NUMERO: 703 DATA: 07/10/2013

# A P P U N T I

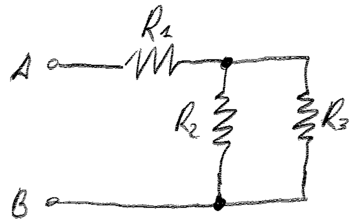
STUDENTE: Orefice

MATERIA: Elettrotecnica Esercizi + temi d'esame
Prof. Ragusa

ELETTROTECNICA

# ① CALCOLO di RESISTENZE EQUIVALENTI

1)




$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 2\Omega$

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = 1\Omega$$

$$R_{eq} = R_1 + R_{23} = 2\Omega$$

OK

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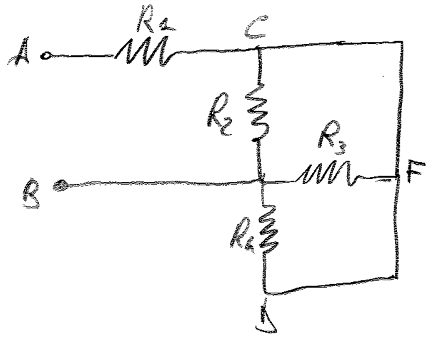
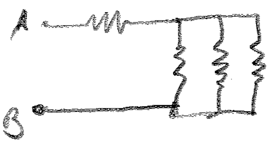
2)




$R_1 = 1\Omega$
$R_2 = 3\Omega$
$R_3 = 3\Omega$
$R_4 = 3\Omega$




$$R_{234} = \frac{R_2 R_3 R_4}{R_2R_3 + R_2R_4 + R_3R_4} = \frac{27}{9+9+9} = 1\Omega$$

$$\Rightarrow R_{eq} = 2\Omega$$

OK

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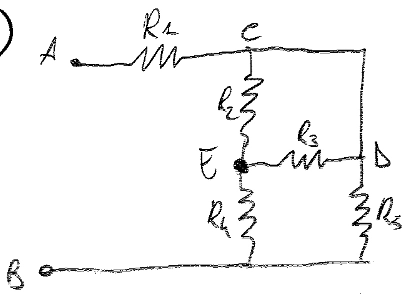
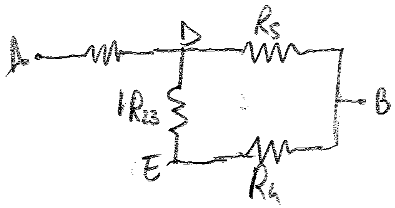
3)




$R_1 = 2\Omega$
$R_2 = 3\Omega$
$R_3 = 6\Omega$
$R_4 = 2\Omega$
$R_5 = 4\Omega$

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = \frac{18}{9} = 2\Omega$$




$$R_{234} = R_{23} + R_4 = 4\Omega$$

$$R_{2345} = \frac{R_{234} R_5}{R_{234} + R_5} = 2\Omega$$

$$R_{eq} = R_1 + R_{2345} = 4\Omega$$

OK

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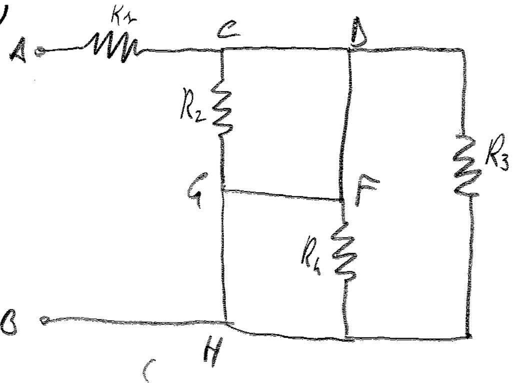
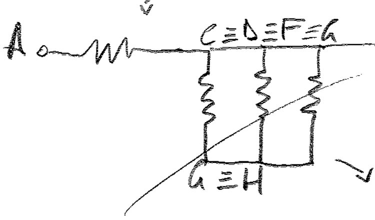
2)



$R_1 = 10\,\Omega$
$R_2 = 20\,\Omega$
$R_3 = 60\,\Omega$
$R_4 = 20\,\Omega$

SEGUO CORRENTE
A - C - D - F - G - H - B

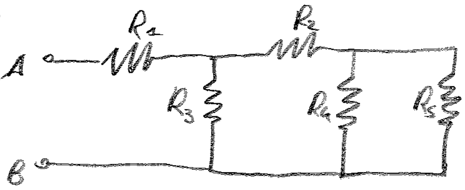


INFLUENTI

$\Rightarrow \; R_{eq} = R_1 = 10\,\Omega$

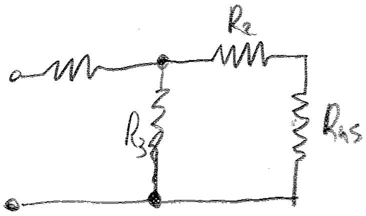
OK

3)



$R_1 = 1\,\Omega$
$R_2 = 3\,\Omega$
$R_3 = 3\,\Omega$
$R_4 = 3\,\Omega$
$R_5 = 1\,\Omega$

$R_{45} = \frac{3}{4}\,\Omega$



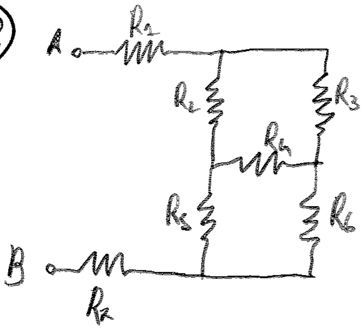
$R_{245} = 3 + \frac{3}{4} = \frac{15}{4}\,\Omega$

$R_{3245} = \dfrac{\frac{45}{4}}{\frac{27}{4}} = \frac{45}{27}\,\Omega$

$\Rightarrow R_{eq} = 1 + \frac{45}{27} = \frac{72}{27} = \frac{8}{3}\,\Omega$

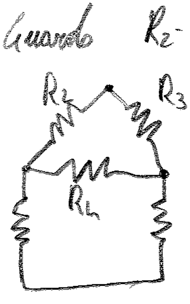
OK

4)



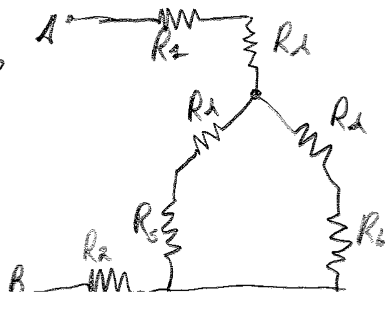
$R_1 = 1\,\Omega$
$R_2 = 3\,\Omega$
$R_3 = 3\,\Omega$
$R_4 = 3\,\Omega$
$R_5 = 2\,\Omega$
$R_6 = 2\,\Omega$
$R_7 = 1\,\Omega$

Guardo $R_2 - R_3 - R_4$



Sostituisco TRIANG $\rightarrow$ STELLA : $R_2 = R_3 = R_4$

$\Rightarrow$



$R_\lambda = \frac{1}{3} R_2 = 1\,\Omega$

$R_{\lambda 5} = 3\,\Omega$

$R_{\lambda 6} = 3\,\Omega$

$R_{1\lambda} = 2\,\Omega$

$R_{\lambda 56} = \frac{3}{2}\,\Omega$

$\Rightarrow R_{eq} = \frac{9}{2}\,\Omega$

OK

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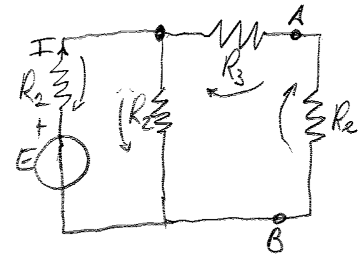
ELETTROTECNICA

# ② RETI RESISTIVE

1)

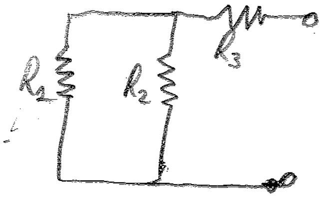



$E = 10\ V$
$R_1 = 4\ \Omega$
$R_2 = 6\ \Omega$
$R_3 = 2\ \Omega$
$R_e = 5\ \Omega$

Tb: a) Equivalente Thevenin AB
b) Corrente in Re

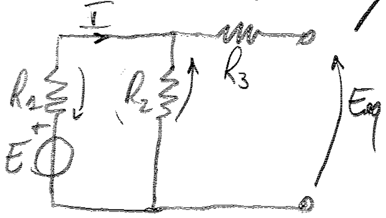
a) Per determinare l'equivalente di Thevenin ai morsetti A-B, si calcolano la TENSIONE a VUOTO e la Req.

* Req: Passiviamo i generatori:




$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} + R_3 = 4{,}4\ \Omega$$

* Eeq = Al posto dell'utilizzatore ai capi dei morsetti A-B, sostituisco Eeq, come se il circuito fosse lì APERTO



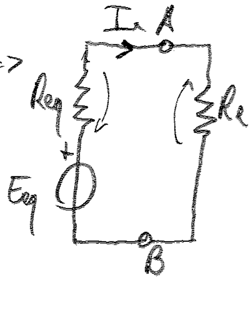

Essendo lì Aperto, NON circola corrente su $R_3$!

LKT1: $E - R_1 I - R_2 I = 0 \Rightarrow I = \frac{E}{R_1 + R_2}$

LKT2: $E - R_1 I - E_{eq} = 0 \Rightarrow E_{eq} = E - R_1 I$

$$\Rightarrow E_{eq} = E - \frac{R_1 E}{R_1 + R_2} = \frac{\cancel{ER_1} + ER_2 - \cancel{ER_1}}{R_1 + R_2} = \frac{E R_2}{R_1 + R_2} = \underline{\underline{6\ V}}$$

=>




LKT: $E_{eq} - R_{eq} I_e - R_e I_e = 0$

$$\Rightarrow I_e = \frac{E_{eq}}{R_{eq} + R_e} = \frac{6}{4{,}4 + 5} = \underline{\underline{0{,}638\ A}}$$

OK



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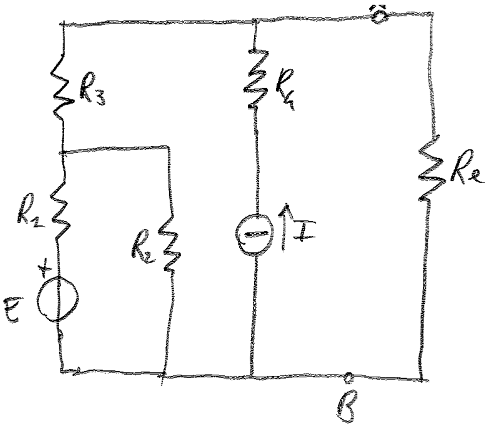
3)



$E = 10V$

$I = 2A$

$R_1 = R_2 = 4\Omega$

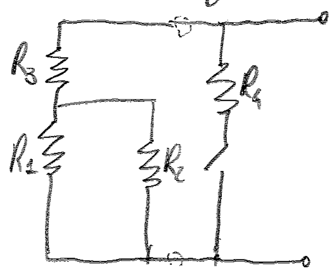
$R_3 = 2\Omega$

$R_4 = 5\Omega$

$R_e = 5\Omega$

Th: Eq Thevenin? Corrente in $R_e$

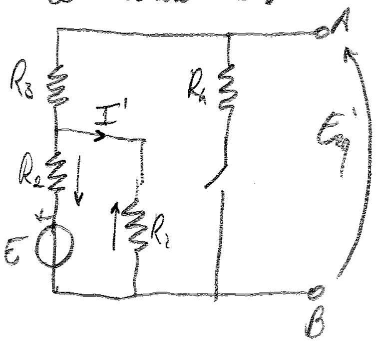
* $R_{eq}$: Passivo i generatori:



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} + R_3 = 4\Omega$$

* $E_{eq}$:

1) Contributo E:



Passivano I, lì il circuito è aperto
=> NON circola corrente su $R_3$ e $R_4$.

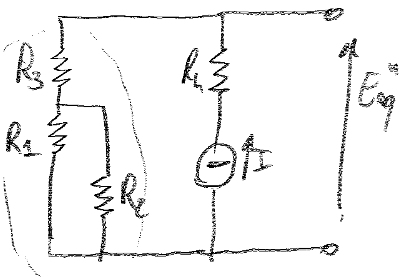
LKT1: $E - R_1 I' - R_2 I' = 0$

$$\Rightarrow I' = \frac{E}{R_1 + R_2}$$

LKT2: $E - R_1 I' - E_{eq}' = 0$

$$\Rightarrow E_{eq}' = E - \frac{R_1 E}{R_1 + R_2} = \frac{E R_2}{R_1 + R_2} =$$

2) Contributo I.

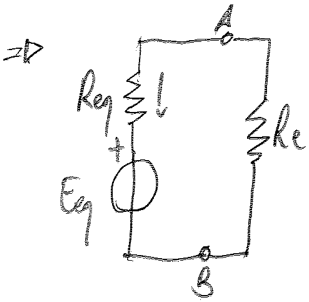


$$R_{eq}'' = \frac{R_1 R_2}{R_1 + R_2} + R_3 = 4$$

$$\Rightarrow E_{eq}'' = R_{eq}'' \cdot I = 8\,V$$

$$\Rightarrow E_{eq} = E_{eq}' + E_{eq}'' = 5 + 8 = 13V$$

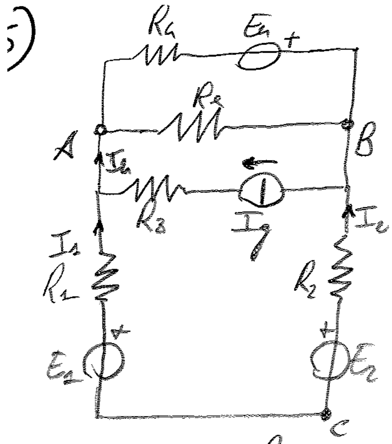
=>



$$I_e = \frac{E_{eq}}{R_{eq} + R_e} = \underline{1{,}444\ A}$$

OK

5)



$E_1 = 1V$

$E_2 = 1V$

$I_g = 2A$

$E_4 = 1V$

$R_1 = 1\Omega$

$R_2 = 1\Omega$

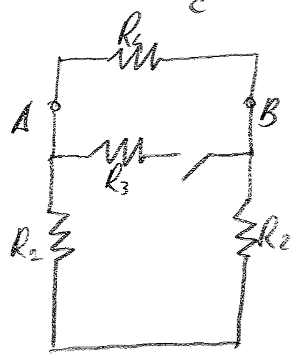
$R_3 = 1\Omega$

$R_4 = 8\Omega$

$R_e = \frac{3}{5}\Omega$

Th: Eq Thevenin AB?
Corrente $R_e$?

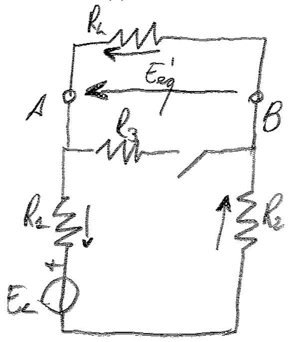
* $R_{eq}$:



$$R_{eq} = \frac{(R_1+R_2)R_4}{R_1+R_2+R_4} = \frac{16}{10} = 1{,}6\ \Omega$$

* $E_{eq}$ =

⊖ Contributo $E_1$:



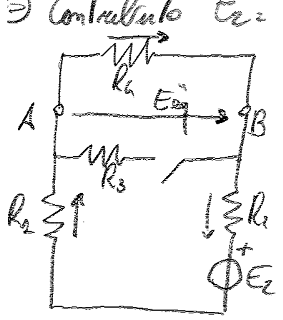
LKT1: $E_1 - R_1 I_1 - R_4 I_1 - R_2 I_1 = 0$

$$\Rightarrow I_1 = \frac{E_1}{R_1+R_2+R_4}$$

LKT2: $E_1 - E_{eq}' - R_1 I_1 - R_2 I_1 = 0$

$$E_{eq}' = E_1 - (R_1+R_2)I_1 = E_1 - \frac{(R_1+R_2)E_1}{R_1+R_2+R_4} = \frac{E_1 R_4}{R_1+R_2+R_4} = 0{,}8$$

⊖ Contributo $E_2$:



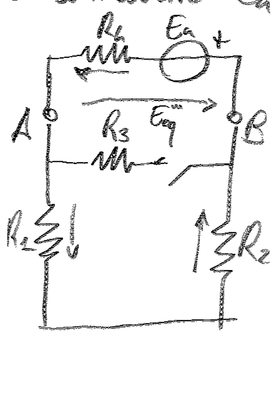
LKT1: $E_2 - R_2 I_2 - R_4 I_2 - R_1 I_2 = 0$

$$\Rightarrow I_2 = \frac{E_2}{R_1+R_2+R_4}$$

LKT2: $E_2 - R_2 I_2 - E_{eq}'' - R_1 I_2 = 0$

$$\Rightarrow E_{eq}'' = \frac{E_2 R_4}{R_1+R_2+R_4} = 0{,}8\ V$$

⊖ Contributo $E_4$:



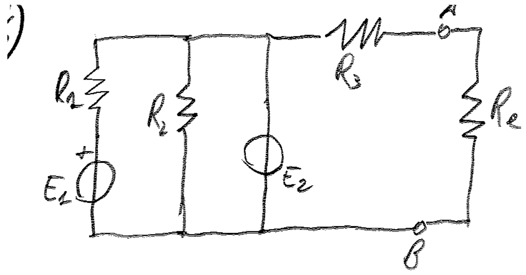
LKT1: $E_4 - R_2 I_4 - R_1 I_4 - R_4 I_4 = 0$

$$\Rightarrow I_4 = \frac{E_4}{R_1+R_2+R_4}$$

LKT2: $E_4 - E_{eq}''' - R_4 I_4 = 0$

$$\Rightarrow E_{eq}''' = E_4 - \frac{R_4 E_4}{R_1+R_2+R_4} = \frac{(R_1+R_2)E_4}{R_1+R_2+R_4} = \frac{2}{10} = 0{,}2\ V$$

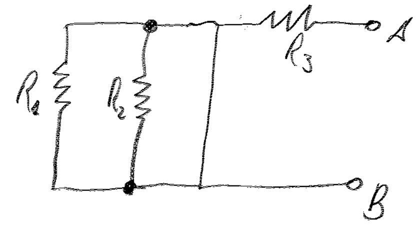
)



$E_1 = 12V$
$E_2 = 10V$
$R_1 = 10\Omega$
$R_2 = 12\Omega$
$R_3 = 1\Omega$
$R_e = 1\Omega$

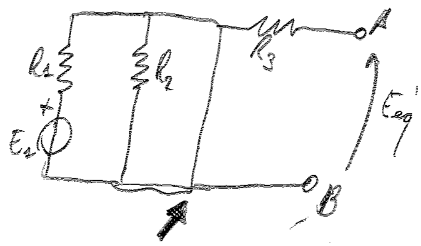
Tb: Eq Thevenin AB?
Corrente in Re?

*Req =



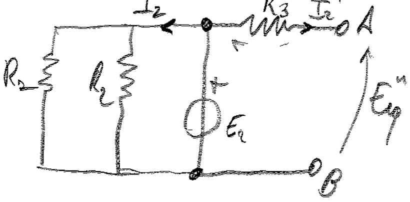
Le Resistenze $R_1$ e $R_2$ sono CORTOCIRCUITATE
$\Rightarrow R_{eq} = R_3$ !

*Eeq: ① Contributo $E_1$



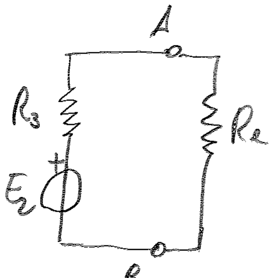
Cortocircuitando $E_2$, l'intera maglia a sx è cortocircuitata $\Rightarrow$ NON dà contributo
$\Rightarrow E'_{eq} = 0$

② Contributo $E_2$:



Essendo i rami in PARALLELO, essi si trovano alla stessa TENSIONE
$\Rightarrow E''_{eq} = E_2$ !

