NUMERO: 598 DATA: 23/07/2013

A P P U N T I

STUDENTE: Marsicovetere

MATERIA: Fondamenti di Meccanica Strutturale + Eserc.
Prof. Carrera

MATTEO

MARSICOVETERE

FONDAMENTI DI MECCANICA STRUTTURALE

ING. AEROSPAZIALE 2012/2013

PROF. CARRERA

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$$\gamma_{xy} = 2\,\varepsilon_{xy} = \frac{1}{G}\,\sigma_{xy}$$

$$\gamma_{xz} = 2\,\varepsilon_{xz} = \frac{1}{G}\,\sigma_{xz}$$

$$\gamma_{yz} = 2\,\varepsilon_{yz} = \frac{1}{G}\,\sigma_{yz}$$

$$\sigma_{xx} = (\lambda + 2G)\,\varepsilon_{xx} + \lambda\,\varepsilon_{yy} + \lambda\,\varepsilon_{zz}$$

$$\sigma_{yy} = \lambda\,\varepsilon_{xx} + (\lambda + 2G)\,\varepsilon_{yy} + \lambda\,\varepsilon_{zz}$$

$$\sigma_{zz} = \lambda\,\varepsilon_{xx} + \lambda\,\varepsilon_{yy} + (\lambda + 2G)\,\varepsilon_{zz}$$

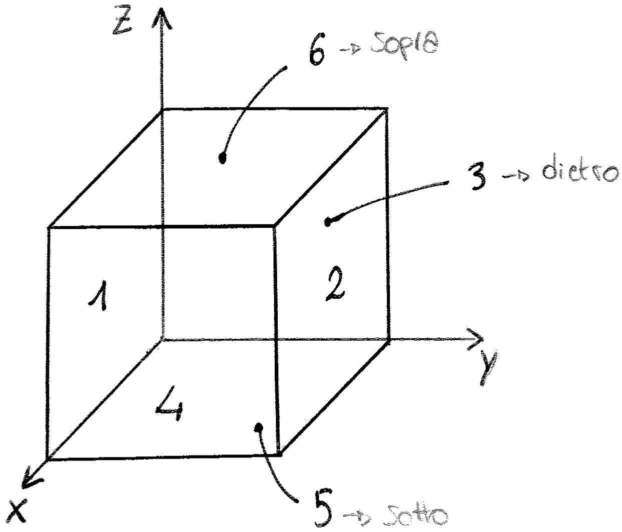
$$\sigma_{xy} = G\,\gamma_{xy}$$

$$\sigma_{xz} = G\,\gamma_{xz}$$

$$\sigma_{yz} = G\,\gamma_{yz}$$

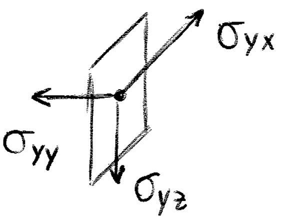
$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

$$G = \frac{E}{2(1+\nu)}$$



Trovare le varie I e gli sforzi sulle facce

1



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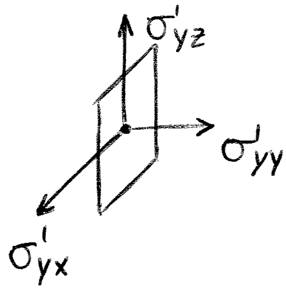

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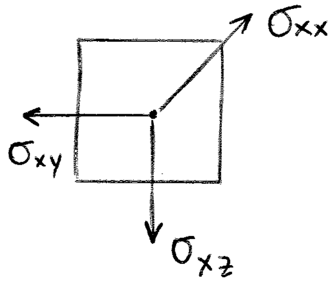

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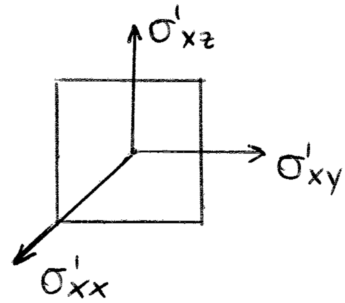

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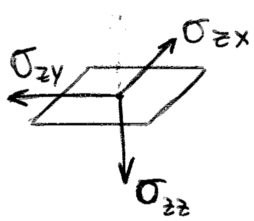

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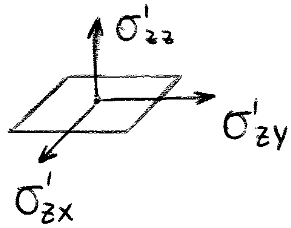

FACCIA		
1	$\sigma_{yx}\, dx\, dz$	$dy/2$
2	$\sigma'_{yx}\, dx\, dz$	$dy/2$
3	$-\sigma_{xy}\, dy\, dz$	$dx/2$
4	$-\sigma'_{xy}\, dy\, dz$	$dx/2$
5	0	
6	0	

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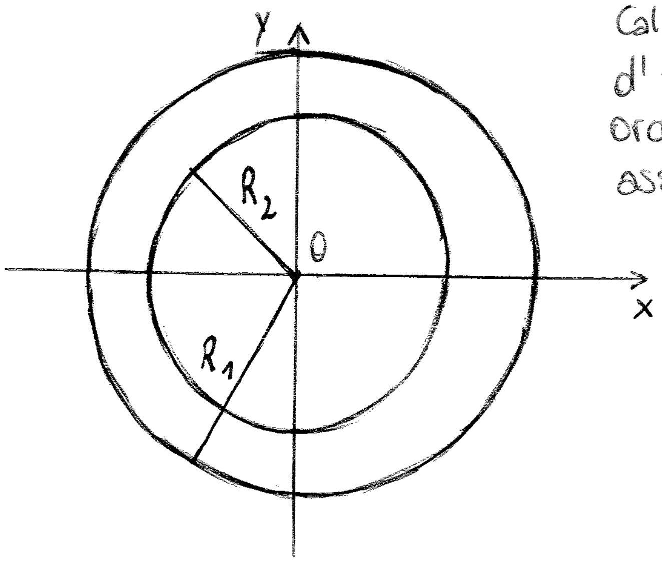
supponendo $I_x > I_y$:

$$I_\xi = (I_x + I_y)\frac{1}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$I_\eta = \frac{1}{2}(I_x + I_y) - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$



Calcolare i momenti d'inerzia del secondo ordine rispetto agli assi x y

$D = D_1 - D_2$

$$\int_D f\,dxdy = \int_{D_1} f\,dxdy - \int_{D_2} f\,dxdy$$

$x = r\cos\theta$
$y = r\,\text{sen}\,\theta$
$\det J = r$

$$I_x = \int_A y^2\,dxdy = \int_{A'} (r\,\text{sen}\,\theta)^2 r\,drd\theta$$

$$I_x = \left[\frac{1}{2}(\theta - \text{sen}\,\theta\cos\theta)\right]_0^{2\pi} \cdot \left\{\left[\frac{r^4}{4}\right]_0^{R_1} - \left[\frac{r^4}{4}\right]_0^{R_2}\right\} =$$

$$= \frac{\pi}{4}\left(R_1^4 - R_2^4\right)$$

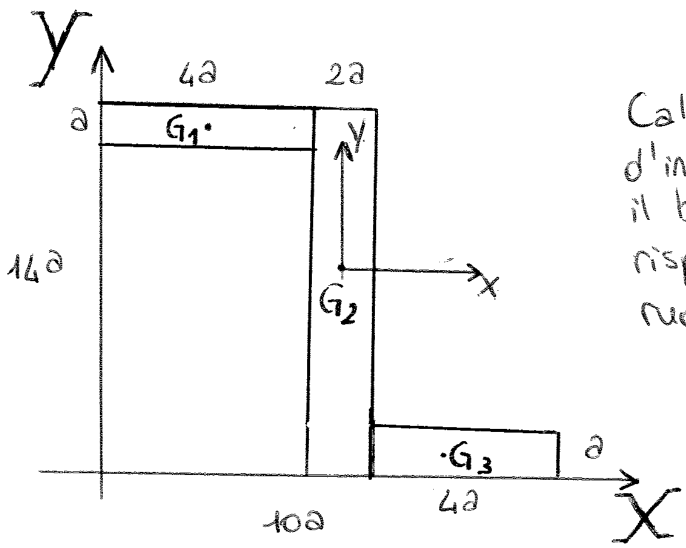
$$I_y = \int_A x^2\,dxdy = \int_{A'} (r\cos\theta)^2 r\,drd\theta$$

$$I_y = \left[\frac{1}{2}(\theta + \text{sen}\,\theta\cos\theta)\right]_0^{2\pi} \cdot \left\{\left[\frac{r^4}{4}\right]_0^{R_1} - \left[\frac{r^4}{4}\right]_0^{R_2}\right\} =$$

$$= \frac{\pi}{4}(R_1 - R_2) = I_x$$

$$I_{xy} = 0$$

2



Calcolare i momenti d'inerzia del secondo ordine, il baricentro e i momenti rispetto ad un riferimento ruotato

Essendo la figura simmetrica rispetto al rettangolo centrale, il baricentro complessivo corrisponde al baricentro del centrale

$$G = \left(4a + \frac{2a}{2}, \frac{14a}{2}\right) = (5a, 7a)$$

$$I_x = \int_A y^2 dxdy = \int_{A_1} y^2 dxdy + \int_{A_2} y^2 dxdy + \int_{A_3} y^2 dxdy =$$

$$= 2\int_{A_1} y^2 dxdy + \int_{A_2} y^2 dxdy$$

$$I_x = 2\left(\frac{4a \cdot a^3}{12} + 4a \cdot a \cdot \left(\frac{13}{2}a\right)^2\right) + \frac{2a \cdot (14a)^3}{12} = 796a^4$$

$$I_y = 2\left[\frac{(4a)^3 \cdot a}{12} + 4a^2(3a)^2\right] + \frac{(2a)^3 \cdot 14a}{12} = 92a^4$$

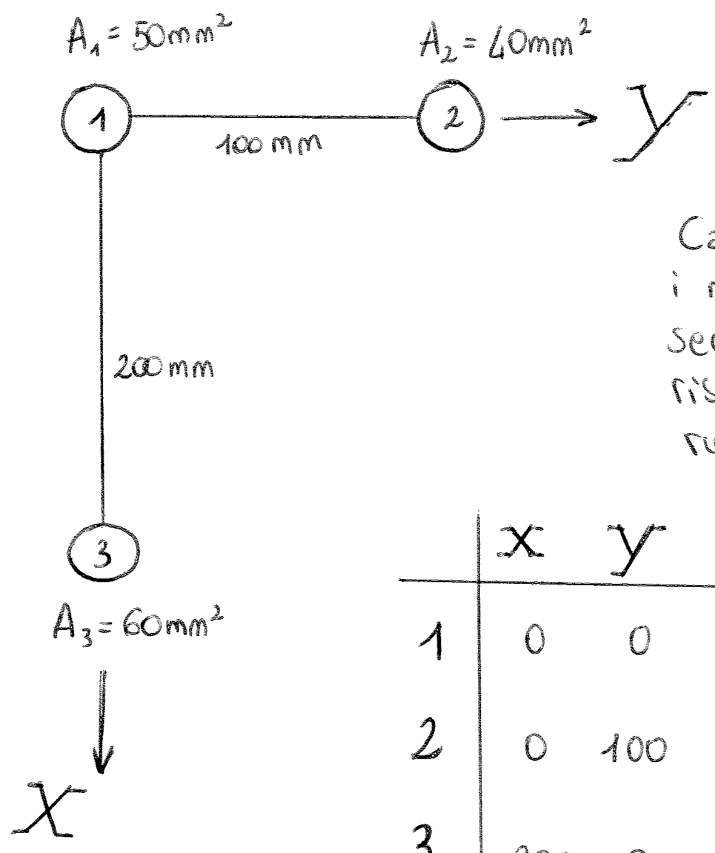
$$I_{xy} = 4a^2(-3a)\left(\frac{13}{2}a\right) + 4a^2(3a)\left(-\frac{13}{2}a\right) = -156a^4$$

