NUMERO: 600

DATA: 23/07/2013

A P P U N T I

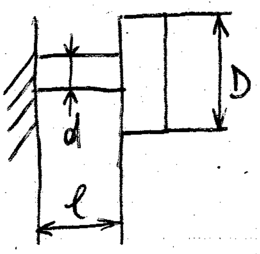
STUDENTE: Bruno

MATERIA: Dinamica dei Sistemi Meccanici Esercizi

Prof. Velardocchia

10/10/12

1)



Sist a 1 gdl

$l = 1\,m$

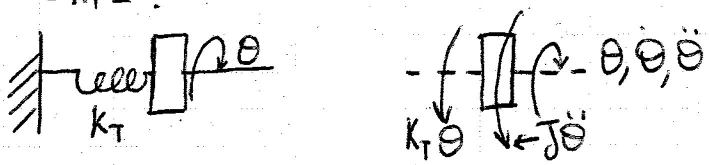
$d = 3\,cm$

$G = 78000\,N/mm^2$

$m = 10\,kg$

$D = 0.2\,m$

$\omega_n = ?$



$$\theta = \frac{M_t}{G I_p}\, l$$

$$K_T = \frac{M_t}{\theta} = G\frac{I_p}{l} = 6203\,\frac{Nm}{rad}$$

$$I_p = \frac{\pi}{32} d^4 = 7.95 \cdot 10^{-8}\, m^4$$

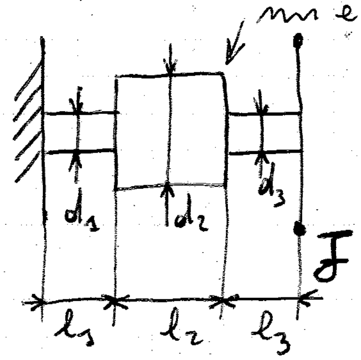
$$J\ddot{\theta} + K_T \theta = 0$$

$$J = m\frac{(r)^2}{2} = m\frac{D^2}{8} = 0{,}05\ kg \cdot m^2$$

$$\omega_n = \sqrt{\frac{K_T}{J}} = 352{,}22\ rad/s$$

L' albero lavora cm 1 molla di torsione

2)



$l_1 = 0.25\,m$

$l_2 = 0.75\,m$

$l_3 = 0.1\,m$

$d_1 = 0.02\,m$

$d_2 = 0.03\,m$

$d_3 = 0.01\,m$

(1)

$y = b \sin\theta \sim b\theta$

$x = l \sin\theta \sim l\theta$

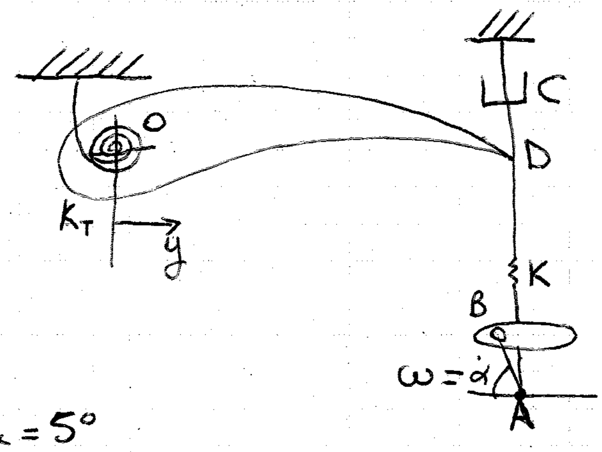
$$I_o \ddot{\theta} + 2Kb^2\theta + mg\frac{l}{2}\theta + Ml^2\ddot{\theta} + Mgl\theta = 0$$

$$(I_o + Ml^2)\ddot{\theta} + (2Kb^2 + mg\frac{l}{2} + Mgl)\theta = 0$$ eq del moto del pendolo fisso

$$\omega_n = \sqrt{\frac{2Kb^2 + mg\frac{l}{2} + Mgl}{I_o + Ml^2}}$$

ESERCITAZIONI:

1° Profilo alare di 1 alettone



$\theta_{max} = 5°$

$\zeta = 60\%$ $\omega = 300$ rad/s

$K = 40$ kg/mm

$AB = R = 5$ cm

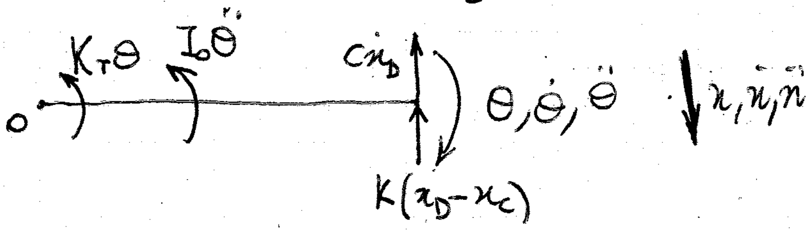
$\mu = \mu_o(1 - \frac{y}{L})$

$\mu_o = 100$ kg/m

$OD = L = 50$ cm $= l$

$x_D \sim l\theta$

$x_C = R \sin\omega t$



$\omega_n, \omega_R, \varphi, K_T$

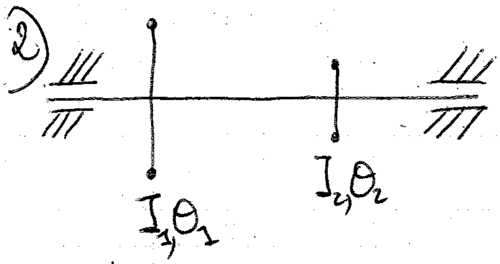
$$I_o = \int_0^l \mu y^2 dy = \int_0^l \mu_o(1 - \frac{y}{l})y^2 dy = \mu_o\left[\frac{y^3}{3} - \frac{y^4}{4l}\right]_0^l$$

$$= \frac{\mu_o}{12} l^3 = \frac{100}{12} \cdot (50 \cdot 10^{-2})^3 = 1.042 \ m^3$$

(3)

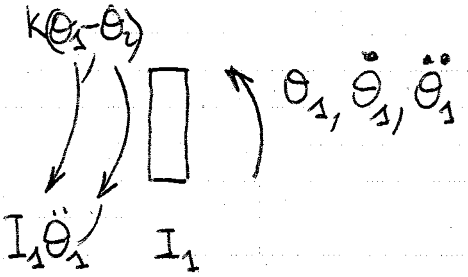
19/10/12

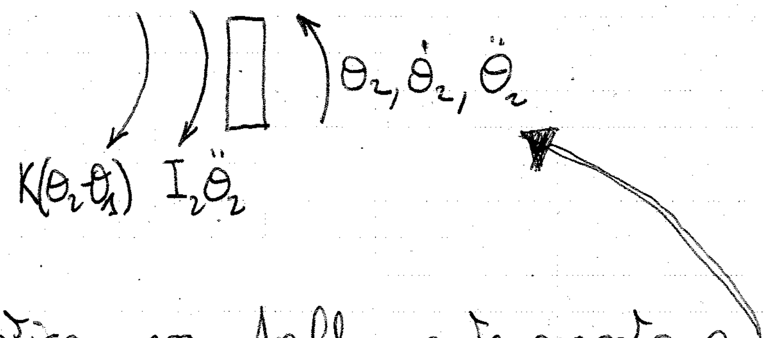
2)



$\omega_n = ?$

$$\begin{cases} I_1 \ddot{\theta}_1 + K(\theta_1 - \theta_2) = 0 & \times I_2 \\ I_2 \ddot{\theta}_2 + K(\theta_2 - \theta_1) = 0 & \times I_1 \end{cases}$$





L'az° elastica va dalla parte opposta a

$$\begin{cases} I_1 I_2 \ddot{\theta}_1 + K I_2 (\theta_1 - \theta_2) = 0 \\ I_1 I_2 \ddot{\theta}_2 + K I_1 (\theta_2 - \theta_1) = 0 \end{cases}$$ eq. del moto

$$I_1 I_2 (\ddot{\theta}_1 - \ddot{\theta}_2) + K(I_1 + I_2)(\theta_1 - \theta_2) = 0$$

$$\theta_1 - \theta_2 = \theta$$

$$(I_1 I_2)\ddot{\theta} + K(I_1 + I_2)\theta = 0$$

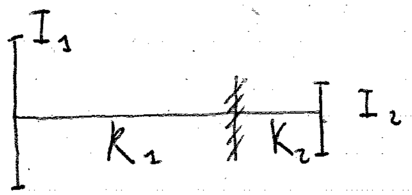
Nn è 1 sist a 2gdl ma a 1gdl (rotaz° relativa dei 2 volani)

$$\omega_n = \sqrt{\frac{I_1 + I_2}{I_1 I_2}}$$

$$I_1 \ddot{\theta}_1 + I_2 \ddot{\theta}_2 = 0$$

le coppie d'I vanno in equilibrio ⇒ i 2 volani vanno in controfase

5



$$K_1 = \frac{G I_p}{l_1}$$

$$K_2 = \frac{G I_p}{l_2}$$

$$K_1 l_1 = K_2 l_2 = K l$$

$$K_1 = K\frac{l}{l_1} = K\frac{I_1+I_2}{I_2} \qquad \omega_{n1} = \sqrt{K\frac{I_1+I_2}{I_1 I_2}}$$

$$K_2 = K\frac{l}{l_2} = K\frac{I_1+I_2}{I_1} \qquad \omega_{n2} = \sqrt{K\frac{I_1+I_2}{I_1 I_2}}$$

Altro modo:

$$\begin{cases} I_1\ddot{\theta}_1 + K(\theta_1-\theta_2)=0 \\ I_2\ddot{\theta}_2 + K(\theta_2-\theta_1)=0 \end{cases}$$

$$\theta_1 = A \sin(\omega_n t + \varphi)$$
$$\theta_2 = B \sin(\omega_n t + \varphi)$$

$$\begin{cases} -A I_1 \omega_n^2 + K(A-B)=0 \\ -I_2 B \omega_n^2 + K(B-A)=0 \end{cases}$$

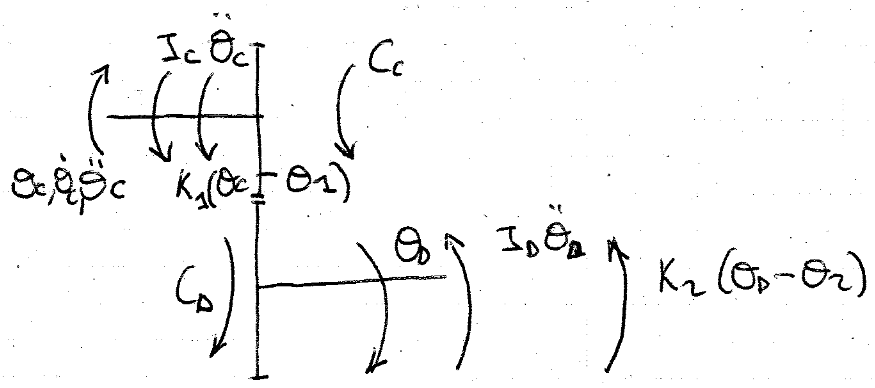
A e B sono le incognite

$$\begin{cases} (K - I_1\omega_n^2)A - KB = 0 \\ -KA + (K - I_2\omega_n^2)B = 0 \end{cases}$$

$$\begin{vmatrix} K - I_1\omega_n^2 & -K \\ -K & K - I_2\omega_n^2 \end{vmatrix} = 0$$

$$\cancel{K^2} - K I_2 \omega_n^2 - K I_1 \omega_n^2 + I_1 I_2 \omega_n^4 - \cancel{K^2} = 0$$

