NUMERO: 824 DATA: 13/02/2014

A P P U N T I

STUDENTE: Casalino

MATERIA: Thermal Machine + Eserc.

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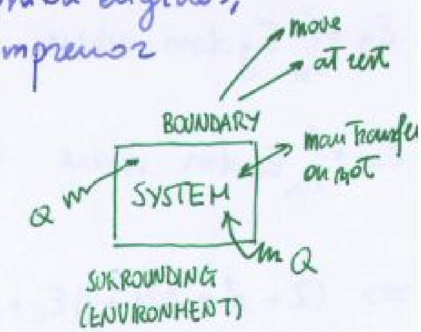
THERMAL MACHINE

Fluid machine → convert energy using a working fluid

MACHINES
- MOTOR MACHINES = power producing machine
- OPERATING MACHINES = power absorbing machine

FLUID MACHINES → HYDRAULIC MACHINES: incompressible fluid, thermal effect negligible
→ THERMAL MACHINES: fluid compressibility taken into account, heat exchange influences the performance

THERMAL MACHINES → TURBOMACHINES: gas and steam turbine (motor machine), turbocompressors, fans (operating machine)
→ VOLUMETRIC MACHINES: internal combustion engines, volumetric compressor



compressor increases the pressure of air
fan increases the kinetic energy of the air

SYSTEM → CLOSED: no mass transfer across boundary
→ OPEN: mass flows through the boundary

STATE OF A SYSTEM → STEADY STATE: none of properties changes through the time
→ CYCLE: series of processes which start and end at the same state

EXTENSIVE PROPERTIES: their value in a system is the sum of the values of subsystems that can be found

INTENSIVE PROPERTIES: we cannot add them in the way we do for extensive properties

ENTHALPY

$$h = U + pv \Rightarrow dh = dU + pdv + vdp \Rightarrow \boxed{dh - vdp = dU + pdv}$$

SIGN CONVENTION FOR OPERATING MACHINE

$dQ + dL = dU + dE_{c,g,w}$; Q, L positive when provided to the system

$dL = -pdv + dE_{c,g,w} + dL_w$

SIGN CONVENTION FOR MOTOR MACHINE

$dQ - dL = dU + dE_{c,g,w}$

$-dL = -pdv + dE_{c,g,w} + dL_w$

II LAW OF THERMODYNAMICS

$\oint \frac{dQ}{T} \le 0$ CLAUSIUS INEQUALITY (for thermodynamic cycle)

REVERSIBLE PROCESS: process that can be followed from state 1 to state 2 and then back to state 1 without seeing any effects on the system itself and on the surroundings

$\oint \frac{dQ + dQ_w}{T} = 0$ where dQ_w takes into account all the irreversibilities of the process due to the friction

Let's introduce the ENTHALPY

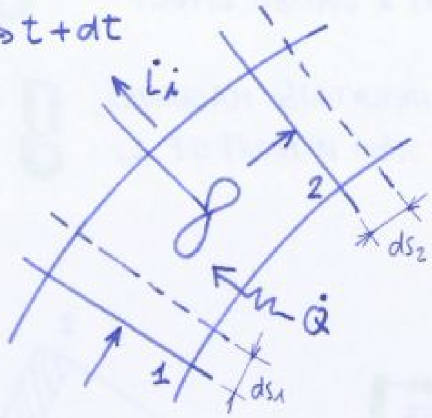
$$\oint \frac{dQ + dQ_w}{T} = \oint dS = 0 \Rightarrow \boxed{dS = \frac{dQ + dQ_w}{T}}$$

SECOND LAW OF THERMODYNAMICS
$Q_w = L_w$

$$\Rightarrow \boxed{TdS = dQ + dL_w = dU + pdV = dh - vdp}$$

CONSERVATION OF ENERGY - OPEN SYSTEMS

$t \rightarrow t+dt$



$$\dot{Q} - \dot{\mathcal{L}} = \frac{d\mathcal{E}_{cs}}{dt} \quad \text{for closed systems}$$

$$\Rightarrow \dot{Q} - \dot{\mathcal{L}} = \underbrace{\frac{d\mathcal{E}_{cv}}{dt}}_{\text{LOCAL TERM}} + \overbrace{\dot{\mathcal{E}}_{FLUX}}^{\text{FLUX TERM}}$$

$$\mathcal{E}_{cv} = \int_V E\rho dV \, ; \quad \dot{\mathcal{E}}_{cv} = \frac{\partial}{\partial t}\int_V E\rho dv$$

let's consider 1-D FLOW, 1-INLET & 1-OUTLET

$$\dot{Q} - \dot{\mathcal{L}} = \frac{\partial}{\partial t}\int_V E\rho dV + \dot{m}_2 E_2 - \dot{m}_1 E_1$$

using the sign convention for motor machine

$$d\mathcal{L} = \underbrace{\dot{\mathcal{L}}_i dt}_{\substack{\text{SYSTEM}\\ \text{ON SURR}}} \underbrace{- p_1 A_1 ds_1}_{\substack{\text{SURROUNDING}\\ \text{ON THE SYSTEM}}} + \underbrace{p_2 A_2 ds_2}_{\substack{\text{SYSTEM}\\ \text{ON SURR}}}$$

$-dL = -pdV + dE_{r,g,w} + dL_w$
$-dL = -[p(v_2 - v_1)] + \dots$
$-dL = +pv_1 - pv_2 + \dots$
$dL = pv_2 - v_1 p + \dots$

$$\Rightarrow d\mathcal{L} = \dot{\mathcal{L}}_i dt - p_1 dm_1 v_1 + p_2 dm_2 v_2$$

$$\Rightarrow \dot{\mathcal{L}} = \dot{\mathcal{L}}_i - \dot{m}_1 p_1 v_1 + \dot{m}_2 p_2 v_2$$

$$\Rightarrow \dot{Q} - \dot{\mathcal{L}}_i - \dot{m}_2 p_2 v_2 + \dot{m}_1 p_1 v_1 = \frac{\partial}{\partial t}\int \rho E dv + \dot{m}_2 (U_2 + E_{c,g,w_2}) - \dot{m}_1 (U_1 + E_{r,g,w_1})$$

$$\Rightarrow \boxed{\dot{Q} - \dot{\mathcal{L}}_i = \frac{\partial}{\partial t}\int_V E\rho dv + \dot{m}_2 (h + E_{r,g,w})_2 - \dot{m}_1 (h + E_{r,g,w})_1}$$ GENERAL FORM

hp: 1-D flow
1-inlet
1-outlet

THERMAL MACHINE - EXERCISES

IDEAL GAS: A GAS WITH NO VISCOSITY ; c_p & c_v ARE CONSTANT

ENTROPY & INTERNAL ENERGY ARE LINEAR FUNCTIONS OF TEMPERATURE

(BECAUSE c_p AND c_v DON'T DEPEND ON TEMPERATURE)

$$dq + dl_w = du + p\,dv \qquad \text{1}^{\text{ST}} \text{ PRINCIPLE + MECHANICAL ENERGY CONSERVATION}$$

$$c_v = \left.\frac{dq + dl_w}{dT}\right|_{v=\text{const}} \qquad c_v = \frac{dq + \overbrace{p\,dv}^{=0 \text{ def of } c_v:\ v=\text{const}}}{dT} \xrightarrow{\text{IDEAL GAS}} u = c_v\,dT$$

$$dq + dl_w = dh - v\,dp$$

$$c_p = \frac{dh - v\,dp}{dT} \xrightarrow{\text{IDEAL GAS}} h = c_p T$$

$$\boxed{pv = NRT}$$

$$pV = NRT\left(\frac{M}{M}\right) \Rightarrow (M \cdot N = m) \Rightarrow pV = m\frac{R}{M}T \Rightarrow pV = mRT \;;\; pv = RT$$

$$\boxed{R = 8.314 \text{ KJ/KmolK}}$$

R depends on type of gas

DRY AIR $R = 287.05$ J/Kg K

QUASI IDEAL GAS $\quad pV = mRT \quad$ ALWAYS HOLDS BUT c_p & c_v ARE NOT CONSTANT (THEY DEPEND ON TEMPERATURE)

$$du = c_v\,dT \;;\; dh = c_p\,dT \xrightarrow{\text{QUASI IDEAL GAS}} u = \int c_v\,dT \;;\; h = \int c_p\,dT$$

SECOND LAW OF THERMODYNAMICS

$$\boxed{T\,ds = dq + dl_w}$$

ISENTROPIC = ADIABATIC + NULL DISSIPATION

$\Rightarrow \quad T ds = dq + l_w \; ;$ BEING $ds = 0 \Rightarrow \begin{matrix} dq = 0 \\ dl_w = 0 \end{matrix}$

A GENERAL TRANSFORMATION IS CALLED **POLITROPIC** $\Rightarrow c = \text{const}$

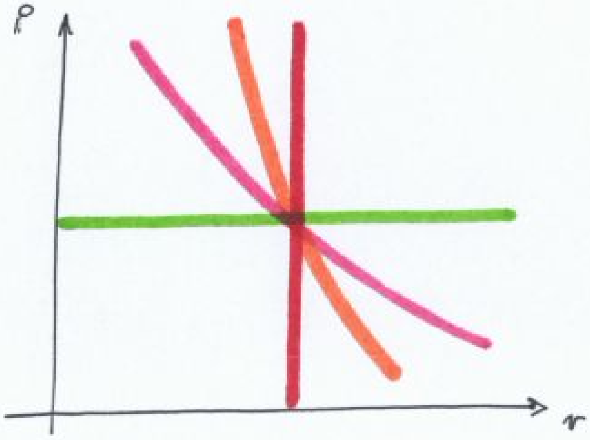
$$p v^m = \text{const} \; ; \; T v^{m-1} = \text{const} \; ; \; T p^{\frac{1-m}{m}} = \text{const} \quad \text{where} \quad m = \frac{c_p - c}{c_v - c}$$

ALL THE OTHER TRANSFORMATIONS ARE PARTICULAR CASES OF POLITROPIC TRANSFORMATION

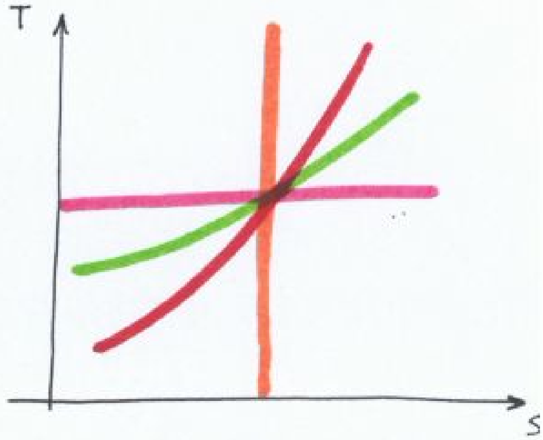
TRANSFORMATION	c	m	
ISOCHORIC	c_v	$+\infty$	$pv^m = \text{const}$; $v = \text{const}$
ISOBARIC	c_p	0	$pv^m = \text{const}$; $p = \text{const}$
ISOTHERMAL	$+\infty$	1	$pv^m = \text{const}$; $T = \text{const}$
ISENTROPIC	0	$c_p/c_v = K$	$pv^m = \text{const}$; $pv^K = \text{const}$

THERMODYNAMIC DIAGRAMS

MOST USED ARE p-v & T-s DIAGRAMS



USEFUL TO COMPUTE WORK



USEFUL TO COMPUTE HEAT TRANSFER AND WASTE WORK
$T ds = dq + dl_w$

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TAB. 5.16 Vapore acqueo saturo allo stato limite.

p bar	t °C	v' m³/kg	v'' m³/kg	ρ'' kg/m³	i' kJ/kg	i'' kJ/kg	r kJ/kg	s' kJ/kg·K	s'' kJ/kg·K
0,010	6,9828	0,0010001	129,20	0,007739	29,34	2514,4	2485,0	0,1060	8,9767
0,015	13,036	0,0010006	87,98	0,01137	54,71	2525,5	2470,7	0,1957	8,8288
0,020	17,513	0,0010012	67,01	0,01492	73,46	2533,6	2460,2	0,2607	8,7246
0,025	21,096	0,0010020	54,26	0,01843	88,45	2540,2	2451,7	0,3119	8,6440
0,030	24,100	0,0010027	45,67	0,02190	101,00	2545,6	2444,6	0,3544	8,5785
0,035	26,694	0,0010033	39,48	0,02533	111,85	2550,4	2438,5	0,3907	8,5232
0,040	28,983	0,0010040	34,80	0,02873	121,41	2554,5	2433,1	0,4225	8,4755
0,045	31,035	0,0010046	31,14	0,03211	129,99	2558,2	2428,2	0,4507	8,4335
0,050	32,898	0,0010052	28,19	0,03547	137,77	2561,6	2423,8	0,4763	8,3960
0,055	34,605	0,0010058	25,77	0,03880	144,91	2564,7	2419,8	0,4995	8,3621
0,060	36,183	0,0010064	23,74	0,04212	151,50	2567,5	2416,0	0,5209	8,3312
0,065	37,651	0,0010069	22,02	0,04542	157,64	2570,2	2412,5	0,5407	8,3029
0,070	39,025	0,0010074	20,53	0,04871	163,38	2572,6	2409,2	0,5591	8,2767
0,075	40,316	0,0010079	19,24	0,05198	168,77	2574,9	2406,2	0,5763	8,2523
0,080	41,534	0,0010084	18,10	0,05523	173,86	2577,1	2403,2	0,5925	8,2296
0,085	42,689	0,0010089	17,10	0,05848	178,69	2579,2	2400,5	0,6079	8,2082
0,090	43,787	0,0010094	16,20	0,06171	183,28	2581,1	2397,9	0,6224	8,1881
0,095	44,833	0,0010098	15,40	0,06493	187,65	2583,0	2395,3	0,6361	8,1691
0,10	45,833	0,0010102	14,67	0,06814	191,83	2584,8	2392,9	0,6493	8,1511
0,11	47,710	0,0010111	13,42	0,07454	199,68	2588,1	2388,4	0,6738	8,1177
0,12	49,446	0,0010119	12,36	0,08089	206,94	2591,2	2384,3	0,6963	8,0872
0,13	51,062	0,0010126	11,47	0,08722	213,70	2594,0	2380,3	0,7172	8,0592
0,14	52,574	0,0010133	10,69	0,09351	220,02	2596,7	2376,7	0,7367	8,0334
0,15	53,997	0,0010140	10,02	0,09977	225,97	2599,2	2373,2	0,7549	8,0093
0,16	55,341	0,0010147	9,433	0,1060	231,59	2601,6	2370,0	0,7721	7,9869
0,17	56,615	0,0010154	8,911	0,1122	236,93	2603,8	2366,9	0,7883	7,9658
0,18	57,826	0,0010160	8,445	0,1184	241,99	2605,9	2363,9	0,8036	7,9460
0,19	58,982	0,0010166	8,027	0,1246	246,83	2607,9	2361,1	0,8182	7,9272
0,20	60,086	0,0010172	7,650	0,1307	251,45	2609,9	2358,4	0,8321	7,9094
0,21	61,145	0,0010178	7,307	0,1368	255,88	2611,7	2355,8	0,8453	7,8925
0,22	62,162	0,0010183	6,995	0,1430	260,14	2613,5	2353,3	0,8581	7,8764
0,23	63,139	0,0010189	6,709	0,1490	264,23	2615,2	2350,9	0,8702	7,8611
0,24	64,082	0,0010194	6,447	0,1551	268,18	2616,8	2348,6	0,8820	7,8464
0,25	64,992	0,0010199	6,204	0,1612	271,99	2618,3	2346,4	0,8933	7,8323
0,26	65,871	0,0010204	5,980	0,1672	275,67	2619,9	2344,2	0,9041	7,8188
0,27	66,722	0,0010209	5,772	0,1732	279,24	2621,3	2342,1	0,9146	7,8058
0,28	67,547	0,0010214	5,579	0,1793	282,69	2622,7	2340,0	0,9248	7,7933
0,29	68,347	0,0010219	5,398	0,1852	286,05	2624,1	2338,1	0,9346	7,7812
0,30	69,124	0,0010223	5,229	0,1912	289,30	2625,4	2336,1	0,9441	7,7695
0,32	70,615	0,0010232	4,922	0,2032	295,55	2628,0	2332,4	0,9623	7,7474
0,34	72,029	0,0010241	4,650	0,2150	301,48	2630,4	2328,9	0,9795	7,7266
0,36	73,374	0,0010249	4,408	0,2269	307,12	2632,6	2325,5	0,9958	7,7070
0,38	74,658	0,0010257	4,190	0,2387	312,50	2634,8	2322,3	1,0113	7,6884
0,40	75,886	0,0010265	3,993	0,2504	317,65	2636,9	2319,2	1,0261	7,6709
0,45	78,743	0,0010284	3,576	0,2796	329,64	2641,7	2312,0	1,0603	7,6307
0,50	81,345	0,0010301	3,240	0,3086	340,56	2646,0	2305,4	1,0912	7,5947
0,55	83,737	0,0010317	2,964	0,3374	350,61	2649,9	2299,3	1,1194	7,5623
0,60	85,954	0,0010333	2,732	0,3661	359,93	2653,6	2293,6	1,1454	7,5327
0,65	88,021	0,0010347	2,535	0,3945	368,62	2656,9	2288,3	1,1696	7,5055
0,70	89,959	0,0010361	2,365	0,4229	376,77	2660,1	2283,3	1,1921	7,4804
0,75	91,785	0,0010375	2,217	0,4511	384,45	2663,0	2278,6	1,2131	7,4570
0,80	93,512	0,0010387	2,087	0,4792	391,72	2665,8	2274,0	1,2330	7,4352
0,85	95,152	0,0010400	1,972	0,5071	398,63	2668,4	2269,8	1,2518	7,4117
0,90	96,713	0,0010412	1,869	0,5350	405,21	2670,9	2265,6	1,2696	7,3954
0,95	98,204	0,0010423	1,777	0,5627	411,49	2673,0	2261,7	1,2865	7,3771
1,0	99,632	0,0010434	1,694	0,5904	417,51	2675,4	2257,9	1,3027	7,3598
1,1	102,32	0,0010455	1,549	0,6455	428,84	2679,6	2250,8	1,3330	7,3277
1,2	104,81	0,0010476	1,428	0,7002	439,36	2683,4	2244,1	1,3609	7,2984
1,3	107,13	0,0010495	1,325	0,7547	449,19	2687,0	2237,8	1,3868	7,2715
1,4	109,32	0,0010513	1,236	0,8088	458,42	2690,3	2231,9	1,4109	7,2465
1,5	111,37	0,0010530	1,159	0,8628	467,13	2693,4	2226,2	1,4336	7,2234
1,6	113,32	0,0010547	1,091	0,9165	475,38	2696,2	2220,9	1,4550	7,2017
1,7	115,17	0,0010563	1,031	0,9700	483,22	2699,0	2215,7	1,4752	7,1813
1,8	116,93	0,0010579	0,9772	1,023	490,70	2701,5	2210,8	1,4944	7,1622
1,9	118,62	0,0010594	0,9290	1,076	497,85	2704,0	2206,1	1,5127	7,1440
2,0	120,23	0,0010608	0,8854	1,129	504,70	2706,3	2201,6	1,5301	7,1268
2,1	121,78	0,0010623	0,8459	1,182	511,29	2708,5	2197,2	1,5468	7,1105
2,2	123,27	0,0010636	0,8098	1,235	517,62	2710,6	2193,0	1,5627	7,0949
2,3	124,71	0,0010650	0,7768	1,287	523,73	2712,6	2188,9	1,5781	7,0800
2,4	126,09	0,0010663	0,7465	1,340	529,64	2714,5	2184,9	1,5929	7,0657
2,5	127,43	0,0010675	0,7184	1,392	535,34	2716,4	2181,0	1,6071	7,0520
2,6	128,73	0,0010688	0,6925	1,444	540,87	2718,2	2177,3	1,6209	7,0389
2,7	129,98	0,0010700	0,6684	1,496	546,24	2719,9	2173,6	1,6342	7,0262
2,8	131,20	0,0010712	0,6460	1,548	551,44	2721,5	2170,1	1,6471	7,0140
2,9	132,39	0,0010724	0,6251	1,600	556,51	2723,1	2166,6	1,6595	7,0023

