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A P P U N T I

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MATERIA: Fundamentals Of Electrical And Electronic System

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CIRCUIT ANALYSIS: GENERALITIES

- ANALYSIS: COMPUTATION OF ALL VOLTAGES AND CURRENTS
  $\Rightarrow$ WRITE CIRCUIT EQUATIONS AND SOLVE

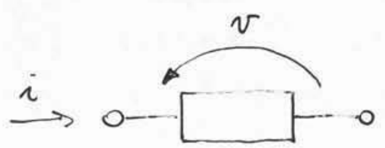
- CIRCUIT EQUATIONS:
  - VOLTAGE & CURRENT EQUATIONS: CIRCUIT TOPOLOGY (LINEAR MEMORYLESS)
  - CONSTITUTIVE RELATIONS: ELEMENT BEHAVIORS (ANY TYPE)

- ANALYSIS METHODS
  - ELEMENTAL: CIRCUIT TRANSFORMATIONS & KNOWN SOLUTIONS FOR SIMPLE TOPOLOGIES
  - GENERAL: VIA EXPEDIENT FORMS OF CIRCUIT EQUATIONS (WORK FOR ANY CIRCUIT)

NODAL ANALYSIS METHOD

CONSTRAINTS

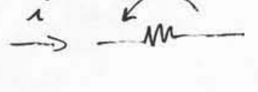
- CONNECTED NETWORK (NO SEPARATE PARTS)
- NATURAL CONDITION: NETWORK ELEMENTS MUST BE VOLTAGE CONTROLLED (NO SHORTS & VOLTAGE SOURCES BETWEEN NODES); IN OTHER WORDS,



2 TERMINAL ELEMENTS: CURRENT i CAN BE EXPRESSED THROUGH VOLTAGE OF THE ELEMENT

$i = F(v) \Longleftrightarrow$ VOLTAGE CONTROLLED

RESISTOR



$i = G \cdot v$ WHERE $G = \frac{1}{R}$ CONDUCTANCE

VOLTAGE CONTROLLED

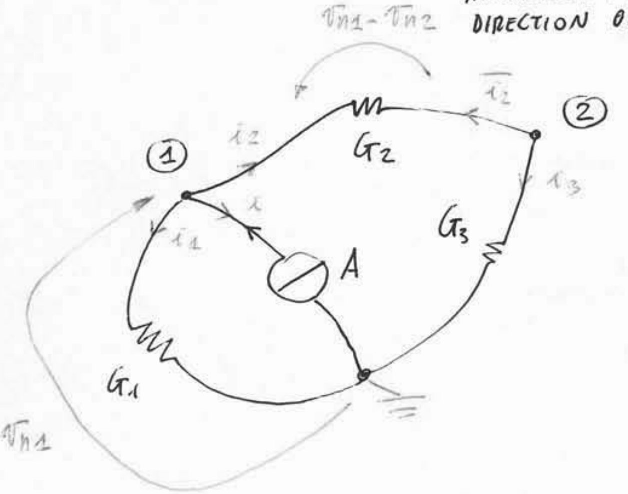
EQUATIONS

NODAL EQUATIONS OF $n-1$ NODES THAT DEFINES THE $n-1$ VOLTAGES

LET'S CONSIDER THIS EXAMPLE

HOW TO WRITE THE NODAL EQUATIONS

ASSOCIATED REFERENCE DIRECTION OF i_2



1) SELECT THE REFERENCE NODE & LABEL THE OTHER $n-1$ NODES FROM 1

2) WRITE NODAL EQUATION FOR EACH OF THE $n-1$ NODES (IN THIS CASE, ① AND ②)

FOR EVERY EQUATION, ASSUME POSITIVE THE CURRENTS EXITING THE NODE, CURRENTS ARE DEFINED AS FLOWING OUT FROM THE NODE

!

① $i_1 + i_2 + i = 0$

② $\overline{i_2} + i_3 = 0$

TO COMPLETE THIS STEP, EXPRESS EACH CURRENT FOR THE NODAL VOLTAGE

!

① $G_1 v_{n1} + G_2 (v_{n1} - v_{n2}) - A = 0$

② $G_2 (v_{n2} - v_{n1}) + G_3 v_{n2} = 0$

$$U_n = \begin{bmatrix} v_{n1} \\ v_{n2} \end{bmatrix}$$

VECTOR OF UNKNOWNS

LET'S USE INVERSE OF CONDUCTANCE MATRIX METHOD TO SOLVE ~~OUR~~ THE PREVIOUS PROBLEM

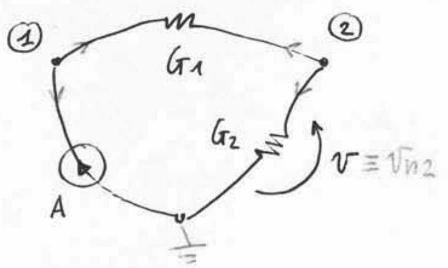
$$\begin{bmatrix} v_{n1} \\ v_{n2} \end{bmatrix} = \frac{1}{(G_1+G_2)(G_2+G_3)-G_2^2} \begin{bmatrix} G_2+G_3 & G_2 \\ G_2 & G_1+G_2 \end{bmatrix} \begin{bmatrix} A \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} v_{n1} \\ v_{n2} \end{bmatrix} = \frac{1}{det} \begin{bmatrix} G_2+G_3 \\ G_2 \end{bmatrix} A$$

LET'S CHECK THE RESULT BY USING CRAMER RULE, SUPPOSE ONE IS INTERESTED TO COMPUTE v_{n2}

$$v_{n2} = \frac{det \begin{bmatrix} G_1+G_2 & A \\ -G_2 & 0 \end{bmatrix}}{det \begin{bmatrix} G_1+G_2 & -G_2 \\ -G_2 & G_2+G_3 \end{bmatrix}} = \frac{G_2 A}{det}$$ AS BEFORE

$\boxed{\text{SAMPLE 2}}$



WRITE THE EQUATION FOR NODAL VOLTAGES AND COMPUTE v

① $G_1(v_{n1}-v_{n2}) - A = 0$

② $G_2 v_{n2} + G_1(v_{n2}-v_{n1}) = 0$

$$\begin{bmatrix} G_1 & -G_1 \\ -G_1 & G_1+G_2 \end{bmatrix} \begin{bmatrix} v_{n1} \\ v_{n2} \end{bmatrix} = \begin{bmatrix} A \\ 0 \end{bmatrix}$$

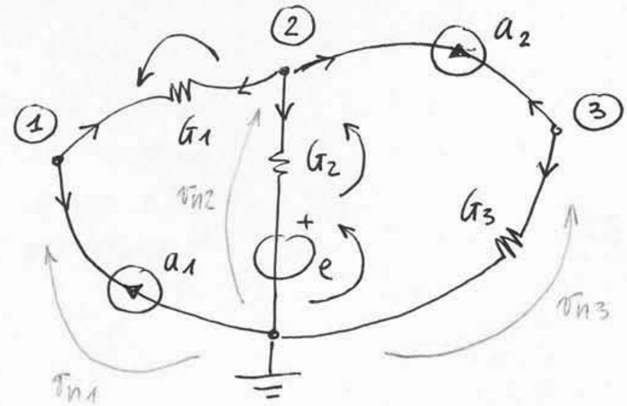
APPLYING CRAMER RULE

$$v \equiv v_{n2} = \frac{G_1 A}{G_1(G_1+G_2)-G_1^2} = R_2 A$$

THE OBTAINED RESULT IS EASY TO BE CHECKED; THE INJECTED CURRENT (CURRENT SOURCE) IS ALSO THE CURRENT FLOWING THROUGH G_1 & G_2 (1 BRANCH)

PROBLEM 2

COMPUTE NODAL VOLTAGES OBTAINED THROUGH NODAL ANALYSIS METHOD



① $-a_1 + G_1(v_{n1} - v_{n2}) = 0$

② $G_1(v_{n2} - v_{n1}) + G_2(v_{n2} - e) + a_2 = 0$

③ $-a_2 + G_3 v_{n3}$

$$\begin{bmatrix} G_1 & -G_1 & 0 \\ -G_1 & G_1+G_2 & 0 \\ 0 & 0 & G_3 \end{bmatrix} \begin{bmatrix} v_{n1} \\ v_{n2} \\ v_{n3} \end{bmatrix} = \begin{bmatrix} a_1 \\ G_2 e - a_2 \\ +a_2 \end{bmatrix}$$

LOOP EQUATION $v_{n1} - v_{n2} = v_{12}$

REPLACE NUMBERS

$$\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{n1} \\ v_{n2} \\ v_{n3} \end{bmatrix} = \begin{bmatrix} 1 \\ e-1 \\ 1 \end{bmatrix}$$

$r_2 \to r_2 + r_1$

$$\begin{bmatrix} 2 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{n1} \\ v_{n2} \\ v_{n3} \end{bmatrix} = \begin{bmatrix} 1 \\ e \\ 1 \end{bmatrix} \Rightarrow$$

$v_{n3} = 1V$

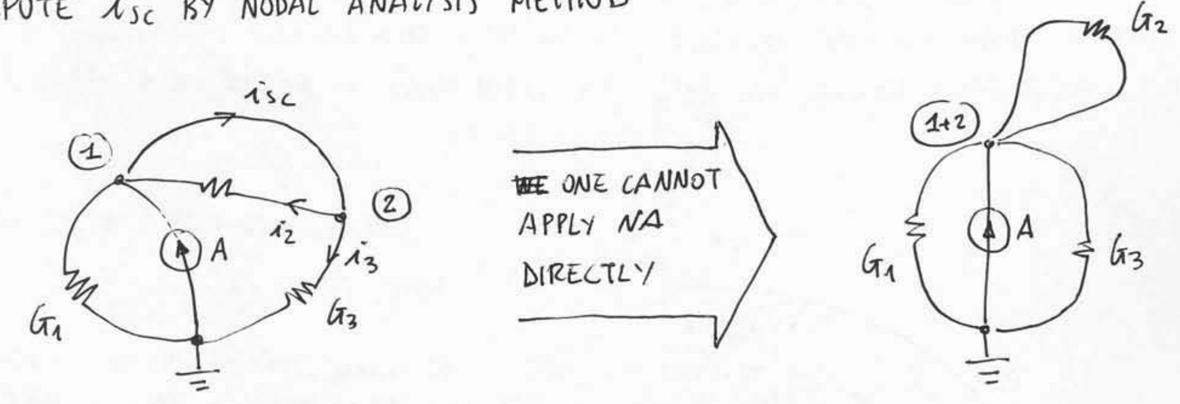
$v_{n2} = e(t)\ V$

$v_{n1} = (1 + 2e(t))\frac{1}{2}\ V$

SAMPLE

1 TERMINAL ELEMENT DOES NOT PLAY ANY ROLE !

- COMPUTE i_{sc} BY NODAL ANALYSIS METHOD



$$G_1 v_{n1} + G_3 v_{n1} = A \Rightarrow v_{n1} = \frac{A}{G_1 + G_3}$$

- IN THE SIMPLIFIED CIRCUIT, ONE CAN COMPUTE i_3 AND i_2, THAT ARE THE SAME FLOWING IN THE ORIGINAL ONE

$$i_3 = G_3 \cdot \frac{1}{G_1 + G_3} A$$

$i_2 = 0$ WHY? BECAUSE IT IS 1 TERMINAL ~~EAN~~ ELEMENT? NOT ENOUGH TO SAY !

$i_2 = 0$ BECAUSE THE VOLTAGE ACROSS THE RESISTOR 2 IS ZERO BY DEFINITION $\Rightarrow$ 0 VOLTAGE = 0 CURRENT !

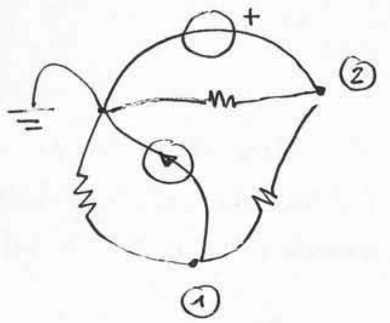
- LET'S COME BACK TO ORIGINAL CIRCUIT

$$i_{sc} = i_3 + \cancel{i_2}^{=0} \Rightarrow \boxed{i_{sc} = i_3}$$

(1+2) $G_1 v_{n1} - A + G_3 (v_{n1} + e) = 0$

$(G_1 + G_3)\, v_{n1} = A - G_3 e$

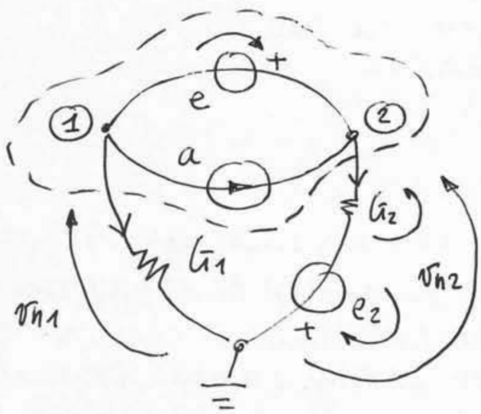
APPENDIX



VOLTAGE SOURCE NO LONGER INVOLVED, EQUATIONS ARE SIMPLER !

PROBLEM 2

COMPUTE NODAL VOLTAGES DEFINED BY LABELS



$v_{n2} = v_{n1} + e$

$G_1 v_{n1} + G_2 (v_{n2} + e_2) = 0$

$G_1 v_{n1} + G_2 (v_{n1} + e + e_2) = 0$

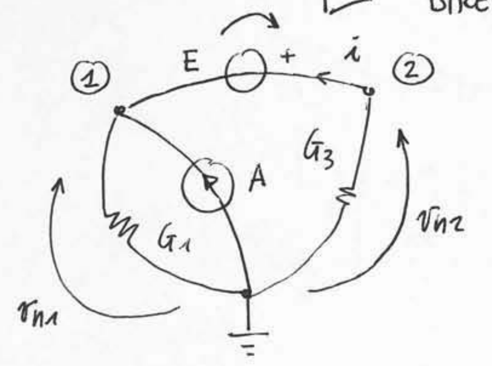
$v_{n1} (G_1 + G_2) = - G_2 (e + e_2)$

$$v_{n1} = - \frac{G_2}{G_1 + G_2} (e + e_2)$$

$$v_{n2} = - \frac{G_2}{G_1 + G_2} (e_2 + e) + e$$

EXAMPLE

ASSOCIATED REFERENCE DIRECTION



ACCORDING TO MNA RULES
UNKNOWNS : $\{v_{n1}, v_{n2}, i\}$

VECTOR OF UNKNOWNS $\begin{bmatrix} v_{n1} \\ v_{n2} \\ i \end{bmatrix}$

① $G_1 v_{n1} - A - i = 0$ WHERE i IS ONE OF THE UNKNOWNS, NO FUNCTION OF NODAL VOLTAGE

② $i + G_3 v_{n2} = 0$

$v_{n2} - v_{n1} = E$ CHARACTERISTIC OF IDEAL VOLTAGE SOURCE

LET'S MOVE TO MATRIX FORM

$$\begin{bmatrix} G_1 & 0 & -1 \\ 0 & G_3 & +1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_{n1} \\ v_{n2} \\ i \end{bmatrix} = \begin{bmatrix} A \\ 0 \\ E \end{bmatrix}$$

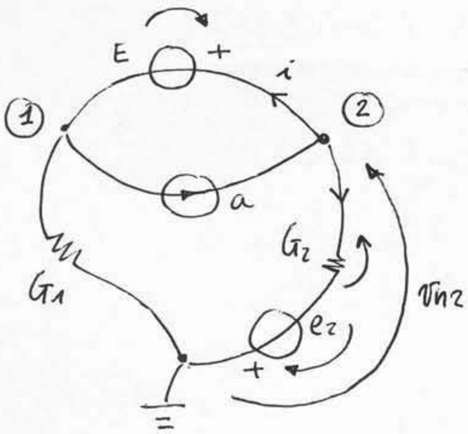
MODIFIED NODAL ANALYSIS IS CONVENIENT

1) FOR NUMERICAL SOLUTION
2) WHERE REQUESTED
3) WHEN ONE WANTS TO PROOVE GENERAL PROPERTIES

IN ALL OTHER CASES, IT IS BETTER TO USE THE OTHER METHOD

PROBLEM 1

WRITE THE MATRIX EQUATION OF THE MNA METHOD



LOOP EQUATION

$v_{G2} = v_{n2} + e_2$

$W^T = [v_{n1}, v_{n2}, i]$

(1) $G_1 v_{n1} + a - i = 0$

(2) $+ i - a + G_2(v_{n2} + e_2) = 0$

(3) $v_{n2} - v_{n1} = e$

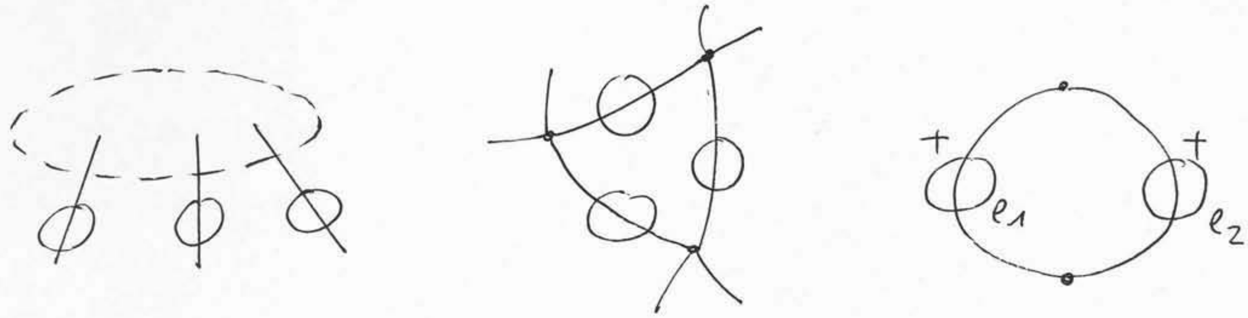
$$G_1 v_{n1} + 0\, v_{n2} + (-1)\, i = -a$$
$$0\, v_{n1} + G_2 v_{n2} + 1\, i = a - G_2 e_2$$
$$-1\, v_{n1} + 1\, v_{n2} + 0\, i = E$$

$$\begin{bmatrix} G_1 & 0 & -1 \\ 0 & G_2 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_{n1} \\ v_{n2} \\ i \end{bmatrix} = \begin{bmatrix} -a \\ a - G_2 e_2 \\ E \end{bmatrix}$$

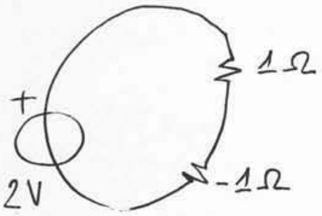
THEOREM 1

- CIRCUITS WITH LOOPS (CUT-SETS) COMPOSED OF VOLTAGE (CURRENTS) SOURCES HAVE EITHER NO OR INFINITELY MANY SOLUTIONS

NECESSARY CONDITION



- NO POSSIBLE INTERPRETATION, MEANINGLESS FOR CIRCUIT POINT OF VIEW
CIRCUIT IS MODEL OF NO PHYSICAL SYSTEM IN THESE CASES



NONE OF THOSE THOPOLOGICAL ERRORS
BUT THIS CIRCUIT HAS AGAIN NO SOLUTIONS
$\Longrightarrow$ THEOREM 1 JUST NECESSARY CONDITION

!

