



Corso Luigi Einaudi, 55 - Torino

Appunti universitari

Tesi di laurea

Cartoleria e cancelleria

Stampa file e fotocopie

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Rilegature

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A P P U N T I

STUDENTE: Taberna

MATERIA: Fisica II

Prof. Barbero

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FISICA II

G. Taberna

CONDIZIONI DI CONSERVATIVITÀ

Ogni campo conservativo è associato a una funzione scalare detta ENERGIA POTENZIALE/POTENZIALE, tale che ogni integrale di linea si può esprimere come differenza di questa funzione scalare.

Ipotesi: \vec{F} è conservativo $\Rightarrow W_{A \rightarrow B} = \int_A^B \vec{F} \cdot d\vec{s}$ non dipende da γ

$$\Rightarrow W_{A \rightarrow B} = \int_A^B \vec{F} \cdot d\vec{s} = f(A, B)$$



$$W_{O \rightarrow B} = f(O, B) \quad W_{O \rightarrow A} = f(O, A) \quad W_{A \rightarrow B} = f(A, B)$$

$$W_{O \rightarrow B} = \int_O^B \vec{F} \cdot d\vec{s} = \int_O^A \vec{F} \cdot d\vec{s} + \int_A^B \vec{F} \cdot d\vec{s} = W_{O \rightarrow A} + W_{A \rightarrow B} \Rightarrow f(O, B) = f(O, A) + f(A, B)$$

$f(A, B) = f(O, B) - f(O, A) \Rightarrow f(x, y) = u(x) - u(y)$: l'integrale di linea tra due punti dipende solo dai punti di partenza e arrivo

$$f(O, B) = u(O) - u(B) \quad f(O, A) = u(O) - u(A) \quad f(O, B) - f(O, A) = u(O) - u(B) - u(O) + u(A) = u(A) - u(B)$$

CONDIZIONI DI CONSERVATIVITÀ

Ipotesi: \vec{F} è conservativo $\Rightarrow W_{A \rightarrow B} = u(A) - u(B)$

$$W_{A \rightarrow B} = \int_A^B \vec{F} \cdot d\vec{s} \quad u(A) - u(B) = - \int_A^B du \quad \Rightarrow \int_A^B \vec{F} \cdot d\vec{s} = - \int_A^B du$$

$\int_A^B (\vec{F} \cdot d\vec{s} + du) = 0 \quad \forall A, B$ dato che deve essere sempre = 0, allora la fz. deve essere nulla: $\vec{F} \cdot d\vec{s} + du = 0$ (*) per ogni linea che possiamo considerare

$$P = (x, y, z) \quad d\vec{s} = \vec{u}_x dx + \vec{u}_y dy + \vec{u}_z dz \quad \vec{F} = F_x \vec{u}_x + F_y \vec{u}_y + F_z \vec{u}_z$$

$$\vec{F} \cdot d\vec{s} = F_x dx + F_y dy + F_z dz \quad du(x, y, z) = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z} dz$$

Quindi (*) diventa:

$$\left(F_x + \frac{\partial u}{\partial x} \right) dx + \left(F_y + \frac{\partial u}{\partial y} \right) dy + \left(F_z + \frac{\partial u}{\partial z} \right) dz = 0$$

$$F_x = - \frac{\partial u}{\partial x} \quad F_y = - \frac{\partial u}{\partial y} \quad F_z = - \frac{\partial u}{\partial z}$$

se un campo è conservativo, le coordinate cartesiane della forza sono le derivate parziali del potenziale rispetto a x, y, z.

$$\vec{F} = - \vec{u}_x \frac{\partial u}{\partial x} - \vec{u}_y \frac{\partial u}{\partial y} - \vec{u}_z \frac{\partial u}{\partial z}$$

$$\vec{\nabla} = \vec{u}_x \frac{\partial}{\partial x} + \vec{u}_y \frac{\partial}{\partial y} + \vec{u}_z \frac{\partial}{\partial z} \quad F$$

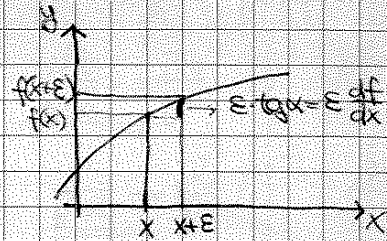
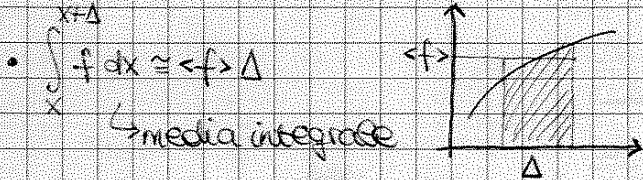
(operatore GRADIENTE, rende una fz. vettoriale)

$$\boxed{\vec{F} = - \vec{\nabla} u} \quad \vec{F} = - \text{grad } u$$

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TEOREMA DI STOKES

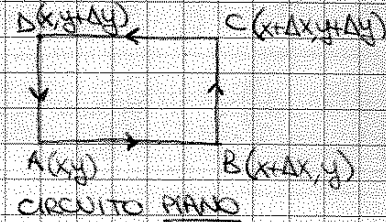
• $f(x+\epsilon) - f(x) \approx \epsilon \frac{df}{dx}$ (*) sviluppo in serie di Taylor



Teorema per il calcolo della circuitazione (indicare sempre il verso!)

$\vec{F}(x,y) = F_x(x,y)\vec{u}_x + F_y(x,y)\vec{u}_y$

$\Delta x, \Delta y$ infinitesimi



Calcolo l'integrale:

$$\oint_{\text{RETT}} \vec{F} \cdot d\vec{s} = \int_A^B \vec{F} \cdot (\vec{u}_x dx) + \int_B^C \vec{F} \cdot (\vec{u}_y dy) + \int_C^D \vec{F} \cdot (-\vec{u}_x dx) + \int_D^A \vec{F} \cdot (-\vec{u}_y dy) =$$

$$= \int_A^B F_x(x,y) dx + \int_B^C F_y(x,y) dy - \int_C^D F_x(x,y) dx - \int_D^A F_y(x,y) dy =$$

$$= \langle F_x \rangle_{AB} \Delta x + \langle F_y \rangle_{BC} \Delta y - \langle F_x \rangle_{CD} \Delta x - \langle F_y \rangle_{DA} \Delta y = (\langle F_x \rangle_{AB} - \langle F_x \rangle_{CD}) \Delta x + (\langle F_y \rangle_{BC} - \langle F_y \rangle_{DA}) \Delta y$$

↳ applico (*)

$$\langle F_x \rangle_{AB} = \frac{F_x(A) + F_x(B)}{2} = \frac{1}{2} [F_x(x,y) + F_x(x+\Delta x, y)] = \frac{1}{2} [F_x(x,y) + F_x(x,y) + \Delta x \frac{\partial F_x}{\partial x}] = F_x(x,y) + \frac{1}{2} \frac{\partial F_x}{\partial x} \Delta x$$

$$\langle F_y \rangle_{BC} = \frac{F_y(B) + F_y(C)}{2} = \frac{1}{2} [F_y(x, y+\Delta y) + F_y(x+\Delta x, y+\Delta y)] = \frac{1}{2} [F_y(x,y) + \frac{\partial F_y}{\partial y} \Delta y + F_y(x,y) + \frac{\partial F_x}{\partial x} \Delta x + \frac{\partial F_y}{\partial y} \Delta y] =$$

$$= F_y(x,y) + \frac{1}{2} \frac{\partial F_x}{\partial x} \Delta x + \frac{\partial F_y}{\partial y} \Delta y$$

$$\langle F_x \rangle_{AB} - \langle F_x \rangle_{CD} = -\frac{\partial F_x}{\partial x} \Delta x$$

$$\langle F_y \rangle_{BC} - \langle F_y \rangle_{DA} = \frac{\partial F_y}{\partial y} \Delta y$$

$$\oint_{\text{RETT}} \vec{F} \cdot d\vec{s} = -\frac{\partial F_x}{\partial x} \Delta x \Delta y + \frac{\partial F_y}{\partial y} \Delta x \Delta y = \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \Delta x \Delta y = \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \Delta S$$

↳ $\Delta S = \Delta x \Delta y$, area circuito

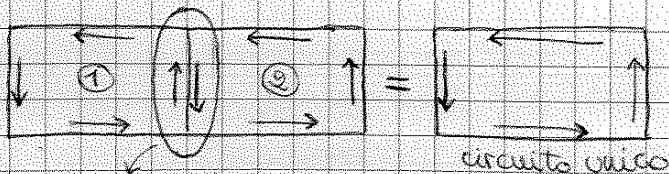
↳ prima condizione di conservatività

CIRCUITO γ GENERICO

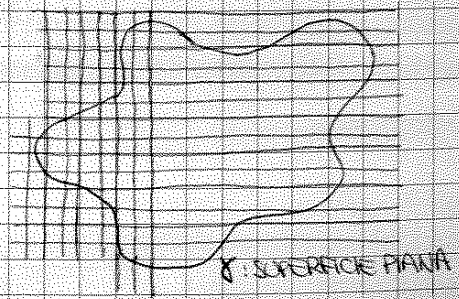
Lo divido in piccoli circuiti rettangolari

$$\oint_{\gamma} \vec{F} \cdot d\vec{s} = \sum_{i=1}^N \oint_{\Gamma_i} \vec{F} \cdot d\vec{s}_i$$

Infatti:



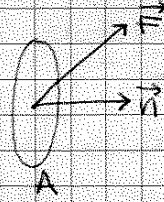
si annullano identicamente!



FLUSSO

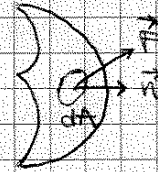
Dato un vettore \vec{F} costante nello spazio e un'area A della ^{piana} spazio, chiamiamo FLUSSO di un vettore attraverso un'area:

$$\Phi_A(\vec{F}) = \vec{F} \cdot \vec{n} \cdot A$$



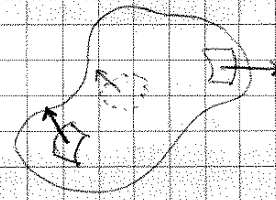
Quando la superficie non è piana e il vettore cambia, allora:

$$\Phi_A(\vec{F}) = \int_A \vec{F} \cdot \vec{n} \, dA \quad \text{se } \vec{F} = \vec{F}(\vec{r})$$



Nel caso di una superficie chiusa, la normale positiva al piano è quella che va da dentro a fuori

$$\Phi_A(\vec{F}) = \oint_A \vec{F} \cdot \vec{n} \, dA$$



↓ pag 58

TEOREMA DI GAUSS

$$P = (x, y, z) \quad P_2 = \left(x + \frac{\Delta x}{2}, y, z\right) \quad \Delta A_1 = \Delta A_2 = \Delta y \Delta z$$

valido per superfici chiuse!

Considero un vettore \vec{F} e calcolo il flusso in direz. x:

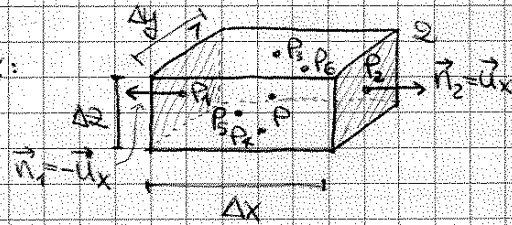
$$\Delta\Phi_x(2) = \vec{F}(2) \cdot \vec{n}_2 \, \Delta y \Delta z = \vec{F}(2) \cdot \vec{u}_x \, \Delta y \Delta z = F_x(2) \, \Delta y \Delta z$$

$$\Delta\Phi_x(1) = \vec{F}(1) \cdot \vec{n}_1 \, \Delta y \Delta z = \vec{F}(1) \cdot (-\vec{u}_x) \, \Delta y \Delta z = -F_x(1) \, \Delta y \Delta z$$

$$\Delta\Phi_x = \Delta\Phi_x(2) + \Delta\Phi_x(1) = [F_x(2) - F_x(1)] \, \Delta y \Delta z$$

$$[F_x(2) - F_x(1)] = F_x\left(x + \frac{\Delta x}{2}, y, z\right) - F_x(x, y, z) = F_x(x, y, z) + \frac{\partial F_x}{\partial x} \Delta x - F_x(x, y, z) = \frac{\partial F_x}{\partial x} \Delta x$$

$$\Delta\Phi_x = \frac{\partial F_x}{\partial x} \Delta x \Delta y \Delta z = \frac{\partial F_x}{\partial x} \Delta \tau \quad \Delta \tau = \Delta x \Delta y \Delta z : \text{elemento di volume}$$



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oppure considerando P_1, P_2 :

$$\Delta\Phi_x = \vec{F}_1 \cdot \vec{n}_1 \, \Delta A_1 + \vec{F}_2 \cdot \vec{n}_2 \, \Delta A_2 = [\vec{F}_1 \cdot (-\vec{u}_x) + \vec{F}_2 \cdot \vec{u}_x] \, \Delta y \Delta z = [F_x(P_2) - F_x(P_1)] \, \Delta y \Delta z =$$

$$= \left[-F_x\left(x - \frac{\Delta x}{2}, y, z\right) + F_x\left(x + \frac{\Delta x}{2}, y, z\right) \right] \, \Delta y \Delta z$$

$$= \left[-F_x + \frac{\partial F_x}{\partial x} \frac{\Delta x}{2} + F_x + \frac{\partial F_x}{\partial x} \frac{\Delta x}{2} \right] \, \Delta y \Delta z = \frac{\partial F_x}{\partial x} \Delta x \Delta y \Delta z = \frac{\partial F_x}{\partial x} \Delta \tau$$

Flusso lungo la direzione z: $\Delta\Phi_z = \frac{\partial F_z}{\partial z} \Delta \tau$ con $P_3 = (x, y, z + \frac{\Delta z}{2})$ $P_4 = (x, y, z - \frac{\Delta z}{2})$

Flusso lungo la direzione y: $\Delta\Phi_y = \frac{\partial F_y}{\partial y} \Delta \tau$ con $P_5 = (x, y - \frac{\Delta y}{2}, z)$ $P_6 = (x, y + \frac{\Delta y}{2}, z)$

Flusso totale: $\Delta\Phi = \Delta\Phi_x + \Delta\Phi_y + \Delta\Phi_z = \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right) \Delta \tau = \nabla \cdot \vec{F} \, \Delta \tau$

$$\Delta\Phi = (\nabla \cdot \vec{F}) \Delta \tau$$

S, Σ superfici aperte

$S+\Sigma$ superficie chiusa \rightarrow posso applicare Teor. GAUSS

se $\vec{\nabla} \cdot \vec{F} = 0$, $\oint_{S+\Sigma} \vec{F} \cdot \vec{n} = 0$

$$\oint_{S+\Sigma} \vec{F} \cdot \vec{n} dA = \iint_S \vec{F} \cdot \vec{n}_S dS + \iint_{\Sigma} \vec{F} \cdot \vec{n}_{\Sigma} d\Sigma = 0$$

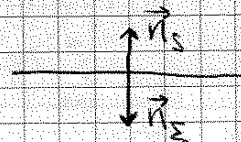
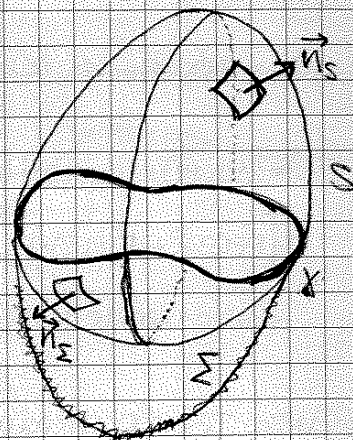
Se S e Σ hanno lo stesso bordo γ

$$\iint_S \vec{F} \cdot \vec{n}_S dS = - \iint_{\Sigma} \vec{F} \cdot \vec{n}_{\Sigma} d\Sigma \quad : \text{flusso uguale e contrario}$$

Immagino di modificare le superfici e portarle a contatto \Rightarrow

$$\rightarrow \iint_S \vec{F} \cdot \vec{n}_S dS = \iint_{\Sigma} \vec{F} \cdot (-\vec{n}_{\Sigma}) d\Sigma$$

\vec{n}_S è concorde con $-\vec{n}_{\Sigma}$ quando $S \equiv \Sigma$



⊗ $\vec{R} = \vec{\nabla} \times \vec{F}$

$$\vec{\nabla} \cdot \vec{R} = \frac{\partial R_x}{\partial x} + \frac{\partial R_y}{\partial y} + \frac{\partial R_z}{\partial z} = \frac{\partial}{\partial x} \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) + \frac{\partial}{\partial y} \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) = 0$$

oppure $\vec{\nabla} \cdot \vec{R} = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = 0$ perchè prodotto scalare di vettori \perp tra loro

Il rotore di un vettore è sempre un vettore ~~scalandale~~ scalandale!

TEOR. DI STOKES: La circuitazione di un vettore lungo una linea γ è uguale al flusso del rotore del vettore attraverso una qualunque superficie di bordo γ .

COORDINATE POLARI SFERICHE

$P \equiv (r, \theta, \phi)$

$x = r \sin \theta \cos \phi$

$y = r \sin \theta \sin \phi$

$z = r \cos \theta$

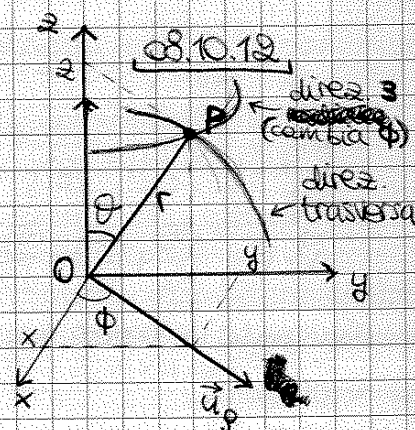
$\vec{r} = x\vec{u}_x + y\vec{u}_y + z\vec{u}_z = r \{ \sin \theta (\vec{u}_x \cos \phi + \vec{u}_y \sin \phi) + \vec{u}_z \cos \theta \}$

$\vec{u}_\rho = \vec{u}_x \cos \phi + \vec{u}_y \sin \phi$: VETTORE TRASVERSO

$dx = dr \sin \theta \cos \phi + r \cos \theta d\theta \cos \phi - r \sin \theta \sin \phi d\phi$

$dy = dr \sin \theta \sin \phi + r \cos \theta d\theta \sin \phi + r \sin \theta \cos \phi d\phi$

$dz = dr \cos \theta - r \sin \theta d\theta$

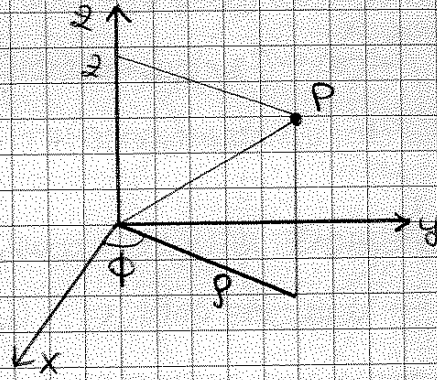


COORDINATE CILINDRICHE

$$P \equiv (\rho, \phi, z)$$

$$\begin{cases} x = \rho \cos \phi \\ y = \rho \sin \phi \\ z = z \end{cases}$$

$$\vec{\nabla} = \vec{u}_\rho \frac{\partial}{\partial \rho} + \vec{u}_\phi \frac{1}{\rho} \frac{\partial}{\partial \phi} + \vec{u}_z \frac{\partial}{\partial z}$$



FORZA CENTRALE

Forza il cui modulo dipende solo dalla distanza punto-origine ed è diretta sempre lungo la direzione radiale

Forza centrale conservativa?

$$W_{A \rightarrow B}^{(F)} = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B f(r) \vec{u}_r \cdot (\vec{u}_r dr + \vec{u}_\phi r d\phi + \vec{u}_z r \sin\theta d\theta) = \int_A^B f(r) dr \Rightarrow \text{dipende solo da } r$$

oppure

$$\vec{F} = \frac{f(r)}{r} \vec{u}_r = g(r) \vec{u}_r$$

$$\begin{cases} F_x = g(r)x \\ F_y = g(r)y \\ F_z = g(r)z \end{cases}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

Dimostro che il rotore di \vec{F} è nullo:

$$\frac{\partial F_x}{\partial y} = \frac{\partial}{\partial y} \{g(r)x\} = x \frac{\partial}{\partial y} g(r) = x \frac{dg}{dr} \frac{\partial r}{\partial y} = \frac{xy}{r} \frac{dg}{dr}$$

$$\frac{\partial r}{\partial y} = \frac{\partial}{\partial y} \sqrt{x^2 + y^2 + z^2} = \frac{1}{2} (x^2 + y^2 + z^2)^{-1/2} \cdot 2y = \frac{y}{r}$$

$$\frac{\partial F_y}{\partial x} = \frac{\partial}{\partial x} \{g(r)y\} = y \frac{\partial}{\partial x} g(r) = \frac{yx}{r} \frac{dg}{dr}$$



PRINCIPIO DI SOVRAPPOSIZIONE

$$\vec{T}_1 = k \frac{Q_1 q}{r_1^2} \vec{u}_1$$

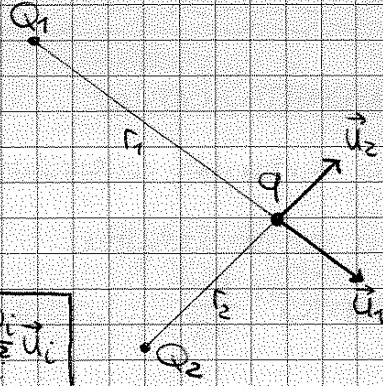
$$\vec{T}_2 = k \frac{Q_2 q}{r_2^2} \vec{u}_2$$

Per le cariche elettriche vale la sovrapposizione degli effetti:

$$\vec{T} = \vec{T}_1 + \vec{T}_2$$

$$\vec{T} = \sum_{i=1}^N k \frac{Q_i q}{r_i^2} \vec{u}_i \quad \bullet \quad \boxed{\vec{F} = q \sum_{i=1}^N k \frac{Q_i}{r_i^2} \vec{u}_i}$$

non dipende da q



$$\boxed{\sum_{i=1}^N k \frac{Q_i}{r_i^2} \vec{u}_i = \vec{E}} = \left[\frac{N}{C} \right] : \text{CAMPO ELETTROSTATICO} \quad \vec{E} = \frac{\vec{T}}{q}$$

$$\vec{R}_i = x_i \vec{u}_x + y_i \vec{u}_y + z_i \vec{u}_z$$

$$\vec{R} = x \vec{u}_x + y \vec{u}_y + z \vec{u}_z$$

$$\vec{r}_i = (x-x_i) \vec{u}_x + (y-y_i) \vec{u}_y + (z-z_i) \vec{u}_z$$

$$\vec{T}_i = k \frac{Q_i}{r_i^2} \vec{u}_i$$

$$\vec{T}(x,y,z) = \sum_{i=1}^N k \frac{Q_i}{r_i^2} \vec{r}_i = \sum_{i=1}^N k Q_i \frac{(x-x_i) \vec{u}_x + (y-y_i) \vec{u}_y + (z-z_i) \vec{u}_z}{[(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2]^{3/2}}$$

P(x,y,z): COORDINATE PUNTO CAMPO

Q_i(x_i, y_i, z_i): COORDINATE PUNTO SORGENTE

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$$E_x = \sum_i k Q \frac{x-x_i}{r_i^3}$$

$$E_y = \sum_i k Q \frac{y-y_i}{r_i^3}$$

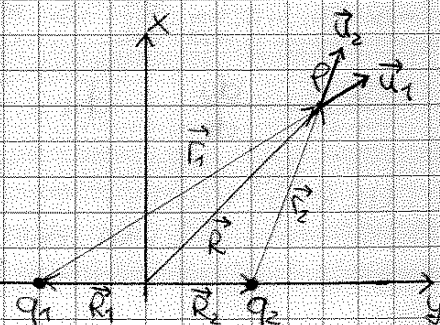
$$E_z = \sum_i k Q \frac{z-z_i}{r_i^3}$$

⊗ Verificare che $\frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x}$, $\frac{\partial E_x}{\partial z} = \frac{\partial E_z}{\partial x}$, $\frac{\partial E_y}{\partial z} = \frac{\partial E_z}{\partial y}$

⊗ $q_1 = q_2$ $P_1 = (0, -\frac{a}{2})$ $P_2 = (0, \frac{a}{2})$

$$\vec{E}(P) = \vec{E}_1(P) + \vec{E}_2(P) = k \frac{q}{r_1^2} \vec{u}_1 + k \frac{q}{r_2^2} \vec{u}_2$$

$$\text{se } q_1 = q_2 = q \quad \vec{E}(P) = k \frac{q}{r_1^2} \vec{u}_1 + k \frac{q}{r_2^2} \vec{u}_2$$



1) P è all'asse y

• se $y > \frac{a}{2}$: $\vec{T} = k \frac{q}{(y+\frac{a}{2})^2} \vec{u}_y + k \frac{q}{(y-\frac{a}{2})^2} \vec{u}_y$

• se $-\frac{a}{2} < y < \frac{a}{2}$: $\vec{T} = k \frac{q}{(y+\frac{a}{2})^2} \vec{u}_y - k \frac{q}{(\frac{a}{2}-y)^2} \vec{u}_y$

• se $y < -\frac{a}{2}$: $\vec{T} = -k \frac{q}{(y+\frac{a}{2})^2} \vec{u}_y - k \frac{q}{(y-\frac{a}{2})^2} \vec{u}_y$

Cosa succede con distribuzioni di carica continue?