$S_{x_1} = S_{x_2}$ → si semplificano

$S_{y_2} = S_{y_1}$ → si semplificano

$I_{xy} = 0$

3

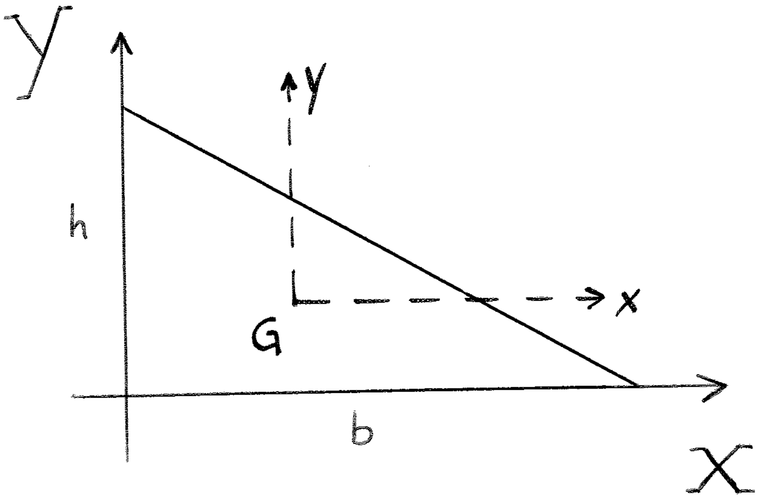


Calcolare il baricentro, i momenti d'inerzia del secondo ordine e i momenti rispetto ad un riferimento ruotato

	x	y	A	Ax	Ay
1	0	0	50	0	0
2	0	100	40	0	4000
3	200	0	60	12000	0

$$G_x = \frac{\sum_{i=1}^{3} x_i A_i}{\sum_{i=1}^{3} A_i} = \frac{12000}{150} = 80\,mm$$

$$G_y = \frac{\sum_{i=1}^{3} y_i A_i}{\sum_{i=1}^{3} y_i} = \frac{4000}{150} = 26,\overline{66}\ mm$$



$$G_x = \frac{\int_A x\,dx\,dy}{\int_A dx\,dy} = \frac{S_y}{A}$$

$$S_y = \int_0^b x\left[\int_0^{-\frac{h}{b}x+h} dy\right]dx = \int_0^b \left(-\frac{h}{b}x^2 + hx\right)dx =$$

$$= \left[-\frac{1}{3}x^3\frac{h}{b} + \frac{1}{2}hx^2\right]_0^b = \frac{hb^2}{6}$$

$$G_x = \frac{hb^2}{6}\cdot\frac{2}{hb} = \frac{b}{3}$$

$$S_x = \int_0^h y\left[\int_0^{-\frac{b}{h}(y-h)} dx\right]dy = \int_0^h \left(-\frac{b}{h}y^2 + by\right)dy =$$

$$= \left[-\frac{1}{3}\frac{b}{h}y^3 + \frac{1}{2}by^2\right]_0^h = \frac{h^2b}{6}$$

$$G_y = \frac{bh^2}{6}\cdot\frac{2}{bh} = \frac{h}{3}$$

$$I_x = I_{\mathcal{X}} - dy^2 A - \underbrace{2 S_x dy}_{}$$

dato che sono nel baricentro è = 0

$$I_x = \frac{bh^3}{12} - \left(\frac{h}{3}\right)^2 \cdot \frac{bh}{2} = \frac{bh^3}{36}$$

$$I_y = I_{\mathcal{Y}} - dx^2 A$$

$$I_y = \frac{b^3h}{12} - \left(\frac{b}{3}\right)^2 \cdot \frac{bh}{2} = \frac{b^3h}{36}$$

$$I_{xy} = I_{\mathcal{XY}} - dx dy A$$

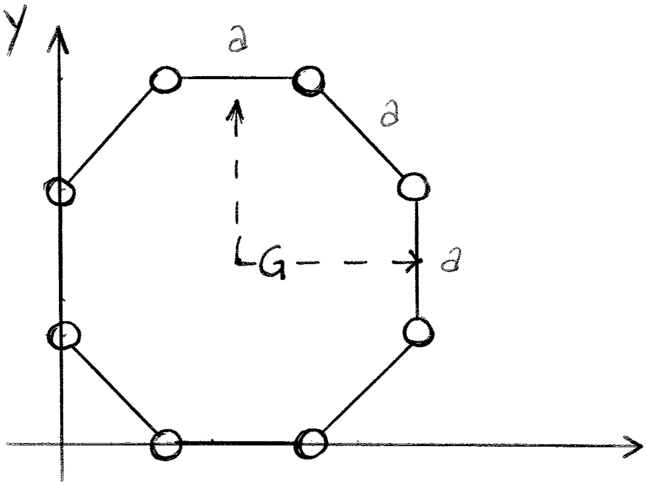
$$I_{xy} = \frac{h^2b^2}{24} - \frac{h}{3} \cdot \frac{b}{3} \cdot \frac{bh}{2} = -\frac{b^2h^2}{72}$$

$$\tilde{\theta} = \frac{1}{2} \operatorname{arctg} \left[-2 \frac{-\frac{b^2h^2}{72}}{\frac{bh^3}{36} - \frac{b^3h}{36}} \right] = \frac{1}{2} \operatorname{arctg} \left[\frac{bh}{h^2 - b^2} \right]$$

$$I_\varepsilon = \frac{1}{2}\left(I_x + I_y\right) + \frac{1}{2}\sqrt{\left(I_x - I_y\right)^2 + 4 I_{xy}^2}$$

$$I_\eta = \frac{1}{2}\left(I_x + I_y\right) - \frac{1}{2}\sqrt{\left(I_x - I_y\right)^2 + 4 I_{xy}^2}$$

5.2



Calcolare il baricentro dando una misura ai lati

Prendo un ottagono regolare → tutti i lati uguali

$$G_x = G_y = \frac{a}{2} + \frac{a}{\sqrt{2}} = \frac{a + a\sqrt{2}}{2} = \frac{a}{2}\left(1+\sqrt{2}\right)$$

$$I_x = \int_A y^2 \, dx \, dy = \frac{bh^3}{12}$$

$$I_y = \int_A x^2 \, dx \, dy = \frac{b^3 h}{12}$$

$$EI_x = E_1 \frac{h_1^3}{3} b + E_2 \frac{h_2^2}{3} b$$

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Determinare la matrice $[?]$ sapendo che

$$[?]^{*}\{\sigma\} = \{0\}$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & 0 \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix}$$

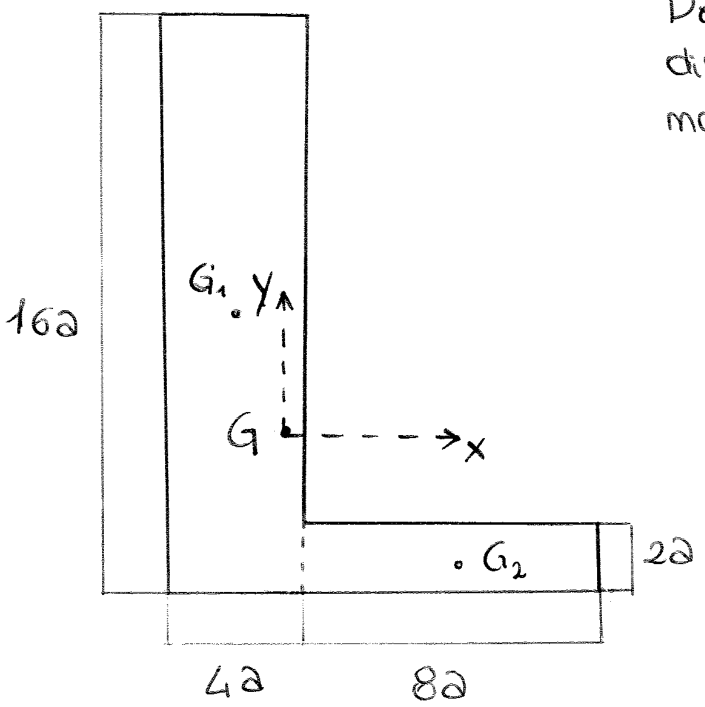
$$[?]^{*} = \left([?^{**}]\right)^{T} \qquad \text{PERCHE'?}$$

$[?^{**}]^{T} = [?^{*}]$ in quanto, moltiplicando le matrici $[?^{**}]$ e $[\sigma]$ ottengo le equazioni indefinite di equilibrio che so essere $=0$ per ipotesi.

②

ESERCITAZIONE 2

Determinare il baricentro, le direzioni principali e il momento centrifugo



$$G = \left(\frac{2a \cdot 16 \cdot 4a^2 + 8a \cdot 16a^2}{16 \cdot 4a^2 + 16a^2} , \frac{8a \cdot 16 \cdot 4a^2 + a \cdot 16a^2}{16 \cdot 4a^2 + 16a^2} \right) =$$

$$= (3{,}2a ; 6{,}6a)$$

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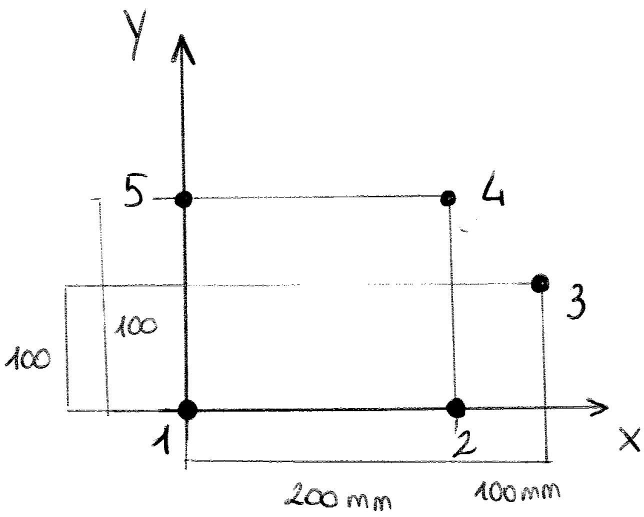
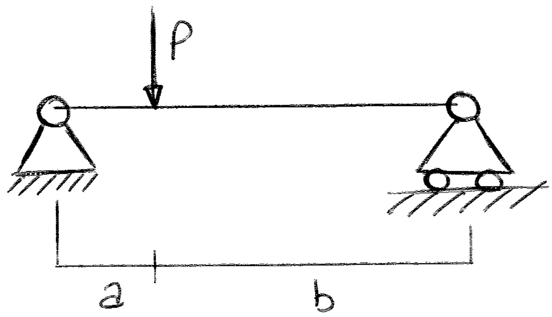


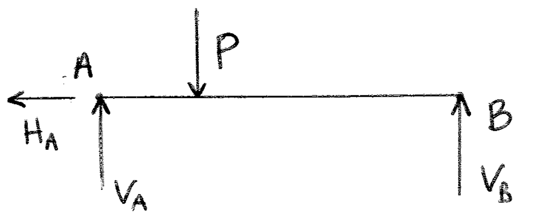
	x_i	y_i	A_i	x_iA_i	y_iA_i
1	0	0	100	0	0
2	200	0	50	10000	0
3	300	100	20	6000	2'000
4	200	200	50	10000	10'000
5	0	200	100	0	20'000

$$G = \left(\frac{26'000}{320} ; \frac{32'000}{320} \right) = \left(81,25\,mm ; 100\,mm \right)$$

