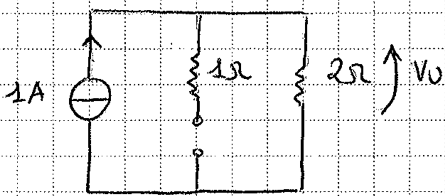
- $t < 0$



$V_u(0^-) = 0\ V$

$i_L(0^-) = 0\ A$

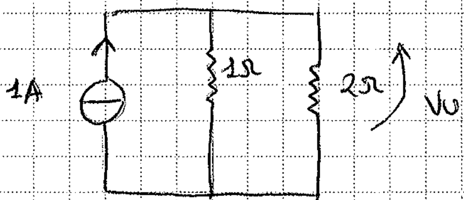
- $t = 0^+$



$i_L(0^-) = i_L(0^+) = 0\ A$

$V_u(0^+) = 2V$

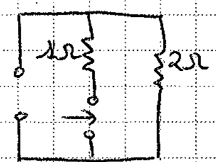
- $t \to +\infty$



$i_{2\Omega} = 1 \cdot \frac{1}{2+1} = \frac{1}{3}\ A$

$V_{u\infty} = \frac{2}{3}\ V$

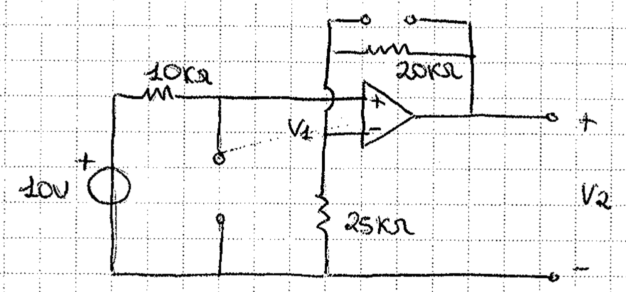
R_T



$R_T = 3\Omega$

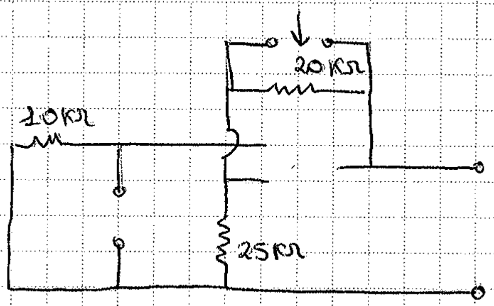
$\tau = \frac{L}{R} = \frac{2}{3}$ sec

$t \to +\infty$



$$V_{2\infty} = \left(1 + \frac{\cancel{20}^{\,4}}{\cancel{25}_{\,5}}\right) \cdot 10 = 18\,V$$

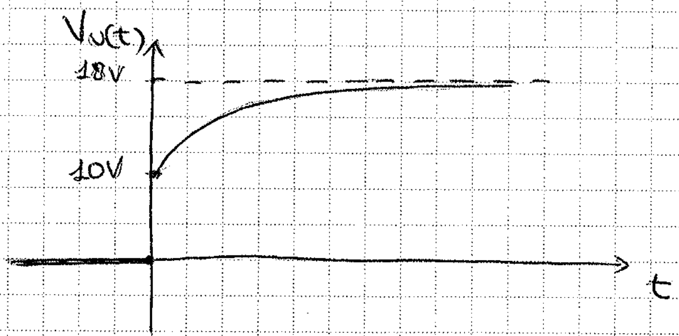
R_T



$R_T = 20\ K\Omega$

$\tau = RC = 1\ sec$

$$V_U(t) = \begin{cases} 0 & t < 0 \\ (10 - 18)e^{-t} + 18 \Rightarrow -8e^{-t} + 18 & t > 0 \end{cases}$$



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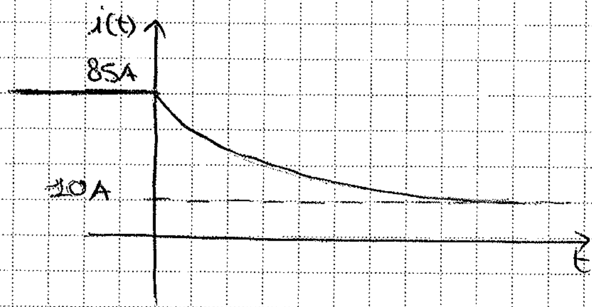
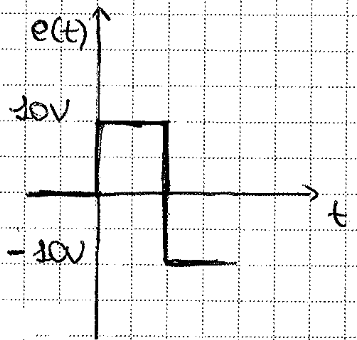
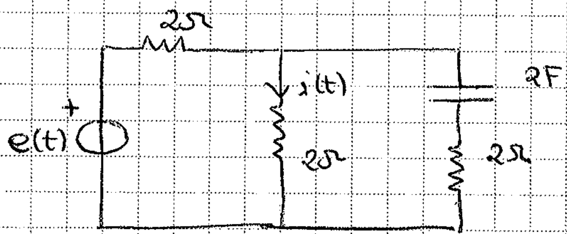
$$i(t) = \begin{cases} 85A & t \leq 0 \\ (85-10)e^{-4t} + 10 & t \geq 0 \\ \downarrow & \\ (75e^{-4t} + 10)A & \end{cases}$$



4.31

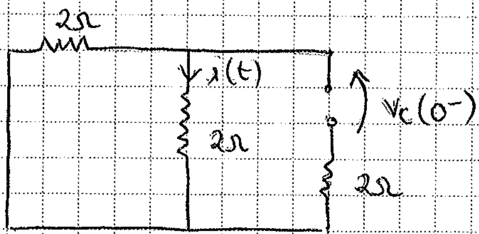
Calcolare e disegnare $i(t)$ $\forall t$





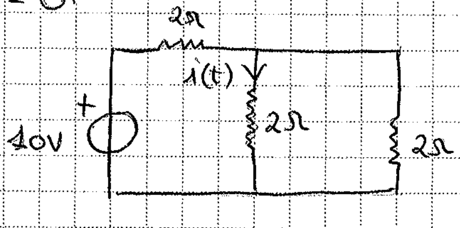
T = 2 sec

- $t<0$



$i(0^-) = 0\,A$
$V_c(0^-) = 0\,V$

- $t = 0^+$

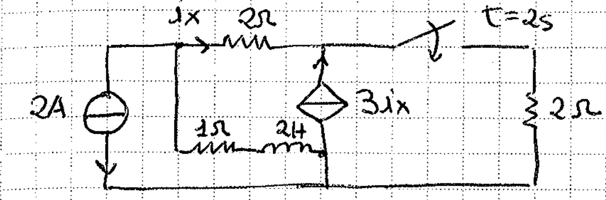


$V_c(0^-) = V_c(0^+) = 0V$

$R_A = 1\Omega$
$R_B = 3\Omega$

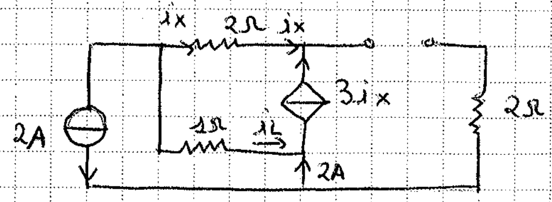
$i(0^+) = \frac{10}{3} A$

4.32



Calcolare e disegnare $i_x(t)\ \forall t$

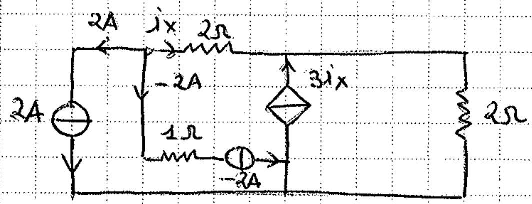
- $t < 2$ sec



$$-i_x - 3i_x = 0 \Rightarrow i_x = 0$$

$$-i_L - 2A + \cancel{3i_x} = 0$$
$$\Rightarrow i_{L_{2^-}} = -2A$$

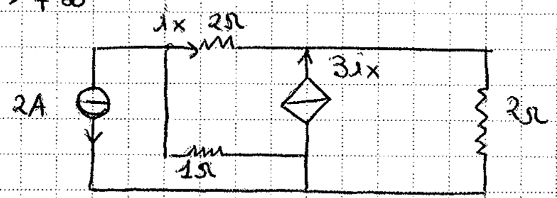
- $t > 2^+$



$$i_L(2^-) = i_L(2^+) = -2A$$

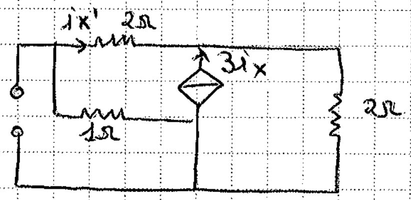
$$i_x + \cancel{2A} - \cancel{2A} = 0 \rightarrow i_x(2^+) = 0$$

- $t \to +\infty$



Sovrapposizione degli effetti: $i_x = i_x' + i_x''$

1)



$$i_x' = -\frac{2}{3+2} \cdot 3i_x = -\frac{2}{5} \cdot 3i_x$$

Esempio 1

Calcolare i fasori delle seguenti grandezze

$i = 6\cos(50t - 40°)\ A \quad \rightarrow \hat{I} = 6 e^{-j40°}$

$\hat{I} = 4{,}59 - j3{,}85$

$v = -4\sin(30t + 50° - 90°)\ V$

$= -4\sin(30t - 40°)\ V \qquad \hat{V} = -4e^{-j40°}\ V$

$\hat{V} = -3{,}06 + j2{,}57$

Esempio 2

Calcolare le grandezze nel dominio del tempo dei seguenti fasori

$\hat{V} = -10\angle 30°\ V \quad \rightarrow v(t) = -10\cos(\omega t + 30°)$

$\hat{I} = j(5 - j12)\ A \rightarrow \hat{I} = 5j + 12$

$\rho = \sqrt{25 + 144} = 13$

$\phi = tg^{-1}\frac{5}{12} = 22{,}62° \quad \rightarrow i(t) = 13\cos(\omega t + 22{,}62°)$

Esempio 3

Calcolare $i(t)$

$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75°)$

$4\hat{I} + \frac{8\hat{I}}{j\omega} - 3j\omega\hat{I} = 50\cos(2t + 75°)$

$4\hat{I} + \frac{8\hat{I}}{2j} - 6j\hat{I} = 50e^{j75°}$

$\hat{I}(4 - 4j - 6j) = 50e^{j75°} \rightarrow \hat{I} = \frac{50e^{j75°}}{4 - j10} = \frac{12{,}94 + j48{,}30}{4 - j10}\frac{(4 + j10)}{(4 + j10)} =$

$50e^{j75°} = 12{,}94 + j48{,}30 \qquad = \frac{51{,}76 + 129{,}4j + 193{,}2j - 483}{16 + 100} = -3{,}71 + j2{,}78$

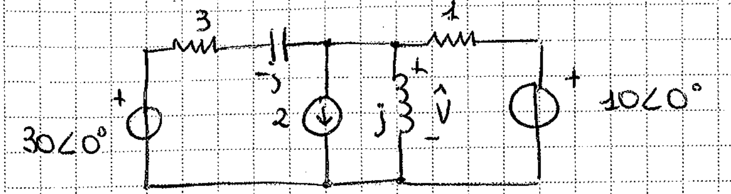
$\qquad a \qquad b$

$\rho = \sqrt{a^2 + b^2} = 4{,}642$

$i(t) = 4{,}642\cos(2t + 143{,}2°)\ A$

Esempio

Ricavare il fasore $\hat{V}$ nel circuito simbolico



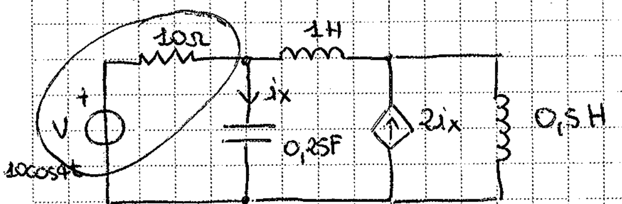
$$\hat{V} = \frac{\frac{30}{3-j} - 2 + 10}{\frac{1}{3-j} + \frac{1}{j} + 1} = \frac{\frac{30+24-8j}{3-j}}{\frac{\cancel{j}+3-\cancel{j}+(3-j)j}{(3-j)j}} = \frac{54-8j}{\cancel{3-j}} \cdot \frac{\cancel{(3-j)}j}{3+3j+1} =$$

$$= \frac{54j+8}{4+3j}$$

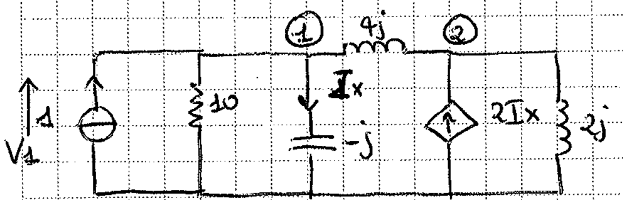
$$|\hat{V}| = \frac{\sqrt{54^2+8^2}}{\sqrt{4^2+3^2}} = 10{,}9\,V \qquad \hat{V} = 10{,}9\,e^{44{,}7°}$$

$$\phi = \operatorname{arctg}\left(\frac{54}{8}\right) - \operatorname{arctg}\left(\frac{3}{4}\right) = 44{,}7°$$

Esempio



Calcolare la corrente $i_x(t)$ in regime



$$-\frac{j}{\omega C} = -j$$

$$\begin{bmatrix} \frac{1}{10} + \frac{1}{-j} + \frac{1}{4j} & -\frac{1}{4j} \\ -\frac{1}{4j} & \frac{1}{2j} + \frac{1}{4j} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2I_x \end{bmatrix} \qquad I_x = \frac{V_1}{-j}$$