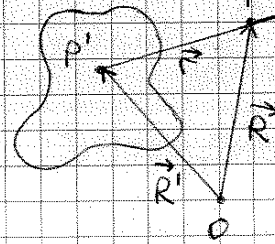
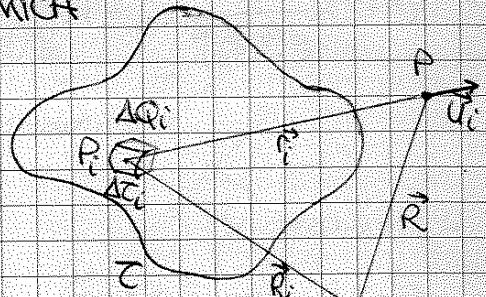
① DISTRIBUZIONE VOLUMICA

$$\vec{E} = \lim_{N \rightarrow \infty} \sum_{i=1}^N k \frac{\Delta Q_i}{r_i^2} \vec{u}_i$$

$$\vec{E} = \int k \frac{dQ}{r^2} \vec{u}$$

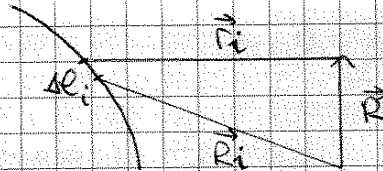
$$\vec{E} = \int k dQ \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

DENSITA' VOLUMICA: $\rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta Q}{\Delta V}$



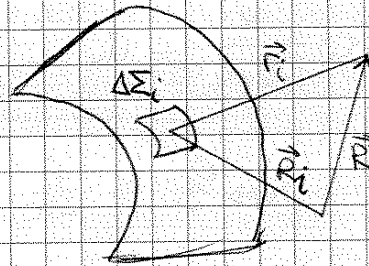
② DISTRIBUZIONE LINEARE

DENSITA' LINEARE: ~~λ = Q/L~~ $\lambda = \lim_{\Delta L \rightarrow 0} \frac{\Delta Q}{\Delta L}$



③ DISTRIBUZIONE SUPERFICIALE

DENS. SUPERFICIALE: $\sigma = \lim_{\Delta S \rightarrow 0} \frac{\Delta Q}{\Delta S}$



④

$$\lambda = \frac{Q}{L} \quad dQ_1 = \lambda dy$$

$$d\vec{E} = 2k\lambda dQ \frac{\cos\theta}{r^2} \vec{u}_x = 2k\lambda dy \frac{\cos\theta}{r^2} \vec{u}_x$$

$$r \cos\theta = x \quad r = \frac{x}{\cos\theta}$$

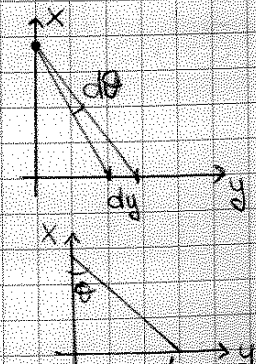
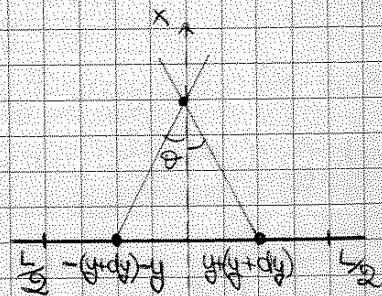
$$x \tan\theta = y$$

Fissato x, quanto vale il campo in P:

$$dy = d(x \tan\theta) = x d(\tan\theta) = x \frac{d\theta}{\cos^2\theta}$$

$$d\vec{E} = 2k\lambda \frac{x}{\cos^2\theta} d\theta \frac{\cos\theta}{r^2} \vec{u}_x \quad d\vec{E} = 2k\lambda \frac{\cos\theta}{x} d\theta \vec{u}_x$$

$$\vec{E} = \int_0^\theta 2k\lambda \frac{\cos\theta}{x} d\theta \vec{u}_x = 2 \frac{k\lambda}{x} [\sin\theta]_0^\theta \vec{u}_x \quad \boxed{\vec{E} = 2 \frac{k\lambda}{x} \sin\theta \vec{u}_x}$$



Disco carico in modo uniforme (sistema 2D)

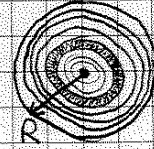
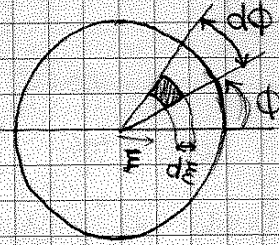
$$\sigma = \frac{Q}{\pi R^2}$$

Approssimo il disco per mezzo di tanti anelli di raggio ξ (spessi di $d\xi$)

$$0 \leq \xi \leq R$$

Per ogni anello vale la formula

$$d\vec{E} = k dQ \frac{y}{(\xi^2 + y^2)^{3/2}} \vec{u}_y$$



$$dQ = \sigma dA \xrightarrow{\text{area della corona circolare di raggi } \xi \text{ e } \xi + d\xi} dA = \frac{2\pi(\xi + d\xi) \cdot d\xi}{2\pi \xi}$$

$$dA = 2\pi \xi d\xi \text{ (approssimata)} \rightarrow dQ = \sigma 2\pi \xi d\xi$$

$$d\vec{E} = k 2\pi \sigma y \frac{\xi d\xi}{(\xi^2 + y^2)^{3/2}} \vec{u}_y$$

$$\vec{E} = \int_0^R k 2\pi \sigma y \frac{\xi d\xi}{(\xi^2 + y^2)^{3/2}} \vec{u}_y \quad k 2\pi \sigma y = \text{cost}$$

$$\vec{E} = k 2\pi \sigma y \left(\int_0^R \frac{\xi d\xi}{(\xi^2 + y^2)^{3/2}} \right) \vec{u}_y$$

$$\int_0^R \frac{\xi d\xi}{(\xi^2 + y^2)^{3/2}} = \frac{1}{2} \int_0^R \frac{d\xi^2}{(\xi^2 + y^2)^{3/2}} = \frac{1}{2} \int_{\eta(0)}^{\eta(R)} \frac{d\eta}{\eta^{3/2}} = \frac{1}{2} \cdot \frac{1}{-\frac{3}{2} + 1} \left[\eta^{-\frac{3}{2} + 1} \right]_{\eta(0)}^{\eta(R)} = - \left[\frac{1}{\eta} \right]_{\eta(0)}^{\eta(R)}$$

$$= - \left\{ \frac{1}{\sqrt{R^2 + y^2}} \right\}_0^R = - \left\{ \frac{1}{\sqrt{R^2 + y^2}} - \frac{1}{|y|} \right\} = \frac{1}{|y|} - \frac{1}{\sqrt{R^2 + y^2}}$$

Sostituendo:

$$\begin{aligned} \eta &= \xi^2 + y^2 \\ \eta(0) &= y^2 \\ \eta(R) &= R^2 + y^2 \end{aligned}$$

$$\vec{E} = k 2\pi \sigma y \left(\frac{1}{|y|} - \frac{1}{\sqrt{R^2 + y^2}} \right) \vec{u}_y$$

C'è $|y|$ perché posso collocare il campo sopra o sotto il disco

$$1) y > 0, |y| = y, \vec{E}(y > 0) = k 2\pi \sigma y \left(\frac{1}{y} - \frac{1}{\sqrt{R^2 + y^2}} \right) \vec{u}_y = k 2\pi \sigma \left\{ 1 - \frac{y}{\sqrt{R^2 + y^2}} \right\} \vec{u}_y$$

$$2) y < 0, |y| = -y, \vec{E}(y < 0) = k 2\pi \sigma y \left\{ -\frac{1}{y} - \frac{1}{\sqrt{R^2 + y^2}} \right\} \vec{u}_y = k 2\pi \sigma \left\{ -1 - \frac{y}{\sqrt{R^2 + y^2}} \right\} \vec{u}_y$$

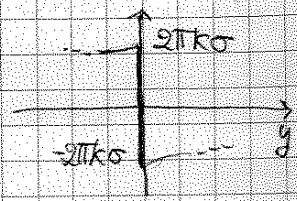
\vec{E} sopra è uguale e opposto a \vec{E} sotto $\vec{E}(y) = -\vec{E}(-y)$!

$$\text{es. } y = 3 \rightarrow \vec{E} = k 2\pi \sigma \left(1 - \frac{3}{\sqrt{R^2 + 9}} \right) \vec{u}_y \quad (\text{sopra})$$

$$y = -3 \rightarrow \vec{E} = k 2\pi \sigma \left(-1 + \frac{3}{\sqrt{R^2 + 9}} \right) \vec{u}_y \quad (\text{sotto})$$

Chiamiamo $\vec{E}_+(y) = \vec{E}(y > 0)$, $\vec{E}_-(y) = \vec{E}(y < 0)$

$$\left. \begin{aligned} \vec{E}_+(0) &= k 2\pi \sigma \vec{u}_y \\ \vec{E}_-(0) &= -k 2\pi \sigma \vec{u}_y \end{aligned} \right\} \vec{E} \text{ è una fz discontinua: } \Delta \vec{E} = \vec{E}_+(0) - \vec{E}_-(0) = \vec{E}(0^+) - \vec{E}(0^-) = k 4\pi \sigma \vec{u}_y$$



LINEA DI FORZA

Linea geometrica che ha tg. in ogni punto un ~~versore~~ versore // al campo.

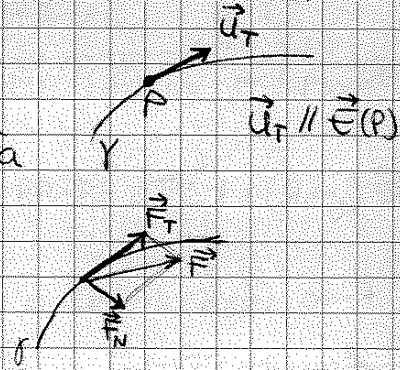
Due linee di forza non si possono mai incontrare.

Se una particella viene lasciata libera su una linea di forza, questa non può muoversi (a meno che non sia una linea retta)

$$\vec{F} = m\vec{a} \quad \vec{a} = \frac{dv}{dt} \vec{u}_T + \frac{v^2}{R} \vec{u}_N$$

$$m \frac{dv}{dt} \vec{u}_T + m \frac{v^2}{R} \vec{u}_N$$

\vec{F} è diretta verso il centro della traiettoria.



MOVIMENTO IN UN CONDENSATORE

$$\vec{E} = 4\pi k \sigma \vec{u}_y = \frac{4\pi}{4\pi \epsilon_0} \sigma \vec{u}_y = \frac{\sigma}{\epsilon_0} \vec{u}_y$$

$$\vec{F} = m\vec{a} = q\vec{E} = qE \vec{u}_y$$

$$m \frac{dv}{dt} = qE \vec{u}_y$$

$$\vec{v} = v_x \vec{u}_x + v_y \vec{u}_y \quad m \frac{dv_x}{dt} \vec{u}_x + m \frac{dv_y}{dt} \vec{u}_y = qE \vec{u}_y$$

$$m \frac{dv_x}{dt} = 0$$

$$m \frac{dv_y}{dt} = qE$$

Condiz. iniziali: $t=0 \begin{cases} v_x(0) = v_0 \\ v_y(0) = 0 \end{cases} \begin{cases} x(0) = 0 \\ y(0) = 0 \end{cases}$

$$v_x = \text{cost} = v_0$$

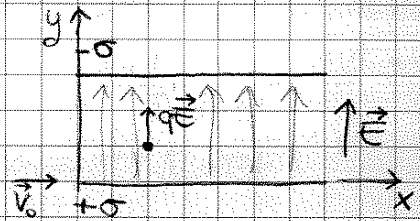
$$\frac{dv_y}{dt} = \frac{q}{m} E$$

$$v_y(t) = v_y(0) + \frac{q}{m} E t = \begin{cases} v_x(t) = v_0 \\ v_y(t) = \frac{q}{m} E t \end{cases}$$

Traiettoria:

$$\frac{dx}{dt} = v_0 \quad x(t) = v_0 t$$

$$\frac{dy}{dt} = \frac{q}{m} E t \quad y(t) = \frac{1}{2} \frac{q}{m} E t^2 \rightarrow y(x) = \frac{1}{2} \frac{q}{m} E \frac{x^2}{v_0^2} : \text{parabola!}$$



e la forza è conservativa, vale che $\vec{F} = -\nabla U$

$$\vec{F} = -\nabla \left(\frac{U}{q_0} \right)$$

$$V = \frac{U}{q_0}$$

POTENZIALE ELETTROSTATICO

un. misura $\frac{J}{C} = V$ (volt)

$$\vec{E} = -\nabla V$$

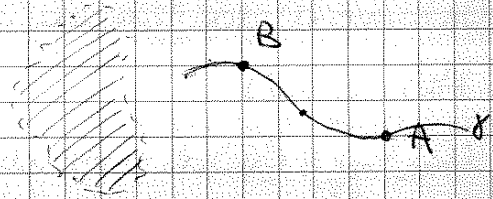
un. misura $\frac{N}{C} = \frac{N \cdot m}{C \cdot m} = \frac{J}{C \cdot m} = \frac{V}{m}$

ENERGIA PROPRIA

Energia che si deve spendere dall'esterno per creare un sistema. (= lavoro) (spostare particelle)

Supponiamo di spostare la carica da A a B lungo γ in modo quasi statico (molto lento)

$$W_{A \rightarrow B}^{(est)} = \int_A^B \vec{F}_{est} \cdot d\vec{s}$$



Se la particella ha $v=0$, vuol dire che in ogni istante $\vec{F}_{est} + \vec{F} = 0$

$$\vec{F}_{est} = -\vec{F}$$

forza dovuta alle cariche che creano il campo

$$W_{A \rightarrow B}^{(est)} = - \int_A^B \vec{F} \cdot d\vec{s} = - (U(A) - U(B))$$

indipendente da γ

$$W_{A \rightarrow B}^{(est)} = U(B) - U(A)$$

Costruisco un sistema

1. Sposto la carica q_1 : $W_{\infty \rightarrow 1}^{(est)} = 0$

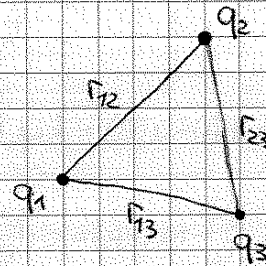
2. Sposto q_2 : $U_2 = k \frac{q_1 q_2}{r_{12}}$ $U_2(A) = 0 \rightarrow$ all'infinito

$$W_{\infty \rightarrow 2}^{(est)} = k \frac{q_1 q_2}{r_{12}}$$

3. Sposto q_3 : $U_3 = U_2 + U_1$ $U_3(B) = k \frac{q_2 q_3}{r_{23}} + k \frac{q_1 q_3}{r_{13}}$ $U_3(A) = 0$

$$W_{\infty \rightarrow 3}^{(est)} = k \frac{q_2 q_3}{r_{23}} + k \frac{q_1 q_3}{r_{13}}$$

$$W^{(est)} = 0 + k \frac{q_1 q_2}{r_{12}} + k \frac{q_2 q_3}{r_{23}} + k \frac{q_1 q_3}{r_{13}}$$



Se si hanno N cariche:

$$W^{(est)} = \frac{1}{2} \sum_{i,j}^* k \frac{q_i q_j}{r_{ij}}$$

(* = "i ≠ j")

ES) Con che velocità q arriva in B?

$$E(A) = E_k(A) + E_p(A) = 0 + qV(A)$$

$$E(B) = E_k(B) + E_p(B) = \frac{1}{2}mv_B^2 + qV(B)$$

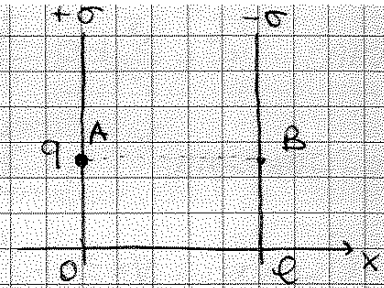
No effetti dissipativi $\Rightarrow E(A) = E(B)$

$$qV(A) = \frac{1}{2}mv_B^2 + qV(B) \quad v_B = \sqrt{\frac{2}{m}q[V(A) - V(B)]} = \sqrt{2 \frac{q}{m} \frac{\sigma}{\epsilon_0} e}$$

$$V = -\frac{\sigma}{\epsilon_0}x + c \quad \left. \begin{array}{l} V(A) = 0 + c = c \\ V(B) = -\frac{\sigma}{\epsilon_0}e + c \end{array} \right\} V(A) - V(B) = \frac{\sigma}{\epsilon_0}e$$

$$F = qE = q \frac{\sigma}{\epsilon_0}$$

$$a = \frac{F}{m} = \frac{q}{m} \frac{\sigma}{\epsilon_0}$$



16.10.12

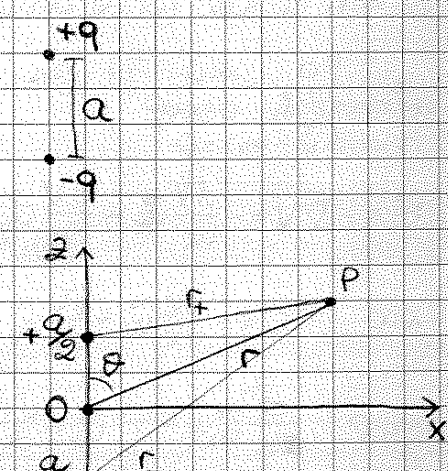
DIPOLO

$$V(P) = V_+(P) + V_-(P) = k \frac{Q}{r_+} - k \frac{Q}{r_-}$$

$$P = (x, z) \quad Q_+ = (0, \frac{a}{2}) \quad Q_- = (0, -\frac{a}{2})$$

$$r_+ = \sqrt{x^2 + (z - \frac{a}{2})^2} \quad r_- = \sqrt{x^2 + (z + \frac{a}{2})^2}$$

$$V(x, z) = kQ \left\{ \frac{1}{\sqrt{x^2 + (z - \frac{a}{2})^2}} - \frac{1}{\sqrt{x^2 + (z + \frac{a}{2})^2}} \right\}$$



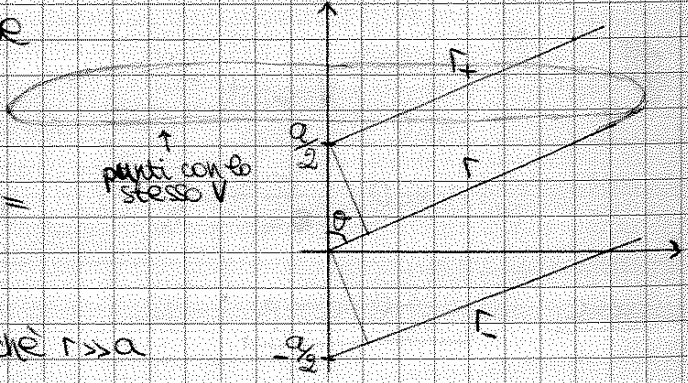
APPROSSIMAZIONE BIPOLARE: calcolo del potenziale e del campo a distanze molto maggiori del dipolo ($r \gg a$)

