



Corso Luigi Einaudi, 55 - Torino

Appunti universitari

Tesi di laurea

Cartoleria e cancelleria

Stampa file e fotocopie

Print on demand

Rilegature

NUMERO: 860

DATA: 12/03/2014

A P P U N T I

STUDENTE: Arcidiacono

MATERIA: Scienza delle Costruzioni temi d'esame

Prof. Chiaia_Surace

Il presente lavoro nasce dall'impegno dell'autore ed è distribuito in accordo con il Centro Appunti.

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**ATTENZIONE: QUESTI APPUNTI SONO FATTI DA STUDENTIE NON SONO STATI VISIONATI DAL DOCENTE.
IL NOME DEL PROFESSORE, SERVE SOLO PER IDENTIFICARE IL CORSO.**

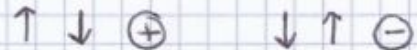
CURVA DELLE PRESSIONI

M	N	T	CDP
mo	mo	mo	tratto scarico
si	mo	mo	non è definita (retta impropria)
mo	si	mo	retta coincidente con l'asse dell'asta
mo	mo	si	IMPOSSIBILE!
si	si	si	<u>CARICO</u> : parabola (trovo rette tangenti)
			<u>NO CARICO</u> : retta inclinata. Compongo N e T e trovo di $x = \frac{M}{N}$ \updownarrow
			Se $M=0$ la retta passa per il vincolo.
si	si	mo	retta parallela all'asta spostata $x = \frac{M}{N}$
si	mo	si	retta perpendicolare all'asta spostata $x = \frac{M}{T}$
			(taglio costante e momento lineare)
si	mo	si	fascio di rette $x = \frac{M}{T} \leftrightarrow$
			(taglio lineare e momento parabolico)
			ATTENZIONE SE SI ANNULLA IL TAGLIO (il fascio va a $+\infty$ o $-\infty$)

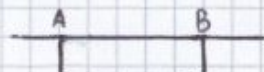
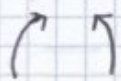
SFORZO NORMALE (N)



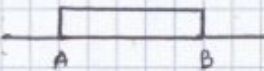
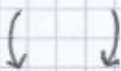
TAGLIO (T)



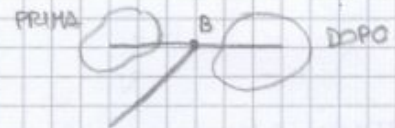
MOMENTO FLETTENTE (M)



FIBRE SOTTO TESI



FIBRE SOPRA TESI



Se ho un momento concentrato su una cerniera interna devo sapere se questo è applicato un infinitesimo prima o dopo

χ = Diagrammi di curvatura

CALCOLO DI $V_B - U_B - \varphi_B$

Considero la struttura sostituita principale e posiziona la forza o il momento unitario in corrispondenza del punto richiesto e ipotizzo un verso.

$$L_e = \pm 1 \cdot V_B$$

$$L_e = \pm 1 \cdot U_B$$

$$L_e = \pm 1 \cdot \varphi_B$$

$$L_e = \pm 1 \cdot \Delta \varphi_B \quad \text{nel caso di una cerniera interna}$$

$$L_i = \frac{1}{EI} \int N^{(s)} \cdot N^{(r)} dz$$

$$\varphi_B = \frac{1}{EI} \int N^{(s)} \cdot N^{(r)} dz \quad \text{se ottengo un risultato negativo ho sbagliato ad ipotizzare il verso}$$

CASI PARTICOLARI:

- Se ho un cedimento anelastico o un cedimento angolare

$$L_e = \pm 1 \cdot V_B \pm 1 \cdot \eta_0$$

$$L_e = \pm 1 \cdot U_B \pm 1 \cdot \eta_0$$

$$L_e = \pm 1 \cdot V_B \pm 1 \cdot \varphi_0$$

$$L_i = \pm 1 \cdot U_B \pm 1 \cdot \varphi_0$$

↳ valore della forza posta dove abbiamo il cedimento.
Se non c'è la forza mi riconduco al caso precedente

- Se ho una variazione termica uniforme

$$L_e = \int \frac{N^{(s)} \cdot N^{(r)}}{EI} dz + \int N^{(s)} \cdot d\Delta T dz$$

Lo sforzo normale va considerato solo sul tratto su cui c'è la variazione termica uniforme

CALCOLO REAZIONI INTERNE

TRAVE 1

$$\rightarrow HA - HD - ql = 0$$

$$HD = HA - ql$$

$$HD = 0$$

$$\uparrow VC + VD = 0$$

$$VD = -\frac{ql}{2}$$

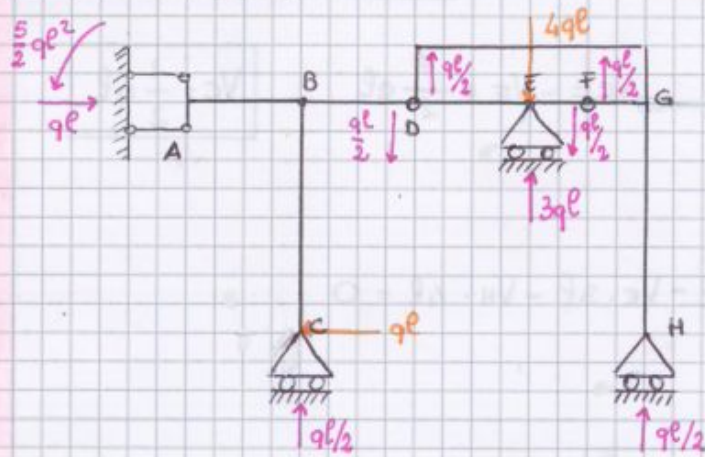
TRAVE 3

$$\uparrow VH - ql - VF = 0$$

$$VF = VH - ql$$

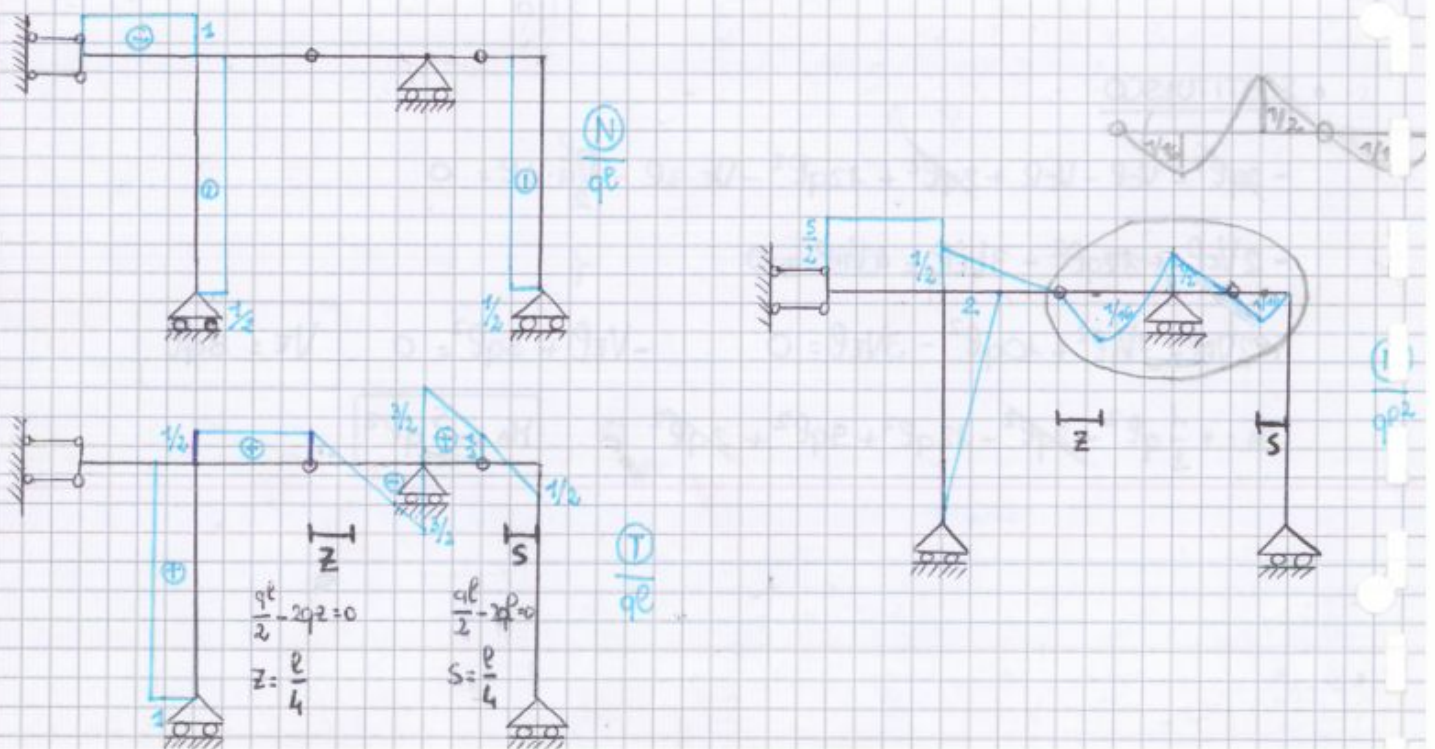
$$VF = -\frac{ql}{2}$$

RIDISEGNO LA TRAVE

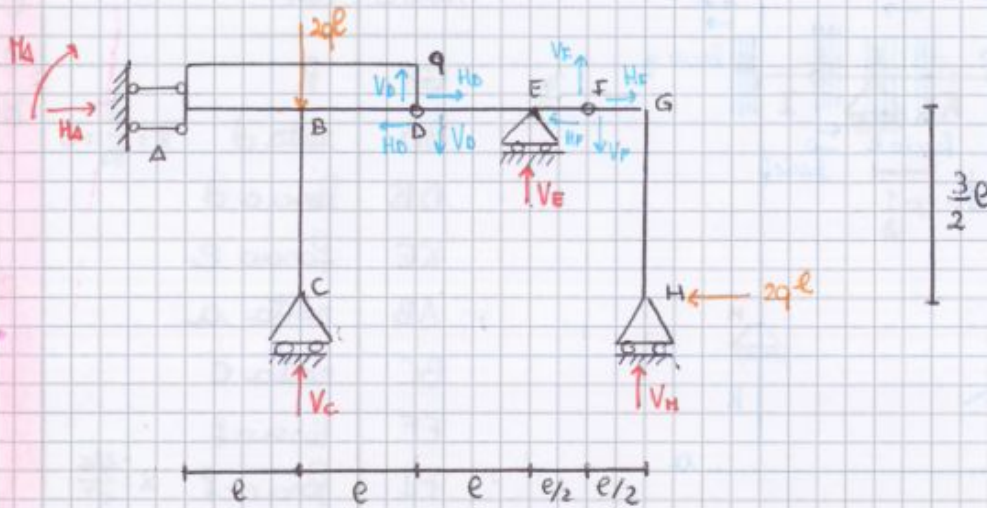


DOVE SI ANNULLA IL TAGLIO AUREMO IL MOMENTO MASSIMO

DISEGNO I DIAGRAMMI



29 GENNAIO 2008 (2)



$$g = 3 \times 3 = 9$$

$$v = 9$$

ISOSTATICA

• TROVO REAZIONI VINCOLARI

$$\uparrow V_c - 2ql + V_E + V_H = 0$$

$$V_c = -4ql - V_E$$

$$\rightarrow H_A - 2ql = 0$$

$$H_A = 2ql$$

$$\uparrow M(A) = H_A \cdot l - V_c \cdot l + 2ql \cdot e - V_E \cdot 3e - V_H \cdot 4e + 2ql \cdot \frac{3}{2}e = 0$$

• USO EQUAZIONI AUSILIARIE

$$\uparrow M(F)^{III} = -V_H \cdot \frac{e}{2} + 2ql \cdot \frac{3}{2}e = 0$$

$$V_H = 6ql$$

$$\uparrow M(D)^I = H_A + V_c \cdot l - 2ql \cdot l = 0$$

$$H_A = -V_c l + 2ql^2$$

• SOSTITUISCO

$$-V_c l + 2ql^2 - V_c \cdot l + 2ql^2 - V_E 3e - 24ql^2 + 3ql^2 = 0$$

$$-2V_c l - 3V_E e - 17ql^2 = 0$$

$$+8ql^2 + V_E e - 3V_E e - 17ql^2 = 0$$

$$-V_E e - 9ql^2 = 0$$

$$V_E = -9ql$$

→

$$V_c = 5ql$$

→

$$H_A = -3ql^2$$

• CALCOLO REAZIONI INTERNE

TRAVE 1

$$\rightarrow H_A - H_D = 0$$

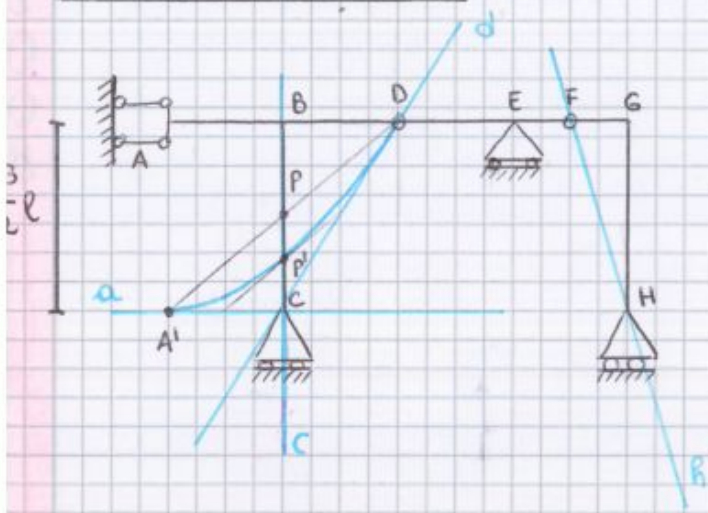
$$2ql - H_D = 0$$

$$H_D = 2ql$$

$$\uparrow V_c - 2ql + V_D = 0$$

$$V_D = -3ql$$

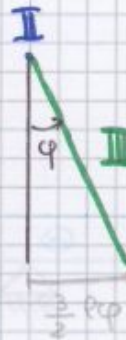
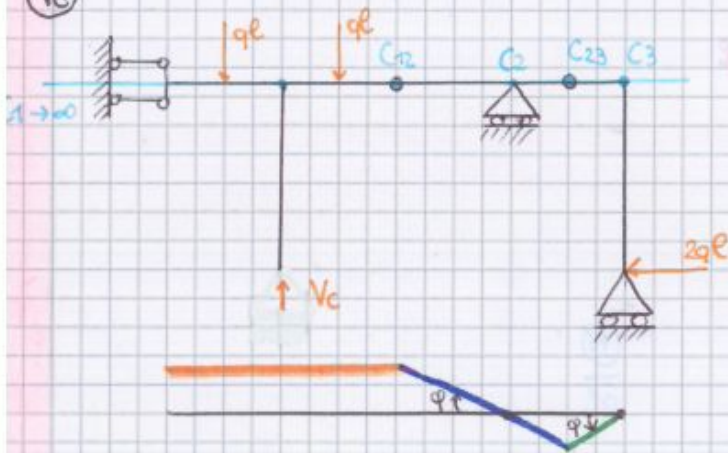
• CURVA DELLE PRESSIONI



TRATTO	CDP
BC	retta c
GH	retta h
FG	retta h
EF	retta h
DE	retta d
AD	parabola

• PRINCIPIO DEI LAVORI VIRTUALI

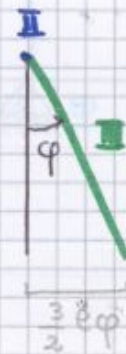
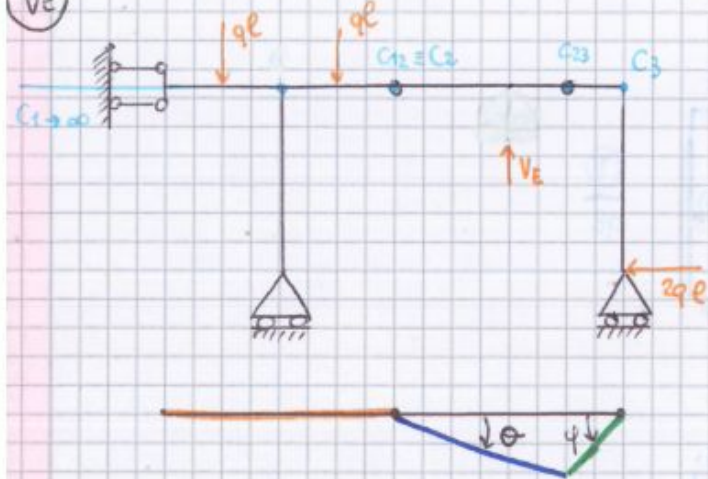
(V_C)



$$L_e = -2ql \cdot \varphi \cdot l + V_c \cdot \varphi \cdot l - 2ql \cdot \varphi \cdot \frac{3}{2}l = 0$$

$$V_c = 5ql$$

(V_E)



$$\frac{3}{2}l\theta = \frac{l}{2}\varphi \quad \theta = \frac{\varphi}{3}$$

$$L_e = -2ql \cdot \frac{3}{2}\varphi l - V_E \cdot l \cdot \frac{\varphi}{3} = 0$$

$$V_E = -9ql$$

$$V_A = -q\ell - \frac{-2 \cdot \frac{q\ell\sqrt{2}}{4} \sqrt{2} + \frac{q\ell\sqrt{2}}{4} \sqrt{2}}{2} = -q\ell - \frac{-q\ell + \frac{q\ell}{2}}{2} = -q\ell + \frac{q\ell}{2} = -q\ell + \frac{q\ell}{4}$$

$$V_A = -\frac{3}{4} q\ell$$

$$V_E = \frac{1}{2} q\ell$$

$$H_A = -\frac{3}{2} q\ell + \frac{q\ell\sqrt{2}}{4} \cdot \frac{\sqrt{2}}{2} = -\frac{3}{2} q\ell + \frac{q\ell}{4}$$

$$H_A = -\frac{5}{4} q\ell$$

$$V_F = \frac{q\ell}{4}$$

$$H_F = \frac{q\ell}{4}$$

• CALCOLO REAZIONI INTERNE

TRAVE 1

$$\rightarrow H_A + \frac{3}{2} q\ell - H_C = 0$$

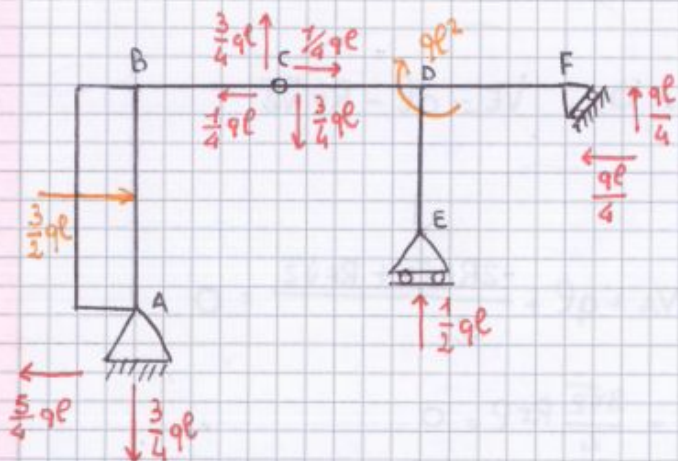
$$-\frac{5}{4} q\ell + \frac{3}{2} q\ell = H_C$$

$$H_C = \frac{1}{4} q\ell$$

$$\uparrow V_A + V_C = 0$$

$$V_C = +\frac{3}{4} q\ell$$

• RIDISEGNO LA TRAVE



PRINCIPIO DEI LAVORI VIRTUALI

V_e

H_e

$$H_A = -\frac{ql}{3} + \frac{2\sqrt{2}ql\sqrt{2}}{3} = -\frac{ql}{3} + \frac{4ql}{3}$$