*

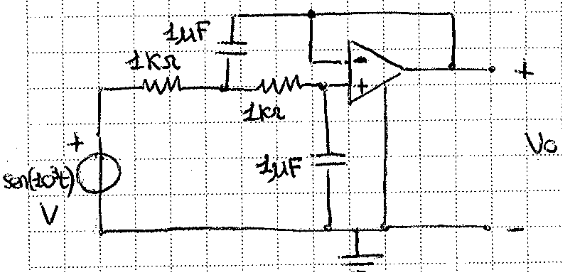
$$|I_x| = \frac{\sqrt{30^2}}{\sqrt{5^2+3^2}} = 5,145\ A$$

$$\phi = -\operatorname{arctg}\left(\frac{-3}{5}\right) = 31°$$

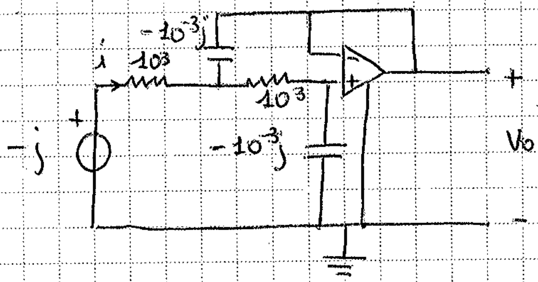
$$i_x(t) = 5,145 \cos(4t + 31°)\ A$$

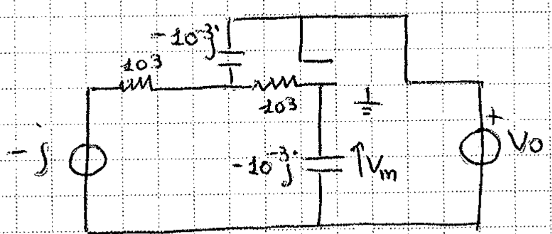
Esempio

Ricavare $V_o(t)$

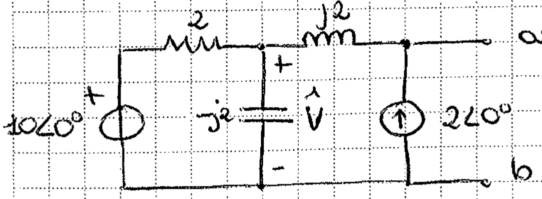


$$\operatorname{sen}(10^3 t) = \cos(10^3 t - 90°) \longrightarrow -j$$





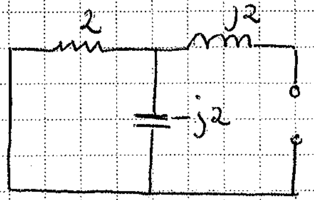
Esempio



Ricavare il circuito equivalente di Thevenin tra i morsetti a e b

$$V_m = V_T = \frac{5\frac{10}{2} + 2}{\frac{1}{2} - \frac{1}{j2}} = 7\left(\frac{2j}{j-1}\right) = \frac{14j}{j-1} \cdot \frac{-j-1}{-j-1} = \left(\frac{14-14j}{1+1}\right) = 7-7j$$

$Z_T \Rightarrow$



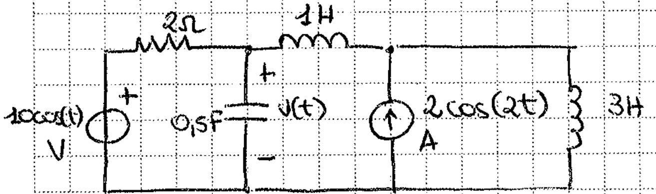
$$Z_T = (2 // -j) + j2$$

$$Z_T = \frac{2(-j2)}{2-j2} + j2 = \frac{-4j}{2-j2} + j2 =$$

$$= \frac{\cancel{-4j} + \cancel{4j} + 4}{2-2j} = \frac{4}{2-2j} \cdot \frac{(2+2j)}{(2+2j)}$$

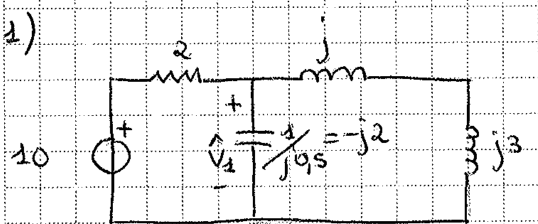
$$= \frac{8+8j}{4+4} = 1+j$$

Esempio - sorgenti sinusoidali non isofrequenziali



calcolare v(t)

1)



$\omega_1 = 1$ rad/s

$$\hat{V}_1 = \frac{5}{\frac{1}{2} + \frac{1}{j+j3} - \frac{1}{j2}} = \frac{5}{\frac{1}{2} + \frac{1}{4j} - \frac{1}{j2}} = 5\left(\frac{4j}{2j+1-2}\right)$$

$$= \frac{20j}{2j+1} \cdot \frac{(-2j-1)}{(-2j-1)} = \frac{40-20j}{4+1} = 8-j4$$

$$|\hat{V}_1| = \sqrt{64+16} = 4\sqrt{5} \text{ V}$$

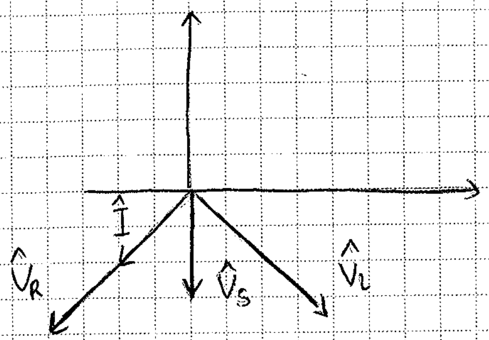
$$\rightarrow \hat{V}_1 = 4\sqrt{5} \angle -26{,}56°$$

$$\phi = \text{arctg}\left(-\frac{41}{82}\right) = -26{,}56°$$

$$V_R = \frac{-j10 \cdot 250}{250+250j} = \frac{-2500j}{250+250j}$$

$$|\hat{V}_R| = 7{,}07\ V$$

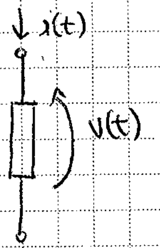
$$\phi = 90° - 45° = -135°$$



$i(t)$ è in ritardo di 45° rispetto a $V_S(t)$

Esempio

Potenza attiva e ~~reattiva~~ istantanea sul seguente bipolo



$$v(t) = 80\cos(10t + \overset{\theta_v}{20°})$$

$$i(t) = 15\sin(10t + 60°)$$

$$i(t) = 15\cos(10t - \overset{\theta_i}{30°})$$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$$= \frac{1}{2}\, 80 \cdot 15 \cos(50°) + \frac{1}{2}\, 80 \cdot 15 \cos(20t - 10°)$$

$$= 385{,}7 + 600\cos(20t - 10°)$$

$$P = 385{,}7\ W$$

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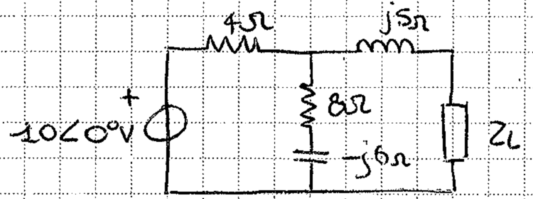
Esempio

Z_L tale da avere la max trasf. di potenza. Potenza max.



$$\hat{Z}_{eq} = [(8-j6) // 4] + j5$$

$$= \frac{(8-j6)\cdot 4}{12-j6} + j5 = \frac{32-24j}{12-j6} + j5 = \frac{32-24j+60j+30}{12-j6} = \frac{62+36j}{12-j6} = \frac{31+18j}{6-j3} =$$

$$= \frac{31+18j}{6-j3} \cdot \frac{(6+j3)}{(6+j3)} = \frac{186+93j+108j-54}{36+9} = \frac{132+201j}{45} = 4,46j + 2,933$$

$$\hat{Z}_{eq} = \hat{Z}_{TH} = \hat{Z}_L$$

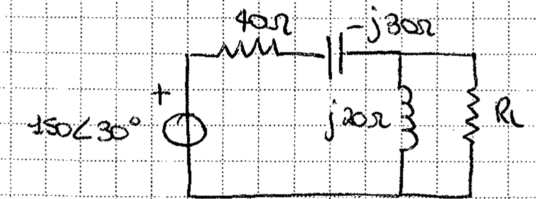
$$V_{TH} = 10 \cdot \frac{8-j6}{4+8-j6} = \frac{80-60j}{12-j6} = \frac{40-30j}{6-j3} \cdot \frac{(6+j3)}{(6+j3)} = \frac{240+120j-180j+90}{45} =$$

$$= \frac{330-60j}{45} \Rightarrow |\hat{V}_{TH}| = 7,453\,V$$

$$P_{max} = \frac{|V_{TH}|^2}{8\,R_{TH}} = 2,368\ W$$

Esempio

R_L tale da avere la max trasf di potenza. Potenza max.



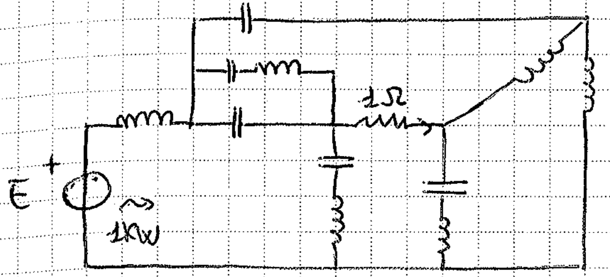
$$\hat{Z}_{TH} = \frac{(40-j30)\,j20}{40-j30+j20} = \frac{800j+600}{40-j10} = \frac{80j+60}{4-j} \cdot \frac{(4+j)}{(4+j)} = \frac{320j-80+240+60j}{17} = \frac{380j+160}{17} =$$

$$= (22,35j + 9,41)\,\Omega$$

$$\hat{V} = 150e^{j30°} = 150(\cos 30° + j\sin 30°) = 75 + j\,75\sqrt{3}$$

Esempio

È possibile determinare la corrente sul resistore di 1Ω?



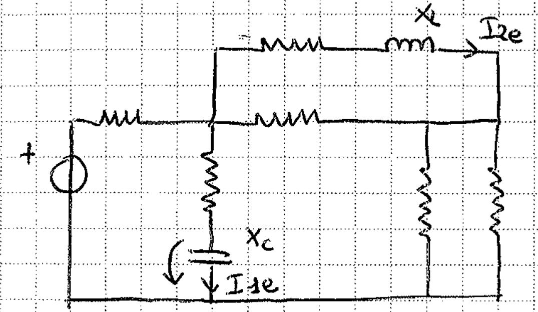
Teorema di Boucherot $\sum_k P_k = 0$ P è sui resistori

$P_{1\Omega} = 0 \rightarrow I = 0$ Oppure $I = \sqrt{\frac{2P}{R}} = 20\sqrt{5}\ A$

Esempio

Un Wattmetro misura una potenza reattiva nulla uscente dal generatore

$X_L = 1\Omega$ $X_C = -2\Omega$ $I_{1e} = 1A$ I_{2e}?



Teorema di Boucherot

$\sum_k Q_K = 0$

$V_C = -2 \cdot 1 = -2V$

$Q_C = + \frac{1}{2} (-2) \cdot (1) = -1\ VAR$

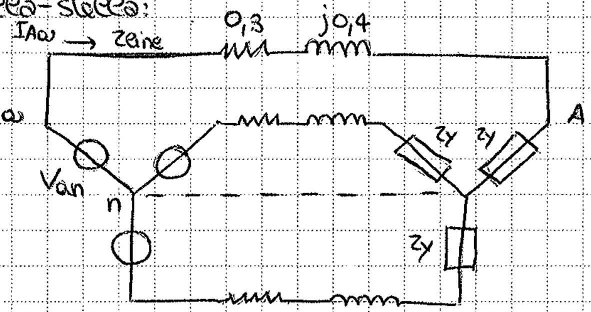
?

$V_L = X_L I_{2e} = I_{2e}$

$Q_L = \frac{1}{2} V I_{2e} = \frac{1}{2} I_{2e}^2$

$Q_L + Q_C = 0 \Rightarrow \frac{1}{2} I_{2e}^2 - 1 = 0 \Rightarrow I_{2e} = \sqrt{2}\ A = 1,44\ A$

Equivalente stella-stella:



$$\hat{V}_{an} = \frac{\hat{V}_{ab}}{\sqrt{3}\angle 30°} = \frac{1000\angle 30°}{\sqrt{3}\angle 30°} = 577,4\angle 0°$$

$$\hat{I}_{aA} = \frac{\hat{V}_{an}}{Zy + Zeine} = \frac{577,4\angle 0°}{0,3 + j0,4 + j2 + 10} = \frac{577,4\angle 0°}{j2,4 + 10,3} = \frac{577,4\angle 0°}{10,58\angle 13,12°} = 54,60\angle -13,12°$$

$$\hat{V}_{An} = \hat{I}_{Aa} Zy = 54,60\angle -13,12° \cdot 10,20\angle 11,31° = 556,9\angle -1,81°$$

$$\hat{V}_{AB} = \hat{V}_{An} \times \sqrt{3}\angle 30° = 964,6\angle 28,19°$$

$$\hat{I}_{AB} = \frac{\hat{V}_{AB}}{Z_\Delta} = \frac{964,6\angle 28,19°}{30 + j6} = \frac{964,6\angle 28,19°}{30,59\angle 11,31°} = 31,53\angle 16,88°$$

$$P_{AB} = I^2_{AB_{rms}} R = \left(\frac{31,53}{\sqrt{2}}\right)^2 \cdot 30 = 14,91 \text{ KW}$$

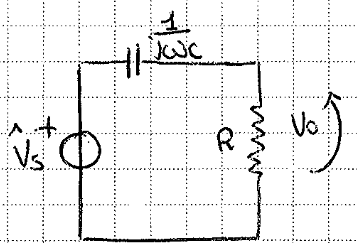
$$P = 3P_{AB} = 44,73 \text{ KW}$$

$$P_{eine\,Aa} = I^2_{Aa_{rms}} R = \left(\frac{54,60}{\sqrt{2}}\right)^2 \cdot 0,3 = 0,447 \text{ KW}$$

$$P_{eine} = 3P_{eine\,A} = 1,341 \text{ KW}$$

Esempio

Circuito RC



$$H(\omega) = \frac{R}{R + \frac{1}{j\omega C}} = \frac{Rj\omega C}{Rj\omega C + 1}$$

$$\frac{1}{1 + \frac{1}{j\omega RC}}$$

$$|H(\omega)|_{\omega \to +\infty} = 1$$

$$|H(\omega)|_{\omega \to 0} = 0$$

passa alto

$\omega_0 = RC$

$$|H(\omega)| = \frac{\omega RC}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$\angle H = \text{arctg}\,\frac{1}{\omega RC}$$

$$\angle H_{\omega \to +\infty} = 0°$$

$$\angle H_{\omega \to 0} = 90°$$

$$\angle H = 90° - \text{arctg}\left(\frac{R\omega C}{1}\right)$$

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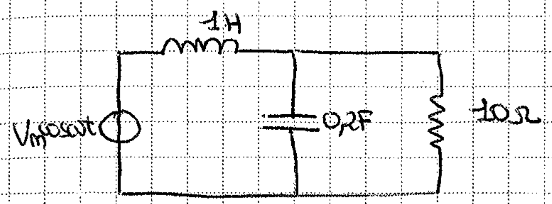
Esempio