$\Rightarrow r_+$ e r_- sono praticamente parallele

$$r_+ \approx r + \frac{a}{2} \cos \theta \quad r_- \approx r - \frac{a}{2} \cos \theta$$

$$V(r, \theta) = kq \left\{ \frac{1}{r_+} - \frac{1}{r_-} \right\} = kq \left\{ \frac{1}{r + \frac{a}{2} \cos \theta} - \frac{1}{r - \frac{a}{2} \cos \theta} \right\} = kq \frac{r - \frac{a}{2} \cos \theta - r - \frac{a}{2} \cos \theta}{r^2 - (\frac{a}{2} \cos \theta)^2}$$

trascurabile perché $r \gg a$



POTENZIALE BIPOLARE:

$$V(r, \theta) = k \frac{qa \cos \theta}{r^2}$$

ha simmetria cilindrica perché dipende solo da θ e non da ϕ , $\propto \frac{1}{r^2}$

(Invece per una carica $V = k \frac{q}{r}$: simmetria sferica, $\propto \frac{1}{r}$)

$$u = q \left\{ V(x,y,z) + \frac{dV}{dx} \cdot \frac{a_x}{2} + \frac{dV}{dy} \cdot \frac{a_y}{2} + \frac{dV}{dz} \cdot \frac{a_z}{2} - V(x,y,z) + \frac{dV}{dx} \cdot \frac{a_x}{2} + \frac{dV}{dy} \cdot \frac{a_y}{2} + \frac{dV}{dz} \cdot \frac{a_z}{2} \right\}$$

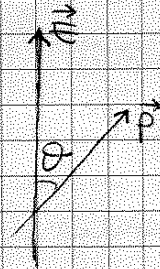
$$= q \left\{ a_x \frac{dV}{dx} + a_y \frac{dV}{dy} + a_z \frac{dV}{dz} \right\}$$

$$\vec{a} \cdot \vec{\nabla} V$$

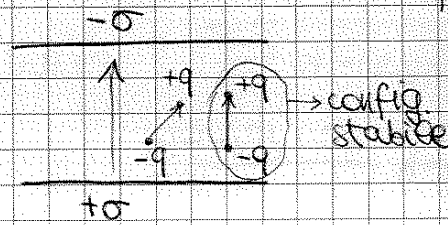
$$u = q \vec{a} \cdot \vec{\nabla} V$$

$$u = -\vec{p} \cdot \vec{E}$$

$$u(\theta) = -pE \cos \theta$$



Stati stabili in un sistema: punti a cui potenziale è minimo ($\theta=0$)



• FORZA ELETTRICA IN UN CAMPO UNIFORME

$$\vec{F} = q\vec{E}(+) - q\vec{E}(-)$$

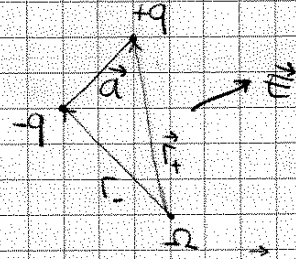
campo uniforme: $\vec{E}(+) = \vec{E}(-) \Rightarrow \vec{F} = 0$

• MOMENTO MECCANICO CHE SI ESERCITA SU UN DIPOLO IN UN C. UNIFORME

$$\vec{M} = \vec{r} \times \vec{F}$$

dato che $\vec{F} = 0$, \vec{M} non dipende dalle polo

$$\vec{M}_p = \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_- = \vec{r}_+ \times (q\vec{E}) + \vec{r}_- \times (-q\vec{E})$$



se \vec{E} è uniforme: $\vec{E}_+ = \vec{E}_- = \vec{E}$

$$\vec{M}_p = (\vec{r}_+ - \vec{r}_-) \times q\vec{E}$$

$$\vec{r}_+ = \vec{r}_- + \vec{a} \Rightarrow \vec{r}_+ - \vec{r}_- = \vec{a}$$

$$\vec{M}_p = \vec{a} \times q\vec{E} = q\vec{a} \times \vec{E}$$

$$q\vec{a} = \vec{p} \Rightarrow \boxed{\vec{M} = \vec{p} \times \vec{E}}$$

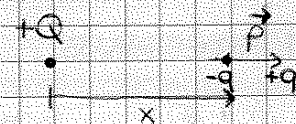
$$M = pE \sin \theta \vec{u}_z$$

$M=0$ quando $\theta=0$ \rightarrow il momento di dipolo cerca di allineare il dipolo lungo il campo
 $\theta=\pi$

[es. 1.10-13-16-18-19-21
 2.14-16-18-19-25]

Es. Campo ed dovuto a Q?

$$\vec{E} = k \frac{Q}{x^2} \vec{u}_x$$

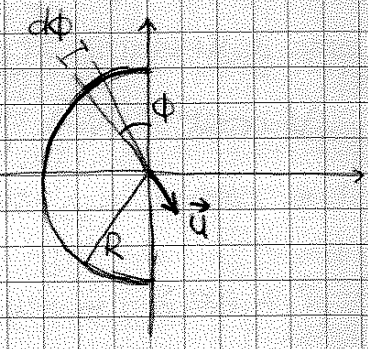


È potenziale? $\vec{u} = -\vec{p} \cdot \vec{E} = -p \vec{u}_x \cdot \left(k \frac{Q}{x^2} \vec{u}_x \right) = -k \frac{pQ}{x^2}$

Forza sul dipolo? $\vec{F} = -\vec{\nabla} u = +\vec{u}_x \frac{d}{dx} \left(k \frac{pQ}{x^2} \right) = +2\vec{u}_x \frac{k p Q}{x^3}$ (forza attrattiva)

Forza del dipolo sulla carica?

$\lambda = \lambda(\phi) \quad dq = \lambda d\ell = \lambda R d\phi$
 $d\vec{E} = k \frac{dq}{R^2} \vec{u} \quad \vec{u} = \sin\phi \vec{u}_x - \cos\phi \vec{u}_y$
 $d\vec{E} = k \frac{\lambda d\phi}{R} (\sin\phi \vec{u}_x - \cos\phi \vec{u}_y)$
 $\vec{E} = \int_0^\pi k \frac{\lambda(\phi) d\phi}{R} (\sin\phi \vec{u}_x - \cos\phi \vec{u}_y)$

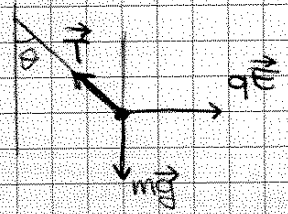


E) λ uniforme : $\lambda(\phi) = \lambda_0$
 $\vec{E} = \int_0^\pi k \frac{\lambda_0 d\phi}{R} (\sin\phi \vec{u}_x - \cos\phi \vec{u}_y) = k \frac{\lambda_0}{R} \left\{ \int_0^\pi (\sin\phi \vec{u}_x - \cos\phi \vec{u}_y) d\phi \right\} =$
 $= -k \frac{\lambda_0}{R} [\cos\phi \vec{u}_x + \sin\phi \vec{u}_y]_0^\pi = -\frac{k\lambda_0}{R} \{-2\vec{u}_x\} = \underline{+2k \frac{\lambda_0}{R} \vec{u}_x}$

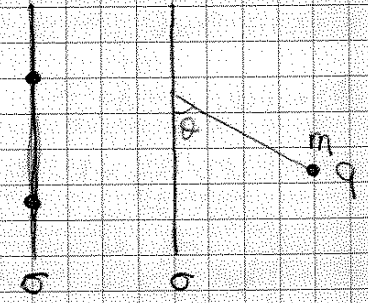
D) $\lambda(\phi) = \begin{cases} +\lambda_0 & 0 \leq \phi \leq \frac{\pi}{2} \\ -\lambda_0 & \frac{\pi}{2} \leq \phi \leq \pi \end{cases}$

$\vec{E} = \int_0^{\frac{\pi}{2}} k \frac{\lambda_0}{R} (\sin\phi \vec{u}_x - \cos\phi \vec{u}_y) d\phi + \int_{\frac{\pi}{2}}^\pi k \frac{(-\lambda_0)}{R} (\sin\phi \vec{u}_x - \cos\phi \vec{u}_y) d\phi = \underline{-2k \frac{\lambda_0}{R} \vec{u}_y}$

Calcolare la posizione di equilibrio della pallina di carica q.



$\vec{T} + q\vec{E} + m\vec{g} = 0$
 x: $qE - T \sin\theta = 0$
 y: $T \cos\theta - mg = 0$
 $\begin{cases} T \sin\theta = qE \\ T \cos\theta = mg \end{cases} \Rightarrow \tan\theta = \frac{qE}{mg}$
 $\Rightarrow T = \sqrt{(qE)^2 + (mg)^2}$



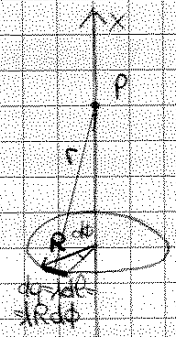
2.16 Anello uniform. carico

$x_1 = 0 \quad x_2 = 2R$

Potenziale creato dall'anello sull'asse x: $V = \int k \frac{dq}{r}$

$dV(P) = k \frac{dq}{r} = k \frac{\lambda R d\phi}{r}$

$V(P) = \int_0^{2\pi} k \frac{\lambda R d\phi}{r} = k \frac{\lambda R}{r} \int_0^{2\pi} d\phi = 2\pi k \frac{\lambda R}{r}$
 $V(x) = 2\pi k \frac{\lambda R}{\sqrt{R^2 + x^2}}$

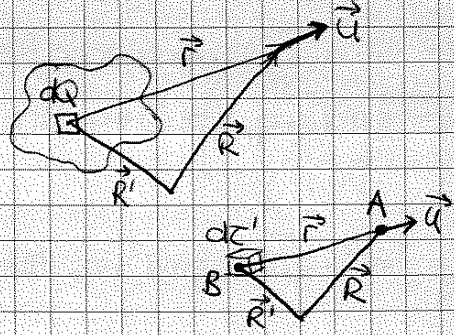


22.10.19

Sviluppo multipolare

$$V = \int_{\text{corpo}} k \frac{dq}{r} = \int_{\text{corpo}} k \frac{\rho(\vec{r}')}{r} d\tau'$$

$$\vec{r} = \vec{r}' + \vec{r} \quad \vec{r} = \vec{R} - \vec{r}' \quad r = \sqrt{\vec{r} \cdot \vec{r}} = \sqrt{(\vec{R} - \vec{r}') \cdot (\vec{R} - \vec{r}')} = \sqrt{R^2 - 2\vec{R} \cdot \vec{r}' + r'^2}$$



$$V = \int_{\tau} k \frac{\rho(\vec{r}')}{\sqrt{R^2 - 2\vec{R} \cdot \vec{r}' + r'^2}} d\tau'$$

Se $R \rightarrow \infty$ ($R \gg$ dimensione tipica del corpo), $\frac{r'}{R} \rightarrow 0$

$$V = \int_{\tau} k \frac{\rho(\vec{r}')}{R \sqrt{1 - 2\frac{\vec{R} \cdot \vec{r}'}{R^2} + \left(\frac{r'}{R}\right)^2}} d\tau' = \frac{1}{R} \left[\frac{1}{\sqrt{1 - 2\epsilon + \epsilon^2}} \right] \int_{\tau} \rho(\vec{r}') d\tau' \quad \epsilon \rightarrow 0$$

$$= \frac{1}{R} \int_{\tau} \frac{\rho(\vec{r}')}{\sqrt{1 - 2\epsilon}} d\tau'$$

Sviluppo di Taylor: $\frac{1}{\sqrt{1 - 2\epsilon}} = 1 - \frac{1}{2}\epsilon + \frac{3}{8}\epsilon^2 - \dots$

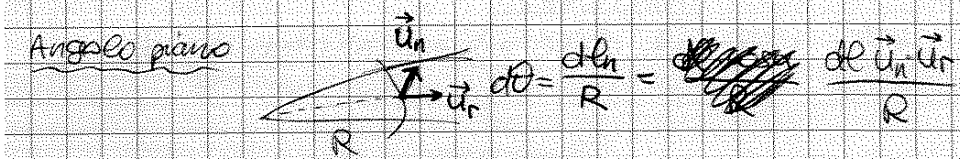
$$V = \frac{k}{R} \int_{\tau} \rho(\vec{r}') \left\{ 1 - \frac{1}{2}\epsilon + \dots \right\} d\tau' = \frac{k}{R} \int_{\tau} \rho(\vec{r}') \left\{ 1 - \frac{1}{2} \left(2 \frac{\vec{R} \cdot \vec{r}'}{R^2} + \dots \right) \right\} d\tau' =$$

$$= \frac{k}{R} \int_{\tau} \left\{ \rho(\vec{r}') + \rho(\vec{r}') \frac{\vec{r}' \cdot \vec{R}}{R^2} + \dots \right\} d\tau' = \frac{k}{R} \underbrace{\int_{\tau} \rho(\vec{r}') d\tau'}_Q + \frac{k}{R^2} \underbrace{\vec{u} \cdot \int_{\tau} \rho(\vec{r}') \vec{r}' d\tau'}_{\vec{p}}$$

$Q = \int_{\tau} \rho(\vec{r}') d\tau'$ $\vec{p} = \int_{\tau} \rho(\vec{r}') \vec{r}' d\tau' = Q \vec{R}$ (quarto polo)
 ↳ termine + impo se il corpo è carico ↳ termine + impo se è un dipolo

TEOREMA DI GAUSS

$$\Phi_E(\vec{r}) = \int_{\Sigma} \vec{F} \cdot \vec{n} d\Sigma$$

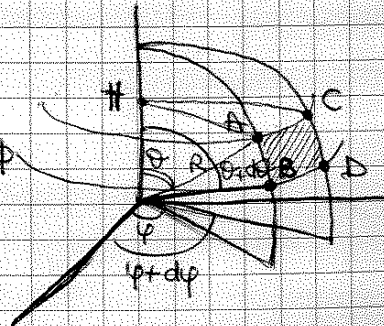


Angolo solido in coord. sferiche

$$d\Sigma_n = \vec{AB} \cdot \vec{AC} \quad \vec{AB} = R d\theta \quad \vec{HA} = R \sin\theta \quad \vec{AC} = \vec{HA} d\phi = R \sin\theta d\phi$$

$$d\Sigma_n = R^2 \sin\theta d\theta d\phi$$

$$d\Omega = \frac{d\Sigma_n}{R^2} \quad \boxed{d\Omega = \sin\theta d\theta d\phi}$$



$$\vec{r} = \frac{\vec{r}}{r} \quad \vec{E} = k \frac{q}{r^2} \vec{u}_r = kq \frac{\vec{r}}{r^3}$$

vettore a divergenza nulla (tranne in $r=0$): solenoide

type $\vec{\nabla} \cdot \vec{E} = 0$



$$\oint_M kq \frac{\vec{r}}{r^2} \cdot \vec{u}_n d\Sigma = -\frac{kq}{r^2} \int_M d\Sigma = -\frac{kq}{r^2} 4\pi r^2$$

$$\oint_{\Sigma+\Sigma'} \vec{E} \cdot \vec{u}_n d\Sigma = 0 \quad \underbrace{\int_M \vec{E} \cdot \vec{u}_n d\Sigma}_{-4\pi kq} + \underbrace{\int_{\Sigma'} \vec{E} \cdot \vec{u}_n d\Sigma}_{\Phi_E(\vec{E})} = 0$$

$$\Phi_E(\vec{E}) = 4\pi kq = \frac{q}{\epsilon_0}$$

EGGE DI POISSON

Teor di Gauss: $\oint_{\Sigma} \vec{E} \cdot \vec{u}_n d\Sigma = \frac{\rho(\Sigma)}{\epsilon_0}$

$$Q(\Sigma) = \iiint_{\tau(\Sigma)} \rho d\tau$$

$$\oint_{\Sigma} \vec{E} \cdot \vec{u}_n d\Sigma = \iiint_{\tau(\Sigma)} (\vec{\nabla} \cdot \vec{E}) d\tau$$

Sostituendo: $\iiint_{\tau(\Sigma)} (\vec{\nabla} \cdot \vec{E}) d\tau = \iiint_{\tau(\Sigma)} \frac{\rho}{\epsilon_0} d\tau \quad \iiint_{\tau(\Sigma)} (\vec{\nabla} \cdot \vec{E} - \frac{\rho}{\epsilon_0}) d\tau = 0 \quad \forall \Sigma$

$$\Rightarrow \vec{\nabla} \cdot \vec{E} - \frac{\rho}{\epsilon_0} = 0 \quad \boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}} : \text{EQ. di POISSON}$$

Eq. locale, lega \vec{E} in un punto con la densità elettrica in quel punto

23.10.12

EQUAZIONI DI MAXWELL

$$\begin{cases} \vec{\nabla} \times \vec{E} = 0 \\ \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \end{cases} \Rightarrow \vec{E} = -\vec{\nabla} V$$

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

$$\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \right) = \frac{\rho}{\epsilon_0}$$

$$\boxed{\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\rho}{\epsilon_0}}$$

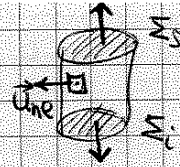
$$V(x,y,z) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(x',y',z')}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} d\tau' \quad \text{soluzione}$$

Se $\rho=0$: $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$

Le funzioni che soddisfano questa equazione sono FUNZIONI ARMONICHE

\vec{u}_{ne} sta nel piano (y, z)

$$\Rightarrow \oint_{\Sigma_e} \vec{E} \cdot \vec{u}_{ne} d\Sigma = \oint_{\Sigma_e} E(x) \underbrace{\vec{u}_x \cdot \vec{u}_{ne}}_{=0} = 0$$



$$\Sigma_s = \Sigma_i = A, \quad \vec{u}_{ns} = \vec{u}_x, \quad \vec{u}_{ni} = -\vec{u}_x$$

$$\rightarrow \iint_{\Sigma_s} E(x) \vec{u}_x \cdot \vec{u}_x d\Sigma + \iint_{\Sigma_i} E(x) \vec{u}_x \cdot (-\vec{u}_x) d\Sigma = E(x) \iint_{\Sigma_s} d\Sigma - E(x) \iint_{\Sigma_i} d\Sigma = [E(x) - E(x)]A$$

$$\Phi_{\Sigma}(\vec{E}) = [E(x) - E(x)]A = [E(x) + E(x)]A = 2E(x)A$$

$$E(x) = -E(x)$$

Per il teor. di Gauss: $\Phi_{\Sigma}(\vec{E}) = \frac{Q(\Sigma)}{\epsilon_0}$

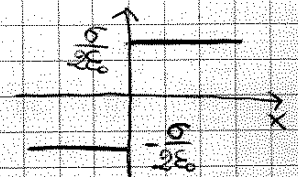
$$Q(\Sigma) = \sigma \cdot A$$

$$\Phi_{\Sigma}(\vec{E}) = \frac{\sigma A}{\epsilon_0}$$

$$\Rightarrow 2E(x)A = \frac{\sigma A}{\epsilon_0}$$

$$E(x) = \frac{\sigma}{2\epsilon_0}$$

$$\begin{cases} \vec{E}(x) = \frac{\sigma}{2\epsilon_0} \vec{u}_x & x > 0 \\ \vec{E}(x) = -\frac{\sigma}{2\epsilon_0} \vec{u}_x & x < 0 \end{cases}$$

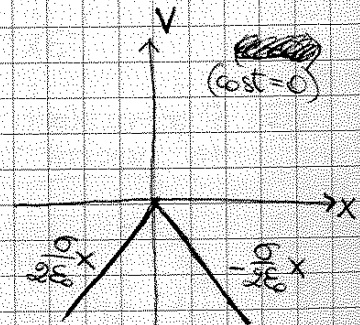


Discontinuità del campo: $\vec{E}(0^+) - \vec{E}(0^-) = \frac{\sigma}{\epsilon_0} \vec{u}_x$ solo nella componente \perp al piano

Potenziale creato dal campo?

$$\vec{E} = -\vec{\nabla}V = -\frac{\partial V}{\partial x} \vec{u}_x - \frac{\partial V}{\partial y} \vec{u}_y - \frac{\partial V}{\partial z} \vec{u}_z = \frac{\sigma}{2\epsilon_0} \vec{u}_x \quad (\text{caso } x > 0)$$

$$\Rightarrow \begin{cases} \frac{\sigma}{2\epsilon_0} = -\frac{\partial V}{\partial x} \\ 0 = -\frac{\partial V}{\partial y} \\ 0 = -\frac{\partial V}{\partial z} \end{cases} \rightarrow \frac{\partial V}{\partial x} = -\frac{\sigma}{2\epsilon_0} \rightarrow V = -\frac{\sigma}{2\epsilon_0} x + \text{cost}$$



$$(\text{caso } x < 0) \quad -\frac{\sigma}{2\epsilon_0} = -\frac{\partial V}{\partial x} \quad \frac{\partial V}{\partial x} = \frac{\sigma}{2\epsilon_0} \quad V = \frac{\sigma}{2\epsilon_0} x + \text{cost}$$

3.21

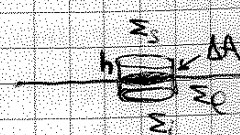
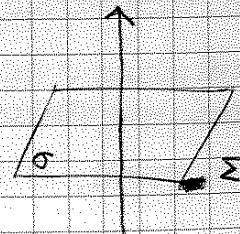
Relazione tra campo appena sopra e appena sotto una superficie carica

$$\textcircled{1} \oint_{\Sigma} \vec{E} \cdot \vec{u}_n d\Sigma = \frac{Q(\Sigma)}{\epsilon_0} \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\textcircled{2} \oint_{\Sigma} \vec{E} \cdot d\vec{S} = 0 \quad \vec{\nabla} \times \vec{E} = 0$$

se ΔA è sufficientemente piccola, la carica si può considerare uniforme: $\Delta Q = \sigma \Delta A$

$$\oint_{\Sigma} \vec{E} \cdot \vec{u}_n d\Sigma = \iint_{\Sigma_s} \vec{E} \cdot \vec{u}_{ns} d\Sigma + \iint_{\Sigma_i} \vec{E} \cdot \vec{u}_{ni} d\Sigma + \iint_{\Sigma_e} \vec{E} \cdot \vec{u}_{ne} d\Sigma$$



$$\lim_{h \rightarrow 0} \oint_{\Sigma} \vec{E} \cdot \vec{u}_n d\Sigma = \iint_{\Sigma_s} \vec{E} \cdot \vec{u}_{ns} d\Sigma + \iint_{\Sigma_i} \vec{E} \cdot \vec{u}_{ni} d\Sigma + 0 =$$

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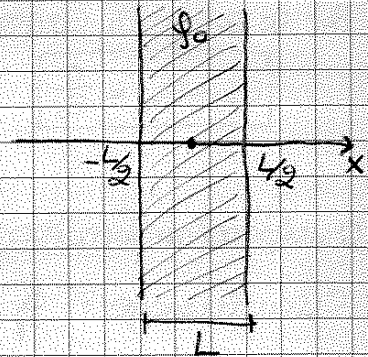
ρ_0 : densità di carica dello strato infinito di materiale