Segue TAB. 5.16 **Vapore acqueo saturo allo stato limite.**

p bar	t °C	v' m³/kg	v'' m³/kg	ρ'' kg/m³	i' kJ/kg	i'' kJ/kg	r kJ/kg	s' kJ/kg · K	s'' kJ/kg · K
25,0	223,94	0,0011972	0,07991	12,51	961,96	2800,9	1839,0	2,5543	6,2536
25,5	225,00	0,0011991	0,07835	12,76	966,87	2801,2	1834,3	2,5640	6,2461
26,0	226,04	0,0012011	0,07686	13,01	971,72	2801,4	1829,6	2,5736	6,2387
26,5	227,06	0,0012031	0,07541	13,26	976,50	2801,6	1825,0	2,5831	6,2315
27,0	228,07	0,0012050	0,07402	13,51	981,22	2801,7	1820,5	2,5924	6,2244
27,5	229,07	0,0012069	0,07268	13,76	985,88	2801,9	1816,0	2,6016	6,2173
28,0	230,05	0,0012088	0,07139	14,01	990,48	2802,0	1811,5	2,6106	6,2104
28,5	231,01	0,0012107	0,07014	14,26	995,03	2802,1	1807,1	2,6195	6,2036
29,0	231,97	0,0012126	0,06893	14,51	999,53	2802,2	1802,6	2,6283	6,1969
29,5	232,91	0,0012145	0,06776	14,76	1004,0	2802,2	1798,3	2,6370	6,1903
30	233,84	0,0012163	0,06663	15,01	1008,4	2802,3	1793,9	2,6455	6,1837
31	235,67	0,0012200	0,06447	15,51	1017,0	2802,3	1785,4	2,6623	6,1709
32	237,45	0,0012237	0,06244	16,02	1025,4	2802,3	1776,9	2,6786	6,1585
33	239,18	0,0012274	0,06053	16,52	1033,7	2802,3	1768,6	2,6945	6,1463
34	240,88	0,0012310	0,05873	17,03	1041,3	2802,1	1760,3	2,7101	6,1344
35	242,54	0,0012345	0,05703	17,54	1049,3	2802,0	1752,2	2,7253	6,1228
36	244,16	0,0012381	0,05541	18,05	1057,5	2801,7	1744,2	2,7401	6,1115
37	245,75	0,0012416	0,05389	18,56	1065,2	2801,4	1736,2	2,7547	6,1004
38	247,31	0,0012451	0,05244	19,07	1072,7	2801,1	1728,4	2,7689	6,0896
39	248,84	0,0012486	0,05106	19,58	1080,1	2800,8	1720,6	2,7829	6,0789
40	250,33	0,0012521	0,04975	20,10	1087,4	2800,3	1712,9	2,7965	6,0685
41	251,80	0,0012555	0,04850	20,62	1094,5	2799,9	1705,3	2,8099	6,0583
42	253,24	0,0012589	0,04731	21,14	1101,5	2799,4	1697,8	2,8231	6,0482
43	254,66	0,0012623	0,04617	21,66	1108,5	2798,9	1690,3	2,8360	6,0383
44	256,05	0,0012657	0,04508	22,18	1115,4	2798,3	1682,9	2,8487	6,0286
45	257,41	0,0012691	0,04404	22,71	1122,1	2797,7	1675,6	2,8612	6,0191
46	258,75	0,0012725	0,04304	23,24	1128,8	2797,0	1668,3	2,8735	6,0097
47	260,07	0,0012758	0,04208	23,76	1135,3	2796,4	1661,1	2,8855	6,0004
48	261,37	0,0012792	0,04116	24,29	1141,8	2795,7	1653,9	2,8974	5,9913
49	262,65	0,0012825	0,04028	24,83	1148,2	2794,9	1646,8	2,9091	5,9824
50	263,91	0,0012858	0,03943	25,36	1154,5	2794,2	1639,7	2,9206	5,9735
51	265,15	0,0012891	0,03861	25,90	1160,7	2793,4	1632,7	2,9320	5,9648
52	266,37	0,0012924	0,03782	26,44	1166,8	2792,6	1625,7	2,9431	5,9561
53	267,58	0,0012957	0,03707	26,98	1172,9	2791,7	1618,8	2,9541	5,9476
54	268,76	0,0012990	0,03633	27,52	1178,9	2790,8	1611,9	2,9650	5,9392
55	269,93	0,0013023	0,03563	28,07	1184,9	2789,9	1605,0	2,9757	5,9309
56	271,09	0,0013056	0,03495	28,62	1190,8	2789,0	1598,2	2,9863	5,9227
57	272,22	0,0013089	0,03429	29,16	1196,6	2788,0	1591,4	2,9967	5,9146
58	273,35	0,0013121	0,03365	29,72	1202,3	2787,0	1584,7	3,0071	5,9066
59	274,46	0,0013154	0,03303	30,27	1208,0	2786,0	1578,0	3,0172	5,8986
60	275,55	0,0013187	0,03244	30,83	1213,7	2785,0	1571,3	3,0273	5,8908
61	276,63	0,0013219	0,03186	31,39	1219,3	2784,0	1564,7	3,0372	5,8830
62	277,70	0,0013252	0,03130	31,95	1224,8	2782,9	1558,0	3,0471	5,8753
63	278,75	0,0013285	0,03076	32,51	1230,3	2781,8	1551,5	3,0568	5,8677
64	279,79	0,0013317	0,03023	33,08	1235,7	2780,6	1544,9	3,0664	5,8601
65	280,82	0,0013350	0,02972	33,65	1241,1	2779,5	1538,4	3,0759	5,8527
66	281,84	0,0013383	0,02922	34,22	1246,5	2778,3	1531,9	3,0853	5,8452
67	282,84	0,0013415	0,02874	34,79	1251,8	2777,1	1525,4	3,0946	5,8379
68	283,84	0,0013448	0,02827	35,37	1257,0	2775,9	1518,9	3,1038	5,8306
69	284,82	0,0013481	0,02782	35,95	1262,2	2774,7	1512,5	3,1129	5,8233
70	285,79	0,0013515	0,02737	36,53	1267,4	2773,5	1506,0	3,1219	5,8162
71	286,75	0,0013548	0,02694	37,12	1272,5	2772,2	1499,8	3,1308	5,8090
72	287,70	0,0013579	0,02652	37,70	1277,6	2770,9	1493,3	3,1397	5,8020
73	288,64	0,0013611	0,02611	38,29	1282,7	2769,6	1486,9	3,1484	5,7949
74	289,57	0,0013644	0,02572	38,89	1287,7	2768,3	1480,5	3,1571	5,7880
75	290,50	0,0013677	0,02533	39,48	1292,7	2766,9	1474,2	3,1657	5,7811
76	291,41	0,0013710	0,02495	40,08	1297,6	2765,5	1467,9	3,1742	5,7742
77	292,31	0,0013743	0,02458	40,68	1302,6	2764,2	1461,6	3,1827	5,7673
78	293,21	0,0013776	0,02422	41,29	1307,4	2762,8	1455,3	3,1911	5,7606
79	294,09	0,0013809	0,02387	41,90	1312,3	2761,3	1449,1	3,1994	5,7538
80	294,97	0,0013842	0,02353	42,51	1317,1	2759,9	1442,8	3,2076	5,7471
81	295,84	0,0013876	0,02319	43,12	1321,9	2758,4	1436,6	3,2158	5,7404
82	296,70	0,0013909	0,02286	43,74	1326,6	2757,0	1430,3	3,2239	5,7338
83	297,55	0,0013942	0,02254	44,36	1331,4	2755,5	1424,1	3,2320	5,7272
84	298,39	0,0013976	0,02223	44,98	1336,1	2754,0	1417,9	3,2399	5,7207
85	299,23	0,0014009	0,02193	45,61	1340,7	2752,5	1411,7	3,2479	5,7141
86	300,06	0,0014043	0,02163	46,24	1345,4	2750,9	1405,5	3,2558	5,7076
87	300,88	0,0014077	0,02133	46,87	1350,0	2749,4	1399,3	3,2636	5,7012
88	301,70	0,0014111	0,02105	47,51	1354,6	2747,8	1393,2	3,2713	5,6948
89	302,51	0,0014145	0,02077	48,15	1359,2	2746,2	1387,0	3,2790	5,6884
90	303,31	0,0014179	0,02050	48,79	1363,7	2744,6	1380,9	3,2867	5,6820
91	304,10	0,0014213	0,02023	49,44	1368,3	2743,0	1374,7	3,2943	5,6757
92	304,89	0,0014247	0,01996	50,09	1372,8	2741,4	1368,6	3,3018	5,6694
93	305,67	0,0014281	0,01971	50,74	1377,2	2739,7	1362,5	3,3093	5,6631
94	306,44	0,0014316	0,01945	51,40	1381,7	2738,0	1356,3	3,3168	5,6568

POLITECNICO DI TORINO

Dipartimento Energia

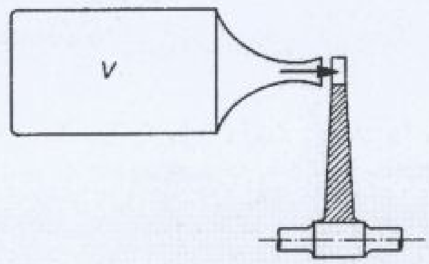
BSc in Automotive Engineering

APPLIED LECTURE 1 - Thermal machines

1) A reciprocating compressor displays a clearance volume of 150 cm^3 and a displacement of 1500 cm^3. The compressor sucks air (R = 287 J/kgK, k = 1.4) from the environment (p_e = 1 bar, T_e = 20 °C) and delivers it to a tank at a pressure of 2.3 bar. When the intake valve opens the chamber displays a volume V_1 = 293.5 cm^3 and the air has reached a pressure p_1 = 95 kPa and a temperature T_1 = 295.4 K. The overall heat to the wall from the opening to the closure of the inlet valve amounts to 19.55 J and the cylinder internal pressure can be assumed as constant and equal to p_1. Determine the amount of air mass sucked by the compressor and its in-cylinder temperature at the end of the intake stroke.

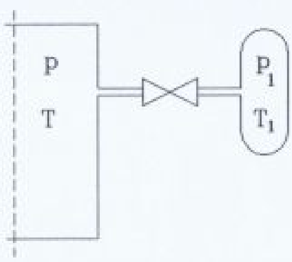
<u>Proposed exercises:</u>

2) A canister displaying a capacity of 0.1 m^3 contains air under a pressure p_1 = 30 bar and at a temperature T_1 = 800 K. The canister is connected to the environment (p_e = 1 bar) by means of a nozzle. The outgoing air is hence admitted to an action turbine. Determine the maximum turbine work admitted that all the kinetic energy of the fluid is converted to work (no kinetic energy losses at the turbine outlet) and that the air expands isentropically from pressure p_1 to pressure p_2 (reversible adiabatic expansion).



Solution: 363 kJ

3) A gas cylinder displaying a capacity of 5 liters contains air under a pressure p_1 = 1 bar and at a temperature T_1 = 300 K. The cylinder communicates with a great tank hosting air under a pressure p = 15 MPa and at a temperature T = 290 K by means of a valve. When the valve opens, the cylinder is filled up with air as long as its pressure reaches the value of 15 Mpa. Neglecting the heat exchanges to the surroundings during the filling process, determine the air mass admitted in the cylinder and the in-cylinder mean temperature at the end of the process.



Solution: M = 0.639 kg; T_{mean} = 405.1 K

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EXERCISE 1.3

$V_{CYL} = 5\,dm^3 = 0.005\ m^3$

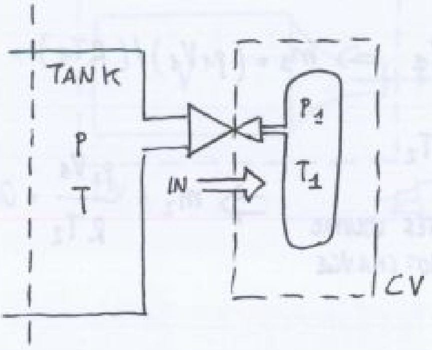
$p_1 = 10^5\ Pa$

$T_1 = 300\ K$

$p_{TANK} = 15 \cdot 10^6\ Pa$

$T_{TANK} = 290\ K$

$q = 0$



AIR FLOWS FROM TANK TO CYLINDER UP TO $p_2 = 15 \cdot 10^6\ Pa \Rightarrow p_2 = p_{TANK}$

CONTINUITY EQUATION $\left(\frac{dm}{dt}\right)_{CV} = \dot m_{IN} - \underbrace{\dot m_{OUT}}_{=0\ \text{NO OUTLET}} \Longrightarrow \int_1^2 \dot m_{IN}\, dt = m_2 - m_1$

1^{ST} LAW OPEN SYSTEMS $\underbrace{\dot Q}_{=0\ \text{adiabatic}} - \underbrace{\dot L_i}_{=0} = \frac{d}{dt}(U + \cancel{E_c} + \cancel{E_g} + \cancel{E_w})_{CV} + \underbrace{\dot m_{OUT}}_{=0} (h + e_c + e_g + e_w)_{OUT} - \dot m_{IN} (h + \cancel{e_c} + \cancel{e_g} + \cancel{e_w})_{IN}$

$\Rightarrow \Delta U - m_{IN} h_{IN} = 0 \Rightarrow m_2 c_v T_2 - m_1 c_v T_1 - (m_2 - m_1) c_p T_{TANK} = 0$

$h_{IN} = c_p T_{TANK}$ BECAUSE THE AIR FLOW ENTERING HAS THE PROPERTIES OF THE TANK (CAREFUL TO IN & OUT PROPERTIES) !