Freq. di risonanza

$$\hat{Z}_{eq} = \frac{10 \cdot \frac{1}{j0{,}2\omega}}{10 + \frac{1}{j0{,}2\omega}} + j\omega = \frac{10}{\cancel{j0{,}2\omega}} \cdot \frac{\cancel{j0{,}2\omega}}{2j\omega + 1} + j\omega = \frac{10 - 2\omega^2 + j\omega}{2j\omega + 1} \cdot \frac{1 - 2j\omega}{1 - 2j\omega}$$

$$= \frac{(10 - 2\omega^2 + j\omega)(1 - 2j\omega)}{4\omega^2 + 1} = \frac{10 - 20j\omega - \cancel{2\omega^2} + 4j\omega^3 + j\omega + \cancel{2\omega^2}}{4\omega^2 + 1} =$$

$$= \frac{10 - 19j\omega + 4j\omega^3}{4\omega^2 + 1}$$

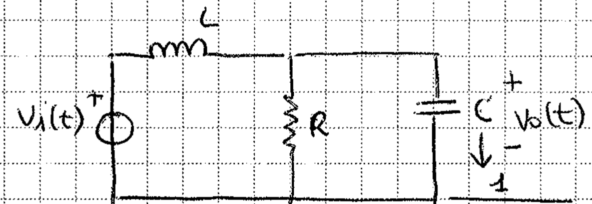
Parte immaginaria $= 0$

$$\frac{4j\omega^3 - 19j\omega}{4\omega^2 + 1} = 0 \rightarrow 4j\omega^3 - 19j\omega = 0 \qquad \omega = 0 \quad N.A.$$

$$j\omega(4\omega^2 - 19) = 0 \qquad \omega = \pm \frac{\sqrt{19}}{2} = + \frac{\sqrt{19}}{2}$$

Esempio

Stabilire il tipo di filtro e calcolare la freq. di taglio (3dB)



$R = 2k\Omega$, $L = 2H$, $C = 2\mu F$

$$\frac{1}{j2 \cdot 10^{-6}\omega}$$

$$Z_{eq} = \frac{R \cdot \frac{1}{j\omega 2 \cdot 10^{-6}}}{R + \frac{1}{j\omega 2 \cdot 10^{-6}}} = \frac{R}{\cancel{j\omega 2 \cdot 10^{-6}}} \cdot \frac{\cancel{j\omega 2 \cdot 10^{-6}}}{Rj\omega 2 \cdot 10^{-6} + 1} = \frac{R}{Rj\omega 2 \cdot 10^{-6} + 1}$$

$$\hat{V}_0 = \hat{V}_1 \frac{R}{Rj\omega C + 1} \cdot \frac{1}{\frac{R}{Rj\omega C + 1} + j\omega L}$$

$$H(\omega) = \frac{R}{\cancel{Rj\omega C + 1}} \cdot \frac{1}{\frac{R + j\omega L(Rj\omega C + 1)}{\cancel{Rj\omega C + 1}}} = \frac{R}{R + j\omega L - \omega^2 RCL}$$

$$|H(\omega)| = \frac{R}{\sqrt{(R - \omega LRC)^2 + \omega^2 L^2}}$$

$$|H(\omega)|_{\omega \to +\infty} = 0$$

$$|H(\omega)|_{\omega \to 0} = 1$$

passa basso

Interflora

Disegnare il diagramma di Bode

$$H(\omega) = \frac{j\omega + 10}{j\omega(j\omega+5)^2} = \frac{2\,\cancel{10}\left(\frac{j\omega}{10}+1\right)}{5\,\cancel{25}\,j\omega\left(\frac{j\omega}{5}+1\right)^2}$$

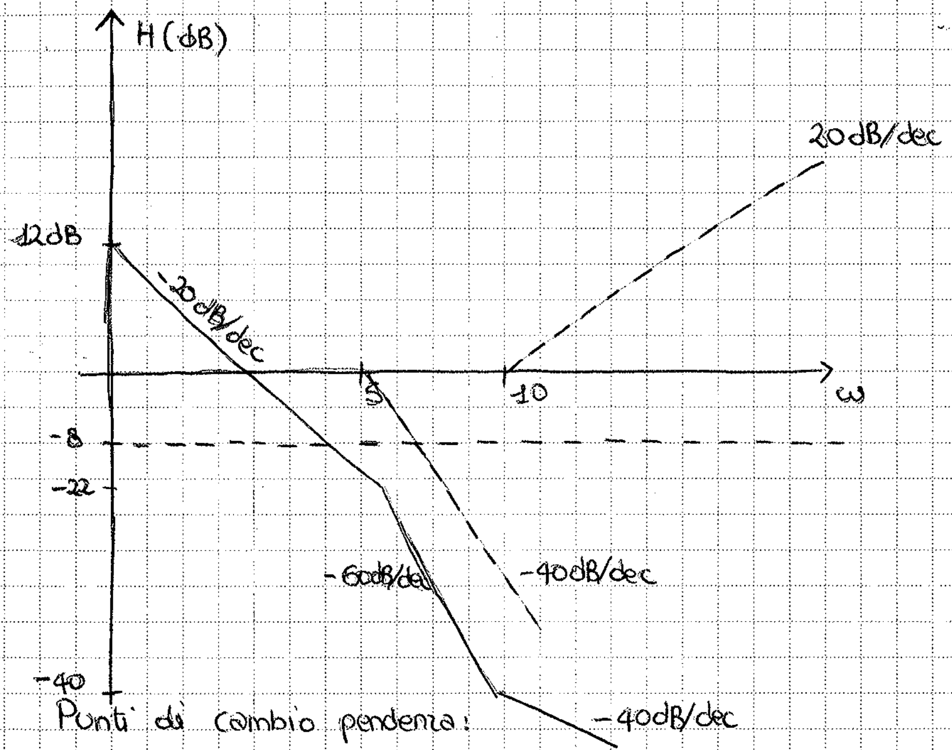
$$\frac{2}{5} = 20\log\frac{2}{5} = -8$$

$$\frac{1}{j\omega} = -20\ dB/dec$$

$$\left(\frac{j\omega}{10}+1\right) = 20\ dB/dec$$

$$\left(\frac{j\omega}{5}+1\right)^2 = -40\ dB/dec$$

S = 10 zero semplice
S = 0 polo semplice
S = 5 polo doppio



Punti di cambio pendenza:

① $\tilde{H}(\omega) \underset{\omega \ll 5}{\simeq} \frac{2}{5}\,\frac{1}{j\omega}$

$|H(\omega = \omega_{1c})| = \left|\frac{2}{5}\cdot\frac{1}{5}\right|_{dB} = -22\ dB$

② $|H(\omega=10)|_{dB} = |H(\omega=5)|_{dB} - 60\log\left(\frac{10}{5}\right)^2$

$= -40\ dB$

② $j40 I_2 + j5 I_1 + I_2(30+j40) + I_2(80+j60) = 0$

$$I_2 = \frac{-j5 I_1}{j40 + 30 + j40 + 80 + j60}$$

① $\underbrace{25 + j25\sqrt{3}}_{\hat{V}} - I_1(60 - j100) - j20 I_1 - \dfrac{25 I_1}{140j + 110} = 0$

$$\frac{\hat{V}}{\hat{I}} = Z_{in} = 60 - j100 + j20 + \frac{25}{140j + 110}$$

$$= 60 - j80 + \frac{25}{140j + 110} = \frac{8400j + 6600 + 11200 - 8800j + 25}{140j + 110}$$

$$= \frac{17825 - 400j}{140j + 110} = 100{,}14 \angle -53{,}13°$$

$$\hat{I} = \frac{\hat{V}}{Z_{in}} = \frac{50\angle 60°}{100{,}14 \angle -53{,}13°} = 0{,}5 \angle 113{,}13°$$

Temi d'esame ~~FEB 2013 - LUG 2013~~

② $V_F = I_F R_F = 150\ V$

$P_{dev} = 9\,hp = 6714\ W \Rightarrow T_{dev} = \dfrac{P_{dev}}{\omega_m} = 45{,}82\ Nm$

$\omega_m = n_m \cdot \dfrac{2\pi}{60} = 146{,}53\ rad/s$

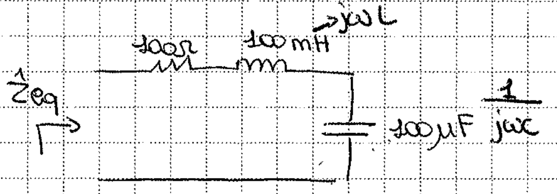
$\dfrac{E_A^{1400}}{E_A^{1200}} = \dfrac{n_1}{n_2} \Rightarrow E_A^{1400} = E_A^{1200} \dfrac{n_1}{n_2} = 186{,}67\ V \qquad E_A^{1200} = 160\ V$

$I_A = \dfrac{P_{dev}}{E_A} = 35{,}97\ A$

$V_T = E_A + R_A I_A = 204{,}66\ V$

S.11

Calcolare l'impedenza equivalente alle pulsazioni $\omega_1 = 10^3$ rad/s
$\omega_2 = 10^2$ rad/s
$\omega_3 = 100\sqrt{10}$ rad/s



$$\hat{Z}_{eq} = 100 + j\,100 \cdot 10^{-3}\,\omega - \frac{j}{100 \cdot 10^{-6}\,\omega}$$

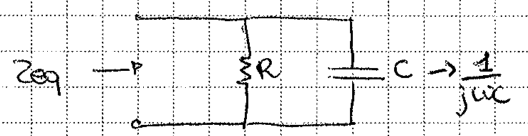
$$\hat{Z}_{eq}(\omega_1) = (100 + j90)\,\Omega$$

$$\hat{Z}_{eq}(\omega_2) = (100 - j90)\,\Omega$$

$$\hat{Z}_{eq}(\omega_3) = 100\,\Omega$$

S.12

Calcolare l'impedenza del bipolo alla freq. $f = 1$ MHz, sapendo che fino $f = 100$ kHz vale $Z_{eq} = (716.975 - j450.457)\,\Omega$



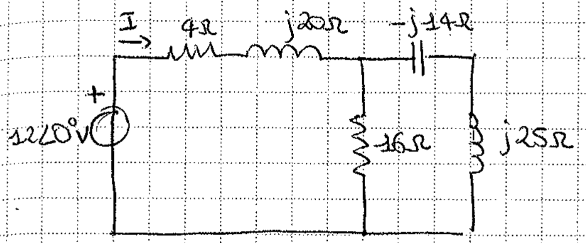
$$f = 100\,\text{KHz} \Rightarrow 1 \cdot 10^5\,\text{Hz}$$

$$f = 1\,\text{MHz} \Rightarrow 1 \cdot 10^6\,\text{Hz}$$

$$Z_{eq} = \frac{R \cdot C}{R + C} = R \cdot \frac{1}{j\omega C} \cdot \frac{1}{R + \frac{1}{j\omega C}} = \frac{R}{\cancel{j\omega C}} \cdot \frac{1}{\frac{j\omega CR + 1}{\cancel{j\omega C}}} = \frac{R}{j\omega CR + 1}$$

$$= \frac{R}{j\omega CR + 1} \cdot \frac{(j\omega CR - 1)}{(j\omega CR - 1)} = \frac{j\omega CR^2 - R}{-\omega^2 C^2 R^2 - 1}$$

S.14



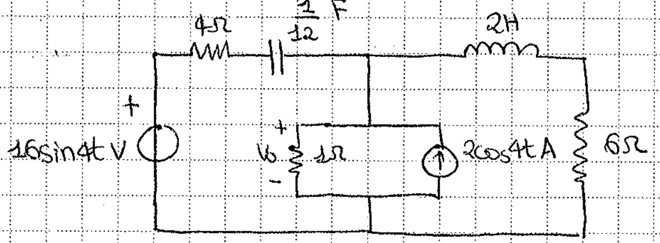
$$\hat{Z}_{eq} = j25 - j14\Omega \Rightarrow (j11\Omega \,//\, 16\Omega) + (4\Omega + j20\Omega)$$

$$\frac{16\Omega \cdot j11\Omega}{16\Omega + j11\Omega} + 4\Omega + j20\Omega = 9,14 + 27,47j$$

$$\hat{I} = 0,13 - 0,39j$$

$$i(t) = 0,4145 \cos(10t - 71,605°)\ A$$

S.15

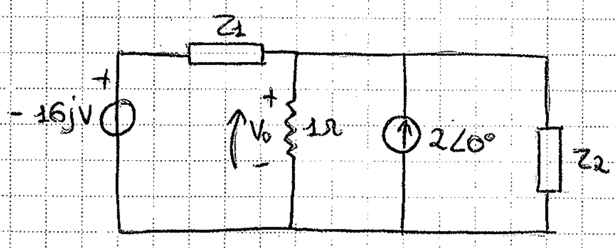


Calcolare $V_o(t)$

$$16\sin 4t \rightarrow 16\cos\left(4t - \frac{\pi}{2}\right) \rightarrow 16\angle -90° \rightarrow 16e^{-j90°}$$

$$16\,(\cos(-90°) + j\sin(-90°)) = -16j\ V$$

$$2\cos 4t \rightarrow 2\angle 0°\ A$$



$$\frac{1}{12}F = \frac{1}{j\omega C} = -\frac{j}{\omega C} = -3j$$

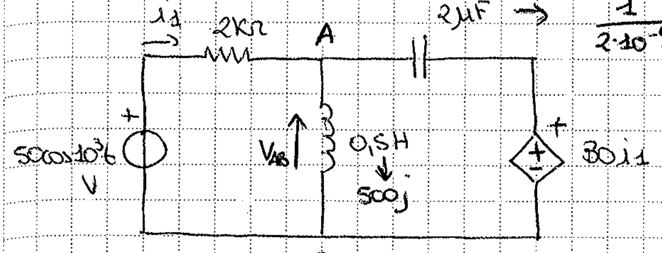
$$Z_1 = 6 + 8j$$

$$Z_2 = 4 - 3j$$

$$2H = j\omega L = j8$$

S. 17

Calcolare $i_1(t)$



$2\mu F \rightarrow \frac{1}{2\cdot 10^{-6}\cdot 10^3} = -500j$

$$V_{AB} = \frac{\frac{50}{2\cdot 10^3} + \frac{30\hat{I}_1}{-500j}}{\frac{1}{2\cdot 10^3} + \cancel{\frac{1}{500}j} - \cancel{\frac{1}{500}j}}$$

$$i_1 = \frac{V_{AB} - 50}{2\cdot 10^3} \Rightarrow \frac{V_{AB} - 50}{2\cdot \cancel{10^3}} - I_1 = 0$$