7 $$I_1 I_2 \omega_n^{4\,2} - K(I_1+I_2)\cancel{\omega_n^2} = 0$$



$$I_c\ddot{\theta}_c + K_1(\theta_c - \theta_1) = -C_c$$

$$I_D\ddot{\theta}_D + K_2(\theta_D - \theta_2) = C_D$$

Il rendimento di 1 trasmissione è unitario

$$\eta = 1 = \frac{C_D\theta_D}{C_c\theta_c} = 1 \Rightarrow C_c = C_D\tau$$

$$I_c\ddot{\theta}_c + K_1(\theta_c - \theta_1) + \tau I_D\ddot{\theta}_D + K_2\tau(\theta_D - \theta_2) = 0$$

$$\theta_D = \tau\theta_c$$

$$I_c\ddot{\theta}_c + K_1(\theta_c - \theta_1) + \tau^2(I_D\ddot{\theta}_c) + K_2\tau(\tau\theta_c - \theta_2) = 0$$

$$(I_c + \tau^2 I_D)\ddot{\theta}_c + K_1(\theta_c - \theta_1) + K_2\tau(\tau\theta_c - \theta_2) = 0$$

3 eq del moto, 3 gdl, 3 pulsaz° proprie:

$$\begin{cases} I_1\ddot{\theta}_1 + K_1(\theta_1 - \theta_c) = 0 \\ I_2\ddot{\theta}_2 + K_2(\theta_2 - \theta_D) = 0 \\ (I_c + \tau^2 I_D)\ddot{\theta}_c + K_1(\theta_c - \theta_1) + K_2\tau(\tau\theta_c - \theta_2) = 0 \end{cases}$$

$$I_c = I_D = 0$$

$$\tau = \frac{\dot{\theta}_D}{\dot{\theta}_c} = \frac{\theta_c}{\theta_D}$$

$$\theta_1 = A_1 \sin(\omega_n t + \varphi)$$

$$\theta_2 = A_2 \sin(\omega_n t + \varphi)$$

$$\theta_c = A_c \sin(\omega_n t + \varphi)$$

Derivando le ip di risoluz° e sostituendo si ottiene:

9

$I_e = I_C + \tau^2 I_D$

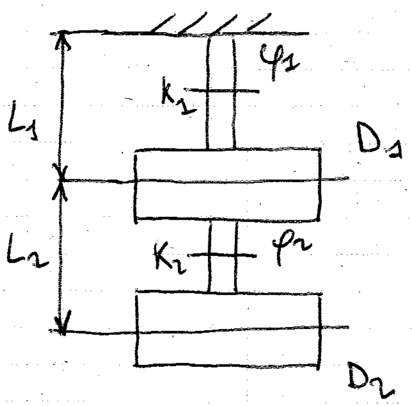
$A\omega^6 + B\omega^4 + C\omega^2 = 0$

eq cn ω di 6° grado

$\left.\begin{array}{l} \omega_{n1}^2 = 0 \\ \omega_{n2}^2 \neq 0 \\ \omega_{n3}^2 \neq 0 \end{array}\right\}$ 3 pulsaz° proprie, ma 1 = 0 ⇒ 2 pulsaz° proprie

Qnd c'è 1 fenomeno inerziale nella trasmissione del moto i mmt d'I nn sn trascurabili

Ex2:



$m_1 = 20 kg$

$m_2 = 10 kg$

$\varphi_1 = 40 mm$

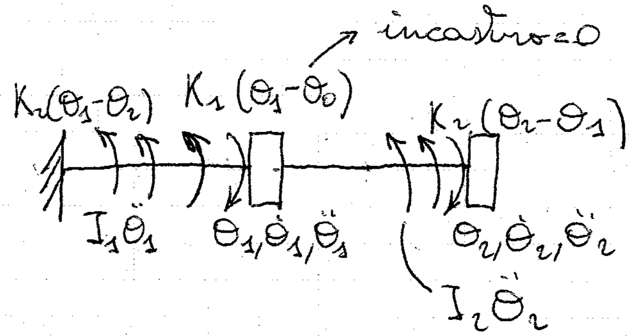
$\varphi_2 = 20 mm$

$L_1 = 100 mm$

$L_2 = 25 mm$

$D_1 = D_2 = 300 mm$

$G = 8 \cdot 10^{10} \frac{N}{m^3}$



I_1, I_2 $\quad I_1 = m_1 \frac{D_1^2}{8} = 0.225 \, kg/m^2$

$I_2 = m_2 \frac{D_2^2}{8} = 0{,}112 \, kg/m^2$

$$\omega_n = \omega_{n1} \rightarrow \begin{cases} 1.78 \cdot 10^5 A_1 - 5 \cdot 10^4 A_2 = 0 \\ -5 \cdot 10^4 A_1 + 1.411 \cdot 10^4 \cdot A_2 = 0 \end{cases}$$

$$\Rightarrow \left(\frac{A_1}{A_2}\right)^{I} = 3.57$$

↳ secondo il 1° modo di vibrare

$$-2.94 \cdot 10^4 \cdot A_1 - 5 \cdot 10^4 \cdot A_2 = 0$$

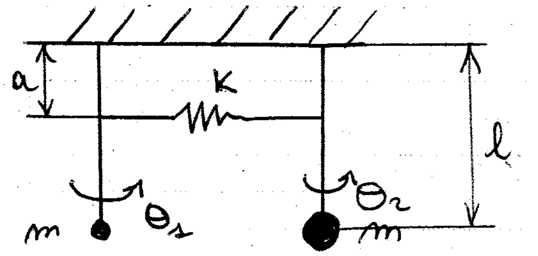
$$\left(\frac{A_2}{A_1}\right)^{II} = -0.561$$

$$\begin{Bmatrix} \theta_1(t) \\ \theta_2(t) \end{Bmatrix} = \begin{Bmatrix} A_1 \\ 3.57 A_2 \end{Bmatrix} \cos(566.7t + \varphi)$$

$$\begin{Bmatrix} \theta_1(t) \\ \theta_2(t) \end{Bmatrix} = \begin{Bmatrix} A_1 \\ -0.561 A_2 \end{Bmatrix} \cos(114t + \varphi)$$

26/10/12

1)



$a < \frac{l}{2}$

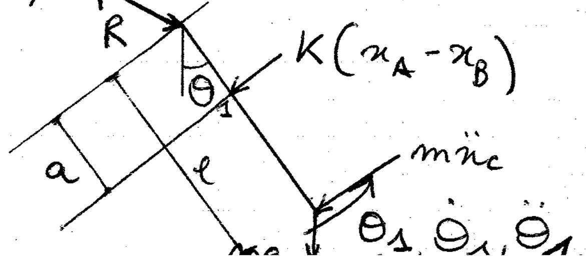
$\theta_1(0) = \theta_0$

$\dot{\theta}_1(0) = 0$

$\theta_2(0) = 0$

$\dot{\theta}_2(0) = 0$

2gdl, fenomeni inerziali ed elastici



$x_A = a\theta_1$

$x_B = a\theta_2$

$x_c = l\theta_1$

(13)

$\theta_{10} = \theta_{20}$

$\{\psi\}_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

$\omega_n = \omega_{n2}$

$\left(\cancel{mgl} + Ka^2 - \cancel{ml^2}\frac{2Ka^2}{\cancel{ml^2}} - \cancel{ml^2\frac{g}{l}}\right)\theta_{10} - Ka^2\theta_{20} = 0$

$-Ka^2\theta_{10} = Ka^2\theta_{20}$

$\theta_{10} = -\theta_{20}$

$\{\psi\}_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$

Si è identificata la matrice dei modi:

$[\psi] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

X passare dalle coordinate fisiche a qlle principali

$[\psi]^T[m][\psi]\{\ddot{\eta}\} + [\psi]^T[K][\psi]\{\eta\} = 0$

$m\ddot{x} + \beta\dot{x} + kx = 0$

$\eta_1 = A_1 \cos\omega_1 t + B_1 \sin\omega_1 t$

$\eta_2 = A_2 \cos\omega_2 t + B_2 \sin\omega_2 t$

$$A_r = \frac{\{\psi_r\}^T[m]\{\theta_0\}}{m_r}$$

$$B_r = \frac{\{\psi_r\}^T[m]\{\dot{\theta}_0\}}{\omega_r m_r}$$

ψ_r se $x = 1 \Rightarrow = 1$

$m_r = \{\psi_r\}^T[m]\{\psi_r\}$

$m_1 = \{1 \quad 1\}\begin{bmatrix} ml^2 & 0 \\ 0 & ml^2 \end{bmatrix}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} =$

15

$$\frac{\omega_1+\omega_2}{2} = \bar{\omega}$$

$$\frac{\omega_2-\omega_1}{2} = \varepsilon$$

eq. matematiche di 1 battimento

$$\begin{cases} \theta_1 = \theta_0 \cos \varepsilon t \cos \bar{\omega} t \\ \theta_2 = \theta_0 \sin \varepsilon t \sin \bar{\omega} t \end{cases}$$

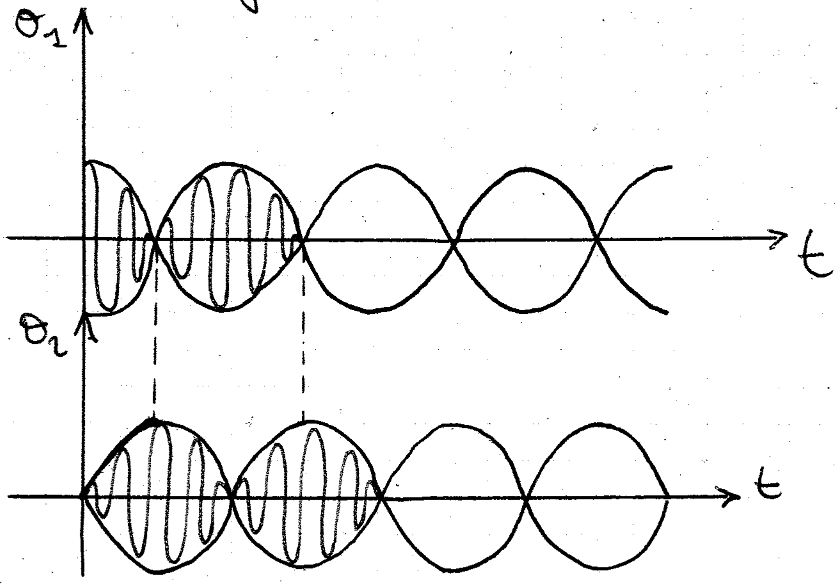
evoluz° nel tempo delle 2 barrette che sn sfasate di 90°