- ## THEOREM 2

CIRCUITS COMPOSED OF RESISTORS WITH POSITIVE RESISTANCE AND INDEPENDENT SOURCES WITHOUT LOOPS OF VOLTAGE SOURCES AND CUT-SET OF CURRENT SOURCES HAVE ONE SOLUTION

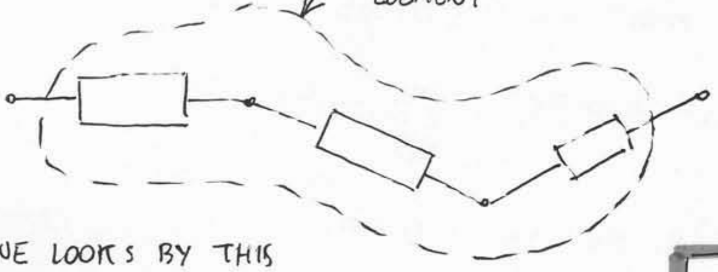
SUFFICIENT CONDITION

WITHOUT
- FORBIDDEN TOPOLOGY
- NEGATIVE RESISTORS

- ONLY NECESSARY & SUFFICIENT CONDITION IS INVOLVING ?

SERIES AND PARALLEL CONNECTIONS

2-TERMINAL ELEMENTS ARE CONNECTED IN **SERIES** IFF THEY ONLY FORM A SINGLE PATH SO THAT THE SAME CURRENT FLOWS THRU EACH ELEMENT

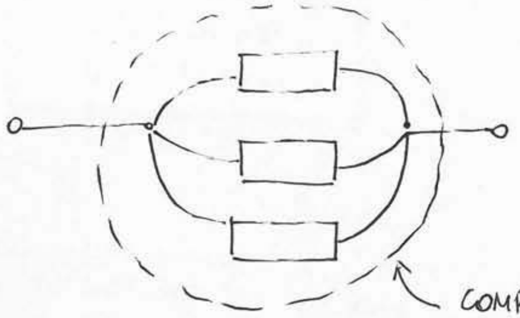


TOPOLOGY: TERMINALS OF ELEMENTS MUST BE JOINED, 1 DIMENSIONAL CHAIN; CURRENT FLOW IS THE SAME FOR ALL ELEMENTS OF THE CONNECTION

IF ONE LOOKS BY THIS CONNECTION, WHAT HE GETS IS A NEW ELEMENT, COMPOSED 2 TERMINAL ELEMENT

> DUALITY
>
> SERIES CONNECTION ⇄ PARALLEL CONNECTION (SHUNT CONNECTION)

2-TERMINAL ELEMENTS ARE CONNECTED IN **PARALLEL** IFF THEIR TERMINALS ARE CONNECTED TO THE SAME TWO NODES SO THAT THE SAME VOLTAGE IS APPLIED TO EACH ELEMENT



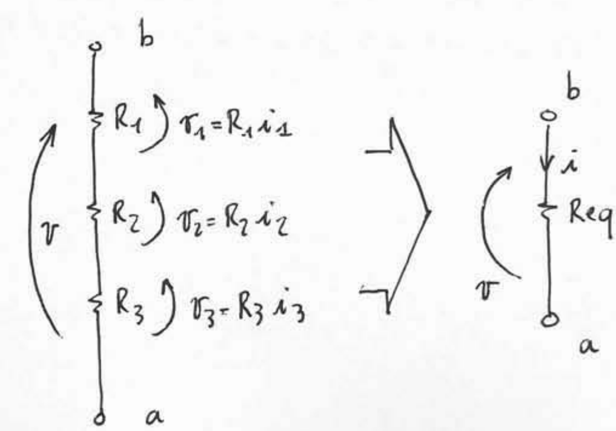
EACH ELEMENT HAS THE SAME VOLTAGE ACROSS BECAUSE THE TERMINALS ARE CONNECTED TO THE SAME 2 NODES

RESISTORS IN SERIES

TO DESCRIBE THE BEHAVIOR OF NEW ELEMENTS, THE CHARACTERISTIC IS NEEDED; IN THIS CASE, $\boxed{v = v(i)}$

LET'S TRANSFORM THE 3 RESISTORS AS A NEW 2-TERMINAL ELEMENTS

POWERFUL TOOL TO SIMPLIFY CIRCUITS



CURRENT i IS THE SUM OF THE 3 CURRENTS FLOWING THROUGH THE RESISTORS

BY DEFINITION OF PARALLEL CONNECTION, VOLTAGE ACROSS IS THE SAME OF THE COMPOSITE ELEMENT

KCE

$$\begin{cases} i = i_1 + i_2 + i_3 = G_1 v_1 + G_2 v_2 + G_3 v_3 \\ v = v_1 = v_2 = v_3 \end{cases}$$

$$\boxed{i = (G_1 + G_2 + G_3)\, v = G_{eq}\, v = 1/R_{eq}\, v}$$

THE COMPOSITE ELEMENT CAN BE REPLACED BY A SINGLE RESISTOR WHOSE CONDUCTANCE IS THE SUM OF CONDUCTANCES !

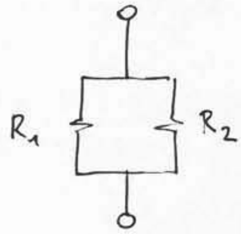
$$G_{EQ} = \sum_K G_K = \sum_K \frac{1}{R_K}$$

DUALITY

SERIES CONNECTION RESISTANCE $\leftrightarrow$ PARALLEL CONNECTION RESISTANCE

$R_{EQ} = \sum R_K$ $\qquad$ $G_{EQ} = \sum G_K$

SPECIAL CASE OF 2 RESISTORS



$$G_{EQ} = G_1 + G_2 = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\boxed{R_{EQ} = \frac{R_1 R_2}{R_1 + R_2}}$$

USEFUL PROPERTIES

IF ONE HAS A SERIES CONNECTION OF RESISTORS WHOSE RESISTANCES R_K ARE ALL POSITIVE RESISTANCES, THE EQUIVALENT RESISTANCE IS FOR SURE GREATER THAN THE RESISTANCE OF EACH COMPONENT

SERIES CONNECTION $\qquad$ $\{R_K\}$ $R_K \geq 0$ $\forall K$ $\quad R_{EQ} \geq R_K$ $\forall K$

DUALITY

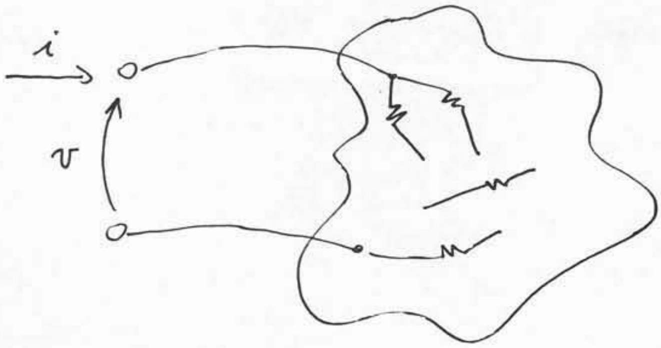
PARALLEL CONNECTION $\qquad$ $\{G_K\}$ $G_K \geq 0$ $\forall K$ $\quad G_{EQ} \geq G_K$ $\forall K$

LET'S DEAL WITH PARALLEL CONNECTIONS NOW

$$R_{EQ} = \frac{1}{G_{EQ}} \; ; \quad \frac{1}{G_{EQ}} \leq \frac{1}{G_K}$$

THAT MEANS $\boxed{R_{EQ} \leq R_K \;\; \forall K}$ PARALLEL CONNECTION

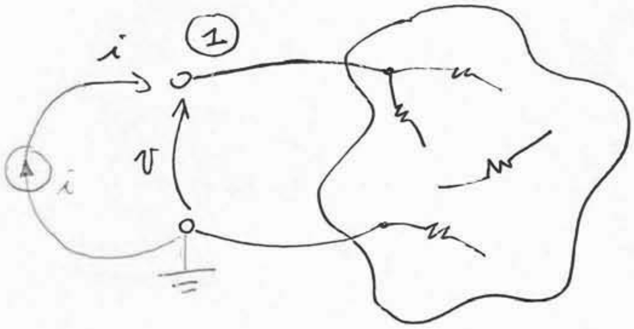
LET'S CONSIDER THIS PROBLEM



WHAT IS THE RELATION BETWEEN v AND i?
FOR WRE $v = R_{EQ} \cdot i$

ANY NETWORK COMPOSED BY RESISTORS BEHAVES LIKE THE REQUIVALENT RESISTOR

LET'S PROOVE IT BY USING N.A. METHOD



$$\mathbb{P} \cdot \mathbb{W} = \mathbb{U}$$

$$\mathbb{P} \begin{bmatrix} v_{n1} \\ \vdots \\ \cdot \end{bmatrix} = \begin{bmatrix} i \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

$$\begin{bmatrix} v_{n1} \\ \vdots \\ \cdot \end{bmatrix} = \mathbb{P}^{-1} \begin{bmatrix} i \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

ONE IS INTERESTED IN v_{n1}

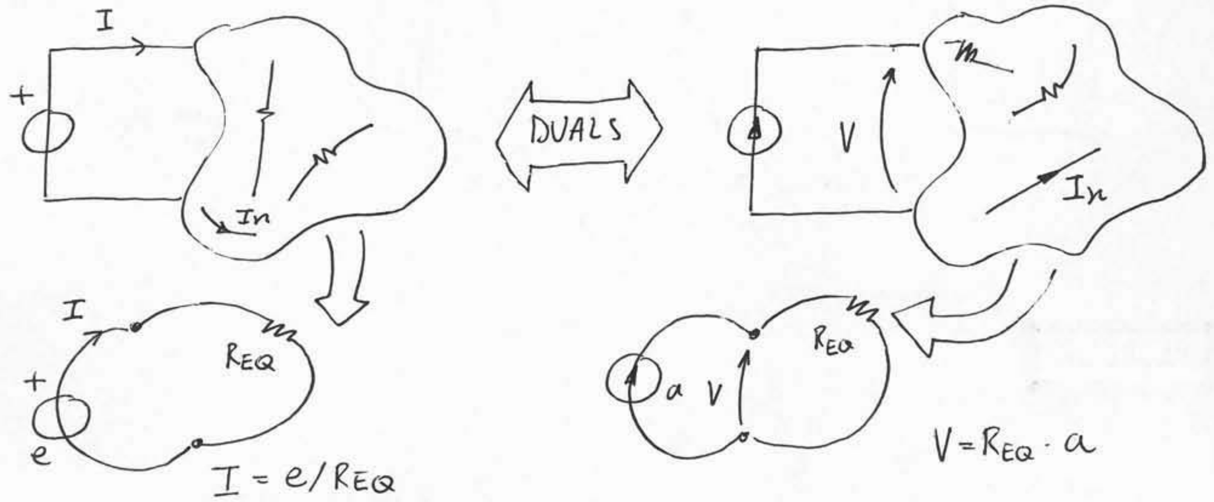
$v_{n1} = v = (\mathbb{P}^{-1})_{1i} \cdot i$ WHERE

v = VOLTAGE ACROSS THE ELEMENT

$(\mathbb{P}^{-1})_{1i}$ = EQUIVALENT RESISTANCE

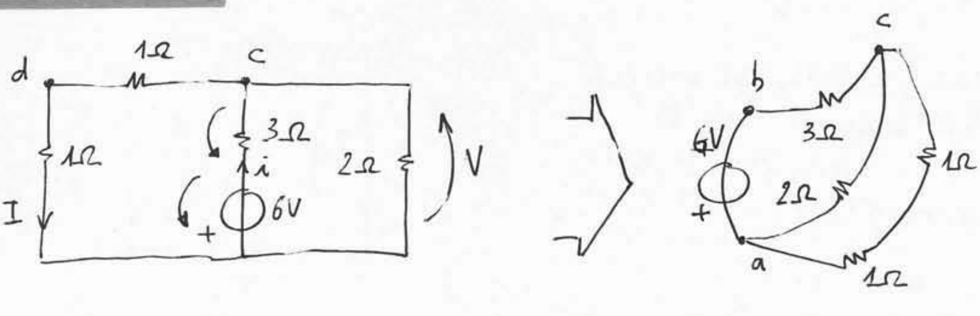
ANALYSIS OF CIRCUITS WITH 1-SOURCE ONLY

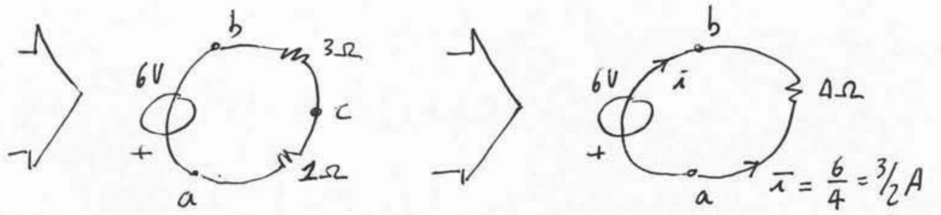
HOW TO COMPUTE THE EXTERNAL (I AND V) OR INTERNAL (I_x) UNKNOWN VARIABLES?