2



Calcolare le reazioni vincolari



$H_A = 0$

$V_A + V_B = P$

A) $V_B(a+b) = Pa$

3

TEORIA 3

$$
\begin{cases}
\dfrac{\sigma_{xx}}{\partial x} + \dfrac{\sigma_{xy}}{\partial y} + \dfrac{\sigma_{xz}}{\partial z} = t_x \\[2ex]
\dfrac{\sigma_{yx}}{\partial x} + \dfrac{\sigma_{yy}}{\partial y} + \dfrac{\sigma_{yz}}{\partial z} = t_y \\[2ex]
\dfrac{\sigma_{zx}}{\partial x} + \dfrac{\sigma_{zy}}{\partial y} + \dfrac{\sigma_{zz}}{\partial z} = t_z
\end{cases}
$$

$$
\sigma_{xx,x} = (\lambda + 2G)\,\varepsilon_{xx,x} + \lambda\,\varepsilon_{yy,x} + \lambda\,\varepsilon_{zz,x} =
$$

$$
= (\lambda + 2G)\,u_{,x,x} + \lambda\,v_{,y,x} + \lambda\,w_{,z,x}
$$

$$
\sigma_{xy,y} = G\,\gamma_{xz,z} = G(u_{,y} + v_{,x})_{,y}
$$

$$
\sigma_{xz,z} = G\,\gamma_{xz,z} = G(u_{,z} + w_{,x})_{,z}
$$

$$
\sigma_{yx,x} = G\,\gamma_{xy,x} = G(u_{,y} + v_{,x})_{,x}
$$

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MATERIALE	E [GPa]	λ [GPa]	G [GPa]	ν [–]	C_{11}^{*} [GPa]	C_{11}^{**} [GPa]	diff %
OTTONE	96	76	36	0,34	148	96	54,1
NIKEL	210	130,7	80	0,31	290,7	210	38,43
FERRO	83	23	35	6,2	93	83	12,05
ACCIAIO	190	55	75	0,27	205	190	7,9
TUNGSTENO	340	94	142	0,2	378	340	11,2
BRONZO	96	76	36	0,34	148	96	54,2
LEGA Mg	41	35	15	0,35	65	41	58,5
LEGA Ti	100	73	38	0,33	149	100	49
LEGA Al (2014-T6)	73	53	27	0,33	107	73	46,6
VETRO	48	10,6	20,5	0,17	51,6	48	7,5

2

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Dimostrare che le equazioni di equilibrio per l'asta sono uguali a zero:

$$R_{xx} = \frac{\partial^2 \varepsilon_{yy}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial y^2} - \frac{\partial^2 \gamma_{yz}}{\partial y \partial z} = 0$$

$$R_{yy} = \frac{\partial^2 \varepsilon_{xx}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial x^2} - \frac{\partial^2 \gamma_{xz}}{\partial x \partial z} = 0$$

$$R_{zz} = \frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} - \frac{\partial \gamma_{xy}}{\partial x \partial y} = 0$$

$$R_{yz} = \frac{\partial}{\partial x}\left[\frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} - \frac{\partial \gamma_{yz}}{\partial x}\right] - \frac{2\partial^2 \varepsilon_{xx}}{\partial y \partial z} = 0$$

$$R_{xz} = \frac{\partial}{\partial y}\left[\frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y}\right] - \frac{2\partial^2 \varepsilon_{yy}}{\partial x \partial z} = 0$$

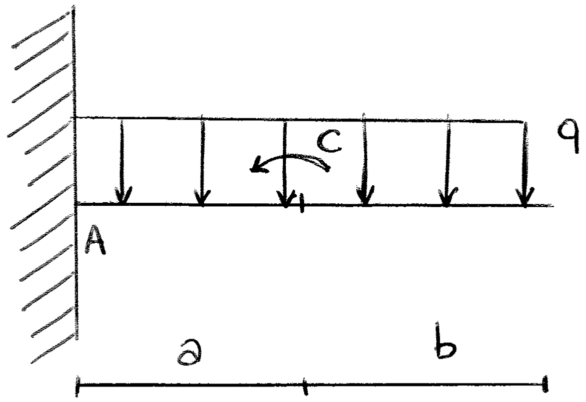
$$R_{yz} = \frac{\partial}{\partial z}\left[\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right] - \frac{2\partial^2 \varepsilon_{zz}}{\partial y \partial x} = 0$$

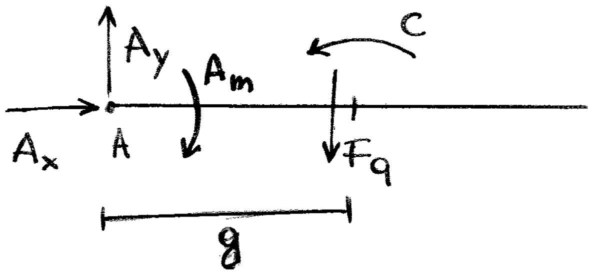
per ipotesi nell'asta $\varepsilon_{xx} = 0$ $\gamma_{xy} = 0$
$\varepsilon_{yy} = 0$ $\gamma_{xz} = 0$
$\varepsilon_{zz} = 0$ $\gamma_{yz} = 0$

ne consegue che:

$R_{xx} = 0$ $R_{xy} = 0$
$R_{yy} = 0$ $R_{xz} = 0$
$R_{zz} = 0$ $R_{yz} = 0$

3





$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_A = 0 \end{cases} \qquad \begin{cases} A_x = 0 \\ A_y + F_y = 0 \\ F_q \cdot g - C + A_m = 0 \end{cases}$$

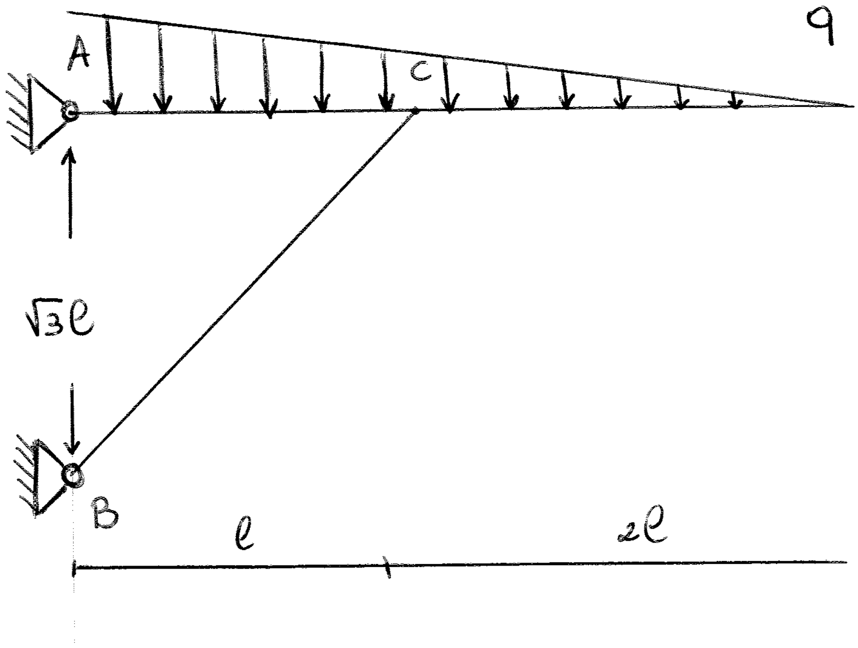
$$F_q = q \cdot (a+b)$$

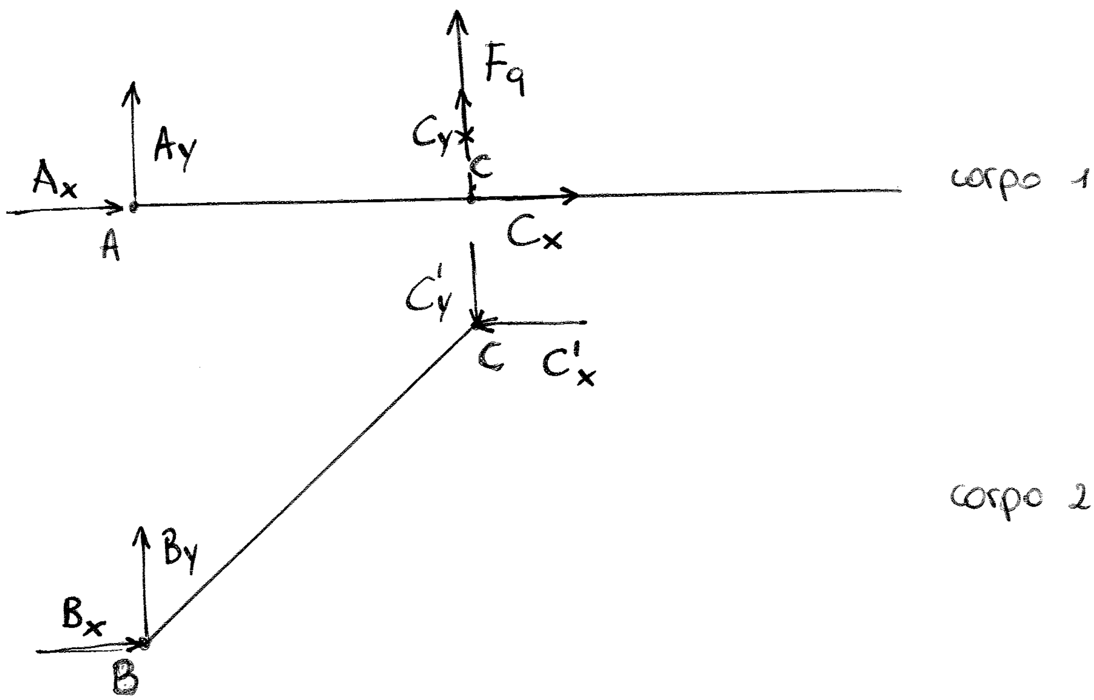
$$g = \frac{a+b}{2}$$

$$q \cdot (a+b)^2 \cdot \frac{1}{2} - C + A_m = 0$$

$$A_m = C - \frac{q}{2}(a+b)^2$$

2





4

Il lavoro esterno è pari a:

$$L_e^{ab} = \int_S \vec{p}^a \cdot \vec{s}^b dS = \int_S \hat{\sigma}^a \vec{n} \vec{s}^b dS =$$

$$= \int_S u^b(\sigma_{xx}^a n_x + \sigma_{xy}^a n_y + \sigma_{xz}^a n_z) + v^b(\sigma_{yx}^a n_x + \sigma_{yy}^a n_y + \sigma_{yz}^a n_z) + w^b(\sigma_{zx}^a n_x + \sigma_{zy}^a n_y + \sigma_{zz}^a n_z)\, dS$$