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ $\vec{E} = \vec{E}(x)$ per le proprietà simmetriche

$\vec{E} = E(x) \vec{u}_x$

$E(x) = -E(-x)$ perchè la fa deve essere dispari

$\vec{\nabla} \cdot \vec{E} = \frac{\partial E}{\partial x} + \frac{\partial E}{\partial y} + \frac{\partial E}{\partial z} = \frac{\partial E}{\partial x}$ $\frac{\partial E}{\partial x} = \frac{\rho}{\epsilon_0}$



$\frac{\partial E}{\partial x} = \frac{\rho_0}{\epsilon_0}$

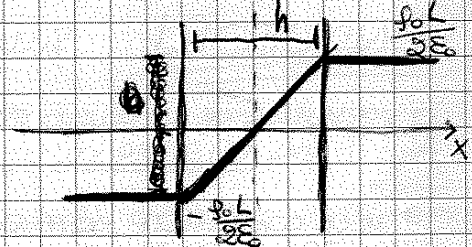
}	$x > L/2$: $\frac{\partial E}{\partial x} = 0 \Rightarrow E_D = \alpha_D$ (costante)
	$-L/2 < x < L/2$: $\frac{\partial E}{\partial x} = \frac{\rho_0}{\epsilon_0}$
	$x < -L/2$: $\frac{\partial E}{\partial x} = 0 \Rightarrow E_S = \alpha_S$ (costante)

$E_S = -E_D \Rightarrow \alpha_S = -\alpha_D$ perchè il campo è dispari

$E(x > L/2) = \alpha_D$, $E(x < -L/2) = -\alpha_D$

$dE_c = \frac{\rho_0}{\epsilon_0} dx$ $E_c = \frac{\rho_0}{\epsilon_0} x + \alpha_c$ $\alpha_c = 0$ perchè $E_c(x) = -E_c(-x) \Rightarrow \alpha_c = 0$

$E(x > L/2) = \alpha_D$
 $E(-L/2 < x < L/2) = \frac{\rho_0}{\epsilon_0} x$
 $E(x < -L/2) = -\alpha_D$



E_n è continuo all'interfaccia $\rightarrow E_c(L/2) = E_D(L/2)$ $\frac{\rho_0}{\epsilon_0} \cdot \frac{L}{2} = \alpha_D$

$E_D - E_S = \frac{\rho_0 L}{\epsilon_0}$

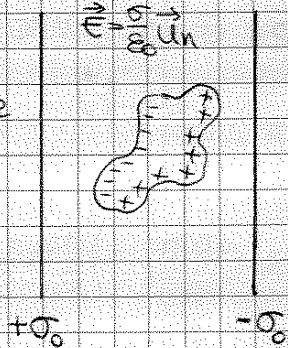
$E(+0) - E(-0) = \frac{\rho_0}{\epsilon_0}$: caso limite con $h \rightarrow 0$

La carica è confinata in una regione di spessore microscopico

CONDUTTORE SCARICO IN UN CAMPO EL

In conduttore messo in un campo elettrico esterno è interessato al suo interno dalla separazione delle cariche, dovuta alla presenza del campo esterno. La separazione continua finchè non si raggiunge l'equilibrio:

$\vec{E} = \vec{E}_{est} + \vec{E}_{cariche separate}$



$$\vec{u} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \vec{u}_r \quad V=?$$

$$\vec{u} = -\vec{\nabla}V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \vec{u}_r = -\frac{dV}{dr} \vec{u}_r = -\frac{1}{r} \frac{dV}{d\theta} \vec{u}_\theta - \frac{1}{r \sin\theta} \frac{dV}{d\phi} \vec{u}_\phi$$

$$\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = -\frac{dV}{dr}$$

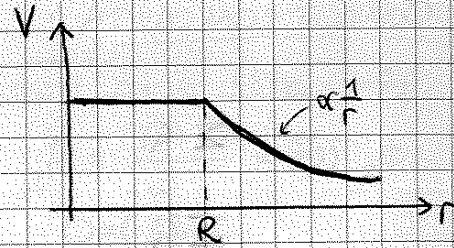
$$0 = -\frac{1}{r} \frac{dV}{d\theta}$$

$$0 = -\frac{1}{r \sin\theta} \frac{dV}{d\phi}$$

$$\Rightarrow V = V(r)$$

$$\frac{dV}{dr} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$$

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r} + B$$



CONDUTTORE CON GUSCIO SPERICO (INDUZIONE COMPLETA)

- $r < R_1$ (interno del conduttore)

$$E_1 = E(r < R_1) = 0 \Rightarrow V = \text{cost}$$

- $R_1 < r < R_2$

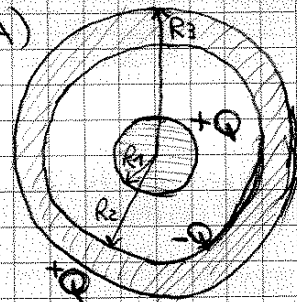
$$E_2 = E(R_1 < r < R_2) = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2} \Rightarrow V \propto \frac{1}{r}$$

- $R_2 < r < R_3$ (interno del guscio)

$$E_3 = E(R_2 < r < R_3) = 0$$

- $r > R_3$

$$E_4 = E(r > R_3) = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$$



$$\bullet r = R_1 : V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R_1} + B = V_1$$

$$\bullet r = R_2 : V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R_2} + B = V_2$$

$$\Delta V = V_1 - V_2 = \frac{1}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) Q \quad V \propto Q$$

CAPACITA'

$$C = \frac{Q}{\Delta V}$$

un. misura $\frac{C}{V} = \text{Farad}$ dipende dalle caratteristiche geometriche!

$$\Delta V = \frac{1}{4\pi\epsilon_0} \cdot \frac{R_2 - R_1}{R_1 R_2} Q$$

$$C = \frac{Q}{\Delta V} = 4\pi\epsilon_0 \frac{R_2 R_1}{R_2 - R_1}$$

capacità di un conduttore sferico

Conduttore sferico isolato ($R_2 \rightarrow \infty$): $C = 4\pi\epsilon_0 \frac{R_2 R_1}{R_2} = 4\pi\epsilon_0 R_1$

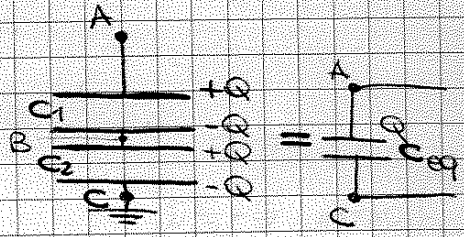
Condensatore piano ($R_1 \rightarrow \infty, R_2 \rightarrow \infty, R_2 - R_1 = h$)

$$R_1 R_2 = R_1 + (R_1 + h) \approx R_1^2$$

$$C = 4\pi\epsilon_0 \frac{R_1^2}{h} = \epsilon_0 \frac{\Sigma}{h} \quad (\Sigma = 4\pi R_1^2)$$

CONDENS. IN SERIE

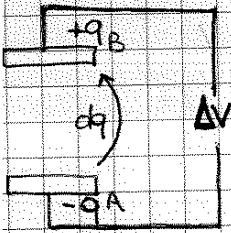
$$\left. \begin{aligned} V_A - V_B &= \frac{Q}{C_1} \\ V_B - V_C &= \frac{Q}{C_2} \end{aligned} \right\} V_A - V_C = (V_A - V_B) + (V_B - V_C) = \frac{Q}{C_1} + \frac{Q}{C_2}$$



$$V_A - V_C = \frac{Q}{C_{eq}} \rightarrow \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} \quad \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

C_{eq} è sempre più piccola delle varie capacità.

ENERGIA PER CARICARE UN CONDENSATORE



$$dW_{ext} = \int_A^B \vec{F}_{ext} \cdot d\vec{s} = \int_A^B -\vec{F}_c \cdot d\vec{s} = - \int_A^B \vec{E} \cdot d\vec{s} = du = dq \cdot \Delta V$$

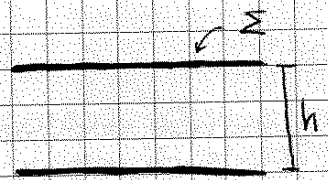
forza del campo

$$C = \frac{Q}{\Delta V} \quad \Delta V = \frac{Q}{C} \quad \rightarrow dW_{ext} = dq \frac{Q}{C}$$

$$W_{ext} = U = \int_0^Q \frac{q dq}{C} = \frac{Q^2}{2C} \quad \boxed{U = \frac{Q^2}{2C}}$$

$$C = \frac{Q}{\Delta V} \quad \boxed{U = \frac{1}{2} Q \Delta V} \quad \boxed{U = \frac{1}{2} C (\Delta V)^2}$$

oppure Ip condensatore piano ($C = \epsilon_0 \frac{\Sigma}{h}$)



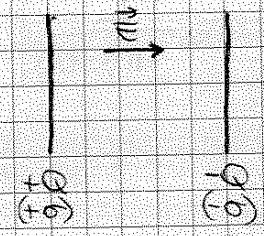
volume $\tau = \Sigma h$

$$U = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} \epsilon_0 \frac{\Sigma}{h} \Delta V^2 = \frac{1}{2} \epsilon_0 \Sigma h \left(\frac{\Delta V}{h} \right)^2 \quad \text{moltiplico e divido per h}$$

$$\boxed{U = \frac{1}{2} \epsilon_0 \tau E^2} \quad \boxed{\frac{U}{\tau} = \frac{1}{2} \epsilon_0 E^2} : \text{DENSITA' DI ENERGIA}$$

FORZA DI ATTRAZIONE TRA LE DUE PIASTRE

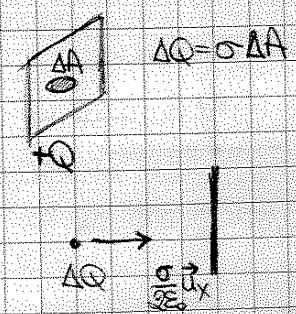
Forza su ΔA : $\Delta \vec{F} = \Delta \vec{F}_+ + \Delta \vec{F}_-$
 forze di repulsione delle altre cariche sulla piastra+ (in totale $\Delta \vec{F}_+ = 0$)
 forze delle cariche negative sull'altra piastra.



$$\Delta \vec{F} = \Delta \vec{F}_- = \Delta Q \vec{E}_- = \Delta Q \frac{\sigma}{2\epsilon_0} \vec{u}_x = \sigma \Delta A \frac{\sigma}{2\epsilon_0} \vec{u}_x$$

$$\Delta \vec{F} = \frac{\sigma^2}{2\epsilon_0} \Delta A \vec{u}_x$$

$$\frac{\Delta \vec{F}}{\Delta A} = \frac{\sigma^2}{2\epsilon_0} \vec{u}_x = \frac{1}{2} \epsilon_0 \left(\frac{\sigma}{\epsilon_0} \right)^2 \vec{u}_x \quad \boxed{\frac{\Delta \vec{F}}{\Delta A} = \frac{1}{2} \epsilon_0 E^2 \vec{u}_x} = \frac{U}{\tau} \vec{u}_x$$



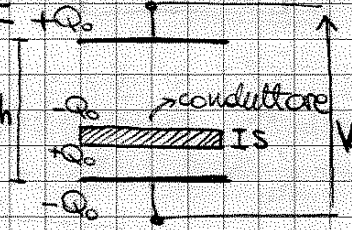
30.10.12

ESPERIENZE DI FARADAY

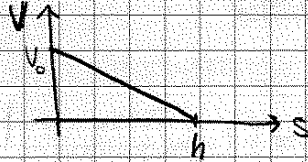
CONDUTTORE

① Senza conduttore: $V_0 = E_0 h$

• Con conduttore: $V = \int \vec{E} \cdot d\vec{s} = E_0(h-s) = E_0 h (1 - \frac{s}{h}) = V_0 (1 - \frac{s}{h}) \rightarrow V < V_0$



$$V(s) = V_0 \left(1 - \frac{s}{h}\right)$$

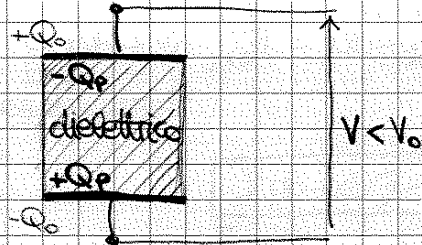


La carica che viene separata all'interno del conduttore per effetto del campo esterno è una carica asportabile

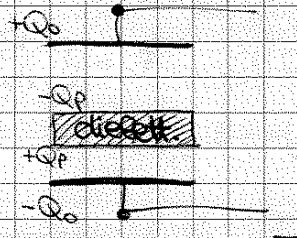
ISOLANTE

② $k = \frac{V_0}{V} > 1$: COSTANTE DIELETTRICA RELATIVA DEL MEZZO

$V(s)$ diminuisce linearmente con s , ma per $s=h$, $V(s) \neq 0$



isolante: $Q_p < Q_0$ (carica non asportabile)
conduttore: $Q_p = Q_0$



$$\left. \begin{aligned} V &= \frac{Q}{\pi \epsilon_0 h} & \sigma_0 &= \frac{Q_0}{\pi} & E &= \frac{V}{h} = \frac{V_0}{kh} = \frac{Q_0}{\pi \epsilon_0 kh} \\ \sigma_p &= \frac{Q_p}{\pi} & E &= \frac{Q_p}{\pi \epsilon_0} & \frac{\sigma_0 - \sigma_p}{\epsilon_0} &= \frac{1}{\pi \epsilon_0 kh} & k\sigma_0 - k\sigma_p &= \sigma_0 \end{aligned} \right\}$$

$$\sigma_p = \frac{k-1}{k} \sigma_0$$

σ_0 : carica libera

σ_p : carica "legata" (non asportabile), dovuta alla polarizzazione

• $C = \frac{Q}{V} = \frac{Q_0}{\frac{V_0}{k}} = \frac{kQ_0}{V_0} = kC_0 = C$ CAPACITÀ DI UN CONDENSATORE CON DIELETTRICO

caso del condens. piano a facce parallele: $C = k\epsilon_0 \frac{S}{h} = \frac{\epsilon S}{h} = C$

$\epsilon = k\epsilon_0$ COSTANTE DIELETTRICA ASSOLUTA

• $u = \frac{Q^2}{2C} = \frac{Q^2}{2kC_0} = \frac{u_0}{k}$ u diminuisce in presenza del dielettrico

: densità atomica ($\frac{1}{m^3}$)



di atomi in $d\tau$: $dN = n d\tau$ $d\vec{p} = dN \cdot \vec{p}_A = n \cdot \vec{p}_A d\tau$

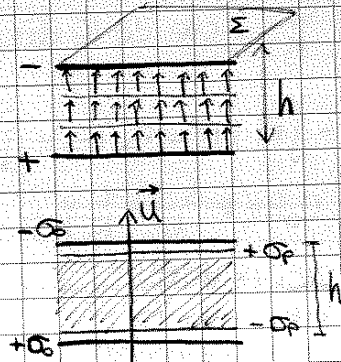
$$\boxed{\frac{d\vec{p}}{d\tau} = n \vec{p}_A} \quad \text{POLARIZZAZIONE } \vec{P}$$

Polazzazione = densità volumetrica di dipolo

POLARIZZ. OMOGENEA

mezzo omogeneo $\rightarrow P = \text{cost}$

Momento di dipolo totale: $\vec{p} = \vec{P} \cdot \tau = \vec{P} \cdot \Sigma h$



$$Q_p = \sigma_p \Sigma$$

$$\Rightarrow \sigma_p \Sigma h \vec{u} = \vec{P} \Sigma h$$

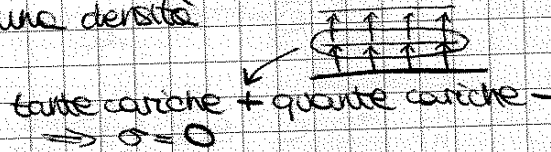
$$p = Q_p h = \sigma_p \Sigma h$$

$$\sigma_p \vec{u} = \vec{P}$$

↳ densità di carica polarizzata

Quando un mezzo è uniformemente polarizzato: $\boxed{\sigma_p = \vec{P} \cdot \vec{u}}$ $[P] = \frac{C}{m^2}$

Una polarizz. uniforme non dà origine a una densità di carica di volume (solo in superficie)



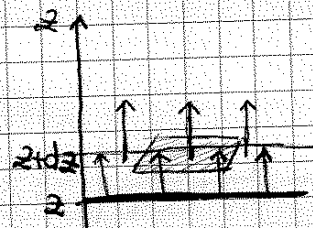
POLARIZZ. NON OMOGENEA

$q(z) = \int_{\text{area}} P_z(z) dx dy$: positiva

$q(z+dz) = \int_{\text{area}} P_z(z+dz) dx dy$: negativa

$$dq = q(z) + q(z+dz) = \int P_z(z) dx dy - \int P_z(z+dz) dx dy = [P_z(z) - P_z(z+dz)] dx dy =$$

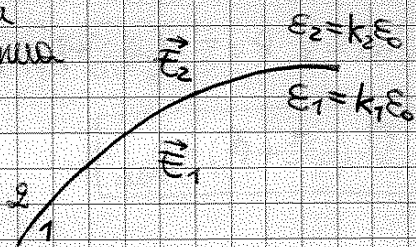
$$= (P_z(z) - P_z(z) - \frac{dP_z}{dz} dz) dx dy = - \frac{dP_z}{dz} dx dy dz \quad dq = - \frac{dP_z}{dz} d\tau$$



Se considero le 3 direzioni: $\frac{dq}{d\tau} = - \frac{dP_x}{dx} - \frac{dP_y}{dy} - \frac{dP_z}{dz} = - \left(\frac{dP_x}{dx} + \frac{dP_y}{dy} + \frac{dP_z}{dz} \right)$

$$\boxed{\frac{dq}{d\tau} = - \vec{\nabla} \cdot \vec{P}} \quad - \rho_{sp}$$

\vec{E} irrotazionale \rightarrow la componente t_g all'interfaccia tra due mezzi dielettrici è continua
 ($\nabla \times \vec{E} = 0$)
 ($E_{t2} = E_{t1}$)

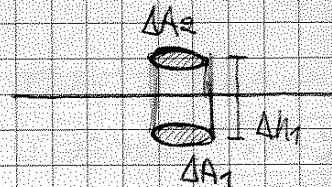


Componente normale

Ip. non c'è densità superficiale di carica ($\sigma = 0$)

$\Delta A_1 = \Delta A_2 = \Delta A$

$\iiint_{cilindro} \vec{\nabla} \cdot \vec{D} d\tau = \iiint_{cilindro} \rho_{ext} d\tau$



$\lim_{\Delta h \rightarrow 0} \iiint_{cilindro} \vec{\nabla} \cdot \vec{D} d\tau = \lim_{\Delta h \rightarrow 0} \iiint_{cilindro} \rho_{ext} d\tau = 0$

perché nel limite $\Delta h \rightarrow 0$ il volume del cilindro ($\tau = \Delta A \cdot \Delta h$) $\rightarrow 0$

$\lim_{\Delta h \rightarrow 0} \iiint_{cilindro} \vec{\nabla} \cdot \vec{D} d\tau = \lim_{\Delta h \rightarrow 0} \left\{ \oint \vec{D} \cdot \vec{u}_n d\Sigma \right\} = \lim_{\Delta h \rightarrow 0} \left\{ \iint_{\Delta A_2} \vec{D}_2 \cdot \vec{u}_{n2} d\Sigma + \iint_{\Delta A_1} \vec{D}_1 \cdot \vec{u}_{n1} d\Sigma + \right.$

$\left. + \iint_{superficie laterale} \vec{D} \cdot \vec{u}_{ne} d\Sigma \right\} = 0$
 perché se $\Delta h \rightarrow 0$, la superficie laterale $\rightarrow 0$

$(\vec{D}_2 \cdot \vec{u}_{n2} + \vec{D}_1 \cdot \vec{u}_{n1}) \Delta A = (\vec{D}_2 \cdot \vec{u}_{n2} - \vec{D}_1 \cdot \vec{u}_{n1}) \Delta A = 0$

$\vec{u}_{n2} = \vec{u}_{n1} \Rightarrow \vec{D}_2 \cdot \vec{u}_n - \vec{D}_1 \cdot \vec{u}_n = 0 \Rightarrow D_{2n} = D_{1n}$

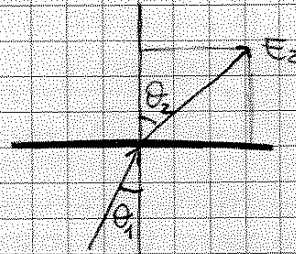
1) $\nabla \times \vec{E} = 0 \Rightarrow E_{t2} = E_{t1}$

2) $\nabla \cdot \vec{D} = \rho_{ext} \Rightarrow D_{2n} = D_{1n}$

Se i mezzi sono lineari e isotropi, $\vec{D} = \epsilon \vec{E}$ con ϵ indipendente da \vec{E}

$\begin{cases} \vec{D}_1 = \epsilon_1 \vec{E}_1 \\ \vec{D}_2 = \epsilon_2 \vec{E}_2 \end{cases}$

$\begin{cases} E_{t1} = E_1 \sin \theta_1 \\ E_{n1} = E_1 \cos \theta_1 \end{cases} \quad \begin{cases} E_{t2} = E_2 \sin \theta_2 \\ E_{n2} = E_2 \cos \theta_2 \end{cases}$



$E_{t1} = E_{t2} \Rightarrow E_1 \sin \theta_1 = E_2 \sin \theta_2$

$D_{n1} = D_{n2} \Rightarrow \epsilon_1 E_1 \cos \theta_1 = \epsilon_2 E_2 \cos \theta_2$

$\frac{1}{\epsilon_1} \tan \theta_1 = \frac{1}{\epsilon_2} \tan \theta_2 \Rightarrow \frac{\tan \theta_1}{\tan \theta_2} = \frac{\epsilon_1}{\epsilon_2}$

LEGGE DI RIFRAZIONE DELLE LINEE DI FORZA DEL CAMPO

05.11.12

$$\rho = 8,96 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}$$

$$A = 63,35$$

$$N_A = 6,02 \cdot 10^{26}$$

$$n = \frac{N_A \cdot \rho}{A} \approx 8,5 \cdot 10^{28} \text{ e}^-/\text{m}^3 \quad \leftarrow \text{densità elettronica del rame}$$