$$H_A = 1 \quad ql$$

$$V_A = ql \frac{3}{2} + ql$$

$$V_A = \frac{5}{2} ql$$

$$V_E = -\frac{ql}{2} - 2\sqrt{2}ql\sqrt{2}$$

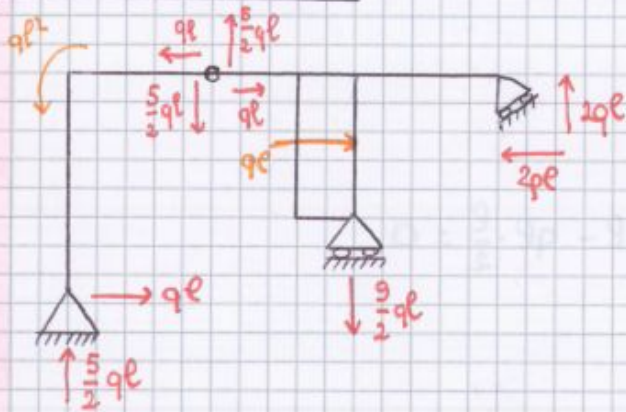
$$V_E = -\frac{ql}{2} - 4ql$$

$$V_E = -\frac{9}{2} ql$$

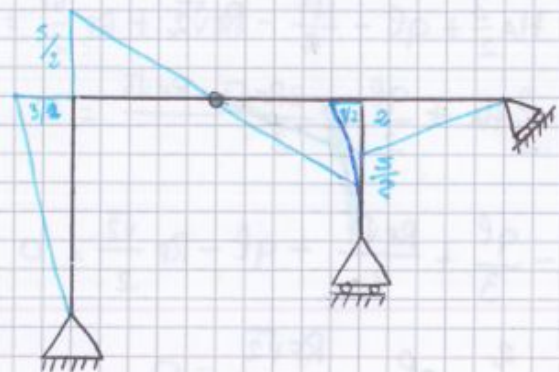
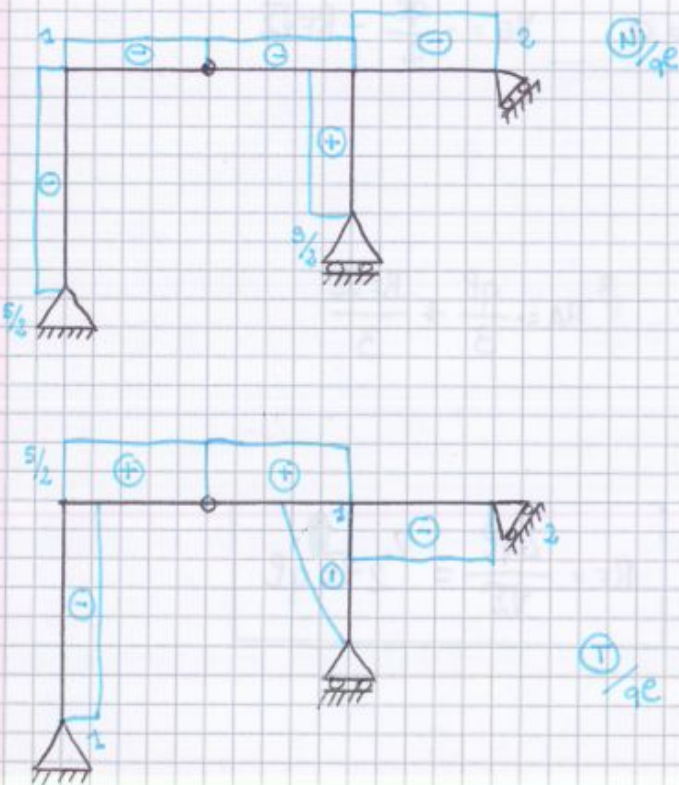
$$V_F = 2ql$$

$$H_F = 2ql$$

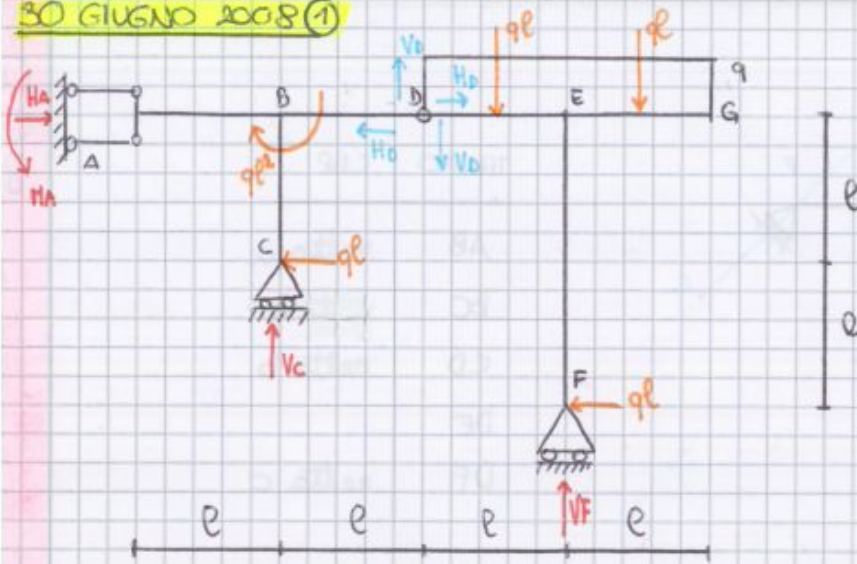
• RIDISEGNO LA TRAVE



• DISEGNO I DIAGRAMMI



30 GIUGNO 2008 (1)



$$g = 3 \times 2 = 6$$

$$V = 6$$

ISOSTATICA

• TROVO REAZIONI VINCOLARI

$$\uparrow V_C + V_F - qe - qe = 0$$

$$V_C = -V_F + 2qe$$

$$V_C = -2qe$$

$$\rightarrow H_A - qe - qe = 0$$

$$H_A = 2qe$$

$$\curvearrowright M(A) = -H_A - V_C \cdot e + qe \cdot e + qe^2 - V_F \cdot 3e + qe \cdot 2e + 2qe \cdot 3e = 0$$

• USO EQUAZIONI AUSILIARE

$$\curvearrowright M(B) = +2qe \cdot e + qe \cdot 2e - V_F \cdot e = 0$$

$$V_F = 4qe$$

• SOSTITUISCO

$$M_A = 2qe^2 + 2qe^2 - 12qe^2 + 8qe^2 = 0$$

$$M_A = 0$$

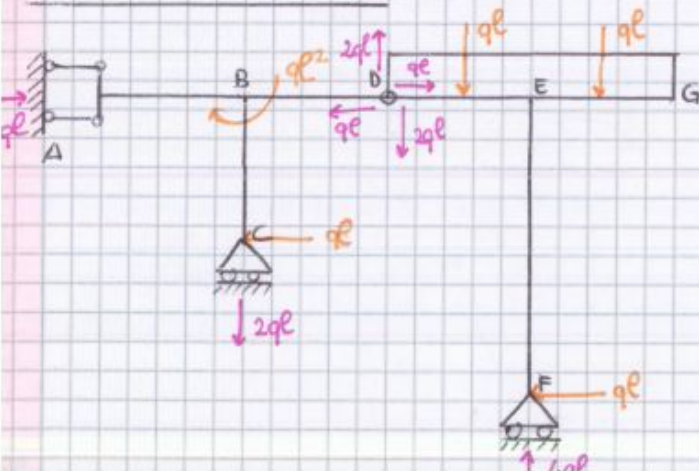
• CALCOLO REAZIONI INTERNE

$$H_D = qe \quad \uparrow V_F - 2qe - V_D = 0$$

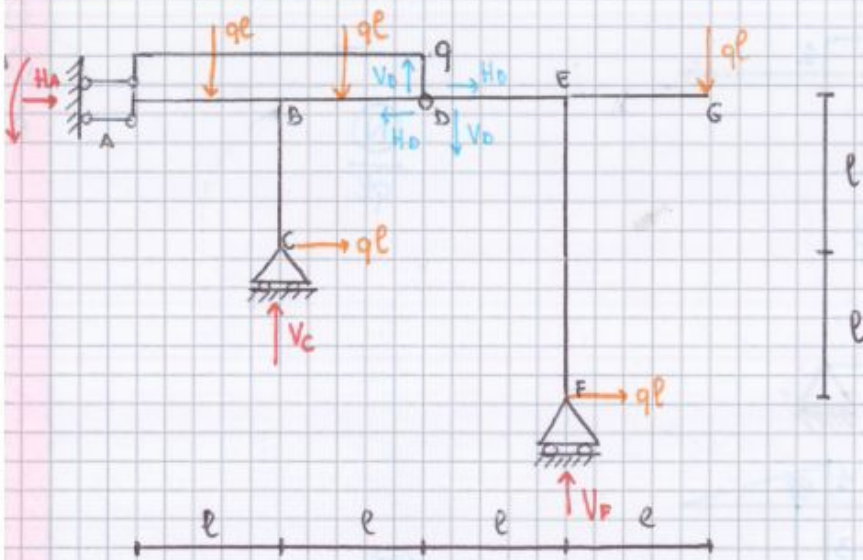
$$4qe - 2qe - V_D = 0$$

$$V_D = 2qe$$

• RIDISEGNO LA TRAVE



30 GIUGNO 2008 (2)



$$g = 3 \times 2 = 6$$

$$V = 6$$

ISOSTATICA

• TROVO REAZIONI VINCOLARI

$$\uparrow + V_c - qe - qe + V_f - qe = 0$$

$$V_c = -V_f + 3qe$$

$$V_c = 3qe$$

$$\rightarrow + H_A + qe + qe = 0$$

$$H_A = -2qe$$

$$\uparrow + M(A) = -H_A + 2qe \cdot l - V_c \cdot l - qe \cdot l - V_f 3l - qe \cdot 2l + qe \cdot 4l = 0$$

$$-H_A + V_f l - 3qe^2 - V_f 3l - qe^2 + 2qe^2 - 2qe^2 + 4qe^2 = 0$$

$$-H_A - 2V_f l = 0$$

• USO EQUAZIONI AUSILIARIE

$$\uparrow + M(C) = qe \cdot 2l - qe \cdot 2l - V_f \cdot l = 0$$

$$V_f = 0$$

$$H_A = 0$$

• CALCOLO REAZIONI INTERNE

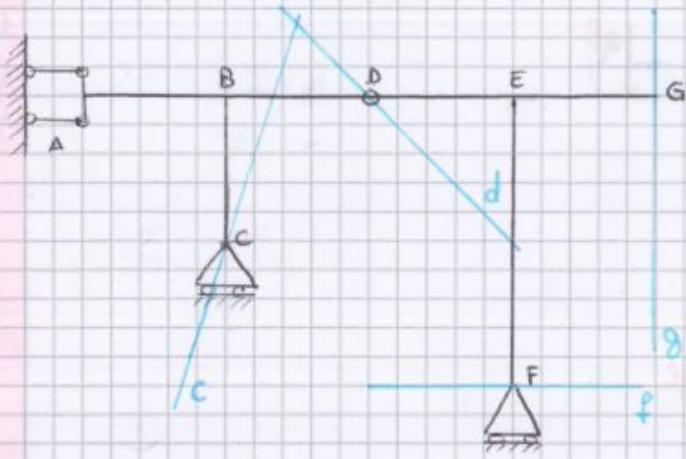
$$H_D = -qe$$

$$\uparrow + V_c - 2qe + V_D = 0$$

$$3qe - 2qe + V_D = 0$$

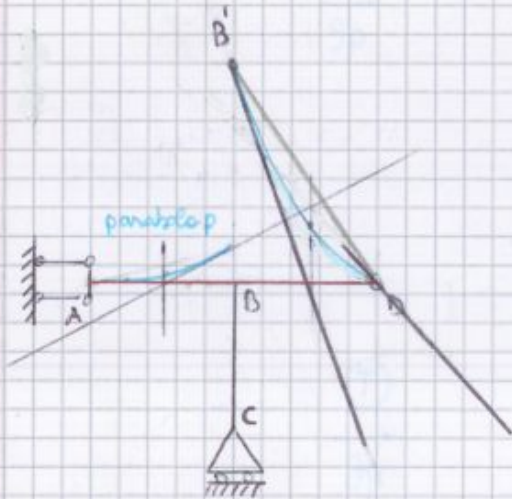
$$V_D = -qe$$

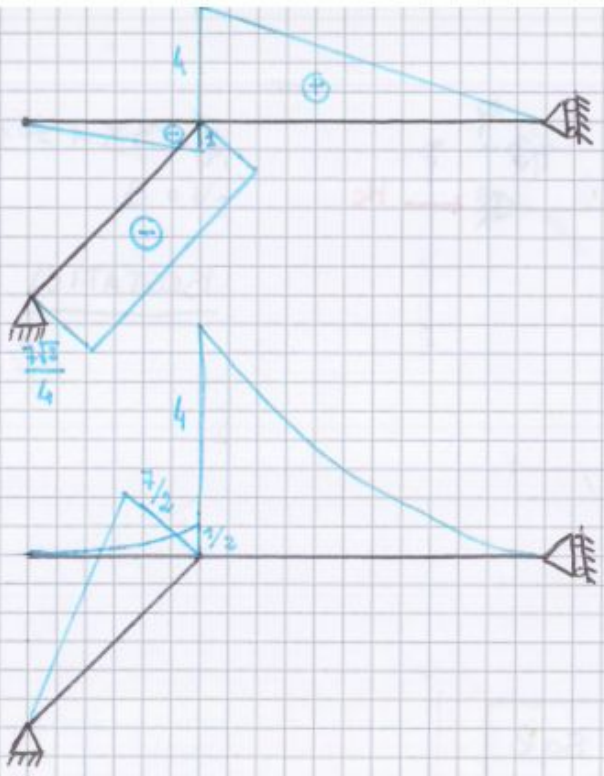
CURVA DELLE PRESSIONI



TRATTO	CDP
AB	parabola p
BD	parabola q
BC	retta c
DE	retta d
EG	retta f
EF	retta g