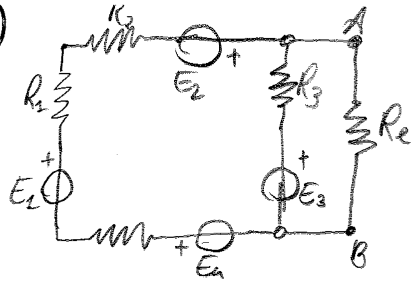
$\Rightarrow$



$$I_e = \frac{E_2}{R_3+R_e} = \frac{10}{1+1} = \underline{\underline{5A}}$$

OK

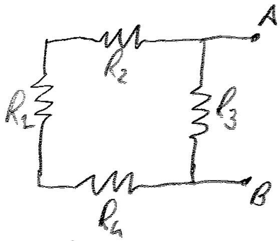
1)




$E_1 = 1V$
$E_2 = 1V$
$E_3 = 2V$
$E_4 = 1V$
$R_1 = 1\Omega$
$R_2 = 1\Omega$
$R_3 = R_4 = 2\Omega$
$R_e = 2\Omega$

Th: Eq. Thevenin AB
Corrente in Re

* $R_{eq}$ =

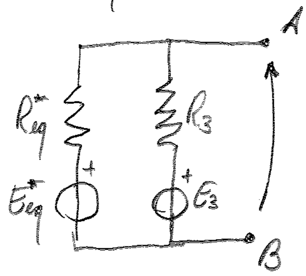



$$R_{eq} = \frac{(R_1+R_2+R_4)R_3}{R_1+R_2+R_3+R_4} = \frac{8}{6} = \underline{1{,}33\,\Omega}$$

* $E_{eq}$ = Ramo 1-2-4 In Serie

$\Rightarrow E^*_{eq} = E_1 + E_2 + E_4 = 3V$
$R^*_{eq} = R_1 + R_2 + R_4 = 4\Omega$

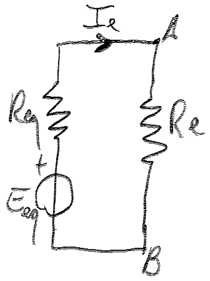
$\Rightarrow$




Ramo * e 3 In Parallelo $\Rightarrow$ MILLMAN:

$$V_{AB} = \frac{\frac{E^*_{eq}}{R^*_{eq}} + \frac{E_3}{R_3}}{\frac{1}{R^*_{eq}} + \frac{1}{R_3}} = \frac{\frac{3}{4} + \frac{2}{2}}{\frac{1}{4} + \frac{1}{2}} = \frac{\frac{7}{4}}{\frac{3}{4}} = \frac{7}{3} = \underline{2{,}33V}$$

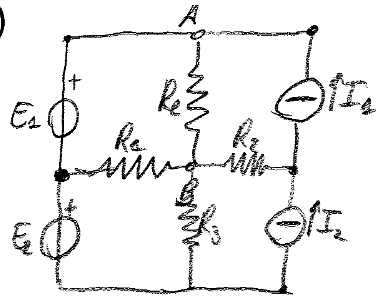
$\Rightarrow \begin{cases} R_{eq} = 1{,}33\,\Omega \\ E_{eq} = 2{,}33\,\Omega \end{cases}$




$$I_e = \frac{E_{eq}}{R_{eq}+R_e} = \frac{2{,}33}{1{,}33+2} = \underline{0{,}69\,A}$$

Ok

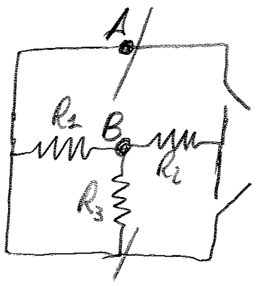
b)




$E_1 = 1V$
$E_2 = 1V$
$I_1 = 1A$
$I_2 = 1A$
$R_1 = 3\Omega$
$R_2 = 3\Omega$
$R_3 = 3\Omega$
$R_e = 2\Omega$

Tb: Eq Thevenin AB
Corrente in $R_e$

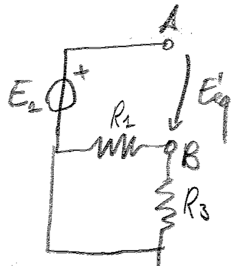
* $R_{eq}$:




$$R_{eq} = \frac{R_1 R_3}{R_1 + R_3} = \underline{\underline{1{,}5\Omega}}$$

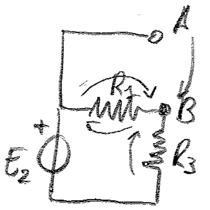
* $E_{eq}$: SOVRAPPOSIZIONE EFFETTI

1) $E_1$:




LKT1: $E_1 - E_{eq}' = 0 \Rightarrow E_{eq}' = E_1$

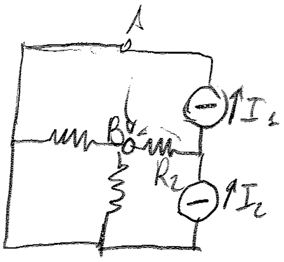
2) $E_2$:




$$E_2 - R_1 i - R_3 i = 0 \Rightarrow i = \frac{E_2}{R_1 + R_3}$$

$$E_{eq}'' - R_1 i = 0 \Rightarrow E_{eq}'' = + i \cdot R_1 = + \frac{R_1 E_2}{R_1 + R_3}$$

3) $I_1, I_2$:



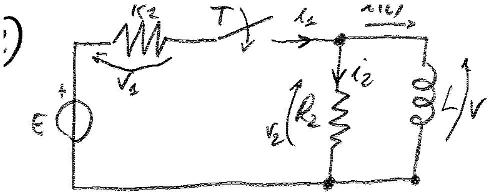

I generatori $I_1$ e $I_2$ trovano una maglia costituita da soli corto circuiti e, come tali, il loro contributo alla tensione AB è NULLO!

$$\Rightarrow E_{eq} = E_1 + \frac{R_1 E_2}{R_1 + R_3} = \underline{\underline{1{,}5V}}$$

$$\Rightarrow \underline{\underline{I_e}} = \frac{E_{eq}}{R_{eq} + R_e} = \frac{1{,}5}{1{,}5 + 2} = \underline{\underline{0{,}43A}}$$

OK

:)




$E = 10V$
$R_1 = R_2 = 1\Omega$
$L = 0{,}1\ H$
$i(0) = 0 \rightarrow$ T Chiuso
Poi T si apre a $t = 1s$ $\rightarrow$ Tb: $i(t)$?

Ta: $i(t)$?
corrente generatore?

• T Chiuso: LKT: $E = v_1 + v_2$ $\quad v_1 = R_1 i_1$
LKC: $i_1 = i_2 + i$ $\quad v_2 = R_2 i_2$
LKT: $v_2 = V$ $\quad v_2 = L\frac{di}{dt}$

$$\Rightarrow \frac{v_1}{R_1} = \frac{v_2}{R_2} + i \rightarrow \frac{E - v_2}{R_1} = \frac{L}{R_2}\frac{di}{dt} + i \rightarrow \frac{E}{R_1} = \frac{L}{R_2}\frac{di}{dt} + \frac{L}{R_1}\frac{di}{dt} + i$$

$$\rightarrow \frac{E}{R_1} = L\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{di}{dt} + i \rightarrow \frac{di}{dt} + \left(\frac{1}{L}\frac{R_1R_2}{R_1+R_2}\right)i = \frac{E R_1 R_2}{L R_1(R_1+R_2)}$$

$$\Rightarrow \text{Polin. Caratt.: } s + \frac{1}{L}\left(\frac{R_1R_2}{R_1+R_2}\right) = 0 \Rightarrow s = -\frac{1}{L}\left(\frac{R_1R_2}{R_1+R_2}\right)$$

$$\Rightarrow \tau = \left|\frac{1}{s}\right| = \frac{L(R_1+R_2)}{R_1R_2}$$

$$\Rightarrow \text{Soluz. Omog.: } i_0(t) = K e^{-t\frac{R_1R_2}{L(R_1+R_2)}}$$

CORRENTE REGIME

• Soluz. Part.: $i_p = \text{cost} \Rightarrow \underbrace{\frac{di_p}{dt}}_{=0}\left(\frac{L}{R_2} + \frac{L}{R_2}\right) + i_p = \frac{E}{R_1} \Rightarrow \boxed{i_p = \frac{E}{R_1}}$

$$\Rightarrow \text{SOLUZ.: } \begin{cases} i(t) = K e^{-\frac{R_1R_2 t}{L(R_1+R_2)}} + \frac{E}{R_1} \\ i(0) = 0 \end{cases} \Rightarrow K = -\frac{E}{R_1}$$

$$\Rightarrow i(t) = \frac{E}{R_1}\left(1 - e^{-\frac{R_1R_2 t}{L(R_1+R_2)}}\right) = \underline{\underline{10(1 - e^{-5t})\ A}}$$

• $i_2 = \frac{v_2}{R_2} = \frac{V}{R_2} = \frac{L}{R}\frac{di}{dt} = \frac{5\cancel{0}}{\cancel{10}}e^{-5t} = \underline{\underline{5e^{-5t}\ A}}$

$\Rightarrow i_1 = i_2 + i = \underline{\underline{5(2 - e^{-5t})\ A}}$

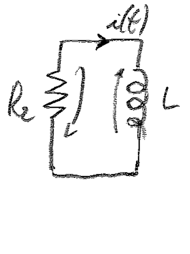
RA: $\tau = 0{,}2s \Rightarrow$ REGIME $= 4\tau = 0{,}8\ s \Rightarrow$ a $t = 1s$ già è in REGIME

$\Rightarrow i(1) = \frac{E}{R_1} = \underline{\underline{10\ A}}$

• T = APERTO:

MANCA GENERATORE = TERMINE FORZANTE

$\Rightarrow$ OMOGENEA

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* Tempo Necessario: $\lim_{t\to\infty} i(t) = 5A \Rightarrow 50\% = 2{,}5A$

$\Rightarrow 2{,}5 = 5(1-e^{-100t}) \rightarrow 2{,}5 = 5e^{-100t}$

$0{,}5 = e^{-100t} \Rightarrow -100t = \ln(0{,}5)$

$\Rightarrow \underline{\underline{t = 69{,}3\ ms}}$

: Energia Immagazzinata:

A Regime: $\lim_{t\to\infty} i(t) = 5A$

$\Rightarrow \underline{\underline{W}} = \frac{1}{2}Li^2 = \frac{1}{2}10\cdot10^{-3}\cdot25 = \underline{\underline{0{,}125\ J}}$

OK

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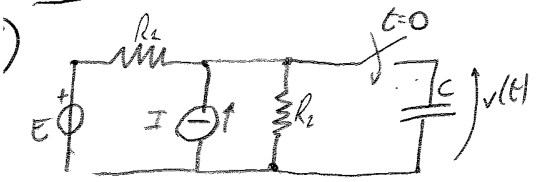
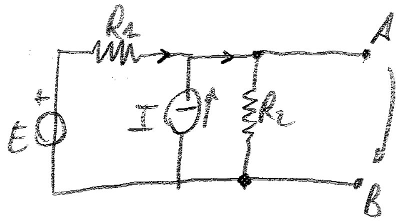



$E = 10V$
$I = 1A$
$R_1 = R_2 = 2\Omega$
$C = 10\mu F$
$v(0) = 0$

Th: • Eq Thevenin ai morsetti di C?
• $v(t)$?
• corrente in $R_2$?
• Energia Immagazzinata a regime?

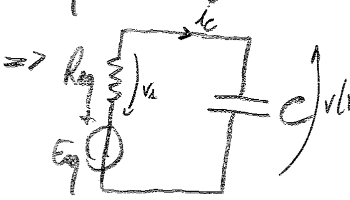



MILLMAN.

$$V_{AB} = \frac{\frac{E}{R_1} + I}{\frac{1}{R_1} + \frac{1}{R_2}} = 6V = E_{eq}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = 1\Omega$$

$\Rightarrow$




$\begin{cases} E_{eq} = 6V \\ R_{eq} = 1\Omega \end{cases}$

LKT: $E_{eq} - R_{eq} i_c - v(t) = 0$

$\Rightarrow E_{eq} - R_{eq} C \frac{dv}{dt} - v = 0 \Rightarrow \frac{dv}{dt} + \frac{1}{R_{eq}C} v = \frac{E_{eq}}{R_{eq}C}$

$\Rightarrow s + \frac{1}{RC} = 0 \Rightarrow s = -\frac{1}{RC} \Rightarrow \tau = \left|\frac{1}{s}\right| = RC$

$\Rightarrow$ Soluz Omog: $v_o(t) = Ke^{-\frac{t}{RC}}$

• Soluz. Particolare: $v_p = cost \Rightarrow \frac{1}{RC} v_p = \frac{E}{RC} \Rightarrow v_p = E$

$\Rightarrow$ SOLUZ: $\begin{cases} v(t) = Ke^{-\frac{t}{RC}} + E \\ v(0) = 0 \end{cases} \Rightarrow K = -E \Rightarrow \underline{v(t) = E(1-e^{-\frac{t}{RC}})}$

$\underline{= 6(1-e^{-\frac{t}{1\cdot10^{-5}}})V}$

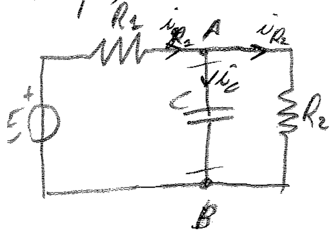
1, 1

Tasto $T_2$ chiuso a $t=10\mu s$:

$\tau = RC = 2\cdot10^{-6} = 2\mu s \Rightarrow t = 10\mu s \gg 4\tau \Rightarrow$ siamo a REGIME RAGGIUNTO!

$v(10\mu s) = 10\,V$

$i(10\mu s) = 0\,A$




LKC: $i_{R_1} = i_C + i_{R_2}$

$$\frac{v_1}{R_1} = C\frac{dv}{dt} + \frac{v_2}{R_2}$$

LKT: $v = v_2$

LKT: $E - v_1 - v = 0 \Rightarrow v_1 = E - v$

$$\Rightarrow \frac{E-v}{R_1} = C\frac{dv}{dt} + \frac{v}{R_2} \rightarrow \frac{E}{R_1} = v\left(\frac{1}{R_1}+\frac{1}{R_2}\right) + C\frac{dv}{dt} \rightarrow \frac{E}{R_1C} = v\left(\frac{1}{CR_1}+\frac{1}{CR_2}\right) + \frac{dv}{dt}$$

$$\Rightarrow s + \frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = 0 \rightarrow s = -\frac{1}{C}\left(\frac{R_1+R_2}{R_1R_2}\right) \Rightarrow \tau = \left|\frac{1}{s}\right| = \frac{CR_1R_2}{R_1+R_2}$$

$\Rightarrow$ Soluz Omog: $v_0(t) = Ke^{-\frac{t(R_1+R_2)}{CR_1R_2}}$

* Soluz Part: $v_p = \text{cost} \Rightarrow v_p = \frac{E}{\cancel{R_1C}} \cdot \frac{\cancel{C}R_1R_2}{R_1+R_2} = \frac{ER_2}{R_1+R_2}$

$$\Rightarrow \text{SOLUZ}: \begin{cases} v(t) = Ke^{-\frac{t(R_1+R_2)}{CR_1R_2}} + \frac{ER_2}{R_1+R_2} \\ v(0) = 10 \end{cases} \Rightarrow K + \frac{ER_2}{R_1+R_2} = 10 \rightarrow K = 5\,V$$

$$\Rightarrow \underline{v(t) = 5\left(1 + e^{-\frac{t}{10^{-6}}}\right)V}$$

$$\underline{i_{R_1}} = \frac{v_1}{R_1} = \frac{E - v(t)}{R_1} = 5 - \frac{5}{2}\left(1+e^{-\frac{t}{10^{-6}}}\right) = \underline{2{,}5\left(1+e^{-\frac{t}{10^{-6}}}\right)A}$$

Energia Immagazzinata a Regime:

COND $\Rightarrow W = \frac{1}{2}CV_{reg}^2$

$$V_{reg} = \lim_{t\to\infty} v(t) = \lim_{t\to\infty}\left(5 + 5e^{-\frac{t}{10^{-6}}}\right) = 5\,V$$

$$\Rightarrow \underline{W} = \frac{1}{2}\cdot10^{-6}\cdot5^2 = \underline{12{,}5\,\mu J}$$

OK



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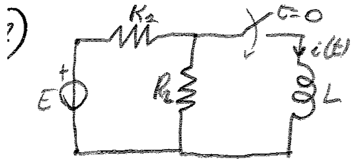
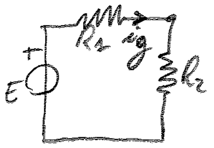



$E = 10V$
$R_1 = R_2 = 10\Omega$
$L = 1mH$

Tb: corrente nel generatore $t<0$
$i(t)$ per $t>0$
corrente nel generatore per $t>0$
Potenza Max erogata dal Generatore

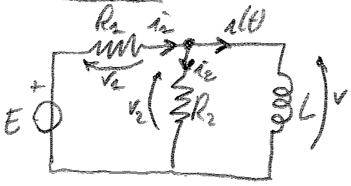
$t<0$ : $i_1$




LKT: $E - R_1 i_g - R_2 i_g = 0$

$\Rightarrow i_g = \dfrac{E}{R_1+R_2} = \underline{\underline{0{,}5\,A}}$

$t>0$ : $i(t)$




LKC: $i_1 = i_2 + i$

$\dfrac{v_1}{R_1} = \dfrac{v_2}{R_2} + i$

LKT: • $E - v_1 - v_2 = 0 \rightarrow v_1 = E - v$

• $v_2 = v$

$\Rightarrow \dfrac{E-v}{R_1} = \dfrac{v}{R_2} + i \rightarrow \dfrac{E}{R_1} = v\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right) + i \rightarrow \dfrac{E}{R_1} = L\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)\dfrac{di}{dt} + i$

$\rightarrow \dfrac{E R_1 R_2}{R_1 L (R_1+R_2)} = \dfrac{di}{dt} + \dfrac{R_1 R_2}{L(R_1+R_2)} i \Rightarrow$ Indico con $R = \dfrac{R_1 R_2}{R_1+R_2}$

$\Rightarrow s + \dfrac{R}{L} = 0 \rightarrow s = -\dfrac{R}{L} \Rightarrow \tau = \left|\dfrac{1}{s}\right| = \dfrac{L}{R}$

$\Rightarrow$ Soluz Omog: $i_0(t) = K e^{-\frac{tR}{L}}$

* Soluz Part: $i_p = cost \Rightarrow i_p = \dfrac{E}{R_1} \Rightarrow$ SOLUZ $\begin{cases} i(t) = K e^{-\frac{tR}{L}} + \dfrac{E}{R_1} \\ i(0) = 0 \end{cases} \Rightarrow K = -\dfrac{E}{R_1}$

$\Rightarrow i(t) = \dfrac{E}{R_1}\left(1 - e^{-\frac{tR}{L}}\right) = \underline{\underline{1 - e^{-\frac{5t}{10^{-3}}}\,A}}$

• $i_1$: $i_1 = \dfrac{v_1}{R_1} = \dfrac{E-v}{R_1} = \dfrac{E}{R_1} - \dfrac{L}{R_1}\dfrac{di}{dt} = 1 - \dfrac{10^{-3}}{10}\left(+5\cdot 10^{3} e^{-\frac{5t}{10^{-3}}}\right) = \underline{\underline{1 - 0{,}5\, e^{-\frac{5t}{10^{-3}}}\,A}}$

• Potenza MAX Erogata dal Generatore

$\Rightarrow P_E = \dfrac{E^2}{R_1} = \dfrac{100}{10} = \underline{\underline{10\,W}}$



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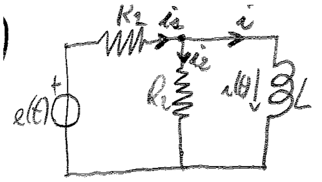
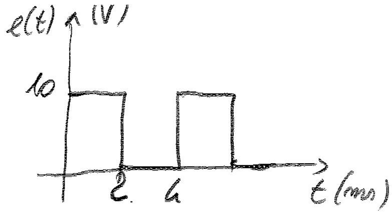
1)