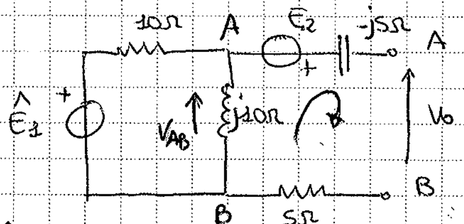
$$V_{AB} - 50 - (2\cdot 10^3) I_1 = 0$$

$$\left(\frac{1}{2\cdot 10^3}\right) \frac{\frac{50}{2\cdot 10^3} + \frac{30 I_1}{-500j}}{\frac{1}{2\cdot 10^3}} - 50 - (2\cdot 10^3) I_1 = 0$$

$$\frac{5\cancel{0}}{2\cdot 10^3} + \frac{30 I_1}{-500j} - \frac{5\cancel{0}}{\cancel{2\cdot 10^3}} - I_1 = 0 \Rightarrow I_1 = 0 \rightarrow i_1(t) = 0 \text{ A}$$

S. 18

Calcolare modello Thevenin dati i fasori $\hat{E}_1 = 10\sqrt{2}\, e^{-j\frac{\pi}{4}}$ V e $\hat{E}_2 = 10$V



$$\hat{E}_1 = 10\sqrt{2}\,(\cos(-45^\circ) + j\,\mathrm{sen}(-45^\circ)) = 10 - 10j$$

$$V_{AB} = E_1 \cdot \frac{j10\Omega}{10\Omega + j10\Omega} = 10V$$

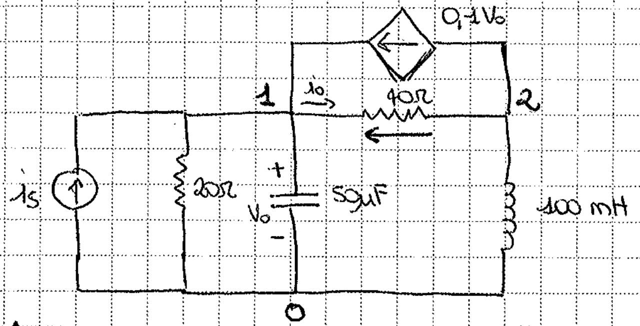
$$V_0 = V_{AB} + \hat{E}_2 = 20V$$

$$Z_{Th} = (10\Omega // j10\Omega) + 5\Omega - j5\Omega$$

$$= \frac{10\cdot j10}{10 + j10} + 5\Omega - j5\Omega = 10\Omega$$

5.21

Calcolare $i_o(t)$ tramite il metodo ai nodi



$i_s(t) = 6\cos(200t + 15°)$ A

$$\hat{I}_s = 6\left(\cos(15)° + j\,\text{sen}(15°)\right) = \frac{3\sqrt{6}+3\sqrt{2}}{2} + j\,\frac{3\sqrt{6}-3\sqrt{2}}{2}$$

$$50\mu F = -\frac{j}{\omega C} = -\frac{j}{200\cdot 50\cdot 10^{-6}} = -\frac{j}{100}$$

$$100mH = j\omega L = j20$$

$$\begin{bmatrix} \frac{1}{20} + 100j + \frac{1}{40} & -\frac{1}{40} \\ -\frac{1}{40} & \frac{1}{40} - \frac{j}{20} \end{bmatrix} \begin{bmatrix} V_o \\ V_2 \end{bmatrix} = \begin{bmatrix} I_s + 0{,}1V_o \\ -0{,}1V_o \end{bmatrix}$$

① $$\frac{V_o}{20} + 100jV_o + \frac{V_o}{40} - \frac{V_2}{40} = I_s + 0{,}1V_o$$

② $$-\frac{V_o}{40} + \frac{V_2}{40} - j\frac{V_2}{20} + 0{,}1V_o = 0$$

① $$\left(-\frac{1}{40} + 100j\right)V_o - I_s = \frac{V_2}{40} \Rightarrow V_2 = 40\left(-\frac{1}{40} + 100j\right)V_o - 40I_s = (-1 + 4000j)V_o - 40I_s$$

Sostituisco in 2 $\Rightarrow$ $$-\frac{V_o}{40} + \left(\frac{1}{40} - \frac{j}{20}\right)\left[(-1 + 4000j)V_o - 40I_s\right] + \frac{V_o}{10} = 0$$

$$-\frac{V_o}{40} + \left(\frac{7999}{40} + \frac{2001}{20}j\right)V_o + (-1+2j)I_s + \frac{V_o}{10} = 0$$

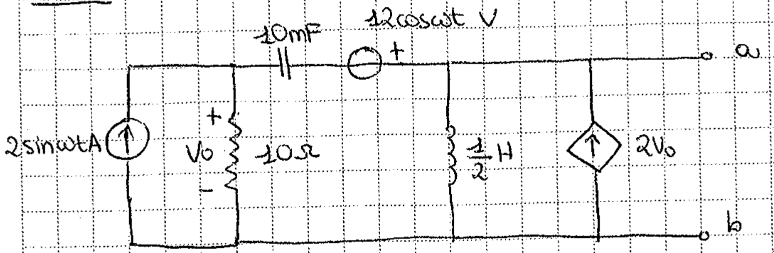
$$\left(\frac{4001}{\cancel{20}} \cancel{+} \frac{\cancel{2001}}{20}j\right)V_o = \frac{-(-1+2j)I_s}{\frac{4001}{20} + \frac{2001}{20}j} = 0{,}02 - 0{,}06j$$

$V_2 = 8{,}16 + 17{,}94j$

$$I_o = \frac{V_1 - V_2}{40}$$

$I_o = 0{,}99 - 7{,}20j$

S.23



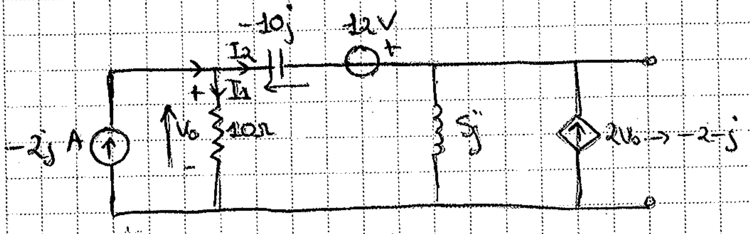
Ricavare modelli Thevenin e Norton nel dominio dei fasori

$\omega = 10$ rad/sec

$2\sin\omega t \rightarrow 2\cos(\omega t - 90°) = -2j$ A

$10mF = \frac{1}{j\omega C} = -10j$

$\frac{1}{2}H = j\omega L = 5j$



$I_1 = -2j \frac{(-10j)}{10-10j} = -1-j$

$V_0 = -10-10j$

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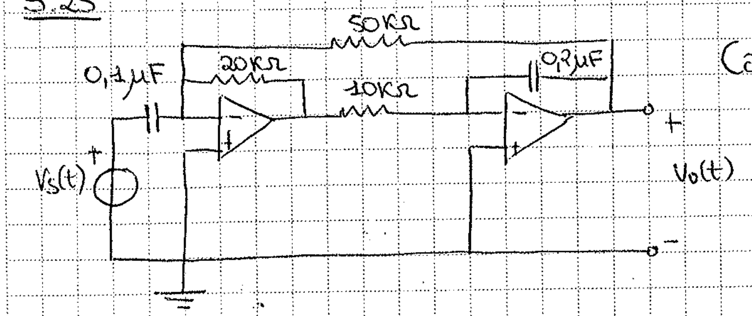
5.25



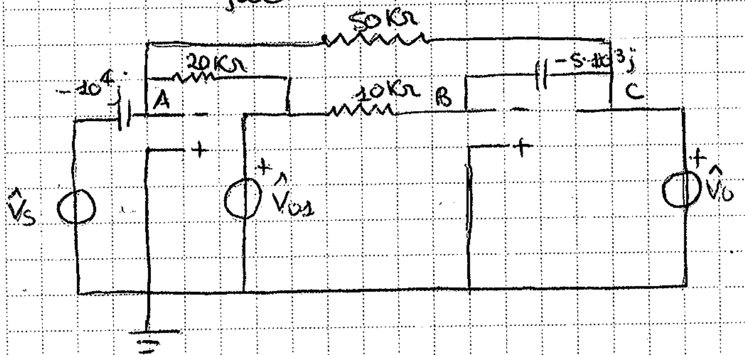
Calcolare $V_0(t)$

$V_S = 4\cos(1000t - 60°)\,V$

$\hat{V}_S = 4(\cos(-60°) + j\,\text{sen}(-60°)) = 2 - 2\sqrt{3}j$

$0{,}1\,\mu F \rightarrow \frac{1}{j\omega C} = -10000j$

$0{,}2\,\mu F \rightarrow \frac{1}{j\omega C} = -5000j$



$$V_A = \frac{\frac{V_S}{-10^4 j} + \frac{V_{01}}{20\cdot10^3} + \frac{V_0}{50\cdot10^3}}{\frac{1}{-10^4 j} + \frac{1}{20\cdot10^3} + \frac{1}{50\cdot10^3}} = 0$$

$$(1)\quad \frac{V_S}{-10^4 j} + \frac{V_{01}}{20\cdot10^3} + \frac{V_0}{50\cdot10^3} = 0$$

$$V_B = \frac{\frac{V_{01}}{10^4} + \frac{V_0}{-5\cdot10^3 j}}{\frac{1}{10^4} + \frac{1}{-5\cdot10^3 j}} = 0 \Rightarrow \frac{V_{01}}{10^4} + \frac{V_0}{-5\cdot10^3 j} = 0 \rightarrow V_{01} = -\frac{V_0\cdot10^4}{-5\cdot10^3 j}$$

$$(1)\quad \frac{V_S}{-10^4 j} - \frac{1}{20\cdot10^3}\cdot\frac{V_0\cdot10^4}{-5\cdot10^3 j} + \frac{V_0}{50\cdot10^3} = 0$$

$$\frac{V_S}{-10^4 j} = (-2\cdot10^{-5} + 1\cdot10^{-4} j)V_0 \rightarrow V_0 = \frac{25+5\sqrt{3}}{13} - \frac{5+25\sqrt{3}}{13}j$$

$$V_0(t) = 3{,}922\cos(1000t - 71{,}31°)\ V$$

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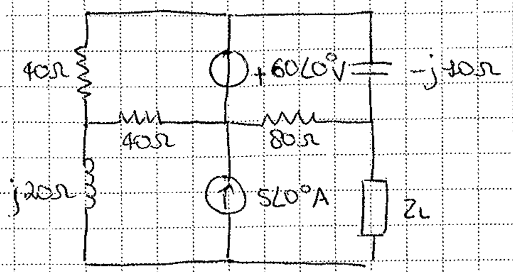
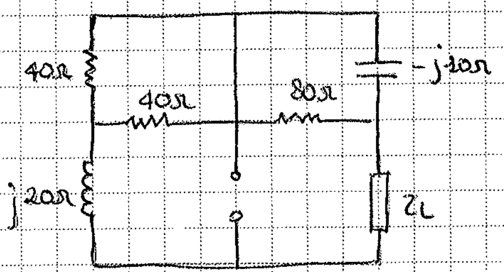
S. 27



Determinare Z_L tale da avere il massimo trasferimento di potenza.

È ohmica, capacitativa, induttiva o ohmica-capacitativa o ohmica-induttiva?



$$R_1 = \frac{40 \cdot 40}{40+40} = 20\ \Omega$$

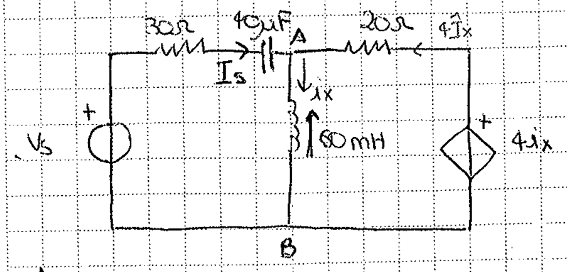
$$Zeq_2 = \frac{80(-j10)}{80-j10} = \frac{16}{13} - \frac{128}{13} j$$

$$Zeq_{tot} = j20 + R_1 + Zeq_2 = 41,23 + j10,15$$

Max trasferimento per: $R_L = R_{TR} = 41,23$
$X_L = -X_{TR} = -10,15\, j$

$Z_L = 41,23 - j10,15\ \Omega$ ohmica capacitativa

5.30



Calcolare la potenza complessa erogata dal gen. Vs

$V_s = 100\cos 2000t$ V

$40\mu F \rightarrow \frac{1}{j\omega C} = -12{,}5j$

$60\,mH \rightarrow j\omega L = 120j$

$\hat{V}_S = 100\angle 0°$

$$\hat{V}_{AB} = \frac{\frac{100}{30-12{,}5j} + \frac{4\hat{I}_x}{20}}{\frac{1}{30-12{,}5j} + \frac{1}{20} + \frac{1}{120j}}$$

$$\hat{I}_x = \frac{\hat{V}_{AB}}{120j} = \frac{\frac{100}{30-12{,}5j} + \frac{4\hat{I}_x}{\cancel{20}5}}{\frac{1}{30-12{,}5j} + \frac{1}{20} + \frac{1}{120j}} \cdot \frac{1}{120j}$$

$$\hat{I}_x\left(-\frac{71}{169} + \frac{1590}{169}j\right) - \frac{\hat{I}_x}{5} = \frac{100}{30-12{,}5j}$$

$$\hat{I}_x\left(-\frac{71}{169} + \frac{1590}{169}j - \frac{1}{5}\right) = \frac{100}{30-12{,}5j} \rightarrow \hat{I}_x = 0{,}105 - 0{,}309j$$