RIGID CYLINDER $\Rightarrow V_1 = V_2 = V_{CYL} \Rightarrow m_2 c_v \left(\frac{p_2 V_{CYL}}{m_2 R}\right) - m_1 c_v \left(\frac{p_1 V_{CYL}}{m_1 R}\right) - (m_2 - m_1) c_p T_{TANK} = 0$

$m_1 = \frac{p_1 V_{CYL}}{R T_1} = \frac{10^5 \cdot 0.005}{287 \cdot 300} = 5.86 \cdot 10^{-3}\ Kg$

$\Rightarrow c_v \frac{V_{CYL}}{R} (p_2 - p_1) - (m_2 - m_1) c_p T_{TANK} = 0 \quad [\dots] \Rightarrow m_2 = 0.645\ Kg$

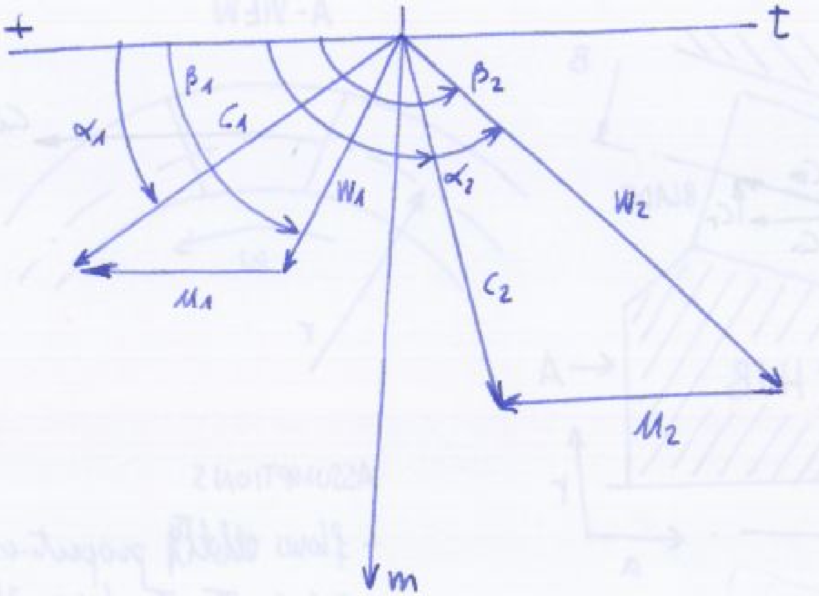
MASS ADMITTED INTO CYLINDER $\Delta m = m_2 - m_1 = 0.639\ Kg$

AT END OF PROCESS $T_2 = \frac{p_2 V_{CYL}}{R\, m_2} = \frac{15 \cdot 10^6 \cdot 0.005}{287 \cdot 0.645} = 405.1\ K$

THERMAL VELOCITIES

$\vec{c} = \vec{w} + \vec{u}$

velocity Triangle using 2 main directions — Tangential t; meridional m



1 = ROTOR INLET

2 = ROTOR OUTLET

Tangential plane of average flow stream, machine rotates toward left

u_1, u_2 have always same direction but we may have $u_1 \neq u_2$ depending on radius of 1 & 2

AXIAL MACHINE	RADIAL MACHINE	MIXED FLOW CENTRIFUGAL
$r_1 = r_2$	$u_1 > u_2$ (CENTRIPETAL FLOW)	$u_2 > u_1$
$u_1 = u_2$	$u_1 < u_2$ (CENTRIFUGAL FLOW)	$c_{m1} = c_{a1}$ $c_{r1} = 0$
$c_m = c_a$	$c_m = c_r$	$c_{m2} = c_{r2}$ $c_{a2} = 0$
$c_r = 0$	$c_a = 0$	

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$$\boxed{L_i = \frac{P_i}{\dot{m}} = u_1 c_{u1} - u_2 c_{u2}}$$ EULER TURBINE EQUATION

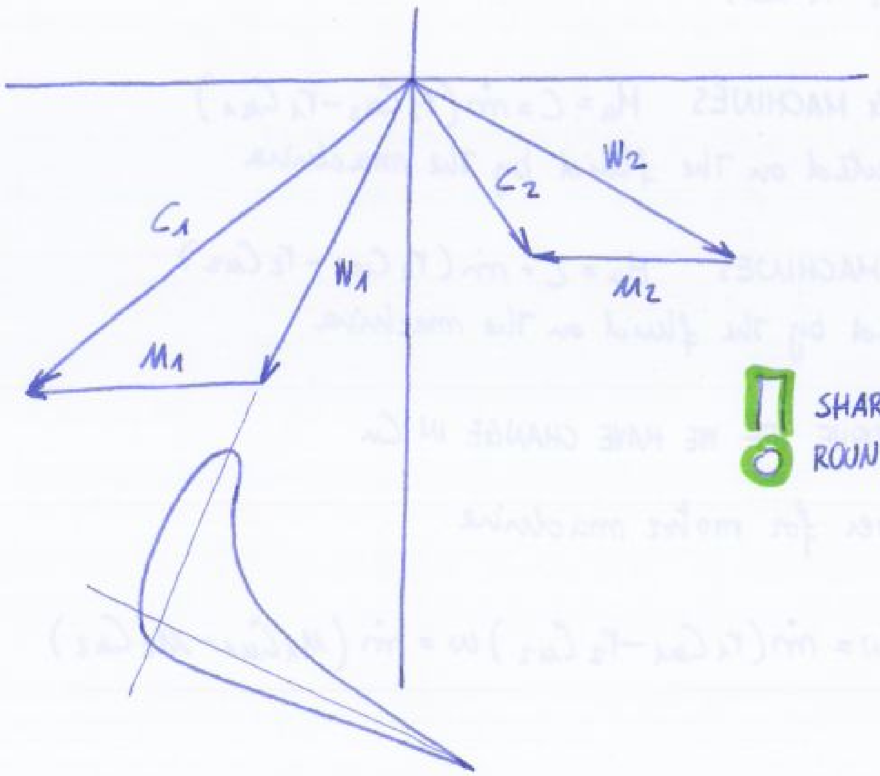
$$\boxed{L_i = \frac{P_i}{\dot{m}} = u_2 c_{u2} - u_1 c_{u1}}$$ EULER PUMP EQUATION

BLADE DESIGN

- velocity triangles ⇒ basic elements to define turbine performances
- starting from velocity triangle ⇒ 1st attempt to design blade profile
- velocity direction given by the blade direction ⇒ $\beta_2 = (\beta_2)_{blade}$

only assumption: 1-D flow $\beta_1 = (\beta_1)_{blade}$

- design stage ⇒ blade direction is the same as velocity direction



SHARP EDGE AT THE OUTLET
ROUNDED ONE AT INLET

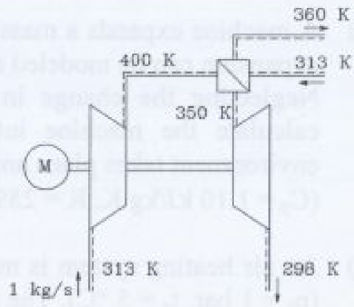
Proposed exercises:

4) A turbo-pump raises water from a well to a tank over a height of 20 m. The duct displays a constant diameter D = 10 cm. The overall viscous losses in the duct and in the pump amount to the 15% of the pump specific work. The water flows out from the duct with a velocity of 2 m/s.
   Determine the power supplied by the engine that operates the pump ($\eta_m = 0.97$).

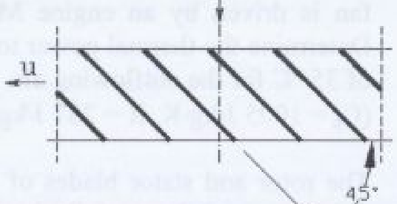
Solution: $P_{ass} = 3.78$ kW

5) Referring to the conditioning plant of the figure, determine the power supplied by the engine M ($\eta_m = 0.95$). The plant exploits air for conditioning purposes and for the intermediate refrigeration.

Solution: $P_{ass} = 37$ kW



6) The rotor blades of a hydraulic axial turbo-machine display a mean diameter d = 1 m and height l = 0.2d which both result to be constant along the axis. The tangential velocity is u = 30 m/s whereas the inlet velocity of the fluid is $c_1 = 60$ m/s along the axis.
   Plot the velocity triangles and determine its internal specific work. Work out whether it behaves as an operating machine or as a motor machine.



Solution: motor machine; $P_i = 33.25$ MW

LET'S GO BACK TO PREVIOUS FORMULA

$$l_i = -\frac{1.5}{0.5}\cdot 289\cdot 773\left(1-\frac{1}{10^{0.5/1.5}}\right) - 62\cdot 10^3 = 297116 \text{ J/Kg}$$

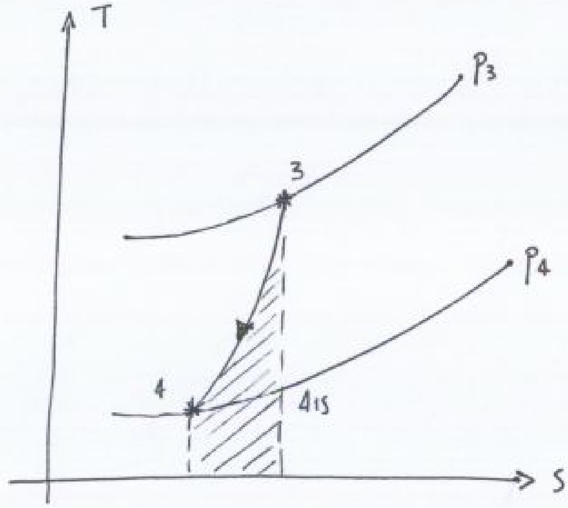
$$\dot{L}_i = \dot{m}\, l_i = 891348 \text{ W}$$ SHAFT POWER = INTERNAL POWER

$$T p^{\frac{1-n}{n}} = \text{const} \Rightarrow T_4 = T_3\left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} = 358.8 \text{ K}$$

LET'S COMPUTE Δh; HAVING l_i, ONE CAN FIND q

$$q - l_i = \Delta h \Rightarrow q - l_i = c_p(T_4 - T_3) \Rightarrow q = l_i + c_p(T_4 - T_3) = 297116 + 1100(358.8 - 773) = -158.5 \text{ KJ/Kg}$$

$\Rightarrow$ HEAT SUBTRACTED TO THE SYSTEM



$$\int T\,ds = q + l_w = -158 + 62 < 0$$

BEING THE INTEGRAL < 0
4 IS ON THE LEFT OF 3

ONE COULD NOTICE THAT 4 IS ON THE LEFT BY NOTING $n = 1.5$ AND $n < k$!!
(k CAN BE COMPUTED KNOWING R & c_p)

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$$\eta_M = \frac{\dot{m}\, l_i}{P_{ABS}} \Longrightarrow l_i = \eta_M \cdot P_{ABS} \cdot \frac{1}{\dot{m}}$$

LET'S COME BACK TO 1ST LAW FOR OPEN SYSTEMS

$$q + l_i = \Delta h$$

$$\frac{\dot{Q}_R}{\dot{m}} + \frac{\eta_M P_{ABS}}{\dot{m}} = c_p (T_{OUT} - T_e)$$

$$\dot{Q}_R + \eta_M P_{ABS} = \dot{m}\, c_p (T_{OUT} - T_e)$$

ONE KNOWS THAT $\dot{m} = \rho_e \dot{V} = \frac{P_e}{R T_e} \cdot \dot{V} = 1.88 \ \mathrm{Kg/s}$ $(R = 287 \text{ air})$

SUBSTITUTING $\dot{m}$, ONE FINDS

$$\dot{Q}_R = 53093 \ W \simeq 53 \ kW$$

$\alpha_1 = 45°$ BECAUSE THE STATOR GIVES THE SHAPE TO THE FLOW (c_1 IS DIRECTED AS STATOR BLADE)

BEING BLADE HEIGHT CONSTANT TROUGH THE STAGE $\Rightarrow$ ONE USES CONTINUITY EQUATION (BECAUSE IT LINKS AXIAL SPEED AT SECTION 0 TO THE ONE AT SECTION 1)

CONTINUITY $0 \to 1$ $\quad \dot m_0 = \dot m_1 \Rightarrow \rho_0 A_0 \underbrace{c_0}_{\perp \text{ TO SECTION}} = \rho_1 A_1 \underbrace{c_{1a}} \Rightarrow \cancel{\rho_0}\, \pi\, \cancel{d}\, \cancel{b_0}\, \cancel{\xi_0}\, c_0 = \cancel{\rho_1}\, \pi\, \cancel{d_1}\, \cancel{b_1}\, \cancel{\xi_1}\, c_1$

ρ = CONST. BECAUSE FLUID USED IS WATER = UNCOMPRESSIBLE FLUID

$\Rightarrow c_{1a} = c_0$ AND $c_{1a} = c_1 \sin 45° \Rightarrow c_1 = c_0 / \sin 45° = 70.711\ m/s$

NOW, ONE CAN USE THE RELATION $\boxed{\vec{c_1} = \vec{w_1} + \vec{u}} \Rightarrow \vec{w_1} = \vec{c_1} - \vec{u}$

ONE CAN TRANSFORM VECTORIAL RELATIONS INTO 2 SCALAR EQUATIONS

$$\begin{cases} w_{1a} = c_{1a} \\ w_{1u} = c_{1u} - u \end{cases}$$ OF COURSE $\vec{u}$ HAS NO AXIAL COMPONENT

$u = \omega \cdot d/2 = 60\ m/s$

BEING $u = 60\ m/s$ AND $c_1 = 70.711\ m/s$, ONE CAN SEE THAT w_1 POINTS TOWARDS THE LEFT

$$\begin{cases} w_{1a} = c_{1a} = 50\ m/s \\ w_{1u} = c_{1u} - u = -10\ m/s \end{cases} \Rightarrow \begin{aligned} w_1 &= \sqrt{50^2 + 10^2} = 50.99\ m/s \\ \beta_1 &= 90 + \operatorname{atan}(10/50) = 101.31° \end{aligned}$$

LET'S CONSIDER VELOCITY TRIANGLE AT ROTOR OUTLET 2

ONE DEFINES β_2 IMMEDIATELY (BEING THE ROTOR ROTATING IT CONVEYS w_2 AS ITS BLADES TO THE FLUID) $\Rightarrow \beta_2 = 135°$

LET'S MAKE AN HYPOTESIS :

$\dot{m}$ IS GIVEN UPSTREAM FROM THE MACHINE AND IT HAS NO CHANGES

IF $\dot{m}$ IS THE SAME AS BEFORE $\Rightarrow$ C_0 AND C_1 ARE THE SAME AS BEFORE

$C_{m1} = C_{m2} \Rightarrow C_1 = C_2 \Rightarrow W_{1u} + \cancel{u} = W_{2u} + \cancel{u}$ BEING AXIAL MACHINE

$W_{1u} = W_{2u} \Rightarrow W_1 = W_2$ BECAUSE $\dot{m} = \text{CONST}$

THIS ENABLES TO COMPUTE $u = 50+50 = 100$ m/s

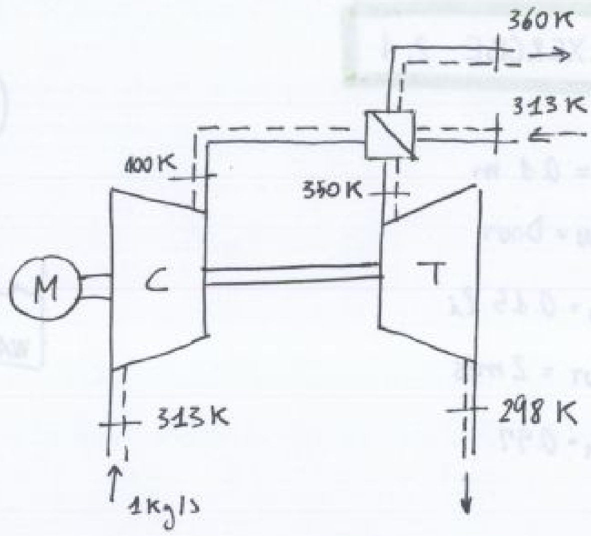
$$\Rightarrow \omega_{DISCR} = \frac{u}{d/2} = \frac{100}{0.25} = 400 \text{ rad/s}$$