$E = 10\,V$
$R_1 = R_2 = 2\Omega$
$L = 1\,mH$
$i(0) = 0$

Ib. $i(t)$: $0 \le t \le 2ms$
$i(t)$: $2 \le t \le 4\,ms$
Corrente generatore: $0 \le t \le 4$

a) $0 \le t \le 2\,ms$:

$e(t) = E$

LKT: $E - R_1 i_1 - R_2 i_2 = 0$

LKC: $i_1 = i_2 + i \Rightarrow i_1 = \frac{L}{R_2}\frac{di}{dt} + i$

LKT: $R_2 i_2 = v_L = L\frac{di}{dt} \Rightarrow i_2 = \frac{L}{R_2}\frac{di}{dt}$

$$\Rightarrow E - R_1\left(\frac{L}{R_2}\frac{di}{dt} + i\right) - R_2\left(\frac{L}{R_2}\frac{di}{dt}\right) = 0$$

$$E - \frac{R_1}{R_2}L\frac{di}{dt} - R_1 i - L\frac{di}{dt} = 0 \rightarrow E - R_1 i - \frac{di}{dt}\left(\frac{LR_1}{R_2} + L\right) = 0$$

$$\rightarrow \frac{ER_2}{LR_1 + LR_2} = \frac{R_1R_2}{L(R_1+R_2)}i + \frac{di}{dt} \Rightarrow s + \frac{R_1R_2}{L(R_1+R_2)} = 0 \Rightarrow s = -\frac{R_1R_2}{L(R_1+R_2)}$$

$$\Rightarrow \tau = \left|\frac{1}{s}\right| = \frac{L(R_1+R_2)}{R_1R_2} \Rightarrow \text{Soluz Omog: } i_0(t) = Ke^{-t\frac{R_1R_2}{L(R_1+R_2)}}$$

Soluz Part: $i_p = \text{cost} \Rightarrow i_p = \frac{E}{R_1}$

$$\Rightarrow \text{SOLUZ} \begin{cases} i(t) = Ke^{-\frac{tR_1R_2}{L(R_1+R_2)}} + \frac{E}{R_1} \\ i(0) = 0 \end{cases} \rightarrow K = -\frac{E}{R_1} \Rightarrow i(t) = \frac{E}{R_1}\left(1 - e^{-\frac{tR_1R_2}{L(R_1+R_2)}}\right)$$

$$\Rightarrow i(t) = 5(1 - e^{-10^3 t})\ A$$

• $i_2 = \frac{v_L}{R_2} = \frac{L}{R_2}\frac{di}{dt} = \frac{\cancel{L}}{R_2}\left(5\cdot\cancel{10^3}e^{-10^3 t}\right) = 2{,}5\,e^{-10^3 t}\ A$

$$\Rightarrow i_1 = i_2 + i = 5 - 5e^{-10^3 t} + 2{,}5e^{-10^3 t} = 5 - 2{,}5\,e^{-10^3 t}\ A$$

b) $2 \le t \le 4\,ms$:

$e(t) = 0$

LKT: $-R_1 i_1 - R_2 i_2 = 0$

LKC: $i_1 = i + i_2$

LKT: $R_2 i_2 = v_L = L\frac{di}{dt} \Rightarrow i_2 = \frac{L}{R_2}\frac{di}{dt}$

$$\Rightarrow -R_1\left(\frac{L}{R_2}\frac{di}{dt} + i\right) - R_2\left(\frac{L}{R_2}\frac{di}{dt}\right) = 0 \rightarrow -R_1 i - \frac{di}{dt}\left(\frac{LR_1}{R_2} + L\right) = 0$$

$$\Rightarrow \frac{R_1R_2}{L(R_1+R_2)}i + \frac{di}{dt} = 0 \Rightarrow \begin{cases} i(t) = Ke^{-\frac{tR_1R_2}{L(R_1+R_2)}} \\ i(2) = 5(1 - e^{-10^3\cdot 2\cdot 10^{-3}})\,A \end{cases} \Rightarrow K = 4{,}32$$

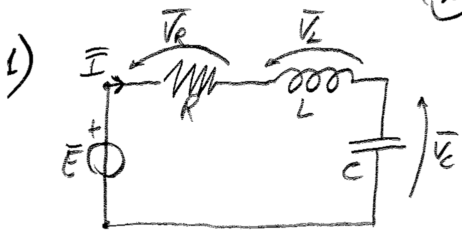
$$= 4{,}32\,A$$

/10/

ELETTROTECNICA

# 7 RETI IN REGIME SINUSOIDALE

1)




$E = 100\,V$
$R = 50\,\Omega$
$C = 100\,\mu F$
$L = 0{,}05\,H$
$f = 50\,Hz$

Ib Fasore Corrente
Diagramma Vettoriale Tensioni

$\omega = 2\pi f = 314{,}16$ rad/s

• LKT: $\overline{E} - \overline{V}_R - \overline{V}_L - \overline{V}_C = 0$

$$E - R\overline{I} - j\omega L\overline{I} - \frac{\overline{I}}{j\omega C} = 0 \rightarrow \overline{E} - \underbrace{R\overline{I} - j\overbrace{\left(\omega L\overline{I} - \frac{1}{\omega C}\overline{I}\right)}^{X}}_{\overline{\overline{Z}}_{eq}\cdot\overline{I}} = 0$$

$$\rightarrow \overline{E} - \overline{Z}_{eq}\overline{I} = 0 \Rightarrow \overline{I} = \frac{\overline{E}}{\overline{Z}_{eq}}$$

$$\Rightarrow I = \frac{E}{Z_{eq}} \quad \text{dove} \quad Z_{eq} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{R^2 + \left(\frac{\omega^2 LC - 1}{\omega C}\right)^2}$$

$$= \underline{52{,}53\,\Omega}$$

$$\Rightarrow I = \frac{100}{52{,}53} = 1{,}9\,A$$

• $\overline{I} = Ie^{j\varphi}$ $\qquad \varphi_Z = \operatorname{arctg}\left(\frac{\frac{\omega^2 LC - 1}{\omega C}}{R}\right) = -12{,}8° \Rightarrow \overline{I} = \frac{Ee^{j0}}{Z_{eq}\,e^{-j12{,}8}} \Rightarrow \varphi_I = 12{,}8°$

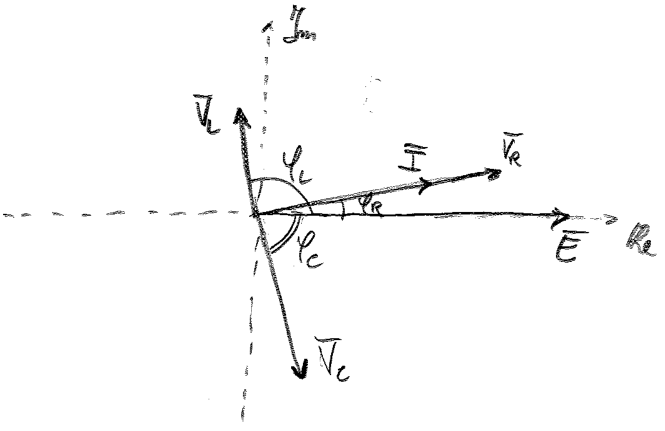
$$\Rightarrow \underline{\underline{\overline{I} = 1{,}9\,e^{+j12{,}8}\,A}}$$

• $\overline{V}_R = R\overline{I} \Rightarrow \begin{cases} V_R = RI \\ \varphi_V = \varphi_I \end{cases} \rightsquigarrow \begin{cases} V_R = 95\,V \\ \varphi_R = +12{,}8° \end{cases}$

$\overline{V}_L = j\omega L\overline{I} \Rightarrow \begin{cases} V_L = \omega LI \\ \varphi_V = \varphi_I + \frac{\pi}{2} \end{cases} \rightsquigarrow \begin{cases} V_L = 29{,}8\,V \\ \varphi_L = +12{,}8 + 90 = 102{,}8° \end{cases}$

$\overline{V}_C = -\frac{j}{\omega C}\overline{I} \Rightarrow \begin{cases} V_C = \frac{I}{\omega C} \\ \varphi_V = \varphi_I - \frac{\pi}{2} \end{cases} \rightsquigarrow \begin{cases} V_C = 60{,}5\,V \\ \varphi_C = -22{,}2° \end{cases}$

$\Rightarrow$

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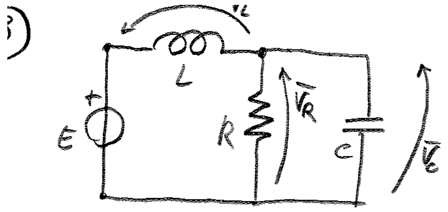
3)




$E = 100\,V$
$R = 50\,\Omega$
$C = 10\,\mu F$
$L = 0{,}1\,H$
$f = 50\,Hz$

Tb = Diagramma Vett Correnti e Tensioni

• $\omega = 2\pi f = 314\ rad/s$

$\bar{Z}_L = j\omega L = 31{,}4j\,\Omega$

$\bar{Z}_R = R = 50\,\Omega$

$\bar{Z}_C = \frac{-j}{\omega C} = -318{,}5j\,\Omega$

$\bar{V}_R = \bar{V}_C \Rightarrow \bar{Z}_R \bar{I}_R = \bar{Z}_C \bar{I}_C$

$\bar{E} - \bar{V}_L - \bar{V}_R = 0$

$\bar{I}_L = \bar{I}_R + \bar{I}_C = \frac{\bar{Z}_C}{\bar{Z}_R}\bar{I}_C + \bar{I}_C$

$\Rightarrow \bar{E} - \bar{Z}_L \bar{I}_L - \bar{Z}_R \bar{I}_R = 0$

$\bar{E} - \bar{Z}_L \bar{I}_L - \bar{Z}_R(\bar{I}_L - \bar{I}_C) = 0 \rightarrow \bar{E} - \bar{Z}_L \bar{I}_L - \bar{Z}_R\left(\bar{I}_L - \frac{\bar{Z}_R \bar{I}_L}{\bar{Z}_C + \bar{Z}_R}\right) = 0$

$\rightarrow \bar{E} - \bar{Z}_L \bar{I}_L - \bar{Z}_R \bar{I}_L + \frac{\bar{Z}_R^2 \bar{I}_L}{\bar{Z}_C + \bar{Z}_R} = 0 \rightarrow \frac{\bar{E}}{\bar{Z}_L + \bar{Z}_R - \frac{\bar{Z}_R^2}{\bar{Z}_C + \bar{Z}_R}} = \bar{I}_L$

$\rightarrow \frac{\bar{E}(\bar{Z}_C + \bar{Z}_R)}{\bar{Z}_L\bar{Z}_C + \bar{Z}_L\bar{Z}_R + \bar{Z}_R\bar{Z}_C + \cancel{\bar{Z}_R^2} - \cancel{\bar{Z}_R^2}} = \bar{I}_L$

$\Rightarrow \bar{I}_L = \frac{100(50 - 318{,}5j)}{-10'000{,}9j^2 + 1570j - 15925j} = \frac{100(50 - 318{,}5j)}{10'000{,}9 - 14355j} =$

$= \frac{100(50 - 318{,}5j)(10'000{,}9 + 14355j)}{306084025{,}8} =$

$= \underline{507211250 - 246753665j} = 1{,}65 - 0{,}80j$

$\Rightarrow$ • $I_L = |\bar{I}_L| = \sqrt{(1{,}65)^2 + (0{,}80)^2} = 1{,}83\,A$ ; $\varphi_{I_L} = \operatorname{arctg}\left(\frac{0{,}80}{1{,}65}\right) = -25{,}8°$

• $\bar{V}_L = \bar{Z}_L \bar{I}_L = 31{,}4j \cdot (1{,}65 - 0{,}80j) = 25{,}12 + 51{,}81j$

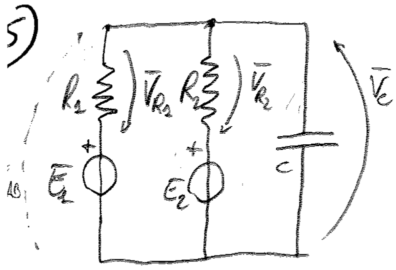
$\Rightarrow \begin{cases} V_L = |\bar{V}_L| = 57{,}6\ V \\ \varphi_{V_L} = \operatorname{arctg}\left(\frac{Im(\bar{V}_L)}{Re(\bar{V}_L)}\right) = 64{,}13° \end{cases}$

$\Rightarrow$ • $\bar{V}_R = \bar{E} - \bar{V}_L = 100 - 25{,}12 - 51{,}81j = (74{,}88 - 51{,}81j)\,V$

$\Rightarrow \begin{cases} V_R = |\bar{V}_R| = 91\,V \\ \varphi_R = \ldots = -34{,}67° \end{cases}$

• $\bar{V}_C = \bar{V}_R \Rightarrow \bar{V}_C = 74{,}88 - 51{,}81j\ V \Rightarrow \begin{cases} V_C = |\bar{V}_C| = 91\,V \\ \varphi_C = \ldots = -34{,}67° \end{cases}$

5)



$\bar{E}_1 = \bar{E}_2 = 100V$

$\varphi_1 = 0°$, $\varphi_2 = 90°$

$R_1 = R_2 = 10\Omega$

$C = 50\mu F$

$f = 50 Hz$

Ib. Corrente

- $\omega = 2\pi f = 314$ rad/s
- Posso sostituire i lati in parallelo con un Equivalente di Norton:

$\bar{Z}_{R_1} = R_1$

$\bar{Z}_{R_2} = R_2$

$\bar{Z}_C = \frac{-j}{\omega C} = -63{,}7j$

$$\Rightarrow \bar{V}_{AB} = \frac{\frac{\bar{E}_1}{\bar{Z}_{R_1}} + \frac{\bar{E}_2}{\bar{Z}_{R_2}}}{\frac{1}{\bar{Z}_{R_1}} + \frac{1}{\bar{Z}_{R_2}} + \frac{1}{\bar{Z}_C}} = \frac{\frac{100e^{j0}}{10} + \frac{100e^{j90}}{10}}{\frac{1}{10} + \frac{1}{10} - \frac{1}{63{,}7j}} = \frac{10+10j}{0{,}2+0{,}016j} = \frac{14{,}14e^{j45}}{0{,}2e^{j4{,}5}} = \underline{70{,}7e^{j40{,}5}\,V}$$

$$\Rightarrow \bullet\ \bar{V}_{R_1} = \bar{E}_1 - \bar{V}_{AB} = 100 - (53{,}76 + 45{,}91j) = 46{,}24 - 45{,}91j = \underline{\underline{65{,}16e^{-44{,}8j}\,V}}$$

$$\bullet\ \bar{V}_{R_2} = \bar{E}_2 - \bar{V}_{AB} = 100j - (53{,}76 + 45{,}91j) = -53{,}76 + 54{,}09j = \underline{\underline{76{,}26e^{j134{,}8}\,V}}$$

$$\bullet\ \bar{I}_1 = \frac{\bar{V}_{R_1}}{\bar{Z}_{R_1}} = \frac{65{,}16e^{-j44{,}8}}{10} = \underline{\underline{6{,}52e^{-j44{,}8}\,A}}$$

$$\bullet\ \bar{I}_2 = \frac{\bar{V}_{R_2}}{\bar{Z}_{R_2}} = \frac{76{,}26e^{j134{,}8}}{10} = \underline{\underline{7{,}63e^{j134{,}8}\,A}}$$

$$\bullet\ \bar{I}_C = \frac{\bar{V}_{AB}}{\bar{Z}_C} = \frac{70{,}7e^{j40{,}5}}{63{,}7e^{-j90}} = \underline{\underline{1{,}11e^{j130{,}5}\,A}}$$

Ok

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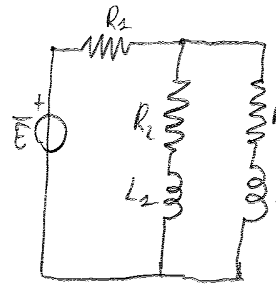
6)



$E = 230V$

$R_1 = 10\Omega$

$R_2 = R_3 = 50\Omega$

$L_1 = L_2 = 0{,}1H$

$f = 50 Hz$

Ib: Corrente

- $\omega = 2\pi f = 314$ rad/s
- 1 solo generatore: Equivalente di Thevenin

$\bar{Z}_{L_1} = \bar{Z}_{L_2} = j\omega L = 31{,}4j$

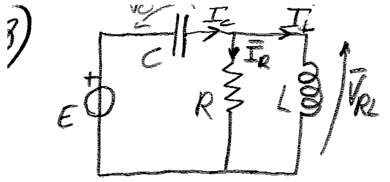
Ramo 1: $\bar{Z}_{eq_1} = \bar{Z}_{R_2} + \bar{Z}_{L_1} = 50 + 31{,}4j$

Ramo 2: $\bar{Z}_{eq_2} = \bar{Z}_{R_3} + \bar{Z}_{L_2} = 50 + 31{,}4j$

Parallelo: $\bar{Z}_{eq_3} = \frac{1}{2}\bar{Z}_{eq_1} = 25 + 15{,}7j$

$$\bar{Z}_{eq} = R_1 + \bar{Z}_{eq_3} = 10 + 25 + 15{,}7j = 35 + 15{,}7j = \underline{\underline{38{,}36e^{j24}\,\Omega}}$$

3)




$I_R = 10\,A$
$R = 10\,\Omega$
$L = 0{,}1\,H$
$C = 50\,\mu F$
$f = 50\,Hz$

Th: Fasore Corrente L, C?
Fasore Tensione E?

- $\omega = 2\pi f = 314\ rad/s$
- $\overline{Z}_C = \frac{-j}{\omega C} = -63{,}7j\ \Omega$

  $\overline{Z}_L = j\omega L = 31{,}4j\ \Omega$

  $\overline{Z}_R = R = 10\ \Omega$
- $\overline{V}_R = \overline{V}_L = \overline{Z}_R \overline{I}_R = 10 \cdot 10 = 100\ V = \underline{\underline{100 e^{j0}\ V}}$
- $\overline{I}_L = \frac{\overline{V}_L}{\overline{Z}_L} = \frac{100 e^{j0}}{31{,}4 e^{j90}} = \underline{\underline{3{,}18 e^{-j90}\ A}}$
- $\overline{I}_C = \overline{I}_R + \overline{I}_L = 10 e^{j0} + 3{,}18 e^{-j90} = (10 + j0) + (-3{,}18j) = \underline{\underline{10{,}49 e^{-j17{,}6}\ A}}$
- $\overline{V}_C = \overline{I}_C \overline{Z}_C = 63{,}7 e^{-j90} \cdot 10{,}49 e^{-j17{,}6} = \underline{\underline{668{,}21 e^{-j107{,}6}\ V}}$
- $\overline{E} = \overline{V}_C + \overline{V}_R = 668{,}21 e^{-j107{,}6} + 100 e^{j0} = -202 - 636{,}9j + 100 = -102 - 636{,}9j =$

  $= \underline{\underline{645 e^{-j80{,}9}\ V}}$

<u>Ok</u>

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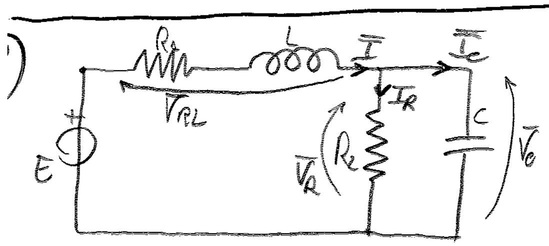



$V_C = 100\,V$
$R_1 = 10\,\Omega$
$R_2 = 10\,\Omega$
$L = 0{,}1\,H$
$C = 50\,\mu F$
$f = 50\,Hz$

Th: Fasore Corrente in $R_1, R_2$?
Fasore Tensione E?