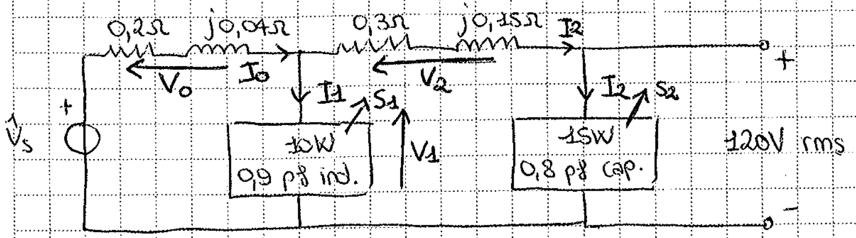
$$\hat{V}_{AB} = \hat{I}_x \cdot 120j = 37{,}06 + 12{,}65j$$

$$I_S = \frac{100 - \hat{V}_{AB}}{30-12{,}5j} = 1{,}94 + 0{,}39j$$

$$S = \frac{1}{2} V_S I_S^* = 98{,}87 - 19{,}28j \text{ VAR}$$

S.32

Calcolare il fasore $\hat{V}_s$



$S_2 = 15W - j\dfrac{15W}{0,8}\sin(\cos^{-1}(0,8)) = 15W - 11,25j\ VA$

$S_2 = \underset{V_e}{V_0} I_{2e}^* \rightarrow I_{2e}^* = \dfrac{S_2}{V_0} = 0,125 - 0,094j$

$I_2 = 0,125 + 0,094j$

$V_2 = I_2 \cdot (0,3 + j0,15\Omega) = 0,023 + 0,047j$

$V_1 - V_2 - 120V = 0 \quad \rightarrow V_1 = 120,023 + 0,047j$

$S_1 = 10W + \dfrac{10}{0,9}\sin(\cos^{-1}(0,9)) = 10W + j4,84\ VA$

$S_1 = \dfrac{1}{2} V_1 I_1^* \Rightarrow I_1^* = \dfrac{2S_1}{V_1} = 0,167 + 0,081j$

$I_1 = 0,167 - 0,081j$

$I_0 = I_1 + I_2 = 0,292 + 0,013j$

$V_0 = I_0 \cdot (0,2 + j0,04)\Omega = 0,058 + 0,014j$

$\hat{V}_s = \hat{V}_1 + \hat{V}_0 = 120,080 + 0,061j$

$\hat{V}_s = 120,08 \angle 0,03° \ V$

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S.34

Due carichi sono collegati in parallelo e assorbono una potenza pari a 2,4 KW con pf = 0,8 induttivo. Sono collegati ad una linea 120 V eff. a 60 Hz. Un carico assorbe 1,5 KW con pf = 0,707 induttivo.

Determinare: 1) pf del 2° carico

2) elemento da inserire in parallelo per correggere il pf totale a 0,9 induttivo → θ_{2T}

$$P = S\cos\theta_T \rightarrow S = \frac{P}{\cos\theta_T} = \frac{2{,}4\ KW}{0{,}8} = 3\ KVA$$

$$Q_{T_1} = P\tan\theta_T = P\tan(\cos^{-1}\theta_T) = 1{,}8\ KVA$$

$$S = 2{,}4\ KW + j1{,}8\ KVA$$

$$P_1 = S_1\cos\theta_1 \rightarrow S_1 = \frac{P_1}{\cos\theta_1} = 2{,}122\ KVA$$

$$Q_1 = P_1\tan\theta_1 = 1{,}5\ KVA$$

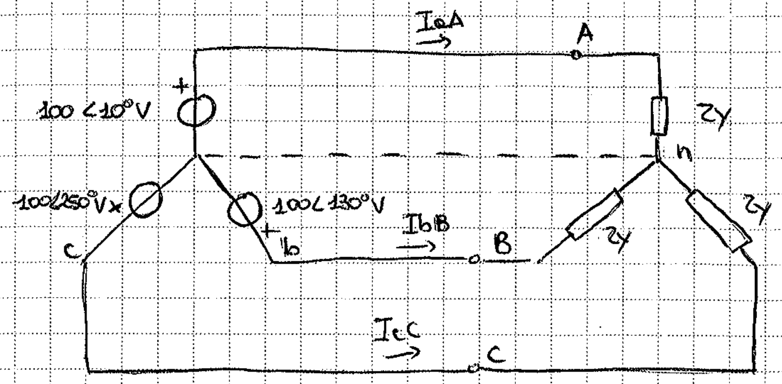
$$S_1 = 1{,}5\ KW + j1{,}5\ KVA$$

$$S = S_1 + S_2 \rightarrow S_2 = S - S_1 = 900 + 300j$$

1) $\cos\theta_2 = \frac{P_2}{S_2} \rightarrow \cos\theta_2 = 0{,}9 - 0{,}3j = 0{,}9487 = pf_2$

2) $Q_{T_2} = P\tan\theta_{2T} = 1162{,}37\ VA$

$$C = \frac{Q_{T_1} - Q_{T_2}}{\omega V_e^2} = 117{,}5\ \mu F$$



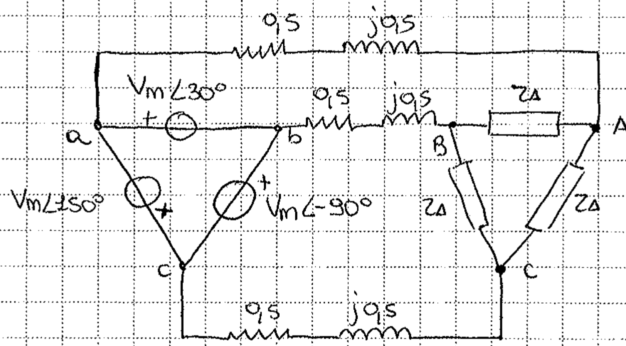
$$I_{aA} = \frac{100\angle 10°\ V}{\frac{8}{3} + \frac{4}{3}j} = 33{,}54\angle -16{,}57°$$

$$I_{bB} = 33{,}54\angle 103{,}43°$$
$$I_{cC} = 33{,}54\angle 223{,}43°$$

$$V_{AB} = V_{An} \cdot \sqrt{3}\angle 30° = 173{,}20\angle 40°$$

$$I_{AB} = 19{,}36\angle 13{,}43°$$
$$I_{BC} = 19{,}36\angle 133{,}43°$$
$$I_{CA} = 19{,}36\angle 253{,}43°$$

5.37



Dati $V_{m\,eff} = 440V$, $Z_\Delta = 10 - 2j\ \Omega$ calcolare $\hat{I}_{aA}$, $\hat{V}_{AB}$, $\hat{I}_{AB}$
Calcolare la potenza complessiva fornita al carico a triangolo e la potenza persa nella linea

$$V_{an} = \frac{V_{ab}}{\sqrt{3}\angle 30°} = 254{,}03\sqrt{2}\ V$$

→ trasformo da triangolo a stella

$$Z_y = \frac{Z_\Delta}{3} = \frac{10}{3} - \frac{2}{3}j$$

$$I_{aA} = \frac{\hat{V}_{an}}{Z_y + Z_{linee}} = 66{,}21\sqrt{2}\angle 2{,}48°$$

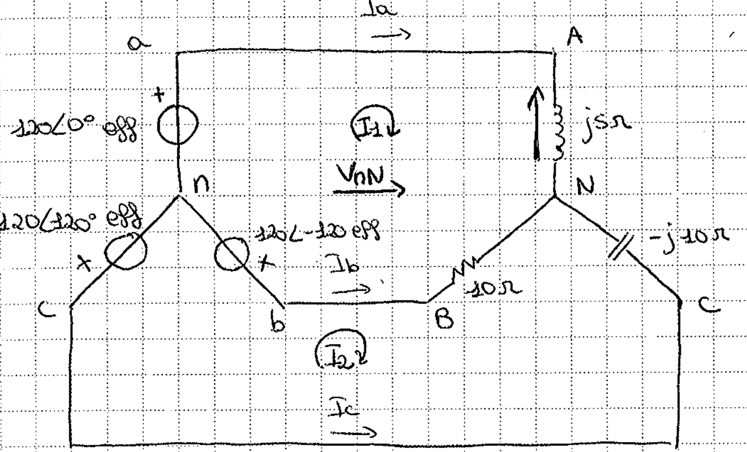
39

motore trifase può essere modellato come un carico trifase a stella. Se il motore assorbe 5,6 kW ad una tensione di linea pari a 220V* e corrente di linea pari a 18,2 A*, calcolare il fattore di potenza. * Valori efficaci

$$P = 3 V_Y I_L \cos\theta \rightarrow \cos(\theta) = \frac{P}{3 V_Y I_L} = 0{,}8075$$

$$V_L = \sqrt{3}\, V_Y \rightarrow V_Y = \frac{V_L}{\sqrt{3}} = 127{,}02$$

40



Calcolare le correnti di linea, la potenza complessa totale assorbita dal carico e la potenza complessa erogata dal generatore

$$V_{nN} = \frac{\frac{120\angle 0°}{j5} + \frac{120\angle -120°}{10} + \frac{120\angle 120°}{-j10}}{\frac{1}{j5} + \frac{1}{10} + \frac{1}{-j10}} = 308{,}34\angle -67{,}09°$$

$$\hat{I}_a = \frac{V_{an} - V_{nN}}{5j} = 56{,}78\ A$$

$$\hat{I}_b = \frac{V_{bn} - V_{nN}}{10} = 25{,}46\angle 135°\ A$$

$$\hat{I}_c = \frac{V_{cn} - V_{nN}}{-j10} = 42{,}75\angle -155{,}1°\ A$$

$S_a = V_{an} I_a^* = 6814\ VA$

$S_b = V_{bn} I_b^* = (-791 + j2951)\ VA$

$S_c = V_{cn} I_c^* = (456 - j5110)\ VA$

$S_s = S_a + S_b + S_c = 6480 - j2156\ VA$

$S_A = (V_{an} - V_{nN}) \cdot I_a^* = 0{,}39 + j16121\ VA$

$S_B = (V_{bn} - V_{nN}) \cdot I_b^* = 6481 - j0{,}16\ VA$

$S_C = (V_{cn} - V_{nN}) \cdot I_c^* = -1{,}55 - j18279\ VA$

$S_L = S_A + S_B + S_C = 6480 - j2156\ VA$

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$$\frac{V_{out}}{V_S} = \frac{\frac{R_L}{j\omega C}}{R_L + \frac{1}{j\omega C}} \cdot \frac{1}{R + R_S + \frac{R_L/j\omega C}{R_L + \frac{1}{j\omega C}}}$$

$$= \frac{\frac{R_L}{j\omega C}}{R_L + \frac{1}{j\omega C}} \cdot \frac{1}{R + R_S + \frac{R_L}{j\omega C} \frac{1}{R_L + \frac{1}{j\omega C}}}$$

$$= \frac{R_L}{\cancel{j\omega C}} \frac{1}{\frac{1}{\cancel{j\omega C}}(R_L j\omega C + 1)} \cdot \frac{1}{R + R_S + \frac{R_L}{\cancel{j\omega C}} \cdot \frac{1}{\frac{1}{\cancel{j\omega C}}(R_L j\omega C + 1)}}$$

$$= \frac{R_L}{(R_L j\omega C + 1)} \cdot \frac{1}{R + R_S + R_L \frac{1}{R_L j\omega C + 1}}$$

$$= \frac{R_L}{(R_L j\omega C + 1)R + (R_L j\omega C + 1)R_S + R_L} = \frac{R_L}{R_L j\omega C R + R + R_L j\omega C R_S + R_S + R_L}$$

$$= \frac{R_L}{(R + R_L + R_S) + [j\omega C R_L (R + R_S)]} \rightarrow \frac{R_L}{1 + \frac{j\omega C R_L (R + R_S)}{R_L + R + R_S}}$$

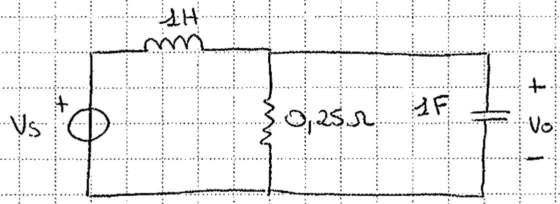
$$\omega_0 = \frac{1}{R_{eq} C} \qquad R_{eq} = \frac{R_L (R + R_S)}{R_L + R + R_S}$$

$$f_B = \frac{1}{2\pi R_{eq} C} \qquad \underset{\omega \to 0}{H(\omega)} = 1 \qquad \underset{\omega \to \infty}{H(\omega)} = 0$$

filtro passa basso del primo ordine

S. 44

Calcolare la funzione di trasferimento Vo/Vs e mostrare che il filtro è un passa basso



$$\hat{Z}_{eq} = \frac{R\,\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}$$

$$V_o = V_s \cdot \frac{\hat{Z}_{eq}}{j\omega L + Z_{eq}} = V_s \cdot \frac{\frac{R/j\omega C}{R+1/j\omega C}}{j\omega L + \frac{R/j\omega C}{R+\frac{1}{j\omega C}}}$$

$$\frac{V_o}{V_s} = \frac{R}{j\omega C} \cdot \frac{1}{R+\frac{1}{j\omega C}} \cdot \frac{1}{j\omega L + \frac{R}{j\omega C}\cdot\frac{1}{R+\frac{1}{j\omega C}}} =$$

$$= \frac{R}{\cancel{j\omega C}} \cdot \frac{1}{\frac{1}{\cancel{j\omega C}}(j\omega CR+1)} \cdot \frac{1}{j\omega L + \frac{R}{\cancel{j\omega C}} \cdot \frac{1}{\frac{1}{\cancel{j\omega C}}(j\omega CR+1)}} =$$

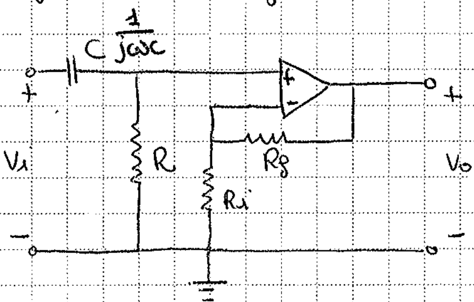
$$= \frac{R}{(j\omega CR+1)\,j\omega L + R} = \frac{R}{-\omega^2 RLC + j\omega L + R}$$

$H(\omega \to 0) = 1$

$H(\omega \to \infty) = 0$

S. 45

Calcolare la funzione di trasferimento. Stabilire le proprietà filtranti.