$\omega_2 \sim \omega_1$ se K nn è tanto grande.

$$\omega_1 = \sqrt{\frac{g}{l}}$$

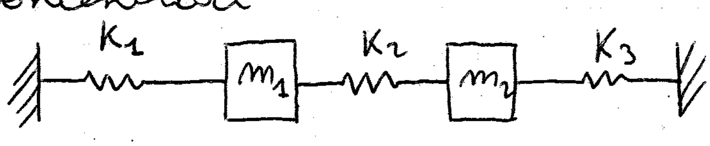
$$\omega_2 = \sqrt{\frac{2ka^2}{ml^2} + \frac{g}{l}}$$

+ K è grande + $\omega_2 \neq \omega_1$



02/11

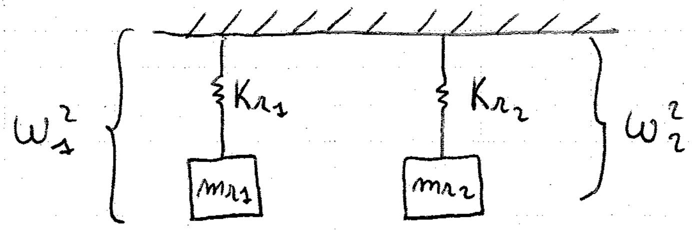
ex: 1) Dinamica dei sist lineari a parametri concentrati



$$[\psi] = \begin{bmatrix} 1 & 1 \\ 1 & -1/2 \end{bmatrix}$$ matrice modale

$$\begin{array}{cc} \uparrow & \uparrow \\ \psi_1 & \psi_2 \end{array}$$

Si ricorda che $\{x(t)\} = [\psi]\{\eta(t)\}$



$$[\psi]^T[m][\psi]\{\ddot{\eta}\} + [\psi]^T[k][\psi]\{\eta\} = \{0\}$$

$$[\,{}^{\ddots}m_r{}_{\ddots}\,]\{\ddot{\eta}\} + [\,{}^{\ddots}k_r{}_{\ddots}\,]\{\eta\} = \{0\}$$

$$m_r = \begin{bmatrix} 1 & 1 \\ 1 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 15 & 0 \\ 0 & 15/2 \end{bmatrix} \text{diag}(m_r)$$

$$k_r = \begin{bmatrix} 1 & 1 \\ 1 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & \frac{15}{2} \end{bmatrix} \text{diag}(K_r)$$

$$\omega_{r1}^2 = \frac{K_{r1}}{m_{r1}} = \frac{2}{5} (rad/s)^2$$

$$\omega_{r2}^2 = \frac{K_{r2}}{m_{r2}} = 1 \, (rad/s)^2$$

$$\begin{cases} 15\,\ddot{\eta}_1 + 6\,\eta_1 = 0 \\ \ddot{\eta}_2 + \eta_2 = 0 \end{cases}$$ xché il sist non è forzato sono le 2 eq modali

$$\eta(t) = A_r \cos \omega_r t + B_r \sin \omega_r t$$

Bisogna capire le cond iniziali del problema

$$\{x_0\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \quad \{\dot{x}_0\} = \{v_0\} = \begin{Bmatrix} 2 \\ 2 \end{Bmatrix}$$

di vibraz°

$$\begin{Bmatrix} x_{10} \\ x_{20} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

$$\{v_0\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$A_r = \frac{\{\psi_r\}^T [m] \{x_0\}}{m_r}$$

$$\psi_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} ; \psi_2 \begin{Bmatrix} 1 \\ -\frac{1}{2} \end{Bmatrix}$$

$$B_r = \frac{\{\psi_r\}^T [m] \{v_0\}}{\omega_r \, m_r}$$

$$A_1 = \frac{\{1 \quad 1\} \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}}{15} =$$

$$= \frac{\{1 \, 1\} \begin{Bmatrix} 5 \\ 0 \end{Bmatrix}}{15} = \frac{5}{15} = \frac{1}{3}$$

$$A_2 = \frac{\{1 \quad -\frac{1}{2}\} \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}}{\frac{15}{2}}$$

$$= 5 \cdot \frac{2}{15} = \frac{2}{3}$$

$$B_1 = \frac{\{1 \, 1\} \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}}{\underbrace{\sqrt{\frac{2}{5}}}_{\omega_1} \cdot \underbrace{15}_{m_{r_1}}} = 0$$

$$B_2 = 0$$

$$\eta_1 = \frac{1}{3} \cos \omega_1 t$$

$$\eta_2 = \frac{2}{3} \cos \omega_2 t$$

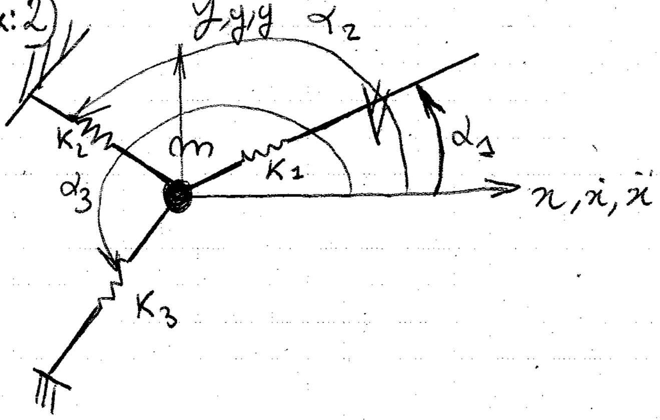
$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = [\psi] \begin{Bmatrix} \eta_1 \\ \eta_2 \end{Bmatrix} = \begin{Bmatrix} 1/3 \cos \omega_1 t + \frac{2}{3} \cos \omega_2 t \\ 1/3 \cos \omega_1 t - \frac{1}{3} \cos \omega_2 t \end{Bmatrix}$$

Supponiamo che al sistema sia applicata 1 forza F (forzante) $\Rightarrow$ $F = 15N$

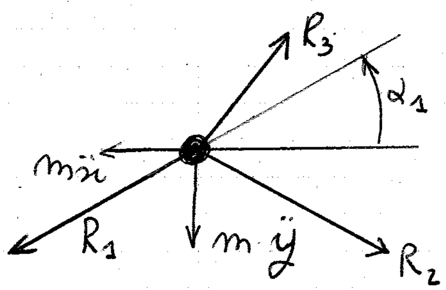
(11)

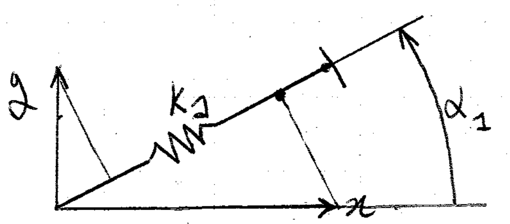
$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -\frac{1}{2} \end{bmatrix} \begin{Bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}$$

Ex: 2)



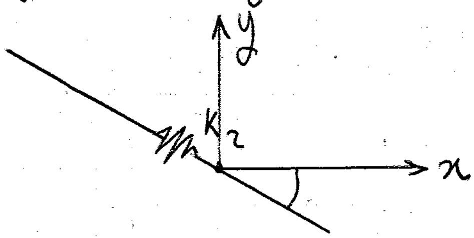
Sist a 2gdl x e y.





$R_1 = K_1 (+x \cos\alpha_1 + y \sin\alpha_1)$

$R_2 = K_2 (x \cos\alpha_2 + y \sin\alpha_2)$ xché $\alpha_2 > 90°$



$m\ddot{x} + (K_1 \cos\alpha_1 + K_2 \cos\alpha_2 + \dots) x = 0$

$m\ddot{y} + (K_1 \sin\alpha_1 + \dots \qquad) y = 0$

cos

(22)

$$\begin{bmatrix} K_1+K_2-m\omega_n^2 & K_2b-K_1a \\ K_2b-K_1a & K_2b^2+K_1a^2-I_G\omega_n^2 \end{bmatrix} \begin{Bmatrix} x_0 \\ \theta_0 \end{Bmatrix} = 0$$

$\omega_{1,2}^2 = 30 \pm \sqrt{900-490} \Rightarrow \omega_1^2 = 9.57 \ (rad/s)^2$

$\omega_2^2 = 50.25 \ (rad/s)^2$

$\omega_{n1} = 3.12 \ rad/s \Rightarrow \sim 0.5 Hz = \frac{3.12}{2\pi}$

$\omega_{n2} = 7.09 \ rad/s \Rightarrow \sim 1.13 Hz = \frac{7.09}{2\pi}$

Se $\omega_n = \omega_{n1}$

$(K_1+K_2-m\omega_{n1}^2)x_0 + (K_2b-K_1a)\theta_0 = 0$

$\left(\frac{\theta_0}{x_0}\right)^1 = \frac{K_1+K_2-m\omega_{n1}^2}{K_1a-K_2b} = -5.16 \ rad/sm$

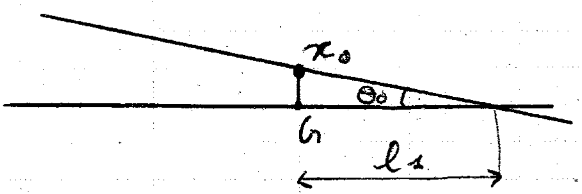
↳ secondo il 1° modo di vibrare

$\omega_n = \omega_{n2}$

$\left(\frac{\theta_0}{x_0}\right)^2 = 0.0194 \ rad/m$

↳ 2° modo di vibrare

1° modo



posiz° indeformata (linea che passa x il bar della macchina)

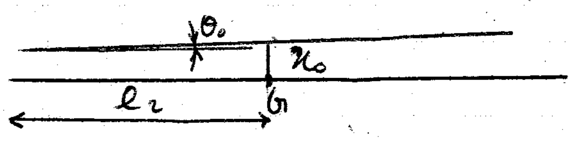
$\frac{\theta_0}{x_0} = -5.16 \ rad/m$ con $x_0 = 0.1 m$

$\theta = -29,5°$

$l_1 = \frac{x_0}{tg\,\theta_0} = 0.18 m$

2° modo

$\frac{\theta_0}{x_0} = 0.019 \ rad/m \quad x_0 = 0.1 m \quad \theta_0 = 0.1°$



(25)