I, V ARE THE ONLY TERMS DEFINED IN BOTH ORIGINAL CIRCUIT AND SIMPLIFIED ONE; IF ONE IS INTERESTED IN INTERNAL QUANTITIES, HE HAS TO STEP BACK TO THE ORIGINAL CIRCUIT

EXAMPLE 1





$\Rightarrow i = -\frac{3}{2} A$

$V_{3\Omega} = 3\Omega \cdot \left(-\frac{3}{2}A\right) = -\frac{9}{2} V$, $V = -6 - \left(-\frac{9}{2}V\right)$ LOOP EQUATION

$G_{EQ} = 1/(1+1)$

$I = V \cdot \frac{1}{2} \Rightarrow I = \left(-6 + \frac{9}{2}\right) V \cdot \frac{1}{2} S = -\frac{3}{4} A$

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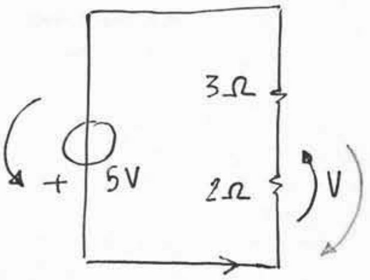
SAMPLE 1

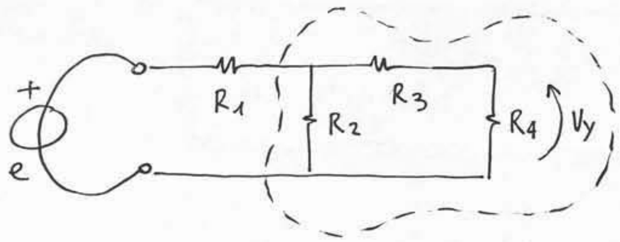
PAY ATTENTION ON SIGNS



$\overline{V} = \frac{2}{3+2} 5 = 2V$ ASSOCIATED REFERENCE DIRECTION $\Rightarrow$ $V = -2V$

SAMPLE 2

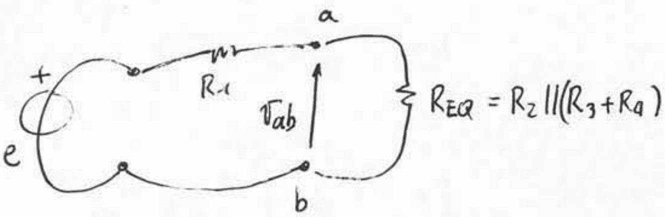
PAY ATTENTION ON TOPOLOGY



! VOLTAGE DIVIDER HOLD ONLY FOR SERIES CONNECTIONS, WE MUST NOT APPLY TO EVERYTHING THAT IS NOT SERIES CONNECTED

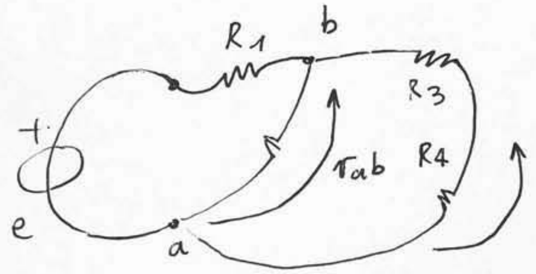
R_1 SERIES CONNECTED TO THE COMPOSITE 2-TERMINAL ELEMENTS

CURRENT FLOWING THROUGH $R_1 \neq$ CURRENT FLOWING TRHOUGH R_2



ACCORDING TO VOLTAGE DIVIDER RULE $\quad V_{ab} = e \frac{R_{EQ}}{R_{EQ} + R_1}$

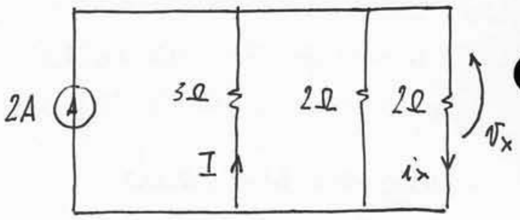
SINCE THE 2 CIRCUITS ARE EQUIVALENT, THE QUANTITIES COMPUTED HAVE THE SAME VALUE IN BOTH THE CIRCUITS



$V_y = V_{ab} \frac{R_4}{R_3 + R_4}$

SAMPLE 1

COMPUTE THE CURRENT I and i_x and THE VOLTAGE v_x



$$i_x = 2A \cdot \frac{1/2}{1/3 + 1/2 + 1/2}$$

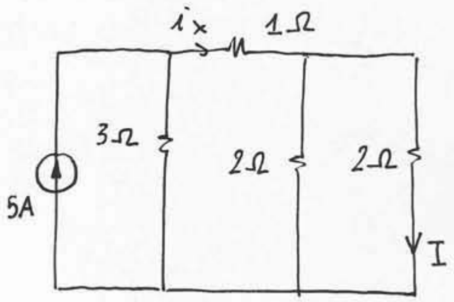
PAY ATTENTION TO THE SIGNS

$$v_x = 2\Omega \cdot i_x$$

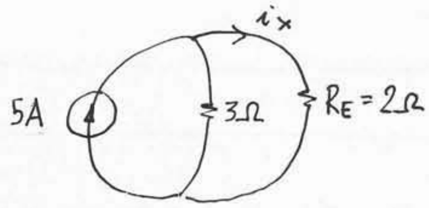
$$v_x = -3\Omega \cdot I \Rightarrow I = v_x / -3$$

SAMPLE 2

COMPUTE CURRENT i_x and I



LET'S COMPUTE THE EQUIVALENT RESISTANCE OF THE SERIES CONNECTION BETWEEN R_1 AND $R_2 \| R_2$; NOW, ONE IS ABLE TO COMPUTE i_x (ONLY PARALLEL CONNECTIONS)



$$i_x = 5 \cdot \frac{3}{3+2} = 3A$$

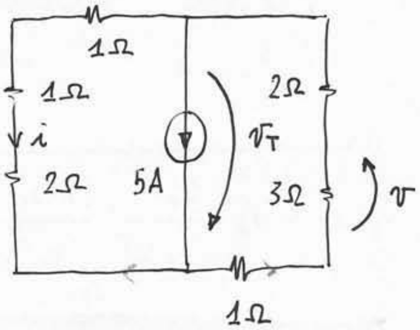
PAY ATTENTION TO THE TOPOLOGY

$$I = i_x \cdot \frac{1}{2} = \frac{3}{2} A$$

PROBLEM 2

DETERMINE i AND v VIA EITHER CURRENT OR THE VOLTAGE DIVISION RULE

$$R_{EQ} = (1+3+2) \| (1+1+2) = 12/5\ \Omega$$



$v_T = \frac{12}{5} \cdot 5 = 12\ V$; $v = -\cancel{3}(-3 \cdot 12)/(1+3+2) = -6\ V$

$$i = -5 \cdot \frac{G_1}{G_E} = -5 \cdot \frac{1/4}{5/12} = -3A$$

PAY ATTENTION TO THE SIGNS

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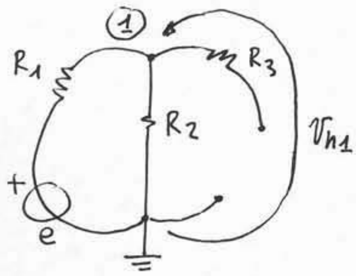
e EFFECT AND a EFFECT CAN BE COMPUTED INDEPENDENTLY BY SETTING ONE OR THE OTHER ONE EQUAL TO ZERO !

e-EFFECT $v_{n1}\big|_{a=0}$

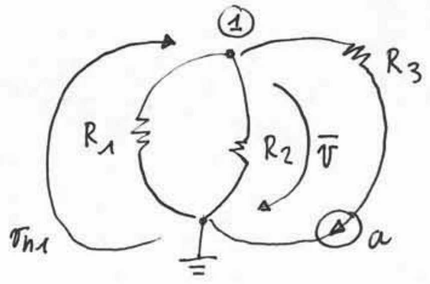


$a = 0 \iff$ OPEN CIRCUIT

LET'S APPLY VOLTAGE DIVIDER RULE

$$v_{n1}\Big|_{a=0} = \frac{R_2}{R_1+R_2}\, e$$

a-EFFECT $v_{n1}\big|_{e=0}$



$e = 0 \iff$ SHORT CIRCUIT

$$\bar{v} = \frac{R_1 R_2}{R_1+R_2}\, a$$

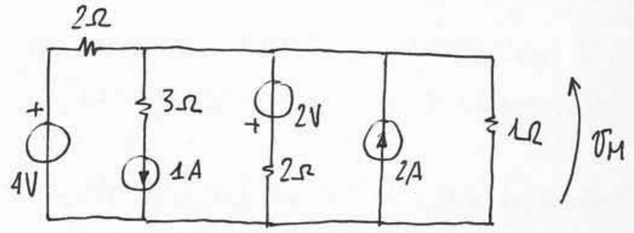
$$v_{n1}\Big|_{e=0} = -\frac{R_1 R_2}{R_1+R_2}\, a$$

$$\boxed{v_{n1} = \frac{R_2}{R_1+R_2}\, e - \frac{R_1 R_2}{R_1+R_2}\, a}$$

NOT EFFECTIVE METHOD !

MILLMAN'S THEOREM, CONT'D

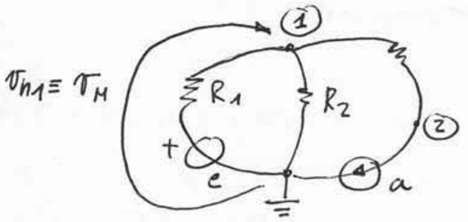
$$v_M = \frac{[A]+[B]}{[C]} = \frac{\left(\frac{4V}{2\Omega} - \frac{2V}{2\Omega}\right) + (2A - 1A)}{\left(\frac{1}{2\Omega} + \frac{1}{2\Omega} + \frac{1}{1\Omega}\right)} = 1V$$



THE ELEMENTS CONNECTED IN SERIES TO A CURRENT SOURCE HAVE NO EFFECTS ON V_M !

SAMPLE 2

$$v_{n1} = \frac{G_1 e - a}{G_1 + G_2} \quad \text{MILLMAN}$$

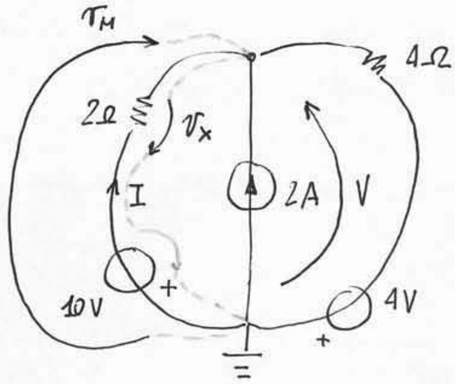


IT IS FASTER THAN SUPERPOSITION METHOD AND IT LEAS TO THE SAME RESULT

PROBLEM 1

DETERMINE VOLTAGE V & CURRENT i

$$v_M = \frac{-1/2 \cdot 10 - 1/4 \cdot 4 + 2}{1/2 + 1/4} = -\frac{16}{3}\ V$$



LOOP EQUATION

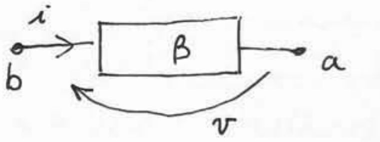
$$v_M + v_x + 10 = 0$$

$$\Rightarrow v_x = -10 + \frac{16}{3} = -\frac{14}{3}\ V$$

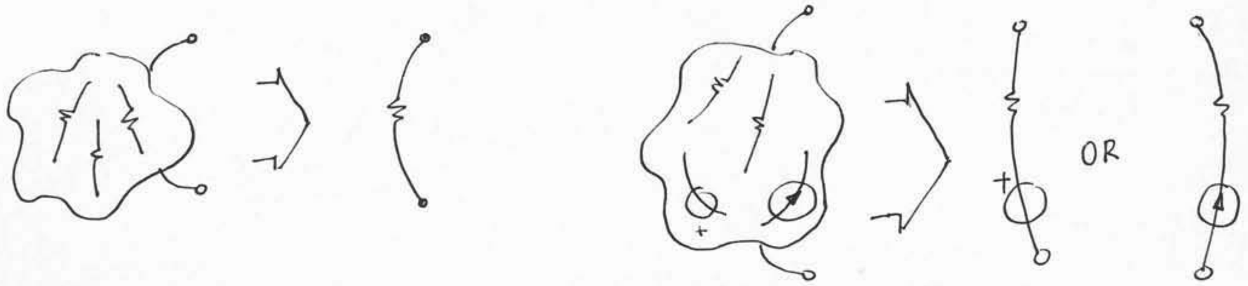
$$\Rightarrow I = \left(-\frac{14}{3}\right) \cdot \frac{1}{2} = -\frac{7}{3}\ A$$

AFTER COMPUTING THE MILLMAN'S VOLTAGE, THE OTHER UNKNOWNS ARE SIMPLY COMPUTED !

THEVENIN'S THEOREM



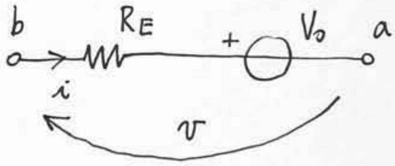
IT IS A GENERALIZATION OF EQUIVALENT RESISTANCE



ANY WELL-DEFINED (IT DOES NOT CONTAIN CIRCUIT ELEMENTS THAT ARE COUPLED TO SOME PHYSICAL VARIABLES OUTSIDE), 2-TERMINAL CIRCUIT ELEMENT β THAT IS COMPOSED BY LINEAR, TIME-INVARIANT, RESISTIVE ELEMENTS AND THAT LEADS TO A ONE-SOLUTION CIRCUIT WHEN DRIVEN BY AN IDEAL CURRENT SOURCE

1) HAS THE FOLLOWING CHARACTERISTIC $\boxed{v = R_E\, i + V_0}$

2) IS EQUIVALENT TO THE FOLLOWING 2-TERMINAL ELEMENT (THEVENIN EQUIVALENT)



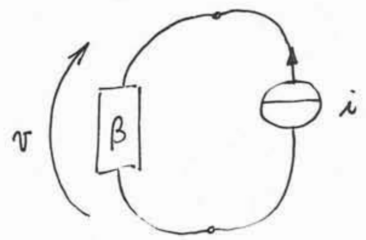
> ALLOW TO REPLACE A COMPOSITE LINEAR RESISTIVE 2-TERMINAL ELEMENT BY THE SERIES CONNECTION OF ONE RESISTOR AND ONE VOLTAGE SOURCE

DEMONSTRATION

IT IS BASED ON SUPERPOSITION THEOREM

β CONTAINS resistors, voltage sources, current sources, ETC. AND IT RESPECTS THEVENIN'S HYPOTHESIS

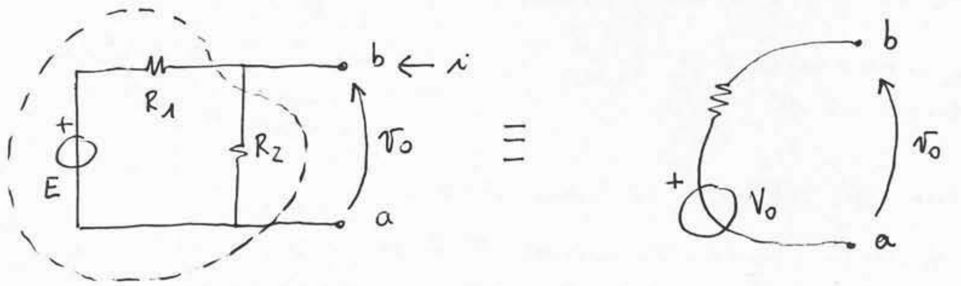
(WELL-DEFINED ≡ CLOSED, 1 SOLUTION, ...)



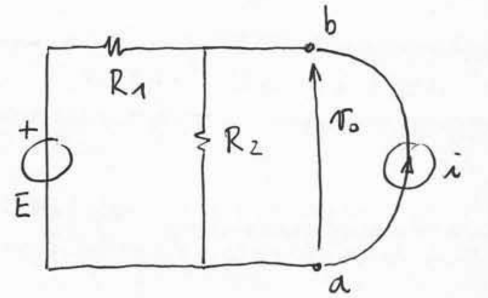
SUPERPOSITION THEOREM : $v = K_i\, i + v_0 \Rightarrow v = R_E\, i + v_0$

SAMPLE 1

DETERMINE THE THEVENIN'S EQUIVALENT CIRCUIT AT TERMINALS b-a



ONE STEP COMPUTATION



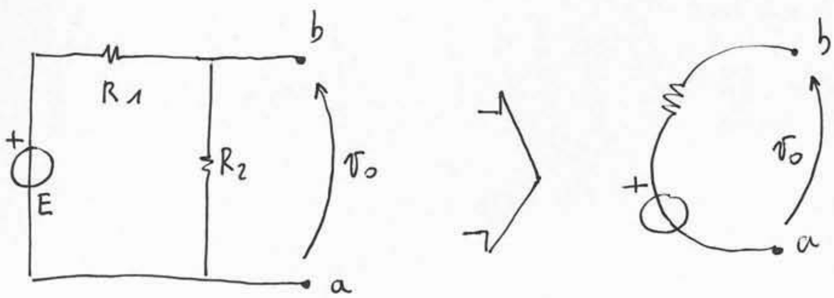
APPLYING MILLMAN

$$v = (G_1 E + i)/(G_1 + G_2)$$

LET'S CHANGE THE FORM TO OBTAIN THEVENIN'S EQUIVALENT

$$v = \underbrace{\frac{1}{G_1+G_2}}_{R_E}\, i + \underbrace{\frac{G_1}{G_1+G_2}\, E}_{v_0}$$

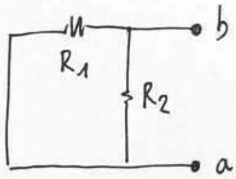
TWO STEP COMPUTATION (MORE EFFICIENT METHOD)



ONE LEAVES THE TERMINALS OPEN (NO CURRENT THROUGH THEM)