$$V_R = V_i\,\frac{R}{R+\frac{1}{j\omega C}} = V_i \cdot \frac{R}{\frac{j\omega CR+1}{j\omega C}} = V_i\,\frac{j\omega CR}{j\omega CR+1}$$

$$V_o = \left(1+\frac{R_f}{R_i}\right)V_R = \left(1+\frac{R_f}{R_i}\right)V_i\,\frac{j\omega CR}{j\omega CR+1}$$

$$H(\omega) = \left(1+\frac{R_f}{R_i}\right)\frac{j\omega CR}{j\omega CR+1}$$

$H(\omega \to 0) = 0$

$H(\omega \to \infty) = 1$

Filtro passa alto

S. 48

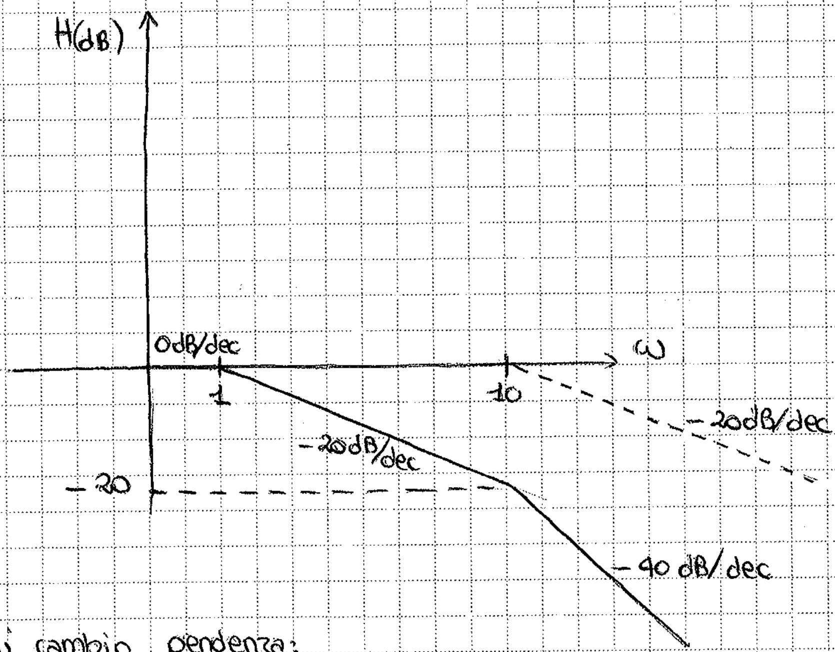
Disegnare il diagramma di Bode dell' ampiezza: $H(\omega) = \dfrac{10}{(1+j\omega)(10+j\omega)}$

$$H(\omega) = \frac{\cancel{10}}{(1+j\omega)\cancel{10}\left(\frac{j\omega}{10}+1\right)} = \frac{1}{(1+j\omega)\left(\frac{j\omega}{10}+1\right)}$$

$\left.\begin{matrix} S = 1 \\ S = 10 \end{matrix}\right\}$ poli semplici

$(1+j\omega) = -20$ dB/dec

$\left(\frac{j\omega}{10}+1\right) = -20$ dB/dec



Punto di cambio pendenza:

$\tilde{H}(\omega) \underset{\omega \ll 1}{\simeq} 1$

$|\tilde{H}(\omega = \omega_{1c})|_{dB} = |1|_{dB} = 0$ dB

$|H(\omega = 10)|_{dB} = |H(\omega = 1)|_{dB} - 20\log\left(\frac{10}{1}\right) = -20$ dB

S.50

Disegnare il diagramma di Bode dell'ampiezza: (quotare il diagramma)

$$H(\omega) = \frac{50(j\omega+1)}{j\omega(-j\omega^2+10j\omega+25)} = \frac{2\,\cancel{50}(j\omega+1)}{\cancel{25}\,j\omega\left(-\frac{j\omega^2}{25}+\frac{\cancel{10}}{\cancel{25}}j\omega+1\right)}$$

$s=-1$ zero semplice
$s=-12$ non occ.
$s=2$ a cc.
$s=0$ polo semplice

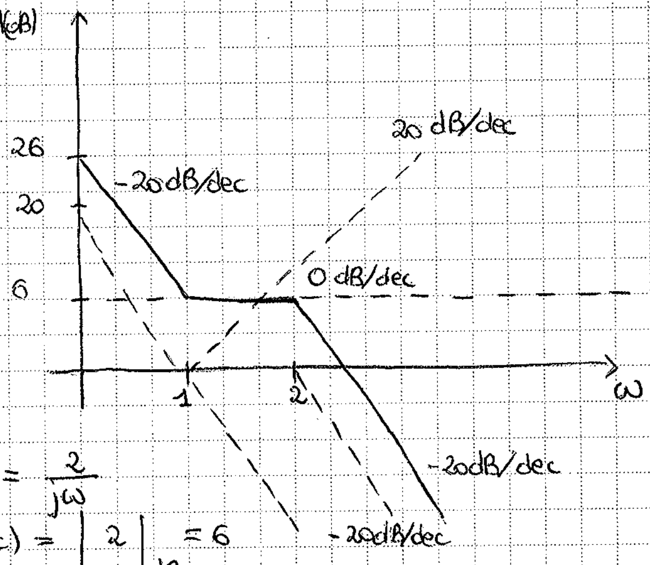
$$s_{1,2} = \frac{-5 \pm \sqrt{25+25}}{-1} = \begin{cases} 5+5\sqrt{2}\ (12) \\ 5-5\sqrt{2}\ (-2) \end{cases}$$ poli semplici

$2 = 20\log(2) = 6$ dB

$(j\omega+1) = 20$ dB/dec

$\left(-\left(\frac{j\omega}{5}\right)^2+\frac{2}{5}j\omega+1\right) = -20$ dB/dec

$\frac{1}{j\omega} = -20$ dB/dec



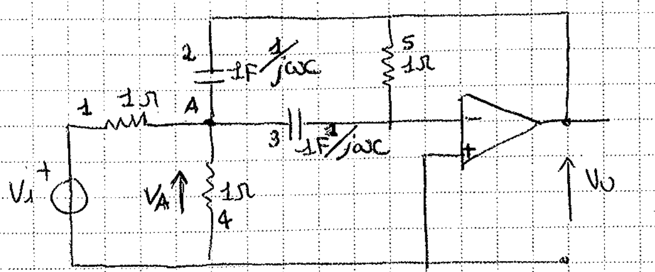
$\tilde{H}(\omega)_{\omega \ll 1} = \frac{2}{j\omega}$

$H(\omega=\omega_{1c}) = \left|2\right|_{dB} = 6$

5.52

Calcolare la funzione di trasf. nel dominio dei fasori ($s = j\omega$).

$$H(s) = \frac{V_U}{V_i}$$



$$\frac{V_A}{\frac{1}{sc}} = -\frac{V_U}{1} \rightarrow V_U = -V_A sc$$

$$V_A \text{ (Millman)} = \frac{\frac{V_i}{1} + \frac{V_U}{1/sc}}{1 + 1 + \frac{1}{1/sc} + \frac{1}{1/sc}}$$

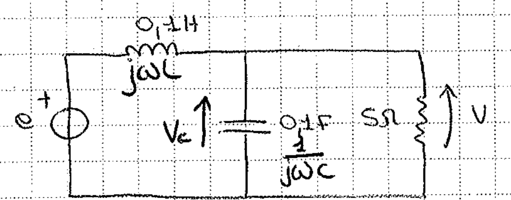
$$\frac{Y_1 Y_3}{Y_2 Y_3 + Y_1 (Y_1 + Y_2 + Y_3 + Y_4)}$$

Sostituendo ottengo: $\frac{V_o}{V_i} = -\frac{sc}{s^2c^2 + 1(1 + sc + sc + 1)} = -\frac{sc}{s^2c^2 + 2 + 2sc}$

$$H(s) = -\frac{s}{s^2 + 2 + 2s}$$

5.53

Calcolare la funzione di trasf. $H(\omega) = V_o(\omega)/E(\omega)$; disegnare il diagramma di Bode dell'ampiezza, calcolare l'uscita $V_o(t)$ per l'ingresso $e(t) = 10 + 10\cos(100t - 1{,}15733°)V$



$$j\omega L = 10j$$

$$\frac{1}{j\omega C} = -10j$$

$$\hat{Z}eq = \frac{\frac{5}{j\omega c}}{5 + \frac{1}{j\omega c}} = \frac{\frac{50}{s}}{5 + \frac{10}{s}} = \frac{\frac{50}{\cancel{s}}}{\frac{5s+10}{\cancel{s}}} = \frac{50}{5s+10}$$

$$V_o = E\,\frac{\hat{Z}eq}{0{,}1s + Zeq} \Longrightarrow \frac{V_o}{E} = \frac{\frac{50}{5s+10}}{0{,}1s + \frac{50}{5s+10}} = \frac{\frac{50}{\cancel{5s+10}}}{\frac{0{,}5s^2 + s + 50}{\cancel{5s+10}}} = \frac{50 \cdot 2}{(0{,}5s^2 + s + 50) \cdot 2}$$

$$\Longrightarrow \frac{V_o}{E} = \frac{100}{s^2 + 2s + 100}$$

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S.54

Disegnare il diagramma di Bode dell'ampiezza.

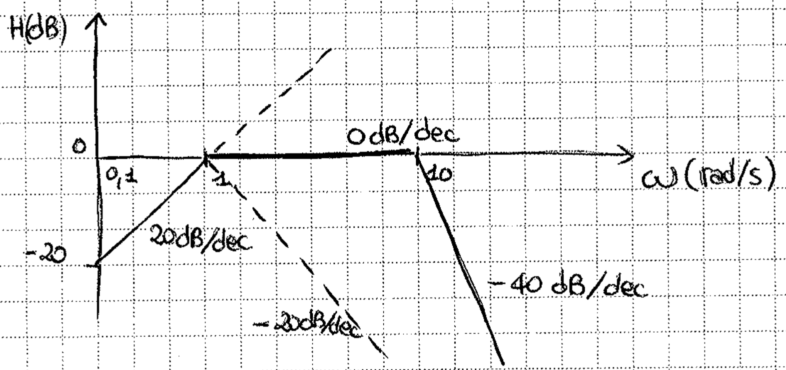
$$H(\omega) = \frac{100 j\omega}{(1+j\omega)(10+j\omega)^2} = \frac{\cancel{100} j\omega}{\cancel{100}(1+j\omega)\left(1+\frac{j\omega}{10}\right)^2}$$

S = 0 zero semplice
S = 1 polo semplice
S = 10 polo doppio

$j\omega = 20$ dB/dec

$(1+j\omega) = -20$ dB/dec

$\left(1+\frac{j\omega}{10}\right)^2 = -40$ dB/dec



S.55

Disegnare il diagramma di Bode dell'ampiezza e calcolare y(t) quando
e(t) = 50 - 20 cos(3t) V

$$H(s) = \frac{Y}{E} = 100\frac{s-3}{(s+1)(s+10)} = \frac{\overset{10}{\cancel{100}}\, 3\left(\frac{s}{3}-1\right)}{\cancel{10}(s+1)\left(\frac{s}{10}+1\right)} = \frac{30\left(\frac{s}{3}-1\right)}{(s+1)\left(\frac{s}{10}+1\right)}$$

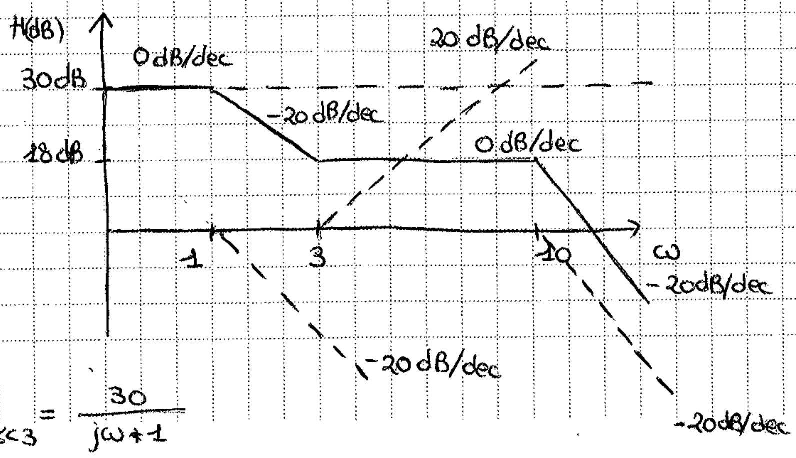
S = 1 polo semplice
S = 10 " "
S = 3 zero semplice

30 = 20log 30 = 30 dB

$\left(\frac{s}{3}-1\right) = 20$ dB/dec

$(s+1) = -20$ dB/dec

$\left(\frac{s}{10}+1\right) = -20$ dB/dec

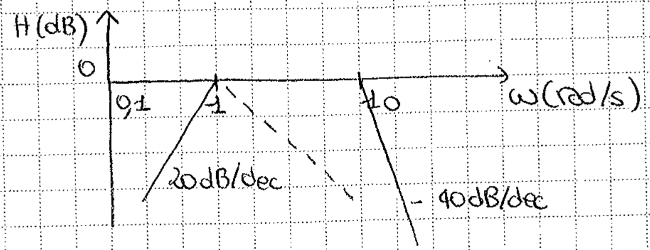


$$|\tilde{H}(\omega)|_{\omega \ll 3} = \frac{30}{j\omega+1}$$

$$H(\omega=\omega_{1c}) = \left|\frac{\cancel{30}\,15}{\cancel{2}\,2}\right|_{dB} = 18 \text{ dB}$$

S.57

Ricavare la funzione di trasf. con diagramma di Bode

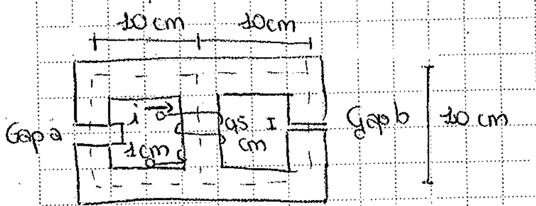


$$H(\omega) = \frac{j\omega}{100\,(j\omega+1)\left(1+\frac{j\omega}{10}\right)^2} = \frac{100\,j\omega}{(j\omega+1)(j\omega+10)^2}$$

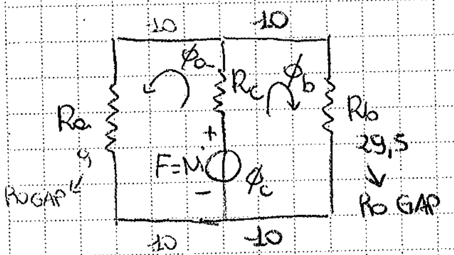
ELETTROMECCANICA

Circuiti magnetici

Esempio



Sezione del nucleo 2×2 cm $\rightarrow 4\ cm^2 \rightarrow 4\cdot10^{-4}$ m
Permeabilità relativa 1000
Bobina da 500 spire
Corrente su bobina di 2A
Calcolare il flusso in ogni traferro