$k_1 = 1.2 \cdot 10 \; N/m$ $\qquad$ $a = 1.3\, m$

$k_2 = 1.3 \cdot 10^4 \; N/m$ $\qquad$ $b = 1,5\, m$

$m = 1000\, kg$

$$[m]\{\ddot{x}\} + [k]\{x\} = \{f\}$$

se $\{f\} = \text{cte}$ risp a t si cerca 1 sol del tipo

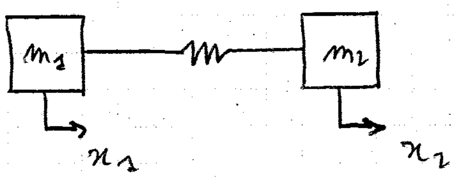
$\{x\} = \{x_0\} = \text{cte}$

$\Rightarrow \{\ddot{x}\} = \{0\} \Rightarrow [k]\{x_0\} = \{f\}$

$\{x_0\} = [k]^{-1}\{f\}$

$= [\alpha]\{f\}$

matrice di flessibilità



$x = x_G$

$\sin\theta \cong \theta$

$\cos\theta \cong 1$

$$\Rightarrow \begin{cases} x_1 = x - \theta a \\ x_2 = x + \theta b \end{cases}$$

$$\begin{cases} \dot{x}_1 = \dot{x} - \dot{\theta} a \\ \dot{x}_2 = \dot{x} + \dot{\theta} b \end{cases}$$

E' 1 sist lineare

$$\begin{aligned} &+\downarrow : \; \begin{cases} m\ddot{x} + K_1 x_1 + c_1\dot{x}_1 + K_2 x_2 + c_2 \dot{x}_2 = 0 \\ I_G\ddot{\theta} + (-K_1 x_1 - C_1\dot{x}_1)a + (K_2 x_2 + c_2\dot{x}_2) b = 0 \end{cases} \\ &\curvearrowleft G : \end{aligned}$$

$$\{x\} = \begin{Bmatrix} x \\ \theta \end{Bmatrix}$$

1) $m\ddot{x} + K_1(x - a\theta) + c_1(\dot{x} - a\dot{\theta}) + K_2(x + b\theta) + c_2(\dot{x} + b\dot{\theta}) = 0$

2) $I_G\ddot{\theta} - k_1 a(x - a\theta) - c_1 a(\dot{x} - a\dot{\theta}) + K_2 b(x + b\theta) + c_2 b(\dot{x} + b\dot{\theta}) = 0$

1) $m\ddot{x} + 0\ddot{\theta} + \dot{x}(c_1 + c_2) + \dot{\theta}(-a c_1 + b c_2) + x(K_1 + K_2) + \theta(-aK_1 + b K_2) = 0$

2) $0\ddot{x} + I_G\ddot{\theta} + \dot{x}(-a c_1 + b c_2) + \dot{\theta}(+a^2 c_1 + b^2 c_2) + x(-a K_1 + b K_2) + \theta(a^2 K_1 + b^2 K_2) = 0$

$$\begin{bmatrix} m & 0 \\ 0 & I_G \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} (c_1 + c_2) & (-a c_1 + b c_2) \\ (-a c_1 + b c_2) & (a^2 c_1 + b^2 c_2) \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} K_1 + K_2 & -aK_1 + bK_2 \\ -aK_1 + bK_2 & a^2 K_1 + b^2 K_2 \end{bmatrix}\begin{Bmatrix} x \\ \theta \end{Bmatrix} = \{0\}$$

$$\left[[D]-\omega^2[I]\right]\{x_0\}=\{0\}$$

$[C]=\alpha[m]+\beta[k]$ smorzamento viscoso proporzionale

$$\{\psi\}^T[C]\{\psi\}_p \begin{cases} \alpha m_p+\beta K_p & \text{se } l=p \\ 0 & \text{se } l\neq p \end{cases}$$

$$\{x\}=\{\psi\}\{\eta\}$$

$$[\psi]=\left[\{\psi_1\}\ \{\psi_2\}\cdots\{\psi_n\}\right]\in\mathbb{C}^{N,N}$$

$$\{\psi\}_1=\begin{Bmatrix}0.9998\\0.0194\end{Bmatrix}=\begin{Bmatrix}1\\0.0194\end{Bmatrix} \quad \text{se } x_0=1$$

$$\{\psi\}_2=\begin{Bmatrix}-0.1903\\0.9817\end{Bmatrix}=\begin{Bmatrix}-0.194\\1\end{Bmatrix} \quad \text{se } \theta_0=1$$

$\omega_1=5.0076$ rad/s $\Rightarrow f_1=0.7970$ Hz

$\omega_2=2.2085$ rad/s $\quad f_2=0.3515$ Hz

$$[\psi]^T[m][\psi]=\begin{bmatrix}m_1 & & 0\\ & m_2 & \\ 0 & \ddots & m_n\end{bmatrix}$$

$$[\psi]^T[K][\psi]=\begin{bmatrix}K_1 & & 0\\ & K_2 & \\ 0 & \ddots & K_n\end{bmatrix}$$

$$\frac{K_p}{m_p}=\omega_p^2$$

$\tilde{c}_1=1.76\cdot10^3$

$\tilde{c}_2=3.303\cdot10^3$

$$[\psi]^T[c][\psi]=\begin{bmatrix}c_1 & & 0\\ & c_2 & \\ 0 & \ddots & c_n\end{bmatrix}$$

$$[\psi]^T[m][\psi]\{\ddot\eta\}+[\psi]^T[c][\psi]\{\dot\eta\}+[\psi]^T[k][\psi]\{\eta\}=\{0\}$$

19) [illegible]

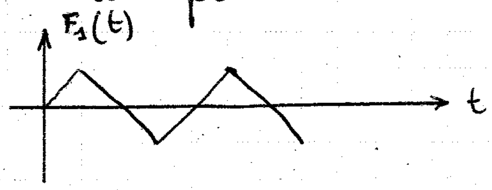
se $\{F_0\} = \begin{Bmatrix} F_{01} \\ F_{02} \\ \emptyset \end{Bmatrix}$

$\{F(t)\} = \begin{Bmatrix} F_{01} \sin\omega_1 t \\ F_{02} \sin\omega_2 t \\ 0 \end{Bmatrix} \rightarrow$ 2 casi : $F(t) = \begin{Bmatrix} F_{01} \\ 0 \end{Bmatrix} e^{j\omega_1 t} \Rightarrow \{x(t)\}_1$

$F(t) = \begin{Bmatrix} 0 \\ F_{02} \\ \dots \end{Bmatrix} e^{j\omega_2 t} \Rightarrow \{x(t)\}_2$

$\{x(t)\}_1$, $\{x(t)\}_2$ $\Rightarrow \{x(t)\} = \underbrace{\{x(t)\}_1}_{\Downarrow\ \omega_1} + \underbrace{\{x(t)\}_2}_{\Downarrow\ \omega_2}$

Se la forsante è 1 onda triangolare:



Si analissa la risp sulle 1e armoniche

ω = pulsaz° forsante

$\{x(t)\} = \{x_0\} e^{j\omega t}$

$\{\dot{x}\} = j\omega \{x_0\} e^{j\omega t}$

$\{\ddot{x}\} = -\omega^2 \{x_0\} e^{j\omega t}$

$-\omega^2 [m]\{x_0\} \cancel{e^{j\omega t}} + j\omega [c]\{x_0\} \cancel{e^{j\omega t}} + [k]\{x_0\} \cancel{e^{j\omega t}} = \{F_0\} \cancel{e^{j\omega t}}$

$-\omega^2 [m]\{x_0\} + j\omega [c]\{x_0\} + [k]\{x_0\} = \{F_0\}$

$(-\omega^2 [m] + j\omega [c] + [k])\{x_0\} = \{F_0\}$

$[Z(\omega)]\{x_0\} = F_0 \Rightarrow \{x_0\} = [Z(\omega)]^{-1}\{F_0\}$, altro modo!

$[m]\{\ddot{x}\} + [k]\{x\} = \{0\}$

$\{x\} = \{x_0\} f(t) = \{x_0\} \cos\omega t$

$\omega = \omega_r \qquad V = 1 \dots n$

$\{x_0\} = \{\psi\}_v \qquad V = 1 \dots n$

(R1)

$$\lfloor c \rfloor = \alpha \lfloor m \rfloor + \beta \lfloor k \rfloor$$

$$\Rightarrow c_v = \alpha m_x + \beta k_x$$

$$m_p \ddot{\eta}_p + c_p \dot{\eta}_p + k_p \eta_p = \underbrace{\{\psi\}_p^T \{F_0\}}_{} e^{j\omega t} \qquad p = 1 \ldots N$$

$F_{0p} \rightarrow$ forza modale che va ad agire sul sist in coordinate modali nella p^{esima} riga delle coordinate modali

$$\eta_p(t) = \eta_{op}\, e^{j\omega t}$$

$$-\omega^2 m_p \eta_{op}\, e^{j\omega t} + j\omega c_p\, \eta_{op}\, e^{j\omega t} + k_p \eta_{op}\, e^{j\omega t} = \{\psi\}_p^T \{F_0\} e^{j\omega t}$$

$$(-\omega^2 m_p + j\omega c_p + k_p)\, \eta_{op} = \{\psi\}_p^T \{F_0\}$$

$$\eta_{op} = \frac{\{\psi\}_p^T \{F_0\}}{-\omega^2 m_p + k_p + j\omega c_p} \qquad p = 1..N$$

$$\eta_p(t) = \eta_{op}\, e^{j\omega t}$$

$$\{x\} = [\psi]\{\eta\} = \sum_{p=1}^{N} \{\psi\}_p\, \eta_p(t)$$

$$\{x\} = \underbrace{\left(\sum_{p=1}^{N} \frac{\{\psi\}_p \{\psi_p\}^T \{F_0\}}{-\omega^2 m_p + k_p + j\omega c_p} \right)}_{\text{vettore delle ampiezze di risposta}} e^{j\omega t}$$

$$\{x\} = \{x_0\}\, e^{j\omega t}$$

$$\{x_0\} = \sum_{p=1}^{N} \frac{\{\psi\}_p \{\psi\}_p^T \{F_0\}}{-\omega^2 m_p + k_p + j\omega c_p}$$

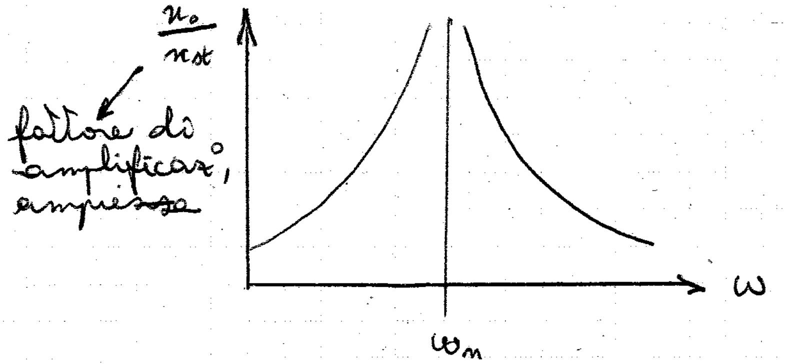
$$\{F_0\} = \begin{Bmatrix} 0 \\ 0 \\ F_{0j} \\ 0 \\ 0 \end{Bmatrix} \Leftarrow \text{componente } j\text{-esima} \neq 0$$

$$\{x_0\} = \sum_{p=1}^{N} \{\psi\}_p \frac{\{\psi\}_p^T \begin{Bmatrix} 0 \\ 0 \\ F_{0j} \\ 0 \\ 0 \end{Bmatrix}}{-\omega^2 m_p + k_p + j\omega c_p}$$

$$\{x_0\} = \sum_{p=1}^{N} \{\psi\}_p \frac{\psi_{jp} F_{0j}}{-\omega^2 m_p + k_p + j\omega c_p}$$