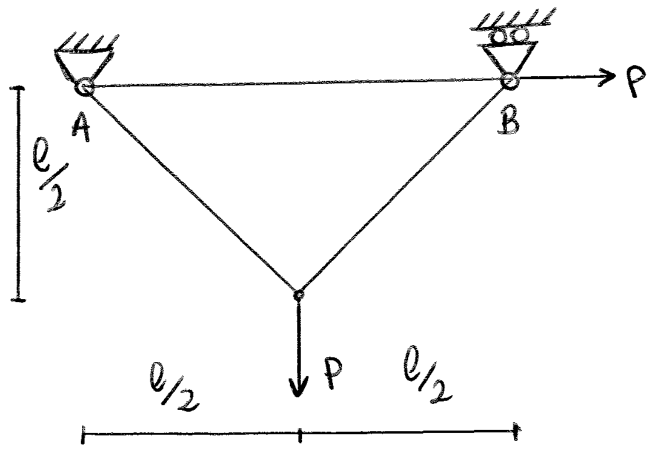
Imponendo l'uguaglianza tra la somma di tutti i lavori interni e quello esterno, e applicando le dovute semplificazioni, si ottiene:

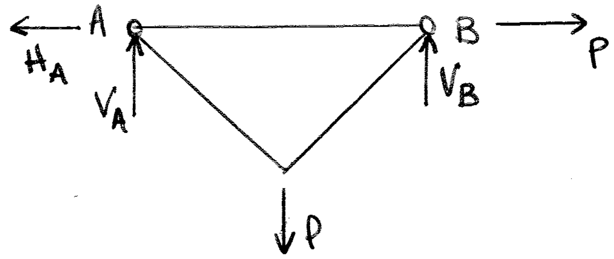
$$\int_V u^b \frac{\partial \sigma_{xx}^a}{\partial x} + u^b \frac{\partial \sigma_{xy}^a}{\partial y} + v^b \frac{\partial \sigma_{xy}^a}{\partial x} + v^b \frac{\partial \sigma_{yy}^a}{\partial y} + v^b \frac{\partial \sigma_{yz}^a}{\partial z} + w^b \frac{\partial \sigma_{yz}^a}{\partial y} + w^b \frac{\partial \sigma_{zz}^a}{\partial z} + w^b \frac{\partial \sigma_{xz}^a}{\partial x} + u^b \frac{\partial \sigma_{xz}^a}{\partial z}\, dV = 0$$

Dal principio dei lavori virtuali è quindi possibile ricavare le equazioni indefinite d'equilibrio.

$$\begin{pmatrix} \frac{\partial \sigma_{xx}^a}{\partial x} + \frac{\partial \sigma_{xy}^a}{\partial y} + \frac{\partial \sigma_{xz}^a}{\partial z} \\ \frac{\partial \sigma_{yx}^a}{\partial x} + \frac{\partial \sigma_{yy}^a}{\partial y} + \frac{\partial \sigma_{yz}^a}{\partial z} \\ \frac{\partial \sigma_{zx}^a}{\partial x} + \frac{\partial \sigma_{zy}^a}{\partial y} + \frac{\partial \sigma_{zz}^a}{\partial z} \end{pmatrix} \begin{pmatrix} u^b \\ v^b \\ w^b \end{pmatrix} = 0$$

ESERCITAZIONE 4





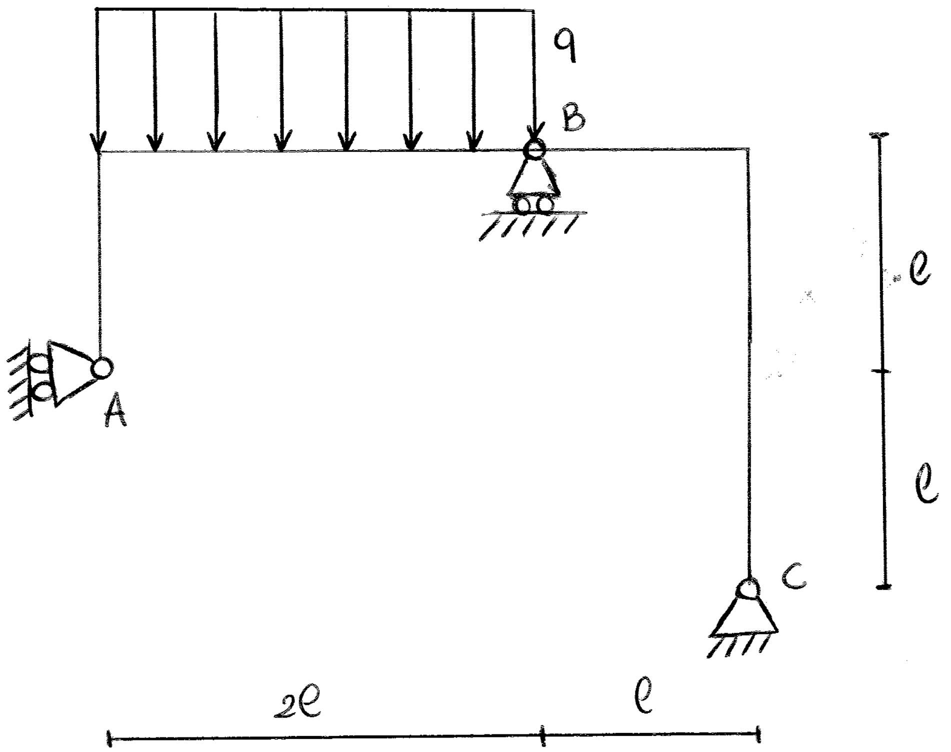
$$\begin{cases} \rightarrow \quad H_A - P = 0 \\ \uparrow \quad V_A + V_B - P = 0 \\ A \circlearrowright \quad P\frac{l}{2} - V_B l = 0 \end{cases}$$

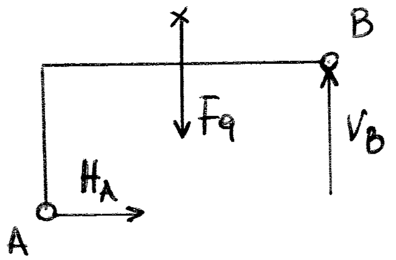
$H_A = P$

$V_B = \frac{P}{2}$

$V_A = \frac{P}{2}$

1

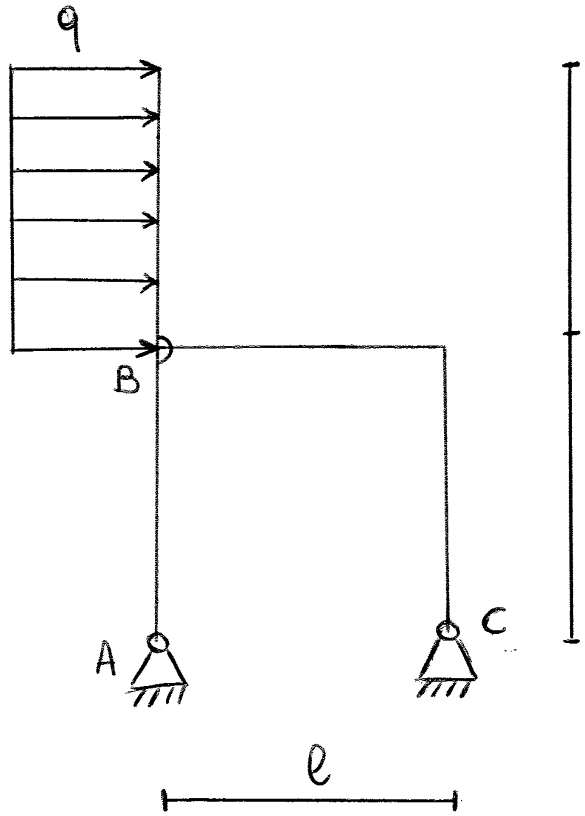


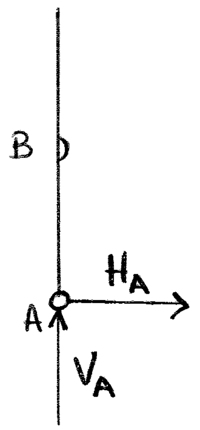


C) $-F_q \cdot l - H_A \cdot l = 0$

$H_A = -2ql$

③

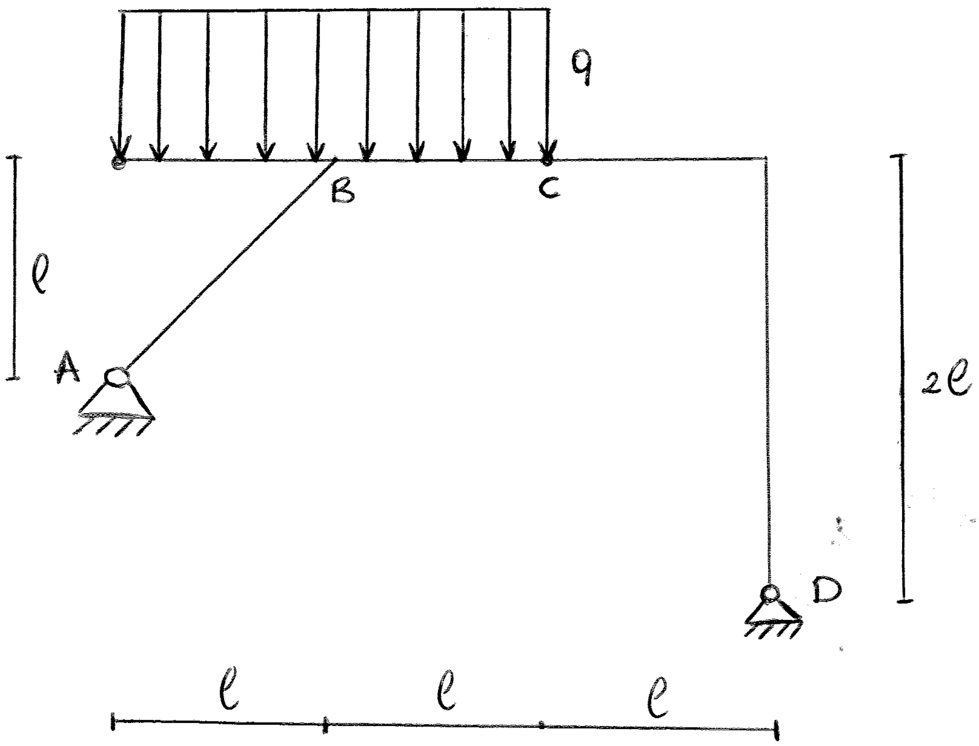




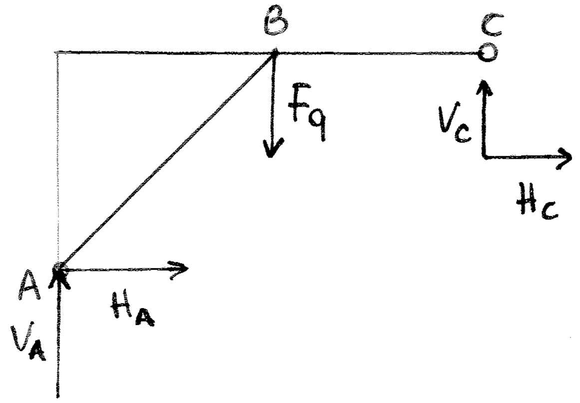
$B)$ $F_q \cdot \frac{\ell}{2} - H_A \cdot \ell$

$$H_A = \frac{F_q}{2} = \frac{q\ell}{2}$$

4



in C è posizionata una cerniera interna



$F_q = q \cdot \ell \cdot 2$

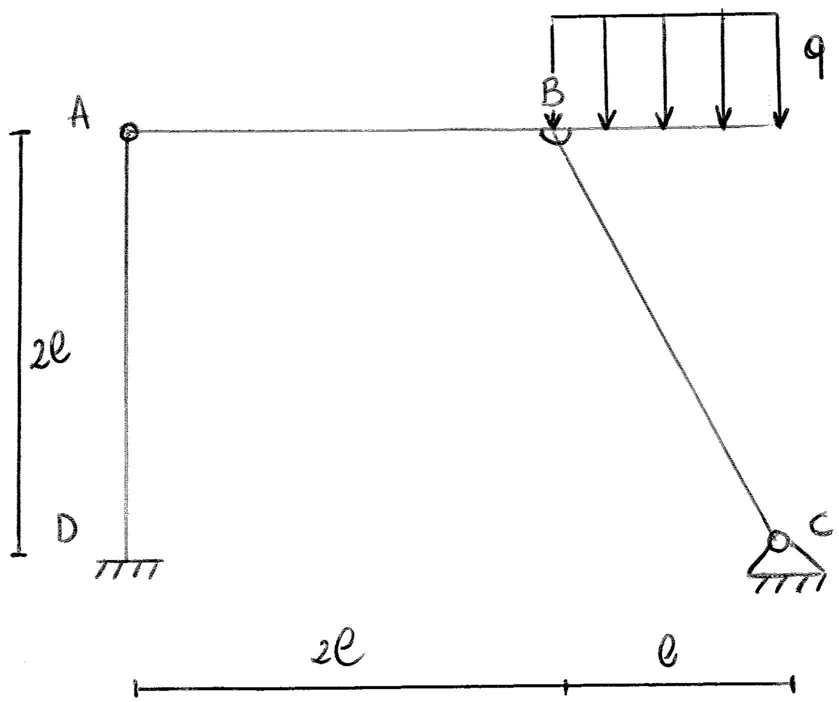
C)