Percorso centrale:

$$R_c = \frac{\ell_c}{\mu_0\mu_r A_{core}} = 1{,}989\cdot10^5 \text{ A spire/Wb}$$

Percorso sx: $R_{gap} + R_{core} = \frac{\ell_{gap\,a}}{\mu_0 A_a} + \frac{\ell_c}{\mu_0\mu_r A_{core}} = 9{,}420 \cdot 10^6$ A spire/Wb

($\ell_c \rightarrow 29\cdot10^{-2}$)

$A_a = 3\times3$ cm $= 9\ cm^2 = 9\cdot10^{-2}$ m

2×2 nucleo $+ 1\times1$ gap $\rightarrow$ SEZIONE CON EFFETTO DI BORDO

$I =$ tensione
$\phi =$ corrente
Rilutt. $=$ resistenza

Stacca il coupon

Qualunque sia

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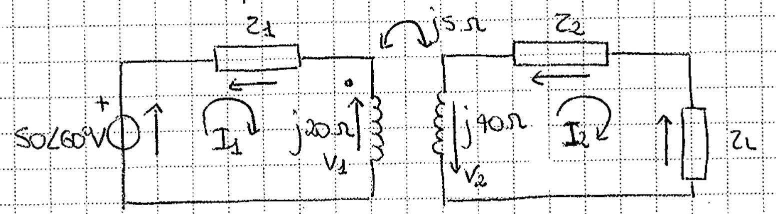
Esempio

Calcolare l'impedenza di ingresso e I_1



$Z_1 = 60 - j100\,\Omega$
$Z_2 = 30 + j40\,\Omega$
$Z_L = 80 + j60\,\Omega$

$$\hat{V}_{in} = 50\,(\cos 60° + j\,\mathrm{sen}\,60°) = 25 + 25\sqrt{3}\,j$$

(1) $\hat{V}_{in} - Z_1\hat{I}_1 - \hat{V}_1 = 0$

(2) $\hat{V}_2 + Z_L\hat{I}_2 + Z_2\hat{I}_2 = 0$

(1) $\hat{V}_{in} - Z_1\hat{I}_1 - j\omega L_1\hat{I}_1 - j\omega M\hat{I}_2 = 0$

(2) $j\omega L_2\hat{I}_2 + j\omega M\hat{I}_1 + Z_L\hat{I}_2 + Z_2\hat{I}_2 = 0$

$$\Rightarrow I_2 = \frac{-j\omega M\hat{I}_1}{j\omega L_2 + Z_L + Z_2}$$

Sostituisco in (1) $\Rightarrow$ $\hat{V}_{in} - Z_1\hat{I}_1 - j\omega L_1\hat{I}_1 - j\omega M\left(\frac{-j\omega M\hat{I}_1}{j\omega L_2 + Z_L + Z_2}\right) = 0$

$$\hat{V}_{in} - Z_1\hat{I}_1 - j\omega L_1\hat{I}_1 - \frac{\omega^2 M^2}{j\omega L_2 + Z_L + Z_2}\hat{I}_1 = 0$$

$$Z_{in} = \frac{\hat{V}_{in}}{\hat{I}_1} = Z_1 + j\omega L_1 + \frac{\omega^2 M^2}{j\omega L_2 + Z_L + Z_2} = 100{,}14\angle -53{,}1°\,\Omega$$

$$I_1 = \frac{\hat{V}_{in}}{Z_{in}} = 0{,}5\angle 113{,}1°\,A$$

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Macchine elettriche

Esempio

Calcolo delle prestazioni di un motore asincrono trifase.
Motore con potenza 5Hp; tensione di linea 440 V efficaci; corrente di linea assorbita $I_1 = 6,8$ A eff, fattore di potenza 0,78. Velocità a pieno carico è 1150 giri/min
Velocità senza carico 1195 giri/min, $I_2 = 1,2$ A eff, pf = 0,3

Calcolare le perdite di potenza, il rendimento a pieno carico, la potenza in entrata senza carico e la regolazione di velocità.

$P_{out} = 5 \cdot 746 = 3730$ W

$P_{in} = \sqrt{3}\, V_{eff} I_{1eff} \cos(\theta) = 4042$ W

$P_{loss} = P_{in} - P_{out} = 312$ W

$\eta = \frac{P_{out}}{P_{in}} \cdot 100 = 92,28\,\%$

$P_{in(no\,load)} = \sqrt{3}\, pf\, V_{eff} I_{2eff} = 274,4$ W

$$\left(\begin{array}{l} P_{out} = 0 \text{ W} \\ P_{loss} = P_{in} = 274,4 \text{ W} \\ \eta = 0\,\% \end{array}\right)$$

$n = \frac{n_{no-load} - n_{full-load}}{n_{full-load}} \cdot 100 = 3,91\,\%$

Esempio – macchina lineare (macchine elettriche DC)

slide 322

Siano dati $B = 1$ T, $l = 0,3$ m, $V_T = 2$ V, $R = 0,05\,\Omega$
Per $t<0$ macchina ferma. Calcolare la corrente d'avvio, la forza iniziale sulla barra, calcolare la velocità stazionaria u senza carico meccanico.

$i_A^{avvio} = \frac{V_T}{R_A} = 40$ A

$f = i_A l B = 12$ N

Senza carico meccanico $e_A = V_T = Blu$

↓

corrente nulla $u = \frac{V_T}{Bl} = 6,667$ m/s

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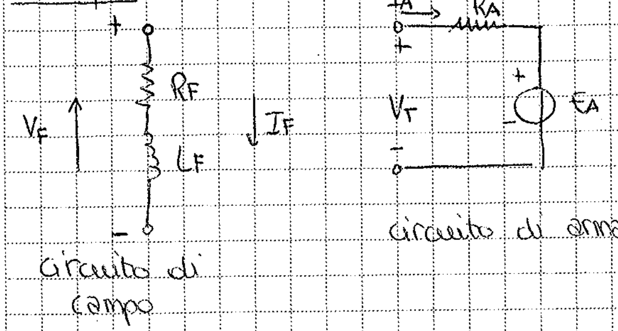
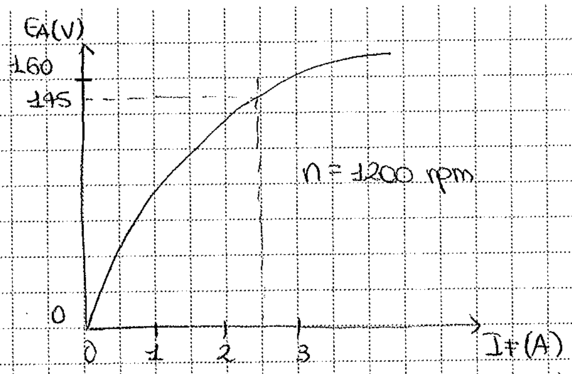
Esempio → macchina rotante D.C.





Curva di magnetizzazione per un motore a corrente continua 200V 10Hp

La macchina, con la curva di magnetizzazione sopra indicata, lavora come motore a 800 giri/min con $I_A = 30A$ e $I_F = 2,5\,A$. La resistenza dell'armatura è $R_A = 0,3\,\Omega$ e quella di campo è $R_F = 50\,\Omega$. Calcolare la tensione V_F e V_T applicate rispettivamente ai circuiti di campo e armatura. Calcolare la coppia sviluppata e la potenza sviluppata.

$$V_F = R_F I_F = 125\ V$$

$$\frac{E_{A\,800}}{E_{A\,1200}} = \frac{n_{800}}{n_{1200}} \qquad \text{dove } E_{A\,1200} = 145\ V$$

$$E_{A\,800} = \frac{n_{800}}{n_{1200}} \cdot E_{A\,1200} = 96,67\ V$$

$$\omega_m = n_{800} \cdot \frac{2\pi}{60} = 83,78\ rad/s$$

$$K\phi = \frac{E_A}{\omega_m} = 1,154\ Wb$$

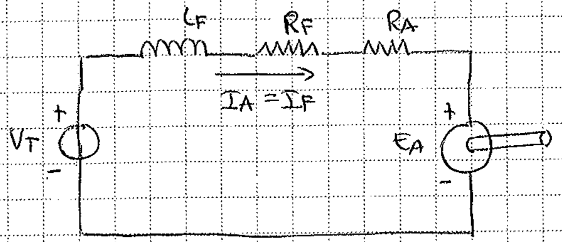
$$T_{dev} = K\phi I_A = 34,62\ Nm$$

$$P_{dev} = \omega_m T_{dev} = 2900\ W \iff P_{dev} = E_A \underset{}{\overset{\text{armatura}}{I_A}} = 2900\ W$$

$$V_T = E_A + R_A I_A = 105,67\ V$$

Esempio → Motori in corrente continua con eccitazione serie

Un motore cc con eccitazione serie funziona a 1200 giri/min azionando un carico che richiede 12 Nm. Si ignorino le resistenze, le perdite rot e gli effetti della saturazione. Calcolare la potenza in uscita. Calcolare la velocità di rotazione e la potenza in uscita per un carico di 24 Nm.



$$\omega_{m_1} = n_{m_1} \cdot \frac{2\pi}{60} = 125,7 \text{ rad/s}$$

$$P_{dev} = P_{out_1} = \omega_{m_1} T_{out_1} = 1508 \text{ W}$$

$$R_A = R_F = 0$$

$$T_{dev} = \frac{K K_F V_T^2}{(R_A + R_F + K K_F \omega_m)^2} = \frac{V_T^2}{K K_F \omega_m^2} \Rightarrow \frac{T_{dev_1}}{T_{dev_2}} = \frac{\omega_{m_2}^2}{\omega_{m_1}^2}$$

$$\omega_{m_2}^2 = \omega_{m_1}^2 \frac{T_{dev_1}}{T_{dev_2}} \Rightarrow \omega_{m_2} = \omega_{m_1} \sqrt{\frac{T_{dev_1}}{T_{dev_2}}} = 88,88 \text{ rad/s}$$

$$n_{m_2} = \omega_{m_2} \frac{2\pi}{60} = 848,5 \text{ rpm}$$

$$P_{out_2} = \omega_{m_2} T_{dev_2} = 2133 \text{ W}$$

Macchine elettriche AC

Esempio -> prestazioni di un motore a induzione

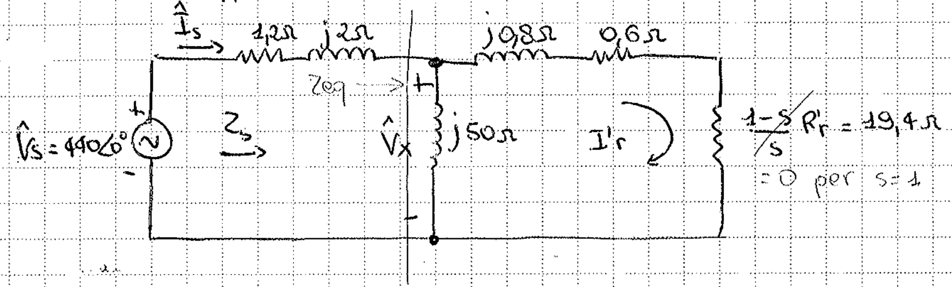
Un certo motore trifase ad induzione con connessione a triangolo da 30 hp, quattro poli, 440 Veff a 60 Hz ha:

$R_s = 1,2\,\Omega$ $\quad R'_r = 0,6\,\Omega$

$X_s = 2,0\,\Omega$ $\quad X'_r = 0,8\,\Omega$

$X_m = 50\,\Omega$

Sotto carico, la macchina opera a 1746 rpm e presenta 900 W di perdite di rotazione. Trovare il fattore di potenza, la corrente di linea, la potenza in uscita, le perdite nel rame, la coppia in uscita ed il rendimento.



$n_s = 1800$ rpm

$s = \frac{n_s - n_m}{n_s} = 0,03$

$\hat{I}_s = \frac{\hat{V}_s}{Z_s} = 21,70\angle -27,59°$ A eff

$Z_s = [(19,4 + 0,6 + 0,8j) // 50j] + 2j + 1,2$

$I_{linea} = I_s\sqrt{3} = \underline{37,59}$ A eff $\qquad \cos\theta = \cos(27,59°) = \underline{0,89}$ (88,63% in ritardo)

$P_{in} = 3 I_s V_s \cos\theta = 25,38$ KW

$\hat{V}_x = \hat{V}_s \cdot \frac{50j // (19,4 + 0,6 + 0,8j)}{50j // (19,4 + 0,6 + 0,8j) + 1,2 + 2j} = 397,8\angle -3,807°$ V rms

$I'_r = 19,88\angle -6,098$ A rms

$P_s = 3 R_s I_s^2 = \underline{1695\ W}$

$P_r = 3 R'_r I'^2_r = \underline{711,4\ W}$

$P_{dev} = 3 \cdot \frac{1-s}{s} R'_r (I'_r)^2 = 23,00$ KW

$P_{in} = P_{dev} + P_s + P_r = 25406$ W

Esercitazione 6

6.1

N spire avvolte
corrente $I(t)$
permeabilità μ
$R \gg r$ → r diametro toro, R raggio linea tratteggiata

flusso totale? flusso concatenato?