$$x = \frac{1}{-1} = -1$$

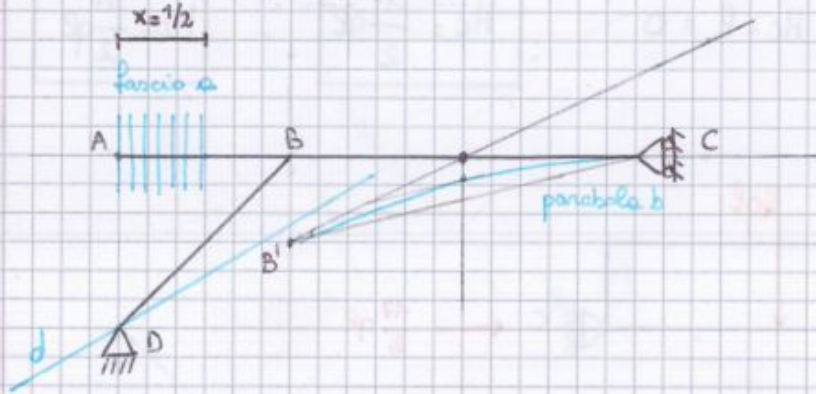




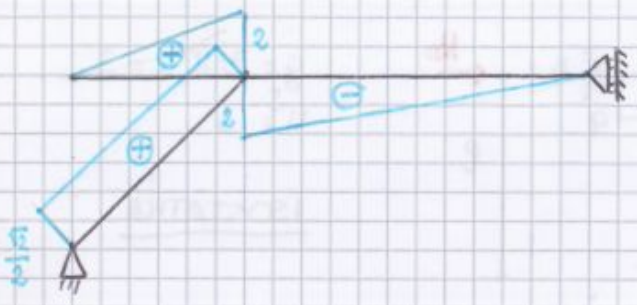
R/T

H/T

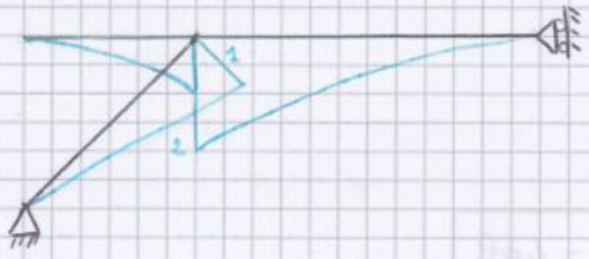
• CURVA DELLE PRESSIONI



TRATTO	CDP
AB	fascio a
BC	parabola b
BD	retta d

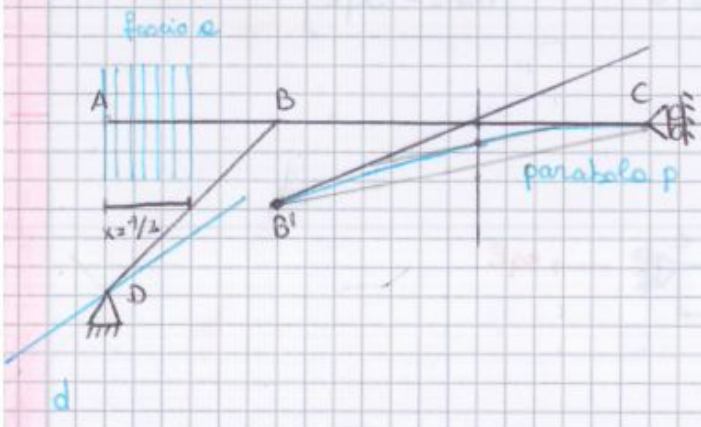


$\frac{F}{Q}$



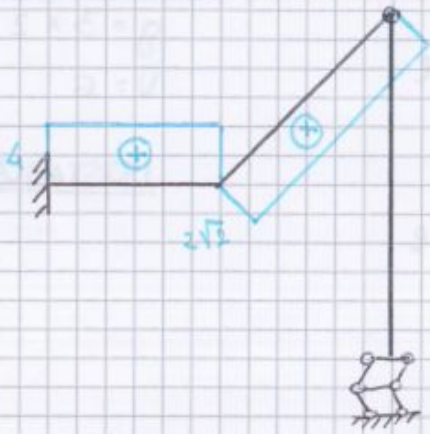
$\frac{M}{Q^2}$

• CURVA DELLE PRESSIONI

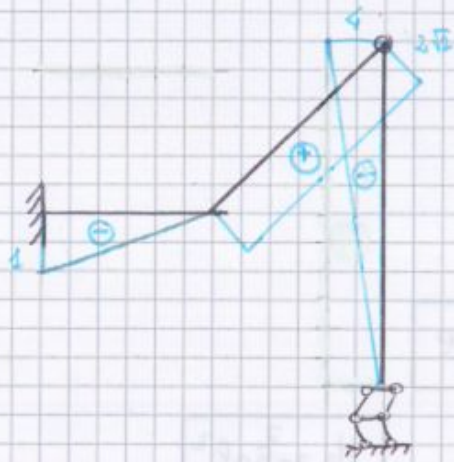


TRATTO	CDP
AB	fascio a
BC	parabola p
BD	retta d

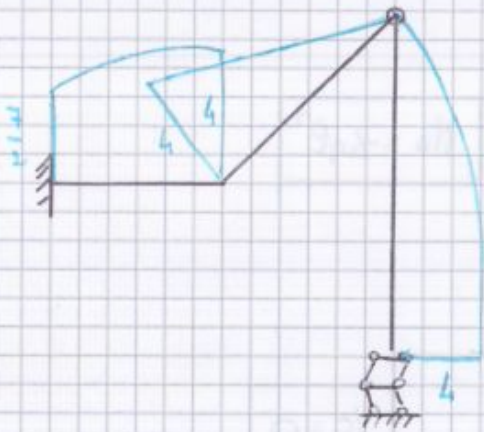
• DISEGNO I DIAGRAMMI



$\frac{N}{q\ell}$

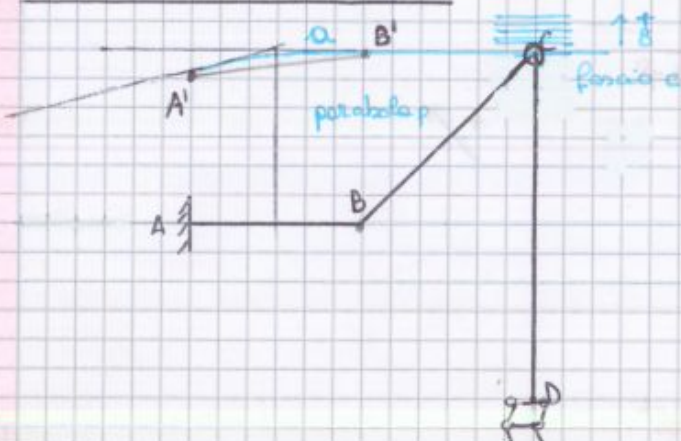


$\frac{T}{q\ell}$



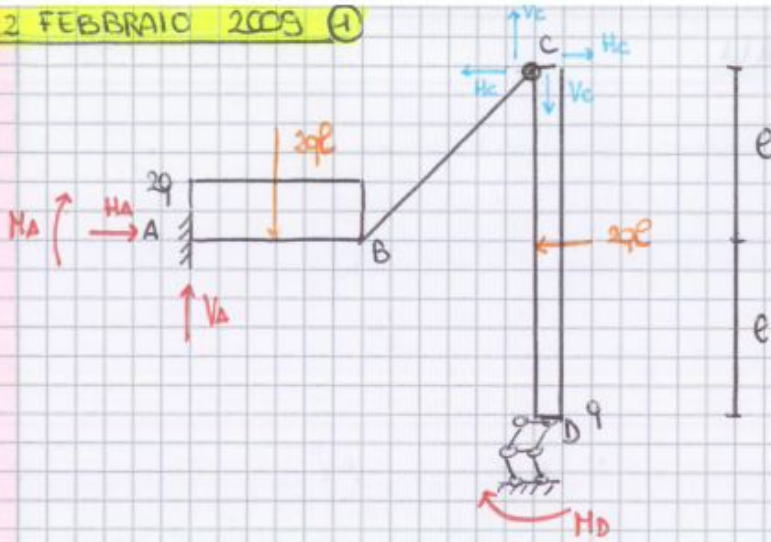
$\frac{M}{q\ell^2}$

• CURVA DELLE PRESSIONI



TRATTO	CDP
AB	parabola p
BC	retta a
CD	fascio c

2 FEBBRAIO 2009 (1)



$$g = 3 \times 2 = 6$$

$$V = 6$$

ISOSTATICA

• CALCOLO REAZIONI VINCOLARI

$$\uparrow + V_A - 2qe = 0 \quad \boxed{V_A = 2qe}$$

$$\rightarrow + H_A - 2qe = 0 \quad \boxed{H_A = 2qe}$$

$$\curvearrowright + M(A) = H_A + 2qe \frac{e}{2} + M_D = 0$$

• USO EQUAZIONI AUXILIARIE

$$\curvearrowright + M(C)^II = M_D + 2qe \cdot e = 0$$

$$\boxed{M_D = -2qe^2}$$

$$\boxed{H_A = qe^2}$$

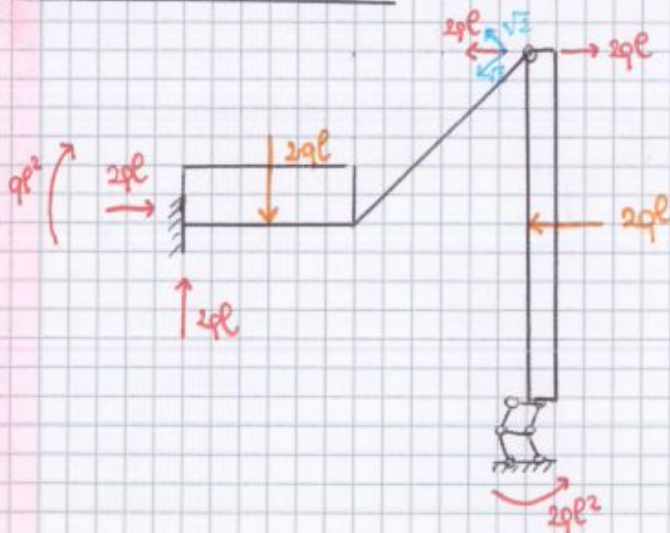
• CALCOLO REAZIONI INTERNE

TRAVE 2

$$H_c - 2qe = 0 \quad \boxed{H_c = 2qe}$$

$$\boxed{V_c = 0}$$

• RIDISEGNO LA TRAVE

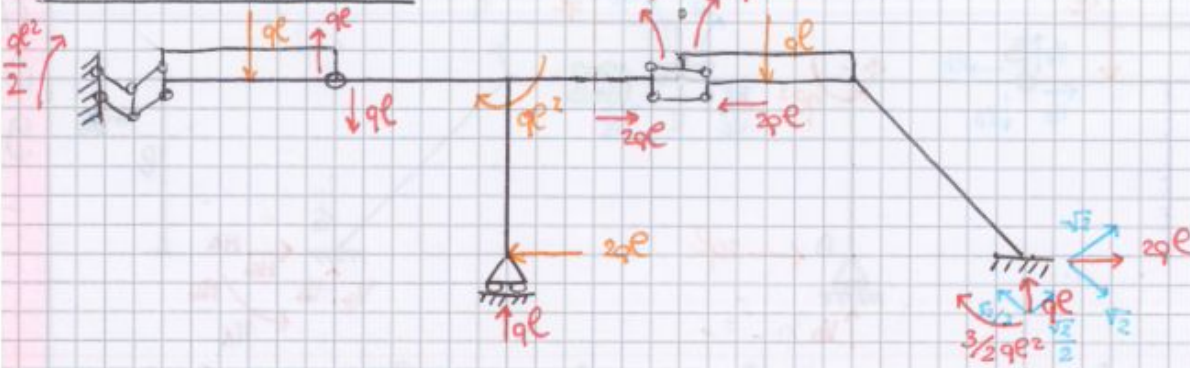


• PRINCIPII DEI LAVORI VIRTUALI

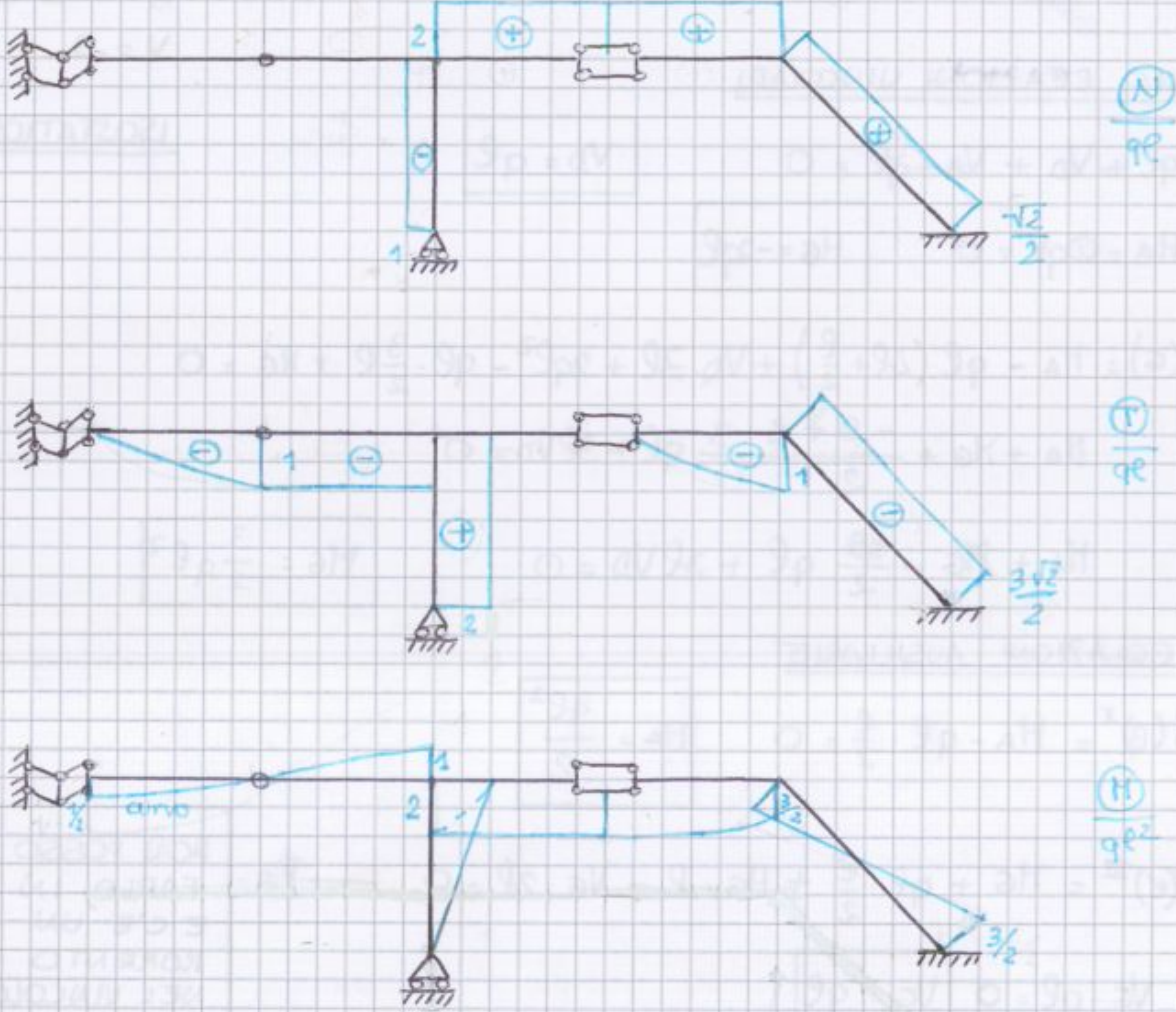
H_c

V_c

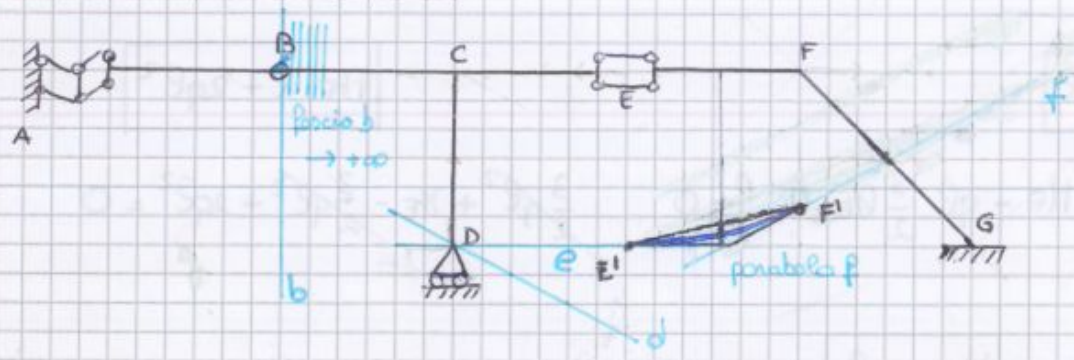
• RIDISEGNO LA TRAVE



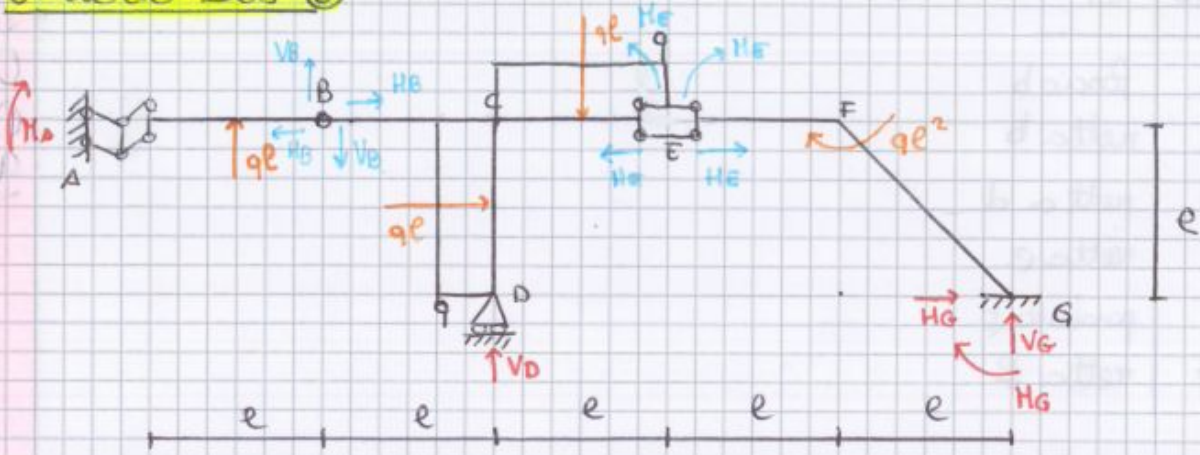
• DISEGNO I DIAGRAMMI



• CURVA DELLE PRESSIONI



21 UGIUO 2003 (2)



$g = 3 \times 3 = 9$
 $V = 9.$

ISOSTATICA

• CALCOLO REAZIONI VINCOLARI

$\uparrow + qe + V_D - qe + V_G = 0 \quad \boxed{V_D = 0}$

$\rightarrow + qe + H_G = 0 \quad \boxed{H_G = -qe}$

$\uparrow + M(G) = H_G + qe^2 + M_A - qe \cdot \frac{5}{2}e + V_D \cdot 3e + qe \cdot \frac{e}{2} + qe \cdot \frac{9}{2}qe = 0$
 $H_G + M_A + 3V_D e + \frac{-5+2+1+9}{2} qe^2 = 0 \quad \boxed{M_G = -3qe^2}$

• USO EQUAZIONI AUSILIARIE

$\uparrow + M(B)^I = M_A + qe \cdot \frac{e}{2} = 0 \quad \boxed{M_A = -\frac{qe^2}{2}}$

$\uparrow + \boxed{V_G = 0}$

• CALCOLO REAZIONI INTERNE

$\boxed{H_B = 0} \quad \boxed{V_B = -qe} \quad \boxed{H_E = qe}$

TRAVE 3

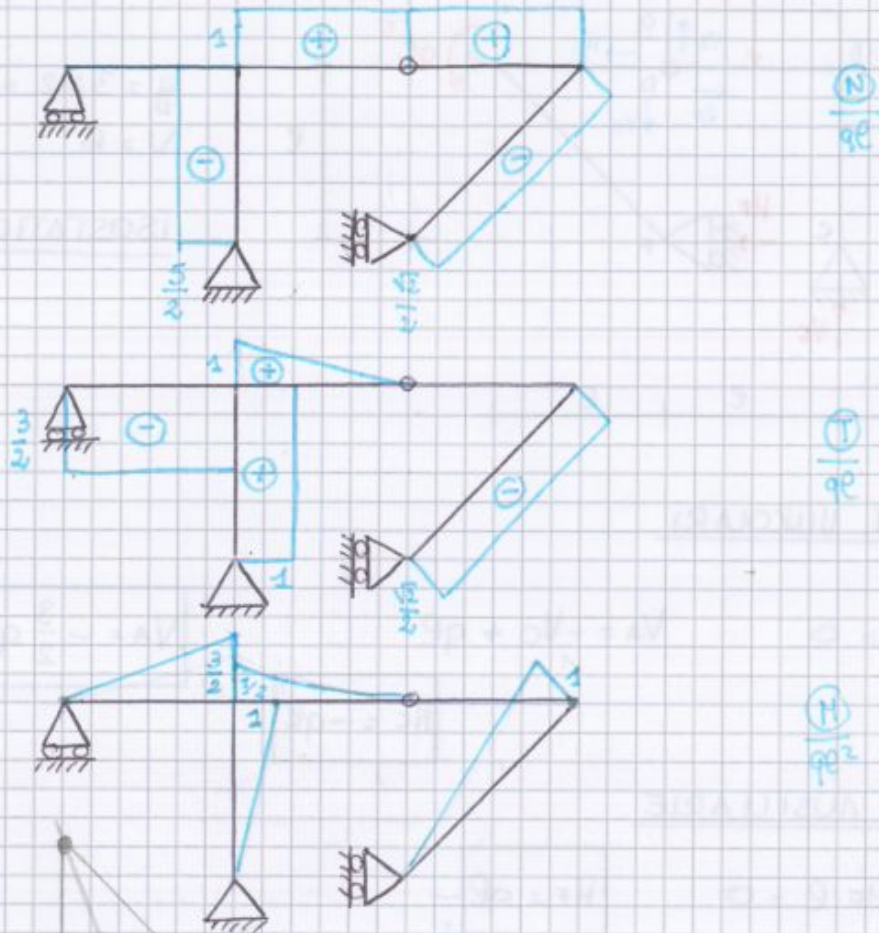
$\uparrow + M(G)^{II} = M_G + qe^2 + H_E + H_e \cdot e = 0$
 $-3qe^2 + qe^2 + qe^2 = -H_E \quad \boxed{M_E = qe^2}$

• PRINCIPIO DEI LAVORI VIRTUALI

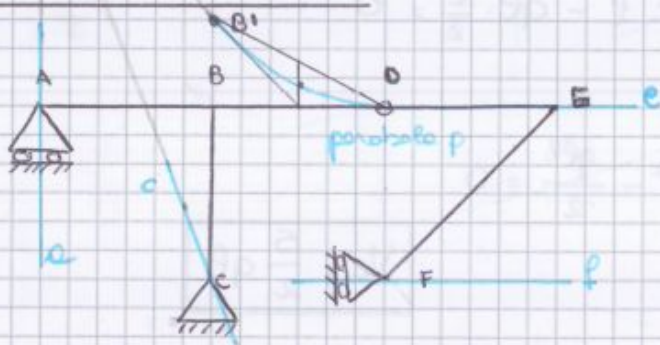
(He)

(He)

• DISEGNO I DIAGRAMMI



• CURVA DELLE PRESSIONI

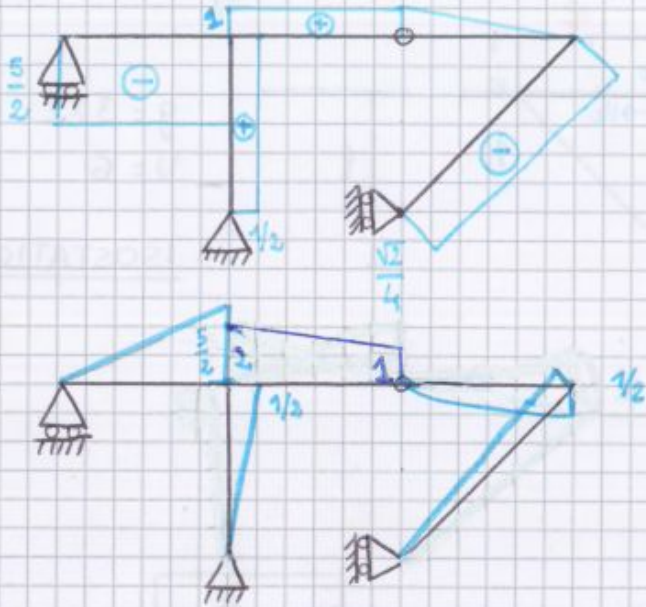


TRATTO	CDP
AB	retta a
BC	retta c
BD	parabola p
EF	retta f
DE	retta e

• PRINCIPIO DEI LAVORI VIRTUALI

(H_D)

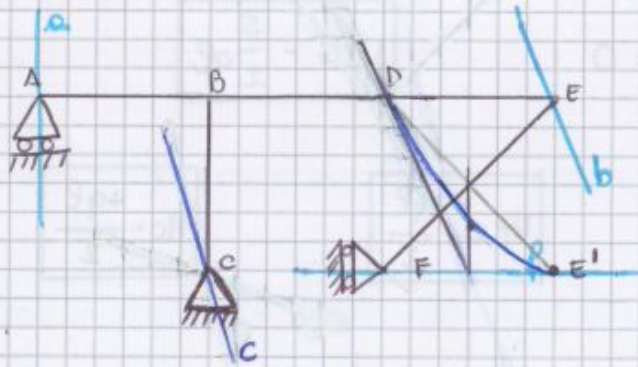
(V_D)



(F)
1/2

(H)
1/2

• CURVA DELLE PRESSIONI



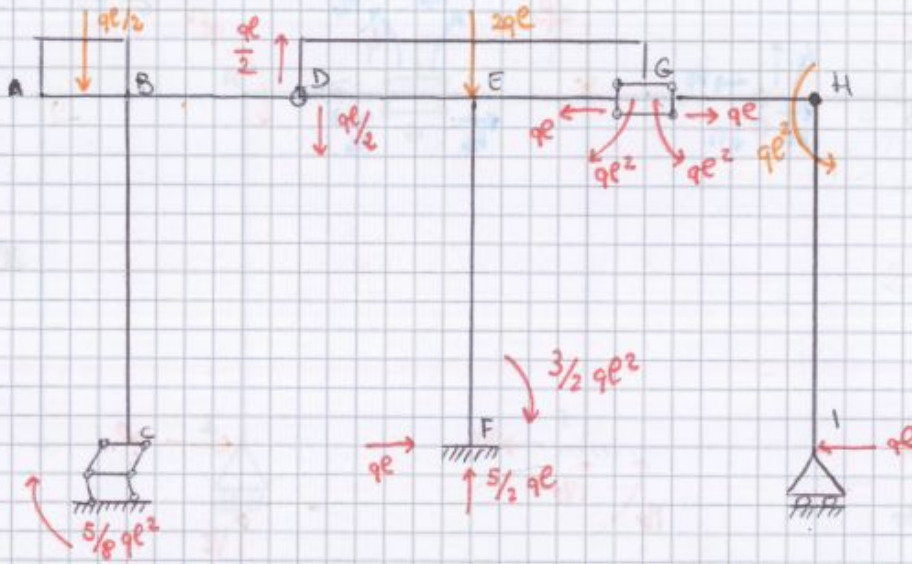
TRATTO	CDP
AB	retta a
BC	retta c
BD	retta b
DE	parabola p
EF	retta f