CORRENTE

Le particelle che passano attraverso $d\Sigma$ hanno la stessa velocità

$$d\vec{\ell} = \vec{v} dt$$

che passano per $d\Sigma$ nel tempo dt

Particelle: $dN = n d\tau$

$$dN = d\ell \cdot \cos\beta$$

$$d\tau = d\Sigma d\ell \cos\beta = d\Sigma v dt \cos\beta = d\Sigma dt \vec{v} \cdot \vec{u}_n$$

$$dN = n \vec{v} \cdot \vec{u}_n d\Sigma dt$$

Densità di corrente: $\frac{dN}{dt} = n \vec{v} \cdot \vec{u}_n d\Sigma$

$$\frac{dq}{dt} = nq \vec{v} \cdot \vec{u}_n d\Sigma$$

$$\frac{dQ}{dt} = \iint_{\Sigma} nq \vec{v} \cdot \vec{u}_n d\Sigma$$

Vettore ~~densità~~ di corrente: $\vec{j} = nq \vec{v}_d$ \vec{v}_d : VELOCITÀ DI DERIVA

$$[j] = [n][q][v] = \frac{1}{\text{m}^3} \cdot \text{C} \cdot \frac{\text{m}}{\text{s}} = \frac{\text{A}}{\text{m}^2}$$

$$A = \frac{\text{C}}{\text{s}} \quad [j] = \frac{\text{A}}{\text{m}^2}$$

$$\frac{dQ}{dt} = \iint_{\Sigma} \vec{j} \cdot \vec{u}_n d\Sigma = I \quad \text{INTENSITÀ DI CORRENTE: flusso del vettore } \vec{j} \text{ attraverso la superficie } \Sigma$$

EQUAZIONI DI CONTINUITÀ

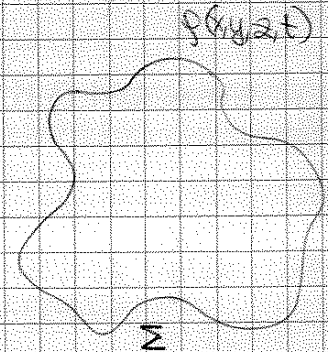
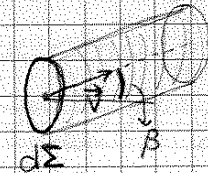
Immagino Σ superficie fissa

$$Q(t) = \iiint_{\tau(\Sigma)} \rho d\tau \quad \frac{dQ}{dt} = \frac{d}{dt} \iiint_{\tau(\Sigma)} \rho d\tau = \iiint_{\tau(\Sigma)} \frac{d\rho}{dt} d\tau$$

$$\left(\frac{dQ}{dt}\right)_{\text{uscita}} = - \oiint_{\Sigma} \vec{j} \cdot \vec{u}_n d\Sigma$$

↘ decremento di particelle

Dato che Q si conserva: $\frac{dQ}{dt} = \left(\frac{dQ}{dt}\right)_u \quad \iiint_{\tau(\Sigma)} \frac{d\rho}{dt} d\tau = - \oiint_{\Sigma} \vec{j} \cdot \vec{u}_n d\Sigma$



legame stazionario:

$$j = \frac{c}{M} \vec{E} = -\vec{\nabla}V$$

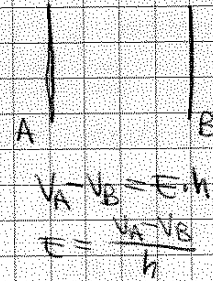
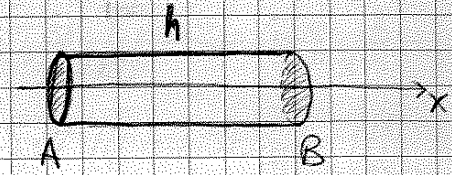
$$\vec{E} = \rho \vec{j} \quad \frac{V_A - V_B}{h} = \rho \frac{c}{M}$$

$$V_A - V_B = \rho \frac{h}{M} i$$

$$R = \rho \frac{h}{M} \quad \text{RESISTENZA } [R] = \Omega$$

(conduttore)

$$V_A - V_B = Ri$$



$$\vec{E} = \rho \vec{j}$$

$$\vec{E} \cdot d\vec{s} = \rho \vec{j} \cdot d\vec{s}$$

$d\vec{s}$ su una linea di corrente

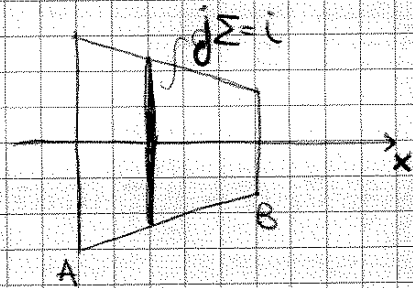
($\vec{u}_r \parallel$ alla velocità dei portatori)

$$-dV = \rho j ds \quad \int_A^B -dV = \int_A^B \rho \frac{c}{M} ds$$

$$V_A - V_B = \int_A^B \rho \frac{ds}{M} j c$$

$$R = \int_A^B \rho \frac{ds}{M}$$

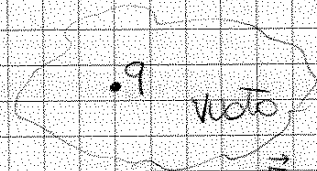
RESISTENZA (caso generico)



EFFETTO JOULE

$$\vec{F} = q\vec{E} \quad \vec{a} = \frac{F}{m} = \frac{q}{m} \vec{E}$$

se \vec{E} è uniforme: $\vec{v}(t) = \vec{v}(0) + \frac{q}{m} \vec{E} t$
moto unif. accelerato



$$\Rightarrow \text{oppure } \vec{v}_d = \frac{\sigma}{nq} q \frac{E}{m} \Rightarrow \vec{v}_d \propto \vec{E}$$

invece nella legge di Ohm.

$$\vec{j} = \sigma \vec{E} \quad \vec{j} = nq \vec{v}_d \quad \rightarrow \quad nq \vec{v}_d = \sigma \vec{E}$$

$$\vec{v}_d = \frac{\sigma}{nq} \vec{E}$$

moto uniforme anche in presenza di una forza

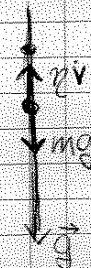
Il mezzo è responsabile di un effetto dissipativo che fa sì che la velocità tenda a un valore costante

Situazione equivalente:

$$m \frac{dv}{dt} = mg - \eta v \quad mg - \eta v_{limite} = 0 \quad v_e = \frac{mg}{\eta}$$

$$\text{Dissipazione di energia: } E_T = E_K + E_P = \frac{1}{2} m v^2 - mgy$$

$$\frac{dE_T}{dt} = \frac{d}{dt} \left(\frac{1}{2} m v^2 - mgy \right) = \frac{1}{2} m \cdot 2v \frac{dv}{dt} - mg \frac{dy}{dt} = \left(m \frac{dv}{dt} - mg \right) v = -\eta v^2$$



MOVIMENTO DEGLI E

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T \quad (\langle v^2 \rangle)^{1/2} = \sqrt{\frac{3k_B T}{m}}$$

$$k_B = 1,38 \cdot 10^{-23} \text{ J/K} \\ T = 300 \text{ K} \quad m = 9,1 \cdot 10^{-31} \text{ kg}$$

Suppongo $i = 8 \text{ A}$, $\Sigma = 4 \text{ mm}^2$, $n = 8,5 \cdot 10^{28} \text{ e}^-/\text{m}^3$

Movimento degli e lungo il campo?

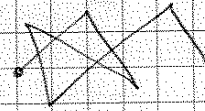
$$j = \frac{i}{\Sigma} = \frac{8 \text{ A}}{4 \cdot 10^{-6} \text{ m}^2} = 2 \cdot 10^6 \text{ A/m}^2$$

$$j = nq v_d \quad v_d = \frac{j}{nq} = \frac{2 \cdot 10^6}{8,5 \cdot 10^{28} \cdot 1,6 \cdot 10^{-19}} = 1,5 \cdot 10^{-4} \text{ m/s}$$

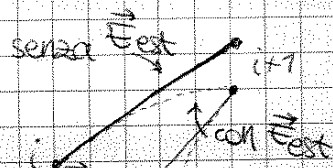


$v_d \ll$ alla velocità degli e

Senza campo esterno, in media: $\frac{1}{N} \sum_{i=1}^N \vec{v}_i = 0$ (*)



Se esiste un \vec{E}_{est} , il moto è uniformemente accelerato:



$$\vec{v}_{i+1} = \vec{v}_i + \frac{q\vec{E}}{m} \Delta t_{(i,i+1)}$$

$$\langle \vec{v} \rangle = \frac{1}{N} \sum \vec{v}_{i+1} = \frac{1}{N} \sum \vec{v}_i + \frac{1}{N} \sum \frac{q\vec{E}}{m} \Delta t_{(i,i+1)} = \frac{q\vec{E}}{m} \left(\frac{1}{N} \sum \Delta t_{(i,i+1)} \right) = \frac{q\vec{E}}{m} \tau$$

(*) tempo medio tra due urti (τ)

$$\langle \vec{v} \rangle = \frac{q\vec{E}}{m} \tau = \vec{v}_d$$

$$\vec{j} = nq\vec{v}_d = \frac{nq^2\tau}{m} \vec{E} \Rightarrow \sigma = \frac{nq^2\tau}{m}$$

Modello di Drude per la conducibilità nei metalli

ES Rame $\rho = 1,7 \cdot 10^8 \Omega \cdot \text{m}$

$$q = 1,6 \cdot 10^{-19} \text{ C} \\ n = 8,5 \cdot 10^{28} \text{ e}^-/\text{m}^3$$

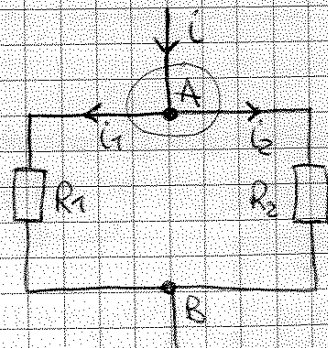
$$m = 9,1 \cdot 10^{-31} \text{ kg}$$

$$\sigma = \frac{1}{\rho} \approx 0,5 \cdot 10^8 \frac{1}{\Omega \cdot \text{m}}$$

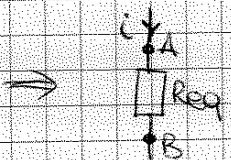
$$\tau = \frac{m\sigma}{nq^2} = 2,5 \cdot 10^{-14} \text{ s}$$

RESISTENZE IN PARALLELO

$$i = i_1 + i_2 \\ V_A - V_B = R_1 i_1 \\ V_A - V_B = R_2 i_2$$



$$i = \frac{V_A - V_B}{R_1} + \frac{V_A - V_B}{R_2} = (V_A - V_B) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad \frac{i}{V_A - V_B} = \frac{1}{R_1} + \frac{1}{R_2}$$



$$R_{eq} = \frac{V_A - V_B}{i}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

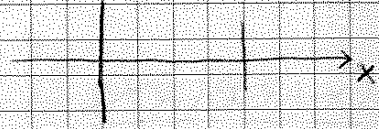
09.11.12

In un conduttore cilindrico in cui scorre una corrente stazionaria, il campo \vec{E} è solenoidale e omogeneo

$$\vec{\nabla} \cdot \vec{j} = 0 \quad \vec{\nabla} \cdot \vec{j} = \vec{\nabla} \cdot (\sigma \vec{E}) = 0 \quad \sigma \vec{\nabla} \cdot \vec{E} = 0 \Rightarrow \vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{E} = E(x) \vec{u}_x \quad \vec{\nabla} \cdot \vec{E} = \frac{dE_x}{dx} + \frac{dE_y}{dy} + \frac{dE_z}{dz}$$

$$\frac{dE}{dx} = 0 \Rightarrow E = \text{cost}$$

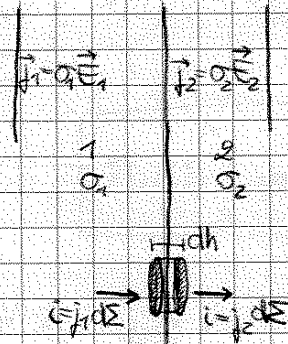


$$\left. \begin{aligned} \vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \\ \vec{\nabla} \cdot \vec{E} &= 0 \end{aligned} \right\} \Rightarrow \rho = 0 \text{ carica netta nulla in quel punto}$$

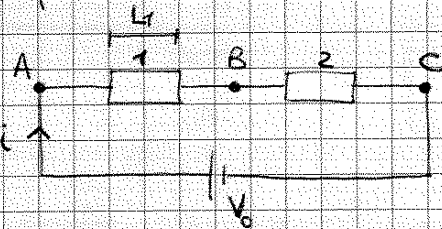
Due conduttori cilindrici affiancati. Condizione al contorno?

Regime stazionario: $\vec{\nabla} \cdot \vec{j}_1 = 0, \vec{\nabla} \cdot \vec{j}_2 = 0$

Deve valere $\int j_1 dS = \int j_2 dS \Rightarrow \sigma_1 E_1 = \sigma_2 E_2$



Verifica:



$$R_1 = \frac{1}{\sigma_1} \frac{L_1}{M/A}$$

$$R_2 = \frac{1}{\sigma_2} \frac{L_2}{M/A}$$

$$V_0 = R_1 i + R_2 i = (R_1 + R_2) i \quad i = \frac{V_0}{R_1 + R_2}$$

$$V_A - V_B = R_1 i = \frac{R_1}{R_1 + R_2} V_0 \quad V_B - V_C = R_2 i = \frac{R_2}{R_1 + R_2} V_0$$

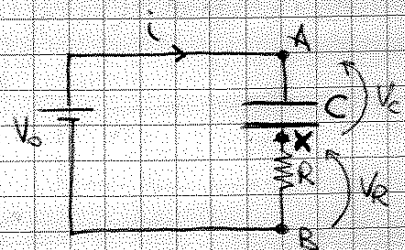
$$E_1 = \frac{V_A - V_B}{L_1} \quad E_2 = \frac{V_B - V_C}{L_2} \quad \frac{E_1}{E_2} = \frac{V_A - V_B}{L_1} \cdot \frac{L_2}{V_B - V_C}$$

$$\frac{E_1}{E_2} = \frac{R_1}{R_1 + R_2} \frac{V_0}{L_1} \cdot \frac{L_2}{\frac{R_2}{R_1 + R_2} V_0} = \frac{L_2}{L_1} \frac{R_1}{R_2} = \frac{L_2}{L_1} \frac{1}{\sigma_1} \frac{M}{A} \cdot \sigma_2 \frac{A}{L_2} = \frac{\sigma_2}{\sigma_1} \Rightarrow \sigma_1 E_1 = \sigma_2 E_2$$

CARICA DI UN CONDENSATORE

$q(0) = 0 \quad C = \frac{Q}{\Delta V} \rightarrow \Delta V = \frac{Q}{C}$

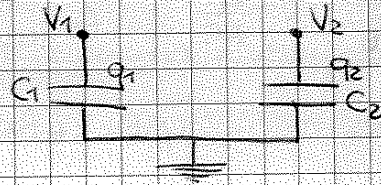
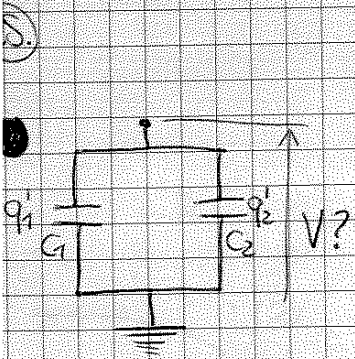
$$V_0 = (V_A - V_x) + (V_x - V_B) = \frac{Q}{C} + Ri \quad \boxed{V_0 = \frac{Q}{C} + R \frac{dq}{dt}}$$



$q = q_{omogenea} + q_{particolare}$

$$R \frac{dq_{om}}{dt} + \frac{1}{C} q_{om} = 0 \quad \frac{dq_{om}}{dt} + \frac{1}{RC} q_{om} = 0$$

$[R] = \Omega = \frac{V}{A} \quad [C] = \text{Farad} = \frac{C}{V} \quad [RC] = \frac{V}{A} \frac{C}{V} = \frac{V \cdot C}{A \cdot V} = \frac{C}{A} \frac{V}{V} = s \quad RC \text{ COSTANTE DI TEMPO}$



$$C = \frac{q}{V} \quad q_1 = C_1 V_1 \quad q = q_1 + q_2 = C_1 V_1 + C_2 V_2$$

$$q_2 = C_2 V_2$$

Condensatori in // : $C = C_1 + C_2$ $C = \frac{q}{V}$ $V = \frac{q}{C} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

$q'_1 = ?$ $q'_1 = C_1 V = C_1 \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

$q'_2 = C_2 V = C_2 \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

$\Delta q_1 = ?$ $\Delta q_1 = q'_1 - q_1 = C_1 \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} - C_1 V_1 = \frac{C_1^2 V_1 + C_1 C_2 V_2 - C_1^2 V_1 - C_1 C_2 V_1}{C_1 + C_2} = \frac{C_1 C_2}{C_1 + C_2} (V_2 - V_1)$

$\Delta q_2 = q'_2 - q_2 = \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2) = -\Delta q_1$

Energia:

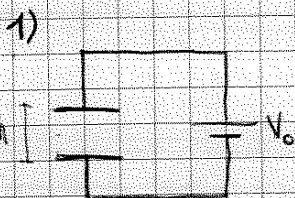
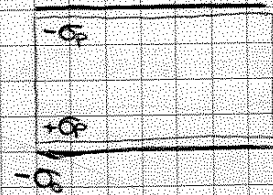
$$U_i = U_{i1} + U_{i2} = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

$$U_f = \frac{1}{2} (C_1 + C_2) V^2 = \frac{1}{2} C V^2$$

$$\Delta U = U_f - U_i = \frac{1}{2} (C_1 + C_2) \left(\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right)^2 - \left(\frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 \right) = -\frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2$$

ES. $E = \frac{E_0}{K}$ $E_0 = \frac{\sigma_0}{\epsilon_0}$ $E = \frac{\sigma_0}{K \epsilon_0}$ (caso sist. isolato) $+Q$

Un condensatore piano con armature di area S distanti h viene collegato a un generatore che mantiene V_0 cost tra le armature. Una foglia di dielettrico viene inserita tra le armature.



CAMPO MAGNETICO

$$[B] = \frac{[F]}{[q][v]} = \frac{N}{A \cdot s \cdot \frac{m}{s}} = \frac{N}{A \cdot m} = \frac{N \cdot m}{A \cdot m^2} = \frac{J}{\frac{C}{s} \cdot m^2} = \frac{J}{C} \frac{s}{m^2} = \frac{Vs}{m^2}$$

1V · 1s = 1Wb (weber) → $[B] = \frac{Wb}{m^2}$

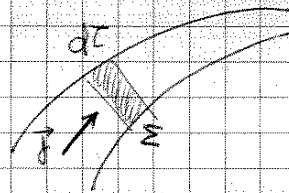
1Tesla = 1 $\frac{Wb}{m^2}$ → $[B] = \text{Tesla}$

$$\vec{F} = q\vec{v} \times \vec{B}$$

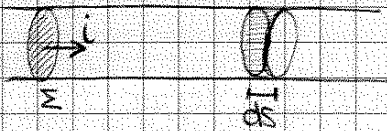
forze per unità di volume: $\frac{d\vec{F}}{d\tau} = nq\vec{v} \times \vec{B}$

$$\frac{d\vec{F}}{d\tau} = \vec{j} \times \vec{B} \quad d\vec{F} = \vec{j} \times \vec{B} d\tau$$

$$\vec{F} = \iiint_V \vec{j} \times \vec{B} d\tau$$



CONDUTTORE CAURRO (stessa sezione)



$$\vec{j} = \frac{i}{M} \vec{u}_T \quad d\tau = \Sigma ds$$

$$\vec{F} = \iiint_{\text{conduttore}} \frac{i}{M} \vec{u}_T \times \vec{B} (\Sigma ds)$$

Dato che siamo in regime stazionario, $i = \text{cost.}$ $\vec{F} = i \int_P^Q (\vec{u}_T ds) \times \vec{B}$

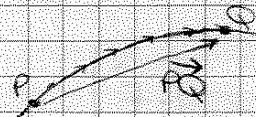
$$\vec{F} = i \int_P^Q ds \times \vec{B}$$

SECONDA LEGGE
di LAPLACE

$$d\vec{F} = i ds \times \vec{B}$$



Se \vec{B} è uniforme: $\vec{F} = i \left(\int_P^Q ds \right) \times \vec{B}$ $\vec{F} = i \vec{PQ} \times \vec{B}$

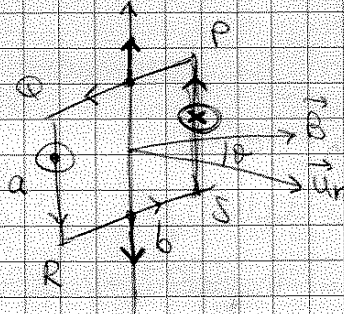


Se un circuito è interamente contenuto in B uniforme, $\vec{PQ} = 0 \Rightarrow \vec{F} = 0$.

$$\vec{F}_{PQ} = i \vec{PQ} \times \vec{B} \quad \vec{F}_{RS} = i \vec{RS} \times \vec{B} \quad \vec{F}_{PQ} = \vec{F}_{RS}$$

$$\left. \begin{aligned} \vec{F}_{SP} &= i \vec{SP} \times \vec{B} \\ \vec{F}_{QR} &= i \vec{QR} \times \vec{B} \end{aligned} \right\} \text{COPPIA}$$

$$\left. \begin{aligned} |\vec{F}_{SP}| &= iaB \\ |\vec{F}_{QR}| &= iaB \end{aligned} \right\}$$

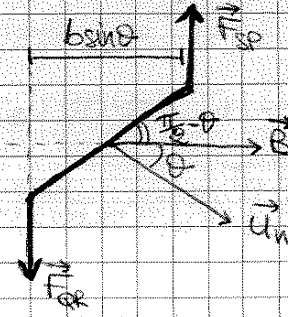


$$M = iaB \cdot b \sin \theta = i \Sigma B \sin \theta$$

$$\vec{m} = i \Sigma \vec{u}_n \Rightarrow \boxed{\vec{M} = \vec{m} \times \vec{B}}$$