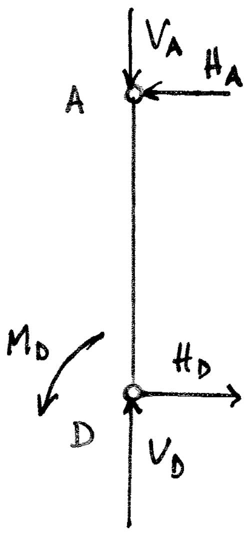
$$V_A \cdot 2\ell - F_q \cdot \ell - H_A \cdot \ell = 0$$

$$2V_A = q \cdot \ell + H_A$$

$$V_A = \frac{2q\ell + H_A}{2}$$

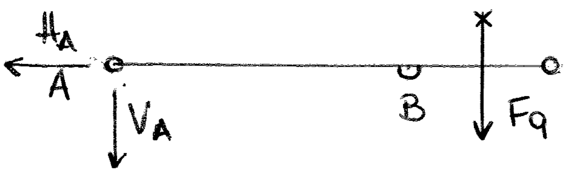
5





B) $2l V_D - 2l H_D - M_D = 0$

$$M_D = 2l V_D - 2l H_D$$



$V_A = V_D$

B) $V_D \cdot 2l + F_q \cdot \frac{l}{2} = 0$

$$V_D = -\frac{F_q}{4}$$

6

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TEORIA 5

Verificare che da $N_{,z} = -P(z)$ si ricava w:

$$N = \sigma_{zz} A = E \varepsilon_{zz} A = EA \frac{\partial w}{\partial z}$$

$$N = \left(EA \frac{\partial w}{\partial z} \right)_{,z} = \underset{\substack{\downarrow \\ = \text{costante}}}{EA} \frac{\partial^2 w}{\partial z^2}$$

$$EA \frac{\partial^2 w}{\partial z^2} = -P(z)$$

Pongo $P_{(z)} = P \operatorname{sen}\left(\frac{n \pi z}{L} \right)$ quindi:

$$EA \frac{\partial^2 w}{\partial z^2} = -P \operatorname{sen}\left(\frac{n \pi z}{L} \right)$$

$$\int EA \frac{\partial^2 w}{\partial z^2} dz = \int -P \operatorname{sen}\left(\frac{n \pi z}{L} \right) dz$$

$$\int EA \frac{\partial w}{\partial z} = \frac{LP}{n\pi} \int \cos\left(\frac{n \pi z}{L} \right) dz$$

$$EA\, w = \frac{L^2}{n^2 \pi^2} \operatorname{sen}\left(\frac{n \pi z}{L} \right) ..$$

$$w_{(z)} = \frac{L^2}{n^2 \pi^2 EA} \operatorname{sen}\left(\frac{n \pi z}{L} \right)$$

$$w_{(0)} = 0$$

$$w_{(L)} = 0$$

1

Ricaviamo Φ_x e Φ_y partendo dalle formule di Timoshenko:

$$u_{(x,y,z)} = u^\circ_{(z)}$$

$$v_{(x,y,z)} = v^\circ_{(z)}$$

$$w_{(x,y,z)} = w^\circ_{(z)} + x\,\Phi_{x(z)} + y\,\Phi_{y(z)}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = u^\circ_{,z}(z) + \Phi_x(z) = 0$$

$$\Phi_x(z) = -u^\circ_{,z}(z)$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = v^\circ_{,z}(z) + \Phi_{y(z)} = 0$$

$$\Phi_y = -\,-\,v^\circ_{,z}(z)$$

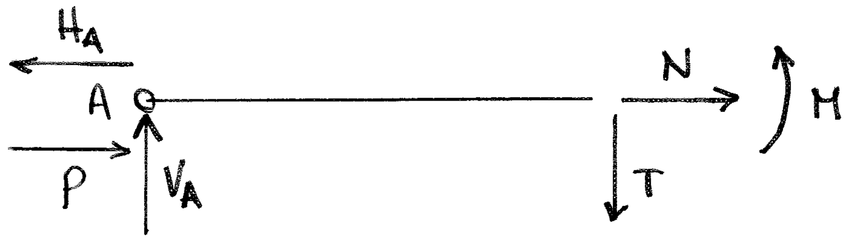
Quindi ricavo:

$$u_{(x,y,z)} = u^\circ_{(z)}$$

$$v_{(x,y,z)} = v^\circ_{(z)}$$

$$w_{(x,y,z)} = w^\circ_{(z)} - x\,u^\circ_{,z}(z) - y\,v^\circ_{,z}(z)$$

tratto $\overline{AC}$: $0<z<a$

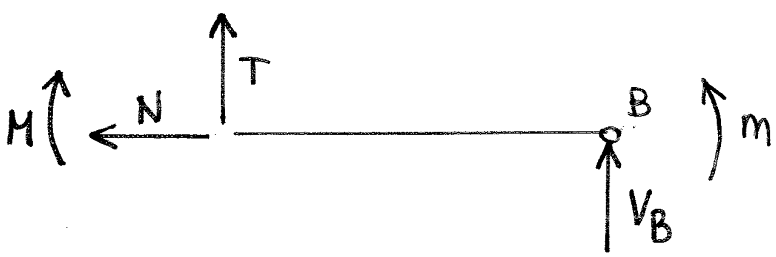


$$N = H_A - P = 0$$

$$T = V_A = \frac{P \cdot b + m}{a+b}$$

$$M = V_A \cdot z = \begin{cases} M_A = 0 \\ M_C = V_A \cdot a = \dfrac{P \cdot b + m}{a+b} \cdot a \end{cases}$$

tratto $\overline{BC}$: $0<z<b$



$$N = 0$$

$$T = -V_B = -\frac{P \cdot a - m}{a+b}$$

$$M = m + V_B \cdot z = \begin{cases} M_C = m + V_B \cdot b = \dfrac{P \cdot b + m}{a+b} \cdot a \\ M_B = m \end{cases}$$

$P = 10N$

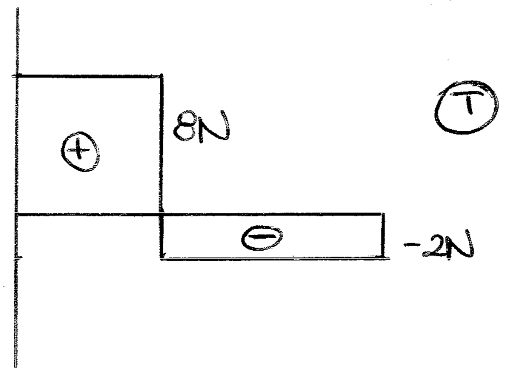
$m = 2000 N \cdot mm$

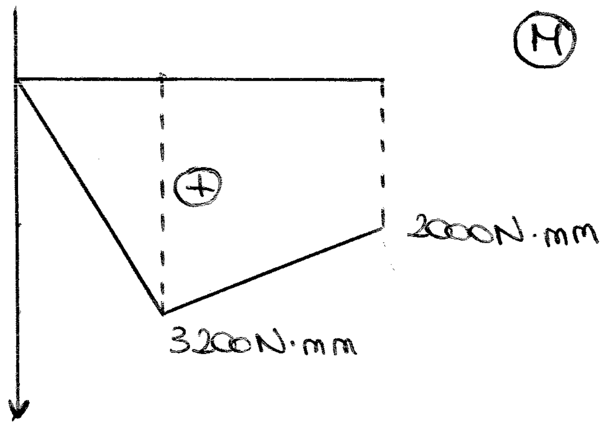
$a = 400 mm$

$b = 600 mm$

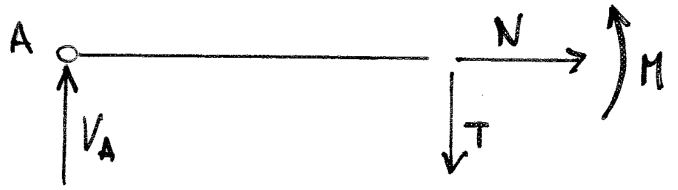
tratto $\overline{AC}$: $\begin{cases} T = 8N \\ M_A = 0 \\ M_C = 3200 N \cdot mm \end{cases}$

tratto $\overline{CB}$: $\begin{cases} T = -2N \\ M_C = 3200 N \cdot mm \\ M_B = 2000 N \cdot mm \end{cases}$





tratto $\overline{AB}$: $0 < z < a$

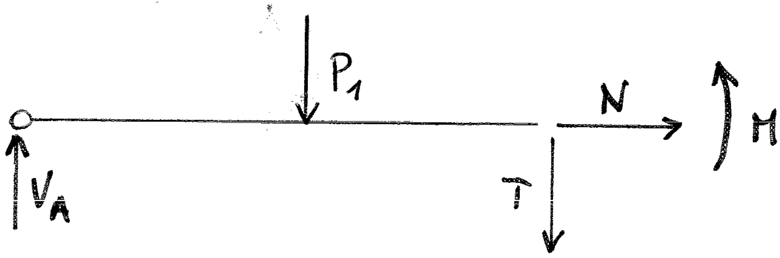


$N = 0$

$T = V_A = 20N$

$$M = V_A \cdot z = \begin{cases} M_A = 0 \\ M_B = V_A \cdot a = 20'000 \; N \cdot mm \end{cases}$$