ω_{DISCR} IS NAMED **RUNAWAY VELOCITY** ; IT IS THE VELOCITY AT WHICH TURBINE WOULD WORK WITH NO LOAD AND NEGLECTING ANY LOSS ; IT SHOULD NOT BE TOO LARGE FOR SAFETY REASONS

IT DISCRIMINATES DRIVEN FROM DRIVING MACHINES

IF $u > 100$ m/s $\Rightarrow$ OPERATING MACHINE BEHAVIOR

EXERCISE 2.5



$\eta_m = 0.95$

1ST LAW OPEN SYSTEMS
OPERATING MACHINE

$$\cancel{q} + l_i = \Delta h + \cancel{\Delta e_c} + \cancel{\Delta e_p} + \cancel{\Delta e_w}$$

2 OPEN SYSTEMS → COMPRESSOR C
→ TURBINE T

$$l_i(C) = \Delta h = c_p(400-313) = 87371.5 \text{ J/Kg}$$

$$l_i(T) = c_p(298-350) = -52234 \text{ J/Kg}$$

$$P_{ABS} = \frac{\dot{m}(l_i(C)+l_i(T))}{\eta_M} = \frac{35157.5}{0.95} \cong 37 \text{ KW}$$

NOZZLE & DIFFUSER

SPEED OF SOUND

when, in a mass of gas at rest, a small disturbance results in a slight local rise of pressure, it can be shown that a pressure wave is propagated throughout the gas with a velocity which depends upon the pressure and density of the gas; this velocity is the speed of sound (or sonic velocity) given by

$$\boxed{c_s = \sqrt{\left(\frac{dp}{d\rho}\right)_{s=const}}}$$

INTENSIVE PROPERTY

for an ideal gas $pv^k = p\rho^{-k} = const$ where $k = c_p/c_v$

$$d(p\rho^{-k}) = \rho^{-k}dp - pk\rho^{-k-1}d\rho = 0 \Rightarrow dp - kp\frac{d\rho}{\rho} = 0$$

$$\Rightarrow dp/d\rho = kp/\rho \Rightarrow \boxed{c_s = \sqrt{kp/\rho} = \sqrt{kRT}}$$

for the steam

$$p_A v_A^k = p_B v_B^k \Rightarrow p_A/p_B = (v_B/v_A)^k$$

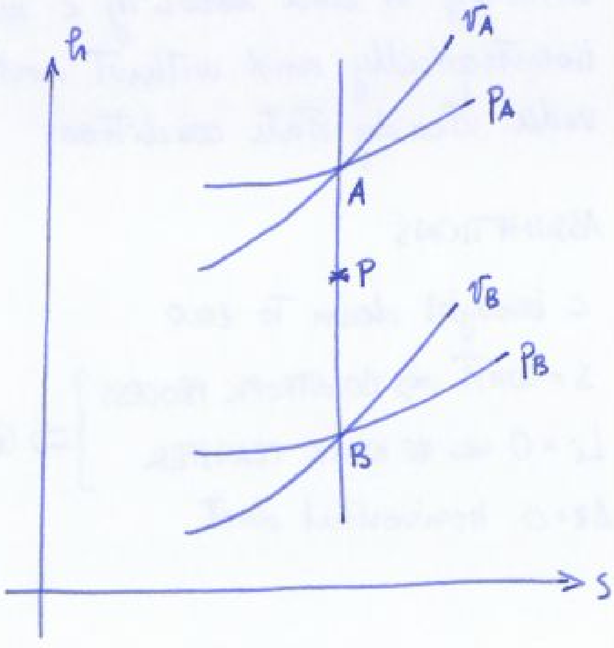
$$\ln(p_A/p_B) = k\ln(v_B/v_A)$$

$$\Rightarrow k = \frac{\ln(p_A/p_B)}{\ln(v_B/v_A)}$$

VALID ONLY BETWEEN POINTS A & B

$k \approx 1.3$ SUPERHEATED STEAM

$k \approx 1.4$ SATURATED STEAM



HOLLIER CHART

$$\Rightarrow h^0 - h + \frac{c_0^2 - c^2}{2} = 0$$

$$\Rightarrow \boxed{h^0 = h + c^2/2}$$ TOTAL ENTHALPY OR STAGNATION ENTHALPY $\Rightarrow \Delta h^0 = \Delta h + \Delta E_c$

$$\Rightarrow \boxed{s^0 = s}$$

One can rewrite I thermodynamic law as follows

$$\cancelto{=0}{Q} + \cancelto{=0}{L_i} = \Delta h^0 + \cancelto{=0}{\Delta E_g} \Rightarrow \boxed{h_A^0 = h_B^0}$$ valid for an HORIZONTAL DUCT

for an ideal gas $\Delta h = c_p \Delta T$

$$\Rightarrow \boxed{T^0 = T + \frac{1}{2} c^2 \cdot \frac{1}{c_p}}$$

$$\Rightarrow \boxed{T_A^0 = T_B^0} \Rightarrow s_B - s_A = R \ln(p_A^0 / p_B^0)$$ horizontal duct

$$p v^K = \text{const} = p^0 v^{0K} \Rightarrow p^0 = p(T^0/T)^{K/(K-1)} \; ; \; \rho^0 = \rho(T^0/T)^{1/(K-1)}$$

FOR STEAM, ONE CAN USE THE SAME FORMULAS BUT INSTEAD OF USING K ONE HAS TO USE $\overline{K}$ WHICH IS THE AVERAGE VALUE !

putting (3) into (1)

$$\frac{dc}{c} + \frac{dA}{A} - Ma^2 \frac{dc}{c} = 0 \Rightarrow \boxed{\frac{dA}{A} = (Ma^2 - 1)\frac{dc}{c}} \quad (4)$$

from (2)

$$\frac{dp}{p} \cdot p \cdot \frac{1}{K} \cdot K \cdot \frac{1}{\rho} \cdot \frac{1}{c^2} + \frac{c\,dc}{c^2} = 0 \quad \text{being } Kp/\rho = c_s^2$$

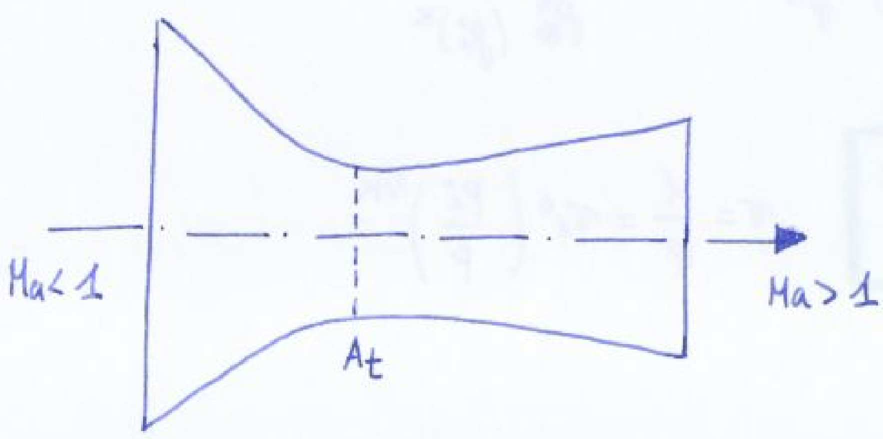
$$\Rightarrow \frac{dp}{p} \cdot \frac{1}{K} \cdot \frac{c_s^2}{c^2} + \frac{dc}{c} = 0 \Rightarrow \boxed{\frac{dp}{p} = -K Ma^2 \frac{dc}{c}} \quad (5)$$

putting (5) into (4) $\Rightarrow \boxed{\frac{dA}{A} = \frac{dp}{p}\left(\frac{1 - Ma^2}{K Ma^2}\right)} \quad (6)$

EFFECTS OF CHANGES IN FLOW AREA (4) & (6)

NOZZLES: considering a continous expansion ($dp<0$, $dc>0$) the changes in flow area of the nozzle must be as follows

- SUBSONIC FLOW ($Ma<1$) $\Rightarrow dA<0$ CONVERGENT DUCT
- SUPERSONIC FLOW ($Ma>1$) $\Rightarrow dA>0$ DIVERGENT DUCT
- $Ma=1 \Rightarrow dA=0$ IT CAN OCCOURR ONLY IN THE THROAT DUCT



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VELOCITY

$$\int_0^c c\,dc = -\int_{p_1^o}^{p} dp/\rho \Rightarrow \frac{c^2}{2} = -\int_{p_1^o}^{p} v\,dp = -v_1^o\, p_1^{o\,\frac{1}{k}} \int_{p_1^o}^{p} p^{-\frac{1}{k}}\,dp = \frac{k}{k-1}\,\frac{p_1^o}{\rho_1^o}\left[1-\left(\frac{p}{p_1^o}\right)^{\frac{k-1}{k}}\right]$$

$$\Rightarrow \boxed{c = \sqrt{2\,\frac{k}{k-1}\,\frac{p_1^o}{\rho_1^o}\left[1-\left(\frac{p}{p_1^o}\right)^{\frac{k-1}{k}}\right]}} \quad (6)$$

$$\rho c = \left[\rho_1^o\left(\frac{p}{p_1^o}\right)^{1/k}\right]\sqrt{2\,\frac{k}{k-1}\,\frac{p_1^o}{\rho_1^o}\left[1-\left(\frac{p}{p_1^o}\right)^{\frac{k-1}{k}}\right]}$$

THE THROAT AREA CAN BE FOUND BY MAXIMIZING (7) AND REVERSING IT !

$$\Rightarrow \boxed{\rho c = \sqrt{2\,\frac{k}{k-1}\,p_1^o\rho_1^o\left[\left(\frac{p}{p_1^o}\right)^{2/k}-\left(\frac{p}{p_1^o}\right)^{\frac{k+1}{k}}\right]}} \quad (7) \qquad \rho c = \frac{\dot m}{A}\ ;\ A \propto \frac{1}{\rho c}$$

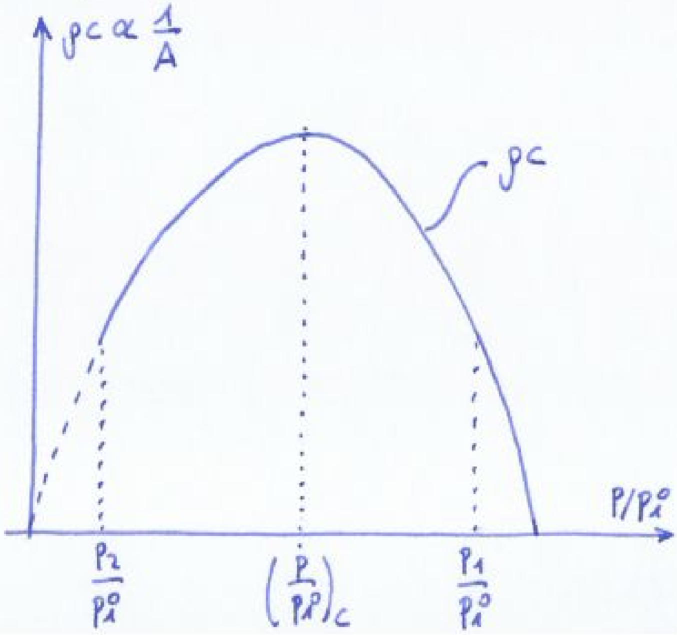
for a fixed flow rate we can find the max of (7)

$$\frac{d\rho c}{d(p/p_1^o)} = 0 \Rightarrow \boxed{\left(\frac{p}{p_1^o}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}} \quad (8)$$

CRITICAL PRESSURE RATIO

putting (8) into (6) it is possible to find the velocity in the throat section

$$\boxed{c_c = \sqrt{\frac{2k}{k+1}\,\frac{p_1^o}{\rho_1^o}}} \quad (9)$$



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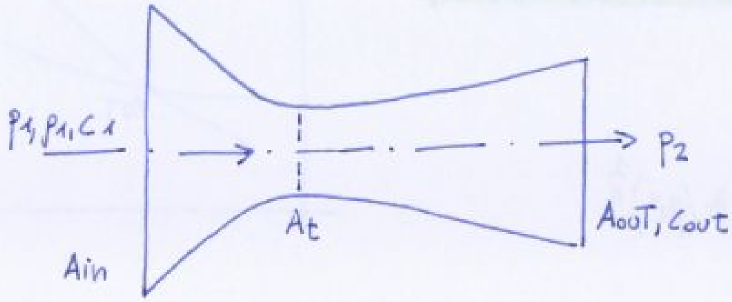
DESIGN OF NOZZLES

DESIGN DATA

- mass flow rate $\dot{m}$
- fluid properties upstream of nozzle $p_1, \rho_1, c_1 \Rightarrow p_1^\circ, \rho_1^\circ$
- back pressure (in the environment downstream from nozzle) p_2



QUANTITIES TO BE EVALUATED

- characteristic nozzle cross section A_{in}, A_t, A_{out}
- fluid velocity at outlet port c_{out}

The inlet section is $A_{in} = A_1$ $\boxed{A_1 = \dot{m}/(\rho_1 c_1)}$

if $c_1 \to 0 \Rightarrow \rho_1 \to \rho_1^\circ \; ; \; A_1 \to +\infty$

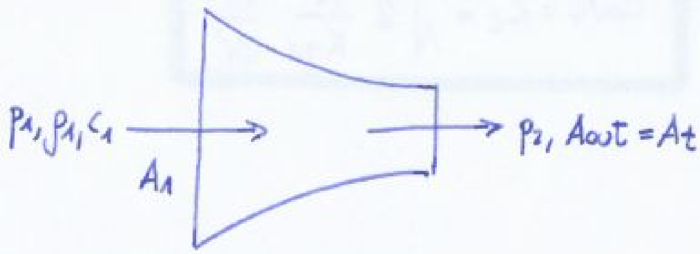
since it can be $\frac{p_2}{p_1^\circ} \gtreqless \left(\frac{p}{p_1^\circ}\right)_c$ we have 3 different cases

CASE I

$$\frac{p_2}{p_1^\circ} > \left(\frac{p}{p_1^\circ}\right)_c = \left(\frac{2}{K+1}\right)^{\frac{K}{K-1}}$$

THE FLOW WILL NOT REACH THE SONIC CONDITION; IT REMAINS IN SUBSONIC CONDITION !