- $\omega = 2\pi f = 314\ rad/s$
- $\overline{Z}_C = \frac{-j}{\omega C} = -63{,}7j\ \Omega$

  $\overline{Z}_L = j\omega L = 31{,}4j\ \Omega$

LKC: $\overline{I} = \overline{I}_R + \overline{I}_C$

LKT: $\overline{V}_R = \overline{V}_C$ $\Rightarrow$

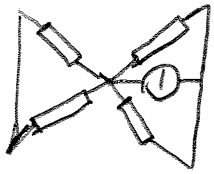
$\overline{I}_C = \frac{\overline{V}_C}{\overline{Z}_C} = \frac{100 e^{j0}}{63{,}7 e^{-j90}} = \underline{\underline{1{,}57 e^{j90}\ A}}$

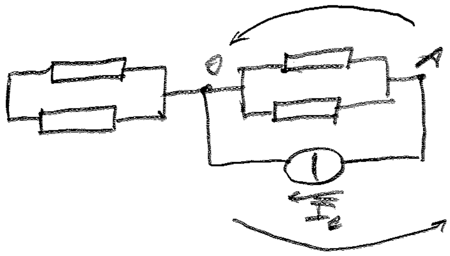
$\overline{I}_R = \frac{\overline{V}_R}{R_2} = \frac{100 e^{j0}}{10 e^{j0}} = \underline{\underline{10\ A}}$

$\Rightarrow \overline{I} = \overline{I}_C + \overline{I}_R = 1{,}57 e^{j90} + 10 e^{j0} = 1{,}57j + 10 = \underline{\underline{10{,}12 e^{j8{,}9}\ A}}$

$$\bar V_{OA}'' = -\bar I_2\left(\frac{\bar Z_2 \bar Z_4}{\bar Z_2+\bar Z_4}\right) = (-1-5j)\,V = \underline{\underline{5{,}1\,e^{j\ldots}\ V}}$$

$\Rightarrow \underline{\bar I_1}$:

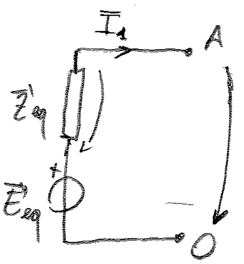





$$\bar V_{OA}''' = -\bar I_1\left(\frac{\bar Z_2 \bar Z_4}{\bar Z_2+\bar Z_4}\right) = -(2+2j)(3+2j) = -6-4j-6j+4 = (-2-10j)\,V$$

$$\Rightarrow \bar E_{eq} = 2(-1-5j)-2-10j = -2-10j-2-10j = -4-20j = \underline{\underline{20{,}4\,e^{-j78{,}7^\circ}\ V}}$$

· **POTENZA**: Per determinare la Potenza Erogata da $\bar I_1$ si può calcolare l'Equivalente di Thevenin ai capi del generatore stesso (cioè in assenza del generatore): Considero il circuito privo di $\bar I_1$, che rappresenta "l'utilizzatore" che di solito trovi negli esercizi sugli Equivalenti di Thevenin.




$\bar Z'_{eq} = \bar Z_{eq} = 3+2j\ \Omega = \underline{3{,}6\,e^{j33{,}7^\circ}\ \Omega}$

$\bar E'_{eq}$ = è dato dai contributi di tutti, tranne che di $\bar I_1$

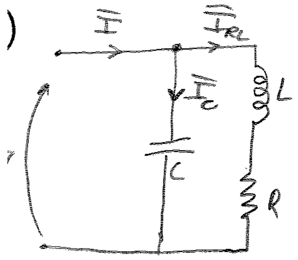
$$\Rightarrow \bar E'_{eq} = 2(-1-5j) = -2-10j = \underline{10{,}2\,e^{-j78{,}7^\circ}\ V}$$

$\Rightarrow$ LKT: $\bar E'_{eq} - \bar I_1 \bar Z'_{eq} + \bar V_{OA} = 0 \Rightarrow \bar V_{OA} = \bar I_1 \bar Z'_{eq} - \bar E'_{eq}$

$\Rightarrow \bar V_{OA} = 4+20j\ V$

$$\Rightarrow \bar S = \bar V_{OA}\cdot \bar I_1^* = (4+20j)(2-2j) = 8-8j+40j+40 = \underline{\underline{48+32j\ VA}}$$

)




$R = 2\,\Omega$
$C = 60\,\mu F$
$L = 10\,mH$
$V = 230\,V$

Tb: P,Q ramo RL
Q condensatore
P,Q totali
corrente Assorbita

- $f = 50\,Hz \Rightarrow \omega = 314\ rad/s$

$\overline{Z}_c = \frac{-j}{\omega C} = -53j\ \Omega$

$\overline{Z}_L = j\omega L = 3{,}14j\ \Omega \longrightarrow \overline{Z}_{RL} = R + \overline{Z}_L = 2 + 3{,}14j = \underline{3{,}72\, e^{j\,57{,}5}\ \Omega}$

$\Rightarrow \overline{I}_{RL} = \frac{\overline{V}}{\overline{Z}_{RL}} = \frac{230\,e^{j0}}{3{,}72\,e^{j\,57{,}5}} = \underline{61{,}8\,e^{-j\,57{,}5}\ A}$

$\overline{S}_{RL} = \overline{V}\,\overline{I}_{RL}^{*} = P_{RL} + jQ_{RL} \Rightarrow \begin{cases} P_{RL} = 230 \cdot 61{,}8 \cdot \cos(+57{,}5) \\ Q_{RL} = 230 \cdot 61{,}8 \cdot \sin(+57{,}5) \end{cases} \Rightarrow \begin{cases} P_{RL} = 7632{,}2\ W \\ Q_{RL} = +11988\ Var \end{cases}$

- $\overline{I}_c = \frac{\overline{V}}{\overline{Z}_c} = \frac{230\,e^{j0}}{53\,e^{-j90}} = \underline{4{,}34\,e^{j90}\ A}$

$\overline{S}_c = \overline{V}\,\overline{I}_c^{*} = \cancel{P_c} + jQ_c \Rightarrow \underline{Q_c} = VI_c \sin(-90) = \underline{-998{,}2\ Var}$

$\overline{I}_{ass} = \overline{I} = \overline{I}_{RL} + \overline{I}_c = 61{,}8\,e^{-j\,57{,}5} + 4{,}34\,e^{j90} =$

$= 33{,}2 - 52{,}1j + 4{,}34j = 33{,}2 - 47{,}28j = \underline{58{,}18\,e^{-j\,55{,}2}\ A}$

$\overline{S}_{TOT} = \overline{V}\,\overline{I}^{*} = P_{TOT} + jQ_{TOT} \Rightarrow \begin{cases} P_{TOT} = 230 \cdot 58{,}18 \cos(55{,}2) \\ Q_{TOT} = 230 \cdot 58{,}18 \sin(55{,}2) \end{cases} \Rightarrow \begin{cases} P_{TOT} = 7632\ W \\ Q_{TOT} = 10988\ Var \end{cases}$

<u>OK</u>

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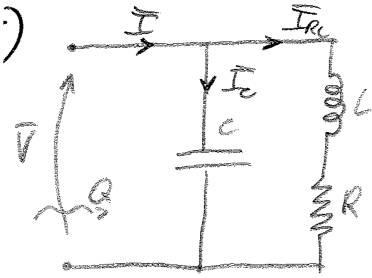


$R = 1\,\Omega$
$L = 10\,mH$
$V = 230\,V$
$Q = 10\,KVar$

**Th**: P, Q carico RL
Q condensatore
C
Corrente Assorbita

• $f = 50\,Hz \rightarrow \omega = 314\,rad/s$

$\bar{Z}_L = j\omega L = 3{,}14j\,\Omega \rightarrow \bar{Z}_{RL} = \bar{Z}_L + \bar{R} = 1 + 3{,}14j = \underline{3{,}29\, e^{j72{,}3°}\,\Omega}$

$\bar{I}_{RL} = \dfrac{\bar{V}}{\bar{Z}_{RL}} = \dfrac{230\, e^{j0}}{3{,}29\, e^{j72{,}3}} = \underline{\underline{69{,}9\, e^{-j72{,}3}\,A}}$

$\bar{S}_{RL} = \bar{V}\bar{I}_{RL}^{*} = P_{RL} + jQ_{RL} \Rightarrow \begin{cases} P_{RL} = 230 \cdot 69{,}9 \cdot \cos(72{,}3) \\ Q_{RL} = 230 \cdot 69{,}9 \cdot \sin(72{,}3) \end{cases} \Rightarrow \begin{cases} P_{RL} = 4882{,}9\,W \\ Q_{RL} = 15315{,}9\,Var \end{cases}$

• TEOREMA BOUCHEROT:

$Q_{TOT} = Q_C + Q_{RL}$
$P_{TOT} = P_{RL}$ $\Rightarrow$ • $P_{TOT} = 4882{,}9\,W$
• $Q_C = Q_{TOT} - Q_{RL} = 10000 - 15315{,}9 = \underline{\underline{-5315\,Var}}$

$|Q_C| = \dfrac{V^2}{Z_C} = V^2\omega C \Rightarrow C = \dfrac{|Q_C|}{V^2\omega} = \dfrac{5315}{230^2 \cdot 314} = 319\,\mu F$

$tg\,\varphi = \dfrac{Q_{TOT}}{P_{TOT}} = 2{,}046 \Rightarrow \cos\varphi = 0{,}44$

$P_{TOT} = VI\cos\varphi \Rightarrow I = \dfrac{P_{TOT}}{V\cos\varphi} = \dfrac{4882{,}9}{230 \cdot 0{,}44} = \underline{\underline{48{,}3\,A}}$

$\bar{I}_C = \dfrac{\bar{V}}{\bar{Z}_C} = \dfrac{230\, e^{j0}}{9{,}98\, e^{-j90}} = 23{,}05\, e^{j90}$

$\bar{Z}_C = \dfrac{-j}{\omega C} = -9{,}98j\,\Omega$

$\Rightarrow \bar{I} = \bar{I}_C + \bar{I}_{RL} = 23{,}05\, e^{j90} + 69{,}9\, e^{-j72{,}3}$

$= 23{,}05j + 21{,}25 - 66{,}59j = 21{,}25 - 43{,}54j$

$= \underline{\underline{48{,}3\, e^{-j63{,}9°}\,A}}$

Ok

41

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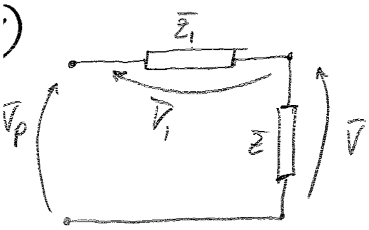


$V = 230\,V$

$\bar Z = 5 + j5\ \Omega$

$\bar Z_1 = 0{,}1 + 0{,}2j\ \Omega$

Tb = Corrente in $\bar Z$

P,Q di $\bar Z_1$

$\bar V_p$

- $\bar I = \frac{\bar V}{\bar Z} = \frac{230\, e^{j0}}{7{,}08\, e^{j45}} = \underline{32{,}48\, e^{-j45}\ A}$

- $\bar S = \bar V \bar I^* = P + jQ \Rightarrow \begin{cases} P = 230 \cdot 32{,}48 \cdot \cos(45) \\ Q = 230 \cdot 32{,}48 \cdot \sin(45) \end{cases} \Rightarrow \begin{cases} P = 5282{,}4\ W \\ Q = 5282{,}4\ VAr \end{cases}$ ✓

$\bar V_1 = \bar Z_1 \cdot \bar I = 32{,}48\, e^{-j45} \cdot 0{,}22\, e^{j63{,}4} = \underline{7{,}15\, e^{j18{,}4}\ V}$

$\Rightarrow \bar V_p = \bar V_1 + \bar V = 230\, e^{j0} + 7{,}15\, e^{j18{,}4} = 230 + 6{,}78 + 2{,}26j = 236{,}78 + 2{,}26j = \underline{\underline{236{,}8\, e^{j0{,}5°}\ V}}$

$\bar S_1 = \bar V_1 \bar I^* = P_1 + jQ_1 \Rightarrow \begin{cases} P_1 = 7{,}15 \cdot 32{,}48 \cdot \cos(63{,}4) \\ Q_1 = 7{,}15 \cdot 32{,}48 \cdot \sin(63{,}4) \end{cases} \Rightarrow \begin{cases} P_1 = 104\ W \\ Q_1 = 207{,}6\ VAr \end{cases}$

Oppure: $P_1 = R_1 I^2 = 0{,}1 \cdot 32{,}48^2 = \underline{105{,}5\ W}$

$Q_1 = X_1 I^2 = 0{,}2 \cdot 32{,}48^2 = \underline{211\ VAr}$ ✓

OK

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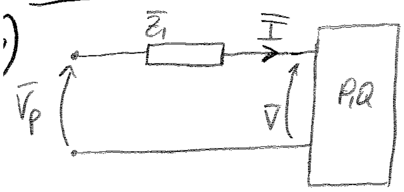



$V = 230\,V$

$P = 10\ kW$

$Q = 10\ kVAr$

$\bar Z_1 = 0{,}1 + 0{,}2j\ \Omega$

Tb corrente $\bar I$

P,Q di $\bar Z_1$

$\bar V_p$

- $tg\,\varphi = \frac{Q}{P} = 1 \Rightarrow \cos\varphi = 0{,}71$

$P = VI\cos\varphi \Rightarrow I = \frac{P}{V\cos\varphi} = \frac{10000}{230 \cdot 0{,}71} = \underline{61{,}23\ A} \Rightarrow \underline{\bar I = 61{,}23\, e^{j45°}\ A}$

$\begin{cases} P_1 = R_1 I^2 = 0{,}1 \cdot 61{,}23^2 = 374{,}9\ W \\ Q_1 = X_1 I^2 = 0{,}2 \cdot 61{,}23^2 = 749{,}8\ VAr \end{cases}$

TEOREMA BOUCHEROT:

$P_p = P_1 + P = 10374{,}9\ W$

$Q_p = Q_p + Q = 10749{,}8\ VAr$

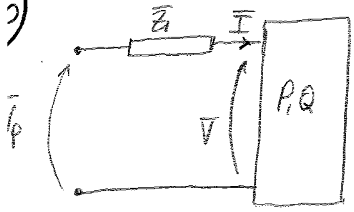
$\Rightarrow tg\,\varphi_p = \frac{Q_p}{P_p} \Rightarrow \cos\varphi_p = 0{,}69$

$\Rightarrow P_p = V_p I \cos\varphi_p \Rightarrow V_p = \frac{P_p}{I\cos\varphi_p} = \frac{10374{,}9}{61{,}23 \cdot 0{,}69} = 245{,}5\,V$

$\Rightarrow \underline{\underline{\bar V_p = 245{,}5\, e^{j46{,}4°}\ V}}$

OK

c)




$V = 230\,V$
$P = 10\,kW$
$Q = 15\,kVAr$
$\bar{Z}_1 = 0.2 + 0.2j\ \Omega$

Tb, $\bar{I}$? $\bar{V}_p$?
Rifasamento $\cos\varphi' = 0.9$
$\Rightarrow \bar{I}'$? $\bar{V}_p'$?

Corrente Assorbita dal Carico:

$tg\,\varphi = \frac{Q}{P} = 1.5 \Rightarrow \cos\varphi = 0.554$

$\Rightarrow I = \frac{P}{V\cos\varphi} = \frac{10'000}{230 \cdot 0.554} = \underline{28.48\ A}$

$P_1 = RI^2 = 0.2 \cdot 28.48^2 = 1231.82\ W$

$Q_1 = XI^2 = 0.2 \cdot 28.48^2 = 1231.82\ VAr$

$\Rightarrow$ BOUCHEROT: $\begin{cases} P_p = P + P_1 = 11231.8\ W \\ Q_p = Q + Q_1 = 16231.8\ VAr \end{cases}$

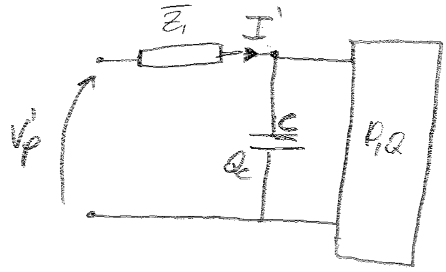
$\Rightarrow tg\,\varphi_p = \frac{Q_p}{P_p} = 1.445 \Rightarrow \cos\varphi_p = 0.569$

$\Rightarrow V_p = \frac{P_p}{I\cos\varphi_p} = \frac{11231.8}{28.48 \cdot 0.569} = \underline{251.5\ V}$

Per realizzare il Rifasamento con Fattore di Potenza 0.9 si deve introdurre una Capacità che assorba la Potenza Reattiva in eccesso:

$Q_c = P(tg\,\varphi - tg\,\varphi') = 10'000(0.484 - 1.5) = \underline{-10160\ VAr}$

$C = \frac{|Q_c|}{\omega V^2} = \frac{10160}{314 \cdot 230^2} = 611\,\mu F$




$I' = \frac{P}{V\cos\varphi'} = \frac{10000}{230 \cdot 0.9} = \underline{48.3\ A}$

$P_1' = RI'^2 = 466.2\ W$

$Q_1' = XI'^2 = 466.2\ VAr$

$P_p' = P + P_1' = 10466.2\ W$

$Q_p' = Q + Q_1' + Q_c = 5306.2\ VAr$

$\Rightarrow tg\,\varphi_p' = \frac{Q_p'}{P_p'} = 0.507 \Rightarrow \cos\varphi_p' = 0.891$

$\Rightarrow V_p' = \frac{P_p'}{I'\cos\varphi_p'} = \underline{243.2\,V}$

OK

$$\bar I_1 = \frac{\bar E_1}{\bar Z_y} = \frac{230{,}9\, e^{j0}}{\sqrt{2}\, e^{j45}} = \underline{163\, e^{-j45°}\, A}$$

$$\bar I_2 = \frac{\bar E_2}{\bar Z_y} = \frac{230{,}9\, e^{-j120}}{\sqrt{2}\, e^{j45}} = \underline{163\, e^{-j165°}\, A}$$

$$\bar I_3 = \frac{\bar E_3}{\bar Z_y} = \frac{230{,}9\, e^{j120}}{\sqrt{2}\, e^{j45}} = \underline{163\, e^{j75°}\, A}$$

• Carico Equilibrato => $P = 3RI^2 = 3 \cdot 1 \cdot 163^2 \simeq 80\ kW$

$Q = 3XI^2 = 3 \cdot 1 \cdot 163^2 \simeq 80\ kVAr$

OK

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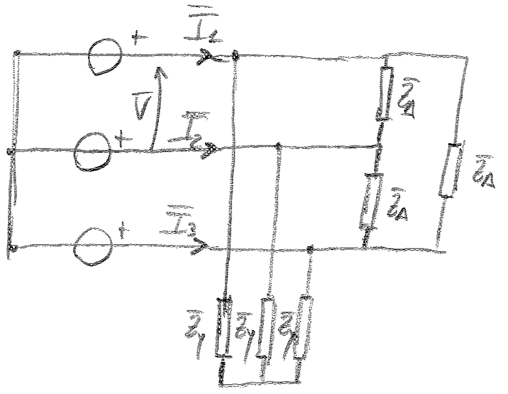
3)




$\bar Z_y = 1 + 1j\ \Omega$

$\bar Z_\Delta = 3 + 3j\ \Omega$

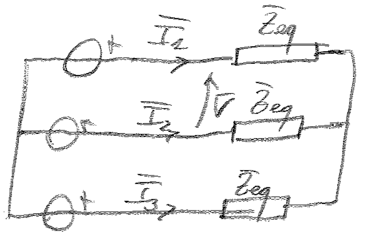
$\bar V = 400\ V$

Ib = $\bar I_1, \bar I_2, \bar I_3$?

P, Q carico?