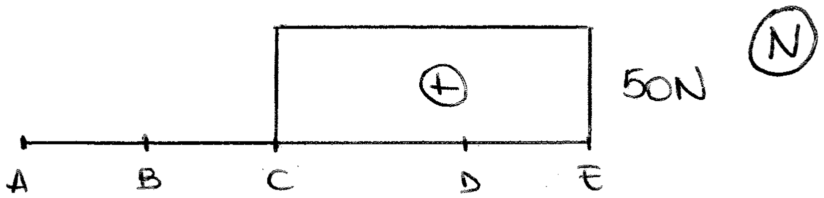
tratto $\overline{AC}$: $a < z < 2a$

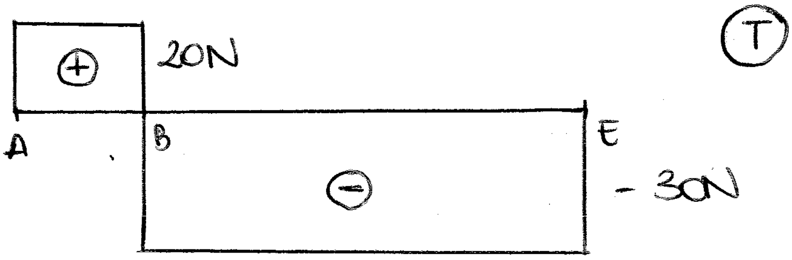


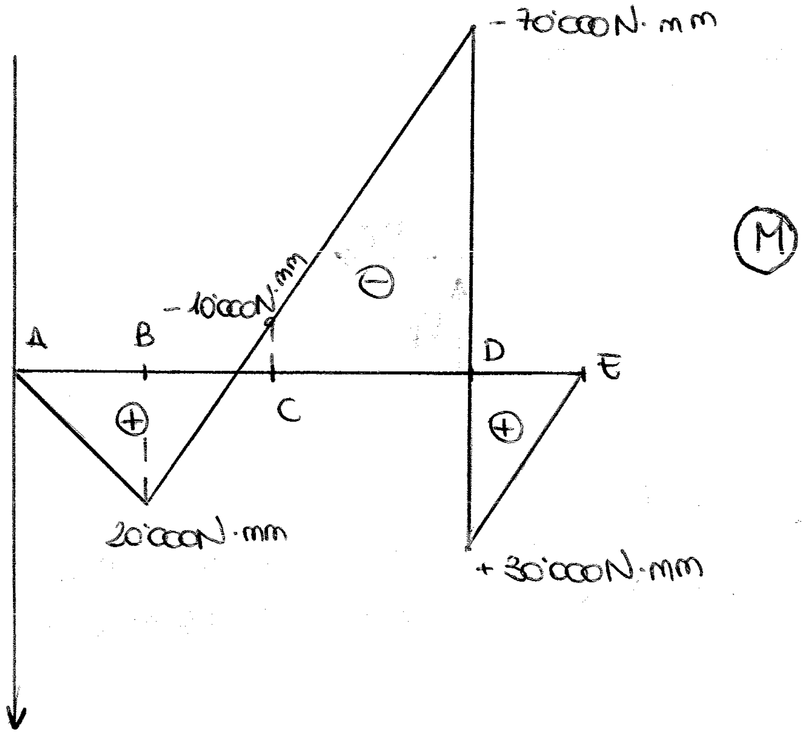
$N = 0$

$T = V_A - P_1 = -30N$

$$M = V_A \cdot z - P_1 \cdot (z - a) = \begin{cases} M_B = 20'000 N \cdot mm \\ M_C = -10'000 \; N \cdot mm \end{cases}$$

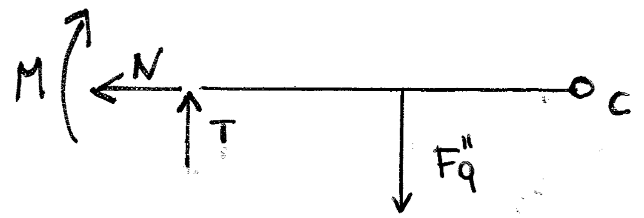

N
+
50N
A
B
C
D
E


T
+
20N
A
B
E
−
- 30N


M
- 70'000N·mm
- 10'000N·mm
−
A
B
C
D
E
+
+
20'000N·mm
+ 30'000N·mm

tratto $\overline{CB}$: $0 < z < b$

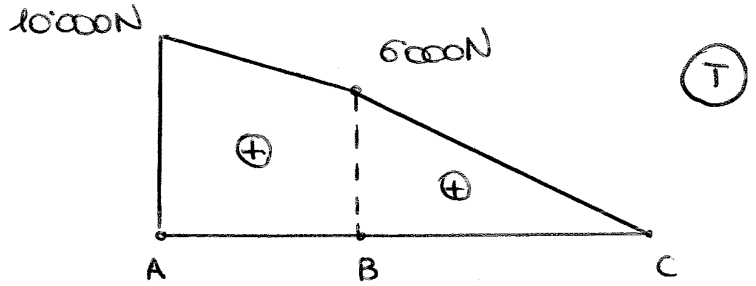
$F''_q = q \cdot z$

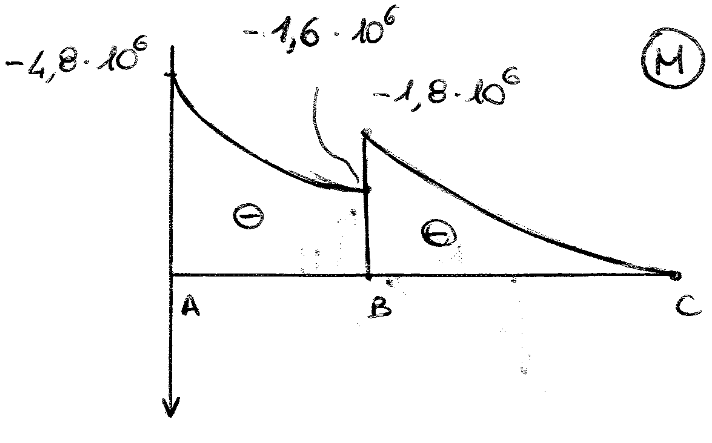


$N = 0\,N$

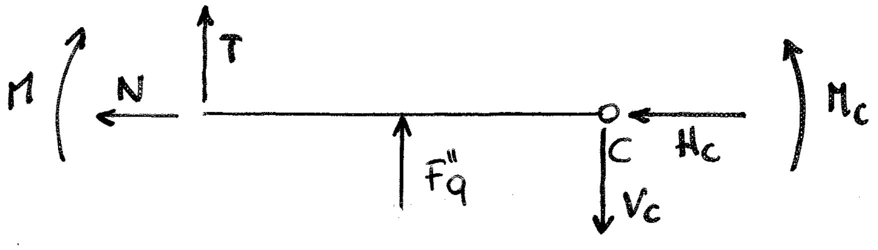
$T = F''_q = \begin{cases} T_C = 0\,N \\ T_B = qb = 6'000\,N \end{cases}$

$M = -F''_q \cdot \frac{z}{2} = \begin{cases} M_C = 0\ N \cdot mm \\ M_B = -1'800'000\ N \cdot mm \end{cases}$





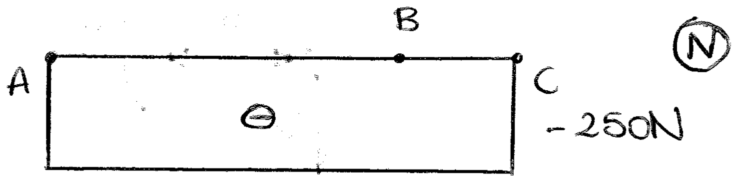
tratto $\overline{CB}$: $\quad 0<z<a$



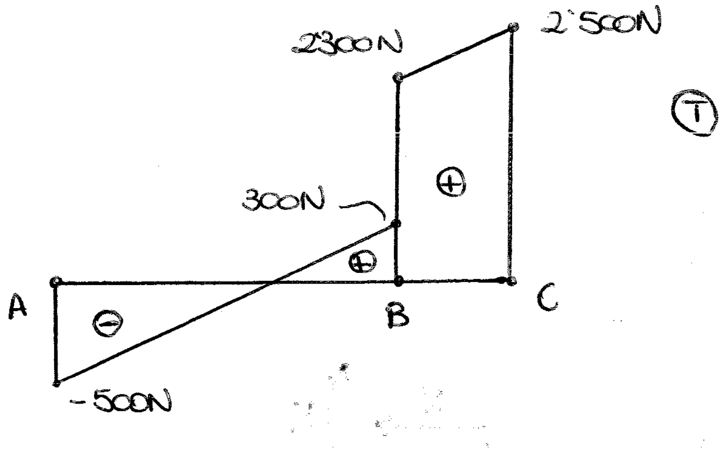
$$N = -H_c = -250N$$

$$T = V_c - F_q'' = \begin{cases} T_C = 2'500N \\ T_B = 2'300N \end{cases}$$

$$M = M_c - V_c \cdot z + F_q'' \cdot \frac{z}{2} = \begin{cases} M_C = M_c = 400'000 N\cdot mm \\ M_B = -80'000 N \cdot mm \end{cases}$$



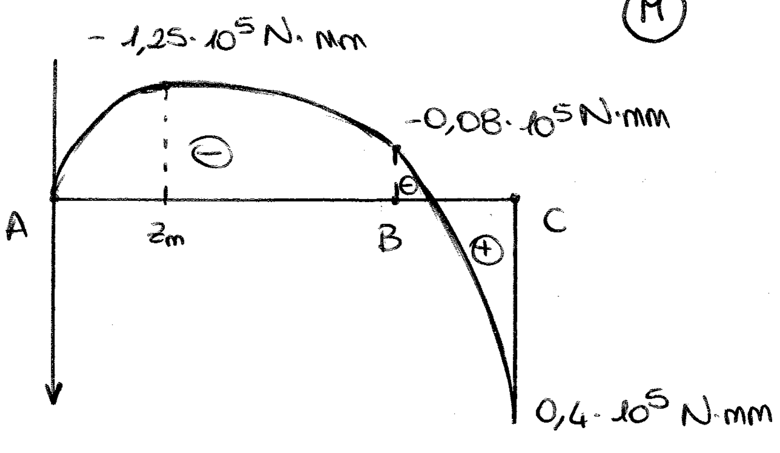
devo trovare il punto di max del momento.



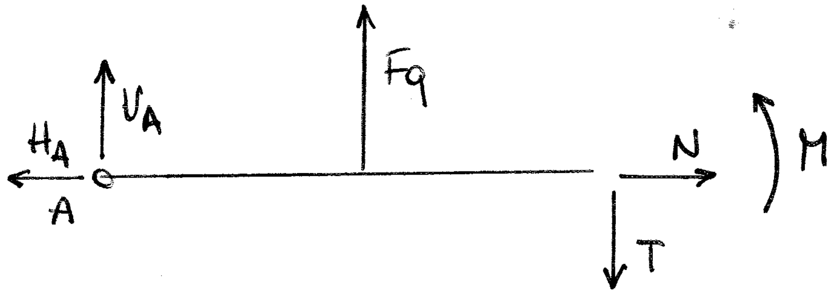
$$\frac{dM_{AB}}{dz} = 0 \rightarrow T_{AB} = 0$$

$$q z_m = P_2 \rightarrow z_m = 500mm$$

$$M = -P_2 \cdot 500 + F_q' \cdot 250 = = -1,25 \cdot 10^5 N \cdot mm$$



tratto $\overline{AB}$: $0 < z < \ell$

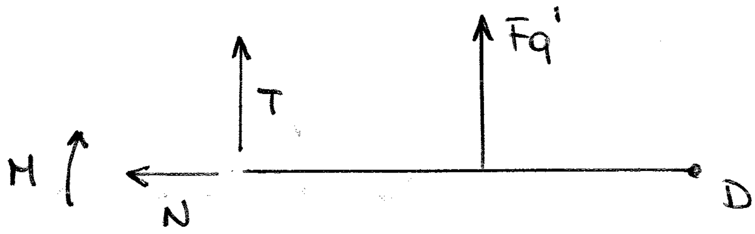


$N = H_A = -3000\,N$

$T = V_A + \frac{Fq}{3} = \begin{cases} T_A = 1000\,N \\ T_B = 1666\,N \end{cases}$