• PRINCIPIO DEI LAVORI VIRTUALI

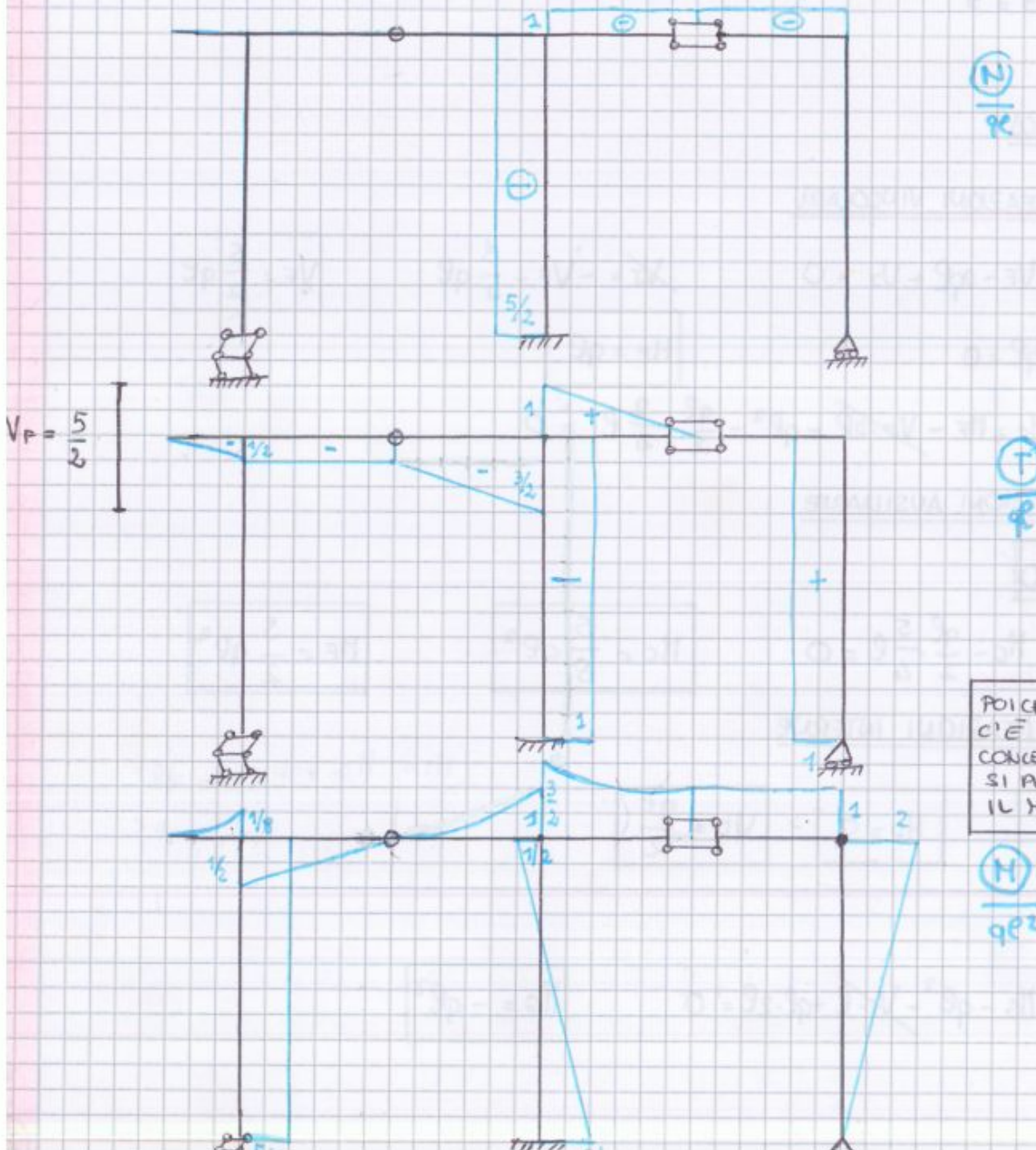
(Hc)

(Mc)

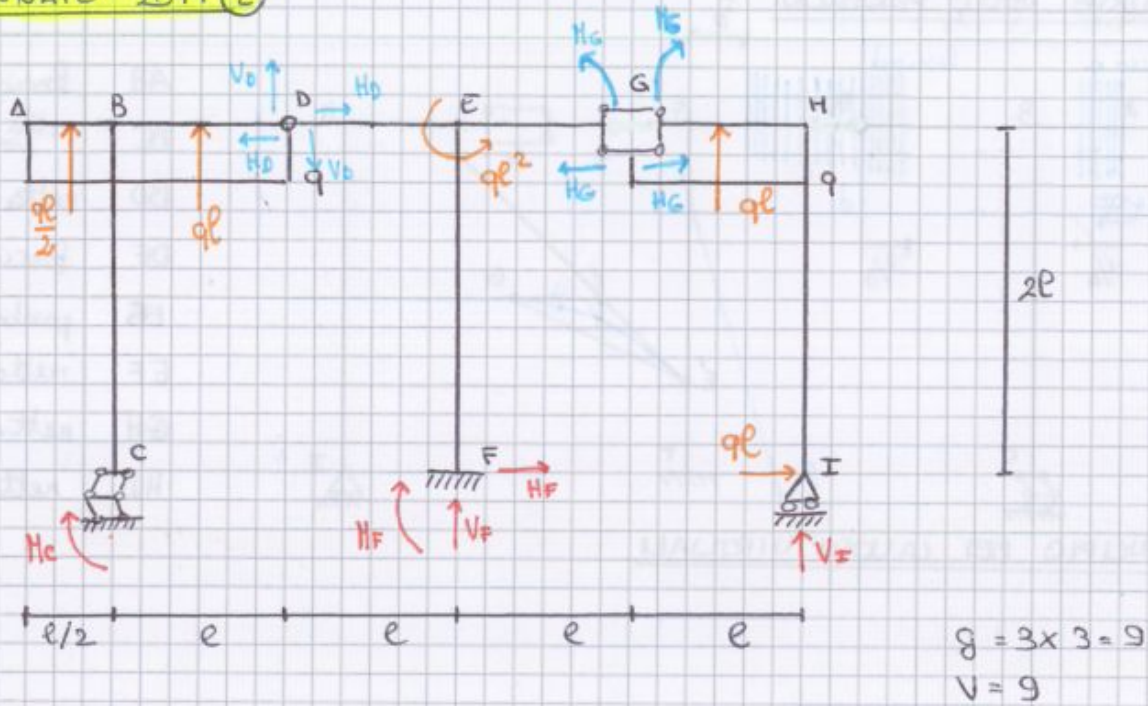
• RIDISEGNO LA TRAVE



• DISEGNO I DIAGRAMMI



7 GENNAIO 2011 (2)



• CALCOLO LE REAZIONI VINCOLARI

$$\uparrow + \frac{qe}{2} + ql + ql + V_F + V_I = 0$$

$$\rightarrow H_F + ql = 0 \quad \boxed{H_F = -ql}$$

$$\uparrow + M_F = M_C + M_F + ql/2 \cdot \frac{9}{4}e + ql \cdot \frac{3}{2}e - ql^2 - V_I \cdot 2e - ql \cdot \frac{3}{2}e = 0$$

$$M_C + M_F - V_I \cdot 2e + \frac{1}{8} ql^2 = 0$$

ISOSTATICA

• USO EQUAZIONI AUSILIARIE

$$\uparrow + V_I + ql = 0 \quad \boxed{V_I = -ql} \quad \boxed{V_F = \frac{-3}{2} ql}$$

$$\uparrow + M(C) = M_C + \frac{1}{2} ql \cdot \frac{5}{4}e + ql \cdot \frac{e}{2} = 0 \quad \boxed{M_C = -\frac{9}{8} ql^2}$$

$$\boxed{M_F = -ql^2}$$

• CALCOLO REAZIONI INTERNE

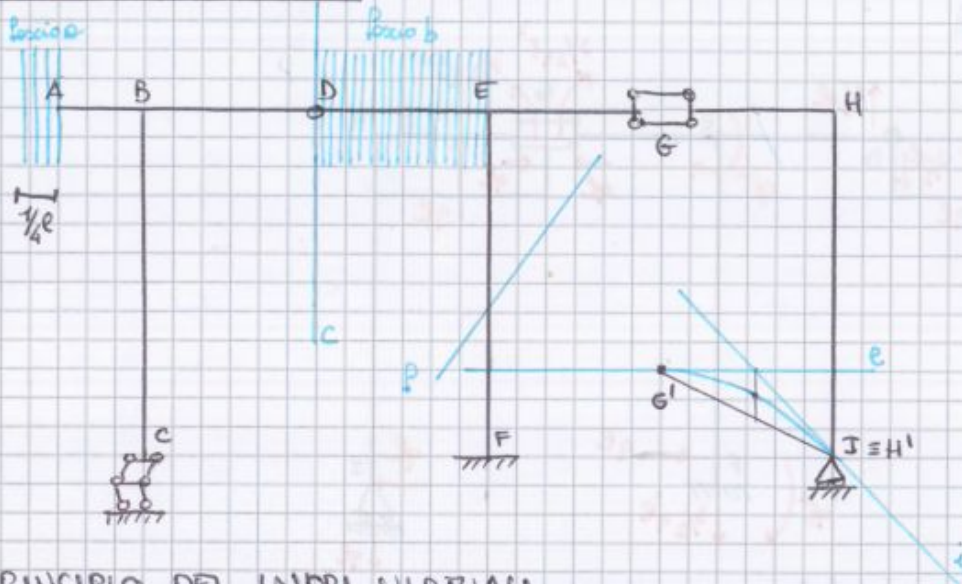
$$\boxed{H_G = -ql} \quad \boxed{H_D = 0} \quad \boxed{V_D = -\frac{3}{2} ql^2}$$

TRAVE 3

$$\uparrow + M(G) = -ql \cdot \frac{e}{2} - V_I \cdot l - ql \cdot 2e + M_G \quad M_G = +\frac{ql^2}{2} - ql^2 + 2ql^2$$

$$\boxed{M_G = +\frac{3}{2} ql^2}$$

• CURVA DELLE PRESSIONI



TRATTO	CDP
AB	fascio a
BC	retta empirica
BD	fascio b
DE	retta c
EF	retta f
EG	retta e
GH	parabola
HI	retta i

• PRINCIPIO DEI LAVORI VIRTUALI

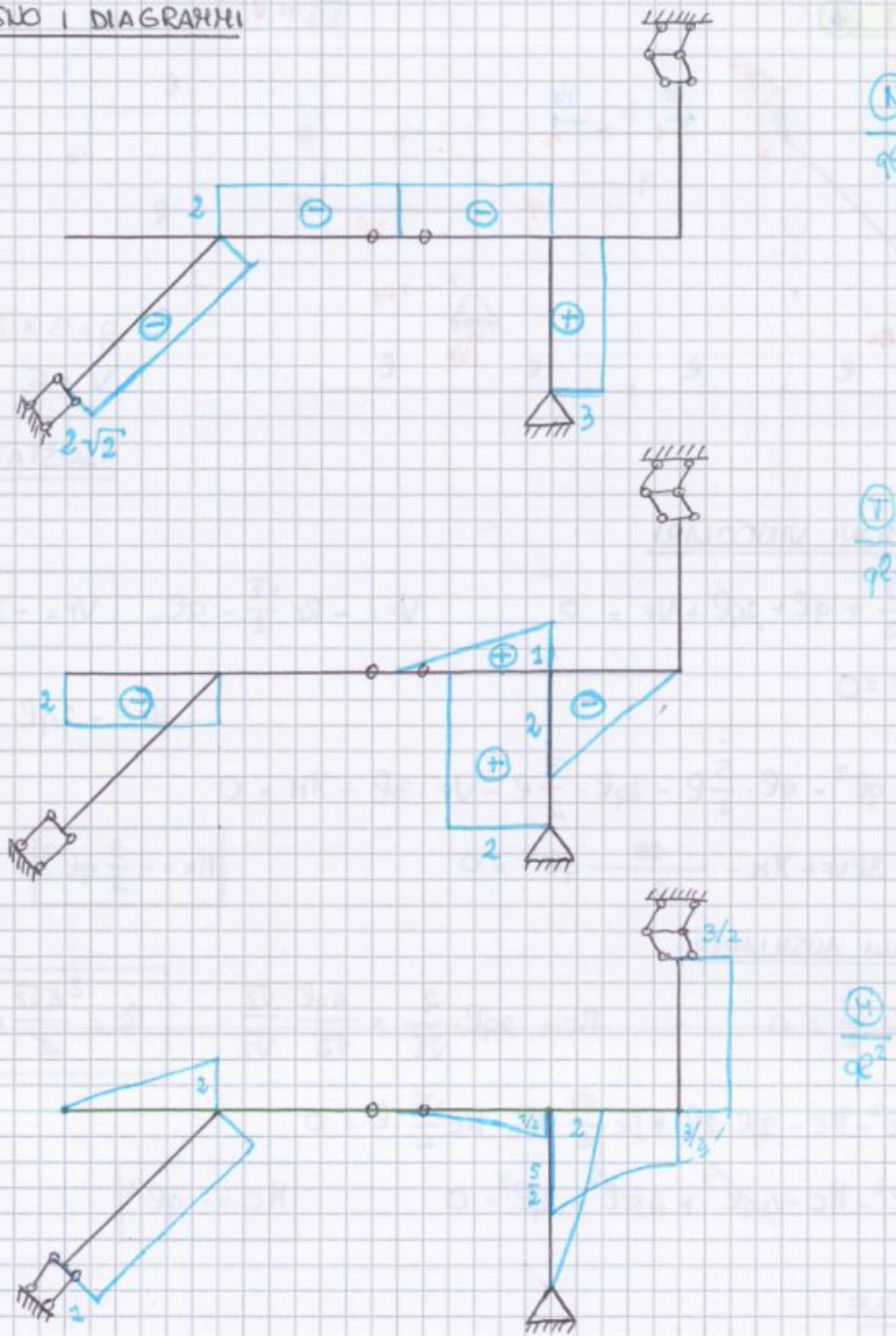
H_D

U_D

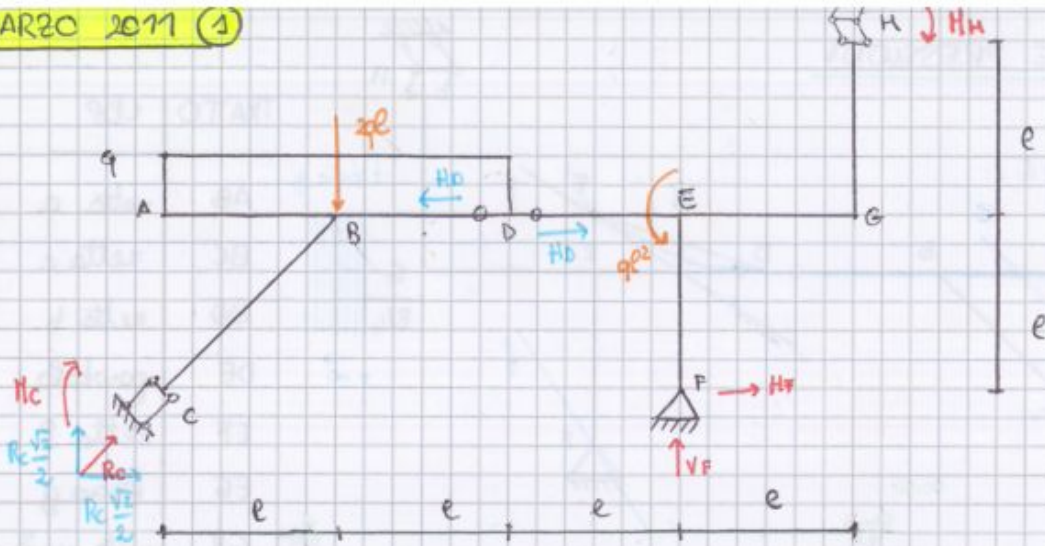
H_G

H_G

• DISEGNO I DIAGRAMMI



3 MARZO 2011 (1)



$$g = 3 \times 2 = 6$$

$$V = 6$$

ISOSTATICA

• CALCOLO REAZIONI VINCOLARI

$$+\uparrow R_c \frac{\sqrt{2}}{2} - 2ql + V_F = 0$$

$$R_c \frac{\sqrt{2}}{2} - 2ql = 0$$

$$R_c = \frac{4}{\sqrt{2}} ql = 2\sqrt{2} ql$$

$$+\rightarrow R_c \frac{\sqrt{2}}{2} + H_F = 0$$

$$H_F = -2ql$$

$$+\curvearrowright M(c) = M_c + 2ql^2 - ql^2 - V_F \cdot 3l + H_H = 0$$

$$H_H = -ql^2$$

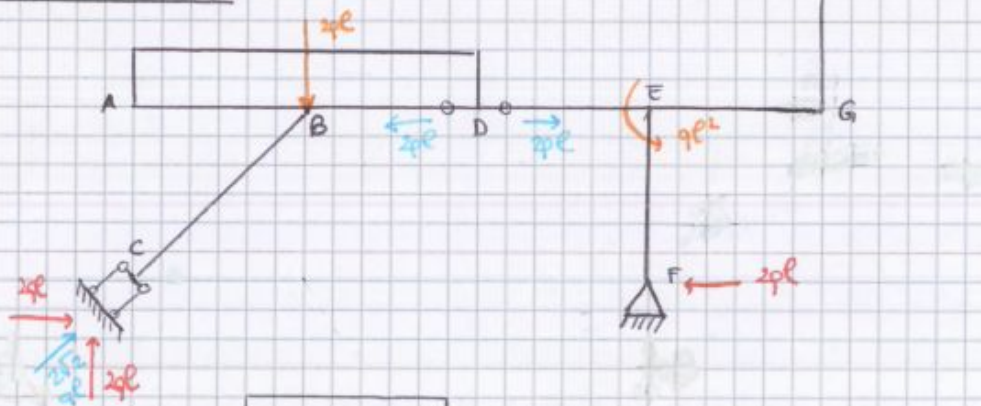
• USO EQUAZIONI AUSILIARIE

$$+\curvearrowright M(o)^I = M_c - 2ql \cdot l + R_c \frac{\sqrt{2}}{2} \cdot 2l - R_c \frac{\sqrt{2}}{2} \cdot l = 0$$

$$M_c = 0 \quad ql^2$$

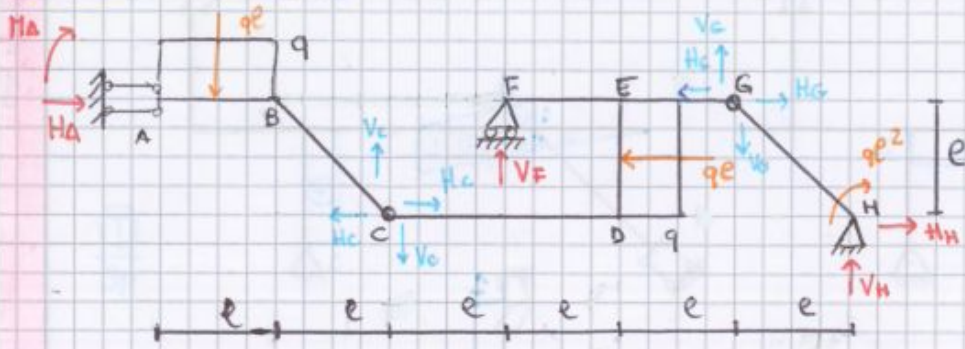
$$+\uparrow^{II} V_F = 0$$

• RIDISEGNO LA TRAVE



$$H_D = 2ql^2$$

26 GIUGNO 2012 (1)



$$g = 3 \times 3 = 9$$

$$v = 9$$

ISOSTATICA

• CALCOLO REAZIONI VINCOLARI

$$\uparrow + -qe + V_F + V_H = 0$$

$$V_H = -\frac{1}{6}qe$$

$$\rightarrow + H_A - qe + H_H = 0$$

$$H_A = -\frac{1}{6}qe$$

$$\curvearrow + M(H) = H_A + H_A \cdot e - qe \cdot \frac{11}{2}e + V_F \cdot 3e - qe \cdot \frac{e}{2} + qe^2 = 0$$

$$H_A = \frac{5}{3}qe^2$$

• USO EQUAZIONI AUSILIARIE

$$\curvearrow + M(C)^I = H_A + H_A \cdot e - qe \cdot \frac{3}{2}e = 0$$

$$H_A = -H_A \cdot e + \frac{3}{2}qe^2$$

$$\curvearrow + M(G)^III = qe^2 - V_H e - H_H e = 0$$

$$H_H = \frac{7}{6}qe$$

• SOSTITUISCO

$$-H_A \cdot e + H_A \cdot e + \frac{3}{2}qe^2 - \frac{11}{2}qe^2 - \frac{qe^2}{2} + qe^2 + V_F \cdot 3e = 0$$

$$V_F = \frac{7}{6}qe$$

• CALCOLO REAZIONI INTERNE

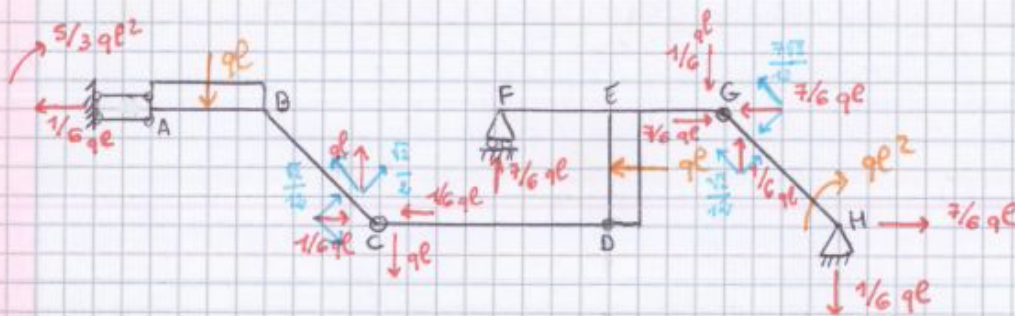
$$H_G = -\frac{7}{6}qe$$

$$V_G = -\frac{1}{6}qe$$

$$H_C = -\frac{1}{6}qe$$

$$V_C = qe$$

• RINSEGNO LA TRAVE



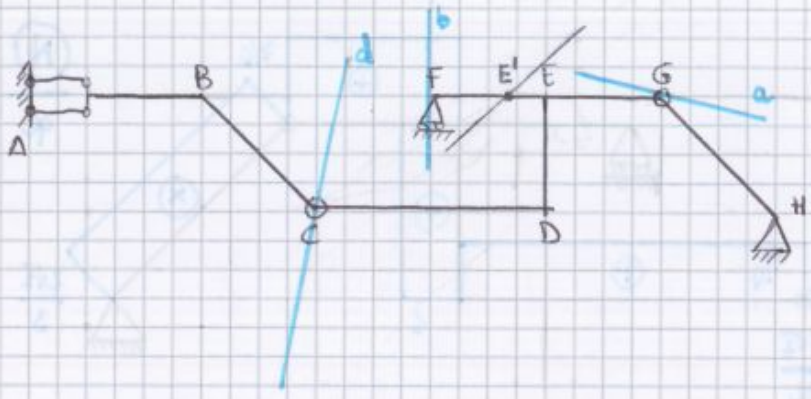
$$\curvearrow + \frac{\sqrt{2}}{2}qe - \frac{\sqrt{2}}{12}qe = \frac{5\sqrt{2}}{12}qe$$

$$\curvearrow + \frac{7\sqrt{2}}{12}qe + \frac{\sqrt{2}}{12}qe = \frac{8\sqrt{2}}{12}qe = \frac{2\sqrt{2}}{3}qe$$

$$\rightarrow + \frac{\sqrt{2}}{2}qe + \frac{\sqrt{2}}{12}qe = \frac{7\sqrt{2}}{12}qe$$

$$\swarrow + \frac{7\sqrt{2}}{12}qe - \frac{\sqrt{2}}{12}qe = \frac{\sqrt{2}}{2}qe$$