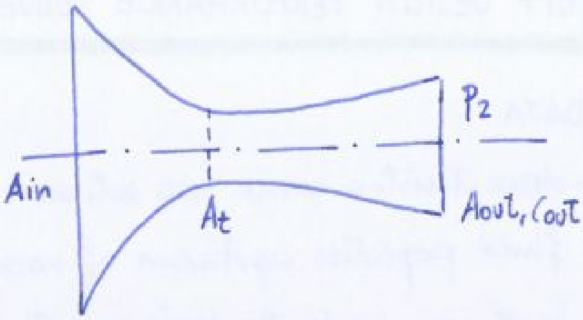
$\Rightarrow$ CONVERGENT NOZZLE



$$\boxed{A_{out} = \frac{\dot{m}}{\rho_{out} c_{out}} = \frac{\dot{m}}{\sqrt{2 \frac{K}{K-1} p_1^\circ \rho_1^\circ \left[\left(\frac{p_2}{p_1^\circ}\right)^{2/K} - \left(\frac{p_2}{p_1^\circ}\right)^{\frac{(K+1)}{K}} \right]}}}$$

CASE III

$$\frac{p_2}{p_1^o} < \left(\frac{p}{p_1^o}\right) = \left(\frac{2}{K+1}\right)^{\frac{K}{K-1}}$$



$\Rightarrow$ CONVERGENT-DIVERGENT NOZZLE

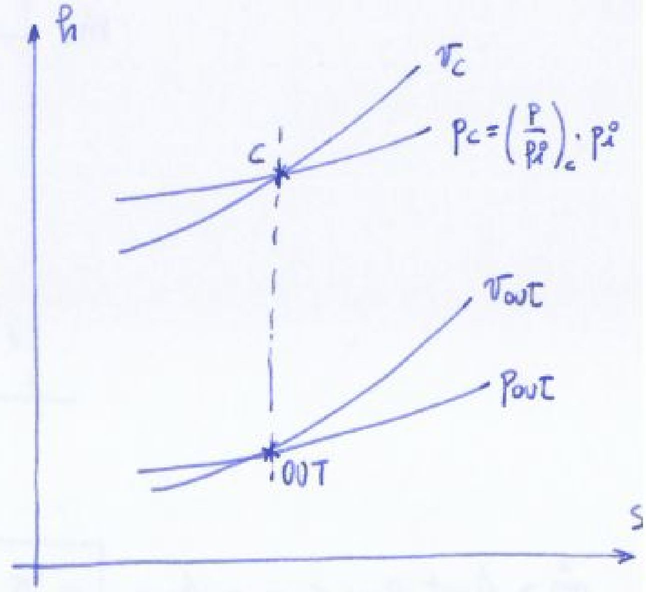
$$\boxed{A_{out} = \dot{m} / (\rho_{out} c_{out})}$$

$\rho_{out} c_{out}$ THE SAME AS CASE I !

$$\boxed{A_t = A_c = \dot{m} / (\rho c)_c}$$

$$\boxed{c_{out} = \sqrt{2 \frac{K}{K-1} \frac{p_1^o}{\rho_1^o} \left[1 - \left(\frac{p_2}{p_1^o}\right)^{\frac{K-1}{K}}\right]} = \sqrt{2(h_1^o - h_{out})}}$$

$$\boxed{c_c = c_{s,c} = \sqrt{2 \frac{K}{K+1} \frac{p_1^o}{\rho_1^o}}}$$



since $p_c/p_1^o = (2/(K+1))^\wedge(K/(K-1))$, $\dot m_c$ can be rewritten as

$$\dot m_c = A_{OUT}\sqrt{\frac{p_1^o}{v_1^o}}\sqrt{K\left(\frac{2}{K+1}\right)^{\frac{K+1}{K-1}}} \Rightarrow \boxed{\dot m_c = A_{OUT}\sqrt{\frac{p_1^o}{v_1^o}}\,\Gamma(K)}$$

it is usefull to express the nozzle performance in terms of non-dimensional parameters

- the expansion ratio
- the reduced mass flow rate

SUBSONIC NOZZLE

for $\frac{p_2}{p_1^o} > \left(\frac{p}{p_1^o}\right)_c \Rightarrow$

$$\boxed{\frac{\dot m\sqrt{p_1^o v_1^o}}{A_{out}\,p_1^o} = \sqrt{2\frac{K}{K-1}\left[\left(\frac{p_2}{p_1^o}\right)^{2/K} - \left(\frac{p_2}{p_1^o}\right)^{\frac{(K+1)}{K}}\right]} = f\left(\frac{p_2}{p_1^o}\right)}$$

for $\left(\frac{p_2}{p_1^o}\right) \le \left(\frac{p}{p_1^o}\right)_c \Rightarrow$

$$\boxed{\frac{\dot m\sqrt{p_1^o v_1^o}}{A_{out}\,p_1^o} = \sqrt{K\left(\frac{2}{K+1}\right)^{\frac{K+1}{K-1}}} = \Gamma(K)}$$

CHOCKED NOZZLE

CHARACTERISTIC DIAGRAM OF CONVERGENT NOZZLES (NON-DIMENSIONAL UNITS)

if $h_1^o = \text{CONST} \Rightarrow \dot m_c \propto p_1^o$

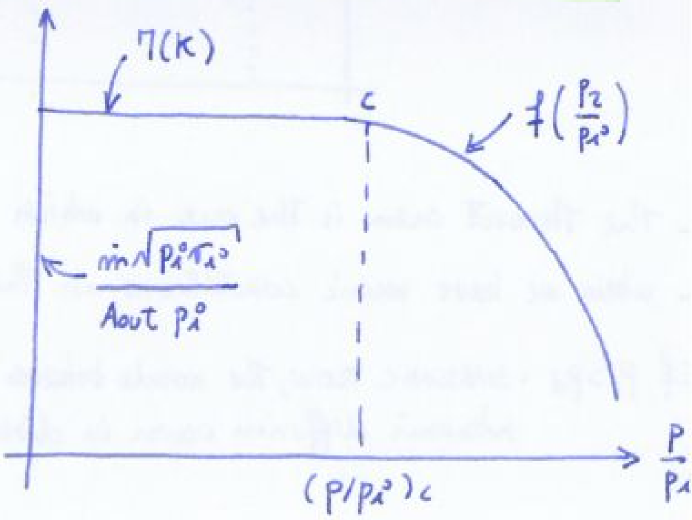
IDEAL GAS

$h_1^o = \text{const} \Rightarrow T_1^o = \text{const} \Rightarrow p_1^o v_1^o = RT_1^o = \text{const}$

$\Rightarrow \dot m_c \propto p_1^o$

STEAM

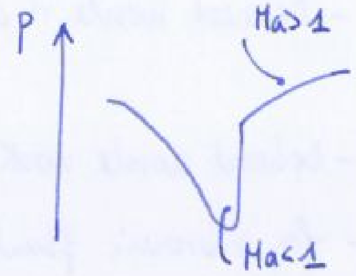
$h_1^o = \text{const} \Rightarrow p_1^o v_1^o \approx \text{const}$



$p_d < p_2 < p_a$: The flow is SUPERSONIC in the THROAT, therefore the nozzle is chocked; however, a shock wave occurs in the divergent duct

BEETWEEN p_d & p_a NON-ISOENTROPIC EVOLUTION WITH NORMAL SHOCK, SUDDEN INCREASE IN PRESSURE WITH THE FLOW THAT CHANGES FROM SUPERSONIC TO SUBSONIC

!



$p_2 = p_e \Rightarrow$ chocked flow with normal shock

$p_2 = p_f \Rightarrow$ chocked flow with shock in outlet section

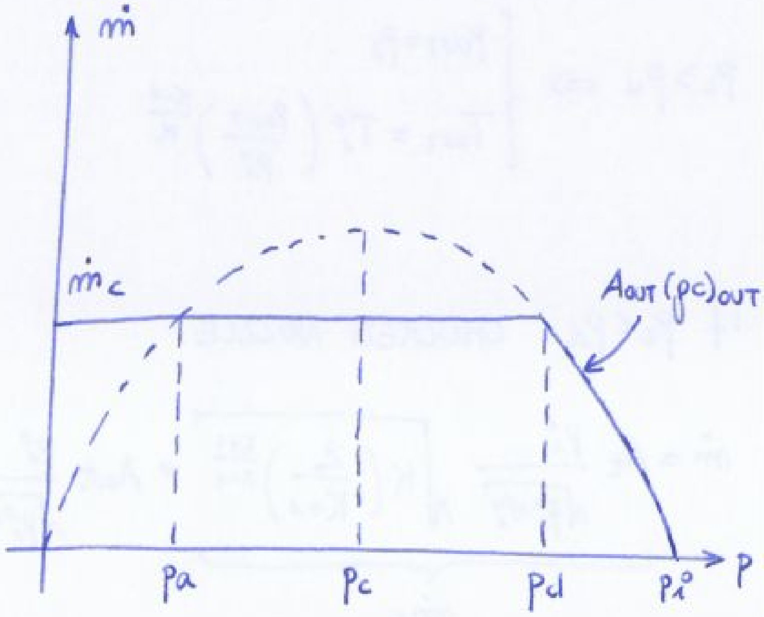
$p_2 < p_a \Rightarrow$ The fluid expands isoentropically through the nozzle and then expands outside the nozzle to the back pressure through oblique expansion waves

$p_2 = p_g \Rightarrow$ chocked flow $p_{OUT} = p_a$; compression wave in downstream environment

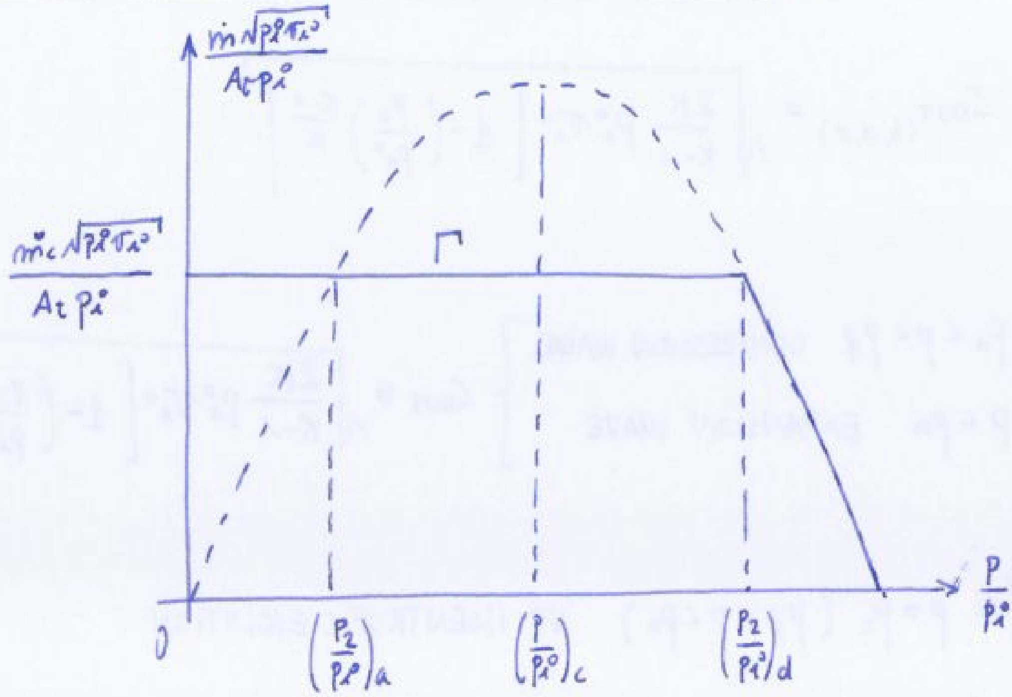
$p_2 = p_h \Rightarrow$ chocked flow $p_{OUT} = p_a$, expansion waves in downstream environment

! AFTER THE CHOCKING CONDITION IS REACH, NOTHING CAN CHANGE IN THE UPSTREAM PART OF THE THROAT SECTION

$$p_c = p_1^\circ \left(\frac{2}{K+1}\right)^{\frac{K}{K-1}}$$



CHARACTERISTIC DIAGRAM OF CONVERGENT - DIVERGENT NOZZLE



if $\frac{p_2}{p_1^0} \geq \left(\frac{p}{p_1^0}\right)_d$ SUBSONIC NOZZLE $\Rightarrow$ $\frac{\dot m \sqrt{p_1^0 v_1^0}}{A_t p_1^0} = \frac{A_{OUT}}{A_t}\sqrt{\frac{2K}{K-1}\left[\left(\frac{p_2}{p_1^0}\right)^{2/K} - \left(\frac{p_2}{p_1^0}\right)^{\frac{K+1}{K}}\right]}$

if $\frac{p_2}{p_1^0} \leq \left(\frac{p}{p_1^0}\right)_d$ CHOCKED NOZZLE $\Rightarrow$ $\frac{\dot m_c \sqrt{p_1^0 v_1^0}}{p_1^0 A_t} = \sqrt{K\left(\frac{2}{K+1}\right)^{\frac{K+1}{K-1}}}$

POLITECNICO DI TORINO

Dipartimento Energia

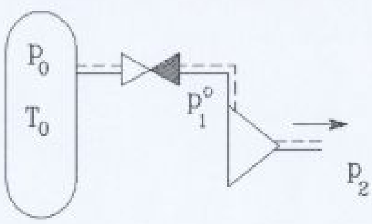
BSc in Automotive Engineering

APPLIED LECTURE 3 - Thermal Machines

1) Design a nozzle so as to expand 3 kg/s of air ($R = 287$ J/kg·K, $k = 1.4$) from the upstream conditions $p_1 = 160$ kPa, $T_1 = 500$ K, $c_1 = 100$ m/s to the downstream conditions $p_2 = 1$ bar, $T_2 = 295$ K. Determine the fluid velocity in the outlet cross section.
Also determine the mass flow rate expanded by the nozzle under the following off-design conditions: $p_1' = 0.5$ MPa, $T_1' = 550$ K, $c_1' \approx 0$, $p_2' = 2$ bar, $T_2' = 310$ K.

2) A convergent-divergent nozzle expands air ($R = 287$ J/kg·K, $k = 1.4$) from the upstream conditions $p_1 = 0.25$ MPa, $T_1 = 543$ K, $c_1 \approx 0$ to pressure $p_2 = 0.16$ Mpa. The nozzle displays an outlet cross section $A_{out} = 5.493$ cm^2 and is characterized by an isentropic expansion ratio $(p_2/p_1°)_a = 0.11$.
Determine the mass flow rate through the nozzle and the fluid velocity in the outlet cross section.

3) A tank containing air under a pressure $p_0 = 300$ kPa and at a temperature $T_0 = 500$ K is connected to the environment ($p_2 = 100$ kPa) by means of a valve and a convergent isentropic nozzle displaying an outlet cross section $A_u = 0.05$ m^2. The fluid conditions in the tank are to be considered constant. Determine:



- the pressure $p_1°$ downstream from the valve so that the maximum fluid velocity c_2 at the nozzle outlet is achieved;
- the above mentioned velocity c_2 and the mass flow rate $\dot{m}$ for such a pressure;
- the maximum mass flow rate which the nozzle could possibly expand.

LET'S DESIGN A_{OUT}

1ST LAW OF THERMODYNAMICS $1 \rightarrow OUT$

$$q - l_i = \Delta h + \Delta e_c \Rightarrow c_p(T_{OUT} - T_1) + \frac{1}{2}(C_{OUT}^2 - C_1^2) = 0$$

$P_{OUT} = P_2$

$T_{OUT} \neq T_2$

T_2 HAS TO BE IGNORED

EVALUATE $C_{OUT} = \sqrt{C_1^2 - 2c_p(T_{OUT} - T_1)}$

ISENTROPIC PROCESS $T_{OUT} = T_1^0 \left(\frac{P_{OUT}}{P_1^0}\right)^{\frac{k-1}{k}} = 437.2\ K$

$$\Rightarrow C_{OUT} = \sqrt{C_1^2 - 2c_p(T_{OUT} - T_1)} = 369\ m/s$$

$$A_{OUT} = \frac{\dot{m}}{\left(\frac{P_2}{RT_{OUT}}\right) C_{OUT}} = 0.010201\ m^2$$

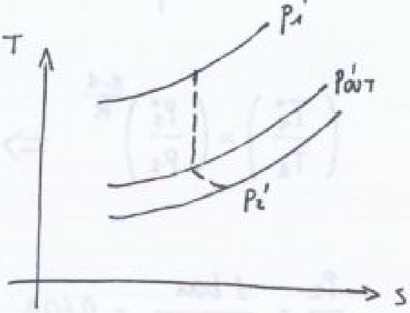
LET'S EXAMINE OFF-DESIGN CONDITIONS: $P_1' = 5\ bar$, $T_1' = 550\ K$, $C_1' \approx 0$, $P_2' = 2\ bar$, $T_2' = 310\ K$

COMPUTE $\dot{m}'$ IN THESE OF DESIGN CONDITIONS

FOR $\frac{P_2}{P_1^0} \geq \left(\frac{P_2}{P_1^0}\right)_{CRIT} \Rightarrow \dot{m} \propto \frac{P_2}{P_1^0} \Rightarrow$ VALID ALSO $P_{OUT} = P_2$

$P_{OUT}' = P_1' \left(\frac{P_2}{P_1^0}\right)_{CRIT}$

FOR $\frac{P_2}{P_1^0} < \left(\frac{P_2}{P_1^0}\right)_{CRIT} \Rightarrow \dot{m} = const$ AND $P_{OUT} > P_2$



$$\frac{P_2'}{P_1^{0\prime}} = \frac{P_2'}{P_1^0} = 0.4 < \left(\frac{P_2}{P_1^0}\right)_{CRIT}$$

$$\Gamma(k) = \sqrt{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} = 0.6847 \Rightarrow \dot{m} = \Gamma(k) A_{OUT} \frac{P_1'}{\sqrt{P_1^0 v_1^0}} = \Gamma(k) A_{OUT} \frac{P_1'}{\sqrt{RT_1'}} = 8.78\ Kg/s$$

P_2' IS NOT REACHED ISENTROPICALLY! IT IS NOT REACHED INSIDE THE NOZZLE (DOWNSTREAM ENVIRONMENT $\neq$ OUTLET OF NOZZLE)

$\Rightarrow$ EXPANSION WAVES FORM IN THE DOWNSTREAM ENVIRONMENT (NON 1-D PHENOMENON THAT CANNOT BE ISENTROPICAL)

THEN ONE COMPUTES $\dot{m}_c$ USING EQ. 2 WITH $A_{th} \Rightarrow \dot{m}_c = 0.13$ Kg/s

AT THIS STAGE, ONE CAN EVALUATE THE "ACTUAL" $\dot{m}$ WITH EQ. 1 USING $p_2/p_1^\circ = 0.64$

$\Rightarrow \dot{m} = 0.232$ Kg/s

SINCE $0.232 > 0.13$ THE FOUND VALUE HAS PHYSICAL MEANING, THUS ONE IS ACTUALLY IN CHOCKED MASS FLOW RATE

$\frac{p_2}{p_1^\circ} = 0.64$ YIELDS A $\dot{m} = \dot{m}_c = 0.13$ Kg/s

$\dot{m}$ CAN NEVER BE $> \dot{m}_c$!