$M = V_A \cdot z + \frac{Fq}{3} \cdot \frac{z}{2} = \begin{cases} M_A = 0\ N \cdot mm \\ M_B = 2666 \cdot 10^3\ N \cdot mm \end{cases}$

tratto $\overline{DB}$: $0 < z < 2\ell$



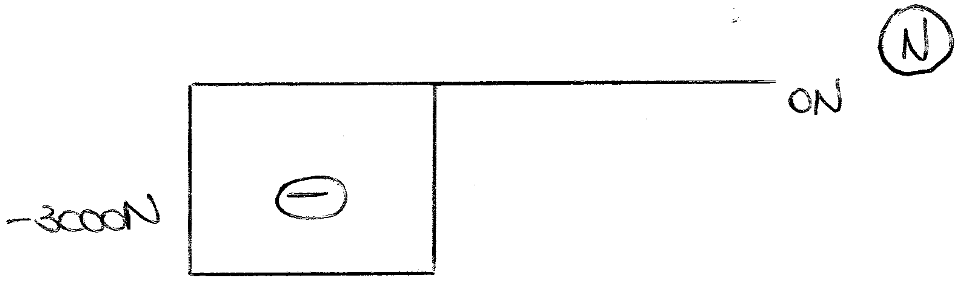
$Fq' = \frac{2}{3} Fq$

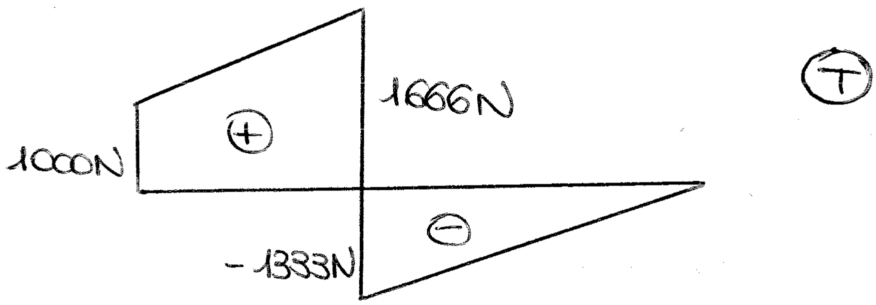
$N = 0$

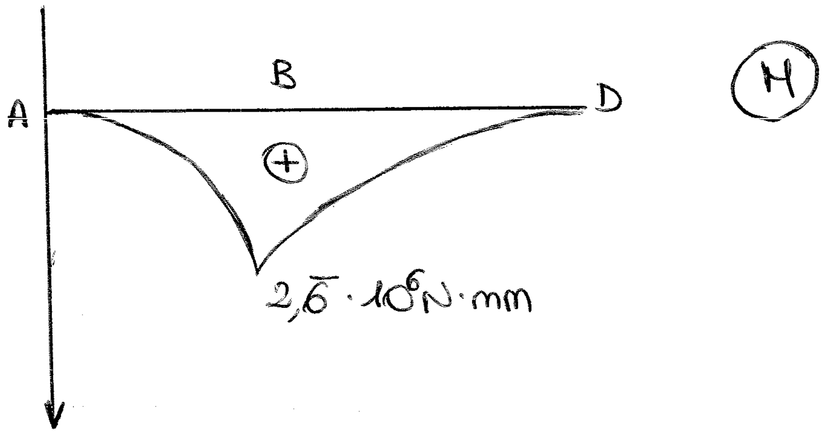
$T = -Fq' = \begin{cases} T_D = 0\,N \\ T_B = -1333\,N \end{cases}$

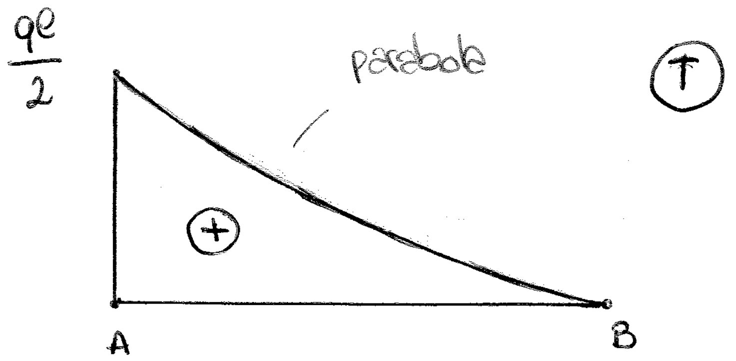
$M = Fq' \cdot \frac{z}{2} = \begin{cases} M_D = 0\ N \cdot mm \\ M_B = 2666 \cdot 10^3\ N \cdot mm \end{cases}$

CORPO 1

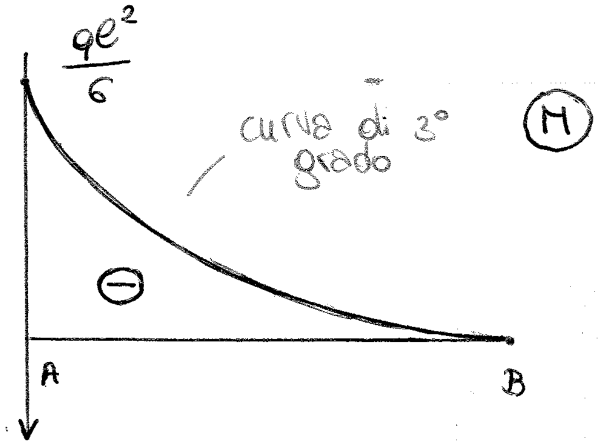






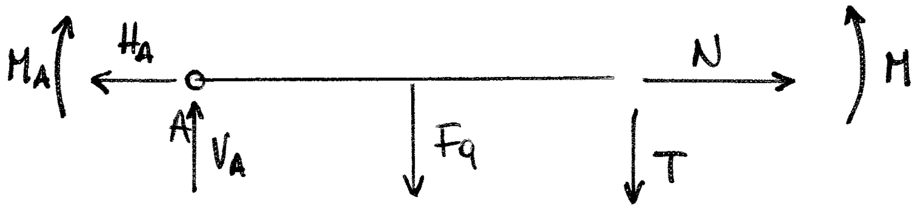


$\frac{ql}{2}$
parabola
T
+
A
B



$\frac{ql^2}{6}$
curva di 3° grado
M
A
B

tratto $\overline{AB}$: $0 < z < \ell$

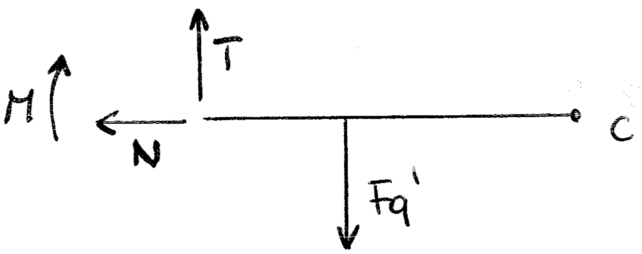


$N = H_A = 0$

$T = V_A - Fq = \begin{cases} T_A = V_A = \frac{3}{2} q\ell \\ T_B = V_A - Fq = \frac{q\ell}{2} \end{cases}$

$M = \underbrace{M_A}_{V_A \cdot z +} - qz \cdot \frac{z}{2} = \begin{cases} M_A = M_A \\ M_B = M_A - \frac{q\ell^2}{2} + \frac{3}{2} q\ell^2 = -\frac{1}{6} q\ell^2 \end{cases}$

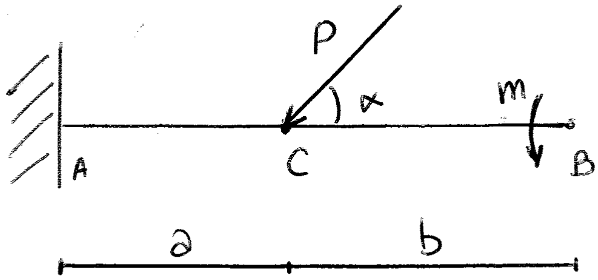
tratto $\overline{CB}$: $0 < z < \ell$



$N = 0$

$T = Fq' \cdot z = \begin{cases} T_C = 0 \\ T_B = \frac{q\ell}{2} \end{cases}$

$M = -Fq' \cdot \frac{z}{3} \cdot z = \begin{cases} M_C = 0 \\ M_B = -\frac{q\ell^2}{6} \end{cases}$



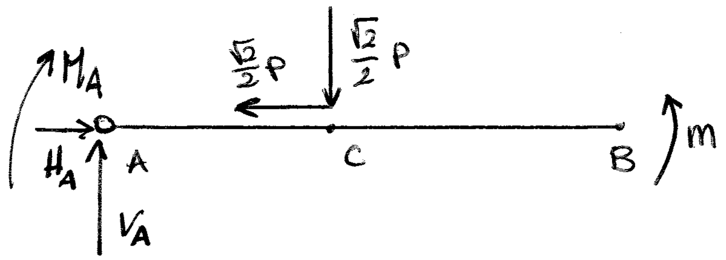
$\alpha = 45°$

$P = 10\,N$

$m = 2000\,N \cdot mm$

$a = 400\,mm$

$b = 600\,mm$



$P\frac{\sqrt{2}}{2} = 7\,N$

$\rightarrow \quad H_A = \frac{\sqrt{2}}{2}P = 7\,N$

$\uparrow \quad V_A = \frac{\sqrt{2}}{2}P = 7\,N$

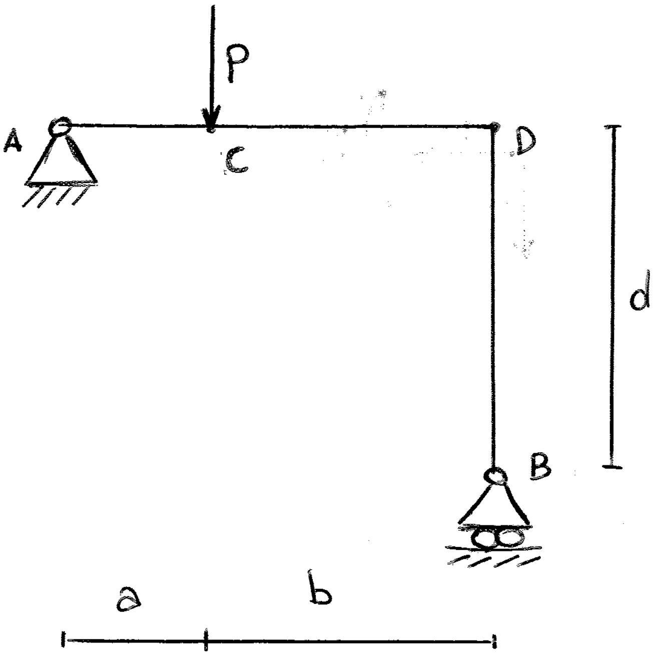
$\curvearrowleft A \quad M_A = m - \frac{\sqrt{2}}{2}P \cdot a = -800\,N \cdot mm$

tratto $\overline{AC}$: $0 < z < a$

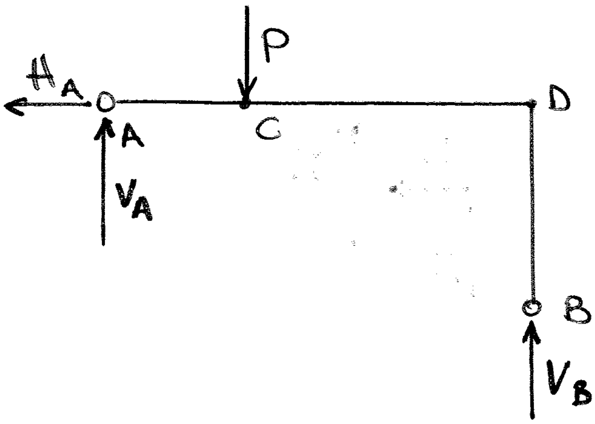
$N = -H_A = -7\,N$

$T = V_A = 7\,N$

$$M = M_A + V_A \cdot z = \begin{cases} M_A = M_A = -800\,N \cdot mm \\ M_C = M_A + V_A \cdot a = 2000\,N \cdot mm \end{cases}$$