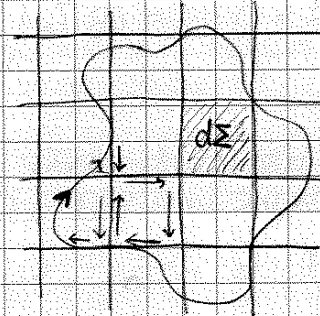
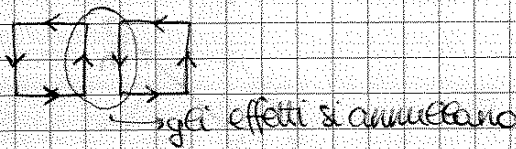
Quando $\vec{m} \parallel \vec{B}$: EQUILIBRIO STABILE

\vec{m} anti $\parallel \vec{B}$: EQ. INSTABILE



13.11.12

Circuito piano generico



$$d\vec{m} = i d\Sigma \vec{u}_n$$

$$\vec{m} = \int d\vec{m} = \int d\vec{m} \times \vec{B} = \int i d\Sigma \vec{u}_n \times \vec{B} = i \left(\int d\Sigma \vec{u}_n \right) \times \vec{B}$$

$$\boxed{\vec{M} = (i \Sigma \vec{u}_n) \times \vec{B}} \quad \text{MOMENTO TOTALE}$$

MOMENTO DI DIPOLO MAGNETICO

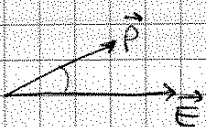
\vec{E} uniforme

$$\vec{M} = \vec{p} \times \vec{E}$$

$$\vec{u} = -\vec{p} \cdot \vec{E}$$

$$\vec{M} = \vec{m} \times \vec{B}$$

$$\boxed{\vec{u} = -\vec{m} \cdot \vec{B}}$$

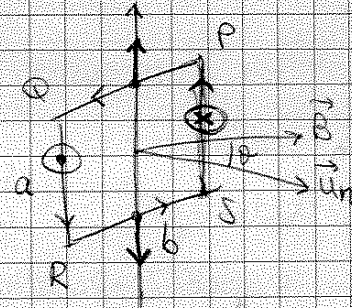


$$\vec{F}_{PQ} = i\vec{PQ} \times \vec{B} \quad \vec{F}_{RS} = i\vec{RS} \times \vec{B} \quad \vec{F}_{PQ} = \vec{F}_{RS}$$

$$\vec{F}_{SP} = i\vec{SP} \times \vec{B} \quad \vec{F}_{SR} = i\vec{SR} \times \vec{B}$$

$$\vec{F}_{QR} = i\vec{QR} \times \vec{B} \quad \vec{F}_{RQ} = i\vec{RQ} \times \vec{B}$$

] COPPIA

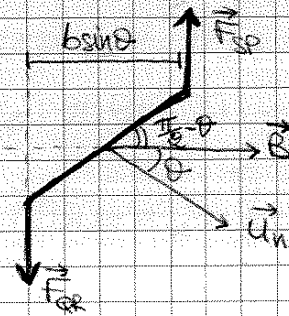


$$M = iaB \cdot b \sin\theta = i \Sigma B \sin\theta$$

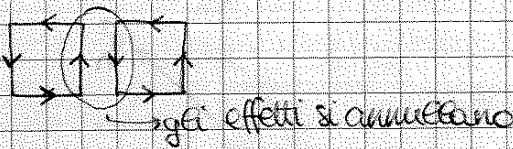
$$\vec{m} = i \Sigma \vec{u}_n \Rightarrow \boxed{\vec{M} = \vec{m} \times \vec{B}}$$

Quando $\vec{m} \parallel \vec{B}$: EQUILIBRIO STABILE

\vec{m} anti $\parallel \vec{B}$: EQ. INSTABILE



Circuito piano generico

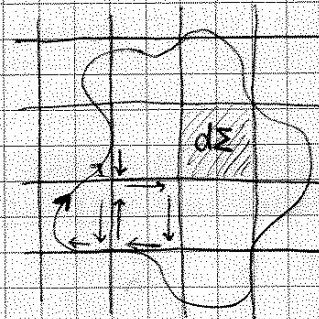


$$d\vec{m} = ia \Sigma \vec{u}_n$$

$$\vec{M} = \int d\vec{m} = \int d\vec{m} \times \vec{B} = \int ia \Sigma \vec{u}_n \times \vec{B} = i \left(\int \Sigma \vec{u}_n \right) \times \vec{B}$$

$$\boxed{\vec{M} = (i \Sigma \vec{u}_n) \times \vec{B}} \quad \text{MOMENTO TOTALE}$$

B.11.12



MOMENTO DI DIPOLLO MAGNETICO

\vec{E} uniforme

$$\vec{M} = \vec{p} \times \vec{E}$$

$$\vec{u} = -\vec{p} \cdot \vec{E}$$

$$\vec{M} = \vec{m} \times \vec{B}$$

$$\boxed{\vec{u} = -\vec{m} \cdot \vec{B}}$$



$$i = \frac{q}{T} = \frac{q}{2\pi R} \sqrt{\frac{kq^2}{mR}} = \frac{q^2}{2\pi R} \sqrt{\frac{k}{mR}}$$

$$m = i \Sigma = i \pi R^2 = \frac{q^2}{2\pi R} \sqrt{\frac{k}{mR}} \cdot \pi R^2$$

$$m = \sqrt{\frac{k}{mR}} \cdot \frac{q^2 R}{2}$$

→ ogni atomo ha un momento magnetico

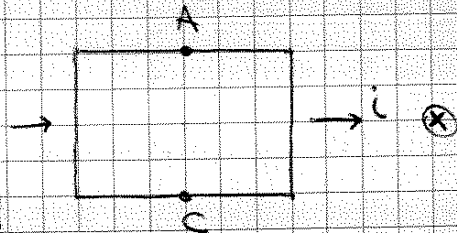
PRINCIPIO DI EQUIVALENZA DI AMPÈRE:

Ogni corpo magnetizzato è equivalente a una spira percorsa da corrente

EFFETTO HALL

Lastra percorsa da corrente

\vec{E} orientato come \vec{i} : ogni piano della lastra è una superficie equipotenziale ($V_{AC} = 0$)



Applico un \vec{B} alla lastra, entrante $\Rightarrow V_{AC} \neq 0!$

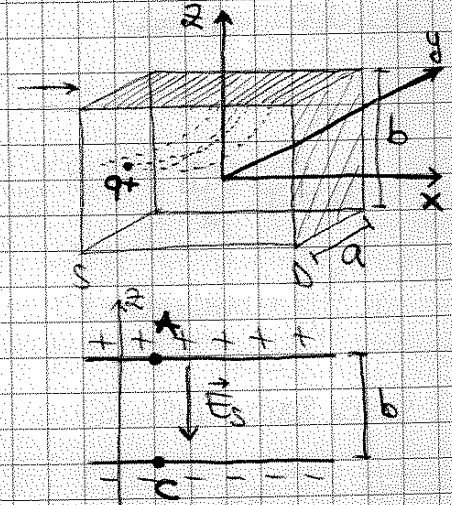
• Caso q positive

$$\vec{B} = B \vec{u}_y$$

$$\vec{j} = nq\vec{v} \quad \vec{v} = v \vec{u}_x$$

$$\vec{F} = q\vec{v} \times \vec{B} = qv(\vec{u}_x \times \vec{u}_y) = qvB \vec{u}_z$$

accumulo di cariche +



L'accumulo di cariche + dà origine a un campo \vec{E}_s che si oppone a un ulteriore accumulo.

L'accumulo continua finché:

$$q\vec{E}_s + \vec{F} = 0 \quad -qE_s + qvB = 0 \quad \boxed{E_s = vB}$$

$$\vec{E}_s = -vB \vec{u}_z$$

$$\int_C^A \vec{E}_s \cdot d\vec{s} = V_C - V_A$$

$$V_C - V_A = \int_C^A (-vB \vec{u}_z) \cdot (dz \vec{u}_z) = \int_C^A -vB dz = -vB \int_C^A dz = -vB [z(A) - z(B)] = -vBb$$

$$\boxed{V_A - V_C = \Delta V = vBb} \quad \text{DIFFERENZA DI POTENZIALE DI HALL}$$

$$j = nqv \quad v = \frac{j}{nq} \quad i = \frac{ic}{\Sigma} = \frac{i}{ab} \rightarrow v = \frac{i}{nqab}$$

$$\Delta V = \frac{i}{nqab} Bb$$

$$\boxed{\Delta V = \frac{iB}{nqa}}$$

→ posso ricavare n, note le altre grandezze

→ posso misurare B (se è noto n)

$$\boxed{B = \frac{nqa}{i} \Delta V} \quad \text{: SONDA HALL}$$

CONSIGLIENZA LU Aula 155 h.11.30
 MA Aula 6D h.10

16.11.12

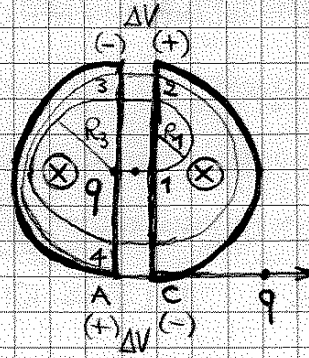
> Utilizzo: CICLOTRONE

Viene applicata una d.d.p. tra A e C ($V_A > V_C$)

La particella viene accelerata secondo: $\frac{1}{2} m v_1^2 = q \Delta V$

$$\Delta t_1 = \frac{\pi m}{qB}$$

$v_1 \perp B$



Quando la particella arriva in 2, scambiamo la polarità di A e C

$$\frac{1}{2} m v_2^2 + q \Delta V = \frac{1}{2} m v_3^2 \quad 2q \Delta V = \frac{1}{2} m v_3^2 \quad v_3 > v_1 \Rightarrow R_3 > R_1$$

perché $v_2 = v_1 \Rightarrow \frac{1}{2} m v_2^2 = \frac{1}{2} m v_1^2 = q \Delta V$

$\Delta t_2 = \Delta t_1 = \frac{\pi m}{qB}$: Δt è lo stesso ma v è maggiore \rightarrow accelerazione!

In 4 inverti di nuovo i poli, ecc...

• Ogni $\Delta t = \frac{\pi m}{qB}$ inverti polarità ($\nu = \frac{1}{\Delta t}$ RADIOTRUFREQUENZA)

• v aumenta finché $R_{max} = \frac{m v_{max}}{qB}$ $v_{max} = \frac{qBR_{max}}{m}$
 R del ciclotrone

• Raggiunti R_{max} e v_{max} , la particella continua accelerata in linea retta

• Caso generale

$$\vec{B} = B \vec{u}_z$$

$$m \frac{d\vec{v}}{dt} = \vec{F} = q \vec{v} \times \vec{B}$$

$$\vec{v} = v_x \vec{u}_x + v_y \vec{u}_y + v_z \vec{u}_z$$

$$\vec{v} \times \vec{B} = (v_x \vec{u}_x + v_y \vec{u}_y + v_z \vec{u}_z) \times (B \vec{u}_z) =$$

$$= v_x B (\vec{u}_x \times \vec{u}_z) + v_y B (\vec{u}_y \times \vec{u}_z) + v_z B (\vec{u}_z \times \vec{u}_z) =$$

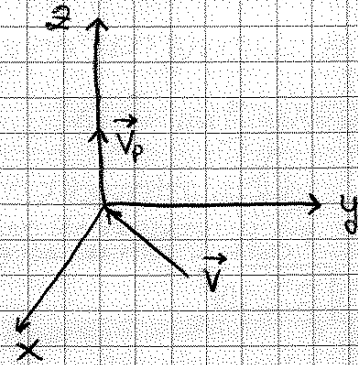
$$= -v_x B \vec{u}_y + v_y B \vec{u}_x$$

$$m \left\{ \frac{dv_x}{dt} \vec{u}_x + \frac{dv_y}{dt} \vec{u}_y + \frac{dv_z}{dt} \vec{u}_z \right\} = -q v_x B \vec{u}_y + q v_y B \vec{u}_x$$

$$m \frac{dv_x}{dt} = q v_y B$$

$$m \frac{dv_y}{dt} = -q v_x B$$

$m \frac{dv_z}{dt} = 0 \rightarrow$ moto con v costante (uniforme) lungo l'asse z

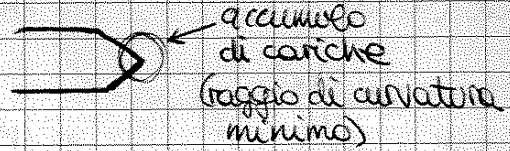


$$E_1 = k \frac{q_1}{R_1^2} = k \frac{R_1}{R_1 R_2} q \frac{1}{R_1^2} = \cancel{k} \frac{q}{R_1(R_1+R_2)}$$

$$E_2 = k \frac{q}{R_2(R_1+R_2)}$$

→ il campo è maggiore per la sfera più piccola: EFFETTO PUNTE

Quando la concentrazione di cariche è molto grande, le cariche cominciano a liberarsi



ES CONDENSATORE CILINDRICO

Legge di Gauss: $\phi_z(\vec{E}) = \frac{Q(\Sigma)}{\epsilon_0}$

$\vec{E} = E(r)\vec{u}_r$ (campo \perp filo)

$$\phi_z(\vec{E}) = \iint_{\Sigma} \vec{E} \cdot \vec{u}_n d\Sigma = \underbrace{\iint_{\Sigma_{sup}} \vec{E} \cdot \vec{u}_n d\Sigma}_0 + \underbrace{\iint_{\Sigma_{inf}} \vec{E} \cdot \vec{u}_n d\Sigma}_0 + \iint_{\Sigma_{lat}} \vec{E} \cdot \vec{u}_n d\Sigma$$

$\vec{u}_n \parallel \vec{u}_r$

→ $\phi_z(\vec{E}) = \iint_{\Sigma_{lat}} \vec{E} \cdot \vec{u}_n d\Sigma = \iint_{\Sigma_{lat}} E(r) d\Sigma$

$\Sigma_{lat} = 2\pi r L$ $\phi_z(\vec{E}) = 2\pi r L E(r)$

Se $r < R_1$: $E(r) = 0$

Se $R_1 < r < R_2$: $Q(\Sigma) = \lambda L \Rightarrow \phi_z(\vec{E}) = 2\pi r L E(r) = \frac{\lambda L}{\epsilon_0}$

$E(r) = \frac{\lambda}{2\pi\epsilon_0 r}$

$\vec{\nabla}V = \frac{\partial V}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial V}{\partial \phi}\vec{u}_\phi + \frac{\partial V}{\partial z}\vec{u}_z$

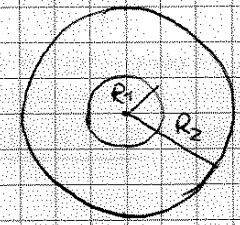
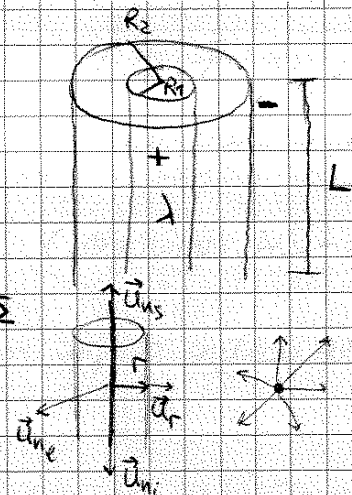
$\vec{E} = -\vec{\nabla}V : \frac{\lambda}{2\pi\epsilon_0 r}\vec{u}_r = -\frac{\partial V}{\partial r}\vec{u}_r - \frac{1}{r}\frac{\partial V}{\partial \phi}\vec{u}_\phi - \frac{\partial V}{\partial z}\vec{u}_z$

$$\left\{ \begin{array}{l} \frac{\lambda}{2\pi\epsilon_0 r} = -\frac{\partial V}{\partial r} \\ 0 = -\frac{1}{r}\frac{\partial V}{\partial \phi} \\ 0 = -\frac{\partial V}{\partial z} \end{array} \right\} \Rightarrow V = V(r)$$

$$\frac{dV}{dr} = -\frac{\lambda}{2\pi\epsilon_0 r} \quad dV = -\frac{\lambda}{2\pi\epsilon_0} \frac{dr}{r}$$

$$\int_{R_1}^{R_2} dV = -\frac{\lambda}{2\pi\epsilon_0} \int_{R_1}^{R_2} \frac{1}{r} dr \quad \Delta V = V(R_1) - V(R_2) = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{R_2}{R_1}$$

$$C = \frac{q}{\Delta V} = \frac{\lambda L}{\frac{\lambda}{2\pi\epsilon_0} \ln \left(\frac{R_2}{R_1}\right)} = 2\pi\epsilon_0 L \frac{1}{\ln \left(\frac{R_2}{R_1}\right)} \quad \frac{C}{L} = \frac{2\pi\epsilon_0}{\ln \left(\frac{R_2}{R_1}\right)}$$

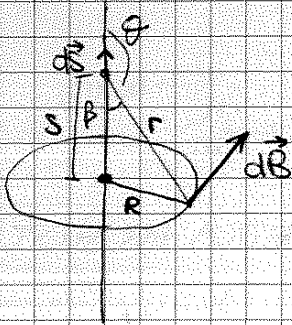
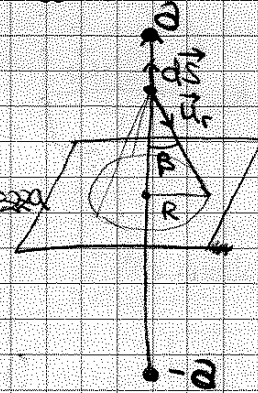


[ES 45-46-49-55-56-57]

55) Determinare \vec{B} creato da un conduttore rettilineo lungo $2a$.

$$d\vec{B} = \frac{\mu_0}{4\pi} i \frac{d\vec{s} \times \vec{u}_r}{r^2}$$

Calcolo \vec{B} per P e piano passante per O, \perp filo, a metà lunghezza.



$$d\vec{B} = \frac{\mu_0}{4\pi} i \frac{ds \sin\beta}{r^2} \vec{u}_\beta$$

$$\beta = \pi - \theta \quad \sin\beta = \sin(\pi - \theta)$$

$$\theta = \pi - \beta$$

$$r \sin\beta = R \quad r = \frac{R}{\sin\beta} = \frac{R}{\sin(\pi - \theta)} = \frac{R}{\sin\theta}$$

$$s \cdot \tan\beta = R \quad \tan\beta = \tan(\pi - \theta) = -\tan\theta \rightarrow s = \frac{R}{\tan\beta} = -\frac{R}{\tan\theta}$$

$$ds = d\left(-\frac{R}{\tan\theta}\right) = R \frac{1}{\tan^2\theta} d(\tan\theta) = R \frac{\cos^2\theta}{\sin^2\theta} \frac{1}{\cos^2\theta} d\theta = \frac{R}{\sin^2\theta} d\theta$$

$$d\vec{B} = \frac{\mu_0}{4\pi} i \frac{R}{\sin^2\theta} d\theta \cdot \sin\theta \cdot \frac{\sin\theta}{R^2} \vec{u}_\beta = \frac{\mu_0}{4\pi} i \frac{1}{R} \sin\theta d\theta \vec{u}_\beta$$

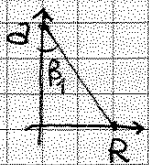
$$d\vec{B} = -\frac{\mu_0}{4\pi} \frac{1}{R} d(\cos\theta) \vec{u}_\beta$$

$$\vec{B}_{1/2} = \int_{-\theta_1}^{\theta_1} -\frac{\mu_0 i}{4\pi R} \vec{u}_\beta d(\cos\theta) = -\frac{\mu_0 i}{4\pi R} \vec{u}_\beta \int_{\cos\theta_1}^{\cos\theta_1} d(\cos\theta) =$$

$$= -\frac{\mu_0 i}{4\pi R} \cos\theta_1 \vec{u}_\beta$$

$$\theta_1 = \pi - \theta_1 \quad \vec{B}_{1/2} = \frac{\mu_0 i}{4\pi R} \cos\theta_1 \vec{u}_\beta$$

$$\vec{B} = 2\vec{B}_{1/2} = \frac{\mu_0 i}{2\pi R} \cos\theta_1 \vec{u}_\beta$$



$$2a \tan\theta_1 = R$$

$$\tan\theta_1 = \frac{R}{2a}$$

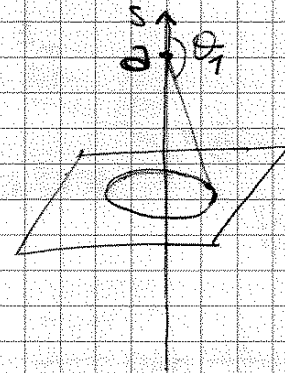
$$\cos\theta_1 = \frac{1}{\sqrt{1 + \tan^2\theta_1}} = \frac{1}{\sqrt{1 + \left(\frac{R}{2a}\right)^2}}$$

$$\vec{B} = \frac{\mu_0 i}{2\pi R} \frac{1}{\sqrt{1 + \left(\frac{R}{2a}\right)^2}} \vec{u}_\beta$$

se $a \rightarrow \infty$ ($a \gg R$)

$$\vec{B} = \frac{\mu_0 i}{2\pi R} \vec{u}_\beta$$

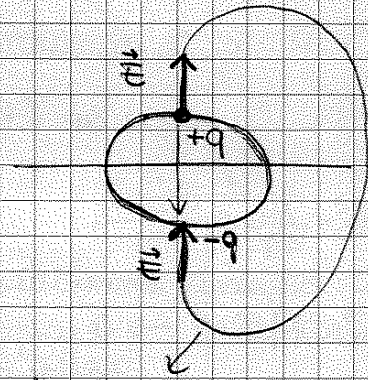
LEGGE DI BIOT-SAVART



$$\vec{B}(z \gg R) = \frac{\mu_0}{2\pi} i \frac{\pi R^2}{z^3} \vec{u}_z$$

$$\vec{m} = \sum i \vec{u}_n \quad \vec{m} = \pi R^2 i \vec{u}_z$$

$$\vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{m}}{z^3}$$



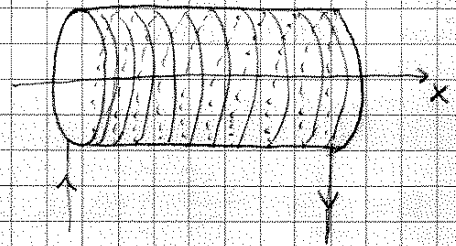
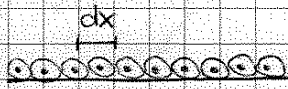
L'unità fondamentale per il magnetismo è il momento di dipolo magnetico, che possiamo considerare come una spirale percorsa da corrente.