LET'S COMPUTE C_{OUT}

ONE USES $\dot{m}$ JUST COMPUTED $\Rightarrow \dot{m} = \rho_{OUT} A_{OUT} C_{OUT}$

ONE HAS TO COMPUTE $\rho_{OUT} = p_{OUT}/(RT_{OUT})$

LET'S COMPUTE T_{OUT} USING 1ST LAW OF TH INLET $\rightarrow$ OUTLET

$$\cancel{q} + \cancel{l_i} = \Delta h + \Delta e_c + \cancel{\Delta e_g} + \cancel{\Delta e_w} \Rightarrow c_p(T_{OUT} - T_{IN}) + \frac{1}{2} C_{OUT}^2 = 0$$

COMBINE THIS EQUATION WITH $\dot{m}_{OUT} = \frac{p_{OUT}}{RT_{OUT}} A_{OUT} C_{OUT}$

2 EQUATIONS IN 2 UNKNOWNS (C_{OUT}, T_{OUT})

T_{OUT}:
- < 0
- 518.9 K ONLY REASONABLE VALUE

ONCE T_{OUT} IS COMPUTED $C_{OUT} = \frac{\dot{m}_{OUT}}{\frac{p_2}{RT_{OUT}} A_{OUT}} = 220.3$ m/s

REDUCED MASS FLOW RATE $\Gamma = \frac{\dot{m}\sqrt{p_1^\circ v_1^\circ}}{p_1^\circ A_{OUT}}$ AND Γ IS FUNCTION ONLY OF K

$$\begin{cases} \Gamma = \sqrt{K\left(\frac{2}{K+1}\right)^{\frac{K+1}{K-1}}} \\ \Gamma = \frac{\dot{m}\sqrt{p_1^\circ v_1^\circ}}{p_1^\circ A_{OUT}} = \frac{\dot{m}\sqrt{RT_1^\circ}}{p_1^\circ A_{OUT}} \end{cases}$$

EQUALIZING THE 2 PREVIOUS EQUATIONS, AND PUTTING $p_1^\circ = p_0^\circ$, MAX PRESSURE UPSTREAM ~~THE~~ OF THE NOZZLE IS REACHED WHEN VALVE IS TOTALLY OPEN

$\Rightarrow$ NO PRESSURE DROP FROM TANK

ACCORDING TO THESE CONSIDERATIONS ONE OBTAINS $\dot{m}_{MAX} = 27.098$ Kg/s

$$\frac{\dot{m}\sqrt{287\cdot 500}}{3\cdot 10^5\cdot 0.05} = \sqrt{1.4\left(\frac{2}{1.4+1}\right)^{\frac{1.4+1}{1.4-1}}} \Rightarrow \dot{m} = \frac{0.685}{0.0253} = 27.12 \text{ Kg/s}$$

CHEMICALLY REACTIVE SYSTEMS

$$\left.\begin{array}{l}\text{Fuel}\\ \text{Oxidant}\end{array}\right\} \Rightarrow \begin{array}{l}\text{OXIDATION}\\ \text{PRODUCTS}\end{array}$$

- COMPLETE COMBUSTION ~~COMBUSTION~~ : all the carbon is converted in carbon dioxide and all the hydrogen in water $C \Rightarrow CO_2$; $H \Rightarrow H_2O$
- INCOMPLETE COMBUSTION: if the air is not enough rich of oxigen, we don't have complete combustion
- AIR TO FUEL RATIO : $\alpha = m_A / m_F = A/F$
- STECHIOMETRIC AIR TO FUEL RATIO : $\alpha_{ST} = m_{A,ST}/m_F$
- RELATIVE AIR TO FUEL RATIO : $\lambda = \alpha / \alpha_{ST}$

  $\lambda = 1 \Rightarrow$ STECHIOMETRIC MIXTURE

  $\lambda < 1 \Rightarrow$ RICH MIXTURE

  $\lambda > 1 \Rightarrow$ LEAN MIXTURE

- EQUIVALENT RATIO $\phi = \alpha_{ST}/\alpha = 1/\lambda$
- AIR COMPOSITION (simplified) : $21\%\ O_2$ & $79\%\ N_2$
- COMBUSTION

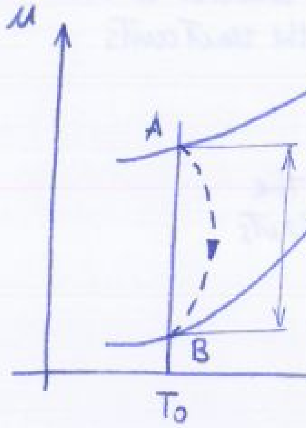
$$CH_4 + a\left(O_2 + \frac{79}{21} N_2\right) \Rightarrow b\,CO_2 + d\,H_2O + e\,N_2$$

- OXYGEN $\Rightarrow 2a = 2b + d \Rightarrow a = 2$
- CARBON $\Rightarrow 1 = b$
- HYDROGEN $\Rightarrow 4 = 2d \Rightarrow d = 2$
- NITROGEN $\Rightarrow 2a \cdot 79/21 = 2e \Rightarrow e = 7.52$

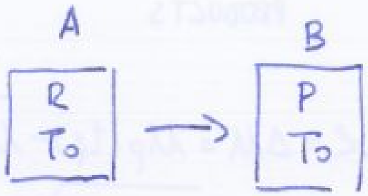
$$\Rightarrow CH_4 + 2\,(O_2 + 79/21\, N_2) \Rightarrow CO_2 + 2H_2O + 7.52\, N_2$$

$$\alpha_{ST} = \frac{m_{A,ST}}{m_f} = \frac{2 \cdot 32 + 2 \cdot 3.76 \cdot 28}{(12+4)} = 17.16$$

HEATING VALUE AT CONSTANT VOLUME



$-Q>0$ since $\alpha>\alpha_{ST} \Rightarrow Q>0$



$$Q-\underbrace{\cancel{L}}_{=0} = \Delta u = u_B - u_A < 0 \Rightarrow -Q = |Q| = -\Delta u = m_f H_{V,T_0}$$

$$\Rightarrow \boxed{H_{V,T_0} = -\Delta u_V / m_f} \qquad \boxed{H_{V,T_0} = |Q| / m_f}$$

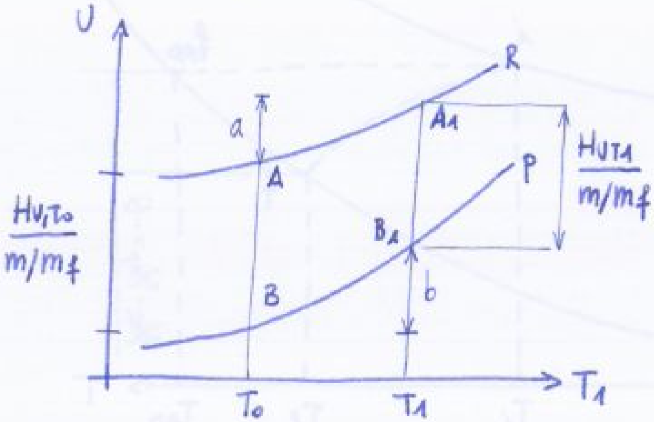
HEATING VALUE OF A FUEL

heat that has to be taken ~~into account~~ away from the fuel at constant volume to remain at the same T at the end of the process !

$\Delta u = m \Delta U$

$\Downarrow$

$$H_{V,T_0} = -\frac{m}{m_f}\Delta U \Rightarrow -\Delta U = U_A - U_B = H_{V,T_0}/(m/m_f)$$



$$\frac{H_{VT1}}{m/m_f} < \frac{H_{V,T_0}}{m/m_f}$$

INCREASING T, HEATING VALUE DECREASES !

$a = U_{A1} - U_A$

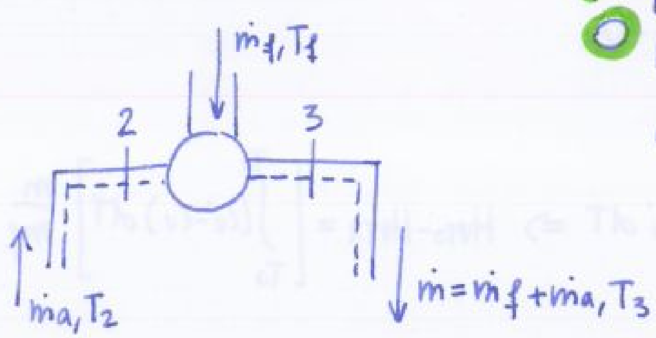
$b = U_{B1} - U_B$

heating values:
- HIGHER (HHV): H_2O liquid
- LOWER (LHV): H_2O gaseous

heating of vaporisation of water causes the difference between the 2 values

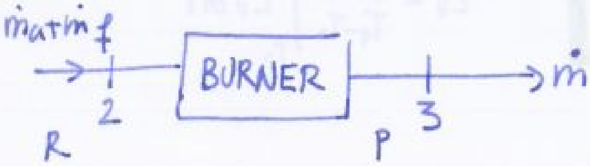
STEADY STATE COMBUSTION PROCESS

GAS TURBINE BURNER



!! WE NEGLECT THE DIFFERENCE BETWEEN T_2 & T_f WITHOUT MAKING A BIG MISTAKE SINCE

$\alpha = \dot m_a / \dot m_f \Rightarrow 30$ [usually $T_f < T_2$]

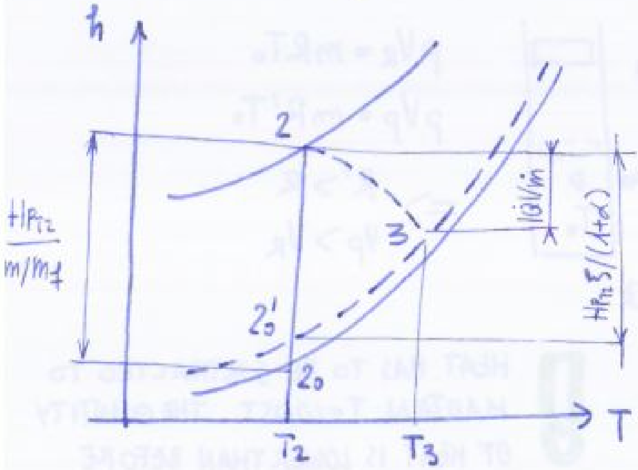


I Thermodynamic law between 2 & 3

$\dot{\mathcal{L}}_i = 0$ (no internal work)

$\Rightarrow \dot Q = \dot m (h_3 - h_2)$

heat losses through the walls $\Rightarrow -|\dot Q| = \dot m (h_3 - h_2)$; $\dot Q < 0$



$$\frac{\dot m}{\dot m_f} = \frac{m}{m_f} = \frac{m_a + m_f}{m_f} = 1 + \frac{\dot m_a}{\dot m_f} = 1 + \alpha$$

$$\Rightarrow \frac{H_{PT_2}}{m/m_f} = \frac{H_{PT_2}}{1+\alpha} \qquad \xi < 1$$

!! IT IS ALMOST IMPOSSIBLE TO CARRY OUT A COMPLETE COMBUSTION DUE TO SOME EFFECTS WE HAVE IN REAL CASES. THERE'LL BE SOME RESIDUAL ENERGY IN THE EXHAUST GASES, SO THE ENTHALPY WILL BE SLIGHTLY HIGHER $\Rightarrow$ PARTIALLY OXYDIZED PRODUCT (DASHED LINE)

$$h_3 - h_2 = -|\dot Q|/\dot m$$

$$h_2 - h_{2'_0} = h_2 - h_3 + h_3 - h_{2'_0}$$

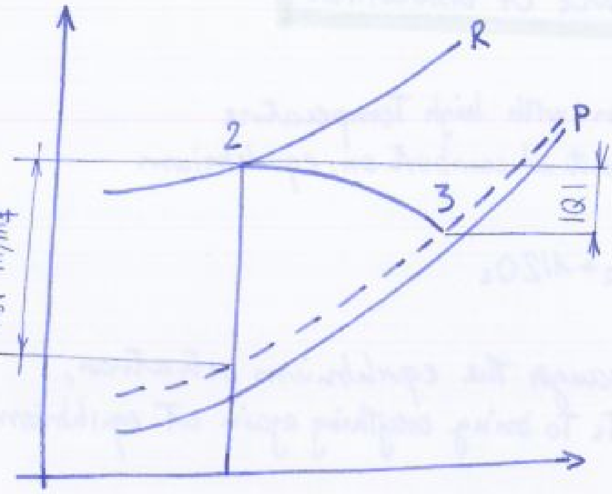
$$\Rightarrow \xi \frac{H_{PT_2}}{1+\alpha} = \frac{|\dot Q|}{\dot m} + \overline{c_p'}(T_3 - T_2)$$

$$\Rightarrow \boxed{\underbrace{\frac{H_{PT_2}}{1+\alpha}\left(\xi - \frac{|\dot Q|}{\dot m}\frac{(1+\alpha)}{H_{PT_2}}\right)}_{\text{BURNER EFFICIENCY } \eta_B} = \overline{c_p'}(T_3 - T_2) = \eta_B \frac{H_{PT_2}}{1+\alpha}}$$

COMBUSTION WITH RICH MIXTURE

$\alpha < \alpha_{ST}$

(constant volume combustion)



NO COMPLETE COMBUSTION, WE HAVE NOT ENOUGH OXYGEN

$\Rightarrow \alpha < \alpha_{ST}$

$m_a = \alpha\, m_f$

- given the air available in the system, we can calculate the stechiometric mass of fuel

$$m_{f,ST} = \frac{m_a}{\alpha_{ST}} < m_f = \frac{m_a}{\alpha}$$

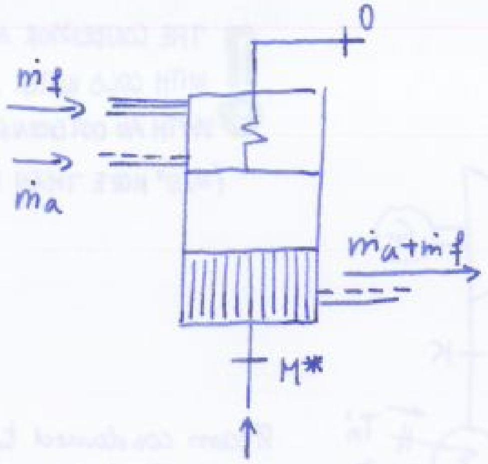
- percentage of energy released $= \dfrac{m_{fST}}{m_f} = \dfrac{\alpha}{\alpha_{ST}}$

- the equation of the combustion becomes

$$\boxed{\frac{\alpha}{\alpha_{ST}} \frac{H_{uT2}}{m/m_f} = \overline{C_v}(T_3 - T_2) + |Q|}$$

ENERGY BALANCES

STEAM GENERATOR



IT is a kind of heat exchanger

$$\dot{Q} - \underbrace{\cancel{\dot{\mathcal{L}}}}_{=0} = \dot{m}(h_0 - h_{M^*})$$

$$Q = \frac{\dot{Q}}{\dot{m}} = h_0 - h_{M^*} = Q_1$$

HEAT PROVIDED TO THE SYSTEM BY THE SURROUNDING $\Rightarrow Q_1$ POSITIVE

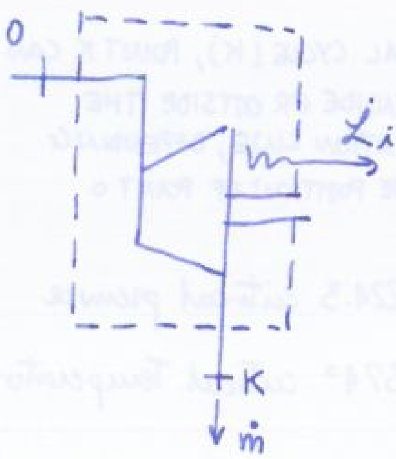
$$\boxed{\dot{Q}_1 = \dot{m}(h_0 - h_{M^*}) = \eta_b \dot{m}_f H_p}$$

H_p = heating value at constant pressure of the fuel

$\eta_b < 1 \approx 0.9 \Rightarrow$
- incomplete combustion
- heat losses
- enthalpy lost with exhaust gases

TURBINE

open system with 1 entry & 1 exit



$$\dot{Q} - \dot{\mathcal{L}}_i = \dot{m}(h_K - h_0)$$ assuming negligible changes in e_c and e_g