• CURVA DELLE PRESSIONI



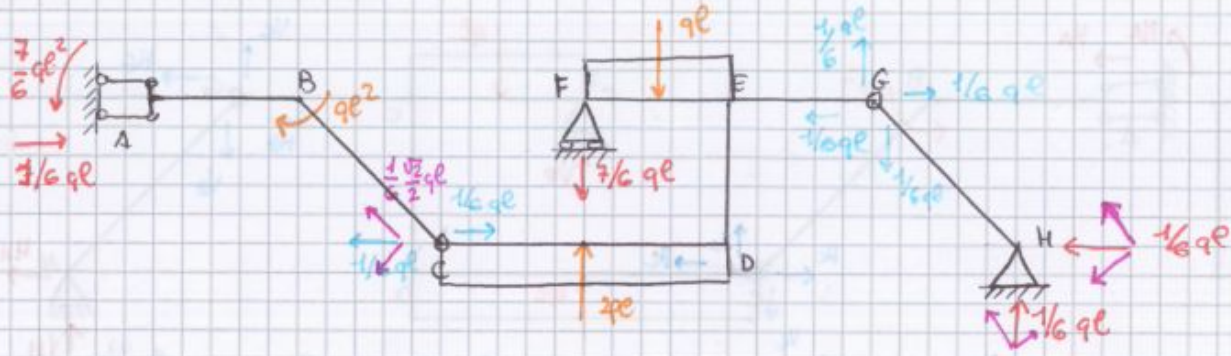
TRATTO	CDP
FE	retta b
AB	parabola
BC	retta d
CD	retta d
ED	parabola
EG	retta a
GH	retta a

• PRINCIPIO DEI LAVORI VIRTUALI

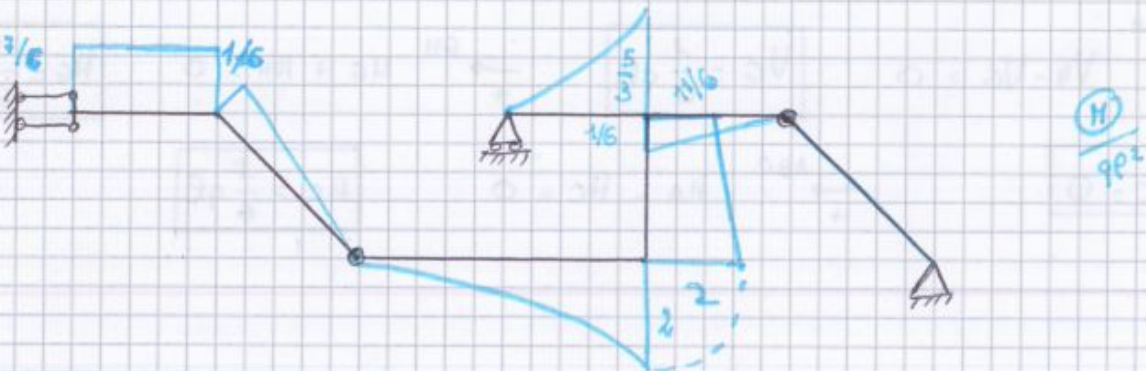
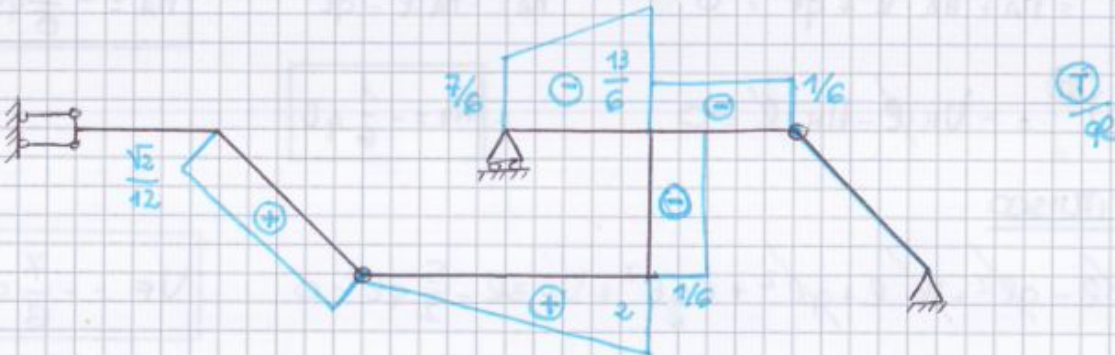
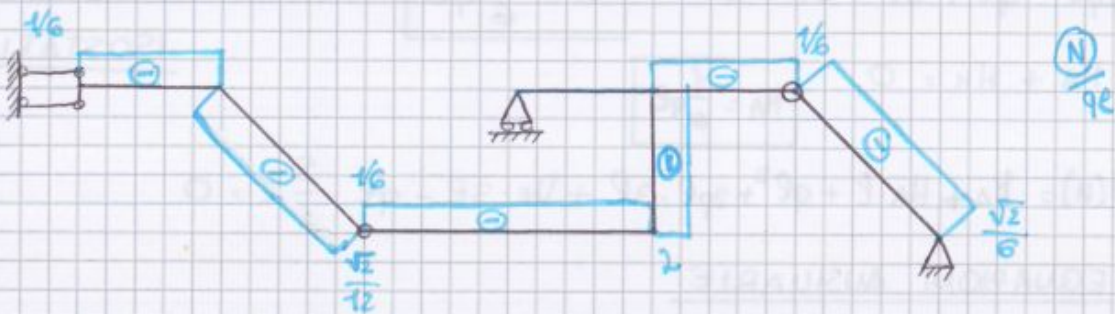
(Vc)

(Hc)

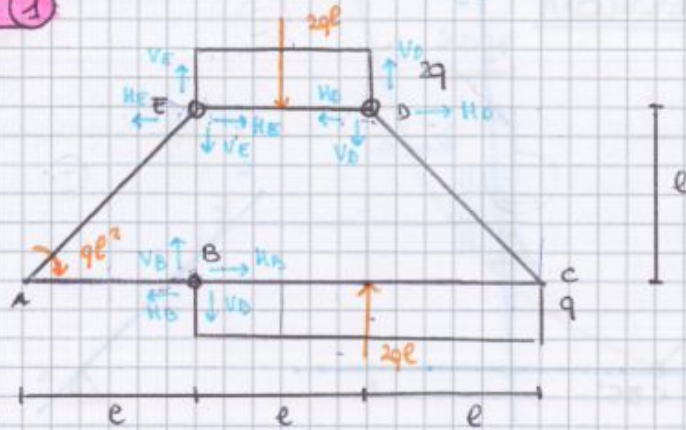
• RIDISEGNO LA TRAVE



• DISEGNO I DIAGRAMMI



15 GENNAIO 2008 (1)



$g = 3 \times 3 = 9$
 $v = 6$
 CARICO AUTOEQUILIBRATO
ISOSTATICA

• CALCOLO REAZIONI INTERNE CON USO DI EQUAZIONI AUSILIARIE

$$\uparrow M(E)^{ED} = 2ql \cdot \frac{e}{2} + V_D \cdot e = 0 \quad \boxed{V_D = -ql}$$

$$\uparrow +^{ED} - V_D - V_E - 2ql = 0 \quad \boxed{V_E = -ql}$$

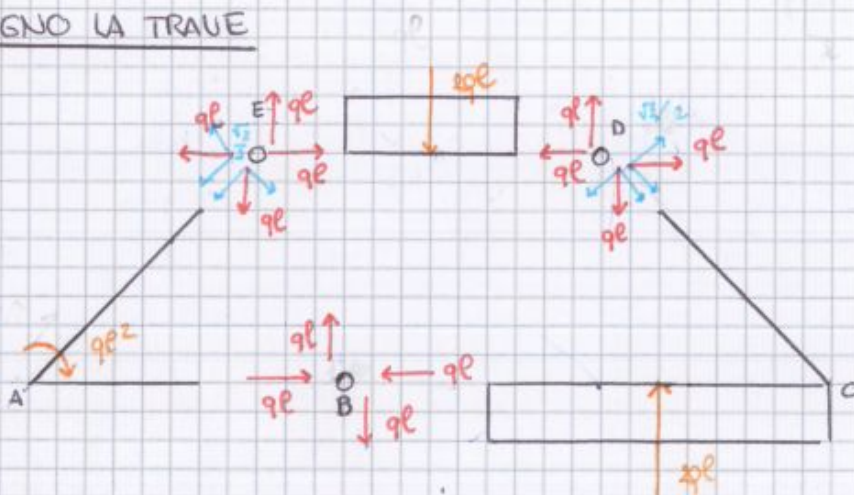
$$\downarrow +^{DCB} - V_D + V_B - 2ql = 0 \quad \boxed{V_B = ql}$$

$$\uparrow M(B)^{EAB} = ql \cdot e^2 - H_E \cdot e = 0 \quad \boxed{H_E = ql}$$

$$\rightarrow +^{EAB} - H_E - H_B = 0 \quad \boxed{H_B = -ql}$$

$$\rightarrow +^{ED} H_E - H_D = 0 \quad \boxed{H_D = ql}$$

• RIDISEGNO LA TRAVE



(AE)

$$\swarrow + \frac{\sqrt{2}}{2} ql + \frac{\sqrt{2}}{2} ql = \sqrt{2} ql$$

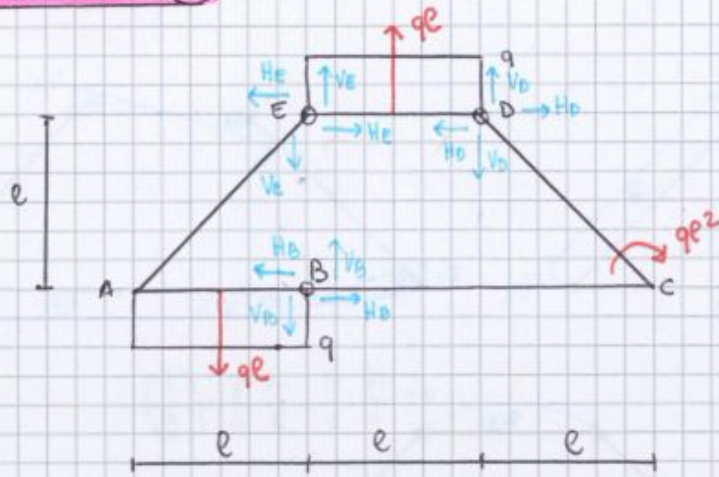
$$\nwarrow + \frac{\sqrt{2}}{2} ql - \frac{\sqrt{2}}{2} ql = 0$$

(DC)

$$\swarrow + \frac{\sqrt{2}}{2} ql + \frac{\sqrt{2}}{2} ql = \sqrt{2} ql$$

$$\nwarrow + \frac{\sqrt{2}}{2} ql - \frac{\sqrt{2}}{2} ql = 0$$

15 GENNAIO 2008 (2)



$q = 3 \times 3 = 9$
 $V = 6$

CARICO AUTOEQUILIBRATO

ISOSTATICA

• CALCOLO REAZIONI INTERNE CON USO DI EQUAZIONI AUSILIARIE

$\overset{+}{\curvearrowright} M_{(E)}^{ED} = -q \cdot e \cdot \frac{e}{2} + V_D \cdot e = 0$

$V_D = \frac{q \cdot e}{2}$

$\uparrow +^{ED} = q \cdot e + V_E - V_D = 0$

$V_E = -\frac{q \cdot e}{2}$

$\uparrow +^{DCB} = +V_D + V_B = 0$

$V_B = -\frac{q \cdot e}{2}$

$\overset{+}{\curvearrowright} M_{(B)}^{EBA} = -H_E \cdot e - q \cdot e \cdot \frac{e}{2} = 0$

$H_E = -\frac{q \cdot e}{2}$

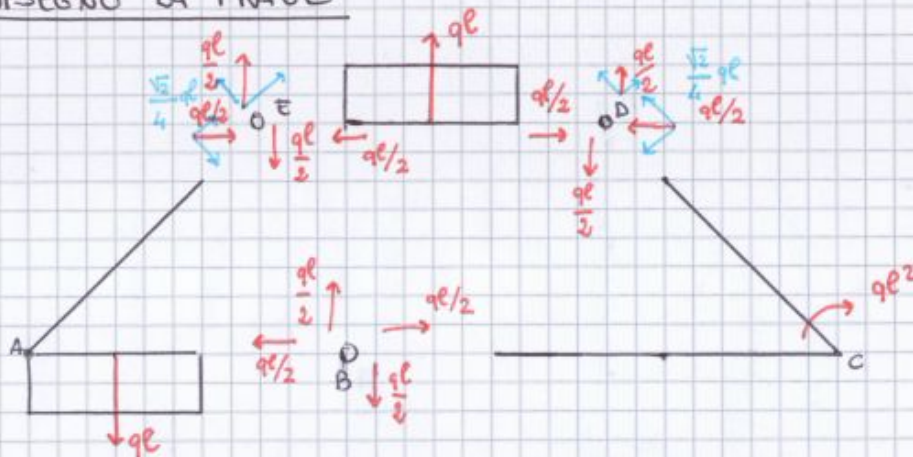
$\rightarrow +^{ED} H_E - H_D = 0$

$H_D = -\frac{q \cdot e}{2}$

$\rightarrow +^{DCB} H_D + H_B = 0$

$H_B = \frac{q \cdot e}{2}$

• RIDISEGNO LA TRAVE



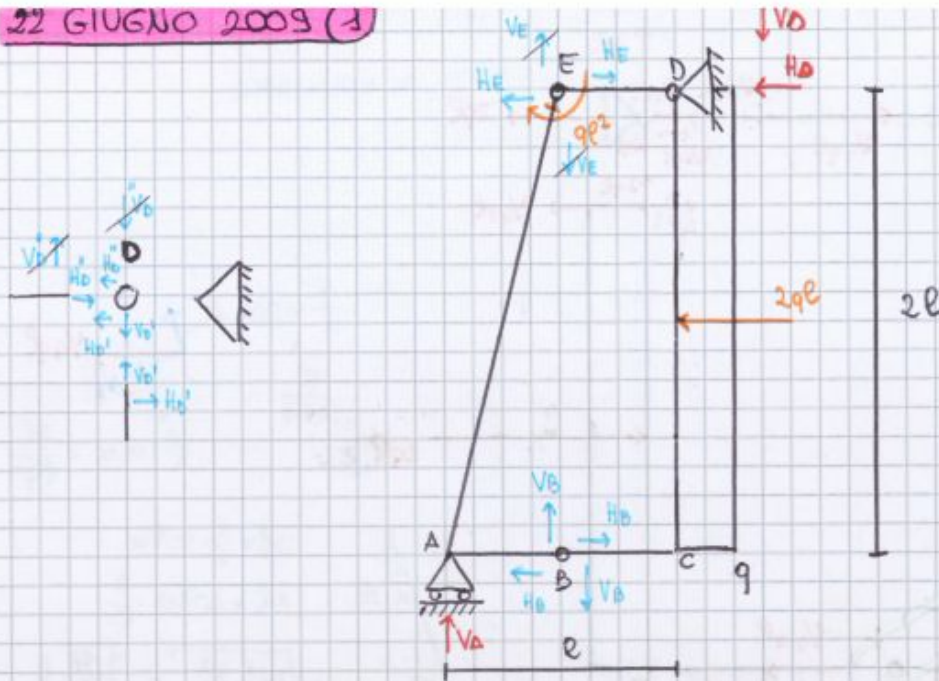
(AE)

$\rightarrow \frac{\sqrt{2}}{4} q \cdot e + \frac{\sqrt{2}}{4} q \cdot e = \frac{\sqrt{2}}{2} q \cdot e$

(DC)

$\rightarrow \frac{\sqrt{2}}{4} q \cdot e + \frac{\sqrt{2}}{4} q \cdot e = \frac{\sqrt{2}}{2} q \cdot e$