Le linee del campo non si chiudono nella zona del dipolo.

SOLENOIDE

20.11.12

Successione di spire



Ipotesi un avvolgimento compatto: $n = \frac{dN}{dx}$

$dN = n dx$ (n di spire nell'intervallo dx)

$$d\vec{B} = \frac{\mu_0}{2} i \frac{R^2}{r^3} dN \vec{u}_x = \frac{\mu_0}{2} i \frac{R^2}{r^3} n dx \vec{u}_x$$

$$r \sin \phi = R \quad r = \frac{R}{\sin \phi}$$

$$(x_p - x) \tan \phi = R \quad x_p - x = \frac{R}{\tan \phi}$$

Fissato P, $x_p = \text{cost} \Rightarrow$

$$-dx = d\left(\frac{R}{\tan \phi}\right) = + \frac{R}{\tan^2 \phi} d(\tan \phi)$$

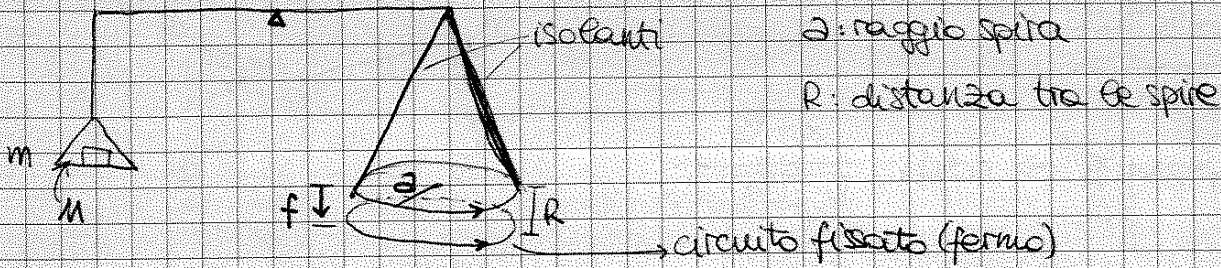
$$dx = \frac{R}{\tan \phi} \frac{1}{\cos^2 \phi} d\phi = R \frac{\sin^2 \phi}{\sin \phi \cos^2 \phi} d\phi = \frac{R}{\sin \phi} d\phi$$

$$\vec{B} = \frac{\mu_0}{2} i R^2 \frac{\sin^2 \phi}{R^3} n \frac{R}{\sin \phi} d\phi \vec{u}_x = \frac{\mu_0}{2} n i \sin \phi d\phi \vec{u}_x = - \frac{\mu_0}{2} n i d(\cos \phi) \vec{u}_x$$

\rightarrow il campo sull'asse non dipende dal raggio del sistema.

$$\phi_1 \leq \phi \leq \phi_2 \Rightarrow \vec{B}(x_p) = - \frac{\mu_0}{2} n i \left[\cos \phi \right]_{\phi_1}^{\phi_2} \vec{u}_x = - \frac{\mu_0}{2} n i (\cos \phi_2 - \cos \phi_1) \vec{u}_x$$

$$\phi_2 = \pi - \phi_1' \quad \cos \phi_2 = -\cos \phi_1' \Rightarrow \boxed{\vec{B}(x_p) = \frac{\mu_0}{2} n i (\cos \phi_1 + \cos \phi_1') \vec{u}_x}$$



se $a \gg R$: $f = \frac{\mu_0}{2\pi R} i_1 i_2 2\pi a = \frac{\mu_0}{R} i_1 i_2 a$

se $i_1 = i_2 = i$

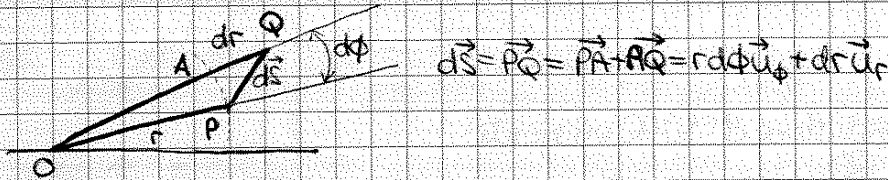
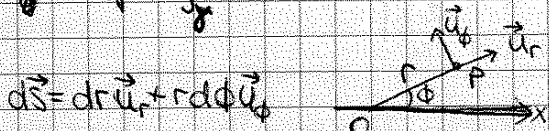
$$f = \frac{\mu_0}{R} i^2 a$$

Per mantenere $R = \text{cost}$, devo aggiungere una massa M : misuro la forza attrattiva tra le spire tramite una forza gravitazionale

LEGGE DI AMPÈRE (INTEGRALE)

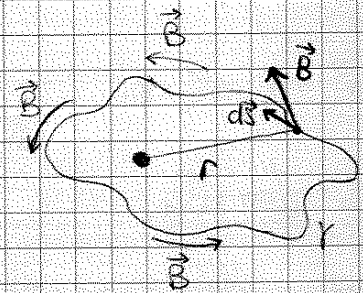
$\vec{B} = \frac{\mu_0}{2\pi r} i \vec{u}_\phi$ (legge di Biot-Savart)

$\Gamma(\gamma) = \oint_\gamma \vec{B} \cdot d\vec{s}$



• Caso: filo contenuto in γ (filo chiuso)

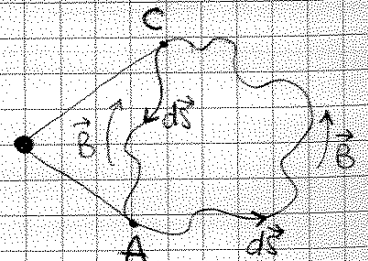
$\vec{B} = \frac{\mu_0}{2\pi r} i \vec{u}_\phi$ $d\vec{s} = dr \vec{u}_r + r d\phi \vec{u}_\phi$



$$\begin{aligned} \Gamma(\gamma) &= \oint_\gamma \vec{B} \cdot d\vec{s} = \oint_\gamma \left(\frac{\mu_0}{2\pi r} i \vec{u}_\phi \right) \cdot (dr \vec{u}_r + r d\phi \vec{u}_\phi) = \oint_\gamma \frac{\mu_0}{2\pi r} i r d\phi = \\ &= \oint_\gamma \frac{\mu_0}{2\pi} i d\phi = \frac{\mu_0}{2\pi} i \oint_\gamma d\phi = \frac{\mu_0}{2\pi} i (2\pi) \end{aligned} \quad \boxed{\Gamma(\gamma) = \mu_0 i}$$

• Caso: filo non chiuso

$$\begin{aligned} \Gamma(\gamma) &= \oint \vec{B} \cdot d\vec{s} = \int_A^C \vec{B} \cdot d\vec{s} + \int_C^A \vec{B} \cdot d\vec{s} = \int_A^C \frac{\mu_0}{2\pi r} i \vec{u}_\phi (dr \vec{u}_r + r d\phi \vec{u}_\phi) + \\ &+ \int_C^A \frac{\mu_0}{2\pi r} (-i \vec{u}_\phi) (dr \vec{u}_r + r d\phi \vec{u}_\phi) \end{aligned} \quad \text{: due contributi uguali e opposti}$$



$$\boxed{\Gamma(\gamma) = 0}$$

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§ Divergenza di B

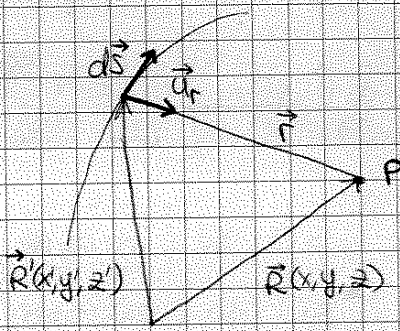
$$d\vec{B} = \frac{\mu_0}{4\pi} i \frac{d\vec{s} \times \vec{u}_r}{r^2} \quad \vec{B} = \frac{\mu_0}{4\pi} i \int \frac{d\vec{s} \times \vec{u}_r}{r^2}$$

$$\vec{r} = \vec{R} - \vec{R}'$$

$$\vec{r} = (x-x')\vec{u}_x + (y-y')\vec{u}_y + (z-z')\vec{u}_z$$

$$r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

\vec{R} : punto campo, \vec{R}' : punto sorgente



$$\vec{\nabla} \left(\frac{1}{r} \right) = \frac{\partial}{\partial x} \left(\frac{1}{r} \right) \vec{u}_x + \frac{\partial}{\partial y} \left(\frac{1}{r} \right) \vec{u}_y + \frac{\partial}{\partial z} \left(\frac{1}{r} \right) \vec{u}_z = -\vec{u}_x \frac{1}{r^2} \frac{dr}{dx} \dots$$

$$\frac{dr}{dx} = \frac{\partial}{\partial x} \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} = \frac{2(x-x')}{2\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} = \frac{x-x'}{r}$$

$$\vec{\nabla} \left(\frac{1}{r} \right) = -\vec{u}_x \frac{x-x'}{r^3} - \vec{u}_y \frac{y-y'}{r^3} - \vec{u}_z \frac{z-z'}{r^3} = -\frac{(x-x')\vec{u}_x + (y-y')\vec{u}_y + (z-z')\vec{u}_z}{r^3} = -\frac{\vec{r}}{r^3} = -\frac{\vec{u}_r}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi} i \int -\frac{\vec{u}_r \times d\vec{s}}{r^2} = \frac{\mu_0}{4\pi} i \int \vec{\nabla} \left(\frac{1}{r} \right) \times d\vec{s}$$

$$d\vec{s} = \vec{u}_x dx' + \vec{u}_y dy' + \vec{u}_z dz'$$

$$\vec{B} = \frac{\mu_0}{4\pi} i \int \vec{A} \times d\vec{s} \quad \text{con } \vec{A} = \vec{\nabla} \left(\frac{1}{r} \right)$$

Applico $\vec{\nabla}$ a entrambi i membri:

$$\vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot \left\{ \frac{\mu_0}{4\pi} i \int \vec{A} \times d\vec{s} \right\} = \frac{\mu_0}{4\pi} i \vec{\nabla} \cdot \int \vec{A} \times d\vec{s} = \frac{\mu_0}{4\pi} i \int \vec{\nabla} \cdot (\vec{A} \times d\vec{s})$$

Dati due vettori \vec{A} e \vec{C} :

$$\vec{\nabla} \cdot (\vec{A} \times \vec{C}) = -\vec{A} \cdot (\vec{\nabla} \times \vec{C}) + \vec{C} \cdot (\vec{\nabla} \times \vec{A})$$

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} i \int (-\vec{A} \cdot (\vec{\nabla} \times d\vec{s}) + d\vec{s} \cdot (\vec{\nabla} \times \vec{A})) = \frac{\mu_0}{4\pi} i \int d\vec{s} \cdot (\vec{\nabla} \times \vec{A})$$

$\vec{\nabla} \times (\vec{\nabla} \cdot \Phi) \equiv \vec{0}$ perché $\vec{\nabla} \times (\vec{\nabla} \cdot \Phi) \equiv \vec{0}$

$$\vec{\nabla} \cdot \vec{B} = \vec{0} \quad \vec{B} \text{ sempre CAMPO SOLENOIDALE}$$

PROPRIETÀ MAGNETICHE DELLA MATERIA

$$\vec{B}_0 = \mu_0 n i \vec{u}_x$$



Se inserisco un materiale nel solenoide: $\vec{B} \neq \vec{B}_0$

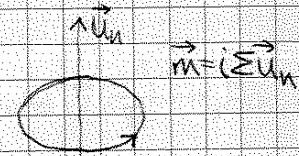
$$\frac{B}{B_0} = k_m \quad k_m \begin{cases} > 1 \\ < 1 \end{cases}$$



$$B = k_m B_0 = B_0 + (k_m - 1) B_0$$

campo nel vuoto

correzione dovuta alle proprietà del mezzo



$$B = \mu_0 n i + \mu_0 \chi_m n i$$

$$\chi_m = k_m - 1$$

$n i = H$: corrente/unità di lunghezza dovuta a correnti libere

$$B = \mu_0 n i + \mu_0 \chi_m n i$$

$$H = n i$$

MAGNETIZZAZIONE

M : corrente/unità di lunghezza dovuta alle proprietà magnetiche

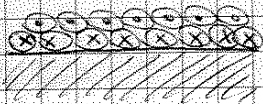
$$B = \mu_0 H + \mu_0 \chi_m H$$

$$M = \chi_m H$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

1) $k_m < 1$ $\chi_m < 0$, M opposto a H

MATERIALE DIAMAGNETICO



\vec{B} diminuisce

k_m, χ_m costanti

2) $k_m > 1$ $\chi_m > 0$, M concorde a H

MAT. PARAMAGNETICO



\vec{B} aumenta

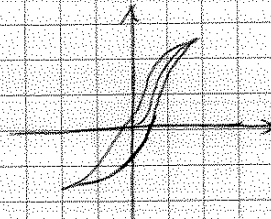
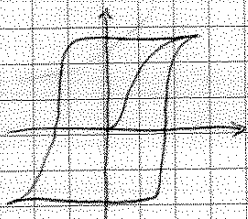
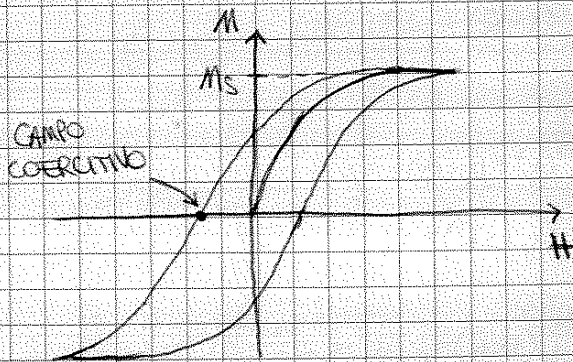
3) $k_m \gg 1$

MAT. FERROMAGNETICI

k_m, χ_m dipendono dal campo

CICLO D'ISTERESI

Per creare un magnete serve materiale con ciclo d'isteresi largo.
Per un elettromagnete, ciclo d'isteresi stretto.



↓ pag 195
ELETROSTATICA (VUOTO)

$$\left\{ \begin{aligned} \oint_S \vec{E} \cdot d\vec{S} &= 0 \\ \oint_S \vec{E} \cdot \vec{u}_n d\Sigma &= \frac{Q(\Sigma)}{\epsilon_0} \end{aligned} \right. \quad \left\{ \begin{aligned} \vec{\nabla} \times \vec{E} &= 0 \\ \vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \end{aligned} \right.$$

ELETROSTATICA (MEZZO)

$$\left\{ \begin{aligned} \oint_S \vec{E} \cdot d\vec{S} &= 0 \\ \oint_S \vec{D} \cdot \vec{u}_n d\Sigma &= Q_{lib}(\Sigma) \end{aligned} \right. \quad \left\{ \begin{aligned} \vec{\nabla} \times \vec{E} &= 0 \\ \vec{\nabla} \cdot \vec{D} &= \rho_{lib} \end{aligned} \right. \quad \vec{D} = \epsilon \vec{E}$$

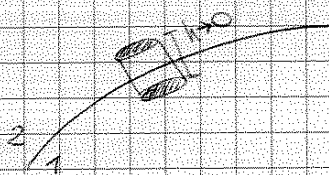
MAGNETOSTATICA (VUOTO)

$$\left\{ \begin{aligned} \vec{B} &= \frac{\mu_0}{4\pi} \int \frac{d\vec{s} \times \vec{u}_r}{r^2} \\ \oint_S \vec{B} \cdot d\vec{S} &= \mu_0 i(t) \end{aligned} \right. \quad \left\{ \begin{aligned} \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{B} &= \mu_0 \vec{j} \end{aligned} \right.$$

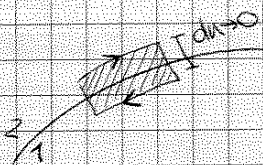
MAGNETOSTATICA (MEZZO)

$$\left\{ \begin{aligned} \vec{B} &= \frac{\mu}{4\pi} \int \frac{d\vec{s} \times \vec{u}_r}{r^2} \\ \oint_S \vec{H} \cdot d\vec{S} &= i_{lib}(\Sigma) \end{aligned} \right. \quad \left\{ \begin{aligned} \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{H} &= \vec{j}_{lib} \end{aligned} \right. \quad \vec{B} = \mu \vec{H}$$

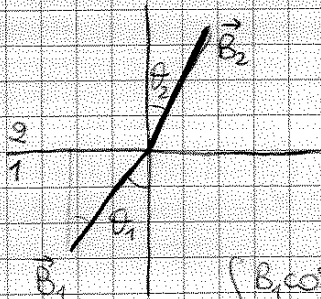
↓ pag 195
CONDIZIONE DI CONTINUITÀ ALL'INTERFACCIA



$$\vec{B}_1 \cdot \vec{u}_n = \vec{B}_2 \cdot \vec{u}_n \Rightarrow \boxed{B_{n1} = B_{n2}}$$



$$H_{t1} = H_{t2}$$



$$\begin{cases} B_{t1} = B_1 \sin \theta_1 \\ B_{n1} = B_1 \cos \theta_1 \end{cases}$$

$$\begin{cases} B_{t2} = B_2 \sin \theta_2 \\ B_{n2} = B_2 \cos \theta_2 \end{cases}$$

$$B_1 = \mu_1 H_1$$

$$B_2 = \mu_2 H_2$$

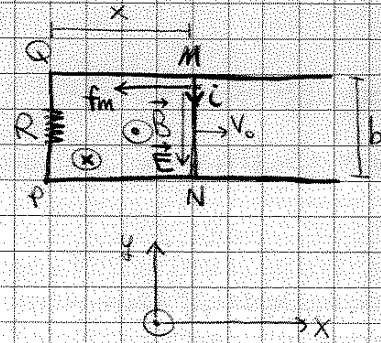
$$\begin{cases} B_1 \cos \theta_1 = B_2 \cos \theta_2 \\ \frac{B_1}{\mu_1} \sin \theta_1 = \frac{B_2}{\mu_2} \sin \theta_2 \end{cases}$$

$$\boxed{\frac{1}{\mu_1} \tan \theta_1 = \frac{1}{\mu_2} \tan \theta_2}$$

$$\int_{\Sigma(p)} (\vec{\nabla} \times \vec{E} + \frac{d\vec{B}}{dt}) \cdot \vec{u}_n d\Sigma = 0 \quad \rightarrow \quad \boxed{\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}} \quad \text{3ª eq. di Maxwell}$$

tutte le volte che esiste un campo magnetico variabile, c'è un campo \vec{E} non conservativo

ES) \vec{B} uscente, $v_0 = \text{cost}$, sbarra in movimento di materiale conduttore



$$\vec{v}_0 = v_0 \vec{u}_x$$

$$\vec{B} = B \vec{u}_z$$

$$\vec{F}_c = q \vec{u}_0 \times \vec{B} \quad \vec{E} = \frac{\vec{F}}{q} = \vec{u}_0 \times \vec{B} = -\vec{u}_y v_0 B$$

La fem indotta crea un campo \vec{B} entrante, che si oppone al campo \vec{B} che l'ha generata

$$fem = \oint \vec{E} \cdot d\vec{s}$$

$$\Phi_z(\vec{B}) = B \times b \quad \frac{d\Phi_z(\vec{B})}{dt} = B \frac{dx}{dt} b = B v_0 b$$

$$fem = - \frac{d\Phi_z(\vec{B})}{dt} = - B b v_0$$

⊖: la corrente circola in modo da compensare l'incremento del flusso dovuto alla parte in moto

$$i = \frac{fem}{R+r} \quad P = (R+r) i^2 = (R+r) \frac{fem^2}{(R+r)^2} = \frac{B^2 b^2 v_0^2}{R+r}$$

Per la 2ª legge di Laplace: $\vec{f}_m = I(\vec{MN} \times \vec{B})$ (forza magnetica)

$$\vec{f}_m = -I l b \vec{B} \quad \vec{f}_{est} + \vec{f}_m = 0 \rightarrow \vec{f}_{est} - \vec{f}_m = I l b \vec{B}$$

$$P_{est} = \vec{f}_{est} \cdot \vec{v}_0 = I l b B v_0 \quad P_{est} = \frac{B b v_0}{R+r} b B v_0 = \frac{B^2 b^2 v_0^2}{R+r} \equiv \text{potenza dissipata nel circuito}$$

La corrente non potrebbe scorrere in verso opposto, altrimenti non ci sarebbe la conservazione dell'energia ($v_0 \rightarrow \infty$)

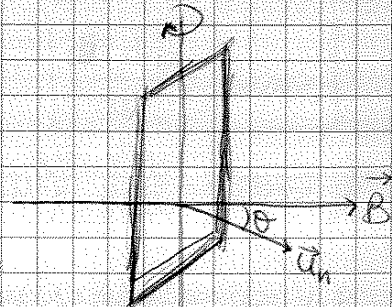
$$\Phi = \vec{B} \cdot \vec{u}_n \Sigma = B \Sigma \cos \theta$$

$$\theta = \omega t \rightarrow \Phi = B \Sigma \cos(\omega t)$$

$$fem = - \frac{d\Phi}{dt} = \omega B \Sigma \sin(\omega t) \quad i = \frac{fem}{R} = \frac{\omega B \Sigma \sin(\omega t)}{R}$$

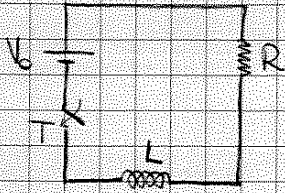
periodica

$$P = R i^2 = R \left(\frac{\omega B \Sigma}{R} \right)^2 \sin^2 \omega t \quad \langle P \rangle = \frac{1}{2} (\omega B \Sigma)^2 \frac{1}{R}$$



$$\langle P \rangle = \frac{1}{2} \frac{(\omega B \Sigma)^2}{R}$$

INDUTTORE



$$V_0 + f_{em} = Ri$$

↳ fem autoindotta ($\phi = Li$)

$$V_0 - \frac{d\phi}{dt} = Ri$$

$$V_0 - \frac{d}{dt}(Li) = Ri$$

$$V_0 = Ri + L \frac{di}{dt}$$

Se $V_0 = \text{cost}$: $i = i_{omogenea} + i_{particolare}$

$$L \frac{di_{om}}{dt} + Ri_{om} = 0 \quad \frac{di_{om}}{dt} + \frac{R}{L} i_{om} = 0 \quad \underline{i_{om}(t) = Ae^{-\frac{R}{L}t}}$$