LEAVING TERMINALS OPEN TRANSFORM CONNECTION INTO SERIES ⇒ VOLTAGE DIVIDER

$$v_0 = E R_2/(R_1+R_2)$$

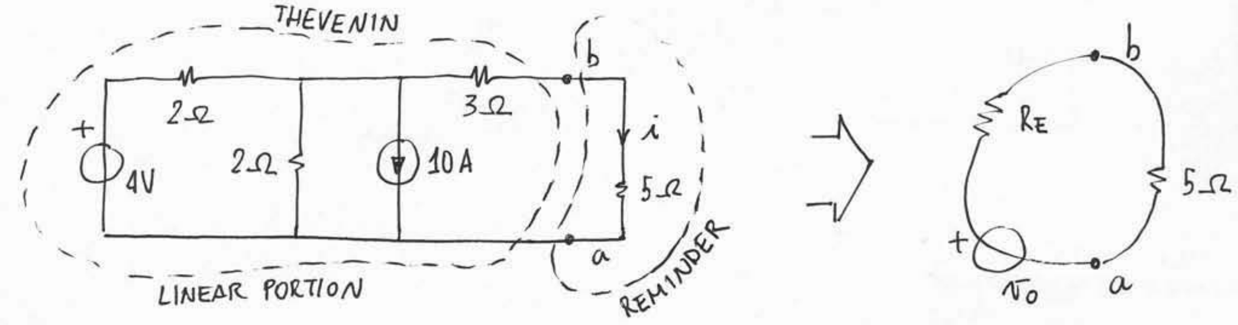


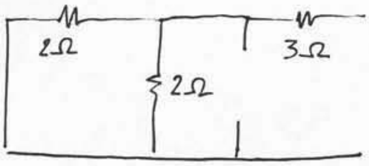
$$R_E = R_1 \| R_2 = \frac{R_1 R_2}{R_1+R_2}$$

$$\boxed{v = \frac{R_1 R_2}{R_1+R_2}\, i + \frac{E R_2}{R_1+R_2}}$$

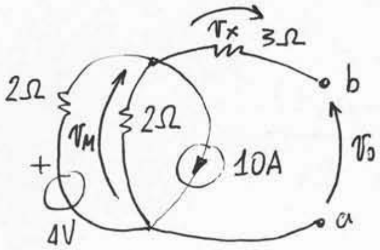
PROBLEM 1

DETERMINE THE CURRENT i BY REDUCING THE 2-TERMINAL ELEMENT ATTACHED TO THE 5Ω RESISTOR TO ITS THEVENIN EQUIVALENT



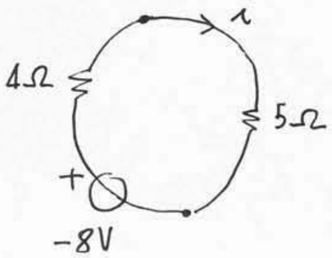


$$\Rightarrow R_E = (2 \| 2) + 3 = 4\,\Omega$$



$$v_M = v_0 = \frac{\frac{1}{2}\cdot 4 - 10}{1/2 + 1/2} = -8\,V$$

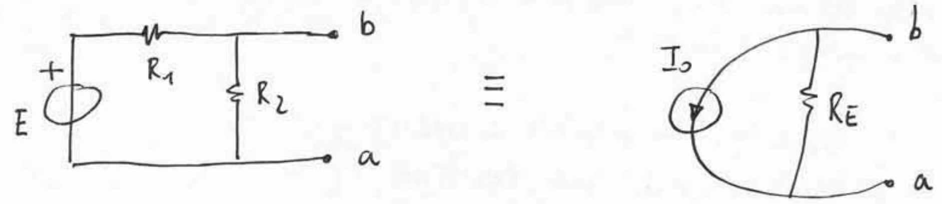
v_x ACROSS $R = 3\Omega$ IS NULL BECAUSE TERMINALS ARE LEFT OPEN



$$i = \frac{-8\,V}{4\Omega + 5\Omega} = -\frac{8}{9}\,A$$

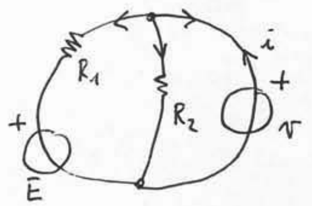
SAMPLE 1

DETERMINE THE NORTON EQUIVALENT CIRCUIT AT TERMINALS b-a



ONE STEP COMPUTATION

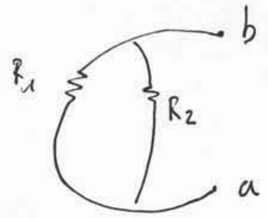
LET'S APPLY NODE EQUATION



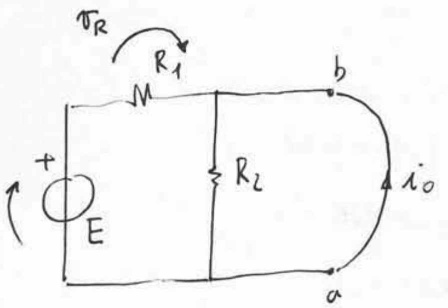
$$i = G_2 v + G_1 (v - E)$$

$$i = \underbrace{(G_1 + G_2)}_{G_E} v - \underbrace{G_1 E}_{I_0}$$

TWO-STEP COMPUTATION (MORE EFFICIENT)



$$G_E = G_1 + G_2 = \frac{1}{R_E}$$



LOOP EQUATION

$$E + v_R = 0 \Rightarrow v_R = -E$$

$$i_R = G_1 v_R$$

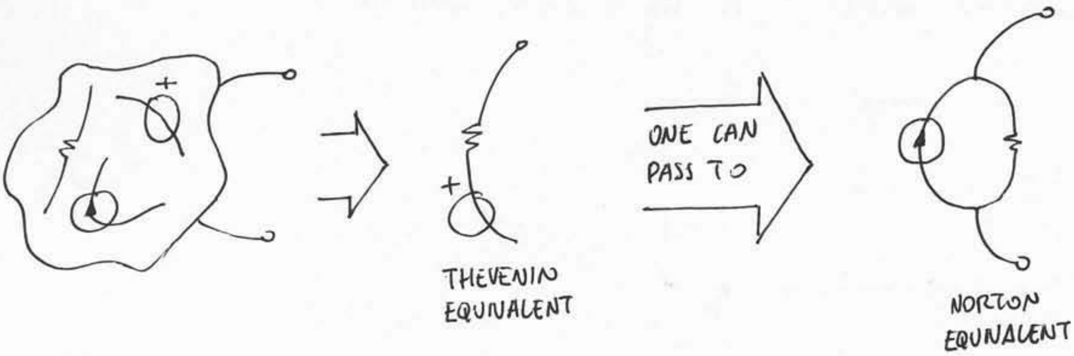
$$\Rightarrow i_o = i_R = -G_1 \cdot E$$

CURRENT THRU R_2 IS NULL BECAUSE R_2 IS SHORT CIRCUITED

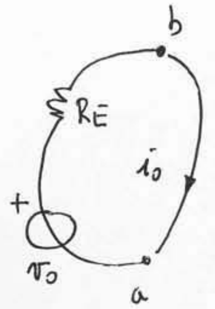
$$i = (G_1 + G_2) v - G_1 E$$

CONVERSION ~~OF~~ OF EQUIVALENTS

BUILD THE DUAL EQUIVALENT OF ONE THAT IS GIVEN

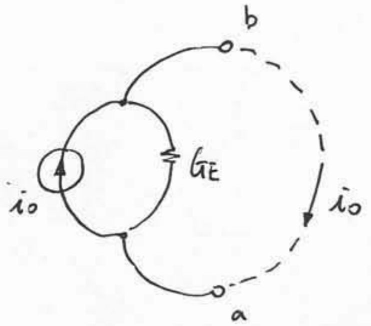


BY A USEFUL TRANSFORMATION



$$\boxed{G_E = \frac{1}{R_E} \qquad i_o = \frac{v_o}{R_E}}$$

ONCE ONE HAS COMPUTED G_E & i_o



EVEN THE OPPOSITE PROCEDURE IS ALLOWED

$R_E = 1/G_E$ & $v_o = i_o/G_E$

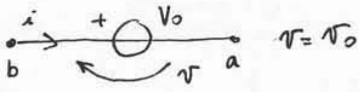
THE CURRENT SOURCE HAS TO BE DIRECTED AS i_o IN THIS FIGURE

R_E MUST BE $\neq 0$ OTHERWISE WE CANNOT COMPUTE ~~the~~ IT

EXISTANCE OF EQUIVALENTS

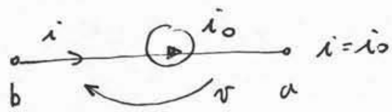
PRACTICAL CONDITIONS FOR ELEMENTS COMPOSED OF RESISTORS & INDEPENDENT SOURCES

- IF $|R_E| < \infty$ & $R_E \neq 0 \Rightarrow$ BOTH THE 2 EQUIVALENTS EXIST
- IF $R_E = 0$, THE THEVENIN EQUIVALENT IS



  NORTON EQUIVALENT DOES NOT EXIST

- IF $|R_E| = \infty$ ($G_E = 0$), THE NORTON EQUIVALENT IS

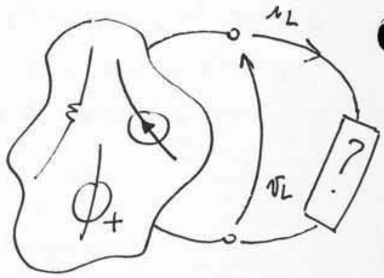


  THEVENIN EQUIVALENT DOES NOT EXIST

APPLICATION OF THEVENIN : MAXIMUM POWER TRANSFER

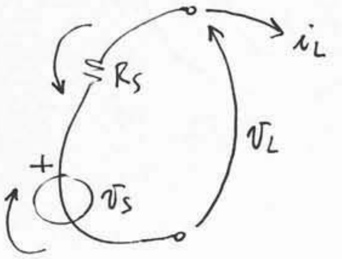
MAXIMUM POWER OBTAINED

APART FROM SPECIAL CASES, ONE USES THEVENIN'S EQUIVALENT



PROBLEM : FIND MAX OF $p(t) = v_L i_L$ WHERE L = LOAD

ONE CAN COMPUTE v_L AND i_L



$v_{RES} = R_S i_L$

$v_L = v_S - R_S i_L$

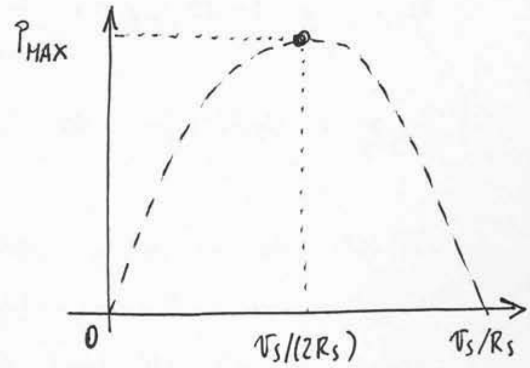
LET'S COMPUTE THE POWER $\Rightarrow$ $\boxed{p = \underbrace{(v - R_S i_L)}_{v_L} i_L}$

POWER ≡ QUADRATIC FUNCTION OF LOAD

IF $i_L = 0 \Rightarrow$ OPEN CIRCUIT AS UTILIZER

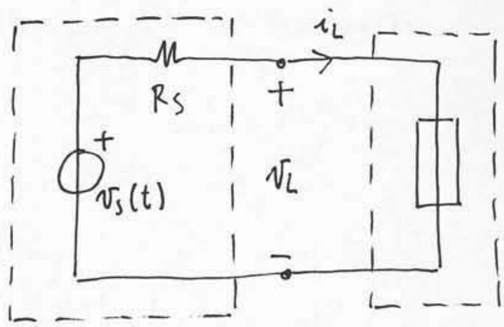
IF $i_L = v_S/R_S \Rightarrow$ SHORT CIRCUIT AS UTILIZER

P IS NULL IN THESE ~~ABOVE~~ 2 POINTS ABOVE MENTIONED



MAXIMUM POWER (READ ON PARABOLIC GRAPH)

$P_{MAX} = \left(v - R_S \frac{v_S}{2R_S}\right) \cdot \frac{v_S}{2R_S}$



$\boxed{P_{MAX} = \frac{v_S^2}{4R_S}}$ MAX POWER

$\boxed{R_L = R_S}$ MATCHING LOAD

WHEN A SOURCE DELIVERS MAX POWER, ONE SAYS THAT IT IS MATCHED

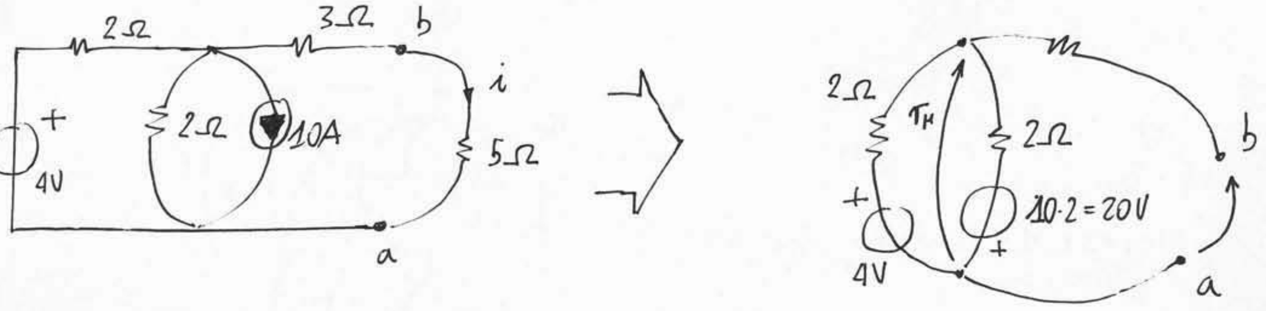
IN OUR CASE, $v_L = v_S/2$: SMALL VALUE OF VOLTAGE; FOR EXAMPLE, IN CASE OF BATTERIES IT IS NOT GOOD TO BE AT LOW V_L IN COMPARISON WITH v_S (NOT IN WORKING ZONE)

ANALYSIS VIA CIRCUIT TRANSFORMATION

- MODIFY THE CIRCUIT TOPOLOGY, GROUP AND REPLACE ELEMENTS TO REDUCE THE CIRCUIT TO A SINGLE LOOP OR SINGLE-NODE-PAIR CIRCUIT (NOT ALWAYS POSSIBLE)
- SOLVE THE SIMPLIFIED CIRCUIT BY DIVIDER EQUATIONS OR MILLMAN'S EQUATION

THERE IS NO ALGHORITHM, IT REQUIRES EXPERIENCE !

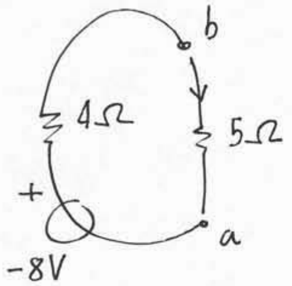
SAMPLE 1



LET'S BUILD THEVENIN EQUIVALENT

1^{st} STEP : $R_E =$ $(2 \| 2) + 3 = 4\Omega$

$$v_M = v_o = \frac{4 \cdot 1/2 - 20 \cdot 1/2}{1} = -8\,V$$ 2^{nd} STEP



$$i = -\frac{8V}{9} = -\frac{8}{9}\,A$$

23/10/2013

4- CONTROLLED SOURCE AND 2-PORT ELEMENTS

- CONTROLLED SOURCES ARE IDEAL SOURCES (VOLTAGE SOURCE OR CURRENT ONE) FORCING VOLTAGES AND CURRENTS THAT ARE FUNCTIONS OF OTHER CIRCUIT VARIABLES; THEY DEPEND ON SOMETHING ELSE, THEY ARE NOT INDEPENDENT

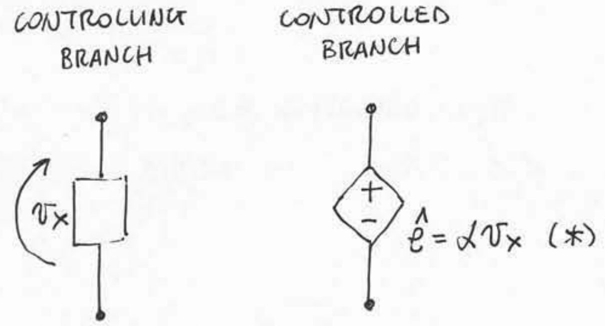
LET'S DEAL WITH LINEAR CONTROLLED SOURCES

4 KINDS

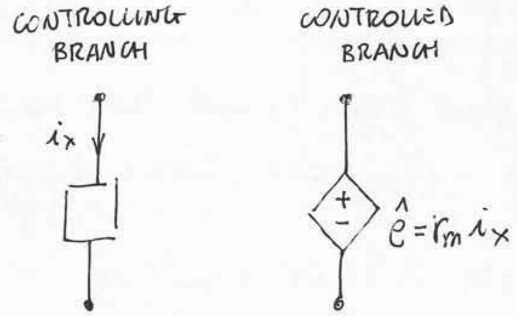
VOLTAGE SOURCES CONTROLLED BY
- VOLTAGE ⇒ VOLTAGE CONTROLLED VOLTAGE SOURCE VCVS
- CURRENT ⇒ CURRENT CONTROLLED VOLTAGE SOURCE CCVS

- THEIR DUAL ENTITIES ARE
  - VCCS
  - CCCS

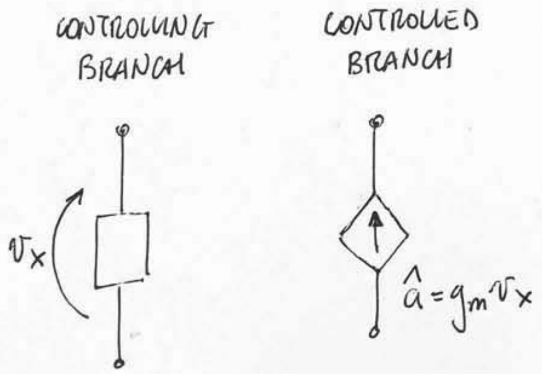
(*) IT DEFINES THE BEHAVIOUR OF THE ELEMENT; VOLTAGE IS LINEAR FUNCTION OF CONTROLLING VOLTAGE v_x