22 GIUGNO 2009 (1)



$$q = 3 \times 3 = 9$$

$$V = 9$$

ISOSTATICA

• CALCOLO REAZIONI VINCOLARI

$$+\uparrow V_A - V_D = 0$$

$$V_D = -3qe$$

$$+\rightarrow -H_D - 2qe = 0$$

$$H_D = -2qe$$

$$+\curvearrowright M_D = ql^2 + V_A \cdot l + 2qe \cdot l = 0$$

$$V_A = -3qe$$

• CALCOLO REAZIONI INTERNE

$$\begin{cases} H_D'' + H_D' = 2qe \\ V_D'' + V_D' = 3qe \end{cases}$$

$$H_D' = \frac{7}{4} qe$$

$$V_D' = 3qe$$

$$+\curvearrowright M_E^{ED} = -V_D'' \cdot \frac{l}{3} = 0$$

$$V_D'' = 0$$

$$V_E = 0$$

ED è una biella

$$+\curvearrowright M_B^{EAB} = ql^2 - H_E \cdot 2l + V_A \cdot \frac{l}{2} = 0$$

$$ql^2 - 3qe \cdot \frac{l}{2} = H_E \cdot 2l$$

$$H_E = -\frac{1}{4} qe$$

$$+\rightarrow^{ED} H_E + H_D'' = 0$$

$$H_D'' = \frac{1}{4} qe$$

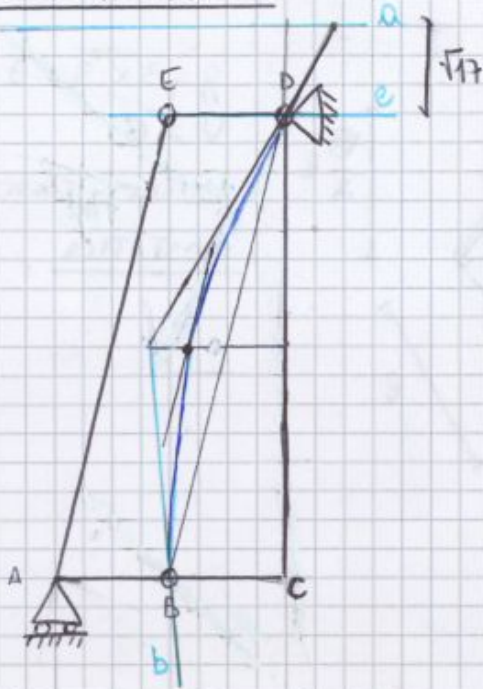
$$+\uparrow^{DCB} -V_B + V_D' = 0$$

$$V_B = 3qe$$

$$+\rightarrow^{DCB} H_B - 2qe + H_D' = 0$$

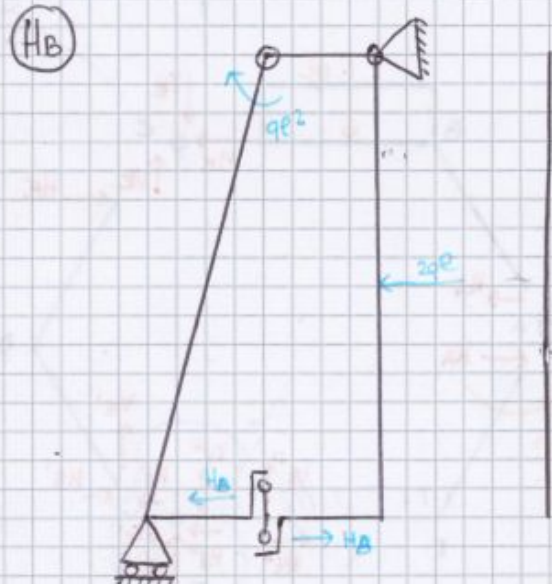
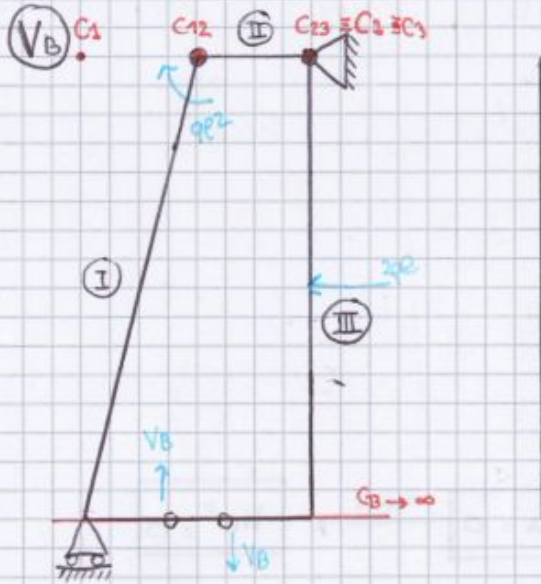
$$H_B = \frac{1}{4} qe$$

• CURVA DELLE PRESSIONI

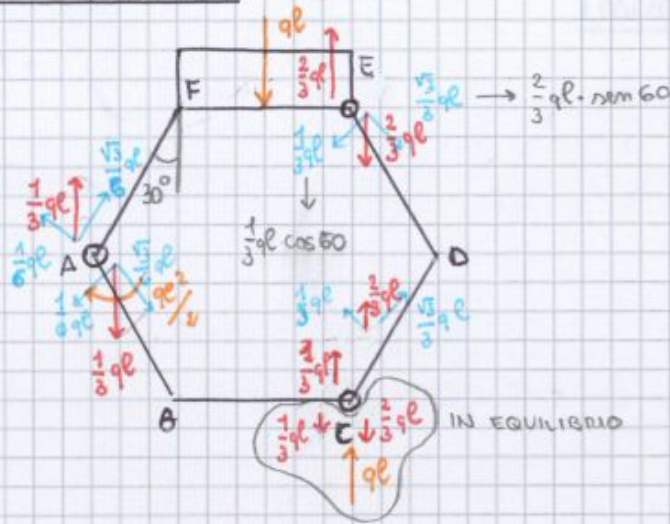


TRATTO	CDP
AB	retta b
BC	retta b
CD	parabola
DE	retta e
AE	retta s

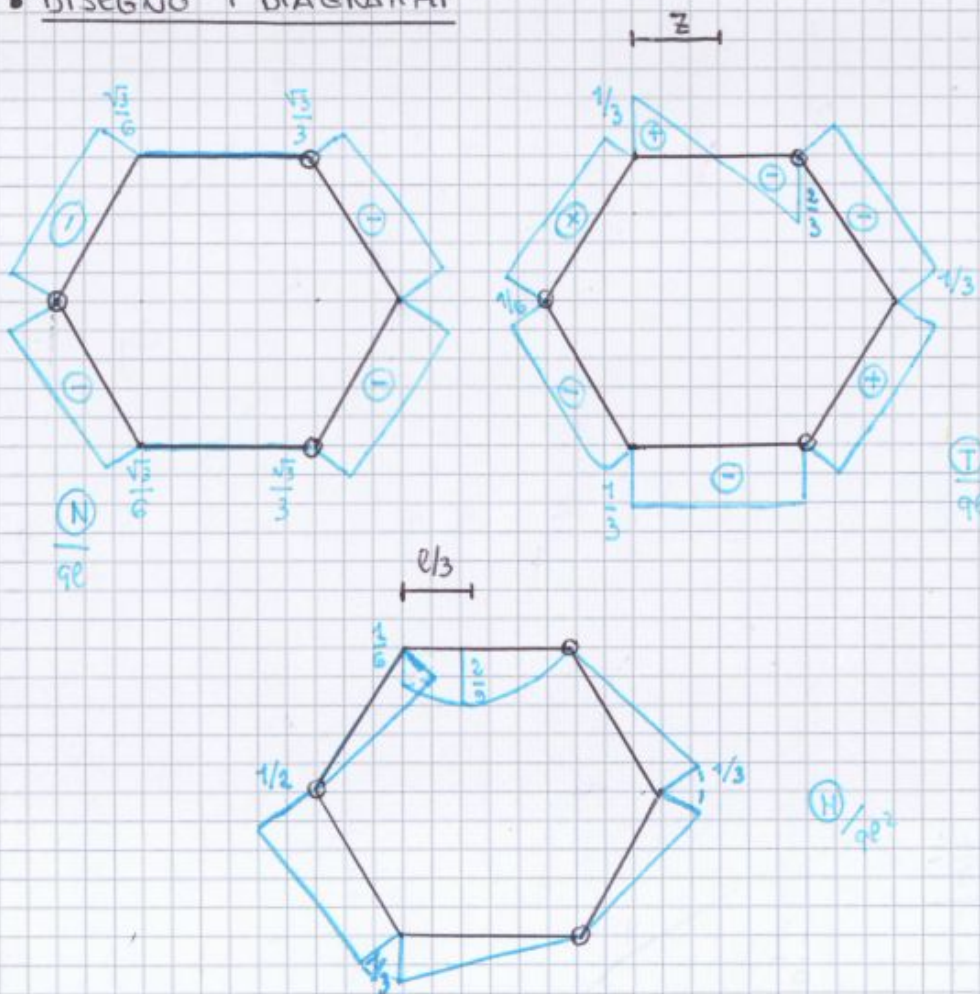
• PRINCIPIO DEI LAVORI VIRTUALI



• RIDISEGNO LA TRAVE



• DISEGNO I DIAGRAMMI



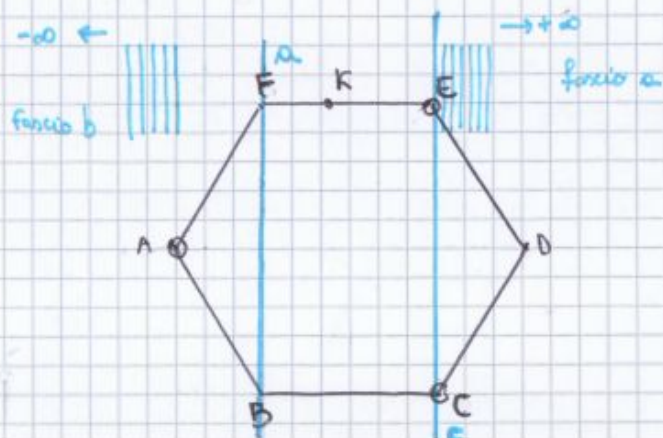
$$\frac{1}{3}ql - qz = 0$$

$$z = \frac{l}{3}$$

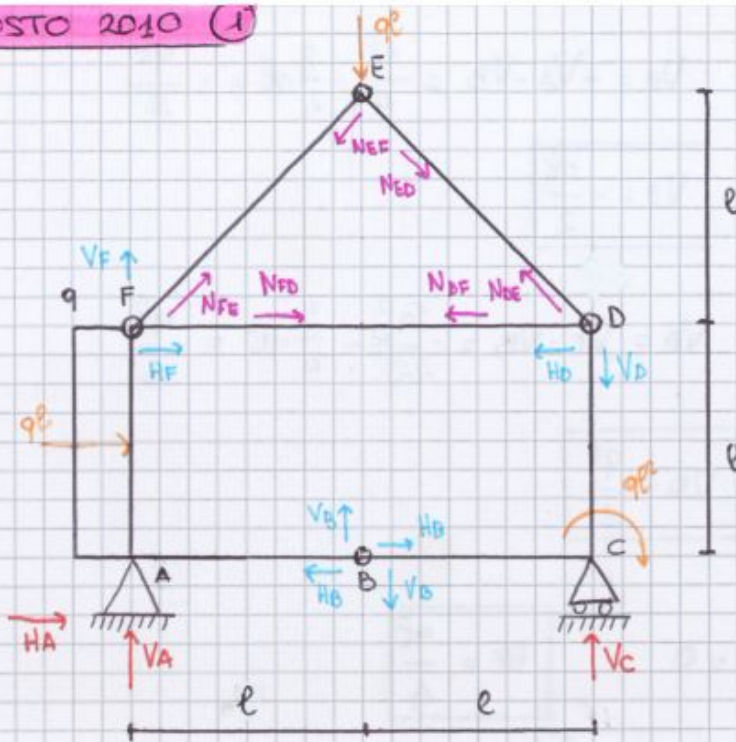
In F posso ribaltare $\frac{1}{3}ql^2$ ortogonalmente ad FA.

• CURVA DELLE PRESSIONI

TRATTO	CDP
AB	retta c
BC	retta c
CD	retta c
ED	retta c
FK	fascio b
EK	fascio a
AE	retta c



31 AGOSTO 2010 (1)



RETICOLARE

ISOSTATICA

ED - FD - FE
sono 3 bielle

• CALCOLO REAZIONI ESTERNE

$$\uparrow V_A + V_C - ql = 0$$

$$\rightarrow ql + H_A = 0$$

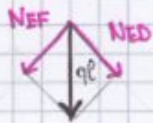
$$H_A = -ql$$

$$V_A = -\frac{ql}{4}$$

$$\uparrow M(A) = ql \cdot l + ql \cdot \frac{l}{2} + ql^2 - V_C \cdot 2l = 0$$

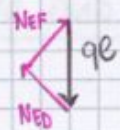
$$V_C = \frac{5}{4}ql$$

• CALCOLO REAZIONI INTERNE



$$N_{FE} = \frac{\sqrt{2}}{2} ql$$

$$N_{ED} = \frac{\sqrt{2}}{2} ql$$



$$\uparrow M(F)^{FAB} = -H_A \cdot l - ql \cdot \frac{l}{2} - V_B \cdot l + H_B \cdot l = 0$$