$P = 20N$
$a = 400mm$
$b = 600\,mm$
$d = 500mm$



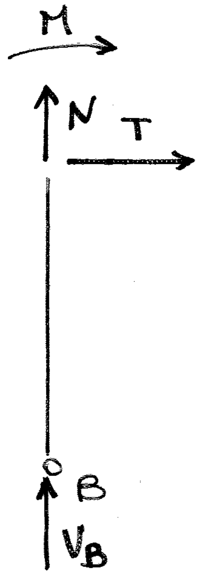
$\leftarrow \quad H_A = 0$ $\qquad H_A = 0N$

$\uparrow \quad V_A + V_B = P$ $\qquad V_A = 12N$

$\curvearrowleft A \quad P \cdot a - V_B \cdot (a+b) = 0$ $\qquad V_B = 8N$

9.1

tratto $\overline{BD}$: $0<z<d$



$N=-V_B=-8N$

$T=0N$

$M=0N$

9.2

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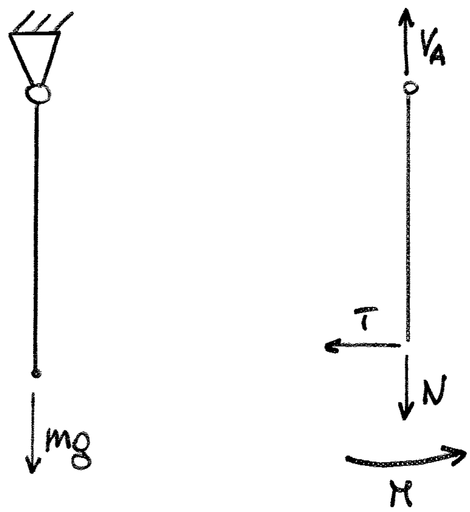
TEORIA 6



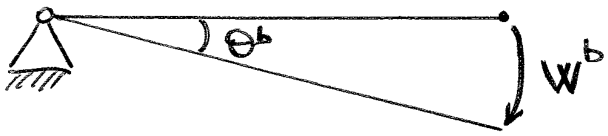
$$N = V_A = mg$$
$$T = 0$$
$$M = 0$$

$$\sigma_R = \frac{N}{A} \rightarrow N = \sigma_R A$$

$$mg = \underbrace{\rho L_R \cancel{A}}_{m} g = \sigma_R \cancel{A}$$

$$\sigma_R = \rho L_R g \qquad L_R = \frac{\sigma_R}{\rho g}$$

sistema "b" $\rightarrow$ congruente (virtuale)



$$w_B^b = (a+b)\,\theta^b$$

$$w_P^b = a\,\theta^b$$

$$\delta w_B^b = (a+b)\,\delta\theta^b$$

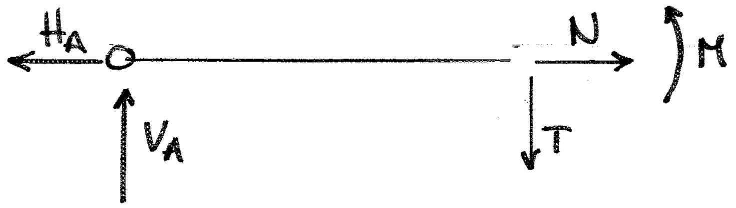
$$\delta w_P^b = a\,\delta\theta^b$$

$$\delta L_i^{ab} = 0 \qquad \delta L_e^{ab} = 0 = P\,\delta w_P^b - V_B^a\,\delta w_B^b$$

$$P \cdot a\,\delta\theta^b = V_B^a \cdot (a+b)\,\delta\theta^b$$

sistema "b"

tratto $\overline{AC}$: $0<z<a$

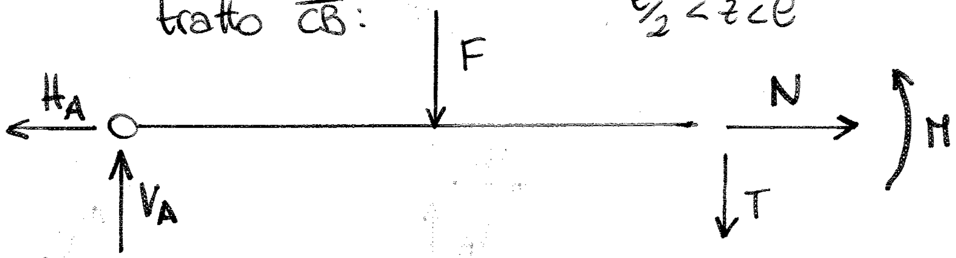


$N = H_A = 0$

$T = V_A = 1/2$

$M^b = V_A z = \left\langle \begin{array}{l} M_A = 0 \\ M_C = \ell/4 \end{array} \right.$

tratto $\overline{CB}$: $\ell/2 < z < \ell$



$N = H_A = 0$

$T = V_A = 1/2$

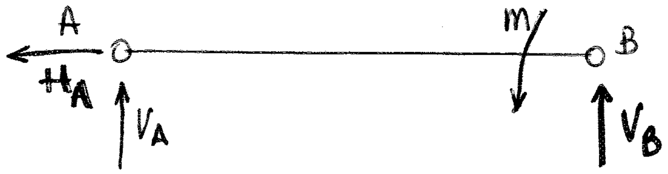
$M^b = V_A \cdot z + 1 \cdot \left(\frac{\ell}{2} - z\right) = \left\langle \begin{array}{l} M_C = \ell/4 \\ M_B = 3/4\,\ell \end{array} \right.$

$M^a = \frac{m}{\ell} \cdot z \qquad 0<z<\ell$

$M^b = \frac{z}{2} \qquad 0<z<\ell/2$

$M^b = \frac{\ell - z}{2} \qquad \ell/2<z<\ell$

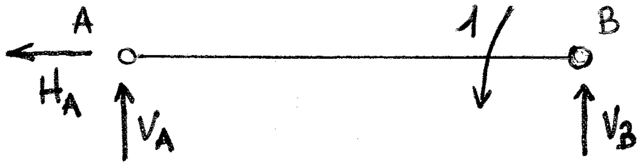
sistema "a": θ_B



$0 < z < \ell$

$$M^a = \frac{m}{\ell} \cdot z$$

sistema "b": → prendo il momento = 1



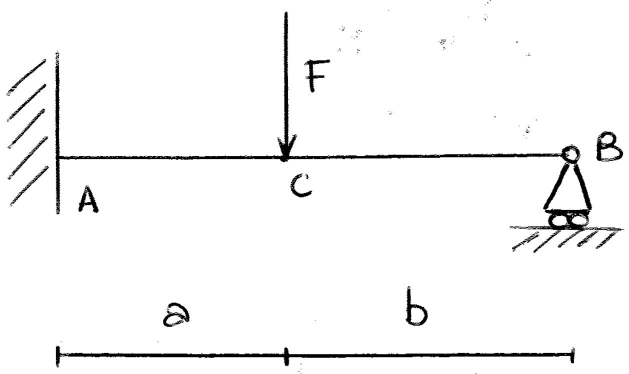
$0 < z < \ell$

$$M^b = \frac{1}{\ell} \cdot z = \frac{z}{\ell}$$

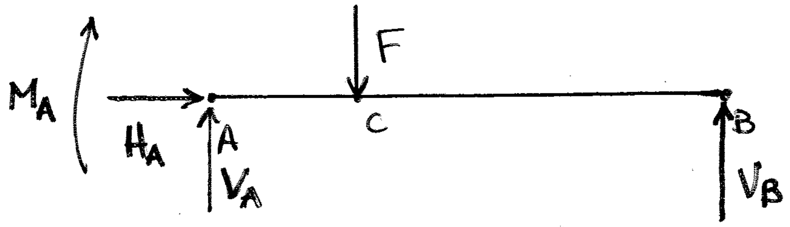
$$L_i^{ab} = L_e^{ab} \qquad L_e^{ab} = 1^b \cdot \theta_B^a$$

$$\theta_B = \int_0^\ell \frac{M^a M^b}{EI} dz = \frac{m}{\ell^2 EI} \int_0^\ell z^2 dz =$$

$$= \frac{m\ell}{3EI}$$



Determinare V_B



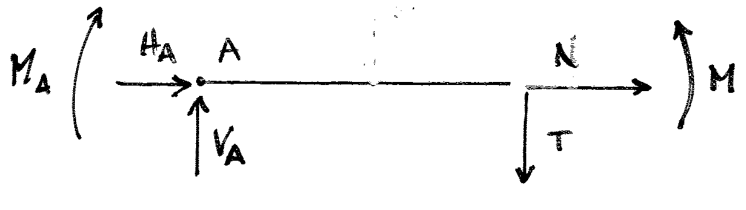
$\rightarrow \; H_A = 0$

$\uparrow \; V_A + V_B = F$

$\curvearrowleft A \; M_A = V_B \cdot (a+b) - F \cdot a$

sistema "0": → sistema in cui togli il vincolo di cui vuoi calcolare le reazioni: valido per le iperstatiche

tratto $\overline{AC}$: $0 < z < a$



$M_A^0 = -Fa$

$H_A^0 = 0$

$V_A^0 = F$

$\uparrow \; V_A^0 = T$

$\rightarrow \; H_A = -N = 0$

$\curvearrowleft \; M^0 = M_A^0 + V_A^0 \cdot z = \begin{cases} M_A^0 = M_A^0 \\ M_C^0 = M_A^0 + V_A^0 \cdot a \end{cases}$

3.1

$$\int_0^{a+b} \frac{M^a(M^0+xM^a)}{EI}\,dz = \int_0^{a+b} \frac{M^0M^a}{EI}\,dz + x\int_0^{a+b}\frac{(M^a)^2}{EI}\,dz$$

$$x = -\frac{\int_0^{a+b} M^0M^a\,dz}{\int_0^{a+b}(M^a)^2\,dz}$$

$$\int_0^{a+b} M^0M^a\,dz = \int_0^{a} M^0M^a\,dz + \underbrace{\cancel{\int_a^{a+b} M^0M^a\,dz}}_{=0} =$$

$$= F\int_0^{a}(a+b-z)(z-a)\,dz = -Fa^2\left(\frac{b}{2}+\frac{a}{3}\right)$$

$$\int_0^{a+b}(M^a)^2\,dz = \left[-\frac{(a+b-z)^3}{3}\right]_0^{a+b} = \frac{(a+b)^3}{3}$$

$$x = \frac{Fa^2\left(\frac{a}{3}+\frac{b}{2}\right)}{\frac{(a+b)^3}{3}} = Fa^2\,\frac{\frac{2a+3b}{2}}{(a+b)^3}$$

$$V_B^b = \underbrace{\cancel{V_B^0}}_{=0} + 1\cdot x = Fa^2\,\frac{2a+3b}{2(a+b)^3}$$

3.2