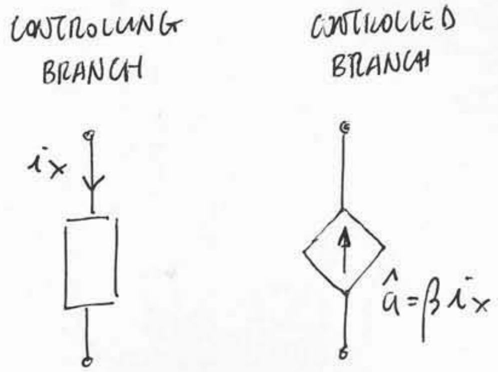
VOLTAGE CONTROLLED VOLTAGE SOURCE



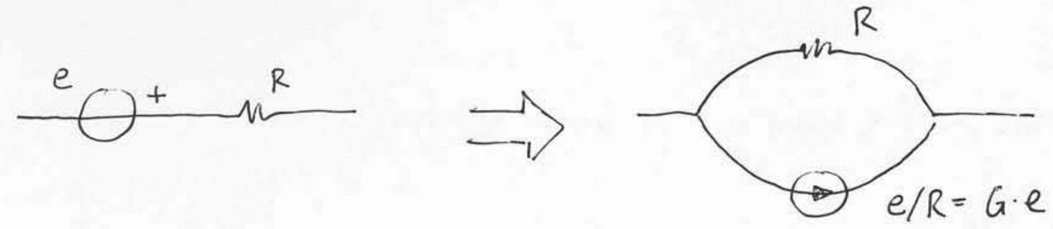
CURRENT CONTROLLED VOLTAGE SOURCE

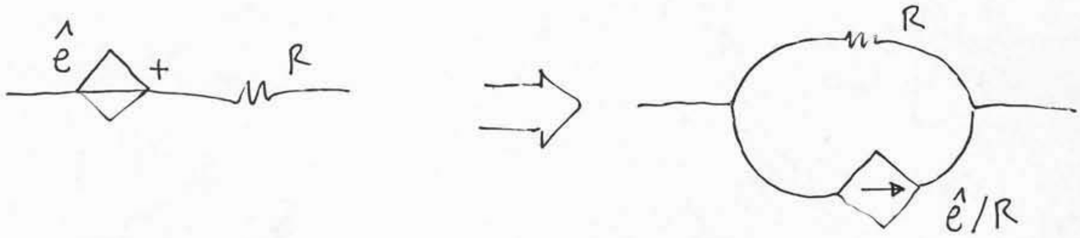


VOLTAGE CONTROLLED CURRENT SOURCE



CURRENT CONTROLLED CURRENT SOURCE





equivalent conductance = 0

2) FOR THE RESULTING RESISTOR C-SOURCE CIRCUIT, USE

$[G_n]_{kk} = \sum G_k$ RESISTORS CONNECTED TO NODE K

$[G_n]_{hk} = [G_n]_{kh} = -\sum G_h$ RESISTORS BETWEEN NODES K & h

$[A]_k = \sum a_k$ CURRENT INJECTED INTO NODE K

NODAL ANALYSIS REVISITED, CONT'D

NODAL MATRIX EQUATION FROM MATRIX ELEMENTS, CURRENT CONTROLLED ELEMENTS CASE

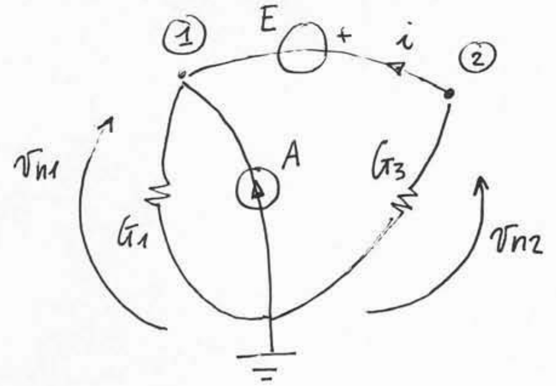
1) SHORT CIRCUITS ONLY $\Rightarrow$ JUST REMOVE THEM

2) VOLTAGE SOURCE: USE MNA (TREAT AUXILIARY CURRENTS AS FORCING TERMS)

$$[A]_k = \sum a_k + \sum i_{ek}$$ CURRENTS INJECTED INTO NODE K

... THEN MOVE AUXILIARY CURRENTS ON LEFT-HAND SIDE AND CAST IN MATRIX FORM !

EXAMPLE



NO SERIES TOPOLOGY

VOLTAGE SOURCE: FORBIDDEN ELEMENT

LET'S DEFINE AUXILIARY CURRENT i THROUGH THE VOLTAGE SOURCE AND THEN APPLY THE RULES ONE HAS JUST SEEN

ACTUAL VECTOR OF SOURCE TERM

$$\begin{bmatrix} G_1 & 0 \\ 0 & G_3 \end{bmatrix}\begin{bmatrix} v_{n1} \\ v_{n2} \end{bmatrix} = \begin{bmatrix} A+i \\ -i \end{bmatrix} = \begin{bmatrix} A \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} i$$

VECTOR OF SOURCES DEPENDING ON THE UNKNOWN i

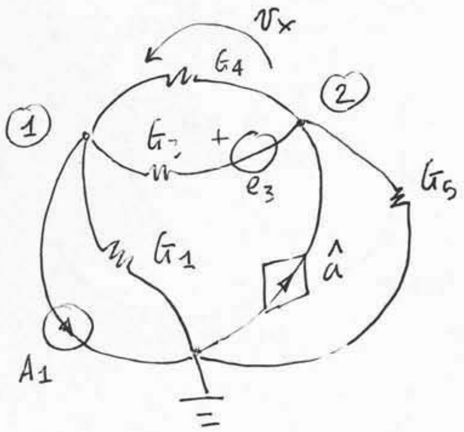
SET OF UNKNOWNS $\{v_{n1}, v_{n2}, i\}$

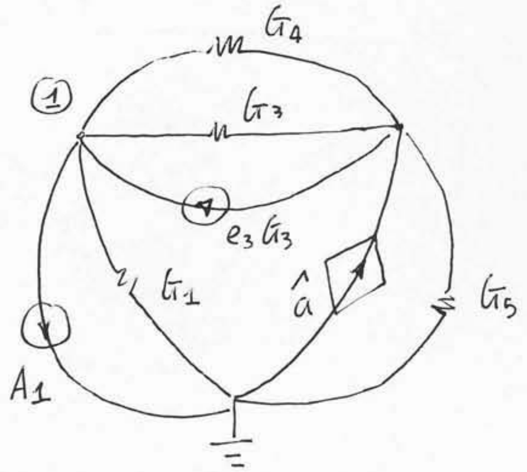
$$\begin{bmatrix} \begin{bmatrix} G_1 & 0 \\ 0 & G_3 \end{bmatrix} & \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} -1 & 1 \end{bmatrix} & \begin{bmatrix} 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} v_{n1} \\ v_{n2} \\ i \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} A \\ 0 \end{bmatrix} \\ \begin{bmatrix} E \end{bmatrix} \end{bmatrix}$$

ANALYSIS OF CIRCUITS WITH CONTROLLED SOURCES

- BY NODAL ANALYSIS OR MODIFIED NODAL ANALYSIS ONLY
- 2 STEPS
  - TREAT CONTROLLED SOURCES FORCING TERMS AS KNOWN: IMPLICIT MATRIX EQUATIONS
  - EXPRESS FORCING TERMS AS FUNCTIONS OF UNKNOWN VARIABLES: EXPLICIT EQUATIONS
- SYMMETRY PROPERTIES ARE LOST
- CURRENT SOURCES AFFECT THE MATRIX OF THE NA EQUATION <u>ONLY</u>

SAMPLE 1





$$\begin{bmatrix} G_1+G_3+G_4 & -(G_3+G_4) \\ -(G_3+G_4) & G_4+G_3+G_5 \end{bmatrix} \begin{bmatrix} v_{n1} \\ v_{n2} \end{bmatrix} = \begin{bmatrix} -a_1 + G_3 e_3 \\ -G_3 e_3 + \hat{a} \end{bmatrix}$$

$$v_x = v_{n1} - v_{n2} \Rightarrow \hat{a} = g_m (v_{n1} - v_{n2})$$

$$\begin{bmatrix} G_1+G_3+G_4 & -(G_3+G_4) \\ -(G_3+G_4) & G_4+G_3+G_5 \end{bmatrix} \begin{bmatrix} v_{n1} \\ v_{n2} \end{bmatrix} = \begin{bmatrix} -a_1 + G_3 e_3 \\ -G_3 e_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ +g_m & -g_m \end{bmatrix} \begin{bmatrix} v_{n1} \\ v_{n2} \end{bmatrix}$$

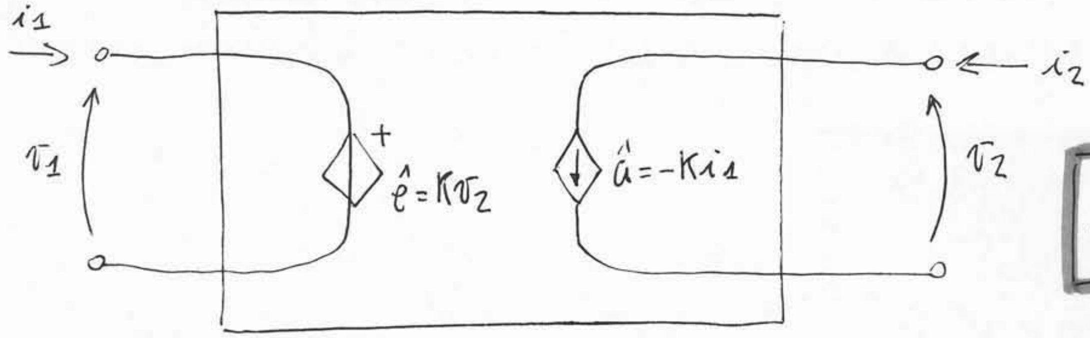
$$\begin{bmatrix} 1/3 + 1/2 & -1/3 \\ -1/3 + 4/3 & 1/4 + 1/2 - 4/3 \end{bmatrix} \begin{bmatrix} v_{n1} \\ v_{n2} \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$ EXPLICIT EQUATION

$$\underbrace{\begin{bmatrix} 5/6 & -1/3 \\ 1 & 5/4 \end{bmatrix}}_{G_{Tn}} \begin{bmatrix} v_{n1} \\ v_{n2} \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} v_{n1} \\ v_{n2} \end{bmatrix} = G_T^{-1} \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

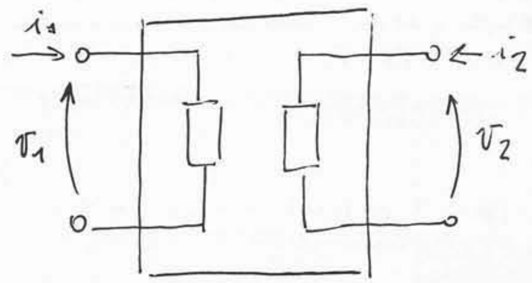
STEP BACK TO THE ORIGINAL CIRCUIT

$$i = \frac{1}{2}\left(v_{n1} - 12 \right)$$

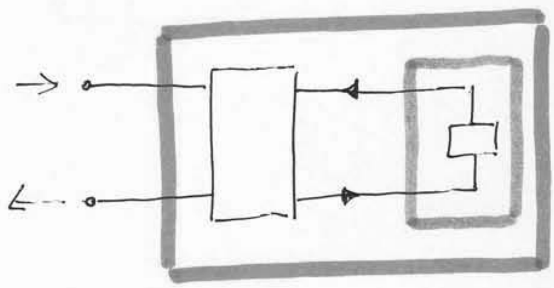


IDEAL TRANSFORMER

- AN INTRINSIC TWO PORT ELEMENT IS A 4-TERMINAL ELEMENT WITH AN INTERNAL STRUCTURE THAT GUARANTIE THE EXISTANCE OF PORT CURRENTS, E.G.

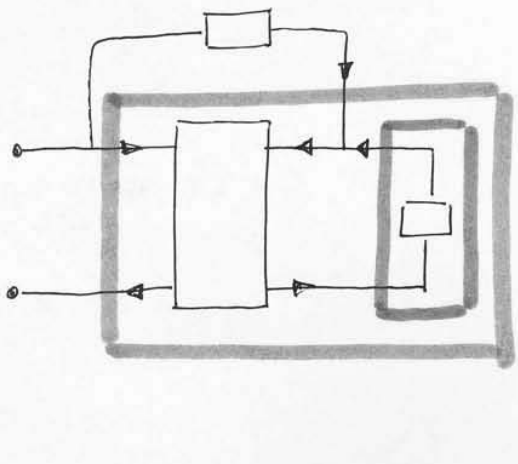


- ANY FOUR-TERMINAL ELEMENT WITH TWO TERMINALS TERMINATED BY A TWO-TERMINAL ELEMENT OPERATED AS TWO PORT



DEFINES A CUT-SET
CURRENT ENTERING = CURRENT EXITING

DEFINES A CUT-SET
CURRENT ENTERING = CURRENT EXITING



CURRENT ARE NOT THE SAME; NO LONGER GUARANTEE THEIR EQUIVALENCE

PROPERTY NO LONGER HOLD
2-TERMINAL NO LONGER FORMS A CUT-SET

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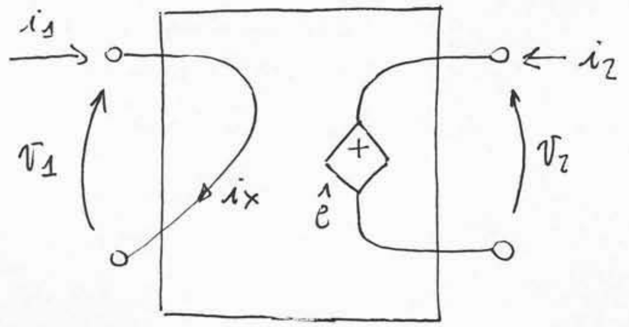
CONTROLLED SOURCE : BASIC 2-PORT ELEMENTS



$\hat{e} = r_m i_x$

$$\begin{cases} v_1 = 0 & \text{REGARDLESS CURRENT} \\ v_2 = r_m i_1 & \text{REGARDLESS } i_2 \text{ SINCE ONE HAS A VOLTAGE SOURCE} \end{cases}$$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbb{A}} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ -r_m & 0 \end{bmatrix}}_{\mathbb{B}} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\mathbb{C}}$$

NEXT STEP: WRITE THE EXPLICIT CHARACTERISTIC

- 6 DIFFERENT POSSIBLE COMBINATIONS FOR 2-PORT ELEMENTS
- 2 DIFFERENT POSSIBLE COMBINATIONS FOR 1-PORT ELEMENT

$$\mathbb{A} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \mathbb{B} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = -\mathbb{C}$$

CONDITION OF EXISTANCE: MATRIX INVERSION IS INVOLVED !

ONCE PROVIDED INVERSE OF MATRIX $\mathbb{A}$ EXISTS

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = -\mathbb{A}^{-1} \mathbb{B} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \mathbb{A}^{-1} \mathbb{C}$$

NOT SURPRISING RESULT: 2 PORT ELEMENT IS GENERALIZATION OF 1 PORT ELEMENT !

$$\mathbb{v} = \mathbb{R} \cdot \mathbb{i} + \mathbb{v}_0$$

NAMING

$$\mathbb{V} = \mathbb{R}\, \boldsymbol{i} + \mathbb{V}_0$$

OPEN CIRCUIT VOLTAGE ($\mathbb{V}_0$)

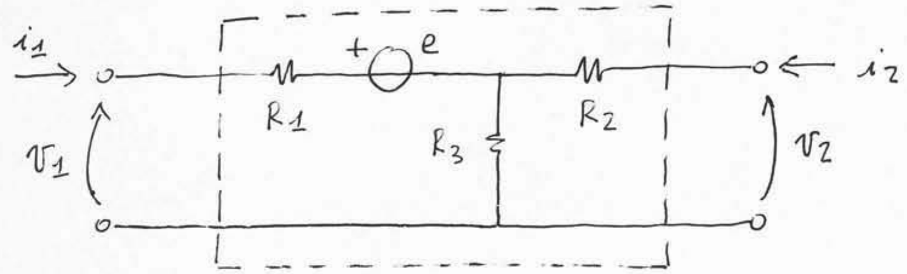
OPEN CIRCUIT MATRIX RESISTANCE ($\mathbb{R}$)

$$\boldsymbol{i} = \mathbb{G}\, \mathbb{V} + \boldsymbol{i}_0$$

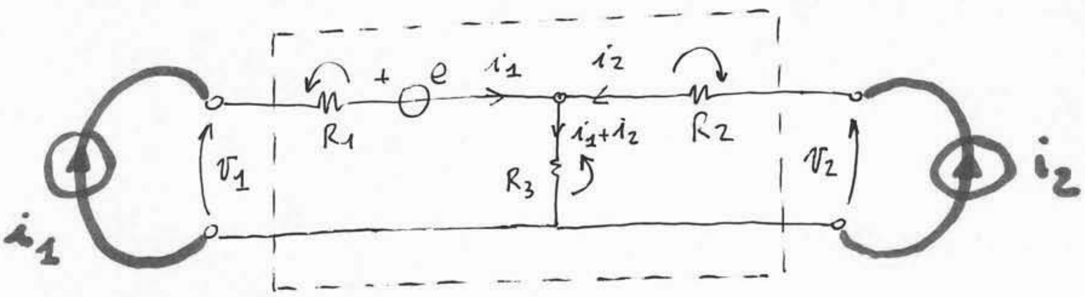
SHORT CIRCUIT CURRENTS ($\boldsymbol{i}_0$)

SHORT CIRCUIT CONDUCTANCE MATRIX ($\mathbb{G}$)

SAMPLE 1



COMPUTE THEVENIN CHARACTERISTIC



$v_{R1} = R_1 i_1$

$v_{R3} = R_3 (i_1 + i_2)$

$v_{R2} = R_2 i_2$

$$\begin{cases} v_1 = R_1 i_1 + e + R_3 (i_1 + i_2) \\ v_2 = R_2 i_2 + R_3 (i_1 + i_2) \end{cases}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \underbrace{\begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix}}_{\mathbb{R}} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \underbrace{\begin{bmatrix} e \\ 0 \end{bmatrix}}_{\mathbb{V}_0}$$

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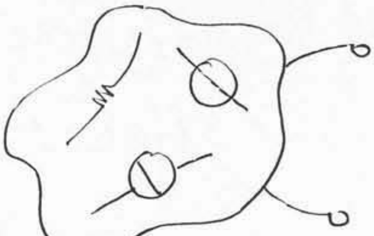
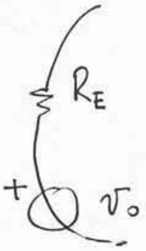
EQUIVALENT CIRCUIT OF 2-PORT ELEMENTS

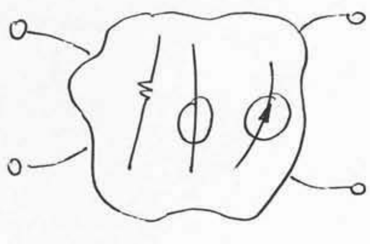
1-PORT ELEMENT



$\Rightarrow$ $v = R_E i + v_0$ $\Rightarrow$



2-PORT ELEMENT



$\Rightarrow$ $\mathbb{V} = \mathbb{R}\,\mathbb{i} + \mathbb{V}_0$ $\Rightarrow$ MANY POSSIBLE EQUIVALENT CIRCUITS !