Noto che i 2 carichi sono collegati IN PARALLELO => Trasformo le impedenze a Triangolo in impedenze a stella:

$\bar Z_y' = \frac{1}{3}\bar Z_\Delta \Rightarrow \bar Z_y' = 1 + 1j\ \Omega$




$$\Rightarrow \bar Z_{eq} = \frac{\bar Z_y \cdot \bar Z_y'}{\bar Z_y + \bar Z_y'} = \frac{1}{2}\sqrt{2}\, e^{j45}\ \Omega$$

Carico Equilibrato => $\bar E_1 = \frac{\bar V}{\sqrt{3}} = \underline{230{,}9\, e^{j0}\, V}$ => $\begin{cases} \bar E_1 = 230{,}9\, e^{j0}\, V \\ \bar E_2 = 230{,}9\, e^{-j120}\, V \\ \bar E_3 = 230{,}9\, e^{j120}\, V \end{cases}$

$$\begin{cases} \bar I_1 = \frac{\bar E_1}{\bar Z_{eq}} = \frac{230{,}9\, e^{j0}}{0{,}7\, e^{j45}} = \underline{328{,}5\, e^{-j45}\, A} \\ \bar I_2 = \frac{\bar E_2}{\bar Z_{eq}} = \underline{328{,}5\, e^{-j165°}\, A} \\ \bar I_3 = \frac{\bar E_3}{\bar Z_{eq}} = \underline{328{,}5\, e^{+j75}\, A} \end{cases}$$

• Carico Equilibrato

$$\Rightarrow \begin{cases} P = 3R_{eq}I^2 = 3 \cdot 0{,}5 \cdot 328{,}5^2 \simeq 162\ kW \\ Q = 3X_{eq}I^2 = 3 \cdot 0{,}5 \cdot 328{,}5^2 \simeq 162\ kVAr \end{cases}$$

OK

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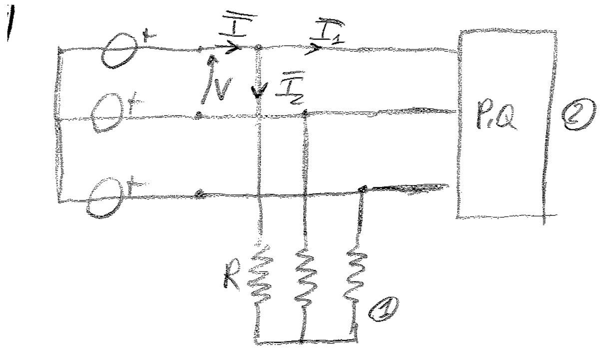


$V = 400\ V$
$R = 10\ \Omega$
$P = 5\ KW$
$Q = 2\ KVAr$

Td: Corrente Assorbita dai carichi?
P,Q dei carichi?

• I due carichi sono collegati in Parallelo => vedono la stessa tensione concatenata $\bar{V}$. Inoltre, la corrente assorbita dai carichi è composta da 2 contributi: ① Corrente assorbita da R; ② Corrente assorbita dal carico di fine linea.

$$\Rightarrow ①\ \ \operatorname{tg}\varphi = \frac{Q}{P} = 0.4 \Rightarrow \cos\varphi = 0.928$$

$$\Rightarrow I_1 = \frac{P}{\sqrt{3}\,V\cos\varphi} = \frac{5000}{\sqrt{3}\,400\cdot 0.928} = \underline{7.77\ A}$$

$$②\ \ \bar{I}_2 = \frac{E}{R} = \frac{V}{\sqrt{3}\,R} = \frac{400}{\sqrt{3}\,10} = \underline{23.09\ A}$$

$$\Rightarrow \bar{I} = I_1 + \bar{I}_2 = \underline{\underline{30.86\ A}}$$

• $P_R = 3RI_2^2 = 16000\ W$

$$\Rightarrow \begin{cases} P_{TOT} = P + P_3 = 21\ KW \\ Q_{TOT} = Q = 2\ KVAr \end{cases}$$

OK

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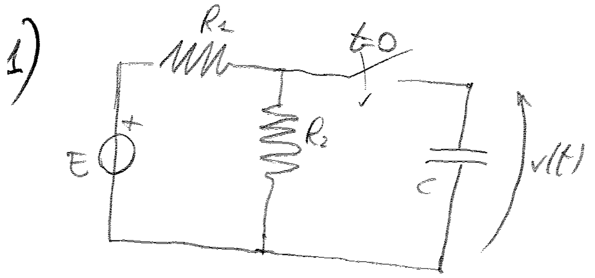
# TRANSITORI

1)



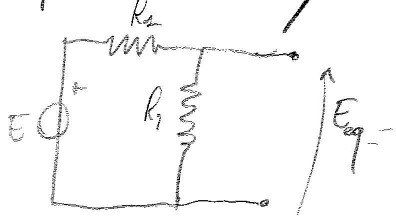

$E = 10V$

$R_1 = R_2 = 1\Omega$

$C = 1\mu F$

$v(0) = 0$

· Eq. Thevenin ai capi di C =



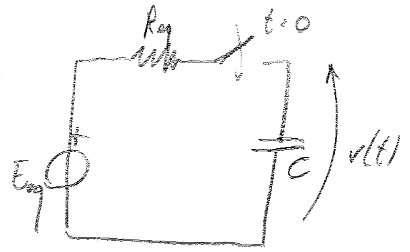

$R_{eq} = \frac{R_1 + R_2}{R_1 R_2}$

$E_{eq} = \quad \underline{LKT1} \quad E - R_1 I - R_2 I = 0$

$\underline{LKT2}, \quad E - R_1 I - E_{eq} = 0$

$\Rightarrow I = \frac{E}{R_1 + R_2} \leadsto E_{eq} = E - \frac{R_1 E}{R_1 + R_2} = \frac{E R_2}{R_1 + R_2}$

$\Rightarrow$




$LKT = E_{eq} - R_{eq} i_c - v(t) = 0$

$E_{eq} - C \frac{dv}{dt} R_{eq} - v = 0$

$\frac{dv}{dt} + \frac{1}{CR_{eq}} v - \frac{E_{eq}}{CR_{eq}} = 0$

· Eq. Omogenea: $\frac{dv}{dt} + \frac{1}{CR_{eq}} v = 0$

da cui $s + \frac{1}{CR_{eq}} = 0 \leadsto s = -\frac{1}{CR_{eq}} \leadsto \tau = \left|\frac{1}{s}\right| = CR_{eq}$

$\Rightarrow$ Sol omog: $v_0(t) = K e^{-\frac{t}{CR_{eq}}}$

· Soluzione Particolare: $v(t) = cost \Rightarrow \frac{dv}{dt} = 0$

$\Rightarrow \frac{1}{CR_{eq}} v - \frac{E_{eq}}{CR_{eq}} = 0 \Rightarrow v = E_{eq}$

QUINDI: $\begin{cases} v(t) = v_0 + v_p = K e^{-\frac{t}{CR_{eq}}} + E_{eq} \\ v(0) = 0 \end{cases}$

$\Rightarrow K e^{-\frac{0}{CR_{eq}}} + E_{eq} = 0 \leadsto K = -E_{eq}$

$\Rightarrow v(t) = E_{eq}\left(1 - e^{-\frac{t}{CR_{eq}}}\right)$

$= 5\left(1 - e^{-\frac{2t}{10^{-6}}}\right) V$



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$$\Rightarrow \begin{cases} i(t) = K e^{-t\frac{R_1R_2}{L(R_1+R_2)}} + \frac{E}{R_1} \\ i(0) = 0 \end{cases}$$

$$\rightarrow K = -\frac{E}{R_1} \Rightarrow i(t) = \frac{E}{R_1}\left(1 - e^{-\frac{tR_1R_2}{L(R_1+R_2)}}\right) = 10\left(1 - e^{-5t}\right) A$$

- $i_2 = \frac{V_2}{R_2} = \frac{V_L}{R_2} = \frac{L}{R_1}\frac{di(t)}{dt} = 5e^{-5t} A$
- $i_1 = i + i_2 = \left(10 + 5e^{-5t}\right) A$
- $\tau = \frac{L(R_1+R_2)}{R_1R_2} = 0{,}2\ s \Rightarrow$ A $t = 1s$, sono ormai passate 5 costanti di tempo $\Rightarrow$ la corrente ha già raggiunto condizioni di Regime

$\Rightarrow i(1s) = 10 A$

- A $t = 1s$, si riapre T:

<u>LKT</u>: $V_2 = V_L \Rightarrow R_2 i(t) - L\frac{di}{dt} = 0$

$$\frac{di}{dt} - \frac{R_2}{L} i = 0$$

da cui $s - \frac{R_2}{L} = 0 \Rightarrow s = \frac{R_2}{L} \Rightarrow \tau = \left|\frac{1}{s}\right| = \frac{L}{R_2}$

$$\Rightarrow \begin{cases} i(t) = K e^{-t\frac{R_2}{L}} \\ i(0) = 10 A \end{cases} \Rightarrow K = 10 \Rightarrow i(t) = 10 e^{-10t}$$

dove ho traslato il tempo in modo da avere l'inizio di questo nuovo transitorio coincidente con $t=0$!

- $i_\infty = 5\,A \Rightarrow 50\%$ di $i_\infty = 2{,}5\,A$

Affinché si raggiunga 50% di $i_\infty$, si ha

$$2{,}5 = 5(1 - e^{-100t})$$

$$2{,}5 = 5e^{-100t} \Rightarrow -100t = \ln\left(\frac{2{,}5}{5}\right) =$$

$$-100t = -0{,}693 \Rightarrow t = 6{,}93\ ms$$

- $W = \frac{1}{2} L i^2$ dove $i$ di regime

$\Rightarrow$ $\underline{W = 0{,}125\ J}$

- $2 \leq t \leq 4\,ms$

Essendo $\tau = 1\,ms$, la corrente NON ha ancora raggiunto le condizioni di regime

$\Rightarrow i(2ms) = 5\left(1 - e^{-1000 \cdot 2 \cdot 10^{-3}}\right) = 4{,}32\ A$

In questo intervallo di tempo, $E = 0\,V$

$\Rightarrow$ l'equazione differenziale diventa omogenea

$$\frac{di}{dt} + \frac{R_1 R_2}{L(R_1+R_2)}\, i = 0$$

$$\Rightarrow \begin{cases} i'(t) = K e^{-t\frac{R_1R_2}{L(R_1+R_2)}} \\ i(2ms) = 4{,}32 \end{cases} \Rightarrow \underline{i'(t) = 4{,}32\, e^{-1000t}\ A}$$

- $i_2' = \frac{V_L}{R_2} = \frac{L}{R_2}\frac{di}{dt} = \frac{10^{-3}}{2} \cdot \left(-4320\, e^{-1000t}\right) = -2{,}16\, e^{-1000t}\ A$

$\Rightarrow \underline{i_1' = i' + i_2' = 2{,}16\, e^{-1000t}\ A}$

• LKT: $E_{eq} - i_C R_{eq} - v(t) = 0$

$$E_{eq} - C\frac{dv}{dt} R_{eq} - v(t) = 0$$

$$\frac{dv}{dt} + \frac{1}{CR_{eq}} v - \frac{E_{eq}}{CR_{eq}} = 0$$

Omog: $\frac{dv}{dt} + \frac{1}{CR_{eq}} v = 0$

$$s + \frac{1}{CR_{eq}} = 0 \rightarrow s = -\frac{1}{CR_{eq}} \Rightarrow \tau = \left|\frac{1}{s}\right| = CR_{eq}$$

$$\Rightarrow v_0(t) = K e^{-\frac{t}{CR_{eq}}}$$

Part: $v_p = cost \Rightarrow \frac{dv_p}{dt} = 0$

$$\Rightarrow \frac{1}{\cancel{CR_{eq}}} v - \frac{E_{eq}}{\cancel{CR_{eq}}} = 0 \Rightarrow v_p = E_{eq}$$

$$\Rightarrow \begin{cases} v(t) = K e^{-\frac{t}{CR_{eq}}} + E_{eq} = 0 \\ v(0) = 0 \end{cases}$$

$\rightarrow K = -E_{eq}$ $\Rightarrow$ $v(t) = E_{eq}\left(1 - e^{-\frac{t}{CR_{eq}}}\right) = 6\left(1 - e^{-10^6 t}\right) V$

• $i_C = C\frac{dv}{dt} = 10^{-6} \cdot 10^{6} \cdot 6 e^{-10^6 t} = e^{-10^6 t} A$

• $i_2 = \frac{v(t)}{R_2} = 3\left(1 - e^{-10^6 t}\right) A$

• $W = \frac{1}{2} C v^2 = \underline{0.18\ mJ}$



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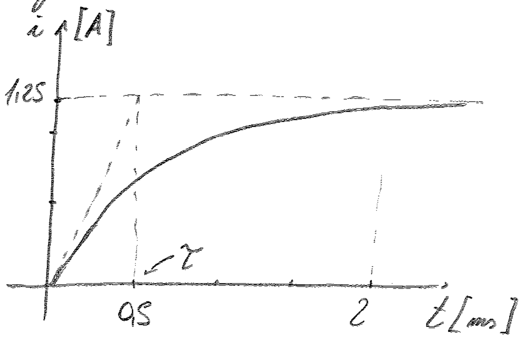
× DIAGRAMMI

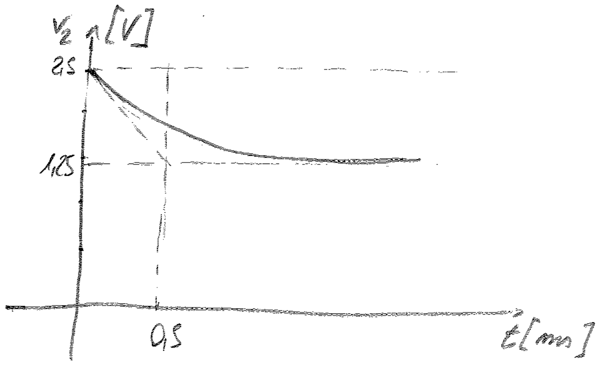
Dopo un tempo pari a $4\tau = 2ms$, la corrente $i$ e la tensione $v_2$ raggiungono il regime

$\Rightarrow$ $i_{reg} = 1.25\ A$

$v_{2reg} = 1.25\ V$

$\Rightarrow$

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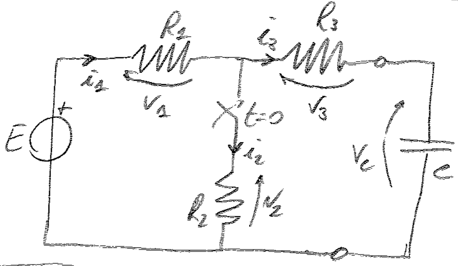


$E = 12V$

$R_1 = R_2 = R_3 = 2\Omega$

$C = 1mF$

Interruttore chiuso a $t = 0$

Th: $v_c$?

$\boxed{t<0}$: LKT: $E - v_1 - v_3 - v_c = 0$

$$v_c = E - v_1 - v_3 = E - (R_1+R_3)i = E - (R_1+R_3)C\frac{dv_c}{dt}$$

$$\Rightarrow v_c + (R_1+R_3)C\frac{dv_c}{dt} = E \Rightarrow \frac{1}{(R_1+R_3)C}v_c + \frac{dv_c}{dt} = \frac{E}{C(R_1+R_3)}$$

$\Rightarrow$ Polin. Caratt.: $s + \frac{1}{(R_1+R_3)C} = 0 \rightsquigarrow s = -\frac{1}{(R_1+R_3)C} \Rightarrow \tau = \left|\frac{1}{s}\right| = C(R_1+R_3) = 4ms$

$\Rightarrow$ Soluz. Omogenea: $v_0(t) = Ke^{-\frac{t}{4\cdot 10^{-9}}}\ V$

Soluz. Part.: $v_p = cost \Rightarrow \frac{dv_p}{dt} = 0 \Rightarrow v_p = E$

$$\Rightarrow v_c(t) = Ke^{-\frac{t}{4\cdot10^{-9}}} + E \rightsquigarrow v_c(0) = \ldots E?$$

?

$\boxed{t>0}$: $\underline{LKT1}$: $E - v_1 - v_3 - v_c = 0$

$\underline{LKT2}$: $v_2 - v_3 - v_c = 0$

$\underline{LKC}$: $i_1 = i_2 + i_c$

$v_1 = R_1 i_1$

$v_2 = R_2 i_2$

$v_3 = R_3 i_c$

$i_c = C\frac{dv_c}{dt}$

$\Rightarrow E - R_1 i_1 - R_3 i_c - v_c = 0$

$E - R_1(i_2 + i_c) - R_3 i_c - v_c = 0$

$E - R_1\left(\frac{v_2}{R_2} + i_c\right) - R_3 i_c - v_c = 0$

$E - \frac{R_1}{R_2}\left(R_3 i_c + v_c\right) - R_1 i_c - R_3 i_c - v_c = 0$

$E - \frac{R_1R_3}{R_2}\cdot C\frac{dv_c}{dt} - \frac{R_1}{R_2}v_c - R_1C\frac{dv_c}{dt} - R_3C\frac{dv_c}{dt} - v_c = 0$

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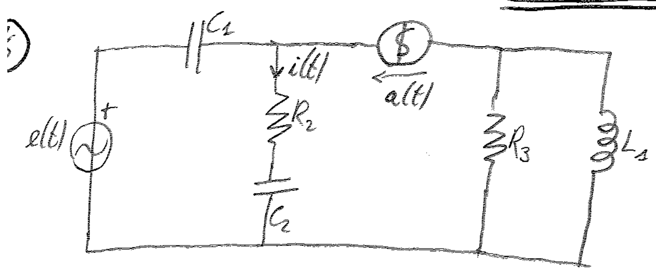
# ELETTROTECNICA
## TECNOS SANDRO

1)




$e(t) = 100\sqrt{2}\,\sin(\omega_1 t)$

$a(t) = 5\sqrt{2}\,\sin(\omega_2 t + \frac{\pi}{3})$

$C_1 = 1\mu F$ $\quad L_1 = 10\ mH$

$C_2 = 10\mu F$ $\quad R_2 = 100\,\Omega$

Tb: $i(t)$?

2) $\omega_1 = \omega_2 = 2\pi \cdot 50\ rad/s = 314\ rad/s$

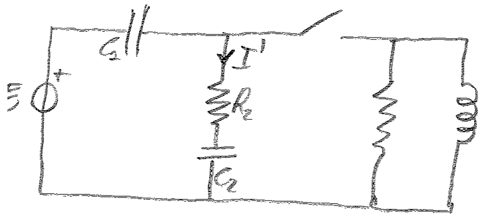
Entrambi i generatori operano secondo funzioni sinusoidali. Li sostituisco con i Fasori corrispondenti:

$$\bar{E} = \frac{100\sqrt{2}}{\sqrt{2}} e^{j0}\ V = 100\, e^{j0}\ V$$

$$\bar{A} = \frac{5\sqrt{2}}{\sqrt{2}} e^{j60}\ A = 5\, e^{j60}\ A$$

Essendoci 2 generatori, uso il Principio di Sovrapposizione degli Effetti.

① $A = 0,\ E \neq 0$:



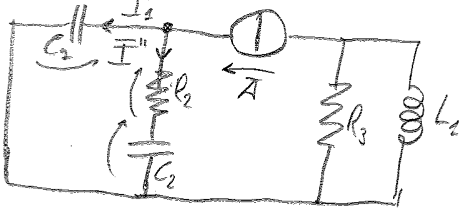

$X_{C_1} = \frac{1}{\omega C_1} = 3184,7\ \Omega$

$X_{C_2} = \frac{1}{\omega C_2} = 318,47\ \Omega$

$\bar{Z}_{eq} = R_2 - jX_{C_1} - jX_{C_2} = 100 - 3503,17j = \underline{3504,6\, e^{-j88,3\ldots}}$

$$\Rightarrow \bar{I}' = \frac{\bar{E}}{\bar{Z}_{eq}} = \frac{100\, e^{j0}}{3504,6\, e^{-j88,36}} = \underline{0,028\, e^{j88,36}\ A}$$

② $E = 0,\ A \neq 0$:




LKT: $\bar{V}_{C_1} = \bar{V}_{R_2} + \bar{V}_{C_2}$

LKC: $\bar{A} = \bar{I}_1 + \bar{I}''$

$\Rightarrow -\bar{I}_1 jX_{C_1} = \bar{I}'' R_2 - \bar{I}'' jX_{C_2}$

$-(\bar{A} - \bar{I}'')jX_{C_1} = \bar{I}''(R_2 - jX_{C_2})$

$$\bar{I}'' = \bar{A} \cdot \frac{-jX_{C_2}}{R_2 - jX_{C_2} - jX_{C_1}} = 5e^{j60} \cdot \frac{3184,7\, e^{-j90}}{3504,6\, e^{-j88,36}} =$$

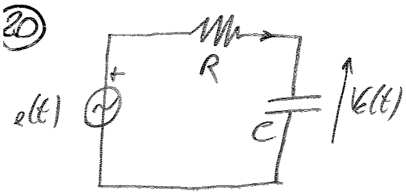
$$= \underline{4,54\, e^{j58,36}\ A}$$

$\Rightarrow \bar{I} = \bar{I}' + \bar{I}'' = 0,028\, e^{j88,36} + 4,54\, e^{j58,36} = 0,0008 + 0,028j + 2,38 + 3,86j =$

$= 2,3808 + 3,888j = \underline{4,56\, e^{j58,52}\ A}$

QUINDI: $i(t) = \sqrt{2}\, I \sin(\omega t + 58,52) = \boxed{4,56 \cdot \sqrt{2}\, \sin(314t + 58,52)\ A}$

20)




Noto: $e(t) = \operatorname{sen} \omega t$

$R, C$

Tb: $V_C(t)$ affinché sia in Ritardo di $\frac{\pi}{4}$ rispetto a $e(t)$?