1) calcolo tramite la legge di Ampère

$$H\ell = H 2\pi R = NI \implies H = \frac{NI}{2\pi R}$$

$$\underset{\mu H}{\underset{\downarrow}{B}} = \frac{\mu N I}{2\pi R}$$

flusso concatenato: $\phi = \underset{\text{area toro}}{\underset{\downarrow}{BA}} = \frac{\mu N I}{2\pi R}\pi r^2 = \frac{\mu N I r^2}{2R}$

flusso totale: $\lambda = N\phi = \frac{\mu N^2 I r^2}{2R}$

2) Calcolo usando il circuito magnetico e il concetto di riluttanza

$$\ell = 2\pi R \qquad A = \pi r^2$$

$$\mathcal{R} = \frac{\ell}{\mu A} = \frac{2\pi R}{\mu \pi r^2} = \frac{2R}{\mu r^2}$$

$$\phi = \frac{\mathcal{F}}{\mathcal{R}} \qquad \text{dove } \mathcal{F} = NI$$

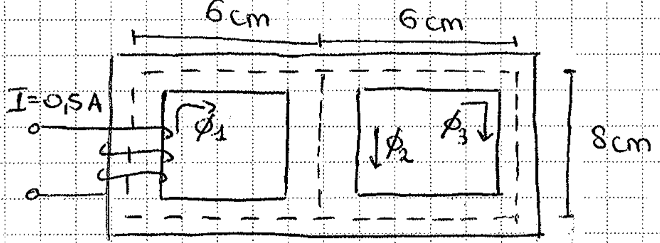
$$\phi = \frac{NI\mu r^2}{2R} \qquad \lambda = N\phi$$

$$\phi_{core} = \frac{N i}{R_{tot}} = \phi_{gap}$$

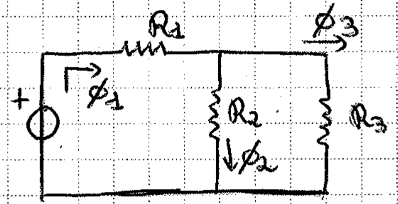
$$B_{gap} = \frac{\phi_{gap}}{A_{gap}} = \frac{N i}{R_{tot} A_{gap}} \Rightarrow i = B_{gap} \cdot \frac{R_{tot} A_{gap}}{500} = 2{,}012 \text{ A}$$

5.4

Calcolare il flusso in ciascuno dei tratti del circuito magnetico in figura



N = 1000
Sezione core 2 x 2 cm
$\mu_r = 5000$



$$R_1 = \frac{\overset{\to 20cm}{e_1}}{\mu_0 \mu_r A_{core}} = 79{,}58 \cdot 10^3 \text{ A spire/Wb}$$

$$R_2 = \frac{\overset{\to 8 \cdot 10^{-2}}{e_2}}{\mu_0 \mu_r A_{core}} = 31{,}83 \cdot 10^3 \text{ A spire/Wb}$$

$$R_3 = R_1 = 79{,}58 \cdot 10^3 \text{ A spire/Wb}$$

$$R_{eq\,tot} = \frac{R_2 \cdot R_3}{R_2 + R_3} + R_1 = 102{,}3 \cdot 10^3$$

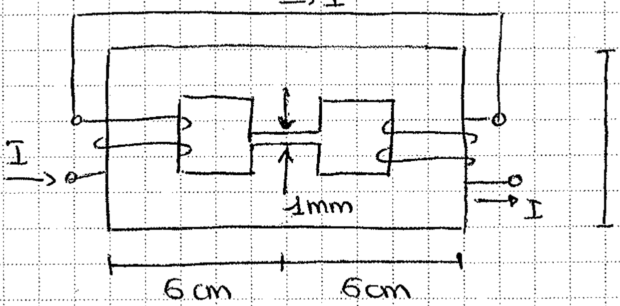
$$\phi_1 = \frac{N i}{R_{tot}} = 4{,}887 \cdot 10^{-3} \text{ Wb}$$

$$\phi_2 = \phi_1 \frac{R_3}{R_3 + R_2} = 3{,}490 \cdot 10^{-3} \text{ Wb}$$

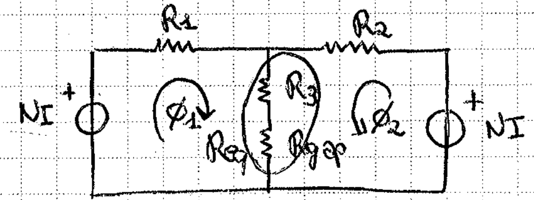
$$\phi_3 = \phi_1 \frac{R_2}{R_3 + R_2} = 1{,}396 \cdot 10^{-3} \text{ Wb}$$

6.6

IN tale che con $I = 2A$ si abbia una B sul traferro di 0,25 T.
Sezione 2x2 cm, si considerino gli effetti di bordo (gli avvolgimenti sono tali da avere effetto di flusso concorde sul tratto centrale del circuito magnetico.)



$\mu_r = 2500$



$$R_1 = \frac{l_1}{\mu_0 \mu_r A_1} = 1{,}43 \cdot 10^5 \text{ Aspire}/Wb = R_2$$

$$R_{eq} = R_3 + R_{gap} = \frac{l_3}{\mu_0 \mu_r A_3} + \frac{l_{gap}}{\mu_0 A_{gap}} = 1{,}85 \cdot 10^6 \text{ Aspire}/Wb$$

$A_{gap} \rightarrow 2{,}1 \cdot 2{,}1$ cm

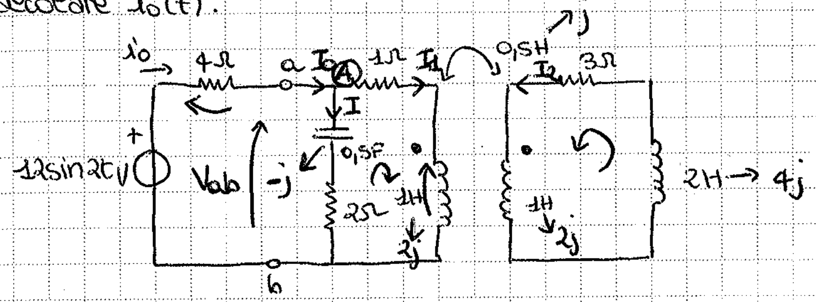
$$\phi_{gap} = B_{gap} A_{gap} = 1{,}1025 \cdot 10^{-4} \; Wb$$

$$\phi_{gap} = \phi_1 + \phi_2 \quad \text{ma} \quad \phi_1 = \phi_2 \Rightarrow \phi_1 = \frac{\phi_{gap}}{2} = 55{,}13 \; \mu Wb$$

$$NI - \phi_1 R_1 - \phi_1 R_{eq} \Rightarrow NI = \phi_1 R_1 + R_{eq}(\phi_1 + \phi_2) \Rightarrow N = 106$$

6.8

Circuito in regime sinusoidale. Calcolare $\hat{Z}_{ab}$ e stabilire il tipo di impedenza. Calcolare $i_0(t)$.



Trasformatore in accoppiamento non perfetto $L_1L_2 \neq M^2$

$$12\sin 2t = 12\cos\left(2t - \frac{\pi}{2}\right) = 12j$$

KCL (A) : $-I_0 + I_1 + I = 0 \Rightarrow I = I_0 - I_1$

KVL (1) : $\underbrace{(I_0 - I_1)(2-j)}_{V_{ab}} - I_1 - 2jI_1 - jI_2 = 0$

KVL (2) : $2jI_2 + jI_1 - (3+4j)I_2 = 0$

$\hookrightarrow$ Ricavo I_2: $I_2 = \dfrac{-jI_1}{2j - 3 - 4j} = \dfrac{-jI_1}{-3-2j}$

Sostituisco in I_1: $\hat{V}_{ab} + \hat{I}_1(-1-2j) - j\dfrac{-jI_1}{-3-2j} = 0$

$$\overbrace{I_0(2-j) - I_1(2-j)}^{\hat{V}_{ab}} + I_1(-1-2j) + \left(\frac{3}{13} - \frac{2}{13}j\right)I_1 = 0$$

$$I_1 = \frac{-I_0(2-j)}{-(2-j) + (-1-2j) + \left(\frac{3}{13} - \frac{2}{13}j\right)}$$

$$\hat{V} - 4I_0 - (2-j)(I_0 - I_1) = 0$$

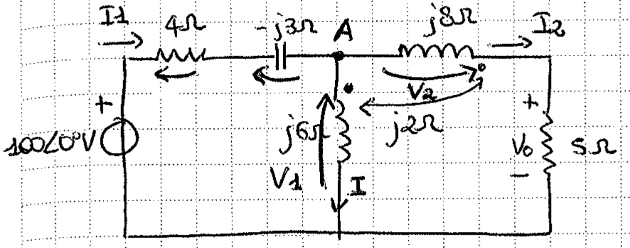
$$12j - 4I_0 - (2-j)I_0 + (2-j)\frac{-I_0(2-j)}{-\frac{36}{13} - \frac{15}{13}j}$$

$$12j - 4I_0 - (2-j)I_0 + \left(\frac{16}{39} - \frac{21}{13}j\right)I_0 = 0$$

$$I_0 = 2{,}13\angle 83°$$

6.10

Calcolare V_0



KCL (A) : $-I_1 + I + I_2 = 0 \rightarrow I = I_1 - I_2$

(1) KVL 1 : $100 - 4I_1 + j3I_1 - j6(I_1 - I_2) + 2jI_2 = 0$

(2) KVL 2 : $j6(I_1 - I_2) - I_2(j8+5) - 8jI_2 + 2j(I_1 - I_2) = 0$

(1) Ricavo $I_1 \Rightarrow 100 - (4-j3)I_1 - j6I_1 + j6I_2 + 2jI_2 = 0$

$$I_1 = \frac{-100 - j6I_2 - 2jI_2}{-(4-j3) - j6}$$

Sostituisco in (2) : $j6\,\dfrac{-100 - j6I_2 - 2jI_2}{-(4-j3)-j6} - I_2(j8+5) - 8jI_2 + 2j\,\dfrac{-100 - j6I_2 - 2jI_2}{-(4-j3)-j6} - 2jI_2 = 0$

$$j6\,\frac{-100 - 8jI_2}{-4-j3} - I_2(j8+5) - 8jI_2 + 2j\,\frac{-100 - 8jI_2}{-4-j3} - 2jI_2 = 0$$

$$72 + 96j + \left(-\frac{192}{25} + \frac{144}{25}j\right)I_2 - I_2(j8+5) - 10jI_2 + 24 + 32j + \left(-\frac{64}{25} + \frac{48}{25}j\right) = 0$$

$$\hat{I}_2 = 8{,}22 + 2{,}83j$$

$\hat{V}_0 = 5\hat{I}_2 = 43{,}465 \angle 19°$

6.12

Un trasformatore reale da 60 Hz, 20 KVA, 8000/240V con i seguenti parametri

$R_1 = 15\,\Omega$
$R_2 = 0{,}02\,\Omega$
$X_1 = 120\,\Omega$
$X_2 = 0{,}15\,\Omega$
$X_m = 30\,K\Omega$
$R_c = 200\,K\Omega$

è connesso ad un carico induttivo con $pf = 0{,}8$ (in ritardo) e assorbe 2KVA (10% del carico nominale)

Calcolare regolazione e rendimento

Rapporto: 33,33 : 1

$\cos\theta = 0{,}8 \rightarrow \theta = 36{,}87°$

$\hat{I}_2 = \frac{2KVA}{V_{load}} = 8{,}333$ A rms $\rightarrow \hat{I}_2 = 8{,}333 \angle -36{,}87°$ A eff

$\hat{V}_2 = V_{load} + I_2(jX_2 + R_2) = 240{,}88 + j0{,}90$

$\hat{I}_1 = \frac{N_2}{N_1}\hat{I}_2 = 0{,}25 \angle -36{,}87°$ A eff

$\hat{V}_1 = \frac{N_1}{N_2}\hat{V}_2 = 8029{,}4 + j30$ Veff

$\hat{V}_s = \hat{V}_1 + \hat{I}_1(R_1 + jX_1) = 8050{,}6 \angle 0{,}368°$ V eff

$P_{loss} = \frac{\hat{V}_s}{R_c} + \hat{I}_1 R_1 + \hat{I}_2 R_2 = 326{,}4\,W$

$P_{load} = V_{load}\hat{I}_2 \cdot pf = 1600\,W$

$P_{in} = P_{load} + P_{loss} = 1926{,}4\,W$

rendimento in potenza: $\frac{P_{load}}{P_{in}} \cdot 100 = 83{,}06\,\%$

$V_{no-load}$: $I_1 = I_2 = 0$

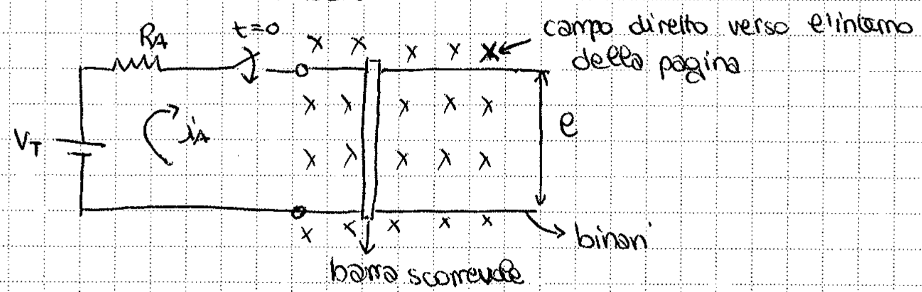
$V_s = V_1 = 8050{,}6$ Veff

$V_2 = \frac{N_2}{N_1}V_1 = 241{,}5$ Veff $= V_{no-load}$

regolazione percentuale: $\frac{V_{no-load} - V_{load}}{V_{load}} = 0{,}625\,\%$

6.15

Data la macchina lineare



In assenza di carico, cosa accade alla velocità stazionaria se:
a) la tensione V_T raddoppia
b) la resistenza R_A raddoppia
c) l'intensità del flusso magnetico B raddoppia

a) $v = \dfrac{e_A}{Bl}$ $\qquad e_A = V_T \rightsquigarrow$ senza carico

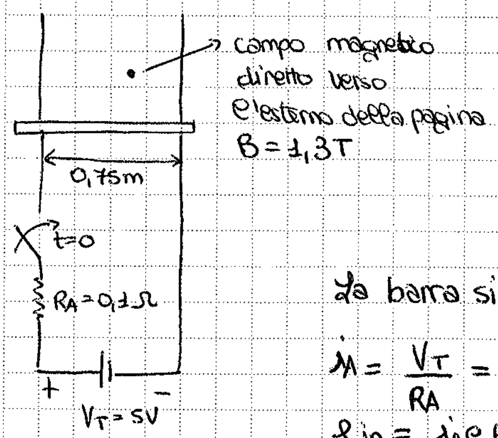
v raddoppia

b) $V_T = i_A R_A \Rightarrow$ se R_A raddoppia i_A dimezza $\rightsquigarrow V_T$ costante

v è invariata

c) v si dimezza

6.16



Data la seguente macchina lineare, quando l'interruttore si chiude in quale direzione la barra si sposta?
Calcolare l'ampiezza della forza iniziale
Determinare il valore della velocità finale trascurando l'attrito.

La barra si sposta verso il basso

$i_A = \dfrac{V_T}{R_A} = 50\ A$

$f_{in} = i_A l B = 48{,}75\ N$

$e_A = V_T$

$v = \dfrac{e_A}{Bl} = 5{,}13\ m/s$