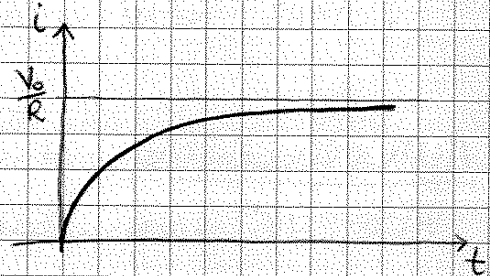
• i_p dello stesso tipo della causa eccitatrice (qui $V = \text{cost}$) : $i_p = \text{cost}$

$$V_0 = Ri_p \quad i_p = \frac{V_0}{R}$$

$$i(t) = \frac{V_0}{R} + Ae^{-\frac{R}{L}t}$$

dato che $i(0) = 0 \rightarrow 0 = \frac{V_0}{R} + A \quad A = -\frac{V_0}{R}$

$$\boxed{i(t) = \frac{V_0}{R} (1 - e^{-\frac{R}{L}t})}$$



$$[L] = \frac{[\Phi]}{[i]} = \frac{[B \cdot S]}{[i]} = \frac{\omega b \cdot m^2}{A} \cdot \frac{1}{A} = \frac{Vs}{A} = \Omega \cdot s = H$$

$$\boxed{\frac{L}{R} = \tau} = \frac{\Omega \cdot s}{\Omega} = s$$

$$\boxed{i(t) = \frac{V_0}{R} (1 - e^{-\frac{t}{\tau}})}$$

$$t \rightarrow \infty : i(t) \rightarrow \frac{V_0}{R}$$

$$\frac{di}{dt} = \frac{V_0}{RC} e^{-\frac{t}{\tau}} \quad t=0 \rightarrow \frac{V_0}{RC} = \frac{V_0 R}{R L} = \frac{V_0}{L}$$

$$f_{em} = -L \frac{di}{dt} = -L \frac{V_0}{RC} e^{-\frac{t}{\tau}} = -k \frac{V_0}{k} e^{-\frac{t}{\tau}} \quad f_{em} = -V_0 e^{-\frac{t}{\tau}}$$

BILANCIO ENERGETICO

$$V_0 = Ri + L \frac{di}{dt} \quad V_0 i = Ri^2 + Li \frac{di}{dt}$$

$$Li \frac{di}{dt} = L \frac{d}{dt} \left(\frac{1}{2} i^2 \right) = \frac{d}{dt} \left(\frac{1}{2} Li^2 \right)$$

$$\underbrace{V_0 i}_{\text{potenza fornita dal generatore}} = \underbrace{Ri^2}_{\text{potenza dissipata dal circuito (calore)}} + \frac{d}{dt} \left(\frac{1}{2} Li^2 \right)$$

potenza fornita dal generatore

↳ potenza immagazzinata nell'induttore (potenza persa dal gen. per vincere la fem)

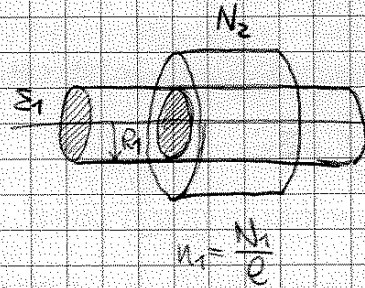
potenza dissipata dal circuito (calore)

Due circuiti vicini:

- la variazione di corrente genera in quel circuito una fem;
- " " " " genera nell'altro circuito una fem di mutua induzione

(55)

$$\begin{cases} B_1 = \mu_0 n_1 i_1 & r < R_1 \\ B_1 = 0 & r > R_1 \end{cases}$$



$$\Phi_{12\text{spira}} = \mu_0 n_1 i_1 \Sigma_1 \quad \Phi_{12\text{tot}} = N_2 \Phi_{12\text{spira}} = \mu_0 n_1 N_2 \Sigma_1 i_1$$

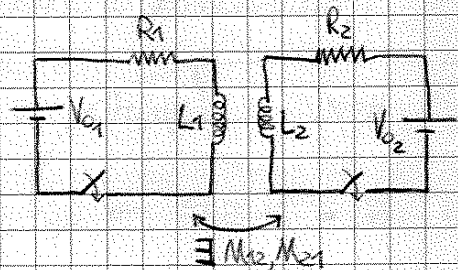
$$\Phi_{12} = M_{12} \cdot i_1 \quad \rightarrow \quad \boxed{M_{12} = \mu_0 N_1 N_2 \Sigma_1}$$

TRASFORMATORE

$$\Phi_1 = \underbrace{\Phi_{11}}_{\text{autoconcatenato}} + \Phi_{21} = L_1 i_1 + L_2 M_{21}$$

$$\Phi_2 = L_2 i_2 + L_1 M_{12}$$

$$\begin{cases} \text{fem}_{i_1}: & V_{01} - \frac{d\Phi_1}{dt} = R_1 i_1 \\ \text{fem}_{i_2}: & V_{02} - \frac{d\Phi_2}{dt} = R_2 i_2 \end{cases}$$



$$V_{01} - L_1 \frac{di_1}{dt} - M_{21} \frac{di_2}{dt} = R_1 i_1$$

$$\begin{cases} V_{01} = R_1 i_1 + L_1 \frac{di_1}{dt} + M_{21} \frac{di_2}{dt} \\ V_{02} = R_2 i_2 + L_2 \frac{di_2}{dt} + M_{12} \frac{di_1}{dt} \end{cases}$$

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30.11.12

$$\begin{cases} V_{01} i_1 = R_1 i_1^2 + L_1 i_1 \frac{di_1}{dt} + M_{21} i_1 \frac{di_2}{dt} \\ V_{02} i_2 = R_2 i_2^2 + L_2 i_2 \frac{di_2}{dt} + M_{12} i_2 \frac{di_1}{dt} \end{cases}$$

$$\underbrace{V_{01} i_1 + V_{02} i_2}_{\text{pot. fornita dai gen.}} = \underbrace{R_1 i_1^2 + R_2 i_2^2}_{\text{pot. dissipate. nelle R}} + \underbrace{\frac{d}{dt} \left\{ \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \right\}}_{\text{derivata dell'En. potenziale magnetica}} + \underbrace{M_{21} i_1 \frac{di_2}{dt} + M_{12} i_2 \frac{di_1}{dt}}_{\text{energia legata alla mutua induzione}}$$

Suppongo che esista un'energia di mutua induzione: $U_M = U_M(i_1, i_2)$ dovuta alle correnti (energia magnetica di accoppiamento)

Per le teor. di Stokes: $\int_{\Sigma(\gamma)} (\vec{\nabla} \times \vec{B}) \cdot \vec{u}_n d\Sigma = \int_{\gamma} \vec{B} \cdot d\vec{s}$

$$\int_{\Sigma(\gamma)} \vec{j} \cdot \vec{u}_n d\Sigma = i(\gamma)$$

$$\int_{\Sigma(\gamma)} \frac{\partial \vec{H}}{\partial t} \cdot \vec{u}_n d\Sigma = \frac{d}{dt} \int_{\Sigma(\gamma)} \vec{E} \cdot \vec{u}_n d\Sigma = \frac{d\phi(\vec{E})}{dt}$$

$$\int_{\gamma} \vec{B} \cdot d\vec{s} = \mu_0 i(\gamma) + \epsilon_0 \mu_0 \frac{d\phi(\vec{E})}{dt}$$



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EQUAZIONI DI MAXWELL

NEL VUOTO, FORMA LOCALE

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t})$$

NEL VUOTO, FORMA INTEGRALE

$$\int_{\Sigma} \vec{E} \cdot \vec{u}_n d\Sigma = \frac{Q(\Sigma)}{\epsilon_0}$$

$$\int_{\Sigma} \vec{B} \cdot \vec{u}_n d\Sigma = 0$$

$$\int_{\Sigma} \vec{E} \cdot d\vec{s} = - \frac{\partial}{\partial t} \int_{\Sigma(\gamma)} \vec{B} \cdot \vec{u}_n d\Sigma$$

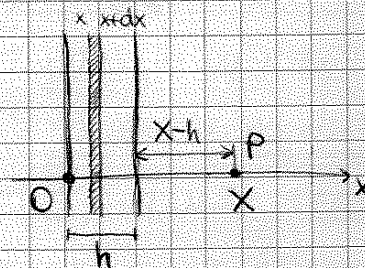
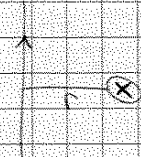
$$\int_{\gamma} \vec{B} \cdot d\vec{s} = \mu_0 i(\gamma) + \epsilon_0 \mu_0 \frac{\partial}{\partial t} \int_{\Sigma(\gamma)} \vec{E} \cdot \vec{u}_n d\Sigma$$

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! ES 7.13

h = 2cm
i = 10A
 $\vec{m} = 0,1 \vec{u}_x$

Formule di Biot-Savart: $\vec{B} = \frac{\mu_0}{2\pi} \frac{i}{r} \vec{u}_\phi$



i uniformemente distribuito $\Rightarrow j = \frac{i}{h}$ (corrente per unità di larghezza)

Posso considerare la parte dx come filiforme: $di = j dx = \frac{i}{h} dx$

$$d\vec{B} = \frac{\mu_0}{2\pi} \frac{di}{(X-x)} \vec{u}_\phi = \frac{\mu_0}{2\pi} \frac{i}{h} \frac{1}{X-x} dx \vec{u}_\phi$$

$$\vec{B} = \int_0^h \frac{\mu_0}{2\pi} \frac{i dx}{h(X-x)} \vec{u}_\phi = \frac{\mu_0}{2\pi} \frac{i}{h} \vec{u}_\phi \int_0^h \frac{dx}{X-x} = \frac{\mu_0}{2\pi} \frac{i}{h} \log\left(\frac{X}{X-h}\right) \vec{u}_\phi$$

$$\int_0^h \frac{dx}{X-x} = -[\log(X-x)]_0^h = -\log \frac{X-h}{X}$$

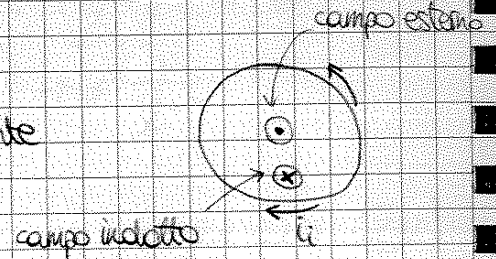
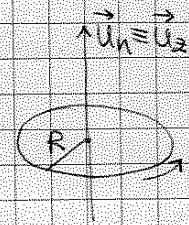
$$\Sigma = TR^2$$

$$\Phi = \int_{\Sigma} \vec{B} \cdot \vec{u}_n = B_0 \left(\frac{t}{t_0}\right)^2 TR^2 \quad \Phi = N\Phi_1$$

$$fem_i = -\frac{d\Phi}{dt} = -2N \frac{B_0}{t_0^2} t TR^2$$

$r =$ resistenza spira: $i_i = \frac{fem_i}{r} = 2N \frac{B_0}{r t_0} t TR^2$

Verso di i_i ? So che B aumenta lungo z (è uscente), quindi il campo indotto dev'essere entrante $\Rightarrow i_i$ in senso antiorario



$$A = 2N \frac{B_0}{r t_0} TR^2 \quad i_i = At$$

$$P_g = r i_i^2 = r A^2 t^2$$

$$\omega = \int_0^{t_0} P_g dt = r A^2 \int_0^{t_0} t^2 dt = \frac{1}{3} r A^2 t_0^3$$

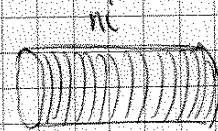
ES 1

! (ES) Calcolare B sull'asse di un cilindro (raggio R , lunghezza d) magnetizzato unif. con magnetizzazione M e in particolare B_0 nel centro.

$$ni \rightarrow M$$

~~$$B(x) = \frac{\mu_0 ni}{2} (\cos \varphi_1 + \cos \varphi_2)$$~~

$$\Rightarrow B(x) = \frac{\mu_0 M}{2} (\cos \varphi_1 + \cos \varphi_2)$$

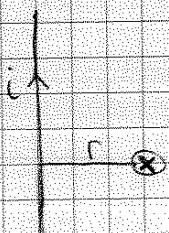


Nel centro, se il magnete è lungo: $\varphi_1 = 0$
 $\varphi_2 = 0$

$$\Rightarrow B(0) = \mu_0 M$$

03.12.12

ES



$$\vec{B} = \frac{\mu_0 i}{2\pi r} \vec{u}_\phi$$

Regime statico: $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$ $\vec{B} = B(r) \vec{u}_\phi$ $\vec{\nabla} \times \vec{B} = \mu_0 j \vec{u}_z$

$$\vec{\nabla} \times \vec{B} = \epsilon_0 \mu_0 \frac{dE}{dt} \quad \vec{B} = B(r) \vec{u}_\phi \quad \vec{\nabla} \times \vec{B} = \epsilon_0 \mu_0 \frac{dE}{dt} \vec{u}_z$$

$$\frac{1}{2C} q^2 + \frac{1}{2} L \left(\frac{dq}{dt} \right)^2 = \frac{1}{2C} q_0^2$$

CONSERVAZIONE DELL'ENERGIA

pag 938

Con \mathcal{E}

$$\mathcal{E} = \frac{q}{C} + L \frac{di}{dt}$$

$$\begin{cases} q(0) = 0 \\ i(0) = 0 \end{cases}$$

$$i = \frac{dq}{dt}$$

$$\mathcal{E} = \frac{1}{C} q + L \frac{d^2 q}{dt^2}$$

$$q(t) = q_{\text{hom}}^{(t)} + q_{\text{part}}^{(t)}$$

$$\frac{1}{C} q_{\text{hom}} + L \frac{d^2 q_{\text{hom}}}{dt^2} = 0$$

$$q(t) = A \cos(\omega_0 t + \phi)$$

se $\mathcal{E} = \text{cost}$ (rispetto a t), $q_p = \text{cost}$

$$\mathcal{E} = \frac{1}{C} q_p \quad q_p = \mathcal{E} C$$

$$q(t) = A \cos(\omega_0 t + \phi) + \mathcal{E} C \quad i(t) = \frac{dq}{dt} = -A \omega_0 \sin(\omega_0 t + \phi)$$

$$\begin{cases} q(0) = A \cos \phi + \mathcal{E} C = 0 \\ i(0) = -A \omega_0 \sin \phi = 0 \end{cases} \rightarrow \begin{cases} A = -\mathcal{E} C \\ \phi = 0 \end{cases} \quad q(t) = \mathcal{E} C \{ 1 - \cos(\omega_0 t) \}$$

Bilancio: $\mathcal{E} dq = \frac{1}{C} q dq + L \frac{di}{dt} i dt \quad \mathcal{E} dq = d(\mathcal{E} q) = d(W_C)$

$$\frac{1}{C} q dq = d\left(\frac{1}{2C} q^2\right) = d(W_C)$$

$$L i di = d\left(\frac{1}{2} L i^2\right) = d(W_L)$$

$$d(W_C - W_C - W_L) = 0$$

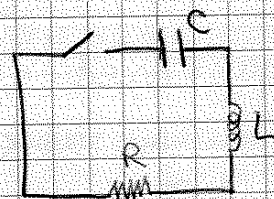
$$W_C - W_C + W_L$$

Con R

$$\begin{cases} q(0) = q_0 \\ i(0) = 0 \end{cases}$$

$$0 = Ri + \frac{q}{C} + L \frac{di}{dt}$$

$$\frac{d^2 q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{1}{LC} q = 0$$



Eq. del tipo: $\frac{d^2 x}{dt^2} + r \frac{dx}{dt} + \omega_0^2 x = 0 \quad \left(m \frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + kx = 0 \right)$

$$x(t) = A e^{-\gamma t} \cos(\omega t + \phi) \quad \omega = \sqrt{\omega_0^2 - \gamma^2}$$

$$r \rightarrow \frac{R}{L} \quad \omega_0^2 \rightarrow \frac{1}{LC} \quad \omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

$$q(t) = A e^{-\frac{R}{L} t} \cos(\omega t + \phi)$$

$$0 = \underbrace{Ri^2 dt}_{R i dq} + \frac{1}{C} q dq + L i di$$

$$0 = d(W_R) + d(W_C) + d(W_L) \quad dW_R = -d(W_C + W_L)$$

$$W_R = \int_0^\infty dW_R$$

$$W_R = - \int d(W_C + W_L) \quad W_R = - [W_C + W_L]_0^\infty$$

$$W_R = W_C(0) + W_L(0) - (W_C(\infty) + W_L(\infty))$$

- ① $\frac{1}{2C} q_0^2$
- ② $\frac{1}{2} L (\omega_0 i_0)^2$
- ③ $\frac{1}{2C} q^2(\infty) = 0$
- ④ $\frac{1}{2} L i^2(\infty) = 0$

LEGGE DI GAULEO FERRARIS

Potenza sviluppata dal circuito?

$$P = \varepsilon(t) i(t) = \varepsilon_0 \cos(\omega t + \phi) i_0 \cos(\omega t)$$

$$\cos(\omega t + \phi) = \cos(\omega t) \cos \phi - \sin(\omega t) \sin \phi$$

$$P(t) = \varepsilon_0 i_0 \left\{ \cos^2(\omega t) \cos \phi - \cos(\omega t) \sin(\omega t) \sin \phi \right\}$$

$$P(t) = \varepsilon_0 i_0 \left\{ \underbrace{\cos^2(\omega t) \cos \phi}_{\text{sempre } > 0} - \frac{1}{2} \sin(2\omega t) \sin \phi \right\}$$

periodo: $2T = 2\pi$ $T = \frac{2\pi}{\omega}$

$$\langle P \rangle = \frac{1}{T} \int_0^T P(t) dt = \varepsilon_0 i_0 \left\{ \cos \phi \cdot \underbrace{\frac{1}{T} \int_0^T \cos^2(\omega t) dt}_{= \frac{1}{2}} - \frac{1}{2} \sin \phi \underbrace{\int_0^T \sin(2\omega t) dt}_0 \right\}$$

$$\langle P \rangle = \frac{1}{2} \varepsilon_0 i_0 \cos \phi = \frac{1}{2} R i_0^2 = \frac{1}{2} R \frac{\varepsilon_0^2}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$\varepsilon_0 \cos \phi = R i_0$

$\langle P \rangle = \frac{1}{2} \varepsilon_0 i_0 \cos \phi$

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LEGGE DI HOPKINSON

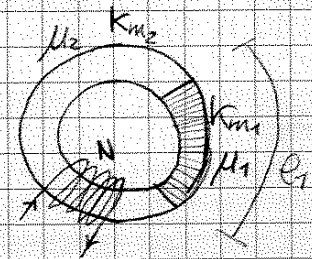
04.12.19

Legge di Ampère (caso con mezzo magnetico): $\oint \vec{H} \cdot d\vec{S} = i(r)$

$$\mu = k_m \mu_0, \quad k_m > 1$$

Suppongo $B = \mu H$ con μ indipendente da H

$$\oint \vec{H} \cdot d\vec{S} = Ni$$



\vec{B} è confinato nel circuito ed è uguale in ogni punto perché $\Phi(\vec{B}) = 0$ ($\vec{B} = \vec{B}_u$)

$$\int_{S_1} \vec{H}_1 \cdot d\vec{S}_1 + \int_{S_2} \vec{H}_2 \cdot d\vec{S}_2 = Ni$$

$$B = \text{cost}, \quad B = \mu H \Rightarrow H = \text{cost} \Rightarrow H_1 l_1 + H_2 l_2 = Ni$$

$\Phi = B S \text{ cost}$ su ogni sezione \Rightarrow se la sez. è costante, $B = \frac{\Phi}{S}$

$$H = \frac{B}{\mu} = \frac{\Phi}{\mu S}$$

$$H_1 = \frac{\Phi}{\mu_1 S} \quad H_2 = \frac{\Phi}{\mu_2 S}$$

← equivalente della legge di Ohm

$$\Rightarrow \frac{\Phi}{\mu_1 S} l_1 + \frac{\Phi}{\mu_2 S} l_2 = Ni$$

$\left(\frac{1}{\mu_1} \frac{l_1}{S} + \frac{1}{\mu_2} \frac{l_2}{S} \right) \Phi = Ni$

$R = \frac{1}{\mu} \frac{l}{S}$

(analoga a $R = \rho \frac{l}{S}$)

Problema: di solito non conosco μ_1, μ_2

$$\vec{u}_x \frac{\partial}{\partial x} (\vec{\nabla} \cdot \vec{F}) + \vec{u}_y \frac{\partial}{\partial y} (\vec{\nabla} \cdot \vec{F}) + \vec{u}_z \frac{\partial}{\partial z} (\vec{\nabla} \cdot \vec{F}) = \vec{\nabla} (\vec{\nabla} \cdot \vec{F})$$

$$\vec{u}_x \nabla^2 F_x + \vec{u}_y \nabla^2 F_y + \vec{u}_z \nabla^2 F_z = \vec{\nabla} (\vec{u}_x F_x + \vec{u}_y F_y + \vec{u}_z F_z) = \nabla^2 \vec{F}$$

$$\boxed{\vec{G} = \vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla} (\vec{\nabla} \cdot \vec{F}) - \nabla^2 \vec{F}}$$

ONDE E.M. NEL VUOTO

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = \frac{\rho_{lib}}{\epsilon_0} \\ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \end{cases} \quad \begin{cases} \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{B} = \mu_0 (\vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}) \end{cases} \quad \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

Se $\rho=0$, $\vec{j}=0$ (no cariche, no correnti):

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = 0 \\ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \end{cases} \quad \begin{cases} \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \end{cases}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\vec{\nabla} \times \left(\frac{\partial \vec{B}}{\partial t} \right)$$

$$\vec{\nabla} (\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B})$$

$$-\nabla^2 \vec{E} = -\frac{\partial}{\partial t} \left(\epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\begin{cases} \nabla^2 \vec{E} - \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \\ \nabla^2 \vec{B} - \epsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \end{cases} \quad c^2 = \frac{1}{\epsilon_0 \mu_0}$$

$$\rightarrow \begin{cases} \nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \\ \nabla^2 \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \end{cases}$$

Eq. delle ONDE: $\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0$ (*) (v parametro)

Caso $\psi = \psi(x,t)$ $\psi(x,t) = \underbrace{f(x-vt)}_{\text{onda progressiva}} + \underbrace{g(x+vt)}_{\text{onda regressiva}}$ (**)

onda progressiva onda regressiva

Voglio dimostrare che (*) ha soluzioni della forma (**)

$$\xi = x - vt \quad \eta = x + vt \quad \psi(x,t) = \psi(\xi, \eta)$$