Trasformo le funzioni sinusoidali nei fasori corrispondenti => indico con $\bar{E}$ e $\bar{V}_C$ la tensione, rispettivamente, del generatore e di C

$X_C = \omega C$ $\qquad \bar{E} - \bar{V}_R - \bar{V}_C = 0$

$\bar{Z}_{eq} = R - jX_C$ $\qquad \bar{E} - R\bar{I} - \bar{V}_C = 0$

$$\bar{I} = \frac{\bar{E}}{\bar{Z}_{eq}} = \frac{\bar{E}}{R - jX_C} \Rightarrow \bar{V}_C = \bar{E} - \frac{R\bar{E}}{R - jX_C} = \bar{E}\left(1 - \frac{R}{R - jX_C}\right) =$$

$$= \frac{\bar{E}(-jX_C)}{R - jX_C} = \frac{-j\bar{E}X_C}{R - jX_C} \longrightarrow \angle(\cdot) = -90°$$

Affinché $\bar{V}_C$ sia in ritardo di $\frac{\pi}{4}$ rispetto a $\bar{E}$ (preso con fase nulla), deve essere

$$\angle \bar{V}_C = -45° \Rightarrow \operatorname{arctg}(-X_C) - \operatorname{arctg}\left(\frac{-X_C}{R}\right) = -45$$

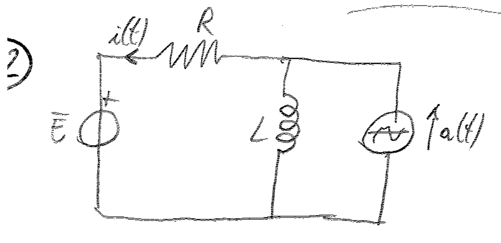
$$-90° - \operatorname{arctg}\left(\frac{-X_C}{R}\right) = -45°$$

$$-\operatorname{arctg}\left(\frac{-X_C}{R}\right) = 45° \Rightarrow \operatorname{arctg}\left(\frac{-X_C}{R}\right) = -45°$$

$$-\frac{X_C}{R} = -1 \Rightarrow X_C = +R \Rightarrow \frac{1}{\omega C} = +R \Rightarrow \boxed{\omega = +\frac{1}{RC}}$$

$\boxed{\text{Ok}}$

2)




$E = 100\,V$

$X_L = 10\,\Omega = R$

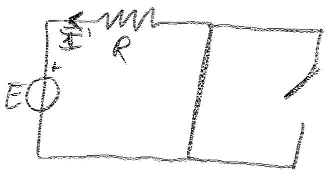
$a(t) = 100\sqrt{2}\,\operatorname{sen}(\omega t)$

$\omega = 314\ rad/s$

Tb: $i(t)$

Ci sono 2 generatori => Principio Sovrapposizione effetti

1) $E \neq 0,\ A = 0$




Essendo $\bar{E}$ un generatore COSTANTE, anche la corrente che circola nel circuito sarà COSTANTE => l'Induttore si comporta come un CORTOCIRCUITO

$$\bar{I}' = -\frac{E}{R} = -\frac{100}{10} = \underline{-10\,A}$$

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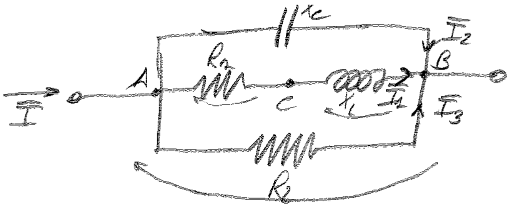



$V_{AC} = 100\,V$
$I = 30\sqrt{2}\,A$
$I_3 = 20\sqrt{2}\,A$
$X_L = 10\,\Omega$
$R_2 = 5\,\Omega$

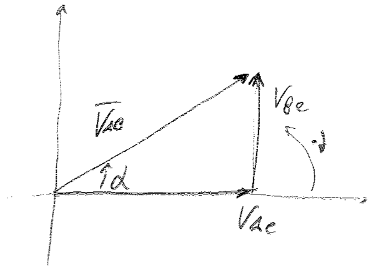
$I_b$: $R_1, X_C, I_1$?

• LKT: $\bar{V}_{AC} + \bar{V}_{BC} = \bar{V}_{AB}$

LKC: $\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$

• $\bar{I}_3 R_2 = -\bar{I}_2 j X_C \rightsquigarrow \bar{I}_2 = j\dfrac{\bar{I}_3 R_2}{X_C}$

• $R_1 \bar{I}_1 + j X_L \bar{I}_1 = \bar{I}_3 R_2$




?mp $\bar{V}_{AC}$ a fase nulla, come riferimento di fase

$\Rightarrow \bar{I}_3 = \dfrac{\bar{I}_1(R_1 + jX_L)}{R_2}$

• $\bar{V}_{AB} = (R_1 + jX_L)\bar{I}_1 = \underbrace{R_1 I_1}_{V_{AC}} + j\underbrace{I_1 X_L}_{V_{BC}}$ $\qquad \angle \bar{V}_{AB} = \operatorname{arctg}\left(\dfrac{X_L}{R_1}\right)$

• $|\bar{V}_{AB}| = |R_2 \bar{I}_3| = R_2 I_3 = 100\sqrt{2}\,V$

$V_{AC} = V_{AB} \cdot \cos\varphi \Rightarrow \cos\alpha = \dfrac{V_{AC}}{V_{AB}} = \dfrac{1}{\sqrt{2}} \Rightarrow \alpha = \dfrac{\pi}{4}$

$\Rightarrow \bar{V}_{AB} = 100\sqrt{2}\, e^{j45°}\,V$

$\bar{I}_3 = 20\sqrt{2}\, e^{j45°}\,V$

$\bar{I}_2 = \dfrac{\bar{V}_{AB}}{-jX_C} \Rightarrow \angle \bar{I}_2 = 45 - (-90) = 135°$

$\angle \bar{V}_{AB} = \operatorname{arctg}\left(\dfrac{X_L}{R_1}\right) \Rightarrow \dfrac{\pi}{4} = \operatorname{arctg}\left(\dfrac{X_L}{R_1}\right) \Rightarrow 1 = \dfrac{X_L}{R_1} \Rightarrow R_1 = X_L = 10\,\Omega$

$\Rightarrow \bar{I}_1 = \dfrac{\bar{V}_{AC}}{R_1 + jX_L} = \dfrac{100}{10 + 10j} = \dfrac{100}{14{,}14\, e^{j45}} = \underline{7{,}1\, e^{-j45°}\,A}$

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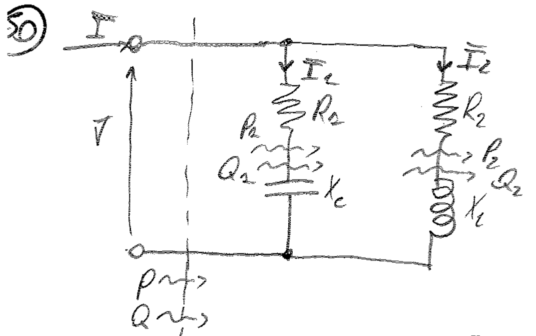
50)



$P_1 = 50\,W$
$R_1 = 5\,\Omega$
$R_2 = 10\,\Omega$
$X_L = 10\,\Omega$
$X_C = 20\,\Omega$

Ib: $\bar{A} = P + jQ$

• $P_1 = R_1 I_1^2 \Rightarrow I_1 = \sqrt{\frac{P_1}{R_1}} = 3,16\,A$

$|Q_1| = I_1^2 \cdot X_C = 3,16^2 \cdot 20 = 199,7\,VAr$

$\bar{V} = I_1 \cdot \bar{Z}_{R_1C} \Rightarrow V = I_1 \cdot \sqrt{R_1^2 + X_C^2} = \sqrt{10} \cdot \sqrt{425} = \sqrt{4250} = 65,2\,V$

MA, $\bar{V} = I_2 \cdot \bar{Z}_{R_2L} \Rightarrow V = I_2 \cdot \sqrt{R_2^2 + X_L^2} \Rightarrow I_2 = \frac{V}{\sqrt{R_2^2+X_L^2}} = \frac{65,2}{14,14} = 4,61\,A$

$\Rightarrow P_2 = R_2 I_2^2 = 4,61^2 \cdot 10 = 212,5\,W$

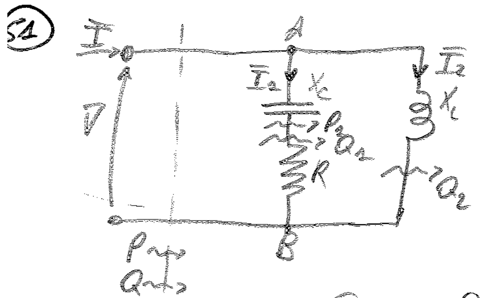
$|Q_2| = X_L \cdot I_2^2 = 212,5\,VAr$

$\Rightarrow P = P_1 + P_2 = 262,5\,W$

$Q = Q_2 - Q_1 = +12,8\,VAr$ $\Rightarrow \boxed{\bar{A} = 262,5 + j12,8\ VA}$

OK

51)



$P = 200\,W$
$Q = 200\,VAr$
$R = 200\,\Omega$
$\bar{V} = 200\sqrt{2}\,V$

Ib: $X_C$? $X_L$?

• C'è un solo Resistore $\Rightarrow P = P_1$

$P = R I_1^2 \leadsto I_1 = \sqrt{\frac{P}{R}} = 1\,A$

$Q = Q_2 - Q_1$

• $\bar{V} = \bar{I}_1 \cdot \bar{Z}_{RC} \Rightarrow V = I_1 \cdot \sqrt{R^2 + X_C^2} \Rightarrow \left(\frac{V}{I_1}\right)^2 = R^2 + X_C^2 \Rightarrow X_C = \sqrt{\left(\frac{V}{I_1}\right)^2 - R^2}$

$\Rightarrow X_C = 200\,\Omega$ ✓

$\Rightarrow |Q_1| = X_C I_1^2 = 200\,VAr$

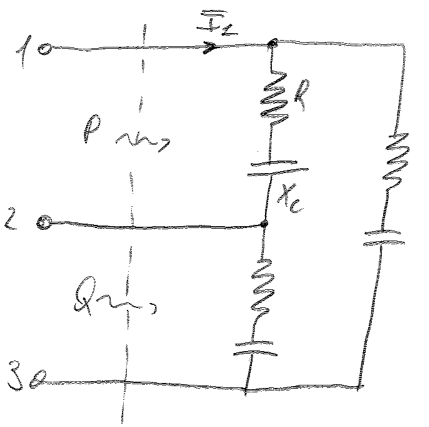
$\Rightarrow Q_2 = Q + Q_1 = 400\,VAr$

MA $Q_2 = V \cdot I_2 \cdot \text{sen}\,\varphi$ dove $\varphi$ = sfasamento tra V e I. Essendoci induttore $\varphi = 90°$

$\Rightarrow I_2 = \frac{Q_2}{V} = \sqrt{2}\,A$

$\Rightarrow Q_2 = X_L I_2^2 \leadsto X_L = \frac{Q_2}{I_2^2} = 200\,\Omega$ !

26)




Terna Simmetrica
Carico Equilibrato
$I_1 = 10\ A$
$Q = 1800\ VAr$
$R = 30\ \Omega$

Ib: $X_c$, S, P?

Carico collegato a triangolo: passo a stella:

$R_1 = \frac{1}{3}R = 10\,\Omega$ $\quad Q = 3X_{c1}I_1^2 = X_c I_1^2$

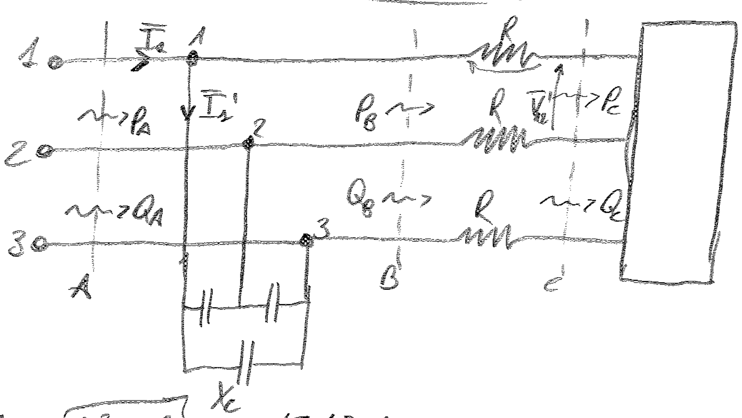
$X_{c1} = \frac{1}{3}X_c$ $\quad \Rightarrow \quad \Rightarrow X_c = \frac{Q}{I_1^2} = 18\,\Omega$

$P = 3R_1 I_1^2 = R I_1^2 = 3000\ W$

$\Rightarrow \bar{S} = P + jQ = 3000 + j1800\ VA = \underline{3498{,}6\, e^{j31}\ VA}$

OK

27)




$P_C = 3000\ W$
$Q_C = 3000\ VAr$
$V_{12}' = 100\ V$
$R = 10\,\Omega$

Ib: $X_c$ affinché $Q_A = 0$

$S_C = \sqrt{P_C^2 + Q_C^2} = 4242{,}64\ VA$

VA $\quad S_C = V_{12}' \cdot I_1'' \Rightarrow I_1'' = \frac{S_C}{V_{12}'} = 42{,}43\ A$

$P_B = 3RI_1''^2 + P_C = 3 \cdot 10 \cdot 42{,}43^2 + 3000 \approx 57\ KW$

$Q_B = Q_C = 3000\ VAr$ xché R NON dà potenza reattiva

$S_B = \sqrt{P_B^2 + Q_B^2} = 52078{,}89\ VA$

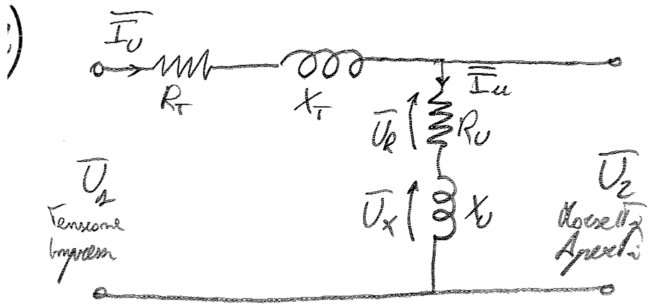
VA $\quad S_B = V_{12} \cdot I_1'' \Rightarrow V_{12} = \frac{S_B}{I_1''} = 1365{,}25\ V$

Bn, $\quad P_A = P_B = 57000\ W$

$Q_A = Q_B - 3\left(\frac{X_c}{3}\right)(I_1')^2 = 0 \Rightarrow Q_B = X_c (I_1')^2$

1121

)




$I_u = 208\ A$

$U_2 = 240\ V \qquad \angle \overline{U_2} = 0$

$R_u = 0{,}9231\ \Omega$

$R_T = 14{,}2\ m\Omega$

$X_T = 18{,}2\ m\Omega$

$I_b = X_u, \overline{I_u}, \overline{U_1}\ ?$

) $\overline{Z_T} = jX_T = 0{,}0182\, e^{j90} = 0{,}0182j$

LKT: $\overline{U_R} + \overline{U_X} = \overline{U_2}$

$U_R = R_u \cdot I_u = 192\ V$

$U_X = \sqrt{U_2^2 - U_R^2} = 144\ V \qquad$ MA $\quad U_X = X_u I_u \Rightarrow X_u = 0{,}6923\ \Omega$

Quindi: $R_u \overline{I_u} + jX_u \overline{I_u} = \overline{U_2}$

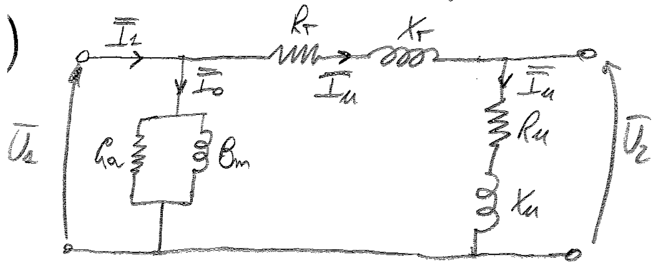
$$\overline{I_u} = \frac{\overline{U_2}}{R_u + jX_u} = \frac{240\, e^{j0}}{0{,}9231 + j0{,}6923} = \frac{240\, e^{j0}}{1{,}154\, e^{j36{,}87}} = \underline{\underline{208\, e^{-j36{,}87}\ A}}$$

LKT: $\overline{U_1} = R_T \overline{I_u} + jX_T \overline{I_u} + \overline{U_2}$

$= 2{,}9536\, e^{-j36{,}87} + 0{,}0182\, e^{j90} \cdot 208\, e^{-j36{,}87} + 240\, e^{j0} =$

$= 2{,}363 - 1{,}772j + 2{,}221 + 3{,}028j + 240 = \underline{\underline{244{,}6 + 1{,}256j\ V}}$

)




$G_a = 3{,}24\ mS$

$B_m = -22{,}4\ mS$

$I_b: \overline{I_0}, \overline{I_1}\ ?$

$G_a$ e $B_m$ mi danno valori di AMMETTENZA Y:

$\overline{Y_{eq}} = G_a + jB_m = 0{,}00324 - 0{,}0224j\ \ S$

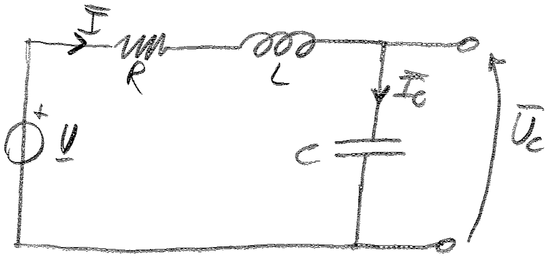
$$\Rightarrow \overline{Z_{eq}} = \frac{1}{\overline{Y_{eq}}} = \frac{1}{0{,}00324 - 0{,}0224j} = 44{,}92\, e^{+j81{,}62}\ \Omega$$

$\overline{I_1} = \overline{I_0} + \overline{I_u} =$

$$\overline{I_0} = \frac{\overline{U_1}}{\overline{Z_{eq}}} = \frac{(244{,}6 + 1{,}256j)}{44{,}92\, e^{+j84{,}62}} = \frac{244{,}6\, e^{j0{,}3}}{44{,}92\, e^{+j84{,}6}} = \underline{5{,}44\, e^{-j82}\ A}$$

$\Rightarrow \overline{I_1} = 166{,}4 - 124{,}8j + 0{,}76 - 5{,}39j = \underline{\underline{167{,}2 - 130{,}2j\ A}}$

3)




Studiare come varia $\bar U_C$ al variare della pulsazione

$R = 50\,\Omega$ $\quad \bar U = U = 500\,V$

$L = 0{,}1\,H$

$C = 10 \cdot 10^{-6}\,F$

$$\bar U - R\bar I - j\omega L \bar I - \bar U_C = 0$$

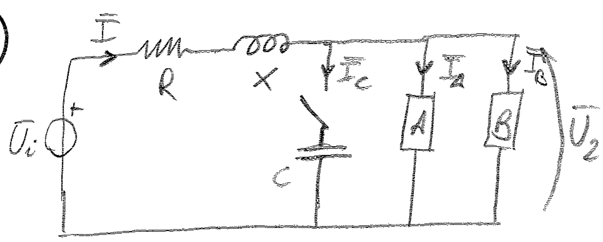
$$\Rightarrow \bar U = R\bar I + j\omega L \bar I + jX_C \bar I = \bar I (R + j\omega L + jX_C)$$

$$\Rightarrow \bar I = \frac{\bar U}{R + j\omega L + jX_C}$$

$$\bar U_C = jX_C \bar I \Rightarrow \bar U_C = \frac{-Uj}{\omega C\left(R + j\omega L + j\left(-\frac{1}{\omega C}\right)\right)} = \frac{-Uj}{\omega CR + j\omega^2 CL + 1}$$