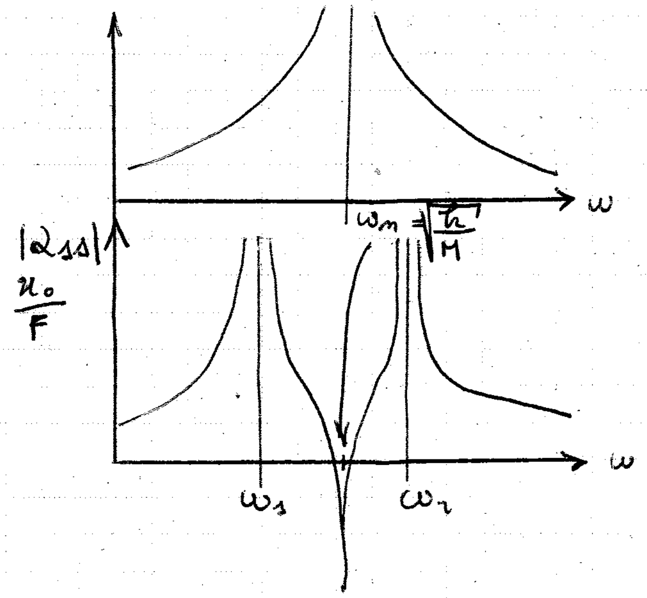
$$x_{0i} = \sum_{p=1}^{N} \psi_{ip} \frac{\psi_{jp} F_{0j}}{-\omega^2 m_p + k_p + j\omega c_p}$$

(33)



Specifica: $x_0(\omega = \omega_0) < x_{0\,max}$

Si introduce ...



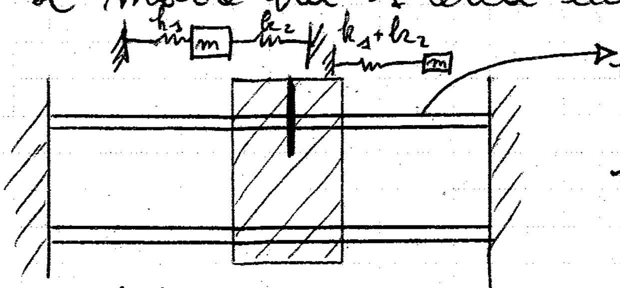
sode
↳ k, M

2 gdl

$\alpha_{11}(\omega) = \frac{x_{01}}{f_{01}} = \frac{x_0}{F_0}$: recettanza

$\alpha_{11}(\omega = \omega_1) = 0$ 1 gdl

Il motore ha 1 certa eccentricità



$k_{ass} = \frac{EA}{(L/2)}$ di mezza trave

$k_{orizzontale} = 4\, k_{ass}$

$E = 2.1 \cdot 10^{11}\ N/m^2$

$L = 3\,m$

$\delta_{statico\ max} = 1\ mm$

$$I_x = \frac{F}{\delta} \cdot \frac{L^3}{192 E} = 6.5 \cdot 10^{-7}\, m^4 = \frac{11 g L^3}{\delta E 192} = \frac{100 \cdot 9.81 \cdot 5^3}{10^{-3} \cdot 2.1 \cdot 10^{11} \cdot 192} = 6.5 \cdot 10^{-7}\, m^4$$

$$= 6.5 \cdot 10^{-7} \cdot (10^2)^4\, cm^4$$

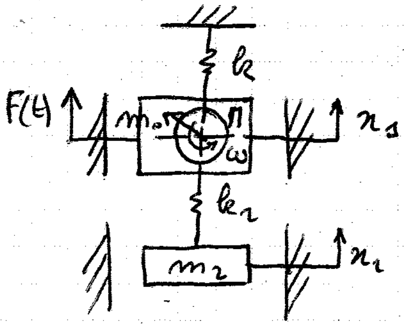
$$= 65\, cm^4$$

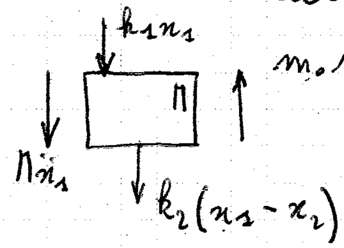
Le 2 travi lavorano insieme, le molle sn messe in //

$$I_{x_1} = \frac{I_x}{2} = 32.5\, cm^4$$

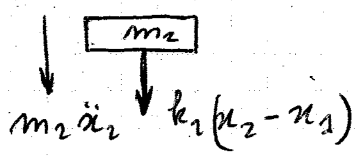
La scelta è il profilato 80×20, $s = 3$, $I_x = 30.10\, cm^4$

$F(t) = m_0 r \cdot \omega^2 \sin \omega t$ è la forzante che va a lavorare sul sist





$m_0 r \omega^2 \sin \omega t$



Eq del moto:

$$\begin{cases} M\ddot{x}_1 + k_1 x_1 + k_2(x_1 - x_2) = m_0 r \omega^2 \sin \omega t \\ m_2 \ddot{x}_2 + k_2(x_2 - x_1) = 0 \end{cases}$$

$$\{x\} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

$$\begin{bmatrix} M & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} m_0 r \omega^2 \sin \omega t \\ 0 \end{Bmatrix}$$

X rimuovere il 2° gdl:

Le $k_2 = 0$; $m_2 = 0$

$$M \ddot{x}_1 + k_1 x_1 = m_0 r \omega^2 \sin \omega t$$

$$\omega_0 = \sqrt{\frac{k_1}{M}} = \sqrt{\frac{2 \cdot 5.6896 \cdot 10^5}{100}} = 106.67\, rad/s$$

(37) $k_{fl} = 192 \frac{EI}{L^3} = 5.6896 \cdot 10^5 \frac{N}{m}$

b) $b = \dfrac{m_0 r k_2 \omega^2}{(-\omega^2 M + k_1 + k_2)(-\omega^2 m_2 + k_2) - k_2^2}$

$$\omega^2 = \frac{k_2}{m_2} = \frac{k_1}{M}$$

$$b = \frac{m_0 r k_2 \frac{k_2}{m_2}}{\left(-\frac{k_2}{m_2} M + k_1 + k_2\right)\left(-\frac{k_2}{\cancel{m_2}} \cancel{m_2} + k_2\right) - k_2^2}$$

$$= -\frac{m_0 r \frac{k_2^2}{m_2}}{+k_2^2}$$

$$= -m_0 \frac{r}{m_2}$$

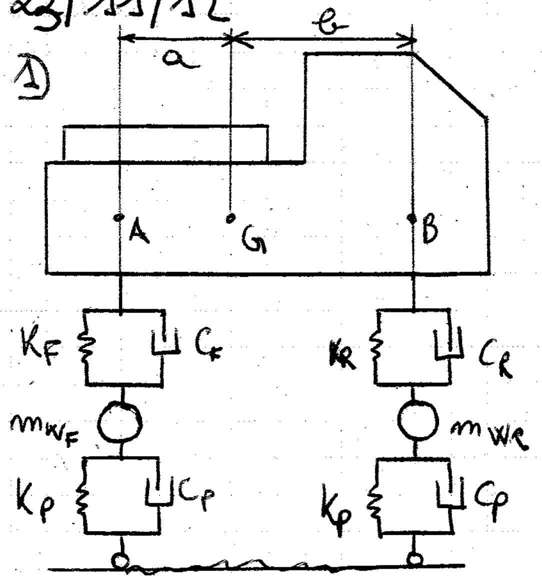
$$b \leq x_{02} = 10^{-2} m$$

$$b = |x_{02}| \Rightarrow m_2 = \frac{m_0 r}{|x_{02}|} = 0.5 \text{ kg}$$

$$k_2 = m_2 \frac{k_1}{M} = 5689 \frac{N}{m}$$

22/11/12

1)



m

J

K_F C_F

K_R C_R

m_{WF}

m_{WR}

Modello semplificato:

(29)

$$L = \frac{1}{2} m \dot{x}_G^2 + \frac{1}{2} J \dot{\theta}^2 + \frac{1}{2} m_{WF} \dot{x}_{WF}^2 + \frac{1}{2} m_{WR} \dot{x}_{WR}^2 - \frac{1}{2} K_F (x_G + - a\theta - x_{WF})^2 - \frac{1}{2} K_R (x_G + b\theta - x_{WR})^2 - \frac{1}{2} K_P x_{WR}^2 - \frac{1}{2} K_P x_{WF}^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_G}\right) = m \ddot{x}_G$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = J \ddot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_{WF}}\right) = m_{WF} \ddot{x}_{WF}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_{WR}}\right) = m_{WR} \ddot{x}_{WR}$$

$$\frac{\partial L}{\partial x_G} = - K_F (x_G - a\theta - x_{WF}) - K_R (x_G + b\theta - x_{WR})$$

$$\frac{\partial L}{\partial \theta} = a K_F (x_G - a\theta - x_{WF}) - b K_R (x_G + b\theta - x_{WR})$$

$$\frac{\partial L}{\partial x_{WF}} = \frac{1}{\cancel{2}} \cancel{2} K_F (x_G - a\theta - x_{WF}) - K_P x_{WF}$$

$$\frac{\partial L}{\partial x_{WR}} = K_R (x_G + b\theta - x_{WR}) - K_P x_{WR}$$

$$Q_K = - \frac{\partial D}{\partial \dot{q}_K}$$

$$D = \frac{1}{2} C_F (\dot{x}_A - \dot{x}_{WF})^2 + \frac{1}{2} C_R (\dot{x}_B - \dot{x}_{WR})^2 + \frac{1}{2} C_P (\dot{x}_{WF})^2 + \frac{1}{2} C_P \dot{x}_{WR}^2$$

$$-Q_{x_G} = \frac{\partial D}{\partial \dot{x}_G} = C_F (\dot{x}_G - a\dot{\theta} - \dot{x}_{WF}) + C_R (\dot{x}_G + b\dot{\theta} - \dot{x}_{WR})$$

$$-Q_\theta = \frac{\partial D}{\partial \dot{\theta}} = - C_F \cdot a (\dot{x}_G - a\dot{\theta} - \dot{x}_{WF}) + b C_R (\dot{x}_G + b\dot{\theta} - \dot{x}_{WR})$$

$$-Q_{WF} = \frac{\partial D}{\partial \dot{x}_{WF}} = - C_F (\dot{x}_G - a\dot{\theta} - \dot{x}_{WF}) + C_P \dot{x}_{WF}$$

$$-Q_{WR} = \frac{\partial D}{\partial \dot{x}_{WR}} = - C_R (\dot{x}_G + b\dot{\theta} - \dot{x}_{WR}) + C_P \dot{x}_{WR}$$