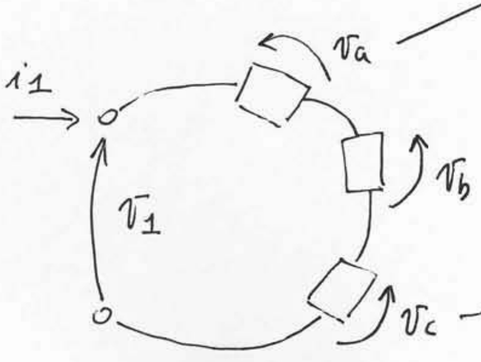
THEVENIN

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} v_{01} \\ v_{02} \end{bmatrix}$$

$$v_1 = \underbrace{R_{11}\, i_1}_{v_a} + \underbrace{R_{12}\, i_2}_{v_b} + \underbrace{v_{01}}_{v_c}$$

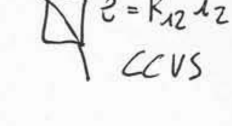
3 VOLTAGE CONTRIBUTIONS SUMMED UP TO OBTAIN v_1

$\Rightarrow$

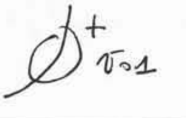


v_a $\rightarrow$ ONE CAN IDENTIFY IT BY A RESISTOR WITH RESISTANCE R_{11}

v_b $\rightarrow$ VOLTAGE DEPENDING ON A CURRENT DEFINED SOMEWHERE



v_c $\rightarrow$ VOLTAGE INDEPENDENT ON EVERYTHING $\Rightarrow$ IDEAL VOLTAGE SOURCE



25/10/2013

5 – FUNDAMENTALS OF DYNAMIC CIRCUITS

LET'S DEAL WITH LINEAR DYNAMIC CIRCUITS

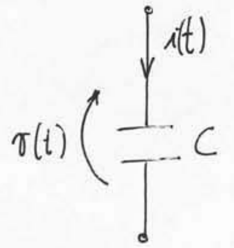
DYNAMIC ELEMENT : ANY CIRCUIT ~~an~~ ELEMENT DEFINED BY A CONSTITUTIVE RELATION WITH MEMORY; IT CHARACTERISTIC HAS A MEMORY AND ALSO THE PREVIOUS STEP OF VOLTAGE AND CURRENT ARE INVOLVED, NOT ONLY THE INSTANTANEOUS VALUES

3 ELEMENTAL DYNAMIC ELEMENTS TIME INVARIANT:
- CAPACITOR
- INDUCTOR
- COUPLED INDUCTOR

DYNAMIC CIRCUIT : ANY CIRCUIT CONTAINING ONE OR MORE DYNAMIC ELEMENTS

DYNAMIC ELEMENTS

CAPACITOR (IDEAL)

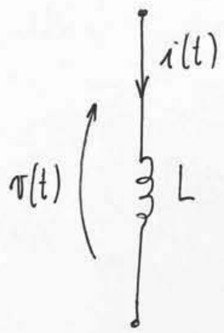


$$\begin{cases} q = C \cdot v(t) \\ i(t) = \dfrac{d}{dt} q(t) = C \dfrac{d}{dt} v(t) \end{cases}$$

EQUATION DESCRIBING BEHAVIOR OF CAPACITOR

C : CAPACITANCE [Farad, F], 1F = 1C/V

INDUCTOR (IDEAL)



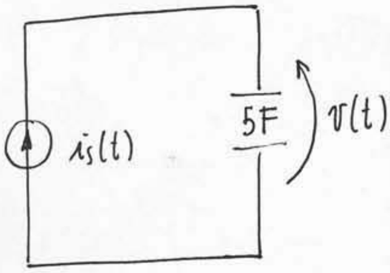
$$\begin{cases} \Psi(t) = L\, i(t) \\ v(t) = \dfrac{d}{dt} \Psi(t) = L \dfrac{d}{dt} i(t) \end{cases}$$

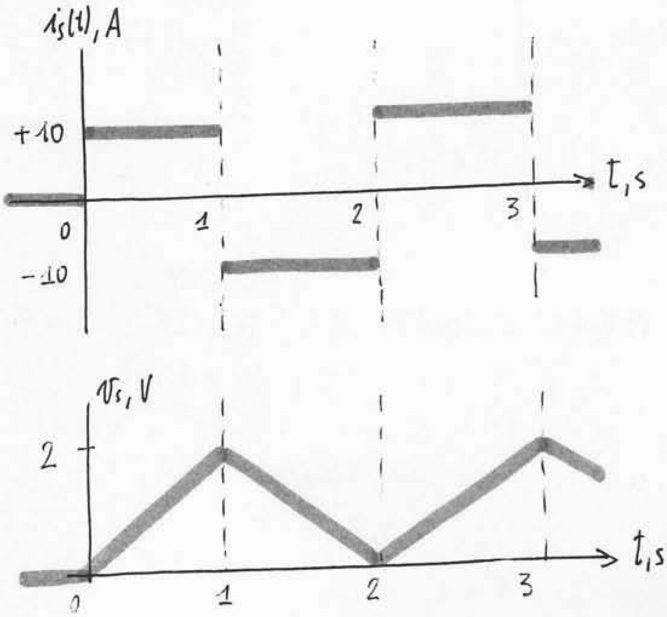
Ψ : MAGNETIC FLUX [Weber, W]

L : INDUCTANCE [henry, H], 1H = 1Wb/A

CONTINUITY OF CAPACITOR VOLTAGE

FOR ANY CURRENT FUNCTION BOUNDED IN A CLOSED INTERVAL $[t_a, t_b]$ THE CAPACITOR VOLTAGE IS A CONTINOUS FUNCTION IN THE OPEN INTERVAL (t_a, t_b)





$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t')\,dt'$$

$$v(t) = \frac{1}{5}\int_{-\infty}^{t} i_s(t')\,dt'$$

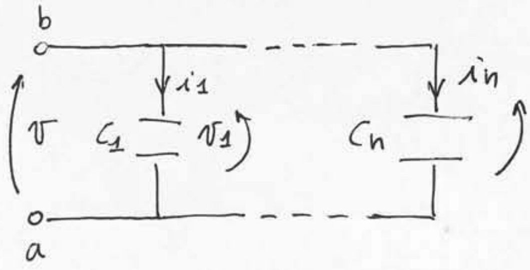
DISCONTINUITY IN CURRENT FLOW BUT CONTINUITY IN VOLTAGE ACROSS THE CAPACITOR

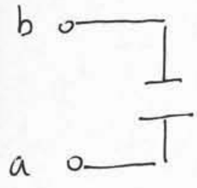
LET'S CARRY OUT THE INTEGRATION GRAPHICALLY

$$v(t) = \begin{cases} 0 & t \le 0 \\ 2t & 0 \le t \le 1 \\ -2t & 1 \le t \le 2 \end{cases}$$

linear function growing in time from 0

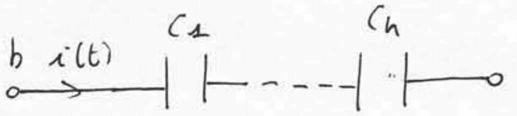
SERIES AND PARALLEL CONNECTIONS OF CAPACITORS

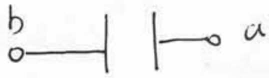




$C_{EQ} = C_1 + \ldots + C_n$

$$\frac{1}{C_{EQ}} = \frac{1}{C_1} + \frac{1}{C_n}$$

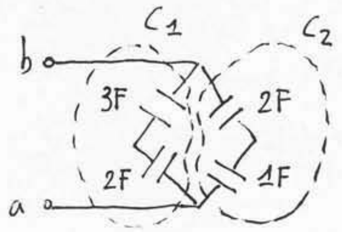


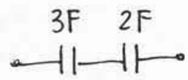


THE CAPACITANCE PARAMETER BEHAVES LIKE THE CONDUCTANCE PARAMETER

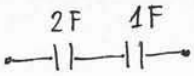
THE INDUCTANCE PARAMETER BEHAVES LIKE THE RESISTANCE PARAMETER

SAMPLE 1

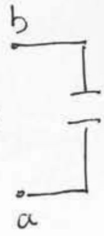




$C_1 = \frac{2 \cdot 3}{3+2} = \frac{6}{5}$ F

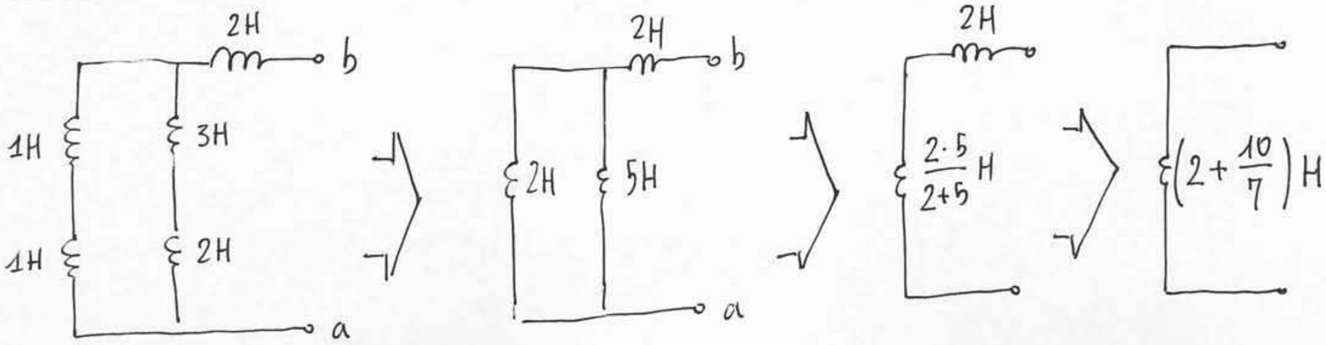


$C_2 = \frac{2 \cdot 1}{1+2} = \frac{2}{3}$ F

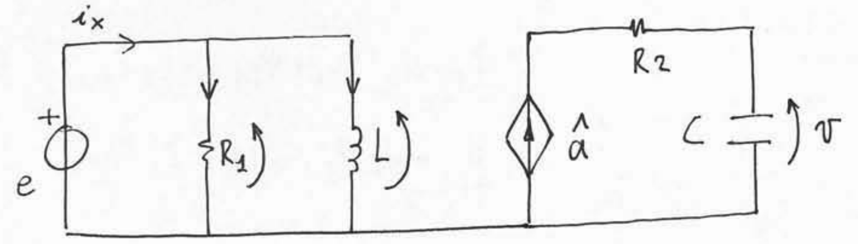


$C_{EQ} = C_1 + C_2$

PROBLEM 1



PROBLEM



$e = u(t)$ V

$\hat{a} = \beta i_x$

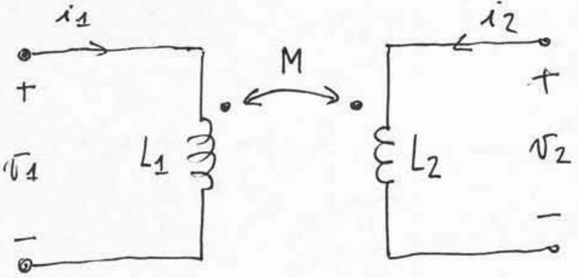
$$i_x = G_1 e + \frac{1}{L}\int_{-\infty}^{t} e(t')dt'$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} \hat{a}(t')dt' = \frac{\beta}{C}\int_{-\infty}^{t}\left(G_1 e + \frac{1}{L}\int e\right) = \beta\frac{G_1}{C}\int_{-\infty}^{t} u(t')dt' + \beta\frac{1}{LC}\iint_{-\infty}^{t} u(t')dt'$$

COUPLED INDUCTORS

COUPLED INDUCTORS ARE A MODEL FOR COUPLED WINDINGS (REAL TRANSFORMERS)

COUPLED INDUCTORS ARE INTRINSIC 2-PORT ELEMENT



$$v_1(t) = L_1\frac{d}{dt}i_1(t) + M\frac{d}{dt}i_2(t)$$

$$v_2(t) = M\frac{d}{dt}i_1(t) + L_2\frac{d}{dt}i_2(t)$$

CONDITIONS: $L_1 \geq 0$; $L_2 \geq 0$; $M \leq \sqrt{L_1 L_2}$

M COUPLES THE TWO PORT; IT IS NAMED MUTUAL INDUCTANCE

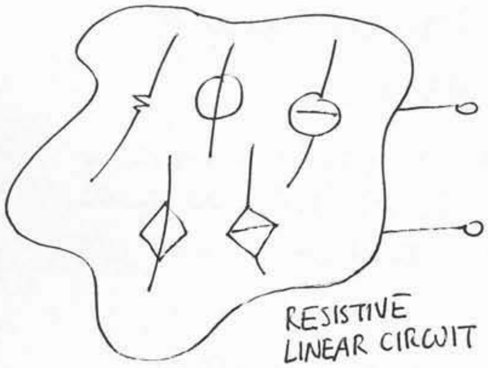
$$\mathbb{L} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}$$ INDUCTANCE MATRIX $\Leftarrow$ USEFULL TO DEFINE IT USING COEFF. L_1, L_2, M IN THIS WAY

$$\mathbb{v}(t) = \mathbb{L}\frac{d}{dt}\mathbb{i}(t)$$

VECTOR EQUIVALENT OF SCALAR EQUATION THAT HOLDS FOR 1 INDUCTOR (1 PORT ELEMENT) $v = L\frac{di}{dt}$

30/10/2013

LTI DYNAMIC CIRCUITS : ANALYSIS



$\sum \alpha_K v_K = 0$ KVL

$\sum \beta_K i_K = 0$ KCL

$a_n v_K + b_n i_K + c_n = 0$ SYSTEM OF LINEAR ALGEBRAIC EQUATIONS

IF THERE IS SOMETHING DYNAMIC LIKE A CAPACITOR

$i = C \frac{dv}{dt}$

IMPORTANT TO DECIDE THE BEST METHOD TO SOLVE THE PROBLEM

LINEAR ALGEBRAIC EQUATIONS
⊕
DIFFERENTIAL EQUATIONS

} DIFFERENTIAL ALGEBRAIC EQUATION

MIXED SYSTEMS ARE DIFFICULT TO BE SOLVED : THE ISSUE IS THE SOLUTION OF DAE

LET'S DEFINE THE DIFFERENT CASES AND THE BEST METHODS FOR EACH ONE

- PROBLEM REDUCIBLE TO PURE ALGEBRAIC OR DECOUPLED DIFFERENTIAL EQUATIONS INVOLVING EITHER THE UNKNOWNS OR THEIR DERIVATIVES/INTEGRALI ONLY
  - ALGEBRAIC SOLUTION OR ALGEBRAIC + INTEGRATION/DIFFERENTIATION (SELDOM HAPPENS)

- REDUCIBLE TO A FIRST ORDER COMPLETE DIFFERENTIAL EQUATION (PARTICULARLY IMPORTAN CASE)
  - PARAMETRIC FORM OF THE SOLUTION : IT HAPPENS WHEN THE CIRCUIT HAS JUST 1 CAPACITOR OR 1 INDUCTOR OR THEIR EQUIVALENTS

USING THE PARAMETER τ

$$\frac{dv}{dt} + \frac{1}{\tau} v = \frac{1}{\tau} v_0(t)$$

THIS IS THE EQUATION TO BE SOLVED

TO SOLVE IT, ONE HAS TO → INCLUDE INITIAL CONDITIONS I.C.