---

4)




$P_A = 20\,kW$ $\quad R = 12{,}2\,m\Omega$

$\cos\varphi_A = 0{,}894$ $\quad X = 34{,}2\,m\Omega$

$P_B = 19936\,W$ $\quad \bar U_2 = U_2 = 240\,V$

$Q_B = 19952\,VAr$ $\quad \angle \bar U_2 = 0$

$\bar I_A$ in ritardo rispetto a $\bar U_2$

1) a Interruttore Aperto:

$$P_A = U_2 I_A \cos\varphi_A \Rightarrow I_A = \frac{P_A}{U_2\cos\varphi_A} = \frac{20000}{240 \cdot 0{,}894} = \underline{93{,}12\,A} \rightsquigarrow \underline{\underline{\bar I_A = 93{,}12\, e^{-j26{,}56°}}}$$

$$S_B = \sqrt{P_B^2 + Q_B^2} = 28205{,}08\ VA$$

$$\Rightarrow I_B = \frac{S_B}{U_2} = \frac{28205{,}08}{240} = \underline{117{,}5\,A}$$

NB) $\bar S_B = \bar U_2 \cdot \bar I_B^* \Rightarrow \bar I_B = \frac{\bar S_B^*}{\bar U_2} \Rightarrow \underline{\underline{\bar I_B = 117{,}5\, e^{-j45{,}02°}\,A}}$

$$\bar I = \bar I_A + \bar I_B = 83{,}34 - 41{,}66j + 83{,}06 - 83{,}11j = 166{,}4 - 124{,}77j \approx \underline{\underline{208\, e^{-j36{,}86}\,A}}$$

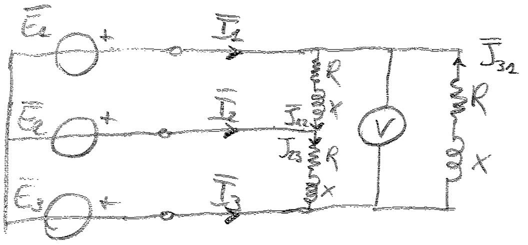
$$\bar Z_{RX} = R + jX = 0{,}0122 + 0{,}0342j\ \Omega = \underline{\underline{0{,}0383\, e^{j63{,}3}\,\Omega}}$$

$$\Rightarrow \bar U_{RX} = \bar Z_{RX} \cdot \bar I = \underline{\underline{7{,}96\, e^{j26{,}44}\,V}}$$

$$\Rightarrow \bar U_i = \bar U_{RX} + \bar U_2 = 7{,}13 + 3{,}54j + 240 = 247{,}13 + 3{,}54j = \underline{\underline{247{,}15\, e^{j0{,}8°}\,V}}$$

OK

5)




Tensioni Simmetriche Destrorse
Carico Simmetrico Equilibrato
$V = 380\ V$
$R = 30\ \Omega$
$X = 8{,}04\ \Omega$

Th: Correnti di Linea e di Fase?
Potenze Attive e Reattive Assorbite?

- V legge una Tensione CONCATENATA

$\Rightarrow E = \dfrac{V}{\sqrt{3}} = 220\ V = E_1 = E_2 = E_3$

Essendo Terna Simmetrica Destrorsa, si ha: $\begin{cases} \bar{E}_1 = 220\, e^{j0}\ V & \rightarrow \text{riferimento di Fase} \\ \bar{E}_2 = 220\, e^{-j120}\ V \\ \bar{E}_3 = 220\, e^{j120}\ V \end{cases}$

Lo stesso vale per le $V_{ij}$, sapendo che
le Tensioni Concatenate $V_{ij}$ sono in ANTICIPO di 30° rispetto alle $E_i$ corrispondenti

$$\Rightarrow \begin{cases} \bar{V}_{12} = 380\, e^{j30}\ V \\ \bar{V}_{23} = 380\, e^{-j90}\ V \\ \bar{V}_{31} = 380\, e^{j150}\ V \end{cases}$$

Ora, per le CORRENTI di FASE, si ha

$\bar{J}_{12} = \dfrac{\bar{V}_{12}}{\bar{Z}_\Delta}$ , dove $\bar{Z}_\Delta = R + jX = 30 + 8{,}04j = \underline{31{,}06\, e^{j15°}\ \Omega}$

$\Rightarrow \bar{J}_{12} = \dfrac{380}{31{,}06} \dfrac{e^{j30}}{e^{j15}} = \underline{12{,}23\, e^{j15°}\ A} \Rightarrow \begin{cases} \bar{J}_{12} = 12{,}23\, e^{j15}\ A \\ \bar{J}_{23} = 12{,}23\, e^{-j105}\ A \\ \bar{J}_{31} = 12{,}23\, e^{j135}\ A \end{cases}$

In un CARICO EQUILIBRATO, si ha che $I_{linea} = \sqrt{3}\, I_{fase}$

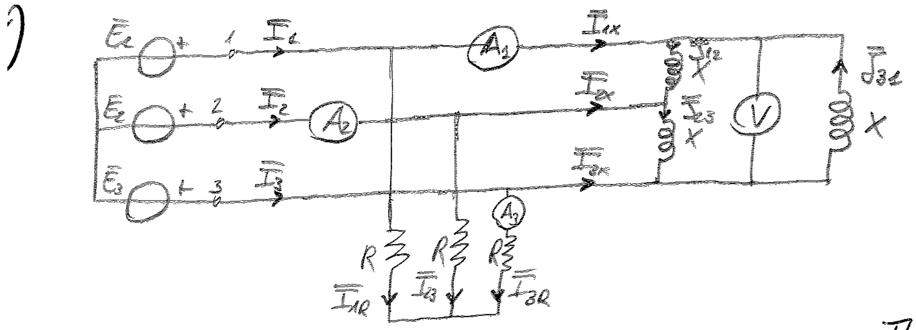
oppure: $\begin{cases} \bar{I}_1 = \bar{J}_{12} - \bar{J}_{31} = 11{,}81 + 3{,}16j + 8{,}65 - 8{,}65j = 20{,}46 - 5{,}49j = \underline{21{,}18\, e^{-j15°}\ A} \\ \bar{I}_2 = \bar{J}_{23} - \bar{J}_{12} = -3{,}16 - 11{,}81j - 11{,}81 - 3{,}16j = -14{,}97 - 14{,}97j = \underline{21{,}18\, e^{-j135}\ A} \\ \bar{I}_3 = \bar{J}_{31} - \bar{J}_{23} = -8{,}65 + 8{,}65j + 3{,}16 + 11{,}81j = -5{,}49 + 20{,}46j = \underline{21{,}18\, e^{j105}\ A} \end{cases}$

Per le Potenze Attive e Reattive, Trasformo il CARICO TRIANGOLO $\rightarrow$ STELLA:

$\bar{Z}_Y = \dfrac{1}{3} \bar{Z}_\Delta = 10 + 2{,}68j\ \Omega = \underline{10{,}35\, e^{j15}\ \Omega}$

$\Rightarrow P_{TOT} = 3 R_Y I_1^2 = 3 \cdot 10 \cdot 21{,}18^2 \simeq 13458\ W$

$Q_{TOT} = 3 X_Y I_1^2 = 3 \cdot 2{,}68 \cdot 21{,}18^2 = 3607\ VAr$



$V = 400\,V$
$R = 52{,}23\,\Omega$
$X = 55{,}62\,\Omega$
Terna Simmetrica Destrorsa
Carichi Equilibrati
$I_b = A_1, A_2, A_3$ ?

I carichi sono collegati in PARALLELO

(V) misura Tensione Concatenata => essendo il Carico Equilibrato, si avrà che le tensioni concatenate $V_{ij}$ sono in ANTICIPO di 30° rispetto alle rispettive Tensioni di Fase $E_i$

$$\Rightarrow\ E_1 = E_2 = E_3 = \frac{V}{\sqrt{3}} = 231\,V \Rightarrow \begin{cases} \bar{E}_1 = 231\, e^{j0}\,V \rightarrow \text{riferimento di fase} \\ \bar{E}_2 = 231\, e^{-j120}\,V \\ \bar{E}_3 = 231\, e^{j120}\,V \end{cases}$$

$$\Rightarrow \begin{cases} \bar{V}_{12} = 400\, e^{j30}\,V \\ \bar{V}_{23} = 400\, e^{-j90}\,V \\ \bar{V}_{31} = 400\, e^{j150}\,V \end{cases}$$

Possiamo determinare le Correnti di Fase $\bar{J}_{ij}$:

$$\bar{J}_{12} = \frac{\bar{V}_{12}}{jX} = \frac{400\, e^{j30}}{55{,}62\, e^{j90}} = \underline{7{,}22\, e^{-j60}\,A} \Rightarrow \begin{cases} \bar{J}_{12} = 7{,}22\, e^{-j60}\,A \\ \bar{J}_{23} = 7{,}22\, e^{-j180}\,A \\ \bar{J}_{31} = 7{,}22\, e^{j60}\,A \end{cases}$$

$$\Rightarrow \begin{cases} \bar{I}_{1X} = \bar{J}_{12} - \bar{J}_{31} = 3{,}61 - 6{,}25j - 3{,}61 - 6{,}25j = -12{,}5j = \underline{12{,}5\, e^{-j90}\,A} \\ \bar{I}_{2X} = \bar{J}_{23} - \bar{J}_{12} = -7{,}22 - 3{,}61 + 6{,}25j = -10{,}83 + 6{,}25j = \underline{12{,}5\, e^{j150}\,A} \\ \bar{I}_{3X} = \bar{J}_{31} - \bar{J}_{23} = 3{,}61 + 6{,}25j + 7{,}22 = 10{,}83 + 6{,}25j = \underline{12{,}5\, e^{j30}\,A} \end{cases}$$

- $$\bar{I}_{1R} = \frac{\bar{E}_1}{R} = \frac{231\, e^{j0}}{52{,}23} = 4\, e^{j0}\,A \Rightarrow \begin{cases} \bar{I}_{1R} = 4\, e^{j0}\,A \\ \bar{I}_{2R} = 4\, e^{-j120}\,A \\ \bar{I}_{3R} = 4\, e^{j120}\,A \end{cases}$$

$$\Rightarrow \begin{cases} \bar{I}_1 = \bar{I}_{1X} + \bar{I}_{1R} = -12{,}5j + 4 = 4 - 12{,}5j = 13{,}12\, e^{-j72{,}25°}\,A \\ \bar{I}_2 = \bar{I}_{2X} + \bar{I}_{2R} = -10{,}83 + 6{,}25j - 2 - 3{,}46j = -12{,}83 + 2{,}79j = 13{,}12\, e^{-j192{,}25}\,A \\ \bar{I}_3 = \bar{I}_{3X} + \bar{I}_{3R} = 10{,}83 + 6{,}25j - 2 + 3{,}46j = 8{,}83 + 9{,}71j = 13{,}12\, e^{j47{,}75°}\,A \end{cases}$$



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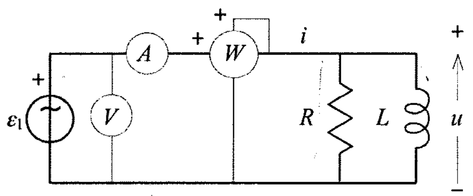
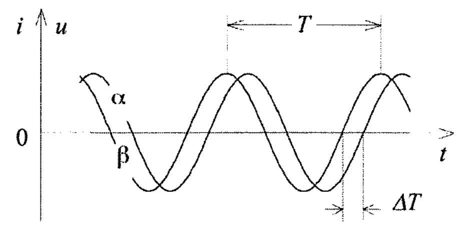
ELETTROTECNICA (C.L.Ing. Ambiente & Territorio) Appello 10° 19/09/2001
(Dip. Ingegneria Elettrica Industriale, prof. E. Barbisio)

## Circuito in regime sinusoidale permanente 1

Dati: $T$=50ms $\Delta T$=2ms max($i$)=8A max($u$)=200V

a) Quale delle due sinusoidi α,β rappresenta la $u(t)$ e la $i(t)$?
b) Quanto valgono le indicazioni dei tre strumenti di misura (voltmetro e amperometro sono a vero valore efficace)?
c) Se si raddoppia la frequenza del generatore, a parità di $R$,$L$ e di indicazione del voltmetro (v. sopra) quanto valgono le indicazioni del wattmetro e dell'amperometro?

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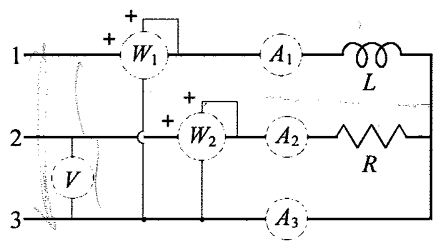
## Sistema trifase simmetrico squilibrato 1

Terna destrorsa.
Dati: $f$=50Hz $V$=400V $A_1$=5A $A_2$=5A




a) A quanto ammontano le indicazioni dei due wattmetri e dell'amperometro $A_3$?
b) Determinare il valore di $R$ e $L$.

---

Soluzioni:

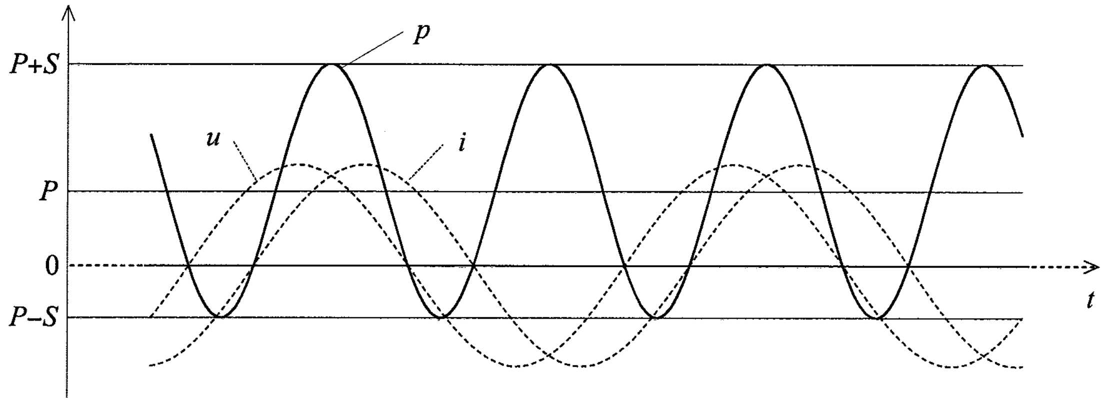
| | | | |
|---|---|---|---|
| 1a) | $u(t)\leftrightarrow\beta$ | $i(t)\leftrightarrow\alpha$ | |
| 1b) | $W\approx$775W | $A\approx$5,66A | $V\approx$141V |
| 1c) | $W'=W\approx$775W | $A'\approx$5,52A | |
| 2a) | $W_1$=0W | $W_2$=2kW | $A_3\approx$9,66A |
| 2b) | $R$=80Ω | $L\approx$255mH | |

**Soluzioni:**

1)

$\varphi = \frac{3}{20} 360° = 54°$ $\quad S = UI = \frac{200}{\sqrt{2}} \frac{8}{\sqrt{2}} = 800\,\text{VA}$ $\quad P = S\cos\varphi \approx 470\,\text{W}$

$\max(p) = P + S \approx 1270\,\text{W}$ $\quad \min(p) = P - S \approx -330\,\text{W}$



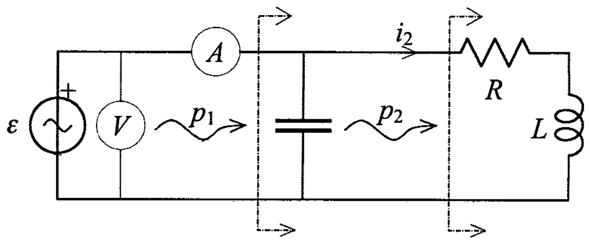

2)

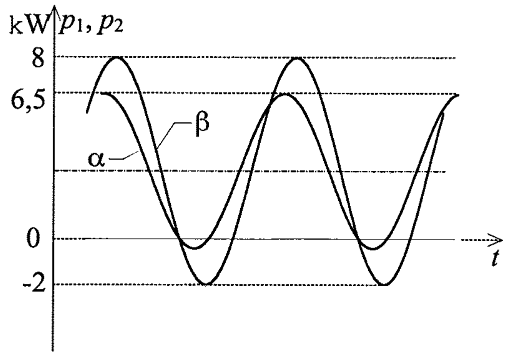
$$P = \sqrt{3} U I_{A_1} = 4\sqrt{3}U \approx 2{,}77\ \text{kW}$$
$$S = \sqrt{3} U I_{A_2} = 5\sqrt{3}U \approx 3{,}46\ \text{kVA}$$
$$Q = \sqrt{3} U I_{A_3} = \sqrt{S^2 - P^2} = 3\sqrt{3}U \approx 2{,}08\ \text{kvar} \qquad \Rightarrow \qquad I_{A_3} = 3\,\text{A}$$
$$W_1 + W_2 = P \quad W_1 - W_2 = Q/\sqrt{3}$$
$$W_1 = \tfrac{1}{2}\left(P + Q/\sqrt{3}\right) \approx 1986\,\text{W}$$
$$W_2 = \tfrac{1}{2}\left(P - Q/\sqrt{3}\right) \approx 786\,\text{W}$$

**1) Circuito in regime sinusoidale permanente**

Frequenza 50Hz. Il carico induttivo è parzialmente rifasato.
Dati: $V$=400V

c) Dopo aver stabilito come vanno associati i diagrammi α e β alle potenze $p_1$ e $p_2$, ricavare i valori di $L$ e $R$.




**Risultati:** $R \approx 19{,}2\,\Omega$ $L \approx 81\,\text{mH}$

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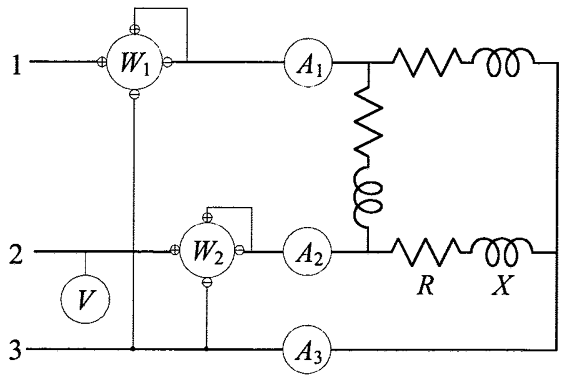
**1) Circuito trifase simmetrico**

Terna delle tensioni di alimentazione destrorsa.
Dati: $f$=50Hz $V$=400V $R = 40\,\Omega$ $X = 30\,\Omega$




a) Ricavare i valori delle indicazioni dei wattmetri e degli amperometri.

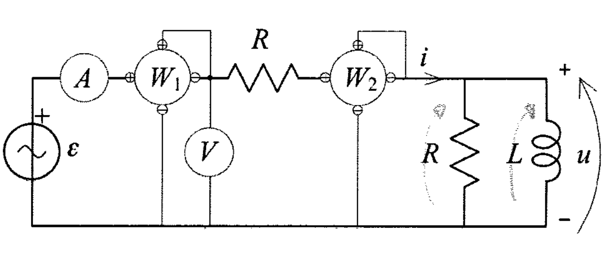
**Risultati:** $A_1 = A_2 = A_3 \approx 13{,}86\,\text{A}$ $W_1 \approx 5{,}5\,\text{kW}$ $W_2 \approx 2{,}18\,\text{kW}$

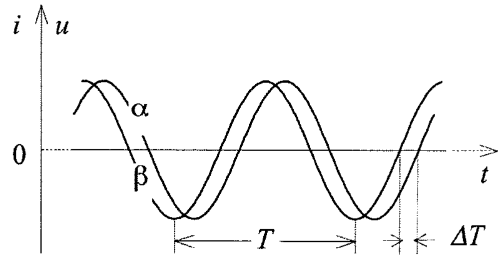
ELETTROTECNICA (C.L.Ing. Ambiente & Territorio) Appello 2° 11/02/2002
(Dip. Ingegneria Elettrica Industriale, prof. E. Barbisio)

## 1) Circuito in regime sinusoidale permanente

Dati: $T$=20ms $\Delta T$=2ms max($i$)=8A max($u$)=200V

a) Ricavare i valori di $L$ e $R$.
b) Quanto valgono le indicazioni dei wattmetri?