Si ottiene:

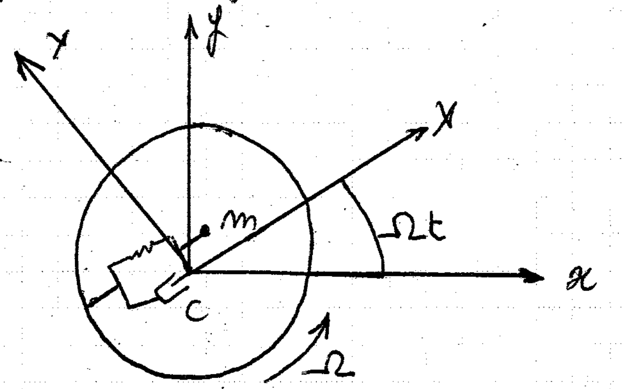
(1) $m \ddot{x}_G + K_F (x_G - a\theta - x_{WF}) + K_R (x_G + b\theta - x_{WR}) + C_F (\dot{x}_G - a\dot{\theta} - \dot{x}_{WF}) + C_R (\dot{x}_G + b\dot{\theta} - \dot{x}_{WR}) = 0$

(1.1)

$$Q_{x_G} = -c_F\,(x_G - a\theta - x_{wF}) - c_R\,(x_G + b\theta - x_{wR})$$

La f° di Railay vale sl x le forze dissipative

2)



$h \rightarrow \begin{matrix} x \\ y \end{matrix}$

$$\left.\begin{aligned} F_x &= -K_x\,(x + \varepsilon_x x^3) \\ F_y &= -K_y\,(y + \varepsilon_y y^3) \end{aligned}\right\}$$ 2 componenti della forza che agiscono sulla massa m a causa della forza di vincolo della molla in la massa m risp al sist di riferimento xy

$$\vec{r} = x\vec{i} + y\vec{j}$$

$$\vec{\omega} = \Omega\,\vec{K}$$

$$\vec{V} = \dot{\vec{r}}_{rel} + \vec{\omega} \wedge \vec{r} = \dot{x}\vec{i} + \dot{y}\vec{j} + \Omega\vec{K} \wedge (x\vec{i} + y\vec{j})$$

$$= \dot{x}\vec{i} + \dot{y}\vec{j} + \Omega x\vec{j} - \Omega y\vec{i}$$

$$= (\dot{x} - \Omega y)\vec{i} + (\dot{y} + \Omega x)\vec{j}$$

$$T = \frac{1}{2}m\left[(\dot{x} - \Omega y)^2 + (\dot{y} + \Omega x)^2\right]$$

$$= \frac{1}{2}m\left[\dot{x}^2 - 2\dot{x}\,\Omega y + \Omega^2 y^2 + \dot{y}^2 + 2\Omega x\dot{y} + \Omega^2 x^2\right]$$

$$= \frac{1}{2}m\left[(\dot{x}^2 + \dot{y}^2) + 2\Omega(\dot{y}x - \dot{x}y) + \Omega^2(x^2 + y^2)\right]$$

$$U = \int_0^x F_x(\xi)\,d\xi + \int_0^y F_y(\eta)\,d\eta$$

$$= \int_0^x K_x(\xi + \varepsilon_x \xi^3)\,d\xi + \int_0^y K_y\,(\eta + \varepsilon_y \eta^3)\,d\eta$$

$$= K_x\left[\frac{\xi^2}{2} + \varepsilon_x \frac{\xi^4}{4}\right]_0^x + K_y\left[\frac{\eta^2}{2} + \frac{\varepsilon_y}{4}\eta^4\right]_0^y$$

(42)

$$\frac{\partial L}{\partial \dot{x}} = \frac{1}{2} m \left[2\dot{x} - 2y\Omega\dot{x} \right]$$

COMPITO:

- Sist di 2gdl ω_n + forme modali
- " mecc applicando Lagrange ricavare eq del moto
- Dinamica dei sist continui
- + Teoria

$$\frac{\partial L}{\partial x} = \frac{1}{2} m \left[2x\Omega^2 \right] - \frac{1}{2}\left[2K_x x + 2x^3 \varepsilon_x K_x \right]$$

$$m\ddot{x} + (c+h)\dot{x} - 2m\Omega\dot{y} + (K_x - m\Omega^2)x - h\Omega y + K_x \varepsilon_x x^3 = 0$$

$$m\ddot{y} + 2m\Omega\dot{x} + h\dot{y} + h\Omega x + (K_y - m\Omega^2)y + K_y \varepsilon_y y^3 = 0$$

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} c+h & -2m\Omega \\ 2m\Omega & h \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} K_x - m\Omega^2 & -h\Omega \\ h\Omega & K_y - m\Omega^2 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = 0$$

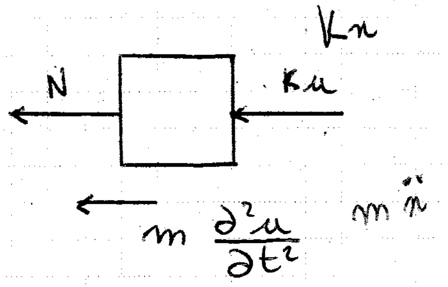
Se $\Omega = 0$ le 2 eq sarebbero accoppiate

$$\begin{bmatrix} 0 & -2m\Omega \\ 2m\Omega & 0 \end{bmatrix}$$ matrice giroscopica

$$\begin{bmatrix} 0 & -h\Omega \\ h\Omega & 0 \end{bmatrix}$$ " di circolaz°

$\varphi(x) = C \cos \gamma x + D \sin \gamma x$

$\varphi(0) = 0 \rightarrow 1^a$ cond. di incastro



$$N(\ell, t) + K u(\ell, t) + m \frac{\partial^2 u(\ell, t)}{\partial t^2} = 0$$

$$N = AE \frac{\partial u}{\partial x}$$

$$EA \frac{\partial u}{\partial x}(\ell, t) + m \frac{\partial^2 u(\ell, t)}{\partial t^2} + K u(\ell, t) = 0$$

ip di sol: $u(x,t) = \varphi(x) e^{j\omega t}$

$$EA \varphi'(\ell) - m \omega^2 \varphi(\ell) + K \varphi(\ell) = 0$$

$(K - m\omega^2)\varphi(\ell) + EA \varphi'(\ell) = 0 \rightarrow 2^a$ cond di BORDO

$\varphi(x) = C \cos \gamma x + D \sin \gamma x$

$\boxed{\varphi(0) = 0 = C}$ $\qquad \varphi(\ell) = 0$

$(K - m\omega^2) \cancel{D} \sin \gamma \ell + EA \cancel{D} \gamma \cos \gamma \ell = 0$ x la cond di bordo

$$\frac{\operatorname{tg} \gamma \ell + EA\gamma}{K - m\omega^2} = 0$$

$$\gamma^2 = \frac{\mu \omega^2}{AE}$$

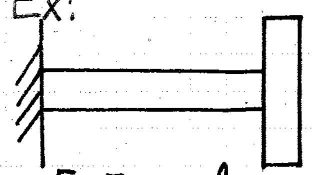
$$\operatorname{tg} \gamma \ell + \frac{AE\gamma}{K - \omega^2 m} = 0 \Rightarrow \operatorname{tg} \gamma \ell + \frac{AE\gamma \ell}{\frac{\ell K}{AE} - \gamma \frac{AE \ell^2}{\mu \ell}} = 0$$

$$\frac{EA}{\ell} = K_0$$

$$m_0 = \mu \ell$$

06/12/12

Ex:



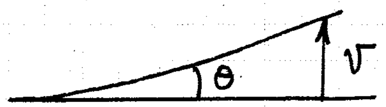
m, J_d $\uparrow v(x,t)$ Det le cond di bordo del sist.

E, I, μ, l

Trave cn massa distribuita

$v(x,t) = \varphi(x) e^{j\omega t}$

$$EI \frac{\partial^4 V}{\partial x^4} + \mu \frac{\partial^2 V}{\partial t^2} = 0$$



$$\theta = \frac{\partial V}{\partial x}$$

$$\frac{\partial^4 V}{\partial x^4} = \varphi^{IV}(x) e^{j\omega t}$$

$$\frac{\partial^2 V}{\partial t^2} = -\omega^2 \varphi(x) e^{j\omega t}$$

$\phi = \varphi$

$$EI\, \phi^{IV}(x) - \mu\omega^2 \varphi(x) = 0$$

$\rightarrow \varphi(x) = A\cos\beta x + B\sin\beta x + C\cosh(\beta x) + D\sinh(\beta x)$

$\downarrow$

sol dell' eq. differenziale

$$\frac{\mu\omega^2}{EI} = \beta^4$$

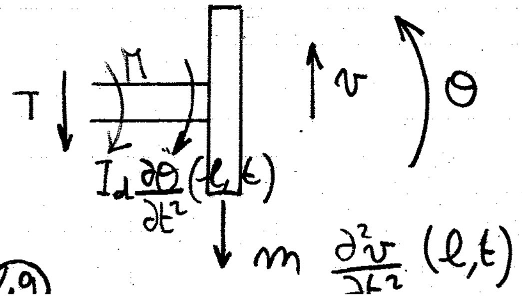
$x = 0$ $\quad v = V$

$v(0,t) = 0$ $\quad \varphi(0) = 0$ (1)

$\frac{\partial V}{\partial x}(0,t) = 0$ $\quad \varphi'(0) = 0$ (2)

$\varphi(0) = 0$

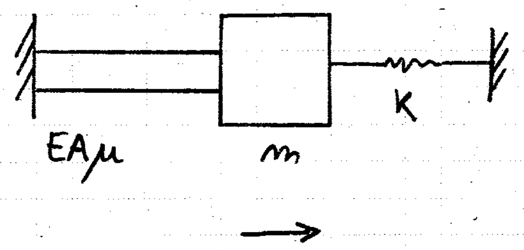
$\phi'(0) = 0$



Az° di taglio: a sx si oppone alla convenz°, a dx è cn la convenz°

$\det\left[\quad\right] = 0 \Rightarrow \omega = \ldots$

Ex:



$$\gamma^2 = \frac{\mu\omega^2}{EA}$$

$$\varphi(x) = C\cos\gamma x + D\sin\gamma x$$

$$\varphi(0) = 0 = C \qquad \Rightarrow C = 0$$

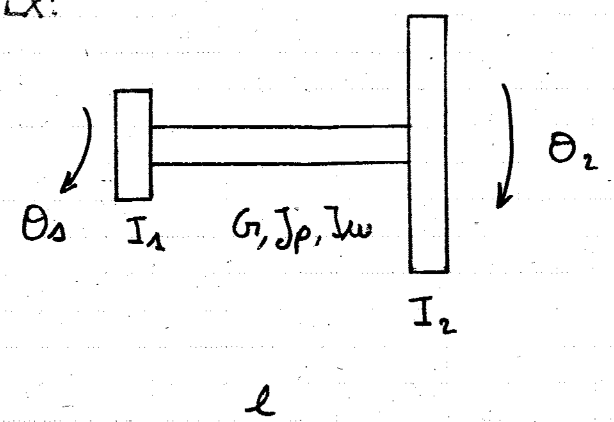
$$\Rightarrow \varphi(x) = D\sin\gamma x$$

$$\operatorname{tg}\gamma l + \frac{\gamma l}{\frac{K}{K_0} - \frac{m}{m_0}(\gamma^2 l^2)} = 0$$