→ SPECIFY $v_0(t)$

$v_0(t) = E\,u(t)$

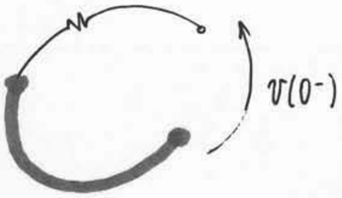
THE PROBLEM CAN BE SPLIT IN THIS WAY

$$\begin{cases} \dfrac{dv}{dt} + \dfrac{1}{\tau} v = \dfrac{1}{\tau} E & t > 0 \\ v(0^+) = v(0^-) \end{cases}$$

INITIAL CONDITION DESCRIBING THE PROBLEM; REMEMBER THAT VOLTAGE ACROSS CAPACITOR IS A CONTINOUS FUNCTION

FOR $t < 0$ VOLTAGE APPLIED BY THE VOLTAGE SOURCE IS 0 ⇒ SHORT CIRCUIT

LET'S REPRESENT THE CIRCUIT



CAPACITOR IS IN STEADY STATE CONDITIONS ⇒ OPEN CIRCUIT

RESISTOR CONNECTED TO OPEN CIRCUIT ⇒ NO CURRENT IS FLOWING

⇒ VOLTAGE ACROSS THE RESISTOR IS 0 ⇒ $v(0^-) = 0$

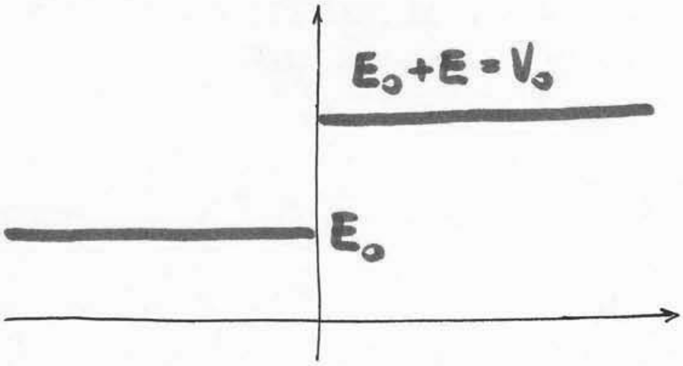
THE SOLUTION OF THE PROBLEM IS

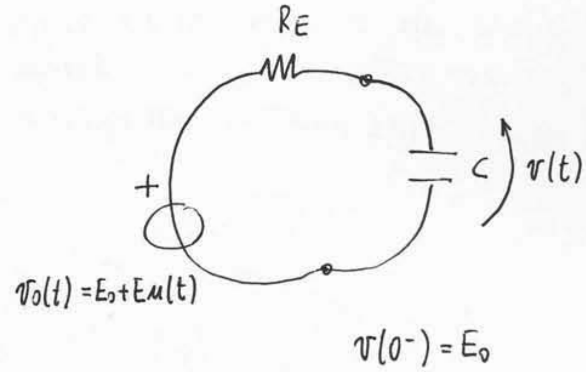
$$v(t) = \begin{cases} 0 & t \leq 0 \\ -E e^{-t/\tau} + E & t \geq 0 \end{cases}$$

ONCE ONE HAS THIS SOLUTION, HE DOES NOT NEED TO WRITE ANYMORE THE DIFFERENTIAL EQUATION

IN MORE COMPACT FORM $\quad v(t) = -E\,u(t)\,e^{-t/\tau} + E\,u(t)$

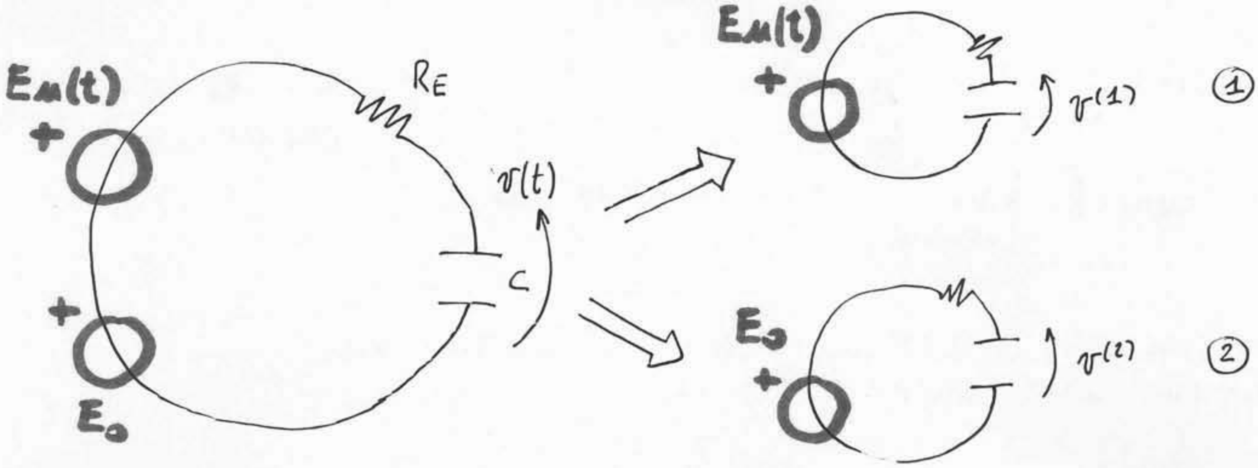
STEP RESPONSE FROM NON ZERO I.C.





ONE CAN SPLIT THE SOURCE AS DEPICTED BELOW; IN THIS CASE, SUPERPOSITION THEOREM HOLDS $\Longrightarrow$ ONE IS GOING TO CONSIDER 2 DIFFERENT PROBLEMS

THE VOLTAGE ACROSS THE CAPACITOR WILL BE THE SUM OF THE VOLTAGES COMPUTED IN THE 2 PROBLEMS !



LET'S CONSIDER PROBLEM 1 : ZERO-STATE RESPONSE COMPUTED BEFORE $v^{(1)} = -E e^{-t/\tau} + E \quad t \geq 0$

LET'S CONSIDER PROBLEM 2 : SOURCE HAS NO DISCONTINUITY: AT A FIXED TIME, ONE IS LOOKING FOR THE FINAL VALUE AND IT IS E_0

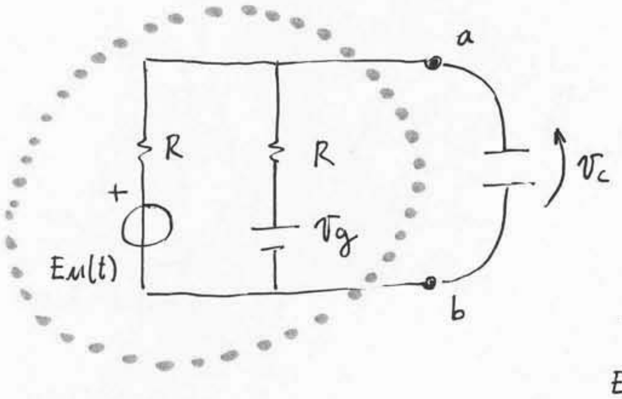
$$\Longrightarrow v^{(2)} = E_0$$

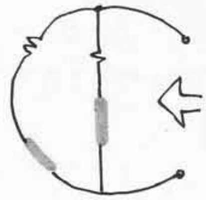
$$\Longrightarrow v(t) = -E e^{-t/\tau} + E + E_0 = -E e^{-t/\tau} + V_0$$

! ONCE IS KNOWN $v(0^-)$ AND TIME CONSTANT PART ONE DOES NOT NEED TO WRITE DIFF. EQUATION

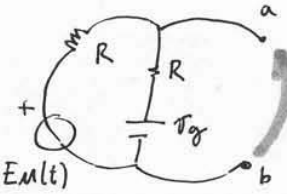
SINCE $E = V_0 - E_0 \Longrightarrow v(t) = (E_0 - V_0)\, e^{-t/\tau} + V_0 \quad t \geq 0$

$\boxed{\text{SAMPLE 1}}$ COMPUTE & PLOT $v_c(t)$





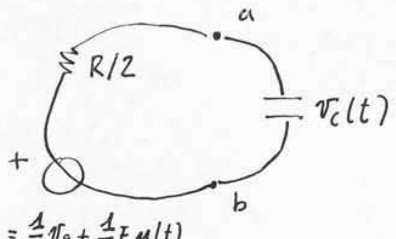
$\Leftarrow R_E = R_1 \| R_2 = \frac{1}{2} R$



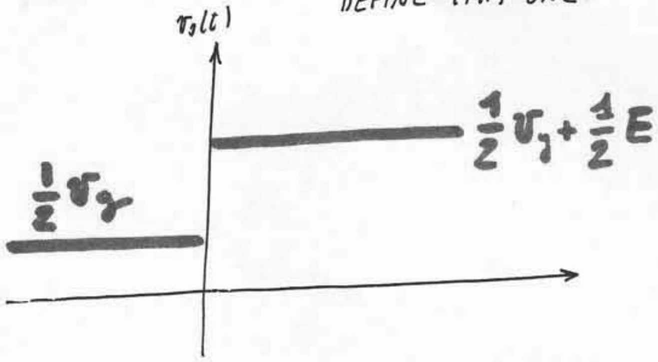
$$v_0(t) = \frac{G\,Em(t) + G v_g}{G + G}$$

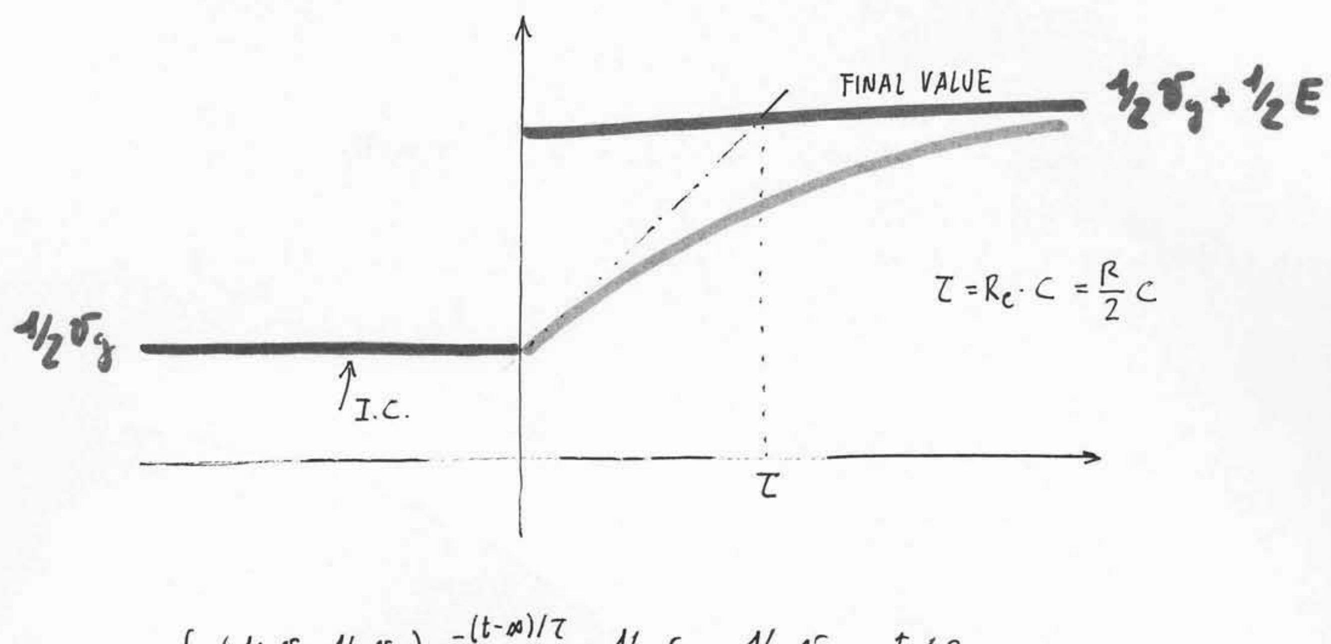
SINCE ONE IS LOOKING FOR v_c, HE IS NOT FREE TO DEFINE THIS ONE

LET'S FOCUS THE ATTENTION ON THE PROBLEM



$v_0(t) = \frac{1}{2} v_g + \frac{1}{2} Em(t)$





$\tau = R_e \cdot C = \frac{R}{2} C$

$$v(t) = \begin{cases} \left(1/2\, v_g - 1/2\, v_g\right) e^{-(t-\infty)/\tau} + 1/2\, v_g = 1/2\, v_g & t < 0 \\ \left[1/2\, v_g - \underbrace{\left(1/2\, v_g + 1/2\, E\right)}_{\text{FINAL VALUE}}\right] e^{-(t-0)/\tau} + \left(1/2\, v_g + 1/2\, E\right) & t \geq 0 \end{cases}$$

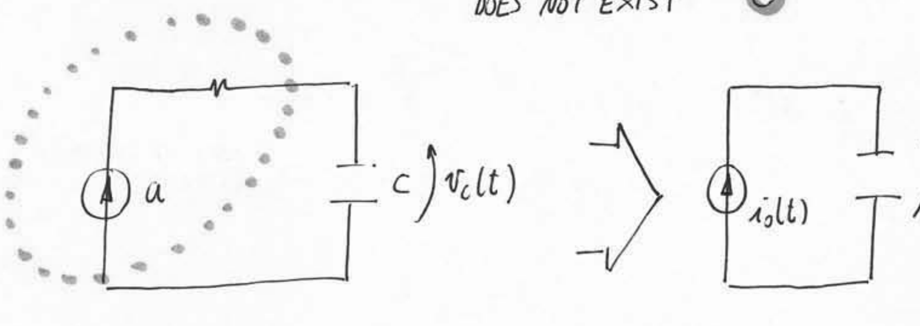
PROBLEM 2 — COMPUTE AND PLOT $v_c(t)$

- $a = 2\left[u(t) - u(t-T)\right]$ A

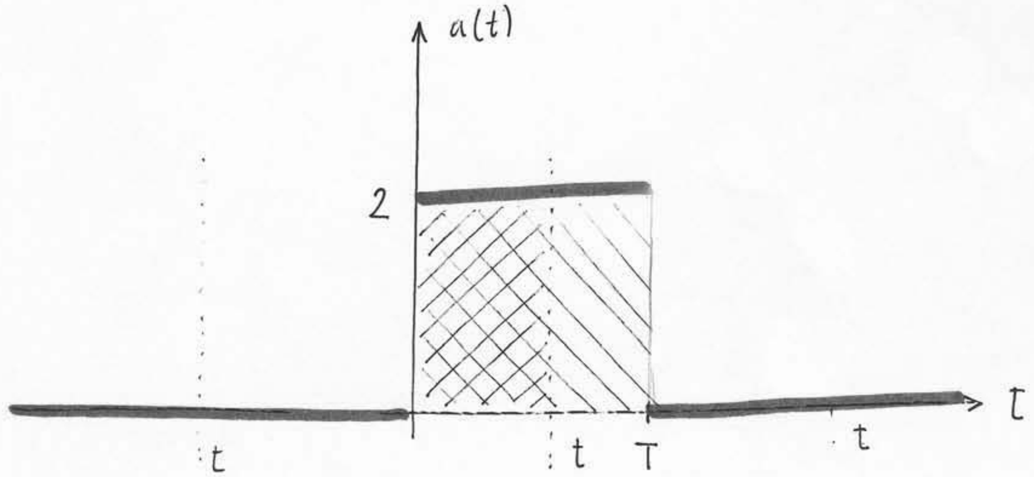
$T > 0$

$v_c(-1) = V_0$

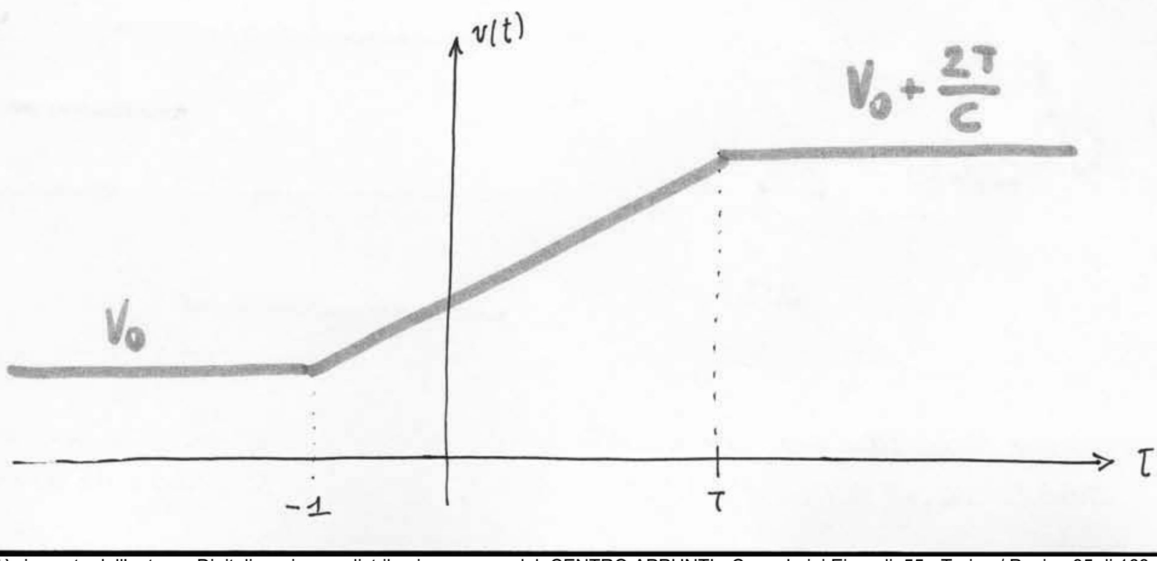
SERIES EQUIVALENT DOES NOT EXIST !!



$$v(t) = V_0 + \frac{1}{C}\int_{-1}^{t} i(t')\,dt'$$

- 

- $$v(t) = V_0 + \frac{1}{C}\int_{-1}^{t} a(t')\,dt' = \begin{cases} V_0 & -1 < t < 0 \\ V_0 + \frac{2}{C}t & 0 \le t \le T \\ V_0 + \frac{2}{C}T & t \ge T \end{cases}$$