$$ql^2 - \frac{ql^2}{2} - V_B \cdot l + H_B \cdot l = 0$$

$$H_B - V_B = -\frac{ql}{2}$$

$$\uparrow M(B)^{DCB} = ql^2 - V_B \cdot l - H_B \cdot l = 0$$

$$H_B + V_B = ql$$

$$H_B = -V_B + ql$$

$$-V_B + ql - V_B + \frac{ql}{2} = 0$$

$$-2V_B + \frac{3}{2}ql = 0$$

$$V_B = \frac{3}{4}ql$$

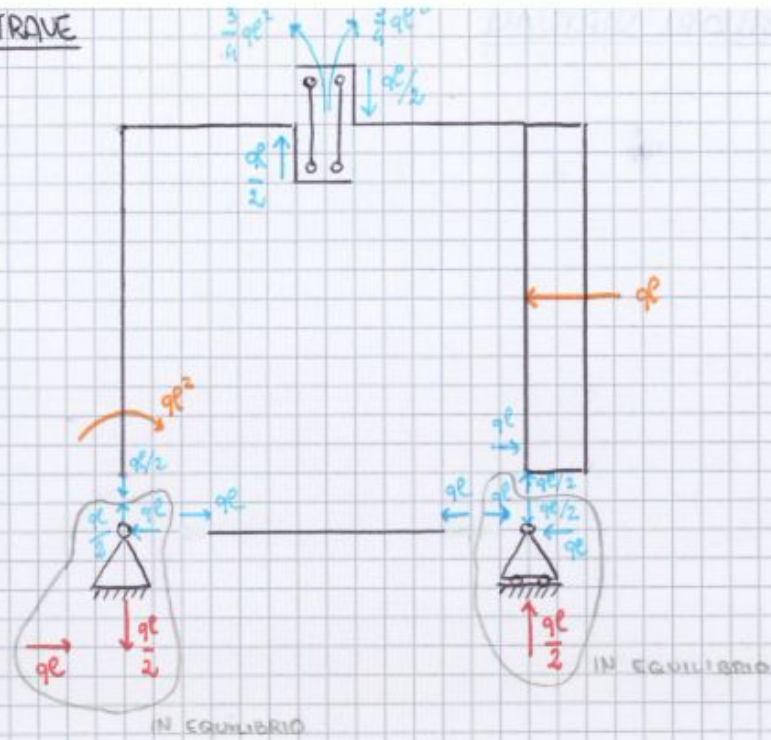
$$H_B = \frac{1}{4}ql$$

● DISEGNO I DIAGRAMMI

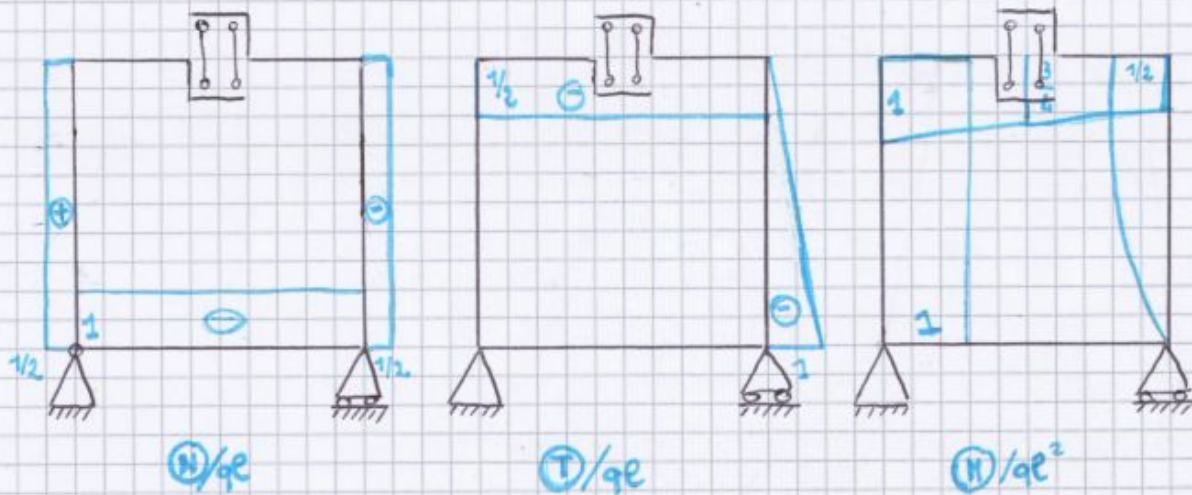
2010

62

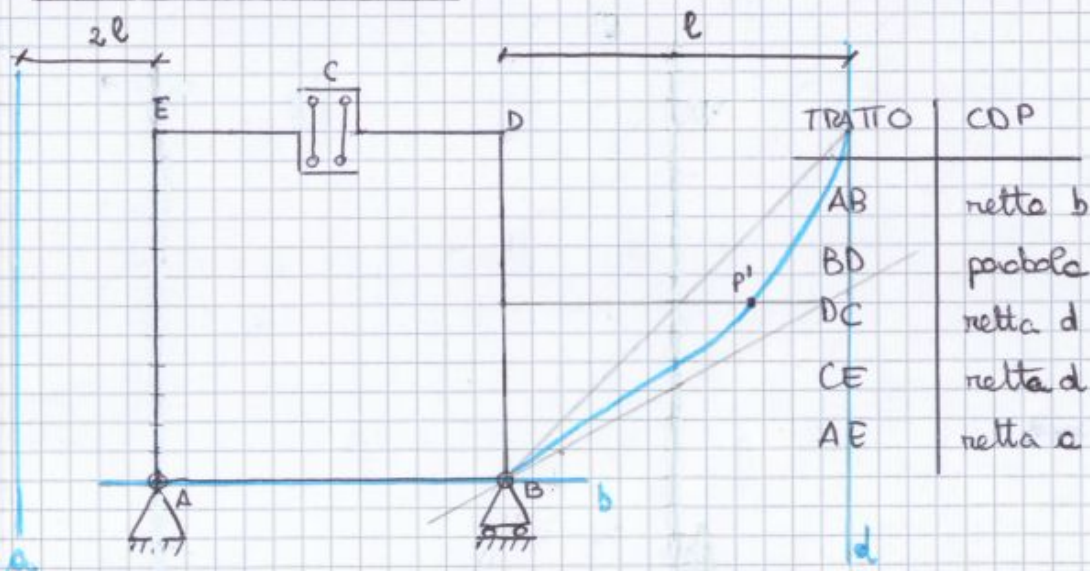
• RIDISEGNO LA TRAVE



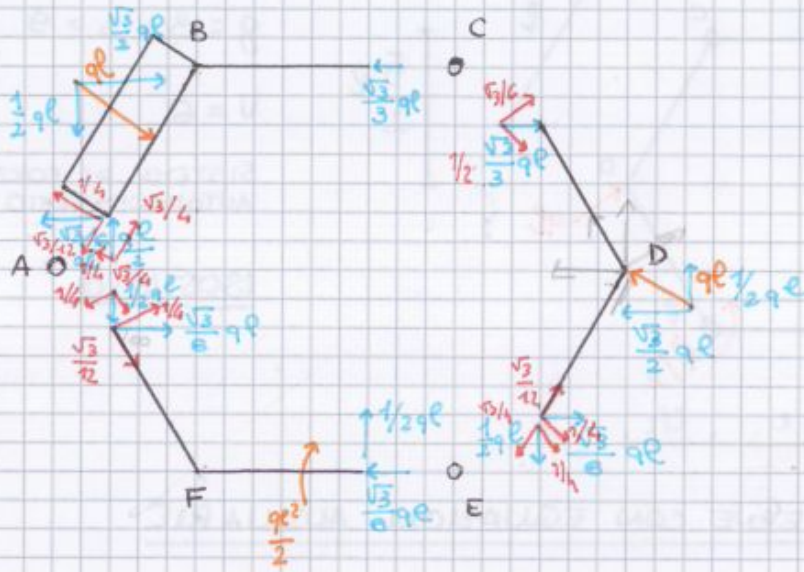
• DISEGNO I DIAGRAMMI



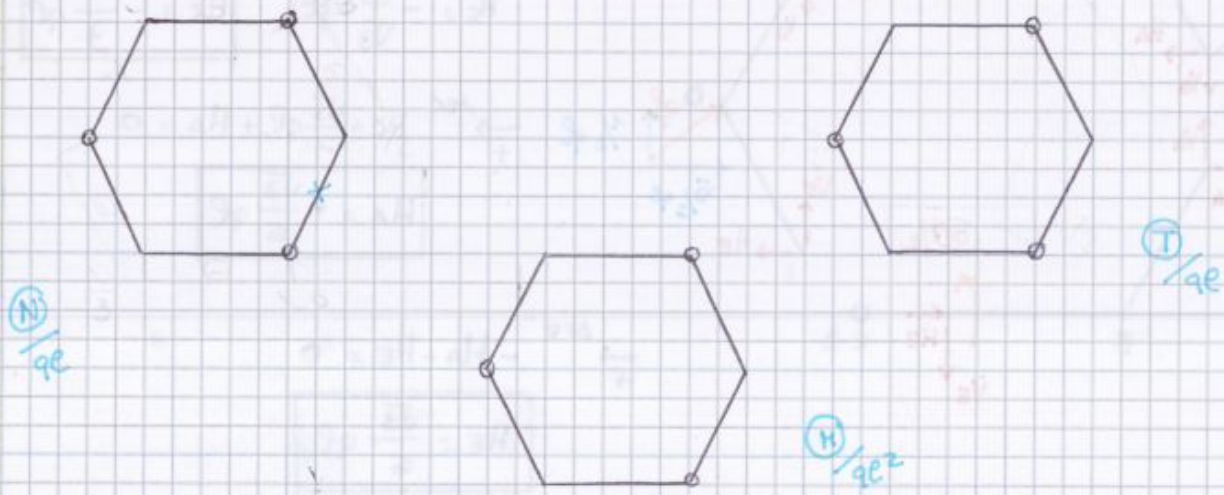
• CURVA DELLE PRESSIONI



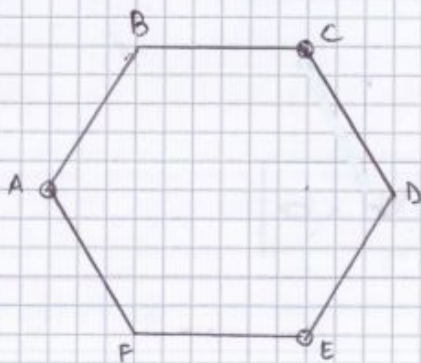
• RIDISEGNO LA TRAVE



• DISEGNO I DIAGRAMMI

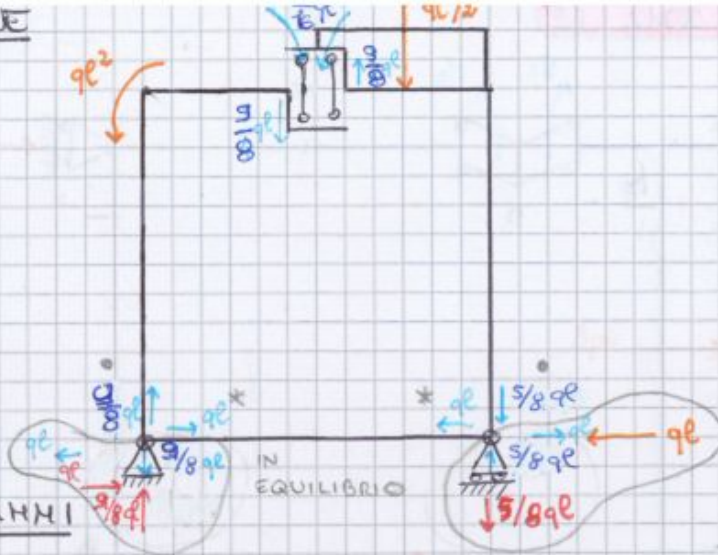


• CURVA DELLE PRESSIONI

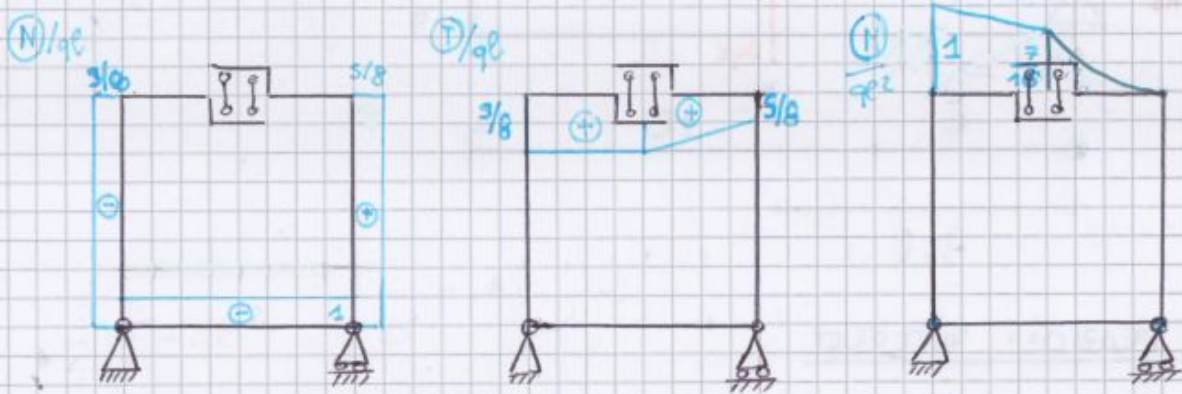


TRATTO	CDP
AB	
BC	
CD	
DE	
EF	
AF	

• RIDISEGNO LA TRAVE

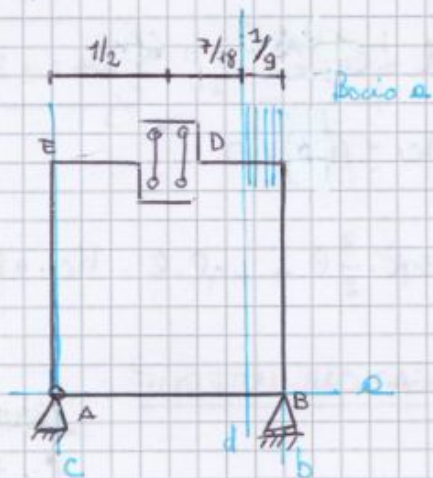


• DISEGNO I DIAGRAMMI

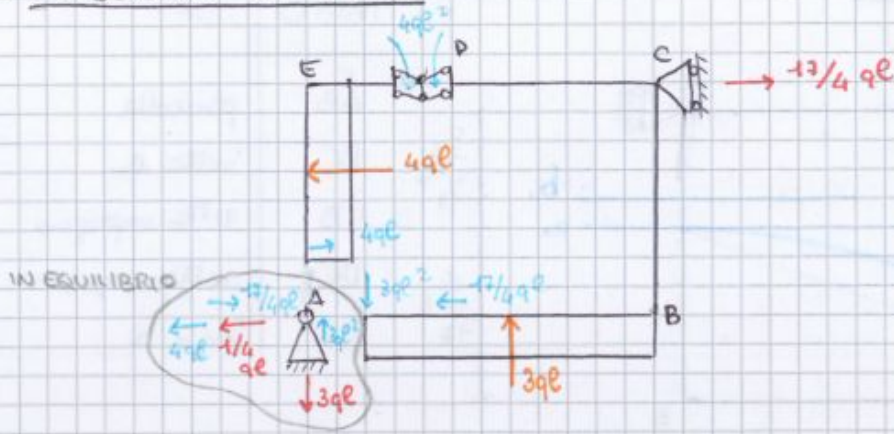


• CURVA DELLE PRESSIONI

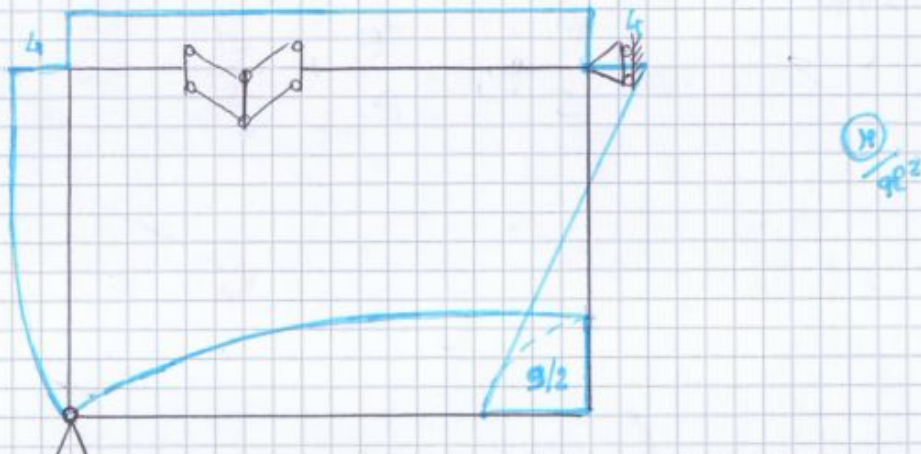
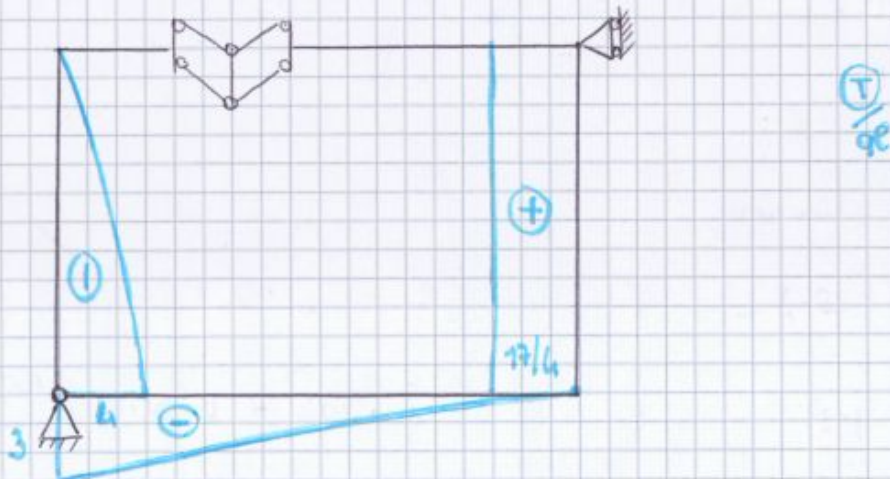
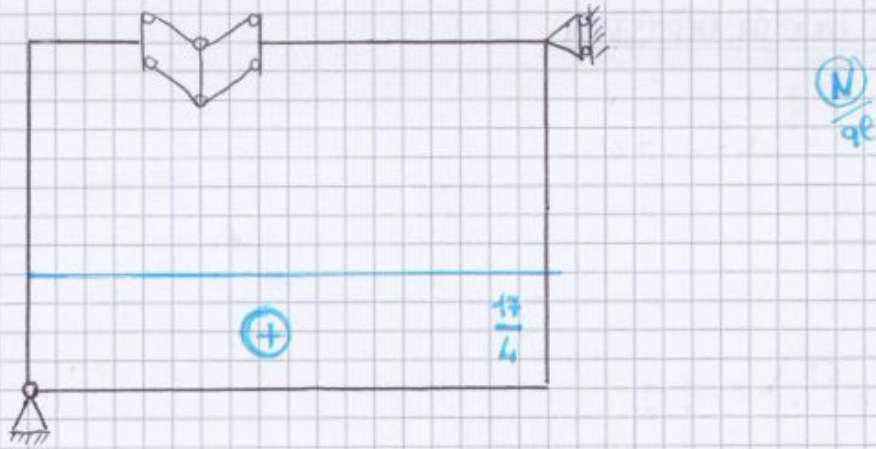
TRATTO	CDP	TRATTO	CDP
AB	retta a	DE	retta d
BC	retta b	AE	retta c
CD	base a		



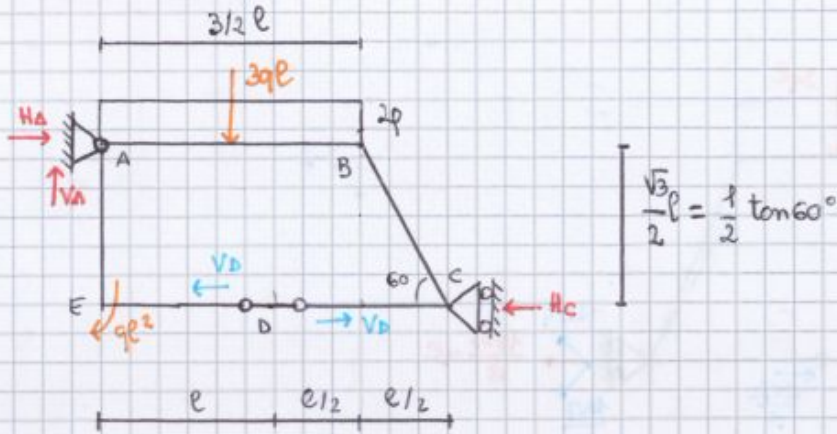
• RIDISEGNO LA TRAVE



• DISEGNO I DIAGRAMMI



29 APRILE 2008 (1)



$$g = 2 \times 3 = 6$$

$$V = 6$$

ISOSTATICA

• CALCOLO REAZIONI VINCOLARI

$$+\uparrow V_A - 3ql = 0$$

$$V_A = 3ql$$

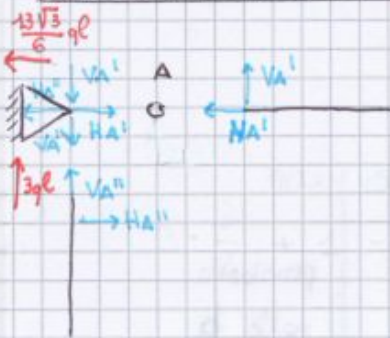
$$\rightarrow H_A - H_C = 0$$

$$H_A = -\frac{13\sqrt{3}}{6} ql$$

$$+\uparrow M(A) = +3ql \cdot \frac{3}{4} l + ql^2 + H_C \cdot \frac{\sqrt{3}}{2} l = 0 \quad -\left(\frac{9}{4} ql^2 + ql^2\right) \cdot \frac{2}{\sqrt{3}} l = H_C$$

$$H_C = -\frac{13\sqrt{3}}{6} ql$$

• CALCOLO REAZIONI INTERNE



$$\begin{cases} H_A' - H_A'' - \frac{13\sqrt{3}}{6} ql = 0 \\ -V_A' - V_A'' + 3ql = 0 \end{cases}$$

$$V_A' = 3ql$$

$$V_A'' = 0$$

$$+\uparrow M(C)^{ABCD} = +H_A' \cdot \frac{\sqrt{3}}{2} l + V_A' \cdot 2l - 3ql \cdot \frac{5}{4} l = 0$$

$$H_A' = \left(6ql^2 - \frac{15}{4} ql^2\right) \frac{2}{\sqrt{3}l} = \left(\frac{9}{4}\right) \cdot \frac{2}{\sqrt{3}} ql = \frac{9}{2\sqrt{3}} ql$$

$$H_A' = \frac{3\sqrt{3}}{2} ql$$

$$H_A'' = -\frac{2\sqrt{3}}{3} ql$$

$$V_D - H_C - H_A' = 0 \quad V_D = -\frac{13\sqrt{3}}{6} ql + \frac{3\sqrt{3}}{2} ql$$

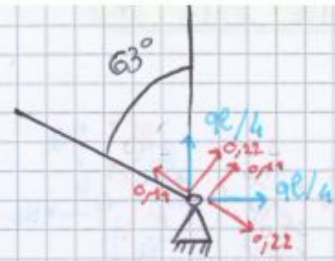
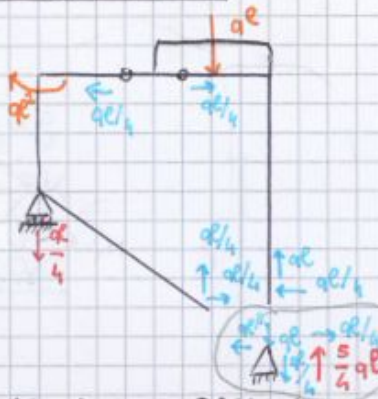
$$V_D = -\frac{2\sqrt{3}}{3} ql$$