↳ essendo 1 eq. in γ si tira fuori ω

$$u(x,t) = D\sin\gamma x\, e^{j\omega t}$$

Ex:



2gdl (le 2 rotaz°)

$I_1 = 8.85 \cdot 10^{-4}$ kg s²m

$I_2 = 1.9 \cdot 10^{-3}$ kg s²m

$I_u = 1.107 \cdot 10^{-3}$ kg s²

$J_p = 4.02 \cdot 10^{-6}$ m⁴

$l = 28$ cm

$G = 2700$ kg/mm²

(51) l'albero ... è ... elastico ⇒ c'è 1 molla torsione

$$\frac{\partial^2\theta}{\partial t^2} = -\omega^2 \varphi(x)\, e^{j\omega t}$$

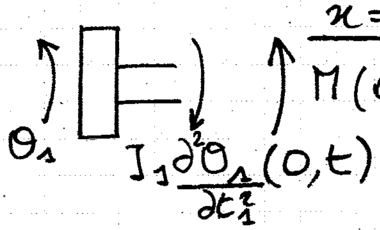
$$\frac{\partial^2\theta}{\partial x^2} = \varphi''(x)\, e^{j\omega t}$$

$$-\omega^2 I_u\, \varphi(x) - G J_p\, \varphi''(x) = 0$$

$$\varphi''(x) + \omega^2 \frac{I_u}{G J_p} \varphi(x) = 0 \rightarrow \varphi(x) = C\cos\gamma x + D \sin\gamma x$$

$$\gamma^2 = \frac{\omega^2 I_u}{G J_p}$$

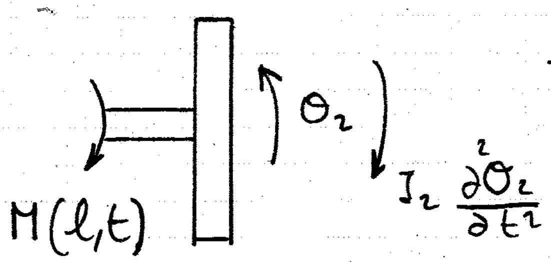
$x=0$



$M(0,t)$ concorde xchè è la faccia dx

$$M(0,t) - I_1 \frac{\partial^2\theta_1}{\partial t_1^2}(0,t) = 0 \qquad (1)$$

$x=\ell$



$$M(\ell,t) + I_2 \frac{\partial^2\theta_2}{\partial t^2}(\ell,t) = 0 \qquad (2) \rightarrow 2 \text{ cond di bordo}$$

$$G J_p \frac{\partial\theta}{\partial x}(0,t) - I_1 \frac{\partial^2\theta_1(0,t)}{\partial t^2} = 0$$

(1) $G J_p\, \varphi'(0) + \omega^2 I_1 \varphi(0) = 0$

$$G J_p \frac{\partial\theta}{\partial x}(\ell,t) + I_2 \frac{\partial^2\theta_2}{\partial t^2}(\ell,t) = 0$$

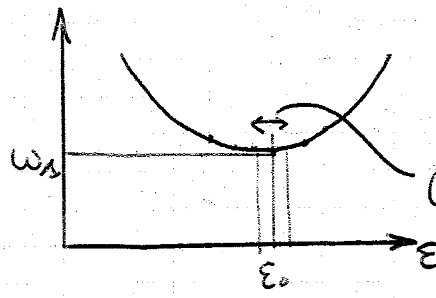
(2) $G J_p\, \varphi'(\ell) - I_2 \omega^2 \varphi(\ell) = 0$

14/12/12 p_88 teoria

Ex: Sist omogeneo nn forzato, dissipativo

$$[m]\{\ddot{x}\} + [k]\{x\} = \{0\}$$

Es:

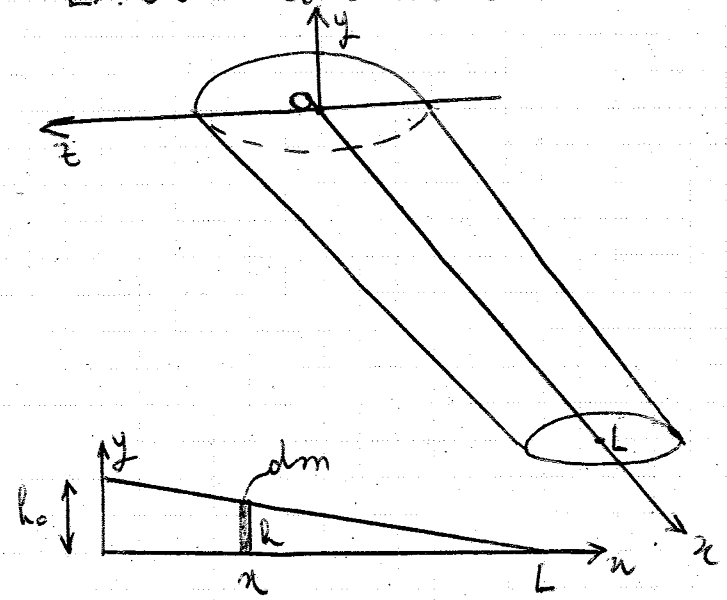


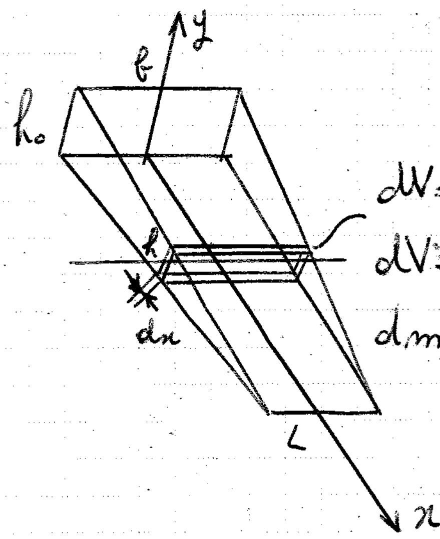
$\{x_0\} = \{\psi\}_1 + \varepsilon \{\psi_{errore}\}$

Se $\varepsilon = \varepsilon_0 \Rightarrow \{x_0\} = \{\psi\}_1$

Ci si può muovere su ε avendo min variaz° di ω

Ex. di 1 sez° alare.





$dV = h(x)\, b \cdot dx$

$dV = h_0\left(1 - \frac{x}{L}\right) b \cdot dx$

$dm = \rho\, dV$

$0 \leq x \leq L$

$h(x) = h_0\left(1 - \frac{x}{L}\right)$

$h_0 = 17\,cm = 0,17\,m$

$b = \text{cte}$

$L = 8\,m$

$\rho = 900\,kg/m^3$

$E = 18148\,N/mm^2$

$E = 18148\,MPa$

Il legno ha 1 densità < a qlla dell'H_2O

$X = \frac{x}{L}$ dv $0 \leq X \leq 1$

Coordinata intrinseca / naturale

$h(x) = h_0(1 - X)$ $\qquad y = y(x, t)$

$T = \int_0^L \frac{1}{2}\left(\frac{\partial y}{\partial t}\right)^2 dm$

$U = \int_0^L \frac{1}{2} M_f\, d\varphi$

$\varphi = \frac{\partial y}{\partial x}$ $\qquad d\varphi = \frac{\partial \varphi}{\partial x}\, dx = \frac{\partial^2 y}{\partial x^2}\, dx$

(55)

$$J(x) = \frac{b h_0^3}{12}(1-x)^3$$

$$U = \frac{1}{2}\frac{E}{L^2}\int_0^1 \left(\frac{\partial^2 y}{\partial x^2}\right)^2 \frac{b h_0^3}{12}(1-x)^3\,dx$$

$$U_{max} = \frac{1}{2}\frac{E}{L^3}\int_0^1 y_0^2\left[f''(x)\right]^2 \frac{b h_0^3}{12}(1-x)^3\,dx$$

$$y(x,t) = y_0 \sin\omega t\, f(x)$$

$$\frac{\partial y}{\partial t} = \omega y_0 \cos\omega t\; f(x)$$

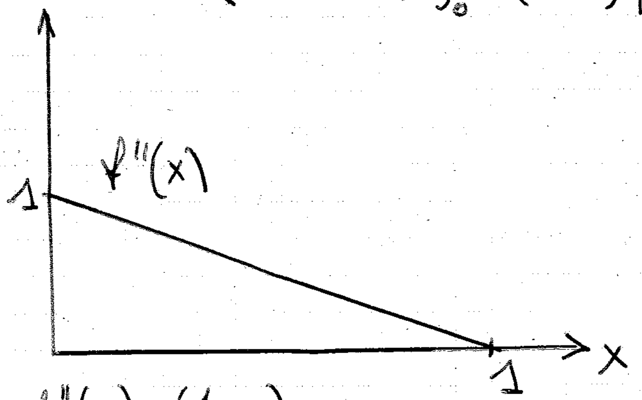
$$T = \frac{\rho h_0 b L}{2}\int_0^1 \omega^2 y_0^2 (\cos(\omega t))^2 (1-x)\,dx\, f(x)$$

$$T_{max} = \omega^2 y_0^2 \frac{\rho h_0 b L}{2}\int_0^1 (1-x)\, f(x)^2\,dx$$

$$\omega^2 = \frac{U_{max}}{T_{max}} =$$

$$= \frac{\cancel{\frac{1}{2}}\frac{E}{L^3}\cancel{y_0^2}\frac{b h_0}{12}\int_0^1 (1-x)^3 (f''(x))^2\,dx}{\cancel{y_0^2}\,\frac{\rho h_0 b L}{\cancel{2}}\int_0^1 (1-x)\, f(x)^2\,dx}$$

$$= \frac{E h_0^2}{12\rho\; L^4}\;\frac{\int_0^1 (1-x)^3\left[f''(x)\right]^2 dx}{\int_0^1 (1-x)\left[f(x)\right]^2 dx}$$