! EVEN IF $\dot{Q}$ IS NOT ZERO, THE RATIO $\dot{Q}/\dot{m}$ IS VERY LOW SINCE $\dot{m}$ IS VERY HIGH

$\Rightarrow$ WE CAN NEGLECT $\dot{Q}$

$$\Rightarrow \dot{\mathcal{L}}_i = \dot{m}(h_0 - h_K)$$

$$\Rightarrow L_i = \frac{\dot{\mathcal{L}}_i}{\dot{m}} = h_0 - h_K$$

Pump ⇒ power absorbing machine

turbine ⇒ power producing machine

Energy balance of the fluid operating in the plant

$\dot{Q} - \dot{\mathcal{L}}_i = \dot{m}\,\Delta h = 0$ since the initial and final state are the same

$\Rightarrow \dot{\mathcal{L}}_i = \dot{Q} = \dot{Q}_1 - \dot{Q}_2$ where

$\dot{\mathcal{L}}_i$ = mechanical power

$\dot{Q}$ = thermal power

$\dot{Q}_1$ = heat power provided in the steam generator

$\dot{Q}_2$ = heat rejected in the condenser

$$\Rightarrow \boxed{\dot{\mathcal{L}}_{it} - \dot{\mathcal{L}}_{i,p} = \dot{Q}_1 - \dot{Q}_2}$$

! POWER ABSORBED BY THE PUMP IS MUCH LOWER THAN THE POWER ABSORBED BY THE TURBINE

$$\dot{\mathcal{L}}_{ip} \ll \dot{\mathcal{L}}_{it} \Rightarrow \dot{\mathcal{L}}_i \approx \dot{\mathcal{L}}_{it} = \dot{Q}_1 - \dot{Q}_2$$

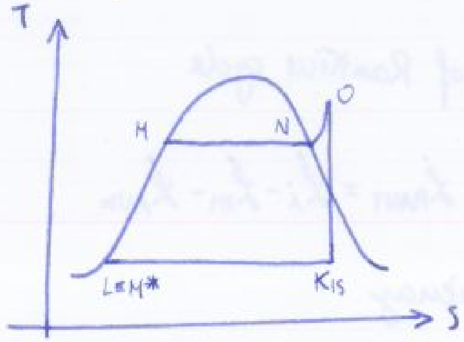
it can be shown that for values

$p_{M^*} = 100$ bar ; $p_L = 0.1$ bar (negligible) ; $\rho = 1000\ Kg/m^3$ (water)

$$L_{ip} = \int_{p_L}^{p_{M^*}} v\,dp + \Delta E_c + \Delta E_p + L_W = \frac{p_{M^*} - p_L}{\rho} \cong 10\ KJ/Kg$$

$\Rightarrow \dot{\mathcal{L}}_{i,p} \approx 0 \Rightarrow h_{M^*} \approx h_L$

WE NEGLECT THE DIFFERENCE IN ENTHALPY ACROSS THE PUMP reference cycle ⇒ Carnot cycle



! RANKINE CYCLE QUITE SIMILAR TO CARNOT CYCLE ⇒ MORE DIFFICULT TO COMPRESS THAN TO EXPAND

CARNOT CYCLE

$$\eta_c = 1 - \frac{Q_{2c}}{Q_{1c}} = 1 - \frac{T_2}{T_1}$$

$$\eta \Uparrow : \begin{cases} T_2 \Downarrow \\ T_1 \Uparrow \end{cases}$$

AVERAGE TEMPERATURE OF HEAT ADDITION $\overline{T_1}$

$$Q_1 = \int_{S_{M*}}^{S_O} T\,ds = \overline{T_1}(S_O - S_{M*}) \Rightarrow \overline{T_1} = \frac{1}{S_O - S_{M*}} \int_{S_{M*}}^{S_O} T\,ds$$

AVERAGE TEMPERATURE OF HEAT REJECTION $\overline{T_2}$

$$\overline{T_2} = \frac{1}{S_{K,IS} - S_L} \int_{S_L}^{S_{K,IS}} T\,dS$$

RANKINE CYCLE (REVERSIBLE)

Reversible ⇒ $S_{KIS} = S_O$; $S_L = S_{M*}$

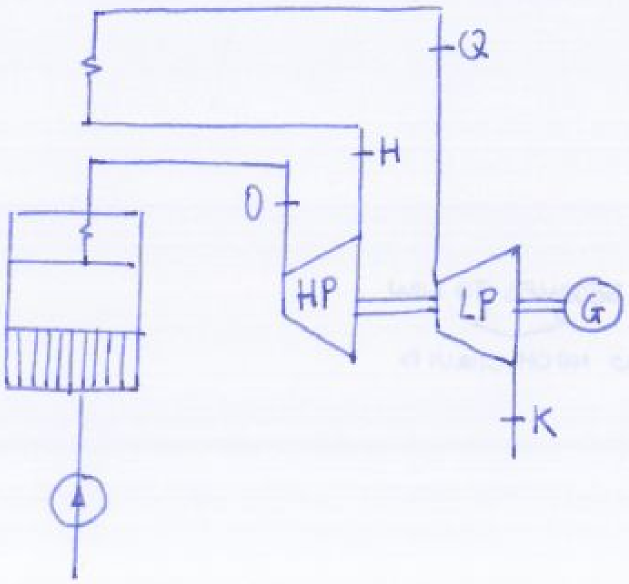
$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{\overline{T_2}(S_{KIS} - S_L)}{\overline{T_1}(S_O - S_{M*})} = 1 - \frac{\overline{T_2}}{\overline{T_1}}$$

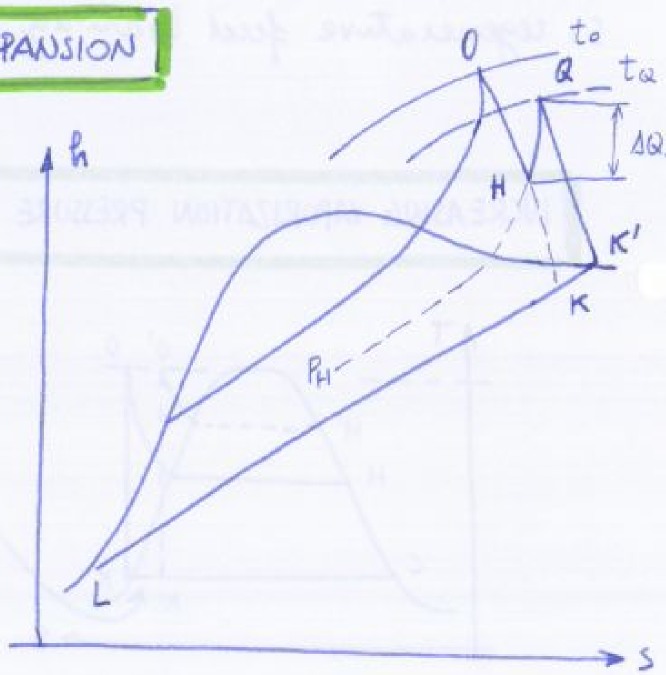
	p_o [bar]	T_o [°C]	η_{PLANT} [%]
BASELINE	170	540	42
TOP LEVEL	315	620	46
RESEARCH *	350	720	48

IMPROVEMENTS GOT WORKING ON ALL POSSIBLE FACTORS !

* not available, only at prototype level

REHEATING STEAM AFTER A PARTIAL EXPANSION

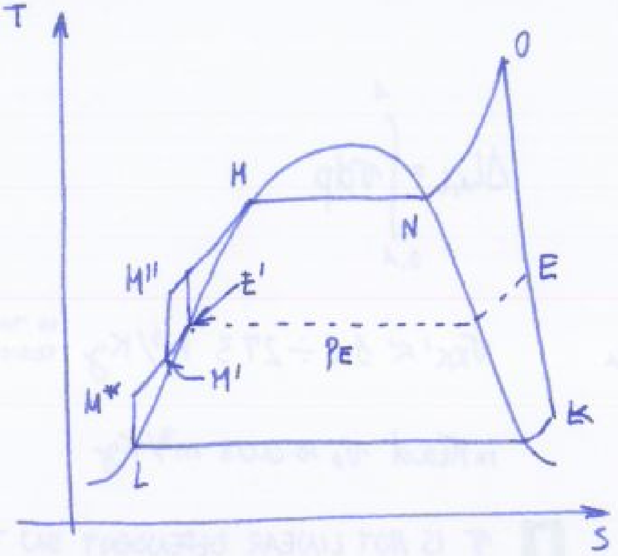


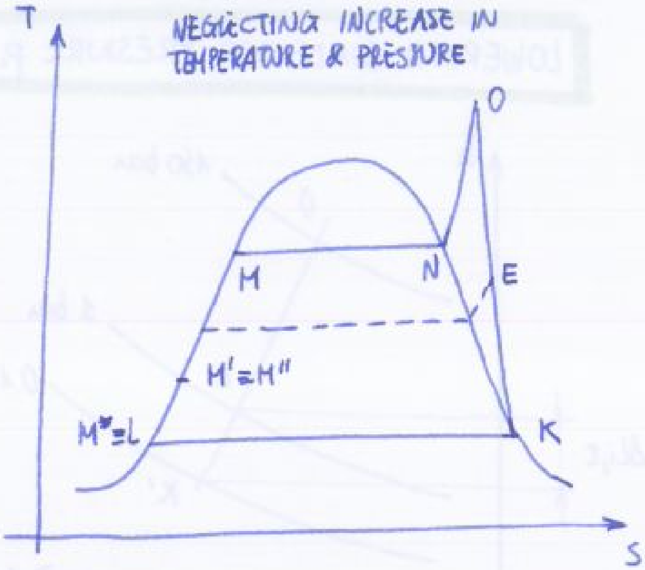


$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$\eta' = 1 - \frac{Q_2'}{Q_1'} = 1 - \frac{Q_2 + \Delta Q_2}{Q_1 + \Delta Q_1} \quad \text{where } \Delta Q_2 < \Delta Q_1$$

$$\eta' = 1 - \left[\frac{Q_2}{Q_1} \underbrace{\left(\frac{1 + \Delta Q_2/Q_2}{1 + \Delta Q_1/Q_1} \right)}_{<1} \right] \Rightarrow \eta' > \eta$$





CYCLE WITHOUT REGENERATION $\quad \eta = L_i / Q_1 = (h_O - h_K)/(h_O - h_L)$

CYCLE WITH REGENERATION $\quad \eta' = P_i / \dot{Q}_1 = \dfrac{(\dot{m}+\dot{m}_e)(h_O - h_E) + \dot{m}(h_E - h_K)}{(\dot{m}+\dot{m}_e)(h_O - h_{M'})}$

IN THE CONTROL VOLUME OF FEEDWATER HEATER

$$\dot{Q} = 0 \;,\; \dot{L}_{i,PUMP} = 0 \Rightarrow \dot{m} h_L + \dot{m}_e h_E = (\dot{m}+\dot{m}_e) h_{M'}$$

$$\Rightarrow \dot{m}_e (h_E - h_{M'}) = \dot{m}(h_{M'} - h_L) \Rightarrow \frac{\dot{m}_e}{\dot{m}} = \frac{h_{M'} - h_L}{h_E - h_{M'}}$$

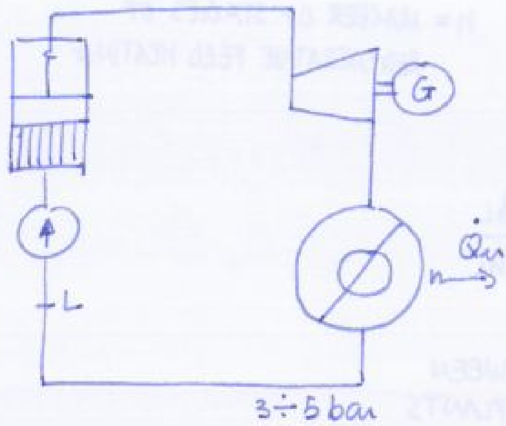
$$\Rightarrow \eta' = \frac{\dot{m}(h_O - h_K) + \dot{m}_e (h_O - h_E)}{(\dot{m}+\dot{m}_e)(h_O - h_M) + \underbrace{\dot{m}_e (h_E - h_{M'}) - \dot{m}(h_{M'} - h_L)}_{=0}}$$

$$\Rightarrow \eta' = \underbrace{\frac{h_O - h_K}{h_O - h_L}}_{\eta} \cdot \underbrace{\frac{1 + \frac{\dot{m}_e}{\dot{m}} \frac{h_O - h_E}{h_O - h_K}}{1 + \frac{\dot{m}_e}{\dot{m}} \frac{h_O - h_E}{h_O - h_L}}}_{>1} \Rightarrow \eta' > \eta$$

COGENERATION STEAM PLANTS

In a cogeneration plant, the steam produced in the steam generator is used not only to produce electric power through the turbine but also to meet process and heating requirements

BACK PRESSURE PLANT



- Pressure is increased
- I law of efficiency $\eta_{PLANT} = \dfrac{P_{PLANT} + \dot{Q}_u}{\dot{m}_f H_p}$
- $\eta = P_i/\dot{Q}_1$ Thermodynamic efficiency
- $\eta_{PLANT} = \dfrac{P_{PLANT}}{\dot{m}_f H_p - \dot{Q}_u/\eta_b}$ "economical" efficiency

$$\dot{Q}_u = \begin{cases} \text{TRADITIONAL STEAM GENERATOR} \rightarrow \dot{m}_{fu} H_p = \dot{Q}_u/\eta_b \\ \text{COGENERATION} \rightarrow \dot{m}_f H_p \quad \dot{m}_f > \dot{m}_{fu} \end{cases}$$

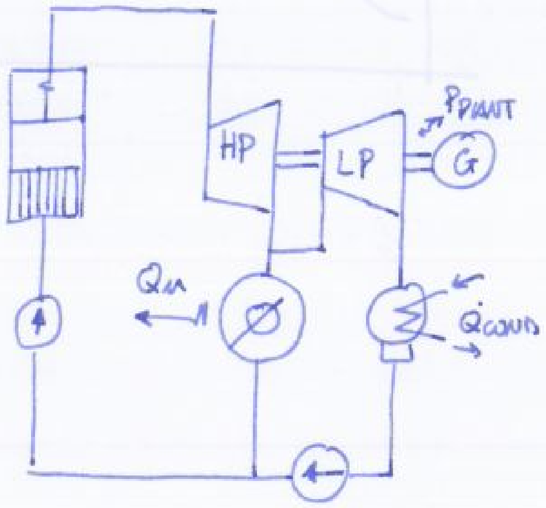
additional positive effect

$$\eta_{PLANT} = \frac{P_{PLANT}}{(\dot{m}_f - \dot{m}_{fu}) H_p} = \frac{P_{PLANT}}{\dot{m}_f H_p - \dot{Q}_u/\eta_b} \simeq \frac{\eta_m (\dot{Q}_1 - \dot{Q}_2)}{\dfrac{\dot{Q}_1}{\eta_b} - \dfrac{\dot{Q}_2}{\eta_b}} = \eta_m \cdot \eta_b \cong 0.86$$

additional expense

! ONE CANNOT DEAL SEPARATELY WITH THE MECHANICAL & THERMAL PARTS OF THE PLANT

EXTRACTION CONDENSING PLANT



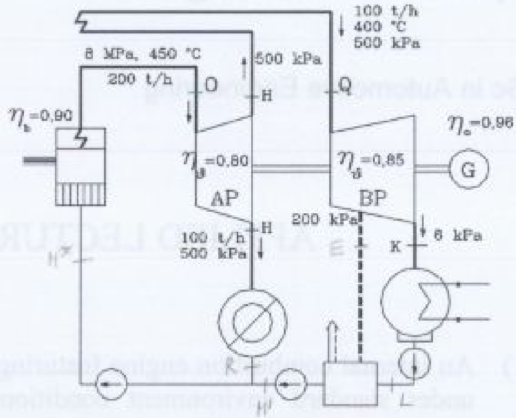
$$\dot{Q}_2 = \dot{Q}_u + \dot{Q}_{COND}$$

$$\eta_{PLANT} = \frac{P_{PLANT}}{\dot{m}_f H_p - \dot{Q}_u/\eta_b} \sim 0.6 \div 0.65$$

! MORE DOFS WITH RESPECT TO THE PREVIOUS SITUATION; THERMAL & MECHANICAL USERS ARE MORE INDEPENDENT

4) Referring to the plant of the figure, determine:
   - the plant mechanical power;
   - the overall efficiency of the plant;
   - the fuel mass flow rate (H_i = 40000 kJ/kg).

   The fluid discharged by the thermal users is saturated liquid (p = 500 kPa) and so is the fluid outflowing from the heat exchanger.