• PRINCIPIO DEI LAVORI VIRTUALI

2008

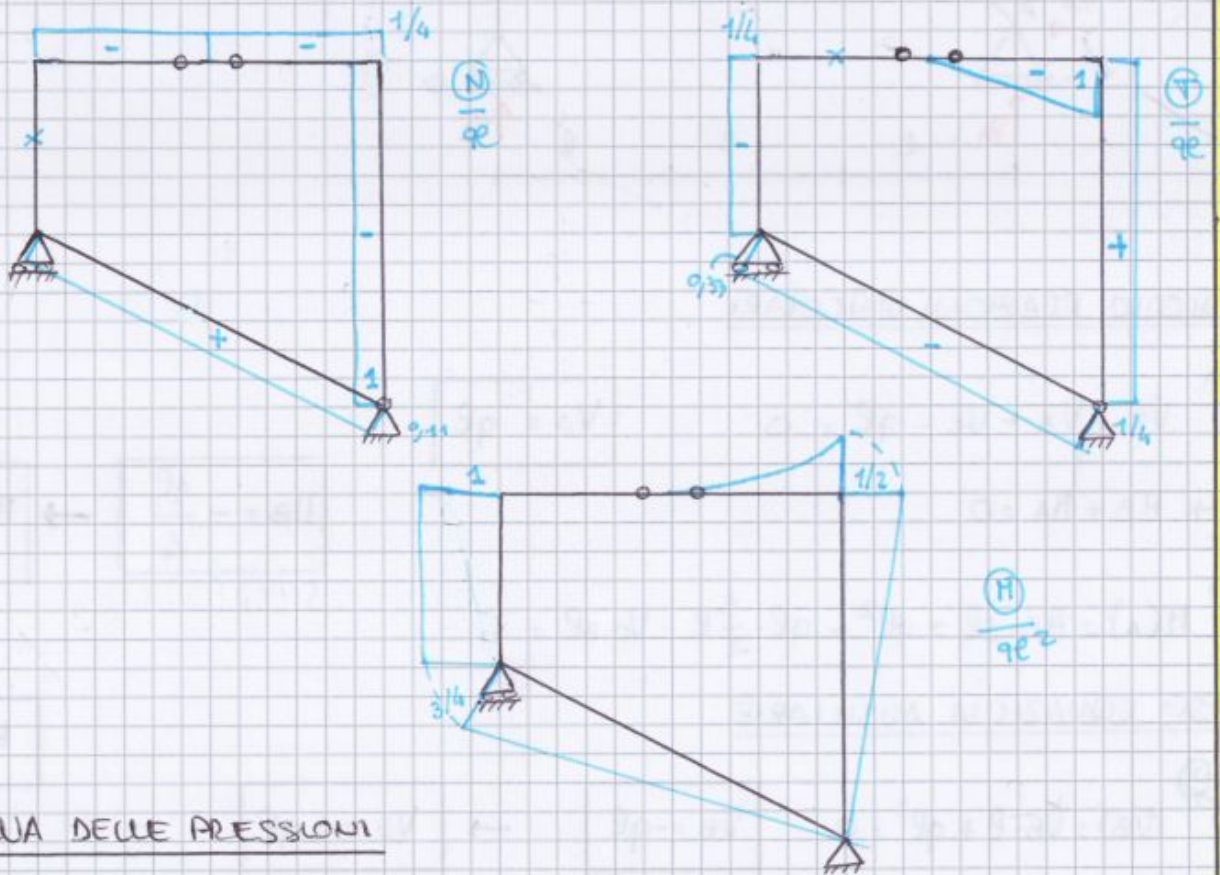
4

• RIDISEGNO LA TRAVE

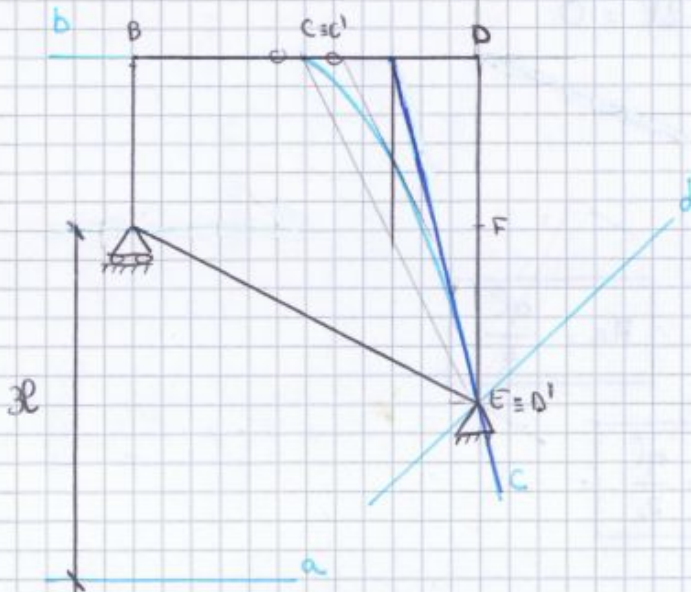


NEQUILIBRIO

• DISEGNO I DIAGRAMMI

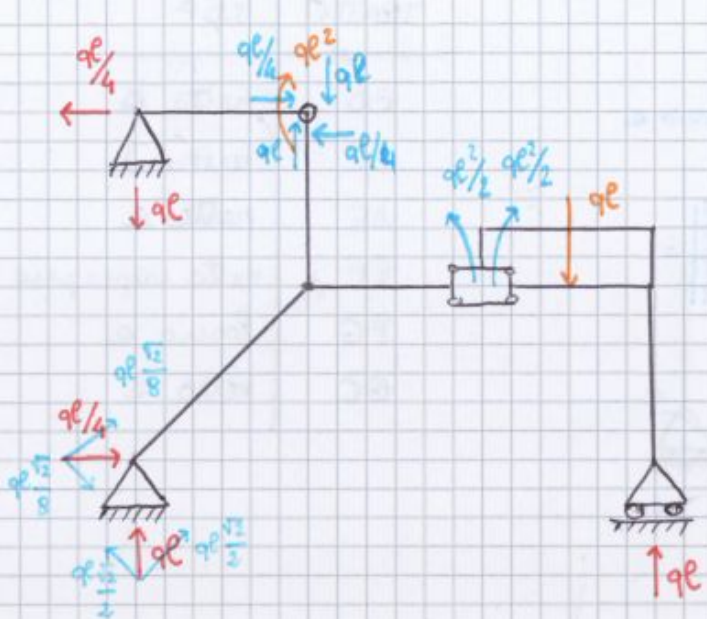


• CURVA DELLE PRESSIONI



TRATTO	CDP
AB	retta a
BC	retta b
CD	parabola
DF	retta c
FE	retta c
AE	retta d

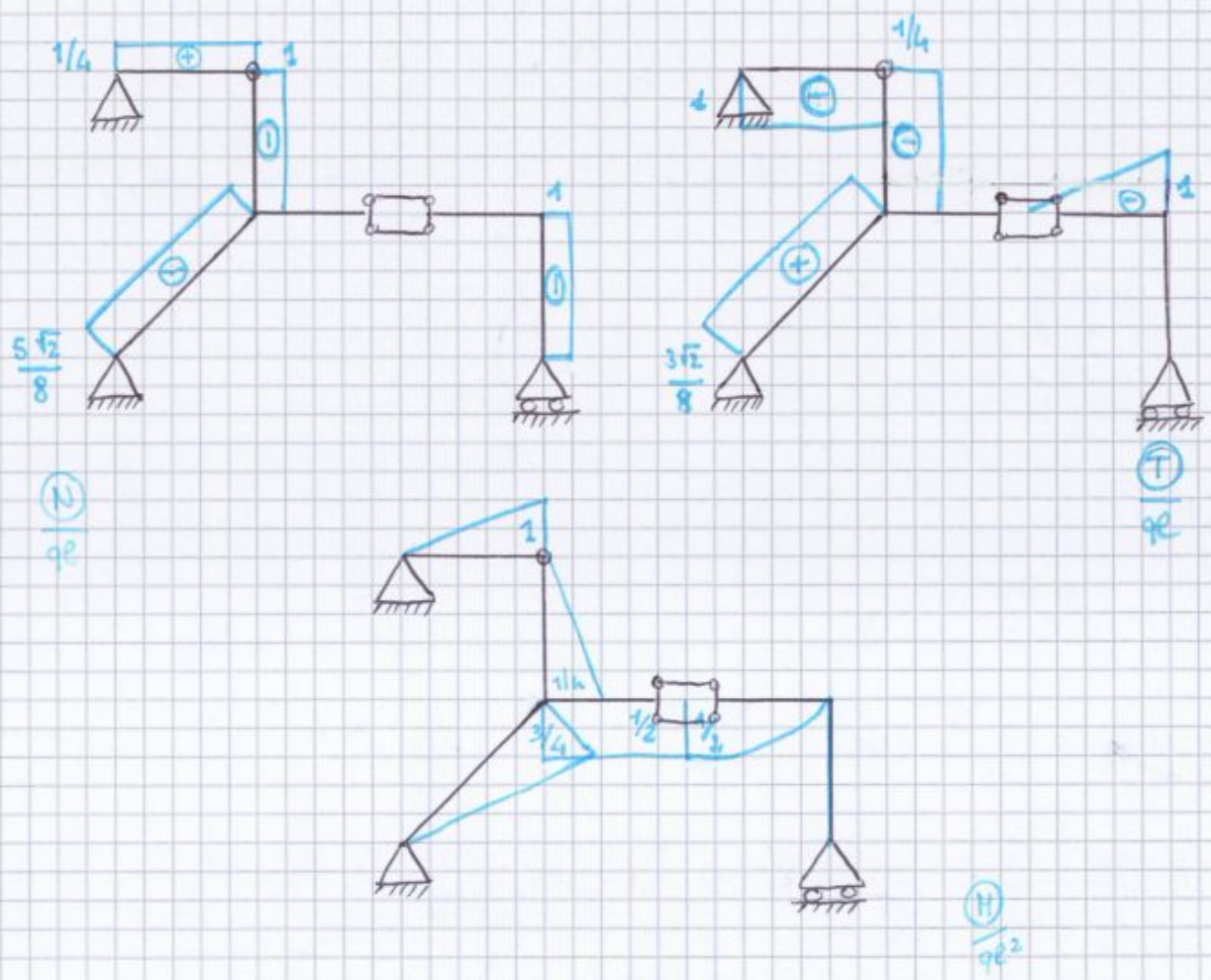
• RIDISEGNO LA TRAVE



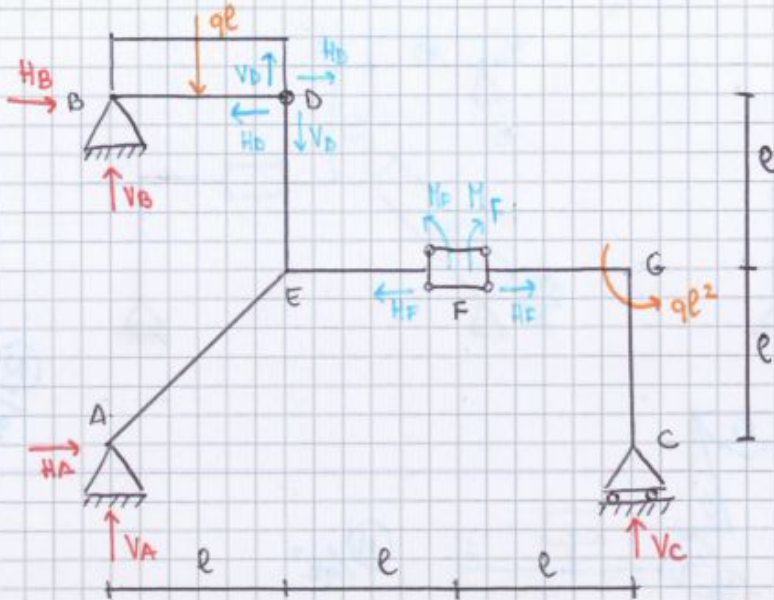
$$+ \nearrow q a \frac{\sqrt{2}}{2} + q a \frac{\sqrt{2}}{8} = \frac{5\sqrt{2}}{8} q a$$

$$+ \searrow q a \frac{\sqrt{2}}{2} - q a \frac{\sqrt{2}}{8} = \frac{3\sqrt{2}}{8} q a$$

• DISEGNO I DIAGRAMMI



16 SETTEMBRE 2013 (1)



$$g = 3 \times 3$$

$$V = 9$$

ISOSTATICA

• CALCOLO REAZIONI ESTERNE

$$+\uparrow V_A + V_B + V_C - qe = 0$$

$$V_A = \frac{qe}{2}$$

$$\rightarrow H_B + H_A = 0$$

$$H_B = \frac{qe}{4}$$

$$+\curvearrowright M(A) = H_B \cdot 2e + qe \cdot \frac{e}{2} - qe^2 - V_C \cdot 3e = 0$$

$$H_D = \frac{qe}{4}$$

• USO EQUAZIONI AUSILIARIE

$$+\curvearrowright M(D)^{\text{I}} = V_B \cdot e - qe \cdot \frac{e}{2} = 0$$

$$V_B = \frac{qe}{2}$$

$$+\curvearrowright M(D)^{\text{II,III}} = -V_C \cdot 2e - qe^2 + V_A \cdot e - H_A \cdot 2e = 0$$

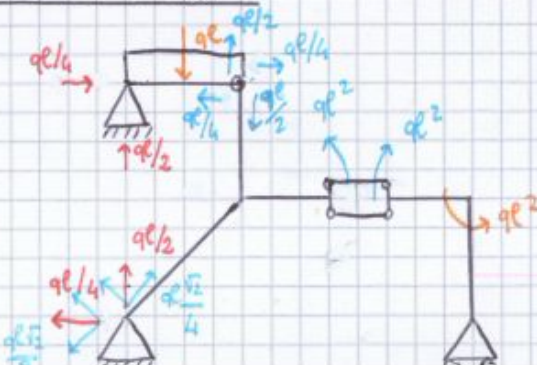
$$H_A = -\frac{qe}{4}$$

$$V_C = 0 \rightarrow V_D = \frac{qe}{2}$$

$$+\curvearrowright M(F)^{\text{III}} = -V_C \cdot e - qe^2 + H_F = 0$$

$$H_F = qe^2$$

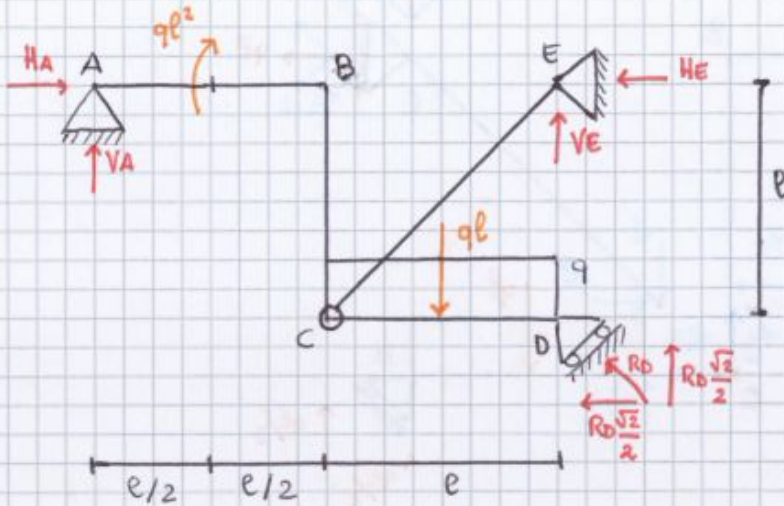
• RIDISEGNO LA TRAVE



$$+\nearrow \frac{qe\sqrt{2}}{4} - \frac{qe\sqrt{2}}{8} = \frac{qe\sqrt{2}}{8}$$

$$+\searrow \frac{qe\sqrt{2}}{4} + \frac{qe\sqrt{2}}{8} = \frac{3\sqrt{2}qe}{8}$$

30 GENNAIO 2013 (1)



$$g = 3 \times 3 = 9$$

$$U = 9$$

ISOSTATICA

• CALCOLO REAZIONI VINCOLARI

$$\uparrow + V_A + R_D \frac{\sqrt{2}}{2} + V_E - ql = 0$$

$$V_A + V_E = ql - \frac{1}{2} ql$$

$$V_A = -\frac{ql}{2}$$

$$\rightarrow + H_A - H_E - R_D \frac{\sqrt{2}}{2} = 0$$

$$H_A - H_E = \frac{1}{2} ql$$

$$H_E = -ql$$

$$\uparrow + H(D) = V_A \cdot 2l + H_A \cdot l + ql^2 - H_E \cdot l - ql \cdot \frac{ql}{2} = 0$$

NON SERVE

• USO EQUAZIONI AUSILIARIE

$$\uparrow + H(C)^I = V_A \cdot l + H_A \cdot l + ql^2 = 0$$

$$H_A = -\frac{ql}{2}$$

$$\uparrow + H(C)^III = -R_D \frac{\sqrt{2}}{2} l + ql \cdot \frac{l}{2} = 0$$

$$R_D \frac{\sqrt{2}}{2} = \frac{ql}{2}$$

$$R_D = \frac{ql}{\sqrt{2}}$$

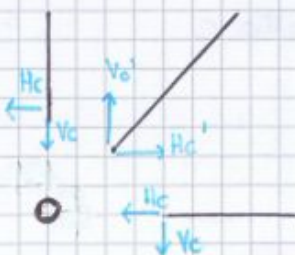
$$R_D = \frac{\sqrt{2}}{2} ql$$

$$\uparrow + H(A) = ql^2 - V_E \cdot 2l + ql \cdot \frac{3}{2} l - R_D \frac{\sqrt{2}}{2} \cdot 2l + R_D \frac{\sqrt{2}}{2} \cdot l = 0$$

$$ql^2 - V_E \cdot 2l + \frac{3}{2} ql^2 - ql^2 + \frac{1}{2} ql^2 = 0$$

$$V_E = +ql$$

• CALCOLO REAZIONI INTERNE



$$\uparrow +^{ABC} -V_C + V_A = 0$$

$$V_C = -\frac{ql}{2}$$

$$\rightarrow +^{ABC} -H_C + H_A = 0$$

$$H_C = -\frac{ql}{2}$$

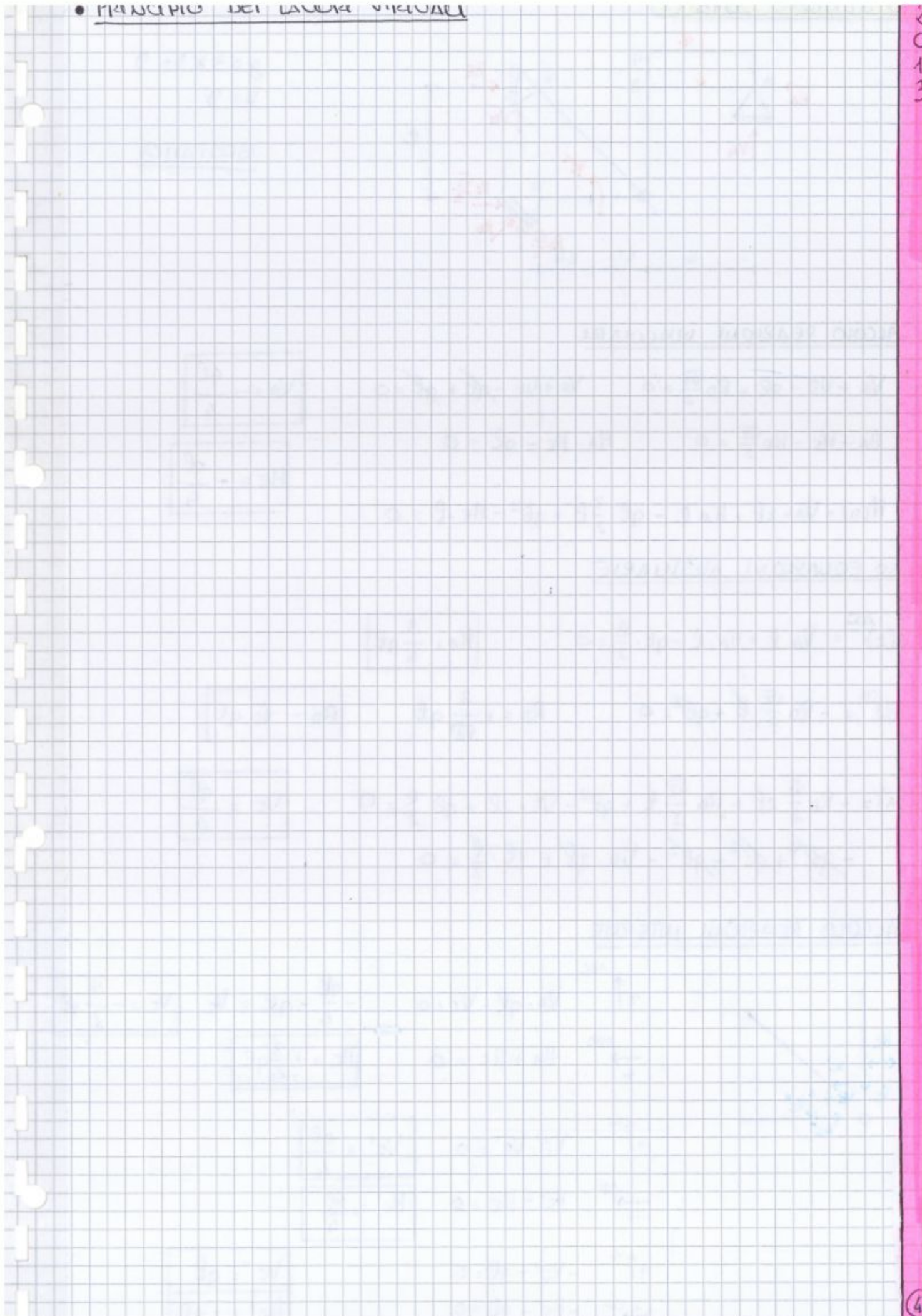
$$\uparrow +^{CE} V_C' + V_E = 0$$

$$V_C' = -ql$$

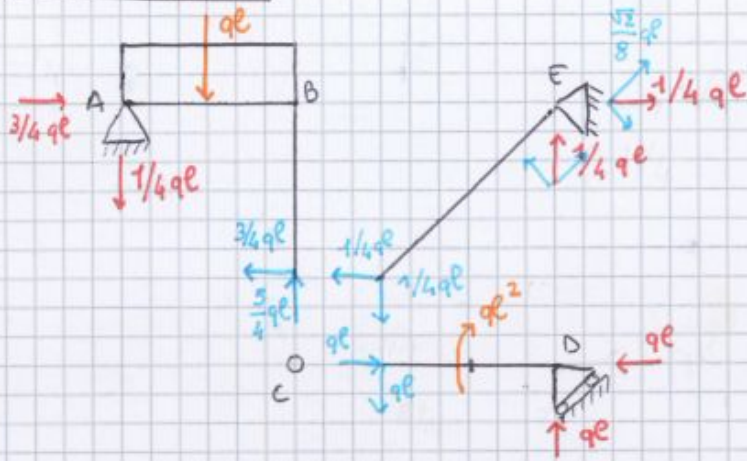
$$\rightarrow +^{CE} H_C' - H_E = 0$$

$$H_C' = -ql$$

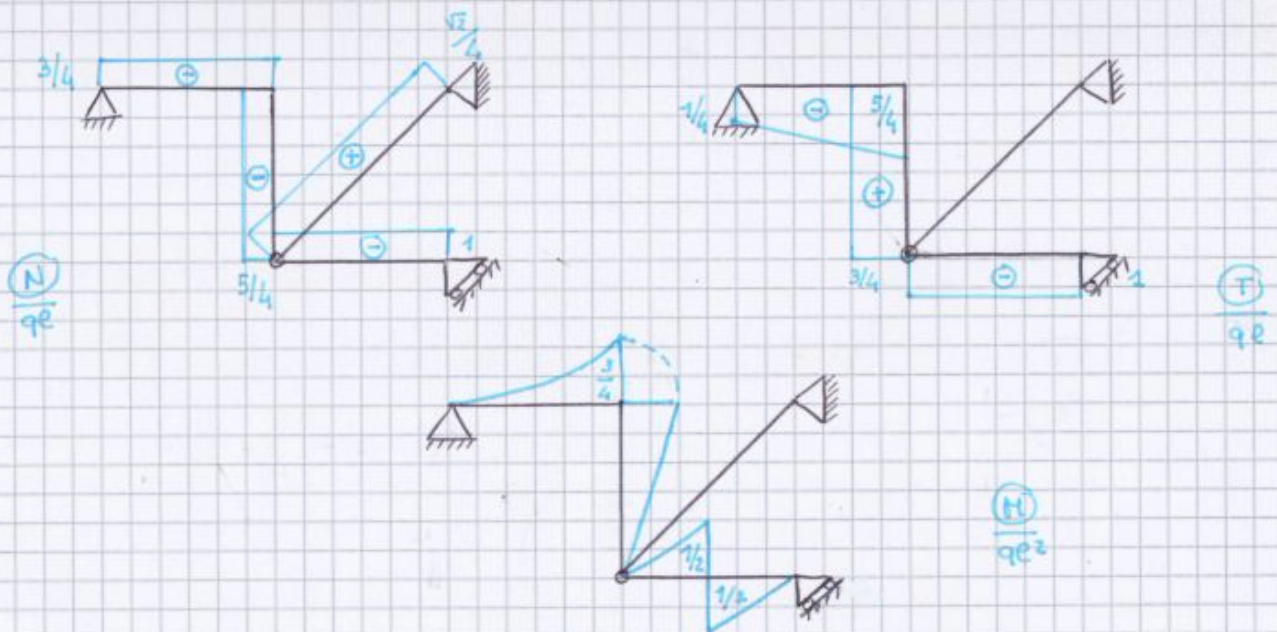
● PRINCIPIO DEL LAVORO VISUALE



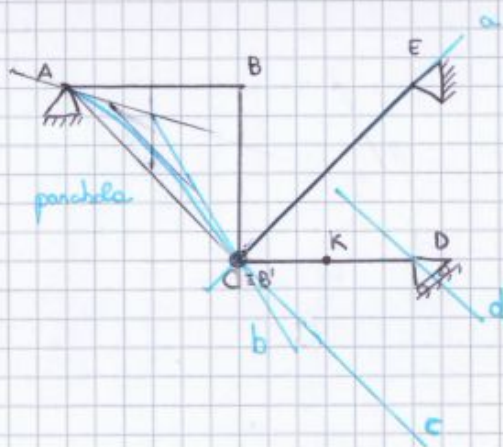
• RIDISEGNO LA TRAVE



• DISEGNO I DIAGRAMMI



• CURVA DELLE PRESSIONI



TRATTO	CDP
AB	parabola
BC	retta b
CE	retta a
KD	retta d
KC	retta c