è l'ip che $f''(x)$ sia lineare

$$f''(x) = (1-x)$$

$$f'(x) = x - \frac{x^2}{2} + a$$

$$f(x) = \frac{x^2}{2} - \frac{x^3}{6} + ax + b$$

$f'(x=0) = 0 \Rightarrow a = 0$ xché è incastrata

$f(x=0) = 0 \Rightarrow b = 0$ " non si muove

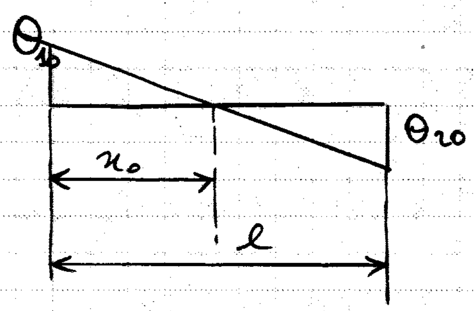
$$\Rightarrow f(x) = \frac{x^2}{2} - \frac{x^3}{6} \quad \text{cn } f'(x) = x - \frac{x^2}{2} \text{ e } f''(x) = 1-x$$

Albero nn inerte

$\omega_1 = 0$

$\omega_2 = \sqrt{\dfrac{K_t(I_1+I_2)}{I_1 I_2}} = 8013 \text{ rad/s}$

$$[\varphi] = \begin{bmatrix} 1 & 1 \\ 1 & -\dfrac{I_1}{I_2} \end{bmatrix}$$



$$\frac{x_0}{l} = \frac{\theta_{10}}{\theta_{10}-\theta_{20}}$$

$x_0 = \dfrac{I_2}{I_1+I_2}\, l$ è molto approssimato xché nn si considera che l'albero abbia 1 massa

2° metodo: metodo dell'E

siamo in assenza di fenomeni dissipativi

$T_{max} = U_{max}$

$$T = \frac{1}{2}\int_0^l I_u \left(\frac{\partial \theta}{\partial t}\right)^2 dx + \frac{1}{2} I_1 \left(\frac{\partial \theta}{\partial t}\right)^2_{x=0} + \frac{1}{2} I_2 \left(\frac{\partial \theta}{\partial t}\right)^2_{x=l}$$

$$U = \frac{1}{2}\int_0^l M_t\, d\theta = \frac{1}{2}\int_0^l G J_p \left(\frac{\partial \theta}{\partial x}\right)^2 dx$$

$\theta(x,t) = \theta_0 \cos \omega t \, f(X)$

$X = \dfrac{x}{l}$ cm evolve nel tempo la deformata

ampiezza

Qd $\cos = 1 \Rightarrow \theta$ è max

" $\cos = 0 \Rightarrow \theta = 0$

$\dfrac{\partial \theta}{\partial t} = -\theta_0\, \omega \sin \omega t\, f(X)$

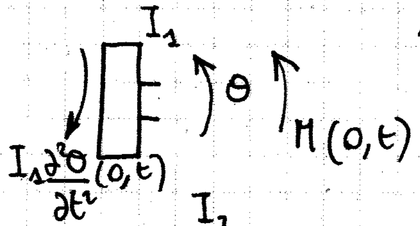
59 $T = \dfrac{1}{2} I_u\, \theta_0^2 \omega^2 \sin^2 \omega t \int_0^1 l f(X)^2 dX + \dfrac{1}{2} I_1 \theta_0^2 \omega^2 \sin^2 \omega t\, f^2(0) +$

$I_u \omega^2 \varphi(x) + G J_p \varphi''(x) = 0$ eq. differenziale del 2° ordine in φ.

$\varphi(x) = C \cos \gamma x + D \sin \gamma x$

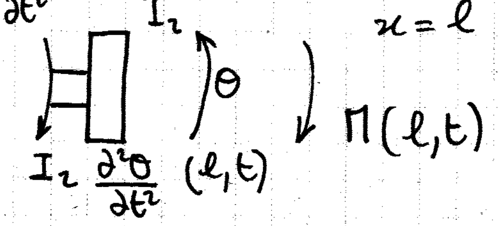
dove $\gamma^2 = \dfrac{I_u \omega^2}{G J_p}$

$x = 0$



$M = G J_p \dfrac{\partial \theta}{\partial x}$

$x = l$



① $M(0,t) - I_1 \dfrac{\partial^2 \theta}{\partial t^2}(0,t) = 0 \Rightarrow G J_p \dfrac{\partial \theta}{\partial x} - I_1 \dfrac{\partial^2 \theta}{\partial t} = 0 \rightarrow G J_p \varphi'(0) + \omega^2 I_1 \varphi(0) = 0$

② $M(l,t) + I_2 \dfrac{\partial^2 \theta}{\partial t^2}(l,t) = 0$

$G J_p \dfrac{\partial \theta}{\partial x}(0,t) - I_1 \dfrac{\partial^2 \theta}{\partial t^2}(0,t) = 0$

$\theta(x,t) = \varphi(x)\, e^{j\omega t}$

$G J_p\, \varphi'(0) + I_1 \omega^2 \varphi(0) = 0$ 1ª cond. di bordo

$G J_p \dfrac{\partial \theta}{\partial x}(l,t) + I_2 \dfrac{\partial^2 \theta}{\partial t^2}(l,t) = 0$

$G J_p\, \varphi'(l) - I_2 \omega^2 \varphi(l) = 0$ 2ª " " "

$\varphi(x) = C \cos \gamma x + D \sin \gamma x$ $\begin{cases} \varphi(0) = C \cos \gamma(0) + D \sin \gamma(0) = C \\ \varphi'(0) = D\gamma \end{cases}$

① $G J_p D \gamma + I_1 \omega^2 C = 0$

② $G J_p(-C\gamma \sin \gamma l + D \gamma \cos \gamma l) - I_2 \omega^2 (\cos \gamma l + D \sin \gamma l) = 0$

$$\begin{bmatrix} I_1 \omega^2 & G J_p \gamma \\ I_2 \omega^2 \cos \gamma l + G J_p \gamma \sin \gamma l & I_2 \omega^2 \sin \gamma l - G J_p \gamma \cos l \end{bmatrix} \begin{Bmatrix} C \\ D \end{Bmatrix} = 0$$

61

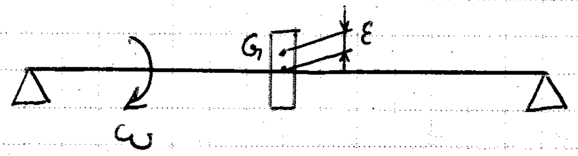
$l = 40\,cm = 0.4\,m$

$d = 2.5\,cm = 0.025\,m$

$G = 3.15\,mm/s$ → grado di bilanciamento: in f° dell' uso che devi fare di quel sist rotante, dell' istallaz° dell' oggetto si deve avere 1 certo grado di bilanciamento → è la max vel periferica ideale del centro di massa sbilanciato del sist rotante

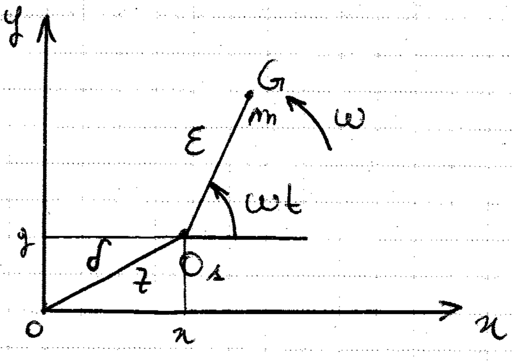
$E = 2.10 \cdot 10^{11}\,N/m^2$

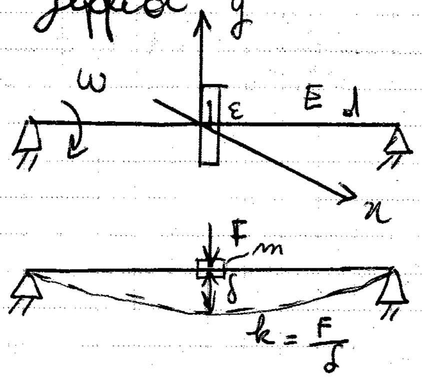
$K = \frac{48EI}{l^3}$



$v = \omega \varepsilon$

Modello del rotore di Jeffcot



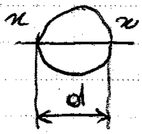


il sist è statico

Che forze agiscono sul rotore?

$K = \frac{F}{\delta} = \frac{48EI}{l^3}$

$I = I_{xx} = \frac{\pi d^4}{64}$



$x_G = x + \varepsilon \cos \omega t$

$y_G = y + \varepsilon \sin \omega t$

63

$$\begin{cases} m\ddot{x} + kx = m\varepsilon\omega^2\cos\omega t \\ jm\ddot{y} + jky = jm\varepsilon\omega^2\sin\omega t \end{cases}$$

$$m\underbrace{(\ddot{x}+j\ddot{y})}_{\ddot{z}} + k\underbrace{(x+jy)}_{z} = m\varepsilon\omega^2\underbrace{(\cos\omega t + j\sin\omega t)}_{e^{j\omega t}}$$

$$m\ddot{z} + kz = m\varepsilon\omega^2 e^{j\omega t}$$

(b) equivalente ad (a) ma + semplice da risolvere

(b) $z(t) = z_0 e^{j\omega t}$

$$\ddot{z} = -\omega^2 z_0 e^{j\omega t}$$

$$-\omega^2 m z_0 \cancel{e^{j\omega t}} + kz_0 \cancel{e^{j\omega t}} = m\varepsilon\omega^2 \cancel{e^{j\omega t}}$$

$$z_0(-m\omega^2 + k) = m\varepsilon\omega^2$$

$$z_0 = \frac{\cancel{m}\varepsilon\omega^2}{(-m\omega^2+k)\frac{\cancel{m}}{m}}$$

$$z_0 = \varepsilon\frac{\omega^2}{\omega_n^2 - \omega^2}$$

$$I = \frac{\pi d^4}{64}$$

$$k = \frac{48EI}{l^3} = 48E\frac{\pi d^4}{64 l^3} = 3.02 \cdot 10^6 \ \frac{N}{m}$$

$$m = 9\,kg \Rightarrow \omega_n = \sqrt{\frac{k}{m}} = 579.5\ rad/s$$

$$n = n_1 = 3200\ rpm \Rightarrow \omega = \omega_1 = \frac{2\pi n_1}{60} = 335.1\ rad/s$$

$$n = n_2 = 7000\ rpm \Rightarrow \omega = \omega_2 = \frac{2\pi n_2}{60} = 733\ rad/s$$

$$G = \omega\varepsilon = 3.15\ mm/s$$

$$= 3.15 \cdot 10^{-3}\ m/s = v$$

$$\omega = \omega_1 \Rightarrow \varepsilon = \varepsilon_1 = \frac{G}{\omega_1} = \frac{30G}{\pi n_1} = 9.4 \cdot 10^{-6}\,m = 9.4\,\mu m$$

$$\omega = \omega_2 \Rightarrow \varepsilon = \varepsilon_2 = \frac{30G}{\pi n_2} = 4.3\,\mu m$$

65 Bisogna soddisfare la + piccola eccentricità