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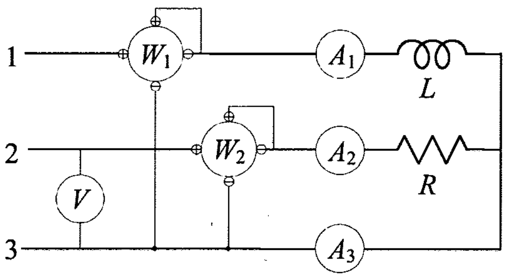
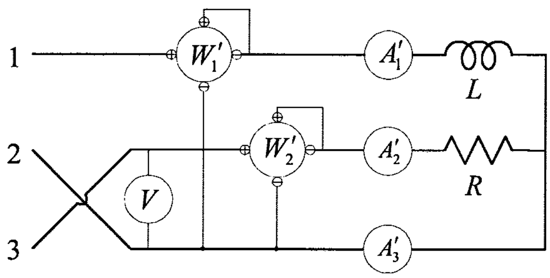
## 2) Sistema trifase simmetrico

Terna delle tensioni di alimentazione destrorsa. Carico squilibrato.
Dati (Fig. a): $f$=50Hz $V$=400V $A_1$=5A $A_2$=5A




(a)




(b)

a) Ricavare le indicazioni di tutti gli strumenti dopo lo scambio di connessioni, come indicato nella Fig. b.

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**Soluzioni:**

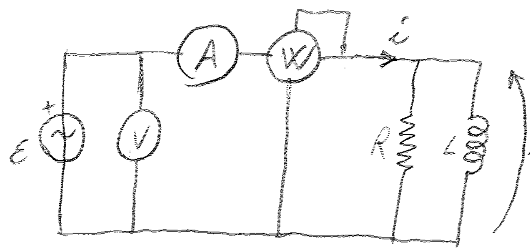
1a: $L \approx 135{,}4\,\text{mH}$ $R \approx 30{,}9\,\Omega$ 1b: $W_1 \approx 1636\,\text{W}$ $W_2 \approx 647{,}2\,\text{W}$

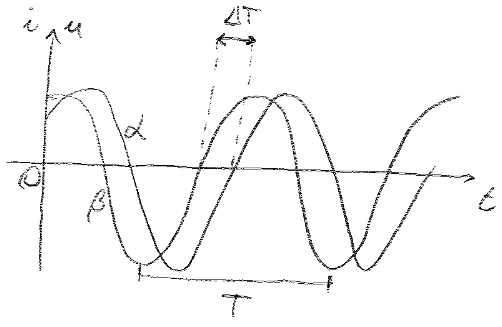
2a: $W_1' = W_1 = 0\,\text{W}$ $W_2' = W_2 = 2\,\text{kW}$ $A_1' = A_1 = 5\,\text{A}$ $A_2' = A_2 = 5\,\text{A}$ $A_3' \approx 2{,}6\,\text{A}$

# ELETTROTECNICA
# TEMI D'ESAME

## 19/09/2001

a)




$T = 50\,ms$

$\Delta T = 2\,ms$

$max(i) = 8\,A$

$max(u) = 200\,V$

a) Il circuito ha carattere Ohmico-Induttivo => la corrente è in RITARDO rispetto alla tensione, che viene assunta come riferimento di fase. Quindi la curva $\beta$ è associata a $u(t)$, la curva $\alpha$ è associata a $i(t)$

b) $I = \dfrac{max(i)}{\sqrt{2}} = 5{,}65\,A$

$U = \dfrac{max(u)}{\sqrt{2}} = 141{,}42\,V$

Calcoliamo differenza di fase tra $i(t)$ e $u(t)$:

$$360 : T = \varphi : \Delta T \Rightarrow \varphi = \frac{360 \cdot \Delta T}{T} = \frac{360 \cdot 2 \cdot \cancel{10^{-3}}}{50 \cdot \cancel{10^{-3}}} = 14{,}4$$

Ora: • Non essendoci impedenze intermedie tra Ⓐ e $i(t)$, il valore letto dall'Amperometro (a Valore Efficace) vale: Ⓐ $= 5{,}65\,A$

• Lo stesso discorso vale per Ⓥ: è collegato in parallelo con le impedenze R e L => "sentirà" la stessa tensione (a Valore Efficace) => Ⓥ $= 141\,V$

• Il Watt-metro legge SOLO il valore della POTENZA ATTIVA:

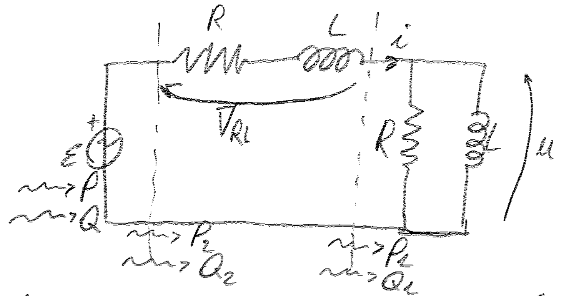
$$P = V \cdot I \cdot \cos\varphi = 141 \cdot 5{,}65 \cdot \cos(14{,}4) = 774\,W$$

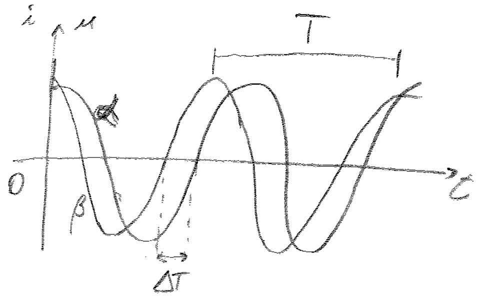
$$\Rightarrow Ⓦ = 774\,W$$

$\boxed{OK}$

28/01/2002

a)




$T = 40$ ms
$\Delta T = 5$ ms
$\max(i) = 8A$
$\max(u) = 200V$

Tr: $\max(p(t))$? $\min(p(t))$? $\langle p(t) \rangle_T$?

• Il circuito ha carattere Ohmico-Induttivo => la corrente è in RITARDO rispetto alla Tensione e, quindi, alla corrente $i(t)$ è associata la curva $\alpha$, alla Tensione $u(t)$ è associata la curva $\beta$.

$$I = \frac{\max(i)}{\sqrt{2}} = 5{,}66\ A$$
$$V = \frac{\max(u)}{\sqrt{2}} = 141{,}4\ V$$

• Cerchiamo differenza di fase $\varphi$ tra $i(t)$ e $u(t)$ considerando $u(t)$ come riferimento di fase

$$360 : T = \varphi : \Delta T$$
$$\Rightarrow \varphi = \frac{360 \cdot \Delta T}{T} = \frac{360 \cdot 5 \cdot 10^{-3}}{40 \cdot 10^{-3}} = 45°$$

• $P_1 = V \cdot I \cdot \cos\varphi = 565\ W$
$Q_1 = VI \operatorname{sen}\varphi = 565\ VAr$

$$\Rightarrow \bar{S}_1 = P_1 + jQ_1 = 565(1+j)\ VA = 799\, e^{j45}\ VA$$

• Considerando il Parallelo:

$\bar{I} = 5{,}66\, e^{j45}\ A$ possiamo scomporla in

PARTE REALE: $I_R = 5{,}66 \cos 45 = 4\ A$
PARTE IMMAGINARIA: $I_L = 5{,}66 \operatorname{sen} 45 = 4A$

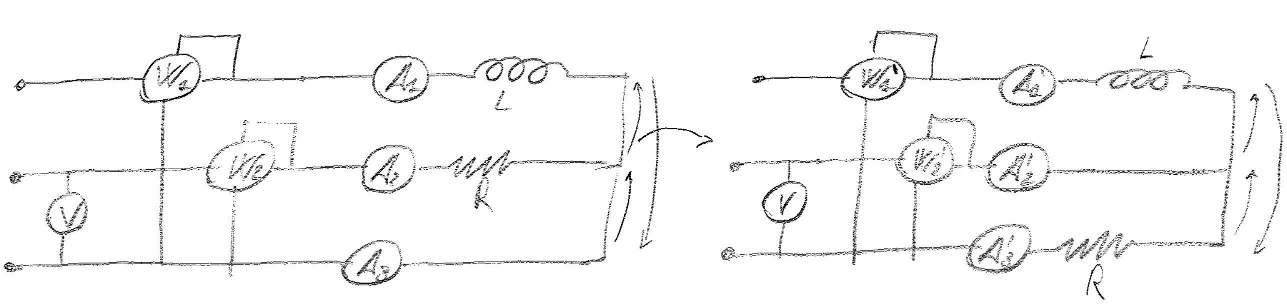
Ma $V = R I_R \Rightarrow R = \frac{V}{I_R} = 35{,}35\ \Omega$
$V = X_L I_L \Rightarrow X_L = \frac{V}{I_L} = 35{,}35\ \Omega$

Considerando R e L in Serie:

$$\bar{Z}_{eq} = R + jX_L = 35{,}35(1+j) = 50\, e^{j45}\ \Omega$$

$$\Rightarrow \bar{V}_{RL} = \bar{Z}_{eq} \cdot \bar{I} = 50\, e^{j45} \cdot 5{,}66\, e^{j45} = 283\, e^{j90}\ V$$

)




$f = 50\,Hz$ $\quad A_1 = 5A$

$V = 400\,V$ $\quad A_2 = 5A$

- Il sistema è SIMMETRICO $\Rightarrow |\bar{E}_1| = |\bar{E}_2| = |\bar{E}_3| = E$

$|\bar{V}_{12}| = |\bar{V}_{23}| = |\bar{V}_{31}| = V$

dove $E = \frac{V}{\sqrt{3}} = 231\,V$

$$\Rightarrow \begin{cases} \bar{E}_1 = 231\, e^{j0}\, V \\ \bar{E}_2 = 231\, e^{-j120}\, V \\ \bar{E}_3 = 231\, e^{j120}\, V \end{cases} \qquad \begin{cases} \bar{V}_{12} = 400\, e^{j30}\, V \\ \bar{V}_{23} = 400\, e^{-j90}\, V \\ \bar{V}_{31} = 400\, e^{j150}\, V \end{cases}$$

$$\bar{I}_1 = \frac{-\bar{V}_{31}}{jX_L} \qquad \bar{I}_2 = \frac{\bar{V}_{23}}{R}$$

dove $X_L = \frac{V}{I_1} = 80\,\Omega$ $\Rightarrow \begin{cases} \bar{I}_1 = \dfrac{400\, e^{-j30}}{80\, e^{j90}} = 5\, e^{-j120}\, A \\ \bar{I}_2 = \dfrac{400\, e^{-j90}}{80} = 5\, e^{-j90}\, A \end{cases}$

$R = \frac{V}{I_2} = 80\,\Omega$

LEGGE CORRENTI: $\bar{I}_1 + \bar{I}_2 + \bar{I}_3 = 0$

$$\Rightarrow \bar{I}_3 = -(\bar{I}_1 + \bar{I}_2) = -(5\cos(-120) + j5\sin(-120) + 5\cos(-90) + j5\sin(-90))$$

$$= -(-2{,}5 - 4{,}33j - 5j) = 2{,}5 + 9{,}33j = 9{,}66\, e^{j25}$$

$\Rightarrow A_3 = 9{,}66\, A$

- $W_1 = -\bar{V}_{31} \cdot \bar{I}_1 = V_{31} I_1 \cos(-30+120) = 0$
- $W_2 = \bar{V}_{23} \cdot \bar{I}_2 = V_{23} I_2 \cos(-90+90) = 2\,KW$



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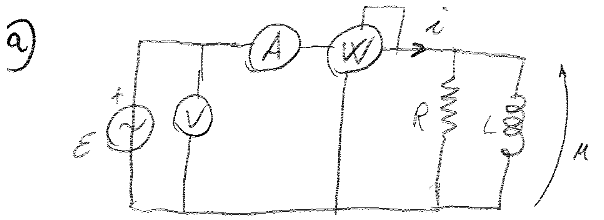
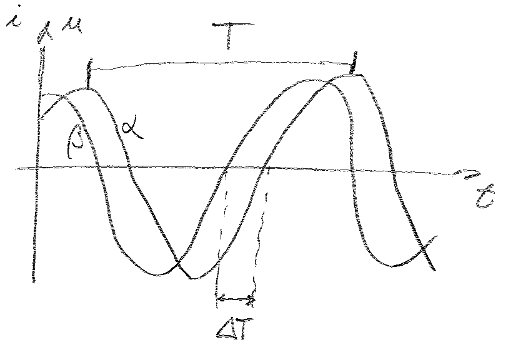
28/01/2002

a)




$T = 20\,ms$ $\quad max(i) = 8A$

$\Delta T = 3\,ms$ $\quad max(u) = 200V$ $\quad$ Th. Disegnare $p(t)$?

• Il circuito è Ohmico-Induttivo => la corrente è in RITARDO rispetto alla tensione $u$, che viene presa come riferimento di fase. Quindi, la curva $\alpha$ è associata a $i(t)$, la curva $\beta$ a $u(t)$.

• $I = \frac{max(i)}{\sqrt{2}} = 5{,}66\,A$

$V = \frac{max(u)}{\sqrt{2}} = 141{,}4V$

Cerchiamo differenza di fase tra $i(t)$ e $u(t)$:

$360 : T = \varphi : \Delta T$

$\Rightarrow \varphi = \frac{360 \cdot \Delta T}{T} = \frac{360 \cdot 3 \cdot \cancel{10^{-3}}}{20 \cdot \cancel{10^{-3}}} = 54°$

$\Rightarrow \bar{I} = 5{,}66\, e^{j54} A$

$\Rightarrow I_R = 5{,}66 \cos 54 = 3{,}33\,A$

$I_L = 5{,}66 \,\text{sen}\, 54 = 4{,}58\,A$

$\Rightarrow R = \frac{V}{I_R} = 42{,}46\,\Omega$ $\quad P = VI\cos\varphi = 420{,}42\,W$

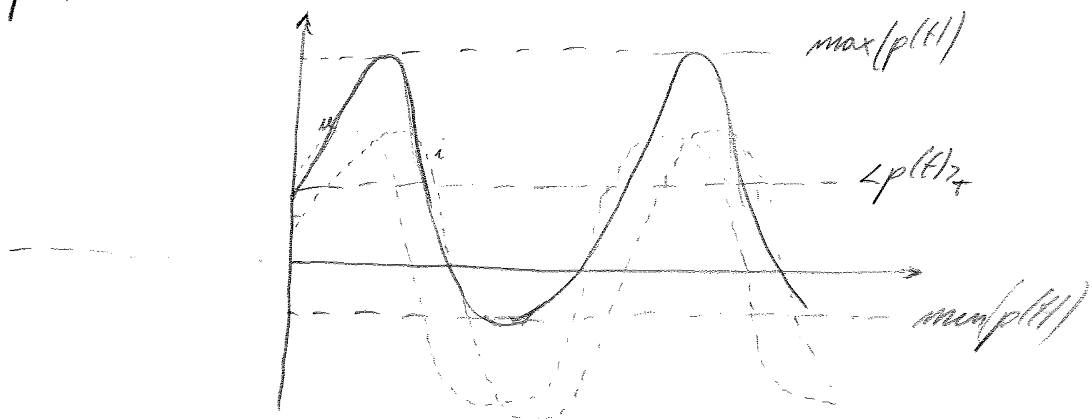
$X_L = \frac{V}{I_L} = 30{,}82\,\Omega$ $\quad Q = VI \,\text{sen}\, \varphi = 642{,}47\,VAr$

$\Rightarrow S = P + jQ = 420{,}42 + j\,642{,}42 = 800\, e^{j54}\,VA$

$\Rightarrow \langle p(t) \rangle_T = P = 420\,W$

$max(p(t)) = P + |S| = 1220\,W$

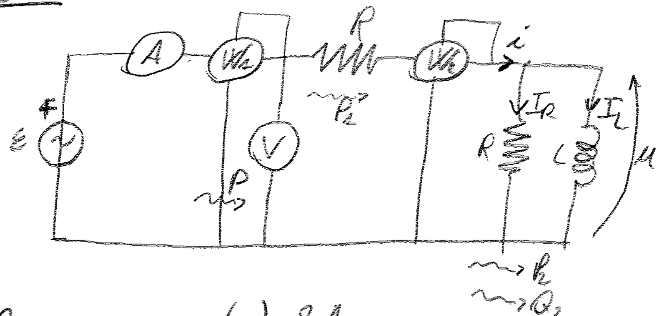
$min(p(t)) = P - |S| = -380\,W$

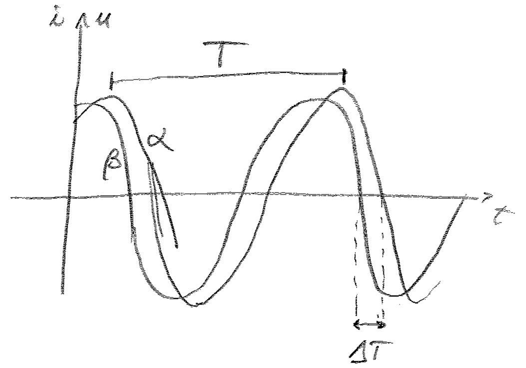



OK

11/02/2002

(a)




$T = 20\ ms$ $\quad max(i) = 8\ A$

$\Delta T = 2\ ms$ $\quad max(u) = 200\ V$

Tb: L, R? Indicazioni strumenti di misura?

• Il circuito è Ohmico-Induttivo => la corrente è in RITARDO rispetto alla tensione, che viene presa come riferimento di fase. Quindi la curva $\alpha$ è associata a $i(t)$ mentre la curva $\beta$ è associata a $u(t)$.

• $I = \dfrac{max(i)}{\sqrt{2}} = 5,66\ A$

$U = \dfrac{max(u)}{\sqrt{2}} = 141,4\ V$

• Calcoliamo differenza di fase tra $i(t)$ e $u(t)$:

$360 : T = \varphi : \Delta T$

$\Rightarrow \varphi_1 = \dfrac{360 \cdot \Delta T}{T} = \dfrac{360 \cdot 2 \cdot \cancel{10^{-3}}}{20 \cdot \cancel{10^{-3}}} = 36°$

$\Rightarrow \bar{U} = 141,4\ e^{j0}\ V$

$\bar{I} = 5,66\ e^{j36}\ A$

Ma $\bar{I} = I_R + j I_L \Rightarrow I_R = I \cos 36 = 4,58\ A$

$I_L = I \,\text{sen}\, 36 = 3,33\ A$

$\Rightarrow R = \dfrac{U}{I_R} = 30,9\ \Omega$

$X_L = \dfrac{U}{I_L} = 42,46\ \Omega$

Ma $X_L = \omega L \Rightarrow L = \dfrac{X_L}{\omega} = \dfrac{X_L}{2\pi} \cdot T = 135,2\ mH$

• $W_2 = |U \cdot I \cos(0 - 36)| = 647,5\ W$

• $P_1 = R I^2 = 989,9\ W$

$\Rightarrow W_1 = P_1 + P_2 = 1637,4\ W$

Oppure: $\bar{V}_R = R\bar{I} = 174,9\ e^{j36}\ V$

$\Rightarrow \bar{V} = \bar{V}_R + \bar{U} = 174,9\ e^{j36} + 141,4\ e^{j0} = 141,5 + j102,8 + 141,4 = 282,9 + j102,8$

$= 301\ e^{j20}\ V$

$\Rightarrow W_1 = |V I \cos(20 - 36)| = 1637,6\ W$ OK