Proposed exercises:

5) A regenerative steam turbine plant features a fuel consumption of 63 t/h (H_i = 40 MJ/kg). The water mass flow rate required for the condensation process is 26650 m^3/h and the water increases its temperature of 10°C while flowing through the condenser.
   The steam generator efficiency is η_b = 0.88. The ancillaries absorb an overall power of 8 MW whereas the turbine mechanical losses amount to 2 MW.
   Determine the plant mechanical power and its overall efficiency.

Solution: P_u = 296.1 MW, η_g = 0.423

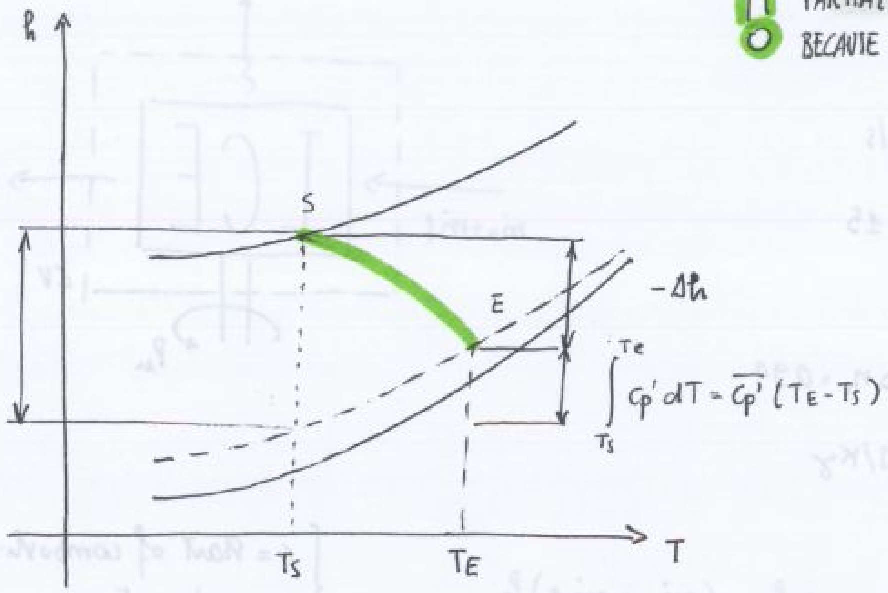
6) An energetic system is given, in which combustion of methane with air occurs with an air-to-fuel ratio α = 18.96.
   The temperature at the start of combustion is T_2 = 780 K and the heating values for methane are $H_{v,T2}$ = 48081 kJ/kg (constant volume) and $H_{p,T2}$ = 47916 kJ/kg (constant pressure).
   Determine the end-of-combustion temperature for both the constant-volume and the constant-pressure combustion, under each of the following conditions:
   a. adiabatic combustion;
   b. adiabatic combustion in the presence of dissociation, by representing the heat hidden by dissociation with $Q_{diss} = 6.5 \cdot 10^{-4}(T-1850)^2$ [kcal/kg], if necessary;
   c. combustion with heat subtraction and dissociation, should the heat power to the surroundings be given by 5% of the fuel energy at the combustion start.

(c_p = 1008.7, R = 287.1, c_p' = 1238.1, R' = 298.6 J/kg K)

Solution:
a. 3343,6 K (constant volume), 2718,6 K (constant pressure)
b. 2415,9 K (constant volume), 2291,0 K (constant pressure)
c. 2385,3 K (constant volume), 2257,2 K (constant pressure)

EVALUATION OF $h_E - h_S$

PARTIAL OXIDATION BECAUSE $\eta_c = 0.94$



GRAPHICALLY $$\eta_c \frac{H_L}{1+\alpha} = \underbrace{\frac{2.4 \, P_u}{\dot m_{EX}}}_{-\Delta h} + \overline{c_p'}(T_E - T_S)$$

$$\begin{cases} T_S = 293 \text{ K} \quad \Leftarrow \text{STANDARD } T_i \text{ FOR AIR} \\ \dot m_{EX} = \dot m_f (1+\alpha) = 0.0682 \text{ Kg/s} \end{cases}$$

$$0.94 \frac{H_L}{1+\alpha} = \frac{(1+1.4) P_u}{\dot m_{EX}} + \overline{c_p'}(T_E - 293) \Rightarrow T_E = 734.3 \, °C$$

$$\left.\begin{array}{l}5\ bar\\ sll\end{array}\right\} h_{H1} = 610.594\ KJ/Kg$$

$$\Rightarrow \frac{\delta\dot{m}}{\dot{m}} = \frac{h_{H1} - h_{L1}}{h_E - h_{L1}} = 0.1665$$

BY REPLACING $\frac{\delta\dot{m}}{\dot{m}}$ INTO (A) $\Rightarrow \dot{m} = 104.404\ Kg/s$

EVALUATE GLOBAL EFFICIENCY

$$\eta_{GLOBAL} = \frac{P_u}{\dot{m}_f \cdot H_L} = \eta_B \frac{P_u}{\dot{Q}_1}$$

1ST LAW OF TH ON STEAM GENERATOR

$$\dot{Q}_1 = \dot{m}(h_0 - h_{H1})$$

$$\Rightarrow \eta_{GLOBAL} = \eta_B \frac{P_u}{\dot{m}(h_0 - h_{H1})} = 0.305$$

$$\dot{Q}_{COND} = \dot{m}_{COND}(h_K - h_L) = 90 \cdot \frac{1000}{3600}(2520 - 151.5) = 59.21 \text{ MW}$$

$$\dot{Q}_{USER} = \dot{m}_{USER}(h_H - h_R) = 100 \frac{1000}{3600}(2760 - 590.6) = 60.26 \text{ MW}$$

$$P_i = \dot{Q}_1 - \dot{Q}_2 = 178.47 - (59.21 + 60.26) = 59 \text{ MW}$$

$$P_{PLANT} = \eta_{MECH} \cdot P_i = 56.64 \text{ MW} \qquad \text{WHERE } \eta_{MECH} = 0.96$$

$$\eta_{PLANT} = \eta_B \frac{P_{PLANT}}{\dot{Q}_1 - \dot{Q}_u} = 0.9 \cdot 0.479 = 0.431$$

$$\eta_{0,LP} = \frac{h_Q - h_K}{h_Q - h_{K,IS}} = \frac{3270 - 2520}{3270 - 2395} = 0.86$$

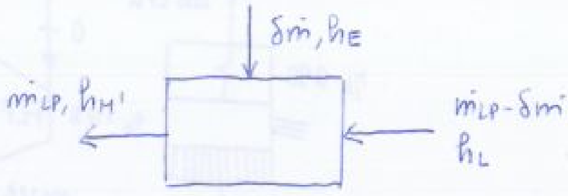
$$\eta_{0,HP} = \frac{h_J - h_H}{h_J - h_{H,IS}} = \frac{3375 - 2760}{3375 - 2655} = 0.85$$

$$h_Q = \begin{cases} 500\ \text{KPa} \\ 400°C \end{cases} \Rightarrow 3276\ \text{KJ/Kg}$$

$$h_{E,IS} \begin{cases} 200\ \text{KPa} \\ s = s_Q\ (\text{ISENTROPIC}) \end{cases} \Rightarrow 3026\ \text{KJ/Kg}$$

$$\eta_{0LP} = \frac{h_Q - h_E}{h_Q - h_{E,IS}} \Rightarrow h_E = 3064\ \text{KJ/Kg}$$

ENTHALPY BALANCE



$$(\dot{m}_{LP} - \delta\dot{m}) h_L + \delta\dot{m}\, h_E = \dot{m}_{LP}\, h_{H'}$$

$$\Rightarrow \delta\dot{m} = 12.13\ t/h$$

$$h_0 \begin{cases} 80\ \text{bar} \\ 450°C \end{cases} \Rightarrow 3276\ \text{KJ/Kg}$$

$$h_{H,IS} \begin{cases} 5\ \text{bar} \\ s = s_0 \end{cases} \Rightarrow 2642\ \text{KJ/Kg}$$

$$h_H = h_0 - \eta_{0HP}(h_0 - h_{HIS}) = 2768\ \text{KJ/Kg}$$

$$h_{K,IS} \begin{cases} 0.06\ \text{bar} \\ s = s_Q \end{cases} \Rightarrow 2408\ \text{KJ/Kg} \Rightarrow h_K = h_Q - \eta_{0LP}(h_Q - h_{KIS}) = 2538\ \text{KJ/Kg}$$

$$h_R = \begin{cases} \text{SLL} \\ 5\ \text{bar} \end{cases} \Rightarrow 640.12\ \text{KJ/Kg}$$

LET'S COMPUTE POWER

$$P_{PLANT} = \eta_M \cdot P_i = \eta_M \left[\dot{m}_{HP}(h_0 - h_H) + \dot{m}_{LP}(h_Q - h_E) + (\dot{m}_{LP} - \delta\dot{m})(h_E - h_K) \right] = 45.07\ \text{MW}$$

LET'S COMPUTE THE EFFICIENCY

$$\eta_{PLANT} = \eta_B \frac{P_{PLANT}}{\dot{Q}_1 - \dot{Q}_u}$$

$\boxed{\text{EXERCISE 4.5}}$

$\dot{m}_f = 63\ t/h = 17.5\ Kg/s$

$\eta_B = 0.88$

$P_{LOSS,T} = 2000 \cdot 10^3\ W$

$H_L = 40000\ 10^3\ J/Kg$

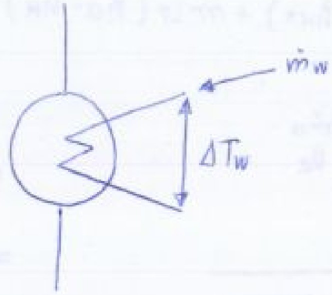
$\dot{m}_w = 26650\ m^3/h$

$P_{AUX} = 8000 \cdot 10^3\ W$

$\rho_{H_2O} = 7402.78\ Kg/s$

$\Delta T_W = 10\ K$

W = WATER OF CONDENSER



$\Delta h = C_p \Delta T_W = 4186 \cdot 10 = 41860\ J/Kg$

$\dot{Q}_2 = \dot{m}_v \Delta h = 309880.4\ KW$

$\dot{Q}_1 = \eta_B \dot{m}_f H_L = 616000\ KW$

Π IN A CYCLE $|\dot{Q}| = |\dot{L}|$

$|\dot{Q}| = \dot{Q}_1 - \dot{Q}_2 = 306119.6\ KW$

$|\dot{L}| = P_u + P_{AUX} + P_{LOSS,T} = 306119 - 8000 - 2000 = 296119\ KW$

BEFORE LOSSES $= |\dot{Q}|$

$\eta_{TOT} = \eta_B \cdot \eta \cdot \eta_{MECH} = 0.88 \cdot \frac{\dot{L}}{\dot{Q}_1} \cdot \frac{P_u}{\dot{L}} = 0.88 \frac{296.1}{616} = 0.423$

$\eta_{TOT} = \frac{P_u}{\dot{m}_f H_L} = \frac{296119.6}{700000} = 0.423$

b) ADIABATIC + DISSOCIATION WITH $q_{DISS} = [6.5(\underset{T_{FIN}}{T} - \underset{T_{th}}{1850})^2]\ 4186$ J/Kg

CONSTANT VOLUME

$$\frac{H_{UT2}}{\alpha+1} = q_{DISS} + C_v'(T_{FIN} - T_{IN}) \Rightarrow 2408867.7 = 2.72\,(T_{FIN}^2 - 3700\,T_{FIN} + 3422500) + 937.5\,T_{FIN} - 732810$$

$$\Rightarrow 2.72\,T_{FIN}^2 - 9124.5\,T_{FIN} + 6167522 = 0 \ ;\ \Delta = 16153860 \Rightarrow T_{FIN} < \begin{matrix} 2416.1\ K \leftarrow \\ 938.5\ K \end{matrix}$$

□ BETWEEN THE 2 POSSIBLE SOLUTIONS, CHOOSE THE ONE ABOVE THRESHOLD T_{th} ; IF
○ NO SOLUTIONS ABOVE ⇒ NO DISSOCIATION

CONSTANT PRESSURE

$$\frac{H_{PT2}}{\alpha+1} = q_{DISS} + C_p'(T_{FIN} - T_{IN}) \Rightarrow T_{FIN} < \begin{matrix} 953.8\ K \\ 2291.2\ K \leftarrow \end{matrix}$$

c) DISSOCIATION AND 5% OF FUEL ENERGY SUBTRACTED AS HEAT $(|q|)$
AT COMBUSTION START

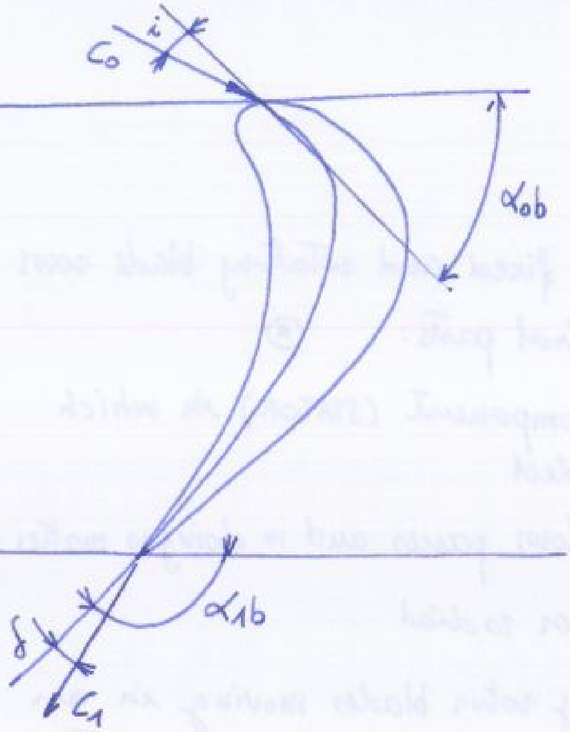
$$|q| = \frac{5}{100}\,\frac{H_{UT1}}{\alpha+1} = 120443\ J/Kg$$

$$\frac{H_{UT2}}{\alpha+1} = q_{DISS}' + C_v'(T_{FIN} - T_{IN}) + |q| \Rightarrow T_{FIN} < \begin{matrix} 969.3\ K \\ 2385.5\ K \end{matrix}$$

CONSTANT PRESSURE

$$|q| = \frac{5}{100}\,\frac{H_{PT2}}{1+\alpha} = 120030\ J/Kg$$

$$\frac{H_{PT2}}{\alpha+1} = q_{DISS} + C_p'(T_{FIN} - T_{IN}) + |q| \Rightarrow T_{FIN} < \begin{matrix} 987.6\ K \\ 2257.3\ K \end{matrix}$$



STATOR BLADES ANGLES

$\theta = \alpha_{b0} - \alpha_{b1}$ CAMBER ANGLE

$\varepsilon = \alpha_0 - \alpha_1$ DEFLECTION ANGLE

$i = \alpha_0 - \alpha_{b0}$ INCIDENCE ANGLE

$\delta = \alpha_1 - \alpha_{b1}$ DEVIATION ANGLE

ROTOR BLADES ANGLES

$\theta = \beta_{b1} - \beta_{b2}$

$\varepsilon = \beta_1 - \beta_2$

$i = \beta_1 - \beta_{b1}$

$\delta = \beta_2 - \beta_{b2}$

Within 1-D approach, the usual approach is to set

$\delta = 0$ for both stator & rotor blades (all operating conditions)

$i = 0$ for both stator & rotor blades (design conditions only)

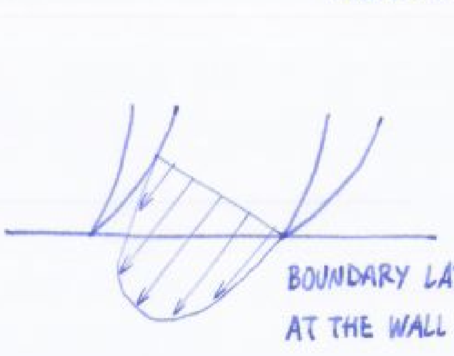
$\varepsilon = \theta$ (1D flow and in design conditions)

! IN DESIGN CONDITION FLOW LOSSES HAVE TO BE TAKEN INTO ACCOUNT.

AERODYNAMIC LOSSES

- PROFILE LOSSES: due to skin friction on the blade surface (VISCOSITY EFFECT) and to impingement losses
- ANNULUS LOSSES: caused by friction on the endwall surface
- SECONDARY FLOWS: are vortices that occur as a result of boundary layers and the curvature of the passage, and cause some parts of the fluid to move in directions other than the principal direction of the flow

! THERE ARE ALSO LOSSES DUE TO THE DISCHARGE OF THE KINETIC ENERGY



BOUNDARY LAYER AT THE WALL $C=0$