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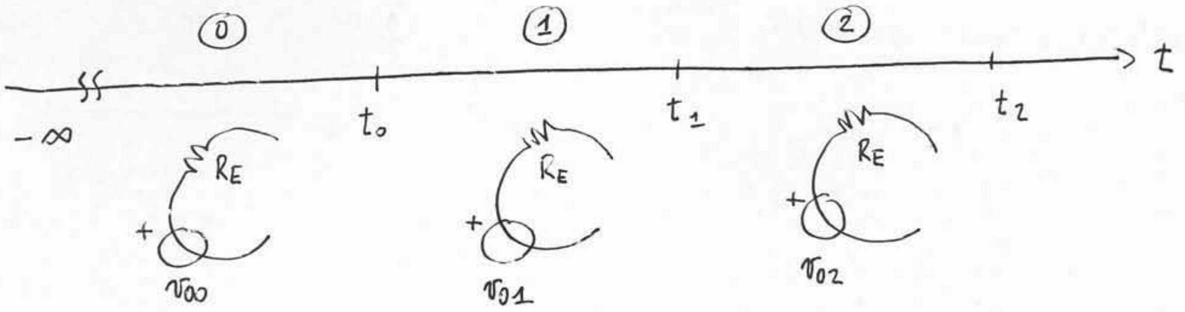


$$v(t) = v_{00}$$

$$v_1(t) = (v_{00} - v_{01})\, e^{-\frac{(t-t_0)}{\tau}} + v_{01} \qquad t_0 \le t \le t_1$$

$$v_2(t) = (v_1(t_1) - v_{02})\, e^{-(t-t_1)/\tau} + v_{02}$$

! DEPENDING ON THE LENGTH OF INTERVAL AND ON THE VALUE OF τ, THE FUNCTION CAN REACH OR NOT THE FINAL VALUE

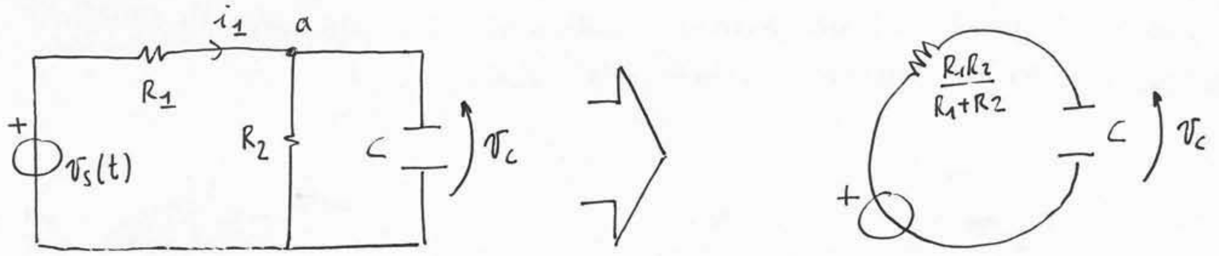
FIRST ORDER DYNAMIC CIRCUIT, RECIPE

1) PLOT SOURCE WAVEFORMS AND SUBDIVIDE TIME AXIS IN INTERVALS WHERE SOURCES ARE CONSTANT

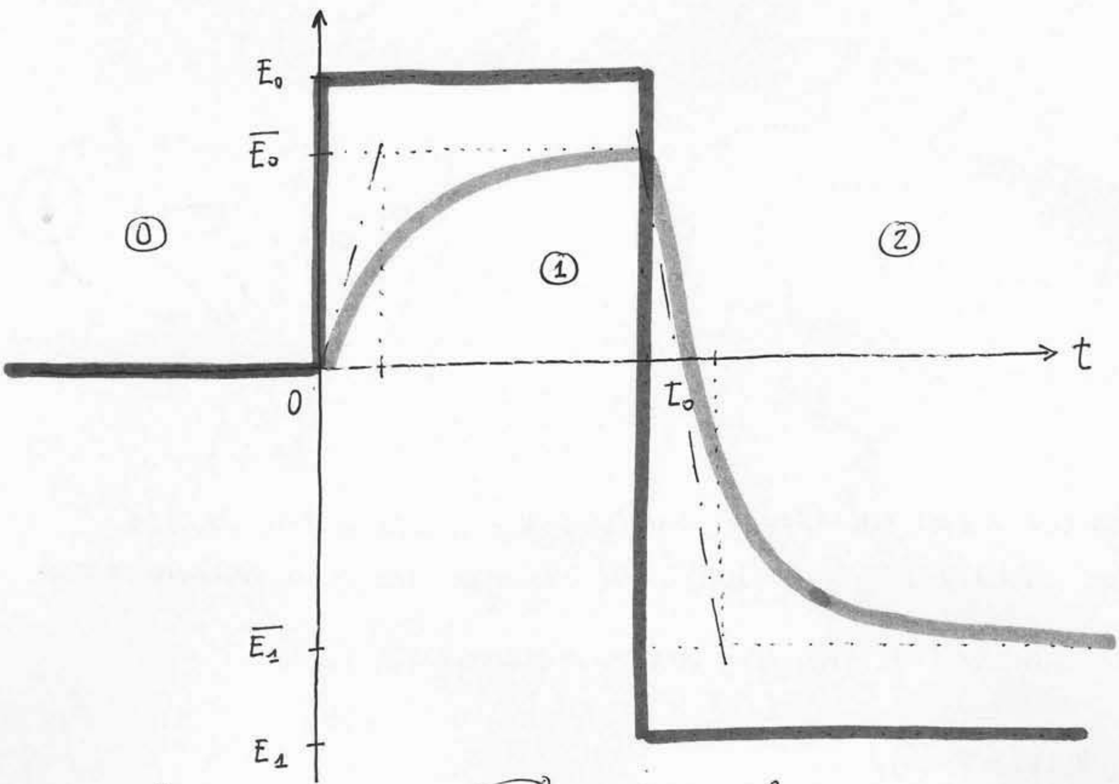
2) FOR EACH TIME INTERVAL, BUILD THE SERIES/PARALLEL EQUIVALENT OF THE RESISTIVE COMPOSITE ELEMENT DRIVING THE CAPACITOR/INDUCTOR

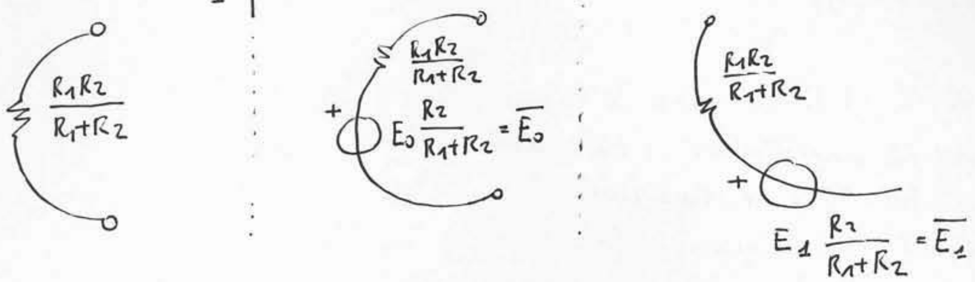
3) FOR EACH TIME INTERVAL, WRITE THE SOLUTION USING THE PARAMETERS OF THE EQUIVALENT OF THE INTERVAL

PROBLEM COMPUTE AND PLOT $v_c(t)$



$$\frac{G_1 v_s}{G_1+G_2} = v_s \frac{R_2}{R_1+R_2}$$





$\tau = R_E \cdot C$

let's suppose $\tau = \frac{1}{5} t_0$

⓪ $v_{00} = 0$

① $v_{c1} = \left(0 - E_0 \frac{R_2}{R_1+R_2}\right) e^{-\frac{(t-t_0)}{\tau}} + E_0 \frac{R_2}{R_1+R_2}$

② $v_{c2} = \left(v_{c1}(t=t_0) - E_1 \frac{R_2}{R_1+R_2}\right) e^{-\frac{(t-t_0)}{\tau}} + E_1 \frac{R_2}{R_1+R_2}$

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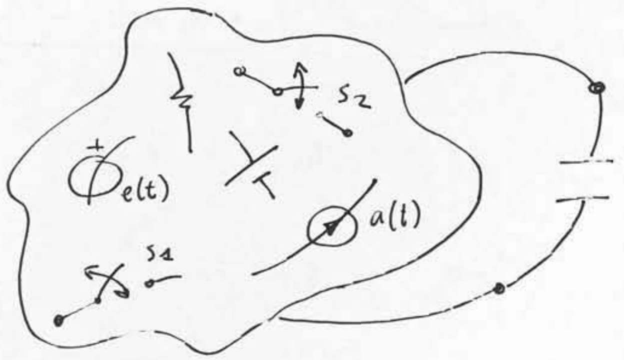
IDEAL SWITCHES



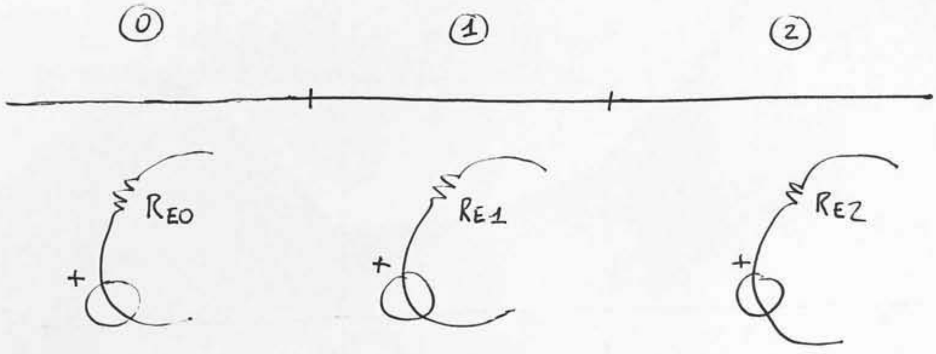
EVERYTHING AS BEFORE
SAME CONSIDERATIONS
SAME PROCEDURES

!!

ONE THING TO BE UPDATED: ONE HAS TO DIVIDE TIME AXIS INTO INTERVALS WITH → CONSTANT SOURCE VALUE
→ CONSTANT SWITCH STATE

ONE MINOR DIFFERENCE: WHEN SWITCH CHANGES ITS STATE, ALSO TOPOLOGY OF DRIVING ELEMENT CHANGES; CONSEQUENTLY, ALSO EQUIVALENT RESISTANCE CAN CHANGE

TIME CONSTANT IS DEFINED FOR EACH TIME INTERVAL; NO LONGER CONSTANT VALUE; R_E CHANGES ⇒ τ CHANGES



SOLUTION HAS THE SAME FORM AS BEFORE

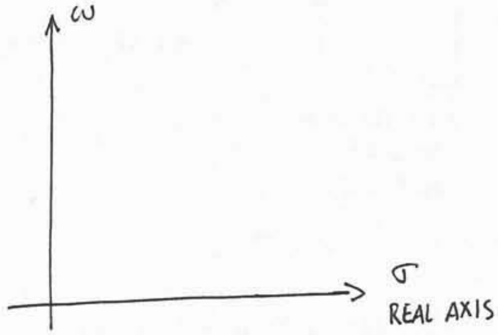
6- GENERAL DYNAMIC CIRCUITS

SINGLE SIDED LAPLACE TRANSFORM

ARBITRARY # OF CAPACITOR & INDUCTORS

INTEGRAL LINEAR TRANSFORMATION

$$F(s) = \mathcal{L}[f(t)] = \int_{0^-}^{\infty} f(t)e^{-st}\,dt$$

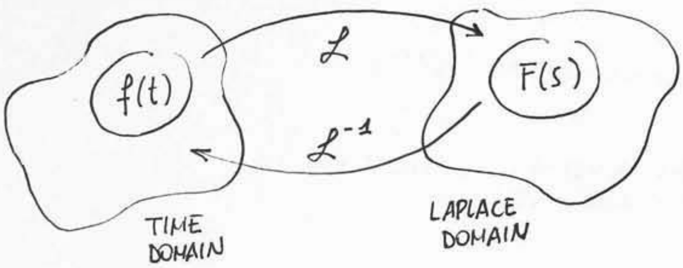


$s = \sigma + j\omega$

COMPLEX NUMBER: COMPLEX FREQUENCY

COMPLEX VARIABLE ⇒ COMPLEX RESULTS

WHEN THE INTEGRAL IS DEFINED, 1 TO 1 CORRESPONDENCE
THE ELEMENTS ARE PAIRED



SINGLE SIDED BECAUSE THE INTEGRAL IS DEFINED $[0, +\infty)$

0^- INCLUDED TO ALLOW FUNCTION TO WORK PROPERLY WITH THE TRANSFORMATION OF dt

$0^- = 0 - \varepsilon$

$\{f(t), F(s)\}$ = LAPLACE TRANSFORM PAIR

LOWER CASE LETTER FUNCTION OF TIME

UPPER CASE FUNCTION OF FREQUENCY

KEY PROPERTY

$\mathcal{L}[f(t)] = F(s)$ TRANSFER OF THE FUNCTION

DIFFERENTIAL OPERATOR IN TIME DOMAIN BECOMES LINEAR ALGEBRAIC IN FREQUENCY DOMAIN

$$\boxed{\mathcal{L}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0^-)}$$

SIMPLY LINEAR ALGEBRAIC EQUATION

REAL RATIONAL FUNCTIONS

- RATIO OF POLYNOMIALS
- $a_m, b_n \dots$ ARE <u>REAL</u> COEFFICIENTS

$$F(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_0}$$

$n > m$

$\{a_m, b_n\}$ Real

$$F(s) = \frac{N(s)}{D(s)}$$

$N(s) \leftarrow$ REPRESENTING THE POLYNOMIAL ON NUMERATOR

$D(s) \leftarrow$ REPRESENTING THE POLYNOMIAL ON DENOMINATOR

IF $n < m$, NUMERATOR CAN BE DIVIDED BY DENOMINATOR AND THE FUNCTION CAN BE WRITTEN IN THIS WAY

$$F(s) = \underbrace{\sum_K C_K s^K}_{\text{RESULT OF DIVISION}} + G(s)$$

$G(s) \leftarrow$ REAL RATIONAL FUNCTION PROPER FUNCTION

ROOTS OF POLYNOMIALS : n-ROOTS IN COMPLEX PLANE FOR BOTH NUMERATOR AND DENOMINATOR

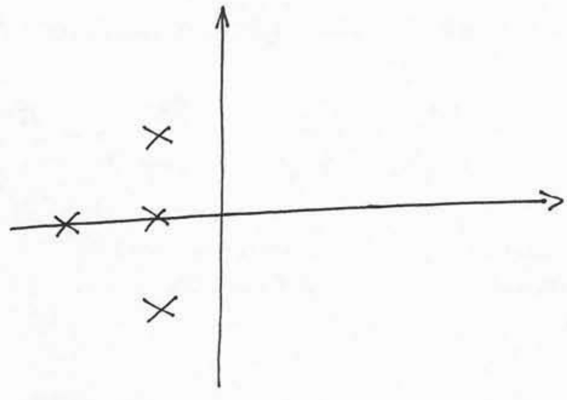
ROOTS OF POLYNOMIALS WITH REAL COEFFICIENTS CAN BE REAL OR IN PAIR COMPLEX CONJUGATES

THE ZEROS OF $N(s)$ ARE ALSO ZEROS OF THE FUNCTION

THE ZEROS OF $N(s)$ $\Rightarrow$ ZEROS OF FUNCTION NAMED <u>ZEROS</u>

THE ZEROS OF $D(s)$ $\Rightarrow$ POINTS OF COMPLEX PLANE WHERE FUNCTION DIVERGES, GOES TO INFINITY ; THEY ARE NAMED <u>POLES</u>

AT EVERY ZERO OF $D(s)$, ONE HAS SINGULARITY OF $F(s)$



THE 2 CONTRIBUTION ARE LUMPED IN SINGLE CONTRIBUTION OF THIS FORM

$$\frac{K_3}{s-p_3} + \frac{K_3^*}{s-p_3^*} \Rightarrow \frac{K_{31} s + K_{30}}{(s-\sigma_3)^2 + \omega_3}$$

DIMOSTRABILE COME DIFFERENZA DI QUADRATI

ONE KEEPS CONJUGATES TOGETHER $\Rightarrow$ THIS PRODUCT IS REAL

TO COMPUTE THE COEFFICIENT OF THE RESIDUES $\Rightarrow$ MANY TECNIQUES

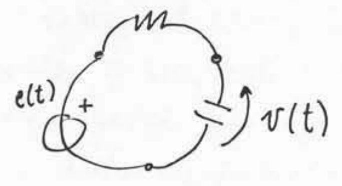
THE SIMPLEST ONE IS TO EQUATING $\frac{K}{s-p_1}$ $\frac{K_{21}}{s-p_2}$

ONE UTILITY CAN BE FOUND ON www.quickmath.com

THE LAPLACE METHOD (EAVISIDE ~ 1900)

SOLUTION PROCEDURE

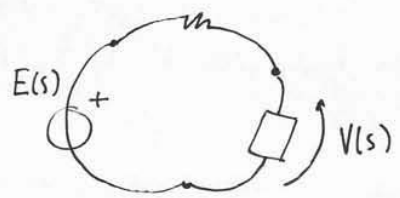
SUPPOSE THIS IS THE GIVEN PROBLEM: ONE DOES NOT NEED TO WRITE EQUATIONS IN TIME DOMAIN



(1) TRANSFORMED CIRCUIT

REPLACE ORIGINAL CIRCUIT WITH THE TRANSFORMED ONE; IN TRANSFORMED CIRCUIT, EVERY ELEMENT HAS ALGEBRAIC CHARACTERISTIC

$\Rightarrow$ PLANE LINEAR ALGEBRAIC



(2) ANALYSIS $V(s) = \dots$

ANY METHOD FOR LINEAR RESISTIVE CIRCUIT (OF COURSE, THE MOST CONVENIENT FOR THE PROBLEM ONE HAS)

(3) INVERSE TRANSFORMATION OF SOLUTIONS

$$\mathcal{L}^{-1}[V(s)] = v(t)$$